

Deformation and Gravity Change: Indicators of **Isostasy, Tectonics, Volcanism, and Climate Change**

Edited by
Detlef Wolf
José Fernández

BIRKHAUSER



pageoph topical volumes

Deformation and Gravity
Change: Indicators of
**Isostasy, Tectonics,
Volcanism, and
Climate Change**

Edited by
Detlef Wolf
José Fernández

Birkhäuser
Basel · Boston · Berlin

Reprint from Pure and Applied Geophysics
(PAGEOPH), Volume 164 (2007) No. 4

Editors:

Detlef Wolf
Geoforschungszentrum Potsdam
Abteilung Geodäsie und Fernerkundung
Telegrafenberg
14473 Potsdam
Germany
dasca@gfz-potsdam.de

José Fernández
Instituto de Astronomía y
Geodesia (CSIC-UCM)
Spanish Council for Scientific Research
Fac. C. Matemáticas
Plaza de Ciencias, 3
28040 Madrid
Spain
jose_fernandez@mat.ucm.es

Library of Congress Control Number: 2007925494

Bibliographic information published by Die Deutsche Bibliothek:
Die Deutsche Bibliothek lists this publication in the Deutsche Nationalbibliografie; detailed
bibliographic data is available in the Internet at <<http://dnb.ddb.de>>

ISBN 978-3-7643-8416-6 Birkhäuser Verlag AG, Basel · Boston · Berlin

This work is subject to copyright. All rights are reserved, whether the whole or part of the
material is concerned, specifically the rights of translation, reprinting, re-use of illustra-
tions, recitation, broadcasting, reproduction on microfilms or in other ways, and storage in
data banks. For any kind of use permission of the copyright owner must be obtained.

© 2007 Birkhäuser Verlag AG
Basel · Boston · Berlin
P.O. Box 133, CH-4010 Basel, Switzerland
Part of Springer Science+Business Media
Printed on acid-free paper produced from chlorine-free pulp. TCF ∞
Printed in Germany

ISBN 978-3-7643-8416-6
9 8 7 6 5 4 3 2 1

e-ISBN 978-3-7643-8417-3
www.birkhauser.ch

Contents

- 633 Deformation and Gravity Change: Indicators of Isostasy, Tectonics, Volcanism and Climate Change. Introduction
D. Wolf, J. Fernández
- 637 An Overview of the Small BASeline Subset Algorithm: A DInSAR Technique for Surface Deformation Analysis
R. Lanari, F. Casu, M. Manzo, G. Zeni, P. Berardino, M. Manunta, A. Pepe
- 663 Propagator-matrix Technique for the Viscoelastic Response of a Multi-layered Sphere to Surface Toroidal Traction
Z. Martinec
- 683 Using Fuzzy Logic for the Analysis of Sea-level Indicators with Respect to Glacial-isostatic Adjustment: An Application to the Richmond-Gulf Region, Hudson Bay
V. Klemann, D. Wolf
- 697 Shear-wave Velocity Structure around Teide Volcano: Results Using Microtremors with the SPAC Method and Implications for Interpretation of Geodetic Results
F. J. Chávez-García, F. Luzón, D. Raptakis, J. Fernández
- 721 Steric Sea-level Variations Inferred from Combined Topex/Poseidon Altimetry and GRACE Gravimetry
D. García, G. Ramillien, A. Lombard, A. Cazenave
- 733 Interpretation of 1992–1994 Gravity Changes around Mayon Volcano, Philippines, Using Point Sources
A. G. Camacho, J. Fernández, M. Charco, K. F. Tiampo, G. Jentzsch
- 751 Glacial-isostatic Adjustment and the Viscosity Structure Underlying the Vatnajökull Ice Cap, Iceland
K. Fleming, Z. Martinec, D. Wolf
- 769 Review of Microgravity Observations at Mt. Etna: A Powerful Tool to Monitor and Study Active Volcanoes
D. Carbone, F. Greco

- 791 An Estimate of Global Mean Sea-level Rise Inferred from Tide-gauge Measurements Using Glacial-isostatic Models Consistent with the Relative Sea-level Record
J. M. Hagedoorn, D. Wolf, Z. Martinec
- 819 Modeling of Stress Changes at Mayon Volcano, Philippines
K. F. Tiampo, J. Fernández, T. Hayes, G. Jentzsch
- 837 A Hybrid Model for the Summit Region of Merapi Volcano, Java, Indonesia, Derived from Gravity Changes and Deformation Measured between 2000 and 2002
C. Tiede, J. Fernández, C. Gerstenecker, K. F. Tiampo
- 851 Vertical Crustal Motion along the Mediterranean and Black Sea Coast Derived from Ocean Altimetry and Tide Gauge Data
D. García, I. Vigo, B. F. Chao, M. C. Martínez
- 865 Some Insights into Topographic, Elastic and Self-gravitation Interaction in Modelling Ground Deformation and Gravity Changes in Active Volcanic Areas
M. Charco, J. Fernández, F. Luzón, K. F. Tiampo, J. B. Rundle

Deformation and Gravity Change: Indicators of Isostasy, Tectonics, Volcanism and Climate Change. Introduction

DETLEF WOLF^{1,2} and JOSÉ FERNÁNDEZ³

The workshop on *Deformation and Gravity Change: Indicators of Isostasy, Tectonics, Volcanism and Climate Change* took place at the Casa de los Volcanes on Lanzarote, Spain, during March 1–4, 2005 (see Fig. 1). It was jointly organized and supported by the International Association of Geodesy (IAG), the Spanish Ministry of Education and Science, the Spanish Council for Scientific Research and the Cabildo Insular de Lanzarote. From an organizational point of view, the workshop also served as the first meeting of the members the IAG Working Group ICCT2 on *Dynamic Theories of Deformation and Gravity Fields* existing since late 2003, but was open to all “external” colleagues interested in its topic.

Furthermore, it continued a tradition established by a series of preceding workshops organized under the auspices of the IAG and devoted to the themes “Models of Temporal Variations of the Gravity Field” (Walferdange, Luxembourg, March 17–19, 1997 and Potsdam, Germany, November 23–25, 1998) and “Dynamic Theories of Deformation and Gravity Fields” (Sopron, Hungary, February 19–23, 2001 and Lanzarote, Spain, February 18–21, 2003).

The common subject of these workshops reflects the major developments in geodynamics during the last decade, when temporal variations of the deformation and gravity fields recorded by the geodetic measuring techniques and reflecting isostatic, tectonic or volcanic processes in the Earth’s interior as well as hydrologic or oceanographic processes on its surface have gained ever increasing importance. On the one hand, this development is the consequence of the introduction of novel and highly precise measuring techniques as a prerequisite for monitoring these variations.

¹ GeoForschungsZentrum Potsdam, Department 1, Geodesy and Remote Sensing, D-14473 Potsdam, Germany. E-mail: dasca@gfz-potsdam.de

² Geodetic Institute, University of Stuttgart, D-70174 Stuttgart, Germany.

³ Instituto de Astronomía y Geodesia (CSIC-UCM), Fac. C. Matemáticas, Ciudad Universitaria, Plaza de Ciencias, 3, 28040 Madrid, Spain. E-mail: jose_fernandez@mat.ucm.es.



Figure 1

Some of the participants in the workshop *Deformation and Gravity Change: Indicators of Isostasy, Tectonics, Volcanism and Climate Change* before their visit of the Cueva de los Verdes on Lanzarote.

On the other hand, continuing improvements in the theoretical formulation of viscoelastic field theories and their numerical implementations are now reaching a level that allows us to model the various geodynamic processes in a more realistic way than has yet been possible and also to gain understanding of their intricate interference and coupling. The workshop held on Lanzarote during March 1–4 and the resulting articles collected in this special issue reflect this duality of the recent evolution: Whereas the preceding workshops had focussed on theoretical developments, equal weight has been given this time to the two aspects of our current understanding of geodynamics.

We would like to thank the Consejería de Ciencia y Tecnología of the Cabildo Insular de Lanzarote, the Lanzarote Laboratory for Geodynamics (CSIC-UCM), the staff of the Casa de los Volcanes—in particular, its director Joaquín Naverán as well as Orlando Hernández and Jaime Arranz—and Cristina Bernal of the Timanfaya National Park administration for their support during the workshop.

Useful suggestions by Renata Dmowska during the preparation of this topical issue are appreciated. We thank all authors for their contributions and acknowledge the assistance of the following reviewers: F. Amelung, O. Bennike, A. Bonforte, C. Braitenberg, A. Braun, C. Gerstenecker, M. Ishii, E. Ivins, W. Jacoby,

T. Johnson, C. Kisslinger, V. Klemann, C.A. Locke, B. Lühr, P. Lundgren, J.J. Mallorqu, Z. Martinec, G. Milne, G.T. Mitchum, J. Morales, S. Okubo, M. Poland, G. Puglisi, G. Rodriguez-Velasco, F. Roth, H. Rymer, F. Sánchez-Sesma, E. Schrama, G. Spada, K.F. Tiampo, C. Tiede and B. Vermeersen.

An Overview of the Small BAseline Subset Algorithm: A DInSAR Technique for Surface Deformation Analysis

RICCARDO LANARI,¹ FRANCESCO CASU,^{1,2} MARIAROSARIA MANZO,^{1,3} GIOVANNI ZENI,^{1,3}
PAOLO BERARDINO,¹ MICHELE MANUNTA,² and ANTONIO PEPE⁴

Abstract—We present an overview of the Differential SAR Interferometry algorithm referred to as Small BAseline Subset (SBAS) technique, that allows us to detect surface deformation and to analyze their space-time characteristics. Following the description of the main theoretical aspects of the algorithm, we present several results obtained by applying the SBAS approach in selected case studies relevant to phenomena affecting volcanic areas (Campi Flegrei caldera and Somma-Vesuvio complex, Italy), aquifers (Santa Clara area within the San Francisco bay, California) and landslides (Maratea Valley, Italy). The overall analysis is carried out by using multilook DInSAR interferograms with a spatial resolution of the order of 100×100 m, computed from SAR data acquired by the ERS-1 and ERS-2 sensors. In particular, we highlight the peculiarities of the SBAS technique and its surface deformation retrieval capability for what concerns both large-scale deformation phenomena and more localized displacement effects.

Key words: Differential SAR Interferometry, SBAS, surface deformation.

1. Introduction

Differential Synthetic Aperture Radar Interferometry (DInSAR) is a microwave remote sensing technique that allows us to investigate surface deformation phenomena with a centimeter to millimeter accuracy and with a large spatial coverage capacity (GABRIEL *et al.*, 1989). In particular, the DInSAR technique exploits the phase difference, often referred to as interferogram, between two SAR images relevant to temporally separated observations of an investigated area and provides a measurement of the ground deformation projection along the radar line of sight (LOS).

¹ Istituto per il Rilevamento Elettromagnetico dell'Ambiente, National Research Council, via Diocleziano 328, 80124 Napoli, Italy.

² Dipartimento di Ingegneria Elettrica ed Elettronica, Università degli studi di Cagliari, Piazza d'Armi, 09123 Cagliari, Italy.

³ Dipartimento di Ingegneria e Fisica dell'Ambiente, Università degli Studi della Basilicata, Viale dell'Ateneo Lucano 10, 85100 Potenza, Italy.

⁴ Dipartimento di Ingegneria Elettronica e delle Telecomunicazioni, Università degli Studi di Napoli, Federico II, via Claudio 21, 80125 Napoli, Italy.

The DInSAR technology already plays an important role for surface displacement analysis and monitoring but we must remark that it also has key limitations, the most relevant being those due to the noise effects on the interferograms, referred to as decorrelation phenomena (ZEBKER and VILLASENOR, 1992). Intuitively, we can say that these noise phenomena are the consequence of any effect leading to differences in the “radar observation” of the imaged area, for what attains its electromagnetic response (reflectivity). The above mentioned differences may be caused by changes of the reflectivity of the zone which occurred between the two sensor passes (temporal decorrelation), or by the different imaging directions due to the spatial separation between the two acquisition orbits (spatial decorrelation). We further remark that the actual SAR scenarios mostly involve sensors operating at C-Band (5.3 GHz central frequency corresponding to a 5.6 cm wavelength) with a revisiting time of about 20–40 days and with a typical separation between the sensor orbits in the range of 0–2 km. Accordingly, we may expect low decorrelation effects mostly in urbanized and rocky areas while zones characterized by vegetation, agricultural fields, snow or water are drastically affected by noise (ROSEN *et al.*, 2000).

The DInSAR methodology has been applied first to investigate single deformation events (MASSONET *et al.*, 1993; PELTZER and ROSEN, 1995; RIGNOT, 1998). However, more recently, it also has been exploited to analyze the temporal evolution of the detected displacements via the generation of deformation time series. In this case, a time series of deformation can be solved for through the inversion of an appropriate sequence of DInSAR interferograms. The interest in the development of these methodologies is testified by several approaches which already have been presented or that are under development (FERRETTI *et al.*, 2000; BERARDINO *et al.*, 2002; MORA *et al.*, 2003; USAI, 2003; WERNER *et al.*, 2003; HOOPER *et al.*, 2004; CROSETTO *et al.*, 2005).

In this work we concentrate on the technique referred to as Small BASeline Subset (SBAS) approach proposed by BERARDINO *et al.* (2002), whose capability to detect and investigate deformation phenomena has been already shown in different applications, mostly based on exploiting the European Remote Sensing (ERS) SAR data, acquired in C-Band (LANARI *et al.*, 2002; LANARI *et al.*, 2004a; LANARI *et al.*, 2004b; LUNDGREN *et al.*, 2004; BORGIA *et al.*, 2005; MANZO *et al.*, 2006; CASCINI *et al.*, 2006). In particular, we focus on the basic SBAS technique that has been originally developed to investigate large spatial scale displacements with relatively low resolution (typically of the order of 100×100 m), by using low-pass filtered (multilook) DInSAR interferograms (ROSEN *et al.*, 2000). The key point of the SBAS technique, in addition to the previously mentioned use of multilook interferograms, is that the data pairs involved in the generation of the interferograms are properly selected in order to minimize the spatial and temporal separation (baseline) between the acquisition orbits, thus mitigating the decorrelation phenomena.

The SBAS procedure allows us to produce mean deformation velocity maps and, at the same time, deformation time series for each coherent pixel¹ of the investigated area. However, we also stress that the SBAS approach suffers some of the previously discussed limitations of the actual C-Band DInSAR technology. Indeed, despite the applied small (spatial and temporal) baseline constraints on the selected SAR data pairs, the density of the coherent pixels is typically not homogeneous in the scene and the technique can provide spatially dense results mostly in urban and rocky areas where the decorrelation phenomena have a very limited impact.

The key purpose of the work is to first provide a discussion of the SBAS algorithm basic theory and, more widely, to present results achieved by applying the approach to ERS SAR data, in selected case studies relevant to deformation phenomena affecting volcanic areas, aquifers and landslides.

For what concerns the volcano displacement analysis, we consider the Campi Flegrei caldera and the Vesuvio volcanic complex, both located in the Napoli Bay area (Italy). A second set of results is relevant to the San Francisco zone (California) with the analysis we present, focused on the Santa Clara aquifer. Finally, we investigate the gravitational phenomena affecting the Maratea Valley, which is located in southern Italy.

The overall analysis provides a quite large overview of the SBAS algorithm rationale and of its deformation retrieval capability in the various investigations.

2. Basic Theory of the SBAS Algorithm

The SBAS technique is a DInSAR approach allowing us to detect Earth's surface deformation and, above all, to analyze their temporal evolution. In particular, it relies on the use of a large number of SAR acquisitions and implements an easy combination of the multilook DInSAR interferograms computed from these data, finally leading to the generation of mean deformation velocity maps and time series. A detailed discussion on the SBAS approach is clearly outside the scope of this work and can be found in BERARDINO *et al.* (2002). Accordingly, we will present in the following the key points of the algorithm².

We start our discussion by providing first an intuitive explanation of the approach implemented by the SBAS procedure for retrieving the deformation signals. To achieve this task the algorithm first implements a selection of the coherent pixels in which the noise effects can be assumed negligible. On these pixels a decoupling of the deformation signal component from the undesired patterns,

¹ Note that we refer to coherent SAR pixels of the interferograms if they are characterized by a significantly high signal to noise ratio (SNR); thus, this implies low decorrelation effects.

² Note that we will maintain the same notation of BERARDINO *et al.* (2002).

referred to as topographic and atmospheric artifacts, is carried out by exploiting the characteristics of these patterns. In particular, the interferometric phase component relevant to the topographic artifacts is correlated with the vector of the spatial baselines (more specifically of the perpendicular baselines component with respect to the radar line of sight) of the interferograms sequence; moreover, the atmospheric phase signals are highly correlated in the space but poorly in time (FERRETTI *et al.*, 2000). Based on these characteristics, the SBAS procedure performs an estimation of these undesired signals that are subsequently filtered out from the measured interferometric phase, finally leading to the generation of the deformation time series.

We now provide more analytical details of the SBAS approach. Accordingly, we start by considering a set of $N + 1$ SAR images relative to the same area, acquired at the ordered times $(t_0, \dots, t_N)$. We also assume, for sake of simplicity, that all the images are coregistered with respect to a single one (referred to as master) in order to have a common reference grid.

The starting point of the proposed technique is represented by the generation of a number, say M , of differential interferograms involving the previously mentioned set of $N + 1$ SAR acquisitions, properly generated in order to mitigate the decorrelation phenomena. To achieve this task, the SAR image pairs selected for the interferograms generation are characterized by a small spatial and temporal baseline as well as by a small frequency shift between the Doppler centroids (FRANCESCHETTI and LANARI, 1999). Note also that, as a consequence of these constraints, the SAR images involved in the interferograms generation could be grouped in several independent small baseline subsets that must be appropriately “linked” in order to retrieve the deformation time series.

We further remark that, since the phase information in the computed interferograms represents the modulo- 2π restriction (wrapped) of the original (unwrapped) interferometric signal, each interferogram is processed in order to retrieve the original phase. This operation is referred to as phase unwrapping and is carried out, in our case, via a two-step approach. In particular, we first apply the minimum cost flow algorithm proposed by COSTANTINI and ROSEN (1999); the second step is represented by a region growing procedure allowing us to expand the unwrapped data in areas with lower coherence, i.e., lower SNR.

Let us now refer to a generic pixel of our SAR images with azimuth and range coordinates (x, r) and assume that the phase signal of each unwrapped interferogram is referenced to a pixel characterized by high coherence, whose deformation behavior is known *a priori* (typically, it is located in a non-deforming zone). Note that the selection of this reference SAR pixel must be carefully carried out because any error affecting this point will influence the overall results. In this context, particularly critical can be the impact of the atmospheric phase artifacts that, as already remarked, are spatially correlated, thus can be confused with deformation signals (BEAUDUCEL *et al.*, 2000).

The expression of the generic j -th interferogram computed from the SAR acquisitions at times t_B and t_A is, according to BERARDINO *et al.* (2002), the following:

$$\begin{aligned} \delta\phi_j(x, r) &= \phi(t_B, x, r) - \phi(t_A, x, r) \\ &\approx \frac{4\pi}{\lambda} [d(t_B, x, r) - d(t_A, x, r)] + \frac{4\pi B_{\perp j}}{\lambda} \frac{\Delta z}{r \sin \vartheta} + \frac{4\pi}{\lambda} [d_{atm}(t_B, x, r) - d_{atm}(t_A, x, r)] \\ &\quad + \Delta n_j, \quad \forall j = 1, \dots, M, \end{aligned} \quad (1)$$

wherein λ is the transmitted signal central wavelength, $\phi(t_B, x, r)$ and $\phi(t_A, x, r)$ represent the phases of the two images involved in the interferogram generation and $d(t_B, x, r)$ and $d(t_A, x, r)$ are the radar line of sight projections of the cumulative deformation at times t_B and t_A , with respect to the instant t_0 assumed as a reference, i.e., implying $\phi(t_0, x, r) = 0, \forall(x, r)$. Moreover, concerning the right-hand side in equation (1), the second term accounts for possible topographic artifacts Δz that can be present in the Digital Elevation Model (DEM) used for the interferograms generation; it depends on the perpendicular baseline component $B_{\perp j}$ (also referred to as spatial baseline) as well as on the sensor-target distance r and on the look angle ϑ . The terms $d_{atm}(t_A, x, r)$ and $d_{atm}(t_B, x, r)$ account for possible atmospheric phase artifacts (GOLDSTEIN, 1995) and the last term Δn_j for the decorrelation effects.

The expression (1) allows us to define a system of M equations in the N unknowns $\phi(t_i, x, r), \forall i = 1, \dots, N$, which can be reorganized, by using a matrix formalism, as follows:

$$A\phi = \delta\phi, \quad (2)$$

wherein A is an incidence-like matrix directly related to the set of interferograms generated from the available data, see BERARDINO *et al.* (2002). Note also that the SBAS technique implies a pixel-by-pixel temporal analysis; accordingly, the dependence on the variables (x, r) has been neglected in (2) and this simplification will be maintained hereafter in the overall matrix representation.

We may now manipulate the system of equation (2) in such a way to replace the present unknowns with the mean phase velocity between time adjacent acquisitions. Accordingly, the new unknowns become:

$$v = \left[v_1 = \frac{\phi(t_1, x, r)}{t_1 - t_0}, \dots, v_N = \frac{\phi(t_N, x, r) - \phi(t_{N-1}, x, r)}{t_N - t_{N-1}} \right] \quad (3)$$

and, by substituting (3) in (2), we finally get the new system of equations:

$$Bv = \delta\phi, \quad (4)$$

wherein B represents an $M \times N$ matrix whose expression can be found in BERARDINO *et al.* (2002).

A few considerations of the system of equation (4) are in order. First of all, we remark that the previously mentioned (possible) separation of the SAR data into independent subsets causes a rank deficiency of the matrix B , leading the system (4) to have infinite solutions. However, the application of the Singular Value Decomposition (SVD) method, carried out within the SBAS procedure, allows us to evaluate the pseudo-inverse of the matrix B , which gives the minimum norm Least-Squares (LS) solution of the system (4). In this context, the mentioned minimum norm constraint for the velocity vector v allows us to mitigate the presence of large discontinuities in the final result, thus guaranteeing a physically sound solution as shown by LANARI *et al.* (2004c); this consideration is at the base of the data manipulation leading to (4). Obviously, an additional integration step is necessary to compute the solution ϕ from the estimated vector v , although this represents a trivial operation.

As a further remark we observe that, after resolving the system of equation (4), an estimation of the topographic artifacts Δz and of the atmospheric signal component $d_{atm}(\cdot)$, see equation (1), is carried out. In particular, the evaluation of Δz benefits from the characteristics of the topographic phase component that is correlated with the perpendicular baseline vector $(B_{\perp 1}, \dots, B_{\perp M})$; note also that the need for the estimation of the topographic factor in equation (1) could appear irrelevant due to the hypothesis of small spatial baseline interferograms. However, since the amplitude of local topographic artifacts may significantly exceed the expected DEM accuracy, it can cause, if uncompensated, a remarkable quality degradation of the produced DInSAR fringes.

For that which concerns the mitigation of possible atmospheric phase artifacts, a filtering operation is performed that, as already highlighted, is based on the observation that the atmospheric signal component is highly correlated in space but poorly in time (FERRETTI *et al.*, 2000). Accordingly, the undesired atmospheric phase signals are estimated through the cascade of a low-pass filtering step in the two-dimensional spatial domain and a temporal high-pass filtering operation. Moreover, this procedure also allows us to detect possible orbital ramps caused by inaccuracies in the SAR sensors orbit information (LANARI *et al.*, 2004b). Finally, a filtering step is carried out in order to remove the detected atmospheric artifacts and orbital ramps.

In summary, the SBAS technique, whose block diagram is depicted in Figure 1, allows us to satisfy two key requirements:

- maximize the “temporal sampling rate” of the retrieved displacements signals by using nearly all the available SAR acquisitions;
- preserve the capability of the system to provide spatially dense deformation maps, which is a key feature of conventional DInSAR interferometry, by exploiting SAR data pairs with limited baseline values.

Accordingly, the SBAS approach permits us, by using standard multilook DInSAR interferograms, to detect and follow the temporal evolution of surface deformation with a high degree of temporal and spatial coverage.

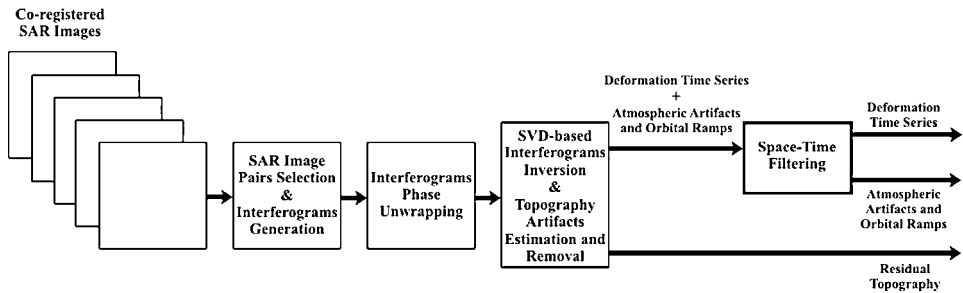


Figure 1
Block diagram of the SBAS algorithm.

3. SBAS-DInSAR Results

The following section presents a selection of results that have been recently achieved by applying the SBAS algorithm. In particular, we analyze the surface deformation effects relevant to different phenomena affecting volcanic areas, aquifers, and landslides. The overall set of results is based on processing data acquired by the ERS sensors (see Table 1 for a description of the sensors characteristics) and provides a clear idea of the surface deformation retrieval capability of the SBAS procedure.

3.1. Surface Deformation Analysis in Volcanic Areas: The Campi Flegrei and Vesuvio Case Study

3.1.1. Site description

The Neapolitan Volcanic area, localized in the southern sector of the Piana Campana, includes three active volcanoes: the Campi Flegrei caldera, Ischia Island

Table 1
Main characteristics of the ERS-1 and ERS-2 SAR sensors

Organization	European Space Agency (ESA)
Platform	Satellite
Launch Date	7/1991 (ERS-1), 4/1995 (ERS-2)
Frequency (GHz)	5.3 (C-band)
Polarization	VV
Orbit Altitude (km)	780
Orbit Inclination (deg)	98.5
Look Angle (deg)	23
Swath Width (km)	100
Antenna Dimensions (m)	10. × 1.
Pulse Duration (μS)	37.1
Pulse Bandwidth (MHz)	15.5
Pulse Repetition Frequency (Hz)	1640–1720
Transmitted Peak Power (kW)	4.8
Data Rate (Mbps)	105 (5 bits/sample, I/Q)

and the Somma-Vesuvio complex, that represent high sources of risk to the surrounding population. In the following analysis we focus on the portion of the Napoli Bay area that includes the two largest volcanoes of the zone, i.e., the Campi Flegrei caldera and the Somma-Vesuvio volcanic complex (see Fig. 2.).

Campi Flegrei caldera, located to the west of the City of Napoli within a densely populated urban area, is a volcanic field where numerous, different eruptive centers have been active over the past 39 ka. The beginning of volcanic activity of Campi Flegrei is not known precisely: sequences of lavas and pyroclasts approximately 2 million years in age have been found in several boreholes. However, the caldera is mainly the result of two principal collapse events related to the Campanian Ignimbrite eruption (39 ka) and to the Neapolitan Yellow Tuff eruption (15 ka), respectively, see ORSI *et al.* (1996) and DI VITO *et al.* (1999). Its evolution has been characterized by intense deformation phenomena (long and short-term deformation) with strong changes of the ground level. The Campi Flegrei magmatic system is still active and over the past 30 years the caldera has experienced episodes of rapid and large amplitude uplift with displacements of several meters. During the two bradyseismic crises of 1969–1972 and 1982–1984, the maxima ground uplifts were of about 170 cm and 180 cm, respectively, see

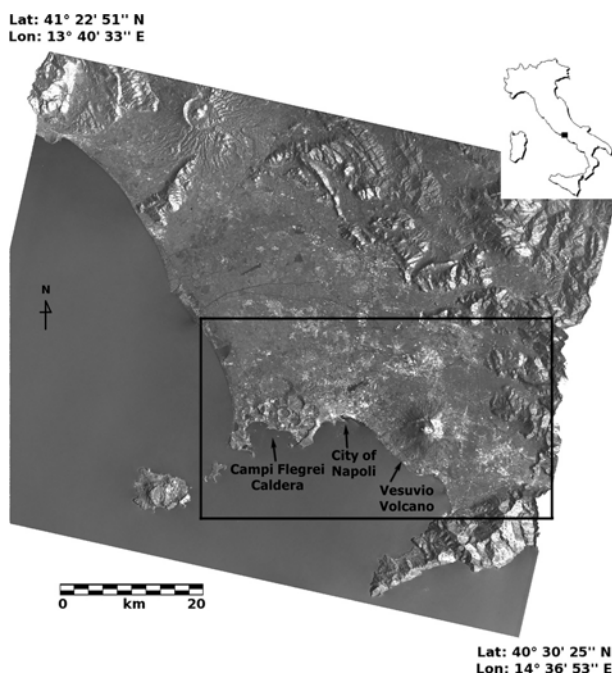


Figure 2

SAR image of the Campania Region (Italy). The black box identifies the investigated Napoli Bay area; the main sites of the zone have been also highlighted. The inset in the upper right corner shows the location of the area.

BIANCHI *et al.* (1987), DVORAK and GASPARINI (1991). These phenomena have been followed by longer term subsidence investigated also via DInSAR techniques (AVALLONE *et al.*, 1999; LUNDGREN *et al.*, 2001; BEAUDUCEL *et al.*, 2004); in particular, the results of the DInSAR data inversion presented by LUNDGREN *et al.* (2001) support the interpretation of subsidence due to hydrothermal diffusion as the main deformation mechanism during this phase of caldera deflation. We further remark that the caldera long-term subsidence phenomenon has been interrupted during 2000 by an uplift event whose deformation also has been analyzed by jointly applying SAR and classical geodetic techniques (LANARI *et al.*, 2004a); after 2000 a restart of the deflation has been recorded.

The Somma-Vesuvio volcano complex (maximum height 1281 m above sea level), located to the southeast of the City of Napoli, is one of the best known volcanoes in the world due to the historic Plinian eruption of 79 A.D. The volcanic activity in the area began at least 400 ka ago, as testified by lavas found (in boreholes) at 1350 m depth, while the history of the Somma-Vesuvio volcanic edifice began almost 25 ka ago with the growth of Mt. Somma. The first Plinian eruption of Pomici di Base (18.3 ka ago) marked the beginning of severe change in the shape of the Somma volcanic edifice, with the formation of a caldera due to the collapse of its summit. The next activity contributed to the growth of the more recent Vesuvio volcano inside this caldera. Its activity has been characterized by great variability both in styles of eruptions and in chemical composition of the emitted magmas. Since the last eruption in 1944, the volcano is in a quiescent stage, its activity limited to the occurrence of local seismicity consisting of some hundreds of events per year reaching maximum magnitudes of 3.7 (DE NATALE *et al.*, 2004). We further remark that deformation phenomena of the Somma-Vesuvio complex have been detected via DInSAR techniques (LANARI *et al.*, 2002) and an interpretation of these effects in terms of volcanic spreading has been recently given (BORGIA *et al.*, 2005).

3.1.2. Results

In order to exploit the capability of the SBAS approach to investigate the deformation of the Campi Flegrei caldera and of the Vesuvio volcano we have processed 116 SAR data acquired by the European Space Agency (ESA) ERS-1/2 sensors on both ascending (track 129, frame 809) and descending (track 36, frame 2781) passes. In particular, starting from the available SAR data set, we have computed 133 interferograms from 58 ERS acquisitions relevant to the ascending orbits spanning the January 1993 to May 2003 time interval. Moreover, 148 interferograms have been produced from 58 ERS acquisitions from descending orbits, collected between June 1992 and October 2002. Each interferometric SAR image pair has been chosen with a perpendicular baseline value smaller than 300 m and with a maximum time interval of 4 years; precise satellite orbital information and a DEM relevant to the Shuttle Radar Topography Mission (SRTM) (ROSEN *et al.*, 2001) have also been used. Moreover, all the DInSAR products have been obtained

following a complex multilook operation (ROSEN *et al.*, 2000) with four looks in the range direction and twenty looks in the azimuth one, with a resulting pixel dimension of about 100×100 m. Note also that the investigated SAR data set is the same exploited by BORGIA *et al.* (2005) for studying the Vesuvio volcano although in this case the processed area extends to the entire Napoli Bay zone.

A first result relevant to this area is presented in Figure 3, where the mean deformation velocity maps, computed (in SAR coordinates) from the ascending and the descending orbits data set, are shown. It is interesting to note the differences between the two geometries that allow also to understand how the radar sensor is imaging the investigated area.

The availability, in this study, of both ascending and descending data allows us to estimate, in addition to the conventional LOS displacements, also the east-west and vertical deformation components. Indeed, we can achieve this result by combining, equivalently to that shown by LUNDGREN *et al.* (2004), BORGIA *et al.* (2005) and MANZO *et al.* (2006), the mean displacement velocity maps computed from the ascending and descending orbits on pixels that are coherent in both maps (67% and 75% of the coherent pixels of the ascending and descending passes, respectively, for a total amount of about 56,000 points). In particular, the difference between these two patterns effectively eliminates their vertical and north velocity components and allows us to obtain an estimate of the surface deformation velocities in the east-west direction (see Fig. 4a). In contrast, the sum of the ascending and descending deformation patterns allows us to get a picture that is mostly vertical motion (see Fig. 4b). They have been computed with respect to a reference pixel that is located in the Napoli Harbor zone (black squares in Fig. 4), in correspondence to the reference

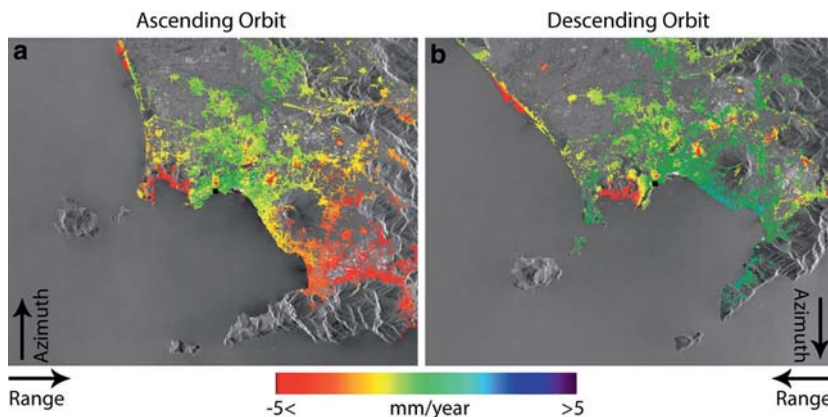


Figure 3

LOS mean deformation velocity maps of the Campania Region, represented in radar geometry, computed in coherent zones and superimposed to the SAR amplitude images: a) ascending orbit; b) descending orbit. Note that the red color corresponds to a sensor-target range increase; the azimuth and range directions are also highlighted. The black squares indicate the homologous reference pixels for the SAR measurements.

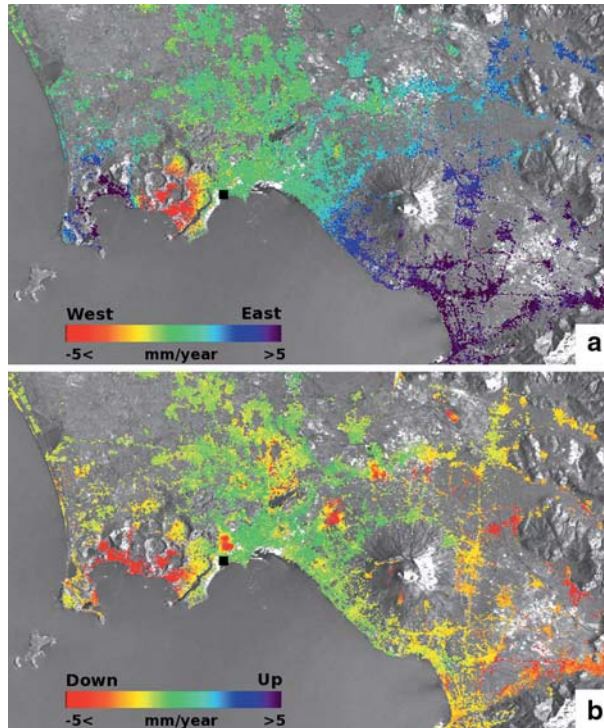


Figure 4

Mean deformation velocity maps relevant to the area highlighted by the black box in Figure 2, computed in coherent areas and superimposed to the SAR amplitude image: a) east-west component of the mean deformation velocity generated from the ascending and descending velocity patterns for pixels which are coherent in both maps; b) vertical component of the mean deformation velocity. Note that the red color corresponds in a) to a displacement toward west, in b) to a subsidence effect. The reference pixel for the SAR measurements, marked by a black square, is located in correspondence of the leveling benchmark referred to as LNA001, see (INGV-OV, 2001; 2002; 2003).

leveling benchmark LNA001. Note also that the DInSAR products shown in Figures 4a and 4b have been geocoded and superimposed on a SAR amplitude image of the zone; moreover, the noisy areas (non-coherent zones) with low accuracy measurements have been excluded.

By considering Figures 4a and 4b we note that reliable information is only available for the urbanized zones and rocky areas. This is particularly critical in the Vesuvio case where only a small part of the volcanic complex can be investigated due to extended vegetated zones. On the contrary, the highly urbanized characteristics of the Campi Flegrei caldera lead to a highly coherent site. Based on the results shown in Figures 4a and 4b, it is evident that the approach can provide information on the mean surface velocity displacements with a rather large spatial coverage. More specifically, by considering Figure 4a, two effects appear evident: the first corresponds to the peculiar deformation pattern of the Campi Flegrei caldera, related to

the long-term deflation, with the eastern and western sectors moving to the west and east, respectively; note also that the central area of the caldera, located in the Pozzuoli Harbor, is characterized by nearly no horizontal displacement. The second effect is related to a regional trend, showing the SE sector moving to the E relatively to the NW sector, as originally shown by BORGIA *et al.* (2005); in this case the transition between W and E displacements corresponds to an active regional tectonic structure previously highlighted by MAINO *et al.* (1963).

Regarding the vertical displacement signals shown in Figure 4b, we remark that also in this case several effects are visible. The first is a rather large displacement pattern relative to the Campi Flegrei caldera that is a peculiar feature relevant to the long-term subsidence. Other effects are several deformations found within the Napoli urban area (the largest one located in the Vomero Quarter, see TESAURO *et al.*, 2000; LANARI *et al.*, 2004d) and the occurrence of displacements on the summit of the Vesuvio volcano and in a sort of semi-corona located at the NW, N, NE, E, and SE base of the volcanic complex; these latter effects relevant to the Vesuvio volcano have been recently interpreted by BORGIA *et al.* (2005) in terms of a volcanic spreading phenomenon. Finally, the last detected effect corresponds to the regional subsidence of the eastern and southeastern sectors of the image.

We remark that the availability of the estimated vertical deformation component shown in Figure 4b is not only important for the analysis of the detected phenomena but also allows us to investigate the quality of the achieved DInSAR results. Indeed, in this case we may benefit from the presence in the Napoli Bay zone of one of the more extended leveling networks of the world with about 600 benchmarks and more than 350 km linear extension (INGV-OV, 2001, 2002, 2003). This geometric leveling network, that is managed by the Osservatorio Vesuviano (OV) belonging to the Italian National Institute of Geophysics and Volcanology (INGV), is a key element of the surveillance system of this area; indeed, it allows the achievement of high accuracy vertical measurements of deformation (mean square deviation of 0.1–0.2 mm/km are typical). Accordingly, we have compared the estimated vertical displacements available from the DInSAR measurements with the corresponding ones computed from the leveling data that are assumed as a reference. In particular, our comparison is focused on the leveling benchmarks identified by the squares shown in Figure 5a; note that this selection of benchmarks, which represents only a portion of the entire network, has been chosen because it permits us to “connect” the central deforming zones of the two investigated volcanoes. Moreover, the related leveling measurements, provided to us by OV-INGV, span the 1990–2003 time interval thus ensuring the temporal overlap with the available SAR data.

The achieved results of the DInSAR/leveling mean deformation velocity comparison are presented in the plot of Figure 5b. It clearly shows the good agreement between the two mean velocity measures (leveling: black stars, SAR: red triangles). Moreover, it also allows us to provide some interesting numbers relevant to the DInSAR products accuracy. To achieve this task, we have computed the

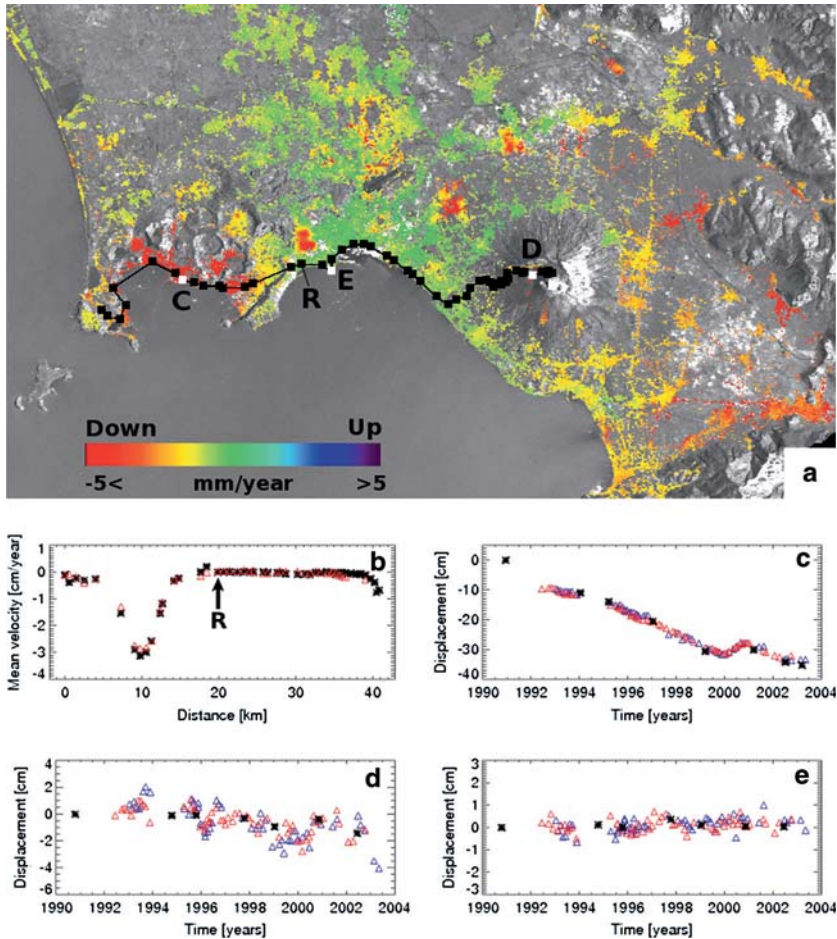


Figure 5

DInSAR/leveling comparison: a) vertical component of the mean deformation velocity (see Fig. 4b) superimposed with a selection of leveling benchmarks (black and white squares). The location of the reference benchmark LNA001, coincident with the reference SAR pixel, is identified by R; b) plot of the mean vertical deformation velocity computed from SAR data (red triangles) and from leveling data (black stars) corresponding with the benchmarks identified in Figure 5a; c–e) comparison between the DInSAR LOS deformation time series (ERS ascending data: blue triangles, ERS descending data: red triangles) and the leveling measurements (black stars) projected on the radar line of sight for the three pixels marked by white squares and labeled C, D and E in Figure 5a, respectively.

standard deviation value of the DInSAR/leveling difference that is equal to 0.11 cm/year. This result is consistent with those obtained through extended studies on the SBAS algorithm performance presented by CASU *et al.* (2005, 2006).

We further remark that the SBAS technique allows us to generate, in addition to the mean deformation velocity patterns, also deformation time series for each coherent pixel. This is a crucial information if we want to study the temporal

evolution of the displacements. To analyze this capability of the procedure, we have investigated the time series achieved from the SAR data relevant to the ascending and descending orbits. Moreover, in order to also compare in this case the DInSAR results with those available from the leveling measurements (the latter projected in the radar LOS with 23° mean look angle to allow the comparison with the ascending and descending DInSAR data), we focused our analysis on areas characterized by nearly vertical deformation only. Accordingly, we have identified first, from the result shown in Figure 4a, a selection of benchmarks with horizontal velocity amplitude below 0.25 cm/year and characterized by different deformation features in their time series; they are represented by the three white squares in Figure 5a and are labeled C, D and E (corresponding to the leveling benchmarks LCF/25A, LVE010 and LVE64B/18), respectively. For each of these benchmarks we have superimposed the line of sight projected leveling measurements (available in the SAR observation time window) and the deformation time series computed from the ascending and descending data, see Figures 5c, 5d and 5e, respectively. By observing the different plots shown in Figures 5c–5e, a strong similarity is clear between the temporal evolution of the ascending (blue triangles) and descending (red triangles) measurements as well as their agreement with the LOS projected leveling (black stars) deformation time series; the capability to follow the temporal evolution of the detected displacements via the SBAS retrieved time series is evident. In particular, by considering Figure 5c the presence is clear of a rather continuous subsidence phenomenon of the Campi Flegrei from 1992 until the beginning of 2000, when a change of the deformation trend occurred resulting in an uplift phase. For what concerns Figure 5d, it shows that the subsidence effect of the Vesuvio summit is rather continuous during the overall investigated time period. Finally, Figure 5e is relevant to a benchmark located in a non-deforming zone which represents a sort of control case in our set of experiments. Similar to what was done before we have computed, for all the benchmarks of Figure 5a, the difference between the DInSAR time series and the leveling data, the latter interpolated via a linear regression within the interval common to SAR data. The average values of the standard deviation of the computed differences amount to 0.44 cm for the ascending data and 0.47 cm for the descending ones, respectively. This is again consistent with what is expected from CASU *et al.* (2005, 2006).

3.2. Surface Deformation Analysis in Aquifer Areas: The Santa Clara Case Study

3.2.1. Site description

The Santa Clara valley aquifer is located within the San Francisco Bay area (California) (see Fig. 6), and represents a complex network of permeable, water-bearing units that exchange groundwater under different confining pressures. This area has experienced a long history of subsidence caused by an excessive pumping of groundwater (USGS Ground Water Atlas). In particular, starting from the

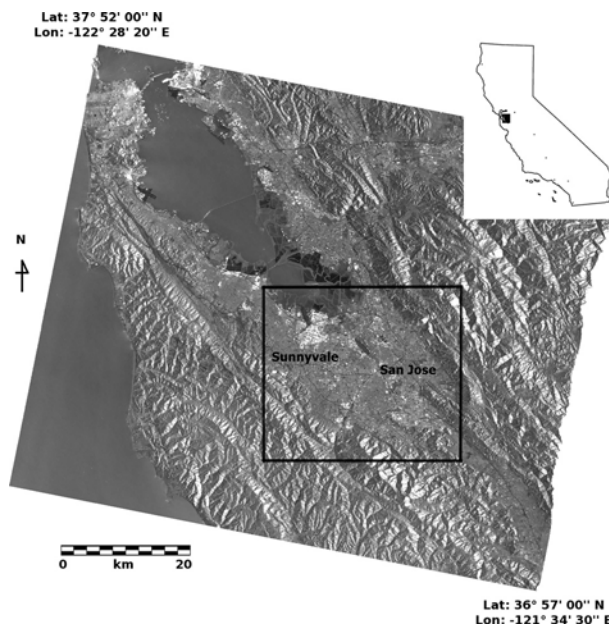


Figure 6

SAR image of the San Francisco area (California); the black box identifies the investigated zone relevant to the Santa Clara basin whose main sites have been also highlighted. The inset in the upper right corner shows the location of the area.

beginning of the twentieth century, extensive use of the groundwater resources has been carried out for agriculture purposes first, and then for urban and industrial developments. A decrease in rainfall during the first half of the twentieth century, coupled with an increase in the pumpage rate, finally led to subsidence phenomena reaching 5 m deformation (POLAND and IRELAND, 1988). Only in the mid-1960s effective actions for mitigating this subsidence were carried out; in particular, the importation of outside sources of water through the construction of various aqueducts represented a key element to offset the effects of groundwater withdrawal (USGS Ground Water Atlas). Moreover, also some percolation ponds were constructed on the valley margins in order to increase the recharge from winter runoff (SCHMIDT and BÜRGMANN, 2003). As a result of these actions, it was recorded through the late 1980s only a small amount of residual aquifer system compaction and subsidence (POLAND and IRELAND, 1988). Since then, no significant inelastic compaction (permanent subsidence) has been detected, while elastic displacements have been measured in response to seasonal variations in groundwater levels that also have been investigated via DInSAR techniques (GALLOWAY *et al.*, 2000; SCHMIDT and BÜRGMANN, 2003).

3.2.2. Results

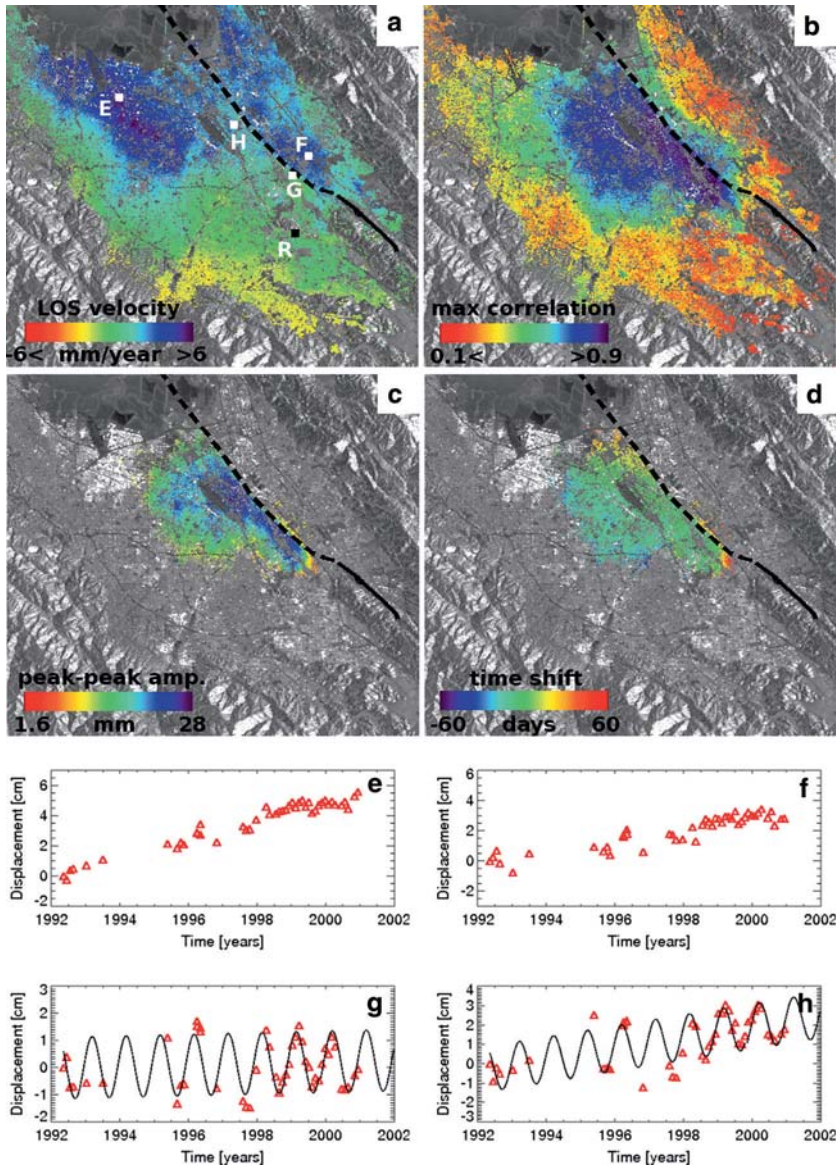
We have applied the SBAS algorithm to a data set of 45 SAR images acquired on descending orbits by the ERS satellites, from mid 1992 to the end of 2000 (track 70, frame 2853), paired in 109 interferograms with a perpendicular baseline always smaller than 300 m and a maximum time interval of 4 years; precise satellite orbital information and an SRTM DEM of the study area have also been used. Moreover, all the interferograms have been obtained following a complex multilook operation, with four looks in the range direction and twenty looks in the azimuth one, with a resulting pixel dimension of about 100×100 m. These processing parameters are consistent with those applied for the generation of the DInSAR products relevant to the previous analysis of the volcanoes located in the Napoli Bay area (see section 3.1.2).

In order to provide an overall picture of the detected displacements, we present in Figure 7a the estimated LOS mean deformation velocity map that has been geocoded and superimposed on the SAR amplitude image of the investigated zone. By considering Figure 7a, two main uplift features appear in correspondence to the Sunnyvale zone (see Fig. 6) and of the area to the east of the Silver Creek fault highlighted in Figure 7a by a black line. The former has a maximum uplift velocity of 0.6 cm/year, while the latter of 0.4 cm/year. The deformation time series of two pixels selected (identified in Figure 7a by the white squares labeled E and F, respectively) are presented in the plots of Figures 7e and 7f, respectively. They clearly show a rather continuous trend that is related to artificial and/or natural recharge phenomena. However, the availability for each coherent pixel of Figure 7a of a deformation time series allows us to further expand our analysis, similar to that done by LANARI *et al.* (2004b) for the Santa Ana basin in southern California. Indeed, in order to investigate the seasonal oscillations of the Santa Clara basin, we have computed the correlation coefficient of each pixel's detrended time series, with a sinusoid within ± 60 days of March 10 in ten-day time bins. As a result of this analysis, we found a significant correlation with an annual oscillation (see Fig. 7b). Moreover, we have also computed the best-fitting sinusoid amplitudes (Fig. 7c) and the maximum correlation time centered on March 10 (Fig. 7d), in areas with correlation greater than 0.6. We found an interesting set of patterns: first of all, we noticed that the areas where deformation is highly correlated with the seasonal variations (see the two sample plots in Figs. 7g and

Figure 7

DInSAR results relevant to the investigated Santa Clara basin: a) LOS mean deformation velocity map computed in coherent areas and superimposed to the SAR amplitude image. The location of the Silver Creek fault (continuous and dashed black lines), of the reference SAR pixel labeled R and of the sample pixels labeled E, F, G, and H, also have been highlighted; b) maximum correlation map between the SAR time series and an annual sinusoid (falling within ± 60 days of March 10); c) peak to peak amplitude of the sinusoid computed in the area where the correlation map shown in Figure 7b is greater than 0.6; d) time shift of the sinusoid relative to the center date of March 10; e-h) deformation time series relevant to the pixels labeled E, F, G and H in Figure 7a, respectively. Note that in Figures 7g and 7h the trended sinusoid (black line) corresponding to the maximum correlation also has been plotted.

7h) are sharply bounded by the northward extension of the Silver Creek fault, which is clearly detectable in Figure 7b. This result is consistent with the interpretation of the fault as a hydrological barrier to groundwater flow, which was previously given by GALLOWAY *et al.* (2000) and SCHMIDT and BÜRGMANN (2003) through alternative DInSAR analysis. Moreover, Figures 7c and 7d show additional intriguing patterns. The former suggests that the zone with higher amplitude oscillations is close to the sharp discontinuity of the Silver Creek fault.



The latter highlights the area in the western part of the aquifer to have oscillations that peak earlier than those in the eastern part, this effect is likely related to groundwater flow. Although these measurements need further investigation and integration with additional hydrological information, they clearly show the capability of the SBAS technique to provide key results for the study of the basin characteristics and for the analysis of the groundwater flow.

3.3. Surface Deformation Analysis in Areas Affected by Landslides: The Maratea Valley Case Study

3.3.1. Site description

The Maratea Valley (see Fig. 8), is located on the Tyrrhenian Coast of the Basilicata region (southern Italy) and is a Quaternary pull-apart basin. This site is characterized by important gravitational phenomena, formerly indicated as sackung (GUERRICCHIO and MELIDORO, 1979), and by strong superficial debris landslide affecting, respectively, the Mesozoic carbonate-dolomite formations and clayey-marly flyschoid terrains (GUERRICCHIO *et al.*, 1987; D'ECCLESIIIS *et al.*, 1993; RIZZO, 1995).

The most important tectonic lineament is the overthrust of the Bulgheria-Verbicaro Unit (BVU) (GUERRICCHIO and MELIDORO, 1979, 1981) on the highly tectonized and chaotic clayey-marly flysch, in black-shales facies (Crete Nere Formation, Cretaceous-lower Eocene). The Crete Nere argillitic flysch, under the assumption of a plastic behavior caused by high water content, is in the presence of tectonic stresses and represents a lubricant for deep gravitational movements (BERARDINO *et al.*, 2003). Geological surveys, previously performed in the area, show

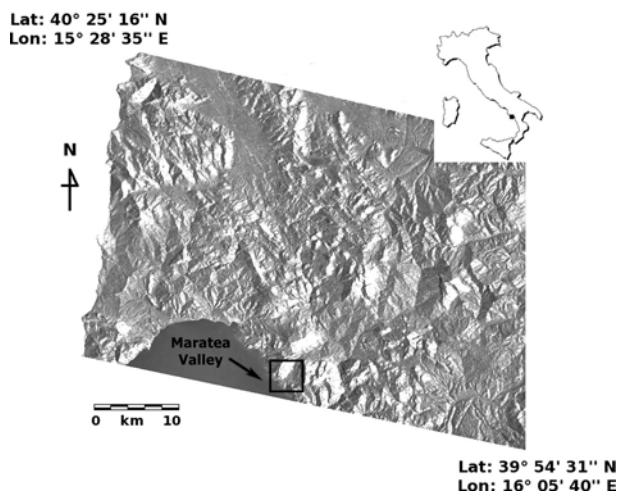


Figure 8

SAR image of the southern part of the Basilicata Region (Italy) where the black box identifies the investigated Maratea Valley. The inset in the upper right corner shows the location of the area.

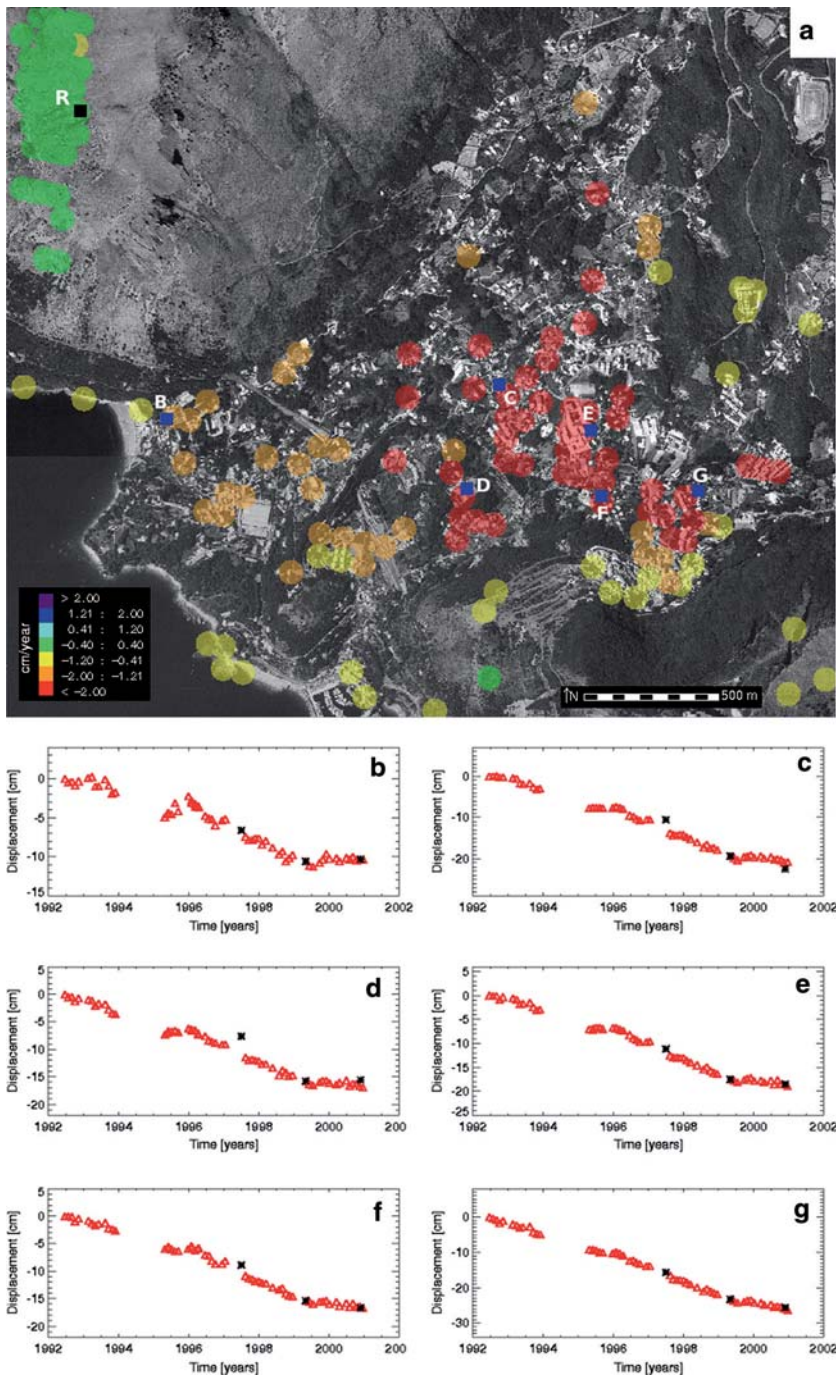
a lithostratigraphic setting characterized by calcareous materials of the BVU, dislocated by gravitational phenomena and positioned on a clayey-marly substratum. The thickness changes from 3 to 148 m and the piezometers show the presence of a ground water flow confined at the bottom by the Liguride Unit.

3.3.2. Results

The presented study extends the DInSAR analysis on the Maratea Valley deformation presented by BERARDINO *et al.* (2003). In particular, in this work we have applied the SBAS technique to a data set of 60 ERS-1/2 SAR images acquired on descending orbits (track 222, frame 2799) and spanning the 1992–2000 time interval. From these data, 119 interferograms have been produced using the same criteria described in the previous sections 3.1.2 and 3.2.2.

We present first in Figure 9a the estimated mean deformation velocity map in the radar LOS that provides key information on the detected displacement pattern. In this case the coherent SAR pixels, represented by circles, have been superimposed on an orthophoto of the zone; note also that the color of each SAR pixel identifies its mean deformation velocity amount. By observing Figure 9a, it is evident that the investigated area shows slope instability movements of significant magnitude reaching a maximum deformation velocity of about 3.3 cm/year during the time interval studied. In order to analyze, in addition to the mean deformation characteristics, also the temporal evolution of the detected displacements, we present in Figures 9b–9g the plots relevant to the deformation time series for a selection of coherent SAR pixels. These sites have been chosen due to their features in the corresponding time series and also because they are relevant to ground points, identified by blue squares in Figure 9a, where GPS measurements were available from campaigns carried out in June 1997, November 1999 and March 2000. Accordingly, we could carry out a comparison between the geodetic and the SAR measurements by projecting the former in the radar LOS and superimposing the results on the corresponding DInSAR measurements (see Figs. 9b–9g).

Although a quantitative analysis is not appropriate in this case, due to the limited amount of GPS data, it is rather clear from Figures 9b–9g that the geodetic and DInSAR measurements are consistent. By further observing these time series, an interesting observation can be also carried out. Indeed, the deformation time series generally reveal the presence of temporal changes in the velocity of the landslide movements. In particular, it appears that after a nearly constant deformation rate of up to a few cm/year in the mid-1990s, during the 1995–1996 and 1999–2000 time intervals the landslide movements significantly slowed; a similar trend is also generally revealed by the available GPS observations. We remark that a hypothesis on the correlation between this apparent stabilization and the rainfall precipitation in the Maratea Valley has been presented by BERARDINO *et al.* (2005), however this issue needs more extensive investigation; moreover, a



deeper analysis clarifying which is the behavior of the landslide and in particular which are the measurements of the landslide itself and what are its consequences, is still in progress.

Finally we must stress that, particularly in the context of landslide monitoring, the orientation and/or the slope of the investigated area may play a key role in the analysis. Indeed, the landslide could even be not “visible” by the SAR sensor just because of the geometric distortions, intrinsic to the side-looking radar imaging characteristic of the system (FRANCESCHETTI and LANARI, 1999).

In any case, it is evident from the results presented, the capability of the SBAS technique to identify changes in mobility for landslides with the characteristics of the Maratea Valley where a very significant urbanization is present. In these scenarios, the SBAS technique can provide very useful information on the temporal landslide hazard assessment.

4. Conclusions and Future DInSAR Developments

The presented overview demonstrates the effectiveness of the SBAS algorithm for the analysis of deformation relevant to the investigated geophysical phenomena. A key element is certainly represented by the capability of the approach to provide information on the space-time characteristics of the detected deformation. More specifically, the availability of measurements relevant to the mean deformation (for instance, the mean deformation velocity patterns) with significantly extended spatial coverage, allows us to identify peculiar features and to implement advanced post-processing operations; they may include, for example, the combination of the information retrieved from multiple radar LOS observations, finally leading to retrieve different components of the detected displacements.

Moreover, the capability of the SBAS technique to produce deformation time series represents another element of key importance because it leads to the detection of signal patterns that yield additional insight into the investigated signal process.

For what concerns the accuracy of the retrieved deformation, the presented analysis is consistent with what was previously shown in an extended study focused on the assessments of the SBAS algorithm performance (CASU *et al.*, 2006). In this work, based on a comparison between DInSAR and geodetic (leveling and GPS) measurements, it is shown that the SBAS technique, for a typical SAR data set involving between 40 and 60 SAR images, can provide measurements with an



Figure 9

DInSAR results relevant to the Maratea Valley: a) orthophoto of the investigated zone superimposed with the LOS mean deformation velocity map in correspondence of the coherent SAR pixels (color circles); the locations of the points where GPS measurements have been investigated (blue squares in correspondence of the pixels labeled B, C, D, E, F and G) also have been highlighted. The SAR reference pixel, labeled R, has been marked; b-g) comparisons between the DInSAR time series (red triangles) and the GPS data (black stars) for the pixels labeled B, C, D, E, F and G in Figure 9a, respectively. Note that the GPS measurements have been projected on the radar LOS.

accuracy of about 1 mm/year and 5 mm for the mean deformation velocity and the displacement time series measurements, respectively.

Regarding the main limitations of the SBAS technique, we can say that the phase unwrapping errors and the unfiltered atmospheric phase artifacts represent the main error sources in the deformation retrieval process; indeed, they may affect the accuracy and also the coherent pixel density of the obtained results; the development of advanced techniques for improving the performances of the available algorithms is still an open research topic.

In addition to these critical issues there are also limitations related to the actual DInSAR technology that is essentially based on SAR sensors operating at C-band. In this case, as discussed and clearly shown in the results of the presented overview, the coherence is essentially maintained only in urbanized and rocky areas. In this context a key role can be played by the next generation sensors like the Japanese ALOS-PALSAR system (launched January, 24th 2006), see SHIMADA *et al.* (2005), that will operate at lower frequencies of the electromagnetic spectrum and in particular within the L-band (1.27 GHz central frequency corresponding to a 23.6 cm wavelength). In this case, although a sensitivity reduction in the displacement detection will be caused by the wavelength increase, a significant improvement of the coherence characteristics is also expected in the generated DInSAR interferograms.

Another promising element is represented by the development of SAR sensor constellations such as the COSMO-SKYMED system (RUM, 2003) that should allow a significant reduction in the SAR sensor revisit time with a drastic, positive impact on the temporal decorrelation phenomena.

Overall, future scenarios involving multi-frequency and multi-LOS SAR observation, coupled with reduced revisit time options, seem extremely promising for the development of advanced applications of the SBAS methodology and more generally for the DInSAR techniques. This may finally lead to the use of the DInSAR methodologies as part of routine surveillance scenarios.

Acknowledgements

This work has been partially sponsored by the European Community on Provision 3.16, under the project of the Regional Center of Competence “Analysis and Monitoring of the Environmental Risk” (CRdC-AMRA) and the (Italian) National Group for Volcanology (GNV). We thank the European Space Agency, which has provided SAR data relevant to the Napoli Bay area within the Cat-1 1065 and 1318 and to the Maratea Valley within the Cat-1 ERC-120, in collaboration within V. Lapenna and A. Perrone of IMAA-CNR. The ERS data of the San Francisco zone have been provided through the WInSAR data archive in collaboration with P. Lundgren, JPL, Caltech. We also thank V. Rizzo for providing the GPS measurements relevant to the Maratea Valley. Moreover, the DEMs of the investigated zones

have been obtained through the SRTM archive while precise ERS-1/ERS-2 satellite orbit state vectors are courtesy of the Technical University of Delft, The Netherlands. The optical images (orthophotos) of the Maratea Valley have been made available by the River Basin Authority of the Basilicata Region. The authors also thank the Osservatorio Vesuviano which has provided the leveling measurements of the Napoli Bay site and, in particular, G. P. Ricciardi for his continuous help and support.

REFERENCES

- AVALLONE, A., ZOLLO, A., BRIOLE, P., DELACOURT, C., and BEAUDUCEL, F. (1999), *Subsidence of Campi Flegrei (Italy) detected by SAR interferometry*, *Geophys. Res. Lett.* 26, 2303–2306.
- BEAUDUCEL, F., BRIOLE, P., and FROGER, J. L. (2000), *Volcano-wide fringes in ERS synthetic aperture radar interferograms of Etna (1992–1998): Deformation or tropospheric effect?* *J. Geophys. Res. - Solid Earth* 105(B7), 16391–16402.
- BEAUDUCEL, F., DE NATALE, G., OBRIZZO, F., and PINGUE, F. (2004), *3-D modelling of Campi Flegrei ground deformation: Role of caldera boundary discontinuities*, *Pure Appl. Geophys.* 161(7), 1329–1344.
- BERARDINO, P., FORNARO, G., LANARI, R., and SANSOSTI, E. (2002), *A new Algorithm for Surface Deformation Monitoring based on Small Baseline Differential SAR Interferograms*, *IEEE Transact. Geoscience and Remote Sensing*, 40(11), 2375–2383.
- BERARDINO, P., COSTANTINI, M., FRANCESCHETTI, G., IODICE, A., PIETRANERA, L., and RIZZO, V. (2003), *Use of differential SAR interferometry in monitoring and modelling large slope instability at Maratea (Basilicata, Italy)*, *Engin. Geol.* 68, 31–51.
- BERARDINO, P., LANARI, R., PEPE, A., ZENI, G., CASARANO, D., WASOWSKI, J., and GUZZETTI, F. (2005), *A space/time deformation analysis of unstable slopes at Maratea (Italy) based on satellite radar interferometry*, *Proc. of EGU General Assembly 2005*, Vienna (Austria).
- BIANCHI, R., CORADINI, A., FEDERICO, C., GIBERTI, G., LANCIANO, P., POZZI, J. P., SARTORIS, G., and SCANDONE, R. (1987), *Modeling of surface ground deformation in volcanic areas: The 1970–1972 and 1982–1984 crises of Campi Flegrei, Italy*, *J. Geophys. Res.* 92(B13), 14,139–14,150.
- BORGIA, A., TIZZANI, P., SOLARO, G., MANZO, M., CASU, F., LUONGO, G., PEPE, A., BERARDINO, P., FORNARO, G., SANSOSTI, E., RICCIARDI, G. P., FUSI, N., DI DONNA, G., and LANARI, R. (2005), *Volcanic spreading of Vesuvius, a new paradigm for interpreting its volcanic activity*, *Geophys. Res. Lett.* 32, L03303, doi: 10.1029/2004GL022155.
- CASCINI, L., FERLISI, S., FORNARO, G., LANARI, R., PEDUTO, D., and ZENI, G. (2006), *Subsidence monitoring in Sarno urban area via multi-temporal DInSAR technique*, *Internat. J. Remote Sensing* 27(8), 1709–1716.
- CASU, F., MANZO, M., and LANARI, R. (2005), *Performance analysis of the SBAS algorithm for surface deformation retrieval*, *Proc. Fringe 2005 Workshop*, 28 November–2 December 2005, Frascati, Italy.
- CASU, F., MANZO, M., and LANARI, R. (2006), *A quantitative assessment of the SBAS algorithm performance for surface deformation retrieval*, *Remote Sensing of Environ.* 102(3–4), 195–210, doi: 10.1016/j.rse.2006.01.023.
- COSTANTINI, M. and ROSEN, P. A. (1999), *A generalized phase unwrapping approach for sparse data*, *Proc. of IGARSS'99*, 267–269.
- CROSETTO, M., CRIPPA, B., and BIESCAS, E. (2005), *Early detection and in-depth analysis of deformation phenomena by radar interferometry*, *Engin. Geol.* 79(1–2), 81–91.
- D'ECCLESII, G., GRASSI, D., and SDAO, F. (1993), *Espandimenti laterali in corrispondenza di due opposti versanti dei monti di Maratea (Basilicata)*, *Atti del 2° Convegno Internazionale di Geoidrologia* 49, 1–17.
- DE NATALE, G., KUZNETZOV, I., KRONROD, T., PERESAN, A., SARAO, A., TROISE, C., and PANZA, G. F. (2004), *Three decades of seismic activity at Mt. Vesuvius: 1972–2000*, *Pure Appl. Geophys.* 161(1), 123–144.
- DI VITO, M. A., ISAIA, R., ORSI, G., SOUTHON, J., DE VITA, S., D'ANTONIO, M., PAPPALARDO, L., and PIOCHI, M. (1999), *Volcanism and deformation since 12,000 years at the Campi Flegrei caldera (Italy)*, *J. Volcanol. Geotherm. Res.* 91, 221–246.

- DVORAK, J. J. and GASPARINI, P. (1991), *History of earthquakes and vertical ground movement in Campi Flegrei caldera, southern Italy: Comparison of precursory events to the A.D. 1538 eruption of Monte Nuovo and of activity since 1968*, J. Volcanol. Geotherm. Res. 48, 77–92.
- FERRETTI, A., PRATI, C., and ROCCA, F. (2000), *Non-linear subsidence rate estimation using permanent scatterers in differential SAR interferometry*, IEEE Transact. Geoscience and Remote Sensing 38, 5.
- FRANCESCHETTI, G. and LANARI, R., *Synthetic Aperture Radar Processing* (CRC Press, Boca Raton FL 1999).
- GABRIEL, A. K., GOLDSTEIN, R. M., and ZEBKER, H. A. (1989), *Mapping small elevation changes over large areas: Differential interferometry*, J. Geophys. Res. 94, 9183–9191.
- GALLOWAY, D. L., BÜRGEMANN, R., FIELDING, E., AMELUNG, F., and LACZNIAK, R. J. (2000), *Mapping recoverable aquifer-system deformation and land subsidence in Santa Clara valley, California, USA, using space-borne synthetic aperture radar*, Proc. of the Sixth Internat. Symp. on Land Subsidence 2, 229–236.
- GOLDSTEIN, R. M. (1995), *Atmospheric limitations to repeat-track radar interferometry*, Geophys. Res. Lett. 22, 2517–2520.
- GUERRICCHIO, A. and MELIDORO, G. (1979), *Deformazioni gravitative profonde del tipo sackung nei monti di Maratea (Lucania)*, Geol. Appl. Idrogeol. XIV, parte I.
- GUERRICCHIO, A. and MELIDORO, G. (1981), *Movimenti di massa pseudotettonici nell'Appennino dell'Italia Meridionale*, Geol. Appl. Idrogeol. 16, 251–294.
- GUERRICCHIO, A., MELIDORO, G., and RIZZO, V. (1987), *Sulla dinamica geomorfologica recente ed attuale della Valle di Maratea (Lucania)*, Boll. Soc. Geol. It. 106, 293–302.
- HOOPER, A., ZEBKER, H., SEGALL, P., and KAMPES, B. (2004), *A new method for measuring deformation on volcanoes and other natural terrains using InSAR persistent scatterers*, Geophys. Res. Lett. 31, L23611, doi:10.1029/2004GL021737.
- INGV - Osservatorio Vesuviano (2001), *Rendiconto sull'attività di sorveglianza*.
- INGV - Osservatorio Vesuviano (2002), *Rendiconto sull'attività di sorveglianza*.
- INGV - Osservatorio Vesuviano (2003), *Rendiconto sull'attività di sorveglianza*.
- LANARI, R., DE NATALE, G., BERARDINO, P., SANSOSTI, E., RICCIARDI, G. P., BORGSTROM, S., CAPUANO, P., PINGUE, F., and TROISE, C. (2002), *Evidence for a peculiar style of ground deformation inferred at Vesuvius volcano*, Geophys. Res. Lett. 29.
- LANARI, R., BERARDINO, P., BORGSTRÖM, S., DEL GAUDIO, C., DE MARTINO, P., FORNARO, G., GUARINO, S., RICCIARDI, G. P., SANSOSTI, E., and LUNDGREN, P. (2004a), *The use of IFSAR and classical geodetic techniques for caldera unrest episodes: Application to the Campi Flegrei uplift event of 2000*, J. Volcanol. Geotherm. Res. 133, 247–260.
- LANARI, R., LUNDGREN, P., MANZO, M., and CASU, F. (2004b), *Satellite radar interferometry time series analysis of surface deformation for Los Angeles, California*, Geophys. Res. Lett. 31, L23613, doi:10.1029/2004GL021294.
- LANARI, R., MORA, O., MANUNTA, M., MALLORQUÍ, J. J., BERARDINO, P., and SANSOSTI, E. (2004c), *A Small Baseline Approach for Investigating Deformation on Full resolution Differential SAR Interferograms*, IEEE Transact. Geoscience and Remote Sensing 42, 7.
- LANARI, R., ZENI, G., MANUNTA, M., GUARINO, S., BERARDINO, P., and SANSOSTI, E. (2004d), *An integrated SAR/GIS approach for investigating urban deformation phenomena: The city of Napoli (Italy) case study*, Int. J. Remote Sens. 25, 2855–2862.
- LUNDGREN, P., USAI, S., SANSOSTI, E., LANARI, R., TESAURO, M., FORNARO, G., and BERARDINO, P. (2001), *Modeling surface deformation observed with synthetic aperture radar interferometry at Campi Flegrei caldera*, J. Geophys. Res. 106, 19, 355–19, 366.
- LUNDGREN, P., CASU, F., MANZO, M., PEPE, A., BERARDINO, P., SANSOSTI, E., and LANARI, R. (2004), *Gravity and magma induced spreading of Mount Etna volcano revealed by satellite radar interferometry*, Geophys. Res. Lett. 31, L04602, doi:10.1029/2003GL018736.
- MAINO, A., SEGRE, A. G., and TRIBALTO, G. (1963), *Rilevamento gravimetrico dei Campi Flegrei e dell'Isola d'Ischia*, Ann. Oss. Vesuviano, Ser. VI, 5, 229–313.
- MANZO M., RICCIARDI, G. P., CASU, F., VENTURA, G., ZENI, G., BORGSTRÖM, S., BERARDINO, P., DEL GAUDIO, C., and LANARI, R. (2006), *Surface deformation analysis in the Ischia island (Italy) based on spaceborne radar interferometry*, J. Volcanol. Geotherm. Res. 151, 399–416, doi:10.1016/j.jvolgeores.2005.09.010.

- MASSONNET, D., ROSSI, M., CARMONA, C., ARDAGNA, F., PELTZER, G., FEIGL, K., and RABAUTE, T. (1993), *The displacement field of the Landers earthquake mapped by radar interferometry*, *Nature* 364, 138–142.
- MORA, O., MALLORQUÍ, J. J., and BROQUETAS, A. (2003), *Linear and nonlinear terrain deformation maps from a reduced set of interferometric SAR images*, *IEEE Transact. Geoscience Remote Sensing* 41, 2243–2253.
- ORSI, G., DE VITA, S., and DI VITO, M. (1996), *The restless, resurgent Campi Flegrei nested caldera (Italy): Constraints on its evolution and configuration*, *J. Volcanol. Geotherm. Res.* 74, 179–214.
- PELTZER, G. and ROSEN, P. A. (1995), *Surface displacement of the 17 May 1993 Eureka Valley, California, earthquake observed by SAR interferometry*, *Science* 268, 1333–1336.
- POLAND, J. F. and IRELAND, R. L. (1988), *Land subsidence in the Santa Clara valley, California, as of 1982, mechanics of aquifer systems*, *U. S. Geol. Surv. Prof. Pap.*, 497–F, 61.
- RIGNOT, E. (1998), *Fast recession of a west Antarctic glacier*, *Science* 281, 549–551.
- RIZZO, V. (1995), *Processi morfodinamici e movimenti del suolo, in atto e storici, nella valle di Maratea (Lucania)*, *Geogr. Fis. Dinam. Quat.* 20, 71–78.
- ROSEN, P. A., HENSLEY, S., JOUGHIN, I. R., LI, F. K., MADSEN, S. N., RODRIGUEZ, E., and GOLDSTEIN, R. (2000), *Synthetic Aperture Radar Interferometry*, *IEEE Proc.* 88, 333–376.
- ROSEN, P. A., HENSLEY, S., GURROLA, E., ROGEZ, F., CHAN, S., and MARTIN, J. (2001), *SRTM C-band topographic data quality assessment and calibration activities*, *Proc. of IGARSS'01*, 739–741.
- RUM, G. (2003), *COSMO-SkyMed: Mission definition and main applications and products*, *Proc. of POLinSAR 2003*.
- SCHMIDT, D. A. and BÜRGMANN, R. (2003), *Time-dependent land uplift and subsidence in the Santa Clara valley, California, from a large interferometric synthetic aperture radar data set*, *J. Geophys. Res.* 108(B9), 2416, doi: 10.1029/2002JB002267.
- SHIMADA, M., ROSENQVIST, A., WATANABE, M., and TADONO, T. (2005), *Polarimetric and interferometric potential of the PALSAR/ALOS*, *Proc. of POLinSAR 2005*.
- TESAURO, M., BERARDINO, P., LANARI, R., SANSOSTI, E., FORNARO, G., and FRANCESCHETTI, G. (2000), *Urban Subsidence inside the City of Napoli (Italy) observed by satellite radar interferometry*, *Geophys. Res. Lett.* 27(13), 1961–1964.
- USAI, S. (2003), *A least squares database approach for SAR interferometric data*, *IEEE Transact. Geoscience and Remote Sensing* 41(4), 753–760.
- USGS Ground Water Atlas of the United States web site - <http://capp.water.usgs.gov/gwa/index.html>.
- WERNER, C., WEGMULLER, U., STROZZI, T., and WIESMANN, A. (2003), *Interferometric point target analysis for deformation mapping*, *Proc. of IGARSS '03*, 7, 4362–4364.
- ZEBKER, H. A. and VILLASENOR, J. (1992), *Decorrelation in interferometric radar echoes*, *IEEE Transact. Geoscience and Remote Sensing* 30, 950–959.

(Received October 24, 2005, revised June 9, 2006, accepted June 19, 2006)

Published Online First: April 16, 2007

To access this journal online:
www.birkhauser.ch/pag

Propagator-matrix Technique for the Viscoelastic Response of a Multi-layered Sphere to Surface Toroidal Traction

ZDENĚK MARTINEC

Abstract—We derive the matrix propagator for the response of a multi-layered, Maxwell viscoelastic sphere to a surface toroidal force in analytical form. Several numerical methods for 2-D and 3-D glacial-isostatic adjustment (GIA) modelling have been applied recently. The present analytical approach may be used as an aid for testing more general algorithms of GIA. The propagator-matrix solution has been tested by comparing it with the analytical solution of two-layer and three-layer spheres, with full agreement obtained.

Key words: Matrix propagator, normal modes, toroidal traction, toroidal vector spherical harmonics, viscoelasticity.

1. Introduction

The time-varying distribution of ice and water over the Earth's surface, with the accompanying changes in sea level, causes glacial-isostatic adjustment (GIA) of the solid Earth. If the Earth is modelled as a linear viscoelastic body with a spherically symmetric distribution of elastic and viscous parameters, the induced deformations are described by the spheroidal displacements only. For a 3-D viscoelastic earth model, in which the elastic and viscous parameters vary in both the radial and lateral directions, the induced deformations are described by both the spheroidal and toroidal displacements. The toroidal displacements, therefore, complete the description of induced deformations for a 3-D earth model and, hence, must be considered in GIA modelling.

The toroidal displacements and associated deformations have not yet been studied in GIA modelling, since spherically symmetric viscoelastic earth models have mostly been applied. Nevertheless, methods for the forward modelling of viscoelastic relaxation in a spherical earth model with a 3-D viscosity distribution are under

GeoForschungsZentrum Potsdam, Department 1: Geodesy and Remote Sensing, Telegrafenberg, D-14473, Potsdam, Germany.

active development. A 3-D finite-element approach is mostly applied to model viscoelastic deformations and perturbations of the gravitational potential (e.g., ZHONG *et al.*, 2003; WU, 2004). Since the 3-D finite-element approach does not use the decomposition of displacement into spheroidal and toroidal parts, the deformations due to toroidal displacements are not computed explicitly, but are involved implicitly in the 3-D finite-element solution.

The above two reasons imply that there is no simple 1-D or 2-D distribution of viscosity and rigidity, whose induced viscoelastic response has an analytical form for toroidal displacements. Such solutions would be useful for validating methods and numerical codes of 2-D and 3-D viscoelastic-relaxation modelling. One configuration that has an analytical solution for toroidal displacements is a multi-layered model of concentrically nested spheres. The purpose of the present paper is to adopt a multi-layered model and apply the propagator-matrix technique to derive the solution for the viscoelastic response to surface toroidal traction, with the aim of finding a rigorous control with respect to which 2-D and 3-D numerical forward solutions can be checked. We also present the explicit analytical expressions for the toroidal relaxation eigenmodes of two-layer and three-layer spherical earth models.

We should note that toroidal motions occur spontaneously when a spherically-layered Earth is perturbed by an internal seismic dislocation (POLLITZ, 1997; RIVA, 2004; SABADINI and VERMEERSEN, 2004). Thus, the results reported here may be used in post-seismic deformation modelling.

2. Viscoelastic Response of a Spherically Symmetric Model to a Surface Toroidal Force

2.1. Formulation of the Problem

We consider a viscoelastic sphere loaded at $t = 0+$ by a time-varying toroidal force that is applied at the external surface $r = a$. Representing the toroidal force by a surface toroidal traction $\mathbf{T}$, the boundary condition at the external surface for $t > 0$ is

$$\boldsymbol{\tau} \cdot \mathbf{e}_r - (\mathbf{e}_r \cdot \boldsymbol{\tau} \cdot \mathbf{e}_r) \mathbf{e}_r = \mathbf{T} \quad \text{for } r = a, \quad (1)$$

where $\boldsymbol{\tau}$ is the Lagrangian increment of the Cauchy stress tensor and $\mathbf{e}_r$ is the unit vector in the radial direction. The traction $\mathbf{T}$ is assumed to be a toroidal vector field depending on colatitude ϑ , longitude φ and time t , that is $\mathbf{T} = \mathbf{T}(\Omega, t)$, where $\Omega := (\vartheta, \varphi)$.

In the Laplace-transform domain s , the constitutive relation of a linear viscoelastic material is

$$\boldsymbol{\tau} = \left(K - \frac{2}{3} \mu \right) \text{div } \mathbf{u} \mathbf{I} + \mu (\text{grad } \mathbf{u} + \text{grad}^T \mathbf{u}), \quad (2)$$

where $\mathbf{u} = \mathbf{u}(r, \Omega, s)$ is the displacement, r is radial distance and $\varepsilon = (\text{grad} \mathbf{u} + \text{grad}^T \mathbf{u})/2$ is the strain tensor, K is the bulk modulus and $\mathbf{I}$ is the second-order identity tensor. For the Maxwell viscoelasticity considered here, the s -dependent shear modulus μ has the form

$$\mu := \frac{\mu^E s}{s + \mu^E/\nu}, \quad (3)$$

where μ^E and ν are the elastic shear modulus and dynamic viscosity, respectively. Hereafter, we assume that μ^E and ν are radially dependent functions only:

$$\mu^E = \mu^E(r), \quad \nu = \nu(r). \quad (4)$$

This implies that the s -dependent shear modulus μ is also spherically symmetric, that is $\mu = \mu(r, s)$. Under the restrictions that the viscoelastic model is loaded by a surface toroidal force and that the elastic shear modulus and the viscosity are r -dependent functions only, ALTERMAN *et al.* (1959) showed that the induced displacement field $\mathbf{u}$ is a toroidal vector, $\mathbf{u} = \mathbf{u}_T$, and has only the ϑ - and φ -components, $\mathbf{u}_T = u_\vartheta(r, \Omega, s)\mathbf{e}_\vartheta + u_\varphi(r, \Omega, s)\mathbf{e}_\varphi$, where $\mathbf{e}_\vartheta$ and $\mathbf{e}_\varphi$ are the unit vectors along colatitude ϑ and longitude φ , respectively. Consequently, the strain tensor has the following spherical components:

$$\begin{aligned} \varepsilon = & \frac{1}{r} \frac{\partial u_\vartheta}{\partial \vartheta} \mathbf{e}_{\vartheta\vartheta} + \frac{1}{r} \left(\frac{1}{\sin \vartheta} \frac{\partial u_\varphi}{\partial \varphi} + \cot \vartheta u_\vartheta \right) \mathbf{e}_{\varphi\varphi} + \left(\frac{\partial u_\vartheta}{\partial r} - \frac{u_\vartheta}{r} \right) \mathbf{e}_{r\vartheta} \\ & + \left(\frac{\partial u_\varphi}{\partial r} - \frac{u_\varphi}{r} \right) \mathbf{e}_{r\varphi} + \frac{1}{r} \left(\frac{\partial u_\varphi}{\partial \vartheta} + \frac{1}{\sin \vartheta} \frac{\partial u_\vartheta}{\partial \varphi} - \cot \vartheta u_\varphi \right) \mathbf{e}_{\vartheta\varphi}, \end{aligned}$$

where $\mathbf{e}_{ij}$, $i, j \in \{r, \vartheta, \varphi\}$, are the symmetric parts of the dyadic products of the spherical base vectors. Moreover, $\mathbf{u}_T$ is divergence-free, that is $\text{div} \mathbf{u}_T = 0$. Consequently, the part of the constitutive relation (2) that is proportional to $\text{div} \mathbf{u}$ is identically zero. The traction on any spherical surface, that is a vector $\mathbf{t}_r := \boldsymbol{\tau} \cdot \mathbf{e}_r$, is then expressed as

$$\mathbf{t}_r = \mu \left[\left(\frac{\partial u_\vartheta}{\partial r} - \frac{u_\vartheta}{r} \right) \mathbf{e}_\vartheta + \left(\frac{\partial u_\varphi}{\partial r} - \frac{u_\varphi}{r} \right) \mathbf{e}_\varphi \right]. \quad (5)$$

In view of this, the boundary condition (1) reduces to

$$\mathbf{t}_r = \mathbf{T} \quad \text{for } r = a. \quad (6)$$

In the case of toroidal motion, the equation of linear-momentum balance simplifies to $\text{div} \boldsymbol{\tau} = \mathbf{0}$ or written more explicitly:

$$-\mu \text{rot rot } \mathbf{u}_T + \frac{d\mu}{dr} (\text{grad } \mathbf{u}_T + \text{grad}^T \mathbf{u}_T) \cdot \mathbf{e}_r = \mathbf{0}. \quad (7)$$

Let us make two remarks. First, the spheroidal part of displacement is not induced in this case. However, this decoupling between the toroidal and spheroidal parts of displacement is broken if the basic assumption on the spherically symmetric geometry of the problem is violated.

Second, equations (1)–(7) can be derived also in the case where the viscoelastic sphere is assumed to be incompressible. Hence, under the assumption that the elastic rigidity and viscosity are radially dependent functions only, the incompressibility constraint does not influence the induced toroidal motion. However, this constraint influences the character of the spheroidal motion, therefore, in the case where the elastic rigidity and viscosity vary in lateral direction, that is, when the toroidal and spheroidal displacements are coupled, the incompressibility constraint will also influence the character of toroidal motion.

The interface conditions at internal discontinuities are characterized by the continuity of the displacement $\mathbf{u}_T$ and the continuity of the surface traction $\mathbf{t}_r$. When an inviscid fluid core is included, free-slip behavior, that is $\mathbf{t}_r = \mathbf{0}$ is appropriate.

The displacement field $\mathbf{u}_T$ resulting from solving the initial- and boundary-value problem described by equations (1)–(7) is determined apart from a rigid-body rotation. This fact implies two consequences. First, a solution to the problems (1)–(7) does not, in general, exist. To guarantee the existence of a solution, the applied surface toroidal traction $\mathbf{T}$ must satisfy the so-called equilibrium condition (e.g., NEČAS and HLAVÁČEK, 1981, Sect. 7.3):

$$\int_{\Omega_0} (\mathbf{r} \times \mathbf{T}) \Big|_{r=a} d\Omega = \mathbf{0}, \quad (8)$$

where Ω_0 is the full solid angle and $d\Omega$ its infinitesimal element. This condition means that the surface toroidal traction $\mathbf{T}$ cannot be chosen arbitrarily, but only such that its surface net torque vanishes. Second, the displacement field $\mathbf{u}_T$ is not determined uniquely. To guarantee the uniqueness of the solution, an additional constraint on the solution must be imposed. We will return to this point in Section 3.5.

2.2. Spherical Harmonic Representation

We now apply the formalism of vector and tensor spherical harmonics to parameterize the field variables. We first represent the displacement vector $\mathbf{u}_T$ in terms of toroidal vector spherical harmonics $\mathbf{S}_{jm}^{(0)}(\Omega)$:

$$\mathbf{u}_T(r, \Omega, s) = \sum_{j=1}^{\infty} \sum_{m=-j}^j W_{jm}(r, s) \mathbf{S}_{jm}^{(0)}(\Omega), \quad (9)$$

where the vector functions $\mathbf{S}_{jm}^{(0)}(\Omega)$ can be expressed in terms of the scalar spherical harmonics $Y_{jm}(\Omega)$ as (e.g., MARTINEC, 2000, Appendix B)

$$\mathbf{S}_{jm}^{(0)}(\Omega) = \frac{\partial Y_{jm}(\Omega)}{\partial \vartheta} \mathbf{e}_\varphi - \frac{1}{\sin \vartheta} \frac{\partial Y_{jm}(\Omega)}{\partial \varphi} \mathbf{e}_\vartheta. \quad (10)$$

Introducing the spherical harmonic representation of $\mathbf{u}_T$, the strain tensor ε can be expressed in the form

$$\varepsilon = \sum_{j=1}^{\infty} \sum_{m=-j}^j \left[\left(\frac{dW_{jm}}{dr} - \frac{W_{jm}}{r} \right) \mathbf{Z}_{jm}^{(3)}(\Omega) + \frac{W_{jm}}{r} \mathbf{Z}_{jm}^{(4)}(\Omega) \right], \quad (11)$$

where $\mathbf{Z}_{jm}^{(\lambda)}(\Omega)$, $\lambda = 3, 4$, are the tensor spherical harmonics defined by (Martinec, 2000, Appendix B)

$$\begin{aligned} \mathbf{Z}_{jm}^{(3)}(\Omega) &:= -\frac{1}{\sin \vartheta} \frac{\partial Y_{jm}(\Omega)}{\partial \varphi} \mathbf{e}_{r\vartheta} + \frac{\partial Y_{jm}(\Omega)}{\partial \vartheta} \mathbf{e}_{r\varphi}, \\ \mathbf{Z}_{jm}^{(4)}(\Omega) &:= \left(\frac{\partial^2}{\partial \vartheta^2} - \cot \vartheta \frac{\partial}{\partial \vartheta} - \frac{1}{\sin^2 \vartheta} \frac{\partial^2}{\partial \varphi^2} \right) Y_{jm}(\Omega) \mathbf{e}_{\vartheta\varphi} \\ &\quad - \frac{\partial}{\partial \vartheta} \left(\frac{1}{\sin \vartheta} \frac{\partial Y_{jm}(\Omega)}{\partial \varphi} \right) (\mathbf{e}_{\vartheta\vartheta} - \mathbf{e}_{\varphi\varphi}). \end{aligned} \quad (12)$$

Since $\mathbf{e}_r \cdot \mathbf{Z}_{jm}^{(3)}(\Omega) = \frac{1}{2} \mathbf{S}_{jm}^{(0)}(\Omega)$ and $\mathbf{e}_r \cdot \mathbf{Z}_{jm}^{(4)}(\Omega) = \mathbf{0}$, the vector spherical harmonic representation of the traction $\mathbf{t}_r$ is

$$\mathbf{t}_r = \sum_{j=1}^{\infty} \sum_{m=-j}^j t_{jm}(r) \mathbf{S}_{jm}^{(0)}(\Omega), \quad (13)$$

where

$$t_{jm}(r) = \mu \left(\frac{dW_{jm}(r)}{dr} - \frac{W_{jm}(r)}{r} \right). \quad (14)$$

We next make use of the differential relation (VARSHALOVICH *et al.*, 1989)

$$\text{rot rot} [f(r) \mathbf{S}_{jm}^{(0)}(\Omega)] = - \left(\frac{d^2}{dr^2} + \frac{2}{r} \frac{d}{dr} - \frac{j(j+1)}{r^2} \right) f(r) \mathbf{S}_{jm}^{(0)}(\Omega), \quad (15)$$

where $f(r)$ is a differentiable function. With equation (15), the partial differential equation (7) can be reduced to the ordinary second-order differential equation with respect to the radial variable r :

$$\mu \left(\frac{d^2}{dr^2} + \frac{2}{r} \frac{d}{dr} - \frac{j(j+1)}{r^2} \right) W_{jm}(r) + \frac{d\mu}{dr} \left(\frac{dW_{jm}(r)}{dr} - \frac{W_{jm}(r)}{r} \right) = 0. \quad (16)$$

Finally, transforming the surface toroidal traction $\mathbf{T}(\Omega, t)$ into the Laplace domain and expanding its Laplace image $\mathbf{T}(\Omega, s)$ into a series of the toroidal vector spherical harmonics, we obtain

$$\mathbf{T}(\Omega, s) = \sum_{j=1}^{\infty} \sum_{m=-j}^j T_{jm}(s) \mathbf{S}_{jm}^{(0)}(\Omega). \quad (17)$$

The expansion coefficients can be derived by the spherical-harmonic analysis of $\mathbf{T}(\Omega, s)$:

$$T_{jm}(s) = \frac{1}{j(j+1)} \int_{\Omega_0} \mathbf{T}(\Omega, s) \cdot \left[\mathbf{S}_{jm}^{(0)}(\Omega) \right]^* d\Omega, \quad (18)$$

where the asterisk denotes complex conjugation.

3. Propagator-matrix Technique

3.1. Matrix of Fundamental Solution

The differential equation (16) can alternatively be expressed as a set of two simultaneous first-order differential equations in the form

$$\frac{d\mathbf{Y}}{dr} = \mathbf{A}\mathbf{Y}, \quad (19)$$

where the elements of the vector $\mathbf{Y}(r, s) = (W_{jm}, t_{jm})^T$ are the spherical-harmonic coefficients of toroidal displacement $\mathbf{u}_T$ and toroidal traction $\mathbf{t}_r$, respectively. The 2×2 matrix $\mathbf{A}$ has the form

$$\mathbf{A} = \begin{pmatrix} \frac{1}{r} & \frac{1}{\mu} \\ \frac{\mu(j-1)(j+2)}{r^2} & -\frac{3}{r} \end{pmatrix}. \quad (20)$$

This matrix has been derived by ALTERMAN *et al.* (1959) for an elastic material.

A general solution to equation (19) can be expressed as the sum of two linearly independent fundamental solution vectors:

$$\mathbf{Y}(r, s) = \mathbf{M}(r, s) \mathbf{C}(s), \quad (21)$$

where $\mathbf{M}(r, s)$ is the 2×2 matrix whose columns are the two fundamental solution vectors, and $\mathbf{C}(s)$ is a 2×1 vector of r -independent constants. For a homogeneous spherical solid, the matrix $\mathbf{M}$ was given in explicit analytical form by, e.g., SABADINI and VERMEERSEN (2004):

$$\mathbf{M}(r, s) = \begin{pmatrix} r^j & r^{-j-1} \\ \mu(s)(j-1)r^{j-1} & -\mu(s)(j+2)r^{-j-2} \end{pmatrix}. \quad (22)$$

To construct the propagator matrix for a multi-layered, Maxwell viscoelastic sphere, the inverse of matrix $\mathbf{M}$ must be found, which, after some algebraic manipulation, results in (SABADINI and VERMEERSEN, 2004)

$$\mathbf{M}^{-1}(r, s) = \frac{1}{2j+1} \begin{pmatrix} (j+2)r^{-j} & \frac{1}{\mu(s)}r^{-j+1} \\ (j-1)r^{j+1} & -\frac{1}{\mu(s)}r^{j+2} \end{pmatrix}. \quad (23)$$

3.2. Layer-propagator Matrix

Let the spherical earth model be composed of N homogeneous viscoelastic spherical layers bounded by spheres of radii $r_1 = b < r_2 < \dots < r_N = a$. In the k -th ($k \geq 2$) layer, the relation between $\mathbf{Y}(r, s)$ at inner boundary r_{k-1} and outer boundary r_k can be expressed in the form (GANTMACHER, 1959; GILBERT and BACKUS, 1966)

$$\mathbf{Y}(r_k, s) = \mathbf{P}(r_k, r_{k-1}, s) \mathbf{Y}(r_{k-1}, s), \quad (24)$$

where the layer-propagator matrix $\mathbf{P}(r_k, r_{k-1}, s)$ is given in terms of matrices $\mathbf{M}$ and $\mathbf{M}^{-1}$ as

$$\mathbf{P}(r_k, r_{k-1}, s) = \mathbf{M}(r_k, s) \mathbf{M}^{-1}(r_{k-1}, s). \quad (25)$$

Making use of expressions (22) and (23) for the matrices $\mathbf{M}$ and $\mathbf{M}^{-1}$, respectively, the matrix $\mathbf{P}$ is

$$\mathbf{P}(r_k, r_{k-1}, s) = \frac{1}{2j+1} \begin{pmatrix} (j+2)h^j + (j-1)h^{-j-1} & \frac{r_{k-1}}{\mu_k(s)}(h^j - h^{-j-1}) \\ \frac{(j-1)(j+2)\mu_k(s)}{r_{k-1}}(h^{j-1} - h^{-j-2}) & (j-1)h^{j-1} + (j+2)h^{-j-2} \end{pmatrix}, \quad (26)$$

where $h := r_k/r_{k-1}$.

The solution in the inner sphere ($0 \leq r \leq b$) is given by equation (21), which takes the particular form

$$\mathbf{Y}(r, s) = \mathbf{Z}(r, s) K(s), \quad (27)$$

where $\mathbf{Z}(r, s)$ is the 2×1 vector composed of the first column of matrix $\mathbf{M}$ and $K(s)$ is an r -independent constant. In particular, for a homogeneous inviscid fluid core, the toroidal stresses vanish and the vector $\mathbf{Z}(r, s)$ reduces to the vector $(r^j, 0)^T$. The solution in the k th spherical layer then follows from the upward continuation of the solution from the inner sphere and the continuity of the vector $\mathbf{Y}(r, s)$ at the interfaces $r_1, \dots, r_N$. This corresponds to the multiplication of the product of the layer-propagator matrices with the inner-sphere solution taken at $r = b$. In the k th layer, $r_{k-1} \leq r \leq r_k$, $k \geq 2$, we obtain

$$\mathbf{Y}(r, s) = \mathbf{L}(r, b, s) \mathbf{Y}(b, s), \quad (28)$$

where

$$\mathbf{L}(r, b, s) = \mathbf{P}(r, r_{k-1}) \mathbf{P}(r_{k-1}, r_{k-2}) \cdots \mathbf{P}(r_2, b). \quad (29)$$

Substituting for $\mathbf{Y}(b, s)$ from equation (27) yields

$$\mathbf{Y}(r, s) = \mathbf{L}(r, b, s) \mathbf{Z}(b, s) K(s). \quad (30)$$

The constant $K(s)$ can be determined from the boundary condition (6). In view of equations (13) and (17), it takes the form

$$t_{jm}(a, s) = T_{jm}(s), \quad (31)$$

where $T_{jm}(s)$ is the s -dependent spherical-harmonic coefficient of the applied surface toroidal traction. Considering the second equation in system (30) for $r = a$, the boundary condition takes the form

$$B(s) = L(s) K(s), \quad (32)$$

where, for brevity, $B(s)$ and $L(s)$ denote $T_{jm}(s)$ and the second component of vector $\mathbf{L}(a, b, s) \mathbf{Z}(b, s)$, respectively:

$$L(s) = L_{21}(a, b, s) Z_1(b) + L_{22}(a, b, s) Z_2(b, s). \quad (33)$$

Determining $K(s)$ from equation (32) and substituting the result into equation (30) gives the solution in the k -th layer ($r_{k-1} \leq r \leq r_k$):

$$\mathbf{Y}(r, s) = \frac{\mathbf{L}(r, b, s) \mathbf{Z}(b, s) B(s)}{L(s)}. \quad (34)$$

3.3. Spherical Harmonics of Degree $j = 1$

We now discuss the solvability of equation (32) for angular degree $j = 1$. In this case, the 2,1 element of the layer-propagator matrix $\mathbf{P}(r_k, r_{k-1}, s)$ is equal to zero, i.e., $P_{21} = 0$. Consequently, the 2,1 element of the model-propagator matrix $\mathbf{L}(a, b, s)$ vanishes, i.e., $L_{21} = 0$. Moreover, the second element of vector $\mathbf{Z}(b, s)$ is equal to zero, i.e., $Z_2 = 0$. These facts imply that $L(s) = 0$ and equation (32) is inconsistent, meaning that, in general, there is no solution $K(s)$ for $j = 1$. To circumvent this problem, the existence condition (8) is taken into consideration. Employing the orthogonality property of vector spherical harmonics (e.g., MARTINEC and WOLF, 1999), equation (8) reduces to the form

$$T_{1m} = 0 \quad \text{for } m = -1, 0, 1. \quad (35)$$

Hence, the quantity $B(s) = 0$ for $j = 1$ and a solution $K(s)$ of equation (32) exists, but, in general, is not unique. To define the solution uniquely, we impose an additional constraint on the solution. From an infinite number of possibilities, we will require that the surface net rotation of the viscoelastic sphere vanishes, that is

$$\int_{\Omega_0} (r \times \mathbf{u}_T) \Big|_{r=a} d\Omega = \mathbf{0}. \quad (36)$$

Employing again the orthogonality property of vector spherical harmonics, this constraint reduces to the form

$$W_{1m}(a) = 0 \quad \text{for } m = -1, 0, 1. \quad (37)$$

The first equation in (30) for $r = a$ then implies that $K(s) = 0$ and

$$W_{1m}(r) = 0 \quad \text{for } m = -1, 0, 1, \quad (38)$$

and $b \leq r \leq a$. In summary, provided that the applied surface toroidal traction $\mathbf{T}$ does not contain the first-degree spherical harmonics and assuming that the surface net rotation of the model is equal to zero, the first-degree spherical harmonics of induced toroidal displacement $\mathbf{u}_T$ vanish.

3.4. Toroidal Load-deformation Love Number

In analogy with the surface-mass loading problem for a viscoelastic sphere, it is convenient to describe the toroidal response of a viscoelastic sphere in terms of Love numbers. To determine the response, let us express the spherical-harmonic coefficient of the induced toroidal displacement from equation (34):

$$W_{jm}(r, s) = \frac{L_{11}(r, b, s)Z_1(b, s) + L_{12}(r, b, s)Z_2(b, s)}{L(s)} T_{jm}(s). \quad (39)$$

The toroidal load-deformation Love number $\tau_j(r, s)$ may be defined by the following implicit equation:

$$W_{jm}(r, s) =: \tau_j(r, s) \frac{Ga}{g_0^2} T_{jm}(s). \quad (40)$$

Making use of equation (39), we obtain

$$\tau_j(r, s) = \frac{g_0^2}{Ga} \frac{L_{11}(r, b, s)Z_1(b, s) + L_{12}(r, b, s)Z_2(b, s)}{L(s)}, \quad (41)$$

where G is Newton's gravitation constant, $g_0 = GM/a^2$ and M is the mass of the Earth. Note that the Love number $\tau_j(r, s)$ is dimensionless due to scaling by the factor g_0^2/Ga .

3.5. Time-domain Solution

In the case that the surface toroidal traction is applied as the Heaviside step function $H(t)$, the solution (34) in the Laplace-transform domain can, according to PELTIER (1985), be transformed to the time domain for $t \geq 0$ as

$$\mathbf{Y}(r, t) = \mathbf{Y}^E(r)H(t) - \sum_j \frac{\mathbf{Y}_j^V(r)}{s_j} (1 - e^{s_j t}), \quad (42)$$

where $\mathbf{Y}^E(r)$ is the elastic amplitude:

$$\mathbf{Y}^E(r) = \lim_{s \rightarrow -\infty} \mathbf{Y}(r, s). \quad (43)$$

$\mathbf{Y}_j^V(r)$ is the viscous amplitude spectrum and $-s_j$ is the inverse relaxation-time spectrum. The inverse relaxation-time spectrum can be determined from the roots of the secular determinant $L(s_j) = 0$. The viscous amplitudes $\mathbf{Y}_j^V(r)$ are obtained using the residue theorem:

$$\mathbf{Y}_j^V(r) = \frac{\mathbf{L}(r, b, s_j) \mathbf{Z}(b, s_j) B}{\left. \frac{dL(s)}{ds} \right|_{s=s_j}}, \quad (44)$$

where simple zeros s_j are assumed.

3.6. The s Derivative of the Secular Determinant

The analytical form (26) of the layer-propagator matrix allows us to compute the s derivative of the secular determinant $L(s)$ in equation (44) analytically. The differentiation of equation (33) with respect to s yields

$$\frac{dL(s)}{ds} = \frac{dL_{21}(a, b, s)}{ds} Z_1(b) + \frac{dL_{22}(a, b, s)}{ds} Z_2(b, s) + L_{22}(a, b, s) \frac{dZ_2(b, s)}{ds}. \quad (45)$$

In particular, for a viscoelastic sphere with an inviscid fluid core, $Z_2(b, s) = 0$, and equation (45) reduces to

$$\frac{dL(s)}{ds} = \frac{dL_{21}(a, b, s)}{ds} Z_1(b). \quad (46)$$

The derivative of the model-propagator matrix $\mathbf{L}(a, b, s)$ can be obtained by differentiating equation (29) with respect to s :

$$\begin{aligned} \frac{d\mathbf{L}(a, b, s)}{ds} &= \frac{d\mathbf{P}(r_N, r_{N-1})}{ds} \mathbf{P}(r_{N-1}, r_{N-2}) \cdots \mathbf{P}(r_2, r_1) \\ &\quad + \mathbf{P}(r_N, r_{N-1}) \frac{d\mathbf{P}(r_{N-1}, r_{N-2})}{ds} \cdots \mathbf{P}(r_2, r_1) \\ &\quad + \cdots + \mathbf{P}(r_N, r_{N-1}) \mathbf{P}(r_{N-1}, r_{N-2}) \cdots \frac{d\mathbf{P}(r_2, r_1)}{ds} \\ &= \sum_{k=2}^N \mathbf{L}(r_N, r_k, s) \frac{d\mathbf{P}(r_k, r_{k-1})}{ds} \mathbf{L}(r_{k-1}, r_1, s). \end{aligned} \quad (47)$$

Note that, for the last step, we have used again the definition (29) and considered that $\mathbf{L}(r_k, r_k, s)$ is equal to the 2×2 unit matrix. Differentiating equation (26) with respect to s yields

$$\frac{d\mathbf{P}(r_k, r_{k-1}, s)}{ds} = \frac{1}{\mu_k(s)} \frac{d\mu_k(s)}{ds} \begin{pmatrix} 0 & -P_{12} \\ P_{21} & 0 \end{pmatrix}. \quad (48)$$

Moreover, the s derivative of the shear modulus can be obtained from equation (3):

$$\frac{d\mu_k(s)}{ds} = \frac{\mu_k^2(s)}{v_k s^2}. \quad (49)$$

Hence, equation (48) takes the form

$$\frac{d\mathbf{P}(r_k, r_{k-1}, s)}{ds} = \frac{\mu_k(s)}{v_k s^2} \begin{pmatrix} 0 & -P_{12} \\ P_{21} & 0 \end{pmatrix}. \quad (50)$$

Likewise, the s derivative of $Z_2(b, s)$ is

$$\frac{dZ_2(b, s)}{ds} = \frac{\mu_1(s)}{v_1 s^2} Z_2(b, s). \quad (51)$$

Substituting equation (51) and (47) for the 2,1 and 2,2 elements into equation (45) provides us with the analytical form of the s derivative of the secular determinant $L(s)$.

4. Analytical Solutions for Two-layer and Three-layer Spheres

4.1. Two-layer Sphere

Let a spherical earth model consist of two spherical layers: lithosphere L and mantle M of external radii a and b , respectively. The elastic shear modulus and the viscosity of the two layers are assumed to be constant and equal to μ_L^E and v_L , respectively, inside the lithosphere L , and μ_M^E and v_M , respectively, inside the mantle M .

The model-propagator matrix $\mathbf{L}(a, b, s)$ reduces to the layer-propagator matrix $\mathbf{P}(a, b, s)$:

$$\mathbf{P}(a, b, s) = \frac{1}{2j+1} \begin{pmatrix} (j+2)\alpha^j + (j-1)\alpha^{-j-1} & \frac{b}{\mu_L(s)}(\alpha^j - \alpha^{-j-1}) \\ \frac{(j-1)(j+2)\mu_L(s)}{b}(\alpha^{j-1} - \alpha^{-j-2}) & (j-1)\alpha^{j-1} + (j+2)\alpha^{-j-2} \end{pmatrix}, \quad (52)$$

the vector $\mathbf{Z}(b, s)$ is equal to $\mathbf{Z}(b, s) = \left(b^j, \mu_M(s)(j-1)b^{j-1} \right)^T$, where $\alpha := a/b$ and $\mu_{L,M}(s) := \mu_{L,M}^E s / (s + \mu_{L,M}^E / v_{L,M})$, and the secular determinant $L(s)$ can be expressed in the form

$$L(s) = \frac{(j-1)b^{j-1}}{2j+1} \{ (j+2)[\alpha^{j-1} - \alpha^{-j-2}]\mu_L(s) + [(j-1)\alpha^{j-1} + (j+2)\alpha^{-j-2}]\mu_M(s) \}. \quad (53)$$

Making use of the definition (41), the surface toroidal load-deformation Love number $\tau_j(a, s)$ has the form

$$\tau_j(a, s) = \frac{g_0^2}{G\mu_L(s)} \frac{1}{j-1} \frac{[(j+2)\alpha^{j-1} + (j-1)\alpha^{-j-2}]\mu_L(s) + (j-1)[\alpha^{j-1} - \alpha^{-j-2}]\mu_M(s)}{(j+2)[\alpha^{j-1} - \alpha^{-j-2}]\mu_L(s) + [(j-1)\alpha^{j-1} + (j+2)\alpha^{-j-2}]\mu_M(s)}. \quad (54)$$

Obviously, there is a root of the secular determinant $L(s)$ associated with the vanishing of $\mu_L(s)$ and $\mu_M(s)$ for $s = 0$. Since

$$\lim_{s \rightarrow 0} \frac{1 - e^{st}}{s} = -t, \quad (55)$$

the time evolution of the solution $\mathbf{Y}(r, t)$ is, according to equation (42), described by the function

$$\mathbf{Y}(r, t) = \mathbf{Y}^E(r) + \mathbf{Y}^V(r)t, \quad (56)$$

where the elastic and viscous amplitudes, $\mathbf{Y}^E(r)$ and $\mathbf{Y}^V(r)$, respectively, are finite. Equation (56) shows that, for Heaviside loading, the magnitude of the solution $\mathbf{Y}(r, t)$ grows linearly in time without approaching a finite limit for $t \rightarrow \infty$. The physical explanation is that there is no internal force balancing the applied surface toroidal traction and induced toroidal viscoelastic stresses inside the model, such as the gravitational force balancing the viscoelastic stresses induced by a surface-mass load.

To avoid an unbounded response, we will now assume that the lithosphere is composed of a purely elastic material with the elastic rigidity μ_L^E . In this case, we have

$$\mu_L(s) = \mu_L^E, \quad (57)$$

that is, $\mu_L(s)$ is independent of the s variable, and the root $s = 0$ does not occur in the secular determinant $L(s)$. Hence, there is a finite limit of $\mathbf{Y}(r, t)$ for $t \rightarrow \infty$. In contrast to the previous case, a finite strength of the elastic lithosphere for $t \rightarrow \infty$ and the continuity of the horizontal displacement at the interface between the lithosphere and mantle prevent an unbounded deformation of the viscoelastic mantle.

On the assumption (57), there is another root of the secular equation. Defining

$$A_j := \frac{1}{j+2} \frac{(j-1)\alpha^{j-1} + (j+2)\alpha^{-j-2}}{\alpha^{j-1} - \alpha^{-j-2}}, \quad (58)$$

$L(s) = 0$ is a linear equation for one set of roots:

$$s_j^{\text{TM}} = -\kappa_M \frac{p^E}{A_j + p^E}, \quad (59)$$

where $p^E := \mu_L^E/\mu_M^E$ and $\kappa_M := \mu_M^E/\nu_M$ is the inverse Maxwell time of the mantle. Since $\alpha > 1$, equation (58) shows that $A_j > 0$ and, thus, $s_j^{\text{TM}} < 0$. Hence, $-s_j^{\text{TM}}$ is the inverse relaxation-time spectrum of the toroidal viscoelastic modes for a two-layer sphere. Equation (59) also shows that $-1/s_j^{\text{TM}} > 1/\kappa_M$, that is all relaxation times of a two-

layer sphere are longer than the Maxwell time of the mantle. Letting $j \rightarrow \infty$, we have $A_j = 1$, and the roots of the toroidal relaxation modes TM approach the value

$$s_{j \rightarrow \infty}^{\text{TM}} = -\kappa_M \frac{p^E}{1 + p^E}. \quad (60)$$

To calculate the viscous amplitude $\mathbf{Y}^V(r)$, the derivative of $L(s)$ with respect to s must be determined. The differentiation of equation (53) yields

$$\frac{dL(s)}{ds} = \frac{(j-1)b^{j-1}}{(2j+1)} [(j-1)\alpha^{j-1} + (j+2)\alpha^{-j-2}] \frac{d\mu_M(s)}{ds}, \quad (61)$$

where the s derivative of the shear modulus is given by equation (49). The elastic amplitudes $\mathbf{Y}^E(r)$ are determined by $\mathbf{Y}(r, s)$ for $\mu_M(s) = \mu_M^E$ and considering the assumption (57).

4.1.1. Homogeneous sphere

Let us consider the special case of a two-layer spherical earth model, where the elastic and viscous properties of the lithosphere and the mantle are identical, that is the two-layer spherical model is reduced to a homogeneous viscoelastic sphere. Letting $\mu_M^E = \mu_L^E$ and $v_M = v_L$, the secular determinant reduces to $L(s) = (j-1)a^{j-1}\mu(s)$ and the surface toroidal load-deformation Love number becomes $\tau_j(a, s) = \frac{g_0^2}{G\mu_L(s)} \frac{1}{j-1}$. The secular determinant $L(s)$ vanishes only when $\mu(s) = 0$, that is for $s = 0$. Hence, the response of a homogeneous viscoelastic sphere to the Heaviside surface toroidal loading grows linearly in time without approaching a finite limit for $t \rightarrow \infty$.

4.2. Two-layer Sphere with an Inviscid Fluid Core

Let us consider another spherical earth model consisting of an elastic lithosphere L , a viscoelastic mantle M and a liquid core C of external radii a , b and c ($a > b > c$), respectively. The elastic shear moduli of the lithosphere and mantle are assumed to be constant and equal to μ_L^E and μ_M^E , respectively, and the viscosity of the mantle is v_M .

The model-propagator matrix $\mathbf{L}(a, c, s)$ is generated by the multiplication of two layer-propagator matrices $\mathbf{L}(a, c, s) = \mathbf{P}(a, b)\mathbf{P}(b, c, s)$, where the matrix $\mathbf{P}(a, b)$ does not depend on the variable s since the lithosphere consists of elastic material only:

$$\mathbf{P}(a, b) = \frac{1}{2j+1} \begin{pmatrix} (j+2)\alpha^j + (j-1)\alpha^{-j-1} & \frac{b}{\mu_L^E}(\alpha^j - \alpha^{-j-1}) \\ \frac{(j-1)(j+2)\mu_L^E}{b}(\alpha^{j-1} - \alpha^{-j-2}) & (j-1)\alpha^{j-1} + (j+2)\alpha^{-j-2} \end{pmatrix}, \quad (62)$$

while the layer-propagator matrix for the mantle is s dependent:

$$\mathbf{P}(b, c, s) = \frac{1}{2j+1} \begin{pmatrix} (j+2)\beta^j + (j-1)\beta^{-j-1} & \frac{c}{\mu_M(s)}(\beta^j - \beta^{-j-1}) \\ \frac{(j-1)(j+2)\mu_M(s)}{c}(\beta^{j-1} - \beta^{-j-2}) & (j-1)\beta^{j-1} + (j+2)\beta^{-j-2} \end{pmatrix}, \quad (63)$$

where $\alpha := a/b$ and $\beta := b/c$. The vector $\mathbf{Z}(c, s)$ reduces to the vector $(c^j, 0)^T$, since the core consists of an inviscid fluid. Consequently, the secular determinant reduces to $L(s) = L_{21}(a, c, s)c^j$. In view of equations (62) and (63), $L(s)$ can be expressed in the form

$$L(s) = \frac{(j-1)(j+2)c^j}{(2j+1)^2} \left\{ \frac{\mu_L^E}{b} [\alpha^{j-1} - \alpha^{-j-2}] [(j+2)\beta^j + (j-1)\beta^{-j-1}] \right. \\ \left. + \frac{\mu_M(s)}{c} [(j-1)\alpha^{j-1} + (j+2)\alpha^{-j-2}] [\beta^{j-1} - \beta^{-j-2}] \right\}. \quad (64)$$

Defining

$$B_j := A_j \frac{(j+2)\beta^j - (j+2)\beta^{-j-1}}{(j+2)\beta^j + (j-1)\beta^{-j-1}}, \quad (65)$$

where A_j is defined by equation (58), $L(s) = 0$ is a linear equation for one set of roots:

$$s_j^{\text{TM}} = -\kappa_M \frac{p^E}{B_j + p^E}. \quad (66)$$

Since $A_j > 0$ and the fraction on the right-hand side of equation (63) is positive but smaller than 1, we have $A_j > B_j > 0$. This means that, for a fixed j , the inverse relaxation time given by equation (64), that is $-s_j^{\text{TM}}$, is larger than the inverse relaxation time given by equation (59), meaning that the two-layer model with a liquid core relaxes faster than the same model with a viscoelastic core.

4.3. Three-layer Sphere

Let us consider yet another spherical earth model consisting of three spherical layers: Elastic lithosphere L , viscoelastic mantle M and viscoelastic core C of external radii a , b and c ($a > b > c$), respectively. The elastic shear moduli of the lithosphere, mantle and core are assumed to be constant and equal to μ_L^E , μ_M^E and μ_C^E , respectively, and the viscosities of the mantle and core are ν_M and ν_C , respectively.

The model-propagator matrix $\mathbf{L}(a, c, s)$ is the same as for the model investigated in the previous section, that is $\mathbf{L}(a, c, s) = \mathbf{P}(a, b)\mathbf{P}(b, c, s)$, where the matrices $\mathbf{P}(a, b)$ and $\mathbf{P}(b, c, s)$ are given by equations (62) and (63), respectively. In contrast to the previous case, the vector $\mathbf{Z}(c, s)$ is equal to

$$\mathbf{Z}(c, s) = \left(c^j, \mu_C(s)(j-1)c^{j-1} \right)^T. \quad (67)$$

An inspection of equations (62) and (63) shows that only the (1,2) and (2,1) elements of matrices $\mathbf{P}(a, b)$ and $\mathbf{P}(b, c, s)$ depend on the shear moduli μ_L^E and $\mu_M(s)$, respectively. Likewise, equation (67) shows that only the second element of vector $\mathbf{Z}(c, s)$ depends on $\mu_C(s)$. To express these dependencies in the matrix $\mathbf{L}(a, c, s)$ explicitly, we define new elements that are independent of shear moduli:

$$\begin{aligned}
R_{12}(a, b) &:= \mu_L^E P_{12}(a, b), \\
R_{21}(a, b) &:= \frac{1}{\mu_L^E} P_{21}(a, b), \\
R_{12}(b, c) &:= \mu_M(s) P_{12}(b, c, s), \\
R_{21}(b, c) &:= \frac{1}{\mu_M(s)} P_{21}(b, c, s), \\
Z_2(c) &:= \frac{1}{\mu_C(s)} Z_2(c, s).
\end{aligned} \tag{68}$$

The secular determinant is equal to

$$L(s) = L_{21}(a, c, s) Z_1(c) + L_{22}(a, c, s) Z_2(c, s). \tag{69}$$

Making use of equations (62), (63) and (68), we have

$$L_{21}(a, c, s) = \mu_L^E R_{21}(a, b) P_{11}(b, c) + \mu_M(s) P_{22}(a, b) R_{21}(b, c), \tag{70}$$

$$L_{22}(a, c, s) = \frac{\mu_L^E}{\mu_M(s)} R_{21}(a, b) P_{12}(b, c) + P_{22}(a, b) P_{22}(b, c), \tag{71}$$

and $L(s)$ can be written in a form in which the dependence on the shear moduli is expressed explicitly:

$$L(s) = f_1 \mu_C(s) + f_2 \mu_L^E \frac{\mu_C(s)}{\mu_M(s)} + f_3 \mu_M(s) + f_4 \mu_L^E, \tag{72}$$

where f_i , $i = 1, \dots, 4$, are s -independent variables defined as follows:

$$\begin{aligned}
f_1 &:= P_{22}(a, b) P_{22}(b, c) Z_2(c), \\
f_2 &:= R_{21}(a, b) R_{12}(b, c) Z_2(c), \\
f_3 &:= P_{22}(a, b) R_{21}(b, c) Z_1(c), \\
f_4 &:= R_{21}(a, b) P_{11}(b, c) Z_1(c).
\end{aligned} \tag{73}$$

Substituting for the quantities on the right-hand sides from equations (62), (63) and (68), we obtain

$$\begin{aligned}
f_1 &= \gamma_j \left[(j-1) \alpha^{j-1} + (j+2) \alpha^{-j-2} \right] \left[(j-1) \beta^{j-1} + (j+2) \beta^{-j-2} \right], \\
f_2 &= \gamma_j (j-1)(j+2) \left[\alpha^{j-1} - \alpha^{-j-2} \right] \left[\beta^{j-1} - \beta^{-j-2} \right], \\
f_3 &= \gamma_j (j+2) \left[(j-1) \alpha^{j-1} + (j+2) \alpha^{-j-2} \right] \left[\beta^{j-1} - \beta^{-j-2} \right], \\
f_4 &= \gamma_j (j+2) \left[\alpha^{j-1} - \alpha^{-j-2} \right] \left[(j+2) \beta^j + (j-1) \beta^{-j-1} \right],
\end{aligned} \tag{74}$$

where $\gamma_j := (j-1)c^{j-1}/(2j+1)^2$.

Finally, considering $\mu_{M,C}(s) := \mu_{M,C}^E s / (s + \mu_{M,C}^E / \nu_{M,C})$ and denoting

$$\begin{aligned} p^E &:= \frac{\mu_L^E}{\mu_M^E}, & \kappa_M &:= \frac{\mu_M^E}{\nu_M}, \\ q^E &:= \frac{\mu_M^E}{\mu_C^E}, & \kappa_C &:= \frac{\mu_C^E}{\nu_C}, \end{aligned} \quad (75)$$

the secular determinant $L(s)$ can be written as a ratio of quadratic functions of variable s :

$$L(s) = \mu_C^E \frac{As^2 + Bs + C}{(s + \kappa_M)(s + \kappa_C)}, \quad (76)$$

where

$$\begin{aligned} A &:= f_1 + f_2 p^E + f_3 q^E + f_4 p^E q^E, \\ B &:= f_1 \kappa_M + 2f_2 p^E \kappa_M + f_3 q^E \kappa_C + f_4 p^E q^E (\kappa_M + \kappa_C), \\ C &:= f_2 p^E \kappa_M^2 + f_4 p^E q^E \kappa_M \kappa_C. \end{aligned} \quad (77)$$

Hence, $L(s) = 0$ is a quadratic equation for two sets of roots:

$$s_j^{\text{TM,TC}} = \frac{-B \mp \sqrt{B^2 - 4AC}}{2A}, \quad (78)$$

where the labels TM and TC denote two sets of toroidal viscoelastic relaxation modes.

We next check the formulae derived for a three-layer spherical model by comparing them with the solutions for a two-layer model and a two-layer model with a liquid core as derived in Sections 4.1 and 4.2, respectively.

4.3.1. Core = mantle case

Let us consider the special case of a three-layer spherical earth model where the viscoelastic properties of the mantle and core are identical, that is

$$\begin{aligned} \mu_M^E &= \mu_C^E, \\ \nu_M &= \nu_C, \end{aligned} \quad (79)$$

but where the radius of the mantle differs from the radius of the core, that is $b \neq c$. In this case, $q^E = 1$ and $\kappa_M = \kappa_C$ by equation (75). The auxiliary quantities defined by equation (77) are reduced to

$$\begin{aligned} A &= f_1 + f_2 p^E + f_3 + f_4 p^E, \\ B &= \kappa_M (f_1 + 2f_2 p^E + f_3 + 2f_4 p^E), \\ C &= p^E \kappa_M^2 (f_2 + f_4), \end{aligned} \quad (80)$$

which causes the reduction of the discriminant to

$$B^2 - 4AC = \left[\kappa_M (f_1 + f_3) \right]^2. \quad (81)$$

The inverse relaxation times of the toroidal relaxation modes given by equation (78) are reduced to

$$s_j^{\text{TM,TC}} = \begin{cases} -2\kappa_M, \\ -\kappa_M \frac{p^E}{\frac{f_1 + f_3}{f_2 + f_4} + p^E}. \end{cases} \quad (82)$$

The first relaxation time is independent of the angular degree j and must therefore be excluded from the solution. Evaluating the ratio $(f_1 + f_3)/(f_2 + f_4)$ by means of equation (74), the second relaxation time is equal to that given by equation (59). We can therefore conclude that the solution for this special case of a three-layer model is identical to that of the two-layer model.

4.3.2. Inviscid-fluid-core case

Let us consider another special case of a three-layer spherical earth model in which the core is composed of an inviscid fluid. For the Maxwell viscoelasticity considered here, this corresponds to the case where both the elastic shear modulus and the inverse Maxwell time of the core vanish:

$$\mu_C^E = 0, \quad (83)$$

$$\kappa_C = 0.$$

Consequently, $q^E \rightarrow \infty$ by equation (75). The auxiliary quantities defined by equation (77) and divided by q^E are reduced to

$$\begin{aligned} \lim_{q^E \rightarrow \infty} \frac{A}{q^E} &= f_3 + f_4 p^E, \\ \lim_{q^E \rightarrow \infty} \frac{B}{q^E} &= f_4 p^E \kappa_M, \\ \kappa_C &\rightarrow 0 \\ \lim_{q^E \rightarrow \infty} \frac{C}{q^E} &= 0. \\ \kappa_C &\rightarrow 0 \end{aligned} \quad (84)$$

The inverse relaxation times of the two toroidal relaxation modes given by equation (78) are reduced to

$$s_j^{\text{TM,TC}} = \begin{cases} 0, \\ -\kappa_M \frac{p^E}{f_3 + p^E}. \end{cases} \quad (85)$$

The first relaxation time $s_j = 0$ equivalently describes the fact that the model contains a fluid core. Evaluating the ratio f_3/f_4 by means of equation (74) establishes that the second relaxation time is equal to that given by equation (66). We conclude that the solution for this special case of a three-layer sphere model is identical to that of the two-layer model with an inviscid fluid core.

5. Conclusion

This paper has been motivated by the effort to derive analytical solutions for a class of multi-dimensional forward problems of viscoelastic relaxation in a multi-layered sphere. The objective behind this effort is to provide a standard solution that may be used to validate more general 2-D and 3-D forward-modelling codes, e.g., the finite-element approach (e.g., ZHONG *et al.*, 2003; WU, 2004) for computing the viscoelastic relaxation in 3-D spherical earth models.

In this paper, the model-propagator matrix for the response of a multi-layered Maxwell viscoelastic sphere to a surface toroidal force has been derived. Its analytical form of the dependence on the Laplace variable s has allowed us to determine the s derivative of the secular determinant analytically. This is superior to a numerical determination of this derivative, particularly when its poles and roots are located close to each other. For two-layer and three-layer spherical models, we have, in addition, found analytical expressions for the toroidal relaxation eigenmodes.

Acknowledgments

The author thanks Giorgio Spada and Bert Vermeersen for constructive reviews and Kevin Fleming for his comments on the original version of the manuscript. Support for this research is provided by the Grant Agency of the Czech Republic through Grant No. 205/06/580 is acknowledged.

REFERENCES

- ALTERMAN, Z., JAROSCH, H., and PEKERIS, C.L. (1959), *Oscillations of the Earth*, Proc. Roy. Soc. A252, 80–95.
- GANTMACHER, F.R. *The Theory of Matrices* (Chelsea Publishing Company, New York, 1959) (translated from Russian by K. A. Hirsch).
- GILBERT, F. and BACKUS, G.E. (1966), *Propagator matrices in elastic waves and vibration problems*, Geophysics 31, 326–332.

- MARTINEC, Z. (2000), *Spectral-finite element approach to three-dimensional viscoelastic relaxation in a spherical Earth*, *Geophys. J. Int.* 142, 117–141.
- MARTINEC, Z. and WOLF, D. (1999), *Gravitational viscoelastic relaxation of eccentrically nested spheres*, *Geophys. J. Int.* 138, 45–66.
- NEČAS, J. and HLAVÁČEK, I., *Mathematical Theory of Elastic and Elasto-Plastic Bodies: An Introduction* (Elsevier, Amsterdam, 1981).
- PELTIER, W.R. (1985), *The LAGEOS constraint on deep mantle viscosity: Results from a new normal mode method for the inversion of viscoelastic relaxation spectra*, *J. Geophys. Res.* 90, 9411–9421.
- POLLITZ, F.F. (1997), *Gravitational viscoelastic postseismic relaxation on a layered spherical Earth*, *J. Geophys. Res.* 102 (B8), 17921–17941.
- RIVA, R.E.M., *Crustal Rheology and Postseismic Deformation: Modeling and Application to the Apennines* (Delft University Press) Amsterdam. pp. 126.
- SABADINI, R. and VERMEERSEN, B., *Global Dynamics of the Earth. Applications of Normal Mode Relaxation Theory to Solid-Earth Geophysics* (Kluwer Academic Publishers, Dordrecht, 2004).
- VARSALOVICH, D.A., MOSKALEV, A.N., and KHERSONSKII, V.K., *Quantum Theory of Angular Momentum* (World Scientific Publ., Singapore, 1989).
- WU, P., (2004), *Using commercial finite element packages for the study of earth deformations, sea levels and state of stress*, *Geophys. J. Int.* 158, 401–408.
- WU, P. and PELTIER, W.R. (1982), *Viscous gravitational relaxation*, *Geophys. J. R. Astr. Soc.* 70, 435–485.
- ZHONG, S., PAULSON, A., and WAHR, J. (2003), *Three-dimensional finite-element modelling of earth's viscoelastic deformation: effect of lateral variations in lithospheric thickness*, *Geophys. J. Int.* 155, 679–695.

(Received August 5, 2005, revised April 19, 2006, accepted May 15, 2006)

To access this journal online:
www.birkhauser.ch/pag

Using Fuzzy Logic for the Analysis of Sea-level Indicators with Respect to Glacial-isostatic Adjustment: An Application to the Richmond-Gulf Region, Hudson Bay

VOLKER KLEMMANN¹ and DETLEF WOLF¹

Abstract—An important constraint for the inference of mantle viscosity is the variation of the Holocene relative sea-level (RSL) height (with respect to today) following the last deglaciation. As a measure of this variation, sea-level indicators (SLIs) related to the RSL heights at specific past time epochs are used. For the inversion of the RSL-height change in terms of mantle viscosity, neighbouring SLIs may be grouped into an RSL diagram taken as representative for the region considered. Usually, the nominal height and age of a particular SLI are the only characteristics considered when determining the former RSL height. However, only SLIs based on isolation basins yield a narrow range for this height, whereas SLIs based on fossil samples provide a lower bound (shells), an upper bound (driftwood) or a finite interval (basal peat) for it. To also use fossil samples objectively, we develop a classification scheme of the depositional conditions based on fuzzy logic. After the definition of appropriate membership functions, this method leads to a systematic interpretation of the large number of SLIs available. We apply this method to SLIs from the Richmond-Gulf region, southeastern Hudson Bay, near the former glaciation center of Canada and derive a decay time of 5 ka for the exponential function best fitting the RSL diagram for this region.

Key words: Canada, data analysis, fuzzy logic, glacial-isostatic adjustment, sea-level indicator, sea-level change.

1. Introduction

For our study, several concepts are required and must be defined. By sea-level indicator (SLI), we denote some fossil sample, such as shell, peat or driftwood, that is indicative, usually in an indirect way, of the position of the relative sea-level height (RSL) at the time of deposition. The age of an SLI is commonly determined by radiocarbon dating and its observational SLI height determined as the elevation relative to the present mean sea-level height. Commonly, neighbouring SLIs are combined to construct a sea-level diagram taken as representative of the average location of the SLIs considered. Implicit in this construction is that

¹ GeoForschungsZentrum Potsdam, Sekt. 1.3, Gravity Field and Earth Models, Telegrafenberg, D-14473, Potsdam, Germany. E-mail: volkerk@gfz-potsdam.de

the observational RSL height agrees with the observational SLI height or is offset from it by a constant amount. Assuming that the glacial-isostatic adjustment (GIA) process following the last deglaciation and the associated global rise of the sea-level height, governs the observational RSL height during the Holocene, such diagrams may then be used to infer the viscosity distribution in the earth's interior or the deglaciation history. Near the centers of the former ice sheets, the SLIs indicate that GIA has proceeded nearly exponentially in time after deglaciation. A useful quantity for further analysis is therefore the decay time of the exponential function that best fits the observational RSL heights.

The main problem of this procedure is the varied quality of the SLIs used. Usually, this problem is solved by arbitrarily excluding SLIs whose location seems to be anomalous in some way. Related to this is the question of which SLI types should be included or excluded when fitting the exponential function and determining the decay time. Thus, LAMBECK *et al.* (1998) questioned comprehensive compilations comprising any type, such as that by TUSHINGHAM and PELTIER (1993), and suggested to use as SLIs only basal peats or lake sediments indicative of marine-lacustrine transitions (isolation events). The disadvantage of this restrictive selection is that it excludes all coasts where the quality of SLIs is poor. Moreover, isolation events have been mapped extensively only in Europe.

An alternative solution to this problem is based on the use of fuzzy logic (*e.g.*, DEMICCO and KLIR, 2004). In the sense employed here, it encompasses a system of concepts, principles and methods for approximate, rather than exact, modes of reasoning (NOVÁK and PERFILEVA, 2000 in KLIR, 2004). Thus, instead of transforming the observational SLI height into an observational RSL height according to some standard scheme, we merely assume that the depositional conditions of the SLI type—where the fossil sample representing the SLI lived or settled—indicate some RSL-height range. In accordance with this, we **assume** that the predicted RSL height indicates a height range, expressed by the fuzzy set appropriate to the particular SLI type considered, and take this into account when defining the misfit between observational SLI height and predicted RSL height.

Our study proceeds as follows. First, in presenting the methodology, we will define a fuzzy set, $\mathbf{P}_i$, for the prediction of the depositional condition of the SLI considered by combining the fuzzy set, $\mathbf{E}_i$, containing the information about the depositional condition of the SLI type with the predicted RSL height, h_i^P . After this, we will define a fuzzy set, $\mathbf{O}_i$, for the observational SLI height, h_i^O , where the statistics related to the errors of levelling and dating are taken into account. Based on these two fuzzy sets, we will calculate the fit, $\mathbf{F}_i$, by their intersection, and derive as a robust measure an algebraic number expressing the possibility, p_i , that the prediction fits the observation. As an example, our method will be employed to determine the decay time of GIA for the Richmond-Gulf region, southeastern Hudson Bay, using the SLI evidence of this region.

2 Application of Fuzzy Logic

Fuzzy logic is based on fuzzy sets allowing us to quantify the imprecision of a statement. For our application, this means that we replace the predicted RSL height, h_i^P , associated with the i -th SLI by a fuzzy set representing the depositional condition of the sample:

$$\begin{aligned} \mathbf{P}_i : \quad X_i^P &\rightarrow [0, 1], \\ h &\rightarrow \mu_i^P(h). \end{aligned} \quad (1)$$

Here, the depositional condition is defined by the support, X_i^P , of the fuzzy set (the height range where the SLI lived or settled) and by the membership function, μ_i^P , indicating the possibility, in the range $[0, 1]$, that the species of the SLI lived or settled at the height, h , relative to the present mean sea-level height (Fig. 1).

2.1. Fuzzification

The predicted RSL height, either based on the assumption of exponential decay, as studied here, or on a forward calculation for a particular earth-ice-model combination, as studied by WOLF *et al.* (2006), is

$$h_i^P = m(\mathbf{x}, t_i), \quad \text{SLI}_i \in \mathcal{S}, \quad (2)$$

where m denotes the model used for prediction, $\mathbf{x}_i$ and t_i the location and calibrated age, respectively, of the i -th SLI in the set, $\mathcal{S}$, considered and h_i^P its predicted RSL height. The latter is used to predict the depositional condition of the SLI relative to the present mean sea-level height as follows.

First, we define the depositional condition of the i -th SLI relative to the contemporaneous mean sea level by the fuzzy set

$$\begin{aligned} \mathbf{E}_i : \quad X_i &\rightarrow [0, 1], \quad \text{SLI}_i \in \mathcal{S}, \\ h &\rightarrow \mu_i^E, \end{aligned} \quad (3)$$

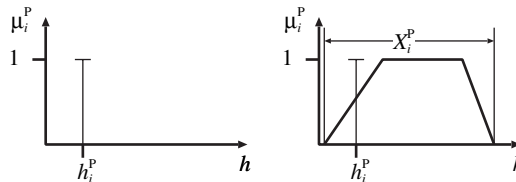


Figure 1

Transformation of the predicted RSL height, h_i^P , (left) into the fuzzy set, μ_i^P , (right) for the prediction of the depositional condition of the SLI considered. The height, h , is with respect to present mean sea-level height.

where μ_i^E is the membership function describing the depositional condition and X_i its support. The prediction of the depositional condition of the i -th SLI is then defined by the shift associated with the predicted RSL height:

$$\begin{aligned} \mathbf{P}_i &:= \mathbf{E}_i + h_i^P : X_i^P := X_i + h_i^P \rightarrow [0, 1], \\ h &\rightarrow \mu_i^P(h) := \mu_i(h - h_i^P). \end{aligned} \quad (4)$$

To consider the errors of the observational SLI height, h_i^O , we define a fuzzy set based on their statistics. We assume here that the measurements are normally distributed with the probability-density function

$$n(h, h_i^O, \sigma_i^2) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(h - h_i^O)^2}{2\sigma^2}\right]. \quad (5)$$

The standard deviation in height, σ_i , is calculated from the height error, Δh_i , and the error in dating and its calibration, Δt_i :

$$\sigma_i = \sqrt{\Delta h_i^2 + \left(\frac{dh^P}{dt}\bigg|_{(x_i, t_i)} \Delta t_i\right)^2}, \quad (6)$$

where the contribution due to the error in the location of the SLI has been neglected. Instead of using probabilities, we will, in the following, employ possibilities. Based on the possibility-probability principle (ZADEH, 1978) stating that the degree of possibility of an event is at least equal to its degree of probability, we define the fuzzy set for the observational SLI height, h_i^O , with the probability-density function given by equation (5) according to CIVANLAR and TRUSSELL (1986) as

$$\begin{aligned} \mathbf{O}_i : (-\infty, \infty) &\rightarrow [0, 1], \\ h &\rightarrow \mu_i^O := \begin{cases} 1 & \text{if } \left|\frac{h - h_i^O}{\sigma_i}\right| < a, \\ \exp\frac{a^2}{2} \exp\left[-\frac{(h - h_i^O)^2}{2\sigma_i^2}\right] & \text{otherwise,} \end{cases} \end{aligned} \quad (7)$$

where μ_i^O is the membership function of the observational height and the support is infinite. The parameter a is related to the confidence level, which is $\sim 90\%$ if we assume that $a = 1$.

As the next step, we define the fuzzy set representing the fit to the observation associated with the i -th SLI achieved by the modelling as the standard intersection (KLIR, 2004):

$$\begin{aligned} \mathbf{F}_i &:= \mathbf{P}_i \cap \mathbf{O}_i : X_i^P \rightarrow [0, 1], \\ h &\rightarrow \mu_i^F := \min\{\mu_i^P(h), \mu_i^O(h)\}, \end{aligned} \quad (8)$$

where μ_i^F is the respective membership function and its support is equal to X_i^P because $X_i^F = (-\infty, \infty) \cup X_i^P = X_i^P$ holds.

2.2. Defuzzification

For the appraisal of the modelling, we derive from the fuzzy set, $\mathbf{F}_i$, of the fit the possibility

$$p_i := \sup_{h \in X_i^P} \mu_i^F(h) \quad (9)$$

that the prediction represents the observation. In particular, $p_i = 1$ means that the model fits the data. A further quantity of interest is the revised nominal height, h_i^F , of the SLI defined as the height where p_i holds and the difference from h_i^O has a minimum:

$$h_i^F = \min_{\mu_i^F(h)=p_i} |h - h_i^O|. \quad (10)$$

Figure 2, top, shows the possibility, p_i , calculated from equation (9) for variable observational SLI height, h_i^O , constant standard deviation, $\sigma_i = 1$, and trapezoidal membership function, μ_i^P , of the prediction with a core ($\mu_i^P = 1$) in the range $h \in [4, 8]$ and ramps on both sides with widths of 1. For $h_i^O = 2$, μ_i^O , μ_i^P and their intersection μ_i^F , are shown in Figure 2, bottom. Also plotted are h_i^F and h_i^O . The figure shows that $p_i = 1$ inside the core of μ_i^P extended by the standard error of the measurement and that p_i decays with distance from it.

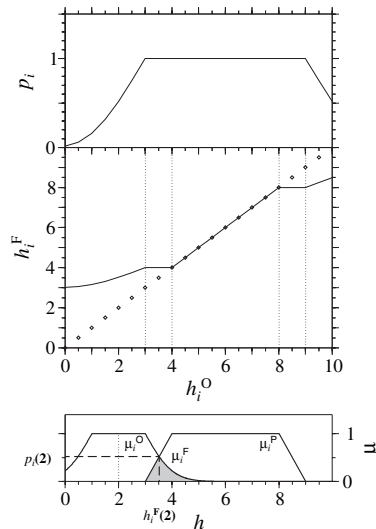


Figure 2

Possibility, p_i , (top) and revised nominal height, h_i^F , (middle) as functions of the observational RSL height, h_i^O , according to equations (9) and (10), respectively, (black lines) and the deviation of h_i^F from h_i^O (black diamonds). Also shown (bottom) is the construction of p_i and h_i^F for $h_i^O = 2$ from the intersection μ_i^F (grey area) of the membership functions of observation, μ_i^O and the prediction, μ_i^P .

In the following, we define the misfit of a set of SLIs by

$$\chi := \frac{1}{N} \sum_{i=1}^N (1 - p_i), \quad \text{SLI}_i \in \mathcal{S}, \quad (11)$$

where N is the total number of SLIs considered.

2.3. Definition of Membership Functions for SLIs

Art Dyke (pers. comm.) suggested to distinguish three classes of SLIs. The first class, where the observational SLI height, h_i^O , defines an upper bound for the former mean sea-level height, mainly consists of marine species, such as shells found in their living position. The second class, where h_i^O defines a lower bound for the former mean sea-level height, mainly consists of terrestrial species, such as tree roots. The third class, where h_i^O defines a finite range for the former mean sea-level height, comprises isolation events, basal peats and coastal regions where, in addition to samples of the first class, the highest former mean sea-level height (marine limit) is known. Here, we refine this classification and consider the particular depositional conditions of SLI. This means that the tidal range, the probable height of storm beaches, the support of the depositional condition, or the probability that material usually washed to the shore sank can be recovered from supplementary information associated with the SLIs considered.

The advantage of using fuzzy logic is that such refinements can be quantified by combining the corresponding fuzzy sets. Thus, for a shallow water shell also able to live in the tidal range, we obtain

$$\mathbf{E}^{\text{SW}} = \mathbf{E}^{\text{SW}} \cup \mathbf{E}^{\text{MTR}} \quad (12)$$

as the union of the fuzzy set describing a shallow-water habitat, $\mathbf{E}^{\text{SW}}$, and the fuzzy set describing the mean tidal range, $\mathbf{E}^{\text{MTR}}$, for this region (Fig. 3). For simplicity, we assume the membership functions to be of trapezoidal type.

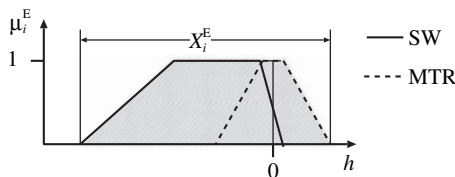


Figure 3

Example of the refinement of a fuzzy set defining the depositional condition of a shell living in shallow water, SW, extended by a littoral habitat defined by the mean tidal range (MTR). The fuzzy sets of the SW and MTR depositional conditions are indicated by lines, the union fuzzy set by the hatched area.

3 Decay Time for the Richmond-Gulf Region

An example of the use of SLIs in GIA is the derivation of the decay time of GIA from selected sea-level diagrams for formerly glaciated regions. In particular, Ångermanland, central Sweden, and Richmond Gulf, southeastern Hudson Bay, display the pronounced RSL-height fall typical of regions near the former glaciation centers. The SLIs from Ångermanland are based on Holocene varves deposited near the mouth of Ångerman River exactly at the contemporaneous mean sea-level height (see CATO, 1998, for an overview of the Ångermanland data). The dating of the varves is carried out by counting them and relating gaps in the stratigraphy to ^{14}C chronology (e.g., ANDRÉN *et al.*, 1999; WOHLFARTH and POSSNERT, 2000).

The data reflecting the RSL-height fall in the Richmond-Gulf region (Figs. 4 and 5) are less directly related to the contemporaneous sea-level height and, therefore, discussed more controversially (e.g., WALCOTT, 1980; PELTIER, 1999; MITROVICA *et al.*, 2000; FANG and HAGER, 2002; MITROVICA and FORTE, 2004). Following FANG and HAGER (2002), we use here the decay time, T , and the decay amplitude, A , according to the fit of

$$h_i^P = A (e^{t_i/T} - 1) \quad (13)$$

to the SLIs of the Richmond-Gulf region considered. The estimates of A and T proposed so far are listed in Table 1. The differences between the values are essentially due to two reasons: the consideration of different subsets of the complete set of SLIs for this region and the use of different methods for the inference of the

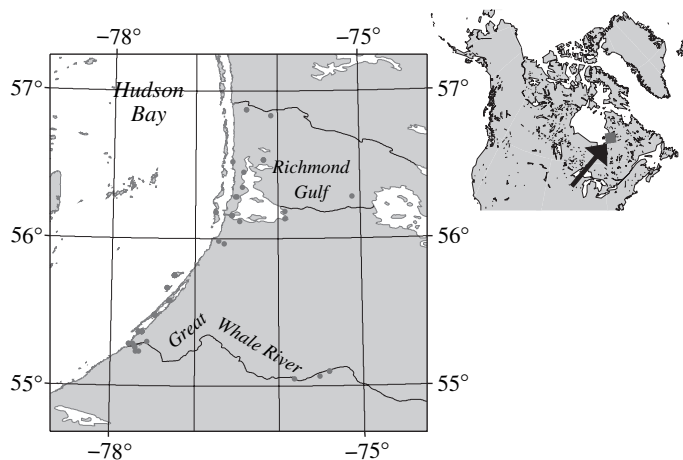


Figure 4

Maps showing the locations (circles) of the SLIs of the Richmond-Gulf region (left) and its setting in North America (right). Note that SLI locations may be too close to be distinguishable on the scale of the map.

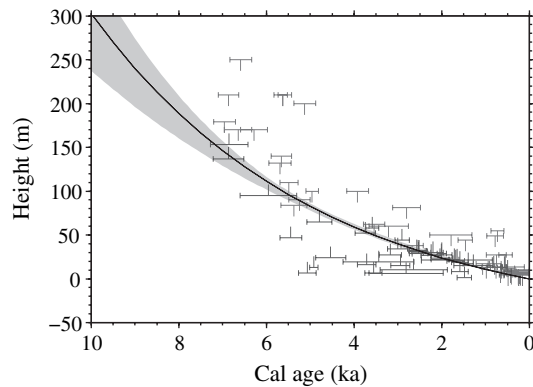


Figure 5

Sea-level diagram for the Richmond-Gulf region showing the observational SLI height, h_i^O , as a function of the calibrated age according to the compilation by Art Dyke (pers. comm.). Lower bounds denote SW- and W-type samples, Upper bounds denote D- and T-type samples. The solid line shows the best-fitting exponential function, equation (13), with a decay amplitude of $A = 50$ m and a decay time of $T = 5.1$ ka. The grey band shows the range of exponential functions with a misfit of $\chi < 0.01$.

Table 1

Estimates of the decay amplitude, A , and decay time, T , for the Richmond-Gulf region

Reference	Number of SLIs	(A , T) (m, ka)
WALCOTT (1980)	9	(-, > 5.0)
MITROVICA and PELTIER (1995)	8	(96.2, 7.6)
PELTIER (1996)	-	(-, 7.23 ± 3.18)
PELTIER (1998)	48	(21.9, 3.426)
PELTIER (1999)	8	(53.81, 4.68)
DYKE and PELTIER (2000)	-	(-, 2.0)
MITROVICA <i>et al.</i> (2000)	78	(-, 5.3 ± 1.3)
FANG and HAGER (2002)	48	(70.6 ± 12 , 6.32 ± 1.0)
This study	93	(50, 5.1)

best-fitting exponential function. Thus, WALCOTT (1980) used only SLIs based on shells of *Mytilus edulis*, but included data from Cape Henrietta Maria on the west coast of James Bay. MITROVICA and PELTIER (1995) employed the data set provided by HILLAIRE-MARCEL (1980), which is based on SLIs from storm beaches. PELTIER (1996) did not provide any details about the data used, but it is reasonable to assume that they are similar to those of MITROVICA and PELTIER (1995). In contrast to this, PELTIER (1998) considered a new compilation based on a larger number of SLIs from Richmond Gulf and James Bay. Later, PELTIER (1999) proposed another estimate of the decay time, now based on the SLIs of HILLAIRE-MARCEL (1980) for Richmond Gulf alone. The estimate of DYKE and PELTIER (2000) is based on a subjective fit of an exponential curve to the data. MITROVICA *et al.* (2000) reviewed the data base used

by PELTIER (1998), pointed out some errors in the data and proposed a revised estimate for Richmond Gulf only. FANG and HAGER (2002) also discussed the data set of PELTIER (1998) and estimated a decay time similar to that determined by MITROVICA *et al.* (2000). Except for PELTIER (1998) and DYKE and PELTIER (2000), the decay times obtained range from 5 to 8 ka (Table 1).

The new estimate presented here is based on a larger set of 93 SLIs (Table 3, Appendix) provided by Art Dyke (pers. comm.), where 7000 a has been adopted as the upper limit of the calibrated age. FANG and HAGER (2002) used a similar limit and justified it by the complex deglaciation history of Hudson Bay. Another argument is that, after 7000 a BP, the eustatic contribution of the RSL-height change is negligible in regions covering the former ice-sheet centers (FLEMING *et al.*, 1998).

Table 2 lists the membership functions of the SLI types considered. The depositional condition of the type shallow water (SW), is represented by trapezoidal membership functions representing the shallow-water habitat of *Mytilus edulis*. The depositional condition of the type water (W) only provides an upper limit. The corresponding membership function represents the habitat of ordinary shells, potentially sunk driftwood (GSC-27, L-441A, Qu-1291) and a beluga remnant (TO-3713). In contrast to this, SLIs based on driftwood (D) were usually deposited above the mean sea-level height, extending the lower bound to -2 m. Terrestrial (T) types, such as charred fat or charcoal from forest fires or archeological sites, peat, soil and lake sediments, such as gyttja, are assumed to have formed above the high-tide level (> 2 m above the mean sea-level height). Storm beaches along the east coast of Hudson Bay with a proposed 45-a periodicity (HILLAIRE-MARCEL, 1976) are prominent features and many SLIs related to them have been collected (e.g., WALCOTT, 1980). MITROVICA *et al.* (2000) allowed for the resulting offset in the RSL heights by introducing a constant term in equations (13). In contrast to this, we consider the fact that shells and driftwood may be related to the storm beaches by extending the upper bound to 6 m above the contemporaneous RSL height (ALLARD and TREMBLAY, 1983).

Table 2

Membership functions, $\mu^E(h)$, of SLI types considered for selected heights, h . Between the pairs of values indicated, membership functions are linearly interpolated. The relation to storm beaches has been taken into account for the SLI types SW, W and D

SLI type	$(h, \mu^E(h))$
SW	$(-50, 0); (-20, 1); (6, 1); (8, 0)$
W	$(-\infty, 1); (6, 1); (8, 0)$
D	$(-4, 0); (-2, 1); (\infty, 1)$
T	$(0, 0); (2, 1); (\infty, 1)$

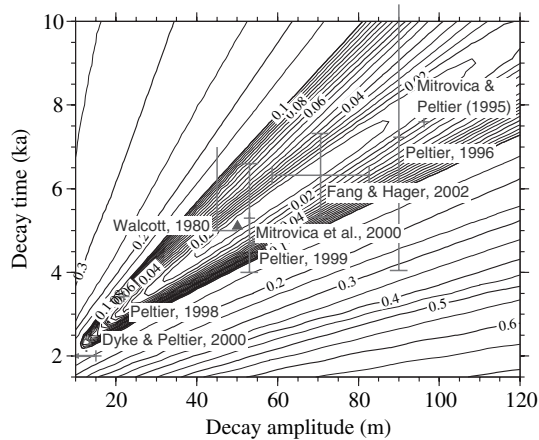


Figure 6

Misfit of the exponential function for SLIs from the Richmond-Gulf region as a function of the decay amplitude, A , and decay time, T . The triangle indicates the best-fitting pair of values obtained in the present study. The estimates and uncertainties found in previous studies are also shown (Table 2).

Figure 6 shows the misfit defined by equation (11) as a function of the decay amplitude and decay time. The minimum applies to $(A, T) = (50 \text{ m}, 5.1 \text{ ka})$ which corresponds to a present-time sea-level fall of 9.8 mm/a . Clearly visible is the trade-off between decay amplitude and decay time, which is partly responsible for the variability of the previous estimates (Table 1). In Figure 5, the best-fitting exponential function and the range of exponential functions with a misfit $\chi < 0.01$ have been added to the data.

4. Summary

We have introduced fuzzy logic for the specification of the depositional conditions of SLIs expressed by suitably defined membership functions. This innovation allows us to consider a larger variety of SLIs in the interpretation of Holocene RSL-height change caused by GIA and to construct sea-level diagrams in an objective and reproducible way. As an application of the method, we have redetermined the decay time for the Richmond-Gulf region using all SLIs available for this region and obtained a value of 5.1 ka and a range of acceptable fits from 4 to 8 ka reflecting the poor quality of the SLIs available from this region.

Acknowledgements

We thank Carola Tiede and an anonymous reviewer for their constructive comments. We also thank Ludwig Ballani (GeoForschungsZentrum Potsdam), who

improved the mathematical formulation of the problem, and Art Dyke (Geological Survey of Canada), who provided the SLIs for the Richmond-Gulf region. The Graphics program GMT (WESSEL and SMITH, 1991) was employed. This study was supported by the SEAL project, BMBF Project No. SF2000/13.

Appendix

Table 3

Characteristics of SLIs for the Richmond-Gulf region. For each sample are given the laboratory number with the superscript indicating the reference (a: ALLARD and SEGUIN (1985), b: ALLARD and TREMBLAY (1983), c: FILION et al. (1991), d: HARP (1976), e: HILLAIRE-MARCEL (1976), f: LEE (1960), g: LOWDON and BLAKE (1980), h: PLUMET (1974), i: WALCOTT (1972), k: WALCOTT and CRAIG (1975)), the conventional ^{14}C age with the standard error (the basin correction is applied to marine samples only if the age is corrected for $\delta^{13}\text{C}$ (e.g., VOGEL and MARAIS, 1971)), the dated material (char fat: charred fat, dl: Delphinapterus leucas, ha: Hiatella arctica, lake sed: lake sediment, me: Mytilus edulis), the SLI type (T: terrestrial, D: driftwood, SW: shallow water, W: water), the observational SLI height, h^{O} , and the 2σ -calibrated age range with CALIB 4.1 (STUIVER and REIMER, 1993)

Lab. no.	^{14}C age (a)	Dated material	SLI type	h^{O} (m)	Cal.-age range (a)
UL-409 ^c	6010 $\pm$ 100	Charcoal	T	210	6635–7105
Beta-28576 ^f	4900 $\pm$ 60	Charcoal	T	210	5525–5735
UL-411 ^c	4880 $\pm$ 90	Charcoal	T	210	5430–5825
Beta-28587 ^h	4520 $\pm$ 80	Charcoal	T	200	4880–5380
I-11754 ^c	3610 $\pm$ 95	Charcoal	T	100	3670–4170
Gif-1567 ^h	3300 $\pm$ 110	Charcoal	T	62	3305–3790
I-10883 ^c	3040 $\pm$ 215	Charcoal	T	60	2765–3665
I-11730 ^c	2375 $\pm$ 85	Charcoal	T	30	2255–2675
\#	1050 $\pm$ 50	Charcoal	T	13	885–1085
GX-2083 ^d	845 $\pm$ 120	Charcoal	T	49	590–975
GX-2065 ^d	795 $\pm$ 160	Charcoal	T	13	535–980
GX-2070 ^d	1130 $\pm$ 95	char fat	T	17	910–1215
GX-2071 ^d	855 $\pm$ 95	char fat	T	17	640–940
GX-2069 ^d	780 $\pm$ 160	char fat	T	27	525–980
I-13406 ^a	6080 $\pm$ 120	Peat	T	179	6710–7210
I-13130 ^a	5840 $\pm$ 150	Peat	T	170	6335–6960
UQ-622 ^a	5520 $\pm$ 160	Peat	T	170	5980–6590
Qu-140 ^a	4960 $\pm$ 120	Peat	T	132	5435–5945
Lv-698 ^a	4920 $\pm$ 120	Peat	T	140	5425–5895
GSC-2354 ^g	4780 $\pm$ 110	Peat	T	110	5285–5695
Qu-142	4720 $\pm$ 120	Peat	T	84	5075–5675
GSC-2432 ^g	4580 $\pm$ 80	Peat	T	90	5000–5495
Lv-797	4370 $\pm$ 70	Peat	T	100	4810–5105
I-8367 ^c	2630 $\pm$ 90	Peat	T	38	2425–2930
Qu-1289 ^b	770 $\pm$ 110	Peat	T	54.7	580–875
GSC-4655	5790 $\pm$ 130	Gyttja	T	250	6340–6840
UQ-737	2710 $\pm$ 120	Soil	T	81	2485–3120
Qu-1297 ^b	1830 $\pm$ 240	lake sed	T	50	1310–2270
UPS-2240	1560 $\pm$ 100	Soil	T	44	1300–1625
Qu-1107 ^b	720 $\pm$ 80	Wood	D	8.53	520–795
Qu-1202 ^b	720 $\pm$ 80	Wood	D	12.86	520–795

Table 3

(Contd.)

Lab. no.	^{14}C age (a)	Dated material	SLI type	h^{O} (m)	Cal.-age range (a)
Qu-1203 ^b	680 $\pm$ 80	Wood	D	12.86	540–730
Qu-1204 ^b	570 $\pm$ 90	Wood	D	12.86	515–645
GX-2068 ^d	550 $\pm$ 120	Wood	D	27	390–700
Qu-1299 ^b	480 $\pm$ 90	Wood	D	4.4	380–585
Qu-1088 ^b	410 $\pm$ 80	Wood	D	11.76	335–535
Qu-1207 ^b	400 $\pm$ 90	Wood	D	8.9	305–560
Qu-1205 ^b	350 $\pm$ 80	Wood	D	6.32	285–530
Qu-1293 ^b	330 $\pm$ 100	Wood	D	13.5	110–545
Qu-1201 ^b	210 $\pm$ 80	Wood	D	9.12	0–510
Qu-1093 ^b	190 $\pm$ 90	Wood	D	6.2	0–515
Qu-1208 ^b	180 $\pm$ 80	Wood	D	6.76	0–505
Qu-1209 ^b	160 $\pm$ 100	Wood	D	4.83	0–510
Qu-1206 ^b	150 $\pm$ 80	Wood	D	5.45	0–480
Qu-1212 ^b	150 $\pm$ 100	Wood	D	10.3	0–510
Qu-1210 ^b	130 $\pm$ 100	Wood	D	9.12	0–505
Qu-1090 ^b	130 $\pm$ 80	Wood	D	7	0–470
Qu-1108 ^b	120 $\pm$ 90	Wood	D	7.55	0–475
Qu-1094 ^b	70 $\pm$ 90	Wood	D	6.2	0–430
Qu-1095 ^b	60 $\pm$ 80	wood	D	4.51	0–415
Qu-1092 ^b	40 $\pm$ 80	wood	D	6.2	0–310
Qu-1096 ^b	20 $\pm$ 80	wood	D	4.51	0–305
GSC-1287 ⁱ	6000 $\pm$ 160	me	SW	137	6515–7215
Qu-1294 ^b	4270 $\pm$ 100	ha, me	SW	65	4510–5075
Qu-1290 ^b	3480 $\pm$ 100	me	SW	52.3	3515–3980
GSC-2070 ^k	3360 $\pm$ 60	me	SW	58	3435–3735
Qu-1288 ^b	2860 $\pm$ 100	me	SW	34.25	2780–3235
GSC-2348 ^k	2760 $\pm$ 80	me	SW	44	2740–3070
Qu-1296 ^b	2510 $\pm$ 80	me	SW	32	2380–2755
Qu-1097 ^b	2470 $\pm$ 100	me	SW	31.71	2345–2740
Qu-1099 ^b	2430 $\pm$ 100	me	SW	28.91	2320–2740
Qu-1295 ^b	2410 $\pm$ 90	me	SW	31.5	2330–2695
Qu-1101 ^b	2260 $\pm$ 100	me	SW	26.57	2010–2480
Qu-1098 ^b	2230 $\pm$ 100	me	SW	29.94	1970–2425
Qu-1103 ^b	2050 $\pm$ 100	me	SW	21.94	1835–2245
GSC-2129 ^k	2030 $\pm$ 60	me	SW	29	1890–2100
Qu-1100 ^b	2026 $\pm$ 100	me	SW	27.26	1770–2245
Qu-1102 ^b	2020 $\pm$ 100	me	SW	23.69	1750–2245
GSC-2074 ^k	1790 $\pm$ 60	me	SW	22	1610–1825
Qu-1104 ^b	1760 $\pm$ 90	me	SW	20.38	1485–1885
Qu-1087 ^b	1680 $\pm$ 390	me	SW	21.43	845–2460
Qu-1105 ^b	1680 $\pm$ 90	me	SW	18.8	1400–1775
Qu-1106 ^b	1490 $\pm$ 90	me	SW	17.26	1280–1515
Qu-1298 ^b	670 $\pm$ 80	me	SW	4.4	575–685
Qu-1064 ^b	890 $\pm$ 100	me	SW	13.31	660–960
Qu-1081 ^b	580 $\pm$ 70	me	SW	13.21	530–610
Qu-1068 ^b	490 $\pm$ 80	me	SW	8.89	410–585
GSC-1725 ⁱ	6000 $\pm$ 210	ha	W	153	6420–7285
UQ-863 ^a	5200 $\pm$ 300	shells	W	95	5315–6605

Table 3

(Contd.)

Lab. no.	^{14}C age (a)	Dated material	SLI type	h^0 (m)	Cal.-age range (a)
GSC-27 ^c	4740 $\pm$ 110	wood	W	3	5200–5695
Qu-1067 ^b	4420 $\pm$ 100	shells	W	6.41	4870–5265
GSC-1326 ⁱ	4070 $\pm$ 140	ha	W	24	4190–4880
TO-3713	4320 $\pm$ 50	dl	W	11	4825–5015
Qu-1065 ^b	3460 $\pm$ 90	shells	W	16.02	3505–3920
Qu-1066 ^b	3310 $\pm$ 100	shells	W	6.41	3365–3755
Qu-1082 ^b	3160 $\pm$ 360	shells	W	19.17	2635–4250
L-441A ^c	3020 $\pm$ 120	wood	W	6	2920–3430
Qu-1083 ^b	2780 $\pm$ 110	shells	W	15.24	2725–3160
Qu-1086 ^b	2720 $\pm$ 370	shells	W	6.32	1970–3660
Qu-1085 ^b	2560 $\pm$ 350	shells	W	10.41	1870–3410
Qu-1292 ^b	1680 $\pm$ 90	shells	W	8	1400–1775
Qu-1291 ^b	1590 $\pm$ 90	wood	W	1.5	1320–1645
Qu-1084 ^b	1360 $\pm$ 230	shells	W	15.24	830–1725

REFERENCES

- ALLARD, M. and SEGUIN, M. K. (1985), *La déglaciation d'une partie du versant hudsonien québécois: bassins des rivières Nastapoca, sheldrake et à l'Eau Claire*, Geogr. Phys. Quat. 39, 13–24.
- ALLARD, M. and TREMBLAY, G. (1983), *La dynamique littorale des îles Manitounuk durant l'Holocène*, Z. Geomorphol. suppl. 47, 61–95.
- ANDRÉN, T., BJÖRCK, J. and JOHNSEN, S. (1999), *Correlation of Swedish glacial varves with the Greenland (GRIP) oxygen isotope record*, J. Quat. Sci. 14, 361–371.
- CATO, I. (1998), *Ragnar Liden's postglacial varve chronology from the Angermanälven Valley, northern Sweden*, Sverig. Geol. Unders. 88, 1–82.
- CIVANLAR, M. R. and TRUSSELL, H. J. (1986), *Constructing membership functions using statistical data*, Fuzzy Sets Syst. 18, 1–13.
- DEMICO, R. V. and KLIR, G. J. (eds.), *Fuzzy Logic in Geology* (Elsevier Academic Press, San Diego 2004).
- DYKE, A. S. and PELTIER, W. R. (2000), *Forms, response times and variability of relative sea-level curves, glaciated North America*, Geomorph. 32, 315–333.
- FANG, M. and HAGER, B. F., *On the apparent exponential relaxation curves at the central regions of the last Pleistocene ice sheets*, In *Ice Sheets, Sea Level, and the Dynamic Earth* (eds. Mitrovica, J. X. and Vermeersen, B. L. A.) (Am. Geophys. Union, Washington 2002), pp. 201–218.
- FILION, L., SAINT-LAURENT, D., DESPONT, M., and PAYETTE, S. (1991), *The late Holocene record of aeolian and fire activity in northern Québec, Canada*, Holocene 1, 201–208.
- FLEMING, K., JOHNSTON, P., ZWARTZ, D., YOKOHAMA, Y., LAMBECK, K., and CHAPPELL, J. (1998), *Refining the eustatic sea-level curve since the Last Glacial Maximum using far- and intermediate-field sites*, Earth Planet. Sci. Lett. 163, 327–342.
- HARP, E., J. (1976), *Dorset settlement patterns in Newfoundland and southeastern Hudson Bay*, Mem. Soc. Amer. Arch. 31, 119–138.
- HILLAIRE-MARCEL, C. (1976), *La déglaciation et le relèvement isostatique sur la côte est de la baie d'Hudson*, Cah. Geogr. Quebec 20, 185–220.
- HILLAIRE-MARCEL, C., *Multiple component postglacial emergence, Eastern Hudson Bay, Canada*, In *Earth Rheology, Isostasy, and Eustasy* (ed. Mörner, N.) (John Wiley and Sons, New York 1980) pp. 215–230.
- KLIR, G. J., *Fuzzy logic: A specialized tutorial*, In *Fuzzy Logic in Geology* (eds. Demicco, R. V. and Klir, G. J.) (Elsevier Academic Press, San Diego 2004) pp. 11–61.

- LAMBECK, K., SMITHER, C. and JOHNSTON, P. (1998), *Sea-level change, glacial rebound and mantle viscosity for northern Europe*, *Geophys. J. Int.* 134, 102–144.
- LEE, H. A. (1960), *Late glacial and postglacial Hudson Bay sea episode*, *Science* 131, 1609–1611.
- LOWDON, J. A. and BLAKE, W. (1980), *Radiocarbon dates XX*, *Geol. Surv. Can., Paper* 80–7, 1–28.
- MITROVICA, J. X. and FORTE, A. M. (2004), *A new inference of mantle viscosity based upon joint inversion of convection and glacial isostatic adjustment data*, *Earth Planet. Sci. Lett.* 225, 177–189.
- MITROVICA, J. X. and PELTIER, W. R. (1995), *Constraints on mantle viscosity based upon the inversion of post-glacial uplift data from the Hudson Bay region*, *Geophys. J. Int.* 122, 353–377.
- MITROVICA, J. X., FORTE, A. M., and SIMONS, M. (2000), *A reappraisal of postglacial decay times from Richmond Gulf and James Bay, Canada*, *Geophys. J. Int.* 142, 783–800.
- PELTIER, W. R. (1996), *Mantle viscosity and ice-age ice sheet topography*, *Science* 273, 1359–1364.
- PELTIER, W. R. (1998), *Postglacial variations in the level of the sea: implications for climate dynamics and solid-earth geophysics*, *Rev. Geophys.* 36, 603–689.
- PELTIER, W. R. (1999), *Global sea-level rise and glacial isostatic adjustment*, *Global Planet. Change* 20, 93–123.
- PLUMET, P. (1974), *L'archeologie et le relevement glacio-isostatique de la region de Poste-de-la-Baleine*, *Rev. Geogr. Montreal* 28, 443–447.
- STUIVER, M. and REIMER, P. J. (1993), *Extended ^{14}C data base and revised CALIB 3.0 ^{14}C age calibration program*, *Radiocarbon* 35, 215–230.
- TUSHINGHAM, A. M. and PELTIER, W. R., *Relative Sea Level Database*, 93-016 (NOAA/NGDC Paleoclimatology Program, Boulder, CO 1993).
- VOGEL, J. C. and MARAIS, M. (1971), *Pretoria radiocarbon dates I*, *Radiocarbon* 13, 378–394.
- WALCOTT, R. I. (1972), *Late Quaternary vertical movements in eastern North America: Quantitative evidence of glacio-isostatic rebound*, *Rev. Geophys. Space Phys.* 10, 849–884.
- WALCOTT, R. I., *Rheological models and observational data of glacio-isostatic rebound*. In *Earth Rheology, Isostasy, and Eustasy* (ed. Mörner, N.-A.) (Wiley, Chichester 1980), pp. 3–10.
- WALCOTT, R. I. and CRAIG, B. G. (1975), *Uplift studies, southeastern Hudson Bay*, *Geol. Surv. Can., Paper* 75-1 Part A, 455–456.
- WESSEL, P. and SMITH, W. H. F. (1991), *Free software helps map and display data*, *EOS, Trans. Am. Geophys. Union* 72, 441–446.
- WOHLFARTH, B. and POSSNERT, G. (2000), *AMS radiocarbon measurements from the Swedish varved clays*, *Radiocarbon* 42, 323–333.
- WOLF, D., KLEMMANN, V., WÜNSCH, J. and ZHANG, F.-P. (2006), *A reanalysis and reinterpretation of geodetic and geomorphologic evidence of glacial-isostatic uplift in the Churchill region, Hudson Bay*, *Surv. Geophys.* 27, 19–61.
- ZADEH, L. A. (1978), *Fuzzy sets as a basis for a theory of possibility*, *Fuzzy Sets Syst.* 1, 3–28.

(Received July 6, 2005, revised February 15, 2006, accepted March 16, 2006)

Published Online First: April 16, 2007

To access this journal online:
www.birkhauser.ch/pag

Shear-wave Velocity Structure around Teide Volcano: Results Using Microtremors with the SPAC Method and Implications for Interpretation of Geodetic Results

FRANCISCO J. CHÁVEZ-GARCÍA,¹ FRANCISCO LUZÓN,^{2,3} DIMITRIS RAPTAKIS,⁴ and JOSÉ FERNÁNDEZ⁵

Abstract—Deformation analysis and simulation of volcanic edifices require the construction of models of elastic properties of those structures. In this paper we present an analysis of microtremor measurements recorded during the performance tests of two temporary seismic arrays installed in the eastern portion of the Teide caldera in 1994. We take advantage of recent developments of the SPAC method and use spatial cross-correlation computations to estimate phase velocity dispersion of Rayleigh waves at the location of the arrays. We show that the extension of the standard SPAC method is valid in the case of our data, justifying its use and supporting the generalization of the SPAC method to single station pairs. The phase velocity dispersion curve obtained was inverted to recover the shear-wave profile at the site of the arrays. Our results indicate that the subsoil structure of the caldera is laterally homogeneous at the scale of a few km about the location of the arrays. We obtained about 315 m of volcanic sediments overlying rocks with a shear-wave velocity of 2 km/s. These results are robust and are a starting point to further modelling of deformation, permanent or transient, at this volcanic edifice, which can be useful in the interpretation of different observed fields. In fact, the computation of deformations and gravity changes due to possible volcanic intrusions in two models; one considering the volcanic sediments and the other without considering them, provided different results in the near field.

Key words: Teide volcano, SPAC method, microtremors, spatial correlation, phase velocity dispersion, Vs subsoil structure.

1. Introduction

The Canary Islands consist of seven islands and several islets located at about 28°N, 16°W in the Atlantic Ocean (Fig. 1). They lie in an intraplate setting on the

¹ Instituto de Ingeniería, Universidad Nacional Autónoma de México, Ciudad Universitaria, Coyoacan, 04510, Mexico, D.F., Mexico.

² Departamento de Física Aplicada, Universidad de Almería, Cañada de San Urbano s/n, 04120, Almería, Spain.

³ Instituto Andaluz de Geofísica, Universidad de Almería, Cañada de San Urbano s/n, 04120, Almería, Spain. E-mail: fluzon@ual.es

⁴ Laboratory of Soil Mechanics and Foundation Engineering, School of Engineering, Aristotle University of Thessaloniki, P.O. Box 450, Thessaloniki, Greece.

⁵ Instituto de Astronomía y Geodesia (CSIC-UCM), Fac. C. Matemáticas, Ciudad Universitaria, Pza. de Ciencias, 3, 28040-Madrid, Spain.

northwestern African continental shelf. Several theories have been proposed to explain the origin of the Canarian volcanism (ANGUITA and HERNÁN, 2000), including a mantle hotspot (MORGAN, 1983; CARRACEDO *et al.*, 1998; DAÑOBEITIA and CANALES, 2000) and the uplift of tectonic blocks (ARAÑA and ORTIZ, 1991) among others. In Tenerife, subaerial volcanism started more than 12 Ma ago (ANCOCHEA *et al.*, 1990). The activity was concentrated mainly along the NE-SW and NW-SE faults that cross the center of the island, where a massive volcanic edifice was built some 3 Ma ago (MARTÍ *et al.*, 1994). The collapse of this edifice, most likely by multiple collapses rather than by a single giant event (MARTÍ and GUDMUNDSSON, 2000) produced the Las Cañadas caldera, which constitutes one of the best exposed calderas in the world. It has an approximately elliptical shape, with dimensions of 16 and 9 km in east-west and north-south directions, respectively. The caldera floor has an elevation of about 2 km, and is surrounded by walls clearly visible for a length of 27 km. A gravitational failure of part of the north wall left the caldera open in this direction. Subsequent episodes of volcanism on the northwest of the caldera produced the Teide-Pico Viejo stratovolcano (ABLAY and MARTÍ, 2000) and contributed to the filling of the caldera. There is also evidence of the importance

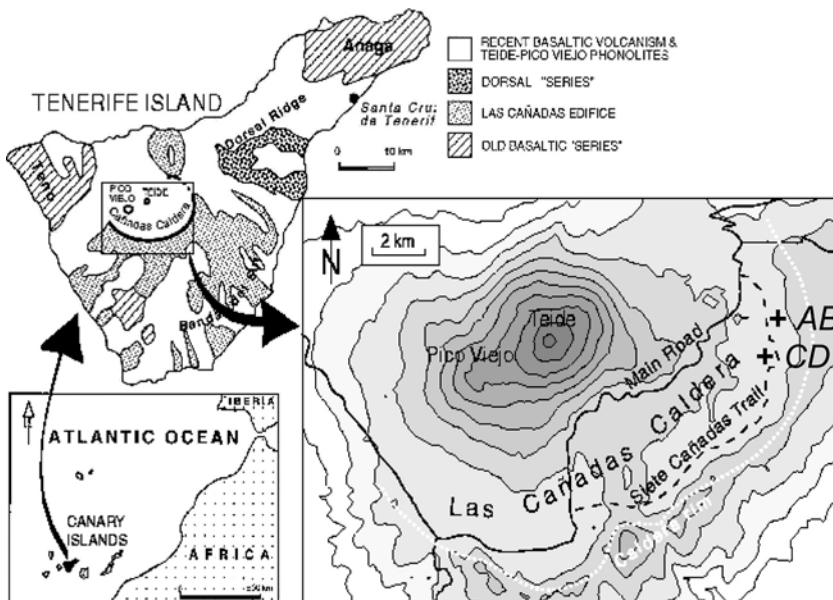


Figure 1

Composite map showing the region under study (modified from ALMENDROS *et al.* 2004). Lower left: Setting of the Canary Islands near the northwestern African coast. Upper left: Simplified geological map of Tenerife Island, showing the position of Teide Volcano and Las Cañadas caldera. Lower right: Las Cañadas Caldera. Microtremor records were obtained from arrays AB and CD (marked with crosses).

of landslides in the volcanic evolution of Tenerife (ANCOCHEA *et al.*, 1990; ABLAY and HÜRLIMANN, 2000).

Teide is considered as potentially hazardous due to the proximity of populated areas such as the Orotava Valley. This may be one of the reasons for its selection as the Decade Volcano and the European Laboratory Volcano by the IAVCEI and the European Science Foundation, respectively. Several eruptions have occurred in Tenerife within historical times, as for example that of Siete Fuentes (1704), the Fasnía and Montaña Arenas (1705), the Montaña Negra (1706), the Chahorra (1798), or the Chinyero (1909) on the northwestern flank of the volcano. All these eruptions were preceded by premonitory seismicity (ARAÑA and CARRACEDO, 1978), which emphasizes the importance of systematic geophysical monitoring.

Volcanic activity is often manifested on the surface as ground deformation and/or gravity changes, which are eruption precursors in some cases. This fact and the high levels of accuracy attainable today with geodetic techniques provide a powerful tool for monitoring the volcanic activity. It is possible to detect crustal deformation and gravity changes that reflect magma rising from depth, sometimes months or weeks before the magma flow leads to earthquakes or other eruption precursors (e.g., BURGMANN *et al.*, 2000; STEIN *et al.*, 2000; DZURISIN, 2003; FERNÁNDEZ *et al.*, 2005). Therefore, traditional monitoring systems based on the use of seismic, hydrologic or fumarolic activity observations are complemented with geodetic observations (e.g., SIGURDSSON *et al.*, 2000), extending the study of volcanic phenomena from seconds (seismic monitoring) to years and providing details on the growth of magma bodies within the volcano (STEIN *et al.*, 2000). Geodetic volcano monitoring includes the deployment of long-term instrumentation, such as clinometers and extensometers, and regular geodetic network surveying campaigns using classic terrestrial techniques or GPS (Global Positional System), the use of gravimetry as well as the more recent techniques PGPS (Permanent GPS observation) and radar interferometry technology (DIXON *et al.*, 1997; SEGALL *et al.*, 2001; BARTEL *et al.*, 2003; DZURISIN, 2003). As an obvious consequence, these long-term geodetic monitoring techniques involve subsequent interpretation of observed deformation and gravity changes. In this step appropriate mathematical deformation models are the essential tool for the latter task.

In this final part, to be used as parameterization of the models, the interpretation of deformation or gravity measurements requires a detailed knowledge of the crustal structure that is not usually available. A detailed subsoil model is also necessary in any modelling of the dynamics of a volcanic structure. For this reason, in addition to its usefulness to monitor volcanic activity, many seismological studies have been conducted to investigate the subsoil structure of volcanic edifices (e.g., DELUCA *et al.*, 1997; CHOUET *et al.*, 1998). Given the cost of installing and maintaining dense seismic networks, these efforts are sparse. This problem requires deriving as much information as possible from the available few studies. This reasoning is behind the

present study. Here, we have recovered data from a seismic experiment carried out in the eastern part of Las Cañadas Caldera in 1994. Although the aim of that experiment was to record earthquakes, simultaneous records of ambient vibration were obtained periodically, to test the performance of the seismic arrays. We take a fresh look at these records and investigate the applicability of a recent idea concerning the use of the correlation between records to obtain the subsoil structure. In the following, a description of the geology of the area under study is given. Then, a description of the experiment and of the data that were obtained is presented. After this, we describe the SPAC method, both as it has been traditionally applied and the modifications that have been recently proposed by CHÁVEZ-GARCÍA *et al.* (2005). The application of the method to a dataset recorded more than 10 years ago allows us to recover phase velocity dispersion curves at the site where the array operated. Finally, the inversion of the dispersion curve allows us to propose a shear-wave velocity (V_s) profile at the site. This study shows the large interest in the re-analysis of old datasets, taking advantage of progress in processing techniques. In addition, our results provide a reliable estimate of the shear-wave velocity structure in the Eastern portion of Las Cañadas Caldera: a contribution that will be useful in the studies of deformation at this volcanic edifice.

2. Geology of the Area under Study

Exhaustive studies of the surface geology in Las Cañadas (ANCOCHEA *et al.*, 1990, 1999; MARTÍ *et al.*, 1994; ABLAY and MARTÍ, 2000) have yielded detailed information about the composition, location, and extent of each exposed rock type and its role in the volcanic evolution of the region. Borehole data provide additional constraints on the subsurface structure of Las Cañadas. Current interpretations (e.g., ABLAY and MARTÍ, 2000) assume that the caldera fill is made by superposition of a variety of eruptive products produced by different events related to the Teide-Pico Viejo edifice. These deposits are thick (> 500 m) in the central and eastern parts of the caldera and thin (< 100 m) in the western part. Although the different layers are assumed to be essentially horizontal, no detailed information about their thickness or horizontal extent is available.

The diversity of volcanic materials that forms the caldera prevents us from assuming a homogeneous seismic velocity for the entire caldera. Therefore in this sense, several geophysical studies have been carried out to investigate the structure and evolution of the Las Cañadas edifice. For example, gravimetric measurements have revealed the presence of density anomalies (VIEIRA *et al.*, 1986; CAMACHO *et al.*, 1991, 1996; ARAÑA *et al.*, 2000; ABLAY and KEAREY, 2000); a shallow, low-density region is located under the Teide-Pico Viejo edifice in the northern part of the caldera. This anomaly could be due to the presence of a low-density caldera

fill and the abundance of light materials erupted during recent volcanism. High-density anomalies are observed near the caldera walls. Gravity modeling suggests that these anomalies could extend several kilometers in depth (ABLAY and KEAREY, 2000) and that they could be caused by the influence of dense materials related to the old basaltic plateau (CAMACHO *et al.*, 1996). A dense fill could also be responsible for the positive gravity anomalies in the southern part (ABLAY and KEAREY, 2000).

Magnetic surveys have revealed the existence of anomalies (BLANCO, 1997; GARCÍA *et al.*, 1997). A negative anomaly coinciding with the Teide summit has been interpreted as an effect of hydrothermal alteration due to fluids at high temperatures (ARAÑA *et al.*, 2000). The presence of volcanic fluids has been supported by measurements of gas emission rates (HERNÁNDEZ *et al.*, 2000). Another negative anomaly, coinciding with the Diego Hernández caldera, has been attributed to an accumulation of phonolitic materials in filling the caldera, which show very low magnetization compared with basalts (ARAÑA *et al.*, 2000). Seismic studies have also been performed to investigate the structure of Tenerife. Recent works have established the presence of a high-velocity zone southwest of Las Cañadas (WATTS *et al.*, 1997; CANALES *et al.*, 2000). This zone coincides with a high-density body inferred from gravimetric studies and interpreted as the core of an old, large mafic volcano (CANALES *et al.*, 2000; ABLAY and KEAREY, 2000). The eastern part of Las Cañadas was the site of a seismic experiment by DEL PEZZO *et al.* (1997), who analyzed the coda on the seismograms of local earthquakes using two seismic arrays. They showed that the area is characterized by a highly heterogeneous shallow structure that produces strong scattering of the seismic waves. ALMENDROS *et al.* (2004), in another experiment, used time-dependent horizontal-to-vertical spectral ratios (HVSr) of microtremors to determine the dominant frequencies of vibration of the geological structures beneath several recording sites in the vicinity of Teide volcano. Dominant frequencies in the caldera varied sharply, even for nearby locations just a few hundred meters apart. All these studies are contributions to our understanding of the general structure and volcanic evolution of Las Cañadas, nonetheless they fall short of providing a description of the elastic properties of the zone.

In previous works, a layered crustal model for the Tenerife Island was used in deformation models (see e.g., FERNÁNDEZ and DíEZ, 1995; FERNÁNDEZ *et al.*, 1999). It was based on the velocity model by BOSSHARD and MACFARLANE (1970) determined by seismic experiments. In the central part of the Island of Tenerife, this model consists of three layers overlying the mantle. FERNÁNDEZ and DíEZ (1995) assign values to the elastic properties and densities of the layers and mantle using values of V_P velocities and pressure values from PREM (Preliminary Reference Earth Model) (DZIEWONSKI and ANDERSON, 1981). The values used are listed in Table 1.

Table 1

Properties of Tenerife crustal model (FERNÁNDEZ and DíEZ, 1995). ρ is density; λ and μ are the Lamé's parameters

Layer	Thickness (km)	ρ (10^3 kg m $^{-3}$)	μ (10^{10} Pa)	λ (10^{10} Pa)
1	3.5	2.1	0.7	0.8
2	2.5	2.3	1.3	1.7
3	19	2.9	4.0	4.7
Half space	∞	3.3	6.4	8.3

3. Instruments and Data

We use the microtremor records obtained by two clusters of seismometers or seismic arrays (Fig. 2) These arrays were installed with the objective of monitoring the seismic activity of the Teide volcano. The first antenna, named array *AB*, was deployed in the Diego Hernández Formation, and included ten vertical seismometers and two three-component sensors. Its geometry was approximately circular with a diameter of about 600 m. The second cluster, array *CD*, was deployed in the Las Pilas Formation consisting of eight vertical seismometers and two three-component sensors. The geometry of this array was triangular with an aperture of about 300 m. Each seismic array was composed of two independent eight-channel modules. Each module had eight seismometers with preamplifiers close to them, a 16-bit data acquisition system and a GPS time receiver. All seismometers used in the cluster *AB* were of the type Mark L15 and, in the array *CD*, were of the type Mark L25, all of them with natural frequency of 4.5 Hz. Their response was electronically extended to 1 Hz. Each sensor had a preamplifier that permitted signal transmission via cable to the data logger. The data acquisition system was composed of (a) an eight-channel anti-alias Butterworth multipole filter at 48 Hz; (b) a multiplexer that sampled the eight-channels every 5 ms; (c) a 16-bit A/D converter. The internal clock was synchronized by GPS time every second, and the sampling rate was 200 samples per second. The control of the whole system and the storage of the data were achieved by a portable PC. Station Cartesian coordinates were measured with a laser theodolite with respect to a reference benchmark, whose absolute geographic coordinates were obtained through GPS positioning. The reference system had the *y*-axis oriented toward north and the *x*-axis toward east. A detailed presentation of these arrays, as well as the analysis of the earthquakes that were recorded was presented in DEL PEZZO *et al.* (1997) and by ALMENDROS *et al.* (2000).

During the time these arrays were in operation, periodic simultaneous triggering of the recorders was carried out as part of the performance procedures. Thus, simultaneous microtremor records at all the locations were obtained. From these records we selected samples of noise obtained for six days. Two samples were

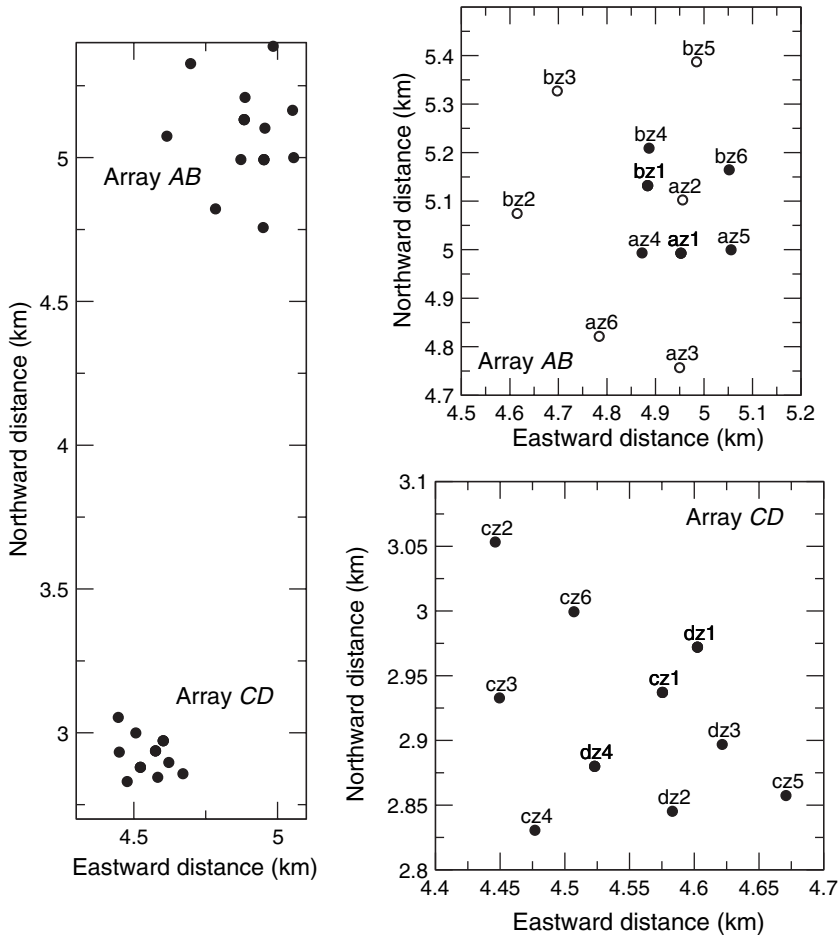


Figure 2

Distribution of stations of arrays *AB* and *CD* installed at Teide Caldera. The open circles for some of the stations of array *AB* highlight those that were used to test the equivalence between azimuthal average and temporal average of the correlation computations.

obtained each day, one at midnight and another one hour later. The length of the records varies between 48 and 64 s. We will use only the vertical component records.

4. Method

The spatial autocorrelation (SPAC) technique was introduced by AKI (1957). The SPAC method is based on array measurements of microtremors to determine a subsoil profile using the cross-correlation properties of ambient vibration. From the start we adopt two hypotheses on the nature of the microtremor wavefield. The first

is that the seismic noise is stationary both in time and frequency. The second is that the wavefield consists of surface waves propagating along the free surface, with power distributed equally in all directions. These two hypotheses are not restrictive in the light of a large number of publications on the nature of microtremors (see for example BARD, 1999, for a review).

Let us consider microtremors recorded at the free surface, using an array of stations sampling different orientations, at the same interstation distance. AKI (1957) defined the spatial cross-correlation coefficient as the cross-correlation between two simultaneous microtremor measurements among two stations, normalized by the autocorrelation function of one of them. He showed that the spatial cross-correlation coefficient as a function of frequency, $\rho(r, \omega)$, for a given interstation distance, r , and angular frequency, ω , averaged over many different azimuths, θ , can be written as

$$\rho(r, \omega) = \frac{1}{2\pi\phi(0, \omega)} \int_0^{2\pi} \phi(r, \theta, \omega) d\theta = J_0\left(\frac{r\omega}{c(\omega)}\right), \quad (1)$$

where $\phi(0, \omega)$ is the autocorrelation function at the center of the array, $\phi(r, \theta, \omega)$ is the cross-correlation function between the record at a site with coordinates (r, θ) and the record obtained at the station at the origin, $c(\omega)$ is the phase velocity at the site, and J_0 is the Bessel function of first kind and order zero. The details of the derivation of this equation have been published repeatedly (e.g., CHOUET *et al.*, 1998). The previous equation shows that, when enough measurements at the same interstation distance, sampling different azimuths, are averaged, the cross-correlation coefficients should reproduce a Bessel function of first kind and order zero. The variables in the argument of this Bessel function are known, with the exception of the phase velocity dispersion function, $c(\omega)$, which can be obtained by inversion of the cross-correlation coefficients. In recent applications of the SPAC method (FERRAZZINI *et al.*, 1991; METAXIAN, 1994; CHOUET *et al.*, 1998), the seismographs have been disposed on a half circle, with a central station recording simultaneously. This array satisfies the requirements of the method, as it allows us the sampling of different azimuths between pairs of stations at the same distance to compute the azimuthal average. Once we have a reliable phase velocity dispersion curve, it is possible to invert it to obtain a Vs profile using standard techniques (see for example HERRMANN, 1987).

Recently, CHÁVEZ-GARCÍA *et al.* (2005) have proposed a generalization of the SPAC method as described above, based on the temporal stationarity of long microtremor records. The approach used in that paper is similar to that described here with the exception of the azimuthal averaging of the cross-correlation coefficients. AKI (1957) showed that, at least for one case, microtremors were propagating along different directions with comparable power. If the microtremor wavefield includes waves propagating in all directions, the average cross-correlation between any two stations would be equivalent to the azimuthal average of the cross-correlation at that interstation distance. In this paper we will use the approach to

SPAC presented in CHÁVEZ-GARCÍA *et al.* (2005). We will first evaluate the validity of our hypotheses, and then we will use the method to determine phase velocity dispersion at the eastern part of Las Cañadas caldera. Finally, we will invert this dispersion curve to obtain the Vs profile at the instrumented sites, and discuss its implication in the elastic models of the area.

Our proposed generalization of the SPAC method to a single station pair has experienced a parallel development in time domain. First progress was accomplished in the seismic exploration field (*e.g.*, BASKIR and WELLER, 1975; COLE, 1995; RICKETT, 2001). Later, the idea was extended to coda waves and noise records (NIKOLAEV and TROITSKIY, 1987), and to solar seismology (DUVALL *et al.*, 1993; RICKETT and CLAERBOUT, 1999). In recent years, a few papers have presented theoretical demonstrations of the relation between the temporal correlation of noise records at two stations and the Green's function of the medium (LOBKIS and WEAVER, 2001; WEAVER and LOBKIS, 2001, 2004; ROUX *et al.*, 2004; SABRA *et al.*, 2005a, b; SÁNCHEZ-SESMA and CAMPILLO, 2006). However, applications of correlation in the time domain to obtain the subsoil local structure are very recent. For example, CHÁVEZ-GARCÍA and LUZÓN (2005) presented a study in which they compared correlation results in time and frequency domains. In this paper we will restrict ourselves to the application of the correlation technique in the frequency domain. The reason is twofold. First, CHÁVEZ-GARCÍA and LUZÓN (2005) showed that the information that can be retrieved from the cross-correlation of noise records is the same in time domain and in frequency domain. Second, our use of a single station pair to compute correlation coefficients does not change the foundations of the SPAC method. Therefore, the inversions required to pass from the correlation coefficients to the site dispersion curve and from this to the subsoil structure are supported by the many papers that have dealt with this problem during the last 50 years (see CHÁVEZ-GARCÍA *et al.*, 2005, for a thorough review).

5. Results

Before presenting the results of the correlation analysis of our data, we have computed horizontal-to-vertical spectral ratios (HVSr) using the microtremor data. These spectral ratios have been shown to give a reliable estimate of the dominant frequency and maximum amplification, due to shear-wave velocity contrast in the shallow subsoil structure (LERMO and CHÁVEZ-GARCÍA, 1994). A large peak in HVSr is a good indicator of a significant impedance contrast in the subsoil structure. A flat HVSr, on the contrary, indicates either that no large impedance contrast exists or that this contrast does not occur at a single interface (a shear-wave velocity gradient could have a large velocity contrast and it would not show a large peak in the HVSr). We have used five 48 s windows from the data available at each of the four available three-component stations. The results indicated no significant

amplification at any of the stations, all of which showed similar results. This suggests that, if a large impedance contrast exists in the subsoil structure, it occurs as a velocity gradient, without the impedance contrast being concentrated at a single interface. In addition, it also suggests that the locations of arrays *AB* and *CD* have a similar subsoil structure.

We have chosen 12 sets of simultaneous microtremor recordings from the data obtained by arrays *AB* and *CD*. The chosen windows are simultaneous for all sites but not all of them have the same length, which varies between 48 and 64 s. Figure 3 shows an example of the six records obtained at stations *b* of array *AB* for a single microtremor record. We observe that the signals are dominated by low frequency signals. From each record, we have selected different windows for the analysis. We have tested 5, 10 and 20 s windows, with different overlapping between successive windows. For each selected window we applied a series of band-pass Butterworth filters, with central frequencies between 0.9 and 4.5 Hz and a width of 0.4 Hz. The filtered traces were used to compute the correlation coefficients, using the definition of AKI (1957). The available 22 stations would theoretically allow us to compute results for 231 different interstation distance values, between 44 and 2606 m. The advantages of this approach are evident when we recall that conventional studies using SPAC are limited to 2 or 3 different interstation distance values (see for example MORIKAWA *et al.*, 1998).

The use of overlapping windows allowed us to compute between 8 and 12 cross-correlations for each record, i.e., between 96 and 144 correlations for each station pair. We computed average and standard deviation values for each frequency from all correlations computed for a given station pair. The average curves were carefully considered. Because of the limitations of the data, many of them were not useful to obtain a phase velocity dispersion curve. Better results were obtained with larger distances between stations. The reason for this is shown in Figure 4. This figure shows the Fourier amplitude spectra of the traces shown in Figure 3. The maximum amplitude of the signals occurs at a frequency smaller than 0.5 Hz, with the amplitude of the signals decreasing fast for frequencies larger than 1 Hz. A similar amplitude spectrum was observed for all measurements. Thus, our results are stringently constrained in the frequency domain; at the low frequency end we are limited by the passing band of the seismometers used, while at the high frequency end we are limited by the low amplitude of the signals.

At this point we can test the validity of our main hypothesis, namely that the temporal averaging of cross-correlation coefficients is equivalent to an azimuthal average of the same. Our stations were not disposed along a circle, however, several stations are at comparable distance from a “central” station. Consider station *az2* in Figure 2. This station is at similar distances from stations *az3* (distance of 346 m), *az6* (329 m), *bz2* (342 m), *bz3* (342 m), and *bz5* (286 m). The average interstation distance is 329 m. Thus, station *az2* and each of the other five stations are 5 station pairs at similar interstation distances and with azimuths varying slightly more than

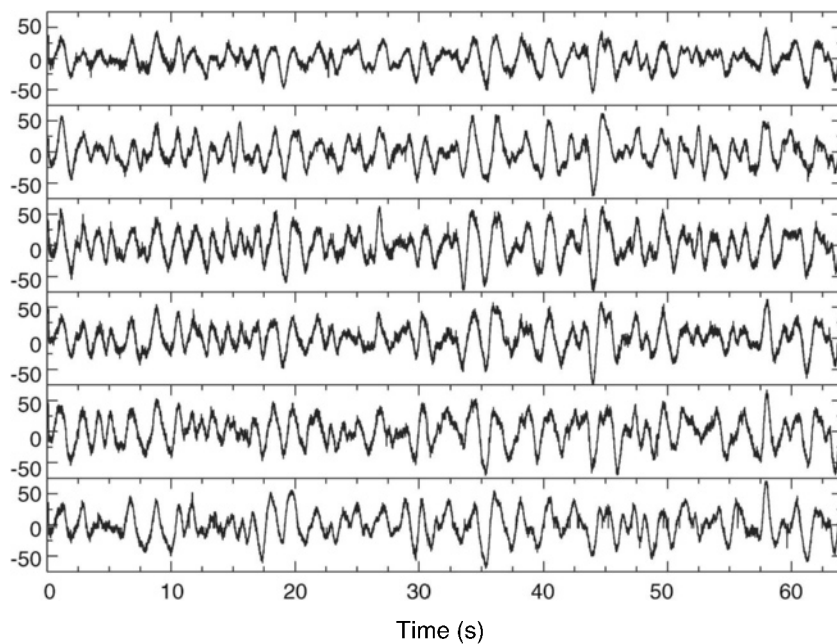


Figure 3

An example of the microtremor records analyzed in this paper. Shown are the six records for the vertical component obtained at stations b of array *AB*. The records demonstrate that ambient vibration is dominated by low frequency signals.

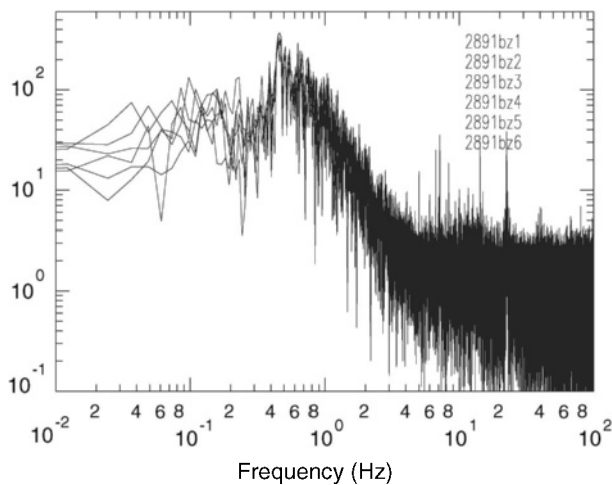


Figure 4

Fourier amplitude spectra of the traces shown in Figure 3. The maximum amplitudes occur around 0.5 Hz. At larger frequencies amplitudes decrease sharply.

180°. For these station pairs, we have computed the azimuthally averaged correlation coefficients, following precisely the standard SPAC method. We used a single 48 s window from each of the available 12 simultaneous microtremor measurements. For each of these 12 time windows we computed the azimuthal average of the cross-correlation coefficients for those 5 station pairs. The result is shown by the 12 solid lines in Figure 5. We observe good agreement between the estimates, demonstrating that the recorded seismic noise is stationary in time and that the differences between the interstation distances included are not significant. The open circles, with their associated error bars shown in Figure 5, show the cross-correlation coefficients estimated using the single station pair az2-az6 and the 12 windows available. We observe a very good agreement between the cross-correlation coefficients estimated using a single pair with those estimated using an azimuthal average. This result strongly supports the generalization of the SPAC method proposed in CHÁVEZ-GARCÍA *et al.* (2005).

We processed data for all available station pairs. However, not all results could be used. A careful selection led us to retain results for 94 interstation distances, ranging from 44 to 631 m. All of them correspond to combinations of the stations within either array *AB* or *CD*. Results for some station pairs had to be dropped

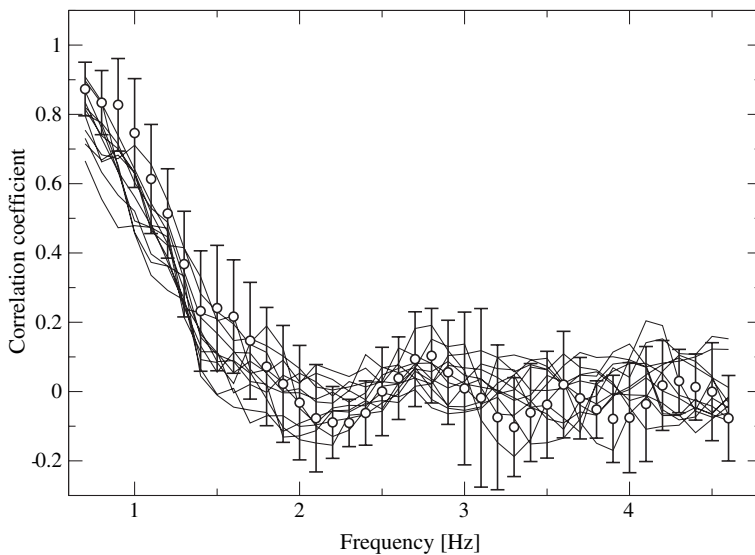


Figure 5

Comparison between correlation coefficients computed using the azimuthal average with those computed for a single station pair. Each solid line shows the average of the cross correlation coefficients computed between stations az2-az3 (346 m of inter station distance), az2-az6 (329 m), az2-bz2 (342 m), az2-bz3 (342 m), and az2-bz5 (286 m) for a particular 48 s record of simultaneous ambient vibration at all these stations. The solid lines are as close as possible to an azimuthal average at the average inter-station distance of 329 m using the stations of array *AB*. The solid symbols with their associated error bars show the average of the correlation coefficients computed for the station pair az2-az6 only. The agreement is excellent.

because transient perturbations close to a single receiver clearly violated the underlying hypotheses, and the correlation coefficients were far from the expected shape of a J_0 function. In addition, all station pairs involving one station from array AB and one from array CD had to be discarded. The reason is that distances in this case ranged from 1776 to 2606 m. The result for 631 m showed that cross-correlation could be used only close to the lower frequency limit imposed by the seismometer response. Thus, for interstation distances larger than 1776 m, if correlation between stations exists, it does only at low frequencies outside the range of our seismometer's usefulness.

Once having chosen the useful correlation coefficients, we selected a frequency range of validity for each of them, attending to the restrictions discussed by HENSTRIDGE (1979) and CHÁVEZ-GARCÍA *et al.* (2005). An example of the final selection is shown in Figure 6, for three different values of interstation distance. As expected, the frequency of the first zero-crossing of the correlation coefficients decreases with increasing distance, signalling the frequency range where this particular cross-correlation is able to provide useful information. The thin lines in Figure 6 are the expected shape of the cross-correlation coefficients at these distances, using the phase velocity estimate discussed below.

The correlation coefficients were inverted using the scheme described in detail in CHÁVEZ-GARCÍA *et al.* (2005). Given that the inversion of the correlation coefficients to obtain a phase velocity dispersion curve is a nonlinear process, we made several tests to insure the independence of the final result of the initial model chosen. In addition to the variance reduction produced by the iterative inversion procedure, we can trust the results only where they are independent of the initial model used for the inversion, i.e., where the data are able to constrain the result well. We also analyzed the compatibility of the results obtained for the different windows used, 5, 10 and 20 s in length. An additional test involved exploring the possibility that soil structure was different at the location of array CD from that at the location of array AB . To this end, we inverted separately the correlation coefficients obtained from the two arrays. The results did not differ significantly, showing that the subsoil structure at Teide caldera is homogeneous, at least at the scale of the distance between arrays AB and CD .

Finally, we elaborated a composite phase velocity dispersion curve from all results. As expected, the longer windows provide reliable results at lower frequencies, while the shorter windows provide stable estimates at higher frequencies. The final estimate is shown in Figure 7. The composite phase velocity dispersion curve was used to compute the expected correlations shown with thin lines in Figure 6.

The resulting phase velocity dispersion curve was inverted to obtain a V_s profile. This is, once again, a nonlinear problem, solved by an iteration procedure. We used the inversion codes of HERRMANN (1987). The phase velocity dispersion curve computed for the final inverted model is shown in Figure 8, together with the

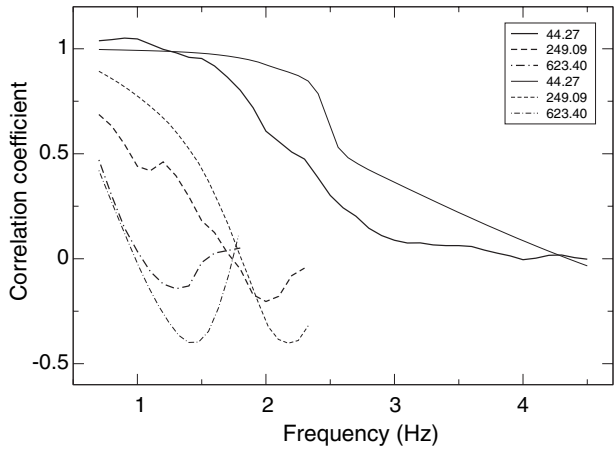


Figure 6

Correlation coefficients as a function of frequency. We show the observed correlations for three different inter station distances with thick lines. The corresponding distances in meters are indicated in the legend. The thin lines show the predicted shape of the coefficients, using the phase velocity dispersion curve inferred from all measurements.

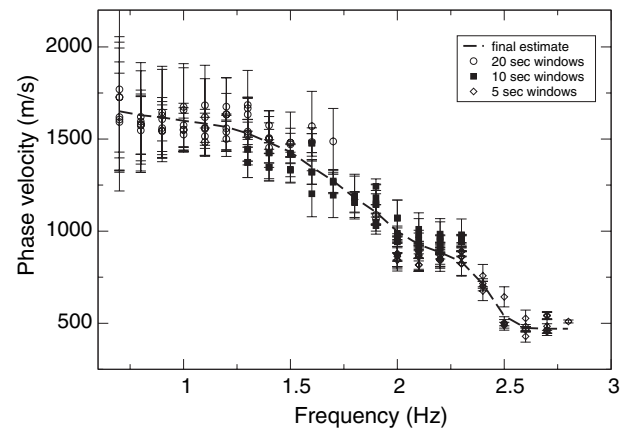


Figure 7

Final estimate of the phase velocity dispersion curve in the region of the measurements. The symbols show the results obtained from the inversion of the correlation coefficients with their associated uncertainty. Different symbols correspond to results obtained using different windows in the analysis. The thick dashed line shows the composite final estimate of the phase velocity dispersion curve.

estimated phase velocity dispersion curve. We observe that the fit is very good. Figure 9 displays the final model together with the resolving kernels of the inversion procedure. The resolving kernels should manifest a single peak at the depth of the layer they correspond to. A resolving kernel that shows this single peak is an

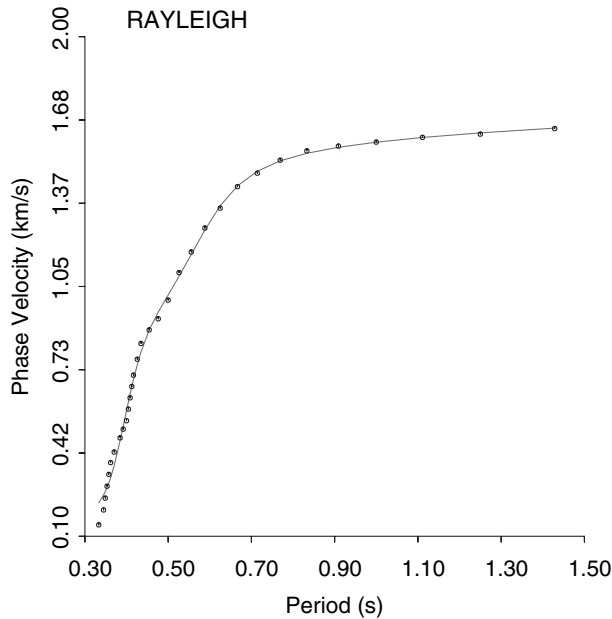


Figure 8

The circles show the final estimate of the phase velocity dispersion curve (see also dashed line in Figure 7). The solid line shows the predicted phase velocity dispersion curve computed for the best-fitting model. The agreement between observed and predicted curves is very good.

indication that the V_s value inverted of the corresponding layer has been well resolved. The kernels of Figure 9 indicate that the shear-wave velocity for all 7 layers could be well constrained by the inversion scheme.

The final model shows a surficial layer of very soft material. This layer is very thin and probably corresponds to the loose material at the surface. Below this layer we obtain two layers with a shear-wave velocity around 300 m/s and a total thickness of 30 m. Below there is a layer 180 m thick with a V_s of almost 1000 m/s. Finally, below this, there is a 100 m thick layer with $V_s = 1360$ m/s overlying a half space with V_s around 2 km/s. Clearly, we have then about 35 m of light materials, overlying harder volcanic products. The total thickness of the volcanic sediments at the location of the arrays is about 415 m, with V_s values increasing to 1360 m/s. The shear-wave velocity increases steadily with depth, without any interface with a large impedance contrast, in agreement with the observed horizontal-to-vertical spectral ratios. We computed the theoretical transfer function for the inverted profile, excluding the 5-m thick soft surficial layer; attenuation in this layer must be large, but we are unable to estimate it from our data. The computed transfer function shows a very small amplification (a factor smaller than 2.5) at 1 Hz, and it is thus compatible with the observed HVSR.

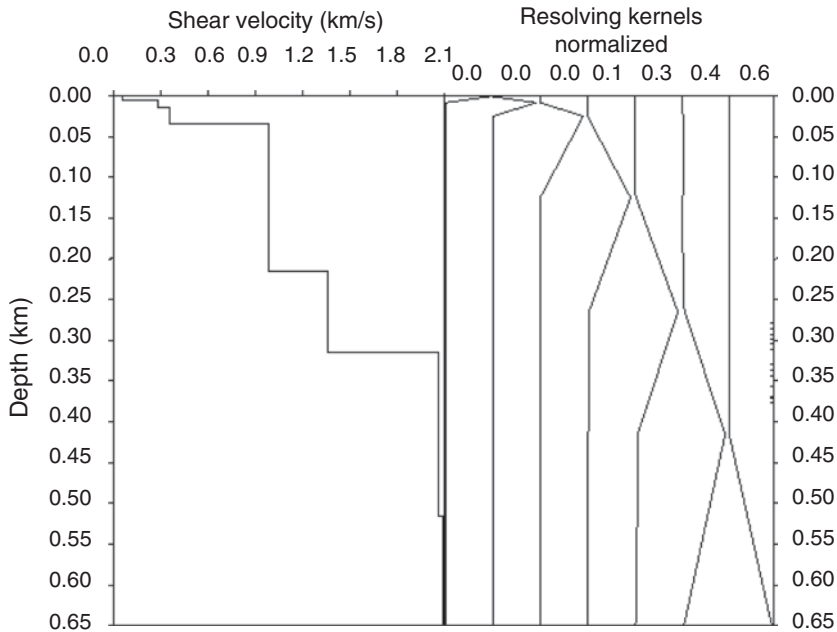


Figure 9

Left diagram: Shear-wave velocity profile obtained from the inversion of the phase velocity dispersion curve. Results are reliable down to at least 500 m depth. Right diagram: Resolving kernels for each layer in the model. The resolving kernels show a single peak at the depth of the corresponding layer, if this layer is well resolved by the inversion procedure.

6. Implications for the Interpretation of Geodetic Monitoring Results

Now we are interested in the evaluation of the effect of considering these properties of the shallower upper crust in the prediction of geodetic anomalies (displacements and gravity changes) on Tenerife Island. For this purpose we will use the methodology developed by RUNDLE (1982) and extended by Fernández, Rundle and co-workers in more recent years (e.g., FERNÁNDEZ and RUNDLE, 1994a,b; FERNÁNDEZ *et al.*, 2005, 2006). In this method, the Earth model is represented as a set of p homogeneous flat layers, each having its own Lamé parameters, λ_i , μ_i , density, ρ_i and thickness, d_i , overlying a homogeneous half space. The bottom half space is considered to be layer $p+1$ with properties λ_{p+1} , μ_{p+1} , ρ_{p+1} . A system of cylindrical coordinates (r, θ, z) with unit vectors (e_r, e_θ, e_z) , where the origin is located at the projection of the intrusion onto the surface, is used, such that the z axis points downward into the medium. Displacements, gravity and potential changes are considered to be caused by a spherical intrusion which is modeled as a point source, located at $(0,0,c)$ with c the depth of the intrusion. The point source is the combined effect of a center of expansion (ce) and a point mass (pm) located at the same

position. Therefore, the source represents both the pressure change inside the intrusion and its mass, respectively. The equations that satisfy the displacement vector u and the disturbing gravitational potential ϕ were given by RUNDLE (1980). ϕ represents the change in gravitational potential due to the intruded mass and due to the resulting deformation, evaluated at the original (undeformed) surface position. The solution (RUNDLE, 1980, 1982; FERNÁNDEZ *et al.*, 1997; FERNÁNDEZ *et al.*, 2005) for the free surface ($z = 0$) deformation, gravitational potential and gravity changes was obtained using dislocation theory and the propagator matrix technique. A full description of the general numerical formulation and the computer programs to obtain this solution can be found in FERNÁNDEZ *et al.* (1997). We will use the GRAVW software package described by FERNÁNDEZ *et al.* (1997, 2006).

Using the same methodology as FERNÁNDEZ and DÍEZ (1995) with the results obtained above using the SPAC methodology, we obtain for the upper part of the Tenerife crustal model the results shown in Table 2. Below the maximum depth of the model obtained, the properties agree with those in FERNÁNDEZ and DÍEZ (1995) which used the results by BOSSHARD and MACFARLANE (1970).

We include the layers described in Table 2 into the global crustal model given in Table 1 to obtain a layered crustal model composed of 4 layers over a half space to use the program GRAVW4 (FERNÁNDEZ *et al.*, 2006). Considering that the thickness of layer 1 in Table 2 is extremely small, we combine layers one and two to obtain a new layer 1 for the global crustal model with a thickness of 215 m. We assign density and Lamé parameters to the material in this layer using a weighted average of the properties of the original layers: the weights being the thickness of each layer. Layer 2 of the new crustal model has the same characteristics as layer 3 defined in Table 2. To obtain the properties of layer 3, we apply the same methodology used to obtain the new layer 1, but now to the layers 1 and 2 described in Table 1. Layer 4 and the bottom half space are equal to layer 3 and half space of Table 1, respectively. This new crustal model is given in Table 3(a). It is clear that this crustal model is only an approximation to study the effects of including the variation of properties in the upper hundreds of meters of the crust. To be more realistic, we should consider two aspects: the local more than a global model and a crustal model composed of 6 layers over a half space. To check the effect of the upper layering, we will show results for a medium composed of two layers over a half space. The first layer with the same

Table 2

Properties of the upper part of the Tenerife crustal model (top 315 m of the crust). ρ is density; λ and μ are the Lamé's parameters

Layer	Thickness (m)	ρ (10^3 kg m^{-3})	μ (10^{10} Pa)	λ (10^{10} Pa)
1	35	1.0	0.01	0.01
2	180	1.4	0.14	0.16
3	100	1.6	0.30	0.35

Table 3

Properties of media used for testing. See text for more details. ρ is density; λ and μ are the Lamé's parameters

Layer	Thickness (km)	ρ (10^3 kg m $^{-3}$)	μ (10^{10} Pa)	λ (10^{10} Pa)
(a)				
1	0.215	1.3	0.12	0.14
2	0.100	1.6	0.30	0.35
3	5.686	2.2	1.0	1.2
4	19.00	2.9	4.0	4.7
Half space	∞	3.3	6.4	8.3
(b)				
1	6.0	2.2	1.0	1.2
2	19	2.9	4.0	4.7
Half space	∞	3.3	6.4	8.3

properties as those of layer 3 in table 3(a). The second layer and the half space with the properties of layer 4 and half space of the model in Table 3(a), respectively. see Table 3(b).

We locate in both crustal models two kinds of intrusion. The first source is a *ce* source with a radius of 1 km and a pressure change of 100 MPa. The second one is a *pm* source of 1 MU ($= 10^{12}$ kg) of mass. In Figures 10 and 11 we compare vertical displacements and surface gravity changes obtained for these sources in media (a) and (b) located at a depth of 2 and 7 km, respectively.

Looking at Figure 10, we observe that, for shallow *ce* intrusions, the effect of considering one or the other crustal model can extend up to radial distances of a few km. The magnitude of the differences is measurable considering the accuracy of current geodetic techniques. For a *pm* source, the effects are close to the observation error level for vertical displacement and gravity changes, being in practice not detectable.

When the depth of the source increases, we see that for *ce* sources (pressure changes effects), Figure 11A, the magnitude of the differences decreases but is still measurable. For a *pm* source, Figure 11B, the differences have similar values as for the shallow case shown in Figure 10B. We must, however, consider that deformation and gravity changes are proportional to the mass of *pm* and intensity of *ce* sources.

7. Conclusions

We have presented an analysis of microtremors recorded using an array of stations in the eastern portion of Las Cañadas Caldera. The data we use originate from a previous experiment using an array of stations whose purpose was to record earthquakes. Fortunately, for calibration purposes, the array was made to operate periodically for about 60 s several times. The microtremor records thus obtained

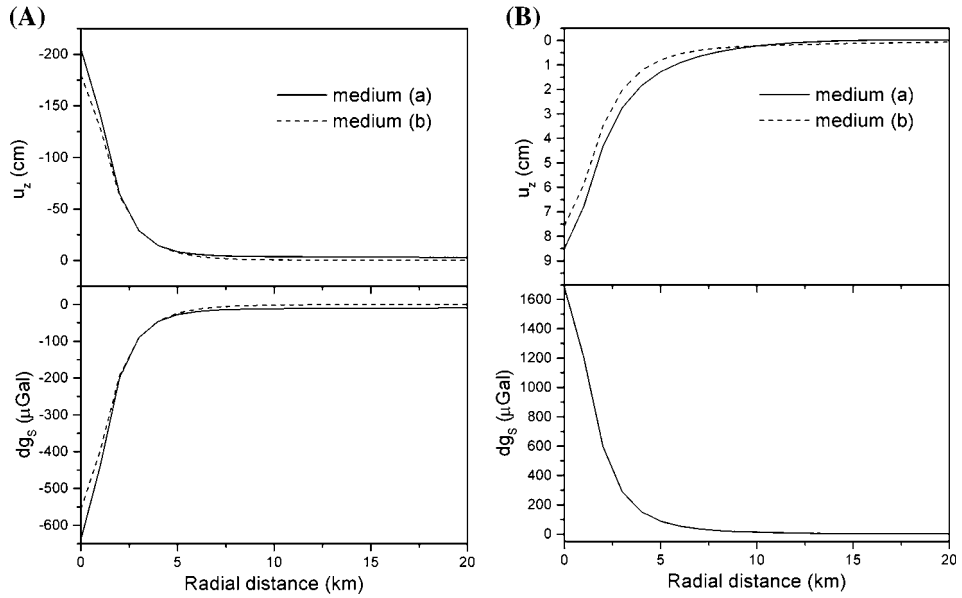


Figure 10

Vertical displacement and surface gravity change produced by **A** a *ce* source with a radius of 1 km and a pressure change of 100 MPa, and **B** a *pm* source of $1 \text{ MU} (= 10^{12} \text{ kg})$ located at a depth of 2 km in the media described in Table 3.

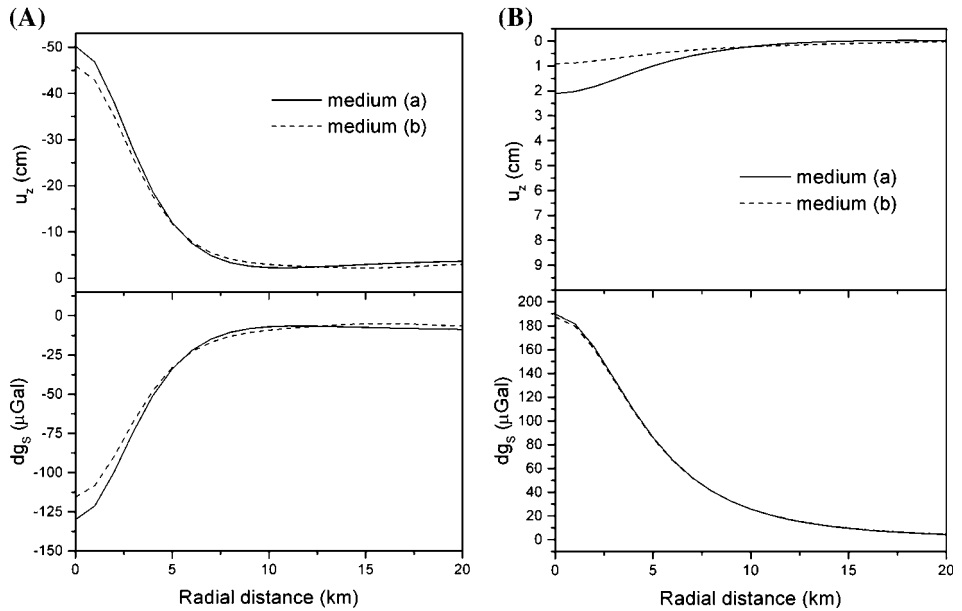


Figure 11

The same as Figure 10, but for sources located at 7 km depth.

were analyzed in this paper. We have taken advantage of a recent idea concerning the use of the SPAC method, in which the need for a particular geometry of the array is obviated. The correlation between pairs of stations, averaged for many different windows, substitutes the azimuthal average required in the original version of the SPAC method. The results show that, in the case of the microtremor records recorded around Teide, the substitution of the temporal average for the azimuthal average is valid, suggesting that the SPAC method is more general than assumed previously.

We have obtained a reliable estimate of the Vs profile for the eastern portion of the Teide Caldera. The result indicates a series of eruptive products about 315-m thick overlying a half space with Vs around 2 km/s. In spite of the expected heterogeneity of the subsoil structure, we observed no differences between arrays *AB* and *CD*, suggesting that the Vs structure is laterally homogeneous in this region on a scale of a few km. In fact, if the Vs structure were laterally heterogeneous, we would have been unable to determine phase velocity dispersion of Rayleigh waves, which requires a layered medium to propagate. The resulting Vs profile shows a good agreement with the expected thickness of volcanic sediments in the region of the arrays and is useful to model transient or permanent deformation near the volcanic edifice. Important conclusions have been derived from the computation of deformations and gravity changes due to a center of expansion (*ce*) plus a point mass (*pm*) in earth models with and without discrimination of the eruptive products existing in the upper 315 m of the Tenerife crustal model. Our results demonstrate that the consideration of the variation of properties in the upper hundreds of meters of the crust can be important in the very near field to distinguish between the properties of the intrusion, especially when there are important pressure changes attendant to the magma recharge or movement.

Acknowledgments

Gratitude is extended to A. García-Jerez for the careful reading of the manuscript and his constructive remarks, and to Dr. J. Almendros for his assistance with the data. The comments by two anonymous reviewers and D. D. Wolf enhanced the manuscript. This work was partially supported by CICYT, Spain, under grants CGL2005-05500-C02-02/RIES, by the EU with FEDER, and by the research teams RNM-194 of Junta de Andalucía, Spain. Signal processing benefited significantly from the availability of SAC (GOLDSTEIN *et al.*, 1998). We thank the Teide Natural Park authorities and all institutions and individuals that collaborated in the microtremor experiments. Data were collected in the frame of the European Project EVOV-CT93-0283 CEE. This research was carried out while one of us (FJCG) enjoyed a sabbatical leave from UNAM at University of Almería, Spain, and at Thessaloniki, Greece. This sabbatical leave benefited from the financial support of

DGAPA, UNAM. Research by JF has been supported by grants CGL2005-05500-C02-01/RIES, REN2002-03450 and CGL2004-21643E.

REFERENCES

- ABLAY, G. and HÜRLIMANN, M. (2000), *Evolution of the north flank of Tenerife by recurrent giant landslides*, J. Volcan. Geotherm. Res. 103, 135–159.
- ABLAY, G. and KEAREY, P. (2000), *Gravity constraints on the structure and volcanic evolution of Tenerife, Canary Islands*, J. Geophys. Res. 105, 5783–5796.
- ABLAY, G.J. and MARTÍ, J. (2000), *Stratigraphy, structure, and volcanic evolution of the Pico Teide-Pico Viejo formation, Tenerife, Canary Islands*, J. Volcan. Geotherm. Res. 103, 175–208.
- AKI, K. (1957), *Space and time spectra of stationary stochastic waves, with special reference to microtremors*, Bull. Earthq. Res. Inst. Tokyo University, 25, 415–457.
- ALMENDROS, J., IBÁÑEZ, J., ALGUACIL, G., MORALES, J., DEL PEZZO, E., LA ROCCA, M., ORTIZ, R., ARAÑA, V., and BLANCO, M.J. (2000), *A double seismic antenna experiment at Teide Volcano: Existence of local seismicity and lack of evidences of volcanic tremor*, J. Volcan. Geotherm. Res. 103, 439–462.
- ALMENDROS, J., LUZÓN, F., and POSADAS, A. (2004) *Microtremors analysis at Teide Volcano (Canary Islands, Spain): Assessment of natural frequencies of vibration using time-dependent horizontal-to-vertical spectral ratios*, Pure Appl. Geophys. 161, 1579–1596.
- ANCOCHEA, E., FÜSTER, J., IBARROLA, E., CENDRERO, A., COELLO, J., HERNÁN, F., CANTAGREL, J., and JAMOND, C. (1990), *Volcanic evolution of the Island of Tenerife (Canary Islands) in the light of new K-Ar data*, J. Volcan. Geotherm. Res. 44, 231–249.
- ANCOCHEA, E., HUERTAS, M.J., CANTAGREL, J.M., COELLO, J., FÜSTER, J.M., ARNAUD, N., and IBARROLA, E. (1999), *Evolution of the Cañadas edifice and its implications for the origin of the Cañadas Caldera (Tenerife, Canary Islands)*, J. Volcan. Geotherm. Res. 88, 177–199.
- ANGUITA, F. and HERNÁN, F. (2000), *The Canary Islands: A unifying model*, J. Volcan. Geotherm. Res. 103, 1–26.
- ARAÑA, V. and CARRACEDO, J.C., *Tenerife*, in *Los volcanes de las Islas Canarias* (ed. Rueda) (Madrid 1978).
- ARAÑA, V. and ORTIZ, R. (1991), *The Canary Islands: Tectonics, magmatism and geodynamic framework*. In *Magmatism in Extensional Structural Settings: The Phanerozoic African Plate*, (ed. Kampunzu, A., Lubala, R.) (Springer-Verlag 1991) pp. 209–249.
- ARAÑA, V., CAMACHO, A.G., GARCÍA, A., MONTESINOS, F.G., BLANCO, I., VIEIRA, R., and FELPETO, I. (2000), *Internal structure of Tenerife (Canary Islands) based on gravity, aeromagnetic, and volcanological data*, J. Volcan. Geotherm. Res. 103, 43–64.
- BARD, P.-Y. *Microtremor measurements: A tool for site effect estimation?* In *The Effects of Surface Geology on Seismic Motion* (eds. Irikura, Kudo, Okada and Sasatani) (Balkema, Rotterdam (1999) pp. 1251–1279.
- BARTEL, B.A., HAMBURGER, M.W., MERTERNS, C.M., and LOWRY, A.R. (2003), *Dynamics of active magmatic and hydrothermal systems at Taal volcano, Philippines, from continuous GPS measurements*. J. Geophys. Res. 108, 2475, doi: 10.1029/2002JB002194.
- BASKIR, E. and WELLER, C.E. (1975), *Sourceless reflection seismic exploration*, Geophysics 40, 1, 158–159.
- BLANCO, I. (1997), *Análisis e interpretación de las anomalías magnéticas de tres calderas volcánicas: Decepción (Shetland del Sur, Antártida), Furnas (San Miguel, Azores) y Las Cañadas del Teide (Tenerife, Canarias)* (Ph. D. Thesis, Universidad Complutense de Madrid).
- BOSSHARD, E. and MACFARLANE, D.J., (1970), *Crustal structure of the western Canary Islands from seismic refraction and gravity data*, J. Geophys. Res. 75, 4901–4918.
- BURGMANN, R., ROSEN, P.A., and FIELDING, E.J. (2000), *Synthetic aperture radar interferometry to measure Earth's surface topography and its deformation*. Annu. Rev. Earth Planet. Sci. 28, 169–209.
- CAMACHO, A.G., MONTESINOS, F. G., and VIEIRA, R., (1991), *Microgravimetric model of the Las Cañadas caldera (Tenerife)*, J. Volcan. Geotherm. Res. 47, 75–88.

- CAMACHO, A.G., MONTESINOS, F.G., and VIEIRA, R. (1996), *Gravimetric structure of the Teide Volcano environment*. In *Proceed. of the Second Workshop on European Lab. Volcanoes* (Santorini, Greece 1996) pp. 605–613.
- CANALES, J.P., DAÑOBEITIA, J.J., and WATTS, A.B. (2000), *Wide-angle seismic constraints on the internal structure of Tenerife, Canary Islands*, J. Volcan. Geotherm. Res. 103, 65–81.
- CARRACEDO, J.C., DAY, S., GUILLLOU, H., RODRÍGUEZ, E., CANAS, J.A., and PÉREZ, F.J. (1998), *Hotspot volcanism close to a passive continental margin*, Geol. Mag. 135, 591–604.
- CHÁVEZ-GARCÍA, F.J. and LUZÓN, F. (2005), *On the correlation of seismic microtremors*, J. Geophys. Res. 110, B11, B11313, doi: 10.1029/2005JB003671.
- CHÁVEZ-GARCÍA, F.J., RODRÍGUEZ, M. and STEPHENSON, W.R. (2005), *An alternative approach to the SPAC analysis of microtremors: Exploiting the stationarity of noise*, Bull. Seism. Soc. Am. 95, 277–293.
- CHOUET, B., DE LUCA, G., MILANA, G., DAWSON, P., MARTINI, M., and SCARPA R. (1998), *Shallow velocity structure of Stromboli Volcano, Italy, derived from small-aperture array measurements of Strombolian tremor*, Bull. Seismol. Soc. Am. 88, 653–666.
- COLE, S. (1995), *Passive seismic and drill-bit experiments using 2-D arrays*, Ph.D. Thesis, Stanford University.
- DAÑOBEITIA, J.J. and CANALES, J.P. (2000), *Magmatic underplating in the Canary Archipelago*, J. Volcan. Geotherm. Res. 103, 27–42.
- DEL PEZZO, E., LA ROCCA, M., and IBÁÑEZ, J. (1997), *Observations of high-frequency scattered waves using dense arrays at Teide Volcano*, Bull. Seismol. Soc. Am. 87, 1637–1647.
- DE LUCA, G., SCARPA, R., DELPEZZO, E., and SIMINI, M. (1997), *Shallow structure of Mt. Vesuvius Volcano, Italy, from seismic array analysis*, Geophys. Res. Lett. 24, 481–484.
- DIXON, T.H., MAO, A., BURSIK, M., HEFLIN, M., LANGBEIN, J., STEIN, R., and WEBB, F. (1997), *Continuous monitoring of surface deformation at Long Valley Caldera, California, with GPS*, J. Geophys. Res. 102, 12017–12034.
- D’SILVA, S. (1996), *Theoretical foundations of time-distance heliosismology*, The Astrophys. J. 469, 964–975.
- DUVALL Jr. T.L., JEFFERIES, S.M., HARVEY, J.W. and POMERANTZ, M.A. (1993), *Time-Distance Helioseismology* Nature 362, 430–432.
- DZIEWONSKI, A.M. and ANDERSON, D.L. (1981), *Preliminary reference Earth model*, Phys. Earth Planet. Inter. 25, 297–356.
- DZURISIN, D., (2003), *A comprehensive approach to monitoring volcano deformation as a window on the eruptive cycle*, Rev. Geophys. 41(1), 1001, doi: 10.1029/2001RG000107 (Correction: 2003, 41(2), 1009, doi: 10.1029/2003RG000134).
- FERNÁNDEZ, J. and RUNDLE, J.B. (1994a), *Gravity changes and deformation due to a magmatic intrusion in a two-layered crustal model*, J. Geophys. Res. 99, 2737–2746.
- FERNÁNDEZ, J. and RUNDLE, J.B. (1994b), *FORTRAN program to compute displacement, potential and gravity changes due to a magma intrusion in a multilayered Earth model*, Comput. Geosci. 20, 461–510.
- FERNÁNDEZ, J. and Díez, J.L., (1995), *Volcano monitoring design in Canary Islands by deformation model*, Cahiers du Centre Européen de Géodynamique et de Séismologie 8, 207–217.
- FERNÁNDEZ, J., RUNDLE, J.B., GRANELL, R.D.R., and YU, T.-T. (1997), *Programs to compute deformation due to a magma intrusion in elastic-gravitational layered Earth models*, Comput. Geosci. 23, 231–249.
- FERNÁNDEZ, J., CARRASCO, J.M., RUNDLE, J.B., and ARAÑA, V. (1999), *Geodetic methods for detecting volcanic unrest. A theoretical approach*, Bull. Volcanol. 60, 534–544.
- FERNÁNDEZ, J., TIAMPO, K.F., RUNDLE, J.B., and JENTZSCH, G., (2005), *On the interpretation of vertical gravity gradients produced by magmatic intrusions*, J. Geody. 39/5, 475–492. doi: 10.1016/j.jog.2005.04.005.
- FERNÁNDEZ, J., CHARCO, M., RUNDLE, J.B., and TIAMPO, K.F., (2006), *A revision of the FORTRAN codes GRAVW to compute deformation produced by a point magma intrusion in elastic-gravitational layered Earth models*, Comp. Geosci. 32/2, 275–281. doi: 10.1016/j.cageo.2005.06.015.
- FERRAZZINI, V., AKI, K., and CHOUET, B. (1991), *Characteristics of seismic waves composing Hawaiian volcanic tremor and gas-piston events observed by a near-source array*, J. Geophys. Res. 96, 6199–6209.
- GARCÍA, A., BLANCO, I., TORTA, M., and SOCÍAS, I. (1997), *High-resolution aeromagnetic survey of the Teide Volcano (Canary Islands): A preliminary analysis*, Annali di Geofisica 40, 329–340.

- GOLDSTEIN, P., D. DODGE, FIRPO, M., and STAN, R. (1998), *Electronic seismologist: what's new in sac2000? Enhanced processing and database access*, Seism. Res. Lett. 69, 202–205.
- HENSTRIDGE, D.J. (1979), *A signal processing method for circular arrays*, Geophysics, 44, 179–184.
- HERNÁNDEZ, P., PÉREZ, N., SALAZAR, J., SATO, M., NOTSU, K., and WAKITA, H. (2000), *Soil gas CO₂, CH₄, and H₂ distribution in and around Las Cañadas caldera, Tenerife, Canary Islands, Spain*, J. Volcan. Geotherm. Res. 103, 425–438.
- HERRMANN, R.B. (1987), *Computer programs in Seismology*, Saint Louis University, 7 vols.
- LERMO, J. and CHÁVEZ-GARCÍA, F.J. (1994), *Are microtremors useful in site response evaluation?* Bull. Seism. Soc. Am., 84, 1350–1364.
- LOBKIS, O. I. and WEAVER, R. L. (2001), *On the emergence of the Green's function in the correlations of a diffuse field*, J. Acoust. Soc. Am. 110, 3011–3017.
- MARTÍ, J., MITJAVILA, J. and ARAÑA, V. (1994), *Stratigraphy, structure and geochronology of the Cañadas Caldera*, Geolog. Magazine 131, 715–727.
- MARTÍ, J. and GUDMUNDSSON, A. (2000), *The Las Cañadas Caldera (Tenerife, Canary Islands): An overlapping collapse caldera generated by magma-chamber migration*, J. Volcan. Geotherm. Res. 103, 161–173.
- METAXIAN, J.-P. (1994), *Etude sismologique et gravimétrique d'un volcan actif: Dynamisme interne et structure de la Caldeira Masaya, Nicaragua*, Ph. D. Thesis, Université de Savoie, 319 pp, in French.
- MORGAN, W.J. (1983), *Hotspot tracks and the early rifting of the Atlantic*, Tectonophysics 94, 123–139.
- MORIKAWA, H., TOKI, K., SAWADA, S., AKAMATSU, J., MIYAKOSHI, K., EJIRI, J., and NAKAJIMA, D. *Detection of dispersion curves from microseisms observed at two sites*. In (K. Irikura, K. Kudo, H. Okada, and T. Sasatani, eds.), *The effects of Surface Geology on Seismic Motion*; Proc. of the 2nd. Intl. Symp. on the effects of surface geology on seismic motion, Yokohama, 1–3 December, (Rotterdam, Balkema 1998). 2, 719–724.
- NIKOLAEV, A.V. and TROITSKIY, P.A. (1987), *Lithospheric studies based on array analysis of P-coda and microseisms*, Tectonophysics, 140, 103–113.
- RICKETT, J. and CLAERBOUT, J. (1999), *Acoustic daylight imaging via spectral factorization: Helioseismology and reservoir monitoring*, Leading Edge, 18(8), 957–960.
- RICKETT, J.E. (2001), *Spectral Factorization of wavefields and wave operators*, Ph.D. Thesis, Stanford University.
- ROUX, P., KUPERMAN, W.A. and the NPAL group (2004), *Extracting coherent wavefronts from acoustic ambient noise in the ocean*, J. Acoust. Soc. Am. 116, 1995–2003.
- RUNDLE, J.B., (1980), *Static elastic–gravitational deformation of a layered half space by point couple sources*, J. Geophys. Res. 85, 5355–5363.
- RUNDLE, J.B. (1982), *Deformation, gravity and potential changes due to volcanic loading of the crust*, J. Geophys. Res. 87, 10729–10744 (Correction: 1983, 88, 10647–10652).
- SABRA, K.G., P. ROUX, and KUPERMAN W.A. (2005a), *Arrival-time structure of the time-averaged ambient noise cross-correlation function in an oceanic guide*, J. Acoust. Soc. Am. 117, 164–174.
- SABRA, K.G., GERSTOFT, P., ROUX, P., KUPERMAN, W.A., and Fehler, M.C. (2005b), *Extracting time-domain Greens function estimates from ambient seismic noise*, Geophys. Res. Lett. 32, L03310, doi:10.1029/2004GL021862.
- SÁNCHEZ-SESMA, F. J. and CAMPILLO, M. (2006), *Retrieval of the Green function from cross-correlation: The canonical elastic problem*, Bull. Seismol. Soc. Am. 96, 1182–1191.
- SEGALL, P., CERVELLI, P., OWEN, S., LISOWSKI, M., and MIKLIUS, A., (2001), *Constraints on dike propagation from continuous GPS measurements*. J. Geophys. Res. 106, 19301–19317.
- SIGURDSSON, H., HOUGHTON, B., McNUTT, S.R., RYMER, H., and STIX, J. (Eds.) *Encyclopedia of Volcanoes* (Academic Press, San Diego, CA, USA. 2000).
- STEIN, S., LANE, M., HAMBURGER, M., MEERTENS, C., DIXON, T., and OWEN, S., (2000), *UNAVCO Conference explores advance in volcanic geodesy*. EOS Trans. AGU 81, 121, 126.
- VIEIRA, R., TORO, C., and ARAÑA, V. (1986), *Microgravimetric survey in the caldera of Teide, Tenerife, Canary Islands*, Tectonophysics, 130, 249–257.
- WATTS, A.B., PEIRCE, C., COLLIER, J., DALWOOD, R., CANALES, J.P., and HENSTOCK, T.J. (1997), *A seismic study of lithospheric flexure in the vicinity of Tenerife, Canary Islands*, Earth Planet. Sci. Lett. 146, 431–447.

WEAVER, R.L. and LOBKIS, O.I. (2001), *Ultrasonics without a source: Thermal fluctuation correlations at MHz*, Phys. Rev. Lett. 87, 134301.

WEAVER, R.L. and LOBKIS, O.I. (2004), *Diffuse fields in open systems and the emergence of the Green's FUNCTION*, J. Acoust. Soc. Am. 116, 2731–2734.

(Received September 28, 2005, revised April 28, 2006, accepted May 15, 2006)

Published Online First: April 16, 2007

To access this journal online:
www.birkhauser.ch/pag

Steric Sea-level Variations Inferred from Combined Topex/Poseidon Altimetry and GRACE Gravimetry

D. GARCÍA,^{1,2} G. RAMILLIEN,¹ A. LOMBARD,¹ and A. CAZENAVE¹

Abstract—In this study, we propose to estimate the steric sea-level variations over a < 2 -year period (April 2002 through December 2003) by combining global mean sea level (GMSL) based on Topex/Poseidon (T/P) altimetry with time-variable geoid averaged over the oceans, as observed by the GRACE (Gravity Recovery and Climate Experiment) satellite. In effect, altimetry-derived GMSL changes results from two contributions: Steric (thermal plus salinity) effects due to sea water density change and ocean mass change due to water exchange with atmosphere and continents. On the other hand, GRACE data over the oceans provide the ocean mass change component only. The paper first discusses the corrections to apply to the GRACE data. Then the steric contribution to the GMSL is estimated using GRACE and T/P data. Comparison with available thermal expansion based on in situ hydrographic data is performed.

Key words: Global Ocean, Steric sea level, ocean mass change, GRACE time-variable gravity, TOPEX/poseidon altimetry, hydrographic data.

1. Introduction

Global mean sea-level (GMSL) variations results from two processes: (1) Steric height variations (hereafter denoted $\text{GMSL}_{\text{steric}}$) produced by dilatation/contraction of sea water via temperature and salinity change, and (2) variations of ocean mass due to water exchange with atmosphere, continental water reservoirs and land ice bodies. The ocean mass component is in the following denoted by $\text{GMSL}_{\text{ocean mass}}$. It results in: $\text{GMSL} = \text{GMSL}_{\text{steric}} + \text{GMSL}_{\text{ocean mass}}$.

Long-term sea-level rise measured by tide gauges over the past decades (e.g., CHURCH *et al.*, 2004; HOLGATE and WOODWORTH, 2004) and satellite altimetry since 1993 (e.g., NEREM and MITCHUM, 2001; LEULIETTE *et al.*, 2004) has been interpreted in terms of thermal expansion and ocean mass change using models and various observations (see CHURCH *et al.*, 2001 and CAZENAVE and NEREM, 2004, for reviews).

¹ LEGOS, Observatoire Midi-Pyrénées, Toulouse, France.

² On leave from Space Geodesy Laboratory, Applied Mathematics Department, EPS, University of Alicante, Alicante, Spain.

Other studies, focusing on the annual mean sea level (CHEN *et al.*, 1998; MINSTER *et al.*, 1999; CAZENAVE *et al.*, 2000; MILLY *et al.*, 2003), showed that the non-steric annual GMSL based on Topex/Poseidon (T/P) altimetry corrected for thermal expansion, is well explained by seasonal variations in atmospheric water vapor content and total land water storage (as estimated from outputs of global land surface models).

The newly launched GRACE (Gravity Recovery and Climate Experiment) mission, devoted to measure tiny variations of the Earth's gravity field (TAPLEY *et al.*, 2004a,b; WAHR *et al.*, 2004; SCHMIDT *et al.*, 2005), now allows direct estimates of land water storage change over continental areas as well as ocean mass change over oceanic areas. The latter application has recently been presented by CHAMBERS *et al.* (2004) who showed that the GRACE data averaged over the oceans closely follow the altimetry-derived GMSL after correcting for steric effects, thus representing the ocean mass component due to change in atmospheric water, land hydrology and land ice mass. In the present study, we will estimate over a short time span (April 2002–December 2003), the steric sea level by combining GRACE data over the oceans with T/P-derived GMSL.

2. Data and Processing

2.1. Topex/Poseidon Altimetry Data

GMSL is estimated from T/P altimetry data distributed by the AVISO (Archiving, Validation and Interpretation of Satellite Oceanographic) data centre. We consider the period from April 2002 to December 2003. Conventional geophysical and atmospheric corrections (tides, wet and dry tropospheric corrections, ionospheric correction, sea state bias as well as instrumental drifts and bias) are applied to the data, as recommended by AVISO. We also apply the inverted barometer (IB) correction to account for atmospheric loading on sea surface, using the time-variable mean surface pressure averaged over the oceanic domain (e.g., MINSTER *et al.*, 1999). The IB correction, at latitude φ , longitude λ and time t , is expressed as: $-0.9948 [p(\varphi, \lambda, t) - P(t)]$, where $p(\varphi, \lambda, t)$ is the instantaneous local surface pressure and $P(t)$ is the mean surface pressure spatially averaged over the entire oceanic domain. In this equation, the IB correction is in cm if p is in mbar. IB-corrected T/P sea-level data are interpolated onto $1^\circ \times 1^\circ$ regular grids at 10-day intervals (the duration of the T/P orbital cycle), then spatially averaged between latitudes 60°S and 60°N , using equi-area weighting, and further expressed as monthly means between April 2002 and December 2003. Errors associated with 10-day T/P GMSL values are estimated to be ~ 4 mm (NEREM and MITCHUM, 2001). For monthly values, we assume that the uncertainty is $4/\sqrt{3}$ mm, thus ~ 2.5 mm.

2.2. *Grace*

Launched in March 2002, the GRACE mission measures spatio-temporal gravity variations, providing direct monitoring of water mass change in the Earth system (TAPLEY *et al.*, 2004a,b; WAHR *et al.*, 2004). Here we focus on the ocean mass component that can be determined by considering the GRACE data over the oceanic domain. We use the Release 01 of GRACE L-2 products made available by the GRACE project (Level-2 Gravity Field Product User Handbook, S. BETTADPUR, 2004a). These consist of monthly solutions of geoid spherical harmonic coefficients, up to degree and order 120, from April 2002 through December 2003. Note that several monthly solutions are missing (June 2002, July 2002, December 2002, January 2003 and June 2003).

During the GRACE data processing, several processes are modelled and their effects removed (see BETTADPUR, 2004b; FLETCHNER, 2003). In addition to solid Earth and ocean tides, other processes include an atmospheric loading model based on ECMWF (European Centre for Medium-Range Weather Forecast) surface pressure data and high-frequency ocean mass variations computed with a mass-conserving barotropic ocean model driven by high-frequency winds and atmospheric pressure (see FLETCHNER, 2003). Since we are interested into the total ocean mass signal, we restored the barotropic ocean model removed during the GRACE data processing, as recommended in FLETCHNER (2003). As mentioned in CHAMBERS *et al.* (2004), the barotropic model is mass-conserving. Thus it cannot introduce a spurious signal into GRACE $\text{GMSL}_{\text{ocean mass}}$ as far as global average is concerned. However, we checked that missing the high-latitude ocean domain above 60°N/S , has a small, but non-negligible effect on the $\text{GMSL}_{\text{ocean mass}}$ curve. We also restored the atmospheric loading model. Thus, over the ocean, the total GRACE signal, which now includes both atmospheric and oceanic loads, accounts for the IB effect (i.e., locally, any atmospheric pressure anomaly is cancelled by a corresponding ocean mass anomaly). However, for comparison with the IB-corrected sea-level change based on T/P altimetry, we have to remove from the GRACE signal the $P(t)$ term (i.e., mean atmospheric surface pressure averaged over the whole oceanic domain). In effect, this term is removed from the altimetry data when applying the IB correction. This is different from the GRACE data: when averaged over the oceanic domain, the sum of the restored atmosphere plus ocean loads, while cancelling locally the ocean response to atmospheric mass anomalies, leads to a non-zero term, equal to $P(t)$. It has thus to be removed to the GRACE data, too.

GRACE data do not include degree 1 spherical harmonic coefficients. These terms represent the position of the Earth's center of mass in a terrestrial reference frame. To be consistent with the reference frame used for T/P, the degree 1 coefficients, which are estimated from the variations of the Earth's center of mass proposed by CHEN *et al.* (1999), are added to the GRACE data.

Figure 1

Global mean sea level (GMSL) over 2002–2004. (a) T/P GMSL. (b) GRACE GMSL_{ocean mass}. (c) (T/P GMSL–GRACE GMSL_{ocean mass}).

The degree 2, order 0 coefficient, representing the Earth's oblateness (COX and CHAO, 2002), is not well observed by GRACE (TAPLEY *et al.*, 2004a). Therefore, we use a better estimate of the degree 2, order 0 coefficient based on Satellite Laser Ranging observations (updated from COX and CHAO, 2002; COX, personal communication, 2005).

Assuming that the gravity changes observed by GRACE are produced by mass changes at the surface of the Earth, the GRACE monthly geoid data can be converted into surface mass variations and expressed by equivalent water height (WAHR *et al.*, 1998; CHAO, 2005; RAMILLIEN *et al.*, 2005). In order to minimize the leakage error due to land water mass signal over the continents, a Gaussian filter as described by SWENSON and WAHR (2002) was applied to the GRACE data over the 60°S–60°N ocean domain for each month from April 2002 through December 2003. We tested two Gaussian filter radii, 300 km and 1000 km. In principle, the smaller the radius, the smaller the leakage. However, because of noise affecting higher degree spherical harmonic coefficients of the current GRACE solutions (TAPLEY *et al.*, 2004a; WAHR *et al.*, 2004), the 1000 km radius was chosen, which by definition removes all signals beyond degree 30. Alternative approaches to process the GRACE data that aim at reducing the noise at high degrees, in order to improve spatial resolution, are currently explored by several groups. For example, ROWLANDS *et al.* (2005) reported an improvement by a factor of 3 in both temporal and spatial resolutions through a “mascon” analysis of GRACE data.

Errors associated with each spherical harmonics coefficient are released with the GRACE L-2 data. These errors are weighted similarly to the coefficients during the Gaussian filter processing. Then, the error associated to GRACE-derived GMSL_{ocean mass} estimate is computed as described in SWENSON and WAHR (2002), which is estimated around ~ 2 mm.

3. Result

3.1. Combining GRACE and T/P Altimetry Data

Figures 1a, 1b and 1c present the observed T/P GMSL, GRACE GMSL_{ocean mass} and the differences between the two time series (noted T/P GMSL–GRACE GMSL_{ocean mass}). These curves are based on a spatial averaging between 60°S and 60°N. The least-squares fitted annual amplitudes and phases of the three time series are presented in Table 1. The T/P GMSL has an annual amplitude of 6.1 ± 1.2 mm,

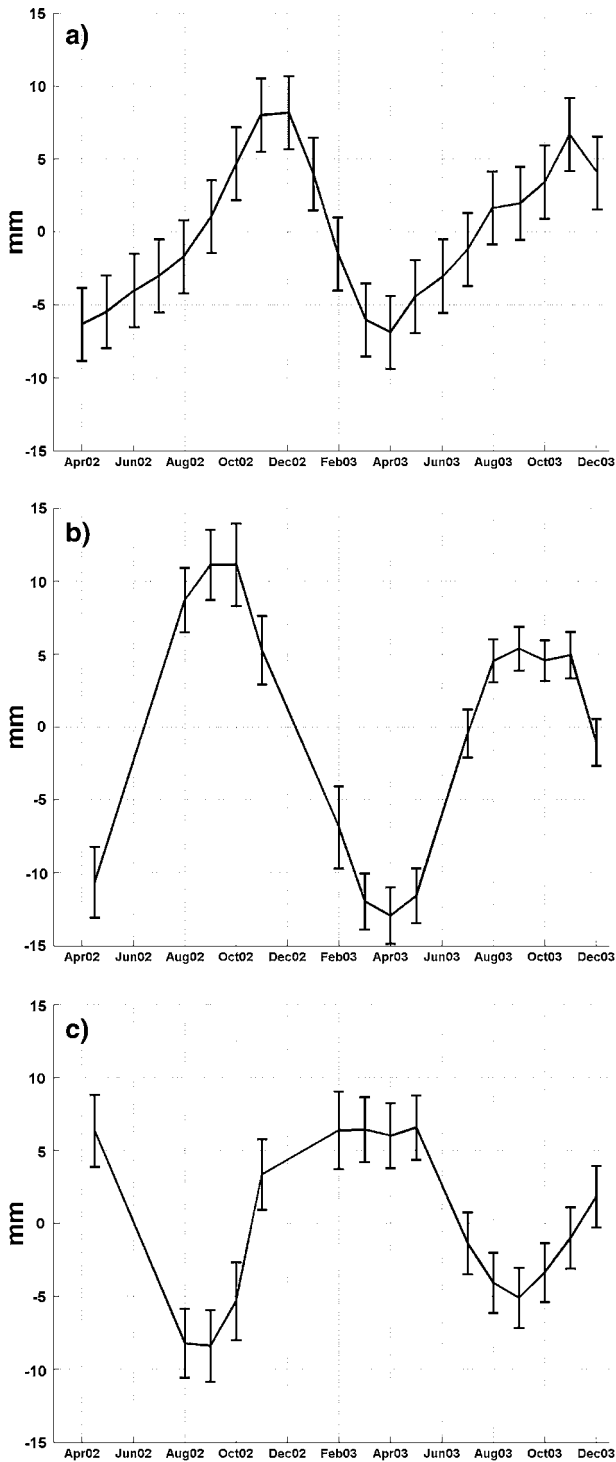


Table 1

Annual amplitudes and phases of the best fitting sinusoids to the time series shown in Figures 1 and 2. The phase φ is defined in degrees from 1 January using a $\cos(2\pi t/T - \varphi)$ definition ($2\pi/T$ is the annual frequency and t is time). Adjustments are performed over the April 2002–December 2003 time span

	Annual Amplitude (mm)	Annual Phase (°)
(T/P GMSL) ($\pm 60^\circ$)	6.1 ± 1.2	309.7 ± 11.3
GRACE GMSL _{ocean mass} ($\pm 60^\circ$)	10.9 ± 1.1	276.5 ± 7.8
(T/P GMSL – GRACE GMSL _{ocean mass}) ($\pm 60^\circ$)	6.4 ± 2.1	72.7 ± 22.3
GMSL _{steric} ($\pm 60^\circ$)	5.3 ± 1.0	101.4 ± 11.3

and a phase of $309.7^\circ \pm 11.3^\circ$ (maximum signal in early November). The GRACE GMSL_{ocean mass} has an annual amplitude of 10.9 ± 1.1 mm and a phase of $276.5^\circ \pm 7.8^\circ$ (maximum signal in early October). Note that the GRACE GMSL_{ocean mass} computed over the entire oceanic domain, which represents the total water budget exchanged between the ocean and the atmosphere/continents, has an annual amplitude of 10.0 ± 2.3 mm and a phase of $282.1^\circ \pm 16.3^\circ$ (maximum signal in mid-October). Accounting for the global ocean surface, this signal represents a water mass variation of $6.86 \cdot 10^{12} \text{ m}^3$ from peak-to-peak.

The annual amplitude of the difference curve is 6.4 ± 2.1 mm, with a phase of $72.7^\circ \pm 22.3^\circ$ (maximum signal on 15 March). In principle this difference curve represents the steric contribution to GMSL. The annual amplitude and phase of the residual curve compare well with published estimates of the steric annual sea level based on climatologies. For example, CHEN *et al.* (1998), using WOA94 objectively analyzed temperature fields (LEVITUS *et al.*, 1994), report an amplitude of 5.5 mm and a phase of 79° (corresponding to 20 March) for the annual steric sea-level cycle. MINSTER *et al.* (1999) and CAZENAVE *et al.* (2000), also using the LEVITUS *et al.* (1994) climatology, found an amplitude of 5 mm and a maximum signal on 12 March. Thus the result reported here by combining T/P GMSL and GRACE ocean mass appears to be consistent with values based on ocean data climatologies.

3.2. Steric Sea-Level Data

We have compared the residual (T/P GMSL – GRACE GMSL_{ocean mass}) time series with the steric sea level computed using the ISHII *et al.* (2006) global ocean temperature data. The latter data set consists of $1^\circ \times 1^\circ$ gridded temperature fields given as monthly means to 700 m, from 1945 to 2003 (see ISHII *et al.* 2003, 2006, for detailed descriptions). Unless using an ocean temperature climatology (i.e., based on data averaged over many years) as done in CHAMBERS *et al.* (2004), the ISHII *et al.* data base is the only one providing monthly mean temperatures suitable for the short time span of overlap with the GRACE data (April 2002

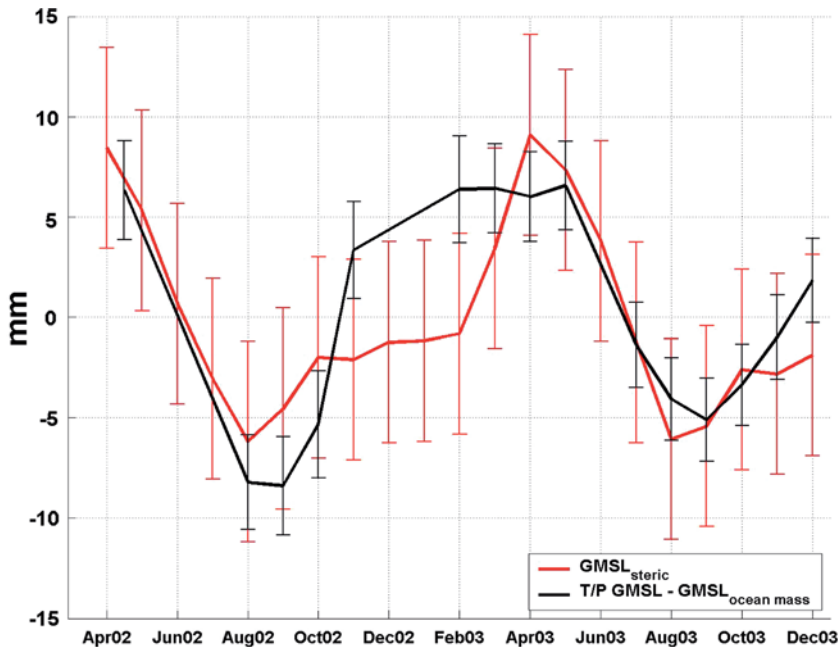


Figure 2

Red curve: Steric sea level based on ISHII *et al.* (2006) data; black curve: (T/P GMSL - GMSL_{ocean mass}).

through December 2003). Indeed other available global ocean data bases (e.g., WILLIS *et al.*, 2004; LEVITUS *et al.*, 2005) provide only yearly-mean temperatures. We have computed monthly steric sea-level maps by vertically integrating density anomalies based on the ISHII *et al.* (2006) temperature anomalies, through the equation of state of sea water (see LOMBARD *et al.*, 2005). The monthly steric sea-level data were further spatially averaged between 60°S and 60°N, using equi-area weighting. The resultant time series is denoted by GMSL_{steric} and is shown in Figure 2. Error bars of the monthly values are crudely estimated to be ± 5 mm, based on the errors provided in ISHII *et al.* (2006), for the annual means. The corresponding best-fitting annual sinusoid has an amplitude of 5.3 mm and a phase of 101.4° (maximum on 10 April). The amplitude agrees very well with that of the WOA94 climatology, while a 22° phase difference is noted. The latter difference is probably due to the difference in time span between the two different data sets.

On Figure 2 the (T/P GMSL – GRACE GMSL_{ocean mass}) curve is superimposed (i.e., Fig. 1c). We observe a good agreement between the two curves, within the error bars, for all studied months, with two exceptions: November 2002 and February 2003. The annual amplitudes of the two signals also show a very good agreement, with only

1 mm difference (see Table 1). Regarding the phases, we note a 30° difference, with the phase of (T/P GMSL – GRACE GMSL_{ocean mass}) closely coinciding with the phase of the steric sea level deduced from the WOA94 climatology. This result validates the applied methodology, which allows us to estimate steric sea-surface height variations independently from *in situ* hydrographic data.

4. Conclusion

In this study we combined GMSL data from T/P altimetry and GRACE data averaged over the same oceanic domain (60°N – 60°S) and compared the difference time series to the steric sea level estimated over the same time span (and same oceanic domain) from *in situ* hydrographic data. An overall good agreement between the two data sets has been recognized. However, the observed steric sea level and that deduced from T/P-GRACE do not exactly coincide. Several reasons may be invoked: (1) Errors of the different data sets, (2) deep ocean contribution, not accounted for in the thermal expansion curve (based on the upper 700 m ocean layer), and (3) effect of salinity. However, the annual cycle of thermal expansion mainly occurs in the upper ocean layers (0–500 m), so that unaccounted deep thermal expansion is unlikely to be large. The effect of salinity on the GMSL is negligible compared to thermal expansion, on interannual to decadal time scales (ANTONOV *et al.*, 2002; ISHII *et al.*, 2006). To check whether on an annual time scale, the salinity contribution to sea level is also negligible in terms of global mean, we have also estimated the GMSL_{steric} signal accounting for both temperature and salinity, using for the latter, the monthly salinity variations of a standard year based on the WOA01 climatology. In that case, a negligible difference of 0.2 mm and 1.2° was observed in the annual amplitude and phase, respectively. Besides, the salinity curve does not show any annual signal. Finally data errors, both in GRACE data and in the steric sea level, may be the dominant factors explaining the phase difference between the T/P-GRACE curve and the steric sea level. For the latter, poor sampling of the southern oceans may be the largest source of uncertainty. The fact that the phase of the T/P-GRACE curve agrees well with the mean steric sea level curves based on ocean temperature climatologies (which provide better coverage of the oceans over the long time spans of data averaging) suggests that this is indeed the case.

It is important to emphasize that the maximum of the ocean mass and steric annual GMSL cycles is shifted by about 150° – 180° (5–6 months). A similar behavior has also been observed in previous studies of the global ocean (CHEN *et al.*, 1998; MINSTER *et al.*, 1999; CAZENAVE *et al.*, 2000), as well as in local studies like that of the Mediterranean Sea (GARCIA *et al.*, 2006). This difference in phase means that the water budget of the ocean is at a minimum when the steric sea level, and thus the heat content in the ocean, is at a maximum. Depending on

the estimate, the maximum $\text{GMSL}_{\text{steric}}$ is reached between 72.7° (mid-March) and 101.4° (early-April), following the austral summer when the heat stored is at a maximum in the southern ocean. The annual cycle of the ocean mass variation results mainly from land water storage variations, plus a small contribution from the atmospheric water vapor (CHAMBERS *et al.*, 2004; CHEN *et al.*, 1998; MINSTER *et al.*, 1999; CAZENAVE *et al.*, 2000; MILLY *et al.*, 2003). These studies, based on comparisons between T/P GMSL (corrected for steric effects) and global land model outputs, showed that seasonal change in the snow pack is the main driver of the annual ocean mass variation. But in addition, NGO-DUC *et al.* (2005), showed that underground water stored in the major tropical basins (Amazon, Congo, etc.) in response to seasonal precipitation plays a non-negligible role.

The present study is limited to a very short time span of less than two years. However, when the GRACE time series extends in the future, this approach will provide new information on the global ocean heat content change, with important perspectives on a better estimate of the oceans' contribution to the net radiative imbalance of the Earth's climate system (LEVITUS *et al.*, 2005) and to sea level rise.

Acknowledgements

We are very grateful to S. Bettadpur and F. Fletchner from the GRACE project for clarifications regarding the GRACE data processing. We also thank D. Chambers, J.M. Lemoine and F. Lyard for helpful discussions pertaining to the GRACE and T/P inverted barometer correction. Finally we are grateful to C. Cox from GSFC/NASA for supplying J2 SLR time series. This study is part of David Garcia's Ph.D. dissertation.

REFERENCES

- ANTONOV, J. I., LEVITUS, S., and BOYER, T. P. (2002), *Steric sea level variations during 1957–1994: Importance of salinity*, J. Geophys. Res. 107 (C12), 8013, doi:10.1029/2001JC000964.
- AVISO/Altimetry (1996), *AVISO User Handbook for merged TOPEX/Poseidon Products*, AVI-NT-02-101, Edition 3.0. Available at www.aviso.oceanobs.com.
- BETTADPUR, S. (2004a), *GRACE 327-734, Level-2 Gravity Field Product User Handbook*, online available from http://podaac.jpl.nasa.gov/grace/documentation_test.html.
- BETTADPUR, S. (2004b), *GRACE 327-742, UTCSR Level-2 Processing Standards Document for Level-2 Product Release 0001*, online available from http://podaac.jpl.nasa.gov/grace/documentation_test.html.
- CAZENAVE, A., REMY, F., DOMINH, K., and Douville, H. (2000), *Global ocean mass variations, continental hydrology and the mass balance of Antarctica ice sheet at seasonal timescale*, Geophys. Res. Lett. 27(22), 3755–3758.
- CAZENAVE, A. and Nerem, R. S. (2004), *Present-day sea level change: Observations and causes*, Rev. Geophys. 42, RG3001.
- CHAMBERS, D. P., WAHR, J., and Nerem, R. S. (2004), *Preliminary observations of global ocean mass variations with GRACE*, Geophys. Res. Lett. 31, L13310.

- CHAO, B. F. (2005), *On inversion for mass distribution from global (time-variable) gravity field*, J. Geodynamics 39, 223–230, doi:10.1016/j.jog.2004.11.001.
- CHEN, J. L., WILSON, C. R., Chambers, D. P., Nerem, R. S., and Tapley, B. D. (1998), *Seasonal global water mass budget and mean sea level variations*, Geophys. Res. Lett. 32(19), 3555–3558.
- CHEN, J. L., WILSON, C. R., EANES, R. J., and NEREM, R. S. (1999), *Geophysical interpretation of the observed geocenter variations*, J. Geophys. Res. 104, B2, 2683–2690.
- CHURCH, J. A., GREGORY, J. M., HUYBRECHTS, P., KUHN, M., LAMBECK, K., NHUAN, M. T., QIN, D., and WOODWORTH, P. L. (2001), *Changes in sea level. Climate Change 2001: The Scientific Basis* (eds. Houghton and Ding), (Cambridge Univ. Press.).
- CHURCH, J. A., WHITE, N. J., COLEMAN, R., LAMBECK, K., and MITROVICA, J. (2004), *Estimates of the regional distribution of sea level rise over the 1950–2000 period*, J. climate 17, 2609–2625.
- COX, C. and CHAO, B. F. (2002), *Detection of a large-scale mass redistribution in the terrestrial system since 1998*, Science 297.
- FLECHTNER, F. (2003), *GRACE AOD1B Product description document*, online available at http://podaac.jpl.nasa.gov/grace/documentation_test.html.
- GARCIA, D., CHAO, B. F., DEL RIO, J., VIGO, I., and GARCIA-LAFUENTE, J. (2006), *On the steric and mass-induced contributions to the annual sea-level variations in the Mediterranean Sea*, J. Geophys. Res. in press.
- HOLGATE, S. J. and WOODWORTH, P. L. (2004), *Evidence for enhanced coastal sea-level rise during the 1990s*, Geophys. Res. Lett. 31, L07305.
- ISHII, M., KIMOTO, M., and KACHI, M. (2003), *Historical ocean subsurface temperature analysis with error estimates*, Monthly Weather Rev. 131, 51–73.
- ISHII, M., KIMOTO, M., SAKAMOTO, K., and IWASAKI, S. I. (2006), *Steric sea-level changes estimated from historical ocean subsurface temperature and salinity analyses*, J. Oceanog. 62(2), 155–170.
- LEULIETTE, E. W., NEREM, R. S., and MITCHUM, G. T. (2004), *Calibration of TOPEX/Poseidon and Jason altimeter data to construct a continuous record of mean sea level change*, Mar. Geod. 27, 79–94.
- LEVITUS, S., BURGETT, R., and BOYER, T. P. (1994), *World Ocean Atlas*, vol. 3, 4.
- LEVITUS, S., ANTONOV, J., and BOYER, T. (2005), *Warming of the world ocean, 1955–2003*, Geophys. Res. Lett. 32, L02604.
- LOMBARD, A., CAZENAVE, A., LE TRAON, P. Y., and ISHII, M. (2005a), *Contribution of thermal expansion to present-day sea level rise revisited*, Global Planet. Change 47, 1–16.
- MINSTER, J. F., CAZENAVE, A., SERAFINI, Y. V., MERCIER, F., GENNERO, M. C., and ROGEL, P. (1999), *Annual cycle in mean sea level from Topex-Poseidon and ERS-1: inference on the global hydrological cycle*, Global Planet. Change 20, 57–66.
- MILLY, P. C. D., CAZENAVE, A., and GENNERO, M. C. (2003), *Contribution of climate-driven change in continental water storage to recent sea-level rise*, Proc. Natl. Acad. Sci. U. S. A. 100, 13, 158–161.
- NEREM, R. S. and MITCHUM, G. T. *Sea level change*. In *Satellite Altimetry and Earth Sciences* (Fu and Cazenave, ed.) (Academic Press, San Diego, 2001).
- NGO-DUC, T., LAVAL, K., POLCHER, Y., and CAZENAVE, A. (2005), *Analyses of the contribution of continental water to sea level variations during the 1997–1998 ENSO event; Comparison between the AMIP simulations and the Topex/Poseidon satellite data*, J. Geophys. Res. 110, D09103, doi:10.1029/2004JDO04940.
- RAMILLIEN, G., FRAPPART, F., and CAZENAVE, A., (2005), *Change in land water storage from 2 years of GRACE satellite data*, Earth Planet. Sci. Lett. 235, 283–301.
- ROWLANDS, D. D., LUTHCKE, S. B., KLOSKO, S. M., LEMOINE, F. R., CHINN, D. S., MCCARTHY, J. J., COX, C. M., and ANDERSON, O. B. (2005), *Resolving mass flux at high spatial and temporal resolution using GRACE intersatellite measurements*, Geophys. Res. Lett. 32, L04310.
- SCHMIDT, R., FLECHTNER, F., REIGBER, Ch., SCHWINTZER, P., GUNTER, A., DOLL, P., RAMILLIEN, G., CAZENAVE, A., PETROVIC, S., JOCHMAN, H., and WUNSCH, J. (2005), *GRACE observations of changes in continental water storage*, Global Planet. Change, in press.
- SWENSON, S. and WAHR, J. (2002), *Methods for inferring regional surface-mass anomalies from Gravity Recovery and Climate Experiment (GRACE) measurements of time-variable gravity*, J. Geophys. Res. 107, B9, 2193, doi:10.1029/2001JB000576.

- TAPLEY, D. B., BETTADPUR, S., WATKINS, M., and REIGBER, C. (2004a), *The gravity recovery and climate experiment: Mission overview and early results*, *Geophys. Res. Lett.* *31*, L09607.
- TAPLEY, D. B., BETTADPUR, S., RIES, J. C., THOMPSON, P. F., and WATKINS, M. M. (2004b), *GRACE measurements of mass variability in the earth system*, *Science* 305.
- WAHR, J., MOLEENAR, M., and BRYAN, F. (1998), *Time variability of the Earth's gravity field: Hydrological and oceanic effects and their possible detection using GRACE*, *J. Geophys. Res.* *103*, B12, 30205–30229.
- WAHR, J., SWENSON, S., ZLOTNICKI, V., and VELICOGNA, I. (2004), *Time-variable gravity from GRACE: First results*, *Geophys. Res. Lett.* *31*, L11501.
- WILLIS, J. K., ROEMMICH, D., and CORNUELLE, B. (2004), *Interannual variability in upper ocean heat content, temperature, and thermosteric expansion on global scales*, *J. Geophys. Res.* *109*, C12036.

(Received November 23, 2005, revised July 5, 2006, accepted July 6, 2006)

To access this journal online:
www.birkhauser.ch/pag

Interpretation of 1992–1994 Gravity Changes around Mayon Volcano, Philippines, Using Point Sources

A. G. CAMACHO,¹ J. FERNÁNDEZ,¹ M. CHARCO,¹ K. F. TIAMPO,² and G. JENTZSCH³

Abstract—Significant gravity changes observed around the Mayon Volcano (Philippines) between 1992 and 1994 at 26 stations are interpreted in terms of an increase of mass and pressure changes at several point sources modelled using a fast inversion process. This inversion approach attempts to fit gravity and elevation changes by combining a random search for the positions of the sources and a linear least-squares fit for the incremental mass, pressure and possible common regional values for gravity or elevation changes. Some stabilizer terms are included in the misfit function. Models with one and two sources were tested against the observed changes at Mayon. Models with only one-source give a best fit for a shallow source with a positive mass increment, horizontally displaced far from the summit. The study using two sources gives a best fit that is similar to the one-source model, but in addition indicates anomalous behavior at stations in the SW. Neglecting the stations located southward from a local fracture, the best-fitting model suggests one central positive mass change source, which is likely to be an intrusion of about 0.5 MU with a depth of about 5 km beneath the volcano. Standard deviation for the residuals ranges from 7–8 μGal for one-source models to 6–7 μGal for models with two sources. Both of the cases are below the error value of 9.4 μGal estimated for the gravity data, so that it is not possible to discriminate between both possible interpretations without additional information.

Key words: Gravity changes, Mayon Volcano, inversion, point sources.

1. Introduction

Temporal gravity changes dg can provide insights into mass and/or density variations within the volcano edifice (e.g., RYMER and WILLIAMS-JONES, 2000; WILLIAMS-JONES and RYMER, 2002; GOTTMANN and RYMER, 2002; GOTTMANN *et al.*, 2003). Devesiculation processes combined with mass redistribution, dependent upon the internal structure of vents and reservoirs and the subsurface mechanical properties, give rise to changes in surface gravity and to deformation phenomena. Accurate gravity data for control networks are collected in a timely fashion and then significant temporal gravity changes can be detected. An inversion process,

¹ Instituto de Astronomía y Geodesia (CSIC-UCM), Facultad CC Matemáticas, 28040 Madrid, Spain.
E-mail: antonio_camacho@mat.ucm.es

² Dept. of Earth Sciences, University of Western Ontario, London, Ontario, Canada.

³ Institute for Geosciences, FSU Jena, Burgweg 11, D-07749, Jena, Germany.

constrained by general hypotheses, can lead to further conclusions about changes in density structure taking place in the sub-surface domain. The interpretative discussion of results aims to offer a better understanding of the mechanism of the volcano, a basic aspect in a crisis situation to help decision-making for civil protection agencies. For example, magma replenishment and withdrawal as well as dynamic changes in hydrothermal reservoirs have been deduced at caldera-type volcanoes undergoing periodic unrest (e.g., EGGERS, 1987; BONAFEDE and MAZZANTI, 1998).

Global Positioning System (GPS) data are usually collected simultaneously with the gravity data to allow for a joint interpretation of elevation changes and gravity changes. Accuracies of observed gravity changes are usually about 10 μGal (1 mGal = $10^{-5}\text{m}\cdot\text{s}^{-2}$) and an equivalent level of accuracy comes from altitude changes.

The source approach for the data fit is usually based on spherical, ellipsoidal or other symmetrical bodies. Faults, dikes, laminar or cylindrical structures are also sometimes employed. Moreover, mathematical deformation models in addition to the source structure must consider the elastic properties of the surrounding material to account for the deformation phenomena. The elastic-gravitational model by Rundle and coworkers (see e.g., RUNDLE, 1980, 1982; FERNÁNDEZ and RUNDLE, 1994a,b; FERNÁNDEZ *et al.*, 2005) allows for more realistic interpretations than purely elastic models. Furthermore, the gravitational effects produced by changes in the groundwater level or soil moisture must be taken into account in the fitting process.

Once a mathematical model is adopted, the model parameters (characteristics) for the source can be determined by means of an inverse approach. Usually, nonlinear formulations are involved requiring iterative or exploratory processes to reach optimal values satisfying an objective function as defined by the model structure, the adopted constraints and the statistical pattern of the parameters (see e.g., TARANTOLA, 1987).

Mayon Volcano is the most active volcano in the Philippines. It is located at 13.26 N, 123.69 E, (see Fig. 1) with a summit elevation of 2462 m. Mayon Volcano has the classic conical shape of a stratovolcano. Its symmetric cone was formed through alternate pyroclastic and lava flows. Lava varies from basalts through basaltic andesites into the andesitic region, with a pattern of three basaltic flows followed by six to ten andesitic flows (NEWHALL, 1979). According to AURELIO (2000), the activity of the Mayon Volcano may be partially controlled by the Legazpi Lineament, which runs NW-SE across Legazpi City. The San Vicente-Linao Fault is the most prominent regional structure close to Mayon. It's considered as part of the Legazpi Lineament identified by AURELIO *et al.* (1997).

Almost one million people live in the vicinity of Mayon Volcano, and Legazpi City is situated only 12 km away from the crater. Since 1616, it has erupted 47 times. The eruption in 1993 began unexpectedly with an explosion. The initial eruption lasted only 30 minutes, but it generated pyroclastic flows that killed 68 people and

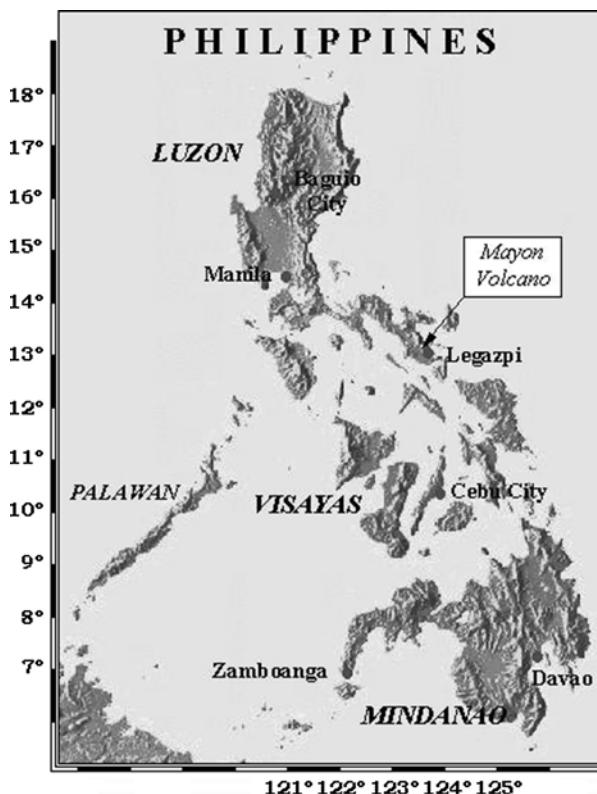


Figure 1

Location of Mayon stratovolcano, 13.3N, 123.7E, 12 km away from Legazpi City, Philippines. Elevation: 2,460 m.

prompted the evacuation of 60,000 others. The volcano erupted again in 2000 and 2001 (JENTZSCH *et al.*, 2004; TIAMPO *et al.*, 2004a).

A local network of 26 stations was installed around Mayon Volcano in 1992. From November 1992 until December 1996, five campaigns of repeated gravity and GPS observation were carried out (JENTZSCH *et al.*, 2001). Just after the start of gravity measurements in December 1992, the eruption in early 1993 occurred. The collected data for the interval 1992–1994 show significant gravity changes, which are analyzed here to allow insights into the Mayon Volcano mechanism.

2. Gravity and GPS Data

The local network is composed of 26 stations installed around Mayon Volcano, including two profiles on the slope (see Figs. 2 and 3), arriving at elevation 850 m

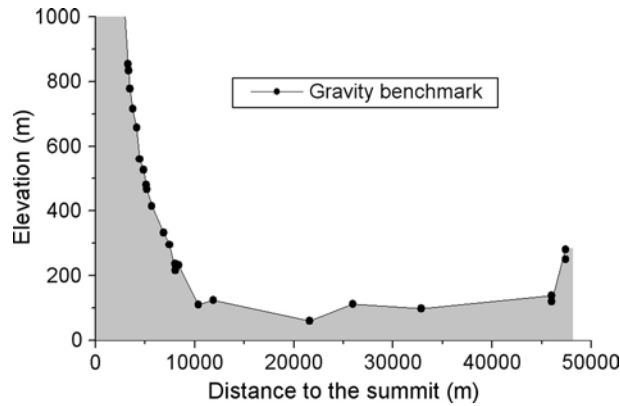


Figure 2
Altimetric radial profile of the gravity-GPS stations.

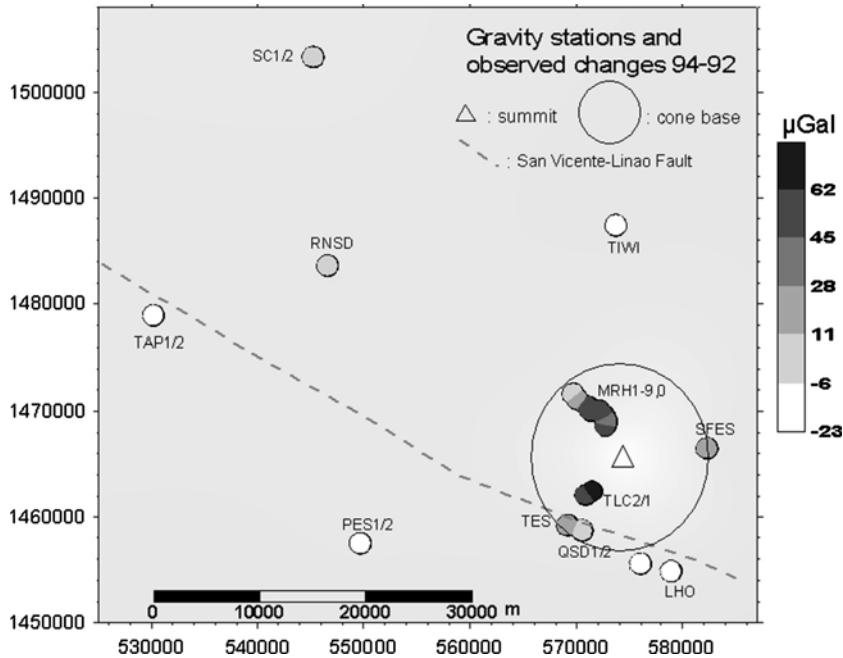


Figure 3
Planar location of the gravity stations around the volcano and gravity changes (μGal) dg 94-92. (UTM planar coordinates in meters).

towards the summit (JAHR *et al.*, 1995) and connected to a regional network around the volcano. Distances to the summit range from 3 km (highest point of the profiles) to 50 km for station SC1 north of the San Vicente-Linao Fault. The gravity points

are located in areas of sand (lahars), pasture ground and basaltic rocks. The elevation range is 794 m and the gravity range is 180 mGal.

The dates of the five campaigns are: November 1992, May 1993, May 1994, November 1994, and November 1996. The surveys were carried out with three LaCoste and Romberg gravimeters of the types G and D equipped with electrostatic feedback systems and electronic levels. They were calibrated several times at the gravimeter calibration lines Hannover-Harz Mountains in Germany. During each campaign the gravity differences were observed at least three times with each of the three gravimeters (minimum nine differences). The least-squares fit for the traverses gives an estimated mean standard deviation of the resulting gravity differences of about $\pm 12 \mu\text{Gal}$ (JENTZSCH *et al.*, 2001). In the observation lapse (1992–1996), gravity increased, mostly for stations close to the summit. But, despite the activity in February/March 1993, there was no significant gravity change between the first and the second campaign (November 1992, May 1993), while the gravity increase between May 1993 and November 1996 was almost 150 μGal ($\pm 14 \mu\text{Gal}$). Considerably smaller changes were registered far from the summit. Since it is unusual for gravity to increase with decreasing activity, a model was developed to explain this effect by mass redistribution (JENTZSCH *et al.*, 2001). But the following activity gave rise to the assumption that also additional mass could be the cause of these changes.

For the present work we will consider the gravity differences corresponding to the period November 1992–November 1994, which includes significant changes and offers a large dataset with the smallest standard deviations of the entire period 1992–1996. Table 1 shows the coordinates, gravity changes 1994–1992 and respective standard deviations for the 26 stations of the network as estimated from the traverse fits.

Height control was provided by nearly simultaneous GPS measurements (VÖLKSEN and SEEBER, 1995). It is noticeable that the differential GPS measurements carried out at the same locations with three parallel receivers revealed no significant elevation changes during this time, within their accuracy of $\pm 4 \text{ cm}$. As a result, the observed gravity increase, unaccompanied by significant changes in elevation, cannot be explained with the classical Mogi model (e.g., JAHR *et al.*, 1995).

3. Gravity Data Fit and Inversion Approach

Originally, the data were inverted upon the assumption of an extended source located some distance below the crater. JAHR *et al.* (1998) and JENTZSCH *et al.* (2001) suggested that the gravity changes can be explained in terms of density changes within a vent system. This model is based on a redistribution of the mass via the migration of magma down the vent system and can explain about 50% of the magnitude and distribution of the observed gravity changes. FERNÁNDEZ *et al.* (2001) and TIAMPO *et al.* (2004a) used a genetic algorithm for an inversion based on an

Table 1

Coordinates (latitude, longitude and elevation), observed gravity changes between 1992 and 1994 and corresponding estimated standard deviations for the control stations

Name	Latitude N (deg)	Longitude E (deg)	Elevation (m)	dg 94–92 (μGal)	St.dv. (μGal)
MRH1	13.310983	123.643219	295.04	6.3	7.1
MRH2	13.306520	123.646429	332.82	23.6	10.2
MRH3	13.300150	123.655841	414.89	61.2	10.5
MRH4	13.296547	123.658367	467.10	55.0	11.1
MRH5	13.296941	123.664896	527.01	51.2	11.7
MRH6	13.292311	123.669681	657.53	49.4	12.2
MRH7	13.289117	123.671577	715.90	38.7	12.5
MRH8	13.286501	123.671983	777.78	41.4	12.7
MRH9	13.283851	123.670849	854.78	54.1	12.7
MRH0	13.284927	123.671112	833.79	36.3	13.8
LHO	13.159138	123.728385	124.08	−15.7	7.9
CAG0	13.165515	123.701006	110.16	−12.2	7.2
QSD1	13.193788	123.650666	237.38	3.4	9.8
QSD2	13.193790	123.650700	234.00	8.1	8.9
TES0	13.198571	123.638106	232.80	16.0	9.7
SFES	13.264276	123.760044	215.97	19.0	7.6
TIWI	13.453205	123.680344	60.55	−20.3	8.8
TLC1	13.227806	123.658997	559.69	68.6	11.8
TLC2	13.224463	123.653490	480.02	53.6	10.9
RNSD	13.419566	123.430811	97.51	4.1	5.7
PES1	13.183580	123.458700	112.00	−21.4	8.7
PES2	13.183575	123.458645	112.44	−22.5	8.7
TAP1	13.377500	123.278800	138.01	−17.7	8.4
TAP2	13.377500	123.278800	120.00	−6.8	8.6
SC1	13.597812	123.418533	279.58	.0	1.4
SC2	13.597800	123.418500	250.00	−1.2	5.7

elastic-gravitational model, which introduces a long-wavelength effect and can combine the gravity and deformation fields.

JENTZSCH *et al.* (2001) considered the effects of changes in the groundwater level or soil moisture for tropical regions. For instance, for a porosity of 50%, a 50% change in the filling of the pore space produces a maximum attraction effect of 42 μGal . Significant effects could only be observed at 3 stations, mostly for gravity campaigns corresponding to different seasons of the year. These three stations showed significant variations, although being outside the observed spectrum of the changes due to volcanic activities.

Usually, the reduced quantity and quality of the geodetic data in active volcanic regions do not allow for the proper quantification of the constitutive parameters for sophisticated source models (extended sources) unless additional information can be provided. In this work, to model the observed gravity and elevation changes, we use simple spherical point sources, considered as a combination of pressure and mass changes, in an elastic medium. One or more sources may be adjusted for 26 gravity

values using a system of equations with few unknowns. Moreover, some parameters for long wavelength or common offset values are also included. This rather simple approach will produce an overdetermined fit with valuable results.

The observed elevation changes uz_i ($i = 1, \dots, n$) at n stations (x_i, y_i, z_i) are related by the equation system given by:

$$\begin{cases} dg_i = dg_i^c + v_i \\ uz_i = uz_i^c + w_i \\ (i = 1, \dots, n) \end{cases} \quad (1)$$

where dg_i^c and uz_i^c are calculated values for gravity and elevation changes for the model and v_i w_i are residual values.

In a simple form, we calculate the changes produced by m spherical buried sources located at (X_j, Y_j, Z_j) ($j = 1, \dots, m$), and with respective anomalous mass, radius and pressure, M_j, R_j and P_j , according to a Mogi model (MOGI, 1958) and Boussinesq's problem solution (e.g., DAVIS and SELVADURAI, 1996). Moreover, we suppose a possible common term uz_0 for the vertical elevations (it can represent for instance an unknown change of the levelling origin), a constant term dg_0 (a simple form to represent a possible long wavelength component or a common groundwater effect) plus a term $dgz \cdot (z_i - \bar{z})$ proportional to relative elevation ($z_i - \bar{z}$) with respect to a mean value $\bar{z}$ (simplest form for representing the effect of a horizontal layer) for the gravity changes. Then, we adopt the following expressions for the modelled changes:

$$uz_i^c = - \sum_{j=1}^m \frac{g_0}{4\pi\mu} \frac{1}{d_j} \left(2 \cdot (1 - \sigma) + \frac{(z_i - Z_j)^2}{d_j^2} \right) M_j + \sum_{j=1}^m \frac{1 - \sigma}{\mu} \frac{z_i - Z_j}{d_j^3} P_j R_j^3 + uz_0 \quad (2)$$

$$dg_i^c = G \sum_{j=1}^m M_j \frac{z_i - Z_j}{d_j^3} + dg_0 + dgz \cdot (z_i - \bar{z}) + D \cdot uz_i^c \quad (3)$$

where

$$d_j = ((x_i - X_j)^2 + (y_i - Y_j)^2 + (z_i - Z_j)^2)^{1/2}$$

and D is the coefficient for the free-air correction, $D \cong -3.08 \mu\text{Gal/cm}$, corresponding to modelled vertical displacements uz_i^c of the observing stations. G is the gravitational constant, μ and σ are shear modulus and Poisson ratio, respectively, and g_0 is the mean surface gravity value.

Parameters P_j and R_j appear combined as $P_j R_j^3$ (strength of the source). Assumptions for the density of magmatic bodies, combined with the estimated values of $P_j R_j^3$ and M_j , allow us to assign individual values to P_j and R_j .

If we estimate a standard Gaussian uncertainty for the gravity and elevation data, dg_i and uz_i , a minimization condition

$$f = \min \quad (4)$$

can be established using a misfit function f given by:

$$f = f_1 + f_2 + f_3 + f_4 = \sum_{i=1}^n \frac{v_i v_i}{\sigma_i^2} + \alpha \sum_{i=1}^n \frac{w_i w_i}{\theta_i^2} + \beta \sum_{j=1}^m \frac{|M_j|}{(\bar{z} - Z_j)^2} + \gamma \sum_{j=1}^m \frac{|P_j R_j^3|}{(\bar{z} - Z_j)^2}, \quad (5)$$

where σ_i^2 ($i = 1, \dots, n$) are variances for the gravity data values, as estimated in the previous least-squares adjustment of traverses for the gravity network, and where θ_i^2 ($i = 1, \dots, n$) are variances for the elevation changes, as estimated in the GPS or levelling adjustment. $\alpha > 0$ is a positive factor for optional balance between the gravity fit and the elevation fit, usually $\alpha \cong 1$. The terms f_3 and f_4 in (5) are stabilization terms (e.g., FARQUHARSON and OLDENBOURG, 1998; BERTETE-AGUIRRE *et al.*, 2002) introduced as controls on the total incremental mass and the total pressure for the m bodies (weighted according to the depths of the sources). These terms are necessary to prevent unreasonably large values of mass and/or pressure resulting from a rather poorly determined model (for instance due to coupling of positive and negative sources, aligned stations, peripheral sources, etc.). Factors $\beta, \gamma > 0$ play the role of homogenization units and the role of suitable balance factors between data fit (minimum residual values) and model magnitude (minimum absolute incremental mass and/or pressure), similar to Marquardt factors (e.g., BOULANGER and CHOUTEAU, 2001). Values for α, β, γ were selected before fitting according to external criteria: α according to additional information about external relative quality, different from the internal precision, of the gravity and elevation data, and β, γ according to an estimation of possible maximum magnitudes of mass and pressure (values for β and γ can be also introduced *ad hoc* just to avoid instability problems in the solution).

The model system, presented by equations (1)–(3), must be satisfied within the minimization constraints, (4) and (5). It constitutes a nonlinear problem of optimization. Iterative or explorative methods are commonly used for this purpose (e.g., TARANTOLA, 1987; SAMBRIDGE and MOSEGAARD, 2002). Iterative methods (gradient methods) are supported by a large tradition, however they require a good initial solution. More versatile, the explorative methods can be advantageously applied when the model space to explore is limited (small number and range of the parameters). In this case, random explorative processes (e.g., genetic algorithms or simulated annealing) are mostly adopted. Taking into account the very small size of our problem (26 values of observed gravity changes and six unknown parameters to explore per source) we select a direct hard random exploration of the model space.

We observe that the dependence on parameters $dg_0, dgz, P_j R_j^3$ and M_j ($j = 1, \dots, m$) is linear in equations (1)–(3). Then, they can be partially derived by standard linear fit. In contrast to this, the dependence on X_j, Y_j, Z_j ($j = 1, \dots, m$) (six parameters for $m = 2$) is nonlinear. They can be determined by trying values randomly selected

across the subsurface volume $X \in (x_{\min}, x_{\max})$, $Y \in (y_{\min}, y_{\max})$, $Z \in (z_{\min}, z_{\max})$ in the form:

$$\begin{aligned} X_j &= x_{\min} + \text{ran1} \cdot (x_{\max} - x_{\min}), \\ Y_j &= y_{\min} + \text{ran2} \cdot (y_{\max} - y_{\min}), \\ Z_j &= z_{\min} + \text{ran3} \cdot (z_{\max} - z_{\min}), \end{aligned} \quad (6)$$

where *ran1*, *ran2* and *ran3* are arbitrary numbers randomly selected in the interval (0,1) (different for each source j). Therefore, once the former X_j , Y_j , Z_j ($j = 1, \dots, m$) have been selected and substituted into (1)–(3), the resulting linear system can be solved by a least-squares fit for parameters $\mathbf{x} \equiv (dg_0, dgz, P_j R_j^3, M_j; j = 1, \dots, m)$ and the minimization condition $f = \min$. Positive and/or negative masses or pressure changes can arise in the adjustment. Next, the resulting values v_i , w_i , $P_j R_j^3$ and M_j are substituted in (5) in order to determine the misfit value f corresponding to this particular solution.

The process is repeated sufficiently to ensure a good resolution in the subsurface volume. We chose an optimal solution corresponding to the minimum value of the misfit function f . A secondary exploration is conducted in the close neighborhood of the previous optimal solution to look for a more precise definition of its values. This final solution (with minimum f value) is denoted as $\hat{\mathbf{x}} = (d\hat{g}_0, d\hat{g}z, d\hat{z}_0, \hat{X}_j, \hat{Y}_j, \hat{Z}_j, \hat{M}_j, (P_j \hat{R}_j^3); j = 1, \dots, m)$ and determines the model that fits the observed data best.

As a further step, a linear approximation $\mathbf{A}\mathbf{x} - \mathbf{t} = \mathbf{v}$ of (1)–(3) is used for the final locations $\hat{X}_j$, $\hat{Y}_j$, $\hat{Z}_j$ ($j = 1, \dots, m$), with $\mathbf{v} = (v_i, w_i; i = 1, \dots, n)$ as the residual vector and $\mathbf{x} \equiv (dg_0, dgz, P_j R_j^3, M_j; j = 1, \dots, m)$ as unknown parameters. Then, we can evaluate the covariance matrix of the final solution $\hat{\mathbf{x}}$ as $\mathbf{Q}_{\hat{\mathbf{x}}\hat{\mathbf{x}}} = (\mathbf{A}^T \mathbf{P} \mathbf{A})^{-1}$, with $\mathbf{P} = \text{diag}(\frac{1}{\sigma_i^2}, \frac{1}{\sigma_i^2}; i = 1, \dots, n)$ the diagonal matrix of weights.

By means of simulation at the benchmark points of Mayon, we tested the effectiveness of the inversion method with good results. For exactly simulated gravity changes, the agreement between the inversion model and the simulated one for position and mass is approximately 99%. Nevertheless, the presence of observational noise has a predicted smoothing effect, giving rise to smaller adjusted mass and shallower adjusted positions. The reduction depends on the ratio noise level/data level. Let us note finally that gravity data were reduced using the free-air effect corresponding to the adjusted rather than the observed elevation changes.

4. Results and Discussion

We apply the process described in Section 3 to the displacement and gravity changes observed around Mayon Volcano between November 1992 and November 1994 (N94–N92). As pointed out before, significant elevation changes observed are within ± 4 cm. Thus, the problem primarily considers gravity effects due to

anomalous mass bodies, with some contribution due to the elastic deformation. Table 1 shows the observed gravity changes N94–N92 considered for the source modelling. The *a priori* mean standard deviation for the gravity data in Table 1 is 9.4 μGal , but a more realistic value of 12 μGal is suggested by JENTZSCH *et al.* (2001).

Apart from considering one or several sources (positive and/or negative), a uniform change for seasonal variations and the effect of a laminar body can be tested. Thus, for a more efficient approach, we apply the inversion process for eight different models corresponding to eight possible combinations of model parameters P_j , R_j^3 , M_j , dg_0 , dgz and X_j , Y_j , Z_j . To start, we consider j equal to 1 or 2. Table 2 gives some representative results for possible models.

We note again that adjusted elevation changes are used for the free-air reduction. Using the observed elevation changes, the fit of the model would deteriorate by about 30%. This is due to the poor quality of the GPS data compared to the gravity data.

Model A (where we only consider dg_0 and dgz) results from direct linear least-squares adjustment without exploration and gives a pattern not completely satisfying the data (with a root-mean-square (r.m.s.) residual equal to 10.3 μGal , greater than the error of the measurements). Models B (one source $\hat{M}_1$), C (dg_0 and one source $\hat{M}_1$) and D (dg_0 , dgz and one source M_1) correspond to one-source configurations. These three models suggest a positive mass source located 5 km west of the summit, with depths of 1.8, 1.9 and 2.4 km, respectively. Figure 4 shows a sketch of the source location. These models display similar source characteristics and a good fit (residuals of about 8 and 7 μGal , within the data accuracy). Figure 5 shows these profiles of data fitting. They suggest an increasing mass about 0.3 ± 0.2 MU (1 MU = 10^{12} kg) at 2 km depth under the western limit of the volcanic cone. Let us note that the estimated standard deviations for the solution (about ± 2 km for x , ± 0.3 km for y and ± 2 km for z) provide some confidence in the north component, although a rather poor determination of the depth and east component. The north component is nearly coincident with that of the summit, however the east component suggests an unplausible westward deviation. The different accuracies of the three components of the source result from the relative geometrical configuration of the observation points and the source. If we consider the estimated errors for location, these results are rather similar to those obtained by FERNÁNDEZ *et al.* (2001). TIAMPO *et al.* (2004a,b) and TIEDE *et al.* (2005) developed a different kind of sensitivity analysis for the source fit based on the simulated effects of changes in the source parameters. In their sensitivity analysis, they noted that, for a source of relatively small mass, while the pressure and radius are the parameters most difficult to determine, the source depth is difficult to determine below the real depth, resulting in a tendency to overestimate the depth.

Models E and F correspond to attempts to find a fit with two sources (see E (b) in Fig. 4). We observe that the data do not allow a clear conclusion about a double-source configuration. In fact, we can obtain several double-source models (with positive and/or negative mass sources) with very different characteristics and a

Table 2

Some results for the fit of seven possible models (*A, B,...*) using different combinations of unknown parameters. *N*^o is the number of stations considered, *x*₁ and *y*₁ correspond to the horizontal position of the main source, *z*₁ is the depth below the sea level, *M*₁ the mass (1 MU = 10¹² kg), *z*₂ and *M*₂ refer to the second source, and the last column (r.m.s.) gives the root-mean-square residual of the fit (1 μGal = 10⁻⁸ m s⁻²). 'W' means a source located far west of the summit, 'C' means a nearly centered source under the summit

Mod.	N ^o	<i>d</i> <i>q</i> ₀ (μGal)	<i>d</i> <i>q</i> _z (μGal/km)	<i>x</i> ₁	<i>y</i> ₁	<i>z</i> ₁ (m b.s.l.)	<i>M</i> ₁ (MU)	<i>z</i> ₂ (mb.s.l.)	<i>M</i> ₂ (MU)	r.m.s. resid. (μGal)
A	26	12 ± 2	104 ± 13	—	—	—	—	—	—	10.3
B	26	—	—	W ± 2134	± 320	1773 ± 3090	+ .330 ± .270	—	—	8.3
C	26	-3 ± 2	—	W ± 1915	± 293	1879 ± 2848	+ .356 ± .247	—	—	7.9
D	26	+ 3 ± 3	47 ± 20	W ± 2689	± 464	2393 ± 3700	+ .290 ± .220	—	—	7.2
E	26	—	—	C ± 1305	± 1228	7045 ± 3140	+ .935 ± 1.022	13393	-2.88	6.1
F	26	-29 ± 3	—	C ± 877	± 447	6992 ± 1131	1.061 ± .311	11447	1.48	6.5
G	19	—	—	C ± 1731	± 645	5309 ± 1612	+ .469 ± .166	—	—	6.5
H	19	-2 ± 2	—	C ± 1558	± 653	5714 ± 1698	+ .538 ± ±.188	6.4	—	6.5

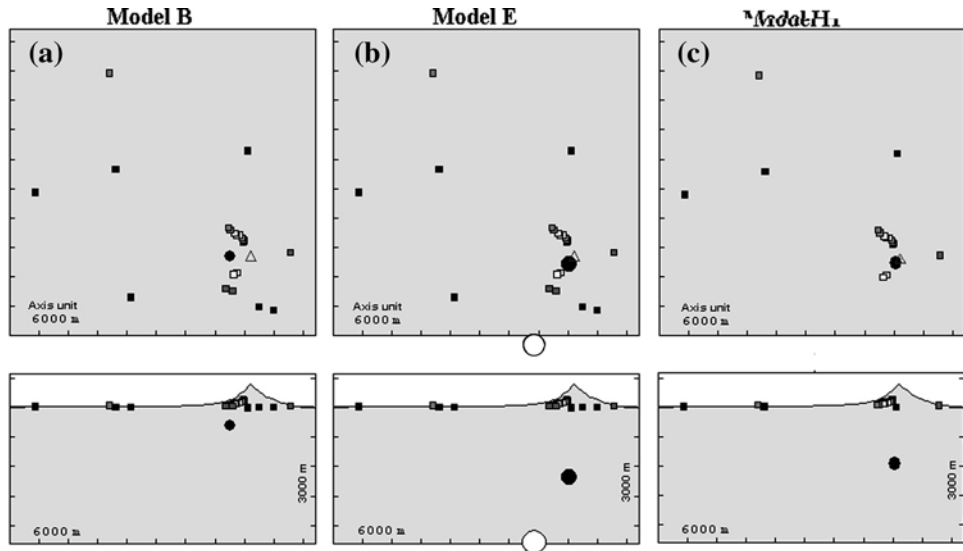


Figure 4

Sketch of the location of the stations (squares), the summit (triangle) and the sources (black circle for positive mass and white, see bottom in part b, for negative mass) resulting by fitting models B (a), E (b) and H (c) from the observed data (26 points for B and E, and 19 for H). Top with plan views and bottom with side views. In part (c) the SW stations are not included. (a) gives a horizontally displaced position of the main source, (b) and (c) give centered positions under the summit.

similar fit (residuals of about $6 \mu\text{Gal}$) (see Fig. 5). TIAMPO *et al.* (2004a) also considered a two-source model, but the inferred sources were at depth of 1 km and 200 m. These shallow sources, hypothesized to be magmatic and hydrothermal respectively, resolve the local gravity changes on different wavelengths.

The data accuracy level ($12 \mu\text{Gal}$ according to JENTZSCH *et al.*, 2001) does not allow us to distinguish between the models. Nevertheless, interesting conclusions can be drawn from the source models. The two-source configurations, as expected, are composed of a main body rather similar to that of the single-source models (now larger, deeper and more centered) and a secondary deep and far body of variable location and characteristics (positive or negative). We observe that, by including a secondary body, the main body now adopts a nearly central position. Considering the unstable results obtained for this secondary body, it appears to be potentially fictitious, although it plays a role in presenting the distinct gravity pattern for the southwestern stations. In fact, Figure 6 shows that, with this secondary body, the modelled gravity displays a pattern for the stations located northward from the San Vicente-Linao Fault that is rather different from the pattern for the southern stations. Therefore, we can conclude that this two-source model suggests the existence of a significant discontinuity effect due to the San Vicente-Linao Fault (LAGMAY *et al.*, 2005) on the deformation pattern determined by the volcanic activity

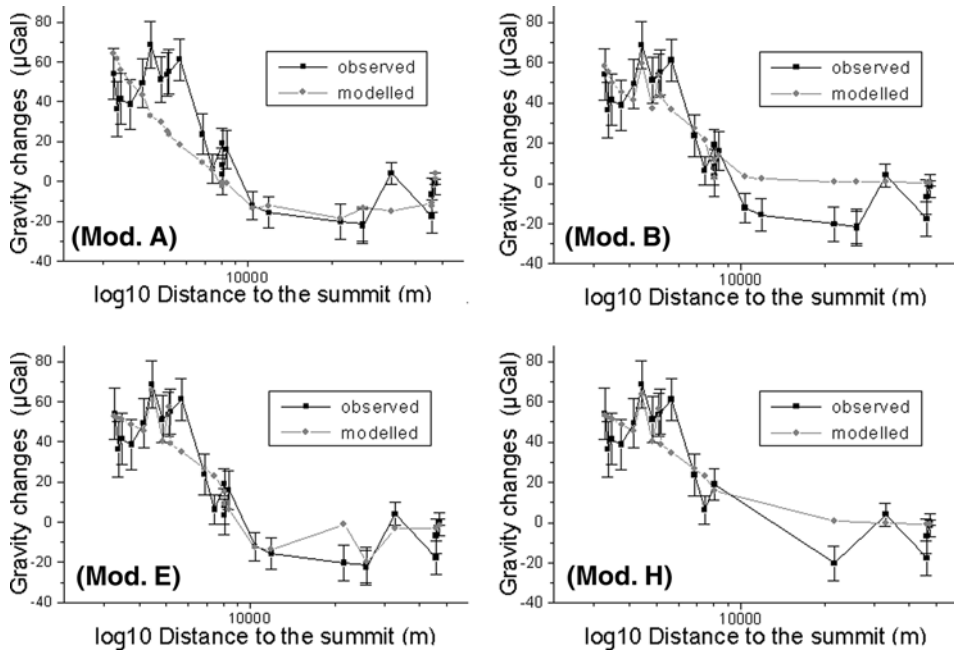


Figure 5

Profiles demonstrating the fit between observed and modelled gravity changes for model A, B, E and H of Table 2. For better representation, the logarithm of the radial distance of the stations from Mayon, independent of their azimuth, is used.

in Mayon Volcano. The San Vicente-Linao Fault, SW of Mayon, appears as a northwest-trending left-handed normal-slip fault (LAGMAY *et al.*, 2005). We have observed that when we try to obtain a one-source model, without considering a second source, the discontinuity of data can produce (as is typical in these situations) sources that are displaced and too shallow. A two-source model permits the secondary source to absorb the effect of the discontinuity, allowing the primary source to be adjusted to its more likely position. Taking into account this phenomenon, we finally propose several one-source fits for the stations north of the San Vicente-Linao Fault (19 stations) and then discard the southern stations (as corresponding to a rather different pattern of deformation). The resulting models (G and H in Table 2) suggest a positive mass source of about 0.5 ± 0.18 MU located nearly under the summit, at a depth of about 5.5 ± 1.6 km, and with residuals of about 6.5 μ Gal.

Parameters dg_0 and dg_z reach small adjusted values (except for model A without mass sources). They allow a more flexible fit able to absorb uniform effects due to changes of data origin, changes in the ground water levels, deep horizontal laminar structures and regional changes. The inclusion of these terms permits a better fit for the distant stations.

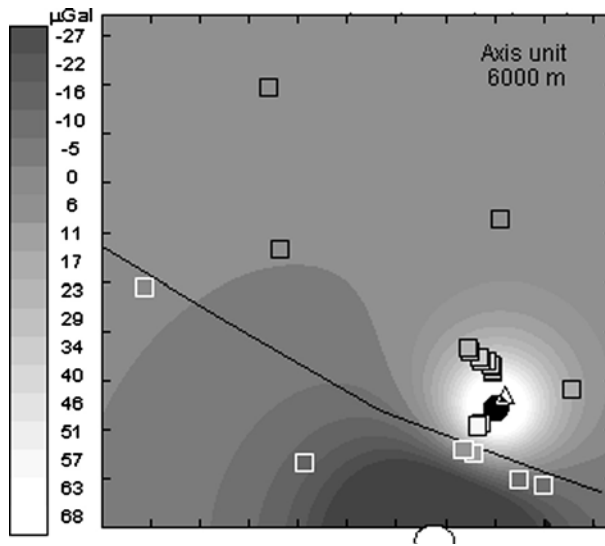


Figure 6

Calculated pattern of surface gravity effect for the two-source model E (see Fig. 4 and Table 2). We observe that the San Vicente-Linao Fault (continuous line) is very significant, dividing the modelled gravity changes into two different parts and suggesting a discriminatory role of the fracture system.

Concerning possible pressure changes resulting from the joint inversion of gravity and elevation changes, results for $P_j R_j^3$ (strength of the source) are not significant, as expected for small deformations. For instance, for the main source, the joint inversion produces the following results: $4 \pm 11 \text{ MPa} \cdot \text{km}^3$ (model B), $5 \pm 10 \text{ MPa} \cdot \text{km}^3$ (model C), $8 \pm 11 \text{ MPa} \cdot \text{km}^3$ (model G). Again, while the sensitivity analysis of TIAMPO *et al.* (2004a,b) suggested that the pressure and radius are the most difficult parameters to resolve for a source of small mass and with negligible deformation, they determined that the main source has a source strength of less than $1 \text{ MPa} \cdot \text{km}^3$.

5. Conclusions

We have interpreted gravity and elevation changes observed around Mayon Volcano between November 1992 and November 1994. Significant gravity changes determined with a mean accuracy of $9.4 \mu\text{Gal}$ are modelled, taking into account that significant elevation changes were not detected within 4 cm. A new, simple method to fit gravity and elevation changes simultaneously is applied. This method is based on a combination of an explorative search for location of the source and a linear least-squares fit for the anomalous mass, pressure, and parameters for regional common values in the observed values. The adjustment conditions include

a factor to regulate the balance between gravity and elevation fits and some terms to reduce instabilities based on a control of the total mass and pressure magnitude of the model.

Several models are adjusted using one and two sources. Models with one source produce a fit level of about 7–8 μGal , which is within the data accuracy level. They suggest an intrusion of about 0.3 MU at a depth of 2 km below sea level, located 5 km west of the summit. It is perhaps too small, shallow and displaced from the summit. FERNÁNDEZ *et al.* (2001) and TIAMPO *et al.* (2004b) suggested a similar source (0.47 MU with a depth of 1.95 km below the base of the volcano). By trying a two-source model, we observe that now a nearly centered main positive source is inferred and a secondary unclear deep source is introduced, which explains the existence of a discontinuity pattern in the gravity variations, nearly coincident with the San Vicente-Linao Fault. A final fit is attempted using only data from the 19 stations located north of the fracture. It produces a positive source of about 0.5 ± 0.18 MU located nearly under the summit at a depth of 5.5 ± 1.6 km. Nevertheless, the mean accuracy of 9.4 Gal assumed for the gravity data does not permit better discrimination between the models considered, which (except for model A in Table 2 that only considers constant variation in vertical displacement and gravity change and no mass and/or pressure changes) produce residuals smaller than this accuracy threshold. TIAMPO *et al.* (2004a), working with the entire set of stations and using a genetic algorithm as inversion method, obtained shallower sources. We conclude, however, that the point-source inversion suggests a discontinuity phenomenon whose consideration as a model of the San Vicente-Linao Fault provides an optimal fit of the Mayon data, using one very plausible source. Seismic data suggest that the San Vicente-Linao Fault system is active (LAGMAY *et al.*, 2005). The consideration of the fault influence on the interpretation of geodetic observations and their time variations may allow us to detect possible flank instability hazards. The inversion methodology used here in combination with continuous geodetic observation will be very useful in this task.

Acknowledgements

The research by AGC, JF and MC was funded under MCyT projects REN2002-03450 and CGL2005-05500-CO2/BTE. Work by KT was performed under NASA grant NAG5-3054 and an NSERC Discovery grant. The work at Mayon volcano was supported by the German Research Foundation (DFG) and the “Gesellschaft für Technische Zusammenarbeit” (GTZ) and partly by the European Commission (Grant ENV4 - CT96 - 0259) as well as by PHIVOLCS. Thoughtful comments from two reviewers, H. Rymer and F. Roth, helped us to improve the text.

REFERENCES

- AURELIO, M.A. (2000), *Tectonics of the Philippines revisited*, J. Geol. Soc. Philippines 55, 119–183.
- AURELIO, M., BARRIER, E., GOULON, R. and RANGIN, C. (1997), *Deformation and stress states along the central segment of the Philippine Fault: Implications to wrench fault tectonics*, J. Asian Sci. 15(2–3), 107–119.
- BERTETE-AGUIRRE, H., CHERKAEV, E. and ORISTAGLIO, M. (2002), *Non-smooth gravity problem with total variation penalization functional*, Geophys. J. Int. 149, 499–507.
- BONAFEDE, M. and MAZZANTI, M. (1998), *Modeling gravity variations consistent with ground deformation in the Campi Flegrei caldera (Italy)*, J. Volcanol. Geoth. Res. 81, 137–157.
- BOULANGER, O. and CHOUTEAU, M. (2001), *Constraints in 3D gravity inversion*, Geophys. Prospecting 49, 265–280.
- DAVIS, R.O. and SELVADURAI, A.P.S., *Elasticity and Geomechanics* (Cambridge Univ. Press 1996) 201 pp.
- EGGERS, A. (1987), *Residual gravity changes and eruption magnitudes*, J. Volcanol. Geotherm. Res. 33, 201–216.
- FARQUHARSON, C.G. and OLDENBOURG, D.W. (1998), *Non-linear inversion using general measures of data misfit and model structure*, Geophys. J. Int. 134, 213–227.
- FERNÁNDEZ, J. and RUNDLE, J. (1994a), *Gravity changes and deformation due to a magmatic intrusion in a two-layered crustal model*, J. Geophys. Res. 99, 2737–2746.
- FERNÁNDEZ, J. and RUNDLE, J. (1994b), *FORTRAN program to compute displacement, potential and gravity changes due to a magma intrusion in a multilayered Earth model*, Comp. Geosci. 20, 461–510.
- FERNÁNDEZ, J., TIAMPO, K. F., JENTZSCH, G., CHARCO, M. and RUNDLE, J. B. (2001), *Inflation or deflation? New results for Mayon Volcano applying elastic-gravitational modeling*, Geophys. Res. Lett. 28, 2349–2352.
- FERNÁNDEZ, J., TIAMPO, K.F., RUNDLE, J.B. and JENTZSCH, G. (2005), *On the interpretation of vertical gravity gradients produced by magmatic intrusions*, J. Geodyn. 39, 475–492. doi: 10.1016/j.jog.2005.04.005.
- GOTTMANN, J. and RYMER, H. (2002), *Deflation during caldera unrest: Constraints on subsurface processes and eruption prediction from gravity-height data*, Bull. Volcanol. 64, 338–348.
- GOTTMANN, J., BERRINO, G., RYMER, H. and WILLIAMS-JONES, G. (2003), *Hazard assessment during caldera unrest at the Campi Flegrei: a contribution from gravity-height gradients*, Earth Planet. Sci. Lett. 211, 295–309.
- JAHR, T., JENTZSCH, G. and DIAO, E. (1995), *Microgravity measurements at Mayon Volcano, Luzon, Philippines*, Cahiers du Centre Européen de Géodynamique et de Séismologie, vol. 8, Proc. of the Workshop: *New Challenges for Geodesy in Volcanoes Monitoring*, June 14–16, 1993, Walferdange, Luxembourg, 307–317.
- JAHR, T., JENTZSCH, G., PUNONGBAYAN, R.S., SCHREIBER, U., SEEGER, G., VÖLKSEN, C. and WEISE, A. (1998), *Mayon Volcano, Philippines: Improvement of hazard assessment by microgravity and GPS?* Proc.Int. Symp. *On Current Crustal Movement and Hazard Assessment* (IUGG, IAG), Wuhan. Seismological Press, Beijing, pp. 599–608.
- JENTZSCH, G., PUNONGBAYAN, R.S., SCHREIBER, U., SEEGER, G., VÖLKSEN, C. and WEISE, A. (2001), *Mayon volcano, Philippines: Change of monitoring strategy after microgravity and GPS measurements*, J. Volcanol. Geotherm. Res. 109/1–3, 219–234.
- JENTZSCH, G., WEISE, A., REY, C. and GERSTENECKER, C. (2004), *Gravity changes and internal processes: Some results obtained from observations at three volcanoes*, Pure Appl. Geophys. 161, 1415–1431. DOI 10.1007/s00024-004-2512-7.
- LAGMAY, A.M.F., TENONCIANG, A.M.P. and UY, H.S. (2005), *Structural setting of the Bicol Basin and kinematic analysis of fractures on Mayon Volcano, Philippines*, J. Volcanol. Geotherm. Res. Doi:10.1016/j.jvolgeores.2004.11.015
- MOGI, K. (1958), *Relations between the eruptions of various volcanoes and the deformations of the ground surfaces around them*, Bull. Earth. Res. Inst. Tokyo 36, 99–134.
- NEWHALL, C.G. (1979), *Temporal variation in the lavas of Mayon Volcano, Philippines*, J. Volcanol. Geotherm. Res. 6, 61–83.

- RUNDLE, J.B. (1980), *Static elastic-gravitational deformation of a layered half space by point couple sources*, J. Geophys. Res. 85, 5355–5363.
- RUNDLE, J.B. (1982), *Deformation, gravity, and potential changes due to volcanic loading in the crust*, J. Geophys. Res. 87(B13), 10,729–10,744.
- RYMER, H. and WILLIAMS-JONES, G. (2000), *Volcanic eruption prediction: Magma chamber physics from gravity and deformation measurements*, Geophys. Res. Lett. 27, 2389–2392.
- SAMBRIDGE, M. and MOSEGAARD, K. (2002), *Monte Carlo methods in geophysical inverse problems*, Rev. Geophys. 40, 3, doi:10.1029/2001GC000213.
- TARANTOLA, A., *Inverse Problem Theory* (Elsevier, Amsterdam 1987)
- TIAMPO, K., FERNÁNDEZ, J., JENTZSCH, G., CHARCO, M. and RUNDLE, J. (2004a), *New results at Mayon, Philippines from a joint inversion of gravity and deformation measurements*, Pure Appl. Geophys. 161, 1433–1452.
- TIAMPO, K., FERNÁNDEZ, J., JENTZSCH, G., CHARCO, M. and RUNDLE, J. (2004b), *Volcanic source inversion using a genetic algorithm and an elastic-gravitational layered earth model for magmatic intrusions*, Comp. Geosci. 30, 985–1001.
- TIEDE, C., TIAMPO, K., FERNÁNDEZ, J. and GERSTENECKER, C. (2005), *Deeper understanding of nonlinear geodetic data inversion using a quantitative sensitivity analysis*, Nonlinear Processes in Geophys. 12, 373–379.
- VÖLKSEN, C. and SEEGER, G. (1995), *Establishment of a GPS based control network at Mayon Volcano*, Cahiers du Centre Européen de Géodynamique et de Séismologie, vol. 8, Proc. of the Workshop: *New Challenges for Geodesy in Volcanoes Monitoring*, June 14–16, 1993, Walferdange, Luxembourg, 307–317.
- WILLIAMS-JONES, G. and RYMER, H. (2002), *Detecting volcanic eruption precursors: A new method using gravity and deformation measurements*, J. Volcanol. Geotherm. Res. 113, 379–389.

(Received June 21, 2005, revised April 10, 2006, accepted May 15, 2006)

To access this journal online:
www.birkhauser.ch/pag

Glacial-isostatic Adjustment and the Viscosity Structure Underlying the Vatnajökull Ice Cap, Iceland

KEVIN FLEMING, ZDENĚK MARTINEC, and DETLEF WOLF

Abstract—We examine the dependence of glacial-isostatic adjustment (GIA) due to changes in the Vatnajökull Ice Cap, Iceland, on the underlying viscosity structure. Iceland offers a unique case study for GIA research, with a thinner elastic lithosphere underlain by a low-viscosity zone or asthenosphere, as opposed to regions such as Fennoscandia or North America described by a thicker lithosphere, while not necessarily featuring an asthenosphere.

A laterally homogeneous spherical earth model is used consisting of an elastic lithosphere, a viscoelastic asthenosphere, a viscoelastic upper and lower mantle and a fluid core. We examine the response of the earth model to three ice models with circular plans and cross-section profiles based on the assumption of perfectly plastic material, but with different load histories. These are: (1) A history where the ice cap grows from a AD 900 minimum to a maximum at 1890, followed by a uniform decrease until 1991, continuing to the present day at an average rate based on recent mass-balance measurements, (2) a history that is the same as the first, except for constant ice volumes prior to 1890, and (3) a history that is again the same as the first model, except that the post-1991 changes correspond to the measured mass-balance values. We first compare the response to each ice model using typical earth-model parameters for Iceland presented in the literature. We then undertake a parameter-space search, where we assess the importance of lithosphere thickness, asthenosphere viscosity and basal asthenosphere depth, to predicted vertical-displacement rates, and compare them to rates determined from GPS measurements obtained from campaigns conducted between 1991 and 1999.

The earth-viscosity structure that provides the optimum predictions with respect to the GPS-derived vertical-displacement rates consists of an elastic lithosphere with a thickness of between 20 and 30 km, an asthenosphere viscosity between 1 and 2×10^{18} Pa s, and a basal asthenosphere depth between 250 km and possibly greater than 400 km. We find that the very low asthenosphere viscosity values of *ca.* 10^{17} Pa s sometimes suggested in the literature are not necessary to account for the rapid vertical-displacement rates observed, which are the result of the contemporary decrease in the mass of the ice cap not considered previously.

Key words: Glacial-isostatic adjustment, GPS, Mass balance, Vatnajökull Ice Cap.

1. Introduction

Iceland's complex geological and geophysical environment is a result of the interaction between the Mid-Atlantic Ridge, upon which it is located, and a hotspot. The expected viscosity structure for Iceland, a thin elastic lithosphere and a pronounced low-viscosity zone or asthenosphere (e.g., SIGMUNDSSON, 1991;

SIGMUNDSSON and EINARSSON, 1992), with substantial lateral variability in the properties of the crust and mantle (e.g., DARBYSHIRE *et al.*, 2000), therefore provides a case for glacial-isostatic adjustment (GIA) studies different from that of regions that experienced more intensive late-Pleistocene glaciation, such as Fennoscandia and North America, that feature a thicker elastic lithosphere and not everywhere necessarily requiring a lower-viscosity asthenosphere (e.g., MITROVICA, 1996; LAMBECK *et al.*, 1998).

Iceland experienced a complex glacial history following the Last Glacial Maximum (LGM, *ca.* 21 ka BP), involving a series of retreats and readvances of the ice sheet (NORDDAHL, 1990; INGÓLFSSON and NORDDAHL, 1994; GUDMUNDSSON, 1997). The Earth's viscosity structure is one of the fundamental unknowns in GIA studies, hence resolving it will assist in constraining the behavior of Iceland's late-Pleistocene ice sheet when, for example, carrying out sea-level change analyses (e.g., SIGMUNDSSON, 1991; RUNDGREN *et al.*, 1997; NORDDAHL and EINARSSON, 2001). There is also the need to reconcile interpretations drawn from the different methodologies used to study the underlying structure of Iceland, in particular gravity surveys (e.g., JACOBY *et al.*, 2000; BÜRGER *et al.*, 2005), various seismic techniques (e.g., WOLFE *et al.*, 1997; KUMAR *et al.*, 2005), gravity methods combined with seismology (e.g., DARBYSHIRE *et al.*, 2000; KABAN *et al.*, 2002; FEDOROVA *et al.*, 2005), and GPS, especially campaign surveys (e.g., SJÖBERG *et al.*, 2004; ÁRNADÓTTIR *et al.*, 2005). In addition, glacial-load changes influence the frequency, location, magnitude and chemistry of volcanic eruptions (e.g., SIGVALDASON *et al.*, 1992; JULL and MCKENZIE, 1996; SLATER *et al.*, 1998; GEE *et al.*, 1998; BOURGEOSIS *et al.*, 1998; MACLENNAN *et al.*, 2002).

This work deals with the GIA response in the vicinity of the Vatnajökull Ice Cap, the largest ice mass in Iceland, as recorded by a series of GPS campaigns conducted between 1991 and 1999. We examine the effect of its changing extent over the last 1000 years, while focusing on the influence of the period since *ca.* 1991, for which there are detailed mass-balance estimates available. Section 2 describes the present-day glaciation of Iceland, in particular the current state of the Vatnajökull Ice Cap. Section 3 discusses the underlying viscosity structure of Iceland, as determined from GIA and other studies, especially seismology and gravity. This is followed in Section 4 by results from a sensitivity study that investigates the relative importance to predictions of GIA-induced vertical-displacement rates of the various parameters describing the viscosity structure below the Vatnajökull Ice Cap, with the predicted rates being compared to those obtained from the GPS campaigns. The results are discussed and summarized in Section 5.

2. Iceland's Present-day Glaciation

Glaciers and ice caps currently cover *ca.* 11260 km² or *ca.* 11% of Iceland (Fig. 1, BJÖRNSSON, 1978, 1979). All of Iceland's ice masses are temperate, dynamically active

and very responsive to climatic change. The latter point is particularly important since Iceland's location near the Arctic/Boreal climatic boundary means that its climate, and therefore its glacial regime, is very sensitive to changes in the oceanic and atmospheric circulation patterns of the North Atlantic (BJÖRNSSON, 1979; EINARSSON and ALBERTSSON, 1988; INGÓLFSSON and NORDDAHL, 1994; GUDMUNDSSON, 1997; INGÓLFSSON *et al.*, 1997; JENNINGS *et al.*, 2000).

Changes in the extent of Iceland's glaciers have been observed since the European settlement of Iceland (AD 874, BJÖRNSSON, 1979), as noted from the movement of the fronts of outlet glaciers that drain the major ice caps (e.g., Vatnajökull and Myrdalsjökull) and of mountain glaciers in the north (e.g., Tröllaskagi). At least two periods of major glacier readvance have been identified; the Medieval Glacier Advance, which started around 1200, and the Little Ice Age Glacier Advance, which started around the middle of the 18th and 19th centuries for northern and southern Iceland, respectively, lasting until the early 20th century (e.g., GUDMUNDSSON, 1997). Many glaciers began retreating from their maximum extent between 1850 and 1900, with rates of retreat increasing after 1930. From 1940, a cooler period caused the rate of retreat to decrease, with glaciers readvancing after 1970. Since 1985, the warming climate has caused many glaciers to retreat again. The ice-covered area of Iceland shrank by *ca.* 3% between 1903 and 1979, the most pronounced changes occurring in the smaller ice masses. However, even during times when the majority of glaciers are

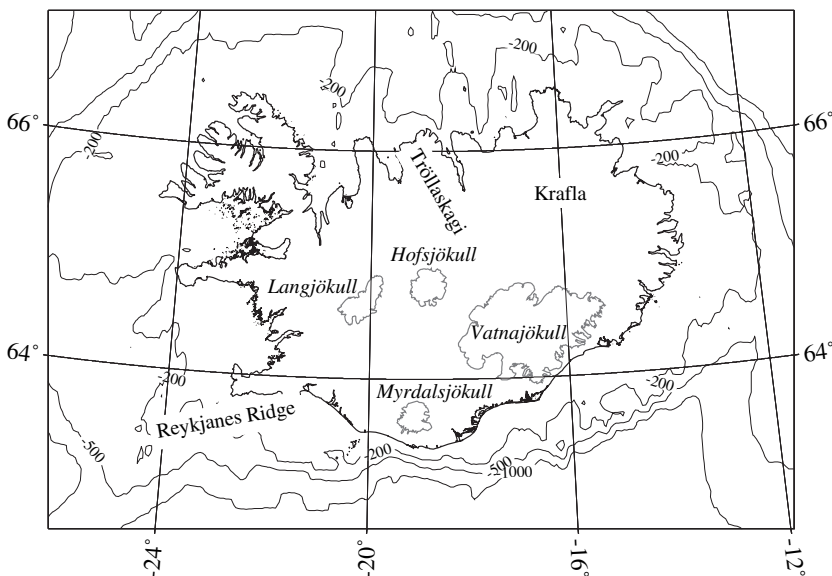


Figure 1

Map of Iceland showing the areas of present-day glaciation and the locations and geological features discussed in the text.

retreating (or advancing), a sizable minority may behave in the opposite manner (JOHANNESSON and SIGMUNDSSON, 1998).

The Vatnajökull Ice Cap is the largest ice mass in Iceland, with an area of *ca.* 8100 km² (BJÖRNSSON *et al.*, 1998; 2002). The average ice thickness is *ca.* 420 m, with a maximum of *ca.* 1000 m (BJÖRNSSON, 1979). The ice cap and its surroundings have been the subject of a number of studies. Of particular relevance to this work are the GPS campaigns conducted between 1991 and 1999 (EINARSSON *et al.*, 1996; SJÖBERG *et al.*, 2000, 2004). Mass-balance measurements have been carried out along the flowlines of several outlets of the ice cap since 1991, using snow cores and stake measurements (BJÖRNSSON *et al.*, 1998) as well as airborne surveys using synthetic aperture radar (MAGNÚSSON *et al.*, 2005a,b). Such studies are complicated owing to Vatnajökull overlying areas of active volcanism that undergo periodic eruptions, resulting in localized basal melting, the formation of meltwater lakes and sudden outbursts of meltwater termed *jökulhlaups* from the ice cap's base (e.g., GUDMUNDSSON and BJÖRNSSON, 1991; GUDMUNDSSON *et al.*, 1997; BJÖRNSSON *et al.*, 2001).

Changes in the extent and mass of Vatnajökull occur over a number of time scales. For example, between 1890 and 1973, the ice cap shrank from an area of 8600 km² to 8300 km², corresponding to an average loss of *ca.* 2 km³ of ice per year (SIGMUNDSSON and EINARSSON, 1992) (note that the latter year quoted by SIGMUNDSSON and EINARSSON (1992) is 1978, which is an error as the area they use is based on a 1973 Landsat image discussed by BJÖRNSSON (1978)). However, during the 1996 Gjálp eruption, it is estimated that *ca.* 3 km³ of ice was lost over 13 days (GUDMUNDSSON *et al.*, 1997). BJÖRNSSON *et al.* (2002) reported that between 1991 and 1994 Vatnajökull experienced a positive net balance, between 1994 and 1995 it was almost balanced, and following this the mass balance was negative, with a total loss between 1991 and 2001 of 19.4 km³ of ice or 2.37 m water equivalent over its area. MAGNÚSSON *et al.* (2005a) found that between 1985 and 1998, outlet glaciers in western Vatnajökull lost 14 ± 5 km³ of ice, although some glaciers also experienced thickening, with a decrease in area between the late 19th-century glacial maximum and 1998 of *ca.* 109 km² (MAGNÚSSON *et al.* 2005b). While summer temperatures are the primary controller of glacier fluctuations (JOHANNESSON and SIGURÁSSON, 1998), other non-climatic factors also play a role. For example, higher summer ablation may arise from the decrease in ice-surface albedo due to the deposition of ash from volcanic eruptions (e.g., GUDMUNDSSON and BJÖRNSSON, 1991; BJÖRNSSON, 2002).

3. Iceland's Underlying Viscosity Structure

The complexity of the crustal and mantle structure of Iceland is the result of the interaction between the Mid-Atlantic Ridge and a hotspot located approximately

under Vatnajökull. This has given rise to a region characterized by a large-scale topographic high, positive regional free-air gravity and geoid anomalies, and anomalously thick crust (e.g., DARBYSHIRE *et al.*, 2000; KABAN *et al.*, 2002; FEDOROVA *et al.*, 2005).

The crust and mantle display considerable lateral variation, with the term *Icelandic-type crust* being used to distinguish this special type of crust from that in oceanic or continental areas (e.g., FOULGER *et al.*, 2003). Icelandic crust is formed by plate tectonic spreading in the neovolcanic zones, with sporadic jumps in the position of these zones. There is evidence that the location of the various rift zones may be influenced by changes in glacial loading, as seen by the distribution of subglacial eruptive features, such as tabletop mountains and hyaloclastic ridges (BOURGOIS *et al.*, 1998). There are various models describing the nature of the Icelandic crust, a result of the variety of methodologies used (e.g., gravity, teleseismic and explosion-seismic studies). However, in broad terms, it may be divided into an upper crust with a thickness of $ca. 7 \pm 1$ km that displays pronounced lateral variability, and a lower crust which is typically 15 to 30 ± 5 km thick and is laterally more uniform. The thickest crust underlies central Iceland, around the northwest of Vatnajökull, with thinner crust along the Reykjanes Ridge in the southwest and towards the north (e.g., MENKE, 1999; DARBYSHIRE *et al.*, 2000; KABAN *et al.*, 2002; ALLEN *et al.*, 2002a; FOULGER *et al.*, 2003; GUDMUNDSSON, 2003). Another aspect of the crustal structure of Iceland is the proposed existence of a buried fragment of older crust, possibly continental, $ca. 200$ km wide (FEDOROVA *et al.*, 2005; FOULGER, 2006).

The viscosity of the uppermost mantle has been estimated to be between 10^{17} and 10^{19} Pa s. There are also magma chambers and the occurrence of partial melting, as evident by the extensive volcanism about the island. An anomalous low-density contrast of $ca. 100 \text{ kg m}^{-3}$ exists between the lower crust and the upper mantle, much less than the usual value of 330 kg m^{-3} for the mid-Atlantic Ridge (e.g., MENKE, 1999; FOULGER *et al.*, 2003; GUDMUNDSSON, 2003; FEDOROVA *et al.*, 2005). The reason for this is another topic of debate. For example, based on an assessment of the crustal isostatic equilibrium of Iceland, MENKE, (1999) suggested that the low-density contrast is the result of the uppermost mantle being anomalously light, although conceding that chemical depletion, thermal expansion and melt entrainment cannot fully account for this situation. In contrast, GUDMUNDSSON (2003) argued that the bulk of the density contrast is due to a denser lower crust, resulting from geochemical changes involving olivine enrichment.

Geologically, Iceland is classified as a *transition zone hotspot*, based on the magnitude and depth extent of the underlying seismic-velocity anomalies (e.g., RITSEMA and ALLEN, 2003). The source of the hotspot is another contentious issue, with the extent of the associated mantle plume, the generally accepted cause, being as yet unresolved (e.g., WOLFE *et al.*, 1997; SHEN *et al.*, 1998; ALLEN *et al.*, 1999; FOULGER *et al.*, 2000; FOULGER and PEARSON, 2001), while there is even some doubt as to the very existence of a plume (e.g., FOULGER *et al.*, 2003). Regardless of its

Table 1

*Estimates of lithosphere thickness and asthenosphere viscosity for Iceland as found from various GIA studies. In KUMAR *et al.* (2005), $H_L = 10$ km refers to the upper part of a 80-km-thick lithosphere divided by a channel with a viscosity η_A and a thickness of at least 50 km*

Source	H_L (km)	η_A (Pa s)	Model	Observational technique
SIGMUNDSSON (1991)	10	$< 10^{19}$	Planar	Holocene sea level
SIGMUNDSSON and EINARSSON (1992)	5–25	1×10^{18} – 5×10^{19}	Planar	Lake tilt, historical sea level
SJÖBERG <i>et al.</i> (2000)	30	5×10^{18}	Planar	GPS
THOMA <i>et al.</i> (2001)	10–20	7×10^{16} – 3×10^{18}	Spherical	GPS
Bürger <i>et al.</i> (2005)	10–15	3×10^{17} – 1×10^{18}	Spherical	Gravity
KUMAR <i>et al.</i> (2005)	10	1×10^{18}	Spherical	Seismic and GPS
HARTMANN (2005)	≤ 6	7×10^{17} – 1×10^{18}	Spherical	Gravity and GPS
This work	20–30	1×10^{18} – 2×10^{18}	Spherical	GPS

origin, determining the nature of the hotspot has received considerable attention. For example, WOLFE *et al.* (1997) proposed an underlying cylindrical zone of low seismic velocities *ca.* 150 km wide, extending from 100 km to at least 400 km depth, the deeper value also proposed by SHEN *et al.* (1998). ALLEN *et al.* (1999) suggested that a thin hot plume underlies Iceland whose radius is *ca.* 100 km, the distance over which the seismic anomaly decreases to $1/e$ of its maximum value, while ALLEN *et al.* (2002b) reported anomalous seismic velocities of a radius of 150 km under central Iceland.

The relationship between the displacement of the Earth's surface due to changes in ice loading for Iceland was first discussed several decades ago (e.g., EINARSSON, 1953; 1966). Table 1 summarizes the values inferred for lithosphere thickness and asthenosphere viscosity from interpretations of GIA. These values are the result of efforts to resolve the underlying structure of Iceland and Vatnajökull, since the vertical-displacement rates are primarily dependent upon the asthenosphere viscosity, and the shape of the vertical-displacement rate curve crossing the center of the ice cap depends strongly upon the lithosphere thickness (e.g., SJÖBERG *et al.*, 2004). SIGMUNDSSON (1991) used sea-level observations from the period following the retreat of the late-Pleistocene ice sheet, assuming a 10 km-thick elastic lithosphere (the maximum depth to earthquakes in Southwest Iceland and to a low-resistivity layer found by magnetotellurics). An upper value for the asthenosphere viscosity of 10^{19} Pa s was inferred, meaning that the rebound following the end of the late-Pleistocene ice sheet's melting had been completed in *ca.* 400 years. SIGMUNDSSON and EINARSSON (1992) examined vertical displacement resulting from changes in Vatnajökull and the resulting tilt in the nearby Lake Langisjör, inferring an asthenosphere viscosity between 1×10^{18} and 5×10^{19} Pa s. Predictions of present-day vertical displacement and gravity changes about Vatnajökull were presented by WOLF *et al.* (1997a,b), who used an earth model described by a laterally homogeneous, incompressible, layered Maxwell-viscoelastic half space.

Considering the available data, they concluded that the asthenosphere viscosity does not exceed 10^{19} Pa s. SJÖBERG *et al.* (2000), using GPS data, inferred a lithosphere thickness of 30 km and an asthenosphere viscosity of 5×10^{18} Pa s. In contrast to this, THOMA *et al.* (2001) inferred values of 10 to 20 km for the lithosphere thickness and 7×10^{16} to 3×10^{18} Pa s for the asthenosphere viscosity, based on the GPS-derived vertical-displacement rates of SJÖBERG *et al.* (2000) and the use of an ice model with an elliptical plan, which more closely resembles the shape of the present-day Vatnajökull Ice Cap than the more commonly used circular-plan models. Making use of the latest GPS-campaign results, SJÖBERG *et al.* (2004) proposed a lithosphere thickness of 10 to 20 km, and an asthenosphere viscosity as low as 1×10^{17} Pa s, although they acknowledged that assumptions about when the ice cap ceased retreating significantly influences the viscosities inferred. Making use of gravity-change data, BÜRGER *et al.* (2005) inferred the lithosphere thickness and asthenosphere viscosity to be between 10 and 15 km, and 3×10^{17} and 1×10^{18} Pa s, respectively, with the asthenosphere having a thickness of between 165 km to 170 km. Using GPS and gravity observations, HARTMANN (2005) inferred a lithosphere thickness of 6 km at most and an asthenosphere viscosity between 7×10^{17} and 1×10^{18} Pa s. By contrast, KUMAR *et al.* (2005) identified a discontinuity beneath Iceland at a depth of *ca.* 80 km, based on seismic S-receiver functions, that they interpreted as the lithosphere-asthenosphere boundary. This result was reconciled with observations from GPS campaigns by placing a low-viscosity channel within the lithosphere, with a viscosity of 1×10^{18} Pa s and a thickness of at least 50 km.

The values discussed above for the asthenosphere viscosity are consistent with those suggested by non-GIA studies. For example, from an analysis of displacement measured by GPS in Northeast Iceland following the 1975 to 1984 Krafla rifting episode, values of 0.3 to 2×10^{19} Pa s and 3×10^{18} Pa s were proposed by FOULGER *et al.* (1992) and POLLITZ and SACKS (1996), respectively. In Southeast Iceland, ÁRNADÓTTIR *et al.* (2005) inferred a lower-crust viscosity of between 0.5 and 1×10^{19} Pa s and an upper-mantle viscosity of *ca.* 3×10^{18} Pa s.

4. Present-day Glacial-isostatic Response around Vatnajökull

To investigate the GIA response of the Earth to changes in the Vatnajökull Ice Cap, we use a computer program based on the spectral finite-element method that calculates the vertical and horizontal displacements, as well as the geoid-height changes arising from variations in an axisymmetric surface load on a radially symmetric, incompressible, Maxwell viscoelastic Earth (MARTINEC, 2000). While there may be significant lateral variability in the lithosphere thickness and upper-mantle viscosity of Iceland, a 3-D code that is appropriate for the large number of calculations and high resolution necessary for this work is currently unavailable to

the authors. Therefore, the results presented in this section must be viewed within the context of this limitation.

The earth model is parameterized by the lithosphere thickness H_L , the asthenosphere viscosity η_A , the basal asthenosphere depth D_A , the upper- and lower-mantle viscosities η_{UM} and η_{LM} , respectively, and a fluid core. The bulk density and elastic properties are taken from PREM (DZIEWONSKI and ANDERSON, 1981). Because Vatnajökull's relatively small size means that no information about the lower mantle can be inferred from GIA analyses, we set $\eta_{LM} = 10^{22}$ Pa s, which is a typical value used in GIA studies (e.g., LAMBECK *et al.*, 1998).

The ice models are, as mentioned above, axisymmetric, whose cross sections are consistent with the assumption that the ice cap behaves as a perfectly plastic material, their profiles given by

$$h = \sqrt{\frac{2\tau}{\rho g}} \sqrt{R - r} = 4.0 \sqrt{R - r}, \quad (1)$$

where τ is the ice-cap basal shear stress, ρ is the ice density, g is the acceleration due to gravity, with h being the ice thickness at a radial distance r from its center, and R is the ice-cap radius (SIGMUNDSSON and EINARSSON, 1992).

We examine two issues with regards to the load history of Vatnajökull. These are the effect of the ice history prior to Vatnajökull's maximum extent in 1890 and its more recent mass-balance history since 1991 (Fig. 2). Regarding the first issue, we use VATNA-1 and VATNA-2, essentially taken from THOMA *et al.* (2001). VATNA-1 has its maximum extent in 1890, with an area of 8600 km² ($R = 52320$ m), decreasing until 1973 to an area of 8300 km² ($R = 51400$ m), with central thicknesses of 915 and 907 m, respectively (SIGMUNDSSON and EINARSSON, 1992). Its minimum extent is at AD 900, with an area of 6453 km² ($R = 45320$ km), and a central thickness of 852 m, with linear growth between AD 900 and 1890. Prior to AD 900 we assume isostatic equilibrium. For the period after 1973, we apply the same rate of ice loss until 1991 (*ca.* 2 km³a⁻¹), after which we use the estimates of BJÖRNSSON *et al.* (2002) who, as discussed in the previous section, proposed a loss of 19.4 km³ between 1991 and 2001, leading to a final radius and central thickness of 51098 m and 904 m, respectively. The second model, VATNA-2, is characterized by a constant ice volume prior to 1890, with the same load history after 1890 as VATNA-1. The third ice model, VATNA-3, uses the history of VATNA-1 prior to 1991, but then considers the annual mass-balance values of BJÖRNSSON *et al.* (2002). Figure 2 describes the variations in area and central thickness for each ice model, while Figure 3 shows the present-day outline with respect to the Vatnajökull Ice Cap. We neglect the contribution from the ongoing GIA response of the Earth due to the disappearance of the late-Pleistocene ice sheet, since a series of tests (not shown here) indicate that the resulting signal is less than the uncertainties associated with the GPS measurements, as also found by THOMA *et al.* (2001).

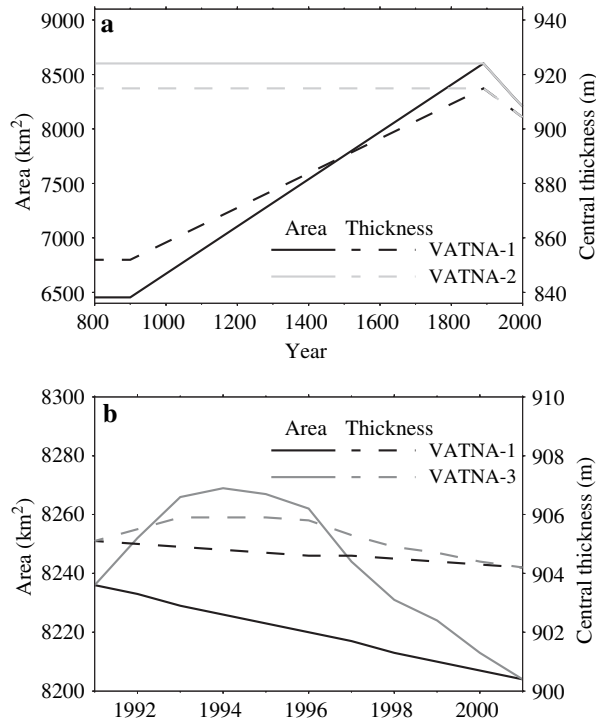


Figure 2

The change in area and central thickness for the ice models of the Vatnajökull Ice Cap considered (SIGMUNDSSON and EINARSSON, 1992; BJÖRNSSON *et al.*, 1998, 2002; THOMA *et al.*, 2001). (a) The ice models used to assess the importance of Vatnajökull's history prior to its maximum at 1890. The post-1991 histories for these models are identical. (b) The ice models used to assess the importance of Vatnajökull's post-1991 mass balance. The pre-1890 histories for these models are identical.

We compare our predicted vertical-displacement rates with results from a series of GPS campaigns undertaken in the vicinity of the Vatnajökull Ice Cap since 1991 (EINARSSON *et al.*, 1996; SJÖBERG *et al.*, 2000, 2004). The locations of the GPS sites are shown in Figure 3, with details about the sites, including the vertical-displacement rates inferred by different authors, listed in Table 2. Our first results concern the importance of the pre-1890 load history. Figure 4 presents the predicted vertical-displacement rates as functions of the radial distance from the ice cap's center for the VATNA-1 and VATNA-2 models (Fig. 2a). We test several earth-model pairs with $H_L = 10$ km and $\eta_A = 1 \times 10^{18}$ Pa s, and $H_L = 30$ km and $\eta_A = 5 \times 10^{18}$ Pa s. The upper-mantle viscosity is set to $\eta_{UM} = 1 \times 10^{20}$ Pa s, with $D_A = 400$ km (Fig. 4a) and 100 km (Fig. 4b). For all combinations, the differences between the responses for the two ice models are less than the uncertainty in the GPS measurements, with smaller differences arising from the examples with lower values of η_A . If we use the higher value of $\eta_{UM} = 5 \times 10^{20}$ Pa s (results not shown here), the

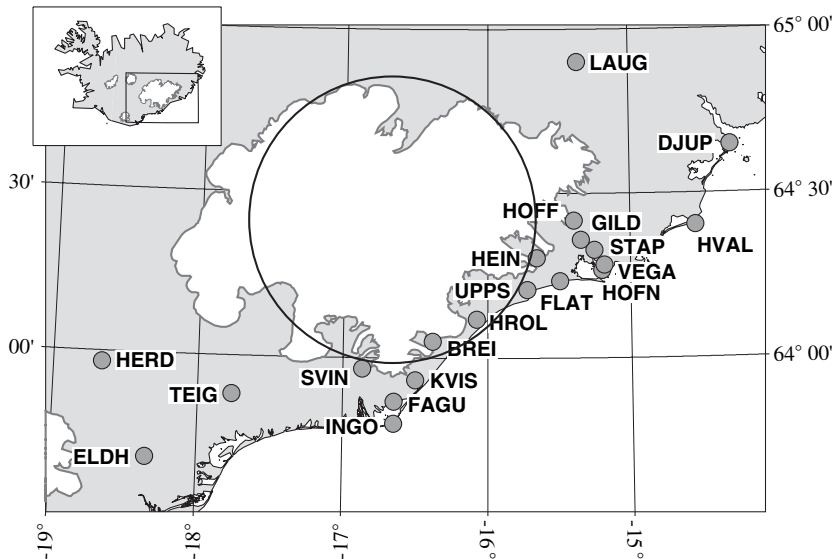


Figure 3

Locations of GPS stations (Table 2) around the Vatnajökull Ice Cap (SJÖBERG *et al.*, 2004). Also shown is the outline of the axisymmetric ice model (black circle) representing the equivalent present-day area of Vatnajökull.

differences between the responses for the ice models are still small for $D_A = 400$ km, but become more pronounced for $D_A = 100$ km. However, as will be shown below, this D_A value is not realistic. We can therefore conclude that the pre-1890 load history is of minor importance to present-day vertical displacement, given the uncertainties of the observed rates.

To examine the dependence on the earth-model parameters more thoroughly, we conduct a parameter-space search and compare the predicted vertical-displacement rates to the GPS-derived values using a least-squares criterion:

$$S^2 = \frac{1}{N-1} \sum_{n=1}^N \left(\frac{\dot{u}_P^n - \dot{u}_O^n}{\delta \dot{u}_O^n} \right)^2, \quad (2)$$

where S^2 is the misfit function, N is the number of observations, n refers to the particular observation, $\dot{u}_P$ and $\dot{u}_O$ are the predicted and observational vertical-displacement rates, respectively, and $\delta \dot{u}_O$ is the uncertainty in the GPS-derived vertical-displacement rate. We now compare the results for the VATNA-1 and VATNA-3 ice models to examine the effect of the post-1991 load history (Fig. 2b).

We first conduct a parameter-space search over a range of lithosphere thicknesses and basal asthenosphere depths for $\eta_A = 1$ and 5×10^{18} Pa s. The misfit function S^2 is plotted in Figure 5 as a function of H_L (varied between 10 and 50 km) and D_A (varied between 100 and 400 km) with $\eta_{UM} = 1 \times 10^{20}$ Pa s. The most obvious result

Table 2

Details of the GPS stations used to measure surface displacement in the vicinity of the Vatnajökull Ice Cap. Also listed are the estimates of vertical-displacement rates derived from the GPS data. Rates are from Sjöberg et al. (2000) (S2000), THOMA et al. (2001) (TW2001) and Sjöberg et al. (2004) (S2004). The rates from THOMA et al. (2001) are relative to HOFN. The radial distances refer to the center of the ice cap

Station	Location	Radial distance (km)	GPS campaigns	Vertical-displacement rate (mm a ⁻¹)		
				S2000	TW2001	S2004
BREI	64°03'N	16°23'W	1996, 1999	8 ± 2	5 ± 2	15 ± 4
HROW	64°19'N	16°09'W				13 ± 4
HROL	64°07'N	16°05'W	1996, 1999	8 ± 2	5 ± 2	14 ± 3
SVIN	63°58'N	16°52'W	1996, 1999	5 ± 3	2 ± 3	10 ± 3
KVIS	63°56'N	16°30'W	1996, 1999	5 ± 2	2 ± 2	12 ± 3
HEIN	64°18'N	15°39'W	1991, 1992 (data later rejected), 1996, 1999	4 ± 2	1 ± 2	15 ± 4
UPPS	64°12'N	15°43'W				10 ± 3
FAGU	63°52'N	16°39'W	1996, 1999	8 ± 2	5 ± 2	10 ± 3
HOFF	64°25'N	15°24'W	1991, 1992, 1996, 1999	5 ± 1	2 ± 1	13 ± 3
FLAT	64°14'N	15°29'W				11 ± 3
INGO	63°48'N	16°39'W	1996, 1999	8 ± 3	5 ± 3	15 ± 3
SKIN	64°08'N	15°27'W	1996, 1999	5 ± 1	2 ± 1	14 ± 4
GILD	64°22'N	15°21'W	1991, 1992, 1996, 1999			12 ± 4
TEIG	63°53'N	17°46'W	1996, 1999	5 ± 3	2 ± 3	13 ± 3
STAP	64°20'N	15°15'W	1991, 1992, 1996, 1999	5 ± 1	2 ± 1	9 ± 3
HOFN	64°16'N	15°12'W	1991, 1992, 1996, 1999	3 ± 1	0 ± 1	10 ± 2
VEGA	64°17'N	15°11'W	1996, 1999	3 ± 1	0 ± 1	6 ± 3
LAUG	64°54'N	15°22'W	1996, 1999	5 ± 2	2 ± 2	12 ± 3
HERD	63°58'N	18°40'W	1996, 1999	8 ± 3	5 ± 3	7 ± 3
ELDH	63°41'N	18°21'W	1996, 1999	8 ± 3	5 ± 3	4 ± 3
DJUP	64°39'N	14°17'W	1991, 1992 (data later rejected), 1996, 1999	8 ± 3	5 ± 3	4 ± 3
HVAL	64°24'N	14°33'W				4 ± 3

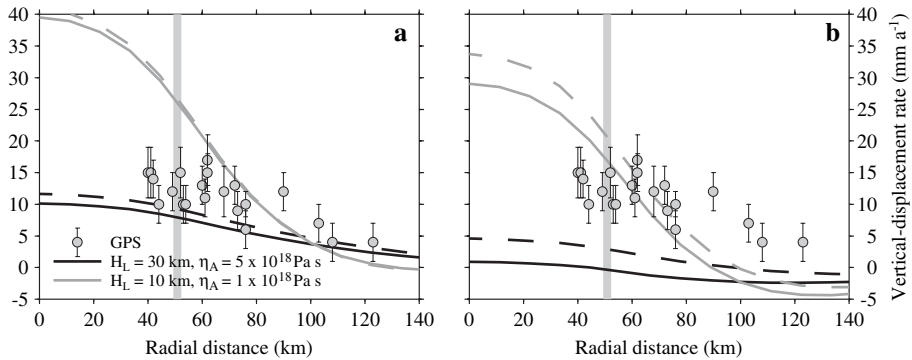


Figure 4

Vertical-displacement rates about the Vatnajökull Ice Cap as functions of radial distance from the ice cap's center for VATNA-1 (solid lines), VATNA-2 (dashed lines) and several earth models. The results are compared with GPS-based vertical-displacement rates (Sjöberg *et al.*, 2004). The basal asthenosphere depth is (a) 400 km and (b) 100 km. The vertical lines indicate the ice models' present-day radius.

is that the choice of the lower η_A value (Figs. 5a and b) gives considerably smaller values of S^2 than the higher η_A value (Figs. 5c and d). The second point is that the basal asthenosphere depth is at least 250 km (Fig. 5b), but may reach depths greater than 400 km. This is especially obvious from the results for VATNA-3 and $\eta_A = 1 \times 10^{18}$ Pa s, where S^2 is fairly insensitive to D_A values greater than *ca.* 200 km. Additional calculations using $\eta_{UM} = 5 \times 10^{20}$ Pa s (not presented here) show very little difference, with the resulting S^2 values being almost identical to those from $\eta_{UM} = 1 \times 10^{20}$ Pa s. Hence we conclude that the optimal earth model consists of a lithosphere thickness between 20 and 30 km, a basal asthenosphere depth ranging between 250 km and at least 400 km, and an asthenosphere viscosity of *ca.* 1×10^{18} Pa s.

The next parameter-space search is concerned with better constraining the lithosphere thickness and asthenosphere viscosity. The resulting misfit is shown in Figure 6, where we present S^2 as a function of lithosphere thickness and asthenosphere viscosity. A smaller misfit arises for the VATNA-3 model (Fig. 6b) when compared to the previous results (Fig. 5b), although not significantly. For both ice models, their 1σ uncertainty ranges overlap, with the optimal lithosphere thickness again between 20 km (VATNA-3) and 30 km (VATNA-1). The results for VATNA-3, where a more rapid rate of ice-mass loss in the last few years is considered, are consistent with a higher asthenosphere viscosity. While asthenosphere viscosities lower than 1×10^{18} Pa s cannot be ruled out, we find the low values of *ca.* 10^{17} Pa s discussed in the literature unnecessary (e.g., Sjöberg *et al.*, 2004).

We finally show in Figure 7 the predicted vertical-displacement rates as functions of the radial distance from the center of the ice cap, resulting from the optimal earth/ice model combinations presented in Figure 6. These results illustrate the problem of

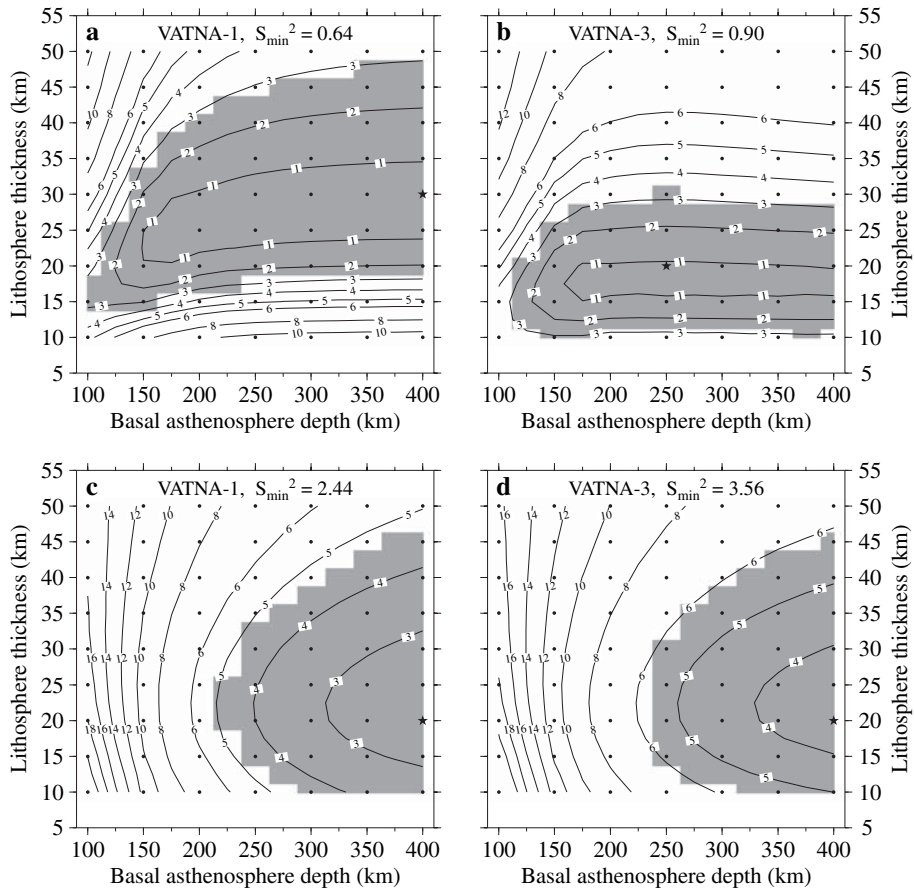


Figure 5

Misfit function S^2 for predicted and GPS-based vertical-displacement rates as functions of lithosphere thickness and basal asthenosphere depth. The asthenosphere viscosity is (a and b) 1×10^{18} Pa s and (c and d) 5×10^{18} Pa s. The ice models are (a and c) VATNA-1 and (b and d) VATNA-3. The stars represent the optimum parameter combinations for which $S_{\min}^2$ apply, and the grey shading indicates the 1σ uncertainty.

the non-uniqueness of the solution, with both curves fitting the observational vertical-displacement rates almost equally well.

5. Discussion and Conclusions

The objective of this study has been to assess the influence of the viscosity structure below the Vatnajökull Ice Cap on GIA. Based on the use of simple axisymmetric ice models with realistic load histories, and 1-D earth models, we find that the viscosity structure that provides predictions best matching the GPS-derived

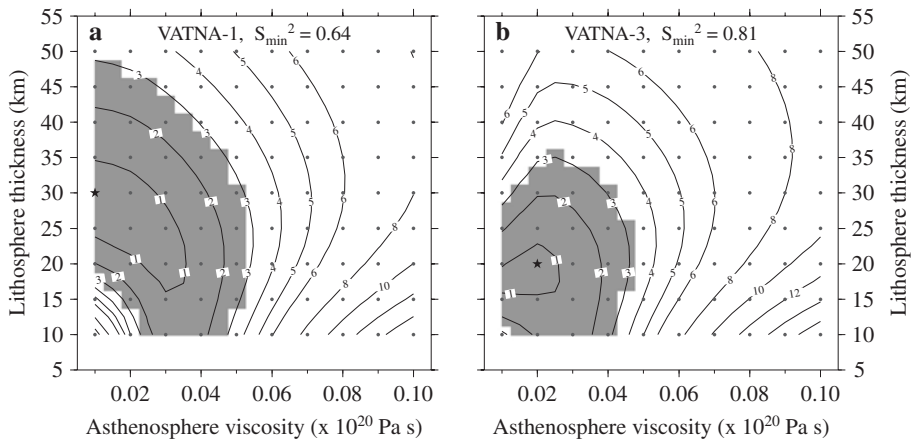


Figure 6

Misfit function S^2 for predicted and GPS-based vertical-displacement rates as functions of lithosphere thickness and asthenosphere viscosity. The basal asthenosphere depth is 400 km. The ice models are (a) VATNA-1 and (b) VATNA-3. The stars represent the optimum parameter combinations for which $S_{\min}^2$ apply, and the grey shading indicates the 1σ uncertainty.

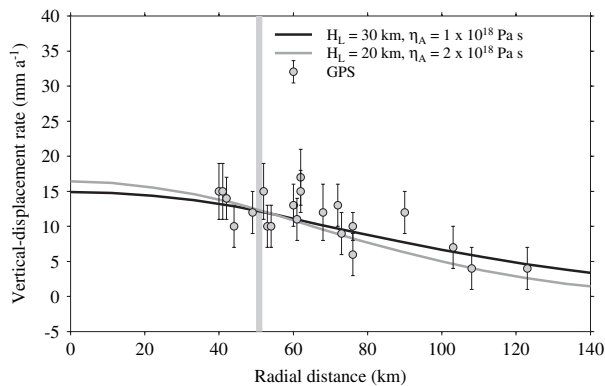


Figure 7

Vertical-displacement rates as functions of radial distance resulting from the optimum earth and ice model combinations shown in Figure 6. The vertical line indicates the ice models' present-day radius.

vertical-displacements rates is described by a lithosphere thickness between 20 and 30 km, an asthenosphere viscosity between 1 and 2×10^{18} Pa s, and a basal asthenosphere depth between *ca.* 250 km to possibly greater than 400 km. Therefore, the need for a low asthenosphere viscosity to give rise to rapid vertical displacement is reaffirmed. However, it need not be as low as suggested by some workers (e.g., THOMA *et al.*, 2001) due to the ice cap's ongoing decrease in mass (compare Figs. 6a and b), a factor neglected in previous studies (e.g., THOMA *et al.*, 2001; SJÖBERG *et al.*, 2004). Our inferred values are therefore consistent with those published previously

(Table 1). In addition, it must be emphasized that the asthenosphere's viscoelastic response is very important to vertical displacement, and it would be insufficient to simply consider the elastic response of the lithosphere to contemporary mass changes (see Figs. 3 and 7 of IVINS *et al.*, 2000).

Such a viscosity structure is similar to that found in other regions of the world that also combine complex tectonic regimes and significant contemporary ice-load changes. For example, the coastal area around the Alaskan Gulf, which is underlain by a subduction zone and is the largest non-polar area of glaciation, is experiencing rapid ice-mass loss (ARENDR *et al.*, 2002), resulting in the fastest uplift measured by GPS (LARSEN *et al.*, 2005). The inferred viscosity structure for this region consists of a lithosphere of thickness between 60 and 70 km, an asthenosphere extending to a depth of 110 km with a viscosity of 2.5 to 4×10^{18} Pa s, and an upper-mantle viscosity of 4×10^{20} Pa s. Another region that combines complex tectonics with a glacial environment is the southernmost Andes, where GPS measurements revealed significant uplift due to substantial ice loss (RIGNOT *et al.*, 2003; IVINS and JAMES, 2004). The viscosity structure resolved for this region consists of a 35-km-thick lithosphere and an asthenosphere extending to over 100 km depth with a viscosity less than 10^{19} Pa s.

Future work on this topic will deal with the following themes:

- The incorporation of time-varying gravity data.
- The expansion of the GIA modeling tools, specifically the development of a 3-D code.
- The incorporation of more realistic ice models that better resemble Vatnajökull's shape.

Acknowledgments

This work was undertaken with support from the SEAL (Sea Level Change) project of the Hermann von Helmholtz Association of German Research Centers and the Grant Agency of the Czech Republic (Grant No. 205/03/0778). We wish to thank Erik Ivins of JPL and Wolfgang Jacoby of the University of Mainz, whose comments have greatly improved this work, and Gudfinna Adalgeirsdottir of the University of Wales-Swansea for her advice and suggestions.

REFERENCES

- ALLEN, R. M., NOLET, G., MORGAN, W. J., VOGFJÖRD, K., BERGSSON, B. H., ERLENDSSON, P., FOULGER, G. R., JAKOBSDÓTTIR, S., JULIAN, B. R., PRITCHARD, M., RAGNARSSON, S., and STEFANSSON, R. (1999), *The thin hot plume beneath Iceland*, *Geophys. J. Internat.* **137**, 51–63.
- ALLEN, R. M., NOLET, G., MORGAN, W. J., VOGFJÖRD, K., NETTLES, M., EKSTRÖM, G., BERGSSON, B., ERLENDSSON, P., FOULGER, G., JAKOBSDÓTTIR, S., JULIAN, B. R., PRITSCHARD, M., RAGNARSSON, S.,

- and STEFÁNSSON, R. (2002a), *Plume-driven plumbing and crustal formation in Iceland*, J. Geophys. Res. 107(B8), 10.1029/2001JB000584.
- ALLEN, R., NOLET, G., MORGAN, W., VOGFJÖRD, K., BERGSSON, B., ERLENDSSON, P., FOULGER, G., JAKOBSDÓTTIR, S., JULIAN, B., PRITCHARD, M., RAGNARSSON, S., and STEFÁNSSON, R. (2002b), *Imaging the mantle beneath Iceland using integrated seismological techniques*, J. Geophys. Res. 107(B12), 2325–2340.
- ARENDE, A. A., ECHELMAYER, K. A., HARRISON, W. D., LINGLE, C. S., and VALENTINE, V. B. (2002), *Rapid Wastage of Alaska Glaciers and their contribution to rising sea level*, Science 297, 382–386.
- ÁRNADÓTTIR, T., JÓNSSON, S., POLLITZ, F., JIANG, W., and FEIGL, K. (2005), *Postseismic deformation following the June 2000 earthquake sequence in the south Iceland seismic zone*, J. Geophys. Res. 110(B12308), 10.1029/2005JB003701.
- BJÖRNSSON, H. (1978), *The surface area of glaciers in Iceland*, Jökull 28, 31.
- BJÖRNSSON, H. (1979), *Glaciers in Iceland*, Jökull 29, 74–80.
- BJÖRNSSON, H., PÁLSSON, F., GUDMUNDSSON, M., and HARALDSSON, H. (1998), *Mass balance of western and northern Vatnajökull, Iceland, 1991–1995*, Jökull 45, 35–58.
- BJÖRNSSON, H., ROTT, H., GUDMUNDSSON, S., FISCHER, A., SIEGEL, A., and GUDMUNDSSON, M. (2001), *Glacier-volcano interactions deduced by SAR interferometry*, J. Glaciol. 47(156), 58–70.
- BJÖRNSSON, H., PÁLSSON, F., and HARALDSSON, H. (2002), *Mass balance of Vatnajökull (1991–2001) and Langjökull (1996–2001)*, Iceland, Jökull 51, 75–78.
- BOURGEOIS, O., DAUTEUIL, O., and VAN VLIET-LANOË, B. (1998), *Pleistocene subglacial volcanism in Iceland: Tectonic implications*, Earth Planet. Sci. Lett. 164, 165–178.
- BÜRGER, S., JACOBY, W., HAGEDOORN, J., and WOLF, D. (2005), *Zeitliche Schwereänderungen und glazio-isostatische Ausgleichsprozesse am Vatnajökull, Südostisland*, Scientific Technical Report ISSN 1610-0956, GeoForschungsZentrum Potsdam, 69 pp., in German.
- DARBYSHIRE, F., WHITE, R., and PRIESTLY, K. (2000), *Structure of the crust and uppermost mantle of Iceland from a combined seismic and gravity study*, Earth Planet. Sci. Lett. 181, 409–428.
- DZIEWONSKI, A., and ANDERSON, D. (1981), *Preliminary reference Earth model*, Phys. Earth Planet. Int. 125, 297–356.
- EINARSSON, P., SIGMUNDSSON, S., HOFTON, M. A., FOULGER, G. R., and JACOBY, W. (1996), *An experiment in glacio-isostasy near Vatnajökull, Iceland, 1991*, Jökull 44, 29–40.
- EINARSSON, T. (1953), *Depression of the Earth's crust under glacier load. Various aspects*, Jökull 3, 2–5.
- EINARSSON, T. (1966), *Late- and post-glacial rise in Iceland and sub-crustal viscosity*, Jökull 16, 157–166.
- EINARSSON, T., and ALBERTSSON, K. (1988), *The glacial history of Iceland during the past three million years*, Philos. Trans. Roy. Soc. London B318, 637–644.
- FEDOROVA, T., JACOBY, W., and WALLNER, H. (2005), *Crust-mantle transition and Moho model for Iceland and surroundings from seismic, topography, and gravity data*, Tectonophysics 396, 119–140.
- FOULGER, G. (2006), *Older crust underlies Iceland*, Geophys. J. Internat. 165, 672–676.
- FOULGER, G., and PEARSON, D. (2001), *Is Iceland underlain by a plume in the lower mantle? Seismology and helium isotopes*, Geophys. J. Internat. 145, F1–F5.
- FOULGER, G., JAHN, C.-H., SEEGER, G., EINARSSON, P., JULIAN, B., and HEK1, K. (1992), *Post-rifting stress relaxation at the divergent plate boundary in Northeast Iceland*, Nature 358, 488–490.
- FOULGER, G., PRITCHARD, M., JULIAN, B., EVANS, J., ALLEN, R., NOLET, G., MORGAN, W., BERGSSON, B., ERLENDSSON, P., JAKOBSDÓTTIR, S., RAGNARSSON, S., STEFÁNSSON, R., and VOGFJÖRD, K. (2000), *The seismic anomaly beneath Iceland extends down to the mantle transition zone and no deeper*, Geophys. J. Internat. 142, F1–F5.
- FOULGER, G., DU, Z., and JULIAN, B. (2003), *Icelandic-type crust*, Geophys. J. Internat. 155, 567–590.
- GEE, M., TAYLOR, R., THIRLWALL, M., and MURTON, B. (1998), *Glacioisostasy controls chemical and isotopic characteristics of tholeiites from the Reykjanes Peninsula, SW Iceland*, Earth Planet. Sci. Lett. 164, 1–5.
- GUDMUNDSSON, H. (1997), *A review of the Holocene environmental history of Iceland*, Quaternary Sci. Rev. 16, 81–91.
- GUDMUNDSSON, M. T., SIGMUNDSSON, F., and BJÖRNSSON, H. (1997), *Ice-volcano interaction of the 1996 Gjálp subglacial eruption, Vatnajökull, Iceland*, Nature 389, 954–957.
- GUDMUNDSSON, O. (2003), *The dense root of the Iceland crust*, Earth Planet. Sci. Lett. 206, 427–440.

- GUDMUNDSSON, M., and BJÖRNSSON, H. (1991), *Eruptions in Grímsvötn, Vatnajökull, Iceland, 1934–1991*, Jökull 41, 21–45.
- HARTMANN, O. (2005), *Glazio-isotatische Ausgleichsprozesse im Südosten Islands*, Diplomarbeit Johannes Gutenberg-Universität Mainz, Mainz, 96 pp., in German.
- INGÓLFSSON, Ó., BJÖRCK, S., HAFLIDASON, H., and RUNDGREN, M. (1997), *Glacial and climatic events in Iceland reflecting regional North Atlantic climatic shifts during the Pleistocene-Holocene transition*, Quaternary Sci. Rev. 16, 1135–1144.
- INGÓLFSSON, O., and NORDDAHL, H. (1994), *A review of the environmental history of Iceland, 13000–9000 yr BP*, J. Quaternary Sci. 9(2), 147–150.
- IVINS, E., RAYMOND, C., and JAMES, T. (2000), *The influence of 5000 year-old and younger glacial mass variability on present-day crustal rebound in the Antarctic Peninsula*, Earth Planets and Space 52, 1023–1029.
- IVINS, E. and JAMES, T. (2004), *Bedrock response to Llanquihue Holocene and present-day glaciation in southern South America*, Geophys. Res. Lett. 31(L24613), 10.1029/2004GL021500.
- JACOBY, W., BÜRGER, S., SMILDE, P., and WALLNER, H. (2000), *Temporal gravity variations observed in SE Iceland*, InterRidge News 10(1), 52–55.
- JENNINGS, A., SYVITSKI, J., GERSON, L., GRÖNVOLD, K., GEIRSDÓTTIR, A., HARDARDÓTTIR, J., ANDREWS, J., and HAGEN, S. (2000), *Chronology and paleoenvironments during the late Weichselian deglaciation of the southwest Iceland shelf*, Boreas 29, 167–183.
- JOHANNESSEN, T. and SIGURÖSSON, O. (1998), *Interpretation of glacier variations in Iceland 1930–1995*, Jökull 45, 27–33.
- JULL, M. and MCKENZIE, D. (1996), *The effect of deglaciation on mantle melting beneath Iceland*, J. Geophys. Res. 101(B10), 21815–21828.
- KABAN, M., FLOVENZ, O., and PALMASON, G. (2002), *Nature of the crust-mantle transition zone and the thermal state of the upper mantle beneath Iceland from gravity modelling*, Geophys. J. Internat. 149, 281–299.
- KUMAR, P., KIND, R., HANKA, W., WYLEGALLA, K., REIGBER, C., YUAN, X., WOELBERN, I., SCHWINTZER, P., FLEMING, K., DAHL-JENSEN, T., LARSEN, T., SCHWEITZER, J., PRIESTLY, K., GUDMUNDSSON, O., and WOLF, D. (2005), *The lithosphere-asthenosphere boundary in the North-West Atlantic region*, Earth Planet. Sci. Lett. 236, 249–257.
- LAMBECK, K., SMITHER, C., and JOHNSTON, P. (1998), *Sea-level change, glacial rebound and mantle viscosity for northern Europe*, Geophys. J. Internat. 134, 102–144.
- LARSEN, C., MOTYKA, R., FREYMUELLER, J., ECHELMAYER, K.A., and IVINS, E. (2005), *Rapid viscoelastic uplift in southeast Alaska caused by post-little Ice Age glacial retreat*, Earth Planet. Sci. Lett. 237, 548–560.
- MACLENNAN, J., JULL, M., MCKENZIE, D., SLATER, L., and GRÖNVOLD, K. (2002), *The link between volcanism and deglaciation in Iceland*, Geochim. Geophys. Geosyst. 3(11), 1–25.
- MAGNÚSSON, E., BJÖRNSSON, H., DALL, J., and PÁLSSON, F. (2005a), *Volume changes of Vatnajökull ice cap, Iceland, due to surface mass balance, ice flow, and subglacial melting at geothermal areas*, Geophys. Res. Lett. L05504, 10.1029/2004GL021615.
- MAGNÚSSON, E., BJÖRNSSON, H., DALL, J., and PÁLSSON, F. (2005b), *The 20th century retreat of ice caps in Iceland derived from airborne SAR: W-Vatnajökull and N-Mýrdalsjökull*, Earth Planet. Sci. Lett. 237, 508–515.
- MARTINEC, Z. (2000), *Spectral-finite element approach to three-dimensional viscoelastic relaxation in a spherical earth*, Geophys. J. Internat. 142, 117–141.
- MENKE, W. (1999), *Crustal isostasy indicates anomalous densities beneath Iceland*, Geophys. Res. Lett. 26(9), 1215–1218.
- MITROVICA, J. (1996), HASKELL (1935) revisited, J. Geophys. Res. 101(B1), 555–569.
- NORDDAHL, H. (1990), *Late Weichselian and early Holocene deglaciation history of Iceland*, Jökull 40, 27–50.
- NORDDAHL, H. and EINARSSON, T. (2001), *Concurrent changes of relative sea-level and glacier extent at the Weichselian-Holocene boundary in Berufjordur, Eastern Iceland*, Quaternary Sci. Rev. 20, 1607–1622.
- POLLITZ, F. F. and SACKS, I. S. (1996), *Viscosity structure beneath northeast Iceland*, J. Geophys. Res. 101, 17771–17793.

- RIGNOT, E., RIVERA, A., and CASASSA, G. (2003), *Contribution of the Patagonia Icefields of South America to Sea Level Rise*, *Science* 302, 434–437.
- RITSEMA, J. and ALLEN, R. (2003), *The elusive mantle plume*, *Earth Planet. Sci. Lett.* 207, 1–12.
- RUNDGREN, M., INGÓLFSSON, Ó., BJÖRCK, S., JIANG, H., and HAFLIÓASON, H. (1997), *Dynamic sea-level change during the last deglaciation of northern Iceland*, *Boreas* 26, 201–215.
- SHEN, Y., SOLOMON, S., BJARNASON, I., and WOLFE, C. (1998), *Seismic evidence for a lower-mantle origin of the Iceland plume*, *Nature* 395, 62–65.
- SIGMUNDSSON, F. (1991), *Post-glacial rebound and asthenosphere viscosity in Iceland*, *Geophys. Res. Lett.* 18(6), 1131–1134.
- SIGMUNDSSON, F. and EINARSSON, (1992), *Glacial-isostatic crustal movements caused by historical volume change of the Vatnajökull ice cap, Iceland*, *Geophys. Res. Lett.* 19(21), 2123–2126.
- SIGVALDASON, G., ANNERTZ, K., and NILSSON, M. (1992), *Effect of glacier loading/deloading on volcanism: postglacial volcanic production rate of Dynjufjöll area, central Iceland*, *Bull. Volcanol.* 54, 385–392.
- SJÖBERG, L., PAN, M., ASENJO, E., and ERLINGSSON, S. (2000), *Glacial rebound near Vatnajökull, Iceland, studied by GPS campaigns in 1992 and 1996*, *J. Geodyn.* 29, 63–70.
- SJÖBERG, L. E., PAN, M., ERLINGSSON, S., ASENJO, E., and ARNASON, K. (2004), *Land uplift near Vatnajökull, Iceland, as observed by GPS in 1992, 1996 and 1999*, *Geophys. J. Internat.* 159, 943–948.
- SLATER, L., JULL, M., MCKENZIE, D., and GRONVÖD, K. (1998), *Deglaciation effects on mantle melting under Iceland: Results from the northern volcanic zone*, *Earth Planet. Sci. Lett.* 164, 151–164.
- THOMA, M., WOLF, D., and NEUMEYER, J. (2001) *Inverting land uplift near Vatnajökull, Iceland, in terms of lithosphere thickness and viscosity stratification*. In *Gravity, Geoid and Geodynamics* (ed. M. Sideris), pp. 97–102 (Springer-Verlag, Berlin 2001).
- WOLF, D., BARTHELMES, F., and SIGMUNDSSON, F. (1997b), *Predictions of deformation and gravity changes caused by recent melting of the Vatnajökull ice cap, Iceland*. In *Gravity, Geoid and Marine Geodesy* (eds. J. Segawa, H. Jijimoto, and S. Okuba) pp. 311–319, (Heidelberg 1997a).
- WOLF, D., BARTHELMES, F., and SIGMUNDSSON, F. (1997b), *Predictions of deformation and gravity variation caused by recent change of the Vatnajökull ice cap, Iceland*, *Comptes Rendus Journées Luxembourgeoises Géodynamique* 82, 36–42.
- WOLFE, C., BJARNASON, I., VANDECAR, J., and SOLOMON, S. (1997), *Seismic structure of the Iceland mantle plume*, *Nature* 385, 245–247.

(Received July 16, 2005, revised June 17, 2006, accepted July 6, 2006)

To access this journal online:
www.birkhauser.ch/pag

Review of Microgravity Observations at Mt. Etna: A Powerful Tool to Monitor and Study Active Volcanoes

DANIELE CARBONE and FILIPPO GRECO

Abstract—Microgravity observations at Mt. Etna have been routinely performed as both discrete (since 1986) and continuous (since 1998) measurements. In addition to describing the methodology for acquiring and reducing gravity data from Mt. Etna, this paper provides a collection of case studies aimed at demonstrating the potential of microgravity to investigate the plumbing system of an active volcano and detect forerunners to paroxysmal volcanic events. For discrete gravity measurements, results from 1994–1996 and 2001 are reported. During the first period, the observed gravity changes are interpreted within the framework of the Strombolian activity which occurred from the summit craters. Gravity changes observed during the first nine months of 2001 are directly related to subsurface mass redistributions which preceded, accompanied and followed the July–August 2001 flank eruption of Mt. Etna. Two continuous gravity records are discussed: a 16-month (October 1998 to February 2000) sequence and a 48-hour (26–28 October, 2002) sequence, both from a station within a few kilometers of the volcano’s summit. The 16-month record may be the longest continuous gravity sequence ever acquired at a station very close to the summit zone of an active volcano. By cross analyzing it with contemporaneous discrete observations along a summit profile of stations, both the geometry of a buried source and its time evolution can be investigated. The shorter continuous sequence encompasses the onset of an eruption from a location only 1.5 km from the gravity station. This gravity record is useful for establishing constraints on the characteristics of the intrusive mechanism leading to the eruption. In particular, the observed gravity anomaly indicates that the magma intrusion occurred “passively” within a fracture system opened by external forces.

Key words: Mt. Etna, Microgravity, magma sources, modeling.

1. Introduction

Temporal gravity changes in volcanic regions are related to subsurface mass-redistributions and/or surface elevation changes in response to magmatic activity, and their amplitude, wavelength and duration are a function of the size, depth and evolution of the magma bodies involved. The associated gravity anomalies can thus vary significantly in both space (wavelengths ranging from hundreds of meters to tens of kilometers) and time (periods ranging from minutes to years). The choice of gravity monitoring strategy is therefore crucial for investigating the anomalies of interest.

Usually, microgravity measurements are collected in a “discrete” fashion: a single instrument is used to make a series of measurements at different locations, with each

gravity value referenced to a stable site (i.e., where changes in the gravity field over time, due to magmatic activity, are not expected). Repeat surveys of the network will expose any time variations in the difference between the base and other stations. After removing the effect of Earth tides and instrumental drift (TORGE, 1989), the observed variations of the gravity field over time can be interpreted in terms of subsurface mass-redistributions.

A disadvantage of discrete gravity surveys is the lack of information regarding the rate at which the volcanic processes occur, since changes in subsurface mass distribution can only be determined for the time between successive surveys (which usually ranges from a week to a year). As a result, there will remain some ambiguity as to the nature of the processes causing the changes. Further, logistical problems (for example, snow coverage during winter months) can prevent discrete gravity surveys from being performed at regular intervals. Coupled with the desire to reduce the exposure of personnel in hazardous areas, there is a need to supplement the repeated surveys with continuous observations at key sites. However, given the high costs of gravimeters, only a few instruments can be deployed to operate in continuous mode, limiting the achievable spatial resolution.

Discrete microgravity surveys have been carried out at Mt. Etna since 1986 using a network of up to 70 stations. Measurements at these stations have identified mass redistributions occurring at depths between about 6 km below sea level to within a few hundred meters of the surface. These results have been correlated with the ensuing volcanic activity (BUDETTA *et al.*, 2004).

From 1998, the Etna discrete gravity network has been supplemented by three continuously running gravity stations, which worked intermittently ever since. Continuous microgravity observations at Etna have allowed high-frequency variations in gravity to be observed and interpreted as a consequence of shallow intrusive processes or changes in the gas content within the upper levels of the plumbing system (CARBONE *et al.*, 2003a; BRANCA *et al.*, 2003).

In the following, some relevant case studies from Etna are presented to show how discrete and continuous gravity studies can be used separately or, better, in conjunction to gather a more complete picture of those volcanic phenomena which can produce underground mass redistributions, and thus variation of the gravity field, at volcanic zones.

2. Discrete Gravity Measurements at Mt. Etna

2.1. Network Description

The Etna network for discrete microgravity measurements was established in 1986 as an array of 20 stations located 3–4 km apart (BUDETTA *et al.*, 1989; BUDETTA and CARBONE, 1998). By 2006, the network had been expanded to include 70 stations

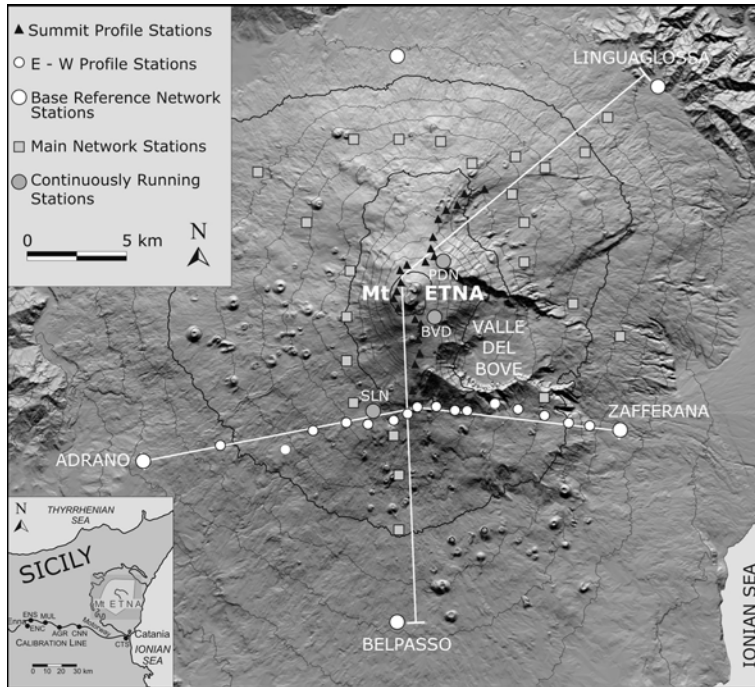


Figure 1

Location map of Mt. Etna showing the positions of both the benchmarks for discrete microgravity measurements and the continuously running microgravity stations. The position of the Catania - Enna calibration line is shown in the bottom left inset.

(Fig. 1) located 0.5 to 4 km apart and covering an area of about 400 km². Four array subsets have been distinguished (dates of installation in brackets): (1) the Main Network (1986); (2) the Summit Profile (1992); (3) the East-West (E-W) Profile (1994) and (4) a four-station Base Reference Network (1994). The subarrays differ from one another in station density, access (determined by snow coverage), and the time required to collect gravity measurements. Each subarray can be occupied independently, optimizing the flexibility in data collection to accommodate variations in activity and accessibility of the volcano.

A calibration line was also established in February 1995 to investigate systematic variations in instrumental calibration factors with time. The line runs 80 Km along the motorway between CATANIA and ENNA (Fig. 1) and consists of six stations (BUDETTA and CARBONE, 1997).

2.2. Data Collection

Measurements over the entire Etna microgravity network are generally conducted every six months. To keep the instrumental drift under control, the measurement schedule called “step method” (TORGE, 1989) is followed. The instrument is moved

between each pair of adjacent stations until at least three coherent gravity differences (with standard deviation within a predicted value) are assessed. Successively, the same operation is repeated for the next pair of stations. Some parts of the array are reoccupied more frequently (approximately monthly measurements along the E-W and Summit Profiles, although snow coverage restricts measurements along the Summit Profile to summer months). Due to logistic reasons, a different measurement schedule is followed for the stations along the calibration line and Summit Profile. These stations are occupied in sequence and the arrays are traversed at least two (Summit Profile) or three (calibration line) times for each survey (a procedure that is termed the “profile method”; TORGE, 1989).

Since 1994, discrete gravity measurements at Mt. Etna have been accomplished using a Scintrex CG3-M gravimeter (serial num. 9310234). A real-time compensator fitted to the gravimeter allows instrumental drift to be corrected to a few tens of $\mu\text{Gal/day}$ (BUDETTA and CARBONE, 1997). Variations in the CG-3M calibration factor over time are determined to a precision of about 30 ppm (BUDETTA and CARBONE, 1997) by repeatedly occupying the Catania - Enna calibration line (Fig. 1).

Even when subjected to the challenging conditions encountered at Mt. Etna (rough unpaved roads, large elevation differences, etc.), the Scintrex CG-3M gravimeter yields a high precision thanks to its low sensitivity to both shocks and external temperature changes. BUDETTA and CARBONE (1997) and CARBONE *et al.* (2003b) evaluated the uncertainties in measurements collected with the gravimeter on Etna to be $\pm 7 \mu\text{Gal}$ ($\pm 11 \mu\text{Gal}$ at stations along the Summit Profile, connected by very dirt tracks and measured using the profile method; $1 \mu\text{Gal} = 10 \text{ nm s}^{-2}$) at the 95% confidence interval. According to RYMER (1989), the error on gravity change measurements obtained by surveys completed at different times is given by $\sqrt{2} * e$ (where e is the error of a single survey). Thus, at the 95% confidence interval, the error on temporal gravity differences on Etna is $10 \mu\text{Gal}$ ($15 \mu\text{Gal}$ along the Summit Profile).

2.3. Case Studies

2.3.1. The 1994–1996 summit anomalies. Between September 1994 and October 1995, a positive gravity change of about $40 \mu\text{Gal}$ was recorded along the Etna Summit Profile (Fig. 2b) near the summit craters. This anomaly reversed between October 1995 and July 1996, while a new positive anomaly, also of about $40 \mu\text{Gal}$ (Fig. 2c), appeared further south, centered upon CS and CZ. These stations are the closest ones to the fracture system which opened in 1989 and was reactivated during the 1991–1993 eruption (Fig. 2a). Elevation changes able to cause significant gravity effects did not occur between 1994 and 1996 (PUGLISI and BONFORTE, 1999). Thus, the gravity changes observed during the same period can be assumed to be due only to subsurface mass redistribution. Accordingly, they are not inverted through elastic modeling. Rather, to assess the geometrical characteristic of the source and the amount of mass involved in the redistribution process, a 3-D program, able to

calculate at any observation point (in its actual position on the topographic surface) the gravity effect of buried masses, is utilized. This program is called GRAVERSE and was designed on-purpose under the LabVIEW[®] environment (CARBONE, 2002). It results that the September 1994 to October 1995 gravity variation can be related to a spherical source beneath the Central Craters at an elevation of about 1000 m a.s.l. (2000 m below the surface; Source 1 — Figs. 2a and 3). The spherical source is also

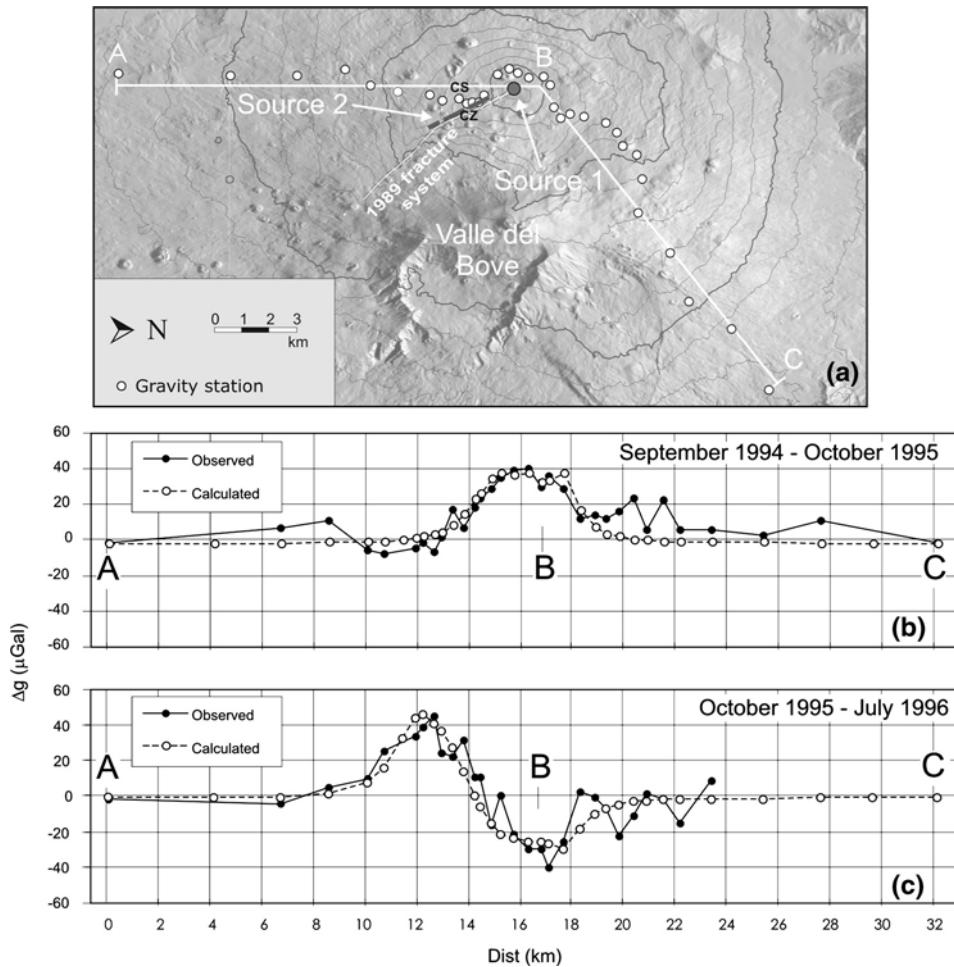


Figure 2

Gravity changes along the Summit Profile (a) between 1994 and 1996. The September 1994–October 1995 variations (b) are attributed to the arrival of new magma in a source zone modeled as a sphere whose projection onto the surface (Source 1) is shown as a gray circle in (a). The October 1995–July 1996 variations (c) are associated with a withdrawal of magma from the source zone responsible for the previous change and the contemporary arrival of new magma in a source zone that is modeled as a prism-shaped body, whose projection onto the surface (Source 2) is also shown in (a) as a gray line (modified after BUDETTA *et al.*, 1999).

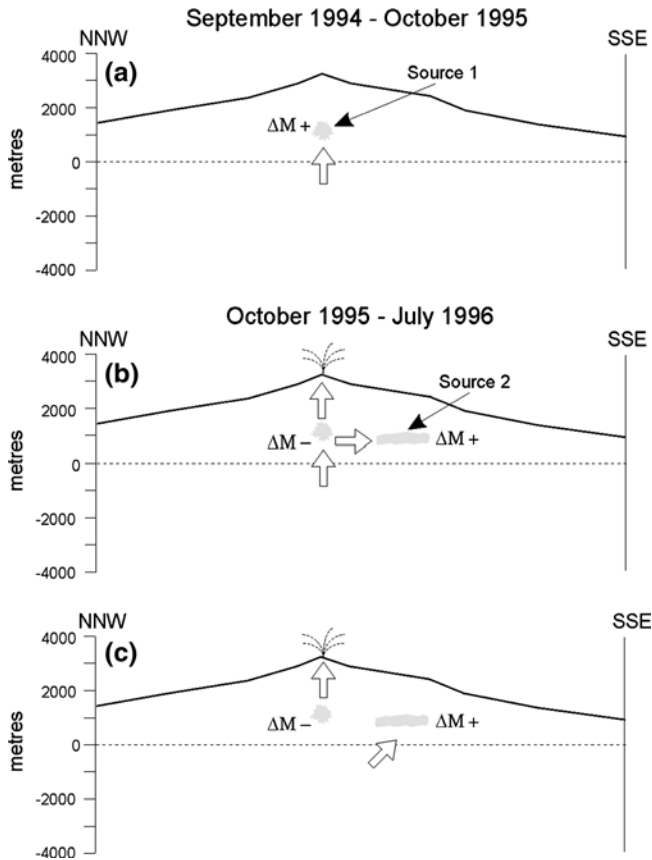


Figure 3

Schematic representation showing mass redistributions detected by gravity changes during the September 1994–July 1996 interval. (a) During the September 1994–October 1995 period, a mass increase occurs 2000 m beneath the summit (Source 1). Two possible sequences of events can explain the October 1995–July 1996 gravity variations: (b) shallow-level magma is intruded laterally (towards Source 2), allowing the ascent of magma to the summit from deeper regions; (c) magma from depth feeds the lateral intrusion (Source 2) directly, as well as replacing shallow-level magma which is erupted at the summit. $\Delta M+$ and $\Delta M-$ indicate mass increase and decrease, respectively (modified after BUDETTA *et al.*, 1999).

consistent with the central part of the October 1995 to July 1996 gravity decrease (Fig. 2c), while an elongate source (Source 2 — Figs. 2a and 3) can be used to explain the positive change centered on stations CS and CZ (BUDETTA *et al.*, 1999). A prismatic model, following the track of the northernmost part of the 1989 fracture, yields a good fit to the observations for a length of 2.5 km, a vertical extent of 0.5, and a top about 800–1000 m above the sea level (1500 m below the surface; Source 2 — Figs. 2a and 3).

The central gravity increase (1994–1995)/ decrease (1995–1996) cycle reflects either voids around the central conduit being filled and drained, or changes in

magmatic vesicularity (DZURISIN *et al.*, 1980). The occurrence of the eccentric 1995–1996 gravity increase, close to the 1989 fracture system, suggests a passive mode of intrusion, perhaps with magma filling voids in the fracture system. This hypothesis is supported by the absence of significant accompanying seismicity at shallow depth (SPAMPINATO *et al.*, 1998).

The magnitudes of the mass changes indicated by 3-D modelling for gravity changes along the central part of the transect are an increase of 2×10^{10} kg (corresponding to a volume of about 10^7 m³, assuming a density of the vesiculated magma of 2000 kg/m³) between September 1994 and October 1995, and a comparable decrease between October 1995 and July 1996. For example, a 10 vol.% of melt in the source zone implies that the inferred mass would correspond to a sphere with a radius of about 300 m.

A mass increase of about the same magnitude seems to have occurred within Source 2, suggesting a mean width of 8 m for the intrusion. Such a width is greater than that of classical Etnean dykes, whose mean thickness is about 2 m (FERRARI, 1991). Possibly, therefore, the intrusion occurred as a collection of parallel dykes.

Between November 1995 and February 1996, at least ten episodes of paroxysmal Strombolian activity occurred at the summit craters (ARMIENTI *et al.*, 1996), their explosive nature indicating the eruption of fresh, gas-rich magma. The largest eruptive episode occurred on 23 December 1995 and expelled about 3×10^6 m³ of scoria and lapilli (ARMIENTI *et al.*, 1996), corresponding to 5×10^9 kg, if a density of 1800 kg/m³ is assumed (ANDRONICO *et al.*, 2001). It is feasible, therefore, that the total mass ejected by the entire sequence of eruptions was similar to that of the shallow intrusion detected in 1994–1995 by summit gravity changes (Fig. 2b).

The gravity data are consistent with both (a) the lateral injection of the first shallow intrusion (1994–1995) into the 1989 fracture system (Fig. 3b), a process allowing the ascent and eruption of new magma and (b) the summit activity having been fed by the first shallow intrusion, while new magma entered the 1989 fracture system (Fig. 3c).

2.3.2. Gravity changes related to the 2001 eruption. Activity at Mt. Etna during late 2000 and early 2001 consisted of episodic Strombolian and ash explosions, and lava emission from the summit craters. After late January, lava emission concentrated at the Southeast Crater (LAUTZE *et al.*, 2004). Effusive activity escalated during the months that followed, with many paroxysmal episodes of fire fountaining accompanied by lava effusion. The 2001 flank eruption started on 17 July, when an eruptive fissure opened at the base of the Southeast Crater, producing a lava flow that spread into the Valle del Bove. The fissure system propagated to both the north and south until late July. On 19 July, an eruptive vent opened at 2570 m on the upper southern flank of the volcano with a series of violent phreatomagmatic explosions. The rate of lava effusion rapidly diminished beginning in late July, and by 9 August the eruption ended, having produced an estimated volume of ca. 48 million m³ (INGV-CT, 2001).

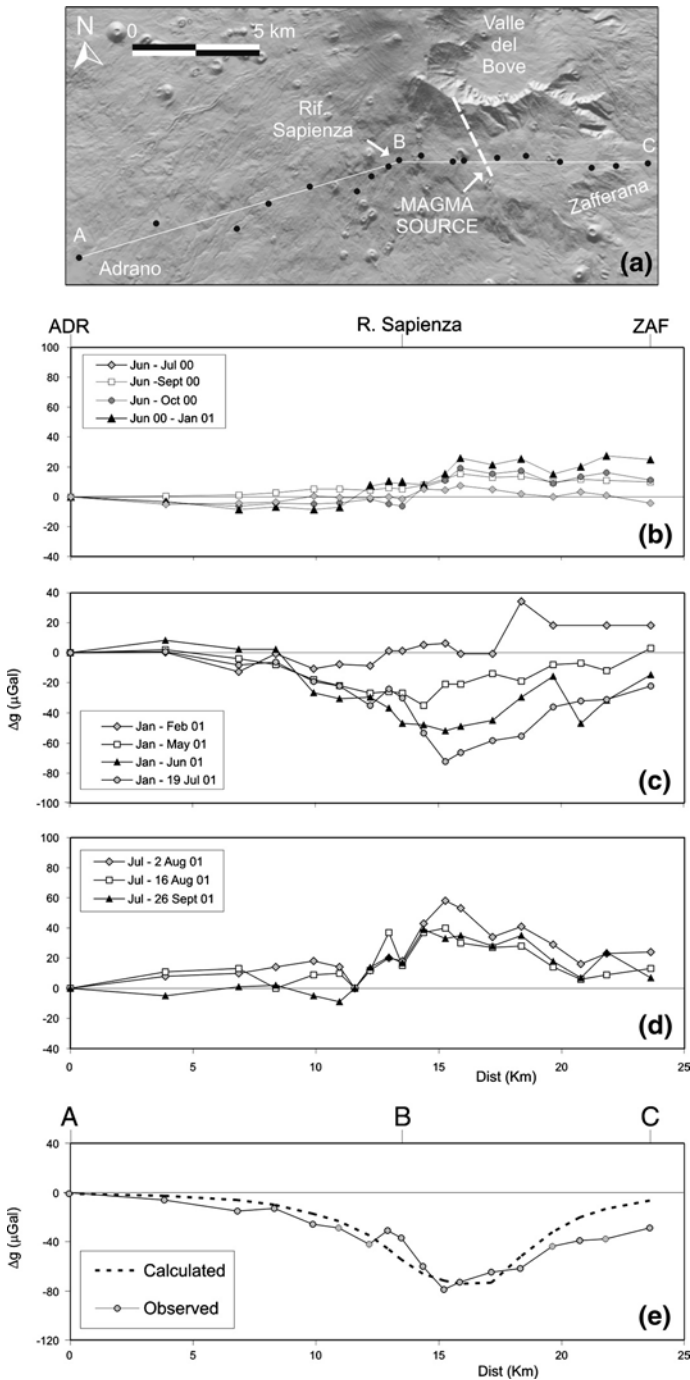
During the early part of this activity (June 2000 – January 2001), only a weak gravity increase (within about 20 μGal ; CARBONE *et al.*, 2003c) was observed (Fig. 4b). A progressive gravity decrease along the E-W profile (Fig. 1) began sometime between the surveys in February and May 2001 (Fig. 4c), and continued until July 2001. The maximum amplitude of the decrease was about 80 μGal , with a wavelength of the anomaly of the order of 15 km (Fig. 4c). Between 19 July and 2 August 2001, a period encompassing the early stages of the eruption, a gravity increase was observed along the E-W Profile (Fig. 4d), which partially recovers the previous gravity decrease. No further significant gravity changes were observed until September 2001. GPS measurements taken between 2000 and 2001 indicated that elevation changes along the E-W Profile were less than 5 cm (BONFORTE *et al.*, submitted). Thus, it is reasonable to assume that the majority of the 2000–2001 gravity changes are caused by subsurface mass redistributions.

CARBONE *et al.* (2003c) assumed that the source of the January–July 2001 gravity decrease had the same geometry as that predicted for previously observed gravity changes at Mt. Etna, i.e., elongated and oriented NNW–SSE (BUDETTA and CARBONE, 1998; BUDETTA *et al.*, 1999; CARBONE *et al.*, 2003b). This source-volume is thought to play a key role in regulating the flux of magma from the deeper mantle source to the upper plumbing system of Etna (CARBONE *et al.*, 2003b), and coincides with a location of partial melt and/or unfilled fractures identified in seismic velocity studies (LAIGLE *et al.*, 2000). A 3-D calculation performed through GRAVERSE (CARBONE, 2002; see previous section) shows that the observed data can be approximated (Fig. 4e) by a body with a length of about 3 km, a top depth of about 2 km b.s.l. (about 4.5 km below the surface), and a mass change of about $2.5 \cdot 10^{11}$ kg (the projection of the source onto the surface is shown in Fig. 4a).

Gradual changes in mass, with periods of about three years, are believed to have taken place approximately within the same volume between 1994 and 1999 due to fluctuations in the magma/host-rock ratio (BUDETTA *et al.*, 1999; CARBONE *et al.*, 2003b). Conversely, the relatively fast (4–5 month) pre-eruptive gravity decrease and the even faster (14 days) increase in late July, after the start of the 2001 eruption, are thought to represent a rapid event, which left open voids in the source region, followed by collapse and/or refilling of the open spaces by new magma from depth (CARBONE *et al.*, 2003c). Seismic data indicate a progressive renewal of tensile stresses during the first months of 2001, with hundreds of earthquakes clustering on the southeastern sector of the volcano, primarily along a NNW–SSE trend and at a depth of 1–5 km. This pattern coincides with the inferred source of the January–July 2001 gravity

Figure 4

Gravity changes observed along the E-W profile (a) between June 2000 and January 2001 (b), between January and 19 July, 2001 (c), and between 19 July, 2001 and 26 September, 2001 (d). In (e) the gravity effect of the best-fit model source, whose projections onto the surface are shown in (a) as a dashed line, is shown (modified after CARBONE *et al.*, 2003b).



decrease (BONACCORSO *et al.*, 2004) and suggests that the observed initial gravity variations could have been caused by opening of new void space by tectonic activity.

Petrological and volcanological evidence (INGV-CT, 2001; POMPILIO *et al.*, 2001, CORSARO *et al.*, 2007) imply that during the 2001 eruption, vents below an elevation of 2750 m were fed by a magma that evolved at high pressures and may have ascended rapidly. The gravity increase observed during 19 July – 2 August, 2001 along the E-W Profile could have been caused by the emplacement of magma from the deep storage system into the new path opened by tectonic stresses. Although this process is likely to have started before 19 July, the gravity increase could have been overwhelmed by the effective density decrease of the host rock caused by the tectonic extension. This model, which requires relatively rapid magma ascent, agrees with the petrological and volcanological characteristics of the erupted products.

Using continuous tilt and GPS measurements, BONACCORSO *et al.* (2002) detected the emplacement of a N-S trending dyke whose modeled volume is about one third that which was erupted. Since no other intrusion was inferred to have formed inside the volcanic edifice during the months preceding the eruption, they concluded that the 2001 eruption must have been fed by material coming directly from a deeper region. This result agrees with our model of rapid magma flow from a deep storage system to the surface, mainly occurring while the eruption was in progress.

3. Continuous Gravity Measurements at Mt. Etna

The three continuous Etna gravity stations (Fig. 1) are equipped with LaCoste and Romberg (L&R) spring gravimeters, each featuring an electronic feedback system. The stations are located (Fig. 1): ca. 10 km south of the active craters and outside the Serra la Nave Astrophysical Observatory (SLN; 1740 m a.s.l.), 2 km NE of the summit North-East Crater at the Pizzi Deneri Volcanological Observatory (PDN; 2920 m a.s.l.), and just 600 m S of the summit South-East Crater (BVD; 2920 m a.s.l.). At SLN and BVD the sensors are installed in partially buried concrete boxes (Fig. 5a), while at PDN the gravity station is inside the observatory building.

3.1. Setup of the Continuous Gravity Stations

The conditions at a gravity site close to the summit of an actively erupting vent are far from a clean, ideal laboratory setting; thus, it is quite difficult to attain the

Figure 5

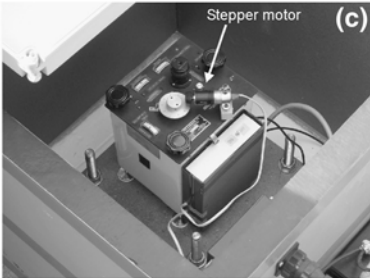
Continuously running gravity station at Mt. Etna. (a) Panoramic view showing the relative position of BVD station and the Southeast Crater (one of the summit craters of Etna), 600 m away from the station. The sensor is placed inside a partially buried concrete box. (b) The interior of the concrete box. The polystyrene thermo-insulating box containing the gravimeter, batteries, and waterproof plastic boxes hosting the electronics are visible. (c) Particular of the gravimeter. The stepper motor to remotely reset the meter is mounted on its black lead. (d) Schematic of the three-component setup of Etna's continuous gravity stations.



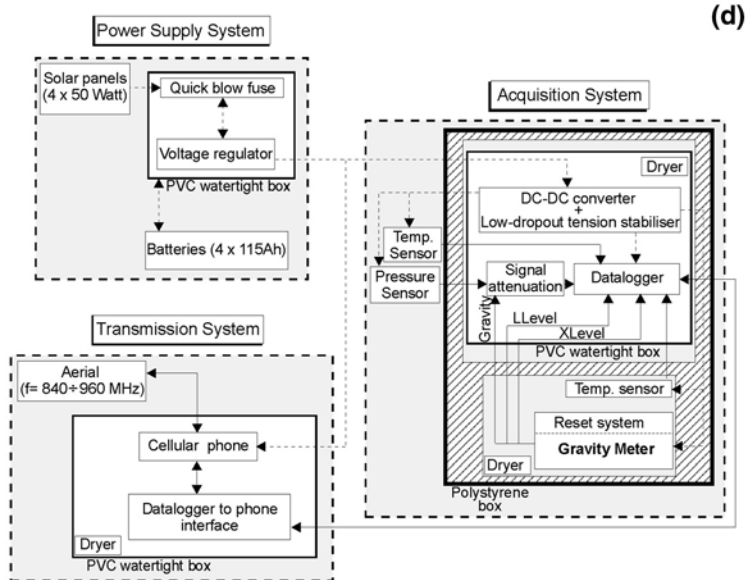
(a)



(b)



(c)



(d)

high precision necessary for reliable gravity results. At Mt. Etna, we have developed a three-component setup (Fig. 5d) for field stations that is robust, easy to remove and re-establish, and relatively inexpensive.

The station is powered by solar panels connected to trickle-charged batteries (Fig. 5b). To provide a constant power supply (within a few hundredths of a volt) to the feedback systems of the recording L&R meters, a DC-DC converter coupled with a low-dropout tension stabilizer is used.

The acquisition system includes the gravity meter itself, which outputs analog signals representing the gravity field, and tilt changes. Changes in the ground tilt are recorded through the two levels fitted to each L&R instrument which measure along a direction parallel (long level) and perpendicular (cross level) to the meter's beam, respectively (resolution = 2.5 μ rad; LACOSTE and ROMBER, 1997). Sensors that record the atmospheric temperature and pressure are also installed at each station. The entire system is placed inside a thermally insulating polystyrene container (Figs. 5b and c). Data are acquired every second. The average over 60 measurements is then calculated and stored in the solid-state memory of the data logger (at 1 datum/min).

Data are automatically downloaded to the INGV - Sezione di Catania (Catania, Italy) every 24 hours by a transmission system that employs a cellular or wireless connection. Using a computer in Catania it is also possible to (a) remotely activate the stepper motor, which turns the meter dial and allows the meter to be reset (Fig. 5c), and (b) monitor in real time all the parameters that are recorded.

3.2. Data Reduction

Once the gravity data are transmitted to the INGV in Catania, they are pre-analyzed in real-time using the GraVisual software, designed in the LabView® environment (CARBONE, 2002). In addition to correcting for instrumental resets and interpolating gaps in the data sequences, this software allows the data to be compensated for the effects of:

1. Earth tide;
2. instrumental drift;
3. ground tilt;
4. atmospheric pressure

The effect of Earth tides (amplitude up to 200 μ Gal peak-to-peak depending on latitude, elevation and stage in the tidal cycle) is modeled following the tidal potential catalog from TAMURA (1987). We computed the synthetic tide through the Eterna 3.30 data processing package (WENZEL, 1996), using the local tidal parameters deduced from local recordings and the Wahr-Dehant-Zschau inelastic Earth model (DEHANT, 1987).

For gravity sequences shorter than about one month, the instrumental drift is usually modeled as a first degree curve, while low degree polynomial curves are utilized to model the instrumental drift over longer sequences (BONVALOT *et al.*, 1998).

Ground tilt changes are measured using the electronic levels fitted to L&R gravimeters (resolution = 2.5 μ rad; LACOSTE and ROMBERG, 1997). The gravity effect of the tilt changes is calculated using the formula (SCINTREX LIMITED, 1992):

$$\delta g = g(1 - \cos X \cos Y), \quad (1)$$

where g is an average gravity value (980.6 Gal).

The correction for the effect of atmospheric pressure is performed using the value of the theoretical local admittance (-0.365μ Gal/mbar), which is a combination of (1) the gravitational attraction of the air column, and (2) the distortion of the Earth's surface resulting from barometric changes (SPRATT, 1982; NIEBAUER, 1988; MERRIAM, 1992).

Changes in the atmospheric temperature have been shown to influence the signal from continuously recording gravimeters (ANDÒ and CARBONE, 2001, 2004). It is now well established that apparent gravity changes depend on the temporal development and magnitude of the temperature change that caused them as well as on the insulation and compensation of the spring gravimeter utilized (CARBONE *et al.*, 2003a). Thus, the correction formulas are instrument-specific and often frequency-dependent. Accordingly, case-by-case nonlinear approaches must be followed in order to reduce the signal from continuously running gravity meters for the effect of temperature. As a result, the reduction for the temperature effects is not included as a standardized procedure through the GraVisual software. It should also be noted that the most important temperature effects occur over periods greater than about 1 month (CARBONE *et al.*, 2003a). Thus, if the gravity sequence being analyzed is less than 1 month in duration, the effects of atmospheric temperature changes can be neglected.

3.3. Case Studies

In the following, two continuous gravity sequences coming from PDN station (section 3 and Fig. 1) will be presented and discussed. Unfortunately, the other two stations (TDF and SLN section 3 and Fig. 1); were not working during the two selected periods.

3.3.1. The October 1998 – February 2000 gravity sequence. Between October 1998 and February 2000, an almost continuous gravity sequence was acquired at PDN station. Only 8 interruptions of the continuous record, with lengths ranging between 5 and 90 hours, occurred as a result of temporary power failures, seismic shocks sending the proof mass outside the measurement range of the feedback system, and other causes. The first degree instrumental drift rate of the data sequence is about 285 μ Gal/month, while the mean background noise is less than 1 μ Gal.

Raw gravity data are presented in Figure 6a. The same sequence, reduced for the effect of Earth tide, instrumental drift, ground tilt, atmospheric pressure, and atmospheric temperature is shown in Figure 6b. The first four corrections were accomplished as noted in the previous section, while the effect of temperature was corrected following the procedure outlined by ANDÒ and CARBONE (2001), which led to a reduction in the amplitude of the signal of about 90% (Carbone *et al.*, 2003a). The main features of the reduced gravity sequence (Fig. 6b) are cycles with periods of about 6 months and peak-to-peak amplitudes of about 100 μGal . The 6-month periodicity is not reflected neither in the pressure nor in the tilt data.

In Figure 6b data acquired through discrete measurements at station CO (about 500 m from PDN; Fig. 7a) in June, July, September, and October 1999 are also shown (black dots). The excellent agreement between continuous and discrete data suggests that the variations in both were caused by the same source.

The geometrical characteristics of this source are assessed through a 3-D inversion on the discrete measurements along the Summit Profile between June and September 1999 (Fig. 7c). Assuming a prismatic shape and an orientation following the trend of Etna's Northeast rift (GARDUÑO *et al.*, 1997; Fig. 7a), a good fit with

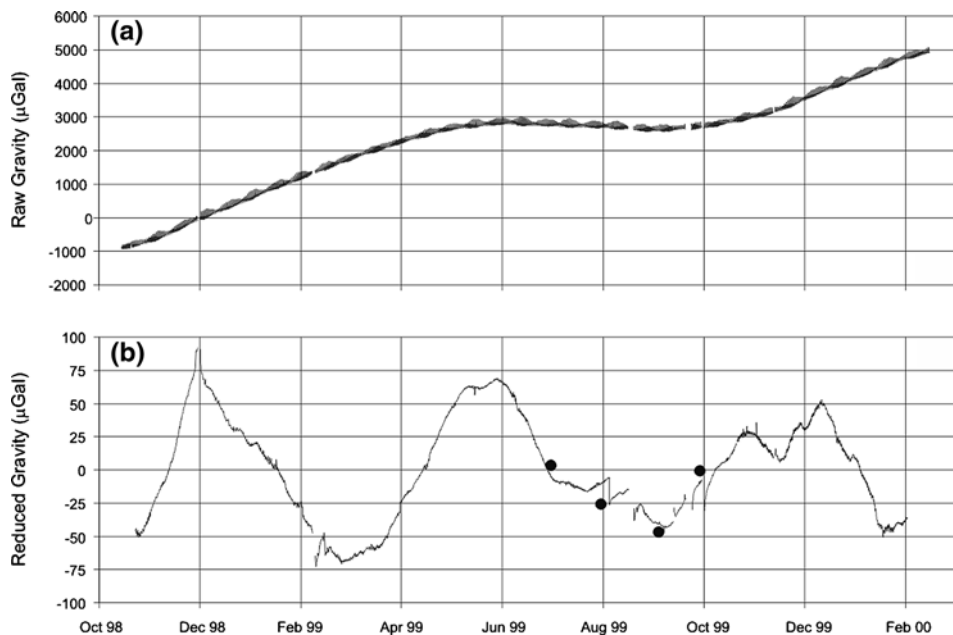


Figure 6

(a) Raw gravity data recorded at station PDN station during the October 1998–February 2000 time interval. (b) The same sequence as in (a), reduced for the effects of Earth tide, instrumental drift, ground tilt, atmospheric pressure, and atmospheric temperature. Filled circles in (b) are gravity changes from discrete relative observations at CO, a site very close to the continuous station (also see Fig. 7; modified after CARBONE *et al.*, 2003c).

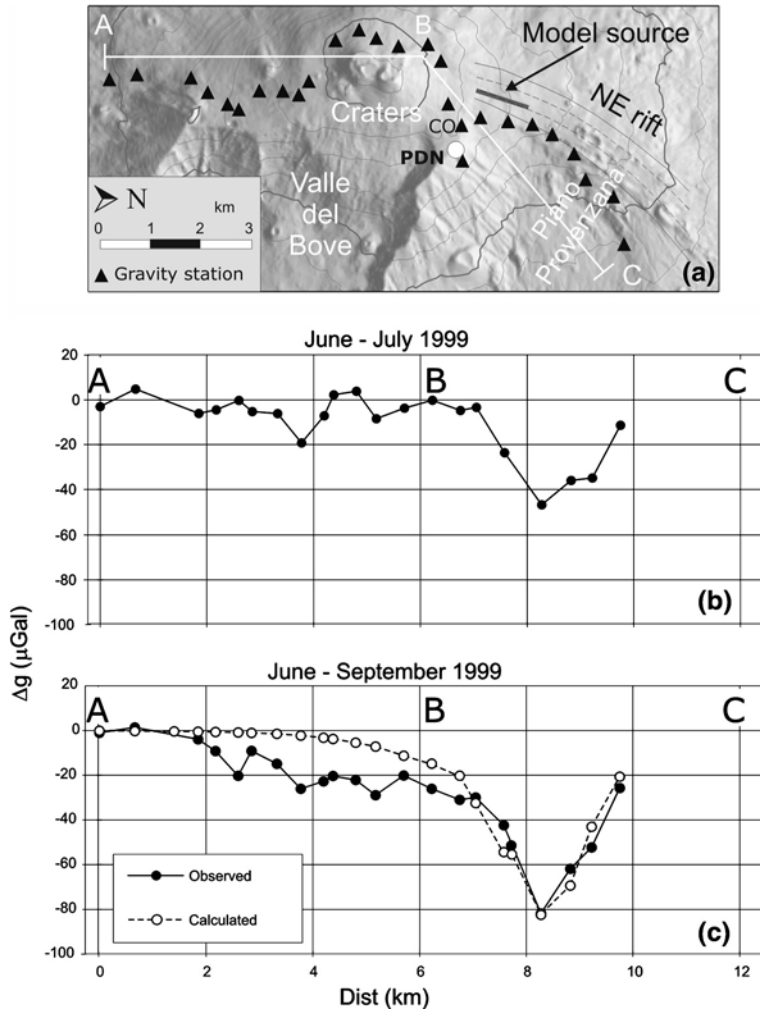


Figure 7

Gravity changes along the Summit Profile (a) during the June–July 1999 (b) and June–September 1999 (c) periods. The variations are attributed to a mass decrease in a source region that is modeled as a prism-shaped body and whose projection onto the surface is shown in (a) as a gray line (modified after CARBONE *et al.*, 2003c).

observation is achieved for a source 1.5 km long, 0.5 km in vertical extent, with its top about 1.8–2 km above sea level (the projection onto the surface of this source is shown in Fig. 7a), and with a mass decrease of about 2×10^{10} kg.

CARBONE *et al.* (2003a) assumed that the cyclic variations in mass within the inferred dike were caused by fluctuations in the degassing processes. This hypothesis is supported by a negative correlation between the gravity sequence recorded at PDN and the volcanic tremor observed at both PDN and ESP stations (placed respectively

within a few meter from PDN gravity station and in the southeastern sector of the volcano). Given that volcanic tremor is generated by fluid flow as gases escape through open magma-filled conduits (SEIDL *et al.*, 1981; SCHICK, 1988) and has an amplitude related to the intensity of turbulent motions (LEONARDI *et al.*, 1999), it is likely that increased degassing in the inferred source leads to both a gravity (mass) decrease and an increase in the spectral amplitude of the volcanic tremor, giving rise to the observed negative correlation.

3.3.2. The gravity sequence encompassing the start of the 2002 NE-Rift eruption. The 2002–2003 Etna eruption began on the night of October 26, 2002 with lava flows issued from fissure systems on both the southern and northeastern flanks of the volcano (ANDRONICO *et al.*, 2005). The activity on the southern flank lasted 93 days with a variable eruptive style. It concentrated mostly at a vent located at 2750 m a.s.l., with intense fire fountains, Strombolian activity, and lava flows from different vents at its base.

On the northern flank, the activity occurred along the eastern border of Etna's Northeast Rift (GARDUÑO *et al.*, 1997; Fig. 7a). It lasted nine days and was characterized by fire fountaining, Strombolian and effusive activity. The explosive activity was less intense than that occurring on the southern flank.

A gravity sequence, encompassing the start of the eruption on the northeastern flank (2002 NE-Rift eruption), was acquired at PDN station (Fig. 1). Before the start of the eruption, the mean background noise of the gravity signal was less than 1 μGal . The noise level began to increase starting at 21:36 GMT on 26 October. At 00:07 GMT of 27 October, a very strong and rapid gravity decrease began (Fig. 8). In less than one hour, the amplitude of the change reached about 400 μGal . Afterwards, the mean value of the gravity signal started rising again at a high rate (roughly 100 $\mu\text{Gal}/\text{hour}$).

Unfortunately, it is not possible to evaluate the influence of elevation changes on this gravity signal since, during the period of interest, the GPS stations operating on the summit zone of Mt. Etna acquired data for only three hours every day, i.e., between 10:00 and 13:00 GMT, a time interval which does not cover the gravity anomaly under study. Changes in the ground tilt were recorded through the two levels fitted to the L&R. Peak-to-peak fluctuations in ground tilt of up to 120 (long) and 350 (cross) μrad occurred during the 26–27 October anomaly, whose calculated gravity effect is less than 30 μGal (BRANCA *et al.*, 2003). Obviously, tilt changes that move the gravimeter away from a horizontal position (where the full force of gravity can be measured) produce a negative effect. Other possible perturbations to the gravity signal (for example, atmospheric temperature and pressure) are expected to be small over periods on the order of a few hours (ANDÒ and CARBONE, 2001). In particular, temperature, pressure and relative humidity changes contemporaneous to the 26–27 October gravity anomaly were surely too minor (within 0.1 °C, 2% RH and 4 mbar, respectively) to produce any significant effect.

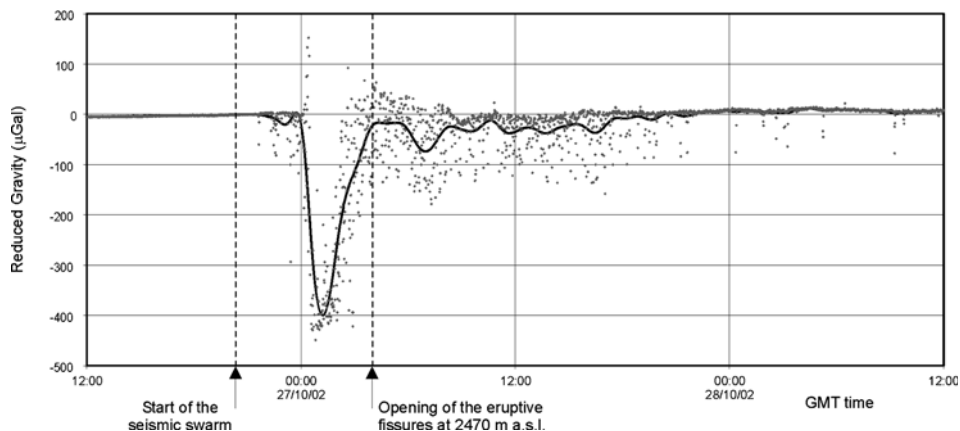


Figure 8

Reduced gravity, after removal of the best linear fit and the theoretical Earth tide effects, observed at the PDN station during a 48-hour period encompassing the start of the 2002–2003 eruption. The solid curve shows the result of a low-pass filter (see text for details). Dashed lines mark (left) the start of the seismic swarm heralding the eruption and (right) the first opening of the eruptive fissures on the northeastern slope of the volcano. Dots show unfiltered gravity values. Note the increase in the background noise of the measurements a few hours following the onset of seismicity (modified after BRANCA *et al.*, 2003).

As stated before, a part of the 400 μGal gravity decrease observed during the onset of the 2002 NE-Rift eruption is due to ground tilt and, possibly, height changes. However, it cannot be ruled out that much of the 26–27 October gravity anomaly may have been linked to subsurface mass redistributions. The most likely mechanism for the rapid and high-magnitude gravity decrease is the opening of new voids. Accordingly, the negative (first) part of the anomaly could have been caused by the opening of a dry fracture system on the northeastern slope of the volcano, within 1 km from the station, during the early stage of the 2002 NE-Rift eruption (BRANCA *et al.*, 2003). By assuming that the newly forming fractures are dry, magma overpressure is ruled out as a cause of their opening; thus, it is possible that the fractures were opened by external forces. The right-stepping *en-echelon* arrangement of the new fracture system could reflect the eastward gravitational sliding of a mega block delimited by the Provenzana-Pernicana fault system (BORGIA *et al.*, 1992; LO GIUDICE and RASÀ, 1992; FROGER *et al.*, 2001). Magma from the central conduit would then have used the new fracture system as a path to the eruptive vents downslope and, filling the newly formed voids, it provoked the observed gravity increase which roughly compensated the previous decrease. It is noteworthy that some characteristics of the 2002 NE-Rift eruption, namely (1) its short duration (9 days) and (2) the effusion rate decreasing rapidly with time, agree with the inferred intrusive mechanism.

BRANCA *et al.* (2003) calculated that a dry fracture at a distance of about 1 km from the gravity station should have a width of 2 to 5 m to produce the observed

gravity decrease. Accordingly, through a pit crater along the fissure system, it was possible to estimate the width of the intruded dike to be 3–4 m.

Studies based on the deformation data encompassing the start of the 2002 NE-Rift eruption of Mt. Etna yielded controversial results about the geometrical characteristics of the feeding dyke. Using continuous GPS and ground tilt data from stations at elevations below 2000 m, ALOISI *et al.* (2003) calculated an opening of the dyke feeding the 2002 NE-Rift eruption of 1 m. Using the same dataset and cross-analyzing it with accurate 3-D hypocentral locations, PATANÈ *et al.* (2005) came to the same result. Conversely, the analysis of ground tilt data from seven stations (elevations between 1400 and 2800 a.s.l.), pointed towards an opening of about 3.5 m (ALOISI *et al.*, 2006). In a more recent study, based on discrete GPS data from a summit network of 13 stations (elevations between 1800 and 3100 m a.s.l.), an opening of the feeding dike below 1 m could be evaluated (BONFORTE *et al.*, 2007).

None of the above studies took into account the information coming from the continuous gravity station and the observed deformation was always interpreted as due to magma forcing its way forward from the central conduit to the eruptive vents downslope. Thus, the above reported results were obtained using an analytical elastic model (OKADA, 1985) which accounts for the effect arising from a tensile fault buried in a homogeneous half space. As stated before, if the eruptions were caused by an “active” intrusion of magma, the observed decrease/increase gravity anomaly would not have developed. This observation, coupled with the fact that the same analytical formulation, thought it allows to satisfactorily invert different datasets, yields different results depending on the data in input, indicates that the assumption of a forceful intrusion within a medium homogeneous and isotropic is fallacious and must be reconsidered in light of all the available deformation and gravity data.

4. Conclusive Remarks

During the 1994–1996 period, discrete gravity measurements across the summit zone of Mt. Etna allowed the detection of mass redistributions which were interpreted to be linked to the ensuing explosive activity. The observed gravity changes were not accompanied by significant deformation and thus it was assumed that the magma feeding the explosive activity was temporally stored within pre-existing open voids about 2000 m below the surface.

For five months before the 2001 Mt. Etna eruption, a mass decrease was evidenced within a volume where many tensional earthquakes clustered during the same period. Coupled with the absence of important deformation, the sign and amplitude of the gravity change point towards a mechanism triggering a mass (density) decrease in the source region, rather than to a pre-eruptive intrusion of magma. In our view, a new

fracture zone formed below the southeastern sector of volcano, causing both the gravity decrease and the ensuing seismicity. The new-formed voids were used by the magma as a path to quickly reach the surface from a deep storage and give rise to the 2001 flank eruption. Accordingly, (a) a positive gravity change almost compensated the previous decrease soon after the start of the eruption and (b) petrological and volcanological data agree in indicating a quick rise of the magma feeding the eruption.

Both the above case studies prove the potential of discrete microgravity observations to provide valuable information about volcanic processes preceding potentially dangerous paroxysms. The same information, being intimately linked to the redistribution of underground masses, would have been difficult (if not impossible) to retrieve through other techniques.

To overcome the shortcoming typical of any discrete measurement of the time resolution being limited by the repeat time of the measurement campaigns, experiments of continuous gravity observations were started at Mt. Etna in 1998. A special station setup has been designed which, in conjunction with suitable procedures aimed at reducing the instrumental effect of atmospheric parameters, has allowed a good signal-to-noise ratio to be achieved. Beside the possibility of detecting meaningful gravity changes due to processes occurring over short time intervals (between a few hours and a few days), the availability of continuously running stations in the area covered by a network for discrete measurements allows to better describe the time evolution of the long-period changes, detectable through discrete network monitoring.

The strong gravity decrease, marking the start of the 2002 NE-Rift paroxysmal event and interpreted as the effect of the sudden opening of new voids, would prove that the magma did not open the eruptive fissures by itself, but, rather, intruded after the development of the fracture system, which was driven by the large-scale collapse of Mt. Etna's northeastern flank.

Conversely, the cross analysis of the 1998–2000 discrete and continuous gravity data from the summit zone of the volcano, enabled cyclic gravity changes, due to mass changes under the NE-Rift to be discovered.

The comparison between long-lasting continuous data sequences with discrete gravity data from the same area carries two advantages: (a) if common anomalies are discovered, a wide range of information on the location, size and time evolution of the source can be retrieved; (b) the algorithm used to reduce long continuous gravity sequences for the effect of atmospheric parameters, which is often very difficult to define, can be “calibrated” by comparing the results of the filtering process with the gravity data acquired through discrete gravity measurement at a site close to the continuously running station.

Acknowledgements

The authors are very grateful to M. Poland, K. Tiampo and another anonymous reviewer for their critical revision of the manuscript that contributed to significantly improve the paper.

REFERENCES

- ALOISI, M., BONACCORSO, A., GAMBINO, S., MATTIA, M., and PUGLISI, G. (2003), *Etna 2002 eruption imaged from continuous tilt and GPS data*, Geophys. Res. Lett. 30, 2214–2218.
- ALOISI, M., BONACCORSO, A., and GAMBINO, S. (2006), *Imaging composite dike propagation (Etna, 2002 case)*, J. Geophys. Res. 111(B6), B06404 10.1029/2005JB003908.
- ANDÒ, B. and CARBONE, D. (2001), *A Methodology for Reducing the Signal from a Continuously Recording Gravity Meter for the Effect of Meteorological Parameters*, IEEE Transact. Instrumentation and Measurement 50(5), 1248–1254.
- ANDÒ, B. and CARBONE, D. (2004), *A test on a Neuro-Fuzzy algorithm used to reduce continuous gravity records for the effect of meteorological parameters*, Phys. Earth and Planet. Int. 142, 37–47.
- ANDRONICO, D., BRANCA, S., and DEL CARLO, P. (2001), *The 18.7 ka phreatomagmatic flank eruption on Etna (Italy): Relationship between eruptive activity and sedimentary basement setting*, Terra Nova 13, 235–240.
- ANDRONICO, D., BRANCA, S., CALVARI, S., BURTON, M.B., CALTABIANO, T., CORSARO, R.A., DEL CARLO, P., GARFI, G., LODATO, L., MIRAGLIA, L., MURÈ, F., NERI, M., PECORA, E., POMPILIO, M., SALERNO, G., and SPAMPINATO, L. (2005), *A multi-disciplinary study of the 2002–03 Etna eruption: Insights into a complex plumbing system*, Bull. Volcanol. 67, 314–330.
- ARMIENTI, P., D'ORAZIO, M., INNOCENTI, F., TONARINI, S., and VILLARI, L. (1996), *October 1995–February 1996 Mt. Etna explosive activity: Trace element and isotopic constraints on the feeding system*, Acta Vulcanol. 8, 1–6.
- BONACCORSO, A., ALOISI, M., and MATTIA, M. (2002), *Dike emplacement forerunning the Etna July 2001 eruption modelled through continuous tilt and GPS data*, Geophys. Res. Lett. 29, 13.
- BONACCORSO, A., D'AMICO, S., MATTIA, M., and PATANÈ, D. (2004), *Intrusive mechanism at Mt. Etna forerunning the July–August 2001 eruption*, Pure Appl. Geophys. 161(7), 1469–1487.
- BONFORTE, A., CARBONE, D., GRECO, F., and PALANO, M. (2007), *Intrusive mechanism of the 2002 NE-rift eruption at Mt Etna (Italy) modelled using GPS and gravity data*, Geophys. J. Int. 169, 339–347.
- BONFORTE, A., GUGLIELMINO, F., PALANO, M., PUGLISI, G., FERRETTI, A., and COLESANTI, C., *Dynamics of Mt. Etna before, during and after the 2001 eruption inferred from GPS and InSAR data*, J. Geoph. Res., submitted.
- BONVALOT, S., DIAMENT, M., and GERMINAL, G. (1998), *Continuous gravity recording with Scintrex CG-3M meters: A promising tool for monitoring active zones*, Geophys. J. Int. 135, 470–494.
- BORGIA, A., FERRARI, L., and PASQUARÈ, G. (1992), *Importance of gravitational spreading in the tectonic and volcanic evolution of Mount Etna*, Nature 357, 231–235.
- BRANCA, S., CARBONE, D., and GRECO, F. (2003), *Intrusive mechanism of the 2002 NE-Rift eruption at Mt. Etna (Italy) inferred through continuous microgravity data and volcanological evidences*, Geophys. Res. Lett. 30, 20, 2077, 10.1029/2003GL018250.
- BUDETTA, G., GRIMALDI, M., and LUONGO, G. (1989), *Variazioni di gravità nell'area etnea (1986–1989)*, Boll. GNV 5, 137–146.
- BUDETTA, G. and CARBONE, D. (1997), *Potential application of the Scintrex CG-3M gravimeter for monitoring volcanic activity: Results of field trials on Mt. Etna, Sicily*, J. Volcanol. Geotherm. Res. 66, 199–214.
- BUDETTA, G. and CARBONE, D. (1998), *Temporal variations in gravity at Mt. Etna (Italy) associated with the 1989 and 1991 eruptions*, Bull. Volcanol. 59, 311–326.

- BUDETTA, G., CARBONE, D., and GRECO, F. (1999), *Subsurface mass redistribution at Mount Etna (Italy) during the 1995–96 explosive activity detected by microgravity studies*, *Geophys. J. Int.* 138, 77–88.
- BUDETTA, G., CARBONE, D., GRECO, F., and RYMER, H. (2004), *Microgravity Studies at Mount Etna (Italy)*, in “Mt. Etna: Volcano Laboratory”, *Geophys. Monogr. Ser.* 143, (Calvari S., Bonaccorso A., Coltelli M., Del Negro C., Falsaperla S. Eds), AGU (Geophysical Monograph series; 143), 221–240.
- CARBONE, D., (2002), *Gravity monitoring of Mount Etna (Italy) through discrete and continuous measurements*, Ph.D. Thesis, The Open University, Milton Keynes, UK.
- CARBONE, D., BUDETTA, G., and GRECO, F. (2003a), *Possible mechanisms of magma redistribution under Mt. Etna during the 1994–1999 period detected through microgravity measurements*, *Geophys. J. Int.* 152, 1–14.
- CARBONE, D., BUDETTA, G., and GRECO, F. (2003b), *Bulk processes some months before the start of the 2001 Mt Etna eruption, evidenced through microgravity studies*, *J. Geophys. Res.* 108(B12), 2556.
- CARBONE, D., BUDETTA, G., GRECO, F., and RYMER, H. (2003c), *Combined discrete and continuous gravity observations at Mt. Etna*, *J. Volcanol. Geoth. Res.* 2581, 1–13.
- CORSARO, R., MIRAGLIA, L., and POMPILIO, M. (2007), *Petrologic evidence of a complex plumbing system feeding the July–August 2001 eruption of Mt. Etna, Sicily, Italy*, *Bull. Volcanol.* 69, 401–421.
- DEHANT, V. (1987), *Tidal parameters for an inelastic Earth*, *Phys. Earth Planet. Inter.* 49, 97–116.
- DZURISIN, D., ANDERSON, L. A., EATON, G. P., KOYANAGI, R. Y., LIPMAN, P. W., LOCKWOOD, J. P., KAMURA, R. T., PUNIWI, G. S., SAKO, M. K., and YAMASHITA, K. M. (1980), *Geophysical observations of Kilauea volcano, Hawaii. 2. Constraints on the magma supply during November 1975–September 1977*, *J. Volcanol. Geotherm. Res.* 7, 241–269.
- Ferrari, L. (1991), *Evoluzione vulcanologica e strutturale del Monte Etna e i suoi rapporti con il vulcanismo ibileo*, Ph.D. thesis, Dipartimento di Scienze della Terra, Università di Milano, Milano.
- FRÖGER, J.L., MERLE, O., and BRIOLE, P. (2001), *Active spreading and regional extension at Mount Etna imaged by SAR interferometry*, *Earth Planet. Sci. Lett.* 187, 245–258.
- GARDUÑO, V.H., NERI, M., PASQUARÈ, G., BORGIA, A., and RIBALDI, A. (1997), *Geology of the NE-Rift of Mount Etna (Sicily, Italy)*, *Acta Vulcanol.* 9(1/2), 91–100.
- INGV-SEZIONE DI CATANIA RESEARCH STAFF (2001), *Multidisciplinary approach Yields insight into Mt. Etna eruption*, EOS, Transactions, Am. Geophys. Union 82(50), 653–656.
- LACOSTE and ROMBER, (1997), *General Catalog*, Austin, Texas (U.S.A).
- LAIGLE, M., HIRN, A., SAPIN, M., and LEPINE, J.C. (2000), *Mount Etna dense array local earthquakes P and S tomography and implications for volcanic plumbing*, *J. Geophys. Res.* 105, 21633–21646.
- LAUTZE, N.C., HARRIS, A.J.L., BAILEY, J.E., RIPEPE, M., CALVARI, S., DEHN, J., ROWLAND, S., and EVANS-JONES, K. (2004), *Pulsed lava effusion at Mount Etna during 2001*, *J. Volcanol. Geotherm. Res.* 137, 231–246.
- LEONARDI, S., CALTABIANO, T., GRESTA, S., MULARGIA, F., PRIVITERA, E., ROMANO, R., and ROSSI, P.L. (1999), *Cross-correlation between volcanic tremor and SO₂ flux data from Mt. Etna volcano, 1987–1995*, *Acta Vulcanologica* 11(2), 225–258.
- LO GIUDICE, E. and RASÀ, R. (1992), *Very shallow earthquakes and brittle deformation in active volcanic areas: The Etnan region as an example*, *Tectonophysics* 202, 257–268.
- MERRIAM, J.B. (1992), *Atmospheric pressure and gravity*, *Geophys. J. Int.* 109, 488–500.
- NIEBAUER, T.M. (1988), *Correcting gravity measurements for the effect of local air pressure*, *J. Geophys. Res.* 93, 7989–7991.
- PATANÈ, D., CHIARABBA, C., COCINA, O., DE GORI, P., MORETTI, M., and BOSCHI, E. (2002), *Tomographic images and 3-D earthquake locations of the seismic swarm preceding the 2001 Mt. Etna eruption: Evidence for a dyke intrusion*, *Geophys. Res. Lett.* 29(10), 10.1029/2001GL014391.
- PATANÈ, D., MATTIA, M., and ALOISI, M. (2005), *Shallow intrusive processes during 2002–2004 and current volcanic activity on Mt. Etna*, *Geophys. Res. Lett.* 32, L06302, doi:10.1029/2004GL021773.
- POMPILIO, M., CORSARO, R.A., FREDA, C., MIRAGLIA, L., SCARLATO, P., and TADDEUCCI, J. (2001), *Petrological Evidences of a Complex Plumbing System Feeding the July–August 2001 Eruption of Mt. Etna*, EOS Trans. Am. Geophys. Union, Fall Meeting Suppl. Abstract 82(47) F1412.
- PUGLISI, G. and BONFORTE, A. (1999), *Monitoring active deformation of volcanoes by interferometry as an early warning system*, in: MADVIEWS, EC Project n. ENV4-CT96-0294, contribution of CNR – IIV, Final Report.

- OKADA, Y. (1985), *Surface deformation due to shear and tensile fault in half space*, Bull. Seismol. Soc. Am. 75, 1135–1154.
- RYMER, H. (1989), *A contribution to precision microgravity data analysis using LaCoste and Romberg gravity meters*, Geophys. J. 97, 311–322.
- SEIDL, D., SCHICK, R., and RIUSCETTI, M. (1981), *Volcanic tremors at Etna: A model for hydraulic origin*, Bull. Volcanol. 44, 43–56.
- SCHICK, R. (1988), *Volcanic tremor-source mechanism and correlation with eruptive activity*, Natural Hazard 141, 125–144.
- SCINTREX LIMITED, (1992), *Autograv Operator Manual Version 4.4*, Concord, Ontario (Canada).
- SPAMPINATO, S., GAMBINO, S., PATANE, D., PRIVITERA, E., D'AMICO, G., DI PRIMA, S., PELLEGRINO, A., SCUDERI, L., and TORRISI, O. (1998), *Seismic activity*. In *Data Related to Eruptive Activity, Unrest Phenomena and other Observations on the Italian Active Volcanoes*, Geophys. Monitoring of the Italian Active Volcanoes 1993–1995—Etna (P. Gasperini, ed.), Acta Volcanol. 10, 149–153.
- SPRATT, R.S. (1982), *Modelling the effect of atmospheric pressure variations on gravity*, Geophys. J. R. Astron. Soc. 71, 173–186.
- TAMURA, Y. (1987), *A harmonic development of the tide-generating potential.*, Bull. Inf. Marees Terr. 99, 6813–6855.
- TORGE, W., *Gravimetry* (ed. Walter de Gruyter) (Berlin-New York 1989).
- WENZEL, H.G. (1996), *The nanogal software: Earth tide data processing package ETERNA 3.30*. Inf. Marées Terrestres. 124, 9425–9439.

(Received November 30, 2005, revised/accepted September 21, 2006)

To access this journal online:
www.birkhauser.ch/pag

An Estimate of Global Mean Sea-level Rise Inferred from Tide-gauge Measurements Using Glacial-isostatic Models Consistent with the Relative Sea-level Record

J. M. HAGEDOORN, D. WOLF, and Z. MARTINEC

Abstract—This study is concerned with the influence of the glacial-isostatic adjustment caused by the last Pleistocene deglaciation on the present-day sea level. The viscoelastic deformation caused by the time-variable ice and ocean loads is simulated by computing the resulting perturbations for a spherical, self-gravitating, incompressible, Maxwell-viscoelastic earth model. The associated variation of the earth rotation is described in terms of the Liouville equation, which is solved by means of the MacCullagh formulae. This allows the determination of the vertical displacement and geoid height and, thus, the solution of the sea-level equation. We test several viscosity and ice models and evaluate them by comparison of the computed response with the Holocene relative sea-level record. Using the optimum combination of viscosity and ice models, we then estimate the influence of the last Pleistocene deglaciation on the tide-gauge measurements. A comparison between the observational and residual linear trends for the tide-gauge measurements shows a significant reduction of the variance and geographical variability for the latter, in particular for the formerly ice-covered regions of North America and Scandinavia. The favoured value determined for the global mean sea-level rise is $(1.46 \pm 0.2) \text{ mm a}^{-1}$.

Key words: Sea-level rise, glacial-isostatic adjustment, tide-gauge measurements

1. Introduction

The present-day changes of the relative sea-level (RSL) height can be observed by several techniques distinguished by differing time intervals and differing spatial resolution, e.g. tide-gauge measurements, satellite altimetry and GPS buoys. An important application of these measurements is the determination of the global mean sea-level (GMSL) rise, which is a measure of the volume change of the ocean water. There are two contributions to this variation: Changes due to variations in the ocean mass and changes due to variations in the ocean temperature or salinity. The former contribution is mainly due to the influx of melt water from ice sheets, ice caps or mountain glaciers and the variation of the terrestrial water storage. The latter is largely related to the changes of the global climate. In the present study, we use tide-gauge measurements to determine the GMSL rise, because only this type

of measurement provides estimates that are representative of the 20th century, which, in turn, allow us to discuss whether there has been an acceleration of the sea-level rise during the more recent past. In obtaining such estimates, it is necessary to consider that a tide gauge only records the change of the distance between the sea level and some reference, usually a benchmark fixed to bedrock. Therefore, any vertical displacement of the reference is also recorded by the tide gauge, such that it is impossible to separate this displacement from the sea-level change related to ocean-volume change. Moreover, since the sea level coincides with the geoid, any variation of the geoid height also affects the tide-gauge record. Finally, tide-gauge measurements are also influenced by the deformation of the sea level resulting from air-pressure variations (inverse-barometer (IB) effect) or changes of the dynamic ocean topography.

An important global process causing vertical displacement is the glacial-isostatic adjustment (GIA) following the last Pleistocene deglaciation. For this load redistribution, the earth responds as a viscoelastic continuum. The ongoing relaxation of the earth after the end of deglaciation causes additional redistributions of mass in its interior and in the oceans, which both affect the tide-gauge measurements. Simultaneously, these redistributions also force variations in the inertia tensor of the earth, which, in turn, lead to variations in the earth rotation. Furthermore, the centrifugal potential associated with the earth rotation also changes, leading to variations of the geoid height.

The main objective of the present study is the development of a method to reduce tide-gauge measurements for the influence of the last glacial cycle during the Pleistocene. Since we restrict our investigation to this particular influence, we must exclude tide-gauge measurements strongly affected by other contributions, e. g. the IB effect, the dynamic ocean topography and vertical displacements due to tectonic processes, sediment compaction or groundwater extraction. This requires the development of criteria that allow us to detect such influences and to exclude the corresponding tide-gauge measurements.

In Section 2, we will sketch the theory required for the determination of the influence of GIA on tide-gauge measurements, i.e., the gravitational-viscoelastic equations (GVEs), the Liouville equation (LE) and the sea-level equation (SLE), and outline the method of solution. In Section 3, we will introduce the input models considered, i.e., the parameterizations of the viscosity and ice models. The input models will be evaluated using sea-level indicators (SLIs) representative of the Holocene RSL history. Section 4 will be concerned with criteria allowing us to identify tide-gauge stations which are influenced predominantly by GIA and with the estimation of the observational linear trends of the RSL-height change for these stations. In Section 5, we will reduce these trends for contributions due to GIA and determine the GMSL rise from the residual trends. The results obtained will be discussed in Section 6.

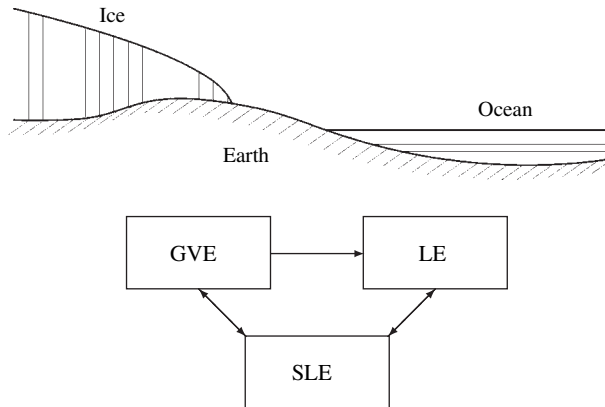


Figure 1

Top: The earth-ice-ocean system involved in GIA. Bottom: The basic components of the associated theoretical description, i.e., the viscoelastic field equations (GVEs), the Liouville equation (LE) and the sea-level equation (SLE). The arrows indicated the couplings considered.

2. Theory

Figure 1 shows the basic components of the theoretical description of the RSL-height change associated with GIA and their mutual coupling. These components are the GVEs governing the load-induced perturbation of the hydrostatic reference state, the LE describing the variation of the earth rotation and the SLE controlling the redistribution of water in the ice-ocean system.

For the solution of the GVEs describing the load-induced perturbation of a spherical, self-gravitating, incompressible, Maxwell-viscoelastic continuum, the spectral finite-element representation (SFER) developed by MARTINEC (2000) is used. In this representation, the equations are stated in the weak formulation, with the angular dependence represented by spherical harmonics and the radial dependence by finite elements. The characteristic feature of the SFER is that the GVEs are solved in the time domain. For given ice and ocean loads, this allows us to compute the perturbation of the earth as a function of the time epoch, t , and the position, $\Omega = (\vartheta, \varphi)$, where ϑ and φ are the colatitude and longitude, respectively.

The theoretical formulation for the variation of earth rotation is based on the conservation of the moment of inertia described by the LE. Since the GVEs are solved in the time domain, the same is convenient for the LE, which is implemented using the MacCullagh formulae. This approach was studied in detail by MARTINEC and HAGEDOORN (2005).

For a self-consistent implementation of the SLE in the space and time domains, we proceed as follows. Using the GVEs and the LE, we first compute the vertical displacement, u , and the geoid height, e , where, for the latter, we use the Bruns formula (HEISKANEN and MORITZ, 1967):

$$e(\Omega, t) = -\frac{\phi(\Omega, t)}{g^{(0)}(a)}, \quad (1)$$

with ϕ the incremental gravity potential, $g^{(0)}(a)$ the initial surface gravity and a the earth radius. The SLE satisfies the following principles: (i) Mass is conserved in the ice-ocean system, i.e., accumulated ice is extracted from the ocean and melting ice returns into the ocean; (ii) the sea surface is an equipotential surface and, therefore, coincides with the geoid. We may split the RSL height, s , into a uniform, s^{UF} , and a non-uniform, s^{NU} , contribution:

$$s(\Omega, t) = s^{\text{UF}}(\Omega, t) + s^{\text{NU}}(\Omega, t). \quad (2)$$

Usually, the SLE is stated in integral form (FARRELL and CLARK, 1976; CLARK *et al.*, 1978; PELTIER *et al.*, 1978; MITROVICA and PEITIER 1991; JOHNSTON, 1993; LAMBECK, 1993; PELTIER, 1994; MILNE and MITROVICA, 1998; JOHNSTON and LAMBECK 1999; MILNE *et al.*, 1999). In Equation (2), the integral formulation is implicit in both contributions.

The uniform contribution is given by

$$s^{\text{UF}}(\Omega, t) = h^{\text{UF}}(t)\mathcal{O}(\Omega, t), \quad (3)$$

where h^{UF} is the thickness of the uniform water layer and $\mathcal{O}$ the ocean function defined by

$$\mathcal{O}(\Omega, t) := \begin{cases} 1 & \text{for ocean} \\ 0 & \text{for land} \end{cases}. \quad (4)$$

The determination of h^{UF} is governed by the principle of mass conservation in the ice-ocean system. Since we use the SFER, this conservation can be expressed by means of a constant degree-zero term, σ_{00} , of the spherical-harmonic (spectral) representation of the surface-mass density associated with the ice-ocean load. We choose

$$\sigma_{00}(t) = 0. \quad (5)$$

For h^{UF} , we find the following expression:

$$h^{\text{UF}}(t) = -\frac{\sigma_{00}^{\text{I}}(t) + \sigma_{00}^{\text{NU}}(t)}{\rho^{\text{O}}\mathcal{O}_{00}(t)}, \quad (6)$$

where $\sigma_{00}^{\text{I|NU}}$ is the degree-zero term of the spectral representation associated with the ice load and the non-uniform ocean load, respectively, and $\mathcal{O}_{00}$ the degree-zero term of the spectral representation of the ocean function. For the ocean-water density, we choose $\rho^{\text{O}} = 1020 \text{ kgm}^{-3}$.

The next step is the determination of the non-uniform contribution to the RSL height. To ensure that the sea surface is an equipotential surface, the vertical displacement, u , and the geoid height, e , must be known. We find

$$s^{\text{NU}}(\Omega, t) = [e(\Omega, t) - u(\Omega, t)]\mathcal{O}(\Omega, t) - \zeta^{(0)}(\Omega) [\mathcal{O}(\Omega, t) - \mathcal{O}^{(0)}(\Omega)], \quad (7)$$

where $\zeta^{(0)}$ is the topography of the undeformed earth and $\mathcal{O}^{(0)}$ the associated ocean function. According to Equation (7), the influence of the topography becomes important only for locations near the coast where the ocean function is time dependent. Figure 2a shows a transgression of the ocean. During one time step, the RSL height increases and, therefore, the ocean function changes from zero to one for a location in the flooded area. The second part of Equation (7) ensures that the RSL height is reckoned from the ocean bottom displaced by $\zeta^{(0)} + u$ to the sea surface given by e . Figure 2b shows a regression of the ocean. During one time step, the RSL height decreases and, therefore, the ocean function changes from one to zero for a location in the emerged area. As before, the second part of Equation (7) accounts for the fact that the RSL height is reckoned from the deformed ocean bottom to the deformed sea surface.

We now define the time-dependent ocean function allowing for transgression and regression more closely. For this, we take into account that such changes can only occur at locations and times without ice. In case of an ice cover, it is necessary to compare the ice thickness with the water depth. If the former is smaller than the latter, the ice cover is floating shelf ice, and the corresponding area is treated as ocean. The different scenarios are formalized by the following definition:

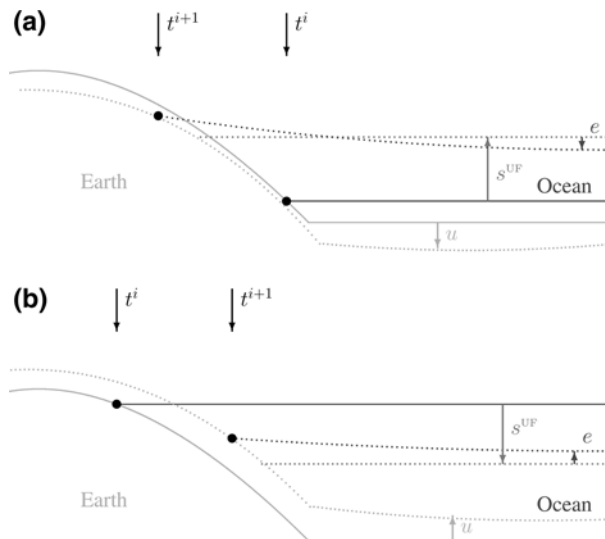


Figure 2

Contributions to RSL height for time-dependent ocean function, $\mathcal{O}$, consisting of uniform RSL height, s^{UF} (red), vertical displacement, u (green), and geoid height, e (blue) for (a) transgression and (b) regression. Solid lines apply to time epoch t^i and dotted lines to time epoch t^{i+1} . The corresponding locations of the coastline are marked by black dots.

$$\mathcal{O}(\Omega, t) := \begin{cases} 1 & \text{for } e(\Omega, t) + h^{\text{UF}}(t) \geq \zeta^{(0)}(\Omega) + u(\Omega, t) \wedge s^{\text{E}}(\Omega, t) \geq \frac{\rho^{\text{O}}}{\rho^{\text{F}}} s^{\text{O}}(\Omega, t) \\ 0 & \text{for } e(\Omega, t) + h^{\text{UF}}(t) < \zeta^{(0)}(\Omega) + u(\Omega, t) \vee s^{\text{E}}(\Omega, t) \geq \frac{\rho^{\text{O}}}{\rho^{\text{F}}} s^{\text{O}}(\Omega, t) \end{cases} \quad (8)$$

Taken together, Equations (3), (7) and (8) describe consistently all contributions to the RSL height given by Equation (2).

The evaluation of the coupled equations between time epochs t^i and t^{i+1} is implemented in three steps (Fig. 3). In the first step, the GVEs are solved in the spectral domain and $U_{jm}(t^i)$ and $E_{jm}(t^i)$ for given ice and ocean loads represented by $\sigma_{jm}^{\text{I}}(t^i)$ and $\sigma_{jm}^{\text{O}}(t^{i-1})$, respectively, are computed. In the second step, the incremental inertia tensor, $\mathbf{C}^{(1)}(t^i)$, and the non-dimensional incremental earth-rotation vector, $\mathbf{m}(t^i)$, are determined from the spectral solution of the LE in terms of the MacCullagh formulae (MARTINEC and HAGEDOORN, 2005). Also, the centrifugal potential, $\Psi_{jm}(t^i)$, is computed. After transformation into the spatial domain, the SLE is solved using Equations (3), (7) and (8) in the third step and the next spectral representation of the ocean load, $\sigma_{jm}^{\text{O}}(t^i)$, is computed.

The significance of the variation of earth rotation to the present-day rate of RSL-height change was shown before (MITROVICA *et al.*, 2001, 2005). The influence of a time-dependent ocean function was also discussed previously (e.g., MILNE *et al.*, 1999; LAMBECK *et al.*, 2003; MITROVICA, 2003; KENDALL *et al.*, 2005).

3. Input Models

To determine the influence of the Pleistocene glaciation on tide-gauge measurement, it is necessary to select an appropriate combination of viscosity

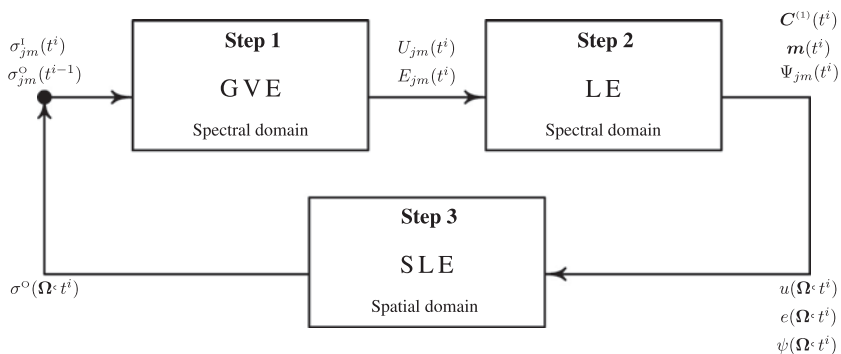


Figure 3

Implementation of the solution between time epochs t^i and t^{i+1} . In the first step, the GVEs and, in the second step, the LE are solved in the spectral domain. Then, the output quantities are transformed into the spatial domain and the SLE is solved in the third step (see text for further details).

Table 1
Parameterization of the viscosity models

Radial distance r (km)	Viscosity $\nu(r)$ (Pas)			
	VM1	VMA	VMF	VMM
0–3480	0	0	0	0
3480–5701	2.0×10^{21}	2.0×10^{21}	5.9×10^{21}	5.0×10^{21}
5701–5951	1.0×10^{21}	1.0×10^{20}	5.2×10^{20}	5.5×10^{20}
5951–6271	1.0×10^{21}	1.0×10^{21}	5.2×10^{20}	5.5×10^{20}
6271–6371	∞	∞	∞	∞

and ice models (input model). The earth models used are parameterized by the volume-mass density and shear modulus according to PREM (DZIEWONSKI and ANDERSON, 1981) and some radially symmetric viscosity model with liquid core. For the 109 finite elements in the radial direction, the values of the volume-mass density and the shear modulus are computed from the PREM polynomial according to their radial distance.

In Table 1, we list the viscosity models considered. Viscosity model VM1 is taken from PELTIER (1999), who used a relaxation-time spectrum from McCONNELL (1968) for Scandinavia, RSL data from 25 locations and the variation of earth rotation to infer the viscosity. Viscosity model VMA features a low-viscosity shell between $r = 5701$ km and 5951 km. Indications of the existence of such a shell were found on the basis of seismic tomography for many regions, e.g. North America. The values for viscosity model VMA are taken from WU and VAN DER WAL (2003). Viscosity model VMF agrees with that of FLEMING *et al.*, (2003), who inverted the relaxation-time spectrum for Scandinavia derived by WIECZERKOWSKI *et al.*, (1999). Viscosity model VMM was employed by MARTINEC and WOLF (2005) for a comparison with an axially symmetric viscosity model and is also based on the Scandinavian relaxation-time spectrum. It differs only slightly from viscosity model VMF, but the evaluation of the individual input models results in a better fit of the former model for some regions.

In principle, we can choose between two types of ice model: geomorphological and thermomechanical. The first uses geomorphological evidence of the extensions of the Pleistocene ice sheets and records of the GMSL rise since the last glacial maximum (LGM) for estimates of the total ice volume. In contrast to this, the second is based on a thermomechanically consistent description of the ice-sheet evolution, as forced by climatic and other parameters. In the following study of the influence of the Pleistocene glaciation on the Holocene RSL-height change, we will consider three ice models (Fig. 4).

The geomorphological ice model ICE-3G-M is based on ice model ICE-3G from TUSHINGHAM and PELTIER (1991) and consists of 18 global ice-thickness distributions

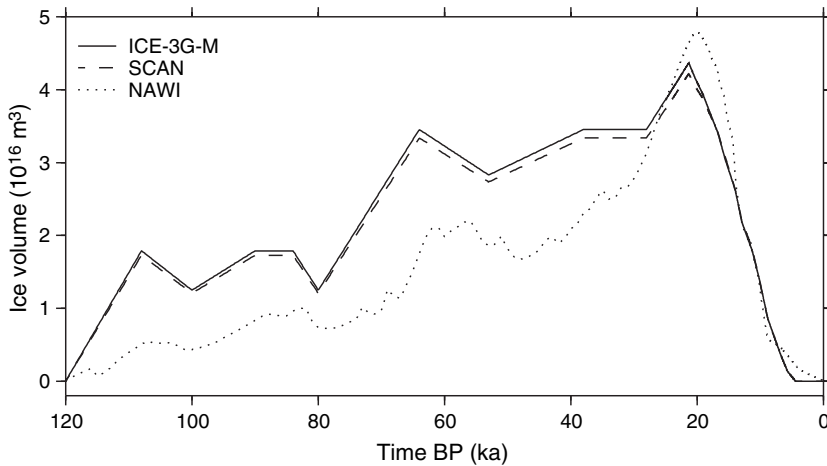


Figure 4
Ice-volume evolution for ice models ICE-3G-M, SCAN and NAWI.

representing the deglaciation history from the LGM at 21 ka BP (before present) until today. We extend this model by a glaciation history starting at 120 ka BP, with the LGM ice-thickness distribution scaled according to the RSL-height records for this time epoch from far-field sites, such as Huon Peninsula, Papua-Neuguinea, Bonaparte Archipelago and western Australia (LAMBECK and CHAPPELL, 2001; YOKOYAMA *et al.*, 2001; BINTANJA *et al.*, 2002). The LGM ice volume is $4.3687 \times 10^{16} \text{m}^3$, which is equivalent with a GMSL fall of 107.2 m.

The geomorphological ice model SCAN consists of a northern European component (LAMBECK *et al.*, 1998) embedded into ice model ICE-3G-M, with the European component largely based on RSL-height observations in northwestern Europe. The maximum ice volume of $4.2213 \times 10^{16} \text{m}^3$ is reached at 21.4 ka BP and is equivalent with a GMSL fall of 103.6 m.

The thermomechanical ice model NAWI is a combination of a northern-hemisphere component (ZWECK and HUYBRECHTS, 2005) and an Antarctic component (HUYBRECHTS, 2002). We have discretized the northern-hemisphere model from 120 ka BP to the LGM in 1-ka time steps and from the LGM to the present in 0.2-ka time steps and combined this with corresponding ice-thickness distributions for Antarctica. The maximum ice volume reached at ~ 20 ka BP is $4.7841 \times 10^{16} \text{m}^3$ and corresponds to a GMSL fall of 117.41 m.

For the following, it is necessary to develop a method for the evaluation of the input models which is independent from the tide-gauge data. For this, we use the RSL record of the past ~ 20 ka, which can be reconstructed from a global set of sea-level indicators (SLIs) (KLEMMANN *et al.*, 2007). The SLIs are based on geomorphological features or sediments with organic materials that can be dated

Table 2
Optimum input models for regions R1,..., R5.

Region	R1	R2	R3	R4	R5
Viscosity model	VMM	VM1	VMA	VMF	VM1
Ice model	SCAN	NAWI	NAWI	SCAN	SCAN

and are related to the former RSL (e.g. PIRAZZOLI, 1996). For the evaluation, we compute the RSL heights for all input models at the positions and times associated with the ~ 8000 SLIs available. This is then compared with the corresponding observational RSL heights. For testing the consistency of the input model with the RSL record, we use membership functions (HAGEDOORN, 2005; KLEMMANN and WOLF, 2006; WOLF *et al.*, 2006), which take into account that, usually, the SLIs only allow us to infer a lower bound, an upper bound or a finite interval for the former RSL height. Table 2 shows the optimum input models for the regions R1,..., R5 considered for the tide-gauge stations (Sec. 4 and Fig. 6). For the five regions four different viscosity models are found to be optimum input models. This indicates lateral inhomogeneities in the viscosity stratification (e.g. SABADINI, 2002; WU and VAN DER WAL, 2003; WU *et al.*, 2005) or variations in the lithospheric thickness, e.g. the change between continental and oceanic lithospheres (e.g. KAUFMANN *et al.*, 1997; OKUNO and NAKADA, 2001; MARTINEC and WOLF, 2005).

4. Tide-gauge Data

Before correcting tide-gauge measurements for GIA in order to determine the GMSL rise, we develop criteria for the selection of tide-gauge stations suitable for this objective. Here, we use time series of monthly-mean values archived by the PSMSL¹. This data base comprises several hundred time series varying in length between a few years and ~ 180 a. The associated tide-gauge stations are mainly concentrated along the coastlines of Europe and North America.

The problem of defining criteria for the selection of tide-gauge stations was discussed by DOUGLAS (1997). In the following, we will adopt some of his criteria and modify others. In the context of the present study, the main difficulty is that tide gauges also record the non-GIA contributions mentioned above. To eliminate most of these contributions, such as tides, wind and air-pressure effects, we demand a minimum time interval of 45 a for the tide-gauge records to be considered. This is in

¹ Permanent Service for Mean Sea Level at the Proudman Oceanographic Laboratory, see also <http://www.pol.ac.uk/psmsl/>.

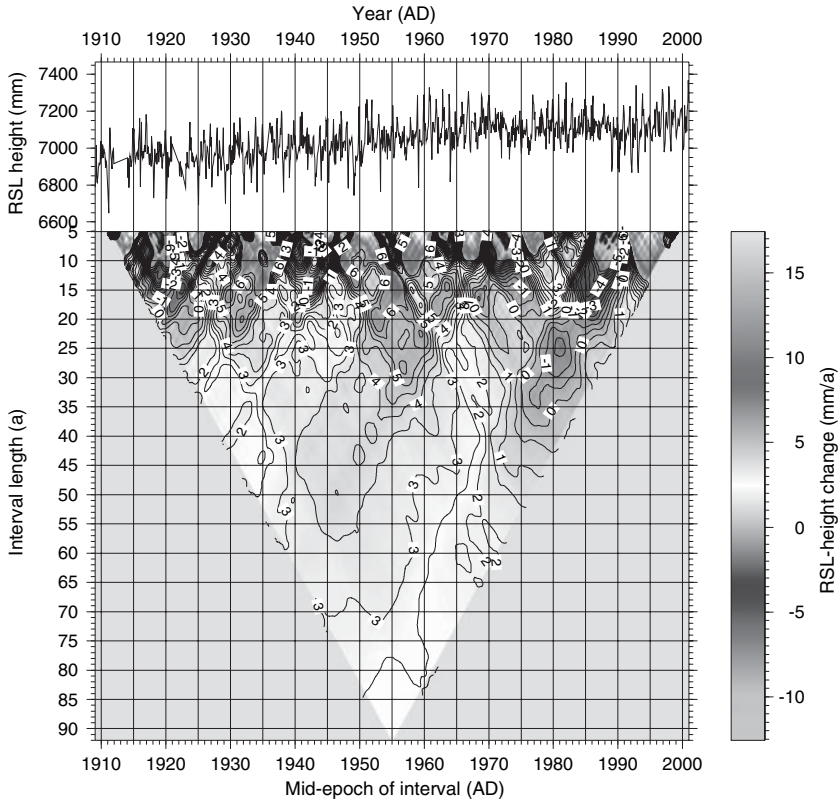


Figure 5
LTA diagram for tide-gauge station Venezia (PSMSL-ID 270054).

accordance with DOUGLAS (1997), who assumed an averaging of such effects over this time interval. As a further control, we use the linear-trend-analysis (LTA) diagram developed by WOLF *et al.* (2006), which displays the time dependence of the linear trends for sub-intervals between 5 a and the maximum length of the time series.

Figure 5 shows the LTA diagram for the station Venezia (PSMSL-ID 270054). For an interval length of $\Delta t = 40$ a, there is an increase of $\dot{s}^L$ from $\sim 2 \text{ mm a}^{-1}$ to $\sim 4 \text{ mm a}^{-1}$ for mid-epochs from 1930 to 1960, after which the value reduces again. TOSI *et al.* (2002) investigated the Venezia time series and explained the variability of the linear trend by ground water extraction before 1969. In view of this variability, Venezia is an example of a station to be excluded in the following. After similar controls for the 168 tide-gauge station with time series longer than 45 a, only 121 show sufficient stability of the linear trends.

For the estimation of the observational rate of RSL-height change, $\dot{s}^L$, we find (e.g. BEVINGTON, 1969, p. 114)

$$\dot{s}^L = \frac{N \sum_{n=1}^N s_n^{\text{TG}} t^n - \sum_{n=1}^N s_n^{\text{TG}} \sum_{n=1}^N t^n}{N \sum_{i=n}^N (t^n)^2 - \left(\sum_{i=n}^N t^n \right)^2}, \quad (9)$$

$$\varepsilon^L = \sqrt{\frac{N \sigma^2}{N \sum_{n=1}^N (t^n)^2 - \left(\sum_{n=1}^N t^n \right)^2}}, \quad (10)$$

$$\sigma^2 = \frac{\sum_{n=1}^N (s_n^{\text{TG}} - \dot{s}^L t^n - \alpha)^2}{N - 2}, \quad (11)$$

$$\alpha = \frac{\sum_{n=1}^N s_n^{\text{TG}} - \dot{s}^L \sum_{n=1}^N t^n}{N}, \quad (12)$$

where s_n^{TG} is the monthly mean RSL height referred to the mid-epoch, t^n , of the n th month, N the total number of months considered and ε^L the standard deviation of $\dot{s}^L$.

As mentioned above, most tide-gauge stations are located along the European and North American coasts. To compensate partly for this geographical non-uniformity, we group the tide-gauge stations selected into five major regions: R1: Scandinavia, R2: North America, R3: northern Atlantic, R4: Indic and northern Pacific and R5: Southern Hemisphere (Fig. 6). The PSMSL-ID numbers, geographical coordinates and linear trends of the stations are listed in Tables 6–10.

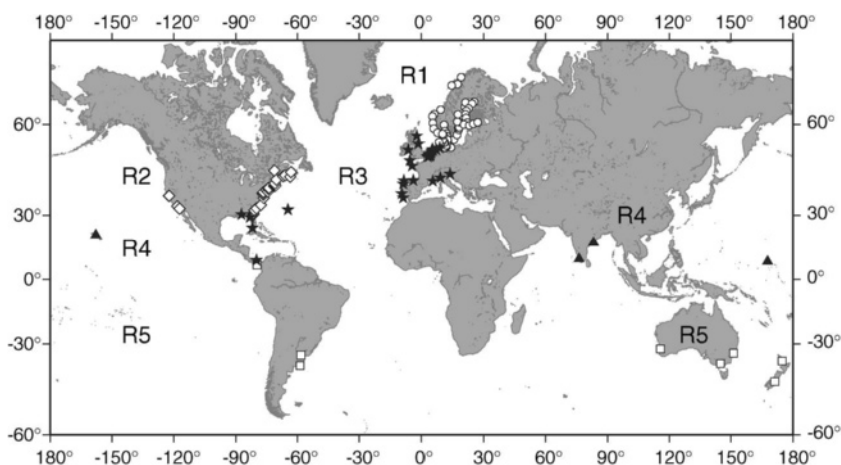


Figure 6

Locations of the 121 tide-gauge stations selected. The stations are assigned to five regions distinguished by different types of symbol: R1 (circles), R2 (diamonds), R3 (stars), R4 (triangles) and R5 (squares).

Table 3

Mean of observational rate of RSL-height change, $\bar{s}^L$, and associated mean standard deviation, $\bar{e}$, for stations in regions R1, ..., R5, respectively, and for all stations in region R1–R5. The values in parentheses apply to the standardized time interval 1931–2000. For the determination of $\bar{s}^L$ see Equation (14)

Region	Number of stations	$\bar{s}^L$ (mm a ⁻¹)	$\bar{e}$ (mm a ⁻¹)
R1	50 (24)	-2.08 (-2.99)	0.19 (0.28)
R2	29 (13)	2.55 (2.75)	0.14 (0.14)
R3	30 (14)	1.59 (1.78)	0.13 (0.20)
R4	4 (1)	1.16 (1.28)	0.25 (0.10)
R5	8 (3)	1.42 (1.14)	0.12 (0.16)
R1–R5	121 (55)	0.28 (-0.13)	0.17 (0.22)

In Table 3, we present the mean of the observational rate of RSL-height change, $\bar{s}^L$, and the associated standard deviation, $\bar{e}$, for stations in regions R1, ..., R5, respectively, and for all stations in region R1–R5. The values are estimated from the observational RSL-height change, s^L , for the individual tide-gauge stations according to Equation (9) and from the standard deviation given by Equation (10), respectively. The same results are given for the standardized time interval 1931–2000. The mean values, $\bar{s}^L$, demonstrate the influence of GIA on the tide-gauge data. Most tide-gauge stations of region R1 are located in the uplifting regions of Scandinavia, resulting in the negative value of $\bar{s}^L$, whereas the stations of region R2 are mainly located on the collapsing peripheral bulge associated with the former North American ice sheets, leading to the largest positive value. The restriction to the standardized time interval 1931–2000 does not cause any significant differences.

5. Residual RSL Height

To reduce the observational trends with respect to the influence of the Pleistocene glaciation, we use

$$\dot{s}^R := \dot{s}^L - \dot{s}, \quad (13)$$

where $\dot{s}$ denotes the computational rate of RSL-height change and $\dot{s}^R$ the residual rate of RSL-height change. The mean values and associated variances are estimated using

$$\bar{s}^{L|R} := \frac{1}{K} \sum_{k=1}^K \dot{s}_k^{L|R}, \quad (14)$$

$$\bar{\sigma}^{L|R} := \sqrt{\frac{1}{K-1} \sum_{k=1}^K \left(\dot{s}_k^{L|R} - \bar{s}^{L|R} \right)^2}, \quad (15)$$

where $\dot{s}_k^{L|R}$ denotes the observational value and the residual values for the k th station, respectively, and K the total number of stations in the region considered. The variances, $\bar{\sigma}^{L|R}$, are used in lieu of standard deviations.

For region R1, where input model VMM/SCAN is favored, the computational RSL-height change is shown in Figure 7(a). The pattern of the contour map is controlled by the Scandinavian ice sheet. In the Gulf of Bothnia, $\dot{s}$ reaches $\sim -8 \text{ mm a}^{-1}$, whereas further south the value approaches zero along the German coast. Off the Norwegian coast, the zero line runs at a distance of $\sim 200 \text{ km}$. Figure 7(b) shows $\dot{s}^L$, $\dot{s}$ and $\dot{s}^R$ for the tide-gauge stations of region R1. The values of $\dot{s}^L$ and $\dot{s}$ are negative for tide-gauge stations located near the center of the Gulf of Bothnia. In contrast to this, $\dot{s}^R$ is positive for nearly all stations. This is also reflected in the observational mean, $\bar{\dot{s}}^L = -2.08 \text{ mm a}^{-1}$, and the residual mean, $\bar{\dot{s}}^R = 1.23 \text{ mm a}^{-1}$. The reduction of variability is also evident in the corresponding variances, $\bar{\sigma}^L = 3.09 \text{ mm a}^{-1}$ and $\bar{\sigma}^R = 0.68 \text{ mm a}^{-1}$.

The computational RSL-height change for input model VM1/NAWI indicated for region R2 is shown in Figure 8(a). The tide-gauge stations on the west coast of North America are located close to the zero contour of $\dot{s}$. For the stations on the North American east coast, $\dot{s}$ increases from $\sim 0 \text{ mm a}^{-1}$ in the southwest to $\sim 3 \text{ mm a}^{-1}$ in the northeast. In the Canadian Arctic, $\dot{s}$ reaches a maximum of $\sim 5 \text{ mm a}^{-1}$, whereas a minimum of $\sim -9 \text{ mm a}^{-1}$ is reached in Hudson Bay. The influence of the collapsing peripheral bulge is also evident in Figure 8(b), where the residual mean, $\bar{\dot{s}}^R = 1.77 \text{ mm a}^{-1}$, is slightly smaller than the observational mean, $\bar{\dot{s}}^L = 2.55 \text{ mm a}^{-1}$.

Figure 9(a) shows the computational RSL-height change for input model VMA/NAWI favored for region R3. All tide-gauge stations selected for this region are located where $\dot{s}$ differs only slightly from zero. The weak influence of GIA on the observational RSL height in region R3 is also visible in Figure 9(b), which shows that the differences between the observational mean, $\bar{\dot{s}}^L = 1.59 \text{ mm a}^{-1}$, and the residual mean, $\bar{\dot{s}}^R = 1.47 \text{ mm a}^{-1}$, are minor. This is also confirmed by the nearly identical variances, $\bar{\sigma}^L = 0.56 \text{ mm a}^{-1}$ and $\bar{\sigma}^R = 0.53 \text{ mm a}^{-1}$.

Figure 10(a) shows the computational RSL-height change for input model VMF/SCAN selected for region R4. The four tide-gauge stations are located in the Indic and northern Pacific Oceans, where the computational RSL height is slightly falling. This is confirmed in Figure 10(b), showing that the residual mean, $\bar{\dot{s}}^R = 1.40 \text{ mm a}^{-1}$, is barely larger than the observational mean, $\bar{\dot{s}}^L = 1.16 \text{ mm a}^{-1}$.

Figure 11(a) shows the computational RSL height for the optimum input model VM1/SCAN for region R5. The associated tide-gauge stations are located on the coasts of Australia, New Zealand and South America, where the computational RSL-height change is close to zero. Around Antarctica, $\dot{s}$ reaches $\sim 2 \text{ mm a}^{-1}$, whereas close to the Antarctic Peninsula $\dot{s}$ drops to -6 mm a^{-1} . Figure 11(b) shows that only for Fremantle (PSMSL-ID 680471) and Quequen (PSMSL-ID 860081) $\dot{s}$ differs significantly from zero. The residual mean, $\bar{\dot{s}}^R = 1.45 \text{ mm a}^{-1}$, is only slightly

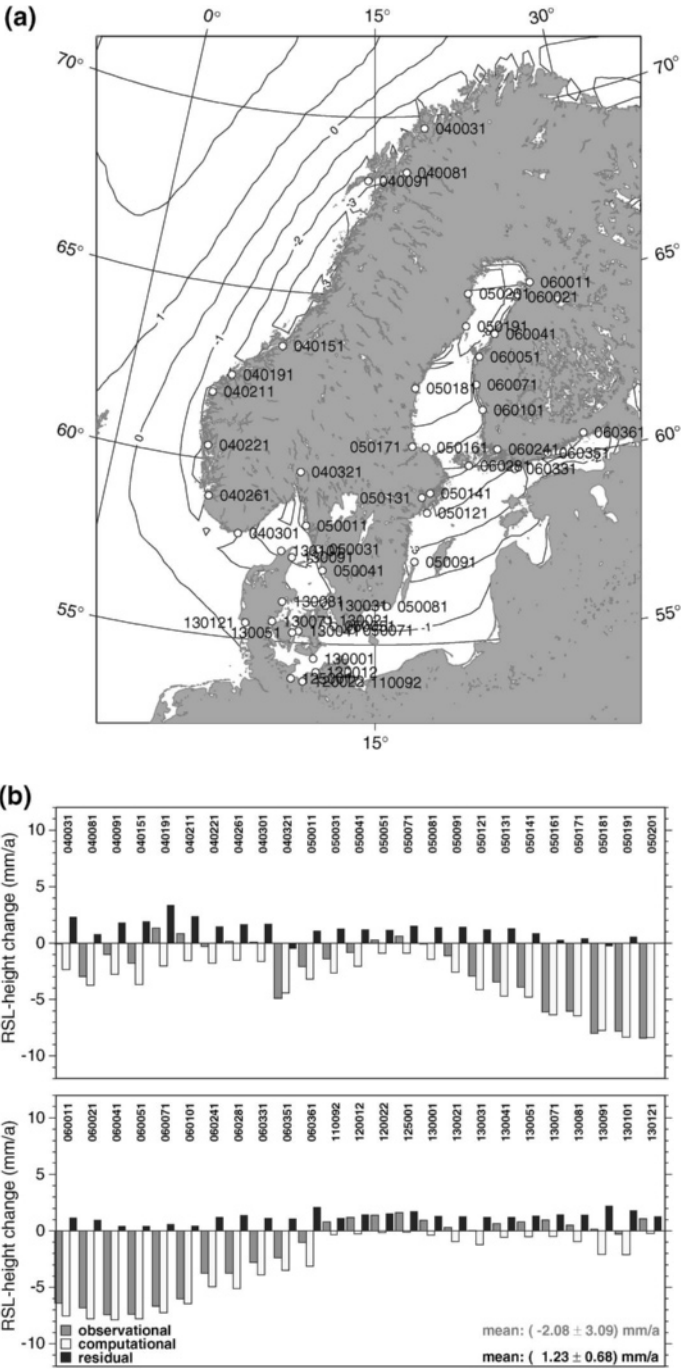




Figure 7

(a) Geographical distribution of tide-gauge stations in region R1 (northern Europe). The stations are identified by their PSMSL-ID. Isolines show the computational rate of RSL-height change in mm a^{-1} . (b) Observational, computational and residual rates of RSL-height changes for region R1. The computations are based on input model VMM/SCAN. The corresponding numerical values are given in Table 6.

larger than the observational mean, $\bar{s}^L = 1.42 \text{ mm a}^{-1}$. The variance $\bar{\sigma}^R = 0.76 \text{ mm a}^{-1}$ is greater than the variance $\bar{\sigma}^L = 0.59 \text{ mm a}^{-1}$ due to the relatively large value of $\dot{s}^R = 0.18 \text{ mm a}^{-1}$ for station Quequen (PSMSL-ID 860081).

Figure 12 shows $\dot{s}^L$, $\dot{s}$ and $\dot{s}^R$ for the 55 tide-gauge stations available for the standardized time interval 1931–2000. Similar to before, large negative values of $\sim -7 \text{ mm a}^{-1}$ apply to the observational RSL-height change for the stations of region R1 in the Gulf of Bothnia. In contrast to this, the residual RSL-height change is positive for all stations of region R1 with the exception of Oslo (PSMSL-ID 040321). This is reflected in the values $\bar{s}^R = 1.26 \text{ mm a}^{-1}$ and $\bar{\sigma}^R = 0.60 \text{ mm a}^{-1}$. For region R2 and stations PSMSL-ID 823031–960041 and 960091 located mainly on the North American west coast, $\dot{s}$ is close to zero and, therefore, the observational and residual values are nearly identical. For stations PSMSL-ID 960076–960087 and 960121–960161 on the North American east coast, the residual RSL-height change is close to zero. Therefore, the residual mean, $\bar{s}^R = 1.51 \text{ mm a}^{-1}$, is smaller than the observational mean, $\bar{s}^L = 2.75 \text{ mm a}^{-1}$. For the tide-gauge stations of region R3 largely located on the Dutch and German North Sea coasts, $\dot{s}$ is in the range of -0.27 mm a^{-1} to 0.55 mm a^{-1} . The residual mean, $\bar{s}^R = 1.58 \text{ mm a}^{-1}$, differs only by 0.20 mm a^{-1} from the observational mean. For region R4, Honolulu (PSMSL-ID 760031) is the only station left for the standardized time interval 1931–2000. Its residual rate of RSL-height change is $\bar{s}^R = 1.51 \text{ mm a}^{-1}$. For the tide-gauge stations of region R5 located in Australia and New Zealand, the corresponding value is $\bar{s}^R = 1.37 \text{ mm a}^{-1}$.

The values of $\bar{s}^L$ and $\bar{s}^R$ obtained for regions R1, ..., R5 are listed in Table 4. In addition, we give these results for the total number of tide-gauge stations in regions R1–R5 and also determine the mean of the regional mean values. The corresponding results for the standardized time interval 1931–2000 are also shown. The second type of averaging is considered to be superior in view of the large differences in the numbers of tide-gauge stations in the individual regions (Table 3). Based on it, we obtain a GMSL rise of $\bar{s}^R = (1.46 \pm 0.20) \text{ mm a}^{-1}$ for the complete time intervals and of $\bar{s}^R = (1.45 \pm 0.13) \text{ mm a}^{-1}$ for the standardized time interval. The GMSL rise obtained by direct averaging leads to $\bar{s}^R = (1.44 \pm 0.84) \text{ mm a}^{-1}$ and $\bar{s}^R = (1.43 \pm 0.80) \text{ mm a}^{-1}$ for the complete and standardized time intervals, respectively. Notable are the small differences between the estimates of the GMSL rise obtained on the basis of different time intervals and averaging procedures relative to the deviations (variances).

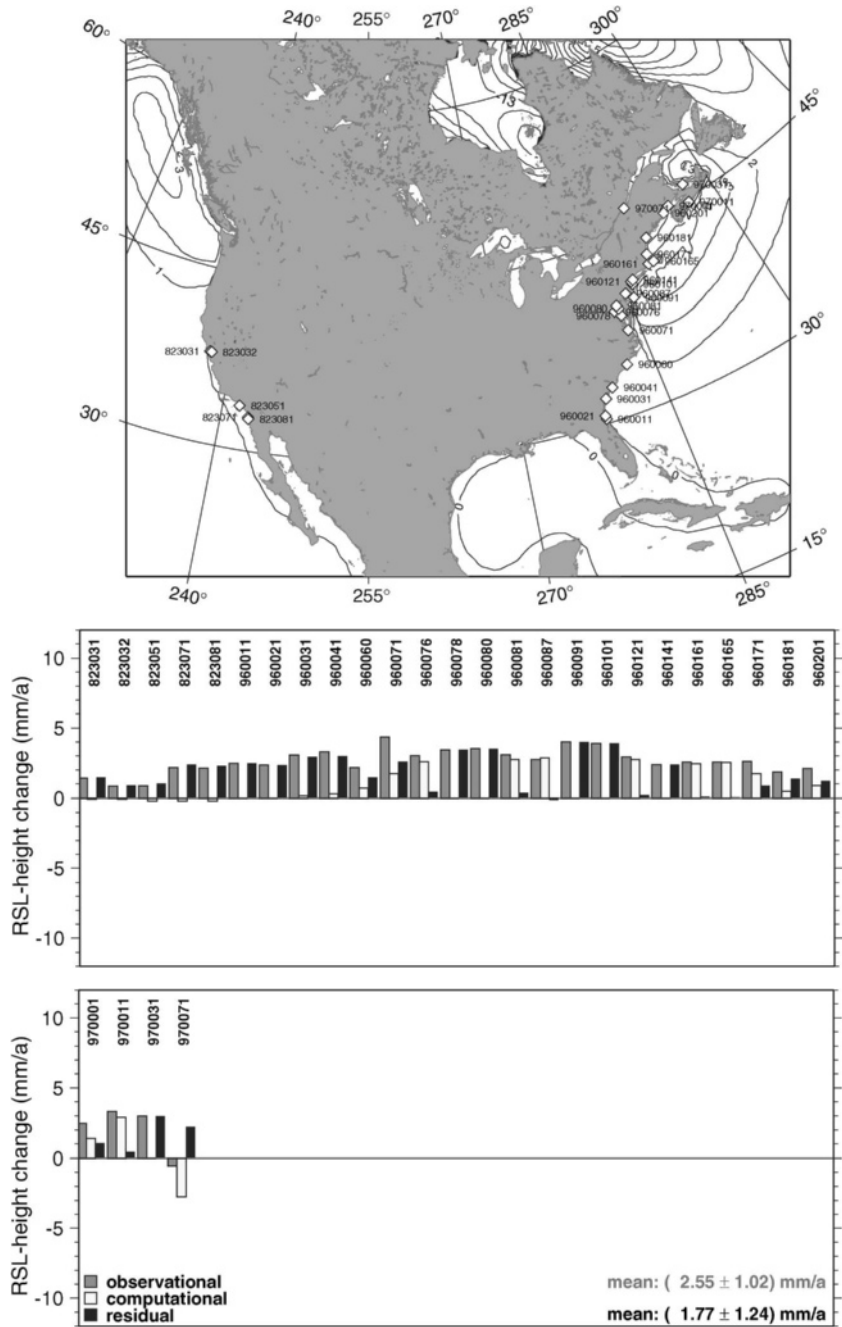




Figure 8

(a) Geographical distribution of tide-gauge stations in region R2 (North America). The stations are identified by their PSMSL-ID. Isolines show the computational rate of RSL-height change in mm a^{-1} . (b) Observational, computational and residual rates of RSL-height changes for region R2. The computations are based on input model VM1/NAWI. The corresponding numerical values are given in Table 7.

Table 5 compares several previous estimates of the GMSL rise with our result. With one exception, the value obtained in this work is less than the other values listed. However, five values (DAVIS and MITROVICA, 1996; PELTIER and JIANG, 1997; CHURCH *et al.*, 2001, 2004; HOLGATE and WOODWORTH, 2004) agree within their deviations with our result and two others (TRUPIN and WAHR, 1990; DOUGLAS, 1997) differ by only $\sim 0.3 \text{ mm a}^{-1}$. Exceptions are the estimates of $(3.20 \pm 0.20) \text{ mm a}^{-1}$ by CABANES *et al.* (2001) and $(3.10 \pm 0.40) \text{ mm a}^{-1}$ by CAZENAVE and NEREM (2004). In contrast to the others, which were derived from tide-gauge data, their estimates were based on satellite-altimetry data, which cover a time interval of only $\sim 10 \text{ a}$ and have a spatial resolution different from that of the tide-gauge data. This problem will be discussed in the following section.

6. Discussion

When discussing our result, we must consider the fact that the GMSL rise is composed of several contributions, where the largest are the ocean-mass change due to melt water from glaciers, ice caps and ice sheets, and the thermosteric effect, i.e. the volume change of the ocean water due to thermal expansion. For the first contribution, CHURCH *et al.* (2001) estimated $\sim 0.5 \text{ mm a}^{-1}$, whereas DOUGLAS and PELTIER (2002) reported a value of $\sim 0.3 \text{ mm a}^{-1}$. Based on interpolated ocean-temperature data recorded during the time interval 1955–1996, CABANES *et al.* (2001) estimated the thermosteric contribution to be $(0.50 \pm 0.05) \text{ mm a}^{-1}$. However, if the restricted time interval 1993–1998 was considered, a much larger thermosteric contribution of $(3.1 \pm 0.4) \text{ mm a}^{-1}$ was obtained. LOMBARD *et al.* (2005a) demonstrated that this large value is partly due to inconsistencies in the ocean-temperature data set used. By comparing two data sets, LOMBARD *et al.* (2005b) found an oscillatory behavior of the thermosteric sea-level contribution with periods on the decadal time scale. Moreover, they showed the influence of dynamic processes, such as the El Niño Southern Oscillation, the Pacific Decadal Oscillation and the North Atlantic Oscillation, on the thermosteric contribution to the sea level. The global mean of the linear thermosteric sea-level trend for a moving 10-a window (see Fig. 9 of LOMBARD *et al.*, 2005) varied between -1.5 mm a^{-1} and 1.5 mm a^{-1} during 1955–1995. For the complete time interval,

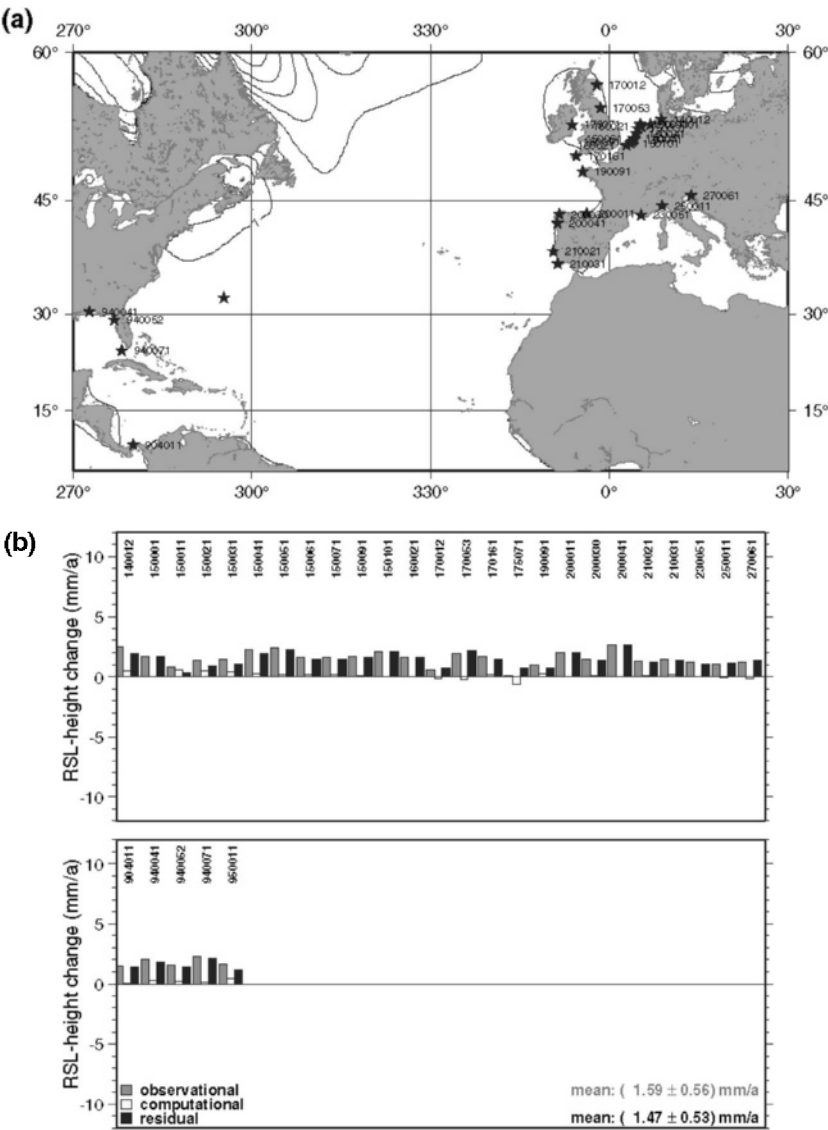


Figure 9

(a) Geographical distribution of tide-gauge stations in region R3 (northern Atlantic). The stations are identified by their PSMSL-ID. Isolines show the computational rate of RSL-height change in mm a⁻¹. (b) Observational, computational and residual rates of RSL-height changes for region R3. The computations are based on input model VMA/NAWI. The corresponding numerical values are given in Table 8.

the mean values for the two data sets are 0.02 mm a⁻¹ and 0.22 mm a⁻¹. These results indicate the complicated time dependence of the thermosteric contribution and the uncertainties in the data sets.

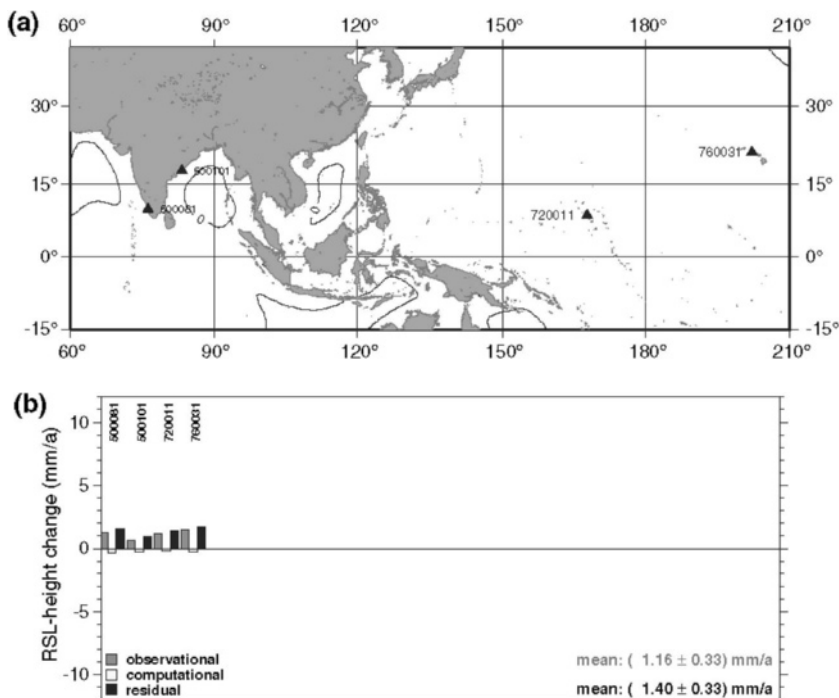


Figure 10

(a) Geographical distribution of tide-gauge stations in region R4. The stations are identified by their PSMSL-ID. Isolines show the computational rate of RSL-height change in mm a^{-1} . (b) Observational, computational and residual rates of RSL-height changes for region R4. The computations are based on input model VMF/SCAN. The corresponding numerical values are given in Table 9.

Based on TOPEX/Poseidon satellite altimetry, CABANES *et al.* (2001) and CAZENAVE and NEREM (2004) estimated a GMSL rise of $(3.2 \pm 0.2) \text{ mm a}^{-1}$ for the interval 1993–1998, whereas CAZENAVE and NEREM (2004) arrived at an estimate of $(2.8 \pm 0.4 \text{ mm a}^{-1})$ for the interval 1993–2003 after applying a GIA reduction of -0.3 mm a^{-1} . In addition to the short time intervals of satellite-altimetry measurements in comparison to those of tide-gauge data, the spatial distribution of the two types of measurement is also different. Tide-gauge stations are mostly located on the coast lines of North America and northern Europa. In contrast this, satellite-altimetry data cover the complete ocean surface between 66°N and 66°S , with the exception of coastal regions.

The value of $\bar{s}^R = (1.46 \pm 0.20) \text{ mm a}^{-1}$ estimated in this study is representative of the 20th century. To explain the difference between the GMSL rise based on tide-gauge and satellite-altimetry measurements, we propose the following hypotheses.

- The linear trends based on tide-gauge and satellite-altimetry measurements are only representative of the time interval considered. The difference between the

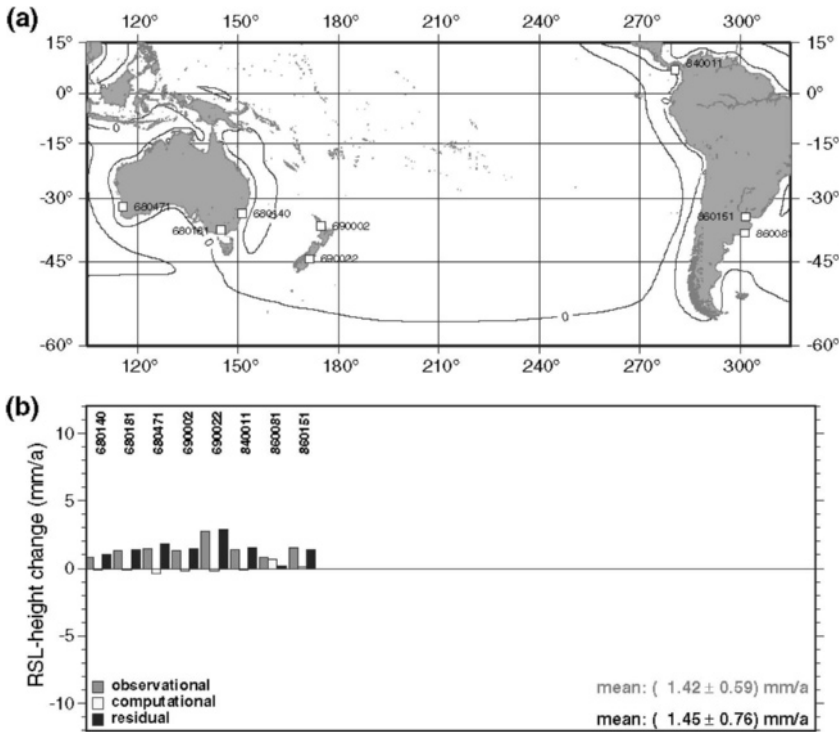


Figure 11

(a) Geographical distribution of tide-gauge stations in region R5. The stations are identified by their PSMSL-ID. Isolines show the computational rate of RSL-height change in mm a^{-1} . (b) Observational, computational and residual rates of RSL-height changes for region R5. The computations are based on input model VM1/SCAN. The corresponding numerical values are given in Table 10.

- values thus reflects a substantial acceleration of the GMSL rise during the last decade. The question remains, however, as to whether the thermosteric or the ocean-mass contribution is the main cause of this increase.
- The linear trend based on satellite-altimetry measurements does not reflect the GMSL rise. This hypothesis is supported by the study of LOMBARD *et al.* (2005b), which demonstrated the oscillatory behavior of the thermosteric sea level. Thus, for a 10-a time interval, its linear trend increases from -1.5 mm a^{-1} at the beginning of the 1980s to 1.5 mm a^{-1} at the end of the 1990s. In contrast to this, the linear trend based on long-term tide-gauge data does reflect the GMSL rise over several decades. In agreement with this, the long-term thermosteric contribution has been estimated to be between 0.02 mm a^{-1} and 0.5 mm a^{-1} .
 - The linear trend based on satellite-altimetry measurements is representative of the GMSL rise even for the 20th century, whereas the linear trend based on tide-gauge data is biased by the restriction of the stations to the coast. This means that tide-gauge records systematically underestimate the GMSL rise.

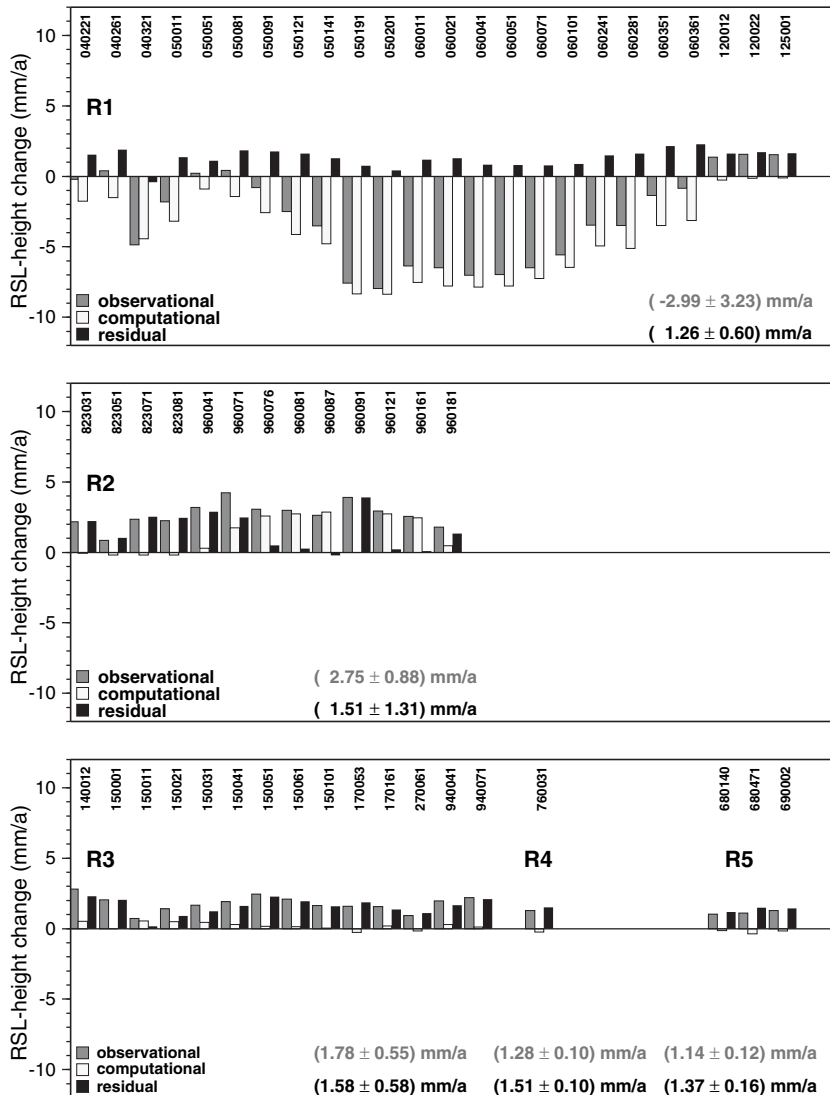


Figure 12

Observational, computational and residual rates of RSL-height changes for regions R1, ..., R5 and the standardized time interval 1931–2000 based on the input models selected for each region. The stations are identified by their PSMSL-ID. The corresponding numerical values are given in Tables 6–10.

Although we favor the second hypothesis, we cannot rule out the other two. Obviously, our discussion highlights the increasing importance of satellite-altimetry data for the determination of the GMSL rise. CAZENAVE and NEREM (2004) claimed that, on this basis and after a time interval of 30 a, it should become possible to determine time variations of the linear trend in the GMSL rise.

Table 4

Mean of observational rate of RSL-height change, $\bar{s}^L$, and mean of residual rate of RSL-height change, $\bar{s}^R$, for stations in regions $R_1, \dots, R_5$, respectively, and for all stations in region R1–R5. The associated variances, $\bar{\sigma}^L$ and $\bar{\sigma}^R$, are used as deviations. The values in parentheses apply to the standardized time interval 1931–2000, those on the last line are the means of the regional mean values for regions $R_1, \dots, R_5$.

Region	Input model	$\bar{s}^L$ (mm a ^{−1})		$\bar{s}^R$ (mm a ^{−1})	
R1	VMM/SCAN	−2.08 ± 3.09	(−2.99 ± 3.23)	1.23 ± 0.68	(1.26 ± 0.60)
R2	VM1/NAWI	2.55 ± 1.02	(2.75 ± 0.88)	1.77 ± 1.24	(1.51 ± 1.31)
R3	VMA/NAWI	1.59 ± 0.56	(1.78 ± 0.55)	1.47 ± 0.53	(1.58 ± 0.58)
R4	VMF/SCAN	1.16 ± 0.33	(1.28 ± 0.10)	1.40 ± 0.33	(1.51 ± 0.10)
R5	VM1/SCAN	1.42 ± 0.59	(1.14 ± 0.12)	1.45 ± 0.76	(1.37 ± 0.16)
R1–R5		0.28 ± 2.89	(−0.13 ± 3.36)	1.44 ± 0.84	(1.43 ± 0.80)
		0.93 ± 1.76	(0.79 ± 2.21)	1.46 ± 0.20	(1.45 ± 0.13)

Table 5

Previous estimates of GMSL rise.

Publication	GMSL rise $\bar{s}^R$ (mm a ^{−1})
TRUPIN and WAHR (1990)	1.75 ± 0.13
MITROVICA and DAVIS (1995); DAVIS and MITROVICA (1996)	1.40 ± 0.40
DOUGLAS (1997)	1.80 ± 0.10
PELTIER and JIANG (1997)	1.80 ± 0.60
CABANES <i>et al.</i> (2001)	3.20 ± 0.20
CHURCH <i>et al.</i> (2001)	1.50 ± 0.50
CAZENAVE and NEREM (2004)	3.10 ± 0.40
CHURCH <i>et al.</i> (2004)	1.80 ± 0.30
HOLGATE and WOODWORTH (2004)	1.70 ± 0.20
This study (Table 4)	1.46 ± 0.20

Unfortunately, satellite altimetry does not allow us to trace the RSL-height changes through the 20th century. Therefore, tide-gauge records remain the basis for the determination of the GMSL rise during the 20th century and continue to provide the background for the interpretation of satellite-altimetry measurements. In accordance with this, we propose further improvements of the tide-gauge data processing and the modelling of the GIA contribution to RSL-height change.

Thus, tide-gauge data processing may be optimized using statistical tests in the linear-trend analysis to provide more advanced criteria for the quality of the estimates. Furthermore, the inverted-barometer reduction should routinely be applied to tide-gauge stations. In addition, GPS-based vertical-displacement rates should be considered.

Apart from this, the modelling of GIA should be extended to a truly three-dimensional parameterization of the mantle viscosity. Moreover, the contribution of the incremental centrifugal potential to the linear-momentum conservation has been neglected so far and should be included in the future.

Appendix

A Tide-gauge Data

Table 6

Tide-gauge stations in region R1 with PSMSL-ID, name of station, geographical coordinates (latitude, λ , longitude, φ), observational, $\dot{s}^L$, computational, $\dot{s}$, and residual, $\dot{s}^R$, rate of RSL-height change. For $\dot{s}$, input model VMM/SCAN is used. The values in parentheses apply to the standardized time interval 1931–2000

PSMSL-ID	Name of station	λ (° N)	φ (° E)	$\dot{s}^L$ (mm a ⁻¹)	$\dot{s}$ (mm a ⁻¹)	$\dot{s}^R$ (mm a ⁻¹)
040031	Tromsø	69.650	18.967	-0.06	-2.36	2.30
040081	Narvik	68.433	17.417	-2.97	-3.75	0.78
040091	Kabelvåg	68.217	14.483	-1.01	-2.79	1.78
040151	Heimsjø	63.433	9.117	-1.77	-3.70	1.93
040191	Ålesund	62.467	6.150	1.32	-2.05	3.37
040211	Maløy	61.933	5.117	0.83	-1.55	2.38
040221	Bergen	60.400	5.300	-0.30 (-0.22)	-1.76	1.46 (1.54)
040261	Stavanger	58.967	5.733	0.17 (0.40)	-1.50	1.67 (1.90)
040301	Tregde	58.000	7.567	0.07	-1.63	1.70
040321	Oslo	59.900	10.750	-4.92 (-4.85)	-4.42	-0.50 (-0.43)
050011	Smögen	58.367	11.217	-2.10 (-1.81)	-3.18	1.08 (1.37)
050031	Göteborg	57.717	11.950	-1.40	-2.65	1.25
050041	Varberg	57.100	12.217	-0.85	-2.06	1.21
050051	Klagshamn	55.517	12.900	0.27 (0.22)	-0.89	1.16 (1.11)
050071	Ystad	55.417	13.817	0.60	-0.91	1.51
050081	Kungholmsfort	56.100	15.583	-0.06 (0.41)	-1.44	1.38 (1.85)
050091	Ölands n. dde	57.367	17.100	-1.13 (-0.81)	-2.57	1.44 (1.76)
050121	Landsort	58.750	17.867	-2.91 (-2.51)	-4.12	1.21 (1.61)
050131	Nedreödertalje	59.200	17.617	-3.43	-4.71	1.28
050141	Stockholm	59.317	18.083	-3.91 (-3.51)	-4.79	0.88 (1.28)
050161	Björn	60.633	17.967	-6.10	-6.37	0.27
050171	Nedre Gävle	60.667	17.167	-6.03	-6.44	0.41
050181	Draghallan	62.333	17.467	-7.99	-7.73	-0.26
050191	Ratan	64.000	20.917	-7.80 (-7.59)	-8.34	0.54 (0.75)
050201	Furuogrund	64.917	21.233	-8.41 (-7.96)	-8.37	-0.04 (0.41)
060011	Oulu	65.033	25.417	-6.39 (-6.36)	-7.54	1.15 (1.18)
060021	Raahe	64.667	24.417	-6.83 (-6.49)	-7.78	0.95 (1.29)
060041	Pietarsaari	63.700	22.700	-7.43 (-7.02)	-7.85	0.42 (0.83)
060051	Vaasa	63.100	21.567	-7.37 (-6.98)	-7.78	0.41 (0.80)
060071	Kaskinen	62.333	21.217	-6.67 (-6.49)	-7.26	0.59 (0.77)
060101	Mantyluoto	61.600	21.467	-6.03 (-5.58)	-6.47	0.44 (0.89)
060241	Turkur	60.433	22.100	-3.74 (-3.46)	-4.94	1.20 (1.48)
060281	Degerby	60.033	20.383	-3.74 (-3.50)	-5.12	1.38 (1.62)
060331	Hanko	59.817	22.983	-2.78	-3.91	1.13
060351	Helsinki	60.150	24.967	-2.41 (-1.35)	-3.50	1.09 (2.15)

Table 6

(Contd.)

PSMSL-ID	Name of station	λ (° N)	φ (° E)	$\dot{s}^L$ (mm a ⁻¹)	$\dot{s}$ (mm a ⁻¹)	$\dot{s}^R$ (mm a ⁻¹)
060361	Hamina	60.567	27.183	-1.03 (-0.86)	-3.13	2.10 (2.27)
110092	Swinoujscie	53.917	14.233	0.81	-0.33	1.14
120012	Warnemunde 2	54.183	12.083	1.19 (1.35)	-0.26	1.45 (1.61)
120022	Wismar 2	53.900	11.467	1.39 (1.57)	-0.15	1.54 (1.72)
125001	Travemunde	53.967	10.883	1.62 (1.54)	-0.11	1.73 (1.65)
130001	Gedser	54.567	11.933	0.94	-0.36	1.30
130021	Kobenhavn	55.683	12.600	0.32	-0.96	1.28
130031	Hornbæk	56.100	12.467	0.01	-1.23	1.24
130041	Korsør	55.333	11.133	0.66	-0.58	1.24
130051	Slipshavn	55.283	10.833	0.81	-0.52	1.33
130071	Fredericia	55.567	9.767	0.95	-0.49	1.44
130081	Århus	56.150	10.217	0.49	-0.92	1.41
130091	Frederikshavn	57.433	10.567	0.15	-2.08	2.23
130101	Hirtshals	57.600	9.967	-0.28	-2.09	1.81
130121	Esbjerg	55.467	8.433	1.07	-0.21	1.28

Table 7

Tide-gauge stations in region R2 with PSMSL-ID, name of station, geographical coordinates (latitude, λ , longitude, φ), observational, $\dot{s}^L$, computational, $\dot{s}$, and residual, $\dot{s}^R$, rate of RSL-height change. For $\dot{s}$, input model VMM1/NAWI is used. The values in parentheses apply to the standardized time interval 1931–2000

PSMSL-ID	Name of station	λ (° N)	φ (° E)	$\dot{s}^L$ (mm a ⁻¹)	$\dot{s}$ (mm a ⁻¹)	$\dot{s}^R$ (mm a ⁻¹)
823031	San Francisco	37.800	237.533	1.41 (2.17)	-0.05	1.46 (2.22)
823032	Alameda	37.767	237.700	0.83	-0.07	0.90
823051	Los Angeles	33.717	241.733	0.85 (0.86)	-0.18	1.03 (1.04)
823071	La Jolla	32.867	242.750	2.17 (2.34)	-0.20	2.37 (2.54)
823081	San Diego	32.717	242.833	2.11 (2.25)	-0.19	2.30 (2.44)
960011	Mayport	30.400	278.567	2.47	-0.02	2.49
960021	Fernandina	30.683	278.533	2.35	-0.01	2.36
960031	Fort Pulaski	32.033	279.100	3.07	0.13	2.94
960041	Charleston I	32.783	280.067	3.28 (3.18)	0.30	2.98 (2.88)
960060	Wilmington	34.233	282.267	2.17	0.71	1.46
960071	Hampton Roads	36.950	283.667	4.36 (4.24)	1.75	2.61 (2.49)
960076	Washington DC	38.867	282.983	3.02 (3.06)	2.57	0.45 (0.49)
960078	Solomons Island	38.317	283.550	3.45	0.00	3.45
960080	Annapolis	38.983	283.517	3.52	0.00	3.52
960081	Baltimore	39.267	283.417	3.10 (2.98)	2.72	0.38 (0.26)
960087	Philadelphia	39.950	284.867	2.73 (2.64)	2.85	-0.12 (-0.21)
960091	Atlantic City	39.350	285.583	3.99 (3.90)	0.00	3.99 (3.90)
960101	Sandy Hook	40.467	285.983	3.90	0.00	3.90
960121	New York	40.700	285.983	2.93 (2.93)	2.72	0.21 (0.21)
960141	Willets Point	40.800	286.217	2.38	0.00	2.38
960161	Newport	41.500	288.667	2.56 (2.55)	2.46	0.10 (0.09)
960165	Woods Hole	41.533	289.333	2.56	2.52	0.04
960171	Boston	42.350	288.950	2.61	1.73	0.88
960181	Portland	43.667	289.750	1.84 (1.80)	0.46	1.38 (1.34)

Table 7

(Contd.)

PSMSL-ID	Name of station	λ ($^{\circ}$ N)	φ ($^{\circ}$ E)	$\dot{s}^L$ (mm a $^{-1}$)	$\dot{s}$ (mm a $^{-1}$)	$\dot{s}^R$ (mm a $^{-1}$)
960201	Eastport	44.900	293.017	2.09	0.89	1.20
970001	Saint John	45.267	293.933	2.44	1.38	1.06
970011	Halifax	44.667	296.417	3.31	2.90	0.41
970031	Charlottetown	46.267	296.883	2.99	0.00	2.99
970071	Quebec	46.833	288.833	-0.54	-2.76	2.22

Table 8

Tide-gauge stations in region R3 with PSMSL-ID, name of station, geographical coordinates (latitude, λ , longitude, φ), observational, $\dot{s}^L$, computational, $\dot{s}$, and residual, $\dot{s}^R$, rate of RSL-height change. For $\dot{s}$, input model VMA/NAWI is used. The values in parentheses apply to the standardized time interval 1931–2000

PSMSL-ID	Name of station	λ ($^{\circ}$ N)	φ ($^{\circ}$ E)	$\dot{s}^L$ (mm a $^{-1}$)	$\dot{s}$ (mm a $^{-1}$)	$\dot{s}^R$ (mm a $^{-1}$)
140012	Cuxhaven 2	53.867	8.717	2.44 (2.80)	0.51	1.93 (2.29)
150001	Delfzijl	53.333	6.933	1.66 (2.05)	0.00	1.66 (2.05)
150011	West Terschelling	53.367	5.217	0.86 (0.72)	0.55	0.31 (0.17)
150021	Harlingen	53.167	5.417	1.37 (1.40)	0.50	0.87 (0.90)
150031	Den Helder	52.967	4.750	1.46 (1.67)	0.44	1.02 (1.23)
150041	IJmuiden	52.467	4.583	2.23 (1.92)	0.30	1.93 (1.62)
150051	Hoek van Holland	51.983	4.117	2.39 (2.44)	0.17	2.22 (2.27)
150061	Maassluis	51.917	4.250	1.65 (2.10)	0.15	1.50 (1.95)
150071	Hellevoetsluis	51.817	4.133	1.59	0.13	1.46
150091	Zierikze	51.633	3.917	1.70	0.08	1.62
150101	Vlissingen	51.450	3.600	2.11 (1.63)	0.04	2.07 (1.59)
160021	Oostende	51.233	2.917	1.61	-0.01	1.62
170012	Aberdeen II	57.150	357.917	0.59	-0.16	0.75
170053	North Shields	55.000	358.567	1.90 (1.59)	-0.27	2.17 (1.86)
170161	Newlyn	50.100	354.450	1.69 (1.56)	0.20	1.49 (1.36)
175071	Dublin	53.350	353.783	0.12	-0.62	0.74
190091	Brest	48.383	355.500	1.00	0.27	0.73
200011	Santander I	43.467	356.200	2.00	-0.01	2.01
200030	La Coruna I	43.367	351.600	1.44	0.11	1.33
200041	Vigo	42.250	351.267	2.63	0.01	2.62
210021	Cascais	38.683	350.583	1.28	0.06	1.22
210031	Lagos	37.100	351.333	1.50	0.11	1.39
230051	Marseille	43.300	5.350	1.22	0.14	1.08
250011	Genova	44.400	8.900	1.07	-0.04	1.11
270061	Trieste	45.650	13.750	1.17 (0.94)	-0.17	1.34 (1.11)
904011	Cristobal	9.350	280.083	1.43	0.07	1.36
940041	Pensacola	30.400	272.783	2.08 (1.96)	0.29	1.79 (1.67)
940052	Cedar Key II	29.133	276.967	1.57	0.18	1.39
940071	Key West	24.550	278.200	2.26 (2.21)	0.11	2.15 (2.10)
950011	St Georges	32.367	295.300	1.61	0.44	1.17

Table 9

Tide-gauge stations in region R4 with PSMSL-ID, name of station, geographical coordinates (latitude, λ , longitude, ϕ), observational, $\dot{s}^L$, computational, $\dot{s}$, and residual, $\dot{s}^R$, rate of RSL-height change. For $\dot{s}$, input model VMF/SCAN is used. The values in parentheses apply to the standardized time interval 1931–2000

PSMSL-ID	Name of station	λ (° N)	ϕ (° E)	$\dot{s}^L$ (mm a ⁻¹)	$\dot{s}$ (mm a ⁻¹)	$\dot{s}^R$ (mm a ⁻¹)
500081	Cochin	9.967	76.267	1.27	-0.33	1.60
500101	Vishakhapatnam	17.683	83.283	0.68	-0.26	0.94
720011	Kwajalein	8.733	167.733	1.22	-0.16	1.38
760031	Honolulu	21.317	202.133	1.46 (1.28)	-0.23	1.69 (1.51)

Table 10

Tide-gauge stations in region R5 with PSMS-ID, name of station, geographical coordinates (latitude, λ , longitude, ϕ), observational, $\dot{s}^L$, computational, $\dot{s}$, and residual, $\dot{s}^R$, rate of RSL-height change. For $\dot{s}$, input model VM1/SCAN is used. The values in parentheses apply to the standardized time interval 1931–2000

PSMSL-ID	Name of station	λ (° N)	ϕ (° E)	$\dot{s}^L$ (mm a ⁻¹)	$\dot{s}$ (mm a ⁻¹)	$\dot{s}^R$ (mm a ⁻¹)
680140	Sydney	-33.850	151.233	0.86 (1.04)	-0.14	1.00 (1.18)
680181	Williamstown	-37.867	144.917	1.29	-0.11	1.40
680471	Fremantle	-32.050	115.733	1.43 (1.11)	-0.37	1.80 (1.48)
690002	Auckland II	-36.850	174.767	1.30 (1.28)	-0.16	1.46 (1.44)
690022	Lyttelton II	-44.400	171.267	2.72	-0.16	2.88
840011	Balboa	8.967	280.433	1.42	-0.09	1.51
860081	Quequen	-38.583	301.300	0.82	0.64	0.18
860151	Buenos Aires	-34.600	301.633	1.55	0.15	1.40

REFERENCES

- BEVINGTON, P., *Data Reduction and Error Analysis for the Physical Sciences* (McGraw-Hill, New York 1969).
- BINTANJA, R., VAN DER WAL, R. and OERLEMANS, J. (2002), *Global ice volume variations through the last glacial cycle simulated by a 3-d ice-dynamical model*, *Quat. Int.* 95–96, 11–23.
- CABANES, C., CAZENAVE, A., and LE PROVOST, C. (2001), *Sea level rise during past 40 years determined from satellite and in situ observations*, *Science* 294, 840–842.
- CAZENAVE, A. and NEREM, R. (2004), *Present-day sea level change: Observations and causes*, *Rev. Geophys.* 42, doi:10.1029/2003RG000139.
- CHURCH, J., GREGORY, J., HUYBRECHTS, P., KUHN, M., LAMBECK, K., NHUAN, M., QIN, D. and WOODWORTH, P. (2001), *Changes in sea level*. In (Houghton, J. E. A. Ed.), *Climate Change 2001: The Scientific Basis*, Contributions of the Working Group I to the Third Assessment Report of the Intergovernmental Panel on Climate Change (Cambridge University Press, New York 2001) pp. 641–684.
- CHURCH, J., WHITE, N., COLEMAN, R., LAMBECK, K., and MITROVICA, J. (2004), *Estimates of the regional distribution of sea level rise over the 1950–2000 period*, *J. Climate* 17, 2609–2625.
- CLARK, J., FARRELL, W., and PELTIER, W. (1978), *Global change in postglacial sea level: A numerical calculation*, *Quat. Res.* 9, 265–287.
- DAVIS, J. and MITROVICA, J. (1996), *Glacial isostatic adjustment and the anomalous tide gauge record of eastern North America*, *Nature* 379, 331–333.
- DOUGLAS, B. (1997), *Global sea rise: A redetermination*, *Surv. Geophys.* 18, 279–292.
- DOUGLAS, B. and PELTIER, W. (2002), *The puzzle of global sea-level rise*, *Physics Today* 55(3), 35–40.

- DZIEWONSKI, A. and ANDERSON, D. (1981), *Preliminary reference earth model*, Phys. Earth Planet. Inter. 25, 297–356.
- FARRELL, W. and CLARK, J. (1976), *On postglacial sea level*, Geophys. J. Astr. Soc. 46, 647–667.
- FLEMING, K., MARTINEC, Z., and WOLF, D., *A reinterpretation of the Fennoscandian relaxation-time spectrum for a viscoelastic lithosphere*. In (Tziavoz, I., Ed.), *Gravity and Geoid 2002* (Ziti Publishing 2003) pp. 432–438.
- HAGEDOORN, J. (2005), *Glaziale Isostasie und rezente Meeresspiegeländerung*, Sci. Techn. Rep. GFZ-Potsdam, STR 05/13.
- HEISKANEN, W. and MORITZ, H., *Physical Geodesy* (W.H. Freeman and Co., San Francisco 1967).
- HOLGATE, S. and WOODWORTH, P. (2004), *Evidence for enhanced coastal sea level rise during the 1990s*, Geophys. Res. Lett. 31, doi:10.1029/2004GL019626.
- HUYBRECHTS, P. (2002), *Sea-level changes at the LGM from ice-dynamic reconstructions of the Greenland and Antarctic ice sheets during the glacial cycles*, Quat. Sci. Rev. 21, 203–231.
- JOHNSTON, P. (1993), *The effect of spatially non-uniform water loads on prediction of sea-level change*, Geophys. J. Int. 114, 615–634.
- JOHNSTON, P. and LAMBECK, K. (1999), *Postglacial rebound and sea level contributions to changes in the geoid and the earth's rotation axis*, Geophys. J. Int. 136, 537–558.
- KAUFMANN, G., WU, P., and WOLF, D. (1997), *Some effects of lateral heterogeneities in the upper-mantle on postglacial land uplift close to continental margins*, Geophys. J. Int. 128, 175–187.
- KENDALL, R., MITROVICA, J., and MILNE, G. (2005), *On post-glacial sea level – II. numerical formulation and comparative results on spherically symmetric models*, Geophys. J. Int. 161, 679–706.
- KLEMMANN, V. and WOLF, D. (2007), *Using fuzzy logic for the analysis of sea-level indicators with respect to glacial-isostatic adjustment*, Pure Appl. Geophys. 164, doi: 10.1007/s00024-007-0191-x.
- KLEMMANN, V., WOLF, D., FLEMING, K., and SHENNEN, I. (2007), *A data bank for late Peistocene and Holocene sea-level indicators*, Sci. Techn. Rep. GFZ Potsdam, in preparation.
- LAMBECK, K. (1993), *Glacial rebound of the British Isles–II. A high-resolution, high-precision model*, Geophys. J. Int. 115, 960–990.
- LAMBECK, K. and CHAPPELL, J. (2001), *Sea level change through the last glacial cycle*, Science 292, 697–686.
- LAMBECK, K., PURCELL, A., JOHNSTON, P., NAKADA, M. and YOKOYAMA, Y. (2003), *Water-load definition in the glacio-hydro-isostatic sea-level equation*, Quat. Sci. Rev. 22, 309–318.
- LAMBECK, K., SMITHER, C. and JOHNSTON, P. (1998), *Sea-level change, glacial rebound and mantle viscosity for northern Europe*, Geophys. J. Int. 134, 102–144.
- LOMBARD, A., CAZENAVE, A., DOMINH, K., CABANES, C. and NEREM, R. (2005a), *Thermosteric sea level rise for the past 50 years; Comparison with tide gauges and inference on water mass contribution*, Global Planet. Change 48, 303–312.
- LOMBARD, A., CAZENAVE, A., LE TRAON, P.-Y., and ISHII, M. (2005b), *Contribution of thermal expansion to present-day sea-level change revisited*, Global Planet. Change 47, 1–16.
- MARTINEC, Z. (2000), *Spectral-finite element approach to three-dimensional viscoelastic relaxation in a spherical earth*, Geophys. J. Int. 142, 117–141.
- MARTINEC, Z. and HAGEDOORN, J. (2005), *Time-domain approach to linearized rotational response of a three-dimensional viscoelastic earth model induced by glacial isostatic adjustment: I. Inertia-tensor perturbations*, Geophys. J. Int. 163, 443–462.
- MARTINEC, Z. and WOLF, D. (2005), *Inverting the Fennoscandian relaxation-time spectrum in terms of an axisymmetric viscosity distribution with a lithospheric root*, J. Geodyn. 39, 143–163.
- MCCONNELL, R. (1968), *Viscosity of the mantle from relaxation time spectra of isostatic adjustment*, J. Geophys. Res. 73, 7089–7105.
- MILNE, G. and MITROVICA, J. (1998), *Postglacial sea-level change on a rotating earth*, Geophys. J. Int. 133, 1–19.
- MILNE, G., MITROVICA, J., and DAVIS, J. (1999), *Near-field hydro-isostasy: The implementation of a revised sea-level equation*, Geophys. J. Int. 139, 464–482.
- MITROVICA, J. (2003), *Recent controversies in predicting post-glacial sea-level change*, Quat. Sci. Rev. 22, 127–133.
- MITROVICA, J. and DAVIS, J. (1995), *Present-day post-glacial sea level change far from the Late Pleistocene ice sheets: implications for recent analysis of tide gauge records*, Geophys. Res. Lett. 22, 2529–2532.

- MITROVICA, J., MILNE, G., and DAVIS, J. (2001), *Glacial isostatic adjustment on a rotating earth*, *Geophys. J. Int.* 147, 562–578.
- MITROVICA, J. and PELTIER, W. (1991), *On postglacial geoid subsidence over the equatorial oceans*, *J. Geophys. Res.* 96(B12), 20053–20071.
- MITROVICA, J., WAHR, J., MATSUYAMA, I., and PAULSON, A. (2005), *The rotational stability of an ice-age earth*, *Geophys. J. Int.* 161, 491–506.
- OKUNO, J. and NAKADA, M. (2001), *Effects of water load on geophysical signals due to glacial rebound and implications for mantle viscosity*, *Earth Planets Space* 53, 1121–1135.
- PELTIER, W. (1994), *Ice age paleotopography*, *Science* 265, 195–201.
- PELTIER, W. (1999), *Global sea level rise and glacial isostatic adjustment*, *Global Planet. Change* 20, 93–123.
- PELTIER, W., FARRELL, W., and CLARK, J. (1978), *Glacial isostasy and relative sea level: A global finite element model*, *Tectonophysics* 50, 81–110.
- PELTIER, W. and JIANG, X. (1997), *Mantle viscosity, glacial isostatic adjustment and the eustatic level of the sea*, *Surv. Geophys.* 18, 239–277.
- PIRAZOLLI, P., *Sea-Level Change, The last 20000 years. Coastal Morphology and Research* (John Wiley 2377).
- TOSI, L., CARBOGNIN, L., TEATINI, P., STROZZI, T., and WEGMÜLLER, U. (2002), *Evidence of the present relative land stability of Venice, Italy, from land, sea, and space observations*, *Geophys. Res. Lett.* 29, doi:10.1029/2001GL013211.
- TRUPIN, A. and WAHR, J. (1990), *Spectroscopic analysis of global tide gauge sea level data*, *Geophys. J. Int.* 100, 441–453.
- TUSHINGHAM, A. and PELTIER, W. (1991), *Ice-3G: A new global model of late Pleistocene deglaciation based upon geophysical predictions of post-glacial relative sea level change*, *J. Geophys. Res.* 96(B3), 4497–4523.
- WIECZKOWSKI, K., MITROVICA, J., and WOLF, D. (1999), *A revised relaxation-time spectrum for Fennoscandia*, *Geophys. J. Int.* 139, 69–86.
- WOLF, D., KLEMAN, V., WÜNSCH, J., and ZHANG, F.-P. (2006), *A reanalysis and reinterpretation of geodetic and geological evidence of glacial-isostatic adjustment in the Churchill region, Hudson Bay*, *Surv. Geophys.* 27, 19–61.
- WU, P. and VAN DER WAL, W. (2003), *Postglacial sealevels on a spherical, self-gravitating viscoelastic earth: Effects of lateral viscosity variations in the upper mantle on the inference of viscosity contrasts in the lower mantle*, *Earth Planet. Sci. Lett.* 211, 57–68.
- WU, P., WANG, H., and SCHOTMAN, H. (2005), *Postglacial induced surface motions, sea-levels and geoid rates on a spherical, self-gravitating laterally heterogeneous earth*, *J. Geodyn.* 39, 127–142.
- YOKOYAMA, Y., DE DEKKER, P., LAMBECK, K., JOHNSTON, P., and FIFIELD, L. (2001), *Sea-level at the last glacial maximum: Evidence from northwestern Australia to constrain ice volumes for oxygen isotope stage 2*, *Palaeogr., Palaeoclim., Palaeoecol.* 165, 281–297.
- ZWECK, C. and HUYBRECHTS, P. (2005), *Modelling of the northern hemisphere ice sheet during the last glacial cycle and glaciological sensitivity*, *J. Geophys. Res.*, 110, doi: 10.1029/2004JD005489.

(Received August 26, 2005, revised June 26, 2006, accepted June 29, 2006)

Published Online First: April 16, 2007

To access this journal online:
www.birkhauser.ch/pag

Modeling of Stress Changes at Mayon Volcano, Philippines

K. F. TIAMPO,¹ J. FERNÁNDEZ,² T. HAYES,¹ and G. JENTZSCH³

Abstract—In this paper, we analyze the stress change associated with the inverted volcanic source models at Mayon volcano, Philippines, where there are gravity changes without significant deformation after the 1993 eruption. We detail the applicable data and the associated inversion techniques and models prior to calculating the appropriate stress changes. It is determined that the stress change associated with the central magmatic source produces compressional stress changes at a secondary source to the northwest, prompting a change in the local water storage in the underlying fractured rock.

Key words: Mayon volcano, genetic algorithm, elastic-gravitational layered earth model, volcanic source, gravity, displacement.

1. Introduction

Volcanic activity almost inevitably produces geodetic effects such as deformation and gravity changes. As a result, the most promising methods for studying volcano- and earthquake-related phenomena today include the use of high-precision crustal deformation data. On the basis of this fact and the high levels of precision attainable, different geodetic techniques have proven to be a powerful tool in the monitoring of volcanic activity, making it possible to detect ground motion and gravity changes that reflect magma rising from depth, at times months or weeks before the magma flow leads to earthquakes or other eruption precursors (e.g., DELANEY and MCTIGUE, 1994; RYMER, 1996; DVORAK and DZURISIN, 1997; MASSONET and FEIGL, 1998; DZURISIN *et al.*, 1999; FERNÁNDEZ *et al.*, 1999; DZURISIN, 2000; RYMER and WILLIAMS-JONES, 2000). However, applying these long-term geodetic monitoring techniques to volcanically active zones inevitably involves the subsequent interpretation of the deformations and gravity changes using a variety of modeling techniques.

Physical deformation models are essential tools for the latter task, and include spherical and ellipsoidal point sources, finite sources, magma migration, collapse

¹ Dept. of Earth Sciences, University of Western Ontario, London, Ontario, Canada.
E-mail: ktiampo@uwo.ca

² Instituto de Astronomía y Geodesia (CSIC-UCM). Fac. C. Matemáticas, Ciudad Universitaria, Plaza de Ciencias, 3., 28040, Madrid, Spain.

³ Institute for Geosciences, FSU Jena, Burgweg 11, D-07749, Jena, Germany.

structures and fluid migration (e.g., MOGI, 1958; RUNDLE, 1982; DAVIS, 1986; BONAFEDE, 1991; FERNÁNDEZ and RUNDLE, 1994; LANGBEIN *et al.*, 1995; DE NATALE and PINGUE, 1996; DVORAK and DZURISIN, 1997; TIAMPO *et al.*, 2000). Such models are also useful for analyzing precursory phenomena and optimizing the development of observation networks. Normally, simple deformation models are used because of the ease of calculating the inverse problem. However, sometimes it is better to use models that are as realistic as possible, particularly when it is desirable to learn about the physics of the source process.

Obviously, quantitative models cannot purport to cover all different physical and chemical aspects of volcanoes. It is important to select and focus on key phenomena. Present knowledge of the critical stages prior to volcanic eruption in large part is based on elastostatic views; by studying volcanic unrest in terms of mechanical models involving overpressure in magma chambers and conduits. MOGI (1958) applied a center of dilatation (point pressure source) in an elastic half-space to interpret the ground deformation produced in volcanic areas. The Mogi model has been extensively applied in modeling ground deformations in volcanic areas and has been successful primarily in explaining the vertical component of the ground deformations. However, this model often poses difficulties in simultaneously modeling all components of the observed displacements in conjunction with gravity changes (KISSLINGER, 1975; RYMER *et al.*, 1993; RYMER, 1996), and there is a large body of evidence for ground deformations and seismicity at calderas and other volcanic areas that cannot be modeled by these purely elastic effects (e.g., EGGERS, 1987; BERRINO and CORRADO, 1991; BONAFEDE, 1991; RYMER *et al.*, 1993; DE NATALE *et al.*, 1997; GAETA *et al.*, 1998; JAHR *et al.*, 1998; WATANABE *et al.*, 1998; JENTZSCH *et al.*, 2001b; PRITCHARD and SIMONS, 2002).

In this paper, we detail the modeling of stress at Mayon volcano, based upon the results from a prior inversion of the geodetic data from Mayon volcano in the Philippines, using a nonlinear genetic algorithm (GA) inversion technique and the elastic-gravitational model originally developed by RUNDLE (1980, 1982) and subsequently refined by FERNÁNDEZ and RUNDLE (1994) and FERNÁNDEZ *et al.* (1997, 2005). Mayon volcano provides an example of the case discussed above, where there are gravity changes without significant deformation and where the more complicated inversion provides new insights into the underlying source mechanisms. We perform this analysis of the stress changes at Mayon volcano as one possible explanation of the observed phenomena.

2. Background

Mayon volcano is part of the Bicol volcanic chain southeast of the Island of Luzon, Philippines (Fig. 1), part of the Legazpi Lineament of the central Philippine fault system, which runs NW-SE across Legazpi City. The Bicol chain, ten volcanic

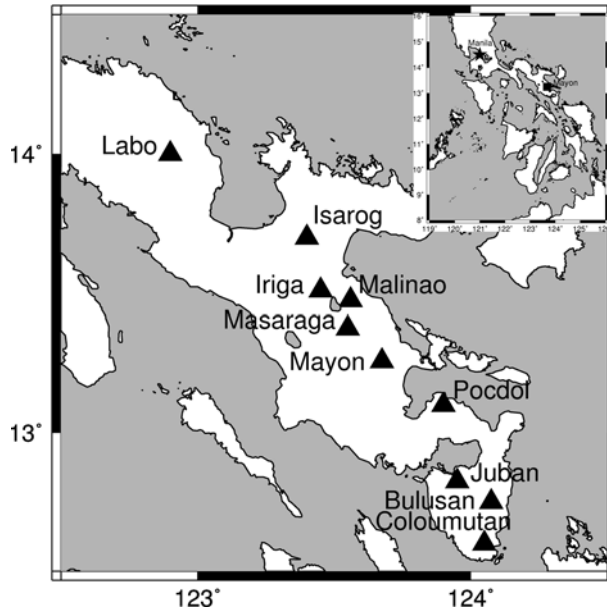


Figure 1

Location of Mayon volcano, Philippines, (Previously published in pageoph 161/7, 2001).

peaks, of which Mayon and Bulusan are still active, is made up of volcanic rocks, accompanied by Tertiary and Pleistocene sediments (JENTZSCH *et al.*, 2001a). Mayon itself is a nearly perfect cone with a height of 2462 m. Almost one million people live in the vicinity of Mayon volcano, and Legazpi City is situated only 12 km away from the crater. Mayon lava varies from basaltic through basaltic andesites, and into the andesitic region, with a pattern of three basaltic flows followed by six to ten andesitic flows (NEWHALL, 1979). Its activity is quasiperiodic; in the last century it erupted nearly every 10 years, with the last eruption in 2001. In early 1993, just after the start of gravity measurements in December 1992, an eruption occurred, with ash fall and a lava flow of approximately 10 million m³ (VÖLKSEN and SEEGER, 1995; JAHR *et al.*, 1998; JENTZSCH *et al.*, 2001b).

After the 1993 eruption, a second campaign was carried out in May 1993 to monitor the decrease in activity. Height control was provided by parallel global positioning system (GPS) measurements. Five microgravity and differential GPS campaigns were carried out within four years, designated as campaigns one through five (December 1992, May 1993, May 1994, December 1994, and December 1996). Differences between each campaign are designated as epochs, e.g. epoch 2–1 is the difference in gravity measurements between May 1993, campaign 2, and December 1992, campaign 1. The error of the gravity differences between the points derived from a least-squares adjustment of the data of each campaign is approximately $\pm 12 \mu\text{Gal}$ (JENTZSCH *et al.*, 2001b). Note that, while the measurements in the

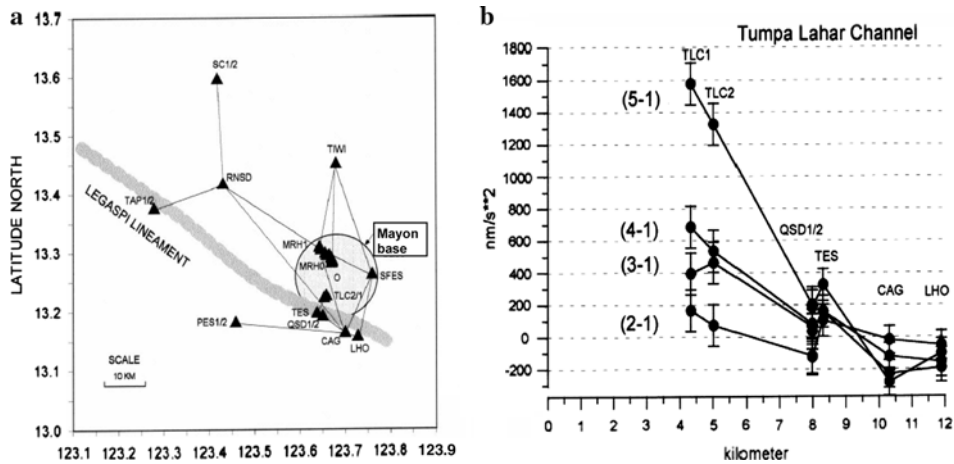


Figure 2

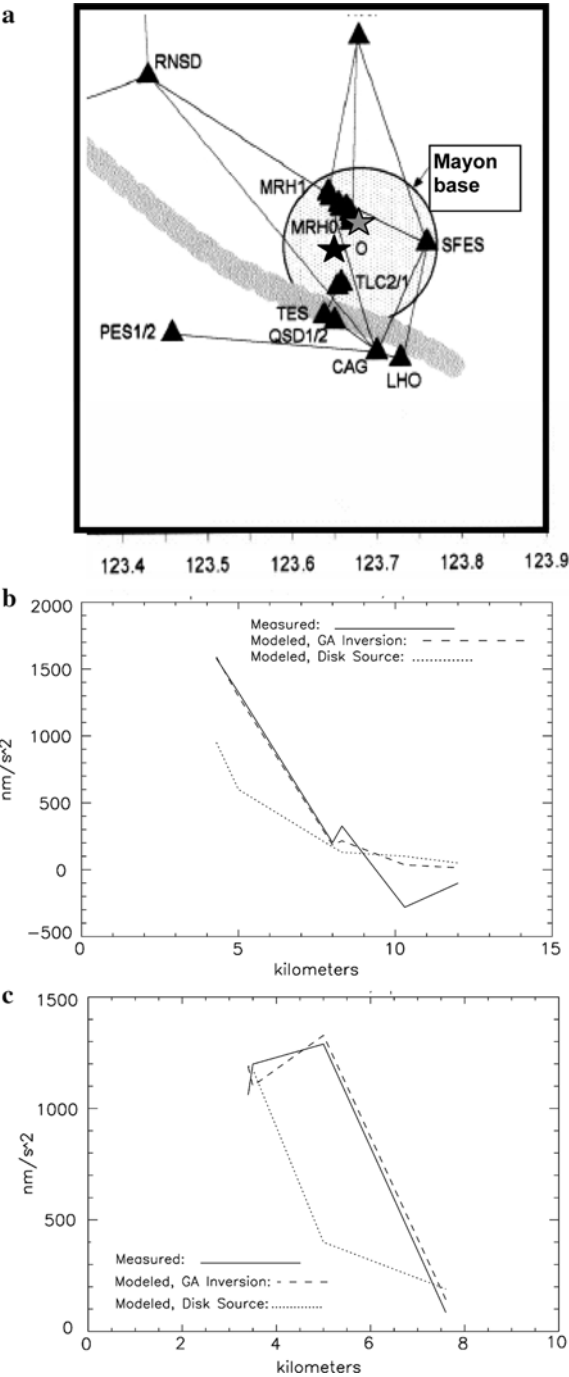
a) Gravity and GPS network around Mayon crater and b) microgravity measurements versus horizontal distance from the crater, for the Tumpa-Lahar-Channel profile, epochs 2-1, 3-1, 4-1, and 5-1 (JENTZSCH *et al.*, 2001b), (Previously published in pageoph 161/7, 2001).

following figures are in SI units, the conversion to gravimetric units of gals is that $10 \text{ nm/s}^2 = 1 \mu\text{Gal}$. The gravity and GPS measurements covered two profiles towards the summit (one of them at a maximum elevation of 850 m) in a network of 26 points, and connected to a local and a regional network around the volcano with an extension of 40 km by 50 km, via two points at the opposite side of the Legazpi lineament (Fig. 2), although the closest gravity measurements are along the Mayon-Resthouse profile, more than 3 km from the crater (Figs. 2a and 3a).

There was no significant gravity change between the first and the second campaign (December 1992, May 1993), despite the activity in February/March 1993. The gravity increase between May 1993 and December 1996 reached almost $150 \mu\text{Gal}$ ($\pm 14 \mu\text{Gal}$), or 1500 nm/s^2 , increasing with elevation and decreasing with distance from the crater. Note that a detailed discussion of the effect of groundwater on the gravity measurements can be found in JENTZSCH *et al.* (2001b) but, based on the lack of seasonal variation in the data, the location of the preponderance of measurements on consolidated rock, and the steep slopes of the upper reaches of the network, they estimated that the maximum effect due to water level changes is only on the order of $50 \mu\text{Gal}$ on the lower slopes of the volcano, and only for three or fewer stations

Figure 3

a) Location of the two sources relative to the crater and the monitoring stations (grey star is the shallower, low density source, the black star is the deeper magmatic source); b) inversion results for the Tumpa-Lahar-Channel profile, epoch 5-1, compared to the measure values and the vent motion model; and c) the same results for the Mayon-Resthouse profile (MRH# stations) (TIAMPO *et al.*, 2004), (Previously published in pageoph 161/7, 2001).



(JENTZSCH *et al.*, 2001b). There remained, therefore, a significant residual gravity signal to be explained near the volcanic crater.

The microgravity measurements for the Tumpa-Lahar-Channel profile are shown in Figure 2b. The gravity changes are restricted to a radius of 8 km around the volcano summit, a variation of around 30 μGal per year if a constant process is assumed. Yet, the differential GPS measurements carried out at the same locations with three parallel receivers revealed no significant changes of elevation during this time, within their accuracy of ± 3 to 4 cm.

The connection between the variation of gravity and elevation due to mass and density changes is usually interpreted in relation to two gradients. If gravity follows the free air gradient (FAG), no subsurface change in the mass has occurred, while data following the gradient after the standard Bouguer correction (BCFAG) implies mass changes (e.g., BROWN and RYMER, 1991; RYMER *et al.*, 1993; RYMER, 1996). Departures from these gradients are used to model volcanic processes. The results at Mayon cannot be related to the gravity gradients FAG and BCFAG because the gravity changes drawn against height differences from one campaign to the next show no gradient, while it is unusual for gravity to increase with decreasing activity (JENTZSCH *et al.*, 2001b).

As a result, the observed gravity increase, unaccompanied by significant changes in elevation, could not be explained with the classical Mogi model. Originally, JAHR *et al.* (1998) and JENTZSCH *et al.* (2001b) explained the gravity changes as density changes within a vent system. This model is based on a redistribution of the mass via the migration of magma down the vent system, and explains about 50% of the observed signal as well as the pattern of observed gravity changes (see the computed results, dotted lines in Fig. 3b). At that time such a model was in accordance with the assumed decrease of activities, whereas later it turned out that the activity continued. This prompted further analysis using a more complicated elastic-gravitational model.

The elastic-gravitational model has several advantages for this particular inversion. First, the numerical formulation allows for the joint inversion of deformation and gravity. Second, the incorporation of the elastic-gravitational effect introduces a long-wavelength effect to the gravity and deformation field that might prove important in an area with significant topography. Finally, with the correct combination of mass, pressure, and radius, inside a volcano that likely contains a complicated, preexisting vent system, it is possible to model the unusual measurements recorded at Mayon and originally presented in TIAMPO *et al.* (2004).

3. Inversion Methodology

3.1. The Genetic Algorithm

Many geophysical optimization problems are nonlinear and result in objective functions with a rough fitness landscape and several local minima. Consequently,

local optimization techniques such as linearized matrix inversion and steepest descent can converge prematurely to a local minimum. In addition, the success in obtaining an optimum solution can depend strongly on the choice of the starting model. GAs have proven themselves an attractive global search tool suitable for the irregular, multimodal fitness functions typically observed in nonlinear optimization problems in the physical sciences. Because of their initially random and progressively more deterministic sampling of the parameter space, these algorithms offer the possibility of efficient and relatively rapid location of the most promising regions of the solution space. The GA is different from other optimization and search procedures in four ways: (1) the GA works with a *coding* of the parameter set, not the parameters themselves; (2) the GA searches a population of points, not a single point; (3) the GA uses the objective function information in a forward model, *not* derivatives, in its auxiliary knowledge; and (4) the GA makes use of probabilistic transition rules, not deterministic ones (GOLDBERG, 1989).

In general, geophysical inverse problems involve employing large quantities of measured data, in conjunction with an efficient computational algorithm that explores the model space to find the global minimum associated with the optimal model parameters. In the GA, the parameters to be inverted for are coded as genes, and a large population of potential solutions is searched for the optimal solution. Several recent studies have employed GAs to invert for seismic structure (STOFFA and SEN, 1991; BOSCHETTI *et al.*, 1996), hypocenter relocation (BILLINGS *et al.*, 1994), seismic phase alignment (WINCHESTER *et al.*, 1993), mantle velocity structure (CURTIS *et al.*, 1995; NEVES *et al.*, 1996), crustal velocity structure (JIN and MADARIAGA, 1993; BATTACHARYYA *et al.*, 1999), mantle viscosity (KIDO *et al.*, 1998), fault zone geometry (YU *et al.*, 1998), and a variety of volcanic sources (TIAMPO *et al.*, 2000; FERNÁNDEZ *et al.*, 2001a).

The basic structure of the GA code used here is modified from MICHALEWICZ (1992, appendix). The process begins by representing the model to be optimized as a real-value string. Starting with an initial range of models, these algorithms progressively modify the solution by incorporating the evolutionary behavior of biological systems. The fitness of each solution is measured by a quantitative, objective function, the fitness function, *FV*. Next, the fittest members of each population are combined using probabilistic transition rules to form a new offspring population. Copying strings according to their *FV* means that strings with a better value of fitness have a higher probability of contributing one or more offspring in the next generation. This procedure is repeated through numerous generations until the best solution is obtained, based on this fitness measure, *FV* (MICHALEWICZ, 1992). It has been demonstrated that those members of the population with an *FV* greater than the average fitness of the population itself will increase in number exponentially, effectively accelerating the convergence of the inversion process (HOLLAND, 1975; GOLDBERG, 1989).

Our program employs a random number generator to produce an initial set of 100 potential values for each of the model parameters, which are coded as genes. For example, if the model to be fitted is the spherical Mogi source, then there are four parameters to be optimized— x and y location, depth and expansion volume. One gene for each model parameter is assigned to a particular member of that initial population, creating 100 potential solutions to the inversion problem. These members are ranked, from best to worst, according to an external fitness function. The fitness value, FV , is obtained from a fitness function consisting of the chi-square fit to the difference between the measurements (gravity and or deformation) and the appropriate values calculated for each potential solution. The members with the lowest chi-square value are the fittest and are selected to contribute to the next generation, where the genetic operations of crossover and mutation take place.

Crossover and mutation alter the new, fitter population in a process of controlled yet random information exchange (WRIGHT, 1991). The crossover, or mating, process is repeated for a large portion of the entire population, while mutation, the random alteration of a small number of genes, is introduced to ensure against the occasional loss of valuable genetic material (MICHALEWICZ, 1992). After completion of both crossover and mutation, the population is re-evaluated as above, and the process is repeated, exploiting information in past generations to search the parameter space with improved performance. Specific features of this GA include real-valued genes, an elitist function, and a windowing fitness function, in order to ensure higher precision, continuous incorporation of the fittest parameter into the gene pool, and to prevent search stagnation.

This process modifies the initial population over subsequent generations, where each member's fitness is calculated in the evaluate subroutine. In this work, we inverted for six parameters—the x and y location of the volcanic source, the depth to the source, and its radius, mass, and internal pressure change.

The algorithm seeks that model with the best fit, i.e., the model that produces solutions to the given problem that are closest to the observed measurements. In the case of one or more volcanic sources, it is the model whose parameters, when input into the forward source model, produce surface deformations and gravity changes that best match the observations. Given the expected range of values for the model parameters, the algorithm randomly selects a set of models and proceeds to evolve them to produce better, fitter models. The FV for any one model is calculated in the fitness subroutine, which contains the geophysical information specific to this problem. The gravity or deformation pattern for the expected volcanic source(s) is calculated in this subroutine and then the value of chi-square is calculated for that deformation pattern as it relates to the measurement pattern:

$$\chi^2 = \sum_k \frac{(c_k - e_k)^2}{\sigma_k^2}, \quad (1)$$

where E_k are the expected or measured data, C_k are the calculated data, σ_k is measurement standard deviation, and k is total number of measurements (TAYLOR, 1982; BEVINGTON and ROBINSON, 1992). The GA picks the fittest members of the population based upon the maximum FV , yet the value of chi-square decreases as the model approaches the correct solution. We therefore chose the FV to be $FV = 1/\chi^2$, effectively converting the fitness to a continuously increasing function.

3.2. The Elastic-Gravitational Layered Earth Model

In this problem, the forward model employed by the GA contains the geophysical information relating the gravity changes and surface deformations for the particular volcanic source model.

While in the past the simple Mogi model has been used for the computation of surface deformations due to volcanic intrusion, it often poses difficulties in simultaneously modeling observed displacements and gravity changes (JAHR *et al.*, 1995; RYMER *et al.*, 1993; RYMER, 1996). In addition, gravity calculations must be modeled separately, requiring an iterative inversion process. More complicated models have been proposed for the modeling of deformation (DAVIS, 1986; YANG *et al.*, 1988; BONAFEDE, 1991; LANGBEIN *et al.*, 1995; DE NATALE *et al.*, 1997; FOLCH *et al.*, 2000; TIAMPO *et al.*, 2000; FERNÁNDEZ *et al.*, 2001b), but the joint modeling of gravity and deformation has remained elusive, despite the obvious benefits (BONAFEDE and MAZZANTI, 1998; JENTZSCH *et al.*, 2001b; BATTAGLIA and SEGALL, 2007).

In this inversion we employed a model in which the Earth is represented as a stack of plane layers having both elastic properties and mass (RUNDLE, 1980, 1982); hence both elasticity and self-gravitation are potentially important effects. The numerical formulation required to obtain the integration kernels for a stack of layers overlying a half-space is described in RUNDLE (1980, 1982), and FERNÁNDEZ *et al.* (1994a,b). FERNÁNDEZ *et al.* (1997, 2005) presented an elastic-gravitational model consisting of a stratified half-space of homogeneous layers that takes into account the interaction between the mass of the intrusion, the ambient gravity field, and the effect caused by the change of pressure in the magmatic system. The magmatic source can be located in any of the layers or the half-space (FERNÁNDEZ, 1997, 2005).

The required input data include the characteristics of the crustal model (layer thicknesses, Lamé constants and densities), the x and y coordinates of the points where we want to compute the effects, and the magma intrusion characteristics (radius, mass, pressure and depth). The output includes various effects computed for each measurement location, including vertical and radial displacement, tilt, vertical strain, surface gravity change, free air gravity change, Bouguer gravity change, and sea level change (FERNÁNDEZ *et al.*, 1997).

4. Source Model

The recent eruptions of the Mayon volcano suggest that a process of injection might have taken place, instead of deflation. We tested this hypothesis by modeling the observed data, including both the measured gravity changes and the GPS deformation measurements, with the elastic-gravitational Earth model detailed above, using the GA inversion technique (TIAMPO *et al.*, 2004). Originally, the data were inverted based upon the assumption of a single source, located some distance below the crater (FERNÁNDEZ *et al.*, 2001b). However, while the results for the Tumpa-Lahar-Channel profile reasonably reproduced the measured signal for epoch 5–1 and the Mayon-Resthouse profile for the same epoch reproduced the total magnitude and character of the measured signal, they did not extend as far from the vent, nor did the location of the maximum for the modeled source coincide with that seen in the data. In addition, the need to match gravity measurements far from the crater resulted in a source radius that is unreasonably large, and a resulting magma density that is too low. As a result, a GA inversion for a two-source model was implemented, in order to study the possible asymmetries in the source mechanism (TIAMPO *et al.*, 2004).

Results for the two-source inversion to the epoch 5–1 data are shown in Figure 3. The elastic-gravitational model provided a much better solution than either the simple Mogi source or the vent magma model for both the Tumpa-Lahar-Channel profile (Fig. 3b) and the Mayon-Resthouse profile (Fig. 3c). While the fit to the Tumpa-Lahar-Channel profile data is similar to the single-source inversion, the fit to the data along the Mayon-Resthouse profile increased significantly, with the maximum calculated gravity now matching the location of the data maximum. Interestingly, the two sources are quite different. The first source, located at a depth of 1.02 km below the base of the volcano, with a pressure of 24 bar, a radius of 0.312 km, and a mass of 0.629 MU (1 MU = 1 Mass Unit = 10^{12} kg), has a much smaller mass and radius than that of the single source inversion.

The second source is located north of the caldera and is much shallower and broader than the first source, at a depth of only 0.22 km below the base of the volcano, with 75 bar pressure, a radius of 1.54 km, and a mass of 0.378 MU. With a density much less than that of magma, this suggests that it results from significant groundwater changes. In support of this hypothesis, changes in well water and outflow have been noticed by the local inhabitants both prior and subsequent to Mayon eruptions (JENTZSCH *et al.*, 2001b). In particular, the local residents had to deepen their wells prior to the 1993 eruption, although measurements of the wells are not available after the eruption (ALBANO, *et al.* 2001). However, these changes were difficult to correlate with the existing gravity measurements (JENTZSCH *et al.*, 2001b). In addition, while the steep slopes of the Mayon-Resthouse profile would seem to preclude a significant water table (JENTZSCH *et al.*, 2001b), there are recorded instances where dilatation, due to changes in the local strain field, results in the

storage of large quantities of groundwater, with an accompanying decrease in gravity (KISSLINGER, 1975). Following the eruption, crack closure results in the expulsion of large amounts of water downslope and an *increase* in the relative gravity accompanied by no uplift of the ground surface (KISSLINGER, 1975). Similar models could account for this second source along the volcanic slope, where the gravity measurements recorded not a change in the groundwater table per se, but a temporary storage of water as a result of underlying tectonic and volcanic stress changes.

Interpretation of the above results suggests that the primary gravity changes at Mayon are better explained by reinjection of magma below the volcano following the 1993 eruption rather than mass redistribution in the volcanic vent. Note that these results do not preclude some combination of mass redistribution down the vent, as proposed by JENTZSCH *et al.* (2001b), in conjunction with additional mass injection below the vent. Concurrent with this intrusion, localized changes in the strain field caused the expulsion of water stored in local fractures at some point during and/or after the previous eruption, increasing the gravity signal. This prompted additional analysis to explore the potential stress changes in the area of the second source.

5. Stress Change

In order to analyze the potential for fluctuations in water storage in underground fractures at the location of the second source as a result of stress changes from the deeper magmatic source, the stress change due to that first source was calculated using the point source method of OKADA (1992). Here we analyze the dilatational stress change from the first source at the same depth as the second source, in MPa (Fig. 4). The figure origin is centered at the location of the first source, and the second source is denoted by a star. It is located in a region of broad compressional stress change, nonetheless that change is small, only 0.19×10^{-3} MPa, due to the slight pressure change of the central magmatic source, only 2.4 MPa. If one calculates the normal stress change on optimally oriented planes, in this case 77 degrees clockwise from horizontal, as seen in Figure 5, the stress change is again compressional, and of the same order as that seen in Figure 4.

The low magnitude of these stress changes suggests that the regional stress field dominates, a trans-tensional stress field due to the bending of the central segment of the Philippine fault in the Bicol region, as signified by the formation of the Legazpi fault to the northeast and the San Vicente-Linao fault to the southwest of Mayon (MARCOS *et al.*, 2001). Note that the optimal fault direction here is roughly perpendicular to the Legazpi lineament and this extensional regime. The hypothesis is that the tensional tectonics allows for long-term water storage in underground fracture systems. However, small compressional fluctuations due to filling of the magma chamber cause these fractures to close up, expelling the groundwater,

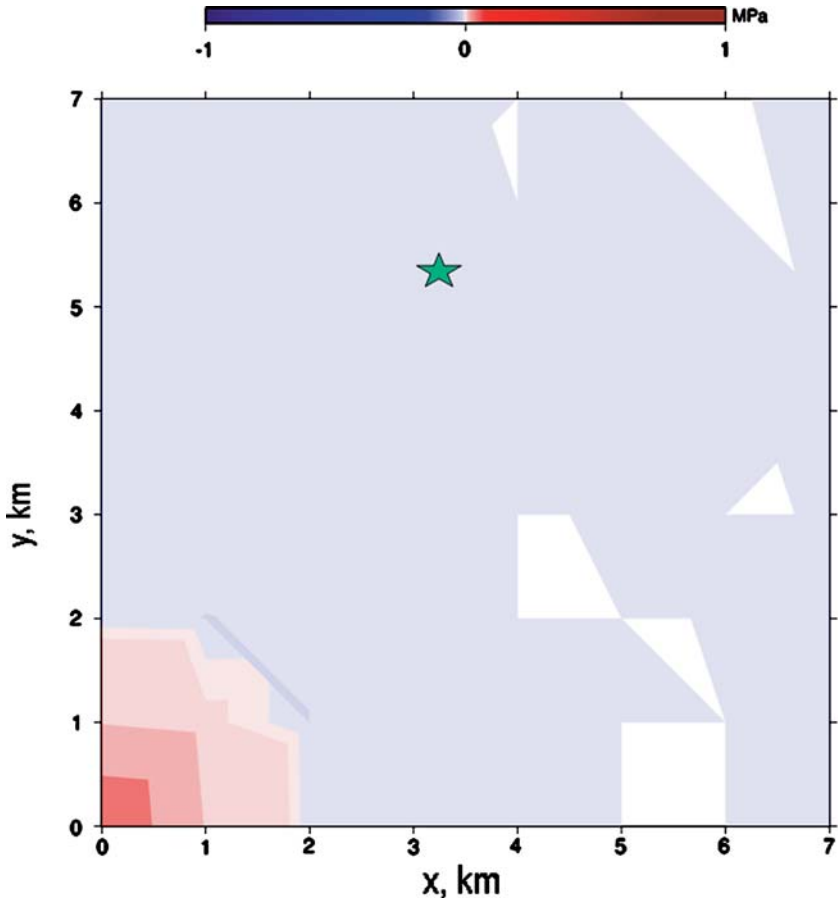


Figure 4
Net dilatational stress change, in MPa. The origin is centered at the primary source, under the central cone, and the location of the secondary source is denoted by the star. Blue colors indicate compression, in this case a negative stress change.

simultaneously increasing water levels in local wells, and causing a short-wavelength increase in the gravity signal (KISSLINGER *et al.*, 1975; ALBANO *et al.*, 2001; TIAMPO *et al.*, 2004).

6. Discussion and Conclusions

This paper details the two-source volcanic model derived from the integration of a GA inversion technique with a semi-analytical technique for the calculation of ground deformation and gravity changes caused by a point source magma intrusion into an elastic-gravitational multilayered earth model. Here we analyzed the optimal

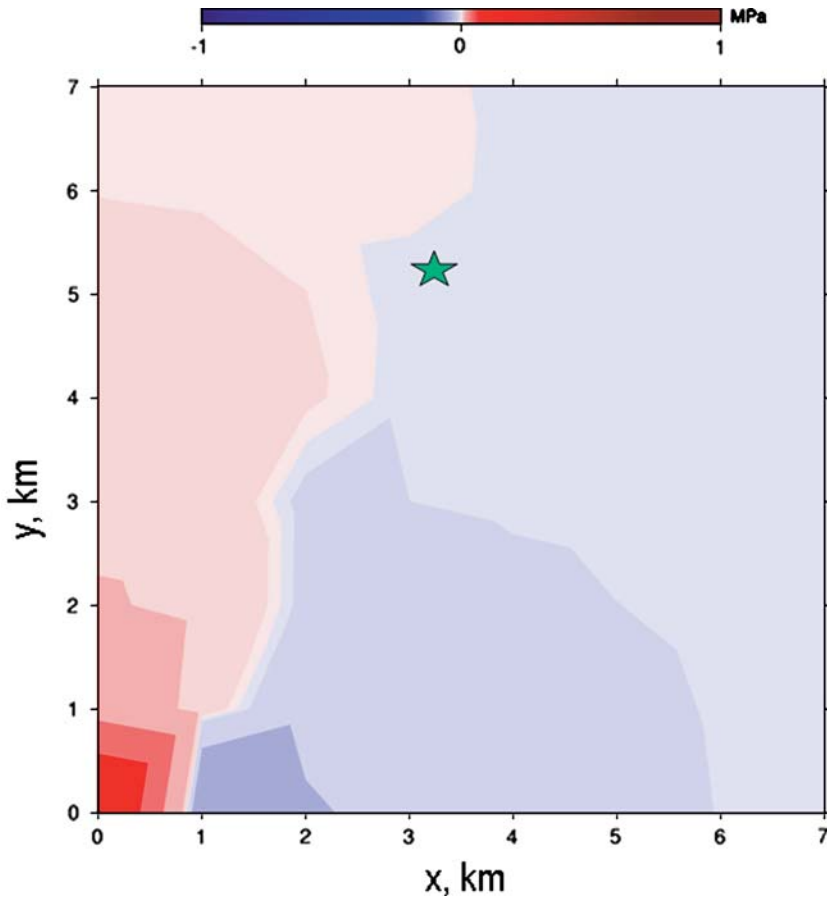


Figure 5
Net normal stress change, in MPa. Details are as in Figure 4.

solution, as determined by TIAMPO *et al.* (2004), for the associated stress change in order to better understand the dynamics of a two-source system, one of which is likely to be magmatic and the other hydrogeologic.

In order to properly interpret cases in which gravity changes exist without deformation, and *vice versa*, analysis should include adequate mass and pressure combinations for the intrusion. While simpler models were unable to explain gravity changes without displacements or *vice versa*, the elastic-gravitational model took into account the interaction of the mass of the intrusion with the ambient gravity field and the resulting redistribution of densities inside the crust. As a result, it explained the observations at more complex volcanic regions.

Our best inversion suggested that the system contains two sources, one deeper and magmatic in nature and a second shallower source that is hydrogeologic

(TIAMPO *et al.*, 2004). Here we studied the stress changes from the first on the region of the second in order to determine the likelihood that one had an important effect on the other. There was a small, compressional stress increase from the first source on the second, suggesting that stress change may have had an effect on the local storage of water. Additional monitoring of water level changes in wells below the volcano would potentially help to better characterize the temporal nature of this signal and its correlation with volcanic eruptions.

Note that there are a number of important effects not considered here. For example, we did not consider topographic effects, although these can be important under certain conditions (PINEL and JAUPART, 2003; CHARCO *et al.*, 2007 this volume). In future work, effects such as source geometry, topography, and multiple layer rheologies will be modeled as well for an even more realistic interpretation. Finally, better characterization of both the local fault structure and the regional stress field would assist in determining whether this stress change is affecting a network of randomly oriented fractures, or a distinct fault or group of faults with a particular orientation, and could provide important information on the subsurface processes at Mayon.

Acknowledgements

The research by KFT was conducted under an NSERC discovery grant and an ADF grant from the University of Western Ontario. The research by JF was funded under MEC REN2002-03450 and CGL 2005-05500-C02 research-projects. The research by TH was funded by an OGS grant from the province of Ontario. The research by GJ at Mayon volcano was supported by the German Research Foundation (DFG) and the "Gesellschaft für Technische Zusammenarbeit" (GTZ) and partly by the European Commission (Grant ENV4 - CT96 - 0259), as well as by PHIVOLCS. KFT would also like to acknowledge the Instituto de Astronomia y Geodesia, Universidad Complutense de Madrid, for its hospitality during the bulk of this research. The researchers would also like to thank two anonymous reviewers for their assistance in improving this manuscript.

REFERENCES

- ALBANO, S.E., SANDOVAL, T., and TOLEDO, R. (2001), *Groundwater at Mayon volcano*, EOS Trans. AGU 82, 47.
- BERRINO, B. and CORRADO, G. (1991), *Gravity changes and volcanic dynamics*, Cahier du Centre Européen de Géodynamique et de Séismologie 4, Proc. of the workshop: *Geodynamical Instrumentation Applied to Volcanic Areas*, Oct. 1–3, 1990, Walferdange, Luxembourg, 305–323.
- BATTAGLIA, M., ROBERTS, C., and SEGALL, P. (1999), *Magma intrusion beneath Long Valley caldera confirmed by temporal changes in gravity*, Science 285, 2119–2122.

- BATTAGLIA, M. and SEGALL, P. (2007), *The interpretation of gravity changes and crustal deformation in active volcanic areas*, Pure Appl. Geophys., this issue.
- BEVINGTON, P.R. and ROBINSON, D.K., *Data Reduction and Error Analysis for the Physical Sciences* (McGraw-Hill, Inc., N.Y. 1992).
- BHATTACHARYA, J., SHEEHAN, A.F., TIAMPO, K.F., and RUNDLE, J.B. (1999), *Using genetic algorithms to model regional waveforms for crustal structure in the western United States*, BSSA 89, 202–214.
- BILLINGS, S., KENNETT, B., and SAMBRIDGE, M. (1994), *Hypocenter location: Genetic algorithms incorporating problem specific information*, Geophys. J. Int. 118, 693–706.
- BONAFEDE, M. (1991), *Hot fluid migration, an efficient source of ground deformation: Application to the 1982–1985 crisis at Campi Flegrei-Italy*, J. Volc. Geoth. Res. 48, 187–198.
- BONAFEDE, M. and MAZZANTI, M. (1998), *Modeling gravity variations consistent with ground deformation in the Campi Flegrei caldera (Italy)*, J. Volc. Geoth. Res. 81, 137–157.
- BOSCHETTI, F., DENTITH, M.C., and LIST, R.D. (1996), *Inversion of seismic refraction data using genetic algorithms*, Geophysics 61, 1715–1727.
- BROWN, G.C. and RYMER, H. (1991), *Microgravity monitoring at active volcanoes, a review of theory and practice*, Cahiers du Centre Européen de Geodynamique et Seismologie 4, 279–304.
- CHARCO, M., FERNÁNDEZ, J., LUZÓN, F., TIAMPO, K.F., and RUNDLE, J.B. (2007), *Some insights about topographic, elastic and self-gravitation interaction in modelling ground deformation and gravity changes in active volcanic areas*, Pure Appl. Geophys., this issue.
- CURTIS, A., DOST, B., TRAMPERT, J., and SNIEDER, R. (1995), *Shear-wave velocity structure beneath Eurasia from surface wave group and phase velocities in an inverse problem reconditioned using the genetic algorithm*, EOS Trans. AGU 76, 386.
- DAVIS, P.M. (1986), *Surface deformation due to inflation of an arbitrarily oriented triaxial ellipsoidal cavity in an elastic half-space, with reference to Kilauea Volcano, Hawaii*, J. Geophys. Res. 91, 7429–7438.
- DE NATALE, G., PETRAZZUOLI, S.M., and PINGUE, F. (1997), *The effect of collapse structure on ground deformations in calderas*, Geophys. Res. Lett. 24, 1555–1558.
- DE NATALE, G., PINGUE, F., 1996. *Ground deformation modeling in volcanic areas*. In (Scarpa, R., Tilling, R.I., Scarpa, R., (Eds), *Monitoring and Mitigation of Volcano Hazards*, (Springer-Verlag, Telos 1996) pp. 365–388.
- DELANEY, P.T. and MCTIGUE, D.F. (1994), *Volume of magma accumulation or withdrawal estimated from surface uplift or subsidence, with application to the 1960 collapse of Kilauea Volcano*, Bull. Volc. 56, 417–424.
- DVORAK, J.J. and DZURISIN, D. (1997), *Volcano geodesy, the search for magma reservoirs and the formation of eruptive vents*, Rev. Geophys. 35, 343–384.
- DZURISIN, D. (2000), *Volcano geodesy: Challenges and opportunities for the 21st century*, Phil. Trans. Roy. Soc. Lond. A 358, 1547–1566.
- DZURISIN, D., WICKS, C. Jr., and THATCHER, W. (1999), *Renewed uplift at the Yellowstone caldera measured by leveling surveys and satellite radar interferometry*, Bull. Volc. 61, 349–355.
- EGGERS, A.A. (1987), *Residual gravity changes and eruption magnitudes*, J. Vol. Geoth. Res. 33, 201–216.
- FERNÁNDEZ, J., CARRASCO, J.M., RUNDLE, J.B., and ARAÑA, P. (1999), *Geodetic methods for detecting volcanic unrest, A theoretical approach*, Bull. Volc. 60, 534–544.
- FERNÁNDEZ, J., CHARCO, M., RUNDLE, J. B., and TIAMPO, K. F. (2005), *A revision of the FORTRAN codes GRAVW to compute deformation produced by a point magma intrusion in elastic-gravitational layered Earth models*, Comp. and Geosc., in press.
- FERNÁNDEZ, J., CHARCO, M., TIAMPO, K.F., JENTZSCH, G., and RUNDLE, J.B. (2001a), *Joint interpretation of displacement and gravity data in volcanic areas. A test example: Long Valley caldera, California*, Geophys. Res. Lett. 28(6), 1063–1066.
- FERNÁNDEZ, J. and RUNDLE, J.B. (1994a), *Gravity changes and deformation due to a magmatic intrusion in a two-layered crustal model*, J. Geophys. Res. 99, 2737–2746.
- FERNÁNDEZ, J. and RUNDLE, J.B. (1994b), *FORTRAN program to compute displacement, potential and gravity changes resulting from a magma intrusion in a multilayered Earth model*, Comp. and Geosc. 20, 461–510.
- FERNÁNDEZ, J., RUNDLE, J.B., GRANELL, R., and YU, T.-T. (1997), *Programs to compute deformation due to a magma intrusion in elastic-gravitational layered Earth models*, Comp. and Geosc. 23, 231–249.

- FERNÁNDEZ, J., TIAMPO, K.F., JENTZSCH, G., CHARCO, M., and RUNDLE, J.B. (2001b), *Inflation or deflation? New results for Mayon volcano applying elastic-gravitational modeling*, Geophys. Res. Lett. 28, 2349–2352.
- FOLCH, A., FERNÁNDEZ, J., RUNDLE, J.B., and MARTI, J. (2000), *Ground deformation in a viscoelastic medium composed of a layer overlying a half-space: a comparison between point and extended sources*, Geophys. J. Int. 140, 37–50.
- GAETA, F.S., DE NATALE, G., and ROSSANO, S. (1998), *Genesis and evolution of unrest episodes at Campi Flegrei caldera, the role of the thermal fluid-dynamical processes in the geothermal system*, J. Geophys. Res. 103, 20921–20933.
- GOLDBERG, D.E., *Genetic Algorithms in Search, Optimization, and Machine Learning* (Addison Wesley, Reading, MA 1989).
- HOLLAND, J.H., *Adaptation in Natural and Artificial Systems* (MIT Press, Cambridge, MA 1975).
- JAHR, T., JENTZSCH, G., PUNONGBAYAN, R.S., SCHREIBER, U., SEEGER, G., VÖLKSEN, C., and WEISE, A., *Mayon Volcano, Philippines: Improvement of hazard assessment by microgravity and GPS?* Proc. Int. Symp. On Current Crustal Movement and Hazard Assessment (IUGG, IAG) (Wuhan. Seismological Press, Beijing 1998) 599–608.
- JENTZSCH, G., HAASE, O., KRONER, C., SEEGER, G., and WINTER, U. (2001a), *Mayon volcano, Philippines: Some insights into stress balance*, J. Volcan. Geotherm. Res. 109, 205–217.
- JENTZSCH, G., PUNONGBAYAN, R.S., SCHREIBER, U., SEEGER, G., VÖLKSEN, C., and WEISE, A. (2001b), *Mayon volcano, Philippines: Change of monitoring strategy after microgravity and GPS measurements*, J. Volc. Geotherm. Res. 109, 219–234.
- JIN, S. and MADARIAGA, R. (1993), *Background velocity inversion with a genetic algorithm*, Geophys. Res. Lett. 20, 93–96.
- KIDO, M., YUEN, D.A., ČADEK, O., and NAKAKUKI, T. (1998), *Mantle viscosity derived by genetic algorithm using oceanic geoid and seismic tomography for whole-mantle versus blocked-flow situations*, Phys. Earth Plan. Int. 107, 307–326.
- KISSLINGER, C. (1975), *Processes during the Matsushiro, Japan earthquake swarm as revealed by leveling, gravity, and spring-flow observations*, Geology 3(2), 57–62.
- LANGBEIN, J., DZURISIN, D., MARSHALL, G., STEIN, R., and RUNDLE, J. (1995), *Shallow and peripheral volcanic sources of inflation revealed by modeling two-color geodimeter and leveling data from Long Valley Caldera, California, 1988–1992*, J. Geophys. Res. 100, 12487–12495.
- MARCOS, H., TENGONCIANG, A., OBILLE, E., and LAGMAY, A.M.F. (2001), *Newly identified faults on Mount Mayon, Albay, Philippines*, Proc. Internat. Symp. on Interaction between Volcanoes and their Basement and Related Geological Hazards, Abstract Volume, UNESCO-IUGS-IGCP, Santiago, Chile.
- MASSONET, D. and FEIGL, K.L. (1998), *Radar interferometry and its application to changes in the Earth's surface*, Rev. Geophys. 36, 441–500.
- MICHALEWICZ, Z., *Genetic Algorithms + Data Structures = Evolution Programs* (Springer-Verlag, New York, NY 1992).
- MOGI, K. (1958), *Relations between the eruptions of various volcanoes and the deformations of the ground surfaces around them*, Bull. Earth. Res. Inst. Tokyo 36, 99–134.
- NEVES, F.A., SINGH, S.C., and PRIESTLEY, K.F. (1996), *Velocity structure of upper-mantle transition zones beneath central Eurasia from seismic inversion using genetic algorithms*, Geophysics 22, 523–552.
- NEWHALL, C.G. (1979), *Temporal variation in the lavas of Mayon volcano, Philippines*, J. Vol. Geoth. Res. 6, 61–83.
- OKADA, Y. (1992), *Internal deformation due to shear and tensile faults in a half-space*, BSSA 82, 1018–1040.
- PINEL, V. and JAUPART, C. (2003), *Magma chamber behavior beneath a volcanic edifice*, J. Geophys. Res. 108, doi: 10.1029/2002JB001751.
- PRITCHARD, M.E. and SIMONS, M. (2002), *A satellite geodetic survey of large-scale deformation of volcanic centers in the central Andes*, Nature 418, 167–171.
- RUNDLE, J.B. (1980), *Static elastic-gravitational deformation of a layered half space by point couple sources*, J. Geophys. Res. 85, 5355–5363.
- RUNDLE, J.B. (1982), *Deformation, gravity and potential changes due to volcanic loading of the crust*, J. Geophys. Res. 87, 10729–10744.

- RYMER, H., *Microgravity monitoring, In monitoring and mitigation of volcano hazards* (Springer 1996) pp. 169–197.
- RYMER, H., MURRAY, J.B., BROWN, G.C., FERRUCCI, F., and MCGUIRE, W.J. (1993), *Mechanism of magma eruption and emplacement at Mt Etna between 1989 and 1992*, *Nature* 361, 439–441.
- RYMER, H. and WILLIAMS-JONES, G. (2000), *Volcanic eruption prediction, magma chamber physics from gravity and deformation measurements*, *Geophys. Res. Lett.* 27, 2389–2392.
- STOFFA, P.L. and SEN, M.K. (1991), *Nonlinear multiparameter optimization using genetic algorithms, Inversion of plane-wave seismograms*, *Geophysics* 56, 1794–1810.
- TAYLOR, J.R., *An introduction to error analysis* (University Science Books, USA 1982).
- TIAMPO, K.F., RUNDLE, J.B., FERNÁNDEZ, J. and LANGBEIN, J. (2000), *Spherical and ellipsoidal volcanic sources at Long Valley Caldera, California using a genetic algorithm inversion technique*, *J. Volc. Geotherm. Res.* 102, 189–206.
- TIAMPO, K.F., FERNÁNDEZ, J., CHARCO, M., JENTZSCH, G., and RUNDLE, J.B. (2004), *New results at Mayon, Philippines, from a joint inversion of gravity and deformation measurements*, *Pure Appl. Geophy.* 161(7), 1433–1452.
- VÖLKSEN, C. and SEEGER, G. (1995), *Establishment of a GPS based control network at Mayon volcano*, *Cahiers du Centre Européen de Géodynamique et de Séismologie* 8, 99–113.
- WATANABE, H., OKUBO, S., SAKASHITA, S., and MAEKAWA, T. (1998), *Drain back process of basaltic magma in the summit conduit detected by microgravity observation at Izu-Oshima volcano, Japan*, *Geophys. Res. Lett.* 25, 2865–2868.
- WINCHESTER, J.P., CREAGER, K.C., and MCSWEENEY, T.J. (1993), *Better alignment through better breeding: Phase alignment using genetic algorithms and cross-correlation techniques*, *EOS, AGU* 74, 394.
- WRIGHT, A.H., *Genetic algorithms for real parameter optimization. In Foundations of Genetic Algorithms* (Morgan Kaufmann Publishers, San Mateo, CA 1991).
- YANG, X.M., DAVIS, P.M., and DIETERICH, J.H. (1988), *Deformation from inflation of a dipping finite prolate spheroid in an elastic half-space as a model for volcanic stressing*, *J. Geophys. Res.* 93, 4249–4257.
- YU, T.T., FERNÁNDEZ, J., and RUNDLE, J.B. (1998), *Inverting the parameters of an earthquake-ruptured fault with a genetic algorithm*, *Comp. and Geosc.* 24, 173–182.

(Received June 23, 2005, revised December 12, 2005, accepted February 20, 2006)

To access this journal online:
www.birkhauser.ch/pag

A Hybrid Model for the Summit Region of Merapi Volcano, Java, Indonesia, Derived from Gravity Changes and Deformation Measured between 2000 and 2002

C. TIEDE,¹ J. FERNÁNDEZ,² C. GERSTENECKER,³ and K. F. TIAMPO⁴

Abstract—Three-dimensional displacements and gravity changes were measured at 20 points between 2000 and 2002 at Merapi volcano, Java, Indonesia. Former models focused on the modeling of a single magmatic source located in the summit region of the volcano. Such models do not fit to our measurements between 2000 and 2002. A new hybrid model approach is developed consisting of an elastic-gravitational source described by a mass and energy term as well as a combined dip-slip/strike-slip fault zone in the summit region. Both nonlinear optimization problems, given by the common inversion of three-dimensional displacements and gravity changes, are solved by applying a genetic algorithm. The hybrid model fits the measurements accurately, tested by Fisher test statistics. Furthermore, our model for Merapi volcano confirms previous structural models for this region so that the new model is statistically proven as well as physically reliable.

Key words: Hybrid model, Merapi volcano, fault model, elastic-gravitational model, genetic algorithm.

1. Introduction

Merapi ($\phi = -7^{\circ}32'25.64''$, $\lambda = 110^{\circ}26'47.88''$, $h = 2969$ m (ellipsoidal height referenced to World Geodetic System 1984)) is an andesitic strato volcano on Java, Indonesia. The volcano is located near a subduction zone, which forms the 6000 to 7000-m deep Java trench approximately 250 km south of the coastline of Indonesia, where the Indo-Australian plate is moving toward the Eurasian plate and is subducted below it. This subduction zone is responsible for the high volcanism and seismic activity along the Sunda arc between Sumatra and Flores (GERTISSER, 2002).

¹ Max-Planck Institute for Astronomy, Königstuhl 17, 69117 Heidelberg, Germany.
E-mail: tiede@mpia.de

² Instituto de Astronomía y Geodesia (CSIC-UCM), Facultad de Ciencias Matemáticas, Plaza de Ciencias 3, Ciudad Universitaria, 28040 Madrid, Spain.

³ Institute of Physical Geodesy, Darmstadt University of Technology, Petersenstrasse 13, 64287 Darmstadt, Germany.

⁴ Department of Earth Sciences, Biological and Geological Sciences, University of Western Ontario, Rm. 9, London, ON, N6A 5B7 Canada.

The topography of the volcano is bell-shaped with a mean dip angle of 5° up to 1300 m, 15° up to 1700 m and 26° up to the summit (BERTHOMMIER, 1990). The summit structure of Merapi itself is characterized by a crater which has moved westwards in recent years and is occupied by an unstable lava dome that is composed of very viscous lava. This area is essentially formed by unpacked rock as well as fractured lava blocks and ash matrix breccia (BEAUDUCEL *et al.*, 2000).

The activity of Merapi is characterized by frequent eruptions within one to five-year intervals, with weak explosions and low gas pressure.

RATDOMOPURBO and POUPINET (2000) interpreted a zone at a depth of about 1500–2500 m below the summit which lacks seismicity as a shallow magma reservoir. According to CAMUS *et al.* (2000), the eruption history of the period starting at the end of the 18th century also suggests such a reservoir. Seismic data from OHRNBERGER *et al.* (2000) do not show clear indications for an aseismic zone. Also LOTEM (Long Offset Transient Electromagnetics) observations neither support a vertically extended conduit nor a shallow or deep magma reservoir, which would explain very low resistivity (COMMER, 2003).

On the basis of GPS and tilt data, BEAUDUCEL and CORNET (1999) modeled magma sources located 6000 and 9000 m below the summit. This result, as well as results of WASSERMANN *et al.* (1998), do not confirm the anticipated shallow magma source of RATDOMOPURBO and POUPINET (2000). KÖRNER (2000) modeled shallow magma chambers with different shapes on the basis of GPS and tilt data but he could not derive a definite geometry and pressure. He assumed pressure of $30\text{--}40 \cdot 10^5$ Pa for a magmatic source located a few hundred meters below the surface. Furthermore, the dome weight approximations lead to the assumption that the pressure for the transport of magmatic material must be smaller than $30 \cdot 10^5$ Pa, whereas magmatic masses are estimated as too small to influence the deformation field. According to FRIEDEL and JACOBS (2004) overpressure around several 10 MPa can be considered in the shallow subsurface.

SETIAWAN (2002) computed magma chamber and conduit models in order to explain measured gravity changes between 1997 and 2000. The conduit system was modeled by a cylinder, the magma chamber by a sphere or ellipsoid. The magma chamber was located at a depth of 8000 m, 2000 m SE of the summit. The observed gravity changes at points near the crater rim could be explained by the flow of the magma in the conduit. Gravity changes distant from the summit were modeled by hydrothermal system changes. Furthermore, it turned out that Merapi must be an open system due to the small displacements and gravity changes which could be measured around Merapi.

JENTZSCH *et al.* (2004) also analyzed the gravity changes and displacements observed between 1997 and 2000, but interpreted the observed changes as produced by a water saturated layer at a depth of 1000 m with varying density, using a two-dimensional model.

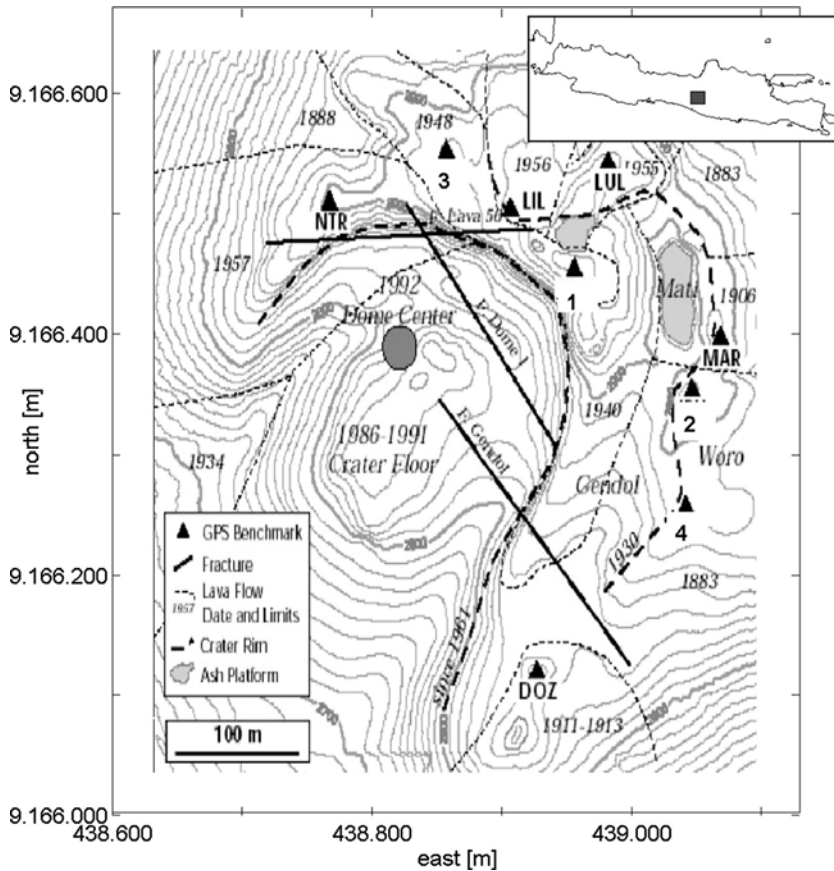


Figure 1

Structural map of the summit region of Merapi from BEAUDUCÉL *et al.* (2000). In the east of the fumarolic fields Gendol and Woro are displayed. The dark lines indicate the fracture zones in the summit region. In addition the four stations closest to the crater are displayed (stations 1–4).

Gas emission was monitored by ZIMMER and ERZINGER (2003) between 2000 and 2001 at two fumarolic fields, Gendol and Woro, located near the summit, Figure 1. Gendol, with temperatures $> 800^{\circ}\text{C}$, is located 150 m SE of the summit, and Woro, with temperatures $> 600^{\circ}\text{C}$, 250 m SE of the summit. The purpose of this study was the determination of the fumarole temperature as well as the study of water vapor and carbon dioxide contents. Results show that gas concentrations and temperatures in the fumaroles change relatively fast, which may be due to periodic changes of carbon dioxide, water vapor and the temperature related to the regularly fluctuating input of meteoric or hydrothermal water to the fumaroles.

2. Data and Model Assumptions

The values to be modeled are given by 20 gravity changes (dg) and displacements in the east (ux), north (uy) and vertical direction (uz), positive definition for increasing depth.

GPS data were measured with up to 11 dual frequency Trimble GPS receivers, with five installed permanently and the other six serving as rover and reference stations. The mean standard deviation of the GPS measurements in the horizontal direction is about ± 0.004 m and in the vertical direction about ± 0.02 m.

Gravity was measured by four relative gravimeters simultaneously in order to minimize systematic errors of the instruments. The mean standard deviation is about ± 0.0077 mGal ($1 \text{ mGal} = 1 \cdot 10^{-5} \text{ m/sec}^2$). A detailed discussion of the data used can be found in TIEDE (2005).

Figure 2 displays the observed gravity changes and three-dimensional displacements for the network where measurements were made in the years 2000 and 2002. The data are shown as a thick solid line, including 1σ -error bars. The data points are sorted with respect to their distance to the top of the volcano. Data points 1–4 are also shown in Figure 1.

Two different models are applied both of which are based on a homogeneous half space:

a) Due to the large horizontal and vertical displacements in the summit region measured between 2000 and 2002 a combined dip-slip and strike-slip model is used for the explanation of the vertical and horizontal movements. MANSINHA *et al.* (1971) computed the displacements due to strike-slip and dip-slip faults. The expressions are derived from Volterra's formula, which describes the displacement field produced by a dislocation across a surface in a uniform elastic half space.

The gravity changes caused by the faults are computed as free air gradients. In the model both effects are superimposed and result in the computed three-dimensional displacements in east (ux), north (uy), height (uz) as well as in the computed gravity change (dg). The unknown parameters of the combined strike-slip/dip-slip fault are given by the east coordinate of the midpoint (ξ_f), the north coordinate of the midpoint (ψ_f), the dip-slip (ds_f), the semi-length (l_f), the maximum depth from the top (ζ_f), the dip angle (dx_f), the strike angle (sx_f) and the strike-slip (ss_f). The minimum depth from the top is assumed to be zero.

For the modeling of the magmatic source the elastic-gravitational model approach (e.g., RUNDLE, 1980, 1982; FERNÁNDEZ and RUNDLE, 1994) is used. The approach is based on the generalized static Navier equations which couple elastic and gravitational effects. They are satisfied by a vector of three-dimensional displacements as well as by the perturbation potential, both produced by a magmatic intrusion in an elastic, self-gravitating uniform, semi-infinite medium.

The unknown parameters of the model are given by the east coordinate (ξ_c), the north coordinate (ψ_c), the depth (ζ_c), the mass (m_c) and the energy of the intrusion (ϵ_c).

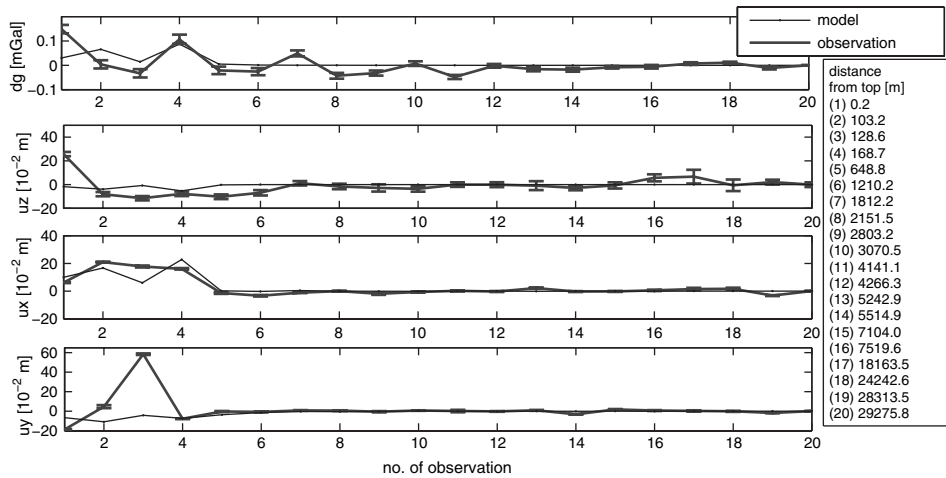


Figure 2

Measured gravity changes (dg) and three-dimensional displacements (ux , uy , uz) between 2000 and 2002 are shown with 1σ -error bars as thick solid line. Points are sorted with respect to their distance to the top of the volcano (distances given in the legend). The effect of the elastic-gravitational source is shown as a thin solid line.

3. Optimization

The optimization is implemented by applying a genetic algorithm (GA) based on the natural selection of the biological evolution, as inspired by DARWIN (1859). The algorithm we use is described in detail by TIAMPO *et al.* (2000, 2004). The algorithm works by the generation of a population, the selection of the fittest members, a crossover scheme and the application of mutation. The probability for mutation is given by 45% and the probability for crossover by 95%. The selection scheme is chosen as proportional to fitness and is based on the rank selection including Elitism principle. As coding scheme we apply real-coding. The fitness is given by the inverse of the χ^2 value

$$1/\chi^2 = \frac{n-u}{\mathbf{v}^T \mathbf{Q}_{\parallel}^{-1} \mathbf{v}}, \quad (1)$$

where $n-u$ are the degrees of freedom (n is the number of observations, u the number of unknown parameters), $\mathbf{v}$ are residuals between observed and modeled values, $\mathbf{Q}_{\parallel}$ the covariance matrix, with the variances of the observations on its diagonal, assuming uncorrelated data.

The population size is set to 200 chromosomes for the optimization of the combined dip-slip/strike-slip fault model and to 100 for the elastic-gravitational source model following former optimization approaches by TIAMPO *et al.* (2000, 2004).

The GA for the computation of the fault model parameters is initialized with the bounds given in Table 1a. 30 GA runs are computed, each with a generation size of 30,000. The elastic-gravitational source is computed using the bounds given in Table 1b by generating 30 GA runs, each with a generation size of 1000.

4. Modeling and Results

The assimilation of the computed models to the observed three-dimensional displacements and gravity changes is given by applying Fisher test statistics. Here, the H_0 -hypothesis states that the variance of the modeled values is equal to the variance of the observed ones:

$$T = \frac{\mathbf{v1}^T \mathbf{Q_{II}}^{-1} \mathbf{v1} \cdot r2}{\mathbf{v2}^T \mathbf{Q_{II}}^{-1} \mathbf{v2} \cdot r1} \sim F(1 - \alpha, r1, r2) \tag{2}$$

where $r1$ and $r2$ are degrees of freedom of $\mathbf{v1}$, $\mathbf{v2}$, $\mathbf{Q_{II}}$ is the covariance matrix and α is the significance level. $\mathbf{v1}$ and $\mathbf{v2}$ indicate the modeled and observed data, respectively, where the larger squared sum is set as numerator. The models with the best fit according to Equation (1), are tested by applying the Fisher test, considering a significance level of 10% for the combined fault and the hybrid model as well as 0.01% for the elastic-gravitational source.

Computing a model based on a single elastic-gravitational source, our measured three-dimensional displacements and gravity changes support very shallow magma

Table 1

a) Parameter bounds for the strike-slip/dip-slip fault. b) Parameter bounds for the elastic-gravitational source

Parameter	Lower bound	Upper bound
a) Dip-slip/strike-slip fault		
ξ_f [10 ³ m]	437.5	440.5
ψ_f [10 ³ m]	9165	9168
ds_f [m]	-1	1
l_f [m]	0	500
ζ_f [m]	0	500
$d\alpha_f$ [rad]	0	< $\pi/2$
$s\alpha_f$ [rad]	0	2π
ss_f [m]	-1	1
b) Elastic-gravitational source		
ξ_c [10 ³ m]	435	441
ψ_c [10 ³ m]	9160	9170
ζ_c [10 ³ m]	0.01	3
ε_c [10 ¹⁴ kg · m ² /sec ²]	-80	800
m_c [10 ¹² kg]	-1	2

chambers as anticipated by KÖRNER (2000). Nevertheless, these models, such as shown in Figure 2, are characterized by very small significance levels of about 0.01% (Fisher test statistics results in $T = 2.31 < F(0.9999, 80, 74) = 2.39$). In Figure 2 the measurements are shown as thick solid lines, including 1σ -error bars. Especially the large three-dimensional displacements in the summit region cannot be modeled by a single elastic-gravitational source.

Figure 3 shows the three-dimensional displacements of the summit region between 2000 and 2002. The large dots indicate the observation points. For each point, the three-dimensional displacements are given in the bar plots in [m]. Due to the NE movement of observation point no. 3, whereas all other points move into SE direction or show no significant horizontal movement, a strike-slip fault is assumed, which separates observation point no. 3 from the other observation points. In addition, a downward motion at observation point no. 1 is observed, whereas all other points show an uplift movement. In order to model this behavior, an additional dip-slip fault is assumed. The direction of this combined fault model is shown as a dotted line in Figure 3. In the computation both types of fault are supposed to interact at the same location in order to avoid the modeling of local effects and the decrease of the degrees of freedom in the optimization.

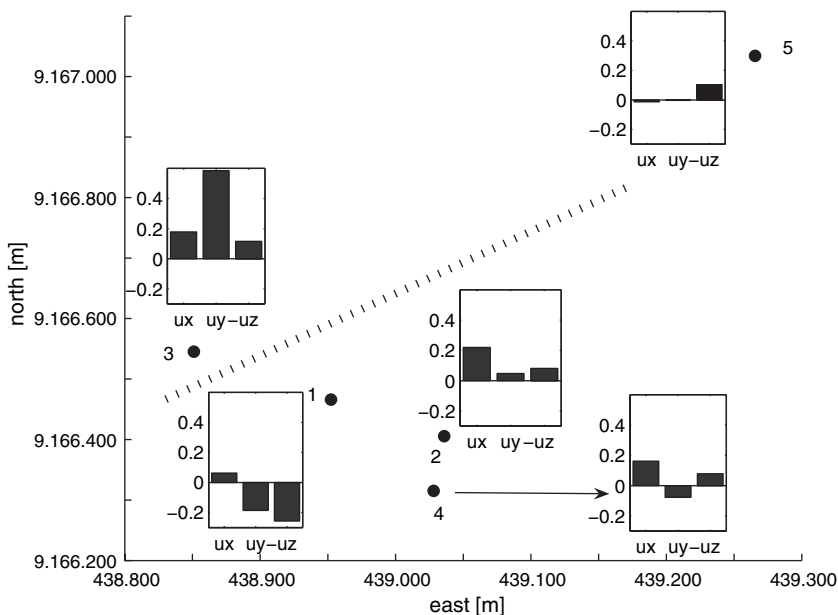


Figure 3

Overview of the summit region with three-dimensional displacements (u_x , u_y , u_z) measured between 2000 and 2002 at each point and given in bar plots. The inferred direction of the fault zone is shown as a dotted line.

The main effect responsible for the measured displacements is assumed to act from the combined fault model. This effect is subtracted in a first step from the measurements. In a second step, the residuals are modeled as effects caused by an elastic-gravitational source. Superimposing both effects results in the hybrid model, where we assume the independence of the two models. The parameters of the described hybrid model are computed by the optimization method given in Section 3.

The distribution of each of the thirty computed model parameters from the two models is shown as bar plots in Figure 4.

From these plots it is obvious that the parameters of the combined fault zone can be determined with high stability; the dispersions of the parameters are very small, whereas those of the parameters regarding the elastic-gravitational source show a

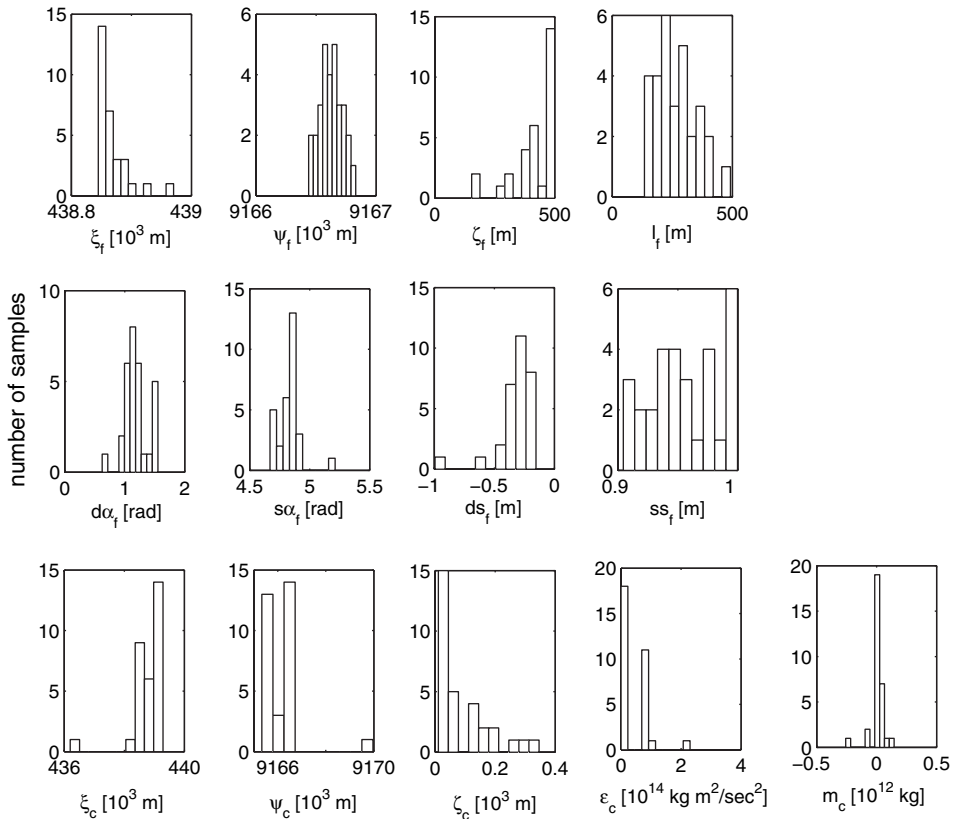


Figure 4

Bar plots showing the distribution of each computed model parameter. The parameters are optimized by applying the GA approach described in Section 3. The bar plots in the first two rows show the distribution of the parameters of the combined fault model; the bar plots in the last row showing the distribution of the elastic-gravitational source parameters.

large spreading. The larger dispersions of the parameters are caused by the small residuals that serve as input values of the optimization approach.

From these generations, the best sample according to the fitness given in Equation (1) is taken and the Fisher test is applied in order to compute the fit of the model to the measurements, according to Equation (2).

The statistics results in the acceptance of the combined fault model whose parameters are given in Table 2 within a significance level of $\alpha = 0.1$ and degrees of freedom of 72, $T_f = 1.16 < F(1-\alpha, 80, 72) = 1.35$. By choosing the best result of the generated elastic-gravitational source models the test statistics show that this model does not fit the residuals within the significance level of $\alpha = 0.0001$ and degrees of freedom of 75, $T_c = 3.52 > F(1-0.0001, 80, 75) = 2.38$.

By superimposing the effects caused by the combined fault model and the elastic-gravitational source model and applying the Fisher test, the hybrid model results in the acceptance of the H_0 -hypothesis which states that the model fits the observed data with a significance level of $\alpha = 0.1$, $T_{\text{hybrid}} = 1.14 < F(1-0.1, 67, 80) = 1.35$.

Table 2

Parameters for the hybrid model for Merapi volcano. The model includes a combined dip-slip/strike-slip fault in the summit region of the volcano as well as an elastic-gravitational source

Model	Parameter
Dip-slip/strike-slip fault	
ξ_f [10^3 m]	438.847
ψ_f [10^3 m]	9166.437
ζ_f [10^3 m]	0.398
l_f [10^3 m]	0.142
$d\alpha_f$ [rad]	1.552944
$s\alpha_f$ [rad]	4.748854
ds_f [m]	-0.588
ss_f [m]	0.993
$T_f < F(0.9, 72, 80)$	$1.16 < 1.35$
Elastic-gravitational source	
ξ_c [10^3 m]	439.183
ψ_c [10^3 m]	9166.453
ζ_c [10^3 m]	0.020
ε_c [10^{14} kg·m ² /sec ²]	0.2730
m_c [10^{12} kg]	0.003326
$T_c < F(0.9999, 80, 75)$	$3.52 < 2.38$
Hybrid model	
$T_{\text{hybrid}} < F(0.9, 67, 80)$	$1.14 < 1.35$
Single elastic-gravitational source	
ξ_c [10^3 m]	439.320
ψ_c [10^3 m]	9166.222
ζ_c [10^3 m]	0.070
ε_c [10^{14} kg·m ² /sec ²]	3.8846
m_c [10^{12} kg]	0.006220
$T_{\text{single source}} < F(0.9999, 80, 75)$	$2.28 < 2.38$

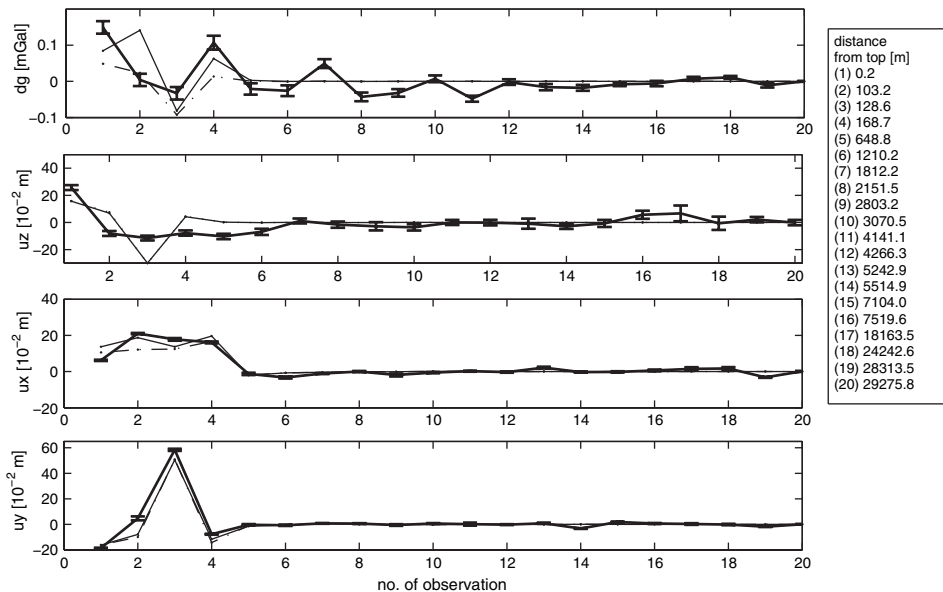


Figure 5

Measured gravity changes (dg) and three-dimensional displacements (ux , uy , uz) between 2000 and 2002 are shown with 1σ -error bars as a thick solid line. Points are sorted with respect to their distance to the top of the volcano (distances given in the legend). The dashed line shows the effects of the combined fault, the thin solid line shows the superposed effects of the combined fault and the elastic-gravitational model.

Furthermore, the hybrid model shows a slightly better fit to the measurements than the fit using the fault model alone.

The modeled effects originating from the combined dip-slip/strike-slip fault are shown in Figure 5 as a dashed line. The effects of the hybrid model are given by the solid line. The observed data are displayed as a thick solid line with 1σ -error bars.

Figure 5 shows clearly that the main effects are modeled by the fault and that the inclusion of the elastic-gravitational source model causes only a small increase in fitness, which confirms the Fisher test statistics result.

5. Interpretations and Conclusions

Figure 6 shows the summit region of the Merapi volcano with the topography from BEAUDUCEL *et al.* (2000). The combined fault is shown as a black area. In addition the elastic-gravitational source is shown as a black sphere. In the middle of the body the mass acts as a point source, but no conclusions can be drawn about the shape and the pressure of the body. In the three-dimensional view the orientation of the fault is shown in detail.

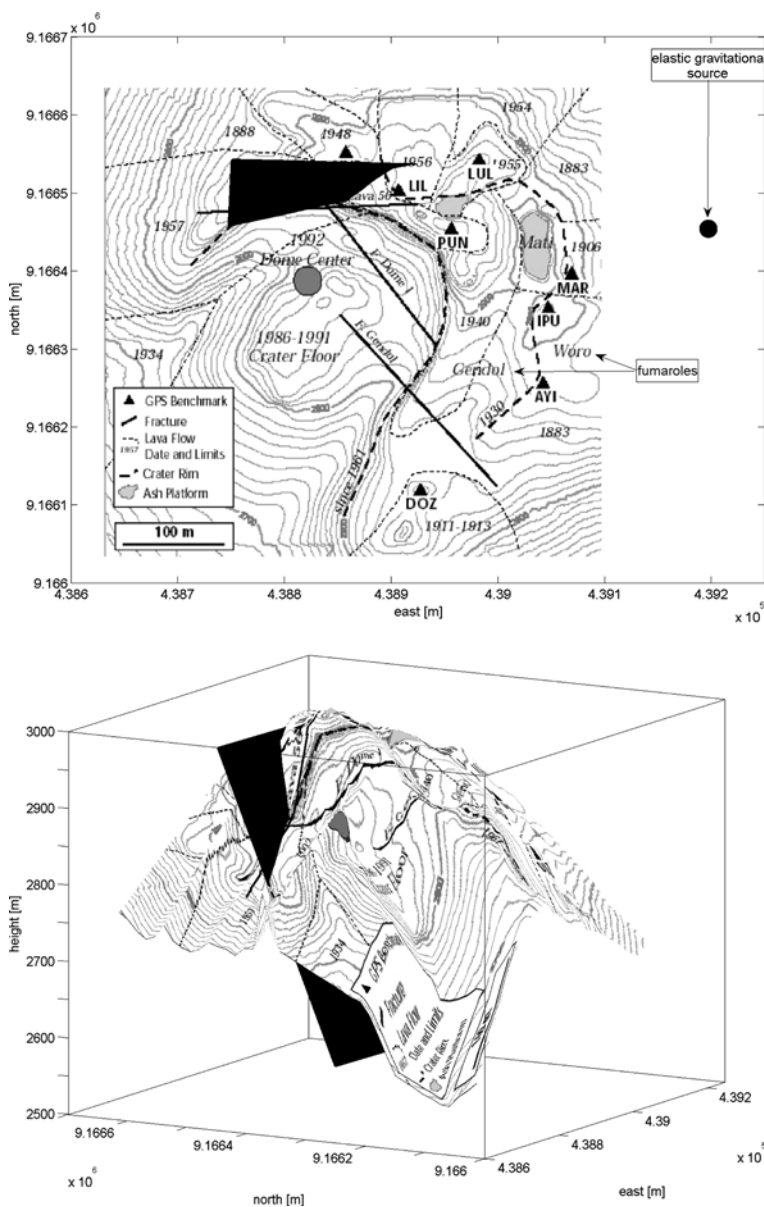


Figure 6

Plan view and three-dimensional view of Merapi. The plan view shows the hybrid model including an elastic-gravitational source (black sphere) in the east as well as a normal fault (black area). The structural map of BEAUDUCEL *et al.* (2000) including fracture zones as solid lines is merged on the digital elevation model derived by LÄUFER (2003). In the three-dimensional view the dip angle of the fault is shown.

Comparing the parameters of the single elastic-gravitational source with the parameters of the elastic-gravitational source of the hybrid model, the parameters do not change significantly at the location. The elastic-gravitational source moves slightly NW, the energy ε_c and also the mass m_c can be reduced (see Table 2). The magnitude of $\varepsilon_c = 0.2730 \cdot 10^{14} \text{ kg} \cdot \text{m}^2/\text{sec}^2$ of the energy term is consistent with, although slightly smaller than, previous values of $0.624\text{--}0.832 \cdot 10^{14} \text{ kg} \cdot \text{m}^2/\text{sec}^2$, which are derived from pressure and radius estimates given by KÖRNER (2000). Nevertheless, the increase of fitness due to the elastic-gravitational model is very small but significant. The elastic-gravitational source is located in the east of the fault zone, in the vicinity of the fumarole fields Woro and Gendol as well as near the ash platform shown in Figure 6. The small mass change of the elastic-gravitational source as well as the location near the fumarole fields suggest sources other than a magmatic chamber. The elastic-gravitational source model may reflect changes in the fumaroles, like an increase in the amount of fluids and also an increase of the gas pressure in the field.

The combined fault coincides very well with the fault zone, which was previously determined by BEAUDUCEL *et al.* (2000) and is shown in Figure 1. The combined fault is explained by a normal fault, with a horizontal opening of the fracture to the north.

The hybrid model shows that we can model very well the summit region of the Merapi volcano; the existence of a deep or shallow magmatic source below the summit, as postulated by BEAUDUCEL and CORNET (1999), RATDOMOPURBO and POUPINET (2000), CAMUS *et al.* (2000) or SETIAWAN (2002), is not necessary for the explanation of our measurements, which also coincides with results from LOTEM observations (COMMER, 2003).

Hybrid models consisting of several or different types of sources responsible for observations have been suggested as potential explanations for other volcanoes. For example, models for Long Valley caldera, California, have included spherical and elliptical sources, as well as zones of faulting, in a variety of deep and shallow combinations, in order to better interpret leveling, GPS, and two-color geodimeter measurements (TIAMPO *et al.* (2000); LANGBEIN, 2003). Model approaches by CAMACHO *et al.* (2007) derived from deformation and gravity measurements at the Mayon volcano also suggest that regional fault and volcanic sources should be considered in order to optimally fit the data.

Acknowledgements

The research by CT has been supported by a scholarship financed by DAAD (Deutscher Akademischer Austauschdienst). The Deutsche Forschungsgemeinschaft (DFG) supported this work under GE381/12. The research by JF has been funded under MCyT project REN2002-03450. The work by KFT was supported by an

NSERC Discovery grant, and an ADF grant from the University of Western Ontario. The research by JF has been funded under MCyT Projects REN2002-03450 and ICGL2005-05500-C02. We thank C. Braitenberg and B. Lühr for their constructive reviews of this manuscript.

REFERENCES

- BEAUDUCEL, F. and CORNET, F. (1999), *Collection and three-dimensional modeling of GPS and tilt data at Merapi volcano, Java*, J. Geophys. Res. 104, 725–736.
- BEAUDUCEL, F., CORNET, F., SUHANTO, E., DUQUESNOY, T., and KASSER, M. (2000), *Constraints on magma flux from displacements data at merapi volcano, Java, Indonesia*, J. Geophys. Res. 105, 8193–8204.
- BERTHOMMIER, P. (1990), *Etude Volcanologique du Merapi (Central Java): Téphrostratigraphie et chronologie-produits éruptifs*, Ph.D. Thesis, Université Blaise Pascal, Clément Ferrand, France.
- CAMACHO, A., FERNÁNDEZ, J., CHARCO, M., TIAMPO, K., and JENTZSCH, G. (2007), *Interpretation of 1992–1994 gravity Changes in Mayon volcano, Philippines, using point sources*, Pure Appl. Geophys., this volume.
- CAMUS, G., GOURGAUD, A., MOSSAND-BERTHOMMIER, P.-C., and VINCENT, P.-M. (2000), *Merapi (Central Java, Indonesia): An outline of the structural and magmatological evolution, with a special emphasis to the major pyroclastic events*, J. Volcanol. and Geotherm. Res. 100, 139–163.
- COMMER, M. (2003), *Three-dimensional inversion of transient electromagnetic data: A comparative study*, Ph.D. Thesis, University of Cologne, Germany.
- DARWIN, C. (1859), *The origin of species, by means of natural selection or the preservation of favored races in the struggle for life*, The Modern library, New York.
- FERNÁNDEZ, J. and RUNDLE, J. (1994), *Gravity changes and deformation due to a magmatic intrusion in a two-layered crustal model*, J. Geophys. Res. 99(B2), 2737–2746.
- FRIEDEL, S. and JACOBS, F. (2004), *Blick in den Vulkan*, Physik J. 3(11), 62–63.
- GERTISSER, R. (2002), *Gunung Merapi (Java, Indonesien): Eruptionsgeschichte und magmatische Evolution eines Hoch-Risikovolkans*, Ph.D. Thesis, Geowissenschaftliche Fakultät, Albert-Ludwigs-Universität Freiburg i.Br., Germany.
- JENTZSCH, G., WEISE, A., REY, C., and GERSTENECKER, C. (2004), *Gravity changes and internal processes: Some results obtained from observations at three volcanoes*, Pure Appl. Geophys. 161, 1415–1431.
- KÖRNER, A. (2000), *Deformationsmodelle nach Auswertung von Tiltmeter- und GPS-Daten am Vulkan Merapi (Indonesien)*, Ph.D. Thesis, University of Potsdam, Germany.
- LANGBEIN, J. (2003), *Deformation of the Long Valley caldera, California: Inferences from Measurements from 1988 to 2001*, J. Volcano. Geother. Res. 127, 247–267.
- LÄUFER, G. (2003), *Erzeugung hybrider digitaler Höhenmodelle aktiver Vulkane am Beispiel des Merapi, Indonesien*, Ph.D. Thesis, Darmstadt University of Technology, Darmstadt, Germany.
- MANSINHA, L., SMYLLIE, D., and ORPHAL, D. (1971), *The displacement fields of inclined faults*, Bull. Seismol. Soc. Am. 61, 1433–1440.
- OHNRBERGER, M., WASSERMANN, J., BUDI, E., and GÖSSLER, J. (2000), *Continuous automatic monitoring of Mt. Merapi's seismicity*, 2. Merapi-Galeras Workshop 1999, Deutsche Geophysikalische Gesellschaft, 4/2000, 103–108.
- RATDOMOPURBO, A. and POUPINET, G. (2000), *An overview of the seismicity of Merapi volcano (Java, Indonesia) 1983–1994*, J. Volcano. Geother. Res. 100, 193–214.
- RUNDLE, J. (1980), *Static elastic-gravitational deformation of a layered half-space by point couple sources*, J. Geophys. Res. 85, 5355–5363.
- RUNDLE, J. (1982), *Deformation, gravity, and potential changes due to volcanic loading of the crust*, J. Geophys. Res. 88(10), 647–652.
- SETIAWAN, A. (2002), *Modeling of gravity changes on Merapi volcano observed between 1997–2000*, Ph.D. Thesis, Darmstadt University of Technology, Darmstadt, Germany.

- TIAMPO, K., FERNÁNDEZ, J., JENTZSCH, G., CHARCO, M., TIEDE, C., GERSTENECKER, C., CAMACHO, A., and RUNDLE, J. (2004), *Elastic-gravitational modeling of geodetic data in active volcanic areas*, Recent Res. Devel. Geophys. 6, 37–58.
- TIAMPO, K., RUNDLE, J., FERNÁNDEZ, J., and LANGBEIN, J. (2000), *Spherical and ellipsoidal volcanic sources at long Valley Caldera, California, Using a genetic algorithm inversion technique*, J. Volcanol. Geotherm. Res. 102, 189–206.
- TIEDE, C. (2005), *Integration of optimization algorithms with sensitivity analysis, with application to volcanic regions*, Ph.D. Thesis, Darmstadt University of Technology, Darmstadt, Germany.
- WASSERMANN, J., OHRNBERGER, M., SCHERBAUM, F., GOSSLER, J., and ZSCHAU, J. (1998), *Kontinuierliche seismologische Netz- und Arraymessungen am Dekadenvulkan Merapi (Java, Indonesien)*, Mitteilungen der Deutschen Geophysikalischen Gesellschaft, 1(3).
- ZIMMER, M. and ERZINGER, J. (2003), *Continuous H_2O , CO_2 , ^{222}Rn and temperature measurements on Merapi volcano, Indonesia*, J. Geophys. Res. 125, 25–38.

(Received October 4, 2005, revised December 22, 2005, accepted February 16, 2006)

Published Online First: April 16, 2007

To access this journal online:
www.birkhauser.ch/pag

Vertical Crustal Motion along the Mediterranean and Black Sea Coast Derived from Ocean Altimetry and Tide Gauge Data

D. GARCÍA,¹ I. VIGO,¹ B. F. CHAO,² and M. C. MARTÍNEZ¹

Abstract—Tide gauge (TG) data along the northern Mediterranean and Black Sea coasts are compared to the sea-surface height (SSH) anomaly obtained from ocean altimetry (TOPEX/Poseidon and ERS-1/2) for a period of nine years (1993–2001). The TG measures the SSH relative to the ground whereas the altimetry does so with respect to the geocentric reference frame; therefore their difference would be in principle a vertical ground motion of the TG sites, though there are different error sources for this estimate as is discussed in the paper. In this study we estimate such vertical ground motion, for each TG site, from the slope of the SSH time series of the (non-seasonal) difference between the TG record and the altimetry measurement at a point closest to the TG. Where possible, these estimates are further compared with those derived from nearby continuous Global Positioning System (GPS) data series. These results on vertical ground motion along the Mediterranean and Black Sea coasts provide useful source data for studying, contrasting, and constraining tectonic models of the region. For example, in the eastern coast of the Adriatic Sea and in the western coast of Greece, a general subsidence is observed which may be related to the Adriatic lithosphere subducting beneath the Eurasian plate along the Dinarides fault.

Key words: Mediterranean and Black Seas, sea level, altimetry, tide gauge, vertical crustal motion, continuous GPS.

1. Introduction

Understanding the sea-level variations is a complex interdisciplinary scientific challenge, for which a first step is to monitor the sea-level change. For many decades, some of the most reliable and useful data series available are the sea-level records taken at tide gauges (TG) at many places along the world's continental as well as island coasts. In the space age, to complement these observations and to acquire a more complete view of the open ocean, the technique of satellite altimetry was implemented and has seen great success ever since the Seasat mission in 1978. In this study we use altimetry data from a merged solution from European Remote Sensing 1-2 (ERS-1, 1991–1996; ERS-2, 1996–Present) of European Space Agency (ESA),

¹ Space Geodesy Laboratory, Department of Applied Mathematics, University of Alicante, Alicante, Spain.

² College of Earth Sciences, National Central University, Chung-li, Taiwan, ROC.

and the U.S. - French TOPEX/Poseidon (T/P, 1992–2005). T/P has achieved an unprecedented accuracy at 2–3 cm, and its follow-on mission Jason-I (2002–Present) is seeking the 1 cm accuracy (LUTHCKE *et al.*, 2003).

The altimetry measurement of the sea-surface height (SSH) consists of (i) the precise satellite location in relation to a reference ellipsoid, and (ii) the height of satellite over the surface measured by means of laser or radar pulses sent from the satellite and bounced back from the target surface. The difference between the two thus gives the SSH relative to the reference ellipsoid at the given location and time. In this sense the altimetry measurement gives the absolute geocentric SSH in the terrestrial reference frame.

On the other hand, the TG measurement gives the SSH relative to the solid ground on which the TG is fixed. Hence any vertical ground motion, for whatever reasons, will masquerade as an apparent sea-level variation: An apparent sea-level rise would be added to the absolute sea-level motion as the land sinks, and conversely. Therefore, in principle the difference between the two data types, altimetry and TG, is the absolute vertical ground motion (CAZENAVE *et al.*, 1999; NEREM and MITCHUM, 2002). Let $TG(t)$ denote the time series of the TG record of SSH, and $Alt(t)$ be that measured by altimetry at the closest available point to the TG site. Ideally, apart from measurement noise and the fact that the actual locales are not exactly the same (especially when coastal geometry influences are significant), $Alt(t)$ and $TG(t)$ should be found to fluctuate in unison, with the only difference being the actual vertical motion at the TG site. At places where this motion is known or negligible, these sites have been the basis for the *in situ* calibration/validation for altimetric measurements (CHAMBERS *et al.*, 1998; MITCHUM 1994, 1998, 2000). In this paper we will form the difference time series $Alt(t)-TG(t)$ and examine its long-term behavior on at least decadal time scales, where the local/geographic differences become less important and the effect of measurement noise can also be greatly reduced. It is hoped that a decade-long record of $Alt(t)-TG(t)$ should be able to give unequivocal information about the long-term vertical ground motion in the absolute sense. We will not be concerned about time scales shorter than interannual and seasonal periods, for which $Alt(t)-TG(t)$ signifies primarily the surface mass loading effects on the ground, both tidal and non-tidal.

We shall estimate the linear trend, or slope, of the $Alt(t)-TG(t)$ record for each of the available TG locations along the northern coast of the Mediterranean Sea and the Black Sea. In this region, the post-glacial rebound effect in the vertical ground motion is minimal according to geophysical models, around $\pm 0.1/0.2$ mm/year (PELTIER, 2001), so our result reflects the vertical ground motion arising primarily from tectonics, except at the sites where local “noises” are evident, for example ground subsidence due to land settlement or groundwater extraction. This provides for regional tectonics studies source information that is independent of, and complementary to, other data types such as gravity and GPS. In fact, the series of vertical motion recorded at various continuous GPS stations will be compared with our results for further insight.

2. Data

Figure 1 shows the locations of all data used in this study as described below. The numbering of the TG sites, which is in the west-to-east order along the coast, corresponds to the numbering in Table 1; the same goes with the GPS stations with respect to Table 2. The altimetry grid points are indicated by the dots; those closest to each TG are indicated by the larger dots.

2.1. Altimetry Data

The altimetry data used in this study are monthly Sea Level Anomalies (SLA) maps, on a 1° by 1° grid, solved from the ocean radar altimetry data from satellite missions of T/P (FU and CAZENAVE, 2001) and ERS-1/2 for a 9-year period of 01/1993–12/2001. Data are a combination of T/P + ERS, except for the period 01/1994–03/1995 (ERS-1 geodetic phase). These gridded multimission data have been used, opposed to using the along track data from one single satellite, because of its higher accuracy and resolution. SLA are given in units of mm, where a 7-year (01/1993–01/1999) T/P data representing the static geoid is removed from the altimeter data. There are about 310 grid points in the Mediterranean and about 61 grid points in the Black Sea. Several corrections have been applied to the data: Orbit error reduction of ERS via the precise orbit of T/P, geophysical (dry and wet troposphere, ionosphere and inverse barometer effect), sea state bias and tides (ocean and load tides, solid earth tide and pole tide). For further details see AVISO (1996; 1998). Whether one should allow inverse barometer (IB) correction or not in this case is debatable as it was noted that IB is not prevalent for the Mediterranean Sea at relatively short

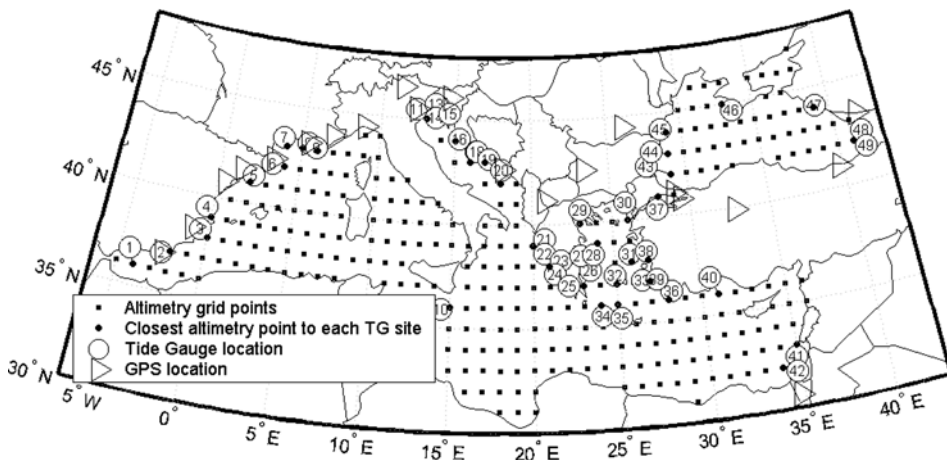


Figure 1
Data locations used in this study.

Table 1

Linear Rate of Change, or slope, (mm/year) of the Alt–TG Series. Negative values (indicated in Bold) indicate land subsidence, positive values (indicated in Italics) indicate land uplift, the underlined are values indistinguishable from zero within 1 sigma. The numbering of the sites corresponds to Figure 1

LINEAR VERTICAL CRUSTAL MOTION RATE OF CHANGE (mm/year)				
MEDITERRANEAN SEA				
	Tide Gauge site (Country)	Location	Trend	Sigma
1	Málaga ii (Spain)	36 43 N 04 25 W	−3.8	2.5
2	Almeria (Spain)	36 50 N 02 29 W	−10.1	3.7
3	Alicante ii (Spain)	38 20 N 00 29 W	−21.3	3.9
4	Valencia (Spain)	39 28 N 00 20 W	<u>2.7</u>	2.9
5	Barcelona (Spain)	41 21 N 02 10 E	<u>2.5</u>	2.5
6	L'Estartit (Spain)	42 03 N 03 12 E	<u>−0.5</u>	1.9
7	Sete (France)	43 24 N 03 42 E	<u>6.3</u>	5.2
8	Marseille (France)	43 18 N 05 21 E	−3.0	2.1
9	Toulon (France)	43 07 N 05 55 E	<u>0.9</u>	1.9
10	Valletta (Malta)	35 54 N 14 31 E	−7.5	2.7
11	Venezia P.S. (Italy)	45 26 N 12 20 E	<u>−0.8</u>	1.9
12	Trieste (Italy)	45 39 N 13 45 E	<u>−0.5</u>	1.7
13	Luka Koper	45 34 N 13 45 E	<u>10.1</u>	2.0
14	Rovinj (Croatia)	45 05 N 13 38 E	<u>−2.1</u>	2.3
15	Bakar (Croatia)	45 18 N 14 32 E	−5.5	2.6
16	Zadar (Croatia)	44 07 N 15 14 E	<u>3.9</u>	4.0
17	Split RT Marjana	43 30 N 16 23 E	<u>−2.1</u>	2.1
18	Split Harbour	43 30 N 16 26 E	−2.4	2.2
19	Sucuraj (Croatia)	43 08 N 17 12 E	−5.0	3.0
20	Dubrovnik (Croatia)	42 40 N 18 04 E	−5.8	2.0
21	Preveza (Greece)	38 57 N 20 46 E	−16.8	2.1
22	Levkas (Greece)	38 50 N 20 42 E	−14.6	1.7
23	Patrai (Greece)	38 14 N 21 44 E	−9.6	2.0
24	Katakolon (Greece)	37 38 N 21 19 E	−9.4	1.8
25	Kalamai (Greece)	37 01 N 22 08 E	−4.5	1.7
26	North Salaminos	37 57 N 23 30 E	<u>4.8</u>	2.0
27	Khalkis South	38 28 N 23 36 E	<u>9.5</u>	2.2
28	Khalkis North	38 28 N 23 36 E	<u>4.3</u>	1.6
29	Tessaloniki	40 37 N 23 02 E	<u>0.9</u>	1.6
30	Alexandroupolis	40 51 N 25 53 E	<u>8.3</u>	2.0
31	Khios (Greece)	38 23 N 26 09 E	<u>4.7</u>	2.0
32	Siros (Greece)	37 26 N 24 55 E	<u>−1.5</u>	4.2
33	Leros (Greece)	37 05 N 26 53 E	<u>11.0</u>	1.4
34	Soudhas (Greece)	35 30 N 24 03 E	<u>1.9</u>	1.4
35	Iraklion (Greece)	35 20 N 25 08 E	−7.5	6.6
36	Rodhos (Greece)	36 26 N 28 14 E	−6.7	2.1
37	Erdek (Turkey)	40 23 N 27 51 E	<u>−1.1</u>	5.5
38	Mentes/Izmir	38 26 N 26 43 E	−12.8	3.8
39	Bodrum ii (Turkey)	37 02 N 27 25 E	<u>21.6</u>	5.6
40	Antalaya ii (Turkey)	36 50 N 30 37 E	−17.6	4.0
41	Hadera (Israel)	32 28 N 34 53 E	−2.4	1.5
42	Tel Aviv (Israel)	32 05 34 46 E	<u>−2.5</u>	3.1

BLACK SEA				
43	Bourgas (Bulgaria)	42 29 N 27 29 E	-12.3	7.4
44	Varna (Bulgaria)	43 11 N 27 55 E	-25.5	6.1
45	Constanza (Romania)	44 10 N 28 40 E	<u>3.6</u>	4.5
46	Sevastopol (Ukraine)	44 37 N 33 32 E	<u>-0.2</u>	9.4
47	Tuapse (Russian Federation)	44 06 N 39 04 E	<u>6.2</u>	1.7
48	Poti (Georgia)	42 10 N 41 41 E	3.5	2.7
49	Batumi (Georgia)	41 38 N 41 42 E	31.6	8.6

Table 2

Vertical velocities (mm/year) of the GPS time series up to ~1199 GPS week (late 2002). Negative values indicate land subsidence, positive values indicate land uplift. The sign of these values is represented in Figure 2

VERTICAL VELOCITIES AT GPS STATIONS (mm/year)				
MEDITERRANEAN SEA				
	GPS site (Country)	Location	Trend	Sigma
1	Alicante (Spain)	38 20 N 00 29 W	-3.21	0.19
2	Valencia (Spain)	39 29 N 00 20 W	-0.68	0.91
3	Ebre (Spain)	41 49 N 00 30 E	-1.33	0.10
4	Bellmunt (Spain)	41 36 N 01 24 E	-0.22	0.18
5	Cap de Creus (Spain)	42 19 N 03 18 E	-0.44	0.31
6	Marseille (France)	43 17 N 05 21 E	-2.35	0.16
7	Calern (France)	43 45 N 06 55 E	-0.78	0.11
8	Genova (Italy)	44 25 N 08 55 E	-1.66	0.21
9	Bolzano (Italy)	46 29 N 11 20 W	1.33	0.23
10	Venezia (Italy)	45 26 N 12 20 E	-5.95	0.74
11	Padova (Italy)	45 24 N 01 15 E	-6.32	0.75
12	Geoservis GR.1 (Slovenija)	46 03 N 14 33 E	-1.18	0.96
13	Dubrovnik (Croatia)	42 39 N 18 07 E	-5.31	0.52
14	Orhid (Macedonia)	41 08 N 20 48 E	-0.68	0.47
15	Sofia (Bulgaria)	42 33 N 23 24 E	-0.17	0.18
16	Mitzpe Ramon (Israel)	30 36 N 34 46 E	-2.25	0.48
BLACK SEA				
17	Bucuresti (Romania)	44 28 N 26 08 E	0.11	0.18
18	Istambul (Turkey)	41 06 N 29 01 E	-0.52	0.28
19	Gebze (Turkey)	40 47 N 29 27 E	-4.33	0.23
20	Ankara (Turkey)	39 53 N 32 46 E	-1.70	0.17
21	Trabzon (Turkey)	41 00 N 39 47 E	-3.27	0.48
22	Zelenchukskaya (Russia)	43 17 N 41 34 E	0.68	0.26

periods (LE TRAON and GAUZELIN, 1997); we here use IB corrected data as we are interested in long-period variations.

The region we examine for the Mediterranean and Black Seas is between 5°W and 42°W in longitude, 30°N and 48°W in latitude. We choose the altimetry measurement at the closest point to the tide gauge as stated.

2.2. Tide Gauge Data

In this study we use TG data available from the Permanent Service for Mean Sea Level (PSMSL) (SPENCER and WOODWORTH, 1993). For the present study we choose, for 42 gauges along the northern coast of the Mediterranean and 7 along the Black Sea coast, the so-called RLR (Revised Local Reference) series that provide precise information about the TG benchmark. Data are monthly time series that cover most of the 9-year altimeter period of this study. When needed, data gaps in the TG data series are linearly interpolated to facilitate data processing.

Barring some limited earlier TG activities in Egypt (predating our study period), no data are available from the southern coast of the Mediterranean. Neither are the many TG records from the Italian peninsula reported to the PSMSL in recent years (TSIMPLIS and SPENCER, 1997) (but see (FENOGLIO-MARC *et al.*, 2004) for analysis of the Italian records). Otherwise, the spatial distribution of the TG that we use (those with data after 1993 in PSMSL) is quite dense and homogeneous along the northern Mediterranean and Black Sea coasts, as shown in Figure 1.

3. Analysis and Results

It is clear that all the data series above are dominated by very strong seasonal signals, primarily annual and semiannual. There are of course various meteorological as well as geophysical reasons why this is so, but whatever the reasons, they are outside of our present interest. It is thus desirable to remove the seasonal signals from our data series, as their presence would skew our later estimates in various ways and render our statistics intractable due to the strong departure from broadband or “white” statistics.

We remove the seasonal signals (annual and semiannual) from the original time series by simple subtraction of estimates obtained by least-squared fitting of seasonal sinusoids with annual and semiannual periods. We then compare the altimetry observed SSH anomaly, $Alt(t)$, at the closest grid point to a TG location with the given $TG(t)$. The chosen altimetry point is just the closest one to the TG and it can be as far as $\sqrt{2}/2$ degrees. The distance between them, which can be estimated from the altimetry point locations in Figure 1 and the TG point locations in Table 1, and the coastal geometry of each TG, has not been taken into account on the computations and they are considered as a source of error. The corrections applied to the altimetry data are made over the orbit groundtrack of the satellites, some of them being highly independent of measurements taken along-track by other instruments onboard the satellite, and most of them were developed to be applied in the open ocean. Some of those corrections present problems in the coast, and for that reason they cannot be easily applied to TG measurements. The resolution of those problems and the implementation of the altimetry corrections in the TG data require further

study and are beyond the scope of this paper. Then, the different corrections applied to each data represent another source of error. Nevertheless, a general good agreement can be observed between the two time series, showing the same temporal behavior in much of the SSH fluctuations. In particular, for example, in VIGO *et al.* (2005) a prominent and abrupt change in the slope of the Mediterranean and Black Sea SSH during 1999 was found to exist in both $Alt(t)$ and $TG(t)$. However, when examining the long-term behavior of SSH, discrepancies between the two, which are our sought signal, arise. In Figure 2, three examples of such a comparison are shown,—Marseille and Trieste in the Mediterranean, and Tuapse in the Black Sea. The time series on the left are the original observation data from the two sources; the middle column are those after the removal of the nominal seasonal signals as just described. The right column shows the respective non-seasonal $Alt(t)-TG(t)$ difference series. As stated, a TG measures the SSH relative to the ground on which the TG sits, whereas the altimetry measures the absolute SSH relative to the

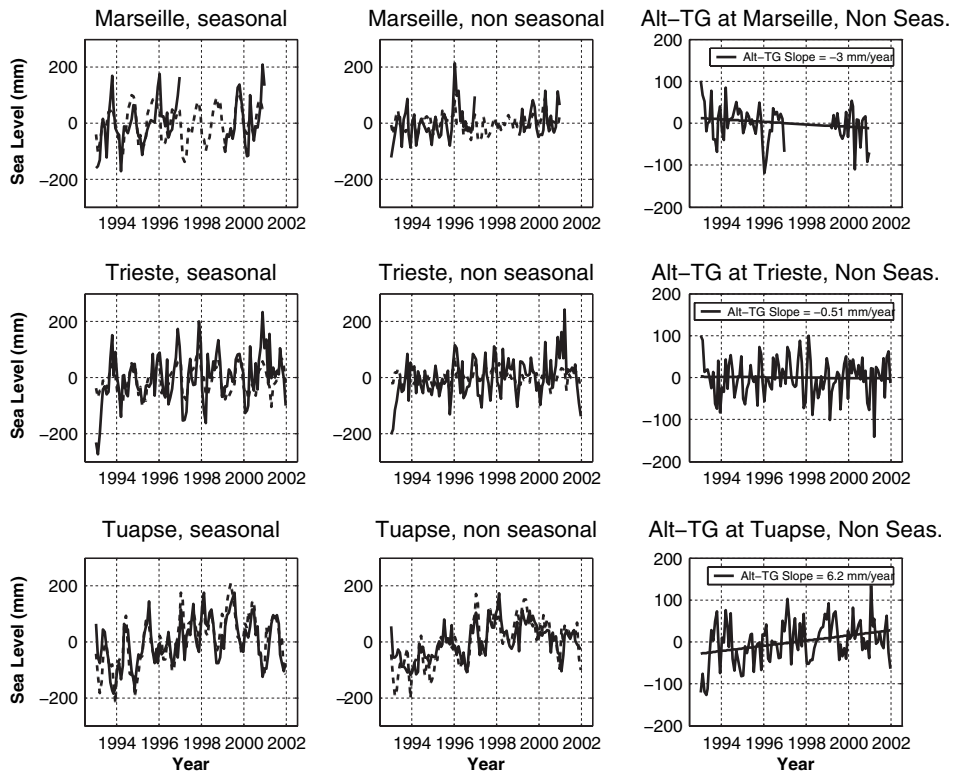


Figure 2

The first and second columns show seasonal and not seasonal Alt (solid line) and TG (dashed line) time series at Marseille (first row), Trieste (second row) and Tuapse (third row). The third column shows the non-seasonal Alt-TG time series and the least-squared linear fit to give the slope.

terrestrial reference frame with respect to which the satellite orbit is defined. Their difference then contains all information about the vertical ground motion of the tide gauge location in the terrestrial frame. We form the (non-seasonal) $\text{Alt}(t) - \text{TG}(t)$ for the 49 PSMSL TG sites as described above and estimate their slopes using least-squares linear fit—a positive slope means land uplift while a negative slope means land subsidence. Such linear slopes are plotted in the right columns of Figure 2.

Table 1 shows our estimates for the $\text{Alt}(t) - \text{TG}(t)$ linear slopes for all stations. Positive values are in *italic* (land uplift), negative values in **bold** (land subsidence), while underlined indicates values statistically indistinguishable from zero within 1 standard deviation (given in the last column). Figure 3 gives a map view of the same results as Table 1, where the “up” triangles represent land uplift, the “down” triangles represent land subsidence, circle means indistinguishable from zero, all color-scaled. It is seen that, except for a few near-zero ones and those that are known to have large local influences, the uplift-subsidence has definite spatial grouping, signifying regional patterns that are most likely of tectonic origin. This will be discussed below.

Several error sources are considered in estimating vertical land movements from the $\text{Alt} - \text{TG}$ time series. For instance, we compare SSH at geographical points that are not exactly the same location but they are the closest grid point at which the seasonal and shorter responses can be significantly different depending on the coastal geometry. Also, corrections for the solid-Earth and ocean tides, solid-Earth pole tides, and the equilibrium inverted barometer effects are all applied in the altimetry

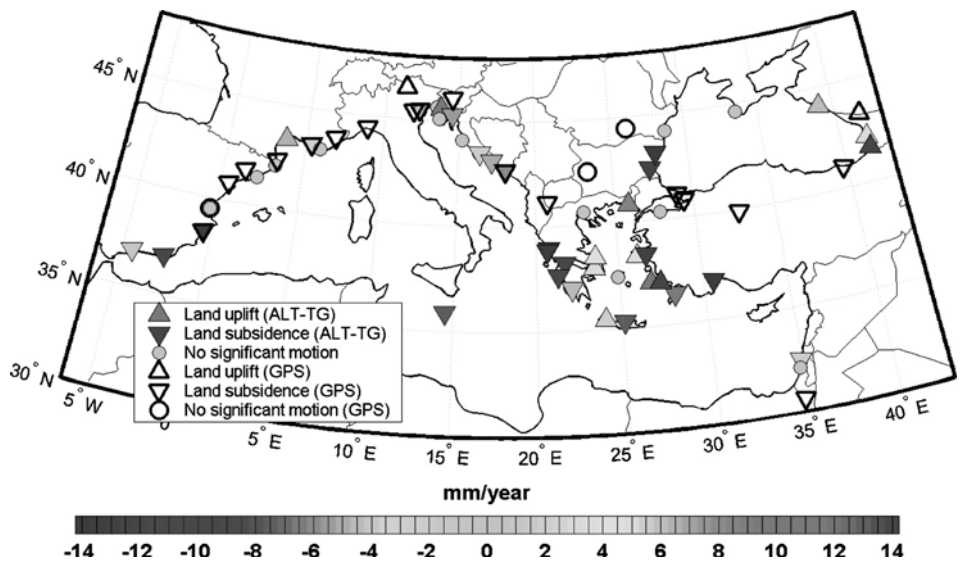


Figure 3

Vertical land movement along the Mediterranean and Black Sea coasts derived from $\text{Alt} - \text{TG}$ time series.

data, but not in the TG data. However, errors due to such differences are insignificant as far as the long-term trend is concerned and in any case small in magnitude. On the other hand, errors have been made also in the preparation and processing of data. Some gaps in the TG data were interpolated over; that if not treated in an optimal way could somewhat skew the slope estimates. Further errors come from the secular drift of the T/P instruments (altimeter as well as radiometer for atmospheric correction) although such drifts have been corrected to the extent possible by the T/P Projects based on *in situ* calibrations (RUF, 2000; KEIHM *et al.*, 2000). These error sources mainly impact our estimates of the vertical land movement that are close to zero, which we have indicated in our results.

For gaining more insight and for further validation we can compare our linear trend estimates for the vertical ground motion derived from $\text{Alt}(t)\text{--TG}(t)$ with corresponding data from permanent, continuous GPS stations that are in reasonable proximity to the TG stations. About 150 GPS stations from the EUREF Permanent Network (EPN), distributed over 32 European countries, acquire high quality GPS data in near real-time. In particular we use the cleaned GPS time series for 22 stations, close to the TG (Fig. 1), produced by the EPN Time Series Monitoring Project (A. KENYERES, personal communication, 2005). These GPS time series did not start until the late 1990s or early 2000s, so they generally only have limited overlap with the available TG data. For that reason, we only use the GPS data (until late 2002) in a rudimentary comparison with respect to the sign of the vertical velocity, where a generally good agreement in sign is observed. This result is shown in Figure 3, whereas the numerical values are given in Table 2.

We also compared our linear trend estimates for the vertical ground motion with those obtained via several GPS campaigns from the SELF projects during the years 1993–1998 as reported by BECKER *et al.* (2002). We note the following: (i) BECKER *et al.* found no significant vertical motion in Spain, whereas our $\text{Alt}(t)\text{--TG}(t)$ result at face value indicates a coastal subsidence in Southern Spain. The reason is presently unclear (but see Section 4). (ii) BECKER *et al.* reported a slight uplift at Venice and a subsidence at Trieste in Italy, whereas we found no significant motion at both sites. (iii) In Greece, a general agreement is observed at Kalamai, Katakolon, Patrai, Preveza, Rodhos and Iraklion where subsidence is reported, unlike the disagreement at Soudhas and Alexandroupolis where we found uplift, and at Siros where we found no significant motion with little TG data available. (iv) In the Black Sea, a good agreement in subsidence is found at Varna and Bourgas.

4. Discussions and Conclusions

The absolute rates of vertical ground motion obtained in this work (Fig. 3 and Table 1) are in accordance with the general notion that the Mediterranean is a location of plate subduction boundary, and the seismicity is more active in the

eastern Mediterranean and becomes quieter towards the north. Here in order to establish the framework to draw further conclusions, we need to delve deeper into the complexity of the Mediterranean tectonics. Prior to that, however, we should assess the impact of the glacial isostatic adjustment (GIA) on the vertical ground motion. Model of GIA by PELTIER (2001) puts the vertical ground motion at around $\pm 0.1/0.2$ mm/year in the Mediterranean and Black Sea region. That amounts to just a small fraction of the land uplift/subsidence rates reported here. Thus, the vertical ground motion due to GIA does add to or subtract from the otherwise tectonic ground motion detected in our results, however the magnitude of that ‘contamination’ is relatively small and has generally little, if any, impact on the sign of our results. Therefore we shall ignore the GIA influence in the discussion on tectonics below.

Figure 4 depicts the main tectonic processes in the Mediterranean and Black Seas. In the Mediterranean the collisions between the African, Eurasian and Arabic plates have produced a very complex tectonic regime of microplates that is far from resolved, especially with respect to vertical motions (R. SABADINI, H. DREWES, personal communication, 2003). Some of the present plate boundaries, especially in the eastern Mediterranean, appear to be so diffuse and so anomalous that they cannot be categorized into the three classical types of plate boundaries—subduction, spreading, and transform. The global tectonic model NUVEL1 (DEMETS *et al.*, 1990)

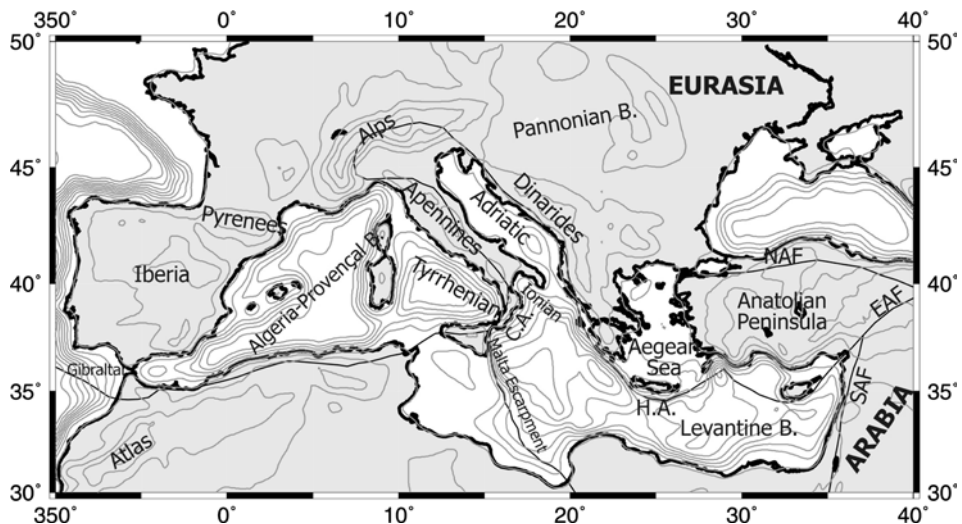


Figure 4

Black lines represent the plates boundaries in the Mediterranean and Black Seas from JIMENEZ-MUNT *et al.* (2003) (with permission from the author). H. A., Hellenic Arc; C. A. Calabrian Arc; B., Basin; NAF, North Anatolian Fault; SAF, South Anatolian Fault; EAF, East Anatolian Fault. Grey lines represent the topography of the region every 500 m. Note that the grey lines on the continents are positives and those on the sea are negatives, being zero on the coast line.

and geodetic data establish that the Eurasian and African plates converge along NW-SE direction, both rotating anticlockwise (JIMENEZ-MUNT *et al.*, 2003).

As one of the most seismically active and rapidly deforming continental regions in the world, the Aegean Sea area in the eastern Mediterranean should be taken in special consideration. In this area, including western Turkey and Greece, many destructive earthquakes have occurred throughout history. The Crete Island situates on the north side of the Hellenic Arc, which is the result of the ongoing collision between the African and Eurasian plates, causing the Anatolian plate (Turkey) to move westward (MCKENZIE, 1972; JIMENEZ-MUNT and SABADINI, 2002). As a result, the African block is subducted northward, producing an uplift in Crete Island (RAHL *et al.*, 2004), which is in agreement with our results found in Soudhas, but not in Iraklion.

A general and distinct land subsidence on the west coast of Greece and the east coast of the Adriatic Sea, is reported in this study. This finding may be related to the Adriatic lithosphere subducting beneath the Eurasian plate along the Dinarides fault (Fig. 4). In particular, although Venice has seen geological evidence of subsidence (due to the rollback of the Apennines subduction, the contemporary effect of the load of the Alpine belt on its foreland and the river sediments, and the exploitation of subsurface water) (RUTIGLIANO *et al.*, 2000), neither this study or BECKER *et al.* (2002) show such a subsidence. Consequently, further studies are needed to clarify this discrepancy. Similarly, the uplift on the eastern coast of Greece may be explained as a consequence of the same but converse associated movements.

Contrastingly, we find conflicting behavior (in terms of uplift versus subsidence) in the TGs located on the eastern (Turkish) coast of the Aegean Sea. It is unclear whether or how much of it is due to data errors, local motions, or any interesting regional tectonics. This awaits further study. Similarly, towards the west, noteworthy is the fact that our study reflects opposite behavior in the case of Sete and Marsella, France. Given their geographical proximity and if the results are correct, it might indicate the existence of a tectonic fault separating the two sites, although the exact location might not be clearly identified.

Finally, the western Mediterranean basins are mostly surrounded by mountain belts that for the most part remain tectonically and seismically active. The three sub-basins of different ages reflect different stages in the development of the western Mediterranean by backarc spreading in response to subduction of the African lithosphere beneath southern Europe. Our finding that the general area in southern Spain is subsiding is at odds with this subducting African plate model because such subduction should predict an uplift for the region. This appears quite interesting, however we should note that in the case of Alicante our estimate is strongly contaminated by the settling of the harbor construct on which the TG is situated. An individualized study for each TG should be done to identify additional or similar sources of error. However, such a study is beyond the scope of this work.

In summary, in spite of several error sources in the Alt-TG series, with a sufficiently long time span reaching 9 years we are able to obtain reliable estimates for the vertical ground motion at the TG sites along the northern Mediterranean and Black Sea coasts. These results provide useful source data for studying, contrasting, and constraining tectonic models of the region.

Acknowledgments

We are grateful for the organizations that provided the source data used here, including NASA Ocean Altimeter Pathfinder Project, the Permanent Service for Mean Sea Level, and EPN time series monitoring project, and for I. Jimenez-Munt that provided Figure 4. We have also benefited from discussions with H-P. Plag, S. Zerbini, and software assistance from A. Au. This work is supported by the Spanish Science and Technology Ministry (Project ESP2001-4533-PE, Project ESP2005-02212), and NASA's Physical Oceanography Program.

REFERENCES

- AVISO, User Handbook: Merged Topex/Poseidon Products, *AVI-NT-02-101-CN*, Edition 3.0, July 1996. Online available from <<http://www.jason.oceanobs.com/>>.
- AVISO, User Handbook: Sea Level Anomalies, *AVI-NT-011-312-CN*, Edition 3.1, April 1998. Online available from <<http://www.jason.oceanobs.com/>>.
- BECKER, M., ZERBINI, S., BAKER, T., BRKI, B., GALANIS, J., GARATE, J., GEORGIEV, I., KAHLE, H.-G., KOTZEV, V., LOBAZOV, V., MARSON, I., NEGUSINI, N., RICHTER, B., VEIS, G., and YUZEFOVICH, P. (2002), *Assessment of height variations by GPS at Mediterranean and Black Sea coast tide gauges from the SELF projects*, *Global and Planetary Change* 34, 5–35.
- CAZENAVE, A. *et al.* (1999), *Sea level changes from Topex-Poseidon altimetry and tide gauges, and vertical crustal motions from DORIS*, *Geophys. Res. Lett.* 26, 2077–2080.
- CHAMBERS, D. P., RIES, J. C., SHUM, C. K., and TAPLEY, B. D. (1998), *On the use of tide gauges to determine altimeter drift*, *J. Geophys. Res.* 103(C6), 12,885–12,890.
- DEMETS, C., GORDON, R.G., ARGUS, D.F., and STEIN, S. (1990), *Current plate motions*, *Geophys. J. Int.* 101, 425–478.
- FENOGLIO-MARC, L., DIETZ, L., and GROTEN, E. (2004), *Vertical land motion in the mediterranean sea from altimetry and tide gauge stations*, *Marine Geodesy* 27, 1–19.
- JIMENEZ-MUNT, I. and SABADINI, R. (2002), *The block-like behavior of Anatolia envisaged in the modeled and geodetic strain rates*, *Geophys. Res. Lett.* 29(20), 1978, doi:10.1029/2002GL015995.
- JIMENEZ-MUNT, I., SABADINI, R., GARDI, A., and BIANCO, G. (2003), *Active deformation in the Mediterranean from Gibraltar to Anatolia inferred from numerical modeling and geodetic and seismological data*, *J. Geophys. Res.* 108(B1), 2006, doi:10.1029/2001JB001544.
- KEIHM, S. J., ZLOTNIKI, V., and RUF, C. S. (2000), *TOPEX Microwave Radiometer Performance Evaluation, 1992–1998*, *IEEE Trans. Geosci. Remote Sensing*, Vol. 38, pp. 1396–1386.
- LE TRAON, P. and GAUZELIN, P. (1997), *Response of the Mediterranean mean sea level to atmospheric pressure forcing*, *J. Geophys. Res.* 102(C1), 973–984.
- LUTCKE, S. B., ZELENKY, N. P., ROWLANDS, D. D., LEMOINE, F. G., and WILLIAMS, T. A. (2003), *The 1-Centimeter Orbit: Jason-1 Precision Orbit Determination Using GPS, SLR, DORIS, and Altimetry Data*, *Marine Geodesy* 26, 399–421.

- McKENZIE, D.P. (1972), *Active tectonics of the Mediterranean region*, Geophys. J. Roy. Astronom. Soc. 30, 109–185.
- MITCHUM, G.T. (1994), *Comparison of TOPEX sea surface heights and tide gauge sea levels*, J. Geophys. Res. 99, 24,541–24,553.
- MITCHUM, G.T. (1998), *Monitoring the stability of satellite altimeters with tide gauges*, J. Atmos. Oceanic Tech. 15, 721–730.
- MITCHUM, G.T. (2000), *An improved calibration of satellite altimetric heights using tide gauge sea levels with adjustment for land motion*, Marine Geodesy 23, 145–166.
- NEREM, R. S. and MITCHUM, G. T. (2002), *Estimates of vertical crustal motion derived from differences of TOPEX/Poseidon and Tide Gauge Sea Level Measurements*, Geophys. Res. Lett. 29(19), art. no. 1934.
- PELTIER, W.R. (2001), ICE4G (VM2) *Glacial Isostatic Adjustment Corrections*. In *Sea Level Rise; History and Consequences* (Douglas, B.C., Kearney, M.S., and S.P. Leatherman, Eds.), Academic Press. etc.
- RAHL, J. M., FASSOULAS, C., and BRANDON, M. T. (2004), *Exhumation of high-pressure metamorphic rocks within an active convergent margin, Crete, Greece: A field guide*, 32th Int. Geolog. Congress, Vol. 2 – from B16 to B33, art. No B32.
- RUF, C.S. (2000), *Detection of calibration drifts in spaceborne microwave radiometers using a vicarious cold referenc*. IEEE Trans. Geosci. Remote Sensing 38, pp. 44–52.
- RUTIGLIANO, P., FERRARO, C., DEVOTI, R., LANOTTE, R., LUCERI, V., NARDI, A., PACIONE, R., and SCIARRETTA, C. (2000), *Vertical motions in the Western Mediterranean area from geodetic and geological data*. In the Proceedings of *The Tenth General Assembly of the Wegener Project*.
- SPENCER, N. E. and WOODWORTH, P. L. (1993), *Data Holdings of the Permanent Service for Mean Sea Level*, Bidston, Birkenhead: Permanent Service for Mean Sea Level. 81pp.
- TSIMPLIS, M. N. and SPENCER, N. E. (1997), *Collection and analysis of monthly mean sea level data in the Mediterranean and Black Sea*, J. Coastal Res. 13(2), 534–544, Fort Lauderdale, Florida.
- VIGO, I., GARCÍA, D., and CHAO, B. F. (2005), *Change of sea level trend in the Mediterranean and Black Seas*, J. Marine Res. 63, 6.

(Received November 15, 2005, revised September 27, 2006, accepted October 11, 2006)

To access this journal online:
www.birkhauser.ch/pag

Some Insights into Topographic, Elastic and Self-gravitation Interaction in Modelling Ground Deformation and Gravity Changes in Active Volcanic Areas

MARÍA CHARCO,¹ JOSÉ FERNÁNDEZ,¹ FRANCISCO LUZÓN,² KRISTY F. TIAMPO,³ and JOHN B. RUNDLE⁴

Abstract—Surface displacements and gravity changes due to volcanic sources are influenced by medium properties. We investigate topographic, elastic and self-gravitation interaction in order to outline the major factors that are significant in data modelling. While elastic-gravitational models can provide a suitable approximation to problems of volcanic loading in areas where topographic relief is negligible, for prominent volcanoes the rough topography could affect deformation and gravity changes to a greater extent than self-gravitation. This fact requires the selection, depending on local relief, of a suitable model for use in the interpretation of surface precursors of volcanic activity. We use the three-dimensional Indirect Boundary Element Method to examine the effects of topography on deformation and gravity changes in models of magma chamber inflation/deflation. Topography has a significant effect on predicted surface deformation and gravity changes. Both the magnitude and pattern of the geodetic signals are significantly different compared to half-space solutions. Thus, failure to account for topographic effects in areas of prominent relief can bias the estimate of volcanic source parameters, since the magnitude and pattern of deformation and gravity changes depend on such effects.

Key words: Topography, self-gravitation, displacement, gravity change.

Introduction

Surface ground deformation and gravity changes are associated with mass movement and stresses in volcanic regions. An important topic in the study of active volcanoes is the correlation of the subsurface magma movement with such surface effects, not only in order to understand the internal volcano processes, but also to detect precursors of volcanic activity. Volcano monitoring includes analysis and interpretation of such precursors through theoretical models. These are key methods for inferring the geometrical and dynamical parameters characterizing buried magma

¹ Instituto de Astronomía y Geodesia (CSIC-UCM), Facultad de C. Matemáticas, Universidad Complutense de Madrid, Madrid, Spain

² Departamento de Física Aplicada, Universidad de Almería, Almería, Spain

³ Department of Earth Sciences, University of Western Ontario, London, ON, Canada

⁴ Center of Computational Science and Engineering, University of California, Davis, CA 95616, USA

bodies. Therefore, realistic models of magma-induced deformation and gravity changes are of great importance for the forecasting of volcanic hazards (GUDMUNDSSON, 1986; BIANCHI *et al.*, 1987; DVORAK and DZURISIN, 1997).

The deformation due to an expanding/contracting magma chamber has frequently been modelled as a dilatational source in an elastic half space. The most commonly used volcanic deformation model corresponds to a spherically symmetric point source of dilatation in a homogeneous elastic half space (MOGI, 1958). The Mogi model does a reasonably good job in explaining vertical displacement in many cases. However, this model poses difficulties in simultaneously modelling the observed displacement and gravity changes (e.g., KISSLINGER, 1975; RYMER, 1996). RUNDLE (1980) studied the effect of the medium properties in modelling and solved the equations that represent the coupled elastic-gravitational problem for a stratified half-space of homogeneous layers. This type of model goes one step further since it allows considering the geologically meaningful superposition of a pressurized cavity and the mass intrusion within an elastic-gravitational medium. Moreover, most deformation models assume the Earth surface as flat and use half-space solutions. While such models have been quite successful at fitting surface measurements, it is clear that local topographic relief can introduce misinterpretation into source parameter estimation.

McTIGUE and STEIN (1984) and McTIGUE and SEGALL (1988) studied the effect of topography by means of a two-dimensional model. CAYOL and CORNET (1998) and WILLIAMS and WADGE (1998, 2000) calculated the effect of the volcanic relief caused by a dilatational source in a purely elastic medium. FOLCH *et al.* (2000) considered a viscoelastic medium in order to study the effect of topography. TRASATTI *et al.* (2003) considered topographic effects on an axisymmetric volcano shape applied to Mt. Etna. LUNGARINI *et al.* (2005) used a numerical finite-element technique to model realistic 3-D topographic effects of ground displacement at Mt. Etna. In general, the conclusion reached by these authors is that topography has a significant effect on the deformation field. Thus, approximating the Earth by a flat horizontal half space can lead to an erroneous interpretation of observed ground deformation.

The following is a study of the relative importance of elasticity, self-gravitation and topography in modelling volcanic ground deformation and gravity changes before discussing a numerical technique for including realistic topographic relief. It is expected that although self-gravitation could produce significant changes in the value and pattern of predicted gravity changes, the topographic effect can mask elastic-gravitational coupling. Thus, while elastic-gravitational models can be a far more suitable approximation to problems of volcanic loading in which topographic relief is negligible, for prominent volcanoes the ground irregularities could affect deformation and gravity changes to a more significant extent than the self-gravitation effects. This fact allows one to simplify the elastic-gravitational approach when topography is significant.

Most subaerial volcanoes are associated with axisymmetrical geometries. Finally, we determine the effects of topography on deformation and gravity changes created at the surface of axisymmetric volcanoes. The modelling method used for our study is a three-dimensional Indirect Boundary Element Method.

Elastic-gravitational Model

The theory of elastic-gravitational deformation of a layered half space caused by point sources was given by RUNDLE (1980). The appropriate equations in which the fully coupled gravity and displacement changes are obtained as a part of one solution are (LOVE, 1911; RUNDLE, 1980):

$$\rho_0 g \nabla(\mathbf{u} \cdot \mathbf{e}_z) - \rho_0 \nabla \phi - \rho_0 g \mathbf{e}_z \nabla \cdot \mathbf{u} + \nabla \cdot \boldsymbol{\sigma} + \rho_0 \mathbf{F}_p = 0, \quad (1)$$

$$\nabla^2 \phi = -4\pi \rho_0 G \nabla \cdot \mathbf{u} + \nabla \cdot \mathbf{F}_m. \quad (2)$$

Here, $\mathbf{u}$ is the vector displacement in a local cylindrical coordinate system (r, θ, z) with the origin at the surface and with the z axis pointing down into the medium, $\mathbf{e}_z$ is the unit vector in the z direction, ϕ is the gravitational potential, ρ_0 the unperturbed density, G is the gravitational constant and g is the unperturbed gravity acceleration. The state of equilibrium is slightly disturbed by the emplacement of a mass source, a pressure source, or some combination of both, at some depth within the crust that will set up an additional stress field, $\boldsymbol{\sigma}$, correlated with displacement field, $\mathbf{u}$, as in the ordinary theory of elasticity.

The elastic-gravitational approach models pressurization together with mass intrusion as two different sources that can be treated with the same formalism. We have approximated the intrusion by the superposition of a point source of dilatation and a mass point source. The term $\mathbf{F}_p$ in equation (1) represents the body force per unit mass equivalent to a point source of dilatation. To define the mass source, we consider the elastically perturbed gravitational potential, ϕ , that is given by

$$\phi = \phi_1 + \phi_2, \quad (3)$$

where ϕ_1 is the potential due to mass redistribution for perturbed density, $\rho_0 \nabla \cdot \mathbf{u}$. Thus, $\nabla^2 \phi_1$ is the first term of the right-hand side of equation (2). ϕ_2 is the gravitational potential due to the influx of magma into a subvolcanic storage system, i.e., $\mathbf{F}_m = \nabla \phi_2$ is the body force per unit mass. Thus, the body force generated by the mass of the intrusion, $-\rho_0 \nabla \phi_2$, is implicit in the self-gravitation term $-\rho_0 \nabla \phi$ of equation (1). The method of solution is to solve the homogeneous version of the equations (1) and (2). The sources are then inserted at an artificially introduced boundary as surface tractions. The boundary conditions are that all perturbation quantities go to zero at infinity. We also need to require the existence of an external gravitational potential to which the internal potential must smoothly join. Furthermore, we demand the continuity of all fields with respect to z , except at singular

planes, $z = \text{constant}$, corresponding to the location of the internal sources. The surface, $z = 0$, is assumed to be stress-free.

RUNDLE (1980) solved the equations (1) and (2) using the propagator matrix technique (THOMPSON, 1950; HASKELL, 1953) for a layered half space to obtain surface gravity change, deformation and potential change arising from volcanic loading. RUNDLE (1981) developed the numerical formulation for the case of a single layer welded with a half space. In subsequent work, the elastic-gravitational approach was refined (e.g., FERNÁNDEZ and RUNDLE, 1994; FERNÁNDEZ *et al.*, 1997; CHARCO *et al.*, 2002).

Mass, Coupling and Topographic Interaction

BATTAGLIA and SEGALL (2004) and CHARCO *et al.* (2006) performed a dimensional analysis and scaling of the elastic-gravitational problem in order to gain insight into the approximations and solutions without actually attempting a formal solution. From dimensionalized parameters and using typical parameter values for volcanic areas, they found that terms depending on G and g in the equations (1) and (2) are negligible in the computation of the displacement field.

As detailed above, the elastic-gravitational model takes into account a pressurized cavity with no mass change (dilatational source, $\mathbf{F}_p$) and a mass intrusion with no magma chamber overpressure (mass source, $\mathbf{F}_m$). We perform a theoretical experiment to examine source effects on both displacements and changes in gravity. Figure 1 shows vertical displacements and surface gravity changes caused by the superposition of both types of sources located at 4-km depth in a homogeneous medium with Lamé parameters $\lambda = \mu = 30$ GPa and density $\rho = 3000$ kg m⁻³, i.e., Poisson's ratio $\nu = 0.25$. For the dilatational source, we consider a pressure increment of 10 MPa and a radius of 1 km, while we consider 10^{12} kg for the mass source. Maximum vertical displacement is of millimeter order, while the maximum surface gravity change reaches a magnitude of around 420 μGal .

RUNDLE (1978) and WALSH and RICE (1979) pointed out that the surface gravity change due to a spherical source of dilatation within an elastic half space is the free-air effect due to uplift. OKUBO (1997) considered the effect of the gravitational attraction of material which intrudes into the dilatational cavities as well as the uplift of the observation point. Thus, surface gravity change, g_s , is given by

$$g_s = \gamma_{FA} u_z + \frac{GMc}{R^3}, \quad (4)$$

where γ_{FA} is the free-air gradient ($= 0.3086$ mGal m⁻¹), u_z is the vertical displacement, and R is the radial distance to the source located at (s_1, s_2, c) . BATTAGLIA and SEGALL (2004) pointed out that the contribution of the intrusion mass is almost zero compared to the pressurization contribution in the displacement field. However,

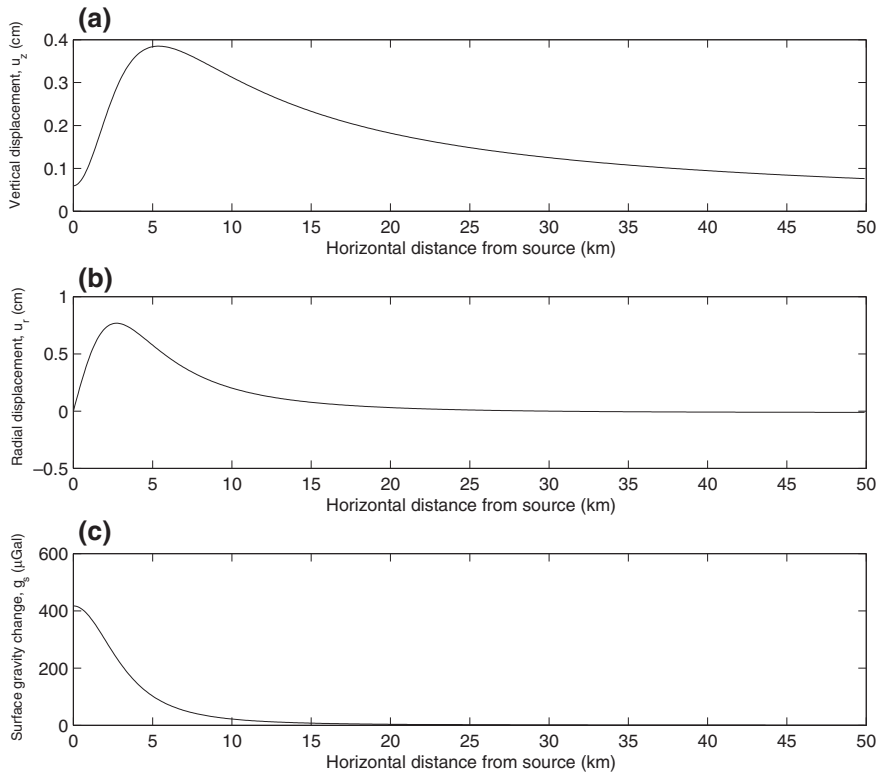


Figure 1

(a) Vertical displacement (cm) due to spherical mass intrusion of 10^{12} kg (radius 1 km) and pressure change of 10 MPa located at 4-km depth in a homogeneous elastic-gravitational medium with $\lambda = \mu = 30$ GPa. (b) and (c) radial displacement (cm) and surface gravity change (μ Gal) for the same source.

Figure 1 demonstrates that with two sources of loading, a point mass and a pressurized cavity, it is possible to obtain changes in gravity without any significant deformation, since total uplift may vanish without total gravity changes vanishing when the mass plays the principal role in modelling, as can be seen from equation (4). Therefore, we cannot neglect the mass source in this modelling approach.

Figure 2 displays the displacement and surface gravity changes produced by a mass point of $0.17 \cdot 10^{12}$ kg located at 4-km depth and the effect caused by a center of dilatation of 150 MPa km^3 strength at the same depth. Pressurization of the magma chamber is responsible for the volcanic edifice inflation (Fig. 2a, u_z) while mass injection causes ground subsidence (Fig. 2b, u_z). Gravity decreases because of inflation at constant mass (Fig. 2a, g_s) and increases because of mass addition at constant volume (Fig. 2b, g_s). In this example, the mass source and dilatational source produce significant gravity changes. In fact, predicted gravity changes caused by both sources are quantitatively and qualitatively similar, i.e., the upward moving

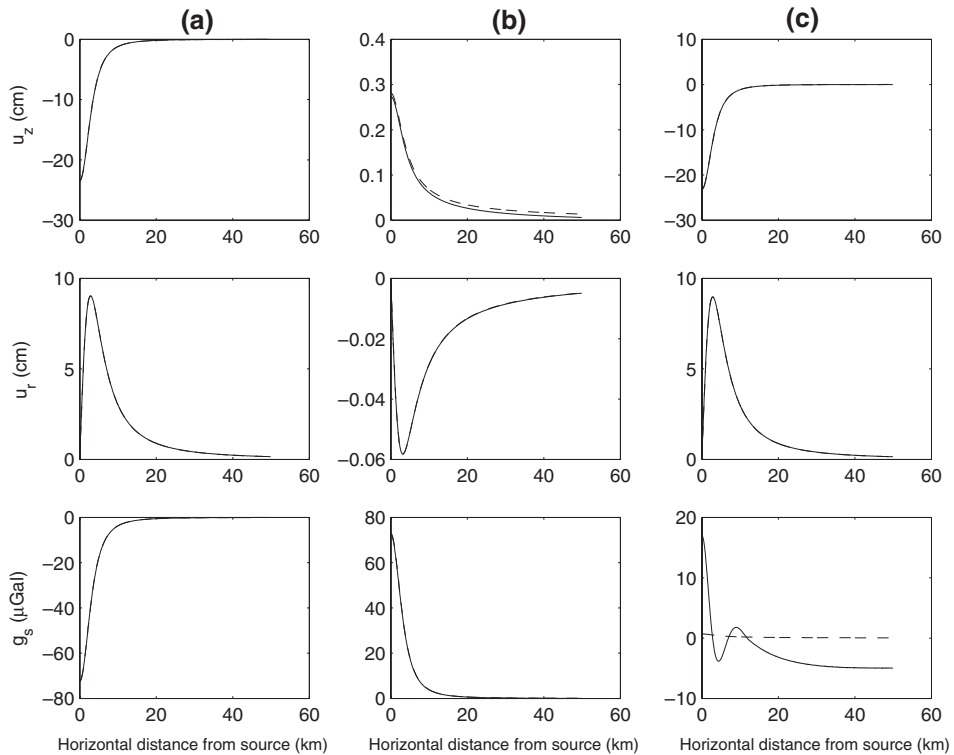


Figure 2

Displacements (cm) and surface gravity change (μGal) caused by (a) a center of dilatation with 150 MPa km^3 strength, (b) a spherical mass source of 0.17 MU and (c) the superposition of both sources in elastic (dashed line) and elastic-gravitational (solid line) homogeneous medium with $\lambda = \mu = 30 \text{ GPa}$.

surface compensates for the gravity changes due to increase of mass in a purely elastic medium (Fig. 2c, g_s , dashed line). However, the superposition of both sources in the elastic-gravitational medium results in self-gravitation effects. The coupling effect due to perturbed potential slightly reduces the vertical displacement caused by the gravitational force acting on the material above the source (Fig. 2b, u_z , solid line). This effect is shown at horizontal distance of the order of source depth in the surface gravity change (Fig. 2c, g_s , solid line). The error caused by neglecting terms depending on g and G in equation (1) is spread through the gravity change calculations because the model equations, (1) and (2), are coupled. In this example, predicted gravity signal is larger than the microgravity accuracy that is attainable presently (e.g., RYMER, 1996). Therefore, terms depending on g and G may contribute to the magnitude of the gravity changes although they are negligible in computing ground displacements.

Data interpretation using elastic-gravitational models assumes that magma intrusion is embedded in a flat half space. However, there are many volcanoes with

prominent relief. In the following, we propose qualitative arguments to assess the influence of topography as well as self-gravitational coupling when modelling displacements and gravity changes.

Elastic and gravitational forces in the elastic gravitational model depend on source depth. In the equation (1), g dependent terms scale as $g\rho_0|\mathbf{u}|/c$, whereas the G dependent term scales as $4\pi G\rho_0^2|\mathbf{u}|$. $|\mathbf{u}|$ is the characteristic length scale for the displacement field and c is the depth of the source, characteristic length scale for the half-space. The elastic stress tensor, σ , scales as $\mu|\mathbf{u}|/c$ and its gradient as $\mu|\mathbf{u}|/c^2$. Therefore, the effect of gravitational forces on displacement depends on source depth in such a way that it can become more significant than elastic forces effect for deep sources (CHARCO *et al.*, 2006). The magnitude of ϕ_1 depends on $|\mathbf{u}|/c$ and the magnitude of ϕ_2 depends on $a^2\Delta a/c^3$, where $a^2\Delta a$ is related to the volume change of the magma chamber. CHARCO *et al.* (2007a) used these scaling relations to describe topographic effects. They assumed that the topographic effect is primarily due to differences in the distance between surface and source location. Thus, the higher the topography the deeper the source relative to the observational point. Topography was included in the scaling analysis by replacing source depth by c' , where $c' = c + f(r, \theta)$. The term f , which is a function of radial distance, r , and azimuth angle, θ , is the elevation above mean sea level. Considering mean heights above mean sea level of 5 km, c and c' are of the same order of magnitude. Thus, terms depending on G and g are still negligible in the computation of displacements when topographic relief is considered.

To investigate the relative importance of topography and self-gravitation in the interpretation of gravity changes we distinguish between sources located between 1 and 10 km depth ($c/c' \ll 1$), and sources located at $c > 10$ km, i.e., ($c/c' \simeq 1$). For sources located between 1 and 10-km depth, the potential ϕ_2 is of prime importance (CHARCO *et al.*, 2007a). Thus, the coupling effect can be neglected relative to the mass effect. However, topography interacts with both potentials causing significant effects on gravity changes (Fig. 3). For a deeper source, $c > 10$ km, the topographic effect is not significant (Fig. 4). Nevertheless, in this case the magnitude of gravity changes is not significant in comparison to the accuracy of microgravity measurements. Therefore, topography can mask self-gravitation effects particularly for magma reservoirs at relatively shallow depths that could produce coupled signals inside the accuracy attainable today in microgravity surveys. In this way, when topographic relief takes part in modelling, we can reduce equation (1) to the vector static Navier equation:

$$\nabla \cdot \sigma - \rho_0 \nabla \phi_2 + \mathbf{F}_p = \nabla^2 \mathbf{u} + \frac{1}{1-2\nu} \nabla \nabla \cdot \mathbf{u} - \rho_0 \nabla \phi_2 + \rho_0 \mathbf{F}_p = 0. \quad (5)$$

Topographic contributions to gravitational potential gradient are numerically implemented in the equation (4) through the change in vertical displacement due to inflation/contraction and the change in vertical displacement due to the deformation

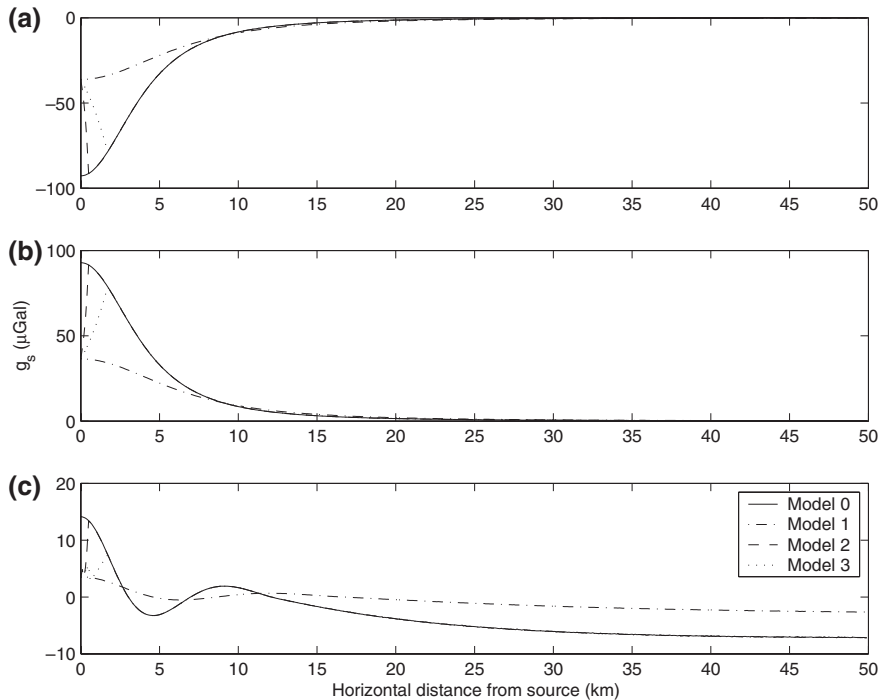


Figure 3

Source located at $c = 5$ km depth in a homogeneous medium with $\lambda = \mu = 30$ GPa. (a) Surface gravity change (μGal) due to a center of dilatation of 300 MPa km^3 strength. (b) Surface gravity change (μGal) for a spherical mass source of 0.35 MU . (c) Surface gravity change caused by the superposition of both sources. As approximation we assume that Model 0 represents a flat homogeneous half space; Model 1 represents a fixed height of 3 km for the top of the half space above the reference system origin, i.e., source is located at $c = 8$ km; Model 2 represents an axisymmetric volcano with average slope of its flanks of 25° and height of 3 km; Model 3 represents an axisymmetric volcano with average slope of its flanks of 60° and height of 3 km. The results for models 2 and 3 are computed considering an analytical solution in which the depth of the source varies with elevation.

induced by a point mass load within the medium. The topographic effect on gravity changes caused by the change in mass can be represented by varying the source location with the point elevation, $f(r, \theta)$, since gravitational attraction primarily depends on distance between the point at which the solution is desired and the gravitational source (mass distribution).

Effects of Topography on Surface Deformation and Gravity Changes

We now perform numerical experiments that examine the effects of topography on ground deformation and gravity changes created at the surface of prominent relief. The modelling method used for our study is the three-dimensional Indirect Boundary

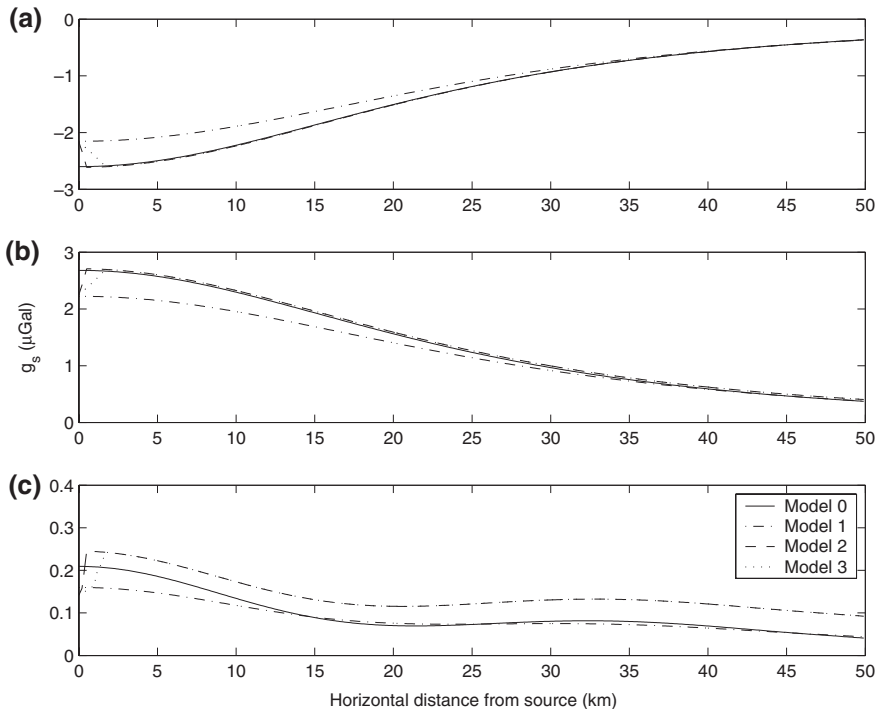


Figure 4

The same as Figure 3 but the source is located at 30-km depth.

Element Method. In general, Boundary Element Methods are based on the approximation of the boundaries of a body by a finite number of elements and on the use of known analytical solutions to singular boundary value problems. In this work, we use a single-layer boundary integral representation for the displacement field perturbed by topographic relief. This method can be classified as the Indirect Boundary Element Method because the problem is formulated in terms of boundary force densities instead of the boundary values of displacements and/or tractions as in the Direct Boundary Element Methods. The integral representation used in our technique is derived from Betti's reciprocal theorem and the solution to Kelvin's problem of a point load in an infinite body (see CHARCO *et al.*, 2007b for more details).

We consider surface displacements and gravity changes due to a pressurized cavity with no mass change and due to mass intrusion with no magma chamber overpressure located beneath axisymmetric volcanoes. We use a homogeneous medium in the present study in order to focus on topographic effects. The effect of topography is represented by a volcano of altitude H and average slope, α , of the flanks of 0° , 15° , 20° or 30° (Fig. 5). The height of the volcanoes is $H = 0$, 1340, 1820 or 2886 m, respectively, in order to keep the basal radius constant at 5 km. The plane

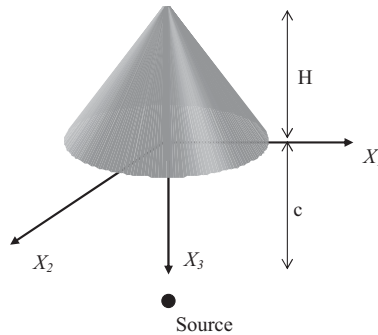


Figure 5

Schematic illustration of the problem. The height of the volcano is H and c is the depth of the volcanic source.

half space is represented by the Lamé parameters $\lambda = \mu = 30$ GPa, i.e., Poisson's ratio is 0.25. The topographic relief is characterized by the same set of parameters.

Figure 6 displays displacement and gravity changes caused by a spherical pressurized point source and a point mass source located at 4-km depth. The source represents a pressure increase of 10 MPa and a mass increment of 10^{12} kg. The results of changing surface slope, α , are compared with the flat solution ($\alpha = 0^\circ$, $H = 0$ m). Topography has a significant effect on the magnitude of the predicted displacements and gravity changes. Vertical displacements and gravity changes considerably decrease in magnitude close to the summit of the volcano. Including the topography effect decreases the ground subsidence caused by the mass source (Fig 6(b)). Therefore, the addition of mass cancels some of the inflation caused by the pressure source, but by less than if the topographic effect is neglected (Fig 6(c)). For gravity changes we can notice the inverse phenomena: Gravity reduction due to volcano inflation is smaller when topographic relief is considered. The reduction of the magnitude of the radial displacement is smaller than for the vertical displacement. The reduction of the magnitude implies that the flat half space solution gives a large overestimation/underestimation (depending on the source) of changes in gravity and deformation.

CAYOL and CORNET (1998) pointed out that the interpretation of surface displacements can lead to erroneous estimations. They found the steeper the volcano the flatter the vertical displacement field. This result can be extrapolated to gravity changes as shown in Figure 6. Displacement and gravity changes vary in a narrow region with a half width similar in magnitude to source depth (RUNDLE, 1981). Topography has a small but noticeable effect in changing the pattern of displacements and gravity changes in this region. The curves corresponding to vertical displacement and gravity changes are slightly flatter in the proximity of the symmetry axis. The location of the radial displacement maximum/minimum value varies depending on the slope model. Vertical displacements and gravity changes caused by

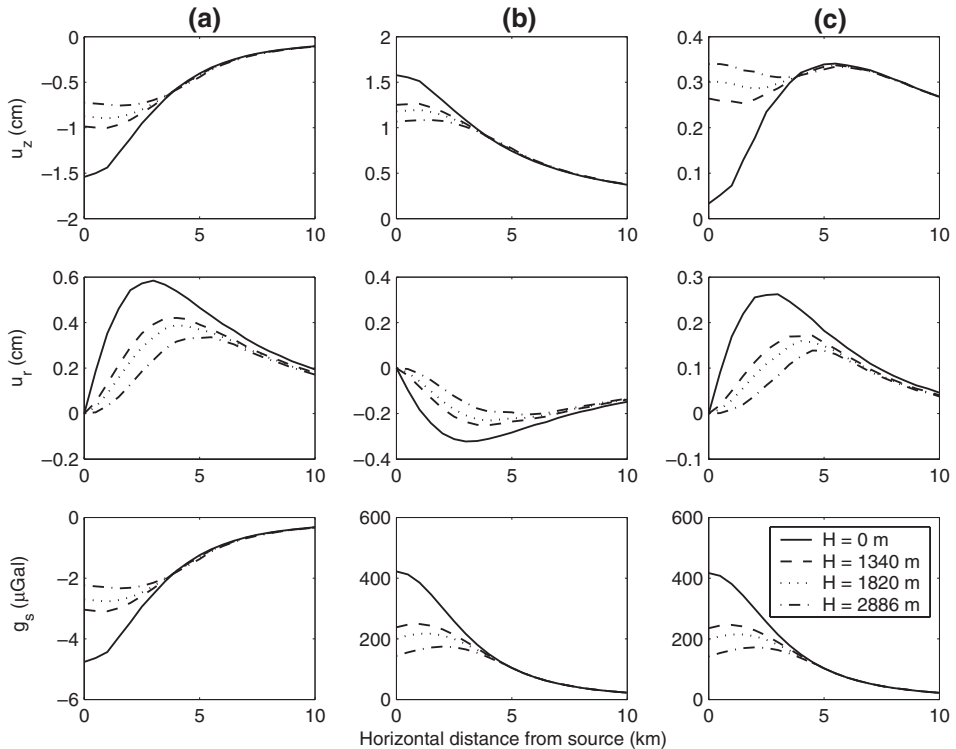


Figure 6

Vertical (u_v) and radial (u_r) components of displacement field and surface gravity change (g_s) caused by (a) a center of dilatation of 10 MPa km³ strength, (b) a point of mass of 1 MU and (c) superposition of both sources. H is the height of the axisymmetric volcano shown in Figure 5. The source is located at 4-km depth.

a center of dilatation vanish faster at large horizontal distances ($r > c$) when topography is neglected.

Variations in the magnitude and pattern of surface deformation and gravity changes indicate that neglecting topography could significantly affect data interpretation in areas of prominent relief.

Summary and Conclusions

We investigate elastic-gravitational model in order to gain insight into the relative importance of elasticity, self-gravitation and topography. We show that while displacements are mainly caused by overpressure, we cannot neglect the mass intrusion effect in the modelling of gravity changes. With two sources of loading, a point mass and a pressurized cavity, the elastic-gravitational model allows for the interpretation of changes in gravity without any significant deformation or *vice versa*.

BATTAGLIA and SEGALL (2004) and CHARCO *et al.* (2006) showed that coupling between gravity and elasticity is negligible for deformations with characteristic length scales of 1–10 km. However, as μ decreases in magnitude the effects of gravity become relatively more important. RUNDLE (1981), WOLF (1991) and AMELUNG and WOLF (1994) pointed out that this particular property implies that, for time-dependent displacements which are a result of a source embedded in a viscoelastic medium, the effects of gravity will become important at sufficiently long times, i.e., when μ has relaxed to some small value. Furthermore, the coupling cannot be ignored in flat half-space models when the mass source term is present in the modelling (Fig. 2). The coupling effect is seen in the gravity change calculations at horizontal distances from the source on the order of the source depth when the mass and dilatational sources have the same weight in expression (4). Coupling has a significant effect on the magnitude of the predicted gravity changes, particularly for magma chambers at relatively shallow depths, as the mass effect (ϕ_2) decreases faster than the dilatation effect (ϕ_1) with depth.

Topographic relief is another factor to take into account for interpreting ground deformation and gravity changes. In this work, we provide insights into the relative importance of topography and self-gravitation. It is well known that deformation and gravity curves depend on source depths. WILLIAMS and WADGE (1998) pointed out that topography has a significant effect on predicted displacements, particularly for magma chambers at relatively shallow depths. This result can be generalized to changes in gravity. Topography reduces the gravity magnitude as source depth increases with relief elevation and modifies the width of the curves that depend on source depth (Figs. 3 and 4). In this way, the magnitude and pattern of changes in gravity not only depend on c . The relief elevation also has a significant effect.

For sources located at 1–10-km depth ($c/c' \ll 1$) the potential due to the intruded mass dominates the equation (2). While topography has a significant effect on the magnitude of the mass interaction with the gravity field for magma reservoirs at relatively shallow depths ($c/c' \ll 1$), the coupling effect is too small to be distinguished from errors associated with gravity change data (RYMER, 1996) for deeper sources ($c/c' \simeq 1$). Therefore, in areas of prominent relief, the neglect of topography in the estimation of source parameters may cause larger errors than that of neglecting the elastic-gravitational coupling. Consequently, the use of the simplified system formed by equations (2)–(5) is sufficient for modelling, if topography is considered.

Finally, we investigate numerically the effects of 3-D topographies on ground deformation and gravity changes. We show that topography has a significant effect on the deformation and gravity changes predicted by point sources. The main effect is a reduction in magnitude of the displacements and gravity changes in regions of higher relief. This result corresponds to that obtained by WILLIAMS and WADGE (1998, 2000) and CAYOL and CORNET (1998) for ground deformation alone. The second striking feature of the topographic effect is that, as in the case of displacements, gravity changes also display a change in pattern. Therefore, topography influence introduces

some variations in theoretical modelling. While the shape of the displacements and gravity curves is controlled by the depth of the source and their magnitude by mass and pressure, we demonstrate in this study that a joint interpretation of displacements and gravity data may be improved if topographic relief is considered. Finally, we should emphasize that it remains to be seen whether this study may be extended to nonspherical sources or those that do not have axial symmetry since displacements and gravity change depend on source geometry.

Acknowledgments

We wish to thank Volker Klemann and Alessandro Bonforte for their thorough and constructive reviews and comments. The research of MC and JF was supported by MCyT projects REN2002-03450 and CGL2005-05500-C02. The research of FL was supported by MCyT projectS REN2002-04198-C02-02/RIES, CGL2005-05500-C02 and by the European community with FEDER funds, the research team RNM-194 of Junta de Andalucía, Spain. The work of KFT was performed under an NSERC Discovery Grant. The research of JBR was supported by US DOE grant DE-FG03-95ER14499.

REFERENCES

- AMELUNG, F. and WOLF, D. (1994), *Viscoelastic perturbations of the Earth: significance of the incremental gravitational force in models of glacial isostasy*, *Geophys. J. Int.* 117, 864–879.
- BATTAGLIA, M. and SEGALL, P. (2004), *The interpretation of gravity changes and crustal deformation in active volcanic areas*, *Pure Appl. Geophys.* 23(3), 231–247.
- BIANCHI, R., CORADINI, A., FEDERICO, GIBERTI, G., LANCIANO, P., POZZI, J., SARTORIS, G., and SCRADRONE, R. (1987), *Modeling of surface deformation in volcanic areas: The 1970–1972 and 1982–1984 crises at Campi Flegrei*, *J. Geophys. Res.* 92, 14139–14150.
- CAYOL, V. and CORNET, F. (1998), *Effects of topography on the interpretation of the deformation field of prominent volcanoes—Application to Etna*, *Geophys. Res. Lett.* 25(11), 1979–1982.
- CHARCO, M., FERNÁNDEZ, J., LUZÓN, F., and RUNDLE, J. B. (2006), *On the relative importance of self-gravitation and elasticity in modeling volcanic ground deformation and gravity changes*, *J. Geophys. Res.* 111, B03404, doi: 10.1029/2005JB003754.
- CHARCO, M., FERNÁNDEZ, J., SEVILLA, J. B., and RUNDLE, J. B. (2002), *Modeling magmatic intrusion's effects on geoid and vertical deflection. Application to Lanzarote, Canary Islands, and Long Valley Caldera, California*, *Física de la Tierra* 14, 11–31.
- CHARCO, M., LUZÓN, F., FERNÁNDEZ, J., TIAMPO, K. F. (2007a), *Topography and self-gravitation interaction in elastic-gravitational modeling*, *Geochem. Geophys. Geos.* 8(1), Q01001, doi: 10.1029/2006 GC001412.
- CHARCO, M., LUZÓN, F., FERNÁNDEZ, J., TIAMPO, K. F., and SÁNCHEZ-SESMA, F. J. (2007b), *3D Indirect Boundary Element Method for deformation and gravity changes in volcanic areas: Application to Teide volcano*, (Tenerife Canary Islands), *J. Geophys. Res.* (in press).
- DVORAK, J. and DZURISIN, D. (1997), *Volcano geodesy: The search for magma reservoirs and the formation of eruptive vents*, *Rev. Geophys.* 35, 343–384.
- FERNÁNDEZ, J. and RUNDLE, J. B. (1994), *Gravity changes and deformation due to a magmatic intrusion in a two-layered crustal model*, *J. Geophys. Res.* 99, 2737–2746.

- FERNÁNDEZ, J., RUNDLE, J. B., GRANELL, R., and YU, T. T. (1997), *Programs to compute deformation due to magma intrusion in elastic-gravitational layered earth models*, *Comp. Geosc.* 23(3), 231–247.
- FOLCH, A., FERNÁNDEZ, J., RUNDLE, J. B., and MARTI, J. (2000), *Ground deformation in viscoelastic medium composed of a layer overlying a half-space. A comparison between point and extended sources*, *Geophys. J. Int.* 140, 37–50.
- GUDMUNDSSON, A. (1986), *Possible effect of aspect ratios of magma chambers on eruption frequency*, *Geology* 14, 991–994.
- HASKELL, N. (1953), *The dispersion of surface waves on multilayered media*, *Bull. Seismol. Soc. Am.* 43, 17–34.
- KISSLINGER, C. (1975), *Processes during the Matsushiro, Japan, earthquake swarm as revealed by leveling, gravity, and spring-flow observations*, *Geology* 3, 57–62.
- LOVE, A. E. H., *Some problems in Geodynamics*, Oxford Applied Mathematics and Computing Science Series, 180 pp. (Cambridge University Press, New York 1911).
- LUNGARINI, L., TROISE, C., MEO, M., and DE NATALE, G. (2005), *Finite element modelling of topographic effects on elastic ground deformation at Mt. Etna*, *J. Volcanol. Geotherm. Res.*, in press.
- MCTIGUE, D. F. and SEGALL, P. (1988), *Displacements and tilts from dip-slip faults and magma chambers beneath irregular surface topography*, *Geophys. Res. Lett.* 15, 601–604.
- MCTIGUE, D. F. and STEIN, D. (1984), *Topographic amplification of tectonic displacements: Implications for geodetic measurement of strain changes*, *J. Geophys. Res.* 89, 1123–1131.
- MOGI, K. (1958), *Relations of eruptions of various volcanoes and deformation of the ground surface around them*, *Bull. Earthquake Res.* 36, 99–134.
- OKUBO, S. (1997), *Potential and gravity changes raised by point dislocations*, *Geophys. J. Int.* 105, 573–586.
- RUNDLE, J. B. (1978), *Gravity changes and the Palmdale uplift*, *Geophys. Res. Lett.* 5, 41–44.
- RUNDLE, J. B. (1980), *Static elastic-gravitational deformation of a layered half space by point couple-sources*, *J. Geophys. Res.* 85(B10), 5355–5363.
- RUNDLE, J. B. (1981), *Numerical evaluation of static elastic-gravitational deformation in a layered half space by point coupled sources*, Internal Rep. SAND81-2058, Sandia Nat. Lab., Albuquerque, New Mexico.
- RYMER, H. (1996), *Microgravity monitoring*. In *Monitoring and Mitigation of Volcanic Hazards* (Springer-Verlag, Berlin Heidelberg 1996), 169–198 pp.
- THOMPSON, W. T. (1950), *Transmission of elastic waves through stratified medium*, *J. Appl. Phys.* 21, 89–93.
- TRASATTI, E., GIUNCHI, C., and BONAFEDE, M. (2003), *Effects of topography and rheological layering on ground deformation in volcanic regions*, *J. Volcanol. Geotherm. Res.* 122, 89–110.
- WALSH, J. B. and RICE, J. R. (1979), *Local changes in gravity resulting from deformation*, *J. Geophys. Res.* 84(B1), 165–170.
- WILLIAMS, C. and WADGE, G. (1998), *The effects of topography on magma chamber deformation models: Application to Mt. Etna and radar interferometry*, *Geophys. Res. Lett.* 25(10), 1549–1552.
- WILLIAMS, C. and WADGE, G. (2000), *An accurate and efficient method for including the effects of topography in three-dimensional elastic models of ground deformation with application to radar interferometry*, *J. Geophys. Res.* 105(B4), 8103–8120.
- WOLF, D. (2000), *Boussinesq's problem of viscoelasticity*, *Terra Nova* 3, 401–407.

(Received July 4, 2005, revised December 12, 2005, accepted February 15, 2006)