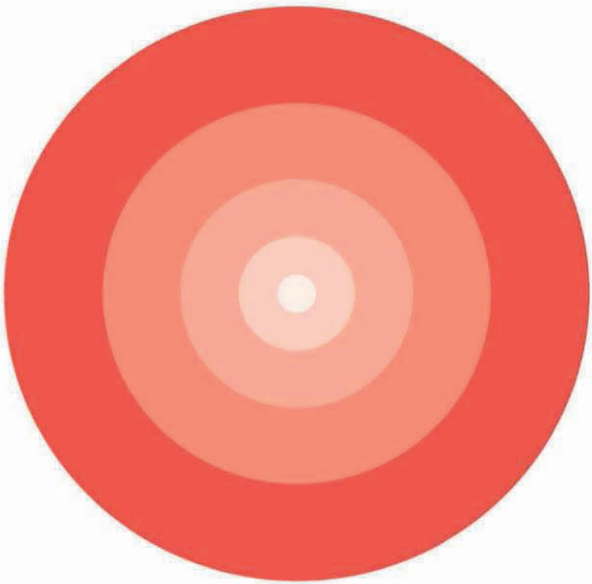
Isodual Theory of Antimatter

with applications to Antigravity, Grand Unification and Cosmology

by

Ruggero Maria Santilli

Springer



Fundamental Theories of Physics

Isodual Theory of Antimatter

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with applications to Antigravity, Grand Unification and Cosmology

by

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This monograph is dedicated to

Prof. M. C. Duffy,

University of Sunderland, England, because of his long commitment to true scientific democracy for qualified inquiries.

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Preface

After being conjectured by A. Schuster in 1898 (for historical aspects on antimatter see, e.g., R. L. Forward [1]), antimatter was predicted by P. A. M. Dirac in the late 1920s in the negative–energy solutions of his celebrated equation [2]. The existence of antimatter was subsequently confirmed via the Wilson chamber and became an established part of science (for a technical review prior to the studies presented in this monograph, see, e.g., M. M. Nieto and T. Goldman [3], and for more recent studies including those of this monograph see, e.g., proceedings [4]).

Dirac soon discovered that particles with negative energy do not behave in a physically acceptable way and, for this reason, he developed his celebrated "hole theory" [2]. This occurrence restricted the study of antimatter to the sole level of second quantization. As a result, antimatter created a scientific imbalance that lasted for the rest of the 20-th century, because matter was treated at *all* levels of study, from Newtonian mechanics to second quantization, while antimatter was solely treated at the level of second quantization.

Studies reviewed in this monograph have shown that the imbalance was not due to insufficient physical insights, but rather to *insufficient mathematics*. Stated differently, the use of conventional mathematics used for matter (such as conventional numbers, conventional spaces, conventional functional analysis, etc.) simply cannot permit a *classical* formulation of antimatter compatible with its quantum description.

The latter occurrence mandated the search of a new mathematics specifically conceived for the resolution of the scientific imbalance created by antimatter. After numerous efforts, the imbalance was finally resolved by a new mathematics today known as *Santilli's isodual mathematics* [5].

The availability of the new mathematics permitted the construction of the *isodual classical mechanics* [6], the new *isodual quantization* and

the resulting *isodual quantum mechanics* [7], the latter resulting to be equivalent, although not identical, to the conventional quantum treatment of antimatter via charge conjugation, thus ensuring compatibility of the isodual theory of antimatter with available experimental evidence.

This monograph is devoted to the resolution of the indicated scientific imbalance caused by antimatter in the 20-th century as permitted by the isodual mathematics and its consequential classical and operator theories. A main scope of the monograph is to show that, rather than being at its final stage, our classical, quantum and cosmological knowledge of antimatter is at its beginning with so much yet to be discovered.

Above all, a primary objective of this monograph is to show that a commitment to antimatter by experimentalists deeper than that granted until now can advance science beyond our imagination, with possible implications such as: experimental detection of antigravity by antimatter in the field of matter (or vice-versa); a consequential fully causal space-time machine; new cosmological vistas of the universe; and other far reaching advances.

[1] Forward, R. L. (1988). In *Antiprotons Science and Technology*, B. W. Augenstein, B. E. Bonner, F. E. Mills and M. M. Nieto, Editors, World Scientific, Singapore.

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Chapter 1

INTRODUCTION

1.1 THE SCIENTIFIC IMBALANCE CAUSED BY ANTIMATTER

1.1.1 Needs for a Classical Theory of Antimatter

The large scientific imbalance of the 20-th century studied in this monograph is that caused by the treatment of *matter* at all possible levels, from Newtonian to quantum mechanics, while *antimatter* was solely treated at the level of *second quantization* [1].

Besides an evident lack of scientific democracy in the treatment of matter and antimatter, the lack of a consistent *classical* treatment of antimatter left open a number of fundamental problems, such as the inability to study whether a faraway galaxy or quasar is made up of matter or of antimatter, because such a study requires first a classical representation of the gravitational field of antimatter, as an evident prerequisite for the quantum treatment (see Figure 1.1).

It should be indicated that classical studies of antimatter simply cannot be done by merely reversing the sign of the charge, because of inconsistencies due to the existence of only one quantization channel. In fact, the quantization of a classical antiparticle merely characterized by the reversed sign of the charge leads to a *particle* (rather than a charge conjugated antiparticle) with the wrong sign of the charge.

It then follows that the treatment of the gravitational field of suspected antimatter galaxies or quasars cannot be consistently done via the Riemannian geometries in which there is a simple change of the sign of the charge, as rather popularly done in the 20-th century, because such a treatment would be structurally inconsistent with the quantum formulation.

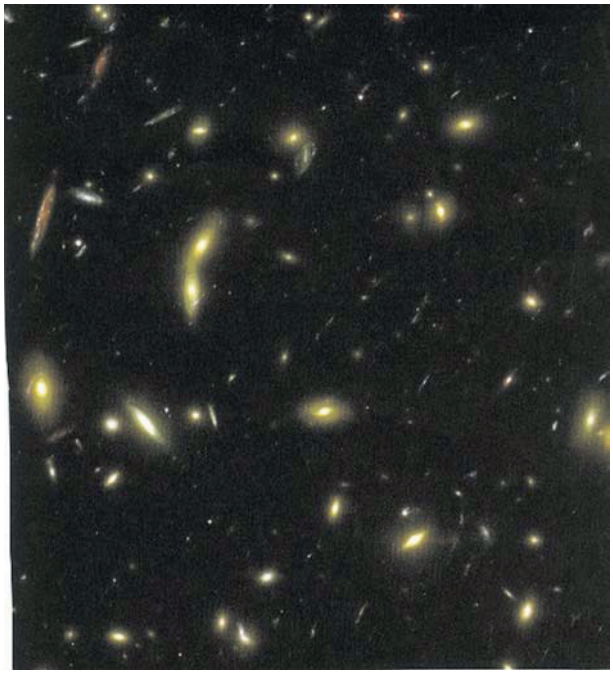

Figure 1.1. An illustration of the first major scientific imbalance of the 20-th century studied in this monograph, the inability to conduct classical quantitative studies as to whether faraway galaxies and quasars are made up of matter or of antimatter. In-depth studies have indicated that the imbalance was not due to insufficient physical information, but instead it was due to the lack of a mathematics permitting the classical treatment of antimatter in a form compatible with charge conjugation at the quantum level.

At any rate, the most interesting astrophysical bodies that can be made up of antimatter are *neutral.* In this case general relativity and its underlying Riemannian geometry can provide no difference at all between matter and antimatter stars with null total charge. The need for a suitable new theory of antimatter then becomes beyond reasonable doubt.

1.1.2 The Mathematical Origin of the Imbalance

The origin of the above scientific imbalance is not of physical nature, because it is due to the *lack of a mathematics suitable for the classical treatment of antimatter in such a way as to be compatible with charge conjugation at the quantum level.*

Charge conjugation is an *anti-homomorphism.* Therefore, a necessary condition for a mathematics to be suitable for the classical treatment of antimatter is that of being anti-homomorphic, or, better, anti-isomorphic to conventional mathematics.

Therefore, the classical treatment of antimatter requires *numbers, fields, functional analysis, differential calculus, topology, geometries, algebras, groups, symmetries, etc. that are anti-isomorphic to their conventional formulations for matter.*

The absence in the 20-th century of such a mathematics is soon established by the lack of a formulation of trigonometric, differential calculus and elementary transforms, let alone complex topological structures, that are anti-isomorphic to the conventional ones.

In the early 1980s, due to the absence of the needed mathematics, the author was left with no other alternative than its construction along the general guidelines of this monograph, namely, the construction of the needed mathematics from the physical reality of antimatter, rather than adapting antimatter to pre-existing insufficient mathematics.[1]

After considerable search, the needed new mathematics resulted in being characterized by the most elementary and, therefore, most fundamental possible assumption, that of a *negative basic unit*,

$$-1, \tag{1.1.1}$$

and then the reconstruction of the entire mathematics and physical theories of matter in such a way as to admit -1 as the correct left and right unit at all levels.

In fact, such a mathematics resulted in being anti-isomorphic to that representing matter, applicable at all levels of study, and resulting in being equivalent to charge conjugation after quantization.[2]

1.1.3 Basic Assumptions of Isodual Mathematics

The central idea of isodual mathematics and related theory of antimatter for the case of *point-like antiparticles* (see Section 1.3 for the treatment of extended antiparticles, such as antiprotons and antineutrons) is the *lifting of the conventional, trivial units $I = +1$ for matter into the negative-definite units $I^d = -1$* for all levels of treatments

$$I > 0 \to I^d = -I < 0, \tag{1.1.2}$$

while jointly changing the conventional product $A \times B$ among generic quantities A, B (such as numbers, vector fields, matrices, etc.) into the form[3]

$$A \times B \to A \times^d B = A \times (I^{d-1}) \times B = A \times (-I) \times B, \tag{1.1.3}$$

under which I^d is the correct left and right unit of the theory,

$$I^d \times^d A = A \times^d I^d = A, \tag{1.1.4}$$

for all elements A of the set considered.

For certain technical reasons reviewed in Chapter 2, I^d is called the *isodual unit*, and the new product $A \times^d B$ is called the *isodual multiplication*. The liftings $I \to I^d$ and $A \times B \to A \times^d B$ are called *isodual maps.*

As we shall see, despite its elementary character, the lifting of the conventional positive-definite units of matter into negative-definite forms implies a revision of the totality of mathematical and physical formulations, with far reaching physical implications, such as: antimatter evolves in time in a direction opposite to ours; negative energies measured with negative units behave in a fully physical way; motion backward in time measured with negative units of time is as causal as motion forward in time measured with positive units of time; etc.

The reader should keep in mind that, for consistency, the isodual maps must be applied to the totality of the mathematical formulations of the conventional theory of matter, including numbers, fields, spaces, geometries, algebras, symmetries, etc. This results in a new mathematics, today known as *Santilli's isodual mathematics*, that is at the foundation of the isodual theory of antimatter.

In particular, the application of map (1.1.2) such as

$$t \to t^d, \;\; x \to x^d, \;\; E \to E^d \tag{1.1.5}$$

generates new notions called *isodual time, isodual space and isodual energy.* Their nontriviality is illustrated by the fact that, while conventional time, space and energy are measured with respect to *positive units*, their isodual images are computed with respect to *negative units.* As we shall see, the latter occurrence is crucial for the resolution of existing inconsistencies for motion backward in time, negative energies and antigravity.

It should be noted that the correct formulation of Santilli's isodual map for an arbitrary quantity Q (e.g., a function, a matrix, an operator, etc.) with an arbitrary dependence is given by

$$Q(x, \phi, \ldots) \;\; \to \;\; Q^d(x^d, \phi^d, \ldots) \;\; = \;\; -Q^\dagger(-x^\dagger, -\phi^\dagger, \ldots). \tag{1.1.6}$$

of which map (1.1.2) is an evident particular case. As we shall see, the above broader form is crucial for the proof of the equivalence between isoduality and charge conjugation.

As recalled earlier, a main characteristic of charge conjugation is that of being an *anti–automorphic map* (i.e., characterizing a map of a given space onto itself). By comparison, that both liftings (1.1.2) and its general form (1.1.6) are *anti–isomorphic maps* (i.e., characterizing maps from one space onto a *different* space with the same dimension).

Note finally that, while charge conjugation solely applies to operator theories, isodual maps are applicable to all possible theories, irrespectively of whether classical or operator.

1.2 GUIDE TO THE MONOGRAPH

1.2.1 Consistency and Limitations of Special Relativity and Quantum Mechanics

As it is well known, *special relativity*[4] continues to dominate the advancement of scientific knowledge in virtually all sections.

Unfortunately, only a few researchers are aware of the fact that, while possessing an axiomatic structure called "majestic" by this author in his writing, special relativity also has very clear limitations, some of which are known to the scientific community at large (such as the inability to represent gravitation), but others remain generally unknown.

There cannot be a scientific process beyond that of academic religion without the identification of the majestic axiomatic structure of special relativity as well as, most importantly for this monograph, the identification of their boundaries of applicability beyond which special relativity is *inapplicable* and not "violated" because not conceived for the related applications.

Thanks to historical contributions by Lorentz, Poincaré, Einstein, Minkowski, Weyl and others conducted for over one century, we can now safely state that the majestic axiomatic consistency of special relativity is due to the following features:

1) Special relativity is formulated in the Minkowski spacetime $M(x,\eta,R)$ with local spacetime coordinates, metric, line element and and basic unit given respectively by

$$x = \{x^\mu\} = (r^k, t),\ \mu = 1,2,3,\ \ k = \mu = 1,2,3,0,\ \ c_0 = 1, \qquad (1.2.1a)$$

$$\eta = Diag.(1,1,1,-1) \qquad (1.2.1b)$$

$$(x-y)^2 = (x^\mu - y^\mu) \times \eta_{\mu\nu} \times (x^\nu - y^\nu); \qquad (1.2.1c)$$

$$I = Diag.(1,1,1,1,1), \qquad (1.2.1d)$$

over the field of real numbers R;

2) All laws of special relativity, beginning with the above line element, are *invariant* (rather than covariant) under the fundamental *Poincaré symmetry*

$$\mathcal{P}(3.1) = \mathcal{L}(3.1) \times \mathcal{T}(3.1) \tag{1.2.2}$$

where $\mathcal{L}(3.1)$ is the *Lorentz group* and $\mathcal{T}(3.1)$ is the *Abelian group of translations* in spacetime; and

3) The *Poincaré transformations are canonical* with implications crucial for physical consistency, such as the invariance of the assumed basic units (as per the very definition of a canonical transformation),

$$\begin{aligned} &\mathcal{P} \times [Diag.(1cm, 1cm, 1cm, 1sec)] \times \mathcal{P}^t \\ &\quad \equiv \; Diag.(1cm, 1cm, 1cm, 1sec), \end{aligned} \tag{1.2.3}$$

with the consequential fundamental property that *special relativity admits basic units and numerical predictions that are invariant in time.* In fact, the quantities characterizing the dynamical equations are the *Casimir invariants* of the Poincaré symmetry.

As a result of the above features, special relativity has been and can be confidently applied to experimental measurements because the units selected by the experimenter do not change in time, and the numerical predictions of the theory can be tested at any desired time under the same conditions without fear of internal axiomatic inconsistencies.

Thanks to these historical results, special relativity is indeed applicable to the area of its original conception, the classical and quantum representation of electromagnetic waves and point-particles when moving in vacuum.

Nevertheless, it should be stressed that, as it was the fate for Galileo's relativity and it is the fate for all theories at large, *special relativity has its own well defined limits of applicability.* Among the numerous limitations, those relevant for this monograph are the following:

I) The inability by special relativity to represent gravitation (as well as accelerations in general) because of the lack of curvature in the Minkowski space as well as numerous other dynamical reasons;

II) The inability by special relativity to provide a *classical* representation of *antimatter* in a way consistent with quantum formulations because the only possible classical representation of antimatter is that via the sole change of the sign of the charge that, under quantization, leads to a "particle", rather than the correct charge conjugated antiparticle, with the wrong sign of the charge;

III) The inability by special relativity to represent both, particles and antiparticles as they are in the physical reality, extended, generally

nonspherical and deformable since such a representation would imply the breaking of the central pillar of the theory, the rotational symmetry;

IV) The inability to provide a classical and/or quantum representation of the motion of extended particles and/or antiparticle within physical media, due to the impossibility of admitting resistive nonpotential forces and, in any case, the reduction of all objects to point-like constituents under which reduction all resistive forces disappear because resistive forces are only experienced by extended particles); and

V) The inability by special relativity to represent the now vast experimental evidence that, far from being a "universal constant", the speed of light is a local variable whose value depends on the characteristics of the medium in which it propagates and can be smaller or bigger than the speed of light in vacuum.

The scientific scene for quantum mechanics is essentially equivalent to that of special relativity. Quantum mechanics also possesses a majestic axiomatic consistency because it is an operator realization of Lie's theory on Hilbert spaces. The experimental verifications of quantum mechanics *in the arena of its original conception, the structure of the hydrogen atom*, are also beyond scientific doubt.

Despite these achievements of clearly historical proportions, quantum mechanics possesses clear limitations whose quantitative study has been vastly suppressed in the 20-th century by organized academic interests in the field.

As an illustration, the spectral lines of the hydrogen atom are represented with astonishing accuracy. Nevertheless, deviations from the predictions of the theory begin to appear for the spectral lines of the helium, to become truly embarrassing in value for the spectral lines of heavy atoms, such as the zirconium. In any case, after one century of attempts, quantum mechanics has been unable to represent the spectral emissions of the Sun.

In particle physics we have a similar occurrence. Quantum mechanics has extremely accurate predictions for the conditions of its original conception, point-like particles moving in vacuum under local-differential, potential interactions *only*, as it is the case for quantum electrodynamics.

However, when passing to broader physical conditions (see next section for an outline), the predictions of quantum mechanics depart from experimental data in a progressively growing way. These departures are usually accommodated by organized interests with the introduction of free *ad hoc* parameters of unknown physical origin or meaning, while in reality these parameters are a measure of the deviations of physical reality from the basic axioms of the theory, as we shall see.

As an illustration, the representation of the Bose-Einstein correlation with relativistic quantum mechanics requires the use of *four different arbitrary parameters* (called "chaoticity parameters"). In reality, the fit of the data requires the presence in the two-point correlation function of *off-diagonal elements* that are simply outside the quantum axiom of expectation value for an observable, thus diagonal, operator. The claim of the exact validity of relativistic quantum mechanics under these extreme deviations then reduces science to a mere academic religion for.

1.2.2 Outline of the Monograph

Recall that "science" requires rigorous mathematical treatments producing numerical values that are consistently confirmed by experiments. Along these lines, Section 1.3 is devoted to an in-depth study of Limitations II-V of special relativity, since that is a premise for any true scientific study beyond the level of academic politics.

Section 1.4 is devoted to the identification of structural inconsistencies of the Riemannian treatment of gravitation because they have a direct impact on the experimental verification of the isodual theory of antimatter, besides forcing the study of gravitation into a sort of religious belief rather than rigorously proved scientific setting.

Section 1.5 is devoted to a guide to the covering of quantum mechanics known as *hadronic mechanics* since the studies presented in this monograph, the isodual theory of antimatter, belong to and can be fully understood only within the broader setting of the covering hadronic mechanics.

Chapter 2 is devoted to the presentation of the new isodual mathematics with Eq. (1.1.1) as its fundamental left and right isodual unit. Chapter 2 then presents the classical isodual treatment of antimatter, beginning with a reformulation of Newton's equations and then passing to the needed analytic theory. Chapter 2 then presents the operator formulation known under the name of isodual quantum mechanics. Particular attention is devoted in Chapter 2 to the experimental verification of the isodual theory of antimatter at both the classical and operator levels.

Chapter 3 is devoted to the presentation of the isodual theory for extended particles and antiparticles in reversible, irreversible and multivalued conditions. This presentation is crucial for an understanding of the implications of the theory, such as the first known axiomatically consistent grand unification permitted by isoduality.

Following these necessary foundational studies, Chapter 4 is devoted to a comprehensive theoretical and experimental study of the prediction

of antigravity for antiparticles in the field of matter or vice-versa, as well as to the "spacetime machine" that is implied by the said antigravity.

Chapter 5 is devoted to the axiomatically consistent grand unification of electroweak and gravitational interactions permitted by isoduality, the novel isoselfdual cosmology with identically null total characteristics of the universe, and other far reaching implications.

1.2.3 Literature on Isoduality

The basic reference of this monograph remains Dirac's historical contributions [1,2]. In fact, isodual mathematics was discovered while conducting a deeper inspection of Dirac's celebrated equation and, more particularly, in the identification of the fact that the two-dimensional unit of the antiparticle component of Dirac's equation is indeed negative-definite.

To the best of our knowledge, isodual mathematics was first presented by Santilli in Ref. [5] of 1985 for the identification of all possible equivalence classes of the group of rotations, although without any reference to antimatter. The fundamental numbers of the new mathematics, today known as *Santilli's isodual numbers*, were studied in details for the first time in Ref. [6] of 1993.

The first treatment of antimatter via the isodual mathematics was presented in Ref. [7] of 1993, which established the equivalence at the operator level of isoduality and charge conjugation. The isoduality of the Minkowski spacetime, the Poincaré symmetry and special relativities were studied for the first time in Ref. [8] also of 1993.

Ref. [9] of 1994 presented the first far-reaching implications of the isodual theory of antimatter, the prediction of *antigravity* for antimatter in the gravitational field of matter, or vice-versa while resolving all known objections against antigravity (which, as we shall see, become inapplicable precisely in view of the new mathematics). Ref. [10] also of 1994 presented another far-reaching implication, the existence of a (mathematical) "causal spacetime machine" as a direct consequence of the possible existence of antigravity.

A systematic study of the new *isodual quantum mechanics*, including the new channel of *isodual quantization*, was presented in the second edition of monographs [11] of 1995. A systematic mathematical treatment of isodual mathematics can be found in memoir [12] of 1996, that also contains the first known formulation of Newton's equations for antimatter thanks to the new *isodual differential calculus*.[5] Ref. [13] of 1997 presented the first known proof that isodual antimatter emits a new photon, called the *isodual photon*, with experimentally detectable features different than those of the photon emitted by matter, thus per-

mitting the first known scientific (i.e., quantitative) study as to whether far-away galaxies and quasars are made up of matter or of antimatter.

Refs. [14] of 1997, [18] of 1999 and [19] of 2003 have established the crucial role of the isodual theory of antimatter for the achievement of an *axiomatically consistent grand unification* of electroweak and gravitational interactions. Ref. [15] of 1998 presented a rigorous study of the *isodual Minkowskian and Riemannian geometries.* Ref. [16] of 1998 presented the novel *isoselfdual cosmology* with identically null total physical characteristics of time, mass, energy, etc. Ref. [17] of 1999 presented the first known *classical theory of antimatter* developed in such a form as to be compatible with charge conjugation at the operator level. Volumes [11,22] of 1995 provided the first comprehensive presentation of the isodual theory of antimatter. Memoir [21] of 1996 provided an upgrade and memoir [22] of 2003 provided a further upgrade for the treatment of antimatter via isodual, isodual isotopic, isodual genotopic and isodual hyperstructural methods. This monograph provides an upgrade of the isodual theory of antimatter as of 2005 that is warranted in view of recent advances in isodual mathematics and physics.

The isodual theory of antimatter has also been independently studied by a number of scholars. The theoretician J. V. Kadeisvili presented in Ref. [23] of 1992 a study of the *isodual functional analysis* and in Ref. [24] of 1996 a study of the *Poincaré-Santilli isodual symmetry.* The experimentalist J. P. Mills, jr., proved in Ref. [25] of 1996 that the experimental test of the gravity of antiparticles in horizontal flight proposed by Santilli in Ref. [9] of 1994 is indeed feasible with current technologies and the experimental results will be resolutory. The thermodynamics of antimatter stars was first formulated by theoreticians J. Dunning Davies in Ref. [26] of 1999. A systematic study of *Santilli's isodual number theory* was done by the mathematician C.-X. Jiang in his monograph [27] of 2001. Monographs [28-31] by various authors provide reviews of the isodual mathematics and/or its treatment of antimatter.

The isodual theory of antimatter is deeply connected to a variety of pre-existing research. First, isodual particles emerge as possessing a *negative time* precisely along the historical conception by Stueckelberg, Feynman and others [1]. The equivalence of treatment between particles and antiparticles at all levels of study can also be seen in the Stueckelberg-Feynman path integral theory.

Similarly, the isodual theory of antimatter is deeply connected to various additional contributions, such as those by E. Majorana [32], J. A. McLennan [33], K. M. Case [34], D. V. Ahluwalia [35], V. V. Dvoeglazov [36] and others identified later on. In fact, the latter studies

admit an immediate and intriguing re-interpretation in terms of the iso-dual theory, by extending their applicability to the classical level too.

The author would gratefully appreciate the indication of additional contributions directly relevant for the content of this monograph, that is, formulations based on the assumption of a negative-definite basic unit.

1.3 THE SCIENTIFIC IMBALANCE CAUSED BY SPECIAL RELATIVITY AND QUANTUM MECHANICS FOR MATTER AND ANTIMATTER

1.3.1 Foundations of the Imbalance

The second large scientific imbalance of the 20-th century studied in this monograph is the abstraction of particles and antiparticles as dictated by special relativity and quantum mechanics, that of being point-like with consequential reduction of all admitted interactions as being of local-differential and potential type.

In the physical reality, particles are generally extended, nonspherical, deformable[6] and hyperdense as it is the case for protons and antiprotons, whose size is actually quite large for particle standards. Even electrons and positrons do have a *point-like charge*, but *point-like wavepackets do not exist in nature.*

With the understanding that point-like abstractions remain valid in a first approximation of nature, no serious advance in the study of both particles and antiparticles can be claimed without a quantitative representation of their actual, extended and deformable shapes as well as of their density.

The scientific imbalance here considered is first caused by the fact that, as expected to be known and admitted by experts to qualify as such, the representation of these features is beyond the capability of special relativity and quantum mechanics.

In fact, nonspherical shapes cause the loss of the basic symmetry of these theories, the rotational symmetry $O(3)$; special relativity and quantum mechanics are known to be incompatible with deformations; and the vast literature in hadron physics of the 20-th century contains no consideration at all of the *densities* of hadrons, as readers are encouraged to verify.[7]

As we shall see shortly, the very notions of shapes and densities are beyond the representational capability of a Lagrangian or a Hamiltonian, thus being structurally beyond the representational capabilities of special relativity and quantum mechanics.

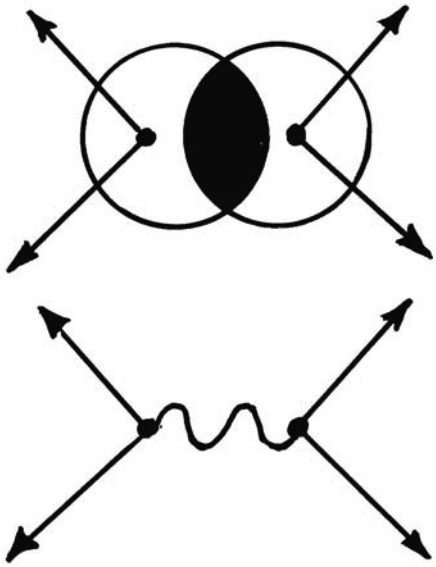

Figure 1.2. A first illustration of the second major scientific imbalance of the 20-th century studied in this monograph, the abstraction of extended and hyperdense particles, such as protons and antiprotons (as well as their constituents) to points, with consequential dismissal of the contact, nonlocal-integral and nonpotential effects caused by the deep overlapping and mutual penetration of hyperdense media. As we shall see, the implications for such an abstraction have been rather serious, such as the emergence of divergences that are eliminated by a quantitative representation of the indicated contact effects. More serious implications deal with the inability to predict new clean energies and fuels that, regrettably, we cannot treat in this monograph but are studied elsewhere [22].

The imbalance here considered is further caused by the fact that the assumption of the extended character of particles and antiparticles implies the necessary emergence of *contact interactions*, that is, interactions occurring within a finite volume or surface that, as such, are strictly *nonlocal-integral* (in the sense of not being reducible to a finite set of isolated points), are of *zero range* by conception and, thus, are not derivable from a potential (see Figure 1.2).

In turn, *the contact, nonlocal-integral and zero range interactions are structurally incompatible with special relativity and quantum mechanics*, because the interactions here considered cause the catastrophic collapse

of the mathematics underlying the said theories, let alone the inapplicability of their physical laws.

In fact, the local-differential topology, calculus, geometries, symmetries, and other mathematical methods underlying special relativity and quantum mechanics permit the sole consistent description of *a finite number of isolated point-like particles moving in vacuum (empty space).* Since points have no dimension and, consequently, cannot experience collisions or contact effects, the only possible interactions are at-a-distance, thus being derivable from a potential. The entire machinery of special relativity and quantum mechanics then follows.

For systems of particles at large mutual distances for which the size of the particle is ignorable, such as for the structure of the hydrogen atoms (anti-hydrogen atom), special relativity and quantum mechanics (their isoduals) are then *fully valid*, as we shall see in Chapter 2.

However, for all systems of particles at short mutual distances, such as the structure of hadrons, nuclei and stars, we have the inevitable emergence of contact, nonlocal-integral, nonpotential and zero-range interactions for which special relativity and quantum mechanics cannot be claimed to be exactly valid.

In Chapter 2 we show the structural inability of special relativity for a *classical* representation of antimatter in a form compatible with charge conjugation. In Chapter 3, we show the inability of special relativity to represent extended, nonspherical and deformable particles or antiparticles and/or their wavepackets under nonlocal-integral interactions at short distances.

The third scientific imbalance of the 20-th century studied in this monograph is the *treatment of irreversible processes for particles and antiparticles via strictly reversible mathematical and physical methods.* In fact, as it is well known, the very axioms of special relativity and quantum mechanics are strictly reversible in time. Consequently, these theories are indeed adequate for the study of reversible systems, such as planetary or atomic structures, but the same theories became manifestly insufficient for the representation of irreversible processes, such as the growth of a sea shell (see Figure 1.4).

It is evident that, under the indicated premises, the results will be compatible with the assumptions, thus implying a *de facto* reduction of irreversible processes to reversible abstractions. Rather than adapting nature to pre-existing mathematical and physical theories, in this monograph we shall do the opposite, that is, adapting mathematical and physical theories to nature.

1.3.2 Limitations of Special Relativity and Quantum Mechanics

A technical study of the scientific imbalances of the preceding section requires the knowledge of the following fundamental distinction:

DEFINITION 1.3.1: Dynamical systems of matter and, separately, of antimatter can be classified into:

IA: CLOSED SYSTEMS, comprising systems of particles isolated from the rest of the universe, thus verifying the known ten total conservation laws of Galilei's and Special relativities;

IB: OPEN SYSTEMS, comprising systems of particles that are not isolated from the rest of the universe, thus generally possessing nonconserved energy, linear momentum, angular momentum or other physical quantities;

IIA: EXTERIOR SYSTEMS, comprising particles at sufficiently large mutual distances to permit their point-like approximation under sole action-at-a-distance interactions;

IIB: INTERIOR SYSTEMS, comprising extended and deformable particles at mutual distances of the order of their size under action-at-a-distance interactions as well as contact nonpotential interactions;

IIIA: REVERSIBLE SYSTEMS, comprising systems for which the behaviour of their physical characteristics is invariant under time reversal;

IIIB: IRREVERSIBLE SYSTEMS, comprising systems whose behavior is not invariant under time reversal.

Typical examples of closed exterior systems are given by planetary and atomic structures of matter or antimatter when isolated from the rest of the universe. Typical examples of closed interior systems are given by the structure of planets at the classical level and by the structure of hadrons, nuclei, and stars of matter or antimatter at the operator level, when also isolated from the rest of the universe.

Planetary and atomic systems also constitute examples of reversible systems, while interior systems are generally irreversible even when closed. This is typically the case in the structure of a planet, such as Jupiter, at the classical level, where irreversibility is established by the internal

increase of entropy, or the structure of a star, such as the Sun, where irreversibility is established by its open character (due to the emission of energy, finite life and other reasons).

Note that open-nonconservative systems are generally irreversible, but the verification of the ten total conservation laws *does not* assure reversibility or internal conservation laws. For instance, when considered as isolated from the rest of the universe, Jupiter does verify indeed the ten total conservation laws of Galilei's or special relativities. Despite that, one can see in a telescope the existence in the interior of Jupiter of *vortices with continuously varying angular momenta*, and similar nonconservative effects. We merely have internal exchanges of angular momenta in such a way to verify the conservation of the total angular momentum.

It is now established that special relativity does apply for classical systems of point-particles in vacuum, that are closed, external and reversible, but it is inapplicable (rather than violated) for the same system when composed of antimatter because of inconsistencies in quantization indicated earlier (the map into particles, rather than antiparticles with the wrong sign of the charge). It is also established that quantum mechanics does apply for operator systems of point particles that are closed, external and reversible, but it solely applies for systems of antiparticles in second quantization.

It is important for the studies of this monograph to see from these introductory notes that special relativity and quantum mechanics *do not* apply for all remaining systems as per Definition 1.3.1. The inapplicability (rather than violation) of these theories for open-nonconservative systems is established by the following:

THEOREM 1.3.1 [22b] *A (relativistic or nonrelativistic) classical, open-nonconservative system cannot be consistently reduced to a finite number of quantum particles and, vice-versa, a finite ensemble of quantum particles cannot consistently produce a classical open-nonconservative system under the correspondence or other principles.*

Proof. Quantum systems are strictly closed, external and reversible, thus based on conservation laws. Consequently, there is no consistent possibility for quantum systems to produce classical nonconservation laws. The condition of a finite number of quantum particles is introduced to avoid divergencies under which things can be adjusted. **q.e.d.**

Note that the inapplicability of quantum mechanics at the operator level implies that of special relativity at both the operator and classical levels. Note also that Theorem 1.3.1 includes the inapplicability of spe-

cial relativity and quantum mechanics for interior systems, because the latter are based on nonconservative forces (see Chapter 3 for details).

The validity of special relativity and quantum mechanics for irreversible systems is prohibited by the following:

THEOREM 1.3.2 [22b]: *A classical, closed or open irreversible system cannot be consistently reduced to a finite number of quantum particles and, vice-versa, a finite ensemble of quantum particles cannot consistently produce an irreversible system under the correspondence or other principles.*

Proof. By recalling that all known, or otherwise physically established potentials are reversible in time, quantum particles are strictly reversible and, therefore, they cannot possibly represent an irreversible system. **q.e.d.**

The ultimate origin of the inapplicability of special relativity and quantum mechanics as per Theorems 1.3.1 and 1.3.2 is the classical existence of the contact, nonlocal, nonpotential and zero range interactions among extended particles indicated earlier. Therefore, the ultimate meaning of Theorems 1.3.1 and 1.3.2 is that of rendering mandatory the emergency of said contact, nonlocal and nonpotential forces at the ultimate elementary level of nature or, equivalently, of the necessity of representing elementary particles and their wavepackets as they are in nature, extended, generally nonspherical and deformable from which feature said novel interactions follows.

Because of the above occurrences, a main objective of Chapter 2 is the construction of *isodual special relativity and isodual quantum mechanics* to reach a consistent classical treatment of closed, external and reversible systems of antiparticles with a consistent operator map, as well as a consistent treatment of antiparticles at the level of first quantization.

In Chapter 3 we shall then study the so-called *isotopic and genotopic liftings of special relativity and quantum mechanics and their isoduals* for a consistent treatment of the remaining systems of matter and antimatter of Definition 1.3.1.

A few introductory comments appear recommendable for the self-sufficiency of these introductory lines. The distinction of systems into exterior and interior forms was introduced by the founders of analytic dynamics, such as Lagrange, Hamilton, Jacobi (see Ref. [54a] for historical accounts and references).[8]

Well written treatises on mechanics up to the early part of the 20-th century (see, e.g., Whittaker [56]) present the distinction between exterior and interior dynamical problems. The greatest majority of scientific

papers, also up to the first part of the 20-th century, treat separately exterior and interior problems.

For instance, Schwartzschild's [57] solution of gravitational field equations has been quoted countless times throughout the 20-th century, but without a mention that Ref. [57] was specifically written for the *exterior gravitational problem*, and that Schwartzschild wrote a perhaps more interesting separate paper [58] dedicated to the *interior gravitational problem*, that has remained largely ignored to this day.

The reasons for ignoring the above distinction are numerous, and have yet to be studied by historians. A first reason indicated earlier is due to the widespread abstraction of particles as being point-like, in which case all distinctions between interior and exterior systems are lost since all systems are reduced to point-particles moving in vacuum.

An additional reason for ignoring interior dynamical systems is due to the great successes of the planetary and atomic structures, thus suggesting the reduction of all structures in the universe to exterior conditions in vacuum.

In the author's view, the primary reason for ignoring interior dynamical systems is that they imply the inapplicability of the virtual totality of the mathematics and physics developed during the 20-th century, including classical and quantum mechanics, special and general relativities, etc., as we shall see.

The most salient distinction between exterior and interior systems is the following. Exterior dynamical systems can be entirely described via the sole knowledge of a Lagrangian or Hamiltonian and the *truncated Lagrange and Hamilton analytic equations, those without external terms*, also known as the *truncated analytic equations*,

$$\frac{d}{dt}\frac{\partial L(t,r,v)}{\partial v_a^k} - \frac{\partial L(t,r,v)}{\partial r_a^k} = 0, \tag{1.3.1a}$$

$$\frac{dr_a^k}{dt} = \frac{\partial H(t,r,p)}{\partial p_{ak}}, \quad \frac{dp_{ak}}{dt} = -\frac{\partial H(t,r,p)}{\partial r_a^k}, \tag{1.3.1b}$$

$$L = \Sigma_a \frac{1}{2} \times m_a \times \mathbf{v}_a^2 - V(t,r,v), \tag{1.3.1c}$$

$$H = \Sigma_a \frac{\mathbf{p}_a^2}{2 \times m_a} + V(t,r,p), \tag{1.3.1d}$$

$$V = U(t,r)_{ak} \times v_a^k + U_0(t,r); \quad k = 1,2,3; \quad a = 1,2,3,\ldots,n, \tag{1.3.1e}$$

where t is the time of the observer, r, v and p represent the coordinates, velocities and momentum, respectively, of a system of n particles, and the convention of the sum of repeated indices is hereon assumed.

By comparison, interior dynamical systems admit additional interactions that simply cannot be represented with a Lagrangian or a Hamiltonian and, for this reason, Lagrange, Hamilton Jacobi and other founders of analytic dynamics presented their celebrated equations with *external terms representing precisely the contact, zero-range, nonpotential forces among extended particles.*

Therefore, the treatment of interior systems requires the *true Lagrange and Hamilton analytic equations, those with external terms*

$$\frac{d}{dt}\frac{\partial L(t,r,v)}{\partial v_a^k} - \frac{\partial L(t,r,v)}{\partial r_a^k} = F_{ak}(t,r,v), \tag{1.3.2a}$$

$$\frac{dr_a^k}{dt} = \frac{\partial H(t,r,p)}{\partial p_{ak}}, \quad \frac{dp_{ak}}{dt} = -\frac{\partial H(t,r,p)}{\partial r_a^k} + F_{ak}(t,r,p), \tag{1.3.2b}$$

$$L = \Sigma_a \frac{1}{2} \times m_a \times v_a^2 - V(t,r,v), \tag{1.3.2c}$$

$$H = \Sigma_a \frac{\mathbf{p}_a^2}{2 \times m_a} + V(t,r,p), \tag{1.3.2d}$$

$$V = U(t,r)_{ak} \times v_a^k + U_0(t,r), \tag{1.3.2e}$$

$$F(t,r,v) = F(t,r,p/m). \tag{1.3.2f}$$

The necessary and sufficient conditions for the existence of a Lagrangian or a Hamiltonian, originally studied by Helmholtz, are known as the *conditions of variational selfadjointness.* Their comprehensive study was conducted in monograph [54a]. These studies permitted the separation of all acting forces into those derivable from a potential, or variationally selfadjoint (SA) forces, and those not derivable from a potential, or variationally nonselfadjoint (NSA) forces,

$$F^{Tot} = F^{SA}(t,r,v) + F^{NSA}(t,r,v,a,\ldots). \tag{1.3.3}$$

In particular, the reader should keep in mind that, while selfadjoint forces are of Newtonian type, *nonselfadjoint forces are generally non-Newtonian*, in the sense of having an unrestricted functional dependence, including that on accelerations a and other non-Newtonian forms.

Nonselfadjoint forces generally have a nonlocal-integral structure that is usually reduced to a local-differential form via power series expansions in the velocities.

For instance, the contact, zero-range, resistive force experiences by a missile moving in our atmosphere (see Figure 1.3) is characterized by an integral over the surface of the missile and it is usually approximated by a power series in the velocities that, in view of the current

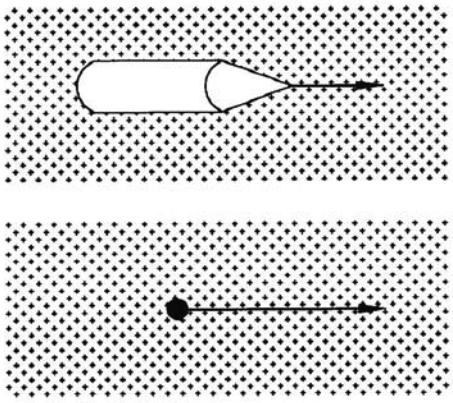

Figure 1.3. Another illustration of the second scientific imbalance studied in this monograph, the impossibility of reducing nature to point-like elementary constituents. The top view depicts a typical open nonconservative and irreversible Newtonian system with nonlocal and nonpotential forces, such as a missile moving in atmosphere, while the bottom view depicts its reduction to point-like constituents. Such a reduction is now known to be inconsistent because no finite ensemble of elementary quantum particles can reproduce a missile in atmosphere, evidently because the former system is closed, reversible and conservative, while the latter system is open nonconservative and irreversible. This establishes that the nonlinear, nonlocal and nonpotential interactions responsible for classical Newtonian systems such as a missile in atmosphere originate at the level of the ultimate elementary constituents of nature.

high speeds, can reach up to the 10-th power of the velocity, thus being irreconcilably beyond the representational capabilities of a Lagrangian or a Hamiltonian[9],

$$F^{NSA} = \int_{\sigma} dr^3 \times \Gamma(r, v, \ldots) = k_1 \times v + k_2 \times v^2 + k_3 \times v^3 + \ldots \quad (1.3.4)$$

where σ is the surface or volume of mutual contact and Γ is a suitable kernel.

Moreover, the studies of monographs [54a] establish the following important:

THEOREM 1.3.3: Nonconservative Newtonian systems in more than one dimension are variationally nonselfadjoint and do not admit a Lagrangian or a Hamiltonian representation in the fixed frame of the observer. Only one-dimensional nonconservative systems admit indirect

Lagrangian or Hamiltonian representations via suitable integrating factors.

As also studied in detail in Refs. [54], under sufficient continuity and regularity conditions and under the necessary reduction of nonlocal external terms to local approximations such as that in Eq. (1.3.4), the *Darboux's theorem* of the symplectic geometry or, equivalently, the *Lie-Koening theorem* of analytic mechanics assure the existence of coordinate transformations

$$\{r, p\} \to \{r'(r, p),\ p'(r, p)\}, \tag{1.3.5}$$

under which nonselfadjoint systems (1.3.2) can be turned into the self-adjoint form (1.3.1), thus eliminating external terms.

However, coordinate transforms (1.3.5) are *necessarily nonlinear.* Consequently, the new reference frames are *necessarily noninertial.* Therefore, the elimination of the external nonselfadjoint forces via coordinate transforms cause the necessary loss of Galileo's and Einstein's relativities.

Moreover, it is evidently impossible to place measuring apparata in new coordinate systems of the type

$$r' = \exp(\alpha \times p),\ \ p' = \ln(\beta \times r^3), \tag{1.3.6}$$

where α and β are constants. For these reasons, the use of Darboux's theorem or of the Lie-Koening theorem was strictly prohibited in monographs [54] and, to avoid misrepresentations, we shall use throughout this monograph the same:

ASSUMPTION 1.3.1: The sole admitted analytic representations are those in the fixed references frame of the experimenter without the use of integrating factors or the transformation theory, called "direct analytic representations". Only after these representations have been identified, the use of the transformation theory may have physical relevance.

As an illustration, the admission of integrating factors within the fixed coordinates of the experimenter does indeed allow the achievement of an analytic representation without external terms of a restricted class of *one-dimensional nonconservative systems*, although the approach results in mathematical Hamiltonians of the type

$$H = e^{f(t,r,...)\times p^2/2\times m.} \tag{1.3.7}$$

The above Hamiltonian has the fully valid *canonical meaning* of representing the time evolution. However, the above Hamiltonian loses its

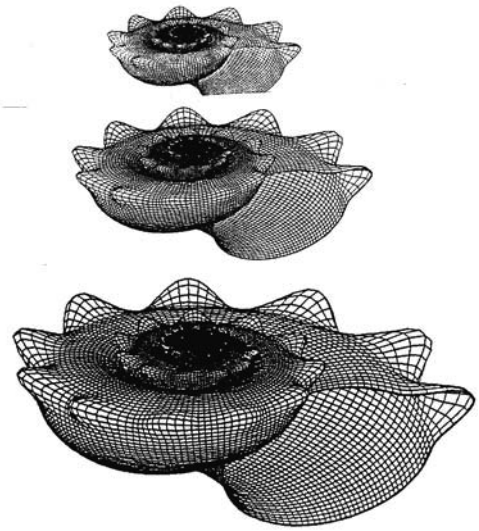

Figure 1.4. A pictorial view of the impossibility for quantum mechanics to be exactly valid in nature: the growth of a seashell. In fact, quantum mechanics is structurally irreversible, in the sense that all its axioms, geometries and symmetries are fully reversible in time, while the growth of a seashell is structurally irreversible. The need for an irreversible generalization of quantum mechanics is out credible reason, as studied in detail in Chapter 4.

meaning as representing the energy of the system (in fact, the energy is nonconserved in this case, while the above Hamiltonian is a constant of motion).

The quantization of such a Hamiltonian then leads to a plethora of illusions, such as the belief that the uncertainty principle is valid for nonconservative systems, an issue basically open to this writing, while in reality the uncertainty principle can be solely formulated for the *mathematical* Hamiltonian (1.3.7) (see Ref. [22] for details).

Under the strict adoption of Assumption 1.3.1, all these ambiguities are prevented because *H will always represent the energy*, irrespective of whether conserved or nonconserved, thus setting up solid foundations for correct physical interpretations.

1.3.3 Limitations of Conventional Mathematical and Physical Methods

The next aspect important for these introductory lines is that the limitations of special relativity and quantum mechanics of the preceding section *are not* due to insufficient physical insight, but rather to *insufficient mathematics.*

This author has repeatedly voiced the opinion that protracted physical controversies are generally due to the use of inadequate mathematics, and they are generally resolved via the use of a broader mathematics more appropriate for the problem at hand.

For instance, the resolution of the century old controversies on general relativity indicated in Section 1.2 appears to be resolved via the construction of a *new geometry*, such as the novel *iso-Minkowskian geometry* [15] that unifies the Minkowskian and Riemannian geometries, thus unifying special and general relativities, with consequential resolution of the controversies not only on general relativities, but also on quantum gravity and grand unifications (see Chapter 4).

It is important for the reader to see from these introductory lines that, along fully similar lines, the limitations of special relativity and quantum mechanics established by Theorems 1.3.1 and 1.3.2 *are not* due to insufficient physical insights, but also to *insufficient mathematics.*

After one century of failed attempts it is time to admit that the *mathematics* underlying quantum mechanics, rather than the quantum axioms themselves, cannot possibly permit a meaningful representation of nonconservations, irreversibility and all non-Hamiltonian effects, including shapes and densities.

At this stage of our studies it is important to see the insufficiency of conventional mathematics at the primitive classical level, since operator formulations merely follow.

To begin, the presence of irreducible nonselfadjoint external terms in the analytic equations causes the loss of their derivability from a variational principle. In turn, the lack of an action principle and related Hamilton-Jacobi equations causes the lack of a consistent quantization, thus illustrating the reasons why the voluminous literature in quantum mechanics of the 20-th century carefully avoids the treatment of analytic equations with external terms.

By contrast, one of the central objectives of chapter 3 is to review the studies that have permitted the achievement of a reformulation of Eq. (1.3.2) fully derivable from a variational principle in conformity with Assumption 1.3.1, while permitting a consistent operator version of Eq. (1.3.2) as a covering of conventional quantum formulations.

Recall that *Lie algebras* are at the foundations of all classical and quantum theories of the 20-th century. This is due to the fact that the brackets of the time evolution as characterized by the truncated Hamilton's equations,

$$\frac{dA}{dt} = \frac{\partial A}{\partial r_a^k} \times \frac{dr_a^k}{dt} + \frac{\partial A}{\partial p_{ak}} \times \frac{dp_{ak}}{dt}$$

$$= \frac{\partial A}{\partial r_a^k} \times \frac{\partial H}{\partial p_{ak}} - \frac{\partial H}{\partial r_a^k} \times \frac{\partial A}{\partial p_{ak}} = [A, H], \tag{1.3.8}$$

firstly, verify the conditions to characterize an *algebra* as currently understood in mathematics, that is, the brackets $[A, H]$, verify the right and left scalar and distributive laws,

$$[n \times A, H] = n \times [A, H], \tag{1.3.9a}$$

$$[A, n \times H] = [A, H] \times n, \tag{1.3.9b}$$

$$[A \times B, H] = A \times [B, H] + [A, H] \times B, \tag{1.3.9c}$$

$$[A, H \times Z] = [A, H] \times Z + H \times [A, Z], \tag{1.3.9d}$$

and, secondly, the brackets $[A, H]$ verify the *Lie algebra axioms*

$$[A, B] = -[B, A], \tag{1.3.10a}$$

$$[[A, B], C] + [[B, C], A] + [[C, A], B] = 0. \tag{1.3.10b}$$

The above properties then persist following quantization into the operator brackets $[A, B] = A \times B - B \times A$, as well known.

In 1978, Santilli [103] showed that, when adding external terms, the resulting new brackets,

$$\frac{dA}{dt} = \frac{\partial A}{\partial r_a^k} \times \frac{dr_a^k}{dt} + \frac{\partial A}{\partial p_{ak}} \times \frac{dp_{ak}}{dt}$$

$$= \frac{\partial A}{\partial r_a^k} \times \frac{\partial H}{\partial p_{ak}} - \frac{\partial H}{\partial r_a^k} \times \frac{\partial A}{\partial p_{ak}} + \frac{\partial A}{\partial r_a^k} \times F_a^k$$

$$= (A, H) = [A, H] + \frac{\partial A}{\partial r_a^k} \times F_a^k, \tag{1.3.11}$$

violate the right scalar law (1.3.9*b*) and the right distributive law (1.3.9*d*) and, therefore, the brackets (A, H) do not constitute any algebra at all, let alone violate the basic axioms of the Lie algebras.

This occurrence should not be surprising because, while the mathematics and physics of the 20-th century were intent in representing *conservation laws*, Lagrange, Hamilton and Jacobi were primarily intent

in representing *nonconservation laws*, trivially, because the former are a particular case of the latter, but *not* vice-versa. In fact, a central feature of analytic equations (1.3.2) is that of representing the *time-rate of variation of the energy and other physical quantities* according to the familiar law

$$\frac{dA}{dt} = [A, H] + \frac{\partial A}{\partial r_a^k} \times F_a^k, \tag{1.3.12}$$

for which

$$\frac{dH}{dt} = \frac{\partial H}{\partial r_a^k} \times F_a^k \neq 0. \tag{1.3.13}$$

Unfortunately, in the transition from conservation to nonconservation laws there is the loss of all algebras that, in turn, causes the loss of *all* mathematical and physical formulations built in the 20-th century without physically significant exceptions known to this author.

The loss of basic methods constitutes the main reason for the abandonment of the study of interior dynamical systems. In fact, external terms in the analytic equations were essentially ignored through the 20-th century, by therefore adapting the universe to the truncated analytic equations (1.3.1).

By contrast, another central objective of Chapter 3 is to review the studies that have permitted the achievement of a reformulation of the historical analytic equations with external terms, into a form not only derivable from an action principle with related operator map, but also characterizing brackets in the time evolution that, firstly, constitute an algebra and, secondly, that algebra results to be a covering of Lie algebras.

1.3.4 Inapplicability of the Galilean and Special Relativities for Matter and Antimatter Dynamical Systems with Resistive Forces

The scientific imbalance caused by the reduction of interior dynamical systems to point-like particles moving in vacuum, is of historical proportion because it caused the belief of the exact applicability of special relativity and quantum mechanics for all conditions of particles existing in the universe, thus implying their applicability under conditions for which these theories were not intended for by their originators.

At the classical level, the "inapplicability" (rather then the "violation") of the Galilean and special relativities for the description of an interior system such as a missile in atmosphere is beyond credible doubt, as any expert should know to qualify as such, because said relativities can only describe systems with action-at-a-distance potential forces, while

the force acting on a missile in atmosphere are of contact-zero-range nonpotential type.

When faced with the above evidence, a rather general posture intended to adapt nature to pre-existing theories is, that the resistive forces are "illusory" because, when the missile in atmosphere is reduced to its elementary point-like constituents, all resistive forces "disappear".

Such a belief has been disproved by Theorem 1.3.1. Therefore, the inapplicability of the Galilean and special relativities to classical systems with resistive forces also extends to its elementary constituents due to the lack of a credible reduction of a nonconservative classical system to a finite ensemble of elementary constituents all in conservative conditions, as necessary for the validity of the relativities herein considered.

Rather than adapting nature to pre-existing doctrines, a main scope of this monograph is that of adapting doctrines to nature, as requested by scientific ethics and accountability.

1.3.5 Inapplicability of Special Relativity for the Propagation of Light within Physical Media of Matter or Antimatter

Among the various cases of interior systems, a most important one is the propagation of light within physical media described by the law we learn in high school

$$C = \frac{c}{n}, \tag{1.3.14}$$

where c is the speed of light in vacuum and n is the familiar *index of refraction.*

As an illustration, it is known that, for the case of water, light propagates at a speed much smaller than the speed in vacuum and approximately given by the value

$$C = \frac{c}{n} = \frac{2}{3} \times c < c, \quad n = \frac{3}{2} > 1. \tag{1.3.15}$$

It is equally known that electrons can propagate in water at speeds bigger than the local speed of light, and actually approaching the speed of light in vacuum. In fact, the propagation of electrons faster than the local speed of light is responsible for the blueish of light, called *Cerenkov light*, that can be seen in the pools of nuclear reactors.

Finally, it is also well known that *special relativity was built to describe the propagation of light in vacuum, and certainly not within physical media.* In fact, the setting of a massive particle travelling faster than the local speed of light is in violation of the basic axioms of special relativity.

In an attempt to adapt nature to special relativity, the following three beliefs are generally voiced. Firstly, it is believed that "the speed of light *in vacuum* c is the maximal causal speed *within water*" (see Figure 1.5).

However, in this case there is the *violation of the axiom of relativistic addition of speeds, because the sum of two speeds of light in water does not yield the speed of light*, as required by a fundamental axiom of special relativity,

$$V_{tot} = \frac{C + C}{1 + \frac{C^2}{c^2}} = \frac{12}{13} \times c \neq C. \tag{1.3.16}$$

Secondly, it is believed that "the speed of light *in water* C is the maximal causal speed *in water*." In this case the axiom of relativistic compositions of speeds is verified,

$$V_{tot} = \frac{C + C}{1 + \frac{C^2}{C^2}} = C, \tag{1.3.17}$$

but there is *the violation of the principle of causality* evidently due to the fact that ordinary massive particles such as the electron (and not hypothetical *tachyons*) can travel faster than the assumed maximal causal speed.

Thirdly, it is believed that "the reduction of the speed of light in water is illusory" because the reduction of light to photons scattering among the atoms constituting water re-establishes the full validity of special relativity.

The latter belief is essentially nonscientific because it is not substantiated by credible calculations proving not only the actual reduction of the speed of light by one third but also the validity of such a large reduction to a sufficient number of different frequencies or wavelengths.

To begin, the nonscientific nature of the third belief is soon proved for the case of electromagnetic waves with large wavelength, since the latter do not admit a credible reduction to photons for the case here considered. Even for the case of wavelengths of atomic size, the cross section of the Compton Scattering herein considered is known to be small, and definitely insufficient to generate a 33% reduction of the speed of light, as first year graduate students can easily prove.

All the preceding aspects refer to the propagation of light at speeds C *smaller* than the speed in vacuum c. In addition, there exist today a large volume of experimental evidence reviewed later on establishing that light propagates within hyperdense media, such as those in the interior of hadrons, nuclei and stars, at speed much bigger than the speed in vacuum,

$$C = \frac{c}{n} \gg c, \quad n \ll 1. \tag{1.3.18}$$

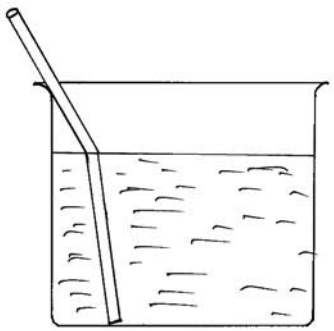

Figure 1.5. A further visual evidence of the lack of applicability of Einstein's doctrines within physical media, the refraction of light in water due to the decrease of its speed, contrary to the axiom of the "universal constancy of the speed of light". Organized academic interests on Einsteinian doctrines have claimed throughout the 20-th century that this effect is "illusory" because Einsteinian doctrines are fully recovered by reducing light to the scattering of photons among atoms. The political nature of the argument, particularly when proffered by experts, is established by the impossibility of achieving numerical representations of the occurrence, such as the 33% reduction of the speed of light in water, experimental evidence for speeds bigger than the speed of light in water (for which the reduction of light to photons scattering among atoms has no physical sense), and other data.

in which case the reduction of light to photons scattering among atoms loses any scientific credibility (because such a propagation can never reach speeds bigger than c).

In conclusion, experimental evidence establishes beyond credible doubt that special relativity is indeed valid for point-particles and electromagnetic waves moving in vacuum, but special relativity is inapplicable (rather than violated) for the propagation of particles and electromagnetic waves within physical media because *the speed of light C is a local variable dependent on the characteristics of the medium in which it propagates, with speed $C = c$ in vacuum, speeds $C \ll c$ within physical media of low density and speeds $C \gg c$ within media of very high density.*

The variable character of the speed of light then establishes the lack of universal applicability of Einsteinian doctrines, since the latter are notoriously based on the philosophical assumption of "universal constancy of the speed of light".

Unreassuringly, the experimental evidence on speeds of light greater than that in vacuum has been confirmed by German experimentalists who have transmitted an entire Beethoven symphony with electromag-

netic waves propagating between certain guides at speeds bigger than that of light in vacuum, thus establishing the limitations of special relativity herein considered.

Nevertheless, this so important an event continues to be ignored by organized interests on Einsteinian doctrines with rather sinister implications for mankind because, as today known, the restriction of nature to comply with special relativity prevents the prediction, let alone study, of much needed new clean energies and fuels [22].

1.3.6 Inapplicability of the Galilean and Poincaré symmetries for Interior Dynamical Systems of Matter or Antimatter

By remaining at the classical level, the inapplicability of Einsteinian doctrines within physical media is additionally established by the dramatic dynamical differences between the structure of planetary systems (such as our Solar system), and the structure of planets (such as Jupiter).

Planetary systems are *Keplerian systems*, that is, a systems in which the heaviest component is at the center (actually in one of the two foci of elliptical orbits) and the other constituents orbit around it. By contrast, planets absolutely do not constitute Keplerian systems, because they do not have a Keplerian center with lighter constituents orbiting around it (Figure 1.6).

Moreover, for a planetary system isolated from the rest of the universe, the total conservation laws for the energy, linear momentum and angular momentum are verified by each individual constituent. For instance, the conservation of the intrinsic and orbital angular momentum of Jupiter is crucial for the stability of its orbit and the same holds for all remaining planets of the solar system.

On the contrary, for the interior dynamical problem of Jupiter, conservation laws hold only globally, while no conservation law can be formulated for individual constituents.

For instance, in Jupiter's structure we can see in a telescope with our naked eye the existence in Jupiter's atmosphere of *interior vortices with variable angular momentum*, yet always in such a way to verify total conservation laws. We merely have internal exchanges of energy, linear and angular momentum but always in such a way that they cancel out globally resulting in total conservation laws.

In the transition to particles, the situation remains the same as that at the classical level. For instance, *nuclei do not have nuclei* and, therefore, nuclei do not constitute Keplerian systems. Similarly, the Solar system is a Keplerian system, but the Sun is not.

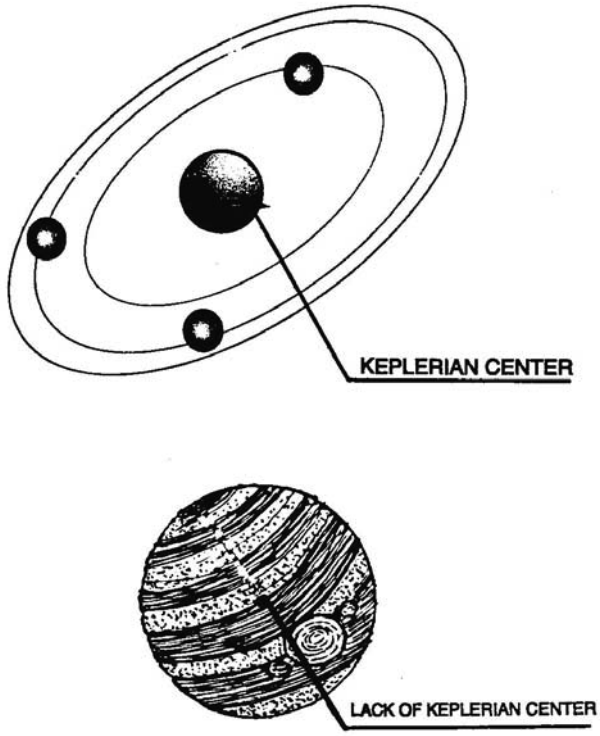

Figure 1.6. Another illustration of the second major scientific imbalance of the 20-th century studied in this monograph, the dramatic structural differences between exterior and interior dynamical systems, here represented with the Solar system (top view) and the structure of Jupiter (bottom view). Planetary systems have a Keplerian structure with the exact validity of the Galilean and Poincaré symmetries. By contrast, interior systems such as planets (as well as hadrons, nuclei and stars) do not have a Keplerian structure because of the lack of the Keplerian center. Consequently, the Galilean and Poincaré symmetries cannot possibly be exact for interior systems in favor of covering symmetries and relativities studied in this monograph.

Any reduction of the structure of the Sun to a Keplerian system directly implies the belief in the perpetual motion within a physical medium, because the former belief implies that electrons and protons move in the hyperdense medium in the core of a star with a locally conserved angular momentum, namely, a belief outside all boundaries of credibility, let alone science.

If the above aspects applies for matter, there is no need to treat antimatter because the problems multiply due to inconsistent quantization of a classical Galilean or Poincaré system into a charge conjugate state. The above evidence establishes beyond credible doubt the following:

THEOREM 1.3.4 [22]: *Galileo's and Poincaré symmetries are inapplicable for classical and operator interior dynamical systems of matter or of antimatter due to their lack of Keplerian structure originating from the presence of contact, zero-range, non-potential interactions, and other reasons.*

Note the use of the word "inapplicable", rather than "violated" or "broken". This is due to the fact that, as clearly stated by the originators of the basic spacetime symmetries (rather than their followers of the 20-th century), Galileo's and Poincaré symmetries were not built for interior dynamical conditions.

1.3.7 Lack of Exact Character of Special Relativity and Quantum Mechanics for the Structure of Hadrons, Nuclei and Stars

Perhaps the biggest scientific imbalance of the 20-th century has been the abstraction of hadronic constituents to point-like particles as a necessary condition to use conventional spacetime symmetries, relativities and quantum mechanics for interior conditions. In fact, such an abstraction is at the very origin of the conjecture that the undetectable *quarks* are the physical constituents of hadrons.

As repeatedly shown later on in this monograph, the unitary symmetries for the Mendeleev-type classification of hadrons into family has indeed a final character. However, the belief that the same unitary symmetries for the classification of hadrons can jointly provide the structure of each individual member of a unitary multiplet, is afflicted by a plethora of catastrophic inconsistencies that, after decades of attempts, have remained unresolved.

It is sufficient to recall at this point that *quarks cannot have any gravity at all*, because gravity can only be defined in spacetime while quarks can only be define on a unitary internal space without connection to our spacetime (due to O'Rafearthaigh's theorem[10]). Note that, under the conjecture that quarks are the constituents of nucleons, thus of nuclei, our bodies should float in the air because of the proven lack of gravity.

Also, *quark masses cannot have a credible inertia*, because to have inertial masses must be defined in our spacetime, that is, masses must

be the eigenvalues of the second order Casimir invariant of the Poincaré symmetry, while quarks cannot be characterized at all with such a spacetime symmetry. As a consequence, *quark masses are purely mathematical parameters solely definable in a purely mathematical unitary space of the classification (and not the structure) of hadrons.* Additional catastrophic inconsistencies of quark conjectures will be pointed out later on.

Even assuming that quarks are physical particles, their structure for protons, neutrons and all baryons does not constitute a Keplerian system, under which condition the use of nonrelativistic or relativistic quantum mechanics is a pure religion and not a scientific truth.

Another main objective of this monograph is to show that the assumption of the unconfinable and undetectable quarks without gravity and inertia are the physical constituents of hadrons prevents any possible utilization of the immense reservoir or energy inside nucleons. On the contrary, if physical particles with proven gravity and inertial are assumed as the physical constituents of hadrons that can be expelled and detected under certain conditions, new clean energies are indeed possible, provided that special relativity and quantum mechanics are abandoned in favor of covering theories.

Irrespective of whether we consider quarks or other more credible particles, all particles have a wavepacket of the order of $1F = 10^{-13}$ cm, that is, a wavepacket of the order of the size of all hadrons. Therefore, *the hyperdense medium in the interior of hadrons is composed of particles with extended wavepackets in conditions of total mutual penetration.* Under these conditions, the belief that Galileo's and Poincaré symmetries are *exactly* valid in the interior of hadrons implies the exiting from all boundaries of credibility, let alone of science.

The inapplicability of the fundamental spacetime symmetries then implies the inapplicability of Galilean and special relativities as well as of classical and quantum nonrelativistic and relativistic mechanics. We can therefore conclude with the following:

THEOREM 1.3.5 [22]: *Classical Hamiltonian mechanics and related Galilean and special relativities are not exactly valid for the treatment of interior classical systems such the structure of Jupiter, while nonrelativistic and relativistic quantum mechanics and related Galilean and special relativities are not exactly valid for interior particle systems, such as the structure of hadrons, nuclei and stars.*

Another important scope of this monograph is to show that the problem of the exact spacetime symmetries applicable to interior dynamical systems is not a mere academic issue, because it carries a direct societal

relevance. In fact, we shall show that broader spacetime symmetries specifically built for interior systems predict the existence of new clean energies and fuels that are absolutely prohibited by the spacetime symmetries of the exterior systems.

More particularly, as we shall see, the assumption that the undetectable quarks are physical constituents of hadrons *prohibits* any possible new energy based on processes occurring in the interior of hadrons (rather than in the interior of their ensembles such as nuclei). On the contrary, the assumption of hadronic constituents that can be fully defined in our spacetime directly implies new clean energies.

1.4 THE SCIENTIFIC IMBALANCE CAUSED BY GENERAL RELATIVITY AND QUANTUM GRAVITY FOR MATTER AND ANTIMATTER

1.4.1 The Negative Impact of General Relativity on Antimatter

An important requirement of any consistent classical theory of antimatter is the existence of *antigravity* defined as a gravitational repulsion experienced by antimatter in the field of matter, and vice-versa.

As we shall see in detail in Chapter 4, this is essentially due to the achievement in the classical treatment of antimatter of a full dynamical equivalence between electromagnetic and gravitational interactions, thus including the existence of both attraction and repulsion, which classical equivalence is generally lost when antimatter is solely treated in second quantization owing to the notorious absence of a consistent gravitational theory for that level of study.

By contrast, in its current formulation, *general relativity* strictly prohibits the existence of antigravity, thus casting a severe constraint in the scientific development of our knowledge on antimatter. For instance, the experimental verification of the prediction of antigravity experienced by very low energy positrons in horizontal flight on Earth [9] is fully feasible with current technology, resolutory (because the displacement can be seen by the naked eye on an scintillator), and transparently more important than various currently preferred, yet sterile particle experiments.

Despite all this relevance, said test of antigravity is often dismissed by experimentalists and theoreticians alike on grounds that "general relativity does not predict antigravity". Consequently, basic advances in antimatter (as well as on numerous other fields) are in jeopardy without an in depth appraisal of general relativity.

In this section we outline a number of inconsistencies of general relativity published in refereed technical journals that are so serious to be known as "catastrophic inconsistencies". The lack of resolution of these inconsistencies, also published in refereed technical journals, establishes that general relativity simply cannot be used as an argument to suppress basic new experiments in antigravity.

Independently from these inconsistencies, recall that, according to Dirac's teaching, antimatter requires negative-energy solutions. Consequently, general relativity cannot have any serious impact on antimatter because it does not allow for negative-definite energies. A detailed study of the content of this section was conducted in Ref. [109].

1.4.2 Catastrophic Inconsistencies of General Relativity due to Lack of Sources

While special relativity has a majestic axiomatic consistency indicated in Section 1.2.1, there is no doubt that, despite widespread popular support, *general relativity* has been the most controversial theory of the 20-th century. In this section we shall review some of the major, mathematical, theoretical and experimental inconsistencies of general relativity published in the refereed technical literature, yet generally ignored by scientists in the field, and today known as "catastrophic inconsistencies".[11]

There exist subtle distinctions between "general relativity", "Einstein's Gravitation", and "Riemannian treatment of gravity".[12] For our needs, we here define *Einstein's gravitation* the reduction of exterior gravitation in vacuum to pure geometry, namely, gravitation is solely represented via curvature in a Riemannian space $\mathcal{R}(x, g, R)$ with spacetime coordinates $x = (x^\mu,\ \mu = 1, 2, 3, 4)$ and nowhere singular real-valued and symmetric metric $g(x)$ over the reals R, with field equations

$$G_{\mu\nu} = R_{\mu\nu} - g_{\mu\nu} \times R/2 = 0, \tag{1.4.1}$$

in which, as a central condition to have "Einstein's gravitation", *there are no sources for the exterior gravitational field in vacuum for a body with null total electromagnetic field (null total charge and magnetic moment).*

For our needs, we define as *general relativity* any description of exterior gravity on a Riemannian space over the reals with Einstein-Hilbert field equations including a source due to the presence of electric and magnetic fields,

$$G_{\mu\nu} = R_{\mu\nu} - g_{\mu\nu} \times R/2 = k \times t_{\mu\nu}, \tag{1.4.2}$$

where k is a constant depending on the selected unit whose value is here irrelevant. For the scope of this monograph, it is sufficient to assume that the *Riemannian description of gravity* coincides with general relativity according to the above definition.[13]

In the following, we shall study in this section the inconsistencies of Einstein gravitation, that is, the inconsistencies in the entire reduction of gravity to curvature without source. We shall then study in the next section the inconsistencies caused by curvature itself even in the presence of sources.

It should be stressed that a technical appraisal of the content of this section can only be reached following the study in Chapter 5 of the axiomatic inconsistencies of grand unified theories of electroweak and gravitational interactions whenever gravity is represented with curvature on a Riemannian space, and irrespective of whether with or without sources [3].

THEOREM 1.4.1 [41]: *Einstein's gravitation and general relativity at large are incompatible with the electromagnetic origin of mass established by quantum electrodynamics, thus being inconsistent with experimental evidence.*

Proof. Quantum electrodynamics has established that the mass of all elementary particles, whether charged or neutral, has a primary electromagnetic origin, that is, all masses have a first-order origin given by the volume integral of the 00-component of the energy-momentum tensor $t_{\mu\nu}$ of electromagnetic origin,

$$m = \int d^4x \times t_{00}^{elm}. \tag{1.4.3a}$$

$$t_{\alpha\beta} = \frac{1}{4\pi}(F_\alpha^\mu F_{\mu\beta} + \frac{1}{4} g_{\alpha\beta} F_{\mu\nu} F^{\mu\nu}), \tag{1.4.3b}$$

where $t_{\alpha\beta}$ is the *electromagnetic tensor*, and $F_{\alpha\beta}$ is the *electromagnetic field* (see Ref. [11a] for explicit forms of the latter with retarded and advanced potentials).

Therefore, quantum electrodynamics requires the presence of a *first-order source tensor* in the *exterior field equations* in vacuum as in Eq. (1.4.2). Such a source tensor is absent in Einstein's gravitation (1.4.1) by conception. Consequently, Einstein's gravitation is incompatible with quantum electrodynamics.

The incompatibility of general relativity with quantum electrodynamics is established by the fact that the source tensor in Eq. (1.4.2) is of *higher order in magnitude*, thus being ignorable in first approximation with respect to the gravitational field, while according to quantum electrodynamics said source tensor is of first order, thus not being ignorable in first approximation.

The inconsistency of both Einstein's gravitation and general relativity is finally established by the fact that, when the total charge and magnetic moment of the body considered are null, Einstein's gravitation and general relativity allow no source at all. By contrast, as illustrated in Ref. [41], quantum electrodynamics requires a source tensor of first order in magnitude even when the total charge and magnetic moments are null due to the charge structure of matter. **q.e.d.**

The first consequence of the above property can be expressed via the following:

COROLLARY 1.4.1A [41]: *Einstein's reduction of gravitation in vacuum to pure curvature without source is incompatible with physical reality.*

A few comments are now in order. As is well known, the mass of the electron is entirely of electromagnetic origin, as described by Eq. (1.4.3), therefore requiring a first-order source tensor in vacuum as in Eq . (1.4.2). Thus, Einstein's gravitation for the case of the electron is inconsistent with nature. Also, the electron has a point charge. Consequently, *the electron has no interior problem at all, in which case the gravitational and inertial masses coincide*,

$$m^{Grav.}_{Electron} \equiv m^{Iner}_{Electron}. \tag{1.4.4}$$

Next, Ref. [41] proved Theorem 1.4.1 for the case of a neutral particle by showing that the π° meson needs a first-order source tensor in the exterior gravitational problem in vacuum since its structure is composed of one charged particle and one charged antiparticle in high dynamical conditions.

In particular, the said source tensor has such a large value to account for the entire *gravitational mass* of the particle [41]

$$m^{Grav.}_{\pi^\circ} = \int d^4x \times t^{Elm}_{00}. \tag{1.4.5}$$

For the case of the interior problem of the π°, we have the additional presence of short range weak and strong interactions representable with a new tensor $\tau_{\mu\nu}$. We, therefore, have the following

COROLLARY 1.4.1B [41]*: In order to achieve compatibility with electromagnetic, weak and strong interactions, any gravitational theory must admit two source tensors, a traceless tensor for the representation of the electromagnetic origin of mass in the exterior gravitational problem, and*

a second tensor to represent the contribution to interior gravitation of the short range interactions according to the field equations

$$G_{\mu\nu}^{Int.} = R_{\mu\nu} - g_{\mu\nu} \times R/2 = k \times (t_{\mu\nu}^{Elm} + \tau_{\mu\nu}^{ShortRange}). \tag{1.4.6}$$

A main difference of the two source tensors is that the electromagnetic tensor $t_{\mu\nu}^{Elm}$ is notoriously traceless, while the second tensor $\tau_{\mu\nu}^{ShortRange}$ is not. A more rigorous definition of these two tensors will be given shortly.

It should be indicated that, for a possible solution of Eq. (1.4.6), various explicit forms of the electromagnetic fields as well as of the short range fields originating the energy-momentum tensors are given in Ref. [41].

Since both sources tensors are positive-definite, Ref. [41] concluded that the interior gravitational problem characterizes the *inertial mass* according to the expression

$$m^{Iner} = \int d^4x \times (t_{00}^{Elm} + \tau_{00}^{ShortRange}), \tag{1.4.7}$$

with consequential general law

$$m^{Inert.} \geq m^{Grav.}, \tag{1.4.8}$$

where the equality solely applies for the electron.

Finally, Ref. [41] proved Theorem 1.4.1 for the exterior gravitational problem of a neutral massive body, such as a star, by showing that the situation is essentially the same as that for the π°. The sole difference is that the electromagnetic field requires the sum of the contributions from *all* elementary constituents of the star,

$$m_{Star}^{Grav.} = \Sigma_{p=1,2,...} \int d^4x \times t_{p00}^{Elem.}. \tag{1.4.9}$$

In this case, Ref. [41] provided methods for the approximate evaluation of the sum that resulted to be of first-order also for stars with null total charge.

When studying a charged body, there is no need to alter equations (1.4.6) since that particular contribution is automatically contained in the indicated field equations.

Once the incompatibility of general relativity at large with quantum electrodynamics has been established, the interested reader can easily prove the incompatibility of general relativity with quantum field theory and quantum chromodynamics, as implicitly contained in Corollary 1.4.1B.

An important property apparently first reached in Ref. [41] of 1974 is the following:

COROLLARY 1.4.1C [41]: *The exterior gravitational field of a mass originates entirely from the total energy-momentum tensor (1.4.3b) of the electromagnetic field of all elementary constituents of said mass.*

In different terms, a reason for the failure to achieve a "unification" of gravitational and electromagnetic interactions, failures initiated by Einstein himself, is that the said interactions can be "identified" with each other and, as such, they cannot be unified. In fact, in all unifications attempted until now, the gravitational and electromagnetic fields preserve their identity, and the unification is attempted via geometric and other means resulting in redundancies that eventually cause inconsistencies (see Chapter 5 for details).

Note that conventional electromagnetism is represented with the tensor $F_{\mu\nu}$ and related Maxwell's equations. When electromagnetism is identified with exterior gravitation, it is represented with the energy-momentum tensor $t_{\mu\nu}$ and related equations (1.4.6).

In this way, *gravitation results to be a mere additional manifestation of electromagnetism.* The important point is that, besides the transition from the field tensor $F_{\mu\nu}$ to the energy-momentum tensor $t_{\mu\nu}$, there is no need to introduce a new interaction to represent gravity.

Note finally the irreconcilable alternatives emerging from the studies herein considered:

ALTERNATIVE I: Einstein's gravitation is assumed as being correct, in which case quantum electrodynamics must be revised in such a way as to avoid the electromagnetic origin of mass; or

ALTERNATIVE II: Quantum electrodynamics is assumed as being correct, in which case Einstein's gravitation must be irreconcilably abandoned in favor of a more adequate theory.

By remembering that quantum electrodynamics is one of the most solid and experimentally verified theories in scientific history, it is evident that the rather widespread assumption of Einstein's gravitation as having a final and universal character is non-scientific.

THEOREM 1.4.2 [42,11]: *Einstein's gravitation (1.4.1) is incompatible with the Freud identity of the Riemannian geometry, thus being inconsistent on geometric grounds.*

Proof. The Freud identity [11b] can be written

$$R^\alpha_\beta - \frac{1}{2}\times\delta^\alpha_\beta\times R - \frac{1}{2}\times\delta^\alpha_\beta\times\Theta \;=\; U^\alpha_\beta + \partial V^{\alpha\rho}_\beta/\partial x^\rho \;=\; k\times(t^\alpha_\beta+\tau^\alpha_\beta), \quad (1.4.10)$$

where

$$\Theta \;=\; g^{\alpha\beta}g^{\gamma\delta}(\Gamma_{\rho\alpha\beta}\Gamma^\rho_{\gamma\beta} - \Gamma_{\rho\alpha\beta}\Gamma^\rho_{\gamma\delta}), \quad (1.4.11a)$$

$$U^\alpha_\beta \;=\; -\frac{1}{2}\frac{\partial\Theta}{\partial g^{\rho\alpha}_{|_\rho}} g^{\gamma\beta}\uparrow_\gamma, \quad (1.4.11b)$$

$$V^{\alpha\rho}_\beta \;=\; \frac{1}{2}[g^{\gamma\delta}(\delta^\alpha_\beta\Gamma^\rho_{\alpha\gamma} - \delta^\rho_\beta\Gamma^\rho_{\alpha\delta}) +$$

$$+(\delta^\rho_\beta g^{\alpha\gamma} - \delta^\alpha_\beta g^{\rho\gamma})\Gamma^\delta_{\gamma\delta} + g^{\rho\gamma}\Gamma^\alpha_{\beta\gamma} - g^{\alpha\gamma}\Gamma^\rho_{\beta\gamma}]. \quad (1.4.11c)$$

Therefore, the Freud identity requires two first order source tensors for the exterior gravitational problems in vacuum as in Eq. (3.6) of Ref. [11a]. These terms are absent in Einstein's gravitation (1.4.1) that, consequently, violates the Freud identity of the Riemannian geometry. **q.e.d.**

By noting that trace terms can be transferred from one tensor to the other in the r.h.s. of Eq. (1.4.10), it is easy to prove the following:

COROLLARY 1.4.2A [11]: *Except for possible factorization of common terms, the t- and τ-tensors of Theorem 1.4.2 coincide with the electromagnetic and short range tensors, respectively, of Corollary 1.4.1B.*

A few historical comments regarding the Freud identity are in order. It has been popularly believed throughout the 20-th century that the Riemannian geometry possesses only *four identities* (see, e.g., Ref. [43]). In reality, Freud [42] identified in 1939 a *fifth identity* that, unfortunately, was not aligned with Einstein's doctrines and, as such, the identity was ignored in virtually the entire literature on gravitation of the 20-th century.

However, as repeatedly illustrated by scientific history, structural problems simply do not disappear with their suppression, and actually grow in time. In fact, the Freud identity did not escape Pauli who quoted it in a footnote of his celebrated book of 1958 [45]. Santilli became aware of the Freud identity via an accurate reading of Pauli's book (including its important footnotes) and assumed the Freud identity as the geometric foundation of the gravitational studies presented in Ref. [11].[14]

Subsequently, in his capacity as Editor in Chief of *Algebras, Groups and Geometries*, Santilli requested the mathematician Hanno Rund, a

known authority in Riemannian geometry [46], to inspect the Freud identity for the scope of ascertaining whether the said identity was indeed a new identity. Rund kindly accepted Santilli's invitation and released paper [46] of 1991 (the last paper prior to his departure) in which Rund confirmed indeed the character of Eq. (1.4.10) as a genuine, independent, fifth identity of the Riemannian geometry.

The Freud identity was also rediscovered by Yilmaz (see Ref. [47] and papers quoted therein) who used the identity for his own broadening of Einstein's gravitation via an external *stress-energy tensor* that is essentially equivalent to the source tensor with non-null trace of Ref. [41] of 1974, Eq. (1.4.6).

Despite these efforts, the presentation of the Freud identity to various meetings and several personal mailings to colleagues in gravitation, the Freud identity continues to remain vastly ignored to this day, with very rare exceptions.[15]

Theorems 1.4.1 and 1.4.2 complete our presentation on the catastrophic inconsistencies of Einstein's gravitation due to the lack of a first-order source in the exterior gravitational problem in vacuum. These theorems, by no means, exhaust all inconsistencies of Einstein's gravitation, and numerous additional inconsistencies do indeed exist.

For instance, Yilmaz [47] has proved that Einstein's gravitation explains the $43''$ of the precession of the perihelion of Mercury, but cannot explain the basic Newtonian contribution. This result can also be seen from Ref. [41] because the lack of source implies the impossibility of importing into the theory the basic Newtonian potential. Under these conditions the representation of the Newtonian contribution is reduced to a religious belief, rather than a serious scientific statement.

For these and numerous additional inconsistencies of Einstein's gravitation or general relativity at large we refer the reader to Yilmaz [47], Wilhelm [48], Santilli [53], Alfvén [49,50], Fock [51], Nordensen [52], and large literature quoted therein.

1.4.3 Catastrophic Inconsistencies of General Relativity due to Curvature

We now pass to the study of the structural inconsistencies of general relativity caused by the very use of the Riemannian *curvature*, irrespective of the selected field equations, including those fully compatible with the Freud identity.

THEOREM 1.4.3 [53]: *Gravitational theories on a Riemannian space over a field of real numbers do not possess time invariant basic units*

and numerical predictions, thus having catastrophic mathematical and physical inconsistencies.

Proof. The map from Minkowski to Riemannian spaces is known to be *noncanonical*,

$$\eta = Diag.(1,1,1,-1) \ \rightarrow \ g(x) = U(x) \times \eta \times U(x)^{\dagger}, \qquad (1.4.12a)$$

$$U(x) \times U(x)^{\dagger} \neq I. \qquad (1.4.12b)$$

Thus, the time evolution of Riemannian theories is necessarily noncanonical, with consequential lack of invariance in time of the basic units of the theory, such as

$$I_{t=0} = Diag.(1\,\text{cm},\ 1\,\text{cm},\ 1\,\text{cm},\ 1\,\text{sec}) \rightarrow I'_{t>0} = U_t \times I \times U_t^{\dagger} \ \neq I_{t=0}. \qquad (1.4.13)$$

The lack of invariance in time of numerical predictions then follows from the known "covariance", that is, lack of time invariance of the line element. **q.e.d.**

As an illustration, suppose that an experimentalist assumes at the initial time $t = 0$ the units 1 cm and 1 sec. Then, all Riemannian formulations of gravitation, including Einstein's gravitation, predict that at the later time $t > 0$ said units have a different numerical value.

Similarly, suppose that a Riemannian theory predicts a numerical value at the initial time $t = 0$, such as the $43''$ for the precession of the perihelion of Mercury. One can prove that the same prediction at a later time $t > 0$ is numerically different precisely in view of its "covariance", rather than invariance as intended in special relativity, thus preventing a serious application of the theory to physical reality. We therefore have the following:

COROLLARY 1.4.3A [53]: *Riemannian theories of gravitation in general, and Einstein's gravitation in particular, can at best describe physical reality at a fixed value of time, without a consistent dynamical evolution.*

Interested readers can independently prove the latter occurrence from the *lack of existence of a Hamiltonian in Einstein's gravitation.* It is known in analytic mechanics (see, e.g., Refs. [55,56]) that Lagrangian theories not admitting an equivalent Hamiltonian counterpart, as is the case for Einstein's gravitation, are inconsistent under time evolution, unless there are suitable subsidiary constraints. But the latter are absent in general relativity.

It is therefore surprising at best to note that thousands of physicists during the 20-th century, who are expected from their academic ranks

to have a serious knowledge analytic mechanics, have believed Einstein's gravitation to be correct despite the characterization by a Lagrangian formulation without a consistent Hamiltonian counterpart or suitable subsidiary constraints.

It should be indicated that the inconsistencies caused by curvature are much deeper than those indicated above. For consistency, the Riemannian geometry must be defined on the field of numbers $R(n, +, \times)$ that, in turn, is fundamentally dependent on the basic unit I. But the Riemannian geometry does not leave time invariant the basic unit I due to its noncanonical character. The loss in time of the basic unit I then implies the consequential loss in time of the base field R, with consequential catastrophic collapse of the entire mathematical structure of the geometry [53].

In conclusion, not only is Einstein's reduction of gravity to pure curvature is inconsistent with nature because of the lack of sources, but also the ultimate origin of the inconsistencies rests in the curvature itself when assumed to represent gravity, due to its inherent noncanonical character at the classical level with consequential nonunitary structure at the operator level.

Serious mathematical and physical inconsistencies are then unavoidable under these premises, thus establishing the impossibility of any credible use of general relativity, for instance, as an argument against the test on antigravity predicted for antimatter in the field of matter [9], as well as establishing the need for a profound revision of our current views on gravitation.

THEOREM 1.4.4: Einstein's gravitation is incompatible with experimental evidence because it predicts the bending of light as due to curvature, thus ignoring that due to Newton's attraction.

Proof. Light carries energy, thus being subjected to a bending due to the conventional Newtonian gravitational attraction, while Einstein's gravitation predicts that the bending of light is entirely due to curvature. The assumption that Einstein's graviutation includes Newtonian attraction leads to a bending twice that experimentally measured. If the bending of light is reducved to pure curvature to comply with experimental values, this requires the inability of Einstein's gravitation to include the Newtonian attraction with consequential other catastrophic inconsistencies, such as the inability to represent the base Newtonian contribution in planetary motion as shown by Yilmaz [47] and other inconsistencies [48-52]. **q.e.d.**

THEOREM 1.4.5: The lack of curvature in gravitation is established by the free fall of masses that necessarily occurs along a straight radial line.

In fact, a consistent representation of the free fall of a mass along a straight radial line requires that the Newtonian attraction be represented the field equations necessarily without curvature, thus disproving the customary belief needed to avoid Corollary 1.4.2.A that said Newtonian attraction emerges at the level of the post-Newtonian approximation (the PPN approximation) [43,44] of Eq. (1.4.1).

THEOREM 1.4.6. Gravitational experimental measurements do not verify Einstein's gravitation uniquely.

Proof. All claimed "experimental verifications" of Einstein's gravitation are based on the PPN "expansion" (or linearization) of the field equations that, as such, is not unique. In fact, Eq. (1.4.1) admit a variety of inequivalent expansions depending on the selected parameter, the selected expansion and the selected truncation. It is then easy to show that the selection of an expansion of the same equations (1.4.1) but different from the PPN approximation leads to dramatic departures from experimental values. **q.e.d.**

A comparison between special and general relativities is here in order. Special relativity can be safely claimed to be "verified by experiments" because the said experiments verify numerical values uniquely and unambiguously predicted by special relativity. By contrast, no such statement can be made for general relativity since the latter does not uniquely and unambiguously predict given numerical values due, again, to the variety of possible expansions and linearization.

The origin of such a drastic difference is due to the fact that *the numerical predictions of special relativity are rigorously controlled by the basic Poincaré "invariance". By contrast, one of the several drawbacks of the "covariance" of general relativity is precisely the impossibility of predicting numerical values in a unique and unambiguous way, thus preventing serious claims of true "experimental verifications" of general relativity.*

By no means, the inconsistencies expressed by the above theorems constitute all inconsistencies of general relativity. In the author's opinion, additional deep inconsistencies are caused by the fact that *general relativity does not possess a well defined Minkowskian limit*, while the admission of the Minkowski space as a tangent space is basically insuf-

ficient on dynamical grounds (trivially, because on said tangent space gravitation is absent).

As an illustration, we should recall the controversy on conservation laws that raged during the 20-th century [11]. Special relativity has rigidly defined total conservation laws because they are the Casimir invariants of the fundamental Poincaré symmetry. By contrast, there exist several definitions of total conservation laws in a Riemannian representation of gravity due to various ambiguities evidently caused by the absence of a symmetry in favor of covariance, the absence of sources and other structural problems.

Moreover, none of the gravitational conservation laws yields the conservation laws of special relativity in a clear and unambiguous way, precisely because of the lack of any limit of a Riemannian into the Minkowskian space. Under these conditions, the compatibility of general relativity with the special reduces to personal beliefs outside a rigorous scientific process. Therefore, we have the following

THEOREM 1.4.7 [11]: *The total conservation laws of general relativity are incompatible with those of special relativity.*

Proof. The lack of compatibility of total conservation laws of general and special relativity is established by the fact that the latter is based on the Poincaré invariance while the former is merely a covariant theory, since total conservation laws are the generator of the Poincaré symmetry while having no role in covariance. **q.e.d.**

The above occurrence will be better illustrated in Chapter 3 where we shall show that the achievement of a rigorous compatibility between total conservation laws of special and general relativity will require their assumption as the generators of a symmetry for both relativities, a task that can only be accomplished via a new mathematics specifically conceived for gravitation.

Another controversy that remained unresolved in the 20-th century (primarily because of lack of sufficient consideration by scholars in the field) is that, during its early stages, gravitation was divided into the *exterior and interior problems.* For instance, Schwartzschild wrote *two* articles on gravitation, one on the exterior [57] and one on the interior problem [58].

However, it soon became apparent that general relativity was structurally unable to represent interior problems for numerous reasons, such as the impossibility of incorporating shape, density, local variations of the speed of light within physical media via the familiar law we study in high school $c = c_\circ/n$ (which variation cannot be ignored classically),

inability to represent interior contact interactions with a first-order Lagrangian, structural inability to represent interior nonconservation laws (such as the vortices in Jupiter's atmosphere with variable angular momenta), structural inability to represent entropy, its increase and other thermodynamical laws, etc. (see Ref. [11] for brevity).

Consequently, Schwartzschild's solution for the *exterior* problem became part of history (evidently because aligned with organized interests in general relativity), while his *interior* solution has remained vastly ignored to this day (evidently because not aligned with said organized interests). In particular, the constituents of all astrophysical bodies have been abstracted as being point-like, an abstraction that is beyond the boundaries of science for classical treatments; all distinctions between exterior and interior problems have been ignored in the majority of the vast literature in the field; and gravitation has been tacitly reduced to one single problem.

Nevertheless, as indicated earlier, major structural problems grow in time when ignored, rather than disappearing. The lack of addressing the interior gravitational problem is causing major distortions in astrophysics, cosmology and other branches of science (see also next section). We have, therefore, the following important result:

THEOREM 1.4.6 [11]: *General relativity is incompatible with the experimental evidence on interior gravitational problems.*

By no means the above analysis exhaust all inconsistencies of general relativity, and numerous additional ones do indeed exist, such as that expressed by the following:

THEOREM 1.4.7 [11b]: *Operator images of Riemannian formulations of gravitation are inconsistent on mathematical and physical grounds.*

Proof. As established by Theorem 1.4.1, classical formulations of Riemannian gravitation are noncanonical. Consequently, all their operator counterparts must be nonunitary for evident reasons of compatibility. But nonunitary theories are known to be inconsistent on both mathematical and physical grounds [11b]. In fact, on mathematical grounds, nonunitary theories of quantum gravity (see, e.g., Refs. [78–80]) do not preserve in time the basic units, fields and spaces, while, on physical grounds, the said theories do not possess time invariant numerical predictions, do not possess time invariant Hermiticity (thus having no acceptable observables), and violate causality. **q.e.d**

The reader should keep in mind the additional well known inconsistencies of quantum gravity, such as its historical incompatibility with quantum mechanics, the lack of a credible PCT theorem, etc.

To avoid raising issues of scientific ethics, all these inconsistencies establish beyond a scientific, or otherwise credible, doubt, the need for a profound revision of the gravitational views of the 20-th century. Studies along these lines will be presented in Chapter 3.

1.5 HADRONIC MECHANICS

1.5.1 Foreword

The isodual theory of antimatter is a particular case of a broadening of quantum mechanics known as *hadronic mechanics.* A knowledge of the latter mechanics is mandatory for any treatment of antiparticles, such as antiprotons and antineutrons, beyond the academic abstraction as being point-like, since the latter abstraction is necessary for the applicability of quantum mechanics.

In turn, the representation of antiprotons and antineutrons as they are in the physical reality, extended, nonspherical, deformable and hyperdense, can be best achieved via the study, first, of the representation of extended protons and neutrons within the context of hadronic mechanics, and then the transition to their antiparticle forms via isoduality.

When facing the limitations of special relativity and quantum mechanics for the representation of extended, nonspherical, deformable and hyperdense particles and antiparticles under linear and nonlinear, local and nonlocal as well as potential and nonpotential forces, a rather general attitude is that of attempting their generalization via the broadening into noncanonical and nonunitary structures, respectively, while preserving the mathematics of their original formulation.

Despite the widespread publication of papers on theories with noncanonical or nonunitary structures in refereed journals, including those of major physical societies, it is not generally known that these broader theories too are afflicted by inconsistencies so serious to be equally called catastrophic.

A central scope of this monograph is the detailed identification of these inconsistencies because their only known resolution is that presented in the next chapters, that permitted by *new mathematics* specifically constructed from the physical conditions considered.

In fact, the broadening of special relativity and quantum mechanics into noncanonical and nonunitary forms, respectively, is necessary to exit form the class of equivalence of the conventional formulations. The resolution of the catastrophic inconsistencies of these broader for-

mulations when treated via the mathematics of canonical and unitary theories, then leaves no other possibility than that of it broadening the basic mathematics.

Therefore, in the next two sections we shall review the inconsistencies of noncanonical and nonunitary theories. The remaining sections of this chapter are devoted to an outline of hadronic mechanics so as to allow the reader to enter in a progressive way into the advanced formulations presented in the next chapters.

1.5.2 Catastrophic Inconsistencies of Noncanonical Theories

As recalled in Section 1.3, the research in classical mechanics of the 20-th century has been dominated by *Hamiltonian systems*, that is, systems admitting their complete representation via the *truncated Hamilton equations* (1.3.1), namely, the historical equations proposed by Hamilton in which the external terms have been cut out.

For the scope of this section, it is best to rewrite Eq. (1.3.1) in the following unified form[16]

$$b = (b^\mu) = (r, p) = (r^k, p_k), \tag{1.5.1a}$$

$$\frac{db^\mu}{dt} = \omega^{\mu\nu} \times \frac{\partial H(t, b)}{\partial b^\nu}, \tag{1.5.1b}$$

$$H = K(p) + V(t, r, p), \tag{1.5.1c}$$

$$\mu = 1, 2, 3, \ldots, 6, \quad k = 1, 2, 3,$$

where H is the Hamtiltonian, K is the kinetic energy, V is the potential energy, $\omega^{\mu\nu}$ is the canonical Lie tensor with explicit form

$$\omega^{\mu\nu} = \begin{pmatrix} 0 & I_{3\times 3} \\ -I_{3\times 3} & 0 \end{pmatrix} \tag{1.5.2}$$

and $I_{3\times 3} = Diag(1, 1, 1)$ is the unit matrix.

In the above unified notation, the brackets of the time evolution can be written

$$\frac{dA}{dt} = [A, H] = \frac{\partial A}{\partial b^\mu} \times \omega^{\mu\nu} \times \frac{\partial H}{\partial b^\nu}, \tag{1.5.3}$$

and they characterize a *Lie algebra*, as well known.

The above equations have a *canonical structure*, namely, their time evolution characterizes a *canonical transformation*[17],

$$b^\mu \to b'^\mu(b), \tag{1.5.4a}$$

$$\omega^{\mu\nu} \to \frac{\partial b'^\mu}{\partial b^\rho} \times \omega^{\rho\sigma} \times \frac{\partial b'^\nu}{\partial b^\sigma} \equiv \omega^{\mu\nu}; \tag{1.5.4b}$$

and the theory possesses the crucial property of predicting the same numbers under the same conditions at different times, a property generically referred to as *invariance*, such as the invariance of the basic analytic equations under their own time evolution

$$\frac{db^\mu}{dt} - \omega^{\mu\nu} \times \frac{\partial H(t,b)}{\partial b^\nu} = 0 \rightarrow \frac{db'^\mu}{dt} - \omega^{\mu\nu} \times \frac{\partial H(t',b')}{\partial b'^\nu} = 0, \qquad (1.5.5)$$

where the invariance is expressed by the preservation of the Lie tensor $\omega^{\mu\nu}$ and of the Hamiltonian H.

It is easy to predict that future research in classical mechanics will be dominated by *non-Hamiltonian systems*, that is, systems that cannot be entirely described by the Hamiltonian and require at least a second quantity for their complete description.

Alternatively, we are referring to systems with internal forces that are partly of potential type, represented by V, and partly of nonpotential type, thus requiring new quantities for their representation.

Also equivalently, we are referring to the transition from *exterior dynamical systems* recalled in Section 1.3 (systems of point-like particles moving in vacuum without collisions under sole action-at-a-distance potential interactions) to *interior dynamical systems* (extended, nonspherical and deformable particles moving within a resistive medium with action-at-a-distance potential forces plus contact, nonpotential, nonlocal, and integral forces).

As also recalled in Section 1.3, exterior dynamical systems can be easily represented with the truncated Hamilton equations, while the first representation of the broader non-Hamiltonian systems is given precisely by the historical analytic equations with external terms, Eq. (1.3.2) that we now rewrite in the unified form

$$\frac{db^\mu}{dt} = \omega^{\mu\nu} \times \frac{\partial H(t,b)}{\partial b^\nu} + F^\mu(t, b, \dot{b}, \ldots), \qquad (1.5.6a)$$

$$F^\mu = (0, F_k), \quad \mu = 1, 2, \ldots, 6, \quad k = 1, 2, 3. \qquad (1.5.6b)$$

Nevertheless, as also recalled in Section 1.3, the addition of the external terms creates serious structural problems since the brackets of the new time evolution

$$\frac{dA}{dt} = (A, H, F) = \frac{\partial A}{\partial b^\mu} \times \omega^{\mu\nu} \times \frac{\partial H}{\partial b^\nu} + \frac{\partial A}{\partial b^\mu} \times F^\mu, \qquad (1.5.7)$$

violate the conditions to characterize an algebra (since they violate the right distributive and scalar laws), let alone violate all possible Lie algebras, thus prohibiting the studies of basic aspects, such as spacetime symmetries under nonpotential forces.

As experienced by the author, when facing the latter problems, a rather natural tendency is that of using coordinate transforms $b \to b'(b)$ to turn a systems that is non-Hamiltonian in the b-coordinates into a Hamiltonian form in the b'-coordinates,

$$\frac{db^\mu}{dt} - \omega^{\mu\nu} \times \frac{\partial H(t,b)}{\partial b^\nu} - F^\mu(t,b,\dot{b},\ldots) = 0$$

$$\to \frac{db'^\mu}{dt} - \omega^{\mu\nu} \times \frac{\partial H'(t,b')}{\partial b^\nu} = 0. \tag{1.5.8}$$

These transformations always exist under the necessary continuity and regularity conditions, as guaranteed by *Lie-Koening theorem* of analytic mechanics or the *Darboux theorem* of the symplectic geometry) [54,55].

This first attempt has no physical value because of excessive problems, such as: the lack of physical meaning of quantum formulations in the b'-coordinates; the impossibility of placing a measuring apparatus in the transformed coordinates; the loss of all known relativities, due to the necessarily nonlinear character of the transforms with consequential mapping of inertial into noninertial frames; and other problems.

The above problems force the restriction of analytic representations of non-Hamiltonian systems within the fixed coordinates of the experimenter, the so-called *direct analytic representations* [54].

Under the latter restriction, the second logical attempt for quantitative treatments of non-Hamiltonian systems is that of broadening conventional canonical theories into a noncanonical form at least admitting a consistent algebra in the brackets of the time evolution, even though the resulting the time evolution of the broader equations cannot characterize a canonical transformation.

As an illustration of these second lines of research, in 1978 the author wrote for Springer-Verlag his first volume of *Foundations of Theoretical Mechanics* [54a] devoted to the integrability conditions for the existence of a Hamiltonian representation (the so-called *Helmholtz's conditions of variational selfadjointness*). The evident scope was that of identifying the limits of applicability of the theory within the fixed coordinates of the experimenter.

A main result was the proof that *the truncated Hamilton equations admit a direct analytic representation in three space dimensions only of systems with potential (variationally selfadjoint) forces*,[18] thus representing only a small part of what are generally referred to as *Newtonian systems.*

In this way, monograph [54a] confirmed the need to enlarge conventional Hamiltonian mechanics within the fixed frame of the experimenter

in such a way to admit a direct representation of all possible Newtonian systems verifying the needed regularity and continuity conditions.

Along the latter line of research, in 1982 the author published with Springer-Verlag his second volume of *Foundations of Theoretical Mechanics* [54b] for the specifically stated objective of broadening conventional Hamiltonian mechanics in such a way to achieve *direct universality*, that is, the capability of representing *all* Newtonian systems (universality) in the fixed frame of the experimenter (direct universality), while jointly preserving not only an algebra, but actually the *Lie algebra* in the brackets of the time evolution.

These efforts gave birth to a broader mechanics called by the author *Birkhoffian mechanics* in honor of the discoverer of the basic equations, G. D. Birkhoff [59], which equations can be written in the unified form

$$\frac{db^\mu}{dt} = \Omega^{\mu\nu}(b) \times \frac{\partial B(t,b)}{\partial b^\nu}, \tag{1.5.9}$$

where $B(t,b)$ is called the *Birkhoffian* in order to distinguish it from the Hamiltonian (since B does not generally represent the total energy), and $\Omega^{\mu\nu}$ is a *generalized Lie tensor*, in the sense that the new brackets

$$\frac{dA}{dt} = [A,B]^* = \frac{\partial A}{\partial b^\mu} \times \Omega^{\mu\nu} \times \frac{\partial B}{\partial b^\nu}, \tag{1.5.10}$$

still verify the Lie algebra axioms (see Ref. [54b] for details).

Stated in different terms, the main efforts of monograph [54b] were to show that, under the necessary continuity and regularity properties, the historical Hamilton's equations with external terms always admit a reformulation within the fixed frame of the experimenter with a consistent Lie algebra in the brackets of the time evolution,

$$\frac{db^\mu}{dt} = \omega^{\mu\nu} \times \frac{\partial H(t,b)}{\partial b^\nu} + F^\mu(t,b,\ldots) \equiv \Omega^{\mu\nu}(b) \times \frac{\partial B(t,b)}{\partial b^\nu}. \tag{1.5.11}$$

In this case, rather than being represented with H and F, non-Hamiltonian systems are represented with B and Ω.

Monograph [54b] achieved in full the intended objective with the proof that *Birkhoffian mechanics is indeed directly universal for Newtonian systems*, and admits the following *generalized canonical transformations*,

$$\Omega^{\mu\nu}(b) \to \frac{\partial b'^\mu}{\partial b^\rho} \times \Omega^{\rho\sigma}(b(b')) \times \frac{\partial b'^\nu}{\partial b^\sigma} \equiv \Omega^{\mu\nu}(b'). \tag{1.5.12}$$

Monograph [54b] concluded with the indication of the apparent full equivalence of the Birkhoffian and Hamiltonian mechanics, since the latter is admitted as a particular case of the former (when the generalized

Lie tensor acquires the canonical form), both theories are derivable from a variational principle, and both theories admit similar transformation properties.

Since the generalized Lie tensor $\Omega^{\mu\nu}$ and related brackets $[A, B]^*$ are antisymmetric, we evidently have conservation laws of the type

$$\frac{dB}{dt} = [B, B]* \equiv 0, \tag{1.5.13}$$

Consequently, Birkhoffian mechanics was suggested in monograph [54b] for the representation of *closed-isolated non-Hamiltonian systems* (such as Jupiter).

The representation of *open-nonconservative non-Hamiltonian systems* required the identification of a yet broader mechanics with a consistent algebra in the brackets of the time evolution, yet such that the basic brackets are not antisymmetric. The solution was reached in monographs [54b] via the *Birkhoffian-admissible mechanics* with basic analytic equations

$$\frac{db^\mu}{dt} = \omega^{\mu\nu} \times \frac{\partial H(t, b)}{\partial b^\nu} + F^\mu(t, b, \ldots) \equiv S^{\mu\nu}(b) \times \frac{\partial B(t, b)}{\partial b^\nu}, \tag{1.5.14}$$

where the tensor $S^{\mu\nu}$ is *Lie-admissible*. According to Santilli's [61–63] realization of Albert [60] abstract formulation, namely, in the sense that the generalized brackets of the time evolution

$$\frac{dA}{dt} = (A, B) = \frac{\partial A}{\partial b^\mu} \times S^{\mu\nu}(b) \times \frac{\partial B}{\partial b^\nu}, \tag{1.5.15}$$

do verify all conditions to characterize an algebra, and their attached antisymmetric brackets

$$[A, B]^* = (A, B) - (B, A), \tag{1.5.16}$$

characterize a generalized Lie algebra as occurring in Birkhoffian mechanics.

The representation of the open-nonconservative character of the equations was then consequential, since the lack of antisymmetry of the brackets yields the correct *time rate of variation of the energy* $E = B$

$$\frac{dE}{dt} = (E, E) = F_k \times v^k, \tag{1.5.17}$$

and the same occurs for all other physical quantities.

Monographs [54b] then proved the direct universality of Birkhoffian-admissible mechanics for all open-nonconservative systems, identified its

transformation theory and provided the following elementary, yet universal realization of the Lie-admissible tensor S for $B = H$ representing the total *nonconserved* energy

$$S^{\mu\nu} = \begin{pmatrix} 0 & I \\ -I & F/(\partial H/\partial p) \end{pmatrix}. \tag{1.5.18}$$

However, studies conducted after the publication of monographs [54] revealed the following seemingly innocuous feature:

LEMMA 1.5.1 [11b]: *Birkhoffian and Birkhoffian-admissible mechanics are noncanonical theories, i.e., the generalized canonical transformations, are noncanonical*,

$$\omega^{\mu\nu} \to \frac{\partial b'^{\mu}}{\partial b^{\rho}} \times \omega^{\rho\sigma} \times \frac{\partial b'^{\nu}}{\partial b^{\sigma}} \equiv \Omega^{\mu\nu}(b') \neq \omega^{\mu\nu}. \tag{1.5.19}$$

It is important to understand that *Birkhoffian and Birkhoffian-admissible mechanics are mathematically impeccable, but they are not recommended for physical applications, both classically as well as foundations of operator theories.*

The canonical Lie tensor has the well known explicit form (1.5.2). Therefore, the diagonal matrix $I_{3\times 3}$ is left invariant by canonical transformations. But $I_{3\times 3}$ is the *fundamental unit of the basic Euclidean geometry.* As such, it represents in an abstract and dimensionless form the basic units of measurement, such as

$$I_{3\times 3} = Diag.(1\text{cm}, 1\text{cm}, 1\text{cm}). \tag{1.5.20}$$

By their very definition, noncanonical transformations do not preserve the basic unit, namely, they are transformations of the representative type (with arbitrary new values)

$$I_{3\times 3} = Diag.(1\text{cm}, 1\text{cm}, 1\text{cm})$$

$$\to U \times I_{3\times 3} \times U^t = Diag.(3.127\ \text{cm},\ e^{-212}\ \text{cm},\ \log 45\ \text{cm}), \tag{1.5.21a}$$

$$U \times U^t \neq I, \tag{1.5.21b}$$

where t stands for transposed. We therefore have the following important:

THEOREM 1.5.1 [53]: *Whether Lie or Lie-admissible, all classical noncanonical theories are afflicted by catastrophic mathematical and physical inconsistencies.*

Proof. Noncanonical theories do not leave invariant under time evolution the basic unit. This implies the loss under the time evolution of the base field on which the theory is defined. Still in turn, the loss in time of the base field implies catastrophic mathematical inconsistencies, such as the lack of preservation in time of metric spaces, geometries, symmetries, etc., since the latter are defined over the field of real numbers.

Similarly, noncanonical theories do not leave invariant under time evolution the basic units of measurements, thus being inapplicable for consistent measurements. The same noncanonical theories also do not possess time invariant numerical predictions, thus suffering catastrophic physical inconsistencies. **q.e.d.**

In conclusion, the regaining of a consistent algebra in the brackets of the time evolution, as it is the case for Birkhoffian and Birkhoffian-admissible mechanics, is not sufficient for consistent physical applications because the theories remain noncanonical. In order to achieve a physically consistent representation of non-Hamiltonian systems, it is necessary that

1) The analytic equations must be derivable from a first-order variational principle, as necessary for quantization;

2) The brackets of the time evolution must characterize a consistent algebra admitting exponentiation to a transformation group, as necessary to formulate symmetries; and

3) The resulting theory must be invariant, that is, must admit basic units and numerical predictions that are invariant in time, as necessary for physical value.

Despite the large work done in monographs [54], the achievement of all the above conditions required the author to resume classical studies from their foundations.

These third efforts finally gave rise to the new *Hamilton-Santilli iso-, geno- and hypermechanics* that do verify all conditions 1), 2) and 3), thus being suitable classical foundations of hadronic mechanics, as reviewed in Chapter 3.

However, the joint achievement of conditions 1), 2) and 3) for non-Hamiltonian systems required the prior construction of *basically new mathematics*, today known as *Santilli's iso-, geno- and hyper-mathematics*, as also reviewed in Chapter 3.

This section would be grossly incomplete and potentially misleading without a study of requirement 1), with particular reference to the derivability of analytic equations from a "first-order" variational principle.

Classical studies of non-Hamiltonian systems are essential, not only to identify the basic methods for their treatment, but above all to iden-

tify quantization channels leading to unique and unambiguous operator formulations.

Conventional Hamiltonian mechanics provides a solid foundation of quantum mechanics because it is derivable from the variational principle that we write in the unified notation

$$\delta A^{\circ} = \delta \int [R^{\circ}_{\mu}(b) \times db^{\mu} - H \times dt]$$

$$= \delta \int (p_k \times dr^k - H \times dt), \tag{1.5.22}$$

where the functions R°_{μ} have the canonical expression

$$(R^{\circ}_{\mu}) = (p_k, 0), \tag{1.5.23}$$

under which expression the canonical tensor assumes the realization

$$\omega_{\mu\nu} = \frac{\partial R^{\circ}_{\nu}}{\partial b^{\mu}} - \frac{\partial R^{\circ}_{\mu}}{\partial b^{\nu}}, \tag{1.5.24a}$$

$$(\omega_{\mu\nu}) = (\omega^{\alpha\beta})^{-1}. \tag{1.5.24b}$$

As it is well known, the foundations for quantization are given by the *Hamilton-Jacobi equations*

$$\frac{\partial A^{\circ}}{\partial t} = -H, \quad \frac{\partial A^{\circ}}{\partial b^{\mu}} = R^{\circ}_{\mu}, \tag{1.5.25}$$

that can be written explicitly in the familiar forms

$$\frac{\partial A^{\circ}}{\partial t} + H = 0, \tag{1.5.26a}$$

$$\frac{\partial A^{\circ}}{\partial r^k} - p_k = 0, \tag{1.5.26b}$$

$$\frac{\partial A^{\circ}}{\partial p_k} = 0, \tag{1.5.26c}$$

The use of the *naive quantization*

$$A^{\circ} \rightarrow -i \times \hbar \times \ell n\, \psi, \tag{1.5.27}$$

yields *Schrödinger's equations* in a unique and unambiguous way

$$\frac{\partial A^{\circ}}{\partial t} + H = 0 \rightarrow -i \times \hbar \frac{\partial \psi}{\partial t} - H \times \psi = 0, \tag{1.5.28a}$$

$$\frac{\partial A^{\circ}}{\partial r^k} = p_k \rightarrow -i \times \hbar \times \frac{\partial \psi}{\partial r^k} - p_k \times \psi = 0, \tag{1.5.28b}$$

$$\frac{\partial A^\circ}{\partial p_k} = 0 \rightarrow \frac{\partial \psi}{\partial p_k} = 0. \tag{1.5.28c}$$

The much more rigorous *symplectic quantization* yields exactly the same results and, as such, it is not necessary for these introductory notes.

A feature crucial for quantization is Eq. (1.5.26c) from which it follows that *the canonical action A° is independent from the linear momentum* and, consequently,

$$A^\circ = A^\circ(t, r). \tag{1.5.29}$$

an occurrence generally (but not universally) referred in the literature as characterizing a *first-order action functional.*

From the naive quantization it follows that, in the configuration representation, *the wave function originating from first-order action functionals is independent from the linear momentum* (and, vice-versa, in the momentum representation it is independent from the coordinates),

$$\psi = \psi(t, r), \tag{1.5.30}$$

which property is crucial for the axiomatic structure of quantum mechanics, e.g., for the correct formulation of Heisenberg's uncertainty principle, causality, Bell's inequalities, etc.

A serious knowledge of hadronic mechanics requires the understanding of the reason why *Birkhoffian mechanics* cannot be assumed as a suitable foundations for quantization. Birkhoff's equations can indeed be derived from the variational principle (see monograph [54b] for details)

$$\delta A = \delta \int [R_\mu(b) \times db^\mu - B \times dt] = 0, \tag{1.5.31}$$

where the new functions $R_\mu(b)$ have the general expression

$$(R_\mu(b)) = (A_k(t, r, p), B^k(t, r, p)), \tag{1.5.32}$$

subject to the regularity condition that Det. $\Omega \neq 0$, under which Birkhoff's tensor assumes the realization

$$\Omega_{\mu\nu}(b) = \frac{\partial R_\nu}{\partial b^\mu} - \frac{\partial R_\mu}{\partial b^\nu}, \tag{1.5.33a}$$

$$(\Omega_{\mu\nu}) = (\Omega)^{\alpha\beta})^{-1}, \tag{1.5.33b}$$

with *Birkhoffian Hamilton-Jacobi equations*

$$\frac{\partial A}{\partial t} = -B, \quad \frac{\partial A}{\partial b^\mu} = R_\mu. \tag{1.5.34}$$

As one can see, Birkhoffian expressions (1.5.31)–(1.5.34) appear to be greatly similar to the corresponding Hamiltonian forms (1.5.22)–(1.5.26).

Nevertheless, there is a fundamental structural difference between the two equations given by the fact that *the Birkhoffian action does indeed depend on the linear momenta*,

$$A = A(t, r, p), \tag{1.5.35}$$

a feature generally referred to as characterizing a *second-order action functional.*

As a consequence, *the "wavefunction" resulting from any quantization of Birkhoffian mechanics also depends on the linear momentum*,

$$\psi = \psi(t, r, p), \tag{1.5.36}$$

resulting in an operator mechanics that is beyond our current technical knowledge for quantitative treatment, since such a dependence would require a dramatic restructuring of all quantum axioms.

In fact, the use of a naive quantization,

$$A(t, r, p) \rightarrow -i \times \hbar \times \ell n\ \psi(t, r, p), \tag{1.5.37}$$

characterizes the following maps

$$\frac{\partial A}{\partial t} + B = 0 \rightarrow -i \times \hbar \frac{\partial \psi}{\partial t} - B \times \psi = 0, \tag{1.5.38a}$$

$$\frac{\partial A}{\partial b^{\mu}} - R_{\mu} = 0 \rightarrow -i \times \hbar \times \frac{\partial \psi}{\partial b^{\mu}} - R_{\mu} \times \psi = 0, \tag{1.5.38b}$$

A first problem is that the latter equations are generally nonlinear and, as such, they cannot be generally solved in the r- and p-operators. This causes the emergence of an operator mechanics in which it is impossible to define basic physical quantities, such as the linear momentum or the angular momentum, with consequential lack of currently known physical relevance at this moment.

On more technical terms, in the lifting of Hamiltonian into Birkhoffian mechanics, there is the replacement of the r-coordinates with the R-functions. In fact, the Birkhoffian action has the explicit dependence on the R-functions, $A = A[t, R(b)] = A'(t, r, p)$. As such, the Birkhoffian action can indeed be interpreted as being of first-order, but in the R-functions, rather than in the r-coordinates.

Consequently, a correct operator image of the Birkhoffian mechanics is given by the expressions first derived in Ref. [11b]. The correct operator image of Birkhoffian mechanics is then given by the equations

$$i \times \hbar \times \frac{\partial \psi[t, R(b)]}{\partial t} = B \times \psi[t, R(b)], \tag{1.5.39a}$$

$$-i \times \hbar \times \frac{\partial \psi[t, R(b)]}{\partial b^\mu} = R_\mu(b) \times \psi[t, R(b)]. \tag{1.5.39b}$$

As we shall see in Chapter 3, the above equations characterize a *covering of hadronic mechanics*, in the sense of being structurally more general, yet admitting hadronic mechanics as a particular case.

Even though mathematically impeccable, intriguing, and deserving further studies, the mechanics characterized by Eq. (1.4.39) is excessively general for our needs, and its study will be left to the interested reader.

The above difficulties identify quite precisely the first basic problem for the achievement of a physically consistent and effective formulation of hadronic mechanics, consisting in the *need of constructing a new mathematics capable of representing closed non-Hamiltonian systems via a first-order variational principle (as required for consistent quantization), admitting antisymmetric brackets in the time evolution (as required by conservation laws), and possessing time invariant units and numerical predictions (as required for physical value).*

The need to construct a new mathematics is evident from the fact that no pre-existing mathematics can fulfill the indicated needs. As we shall see in Chapter 3, *Santilli's isomathematics* has been constructed precisely for and does indeed solve these specific problems.

The impossibility of assuming the *Birkhoffian-admissible mechanics* as the foundation of operator formulation for open non-Hamiltonian systems is clearly established by the fact that said mechanics *is not derivable from a variational principle.*[19]

The latter occurrence identifies a much more difficult task given by the *need to construct a yet broader mathematics capable of representing open non-Hamiltonian systems via a first-order variational principle (as required for consistent quantization), admitting non-antisymmetric brackets in the time evolution (as required by non-conservation laws), and possessing time invariant units and numerical predictions (as required by physical value).*

The lack of any pre-existing mathematics for the fulfillment of the latter tasks is beyond credible doubt. Rather than adapting nature to pre-existing mathematics, the author has constructed a yet broader mathematics, today known as *Santilli's genomathematics*, that does indeed achieve all indicated objectives, as outlined in Chapter 4.

Readers interested in the depth of knowledge are suggested to meditate a moment on the implications of the above difficulties. In fact, these

difficulties have caused the impossibility in the 20-th century to achieve a meaningful operator formulation of contact, nonconservative and nonpotential interactions. A consequence has been the widespread belief that nonpotential interactions "do not exist" in the particle world, a view based on the lack of existence of their operator representation, with negative implications at all levels of knowledge, such as the impossibility of achieving a meaningful understanding of the origin of irreversibility.

As a consequence, the resolution of the difficulties in the quantization of nonpotential interactions achieved by hadronic mechanics implies a rather profound revision of most of the scientific views of the 20-th century, as we shall see in Chapters 3, 4, and 5.

1.5.3 Catastrophic Inconsistencies of Nonunitary Theories

Once the limitations of quantum mechanics are understood (and admitted), another natural tendency is to exit from the class of equivalence of the theory via suitable generalizations, while keeping the mathematical methods used for quantum mechanics.

It is important for these studies to understand that these efforts are afflicted by catastrophic mathematical and physical inconsistencies equivalent to those suffered by classical noncanonical formulations based on the mathematics of canonical theories.

The author has dedicated his research life to the construction of axiomatically consistent and invariant generalizations of quantum mechanics for the treatment of nonlinear, nonlocal, and nonpotential effects because they are crucial for the prediction and treatment of new clean energies and fuels.

In this section we review the foundations of these studies with the identification, most importantly, of the failed attempts in the hope of assisting receptive colleagues in avoiding the waste of their time in the study of theories that are mathematically significant, yet cannot possibly have real physical value.

To begin, let us recall that a theory is said to be *equivalent to quantum mechanics* when it can be derived from the latter via any possible *unitary transform* on a conventional Hilbert space $\mathcal{H}$ over the field of complex numbers $C = C(c, +, \times)$,

$$U \times U^\dagger = U^\dagger \times U = I, \tag{1.5.40}$$

under certain conditions of topological smoothness and regularity hereon ignored for simplicity, where "$\times$" represents again the conventional associative product of numbers or matrices, $U \times U^\dagger \equiv UU^\dagger$.

As a consequence, *a necessary and sufficient condition for a theory to be inequivalent to quantum mechanics is that it must be outside its class of unitary equivalence*, that is, the new theory is connected to quantum mechanics via a *nonunitary transform*

$$U \times U^\dagger \neq I. \tag{1.5.41}$$

generally defined on a conventional Hilbert space $\mathcal{H}$ over C.

Therefore, true generalized theories must have a *nonunitary structure*, i.e., their time evolution must verify law (1.5.41), rather than (1.5.40).[20]

During his graduate studies in physics at the University of Torino, Italy, and as part of his Ph. D. thesis, Santilli [61] published in 1967 the following *parametric deformation of the Lie product* $A \times B - B \times A$, the first in scientific records

$$\begin{aligned}(A, B) &= p \times A \times B - q \times H \times A \\ &= m \times (A \times B - B \times A) + n \times (A \times B + B \times A) \\ &= m \times [A, B] + n \times \{A, B\},\end{aligned} \tag{1.5.42}$$

where $p = m + n, q = n - m$ and $p \pm q$ are non-null parameters.

By remembering that the Lie product characterizes *Heisenberg's equations*, the above generalized product was submitted as part of the following *parametric generalization of Heisenberg's equations* in its finite and infinitesimal forms [61–63]

$$A(t) = U \times A(0) \times U^\dagger = e^{i \times H \times q \times t} \times A(0) \times e^{-i \times t \times p \times H}, \tag{1.5.43a}$$

$$i\, dA/dt = (A, H) = p \times A \times H - q \times H \times A, \tag{1.5.43b}$$

with classical counterpart studied in Ref. [62]. After an extensive research in European mathematics libraries (conducted prior to the publication of Ref. [61] with the results listed in the same publication), the brackets $(A, B) = p \times A \times B - q \times B \times A$ resulted to be *Lie-admissible* according to A. A. Albert [60], that is, the brackets are such that their attached antisymmetric product

$$[A\hat{,}B] = (A, B) - (B, A) = (p + q) \times [A, B], \tag{1.5.44}$$

characterizes a *Lie algebra.*

Jointly, brackets (A, B) are *Jordan admissible* also according to Albert, in the sense that their attached symmetric product,

$$\{A\hat{,}B\} = (A, B) + (B, A) = (p + q) \times \{A, B\}, \tag{1.5.45}$$

characterizes a *Jordan algebra.*[21]

At that time (1967), only three articles on this subject had appeared in Lie- and Jordan-admissibility solely in the sole mathematical literature (see Ref. [61]).

In 1978, when at Harvard University, Santilli proposed the following *operator deformation of the Lie product* [Ref. [104], Eq. (4.15.34) and (4.18.11)],

$$\begin{aligned}(A\hat{,}B) &= A \lhd B - B \rhd A \\ &= A \times P \times B - B \times Q \times A \\ &= (A\times T\times B - B\times T\times A) + (A\times W\times B + B\times W\times A) \\ &= [A\hat{,}B] + \{A\hat{,}B\}, \end{aligned} \tag{1.5.46}$$

where $P = T + W, Q = W - T$ and $P \pm Q$ are, this time, fixed non-null matrices or operators.

Evidently, product (1.5.46) remains jointly Lie-admissible and Jordan-admissible because the attached antisymmetric and symmetric brackets,

$$[A\hat{,}B] = (A\hat{,}B) - (B\hat{,}A) = A \times T \times B - B \times T \times A, \tag{1.5.47a}$$

$$\{A\hat{,}B\} = (A\hat{,}B) + (B\hat{,}A) = A \times W \times B + B \times W \times A, \tag{1.5.47b}$$

characterizes a *Lie-Santilli and Jordan-Santilli isoalgebra* (see Chapter 3 for details).

The reader should be aware that the following alternative versions of product (1.5.46),

$$P \times A \times B - Q \times B \times A, \tag{1.5.48a}$$

$$A \times B \times P - B \times A \times Q, \tag{1.5.48b}$$

do not constitute an algebra since the former (latter) violates the left (right) distributive and scalar laws.

The above operator deformations of the Lie product was also submitted for the following broader *operator Lie-admissible and Jordan-admissible generalization of Heisenberg's equations* in its finite and infinitesimal forms [104]

$$A(t) = U \times A(0) \times U^\dagger = e^{i\times H\times Q\times t} \times A(0) \times e^{-i\times t\times P\times H}, \tag{1.5.49a}$$

$$\begin{aligned} i\ dA/dt = (A\hat{,}H) &= A \lhd H - H \rhd A \\ &= A \times P \times H - H \times Q \times A, \end{aligned} \tag{1.5.49b}$$

$$P = Q^\dagger, \tag{1.5.49c}$$

which equations, as we shall see in Chapter 3, are the fundamental equations of hadronic mechanics following proper mathematical treatment.

It is an instructive exercise for the reader interested in learning the foundation of hadronic mechanics to prove that:

1) Time evolutions (1.5.43) and (1.5.49) are *nonunitary*, thus being outside the class of unitary equivalence of quantum mechanics;

2) The application of a nonunitary transform $R \times R^\dagger \neq I$ to structure (1.5.43) yields precisely the broader structure (1.5.49) by essentially transforming the parameters p and q into the operators

$$P = p \times (R \times R^\dagger)^{-1}, Q = q \times (R \times R^\dagger)^{-1}; \tag{1.5.50}$$

3) The application of additional nonunitary transforms $S \times S^\dagger \neq I$ to structure (1.5.49) preserves its Lie-admissible and Jordan-admissible character, although with different expressions for the P and Q operators.

The above properties prove the following:

LEMMA 1.5.2 [11b]: *General Lie-admissible and Jordan-admissible laws (1.5.49) are "directly universal" in the sense of containing as particular cases all infinitely possible nonunitary generalizations of quantum mechanical equations ("universality") directly in the frame of the observer ("direct universality"), while admitting a consistent algebra in their infinitesimal form.*

The above property can be equally proven by noting that the product $(A\hat{,}B)$ is the most general possible "product" of an "algebras" as commonly understood in mathematics (namely, a vector space with a bilinear composition law verifying the right and left distributive and scalars laws).

In fact, the product $(A\hat{,}B)$ constitutes the most general possible combination of Lie and Jordan products, thus admitting as particular cases *all* known algebras, such as associative algebras, Lie algebras, Jordan algebras, alternative algebras, supersymmetric algebras, Kac-Moody algebras, *etc.*

Despite their unquestionable mathematical beauty, theories (1.5.43) and (1.5.49) possess the following catastrophic physical and mathematical inconsistencies:

THEOREM 1.5.2 [53] *(see also Refs.* [70-77]*): All theories possessing a nonunitary time evolution formulated on conventional Hilbert spaces $\mathcal{H}$ over conventional fields of complex numbers $C(c, +, \times)$ do not admit consistent physical and mathematical applications because:*

1) They do not possess invariant units of time, space, energy, etc., thus lacking physically meaningful application to measurements;

2) They do not conserve hermiticity in time, thus lacking physically meaningful observables;

3) They do not possess unique and invariant numerical predictions;

4) They generally violate probability and causality laws; and

5) They violate the basic axioms of Galileo's and Einstein's relativities.

Nonunitary theories are also afflicted by catastrophic mathematical inconsistencies.

The proof of the above theorem is essentially identical to that of Theorem 1.5.1 (see Ref. [53] for details). Again, the basic unit is not an abstract mathematical notion, because it embodies the most fundamental quantities, such as the units of space, energy, angular momentum, etc.

The nonunitary character of the theories here considered then causes the lack of conservation of the numerical values of such units with consequential catastrophic inapplicability of nonunitary theories to measurements.

Similarly, it is easy to prove that the condition of Hermiticity at the initial time,

$$(\langle\phi| \times H^\dagger) \times |\psi\rangle \equiv \langle\phi| \times (H \times |\psi\rangle), \;\; H = H^\dagger, \tag{1.5.51}$$

is violated at subsequent times for theories with nonunitary time evolution when formulated on $\mathcal{H}$ over C. This additional catastrophic inconsistency (known as *Lopez's lemma* [71,72]), can be expressed by

$$\begin{aligned} &[\langle\psi| \times U^\dagger \times (U \times U^\dagger)^{-1} \times U \times H \times U^\dagger] \times U|\psi\rangle \\ &= \langle\psi| \times U^\dagger \times [(U \times H \times U^\dagger) \times (U \times U^\dagger)^{-1} \times U|\psi\rangle] \\ &= (\langle\hat{\psi} \times T \times H'^\dagger) \times |\hat{\psi}\rangle = \langle\hat{\psi}| \times (\hat{H} \times T \times |\hat{\psi}\rangle), \qquad (1.5.52a) \\ &|\hat{\psi}\rangle = U \times |\psi\rangle, \;\; T = (U \times U^\dagger)^{-1} = T^\dagger, \qquad (1.5.52b) \\ &H'^\dagger = T^{-1} \times \hat{H} \times T \neq H. \qquad (1.5.52c) \end{aligned}$$

As a result, nonunitary theories do not admit physically meaningful observables.

Assuming that the preceding inconsistencies can be by-passed with some manipulation, nonunitary theories still remain with additional catastrophic inconsistencies, such as the lack of invariance of numerical predictions.

To illustrate this additional inconsistency, suppose that the considered nonunitary theory is such that, at $t = 0$ sec, $U \times U^\dagger_{[t=0]} = 1$, at $t = 15$ sec,

$U \times U^{\dagger}_{[t=15]} = 15$, and the theory predicts at time $t = 0$ sec, say, the eigenvalue of 2 eV,

$$H|_{t=0} \times |\psi >= 2\ eV \times |\psi > . \tag{1.5.53}$$

It is then easy to see that the same theory predicts under the same conditions the *different* eigenvalue 30 eV at $t = 15$ sec, thus having no physical value of any type. In fact, we have

$$U \times U^{\dagger}|_{t=0} = I, \quad U \times U^{\dagger}|_{t=15} = 15, \tag{1.5.54a}$$

$$\begin{aligned} U \times H \times |\psi\rangle &= (U \times H \times U^{\dagger}) \times (U \times U^{\dagger})^{-1} \times (U \times |\psi\rangle) \\ = H' \times T \times |\hat{\psi}\rangle &= U \times E \times |\psi\rangle = E \times (U \times |\psi\rangle) = E \times |\hat{\psi}\rangle, \end{aligned} \tag{1.5.54b}$$

$$H' = U \times H \times U^{\dagger}, \quad T = (U \times U^{\dagger})^{-1},$$

$$H' \times |\hat{\psi}\rangle\ |_{t=0} = 2\ \text{eV} \times |\hat{\psi}\rangle\ |_{t=0}, \quad T = 1\ |_{t=0}, \tag{1.5.54c}$$

$$\begin{aligned} H' \times |\hat{\psi}\rangle\ |_{t=15}\ 2\ \text{eV} \times (U \times U^{\dagger}) \times |\hat{\psi}\rangle\ |_{t=15} \\ = 30\ \text{eV} \times |\hat{\psi}\rangle\ |_{t=15}\ . \end{aligned} \tag{1.5.54d}$$

Probability and causality laws are notoriously based on the unitary character of the time evolution and the invariant decomposition of the unit.

Their violation for nonunitary theories is then evident. It is an instructive exercise for the reader interested in learning hadronic mechanics, superconductivity and chemistry to identify a specific example of nonunitary transforms for which the effect *precedes* the cause.

The violation by nonunitary theories of the basic axioms of Galileo's Einstein's relativities is so evident to require no comment.

An additional, most fundamental inconsistency of the theories considered is their *noninvariance*, that can be best illustrated with the lack of invariance of the general Lie-admissible and Jordan-admissible laws (1.5.49).

In fact, under nonunitary transforms, we have, e.g., the lack of invariance of the Lie-admissible and Jordan-admissible product,

$$U \times U^{\dagger} \neq I \tag{1.5.55a}$$

$$\begin{aligned} U \times (A \hat{,} B) \times U^{\dagger} &= U \times (A \lhd B - B \rhd A) \times U^{\dagger} = (U \times A \times U^{\dagger}) \\ &\times [(U \times U^{-1}) \times (U \times P \times U^{\dagger}) \times (U \times U^{\dagger})^{-1}] \times (U \times B \times U^{\dagger}) \\ &- (U \times B \times U^{\dagger}) \times [(U \times U^{-1}) \times (U \times Q \times U^{\dagger}) \times (U \times U^{\dagger})^{-1}] \\ &\times (U \times A \times U^{\dagger}) = A' \times P' \times B' - B' \times Q' \times A' \\ &= A' \lhd' B' - B' \rhd' A'. \end{aligned} \tag{1.5.55b}$$

The above rules confirm the preservation of a Lie-admissible structure under the most general possible transforms, thus confirming the direct universality of laws (1.4.49) as per Theorem 1.4.2. The point is that *the formulations are not invariant* because

$$P' = (U \times U^{-1}) \times (U \times Q \times U^{\dagger}) \times (U \times U^{\dagger})^{-1} \neq P, \qquad (1.5.56a)$$

$$Q' = (U \times U^{-1}) \times (U \times Q \times U^{\dagger}) \times (U \times U^{\dagger})^{-1} \neq Q, \qquad (1.5.56b)$$

that is, because *the product itself is not invariant.*

By comparison, the invariance of quantum mechanics follows from the fact that the associative product "$\times$" is not changed by unitary transforms

$$U \times U^{\dagger} = U^{\dagger} \times U = I, \qquad (1.5.57a)$$

$$A \times B \to U \times (A \times B) \times U^{\dagger}$$

$$= (U \times A \times U^{\dagger}) \times (U \times U^{\dagger})^{-1} \times (U \times B \times U^{\dagger}) = A' \times B'. \qquad (1.5.57b)$$

Therefore, generalized Lie-admissible and Jordan-admissible theories (1.5.49) are not invariant because the generalized products "$\lhd$" and "$\rhd$" are changed by nonunitary transformations, including the time evolution of the theory itself. The same results also holds for other nonunitary theories, as the reader is encouraged to verify.

The mathematical inconsistencies of nonunitary theories are the same as those of noncanonical theories. Recall that mathematics is formulated over a given field of numbers. Whenever the theory is nonunitary, the first noninvariance is that of the basic unit of the field.

The lack of conservation of the unit then causes the loss of the basic field of numbers on which mathematics is constructed. It then follows that the entire axiomatic structure as formulated at the initial time, is no longer applicable at subsequent times.

For instance, the formulation of a nonunitary theory on a conventional Hilbert space has no mathematical sense because that space is defined over the field of complex numbers.

The loss of the latter property under nonunitary transforms then implies the loss of the former. The same result holds for metric spaces and other mathematics based on a field.

In short, the lack of invariance of the fundamental unit under nonunitary time evolutions causes the catastrophic collapse of the entire mathematical structure, without known exception.

The reader should be aware that the above physical and mathematical inconsistencies apply not only for Eq. (1.5.49) but also for a large number of generalized theories, as expected from the direct universality of the former.

It is of the essence to identify in the following at least the most representative cases of physically inconsistent theories, to prevent their possible application (see Ref. [53] for details):

1) Dissipative nuclear theories [87] represented via an imaginary potential in non-Hermitean Hamiltonians,

$$H = H_0 = iV \neq H^\dagger \tag{1.5.58}$$

lose all algebras in the brackets of their time evolution (requiring a bilinear product) in favor of the triple system,

$$i \times \ dA/dt = A \times H - H^\dagger \times A = [A, H, H^\dagger] \tag{1.5.59}$$

This causes the loss of nuclear notions such as "protons and neutrons" as conventionally understood, e.g., because the definition of their spin mandates the presence of a consistent algebra in the brackets of the time evolution.

2) Statistical theories with an external collision term C (see Ref. [88] and literature quoted therein) and equation of the density

$$i \ d\rho/dt = \rho \odot H = [\rho, H] + C, \ \ H = H^\dagger, \tag{1.5.60}$$

violate the conditions for the product $\rho \odot H$ to characterize any algebra, as well as the existence of exponentiation to a finite transform, let alone violating the conditions of unitarity.

3) The so-called "q-deformations" of the Lie product (see, e.g., [64–69] and very large literature quoted therein)

$$A \times B - q \times B \times A, \tag{1.5.61}$$

where q is a non-null scalar, that are a trivial particular case of Santilli's (p, q)-deformations (1.5.42).

4) The so-called "k-deformations" [81–84] that are a relativistic version of the q-deformations, thus also being a particular case of general structures (1.4.42).

5) The so-called "star deformations" [89] of the associative product

$$A \star B = A \times T \times B, \tag{1.5.62}$$

where T is fixed, and related generalized Lie product

$$A \star B - B \star A, \tag{1.5.63}$$

are manifestly nonunitary and *coincide* with Santilli's Lie-isotopic algebras [104].

6) Deformed creation-annihilation operators theories [99,100].

7) Nonunitary statistical theories [92].

8) Irreversible black holes dynamics with Santilli's Lie-admissible structure (1.4.46) [93].

9) Noncanonical time theories [94–96].

10) Supersymmetric theories [101,102] with product

$$(A, B) = [A, B] + \{A, B\}$$

$$= (A \times B - B \times A) + (A \times B + B \times A), \tag{1.5.64}$$

are an evident particular case of Santilli's Lie-admissible product (1.4.46) with $T = W = I$.

11) String theories [77] generally have a noncanonical structure due to the inclusion of gravitation with additional catastrophic inconsistencies when including supersymmetries.

12) The so-called squeezed states theories [85,86] due to their manifest nonunitary character.

13) Kac-Moody superalgebras [105] are also nonunitary and a particular case of Santilli's Lie-admissible algebra (1.5.46) with $T = I$ and W a phase factor.

Numerous additional theories are also afflicted by the catastrophic inconsistencies of Theorem 1.5.2, such as quantum groups, quantum gravity, and other theories the reader can easily identify from the *departures* of their time evolution from the unitary law.

All the above theories have a nonunitary structure formulated via conventional mathematics and, therefore, are afflicted by the catastrophic physical and mathematical inconsistencies of Theorem 1.5.2.

Additional generalized theories were attempted via *the relaxation of the linear character of quantum mechanics* [75]. These theories are essentially based on eigenvalue equations with the structure

$$H(t, r, p, |\psi\rangle) \times |\psi\rangle = E \times |\psi\rangle, \tag{1.5.65}$$

(i.e., H depends on the wavefunction).

Even though mathematically intriguing and possessing a seemingly unitary time evolution, these theories also possess rather serious physical drawbacks, such as: they violate the superposition principle necessary for composite systems such as a hadron; they violate the fundamental Mackay imprimitivity theorem necessary for the applicability of Galileo's and Einstein's relativities and possess other drawbacks [11b] so serious to prevent consistent applications.

Yet another type of broader theory is *Weinberg's nonlinear theory* [90] with brackets of the type

$$A \odot B - B \odot A = \frac{\partial A}{\partial \psi} \times \frac{\partial B}{\partial \psi^\dagger} - \frac{\partial B}{\partial \psi} \times \frac{\partial A}{\partial \psi^\dagger}, \tag{1.5.66}$$

where the product $A \odot B$ is *nonassociative.*

This theory violates Okubo's No-Quantization Theorem [70], prohibiting the use of nonassociative envelopes because of catastrophic physical consequences, such as the loss of equivalence between the Schrödinger and Heisenberg representations (the former remains associative, while the latter becomes nonassociative, thus resulting in inequivalence).

Weinberg's theory also suffers from the absence of any unit at all, with consequential inability to apply the theory to measurements, the loss of exponentiation to a finite transform (lack of Poincaré-Birkhoff-Witt theorem), and other inconsistencies studied in Ref. [74].

These inconsistencies are not resolved by the adaptation of Weinberg's theory proposed by Jordan [91] as readers seriously interested in avoiding the publication of theories known to be inconsistent *ab initio* are encouraged to verify.

Several authors also attempted *the relaxation of the local-differential character of quantum mechanics* via the addition of "integral potentials" in the Hamiltonian,

$$V = \int d\tau \Gamma(\tau, \ldots). \tag{1.5.67}$$

These theories are structurally flawed on both mathematical and physical grounds.

In fact, the nonlocal extension is elaborated via the conventional mathematics of quantum mechanics which, beginning with its topology, is strictly local-differential, thus implying fundamental *mathematical* inconsistencies. Nonlocal interactions are in general of contact type, for which the notion of a potential has no physical meaning, thus resulting in rather serious *physical* inconsistencies.

In conclusion, by the early 1980's Santilli had identified classical and operator generalized theories [103,104] that are directly universal in their fields, with a plethora of simpler versions by various other authors.

However, all these theories subsequently resulted in being mathematically significant, but having no physical meaning because they are noninvariant when elaborated with conventional mathematics.

As we shall see in Chapter 3, 4 and 5, thanks to the construction of new mathematics, hadronic mechanics does indeed solve all the above inconsistencies. The clear difficulties in the solutions then illustrate the value of the result.

1.5.4 The Birth of Isomathematics, Genomathematics and their Isoduals

As it is well known, the basic equations of quantum mechanics, *Heisenberg's time evolution* of a (Hermitean) operator A ($\hbar = 1$),

$$i \times \frac{dA}{dt} = A \times H - H \times A = [A, H], \tag{1.5.68a}$$

$$H = p^2/2 \times m + V(r), \tag{1.5.68b}$$

can only represent the *conservation* of the total energy H (and other quantities) under action-at-a-distance interactions derivable from a potential $V(r)$,

$$i \times \frac{dH}{dt} = [H, H] = H \times H - H \times H \equiv 0. \tag{1.5.69}$$

Consequently, the above equations are basically insufficient to provide an operator representation of *closed non-Hamiltonian systems*, namely, systems of extended particles verifying conventional total conservation laws yet possessing internal potential; and nonpotential interactions, as it is the case for all interior problems, such as the structure of hadron, nuclei and stars.

The central requirement for a meaningful representation of closed, classical or operator interior systems of *particles* with internal contact interactions is the achievement of a *generalization of Lie's theory* in such a way to admit broader brackets, hereon denoted $[A\hat{,}B]$, verifying the following conditions:

1) The new brackets $[A\hat{,}B]$ must verify the distributive and scalars laws (1.3.9) in order to characterize an algebra;

2) Besides the Hamiltonian, the new brackets should admit a new Hermitean operator, hereon denoted with $\hat{T} = \hat{T}^\dagger$, and we shall write $[A\hat{,}B]_{\hat{T}}$, as a necessary condition for the representation of all non-Hamiltonian forces and effects.

3) The new brackets must be anti-symmetric in order to allow the conservation of the total energy under contact nonpotential internal interactions

$$i \times \frac{dH}{dt} = [H\hat{,}H]_{\hat{T}} \equiv 0; \tag{1.5.70}$$

For the case of *open*, classical or operator irreversible interior systems of *particles* there is the need of a *second generalization of Lie's theory* characterizing broader brackets, hereon denoted $(A\hat{,}B)$ verifying the following conditions:

1') The broader brackets (A, B) must also verify the scalar and distributive laws (1.3.9) to characterize an algebra;

2') The broader brackets must include *two* non-Hermitean operators, hereon denoted $\hat{P}$ and $\hat{Q}, \hat{P} = \hat{Q}^\dagger$ to represent the two directions of time, and the new brackets, denoted $_{\hat{P}}(\hat{A,B})_{\hat{Q}}$, must be neither antisymmetric nor symmetric to characterize the time rate of variation of the energy and other quantities,

$$i \times \frac{dH}{dt} \; = \; _{\hat{P}}(\hat{H,H})_{\hat{Q}} \; \neq \; 0; \tag{1.5.71}$$

3') The broader brackets must admit the antisymmetric brackets $[\hat{A,B}]$ and $[A, B]$ as particular cases because conservation laws are particular cases of nonconservation laws.

For the case of closed and open interior systems of *antiparticles*, it is easy to see that the above generalizations of Lie's theory will not apply for the same reason that the conventional Lie theory cannot characterize exterior systems of point-like antiparticles at classical level studied in Section 1.1 (due to the existence of only one quantization channel, the operator image of classical treatments of antiparticles can only yield particles with the wrong sign of the charge, and certainly not their charge conjugate).

The above occurrence requires a *third generalization of Lie's theory* specifically conceived for the representation of closed or open interior systems of antiparticles at *all* levels of study, from Newton to second quantization. As we shall see, the latter generalization is provided by the isodual map.

In an attempt to resolve the scientific imbalances of the preceding section, when at the Department of Mathematics of Harvard University, Santilli [103] proposed in 1978 an axiom-preserving generalization of conventional mathematics verifying conditions 1), 2) and 3), that he subsequently studied in various works (see monographs [11,54] and references quoted therein).

The new mathematics is today known as *Santilli's isotopic and genotopic mathematics* or *isomathematics and genomathematics* for short [27–31], where the word "isotopic" or the prefix "iso" are used in the Greek meaning of preserving the original axioms, and the word "geno" is used in the sense of inducing new axioms.

Proposal [103] for the new isomathematics was centered in the generalization (called *lifting*) of the conventional, N-dimensional unit, $I = Diag.(1, 1, \ldots, 1)$ into an $N \times N$-dimensional matrix $\hat{I}$ that is nowhere singular, Hermitean and positive-definite, but otherwise possesses an unrestricted functional dependence on local coordinates r, velocities v, accelerations a, dimension d, density μ, wavefunctions ψ, their deriva-

tives $\partial\psi$ and any other needed quantity,

$$I = Diag.(1, 1, \ldots, 1) > 0 \rightarrow \hat{I}(r, v, a, d, \mu, \psi, \partial\psi, \ldots) = \hat{I}^{\dagger} = 1/\hat{T} > 0 \tag{1.5.72}$$

while jointly lifting the conventional associative product $A \times B$ among generic quantities A and B (numbers, vector fields, matrices, operators, etc.) into the form

$$A \times B \rightarrow A\hat{\times}B = A \times \hat{T} \times B, \tag{1.5.73}$$

under which $\hat{I}$, rather than I, is the correct left and right unit,

$$I \times A = A \times I \equiv A \rightarrow \hat{I}\hat{\times}A = A\hat{\times}\hat{I} \equiv A, \tag{1.5.74}$$

for all A of the set considered, in which case $\hat{I}$ is called *Santilli's isounit*, and $\hat{T}$ is called the *isotopic element.*

Eqs. (1.5.72)–(1.5.74) illustrate the isotopic character of the lifting. In fact, $\hat{I}$ preserves all topological properties of I; the isoproduct $A\hat{\times}B$ remains as associative as the original product $A \times B$; and the same holds for the preservation of the axioms for a left and right identity.

More generally, the lifting of the basic unit required, for evident reasons of consistency, a corresponding compatible lifting of *all* mathematics used by special relativity and quantum mechanics, with no exception known to this author, thus resulting in the new *isonumbers*, isospaces, isofunctional analysis, isodifferential calculus, isotopologies, isogeometries, etc. (for mathematical works see Refs. [54,105–108,19]).

Via the use of the above liftings, Santilli presented in the original proposal [103] a step-by-step isotopic (that is, axiom-preserving) lifting of all main branches of Lie's theory, including the isotopic generalization of universal enveloping associative algebras, Lie algebras, Lie groups and the representation theory. The new theory was then studied in various works and it is today known as the *Lie-Santilli isotheory* [28–31]. Predictably, from Eq. (1.5.44) one can see that the new isobrackets have the form

$$\begin{aligned}[A\hat{,}B]_{\hat{T}} &= A\hat{\times}B - B\hat{\times}A \\ &= A \times \hat{T} \times B - B \times \hat{T} \times A = [A\hat{,}B],\end{aligned} \tag{1.5.75}$$

where the subscript $\hat{T}$ shall be dropped hereon, whose verification of conditions 1), 2), 3) is evident.

The point important for these introductory lines is that *isomathematics does allow a consistent representation of extended, nonspherical, deformable and hyperdense particles under local and nonlocal, linear and nonlinear, and potential as well as nonpotential interactions.*

In fact, all conventional linear, local and potential interactions can be represented with a conventional Hamiltonian, while the shape and density of the particles and their nonlinear, nonlocal and nonpotential interactions can be represented with Santilli's isounits via realizations of the type

$$\hat{I} = \Pi_{k=1,2,...,n} Diag(n_{k1}^2, n_{k2}^2, n_{k3}^2, n_{k4}^2) \times e^{\Gamma(\psi,\psi^\dagger)\times \int d^3 r \psi^\dagger(r)_k \times \psi(r)_k}, \tag{1.5.76}$$

where: the $n_{k1}^2, n_{k2}^2, n_{k3}^2$ allow the representation, for the first time, the actual, extended, nonspherical and deformable shapes of the particles considered (normalized to the values $n_k = 1$ for the perfect sphere); n_{k4}^2 allows one to represent, also for the first time, the density of the interior medium (normalized to the value $n_4 = 1$ for empty space); the function $\Gamma(\psi, \psi^\dagger)$ represents the nonlinear character of the interactions; and the integral $\int d^3 r \psi^\dagger(r)_k \times \psi(r)_k$ represents nonlocal interactions due to the overlapping of particles or of their wave packets.

When the mutual distances of the particles are much greater than 10^{-13} cm = 1 F, the integral in Eq. (1.5.76) is identically null, and all nonlinear and nonlocal effects are null. When, in addition, the particles considered are reduced to points moving in vacuum, all the n-quantities are equal to 1, generalized unit (1.3.22) recovers the trivial unit, and isomathematics recovers conventional mathematics identically, uniquely and unambiguously.

In the same memoir [103], in order to represent irreversibility, Santilli proposed a broader genomathematics based on the following differentiation of the product to the right and to the left with corresponding generalized units

$$A > B = A \times \hat{P} \times B, \quad \hat{I}^{>} = 1/\hat{P}; \tag{1.5.77a}$$

$$A < B = A \times \hat{Q} \times B, \quad {}^{<}\hat{I} = 1/\hat{Q}, \tag{1.5.77b}$$

$$\hat{I}^{>} = {}^{<}\hat{I}^\dagger, \tag{1.5.77c}$$

where evidently the product to the right, $A > B$, represents motion forward in time and that to the left, $A < B$, represents motion backward in time. Since $A > B \neq A < B$, the latter mathematics represents irreversibility from the most elemental possible axioms.

The latter mathematics was proposed under a broader lifting called "genotopy" in the Greek meaning of inducing new axioms, and it is known today as *Santilli genotopic mathematics*, or *genomathematics* for short [28–31].

It is evident that genoliftings (1.5.77) require a step by step generalization of all aspects of isomathematics, resulting in *genonumbers,*

genofields, genospaces, genoalgebras, genogeometries, genotopologies, etc. [54,105–108,19].

Via the use of the latter mathematics, Santilli proposed also in the original memoir [103] a genotopy of the main branches of Lie's theory, including a genotopic broadening of universal enveloping isoassociative algebras, Lie-Santilli isoalgebras, Lie-Santilli isogroup, isorepresentation theory, etc. and the resulting theory is today known as the *Lie-Santilli genotheory* with basic brackets

$$\begin{aligned} {}_{\hat{P}}(A\hat{,}B)_{\hat{Q}} &= A < B - B > A \\ &= A \times P \times B - B \times Q \times B = (A\hat{,}B), \end{aligned} \tag{1.5.78}$$

and the subscripts $\hat{P}$ and $\hat{Q}$ shall be dropped from now on.

It should be noted that the main proposal of memoir [59] is genomathematics, while isomathematics is presented as a particular case for

$$(A\hat{,}B)_{\hat{P}=\hat{Q}=\hat{T}} = [A\hat{,}B]. \tag{1.5.79}$$

as we shall see in Chapter 3, the *isodual isomathematics* and *isodual genomathematics* for the treatment of antiparticles are given by the isodual image (1.1.6) of the above iso- and geno-mathematics, respectively.

1.5.5 Hadronic Mechanics

Thanks to the prior discovery of isomathematics and genomathematics, in memoir [104] also of 1978 Santilli proposed a generalization of quantum mechanics for closed and open interior systems, respectively, under the name of *hadronic mechanics*, because hyperdense hadrons, such as protons and neutrons, constitute the most representative (and most difficult) cases of interior dynamical systems.

For the case of closed interior systems of particles, hadronic mechanics is based on the following *isotopic generalization of Heisenberg's equations* (Ref. [104], Eqs. (4.15.34) and (4.18.11))

$$i \times \frac{dA}{dt} = [A\hat{,}H] = A\hat{\times}H - H\hat{\times}A. \tag{1.5.80}$$

while for the broader case of open interior systems hadronic mechanics is based on the following *genotopic generalization of Heisenberg's equations* (Ref. [104], Eq. (4.18.16))

$$\begin{aligned} i \times \frac{dA}{dt} &= (A\hat{,}H) = A < H - H > A \\ &= A \times P \times H - H \times Q \times A. \end{aligned} \tag{1.5.81}$$

The isodual images of Eqs. (1.5.80) and (1.5.81) for antiparticles as we as their multivalued hyperformulations significant for biological studies, were added more recently [106].

A rather intense scientific activity followed the original proposal [104], including five *Workshops on Lie-admissible Formulations* held at Harvard University from 1978 to 1982, fifteen *Workshops on Hadronic Mechanics*, and several formal conferences held in various countries, plus a rather large number of research papers and monographs written by various mathematicians, theoreticians and experimentalists, for an estimated total of some 15,000 pages of research published refereed journals (see the references at the end of Chapter 3).

As a result of these efforts, hadronic mechanics is today a rather diversified discipline conceived and constructed for quantitative treatments of all classical and operator systems of particles according to Definition 1.3.1 with corresponding isodual formulations for antiparticles.

It is evident that in Chapter 3 we can review only the most salient foundations of hadronic mechanics and have to defer the interested reader to the technical literature for brevity.

As of today, hadronic mechanics has experimental verifications in particle physics, nuclear physics, atomic physics, superconductivity, chemistry, biology, astrophysics and cosmology [22].

Hadronic mechanics can be classified into sixteen different branches, including: four branches of classical treatment of particles with corresponding four branches of operator treatment also of particles, and eight corresponding (classical and operator) treatments of antiparticles.

An effective classification of hadronic mechanics is that done via the main topological features of the assumed basic unit, since the latter characterizes all branches according to (see Figure 1.7):

$I = 1 > 0$:
HAMILTONIAN AND QUANTUM MECHANICS
Used for the description of closed and reversible systems of point-like particles in exterior conditions in vacuum;

$I^d = -1 < 0$:
ISODUAL HAMILTONIAN AND ISODUAL QUANTUM MECHANICS
Used for the description of closed and reversible systems of point-like antiparticles in exterior conditions in vacuum;

$\hat{I}(r, v, \ldots) = \hat{I}^\dagger > 0$:
CLASSICAL AND OPERATOR ISOMECHANICS

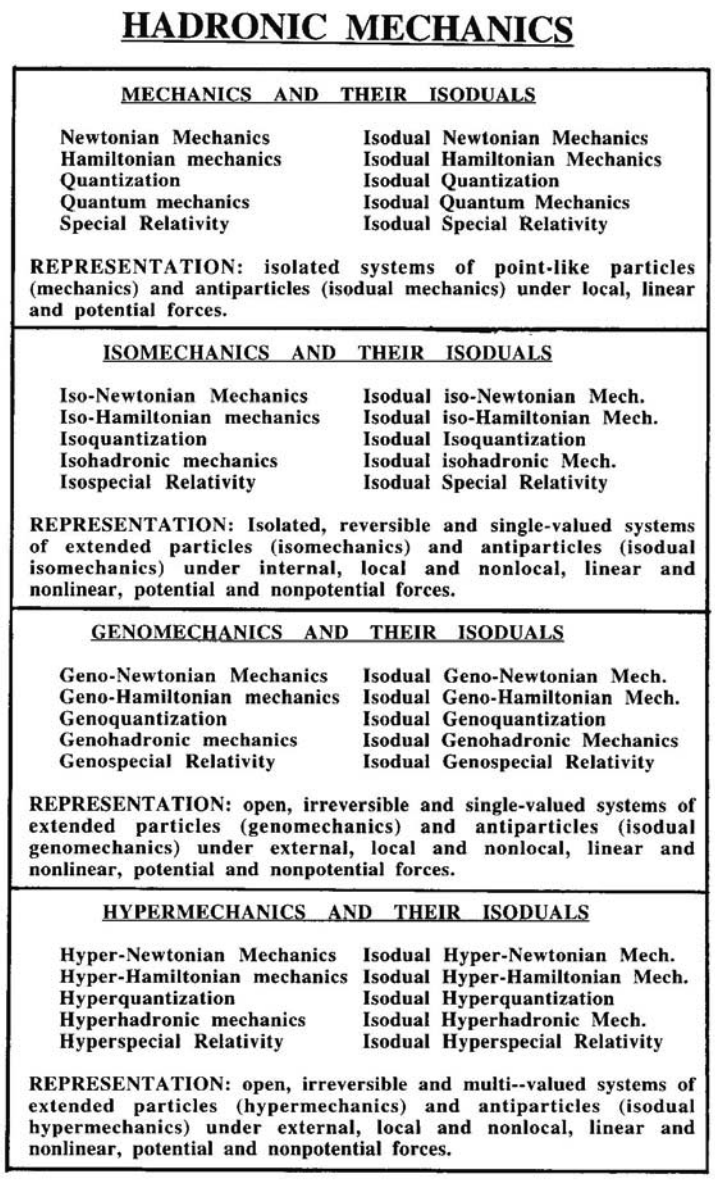

Figure 1.7. The structure of hadronic mechanics.

Used for the description of closed and reversible systems of extended particles in interior conditions;

$\hat{I}^d(r^d, v^d, \ldots) = \hat{I}^{d\dagger} < 0$:
ISODUAL CLASSICAL AND OPERATOR ISOMECHANICS
Used for the description of closed and reversible systems of extended antiparticles in interior conditions;

$\hat{I}^{>}(r^{>}, v^{>}, \ldots) = ({}^{<}\hat{I})^{\dagger}$:
CLASSICAL AND OPERATOR GENOMECHANICS
Used for the description of open and irreversible systems of extended particles in interior conditions;

$\hat{I}^{d>}(r^{d>}, v^{dv}, \ldots\} = ({}^{<}\hat{I})^{d\dagger}$:
ISODUAL CLASSICAL AND OPERATOR GENOMECHANICS
Used for the description of open and irreversible systems of extended particles in interior conditions;

$\hat{I}^{>} = (\hat{I}_1^{>}, \hat{I}_2^{>}, \ldots) = ({}^{<}\hat{I})^{\dagger}$:
CLASSICAL AND OPERATOR HYPERMECHANICS
Used for the description of multivalued open and irreversible systems of extended particles in interior conditions;

$\hat{I}^{d>} = \{\hat{I}_1^{>}, \hat{I}_2^{>}, \ldots\} = ({}^{<}\hat{I})^{\dagger}$:
ISODUAL CLASSICAL AND OPERATOR HYPERMECHANICS
Used for the description of multivalued open and irreversible systems of extended antiparticles in interior conditions.

In summary, a serious study of antiparticles requires its study beginning at the classical level and then following at all subsequent levels, exactly as it is the case for particles.

In so doing, the mathematical and physical treatments of antiparticles emerge as being deeply linked to that of particles since, as we shall see, the former are an anti-isomorphic image of the latter.

Above all, a serious study of antiparticles requires the admission of their existence in physical conditions of progressively increasing complexity, that consequently require mathematical and physical methods with an equally increasing complexity, resulting in the various branches depicted in Figure 1.7.

All in all, young minds of any age will agree that, rather than having reached a terminal character, our knowledge of nature is still at its first infancy and so much remains to be discovered.

Notes

1 In the early 1980s, when the absence of a mathematics suitable for the classical treatment of antimatter was identified, the author was (as a theoretical physicist) a member of the Department of Mathematics at Harvard University. When seeing the skepticism of colleagues toward such an absence, the author used to suggest that colleagues should go to Harvard's advanced mathematics library, select any desired volume, and open any desired page at random. The author then predicted that the mathematics presented in that page resulted to be fundamentally inapplicable to the classical treatment of antimatter, as it did indeed result to be the case without exceptions. In reality, the entire content of advanced mathematical libraries of the early 1980s did not contain the mathematics needed for a consistent classical treatment of antimatter.

2 In 1996, the author was requested to make a 20 minutes presentation at a mathematical meeting held in Sicily. The presentation initiated with a transparency solely containing the number -1 and the statement that such a number was assumed as the basic left and right unit of the mathematics to be presented. Unfortunately, this first transparency created quite a reaction by most participants who bombarded the author with questions advancing his presentation, questions often repeated with evident waste of precious time without the author having an opportunity to provide a technical answer. This behavior continued for the remaining of the time scheduled for the talk to such an extent that the author could not present the subsequent transparencies proving that isodual numbers verify all axioms of a field (see Chapter 2). The case illustrates that the conviction of absolute generality is so engraved among most mathematicians to prevent their minds from admitting the existence of *new* mathematics.

3 The ordinary associative product AB of functions, matrices, operators, etc. will be denoted throughout this monograph with the symbol $\times$ to distinguish it with various other products we shall introduce, such as the isoproduct $\hat{\times}$, the genoproduct to the rights $>$, that to the left $<$ and their isoduals $\times^d$, $\hat{\times}^d$, $>^d$ and $<^d$.

4 It should be indicated that the name "Einstein's special relativity" is political, since a scientifically correct name should be "Lorentz-Poincaré-Einstein relativity". Also, it is appropriate to recall that

Einstein ended up divorcing his first wife Mileva Maric because she was instrumental in writing the celebrated paper on special relativity of 1905 and, for that reason, she had been originally listed as a co-author of that article, co-authorship that was subsequently removed when the article appeared in print. In fact, Einstein donated all funds received from his Nobel Prize on that article to Mileva. Similarly, it should be recalled that Einstein contacted Poincaré prior to his article of 1905, but abstained from quoting Poincaré work in said article in documented knowledge that Poincaré had preceded him in various features of special relativity (see, e.g., the historical account by Logunov [37]). For an instructive reading of these historical aspects, one may inspect, e.g., Ref. [38].

5 Recall that Newton had to discover first the differential calculus as a condition to formulate his celebrated equations. A similar case occurred for antimatter, because the correct formulation of Newton's equations for antimatter required the prior discovery of the new isodual differential calculus [12]. As we shall see in the subsequent chapters, additional broadening of Newton's equations required the prior identification of yet broader forms of the differential calculus.

6 We assume the reader admits that absolute rigidity exists in academia but not in the physical reality.

7 We should perhaps recall here that different hadrons generally have different masses and approximately the same size with radius of about 1 F, thus generally having different densities. While a representation of these different densities is irrelevant for the point-like abstract of hadrons and of their constituents, their representation is instead crucial for contact interactions due to deep mutual penetrations, as we shall see in Chapter 3.

8 Contrary to popular belief, the celebrated Jacobi theorem was formulated precisely for the general analytic equations with external terms, while treatises on mechanics of the 20-th century generally present the reduced version of the Jacobi theorem for the equations without external terms. Consequently, the reading of the original work by Jacobi is strongly recommended over that of simplified versions (see, again, Ref. [54a] for historical accounts and references).

9 There are serious rumors that a famous physicist from a leading institution visited NASA in 1998 to propose a treatment of the trajectory of the space shuttle during re-entry essentially based on the truncated Hamilton equations, and that NASA engineers kindly pushed that physicist through the door.

10 The author begs supersymmetry enthusiasts not to mention their theories at this point because, to achieve any credibility, they have first to prove the existence of an additional zoo of predicted particles none of which appears to be detected or detectable in reality.

11 This is another reason the author has stated several times in his writings that **the most ascientific process of contemporary society is the current scientific process.**

12 The literature on general relativity accumulated during the 20-th century is so vast to discourage discriminatory quotations.

13 We should clarify that with the terms "Einstein's gravitation" we specifically refer to the conception of gravitation as entirely represented with curvature without source because the dubbing of Eq. (1.4.2) as "Einstein field equations" is purely political since, on scientifically correct grounds, the same equations are called *Einstein-Hilbert field equations*. In fact, Hilbert published the same equations prior to Einstein, who consulted Hilbert without quoting his work in his gravitational paper of 1915, as Einstein had done in other cases. It is appropriate to recall that the publication of his paper on gravitation caused Einstein the divorce from his second wife, Elsa Lowenstein for essentially the same reason of his first divorce. Unlike Einstein who had no advanced mathematical training, Elsa was a mathematician, had trained Einstein on the Riemannian geometry (a topic for pure mathematicians at that time), and was supposed to be a co-author of his paper on gravitation, a co-authorship Einstein denied as he did it with the suppression of co-authorship with his first wife Mileva for his 1905 paper on special relativity indicated earlier. For instructive readings on these historical aspects one may consult Refs. [38–40].

14 In another important footnote, Pauli [45] quotes another historical paper that was also completely ignored during the 20-th century because not aligned with the academic interests of the time. We are referring to a seminal paper by Lorentz in which he presents the first studies on scientific record to extend his celebrated symmetry from the case of the speed of light in vacuum $c_\circ$ to the speed of light within physical media, $c = c_\circ/n$, which latter problem is a central objective of hadronic mechanics and its underlying new mathematics (see Chapter 3).

15 The indication by colleagues of additional studies on the Freud identify not quoted herein would be appreciated.

16 We continue to denote the conventional associative multiplication of numbers, vector fields, operators, etc. with the notation $A \times B$ rather

than the usual form AB, because the new mathematics necessary to resolve the catastrophic inconsistencies studied in this chapter is based on various different generalizations of the multiplication. As a consequence, the clear identification of the assumed multiplication will soon be crucial for the understanding of the equations of this monograph.

17 For several additional different but equivalent definitions of canonical transformations one may consult Ref. [54a] Pages 187–188.

18 The truncated Hamilton equations admit analytic representations of nonconservative systems but only *in one dimension*, which systems are essentially irrelevant for serious physical applications.

19 Because conventional variations δ can only characterize antisymmetric tensors of type $\omega_{\mu\nu}$ or $\Omega_{\mu\nu}$ and cannot characterize non-antisymmetric tensors such as the Lie-admissible tensor $S_{\mu\nu}$.

20 The reader should be aware that there exist in the literature numerous claims of "generalizations of quantum mechanics" although they have a unitary time evolution and, consequently, do not constitute true generalizations. All these "generalizations" will be ignored in this monograph because they will not resist the test of time.

21 In 1985, Biederharn [64] and MacFairlane [65] published their papers on the simpler q-deformations

$$A \times B - q \times B \times A$$

without a quotation of the origination of the broader form $p \times A \times B - q \times B \times A$ by Santilli [61] in 1967. Regrettably, Biedenharn and MacFairlane abstained from quoting Santilli's origination despite their documented knowledge of such an origination. For instance, Biedenharn and Santilli had applied for a DOE grant precisely on the same deformations two years prior to their paper of 1985, and Santilli had personally informed MacFairlaine of said deformations years before his paper of 1985. The lack of quotation of Santilli's origination of q-deformations resulted in a large number of subsequent papers by numerous other authors that also abstained from quoting said origination (see representative contributions [66–69]), for which reason Santilli has been often referred to as the "most plagiarized physicist of the 20-th century". Ironically, at the time Biedenharn and MacFairlane published their paper on q-deformations, Santilli had already abandoned them because of their catastrophic mathematical and physical inconsistencies studied in this Section.

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Chapter 2

ISODUAL THEORY OF POINT-LIKE ANTIPARTICLES

2.1 ELEMENTS OF ISODUAL MATHEMATICS

2.1.1 Isodual Unit, Isodual Numbers and Isodual Fields

Since the isodual mathematics has been subjected to a number of developments following its first presentation in papers [1] of 1985, it is important to review it in sufficient details to render this monograph selfsufficient.

In this section, we identify only those aspects of isodual mathematics that are essential for the understanding of the physical profiles presented in the subsequent sections of this chapter. We begin with a study of the most fundamental elements of all mathematical and physical formulations, units, numbers and fields, from which all remaining formulations can be uniquely and unambiguously derived via simple compatibility arguments. To avoid un-necessary repetitions, we assume the reader has a knowledge of the basic mathematics used for the classical and operator treatment of matter.

DEFINITION 2.1.1: Let $F = F(a, +, \times)$ be a field (of characteristic zero), namely a ring with elements given by real number $a = n$, $F = R(n, +, \times)$, complex numbers $A = c$, $F = C(c, +, \times)$, or quaternionic numbers $a = q$, $F = Q(q, +, \times)$, with conventional sum $a + b$ verifying the commutative law

$$a + b = b + a = c \in F, \tag{2.1.1}$$

the associative law

$$(a+b)+c=a+(b+c)=d\in F, \tag{2.1.2}$$

conventional product $a\times b$ *verifying the associative law*

$$(a\times b)\times c=a\times(b\times c)=e\in F, \tag{2.1.3}$$

(but not necessarily the commutative law, $a\times b\neq b\times a$ *since the latter is violated by quaternions), and the right and left distributive laws*

$$(a+b)\times c=a\times c+b\times c=f\in F, \tag{2.1.4a}$$

$$a\times(b+c)=a\times b+a\times c=g\in F, \tag{2.1.4b}$$

left and right additive unit 0*,*

$$a+0=0+a=a\in F, \tag{2.1.5}$$

and left and right multiplicative unit I*,*

$$a\times I=I\times a=a\in F, \tag{2.1.6}$$

$\forall a,b,c\in F$*. Santilli's isodual fields (first introduced in* Refs. [1] *and then presented in details in* Ref. [2]*) are rings* $F^d=F^d(a^d,+^d,\times^d)$ *with elements given by isodual numbers*

$$a^d=-a^\dagger,\ a^d\in F, \tag{2.1.7}$$

with associative and commutative isodual sum

$$a^d+^d b^d=-(a+b)^\dagger=c^d\in F^d, \tag{2.1.8}$$

associative and distributive isodual product

$$a^d\times^d b^d=a^d\times(I^d)^{-1}\times b^d=c^d\in F^d, \tag{2.1.9}$$

additive isodual unit $0^d=0$*,*

$$a^d+^d 0^d=0^d+^d a^d=a^d, \tag{2.1.10}$$

and multiplicative isodual unit $I^d=-I^\dagger$*,*

$$a^d\times^d I^d=I^d\times^d a^d=a^d,\forall a^d,b^d\in F^d. \tag{2.1.11}$$

The proof of the following property is elementary.

LEMMA 2.1.1 [1,2]: *Isodual fields are fields, namely, if F is a field, its image F^d under the isodual map is also a field.*

The above lemma establishes the property (first identified in Ref. [1]) that *the axioms of a field do not require that the multiplicative unit be necessarily positive-definite, because the same axioms are also verified by negative-definite units.* The proof of the following property is equally simple.

LEMMA 2.1.2 [1,2]: *Fields F and their isodual images F^d are anti-isomorphic to each other.*

Lemmas 2.1.1 and 1.2.2 illustrate the origin of the name "isodual mathematics". In fact, to represent antimatter the needed mathematics must be a suitable "dual" of conventional mathematics, while the prefix "iso" is used in its Greek meaning of preserving the original axioms.

It is evident that for real numbers we have

$$n^d = -n, \tag{2.1.12}$$

while for complex numbers we have

$$c^d = (n_1 + i \times n_2)^d = -n_1 + i \times n_2 = -\bar{c}, \tag{2.1.13}$$

with a similar formulation for quaternions.

It is also evident that, for consistency, *all operations on numbers must be subjected to isoduality when dealing with isodual numbers.* This implies: the *isodual powers*

$$(a^d)^{n^d} = a^d \times^d a^d \times^d a^d \ldots \tag{2.1.14}$$

(n times, with n an integer); the *isodual square root*

$$a^{d^{(1/2)d}} = -\sqrt{-a^\dagger}^\dagger, a^{d^{(1/2)d}} \times^d a^{d^{(1/2)d}} = a^d, \quad 1^{d^{(1/2)d}} = -i; \tag{2.1.15}$$

the *isodual quotient*

$$a^d /^d b^d = -(a^\dagger / b^\dagger) = c^d, \ b^d \times^d c^d = a^d; \tag{2.1.16}$$

etc.

An important property for the characterization of antimatter is the following:

LEMMA 2.1.3. [2]: *isodual fields have a negative–definite norm, called isodual norm,*

$$|a^d|^d = |a^\dagger| \times I^d = -(aa^\dagger)^{1/2} < 0, \tag{2.1.17}$$

where $|\dots|$ denotes the conventional norm.

For isodual real numbers we therefore have the isodual isonorm

$$|n^d|^d \;=\; -|n| < 0, \tag{2.1.18}$$

and for isodual complex numbers we have

$$|c^d|^d \;=\; -|\bar{c}| \;=\; -(c\bar{c})^{1/2} \;=\; -(n_1^2 + n_2^2)^{1/2}. \tag{2.1.19}$$

LEMMA 2.1.4 [2]: *All quantities that are positive–definite when referred to positive units and related fields of matter (such as mass, energy, angular momentum, density, temperature, time, etc.) became negative–definite when referred to isodual units and related isodual fields of antimatter.*

As recalled Chapter 1, antiparticles have been discovered in the *negative-energy solutions* of Dirac's equation and they were originally thought to evolve *backward in time* (Stueckelberg, Feynman, and others, see Refs. [1,2] of Chapter 1). The possibility of representing antiparticles via isodual methods is therefore visible already from these introductory notions.

The main novelty is that the conventional treatment of negative–definite energy and time was (and still is) referred to the conventional unit +1. This leads to a number of contradictions in the physical behavior of antiparticles.

By comparison, *negative-definite physical quantities of isodual theories are referred to a negative–definite unit $I^d < 0$*. This implies a mathematical and physical equivalence between *positive–definite quantities referred to positive–definite units, characterizing matter, and negative–definite quantities referred to negative–definite units, characterizing antimatter.* These foundations then permit a novel characterization of antimatter beginning at the *Newtonian* level, and then persisting at all subsequent levels.

DEFINITION 2.1.2 [2]: *A quantity is called isoselfdual when it coincides with its isodual.*

It is easy to verify that the imaginary unit is isoselfdual because

$$i^d \;=\; -i^\dagger \;=\; -\bar{i} \;=\; -(-i) \;=\; i. \tag{2.1.20}$$

This property permits a better understanding of the isoduality of complex numbers that can be written explicitly

$$c^d \;=\; (n_1 + i \times n_2)^d \;=\; n_1^d + i^d \times^d n_2^d \;=\; -n_1 + i \times n_2 \;=\; -\bar{c}. \quad (2.1.21)$$

The above property will be important to prove the equivalence of isoduality and charge conjugation at the operator level.

As we shall see, *isoselfduality is a new fundamental view of nature* with deep physical implications, not only in classical and quantum mechanics but also in cosmology. For instance we shall see that Dirac's gamma matrices are isoselfdual, thus implying a basically new interpretation of this equation that has remained unidentified for about one century. We shall also see that, when applied to cosmology, isoselfduality implies equal distribution of matter and antimatter in the universe, with identically null total physical characteristic, such as identically null total time, identically null total mass, etc.

We assume the reader is aware of the emergence here of *new numbers*, those with a negative unit, that have no connection with ordinary negative numbers and are the true foundations of the isodual theory of antimatter.

2.1.2 Isodual Functional Analysis

All conventional and special functions and transforms, as well as functional analysis at large, must be subjected to isoduality for consistent applications, resulting in the simple, yet unique and significant *isodual functional analysis*, studied by Kadeisvili [3], Santilli [4] and others.

We here mention the *isodual trigonometric functions*

$$\sin^d \theta^d \;=\; -\sin(-\theta), \;\; \cos^d \theta^d \;=\; -\cos(-\theta), \quad (2.1.22)$$

with related basic property

$$\cos^{d\,2d} \theta^d +^d \sin^{d\,2d} \theta^d \;=\; 1^d \;=\; -1, \quad (2.1.23)$$

the *isodual hyperbolic functions*

$$\sinh^d w^d \;=\; -\sinh(-w), \; \cosh^d w^d \;=\; -\cosh(-w), \quad (2.1.24)$$

with related basic property

$$\cosh^{d\,2d} w^d -^d \sinh^{d\,2d} w^d \;=\; 1^d \;=\; -1, \quad (2.1.25)$$

the *isodual logarithm* and the *isodual exponentiation* defined respectively by

$$\log^d n^d \;=\; -\log(-n), \quad (2.1.26a)$$

$$e_d^{X^d} = 1^d + X^d/^d 1!^d + X^{d^{2d}}/^d 2!^d + \ldots = -e^X, \tag{2.1.26b}$$

etc. Interested readers can then easily construct the isodual image of special functions, transforms, distributions, etc.

2.1.3 Isodual Differential and Integral Calculus

Contrary to a rather popular belief, the differential calculus is indeed dependent on the assumed unit. This property is not so transparent in the conventional formulation because the basic unit is the trivial number +1. However, the dependence of the unit emerges rather forcefully under its generalization.

The *isodual differential calculus*, first introduced by Santilli in Ref. [5a], is characterized by the *isodual differentials*

$$d^d x^k \; = \; I^d \times dx^k \; = \; -dx^k, \;\; d^d x_k \; = \; -dx_k, \tag{2.1.27}$$

with corresponding *isodual derivatives*

$$\partial^d/^d \partial^d x^k \; = \; -\partial/\partial x^k, \;\; \partial^d/^d \partial^d x_k \; = \; -\partial/\partial x, \tag{2.1.28}$$

and related isodual properties.

Note that *conventional differentials are isoselfdual*, i.e.,

$$(dx^k)^d \; = \; d^d x^{kd} \equiv d\, x^k, \tag{2.1.29}$$

but *derivatives are not isoselfdual*,

$$[\partial f/\partial x^k]^d \; = \; -\partial^d f^d/^d \partial^d x^{kd}. \tag{2.1.30}$$

The above properties explain why the isodual differential calculus remained undiscovered for centuries.

Other notions, such as the *isodual integral calculus*, can be easily derived and shall be assumed as known hereon.

2.1.4 Lie-Santilli Isodual Theory

Let $\mathbf{L}$ be an n–dimensional Lie algebra in its regular representation with universal enveloping associative algebra $\xi(\mathbf{L})$, $[\xi(\mathbf{L})]^- \approx \mathbf{L}$, n-dimensional unit $I \; = \; Diag.(1, 1, \ldots, 1)$, ordered set of Hermitean generators $X \; = \; X^\dagger \; = \; \{X_k\}$, $k = 1, 2, \ldots, n$, conventional associative product $X_i \times X_j$, and familiar Lie's Theorems over a field $F(a, +, \times)$.

The *Lie-Santilli isodual theory* was first submitted in Ref. [1] and then studied in Refs. [4–7] as well as by other authors [23–31]. *The isodual universal associative algebra* $[\xi(\mathbf{L})]^d$ is characterized by the *isodual unit* I^d, *isodual generators* $X^d \; = \; -X$, and isodual associative product

$$X_i^d \times^d X_j^d \; = \; -X_i \times X_j, \tag{2.1.31}$$

with corresponding infinite–dimensional basis characterized by the *Poincaré-Birkhoff-Witt-Santilli isodual theorem*

$$I^d, X_i^d \times^d X_j^d, \quad i \leq j; \quad X_i^d \times^d X_j^d \times X_k^d, \; i \leq j \leq k, \ldots \tag{2.1.32}$$

and related *isodual exponentiation* of a generic quantity A^d

$$e^{d A^d} \; = \; I^d + A^d/^d 1!^d + A^d \times^d A^d/^d 2!^d + \ldots \; = \; -e^{A^\dagger}, \tag{2.1.33}$$

where e is the conventional exponentiation.

The attached *Lie-Santilli isodual algebra* $\mathbf{L}^d \approx (\xi^d)^-$ over the isodual field $F^d(a^d, +^d, \times^d)$ is characterized by the *isodual commutators* [1]

$$[X_i^d, {}^d X_j^d] \; = \; -[X_i, X_j] \; = \; C_{ij}^{k^d} \times^d X_k^d \tag{2.1.34}$$

with classical realizations given in Section 2.2.6.

Let G be a conventional, connected, n–dimensional Lie transformation group on a metric (or pseudo-metric) space $S(x, g, F)$ admitting $\mathbf{L}$ as the Lie algebra in the neighborhood of the identity, with generators X_k and parameters $w \; = \; \{w_k\}$.

The *Lie-Santilli isodual transformation group* G^d admitting the isodual Lie algebra $\mathbf{L}^d$ in the neighborhood of the isodual identity I^d is the n–dimensional group with generators $X^d \; = \; \{-X_k\}$ and parameters $w^d \; = \; \{-w_k\}$ over the isodual field F^d with generic element [1]

$$U^d(w^d) \; = \; e^{d^{i^d \times^d w^d \times^d X^d}} \; = \; -e^{i \times (-w) \times X} \; = \; -U(-w). \tag{2.1.35}$$

The *isodual symmetries* are then defined accordingly via the use of the isodual groups G^d and they are anti–isomorphic to the corresponding conventional symmetries, as desired. For additional details, one may consult Ref. [4,5b].

In this chapter we shall therefore use the *conventional Poincaré, internal and other symmetries* for the characterization of *matter*, and the *Poincaré-Santilli, internal and other isodual symmetries* for the characterization of *antimatter*.

2.1.5 Isodual Euclidean Geometry

Conventional (vector and) metric spaces are defined over conventional fields. It is evident that the isoduality of fields requires, for consistency, a corresponding isoduality of (vector and) metric spaces. The need for the isodualities of all quantities acting on a metric space (e.g., conventional and special functions and transforms, differential calculus, etc.) becomes then evident.

DEFINITION 2.1.3: Let $S = S(x, g, R)$ *be a conventional* N*-dimensional metric or pseudo-metric space with local coordinates* $x = \{x^k\}$, $k = 1, 2, \ldots, N$, *nowhere degenerate, sufficiently smooth, real–valued and symmetric metric* $g(x, \ldots)$ *and related invariant*

$$x^2 = (x^i \times g_{ij} \times x^j) \times I, \tag{2.1.36}$$

over the reals R. *The isodual spaces, first introduced in* Ref. [1] (see also Refs. [4,5] and, for a more recent account, Ref. [22]), *are the spaces* $S^d(x^d, g^d, R^d)$ *with isodual coordinates* $x^d = x^d = -x^t$ *(where* t *stands for transposed), isodual metric*

$$g^d(x^d, \ldots) = -g^\dagger(-x^\dagger, \ldots) = -g(-x^t, \ldots), \tag{2.1.37}$$

and isodual interval

$$\begin{aligned}(x-y)^{d^2\,d} &= [(x-y)^{id} \times^d g^d_{ij} \times^d (x-y)^{jd}] \times I^d \\ &= [(x-y)^i \times g^d_{ij} \times (x-y)^j] \times I^d,\end{aligned} \tag{2.1.38}$$

defined over the isodual field $R^d = R^d(n^d, +^d, \times^d)$ *with the same isodual isounit* I^d.

The basic nonrelativistic space of our analysis is the three–dimensional *isodual Euclidean space* [1,9],

$$E^d(r^d, \delta^d, R^d) : r^d = \{r^{kd}\} = \{-r^k\} = \{-x, -y, -z\}, \tag{2.1.39a}$$

$$\begin{aligned}\delta^d &= -\delta = diag.(-1, -1, -1), \\ I^d &= -I = Diag.(-1, -1, -1).\end{aligned} \tag{2.1.39b}$$

The *isodual Euclidean geometry* is the geometry of the isodual space E^d over R^d and it is given by a step–by–step isoduality of all the various aspects of the conventional geometry (see monograph [5a] for details).

By recalling that the norm on R^d is negative–definite, the *isodual distance* among two points on an isodual line is also negative definite and it is given by

$$D^d = D \times I^d = -D, \tag{2.1.40}$$

where D is the conventional distance. Similar isodualities apply to all remaining notions, including the notions of parallel and intersecting isodual lines, the Euclidean axioms, etc.

The *isodual sphere* with radius $R^d = -R$ is the perfect sphere on E^d over R^d and, as such, it has *negative radius* (Figure 2.1),

$$R^{d2d} = (x^{d2d} + y^{d2d} + z^{d2d}) \times I^d$$

$$= (x^2 + y^2 + z^2) \times I = R^2. \quad (2.1.41)$$

Note that the above expression coincides with that for the conventional sphere. This illustrates the reasons, following about one century of studies, the isodual rotational group and symmetry where identified for the first time in Ref. [1]. Note, however, that the latter result required the prior discovery of *new numbers*, those with a negative unit.

A similar characterization holds for other isodual shapes characterizing antimatter in our isodual theory.

LEMMA 2.1.5: The isodual Euclidean geometry on E^d over R^d is anti–isomorphic to the conventional geometry on E over R.

The group of isometries of E^d over R^d is the *isodual Euclidean group* $E^d(3) = \mathcal{R}^d(\theta^d) \times^d T^d(3)$ where $\mathcal{R}^d(\theta)$ is the isodual group of rotations first introduced in Ref. [1], and $T^(3)$ is the isodual group of translations (see also Ref. [5a] for details).

2.1.6 Isodual Minkowskian Geometry

Let $M(x, \eta, R)$ be the conventional Minkowski spacetime with local coordinates $x = (r^k, t) = (x^\mu), k = 1, 2, 3,\ \mu = 1, 2, 3, 4$, metric $\eta = Diag.(1, 1, 1, -1)$ and basic unit $I = Diag.(1, 1, 1, 1)$ on the reals $R = R(n, +, \times)$.

The *Minkowski-Santilli isodual spacetime*, first introduced in Ref. [7] and studied in details in Ref. [8], is given by

$$M^d(x^d, \eta^d, R^d) : x^d = \{x^{\mu d}\} = \{x^\mu \times I^d\} = \{-r, -c_o t\} \times I, \quad (2.1.42)$$

with isodual metric and isodual unit

$$\eta^d = -\eta = diag.(-1, -1, -1, +1), \quad (2.1.43a)$$

$$I^d = Diag.(-1, -1, -1, -1). \quad (2.1.43b)$$

The *Minkowski-Santilli isodual geometry* [8] is the geometry of isodual spaces M^d over R^d. The new geometry is also characterized by a simple isoduality of the conventional Minkowskian geometry as studied in details in memoir.

The fundamental symmetry of this chapter is given by the group of isometries of M^d over R^d, namely, the *Poincaré-Santilli isodual symmetry* [7,8]

$$P^d(3.1) = \mathcal{L}^d(3.1) \times T^d(3.1) \quad (2.1.44)$$

where $\mathcal{L}^d(3.1)$ is the Lorentz-Santilli isodual group and $T^d(3.1)$ is the isodual group of translations.

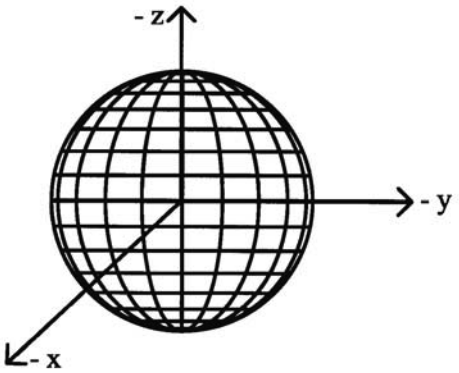

Figure 2.1. A schematic view of the isodual sphere on isodual Euclidean spaces over isodual fields. The understanding of the content of this chapter requires the knowledge that the isodual sphere and the conventional sphere coincide when inspected by an observer either in the Euclidean or in the isodual Euclidean space, due to the identity of the related expressions (2.1.36) and (2.1.38). This identity is at the foundation of the perception that antiparticles "appear" to exist in our space, while in reality they belong to a structurally different space coexisting within our own, thus setting the foundations of a "multidimensional universe" coexisting in the same space of our sensory perception. The reader should keep in mind that the isodual sphere is the idealization of the shape of an antiparticle used in this monograph.

2.1.7 Isodual Riemannian Geometry

Consider a Riemannian space $\Re(x, g, R)$ in $(3 + 1)$ dimensions [32] with basic unit $I = Diag.(1, 1, 1, 1)$, nowhere singular and symmetric metric $g(x)$ and related Riemannian geometry in local formulation (see, e.g., Ref. [27]).

The *Riemannian-Santilli isodual spaces* (first introduced in Ref. [11]) are given by

$$\begin{aligned} \Re^d(x^d, g^d, R^d): \ x^d &= \{-x^\mu\}, \\ g^d &= -g(x), \ g \in \Re(x, g, R), \\ I^d &= Diag.(-1, -1, -1, -1) \end{aligned} \tag{2.1.45}$$

with interval

$$\begin{aligned} x^{2d} &= [x^{dt} \times^d g^d(x^d) \times^d x^d] \times I^d \\ &= [x^t \times g^d(x^d) \times x] \times I^d \in R^d, \end{aligned} \tag{2.1.46}$$

where t stands for transposed.

The *Riemannian-Santilli isodual geometry* [8] is the geometry of spaces $\Re^d$ over R^d, and it is also given by step–by–step isodualities of the conventional geometry, including, most importantly, the isoduality of the differential and exterior calculus.

As an example, an *isodual vector field* $X^d(x^d)$ on $\Re^d$ is given by $X^d(x^d) = -X^t(-x^t)$. The *isodual exterior differential* of $X^d(x^d)$ is given by

$$D^d X^{kd}(x^d) = d^d X^{kd}(x^d) + \Gamma^{dk}_{i\,j} \times^d X^{id} \times^d d^d x^{jd} = DX^k(-x), \quad (2.1.47)$$

where the Γ^d's are the components of the *isodual connection*. The *isodual covariant derivative* is then given by

$$X^{id}(x^d)_{|^{dk}} = \partial^d X^{id}(x^d)/^d \partial^d x^{kd} + \Gamma^{di}_{jk} \times^d X^{jd}(x^d) = -X^i(-x)_{|^k}. \quad (2.1.48)$$

The interested reader can then easily derive the isoduality of the remaining notions of the conventional geometry.

It is an instructive exercise for the interested reader to work out in detail the proof of the following:

LEMMA 2.1.6 [8]: *The isodual image of a Riemannian space* $\Re^d(x^d, g^d, R^d)$ *is characterized by the following maps*:

$$Basic \quad Unit$$

$$I \to I^d = -I,$$

$$Metric$$

$$g \to g^d = -g, \quad (2.1.49a)$$

$$Connection \quad Coefficients$$

$$\Gamma_{klh} \to \Gamma^d_{klh} = -\Gamma_{klh}, \quad (2.1.49b)$$

$$Curvature \quad Tensor$$

$$R_{lijk} \to R^d_{lijk} = -R_{lijk}, \quad (2.1.49c)$$

$$Ricci \quad Tensor$$

$$R_{\mu\nu} \to R^d_{\mu\nu} = -R_{\mu\nu}, \quad (2.1.49d)$$

$$Ricci \quad Scalar$$

$$R \to R^d = R, \quad (2.2.49e)$$

$$Einstein - Hilbert \quad Tensor$$

$$G_{\mu\nu} \to G^d_{\mu\nu} = -G_{\mu\nu}, \quad (2.1.49f)$$

$$Electromagnetic \quad Potentials$$

$$A_\mu \ \rightarrow \ A^d_\mu \ = \ -A_\mu, \tag{2.1.49g}$$

$$Electromagnetic \quad Field$$

$$F_{\mu\nu} \ \rightarrow \ F^d_{\mu\nu} \ = \ -F_{\mu\nu}, \tag{2.1.49h}$$

$$ElmEnergy - Momentum \quad Tensor$$

$$T_{\mu\nu} \ \rightarrow \ T^d_{\mu\nu} \ = \ -T_{\mu\nu}, \tag{2.1.49i}$$

In summary, the geometries significant for this study are: the *conventional Euclidean, Minkowskian and Riemannian geometries* used for the characterization of *matter*; and the *isodual Euclidean, Minkowskian and Riemannian geometries* used for the characterization of *antimatter*.

The reader can now begin to see the achievement of axiomatic compatibility between gravitation and electroweak interactions that is permitted by the isodual theory of antimatter. In fact, the latter is treated via negative-definite energy-momentum tensors, thus being compatible with the negative-energy solutions of electroweak interactions, therefore setting correct axiomatic foundations for a true grand unification studied in the next chapter.

2.2 CLASSICAL ISODUAL THEORY OF POINT-LIKE ANTIPARTICLES

2.2.1 Basic Assumptions

Thanks to the preceding study of isodual mathematics, we are now sufficiently equipped to resolve the scientific impasse caused by the absence of a classical theory of antimatter studied in Section 1.1.

As it is well known, the contemporary treatment of matter is characterized by *conventional mathematics*, here referred to ordinary numbers, fields, spaces, etc. with *positive units and norms*, thus having positive characteristics of mass, energy, time, etc.

In this chapter we study the *characterization of antimatter via isodual numbers, fields, spaces, etc., thus having negative–definite units and norms*. In particular, all characteristics of matter (and not only charge) change sign for antimatter when represented via isoduality.

The above characterization of antimatter evidently provides the correct conjugation of the charge at the desired classical level. However, by no means, the sole change of the sign of the charge is sufficient to ensure a consistent classical representation of antimatter. To achieve consistency, the theory must resolve the main problematic aspect of current classical treatments, the fact that their operator image is not the correct charge conjugate state (Section 2.1).

The above problematic aspect is indeed resolved by the isodual theory. The main reason is that, jointly with the conjugation of the charge, isoduality also conjugates *all* other physical characteristics of matter. This implies *two* channels of quantization, the conventional one for matter and a new *isodual quantization* for antimatter (see Section 2.3) in such a way that its operator image is indeed the charge conjugate of that of matter.

In this section, we study the physical consistency of the theory in its classical formulation. The novel isodual quantization, the equivalence of isoduality and charge conjugation and related operator issues are studied in the next section.

Beginning our analysis, we note that the isodual theory of antimatter resolves the traditional obstacles against negative energies and masses. In fact, *particles with negative energies and masses measured with negative units are fully equivalent to particles with positive energies and masses measured with positive units.* This result has permitted the elimination of sole use of second quantization for the characterization of antiparticles because antimatter becomes treatable at *all* levels, including second quantization.

The isodual theory of antimatter also resolves the additional, well known, problematic aspects of motion backward in time. In fact, *time moving backward measured with a negative unit is fully equivalent on grounds of causality to time moving forward measured with a positive unit.*

This confirms the plausibility of the first conception of antiparticles by Stueckelberg and others as moving backward in time (see the historical analysis in Ref. [1] of Chapter 1), and creates new possibilities for the ongoing research on the so-called "spacetime machine" studied in Chapter 5.

In this section, we construct the classical isodual theory of antimatter at the Newtonian, Lagrangian, Hamiltonian, Galilean, relativistic and gravitational levels; we prove its axiomatic consistency; and we verify its compatibility with available classical experimental evidence (that dealing with electromagnetic interactions only). Operator formulations and their experimental verifications will be studied in the next section.

2.2.2 Need for Isoduality to Represent All Time Directions

It is popularly believed that time has only two directions, the celebrated *Eddington's time arrows.* In reality, *time has four different directions* depending on whether motion is forward or backward and occurs in the future or in the past, as illustrated in Figure 2.2. In turn, the correct

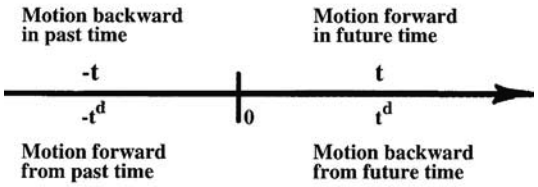

Figure 2.2. A schematic view of the "four different directions of time", depending on whether motion is forward or backward and occurs in the future or in the past. Due to the sole existence of one time conjugation, time reversal, the theoretical physics of the 20-th century missed two of the four directions of time, resulting in fundamental insufficiencies ranging from the lack of a deeper understanding of antiparticles to basic insufficiencies in biological structures and excessively insufficient cosmological views. It is evident that isoduality can indeed represent the two missing time arrows and this illustrates a basic need for the isodual theory.

use of all four different directions of time is mandatory, for instance, in serious studies of bifurcations, as we shall see.

It is evident that theoretical physics of the 20-th century could not explain all four directions of time, since it possessed only one conjugation, time reversal, and this explains the reason the two remaining directions of time were ignored.

It is equally evident that isoduality does indeed permit the representation of the two missing directions of time, thus illustrating its need.

We assume the reader is now familiar with the differences between time reversal and isoduality. Time reversal changes the direction of time while keeping the underlying space and units unchanged, while isoduality changes the direction of time while mapping the underlying space and units into different forms.

Unless otherwise specified, through the rest of this volume time t will be indicate *motion forward toward in future times*, $-t$ will indicate *motion backward in past times*, t^d will indicate motion backward from future times, and $-t^d$ will indicate *motion forward from past times.*

2.2.3 Experimental Verification of the Isodual Theory of Antimatter in Classical Physics

The experimental verification of the isodual theory of antimatter at the *classical* level is provided by the compliance of the theory with the only available experimental data, those on Coulomb interactions.

For that purpose, let us consider the Coulomb interactions under the customary notation that *positive (negative) forces represent repulsion (attraction)* when formulated in conventional Euclidean space.

Under such an assumption, the *repulsive* Coulomb force among two *particles* of negative charges $-q_1$ and $-q_2$ in Euclidean space $E(r, \delta, R)$ is given by

$$F \;=\; K \times (-q_1) \times (-q_2)/r \times r > 0, \tag{2.2.1}$$

where K is a positive constant whose explicit value (here irrelevant) depends on the selected units, the operations of multiplication $\times$ and division / are the conventional ones of the underlying field $R(n, +, \times)$.

Under isoduality to $E^d(r^d, \delta^d, R^d)$ the above law is mapped into the form

$$F^d \;=\; K^d \times^d (-q_1)^d \times^d (-q_2)^d /^d r^d \times^d r^d \;=\; -F < 0, \tag{2.2.2}$$

where $\times^d = -\times$ and $/^d = -/$ are the isodual operations of the underlying field $R^d(n^d, +, \times^d)$.

But the isodual force $F^d \;=\; -F$ occurs in the isodual Euclidean space and it is, therefore, defined with respect to the unit -1. This implies that the reversal of the sign of a repulsive force measured with a negative unit also describes repulsion. As a result, isoduality correctly represents the *repulsive* character of the Coulomb force for two *antiparticles* with *positive* charges, a result first achieved in Ref. [9].

The formulation of the cases of two particles with positive charges and their antiparticles with negative charges is left to the interested reader.

The Coulomb force between a *particle* and an *antiparticle* can only be computed by *projecting the antiparticle in the conventional space of the particle or vice-versa*. In the former case we have

$$F \;=\; K \times (-q_1) \times (-q_2)^d / r \times r < 0, \tag{2.2.3}$$

thus yielding an *attractive* force, as experimentally established. In the projection of the particle in the isodual space of the antiparticle, we have

$$F^d \;=\; K^d \times^d (-q_1) \times^d (-q_2)^d /^d r^d \times^d r^d > 0. \tag{2.2.4}$$

But this force is now measured with the unit -1, thus resulting in being again *attractive*.

The study of Coulomb interactions of magnetic poles and other classical experimental data is left to the interested reader.

In conclusion, the isodual theory of antimatter correctly represents all available classical experimental evidence in the field.

2.2.4 Isodual Newtonian Mechanics

A central objective of this section is to show that the isodual theory of antimatter resolves the scientific imbalance of the 20-th century between matter and antimatter, by permitting the study of antimatter at *all* levels as occurring for matter. Such an objective can only be achieved by first establishing the existence of a *Newtonian* representation of antimatter subsequently proved to be compatible with known operator formulations.

As it is well known, the Newtonian treatment of N *point-like particles* is based on a $7N$–dimensional representation space given by the Kronecker products of the Euclidean spaces of time t, coordinates r and velocities v (for the conventional case see Refs. [33,34]),

$$S(t,r,v) = E(t,R_t) \times E(r,\delta,R_r) \times E(v,\delta,R_v), \tag{2.2.5}$$

where

$$r = (r_a^k) = (r_a^1, r_a^2, r_a^3) = (x_a, y_a, z_a), \tag{2.2.6a}$$

$$v = (v_{ka}) = (v_{1a}, v_{2a}, v_{3a}) = (v_{xa}, v_{ua}, v_{za}) = dr/dt, \tag{2.2.6b}$$

$$\delta = Diag.(1,1,1),\ k = 1,2,3,\ a = 1,2,3,\ldots,N, \tag{2.2.6c}$$

and the base fields are trivially identical, i.e., $R_t = R_r = R_v$, since all units are assumed to have the trivial value $+1$, resulting in the trivial total unit

$$I_{tot} = I_t \times I_r \times I_v = 1 \times 1 \times 1 = 1. \tag{2.2.7}$$

The resulting basic equations are then given by the celebrated *Newton's equations for point-like particles*

$$m_a \times dv_{ka}/dt = F_{ka}(t,r,v),\ k = 1,2,3,\ a = 1,2,3,\ldots,N. \tag{2.2.8}$$

The basic space for the treatment of n antiparticles is given by the $7N$–dimensional *isodual space* [9]

$$S^d(t^d,r^d,v^d) \ = \ E^d(t^d,R_t^d) \times E^d(r^d,\delta^d,R^d) \times E^d(v^d,\delta^d,R^d), \tag{2.2.9}$$

with *isodual unit* and *isodual metric*

$$I_{Tot}^d \ = \ I_t^d \times I_r^d \times I_v^d, \tag{2.2.10a}$$

$$I_t^d \ = \ -1,\ I_r^d \ = \ I_v^d \ = Diag.(-1,-1,-1), \tag{2.2.10b}$$

$$\delta^d = Diag.(1^d,1^d,1^d) = Diag.(-1,-1,-1). \tag{2.2.10c}$$

We reach in this way the basic equations of this chapter, today known as the *Newton-Santilli isodual equations for point-like antiparticles*, first introduced in Ref. [4],[1]

$$m_a^d \times^d d^d v_{ka}^d/{}^d d^d t^d \ = \ F_{ka}^d(t^d,r^d,v^d), \tag{2.2.11}$$

$$k = x, y, z, \quad a = 1, 2, \ldots, n.$$

whose experimental verification has been provided in the preceding section.

It is easy to see that the isodual formulation is anti-isomorphic to the conventional version, as desired, to such an extent that the two formulations actually coincide at the abstract, realization-free level.

Despite this axiomatic simplicity, the physical implications of the isodual theory of antimatter are rather deep. To begin their understanding, note that throughout the 20-th century it was believed that matter and antimatter exist in the same spacetime. In fact, as recalled earlier, charge conjugation is a map of our physical spacetime into itself.

One of the first physical implications of the Newton-Santilli isodual equations is that *antimatter exists in a spacetime co-existing, yet different than our own.* In fact, the isodual Euclidean space $E^d(r^d, \delta^d, R^d)$ co-exist within, but it is physically distinct from our own Euclidean space $E(r, \delta, R)$, and the same occurs for the full representation spaces $S^d(t^d, r^d, v^d)$ and $S(t, r, v)$.

The next physical implication of the Newton-Santilli isodual equations is the confirmation that *antimatter moves backward in time in a way as causal as the motion of matter forward in time* (again, because negative time is measured with a negative unit). In fact, the *isodual time* t^d is necessarily negative whenever t is our ordinary time. Alternatively, we can say that *the Newton-Santilli isodual equations provide the only known causal description of particles moving backward in time.*

Yet another physical implication is that *antimatter is characterized by negative mass, negative energy and negative magnitudes of other physical quantities.* As we shall see, these properties have the important consequence of eliminating the necessary use of Dirac's "hole theory."

The rest of this chapter is dedicated to showing that the above novel features are necessary to achieve a consistent representation of antimatter at all levels of study, from Newton to second quantization.

As we shall see, the physical implications are truly at the edge of imagination, such as: the existence of antimatter in a new spacetime different from our own constitutes the first known evidence of multi-dimensional character of our universe despite our sensory perception to the contrary; the achievement of a fully equivalent treatment of matter and antimatter implies the necessary existence of antigravity for antimatter in the field of matter (and vice-versa); the motion backward in time implies the existence of a causal spacetime machine (although restricted for technical reasons only to isoselfdual states); and other far reaching advances.

2.2.5 Isodual Lagrangian Mechanics

The second level of treatment of *matter* is that via the conventional *classical Lagrangian mechanics.* It is, therefore, essential to identify the corresponding formulation for *antimatter*, a task first studied in Ref. [4] (see also Ref. [9]).

A conventional (first–order) Lagrangian $L(t,r,v) = \frac{1}{2} \times m \times v^k \times v_k + V(t,r,v)$ on configuration space (2.2.5) is mapped under isoduality into the *isodual Lagrangian*

$$L^d(t^d,r^d,v^d) = -L(-t,-r,-v), \tag{2.2.12}$$

defined on isodual space (2.2.9).

In this way we reach the basic analytic equations of this chapter, today known as *Lagrange-Santilli isodual equations*, first introduced in Ref. [4]

$$\frac{d^d}{d^d t^d} d \frac{\partial^d L^d(t^d,r^d,v^d)}{\partial^d v^{kd}} d - \frac{\partial^d L^d(t^d,r^d,v^d)}{\partial^d r^{kd}} d = 0, \tag{2.2.13}$$

All various aspects of the *isodual Lagrangian mechanics* can then be readily derived.

It is easy to see that isodual equations (2.3.13) provide a *direct analytic representation* (i.e., a representation without integrating factors or coordinate transforms) of the isodual equations (2.2.11),

$$\frac{d^d}{d^d t^d} d \frac{\partial^d L^d(t^d,r^d,v^d)}{\partial^d v^{kd}} d - \frac{\partial^d L^d(t^d,r^d,v^d)}{\partial^d x^{kd}} d$$

$$= m_k^d \times^d d^d v_k^d /^d d^d t^d - F_k^{d\,SA}(t,r,v) = 0. \tag{2.2.14}$$

The compatibility of the isodual Lagrangian mechanics with the primitive Newtonian treatment then follows.

2.2.6 Isodual Hamiltonian Mechanics

The *isodual Hamiltonian* is evidently given by [4,9]

$$H^d = p_k^d \times^d p^{dk} /^d 2^d \times^d m^d + V^d(t^d,r^d,v^d) = -H. \tag{2.2.15}$$

It can be derived from (nondegenerate) isodual Lagrangians via a simple isoduality of the Legendre transforms and it is defined on the $7N$–dimensional *isodual phase space (isocotangent bundle)*

$$S^d(t^d,r^d,p^d) = E^d(t^d,R_t^d) \times E^d(r^d,\delta^d,R^d) \times E^d(p^d,\delta^d,R^d). \tag{2.2.16}$$

The *isodual canonical action* is given by [4,9]

$$A^{\circ d} = \int_{t_1}^{t_2} (p_k^d \times^d d^d r^{kd} - H^d \times^d d^d t^d)$$

$$= \int_{t_1}^{t_2} [R_\mu^{\circ d}(b^d) \times^d d^d b^{\mu d} - H^d \times^d d^d t^d], \tag{2.2.17a}$$

$$R^\circ \ = \ \{p, 0\}, \ b \ = \ \{x, p\}, \ \mu \ = \ 1, 2, \ldots, 6. \tag{2.2.17b}$$

Conventional variational techniques under simple isoduality then yield the fundamental canonical equations of this chapter, today known as *Hamilton-Santilli isodual equations* [4,24–31] that can be written in the disjoint r and p notation

$$\frac{d^d x^{kd}}{d^d t^d} = \frac{\partial^d H^d(t^d, x^d, p^d)}{\partial^d p_k^d}, \ \frac{d^d p_k^d}{d^d t^d} = -\frac{\partial^d H^d(t^d, x^d, p^d)}{\partial^d x^{dk}}, \tag{2.2.18}$$

or in the unified notation

$$\omega_{\mu\nu}^d \times^d \frac{d^d b^{d\nu}}{d^d t^d} \ = \ \frac{\partial^d H^d(t^d, b^d)}{\partial^d b^{d\mu}}, \tag{2.2.19}$$

where $\omega_{\mu\nu}^d$ is the *isodual canonical symplectic tensor*

$$(\omega_{\mu\nu}^d) \ = \ (\partial^d R_\nu^{\circ d} /^d \partial^d b^{d\mu} - \partial^d R_\mu^{\circ d} /^d \partial^d b^{d\nu})$$

$$= \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix} \ = \ (\omega^{\mu\nu}). \tag{2.2.20}$$

Note that isoduality maps the canonical symplectic tensor into the canonical Lie tensor, with intriguing geometric and algebraic implications.

The *Hamilton-Jacobi-Santilli isodual equations* are then given by [4,9]

$$\partial^d A^{\circ d} /^d \partial^d t^d + H^d \ = \ 0, \tag{2.2.21a}$$

$$\partial^d A^{\circ d} /^d \partial^d x_k^d - p_k^d \ = \ 0, \ \partial^d A^{\circ d} /^d \partial^d p_k^d \equiv 0. \tag{2.2.21b}$$

The *Lie-Santilli isodual brackets* among two isodual functions A^d and B^d on $S^d(t^d, x^d, p^d)$ then become

$$[A^d, {}^d B^d] \ = \ \frac{\partial^d A^d}{\partial^d b^{d\mu}} d \ \times^d \omega^{d\mu\nu} \times^d \frac{\partial^d B^d}{\partial^d b^{d\nu}} d \ = \ -[A, B] \tag{2.2.22}$$

where

$$\omega^{d\mu\nu} \ = \ (\omega_{\mu\nu}), \tag{2.2.23}$$

is the *Lie-Santilli isodual tensor* (that coincides with the conventional canonical tensor). The direct representation of isodual equations in first–order form is self–evident.

In summary, all properties of the isodual theory at the Newtonian level carry over at the level of isodual Hamiltonian mechanics.

2.2.7 Isodual Galilean Relativity

As it is well known, the Newtonian, Lagrangian and Hamiltonian treatment of matter are only the pre-requisites for the characterization of physical laws via basic relativities and their underlying symmetries. Therefore, no equivalence in the treatment of matter and antimatter can be achieved without identifying the relativities suitable for the *classical* treatment of antimatter.

To begin this study, we introduce the *Galilei-Santilli isodual symmetry* $G^d(3.1)$ [7,5,9,22–31] as the step-by-step isodual image of the conventional *Galilei symmetry* $G(3.1)$ (herein assumed to be known[2]). By using conventional symbols for the Galilean symmetry of a Keplerian system of N point particles with non-null masses m_a, $a = 1, 2, \ldots, n$, $G^d(3.1)$ is characterized by *isodual parameters and generators*

$$w^d = (\theta_k^d, r_o^{kd}, v_o^{kd}, t_o^d) = -w, \tag{2.2.24a}$$

$$J_k^d = \sum\nolimits_a \varepsilon_{ijk} r_{ja}^d \times^d p_{ja}^k = -J_k \tag{2.2.24b}$$

$$P_k^d = \sum\nolimits_a p_{ka}^d = -P_k, \tag{2.2.24c}$$

$$G_k^d = \sum\nolimits_a (m_a^d \times^d r_{ak}^d - t^d \times p_{ak}^d), \tag{2.2.24d}$$

$$H^d = \frac{1}{2}^d \times^d \sum\nolimits_a p_{ak}^d \times^d p_a^{kd} + V^d(r^d) = -H, \tag{2.2.24e}$$

equipped with the *isodual commutator*

$$[A^d, {}^d B^d] = \sum\nolimits_{a,k} [(\partial^d A^d /{}^d \partial^d r_a^{kd}) \times^d (\partial^d B^d /{}^d \partial^d p_{ak}^d)$$

$$-(\partial^d B^d /{}^d \partial^d r_a^{kd}) \times^d (\partial^d A^d /{}^d \partial^d p_{ak}^d)]. \tag{2.2.25}$$

In accordance with rule (2.1.34), the structure constants and Casimir invariants of the isodual algebra $G^d(3.1)$ are negative–definite. If $g(w)$ is an element of the (connected component) of the Galilei group $G(3.1)$, its isodual is characterized by

$$g^d(w^d) = e^{d^{-i^d \times^d w^d \times^d X^d}}$$

$$= -e^{i \times (-w) \times X} = -g(-w) \in G^d(3.1). \tag{2.2.26}$$

The *Galilei-Santilli isodual transformations* are then given by

$$t^d \to t'^d = t^d + t_o^d = -t', \tag{2.2.27a}$$

$$r^d \to r'^d = r^d + r_o^d = -r' \tag{2.2.27b}$$

$$r^d \to r'^d = r^d + v_o^d \times^d t_o^d = -r', \tag{2.2.27c}$$

$$r^d \;\rightarrow\; r'^d \;=\; R^d(\theta^d) \times^d r^d \;=\; -R(-\theta) \times r. \qquad (2.2.27d)$$

where $R^d(\theta^d)$ is an element of the *isodual rotational symmetry* first studied in the original proposal [1].

The desired classical nonrelativistic characterization of antimatter is therefore given by imposing the $G^d(3.1)$ invariance to the considered isodual equations. This implies, in particular, that the equations admit a representation via isodual Lagrangian and Hamiltonian mechanics.

We now confirm the classical experimental verification of the above isodual representation of antimatter already treated in Section 2.2.2. Consider a conventional, classical, massive *particle* and its *antiparticle* in exterior dynamical conditions in vacuum. Suppose that the particle and antiparticle have charge $-e$ and $+e$, respectively (say, an *electron* and a *positron*), and that they enter into the gap of a magnet with constant magnetic field **B**.

As it is well known, visual experimental observation establishes that particles and antiparticles under the same magnetic field have spiral trajectories of *opposite orientation*. But this behavior occurs for the *representation of both the particle and its antiparticle in the same Euclidean space*. The situation under isoduality is different, as described by the following:

LEMMA 2.2.1 [5a]: *The trajectories under the same magnetic field of a charged particle in Euclidean space and of the corresponding antiparticle in isodual Euclidean space coincide.*

Proof: Suppose that the particle has negative charge $-e$ in Euclidean space $E(r, \delta, R)$, i.e., the value $-e$ is defined with respect to the positive unit $+1$ of the underlying field of real numbers $R = R(n, +, \times)$. Suppose that the particle is under the influence of the magnetic field **B**.

The characterization of the corresponding antiparticle via isoduality implies the reversal of the sign of all physical quantities, thus yielding the charge $(-e)^d = +e$ in the isodual Euclidean space $E^d(r^d, \delta^d, R^d)$, as well as the reversal of the magnetic field $B^d = -B$, although now defined with respect to the negative unit $(+1)^d = -1$.

It is then evident that the trajectory of a particle with charge $-e$ in the field B defined with respect to the unit $+1$ in Euclidean space and that for the antiparticle of charge $+e$ in the field $-B$ defined with respect to the unit -1 in isodual Euclidean space coincide (Figure 2.3). **q.e.d.**

An aspect of Lemma 2.2.1, which is particularly important for this monograph, is given by the following:

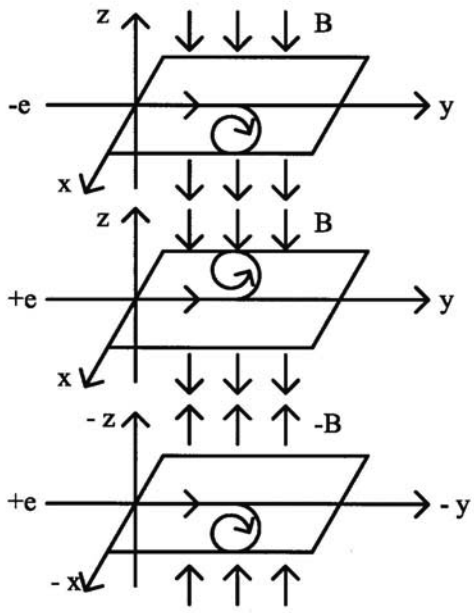

Figure 2.3. A schematic view of the trajectories of an electron and a positron with the same kinetic energy under the same magnetic field. The trajectories "appear" to be the reverse of each other when inspected by one observer, such as that in our spacetime (top and central views). However, when the two trajectories are represented in their corresponding spacetimes they coincide, as shown in the text (top and bottom views).

COROLLARY 2.2.1A: Antiparticles reverse their trajectories when projected from their own isodual space into our own space.

Lemma 2.2.1 assures that isodualities permit the representation of the correct trajectories of antiparticles as physically observed, despite their negative energy, thus providing the foundations for a consistent representation of antiparticles at the level of *first* quantization studied in the next section. Moreover, Lemma 2.2.1 tells us that the trajectories of antiparticles *appear* to exist in our space while in reality they belong to an independent space.

2.2.8 Isodual Special Relativity

We now introduce *isodual special relativity* for the classical relativistic treatment of point-like antiparticles (for the conventional case see Ref. [32]).

As it is well known, conventional special relativity is constructed on the fundamental 4-dimensional unit of the Minkowski space

$$I \ = \ Diag.(1,1,1,1),$$

representing the dimensionless units of space, e.g., $(+1cm, +1cm, +1cm)$, and the dimensionless unit of time, e.g., $+1$ sec, and constituting the basic unit of the conventional *Poincarè symmetry* $P(3.1)$ (hereon assumed to be known).

It then follows that *isodual special relativity* is characterized by the map

$$\begin{aligned} I \ &= \ Diag.(\{1,1,1\},1) > 0 \\ \rightarrow I^d \ &= \ Diag.(\{-1,-1,-1\},-1) < 0. \end{aligned} \tag{2.2.28}$$

namely, the antimatter relativity is based on *negative units of space and time*, e.g., $I^d = Diag.(-1cm, -1cm, -1cm, -1sec)$. This implies the reconstruction of the entire mathematics of the special relativity with respect to the common, isodual unit I^d, including: the *isodual field* $R^d = R^d(n^d, +^d, \times^d)$ of *isodual numbers* $n^d = n \times I^d$; the *isodual Minkowski spacetime* $M^d(x^d, \eta^d, R^d)$ with isodual coordinates $x^d = x \times I^d$, isodual metric $\eta^d = -\eta$ and basic invariant over R^d

$$(x-y)^{d^{2d}} \ = \ [(x^\mu - y^\mu) \times \eta^d_{\mu\nu} \times (x^\nu - y^\nu)] \times I^d \in R^d. \tag{2.2.29}$$

This procedure yields the central symmetry of this chapter indicated in Section 2.2.6, today known as the *Poincaré-Santilli isodual symmetry* [7]

$$P^d(3.1) \ = \ \mathcal{L}^d(3.1) \times^d T^d(3.1), \tag{2.2.30}$$

where $\mathcal{L}^d(3.1)$ is the *Lorentz-Santilli isodual symmetry*, $\times^d$ is the *isodual direct product* and $T^d(3.1)$ represents the *isodual translations.*

The algebra of the connected component $P^{\uparrow d}_+(3.1)$ of $P^d(3.1)$ can be constructed in terms of the isodual parameters $w^d = \{-w_k\} = \{-\theta, -v, -a\}$ and isodual generators $X^d = -X = \{-X_k\} = \{-M_{\mu\nu}, -P_\mu\}$. The isodual commutator rules are given by [7]

$$[M^d_{\mu\nu}, {}^d\, M_{\alpha\beta}]^d$$

$$= i^d \times^d (\eta^d_{\nu\alpha} \times^d M^d_{\mu\beta} - \eta^d_{\mu\alpha} \times^d M^d_{\nu\beta} - \eta^d_{\nu\beta} \times^d M^d_{\mu\alpha} + \eta^d_{\mu\beta} \times^d \hat{M}^d_{\alpha\nu}), \tag{2.2.31a}$$

$$[M^d_{\mu\nu},^d p^d_\alpha] \;=\; i^d \times^d (\eta^d_{\mu\alpha} \times^d p^d_\nu - \eta^d_{\nu\alpha} \times^d p^d_\mu), \tag{2.2.31b}$$

$$[p^d_\alpha, p^d_\beta]^d \;=\; 0. \tag{2.3.31c}$$

The *Poincarè-Santilli isodual transformations* are given by[3]

$$x^{1d\prime} \;=\; x^{1d} = -x^1, \tag{2.2.32a}$$

$$x^{2d\prime} \;=\; x^{2d} = -x^2, \tag{2.2.32b}$$

$$x^{3d\prime} \;=\; \gamma^d \times^d (x^{3d} - \beta^d \times^d x^{4d}) = -x^{3\prime}, \tag{2.2.32c}$$

$$x^{4d\prime} \;=\; \gamma^d \times^d (x^{4d} - \beta^d \times^d x^{3d}) = -x^{4\prime}, \tag{2.2.32d}$$

$$x^{d\mu\prime} \;=\; x^{d\mu} + a^{d\mu} \;=\; -x^{\mu\prime}, \tag{2.3.32e}$$

where

$$\beta^d \;=\; v^d /^d c^d_\circ \;=\; -\beta, \;\; \beta^{d2d} \;=\; -\beta^2, \;\; \gamma^d \;=\; -(1-\beta^2)^{-1/2}. \tag{2.2.33}$$

and the use of the isodual operations (quotient, square roots, etc.), is assumed.

The *isodual spinorial covering*

$$\mathcal{P}^d(3.1) = \mathcal{SL}^d(2.C^d) \times^d \mathcal{T}^d(3.1) \tag{2.2.34}$$

can then be constructed via the same methods.

The basic postulates of the isodual special relativity are also a simple isodual image of the conventional postulates [7]. For instance, the *maximal isodual causal speed in vacuum* is the speed of light in M^d, i.e.,

$$V^d_{max} \;=\; c^d_\circ \;=\; -c_\circ, \tag{2.2.35}$$

with the understanding that it is measured with a *negative–definite unit*, thus being fully equivalent to the conventional maximal speed c_o referred to a positive unit. A similar situation occurs for all other postulates.

The *isodual light cone* is evidently given by (Figure 2.4)

$$x^{d\,2\,d} \;=\; (x^{\mu d} \times^d \eta^d_{\mu\nu} \times^d x^{\nu d}) \times I^d$$

$$= \; (-x \times x - y \times y - z \times z + t \times c^2_\circ \times t) \times (-I) \;=\; 0. \tag{2.2.36}$$

As one can see, the above cone formally coincides with the conventional light cone, although the two cones belong to different spacetimes. The isodual light cone is used in these studies as *the cone of light emitted by antimatter in empty space (exterior problem).*

Note that the *two* Minkowskian metrics $\eta = Diag.(+1,+1,+1,-1)$ and $\eta = Diag.(-1,-1,-1,+1)$ have been popular since Minkowski's times, although both referred to the *same* unit I. We have learned here that these two popular metrics are connected by isoduality.

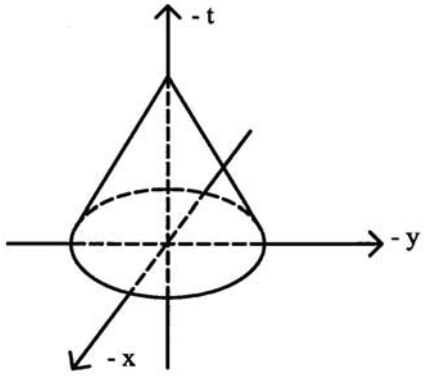

Figure 2.4. A schematic view of the "isodual backward light cone" as seen by an observer in our own spacetime with a time evolution reversed with respect to the "conventional forward light cone."

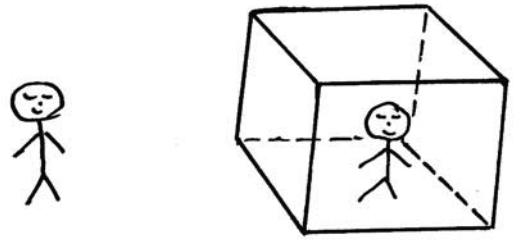

Figure 2.5. A schematic view of the "isodual cube," here defined as a conventional cube with two observers, an external observer in our spacetime and an internal observer in the isodual spacetime. The first implication of the isodual theory is that the same cube coexist in the two spacetimes and can, therefore, be detected by both observers. A most intriguing implication of the isodual theory is that each observer sees the other becoming younger. This occurrence is evident for the behavior of the internal observer with respect to the exterior one, since the former evolves according to a time opposite that of the latter. The same occurrence is less obvious for the opposite case, the behavior of the external observer with respect to the internal one, and it is due to the fact that the projection of our positive time into the isodual spacetime is indeed a motion backward in that spacetime.

We finally introduce the *isodual electromagnetic waves* and related *isodual Maxwell's equations* [9]

$$F^d_{\mu\nu} = \partial^d A^d_\mu /^d \partial^d x^{\nu d} - \partial^d A^d_\nu /^d \partial^d x^{d\mu}, \tag{2.2.37a}$$

$$\partial^d_\lambda F^d_{\mu\nu} + \partial^d_\mu F^d_{\nu\lambda} + \partial^d_\nu F^d_{\lambda\mu} = 0, \tag{2.2.37b}$$

$$\partial^d_\mu F^{d\mu\nu} = -J^{d\nu}. \tag{2.2.37c}$$

As we shall see, the nontriviality of the isodual special relativity is illustrated by the fact that isodual electromagnetic waves experience gravitational repulsion when in the field of matter.

2.2.9 Inequivalence of Isodual and Spacetime Inversions

As it is well known (see, the fundamental spacetime symmetries of the 20-th century are the continuous (connected) component of the Poincaré symmetry plus discrete symmetries characterized by *space reversal* (also called *parity*) and *time reversal.*

As noted earlier, antiparticles are assumed in the above setting to exist in the same representation spacetime and to obey the same symmetries as those of particles. On the contrary, according to the isodual theory, antiparticles are represented in a spacetime and possess symmetries distinct from those of particles, although connected to the latter by the isodual transform.

The latter occurrence requires the introduction of the *isodual spacetime inversions*, that is, the isodual images of space and time inversions, first identified in Ref. [9], that can be formulated in unified coordinate form as follows

$$\begin{aligned} x^{d\mu\prime} &= \pi^d \times^d x^d = -\pi \times x \\ &= (-r, x^4), \quad \tau^d \times^d x^d = -\tau \times x = -(r, -x^4), \end{aligned} \tag{2.2.38}$$

with field theoretical extension (here expressed for simplicity for a scalar field)

$$\pi^d \times^d \phi^d(x^d) \times^d \pi^{d\dagger} = \phi^d(x'^d, x'^d = (-r^d, t^d) = (r, -t), \tag{2.2.39a}$$

$$\tau^d \times^d \phi^d(x^d) \times^d \tau^{d\dagger} = \bar{\phi}^d(x^{"d}, x^{"d} = (r^d, -t^d) = (-r, t), \tag{2.2.39b}$$

where $r^d(= -r)$ is the *isodual coordinate* on space $E^d(r^d, \delta^d, R^d)$, and t^d is the *isodual time* on $E^d(t^d, 1, R^d_t)$.

LEMMA 2.2.2 [9]: *Isodual inversions and spacetime inversions are inequivalent.*

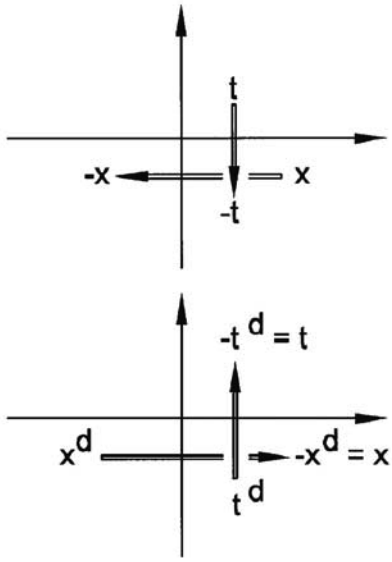

Figure 2.6. A schematic view of the additional peculiar property that the projection in our spacetime of the isodual space inversion appears as a time inversion and vice versa. In fact, a point in the isodual spacetime is given by $(x^d, t^d) = (-x, -t)$. The projection in our spacetime of the isodual space inversion $(x^d, t^d) \to (-x^d, t^d)$ is then given by $(x, -t)$, thus appearing as a time (rather than a space) inversion. Similarly, the projection in our spacetime of the isodual time inversion $(x^d, t^d) \to (x^d, -t^d)$ appears as $(-x, t)$, that is, as a space (rather than time) inversion. Despite its simplicity, the above occurrence has rather deep implications for all discrete symmetries in particle physics indicated later on.

Proof. Spacetime inversions are characterized by the change of sign $x \to -x$ *by always preserving the original metric measured with positive units*, while isodual inversions imply the map $x \to x^d = -x$ *but now measured with an isodual metric* $\eta^d = -\eta$ *with negative units* $I^d = -I$, thus being inequivalent. **q.e.d.**

Despite their simplicity, isodual inversions (or isodual discrete symmetries) are not trivial (Figure 2.6). In fact, all measurements are done in our spacetime, thus implying the need to consider the *projection* of the isodual discrete symmetries into our spacetime which are manifestly different than the conventional forms.

In particular, they imply a sort of interchange, in the sense that the conventional *space* inversion $(r, t) \to (-r, t)$ emerges as belonging to the projection in our spacetime of the isodual *time* inversion, and vice-versa.

Note that the above "interchange" of parity and time reversal of isodual particles projected in our spacetime could be used for experimental verifications, but this aspect is left to interested readers.

In closing this subsection, we point out that the notion of isodual parity has intriguing connections with the parity of antiparticles in the $(j,0)+(0,j)$ representation space more recently studied by Ahluwalia, Johnson and Goldman [10]. In fact, the latter parity results in being opposite that of particles which is fully in line with isodual space inversion (isodual parity).

2.2.10 Isodual Thermodynamics of Antimatter

An important contribution to the isodual theory has been made by J. Dunning-Davies [11] who introduced in 1999 the first, and only known consistent thermodynamics for antimatter with intriguing results and implications.

As conventionally done in the field, let us represent heat with Q, internal energy with U, work with W, entropy with S, and absolute temperature with T. *Dunning-Davies isodual thermodynamics of antimatter* is evidently defined via the isodual quantities

$$Q^d=-Q, U^d=-U, W^d=-W, S^d=-S, T^d=-T \tag{2.2.40}$$

on isodual spaces over the isodual field of real numbers $R^d = R^d(n^d,+^d,\times^d)$ with isodual unit $I^d=-1$.

Recall from Section 2.1.3 that *differentials are isoselfdual* (that is, invariant under isoduality). Dunning-Davies then has the following:

THEOREM 2.2.1 [11]: *Thermodynamical laws are isoselfdual.*

Proof. For the *First Law of thermodynamics* we have

$$dQ=dU-dW \;\equiv\; d^dQ^d=d^dU^d-d^dW^d. \tag{2.2.41}$$

Similarly, for the *Second Law of thermodynamics* we have

$$dQ=T\times dS \;\equiv\; d^dQ^d=T^d\times^d S^d, \tag{2.2.42}$$

and the same occurs for the remaining laws. **q.e.d.**

Despite their simplicity, Dunning-Davies results [11] have rather deep implications. First, the identity of thermodynamical laws, by no means, implies the identity of the thermodynamics of matter and antimatter. In fact, *in Dunning-Davies isodual thermodynamics the entropy must always decrease in time*, since the isodual entropy is always negative and is defined in a space with evolution backward in time with respect

to us. However, these features are fully equivalent to the conventional increase of the entropy tacitly referred to positive units.

Also, Dunning-Davies results indicate that *antimatter galaxies and quasars cannot be distinguished from matter galaxies and quasars via the use of thermodynamics*, evidently because their laws coincide, in a way much similar to the identity of the trajectories of particles and antiparticles of Lemma 2.2.1.

This result indicates that the only possibility known at this writing to determine whether far-away galaxies and quasars are made up of matter or of antimatter is that via the predicted gravitational repulsion of the light emitted by antimatter called *isodual light* (see next section and Chapter 5).

2.2.11 Isodual General Relativity

For completeness, we now introduce the *isodual general relativity* for the classical gravitational representation of antimatter. A primary motivation for its study is the incompatibility with antimatter of the positive-definite character of the energy-momentum tensor of the conventional general relativity studied in Chapter 1.

The resolution of this incompatibility evidently requires a structural revision of general relativity [33] for a consistent treatment of antimatter. The *only* solution known to the author is that offered by isoduality.[4]

It should be stressed that this study is here presented merely for completeness, since the achievement of a consistent treatment of negative-energies, by no means, resolves the serious inconsistencies of gravitation on a Riemannian space caused by curvature, as studied in Section 1.2, thus requiring new geometric vistas beyond those permitted by the Riemannian geometry (see Chapters 3 and 4).

As studied in Section 2.1.7, the *isodual Riemannian geometry* is defined on the isodual field $R^d(n^d, +^d, \times^d)$ for which *the norm is negative–definite*, Eq. (2.1.18). As a result, *all quantities that are positive in Riemannian geometry become negative under isoduality, thus including the energy-momentum tensor.*

In fact, the energy-momentum tensor of isodual electromagnetic waves (2.2.37) is negative-definite [8,9]

$$T^d_{\mu\nu} \;=\; (4\times\pi)^{-1d}\times^d (F^d_{\mu\alpha}\times^d F^d_{\alpha_\nu} + (1/4)^{-1d}\times^d g^d_{\mu\nu}\times^d F^d_{\alpha\beta}\times^d F^{d\alpha\beta}). \tag{2.2.43}$$

The *Einstein-Hilbert isodual equations for antimatter in the exterior conditions in vacuum* are then given by [6, 9]

$$G^d_{\mu\nu} \;=\; R^d_{\mu\nu} - \frac{1}{2}^d \times^d g^d_{\mu\nu}\times^d R^d \;=\; k^d\times^d T^d_{\mu\nu}, \tag{2.2.44}$$

The rest of the theory is then given by the use of the isodual Riemannian geometry of Section 2.1.7.

The explicit study of this gravitational theory of antimatter is left to the interested reader due to the indicated inconsistencies of gravitational theories on a Riemannian space for the *conventional case of matter* (Section 1.2). These inconsistencies multiply when treating antimatter, as we shall see.

2.3 OPERATOR ISODUAL THEORY OF POINT-LIKE ANTIPARTICLES

2.3.1 Basic Assumptions

In this section we study the operator image of the classical isodual theory of the preceding section; we prove that the operator image of isoduality is equivalent to charge conjugation; and we show that isodual mathematics resolves all known objections against negative energies.

A main result of this section is the identification of a simple, structurally new formulation of quantum mechanics known as *isodual quantum mechanics* or, more properly, as the *isodual branch of hadronic mechanics* first proposed by Santilli in Ref. [5]. Another result of this section is the fact that all numerical predictions of operator isoduality coincide with those obtained via charge conjugation on a Hilbert space, thus providing the experimental verification of the isodual theory of antimatter at the operator level.

Despite that, the isodual image of quantum mechanics is not trivial because of a number of far reaching predictions we shall study in this section and in the next chapters, such as: the prediction that antimatter emits a new light distinct from that of matter; antiparticles in the gravitational field of matter experience antigravity; bound states of particles and their antiparticles can move backward in time without violating the principle of causality; and other predictions.

Other important results of this section are a new interpretation of the conventional Dirac equation that escaped detection for about one century, as well as the indication that the isodual theory of antimatter originated from the Dirac equation itself, not so much from the negative-energy solutions, but more properly from their two-dimensional unit that is indeed negative-definite, $I_{2\times 2} = Diag.(-1, -1)$.

As we shall see, Dirac's "hole theory", with the consequential restriction of the study of antimatter to the sole second quantization and resulting scientific imbalance indicated in Section 1.1, were due to Dirac's lack of knowledge of a *mathematics based on negative units.*

Intriguingly, had Dirac identified the quantity $I_{2\times 2} = Diag.(-1,-1)$ as the unit of the *mathematics* treating the negative energy solutions of his equation, the physics of the 20-th century would have followed a different path because, despite its simplicity, the unit is indeed the most fundamental notion of all mathematical and physical theories.

2.3.2 Isodual Quantization

The isodual Hamiltonian mechanics (and its underlying *isodual symplectic geometry* [5a] not treated in this chapter for brevity) permit the identification of a new quantization channel, known as the *naive isodual quantization* [6] that can be readily formulated via the use of the Hamilton-Jacobi-Santilli isodual equations (2.2.21) as follows

$$A^{\circ d} \to -i^d \times^d \hbar^d \times^d Ln^d \psi^d(t^d, r^d), \tag{2.3.1a}$$

$$\partial^d A^{\circ d}/^d \partial^d t^d + H^d \ = \ 0 \to i^d \times^d \partial^d \psi^d/^d \partial^d t^d$$
$$= \ H^d \times^d \psi^d \ = \ E^d \times^d \psi^d, \tag{2.3.1b}$$

$$\partial^d A^{\circ d}/^d \partial^d x^{dk} - \hat{p}_k \ = \ 0 \to p_k^d \times^d \psi^d \ = \ -i^d \times^d \partial_k^d \psi^d, \tag{2.3.1c}$$

$$\partial^d A^{\circ d}/^d \partial^d p_k^d = 0 \to \partial^d \psi^d/^d \partial^d p_k^d = 0. \tag{2.3.1d}$$

Recall that the fundamental unit of quantum mechanics is Planck's constant $\hbar = +1$. It then follows that the fundamental unit of the isodual operator theory is the new quantity

$$\hbar^d = -1. \tag{2.3.2}$$

It is evident that the above quantization channel identifies the new mechanics known as *isodual quantum mechanics*, or the *isodual branch of hadronic mechanics.*

2.3.3 Isodual Hilbert Spaces

Isodual quantum mechanics can be constructed via the anti-unitary transform

$$U \times U^\dagger = \hbar^d = I^d = -1, \tag{2.3.3}$$

applied, for consistency, to the *totality* of the mathematical and physical formulations of quantum mechanics. We recover in this way the isodual real and complex numbers

$$n \to n^d = U \times n \times U^\dagger = n \times (U \times U^\dagger) = n \times I^d, \tag{2.3.4}$$

isodual operators

$$A \to U \times A \times U^\dagger = A^d, \tag{2.3.5}$$

the isodual product among generic quantities A, B (numbers, operators, etc.)

$$A \times B \to U \times (A \times B) \times U^{\dagger}$$

$$= (U \times A \times U^{\dagger}) \times (U \times U^{\dagger})^{-1} \times (U \times B \times U^{\dagger}) = A^d \times^d B^d, \quad (2.3.6)$$

and similar properties.

Evidently, isodual quantum mechanics is formulated in the *isodual Hilbert space* $\mathcal{H}^d$ with *isodual states* [6]

$$|\psi >^d = -|\psi >^{\dagger} = - < \psi|, \quad (2.3.7)$$

where $< \psi|$ is a conventional dual state on $\mathcal{H}$, and *isodual inner product*

$$< \psi|^d \times (-1) \times |\psi >^d \times I^d, \quad (2.3.8)$$

with *isodual expectation values* of an operator A^d

$$< A^d >^d = (< \psi|^d \times^d A^d \times^d |\psi >^d /^d < \psi|^d \times^d |\psi >^d), \quad (2.3.9)$$

and *isodual normalization*

$$< \psi|^d \times^d |\psi >^d = -1. \quad (2.3.10)$$

defined on the *isodual complex field* C^d with unit -1 (Section 2.1.1).

The isodual expectation values can also be reached via anti-unitary transform (2.3.3),

$$< \psi| \times A \times |\psi > \to U \times (< \psi| \times A \times |\psi >) \times U^{\dagger}$$

$$= (< \psi| \times U^{\dagger}) \times (U \times U^{\dagger})^{-1} \times (U \times A \times U^{\dagger}) \times (U \times U^{\dagger})^{-1}$$

$$\times (U \times |\psi >) \times (U \times U^{\dagger}) = < \psi|^d \times^d A^d \times^d |\psi >^d \times I^d. \quad (2.3.11)$$

The proof of the following property is trivial.

LEMMA 2.3.1 [5b]: *The isodual image of an operator A that is Hermitean on $\mathcal{H}$ over* C *is also Hermitean on $\mathcal{H}^d$ over C^d (isodual Hermiticity).*

It then follows that *all quantities that are observables for particles are equally observables for antiparticles represented via isoduality.*

LEMMA 2.3.2 [5b]: *Let H be a Hermitean operator on a Hilbert space $\mathcal{H}$ over* C *with positive-definite eigenvalues E,*

$$H \times |\psi >= E \times |\psi >, H = H^{\dagger}, E => 0. \quad (2.3.12)$$

Then, the eigenvalues of the isodual operator H^d on the isodual Hilbert space $\mathcal{H}^d$ over C^d are negative-definite,

$$H^d \times^d |\psi>^d = E^d \times^d |\psi>^d, H^d = H^{d\dagger^d}, E^d < 0. \tag{2.3.13}$$

This important property establishes an evident compatibility between the classical and operator formulations of isoduality.

We also mention the *isodual unitary laws*

$$U^d \times^d U^{d\dagger} = U^{d\dagger} \times^d U^d = I^d, \tag{2.3.14}$$

the *isodual trace*

$$Tr^d A^d = (TrA^d) \times I^d \in C^d, \tag{2.3.15a}$$

$$Tr^d(A^d \times^d B^d) = Tr^d A^d \times^d Tr^d B^d, \tag{2.3.15b}$$

the *isodual determinant*

$$Det^d A^d = (DetA^d) \times I^d \in C^d, \tag{2.3.16a}$$

$$Det^d(A^d \times^d B^d) = Det^d \times^d Det^d B^d, \tag{2.3.16b}$$

the *isodual logarithm* of a real number n

$$Log^d n^d = -(Log\, n^d) \times I^d, \tag{2.3.17}$$

and other isodual operations.

The interested reader can then work out the remaining properties of the isodual theory of linear operators on a Hilbert space.

2.3.4 Isoselfduality of Minkowski's Line Elements and Hilbert's Inner Products

A most fundamental new property of the isodual theory, with implications as vast as the formulation of a basically new cosmology, is expressed by the following lemma whose proof is a trivial application of transform (2.3.3).

LEMMA 2.3.3 [23]: *Minkowski's line elements and Hilbert's inner products are invariant under isoduality (or they are isoselfdual according to Definition 2.1.2),*

$$x^2 = (x^\mu \times \eta_{\mu\nu} \times x^\nu) \times I$$

$$\equiv (x^{d\mu} \times^d \eta^d_{\mu\nu} \times^d x^{d\nu}) \times I^d = x^{d^{2^d}}, \tag{2.3.18a}$$

$$<\psi| \times |\psi> \times I \equiv <\psi|^d \times^d |\psi>^d \times I^d. \tag{2.3.18b}$$

As a result, *all relativistic and quantum mechanical laws holding for matter also hold for antimatter under isoduality.* The equivalence of charge conjugation and isoduality then follows, as we shall see shortly.

Lemma 2.3.3 illustrates the reason why isodual special relativity and isodual Hilbert spaces have escaped detection for about one century. Note, however, that invariances (2.3.18) require the prior discovery of *new numbers*, those with negative unit.

2.3.5 Isodual Schrödinger and Heisenberg's Equations

The fundamental dynamical equations of isodual quantum mechanics are the isodual images of conventional dynamical equations. They are today known as the *Schrödinger-Santilli isodual equations* [4] (where we assume hereon $\hbar^d = -1$, thus having $\times^d \hbar^d = 1$)

$$i^d \times^d \partial |\psi>^d /^d \partial^d t^d = H^d \times^d |\psi>^d, \tag{2.3.19a}$$

$$p_k^d \times^d |\psi>^d = -i^d \times^d \partial^d |\psi>^d /^d \partial^d r^d, \tag{2.3.19b}$$

and the *Heisenberg-Santilli isodual equations*

$$i^d \times^d d^d A^d /^d d^d t^d = A^d \times^d H^d - H^d \times^d A^d = [A^d, H^d]^d, \tag{2.3.20a}$$

$$[r_i^d, p_j^d]^d = i^d \times^d \delta_j^{di}, [r^d, r^{dj}]^d = [p_i^d, p_j^d]^d = 0. \tag{2.3.20b}$$

Note that, when written explicitly, Eq. (2.3.19a) is based on an associative modular action *to the left*,

$$- <\psi| \times^d H^d = (\partial^d <\psi| \partial^d t^d) \times^d i^d, \tag{2.3.21}$$

It is an instructive exercise for readers interested in learning the new mechanics to prove the equivalence of the isodual Schrödinger and Heisenberg equations via the anti-unitary transform (2.3.3).

2.3.6 Isoselfdual Re-Interpretation of Dirac's Equation

Isoduality has permitted a novel interpretation of the conventional *Dirac equation* (we shall here used the notation of Ref. [12]) in which the negative-energy states are reinterpreted as belonging to the isodual images of positive energy states, resulting in the first known *consistent representation of antiparticles in first quantization.*

This result should be expected since the isodual theory of antimatter applies at the Newtonian level, let alone that of first quantization. Needless to say, the treatment via isodual first quantization does not exclude

that via isodual second quantization. The point is that the treatment of antiparticles is no longer restricted to second quantization, as a condition to resolve the scientific imbalance between matter and antimatter indicated earlier.

Consider the conventional Dirac equation [2]

$$[\gamma^{\mu} \times (p_{\mu} - e \times A_{\mu}/c) + i \times m] \times \Psi(x) = 0, \tag{2.3.22}$$

with realization of Dirac's celebrated gamma matrices

$$\gamma_k = \begin{pmatrix} 0 & -\sigma_k \\ \sigma_k & 0 \end{pmatrix}, \ \gamma^4 = i \times \begin{pmatrix} I_{2\times 2} & 0, \\ 0 & -I_{2\times 2} \end{pmatrix}, \tag{2.3.23a}$$

$$\{\gamma_{\mu}, \tilde{\gamma}_{\nu}\} \ = \ 2\times\eta_{\mu\nu}, \ \ \Psi = i \times \begin{pmatrix} \Phi \\ -\Phi^{\dagger} \end{pmatrix}, \tag{2.3.23b}$$

At the level of first quantization here considered, the above equation is rather universally interpreted as representing an electron under an external electromagnetic field.

The above equations are generally defined in the 6-dimensional space given by the Kronecker product of the conventional Minkowski spacetime and an internal spin space

$$M_{Tot} = M(x, \eta, R) \times S_{spin}, \tag{2.3.24}$$

with total unit

$$\begin{aligned} I_{Tot} &= I_{orb} \times I_{spin} \\ &= Diag.(1,1,1,1) \times Diag.(1,1), \end{aligned} \tag{2.3.25}$$

and total symmetry

$$P(3.1) = SL(2.C) \times T(3.1). \tag{2.3.26}$$

The proof of the following property is recommended to interested readers.

THEOREM 2.3.1 [5b]: *Pauli's sigma matrices and Dirac's gamma matrices are isoselfdual*,

$$\sigma_k \equiv \sigma_k^d, \tag{2.3.27a}$$

$$\gamma_{\mu} \equiv \gamma_{\mu}^d. \tag{2.3.27b).}$$

The above properties imply an important re-interpretation of Eq. (2.3.22), first identified in Ref. [9] and today known as the *Dirac-Santilli isoselfdual equation*, that can be written

$$[\tilde{\gamma}^{\mu} \times (p_{\mu} - e \times A_{\mu}/c) + i \times m] \times \tilde{\Psi}(x) = 0, \tag{2.3.28}$$

with re-interpretation of the gamma matrices

$$\tilde{\gamma}_k = \begin{pmatrix} 0 & \sigma_k^d \\ \sigma_k & 0 \end{pmatrix}, \ \tilde{\gamma}^4 = i \begin{pmatrix} I_{2\times 2} & 0, \\ 0 & I_{2\times 2}^d \end{pmatrix}, \tag{2.3.29a}$$

$$\{\tilde{\gamma}_\mu, \tilde{\gamma}_\nu\} = 2^d \times^d \eta_{\mu\nu}^d, \ \ \tilde{\Psi} = -\tilde{\gamma}_4 \times \Psi = i \times \begin{pmatrix} \Phi \\ \Phi^d \end{pmatrix}, \tag{2.3.29b}$$

By recalling that isodual spaces coexist with, but are different from conventional spaces, we have the following:

THEOREM 3.3.2 [9]: *The Dirac-Santilli isoselfdual equation is defined on the 12-dimensional isoselfdual representation space*

$$M_{Tot} = \{M(x, \eta, R) \times S_{spin}\} \times \{M^d(x^d, \eta^d, R^d) \times^d S_{spin}^d\}, \tag{2.3.30}$$

with isoselfdual total 12-dimensional unit

$$I_{Tot} = \{I_{orb} \times I_{spin}\} \times \{I_{orb}^d \times^d I_{spin}^d\}, \tag{2.3.31}$$

and its symmetry is given by the isoselfdual product of the Poincaré symmetry and its isodual

$$S_{Tot} = \mathcal{P}(3.1) \times \mathcal{P}^d(3.1)$$

$$= \{SL(2.C) \times T(3.1)\} \times \{SL^d(2.C^d) \times^d T^d(3.1)\}. \tag{2.3.32}$$

A direct consequence of the isoselfdual structure can be expressed as follows.

COROLLARY 2.3.2a [9]: *The Dirac-Santilli isoselfdual equation provides a joint representation of an electron and its antiparticle (the positron) in first quantization*,

$$Dirac \ \ Equation = Electron \times Positron. \tag{2.3.33}$$

In fact, the two-dimensional component of the wave function with positive-energy solution represents the electron and that with negative-energy solutions represent the positron without any need for second quantization, due to the physical behavior of negative energies in isodual treatment established earlier.

Note the complete democracy and equivalence in treatment of the electron and the positron in equation (2.3.28), in the sense that the equation can be equally used to represent an electron or its antiparticle. By comparison, according to the original Dirac interpretation, the

equation could only be used to represent the electron [12], since the representation of the positron required the "hole theory".

It has been popularly believed throughout the 20-th century that Dirac's gamma matrices provide a "four-dimensional representation of the SU(2)-spin symmetry". This belief is disproved by the isodual theory, as expressed by the following

THEOREM 3.3.3 [5b]: *Dirac's gamma matrices characterize the direct product of an irreducible two-dimensional (regular) representation of the* SU(2)*-spin symmetry and its isodual*,

$$Dirac's \;\; Spin \;\; Symmetry : SU(2) \times SU^d(2). \tag{2.3.34}$$

In fact, the gamma matrices are characterized by the conventional, 2-dimensional Pauli matrices σ_k and related identity $I_{2\times 2}$ as well as other matrices that have resulted in being the exact isodual images σ_k^d with isodual unit $I_{2\times 2}^d$.

It should be recalled that the isodual theory was born precisely out of these issues and, more particularly, from the incompatibility between the popular interpretation of gamma matrices as providing a "four-dimensional" representation of the SU(2)-spin symmetry and the *lack of existence of such a representation in Lie's theory.*

The sole possibility known to the author for the reconciliation of Lie's theory for the SU(2)-spin symmetry and Dirac's gamma matrices was to assume that $-I_{2\times 2}$ is the unit of a dual-type representation. The entire theory studied in this chapter then followed.

It should also be noted that, as conventionally written, Dirac's equation *is not* isoselfdual because not sufficiently symmetric in the two-dimensional states and their isoduals.

In summary, Dirac's was forced to formulate the "hole theory" for antiparticles because he referred the *negative* energy states to the conventional *positive* unit, while their reformulation with respect to *negative* units yields fully physical results.

It is easy to see that the same isodual reinterpretation applies for Majorana's spinorial representations [13] (see also [14,15]) as well as Ahluwalia's broader spinorial representations $(1/2, 0) + (0, 1/2)$ [16] (see also the subsequent paper [17]), that are reinterpreted in the isoselfdual form $(1, 2, 0) + (1, 2, 0)^d$, thus extending their physical applicability to first quantization.

In the latter reinterpretation the representation (1/2,0) is evidently done conventional spaces over conventional fields with unit +1, while the isodual representation $(1/2, 0)^d$ is done on the corresponding isodual

spaces defined on isodual fields with unit -1. As a result, all quantities of the representation (1/2,0) change sign under isoduality.

It should be finally indicated that Ahluwalia treatment of Majorana spinors has a deep connection with isoduality because the underlying Class II spinors have a *negative norm* [16] precisely as it is the case for isoduality. As a result, the isodual reinterpretation under consideration here is quite natural and actually warranted for mathematical consistency, e.g., to have the topology characterized by a negative norm be compatible with the underlying fields.

2.3.7 Equivalence of Isoduality and charge conjugation

We come now to another fundamental point of this chapter, the proof that isoduality is equivalent to charge conjugation. This property is crucial for the experimental verification of isoduality at the particle level too. This equivalence was first identified by Santilli in Ref. [6] and can be easily expressed today via the following:

LEMMA 2.3.4 [6,5b,18]: *The isodual transform is equivalent to charge conjugation.*

Proof. Charge conjugation is characterized by the following transform of wavefunctions (see, e.g., Ref. [12], pages 109 and 176)

$$\Psi(x) \to C\Psi(x) = c \times \Psi^{\dagger}(x), \tag{2.3.35}$$

where

$$|c| = 1, \tag{2.3.36}$$

thus being manifestly equivalent to the isodual transform

$$\Psi(x) \to \Psi^{d}(x^{d}) = -\Psi^{\dagger}(-x^{t}), \tag{2.3.37}$$

where t denotes transpose.

A reason why the two transforms are equivalent, rather than identical, is the fact that charge conjugation maps spacetime into itself, while isoduality maps spacetime into its isodual. **q.e.d.**

Let us illustrate Lemma 2.3.4 with a few examples. As well known, the Klein-Gordon equation for a free particle

$$\partial^{\mu}\partial_{\mu}\Psi - m^{2} \times \Psi = 0, \tag{2.3.38}$$

is invariant under charge conjugation, in the sense that it is turned into the form

$$c \times [\bar{\Psi}\partial^{\mu}\partial_{\mu} - \bar{\Psi} \times m^{2}] = 0, \;\; |c| = 1, \tag{2.3.39}$$

where the upper bar denotes complex conjugation (since $\bar{\Psi}$ is a scalar), while the Lagrangian density

$$L = -(\hbar \times \hbar/2 \times m) \times \{\partial^\mu \bar{\Psi} - i \times e \times A^\mu/\hbar \times c) \times \bar{\Psi}]$$

$$\times[\partial\Psi + (i \times e \times A_\mu/\hbar \times c) \times \Psi] + m \times m \times \bar{\Psi} \times \Psi, \tag{2.3.40}$$

is left invariant and the four-current

$$J_\mu = -(i \times \hbar/2 \times m) \times [\bar{\psi} \times \partial_\mu \Psi - (\partial_\mu \bar{\Psi}) \times \Psi], \tag{2.3.41}$$

changes sign

$$J_\mu \to C J_\mu = -J_\mu. \tag{2.3.42}$$

By recalling the selfduality of ordinary derivatives, Eq. (2.1.30), under isoduality the Klein-Gordon equation becomes

$$[\partial^\mu \partial_\mu \Psi - m^2 \times \Psi]^d = \Psi^d \partial^{d\mu} \partial_\mu^d - \Psi^d \times^d m^d \times^d m^d$$

$$= -[\bar{\Psi} \partial^\mu \partial_\mu - \bar{\Psi} \times m^2] = 0, \tag{2.3.43}$$

thus being equivalent to Eq. (2.3.39), while the Lagrangian changes sign and the four-current changes sign too,

$$J_\mu^d = -(i \times \hbar/2 \times m) \times [\bar{\Psi} \times \partial_\mu \Psi - (\partial_\mu \bar{\Psi}) \times \Psi]^d$$

$$= (i \times \hbar/2 \times m) \times [\bar{\Psi} \times \partial_\mu \Psi - (\partial_\mu \bar{\Psi}) \times \Psi], \tag{2.3.44}$$

(where we have used the isoselfduality of the imaginary number i).

The above results confirm Lemma 2.3.4 because of the equivalent behavior of the equations of motion and the four-current, while the change of sign of the Lagrangian does not affect the numerical results.

As it is also well known, the Klein-Gordon equation for a particle under an external electromagnetic field [12]

$$[(\partial_\mu + i \times e \times A_\mu/\hbar \times c)$$

$$\times(\partial^\mu + i \times e \times A^\mu/\hbar \times c) - m^2] \times \Psi = 0, \tag{2.3.45}$$

is equally invariant under charge conjugation in which *either* e *or* A_μ change sign, in view of the known invariance

$$C(i \times e \times A_\mu/\hbar \times c) = i \times e \times A_\mu/\hbar \times c, \tag{2.3.46}$$

while the four-current also changes sign. By noting that the preceding invariance persists under isoduality,

$$(i \times e \times A_\mu/\hbar \times c)^d = i \times e \times A_\mu/\hbar \times c, \tag{2.3.47}$$

Eq. (2.3.45) remains invariant under isoduality, while the Lagrangian density changes sign and the four-current, again, changes sign.

Similarly, consider Dirac equation (see also Ref. [12], pp. 176–177)

$$[\gamma^\mu \times (\partial_\mu \Psi - (i \times e \times A_\mu/\hbar \times c) \times \Psi + m \times \Psi = 0, \tag{2.3.48}$$

with Lagrangian density

$$L = (\hbar \times c/2) \times \{\tilde{\Psi} \times \gamma^\mu \times [\partial_\mu \Psi + (i \times e \times A_\mu/\hbar \times c) \times \Psi]$$

$$-(\partial^\mu \tilde{\Psi} - (i \times e \times A^\mu/\hbar \times c) \times \tilde{\Psi}] \times \gamma_\mu - m \times \tilde{\Psi} \times \Psi \tag{2.3.49a}$$

$$\tilde{\Psi} = \Psi^\dagger \times \gamma_4, \tag{2.3.49b}$$

and four-current

$$J_\mu = i \times c \times \tilde{\Psi} \times \gamma_\mu \times \Psi = i \times c \times \Psi^\dagger \times \gamma_4 \times \gamma_\mu \times \Psi \tag{2.3.50}$$

The charge conjugation for Dirac's equations is given by the transform [12]

$$\Psi \to C\Psi = c \times S_C^{-1} \times \tilde{\Psi}^t \tag{2.3.51}$$

where S_C is a unitary matrix such that

$$\gamma_\mu \to -\gamma_\mu^t = S_C \times \gamma_\mu \times S_c^{-1}, \tag{2.3.52}$$

and there is the change of sign *either* of e *or* of A_μ, under which the equation is transformed into the form

$$[\partial_\mu \tilde{\Psi} - (i \times e \times A_\mu/\hbar \times c) \times \tilde{\Psi}] \times \gamma^\mu - m \times \tilde{\Psi} = 0, \tag{2.3.53}$$

while the Lagrangian density changes sign and the four-current remains the same,

$$L \to CL = -L, J_\mu \to CJ_\mu = J_\mu. \tag{2.3.54}$$

It is easy to see that isoduality provides equivalent results. In fact, for Eq. (2.3.48) we have

$$\{[\gamma^\mu \times (\partial_\mu \Psi - i \times e \times A_\mu/\hbar \times c) \times \Psi + m \times \Psi\}^d$$

$$= [\partial_\mu \Psi^\dagger - (i \times e \times A_\mu/\hbar \times c) \times \Psi^\dagger] \times \gamma^\mu - m \times \Psi^\dagger = 0, \tag{2.3.55}$$

that, when multiplied by γ_4 reproduces Eq. (2.3.53) identically. Similarly, by recalling that Dirac' s gamma matrices are isoselfdual (Theorem 2.3.1), and by noting that

$$\tilde{\Psi}^d = (\Psi^\dagger \times \gamma_4)^d = \gamma_4 \times \Psi, \tag{2.3.56}$$

we have

$$L^d = L \tag{2.3.57}$$

while for the four-current we have

$$J^d_\mu = -i \times c \times \Psi^\dagger \times \gamma_\mu \times \gamma_4 \times \psi. \tag{2.3.58}$$

But the γ_μ and γ_4 anticommute. As a consequence, the four-current does not change sign under isoduality as in the conventional case.

Note that the lack of change of sign under isoduality of Dirac's four-current J_μ confirms reinterpretation (2.3.28) since, for the latter equation, the total charge is null.

The equivalence between isoduality and charge conjugation of other equations, such as those by Weyl, Majorana, etc., follows the same lines.

2.3.8 Experimental Verification of the Isodual Theory of Antimatter in Particle Physics

In Section 2.2.3. we have established the experimental verification of the isodual theory of antimatter in classical physics. That in particle physics requires no detailed elaboration since it is established by the equivalence of charge conjugation and isoduality (Lemma 2.3.4), and we can write:

LEMMA 2.3.5 [6,5b,18], [7]: *All experimental data currently available for antiparticles represented via charge conjugation are equally verified by the isodual theory of antimatter.*

2.3.9 Elementary Particles and their Isoduals

We assume the reader is familiar with the conventional definition of *elementary particles* as irreducible unitary representations of the spinorial covering of the Galilei symmetry $G(3.1)$ for nonrelativistic treatments and those of the Poincaré symmetry $P(3.1)$ for relativistic treatments. We therefore introduce the following:

DEFINITION 2.3.1: Elementary isodual particles (antiparticles) are given by irreducible unitary representations of the spinorial covering of the Galilei-Santilli's isodual symmetry $G^d(3.1)$ for nonrelativistic treatments and those of the Poincaré-Santilli isodual symmetry $P^d(3.1)$ for relativistic treatments.

A few comments are now in order. First, one should be aware that "isodual particles" and "antiparticles" do not represent the same notion, evidently because of the negative mass, energy and time of the former

compared to positive mass, energy and time of the latter. In the rest of this chapter, unless otherwise stated, the word "antiparticle" will be referred to the "isodual particle."

For instance the word "positron" e^+ is more appropriately intended to represent the "isodual electron" with symbol e^{-d}. Similarly the, "antiproton" p^- is intended to represent the "isodual proton" p^{+d}.

Second, the reader should note the insistence on the *elementary* character of the antiparticles here admitted. The reason is that the antigravity studied in Chapter 4 is specifically formulated for "elementary" isodual particles, such as the isodual electron, due to a number of unsettled aspects pertaining to composite particles.

Consider, as an illustration, the case of mesons. If the π° is a bound state of a particle and its isodual, the state is isoselfdual and, as such, it *cannot* experience antigravity, as illustrated in the next section. A number of ambiguities then follow for the study of the gravity of the charged mesons $\pi^\pm$, such as the problem of ascertaining which of the two mesons is a particle and which is its isodual or, whether the selected antiparticle is indeed the isodual image of the particle as a necessary condition for meaningful study of their gravity.

Note that essentially the same ambiguities prohibit the use of muons for a serious theoretical and experimental studies of the gravity of antiparticles, again, because of unsettled problems pertaining to the structure of the muons themselves. Since the muons are naturally unstable, they cannot be credibly believed to be elementary. Therefore, serious theoretical and experimental studies on the gravity of muons require the prior identification of their constituents with physical particles.

Finally, the reader should be aware that *Definition 2.3.1 excludes the use of quark conjectures for the gravitational studies of this monograph.* This is due to the well-known basic inconsistency of quark conjecture of not admitting any gravitation at all (see, e.g., the Appendix of Ref. [18]). In fact, gravity can only be defined in our spacetime while quarks can only be defined in their mathematical unitary internal space with no known connection with our spacetime due to the O'Rafearthaigh theorem.[5]

Also, the only "masses" that can be credibly claimed as possessing inertia are the eigenvalues of the second-order Casimir invariant of the Poincaré symmetry $p_\mu \times p^\mu = m^2$. Quarks cannot be characterized via such a fundamental symmetry, as well known. It then follows that "quark masses" are mere mathematical parameters defined in the mathematical internal complex-unitary space that cannot possibly be used as serious basis for gravitational tests.

2.3.10 Photons and their Isoduals

As it is well known, photons have no charge and, therefore, they are invariant under charge conjugation, as transparent from the simple plane-wave representation

$$\Psi(t,r) = N \times e^{i\times(k\times r - E\times t)}, \;\; N \in R, \tag{2.3.59}$$

with familiar relativistic form

$$\Psi(x) = N \times e^{i\times k_\mu \times x^\mu}, \tag{2.3.60}$$

and familiar expression for the energy

$$E = h \times \nu. \tag{2.3.61}$$

As a result, matter and antimatter have been believed throughout the 20-th century to emit the same light. In turn, this belief has left fundamentally unsettled basic questions in astrophysics and cosmology, such as the lack of quantitative studies as to whether far-away galaxies and quasars are made up of matter or of antimatter.

One of the most intriguing and far reaching implications of the isodual theory is that, while remaining evidently invariant under charge conjugation, *the photon is not invariant under isoduality*, thus admitting a conjugate particle first submitted by Santilli in Ref. [18] under the name of *isodual photon.* In particular, the isodual photon emerges as having physical characteristics that can be experimentally measured as being different from those of the photon.

Therefore, the isodual theory offers the first known possibilities of quantitative theoretical and experimental studies as to whether a far-away galaxy or quasar is made of matter or of antimatter due to detectable physical differences of their emitted light.

Note that the term "antiphoton" could be misleading because the prefix "anti" is generally assumed as referring to charge conjugation. For this reason the name of "isodual photon" appears to be preferable, also because it represents, more technically, the intended state.

In fact, the photon is mapped by isoduality into a new particle possessing all negative-definite physical characteristics, with the following simple isodual plane-wave representation

$$\Psi^d(t^d,r^d) = N^d \times^d e_d^{i^d\times^d(k^d\times^d r^d - E^d\times^d t^d)}, \;\; N^d \in R^d, \tag{2.3.62}$$

with relativistic expression on isodual Minkowski space

$$\Psi^d(x^d) = N^d \times^d e_d^{i^d\times^d k_\mu^d \times^d x^{d\mu}}, \tag{2.3.63}$$

and isodual expression for the energy

$$E^d \;=\; h^d \times^d \nu^d, \tag{2.3.64}$$

where e_d is the isodual exponentiation (2.1.26b).

Note that, since i is isoselfdual, Eq. (2.1.20), *the exponent of the plane-wave representation is invariant under both charge conjugation and isoduality*, as illustrated by the following expression

$$i^d \times^d (k^d \times^d r^d - E^d \times^d t^d) \;\equiv\; i \times (k \times r - E \times t), \tag{2.3.65}$$

or its relativistic counterpart

$$i^d \times^d k^d_\mu \times^d x^{d\mu} \;\equiv\; i \times k_\mu \times x^\mu. \tag{2.3.66}$$

thus confirming the lack of contradiction between charge conjugation and isoduality.

Moreover, both the photon and the isodual photon travel in vacuum with the same (absolute) speed $|c|$, for which we have the additional identity

$$k^d_\mu \times^d k^{d\mu} \;\equiv k_\mu \times k^\mu = 0. \tag{2.3.67}$$

Despite the above identities, energy and time are positive-definite for the photon, while they are negative-definite for the isodual photon. As we shall see, the latter property implies that photons are attracted by the gravitational field of matter while isodual photons are repelled, thus providing a physically detectable difference.

Additional differences between light emitted by matter and that emitted by antimatter, such as those pertaining to parity and other discrete symmetries, require additional study.

2.3.11 Electrons and their Isoduals

The next truly elementary particles and antiparticles are the electron e^- and its antiparticle, the positron e^+ or the isodual electron e^{-d}. The differences between the "positron" and the "isodual electron" should be kept in mind. In fact, the former has positive rest energy and moved forward in time, while the latter has negative rest energy and moves backward in time.

Also, the electron is known to experience gravitational attraction in the field of matter, as experimentally established. As conventionally defined, the positron too is predicted to experience gravitational attraction in the field of matter (because its energy is positive).

However, as we shall see in Chapter 4, the isodual electron is predicted to experience antigravity when immersed in the field of matter, and this

illustrates again the rather profound physical differences between the "positron" and the "isodual electron".

Note that, in view of their truly elementary character, isodual electrons are the ideal candidates for the measurement of the gravitational field of antiparticles.

2.3.12 Protons and their Isoduals

The next particles demanding comments are the proton p^+, the antiproton p^- and the isodual proton p^{+d}. In this case the differences between the "antiproton" and the "isodual proton" should be kept in mind to avoid major inconsistencies with the isodual theory, such as the study of the possible antigravity for antiprotons in the field of matter which antigravity cannot exist for the isodual theory (due, again, to the positive mass of the antiproton).

Note that these particles are not elementary and, as such, they are not admitted by Definition 2.3.1. moreover, as stressed earlier [18], when represented in term of quark conjectures both the proton and the antiproton cannot admit any gravity at all, let alone antigravity. As a result, extreme scientific care should be exercised before extending to all antimatter any possible gravitational measurements for antiprotons.

2.3.13 The Hydrogen Atom and its Isodual

The understanding of this chapter requires the knowledge that studies conducted on the *antihydrogen atom* (see, e.g., the various contributions in Proceedings [19]), even though evidently interesting per se, they have no connection with the *isodual hydrogen atom*, because the antihydrogen atom has positive mass, for which antigravity is prohibited, and emits conventional photons. Therefore, it is important to inspect the differences between these two formulations of the simplest possible atom of antimatter.

We assume as exactly valid the conventional quantum mechanical theory of *bound states of point-like particles at large mutual distances*, [6] as available in quantum mechanical books so numerous to discourage even a partial listing.

For the case of two particles denoted with the indices 1, 2, the total state in the Hilbert space is the familiar tensorial product of the two states

$$|\psi >= |\psi_1 > \times |\psi_2 > . \tag{2.3.68}$$

The total Hamiltonian H is the sum of the kinetic terms of each state plus the familiar interaction term $V(r)$ depending on the mutual

distance r,

$$H = p_1 \times p_1/2 \times m_1 + p_2 \times p_2/2 \times m_2 + V(r). \tag{2.3.69}$$

The total angular momentum is computed via the familiar expressions for angular momenta and spins

$$J = J_1 \times I + I \times J_2, S = S_1 \times I + I \times S_2, \tag{2.3.70}$$

where the I's are trivial units, with the usual rules for couplings, addition, etc. One should note that the unit for angular momenta is three-dimensional while that for spin has a generally different dimension.

A typical example of two-body bound states of particles is the *hydrogen atom* that experiences attraction in the gravitational field of matter with the well established emission of conventional photons.

The study of *bound states of point-like isodual particles at large mutual distances* is an important part of isodual quantum mechanics. These bound states can be studied via an elementary isoduality of the corresponding bound states for particles, that is, via the use of the isodual Hilbert spaces $\mathcal{H}^d$ studied earlier.

The *total isodual state* is the tensorial product of the two isodual states

$$|\psi^d(r^d) >^d = |\psi_1^d(r^d) >^d \times^d |\psi_2^d(r^d) >^d = - < \psi_1(-r)| \times < \psi_2(-r)|. \tag{2.3.71}$$

The *total isodual Hamiltonian* is the sum of the isodual kinetic terms of each particle plus the isodual interaction term depending on the isodual mutual distance,

$$H^d = p_1^d \times^d p_1^d/^d 2^d \times^d m_1^d + p_2^d \times^d p_2^d/^d 2^d \times^d m_2^d + V^d(r^d). \tag{2.3.72}$$

The *total isodual angular momentum* is based on the expressions for isodual angular momenta and spin

$$J^d = J_1^d \times^d I^d + I^d \times^d J_2^d, \tag{2.3.73a}$$

$$S^d = S_1^d \times^d I^d + I^d \times^d S_2^d, \tag{2.3.73b}$$

The remaining aspects (couplings, addition theory of angular momenta, etc.) are then given by a simple isoduality of the conventional theory that is here omitted for brevity.

Note that all eigenvalues that are positive for the conventional case measured with positive units become negative under isoduality, yet measured with negative units, thus achieving full equivalence between particle and antiparticle bound states.

The simplest possible application of the above isodual theory is that for the *isodual hydrogen atom* (first worked out in Ref. [18]). The novel

predictions of isoduality over that of the antihydrogen atom is that the isodual hydrogen atom is predicted to experience antigravity in the field of matter and emits isodual photons that are also repelled by the gravitational field of matter.

2.3.14 Isoselfdual Bound States

Some of the most interesting and novel bound states predicted by the isodual theory are the *isoselfdual bound states*, that is, bound states that coincide with their isodual image. The simplest case is the bound state of one elementary particle and its isodual, such as the *positronium.*

The condition of isoselfduality requires that the basic symmetry must be itself isoselfdual, e.g., for the nonrelativistic case the total symmetry must be

$$G_{Tot} = G(3.1) \times G^d(3.1), \tag{2.3.74}$$

where $\times$ is the Kronecker product (a composition of states thus being isoselfdual), with a simple relativistic extension here assumed as known from the preceding sections.

The total unit must also be isoselfdual,

$$I_{Tot} = I \times I^d, \tag{2.3.75}$$

where I represents the space, time and spin units.

The total Hilbert space and related states must also be isoselfdual,

$$\mathcal{H}_{Tot} = \mathcal{H} \times \mathcal{H}^d, \tag{2.3.76a}$$

$$|\psi >_{Tot} = |\psi > + |\psi >^d = |\psi > - < \psi|, \tag{2.3.76b}$$

and so on.

A main feature is that isoselfdual states exist in both the spacetime of particles and that of antiparticles. Therefore, the computation of the total energy must be done *either* in $\mathcal{H}$, in which case the total energy is positive, *or* in $\mathcal{H}^d$, in which case the total energy is negative.

Suppose that a system of one elementary particle and its isodual is studied in our laboratory of matter. In this case the eigenvalues for both particle and its isodual must be computed in $\mathcal{H}$, in which case we have the equation

$$\begin{aligned} i \times \partial_t |\psi > &= (p \times p/2 \times m) \times |\psi > \\ &+ (p^d \times^d p^d/^d 2^s \times^d m^d) \times^d |\psi > + V(r) \times |\psi > \\ &= [p \times p/2 \times m + V(r)] \times |\psi > = E \times |\psi >, \end{aligned} \tag{2.3.77}$$

under which the total energy E is evidently positive.

When the same isoselfdual state is detected in the spacetime of antimatter, it must be computed with respect to $\mathcal{H}^d$, in which case the total energy is negative, as the reader is encouraged to verify.

The total angular momentum and other physical characteristics are computed along similar lines and they also result in having positive values when computed in $\mathcal{H}$, as occurring for the conventional charge conjugation.

As we shall see shortly, the positive character of the total energy of bound states of particles and their antiparticles is crucial for the removal of the inconsistencies of theories with negative energy.

The above properties of the isoselfdual bound states have the following implications:

1) Isoselfdual bound states of elementary particles and their isoduals are predicted to be attracted in both, the gravitational field of matter and that of antimatter because their total energy is positive in our world and negative in the isodual world. This renders necessary an experimental verification of the gravitational behavior of isoselfdual bound states, independently from that of individual antiparticles. Note that the prediction holds only for bound states of truly elementary particles and their isoduals, such as the positronium. No theoretical prediction for the muonium and the pionium is today feasible because the unsettled nature of their constituents.

2) Isoselfdual bound states are predicted to have a null internal total time $t+t^d = 0$ and therefore acquires the time of the matter or antimatter in which they are immersed, although the physical time t of the observer (i.e., of the bound state equation) is not null. This is readily understood by noting that the quantity t of Eq. (2.3.77) is our own time, i.e., we merely study the behavior of the state with respect to our own time. A clear understanding illustrated previously with the "isodual cube" of Section 2.1 is that the description of a state with our own time, by no means, implies that its intrinsic time necessarily coincides with our own. Note that a similar situation occurs for the energy because the intrinsic total energy of the positronium is identically null, $E + E^d = 0$. Yet the energy measured by us is $E_{part.} - E^d_{antipart.} = 2E > 0$. A similar situation occurs for all other physical quantities.

3) Isoselfdual bound states may result in being the microscopic image of the main characteristics of the entire universe. Isoselfduality has in fact stimulated a new cosmology, the *isoselfdual cosmology* [21] studied in Chapter 5, that is patterned precisely along the structure of the positronium or of Dirac's equation in our isoselfdual re-interpretation. In this case the universe results in having null total physical character-

istics, such as null total energy, null total time, etc., thus implying no discontinuity at its creation.

2.3.15 Resolution of the Inconsistencies of Negative Energies

The treatment of antiparticles with negative energies was rejected by Dirac because incompatible with their physical behavior. Despite several attempts made during the 20-th century, the inconsistencies either directly or indirectly connected to negative energies have remained unresolved.

The isodual theory of antimatter resolves these inconsistencies for the reason now familiar, namely, that the inconsistencies emerge when one refers negative energies to conventional numbers with positive units, while the same inconsistencies cannot be evenly formulated when negative energies are referred to isodual numbers and their negative units.

A good illustration is given by the known objection according to which the creation of a photon from the annihilation of an electron-positron pair, with the electron having a positive energy and the positron having a negative energy, would violate the principle of conservation of the energy.

In fact, such a pair could be moved upward in our gravitational field without work and then annihilated in their new upward position. The resulting photon would then have a blueshift in our gravitational field of Earth, thus having more energy than that of the original photon.

Presumed inconsistencies of the above type cannot be even formulated within the context of the isodual theory of antimatter because, as shown in the preceding section, the electron-positron state is isoselfdual, thus having a non-null *positive* energy when observed in our spacetime. Consequently, the lifting upward of the pair does indeed require work and no violation of the principle of conservation of the energy can be expected.

A considerable search has established that all other presumed inconsistencies of negative energy known to the author cannot even be formulated within the context of the isodual theory of antimatter. Nevertheless, the author would be particularly grateful to any colleague who bring to its attention inconsistencies of negative energies that are really applicable under negative units.

Notes

1 Note as necessary pre-requisites of the new Newton's equations, the prior discovery of isodual numbers, spaces and differential calculus.

2 The literature on the conventional Galilei and special relativities and related symmetries is so vast to discourage discriminatory quotations.

3 It should be indicated that, contrary to popular beliefs, the conventional Poincaré symmetry will be shown in Chapter 3 to be *eleven* dimensional, the 11-th dimension being given by a new invariant under change of the unit. Therefore, the isodual symmetry $P^d(3.1)$ is also 11-dimensional.

4 The author would be grateful to colleagues who care to bring to his attention other "classical" gravitational theories of antimatter compatible with the negative-energy solutions needed by antimatter.

5 The possible connection between internal and spacetime symmetries offered by supersymmetric theories cannot be credibly used for gravitational tests due to their highly unsettled character and the prediction of a zoo of new particles none of which has been experimentally detected to the author's best knowledge.

6 We are here referring to the large mutual distances as occurring in the atomic structure and exclude the short mutual distances as occurring in the structure of hadrons, nuclei and stars since a serious study of the latter is dramatically beyond the capabilities of quantum mechanics, as shown beyond scientific doubt in Chapter 3.

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Chapter 3

LIE-ISOTOPIC AND LIE-ADMISSIBLE TREATMENTS OF EXTENDED PARTICLES AND THEIR ISODUALS FOR EXTENDED ANTIPARTICLES

3.1 INTRODUCTION

3.1.1 The Forgotten Legacy of Newton, Lagrange and Hamilton

The mathematics and physics of the 20-th century essentially performed the reduction of the entire universe to *exterior dynamical systems* (Definition 1.3.1), consisting of closed, isolated and reversible systems of constituents abstracted as being point-like while moving in vacuum under sole action-at-a-distance potential interactions.

More technically, we can say that *exterior dynamical systems are characterized by the exact invariance of the Galilean symmetry for the non-relativistic case and the Poincaré symmetry for relativistic treatments*, with the consequential verification of the well known ten total conservation laws.

While the existence of systems admitting an effective abstraction of their constituents as being point like, is beyond doubt, as proved by historical advances in planetary and atomic structures, the reduction of the entire universe to systems of the indicated type was implemented by disregarding the historical legacy of Newton [1], Lagrange [2] and Hamilton [3].

In fact, Newton proposed his celebrated equations for systems of real life, today carrying his name, that is, with forces that are partially of action-at-a-distance type, thus derivable from a potential $V(r)$ and today known as *variationally self-adjoint forces* (SA) [48], and partially of contact, zero-range type, thus not derivable from a potential and today

known as *variationally nonself-adjoint forces* (NSA) [48],

$$m_a \times \frac{dv_{ak}}{dt} = F_{ak}^{SA}(t,r,v) + F_{ak}^{NSA}(t,r,v,\ldots), \tag{3.1.1}$$

$$F_{ak}^{SA} = -\frac{\partial V}{\partial r_a^k}, \quad F_{ak}^{NSA} \neq -\frac{\partial V}{\partial r_a^k}, \tag{3.1.1b}$$

$$a = 1,2,3,\ldots,N; \quad k = 1,2,3.$$

By following Newton's teaching *ad litteram*, Lagrange proposed his celebrated equations, *not* in the form generally used in the 20-th century, but via two quantities, a quantity $L(t,r,v)$, today called the Lagrangian, for the representation of the kinetic energy and all potential forces, as well as *external terms* for the representation of Newton's nonpotential forces, according to the expression known as the *true Lagrange equations*,[1]

$$\frac{d}{dt}\frac{\partial L(t,r,v)}{\partial v_a^k} - \frac{\partial L(t,r,v)}{\partial r_a^k} = F_{ak}^{NSA}(t,r,v,\ldots), \tag{3.1.2a}$$

$$L = \Sigma_a \frac{1}{2} \times m_a \times v_a^2 - V(t,r,v), \tag{3.1.2b}$$

$$V = \Sigma_a U(t,r)_{ak} \times v_a^k + U_o(t,r), \tag{3.1.2c}$$

Hamilton also followed Newton's teaching *ad litteram*, and formulated his celebrated equations via two quantities, the first for the representation of the total energy inclusive of the potential, today known as the Hamiltonian, and the second being given precisely by the external terms representing Newton's nonpotential forces, according to the equations known as the *true Hamilton's equations*

$$\frac{dr_a^k}{dt} = \frac{\partial H(t,r,p)}{\partial p_{ak}}, \quad \frac{dp_{ak}}{dt} = -\frac{\partial H(t,r,p)}{\partial r_a^k} + F_{ak}(t,r,p), \tag{3.1.3a}$$

$$H = \Sigma_a \frac{p_{ak} \times p_{ak}}{2 \times m_a} + V(t,r,p), \tag{3.1.3b}$$

$$F(t,r,v) = F(t,r,p/m). \tag{3.1.3c}$$

When faced with the above so vast and compelling historical teaching, a general attitude by 20-th century physicists is that Newton's nonpotential forces as well as Lagrange and Hamilton external terms are "illusory" (*sic*) because, in their view, when the Newtonian system is reduced to its elementary constituents, all nonpotential forces "disappear" (*sic*) and the systems becomes entirely representable with the "conventional" Lagrange and Hamilton equations, those without external terms.

Particularly when proffered by experts, the above belief is purely political-nonscientific, because essentially intended to adapt nature to Einsteinian doctrines, since the latter are dramatically violated by external terms beginning with the underlying topology, loss of all algebras, let alone of all possible Lie algebras, the loss of Einsteinian physical laws, etc., as studied in detail in Chapter 1.

In fact, it is expected to be known by experts to qualify as such and, in any case, it can be proved by a first year graduate student, that a Newtonian system with nonpotential forces cannot be consistently reduced to a finite number of elementary particles all under potential interactions and, vice-versa, a finite ensemble of elementary particles solely under potential interactions cannot consistently reproduce a Newtonian systems with nonpotential forces (Theorem 1.3.1).

Consequently, rather then being "illusory", Newton's nonpotential interactions originates at the ultimate level of nature, precisely that of elementary particles.

This monograph is devoted to the pursue of scientific knowledge and not of political academic lines. Consequently, we shall ignored the truncated Lagrange and Hamilton equations so much preferred in the 20-th century, and solely study the true analytic equations, those with external terms, in full awareness beginning from these introductory lines, of the consequential need of surpassing Einsteinian doctrines and quantum mechanics in favor of broader theories.

3.1.2 Structural Differences between Exterior and Interior Dynamical Systems of Particles and of Antiparticles

With reference to our fundamental classification of dynamical systems of Definition 1.3.1, a primary scope of this chapter is the study of *close-reversible or open-irreversible interior dynamical systems of extended particles* and, separately, of *extended antiparticles*, admitting internal force of linear and nonlinear type, local-differential and nonlocal-integral type and potential as well as nonpotential type, where the latter originate from actual contact and/or mutual penetration of particles, as it is the case for the structure of planets at the classical level, and the structure of hadrons, nuclei, stars, molecules, Cooper pairs and other systems and other systems at the operator level.

The most important methodological differences between exterior and interior systems of for both particles and antiparticles are the following:

1) Exterior systems are completely represented with the knowledge of only *one* quantity, the Lagrangian or the Hamiltonian, while the representation of interior systems requires the knowledge of the Lagrangian

Figure 3.1. A view of Jupiter, a most representative interior dynamical system, where one can see with a telescope the dramatic differences with exterior systems, such as internal exchanges of linear and angular momentum always in such a way to verify total conservation laws. As repeatedly stated in the literature on hadronic mechanics, the structure of Jupiter has been assumed as fundamental for the construction of new structure models of hadrons, nuclei and stars, and the development of new clean energies and fuels.

or the Hamiltonian for the potential forces, plus additional quantities for the representation of nonpotential forces, as done in the true analytic equations (3.1.2) and (3.1.3).

Consequently, by their very conception, interior systems are structurally beyond the representational capability of classical and quantum Hamiltonian mechanics, in favor of covering disciplines (Figure 3.1).

2) Exterior systems are of *Keplerian type*, while interior systems are not, since they do not admit a Keplerian center (see Section 1.3). Consequently, by their very conception, interior systems cannot be characterized by the Galilean and Poincaré symmetries in favor of covering symmetries (Figure 3.2).

3) Exterior systems are local-differential, that is, they describe a finite set of isolated points, thus being fully treatable with the mathematics of the 20-th century, beginning with conventional local-differential topology. By contrast, interior systems are nonlocal-integral, that is, they admit internal interactions over finite surfaces or volumes that, as such, cannot be consistently reduced to a finite set of isolated points. Consequently, interior systems cannot be consistently treated via the mathematics of classical and quantum Hamiltonian mechanics in favor of a basically new mathematics.

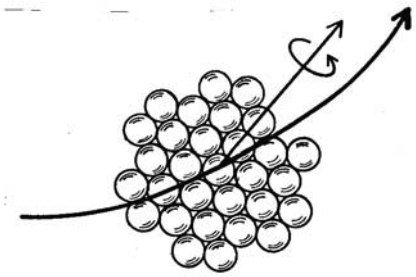

Figure 3.2. A schematic view of nuclei as they are in the physical reality, bound states of extended particles in conditions of partial mutual penetration without a Keplerian center, under which conditions quantum mechanics cannot possibly be exact due to the breaking of the fundamental Galilean and Poincaré symmetries caused by the lack of a Keplerian center. Therefore, the mere inspection of the main features of nuclear structures establishes the said need for covering theories. Even though these breakings are small (because nucleons are in conditions of mutual penetration in nuclei of about 10^{-3} parts of their volumes), said breakings permit the prediction and industrial development of new clean energies and fuels that are prohibited by the exact validity of quantum mechanics [58–61].

4) The time evolution of the Hamiltonian treatment of exterior systems characterizes a *canonical transformation* at the classical level, and a *unitary transformation* at the operator level, that we shall write in the unified form

$$U \times U^\dagger = U^\dagger \times U = I, \tag{3.1.4}$$

where $\times$ represents the usual (associative) multiplication.[2] By contrast, the time evolution of interior systems, being non-Hamiltonian, characterizes *noncanonical transformations* at the classical level and *nonunitary transformations* at the operator level, that we shall jointly write

$$U \times U^\dagger \neq I. \tag{3.1.5}$$

In particular, the noncanonical-nonunitary character is necessary to exit from the class of equivalence of classical and quantum Hamiltonian theories.

5) The *invariance* (rather than "covariance") of exterior systems under the Galilean or Poincaré symmetry has the fundamental implication of preserving the basic units, predicting the same numerical values under the same conditions at different times, and admitting all conditions

needed for consistent applications of the theory to experimental measurements.

By comparison, the loss of the Galilean and Poincaré invariance, combined with the necessary noncanonical-nonunitary structure of interior systems activate the *theorems of catastrophic mathematical and physical inconsistencies* studied in the next section (whenever treated with the mathematics of canonical-unitary theories).

In this chapter we report the rather long scientific journey that lead to a mathematically and physically consistent, classical and operator treatments of interior dynamical systems. Our main methods are the *Lie-isotopic and Lie-admissible branches of hadronic mechanics for extended particles*, and the *isodual Lie-isotopic and isodual Lie-admissible branches for extended antiparticles.*

Besides a number of experimental verifications, the achievement of a consistent treatment of interior systems offers basically new structure models of hadrons, nuclei, stars, Cooper pairs, molecules and other interior structures. In turn, these new models permit quantitative studies of new clean energies and fuels already under industrial, let alone scientific development [58–61].

Stated in a nutshell, a primary aim of this chapter is to show that the assumption of a final character of special relativity and quantum mechanics beyond the conditions of their original conception (isolated point particles in exterior conditions in vacuum) is a primary origin of the current alarming environmental problems [58].

The reader should be aware that, nowadays, the literature on hadronic mechanics is rather vast, having surpassed the mark of 15,000 pages of published research. As such, to avoid a prohibitive length, *the presentation in this chapter is restricted to an outline of the origination of each topic and of the most important developments.* Scholars interested in additional aspects are suggested to inspect the quoted literature.

Also to avoid a prohibitive length, the presentation of this chapter is restricted to studies of direct relevance for hadronic mechanics, namely, research fundamentally dependent on a *generalization of the basic unit.* Related studies not fundamentally dependent on the generalization of the basic unit cannot be reviewed for brevity.

3.1.3 Closed Non-Hamiltonian Systems of Extended Particles and Extended Antiparticles

It is generally admitted that particles at short mutual distances with contact nonconservative interactions can characterize an open and irreversible interior system.

A basic step in the study of interior systems is the dispelling of the belief that total conservation laws necessarily restrict all internal forces to be conservative or, equivalently, that nonconservative internal forces do not permit total conservation laws.

This belief was disproved, apparently for the first time, by Santilli in monographs [48,51]. Ref. [48] presented a comprehensive treatment of the integrability conditions for the existence of a potential or a Hamiltonian, *Helmholtz's conditions of variational selfadjointness* indicated earlier.

Ref. [51] presented the broadest possible realization of the conditions of variational selfadjointness via analytic equations derivable from a variational principle, and included the first known identification of *closed non-Hamiltonian systems* (Ref. [51], pages 233–236), namely, systems that violate the integrability conditions for the existence of a Hamiltonian, yet they verify all ten total conservation laws of conventional Hamiltonian systems.

We should also recall for clarity that, to be Newtonian as currently understood, a force should solely depend on time t, coordinates r and velocity $v = dr/dt$ or momenta $p = m \times v$, $F = F(t, r, v)$ or $F(t, r, p)$. Consequently, forces depending on derivatives of the coordinates of order bigger than the first, such as forces depending on the acceleration $F = F(t, r, v, a)$, $a = dv/dt$, are not generally considered to be Newtonian forces.

Let us begin by recalling the following well known property:

THEOREM 3.1.1: Necessary and sufficient conditions for a system of N particles to be closed, that is, isolated from the rest of the universe, are that the following ten conservation laws are verified along an actual path

$$\frac{dX_i(t,r,p)}{dt} = \frac{\partial X_i}{\partial b^\mu} \times \frac{db^\mu}{dt} + \frac{\partial X_i}{\partial t} = 0, \tag{3.1.6a}$$

$$X_1 = E_{tot} = H = T + V, \tag{3.1.6b}$$

$$(X_2, X_3, X_4) = \mathbf{P}_{Tot} = \Sigma_a \mathbf{p}_a, \tag{3.1.6c}$$

$$(X_5, X_6, X_7) = \mathbf{J}_{tot} = \Sigma_a \mathbf{r_a} \wedge \mathbf{p}_a, \tag{3.1.6d}$$

$$(X_8, X_9, X_{10}) = \mathbf{G}_{Tot} = \Sigma_a (m_a \times \mathbf{r}_a - t \times \mathbf{p}_a), \tag{3.1.6e}$$

$$i = 1, 2, 3, \ldots, 10; \quad k = 1, 2, 3; \quad a = 1, 2, 3, \ldots, N.$$

The isodual version of the above quantities then characterize a closed system of antiparticles.

It is also well known that Galilean or Poincaré invariant systems do verify the above conservation laws since the X_i quantities are the generators of the indicated symmetries. However, in this case all acting forces are derivable from a potential and the systems are Hamiltonian.

Assume now the most general possible dynamical systems, those according to the true Lagrange's and Hamilton equations (3.1.3) where the selfadjoint forces are represented with the Lagrangian or the Hamiltonian and the nonselfadjoint forces are external.

DEFINITION 3.1.1 [51]: *Closed-isolated non-Hamiltonian systems of particles are systems of $N \geq 2$ particles with potential and nonpotential forces characterized by the following equations of motion*

$$\frac{db_a^\mu}{dt} = \begin{pmatrix} dr_a^k/dt \\ dp_{ka}/dt \end{pmatrix} = \begin{pmatrix} p_{ak}/m_a \\ F_{ka}^{SA} + F_{ka}^{NSA} \end{pmatrix}, \tag{3.1.7}$$

verifying all conditions (3.1.5), where the term "non-Hamiltonian" denotes the fact that the systems cannot be entirely represented with the Hamiltonian, thus requiring additional quantities, such as the external terms.

The case $n = 2$ is exceptional, yet it admits solutions, and closed non-Hamiltonian systems with $N = 1$ evidently cannot exist (because a single free particle is always Hamiltonian).

Closed non-Hamiltonian systems can be classified into:

CLASS α: systems for which Eqs (3.1.5) are first integrals;

CLASS β: systems for which Eq. (3.1.5) are invariant relations;

CLASS γ: systems for which Eq. (3.1.5) are subsidiary constraints.

The case of closed non-Hamiltonian systems of antiparticles are defined accordingly via isoduality.

The study of closed non-Hamiltonian systems of Classes β and γ is rather complex. For the limited scope of this presentation it is sufficient to see that interior systems of Class α exist.

THEOREM 3.1.2 [51]: *Necessary and sufficient conditions for the existence of a closed non-Hamiltonian system of Class α are that the nonselfadjoint forces verify the following conditions:*

$$\sum_a \mathbf{F}_a^{NSA} \equiv 0, \tag{3.1.8a}$$

$$\sum_a \mathbf{p}_a \times \mathbf{F}_a^{NSA} \equiv 0, \tag{3.1.8b}$$

$$\sum_a \mathbf{r}_a \wedge \mathbf{F}_a^{NSA} \equiv 0. \tag{3.1.8c}$$

Proof. Consider first the case $N > 2$ and assume first for simplicity that $\mathbf{F}_a^{SA} = 0$. Then, the first nine conservation laws are verified when

$$\frac{\partial X_i}{\partial p_{ka}} \times F_{ka}^{NSA} \equiv 0 \tag{3.1.9}$$

in which case the 10-th conservation law, Eq. (3.1.6e), is automatically verified, and this proves the *necessity* of conditions (3.1.8) for $N > 2$.

The sufficiency of the conditions is established by the fact that Eq. (3.1.7) consist of seven conditions on $3N$ unknown functions F_{ka}^{NSA}. Therefore, a solution always exists for $N \geq 3$.

The case $N = 2$ is special inasmuch as motion occurs in a plane, in which case Eq. (3.1.8) reduce to *five* conditions on *four* functions $\mathbf{F}_{ka}^{NSA}$, and the system appears to be overdetermined. Nevertheless, solutions always exist because the verification of the first four conditions (3.1.6) automatically implies the verification of the last one, Eq. (3.1.6e).

As shown in Ref. [51], Example 6.3, pages 272–273, a first solution is given by the *non-Newtonian force*

$$\mathbf{F}_1^{NSA} = -\mathbf{F}_2^{NSA} = K \times a = K \times \frac{dv}{dt}, \tag{3.1.10}$$

where K is a constant. Another solution is given by

$$\mathbf{F}_1^{NSA} = -\mathbf{F}_2^{NSA}$$

$$= M \times \frac{dr}{dt} \times \phi(M \times \dot{r} + V), \quad M = \frac{m_1 \times m_2}{m_1 + m_2}. \tag{3.1.11}$$

Other solutions can be found by the interested reader. The addition of a non-null selfadjoint force leaves the above proof unchanged. **q.e.d.**

The search for other solutions is recommended to readers interested in acquiring a technical knowledge of hadronic mechanics because such solutions are indeed useful for applications. A general solution of Eq. (3.1.8), as well as of their operator counterpart and of their isodual images for antimatter will be identified later on in this chapter after the identification of the applicable mathematics.

It should be noted that the proof of Theorem 3.1.2 is not necessary because the existence of closed non-Hamiltonian systems is established by visual observations, such as that of Jupiter. At any rate, the representation of Jupiter's structure via one single function, the Lagrangian or the Hamiltonian, necessarily implies the belief in the perpetual motion

within physical media, due to the necessary condition that constituents move inside Jupiter with conserved energy, linear momentum and angular momentum.

3.1.4 Ultimate Elemental Origin of Nonpotential Interactions

As recalled in Chapter 1, whenever exposed to departures from closed Hamiltonian systems, a widespread posture is the claim that the nonpotential interactions are "illusory" (*sic*) because, when the systems are reduced to their elementary constituents, all nonpotential forces "disappear" (*sic*) and conventional Hamiltonian disciplines are recovered in full.

This belief is disproved by Theorem 1.3.1 (whose knowledge is hereon assumed) because, as expected to be known by experts to qualify as such, a classical system with nonpotential forces cannot be consistently reduced to a finite number of particles all in conservative conditions, and, vice-versa, a finite ensemble of particles all under sole potential interactions cannot consistently reproduce a nonconservative classical system under the correspondence or any other principle.

Rather than being "illusory", *nonpotential effect originate at the deepest and most elemental level of nature, that of elementary particles or of antiparticles.* The property also establishes the need for the identification of methods suitable for the invariant treatment of classical and operator non-Hamiltonian systems in such a way to constitute a covering of conventional Hamiltonian treatments.

3.1.5 Basic Conditions to be verified by the Applicable Mathematics

By following the main guidelines of hadronic mechanics, rather than adapting nature to preferred mathematics, we adapt the mathematics to nature. For this purpose, we shall seek a mathematics capable of representing the following main features of interior dynamical systems:

1) Points have no dimension and, consequently, can only have action-at-a-distance potential interactions. Therefore, the first need for the new mathematics is the representation of the *actual, extended, generally nonspherical shape of the wavepackets and/or of the charge distribution of the particles considered*, that we shall assume in this monograph for simplicity to have the shape of spheroidal ellipsoids with diagonal form

$$Shape_a = Diag.(n_{a1}^2, n_{a2}^2, n_{a3}^2), \quad a = 1, 2, 3, \ldots, N, \tag{3.1.12}$$

with more general non-diagonal expressions not considered for simplicity, where $n_{a1}^2, n_{a2}^2, n_{a3}^2$ represent the semiaxes of the spheroidal ellipsoids

assumed as *deviation* from, or normalized with respect to the perfect spheridicity

$$n_{a1}^2 = n_{a2}^2 = n_{a3}^2 = 1. \tag{3.1.13}$$

The n's are called *characteristic quantities* of the particle considered. It should be stressed that, contrary to a rather popular belief, *the n-quantities are not parameters because they represent the actual shape as derived from experimental measurements.*

To clarify this important point, by definition a "parameter" can assume any value as derived form the fit of experimental data, while this is not the case for the characteristic quantities here considered. As an example, the value $n = 10^{-20}$ cm to represent a proton would have no physical value because the proton charge distribution is a spheroidal ellipsoid of the order of 10^{-13} cm= 1 F.

2) Once particles are assumed as being extended, there is the consequential need to represent their *density.* This task can be achieved via a fourth set of quantities

$$Density_a = n_{a4}^2, \tag{3.1.14}$$

representing the *deviation* of the density of the particle considered from the density of the vacuum here assumed to be one,

$$n_{Vacuum,4}^2 = 1. \tag{3.1.15}$$

Again, n_4 is not a free parameter because its numerical value is fixed by experimental data. As an example for the case of a hadron of mass m and radius $r = 1$ F we have the density

$$n_4^2 = \frac{m \times c^2}{\frac{4}{3} \times \pi \times r^3}, \tag{3.1.16}$$

thus establishing that n_{a4} is not a free parameter capable of assuming any desired value.

Predictably, most nonrelativistic studies can be conducted with the sole use of the space components characterizing the shape. Relativistic treatments require the additional use of the density as the forth component, resulting in the general form

$$(Shape - Density)_a = Diag.(n_{a1}^2, n_{a2}^2, n_{a3}^2, n_{a4}^2), \quad a = 1, 2, 3, \ldots, N, \tag{3.1.17}$$

3) Perfectly rigid bodies exist in academic abstractions, but not in the physical reality. Therefore, the next need is for a meaningful representation of the *deformation of shape* as well as *variation of density* that are possible under interior conditions. This is achieved via the appropriate

functional dependence of the characteristic quantities on the energy E_a, linear momentum p_a, pressure P and other characteristics, and we shall write

$$n_{ak} = n_{ak}(E, p, P, \ldots), \quad k = 1, 2, 3, 4. \tag{3.1.18}$$

The reader is suggested to meditate a moment on the fact that Lagrangian or Hamiltonian theories simply cannot represent the actual shape and density of particles. The impossibility of representing deformations of shapes and variations of density are well known, since the pillar of contemporary relativities, the rotational symmetry, is notoriously incompatible with the theory of elasticity.

4) Once particles are represented as they are in the physical reality (extended, nonspherical and deformable), there is the emergence of the following new class of interactions nonexistent for point-particles (for which reason these interactions have been generally ignored throughout the 20-th century), namely, interactions of:

I) *contact type*, that is, due to the actual physical contact of extended particle; consequently, of

II) *zero range type*, since all contacts are dimensionless; consequently of

III) *nonpotential type*, that is, not representable with any possible action-at-a-distance potential; consequently, of

IV) *non-Hamiltonian type*, that is, not representable with any Hamiltonian; consequently, of

V) *noncanonical type* at the classical level and *nonunitary type at the operator level;* as well as of

VI) *nonlinear type*, that is, represented via nonlinear differential equations, such as depending on power of the wavefunction greater than one; and, finally, of

VII) *nonlocal-integral type.* Interactions among point-particles are local-differential, that is, reducible to a finite set of isolated points, while contact interactions among extended particles and/or their wavepackets are, by conception, nonlocal-integral in the sense of being dependent on a finite surface or volume that, as such, cannot be reduced to a finite set of isolated points (see Figure 3.3).

5) Once the above new features of interior systems have been identified, there is the need not only of their mathematical representation, but above all of their *invariant representation* in order to avoid the theorem of catastrophic inconsistencies of Chapter 1.

As an illustration, Coulomb interactions have reached their towering position in the physics of the 20-th century because *the Coulomb potential is invariant under the basic symmetries of physics*, thus predicting the same numerical values under the same conditions at different times

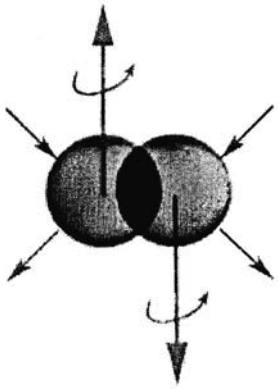

Figure 3.3. A schematic view of the fundamental interactions studied in this monograph, those originating from deep wave-overlappings of the charge distribution as well as of the wavepackets of particles, including particles with point-like charge, as occurring in electron valence bonds, Cooper pairs in superconductivity, Pauli's exclusion principle, and other basic structures. These interactions have been ignored throughout the 20-th century, resulting in the problematic aspects or sheer inconsistencies identified in Chapter 1. As we shall see in this chapter, the representation of the new interactions here depicted with generalized units of type (3.2.4) permits the achievement of the first known, exact and invariant representation of various data in particle physics, nuclear physics, chemistry, astrophysics and other fields [58–61] that have escaped an exact and invariant representation via quantum mechanics for about one century. In addition, the representation of the interactions herein considered permits convergent perturbative series when conventionally convergent.

with consequentially consistent physical applications. The same occurs for other interactions derivable from a potential (except gravitation represented with curvature as shown in Section 1.4).

Along the same lines, any representation of the extended, nonspherical and deformable character of particles, their densities and their novel nonlinear, nonlocal and nonpotential interactions cannot possibly have physical value unless it is also *invariant*, and not "covariant", again, because the latter would activate the theorems of catastrophic inconsistencies of Section 1.5.

3.1.6 Iso-, Geno-, and Hyper-Formulations for Particles and their Isoduals for Antiparticles

Following the identification of closed non-Hamiltonian systems, the author conducted an extensive search during the period 1978–1983 in the advanced libraries of Cambridge, Massachusetts. This search identified numerous integral geometries and other nonlocal mathematics. However, none of them verifies all the following conditions necessary for physical consistency:

CONDITION 1: *The new nonlocal-integral mathematics must admit the conventional local-differential mathematics as a particular case under a well identified limit procedure*, because new physical advances must be a covering of preceding results. This condition alone is not verified by any integral mathematics the author could identify.

CONDITION 2: *The new nonlocal-integral mathematics must permit the clear separation of the contributions of the new nonlocal-integral interactions from those of local-differential interactions.* This second condition too was not met by any of the integral mathematics the author could identify.

CONDITION 3: *The new nonlocal-integral mathematics must permit the invariant formulation of the new interactions.* This latter condition was also violated by all integral mathematics the author could identify, thus ruling them out in a final form for consistent physical applications.

After clarifying that the mathematics needed for the correct treatment of interior systems was absent, the author was left with no other choice than that of constructing the needed mathematics.

After extensive search, Santilli [23] suggested in 1978 as the *only* possible or otherwise known solution, the *invariant* representation of nonlinear, nonlocal and nonpotential interactions via a generalization of the trivial unit of conventional theories. The selection was based on the fact that, whether conventional or generalized, the unit is the basic invariant of any theories. We reach in this way the following:

FUNDAMENTAL ASSUMPTION OF HADRONIC MECHANICS [54,55]: *Interior, closed-reversible and open irreversible systems of extended particles or of extended antiparticles can be represented with two quantities, a conventional Hamiltonians $H(t, r, p)$ for the invariant representation of all action-at-a-distance potential interactions, and a generalization of the trivial N-dimensional unit of Hamiltonian theories, $I = Diag.(1, 1, \ldots, 1)$ into a sufficiently smooth and nowhere singular matrix $\hat{I}(t, r, p, \psi, \partial\psi, \ldots)$, of the same dimension with an arbitrary*

functional dependance on all needed local quantities for the invariant representation of the extended, nonspherical and deformable shape of particles, their variable densities and their nonlinear, nonlocal and non-potential interactions. All conventional mathematical and physical methods are then lifted into a form admitting $\hat{I}$, rather than I, as the correct left and right unit at all levels. Isounits can then be classified as follows:

I. HERMITEAN AND POSITIVE-DEFINITE UNITS $\hat{I} > 0$, CALLED ISOUNITS, AND THEIR ISODUALS $\hat{I}^d < 0$, CALLED ISODUAL ISOUNITS, characterizing the Lie-isotopic representation of interior, closed and reversible systems of extended particles and extended antiparticles, respectively;

II. NONHERMITEAN UNITS $\hat{I} \neq \hat{I}^\dagger$, CALLED GENOUNITS, AND THEIR ISODUALS $\hat{I}^d \neq \hat{I}^{d\dagger}$, CALLED ISODUAL GENOUNITS, characterizing the Lie-admissible representation of interior, open and irreversible systems of extended particles and extended antiparticles, respectively;

III. NONHERMITEAN MULTIVALUED UNITS $\hat{I} \neq \hat{I}^\dagger$, CALLED HYPERUNITS, AND THEIR ISODUALS $\hat{I}^d \neq \hat{I}^{d\dagger}$, CALLED ISODUAL HYPERUNITS, characterizing the Lie-admissible representation of interior, open, irreversible and multi-valued systems of extended particles and extended antiparticles, respectively.

In this chapter we review the long and laborious scientific journey by mathematicians, theoreticians and experimentalists (see the bibliography of Chapter 3) for the achievement of maturity of formulation of hadronic mechanics, its experimental verification, and its novel industrial applications.

3.2 ISOMATHEMATICS FOR EXTENDED PARTICLES AND ITS ISODUAL FOR EXTENDED ANTIPARTICLES

3.2.1 Isounits and their Isoduals

The new mathematics specifically constructed for quantitative invariant treatments of closed non-Hamiltonian systems is today known under the name of *Santilli isotopic mathematics* or *isomathematics* for short (where, as indicated earlier, the prefix "iso" denotes the preservation of conventional axioms). Isomathematics was first proposed by R. M. Santilli in Ref. [23] of 1978 and subsequently studied in various works (see Santilli's monographs [51,59,61], monographs [62–68] by independent authors and references quoted therein).

The main assumption of isomathematics is the lifting of the conventional unit of current formulations, generally given by an N-dimensional unit matrix $I = Diag.(1, 1, \ldots, 1) > 0$, into a quantity $\hat{I}$, called *Santilli isounit*, possessing all topological properties of I (such as positive-definiteness, same dimensionality, etc.), while having an arbitrary functional dependence on time t, mutual distances r, velocities v, wavefunctions ψ, their derivatives $\partial_x \psi$, and any other needed variable [23,51],

$$I = Diag.(1, 1, \ldots, 1) > 0 \to \hat{I}(t, r, v, \psi, \partial_x \psi, \ldots)$$
$$= 1/\hat{T}(t, r, v, \psi, \partial_x \psi, \ldots) > 0. \tag{3.2.1}$$

The conventional associative and distributive product $A \times B$ among generic quantities A, B (such as numbers, vector fields, operators, etc.) is jointly lifted into the more general form

$$A \times B \to A \hat{\times} B = A \times \hat{T} \times B \tag{3.2.2}$$

that remains associative and distributive, thus being called *isoproduct*, under which $\hat{I}$ is the correct left and right unit,

$$I \times A = A \times I = A \to \hat{I} \hat{\times} A = A \hat{\times} \hat{I} = A, \tag{3.2.3}$$

for all elements A of the set considered.

As an illustration, a closed non-Hamiltonian systems of two particles, such as the structure of light mesons, the Cooper pair in superconductivity or the structure of bi-atomic molecules can be characterized by a conventional Hamiltonian for the representation of potential interactions, plus the following nowhere singular, sufficiently smooth, positive-definite, and integro-differential isounit

$$\hat{I} = \hat{I}^\dagger = \hat{I}_{1-2} = Diag.(n_{11}^2, n_{12}^2, n_{13}^2, n_{14}^2)$$
$$\times Diag.(n_{21}^2, n_{22}^2, n_{23}^2, n_{24}^2)$$
$$\times e^{\Gamma(t,r,\psi,\psi^\dagger,\ldots)\times \int dr^3 \times \psi^\dagger(r) \times \psi(r)} = 1/\hat{T} > 0, \tag{3.2.4}$$

with trivial generalizations to multiparticle and nondiagonal forms, where the n_{ak}^2 represents the semiaxes of the spheroidal shape of particle a, n_{a4}^2 represents its density, the expression $\Gamma(t, r, \psi, \psi^\dagger, \ldots)$ represents the nonlinearity of the interaction and $\int dr^3 \times \psi^\dagger(r) \times \psi(r)$ provides a simple representation of its nonlocality.

Antiparticles are then characterized by the isodual map yielding the *isodual isounit*

$$\hat{I}(t, r, \psi, \ldots) \to \hat{I}^d(t^d, r^d, \psi^d, \ldots) = -\hat{I}(-t, -r, -\psi^\dagger, \ldots), \tag{3.2.5}$$

where the Hermiticity of $\hat{I}$ is assumed. Mixed states of particles and antiparticles are represented by the tensorial product of the corresponding units and their isoduals.

As we shall see, the entire structure of hadronic mechanics follows uniquely and unambiguously from the assumption of the above basic unit.

The main features of hadronic mechanics can already be derived from the above basic assumption. For instance, Santilli isounit $\hat{I}$ identifies in full the *covering* nature of isoformulations over conventional formulations, as well as the type of resulting covering.

In fact, at sufficiently large mutual distances of particles the integral in the exponent of Eq. (3.2.4) is null

$$\text{Lim}_{r\gg 1\,\text{Fm}} \int dr^3 \times \psi^\dagger(r) \times \psi(r) = 0. \tag{3.2.6}$$

In this case, the actual size of the particles is irrelevant because terms such as $Diag.(n_{11}^{-2}, n_{12}^{-2}, n_{13}^{-2}, n_{14}^{-2})$ factor out of all equations, resulting in reduced form

$$\text{Lim}_{r\gg 1\,\text{Fm}}\, \hat{I} = I = Diag.(1,1,1,1). \tag{3.2.7}$$

under which limit hadronic mechanics recovers conventional quantum mechanics identically and uniquely.

The above limits also identify the important feature according to which *hadronic mechanics coincides with quantum mechanics for all mutual distances of particles sufficiently bigger than their charge distributions and/or their wavepackets, while at mutual distances below that value hadronic mechanics provides generally small corrections to quantum mechanics.*

Numerous additional examples of isounits exist in the literature. Note that the features represented by the isounits are strictly outside any representational capability by the Hamiltonian.

3.2.2 Isonumbers, Isofields and their Isoduals

The first implication of the lifting of the unit is the need for a corresponding generalization of numbers and fields that can be introduced via the following:

DEFINITION 3.2.1: Let $F = F(a, +, \times)$ be a field as per Definition 2.1.1. Santilli's isofields, first introduced in Ref. [12] *are rings $\hat{F} = \hat{F}(\hat{a}, \hat{+}, \hat{\times})$ whose elements are the isonumbers*

$$\hat{a} = a \times \hat{I}, \tag{3.2.8}$$

with associative, distributive and commutative isosum

$$\hat{a}\hat{+}\hat{b} = (a+b)\times\hat{I} = \hat{c}\in\hat{F}, \tag{3.2.9}$$

associative and distributive isoproduct

$$\hat{a}\hat{\times}\hat{b} = \hat{a}\times\hat{T}\times\hat{b} = \hat{c}\in\hat{F}, \tag{3.2.10}$$

additive isounit

$$\hat{0} = 0,\ \ \hat{a}\hat{+}\hat{0} = \hat{0}\hat{+}\hat{a} = \hat{a}, \tag{3.2.11}$$

and multiplicative isounit

$$\hat{I} = 1/\hat{T} > 0,\ \hat{a}\hat{\times}\hat{I} = \hat{I}\hat{\times}\hat{a} = \hat{a},\ \forall\hat{a},\ \hat{b}\in\hat{F}, \tag{3.2.12}$$

where $\hat{I}$ is not necessarily an element of F. Isofields are called of the first (second) kind when $\hat{I} = 1/\hat{T} > 0$ is (is not) an element of F.

LEMMA 3.2.1 [12]: *Isofields of first and second kind are fields (namely, isofields verify all axioms of a field with characteristic zero).*

The above property establishes the fact (first identified in Ref. [12]) that, by no means, the axioms of a field require that the multiplicative unit be the trivial unit +1, because the unit can be a negative-definite quantity as for the isodual mathematics, as well as an arbitrary positive-definite quantity, such as a matrix or an integrodifferential operator for isonumbers.

Needless to say, the liftings of the unit and of the product requires a corresponding lifting of all conventional operations of a field. In fact, we have the *isopowers*

$$\hat{a}^{\hat{n}} = \hat{a}\hat{\times}\hat{a}\hat{\times}\ldots,\hat{\times}\hat{a}\quad (n\,\text{times}) = a^n\times\hat{I}, \tag{3.2.13}$$

with particular case

$$\hat{I}^{\hat{n}} = \hat{I}; \tag{3.2.14}$$

the *isosquare root*

$$\hat{a}^{\hat{1/2}} = a^{1/2}\times\hat{I}; \tag{3.2.15}$$

the *isoquotient*

$$\hat{a}\hat{/}\hat{b} = (\hat{a}/\hat{b})\times\hat{I} = (a/b)\times\hat{I}; \tag{3.2.16}$$

the *isonorm*

$$\hat{|}\hat{a}\hat{|} = |a|\times\hat{I}, \tag{3.2.17}$$

where $|a|$ is the conventional norm; etc.

Despite their simplicity, the above liftings imply a generalization of the conventional number theory particularly for the case of the first kind (in which $\hat{I} \in F$) with implications for all aspects of the theory. As an illustration, the use of the isounit $\hat{I} = 1/3$ implies that "2 multiplied by 3" $= 18$, while 4 becomes a prime number.

A comprehensive study of Santilli's isonumber theory has been conducted by C.-X. Jiang in monograph [68] with numerous novel developments and applications. Additional studies on isonumbers have been done by N. Kamiya *et al.* [156] and others (see proceedings [69–109]).

3.2.3 Isofunctional Analysis, Isodifferential Calculus and their Isoduals

The lifting of fields into isofields requires a corresponding lifting of functional analysis into a form known as *isofunctional analysis* studied by J. V. Kadeisvili [132,133], A. K. Aringazin *et al.* [144] and other authors. A review of isofunctional analysis up to 1995 with various developments has been provided by Santilli in monographs [54,55]. We here merely recall the *isofunctions*

$$\hat{f}(\hat{x}) = f(x \times I) \times \hat{I}; \tag{3.2.18}$$

the *isologarithm*

$$\hat{log}_{\hat{e}} a = \hat{I} \times log_e a, \ \hat{log}_{\hat{e}} \hat{e} = \hat{I}, \ \hat{log}_{\hat{e}} \hat{I} = 0; \tag{3.2.19}$$

and the *isoexponentiation*,

$$\hat{e}^{\hat{A}} = \hat{I} \hat{+} \hat{A} \hat{/} \hat{1}! \hat{+} \hat{A} \hat{\times} \hat{A} \hat{/} \hat{2}! \hat{+} \ldots = (e^{\hat{A} \times \hat{T}}) \times \hat{I} = \hat{I} \times (e^{\hat{T} \times \hat{A}}). \tag{3.2.20}$$

Particularly important are the *isotrigonometric functions* that cannot be reviewed here for brevity [55]. The reader should be aware that any isocalculation done via conventional functions leads to huge inconsistencies similar to those that would result if quantum mechanical calculations are done via isofunctions.

The conventional differential calculus must also be lifted, for consistency, into the *isodifferential calculus* first identified by Santilli in memoir [14] of 1996, with *isodifferential*

$$\hat{d}\hat{x} = \hat{T} \times d\hat{x} = \hat{T} \times d(x \times \hat{I}), \tag{3.2.21}$$

that, for the case when $\hat{I}$ does not depend on x, coincides with the conventional differential

$$\hat{d}\hat{x} = dx; \tag{3.2.22}$$

the *isoderivatives*

$$\hat{\partial}\hat{f}(\hat{x})/\hat{\partial}\hat{x} = \hat{I} \times [\partial f(\hat{x})/\partial\hat{x}], \tag{3.2.23}$$

and other similar properties.

The reader interested in learning isomathematics should prove that isodifferentials commute when formulated in their isospaces over isofields but they do not generally commute when projected on conventional spaces over conventional fields.

The indicated invariance of the differential under isotopy, $\hat{d}\hat{x} = dx$, illustrates the reason why the isodifferential calculus has remained undetected since Newton's and Leibnitz's times.

3.2.4 Isospaces, Isogeometries, Isotopologies and their Isoduals

The *isotopies of metric or pseudo-metric spaces* (such as the Euclidean, Minkowskian, Riemannian, Finslerian and other spaces), called *Santilli's isospaces*, have a fundamental role in hadronic mechanics. They were first identified in Ref. [26] of 1983 and then studied by Santilli in various works (see Refs. [14,15,29,54,55]) as well as other researchers. We cannot possibly review here these advances in all details for brevity.

We merely mention that any given n-dimensional metric or pseudo-metric space $S(x, m, R)$ with basic unit $I = Diag.(1, 1, \ldots, 1)$, local coordinates $x = (x^i), i = 1, 2, .., n$, $n \times n$-dimensional metric m and invariant

$$x^2 = x^i \times m_{ij} \times x^j \in R \tag{3.2.24}$$

is lifted into the *isospaces* $\hat{S}(\hat{x}, \hat{m}, \hat{R})$ with *isounit* given by

$$I = Diag.(1, 1, \ldots, 1) \to \hat{I}_{n\times n}(x, v, \ldots) = 1/\hat{T}(x, v, \ldots), \tag{3.2.25}$$

isocoordinates

$$x \to \hat{x} = x \times \hat{I}, m \to \hat{m} = \hat{T}(x, v, \ldots)\ m, \tag{3.2.26}$$

isometric

$$m \to \hat{M} = \hat{m} \times \hat{I} = [\hat{T}(x, v, \psi, \partial\psi, \ldots) \times m] \times \hat{I}, \tag{3.2.27}$$

and *isoinvariant*

$$\begin{aligned} x^2 = x^i \times m_{ij} \times x^j \times I \in R \to \hat{x}^{\hat{2}} &= \hat{x}^i \hat{\times} \hat{m}_{ij} \hat{\times} \hat{x}^j \times \hat{I} \\ &= \{x^i \times [\hat{T}(x, v, \ldots) \times m] \times x^j\} \times \hat{I} \in \hat{R}. \end{aligned} \tag{3.2.28}$$

where one should note that $\hat{M}$ is an *isomatrix*, namely, a matrix whose elements are isonumbers (thus being multiplied by $\hat{I}$ to be in $\hat{R}$) and all operation are isotopic.[3]

Santilli's isogeometries are the geometries of isospaces. As such, they are based on the abstract axioms of the original space. For instance, despite an arbitrary functional dependence of the isometric, the iso-Minkowskian geometry verifies the *Minkowskian*, rather than the Riemannian axioms.

An inspection of the functional dependence of the isometric $\hat{m} = \hat{T}(x, v, \ldots) \times m$ then reveals that *isospaces* $\hat{S}(\hat{x}, \hat{m}\hat{R})$ *unify all possible spaces with the same dimension and signature.* As an illustration, the isotopy of the Minkowski space includes as particular case the Riemannian, Finslerian as well as any other space with the same dimension and signature $(+, +, +, -)$ (in view of the positive-definiteness of $\hat{I}$. Broader unifications are possible in the event such positive-definiteness is relaxed.

Since the isotopies preserve the original axioms, the isotopies permit the unification of the Minkowskian and Riemannian geometry, with consequential *unification of special and general relativities* via the axioms of the special, as studied in detail in Ref. [15]. In turn, such a geometric unification has far reaching implications, e.g., for grand unifications and cosmologies (see later on).

It should be mentioned that "deformations" of conventional geometries are rather fashionable these days in the physical and mathematical literature. However, these deformations are generally afflicted by the catastrophic inconsistencies of Theorem 1.5.1 because, when the original geometry is canonical, the deformed geometry is noncanonical, thus losing the invariance needed for consistent applications. The isotopies of conventional geometries were constructed precisely to avoid such inconsistencies by reconstructing invariance on isospaces over isofield while having a fully noncanonical structure, as shown below.

Therefore, for the case of "deformations" the generalized metric $\hat{m} = \hat{T} \times m$ and related invariant are referred to conventional units and fields R, while for "isotopies" the same generalized metric $\hat{m} = \hat{T} \times m$ is referred to a isounit which is the *inverse* of the deformation of the metric, $\hat{I} = \hat{T}^{-1}$.

Moreover, also for the case of "deformations", the deformed geometry verify axioms different then the original ones, because the original metric m has been deformed by the multiplication of the matrix $\hat{T}$ while the basic unit is kept unchanged.

Particularly intriguing are the isotopies of the symplectic geometry, known as *isosymplectic geometry* [14] that is based on the following *fun-*

damental isosymplectic two-isoform

$$\hat{d}\hat{p}\hat{\wedge}\hat{d}\hat{r} = \hat{\omega} \equiv dp \wedge dr = \omega, \tag{3.2.29}$$

due to the fact that, for certain geometric reasons, the isounit of the momentum p in the cotangent bundle (phase space) is the inverse of the isounit of x (i.e., when $\hat{I} = 1/\hat{T}$ is the isounit for x, that for p is $\hat{T} = 1/\hat{I}$). The invariance $\hat{\omega} \equiv \omega$ provide a reason why the isotopies of the symplectic geometry have escaped identification by mathematicians for over one century.

Despite their simplicity, the isotopies of the symplectic geometry have deep implications, e.g., they permit rigorous studies of a broader quantization leading to hadronic mechanics.

We should mention that the isotopies of metric or pseudo-metric spaces are "hidden" in the abstract axioms of conventional spaces. In fact, the conventional line element (3.2.24) remains invariant under the following scaling of the isounit and isotopic element

$$\hat{I} \to \hat{I}' = n^2 \times \hat{I}, \quad \hat{T} \to \hat{T}' = n^{-2} \times \hat{T}, n^2 \in R, \tag{3.2.30}$$

under which we have the new *isoinvariance law*

$$\begin{aligned} x^2 = (x^i \times m_{ij} \times x^j) \times I &\equiv [x^i \times (n^{-2} \times m_{ij}) \times x^j) \times (n^2 \times I) \\ &= (x^i \times \hat{m}_{ij} \times x^j) \times \hat{I}. \end{aligned} \tag{3.2.31}$$

This "hidden" character explains the reason isospaces have remained undetected for centuries. Note, however, that their detection required the prior discovery of *new numbers*, those with arbitrary units.

Despite its simplicity, invariance (3.2.30) has far reaching implications (as it is the case for any new invariance). For instance, the new invariance (3.2.30) establishes that *the Poincaré symmetry is eleven-dimensional*, contrary the popular belief throughout the 20-th century that it is ten-dimensional.

In turn, the additional eleventh dimensionality of the Poincaré symmetry has equally far reaching implications, such as the achievement of an axiomatically consistent quantum gravity, an axiomatically consistent grand unification of electroweak and gravitational interactions, and other advances we shall study later on.

Readers interested in learning the new mathematics are suggested to construct the *isodual isospaces and isodual isogeometries* via the application of the isodual map such as Eq. (3.2.5). Note that the latter formulations are also "hidden" in isospaces and isogeometries. in fact,

the reversal of the sign of $\hat{I}$, with consequential reversal of the sign of $\hat{T}$, leaves invariant (3.2.28) unaffected. In fact, under isoduality

$$\hat{I} \to \hat{I}^d = -\hat{I} = 1/\hat{T}^d = 1/-\hat{T}, \tag{3.2.31}$$

we have the new *isodual isoinvariance law*

$$(x^i \times \hat{m}_{ij} \times x^j) \times \hat{I} \equiv (x^{di} \times \hat{m}^d_{ij} \times x^{dj}) \times \hat{I}^d. \tag{3.2.32}$$

This explains the reason isodual isospaces have also remained undetected for centuries.

Remember that the topology is the ultimate foundation of mathematics, and the same holds also for isomathematics. Therefore, particularly important for these studies are the isotopies of the conventional local-differential topology studied by G. T. Tsagas and D. S. Sourlas [139], R. M. Santilli [14], R. M. Falcón Ganfornina and J. Núñez Valdés [226,227] and today known as the *TSSNC isotopology*. We regret to be unable to review it for brevity.

3.2.5 Lie-Santilli Isotheory and its Isodual

As it is well known, Lie's theory [4] is based on the conventional (left and right) unit $I = Diag.(1, 1, \ldots, 1)$ of the universal enveloping associative algebra, with attached Lie algebras and corresponding (connected) Lie transformation groups achieved via exponentiation.

The lifting $I \to \hat{I}(x, \ldots)$ implies the lifting of the entire Lie theory, first proposed by Santilli in Ref. [23] of 1978 and then studied in numerous works (see, e.g., memoir [14] and monographs [51,54,55]).

The isotopies of Lie's theory are today known as the *Lie-Santilli isotheory* following studies by numerous mathematicians and theoreticians (see the monographs by D. S. Sourlas and Gr. Tsagas [64], J. V. Kadeisvili [66], R. M. Falcón Ganfornina and J. Núñez Valdés [67], proceedings [68–109] and contributions quoted therein).

Let $\xi(L)$ be the universal enveloping associative algebra of an N-dimensional Lie algebra L with (Hermitean) generators $X = (X_i)$, $i = 1, 2, \ldots, n$, and corresponding Lie transformation group G over the reals R. The Lie-Santilli isotheory is characterized by:

(I) The *universal enveloping isoassociative algebra* $\hat{\xi}$ with infinite-dimensional basis characterizing the *Poincaré-Birkhoff-Witt-Santilli isotheorem*

$$\hat{\xi} : \hat{I}, \hat{X}_i, \hat{X}_i \hat{\times} \hat{X}_j, i \leq j; \hat{X}_i \hat{\times} \hat{X}_j \hat{\times} \hat{X}_k, \ldots, i \leq j \leq k; \tag{3.2.34}$$

where the "hat" on the generators denotes their formulation on isospaces over isofields;

(II) The *Lie-Santilli isoalgebras*

$$\hat{L} \approx (\hat{\xi})^- : [\hat{X}_i,\hat{X}_j] = \hat{X}_i\hat{\times}\hat{X}_j - \hat{X}_j\hat{\times}\hat{X}_i$$

$$= \hat{X}_i\times\hat{T}(x,v,\psi,\ldots)\times\hat{X}_j - \hat{X}_j\times\hat{T}(x,v,\psi,\ldots)\times\hat{X}_i = \hat{C}_{ij}^k\hat{\times}\hat{X}_k; \quad (3.2.35)$$

(III) The *Lie-Santilli isotransformation groups*

$$\hat{G} : \hat{A}(\hat{w}) = (\hat{e}^{\hat{i}\hat{\times}\hat{X}\hat{\times}\hat{w}})\hat{\times}\hat{A}(\hat{0})\hat{\times}(\hat{e}^{-\hat{i}\hat{\times}\hat{w}\hat{\times}\hat{X}})$$

$$= (e^{i\times\hat{X}\times\hat{T}\times w})\times A(0)\times(e^{-i\times w\times\hat{T}\times\hat{X}}), \quad (3.2.36)$$

where $\hat{w}\in\hat{R}$ are the *isoparameters;* the *isorepresentation theory;* etc.

The non-triviality of the above liftings is expressed by the appearance of the isotopic element $\hat{T}(x,v,\psi,\ldots)$ at all levels (I), (II) and (III) of the isotheory, such as in the exponentiation. The arbitrary functional dependence of $\hat{T}(x,v,\psi,\ldots)$ then implies the achievement of the desired main features of the isotheory which can be expressed by the following:

LEMMA 3.2.2 [14]: *When formulated on conventional spaces over conventional fields, Lie-Santilli isoalgebras are generally nonlocal, nonlinear and noncanonical, but they reconstruct locality, linearity and canonicity when formulated on isospaces over isofields.*

A main role of the isotheory is then expressed by the following property:

LEMMA 3.2.3 [39]: *Under the condition that $\hat{I}$ is positive-definite, isotopic algebras and groups are locally isomorphic to the corresponding conventional algebras and groups, respectively.*

Stated in different terms, the Lie-Santilli isotheory *was not* constructed to characterize new Lie algebras, because all Lie algebras over a field of characteristic zero are known. On the contrary, the Lie-Santilli isotheory has been built to characterize *new realizations* of known Lie algebras generally of nonlinear, nonlocal and noncanonical character as needed for a deeper representation of valence bonds or, more generally, systems with nonlinear, nonlocal and noncanonical interactions.

The mathematical implications of the Lie-Santilli isotheory are significant. For instance, Gr. Tsagas [142] has shown that *all simple non-exceptional Lie algebras of dimension N can be unified into one single Lie-Santilli isotope of the same dimension*, while studies for the inclusion of exceptional algebras in this grand unification of Lie theory are under way.

In fact, the characterization of different simple Lie algebras, including the transition from compact to noncompact Lie algebras, can be characterized by *different realizations of the isounit while using a unique form of generators and of structure constants* (see the first examples for the SO(3) algebra in Ref. [23] of 1978 and numerous others in the quoted literature).

The physical implications of the Lie-Santilli isotheory are equally significant. We here mention the *reconstruction as exact at the isotopic level of Lie symmetries when believed to be broken under conventional treatment.* In fact, Santilli has proved:

1) the exact reconstruction of the rotational symmetry for all ellipsoidical deformations of the sphere [12];

2) the reconstruction of the exact SU(2)-isospin symmetry under electromagnetic interactions [28,33];

3) the reconstruction of the exact Lorentz symmetry under all (sufficiently smooth) signature-preserving deformations of the Minkowski metric [26];

4) the reconstruction of the exact parity under weak interactions [55].

R. Mignani [180] has studied the reconstruction of the exact SU(3) symmetry under various symmetry-breaking terms. In all these cases the reconstruction of the exact symmetry has been achieved by merely embedding all symmetry breaking terms in the isounit. The positive-definiteness of the latter ensured the local isomorphism of the isotopic and original symmetries.

The construction of the *isodual Lie-Santilli isotheory* for antimatter is an instructive exercise for interested readers.

3.3 CLASSICAL ISO-HAMILTONIAN MECHANICS AND ITS ISODUAL

3.3.1 Newton-Santilli Isomechanics and its Isodual

As it is well known, Newton [1] had to construct the differential calculus as a pre-requisite for the formulation of his celebrated equations.

Today we know that Newton's equations can only represent *point-particles* due to the strictly local-differential character of the underlying Euclidean topology.

The fundamental character of Newtonian Mechanics for all scientific inquiries (such as Hamiltonian mechanics, quantum mechanics, quantum chemistry, quantum field theory, etc.) is due to the preservation at all subsequent levels of study of:

1) The underlying Euclidean topology;

2) The differential calculus; and

3) The abstraction of particles as being point-like.

By keeping in mind Newton's teaching, the author has dedicated primary efforts to the isotopic lifting of the conventional differential calculus, topology and geometries [14] as a pre-requisite for a structural generalization of Newton's equations into a form representing *extended, nonspherical and deformable particles* under action-at-a-distance/potential as well as contact/nonpotential forces.

As studied in detail in Chapter 1, the need for such a lifting is due to the fact that *point particles are dimensionless and, therefore, they cannot experience contact-resistive forces.* This feature has lead to subsequent theories, such as Hamiltonian and quantum mechanics, that solely admit action-at-a-distance/potential forces among point particles.

Such a restriction is indeed valid for a number of systems, such as planetary systems at the classical level and atomic systems at the operator level, because the large distances among the constituents permit an effective point–like approximation of planets of the extended wavepackets of electrons as being massive points.

However, when interactions occur at short distances, as in the case of electron valence bonds or the mutual penetration of the wavepackets of particles in general, the point-like approximation is no longer sufficient and a representation of the actual, extended, generally nonspherical and deformable shape of particles is necessary to admit contact nonpotential interactions.

By recalling the fundamental character of Newtonian mechanics for all of sciences, the achievement of a consistent representation of the contact interactions of valence electron bonds at the *operator* level requires the prior achievement of a consistent *Newtonian* representation.

To outline the needed isotopies, let us recall that Newtonian mechanics is formulated on the Kronecker product

$$S_{tot} = S_t \times S_x \times S_v \tag{3.3.1}$$

of the one dimensional space S_t representing time t, the tree dimensional Euclidean space S_r of the coordinates $r = (r^k_\alpha)$ (where $k = 1, 2, 3$ are the Euclidean axes and $\alpha = 1, 2, \ldots, N$ represents the number of particles), and the velocity space $S_v, v = dr/dt$.

It is generally assumed that all variables t, r, and v are defined on the same field of real numbers R. However, the unit of time is the *scalar* $I = +1$, while the unit of the Euclidean space is the *matrix* $I = Diag.(1, 1, 1)$. Therefore, on rigorous grounds, the representation space of Newtonian mechanics $S_{tot} = S_1 \times S_r \times S_v$ must be defined on the Kronecker product

of the corresponding fields

$$R_{tot} = R_t \times R_r \times R_v \tag{3.3.2}$$

with total unit

$$I_{Tot} = 1 \times Diag.(1,1,1)_r \times Diag.(1,1,1)_v \tag{3.3.3}$$

the Newtonian systems most important for the isotopies are given by the so-called *closed-isolated non-Hamiltonian systems* studied in Section 3.1 [51], namely, systems which are closed-isolated from the rest of the universe, thus verifying all ten Galilean total conservation laws, yet they admit internal non-Hamiltonian forces due to contact interactions, as typically illustrated by the structure of Jupiter. A knowledge of these system is hereon assumed.

The *isotopies of Newtonian mechanics*, also called *Newton-Santilli isomechanics* [63–68], requires the use of: the *isotime* $\hat{t} = t \times \hat{I}_t$ with isounit $\hat{I}_t = 1/\hat{T}_t$ and related isofield $\hat{R}_t$; the *isocoordinates* $\hat{r} = (\hat{r}^k_\alpha) = r \times \hat{I}_x$, with isounit $\hat{I}_x = 1/\hat{T}_r$ and related isofield $\hat{R}_r$; and the *isospeeds* $\hat{v} = (v_{k\alpha}) = v \times \hat{I}_v$ with isounit $\hat{I}_v = 1/\hat{T}_v$ and related isofield $\hat{R}_v$.

Iso-Newtonian Mechanics is then formulated on the Kronecker product of isospaces

$$\hat{S}_{Tot} = \hat{S}_t \times \hat{S}_r \times \hat{S}_v \tag{3.3.4}$$

over the Kronecker product of isofields

$$\hat{R}_t \times \hat{R}_r \times \hat{R}_v \tag{3.3.5}$$

with total isounit

$$\hat{I}_{Tot} = \hat{I}_t \times \hat{I}_r \times \hat{I}_v. \tag{3.3.6}$$

The isospeed is then given by

$$\hat{v} = \frac{\hat{d}\hat{r}}{\hat{d}\hat{t}} = \hat{I}_t \times \frac{d(r \times \hat{I}_r)}{dt} = v \times \hat{I}_t \times \hat{I}_r + r \times \hat{I}_t \times \frac{d\hat{I}_r}{dt} = v \times \hat{I}_v \tag{3.3.7}$$

where

$$\hat{I}_v = \hat{I}_t \times \hat{I}_r \times \Big(1 + r \times \hat{T}_r \times \frac{d\hat{I}_r}{dt}\Big). \tag{3.3.8}$$

The *Newton-Santilli isoequations*, first proposed in memoir [14] of 1996 (following the first identification of the isodifferential calculus) can be written

$$\hat{m}_\alpha \hat{\times} \frac{\hat{d}\hat{v}_{k\alpha}}{\hat{d}\hat{t}} = -\frac{\hat{\partial}\hat{V}(\hat{r})}{\hat{\partial}\hat{r}^k_\alpha}, \tag{3.3.9}$$

namely, *the equations are conceived in such a way to formally coincide with the conventional equations for selfadjoint forces*, $F^{SA} = -\partial V/\partial r$, *while all nonpotential forces are represented by the isounits or, equivalently, by the isodifferential calculus.*

Such a conception is the only one known permitting the representation of extended particles with contact interactions that is invariant (thus avoiding the catastrophic inconsistencies of Theorem 1.5.1) and achieves closure, namely, the verification of all ten Galilean conservation laws.

The above conception is also crucial to permit, apparently for the first time, the derivability from an action principle of sufficiently smooth but otherwise unrestricted nonconservative systems, thus permitting, also for the first time, their treatment via the optimal control theory, as indicated in the next subsection.

An inspection of Eq. (3.3.9) is sufficient to see that

Lemma 3.3.1 [14]: *Iso-Newtonian mechanics reconstructs canonicity on isospace over isofields.*

This property permits Eq. (3.3.9) to avoid Theorem 1.5.1 on the catastrophic inconsistencies of noncanonical theories. Note that this would not be the case if nonselfadjoint forces appear in the right hand side of Eq. (3.3.9) as in Eq. (3.1.1).

THEOREM 3.3.1 [14]: *Under the verification by the non-self-adjoint forces of all conditions (3.1.8), the Newton-Santilli isoequations achieve a consistent representation of closed non-Hamiltonian systems.*

Proof. The verification of all ten Galilean conservation laws is established by a visual inspection of Eq. (3.3.9) since their symmetry is the *Galileo-Santilli isosymmetry*, i.e., the Galilean symmetry, only formulated on isospace over isofields [53]. By recalling that conservation laws are represented by the generators of the underlying symmetry, conventional total conservation laws then follow from the fact that *the generator of the conventional Galilean symmetry and its isotopic lifting coincide.* The admission of internal nonpotential forces is established by the unrestricted functional dependence of the isounit. **q.e.d.**

When projected in the conventional Newtonian space S_{Tot}, Eq. (3.3.9) can be explicitly written

$$\hat{m}\hat{\times}\frac{\hat{d}\hat{v}}{\hat{d}\hat{t}} = m \times \hat{I}_t \times \frac{d(v \times \hat{I}_v)}{dt}$$

$$= m \times a \times \hat{I}_t \times \hat{I}_v + m \times v \times \hat{I}_t \times \frac{d\hat{I}_v}{dt} = -\frac{\hat{\partial}\hat{V}(\hat{r})}{\hat{\partial}\hat{r}} = -\hat{I}_x \times \frac{\partial V}{\partial r}, \quad (3.3.10a)$$

$$m \times a = -\hat{T}_t \times \hat{T}_v \times \hat{I}_r \times \frac{\partial V}{\partial r} - m \times v \times \hat{T}_v \times \frac{d\hat{I}_v}{dt}, \quad (3.3.10b)$$

with necessary and sufficient conditions for the representation of all possible SA and NSA forces

$$\hat{I}_t \times \hat{I}_v \times \hat{I}_x = I, \hat{I}_r = 1/\hat{T}_t \times \hat{T}_r, \quad (3.3.11a)$$

$$m \times v \times \hat{T}_v \times \frac{d\hat{I}_v}{dt} = F^{NSA}(t, r, v), \quad (3.3.11b)$$

which always admit a solution, since they constitute a system of $6N$ *algebraic* (rather than differential) equations in the $6N + 1$ unknowns given by $\hat{I}_t$, and the diagonal $\hat{I}_r$ and $\hat{I}_v$.

As an illustration, we have the following equations of motion of an *extended* particle with the ellipsoidical shape experiencing a resistive force $F^{NSA} = -\gamma \times v$ because moving within a physical medium

$$m \times a = -\gamma \times v \quad (3.3.12a)$$

$$\hat{I}_v = Diag.(n_1^2, n_2^2, n_3^2) \times e^{\gamma \times t/m}. \quad (3.3.12b)$$

The representation of the *density* of the particle considered is done via the time isounit $\hat{I}_t = n_4^2$, as we shall see in our isorelativistic treatments.

Interested readers can construct the representation of *any* desired NSA forces (see also memoir [14] for other examples).

Note the natural appearance of the velocity dependence in the nonself-adjoint forces, as typical of resistive forces.

Note also that the representation of the extended character of particles occurs only in isospace because, when Eq. (3.3.9) are projected in the conventional Newtonian space, all isounits cancel out and the point characterization of particles is recovered.

Note finally the *direct universality* of the Newton-Santilli isoequations, namely, their capability of representing all infinitely possible Newton's equations (universality) directly in the frame of the observer without any need of transforming the local coordinates (direct universality).

As indicated earlier, Eq. (3.3.9) can only describe a system of *particles.* The *isodual Newton-Santilli isoequations* for the treatment of a system of *extended antiparticles* can be written [14]

$$\hat{m}_\alpha^d \hat{\times}^d \frac{\hat{d}^d \hat{v}_{k\alpha}^d}{\hat{d}^d \hat{t}^d} = -\frac{\hat{\partial}\hat{V}^d(\hat{r})}{\hat{\partial}^d \hat{r}_\alpha^{dk}}. \quad (3.3.13)$$

Rather than being a mere mathematical formality, the above isodual equations have deep physical implications. In fact, they are at the foundation of the resolution of the scientific imbalance caused by classical antimatter studied in Chapter 1 also for the case of extended antiparticles.

Moreover, the above isodual equations indicate the *multidimensional character of nature*, not in the popular sense of increasing the dimension of the basic Euclidean space, but rather in the hyperdimensional sense that *different three-dimensional spaces coexisting one inside the other.*

In fact, according to the above isodual theory, extended antiparticle do not exist in the iso-Euclidean space, but rather in their own isodual iso-Euclidean space that is physically distinct from the former.

Note again that classical antiparticles move backward in time, although this time referred to the isotime. the isoduality of other aspects of the Newton-Santilli isomechanics is instructive for readers interested in the field.

3.3.2 Iso-Action Principle and its Isodual

Eq. (3.3.9) admit the analytic representation in terms of the following *isoaction principle* [14]

$$\hat{\delta}\hat{\mathcal{A}}(\hat{t},\hat{r}) = \hat{\delta}\hat{\int}(\hat{p}_{k\alpha}\hat{\times}_p d\hat{r}^k_\alpha) - \hat{H}\hat{\times}_t d\hat{t}$$

$$= \delta\hat{\int}[p_{k\alpha}\times\hat{T}_r(t,r,p,\ldots)\times d(r^k_\alpha\times\hat{I}_r) - H\times\hat{T}_t(t,r,p,\ldots)d(t\times\hat{I}_t) = 0. \tag{3.3.14}$$

Note the main result permitted by the isodifferential calculus, consisting in the *reduction of an action functional of arbitrary power in the linear momentum (arbitrary order) to that of first power in p (first order).*

Since the optimal control theory and the calculus of variation depend on the first order character of the action functional, the above reduction has important implications, such as the treatment, apparently for the first time in scientific records, of extended objects moving within resistive media via the optimal control theory. Note that a first order conventional action is impossible for the systems considered with consequential impossibility to apply the optimal control theory to systems such as the wing of an airplane while moving within air.

Note also that, when the isounits are constant, isoaction and action functional coincide. This illustrates the apparent reason why the isotopies of the action principle creeped in un-noticed for over one century.

3.3.3 Iso-Hamiltonian Mechanics and its Isodual

It is easy to prove that isoaction principle (3.3.14) characterizes the *Hamilton-Santilli isoequations* [14]

$$\frac{\hat{d}\hat{r}}{\hat{d}\hat{t}} = \frac{\hat{\partial}\hat{H}}{\hat{\partial}\hat{p}} = \frac{\hat{p}}{\hat{m}}, \tag{3.3.15a}$$

$$\frac{\hat{d}\hat{p}}{\hat{d}\hat{t}} = -\frac{\hat{\partial}\hat{H}}{\hat{\partial}\hat{r}} = -\hat{I}_r \times \frac{\partial \hat{H}}{\partial \hat{r}} = \hat{F}^{SA} + \hat{F}^{NSA}, \tag{3.3.15b),}$$

under the following isounit is

$$\hat{I}_t = 1, \;\; \hat{I}_r = I + F^{NSA}/F^{SA}, \;\; \hat{I}_p = \hat{T}_r, \tag{3.3.16}$$

and the Hamiltonian with the usual form on isospace over isofields

$$\hat{H} = \sum_{\alpha=1,...,n} \frac{\hat{p}_{k\alpha} \hat{\times}_p \hat{p}^k_\alpha}{\hat{2}\hat{\times}\hat{m}_\alpha} + \hat{V}(\hat{r}), \tag{3.3.17}$$

where the reader should note the real-valued, symmetric and positive-definite character of all isounits.

As one can see, the above analytic equations are *noncanonical* when formulated via conventional mathematics, while they are *isocanonical*, namely, they reconstruct canonicity when formulated via isomathematics, namely, on isospaces over isofields.

Consequently, the projection of the above analytic equations on conventional spaces over conventional field has no invariance, while, when written on isospaces over isofields (that is, without the explicit expression of the isoderivative), the same equations are manifestly invariant under the *Galileo-Santilli isosymmetry* [50], thus resolving the inconsistencies of Theorem 1.5.1.

The above analytic equations characterize the following *Poisson-Santilli isobrackets*

$$[\hat{A},\hat{B}] = \frac{\hat{\partial}\hat{A}}{\hat{\partial}\hat{r}^k_\alpha} \times \frac{\hat{\partial}\hat{B}}{\hat{\partial}\hat{p}_{k\alpha}} - \frac{\hat{\partial}\hat{B}}{\hat{\partial}\hat{r}^k_\alpha} \times \frac{\hat{\partial}\hat{A}}{\hat{\partial}\hat{p}_{k\alpha}}$$

$$= \frac{\partial \hat{A}}{\partial \hat{r}^i_\alpha} \times \hat{I}^k_{ir} \times \hat{I}^j_{pk} \times \frac{\hat{\partial}\hat{B}}{\partial \hat{p}_{j\alpha}} - \frac{\partial \hat{B}}{\partial r^i_\alpha} \times \hat{I}^k_{ir} \times \hat{I}^j_{pk} \times \frac{\partial \hat{A}}{\partial \hat{p}_{j\alpha}}$$

$$= \frac{\partial \hat{A}}{\partial \hat{r}^k_\alpha} \times \frac{\partial \hat{B}}{\partial \hat{p}_{k\alpha}} - \frac{\partial \hat{B}}{\partial \hat{r}^k_\alpha} \times \frac{\partial \hat{A}}{\partial \hat{p}_{k\alpha}}, \tag{3.3.18}$$

where the last identity occurs because $\hat{I}_r = 1/\hat{I}_p$. It is evident that the above brackets constitute a classical realization of the Lie-Santilli

isoproduct. In particular, this is the isoproduct used for the construction of the Galileo-Santilli isoalgebra [50].

Isoaction principle (3.3.14) also characterizes the following *Hamilton-Jacobi-Santilli isoequations* [14]

$$\frac{\hat{\partial}\hat{\mathcal{A}}}{\hat{\partial}\hat{t}} + \hat{H} = 0, \tag{3.3.19a}$$

$$\frac{\hat{\partial}\hat{\mathcal{A}}}{\hat{\partial}\hat{r}^k_\alpha} - \hat{p}_{k\alpha} = 0. \tag{3.3.19b}$$

$$\frac{\hat{\partial}\hat{\mathcal{A}}}{\hat{\partial}\hat{p}_{k\alpha}} = 0. \tag{3.3.19c}$$

As we shall see, a most important property of the iso-Hamilton mechanics is that, when formulated on isospaces over isofields, *the isoaction is independent from the linear momenta*, as proved by Eq. (3.2.19c). this feature has fundamental implications for quantization discussed in the next section.

As it was the case for Eq. (3.3.9), iso-Hamiltonian mechanics has been conceived to coincide at the abstract level with the conventional formulation. Nevertheless, the following main differences occur:

1) Hamiltonian mechanics can only represent point particles while its isotopic covering can represent the actual, extended, nonspherical and deformable shape of particles via the simply identification of isounits (3.3.16);

2) Hamiltonian mechanics can only represent a rather restricted class of Newtonian systems, those with potential forces, while its isotopic covering is directly universal for all possible (sufficiently smooth) SA and NSA Newtonian systems;

3) All non-self-adjoint forces are represented by the isounits or, equivalently, by the isodifferential calculus, thus permitting their *invariant* description, since iso-Hamiltonian mechanics reconstructs canonicity on isospaces over isofields.

As outlined above, iso-Hamiltonian mechanics can only described closed non-Hamiltonian systems of *particles.* The construction of its isodual for *antiparticles* is an instructive exercise for interested readers.

The *Hamilton-Santilli isodual isoequations* for the analytic treatment of extended antiparticles can be written [14]

$$\frac{\hat{d}^d\hat{r}^d}{\hat{d}^d\hat{t}^d} = \frac{\hat{\partial}^d\hat{H}^d}{\hat{\partial}^d\hat{p}^d} = \frac{\hat{p}^d}{\hat{m}^m}, \tag{3.3.20a}$$

$$\frac{\hat{d}^d\hat{p}^d}{\hat{d}^d\hat{t}^d} = -\frac{\hat{\partial}^d\hat{H}^d}{\hat{\partial}^d\hat{r}^d} = \hat{F}^{dSA} + \hat{F}^{dNSA}, \tag{3.3.20b}$$

and they are *directly universal* for the representation of isodual Newton-Santilli isoequations (3.3.13).

The *isodual Hamilton-Jacobi-Santilli isoequations* can be written [14]

$$\frac{\hat{\partial}^d \hat{\mathcal{A}}^d}{\hat{\partial}^d \hat{t}^d} + \hat{H}^d = 0, \tag{3.3.21a}$$

$$\frac{\hat{\partial}^d \hat{\mathcal{A}}^d}{\hat{\partial}^d \hat{r}_\alpha^{dk}} - \hat{p}_{k\alpha}^d = 0. \tag{3.3.21b}$$

$$\frac{\hat{\partial}^d \hat{\mathcal{A}}^d}{\hat{\partial}^d \hat{p}_{k\alpha}^d} = 0. \tag{3.3.21c}$$

rather than being a mere mathematical virtuosity, the above isodual equations are crucial for the resolution of the scientific imbalance caused by classical antimatter studied in Chapter 1. In fact, the above equations permit the creation of a new quantization channel specifically intended for antiparticles in a way independent from that of particles and such that the operator image of the classical treatment is indeed a charge conjugate state.

3.4 LIE-ISOTOPIC BRANCH OF HADRONIC MECHANICS AND ITS ISODUAL

3.4.1 Technical Difficulties in Quantizing Nonpotential Forces

The biggest technical difficulty emerging in the quantization of Newtonian systems with arbitrary nonconservative forces via conventional mathematics is that the action functional generally depends on the linear moments, in addition to the canonical dependence on time and coordinates, $\mathcal{A} = \mathcal{A}(t, r, p)$, as illustrated by Birkhoffian mechanics [51].

In turn, as shown below, the quantization of such type of generalized action leads to "wavefunctions" that are also dependent on time, coordinates and momenta, $\psi = \psi(t, r, p)$, resulting in an operator mechanics that is in dramatic disagreement with quantum mechanics, e.g., on Heisenberg's uncertainty principle, Pauli's exclusion principle, etc.

In fact, all these familiar quantum laws are crucially dependent on the conventional dependence of the wavefunctions either on time and coordinates only, $\psi = \psi(t, r)$, or on time and momenta only, $\psi = \psi(t, p)$, while "wavefunctions" with the joint dependence $\psi = \psi(t, r, p)$ are strictly outside the axioms of quantum mechanics.

In any case, the above studies are afflicted by the catastrophic inconsistencies of Theorem 1.5.1 at the purely classical level and, as such, they

cannot constitute solid classical foundations for new operator theories. In fact, the lack of invariance at the classical level propagates to the operator level resulting in a host of operator inconsistencies.

After studying for decades nonconservative systems at the classical and operator levels (see monographs [48,49,50,51] and papers quoted therein), the author had no other alternative than that of constructing a *new mathematics* specifically conceived for the problem at hand and only thereafter construct a *new classical mechanics* under the condition of resolving the inconsistencies of Theorem 1.5.1 and, in addition, have a generalized action that is independent from the linear momenta, as a condition to avoid wavefunctions not treatable with available methods and catastrophic conflicts with established quantum laws.

Immediately after the original proposal in 1978 the new isomathematics [23], it became possible to propose the construction of hadronic mechanics and to identify its fundamental dynamical equations, the iso-Heisenberg equations reviewed below [38]. Numerous additional developments then followed. However, with the passing of the years hadronic mechanics was still missing the invariance necessary to avoid catastrophic inconsistencies.

By the early 1990s all possible mathematics underlying quantum mechanics had been isotopically lifted, including numbers, fields, spaces, algebras, geometries, etc. Nevertheless, hadronic mechanics was still missing the invariance.

Detailed study of the problem reveals that it originated in the isotopies of the linear momentum operator. In turn, the lack of such a basic realization prevented the finalization of experimental verifications and applications.

It should be indicated that the search for the invariance of the operator isotopic formulations requested the search of a various isotopies of classical Hamiltonian mechanics, as illustrated by Birkhoffian mechanics [51] and the various mechanics of the first edition of Refs. [54,55].

It was finally in 1996, some 18 years following the original proposal of 1978 [23], that the lack of invariance was identified where one would expect it the least, in the absence at of the isotopies of the ordinary differential calculus.

The achievement of the isodifferential calculus in memoir [14] of 1996 finally permitted the achievement of full maturity in the formulation of classical and operator isomechanics, with the achievement of the final form of the classical isomechanics of the preceding section and of the operator isomechanics reviewed below. The final invariant formulation of experimental verifications and applications then followed [58–61].

By looking in retrospective, we can today safely state that the novel *isomathematics* as outlined in Section 3.2 and the new *Hamilton-Santilli isomechanics* of the preceding section do indeed achieve the indicated objectives, a consistent and invariant treatment of all possible nonconservative forces as a covering of conventional treatments.

In fact, the basic isoaction on isospaces over isofields does not depend on linear momenta, as expressed by the crucial Eq. (3.3.19c), while isomathematics permits the invariant formulation of nonconservative interactions, thus resolving the catastrophic inconsistencies of Theorem 1.5.1, and 1.5.2.

Consequently, the novel isomathematics, including most importantly the isodifferential calculus, and the resulting Hamilton-Santilli isomechanics do indeed constitute solid classical foundations for a consistent and invariant operator treatment of all possible contact, nonlinear, nonlocal and nonpotential interactions among extended particles at short mutual distances.

3.4.2 Naive Isoquantization and its Isodual

Following the laborious construction of the above classical foundations, their operator map is straightforward. Recall that the conventional naive (or, more rigorously, the symplectic) quantization

$$\mathcal{A} \to -i \times \hbar \times \mathrm{Ln}\,\psi, \tag{3.4.1}$$

is solely applicable for *first-order action functionals* $\mathcal{A}(t, r)$ and, as such, it is not applicable to the isoaction $\hat{\mathcal{A}}(\hat{t}, \hat{r}) = \mathcal{A}(t \times \hat{I}_t, r \times \hat{I}_r) = \hat{\mathcal{A}}'(t, x, p, \ldots)$ due to its higher order character when formulated on conventional spaces.

Nevertheless, it is easy to show the validity of the following *naive isoquantization*, first formulated by Animalu and Santilli [228] with the conventional differential calculus and by Santilli [14] via the isodifferential calculus

$$\hat{\mathcal{A}}(\hat{t}, \hat{r}) \to -\hat{i}\hat{\times}\hat{\mathrm{Ln}}\,\hat{\psi}(\hat{t}, \hat{r}) = -i \times \hat{I}_r \times \mathrm{Ln}\,\hat{\psi}(\hat{t}, \hat{r}), \tag{3.4.2}$$

that, when applied to Eq. (3.3.19), permits the map here expressed for the case when $\hat{I}_r$ is a constant (see Ref. [55] for the general case)

$$\frac{\hat{\partial}\hat{\mathcal{A}}}{\hat{\partial}\hat{t}} + \hat{H} = 0 \to -\hat{i}\hat{\times}\frac{\hat{\partial}\hat{\psi}}{\partial\hat{t}} + \hat{H}\hat{\times}\hat{\psi}$$

$$= i \times \hat{I}_t \times \frac{\partial\hat{\psi}}{\partial\hat{t}} + \hat{H} \times \hat{T}_r \times \hat{\psi} = 0, \tag{3.4.3a}$$

$$\frac{\hat{\partial}\hat{\mathcal{A}}}{\hat{\partial}\hat{r}^k_\alpha} - \hat{p}_{k\alpha} = 0 \rightarrow -\hat{i}\hat{\times}\frac{\hat{\partial}\hat{\psi}}{\partial\hat{r}^k_\alpha} - \hat{p}_{k\alpha}\hat{\times}\hat{\psi}$$

$$= -i \times \hat{I}^{i\beta}_{k\alpha} \times \frac{\partial\hat{\psi}}{\partial r^{i\beta}} - \hat{p}_{k\alpha} \times \hat{T}_r \times \hat{\psi} = 0. \qquad (3.4.3b)$$

The above equations illustrate the 18 years of research indicated in the preceding section to achieve invariance. In fact, their second line, evidently the form of 1996 prior to the isodifferential calculus, is manifestly noninvariant, while the first line does indeed verify all conditions for invariance. The above occurrence also illustrates the crucial role of the isodifferential calculus for the studies of this monograph.

The most important implication of the above isoquantization can be expressed via the following

LEMMA 3.4.1 [14,55]: *Hadronic mechanics replaces Planck's "constant" with the "integro-differential" isounit* $\hat{I}_r(t, r, v, \psi, \partial\psi, \ldots)$.

Stated in different terms, in the transition from exterior systems (such as the atomic structure) to interior systems (such as the structure of hadrons, nuclei and stars), Planck's constant is turned into a locally varying operator.

This should be expected because the quantization of the energy was conceived and has been established for an electron jumping between different quantized orbits while moving in vacuum. By comparison, the very idea of quantized orbits has no scientific sense for the same electron when in the hyperdense core of a star. The absence of quantized orbits then causes Planck's constant to lose its traditional meaning in favor of a covering quantity.

The reader should be aware that, since the isounit is an operator, conclusions can be solely drawn from its *expectation value*, as studied in the next section.

The *isodual isoquantization* is straightforward, and can be written

$$\hat{\mathcal{A}}^d(\hat{t}^d, \hat{r}^d) \rightarrow -\hat{i}^d\hat{\times}^d\hat{I}^d_{r^d}\hat{\times}^d\hat{L}n^d\hat{\psi}^d(\hat{t}^d, \hat{r}^d), \qquad (3.4.4)$$

that, when applied to Eq. (3.3.21), permits the map [14]

$$\frac{\hat{\partial}^d\hat{\mathcal{A}}^d}{\hat{\partial}^d\hat{t}^d} + \hat{H}^d = 0 \rightarrow -i^d \times^d \hat{I}^d_t \times^d \frac{\partial^d\hat{\psi}^d}{\partial^d t^d} + \hat{H}^{\times}\hat{T}^d_r \times^d \hat{\psi}^d = 0, \quad (3.4.5a)$$

$$\frac{\hat{\partial}^d\hat{\mathcal{A}}^d}{\hat{\partial}^d\hat{r}^{dk}_\alpha} - \hat{p}^d_{k\alpha} = 0 \rightarrow -i^d \times^d \hat{I}^d_r \times^d \frac{\partial^d\hat{\psi}^d}{\partial^d r^d} - \hat{p}^d \times^d \hat{T}^d_r \times^d \hat{\psi}^d = 0. \quad (3.4.5b)$$

The above isodual quantization establishes that the independent channel of quantization for point-like antiparticles of Chapter 2 also applies to the case of extended antiparticles.

3.4.3 Iso-Hilbert Spaces and their Isoduals

Hadronic mechanics formulated over the *iso-Hilbert space*, first introduced by Myung and Santilli [25], $\hat{\mathcal{H}}$ with *isostates* $|\hat{\psi}(\hat{t},\hat{r})>$, *isoinner product*

$$<\hat{\psi}|\hat{\times}|\hat{\psi}>\times\hat{I}=<\hat{\psi}|\times\hat{T}\times|\hat{\psi}>\times\hat{I}\in\hat{C}, \tag{3.4.6}$$

and *isonormalization*

$$<\hat{\psi}|\hat{\times}|\hat{\psi}>=1. \tag{3.4.7}$$

The *isoexpectation values* of an observable $\hat{A}$ are given by

$$\frac{<\hat{\psi}|\hat{\times}\hat{A}\hat{\times}|\hat{\psi}>}{<\hat{\psi}|\hat{\times}|\hat{\psi}>}\in\hat{C}. \tag{3.4.8}$$

Consequently, *the isoexpectation values of the isounit recover Planck's constant*,

$$\frac{<\hat{\psi}|\hat{\times}\hat{I}\hat{\times}|\hat{\psi}>}{<\hat{\psi}|\hat{\times}|\hat{\psi}>}=\frac{<\hat{\psi}|\hat{\times}|\hat{\psi}>}{<\hat{\psi}|\hat{\times}|\hat{\psi}>}=1=\hbar \tag{3.4.9}$$

and the isoeigenvalue of the isounit

$$\hat{I}\,\hat{\times}\,|\hat{\psi}>=1\times|\hat{\psi}>\,. \tag{3.4.10}$$

Since the quantities that can be measured are the isoexpectation values, rather than the iso-operators per se, we see from the above property that, despite the generalization of Planck's constant into an operator (*Lemma 3.4.1*), the sole observable quantity remains $\hbar$.

An important property is given by the fact that, when an operator A is Hermitean on $\mathcal{H}$ over C, mechanics, it is also *iso-Hermitean* of $\hat{\mathcal{H}}$ over $\hat{C}$,

$$<\hat{\psi}|\hat{\times}(\hat{A}\hat{\times}|\hat{\psi}>)\equiv(<\hat{\psi}|\hat{\times}\hat{A}^{\hat{\dagger}})\hat{\times}|\hat{\psi}>\,. \tag{3.4.11}$$

Consequently, all quantities that are observable for quantum mechanics remain observable for hadronic mechanics, and, from here on, we shall drop the "hat" on the sign of Hermiticity,

$$\hat{A}^{\hat{\dagger}}\equiv\hat{A}^{\dagger}. \tag{3.4.12}$$

The theory of *isodual iso-Hilbert spaces* $\mathcal{H}^d$ can be constructed via a simple isoduality of the preceding theory. For instance, the *isodual states* on $\mathcal{H}^d$ are given by

$$|\hat{\psi}>^d=-|\hat{\psi}>^{\dagger}=-<\hat{\psi}|, \tag{3.4.13}$$

with *isodual inner product*

$$< \hat{\psi}|\hat{\times}|\hat{\psi} >^d \times \hat{I}^d =< \hat{\psi}|\hat{\times}|\hat{\psi} > \times \hat{I}^d \in \hat{C}^d, \tag{3.4.14}$$

isodual isoexpectation values of an *isodual observable* $\hat{A}^d$

$$\frac{< \hat{\psi}|\hat{\times}\hat{A}\hat{\times}|\hat{\psi} >^d}{< \hat{\psi}|\hat{\times}|\hat{\psi} >^d} \in \hat{C}^d, \tag{3.4.15}$$

and *isodual isorenormalization* and *isonormalization*

$$< \hat{\psi}|\hat{\times}|\hat{\psi} >^d = -1, \tag{3.4.16}$$

where the isodual quotient is used. Additional aspects of the isodual isotheory can be readily derived by the interested reader.

3.4.4 Isolinearity, Isolocality and Isounitarity

An important property of the theory of iso-Hilbert spaces is that, by conception and construction, the said theory is nonlinear, nonlocal and nonunitary when formulated on conventional spaces over conventional fields. Nevertheless, the theory reconstructs linearity, locality and unitarity when formulated on isospaces over isofields.

In fact, it is easy to see that the theory of iso-Hilbert spaces is *isolinear* because it verifies all needed axioms on $\mathcal{H}$ over $\hat{C}$, such as

$$\hat{A}\hat{\times}(\hat{n}\hat{\times}|\hat{\psi} > +\hat{m}\hat{\times}|\hat{\psi} >) = \hat{n}\hat{\times}\hat{A}\hat{\times}|\hat{\psi} > +\hat{m}\hat{\times}\hat{A}\hat{\times}|\hat{\psi} >), \tag{3.4.17a}$$

$$(\hat{n}\hat{\times}\hat{A} + \hat{m}\hat{\times}\hat{A})\hat{\times}|\hat{\psi} >= \hat{n}\hat{\times}\hat{A}\hat{\times}|\hat{\psi} > +\hat{m}\hat{\times}\hat{A}\hat{\times}|\hat{\psi} >), \tag{3.4.17b}$$

$$(\hat{A} + \hat{B})\hat{\times}|\hat{\psi} >= \hat{A} + (\hat{B})\hat{\times}|\hat{\psi} >). \tag{3.4.17c}$$

Similarly, *isolocality* is given by the property that the theory is everywhere local except at the isounit since, by construction, all nonlocal-integral interactions are embedded in the isounit.

Finally, the theory is *isounitary* in the sense that the basic transformations are given by

$$\hat{H}\hat{\times}U^\dagger = \hat{U}^\dagger\hat{\times}\hat{U} = \hat{I}. \tag{3.4.18}$$

The isotopies identify the following new *isoinvariance of Hilbert's inner product* (here expressed for the case when the isotopic element does not depend on the integration variable)

$$< \psi| \times |\psi > \times I \equiv < \psi| \times \hat{T} \times |\psi > \times \hat{I}, \tag{3.4.19}$$

which invariance explains why the isotopies of Hilbert spaces remained un-discovered since Hilbert's time, although its discovery required the prior generalization of numbers to those with arbitrary units.

Despite its simplicity, the above isoinvariance has the important implication of permitting a structural generalization quantum mechanics. For more details, interested readers may consult Refs. [31,55].

3.4.5 Iso-Schrödinger and Iso-Heisenberg Equations and their Isoduals

The new mechanics is characterized by the *iso-Schrödinger equations* first derived in Refs. [25,179] with ordinary mathematics and first formulated via the isodifferential calculus in Ref. [14]

$$\hat{i}\hat{\times}\frac{\hat{\partial}}{\hat{\partial}\hat{t}}|\hat{\psi}>=\hat{H}\hat{\times}|\hat{\psi}>$$

$$=\hat{H}(\hat{t},\hat{r},\hat{p})\times\hat{T}(\hat{r},\hat{p},\hat{\psi},\hat{\partial}\hat{\psi},....)\times|\hat{\psi}>=\hat{E}\hat{\times}|\hat{\psi}>=E\times|\hat{\psi}>, \quad (3.4.20)$$

with *isomomentum equations* first derived in Ref. [14]

$$\hat{p}_k\hat{\times}|\hat{\psi}>=-\hat{i}\hat{\times}\hat{\partial}_k|\hat{\psi}>=-i\times\hat{I}^i_k\times\partial_i|\hat{\psi}>, \quad (3.4.21)$$

and property

$$\hat{I}\hat{\times}|\hat{\psi}>=|\hat{\psi}>, \quad (3.4.22)$$

confirming that $\hat{I}$ is indeed the isounit of the theory.

The *iso-Heisenberg equations*, first derived in Ref. [38] via conventional mathematics and first formulated via the isodifferential calculus in Ref. [14], can be written

$$\hat{i}\hat{\times}\frac{\hat{d}\hat{A}}{\hat{d}\hat{t}}=[\hat{A}\hat{,}\hat{H}]=\hat{A}\hat{\times}\hat{H}\hat{-}\hat{H}\hat{\times}\hat{A}$$

$$=\hat{A}\times\hat{T}(\hat{t},\hat{r},\hat{p},\hat{\psi},\hat{\partial}\hat{\psi},...)\times\hat{H}-\hat{H}\times\hat{T}(\hat{t},\hat{r},\hat{p},\hat{\psi},\hat{\partial}\hat{\psi},...)\times\hat{A}, \quad (3.4.23)$$

and *isocanonical commutation rules*

$$[\hat{r}^i\hat{,}\hat{p}_j]=\hat{i}\hat{\times}\hat{\delta}^i_j=i\times\delta^i_j\times\hat{I},[\hat{r}^i,\hat{r}^j]=[\hat{p}_i,\hat{p}_j]=0. \quad (3.4.24)$$

A few comments are now in order. Firstly, we should indicate that *the isotopic branch of hadronic mechanics preserves indeed conventional quantum laws.* For instance, *Heisenberg's uncertainty principle* is given by

$$\Delta r\Delta p\geq\frac{1}{2}\times[\hat{r}\hat{,}\hat{]}=\frac{1}{2}, \quad (3.4.25)$$

and coincides with the conventional expression. The same holds for the other uncertainty laws.

Similarly, the preservation of *Pauli's exclusion principle* can be derived from the isomorphism of the isotopic and conventional $SU(2)$-spin symmetry, as well as from the preservation of the conventional spin eigenvalues.

Interested reader can then prove the preservation of other quantum laws.

It is also easy to prove that *all Hermitean quantities that are conserved for quantum mechanics remain conserved under isotopies*, because the symmetries of Schrödinger's and iso-Schrödinger's equations are isomorphic and their generators coincide.

The above results implies the existence of a *new notion of bound state of particles* as the operator image of closed non-Hamiltonian systems of Section 3.1, namely, a bound state admitting internal Hamiltonian as well as nonlinear, nonlocal and nonpotential interactions while preserving conventional total conservation laws.

Note that these are precisely the characteristics needed for quantitative studies of electron valence bonds, as well as, more generally, all bound states of particles at short mutual distances.

In view of the lack of general commutativity of $\hat{H}$ and $\hat{T}$, the *iso-Schrödinger and iso-Heisenberg's equations have a nonunitary time evolution* when formulated on conventional Hilbert spaces over conventional fields,

$$|\hat{\psi}(t)>=(e^{i\times H\times\hat{T}\times t})\times|\hat{\psi}(0)>=U(t)\times|\hat{\psi}(0)>, \tag{3.4.26a}$$

$$U\times U^{\dagger}\neq I. \tag{3.4.26b}$$

However, all nonunitary transforms admit an identical reformulation as isounitary transform on iso-Hilbert spaces,

$$U=\hat{U}\times\hat{T}^{1/2}, \tag{3.4.27a}$$

$$\hat{U}\hat{\times}\hat{U}^{\dagger}=\hat{U}^{\dagger}\hat{\times}\hat{U}=\hat{I}. \tag{3.4.27b}$$

Therefore, as indicated in Section 3.4.2, hadronic mechanics reconstructs unitary on iso-Hilbert spaces over isofields.

This reconstruction is a fundamental property since it resolved the catastrophic inconsistencies of Theorem 1.5.2. The above property is also necessary to exit from the class of equivalence of quantum mechanics, thus illustrating the nontriviality of the lifting.

Another important property is that *nonlinear Schrödinger's equations* on $\mathcal{H}$ over C,

$$H(x,p,\psi,\ldots)\times|\psi>=E\times|\psi>, \tag{3.4.28}$$

cannot represent composite systems because of the violation of the superposition principle, while hadronic mechanics resolves this limitation.

In fact, all nonlinear Schrödinger's equations can be *identically* rewritten in the isotopic form with the embedding of all nonlinear terms in the isotopic element,

$$H(x,p,\psi)\times|\psi>=H'(x,p)\times\hat{T}(x,p,\psi,\ldots)\times|\psi>=E\times|\psi>, \quad (3.4.29a)$$

$$H'=H\times\hat{I}, \quad (3.4.29b)$$

under which we recover the isolinearity of Section 3.4.2.

It is easy to see that the superposition principle does hold for the isotopic reformulation (3.4.29), thus permitting the study of nonlinear composite systems, and this is another illustration of the necessity of isomathematics for the interactions considered in this monograph.

It is easy to prove that the isoexpectation values coincide with the isoeigenvalues, as in the conventional case.

The above properties establish that *the isotopic branch of hadronic mechanics coincides with quantum mechanics at the abstract, realization-free level.* This feature is important to assure the axiomatic consistency of hadronic mechanics, as well as to clarify the fact that *hadronic mechanics is not a new theory, but merely a novel realization of the abstract axioms of quantum mechanics.*

The *isodual isotopic branch of hadronic mechanics* for the treatment of extended antiparticles is given by the image of the preceding theory under the isodual map. We have in this way the *isodual Schrödinger equation*

$$<\hat{\psi}|\frac{\hat{\partial}^d}{\hat{\partial}^d\hat{t}}\hat{\times}^d\hat{i}^d=<\hat{\psi}|\hat{\times}^d\hat{H}^d, \quad (3.4.30)$$

the *isodual iso-Heisenberg equation*

$$\frac{\hat{d}^d\hat{A}}{\hat{d}^d\hat{t}^d}\hat{\times}^d\hat{i}^d=[\hat{H}^{d\hat{}},\hat{A}^d]. \quad (3.4.31)$$

and the *isodual isolinear momentum*

$$<\hat{\psi}|\hat{\times}^d\hat{p}^d=-<\hat{\psi}|\frac{\hat{\partial}^d}{\hat{\partial}^d\hat{r}^d}\hat{\times}^d\hat{i}^d \quad (3.4.32)$$

For additional intriguing features of hadronic mechanics, interested readers can inspect memoir [31] and monograph [55].

3.4.6 Simple Construction of Isotheories

A simple method has been identified in Refs. [14,31] for the construction of the entire isomathematics and nonrelativistic hadronic mechanics. It consists in:

(i) representing all conventional interactions with a Hamiltonian H and all non-Hamiltonian interactions and effects with the isounit $\hat{I}$;

(ii) identifying the latter interactions with a nonunitary transform

$$U \times U^\dagger = \hat{I} \neq I \tag{3.4.33}$$

and

(iii) subjecting the totality of conventional mathematical, physical quantities and all their operations to the above nonunitary transform, resulting in the expressions

$$I \to \hat{I} = U \times I \times U^\dagger = 1/\hat{T}, \tag{3.4.34a}$$

$$a \to \hat{a} = U \times a \times U^\dagger = a \times \hat{I}, \tag{3.4.34b}$$

$$a \times b \to U \times (a \times b) \times U^\dagger$$

$$= (U \times a \times U^\dagger) \times (U \times U^\dagger)^{-1} \times (U \times b \times U^\dagger) = \hat{a}\hat{\times}\hat{b}, \tag{3.4.34c}$$

$$e^A \to U \times e^A \times U^\dagger = \hat{I} \times e^{\hat{T}\times\hat{A}} = (e^{\hat{A}\times\hat{T}}) \times \hat{I}, \tag{3.4.34d}$$

$$[X_i, X_j] \to U \times [X_i X_j] \times U^\dagger$$

$$= [\hat{X}_i \hat{,} \hat{X}_j] = U \times (C^k_{oj} \times X_k) \times U^\dagger = \hat{C}^k_{ij}\hat{\times}\hat{X}_k$$

$$= C^k_{ij} \times \hat{X}_k, \tag{3.4.34e}$$

$$< \psi| \times |\psi > \to U \times < \psi| \times |\psi > \times U^\dagger$$

$$=< \psi| \times U^\dagger \times (U \times U^\dagger)^{-1} \times U \times |\psi > \times (U \times U^\dagger)$$

$$=< \hat{\psi}|\hat{\times}|\hat{\psi} > \times \hat{I}, \tag{3.4.34f}$$

$$H \times |\psi > \to U \times (H \times |\psi >) = (U \times H \times U^\dagger) \times (U \times U^\dagger)^{-1} \times (U \times |\psi >) =$$

$$= \hat{H}\hat{\times}|\hat{\psi} >, etc. \tag{3.4.34g}$$

The above rule permits the explicit construction of any desired application of hadronic mechanics.

3.4.7 Invariance of Isotopic Theories

It is easy to see that the application of an additional nonunitary transform to expressions (3.4.34) implies the *lack of invariance* with consequential activation of the catastrophic inconsistencies of Theorem 1.5.2. As an example, given a new nonunitary transform

$$W \times W^\dagger \neq I, \tag{3.4.35}$$

we have the noninvariance of the isounit

$$\hat{I} = U \times I \times U^{\dagger} \to \hat{I}' = W \times \hat{I} \times W^{\dagger} \neq \hat{I}, \tag{3.4.36}$$

and the same holds for all remaining isotopies.

However, the above noninvariance is based on the use of conventional nonunitary transforms for isotheories, thus being itself inconsistent, since isotheories must be treated with isomathematics.

The use of isounitary, rather than unitary, transforms then readily achieves the needed invariance, as first reached in Refs. [14,31].

In fact, by reformulating any given nonunitary transform in the identical isounitary form,

$$W \times W^{\dagger} = \hat{I}, W = \hat{W} \times \hat{T}^{1/2}, \tag{3.4.37a}$$

$$W \times W^{\dagger} = \hat{W} \hat{\times} \hat{W}^{\dagger} = \hat{W}^{\dagger} \hat{\times} \hat{W} = \hat{I}, \tag{3.4.37b}$$

we have the following *isoinvariance laws*

$$\hat{I} \to \hat{I}' = \hat{W} \hat{\times} \hat{I} \hat{\times} \hat{W}^{\dagger} = \hat{I}, \tag{3.4.38a}$$

$$\hat{A} \hat{\times} \hat{B} \to \hat{W} \hat{\times} (\hat{A} \hat{\times} \hat{B}) \hat{\times} \hat{W}^{\dagger} =$$

$$= (\hat{W} \times \hat{T} \times A \times \hat{T} \times \hat{W}^{\dagger}) \times (\hat{T} \times \hat{W}^{\dagger})^{-1} \times \hat{T} \times (\hat{W} \times$$

$$\times \hat{T})^{-1} \times (\hat{W} \times \hat{T} \times \hat{B} \times \hat{T} \times \hat{W}^{\dagger}) =$$

$$= \hat{A}' \times (\hat{W}^{\dagger} \times \hat{T} \times \hat{W})^{-1} \times \hat{B}' = \hat{A}' \times \hat{T} \times \hat{B}' = \hat{A}' \hat{\times} \hat{B}', \text{ etc.} \tag{3.4.38b}$$

Note that the invariance is ensured by the *numerically invariant values of the isounit $\hat{I}$ and of the isotopic element $\hat{T}$ under nonunitary-isounitary transforms*, namely,

$$\hat{I} \to \hat{I}' = \hat{I}, \hat{T} \to \hat{T}' = \hat{T}, \hat{\times} \to \hat{\times}' = \hat{\times}. \tag{3.4.39}$$

The resolution of the catastrophic inconsistencies of Theorem 1.5.2 is then consequential.

The achievement of invariant for classical noncanonical formulations is equivalent to the preceding nonunitary one and its explicit form is left to the interested reader for brevity.

3.5 ISORELATIVITY AND ITS ISODUAL

3.5.1 Introduction

Special relativity is generally presented in contemporary academia as providing a descriptions of all infinitely possible conditions existing in the universe.

The scientific reality is somewhat different than this academic posture. In fact, in Section 1.1 we have shown that special relativity *cannot* provide a consistent classical description of point antiparticles moving in vacuum because leading to inconsistent quantum images consisting of particles (rather than charge conjugated antiparticles) with the wrong sign of the charge, besides having no distinction for bodies with null total charge.

In Section 1.3 we have established that special relativity *cannot* be exactly valid for extended particles and antiparticles moving within physical media because of an axiomatic inability to represent extended and nonspherical shapes, an incompatibility with the deformation theory, the lack of existence of a consistent reduction of classical nonconservative forces to potential abstractions at the particle level, and other reasons.

In Section 1.3 we have also established that special relativity *cannot* describe irreversible processes for both matter and antimatter due to unavoidable inconsistencies caused by its notoriously reversible axioms and other reasons.

In Section 1.3 we have also establioshed that the complexity of biological systems is immensely beyond the rather limited descriptive capacity of special relativity.[4]

In summary, special relativity can be expected to be exactly valid for the conditions of its original conception and construction, namely, for point-particles and electromagnetic waves propagating in vacuum.[5] All academic claims of exact validity of special relativity for physical conditions beyond those of its original conception are nonscientific.

Particularly damaging the orderly development of scientific knowledge is the widespread statement of the "universal constancy of the speed of light" because contrary to well known experimental evidence that *the speed of light is a local variable depending on the medium in which it propagates*, with well known expression

$$c = c_\circ/n, \tag{3.5.1}$$

where the familiar *index of refraction* is a function of a variety of characteristics of the medium, including time t, coordinates r, density μ, temperature τ, frequency ω, etc., $n = n(t, r, \mu, \tau, \omega, \ldots)$.

In particular, the speed of light is generally smaller than that in vacuum when propagating within media of low density, such as atmospheres or liquids,

$$c \ll c_\circ, \quad n \gg 1, \tag{3.5.2}$$

while the speed of light is generally bigger than that in vacuum when propagating within special guides, or within media of very high density,

such as the interior of stars and quasars,

$$c \gg c_\circ, \quad n \ll 1 \tag{3.5.3}$$

In fact, academic claims of recovering the speed of light in water via photons scattering among the water molecules are afflicted by the following inconsistencies:

1) The said claims have no scientific value for the case of electromagnetic waves with large wavelength, evidently due to the impossibility of a meaningful reduction to photons of wavelength of the order of one meter or so;

2) The said claims cannot recover numerically the reduction of the speed of light in vacuum by about one third due to the notorious low value of the cross section for Compton scattering even when the reduction to photon has scientific sense; and

3) The said claims are vacuous for experimentally established speeds bigger than that of light in vacuum.

It is appropriate here to note that academia continues to ignore the fact that German experimentalists have transmitted an entire Beethoven symphony at speeds bigger than that of light in vacuum via electromagnetic waves propagating within special guides (see Section 1.3).

Assuming that via some unknown manipulation special relativity is shown to represent the propagation of light within physical media, such a representation would activate the catastrophic inconsistencies of Theorem 1.5.1.

This is due to the fact that *the transition from the speed of light in vacuum to that within physical media requires a noncanonical or nonunitary transform.*

This point can be best illustrated by using the metric originally proposed by Minkowski, which can be written

$$\eta = Diag.(1, 1, 1, -c_\circ^2). \tag{3.5.4}$$

Then, the transition from $c_\circ$ to $c = c_\circ/n$ in the metric can only be achieved via a noncanonical or nonunitary transform

$$\eta = Diag.(1, 1, 1, -c_\circ^2) \to \hat{\eta}$$

$$= Diag.(1, 1, 1, -c_\circ/n^2) = U \times \eta \times U^\dagger, \tag{3.5.5a}$$

$$U \times U^\dagger = Diag.(1,\ 1,\ 1,\ 1/n^2) \neq I. \tag{3.5.5b}$$

We should finally mention that the continued insistence in the universal validity of special relativity for whatever condition exist in the universe despite the publications of the above limitations in refereed

journals creates serious problems of scientific ethics and accountability because the said insistence causes the impossibility of predicting much needed new clean energies and fuels [58].

In fact, the latter crucially depend on the admission of the locally varying character of the speed of light and the consequential need of initiating the laborious process of constructing a suitable covering relativity.

In view of the above societal implications, the studies reviewed in this section have been conducted without any consideration of academic interests, and have been inspired by the desire of conducting unobstructed research.

An invariant resolution of the limitations of special relativity for closed and reversible systems of extended and deformable particules under Hamiltonian and non-Hamiltonian interactions has been provided by the lifting of special relativity into a new formulation today known as *Santilli isorelativity*, or *isorelativity* for short, where: the prefix "iso" stands to indicate that relativity principles apply on isospacetime over isofields; and the characterization of "special" or "general" is inapplicable because, as shown below, *isorelativity achieves a geometric unification of special and general relativities.*

Closed and irreversible systems of extended and deformable particles with Hamiltonian and non-Hamiltonian interactions are also characterized by isorelativity under the condition that the isounit is Hermitean and nowhere singular, but explicitly dependent on time, $\hat{I} = \hat{I}(t, \ldots) = \hat{I}^\dagger(t, \ldots) \neq \hat{I}(-t, \ldots)$.

The description of the broader open irreversible systems requires *Santilli's genorelativity* with a Lie-admissible structure studied in the next section.

Isorelativity was first proposed by R. M. Santilli in Ref. [26] of 1983 via the first invariant formulation of *iso-Minkowskian spaces* and related *iso-Lorentz symmetry.*

The studies were then continued in: Ref. [11] of 1985 with the first isotopies of the rotational symmetry; Ref. [28] of 1993 with the first isotopies of the SU(2)-spin symmetry; Ref. [29] of 1993) with the first isotopies of the Poincaré symmetry; Ref. [33] of 1998 with the first isotopies of the SU(2)-isospin symmetries, Bell's inequalities and local realism; and Ref. [30] of 1996 with the first isotopies of the spinorial covering of the Poincaré symmetry.

The studies were then completed with memoir [15] of 1998 presenting a comprehensive formulation of the iso-Minkowskian geometry and its capability to unify the Minkowskian and Riemannian geometries, in-

cluding its formulation via the mathematics of the Riemannian geometry (such iso-Christoffel's symbols, isocovariant derivatives, etc.).

Numerous independent studies on isorelativity are available in the literature, one can inspect in this respect Refs. [63–68] and papers quoted therein; Aringazin's proof [192] of the direct universality of the Lorentz-Poincaré-Santilli isosymmetry for all infinitely possible spacetimes with signature (+, +, +, −); Mignani's exact representation [118] of the large difference in cosmological redshifts between quasars and galaxies when physically connected; the exact representation of the anomalous behavior of the meanlives of unstable particles with speed by Cardone et al. [110,111]; the exact representation of the experimental data on the Bose-Einstein correlation by Santilli [112] and Cardone and Mignani [113]; the invariant and exact validity of the iso-Minkowskian geometry within the hyperdense medium in the interior of hadrons by Arestov et al. [120]; the first known exact representation of molecular features by Santilli and Shillady [125,126]; and numerous other contributions.

Evidently we cannot review isorelativity in the necessary details to avoid a prohibitive length. Nevertheless, to achieve minimal self-sufficiency of this presentation, it is important to outline at least its main structural lines (see monograph [55] for detailed studies).

3.5.2 Iso-Minkowskian Spaces and their Isoduals

The central notion of isorelativity is the lifting of the basic unit of the Minkowski space and of the Poincaré symmetry, $I = Diag.(1, 1, 1, 1)$, into a 4×4-dimensional, nowhere singular and positive-definite matrix $\hat{I} = \hat{I}_{4\times 4}$ with an unrestricted functional dependence on local spacetime coordinates x, speeds v, accelerations a, frequencies ω, wavefunctions ψ, their derivative $\partial\psi$, and/or any other needed variable,

$$I = Diag.(1, 1, 1) \to \hat{I}(x, v, a, \omega, \psi, \partial\psi, \ldots)$$
$$= 1/\hat{T}(x, v, \omega, \psi, \partial\psi, \ldots) > 0. \tag{3.5.6}$$

Isorelativity can then be constructed via the method of Section 3.4.6, namely, by assuming that the basic noncanonical or nonunitary transform coincides with the above isounit

$$U \times U^\dagger = \hat{I} = Diag.(g_{11}, g_{22}, g_{33}, g_{44}),$$
$$g_{\mu\mu} = g_{\mu\mu}(x, v, \omega, \psi, \partial\psi, \ldots) > 0, \mu = 1, 2, 3, 4, \tag{3.5.7}$$

and then subjecting the *totality* of quantities and their operation of special relativity to the above transform.

This construction is, however, selected here only for simplicity in pragmatic applications, since the rigorous approach is the construction of

isorelativity from its abstract axioms, a task we have to leave to interested readers for brevity (see the original derivations [55]).

This is due to the fact that the former approach evidently preserves the original eigenvalue spectra and does not allow the identification of anomalous eigenvalues emerging from the second approach, such as those of the SU(2) and SU(3) isosymmetries [33].

Let $M(x,\eta,R)$ be the Minkowski space with local coordinates $x = (x^\mu)$, metric $\eta = Diag.(1,1,1,-1)$ and invariant

$$x^2 = (x^\mu \times \eta_{\mu\nu} \times x^\nu) \times I \in R. \tag{3.5.8}$$

The fundamental space of isorelativity is the *Minkowski-Santilli isospace* [26,15] and related topology [14,226,227], $\hat{M}(\hat{x},\hat{\eta},\hat{R})$ characterized by the liftings

$$I = Diag.(1,1,1,1) \to U \times I \times U^\dagger = \hat{I} = 1/\hat{T}, \tag{3.5.9a}$$

$$\eta = Diag.(1,1,1,-1) \times I \to (U^{\dagger-1} \times \eta \times U^{-1}) \times \hat{I} = \hat{\eta}$$
$$= \hat{T} \times \eta = Diag.(g_{11}, g_{22}, g_{33}, -g_{44}) \times \hat{I}, \tag{3.5.9b}$$

with consequential isotopy of the basic invariant

$$x^2 = (x^\mu \times \eta_{\mu\nu} \times x^\nu) \times I \in R$$

$$\to U \times x^2 \times U^\dagger = \hat{x}^{\hat{2}} = (\hat{x}^\mu \hat{\times} \hat{m}_{\mu\nu} \times x^\nu) \times I \in R, \tag{3.5.10}$$

whose projection in conventional spacetime can be written

$$\hat{x}^{\hat{2}} = [x^\mu \times \hat{\eta}_{\mu\nu}(x, v, a, \omega, \psi, \partial\psi, \ldots) \times x^\nu] \times \hat{I}, \tag{3.5.11}$$

The nontriviality of the above lifting is illustrated by the following:

THEOREM 3.5.1: The Minkowski-Santilli isospaces are directly universal, in the sense of admitting as particular cases all possible spaces with the same signature $(+,+,+,-)$, such as the Minkowskian, Riemannian, Finslerian and other spaces (universality), directly in terms of the isometric within fixed local variables (direct universality).

Therefore, the correct formulation of the *the Minkowski-Santilli isogeometry* requires the isotopy of all tools of the Riemannian geometry, such as the iso-Christoffel symbols, isocovariant derivative, etc. (see for brevity Ref. [15]).

Despite that, one should keep in mind that, in view of the positive-definiteness of the isounit [34,79], *the Minkowski-Santilli isogeometry*

coincides at the abstract level with the conventional Minkowski geometry, thus having a null isocurvature (because of the basic mechanism of deforming the metric η by the amount $\hat{T}(x,\ldots)$ while deforming the basic unit of the inverse amount $\hat{I} = 1/\hat{T}$).

The geometric unification of the Minkowskian and Riemannian geometries achieved by the Minkowski-Santilli isogeometry constitute the evident geometric foundation for the unification of special and general relativities studied below.

It should be also noted that, following the publication in 1983 of Ref. [26], numerous papers on "deformed Minkowski spaces" have appeared in the physical and mathematical literature (generally without a quotation of their origination in Ref. [26]).

These "deformations" are ignored in these studies because they are formulated via conventional mathematics and, consequently, they all suffer of the catastrophic inconsistencies of Theorem 1.4.1.

By comparison, isospaces are formulated via isomathematics and, therefore, they resolve the inconsistencies of Theorem 1.5.1, as shown in Section 3.5.9. This illustrates again the necessity of lifting the basic unit and related field jointly with all remaining conventional mathematical methods.

3.5.3 Poincaré-Santilli Isosymmetry and its Isodual

Let $P(3.1)$ be the conventional Poincaré symmetry with the well known ten generators $J_{\mu\nu}, P_\mu$ and related commutation rules hereon assumed to be known.

The second basic tool of isorelativity is the *Poincaré-Santilli isosymmetry* $\hat{P}(3.1)$ studied in detail in monograph [55] that can be constructed via the isotheory of Section 3.2, resulting in the isocommutation rules [26,29]

$$[J_{\mu\nu}\hat{,}J_{\alpha\beta}] = i\times(\hat{\eta}_{\nu\alpha}\times J_{\beta\mu} - \hat{\eta}_{\nu\alpha}\times J_{\beta\nu} - \hat{\eta}_{nu\beta}\times J_{\alpha\mu} + \hat{\eta}_{\mu\beta}\times J_{\alpha\nu}), \quad (3.5.12a)$$

$$[J_{\mu\nu}\hat{,}P_\alpha] = i\times(\hat{\eta}_{\mu\alpha}\times P_\nu - \hat{\eta}_{\nu\alpha}\times P_\mu), \quad (3.5.12b)$$

$$[P_\mu\hat{,}P\nu] = 0, \quad (3.5.12c)$$

where we have followed the general rule of the Lie-Santilli isotheory according to which isotopies leave observables unchanged (since Hermiticity coincides with iso-Hermiticity) and merely change the *operations* among them.

The *iso-Casimir invariants* of $\hat{P}(3.1)$ are given by

$$P^{\hat{2}} = P_\mu\hat{\times}P^\mu = P^\mu\times\hat{\eta}_{\mu\nu}\times P^\nu = P_k\times g_{kk}\times P_k - p_4\times g_{44}\times P_4, \quad (3.5.13a)$$

$$W^{\hat{2}} = W_\mu \hat{\times} W^\mu, W_\mu = \hat{\epsilon}_{\mu\alpha\beta\rho} \hat{\times} J^{\alpha\beta} \hat{\times} P^\rho. \tag{3.5.13b}$$

and they are at the foundation of classical and operator *isorelativistic kinematics.*

Since $\hat{I} > 0$, it is easy to prove that *the Poincaré-Santilli isosymmetry is isomorphic to the conventional symmetry.* It then follows that *the isotopies increase dramatically the arena of applicability of the Poincaré symmetry, from the sole Minkowskian spacetime to all infinitely possible spacetimes.*

Next, the reader should be aware that *the Poincaré-Santilli isosymmetry characterizes "isoparticles" (and not particles) via its irreducible isorepresentations.*

A mere inspection of the isounit shows that the Poincaré-Santilli isosymmetry characterizes actual nonspherical and deformable shapes as well as internal densities and the most general possible nonlinear, nonlocal and nonpotential interactions.

Since any interaction implies a renormalization of physical characteristics, it is evident that *the transition from particles to isoparticles, that is, from motion in vacuum to motion within physical media, causes an alteration (called isorenormalization), in general, of all intrinsic characteristics, such as rest energy, magnetic moment, charge, etc.*

As we shall see later on, the said isorenormalization has permitted the first exact numerical representation of nuclear magnetic moments, molecular binding energies and other data whose exact representation resulted to be impossible for nonrelativistic and relativistic quantum mechanics despite all possible corrections conducted over 75 years of attempts.

The explicit form of the *Poincaré-Santilli isotransforms* leaving invariant line element (3.5.11) can be easily constructed via the Lie-Santilli isotheory and are given:

(1) The **isorotations** [11]

$$\hat{O}(3) : \hat{\mathbf{x}}' = \hat{\Re}(\hat{\theta}) \hat{\times} \hat{\mathbf{x}}, \quad \hat{\theta} = \theta \times \hat{I}_\theta \in \hat{R}_\theta, \tag{3.5.14}$$

that, for isotransforms in the (1, 2)-isoplane, are given by

$$x^{1'} = x^1 \times \cos[\theta \times (g_{11} \times g_{22})^{1/2}] - x^2 \times g_{22} \times g_{11}^{-1} \times \sin[\theta \times (g_{11} \times g_{22})^{1/2}], \tag{3.5.15a}$$

$$x^{2'} = x^1 \times g_{11} \times g_{22}^{-1} \times \sin[\theta \times (g_{11} \times g_{22})^{1/2}] + x^2 \times \cos[\theta \times (g_{11} \times g_{22})^{1/2}]. \tag{3.5.15b}$$

For the general expression in three dimensions interested reader can inspect Ref. [55] for brevity.

Note that, since $\hat{O}(3)$ is isomorphic to $O(3)$, Ref. [11] proved, contrary to a popular belief throughout the 20-th century, that

LEMMA 3.5.1: The rotational symmetry remains exact for all possible signature-preserving $(+,+,+)$ *deformations of the sphere.*

The rotational symmetry was believed to be "broken" for ellipsoidal and other deformations of the sphere merely due to insufficient mathematics for the case considered because, when the appropriate mathematics is used, the rotational symmetry returns to be exact, and the same holds for virtually all "broken" symmetries.

The above reconstruction of the exact rotational symmetry can be geometrically visualized by the fact that *all possible signature-preserving deformations of the sphere are perfect spheres in isospace called isosphere.*

This is due to the fact that ellipsoidal deformations of the semiaxes of the perfect sphere are compensated on isospaces over isofields by the *inverse* deformation of the related unit

$$Radius \quad 1_k \to 1/n_k^2, \tag{3.5.16a}$$

$$Unit \quad 1_k \to n_k^2. \tag{3.5.16b}$$

We recover in this way the perfect sphere on isospaces over isofields

$$\hat{r}^{\hat{2}} = \hat{r}_1^2 + \hat{r}_2^2 + \hat{r}_3^2 \tag{3.5.17}$$

with exact $\hat{O}(3)$ symmetry, while its projection on the conventional Euclidean space is the ellipsoid

$$r_1^2/n_1^2 + r_2^2/n_2^2 + r_3^2/n_3^2, \tag{3.5.18}$$

with broken $O(3)$ symmetry.

(2) The **Lorentz-Santilli isotransforms** [26,29]

$$\hat{O}(3.1) : \hat{x}' = \hat{\Lambda}(\hat{v}, \ldots)\hat{\times}\hat{x}, \ \hat{v} = v \times \hat{I}_v \in \hat{R}_v, \tag{3.5.19}$$

that, for isotransforms in the (3,4)-isoplane, can be written

$$x^{1'} = x^1, \tag{3.5.20a}$$

$$x^{2'} = x^2, \tag{3.5.20b}$$

$$x^{3'} = x^3 \times \cosh[v \times (g_{33} \times g_{44})^{1/2}]$$

$$-x^4 \times g_{44} \times (g_{33} \times g_{44})^{-1/2} \times \sinh[v \times (g_{33} \times g_{44})^{1/2}]$$

$$= \hat{\gamma} \times (x^3 - \beta \times x^4), \tag{3.5.20c}$$

$$x^{4'} = -x^3 \times g_{33} \times (g_{33} \times g_{44})^{-1/2} \times \sinh[v(g_{33} \times g_{44})^{1/2}]$$
$$+x^4 \times \cosh[v \times (g_{33} \times g_{44})^{1/2}]$$
$$= \hat{\gamma} \times (x^4 - \hat{\beta} \times x^3), \tag{3.5.20b}$$

where

$$\hat{\beta} = \frac{v_k \times g_{kk} \times v_k}{c_o \times g_{44} \times c_o}, \hat{\gamma} = \frac{1}{(1 - \hat{\beta}^2)^{1/2}}. \tag{3.5.21}$$

For the general expression interested readers can inspect Ref. [55].

Contrary to another popular belief throughout the 20-th century, Ref. [26] proved that

LEMMA 3.5.2: The Lorentz symmetry remains exact for all possible signature preserving $(+,+,+,-)$ *deformations of the Minkowski space.*

Again, the symmetry remains exact under the use of the appropriate mathematics.

The above reconstruction of the exact Lorentz symmetry can be geometrically visualized by noting that the light cone

$$x_2^2 + x_3^2 - c_o^2 \times t^2 = 0, \tag{3.5.22}$$

can only be formulated in vacuum, while within physical media we have the *light isocone*

$$\frac{r_2^2}{n_2^2} + \frac{r_3^2}{n_3^2} - \frac{c_o^2 \times t^2}{n^2(\omega, \ldots)} = 0. \tag{3.5.23}$$

that, when formulated on isospaces over isofield, it is also a perfect cone, as it is the case for the isosphere. This property then explains how the Lorentz symmetry is reconstructed as exact according to Lemma 3.5.2 or, equivalently, that $\hat{O}(3.1)$ is isomorphic to $O(3.1)$.

(3) The **isotranslations** [29]

$$\hat{\mathcal{T}}(4) : \hat{x}' = \hat{\mathcal{T}}(\hat{a}, \ldots)\hat{]} \times x = \hat{x} + \hat{A}(\hat{a}, x, \ldots), \hat{a} = a \times \hat{I}_a \in \hat{R}_a, \tag{3.5.24}$$

that can be written

$$x^{\mu'} = x^\mu + A^\mu(a, \ldots), \tag{3.5.25a}$$

$$A^\mu = a^\mu(g_{\mu\mu} + a^\alpha \times [g_{,i\mu}\hat{,}P_\alpha]/1! + \ldots), \tag{3.5.25b}$$

and there is no summation on the μ indices.

We reach in this way the following important result:

LEMMA 3.5.3 [55]: Isorelativity permits an axiomatically correct extention of relativity laws to noninertial frames.

In fact, noninertial frames are transformed into a forms that are inertial on isospaces over isofields, called *isoinertial*, as established by the fact that isotranslations (3.5.25) are manifestly nonlinear and, therefore, noninertial on conventional spaces while they are isolinar on isospaces, according to a process similar to the reconstruction of locality, lineary and canonicity.

The isoinertial character of the frames can also be seem from the isocommutativity of the linear momenta, Eq. (3.5.12c), while such a commutativity is generally lost in the projection of Eq. (3.5.12c) on ordinary spaces over ordinary fields, thus confirming the lifting of conventional noninertial frames into an isoinertial form.

This property illustrates again the origin of the name "isorelativity" to indicate that conventional relativity axioms are solely applicable in isospacetime.

(4) The novel **isotopic transformations** [29]

$$\hat{\mathcal{I}}(1) : \hat{x}' = \hat{w}^{-\hat{1}} \hat{\times} \hat{x} = w^{-1} \times \hat{x}, \ \hat{I}' = w^{-2} \times \hat{I} \tag{3.5.26}$$

where w is a constant,

$$\hat{I} \to \hat{I}' = \hat{w}^{-2} \hat{\times} \hat{I} = w^{-2} \times \hat{I} = 1/\hat{T}', \tag{3.5.27a}$$

$$\hat{x}^{\hat{2}} = (x^\mu \times \hat{\eta}_{\mu\nu} \times x^\nu) \times \hat{I} \equiv \hat{x}'^{\hat{2}}$$

$$= [x^\mu \times (w^2 \times \hat{\eta}_{\mu\nu}) \times x^\nu] \times (w^2 \times \hat{I}), \tag{3.5.27b}$$

Contrary to another popular belief throughout the 20-th century, we therefore have the following

THEOREM 3.5.2: The Poincaré-Santilli isosymmetry, hereon denoted with

$$\hat{P}(3.1) = \hat{O}(3.1) \hat{\times} \hat{\mathcal{T}}(4) \hat{\times} \hat{\mathcal{I}}(1), \tag{3.5.28}$$

and, therefore, the conventional Poincaré symmetry, are eleven dimensional.

The increase of dimensionality of the fundamental spacetime symmetry as, predictably, far reaching implications, including a basically novel and axiomatically consistent grand unification of electroweak and gravitational interactions studied in Chapter 5.

The simplest possible realization of the above formalism for isorelativistic kinematics can be outlined as follows. The first application of isorelativity is that of providing *an invariant description of locally varying speeds of light propagating within physical media.* For this purpose a realization of isorelativity requires the knowledge of the *density* of the medium in which motion occurs.

The simplest possible realization of the fourth component of the isometric is then given by the function

$$g_{44} = n_4^2(x, \omega, \ldots), \tag{3.5.29}$$

normalized to the value $n_4 = 1$ for the vacuum (note that the density of the medium in which motion occurs *cannot* be described by special relativity). The above representation then follows with invariance under $\hat{P}(3.1)$.

In this case the quantities $n_k,\ k = 1, 2, 3$, represent the *inhomogeneity and anisotropy of the medium considered.* For instance, if the medium is homogeneous and isotropic (such as water), all metric elements coincide, in which case

$$\hat{I} = Diag.(g_{11}, g_{22}, g_{33}, g_{44}) = n_4^2 \times Diag.(1, 1, 1, 1), \tag{3.5.30a}$$

$$\hat{x}^{\hat{2}} = \frac{x^2}{n_4^2} \times n_4^2 \times I \equiv x^2, \tag{3.5.30b}$$

thus confirming that *isotopies are hidden in the Minkowskian axioms*, and this may be a reason why they were not been discovered until recently.

Next, isorelativity has been constricted for the invariant description of systems of extended, nonspherical and deformable particles under Hamiltonian and non-Hamiltonian interactions.

Practical applications then require the knowledge of the actual shape of the particles considered, here assumed for simplicity as being spheroidal ellipsoids with semiaxes n_1^2, n_2^2, n_3^2.

Note that the minimum number of constituents of a closed non-Hamiltonian system is two. In this case we have shapes represented with $n_{\alpha k},\ \alpha = 1, 2, , \ldots, n$.

Specific applications finally require the identification of the nonlocal interactions, e.g., whether occurring on an extended *surface* or *volume.* As an illustration, two spinning particles denoted 1 and 2 in condition of deep mutual penetration and overlapping of their wavepackets (as it is the case for valence bonds), can be described by the following Hamiltonian and total isounit

$$H = \frac{p_1 \times p_1}{2 \times m_1} + \frac{p_2 \times p_2}{2 \times m_2} + V(r), \tag{3.5.31a}$$

$$\hat{I}_{Tot} = Diag.(n_{11}^2, n_{12}^2, n_{13}^2, n_{14}^2) \times Diag.(n_{21}^2, n_{22}^2, n_{23}^2, n_{24}^2)$$
$$\times e^{N\times(\hat{\psi}_1/\psi_1+\hat{\psi}_2/\psi_2)\times\int \hat{\psi}_{1\uparrow}(r)^\dagger\times\hat{\psi}_{2\downarrow}(r)\times dr^3}, \qquad (3.5.31b)$$

where N is a positive constant.

The above realization of the isounit has permitted the first known *invariant and numerically exact* representation of the binding energy and other features of the hydrogen, water and other molecules [59] for which a historical 2% has been missing for about one century. The above isounit has also been instrumental for a number of additional data on two-body systems whose representation had been impossible with quantum mechanics, such as the origin of the spin 1 of the ground state of the deuteron that, according to quantum axioms, should be zero.

Note in isounit (3.5.31) the nonlinearity in the wave functions, the nonlocal-integral character and the impossibility of representing any of the above features with a Hamiltonian.

From the above examples interested readers can then represent any other closed non-Hamiltonian systems.

3.5.4 Isorelativity and its Isodual

The third important part of the new isorelativity is given by the following isotopies of conventional relativistic axioms that, for the case of motion along the third axis, can be written [29] as follows:

ISOAXIOM I. The projection in our spacetime of the maximal causal invariant speed is given by:

$$V_{Max} = c_\circ \times \frac{g_{44}^{1/2}}{g_{33}^{1/2}} = c_\circ \frac{n_3}{n_4} = c \times n_3. \qquad (3.5.32)$$

This isoaxiom resolves the inconsistencies of special relativity recalled earlier for particles and electromagnetic waves propagating within physical media such as water.

In fact, water is homogeneous and isotropic, thus requiring that

$$g_{11} = g_{22} = g_{33} = g_{44} = 1/n^2, \qquad (3.5.33)$$

where n is the index of refraction.

In this case the maximal causal speed for a massive particle is $c_\circ$ as experimentally established, e.g., for electrons, while the local speed of electromagnetic waves is $c = c_\circ/n$, as also experimentally established.

Note that such a resolution requires *the abandonment of the speed of light as the maximal causal speed for motion within physical media, and its replacement with the maximal causal speed of particles.*

It happens that in vacuum these two maximal causal speeds coincide. However, even in vacuum the correct maximal causal speed remains that of particles and *not* that of light, as generally believed.

At any rate, physical media are generally opaque to light but not to particles. Therefore, the assumption of the speed of light as the maximal causal speed within media in which light cannot propagate would be evidently vacuous.

It is an instructive exercise for interested readers to prove that

LEMMA 3.5.4: The maximal causal speed of particles on isominkowski space over an isofield remains $c_\circ$.

In fact, on isospaces over isofields $c_\circ^2$ is deformed by the index of refraction into the form $c_\circ^2/n_4^2$, but the corresponding unit cm^2/sec^2 is deformed by the inverse amount, $n_4^2 \times \text{cm}^2/\text{sec}^2$, thus preserving the numerical value $c_\circ^2$ due to the structure of the isoinvariant studied earlier.

The understanding of isorelativity requires the knowledge that, when formulated on the Minkowski-Santilli isospace over the isoreals, Isoaxiom I coincides with the conventional axiom that is, the maximal causal speed returns to be c. The same happens for all remaining isoaxioms.

ISOAXIOM II. The projection in our spacetime of the isorelativistic addition of speeds within physical media is given by:

$$v_{Tot} = \frac{v_1 + v_2}{1 + \frac{v_1 \times g_{33} \times v_2}{c_\circ \times g_{44} \times c_\circ}} = \frac{v_1 + v_2}{1 + \frac{v_1 \times n_4^2 \times v_2}{c_\circ \times n_3^2 \times c_\circ}} \tag{3.5.34}$$

We have again the correct result that *the sum of two maximal causal speeds in water*,

$$V_{max} = c_\circ \times (n_3/n_4), \tag{3.5.35}$$

yields the maximal causal speed in water, as the reader is encouraged to verify.

Note that such a result is impossible for special relativity. Note also that *the "relativistic" sum of two speeds of light in water*, $c = c_\circ/n$, *does not* yield the speed of light in water, thus confirming that the speed of light within physical media, assuming that they are transparent to light, is not the fundamental maximal causal speed.

ISOAXIOM III. The projection in our spacetime of the isorelativistic laws of dilation of time $t_\circ$ *and contraction of length* $\ell_\circ$ *and the variation of mass* $m_\circ$ *with speed are given respectively by:*

$$t = \hat{\gamma} \times t_\circ, \tag{3.5.36a}$$

$$\ell = \hat{\gamma}^{-1} \times \ell_\circ, \tag{3.5.36b}$$

$$m = \hat{\gamma} \times m_\circ. \tag{3.5.36c}$$

Note that in water these values coincide with the relativistic ones as it should be since particles such as the electrons have in water the maximal causal speed $c_\circ$.

Note again the necessity of avoiding the interpretation of the local speed of light as the maximal local causal speed. Note also that the mass diverges at the maximal local causal speed, but *not* at the local speed of light.

ISOAXIOM IV. The projection in our spacetime of the iso-Doppler law is given by the isolaw (here formulated for simplicity for 90° *angle of aberration):*

$$\omega = \hat{\gamma} \times \omega_\circ. \tag{3.5.37}$$

This isorelativistic axioms permits an *exact, numerical and invariant representation* of the large differences in cosmological redshifts between quasars and galaxies when physically connected.

In this case light simply exits the huge quasar chromospheres already redshifted due to the decrease of the speed of light, while the speed of the quasars can remain the *same* as that of the associated galaxy. Note again as this result is impossible for special relativity.

Isoaxiom IV also permits a numerical interpretation of the internal blue- and red-shift of quasars due to the dependence of the local speed of light on its frequency.

Finally, Isoaxiom IV predicts that *a component* of the predominance toward the red of sunlight at sunset is of iso-Doppler nature. This prediction is based on the different travel within atmosphere of light at sunset as compared to the zenith (evidently because of the travel within a comparatively denser atmosphere).

By contrast, the popular representation of the apparent redshift of sunlight at sunset is that via the scattering of light among the molecules composing our atmosphere. Had this interpretation be correct, the sky at the zenith should be red, while it is blue.

At any rate, the claim of representation of the apparent redshift via the scattering of light is political because of the impossibility of reaching the needed numerical value of the redshift, as serious scholars are suggested to verify.

ISOAXIOM V. The projection in our spacetime of the isorelativistic law of equivalence of mass and energy is given by:

$$E = m \times c_\circ^2 \times g_{44} = m_\times \frac{c_\circ^2}{n_4^2}. \tag{3.5.38}$$

Among various applications, *Isoaxiom V removes any need for the "missing mass" in the universe.* This is due to the fact that all isotopic fits of experimental data agree on values $g_{44} \gg 1$ within the hyperdense media in the interior of hadrons, nuclei and stars [55,120].

As a result, Isoaxiom V yields a value of the total energy of the universe dramatically bigger than that believed until now under the assumption of the universal validity of the speed of light in vacuum.

For other intriguing applications of Isoaxioms V, e.g., for the rest energy of hadronic constituents, we refer the interested reader to monographs [55,61].

The *isodual isorelativity* for the characterization of antimatter can be easily constructed via the isodual map of Chapter 2, and its explicit study is left to the interested reader for brevity.

3.5.5 Isorelativistic Hadronic Mechanics and its Isoduals

The isorelativistic extension of relativistic hadronic mechanics is readily permitted by the Poincaré-Santilli isosymmetry. In fact, iso-invariant (3.5.13a) characterizes the following *iso-Gordon equation* on $\hat{\mathcal{H}}$ over $\hat{C}$ [55]

$$\hat{p}_\mu \hat{\times} |\hat{\psi}> = -\hat{i}\hat{\times}\hat{\partial}_\mu |\hat{\psi}> = -i \times \hat{I}_\mu^\nu \times \partial_\nu |\hat{\psi}>, \tag{3.5.39a}$$

$$(\hat{p}_\mu \hat{\times} \hat{p}^\mu + \hat{m}_\circ^2 \hat{\times} \hat{c}^4) \hat{\times} |\hat{\psi}> = (\hat{\eta}^{\alpha\beta} \times \partial_\alpha \times \partial_\beta + m_\circ^2 \times c^4) \times |\hat{\psi}> = 0. \tag{3.5.39b}$$

The linearization of the above second-order equations into the *Dirac-Santilli isoequation* has been first studied in Ref. [29] and then by other authors (although generally without the use of isomathematics, thus losing the invariance).

By recalling the structure of Dirac's equation as the Kronecker product of a spin 1/2 massive particle and its antiparticle of Chapter 2, the Dirac-Santilli isoequation is formulated on the total isoselfadjoint isospace and related isosymmetry

$$\hat{M}^{Tot} = [\hat{M}^{orb}(\hat{x}, \hat{\eta}, \hat{R}) \times \hat{S}^{spin}(2)]$$

$$\times [\hat{M}^{d\,orb}(\hat{x}^d, \hat{\eta}^d, \hat{R}^d) \times \hat{S}^{d\,spin}(2)] = \hat{M}^{d\,Tot}, \tag{3.5.40a}$$

$$\hat{S}^{Tot} = \hat{P}(3.1) \times \hat{P}^d(3.1) = \hat{S}^{d\,Tot}, \tag{3.5.40b}$$

and can be written [29]

$$[\hat{\gamma}^\mu \hat{\times} (\hat{p}_\mu - \hat{e} \hat{\times} \hat{A}_\mu) + \hat{i} \hat{\times} \hat{m}] \hat{\times} |\phi(x) >= 0, \tag{3.5.41a}$$

$$\hat{\gamma}^\mu = g^{\mu\mu} \times \gamma^\mu \times \hat{I} \tag{3.5.41b}$$

where the γ's are the conventional Dirac matrices.

Note the appearance of the isometric elements directly in the structure of the gamma matrices and their presence also when the equation is projected in the conventional spacetime.

The following generators

$$J_{\mu\nu} = (S_k,\, L_{k4}), P_\mu, \tag{3.5.42a}$$

$$S_k = (\hat{\epsilon}_{kij} \hat{\times} \hat{\gamma}_i \hat{\times} \hat{\gamma}_j)/2, L_{k4} = \hat{\gamma}_k \hat{\times} \hat{\gamma}_4/2, P_\mu = \hat{p}_\mu, \tag{3.5.42b}$$

characterize the *isospinorial covering of the Poincaré-Santilli isosymmetry.*

The notion of "isoparticle" can be best illustrated with the above realization because it implies that,*in the transition from motion in vacuum (as particles have been solely detected and studied until now) to motion within physical media, particles generally experience the alteration, called "mutation", of all intrinsic characteristics*, as illustrated by the following isoeigenvalues,

$$\hat{S}^{\hat{2}} \hat{\times} |\hat{\psi} >= \frac{g_{11} \times g_{22} + g_{22} \times g_{33} + g_{33} \times g_{11}}{4} \times |\hat{\psi} >, \tag{3.5.43a}$$

$$\hat{S}_3 \hat{\times} |\hat{\psi} >= \frac{(g_{11} \times g_{22})^{1/2}}{2} \times |\hat{\psi} > . \tag{3.5.43b}$$

The mutation of spin then characterizes a necessary mutation of the intrinsic magnetic moment given by [29]

$$\tilde{\mu} = \Big(\frac{g_{33}}{g_{44}}\Big)^{1/2} \times \mu, \tag{3.5.44}$$

where μ is the conventional magnetic moment for the same particle when in vacuum. The mutation of the rest energy and of the remaining characteristics has been identified before via the isoaxioms.

Note that the invariance under isorotations allows the rescaling of the radius of an isosphere. Therefore, for the case of the perfect sphere we can always have $g_{11} = g_{22} = g_{33} = g_{44}$ in which case the magnetic moment is not mutated. These results recover conventional classical

knowledge according to which *the alteration of the shape of a charged and spinning body implies the necessary alteration of its magnetic moment.*

The construction of the isodual isorelativistic hadronic mechanics is left to the interested reader by keeping in mind that the iso-Dirac equation is isoselfdual as the conventional equation.

To properly understand the above results, one should keep in mind that *the mutation of the intrinsic characteristics of particles is solely referred to the constituents of a hadronic bound state under conditions of mutual penetration of their wave packets (such as one hadronic constituent) under the condition of recovering conventional characteristics for the hadronic bound state as a whole (the hadron considered)*, much along Newtonian subsidiary constrains on non-Hamiltonian forces, Eq. (3.1.6).

It should be also stressed that *the above indicated mutations violate the unitary condition when formulated on conventional Hilbert spaces, with consequential catastrophic inconsistencies, Theorem 1.5.2.*

As an illustration, the violation of causality and probability law has been established for all eigenvalues of the angular momentum M different than the quantum spectrum

$$M^2 \times |\psi >= \ell(\ell+1) \times |\psi >, \quad \ell = 0, 1, 2, 3, \ldots \tag{3.5.45}$$

As a matter of fact, these inconsistencies are the very reason why the mutations of internal characteristics of particles for bound states at short distances could not be admitted within the framework of quantum mechanics.

By comparison, hadronic mechanics has been constructed to recover unitarity on iso-Hilbert spaces over isofields, thus permitting an invariant description of internal mutations of the characteristics of the constituents of hadronic bound states, while recovering conventional features for states as a whole.

Far from being mere mathematical curiosities, the above mutations permit basically new structure models of hadrons, nuclei and stars, with consequential, new clean energies and fuels [58].

These new advances are prohibited by quantum mechanics precisely because of the preservation of the *intrinsic* characteristics of the constituents in the transition from bound states at large mutual distance, for which no mutation is possible, to the bound state of the same constituents in condition of mutual penetration, in which case mutations have to be admitted in order to avoid the replacement of a scientific process with unsubstantiated personal beliefs one way or the other.

The best illustration of the Dirac-Santilli isoequation is, therefore, that for which it was constructed [30], to describe the transition of

the electron from the hydrogen atom to the interior of the hyperdense medium inside the proton, namely, to achieve a quantitative and invariant representation of the synthesis of the neutron according to Rutherford as a "hydrogen atom compressed in the core of a star".

If special relativity, relativistic quantum mechanics and the conventional Dirac equation are assumed to be exactly valid also for the motion of the electron within the hyperdense medium in the interior of the proton, the neutron *cannot* be a bound state of a proton and an electron at short distances, thus mandating the assumption of undetectable constituents such as the hypothetical quarks that cannot be produced free, as well known.

One of the most important results of hadronic mechanics has been the proof at the nonrelativistic [214] and relativistic level [30] that a hadronic bound state of an *isoprotons* and an *isoelectron* represents exactly and invariantly *all* characteristics of the neutron, including its rest energy, spin, charge, parity, charge radius, anomalous magnetic moment and spontaneous decay. Virtually none of these characteristics can be represented via the hypothetical quarks.

The societal implications of the above alternative are such to require the surpassing of traditional academic interests on pre-established doctrines. In fact, *the neutron is the biggest reservoir of clean energy available to mankind.* This is due to the fact that the neutron is naturally unstable, in its spontaneous decay

$$n \to p^+ + e^- + \bar{\nu}. \tag{3.5.46}$$

that releases a large amount of energy for particle standards (about 0.80 MeV). Such an energy is environmentally acceptable because the electron can be easily trapped with a metal shield and the neutrino is harmless, while the lack of radioactive waste can be achieved via suitable nuclear selections (see web site [60] for details and technical literature quoted therein).

The societal implications originate from the fact that no new energy is conceivably possible under the assumption of the exact validity within a hadron of the Minkowski geometry, special relativity and relativistic quantum mechanics.

On the contrary, the assumption that isorelativity is valid within the hyperdense medium inside hadrons permits the hadronic constituents to be indeed actual physical particles that can be produced free in the spontaneous decay and, therefore, they can be identified among the massive particles released in the spontaneous decays with the lowest mode.

The societal aspect of potentially large dimension is that the political belief that the hypothetical quarks are physical particles in our

spacetime[6] prevents any prediction, let alone any development of new energies from hadrons.

By comparison, the assumption that the hadronic constituents are isoparticles that can be produce free in the spontaneous decays, does indeed permit the prediction of new clean energies by stimulating said decays [58,60].

3.5.6 Isogravitation and its Isodual

As indicated in Section 1.4, there is no doubt that the classical and operator formulations of gravitation on a curved space have been the most controversial theory of the 20-th century because of an ever increasing plethora of problematic aspects remained vastly ignored. By contrast, as also reviewed in Section 1.4, special relativity in vacuum has a majestic axiomatic consistence is its *invariance* under the Poincaré symmetry.

Recent studies have shown that the formulation of gravitation on a curved space or, equivalently, the formulation of gravitation based on "covariance", is necessarily noncanonical at the classical level and nonunitary at the operator level, thus suffering of all catastrophic inconsistencies of Theorems 1.4.1 and 1.4.2 [45,46].

These catastrophic inconsistencies can only be resolved via a new conception of gravity based on a *universal invariance*, rather than covariance.

Additional studies have identified profound axiomatic incompatibilities between gravitation on a curved space and electroweak interactions. These incompatibilities have resulted to be responsible for the lack of achievement of an axiomatically consistent grand unification since Einstein's times [32,35,37] (see Chapter 5).

No knowledge of isotopies can be claimed without a knowledge that isorelativity has been constructed to resolve at least some of the controversies on gravitation. The fundamental requirement is *the abandonment of the formulation of gravity via curvature on a Riemannian space and its formulation instead on an iso-Minkowskian space [15]* via the following steps characterizing *exterior isogravitation in vacuum*:

I) Factorization of any given Riemannian metric representing exterior gravitation $g^{ext}(x)$ into a nowhere singular and positive-definite 4×4-matrix $\hat{T}(x)$ times the Minkowski metric η,

$$g^{ext}(x) = \hat{T}^{ext}_{grav}(x) \times \eta; \tag{3.5.47}$$

II) Assumption of the inverse of $\hat{T}_{grav}$ as the fundamental unit of the theory,

$$\hat{I}^{ext}_{grav}(x) = 1/\hat{T}^{ext}_{grav}(x); \tag{3.5.48}$$

III) Submission of the totality of the Minkowski space and relative symmetries to the noncanonical/nonunitary transform

$$U(x) \times I^{\dagger}(x) = \hat{I}^{ext}_{grav}. \tag{3.5.49}$$

The above procedure yields the isominkowskian spaces and related geometry $\hat{M}(\hat{x}, \hat{\eta}, \hat{R})$ [15], resulting in a new conception of gravitation, exterior isogravity, with the following main features [15,32,35,37,55]:

i) Isogravity is characterized by a universal *symmetry* (and not a covariance), the Poincaré-Santilli isosymmetry $\hat{P}(3.1)$ for the gravity of matter with isounit $\hat{I}^{ext}_{grav}(x)$, the isodual isosymmetry $\hat{P}^d(3.1)$ for the gravity of antimatter, and the isoselfdual symmetry $\hat{P}(3.1) \times \hat{P}^d(3.1)$ for the gravity of matter-antimatter systems;

ii) All conventional field equations, such as the Einstein-Hilbert and other field equations, can be formulated via the Minkowski-Santilli isogeometry since the latter preserves all the tools of the conventional Riemannian geometry, such as the Christoffel's symbols, covariant derivative, etc. [15];

iii) Isogravitation is isocanonical at the classical level and isounitarity at the operator level, thus resolving the catastrophic inconsistencies of Theorems 1.5.1 and 1.5.2;

iv) An axiomatically consistent operator version of gravity always existed and merely creeped in un-noticed through the 20-th century because gravity is imbedded where nobody looked for, in the *unit* of relativistic quantum mechanics, and it is given by isorelativistic hadronic mechanics outlined in the next section.

v) The basic feature permitting the above advances is the abandonment of curvature for the characterization of gravity (namely, curvature characterized by metric $g^{ext}(x)$ referred to the unit I) and its replacement with *isoflatness*, namely, the verification of the axioms of flatness in isospacetime, while preserving conventional curvature in its projection on conventional spacetime (or, equivalently, curvature characterized by the $g(x) = \hat{T}^{ext}_{grav}(x) \times \eta$ referred to the isounit $\hat{I}_{grav}(x)$ in which case curvature becomes null due to the inter-relation $\hat{I}^{ext}_{grav}(x) = 1/\hat{T}^{ext}_{grav}(x)$) [15].

A resolution of numerous controversies on classical formulations of gravity then follow from the above main features, such as:

a) The resolution of the century old controversy on the lack of existence of consistent total conservation laws for gravitation on a Riemannian space, which controversy is resolved under the universal $\hat{P}(3.1)$ symmetry by mere visual verification that the generators of the conven-

tional and isotopic Poincaré symmetry are the same (since they represent conserved quantities in the absence and in the preserve of gravity);

b) The controversy on the fact that gravity on a Riemannian space admits a well defined "Euclidean", but not "Minkowskian" limit, which controversy is trivially resolved by isogravity via the limit

$$\hat{I}^{ext}_{grav}(x) \to I; \tag{3.5.50}$$

c) The resolution of the controversy that Einstein's gravitation predicts a value of the bending of light that is twice the experimental value, one for curvature and one for newtonian attraction, which controversy is evidently resolved by the elimination of curvature as the origin of the bending, as necessary in any case for the free fall of a body along a straight radial line in which no curvature of any type is conceivably possible or credible; and other controversies.

A resolution of the controversies on quantum gravity can be seen from the property that relativistic hadronic mechanics of the preceding section *is* a quantum formulation of gravity whenever $\hat{T} = \hat{T}_{grav}$.

Such a form of operator gravity is as axiomatically consistent as conventional relativistic quantum mechanics because the two formulations coincide, by construction, at the abstract, realization-free level.

As an illustration, whenever

$$\hat{T}^{ext}_{grav} = Diag.(g^{ext}_{11}, g^{ext}_{22}, g^{ext}_{33}, g^{ext}_{44}), \;\; g_{\mu\mu} > 0, \tag{3.5.51}$$

the Dirac-Santilli isoequation (3.5.41) provides a direct representation of the conventional electromagnetic interactions experienced by an electron, represented by the vector potential A_μ, plus gravitational interactions represented by the isogamma matrices.

Once curvature is abandoned in favor of the broader isoflatness, the axiomatic incompatibilities existing between gravity and electroweak interactions are resolved because:

i) isogravity possesses, at the abstract level, the *same* Poincaré invariance of electroweak interactions;

ii) isogravity can be formulated on the *same* flat isospace of electroweak theories; and

iii) isogravity admits positive and negative energies in the *same* way as it occurs for electroweak theories.

An axiomatically consistent *iso-grand-unification* then follows [32–35], as studied in Chapter 5.

Note that the above grand-unification requires the prior *geometric unification of the special and general relativities*, that is achieved precisely by isorelativity and its underlying iso-Minkowskian geometry.

In fact, special and general relativities are merely differentiated in isospecial relativity by the explicit realization of the unit. In particular, *black holes are now characterized by the zeros of the isounit* [55]

$$\hat{I}^{ext}_{grav}(x) = 0. \tag{3.5.52}$$

The above formulation recovers all conventional results on gravitational singularities, such as the singularities of the Schwarzschild's metric, since they are all described by the gravitational content $\hat{T}_{grav}(x)$ of $g(x) = \hat{T}_{grav}(x) \times \eta$, since η is flat.

This illustrates again that *all conventional results of gravitation, including experimental verifications, can be reformulated in invariant form via isorelativity.*

Moreover, the problematic aspects of general relativity mentioned earlier refer to the *exterior gravitational problem.* Perhaps greater problematic aspects exist in gravitation on a Riemannian space for *interior gravitational problems*, e.g., because of the lack of characterization of basic features, such as the density of the interior problem, the locally varying speed of light, etc.

These additional problematic aspects are also resolved by isorelativity due to the unrestricted character of the functional dependence of the isometric that, therefore, permits a direct geometrization of the density, local variation of the speed of light, etc.

The above lines constitute only the initial aspects of isogravitation since its most important branch is *interior isogravitation* as characterized by isounit and isotopic elements of the illustrative type

$$\hat{I}^{int}_{grav} = 1/\hat{T}^{int}_{grav} > 0, \tag{3.5.53a}$$

$$\hat{T}^{int}_{grav} = Diag.(g^{int}_{11}/n_1^2, g^{int}_{22}/n_2^2, g^{int}_{33}/n_3^2, g^{int}_{44}/n_4^2), \tag{3.5.53b}$$

permitting a it geometric representation directly via the isometric of the actual shape of the body considered, in the above case an ellipsoid with semiaxes n_1^2, n_2^2, n_3^2, as well as the (average) interior density n_4^2 with consequential representation of the (average value of the) interior speed of light $C = c/n_4$.

A most important point is that the invariance of interior isogravitation under the Poincaré-Santilli isosymmetry persists in its totality since the latter symmetry is completely independent from the explicit value of the isounit or isotopic element, and solely depends on their positive-definite character.

Needless to say, isounit (3.4.53) is merely illustrative because a more accurate interior isounit has a much more complex functional dependence with a locally varying density, light speed and other characteristics as they occur in reality.

Explicit forms of these more adequate models depends on the astrophysical body considered, e.g., whether gaseous, solid or a mixture of both, and their study is left to the interested reader.

It should also be noted that *gravitational singularities should be solely referred to interior models* evidently because exterior descriptions of type (3.5.52) are a mere approximation or a geometric abstraction.

In fact, *a gravitational singularities existing for exterior models are not necessarily confirmed by the corresponding interior formulations.* Consequently, the current views on black holes could well result to be pseudo-scientific beliefs because the only scientific statement that can be proffered at this time without raising issue of scientific ethics is that *the gravitational features of large and hyperdense aggregations of matter, whether characterizing a "black" or "brown" hole, are basically unresolved at this time.*

Needless to say, exterior isogravitation is a particular case of the interior formulation. Consequently, from now on, unless otherwise specified isogravitation will be referred to the interior form.

The cosmological implications are also intriguing and will be studied in Chapter 5.

It is hoped that readers with young minds of any age admit the incontrovertible character of the limitations of special and general relativities and participate in the laborious efforts toward new vistas because any lack of participation in new frontiers of science, whether for personal academic interest or other reason, is a gift of scientific priorities to others.

3.6 LIE-ADMISSIBLE BRANCH OF HADRONIC MECHANICS AND ITS ISODUAL

3.6.1 The Scientific Imbalance Caused by Irreversibility

As recalled in Chapter 1, physical, chemical or biological systems are called *irreversible* when their images under time reversal $t \to -t$ are prohibited by causality and/or other laws, as it is generally the case for nuclear transmutations, chemical reactions and organism growth.

Systems are called *reversible* when their time reversal images are as causal as the original ones, as it is the case for planetary and atomic structures when considered isolated from the rest of the universe (see reprint volume [81] on irreversibility and vast literature quoted therein).

Another large scientific imbalance of the 20-th century studied in this monograph is the treatment of irreversible systems via the mathematical

and physical formulations developed for reversible systems, since these formulations are themselves reversible, resulting in serious limitations in virtually all branches of science.

The problem is compounded by the fact that all used formulations are of Hamiltonian type, under the awareness that all known Hamiltonians are reversible (since all known potentials, such as the Coulomb potential $V(r)$, etc., are reversible).

This scientific imbalance was generally dismissed in the 20-th century with unsubstantiated statements, such as "irreversibility is a macroscopic occurrence that disappears when all bodies are reduced to their elementary constituents".

These academic beliefs have been disproved by Theorem 1.3.3 according to which a classical irreversible system cannot be consistently decomposed into a finite number of elementary constituents all in reversible conditions and, vice-versa, a finite collection of elementary constituents all in reversible conditions cannot yield an irreversible macroscopic ensemble.

The implications of the above theorem are quite profound because it establishes that, contrary to academic beliefs, *irreversibility originates at the most primitive levels of nature, that of elementary particles, and then propagates all the way to our macroscopic environment.*

3.6.2 The Forgotten Legacy of Lagrange and Hamilton

The scientific imbalance on irreversibility was created in the early part of the 20-th century when the true analytic equations proposed by Lagrange and Hamilton were "truncated" with the removal of the external terms (Section 1.3).

In fact, Lagrange and Hamilton proposed their celebrated equations for the clearly intent of representing all potential interactions with the quantities today called Lagrangians and Hamiltonians, and representing all nonpotential interactions with the external terms,

$$\frac{d}{dt}\frac{\partial L(t,r,v)}{\partial v_a^k} - \frac{\partial L(t,r,v)}{\partial r_a^k} = F_{ak}(t,r,v), \tag{3.6.1a}$$

$$\frac{dr_a^k}{dt} = \frac{\partial H(t,r,p)}{\partial p_{ak}}, \quad \frac{dp_{ak}}{dt} = -\frac{\partial H(t,r,p)}{\partial r_a^k} + F_{ak}(t,r,p), \tag{3.6.1b}$$

$$L = \Sigma_a \frac{1}{2} \times m_a \times v_a^2 - V(t,r,v), \tag{3.6.1c}$$

$$H = \Sigma_a \frac{\mathbf{p}_a^2}{2 \times m_a} + V(t,r,p), \tag{3.6.1d}$$

$$V = U(t,r)_{ak} \times v_a^k + U_o(t,r), \tag{3.6.1e}$$

$$F(t,r,v) = F(t,r,p/m). \tag{3.6.1f}$$

$$k = 1,2,3; \quad a = 1,2,\ldots,N.$$

The forgotten legacy of Lagrange and Hamilton is that irreversibility originates precisely in the truncated external terms because, again, all known potentials are reversible.

Being born and educated in Italy, during his graduate studies at the University of Torino, the author had the opportunity of studying in the late 1960s the original works by Lagrange that there written by Lagrange precisely in Torino most of them in Italian.

In this way, the author had the opportunity of verifying Lagrange's analytic vision on the impossibility of representing all possible physical events via the sole use of the quantity today called the Lagrangian. As the reader can verify, Hamilton had, independently, the same vision.

Consequently, the truncation of the basic analytic equations caused the impossibility of a credible representation of irreversibility at the purely classical level. The lack of a credible representation then propagated at all subsequent levels of study.

3.6.3 Early Representations of Irreversible Systems

As reviewed in Section 1.3.3, the brackets of the time evolution of an observable $A(r,p)$ in phase space according to the analytic equations with external terms,

$$\frac{dA}{dt} = (A,H,F) = \frac{\partial A}{\partial r_a^k} \times \frac{\partial H}{\partial p_{ka}} - \frac{\partial H}{\partial r_a^k} \times \frac{\partial A}{\partial p_{ka}} + \frac{\partial A}{\partial r_a^k} \times F_{ka}, \tag{3.6.2}$$

violate the right associative and scalar laws.

Therefore, the presence of external terms in the analytic equations causes not only the loss of *all* Lie algebras in the study of irreversibility, but actually the loss of all possible algebras as commonly understood in mathematics.

To resolve this problem, the author initiated a long scientific journey beginning with his graduate studies at the University of Torino, Italy, following the reading of Lagrange's papers.

The original argument, still valid today, is to select analytic equations characterizing brackets in the time evolution verifying the following conditions:

(1) The brackets of the time evolution must verify the right and left associative and scalar laws to characterize an algebra;

(2) Said brackets must not be invariant under time reversal as a necessary condition to represent irreversibility *ab initio;*

(3) Said algebra must be a covering of Lie algebras as a necessary condition to have a covering of the truncated analytic equations, namely, as a condition for the selected representation of irreversibility to admit reversibility as a particular case.

Condition (1) requires that said brackets must be bilinear, e.g., of the form (A, B) with properties

$$(n \times A, B) = n \times (A, B), \quad (A, m \times B) = m \times (A, B); \;\; n, m \in C, \;\; (3.6.3a)$$

$$(A \times B, C) = A \times (B, C), \quad (A, B \times C) = (A, B) \times C. \qquad (3.6.3b)$$

Condition (2) requires that brackets (A, B) should not be totally antisymmetric as the conventional Poisson brackets,

$$(A, B) \neq -(B, A), \qquad (3.6.4)$$

becfause time reversal is realized via the use of Hermitean conjugation.

Condition (3) then implies that brackets (A, B) characterize *Lie-admissible algebras* in the sense of Albert [7], namely, the brackets are such that the attached antisymmetric algebra is Lie,[7]

$$[A, B]^* = (A, B) - (B, A) = Lie. \qquad (3.6.5)$$

The latter condition implies that the new brackets are formed by the superposition of totally antisymmetric and totally symmetric brackets,

$$(A, B) = [A, B]^* + \{A, B\}^*. \qquad (3.6.6)$$

It should be noted that the operator realization of brackets (A, B) is also *Jordan-admissible* in the sense of Albert [10], namely, the attached symmetric brackets $\{A, B\}^*$ characterize a *Jordan algebra.* Consequently, *hadronic mechanics provides a realization of Jordan's dream, that of seeing his algebra applied to physics.*

However, the reader should be aware that, for certain technical reasons, the classical realizations of brackets (A, B) are Lie-admissible but not Jordan-admissible. Therefore, Jordan-admissibility appears to emerge exclusively for operator theories.[8]

After identifying the above lines, Santilli [232] proposed in 1967 the following *generalized analytic equations*

$$\frac{dr_a^k}{dt} = \alpha \times \frac{\partial H(t, r, p)}{\partial p_{ak}}, \quad \frac{dp_{ak}}{dt} = -\beta \times \frac{\partial H(t, r, p)}{\partial r_a^k}, \qquad (3.6.7)$$

(where α and β are real non-null parameters) that are manifestly irreversible. The brackets of the time evolution are then given by

$$\frac{dA}{dt} = (A, H)$$

$$= \alpha \times \frac{\partial A}{\partial r_a^k} \times \frac{\partial H}{\partial p_{ka}} - \beta \times \frac{\partial H}{\partial r_a^k} \times \frac{\partial A}{\partial p_{ka}}, \tag{3.6.8}$$

whose brackets are manifestly Lie-admissible, but *not* Jordan-admissible as the interested reader is encouraged to verify.

Analytic equations (3.6.7) characterize the time-rate of variation of the energy

$$\frac{dH}{dt} = (\alpha - \beta) \times \frac{\partial H}{\partial r_a^k} \times \frac{\partial H}{\partial p_{ka}}. \tag{3.6.9}$$

Also in 1967, Santilli [8,9] proposed an operator counterpart of the preceding classical setting consisting in the first known *Lie-admissible parametric generalization of Heisenberg's equation* in the following infinitesimal form

$$i \times \frac{dA}{dt} = (A, B) = p \times A \times H - q \times H \times A$$

$$= m \times (A \times B - B \times A) + n \times (A \times B + B \times A), \tag{3.6.10a}$$

$$m = p + q, \quad n = q - p, \tag{3.6.10b}$$

where $p, q, p \pm q$ are non-null parameters, with finite counterpart

$$A(t) = e^{i \times H \times q} \times A(0) \times e^{-i \times p \times H}. \tag{3.6.11}$$

Brackets (A, B) are manifestly Lie-admissible with attached antisymmetric part

$$[A, B]^* = (A, B) - (B, A) = (p - q) \times [A, B]. \tag{3.6.12}$$

The same brackets are also Jordan-admissible as interested readers are encouraged to verify.

The resulting time evolution is then manifestly irreversible (for $p \neq q$) with nonconservation of the energy

$$i \times \frac{dH}{dt} = (H, H) = (p - q) \times H \times H \neq 0, \tag{3.6.13}$$

as necessary for an open system

Subsequently, Santilli realized that the above formulations are not invariant under their own time evolution because Eq. (3.6.10) is manifestly *nonunitary.*

The application of nonunitary transforms to brackets (3.6.11) then led to the proposal in memoir [39] of 1978 of the following *Lie-admissible operator generalization of Heisenberg equations* in their infinitesimal form

$$\frac{dA}{dt} = A \times P \times H - H \times Q \times A = (A, H)^*, \tag{3.6.14}$$

with finite counterpart

$$A(t) = e^{i \times H \times Q} \times A(0) \times e^{-i \times P \times H}, \tag{3.6.15a}$$

$$P = Q^\dagger. \tag{3.6.15b}$$

where P, Q and $P \pm Q$ are now nonsingular operators (or matrices), and Eq. (3.6.15b) is a fundamental consistency condition explained later on in this section.

Eqs. (3.6.14)–(3.6.15) are the fundamental equations of hadronic mechanics. Their basic brackets are manifestly Lie-admissible and Jordan admissible with structure

$$(A, B)^* = A \times P \times B - B \times Q \times A$$

$$= (A \times T \times B - B \times T \times A) + (A \times R \times B + B \times R \times A), \tag{3.6.16a}$$

$$T = P + Q, \quad R = Q - P. \tag{3.6.16b}$$

It is easy to see that the application of a nonunitary transform to the parametric brackets (3.6.10) leads precisely to the operator brackets (3.5.16) according to the expressions

$$U \times U^\dagger \neq I, \tag{3.6.17a}$$

$$U \times (A, B) \times U^\dagger = A' \times P' \times B' - B' \times Q' \times A', \tag{3.6.17b}$$

where

$$A' = U \times A \times U', \;\; B' = U \times B \times U^\dagger, \tag{3.6.18a}$$

$$P' = U^{\dagger -1} \times P \times U^{-1}, \;\; Q' = U^{\dagger -1} \times Q \times U^{-1}. \tag{3.6.18b}$$

As a result, *all dynamical equations (3.6.10)–(3.6.16) are not invariant under their own time evolution, thus being afflicted by the catastrophic inconsistencies of Theorem 1.4.2.*

Moreover, in the form presented above, dynamical equations (3.6.10) are not derivable from a variational principle. Consequently, they admit no known unique map into their operator counterpart.

In view of these insufficiencies, said equations cannot be assumed in the above given formulation as the basic equations of any consistent physical theory.

3.6.4 Elements of Genomathematics

Recall from Section 1.5 that Eq. (3.6.14) are "directly universal" (Lemma 1.5.2). Therefore, there is no possibility of identifying broader dynamical equations capable of resolving the inconsistencies of Theorem 1.4.2.

To resolve the inconsistencies, Santilli was left with no other choice than that of constructing a *new mathematics*, specifically conceived for the invariant formulation of Eqs. (3.6.14)–(3.6.15).

Following a laborious search conducted over decades, a breakthrough occurred with the discovery, apparently made for the first time by Santilli in Ref. [12] of 1993, that *the axioms of a field hold when the ordinary product of numbers $a \times b$ is ordered to the right or under corresponding generalized units to the right*,

$$a > b = a \times \hat{T}^{>} \times b, \tag{3.6.19}$$

or, separately, to the left,

$$a < b = a \times {}^{>}\hat{T} \times b, \tag{3.6.20}$$

where $\hat{T}^{>}$ and ${}^{<}\hat{T}$ are called *Santilli's genotopic elements to the right and to the left*, respectively, under the following corresponding *generalization of the unit to the right* called *Santilli's genounit to the right*

$$I = Diag.(1, 1, \ldots, 1)$$

$$\to \hat{I}^{>}(t, x, v, \psi, \partial_x \psi, \ldots) = 1/\hat{T}^{>}(t, x, v, \psi, \partial_x \psi, \ldots), \tag{3.6.21}$$

and the *generalization of the unit to the left* called *Santilli's genounit to the left*

$$I = Diag.(1, 1, \ldots, 1)$$

$$\to^{<} \hat{I}(t, x, v, \psi, \partial_x \psi, \ldots) = 1/{}^{<}\hat{T}(t, x, v, \psi, \partial_x \psi, \ldots), \tag{3.6.22}$$

with complementarity condition

$$\hat{I}^{>} = ({}^{<}\hat{I})^{\dagger}. \tag{3.6.23}$$

Under these assumptions we have the preservation of the axiom for a unit, although separately valid for products to the right

$$I \times a = a \times I = a \to \hat{I}^{>} > A = a > \hat{I}^{>} = a \in F, \tag{3.6.24}$$

and to the left

$$I \times a = a \times I = a \to^{<} \hat{I} < a = a <^{<} \hat{I} = a \in F. \tag{3.6.25}$$

The above ordered genoproducts to the right and to the left with corresponding genounits to the right and to the left are at the foundation of the new mathematics needed for an axiomatically consistent and invariant representation of irreversibility, today known as *Santilli's genotopic mathematics*, or *genomathematics* for short, where the prefix "geno" is used in the Greek sense of "inducing" a new mathematics.

Note that in the isomathematics of Section 3.5 the product is indeed generalized,

$$a \times b \to a \hat{\times} b = a \times \hat{T} \times b, \tag{3.6.26a}$$

$$\hat{T} = \hat{T}^\dagger > 0, \tag{3.6.26b}$$

with compatible generalization of the unit,

$$I \to \hat{I} = 1/\hat{T}, \tag{3.6.27}$$

but the isoproduct to the right coincides with that to the left,

$$a \hat{\times} b = b \hat{\times} a, \tag{3.6.28a}$$

as it is the case for ordinary products,

$$a \times b = b \times a. \tag{3.6.28b}$$

A main point in the transition from the iso- to the geno-mathematics is that, in addition to their generalization, products are lifted into different ordered products to the right and to the left

$$a > b = a \times \hat{T}^{>} \times b \neq a < b = a \times {}^{<}\hat{T} \times b, \tag{3.6.29}$$

where

$$\hat{T}^{>} = ({}^{<}\hat{T})^\dagger. \tag{3.6.30}$$

Alternatively, we can say that the identity of the isoproducts to the right and to the left is due to the Hermiticity of the isotopic element, Eq. (3.6.26b). Therefore, the inequivalence of the genoproducts to the right and to the left is due to the lack of Hermiticity of the genotopic element.

The discovery of two complementary orderings of the product while preserving the abstract axioms of a field has truly fundamental implications for irreversibility since it permits the axiomatically consistent and invariant representation of irreversibility via the most ultimate and primitive axioms, those on the product and related unit, as expressed by the following.

FUNDAMENTAL ASSUMPTION ON IRREVERSIBILITY [12,14]: *Dynamical equations for motion forward in time are characterized by*

genoproducts to the right and related genounits, while dynamical equations for the motion backward in time are characterized by genoproducts to the left and related genounits under the condition that said genoproducts and genounits are interconnected by time reversal expressible for generic quantities A, B,

$$(A > B)^\dagger = (A > \hat{T}^> \times B)^\dagger = B^\dagger \times (\hat{T}^>)^\dagger \times A^\dagger, \tag{3.6.31}$$

namely,

$$\hat{T}^> = (^<\hat{T})^\dagger \tag{3.6.32}$$

thus recovering the fundamental complementary conditions (3.6.15b).

As a result, genomathematics permits the embedding of irreversibility in the most fundamental quantities, products and related units, thus assuring *ab initio* the construction of a structurally irreversible mathematics.

DEFINITION 3.6.1: Let $F = F(a, +, \times)$ be a field as per Definition 2.1.1. Santilli's forward genofields (first introduced in Ref. [12] of 1993) are rings $\hat{F}^> = \hat{F}^>(\hat{a}^>, \hat{+}^>, >)$ with *forward genonumbers*

$$\hat{a}^> = a \times \hat{I}^>, \tag{3.6.33}$$

associative, distributive and commutative forward genosum

$$\hat{a}^> \hat{+}^> \hat{b}^> = (a + b) \times \hat{I}^> = \hat{c}^> \in \hat{F}^>, \tag{3.6.34}$$

associative and distributive, but not necessarily commutative, forward genoproduct

$$\hat{a}^> > \hat{b}^> = \hat{a} > \times \hat{T}^> \times \hat{b}^> = \hat{c}^> \in \hat{F}, \tag{3.6.35}$$

additive forward genounit

$$\hat{0}^> = 0, \hat{a}^> \hat{+}^> \hat{0}^> = \hat{0}^> \hat{+}^> \hat{a}^> = \hat{a}^> \in \hat{F}^>, \tag{3.6.36}$$

and multiplicative forward genounit

$$\hat{I}^> = 1/\hat{T}^>, \ \hat{a}^> > \hat{I}^> = \hat{I}^> > \hat{a}^> = \hat{a}^> \in \hat{F}^>, \forall \hat{a}^>, \hat{b}^> \in \hat{F}^>, \tag{3.6.37}$$

where $\hat{I}^>$ is a complex-valued non-Hermitean, or real-value non-symmetric, everywhere invertible quantity generally outside F.

The backward genofields $^<\hat{F}(^<\hat{a}, ^<\hat{+}, <)$, their elements, units and their operations are given by the Hermitean conjugate (or transposed) of the corresponding quantities and their operations in $\hat{F}^>(\hat{a}^>, \hat{+}^>, \hat{\times}^>)$, e.g.,

$$^<\hat{I} = (\hat{I}^>)^\dagger, \ etc. \tag{3.6.38}$$

LEMMA 3.6.1: Forward and backward genofields are fields with characteristic zero (namely, they verify all axioms of a field).

In Section 2.1 we pointed out that the conventional product "2 multiplied by 3" is not necessarily equal to 6 because, depending on the assumed unit and related product, it can be -6.

In Section 3.5 we pointed out that the same product "2 multiplied by 3" is not necessarily equal to $+6$ or -6, because it can also be equal to an arbitrary number, or a matrix or an integrodifferential operator.

In this section we point out that "2 multiplied by 3" can be ordered to the right or to the left, and the result is not only arbitrary, but yielding different numerical results for different orderings, $2 > 3 \neq 2 < 3$, all this by continuing to verify the axioms of a field per each order [12].

Once the forward and backward genofields have been identified, the various branches of genomathematics can be constructed via simple compatibility arguments, resulting in the *genofunctional analysis, genodifferential calculus*, etc. [14,54,55]. We have in this way the *genodifferentials and genoderivatives*

$$\hat{d}^{>}x = \hat{T}_x^{>} \times dx, \quad \frac{\hat{\partial}^{>}}{\hat{\partial}^{>}x} = \hat{I}_x^{>} \times \frac{\partial}{\partial x}, \text{ etc.} \tag{3.6.39}$$

Particularly intriguing are the *genogeometries* [55] because they admit *nonsymmetric metrics.* For instance, the *Minkowski-Santilli genogeometry* admits the metric

$$\eta^{>}(x) = \hat{T}^{>}(x) \times \eta, \tag{3.6.40}$$

where η is the Minkowski metric and $\hat{T}^{>}(x)$ is a real-values, nowhere singular, 4×4 *nonsymmetric* matrix.

Consequently, genomathematics permits, apparently for the first time, to use a nonsymmetric metric under the validity of the abstract Minkowskian axioms, while bypassing known inconsistencies for nonsymmetric metrics since they are referred to the *nonsymmetric genounit*

$$\hat{I}^{>} = 1/\hat{T}^{>}. \tag{3.6.41}$$

In this way, *genogeometries are structurally irreversible* and actually represent irreversibility with their most central geometric notion, the metric.

3.6.5 Lie-Santilli Genotheory and its Isodual

Particularly important for irreversibility is the lifting of Lie's and Lie-Santilli's theories permitted by genomathematics, first identified by

Santilli in Ref. [23] of 1978, today knows as the *Lie-Santilli genotheory*, and characterized by:

(1) The *forward and backward universal enveloping genoassociative algebra* $\hat{\xi}^{>}$, ${}^{<}\hat{\xi}$, with infinite-dimensional basis characterizing the *Poincaré-Birkhoff-Witt-Santilli genotheorem*

$$\hat{\xi}^{>} : \hat{I}^{>}, \hat{X}_i, \hat{X}_i > \hat{X}_j, \hat{X}_i > \hat{X}_j > \hat{X}_k, \ldots, i \leq j \leq k, \tag{3.6.42a}$$

$${}^{<}\hat{\xi} : \hat{I}, {}^{<}\hat{X}_i, \hat{X}_i < \hat{X}_j, \hat{X}_i < \hat{X}_j < \hat{X}_k, \ldots, i \leq j \leq k; \tag{3.6.42b}$$

where the "hat" on the generators denotes their formulation on genospaces over genofields and their Hermiticity implies that $\hat{X}^{>} = {}^{<}\hat{X} = \hat{X}$;

(2) The *Lie-Santilli genoalgebras* characterized by the universal, jointly Lie- and Jordan-admissible brackets,

$${}^{<}\hat{L}^{>} : (\hat{X}_i \hat{,} \hat{X}_j) = \hat{X}_i < \hat{X}_j - \hat{X}_j > \hat{X}_i = C_{ij}^{k} \times \hat{X}_k, \tag{3.6.43}$$

here formulated in an invariant form (see below);

(3) The *Lie-Santilli genotransformation groups*

$$\begin{aligned} {}^{<}\hat{G}^{>} : \hat{A}(\hat{w}) &= (\hat{e}^{\hat{i}\hat{\times}\hat{X}\hat{\times}\hat{w}})^{>} > \hat{A}(\hat{0}) <^{<} (\hat{e}^{-\hat{i}\hat{\times}\hat{w}\hat{\times}\hat{X}}) \\ &= (e^{i\times\hat{X}\times\hat{T}^{>}\times w}) \times A(0) \times (e^{-i\times w\times{}^{<}\hat{T}\times\hat{X}}), \end{aligned} \tag{3.6.44}$$

where $\hat{w}^{>} \in \hat{R}^{>}$ are the *genoparameters*; the *genorepresentation theory*, etc.

The implications of the Lie-Santilli genotheory are significant mathematically and physically. On mathematical grounds, the Lie-Santilli genoalgebras are "directly universal" and include as particular cases all known algebras, such as Lie, Jordan, Flexible algebras, power associative algebras, quantum, algebras, supersymmetric algebras, Kac-Moody algebras, etc. (Section 1.5).

Moreover, when computed on the *genobimodule*

$${}^{<}\hat{M}^{>} = {}^{<}\hat{\xi} \times \hat{\xi}^{>}, \tag{3.6.45}$$

Lie-admissible algebras verify all Lie axioms, while deviations from Lie algebras emerge only in their *projection* on the bimodule

$${}^{<}M^{>} = {}^{<}\xi \times \xi^{>}, \tag{3.6.46}$$

of the conventional Lie theory.

This is due to the fact that the computation of the left action $A < B = A \times^{<} \hat{T} \times B$ on ${}^{<}\hat{\xi}$ (that is, with respect to the genounit ${}^{<}\hat{I} = 1/{}^{<}\hat{T}$) yields the save value as the computation of the conventional product

$A \times B$ on $^<\xi$ (that is, with respect to the trivial unit I), and the same occurs for the value of $A > B$ on $\hat{\xi}^>$.

The above occurrences explain the reason the structure constant and the product in the r.h.s. of Eq. (3.6.43) are those of a conventional Lie algebra.

In this way, thanks to genomathematics, *Lie algebras acquire a towering significance in view of the possibility of reducing all possible irreversible systems to primitive Lie axioms.*

The physical implications of the Lie-Santilli genotheory are equally far reaching. In fact, Noether's theorem on the reduction of reversible conservation laws to primitive Lie symmetries can be lifted to the *reduction, this time, of irreversible nonconservation laws to primitive Lie-Santilli genosymmetries.*

As a matter of fact, this reduction was the very first motivation for the construction of the genotheory in memoir [23] (see also monographs [49,50]). The reader can then foresee similar liftings of all remaining physical aspects treated via Lie algebras.

The construction of the isodual Lie-Santilli genotheory is an instructive exercise for readers interested in learning the new methods.

3.6.6 Geno-Newtonian Mechanics and its Isodual

Recall that, for the case of isotopies, the basic Newtonian systems are given by those admitting nonconservative internal forces restricted by certain constraints to verify total conservation laws (these are the *closed non-Hamiltonian systems* of Chapter 1).

For the case of the genotopies under consideration here, the basic Newtonian systems are the conventional nonconservative systems without subsidiary constraints (*open non-Hamiltonian systems*) with generic expression [48]

$$m \times \frac{dv}{dt} = F^{SA}(r,p) + F^{NSA}(t,r,p,dp/dt,\ldots), \tag{3.6.47}$$

in which case irreversibility is characterized by nonselfadjoint forces, as indicated earlier, since all conservative forces are reversible.

As it is well known, the above equations are not derivable from any variational principle in the fixed frame of the observer [48], and this is the reason why all conventional attempts for consistently quantizing nonconservative forces have failed for about one century. In turn, the lack of achievement of an operator counterpart of nonconservative forces lead to the academic belief that they are illusory (Section 3.6.1).

Hadronic mechanics has achieved the first and only physically consistent operator formulation of nonconservative forces known to the

author.[9] This goal was achieved by rewriting Newton's equations into an identical form derivable from a variational principle. Still in turn, the latter objective was solely permitted by the novel genomathematics.

It is appropriate to recall that Newton was forced to discover new mathematics, the differential calculus, prior to being able to formulated his celebrated equations. Therefore, the need for new mathematics as a condition to represent all Newton's systems from a variational principle should not be surprising.

Recall also from Section 2.3 that, contrary to popular beliefs, there exist *four* inequivalent directions of time, namely, motion forward in future times, motion backward in past time, motion backward from future times and motion forward in past times, each direction having its own unit.

Consequently, time reversal alone cannot represent all these possible motions, and isoduality results to be the only known additional conjugation that, when combined with time reversal, can represent all possible time evolutions of both matter and antimatter.

The above setting implies the existence of four different new mechanics first formulated by Santilli in memoir [14] of 1996, and today known as *Newton-Santilli genomechanics*, namely:

A) *Forward genomechanics* for the representation of forward motion of matter systems;

B) *Backward genomechanics* for the representation of the time reversal image of matter systems;

C) *Isodual backward genomechanics* for the representation of motion backward in time of antimatter systems, and

D) *Isodual forward genomechanics* for the representation of time reversal antimatter systems.

These new mechanics are characterized by:

1) Four different times, *forward and backward genotimes for matter systems and the backward and forward isodual genotimes for antimatter systems*

$$\hat{t}^{>} = t \times \hat{I}_t^{>}, \quad -\hat{t}^{>}, \quad \hat{t^{>}}^{d}, \quad -\hat{t^{>}}^{d}, \tag{3.6.48}$$

with (nowhere singular and non-Hermitean) *forward and backward time genounits and their isoduals*,[10]

$$\hat{I}_t^{>} = 1/\hat{T}_t^{>}, \quad -\hat{I}_t^{>}, \quad \hat{I^{>}}_t^{d}, \quad -\hat{I^{>}}_t^{d}; \tag{3.6.49}$$

2) The *forward and backward genocoordinates and their isoduals*

$$\hat{x}^{>} = x \times \hat{I}_x^{>}, -\hat{x}^{>}, \quad \hat{x^{>}}^{d}, \quad -\hat{x^{>}}^{d}, \tag{3.6.50}$$

with (nowhere singular non-Hermitean) *coordinate genounit*

$$\hat{I}_x^> = 1/\hat{T}_x^>, \quad -\hat{I}_x^>, \quad \hat{I}^>{}_x^d, \quad -\hat{I}^>{}_x^d, \tag{3.6.51}$$

with *forward and backward coordinate genospace and their isoduals* $\hat{S}_x^>$, etc., and related *forward coordinate genofield and their isoduals* $\hat{R}_x^>$, etc.;

3) The *forward and backward genospeeds and their isoduals*

$$\hat{v}^> = \hat{d}^>\hat{x}^>/\hat{d}^>\hat{t}^>, \quad -\hat{v}^>, \quad \hat{v^>}^d, \quad -\hat{v^>}^d, \tag{3.6.52}$$

with (nowhere singular and non-Hermitean) *speed genounit*

$$\hat{I}_v^> = 1/\hat{T}_v^>, \quad -\hat{I}_v^>, \quad \hat{I}^>{}_v^d, \quad -\hat{I}^>{}_v^d, \tag{3.6.53}$$

with related *forward speed backward genospaces and their isoduals* $\hat{S}_v^>$, etc., over *forward and backward speed genofields* $\hat{R}_v^>$, etc.;

The above formalism then leads to the *forward genospace for matter systems* given by the Kronecker product

$$\hat{S}_{Tot}^> = \hat{S}_t^> \times \hat{S}_x^> \times \hat{S}_v^>, \tag{3.6.54}$$

defined over the it forward genofield

$$\hat{R}_{tot}^> = \hat{R}_t^> \times \hat{R}_x^> \times \hat{R}_v^>, \tag{3.6.55}$$

with *total forward genounit*

$$\hat{I}_{tot}^> = \hat{I}_t^> \times \hat{I}_x^> \times \hat{I}_v^>, \tag{3.6.56}$$

and corresponding expressions for the remaining three spaces obtained via time reversal and isoduality.

The basic equations are given by:

I) The *forward Newton-Santilli genoequations for matter systems* [14], formulated via the genodifferential calculus,

$$\hat{m}_a^> > \frac{\hat{d}^>\hat{v}_{ka}^>}{\hat{d}^>\hat{t}^>} = -\frac{\hat{\partial}^>\hat{V}^>}{\hat{\partial}^>\hat{x}_a^{>k}}; \tag{3.6.57}$$

II) The *backward genoequations for matter systems* that are characterized by time reversal of the preceding ones;

III) the *backward isodual genoequations for antimatter systems* that are characterized by the isodual map of the backward genoequations,

$$^<\hat{m}_a^d < \frac{^<\hat{d}^{d<}\hat{v}_{ka}^d}{^<\hat{d}^{d<}\hat{t}^d} = -\frac{^<\hat{\partial}^{d<}\hat{V}^d}{^<\hat{\partial}^{d<}\hat{x}_a^{dk}}; \tag{3.6.58}$$

IV) the *forward isodual genoequations for antimatter systems* characterized by time reversal of the preceding isodual equations.

As one can see, the representation of Newton's equations is done in a way similar to the isotopic case. Note that in Newton's equations the nonpotential forces are part of the applied force, while in the Newton-Santilli genoequations nonpotential forces are represented by the genounits, or, equivalently, by the forward genodifferential calculus, in a way essentially similar to the case of isotopies.

The main difference between iso- and geno-equations is that isounits are Hermitean, thus implying the equivalence of forward and backward motions, while genounits are non-Hermitean, thus implying irreversibility.

Note also that the topology underlying Newton's equations is the conventional, Euclidean, local-differential topology that, as such, can only represent point particles.

By contrast, the topology underlying the Newton-Santilli genoequations is given by a genotopy of the isotopology studied in Refs. [14,139, 226,227] for the representation of extended, nonspherical and deformable particles via forward genounits, e.g., of the type

$$\hat{I}^{>} = Diag.(n_1^2, n_2^2, n_3^2, n_4^2) \times \Gamma^{>}(t, r, v, \ldots), \tag{3.6.59}$$

where $n_k^2, k = 1, 2, 3$ represents the semiaxes of an ellipsoid, n_4^2 represents the density of the medium in which motion occurs (with more general nondiagonal realizations here omitted for simplicity), and $\Gamma^{>}$ constitutes a nonsymmetric matrix representing nonselfadjoint forces, namely, the contact interactions among extended constituents occurring for the motion forward in time.

3.6.7 Lie-Admissible Classical Genomechanics and its Isodual

In this section we show that, once rewritten in their identical genoform (3.6.57), Newton's equations for nonconservative systems are indeed derivable from a variational principle, with analytic equations possessing a Lie-admissible structure and Hamilton-Jacobi equations suitable for the first know consistent and unique operator map studied in the next section.

The most effective setting to introduce real-valued non-symmetric genounits is in the $6N$-dimensional *forward genospace (genocotangent bundle)* with local genocoordinates and their conjugates

$$\hat{a}^{>\mu} = a^{\rho} \times \hat{I}^{>\mu}_{1\,\rho}, (\hat{a}^{>\mu})$$

$$= \begin{pmatrix} \hat{x}^{>k}_{\alpha} \\ \hat{p}^{>}_{k\alpha} \end{pmatrix} \tag{3.6.60}$$

and

$$\hat{R}^{>}_{\mu} = R_{\rho} \times \hat{I}^{>\rho}_{2\,\mu},\ (\hat{R}^{>}_{\mu}) = (\hat{p}_{k\alpha}, \hat{0}), \tag{3.6.61a}$$

$$\hat{I}^{>}_{1} = 1/\hat{T}^{>}_{1} = (\hat{I}^{>}_{2})^{T} = (1/\hat{T}^{>}_{2})^{T}, \tag{3.6.61b}$$

$$k = 1, 2, 3;\ \alpha = 1, 2, \ldots, N;\quad \mu,\ \rho = 1, 2, \ldots 6N,$$

where the superscript T stands for transposed, and nowhere singular, real-valued and non-symmetric genometric and related invariant

$$\hat{\delta}^{>} = \hat{T}^{>}_{1\,6N\times 6N}\delta_{6N\times 6N} \times \delta_{6N\times 6N}, \tag{3.6.62a}$$

$$\hat{a}^{>\mu} > \hat{R}^{>}_{\mu} = \hat{a}^{>\rho} \times \hat{T}^{>\beta}_{1\,\rho} \times \hat{R}^{>}_{\beta} = a^{\rho} \times \hat{I}^{>\beta}_{2\,\rho} \times R_{\beta}. \tag{3.6.62b}$$

In this case we have the following *genoaction principle* [14]

$$\hat{\delta}^{>}\hat{\mathcal{A}}^{>} = \hat{\delta}^{>}\hat{\int}^{>}[\hat{R}^{>}_{\mu} > \hat{d}^{>}\hat{a}^{>\mu} - \hat{H}^{>} >_{t} \hat{d}^{>}\hat{t}^{>}]$$

$$= \delta\int[R_{\mu} \times \hat{T}^{>\mu}_{1\,\nu}(t, x, p, \ldots) \times d(a^{\beta} \times \hat{I}^{>\nu}_{1\,\beta}) - H \times dt] = 0, \tag{3.6.63}$$

where the second expression is the projection on conventional spaces over conventional fields and we have assumed for simplicity that the time genounit is 1.

It is easy to prove that the above genoprinciple characterizes the following *forward Hamilton-Santilli genoequations*, (originally proposed in Ref. [23] of 1978 with conventional mathematics and in Ref. [14] of 1996 with genomathematics (see also Refs. [28,51,52,55])

$$\hat{\omega}^{>}_{\mu\nu} > \frac{\hat{d}^{>}\hat{a}^{\nu>}}{\hat{d}^{>}\hat{t}^{>}} - \frac{\hat{\partial}^{>}\hat{H}^{>}(\hat{a}^{>})}{\hat{\partial}^{>}\hat{a}^{\mu>}}$$

$$= \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \times \begin{pmatrix} dr/dt \\ dp/dt \end{pmatrix} - \begin{pmatrix} 1 & K \\ 0 & 1 \end{pmatrix} \times \begin{pmatrix} \partial H/\partial r \\ \partial H/\partial p \end{pmatrix} = 0, \tag{3.6.64a}$$

$$\hat{\omega}^{>} = \Big(\frac{\hat{\partial}^{>}R^{>}_{\nu}}{\hat{\partial}^{>}\hat{a}^{\mu>}} - \frac{\hat{\partial}^{>}\hat{R}^{>}_{\mu}}{\hat{\partial}^{>}\hat{a}^{\nu>}}\Big) \times \hat{I}^{>} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \times \hat{I}^{>}, \tag{3.6.64b}$$

$$K = F^{NSA}/(\partial H/\partial p). \tag{3.6.64c}$$

The time evolution of a quantity $\hat{A}^{>}(\hat{a}^{>})$ on the forward geno-phase-space can be written in terms of the following brackets

$$\frac{\hat{d}^{>}\hat{A}^{>}}{\hat{d}^{>}t^{>}} = (\hat{A}^{>}, \hat{H}^{>}) = \frac{\hat{\partial}^{>}\hat{A}^{>}}{\hat{\partial}^{>}\hat{a}^{>\mu}} > \omega^{\hat{\mu}\nu>} > \frac{\hat{\partial}^{>}\hat{H}^{>}}{\hat{\partial}\hat{a}^{>\nu}}$$

$$= \frac{\partial \hat{A}^{>}}{\partial \hat{a}^{>\mu}} \times S^{\,\mu\nu} \times \frac{\partial \hat{H}^{>}}{\partial \hat{a}^{>\nu}}$$

$$= \Big(\frac{\partial \hat{A}^{>}}{\partial \hat{r}_{\alpha}^{>k}} \times \frac{\partial \hat{H}^{>}}{\partial \hat{p}_{k\alpha}^{>}} - \frac{\partial \hat{A}^{>}}{\partial \hat{p}_{k\alpha}^{>}} \times \frac{\partial \hat{H}^{>}}{\partial \hat{r}_{\alpha}^{>k}}\Big) + \frac{\partial \hat{A}^{>}}{\partial \hat{p}_{k\alpha}^{>}} \times K_{k}^{k} \times \frac{\partial \hat{H}^{>}}{\partial \hat{p}_{k\alpha}^{>}}, \quad (3.6.65a)$$

$$S^{>\mu\nu} = \omega^{\mu\rho} \times \hat{I}_{\rho}^{2\mu}, \omega^{\mu\nu} = (||\omega_{\alpha\beta}||^{-1})^{\mu\nu}, \quad (3.6.65b)$$

where $\omega^{\mu\nu}$ is the conventional Lie tensor and, consequently, $S^{\mu\nu}$ is Lie-admissible in the sense of Albert [7].

As one can see, the important consequence of genomathematics and its genodifferential calculus is that of turning the triple system (A, H, F^{NSA}) of Eq. (3.6.2) in the bilinear form $(A\hat{,}B)$, thus characterizing a consistent algebra in the brackets of the time evolution.

This is the central purpose for which genomathematics was built (note that the multiplicative factors represented by K are fixed for each given system). The invariance of such a formulation will be proved shortly.

It is an instructive exercise for interested readers to prove that the brackets $(A\hat{,}B)$ are Lie-admissible, although not Jordan-admissible.

It is easy to verify that the above identical reformulation of Hamilton's historical time evolution correctly recovers the *time rate of variations of physical quantities* in general, and that of the energy in particular,

$$\frac{dA^{>}}{dt} = (A^{>}, H^{>}) = [\hat{A}^{>}, \hat{H}^{>}] + \frac{\partial \hat{A}^{>}}{\partial \hat{p}_{k\alpha}^{>}} \times F_{k\alpha}^{NSA}, \quad (3.6.66a)$$

$$\frac{dH}{dt} = [\hat{H}^{>}, \hat{H}^{>}] + \frac{\partial \hat{H}^{>}}{\partial \hat{p}_{k\alpha}^{>}} \times F_{k\alpha}^{NSA} = v_{\alpha}^{k} \times F_{k\alpha}^{NSA}. \quad (3.6.66b)$$

It is easy to show that genoaction principle (3.6.66) characterizes the following *Hamilton-Jacobi-Santilli genoequations*

$$\frac{\hat{\partial}^{>}\mathcal{A}^{>}}{\hat{\partial}^{>}\hat{t}^{>}} + \hat{H}^{>} = 0, \quad (3.6.67a)$$

$$(\frac{\hat{\partial}^{>}\mathcal{A}^{>}}{\hat{\partial}^{>}\hat{a}^{>\mu}}) = (\frac{\hat{\partial}^{>}\mathcal{A}^{>}}{\hat{\partial}^{>}x_{\alpha}^{>k}}, \frac{\hat{\partial}^{>}\mathcal{A}^{>}}{\hat{\partial}^{>}p_{k\alpha}^{>}}) = (\hat{R}_{\mu}^{>}) = (\hat{p}_{k\alpha}^{>}, \hat{0}), \quad (3.6.67b)$$

which confirm the property (crucial for genoquantization as shown below) that the genoaction is indeed independent of the linear momentum.

Note the *direct universality* of the Lie-admissible equations for the representation of all infinitely possible Newton equations (universality) directly in the fixed frame of the experimenter (direct universality).

Note also that, *at the abstract, realization-free level, Hamilton-Santilli genoequations coincide* with Hamilton's equations without external terms, yet represent those with external terms.

The latter are reformulated via genomathematics as the only known way to achieve invariance and derivability from a variational principle while admitting a consistent algebra in the brackets of the time evolution [38].

Therefore, Hamilton-Santilli genoequations (3.6.66) are indeed irreversible for all possible reversible Hamiltonians, as desired. The origin of irreversibility rests in the contact nonpotential forces F^{NSA} according to Lagrange's and Hamilton's teaching that is merely reformulated in an invariant way.

The above Lie-admissible mechanics requires, for completeness, *three* additional formulations, the *backward genomechanics* for the description of *matter moving backward in time*, and the isoduals of both the forward and backward mechanics for the description of *antimatter.*

The construction of these additional mechanics is lefty to the interested reader for brevity.

3.6.8 Lie-Admissible Branch of Hadronic Mechanics and its Isodual

A simple genotopy of the naive or symplectic quantization applied to Eq. (3.6.61) yields the *genotopic branch of hadronic mechanics* defined on the *forward genotopic Hilbert space* $\hat{\mathcal{H}}^>$ with *forward genostates* $|\hat{\psi}^> >$ and *forward genoinner product*

$$<^< \hat{\psi}| > |\hat{\psi}^> > \times \hat{I}^> \in \hat{C}^>. \tag{3.6.68}$$

The resulting genotopy of quantum mechanics is characterized by the *forward geno-Schrödinger equations*, first formulated in Refs. [42,179] via conventional mathematics and in Ref. [14] via genomathematics)

$$\hat{i}^> > \frac{\hat{\partial}^>}{\hat{\partial}^> \hat{t}^>} |\hat{\psi}^> >= \hat{H}^> > |\hat{\psi}^> >$$

$$= \hat{H}(\hat{r}, \hat{v}) \times \hat{T}^>(\hat{t}, \hat{r}, \hat{p}, \hat{\psi}, \hat{\partial}\hat{\psi} \ldots.) \times |\hat{\psi}^> >= E^> > |\psi^> >, \tag{3.6.69}$$

where the time orderings in the second term are ignored for simpolicity of notation, and the forward genomomentum first formulated in Ref. [14] thanks to the advent of said genodifferential calculus

$$\hat{p}_k^> > |\hat{\psi}^> >= -\hat{i}^> > \hat{\partial}_k^> |\hat{\psi}^> >= -i \times \hat{I}_k^{>\, i} \times \partial_i |\hat{\psi}^> >, \tag{3.6.70}$$

with basic expression

$$\hat{I}^> > |\hat{\psi}^> >= |\hat{\psi}^> >, \tag{3.6.71}$$

proving that $\hat{I}^>$ is indeed the correct unit for motion forward in time.

Note in the genoaction principle the crucial independence of isoaction $\hat{\mathcal{A}}^{>}$ in the linear momentum, as expressed by the Hamilton-Jacobi-Santilli genoequations. Such independence assures that genoquantization yields a genowavefunction solely dependent on time and coordinates, $\hat{\psi}^{>} = \hat{\psi}^{>}(t, r)$.

Other geno-Hamiltonian mechanics studied previously [51] do not verify such a condition, thus implying genowavefunctions with an explicit dependence also on linear momenta, $\hat{\psi}^{>} = \hat{\psi}^{>}(t, r, p)$ that violate the abstract identity of quantum and hadronic mechanics whose treatment in any case is beyond our operator knowledge at this writing.

The complementary genotopies of Heisenberg equations, today known as *Heisenberg-Santilli genoequations*, were first formulated in Ref. [38] via conventional mathematics and in Ref. [14] via genomathematics) and can be written in their finite and infinitesimal forms

$$\hat{A}(\hat{t}) = (\hat{e}_{>}^{\hat{i}\hat{\times}\hat{H}\hat{\times}\hat{t}}) > \hat{A}(\hat{0}) < (_{<}\hat{e}^{-\hat{i}\hat{\times}\hat{t}\hat{\times}\hat{H}})$$

$$= (e^{i\times\hat{H}\times\hat{T}^{>}\times t}) \times A(0) \times (e^{-i\times t\times{}^{<}\hat{T}\times\hat{H}}), \tag{3.6.72a}$$

$$\hat{i}\hat{\times}\frac{\hat{d}\hat{A}}{\hat{d}\hat{t}} = (\hat{A}\hat{,}\hat{H}) = \hat{A} < \hat{H} - \hat{H} > \hat{A}$$

$$= \hat{A} \times^{<} \hat{T}(\hat{t}, \hat{r}, \hat{p}, \hat{\psi}, \ldots.) \times \hat{H} - \hat{H} \times \hat{T}^{>}(\hat{t}, \hat{r}, \hat{p}, \hat{\psi}, \ldots.) \times \hat{A}, \tag{3.6.72b}$$

$$(\hat{r}^{i}\hat{,}\hat{p}_{j}) = i \times \delta_{j}^{i} \times \hat{I}^{>}, (\hat{r}^{i}\hat{,}\hat{r}^{j}) = (\hat{p}_{i}\hat{,}\hat{p}_{j}) = 0, \tag{3.6.72c}$$

where Eq. (3.6.72c) are the *fundamental genocanonical rules* and there is no time arrow, since Heisenberg's equations are computed at a fixed time.

Closed irreversible systems are characterized by the Lie-isotopic subcase in which

$$\hat{i}\hat{\times}\frac{\hat{d}\hat{A}}{\hat{d}\hat{t}} = [\hat{A}\hat{,}\hat{H}] = \hat{A} \times \hat{T}(t, \ldots) \times \hat{H} - \hat{H} \times \hat{T}(t, \ldots) \times \hat{A}, \tag{3.6.73a}$$

$${}^{<}\hat{T}(t, \ldots) = \hat{T}^{>}(t, \ldots) = \hat{T}(t, \ldots) = \hat{T}^{\dagger}(t, \ldots) \neq \hat{T}(-t, \ldots). \tag{3.6.73b}$$

for which the Hamiltonian is manifestly conserved. Nevertheless the system is manifestly irreversible. Note also the first and only known observability of the Hamiltonian (due to its iso-Hermiticity) under irrevbersibility.

As one can see, brackets (A, B) are jointly Lie- and Jordan-admissible.

The *genoexpectation values* of an observable for the forward motion $\hat{A}^{>}$ are then given by

$$\frac{<^{<} \hat{\psi}| > \hat{A}^{>} > |\hat{\psi}^{>} >}{<^{<} \hat{\psi}| > |\hat{\psi}^{>} >} \times \hat{I}^{>} \in \hat{C}^{>} \tag{3.6.74}$$

In particular, the genoexpectation values of the genounit recover the conventional Planck's unit as in the isotopic case,

$$\frac{<\hat{\psi}|>\hat{I}^{>}><|\hat{\psi}>}{<\hat{\psi}|>|\hat{\psi}>}=I, \tag{3.6.75}$$

thus confirming that the genotopies are "hidden" in the abstract axioms of quantum mechanics much along the celebrated Einstein-Podolsky-Rosen argument.

Note that *forward geno-Hermiticity coincides with conventional Hermiticity.* As a result, all quantities that are observables for quantum mechanics remain observables for the above formulation.

However, unlike quantum mechanics, physical quantities are generally *nonconserved*, as it must be the case for the energy,

$$\hat{i}^{>}>\frac{\hat{d}^{>}\hat{H}^{>}}{\hat{d}^{>}\hat{t}^{>}}=\hat{H}\times(^{<}\hat{T}-\hat{T}^{>})\times\hat{H}\neq 0. \tag{3.6.76}$$

Therefore, *the genotopic branch of hadronic mechanics is the only known operator formulation permitting nonconserved quantities to be Hermitean, thus being observability.*

Other formulations attempt to represent nonconservation, e.g., by adding an "imaginary potential" to the Hamiltonian. In this case the Hamiltonian is non-Hermitean and, consequently, the nonconservation of the energy cannot be an observable.

Besides, "nonconservative models" with non-Hermitean Hamiltonians are nonunitary and are formulated on conventional spaces over conventional fields, thus suffering all the catastrophic inconsistencies of Theorem 1.5.2. For additional aspects of genomechanics interested readers may consult Ref. [61].

The above formulation must be completed with three additional Lie-admissible formulations, the backward formulation for matter under time reversal and the two additional isodual formulations for antimatter. Their study is left to the interested reader for brevity.

3.6.9 Simple Construction of Lie-Admissible Theories

As it was the case for the isotopies, a simple method has been identified in Ref. [44] for the construction of Lie-admissible (geno-) theories from any given conventional, classical or quantum formulation. It consists in *identifying the genounits as the product of two different nonunitary transforms*,

$$\hat{I}^{>}=(^{<}\hat{I})^{\dagger}=U\times W^{\dagger},\quad ^{<}\hat{I}=W\times U^{\dagger}, \tag{3.6.77a}$$

$$U \times U^{\dagger} \neq 1, W \times W^{\dagger} \neq 1, U \times W^{\dagger} = \hat{I}^{>}, \tag{3.6.77b}$$

and subjecting the totality of quantities and their operations of conventional models to said dual transforms,

$$I \to \hat{I}^{>} = U \times I \times W^{\dagger}, I \to^{<} \hat{I} = W \times I \times U^{\dagger}, \tag{3.6.78a}$$

$$a \to \hat{a}^{>} = U \times a \times W^{\dagger} = a \times \hat{I}^{>}, \tag{3.6.78b}$$

$$a \to^{<} \hat{a} = W \times a \times U^{\dagger} =^{<} \hat{I} \times a, \tag{3.6.78c}$$

$$a \times b \to \hat{a}^{>} > \hat{b}^{>} = U \times (a \times b) \times W^{>}$$
$$= (U \times a \times W^{\dagger}) \times (U \times W^{\dagger})^{-1} \times (U \times b \times W^{\dagger}), \tag{3.6.78d}$$

$$\partial/\partial x \to \hat{\partial}^{>}/\hat{\partial}^{>}\hat{x}^{>} = U \times (\partial/\partial x) \times W^{\dagger} = \hat{I}^{>} \times (\partial/\partial x), \tag{3.6.78e}$$

$$< \psi| \times |\psi > \to <^{<} \psi| > |\psi^{>} >= U \times (< \psi| \times |\psi >) \times W^{\dagger}, \tag{3.6.78f}$$

$$H \times |\psi > \to \hat{H}^{>} > |\psi^{>} >$$
$$= (U \times H \times W^{\dagger}) \times (U \times W^{\dagger})^{-1} \times (U \times \psi > W^{\dagger}), \text{ etc.} \tag{3.6.78g}$$

As a result, any given conventional, classical or quantum model can be easily lifted into the genotopic form.

Note that the above construction implies that *all conventional physical quantities acquire a well defined direction of time.* For instance, the correct genotopic formulation of energy, linear momentum, etc., is given by

$$\hat{H}^{>} = U \times H \times W^{\dagger}, \hat{p}^{>} = U \times p \times W^{>}, \text{ etc.} \tag{3.6.79}$$

In fact, under irreversibility, the value of a nonconserved energy at a given time t for motion forward in time is generally different than the corresponding value of the energy for $-t$ for motion backward in past times.

This explains the reason for having represented in this section energy, momentum and other quantities with their arrow of time $>$. Such an arrow can indeed be omitted for notational simplicity, but only after the understanding of its existence.

3.6.10 Invariance of Lie-Admissible Theories

Recall that a fundamental axiomatic feature of quantum mechanics is the invariance under time evolution of all numerical predictions and physical laws, which invariance is due to the *unitary structure* of the theory.

However, quantum mechanics is reversible and can only represent in a scientific way beyond academic beliefs reversible systems verifying total

conservation laws due to the antisymmetric character of the brackets of the time evolution.

As indicated earlier, the representation of irreversibility and nonconservation requires theories with a *nonunitary structure*. However, the latter are afflicted by the catastrophic inconsistencies of Theorem 1.5.2.

The only resolution of such a basic impasse known to the author has been the achievement of invariance under nonunitarity and irreversibility via the use of genomathematics, provided that such genomathematics is applied to the *totality* of the formalism to avoid evident inconsistencies caused by mixing different mathematics for the selected physical problem.[11]

Such an invariance was first achieved by Santilli in Ref. [44] of 1997 and can be illustrated by reformulating any given nonunitary transform in the *genounitary form*

$$U = \hat{U} \times \hat{T}^{>1/2}, W = \hat{W} \times \hat{T}^{>1/2}, \tag{3.6.80a}$$

$$U \times W^\dagger = \hat{U} > \hat{W}^\dagger = \hat{W}^\dagger > \hat{U} = \hat{I}^> = 1/\hat{T}^>, \tag{3.6.80b}$$

and then showing that genounits, genoproducts, genoexponentiation, etc., are indeed invariant under the above genounitary transform in exactly the same way as conventional units, products, exponentiations, etc. are invariant under unitary transforms,

$$\hat{I}^> \to \hat{I}^{>\prime} = \hat{U} > \hat{I}^> > \hat{W}^\dagger = \hat{I}^>, \tag{3.6.81a}$$

$$\hat{A} > \hat{B} \to \hat{U} > (A > B) > \hat{W}^\dagger$$
$$= (\hat{U} \times \hat{T}^> \times A \times T^> \times \hat{W}^\dagger) \times (\hat{T}^> \times W^\dagger)^{-1} \times \hat{T}^>$$
$$\times (\hat{U} \times \hat{T}^>)^{-1} \times (\hat{U} \times T^> \times \hat{A} \times T^> \times \hat{W}^>)$$
$$= \hat{A}' \times (\hat{U} \times \hat{W}^\dagger)^{-1} \times \hat{B} = \hat{A}' \times \hat{T}^> \times B' = \hat{A}' > \hat{B}', \text{ etc.} \tag{3.6.81b}$$

from which all remaining invariances follow, thus resolving the catastrophic inconsistencies of Theorem 1.5.2.

Note the *numerical invariances of the genounit* $\hat{I}^> \to \hat{I}^{>\prime} \equiv \hat{I}^>$, *of the genotopic element* $\hat{T}^> \to \hat{T}^{>\prime} \equiv \hat{T}^>$, *and of the genoproduct* $>\to>'\equiv>$ that are necessary to have invariant numerical predictions.

3.6.11 Genorelativity and its Isodual

Another important implication of genomathematics is the construction of yet another lifting of special relativity, this time intended for the invariant characterization of irreversible classical, quantum and gravitational processes, today known as *Santilli's genorelativity*.

Studies in the new relativity were initiated with memoir [23] of 1978[12] and continued in monographs [49,50]. The studies were then continued via the genotopies of: the background Euclidean topology [14]; the Minkowski space [15]; the Poincaré symmetry [29]; the physical laws; etc. The geno-Galilean case is treated in monographs [52,53] which appeared prior to the advent of the genodifferential calculus [14]. The relativistic case is outlined in Ref. [29].

Regrettably, we cannot review genorelativity in details to avoid a prohibitive length. For the limited scope of this presentation it is sufficient to indicate that genorelativity can be also constructed from the isorelativity of the preceding section via the lifting of the isounits into time dependent and/or nonsymmetric forms, with consequential selection of an ordering of the product to identify the selection direction of time.

Alternatively, all aspects of genorelativity can be explicitly constructed by subjecting the corresponding aspects of conventional special relativity to the dual noncanonical or nonunitary transform, as of Section 3.6.

The result is a fully invariant description of irreversible and nonconservative processes in classical mechanics, particle physics and gravitation. Note that the latter is achieved thanks to the first known admission of a nonsymmetric metric in the genotopic realization of the Minkowskian axioms, as necessary for a credible representation of irreversible gravitational events, such as the explosion of a star.

Note finally that, as it was the case for isorelativity, all distinctions between special and general relativity are lost also for genorelativity because the two relativities are again unified into one single relativity verifying the same basic axioms, and merely differentiated via different realizations of the basic unit.

As it is well known, throughout the 20-th century thermodynamics has been basically disjoint from Hamiltonian mechanics precisely because the former is strictly irreversible, e.g., for the increase of the entropy in realistic systems, while the latter is strictly reversible.

It appears that the Lie-admissible classical and operator genomechanics presented in this section change the above setting and offer, apparently for the first time, realistic possibility for an interconnection between thermodynamics and mechanics, according to studies left to the interested reader.

3.6.12 Lie-Admissible Hypertheories and their Isoduals

In this author's opinion, the biggest scientific imbalance of the 20-th century has been the treatment of biological systems (herein denoting

DNA, cells, organisms, etc.) via the mathematics, physics and chemistry developed for inanimate matter, such as that of classical and quantum mechanics.

The imbalance is due to the fact that conventional mathematics and related formulations are inapplicable for the treatment of biological systems for various reasons.

To begin, biological events, such as the growth of an organism, are irreversible. Therefore, any treatment of biological systems via reversible mathematics, physical and chemical formulations can indeed receive temporary academic acceptance, but cannot pass the test of time.

Quantum mechanics is ideally suited for the treatment of the structure of the hydrogen atom or of crystals, namely systems that are fully reversible. These systems are represented by quantum mechanics as being ageless. Recall also that quantum mechanics is unable to treat deformations because of incompatibilities with basic formulations, such as the group of rotations.

Therefore, *the rigorous application of quantum mechanics to biological structures implies that all organisms from cells to humans are perfectly reversible, rigid and eternal.*

Even after achieving the invariant formulation of irreversibility outlined in the preceding section, it is easy to see that the underlying genomathematics remains insufficient for in depth treatment of biological systems.

Recent studies conducted by Illert [56] have pointed out that the *shape* of sea shells can certainly be represented via conventional mathematics, such as the Euclidean geometry.

However, the latter is inapplicable for a representation of the *growth in time* of sea shells. Computer simulations have shown that the imposition to sea shell growth of conventional geometric axioms (e.g., those of the Euclidean or Riemannian geometries) causes the lack of proper growth, as expected, because said geometries are strictly reversible, while the growth of sea shells is strictly irreversible.

The same studies by Illert [56] have indicated the need of a mathematics that is not only structurally irreversible, but also *multi-dimensional.* As an example, Illert achieved a satisfactory representation of sea shells via the *doubling of the Euclidean reference axes*, namely, a geometry which appears to be six-dimensional.

A basic problem in accepting such a view is the lack of compatibility with our sensory perception. When holding sea shells in our hands, we can fully perceive their shape as well as their growth with our three Eustachian tubes.

In particular, our senses are fully capable of perceiving deviations from the Euclidean space, as well as the possible presence of curvature.

These occurrences pose a rather challenging problem, the achievement of a representation of the complexity of biological systems via the *most general possible mathematics* that is:

(1) is structurally irreversible (as in the preceding section);
(2) can represent deformations;
(3) is invariant under the time evolution;
(4) is multi-dimensional; and, last but not least,
(5) is compatible with our sensory perception.

A search in the mathematical literature revealed that a mathematics verifying all the above five requirements did not exist and had to be constructed from the main features of biological systems.

As an example, in their current formulations *hyperstructures* (see Ref. [96] lack a well defined left and right unit even under their *weak equalities*, they are not structurally irreversible, and they lack invariance. Consequently, they are not suitable for applications in biology.

After numerous trials and errors, a yet broader mathematics verifying the above five conditions was identified by Santilli in Ref. [14] (see also Refs. [13,47] monograph [57]; it is today known under the name of *Santilli hypermathematics*; and it is characterized by the following *hyperunits* here expressed for the lifting of the Euclidean unit

$$I = Diag.(1,1,1) \to \hat{I}^{>}(t,x,v,\psi,\partial_x\psi,\ldots) = Diag.(\hat{I}_1^{>},\hat{I}_2^{>},\hat{I}_3^{>})$$
$$= Diag.\Big[(\hat{I}_{11}^{>},\hat{I}_{12}^{>},\ldots,\hat{I}_{1m}^{>}),(\hat{I}_{21}^{>},\hat{I}_{22}^{>},\ldots,\hat{I}_{2m}^{>}),$$
$$(\hat{I}_{31}^{>},\hat{I}_{32}^{>},\ldots,\hat{I}_{3m}^{>})\Big], \quad (3.6.82a)$$
$$I = Diag.(1,1,\ldots,1) \to^{<} \hat{I}(t,x,v,\psi,\ldots) = Diag.(^{<}\hat{I}_1,^{<}\hat{I}_2,^{<}\hat{I}_3)$$
$$= Diag.\Big[(^{<}\hat{I}_{11},^{<}\hat{I}_{12},\ldots,^{<}\hat{I}_{1m}),(^{<}\hat{I}_{21},^{<}\hat{I}_{22},\ldots,^{<}\hat{I}_{2m}),$$
$$(^{<}\hat{I}_{31},^{<}\hat{I}_{32},\ldots,^{<}\hat{I}_{3m})\Big], \quad (3.6.82b)$$

with corresponding *ordered hyperproducts to the right and to the left*

$$A > B = A \times \hat{T}^{>} \times B, A < B = A \times^{<} \hat{T} \times B, \quad (3.6.83a)$$
$$\hat{I}^{>} > A = A > \hat{I}^{>} = A, \quad {}^{<}\hat{I} < AA <^{<} \hat{I} = A, \quad (3.6.83b)$$
$$\hat{I}^{>} = (^{<}\hat{I})^{\dagger} = 1/\hat{T}^{>}, \quad (3.6.83c)$$

the only difference with genoforms is that hyperproduct are now multivalued, where all operations are ordinary (and not weak as in conventional hyperstructures).

As one can see, the above mathematics *is not 3m-dimensional, but rather it is 3-dimensional and m-multi-valued.*

Such a feature permits the increase of the reference axes, e.g., for $m = 2$ we have six axes as used by Illert [56], while achieving compatibility with our sensory perception because at the abstract, realization-free level hypermathematics characterized by hyperunit is indeed 3-dimensional.

The various branches of hypermathematics (hypernumbers, hyperspaces, hyperalgebras, etc.) can be constructed via mere compatibility arguments with the selected hyperunit (see monograph [57] for brevity).

A main difference of hypermathematics with the preceding formulations is that in the latter the product of two numbers is indeed generalized but single-valued, e.g., $2 > 3 = 346$.

By comparison, in hypermathematics the product of two numbers yields, by conception, a *set of values*, e.g.,

$$2 > 3 = (12, 341, 891, \ldots). \tag{3.6.84}$$

Such a feature appears to be necessary for the representation of biological systems because the association of two atoms in a DNA (mathematically representable with the hypermultiplication) can yield an organ with an extremely large variety of atoms.

This feature serves to indicate that the biological world has a complexity simply beyond our imagination, and that studies of biological problems conducted in the 20-th century, such as attempting an understanding the DNA code via numbers dating back to biblical times, are manifestly insufficient.

The *isodual hypermathematics* can be constructed via the use of isoduality. The following intriguing and far reaching aspect emerges in biology. Until now we have strictly used isodual theories for the sole representation of antimatter.

As shown in the qupted literature, the complexity of biological systems is such to require the use of both hyperformulations and their isodual for consistent and quantitative representations, as it is the case of bifurcations.

In turn, the above occurrence implies that the intrinsic time of biological structure, here referred to as *hyperbiotime*, is expected to be of a complexity beyond our comprehension because not only multivalued, but also inclusive of all four directions of time.

In conclusion, the achievement of invariant representations of biological structures and their behavior can be one of the most productive frontiers of science with far reaching implications for other branches, including mathematics, physics and chemistry.

As an illustration, the achievement of a mathematically consistent representation of the non-Newtonian propulsion of sap in trees up to big heights will automatically provide a model of *geometric propulsion*, namely propulsion caused via the alteration of the local geometry without any external applied force.

3.7 EXPERIMENTAL VERIFICATIONS AND INDUSTRIAL APPLICATIONS OF HADRONIC MECHANICS

3.7.1 Experimental Verifications of Lie-Isotopic Theories

Nowadays, the Lie-isotopic branch of hadronic mechanics has clear experimental verifications in classical physics, particle physics, nuclear physics, chemistry, superconductivity, astrophysics and cosmology, among which we quote the following representative verifications:

⋆ The first and only known invariant representation of classical closed non-hamiltonian systems [51].

⋆ An axiomatically correct formulation of special relativity in terms of the proper time by T. Gill and his associates [202]–[206].

⋆ The proof by Aringazin [192,197] of the "universality" of Isoaxiom III, namely, its capability of admitting as particular cases all available anomalous time dilations via different expansions in terms of different quantities and with different truncations.

⋆ The exact representation of the anomalous behavior of the mean-lives of unstable particles with speed by Cardone et al [110,111] via Isoaxiom III of isorelativity.

⋆ The exact representation of the experimental data on the Bose-Einstein correlation by Santilli [112] and Cardone and Mignani [113] under the exact Poincaré-Santilli isosymmetry.

⋆ The invariant and exact validity of the Minkowski-Santilli isogeometry within the hyperdense medium in the interior of hadrons by Arestov et al. [120].

⋆ The achievement of an exact confinement of quarks by Kalnay [216] and Kalnay and Santilli [217] thanks to incoherence between the external and internal Hilbert spaces.

⋆ The proof by Jannussis and Mignani [186] of the convergence of isotopic perturbative series when conventionally divergent based on the

property for all isotopic elements used in actual models $\hat{T} \ll 1$, thus implying that perturbative expansions which are divergent when formulated with the conventional associative product $A \times B$ become convergent when re-expressed in terms of the isoassociative product $A\hat{\times}B = A \times \hat{T} \times B$, since all known isounits have resullted to be much bigger than one, thus implying isotopic elements with much smaller then one, $|\hat{I}| >> 1, |\hat{T}| << 1$.

⋆ The initiation by Mignani [182] of a *nonpotential-nonunitary scattering theory* reformulated by Santilli [55] as isounitary on iso-Hilbert spaces over isofields, thus recovering causality and probability laws for the first known description of *scattering among extended particles* with consequential *contact-nonpotential interactions.*

⋆ The first and only known exact and invariant representation by Santilli [114,115] of nuclear magnetic moments and other nuclear characteristics thanks to the mutation of particle characteristics caused by nonpotential interactions, which exact representation has escaped quantum mechanics for about one century.

⋆ The first and only known model by Animalu [170] and Animalu and Santilli [116] of the Cooper pair in superconductivity with an *attractive force between the two identical electrons* in excellent agreement with experimental data.

⋆ The exact representation via isorelativity by Mignani [118] of the large difference in cosmological redshifts between quasars and galaxies when physically connected;

⋆ As it is well known, the exact representation of molecular data could not be achieved by quantum chemistry for about one century because missing about 2% of the data. One of the most important experimental verifications of hadronic mechanics has been *the achievement of a main objective for which it was built: the first exact and invariant representation from unadulterated first axiomatic principles of all experimental data of the hydrogen, water and other molecules, including the missing* 2%.

The representation was achieved by R. M. Santilli and D. D. Shillady [125,126] (see also the comprehensive treatment in monograph [59]) via the use of nonrelativistic hadronic mechanics based on a simple isounit in which, as one can see, *there are no free parameters for ad hoc fits of experimental data*, but only a quantitative description of wave-overlappings, with isorelativistic extension characterized by 4-dimensional isounits.

The above studies confirmed the existence of contact nonpotential interactions at the most ultimate level of nature, that of elementary particles such as the electrons, because the representation of the missing

2% was proved to depend crucially on interactions beyond the capability of a Hamiltonian.

We should recall that, due to the missing 2% and other insufficiencies to represent data, quantum chemistry has lately introduced the so-called "screenings of the Coulomb potential" into forms of the type

$$V^*(r) = e^{k\times r} \times \frac{q_1 \times q_2}{r}, \quad k \in R, \tag{3.7.1}$$

that do indeed improve the representation of experimental data.

However, as stressed in monograph [59], the Coulomb potentials is one of the most fundamental invariants of quantum chemistry. It is then easy to prove that the map from the conventional to screened Coulomb potentials is nonunitary. Consequently, screened Coulomb potentials are outside the class of equivalence of quantum chemistry.

In any case, the quantization of the energy is solely possible for the conventional Coulomb law, and does not exist any more for its screening.

It then follows that the continued use in chemistry of the word "quantum" for screened Coulomb potential is a academic politics deprived of scientific content.

Ref. [59], that screened Coulomb potentials are *particular cases* of the Santilli-Shillady strong valence force, trivially, because hadronic chemistry is nonunitary by conception, thus admitting an infinite class of liftings of the Coulomb law, although this time no longer masqueraded under the name of quantum chemistry.

It should be noted that, whether in valence coupling or not, *electrons repel each other.* Also, the total electric or magnetic forced between neutral atoms are identically null, while exchange, van der Waals and other forces of current use in chemistry are basically insufficient to represent the strength of molecular bonds [59].

Studies [125,126] achieved the *first and only known strongly attractive force between pairs of identical valence electrons in singlet coupling at short distance*, proved to originate from nonlocal, nonlinear and nonpotential interactions due to deep overlappings of electron's wavepackets.[13]

The achievement of a deeper understanding of molecular bonds has far reaching scientific implications. In fact, it confirms that *nonlocal, nonlinear and non potential interactions exist in all interior problems at large, such as the structure of hadrons, nuclei and stars, and imply basically new structure models in which the constituents are isoparticles (irreducible representation of the Poincaré-Santilli isosymmetry), rather than conventional particles in vacuum.*

⋆ The original proposal of 1978 to build hadronic mechanics [38] included the proof that *all* characteristics of the π° meson can be represented in an exact and invariant way via a bound state of one *isoelectron*

$\hat{e}^-$ and its antiparticle $\hat{e}^+$ under conditions of mutual penetration within 10^{-13} cm,

$$\pi^o = (\hat{e}^-, \hat{e}^+)_{HM}; \tag{3.7.2}$$

the $\pi^\pm$ meson can be represented via a bound state of three isoelectrons,

$$\pi^\pm = (\pi^o, e^\pm)_{HM} = (\hat{e}^-, \hat{e}^\pm, \hat{e}^+)_{HM}; \tag{3.7.3}$$

and the remaining mesons can be similarly identified as hadronic bound states of massive isoparticles produced free in the spontaneous decays with the lowest mode.

$\star$ Following the prior achievement of the isotopies of the SU(2) spin [28], Ref. [214] of 1990 achieved for the first time the exact and invariant representation of *all* characteristic of the neutron as a nonrelativistic hadronic bound state of one *isoproton* and one *isoelectron* according to Rutherford's original conception,

$$n = (\hat{p}^+, \hat{e}^-)_{HM}. \tag{3.7.4}$$

Under spontaneous decay, the isoparticles constituting the neutron reacquire their conventional particle configuration,

$$n \to p^+ + e^- + \bar{\nu}, \tag{3.7.5}$$

thus confirming the new law of hadronic mechanics already proved for the structure model of mesons with physical constituents, according to which *the actual particles constituting unstable hadrons can be identified in the massive constituents of the spontaneous decays with the lowest mode.*

The relativistic extension of the model was reached in Ref. [30], jointly with the first invariant isotopies of Dirac's equation. Subsequently, it was easy to see that all unstable baryons can be considered as hadronic bound states of massive isoparticles, again those generally in the spontaneous decays with the lowest modes.

Compatibility of the above new structure models of hadrons and $SU(3)$-color theories was achieved via the assumption that *quarks are composite*, a view first expressed by Santilli [225] in 1981, and the use of hypermathematics with *different units for different hadrons* [31].

This approach essentially yields the hyperrealization $\hat{SU}(3)$ in which *composite hyperquarks* are characterized by the *multivalued* isounit with isotopic element $\hat{T} = (\hat{T}_u, \hat{T}_d, \hat{T}_s)$, resulting in *hypermultiplets* of mesons, baryons, etc.

The compatibility of this hypermodel with conventional theories is established by the isomorphism between conventional $SU(3)$ and the

hyper-$\hat{SU}(3)$, the latter merely being a broader *realization* of the axioms of the former.

The significance of this hypermodel is illustrated by the fact that all perturbative series which are divergent for $SU(3)$ are turned into convergent forms because $\hat{T}_u, \hat{T}_d, \hat{T}_s \ll 1$ under which, as indicated earlier, all divergent perturbative series expressed in terms of the conventional product $A \times B$ become convergent when re-expressed in terms of the hyperproduct $A \times \hat{T} \times B$.

Compatibility with the structure model of hadrons with ordinary massive constituents is evident from the fact that *quarks result to be composed of ordinary massive isoparticles.*

It should be recalled that none of the above hadronic models are possible for quantum mechanics, e.g., because the representation of the rest energies of hadrons would require "positive binding energies"[14]

These and other objections were resolved by the covering hadronic mechanics due to the *isorenormalizations* (also called *mutations*) of the rest energies and other features of the constituents caused by nonlocal, nonlinear and nonpotential interactions.

⋆ Predictably, the reduction of the neutron to a bound state of an isoproton and an isoelectron has permitted a *new structure model of nuclei as hadronic bound states of isoprotons and isoelectron* [113,114], with the conventional quantum models based on protons and neutrons remaining valid in first approximation.

According to quantum mechanics, the ground state of the deuteron

$$D = (p^+, n^\circ)_{QM}, \tag{3.7.6}$$

should have spin zero since it is a *two body system* for which the most stable state is the *singlet.* However, as is well known, experimental data have established that the deuteron ground state has spin 1. This value can only be represented in quantum mechanics as a *triplet coupling* (with parallel spin) that, however, is highly unstable for hadrons at short mutual distances.[15] Consequently, one of the historical insufficiencies of quantum mechanics in the 20-th century has been the inability to understand the spin of the deuteron.

The new *isostructure model of nuclei* has permitted the first known understanding of the reason why the deuteron ground state has spin 1 since it is a *three-body system* for hadronic mechanics,

$$D = (\hat{p}^+, \hat{e}^-, \hat{p}^+)_{HM}, \tag{3.7.7}$$

thus admitting 1 as the lowest possible angular momentum.

The isonuclear model also permitted the exact and invariant representation of all remaining characteristics of the deuteron, as well as of other

nuclear features that have remained unexplained for about one century, such as why the correlation among nucleons is restricted to pairs only.

In particular, the old process of keep adding potentials to the nuclear force without ever achieving an exact representation of nuclear data has been truncated by hadronic mechanics, due to the emergence of a component of the nuclear force that is nonlocal, nonlinear and nonpotential due to the mutual penetration of the charge distribution of nucleons established by nuclear data (e.g., via the ratio between nuclear volumes and the sum of the volume of the nucleon constituents).

⋆ In astrophysics, hadronic mechanics has permitted the formulation of deeper models of neutron stars as being composed of isoprotons and isoelectrons with deeper understanding of a number of events, such as the explosion of stars.

⋆ The exact representation by Santilli [117] of the internal blue- and red-shift of quasar's cosmological redshift.

⋆ The elimination of the need for a missing mass in the universe by Santilli [34] thanks to isoaxiom V.

3.7.2 Experimental Verifications of Lie-Admissible Theories

Hadronic mechanics has reached an additional number of experimental verifications for its broader Lie-admissible branch, among which we note:

⋆ The first identification of the connection between Lie-admissibility and supersymmetries by Adler [211];

⋆ The first and only known optimization of the shape of extended objects moving within resistive media via the optimal control theory [14] following the achievement of the universality of the genoaction for all possible resistive forces;

⋆ The experimental validity of genotheories in classical mechanics is established by the direct representation of all nonconservative and irreversible Newtonian systems by Hamilton-Santilli genoequations via a simple algebraic calculation with Eq. (3.6.64).

⋆ The experimental validity of genotheories in particle physics is established by the fact that all dissipative nuclear models represented via imaginary potentials in the Hamiltonian and other nonunitary theories can be *identically* reformulated in terms of the genotopic branch of hadronic mechanics, while preserving the representation of experimental data identically.

⋆ Above all, genomathematics and its related formulations have indeed achieve the objective for which they were built, namely, *an invariant representation of irreversibility at all levels, from Newtonian to elementary*

particle physics. Such an objective can be achieved via the following main rules:

(i) Identify the classical origin of irreversibility in the contact nonpotential forces among extended particles, much along the historical teaching of Newton [1], Lagrange [2] and Hamilton [3];

(ii) Represent said nonpotential forces via real-valued, nowhere singular, non-symmetric genounits and construct a mathematics which is structurally irreversible for all reversible Hamiltonians in the sense indicated earlier;

(iii) Achieve identical reformulation [4,34] of Hamilton's equations with external terms with a consistent algebra in the brackets of the time evolution of Lie-admissible type according to Albert [7];

(iv) Complement the latter mechanics with the underlying genosymplectic geometry, permitting the mapping of the classical formulations into operator formulations preserving said Lie-admissible character; and

(v) Identify the origin of irreversibility in the most elementary layers of nature, such as elementary particles in their irreversible motion in the interior of stars.

Note that a requirement for the above rules is the *nonconservation* of the energy and other physical quantities, which is readily verified by the geno-Hamilton equations (4.6.64) due to the lack of anti-symmetric character of brackets (A, B) of the time evolution.

$\star$ An important application of genomechanics has been done by J. Ellis *et al.* [122] who have shown that its Lie-admissibility provides an axiomatically consistent, direct representation of irreversibility in interior quasars structures.

In closing, it is hoped that Lagrange's and Hamilton's legacy of representing irreversibility with the external terms in their analytic equations is seriously considered because it implies covering theories with momentous advances in mathematics and all quantitative sciences.

$\star$ The reader should be aware that the complexity of biological structures requires the use of hypermathematics as well as its isodual, e.g., for quantitative interpretations of bifurcations. In fact, a quantitative interpretation of bifurcations, e.g., in sea shells, requires *four different hypertimes and their isoduals*, as indicated below.

In turn, this is sufficient to illustrate the departure from conventional notions of a relativity suitable for quantitative studies on biological systems, known as *hyperrelativity and its isodual* [57]. In fact, such new relativity requires the most general notion of numbers, those with a multi-valued hyperunits characterized by an ordered, yet unlimited num-

ber of non-Hermitean elements, with consequential most general possible geometries and mechanics, plus their isoduals.

This results in an ordered, yet unlimited variety of spaces and their isoduals all coexisting in our three-dimensional Euclidean space, plus corresponding, equally co-existing varieties of time. There is little doubt that such features imply dramatic departures from the simplicity, thus insufficiency, of special relativity.

An illustration of the complexity of hyperformulations and corresponding hyperrelativity is given by the four different notions of hypertime which are needed for the description of complex biological processes, such as bifurcations in seashells, all in a coexisting form and each having a multi-valued character: motion forward in future time $\hat{t}^{>}$; motion backward in past time ${}^{<}\hat{t}$; motion forward from past time ${}^{<}\hat{t}^{d}$; and motion backward from future time $\hat{t}^{>d}$. The necessity of these four directions in time also illustrates the need of the isodual map.

A new conception of biological systems, which constitute a truly fundamental advance over rather simple prior conceptions, has been recently proposed by Erik Trell (see Ref. [164] and contributions quoted therein). It is based on representative blocks which appear in our space to be next to each other, thus forming a cell or an organism, while having in reality hypercorrelations, thus having the structure of hypernumbers, hypermathematics and hyperrelativity, with consequential descriptive capacities immensely beyond those of pre-existing, generally single-valued and reversible biological models. Regrettably, we cannot review Trell's new hyperbiological model to avoid an excessive length, and refer interested readers to the original literature [164].

3.7.3 Industrial Applications to New Clean Energies and Fuels

In closing, it should be indicated that the studies on isotopies have long passed the level of pure scientific relevance, because they now have direct *industrial applications* for new clean energies and fuels so much needed by our contemporary society.

As an **illustration at the particle level**, the synthesis of the neutron from one proton and one electron according to Rutherford, Eq. (3.7.4), has been experimentally confirmed by C. Borghi *et al.* [123] to occur also at low energies, although under a number of conditions studied in monograph [58], and additional tests are under way.

Once Rutherford's original conception of the neutron is rendered acceptable by hadronic mechanics, the electron becomes a physical constituent of the neutron (although in a mutated state). In this case, hadronic mechanics predicts the capability of stimulating the decay of

the neutron via photons with suitable resonating frequencies and other means, thus implying the first known form of "hadronic energy" [58] (that is, energy originating in the structure of hadrons, rather than in their nuclear aggregates), which has already been preliminarily confirmed via an experiment conducted by N. Tsagas *et al.* [124] (see monograph [58] for scientific aspects and the web site www.betavoltaic.com for industrial profiles).

As an **illustration at the nuclear level**, hadronic mechanics predicts a basically new process for controlled nuclear syntheses which is dramatically different than both the "hot" and the "cold" fusions is currently also under industrial development, which condition prohibits its disclosure in this memoir.

As an **illustration at the molecular level**, the deeper understanding of the structure of *molecules* has permitted the discovery and experimental verifications in Ref. [27] (see also the studies by Aringazin and his associates in Refs. [128–130] and monograph [59]) of the new chemical species of *magnecules* consisting of clusters of individual atoms, dimers and molecules under a new bond originating from the electric and magnetic polarization of the orbitals of atomic electrons.

In turn, the new species of magnecules has permitted the industrial synthesis of new fuels without hydrocarbon structure, whose combustion exhaust resolves the environmental problems of fossil fuels by surpassing current exhaust requirement by the U. S. Environmental Protection Agency without catalytic converter or other exhaust purification processes (see monograph [59] for scientific profiles and the web site www.magnegas.com for industrial aspects).

Additional important applications of isotopies have been studied by A. O. E. Animalu, A. K. Aringazin, R. Ashlander, C. Borgji, F. Cardone, J. Dunning-Davies, F. Eder, J. Ellis, J. Fronteau, M. Gasperini, T. L. Gill, J. V. Kadeisvili, A. Kalnay, N. Kamiya, S. Keles, C. N. Ktorides, M. G. Kucherenko, D. B. Lin, C.-X. Jiang, A. Jannussis, R. Mignani, M. R. Molaei, N. E. Mavromatos, H. C. Myung, M. O. Nishioka, D. V. Nanopoulos, S. Okubo, D. L. Rapoport, D. L. Schuch, D. S. Sourlas, A. Tellez-Arenas, Gr. Tsagas, N. F. Tsagas, E. Trell, R. Trostel, S. Vacaru, H. E. Wilhelm, W. Zachary, and others. These studies are too numerous to be effectively reviewed in this memoir.

Notes

1 In order to distinguish Eq. (3.1.2) from those used in the 20-th century, those without external terms, the latter being known as the *truncated Lagrange equations.*

2 We recall from Chapter 1 that several types of multiplications are used in hadronic mechanics. Therefore, to avoid confusions, the literature in the field uses different symbols for their differentiation.

3 In this way the calculation of the value of an *isodeterminant* cancels out all multiplications by $\hat{I}$ except the last, thus correctly producing an isonumber.

4 Even within the arena of the original conception (propagation of point particles and electromagnetic waves in vacuum) there remain doubts on the exact validity of special relativity due to its inability to admit the aether as a universal medium that is necessary not only for the propagation of the electromagnetic waves, but also for the very existence of elementary particles, since the latter are known to be mere oscillations of the aether with a known frequency. Consequently, *when the aether is assumed as the universal medium with consequential privileged reference frame, special relativity has no arena whatever of exact validity.* In this monograph we do not consider these aspects and assume special relativity as being valid in the sole arena of original conception on mere grounds of pragmatic validity. The reader should however bear in mind that the aether is, by far, the most important frontier of the physics of the third millennium with potential advances beyond our most vivid imagination, such as new inextinguishable sources of energy, communications at speeds much bigger than that of the conventional (transversal) electromagnetic waves via conceivable *longitudinal* waves propagating through the aether, and others.

5 Even within the arena of original conception, there remain a number of unresolved epistemological and other aspects. They are ignored here because they have no implication for the content of this chapter, the representation of extended, nonspherical and deformable particles under Hamiltonian of a non-Hamiltonian interaction or the propagation of electromagnetic waves within physical media.

6 Recall that the $SU(3)$-color theory provides the final Mendeleev-type classification of hadrons. However, on scientific grounds outside aca-

demic interests, quarks are purely mathematical representations of a purely mathematical unitary symmetry defined on a purely mathematical internal complex unitary space; quarks cannot be defined in our spacetime due to the Orafearthaigh theorem; and quarks cannot possibly have any gravity, because gravity can solely be defined for inertial masses, that is, for masses characterized by the second order Casimir invariant of the Poincaré symmetry, while it is known by experts to qualify as such that quarks cannot be identified even marginally with the Poincaré symmetry. Therefore, for quark believers to prove themselves to be scientists, they should provide a rigorous prove of the reason their bodies do not float in space due to lack of gravity.

7 More technically, a generally nonassociative algebra U with elements $a, b, c, \ldots$ and abstract product ab is said to be Lie-admissible when the attached algebra U^- characterized by the product $[a, b] = ab - ba$ verifies the *Lie axioms*

$$[a, b] = -[b, a],$$

$$[[a, b], c] + [[b, c], a] + [[c, b], a] = 0.$$

8 More technically, a generally nonassociative algebra U with elements a, b, c, $\ldots$ and abstract product ab is said to be Jordan-admissible when the attached algebra U^+ characterized by the product $\{a, b\} = ab + bA$ verifies the *Jordan axioms*

$$\{a, b\} = \{b, a\},$$

$$\{\{a, b\}, a^2\} = \{a, \{b, a^2\}\}.$$

In classical realizations of the algebra U the first axiom of Jordan-admissibility is verified but the second is generally violated, while in operator realizations both axioms are generally verified.

9 The author would appreciate any indication of operator formulations of nonconservative forces under the conditions verified by hadronic mechanics shown in the next section, namely, that *nonconserved* quantities, such as the Hamiltonian, are *Hermitean* as a necessary condition to be *observable.*

10 Note that, to verify the condition of non-Hermiticity, the time genounits can be *complex valued.*

11 Due to decades of protracted use it is easy to predict that physicists and mathematicians may be tempted to treat the Lie-admissible branch of hadronic mechanics with conventional mathematics, whether

in part or in full. Such a posture would be fully equivalent, for instance, to the elaboration of the spectral emission of the hydrogen atom with the genodifferential calculus, resulting in an evident nonscientific setting.

12 This memoir contains the first generalization of Noether's Theorem on Lie symmetries and conservation laws to Lie-admissible symmetries and nonconservation laws. The indication by interested colleagues of any prior representation of nonconservation laws via any symmetry would be appreciated.

13 The word "strongly" is not evidently referred to strong interactions, but to the strength of the new attractive valence bond.

14 Unlike similar occurrences in nuclear physics, the rest energy of hadronic bound states is much bigger than the sum of the rest energies of the constituents, thus requiring the indicated positive binding energy that is anathema in quantum mechanics because Schrödinger's equations becomes inconsistent.

15 The original proposal of hadronic mechanics [38] suggested the *gear model* for the understanding of couplings of extended particles at short mutual distances. In fact, gears can only couple with antiparallel spins (singlet coupling), while their coupling with parallel spins (triplet couplings) causes extreme repulsive forces, assuming that rotations are allowed.

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Chapter 4

ANTIGRAVITY AND SPACETIME MACHINES

4.1 THEORETICAL PREDICTIONS OF ANTIGRAVITY

4.1.1 Introduction

Antigravity is one of the most ancient dreams of mankind, that has stimulated the imagination of many researchers, from various engineering fields (see, e.g., Refs. [1,2] that also list patents), to the most advanced branches of physics (see the prediction of antigravity in supergravity theories [3,4] and proceedings [5] for other more recent approaches).

An experiment on the gravity of antiparticles was considered by Fairbank and Witteborn [6] via low energy positrons in vertical motion. Unfortunately, the measurements could not conclusive because of interferences from stray fields, excessive upward kinetic energy of the positrons and other reasons.

Additional data on the gravity of antiparticles are those from the LEAR machine on antiprotons at CERN [7], although these data too are inconclusive because of the excessive energy of the antiprotons and other factors, including the care necessary to extend the gravity of antiprotons to all antiparticles pointed out in Chapter 2, the proved impossibility for quarks to experience gravity, let alone antigravity, and other factors.

Additional experiments on the gravity of antiparticles are based on neutron interferometry, such as the experiments by Testera [8], Poggiani [9] and others. These experiments are extremely sensitive and, as such, definite and conclusive results continue to be elusive. In particular, the latter experiments too deal with antiprotons, thus inheriting

the ambiguities of quark conjectures with respect to gravity, problems in the extension to other antiparticles, and other open issues.

All further data on the gravity of antiparticles known to this author are of indirect nature, e.g., via arguments based the equivalence principle (see, e.g., Ref. [10] and papers quoted therein). Note that the latter arguments do not apply under isoduality and will not be considered further.

A review on the status of our knowledge prior to isodual theories is available in Ref. [11], that includes an outline of the arguments against antigravity, such as those by Morrison, Schiff and Good. As we shall see, the latter arguments too cannon even be formulated under isodualities, let alone be valid.

We can therefore conclude by stating that at this writing there exists no experimental or theoretical evidence known to this author that is resolutory and conclusive either against or in favor of antigravity.

One of the most intriguing predictions of isoduality is the existence of *antigravity* conceived as a reversal of the gravitational attraction, first theoretically submitted by Santilli in Ref. [12] of 1994.

The proposal consists of an experiment that is feasible with current technologies and permits a definite and final resolution on the existence or lack of the existence of the above defined antigravity.

These goals were achieved by proposing the test of the gravity of positrons in horizontal flight on a vacuum tube. The experiment is resolutory because, for the case of a 10 m long tube and very low kinetic energy of the positrons (of the order of μeV), the displacement of the positrons due to gravity is sufficiently large to be visible on a scintillator to the naked eye.

Santilli's proposal [12] was studied by the experimentalist Mills [13] to be indeed feasible with current technology, resolutory and conclusive.

The reader should be aware from these introductory lines that *the prediction of antigravity exists, specifically, for the isodual theory of antimatter and not for conventional treatment of antiparticles.*

For instance, no prediction of antigravity can be obtained from Dirac's hole theory or, more generally, for the treatment of antimatter prior to isoduality, that solely occurring in second quantization.

Consequently, antigravity can safely stated to be the ultimate test of the isodual theory of antimatter.

In this chapter, we study the prediction of antigravity under various profiles, we review the proposed resolutory experiment, and we outline some of the far reaching implications that would follow from the possible experimental verification of antigravity, such as the consequential exis-

tence of a fully *Causal Time Machine*, although not for ordinary matter, but for an isoselfdual combination of matter and antimatter.

4.1.2 Newtonian and Euclidean Prediction of Antigravity

It is important to show that the prediction of antigravity can be first formulated at the most primitive possible level, that of Newtonian mechanics and its isodual. All subsequent formulations will be merely consequential.

The current theoretical scene on antigravity is dominated by the fact that, as it is well known, the Euclidean, Minkowskian and Riemannian geometries offer no realistic possibility to reverse the sign of a gravitational mass or of the energy of the gravitational field.

Under these conditions, existing theories can at best predict a decrease of the gravitational force of antiparticles in the field of matter (see Ref. [11] for a review of these conventional studies). In any case the decreased interaction, as such, remains attractive.

Isodual mathematical and physical theories alter this scientific scene. In fact, antigravity is predicted by the interplay between the classical Euclidean geometry and its isodual. The resulting prediction of antigravity persists at all levels, that is, for flat and curved spaces and for classical or quantum formulations, in a fully consistent way without known internal contradictions.

Also, antigravity is a simple consequence of Corollary 2.3.1 according to which the observed trajectories of antiparticles under a magnetic field are the *projection* in our spacetime of inverted trajectories in isodual spacetime.

Once these aspects are understood, the prediction of antigravity becomes so simple to appear trivial. In fact, antigravity merely originates from the projection of the gravitational field of matter in that of antimatter and vice-versa. We therefore have the following:

PREDICTION 4.1.1 [11,15]: *The existence of antigravity, defined as a gravitational repulsion experienced by isodual elementary particles in the field of matter and vice-versa, is a necessary consequence of a consistent classical description of antimatter.*

Let us begin by studying this prediction in Euclidean and isodual Euclidean spaces. Consider the Newtonian gravitational force of two conventional (thus, positive) masses m_1 and m_2

$$F = -G \times m_1 \times m_2/r < 0, \quad G, m_1, m_2 > 0, \tag{4.1.1}$$

where G is the gravitational constant and the minus sign has been used for similarity with the Coulomb law.

Within the context of conventional theories, the masses m_1 and m_2 remain positive irrespective of whether referred to a particle or an antiparticle. This yields the well known "universal law of Newtonian attraction", namely, the prediction that the gravitational force is attractive irrespective of whether for particle-particle, antiparticle-antiparticle or particle-antiparticle.

Again, the origin of this prediction rests in the assumption that antiparticles exist in our spacetime, thus having positive masses, energy and time. Under isoduality the situation is different. For the case of antiparticle-antiparticle under isoduality we have the different law

$$F^d = -G^d \times^d m_1^d \times^d m_2^d/^d r^d > 0, \quad G^d, m_1^d, m_2^d < 0. \tag{4.1.2}$$

But this force exists in the different isodual space and is defined with respect to the negative unit -1. Therefore, isoduality correctly represents the *attractive* character of the gravitational force between two isodual particles.

The case of particle-antiparticle under isoduality requires the *projection* of the isodual particle in the space of the particle (or vice versa), and we have the law

$$F = -G \times m_1 \times m_2^d/r > 0, \tag{4.1.3}$$

that now represents a *repulsion*, because it exists in our spacetime with unit $+1$, and it is opposite to force (4.1.1). This illustrates antigravity as per Prediction 4.1.1 when treated at the primitive Newtonian level.

Similarly, if we project the particle in the spacetime of the antiparticle, we have the different law

$$F^d = -G^d \times^d m_1^d \times^d m_2/^d r^d < 0, \tag{4.1.4}$$

that also represents *repulsion* because referred to the unit -1.

We can summarize the above results by saying that *the classical representation of antiparticles via isoduality renders gravitational interactions equivalent to the electromagnetic ones, in the sense that the Newtonian gravitational law becomes equivalent to the Coulomb law, thus necessarily including both attraction and repulsions.*

The restriction in Prediction 4.1.1 to "elementary" isodual particles will soon turn out to be crucial in separating science from its political conduct, and *de facto* restricts the experimental verification of antigravity to positrons in the field of Earth.

Note also that Prediction 4.1.1 is formulated for "isodual particles" and *not* for antiparticles. This is due to the fact indicated in preceding

sections that, according to current terminologies, antiparticles are defined in our spacetime and have positive masses, energy and time. As such, no antigravity of any type is possible for antiparticles as conventionally understood.

4.1.3 Minkowskian and Riemannian Predictions of Antigravity

It is important to verify the above prediction at the classical relativistic and gravitational levels.

Let $M(x,\eta,R)$ be the conventional Minkowskian spacetime with coordinates $x=(r,t)$ (as a column) and metric $\eta=Diag.(1,1,1,-1)$ over the field of real numbers $R(n,+,\times)$ with unit $I=Diag.(1,1,1,1)$. The *Minkowski-Santilli isodual space* [16] is given by (Section 2.2.8)

$$M^d(x^d,\eta^d,R^d),\ \ x^d=-x^t,\ \ \eta^d=Diag.(-1,-1,-1,+1), \qquad (4.1.5a)$$

$$I^d=Diag.(-1,-1,-1,-1). \qquad (4.1.5b)$$

The *isodual electromagnetic field* on $M^d(x^d,\eta^d,R^d)$ is given by

$$F^d_{\mu\nu}=\partial^d_\nu A^d_\mu-\partial^d_\mu A^d_\nu=-F^d_{\nu\mu},\quad \mu,\nu=1,2,3,4, \qquad (4.1.6)$$

with *isodual energy-momentum tensor*

$$T^d_{\mu\nu}=(1^d/^d4^d\times m^d)\times^d[F^{(}d\alpha_\mu\times^d F^d_{\alpha_\nu}$$

$$+(1^d/^d4^d)\times^d g^d\times^d F^d_{\alpha\beta}\times^d F^{d\alpha\beta}]=-T^t_{\nu\mu}, \qquad (4.1.7)$$

where g is a known constant depending on the selected unit (whose explicit value is irrelevant for this study). Most importantly, the fourth component of the isodual energy-momentum tensor is negative-definite,

$$T^d_{00}<0. \qquad (4.1.8)$$

As such, antimatter represented in isodual Minkowski geometry has negative-definite energy, and other physical characteristics, and evolves backward in time. It is an instructive exercise for the interested reader to prove that the results of the Newtonian analysis of the preceding section carry over in their entirety to the Minkowskian formulation [16].

Consider now a Riemannian space $\mathcal{R}(x,g,R)$ in (3+1)-dimensions with spacetime coordinates x and metric $g(x)$ over the reals R with basic unit $I=diag.(1,1,1,1)$ and related Riemannian geometry as presented, e.g., in Refs. [10,17]. As outlined in Section 2.1.7, the *isodual iso-Riemannian spaces* are given by

$$\mathcal{R}^d(x^d,g^d,R^d):x^d=-x^t,g^d(x^d)=-g^t(-x^t), \qquad (4.1.9a)$$

$$I^d = Diag.(-1, -1, -1, -1). \tag{4.1.9b}$$

Recall that a basic drawback in the use of the Riemannian geometry for the representation of antiparticles is the positive-definite character of its energy-momentum tensor.

In fact, this character causes unsolved inconsistencies at all subsequent levels of study of antimatter, such as lack of a consistent quantum image of antiparticles.

These inconsistencies are resolved *ab initio* under isoduality. In fact, the isodual Riemannian geometry is defined over the isodual field of real numbers R^d for which the norm is negative-definite (Section 2.2.1).

As a result, all quantities that are positive in Riemannian geometry become negative under isoduality, thus including the energy-momentum tensor. In particular, energy-momentum tensors in the Riemannian geometry are given by relativistic expression (2.1.49*i*) and, as such, they remain negative-definite when treated in a Riemannian space.

It then follows that in the isodual Riemannian treatment of the gravity of antimatter, all masses and other quantities are negative-definite, including the *isodual curvature tensor*, Eq. (2.1.49*c*).

Despite that, the gravitational force between antimatter and antimatter remain *attractive*, because said negative curvature is measured with a negative unit.

As it was the case at the preceding Euclidean and Minkowskian levels, the isodual treatment of the gravitation of matter-antimatter systems requires its projection *either* in our spacetime *or* in the isodual spacetime. This again implies a *negative curvature in our spacetime* [16] resulting in Prediction 4.1.1 of antigravity at the classical Riemannian level too.

4.1.4 Prediction of Antigravity from Isodual Einstein's Gravitation

Einstein's gravitation is generally defined (see, e.g., Ref. [10]) as the reduction of gravitation in the exterior problem in vacuum to pure curvature in a Riemannian space $\mathcal{R}(x, g, R)$ with local spacetime coordinates x and metric $g(x)$ over the field of real numbers R *without a source*, according to the celebrated field equations

$$G_{\mu\nu} = R_{\mu\nu} - g_{\mu\nu} \times R/2 = 0, \tag{4.1.10}$$

where $G_{\mu\nu}$ is generally referred to as the *Einstein tensor*, $R_{\mu\nu}$ is the *Ricci tensor*, and R is the *Ricci scalar.*

As it is well known, *Einstein's conception of gravitation as above identified does not permit antigravity*, and this occurrence has been a mo-

tivation for the absence of serious experimental studies in the field, as indicated in Section 1.4.1.

However, we have indicated in preceding chapters that *the problem of antigravity cannot be confidently formulated, let alone treated, in Einstein's gravitation, due to the impossibility of consistently treating antimatter.*

As indicated earlier, the only possible formulation of antimatter is that by *only* changing the sign of the charge. However, this formulation is inconsistent with quantization since it leads to particles, rather than antiparticles, with the wrong sign of the charge.

At any rate, *the most important formulation of the gravity of antimatter is that for astrophysical bodies with null total charge, as expected for an antimatter star or an antimatter neutron star.*

The impossibility for any credible treatment of antimatter is then established by the fact that *according to Einstein's conception of gravitation the gravitational fields equations for matter and antimatter stars with null total charge are identical.*

These inconsistencies are resolved by the isodual theory of antimatter because it implies the novel *isodual field equations for antimatter* defined on the isodual Riemannian space [16] $\mathcal{R}^d(x^d, g^d, R^d)$ with local isodual spacetime coordinates $x^d = -x^t$ and isodual metric $g^d(x^d) = -g^t(-x^t)$ over the isodual field of real numbers R^d

$$G^d_{\mu\nu} = R^d_{\mu\nu} - g^d_{\mu\nu} \times R^d/^d 2^d = 0. \tag{4.1.11}$$

The latter representation is based on a negative-definite energy-momentum tensor, thus having a consistent operator image, as shown in Chapter 3.

We, therefore, conclude this analysis with the following:

THEOREM 4.1.1 : *Antigravity is a necessary and sufficient condition for the existence of a classical formulation of antimatter compatible with its operator counterpart.*

Proof. Assume the validity of Einstein's gravitation for matter and its isodual for antimatter. Then, the former has a positive curvature tensor and the latter has a negative curvature tensor.

Therefore, the projection of the gravitational field of antimatter in the spacetime of matter implies a negative curvature tensor in our spacetime, namely, antigravity, or, vice-versa, a positive curvature tensor in the isodual spacetime, that is also repulsive, and this proves the sufficiency. The necessity comes from the fact that the only formulation of anti-

matter compatible with operator counterparts is that based on negative energies and masses.

In turn, geometric formulations of negative energies and masses necessarily imply, for consistency, a negative curvature tensor. Still in turn, when projected in the space of matter, a negative curvature necessarily implies antigravity and the same occurs for the projection of matter in the field of antimatter. **q.e.d.**

4.1.5 Identification of Gravitation and Electromagnetism

In addition to the above structural inability by Einstein's equations (4.1.10) to represent antimatter, Einstein's gravitation is afflicted by a litany of inconsistencies for the treatment of *matter itself* studied in Section 1.4 whose resolution requires a number of structural revisions of general relativity.

It is important to show that the prediction of antigravity, not only persists, but it is actually reinforced for gravitational theories resolving the inconsistencies of Einstein's gravitation.

The first catastrophic inconsistency of Einstein's gravitation crucial for the problem of antigravity is that of Theorem 1.4.1 on the irreconcilable incompatibility between Einstein's lack of source in vacuum and the electromagnetic origin of mass.

As stressed in Section 1.4, this inconsistency is such that, either one assumes Einstein's gravitation as correct, in which case quantum electrodynamics must be reformulated from its foundation to prevent a first-order source in vacuum, or one assumes quantum electrodynamics to be correct, in which case Einstein's gravitation must be irreconcilably abandoned.

The second catastrophic inconsistency of Einstein's gravitation is that of Theorem 1.4.2 identifying the incompatibility of field equations (4.1.10) and the forgotten Freud identity of the Riemannian geometry,

$$R^\alpha_\beta - \frac{1}{2}\times\delta^\alpha_\beta\times R - \frac{1}{2}\times\delta^\alpha_\beta\times\Theta \;=\; U^\alpha_\beta + \partial V^{\alpha\rho}_\beta/\partial x^\rho \;=\; k\times(t^\alpha_\beta+\tau^\alpha_\beta), \quad (4.1.12)$$

where

$$\Theta \;=\; g^{\alpha\beta}g^{\gamma\delta}(\Gamma_{\rho\alpha\beta}\Gamma^\rho_{\gamma\beta} - \Gamma_{\rho\alpha\beta}\Gamma^\rho_{\gamma\delta}), \quad (4.1.13a)$$

$$U^\alpha_\beta \;=\; -\frac{1}{2}\frac{\partial\Theta}{\partial g^{\rho\alpha}_{|_\rho}}g^{\gamma\beta}\uparrow_\gamma, \quad (4.1.13b)$$

$$V^{\alpha\rho}_\beta \;=\; \frac{1}{2}[g^{\gamma\delta}(\delta^\alpha_\beta\Gamma^\rho_{\alpha\gamma\delta} - \delta^\rho_\beta\Gamma^\rho_{\alpha\delta})$$
$$+(\delta^\rho_\beta g^{\alpha\gamma} - \delta^\alpha_\beta g^{\rho\gamma})\Gamma^\delta_{\gamma\delta} + g^{\rho\gamma}\Gamma^\alpha_{\beta\gamma} - g^{\alpha\gamma}\Gamma^\rho_{\beta\gamma}]. \quad (4.1.13c)$$

The latter inconsistency requires the addition in the right-hand-side of Eq. (4.1.10) of *two source tensors* for astrophysical bodies with null total charge.

As stressed in Section 1.4, the above two inconsistencies are deeply inter-related because complementary to each other, since the inconsistency of Theorem 1.4.2 is the dynamical counterpart of the inconsistency of Theorem 1.4.2 on geometric grounds.

A systematic study of the resolution of these inconsistencies was conducted by Santilli [18] in 1974.

The classical gravitational formulation of antimatter can be done in the *Riemannian-Santilli isodual space* $\mathcal{R}^d(x^d, g^d, R^d)$ studied in Sections 2.1.7 and 2.2.11.

To avoid catastrophic inconsistencies, the field equations of antimatter should be compatible with the basic geometric axioms of the isodual Riemannian geometry, including, most importantly, the *isodual Freud identity* [16], that can be written

$$R^{\alpha d}_{\beta} - \frac{1}{2}^{d} \times^{d} \delta^{\alpha d}_{\beta} \times^{d} R^{d} - \frac{1}{2}^{d} \times^{d} \delta^{\alpha d}_{\beta} \times^{d} \Theta^{d} = k^{d} \times^{d} (T^{d\alpha}_{\beta} + \Upsilon^{d\alpha}_{\beta}). \tag{4.1.14}$$

with corresponding isodualities for Eq. (4.1.13) here assumed as known.

These studies then lead to the following:

PREDICTION 4.1.2: [18] IDENTIFICATION OF GRAVITATION AND ELECTROMAGNETISM. In the exterior problem in vacuum, gravitation coincides with the electromagnetic interactions creating the gravitational mass with field equations

$$G^{Ext.}_{\mu\nu} = R_{\mu\nu} - g_{\mu\nu} \times R/2 = k \times T^{Elm}_{\mu\nu}, \tag{4.1.15}$$

where the source tensor $T^{Elm}_{\mu\nu}$ represents the contribution of all charged elementary constituents of matter with resulting gravitational mass

$$m^{Grav} = \int d^3x \times T^{Elm}_{00}, \tag{4.1.16}$$

while in the interior problem gravitation coincides with electromagnetic interactions plus short range weak, strong and other interactions creating the inertial mass with field equations

$$G^{Int.}_{\mu\nu} = R_{\mu\nu} - g_{\mu\nu} \times R/2 = k \times (T^{Elm}_{\mu\nu} + \Upsilon^{ShortRange}_{\mu\nu}), \tag{4.1.17}$$

where the source tensor $\Upsilon^{ShortRange}_{\mu\nu}$ represents all possible short range interactions in the structure of matter, with inertial mass

$$m^{Inert} = \int d^3x \times (T^{Elm}_{oo} + \Upsilon^{ShortRange}_{00}), \tag{4.1.18}$$

and general law

$$m^{Inert} > m^{Grav}. \tag{4.1.19}$$

The same identification of gravitation and electromagnetism then exists for antimatter with field equations and mass expressions given by a simple isodual form of the preceding ones.

A few comments are in order. All studies on the problem of "unification" of gravitation and electromagnetism prior to Ref. [18] known to this author[1] treated the two fields as *physically distinct*, resulting in the well known historical failures to achieve a consistent *unification* dating back to Albert Einstein (see next chapter for a detailed study). An axiomatically consistent theory emerges if gravitation and electromagnetism are instead "identified", as first done by Santilli [18] in 1974.

Also, Prediction 4.1.2 implies a *theory on the origin of the gravitational field*, rather than a theory providing its "description", as available in standard treatises such as [10]. This is due to the fact that in Identification 4.1.2 *all* mass terms are completely eliminated and replaced with the fields originating mass.

In this way, the use of any mass term in any theory is an admission of our ignorance in the structure of the considered mass.

We should indicate for completeness that the identification of exterior gravitational and electromagnetic fields appears to be disproved by the assumption that quarks are physical constituents of hadrons, owing to the known large value of their "masses".

However, as indicated in Chapter 1, gravitation solely exists in our spacetime and cannot be consistently extended to mathematical unitary symmetries. Also, the only masses that can consistently create gravitation are those defined in our spacetime, thus necessarily being the eigenvalues of the second-order Casimir invariant of the Poincaré symmetry.

Since quarks cannot be defined in our spacetime, they cannot be consistently characterized by the Poincaré symmetry and their masses are not the eigenvalues of the second-order Casimir invariant of the latter symmetry, the use of quark masses has no scientific value in any gravitational profile. This is the reason why quark "masses" have been ignored in Ref. [18] as well as in this chapter.

It is well established in quantum electrodynamics that the mass of the electron is entirely of electromagnetic origin. Therefore, a gravitational theory of the electron in which the source term solely represents the charge contribution is incompatible with quantum electrodynamics. In fact, the latter requires *the entire reduction of the electron mass to electromagnetic fields* according to Eq. (4.1.16).

Note in particular that, since the electron has a point-like charge, we have no distinction between exterior and interior problems with consequential identity

$$m^{Grav}_{Electron} \equiv m^{Inert}_{Electron}. \tag{4.1.20}$$

When considering a neutral, extended and composite particle such as the π°, the absence of a source tensor of electromagnetic nature renders gravitation, again, incompatible with quantum electrodynamics, as established in Ref. [18] and reviewed in Section 1.4.

By representing the π° as a bound state of a charged elementary particle and its antiparticle in high dynamical conditions, quantum electrodynamics establishes the existence not only of a non-null total electromagnetic tensor, but one of such a magnitude to account for the entire gravitational mass of the π° according to Eq. (4.1.16) and gravitational mass

$$m^{Grav}_{\pi^\circ} = \int d^3x \times T^{Elm}_{00\ \pi^\circ}. \tag{4.1.21}$$

Unlike the case of the electron, the π° particle has a very large charge distribution for particle standards. Moreover, the structure of the π° particle implies the additional weak and strong interactions, and their energy-momentum tensor is not traceless as it is the case for the electromagnetic energy-momentum tensor.

Therefore, for the case of the π° particle, we have a well-defined difference between exterior and interior gravitational problems, the latter characterized by Eq. (4.1.18), i.e.,

$$m^{Inert}_{\pi^\circ} = \int d^3x \times (T^{Elm}_{00} + \Upsilon^{ShortRange}_{00}) > m^{Grav}_{\pi^\circ}. \tag{4.1.22}$$

The transition from the π° particle to a massive neutral star is conceptually and technically the same as that for the π°. In fact, the star itself is composed of a large number of elementary charged constituents each in highly dynamical conditions and, therefore, each implying a contribution to the total gravitational mass of the star as well as to its gravitational field.

The separation between exterior and interior problems, the presence of only one source tensor for the exterior problem and two source tensors for the interior problems, and the fact that the inertial mass is bigger than the gravitational mass is the same for both the π° and a star with null total charge.

For the case of a star we merely have an increased number of elementary charged constituents resulting in the expression [18]

$$m^{Grav}_{Star} = \Sigma_{p=1,2,3,...} \int d^3x \times T^{Elem.Constit.}_{00}. \tag{4.1.23}$$

Note that when the star has a non-null total charge there is no need to change field equations (4.1.15) since the contribution from the total charge is automatically provided by the constituents.

As it is well known, there exist numerous other theories on the identity as well as the possible differentiation of gravitational and inertial masses (see, e.g., Ref. [10]). However, these theories deal with exterior gravitational problems while the studies here considered deal with the interior problem, by keeping in mind that inertial masses are a strictly *interior* problem, the exterior problem providing at best a geometric abstraction.

Nevertheless, one should remember that all these alternative theories are crucially based on Einstein's gravitation, while the theory presented in this section is based on quantum electrodynamics. Therefore, *none of the existing arguments on the differences between gravitational and inertial masses is applicable to the theory here considered.*

Note finally that conventional electromagnetism is represented by a *first-order tensor*, the electromagnetic tensor $F_{\mu\nu}$ of type (2.2.37a) and related first-order Maxwell's equations (2.2.37b) and (2.2.37c).

When electromagnetism is identified with exterior gravitation, it is represented with a *second-order tensor*, the energy-momentum tensor $T_{\mu\nu}$ of type (4.1.7) and related second-order field equations (4.1.15).

4.1.6 Prediction of Antigravity from the Identification of Gravitation and Electromagnetism

Another aspect important for this study is that *the identification of gravitation and electromagnetism in the exterior problem in vacuum implies the necessary existence of antigravity.*

In fact, the identification implies the necessary equivalence of the phenomenologies of gravitation and electromagnetism, both of them necessarily experiencing attraction and repulsion.

Note that this consequence is intrinsic in the identification of the two fields and does not depend on the order of the field equations (that is first order for electromagnetism and second order for gravitation as indicated earlier.

Alternatively, for the exterior problem of matter we have the field equations on $\mathcal{R}(x, g, R)$ over R

$$G_{\mu\nu}^{Ext.} = R_{\mu\nu} - g_{\mu\nu} \times R/2 = k \times T_{\mu\nu}^{Elm}, \tag{4.1.24}$$

in which *the curvature tensor is positive*, and for the exterior problem of antimatter we have the isodual equations on $\mathcal{R}^d(x^d, g^d, R^d)$ over R^d

$$G^{d,Ext.}_{\mu\nu} = R^d_{\mu\nu} - g^d_{\mu\nu} \times R^d/2 = k \times T^{d,Elm}_{\mu\nu}, \tag{4.1.25}$$

in which *the curvature tensor is negative.*

The prediction of antigravity, Prediction 4.1.1, follows as a trivial extension of that of the preceding sections and occurs when the gravitational field of antimatter is projected in that of matter, or vice-versa, since such a projection implies a negative curvature in a Riemannian space that, by definition, is antigravity.

The prediction of antigravity is so strong that it is possible to prove that *the lack of existence of antigravity would imply the impossibility of identifying gravitation and electromagnetism.*

In turn, the lack of such identification would necessary require the impossibility for masses to have appreciable electromagnetic origin, resulting in the need for a structural revision of the entire particle physics of the 20-th century.

4.1.7 Prediction of Gravitational Repulsion for Isodual Light Emitted by Antimatter

Another important implication of the isodual theory of antimatter is the prediction that antimatter emits a new light, the *isodual light*, that experiences repulsion when in the vicinity of the gravitational field of matter, or vice-versa [19], where the *isodual electromagnetic waves* emitted by antimatter are given by Eq. (2.3.37), i.e.,

$$F^d_{\mu\nu} \;=\; \partial^d A^d_\mu /^d \partial^d x^{\nu d} - \partial^d A^d_\nu /^d \partial^d x^{d\mu}, \tag{4.1.26a}$$

$$\partial^d_\lambda F^d_{\mu\nu} + \partial^d_\mu F^d_{\nu\lambda} + \partial^d_\nu F^d_{\lambda\mu} = 0, \tag{4.1.26b}$$

$$\partial^d_\mu F^{d\mu\nu} = -J^{d\nu}. \tag{4.1.26c}$$

The gravitational repulsion then emerges from the negative energy of the above isodual waves when in the field of matter. Vice versa, electromagnetic waves emitted by matter are predicted to experience antigravity when in the gravitational field of antimatter because they have a positive energy.

Note that *isodual electromagnetic waves coincide with conventional waves under all known interactions except gravitation.* Alternatively, the isodual electromagnetic waves requires the existence of antigravity at a pure classical level for their proper identification.

In turn, the experimental confirmation of the gravitational repulsion of light emitted by antimatter would have momentous astrophysical and

cosmological implications, since it would permit for the first time theoretical and experimental studies as to whether far away galaxies and quasars are made up of matter or of antimatter.

It is important in this connection to recall that all relativistic quantum field equations admit solutions with positive and negative energies. As it is the case for Dirac's equations, relativistic field equations are generally isoselfdual, thus admitting solutions with both positive and negative energies.

The former are used in numerical predictions, but the negative-energy states are generally discarded because they are believed to be "unphysical."

The isodual theory implies a significant revision of the interpretation of quantum field theory because *the solutions of relativistic equations with positive energy are defined in our spacetime and represent particles, while the joint solutions with negative energy are actually defined on the isodual spacetime and represent antiparticles.*

This re-interpretation cannot be presented in this chapter for brevity. In fact, a systematic study of isodual photons requires the formulation of *isodual quantum field theory* that would render prohibitive the length of this chapter.

It is hoped that interested colleagues will indeed work out the proposed isodual quantum field theory, with particular reference to the isodual re-interpretation of advanced and retarded solutions, Green distributions, Feynman diagrams, and all that, because of various implications, such as those in conjugation of trajectories or in the transition from particles to antiparticles.

In closing, the reader should keep in mind that the isodual theory of antimatter resolves all conventional inconsistencies on negative energies as well as against antigravity (see also Section 2.3.15).

4.2 EXPERIMENTAL VERIFICATION OF ANTIGRAVITY

4.2.1 Santilli's Proposed Test of Antigravity for Positrons in Horizontal Flight

By far the most fundamental experiment that can be realized by mankind with current technologies is the measure of the gravitation of truly elementary antiparticles, such as the positron, in the field of Earth.

Irrespective of whether the outcome is positive or negative, the experiment will simply have historical implications for virtually all of physics, from particle physics to cosmology for centuries to come.

If antigravity is experimentally established, the location of the experiment is predicted to become a place of scientific pilgrimage for centuries, due to the far reaching implications, such as the consequential existence of a Causal Time Machine outlined later on in this chapter.

An inspection of the literature soon reveals that the problem of the *gravity of antiparticles in the field of Earth is fundamentally unsettled at this writing, thus requiring an experimental resolution.*

On theoretical grounds, all arguments based on the weak equivalence principle [10] are dismissed as inconclusive by the isodual theory of antimatter, since the latter predicts that bound states of particles and their isoduals experience *attraction* in the gravitational field of Earth.

At any rate, no argument against antigravity based on general relativity can be considered scientifically valid without first the resolution of the catastrophic inconsistencies of gravitation, such as those expressed by the various inconsistency theorems of Section 1.4.

Similarly, all experiments conducted to date on the test of the *gravity of antiparticles not bounded to matter* are equally inconclusive, to the author's best knowledge.[2] A direct measurement of the gravity of positrons was considered in 1967 by Fairbanks and Witteborn [6] via electrons and positrons in a *vertical* vacuum tube.

However, the test could not be conducted because preliminary tests with electrons discouraged the use of positrons due to excessive disturbances caused by stray fields, impossibility of ascertaining the maximal height of the electrons, and other problems.

Neutron interferometric measurements of the *gravity of antiprotons* have been studied by Testera [8], Poggiani [9] and others. However, these experiments are highly sophisticated, thus implying difficulties, such as those for securing antiprotons with the desired *low energies*, magnetic trapping of the antiprotons, highly sensitive interferometric measurements of displacements, and others.

A number of important proposals to text the gravity of antimatter have been submitted to CERN and at other laboratories by T. Goldman, R. J. Hughes, M. M. Nieto, et al. [22–25], although no resolutory measurement has been conducted to date to the author best knowledge, perhaps in view of the excessive ambiguities for an accurate detection of the trajectories of antiparticles under Earth's gravitational field in existing particle accelerators (see in this respect Figure 4.2).

Additional important references are those studying the connection between antigravity and quantum gravity [26–29], although the latter should be studied by keeping in mind Theorem 1.5.2 on the catastrophic inconsistencies of quantum gravity when realized via nonunitary structures defined on conventional Hilbert spaces and fields.[3]

In view of these unsettled aspects, an experiment that can be *resolutory* with existing technologies, that is, establishing in a final form either the existence of the lack of existence of antigravity, has been proposed by Santilli in Ref. [12] of 1994.

The experiment essentially requires a *horizontal* vacuum tube ranging from 100 meters in length and 0.5 meter in diameter to 10 m in length and 1 m in diameter depending on used energies, with axial collimators at one end and a scintillator at the other end as in Figure 4.1. The proposed test then consists in:

1) Measuring the location in the scintillator of lack of gravitational displacement via a collimated photon beam (since the gravitational displacement on photons at the considered distances is ignorable);

2) Measuring on the same scintillator the downward displacement due to Earth's gravity on an electron beam passing through the same collimators, which downward displacement is visible to the naked eyes for sufficiently small electron energies (for instance, we can have a downward displacement due to gravity of 5 mm, that is visible to the naked eye, for electron kinetic energies of 25 μeV along 100 m horizontal flight, or for electrons with 2 μeV along a 10 m horizontal flight); and

3) Measuring on the same scintillator the displacement due to Earth's gravity on a positron beam passing through the same collimators, which displacement is also visible to the naked eye for positron energies of the order of a few μeV.

If the displacement due to gravity of the positrons is downward, the test would establish the lack of existence of antigravity. On the contrary, the detection of an upward displacement of the positrons would establish the existence of antigravity.

An alternative proposal was submitted by Santilli [20] via the use of the so-called *particle decelerator* in the shape of a doughnut of a diameter of about 10 m and 50 cm in sectional diameter (Figure 4.2). The main idea is that low energy beams of electrons and positrons could be *decelerated* via the use of magnetic fields down to the energy needed to achieve a displacement due to gravity sufficiently larger than the dispersion to be visible to naked eye, at which point the particles are released into a scintillator.

We have stressed throughout this presentation that the only experimental verification of the theoretical prediction of antigravity recommendable at this writing, is that for *truly elementary antiparticles in the gravitational field of matter without any bound to other particles*, such as an isolated beam of positrons under the gravitation field of Earth.

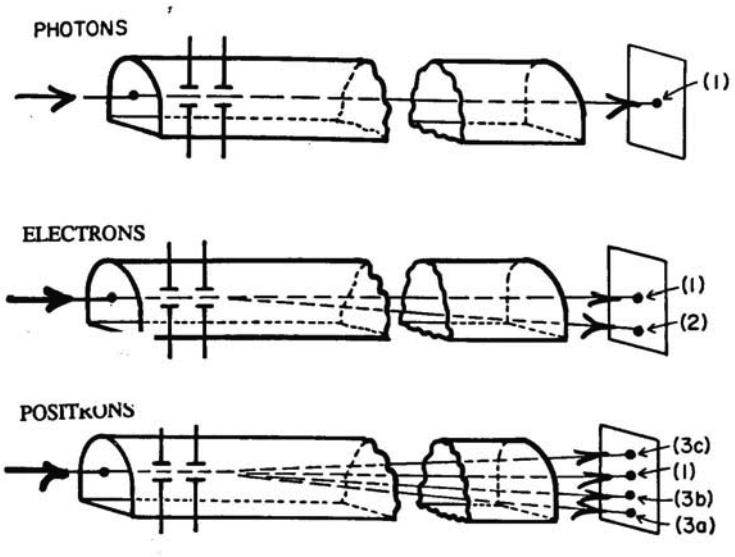

Figure 4.1. A schematic view of the proposal to test the gravity of positrons suggested by Santilli [12] in 1994 via a horizontal vacuum tube with a scintillator at the end in which a collimated beam of photons is used to identify the point in the scintillator of no displacement due to gravity, and collimated beams of very low energy electrons and, separately, positrons are used to measure displacements due to gravity. The latter are indeed visible to the naked eye for sufficiently low kinetic energy of the order of a few μeV. Santilli's proposal [12] was studied by the experimentalist J. P. Mills, jr. [13], as reviewed in the next section.

Other tests of antigravity, if conducted before the above tests with positrons and used for general claims on antigravity, can likely lead to ambiguities or a proliferations of unnecessary controversies.

The reasons for this restriction are numerous. Firstly, the study of the gravity of particle-antiparticle systems, such as a bound state of one electron and one positron at large mutual distances according to quantum mechanics (QM),

$$Positronium \;=\; (e^-,\; e^+)_{QM}, \tag{4.2.1}$$

is *strongly discouraged* for a first "test of antigravity", because all theories, including the isodual theory, predict attraction of the positronium in the field of matter. Therefore, under no condition can any possible experimental verification of this prediction be used as a credible claim on the lack of existence of antigravity at large.

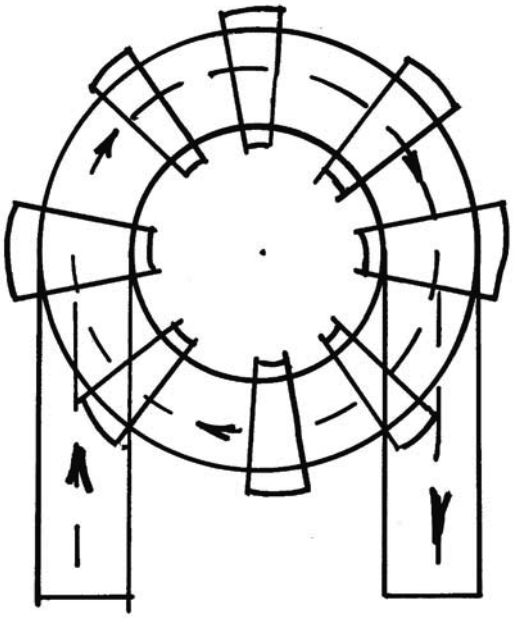

Figure 4.2. A schematic view of the alternative proposal submitted for study by the author [20] at the National High Magnetic Field Laboratory, Tallahassee, Florida, in December 1995. The main idea is to use the established techniques for "particle accelerators" for the construction of a "particle decelerator" that would slow down the initial energy of electron and positron beams down to the amounts needed to produce displacement due to gravity sufficiently bigger than the spread due to stray fields to produce a definite-resolutory answer visible to the naked eye. Suggested dimensions of the "particle decelerator" are 10 m in diameter with a sectional diameter of 0.5 m and two entrances-exits, one used for the entrance-exit of the electron beam and the other for the positron beam. The study conducted by Mills [13] for the horizontal tube indicates that the "particle decelerator" here considered is also feasible and will produce a resolutory answer.

Second, the above restriction eliminates the use of muons for a first test of antigravity, because, in view of their instability and decay modes, and as studied in detail in the next chapter, hadronic mechanics (HM) predicts that muons are a bound state of electrons and positrons in conditions of total mutual penetrations of their wavepackets at very short mutual distances,

$$\mu^{\pm} \ = \ (e^{-}, \ e^{\pm}, \ e^{+})_{HM}, \tag{4.2.2}$$

with consequential highly nonlocal effects structurally beyond any credible treatment by quantum mechanics. Under this structure, *both muons and antimuons are predicted to experience gravitational attraction only* because the possible antigravity of the positron is expected to be less than the gravity of basic electron-positron system.

A similar restriction applies against the use of mesons for first tests of antigravity because they are bound states of particles and antiparticles that, as such, are predicted not to experience antigravity in the field of matter. This is particularly the case for pions. Similarly, a first use of kaons for experiments on antigravity can only result in un-necessary controversies in view of their unsettled structure.

Serious reservation also exist for the first use of antiprotons and antineutrons due to their basically unsettled structure. As stressed earlier, the use of current quark conjecture prevents antiprotons and antineutrons to have any gravity at all, let alone antigravity, as rigorously proved by the fact indicated earlier that gravity can only be defined in our physical spacetime while quarks can only be defined in their internal mathematical unitary space, as well as by the lack of credibly defines "quark masses" as inertial eigenvalues of the second order Casimir invariant of the Poincaré group (see the Appendix of Ref. [8]).

Equally equivocal can be at this stage of our knowledge the conduction of first gravitational measurements via the sole use of the antihydrogen atom for intended general results on antigravity, evidently because its nucleus, the antiproton, is believed to be a bound state of quarks for which no gravity at all can be consistently defined. Any study of antigravity under these unsettled structural conditions can only lead to un-necessary controversies, again, if used for general results on antigravity.

It is evident that, *until baryons theories are afflicted by such fundamental problematic aspects, as the inability even to define gravity in a credible way, no gravitational measurement based on antiprotons and antineutrons can be credibly used as conclusive for all of antimatter.*

After the resolution of the gravitational behavior of unbounded positrons in the field of matter, the tests for the gravitational behavior of positronium, muons, muonium, pions, pionium, antiprotons, antineutrons, antihydrogen atom, etc. become essential to acquire an experimental background sufficiently diversified for serious advances on antimatter beyond the level of personal beliefs one way or the other.

The fundamental test of the gravity of positrons here considered was proposed by the author to the following institutions:

1) Stanford Linear Acceleration Center, Stanford, USA, during and following the Seventh Marcel Grossmann Meeting on General Relativity held at Stanford University in July 1994;

2) The Joint Institute for Nuclear Research in Dubna, Russia, during the International Conference on Selected Topics in Nuclear Physics held there in August 1994;

3) The National High Magnetic Field Laboratory in Tallahassee, Florida, during a meeting there in 1996 on magnetic levitation;

4) CERN, Geneva, Switzerland, during a presentation there of hadronic mechanics;

5) Brookhaven National Laboratories, following the participation at the Sepino meeting on antimatter of 1996 [5];

and to other laboratories as well to universities in various countries.

It is regrettable for mankind that none of these laboratories or universities expressed interest in even considering to date such a fundamental experiment, by preferring to spend much bigger public funds for esoteric experiments manifestly lesser important than that of antigravity.

4.2.2 Santilli's Proposed Tests of Antigravity for Isodual Light

Additionally, in 1997 Santilli [19] predicted that *antimatter emits a new light, the isodual light, that is predicted to be repelled by the gravitational field of matter*, and proposed its experimental verification as the only known (or even conceivable) possibility of ascertaining whether far-away galaxies and quasars are made up of matter or of antimatter.

Measurements as to whether light emitted by the antihydrogen atoms now produced at CERN are attracted or repelled by matter is predictably more delicate than the test of the gravity of the positron, evidently because gravitational displacements for photons in horizontal flight are extremely small, as well know, thus requiring very sensitive interferometric and other measurements.

The experimental detection as to whether far-away galaxies and quasars are made up of matter or of antimatter is predictably more complex and requiring longer periods of time, but with immense scientific implications whatever the outcome.

The test can be done in a variety of ways, one of which consists of measuring the deflection of light originating from far away astrophysical objects when passing near one of our planets. Comparative measurements of a sufficiently large number of galaxies and quasars should permit the detection of possible repulsions, in the event it exists.

Another test has been privately suggested by to the author by an astrophysicist and consists in reinspecting all existing astrophysical data on the deflection of light from far away galaxies and quasars when passing nearby astrophysical bodies.

In the opinion of this astrophysicist, it appears that evidence for the repulsion of light already exists in these data. Such a possible evidence has been ignored so far, and, if found, could not be admitted publicly at the moment, simply because Einstein's gravitation does not allow for any prediction of gravitational repulsion of light.

An understand is that, for these astrophysical measurements to be credible, astrophysicists must conduct the study of a vary large number of galaxies and quasars (of the order of several thousands), and the considered galaxies and quasars must be sufficiently far away to render plausible their possible antimatter structure.

4.2.3 Mills' Studies of Santilli's Proposed Tests of Antigravity

The experimentalist J. P. Mills, jr., [13] conducted a survey of all significant experiments on the gravity of antiparticles in the field of Earth, including indirect tests based on the weak equivalence principle and direct experiments with antiparticles, by concluding that the problem is basically unsettled on theoretical and experimental grounds, thus requiring an experimental resolution.

After considering all existing possible tests, Mills' conclusion is that Santilli's proposed test [12] on the measurement of the gravitational deflection of electrons and positron beams of sufficiently low energy in horizontal flight in a vacuum tube of sufficient length and shielding, is preferable over other possible tests, experimentally feasible with current technology, and providing a resolutory answer as to whether positrons experience gravity or antigravity.

As it is well known, a main technical problem in the realization of Santilli's test is the shielding of the horizontal tube from external electric and magnetic field, and then to have a tube structure in which the internal stray fields have an ignorable impact on the gravitational deflection, or electrons and positrons have such a low energy for which the gravitational deflection is much bigger than possible contributions from internal stray fields, such as the spreading of beams.

The electric field that would cancel the Earth gravitational force on an electron is given by

$$E \;=\; m_e \times g/e = 5.6 \times 10^{-11}\ \mathrm{V/m}. \tag{4.2.3}$$

As it is well known, an effective shielding from stray fields can be obtained via Cu shells. However, our current understanding of the low temperature zero electric field effect in Cu shells does not seem sufficient at this moment to guarantee perfect shielding from stray fields.

Mills [13] then suggested the following conservative basic elements for shielding the horizontal tube.

Assuming that the diameter of the tube is d and the shielding enclosure is composed of randomly oriented grains of diameter λ, the statistical variation of the potential on the axis of the tube of a diameter d would then be [13]

$$\Delta V \ = \ \frac{\lambda}{d \times \sqrt{\pi}}, \tag{4.2.4}$$

As expected, *the effect of stray fields at the symmetry axis of the tube is inversely proportional to the tube diameter.* As we shall see, a tube diameter of 0.5 m is acceptable, although one with 1 m diameter would give better results.

Given a work function variation of 0.5 eV, 1 μm grains and $d = 30$ cm, we would have the following variation of the potential on the axis of the horizontal tube

$$\Delta V \ = 1 \ \mu\text{eV}. \tag{4.2.5}$$

Differences in strain or composition could cause larger variations in stray fields. To obtain significant results without ambiguities for the shielding effect of low temperature Cu shells, Mills [13] suggests *the use of electrons and positrons with kinetic energies significantly bigger than* 1 μeV. As we shall see, this condition is met for tubes with minimal length of 10 m and the diameter of 1 m, although longer tubes would evidently allow bigger accuracies.

The realization of Santilli's horizontal vacuum tube proposed by Mills [13] is the following. As shown in Figure 4.3, the tube would be a long dewar tube, consisting of concentric shells of Al and Mu metals, with Pb and Nb superconducting shells and an inner surface coated with an evaporated Cu film.

There should be two superconducting shells so that they would go superconducting in sequence [Nb (9.25 K), Pb (7.196 K)], evidently for better expulsion of flux. Trim solenoids are also recommended for use within the inner shell and a multitude of connections to the Cu field for trimming electrostatic potentials.

As also shown in Figure 4.3, the flight tube should be configured with an electrostatic lens in its center for use of electron and positron beams in both horizontal directions, as well as to focus particles from a source at one end into a gravity deflection sensitive detector at the other end. The de Broglie wavelength of the particles results in the position resolution

$$d \ = \ 2.4 \times \pi \times \alpha_B \times \frac{c \times L}{v \times D}, \tag{4.2.6}$$

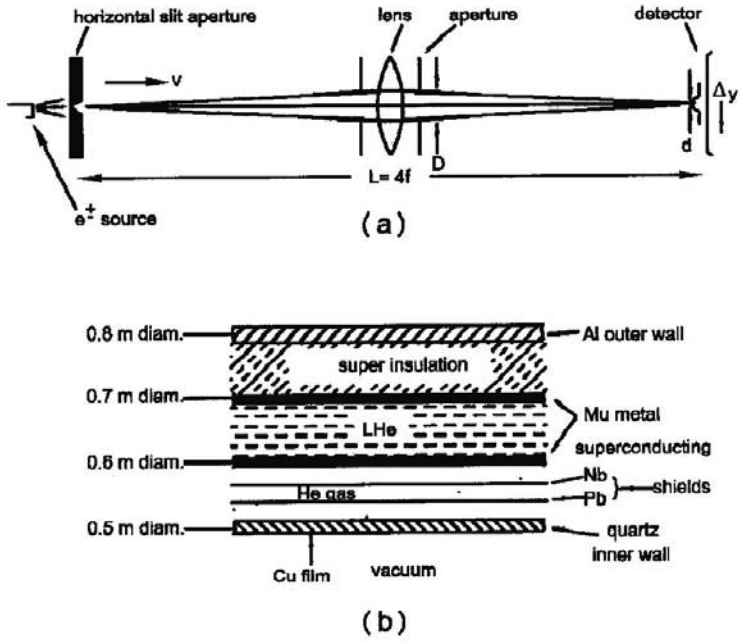

Figure 4.3. A schematic view of the realization suggested by Mills [13] of the horizontal tube proposed by Santilli [12].

where $\alpha = 1/137$ is the fine structure constant, $a_B = 0.529\,\text{Å}$ is the Bohr radius of hydrogen, c is the velocity of light, v is the electron or positron velocity, L is the length of the horizontal path, and D is the diameter of the lens aperture in the center of the flight tube.

The *vertical gravitational deflection* is given by

$$\Delta y \;=\; g \times \frac{L^2}{2 \times v^2}, \tag{4.2.7}$$

Given $L = 100\,\text{m}$, $D = 10\,\text{cm}$, $v/c = 10^{-5}$ (i.e., for 25 μeV particles), we have

$$\Delta y = 5\,\text{mm}. \tag{4.2.8}$$

For $1meV$ particles the resolution becomes

$$\Delta y = 125\,\mu\text{m}. \tag{4.2.9}$$

Therefore, one should be able to observe a meaningful deflection using particles with kinetic energies well above the expected untrimmed fluctuation in the potential.

Mills also notes that the lens diameter should be such as to minimize the effect of lens aberration. This requirement, in turn, dictates the minimum inside diameter of the flight tube to be 0.5 m.

The *electron source* should have a cooled field emission tip. A sufficient *positron source* can be provided, for example, by 0.5 ci of ^{22}Na from which we expect (extrapolating to a source five times stronger) $3 \times 10^7\, e^+/\text{s}$ in a one centimeter diameter spot, namely a positron flux sufficient for the test.

Ideal results are obtained when the positrons should be bunched into pulses of 10^4 e^+ at the rate of 10^3 bunches per second. Groups of 10^3 bunches would be collected into macrobunches containing 10^6 e^+ and 20 nsec in duration. The positrons would be removed from the magnetic field and triply brightness enhanced using a final cold Ni field remoderator to give bunches with 10^4 e^+, 10 meV energy spread, an ellipsoidal emission spot 0.1 μm high and 10 μm wide and a 1 radian divergence.

However, stray fields are notoriously weak and decrease rapidly with the distance. Therefore, there is a diametr of the vacuum tube for which stray fields are expected to have value on the axis insufficient to disrupt the test via a spreding of the beams. Consequently, the proposed tests is also expected to be resolutory via the use of very low energy positrons as available, e.g., from radioactive sources.

As a matter of fact, the detection in the scientillator of the same clear gravitational deflection due to gravity by *a few* positrons would be sufficient to achieve a final resolution, provided, of course, that these few events can be systematically reproduced.

After all, the reader should compare the above setting with the fact that new particles are nowadays claimed to be discovered at high energy laboratories via the use of extremely few events out of hundreds of millions of events on record for the same test.

The beam would then be expanded to 100 μm$\times$1 cm cross section and a 1 mrad divergence, still at 10 meV. Using a time dependent retarding potential Mills would then lower the energy spread and mean energy to 100 μeV with a 2 μs pulse width. Even assuming a factor of 1,000 loss of particles due to imperfections in this scheme, Mills' set-up would then have pulses of about 10 positrons that could be launched into the flight tube with high probability of transmissions at energy of 0 to 100 μeV.

The determination of the gravitational force would require many systematic tests. The most significant would be the measurements of the deflection as a function of the time of flight (enhance the velocity v)

$\Delta v(e\pm, \pm v)$ for both positrons and elections and for both signs of the velocity relative to the lens on the axis of the tube, $v > 0$ and $v < 0$, the vertical gravitational force on a particle of charge q is

$$F_y \ = \ -m \times g + q \times E_y + q \times v_z \times B_x/c. \tag{4.2.10}$$

The deflection is then given by

$$\Delta y = \int_0^L \int_0^{z'} q \times [E(z'') + v \times B(z'')/c] \times dz'' \times dz'/(m \times v^2) - g \times z^2/2 \times v^2. \tag{4.2.11}$$

In lowest order, Mills neglects the transverse variation in E_y and B_x and writes for the average fields

$$\epsilon \ = \ \frac{1}{L^2} \int_0^L \int_0^{z'} E_y(z'') dz'' \times dz', \tag{4.2.12}$$

and

$$\beta \ = \ \frac{1}{L^2} \int_0^L \int_0^{z'} B_x(z'') \times dz'' \times dz'. \tag{4.2.13}$$

Note that these are not simple averages, but the averages of the running averages. They depend on the direction of the velocity. In the approximation that there are not significantly different from simple averages, the average of the four deflection Δy for both positrons and electrons and for both signs of the velocity is independent of ϵ and β and it is given by

$$< \Delta y > = \ (g^+ + g^-) \times \frac{L^2}{v^2}. \tag{4.2.14}$$

where $g^{\pm}$ refers to the gravitational acceleration of $e^{\pm}$. Since we also have the velocity dependence of the Δy's, and can manipulate E and B by means of trim adjustments, it will be possible to unravel the gravitational effect from the electromagnetic effect in this experiment.

In summary, the main features proposed by Mills [13] for Santilli's [12] horizontal vacuum tube are that:

1) The tube should be a minimum of 10 m long and 1 m in diameter, although the length of 100 m (as proposed by Santilli [12]) and 0.5 m in diameter is preferable;

2) The tube should contain shields against internal external electric and magnetic fields and internal stray fields. According to Mills [13], this can be accomplished with concentric shells made of Al, double shells of Mu metal, double shells of superconducting Nb and Pb, and a final internal evaporated layer of fine grain of Cu;

3) Use bright pulsed sources of electrons and, separately, positrons, at low temperature by means of phase space manipulation techniques including brightness enhancement;

4) Time of flight and single particle detection should be tested to determine the displacement of a trajectory from the horizontal line as a function of the particle velocity;

5) Comparison of measurements should be done using electrons and positrons traversing the flight tube in both directions.

The *use of electrons and positrons with 25 μeV kinetic energy would yield a vertical displacement of 5 mm at the end of 100 m horizontal flight, namely, a displacement that can be distinguished from displacements caused by stray fields and be visible to the naked eye*, as insisted by Santilli [12].

Mills [13] then concludes by saying that *"... an experiment to measure the gravitational deflection of electrons and positrons in horizontal flight, as suggested by R. M. Santilli, ... is indeed feasible with current technologies.... and should provide a definite resolution to the problem of the passive gravitational field of the positron"*.

4.3 CAUSAL SPACETIME MACHINE

4.3.1 Introduction

In preceding sections of this monograph we have indicated the far reaching implications of a possible experimental verification of antigravity predicted for antimatter in the field of matter and vice versa, such as a necessary revision of the very theory of antimatter from its classical foundations, a structural revision of any consistent theory of gravitation, a structural revision of any operator formulation of gravitation, and others.

In this section we show that another far reaching implications of the experimental detection of antigravity is the consequential existence of a *Causal Time Machine* [14], that is the capability of moving forward or backward in time without violating the principle of causality, although, as we shall see, this capability is restricted to isoselfdual states (bound states of particles and antiparticles) and *it is not* predicted by the isodual theory to be possible for matter or, separately, for antimatter.

It should be stressed that the Causal Time machine here considered is a *mathematical model*, rather than an actual machine. Nevertheless, science has always surpassed predictions. Therefore, we are confident that, as it has been the cases for other predictions, one the Causal Time Machine is theoretically predicted, science may indeed permits its actual construction, of course, in due time.

As we shall see, once a causal Time Machine has been identified, the transition to a causal SpaceTime Machine with the addition of motion in space is direct and immediate.

4.3.2 Causal Time Machine

As clear from the preceding analysis, *antigravity is only possible if antiparticles in general and the gravitational field of antimatter, in particular, evolve backward in time.* A time machine is then a mere consequence.

Causality is readily verified by the isodual theory of antimatter for various reasons. Firstly, *backward time evolution measured with a negative unit of time is as causal as forward time evolution measured with a positive unit of time.* Moreover, *isoselfdual states evolve according to the time of the gravitational field in which they are immersed.* As a result, no violation of causality is conceivably possible for isoselfdual states.

Needless to say, none of these causality conditions are possible for conventional treatments of antimatter.

The reader should be aware that we are referring here to a "Time Machine," that is, to motion forward and backward in time without space displacement (Figure 4.4). The "Space-Time Machine" (that is, including motion in space as well as in time), requires the isodualities as well as isotopies of conventional geometries studied in Chapter 3 and it will be studied in the next section.

The inability to have motion backward in time can be traced back to the very foundations of special relativity, in particular, to the basic time-like interval between two points 1 and 2 in Minkowski space as a condition to vrify causality

$$(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2 - (t_1 - t_2)^2 \times c^2 < 0. \qquad (4.3.1)$$

defined on the field of real numbers $R(n, \times, I),\ I = Diag.(1, 1, 1, 1)$.

The inability to achieve motion backward in time then prevents the achievement of a *closed loop* in the forward light cone, thus including motion in space and time, since said loop would necessarily rquire motion backward in time.

Consider now an isoselfdual state, such as the positronium or the π° meson (Section 2.3.14). Its characteristics have the sign of the unit of the observer, that is, positive time and energy for matter observers and negative times and negative energies for antimatter observers. Then a closed loop can be achieved as follows [14]:

1) With reference to Figure 4.4, expose first the isoselfdual state to a field of matter, in which case it evolved forward in time from a point

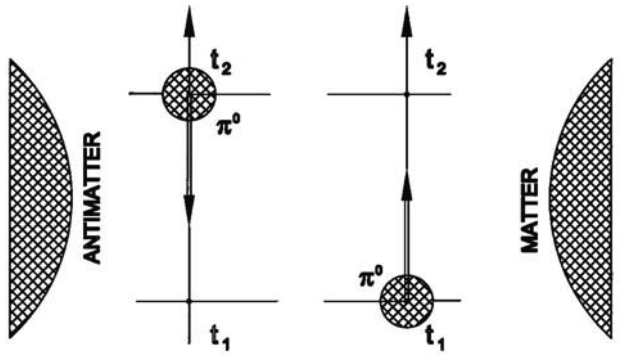

Figure 4.4. A schematic view of the simplest possible version of the "Time Machine" proposed in Ref. [14] via an isoselfdual state such as the positronium or the π° meson that are predicted to move forward (backward) in time when immersed in the gravitational field of matter (antimatter). The Time Machine then follows by a judicious immersion of the same isoselfdual state first in the fields of matter and then in that of antimatter. No causality violation is possible because of the time evolution for isoselfdual states is that of the field in which they are immersed in.

at time t_1 to a point at a later time t_2 where the spacetime coordinates verify the time-like invariant (4.3.1) with $t_2 > t_1$;

2) Subsequently, expose the same isoselfdual state to a field of antimatter in which case, with the appropriate intensity of the field and the duration of the exposure, the state moves backward in time from time t_2 to the original time t_1, where the spacetime coordinates still verify invariant (4.3.1) with $t_2 < t_1$ although in its isodual form.

We, therefore, have the following:

PREDICTION 4.3.1 [14]: *Isoselfdual states can have causal motions forward and backward in time, thus performing causal closed loops in the forward light cone.*

Note that the above causal Time Machine implies gravitational *attraction* for both fields of matter and antimatter, owing to the use of an isoselfdual test particle, in which case we only have the reversal of the sign of time and related unit.

Note also that the use of a particle or, separately, of an antiparticle would violate causality.

Numerous time machines exist in the literature. However, none of them appears to verify causality and, as such, they are ignored.

Other time machines are based on exiting our spacetime, entering into a mathematical space (e.g., of complex unitary character), and then returning into our spacetime to complete the loop.

Other attempts have been based on quantum tunnelling effects and other means.

By comparison, the Causal Time Machine proposed in Ref. [14] achieves a closed loop at the classical level without exiting the forward light cone and verifying causality.[4]

4.3.3 Isogeometric Propulsion

All means of locomotion developed by mankind to date, from prehistoric times all the way to current interplanetary missions, have been based on *Newtonian propulsions*, that is, propulsions all based on *Newton's principle of action and reaction.*

As an example, human walking is permitted by the action generated by leg muscles and the reaction caused by the resistance of the feet on the grounds. The same action and reaction is also the origin of *all* other available locomotions, including contemporary automobiles or rockets used for interplanetary missions.

Following the identification of the principle of propulsion, the next central issue is the displacement that is evidently characterized by the *Euclidean distance.* We are here referring to the conventional Euclidean space $E(r, \delta, R)$ over the reals R with familiar coordinates $r = (x, y, z) \times I$, metric $\delta = Diag.(1, 1, 1)$, units for the three axes $I = I_{3\times 3} = Diag(1\ \text{cm},\ 1\ \text{cm},\ 1\ \text{cm})$ hereon used in their dimensionless form $I = Diag.(1, 1, 1)$, and Euclidean distance that we write in the isoinvariant form

$$D^2 = r^2 \times I = (x^2 + y^2 + z^2) \times I \in R. \tag{4.3.2}$$

The *geometric locomotion* can be defined as the *covering of distances via the alteration (also called deformation) of the Euclidean geometry without any use of action and reaction.* The *only* possible realization of such a geometric locomotion that avoid the theorems of catastrophic inconsistencies of Section 1.5, as well as achieves compatibility with our sensory perception (see below), is the *isogeometric locomotion* [15b] namely, that permitted by the *Euclid-Santilli isogeometry* and relative *isodistance.*

We are here referring to the Euclid-Santilli isospace (Section 3.2) $\hat{E}(\hat{r}, \hat{\delta}, \hat{R})$ over the isoreals $\hat{R}$ with isocoordinates $\hat{r} = (x, y, z) \times \hat{I}$, metric $\hat{\delta} = \hat{T}_{3\times 3} \times \delta$, isounits for the three isoaxes

$$\hat{I} = \hat{I}_{3\times 3} = Diag(n_1^2\,\text{cm},\ n_2^2\,\text{cm},\ n_3^2\,\text{cm}) = 1/\hat{T}_{3\times 3} > 0 \tag{4.3.3}$$

that will also be used hereon in the dimensionless form

$$\hat{I} = Diag(n_1^2,\ n_2^2,\ n_3^2), \tag{4.3.4}$$

and *isodistance* that we write in the isoinvariant form[5]

$$\hat{D}^{\hat{2}} = \hat{r}^{\hat{2}} = (x^2/n_1^2 + y^2/n_2^2 + z^2/n_3^2) \times \hat{I} \in \hat{R}, \tag{4.3.5}$$

in which case the deformation of the geometry is called *geometric mutation.*[6]

It is evident that $\hat{D}$ can be bigger equal or smaller than D. Consequently, the isogeometric locomotion occurs when $\hat{D} < D$, as in the example below

$$\hat{I} = Diag.(n_1^2, 1, 1) \ll I = Diag.(1, 1, 1), \quad \hat{T} \gg I, \tag{4.3.6a}$$

$$\hat{D}^{\hat{2}} = (x^2/n_1^2 + y^2 + z^2) \ll D^2 = (x^2 + y^2 + z^2). \tag{4.3.6b}$$

The understanding of the above locomotion requires a knowledge of the *isobox* of Section 3.2. Consider such an isobox and assume that it is equipped with isogeometric locomotion. In this case, there is no displacement at all that can be detected by the internal observer. However, the external observer detects a displacement of the isobox the amount $x^2 - x^2/n_1^2$.

This type of locomotion is new because it is causal, invariant and occurs without any use of the principle of action and reaction and it is geometric because it occurs via the sole local mutation of the geometry.

The extension to the *causal spacetime machine*, or *spacetime isogeometric locomotion* is intriguing, and can be formulated via the *Minkowski-Santilli isospace* of Section 3.2 with four-isodistance

$$\hat{D}^{\hat{2}} = (x^2/n_1^2 + y^2/n_2^2 + z^2/n_3^2 - c^2 \times t^2/n_4^2) \times \hat{I} \in \hat{R} \tag{4.3.7}$$

where $n_4 > 0$.

The main implications in this case is the emergence of the additional *time mutation* as expected to occur jointly with any *space mutation.* In turn, this implies that the *isotime* $\hat{t} = t/n_4$ (that is, the internal time) can be bigger equal or smaller than the time t (that of the external observer).

More specifically, from the preservation of the original trace of the metric, *isorelativity predicts that the mutations of space and time are inversely promotional to each others.* Therefore, jointly with the motion ahead in space there is a motion backward in time and vice versa.

Consequently, the external observer sees the object moving with his naked eye, and believes that the object evolves in his own time, while

Figure 4.5. An artistic rendering of the "SpaceTime Machine", namely, the "mathematical" prediction of traveling in space and time permitted by the isodual theory of antimatter. The main assumption is that the aether (empty space) is a universal medium characterized by a very high density of positive and negative energies that can coexist because existing in distinct, mutually isodual spacetimes. Virtually arbitrary trajectories and speeds for isoselfdual states (only) are then predicted from the capability of extracting from the aether very high densities of positive and negative energies in the needed sequence. Discontinuous trajectories do not violate the law of inertia, speeds much bigger than the speed of light in vacuum, and similarly apparently anomalous events, do not violate special relativity because the locomotion is caused by the change of the local geometry and not by conventional Newtonian motions.

in reality the object could evolve far in the past. Alternatively, we can say that the inspection of an astrophysical object with a telescope, by no means, implies that said object evolves with our own time because it could evolve with a time dramatically different than that after adjustments due to the travel time of light because, again, light cannot carry any information on the actual time of its source.

To further clarify this important point, *light cannot possibly carry information on the time of its source because light propagates at the speed c at which there is no time evolution.*

As a concrete example, one of the consequences of interior gravitational problems treated via Santilli's isorelativity (see Section 3.5) is that *the time of interior gravitational problems, $\hat{t} = t/n_4$, depends on the interior density n_4^2, rather than the inertial mass, thus varying for astrophysical bodies with different densities.*

This implies that if two identical watches are originally synchronized with each other on Earth, and then placed in the interior gravitational field of astrophysical bodies with different densities, they will no longer

be synchronized, thus evolving with different times, even though light may continue to provide the information needed for their intercommunication.

In particular, *the time evolution of astrophysical bodies slows down with the increase of the density*,

$$\hat{t}_1 < \hat{t}_2, \quad n_{41}^2 > n_{42}^2. \tag{4.3.8}$$

It should also be noted that the above effect has no connection with similar Riemannian predictions because it is structurally dependent on the *change of the units*, rather than geometric features.

A prediction of isospecial relativity is that the bigger the density, the slower the time evolution. Thus, a watch in the interior of Jupiter is predicted to move *slower* than its twin on Earth under the assumption that the density of Jupiter (being a gaseous body) is significantly smaller than that of Earth (that can be assumed to be solid for these aspects).

As stressed in Section 4.3.1, *the above spacetime machine is a purely mathematical model*. To render it a reality, there is the need to identify the *isogeometric propulsion*, namely a source for the geometric mutations of type (4.3.5).

Needless to say, the above problem cannot be quantitatively treated on grounds of available scientific knowledge. However, to stimulate the imagination of readers with young minds of any age, a speculation on the possible mechanism of propulsion should be here voiced.

The only source of geometric mutation conceivable today is the availability of very large energies concentrated in very small regions of space, such as energies of the order of 10^{30} ergs/cm^3. Under these conditions, isorelativity does indeed predict isogeometric locomotion because these values of energy density generate very large values of isounits $\hat{I}$, with very small values of the isotopic element $\hat{T}$, resulting in isogeometric locomotions precisely of type (4.3.5).

The only possible source of energy densities of such extreme value is empty space. In fact, according to current views, space is a superposition of positive and negative energies in equal amounts each having extreme densities precisely of the magnitude needed for isogeometric locomotion.

The speculation that should not be omitted in this section is therefore that, one day in the future, the advancement of science will indeed allow to extract from space at will all needed amounts of both positive and negative energy densities.

In the event such an extraction becomes possible in a directional way, a spaceship would be able to perform all desired types of trajectories, including trajectories with sharp discontinuities (instantaneous 90 degrees turns), instantaneous accelerations, and the like without any violation

of the law of inertia because, as indicated earlier, the spaceship perceives no motion at all. It is the geometry in its surroundings that has changed.

Moreover, such a spaceship would be able to cover interstellar distances in a few of our minutes, although arriving at destination way back in the time evolution of the reached system.

Science has always surpassed science fiction and always will, because there is no limit to the advancement of scientific knowledge. On this ground it is, therefore, easy to predict that, yes, one day mankind will indeed be able to reach far away stars in minutes.

It is only hoped that, when that giant step for mankind is achieved, the theory that first achieved its quantitative and invariant prediction, Santilli isorelativity, will be remembered.

Notes

1 Again, the author would appreciate the indication of similar contributions prior to 1974.

2 The author would appreciate being kept informed by experimentalist in the field.

3 The author would like to express his sincere appreciation to T. Goldman for the courtesy of bringing to his attention the important references [22–29] that could not be reviewed here for brevity, but whose study is recommended as a necessary complement of the presentation of this monograph.

4 The indication by colleagues of other versions of the spacetime machine with a proved verification of causality without existing from our spacetime would be appreciated.

5 By "isoinvariance" we means invariance under conventional space or spacetime symmetries plus the isotopic invariance.

6 According to the contemporary terminology, "deformations" are alterations of the original structure although referred to the original field. As such they are afflicted by the catastrophic inconsistencies of Section 1.5. The term "mutation", first introduced by Santilli in Ref. [21] of 1967, is today referred to an alteration of the original structure under the condition of preserving the original axioms, thus requiring the formulation on isospaces over isofields that avoid said theorems of catastrophic inconsistency.

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Chapter 5

GRAND-UNIFICATION AND COSMOLOGY

5.1 ISO-GRAND-UNIFICATION

5.1.1 The Role of Antimatter in Grand Unifications

As indicated earlier, no conclusive study on antimatter can be conducted without its consistent inclusion in *grand unifications* of gravitational [1–3] and electroweak interactions [4–7]. Vice versa, no grand unification can be considered scientifically valuable without the correct inclusion of antimatter because the latter has a profound impact in the very structure of a consistent grand unification.

All studies on grand unifications conducted until now have been essentially restricted to matter. When antimatter is included, the studies have to be enlarged to *two grand unifications*, one for matter and the other for antimatter with a correct anti-automorphic (or anti-isomorphic) interconnecting map.

Consequently, the inclusion of antimatter in grand unifications introduces severe restrictions on the admissible models, which restrictions are generally absent when antimatter is ignored and grand unifications are restricted to matter alone.

We shall, therefore, avoid the review of the very large number of structurally inconsistent grand unifications published since Einstein's times and leave to the interested reader their re-examination in light of the new advances of this volume.

An in depth study of grand unifications soon reveals the need of formulating antimatter at the purely classical level, the need for abandoning curvature, and the need for a geometric unification of special and general relativities as presented in preceding chapters. It is only at the level

of these broader views on grand unifications that the isodual theory of antimatter emerges as inevitable.

Even though presented at the end of this monograph, the author initiated his studies on grand unification, constructed the needed broadening or modifications of pre-existing methods, and then achieved an invariant, axiomatically consistent grand unification.

This process requires it two decades of research before the publication of the first paper on grand unification, a lapse of time illustrating the complexity of the problem, as known in any case by the failure of the large number of preceding attempts.

The reader should be aware that, in this section, we shall exclusively study *closed-isolated systems of electroweak and gravitational interactions in vacuum* that are treatable via the Lie-isotopic branch of hadronic mechanics and its isodual. Interior problems, such as those inclusive of the origin of gravitation, require the broader Lie-admissible branch of hadronic mechanics and their treatment will be merely indicated at the end of this section for development by interested readers.

5.1.2 Axiomatic Incompatibilities of General Relativity and Electroweak Interactions

The preceding efforts for a grand unification of gauge theories of electroweak interactions and gravitation as described by general relativity are afflicted by the following axiomatic incompatibilities, first presented in Ref. [9] of 1997 (see also the related papers [10,11]):

(1) **Incompatibilities due to antimatter:** electroweak theories are *bona fide* relativistic field theories, thus characterizing antimatter via *negative-energy* solutions, while general relativity characterizes antimatter via *positive-definite* energy-momentum tensors. This first incompatibility renders manifestly inconsistent all attempts at grand unification known to this author.[1]

(2) **Incompatibilities due to curvature:** electroweak theories are essentially flat theories since they are formulated via *Minkowskian* axioms, while general relativity is centrally dependent on curvature since it is based on *Riemannian* axioms. This second incompatibility is another, independent, primary origin of the failure of the vast number of attempts at grand unification existing in the literature and carries profound implications, such as the extension to grand unification of the theorems of catastrophic inconsistencies of Section 1.4.

(3) **Incompatibilities due to spacetime symmetries:** electroweak interactions are based on the axioms of special relativity, thus verifying

the fundamental *Poincaré symmetry* $P(3.1)$, while such a basic symmetry is absent in general relativity and is replaced by a generic covariance. This third incompatibility has additional profound implications for any consistent grand unification because either one abandons the basic symmetries of electroweak interactions in favor of an unknown covariance, or one abandons general relativity for a new theory admitting a universal symmetry.

(4) **Incompatibilities due to the lack of a Minkowskian limit of general relativity:** as it is well known [1–3], general relativity admits a well defined *Euclidean* limit under PPN approximation, but one century of studies have failed to identify a corresponding well defined *Minkowskian* limit. On the other side, electroweak interactions [4–7] are formulated on a Minkowski spacetime. This fourth incompatibility of the two interactions then emerges in a number of aspects, such as irreconcilable ambiguities in the identification of total conservation laws of grand unifications when inclusive of gravitational interactions.

(5) **Incompatibilities due to the nonunitary character of quantum gravity:** as it is also well known, electroweak theories are *operator* field theories with a *unitary* structure, thus having invariant prediction of numerical values permitting meaningful experimental verifications. By comparison, all quantum formulations of general relativity (see, e.g. Ref. [8] and references quoted therein) have a *nonunitary* structure. Besides evident, additional, independent inconsistencies in attempting to combine unitary and nonunitary theories, any attempt of grand unification along contemporary views in general relativity and quantum gravity is afflicted by the theorems of catastrophic inconsistencies of Section 1.4.

It is evident that no significant advance can be achieved in grand unifications without, firstly, a serious addressing of these inconsistencies and, secondly, without their resolution.

Recall that the theory of electromagnetic interactions, when (and only when) restricted to the *vacuum*[2], has a majestic mathematical and physical consistency that eventually propagated to unified theories of electromagnetic and weak interactions.

The view adopted in this monograph, identifiable in more details only now, is that, rather than abandoning the majestic beauty of electroweak theories, we abandon instead the popular views on gravitation of the 20-th century due to their catastrophic inconsistencies and, as a condition to achieve a consistent grand unification, we reconstruct gravitational theories in such a way to have *the same abstract axioms* of electroweak theories.

5.1.3 Resolution of the Incompatibilities via Isotopies and Isodualities

In this chapter we present a resolution of the above incompatibilities first achieved by Santilli in Ref. [9] of 1997 (see also Ref. [10,11] following a number of rather complex and diversified scientific journeys that can be outlined as follows:

(A) Isotopies. The scientific journey to achieve a consistent grand unification started in 1978 with memoirs [12,13] for the classical and operator isotopies. A baffling aspect in the inclusion of gravity in unified gauge theories is their geometric incompatibility.

The view that motivated Refs. [12,13] is that the difficulties experienced in achieving a consistent grand unification are primarily due to *insufficiencies in their mathematical treatment.*

Stated in plain language, the view here considered is that, due to the complexity of the problem, the achievement of an axiomatic compatibility between gravitation and electroweak interactions requires a basically new mathematics, that is, basically new numbers, new spaces, new symmetries, etc.

Following first the verification of the lack of existence in the literature of a mathematics permitting the desired consistent grand unification, and following numerous attempts, the *only* possible new mathematics resulted to be that permitted by the *isotopies* as first proposed in Refs. [12,13], namely, a generalization of the conventional trivial unit $+1$ of electroweak theories into the most general possible, positive-definite unit with an unrestricted functional dependence on local variables, called *Santilli's isounit*,

$$I = +1 > 0 \quad \rightarrow \quad \hat{I} = \hat{I}^\dagger = I(x, v, \psi, \partial\psi, \ldots) > 0, \tag{5.1.1}$$

and consequential compatible reconstruction of all main branches of mathematics.

The uniqueness of the isotopies is due to the fact that, *whether conventional or generalized, the unit is the basic invariant of any theory.* Therefore, the use of the unit for the generalization of pre-existing methods guarantees the preservation of the invariance so crucial for physical consistency (Sections 1.5.2 and 1.5.3).

Another aspect that illustrates the uniqueness of the isotopies for grand unifications is that *the positive-definiteness of the isounit guarantees the preservation of the abstract axioms of electroweak theories*, thus assuring axiomatic consistency of grand unification from the very beginning.

The general lines on isotopies presented in memoirs [12,13] of 1978 were then followed by laborious studies that reached mathematical and physical maturity only in memoir [14] of 1996, as outlined in Chapter 3 (see monographs [15] for a comprehensive presentation).

(B) Isodualities. The achievement of an axiomatically consistent grand unification for *matter* constitutes only *half* of the solution because, as stressed in Section 5.1.1, no grand unification can be considered physically significant without the consistent inclusion of antimatter.

The incompatibility of electroweak theories and general relativity for antimatter identified in Section 5.1.2 is only the symptom of deeper compatibility problems. As now familiar from the studies presented in this monograph, matter is treated at *all* levels, from Newtonian to electroweak theories, while antimatter is treated only at the level of *second quantization.*

Since there are serious indications that half of the universe could well be made up of antimatter (see Section 5.2), it is evident that a more effective theory of antimatter must apply at *all* levels.

Until such a scientific imbalance is resolved, any attempt at a grand unification can well prove to be futile.

Recall that charge conjugation in quantum mechanics is an *anti-automorphic map.* As a result, no classical theory of antimatter can possibly be axiomatically consistent via the mere change of the sign of the charge, because it must be an anti-automorphic (or, more generally, anti-isomorphic) image of that of matter in *all* aspects, including numbers, spaces, symmetries, etc.

The resolution of the above imbalance required a second laborious scientific journey that initiated with the proposal of the *isodual map* in memoirs [16] of 1985, here expressed for an arbitrary quantity

$$Q(x, v, \psi, \ldots) \quad \to Q^d = -Q^\dagger(-x^\dagger, -v^\dagger, -\psi^\dagger, -\partial\psi^\dagger, \ldots), \qquad (5.1.2)$$

proposal that was followed by various studies whose mathematical and physical maturity was only reached years later in memoir [14] of 1996, as reported in Chapters 2 and 3 (see also monographs [15] for a more general presentation).

To illustrate the difficulties, it is appropriate here to note that, following the presentation in papers [16] of 1985 of the main mathematical ideas, it took the author *nine years* before publishing their application to antimatter in paper [17] of 1994.

We are here referring to the original proposal of Refs. [16,17] of mapping isounit (5.1.1) for matter into an *negative-definite* nonsingular

arbitrary unit, known today as *Santilli's isodual isounits*,

$$\hat{I}(x, \psi, \partial\psi, \ldots) > 0$$
$$\rightarrow \hat{I}^d = -\hat{I}^\dagger(-x^\dagger, -\psi^\dagger, -\partial\psi^\dagger, \ldots) < 0 \tag{5.1.3}$$

and its use for the characterization of antimatter at all levels, from Newtonian mechanics to second quantization.

The uniqueness of the isodual representation is given by the fact that isodualities are the *only* known liftings permitting the construction of a mathematic that is anti-isomorphic to the conventional (or isotopic) mathematics, as necessary for a consistent representation of antimatter at all levels, while preserving the crucial invariance needed to avoid catastrophic inconsistencies.

(C) Poincaré-Santilli isosymmetry and its isoduals. The scientific journeys on isotopies and isodualities were only intended as prerequisites for the construction of the *universal symmetry of gravitation for matter and, separately, for antimatter* in such a way to be locally isomorphic to the spacetime symmetry of electroweak interactions, the latter being an evident condition of consistency.

It is easy to see that, without the prior achievement of a new gravitation possessing an invariance, rather than the covariance of general relativity, any attempt at constructing a grand unification will prove to be futile in due time.

The complexity of the problem is illustrated by the fact that, not only gravitation for matter had to be reformulated in a form admitting a symmetry, but that symmetry had to be compatible with the basic Poincaré symmetry of electroweak theories [4–7]. Moreover, a dual compatible symmetry had to be achieved for the gravity of antimatter.

The latter problems called for a third laborious scientific journey on the *isotopies and isodualities of the Poincaré symmetry* $\hat{P}(3.1)$, today called the *Poincaré-Santilli isosymmetry and its isodual* outlined in Section 3.5 (see monographs [15] for comprehensive studies). These studies included:

1) The isotopies and isodualities of the Lorentz symmetry initiated with paper [18] of 1983 on the classical isotopies with the operator counterpart presented in paper [19] of the same year;

2) The isotopies and isodualities of the rotational symmetry first presented in papers [16][3];

3) The isotopies and isodualities of the SU(2)-spin symmetry, first presented in paper [20] of 1993, and related implications for local realist, hidden variables and Bell's inequalities published in Ref. [21] of 1998;

4) The isotopies and isodualities of the Poincaré symmetry including the universal invariance of gravitation, first presented in paper [22] of 1993; and

5) The isotopies and isodualities of the spinorial covering of the Poincaré symmetry first presented in papers [23,24] of 1996.[4]

We are referring here to the reconstruction of the conventional symmetries with respect to an arbitrary nonsingular positive-definite unit (5.1.1) for the isotopies, and with respect to an arbitrary nonsingular negative-definite unit (5.1.3) for the isodualities.

This reconstruction yields the most general known nonlinear, nonlocal and noncanonical or nonunitary liftings of conventional symmetries, while the locally isomorphism for isotopies) (anti-isomorphism for isodualities) with the original symmetries is guaranteed by the positive-definiteness (negative-definiteness) of the generalized units.

One should be aware that the above structures required the prior step-by-step isotopies and isodualities of Lie's theory (enveloping associative algebras, Lie algebras, Lie groups, transformation and representation theories, etc.), originally proposed by Santilli in 1978 [12], studied in numerous subsequent works and today called the *Lie-Santilli isotheory and its isodual* (see Section 3.2 for an outline and Ref. [15] for comprehensive studies).

It is evident that the Poincaré-Santilli isosymmetry and its isodual have fundamental character for these studies. One of their primary applications has been the achievement of the universal *symmetry* (rather than covariance) of all possible Riemannian line elements in their isominkowskian representation [22]

$$ds'^2 = dx'^{\mu} \times g(x')_{\mu\nu} \times dx'^{\nu} \equiv dx^{\mu} \times g(x)_{\mu\nu} \times dx^{\nu} = ds^2, \qquad (5.1.4)$$

Once the unit of gauge theories is lifted to represent gravitation, electroweak interactions will also obey the Poincaré-Santilli isosymmetry for matter and its isodual for antimatter, thus offering realistic hopes for the resolution of the most difficult problem of compatibility between gravitation and electroweak interactions, that for spacetime symmetries.

Perhaps unexpectedly, the fundamental spacetime symmetry of the grand unified theory of Refs. [9–11] is based on the *total symmetry of Dirac's equation*, here written with related spacetime and underlying unit (see Chapter 2 for details)

$$S_{Tot} = \{SL(2.C) \times T(3.1) \times \mathcal{I}(1)\} \times \{SL^d(2.C^d) \times^d T^d(3.1) \times^d \mathcal{I}^d(1)\}, \qquad (5.1.5a)$$

$$M_{Tot} = \{M(x, \eta, R) \times S_{spin}\} \times \{M^d(x^d, \eta^d, R^d) \times^d S^d_{spin}\} \qquad (5.1.5b)$$

$$I_{Tot} = \{I_{orb} \times I_{spin}\} \times \{I^d_{orb} \times^d I^d_{spin}\}, \qquad (5.1.5c)$$

To understand the above occurrence, the reader should be aware that isodualities imply a new symmetry called *isoselfduality* (Section 2.1), given by the invariance under the isodual map (5.1.2).

Dirac's gamma matrices verify indeed this new symmetry (from which the symmetry itself was derived in the first place), i.e.,

$$\gamma_\mu \to \gamma_\mu^d = -\gamma_\mu^\dagger = \gamma_\mu. \tag{5.1.6}$$

Consequently, contrary to a popular belief throughout the 20-th century, the Poincaré symmetry *cannot* be the total symmetry of Dirac's equations, evidently because it is not isoselfdual.

For evident reasons of consistency, the total symmetry of Dirac's equation must also be isoselfdual as the gamma matrices are. This condition identifies the total symmetry (5.1.5a) because that symmetry is indeed isoselfdual.

To understand the dimensionality of symmetry (5.1.5a) one must first recall that isodual spaces are independent from conventional spaces. The doubling of the conventionally believed ten-dimensions of the Poincaré symmetry then yields *twenty* dimensions.

But relativistic invariants possess the novel *isotopic invariance* (3.5.27), i.e.,

$$\begin{aligned}(x^\nu \times \eta_{\mu\nu} \times x^\nu) \times I &\equiv [x^\nu \times (w^{-2} \times \eta)_{\mu\nu} \times x^\nu) \times (w^2 \times I) \\ &= (x^\nu \times \hat{\eta}_{\mu\nu} \times x^\nu) \times \hat{I},\end{aligned} \tag{5.1.7}$$

with corresponding isotopic invariance of Hilbert's inner product

$$\begin{aligned}< \psi| \times |\psi > \times I &\equiv < w^{-1} \times \psi| \times |w^{-1} \times \psi > \times (w^2 \times I) \\ &= < \psi| \hat{\times} |\psi > \times \hat{I}.\end{aligned} \tag{5.1.8}$$

Consequently, the conventional Poincaré symmetry has emerged as being *eleven dimensional* at both the classical and operator levels, as first presented by Santilli in Ref. [22] of 1993 and studied in Section 3.5.3. It then follows that *the total symmetry* (5.1.5*a*) *of Dirac's equations is twenty-two dimensional.*

The grand unification proposed in Refs. [9–11] is based on the axiomatic structure of the conventional Dirac's equations, not as believed throughout the 20-th century, but as characterized by isotopies and isodualities.

In particular, the grand unification here studied is permitted by the new isotopic invariances (5.1.7) and (5.1.8) that are hidden in relativistic invariants [21], thus assuring the operator compatibility of the grand unification, as we shall see.

The reader should not be surprised that the two new invariances (5.1.7) and (5.1.8) remained undetected throughout the 20-th century because their identification required the prior discovery of *new numbers*, first the numbers with arbitrary positive units, and then the additional new numbers with arbitrary negative units for invariances [25].

(D) Classical and operator isogravitation. After a number of (unpublished) attempts, the resolution of numerous inconsistencies of general relativity studied in Section 1.4, plus the inconsistencies for grand unifications, requested the *isotopic reformulation of gravitation*, today known as *Santilli's isogravitation*, first presented at the VII M. Grossman Meeting on General Relativity of 1996 [26], as reviewed in Section 3.5, essentially consisting in the factorization of any given (nonsingular and symmetric) Riemannian metric $g(x)$ into the Minkowskian metric η multiplied by a 4×4-matrix $\hat{T}$,

$$g(x) = \hat{T}_{Grav}(x) \times \eta, \tag{5.1.9}$$

and the reconstruction of gravitation with respect to the isounit

$$\hat{I}_{Grav}(x) = 1/\hat{T}_{Grav}(x), \tag{5.1.10}$$

thus requiring the isotopic reformulation of the totality of the mathematical and physical methods of general relativity.

Despite its simplicity, the implications of isogravitation are far reaching, such as:

1) The isotopic reformulation permits the achievement of the universal Poincaré-Santilli isoinvariance for all possible gravitational models;

2) The isotopic reformulation eliminates curvature for the characterization of gravity, and replaces it with *isoflatness*, thus achieving compatibility with the flatness of electroweak interactions;

3) The isotopic reformulation reconstructs unitarity on iso-Hilbert spaces over isofields via the identical reformulation of nonunitary transform at the foundations of hadronic mechanics (Chapter 3)

$$U \times U^\dagger \neq I \to \hat{U} \hat{\times} \hat{U}^\dagger = \hat{U}^\dagger \hat{\times} \hat{U} = \hat{I}_{Grav} \tag{5.1.11}$$

where

$$U \times U^\dagger = \hat{I}, \quad \hat{U} = U \times \hat{T}^{1/2}_{Grav}, \tag{5.1.12}$$

thus providing the *only* known resolution of the catastrophic inconsistencies of Theorems 1.5.1 and 1.5.2.

Above all, isogravitation achieved the first and only known, axiomatically consistent operator formulation of gravitation provided by relativistic hadronic mechanics of Section 3.5, as first presented in Ref. [27] of 1997.

In fact, gravity is merely imbedded in the *unit* of relativistic operator theories. Since the gravitational isounit is positive-definite from the nonsingular and symmetric character of the metric $g(x)$ in factorization (5.1.9), the abstract axioms of operator isogravity are the conventional axioms of relativistic quantum mechanics, only subjected to a broader realization.

The preservation of conventional relativistic axioms then assures the achievement, for the first time as known by the author, of a consistent operator formulation of gravitation.[5]

(E) Geometric unification of special and general relativities. The resolution of the problems caused by lack of any Minkowskian limit of general relativity requested additional studies. After a number of (unpublished) attempts, the only possible solution resulted to be a geometric unification of special and general relativities, first presented in Ref. [28], in which the two relativities are characterized by the same abstract axioms and are differentiated only by their realization of the basic unit. The trivial realization $I = Diag.(1, 1, 1, 1)$ characterizes special relativity, and broader realization (5.1.10) characterizes general relativity.

The latter final efforts requested the construction *ab initio* of a new geometry, today known as *Minkowski-Santilli isogeometry* [28] in which the abstract axioms are those of the Minkowskian geometry, including the abstract axiom of flatness necessary to resolve the catastrophic inconsistencies of Section 1.4, yet the new geometry admits the entire mathematical formalist of the Riemannian geometry, including covariant derivatives, Christoffel's symbols, etc. (see Section 3.2 for an outline and monographs [15] for comprehensive studies).

The important point is that at the limit

$$Lim\ \hat{I}_{Grav}(x) \to I, \tag{5.1.13}$$

the Minkowskian geometry and conventional special relativity are recovered identically and uniquely.

The reader should be aware that the grand unification presented in this section is centrally dependent on the Minkowski-Santilli isogeometry, the Poincaré-Santilli isosymmetry, and the isotopic formulation of gravitation. Their knowledge is a necessary pre-requisite for the technical understanding of the following sections.

5.1.4 Isotopic Gauge Theories

The isotopies of gauge theories were first studied in the 1980's by Gasperini [29], followed by Nishioka [30], Karajannis and Jannussis [31] and others, and ignored thereafter for over a decade.

These studies were defined on conventional spaces over conventional fields and were expressed via the conventional differential calculus. As such, they are not invariant, as it became shown in memoirs [32], thus suffering of the catastrophic inconsistencies of Theorem 1.5.2.

Refs. [9–11] presented, apparently for the first time, the *invariant isotopies of gauge theories*, or *isogauge theories* for short, and their isoduals,those formulated on isospaces over isofields and characterized by the isodifferential calculus of memoir [14]. For completeness, let us recall that the latter theories are characterized by the following methods:

(1) Isofields [25] of isoreal numbers $\hat{R}(\hat{n}, \hat{+}, \hat{\times})$ and isocomplex numbers $\hat{C}(\hat{c}, \hat{+}, \hat{\times})$ with: additive isounit $\hat{0} = 0$; generalized multiplicative isounit $\hat{I}$ given by Eq. (5.1.9); elements, isosum, isoproduct and related generalized operations,

$$\hat{a} = a \times \hat{I}, \hat{a}\hat{+}\hat{b} = (a + b) \times \hat{I}, \tag{5.1.14a}$$

$$\hat{a}\hat{\times}\hat{b} = \hat{a} \times \hat{T} \times \hat{b} = (a \times b) \times \hat{I}, \tag{5.1.14b}$$

$$\hat{a}^{\hat{n}} = \hat{a}\hat{\times}\hat{a}\hat{\times}...\hat{\times}\hat{a}, \tag{5.1.14c}$$

$$\hat{a}^{\hat{1/2}} = a^{1/2} \times \hat{I}^{1/2}, \hat{a}\hat{/}\hat{b} = (\hat{a}/\hat{b}) \times \hat{I}, \text{ etc.} \tag{5.1.14d}$$

(2) Isominkowski spaces [18] $\hat{M} = \hat{M}(\hat{x}, \hat{\eta}, \hat{R})$ with isocoordinates $\hat{x} = x \times \hat{I} = \{x^\mu\} \times \hat{I}$, isometric $\hat{N} = \hat{\eta} \times \hat{I} = [\hat{T}(x, \ldots) \times \eta] \times \hat{I}$, and *isointerval* over the isoreals $\hat{R}$

$$\begin{aligned}(\hat{x} - \hat{y})^{\hat{2}} &= [(\hat{x} - \hat{y})^\mu \hat{\times} \hat{N}_{\mu\nu} \hat{\times} (\hat{x} - \hat{y})^\nu] \\ &= [(x - y)^\mu \times \hat{\eta}_{\mu\nu} \times (x - y)^\nu] \times \hat{I},\end{aligned} \tag{5.1.15}$$

equipped with *Kadeisvili isocontinuity* [33] and the *isotopology* developed by G. T. Tsagas and D. S. Sourlas [34], R. M. Santilli [14], R. M. Falcón Ganfornina and J. Núñez Valdés [35,36] (see also Aslander and Keles [37]). A more technical formulation of the isogauge theory can be done via the isobundle formalism on isogeometries.

(3) Isodifferential calculus [14] characterized by the following isodifferentials

$$\hat{d}\hat{x}^\mu = \hat{I}^\mu_\nu \times d\hat{x}^\nu, \tag{5.1.16a}$$

$$\hat{d}\hat{x}_\mu = \hat{T}^\nu_\mu \times d\hat{x}_\nu, \tag{5.1.16b}$$

and isoderivatives

$$\hat{\partial}_\mu \hat{f} = \hat{\partial}\hat{f}\hat{/}\hat{\partial}\hat{x}^\mu = (\hat{T}^\nu_\mu \times \partial_\nu f) \times \hat{I}, \tag{5.1.17a}$$

$$\hat{\partial}^\mu \hat{f} = (\hat{I}^\mu_\nu \times \partial_\nu f) \times \hat{I}, \hat{\partial}\hat{x}^\mu\hat{/}\hat{\partial}\hat{x}^\nu = \hat{\delta}^\mu_\nu = \delta^\mu_\nu \times \hat{I}, \text{ etc.} \tag{5.1.17b}$$

where one should note the inverted use of the isounit and isotopic element with respect to preceding formulations.

(4) Isofunctional isoanalysis [15], including the reconstruction of all conventional and special functions and transforms into a form admitting of $\hat{I}_{Grav}$ as the left and right unit. Since the iso-Minkowskian geometry preserves the Minkowskian axioms, it allows the preservation of the notions of straight and intersecting lines, thus permitting the reconstruction of trigonometric and hyperbolic functions for the Riemannian metric $g(x) = \hat{T}(x) \times \eta$.

(5) Isominkowskian geometry [28], i.e., the geometry of isomanifolds $\hat{M}$ over the isoreals $\hat{R}$, that satisfies all abstract Minkowskian axioms because of the joint liftings

$$\eta \to \hat{\eta} = T(x, \ldots) \times \eta \tag{5.1.18a}$$

$$I \to \hat{I} = T^{-1}, \tag{5.1.18b}$$

while preserving the machinery of Riemannian spaces as indicated earlier, although expressed in terms of the isodifferential calculus.

In this new geometry *Riemannian* line elements are turned into identical *Minkowskian* forms via the embedding of gravity in the deferentials, e.g., for the Schwarzschild exterior metric we have the iso-Minkowskian reformulation (Ref. [28], Eq. (2.57)), where the spacetime coordinates are assumed to be covariant,

$$\hat{ds} = \hat{dr}^{\hat{2}} \hat{+} \hat{r}^{\hat{2}} \hat{\times} (\hat{d\theta}^{\hat{2}} \hat{+} isosin^{\hat{2}}\hat{\theta}) \hat{-} \hat{dt}^{\hat{2}}, \tag{5.1.19a}$$

$$\hat{dr} = \hat{T}_r \times dr, \hat{dt} = \hat{T}_t \times dt, \tag{5.1.19b}$$

$$\hat{T}_r = (1 - 2 \times M/r)^{-1}, \quad \hat{T}_t = 1 - 2 \times M/r. \tag{5.1.19c}$$

(6) Relativistic hadronic mechanics [15] characterized by the *iso-Hilbert space* $\hat{\mathcal{H}}$ with *isoinner product and isonormalization* over $\hat{C}$

$$< \hat{\phi}|\hat{\times}|\hat{\psi} > \times \hat{I}, \quad < \hat{\psi}|\hat{\times}|\hat{\psi} >= \hat{I}. \tag{5.1.20}$$

Among various properties, we recall that: the *iso-Hermiticity* on $\hat{\mathcal{H}}$ coincides with the conventional Hermiticity (thus, all conventional observables remain observables under isotopies); the isoeigenvalues of iso-Hermitean operators are real and conventional (because of the identities

$$\hat{H}\hat{\times}|\hat{\psi} >= \hat{E}\hat{\times}|\hat{\psi} >= E \times |\hat{\psi} >); \tag{5.1.21}$$

the condition of *isounitarity* on $\hat{\mathcal{H}}$, over $\hat{C}$ is given by

$$\hat{U}\hat{\times}\hat{U}^{\dagger} = \hat{U}^{\dagger}\hat{\times}\hat{U} = \hat{I}, \tag{5.1.22}$$

(see memoir [27] for details).

(7) The Lie-Santilli isotheory [12] with: conventional (ordered) basis of generators $X = (X_k)$, and parameters $w = (w_k)$, $k = 1, 2, \ldots, n$, only formulated in isospaces over isofields with a common isounit; universal enveloping isoassociative algebras $\hat{\xi}$ with infinite-dimensional basis characterized by the isotopic Poincaré-Birkhoff-Witt theorem [12]

$$\hat{I},\ \hat{X}_i\hat{\times}\hat{X}_j,\ (i \leq j),\quad \hat{X}_i\hat{\times}\hat{X}_j \times \hat{X}_k,\ (i \leq j \leq k, ...) \tag{5.1.23}$$

Lie-Santilli subalgebras [12]

$$[\hat{X}_i\hat{,}\hat{X}_j] = \hat{X}_i\hat{\times}\hat{X}_j - \hat{X}_j\hat{\times}\hat{X}_i = \hat{C}_{ij}^{k}(x, \ldots)\hat{\times}\hat{X}_k, \tag{5.1.24}$$

where the $\hat{C}$'s are the structure disfunctions; and isogroups characterized by isoexponentiation on $\hat{\xi}$ with structure [12]

$$\hat{e}^{\hat{X}} = \hat{I}\hat{+}\hat{X}\hat{/}\hat{1}\hat{!}\hat{+}\hat{X}\hat{\times}\hat{X}\hat{/}\hat{2}\hat{!}\hat{+}... = (e^{\hat{X}\times\hat{T}}) \times \hat{I} = \hat{I} \times (e^{\hat{T}\times\hat{X}}), \tag{5.1.25}$$

Despite the isomorphism between isotopic and conventional structures, the lifting of Lie's theory is nontrivial because of the appearance of the matrix $\hat{T}$ with nonlinear integrodifferential elements in the very *exponent* of the group structure, Eq. (5.1.25).

To avoid misrepresentations, one should keep in mind that the isotopies of Lie's theory *were not* proposed to identify "new Lie algebras" (an impossible task since all simple Lie algebras are known from Cartan's classification), but to construct instead the most general possible nonlinear, nonlocal and noncanonical or nonunitary "realizations" of known Lie algebras.

(8) Isolinearity, isolocality and isocanonicity or isounitarity. Recall from lifting (5.1.25) that isosymmetries have the most general possible nonlinear, nonlocal and noncanonical or nonunitary structure. A main function of the isotopies is that of reconstructing linearity, locality and canonicity or unitarity on isospaces over isofields, properties called *isolinearity, isolocality and isocanonicity or isounitarity.* These are the properties that permit the bypassing of the theorems of catastrophic inconsistencies of Section 1.5.

As a result, the use of the conventional *linear* transformations on M over R, $X' = A(w) \times x$ violates *isolinearity* on $\hat{M}$ over $\hat{R}$.

In general, *any* use of conventional mathematics for isotopic theories leads to a number of inconsistencies which generally remain undetected by nonexperts in the field.[6]

(9) Isogauge theories [9–11]. They are characterized by an n-dimensional connected and non-isoabelian isosymmetry $\hat{G}$ with: basic n-dimensional isounit (4.1.9); iso-Hermitean generators $\hat{X}$ on an iso-Hilbert space $\hat{\mathcal{H}}$ over the isofield $\hat{C}(\hat{c}, \hat{+}, \hat{\times})$; universal enveloping associative algebra $\hat{\xi}$ with infinite isobasis (5.1.23); isocommutation rules (5.1.24); isogroup structure

$$\hat{U} = \hat{e}^{-i\times X_k \times \theta(x)_k} = (e^{-i\times X_k \times \hat{T} \times \theta(x)_k}) \times \hat{I}, \quad \hat{U}^{\dagger}\hat{\times}\hat{U} = \hat{I} \tag{5.1.26}$$

where one should note the appearance of the gravitational isotopic elements in the *exponent* of the isogroup, and the parameters $\theta(x)_k$ now depend on the iso-Minkowski space; isotransforms of the isostates on $\hat{\mathcal{H}}$

$$\hat{\psi}' = \hat{U}\hat{\times}\hat{\psi} = (e^{-i\times X_k \times \hat{T}(x,...)\times\theta(x)_k}) \times \hat{\psi}; \tag{5.1.27}$$

isocovariant derivatives [28]

$$\hat{D}_{\mu}\hat{\psi} = (\hat{\partial}_{\mu} - i\hat{\times}\hat{g}\hat{\times}\hat{A}(\hat{x})^k_{\mu}\hat{\times}\hat{X}_k)\hat{\times}\hat{\psi}; \tag{5.1.28}$$

iso-Jacobi identity

$$[\hat{D}_{\alpha}\hat{,}[\hat{D}_{\beta}\hat{,}\hat{D}_{\gamma}]]\,\hat{+}\,[\hat{D}_{\beta}\hat{,}[\hat{D}_{\gamma}\hat{,}\hat{D}_{\alpha}]]\,\hat{+}\,[\hat{D}_{\gamma}\hat{,}[\hat{D}_{\alpha}\hat{,}\hat{D}_{\beta}]] = 0, \tag{5.1.29}$$

where g and $\hat{g} = g \times \hat{I}$ are the conventional and isotopic coupling constants, $A(x)^k_{\mu} \times X_k$ and $\hat{A}(\hat{x})^k_{\mu}\hat{\times}\hat{X}_k = [A(x)^k_{\mu} \times X_k] \times \hat{I}$ are the gauge and isogauge potentials; isocovariance

$$\begin{aligned}
&(\hat{D}_{\mu}\hat{\psi})' \\
&= (\hat{\partial}_{\mu}\hat{U})\hat{\times}\hat{\psi}\hat{+}\hat{U}\hat{\times}(\hat{\partial}_{\mu}\hat{\psi})\hat{-}\hat{i}\hat{\times}\hat{g}\hat{\times}\hat{A}'(\hat{x})_{\mu}\hat{\times}\hat{\psi} = \hat{U}\hat{\times}\hat{D}_{\mu}\hat{\psi}, && (5.1.30a)\\
&\hat{A}(\hat{x})'_{\mu} = -\hat{g}^{-\hat{1}}\hat{\times}[\hat{\partial}_{\mu}\hat{U}(\hat{x})]\hat{\times}\hat{U}(\hat{x})^{-\hat{1}}, && (5.1.30b)\\
&\hat{\delta}\hat{A}(\hat{x})^k_{\mu} = -\hat{g}^{-\hat{1}} \,\hat{\times}\, \hat{\partial}_{\mu}\hat{\theta}(\hat{x})^k \,\hat{+}\, \hat{C}^k_{ij} \,\hat{\times}\, \hat{\theta}(\hat{x})^i \,\hat{\times}\, \hat{A}(\hat{x})^j_{\mu}, && (5.1.30c)\\
&\hat{\delta}\hat{\psi} = -\hat{i}\hat{\times}\hat{g}\hat{\times}\hat{\theta}(\hat{x})^k\hat{\times}\hat{X}_k\hat{\times}\hat{\psi}; && (5.1.30d)
\end{aligned}$$

non-isoabelian iso-Yang-Mills fields

$$\hat{F}_{\mu\nu} = \hat{i}\hat{\times}\hat{g}^{-\hat{1}}\hat{\times}[\hat{D}_{\hat{\mu}}, \hat{D}_{\nu}]\hat{\psi}, \tag{5.1.31a}$$

$$\hat{F}^k_{\mu\nu} = \hat{\partial}_{\mu}\hat{A}^k_{\nu}\,\hat{-}\,\hat{\partial}_{\nu}\hat{A}^k_{\mu}\,\hat{+}\,\hat{g}\hat{\times}\hat{C}^k_{ij}\hat{\times}\hat{A}^i_{\mu}\hat{\times}\hat{A}^j_{\nu}; \tag{5.1.31b}$$

related isocovariance properties

$$\hat{F}_{\mu\nu} \to \hat{F}'_{\mu\nu} = \hat{U}\hat{\times}\hat{F}_{\mu\nu}\hat{\times}\hat{U}^{-1}, \tag{5.1.32a}$$

$$Isotr(\hat{F}_{\mu\nu'}\hat{\times}\hat{F}^{\mu\nu'}) = Isotr(\hat{F}_{\mu\nu}\hat{\times}\hat{F}^{\mu\nu}), \tag{5.1.32b}$$

$$[\hat{D}_\alpha\hat{,}\hat{F}_{\beta\gamma}]\hat{+}[\hat{D}_\beta\hat{,}\hat{F}_{\gamma\alpha}]\hat{+}[\hat{D}_\gamma\hat{,}\hat{F}_{\alpha'\beta}] \equiv 0; \tag{5.1.32c}$$

derivability from the isoaction

$$\hat{S} = \hat{\int}\hat{d^4}\hat{x}(-\hat{F}_{\mu\nu}\hat{\times}\hat{F}^{\mu\nu}\hat{/}\hat{4}) = \hat{\int}\hat{d^4}\hat{x}(-\hat{F}^k_{\mu\nu}\hat{\times}\hat{F}^{\mu\nu}_k\hat{/}\hat{4}), \tag{5.1.33}$$

where $\hat{\int} = \int \times \hat{I}$, plus all other familiar properties in isotopic formulation.

The *isodual isogauge theory*, first proposed in Refs. [9–11], is the image of the preceding theory under the isodual map (5.1.2) when applied to the totality of quantities and their operations.

The latter theory is characterized by the isodual isogroup $\hat{G}^d$ with isodual isounit

$$\hat{I}^d_{Grav} = -\hat{I}^\dagger_{Grav} = -\hat{I}_{Grav} = -1/\hat{T}_{Grav} < 0. \tag{5.1.34}$$

The elements of the base fields

$$\hat{R}^d(\hat{n}^d, \hat{+}^d, \hat{\times}^d), \tag{5.1.35}$$

are given by the isodual isoreal numbers

$$\hat{n}^d = -\hat{n} = -n \times \hat{I}, \tag{5.1.36}$$

and those of the field

$$\hat{C}^d(\hat{c}^d, \hat{+}^d, \hat{\times}^d), \tag{5.1.37}$$

are the isodual isocomplex numbers

$$\hat{c}^d = -(c \times \hat{I})^\dagger = (n_1 - i \times n_2) \times \hat{I}^d = (-n_1 + i \times n_2) \times \hat{I}. \tag{5.1.38}$$

The carrier spaces are the isodual iso-Minkowski spaces $\hat{M}^d(\hat{x}^d, -\hat{\eta}^d, \hat{R}^d)$ on $\hat{R}^d$ and the isodual iso-Hilbert space $\mathcal{H}^d$ on $\hat{C}^d$ with isodual isostates and isodual isoinner product

$$|\hat{\psi}>^d = -|\hat{\psi}>^\dagger = -<\psi|, \tag{5.1.39a}$$

$$<\hat{\phi}|^d \times \hat{T}^d \times |\hat{\psi}>^d \times \hat{I}^d. \tag{5.1.39b}$$

It is instructive to verify that all eigenvalues of isodual iso-Hermitean operators are *negative − definite* (when projected in our space-time),

$$\hat{H}^d\,\hat{\times}^d\,|\hat{\psi}>^d = <\psi| \times (-E). \tag{5.1.40}$$

$\hat{G}^d$ is characterized by the isodual Lie-Santilli isotheory with isodual generators $\hat{X}^d = -\hat{X}$, isodual isoassociative product

$$\hat{A}^d \hat{\times}^d \hat{B}^d = \hat{A}^d \times \hat{T}^d \times \hat{B}^d, \;\; \hat{T}^d = -\hat{T}, \tag{5.1.41}$$

and related isodual isoenveloping and Lie-Santilli isoalgebra.

The elements of $\hat{G}^d$ are the isodual isounitary isooperators

$$\hat{U}^d(\hat{\theta}^d(\hat{x}^d)) = -\hat{U}^\dagger(-\hat{\theta}(-\hat{x})). \tag{5.1.42}$$

In this way, the isodual isogauge theory is seen to be an anti-isomorphic image of the preceding theory, as desired.

It is an instructive exercise for the reader interested in learning the new techniques to study first the isodualities of the *conventional* gauge theory (rather than of their isotopies), and show that they essentially provide a mere reinterpretation of the usually discarded, advanced solutions as characterizing antiparticles.

Therefore, in the isoselfdual theory with total gauge symmetry $\hat{G} \times \hat{G}^d$, isotopic retarded solutions are associated with particles and advanced isodual solutions are associated with antiparticles.

No numerical difference is expected in the above reformulation because, as shown in Chapter 3, isotopies preserve not only the original axioms but also the original numerical value (when constructd properly).

It is also recommendable for the interested reader to verify that the isotopies are indeed equivalent to charge conjugation for all massive particles, with the exception of the photon (see Section 2.3). In fact, isodual theories predict that the antihydrogen atom emits a new photon, tentatively called by this author the *isodual photon* [38], that coincides with the conventional photon for all possible interactions, thus including electroweak interactions, *except gravitation.* This indicates that the isodual map is inclusive of charge conjugation for massive particles, but it is broader than the latter.

Isodual theories in general, thus including the proposed grand unification, predict that all *stable* isodual particles, such as the isodual photon, the isodual electron (positron), the isodual proton (antiproton) and their bound states (such as the antihydrogen atom), experience *antigravity* in the field of the Earth (defined as the reversal of the sign of the curvature tensor).

If confirmed, the prediction may offer the possibility in the future to ascertain whether far away galaxies and quasars are made-up of matter or of antimatter.

We finally note that isomathematics is a particular case of the broader *genomathematics*, also introduced for the first time in Ref. [12] of 1978

(see Section 3.6), which occurs for non-Hermitean generalized units and is used for an axiomatization of irreversibility.

In turn, genomathematics is a particular case of the *hypermathematics*, that occurs when the generalized units are given by ordered *sets* of non-Hermitean quantities and is used for the representation of multivalued complex systems (e.g. biological entities) in irreversible conditions.

Evidently both the genomathematics and hypermathematics admit an anti-isomorphic image under isoduality (see also Section 3.6).

In conclusion the methods outlined in this note permit the study of *seven* liftings of conventional gauge theories [9–11]:

(1) The *isodual gauge theories* for the treatment of antimatter without gravitation in vacuum;

(2,3) The *isogauge theories and their isoduals*, for the inclusion of gravity for matter and antimatter in reversible conditions in vacuum (exterior gravitational problem);

(4,5) The *genogauge theories and their isoduals*, for the inclusion of gravity for matter and antimatter in irreversible interior conditions (interior gravitational problems); and

(6,7) the *hypergauge theories and their isoduals*, for multivalued and irreversible generalizations.

For brevity this section is restricted to theories of type (1), (2), (3). The development of the remaining genotopies of gauge theories is left to interested readers.

5.1.5 Iso-Grand-Unification

In this section we review the *Iso-Grand-Unification* (IGU) with the inclusion of electroweak and gravitational interactions, first submitted in Refs. [9–11] via the 22-dimensional total isoselfdual isosymmetry given by isosymmetry (3.5.28) and its isodual

$$\hat{S}_{Tot} = (\hat{\mathcal{P}}(3.1)\hat{\times}\hat{G}) \times (\hat{\mathcal{P}}(3.1)^{d}\hat{\times}^{d}\hat{G}^{d})$$

$$= [\hat{SL}(2,\hat{C})\hat{\times}\hat{T}(3.1)\hat{\times}\hat{\mathcal{I}}(1)] \times [\hat{SL}^{d}(2,\hat{C}^{d})\hat{\times}^{d}\hat{T}^{d}(3.1)\hat{\times}^{d}\hat{\mathcal{I}}^{d}(1)], \quad (5.1.43)$$

where $\hat{\mathcal{P}}$ is the Poincaré-Santilli isosymmetry [22] in its isospinorial realization [24], $\hat{G}$ is the isogauge symmetry of the preceding section and the remaining structures are the corresponding isoduals.

Without any claim of a final solution, it appears that the proposed IGU does indeed offer realistic possibilities of resolving the axiomatic incompatibilities (1)–(5) of Section 5.1.2 between gravitational and electroweak interactions.

In fact, IGU represents gravitation in a form geometrically compatible with that of the electroweak interactions, represents antimatter at all

levels via negative-energy solutions, and characterizes both gravitation as well as electroweak interactions via the universal Poincaré-Santilli isosymmetry.

It should be indicated that we are referring here to the *axiomatic* consistency of IGU. In regard to the *physical* consistency we recall that isotopic liftings preserve not only the original axioms, but also the original numerical values [15].

As an example, the image in iso-Minkowskian space over the isoreals of the light cone, the isolight cone, not only is a perfect cone, but a cone with the original characteristic angle, thus preserving the speed of light in vacuum as the maximal causal speed in iso-Minkowskian space.

This peculiar property of the isotopies implies the expectation that the proposed Iso-Grand-Unification preserves the numerical results of electroweak interactions.

The reader should be aware that the methods of the recent memoir [27] permit a truly elementary, explicit construction of the proposed IGU.

As well known, the transition from the Minkowskian metric η to Riemannian metrics $g(x)$ is a *noncanonical transform* at the classical level, and, therefore, a *nonunitary transform* at the operator level.

The method herein considered for turning a gauge theory into an IGU consists in the following representation of the selected gravitational model, e.g., Schwarzschild's model:

$$g(x) = T(x) \times \eta, I(x) = U \times U^\dagger = 1/\hat{T} \tag{5.1.44a}$$

$$= Diag.[(1 - 2 \times M/r) \times Diag(1, 1, 1),\ (1 - 2 \times M/r)^{-1}], \tag{5.1.44b}$$

and then subjecting the *totality* of the gauge theory to the nonunitary transform $U \times U^\dagger$.

The method then yields: the isounit

$$I \to \hat{I} = U \times I \times U^\dagger; \tag{5.1.45}$$

the isonumbers

$$a \to \hat{a} = U \times a \times U^\dagger = a \times (U \times U^\dagger) = a \times \hat{I},\ a = n, c; \tag{5.1.46}$$

the isoproduct with the correct expression and Hermiticity of the isotopic element,

$$\begin{aligned} A \times B &\to U \times (A \times B) \times U^\dagger \\ &= (U \times A \times U^\dagger) \times (U \times U^\dagger)^{-1} \times (U \times B \times U^\dagger) \\ &= \hat{A} \times \hat{T} \times \hat{B} = \hat{A} \hat{\times} \hat{B}; \end{aligned} \tag{5.1.47}$$

the correct form of the iso-Hilbert product on $\hat{C}$,

$$< \phi| \times |\psi > \rightarrow U \times < \phi| \times |\psi > \times U^{\dagger}$$
$$= (< \psi| \times U^{\dagger}) \times (U \times U^{\dagger})^{-1} \times (U \times |\psi >) \times (U \times U^{\dagger})$$
$$=< \hat{\phi}| \times \hat{T} \times |\hat{\psi} > \times \hat{I}; \tag{5.1.48}$$

the correct Lie-Santilli isoalgebra

$$A \times B - B \times A \rightarrow \hat{A} \hat{\times} \hat{B} - \hat{B} \hat{\times} \hat{A}; \tag{5.1.49}$$

the correct isogroup

$$U \times (e^{X}) \times U^{\dagger} = (e^{X \times \hat{T}}) \times \hat{I}, \tag{5.1.50}$$

the Poincaré-Santilli isosymmetry $\mathcal{P} \rightarrow \hat{\mathcal{P}}$, and the isogauge group $G \rightarrow \hat{G}$.

It is then easy to verify that the emerging IGU is indeed invariant under all possible additional nonunitary transforms, provided that, for evident reasons of consistency, they are written in their identical isounitary form,

$$W \times W^{\dagger} = \hat{I}, \tag{5.1.51a}$$

$$W = \hat{W} \times \hat{T}^{1/2}, W \times W^{\dagger} = \hat{W} \hat{\times} \hat{W}^{\dagger} = \hat{W}^{\dagger} \hat{\times} \hat{W} = \hat{I}. \tag{5.1.51b}$$

In fact, we have the invariance of the isounit

$$\hat{I} \rightarrow \hat{I}' = \hat{W} \hat{\times} \hat{I} \hat{\times} \hat{W}^{\dagger} = \hat{I}, \tag{5.1.52}$$

the invariance of the isoproduct

$$\hat{A} \hat{\times} \hat{B} \rightarrow \hat{W} \hat{\times} (\hat{A} \hat{\times} \hat{B}) \hat{\times} \hat{W}^{\dagger} = \hat{A}' \hat{\times} \hat{B}', \text{ etc.} \tag{5.1.53}$$

Note that the isounit is *numerically* preserved under isounitary transforms, as it is the case for the conventional unit I under unitary transform, and that the selection of a nonunitary transform $W \times W^{\dagger} = \hat{I}'$ with value different from $\hat{I}$ evidently implies the transition to a different gravitational model.

Note that the lack of implementation of the above nonunitary-isounitary lifting to only *one* aspect of the original gauge theory (e.g., the preservation of the old numbers or of the old differential calculus) implies the loss of the invariance of the theory [32].

The assumption of the negative-definite isounit $\hat{I}^{d} = -(U \times U^{\dagger})$ then yields the isodual component of the IGU.

Note finally that diagonal realization (5.1.44) has been assumed mainly for simplicity. In general, the isounit is positive-definite but *nondiagonal* 4×4-dimensional matrix. The Schwarzschild metric can then be more effectively represented in its isotropic coordinates as studied, e.g. in Ref. [39], pp. 196–199).

In closing, the most significant meaning of IGU is that *gravitation has always been present in unified gauge theories. It did creep-in unnoticed because embedded where nobody looked for, in the "unit" of gauge theories.*

In fact, the isogauge theory of Section 5.1.4 coincides with the conventional theory at the abstract level to such an extent that we could have presented IGU with exactly the same symbols of the conventional gauge theories without the "hats", and merely subjecting the same symbols to a more general realization.

Also, the isounit representing gravitation as per rule (5.1.9) verifies all the properties of the conventional unit I of gauge theories,

$$\hat{I}^{\hat{n}} = \hat{I}, \quad \hat{I}^{\hat{1/2}} = \hat{I}, \tag{5.1.54a}$$

$$d\hat{I}/dt = \hat{I}\hat{\times}\hat{H} - \hat{H}\hat{\times}\hat{I} = \hat{H} - \hat{H} = 0, \text{ etc.} \tag{5.1.54b}$$

The "hidden" character of gravitation in conventional gauge theories is then confirmed by the isoexpectation value of the isounit recovering the conventional unit I of gauge theories,

$$\hat{<}\hat{I}\hat{>} =< \hat{\psi}| \times \hat{T} \times \hat{I} \times \hat{T} \times |\hat{\psi} > / < \hat{\psi}| \times \hat{T} \times |\hat{\psi} >= I. \tag{5.1.55}$$

It then follows that *IGU constitutes an explicit and concrete realization of the theory of "hidden variables"* [40]

$$\lambda = T(x) = g(x)/\eta, \hat{H}\hat{\times}|\hat{\psi} >= \hat{H} \times \lambda \times |\hat{\psi} >= E_\lambda \times |\hat{\psi} >, \tag{5.1.56}$$

and the theory is correctly reconstructed with respect to the new unit

$$\hat{I} = \lambda^{-1}, \tag{5.1.57}$$

in which von Neumann's Theorem [41] and Bell's inequalities [42] do not apply, evidently because of the nonunitary character of the theory (see Ref. [21] and Vol. II of Ref. [15] for details).

In summary, the proposed inclusion of gravitation in unified gauge theories is essentially along the teaching of Einstein, Podolsky, and Rosen [43] on the "lack of completion" of quantum mechanics, only applied to gauge theories.

5.2 ISO-, GENO-, AND HYPER-SELF-DUAL COSMOLOGIES

A rather popular belief of the 20-th century was that the universe is solely composed of matter. This belief was primarily due to the scientific imbalance pertaining to antimatter as being solely studied at the level of second quantization, without any theoretical, let alone experimental, mean available for the study of antimatter.

In reality, there exists rather strong evidence that the universe is indeed composed of matter as well as antimatter and, more particularly, that some of the galaxies are made up of matter and others of antimatter.

To begin, not only the expansion of the universe, but more particularly the recently detected increase of the expansion itself, can be readily explained via an equal distribution of matter and antimatter galaxies.

In fact, antigravity experienced by matter and antimatter galaxies (studied in the preceding chapter) explains the expansion of the universe, while the continuous presence of antigravity explains the increase of the expansion.

The assumption that the universe originated from a primordial explosion, the "big bang", could have explained at least conceptually the expansion of the universe. However, the "big bang" conjecture is eliminated as scientifically possible by the increase of the expansion itself.

The "big bang" conjecture is also eliminated by the inability to explain a possible large presence of antimatter in the universe, trivially, because it would have been annihilated at the time of the "big bang" because produced jointly with matter, as well as for other reasons.

By comparison, the only plausible interpretation at the current state of our knowledge is precisely the assumption that the universe is made up half of *matter galaxies* and half of *antimatter galaxies* due to the joint explanation of the expansion of the universe and its increase.

Independently from the above, there exists significant evidence that our Earth is indeed bombarded by antimatter particles and asteroids.

Astronauts orbiting Earth in spaceship have systematically reported that, when passing over the dark side, they see numerous flashes in the upper atmosphere that can be only interpreted as *antimatter cosmic rays*, primarily given by high energy antiprotons and/or positrons[7] originating from far away antimatter galaxies, which antiparticles, when in contact with the upper layers of our atmosphere, annihilate themselves producing the flashes seen by astronauts.

Note that the conventional *cosmic rays* detected in our atmosphere are *matter cosmic rays*, that is, high energy *particles*, such as protons and electrons, originating from a matter supernova or other matter astrophysical event.

In any case, it is evident that matter cosmic rays with sufficient energy can indeed penetrate deep into our atmosphere, while antimatter cosmic rays will be stopped by the upper layers of our atmosphere irrespective of their energy.

In addition, there exists evidence that our Earth has been hit by *antimatter meteorites* that, as such, can only originate from an astrophysical body made up of antimatter.

The best case is the very large devastation recorded in 1908 in Tunguska, Siberia, in which over one million acres of forest were completely flattened in a radial direction originating from a common center without any crater whatever, not even at the center.

The lack of a crater combined with the dimension of the devastation, exclude the origination from the explosion of a *matter asteroid*, firstly, because in this case debris would have been detected by the various expeditions in the area and, secondly, because there is no credible possibility that the mere explosion of a matter asteroid could have caused a devastation over such a large area requiring energies computed at about 100 times the atomic bomb exploded over Hiroshima, Japan.

The only plausible interpretation of the *Tunguska explosion* is that it was due to an antimatter asteroid that eventually annihilated after contact deep into our matter atmosphere.

The important point is that the numerical understanding of the Tunguska explosion requires an antimatter mass of the order of a ton, namely, an antimatter asteroid that, as such, can only originate from the supernova explosion of an antimatter star.

Consequently, the evidence on the existence of even one antimatter asteroid confirms the existence in the universe of antimatter stars. Since it is highly improbable that antimatter stars can exist within a matter galaxy, antimatter asteroids constitute significant evidence on the existence in the universe of antimatter galaxies.

But again, the expansion of the universe as well as the increase of the expansion itself are the strongest evidence for an essentially equal distribution of matter and antimatter galaxies in the universe, as well as for the existence of antigravity between matter and antimatter.

In any case, there exist no alternative hypothesis at all known to this author, let alone a credible hypothesis, that could explain quantitatively both the expansion of the universe and the increase of the expansion itself.

In view of the above occurrences, as well as to avoid discontinuities at creation, Santilli [44] proposed the new *Iso-Self-Dual Cosmology*, namely, a cosmology in which the universe has an exactly equal amount of matter

and antimatter, much along the isoselfdual re-interpretation of Dirac's equation of Section 2.3.6.

Needless to say, such a conception of the universe dates back to the very birth of cosmology, although it was abandoned due to various reasons, including the lack of a consistent classical theory of antimatter, inconsistencies for negative energies, and other problems.

The above conception of the universe was then replaced with the "big bang" conjecture implying a huge discontinuity at creation, in which a possible antimatter component in the universe is essentially left untreated.

All the above problems are resolved by the isodual theory of antimatter, and quantitative astrophysical studies on antimatter galaxies and quasars can now be initiated at the purely classical level.

Moreover, the prediction that the *isodual light* emitted by antimatter experiences a repulsion in the gravitational field of matter [38], permits the initiation of actual measurements on the novel *antimatter astrophysics.*

Noticeably, there already exist reports that certain astrophysical events can only be explained via the repulsion experiences by light emitted by certain galaxies or quasars, although such reports could not be subjected to due scientific process since the mere existence of such a repulsion would invalidate Einstein's gravitation, as studied in Section 1.4.

Even though the assumption of an equal distribution of matter and antimatter in the universe dates back to the discovery of antimatter itself in the early 1930s, the Iso-Self-Dual Cosmology is structurally new because it is the first cosmology in scientific records based on a *symmetry*, let alone an *isoselfdual symmetry*, that of Dirac's equation subjected to isotopies, Eq. (5.1.43), i.e.,

$$\hat{S}_{Tot} = (\hat{\mathcal{P}}(3.1)\,\hat{\times}\,\hat{G}) \times (\hat{\mathcal{P}}(3.1)^d\,\hat{\times}^d\,\hat{G}^d)$$

$$= [\hat{SL}(2,\hat{C})\hat{\times}\hat{T}(3.1)\hat{\times}\hat{\mathcal{I}}(1)] \times [\hat{SL}^d(2,\hat{C}^d)\hat{\times}^d\hat{T}^d(3.1)\hat{\times}^d\hat{\mathcal{I}}^d(1)], \quad (5.2.1)$$

In fact, virtually all pre-existing cosmologies are based on Einstein's gravitation, thus eliminating a universal symmetry *ab initio.*

Other novelties of the Iso-Self-Dual Cosmology are given by the implications, that are impossible without the isotopies and isodualities, such as:

1) The direct interpretation of the expansion of the universe, as well as the increase of the expansion itself, since antigravity is permitted by the isodualities but not in general by other theories;

2) The prediction that the universe has absolutely null total characteristics, that is, an absolutely null total time, null total mass, null total energy, null total entropy, etc., as inherent in all isoselfdual states[8];

3) The creation of the universe without any discontinuity at all, but via the joint creation of equal amounts of matter and antimatter, since all total characteristics of the universe would remain the same before and after creation.

We also mention that the isoselfdual cosmology was proposed by Santilli [44] to initiate mathematical and theoretical studies on the creation of the universe, studies that are evidently prohibited by theories with huge discontinuities at creation.

After all, we should not forget that the Bible states the creation first of light and then of the universe, while it is now known that photons can create a pair of a particle and its antiparticle.

Also, there is a mounting evidence that space (the *aether* or the *universal substratum*) is composed of a superposition of positive and negative energies, thus having all pre-requisites needed for the creation of matter and antimatter galaxies.

As one can see, a very simple property of the new number theory, the invariance under isoduality as it is the case for the imaginary unit (Section 2.1.1),

$$i \equiv i^d = -i^\dagger = -\bar{i}, \tag{5.2.2}$$

acquires a fundamental physical character for a deeper understanding of Dirac's gamma matrices (Section 2.3.6),

$$\gamma_\mu \equiv \gamma_\mu^d = -\gamma_\mu^\dagger, \tag{5.2.3}$$

and then another fundamental character for the entire universe.

To understand the power of isodualities despite their simplicity, one should meditate a moment on the fact that the assumed main characteristics of the universe as having an equal amount of matter and antimatter, can be reduced to a primitive abstract axiom as simple as that of the new invariance (5.2.2).

Needless to say, the condition of exactly equal amounts of matter and antimatter in the universe is a *limit case*, since in reality there may exist deviations, with consequential *breaking of the isoselfdual symmetry* (5.2.1). This aspect cannot be meaningfully discussed at this time due to the abyssal lack of knowledge we now have on the antimatter component in our universe.

It should be finally indicated that, in view of the topological features assumed for the basic isounit

$$\hat{I} = \hat{I}^\dagger > 0, \tag{5.2.4}$$

the Iso-Self-Dual Cosmology outlined above can only represent a closed and reversible universe, thus requiring suitable broadening for more realistic theories.

Recall that, from its Greek meaning, "cosmology" denotes the entire universe. Consequently, no theory formulated until now, including the Iso-Self-Dual Theory, can be called, strictly speaking, a "cosmology" since the universe is far from being entirely composed of closed and reversible constituents.

To begin, there is first the need to represent irreversibility, since the behavior in time of all stars, galaxies and quasars in the universe is indeed irreversible.

This first need can be fulfilled with the Iso-Self-Dual Cosmology realized via isounits that are positive-definite, but explicitly time dependent,

$$\hat{I}(t,\ldots) = \hat{I}^{\dagger}(t,\ldots) \neq \hat{I}(-t,\ldots). \tag{5.3.5}$$

which feature assures irreversibility, although the universe remains closed due to the conservation of the total energy of matter and that of antimatter.

The latter model has evident limitations, e.g., in view of the possible continuous creation of matter and antimatter advocated by various researchers as an alternative to the "big bang".

The latter condition, when joint with the necessary representation of irreversibility, requires the broader *Geno-Self-Dual Cosmology*, namely, a cosmology based on the Lie-admissible lifting of symmetry (5.2.1), via the further generalization of generalized units (5.3.4) and (5.2.5) into four genounits, one per each of the four possible directions of time

$$\hat{I}^{>}, \quad -\hat{I}^{>}, \quad (\hat{I}^{>})^{d} = -^{<}\hat{I}, \quad -(\hat{I}^{>})^{d} = ^{<}\hat{I}, \tag{5.2.6}$$

whose explicit construction is left to the interested reader for brevity (see Section 3.6).

Nevertheless, the latter genotopic lifting itself cannot be considered, strictly speaking, a "cosmology" because a basic component of the universe is life, for which genotopic theories are insufficient, as indicated in Section 3.7, due to their single-valuedness.

He latter need inevitably requires the formulation of cosmologies via the most general possible methods studied in this monograph, the multivalued hyperstructure of Section 3.6.12, resulting in the *Hyper-Self-Dual Cosmology*, namely, a cosmology based on the hyperlifting of symmetry (5.2.1) characterized by the ordered multivalued hyperunits

$$\hat{I}^{>} = \{\hat{I}_1^{>}, \hat{I}_2^{>}, \hat{I}_3^{>}, \ldots\} \quad -\hat{I}^{>} = \{-\hat{I}_1^{>}, -\hat{I}_2^{>}, -\hat{I}_3^{>}, \ldots\}, \tag{5.2.7a}$$

$$(\hat{I}^{>})d = \{-^{<}\hat{I}_1, -^{<}\hat{I}_2, -^{<}\hat{I}_3, \ldots\} \quad -(\hat{I}^{>})^d = \{^{<}\hat{I}_1, ^{<}\hat{I}_2, ^{<}\hat{I}_3, \ldots\}. \tag{5.2.6}$$

However, at this point we should remember the limitations of our mind and admit that the foundations of the Hyper-Self-Dual Cosmology, such as the multi-valued hypertime encompassing all four directions of time, is simply beyond our human comprehension.

After all, we have to admit that a final scientific understanding of life will likely require thousands of years of studies.

5.3 CONCLUDING REMARKS

The analysis conducted in this monograph establishes that the isodual theory of antimatter does indeed resolve the scientific imbalance of the 20-th century caused by the treatment of matter at all levels of study, and the treatment of antimatter at the sole level of second quantization.

In fact, the isodual theory of antimatter achieves an absolute democracy of treatment of both matter and antimatter at all levels, from Newton to second quantization.

In particular, the analysis presented in this monograph establishes that the isodual theory of antimatter is verified by all known experimental data on antimatter, since the isodual theory trivially represents all available classical experimental data (Section 2.2.3), while resulting in being equivalent to charge conjugation at the operator level (Section 2.3.7), as a result of which the entire currently available experimental knowledge on antiparticles is verified by the isodual theory.

Despite its simplicity, the isodual theory of antimatter has deep implications for all quantitative sciences, including classical mechanics, particle physics, superconductivity, chemistry, biology, astrophysics and cosmology.

The most salient consequence of the isodual theory is the prediction of antigravity experienced by *elementary* antiparticles in the field of matter and vice-versa.

This prediction is a direct consequence of the very existence of a consistent classical formulation of antimatter, the electromagnetic origin of the gravitational mass with consequential phenomenological equivalence of electromagnetism and gravitation for both attraction and repulsion, the forgotten Freud identity of the Riemannian geometry, and other aspects.

In reality, the prediction of antigravity for truly elementary antiparticles in the field of matter is rooted in so many diversified aspects that the possible experimental disproof of antigravity would likely require the reconstruction of theoretical physics from its foundations.

To minimize controversies, it should be stressed that the prediction of antigravity has been solely and specifically presented for *elementary* antiparticles, that is, for the *positron*, with the careful exclusion for first tests of any unstable or composite particles whose constituents are not seriously established as being all antiparticles.

As an illustration, we have discouraged the use in possible experiments on the gravity of the positronium as claim for final knowledge on the gravity of antimatter, because the positronium is predicted by the isodual theory to be attracted in both fields of matter and antimatter. Similarly we have discouraged the use of leptons because they may eventually result to be composite of particles and antiparticles.

Finally, we have strongly discouraged to assume experimental data on the gravity of antiprotons as final knowledge on the gravity of antiparticles, because antiprotons are today fabricated in high energy laboratories from matter components and are believed to be bound states of quarks for which no gravity at all can be consistently defined [38].

It then follows that, while all experimental data are indeed useful and should be supported, including experimental data on the gravity of antiprotons, their use for general claims on the gravity of antimatter could be deceptive.

Moreover, none of the numerous arguments against antigravity could even be properly formulated for the isodual theory, let alone have any value. As a result, the prediction of antigravity for elementary antiparticles in the field of matter is fundamentally unchallenged at this writing on theoretical grounds.

A test of the gravity of positrons in horizontal flight in a vacuum tube, that is resolutory via gravitational deflections visible to the naked eye, has been proposed by Santilli [45] and proved by the experimentalist Mills [46] to be feasible with current technology and be indeed resolutory (Section 4.2).

A comparative study of other tests has revealed that they are too delicate and require too sensitive measurements to be as resolutory as proposal [45] with current technologies.

It is hoped that the experimental community finally comes to its senses, and conducts fundamental test [45,46], rather than continuing to conduct tests of transparently less relevance at bigger public costs, because in the absence of a final experimental resolution of the problem of antigravity, the entire theoretical physics remains essentially in a state of suspended animation.

In turn, the possible experimental verification of antigravity (as above identified) would have implications so advanced as to be at the edge of our imagination.

One of these implications has been presented in Section 4.3 with the Causal Time Machine, the novel, non-Newtonian *isolocomotion* (propulsion to unlimited speeds without any action and reaction as requested by all currently available propulsions), and other far reaching possibilities.

The experimental resolution of the existence of antigravity for *truly elementary* antiparticles is also crucial to fulfil the original scope for which the isodual theory was built, namely, to conduct quantitative studies as to whether far-away galaxies and quasars are made up of matter or antimatter.

This main scope has been achieved via the *isodual photon*, namely, the discovery that, according to the isodual theory, photons emitted by antimatter appear to have a number of physical differenecs with the photons emitted by matter. In particular, *the simplest possible isodual electromagnetic waves have negative energy, thus experiencing antigravity in the field of matter.*

The above prediction requires the experimental resolution as to whether light emitted by antimatter is attracted or repelled by the gravitational field of matter.

Needless to say, the current availability at CERN of the antihydrogen atom is an ideal source for such a study, with the understanding that gravitational deflections of light at short distances (as attainable in a laboratory on Earth) are extremely small, thus implying extremely sensitive measurements.

More promising is the re-inspection of available astrophysical data privately suggested to the author because said data could already include evidence of light from far-away galaxies and quasars that is repelled by astrophysical objects closer to us.

Such a repulsion could not be publicly disclosed at this time because of known opposition by organized academic interests on Einsteinian doctrines since, as well known, Einstein's gravitation prohibits the existence of antigravity (Section 4.1).

It is hoped that such organized academic interests come to their senses too, if nothing else, to avoid an easily predictable serious condemnation by posterity, in view of the well known catastrophic inconsistencies of Einstein gravitation outlined in Section 1.4.

After all, we should not forget that antiparticles were first experimentally detected in cosmic rays, thus confirming their possible origin from supernova explosions of stars made up of antimatter.

Also, there are reports of huge explosions in Earth's atmosphere before the advent of atomic bombs without any crater on the ground, such as the 1908 Tunguska explosion in Siberia, which explosions can be best

interpreted as antimatter asteroids from far away antimatter galaxies or quasars penetrating in our atmosphere.

Therefore, it should not be surprising if light experiencing gravitational repulsion from matter is discovered first in astrophysics.

Additional tests on the possible gravitational repulsion of light emitted by antimatter can be done via the direct measurement of the deflection of light from far away galaxies and quasars when passing near one of the planets of our Solar system.

Under the assumption of using light originating from far away galaxies and quasars (to render plausible their possible antimatter nature), and for the use of a sufficient number of galaxies and quasars (to have a sufficient probability that at least one of them is made up of antimatter), these astrophysical measurements are potentially historical, and will signal the birth of the new science proposed in thnis monograph under the name of *antimatter astrophysics.*

The reader should be aware that, while the prediction of antigravity for *truly elementary* antiparticles is an absolute necessity for the validity of the isodual theory, the gravitational behavior of light emitted by antimatter is not that simple.

Recall from Section 4.2 that the prediction of antigravity for light emitted by antimatter is based on the negative value of its energy for the selected solution of the electromagnetic wave.

However, the photons is invariant under charge conjugation and travel at the maximal causal speed in vacuum, c. Therefore, the photon could well result to be a superposition of positive and negative energies, perhaps as a condition to travel at the speed c, in which case the photon would be an isoselfdual state, thus experiencing attraction in both fields of matter and antimatter.

As a consequence, the possible disproof of antigravity for light emitted by antimatter stars in the field of matter *would not* invalidate the isodual theory of antimatter, but merely tell us that our conception of light remains excessively simplistic to this day, since it could well be in reality a composite state of photons and their isoduals.

The issue is further complicated by the fact indicated during the analysis of this monograph that *antigravity is predicted between masses with opposite time evolutions*, as it is the case for a positron in the field of Earth. However, the photon travels at the speed of light at which speed time has no meaningful evolution.

As a result, it is not entirely clear to this author whether the sole value of negative energy for the isodual light is sufficient for the existence of a gravitational repulsion, and the issue is suggested for study by interested colleagues.

To express a personal view, it would be distressing if light solely experience gravitational attraction irrespective of whether in the field of matter or antimatter and whether originating from matter or antimatter, because this would imply the impossibility for experimental studies as to whether far-away galaxies and quasars are made up of matter or antimatter, since all other aspects, including thermodynamics, are not detectable at large distances, thus implying the perennial inability for mankind to reach any in depth knowledge of the universe.

The author does not believe so. Advances in human knowledge have no limit, and often go beyond the most vivid imagination, as established by scientific realities that resulted in being beyond the science fiction of preceding generations.

In closing, the author hopes that the studies presented in this monograph have stimulated young minds of any age and confirmed that science will never admit final theories. No matter how precious, beloved and valid a given theory may appear to be at a given time, its surpassing with broader theories more adequate for new scientific knowledge is only a matter of time.

Notes

1 The indication of grand unifications inclusive of antimatter would be greatly appreciated.

2 It is well known by expert, but rarely spoken, that Maxwell's equations have no real physical value for the treatment of electromagnetism within physical media for countless reasons, some of which have been treated in Chapter 1. As an illustration, only to locally varying character of electromagnetic waves within physical media requires a radical revision of electromagnetism in the arena considered as a condition to pass from academic politics to real science.

3 Papers [16] on the lifting of the rotational symmetry were evidently written before paper [19] on the lifting of the Lorentz symmetry, but appeared in print only two years following the latter due to rather unreasonable editorial processing by various journals reported in Ref. [16], which processing perhaps illustrates the conduct of some (but not all) editors when facing true scientific novelty.

4 Ref. [24], which is the most important reference of this entire monograph (because admitting all topics as particular cases), was rejected

for years by all journals of Western Physical Societies because the paper included an *industrial* application currently receiving large investments by the industry – although not by academia, – consisting in the achievement of a numerical, exact and invariant representation of *all* characteristics of the neutron as a bound state of a proton and an electron according to Rutherford. In fact, the resolution of the historical difficulties of Rutherford's conception of the neutron permits the utilization of the large clean energy contained in the neutron's structure, via its stimulated decay caused by a hard photon with a resonating frequency (numerically predicted by hadronic mechanics) that expels Rutherford's electron (the *isoelectron* with an isorenormalized mass generated by the nonlocal and non-Lagrangian interactions in the hyperdense medium inside the proton, see Section 3.7.3 and references quoted therein),

$$\gamma_{reson.} + n \to p^+ + e^- + \bar{\nu}.$$

Despite the undeniable mathematical consistency clear plausibility and evident large societal implications due to the need for new clean energies, Ref. [24] was rejected by all Western Physical Society without any credible scientific motivation because not aligned with organized interests in quantum mechanics and special relativity. Paper [24] was finally published in China in 1996. As a gesture of appreciation for this scientific democracy, the author organized in Beijing the 1997 International Workshop on Hadronic Mechanics (see the Proceedings [104,105,106] listed in the Bibliography of Chapter 3).

5 Note that the use of the words "quantum gravity" for operator formulation of gravitation, whether conventional or characterized by the isotopies, would be merely political. This is due to the fact that, on serious scientific grounds, the term "quantum" can only be referred to physical conditions admitting a quantized emission and absorption of energy as occurring in the structure of the hydrogen atom. By comparison, no such quantized orbits are possible for operator theories of gravity, thus rendering nonscientific its characterization as "quantum gravity". Ironically, the editor of a distinguished physics journal expressed interest in publishing a paper on "operator isogravity" under the condition of being called "quantum gravity", resulting in the necessary withdrawal of the paper by the author so as not to reduce fundamental physical inquiries to political compromises.

6 The use of conventional mathematics for isotheories would be the same as elaborating Balmer's quantum spectral lines in the hydrogen atoms with isofunctional analysis, resulting in evident major inconsistencies.

7 Evidently only *stable antiparticles can travel intergalactic distances without decaying.*

8 We are here referring to intrinsic characteristic of isoselfdual states, and not to the same characteristics when inspected from a matter or an antimatter observer that would be evidently impossible for the universe.

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