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Imagination and Rigor

Essays on Eduardo R. Caianiello's Scientific Heritage

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Foreword

The aim of this volume is to focus some of the unique features of Eduardo's scientific personality debating on what – ten years after his death – is still "alive" of his teachings and research activity in physics and cybernetics. He played an important role also as a scientific organizer. Some unusual and advanced schools that picked up central interdisciplinary aspects of some innovative fields "ahead of time" are still remembered and it seems "natural" to make reference to the problems debated 40 years ago, to the unique scientific climate of these exciting years as well as to many of Eduardo's pioneering papers.

The idea of meeting not "to remember" but to "assess" the present importance of his scientific ideas belongs to Silvia Caianiello. She was right in defending this idea against many "reasonable" objections. The volume reflects most of the content of the workshop held in December 2003 at the Istituto Italiano per gli Studi Filosofici. However, something lacks: first of all, the participatory atmosphere of the overfull conference room of Palazzo Serra di Cassano; secondly, many brief but warm personal remembrances of many (old and young) friends of Eduardo as well as the content of a round table discussion on themes tuned with Eduardo's attitude on the role of science in society.

The essays presented here witness the plurality of scientific interests of Eduardo. For this reason, there is no thematic grouping; the papers are instead presented alphabetically by author name, with the exception of Carla Persico Caianiello's remarks, which conclude the volume.

Hopefully these papers will contribute to diffuse Eduardo's ideas to new generations.

November 2005 *Settimo Termini*

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Pattern Discovery in the Crib of Procrustes*

Alberto Apostolico

Abstract. The study of physics purports to concise descriptors or theories, good at predicting a virtually unlimited set of replicas of a phenomenon of a certain nature. The discovery of patterns or structure in discrete objects pursues a similar goal, but it departs from the inference of physical laws in so far as the ensuing generation of unlimited replicas may be a curse rather than a blessing. Decades after the facts, an engineer turned computer scientist and still struggling with his math speculates about the origins of a physicist's fascination with the essence of complexity and structure; and how they can be inferred from examples. Which led to several and still largely unanswered questions, but ultimately helped shaping many a quest for a lifetime.

1 Introduction

We are taught that the *intensional* definition of a set consists of a list of attributes or qualities that uniquely intercept the elements of that set in a broader population or universe. By contrast, and not without some abuse of language, the *extensional* definition of a set is characterized, intensionally, as consisting of the exhaustive list of the members of that set. Likewise, the *extension* of a term is the collection of elements to which it is correctly applied, while its *intension* is the set of features which are shared by every element to which the term applies. Of course, these two descriptors are meant to be fungible: ideally, the intension of a set or term should accurately determine its extension. This bears pragmatic implications of enormous value: it empowers us with the ability to decide, for each newly-encountered item, whether or not it has all the relevant features shared among the objects defined by a term. Intriguingly, as the intension of a term is increased by the more detailed specification of features, the extension of that term tends to decrease, since fewer items now qualify for its application. In Logic, the collection of the attributes in a term is associated with the notion of *connotation*, whereas the collection of objects designated by that term is associated with *denotation*.

Like the brain, computers appear to be troubled by the denotation of infinite collections or aggregates, a fact perhaps reflected in the school of thought that equates intensionality with intentionality: *intentional* objects (interpreted as the objects of thought) must have intensional properties, the handle for connotation [1]. Thus, in order to serve as effective prostheses of the brain, computers must handle connotations.

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2 The powers of abduction

In order for us to reap the *predictive* powers of intensionality, we must first get to grips with the way it relates to extensionality. It would appear that the nature of this relationship varies with the case. Still speaking in vague terms, the transition from one mode to the other entails some machinery or tool capable of enabling if not implementing the transition itself. But such a machinery incarnates with varying degrees of power, whereby one could say that some intensional definitions are more extensionally oriented than others. When it comes to mathematical objects, for example, an extensional proclivity seems ingrained in that discipline. A theorem tolerates no exception, whereas one single counter-example will suffice to confute it. By contrast, no example can establish the validity of a theorem, unless all cases are checked.

Extensional orientation in physics is generally credited to Galileo, who paid dearly for it. The physical law encapsulates within some concise descriptors or theory a virtually unlimited set of replicas of a phenomenon of a certain type. Thus, a Law of Physics is not only a mathematical relationship among measurable quantities related to the state and properties of bodies. One might also say that there is an actual machinery that makes sure the intensional law is implemented. (One could argue whether this is also intentional with "t" replacing "s", but this goes beyond our scope.)

Disciplines in the natural sciences tend to be more troublesome. The periodic table is full of exceptions, the classification of the species is still being debated, and there is no automated fool proof rule that leads from the set of a patient's symptoms to his diagnosis.

In an extensionally oriented world, the two classical pillars of reasoning, deduction and induction – the second one interpreted here in the non-mathematical sense of probable argument – are not of much help in our utilitarian pursuit of predictability. Deductive inference moves from ensemble to specimen, from population to sample, whereas inductive inference follows the opposite direction, from specimen to ensemble, from sample to population.[1] Neither process is really conducive to discovery. As we know, C.S. Peirce [2, 3] introduced a third way, which he called *abduction* (or retroduction, or hypothesis). This is patterned after a re-shuffling of the terms in a syllogism, which results in a logical fallacy of the form: all M's are P's (rule); all S's are P's (result); therefore, all S's are M's (case). Rephrased in the jargon of sampling theory, the argument reads:

All rabbits came from the magician's hat;
all doves in this particular random sample came from the magician's hat,
and thus all doves are rabbits.

[1]The limitations of deduction appear pretty obvious today, less so in a society still impregnated with the remnants of Scholastics such as the one to which Francis Bacon (1620-1690) introduced induction as an outright rebellion against the principle of authority, and "the tendency of the mind to construct knowledge-claims out of itself", upon which deductive reasoners fed.

As a matter of fact, abduction is not about drawing conclusions as much as it is about building (educated) hypotheses, on the basis of the patterns that can be observed in a phenomenon [3]. Whereas deduction and induction represent two types of symbolic logic, abduction is a form of critical thinking bearing considerable practical yield, as it intertwines with the formation and testing of theories: the unexpected or surprising phenomenon P is observed; among the hypotheses $H_1, H_2, \ldots, H_n$, Hypothesis H_k is capable of explaining P. Therefore, H_k is pursued.

3 To generate and classify

The thriller "Smilla's Sense of Snow" by Peter Hoeg[2] reminds us that the likes of Eskimos, Greenlanders and Lappons enjoy over a dozen distinct nuances of snow.[3] Although, even at our latitudes, it is said that no two snowflakes are identical, we only have one word for snow. One favorite line of E.R. Caianiello was that kids brought up in highly polluted metropolitan areas thought that black was the color of snow. S. Watanabe[4] was not sure how this fitted in with his dismal results in the foundations of statistical classification.

In trying to feed intensional definitions to a computer we realize that these come in two main flavors. In one case, we give some *characteristic* vector of *features*, where each position of the vector is assigned to tag the presence or absence of a specific feature or property. Objects having vectors that are identical or similar by some measure coalesce in a cluster or class. The other way, which we will recapture later, is to give a *generating process* capable of building all and only the objects in a class. The Theorem of the Ugly Duckling states that if all features in our feature vector are given equal value then the Ugly Duckling is just as similar to a swan as another swan [4]. Thus, our ability to classify rests on our bias, which shows itself by way of distributing weights unevenly among the various features, on a subjective basis. Much in line with this philosophy, Watanabe was an advocate of Kahrunen-Loewe transforms [4], due to the ability of that expansion to self-extract the relevant features. One could argue that the Greek knew this (and much more) all along:

[2]Delta; Reprint edition (1995).

[3]Anthony C. Woodbury from University of Texas at Austin set up a compilation (July 1991) from Steven A. Jacobson's Yup'ik Eskimo dictionary (Alaska Native Language Center, University of Alaska, Fairbanks, 1984). This includes: qanuk for 'snowflake', qanir-, qanunge- and qanugglir- for 'to snow'; kaneq for 'frost'; kaner- for 'be frosty/frost'; kanevvluk and the corresponding verb kanevcir- for '(to get) fine snow/rain particles'; natquik 'drifting snow/etc', of which the corresponding action is natqu(v)igte-; nevluk and the verb nevlugte- for 'clinging debris' ... 'lint/snow/dirt ... '; aniu, apun, qanikcaq for 'snow on ground'; to which there correspond the actions qanikcir- and aniu-. We also have muruaneq for 'soft deep snow' qetrar- and qerretrar- 'for snow to crust'; nutaryuk for 'fresh fallen snow on the ground' qanisqineq for 'snow floating on water'. And so on.

[4]Satosi Watanabe, Japanese theoretical physicist and one of the founding fathers of Pattern Recognition. My inquiry: "Would you think that the fact that kids in NYC believe that snow is black is an instance of the theorem of the ugly duckling" was tinted with surrealism: our conversation was taking place in a dusty office atop the temporary site of the University of Salerno, an important Soccer Championship game was being broadcast all over town, and thus hardly any other soul inhabited the place at that moment besides the two of us. NYC was much more polluted at the time than it is today.

"$\Pi\acute{\alpha}\nu\tau\omega\nu\ \xi\rho\eta\mu\acute{\alpha}\tau\omega\nu\ \mu\acute{\epsilon}\tau\rho\nu\ o\ \acute{\alpha}\nu\theta\rho\omega\pi o\sigma$",[5] Protagoras of Abdera (485–415 BC) had clearly spelled this out in his effort to reconcile existence and change, and then sealed it for good measure: "of those that exist as well as of those that *do not* exist". Much later, J.L. Borges would come up with his massively quoted parody of taxonomy[6] which "deeply shattered" Michel Foucault and others, similarly involved in the quest for "the order of things".

As said, an alternative notion of class may be based on generative rules, whereby objects endowed with a similar structure must also be assembled in a similar way. Brillouin [5] devoted considerable attention to the problem of defining and measuring the information embodied in structure with the tools of Information Theory. As is well known, Shannon's theory equates information with surprise, whereby a telegram on a wedding night is not informative [6, 7, 27]. By this measure, however, a library would become more informative if all books were cut into small fragments to be then tossed up in the air. Still, early classifications of genetic sequences based on measures of self-information claimed some success [8], and at any rate the pursuit of similarly global measures is likely to be revived out of necessity in massive comparative analysis of whole genomes. Brillouin argued that structure is better captured by redundancy or *negentropy*, which is formally the opposite of Shannon's information, thereby instituting an interesting duality between the characters of information in storage and in transmission. Kolmogorov [9, 10] proposed alternative measures of information, more akin to the structure embodied in objects. For a string x, this measure is expressed as the length of the shortest program that would take a universal computer to synthesize x from scratch. Thus, strings that are hard to synthesize or unpredictable are more random (for this notion, see in particular [11]) than the easier ones. Along these lines, we have a good theory about a series of observations or phenomena whenever we can come up with a generating mechanism or law that is much shorter than the raw series. Unfortunately, the conclusions reached by Kolmogorov seem pendant to the Ugly Duckling: as long as we don't put our own bias, by way of privileging some regularities over others, we won't be able to come up with a theory that is the most compact possible one. Fortunately, morphology is so finely ingrained and functionally implicated in nature that we can adopt morphological structure as the bias meaningfully in many cases of interest. This leads to privileging *syntactic* regularities such as unveiled by grammatical inference techniques (see, e. g., [12, 13]), where objects are characterized on the basis of the generative power [14] of minimum description grammars and by the corresponding discriminating ability of acceptor automata. In line with the duality of information

[5] Man is the measure of everything.

[6] In "The Analytical Language of John Wilkins," from Otras Inquisiciones (Other Inquisitions 1937-1952, London: Souvenir Press, 1973), Jorge Luis Borges deals with the predicament of classification: "These ambiguities, redundances, and deficiences recall those attributed by Dr. Franz Kuhn to a certain Chinese encyclopedia entitled Celestial Emporium of Benevolent Knowledge. On those remote pages it is written that animals are divided into (a) those that belong to the Emperor, (b) embalmed ones, (c) those that are trained, (d) suckling pigs, (e) mermaids, (f) fabulous ones, (g) stray dogs, (h) those that are included in this classification, (i) those that tremble as if they were mad, (j) innumerable ones, (k) those drawn with a very fine camel's hair brush, (l) others, (m) those that have just broken a flower vase, (n) those that resemble flies from a distance."

mentioned earlier, the very same structural redundancies that are exposed by some such minimal grammars can be exploited in turn to achieve data compression. On these grounds, Lempel and Ziv founded [15] some of the most innovative, effective and elegant data compression methods known to date.

To bridge the statistical and grammatical approaches to classification is an intriguing and so far largely neglected task. Some thought-provoking reflections of the fact that some such bridges might in fact exist arise when we conjugate natural evolution with the formalism of math and computation. This should not come as a surprise. After all, living organisms are the most complex instances of communication, control and computing known to mankind. One of the goals of computational molecular biology is to develop a molecular taxonomy, that is, to derive a classification of species from the structure of their genetic material. In this pursuit, the degree of homology or proximity of a common ancestry may be assessed on the basis of a measure of similarity of genetic sequences [16]. In some relaxed sense, the sequences are treated as feature vectors except that the positions of the individual features are known only approximately. (To compensate for this, the "values" taken by the "attributes" are the same at every position.) In order to account for the similarity or distance among these vectors, a mechanism of *mutation* is now hypothesized which should lead from one sequence to the other within a minimum number of elementary changes or *edit* operations. At the outset, sequences that are connected pairwise by shorter edit scripts are believed to have evolved more closely. Thus, when cytochrome-c molecules[7] are grouped on the basis of their pairwise similarity measured in terms of edit distance, we have a glimpse of the evolutionary process that led to their development, as well as of the differentiation of species that went with it. In conclusion, this grouping of macromolecules – based on a generating process that seems just a restricted or specialized version of Kolmogorov's notion of conditional information – is not far from a taxonomy that regards a bio-sequence as the feature vector representation of self.

Where should we look in an attempt to capture the formation of structure? It must be a spot where some subassemblies aggregate to form more complex units. Under the oxymoron: "The sciences of the artificial", H. Simon [17] tried to pin down the essence of complexity as part of his attempt at reconciling the analytic or descriptive categories of science with the synthetic or prescriptive categories of engineering. He thought that complexity is constituted by the recursive partition of a system into subsystems, which results from the unescapable onset of hierarchy, dictated in turn by necessary constrains of stability[8]: hierarchical systems, such as

[7] An ancient protein, essential to the production of cellular energy, which has undergone little changes in millions of years, so that one can look into yeast, plant or human cells and find similar forms of cytochrome-c. Because of its ubiquity and early availability, cytochrome-c was among the first molecules to be used in studies of molecular taxonomy.

[8] There might be some kind of interpretive leap in going from one level of the hierarchy to another. The failure of the Logic Theorist program to go much beyond the first two pages of *Principia Mathematica* exposed hierarchy in the engineering of math: Simon liked to spend time explaining his efforts to automate the derivation of "meta-theorems", the tools that subliminally guide a mathematician and make him aware of what makes of a mere well formed formula also an actually interesting theorem – something a computer won't grasp as easily.

the social, biological, symbolic, etc., are better at survival because their subsystems can settle more easily into intermediate stable states.

4 Procrustes, the sub-semigroup and the Emperor's new map

The Procrustes[9] algorithm [18] probably reflected a physicist's compulsion to divide forces into "weak" and "strong". Here the strong forces are those that keep aggregates together while the weak ones are the syntax or glue that holds them together. How can we find the strong forces? It seems reasonable to try to distill off subaggregates by looking for unexpectedly high degrees of cohesiveness among the atomic parts. The natural handle for this is statistical analysis. In most natural languages, a strong local cohesiveness among symbols characterizes and actually defines certain substructures such as phonemes or syllables, and thus it is precisely these units that may be expected to be exposed in terms of statistical over-representation. Thus, it can happen that the statistical maneuvers that isolate such subunits in a language also lead to purely syntactic categories of the algebraic theory of formal languages. On the one hand, the syllables or phonemes separate the strong interaction from the weak one. On the other, they form the basis of a peculiar subsemigroup of the semigroup[10] of all strings on the alphabet of the language: like the symbols in the original alphabet, the syllables can generate by concatenation the set of all words in the dictionary and more; however, the spurious sequences generated by the syllables are not nearly as many as those generated by the original alphabet.

Using words repeatedly and interchangeably as the bed and the guest, Procrustes can now set himself up to the less despeakable task of discovering a *basis* for the primitive patterns of the dictionary. Once found, this basis can generate all past and future phrases ever to be proffered in the language. Problem is, the basis also generates concatenations of units, e. g. syllables that do not correspond to well formed words and phrases. This is, because we don't seem to have a strict characterization of the dictionary that would be essentially shorter than the dictionary itself. Ideally, the discovery of patterns and association rules thereof would have to concern itself with the task of deriving, from exposure to hidden specimens and partial extensions, adherent intensional characterizations of objects. Since one of the goals is to apply the findings to tasks of classification and prediction, we should not find ourselves in a position to predict more than can happen. However, this kind of duplicity plagues us still and rather ubiquitously the increasingly reiterated attacks to the task of pattern discovery, and yet we seem to have no other choice but to imagine Sisyphus happy.[11]

[9] Arguably the most interesting of challenges on Theseus's way to fame, Procrustes (literally, "he who stretches") was a bizarre host used to adjust his guests to their bed. He lured his victims to his house promising a great meal and a pleasant sleep in his very special bed which he promised to have the very peculiar property of adjusting in length to fit the occupant to the utmost of comfort. Such a unique "one-size-fits-all" feature, however, was achieved by either stretching or chopping off some of the guest's limbs.

[10] Elementary algebraic structure consisting of a set and a commutative binary operator, e. g. the set of all finite strings over an alphabet together with the concatenation operator.

[11] "Il faut imaginer Sisyphe heureux" is the closing line of Le Mythe de Sisyphe, A. Camus' metaphore of the human experience (Gallimard, Paris, 1985). Cf. also J. Monod, Le Hasard et la Nécessité (Seuil, 1973).

Because strings constitute such primeval structures and a natural habitat or embedding for almost any phenomenon, it is only natural that a formal study of patterns would start with them. Pattern discovery on strings has flourished by building on a repertoire of sophisticated techniques and tools developed since the early 1970s in pattern matching [19, 20]. In the typical problem of pattern matching, e. g. string searching, we are given a text x and a pattern y and the problem is to find all occurrences of y in x. In pattern discovery, we know much less about what we are looking for. Ideally, we would know nothing and yet discover interesting patterns, but we have already argued that this rests on shaky ground methodologically as well as computationally. Some milestones of syntactic string pattern discovery *ante litteram* date back to the beginning of the last century, when Axel Thue showed that over an alphabet of more than two symbols it is possible to write indefinitely long strings not containing any "square", i.e. any pattern of the form ww[12] (see, e. g., [21]). In a pair of sequences or a sequence family, it is of interest sometimes to find the longest common substring, or the longest common subsequence, where the former is a (longest) string w such that any string x in the family may be written as $x = vwz$, and the latter is a (longest) string w that is obtainable from any string in the family by deletion of zero or more consecutive characters.

Whether some prior domain-specific knowledge is given or not, the tenet is that a pattern or association rule that occurs more frequently than one would expect is potentially informative and thus interesting. Accordingly, patterns are sought that are more frequent than one would expect in either one string or a set of strings. To assess the interest of a pattern, measures such as the *quorum* or number of *colors* (i.e. how many sequences contain each one instance of the pattern) or occurrences may be used. Central to these developments is also the notion of an *association rule*, which is an expression of the form $S_1 \rightarrow S_2$ where S_1 and S_2 are sets of data attributes endowed with sufficient *confidence* and *support*. Sufficient support for a rule is achieved if the number of records whose attributes include $S_1 \cup S_2$ is at least equal to some pre-set minimum value. Confidence is measured instead in terms of the ratio of records having $S_1 \cup S_2$ over those having S_1, and is considered sufficient if this ratio meets or exceeds some pre-set minimum. Since the generation of all candidate associations would be prohibitive in most cases, one uses the observation that the terms of a frequently occurring association must be frequent in their own merit. Resort to abduction is widespread there. For example, having determined that some known enactors of protein translation, the so called promoters, contain significantly over-represented patterns, we find it natural to look for over-represented patterns in trying to identify more promoters. Some Web searching engines [22] sift through daunting masses of documents on premises similar to those that drag us to the most overcrowded restaurant when shopping for a good meal in an unfamiliar neighborhood.

The statistical and syntactic approaches mingle in the effort, in ways that are often subtle and almost always dangerous. Suppose that we wanted to build a table

[12] On the binary alphabet, a square is an *unavoidable* regularity, while forming a cube www can be avoided. In view of the many tandem repeats that affect the genetic code, it may sound reassuring that the alphabet there contains four letters.

to report, for all substrings in a textstring of n symbols, the number of occurrences of that substring. Since the number of substrings is of the order of the square of n (denoted $O(n^2)$), then the table would contain more entries that the raw data – a far cry from the concise synopsis we had in mind. Even limiting the table to substrings that are over-represented by some measure is not a guarantee that the table would be smaller than the text. In fact, we typically stipulate our stochastic assumptions before bringing in the observable. Likewise, consider the problem of finding, for a given textstring x of n symbols and an integer constant d, and for any pair (y, z) of subwords of x, the number of times that y and z occur in tandem (i.e., with no intermediate occurrence of either one in between) within a distance of d positions of x. In principle there might be n^4 distinct subword pairs in x! Luckily, an astonishing result of combinatorics on words tells us that the number of states in the finite automaton that recognizes all substrings of a string is linear in the length of the string [23]. A practical consequence of this is that the $O(n^2)$ substrings can be partitioned into a number of equivalence classes that is only linear in n, in such a way that the strings in a class have precisely the same set of occurrences. Clearly, for two strings to be in the same class one must be a prefix of the other. In the light of this, it suffices in the above to consider a family of only n^2 pairs, with the property that for any neglected pair (w', z'), there is a corresponding pair (y, z) contained in our family and such that: (i) w' is a prefix of w and z' is a prefix of z, and (ii) the tandem index of (w', z') equals that of (w, z).

The situation looks more dismal when searching for patterns with "don't care" characters, which are symbols capable of taking up any value from the alphabet. A little reflection establishes that the escalation there is exponential. Assume that on the binary alphabet both *aabaab* and *abbabb* are asserted as interesting patterns. We can give a concise description of both by saying that *a_ba_b* occurs in the string, with "_" denoting the don't care. By this, however, we have immediately generated the spurious patters *aababb* and *abbaab*. This problem is reflected in all approaches that resort to "profiles" or weighed matrices in which the ith column describes the percentage composition of the ith character in a pattern. In these and similar instances, the model seems to introduce more things in our philosophy than are dreamed of in heaven and earth.[13]

In his "Viajes de Varones Prudentes" (Libro Cuarto, Cap. XLV, Lerida, 1658),[14] a J. A. Suàrez Miranda narrated by J.L. Borges writes that

[13] "There are more things in heaven and earth, Horatio, Than art dreamt of in your philosophy" – W. Shakespeare, Hamlet, I, v [76].

[14] The piece was written by Jorge Luis Borges and Adolfo Bioy Casares. English translation quoted from J. L. Borges, *A Universal History of Infamy*, Penguin Books, London, 1975: " ... In that Empire, the craft of Cartography attained such perfection that the map of a single province covered the space of an entire City, and the Map of the Empire itself an entire Province. In the course of time, these extensive maps were found somehow wanting, and so the College of Cartographers evolved a Map of the Empire that was of the same scale as the Empire and that coincided with it point for point. Less attentive to the study of Cartography, succeeding generations came to judge a map of such magnitude cumbersome, and, not without irreverence, they abandoned it to the rigours of Sun and Rain. In the Western Deserts, tattered fragments of the Map are still to be found, sheltering an occasional beast or beggar; in the whole Nation, no other relic is left of the Discipline of Geography."

> ... En aquel Imperio, el Arte de la Cartografia logró tal perfección que el mapa de una sola Provincia ocupaba toda uma Ciudad, y el mapa del Imperio, toda uma Provincia. Con el tiempo, esos mapas desmesurados no satisfacieron y los Colegios de Cartógrafos levantaron um Mapa del Imperio, que tenía el tamaño del Imperio y coincidia puntualmente con él. Menos adictas al Estudio de la Cartografía, las generaciones siguientes entendieron que ese dilatado Mapa era Inútil y no sin impiedad lo entregaron a las inclemencias del Sol y de los Inviernos. En los Desiertos del Oeste perduran despedazadas ruinas del Mapa, habitadas por animales y per mendigos; en todo el País no hay otra reliquia de las Disciplinas Geográficas.

We face widespread and growing risks of building maps bigger than life.

5 Epilogue

There was a time when apples were just fruit and windows a home fixture. Not only Amazon^{TM}, Ebay^{TM} and Google^{TM}, but even the personal computer did not exist at the time of the Procrustes Algorithm. Now, there is at least one department in which the Emperor could use a new map. In the past decade, the framework of human activity has been reshaped forever by technologies ushering in teraflop (i.e. 10^{12} floating point operations per second) machines and data volumes in the terabyte, even petabyte range (1 petabyte is 1 billion times 1 million characters). Enmeshed in a texture of ubiquitous computing, humans are going to be shared among many machines, and face unprecedented problems of knowledge formation, access, management and policy.

The nature and rate of these changes is quantitative on the surface but will induce long qualitative leaps, forcing a transition from paradigms of search "by value" and search "by contents" to a new one of search "by meaning", a paradigm yet to be explored. Automatic or semi-automatic generation of data and relationships thereof will take in an environment that is, like with the famous Heraclitus' river, never twice the same, and a whole new science and engineering of automated discovery will have to take shape, of which the grounds are just beginning to be laid. The mechanics of discovery, and scientific discovery at that, will undergo changes perhaps comparable to the scientific revolution itself. The implications brought about by such a dramatic change in perspectives have barely begun to be perceived. We shall argue next that even making all this data and information *accessible* leaves still an issue of making it *accessed*. Unless of course we wish to take the view that – to paraphrase the central problem of Episteme – Knowledge might form and exist out there without having to reside even once in a human mind.

A recent in-flight entertainment shows the proud new owner of the latest satellite receiver as he wastes his entire day in pushing the next-channel button in search of what he would like to watch best. The message could not be made more clear: gaining access to information requires an investment of time and resources that dominates over fruition, even precludes it altogether. We thus face a completely new scenario and a wild paradigm shift. The new scenario is that data and information accumulate at a pace that makes it no longer fit for direct human digestion. The paradigm shift is that the bottleneck in communication is no longer represented by the channel or medium but rather by the limited perceptual bandwidth of the final user.

"*Μηδέν ϵστί* ", goes the opening line by which Gorgias of Leontini (483–376 BC) recited his threefold nihilism: "Nothing exists, if anything existed it would not be knowable, if anything were knowable it could not be communicated". More than two thousand years worth of episteme have yet to fully sort out the existence of reality outside of us. But if it will become conceivable that the act of knowing is inherently precluded to humans on account of their limited capacity or bandwidth, a modern Gorgias would have to come to terms with an even greater frustration:

> Nothing is known;
> if anything were known it could not be fetched,
> if it could be fetched it could not be digested.

There are very few handles to cope with the information flood. Since there is little hope of implementing any semantics in the common sense of the word, one sure resort is to better understand the syntactic and combinatorial essence of patterns, their structure and organization, and how to go about their discovery [24, 25]. There will be increasing need for new and improved techniques for the extraction of prominent features and relationships in data, for the inference of synthetic descriptions and rules, for the generation of succinct visualizations and digests.

At the end of this loosely organized digression, we come back full circle to problems of finding intensions in an extensionally oriented world. This is significant. Researchers know that their work is ultimately not about looking for answers. The unmistakable mark of talent, the reason most scholars strive day in and day out is not getting to know what is the answer, it is to understand what *was* the question. The recent 50th anniversary issue of the *Journal of the Association for Computing Machinery* opens with an essay by Frederick P. Brooks, jr, entitled "The Great Challenges for Half Century Old Computer Science" [26]. The author gives a list of outstanding problems. Problem Number 1 is as follows:

> Shannon and Weaver [27] performed an inestimable service by giving us a definition of information and a metric for information as communicated from place to place. We have no theory however that gives us a metric for the information embodied in structure ... ... this is the most fundamental gap in the theoretical underpinning of information and computer science. A young information theory scholar willing to spend years on a deeply fundamental problem need look no further.

We must imagine Procrustes happy.

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Close Encounters With Far Ideas

Antonio Barone

1 Introduction

Although the twentieth century has witnessed many advances in particular areas of science, one of the most interesting and important features has been the development of interdisciplinary connections among disparate activities [1]. Thus transfers of established methodologies, the sharing of new technologies, and the hybridization of different materials for advanced devices continue to underscore the importance of closer communications among previously isolated realms of activity. As we move from fundamental studies to applications, therefore, we should recall the advice of Louis Pasteur who suggested that we cease referring to "the applied sciences", speaking instead of the "applications of Science".

In this paper, I shall present a personal view of some of the experimental and theoretical activities that have been carried out at the *Laboratorio di Cibernetica*, now *Istituto di Cibernetica Eduardo R. Caianiello* (*ICIB*),[15] giving, I hope, the flavour of an interdisciplinary approach to science.

Forty years ago, my thesis was in nuclear physics, and, later, aware of the performance promised by a new class of radiation detectors, my interests were directed in general towards solid-state physics and more particularly to superconductive devices and the more fundamental aspects of superconductivity. When with some hesitation I mentioned this scattered trajectory of activities to my teacher of theoretical physics – Professor Eduardo Caianiello – he responded clearly and directly: "Antonio, you are ready to work in a Laboratory of Cybernetics!"

Although in contrast with other authors of this volume, I did not work directly with Eduardo on specific problems, his stimulating suggestions, sage advice and warmhearted support were ever present over many years, helping in countless ways with the development of the research activities described below and with my personal scientific, cultural and human growth.

2 The superconducting Josephson junction

In this section are briefly reported some of the main features of the Josephson effect – see reference [2] for an extensive treatment and bibliography of this subject. Studies of the physics of this effect and of its many applications have been of central interest to the solid-state activity in the *Laboratorio di Cibernetica* and they comprise much of the material presented here.

A fundamental phenomenon in solids is *superconductive tunneling*, which was first reported by Ivar Giaever in 1960 and involves single-electron tunneling through

[15] Although the "R" does not appear in the official name of the *Istituto*, Eduardo was always keeping it.

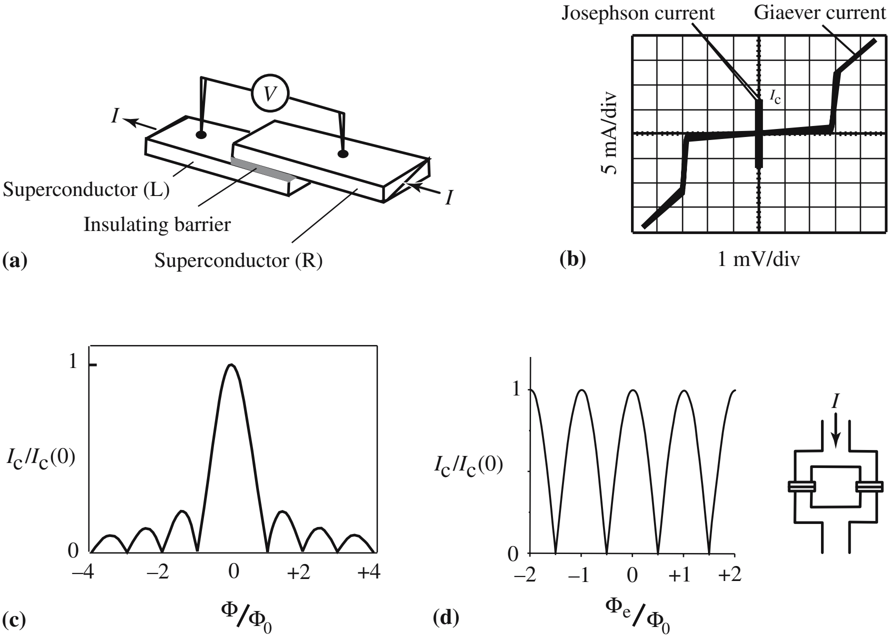

Fig. 1. (**a**) A sketch of the Josephson junction. (**b**) Typical current–voltage characteristics. (**c**) Diffraction pattern of a small junction showing how the maximum Josephson current varies with external magnetic field. (**d**) Interference pattern of two Josephson junctions in a d.c. SQUID configuration

a superconductor–dielectric–superconductor sandwich as shown in Fig. 1(a). In this case, the insulating tunneling barrier is 5–10 nm thick, but in 1962, Bryan Josephson predicted the possibility of tunneling of Cooper pairs (the basic elements of a superconducting wave function) through yet thinner barriers. The essence of Josephson's idea was that, due to quantum-mechanical coherence, it should be possible to realize an overlap of the macroscopic wave functions in the two sides of the junction, leading to weak coupling between the two superconducting regions. A year later John Rowell and Philip Anderson reported experimental confirmation of this prediction. Josephson, Giaever and Leo Esaki (who discovered the tunneling effect in semiconductor devices) were jointly awarded the 1973 Nobel Prize in Physics for their collective efforts toward tunneling in solids.

Thus a Josephson junction comprises two superconducting electrodes (L,R), separated by a thin ($\sim$ 1 nm) oxide layer. The superconducting regions are described by the wave functions $\Psi_{L,R} = \sqrt{\rho_{L,R}}\exp(i\varphi_{L,R})$, where $\rho_{L,R}$ indicates Cooper pair density in the left (L) and right (R) regions.

The Josephson constitutive relations are [2]

$$I = I_c \sin\varphi \tag{1}$$

and

$$\frac{\mathrm{d}\varphi}{\mathrm{d}t} = \frac{2e}{\hbar}V\,, \tag{2}$$

where $\varphi = \varphi_\mathrm{L} - \varphi_\mathrm{R}$ is the relative phase, I_c is the maximum Josephson current, and V is the voltage across the junction. For $V = 0$ it follows that φ is constant, so a supercurrent can flow across the junction with zero voltage drop (d.c. Josephson effect). This is shown in Fig. 1(b), where the finite voltage branch is due to quasiparticle (or Giaever) tunneling, and the zero-voltage current is due to Josephson tunneling.

For $V \neq 0$, integration of Equations (1) and (2) yield

$$I = I_1 \sin\left(\varphi_0 + \frac{2e}{\hbar}Vt\right)$$

implying an oscillating current of frequency

$$\nu_0 = \frac{2e}{\hbar}V\,.$$

This is the a.c. Josephson effect, which in practical units has a frequency of about 480 MHz/μV.

In spatially extended junctions there is also a space modulation related to the magnetic field intensity (**H**) by

$$\nabla_{x,y}\varphi = \frac{2ed}{\hbar}\mathbf{H} \times \mathbf{n}\,,$$

where $\mathbf{n}$ is a unit vector normal to the junction barrier and $d = t + 2\lambda_\mathrm{L}$ is the "magnetic thickness" (t is the barrier thickness and λ_L is the London penetration depth).

Combined with Maxwell's equations, these equations can be written (in one space dimension, neglecting dissipation, and with appropriate normalization) in terms of the magnetic flux (Φ) in the compact form

$$\frac{\partial^2\Phi}{\partial x^2} - \frac{\partial^2\Phi}{\partial t^2} = \sin\Phi\,. \tag{3}$$

This is the celebrated sine-Gordon equation [3] with soliton solutions which are called *fluxons* because they transport quantum units of magnetic flux ($\Phi_0 = hc/2e = 2.064 \times 10^{-15}$ Vs) [4].

In the static case for a small ($L \times L$) rectangular junction, the maximum d.c. Josephson current is

$$I_\mathrm{C}\left(\frac{\Phi}{\Phi_0}\right) = I_\mathrm{C}(0)\left|\frac{\sin\pi(\Phi/\Phi_0)}{\pi(\Phi/\Phi_0)}\right|\,,$$

with the Fraunhofer-like diffraction pattern shown in Fig. 1(c) where $\Phi = H_y L d$. Interference phenomena can occur as well; thus in analogy with optics, the role of the “two slits” is played by two Josephson junctions closed by a superconducting loop, as shown in Fig. 1(d). In this case, the dependence of maximum d.c. current vs. magnetic flux is given by

$$I_{\mathrm{C}}\left(\frac{\Phi}{\Phi_0}\right) = I_{\mathrm{C}}(0)\left|\cos\frac{\Phi_{\mathrm{e}}}{\Phi_0}\right| ,$$

where Φ_{e} is the flux enclosed by the loop and periodicity is given by the flux quantum. On such a system is based a class of devices called SQUIDs (for Superconductive QUantum Interference Devices) which allow the most sensitive measurements of magnetic fields. Under appropriate operating conditions, these devices give an output voltage that is a periodic function of the flux threading the loop (see Chapters 12 and 13 of [2] and references reported therein).

3 Neuristors

At about the same time that Giaever and Josephson were suggesting mechanisms for superconductive tunneling, Hewitt Crane proposed the concept of an electronic analogue of a nerve axon, which he called a *neuristor* [5, 6]. This development was important for the solid-state effort because it offered an interesting way to bring the activities closer to the broad objectives of the *Laboratorio*. The key properties of a neuristor are as follows.

– A neuristor line supports impulse waveforms that propagate with constant speed and shape (attenuationless), which depend only on the characteristics of the line.
– Although active, a neuristor must have a threshold for excitation so it can either rest in a passive state or carry impulses with the above properties.
– The neuristor must possess a refractory period immediately following the impulse during which it cannot be excited.

From a general perspective a nerve axon is a neuristor which has developed in the course of biological evolution, showing that the concept is useful for “communication and control in living organisms” (to use the terms of Norbert Wiener [7]). More specifically, several useful interconnections of neuristor lines are shown in Fig. 2.

In Fig. 2(a) is shown a T-junction, which has the Boolean description C = A OR B. Figure 2(b) shows an R-junction, in which the refractory zone of an impulse on one line blocks the passage of an impulse on an neighbouring line. This provides a logical NOT operation, allowing the synthesis of any Boolean function in the context of neuristor lines. In Fig. 2(c), for example, an R junction is used to construct the more complicated Boolean function C = (A AND NOT B) OR (B AND NOT A). Finally, the arrangement in Fig. 2(d) shows how neuristors can be used to synthesize the equivalent of a transistor – thus signals at C (base) can block transmission between A and B (emitter and collector).

In the spring of 1968, Eduardo Caianiello became aware of the research on superconductive neuristors at the University of Wisconsin (Madison) which was

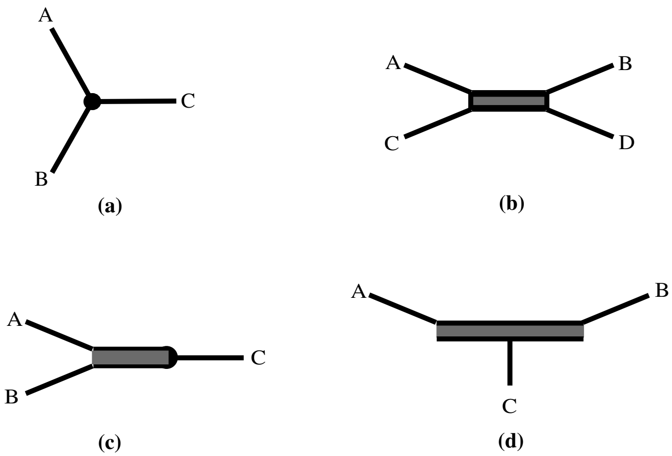

Fig. 2. Some basic neuristor structures. (**a**) T-junction. (**b**) R-junction. (**c**) T–R junction. (**d**) Analogue of a relay or transistor

guided by Alwyn Scott, and he visited Madison both to describe the newly formed *Laboratorio di Cibernetica* and to evaluate the character of the UW research. As a result of this visit, a Naples–Madison collaboration was begun that would last for years, involving visits by many researchers, carrying much technical and scientific knowledge across the Atlantic in both directions and providing cultural enrichment to all who participated.

To appreciate the nature of these activities, consider that the most simple description of a long device with negative conductance generated by Giaever-type superconductive tunneling is the nonlinear diffusion equation

$$\frac{\partial^2 V}{\partial x^2} - \frac{\partial V}{\partial t} = F(V)\,. \tag{4}$$

If $F(v)$ is a cubic polynomial of the form

$$F(V) = V(V - V_1)(V - V_2)\,, \tag{5}$$

this equation provides a qualitative description of the burning of a candle, with energy being both released by and consumed by a travelling impulse. Interestingly, this system has a simple analytic solution for the shape and speed of the wave front, which was first derived by Yakov Zeldovich and David Frank-Kamenetsky in the context of flame-front propagation in 1938, then forgotten for several decades [8]. As the leading edge of an impulse largely determines its conduction velocity, an un-normalized version of Equation (4) (with appropriate selection of the voltages V_1 and V_2) can be used to calculate the speed of a nerve impulse [9, 11].

Adding a recovery term to this system (which accounts for the refractory time) leads directly to Crane's neuristor and also to the well-known FitzHugh–Nagumo (FN) model which had been proposed for a nerve axon [12]. A sketch of a nerve axon is shown in the left-hand side of Fig. 3 with the corresponding electrical equivalent circuit. Interestingly, this equivalent circuit is very close to those describing propagation on a long junction where the shunt current is carried by either Esaki-type or Giaever-type negative conductance.

Josephson-style tunneling, on the other hand, leads to the propagation of discrete quanta of magnetic flux, as described by the sine-Gordon equation (Equation (3)). Adding appropriate dissipative and bias terms to Equation (3), one again finds the FN model, showing the close interrelations among all of these activities [9, 13–15]. Another important technical achievement of the 1960s was the development of integrated circuit techniques, allowing the fabrication of superconductive devices with dimensions defined by optical patterns. These studies – negative conductance through tunneling in semiconductor (Esaki) and superconductor (Giaever) diodes, Josephson tunneling in superconducting diodes, neuristor theory, and integrated circuit techniques – conspired to make the research activities of the solid-state group both exciting and relevant to the overall aims of the *Laboratorio di Cibernetica*.

4 Squid vs. SQUID

Amusingly, the term "squid" arises in two different senses. First, it is the common name for the ubiquitous sea animal *Loligo*, one of which is shown at the top of Fig. 3. This lovely little creature has two unusually large axons running along its back, making it a prime candidate for studies in the electrophysiology of nerve impulse propagation, as indicated on the left of Fig. 3. Second, as noted in Sect. 2 above, the term "SQUID" refers to a sensitive detector of magnetic fields using Josephson junctions, as shown on the right of Fig. 3.

Although the aim here is not to go into the deep theoretical investigations of brain models nor the many intense biological, structural, and functional investigations of carried out at the *Laboratorio di Cibernetica* over the past three decades, it is appropriate to mention a collaboration between the *Laboratorio* and the prestigious *Stazione Zoologica* "Anton Dohrn" in Naples. This activity began with explorative studies of nerve impulse propagation on the giant axon of the squid (*Loligo vulgaris*) by Uja Vota–Pinardi and Scott during the 1970s, leading to a yearlong effort in 1979–1980, which was encouraged by Professor Alberto Monroy and supported by a European Molecular Biology Organization (EMBO) fellowship.

The overall aim of the research was to study the propagation of nerve impulses as they pass through branching regions, in order to appreciate the possibilities for impulse blockage – and therefore logical computations – in the dendrites of real neurons. A typical research result of this effort is shown in Fig. 4 where the second of an impulse pair is observed to be blocked at a branch with the geometry indicated in Fig. 4(a) [16]. In general, these experimental results were in accord with extensive theoretical and numerical studies carried out by Boris Khodorov and his colleagues in the Soviet Union during the early 1970s [17]. Interestingly, the possibility of information processing in dendritic trees has been considered more recently by the neuroscience community [18].

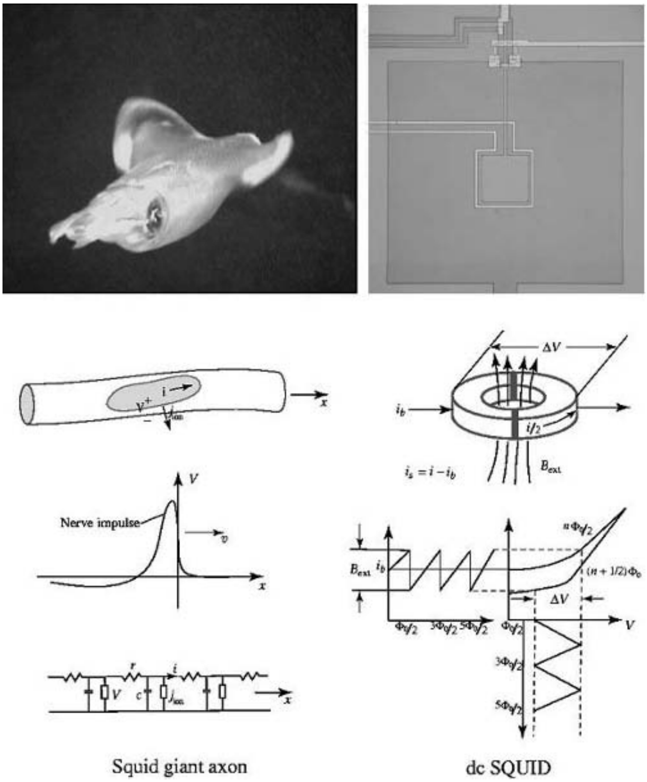

Fig. 3. (*left*) photograph of the squid (*Loligo*) (Courtesy of G. Fiorito). Diagrams of a squid axon, a squid nerve impulse, and an electronic equivalent circuit of a nerve axon. (*right*) Diagrams of magnetic field interacting with a SQUID magnetometer (Courtesy of C. Granata and M. Russo)

As for SQUIDs, these superconductive systems are employed in different fields [19]. In particular such devices are used to obtain magnetoencephalogram (MEG) records of the small magnetic fields generated by the neural activity of the neocortex.

Recently, in the framework of a multidisciplinary collaboration between ICIB, the Zentral Institut für Medizinische Technik of Ulm University (Germany) and the Advanced Technologies Biomagnetics S.r.l., a 500-channel MEG-system has been developed and put into operation at the RKU Hospital in Ulm (see Fig. 5). Such a helmet system, called ARGOS 500, allows the vectorial measurement of the magnetic field at the scalp over 165 different points simultaneously with an active compensation technique to reduce the background electromagnetic noise. The sensor array employs 495 SQUID magnetometers and 14 reference sensors; they have been designed and fabricated at the ICIB and consist of fully integrated SQUID magnetometers, based on niobium technology, showing a magnetic field spectral noise less then $3\text{fT}/\sqrt{\text{Hz}}$. Up to now, ARGOS 500 is the largest multichannel system

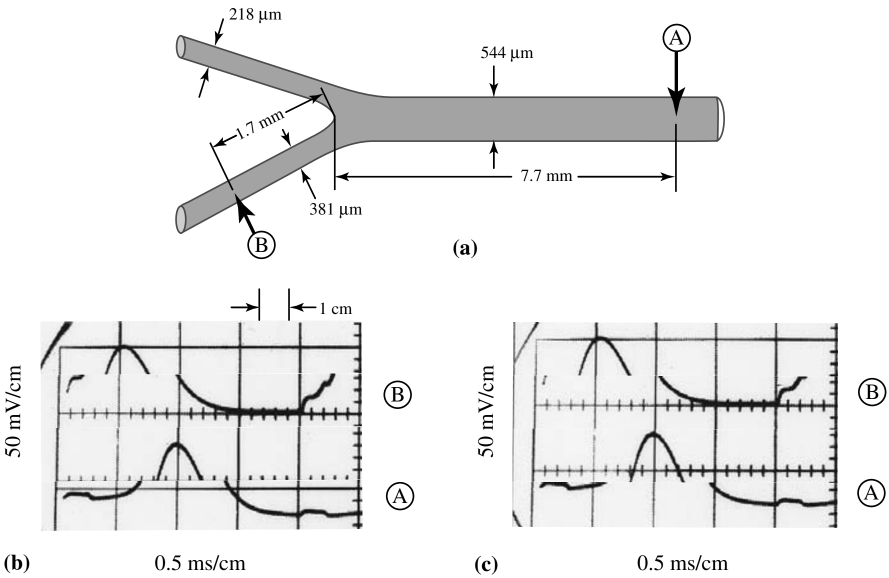

Fig. 4. Threshold action caused by impulse blockage in a branching region of a squid giant axon. The interpulse interval is slightly longer in (c) than in (b)

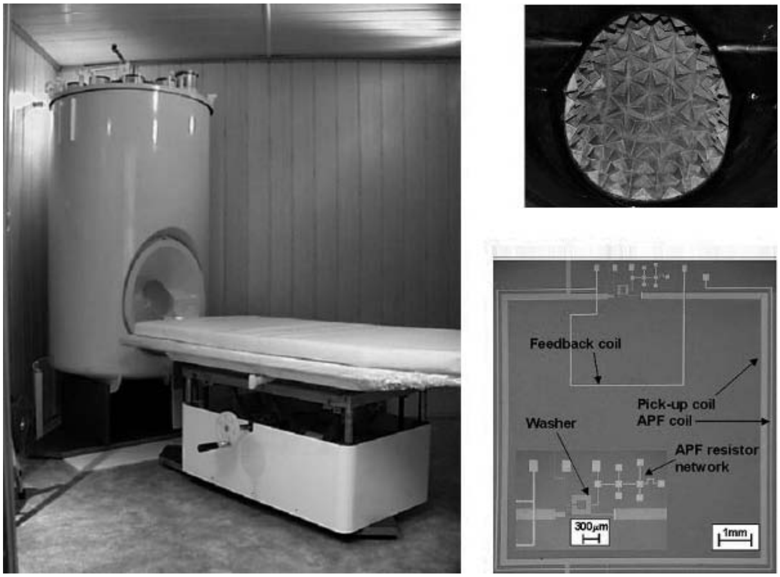

Fig. 5. A 500-channel vectorial system for magnetic brain activity measurements, operating at Hospital RKU of Ulm (Germany). The pictures on the *right* show the pyramidal sensor support and a fully integrated SQUID sensor (Courtesy of C. Granata and M. Russo)

Table 1. Biomagnetic systems employing SQUID sensors projected and fabricated by Istituto di Cibernetica-CNR

System	Location	Type
165-channel helmet	ITAB, University of Chieti	MEG
80-channel planar	ITAB, University of Chieti	MEG
163-channel vectorial	RKU Hospital in Ulm (Germany)	MEG
200-channel planar	F. Schiller Universitat (Germany)	MCG, MEG
28-channel hybrid	Fatebenefratelli Hospital (Roma)	MEG
80-channel planar	ZIMBT University of Ulm (Germany)	MEG
165-channel vectorial*	ITAB, University of Chieti	MEG
165 channel*	CNR Istituto di Cibernetica (Pozzuoli)	MEG

for biomagnetism in the world. Currently, ICIB is involved in projects to develop two new prototype MEG-systems, which will be installed in the Naples area and at Istituto di Tecnologie Avanzate Biomediche (ITAB) of Chieti University. These incoming systems, together with the other six systems for MCG and MEG working in European scientific institutions and hospitals, bear evidence of a technological capability useful for both scientific institutions and industrial companies [20, 21].

5 Information processing systems

Among the new ideas that arose during collaborations between Naples and Madison was that of using arrays of unit cells of Esaki-type or Giaver-type nonlinear conductance to store and process information [9, 10]. The basic structure of such an array is shown in Fig. 6, where the unit cells shown in (a) are connected to form the two-dimensional oscillating structure shown in (b). Each of the nonlinear conductances – indicated as $I(V)$ – correspond to a "cubic-shaped" current–voltage characteristic of the sort indicated in Equation (5). In a sense, this structure can be viewed as an active, vibrating membrane, where transverse displacement corresponds to the voltage V that activates the nonlinear element.

Neglecting for the moment the nonlinear conductances, the oscillating modes of this system are distributed over the entire active membrane, but they are localized in reciprocal space, where each mode could be expected to store a bit of information. This information storage mechanism is thus related to previously proposed holographic storage systems [22–24]. In practice, such a memory scheme would not be permanent because the energy in a mode would decrease with time due to dissipative effects, but this problem is remedied by the presence of nonlinear conductances, $\mathrm{d}I(V)/\mathrm{d}V$, which can pump energy into a mode that has been excited.

As shown in Fig. 6(a), a unit cell of the array consists of a nonlinear conductance $I(V)$ in parallel with a capacitance C, connecting a lattice point to a ground plane. Each lattice point is attached to its four nearest neighbours through an inductance L. If the nonlinear conductance is represented as

$$I(V) = -GV\left(1 - 4V^2/3V_0^2\right) , \qquad (6)$$

the differential conductance (dI/dV) is negative for $-V_0/2 < V < +V_0/2$.

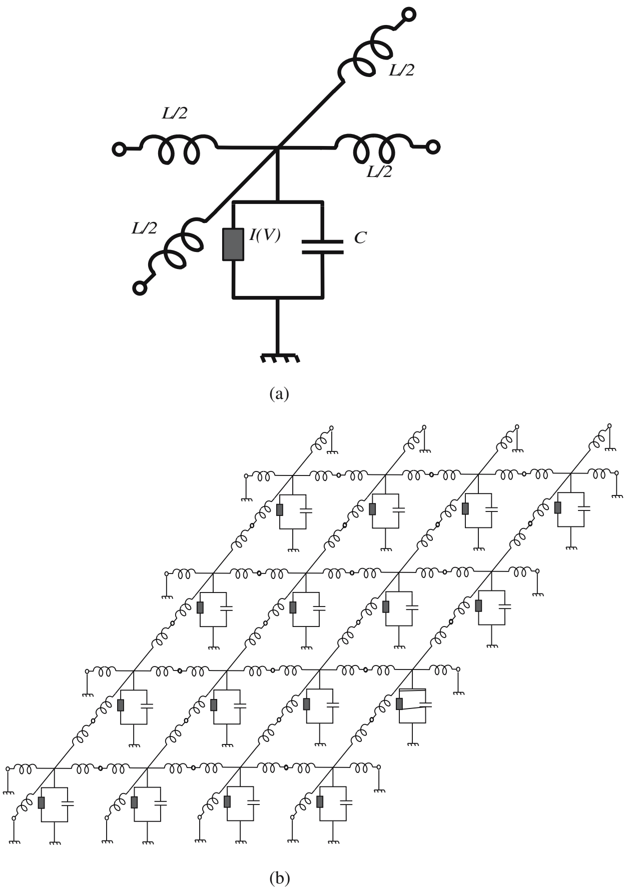

Fig. 6. (**a**) The unit cell of an active membrane. (**b**) A two-dimensional array of the units cells shown in (**a**)

Assume a large, square ($N \times N$) lattice of these unit cells with zero voltage (short circuit) boundary conditions on the edges, as in Fig. 6(b). In a zero-order approximation, the nonlinear conductance can be neglected, and in a continuum approximation, this linear, lossless system supports an arbitrary number (say n) of modes. Thus the total voltage of n modes is

$$V(x, y, t) = \mathcal{V}_1(x, y) \cos\theta_1 + \mathcal{V}_2(x, y) \cos\theta_2 + \cdots + \mathcal{V}_n(x, y) \cos\theta_n , \qquad (7)$$

which depends on space (x and y) and time (t) where

$$\mathcal{V}_i(x, y) \equiv V_i \cos k_{xi} x \cos k_{yi} y \quad \text{and} \quad \theta_i \equiv \omega_i t + \phi_i .$$

Noting that the energy in the ith mode is related to its amplitude by $U_i = (N^2 C/8) V_i^2$ and averaging the product of current (given by Equation (6)) and voltage (given by

Equation (7)) over space and time gives the rate of change in mode energies as functions of mode energies [4, 10].

Using an averaging method, the rate of change of energy (or power) into the first mode is calculated as

$$\frac{dU_1}{d\tau} = U_1 \left[1 - \alpha \left(\frac{9}{8} U_1 + U_2 + \cdots + U_n \right) \right], \tag{8}$$

with $\tau \equiv Gt/C$ and $\alpha \equiv 4/(N^2 C V_0^2)$. For n excited modes, there is a set of n equations – each similar to Equation (8) but with the indices appropriately altered – for the rates of change of mode energies as functions of those energies. These nonlinear, autonomous equations have the same form as those introduced by Vito Volterra to describe the interaction of biological species competing for the same food supply [25], and they are similar to a formulation suggested by Peter Greene [26] to describe interactions between Hebbian cell assemblies of neurons in the neocortex.

Generalizing to a system of d-space dimensions, equations of interacting mode energies become

$$\begin{aligned} \frac{dU_1}{d\tau} &= U_1 \left[1 - \alpha \left(K U_1 + U_2 + \cdots + U_n \right) \right] \\ &\vdots \\ \frac{dU_n}{d\tau} &= U_n \left[1 - \alpha \left(U_1 + U_2 + \cdots + K U_n \right) \right], \end{aligned} \tag{9}$$

where

$$K = 3^d / 2^{d+1} .$$

For $K > 1$ (implying $d \geq 2$), analysis of Equations (9) indicate multimode stability [10]. In other words, several modes can exist stably for two or more spatial dimensions. For one spatial dimension, on the other hand, two modes of equal amplitude are unstable, and only a single mode can be established stably.

Multimode oscillator arrays have been realized using semiconductor tunnel diodes [10], superconductor tunnel diodes [27], and integrated circuits [28]. In these studies, a variety of stable multimode oscillations have been observed, some quasiperiodic and others periodic, indicating mode locking. In a manner qualitatively similar to that proposed by Greene in his model of the neocortex [26], an oscillator array can be induced to switch from one stable multimode configuration to another. I recall the quite stimulating perspective of arrays with optical input employing the light sensitive superconductive junction. Incidentally the technology of light sensitive superconductive structures has represented, in my view, one of the most important stimulating activities developed at the Laboratorio di Cibernetica [29].

At the level of subjective perception, similar switchings in the brain are perceived as one stares at the Necker cube in Fig. 7, where the image seems to jump from one metastable orientation to another, like the flip-flop circuit of a computer engineer.

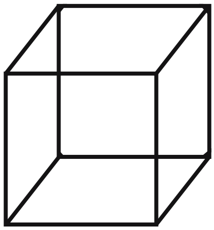

Fig. 7. A Necker cube

Defining order parameters (ξ_1 and ξ_2) to represent neural activities corresponding to the two perceptions of the cube, Hermann Haken has suggested the equations

$$\begin{aligned} \frac{d\xi_1}{dt} &= \xi_1[1 - C\xi_1^2 - (B + C)\xi_2^2] \\ \frac{d\xi_2}{dt} &= \xi_2[1 - C\xi_2^2 - (B + C)\xi_1^2] \end{aligned} \tag{10}$$

as an appropriate dynamic description [30]. With $U_j = \xi_j^2$ ($j = 1, 2$), Equations (10) are identical to Equations (9), showing only single mode stability for $B > 0$. Study of this system in the (ξ_1, ξ_2) phase plane reveals two stable states: $(1/\sqrt{C}, 0)$ and $(0, 1/\sqrt{C})$, and jumping back and forth between these two states models one's subjective experience of Fig. 7.

Of course it is not suggested that the detailed dynamics of the neocortex are like those of Fig. 6, but it is interesting to observe that some global properties of this simple system correspond to those of more realistic brain models.

6 Macroscopic quantum phenomena

Quantum tunnelling on a macroscopic scale has been invoked in cosmology in connection with the decay of a "false vacuum" state through a barrier penetration [32]. This circumstance is related to the possibility of a Universe lying in a relative minimum (Fig. 8(a)) with the possibility of a "transition" to a more stable state. Crudely speaking, the idea is that the Universe is "cold enough" to exclude the possibility of a "jump" by thermal activation, while it is possible to consider a quantum tunnelling activation, which, in this case, would be a Macroscopic Quantum Tunnelling (MQT), including thereby the risky perspective of a "small bang". To study such a ground state metastability one can resort to the possibilities offered by the Josephson effect which would allow more "safe" experiments on a laboratory scale. Let us refer to the junction potential (see Fig. 8(b)) $U(\phi) = -\hbar/2e(I\phi + I_1 \cos\phi)$. The term I indicates the feeding current into the junction whose value determines the average slope of the washboard potential $U(\phi)$. For current values approaching the critical the deepness of the potential valleys progressively decreases up to a value above which the particle falls down the slope. Consequently ϕ will change with time leading thereby (2) to

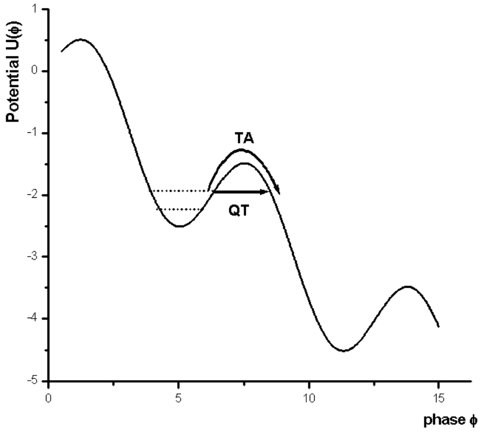

Fig. 8. Junction potential vs phase

a finite voltage state of the junction. For I lower than I_C thermal fluctuations can contribute with an amount of energy kT to overcome the barrier with a probability

$$W_{th} \simeq \omega_o \exp\left(-\frac{\Delta U}{k_B T}\right) \tag{11}$$

ΔU is the barrier height of U(ϕ) and ω_o the natural frequency of oscillation in the metastable state. This problem of thermal activation has been widely investigated in the context of Josephson structures (see [2] Chapter 6 and references reported therein). It has been subject of great attention from both theoretical and experimental point of view also at the Laboratorio di Cibernetica. In the quantum regime [33], MQT will be characterized by a probability

$$W_{MQT} \simeq \omega_o \exp\left(-\frac{\alpha \Delta U}{\hbar \omega_o}\right) . \tag{12}$$

Comparing this expression with W$_{th}$ we can infer a crossover temperature below which quantum activation dominates over the thermal one. The Laboratorio has been active since many years in the area of Macroscopic quantum phenomena [34].

7 Superconductive junctions detectors

As stated in the introduction, one of the main objectives of the activities in superconductivity at the ICIB was and still is the investigation, from both experimental and theoretical point of view, of superconductive junction radiation detectors. Besides the cooperation with the INFN, the Department of Physical Sciences of the University of Naples "Federico II" and the INFM Coherentia, these studies involved through the years joint projects with various Laboratories in Europe, Japan, USA.

The motivation to employ superconductive structures in the field of radiation detectors (for a review see, for instance, [35]) are related to various issues which include high energy resolution, ultrafast time discrimination and processing, spatial resolution, radiation hardness and quite stimulating combinations of these aspects. The most investigated subject was the potential of superconductive tunnel junctions (STJ) structures as high energy resolution detectors. For this purpose the main reason to employ superconductors lies in the low values of the energy gap (say order of 1 meV) with respect to semiconductors (say order of 1 eV). This implies that for a given energy released by the impinging radiation in the detector a higher number of detectable excitations is produced in its sensitive volume (namely quasiparticles resulting from the breaking of Cooper pairs in the superconductors with respect to the electron–hole pairs created in the semiconductor). This results in a reduced statistical fluctuations of the produced excitations and, consequently, in a higher energy resolution. Basic elements of actual realization and circuitry of STJ radiation detectors can be found in the above quoted [35]. As for ultrafast time discrimination it is possible [36] to employ to a Josephson junction structure using the commutation from the zero to the finite voltage namely from the Josephson to the quasiparticle branch of state of an "hysteretic" I–V characteristic of the junction. Such a commutation occurs in a fraction of picosecond in accordance with the uncertainty principle $\Delta E\ \Delta t \simeq \hbar$ (ΔE and Δt corresponding to the gap energy and the switching time respectively). Recently superconductive detectors have been proposed in the frame of time of flight mass spectroscopy. Such devices include also superconductive junction structures. Once again the ICIB has developed frontier solutions proposing the so called "Dual Detector" [37] which combines a Josephson tunnel junction with

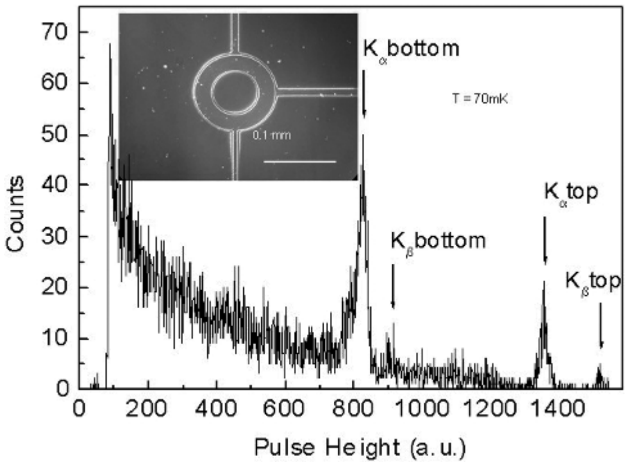

Fig. 9. X-ray energy spectrum collected without external magnetic field by an annular Superconducting Tunnel Junction (STJ) with trapped magnetic flux quanta to suppress the Josephson critical current and Fiske resonances. The inset shows a photograph of the annular STJ fabricated at the CNR Istituto di Cibernetica E.R. Caianiello, Pozzuoli [39]

a STJ detector realizing thereby a merging of fast response with high energy resolution. More recently, in the context of kinetic inductance detectors various structure configurations also including Josephson junction devices have been proposed [38] for mass spectroscopy applications.

Finally I wish to mention the great impact of annular Josephson junction configurations. Such a geometrical shape was chosen for STJ energy high resolution detectors since it offers the important requirement of the intrinsic suppression of the Josephson critical current and Fiske resonances without the drawbacks of an external applied magnetic field [39]. Figure 9 shows a measured X-ray energy spectrum using such an annular structure detector. It is interesting to observe the importance of such annular junction configurations in various studies of different natures (e. g. [40]).

8 High-T_C superconductivity

The discovery of high-T_C cuprates opened a new chapter for superconductivity. In this context, once more the studies related to the Josephson effect were of fundamental importance for both the interpretation of the underlying physical aspects of the new class of superconductors and the potential of devices with less severe stringent cryogenic requirements. Lack of space does not allow us to discuss the various research activities carried out at ICIB on high-T_C superconductors [41]. I wish here just mention the role of the Josephson effect as a probe of the unconventional order parameter symmetry as first proposed in the context of p-wave symmetry in the heavy-fermion systems [42]. It was independently proposed for the high- T_C cuprates to account for the d-wave pairing [43]. For the experimental confirmation and related extended bibliography the reader is referred to the review by Van Harlingen [44]. All aspects of the experiments on the peculiar vortex structures of these unconventional superconductors can be found in the review by Tsuei and Kirtley ([45] and references reported therein). Also in this context, interesting experimental activity has been carried out at the ICIB in a cooperative program with the Coherentia INFM, Universita' di Napoli "Federico II". In particular, unambiguous evidence of d-wave symmetry was obtained for the first time in single junctions not inserted in any loop [46, 47]. Furthermore direct evidence of the 0–π junction transition has been achieved by two phase sensitive tests [48].

9 Concluding comments

Far from an exhaustive review, the above notes provide but a brief sample of activities in superconductive electronics carried out in the early years of the Laboratorio di Cibernetica together with examples of recent achievements and challenging trends. The reader could be stimulated by the enclosed bibliography to satisfy both legitimate scepticism and request a deeper analysis.

I always feel that remembrance is the best way to meet a person. The aim of the present article was to give a contribution to meet once again a great Person a great Friend: Eduardo R. Caianiello.

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Remarks on the Semantics of "Information"

Valentino Braitenberg

Proper and improper uses of the term information

Whatever radiation, field or substance emanates from some piece of matter can be called information only if it induces changes in some other body.

But this is not a sufficient condition. Gravitational attraction together with radiation pressure from the sun determine the shape of a comet. Would you call it information?

On the other hand, a bird-call is certainly information: information produced by a living being and understood by another living being, information about the location of the sender, about the kind of bird it is, about its present sexual or aggressive mood. Implicit in the bird call is the scope of it, which is to be understood by another bird of the same species in the context of a mating strategy. And this ultimately makes sense in the wider context of biological evolution.

However, the bird call is also information to the hungry cat, as well as to the field zoologist and to the huntsman. So the scope is not necessarily implied in the processes which we call information. Or, at least, we could call it information even if the scope intended by the sender is different from that of the receiver. Is the presence of a living being at both ends of the chain of information transmission an essential condition?

It is not. We say quite correctly that the astronomer collects information about the distribution of galaxies in space. Is the receiver then necessarily a living being?

Let us consider situations from engineering, which is where the modern use of this ancient term took its origin. A computer transfers a program to another computer through a wire. We would certainly call this transmission of information and apply to it all the technicalities which we have learned from communication engineers. The question remains whether we are allowed to consider the process in isolation, without reference to the author of the program, to the users of both computers and to the applications of the program. Still, disregarding the human aspects, why don't we feel uncomfortable with the expression "a computer transmits information to another computer?"

It may have something to do with the complexity of the computers. We are less comfortable with "the ignition key transmits information to the engine", than with "the key turns the lock because its complicated shape conveys the right information to it". Apparently one bit is not enough to justify the use of the term information.

But how about the burglar who breaks into the car in a violent action which certainly involves more than one bit of information (and leaves more than one bit for the police to evaluate later): does he transmit information to the lock? The condition of the car after the burglary was not the scope of the burglar's action. This is probably why we hesitate to see transmission of information in the process of wrecking the lock. But we have already seen that scope, at least on the side of the originator of the information, is not essential, since otherwise the astronomer could not collect information about the stars. Besides, scope is not an appropriate category if we want to anchor the concept of information in a physical description of the world.

What is essential then? The easy anwer is: for it to be possible to talk of information transmission, the sender and the receiver must share a common language, or a common alphabet, or at least two codes that are easily and unambiguously translated into each other. But again, language, alphabets and codes are concepts outside the range of physics, and even in the biological world they refer mainly to human communication. Can they be meaningfully applied to more basic facts in biology?

Coding of information in molecular chains, and the translation of this information into different kinds of molecules is of course the mainstay of molecular genetics. Here the language of communication engineering is fully appropriate: there are molecular alphabets of four or twenty-plus symbols, there are translation mechanisms, there is optimal coding and there is redundancy. Only, as in communication engineering, we are left with the feeling that the description is incomplete as long as we do not include the source of the information (is it the world? Darwinian evolution?) as well as the recipient (the processes of development? the structure of the adult organism? its behavior? its strategy of survival?).

The other aspect of biology where information theory can be illuminating is the anatomy and physiology of sense organs. From the physiological properties of the sense cells and from their number we can compute the channel capacity of a sensory organ (in bits per second). In a similar way we get the channel capacity of the sensory nerve leading to the brain, and between the two we can compute the reduction of information due to coding. The coding can be shown to be related to the redundancy in the sensory input: most shapes are continuous and so are most movements, margins are more important than the interior of the shapes etc. The representation of the input in the brain takes advantage of these redundancies. In some cases, as in the eye of the frog, the neurons responsible for the coding have been identified and the corresponding patterns or "redundancies" could be interpreted as relevant objects of the environment. Neurons tuned to little black objects moving through the visual field were called "fly detectors", i.e. prey, others signalling an approaching shadow were interpreted as "stork detectors", i.e. danger from a predator.

There is an intuitive element of interpretation in this, not only on the side of the zoologist. Interpretation is also involved in the frog when he sees a shadow as a predator. We may even say that interpretation by the receiver is an essential constituent of any chain of information transmission worthy of this name. This certifies the astronomer as a true recipient of information, and does away with the doorbell receiving information from the button pressed outside. But what is interpretation? To answer this question we must take a wider view of the relation between living beings and their environment.

What is life?

The most important attribute of life is reproduction. Of course, inanimate things can also multiply, like snowballs rolling down a mountain slope and making more snowballs. However, all snowballs are very similar and their internal structure is quite simple. The point in biological reproduction is the spontaneous generation of exact (or almost exact) copies of exceedingly complicated aggregates. This is in itself a process in which the concept of information comes in handy: determination of form by form.

What is impressive is the enormous variety of the structures that can thus be reproduced. Since reproduction painstakingly conserves the structure within each species, we must explain the variety of structures as a variety of information encoded in each plant or animal species. We have no difficulty in identifying structure with information. But information about what?

For each species of living beings the conditions for survival and for reproduction are keyed to a particular kind of environment, the so-called biological niche. Different niches require different kinds of locomotion, different sense organs, different ways of gathering food, different strategies of defense as well as different kinds of metabolism. All of this information, encoded in genetic molecules, determines the structure and behavior of the organism. From the great number of details in which organisms differ, we must conclude that each niche is also specified in great detail.

In a way, an organism is the answer which self-duplicating living matter gives to one of the accidental configurations arising where physical order leaves some leeway to chance. If we see the structure of an organism as information, we can say: the source of the information is the configuration of the corresponding niche. Organisms are partial images of the environment, where the imaging process involves complicated transformations, mapping food stuff into appetence, danger into avoidance, cavity into shelter etc.

The converse is also true: a biological niche is defined by the existence of an organism which lives in it. Without it, the accidental configuration of the environment remains just that. To make a niche out of it is a process in which we discover, at the most basic level, the concept of interpretation. At the origin of life there is an interpretation of the physical environment as an opportunity to live and multiply.

Knowledge and information

If we see the pair information/interpretation as the hinge between animate and inanimate nature, we can derive from it a philosophy which gets rid of some of the puzzles above. Transmission of information from the world to the living being (the case of the astronomer) is no more a pathological case, but becomes part of the central paradigm, being a continuation of the primordial exchange between the environment and life. Rather, communication between humans becomes the derived case, as an indirect way for the receiver to acquire information about nature. Transmission of information within technical devices, such as computers or communication channels, turns into a partial description of a complex process where ultimately knowledge about reality is gained by human users. To my mind the disturbing question of the "meaning of meaning" becomes meaningless when this view is adopted.

It may be objected that we have shifted the problem of meaning to that of knowledge, a concept which may seem equally unfathomable to a philosopher. I claim that in the description of brains (and, in a wider sense, of organisms) and their relation to the world, the term knowledge may be used in an unambiguous, objectively verifiable way.

There are two attributes which distinguish knowledge: (a) it refers to an outside reality, and (b) it is permanent or at least semi-permanent. Some would add: (c) it conditions behavior, to exlude the most arcane meanings. Quite obviously the idea of knowledge fits the structure of organisms, which is, as we have seen, encoded information about the environment. However, to say that the cat's body is knowledge of how to be a cat clashes with the ordinary usage of the term. On the other hand, there are no objections to the idea that brains embody knowledge of how to deal with the world, or more simply, knowledge about the world.

Two different processes provide the information which is stored as knowledge in the brain. The first is the Darwinian process of natural selection. It prevents brains which do not incorporate a particular knowledge from being reproduced in the progeny. The result is what is called inborn knowledge, and examples, well known to animal behaviorists, are plentiful. The second is individual experience in the course of a lifetime, or what is called learning in the usual sense. Let us consider individual learning in order to examine the relation between information and knowledge.

Not all the information that reaches the brain through the senses turns into knowledge. Most of it is forgotten without leaving any trace. What part of it is stored in the brain? In order to answer this question, we must turn to the physiological processes responsible for learning. Without going into details, we can state with some assurance that neurons in the brain which are often active at the same time strengthen their functional connections, so that later the activation of one will activate (or facilitate the activation of) the other. If neurons in the brain stand for objects or events of the outside world, we have the following consequence: objects or events which are tied together by some causal relation in the world (and therefore often present themselves at the same time), are represented in the brain by neurons connected together by strong synapses (synapses are the functional connections between neurons). The system of synaptic ties between the neurons of the brain approximates the causal ties between things in the world. The learning brain turns more and more into an image, or a dynamic model of the environment. This is what we mean when we say that the brain has knowledge of the world.

Restating this in terms of information theory we get the following: what is stored in the brain as knowledge is not simply the information that reaches it through the senses, but the set of rules which govern that information, or if you wish, the redundancy that is present in it. Just as not every state or succession of states in the sense organs is possible because of the constraints which the rules of physics impose on the world, not every (spatio-temporal) constellation of active neurons in the brain is possible because of the influences which the neurons exert onto each other.

As a measure of the amount of knowledge contained in the brain we may take the reduction of information capacity (of the brain considered as a channel between input and output) due to correlations between neurons caused by the growth of synapses. The amount of knowledge stored is the difference between the channel capacity of the newborn brain and that of the experienced brain. If we want to include inborn knowledge, we must take the difference between the channel capacity of a collection of neurons without any synapses, and that of a brain with the full complement of connections. It is a fact that synapses in mammalian brains appear before learning starts (embodying inborn knowledge?) and then dramatically increase in number during a critical phase of learning. In later life it is more likely an adjustment of the strength of synapses that underlies learning, rather than the growth of new synapses.

Information is the process of updating the knowledge stored in the brain. Knowledge already present is continually compared with the input that comes from the senses.When the input messages carry no surprise, i.e. are identical to what the brain expects on the basis of its internal ruminations, they are immediately forgotten and do not contribute to knowledge. On the contrary, when they differ from the expectation, they are incorporated in the statistics which, little by little, enters the structure of the brain by learning, adapting it to reality.

A source of information which is new to the receiver provides information at a high rate (in bits per second). If the receiver listens long enough, and if he is equipped with a learning brain, he will discover more and more regularities, repetitions, dependancies in the sequence of symbols emitted by the source. With every such discovery the information rate of the source decreases, and the information stored in the receiver's brain increases. Perfect knowledge of a source reduces its information rate to zero. The omniscient is deprived of all information.

On the other hand there is no information where there is no knowledge to receive it, or to be updated by it. Information depends on interpretation, and vice versa.

Appendix

Eduardo never liked the formalism of symbolic logic but was well aware of the fundamental importance of a paper written in that language [1]. Not only was the philosophical impact of this paper enormous, in that it showed that any definition or statement, no matter how complex, corresponds to a network of formalized "neurons" culminating in one neuron that uniquely represents that proposition. It also inspired the terminology that went into the language of computer engineering. Still, it was one of Eduardo's great merits to have replaced the old theory with one definitely housed within the framework of physics and expressed in the corresponding language, and moreover one much wider in its scope [2].

I feel that my friend Eduardo would probably have pardoned the following excursion into the field of formal logic.

La borsa o la vita: an exercise in applied logics

Persons: Northern European lady tourist. Italian brigand.
Place: Lonely region in the Appenine mountains.

TOURIST (*walks onto the scene, stops, inhales deeply and audibly*)
Oh, the happiness of solitude!
The sweet breath of Nature.
God's divine justice in the balance of all things on earth.
Let me dwell here!

BRIGAND (*black cloak, black hat. Appears suddenly and produces a long knife*)
La borsa o la vita

T. (*in a friendly tone*) How do you do. Would you be so kind as to repeat what you just said, but more slowly, please.

B. La – borsa – o – la - vita

T. Just a moment. (*extracts pocket dictionary from her pocket and consults it at length*)

Borsa: bag – stock exchange – scrotum.

Vite: screw, grapevine.

B. Vit-a! No vit-e.

T. Oh, excuse me (*leafs again through the dictionary*) Vita: life. (*with a friendly smile*) Life is beautiful! However, I don't quite see what this has to do with the stock exchange.

B. (*to himself*) Porco cane, siamo in Europa, no? (to her) Speak English, or German?

T. I prefer English. It is the vehicle of international understanding.

B. Your money or your life.

T. (*swooningly*) They are both precious. Money is a necessary condition for life. But without life you can't spend it.

B. Vaff . . .

T. (*consults again her dictionary*) How do you spell it: with one f or with two?

B. (*impatiently*) Basta! You won't find this in the dictionary. I think you got me right: Your money or your life.

T. Excuse me, is that inclusive disjunction, or exclusive?

B. Enough! (*shows his knife*) Either or!

T. So it is exclusive disjunction. But this cannot be: I have both money and life.

B. Listen, lady, if you don't want to understand me, you will find yourself very soon without one or the other, no money and no life.

T. Well, well, then this is no disjunction at all, neither inclusive nor exclusive. No matter how you interpret "your money or your life", for it to be true I must have at least one of them. No money and no life is the negation of an inclusive disjunction. If that is what you intended, you should have said "not – open brackets – your money and/or your live – close brackets"

B. Stop it. I don't care a bit about what you will have in the end. I want your money!

T. Why didn't you say this in the first place? Why this conundrum with the disjunction? In any case, you know I cant't give you my life, since life cannot be transferred in any obvious way. And what good would my life do you!

B. But you can lose it. Your life or your money or both.

T. So you mean inclusive disjunction afterall. Money and/or life. But this is again wrong. In fact, if I don't give you my money and you take away my life, you will certainly also take my money. To lose my life and not my money is an impossible case. This is no disjunction. All you want is money. Whether I also lose my life, depends entirely on me, and does not really interest you. You must express yourself more carefully.

B. Goddamm. You cannot have both, your money and your life!

T. Now this is a different story. What you just said is: to keep my money implies for me to lose my life, and to keep my life implies to lose my money. Mutual exclusion of life and money. This is expressed by a different binary logical function, called "not both", or Sheffer stroke, the negation of the conjunction. But don't think this is correct! To have the money but not to be alive may by a logically correct case, but is actually impossible, as you can easily convince yourself.

B. Fork over the money!

T. And/or my life. The fact of the matter is that I have no money on me. *What is the logical conclusion? Do you want my life? (smiles in a provocating way)*

B. (*confused, pockets his knife*) You can have your life.

T. (*sweetly*) And you can have mine (*comes up close to him*).

B. (*lays his arm around her shoulder*) Where shall we go?

T. To the bankomat.

B. ... and/or ... ?

T.	And.
B.	The money and the life!
T.	This is the conjunction everybody loves.
B.	Felix conjunctio.
B. and T.	(*walk away in a close embrace*).

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Simulation Models of Organism Behavior: Some Lessons from Precybernetic and Cybernetic Approaches

Roberto Cordeschi

1 Introduction

Some recent developments in the machine-simulation methodology of living-organism behavior are discussed. In putting forward these issues, the aim is to isolate recurring themes which help to understand the development of such a machine-simulation methodology, from its, so to speak, *discovery* during the first half of the twentieth century up to the present time. The machine designed by the engineer S. Bent Russell in 1913 seems to share some key points of such a methodology. This machine was designed with the aim of embodying certain hypotheses on the plasticity of nervous connections, pointed out at the time by psychologists in order to explain the physical bases of learning. It is suggested that this machine might be viewed as a case-study of the *discovery* of the above mentioned simulative methodology, later developed by cyberneticians, beginning in the 1940s. Certain present-day steps toward such a methodology are briefly touched upon in the concluding section of the paper.

2 1913: an early attempt at simulation methodology

Let us begin by considering the theories on nervous conduction that Bent Russell took into account for the design of the machine he published in 1913 (further details on both those theories and the machine can be found in [1] and [2], Chapter 2). The authors Russell mentions in his paper are Herbert Spencer, Max Meyer and Edward L. Thorndike – a somewhat mixed bunch.

Meyer, a mechanist psychologist, had formulated his version of what was to become known as the "drainage theory" of nervous conduction. Such a theory rested on the old analogy of the nervous flow running like a liquid through a system of pipes of varied (and modifiable) capacity, connected by one-way valves corresponding to the synapses (although the synapse hypothesis was not universally accepted when introduced). The animal psychologist Thorndike had proposed a theory of the formation and reinforcement of S–R (stimulus–response) connections, soon to find its own analogy in the telephone switchboard, i.e. a network of countless units, corresponding to the neurons, which enable the nerve message to be sorted through a vast range of modifiable connections, corresponding to the synapses.

While drawing largely on Spencer and, more directly, on Meyer and Thorndike, Bent Russell [3] conjectured that:

1. continuous or frequent stimulation of neurons at short time intervals could result in a strengthening of the connections between neurons and in an increase of their conductivity (or in a decrease of their resistance);

2. discontinuous or not frequent stimulation of neurons could result in a weakening of the connections and in a decrease of their conductivity (or an increase of their resistance).

These two statements give us the nervous system image of the time as conceived by Bent Russell. We shall refer to it as the "Standard Theory" of nervous conduction. Basically, the – by no means new – idea was that repetition of stimulation played an important role in the process of habit formation or of learning: the so-called *law of practice*. Let us see how Bent Russell conceived his machine and how he justified his simulative methodology. His general approach is a seminal one, which was later to become popular among psychologists and behavioral scientists. It consists in the following steps:

1. Bent Russell began with a concise exposition of the postulates of the Standard Theory of nervous conduction.
2. He then described the design of a hydraulic machine which "embodied", as he said, the neurological hypotheses of the Standard Theory, before going on to "compare the results obtained with the machine with those given by live nervous connections" ([3], p. 15).
3. This comparison allowed him to conclude that it was actually possible to use a mechanical device to simulate the "essential elements" of the neurological phenomena which, according to the Standard Theory, accompanied inhibition, habit formation and some kinds of associative learning.
4. He then concluded with a brief analysis of some differences between the behavior of the machine and that of the organic nervous system. It was believed that this machine, "with some modification" (p. 34), could simulate other, presumably more complex, kinds of learning responses.

It is interesting to examine in some detail how Bent Russell intended to use his machine to simulate those features (or "essential elements") of the nervous system stated by the Standard Theory, in order to discover the novel, pioneering aspects of his mechanical analogies *vis-à-vis* older, more conventional proposals. Bent Russell's machine worked by compressed air or hydraulic pressure. What could be called the fundamental unit of the machine is the *transmitter*. In fact the transmitter is a valve with an inlet pipe, in which the pressure of a flux of air or water is assumed to remain constant, and an outlet pipe which discharges such a flux. The maximum opening of the transmitter, owing to the sequence of strokes that activate it, is variable with time, and accordingly the intensity of the flux also is variable with time.

Here we have an important novelty with respect to old hydraulic analogies of the nervous system (e.g. those pointed out by drainage theory itself): the simulation of the effect of events ranging over time intervals by an actually working mechanical device. The transmitter has at least a kind of *memory*: it modifies its own behavior according to its previous functioning or history. The time interval between one stroke and the next of the transmitter is of decisive importance for the analogy: it embodies the function of the time lag in determining the variation in the nervous response predicted by the Standard Theory. According to the latter, as we have seen, nervous conductivity is increased by means of repeated stimulations occurring closely together in time and is decreased with longer rest periods.

The analogy between the functioning of the nervous system and that of the machine goes further than this. As a consequence of the Standard Theory, certain initially low-conductivity nervous connections could, by means of repeated stimulation, be "forced" (as both Meyer and Thorndike said) to transmit the impulse, so becoming of high conductivity. In the machine, this type of effect could be simulated through the action of several transmitters working in concert in a variety of combinations. The mechanical device controlling the functioning of several connected transmitters was called by Bent Russell the *coupling gang*: basically, the device functionally reproduces in the machine a network of nervous connections with both high and low conductivity. As Bent Russell concludes, "the result [of the working in concert of the transmitters depends] largely upon what might be called the 'experience' of the transmitters in the combination" (p. 26).

Bent Russell introduced his project with some general remarks on the possibility that engineering might be combined with physiology and psychology in the research on nervous system. Looking back over the age of cybernetics, we detect a prophetic ring in his words, written thirty years before its advent.

It is thought that the engineering profession has not contributed greatly to the study of the nervous system, at least since Herbert Spencer, an engineer, wrote his book on psychology. As the cooperation of workers in different fields of knowledge is necessary in these days of specialists it may be argued that engineers can consistently join in the consideration of a subject of such importance to man ([3], p. 21).

Bent Russell's proposal of such a multidisciplinary study of the nervous system, "in these days of specialists", was bound to be not accepted at the time, and he himself soon abandoned his project of the simulative machine. It was Meyer who discussed Bent Russell's machine in the context of various mechanical analogies of the nervous system current at the time, but mostly as an argument in his own controversy with the vitalist psychologist William McDougall. The latter had censured as "Automaton Theory" any mechanistic interpretation of organism behavior. Meyer's reaction was that of describing McDougall as the champion of the "Ghost Theory" of organism behavior ([4], p. 367), then offering the existence of Bent Russell's "mechanical organism" as an anti-vitalistic argument:

We have here a demonstration of the possibility of an 'organism', capable of learning and forgetting, which obeys no ghost whatsoever, but only the laws of mechanics [...] If it is proved that a mechanical organism can learn and forget without the interaction of a ghost, we have no right to assert that a biological organism can not ([5], p. 559).

In fact Meyer's interpretation of Bent Russell's machine is one of the issues of the simulative methodology pointed out above: if a machine is able to modify its own behavior as an organism does – the argument runs – then there is no need to invoke non-physical (in fact non-mechanical) principles to account for the ability, typical of the organism, to adapt itself to the environment and to learn.

Thus, Bent Russell pointed out some of the points which can be considered as the core of the machine-simulation methodology of organism behavior. It was stated that a so-called “inorganic” or “non-protoplasmatic” machine, when it behaves as predicted by the behavioral theory it embodies, might be considered as a successful *test* for such a theory. The “essential elements”, as Bent Russell calls them, that the learning machine shares with the learning organism reveal a *common functional organization* between them, and this justifies the mechanistic explanation of learning. In fact only *simple* kinds of learning can be simulated by then existing machines; it is hoped anyway that more complex kinds of learning might be simulated through the progress in the construction of simulative machines. In any case, the very existence of those machines did give at least an early argument in support of the *sufficiency* of the principles involved, so that learning might not be a distinctive (non-physical) feature of living organisms alone.

This very simple machine (*not* a mere analogy, but a working artefact or a model) seemed therefore to suggest that learning has not to be viewed as the opposite of automatic or machine behavior: on the contrary, learning is a *particular* kind of automatism, and choice could be mechanized. A *new* kind of machine – in fact, a machine capable of modifying its own internal organization – provides new ideas on how to fill the gap between the organic and the inorganic worlds.

3 Thirty years later: the “turning point”

Marvin Minsky called a “turning point” in the history of the mechanization of thought processes the publication, in 1943, of three seminal papers: the first by Arturo Rosenblueth, Norbert Wiener and Julian Biglelov, the second by Warren McCulloch and Walter Pitts, the third by Kenneth Craik. Even if the word *cybernetics* was introduced by Wiener in 1947, it is usual to date the rise of cybernetics to the works by these authors (see [6]). Rosenblueth, Wiener and Bigelow [7] put forward in their paper, “Behavior, Purpose, and Teleology”, a *unified theory* of organisms and feedback-controlled machines (“the methods of study of the two groups [of systems] are at present similar”). As they concluded, what is at least presently relevant in these systems is the common functional organization they share, not their different structure: “If an engineer were to design a robot, roughly similar in behavior to an animal organism, he would not attempt at present to make it out of proteins and other colloids. He would probably build it out of metallic parts, some dielectrics and many vacuum tubes” (p. 23).

Nevertheless, many of the robots designed during the cybernetic age mimicked animal behaviors without illuminating the underlying functional principles that could justify the analogy between an organism and a machine. For this reason, researchers who were more concerned with the theoretical aspects of simulation considered such robots as mere byproducts of the cybernetic enterprise (see [2], Chapter 5, for further details). The cybernetician Gordon Pask, for example, claimed that almost all cybernetic robots, and especially maze-learners, merely imitate *responses* of organisms, without embodying *functional principles* common to organisms and machines ([8], p. 17).

On the one hand, the interest that some pioneers of cybernetics and control theory had in certain simple artefacts lay in the practical applications that they exemplified in miniature. These simple devices gave them the opportunity to explore the principles of automatic control in general, and negative feedback in particular. In short, these projects were more engineering than psychology lab experiments. On the other hand, the fact that machines were more stringently used for testing theoretical hypotheses – a point I stressed in previous section – was particularly evident in some of the models and robots designed by engineers and psychologists in the 1950s.

In this case, these robots could be seen as "material models" embodying assumptions corresponding to certain "theoretical [formal or intellectual] models", as Rosenblueth and Wiener put it in a further article ([9], p. 317). Important points are the following here: a material model is a simplification of the real system, but "can approach asymptotically the complexity of real situation"; a theoretical model is the basis for the construction of a working material model; the latter is not a "gross analogy", as far as it embodies a theoretical statement or hypothesis (p. 318).

Just to make an example, Wiener raised the problem of explaining the tremor of humans affected by Parkinson's disease. This tremor differs from intention tremor, a pathology resulting from an injury to the cerebellum, present in voluntary or purposeful behavior, which he with the other two authors had interpreted as a malfunction of the negative feedback in their 1943 paper. For example, intention tremor prevents a patient from bringing a glass to his mouth and drinking from it. On the contrary, the tremor caused by Parkinson's disease can manifest itself at rest, in a condition known as the shaking palsy of old men: indeed, if the patient tries to perform a well-defined task, so that there is a purpose, the tremor from Parkinson's disease tends to disappear. This tremor had been construed as the result of a second, so-called "postural", feedback. Therefore, what is manifested in both cases is feedback which is excessive, giving rise to oscillations in behavior. But, and this was the hypothesis endorsed by Wiener, the tendency to amplify excessively, caused by the postural feedback of Parkinson's disease, was countered by voluntary feedback, which was able to bring the postural feedback back out of the zone where oscillations were triggered.

Wiener suggested the possibility of embodying this principle in a machine, so that a "demonstration apparatus which would act according to our theories" would be available, as he said in his popular book *The Human Use of Human Beings* ([10], p. 191), where a machine of the sort is described. This is a simple robot, built by Henry Singleton, a small car moved by a motor, with two driving wheels in the rear and a steering wheel in the front. The robot, equipped with two photocells and appropriate negative feedbacks, could display the two behaviors of positive and negative phototropism, depending on whether its purpose was to seek the light or avoid it. Under the proper conditions, it could even manifest these two antagonistic purposeful behaviors in an oscillatory manner, and the oscillations always increased. This was the analog of the intention tremor. Furthermore, due to the presence of a device that created the second, postural feedback, the robot could manifest a marked oscillatory movement, but this time in the absence of light, i.e. in the absence of the machine's goal. This behavior, which could be made to correspond to the tremor caused by Parkinson's disease, was due to the fact that the second feedback was antagonistic to the first and tended to decrease in amount.

This very stance is at the core of several cybernetic programes. An example is Eduardo Caianiello's theory of thought processes. In his well known 1961 paper, Caianiello gave the following definition of a model:

By "model" or "machine" we mean exclusively a device *that can actually be built*, and which operates according to mathematical equations that *are exactly known and numerically solvable* to any wanted accuracy. Although this necessarily implies drastic schematizations and simplifications, it is hoped that the features essential to thought-production are retained by the model; successive approximations to reality will require improvements in the structure of the machine and in its operational laws, but at each step one must know exactly what is being done ([11], p. 206).

Caianiello's "model" or "machine" corresponds to Rosenblueth and Wiener's "material model", whose actual functioning is justified by the presence of a theoretical model, in this case the system of mathematical equations. As Caianiello pointed out, this system consists of two sets of equations: the "neuronic equations", which describe the instantaneous operation of the machine, and the "mnemonic equations", which describe the growth of memory into it. From these equations, as he concluded, it is possible to predict and study the "mental" phenomena which are typical of such a machine: learning, forgetting, re-integration, conditioning, analysis of patterns and spontaneous formation of new patterns, self-organization into reliable operation.

Here we see the main features of the modeling approach in the study of the organism behavior. I already stressed that the model is a "simplification" of the real phenomenon, but it should be able to grasp its "essential" features. A model is a not a gross analogy, but a working artefact, functioning according to the theoretical statements it embodies. More than this, a model can be improved through "successive approximations" to the real phenomenon. Wiener and Rosenblueth's lesson seems to be present here.

4 More years later: a conclusive statement

I would like to point out briefly a more recent example of the simulative methodology I stressed in previous sections of this paper: the experiments on the neural net models of learning in *Limax*, a simple invertebrate. In my view, it is an enlightening example, because it shows how in this case model-building converged toward better and better approximations of the behavioral phenomenon, and how these approximations were made possible by progress both in learning algorithms and in computational resources (further details on this point are given in [2], Chapter 6).

Tesauro [12] reviews the experiments on *Limax* and shows how certain basic phenomena of classical conditioning (i.e. general features of first and second-order conditioning) could be simulated by a simple one-layer net. This can be viewed as a model that utilizes local representations and Hebb rule as plasticity rule or learning algorithm for modifying the weights.

A first improvement of this simple model led to the simulation of higher forms of conditioning and to good predictions, as regards the behavior of organisms in general during this kind of learning. Anyway the model could not say in detail how conditioning exactly occurs in a particular organism as *Limax*. A more profitable improvement of the model became possible using distributed representations and a Hopfield net with a Hebb rule as learning algorithm [13]. A further improvement of the model was then given by introducing multiple layers and hidden units, with the backpropagation rule as learning algorithm, in the "back-end" of the model (that dealing, in particular, with the generation of the appropriate motor responses). This latter model was proposed by Tesauro himself [14].

Comparison of the behavior of these new models with that of the real animal during learning was actually profitable: the models suggest several behavioral experiments and provide new insights into the bases of conditioning in *Limax* Tesauro points out the common theoretical requirements of these models, i.e. their "simplicity" and "generality", that lead to a grasping of the "essential ingredients", has he puts it, of the behavioral phenomenon ([12], p. 75).

To conclude, here we are again at the core of the methodology put forward by any simulation theorist, starting from the time of the *discovery* of such a methodology. As Tesauro put it, behavior-based models are always a simplification of the phenomenon, but the interaction of modeling studies with cellular and behavioral studies of real animals leads this time both to the design of new experiments and to refinements and improvement in the design of the model itself, based on experimental data that cannot be explained by existing models. This is perhaps a major lesson for the simulation methodology.

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Caianiello and Neural Nets

Paul Cull

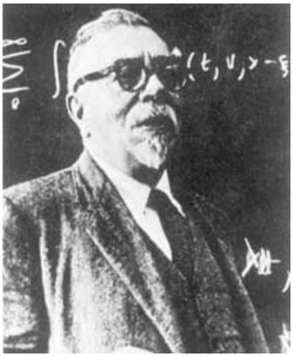
1 Introduction

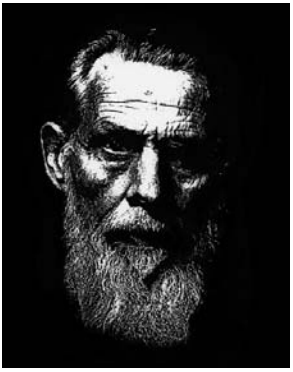
We are gathered here today to celebrate the life and scientific career of Professor Eduardo Caianiello. It is my task to present and assess one small area of his work – neural nets.

I first met Professor Caianiello many years ago when he visited the University of Chicago. After writing my thesis on neural nets, I subsequently worked with Professor Caianiello on various occasions at the Laboratorio di Cibernetica and at the University of Salerno. Hence my discussion will be biased by personal recollections and should count more as scientific heritage rather than as strict history.

2 Pre-history

As most of you know, Caianiello's training and initial scientific work was in physics. But in the post-war period there were forces driving science in new directions. In particular, Norbert Wiener had proposed a new science – Cybernetics [1], which he called the study of control and communication in the animal and the machine. Physics dealt with matter and energy and their various transformations. Cybernetics instead would deal with information. Wiener was not calling for a rejection of physics but for an emphasis on information with energy taking a less important role. For example, he pointed out that the study of biology in terms of energy transformation was all well and good, but such study ignored the central questions of living organisms. How could a mass of atoms be coordinated to behave like a living organism? How could an organism make its way and continue to exist in some environment? These questions could not be answered by looking mainly at energy. The study of information particularly within a biological context was needed.





A start on such a study was made in the 1940s by Warren McCulloch and others. His idea may be paraphrased in syllogistic form as:

> The brain carries out logical thinking.
> Logic describes logical thinking.
> Therefore, the brain's function can be described by logic.

To these ends McCulloch and Pitts [2] created a logical calculus for brains. Their model based on logic and the then available physiology was called a neural net. The neurons in their model could be represented as in Fig. 1 with the following assumptions:

- 0–1 law: inputs and outputs have only two possible values

  OFF = FALSE = 0

  ON = TRUE = 1
- Time delay: the output at time t is a function of the inputs at time $t-1$.
- Weights on inputs:

  $a_j > 0$ input j being ON tends to turn the neuron ON.

  $a_j < 0$ input j being OFF tends to turn the neuron OFF.
- Total input: $Total = \sum_j a_j \; input_j$
- Threshold: h
- Output:

  if $Total - h > 0$ output is ON.

  if $Total - h < 0$ output is OFF.

Such neurons are called linear threshold neurons. (McCulloch and Pitts actually considered the linear threshold neurons as one of several possible models.) Neural nets could be made by connecting a set of these neurons.

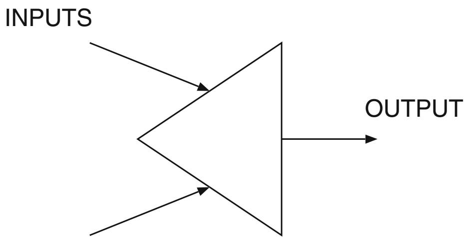


Fig. 1. A neuron

In vector-matrix form a linear threshold net can be described as

$$X_{t+1} = 1(AX_t + BY_t - h)$$

where X_t and X_{t+1} are the vectors of the states of the neurons at times t and $t+1$, Y_t is the vector of external inputs, h is the vector of thresholds, A and B are matrices containing the weights for the connections between neurons and between inputs and neurons, $1(\ )$ is the Heaviside nonlinearity extended to vectors, so that for each vector component

$$1(x) = \begin{cases} 1 \text{ if } x > 0 \\ 0 \text{ if } x \le 0 \end{cases}$$

A neural net has a *finite* number of states with two possible states for each neuron. The behavior of a net can be described by the mappings among these states caused by the input vectors.

McCulloch and Pitts presented a number of small nets and described their behaviors. But there was (and still is) a major problem in describing the behavior of large nets. A neural net with n neurons has 2^n states, so the description of the behavior may be *MUCH BIGGER* than the description of the net.

3 Caianiello's program

The 1960s were a time of foment and upheaval in society and in science. One of the signs of scientific change was the introduction of new journals. The new *Journal of Theoretical Biology* appeared with the new decade. In the first volume of the journal Caianiello published his "An Outline of a Theory of Thought-Processes and Thinking Machines" [3] which laid out his research program on neural nets. As we will see this program led to many advances in the past forty years and continues to be influential.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Outline of a Theory of Thought-Processes and Thinking Machines

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Thought-processes and certain typical mental phenomena are schematized into exact mathematical definitions, in terms of a theory which, with the assumption that learning is a relatively slow process, reduces to two sets of equations: "neuronic equations", with fixed coefficients, which determine the instantaneous behavior, "mnemonic equations", which determine the long-term behavior of a "model of the brain" or "thinking machine". A qualitative but rigorous discussion shows that this machine exhibits, as a necessary consequence of the theory, many properties that are typical of the living brain: including need to "sleep", ability spontaneously to form new ideas (patterns) which associate old ones, self-organization towards more reliable operation, and many others. Future works will deal with the quantitative solution of these equations and with concrete problems of construction—things that appear reasonably feasible. With a transposition of names, this theory could be applied to many sorts of social or, more generally, "collective" problems.

1. Introduction

A. LEVELS OF APPROACH

Attempts at a quantitative understanding and analysis of thought-processes, with or without the explicit aim of devising machines that should reproduce functions typical of the living nervous system, date as far back as Ramon Lull's syllogistic wheels. They have become a recognized and major part of scientific investigation since N. Wiener's celebrated enunciation of the principles of Cybernetics; herein lies indeed clearly, much more than in specialized studies of circuitry or of information theory, the heart and scope of this new science, which aims at synthesis as well as analysis.

The investigation of the mechanism of thought has been undertaken with a variety of methods, ranging e.g. from the study of systems that should mechanize the operations of Aristotelian logic without any requirement of similarity to living structures, to the faithful electronic reproduction of populations of hundreds or thousands of neurons. We shall benefit

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The major innovation in the paper was to break the study of neural nets into two parts – *dynamics* and *learning*. This reductionistic idea may have come from Caianiello's background in physics where breaking systems apart and studying the parts in isolation have proved remarkably fruitful. As in physics, dynamics studies how the state of a system changes over time. Caianiello's insight was to assume that the states of the neurons changed so quickly that the parameters of the net could be assumed to be constant. Also, in most cases, the state changes would be faster than changes in the environment, and thus inputs could also be assumed to be constants.

In contrast to this fast dynamics, Caianiello assumed that neural nets could change and adapt to their environment, but that these adaptive changes would be slow compared to the fast dynamics. In the model these slow changes would occur in the parameters – the interconnection weights and the thresholds.

This dichotomy in time scales could reasonably describe how an organism could react to current inputs but gradually adapt and become more efficient. We will discuss this two-pronged program in the next two sections on *dynamics* and *learning*.

4 Dynamics

The dynamics of neural nets may be quite complicated. There are many ways in which a net may respond to inputs. To simplify matters, Caianiello assumed that a net responded so quickly that the external inputs could be assumed to be constant. Then the inputs could be absorbed into the parameters leading to an *autonomous* neural net equation:

$$X_{t+1} = 1(AX_t - h) .$$

A state of such an autonomous system uniquely determines the next state of the system. So the behavior can be described as a finite directed graph in which each node represents a state and because of the assumed autonomy, there is exactly one arrow out of each node. This limits the types of behavior to a few possibilities. A state may be a *fixed point*, that is, the only out arrow returns directly to the state.

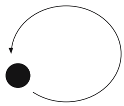


Fig. 2. A fixed point

More generally, a state may be *cyclic*, that is, following the arrows from a cyclic state, one will eventually return to that state.

$$\begin{tikzcd} X_1 \arrow[r] & X_2 \arrow[r] & \cdots \arrow[r] & X_C \arrow[lll, bend left] \end{tikzcd}$$

Fig. 3. A cycle

If a state is not cyclic, it is a *transient* state, that is, following the arrows from a transient state, one will never return to that same state.

The overall behavior of an autonomous net would consist of a number of *basins of attraction*, where a basin is a cycle together with all the transient states which eventually lead to that cycle. A basin could look like:

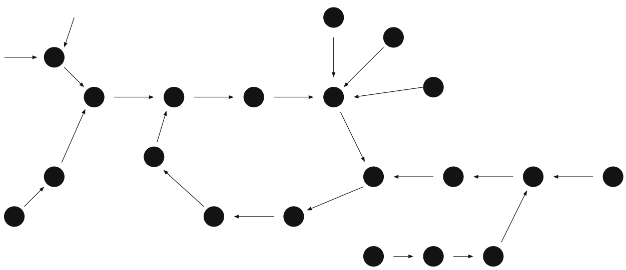


Fig. 4. A basin

4.1 Simple examples

The following are two simple neural nets and the diagrams of their dynamics

$$X_{t+1} = 1\left(\begin{bmatrix} 5 & -4 \\ 7 & 3 \end{bmatrix} X_t - \begin{bmatrix} 5 \\ 5 \end{bmatrix}\right) .$$

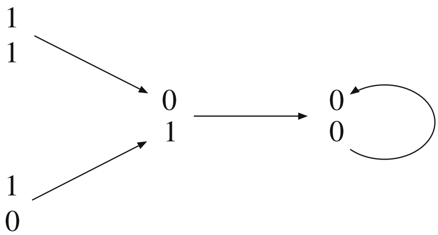


Fig. 5. Example 1

Here there is a single basin of attraction and all states lead to the same fixed point.

$$X_{t+1} = 1\left(\begin{bmatrix} -4 & 6 \\ 7 & 3 \end{bmatrix} X_t - \begin{bmatrix} 5 \\ 5 \end{bmatrix}\right)$$

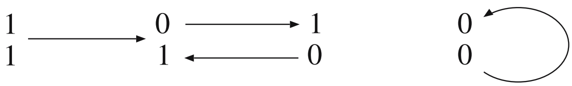


Fig. 6. Example 2

Here there are two basins. The starting state of the net will determine which basin the net is in. In one basin, the net stays fixed at $\begin{smallmatrix} 0 \\ 0 \end{smallmatrix}$ while in the other basin the net will eventually oscillate with period 2.

4.2 Questions about dynamics

Obviously, given any net one can work out its dynamics diagram. But one might also hope to be able to answer more quickly such questions as:

1. How many fixed points are there?
2. What periods occur in cycles in the net dynamics?
3. How many basins of attraction are there?
4. What is the size of the largest basin of attraction?
5. How long is the longest chain of transient states?

Caianiello also recognized that nets with different parameters may behave in the same way. For example, changing a parameter by a small amount will probably leave the dynamics unchanged. So Caianiello said that two nets were *equivalent* iff

$$\forall X \in \{0,1\}^n \quad 1(A_1X - h_1) = 1(A_2X - h_2)\ .$$

While such equivalent nets have exactly the same dynamics diagram, other nets can have the same dynamics diagram except for the name of the states. So Caianiello defined two nets as having *equivalent dynamics* iff there is an invertible function f so that

$$\forall X \in \{0,1\}^n \quad f(1(A_1X - h_1)) = 1(A_2\ f(X) - h_2)\ .$$

5 Linearity and neural nets

Caianiello and co-workers (see the references) set about studying the dynamics of autonomous neural nets. Since linear methods had proved so successful in physics, Caianiello decided to rearrange the neural net equations so that the role of the connection matrix became more prominent.

The equation

$$X_{t+1} = 1(AX_t - h)$$

is replaced by

$$Y_{t+1} = \frac{1}{2}A\ \operatorname{sgn}(Y_t)\ .$$

where

$$Y_t = AX_t - h$$

$$\operatorname{sgn}(Z) = \begin{cases} 1 & \text{if } Z > 0 \\ 0 & \text{if } Z = 0 \\ -1 & \text{if } Z < 0 \end{cases}$$

(Notice that the Heaviside nonlinearity can be written in terms of the sgn nonlinearity $1(Z) = \frac{1}{2}(1 + \mathrm{sgn}(Z))$.)

With the assumption that *no* component of Y_t is ever 0, and the assumption that the threshold vector h satisfies

$$h = \frac{1}{2}A\vec{1}\ ,$$

Caianiello could rewrite neural nets as *normal* nets,

$$Y_{t+1} = \frac{1}{2}A\,\mathrm{sgn}(Y_t)\ .$$

This form has only $+1$ and -1 as components of the Y vector and so in some sense this is more symmetric that the more usual 0,1 vector representations. But the point was to display the matrix A and to exploit the properties of this matrix.

One of the most important properties of a matrix is its *rank*. This rank equals the number of linearly independent columns in A. Vectors $v_1, v_2, \ldots, v_k$ are *linearly independent* iff $\sum c_i v_i = 0$ implies $c_1 = c_2 = \cdots = 0$.

Caianiello and co-workers derived several results from the rank of A:

– If A has rank 1, then for each Y either

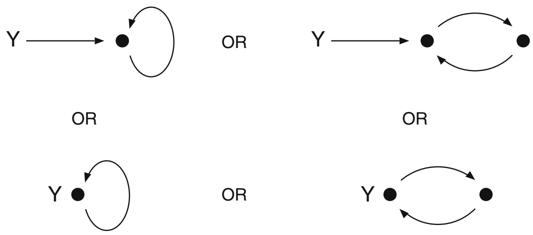


Fig. 7. Dynamics

that is the dynamics of a rank 1 net is extremely simple.

– If an n neuron net has A with rank k, then the number of cyclic states is

$$\leq\ 2^n - 2^{n-k+1} + 2\ .$$

– In particular, if $k = 1$, there are at most 2 cyclic states, while if $k = n$, there may be 2^n cyclic states, i.e. every state may be cyclic.

In dynamics in physics, various quantities, like total energy, do not change even as the physical system changes states. These conserved quantities are called *constants of the motion*. For a neural net with n neurons and $rank(A) = k$, there are $n - k$ constants, $c_1, c_2, \ldots, c_{n-k}$ and $n - k$ vectors $v_1, v_2, \ldots, v_{n-k}$, so that for all t,

$$v_i \cdot X_{t+1} = c_i\ .$$

Further given $c_1, \ldots, c_{n-k}$ and $v_1, \ldots, v_{n-k}$, one can construct a net with these constants of motion.

6 Linearizations

Some neural nets are equivalent to *linear systems*

$$X_{t+1} = A\ X_t$$

where A is a 0,1 matrix and the operations are MOD 2 (e.g. $1 + 1 = 0$). For such systems the fixed points are easy to find, the number of transients is easy to compute, but the lengths of the cycles may be more difficult to compute [4].

Are all neural nets equivalent to linear systems?
YES! If one uses a 2^n dimensional linear system, any n neuron can be represented as

$$Y_{t+1} = TY_t$$

where T is a $2^n \times 2^n$ {0,1}-matrix which represents the permutations and projections of the states. Each state is represented by a {0,1}-vector with exactly one 1. This is called the *transitional* representation. Other linearizations are possible.

Every neural net can be represented as

$$X_{t+1} = F(X_t)$$

where F is a nonlinear function from $\{0, 1\}^n$ to $\{0, 1\}^n$. Any function from $\{0, 1\}^n$ to $\{0, 1\}^n$ can be represented as a polynomial in n variables in which no variable has degree greater than one. A product of such polynomials is also such a polynomial. Let

$$F(X_t) = \begin{pmatrix} f_1(X_t) \\ f_2(X_t) \\ \vdots \\ f_n(X_t) \end{pmatrix} \qquad \text{where} \qquad X_t = \begin{pmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{pmatrix}$$

then if products of variables are taken in the order

$$1,\ x_1,\ x_2,\ x_1x_2,\ x_3,\ x_1x_3,\ \ldots,\ x_1 \ldots x_n$$

and the products of the functions are taken in a similar order

$$1,\ f_1,\ f_2,\ f_1f_2,\ f_3,\ f_1f_3,\ \ldots,\ f_1 \ldots f_n$$

then

$$X_{t+1} = F(X_t)$$

can be represented as

$$\mathcal{X}_{t+1} = \mathcal{F}\mathcal{X}_t$$

where $\mathcal{X}$'s are 2^n dimensional vectors and $\mathcal{F}$ is a $2^n \times 2^n$ matrix. Further $T \simeq \mathcal{F}$ over the field with two elements GF(2). There is a self-inverse matrix H so that

$$TH = H\mathcal{F} \qquad \text{and} \qquad HTH = \mathcal{F}\ .$$

The *trace* of a matrix is the sum of the diagonal elements of the matrix. For the transition matrix T,

$$\text{trace}(T) \;=\; \text{tr}(T) \;=\; \sum \mathrm{T}_{ii}$$

is the number of fixed points in the dynamics of the neural net. Since trace is invariant under similarity transformations, tr(T) $= \text{tr}(\mathcal{F})$. Unfortunately, the similarity is over GF(2) so this equality is a MOD 2 equality and as such it will only allow one to discover whether the number of fixed points is odd or even.

To circumvent these MOD 2 limitations, Caianiello and co-workers sought and found a linearization over a much bigger field -– the rationals. As before, they replaced the state set {0, 1} by the state set $\{-1, 1\}$ and then considered functions from $\{-1, 1\}^n$ to $\{-1, 1\}$. All such functions can be represented as polynomials over the rationals with rational coefficients where denominators are at most 2^n. Because of the closure of the set of polynomials under addition and multiplication, there is a rational field linearization

$$\mathcal{X}_{t+1} = \mathcal{F}\mathcal{X}_t$$

for every neural net.

Again this function matrix $\mathcal{F}$ is similar to the transition matrix T. Since this similarity is over the rationals,

$$\text{tr}(T) = \text{tr}(\mathcal{F})$$

over the rationals, and $\text{tr}(\mathcal{F})$ gives an exact count of the number of fixed points in the net‘s dynamics.

7 Expected behavior

Even though it is algorithmically possible to compute the dynamic behavior of a neural net, the computational effort grows like 2^n, making such computation practically impossible even for moderate values of n. But it might be possible to average over a class of nets and say something about the expected behavior of nets in this class. The obvious hope is that such average computations could be done in reasonable amounts of time.

The effect of inter-neuron connectivity on dynamic behavior was investigated by Kauffman [5]. Using a combination of simulations and theoretical calculations, he investigated the following problem:

> Pick k and let each neuron compute a function of the states of k neurons. Assign these functions at random. How does the length of cycles depend on k?

He found that for $k \approx 2$ the cycle lengths were proportional to n the number of neurons, but for large values of k, the cycle lengths were about $2^{n/2}$, that is about the square root of the number of states.

Since taking averages over the rationals makes sense, the $\{-1, 1\}$ linearization seemed ideal to address this problem.

Specifically,

$$\mathrm{tr}(T^m) = \mathrm{tr}(\mathcal{F}^m) = \begin{cases} \text{Number of states in cycles} \\ \text{whose lengths divide m .} \end{cases}$$

$$E(\mathrm{tr}(T^m)) = E(\mathrm{tr}(\mathcal{F}^m)) = \begin{cases} \text{Expected number of states in cycles} \\ \text{whose lengths divide m .} \end{cases}$$

Some of the results obtained are the following:

Theorem 1 *For any probability distribution which is symmetric*

$$\mathrm{Prob}(f_i = 1) = \mathrm{Prob}(f_i = -1)$$

and has functions assigned independently,

$$E(\mathrm{tr}(T)) = 1\ ,$$

i.e. expect one fixed point.

Theorem 2 *If the functions are assigned using the uniform distribution over functions,*

$$E(\mathrm{tr}(T^m)) = \frac{2^n!}{2^{nm}(2^n - m)!}\ .$$

From this latter theorem, Kauffman's results for $k = n$ nets could be derived. This linearization technique can probably be used to answer some other questions about expected behavior of certain classes of neural nets.

8 Learning

The second prong of Caianiello's program for neural nets was the study of learning in such nets. Specifically, Caianiello suggested a *mnemonic equation* which described how nets could learn by association. The basic idea is that the coupling from neuron j to neuron i increases if neuron i is ON at time t and neuron j was ON at time $t-1$. Caianiello was able to give examples of learning using his mnemonic equation.

Caianiello's emphasis on association was heavily influenced by Valentino Braitenberg. Braitenberg's delightful book "Vehicles" [6] is a must-read for all those interested in the potential of learning in neural nets.

Learning has become the major focus of workers in neural nets. For example, conferences and journals like COLT, *Machine Learning*, NIPS, INNS, etc. attest to the great activity in this area. (Dynamics of neural nets are almost never mentioned in these venues.) A myriad of learning algorithms have now been proposed and studied. The breakthrough which produced the flowering was the replacement of the "hard" nonlinearity with a "soft" nonlinearity, that is,

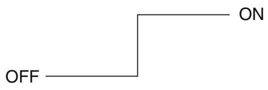


Fig. 8. Heaviside function

was replaced by

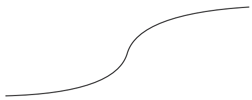


Fig. 9. Sigmoidal function

The sigmoidal nonlinearities were differentiable and this allowed algorithm designers to use calculus based optimization techniques to find "best" weights for a neural net. Many of these algorithms are not based on association, so in some sense they generalize Caianiello's ideas for neural net learning.

Active work over the last 20 years has shown that even simple feedforward networks can learn the solutions to some problems. More recent theoretical work has argued that for many problems no practical learning algorithms can exist.

9 Computational complexity

Computational Complexity [7–9] was developed as a subfield of computer science in an attempt to classify which problems have reasonable algorithms and which problems could have only unreasonable algorithms. In particular, the idea of reasonable algorithms was identified with algorithms whose running time was bounded by a polynomial in the size of its input. If n was the size of the input then an algorithm with n^3 run time would be reasonable, but an algorithm with run time 2^n would be unreasonable. Various complexity classes like $\mathcal{NP}$, co-$\mathcal{NP}$, and PSPACE were defined, and it was possible to show that certain problems were the hardest problems within one of these classes. Although there are still some unsolved questions about this theory, it is generally accepted that these hardest problems will not have reasonble algorithms.

The following diagram shows these classes and indicates the positions of some problems within these classes.

Problems with reasonable algorithms are within the inner circle in this diagram. Among the harder problems for neural nets are:

- EQUI: Do two neural nets have the same behavior?
- DYNAMIC EQUI: Do two neural nets after renaming the states have the same behavior?
- FIXED POINT: Does a neural net have a fixed point in its dynamics?
- LEARN: Can a neural net of a particular type learn to solve a particular problem?

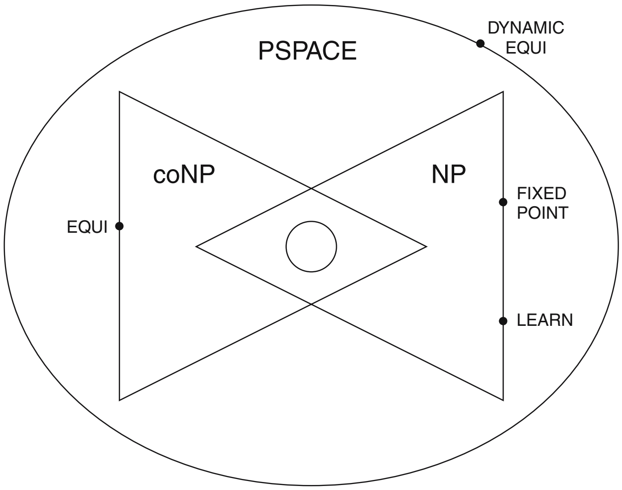


Fig. 10. Complexity classes

The diagram indicates that these problems are among the hardest problems for various complexity classes. Thus, it is unlikely that there are any reasonable algorithms for these problems from neural net dynamics and learning. The challenge is to put enough meaningful restrictions on the type of neural nets so that these questions can be reasonably answered for these restricted neural networks.

In summary there are several major lessons from this line of research:

1. Finite state models are not easy

One of the most attractive features about finite state models is that all questions are algorithmic, that is, it is always possible to create computer programs which are guaranteed to correctly answer the questions. Over the course of this research, it has become clear that while such programs are possible they may not be practical. Research in computational complexity has shown that various questions about neural nets are the hardest problems for various complexity classes. As such, any programs for these problems will require too much storage space and too much running time to be practical. Even answering questions for nets that are significantly smaller than vertebrate brains would require resources that would dwarf the current estimates of the size and age of the universe.

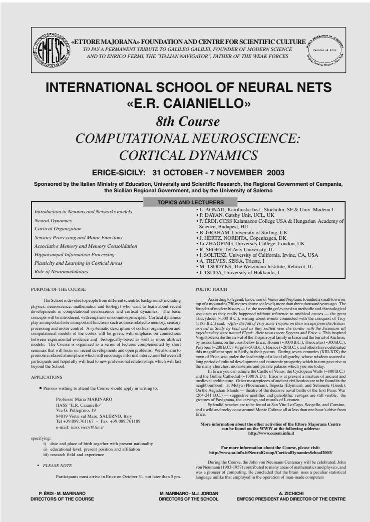
2. Continuous differentiable models *may* be easier

Both dynamics and learning may be studied using continuous models which may use continuity at the neuron and subneuron level, or may model a large neural net as a continuous system. While these continuous models give up the algorithmic character of finite models, continuous models are often amenable to approximation methods. Calculating the dynamic behavior of discrete neural nets may be difficult, but using continuous models and approximation methods like "mean field" may allow an easy estimate of a net's expected behavior. Replacing strict nonlinearities with continuous approximations has given rise to learning rules which seem to behave well in some circumstances. Assumptions of continuity, differentiability, and smoothness give easier-to-understand continuous models. We do not have corresponding simplifying assumptions for discrete models.

3. Ideas from biology still needed

One of the reasons for studying neural nets in the first place was that they would be models of biological systems. Conversely, the actual biological nets serve as an existence proof that there are neural nets with interesting properties. It seems that in pursuit of full mathematical generality we have ignored the constraints that biology places on real nets. For example, real nets can be large but not too large, the connections between neurons show some sort of locality, brains are organized into parts with heirarchical arrangement of these parts. Such information about real biological nets will be needed to create artificial nets that are as useful as brains.

4. The work continues



One of the strongest proofs of the importance of a research program is that research in the program outlives its founder. Before his death, Caianiello established the Institute for Advanced Scientific Studies at Vietri-sul-Mare. This institute with the leadership of Professor Marinaro has continued to host and run meetings in the field of neural nets. The latest conference occured last November. Even after his death, neural net papers with Caianiello as one of the authors continued to appear. For example, "Outline of a Linear Neural Network" by Caianiello and others appeared in 1996 [10]. Caianiello's ideas continue to be referenced in current research papers and books. While Eduardo is no longer with us, the work he started is being continued by other workers. His prescient paper of 1961 is still bearing fruit 40 years later.

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A Little Story

Many years ago, I visited Caianiello in Naples. At that time there was a severe coin shortage. The shopkeepers had to make change by giving you a small item, because they had no coins. Among the scarce coins was the 10 Lire coin: a coin of so little value that it could be ignored. But the 10 Lire coin did have some worth. Most of the apartment buildings in Naples that had elevators required this coin to operate the elevator. Having the coin could save you a long trek upstairs.

One day, Professor Caianiello invited us to his home to celebrate the holidays. With the invitation, he gave me a 10 Lire coin for the elevator.
I was so surprised that Caianiello had these scarce coins that I mentioned it to one of my colleagues at the lab. He replied: "The coins you put into the elevator go into a little locked box – and who do you think has the key?"

I recalled this story because of an announcement from the Vatican in March 2003 that the traditional *Keys of the Kingdom* which had hung on the statue of St. Peter in the basilica had disappeared.

I had to wonder: Who do you think has the keys?

Computability, Computation, and the Real World

Martin Davis

In 1937, Alan Turing [1] in a fundamental paper, not only set forth the foundations of what was to become theoretical computer science, but also hinted at the prospect of the "all-purpose" computers that have since become so ubiquitous [2–6]. Eduardo Caianello recognized, earlier than most, the rich interconnections between the study of theoretical models of computation and their physical implementation. In 1964 he organized a now famous conference in Ravello bringing together a number of researchers from varied disciplines to facilitate this interaction. I feel honored to have been a participant.

Today more than ever, the connections between theory and practice in computer science loom large. There is the promise of models of computation based on quantum mechanics to overcome some of the difficulties associated with computational problems that seem to lead to exponential explosions. But also there are confused efforts to overcome the bounds on what is computable that Turing and other logicians established.

1 Solving the unsolvable

Although algorithms are intended to deal only with finite data, and will produce finite results, in general they have the property that, in principle, they will behave correctly regardless of the size of the data. Their performance is ordinarily specified in terms of their asymptotic behavior as the size of the input increases without bound. The work by Turing and other logicians in the 1930s made precise what it means to say that a problem is algorithmically solvable and then were able to establish the existence of problems for which there is no algorithmic solution [3]. These problems are *unsolvable* in a very strong sense: even computers supplied with arbitrarily large amounts of mamory and given unlimited time cannot algorithmically solve these problems.

As computer scientists struggled with problems that are "solvable" in this sense, but for which no practically feasible algorithms exist, typically problems which seem to require a step-by-step search through all possibilities with no shortcut being available, it was accepted that unsolvable problems are entirely beyond the pale. Nevertheless, recently a number of researchers have argued that this limiation can be overcome, that there are methods to solve the unsolvable. In fact this work is highly dubious. In my article [7], I discuss two of these in some detail; here I will be more brief. In addition I will mention two other recent attempts.

2 Hava Siegelmann's neural nets: "beyond the Turing limit"

In [8, 9], Hava Siegelmann studies nets consisting of interconnected "neurons". These neurons are mathematical abstractions which output data determined by a weighted average of incoming data. The weights are real numbers. Using terminology usual in computer science, she sees each of her her nets as "accepting" a "language" where a *language* is simply a set of strings on a finite alphabet, for our purposes the two letters $\{a, b\}$. Comparing the classes of languages accepted as she varies the kinds of real numbers permitted to serve as weights, she obtains the results indicated in the following table:

Weights	**Languages Computed**
integers	regular languages
rational numbers	computable languages
real numbers	**all** languages

Since with arbitrary real coefficients, all languages, and in particular non-computable languages, are accepted, Siegelmann claims to have gone beyond the "Turing limit". However, not only is this result hardly surprising, as we shall see, it in no way establishes the claim to have produced a model of computation that, in any non-trivial sense, computes the uncomputable.

On the face of it, languages as sets of strings on a two-letter alphabet and real numbers may appear to be very different sorts of entities. But this is not the case. Such a language can be regarded as a set of positive integers using the coding:

$$\begin{array}{ccccccccccccccc} a & b & aa & ab & ba & bb & aaa & aab & aba & abb & baa & bab & bba & bbb & \dots \\ \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \dots \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & \dots \end{array}$$

Next we can associate with a set A of positive integers the real number between 0 and 1, written in binary:

$$0, c_1 c_2 c_3 c_4 \dots$$

where

$$c_n = \begin{cases} 1 & \text{if } n \in A \\ 0 & \text{otherwise} . \end{cases}$$

So the weights that Siegelmann uses in defining her nets are essentially the same kind of mathematical object as the languages accepted by them. Moreover, if she were to restrict herself to *computable* real numbers as weights, only computable languages would be accepted. Siegelmann goes "beyond the Turing limit" only by building into her devices the very non-computability that she extracts from them.

In fact, it is not difficult to see that the language accepted by a Siegelmann net is a computable function of the weights regarded as real parameters.

Siegelmann maintains that her nets execute a kind of *analog* computation, and that in some way they reflect physical reality. Thus she says:

> In nature, the fact that the constants are not known to us, or cannot even be measured, is irrelevant for the true evolution of the system. For example, the planets revolve according to the exact values of G, π, and their masses.

Ignoring the fact that she seems to be referring to Newtonian gravitation (known after Einstein to be only an approxiamtion), Siegelmann is assuming that the masses of the planets are well-defined real numbers, and (for her "beyond the Turing limit" claim) non-computable ones at that. π, like all the important constants of analysis, is a computable real number.

3 Copeland and Proudfoot pursue an oracle

Whereas Siegelmann hides the non-computable language that she claims her nets accept by coding them into the weights of her neurons, Jack Copeland [10–12], who is responsible for the term *hypercomputation*, is quite forthright. He proposes to seek an "oracle", a physical device which can solve unsolvable problems. In addition to the term "hypercomputation", the only contribution of Copeland (together with his collaborator Proudfoot) to this project is the claim that a proposal in this direction is to be found in Turing's dissertation of 1939, work said to have been largely forgotten. In its published form [13], this is a 68 page article devoted to progressions of systems of logic in which propositions undecidable in certain systems become decidable in subsequent systems. Far from having been forgotten or neglected, Turing's paper has been the source of an enormous quantity of important research.

Copeland and Proudfoot base themselves on Turing's notion of "oracle" which plays a rather minor role in his long article. Turing called a sentence a *number-theoretic theorem* if it is a true sentence of the form $(\forall x)(\exists y)[f(x, y) = 0]$ with x, y ranging over the natural numbers, and f a computable function. He wrote:

> Let us suppose that we are supplied with some unspecified means of solving number-theoretic problems; a kind of oracle as it were. We shall not go any further into the nature of this oracle apart from saying that it cannot be a machine.

By considering Turing machines modified to have access to such an oracle (o-machines), Turing was able to use the diagonal method to produce an example of a problem that is not number-theoretic. This is done in one page, and except for a few sentences later on, is the only mention of such machines in the entire paper.

Copeland and Proudfoot claim [12] that "Outside · mathematical logic, Turing's [oracles] have been largely forgotten". Since Turing's paper was entirely concerned with mathematical logic, it would not be surprising if this were true. But it is not true at all. Oracles have proved to be of great importance in theoretical computer science. Proceedong by analogy with the so-called "arithmetic hierarchy" studied by logicians, computer scientists have introduced the polynomial time hierarchy which uses oracles in its definition. Moreover, the great difficulty in settling the most important unresolved question in computer science, the million dollar $\mathcal{P} \stackrel{?}{=} \mathcal{NP}$ problem, is underlined by the Baker–Gill–Solovay Theorem [14]:

> There exist computable oracles A, B such that: $\mathcal{P}^A = \mathcal{NP}^A$ while $\mathcal{P}^B \neq \mathcal{NP}^B$.

Now, one hardly needs Turing to tell us that if we really possessed an "oracle", a device that would be able to correctly answer arbitrary questions about membership in non-computable sets, then we could solve the unsolvable. The oracle itself would be doing that. The notion that Turing himself was suggesting the possibility of a physical implementation is belied by the term he chose: after all, literally speaking, an "oracle" is a means of obtaining a message from a God. Not concerned with any of this, and limiting their contribution to exhortation, Copeland and Proudfoot proclaim: "So the search is on for some practicable way of implementing an oracle" [12].

4 Testing for false coins

The famous "halting problem" was proved unsolvable in Turing's classic paper [1]. It may be formulated as follows: *Provide a method for testing a Turing machine (given, for example, as a table specifying its behaviour for each possible state and scanned symbol) to determine whether, started on an empty tape, it will ever halt*. In [15] Cristian Calude and Boris Pavlov claim to show that a procedure to solve this problem is "mathematically possible". However, the paper has no discussion of the halting problem as such. Instead, we are provided with a succession of amusing puzzles about distinguishing false from true coins (assumed to have different weights) with the least number of weighings. They continue:

> Let us assume that we have now a countable number of stacks [of coins] all of them, except at most one, containing true coins only. Can we determine whether there is a stack containing false coins? It is not difficult to recognize that [this] is equivalent to the Halting Problem.

We shall see that this claim is dubious. But first, let us clear away some of the debris. Since the physical detail that the items in this countable sequence are stacks of coins is clearly irrelevant; what is significant is that they are of two kinds: true and false. Representing a stack of true coins by 0 and a stack of false coins by 1, their problem comes to this: *we are given a countably infinite sequence of 0s and 1s such that at most one of the terms of the sequence is 1, and we are to determine whether in fact 1 does occur in the sequence*.

The treatment of this problem by the authors is mathematically elaborate involving Wiener measure. They set up a statistical sampling procedure which assumes (without discussion) that the infinite sequence that consists of 0s and possibly a single 1 is presented in such a way that arbitrary elements of the sequence can be sampled. Now, algorithms as they are understood in computability theory (and in computer practice as well) are based on inputs that are finite data objects. Taken as a whole, the infinite sequences that Calude and Pavlov take as their inputs are infinite objects as is seen by the fact that they take as directly accessible arbitrary elements of the sequence. Of course a sequence of this kind can be coded by finite data objects in various ways. But to do so is to completely undercut Calude and Pavlov's approach. For example, such a sequence can be coded by a natural number: 0 if the sequence contains no 1, and n if the nth element of the sequence is 1. But then the problem posed becomes trivial. The sequence contains a 1 if and only if the code is non-zero. Alternatively, the sequence could be coded by an algorithm, perhaps in the form of a Turing machine, that outputs the elements of the sequence in order. Now the problem is no longer trivial, but the Calude–Pavlov methodology is clearly inapplicable. There is no way to uniformly sample the elements of the sequence. One simply must wait patiently as they are displayed one by one. If a 1 appears one has one's answer. Otherwise, one must just continue to wait.

Now the halting problem is quintessentially one involving a finite data object as input, namely a table representing the Turing machine in question. If one wishes to model it in terms of a sequence of the sort studied by Calude and Pavlov, one could consider the sequence of successive configurations of the given Turing machine and write a 0 for each non-halting configuration, a 1 if and when a halting configuration is encountered, and then 0s from then on. One can say that solving the Calude–Pavlov problem for the sequence thus obtained is indeed solving the halting problems. *However, there is no finite way to obtain this sequence as a whole.* There is no way to obtain the nth element of the sequence of configurations without first obtaining all of the preceding configurations. There is no way to uniformly sample this sequence. As in the situation above, one can only wait and watch as the Turing machine proceeds through its successive configurations. Halting will be revealed when and if the machine halts.

It is telling that Calude and Pavlov content themselves with the bare remark that "it is not difficult to recognize" that their problem is equivalent to the halting problem. It is only by first wrapping the problem in the obfuscating camouflage of coin weighing puzzles, and by not directly confronting the equivalence that they claim,that two reputable researchers could manage to convince themselves that what they had done had any relevance to the halting problem.

5 Quantum adiabatic algorithms

The current interest in quantum computation serves to underline Eduardo Caianiello's prescience in pursuing connections between physics and computation. It has been shown that the use of q-bits, that is quantum systems that are always measured to be in one of two states, but whose state vector between measurements is a mixture of the two, can yield fast algorithms for problems thought to be computationally intractable. In effect, this indeterminateness of the state vector permits a kind of parallelism.

More recently, a group at MIT has proposed a different way to make computational use of quantum mechanics. Their method uses adiabatic cooling to put a system in its ground state, and they show that exponential speed-up is possible. However, Tien Kieu [16, 17] proposes to go further and to use adiabatic cooling to solve Hilbert's 10th probem, known to be unsolvable.

In 1900, David Hilbert, addressing an international congress of mathematicians, posed 23 problems as a challenge for the new century. The 10th in the list ("Diophantine Equations") may be expressed as follows:

Find an algorithm that will determine of any given equation

$$p(x_1, x_2, \ldots, x_n) = 0 \tag{1}$$

where p is a polynomial with integer coefficients, whether the equation has a solution in non-negative integers.

The combined efforts of four mathematicians, Martin Davis, Yuri Matiyasevich, Hilary Putnam, and Julia Robinson sufficed to prove that no such algorithm exists [18]. Nevertheless Tien D. Kieu proposes to use quantum mechanics to provide a probabilistic algorithm for solving Hilbert's 10th Problem although the classic paper [19] has made it clear that probabilistic algorithms cannot solve unsolvable problems. Kieu proposes to test a given equation (1) by computing $\min[p(x_1, x_2, \ldots, x_n)]^2$ (which will be 0 just in case the equation has a solution) using "adiabatic cooling" to get into its quantum mechanical ground state a physical system with a corresponding Hamiltonian. The required Hamiltonian function is obtained from p^2 by replacing each unknown by the product of a suitable operator with its adjoint. Kieu proposes to determine whether (1) has non-negative integer solutions, by using a physical system with this Hamiltonian, subjected to "adiabatic cooling" to get it into its quantum mechanical ground state. The equation will be solvable just in case its value in this ground state is 0.

This is not the place for a detailed discussion of Kieu's rather intricate arguments involving matters concerning which I have very limited expertise. However, it may not be amiss to mention that in conversation with some of the leading experts in computation by quantum adiabatic cooling, I was assured that there is no way that these methods enable one to accomplish the miracle of surveying infinitely many tuples of natural numbers in a finite time.

By looking at some simple examples, one can see some of the overwhelming difficulties that Kieu's approach would have to overcome. Following a suggestion of Andrew Hodges, consider the Diophantine equation

$$(x+1)^2 - ay^2 = 0 \,. \tag{2}$$

If $a = 167$, (2) has no solutions because $\sqrt{167}$ is irrational. On the other hand, there are values of a arbitrarily close to 167 for which (2) has solutions. For example, making use of a calculator, we find that for

$$a = 166.99999999977909982224 \,,$$

we have the solution

$$x = 1292284798331, \quad y = 100000000000 .$$

Thus in constructing a physical system with the Hamiltonian corresponding to $[(x+1)^2 - 167y^2]^2$, it will be necessary to measure the constant "167" with **infinite precision**. Once again, the need for infinite precision real numbers is lurking in the background, reminding us of what we saw in connection with Hava Siegelmann's nets.

6 What about relativity theory?

The two pillars of contemporary physics are quantum mechanics and relativity theory. So it was inevitable that relativity theory would be brought to bear on solving the unsolvable. In [20] Etesi and Nemeti argue that conditions in the vicinity of certain kinds of black holes in the context of the equations of general relativity indeed permit an infinite time span to occur that will appear as finite to a suitable observer. Assuming that such an observer can feed problems to a device subject to this compression of an infinite time span, such a device could indeed solve the unsolvable. Of course, even assuming that all this really does correspond to the actual universe in which we live, there is still the question of whether an actual device to take advantage of this phenomenon is possible. But the theoretical question is certainly of interest.

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Structure, Combinatorics, and Complexity of "Words"

Aldo de Luca

Eduardo Caianiello was an eminent scientist in various fields including Physics and Cybernetics. I met him for the first time at the beginning of the 1960s as a student of his course on Theoretical Physics and I was tremendously impressed by his towering personality. In 1964, I became a member of the Group of Cybernetics at the Institute of Theoretical Physics of the University of Naples directed by Eduardo and, subsequently, my scientific activity continued at the Institute of Cybernetics of Arco Felice (Naples) founded by him in 1969.

I had the privilege of collaborating with Eduardo for some years on the theory of neural networks. We wrote a joint paper on this subject [1] in 1965 and, subsequently we published, together with my friend and colleague L. M. Ricciardi, two more papers on the dynamics of neural networks [2, 3].

Even though with the years my scientific interests became more and more oriented towards Automata and Information theory, my research activity was very much influenced by his teaching, his interdisciplinary vision of science, and his love for the *ars combinatoria*, of which Eduardo was a great master (see, for instance [4]).

1 Introduction

"Words" are finite, as well as infinite, sequences of symbols, called *letters*, taken from a finite set called *alphabet*.

The study of structural and combinatorial properties of words is of great interest to various disciplines such as Linguistics (classical and modern philology), Algebra (combinatorial group and semigroup theory), Physics (symbolic dynamics and crystallography), Computer Science (pattern matching, data compression, computer graphics, cryptography), and Molecular Biology (analysis of DNA and proteic sequences).

The theory of words has been developed independently in various fields using often a different language or mathematical techniques. Since M. P. Schützenberger and his school, a considerable effort has been made in these last 30 years by several researchers in order to give a unified treatment of the mathematical theory of words. This collective work culminated with the publication, under the name of M. Lothaire, of two volumes devoted to the subject [5, 6]. Moreover, a third volume of Lothaire, more oriented to applications, is in preparation. At present, *Combinatorics on words* is classified by the *Mathematical Review* as an independent subject of mathematical research.

First of all let us observe that words and sentences that are acceptable in any given ancient or modern lexicon have to satisfy strong grammatical and syntactical constraints. As observed by V. Braitenberg in his delightful booklet *Il gusto della lingua* [7], of the Italian word *mulino* there exist 720 permutations of which only *lumino* has a meaning in Italian. If one considers the sequence of letters *abcdef* no one of its permutations has a meaning in any language with the possible exception of *fedcab* which in American can mean the 'car of federal policemen'.

In each language there exists a maximal length of the words. For instance, the longest Italian word is *precipitevolissimevolmente* whose length is 26. Moreover, the number of words of a given lexicon is very limited. The Webster's New World dictionary of American English contains about 170 000 words.

A natural language like English or Italian has a *redundancy* which is essentially due to the 'structure' of the language. The redundancy of English is about 50%. This means that half of what we write in English is determined by the structure of the language and half is chosen freely. The redundancy can be calculated experimentally as follows: one deletes a certain fraction of letters in a given text and then some one is asked to attempt to restore it. The maximal value of the preceding fraction which allows one to reconstruct the text is the redundancy. Another method of evaluation of the redundancy is obtained by an evaluation of the entropy of a (Markovian) source which gives a good approximation of the given language (cf. [8]).

The redundancy of a language is one of the properties used in communication theory in order to compress long texts. Another property is the *repetitivity*, i.e. the presence of repeated sequences of letters (cf. [9, 10]).

In the following we shall deal with words from a *mathematical-syntactic* point of view leaving out of consideration any *semantic-interpretative* aspect. We recall that the "Procuste" algorithm for the analysis of texts was designed by Caianiello [11] exactly in this frame of ideas. In fact, this program essentially consists of a very general automatic procedure (algorithm) able to determine and extract in a complex structure, such as a text written in an unknown language, meaningful substructures by means of which one can recover the initial text.

As we said before in the mathematical frame a *word* is any finite sequence (string) of symbols, called *letters* belonging to a given finite set $\mathcal{A}$, called *alphabet*. The number of elements of $\mathcal{A}$ will be denoted by d and called the *size* of the alphabet $\mathcal{A}$.

In natural languages which are purely alphabetic the size of the alphabet is between 20 and 40 (for instance, the English alphabet contains 26 letters, the Italian 21, the Russian 36). In the case of ideographic languages the alphabet can contain thousands of symbols (in Chinese there are about 5000 ideograms). In the case of syllabic alphabets the size is between 50 and 100. In computers all information is represented by words over a binary alphabet $\mathcal{A} = \{0, 1\}$, in the Morse telegraphy the messages are sequences of *dots* and *dashes*, in molecular biology, DNA sequences are words over a four letter alphabet.

One, usually, represents a word w over a given alphabet $\mathcal{A}$ as:

$$w = a_1 a_2 \cdots a_n \ ,$$

with $a_i \in \mathcal{A}$, $1 \leq i \leq n$. The integer n is called the *length* of the word w and it is denoted by $|w|$. If $w = a_1a_2 \cdots a_n$ and $v = b_1b_2 \cdots b_m$ are two words on the alphabet $\mathcal{A}$, the *concatenation* or *product* of w and v is the word wv defined as:

$$wv = a_1a_2 \cdots a_nb_1b_2 \cdots b_m \ .$$

The operation of concatenation is associative but not commutative. For some technical reasons it is also convenient to introduce the *empty word*, denoted by ϵ, which is a sequence of length 0, i.e. without letters. The set of all words over the alphabet $\mathcal{A}$ is usually denoted by $\mathcal{A}^*$.

A *factor* (or *subword*) of the word $w = a_1a_2 \cdots a_n$ is either the empty word or any 'block'

$$a_ia_{i+1} \cdots a_j \ , \quad 1 \leq i \leq j \leq n \ ,$$

of consecutive letters occurring in w. More precisely the word u is a factor of w if there exist words p and q such that $w = puq$. If $p = \epsilon$ (resp., $q = \epsilon$), then u is called *prefix* (resp., *suffix*) of w. For instance, in the case of the word *abracadabra*, one has that *braca* and *cada* are factors, *abra* is a prefix and *dabra* is a suffix. The set of the factors of a word w will be denoted by $\mathrm{Fact}(w)$.

If w is a word and n a positive integer by *n-power* of w we mean the word w^n defined inductively as:

$$w^1 = w, \quad w^n = ww^{n-1} \ , \quad \text{for} \quad n > 1 \ .$$

For instance, if $w = abb$ and $n = 3$, then $(abb)^3 = abbabbabb$. A word $w = a_1 \cdots a_n$ is called a *palindrome* if it is equal to its *reversal* $w^{\sim} = a_n \cdots a_1$ (the empty word is considered a palindrome). For instance, *ababbaba* is a palindrome.

Let p be a positive integer. A word $w = a_1 \cdots a_n$ has a *period* p if for all integers i, j such that $1 \leq i < j \leq n$, and $i \equiv j \pmod p$, one has $a_i = a_j$. A word can have several periods; for instance, the word *abaababaaba* has the periods 5, 8, and 10. A word w is called *periodic* if $|w| \geq 2\pi_w$, where π_w is the minimal period of w. For instance, the word *abaababaaba* is periodic.

In the following we shall consider also infinite words. An *infinite word* w over a given alphabet $\mathcal{A}$ is an infinite sequence

$$w = a_1a_2 \cdots a_n \cdots$$

of letters a_i, $i > 0$ taken from the alphabet $\mathcal{A}$. A factor of w is any finite sequence $a_ia_{i+1} \cdots a_j$, $1 \leq i \leq j \leq n$, of consecutive letters occurring in w. An infinite word w is *eventually periodic* if there exist words u and $v \neq \epsilon$ such that w may be written as:

$$w = uvvvvvv \cdots = uv^{\omega} \ .$$

The word u called *antiperiod* may be empty; v^{ω} denotes the *periodic* word $v^{\omega} = vvvv \cdots$ of period $|v|$. We recall that if the alphabet $\mathcal{A}$ has d letters, then any real number can be represented in base d by an infinite word over $\mathcal{A}$. Moreover, a real number is rational if and only if it is represented by an infinite eventually periodic word over $\mathcal{A}$.

A *language* L over a given alphabet $\mathcal{A}$ is any set of (finite) words over $\mathcal{A}$. One says that a word w *is a factor of* L if w is a factor of a word of L. Examples of languages are the sets of factors of a finite or of an infinite word. A language can be finite or infinite. Important examples of finite languages are the *natural languages* which are used at the present (such as English, Italian, etc.) or have been used in the past (like ancient Greek, Latin, Etruscan, etc.). We can identify a natural language such as the American English with its Webster dictionary.

A *text* over a given language L is any finite sequence of words of L. In the case of natural languages the 'acceptable' texts have, of course, to satisfy some strong syntactic constraints. We remark that if one adds to the alpahabet $\mathcal{A}$ a special symbol $\square$ to denote the *empty space*, then any text over the language L can be identified with a single word over the alphabet $\mathcal{A} \cup \{\square\}$, so that for some mathematical aspects the study of the structure of a text over a given language can be reduced to the study of the structure of a single word over the extended alphabet.

2 Structure of the words

The 'structure' of a finite or infinite word is due to the presence in the word of some regularities such as:

- presence of periodicities,
- repetitions of factors,
- presence of palindromic factors,
- the absence in the word of some kind of factors (*forbidden factors*); for instance, digrams such as *zb* or *cb* are forbidden factors in Italian and in English,
- presence (or absence) in the word of special *patterns* like x^2, that is any two consecutive equal blocks of letters,
- the existence of simple procedures of construction of the word,
- the variety and multiplicity (i.e. number of occurrences) of the factors of the words.

We shall refer to these regularities as *structural regularities* since they depend on the structure of the particular word under consideration.

We stress that, on the contrary, there exist several kinds of regularities which appear, whatever the size of the alphabet $\mathcal{A}$ is, in *all* sufficiently long words over $\mathcal{A}$ (cf. [5]). For this reason these regularities have been called *unavoidable*; their existence is a consequence of deep mathematical results of combinatorial or number-theoretic nature such as theorems of Ramsey and of van der Waerden. For instance, this latter theorem can be equivalently formulated in terms of words as follows:

For all positive integers d and n there exists an integer $N(d,n)$ *such that any word* w *over a d-letter alphabet of length greater than or equal to* $N(d,n)$ *has at least* *one arithmetic cadence of order n, i.e. there exists at least one letter, say a, which occurs n times in w at equal distances.*

If the word w has an arithmetic cadence of order n, then it has a factor of the kind

$$au_1au_2 \cdots au_{n-1}a$$

with $|u_1| = |u_2| = \cdots = |u_{n-1}|$. For instance, the word $w = a\underline{a}ba\underline{a}bb\underline{a}$ has an arithmetic cadence of order 3. Indeed, the letter a occurs in w in the positions 2, 5, and 8 with a distance 2 (these occurrences have been underlined).

It is possible to calculate $N(d, n)$ for small values of d and n. For instance, one has that $N(2, 3) = 9$ and $N(3, 3) = 27$ so that any word over a two-letter alphabet of length ≥ 9 has at least one arithmetic cadence of order 3 and any word over a three letter alphabet having a length ≥ 27 has at least one arithmetic cadence of order 3. We mention that unavoidable regularities have important consequences in Algebra and in Formal Language Theory (see, for instance, [5, 12]).

In the following we shall consider only structural regularities. The importance of these regularities will be illustrated by three noteworthy examples. The first is concerned with ancient philology and more precisely with the decipherment of the Cretan-Mycenaean writing called *Linear B*, the second with an important class of infinite words, called *Sturmian words*, showing surprising structural properties, and the third with *square-free words* which are words such that no two consecutive factors are equal.

The first example was intentionally chosen since the method proposed by Caianiello in his "Procuste" program for the analysis of natural languages is, for some aspects, similar to that followed by M. Ventris for the decipherment of Linear B. Moreover, as will appear clear in the subsequent sections, some ideas developed by A. Kober and Ventris, such as the analysis of inflexions in Linear B, which play a crucial role in the decipherment, can be translated in mathematical notions which are of great interest for the study of structural and combinatorial properties of abstract words.

In the analysis of the structure of a single word or of the words of a language L over a given alphabet $\mathcal{A}$ a very important notion is that of *special factor* of L. One says that a word u over the alphabet $\mathcal{A}$ is a *right special factor* of L if there exist at least two distinct letters x and y such that ux and uy are factors of L. If L is a singleton $L = \{w\}$ or coincides with the set $\mathrm{Fact}(w)$ of factors of an infinite word w, then a right special factor of L is simply called a *right special factor of* w. For instance, in the case of the word

$$w = a\underline{aab}ab\underline{aab}ba$$

the factor aab is a right special factor of w since it has two occurrences in w (the occurrences have been underlined); in the first aab is followed by the letter a and in the second by the letter b. Another right special factor of w of length 3 is aba. As one verifies w has no right special factor of length > 3, so that aab and aba are right special factors of w of maximal length.

2.1 The decipherment of Linear B

The decipherment of Linear B was one of the most important and exciting linguistic discoveries of the last century. Linear B was written on fire-hardened clay tablets found in Crete (mainly in Knossos) and later in some towns (Pylos, Mycenae, Thebes, Tiryns) of continental Greece. The Linear B script was unknown *as well as* the underlying language. The following facts about Linear B were observed:

- The inscriptions are always written from left to right.
- No bilingual inscription, such as the Rosetta stone for Egyptian, exists.
- There are some *pictograms*, i.e. ideograms, each of which represents in a pictorial form a single word (such as Man, horse, etc.), signs for numerals, and metric signs.
- The texts in Linear B are sequences of signs of length ≤ 8 which belong to an alphabet of 87 signs. The number of signs is meaningful since it is too small for an ideographic writing and too large for a purely alphabetic writing. Therefore, one can reasonably argue that it is a syllabic writing.
- 7 symbols of Linear B are identical and 3 similar to 10 symbols of the classical *Cypriot script* which was used to write Greek from at least the sixth century to the second century bc., as was discovered by some bilingual inscriptions. Each sign represents an 'open' syllable, i.e. either a plain vowel (a, e, i, o, u) or a consonant plus a vowel. Even though there are these sign similarities the Cypriot script is quite different from Linear B script.

It was a common and diffused opinion sustained very strongly by A. Evans, the famous archaeologist who discovered the tablets in Knossos, that the language underlying Linear B was not Greek. Basque and Etruscan were proposed by some authors as candidates.

The decipherment of Linear B was done by the architect M. Ventris who claimed in 1952 to have found the key to its understanding. A surprising conclusion of its analysis was the statement of the evidence that the symbols of Linear B have to represent syllables in an early version of Greek, five centuries older than classic Greek. For this reason, an essential contribution to the complete decipherment was due to J. Chadwick who was a great expert of Greek philology and Greek dialects.

Here, we shall give only some general ideas about this extraordinary decipherment, stressing the method followed by Ventris which is in many respects similar to those used in Cryptography to break a secret code; it is based on a deep analysis of the structure of linguistic material leaving little space to hypotheses and guessworks. An excellent book by Chadwick [13] presents to the general reader the marvellous story of decipherment with several technical details.

In analogy with classical Cypriot script and for other more technical reasons, one can assume that each symbol of Linear B represents an open syllable, i.e. either a plain vowel or a pair consonant plus vowel. Denoting by $\mathscr{L}$ the Linear B alphabet we may identify any element x of $\mathcal{L}$ with a pair (c, v) where c is a consonant and v a vowel, or with $(-, v)$ where v is a vowel and – denotes the absence of consonant. One can then introduce two equivalence relations $\mathcal{C}$ and $\mathcal{V}$ in $\mathcal{L}$ defined as follows:

$$(c_1, v_1)\ \mathcal{C}\ (c_2, v_2) \text{ if } c_1 = c_2\ , \quad (c_1, v_1)\ \mathcal{V}\ (c_2, v_2) \text{ if } v_1 = v_2\ ,$$

i.e. two syllabic symbols are $\mathcal{C}$-equivalent if they begin with the same consonant and are $\mathcal{V}$-equivalent if they terminate with the same vowel. In this way all syllabic symbols of Linear B, as well as of any script which uses open syllables, can be ideally disposed in a ‘grid’, or matrix, where all the elements of one same horizontal line represent $\mathcal{C}$-equivalent syllables, i.e. syllables beginning with the same consonant and all elements lying in a vertical line represent $\mathcal{V}$-equivalent syllables, i.e. syllables terminating with the same vowel or pure vowels. If one knows the grid, then one can spell correctly any word of the given language. The decipherment of Linear B was done by Ventris in the following three steps:

1. construction of the syllabic grid of Linear B ignoring for each vertical (resp., horizontal) line the phonetic value of the vowel (resp., consonant);
2. determination for each $\mathcal{V}$-class (resp., $\mathcal{C}$-class), of the corresponding vowel (resp., consonant);
3. recognition of the evidence that the language underlying Linear B was a proto-Greek.

The crucial point of the decipherment was just step (1). Ventris began to analyse the Linear B script from the syntactic point of view, i.e. without preconceived ideas on the meaning of the words or on the phonetic value of the symbols. He was interested in the occurrences in the text of words which are very similar since they have an identical common prefix, or *base*, and show some variations at the ends (often only on the last symbol). According to our terminology the base of these words is a *right special factor* of the language.

These different terminations in words having a common base were studied in 1950 in some cases by the American archeologist A. Kober who suggested that they represent *inflexions*, that is, endings of the words used to denote different grammar forms such as *cases*, *genders*, etc. The same inflected form in two different words will be represented by terminal syllables which share the same vowel, whereas two different inflexions of a given base will be represented by syllables sharing the same consonant. For instance, the inflections in declensions is well illustrated in the following example of the declensions of the nouns *dominus* (lord) and *servus* (servant) in Latin:

– do-mi-ni, ser-vi,
– do-mi-no, ser-vo,
– do-mi-na, ser-va.

The great majority of inflexions in Linear B are in the declension of nouns. For instance, Ventris found the following (here the numerals refer to the list of signs of Fig. 1):

$$11 - 02 - 10 - 04 - 10\,, \quad 11 - 02 - 10 - 04 - 42\,,$$
$$11 - 02 - 10 - 04 - 75\,.$$

Hence, the symbols 10, 42, and 75 are $\mathcal{C}$-equivalent symbols. Other inflexions are due to a change of gender. The gender of some nouns is often easily derived by the presence in the text of some pictograms representing a MAN or a WOMAN. By studying these inflexions Ventris derived that symbols 02, 12, 36, and 42 are $\mathcal{V}$-equivalent and also 60, 31, 57, and 54 are $\mathcal{V}$-equivalent even though these two $\mathcal{V}$-equivalence classes are disjoint.

Ventris was also able to determine some pure vowel signs starting from the consideration that in a syllabic script of open syllables all words beginning with a vowel have to begin with a symbol representing a pure vowel. Moreover, the pure vowel signs will occur rarely inside the word (to avoid two consecutive vowels in the word). By a statistical analysis of the frequency of each symbol at the beginning and inside the words he was able to identify some pure vowel signs like 08.

In 1951 Ventris constructed a first syllabic grid for Linear B containing 5 vowels and 15 consonants. This grid was subsequently perfected and some errors were corrected. In this way the first step for the decipherment was done. We stress that the importance of the syllabic grid was enormous. In fact, if one is able to identify by *semantic* considerations the vowel (resp., consonant) of a sign, then all the symbols lying in the same vertical (resp., horizontal) line in the grid will have the same vowel (resp., consonant).

As regards steps (2) and (3) of decipherment we shall not enter into details. Indeed, these steps are based on semantic aspects and philology arguments. Here we mention that step (2) was achieved starting with the natural assumption that names of towns like Knossos or Amnisos in Crete island survived up to the classic period probably pronounced in a very similar way. These names were easily recognized in the Linear B script found in Knossos. In this way the phonetic values of some symbols were obtained. Completion of the decipherment was described in a superb way by Chadwick in [13] as follows: “Cryptography is a science of deduction and controlled experiment; hypotheses are formed, tested and often discarded. But the residue which passes the test grows and grows until finally there comes a point when the experimenter feels solid ground beneath his feet: his hypotheses cohere, and fragments of sense emerge from their camouflage. The code ‘breaks’. Perhaps this is best defined as the point when the likely leads appear faster than they can be followed up. It is like the initiation of a chain-reaction in atomic physics; once the critical threshold is passed, the reaction propagates itself”.

2.2 Sturmian words

Sturmian words have been extensively studied for at least two centuries. They have many applications in different fields such as Algebra, Theory of Numbers, Physics

Fig. 1. Linear B signs with numerical equivalents and phonetic values

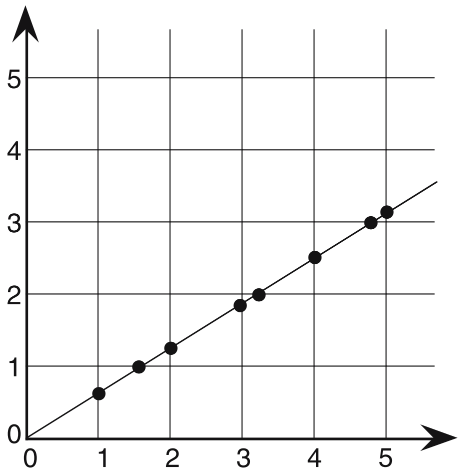

Fig. 2. The cutting sequence with slope $(\sqrt{5}-1)/2$

(Symbolic Dynamics, Quasi-crystals), and Computer Science (Computer Graphics and Pattern Matching). The study of the structure and combinatorics of these words has become a subject of the greatest interest, with a large literature on it (see, for instance, the interesting recent overviews by J. Berstel and P. Séébold in [6, Chapter 2] and J. P. Allouche and J. Shallit in [14, Chapters 9–10]).

Sturmian words can be defined in several different but equivalent ways. Some definitions are 'geometrical' and others, as we shall see in Section 3, of 'combinatorial' nature. A 'geometrical' definition is the following: a Sturmian word is an infinite word which can be generated by considering the sequence of the cuts (*cutting sequence*) in a squared-lattice made by a semi-line having a slope which is an irrational number. A horizontal cut is denoted by the letter b, a vertical cut by a and a cut with a corner by ab or ba. Sturmian words represented by a semi-line starting from the origin are usually called *standard* or *characteristic*.

The most famous standard Sturmian word is the Fibonacci word

$$f = abaababaabaababaababaabaababaabaab\cdots$$

which is obtained when the slope of the semi-line is equal to $g-1$ (see Fig. 2) where g is the *golden ratio*

$$g = \frac{\sqrt{5}+1}{2} = 1{,}61803\cdots \ .$$

This number is of great importance in several parts of Mathematics as well as in the arts. Often g is called the *divine proportion* since it symbolizes perfection in the aesthetic domain (see, for instance, [15]).

The word f can be also introduced as the limit, according to a suitable topology (cf. [5]), of the sequence of finite words $(f_n)_{n\geq 0}$, inductively defined as:

$$f_0 = b\,, \quad f_1 = a\,, \quad f_{n+1} = f_n f_{n-1}\,,$$

for all $n > 0$. The words f_n of this sequence are called the *finite Fibonacci words*. The name Fibonacci is due to the fact that for each n, $|f_n|$ is equal to the $(n+1)$th term of the Fibonacci series:

$$1, 1, 2, 3, 5, 8, 13, \ldots\,.$$

Standard Sturmian words can be defined in the following way which is a natural generalization of the definition of the Fibonacci word. Let $c_0, c_1, ..., c_n, ...$ be any sequence of natural numbers such that $c_0 \geq 0$ and $c_i > 0$ for all $i > 0$. We define, inductively, the sequence of words $(s_n)_{n\geq 0}$, where

$$s_0 = b,\ s_1 = a,\ \text{and}\ s_{n+1} = s_n^{c_{n-1}} s_{n-1},\ \text{for}\ n \geq 1.$$

The sequence $(s_n)_{n\geq 0}$ converges to a limit s which is an infinite standard Sturmian word. Any standard Sturmian word is obtained in this way. The sequence $(s_n)_{n\geq 0}$ is called the *approximating sequence* of s and $(c_0, c_1, c_2, ...)$ the *directive sequence* of s. The Fibonacci word f is the standard Sturmian word whose directive sequence is $(1, 1, ..., 1, ...)$.

Sturmian words satisfy the following two remarkable properties (cf. [6]):

- any Sturmian word s is *uniformly recurrent*, i.e. any factor of s reoccurs infinitely often in the word with bounded gaps between two consecutive occurrences,
- for any integer $n \geq 0$ any Sturmian word s has *a unique right special factor of length* n.

As we shall see with more detail in Section 3 this latter property characterizes infinite words over a binary alphabet which are Sturmian words. Moreover, in the case of a standard Sturmian word s for any $n \geq 0$ the right special factor of s of length n is the reversal of the prefix of s of length n. Thus the right special factors of the Fibonacci words are:

$$\epsilon, a, ba, aba, aaba, baaba, abaaba, babaaba, \ldots\,.$$

The set of all the words s_n, $n \geq 0$ of any standard sequence $(s_n)_{n\geq 0}$ constitutes a language *Stand* whose elements are called *finite standard Sturmian words*, or *generalized Fibonacci words*.

Now we shall analyse some remarkable and surprising properties of finite standard Sturmian words. These properties can be expressed in terms of two basic notions in the theory of words namely *periodicity* and *palindromy*.

As we have previously seen a word can have more than one period. A basic theorem due to Fine and Wilf (cf. [5]) states that *if a word w has two periods p and q and $|w| \geq p + q - \gcd(p, q)$, then w has also the period* $\gcd(p, q)$.

From this theorem one derives in particular that if a word has periods 3 and 7 and its length is 9, then it has the period 1, i.e. it is a power of a letter. This is not more true if a word has length 8, as in the case of the word *aabaabaa*.

Now let us consider the set PER of all words having two periods p and q which are coprimes and such that $|w| \geq p + q - 2$. These words have been called *central Sturmian words*, or simply *central words* in [6]. Hence, a word is central if it is either a power of a single letter or a word, such as *aabaabaa*, of maximal length for which the Fine and Wilf theorem does not apply. It has been proved in [16] that

$$Stand = \{a, b\} \cup PER\{ab, ba\} ,$$

i.e. if we consider any element of $Stand$ of length greater than 1 and cancel the last two letters, we obtain a central word. Conversely, if we concatenate on the right a central word by *ab* or *ba* we obtain an element of $Stand$. For instance, consider the word $f_6 = abaababaabaab$ of the sequence of Fibonacci words; its prefix of length 11 is the word $w = abaababaaba$ which has periods 5 and 8. Therefore, w is a central word.

Now let us consider the equation

$$W = AB = Cxy ,$$

where A, B, C are palindromes and x, y are distinct letters of $\{a, b\}$. It is interesting to determine the words which satisfy the previous equation, i.e. such that W can be factorized as the product of two palindromes and such that cancelling the last two letters one still obtains a palindrome. For instance, the word $W = abaababaabaab$ satisfies this property with $A = abaaba$, $B = baabaab$, and $C = abaababaaba$. One can prove (cf. [16]) that the previous equation with $|A| = r$ and $|B| = s$ has a solution, and this is unique, if and only if $\gcd(r + 2, s - 2) = 1$. If one denotes by Σ the set of all solutions of the above equation, then one can prove [16] that

$$PER\{ab, ba\} = \Sigma .$$

From this one derives that central words are palindromes and that any element of $Stand$ can be factorized into the product of two palindromes. For instance,

$$f_6 = abaababaabaab = (abaababaaba)ab = (abaaba)(baabaab),$$

where the words in brackets are palindromes. Finally, one can prove [16] that the set of central words is equal to the set of the palindromic prefixes of all standard Sturmian words.

2.3 Square-free words

A regularity in the words which is interesting to analyse is the presence or absence of squares, i.e. the presence or absence in a word of two equal and consecutive blocks of letters. For instance, the word *abccabcab* contains two squares namely *cc* and *cabcab*. Let us first consider a binary alphabet $\mathcal{A} = \{a, b\}$. One easily realizes that one cannot construct a word of length > 3 without producing a square. Indeed, if we start with the letter a, then to avoid squares the letter which follows a has to be b. Therefore, we have the square-free word ab. The next letter has to be a, otherwise one produces the square bb. Thus we have the square-free word aba. Now whatever the next letter is we produce a square. In fact, both $abaa$ and $abab$ have a square. Hence, the only square-free words over a two-letter alphabet are the six words

$$a, b, ab, ba, aba, bab .$$

All the other words on the alphabet $\{a, b\}$ contain a square. One may think that this regularity is unavoidable, i.e. it is satisfied by all sufficiently long words whatever the size of the alphabet is. However, this is false. Indeed, one can prove that over a three letter alphabet $\mathcal{A} = \{a, b, c\}$ there exist infinitely many square-free words. Now we shall give a procedure which yields arbitrarily long square-free words over the alphabet $\mathcal{A}$. Consider the map $\phi : \mathcal{A}^* \to \mathcal{A}^*$ (endomorphism of $\mathcal{A}^*$) defined by

$$\phi(\epsilon) = \epsilon \ , \quad \phi(a) = abc \ , \quad \phi(b) = ac \ , \quad \phi(c) = b \ ,$$

and for any word w of length $n > 1$, $w = a_1 \cdots a_n$, $a_i \in \mathcal{A}$, $i = 1, \ldots , n$, one sets

$$\phi(w) = \phi(a_1) \cdots \phi(a_n) \ .$$

We can construct an infinite sequence of words in the following way: one starts with the word a which is square-free. If one applies the map ϕ to the letter a one obtains the word $\phi(a) = abc$ which is square-free. If one applies again ϕ to abc, one produces the word

$$\phi^2(a) = \phi(\phi(a)) = abcacb \ ,$$

which is square-free. By applying again ϕ one obtains

$$\phi^3(a) = abcacbabcbac \ ,$$

which is also square-free. One can prove (cf. [5]) that the word that one constructs at each step is square-free. Moreover, since any of these words is a prefix of the next one, it follows that this sequence of square-free words converges to the infinite square-free word

$$m = abcacbabcbacabcacbacabcb \cdots$$

The word m is uniformly recurrent; moreover, for any $n > 0$, m has *either* 2 *or* 4 *right special factors* [17]. The word m, usually called the *Thue-Morse word* on three symbols, is of interest in various parts of Mathematics as well as in Physics. We mention that the Thue-Morse word played an essential role in Algebra for giving a negative answer to the famous Burnside problem for groups and semigroups, i.e. whether any finitely generated and periodic group (or semigroup) is finite (cf. [5, 12]).

3 Combinatorics of words

The study of the structural properties of words is usually called *algebraic combinatorics* on words, whereas the analysis of the properties which are related to suitable *counting* or *enumeration functions* definable on a word is called *enumerative combinatorics* on words. As we shall see the enumerative properties can strongly condition the structure of a word and *vice versa*.

For any word w over a given alphabet one can introduce a function λ_w defined in the set $\mathbb{N}$ of natural numbers and taking values in $\mathbb{N}$, as follows: for each natural number n, $\lambda_w(n)$ counts the number of distinct factors of w of length n. For instance, in the case of $w = abbab$ one has $\lambda_w(0) = 1$ and,

$$\lambda_w(1) = 2\ , \quad \lambda_w(2) = \lambda_w(3) = 3\ , \quad \lambda_w(4) = 2\ , \quad \lambda_w(5) = 1\ ,$$
$$\lambda_w(n) = 0 \quad \text{for} \quad n > 5\ .$$

The function λ_w is usually called *subword complexity* of w. In the case of a finite word w the *complexity index* $c(w)$ of w is defined as the total number of distinct factors of w, i.e.,

$$c(w) = \sum_{n=0}^{|w|} \lambda_w(n)\ .$$

Let us now show how imposing some constraints on the subword complexity of a word can condition its structure. We shall refer to the case of infinite words. One can easily prove that an infinite word w is eventually periodic if and only if there exists an integer c such that for all n

$$\lambda_w(n) < c\ .$$

Another example is given by Sturmian words. Indeed, it is possible to show [6] that an infinite word w is a Sturmian word if and only if

$$\lambda_w(n) = n + 1\ .$$

It has been proved by Morse and Hedlund (cf. [14]) that if w is an infinite word such that there exists an integer n for which $\lambda_w(n) \le n$, then the word is eventually periodic. Hence, Sturmian words can be defined as *the class of all infinite words having for each n the minimal possible value for the subword complexity without being eventually periodic*.

The preceding examples show that if one imposes strong limits on the values of subword complexity of an infinite word, then these words have a quite simple structure. However, the converse is not in general true. In fact, one can construct an infinite word having a simple structure and such that for any n its subword complexity takes its maximal value. This is, for instance, the case of the *Champernowne sequence* over a two-letter alphabet $\mathcal{A} = \{0, 1\}$

$$011011100101110111 1000 \cdots$$

which is very simply constructed by concatenating successively all binary expansions of all positive integers. In such a case for any n the subword complexity takes the value 2^n.

An important enumeration function which can be associated with a word w is the map ρ_w defined in $\mathbb{N}$ as: for any $n \geq 0$, $\rho_w(n)$ gives the number of right special factors of w of length n. Functions ρ_w and λ_w are related; in the case of an infinite word w in a two-letter alphabet one easily derives that the following relation holds: for all $n \geq 0$

$$\lambda_w(n+1) = \lambda_w(n) + \rho_w(n) .$$

Since $\lambda_w(0) = 1$, one obtains that for all $n \geq 0$

$$\lambda_w(n) = 1 + \sum_{n=0}^{n-1} \rho_w(n) .$$

From this it follows that *the infinite word w is eventually periodic if and only if there exists an integer n_0 such that $\rho_w(n) = 0$ for $n \geq n_0$*. Moreover, it is not difficult to show that *the infinite word w is Sturmian if and only if for any $n \geq 0$ there exists a unique right special factor of w of length n*.

Another natural function which can be introduced for any word w counts the number $\| u \|_w$ of occurrences of any word u in w. For instance, in the case of the word $w = abbab$, one has $\| a \|_w = 2$, $\| b \|_w = 3$, $\| aa \|_w = 0$, $\| ab \|_w = 2$, $\| ba \|_w = \| bb \|_w = 1, \ldots$.

A remarkable class of finite words w can be defined by imposing some constraints on the number of occurrences of any word u in w. More precisely, a word w is called *uniform* if for any n the difference of the number of occurrences in w of any two words of length n is at most 1, i.e. for all $n > 0$ and words u and v of length n one has:

$$| \; \| u \|_w - \| v \|_w \; | \leq 1 .$$

For instance, the word $w = aaababbba$ is a uniform word in a two-letter alphabet.

Uniform words were introduced in [18, 19] and several characterizations of them have been done in terms of some *entropy-like* functionals. The structure of uniform words is quite complex, so that, for instance, their number is known exactly for infinitely many values of their length but not for all. However, it is possible to prove that the number of uniform words of length N goes to infinity when N diverges; moreover, a procedure to construct for any N a uniform word of length N has been given.

Let us observe that a (pseudo)-random finite word is a word which has no 'simple' structural regularities. Several ways for defining the randomness of a finite word have been proposed (cf. [20]). However, in any case a random finite word satisfies the following properties: (1) all 'short' words occur as factors; (2) 'long' factors are unrepeated; (3) words of the same length occur approximately the same number of times. Now from the definition one derives that uniform words satisfy 'at best' the previous conditions. This explains why uniform words have a complicated structure.

4 Complexity of words

As we have seen in the previous sections there are finite and infinite words which have some strong structural regularities so that they are quite simple to describe. On the contrary, there are words such as uniform words or DNA sequences which look very similar to random strings, so that they have a very complicated structure. Natural problems that arise concern the meaning of the 'complexity' of a word and how to measure it. A related problem is what exactly does it mean that a finite word is 'random'. Even though in some cases it is quite simple to recognize that some words are 'more complex' or 'more random' than others, up to now there is no satisfactory or unique answer to the preceding problems.

The subword complexity λ_w of a word w, or the complexity index of w in the case of a finite word, are not good measures of the complexity since, as we have previously seen, there exist words which have the maximal possible value for the subword complexity but are very simple to construct.

As is well known a theory of complexity of words was introduced independently by A. N. Kolmogorov and G. J. Chaitin (cf. [21]). This complexity, called *program complexity*, is measured by the minimal length of a program of a (universal) Turing machine able to compute the given word. However this measure is defined up to an additive constant depending on the Turing machine considered. Therefore, even though very important from the conceptual point of view, the Kolmogorov complexity is not actually utilizable in practice to measure the complexity of finite words; in fact in this case it is meaningful to consider only the quotient of the value of the complexity of a word by its length when the length diverges.

In this section we shall not discuss the preceding problems concerning the complexity and randomness of a finite word (by 'random word' we shall simply mean any word which *appears* to be random even though it can be deterministically produced by a suitable computer program). We shall limit ourselves to consider first biological sequences and pose the problem of recognizing the 'structure' in these very complex words. Subsequently, we shall introduce some ideas and results about a new and interesting notion of *structural complexity (information)* of a finite word.

4.1 Biological words

As is well known the nucleic acids DNA and RNA can be regarded as words on a 4-letter alphabet $\mathcal{A} = \{A, C, G, T\}$, (in RNA the letter T is replaced by U) whereas proteins are words on a 20 letter alphabet, each letter corresponding to a different *aminoacid*. A further biological alphabet is the set $\mathcal{B}$ of all 64 triplets on the alphabet $\{A, C, G, U\}$ which is very important in the genetic coding mechanism. We shall refer to sequences over these biological alphabets also as *biological words*.

The study of biological words can be done according to different approaches. We are mainly interested in the 'syntactic-linguistic' point of view which consists, independently from a semantic-functional analysis of the sequence, in the study of the properties of the sequence itself or in the study of the language of its subwords.

We shall refer to DNA (RNA) macromolecules. With each of these macromolecules one can uniquely associate a quite large word on a four letter alphabet. This word has to contain a very large amount of 'information' (*genetic information*), so that it must have a very complex and sophisticated 'structure'. A DNA sequence at first sight seems to be a random sequence, since there are no evident structural regularities. As stressed by J. Monod: "Biological macromolecules have been designed through molecular evolution, for the performance of highly specific, unique functions. The apparent randomness of these highly ordered sequential structures illustrates and measures the wealth of information, i.e. the precision, with which these various specific functions have had to be defined for optimal performance".

A natural problem that arises is the following. *Is it possible to recognize the 'structure' of a DNA-macromolecule (or of suitable parts of it such as the coding parts of 'genes') only by an analysis of the language of its subwords?*

We stress that the term 'structure' is used in the abstract syntactic-linguistic sense and not in the semantic-functional one. A related problem is the following. Let us compare a gene of length N with a *random word* of length N on the alphabet $\{A, T, C, G\}$ or with a word which is obtained by a random permutation of the letters of the original sequence *(shuffled random word). Does there exist a difference between the structure of the language of the subwords of the given gene and the structure of the language of the subwords of the random (shuffled random) word?*

A positive answer to this problem was given in [22] by making a comparison between the distribution function of right special factors of a gene w and the distribution of right special factors of a sequence $\hat{w}$ on the alphabet $\{A, T, C, G\}$ having the same length and generated by a random source or obtained by making a random permutation (shuffling) of the letters of the original sequence. More precisely, if ρ_w and $\rho_{\hat{w}}$ are the distribution functions of right special factors of w and $\hat{w}$, then in all considered cases, we observed that there exists an integer n_0 (depending on w and $\hat{w}$) such that

$$\rho_w(n) \leq \rho_{\hat{w}}(n) \quad \text{for} \quad n \leq n_0 \ ,$$

and

$$\rho_w(n) \geq \rho_{\hat{w}}(n) \quad \text{for} \quad n \geq n_0$$

(see Table 1).

This kind of difference in the distributions is irrespective of the length and of the phylogenetic origin of the fragments that we have analysed. Moreover, a surprising and unexpected result was that the different behaviour observed for what concerns special factors, between biological and random (random shuffled) sequences seems to be unchanged if one passes from "exonic" (i.e. coding) regions of a gene to larger portions containing "intronic" (i.e., non-coding) parts. It is also worth noting the fact that the complexity index (i.e., the total number of distinct subwords) of the words passing from the biological to the random or to the random shuffled word has only a very small percentage variation.

Table 1. The distribution over the length n of the number of right special factors in the gene ECOPDB (column 2) of length 2198 compared with the corresponding distributions in a random shuffled version of it (column 3) and two random sequences of the same length (columns 4–5)

Length	Native ECOPDB	Reshuffled ECOPDB	Random 1	Random 2
1	4	4	4	4
2	16	16	16	16
3	64	64	64	64
4	240	254	255	256
5	488	565	574	547
6	356	344	311	332
7	160	105	117	112
8	56	21	32	26
9	16	5	9	10
10	6	1	3	4
11	1	0	0	0
12	0			
	1407	1379	1385	1371

4.2 Complexity and information

In general, when one studies a given class of objects one tries to analyse each complex object in terms of simpler components of the same type. For instance, in group theory one analyses the groups in terms of subgroups or by ‘simple groups’ which are the basic components of groups. Therefore, one can ask the following general question: *what information does one have about a complex structure from the knowledge of smaller substructures?* For instance, what information has one about a word, a tree or a graph by knowing a certain set of subwords, subtrees or subgraphs?

Moreover, an important related question is what is the minimal ‘size’ of these substructures capable of determining uniquely the structure itself? We shall refer to this kind of information as ‘structural information’.

Let us observe that this approach is similar to the Kolmogorov approach. The main difference concerns the set of ‘data’. In fact, in this structural approach the ‘data’ about the given structure belong to a class of a certain kind, namely substructures. In the case of a word the substructures are subwords. This kind of problem is of great interest in some applications such as the problem of ‘sequencing’ and ‘assembling’ DNA macromolecules or the transmission of a long message on several and possibly different channels of communication (cf. [23]).

We shall now give some general ideas about this structural information (see [24, 25]) limiting ourselves to the case of ‘words’ even though this approach can be followed for several combinatorial structures such as *trees* or *two-dimensional arrays* (see [26–28]).

In this combinatorial approach an essential role is played by the notions of *extendable* and *special factors* of a given word. A factor u of a word w over a given alphabet $\mathcal{A}$ is called *right extendable* if there exists a letter $x \in A$ such that ux is a factor of w.

As we have seen in Section 2 a factor u of w is called *right special* if there exist two distinct letters x and y such that ux and uy are factors of w. In a similar way one can define *left extendable* and *left special* factors of w. A factor of w is called *bispecial* if it is right and left special. A factor of w of the kind asb, with a and b letters and s bispecial, is called a *proper box*. A proper box is called *maximal* if it is not a factor of another proper box.

The shortest prefix (resp., suffix) of w which is not left (resp., right) extendable in w is denoted by h_w (resp., k_w) and is called the *initial* (resp., *terminal*) *box*. We set $H_w = |h_w|$ and $K_w = |k_w|$. Moreover, we shall denote by R_w (resp., L_w) the minimal natural number such that there is no right (resp., left) special factor of w of length R_w (resp., L_w).

Let us give the following example. Let w be the word $w = abccbabcab$. One has: $h_w = abcc$ and $k_w = cab$. Thus, $H_w = 4$ and $K_w = 3$. Moreover, the set of right special factors is $\{\epsilon, b, c, bc, abc\}$. The set of left special factors is $\{\epsilon, a, b, c, ab\}$. Hence, $R_w = 4$ and $L_w = 3$. The maximal proper boxes are:

$$abc, cba, bcc, ccb, bca.$$

A basic theorem, called *maximal box theorem*, proved in [24] shows that *any word is uniquely determined by the initial box, the terminal box, and the set of maximal proper boxes.*

Moreover, there exist simple algorithms in order to find the initial, the terminal, and the maximal proper boxes of a given word and, conversely, to reconstruct a word from the knowledge of its boxes.

The parameters K_w, H_w, R_w, and L_w, which have been called *characteristic parameters* of w, give much information on the structure of the word w. For instance, the maximal length G_w of a repeated factor of a non-empty word w is given by

$$G_w = \max\{R_w, K_w\} - 1 = \max\{L_w, H_w\} - 1$$

and the minimal period of a word w is not smaller than $\max\{R_w, L_w\} + 1$. As a consequence of the maximal box theorem, one derives the following important result:

Any word w is uniquely determined by the set of its factors up to length $G_w + 2$.

The value $n = G_w + 2$ is *optimal*. Indeed, one can prove [24] that for any word w there exists a word $u \neq w$ which has the same set of factors of w up to length $n - 1$.

An evaluation of the distribution of the characteristic parameters in the set of all words of length n over a d-letter alphabet $\mathcal{A}$, $d > 1$ is given in [29–31]. In particular, G_w satisfies the following noteworthy properties:

– For any word w of length n

$$\lfloor \log_d n \rfloor - 1 \leq G_w \,.$$

– For *almost all* words of length n

$$G_w \leq \lceil 2\log_d n \rceil + \log_d(\log_d n) \ .$$

– The *average value* $\langle G_w \rangle_n$ of G_w over all words of length n has the following logarithmic upper bound

$$\langle G_w \rangle_n \leq \lceil 2\log_d n \rceil - 1/2 \ .$$

Hence, if one takes at random a word w of length n, in the overwhelming majority of cases w is uniquely determined by its factors of a much shorter length (of the order $O(\log n)$).

The above theorems and results have several applications. For instance, some remarkable extensions of the notion of *periodic word* have been given in [32–34]. We shall refer here only to the problem of *sequence assembly* which is one of the most important problems in molecular biology. Indeed, DNA macromolecules are biological words whose length can vary from $\approx 10^6$ to $\approx 10^9$. Current technologies do not allow one to read a very long sequence; for instance, *gel electrophoresis* permits one to determine directly *fragments* of DNA sequences of length ≈ 500. Therefore, the main task consists in reconstructing the entire sequence having only access to short fragments of it. Moreover, a further difficulty is due to the fact that we do not know the exact position of a given fragment in the entire sequence. The problem of sequence assembly can be formulated as follows: given a target (unknown) word w, try to reconstruct it from the knowledge of some suitable factors of w (also called *fragments* or *reads*) $s_1, s_2, \ldots, s_l$.

There exists a large literature on DNA sequencing and assembling. We refer to [35] and references therein. We give here some general ideas on an approach to this problem purely based on the previous mathematical results. The first problem consists in making reasonable assumptions on the target word w and on the set of reads $\{s_1, \ldots, s_l\}$ in order to achieve all the information that uniquely determines the word w. A second problem is to find efficient algorithms that on inputs $s_1, s_2, \ldots, s_l$ reconstruct the word. We assume that:

a) the value of G_w, i.e., of the maximal length of a repeated factor of w, is small,

b) there is an integer $k > G_w$ such that the set $\{s_1, \ldots, s_l\}$ is a *covering* of the factors of length $k+1$ of w, i.e., any factor of w of length $k+1$ occurs in some s_i, $i = 1, \ldots, l$.

Assumption (a) is reasonable, since many micro-organisms such as *prokaryotes* (bacteria) and lower *eukariotes* have DNA sequences without long repeated factors. For instance, in the case of prokaryotes, the maximal length of a repeated factor is ≤ 15. Assumption b) can be achieved taking the reads coming from many random fragmentations of the word w.

As we have previosly seen, the knowledge of the set of factors of w up to $k+1$ is sufficient to reconstruct the word. An efficient algorithm which allows one to reconstruct w with a time complexity of the order $O(kn)$ is described in [23].

In conclusion, we remark that assumptions (*a*) and (*b*) are quite strong if we consider the real situations. In fact, the sequence assembly is much more complicated for superior eukaryotes, whose genes may contain large repeated factors. Moreover, in real situations one has to consider the presence of sequencing errors (1 to 5 % of the sequence), so that the previous results can be considered just as an essential step for a further mathematical research.

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Quantum Field Theory and Renormalization Theory in the Early Scientific Activity of Eduardo R. Caianiello

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Abstract. In this description of the scientific activity of Eduardo R. Caianiello during the 1950s, we point out some specific aspects of his contribution to quantum field theory and renormalization theory. Our main emphasis will be on the characterization of features that, due to the deep cultural basis of the whole activity, appear to be of relevance to present day research.

1 Introduction

On the occasion of this meeting titled "Immaginazione e rigore: Riflessioni sull'opera e sull'eredità scientifica di Eduardo R. Caianiello, a dieci anni dalla sua scomparsa" ("Imagination and Rigor: Reflections on the scientific work and heritage of Eduardo R. Caianiello, ten years after his departure"), we have found it worthwhile to attempt a reconstruction of his early scientific activitity during the 1950s. There are many interesting aspects of this activity, both from a cultural and strictly scientific point of view, that can be a source of inspiration for present day research.

While our main interest will be in quantum field theory and renormalization theory, it is important to put into proper perspective also contributions to elementary particle physics, accelerator physics, and modelization of brain functions.

This paper is organized as follows. In Section 2 we recall some essential biographical information, in particular about the places of activity. Moreover we will attempt a periodization of the early scientific activity. Section 3 is dedicated to activities in quantum field theory and renormalization theory. In Section 4 we try to reconstruct the beginnings of interest in modelling brain functions, finally leading to the celebrated Caianiello neural network equations and their further developments. In Section 5 we give a description of an unpublished manuscript by Eduardo R. Caianiello and Steven Weinberg, about an attempt towards a neutrino theory of light. Finally, Section 6 is dedicated to some conclusions and the outlook for future research.

It is a pleasure to thank the organizing committee, and in particular Settimo Termini, for the invitation to talk in this meeting, in an athmosphere so rich in human, cultural, and scientific content.

2 Some biographical data. Phases of the early research period

The main sources of biographical data and the scientific activity of E.R. Caianiello, together with a fairly complete list of publications, are given in a book by Maria Marinaro and Gaetano Scarpetta [1], and in a long article by Luigi Maria Ricciardi [2]. These two works, besides being excellent sources of information, convey also, from different points of view, a deep feeling of appreciation and understanding of human aspects, particularly important in this case. Therefore, we highly recommend these two sources.

Here, we will recall some basic information related to the period under consideration, by resorting also to unpublished archive documents.

Eduardo Renato Caianiello was born in Naples on June 25th, 1921. He attended High School at the Classical Lyceum Jacopo Sannazzaro, and enrolled in 1938 in the Course for the "Laurea" degree in Physics at the University of Naples. After two years he was drafted into the Army, and resumed his studies only at the end of 1943, after a very intense three year period of military involvement, including training at the Military Officer School in Pavia, participation in the war in North Africa as a volunteer, earning a War Cross, and returning to Italy with severe wounds in 1943. He earned the doctoral degree "Laurea in Fisica" on December 14th, 1944, discussing a research thesis titled "Una verifica sperimentale della teoria di Debye sulla dispersione anomala nei liquidi dipolari" ("Experimental verification of Debye theory on the anomalous dispersion in dipolar liquids"), under the supervision of Professor Antonio Carrelli. Then he became Assistant to the Chair of Rational Mechanics. In June 1948 he left for Boston, as winner of a fellowship of the MIT. Then he was invited to Rochester by Robert E. Marshak, where he earned a PhD degree in Theoretical Physics in 1950, and was Assistant Professor for the academic year 1950–51, teaching two courses on "Mathematical Methods in Physics". In the general archive of the University of Rochester we find the record of the title of his PhD thesis "Investigation of the decay and absorption of mesons. Part I: Beta-decay and the possible electron-decay of the [pi]-meson. Part II: On the spin of the [mu]-meson", under the supervision of R. Marshak.

Intending to come back to Italy, he was offered positions at the Universities of Rome and Turin. First he opted for Turin (1951–52), invited by Gleb Wataghin, then from 1952 to 1955 he was Assistant to the Chair of Theoretical Physics, held by Bruno Ferretti, in Rome. Very important were some long periods of scientific activity abroad, in particular at the CERN in Copenhagen (on leave of absence from Rome), and in Princeton. In 1955 he was winner of a national competition for a Chair of Full Professor in Theoretical Physics (issued by the University of Catania), by ranking first in the winning "triplet", followed by Marcello Cini and Fausto Fumi. Then, he joined the Faculty of Mathematical, Physical and Natural Sciences at the University of Naples as Chair of Theoretical Physics, which was established for Ettore Majorana in 1938, and occupied for short periods after the War by Ezio Clementel and Luigi Radicati di Brozolo. His academic duties should have started on February 1st 1956, but he obtained a delay, until November 1956, in order to complete his fruitful stay in Princeton.

Before leaving for the USA, his scientific activity is documented in three published papers on the integration of complete systems of first order linear equations with partial derivatives [3], on the impulsive motion of a holonomous system with constraints [4], and finally on the Luxembourg effect [5]. Notice that in the paper [5] the affiliation is indicated as “Istituto di Fisica Tecnica dell’Università di Napoli”. This very early scientific activity shows a deep cultural background in Mathematics, Mathematical Physics, and General Physics. This background can be recognized as being present in all his subsequent work.

In the Amaldi Archives at the Department of Physics of the University of Rome “La Sapienza”, we have located a copy of a very important letter written on July 4, 1951 by Robert E. Marshak, and addressed to Gilberto Bernardini (Box 140, Folder 1/3). In the Appendix we offer the complete transcription of the letter. This letter contains a very detailed appreciation of the work done in Rochester by “Dr. Eduardo Caianiello”, on the meson theory and beta decay, on the spin of the μ meson, on the absorption of negative π mesons by tritium, and on the Universal Fermi Interaction.

Let us point out some relevant aspects of the letter. First of all there is a reference to the return to Italy of scientists after prolonged activity in the USA: the well established mature Gilberto Bernardini, the first President of the INFN (Italian National Institute for Nuclear Physics) founded in 1951, and the young very brilliant and further promising Eduardo Caianiello. Marshak makes a lucid comment about the leaving of Bernardini. He says “too bad for Columbia and wonderful for Italy”. Also the return of Caianiello to Italy is seen as very convenient for Italy. After half a century, by looking back in time we can see how many very talented young people were exported to the USA and abroad, and how difficult, or even impossible, it was to get some of them back in the prevailing university and research climate.

The comment of Marshak on the initial raining of the young Caianiello is also interesting. In particular he says “Dr. Caianiello came to Rochester in the fall of 1948 after having received a PhD from Naples (he can inform you about his Italian training). His knowledge of quantum mechanics and modern physics was nil and within the space of two years he learned enough to do a PhD in modern theoretical physics (on “Meson Theory and Beta Decay” and “Spin of the μ Meson”).” Surely, the situation of young people enrolled in Physics in Naples would have been completely different if Ettore Majorana had continued to give his lectures. In any case, the general cultural background of the young Caianiello was enough to enable him to immediately at the highest competitive level research in modern theoretical physics.

Besides PhD thesis, scientific activity carried out at Rochester is documented in a series of six papers. Paper [6], reports a study of the possibility of re-establishing Yukawa’s scheme of β-decay by assuming the decay to occur through the creation of a virtual meson, which subsequently gives rise to an electron–neutrino pair. The conclusion is reached that only a virtual vector meson can fit the Yukawa scheme. The paper [7] is devoted to a detailed analysis of the π^--meson reactions in tritium. This work was done in collaboration with A.M.L. Messiah, then at Rochester, and S. Basri of Columbia University. In [8] the consequences of the assumption of a spin $3/2$ for the μ meson are investigated. The conclusion is that this value of spin can be excluded when the nuclear matrix elements and the nuclear excitation energy are known with greater accuracy. Finally, the papers [9–11] represent his contributions to the search for the universal Fermi-type interaction. This series is completed by [12], sent for publication when Caianiello was already appointed in Rome, but on leave at the CERN in Copenhagen. In the letter by Marshak we find an appreciation of the work on universal Fermi-type interaction, put in proper historical perspective.

The period of activity in Turin includes the papers [13], [14], and [15]. This last paper, in collaboration with Sergio Fubini and dedicated to the evaluation of traces of products of Dirac matrices, is particularly important, because it is the beginning of the shift of interest from elementary particle physics to relativistic quantum field theory, and the first example of the exploitation of combinatorial methods, a very peculiar feature of all future activity in quantum field theory.

We reproduce here also a very interesting letter to Gian Carlo Wick (Fig. 1), concerning a preprint sent by him. The letter is dated in clear handwritten notation "PROCIDA, 18-8-'42", but, by considering the duties developed in 1942, clearly it should be corrected to "1952", as is also evident from the reference to the address in Rome, starting from next October 1st. The letter gives a very vivid picture of the personality of the young Caianiello, and his critical attitude toward scientific research. We give also a free English translation in the Appendix. The paper referred to here is probably the celebrated WWW paper on superselection rules [16], sent for publication in *Physical Review* on June 16th 1952.

The activity of the Rome period, in any case on leave at CERN, includes also studies on strong focusing accelerators, as reported in [17, 18], which extend a previous CERN internal report CERN/T/ERC/2. Note the acknowledgement to Niels Bohr in the first paper.

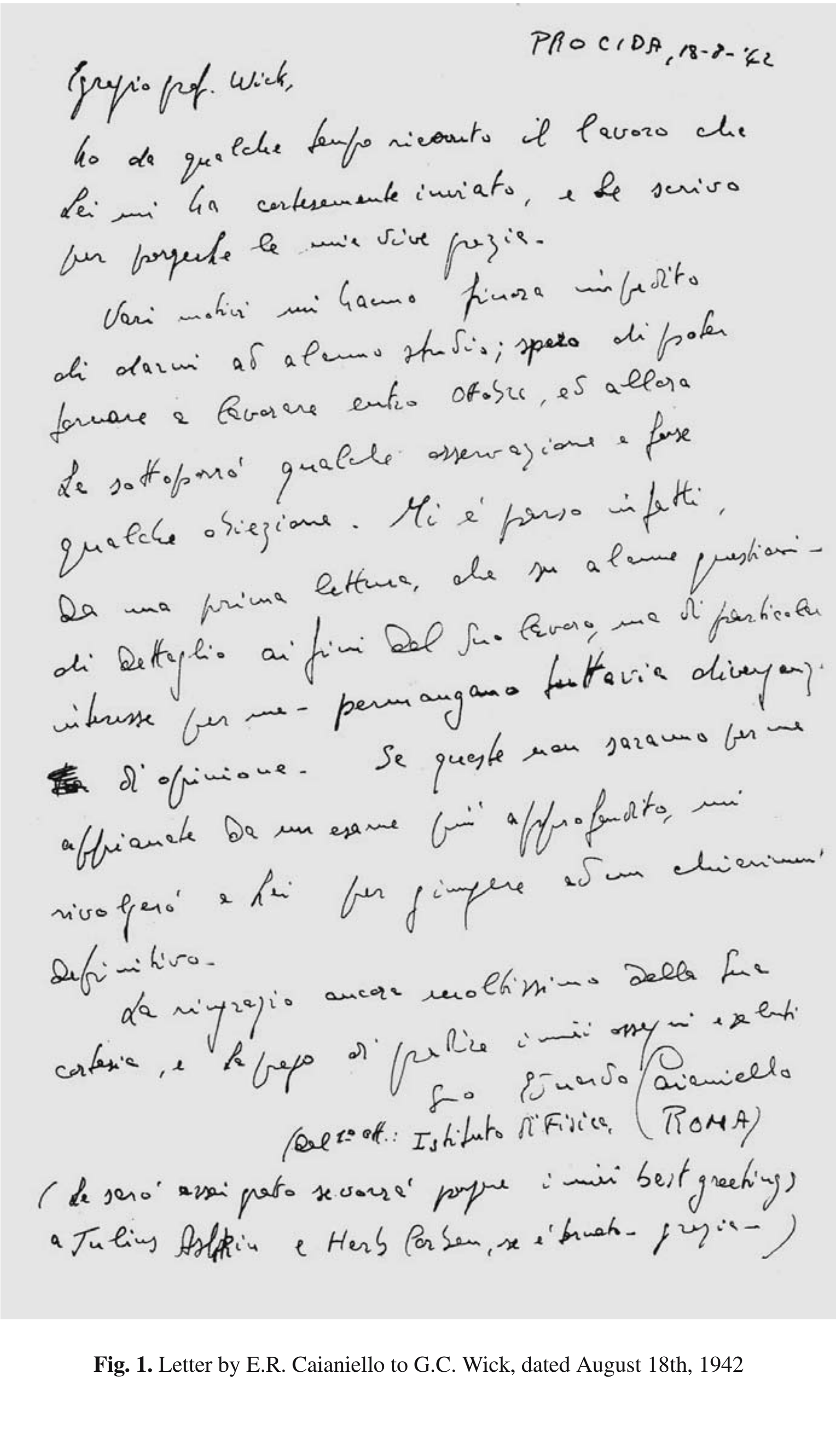
Starting in 1953, Caianiello sent for publication a long series of papers devoted to a detailed explanation of his view on quantum field theory and renormalization theory, which includes, for the period under consideration, around thirty papers over the period 1953–1965.

His interest in implementation of models for brain functions starts with papers [19] and [20], which show very clearly that the modelization was intended in the spirit of theoretical physics, and culminates with the famous article in the *Journal of Theoretical Biology* [21], where his views reach a definitive stage.

3 The early activity in quantum field theory

As we said before, the early activity in quantum field theory includes around thirty papers in the period 1953–1965. We refer to the initial papers [22, 23], and to the extensive reviews in [24–26], for a complete description of the basic methods and results. The last reference [26] is also important because it shows the initial formation of a qualified research group in Naples. We mention also that the book [27] gives an overview of all activity developed to the beginning of the 1970s.

This is not the proper place for a detailed technical description of the relevance and the scientific content of this activity. However, due to the very important ideas and methods put forward, we would like to summarise some of the main aspects, with particular emphasis on ideas and methods which appear of long lasting value, with relevance to present day research.



PROCIDA, 18-8-'42

Egregio prof. Wick,

ho da qualche tempo ricevuto il lavoro che Lei mi ha cortesemente inviato, e Le scrivo per porgerle le mie vive grazie.

Vari motivi mi hanno finora impedito di darmi ad alcuno studio; spero di poter tornare a lavorare entro ottobre, ed allora Le sottoporrò qualche osservazione e forse qualche obiezione. Mi è parso infatti, da una prima lettura, che su alcune questioni – di dettaglio ai fini del Suo lavoro, ma di particolare interesse per me – permangano tuttavia divergenze di opinione. Se queste non saranno per me appianate da un esame più approfondito, mi rivolgerò a Lei per giungere ad un chiarimento definitivo.

La ringrazio ancora moltissimo della Sua cortesia, e La prego di gradire i miei ossequi e saluti

Suo Eduardo Caianiello

(dal 1° ott.: Istituto di Fisica (ROMA)

(Le sarò assai grato se vorrà porgere i miei best greetings a Julius Ashkin e Herb Corben, se è [illegible] – grazie –)

Fig. 1. Letter by E.R. Caianiello to G.C. Wick, dated August 18th, 1942

Regarding the general formulation of quantum field theory, the whole physical content of a particular model is expressed in terms of "propagators" connected with correlation functions of the quantum fields at different points in space-time. Both bosonic fields and fermionic fields are allowed. They correspond to different combinatorial rules, associated with the respective statistics. It is clear that the aim is to develop a detailed local field description in space-time, and not only an asymptotic theory, allowing one to develop the cross-sections of different processes, as in the S-matrix theory.

Propagators satisfy recursive equations, or branching equations, encoding the dynamical aspects of the field space-time evolution, and the dependence of the propagators on the coupling constants and the bare masses of the fields involved in the interaction. All perturbative expansions, as in Feynman graph theory, can be obtained from iteration of the recursive equations, through appropriate combinatorial methods, typically involving determinants, permanents, Pfaffians and Hafnians, constructed with the propagators of the theory, according to the relevant statistics. The "Hafnian" denomination was coined by Caianiello to mark the fruitful period of stay in Copenhagen (Hafnia in Latin). However, the recursive equations have also an independent meaning, outside of the perturbative expansions. In particular, they can be put at the basis of systematic nonperturbative approximations.

In this dynamical-combinatorial approach, regularization and renormalization acquire a peculiar simple canonical form. It is very well known that coupled quantum field theory is affected by ultraviolet divergences. In fact, during iteration of the equations, the difficulty arises of performing integrations without a well defined meaning. These difficulties show up at large integration momenta (this is the reason for the "ultraviolet" denomination), or, in the space-time framework exploited here, at short relative distances in the field propagators. By taking inspiration from similar phenomena appearing in the theory of partial differential equations, Caianiello had the very brilliant idea of introducing a new kind of integral, called the finite part integral, which would make all expressions finite.

A peculiar aspect of these methods is that multiple integrals can be defined recursively from iteration of the simple integral on one space-time relative variable. The general procedure is of course very simple and elegant. By their very definition, finite part integrals are not uniquely defined; there is an intrinsic arbitrariness. However, one combinatorial methods allow one immediately to prove, for the so called renormalizable theories, that the arbitrariness in the definition of finite part integrals can be completely absorbed in the *a priori* arbitrary constants appearing in the theory (coupling constants, bare masses, and overall field rescaling). Therefore, the proof of the renormalizability of the theory is reached in a purely combinatorial frame, far simpler than other renormalizability schemes.

The invariance of the procedure gives rise to a peculiar structure, called renormalization group, which, also in different formulations, has been the basis of the most important developments in quantum field theory in recent times.

Reading again, after so many years, the papers referred to, a vivid impression of freshness and up to date evergreen flavor comes immediately to mind. We hope to dedicate future work to a detailed presentation, in modern form, of the basic ideas contained in the Caianiello approach to regularization and renormalization in relativistic quantum field theory.

4 The early stage of brain function modelling

As far as modelling of the brain functions is concerned, Caianiello is internationally known mostly for his authoritative paper [21], and further developments. Since these topics are largely covered by other contributions to this volume, we find it convenient to concentrate on the early papers and his attempts to find the starting points. In [19], with the very ambitious title "La riproduzione meccanica del pensiero" ("The mechanical reproduction of thinking functions"), there is a synthetic, nevertheless conceptually very deep, description of the ideas forming the basis of relevant activities carried out at the Institute of Theoretical Physics in Naples during the period 1957–58.

First of all there is a very lucid assessment of the possibility of giving a useful description of the activity of a neuron in its interaction with surrounding neutrons, *without* possessing complete knowledge of its behavior. In fact, the model for the single neuron is based on a simple device, able to receive impulses through afferent organs, the dendrites, and consequently, make a simple decision. If the sum of the impulses received in some short time interval is higher than some threshold, then the neuron decides to send its own impulse, which is transmitted through an efferent organ, and only efferent, the axon and its diramations, to the other surrounding neurons.

The other relevant aspect is the crucial recognition that the brain cortex functions, as a coordinating and thinking organ, are dispersed uniformly over all its extension, and not confined to some definite regions. Therefore, the behavior of the cortex is characterized by some redundancy, so that malfunctioning, or suppression, of even a large number of elements does not prevent correct behavior of the whole system.

Moreover, it is recognized that there are various levels of memory, characterized by different timescales, and that systems of this type, made by a large number of interacting simple modular systems, allow for the possibility of categorization, due to the plasticity of behaviour with respect to different timescales.

The description of the phenomenon of categorization is masterful, and particularly fresh, even after fifty years. Since the paper is in Italian, we find it useful to freely translate this part. We warn the reader that, due to the well known peculiarities of the Caianiello writing style, our translation is meant to give only an idea of the description, and by no means can be understood as a valid substitution to reading the original.

> Therefore, let us think about a system made by a large number of elements of this kind [neurons with afferent and efferent organs]. Surely it is not difficult to construct such systems, at least on a small scale, with the present day technological possibilities. Couplings will be characterized by a sensible nonlinearity. We have already written the equations for some ideal model, which we believe is already near enough to the truth (we call them the "equations of the mad

man"). These equations are very difficult to solve by analytical methods, but they can be easily imitated by analogical models. Let us assume that some input, or afferent part, and some output, or efferent part, is established in an organism, or model of this kind. The organism will begin to give answers to the ingoing stimuli in a way completely arbitrary, and uncoordinated, at the beginning. Then, time after time, it will establish preferred paths for those stimuli received more frequently. In particular, for example, if it receives two thousand images, among which one hundred represent always the same triangle, and the others are completely random, after some time it will begin to recognize the image of the triangle, because it is repeated more often than any other random image. If images of different triangles are offered to the analysis of the system, after that the system became able to recognize each of them individually, then its memory will acquire, because of the nonlinearity, a more abstract notion. This notion is what is related to each triangle in that it is a triangle, and not a particular one. Therefore, we have the emergence of the Platonic idea of triangle, if we like so, or equivalently of the mathematical notion of triangle, as an element defined by three vertices, or three sides. And so on.

We can see the dynamical nature of the proposed model, inside a solid frame of modellization according to standard theoretical physics. Moreover, the adiabatic learning hypothesis, very important in future developments, is already firmly established.

Of course, we do not know the explicit form of the "equations of the mad man". However, because it says that they can easily be imitated by analogical models, we infer that they were different from the final Caianiello equations, based on sharp step functions, and may be nearer to the equations resulting from paper [20].

The paper [20] is very important, not only for its scientific content, but also because the four signatures give evidence of the constitution of a valid research group on this subject. It is also relevant that the affiliation of the four authors is denoted as "Scuola di Perfezionamento in Fisica Teorica e Nucleare, Sezione di Cibernetica – Napoli".

The model proposed in [20] is particularly fascinating, and very modern and up to date. The model is presented as promising to share with the living nervous system the properties of *economy*, capacity to recognize *similarity*, and the character of *wholeness* (Gestalt). Here the system is made up of an array of active elements, each represented as a self-coupled nonlinear oscillator, capable of sustained oscillations of prescribed waveform. Each oscillator is coupled with other oscillators in the array by a variable coupling. Moreover, *one* way of implementing the model is to ensure that different *inputs* to this organ are represented by changes in the coupling constants at different sets of points in the array, while different *outputs* are provided by global features of the system, as for example the different frequency spectra of the total oscillation of the array, which can be determined by means of frequency analysis.

It is immediately recognized that this model, due to the arbitrariness in the couplings, and to their different effects on the behavior of each different oscillator, falls in the cathegory of complex dynamical systems, as, for example, spin glasses and neural networks. Due to the work of Giorgio Parisi and others, see for example [28], it is very well known that this class of systems are capable of hierarchical organization and behavior. Therefore, the Braitenberg–Caianiello–Lauria–Onesto model shares a remarkable aspect of modernity, compared to the most advanced research topics in complex dynamical systems. Surely, it would be worth retaking into account this model to investigate its further possibilities on the basis of recent developments in this field.

5 An unpublished manuscript by Caianiello and Weinberg

Recently, I located a manuscript by E.R. Caianiello and S. Weinberg concerning the possibility of describing electromagnetic phenomena through a Fermi interaction. The manuscript was kept for more than thirty years in a remote section of my personal archive, after it was handed to me personally by Eduardo at the beginning of my collaboration with him. The typewritten text has formulae inserted by hand in Caianiello's handwriting. The manuscript is incomplete with endnotes and references, and of course it has never been published.

We find it convenient to mention it, because it gives evidence of the persistent interest of Caianiello in elementary particle physics, in a period when he was moving more towards quantum field theory. In fact, it is very simple to date the manuscript. It refers to the years 1955–56, when Eduardo Caianiello was "Higgins Visiting Professor" at Princeton University, teaching a course on Advanced Topics in Quantum Theory, while Steven Weinberg was a doctoral student, at the beginning of his fantastic career.

We reproduce here the first page. We hope to dedicate some future work to a complete description of the paper in its proper historical perspective concerning the developments of theoretical physics.

The scheme of the paper is very simple. It belongs to a research line, the so called neutrino theory of light, initially developed by Louis De Broglie in [29] and Pascual Jordan in [30], and pushed further by M.H.C. Pryce in [31].

In the paper it is shown that the assumption of a Fermi interaction between electrons and neutrinos leads to results very similar to the results obtained in standard quantum electrodynamics, provided a proper selection of graphs is performed, at least for scattering processes involving electron external lines only. The main difference between the two theories is the fact that the standard photon propagator of quantum electrodynamics is replaced by a new effective propagator resulting from the iteration of neutrino loops. The form of this effective propagator is explicitly evaluated. Of course, due to the four-Fermion interaction, this propagator exhibits a very strong singularity at short distances. This singularity is brilliantly exploited to compensate for the smallness of the coupling constant, through the introduction of a suitable space cutoff, involving a reasonable smallest distance of the order 10^{-16} cm, which is about the magnitude of the classical nucleon radius.

We would like also to remark that the attempt pursued in the paper points toward a kind of unification of the fundamental interactions, in particular the electromagnetic and the weak one. Of course, as is very well known, the development of theoretical physics reached this unification in a completely different way, by keeping the photons and introducing vector mesons, able to transmit the weak force.

We find it also important to recall the following comment by Edoardo Amaldi in [32], about the recurrence of the neutrino theory of light in the published literature: "The idea, although open to much criticism, has been taken up again at later times but never did reach a satisfactory formulation". It would be interesting to see whether the publication of this manuscript would have changed this sharp statement.

6 Conclusions

When reading documents about the early scientific activity of Eduardo Caianiello, one is surprised not only by the deep thinking and originality of ideas, but especially by the extraordinary freshness and evergreen modernity of his approach. There is no doubt that these documents are of value also for future research. We strongly suggest that young people at the start of their careers become fully acquainted with them.

Moreover, they contain in their very essence the source of all further developments, and they permit one to trace the deep cultural and intellectual motives forming the basis of all his subsequent interventions also at the organizational level.

For this reason, *and for many others*, we all say

Grazie, Eduardo!

Acknowledgements

We gratefully acknowledge useful conversations with Bruno Preziosi, Nadia Robotti, Giovanni Battimelli and Matteo Leone, and their help in locating information and documents.

This work was supported in part by MIUR (Italian Minister of Instruction, University and Research), and by INFN (Italian National Institute for Nuclear Physics).

We thank the Department of Physics of the University of Rome "La Sapienza", in particular the Director Guido Martinelli, for permission to publish the transcription of a copy of the letter by R.E. Marshak to G. Bernardini, dated July 4, 1951, and kept in the Amaldi Archives.

We aknowledge also the kindness of Mrs Vanna Wick for permission to reproduce the letter by E.R. Caianiello to G.C. Wick, dated 18-8-'42, and kept in the Gian Carlo Wick Archives at the Library of the Scuola Normale Superiore in Pisa.

Finally, we thank the Family, and in particular Silvia Caianiello, for providing the picture in Fig. 2.

Fig. 2. The young Eduardo on the Atlantic route

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Appendix

This is the transcription of a handwritten letter by R.E. Marshak to G. Bernardini, dated July 4th, 1951. A photographic copy of the letter is kept in the Amaldi Archives at the Department of Physics of the University of Rome "La Sapienza" (Box 140, Folder 1/3).

[start of the letter]

THE UNIVERSITY OF ROCHESTER
COLLEGE OF ARTS AND SCIENCE
ROCHESTER 3, NEW YORK

DEPARTMENT OF PHYSICS July 4th, 1951

Dear Bernardini,

John Tinlot tells me that you are returning to Italy at the end of the summer – all I can say is too bad for Columbia and wonderful for Italy. I hope to see you before you leave the U.S. and to have a nice chat about all the experiments you have been doing during the past few months. However that may be, I should like to tell you something about the bearer of this letter, Dr. Eduardo Caianiello, just in case you find possible to add an American-trained theoretical physicist to your staff in Italy.

Dr. Caianiello came to Rochester in the fall of 1948 after having received a Ph.D. from Naples (he can inform you about his Italian training). His knowledge of quantum mechanics and modern physics was nil and within the space of two years he learned enough to do a Ph.D. in modern theoretical physics (on "Meson Theory and Beta Decay" and "Spin of the μ Meson"). During the past year, he has been an Assistant Professor in the Mathematics Department at the University of Rochester, doing a brilliant and outstanding job teaching the two courses on "Mathematical Methods in Physics". At the same time, he has been carrying on research on the "Absorption of π^- mesons by Tritium" and on "The Universal Fermi Interaction".

(P.T.O.)

[end of page]

GUERRA

NAPOLI

Can Electromagnetic Phenomena be Described by a Fermi Interaction?

by

E. R. Caianiello* and
S. Weinberg
Palmer Physical Laboratory
Princeton University

Abstract

It is shown that the assumption of a Fermi interaction between electrons and neutrinos will lead to results (for scattering processes involving electron external lines only) quite similar to the results obtained in electrodynamics. The major difference between the two theories is that the photon propagator $D^F(X)$ is replaced by another function of X^2 more singular on the light cone. It seems likely that some prescription could be found which would convert this extra singularity into powers of a length chosen small enough to compensate the weakness of the Fermi coupling.

We wish to suggest that a neutrino theory of electrodynamics, free from the inconsistencies of earlier attempts, (1) might be constructed by adopting the view that all interactions among charged particles are due to virtual neutrinos, rather than virtual photons; the smallness of the coupling constant, which we take to be the usual one of beta-decay processes, being compensated by the highly singular character of the interaction. It will be evident that these results can be extended with minor formal changes to the study of bosons other than photons.

We consider only processes involving external electron (or positron) lines, and assume as fundamental interaction: (2)

Fig. 3. The first page of the unpublished manuscript Caianiello–Weinberg

It is the last research, which has already led to two papers to be published in Il Nuovo Cimento, which has shown Caianiello to possess a creative, original and imaginative mind of a very high order. Taking the lead from Yang and Tiomno's paper, which considered the problem of setting up a universal Fermi interaction among proton, neutron, μ meson, electron, neutrino by using the condition of invariance under space reflection, Caianiello has generalized the problem by including invariance under time inversion. He has shown how to allow the processes occurring in nature and to exclude those which do not, at the same time being led to a fairly unique interaction which agrees beautifully with the electron spectrum from the μ meson and, as far as I can see, with the β decay experiments. I have been very much impressed by his latest achievement (you ought to have him tell you something about it) which has revealed his familiarity with the modern and exciting physical problems, his tremendous knowledge of mathematics and his adeptness with mathematical techniques and the rapidity with which he carries out a calculation (as an illustration of this, I asked him a few days ago to calculate the lifetime of the V_0 particle assuming that it decays in accordance with the scheme $V_0 \to P + \pi^-(virtual\ state) \to P + \mu^- + \nu$ where V_0 may possess the spin $3/2$ or $1/2$ – he did the two calculations in *one* day, spectra and all*). His latest paper is very persuasive and I personally am beginning to believe that the universal Fermi interaction probably exists: Caianiello still has

[footnote] * P.S. This theory does not work!

[end of page]

[Rochester letterhead as before]

some important problems to solve before the story is complete but somehow I feel confident that he stands a very good chance of success.

Fig. 4. Mathematics in Naples in the 1940s (on the floor: Eduardo R. Caianiello, Donato Greco; standing *in first row:* Renato Vinciguerra, Vacca, Renato Caccioppoli, Giuseppe Scorza Dragoni, Antonio Colucci, unidentified; *second row:* Guido Stampacchia, Federico Cafiero, unidentified, Carlo Miranda, Paone, Eugenio Moreno, Carlo Ciliberto, unidentified)

To sum up, if you can possibly use an excellent theoretical physicist in Rome, I would urge you to consider Dr. Caianiello very seriously. He is very ambitious and hardworking and wants to try his hand at some of the important problems of modern theoretical physics. If he stays in contact with a good experimental group, this will guarantee that his energies are channeled into fruitful directions. And if he does not lose contact with the realities of present-day physics (a tendency which is always prevalent among mathematically gifted theorists), he may go very far indeed.

I understand that Dr. Caianiello has contacts with Wataghin regarding a position in Turin but I believe he is still undecided. I might add that Dr. Caianiello could have stayed in the U.S. but was compelled to return to Italy for personal reasons.

Cordial regards.

Sincerely,
R.E. Marshak

[end of the letter]

Now we give a free translation of a letter of E.R. Caianiello to G.C. Wick dated 18-8-'42 (sic!). The letter is kept in the Gian Carlo Wick Archives at the Library of the Scuola Normale Superiore in Pisa. The original, in Italian, is reproduced as Fig. 1, at the end of this paper.

[beginning of the letter]

PROCIDA, August 18th, 1942

Dear prof. Wick,
some time ago I have received the paper which you kindly sent me, and I am now writing you in order to give you my thanks.

Various reasons have prevented me from being involved in any study. I hope to be able to resume work by October. Then I will submit you some remarks, and maybe some objections. In fact, it seems to me, at a first reading, that on some questions – of marginal value to the purpose of your work, but of particular interest for me – do persist some divergences of opinion. If these divergences will not be superseded through a more detailed examination, I will get in touch with you in order to arrive at a definitive clarification.

I thank you again very much for your kindness, and send you my best regards and wishes

Yours Eduardo Caianiello
(from October 1st: Institute of Physics, Rome)

(I will be most grateful to you if you are willing to give my best greetings to Julius Ashkin and Herb Corben, if he has been back – thanks –)

[end of the letter]

The Renormalization Group from Bogoliubov to Wilson

Maria Marinaro

1 Foreword

When Eduardo R. Caianiello came back to Naples in 1956 he continued his research in Quantum Field Theory (QFT) in collaboration with some young collaborators: B. Preziosi, A. Campolattaro, G. Scarpetta, F. Guerra and myself.

For some years our interest was concentrated on the problem of regularization and renormalization in QFT, thus to remember that time of our life I have written this paper which, I hope, may be of interest to all those people who, not being experts in the field, wish to know something about this subject.

2 Introduction

In all the fields of science we have to analyze macroscopic phenomena emerging from complex microscopic behaviours. In simple cases the average of microscopic fluctuations allows one to solve the problem, which, at the end, will be described by the averaged quantities, macroscopic variables, satisfying classical continuum equations.

But in more complex systems the fluctuations persist at macroscopic wavelengths, and fluctuations on all the intermediate scale lengths are important. These kinds of problems are very difficult to solve since they involve a large number of degrees of freedom and depend on many scale lengths.

A strategy for dealing with these problems is the Renormalization Group (RG), which uses the symmetries of the solutions of the dynamic equations under scale variation.

From a mathematical point of view the RG in QFT is a group of transformations depending on a continuum real parameter $\mathbf{L}$, the length of the scale. Variation in $\mathbf{L}$ induces a rescaling of the quantities (operators, parameters) which describe the system, when we know the dependence of these quantities on $\mathbf{L}$ we can use it to improve our knowledge of the system. The information obtained in this way is sometimes very relevant and allows solution of the problem, at least in some limits; more usually it is used to improve solutions found making use of approximation methods. It is worth stressing that the dependence on $\mathbf{L}$ is strongly related to the dynamics of the system at microscopic level and therefore (excluding some trivial cases) it does not correspond to the standard dimensional rescaling; the appearance of an anomalous dimension (see next section) is a clear manifestation of the interplay between symmetry and dynamics.

2.1 Renormalization group in quantum electrodynamics

The RG was introduced quite independently by Stückelbery and Peterman [1] and Gell-Man and Low [2] in their works on the perturbative expansion of Quantum Electrodynamics (QED). The year after (1955) Bogoliubov and Shirkov [3] took a relevant step in this subject, establishing the close relations between the above mentioned papers, clarifying the specific symmetry underlying the Group and showing the applicability of RG to improve approximate solutions of QED found perturbatively.

To start from the beginning, it is worth remembering that in the late 1940s physicists faced a difficult problem, the presence in the perturbative expansion of QED of ultraviolet divergences (UD) due to integrations over momentum $\vec{K}$ that diverge in the limit $\left|\vec{K}\right| \to \infty$ (small interaction distance). Nevertheless, many researchers showed that consistent results could be obtained using the following "renormalization procedure":

1. The theory is regularized by introducing a cut-off on $\vec{K}\left(\left|\vec{K}\right| \leq \Lambda\right)$ that eliminates the infinite contribution at large momentum.
2. The arbitrariness introduced by the regularization (a non-justified dependence of the physical results on the value of the cut-off) is eliminated by a suitable rescaling (renormalization) of the parameters and of the electron and electromagnetic fields appearing in the Lagrangian.

Specifically it was shown that the rescaled quantities completely absorb the dependence on the cut-off. Thus all the physical results expressed in terms of these quantities, whose values are fixed by experiment, become independent of the cut-off and free of arbitrariness. The possibility to obtain for the class of QFT, known as renormalizable theories, physical results free of arbitrariness was confirmed by the work of E.R. Caianiello and his co-workers [4]. In this work the regularization of the QFT is obtained in a completely different way, making use of a suitable redefinition of integration.

The above mentioned success is due to the symmetry property exhibited by the QED solutions: these QED solutions are symmetric under the continuum 1-parameter transformation group called RG. In the simplest case (the massless QED, whose solutions are expressed as functions of only two variables, the 4-momentum Q and the electric charge e) the RG operates as follows:

$$R_{\mu^2}\begin{cases} Q^2 & \to R_{\mu^2}\left[Q^2\right] & = \frac{Q^2}{\mu^2} \\ e & \to R_{\mu^2}[e] & = g\left(\mu^2, e\right) \end{cases} \tag{2.1}$$

where μ^2 is the running parameter fixing the 4-momentum scale, $g\left(\mu^2, e\right)$, the renormalized coupling constant, is a function of μ^2 and e determined by the dynamics, it satisfies the group composition law:

$$\begin{cases} g(s, z) = g\left(\frac{s}{\mu^2}, g(\mu^2, z)\right) \\ g(1, z) = z \end{cases} \tag{2.2}$$

The QED solutions are irreducible representations of RG, this symmetry leads to relevant consequences. Two cases are reported: First we consider a quantity $F(Q^2, e)$ invariant under RG:

$$F\left(Q^2,_e\right) = F\left(\frac{Q^2}{\mu^2},_g\left(\mu^2, e\right)\right) \tag{2.3}$$

Equation (2.3) expresses the independence from the scale parameter μ of F:

$$\frac{dF}{d\mu} = 0 \tag{2.4}$$

It puts on a mathematically sound base the renormalization procedure introduced previously, justifying the independence of the QED solutions from the cut off.

In terms of partial derivatives (2.4) is the well-known Callan Symanzik equation

$$\left(x\frac{\partial}{\partial_x} - \beta(g)\frac{\partial}{\partial g}\right) F(x, g) = 0 \tag{2.5a}$$

where $x \equiv \dfrac{Q^2}{\mu^2}$, $g = g\left(\mu^2, e\right)$,

$$\beta(g) = \mu^2 \frac{\partial g\left(\mu^2, e\right)}{\partial \mu^2} \tag{2.5b}$$

The function $\beta(g)$, which expresses the dependence of the renormalized coupling constant on the scale parameter, is fixed by the dynamics of the theory. Equation (2.5) was derived in 1956 by L.V. Ovsyannikov [5] but only at the beginning of the 1970s it become popular and known as the Callan–Symanzik equation [6, 7].

As a second case we consider quantities $\Phi(Q^2, e)$ transforming as a linear representation of RG:

$$\Phi\left(Q^2, e\right) = \Phi\left(\mu^2, e\right)\Phi\left(\frac{Q^2}{\mu^2}, g\left(\mu^2, e\right)\right) \tag{2.6}$$

where $\Phi(1, e) = 1$.

Equation (2.6) can be rewritten in terms of partial derivatives

$$\left(x\frac{\partial}{\partial x} - \beta(g)\frac{\partial}{\partial g} - \gamma(g)\right) \Phi(x, g) = 0 \tag{2.7}$$

where

$$g = g\left(\mu^2, e\right), \gamma(g) = \frac{\partial \Phi\left(\mu^2, e\right)}{\partial \mu^2} \frac{\mu^2}{\Phi\left(\mu^2, e\right)}$$

The quantity: $\gamma \equiv \gamma(g)\big|_{\mu^2=1}$ is the so called *anomalous dimension* of Φ, which is determined, as β, by the dynamics.

2.2 Effective electric charge

The physical meaning of the symmetry properties of QED solutions under RG is obtained by looking at the renormalized coupling constant, the effective electric charge, which depends on the scale parameter, the 4-momentum μ^2. What is the physical meaning of this dependence?

A classical analogy can give some hints on this phenomenon. It is well known that in a polarizable medium the electrical field measured at distance d from a charge Q is smaller than the one given by the Coulomb law in vacuum and that the deviation from the Coulomb law increases with d. Classically, the behavior is explained by observing that there is screening of the charge Q due to the polarization of the medium. The screening increases with the distance between the charge and the point at which the field is measured. Only at $d = 0$ does the screening vanish and the charge take its bare value. To describe an electric field $E(d)$ in a polarizable medium, we can once again use the Coulomb law but we have to substitute the vacuum bare Q with an effective charge $Q(d)$, whose dependence on d is determined by the characteristic of the medium. Explicitly the electric field $E(d)$, at distance d from a charge is given by

$$E(d) = \frac{Q(d)}{d^2} \tag{2.8}$$

where

$$Q(d) \underset{d \to 0}{\longrightarrow} Q \tag{2.9}$$

In QED the role of polarizable medium is played by the quantum vacuum. Quantum vacuum is not physically empty, vacuum polarization and fluctuations are present due to the existence of virtual states and virtual transitions. The polarization of the vacuum produces screening of the electrical charge; the screening is a pure quantum effect and, as a consequence of the complementary principle, depends on the parameter μ^2 the photon 4-momentum used for charge measurement.

In conclusion, the electric charge cannot be fixed once and for all, it is a running parameter whose value depends on the specific experimental conditions. This interpretation becomes very relevant in the study of the unified theories of strong–weak and electromagnetic interactions. At the energy that can be reached in the laboratories, till now, these interactions are very different but RG equations drive the strong and the electroweak couplings to approach each other, making unification possible.

2.3 Application of RG in QFT

Until now we have shown the role played by RG in putting the renormalization procedure used in QED on a sound basis, but the RG has also been used in QFT computations. The first application was performed by Bogoliubov and Shirkov [8] to improve approximate solutions obtained by perturbative expansions. The idea is to combine the approximate solutions with the group symmetry. One computes the group generators $\beta(g)$ and $\gamma(g)$, by using the perturbative approach and then uses them in the differential equations (2.5) and (2.7) to obtain improved solutions which satisfy the group symmetry.

More recently the RG has played a relevant role in the Quantum Cromo Dynamics (QCD), the current theory of quarks and nuclear forces. In this context we report two interesting behaviors of the effective coupling constant as a function of $\frac{Q^2}{\mu^2}$.

The first one occurs when $\beta(g)$ vanishes at $g = g^*$; the second when $\beta(g)$ vanishes to $g = 0$. Keeping in mind equation (2.5b) it is immediate to see that in the first case the effective coupling constant converges to the ultraviolet (UV) fixed point g^* (see Fig. A), while in the second case the coupling constant vanish for $\frac{Q^2}{\mu^2} \to \infty$ and the "so called" *asymptotic freedom* is realized (see Fig. B).

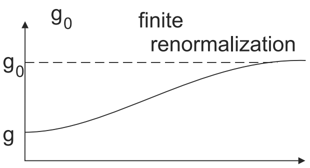

Fig.A

$$\varkappa = \ell n \frac{Q^2}{\mu^2}$$

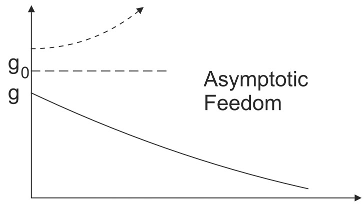

Fig.B

$$\varkappa$$

In the asymptotic freedom regime the interaction between the elementary particles vanishes, the particles are free but have anomalous dimensions. The existence of anomalous dimensions evidences that RG transformations do not correspond to a standard dimensional rescaling; the rescaling, determined by the group generators $\beta(g)$ and $\gamma(g)$, is fixed by the dynamics of the system under consideration.

Politzer [9], Gross and Wilekzev [10], have shown that the non-Abelian gauge theories are asymptotically free, the coupling constant is an increasing function of the distance between the interacting particles that vanish as the distance goes towards zero (infinite energy). This theoretical result is confirmed by measurements: weak coupling to short distance, which drives the deep-inelastic electron scattering; and strong coupling to large distance (nuclear distances), which binds quarks to form protons, neutrons, pions …

2.4 Renormalization group in statistical mechanics

In statistical mechanics (MS) the RG has been largely applied to study complex physical problems. It was used for the first time by Kadanoff and Wilson to explain the physical behavior of systems in the critical region.

Critical regions are characterized by infinite correlation lengths therefore fluctuations of the elementary constitutes of the system of any length scale have to be considered, so the analysis of the system involves infinite degrees of freedom.

To deal with this hard problem L. Kadanoff [11] introduced the so called "blocking procedure". Kadanoff's seminal idea, inspired by the scaling properties of thermodynamic quantities and by the existence of universality classes (independence of the critical behavior from the details of the interaction), was that the critical region could be described as a limit point of a suitable sequence of an effective Hamiltonians.

The sequence of Hamiltonians is constructed by the block procedure, which operates in two steps:

1. decimation: reduction of degrees of freedom;
2. rescaling: the spatial coordinates are changed according to $x \to x' = \frac{x}{b}$

Kadanoff applied the block procedure to the Ising Model on a two-dimensional lattice of spacing a described by the Hamiltonian

$$H(\sigma) = K \sum_{\langle ij \rangle} \sigma_i \sigma_j + h \sum_i \sigma_i, \tag{2.10}$$

where σ_j is the spin operator at lattice site j, $K = \frac{J}{K_B T}$ and h are respectively the adimensional coupling constant and the external magnetic field. $\langle ij \rangle$ means summation on the nearest neighbor sites. In this model, decimation is realized by dividing the lattice up into disjoint blocks of dimension $L \times L (L = ba)$. Each block contains b^2 spin; to the ith block, centered at the point i, is associate an effective spin S_i. The rescaling is obtained by changing the spatial coordinates according to relation $x \to x' = \frac{x}{b}$.

The new lattice of effective spin S_i so obtained looks like the original lattice of spin σ_i.

Kadanoff assumed that the effective Hamiltonian, which describes the block system for large b, is identical to the original one except for a renormalization of the bare parameters and operators, i.e.

$$H_b = K_b \sum_{\langle ij \rangle} S_i S_j + h_b \sum_i S_i \tag{2.11}$$

where K_b and h_b are respectively the renormalized coupling constant and magnetic field depending on the bare parameters and on b while the renormalized operator S_i is the magnetic moment of the averaged block B_i of b^2 spin σ_i:

$$S_i \cong b^{-2} \sum_{i \in B_i} \sigma_i$$

Unfortunately Kadanoff does not succeed in finding the correct transformation between the bare and the renormalized quantities, and does not provide real hints as to the origin of the universality.

The explanation of the critical phenomena universality was found later in 1971 by K. Wilson [12], who implemented Kadanoff's scaling procedure in a concrete computational way.

He showed that the assumption made by Kadanoff, that after decimation and rescaling the effective Hamiltonian can be approximated by a new Hamiltonian identical to the original one except for a rescaling of the parameter, is generally false. The form of the Hamiltonian changes as the scale is changed; at each application of the scaling procedure, new terms appear in the Hamiltonian and new effective parameters have to be added.

So, starting with a two-term Hamiltonian as in (2.11) after m applications of the scaling procedure, an N^m-term reduced Hamiltonian is generated:

$$H^{(m)} = \sum_{n=1}^{N^{(m)}} K_n^{(m)} 0_1^{(m)} \ldots 0_n^{(m)} \tag{2.12}$$

where $N^{(m)}$ generally increases with m and becomes very large. In (2.12) $K_n^{(m)}$ are adimensional parameters, called reduced parameters and $0_1^{(m)} \ldots 0_n^{(m)}$ are renormalized operators. The new set of parameters, $\left\{K_{(n)}^{m}\right\}$ are functions of b and of the previous set $\left\{K_n^{m-1}\right\}$.

A useful picture of Wilson's results is obtained by introducing an N dimensional space $\mathfrak{H}_N$, the space of the reduced Hamiltonians, whose coordinates are the Hamiltonian reduced parameters. Wilson established the following correspondence between the Hamiltonians and the poins in $\mathfrak{H}_N$ space

$$H^{(m)} \to p^{(m)} \equiv \left(K_1^{(m)} \ldots K_{N^{(m)}}^{(m)}\right) \tag{2.13}$$

and described the procedure of rescaling through "trajectories" in $\mathfrak{H}_N$.

In fact, successive applications of the scaling procedure to the original Hamiltonian $H^{(0)}$, which describes a specific physical model, generate the sequence of Hamiltonians $H^{(0)}, H^{(1)} \ldots H^{(m)} \ldots$ whose representative in $\mathfrak{H}_N$ are succession of points $p^{(0)}, p^{(1)} \ldots p^{(m)} \ldots$ i.e. a trajectory in $\mathfrak{H}_N$.

Sets of trajectories are generated by applying the scaling procedure to distinct original Hamiltonians $H_1^{(0)} \ldots H_s^{(0)}$.

Formally the Wilson scaling procedure at the mth step is expressed by the relation

$$H^{(m)} = R_b H^{(m-1)} \tag{2.14}$$

where R_b is a one-parameter operator depending on the scaling parameter b.

The operators R_b satisfy the group composition law

$$R_b R_{b'} = R_{bb'} \tag{2.15}$$

The inverse operation of R_b is not defined thus the Wilson procedure generates a discrete semi-group which induces the following transformation of the parameters appearing in the Hamiltonians: $x \to \frac{x}{b}$, $\left\{K_n^{(m-1)}\right\} \to \left\{K_n^{(m)}\right\}$. The parameters K satisfy the composition law

$$\begin{cases} \overline{K}(x,z) & = \overline{K}\left(\frac{x}{t}, \overline{K}(t,z)\right) \\ \overline{K}(1z) & = \overline{K} \end{cases} \tag{2.16}$$

$$\text{where} \begin{cases} x & = \frac{1}{bb'} \\ t & = \frac{1}{b} \end{cases} \tag{2.17}$$

Equation (2.16) is known as the Kadanoff–Wilson Renormalized Group (KWRG); it looks similar to the RG in QFT (see Equation 2.2). Two main differences distinguish the KWRG from the RG in a QFT:

1. The KWRG is discrete.
2. The KWRG is a semi-group.

2.5 Fixed point and universality

One of the most relevant results obtained by the KWRG is the explanation of universality classes: distinct physical systems having the same critical behavior. The explanation is obtained by putting into correspondence the fixed points of the RG transformation with the critical regions of physic systems. To be specific, let us call H^* the reduced Hamiltonian corresponding to a fixed point: $R_b H^* = H^*$.

H^* describes a physical system which is not affected by the change of scale, i.e the system has lost any characteristic length ℓ. Two types of fixed points exist: the ultraviolet (UV) fixed points which are reached for $\ell \to 0$, and the infrared (IR) fixed points, which are obtained for $\ell \to \infty$.

In the Wilson approach the critical regions, characterized by an infinite correlation length, are described by (IR) fixed points.

It is well known that for $d \geq 4$ the fixed points are trivial, while for dimension $d < 4$, non-trivial fixed points exist. According to Wilson to each fixed point is associated a specific critical behavior.

In this scheme the existence of universality classes is very natural; physical systems described by distinct Hamiltonians exhibit the *same critical behavior* when their Hamiltonians belong to the domain of attraction of the *same fixed point*.

An intuitive picture of this result is obtained looking at the flux generated in the space $\mathcal{H}_N$ by successive applications of RG. Between all the trajectories generated in $\mathcal{H}_N$ there are some, called critical trajectories, which converge at fixed points. The critical trajectories can be separated into sets, each set containing all the trajectories which converge to *the same fixed point*, and describes distinct physical systems which have the *same critical behaviour.*

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Caianiello's Maximal Acceleration. Recent Developments

Giorgio Papini

Abstract. A quantum mechanical upper limit on the value of particle accelerations is consistent with the behavior of a class of superconductors and well known particle decay rates. It also sets limits on the mass of the Higgs boson and affects the stability of compact stars. In particular, type-I superconductors in static conditions offer an example of a dynamics in which acceleration has an upper limit.

1 Introduction

In 1984 Caianiello gave a direct proof that, under appropriate conditions to be discussed below, Heisenberg uncertainty relations place an upper limit $\mathcal{A}_m = 2mc^3/\hbar$ on the value that the acceleration can take along a particle worldline [1]. This limit, referred to as maximal acceleration (MA), is determined by the particle's mass itself. With some modifications [2], Caianiello's argument is the following.

If two observables $\hat{f}$ and $\hat{g}$ obey the commutation relation

$$[\hat{f}, \hat{g}] = -i\hbar\hat{\alpha} \ , \tag{1}$$

where $\hat{\alpha}$ is a Hermitian operator, then their uncertainties

$$\begin{aligned}(\Delta f)^2 &= \langle \Phi \big| (\hat{f} - \langle \hat{f} \rangle)^2 \big| \Phi \rangle \\ (\Delta g)^2 &= \langle \Phi \big| (\hat{g} - \langle \hat{g} \rangle)^2 \big| \Phi \rangle\end{aligned} \tag{2}$$

also satisfy the inequality

$$(\Delta f)^2 \cdot (\Delta g)^2 \geq \frac{\hbar^2}{4} \langle \Phi \mid \hat{\alpha} \mid \Phi \rangle^2 \ , \tag{3}$$

or

$$\Delta f \cdot \Delta g \geq \frac{\hbar}{2} \mid \langle \Phi \mid \hat{\alpha} \mid \Phi \rangle \mid \ . \tag{4}$$

Using Dirac's analogy between the classical Poisson bracket $\{f, g\}$ and the quantum commutator [3]

$$\{f, g\} \rightarrow \frac{1}{i\hbar}[\hat{f}, \hat{g}] \ , \tag{5}$$

one can take $\hat{\alpha} = \{f, g\}\hat{\mathbf{1}}$. With this substitution (1) then yields the usual momentum–position commutation relations. If, in particular, $\hat{f} = \hat{H}$, then (1) becomes

$$[\hat{H}, \hat{g}] = -i\hbar\{H, g\}\hat{\mathbf{1}} \ , \tag{6}$$

(4) gives [3]

$$\Delta E \cdot \Delta g \geq \frac{\hbar}{2} \mid \{H, g\} \mid \tag{7}$$

and

$$\Delta E \cdot \Delta g \geq \frac{\hbar}{2} \left| \frac{dg}{dt} \right| , \tag{8}$$

when $\partial g/\partial t = 0$. Equations (7) and (8) are re-statements of the Ehrenfest theorem. Criteria for its validity are discussed at length in the literature [3–5]. If $g \equiv v(t)$ is the velocity expectation value of a particle whose energy is $E = mc^2\gamma$ and it is assumed with Caianiello [1] that $\Delta E \leq E$, then (8) gives

$$\left| \frac{dv}{dt} \right| \leq \frac{2}{\hbar} mc^2 \gamma \Delta v(t) , \tag{9}$$

where $\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}$. In general and with rigor

$$\Delta v = \left(\langle v^2 \rangle - \langle v \rangle^2\right)^{\frac{1}{2}} \leq v_{max} \leq c . \tag{10}$$

An essential point of Caianiello's argument is that the acceleration is largest in the rest frame of the particle. This follows from the transformations that link the three-acceleration $\vec{a}_p$ in the instantaneous rest frame of the particle to the particle's acceleration $\vec{a}'$ in another frame with instantaneous velocity $\vec{v}$ [6]:

$$\vec{a}' = \frac{1}{\gamma^2} \left[\vec{a}_p - \frac{(1-\gamma)\left(\vec{v} \cdot \vec{a}_p\right)\vec{v}}{v^2} - \frac{\gamma\left(\vec{v} \cdot \vec{a}_p\right)\vec{v}}{c^2} \right] . \tag{11}$$

The equation

$$a'^2 = \frac{1}{\gamma^4} \left(a_p^2 - \frac{\left(\vec{a}_p \cdot \vec{v}\right)^2}{c^2} \right) , \tag{12}$$

where $a^2 \equiv \vec{a} \cdot \vec{a}$, follows from (11) and shows that $a' \leq a_p$ for all $\vec{v} \neq 0$ and that $a' \rightarrow 0$ as $\mid \vec{a}_p \cdot \vec{v} \mid \rightarrow a_p c$. In addition, in the instantaneous rest frame of the particle, $E \leq mc^2$ and $\Delta E \leq mc^2$ if negative rest energies must be avoided as nonphysical. Then (9) gives

$$\left| \frac{dv}{dt} \right| \leq 2 \frac{mc^3}{\hbar} \equiv \mathcal{A}_m . \tag{13}$$

It is at times argued that the uncertainty relation

$$\Delta E \cdot \Delta t \geq \hbar/2 \tag{14}$$

implies that, given a fixed average energy E, a state can be constructed with arbitrarily large ΔE, contrary to Caianiello's assumption that $\Delta E \leq E$. This conclusion is erroneous. The correct interpretation of (14) is that a quantum state with spread in energy ΔE takes a time $\Delta t \geq \frac{\hbar}{2\Delta E}$ to evolve to a distinguishable (orthogonal) state. This evolution time has a lower bound. Margolus and Levitin have in fact shown [7] that the evolution time of a quantum system with fixed average energy E must satisfy the more stringent limit

$$\Delta t \geq \frac{\hbar}{2E}\,, \tag{15}$$

which determines a maximum speed of orthogonality evolution [8, 9]. Obviously, both limits (14) and (15) can be achieved only for $\Delta E = E$, while spreads $\Delta E > E$, that would make Δt smaller, are precluded by (15). This effectively restricts ΔE to values $\Delta E \leq E$, as conjectured by Caianiello [10].

Known transformations now ensure that the limit (13) remains unchanged. It follows, in fact, that in the rest frame of the particle the absolute value of the proper acceleration is

$$\left(\left|\frac{d^2x^\mu}{ds^2}\frac{d^2x_\mu}{ds^2}\right|\right)^{\frac{1}{2}} = \left(\left|\frac{1}{c^4}\frac{d^2x^i}{dt^2}\right|\right)^{\frac{1}{2}} \leq \frac{\mathcal{A}_m}{c^2}\,. \tag{16}$$

Equation (16) is a Lorentz invariant. The validity of (16) under Lorentz transformations is therefore assured.

The uncertainty relation (14) can then be used to extend (13) to include the average length of the acceleration $\langle a\rangle$. If, in fact, $v(t)$ is differentiable, then fluctuations about its mean are given by

$$\Delta v \equiv v - \langle v\rangle \simeq \left(\frac{dv}{dt}\right)_0 \Delta t + \left(\frac{d^2v}{dt^2}\right)_0 (\Delta t)^2 + \dots\,. \tag{17}$$

Equation (17) reduces to $\Delta v \simeq \left|\frac{dv}{dt}\right| \Delta t = \langle a\rangle \Delta t$ for sufficiently small values of Δt, or when $\left|\frac{dv}{dt}\right|$ remains constant over Δt. The inequalities (14) and (10) then yield

$$\langle a\rangle \leq \frac{2c\Delta E}{\hbar} \tag{18}$$

and again (13) follows.

The notion of MA delves into a number of issues and is connected to the extended nature of particles. In fact, the inconsistency of the point particle concept for a relativistic quantum particle is discussed by Hegerfeldt [11], who shows that the localization of the particle at a given point at a given time conflicts with causality.

Classical and quantum arguments supporting the existence of MA have been frequently discussed in the literature [12–21]. MA would eliminate divergence difficulties affecting the mathematical foundations of quantum field theory [22]. It would also free black hole entropy of ultraviolet divergences [23–25]. MA plays a fundamental role in Caianiello's geometrical formulations of quantum mechanics [26] and in the context of Weyl space [27]. A limit on the acceleration also occurs in string theory. Here the upper limit appears in the guise of Jeans-like instabilities that develop when the acceleration induced by the background gravitational field is larger than a critical value for which the string extremities become casually disconnected [28–30]. Frolov and Sanchez [31] have also found that a universal critical acceleration must be a general property of strings.

Incorporating MA in a theory that takes into account the limits (13) or (16) in a meaningful way from inception is a very important question. In Caianiello's reasoning the usual Minkowski line element

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu \,, \tag{19}$$

must be replaced with the infinitesimal element of distance in the eight-dimensional space-time tangent bundle TM

$$d\tau^2 = \eta_{AB} dX^A dX^B \,, \tag{20}$$

where $A, B = 0, \ldots, 7$, $\eta_{AB} = \eta_{\mu\nu} \otimes \eta_{\mu\nu}$, $X^A = \left(x^\mu \frac{c^2}{\mathcal{A}_m} \frac{dx^\mu}{ds} \right)$, $x^\mu = (ct, \vec{x})$ and $dx^\mu/ds \equiv \dot{x}^\mu$ is the four-velocity. The invariant line element (20) can therefore be written in the form

$$d\tau^2 = \eta_{\mu\nu}\, dx^\mu dx^\nu + \frac{1}{\mathcal{A}_m^2} \eta_{\mu\nu} d\dot{x}^\mu d\dot{x}^\nu = \left[1 + \frac{\ddot{x}_\mu \ddot{x}^\mu}{\mathcal{A}_m^2} \right] \eta_{\mu\nu} dx^\mu dx^\nu \,, \tag{21}$$

where all proper accelerations are normalized by $\mathcal{A}_m$. The effective space-time geometry experienced by accelerated test particles contains therefore mass-dependent corrections which, in general, induce curvature and give rise to a mass-dependent violation of the equivalence principle. In the presence of gravity, we replace $\eta_{\mu\nu}$ with the corresponding metric tensor $g_{\mu\nu}$, a natural choice which preserves the full structure introduced in the case of flat space. In the classical limit $(\mathcal{A}_m)^{-1} = \frac{\hbar}{kmc^3} \to 0$ the terms contributing to the modification of the geometry vanish and one returns to the ordinary space-time geometry.

The model of Ref. [26] has led to interesting results that range from particle physics to astrophysics and cosmology [32, 33].

A second, equally fundamental problem stems from Caianiello's original paper "Is there a maximal acceleration?"[34]. In particular, is it possible to find physical conditions likely to lead to a MA? An approach to this problem is illustrated below by means of three examples that are offered here as a tribute to the memory of the great master and to the ever challenging vitality of his thoughts.

The limits (13) and (16) are very high for most particles (for an electron $\mathcal{A}_m \sim 4.7 \times 10^{31}\ \mathrm{cm\,s^{-2}}$) and likely to occur only in exceptional physical circumstances. The examples considered in this work involve superfluids, type-I superconductors in particular, that are intrinsically non-relativistic quantum systems in which ΔE and Δv can be lower than the uncertainties leading to the limit (13). Use of (8) is here warranted because the de Broglie wavelengths of the superfluid particles vary little over distances of the same order of magnitude [3, 5]. Superfluids are particular types of quantum systems whose existence is predicated upon the formation of fermion pairs. They behave, in a sense, as universes in themselves until the conditions for pairing are satisfied. The dynamics of the resulting bosons differs in essential ways from that of "normal" particles. The upper limits of the corresponding dynamical variables like velocity and acceleration must first of all be compatible with pairing and and may in principle differ from c and $\mathcal{A}_m$. If the limit on v is lower, then according to (8) the possibility of observing the effects od MA is greater. It is shown below that indeed type-I superconductors in static conditions offer an example of a dynamics with a MA.

Lepton–lepton interactions with the final production of gauge bosons also seem appropriate choices because of the high energies, and presumably high accelerations, that are normally reached in the laboratory. The application of (8) is justified by the high momenta of the leptons that satisfy the inequality $\Delta x \Delta p \gg \hbar$ [3, 5].

A last example regards the highly unusual conditions of matter in the interior of white dwarfs and neutron stars. In this case the legitimacy of (8) is assured by the inequality $\frac{N}{V}\lambda_T^3 \ll 1$, where $\frac{N}{V}$ is the particle density in the star and $\lambda_T = \frac{2\pi\hbar^2}{mkT}$ is of the same order of magnitude of the de Broglie wavelength of a particle with a kinetic energy of the order of kT [3, 5].

2 Type-I superconductors

The static behaviour of superconductors of the first kind is adequately described by London's theory [35]. The fields and currents involved are weak and vary slowly in space. The equations of motion of the superelectrons are in this case [36]

$$\begin{aligned} \frac{D\vec{v}}{Dt} &= \frac{e}{m}\left[\vec{E} + \left(\frac{\vec{v}}{c} \times \vec{B}\right)\right] \\ &= \frac{\partial \vec{v}}{\partial t} + \left(\vec{v} \cdot \vec{\nabla} \vec{v}\right) \ . \end{aligned} \tag{22}$$

On applying (8) to (22), one finds

$$\sqrt{\left(\frac{1}{2}\vec{\nabla}v^2 - \vec{v} \times (\vec{\nabla} \times \vec{v})\right)^2} \leq \frac{2}{\hbar} \triangle E \cdot \triangle v \ , \tag{23}$$

and again

$$\begin{aligned} &\sqrt{\frac{1}{4}\left(\nabla_i v^2\right)^2 + \frac{e}{mc}\epsilon_{ijk}\left(\nabla^i v^2\right) v^j B^k + \left(\frac{e}{mc}\right)^2 \left[v^2 B^2 - \left(v_i B^i\right)^2\right]} \\ &\leq \frac{2}{\hbar} \triangle E \triangle v \ , \end{aligned} \tag{24}$$

where use has been made of London's equation

$$\vec{\nabla} \times \vec{v} = -\frac{e}{mc}\vec{B} \ , \tag{25}$$

and ϵ_{ijk} is the Levi–Civita tensor. Static conditions, $\partial v/\partial t = 0$, make (23) and (24) simpler. It is also useful, for the sake of numerical comparisons, to apply (24) to the case of a sphere of radius R in an external magnetic field of magnitude B_0 parallel to the polar axis. This problem has an obvious symmetry and can be solved exactly. The exact solutions of London's equations for $r \leq R$ are well known [37] and are reported here for completeness. They are

$$B_r = \frac{4\pi}{\beta^2 c}\frac{1}{r}\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta j_\varphi\right) , \tag{26}$$

$$B_\theta = -\frac{4\pi}{\beta^2 c}\frac{1}{r}\frac{\partial}{\partial r}\left(r j_\varphi\right) , \tag{27}$$

$$j_\varphi = nev_\varphi = \frac{A}{r^2}\left(\sinh\beta r - \beta r\cosh\beta r\right)\sin\theta , \tag{28}$$

where $A = -\frac{c}{4\pi}\frac{3B_0}{2}\frac{R}{\sinh\beta r}$, n is the density of superelectrons and $\beta = \left(\frac{4\pi ne^2}{mc^2}\right)^{\frac{1}{2}}$ represents the reciprocal of the penetration length. On using (22), the inequality (23) can be transformed into

$$| E_r |\leq \frac{| v_\varphi B_\theta |}{c} + \sqrt{\left(\frac{2mc}{e\hbar}\right)^2 (\Delta E)^2 (\Delta v)^2 - \left(\frac{v_\varphi}{c}B_r\right)^2} . \tag{29}$$

For a gas of fermions in thermal equilibrium $\Delta E \sim \frac{3}{5}\mu$, $\Delta v \sim \frac{3}{2}\sqrt{\frac{\mu}{2m}}$ and the chemical potential behaves as $\mu \approx \epsilon_F - \frac{(\pi kT)^2}{12\epsilon_F} \approx \epsilon_F \sim 4.5 \times 10^{-12}$ erg for T close to the transition temperatures of type-I superconductors. The reality of (29) requires that $\Delta E \geq \mu_B B_r$, where $\mu_B = \frac{e\hbar}{2mc}$ is the Bohr magneton, or that $\frac{3}{5}\epsilon_F \geq \mu_B B_r$. This condition is certainly satisfied for values of $B_r \leq B_c$, where B_c is the critical value of the magnetic field applied to the superconductor. From (29) one also obtains

$$| E_r |\leq \frac{3}{2m}\left(\frac{\epsilon_F}{2}\right)^{\frac{1}{2}}\left[\frac{| B_\theta |}{c} + \sqrt{\left(\frac{3\epsilon_F}{5\mu_B}\right)^2 - \left(\frac{B_r}{c}\right)^2}\right] , \tag{30}$$

which is verified by the experimental work of Bok and Klein [38]. More restrictive values for ΔE and Δv can be obtained from B_c. The highest value of the velocity of the superelectrons must, in fact, be compatible with B_c itself, lest the superconductor revert to the normal state. This value is approximately a factor 10^3 smaller than that obtained by statistical analysis. The upper value v_0 of v_φ is at the surface. From $\Delta E \leq \frac{1}{2}mv_0^2$, $\Delta v \leq v_0$ and (29) one finds that at the equator, where $B_r = 0$, E_r satisfies the inequality

$$| E_r |\leq \frac{v_0}{c}\left(| B_\theta | + \frac{v_0^2}{2\mu_B}\right) . \tag{31}$$

For a sphere of radius $R = 1$ cm one finds $v_0 \simeq 4.4 \times 10^4\,\mathrm{cm\,s^{-1}}$ and $E_r \leq 69\,N/C$. If no magnetic field is present, then (31) gives $E_r \leq 4.2\,N/C$. On the other hand

London's equation gives

$$E_r = \frac{m}{2e}\frac{\partial v_\varphi^2}{\partial r} \simeq 0.32\, N/C\ . \tag{32}$$

The inequality (32) agrees with the experimental data [38]. The MA limits (30) and (31) are therefore consistent with (32) and its experimental verification.

3 High energy lepton–lepton interactions

Consider the process $A + B \to D$ by which two particles A and B of identical mass m give rise to particle D. A, or B, or both particles may also be charged. Assume moreover that D is produced at rest in its proper frame. The width of D is $\Gamma(AB)$ and the time during which the process takes place in the centre of mass frame of A and B is $\Delta t \simeq \frac{\hbar}{\Gamma(AB)}$. The acceleration of A and B is thought to remain close to zero until D is produced. If this were not the case, radiation would be produced and the process would acquire different characteristics. The root mean square acceleration of the reduced mass m_r over Δt will in general be $a_r \simeq \frac{c}{\gamma \Delta t}$, where γ refers to the velocity v of m_r [39]. The MA limit (16), applied to a_r, gives $\frac{c}{\gamma\Delta t} \leq \frac{m_D c^3}{\hbar}$, or

$$\Gamma(AB) \leq mc^2\gamma\ , \tag{33}$$

and v obeys the condition $2mc^2\gamma = m_D c^2$, or $\gamma = \frac{m_D}{2m}$.

The limit (33) can also be written in the form

$$\Gamma(AB) = \frac{p_f}{32\pi^2 m_D}\int |M(AB)|^2 \leq \frac{m_D c^2}{2}\ , \tag{34}$$

where $M(AB)$ is the invariant matrix element for the process. Several processes can now be considered. For the process $e^+ + e^- \to Z^0$ one has (in units $\hbar = c = 1$) $\Gamma(e^+e^- \to Z^0) \simeq 0.08391\,\text{GeV}$, $m_{Z_0} = 91.188\,\text{GeV}$ and the inequality (34) is certainly satisfied.

Similarly, $\Gamma(W \to e\nu_e) \simeq 0.22599\,\text{GeV} < \frac{80.419}{2}\,\text{GeV}$.

One expects the limit (34) to be less restrictive at lower energies. In the case of $e^+ + e^- \to J/\psi$ one in fact finds $\Gamma_{ee\to J/\psi} \simeq 5.2\,\text{KeV}$, while the r.h.s. of (34) gives $\frac{3096.87}{2}$ MeV.

Alternatively, (34) can be used to find an upper limit to the value of m_D. These values depend, of course, on theoretical estimates of the corresponding decay widths [40]. From

$$\Gamma(Z^0 \to ee) \simeq \frac{G_F m_{Z0}^3}{12\pi\sqrt{2}} \leq \frac{m_{Z^0}}{2}\ , \tag{35}$$

one obtains $m_{Z^0} \leq (\frac{6\pi\sqrt{2}}{G_F})^{\frac{1}{2}} \simeq 1512\,\text{GeV}$ which is a factor 16.6 larger than the experimental value of the mass of Z^0. Analogously

$$\Gamma(W \to d\bar{u}) \simeq \frac{G_F 4.15 m_W^3}{6\sqrt{2}\pi} \leq \frac{m_W}{2} \tag{36}$$

yields $m_W \leq 525\,\text{GeV}$ which is ~ 6.53 times larger than the experimental value of the boson mass. Finally, from

$$\Gamma(J/\psi \to ee) \simeq \frac{16\pi\alpha^2 0.018}{m_{J/\psi}} \leq \frac{m_{J/\psi}}{2} \,, \tag{37}$$

one finds $m_{J/\pi} \geq 4.6 \times 10^{-2}\,\text{GeV}$, a lower limit which is 1.5×10^{-2} times smaller than the known value of $m_{J/\psi}$.

For the Higgs boson the inequality

$$\Gamma(H^0 \to ee) \simeq \frac{G_F m_e^2 m_H}{4\sqrt{2}\pi} \left(1 - \frac{4m_e^2}{m_H^2}\right)^{\frac{3}{2}} \leq \frac{m_H}{2} \tag{38}$$

is always satisfied for $m_H \geq 2m_e$. Finally, from [40]

$$\Gamma(H^0 \to ZZ) \simeq \frac{G_F m_Z^2 m_H}{16\pi\sqrt{2}x_Z} (1 - x_Z)^{\frac{1}{2}} \left(3x_Z^2 - 4x_Z + 4\right) \leq \frac{m_H}{2} \,, \tag{39}$$

where $x_Z \equiv \frac{4m_Z^2}{m_H^2}$, one finds

$$2m_Z \leq m_H \leq 1760\,\text{GeV}\,. \tag{40}$$

Both lower and upper limits are compatible with the results of experimental searches. The upper limit also agrees with Kuwata's analogous attempt [41] to derive a bound on m_H from the MA constraint, when account is taken of the fact that in Ref. [41] MA is $\mathcal{A}_m/2$ and therefore $m_H \leq 500\,\text{GeV}$.

4 White dwarfs and neutron stars

The standard expression for the ground state energy of an ideal fermion gas inside a star of radius R and volume V is

$$E_0(r) = \frac{m^4 c^5}{\pi^2 \hbar^3} V f(x_F) \,, \tag{41}$$

where $x_F \equiv \frac{p_F}{mc}$, $p_F \equiv \left(3\pi^2 \frac{N}{V}\right)^{\frac{1}{3}} \hbar$ is the Fermi momentum, $0 \leq r \leq R$, N is the total number of fermions and

$$f(x_F) = \int_0^{x_F} dx x^2 \sqrt{1 + x^2} \,. \tag{42}$$

The integral in (42) is approximated by

$$f(x_F) \simeq \frac{1}{3} x_F^3 \left(1 + \frac{3}{10} x_F^2 + ...\right) \,, \quad x_F \ll 1 \tag{43}$$

in the non-relativistic (NR) case, or by

$$f(x_F) \simeq \frac{1}{4}x_F^4 \left(1 + \frac{1}{x_F}^2 + ...\right) , \quad x_F \gg 1 \tag{44}$$

in the extreme relativistic (ER) case. The average force exerted by the fermions a distance r from the centre of the star is [42]

$$\langle F_0 \rangle = \frac{\partial E_0}{\partial r} \simeq \frac{4m^4c^5}{\pi\hbar^3} r^2 f(x_F) . \tag{45}$$

From (45) and the expression

$$N(r) = \frac{4}{9\pi\hbar^3} r^3 {p_F}^3 \tag{46}$$

that gives the number of fermions in the ground state at r, one can estimate the average acceleration per fermion as a function of r:

$$\langle a(r) \rangle = \frac{9c^2}{{x_F}^3 r} f(x_F) . \tag{47}$$

By using (43) and (44) one finds, to second order,

$$\begin{aligned} \langle a(r) \rangle_{NR} &= \frac{3c^2}{r}\left(1 + \frac{3}{10}x_F^2\right) \\ \langle a(r) \rangle_{ER} &= \frac{9c^2}{4r}\left(x_F + \frac{1}{x_F}\right) . \end{aligned} \tag{48}$$

The MA limit (13) applied to $\langle a(r) \rangle$ now yields

$$\begin{aligned} r &\geq (r_0)_{NR} \equiv \frac{3\lambda}{4\pi} \\ r &\geq (r_0)_{ER} \equiv \frac{9}{16\pi}\frac{\lambda p_F}{mc} , \end{aligned} \tag{49}$$

where $\lambda \equiv \frac{h}{mc}$ is the Compton wavelength of m. For a white dwarf, $N/V \simeq 4.6 \times 10^{29}\,\mathrm{cm}^{-3}$ gives $(r_0)_{NR} \simeq 5.8 \times 10^{-11}$ cm and $(r_0)_{ER} \simeq 4 \times 10^{-11}$ cm.

In order to have at least one state with particles reaching MA values, one must have

$$\begin{aligned} Q\left((r_0)_{NR}\right) &= \frac{4}{9\pi}\left((r_0)_{NR}\frac{p_F}{\hbar}\right)^3 \sim 1 \\ Q\left((r_0)_{ER}\right) &= \frac{4}{9\pi}\left((r_0)_{ER}\frac{p_F}{\hbar}\right)^3 \sim 1 . \end{aligned} \tag{50}$$

In the case of a typical white dwarf, the first of (50) gives $(N/V)_{NR} \sim 1.2 \times 10^{30}\,\mathrm{cm}^{-3}$ and the second one $(N/V)_{ER} \sim 1.3 \times 10^{30}\,\mathrm{cm}^{-3}$. On the other hand, the condition

$x_F \ll 1$ requires $(N/V)_{NR} \ll 6 \times 10^{29}\,\mathrm{cm}^{-3}$, whereas $x_F \gg 1$ yields $(N/V)_{ER} \gg 6 \times 10^{29}\,\mathrm{cm}^{-3}$. It therefore follows that the NR approximation does not lead to electron densities sufficient to produce states with MA electrons. The possibility to have states with MA electrons is not ruled out entirely in the ER case.

The outlook is however different if one starts from conditions that do not lead necessarily to the formation of canonical white dwarfs or neutron stars [42]. Pressure consists, in fact, of two terms. The first term represents the pressure exerted by the small fraction of particles that can reach accelerations comparable with MA. It is given by

$$P_{MA} = \frac{2m^2c^3}{\hbar}\frac{Q(r_0)}{4\pi r_0^2} \,. \tag{51}$$

The second part is the contribution of those fermions in the gas ground state that cannot achieve MA

$$-\frac{\partial E_0'}{\partial V} = -\frac{\partial}{\partial V}\left(2\tilde{\gamma} V \int_{r_0}^{\infty} d\epsilon \frac{\epsilon^{3/2}}{e^{\alpha+\beta}+1}\right) \,, \tag{52}$$

where $\tilde{\gamma} \equiv \frac{(2m)^{3/2}}{(2\pi)^2\hbar^3}$ and $V = \frac{4}{3}\pi\left(R^3 - r_0^3\right)$. Since r_0 is small, one can write

$$\frac{\partial E_0'}{\partial V} \simeq \frac{\partial E_0}{\partial V} \sim \frac{4K\tilde{M}^{5/3}}{5\tilde{R}^5} \,, \tag{53}$$

where $\tilde{M} \equiv \frac{9\pi M}{8m_p}$, $\tilde{R} \equiv \frac{2\pi R}{\lambda}$, $K \equiv \frac{m^4c^5}{12\pi^2\hbar^3}$ and m_p is the mass of the proton. In the nonrelativistic case, the total pressure is obtained by adding (51) to (53) and using the appropriate expression for $Q(r_0)$ in (50). The hydrostatic equilibrium condition is

$$\frac{8\pi mc^2}{3\lambda^3}\frac{\tilde{M}}{\tilde{R}^3} + \frac{4K}{5}\frac{\tilde{M}^{5/3}}{\tilde{R}^5} = K'\frac{\tilde{M}^2}{\tilde{R}^4} \,, \tag{54}$$

where $K' \equiv \frac{4\alpha G\pi}{\lambda^4}\frac{64m_p^2}{81}$ and $\alpha \approx 1$ is a factor that reflects the details of the model used to describe the hydrostatic equilibrium of the star. It is generally assumed that the configurations taken by the star are polytropes [43]. Solving (54) with respect to $\tilde{R}$ one finds

$$\tilde{R} = \frac{\tilde{M}\tilde{M}_0^{-2/3}}{8}\left(1 \mp \sqrt{1 - \frac{64}{5}\left(\frac{\tilde{M}_0}{\tilde{M}}\right)^{4/3}}\right) \,, \tag{55}$$

where $\tilde{M}_0 \equiv \left(K/K'\right)^{3/2} = \left(\frac{27\pi\hbar c}{64\alpha G m_p^2}\right)^{\frac{3}{2}} \simeq \frac{9\pi}{8m_p}M_\odot$. Solutions (55) will be designated by $\tilde{R}_-$ and $\tilde{R}_+$. They are real if $M \geq \left(\frac{64}{5}\right)^{3/4} M_0 \sim 6.8M_\odot$. This is a new stability condition. At the reality point the radius of the star is twice that of a canonical white dwarf. This situation persists for the solution R_- up to mass values of the order of $\sim 10M_\odot$. The radii of the R_+ solutions increase steadily. The corresponding electron densities, calculated from $N/V = \frac{3M}{8\pi m_p R^3}$, are $(N/V)_- > \frac{8\pi}{3}\left(\frac{8}{\lambda}\right)^3 (M_0/M)^2 \sim 6.6 \times 10^{30}\,\text{cm}^{-3}$ and $(N/V)_+ \langle 6.6 \times 10^{30}\,\text{cm}^{-3}$. NR stars of the R_- type thus appear to be more compact than those of the R_+ class. The electron density for R_+ is still compatible with that of a canonical NR white dwarf. Equation (54) can also be written in the form

$$\tilde{M}^{1/3}\tilde{R} = \frac{4}{5}\tilde{M}_0^{2/3}\left(1 + \frac{10\pi mc^2}{3\lambda^3 K}\frac{\tilde{R}^2}{\tilde{M}^{2/3}}\right), \tag{56}$$

where the second term on the r.h.s. represents the MA contribution to the usual M-R relation for NR white dwarfs. This contribution can be neglected when $\frac{\tilde{R}}{\tilde{M}^{1/3}} < 1/\sqrt{5}$ which requires $N/V > \frac{8\pi 5^{3/2}}{3\lambda^3} \sim 6.6 \times 10^{29}\,\text{cm}^{-3}$. This condition and the usual M–R relation, $\tilde{M}^{1/3}\tilde{R} = \frac{4\tilde{M}_0^{2/3}}{5}$, are compatible if $R\langle \frac{\lambda}{\pi 5^{3/4}}\left(\frac{9\pi M_0}{8m_p}\right)^{1/3}$, which leads to the density $N/V > \frac{5^{9/4}\pi}{3\lambda^3}(M/M_0) \sim 2.7 \times 10^{30}(M/M_0)\,\text{cm}^{-3}$.

Similarly, one can calculate the MA contribution to the M–R relation for ER white dwarfs. The equation is

$$\frac{m^4c^5}{4\pi^2\hbar^3} + K\left(\frac{\tilde{M}^{4/3}}{\tilde{R}^4} - \frac{\tilde{M}^{2/3}}{\tilde{R}^2}\right) = K'\frac{\tilde{M}^2}{\tilde{R}^4}, \tag{57}$$

where the first term on the l.h.s. represents the MA contribution. From (57) one gets

$$\tilde{R} = \tilde{M}^{1/3}\sqrt{4 - \left(\frac{\tilde{M}}{\tilde{M}_0}\right)^{2/3}}, \tag{58}$$

and the stability condition becomes $M \leq 8M_0 \sim 8M_\odot$. These are very compact objects. For the electron densities determined, the star can still be called a white dwarf. One also finds that for $N/V \sim 3 \times 10^{30}\,\text{cm}^{-3}$ the number of MA states is only $Q(r_0) = \frac{9\lambda^3}{16\pi^2}\frac{N}{V} \simeq 2.2\frac{M}{M_\odot}$. A few MA electrons could therefore be present at this density. However, interactions involving electrons and protons at short distances may occur before even this small number of electrons reaches the MA.

Analogous conclusions also apply to neutron stars, with minor changes if these can be treated as Newtonian polytropes. This approximation may only be permissible, however, for low-density stars [43]. One finds in particular $Q(r_0) = \frac{9\lambda_n^3}{16\pi^2}\frac{N}{V} \simeq 4.5\frac{M}{M_\odot}$. The presence of a few MA neutrons is therefore allowed in this case.

5 Conclusions

A limit on the proper acceleration of particles can be obtained from the uncertainty relations in the following way. The Ehrenfest theorem (8) is first applied to a particle's acceleration $\vec{a}$ in the particle's instantaneous rest frame. The latter is then transformed to a Lorentz frame of instantaneous velocity $\vec{v}$. In any other Lorentz frame the resulting acceleration is $a' \leq a_p$. The absolute value of the proper acceleration satisfies (16). No counterexamples are known to the validity of (16). For this reason (16) has at times been elevated to the status of a principle. It would be only befitting to call it the Caianiello principle.

In most instances the value of MA is so high that it defies direct observation. Nonetheless the role of MA as a universal regulator must not be discounted. It is an intrinsic, first quantization limit that preserves the continuity of space-time and does not require the introduction of a fundamental length, or of arbitrary cutoffs. The challenge is to find situations where MA affects the physics of a system in ways that can be observed.

Though the existence of a MA is intimately linked to the validity of the Ehrenfest theorem, it is not entirely subordinate to it and the limit itself may depend on the dynamical characteristics of the particular system considered. This is the case with superconductors of the first kind which are macroscopic, non-relativistic quantum systems with velocities that satisfy the inequality $\Delta E \Delta v \ll mc^3$. Superfluid particles in static conditions are known to resist acceleration. The MA constraints (23) and (24) lead to a limit on the value of the electric field at the surface of the superconductor that agrees with the value (31) obtained from London's equations and with known experimental results. The MA limit in this case is only ~ 10 times larger than (32). If the pairing condition is satisfied, then superfluids in static conditions obey a dynamics for which a MA *exists* and differs from $\mathcal{A}_m$, as anticipated.

In Section (III) two high-energy particles, typically leptons, produce a third particle at rest. The MA limit applied to the process leads to the constraint (34) on the width of the particle produced. The limit is perfectly consistent with available experimental results. When the end product is Z^0, the acceleration is $a_r = \frac{2mc}{\hbar m_D}\Gamma(AB) \sim 2.8 \times 10^{26}\,\mathrm{cm\,s^{-2}}$. Even at these energies the value of the acceleration is only a factor $\sim 6 \times 10^{-6}$ that of the MA for m_r. Equation (33) and current estimates of $\Gamma(ZZ \to H^0)$ can also be used to derive upper and lower limits (40) on the mass of the Higgs boson.

The last physical situation considered regards matter in the interior of white dwarfs and neutron stars. For canonical white dwarfs, the possibility that states exist with MA electrons can be ruled out in the NR case, but not so for ER stars. On the other hand, the mere presence of a few MA electrons alters the stability conditions of the white dwarf drastically. Equations (56) and (58) represent, in fact, *new stability conditions*. The same conclusions also apply to NR neutron stars, with limitations, however, on the choice of the equation of state. In the collapse of stars with masses larger than the Chandrasekhar and Oppenheimer–Volkoff limits from white dwarfs to neutron stars, to more compact objects, conditions favorable to the formation of states with MA fermions may occur before competing processes take place.

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Modeling Single Neuron Activity in the Presence of Refractoriness: New Contributions to an Old Problem

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Abstract. The inclusion of refractoriness in a model for single neuron activity is the object of the present paper. Differently from previous approaches in which neuronal input was modeled as a point processe, diffusion is invoked here to describe the time course of the neuron's membrane potential, and refractoriness is represented by a sequence of independent and identically distributed random variables. A return process is then constructed, by means of which the probabilities for the neuron to elicit any pre-assigned number of firings up to any pre-assigned time are determined. The presence of refractoriness is included in two different ways, namely by assuming that *(a)* the firing threshold acts as a "partially transparent" elastic barrier, and *(b)* that the neuronal refractoriness period can be modeled by a random variable with a pre-assigned probability density function.

1 Background

In the early 1960s, a small group of young physicists, mathematicians, and biologists, led by Eduardo Caianiello, enthusiastically engaged in a pioneering but possibly hopeless enterprise: to apply the rigorous methods of the hard sciences to the "wet" reality of biological systems. The aim was to understand, to predict, and, above all, to harness for the benefit of mankind the puzzling, highly efficient mechanisms underlying certain features of living systems: namely, their ability to *think* and to *move* using only a tiny amount of energy. Would it be possible to employ such knowledge to design the components of autonomous thinking machines?

With the benefit of over forty years experience, this program now appears to have been much too ambitious to be realized by a few dozen individuals who, though endowed with great energy, imagination, and scientific curiosity, were lacking the necessary financial support from the existing public and private agencies. (Unfortunately, a few years later, when Eduardo was about to sign an adequately funded research agreement with a high-tech multinational company, the well-known events of 1968 precluded its fulfillment on the pretext that ... *the University should not sell out to profit-oriented corporations* ... .) Nevertheless, under the guidance of Eduardo, much valuable work was accomplished. In addition, we (Aldo de Luca and I were the youngest in the group) greatly benefited from the presence of scientists of the calibre of Norbert Wiener, Warren McCulloch, Werner Reichardt, Marc Schutzenberger, Ilya Prigogine and Rudolph Kalman, just to mention a few whose collaboration Eduardo systematically invited.

Together we devoted a great deal of time and energy to what turned out to be very technically hard problems. One such problem of great concern to the international scientific community of that time was how to model the spontaneous activity of neurons embedded in very intricate and complex networks. We produced some such models as contributions to the more general context of “brain theories”design; although at that time our contributions were well received, it is fair to claim that the neuronal spontaneous activity problem has not been satisfactorily resolved up to now, namely some 40 years after our early endeavors.

The object of the present paper is to offer some new contributions to this old problem by focusing specifically on a quantitative method to account for the realistic presence of refractoriness in neuronal activity modeling. Indeed, due to some involved subtle mathematical difficulties, inclusion of refractoriness has not received much attention since my own early approaches [1]. Some brief considerations will now follow to clarify the problem that will be the object of this paper.

Note that modeling neuronal activity, as we attempt to do in the sequel, falls in the large within the realm of *thought processes and thinking machines* [2, 3]. Concerning current endeavors aiming to shed light on the second above-mentioned old project, i.e. the understanding of mechanisms that underlie the ability of living systems to *move* under small amounts of energy consumption, see for instance [4–8], and references therein.

2 Introductory remarks

Even though noteworthy progress has been made towards the understanding of the stochastic properties of model neurons, one physiologically significant parameter has usually been disregarded: the neuron’s intrinsic refractoriness that hampers the release of a spike before a certain time span has elapsed since the release of the previous spike. Although the duration of such a refractory period partly depends on the intensity of the input stimulation, it is known that there exists an intrinsic “dead time” in the neuron’s spike production that is a manifestation of the natural inertia that characterizes the opening and closing of the ion channels responsible for changes of the neuronal membrane electrical potential. Hence, to disregard such a dead time may be justified only if one aims to construct models that are suitable to account exclusively for small firing rates.

The first attempt to investigate the role of refractoriness within neuronal model building dates back to the mid-1960s, when great attention was devoted to hardware implementations of networks of linear switching elements taken to mimic the behavior of physiological neurons. Within this framework, each “neuron” was viewed as a black box, possessing the following distinctive features: *(i)* it is a threshold element, i.e. it releases a standard response at its output at time t if and only if the strength of its input is greater than a certain constant threshold value at time t (delay in the response being ignored as a non-critical first approximation); *(ii)* output response consists of pulses of constant amplitude and width; and *(iii)* there exists a constant dead time. Let τ denote this dead time, i.e. the time interval following every firing during which the neuron cannot fire again. Even for such an oversimplified instance the investigation of the role played by the dead time in determining the distribution of the output when the input is described by a given distribution is a very challenging task, as shown by the following example discussed at length in [1].

Let us assume that the net input to the neuron in time interval $(0, T)$ is modeled by a sequence of positive pulses of standard strength whose times of occurrence are Poisson distributed with rate λ. The problem is to determine the distribution $\pi_n(T, \tau)$ of the number N of output pulses as a function of dead time τ. A rather cumbersome amount of computations lead one to prove that the assumed input distribution

$$P_n(T) = \frac{(\lambda\, T)^n}{n!}\, \mathrm{e}^{-\lambda\, T} \qquad (T > 0,\ n = 0, 1, 2, \ldots) \tag{1}$$

generates the following firing distribution ($n \geq 1$):

$$\begin{aligned} \pi_n(T, \tau) = {}& \vartheta\big[T - (n-1)\tau\big] \left\{1 - \mathrm{e}^{-\lambda\,[T-(n-1)\,\tau]} \sum_{k=0}^{n-1} \frac{\lambda^k\,[T-(n-1)\,\tau]^k}{k!}\right\} \\ & - \vartheta(T - n\,\tau)\left[1 - \mathrm{e}^{-\lambda\,(T-n\,\tau)} \sum_{k=0}^{n} \frac{\lambda^k\,(T-n\,\tau)^k}{k!}\right], \end{aligned} \tag{2}$$

where $\vartheta(x)$ denotes the Heaviside unit step function. From (2) the expected number $E[N(t, \tau)]$ of output pulses follows:

$$E[N(t, \tau)] = \sum_{n=0}^{\lceil T/\tau \rceil} \sum_{k=n+1}^{+\infty} \frac{\lambda^k (T - n\,\tau)^k}{k!}\, \mathrm{e}^{-\lambda\,(T-n\,\tau)}\ . \tag{3}$$

Though not easy to prove, as expected one has $\lim_{\tau\to 0} E[N(t, \tau)] = \lambda\, T$.

Note that if the input sequence consists of Poisson distributed pulses of random strength X with probability density function $f_X(x)$ such that $P\{x : f_X(x) > S\} > 0$, with S denoting the neuron's threshold, formulae (1), (2) and (3) still hold provided one sets

$$\lambda = \int_S^{+\infty} f_X(x)\, dx\ .$$

In [1] extensions to non-stationary Poisson input processes were shown to be feasible, whereas the analysis of other types of input interarrival models at the time appeared to be inaccessible.

Before sketching a mathematically more satisfactory approach to the inclusion of refractoriness in neuronal models, it is interesting to remark that to take into account refractoriness effects, one could for instance model the after firing threshold's time course by suitable functions whose form can be responsible for the inability of the neuron to fire again immediately after the release of a spike. For instance, one could assume that the block of ionic channels after production of each action potential has the effect of making instantly the firing threshold infinitely large, with a subsequent recovery of the reset state. A neuronal threshold such as $S(t) = S - [1 - \exp\{\omega/(t - t_f)\}]$, with a large positive ω and where t_f denotes the last firing time, would do the job, but it is not amenable to analytical or computational purposes. More promising is an approach relying on the explicit dependence of the input random component on the difference between the state variable and the neuronal threshold, as proposed in [9]. Alternatively, one could introduce relative refractoriness effects by assuming that after each firing the membrane potential is not instantly reset to its resting value, but that it bounces back to a state far from the firing threshold, from which it returns to the resting initial state with finite velocity, according to some pre-assigned law. If the neuron is insensitive to input stimulations during such a process, refractoriness automatically occurs. More generally, one could pre-assign a distribution function to characterize the instantaneous after-firing membrane potential state, from which the recovery of the resting potential starts.

In the sequel, we shall outline some new contributions to this old problem. They rely on modeling the time course of the membrane potential by an instantaneous return process constructed from a diffusion process (see [10–14]) under the assumption that after each firing the membrane potential is either reset to a unique fixed value, or that the reset value is characterized by an assigned probability density function (pdf). Within such a context, the presence of refractoriness has been included in two different ways. The first way assumes that the firing threshold acts as a “partially transparent” elastic barrier, i.e. such that its behavior is intermediate between total absorption and total reflection (cf. [15–17]. Alternatively, the return process paradigm for the description of the time course of the membrane potential is analyzed by assuming that the neuronal refractoriness period is described by a random variable with a pre-assigned probability density function (cf. [18, 19]).

3 Notation and basic definitions

We start with some preliminary definitions and the setting up of the notation. Let $\{X(t), t \geq 0\}$ be a regular, time-homogeneous diffusion process, defined over the interval $I = (r_1, r_2)$, characterized by drift and infinitesimal variance $A_1(x)$ and $A_2(x)$, respectively. We assume that Feller conditions [20] are satisfied. Let $h(x)$ and $k(x)$ denote scale function and speed density of $X(t)$:

$$h(x) = \exp\left\{-2\int^x \frac{A_1(z)}{A_2(z)}\,dz\right\}, \qquad k(x) = \frac{2}{A_2(x)\,h(x)}$$

and denote by

$$H(r_1, y] = \int_{r_1}^{y} h(z)\,dz\,, \quad K(r_1, y] = \int_{r_1}^{y} k(z)\,dz$$

scale and speed measures, respectively.

The random variable “first passage time” (FPT) of $X(t)$ through S $(S \in I)$ with $X(0) = x < S$ is defined as follows:

$$T_x = \inf_{t\geq 0}\{t : X(t) \geq S\}\,, \quad X(0) = x < S\,. \tag{4}$$

Then,

$$g(S, t \mid x) = \frac{\delta}{\delta t} P(T < t) \qquad (x < S) \tag{5}$$

is the FPT pdf of $X(t)$ through S conditional upon $X(0) = x$.

In the neuronal modeling context the state S represents the neuron's firing threshold, FPT through S the firing time and $g(S, t \mid x)$ the firing pdf. In the sequel we assume that one of the following cases holds:

(i) r_1 is a natural non-attracting boundary and $K(r_1, y] < +\infty$

(ii) r_1 is a reflecting boundary or it is an entrance boundary.

Under such assumptions the first passage probability $P(S \mid x)$ from x to S is unity and the FPT kth order moments are given by (cf. [21]):

$$\begin{aligned} t_n(S \mid x) :&= \int_0^\infty t^n\, g(S, t \mid x)\, dt \\ &= n \int_x^S h(z)\, dz \int_{r_1}^z k(u)\, t_{n-1}(S \mid u)\, du \quad (x < S)\ , \end{aligned} \tag{6}$$

with $t_0(S \mid x) = P(S \mid x) = 1$.

4 Elastic boundary and refractoriness

We assume that after each firing a period of refractoriness of random duration occurs, during which either the neuron is completely unable to respond, or it only partially responds to the received stimulations. To this end, we look at the threshold S as an elastic barrier being "partially transparent". The degree of elasticity of the boundary depends on the choice of two parameters, α (absorbing coefficient) and β (reflecting coefficient), with $\alpha > 0$ and $\beta \geq 0$. Hence, $p_R := \beta/(\alpha + \beta)$ denotes the reflecting probability at the boundary S, and $1 - p_R = \alpha/(\alpha + \beta)$ the absorption probability at S.

We denote by $\widehat{T_x}$ the random variable describing the "first exit time" (FET) of $X(t)$ through S if $X(0) = x < S$, and by $g_e(S, t \mid x)$ its pdf. The random variable T_r will denote the "refractoriness period" and $g_r(S, t \mid S)$ its pdf. Since $\widehat{T_x}$ can be viewed as the sum of random variable T_x describing the first passage time through S (firing time) and of T_r one has:

$$g_e(S, t \mid x) = \int_0^t g(S, \tau \mid x)\, g_r(S, t \mid S, \tau)\, d\tau\ . \tag{7}$$

Under assumptions *(i)* and *(ii)*, if $\alpha > 0$ the first exit time probability

$$\widehat{P}(S \mid x) := \int_0^{+\infty} g_e(S, t \mid x)\, dt \qquad (x < S)$$

is unity. Furthermore, if $\alpha > 0$ the first exit time moments $\widehat{t_n}(S \mid x) \equiv E(\widehat{T_x^n})$ can be iteratively calculated as

$$\widehat{t_n}(S \mid x) := \int_0^\infty t^n \, g_e(S, t \mid x) \, dt = n \left\{ \int_x^S h(z) \, dz \int_{r_1}^z k(u) \widehat{t_{n-1}}(S \mid u) \, du + \frac{\beta}{\alpha} \int_{r_1}^S k(u) \widehat{t_{n-1}}(S \mid u) \, du \right\} \qquad (n = 1, 2, \ldots ; \; x < S) , \tag{8}$$

where $\widehat{t_0}(S \mid x) = \widehat{P}(S \mid x) = 1$. Note that in the absence of refractoriness, (8) are in agreement with (6). Indeed, if $\beta = 0$ one has $\widehat{t_n}(S \mid x) = t_n(S \mid x)$. We note that the FET moments $\widehat{t_n}(S \mid x)$ are related to the FPT moments $t_n(S \mid x)$ as follows:

$$\widehat{t_n}(S \mid x) = t_n(S \mid x) + n \, \frac{\beta}{\alpha} \sum_{j=0}^{n-1} \binom{n-1}{j} t_{n-1-j}(S \mid x) \int_{r_1}^S k(u) \, \widehat{t_j}(S \mid u) \, du \qquad (n = 1, 2, \ldots ; \; x < S) . \tag{9}$$

Hence, making use of (9), if $\alpha > 0$ and $x < S$ the first two moments and the variance of first exit time are given by

$$\begin{aligned} \widehat{t_1}(S \mid x) &= t_1(S \mid x) + \frac{\beta}{\alpha} \int_{r_1}^S k(u) \, du , \\ \widehat{t_2}(S \mid x) &= t_2(S \mid x) + 2 \, \frac{\beta}{\alpha} \, t_1(S \mid x) \int_{r_1}^S k(u) \, du \\ &\quad + 2 \, \frac{\beta}{\alpha} \int_{r_1}^S k(u) \widehat{t_1}(S \mid u) \, du , \\ \widehat{V}(S \mid x) &= V(S \mid x) + \left(\frac{\beta}{\alpha} \int_{r_1}^S k(u) \, du \right)^2 + 2 \, \frac{\beta}{\alpha} \int_{r_1}^S k(u) \, t_1(S \mid u) \, du , \end{aligned} \tag{10}$$

where $V(S \mid x)$ denotes the FPT variance.

In addition, if $\alpha > 0$ the refractoriness period is doomed to end with certainty, and its moments can be iteratively calculated as

$$E(T_r^n) := \int_0^\infty t^n \, g_r(S, t \mid S) \, dt = n \, \frac{\beta}{\alpha} \int_{r_1}^S k(u) \widehat{t_{n-1}}(S \mid u) \, du \qquad (n = 1, 2, \ldots) . \tag{11}$$

Comparing (8) and (11) we note that

$$E(T_r^n) \equiv \lim_{x \uparrow S} \widehat{t_n}(S \mid x) .$$

In particular, from (11) the first two moments and the variance of the refractoriness period are seen to be:

$$
\begin{aligned}
E(T_r) &= \frac{\beta}{\alpha} \int_{r_1}^{S} k(u)\, du \;, \\
E(T_r^2) &= 2\, \frac{\beta}{\alpha} \int_{r_1}^{S} k(u)\, t_1(S \mid u)\, du + 2 \left(\frac{\beta}{\alpha} \int_{r_1}^{S} k(u)\, du \right)^2 , \\
V(T_r) &= 2\, \frac{\beta}{\alpha} \int_{r_1}^{S} k(u)\, t_1(S \mid u)\, du + \left(\frac{\beta}{\alpha} \int_{r_1}^{S} k(u)\, du \right)^2 .
\end{aligned}
\tag{12}
$$

By virtue of (10) and (12), we have

$$
\begin{aligned}
\widehat{t_1}(S \mid x) &= t_1(S \mid x) + E(T_r) \;, \\
\widehat{V}(S \mid x) &= V(S \mid x) + V(T_r) \;.
\end{aligned}
\tag{13}
$$

The mean (variance) of first exit time through S starting from x is thus the sum of the mean (variance) of first passage time through S starting from x and of the mean (variance) of the refractoriness period.

For the Wiener, Ornstein–Uhlenbeck and Feller neuronal models, an analysis of the features exhibited by the mean and variance of the firing time and of refractoriness period can be performed. Furthermore, steady-state probability densities and asymptotic moments of the neuronal membrane potential can be explicitly obtained.

5 The firing frequency distribution

We now construct the return process $\{Z(t), t \geq 0\}$ in (r_1, S) as follows. Starting at a point $\eta \in (r_1, S)$ at time zero, a firing takes place when $X(t)$ attains the threshold S for the first time, after which a period of refractoriness of random duration occurs. At the end of the period of refractoriness, $Z(t)$ is instantaneously reset at a certain fixed and pre-specified state η. The subsequent evolution of the process then goes as described by $X(t)$, until the boundary S is again reached. A new firing then occurs, followed by the period of refractoriness, and so on.

The process $\{Z(t), t \geq 0\}$, describing the time course of the membrane potential thus consists of recurrent cycles $\mathcal{F}_0, \mathcal{R}_1, \mathcal{F}_1, \mathcal{R}_2, \mathcal{F}_2, \ldots$, each of random duration, where the durations F_i of $\mathcal{F}_i$ $(i = 0, 1, \ldots)$ and the durations of refractoriness period R_i of $\mathcal{R}_i$ $(i = 1, 2, \ldots)$ are independently distributed random variables. Here, F_i $(i = 0, 1, \ldots)$ describes the length of the firing interval, i.e. of the time interval elapsing between the ith reset at the state η and the $(i+1)$th FPT from η to S. Instead, R_i $(i = 1, 2, \ldots)$ describes the duration of ith refractoriness period. Since the diffusion process $X(t)$ is time-homogeneous, the random variables $F_0, F_1, \ldots$ can be safely assumed to be independent and identically distributed, each with pdf $g(S, t \mid \eta)$ depending only on the length of the corresponding firing interval. Furthermore, we assume that $R_1, R_2, \ldots$ are independent and identically distributed random variables, each with pdf $\varphi(t)$ depending only on the duration of the refractoriness period.

Hereafter, we shall provide a description of the random process $\{M(t), t \geq 0\}$ representing the number of firings released by the neuron up to time t. To this purpose, for all $z \in (r_1, S)$, let

$$p_n(t \mid z) = P\{M(t) = n \mid Z(0) = z\} \qquad (n = 0, 1, \dots) \tag{14}$$

be the probability of having n firings up to time t. Recalling that the diffusion process $X(t)$ is time-homogeneous and that $R_1, R_2, \dots$ are independent and identically distributed random variables, the following relations can be seen to hold:

$$\begin{aligned}
p_0(t \mid \eta) &= 1 - \int_0^t g(S, \tau \mid \eta)\, d\tau \\
p_1(t \mid \eta) &= g(S, t \mid \eta) * \varphi(t) * \Big[1 - \int_0^t g(S, \tau \mid \eta)\, d\tau\Big] \\
&\quad + g(S, t \mid \eta) * \Big[1 - \int_0^t \varphi(\tau)\, d\tau\Big] \\
p_k(t \mid \eta) &= \big[g(S, t \mid \eta) * \varphi(t)\big]^{(k)} * \Big[1 - \int_0^t g(S, \tau \mid \eta)\, d\tau\Big] \\
&\quad + g(S, t \mid \eta) * \big[\varphi(t) * g(S, t \mid \eta)\big]^{(k-1)} * \Big[1 - \int_0^t \varphi(\tau)\, d\tau\Big] \\
&\quad (k = 2, 3, \dots)\ ,
\end{aligned} \tag{15}$$

where $(*)$ means convolution, exponent (r) indicates (r)-fold convolution, $g(S, t \mid \eta)$ is the FPT pdf of $X(t)$ through S starting from $X(0) = \eta < S$, and $\varphi(t)$ is the pdf of the refractoriness period.

The probabilities $p_k(t \mid \eta)$ can be used to explore the statistical characteristics of the random variable that describes the number of firings. Now, for $k = 0, 1, 2, \dots,$ let

$$\pi_k(\lambda \mid z) = \int_0^{+\infty} \mathrm{e}^{-\lambda t}\ p_k(t \mid z)\, dt \qquad (\lambda > 0) \tag{16}$$

be the Laplace transform of $p_k(t \mid z)$. Denoting by $g_\lambda(S \mid \eta)$ and $\Phi(\lambda)$ the Laplace transforms of $g(S, t \mid \eta)$ and $\varphi(t)$, respectively, from (15) we have:

$$\begin{aligned}
\pi_0(\lambda \mid \eta) &= \frac{1}{\lambda}\Big[1 - g_\lambda(S \mid \eta)\Big] \\
\pi_k(\lambda \mid \eta) &= \frac{1}{\lambda}\, g_\lambda(S \mid \eta)\Big[g_\lambda(S \mid \eta)\, \Phi(\lambda)\Big]^{k-1}\Big[1 - g_\lambda(S \mid \eta)\, \Phi(\lambda)\Big] \\
&\quad (k = 1, 2, \dots)\ .
\end{aligned} \tag{17}$$

Further, let

$$\psi_n(\lambda \mid \eta) = \int_0^{+\infty} \mathrm{e}^{-\lambda t}\ E\big\{[M(t)]^n \mid \eta\big\}\, dt \qquad (n = 1, 2, \dots) \tag{18}$$

be the Laplace transform of the nth-order moment of the number of firings released by the neuron up to time t. From (17) it then follows:

$$\psi_n(\lambda \mid \eta) = \sum_{k=1}^{+\infty} k^n \, \pi_k(\lambda \mid \eta) = \frac{1}{\lambda} \, g_\lambda(S \mid \eta) \Big[1 - g_\lambda(S \mid \eta) \, \Phi(\lambda) \Big] \times \sum_{k=1}^{+\infty} k^n \Big[g_\lambda(S \mid \eta) \, \Phi(\lambda) \Big]^{k-1} . \tag{19}$$

In particular, using

$$\sum_{k=1}^{+\infty} k \, x^{k-1} = \frac{1}{(1-x)^2} , \qquad \sum_{k=1}^{+\infty} k^2 \, x^{k-1} = \frac{1+x}{(1-x)^3} \qquad (|x| < 1) ,$$

from (19) for $\lambda > 0$ one obtains:

$$\psi_1(\lambda \mid \eta) = \frac{g_\lambda(S \mid \eta)}{\lambda \big[1 - g_\lambda(S \mid \eta) \, \Phi(\lambda) \big]} , \qquad \psi_2(\lambda \mid \eta) = \frac{g_\lambda(S \mid \eta) \big[1 + g_\lambda(S \mid \eta) \, \Phi(\lambda) \big]}{\lambda \big[1 - g_\lambda(S \mid \eta) \, \Phi(\lambda) \big]^2} . \tag{20}$$

Let now $I_0, I_1, \dots$ denote the random variables describing the interspike intervals and $\gamma_k(t)$ be the pdfs of I_k $(k = 0, 1, \dots)$. We note that I_0 identifies with the FPT through the threshold S starting at initial state $X(0) = \eta < S$, so that $\gamma_0(t) \equiv g(S, t \mid \eta)$. Instead, I_k $(k = 1, 2, \dots)$ describes the duration of the interval elapsing between the kth spike and the $(k+1)$th spike. Since $X(t)$ is time-homogeneous and $R_1, R_2, \dots$ are independent and identically distributed:

$$\begin{aligned} P(I_1 > t \mid I_0 = \tau) &= 1 - \int_0^t \varphi(\vartheta) \, d\vartheta \\ &\quad + \int_\tau^{t+\tau} \varphi(\vartheta - \tau) \, P\{M(t + \tau - \vartheta) = 0 \mid \eta\} \, d\vartheta \\ &= 1 - \int_0^t \varphi(\vartheta) \, d\vartheta + \int_0^t \varphi(x) \Big[1 - \int_0^{t-x} g(S, u \mid \eta) \, du \Big] \, dx \\ &= 1 - \int_0^t \varphi(x) \, dx \int_0^{t-x} g(S, u \mid \eta) \, du . \end{aligned} \tag{21}$$

By virtue of the independence of $P(I_1 > t \mid I_0 = \tau)$ on τ, it follows that I_1 is independent of I_0. The iteration of this argument implies that the interspike intervals $I_1, I_2, \dots$ are independent and identically distributed random variables having pdf

$$\gamma(t) \equiv \gamma_k(t) = \int_0^t \varphi(\vartheta) \, g(S, t - \vartheta \mid \eta) \, d\vartheta \qquad (k = 1, 2, \dots) . \tag{22}$$

Hence, the mean and variance of the interspike intervals $I_0, I_1, \ldots$ are given by:

$$E(I_k) = \begin{cases} t_1(S \mid \eta), & k = 0 \\ t_1(S \mid \eta) + E(R), & k = 1, 2, \ldots \end{cases}$$
$$V(I_k) = \begin{cases} V(S \mid \eta), & k = 0 \\ V(S \mid \eta) + V(R), & k = 1, 2, \ldots, \end{cases} \tag{23}$$

where $t_1(S \mid \eta)$ and $V(S \mid \eta)$ are the mean and variance of the FPT pdf of $X(t)$ through S conditional upon $X(0) = \eta$, and where $E(R)$ and $V(R)$ are the mean and variance of refractory periods.

Denoting by $\Gamma(\lambda)$ the Laplace transform of $\gamma(t)$, from (22) one has $\Gamma(\lambda) = g_\lambda(S \mid \eta)\, \Phi(\lambda)$, so that (20) lead to

$$\psi_1(\lambda \mid \eta) = \frac{g_\lambda(S \mid \eta)}{\lambda\big[1 - \Gamma(\lambda)\big]}\,,$$
$$\psi_2(\lambda \mid \eta) = \frac{g_\lambda(S \mid \eta)\big[1 + \Gamma(\lambda)\big]}{\lambda\big[1 - \Gamma(\lambda)\big]^2}\,. \tag{24}$$

Making use of (24), for large times the asymptotic behaviors of the mean and variance of the number of firings released by the neuron can be determined as

$$E\big\{M(t) \mid \eta\big\} \simeq \frac{1}{E(I)}\, t + \frac{1}{2}\, \frac{E(I^2)}{E^2(I)} - \frac{t_1(S \mid \eta)}{E(I)}\,,$$
$$V\big\{M(t) \mid \eta\big\} \simeq \frac{V(I)}{E^3(I)}\, t + \frac{5}{4}\, \frac{[E(I^2)]^2}{E^4(I)} - \frac{2}{3}\, \frac{E(I^3)}{E^3(I)} - \frac{1}{2}\, \frac{E(I^2)}{E^2(I)}$$
$$+ \frac{t_1(S \mid \eta)}{E(I)} + \frac{t_2(S \mid \eta)}{E^2(I)} - \frac{E(I^2)}{E^3(I)}\, t_1(S \mid \eta) - \frac{t_1^2(S \mid \eta)}{E^2(I)}\,, \tag{25}$$

where $E(I^r)$ and $V(I)$ are the rth-order moment and the variance of the interspike intervals, $t_r(S \mid \eta)$ is the rth-order moment of the FPT of $X(t)$ through S conditional upon $X(0) = \eta$ and where $E(R^r)$ is the rth-order moment of refractory periods.

If $\{X(t), t \geq 0\}$ possesses a steady state distribution, we expect that for large firing thresholds an exponential behavior of the firing time pdf $g(S, t \mid \eta)$ takes place (cf. [22]):

$$\lim_{S \to +\infty} t_1(S \mid \eta)\, g\big\{S, t\, t_1(S \mid \eta) \mid \eta\big\} = \mathrm{e}^{-t}\,, \tag{26}$$

where $t_1(S \mid \eta)$ is the mean of the firing time. Hence, for large firing thresholds from (26) one has:

$$g(S, t \mid \eta) \simeq \frac{1}{t_1(S \mid \eta)}\, \exp\Big\{-\frac{t}{t_1(S \mid \eta)}\Big\}. \tag{27}$$

Finally, for long times and large firing thresholds, the mean and variance of the number of firings released by neuron up to time t are approximatively:

$$
\begin{aligned}
E\{M(t) \mid \eta\} &\simeq \frac{1}{E(I)}\, t + \frac{1}{2}\,\frac{E(R^2)}{E^2(I)}\,,\\
V\{M(t) \mid \eta\} &\simeq \frac{V(R) + t_1^2(S \mid \eta)}{E^3(I)}\, t + \frac{1}{E^4(I)}\left\{\frac{5}{4}\left[E(R^2)\right]^2\right.\\
&\quad - \frac{1}{2}\, E^2(R)\, E(R^2) + \frac{3}{2}\, E(R^2)\, t_1^2(S \mid \eta)\\
&\quad \left. + E(R)\, E(R^2)\, t_1(S \mid \eta) - \frac{2}{3}\, E(R^3)\, E(I)\right\}.
\end{aligned}
\tag{28}
$$

6 Requiescam in pace

Of course, the story does not end here. Several further developments are envisaged:

1. use of the foregoing methods to analyse specific neuronal models such as the classical ones based on Wiener, Ornstein–Uhlenbeck and Feller processes;
2. analysis of the role of various types of random refractoriness in models characterized by reversal potentials;
3. determination of the distribution of the number of firings released in a pre-assigned time interval and its asymptotic behavior;
4. determination of the interspike intervals density for the most popular neuronal models under various choices of refractoriness distributions.

Since endeavours along these directions are in progress under the leadership of Amelia Nobile and Nivia Giorno, this author is confident that all the above itemized tasks will be satisfactorily accomplished in the not too distant future. Quite differently from the neuronal refractoriness case discussed in this paper, he shall thus presumably be spared from having to reconsider them in 40 years; surely a very difficult enterprise!

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Cosmological Implications of Caianiello's Quantum Geometry*

Gaetano Scarpetta

1 Introduction

The formulation of a consistent theory of quantum gravity is one of the key questions still open today. Attempts to reconcile the principles of general relativity and quantum theory face severe difficulties, suggesting that a considerable departure from the standard space-time picture may be inevitable, taking into account that at distances comparable to the Planck length $\sqrt{\hbar G/c^3}$, fluctuations of the geometry dissolve the classical smooth manifold picture of space-time.

In 1980 Caianiello [1, 2] approached the problem of unification of quantum mechanics and general relativity from a nonstandard point of view, interpreting quantization as curvature of the relativistic eight-dimensional space-time tangent bundle $TM = M_4 \otimes TM_4$, where M_4 is the usual flat space-time manifold of metric $\eta_{\mu\nu}$ and signature -2. In this space the standard operators of the Heisenberg algebra are represented as covariant derivatives and the quantum commutation relations are interpreted as components of the curvature tensor. The Born reciprocity principle or, equivalently, the symmetry between configuration and momentum space representation of field theory, is thus automatically satisfied in this scheme.

Trying to obtain space-time geometry from quantum commutation relations, Caianiello was driven to modify properly and radically the metric structure of space-time, replacing the four-dimensional space-time invariant, $ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu$ by a new physical invariant, i.e. $d\tau^2 = g_{AB} dx^A \, dx^B$, representing the infinitesimal distance in the relativistic eight-dimensional space-time tangent bundle TM, whose coordinates are $X^A = \left(x^\mu, \frac{c^2}{\mathcal{A}} \frac{dx^\mu}{ds}\right)$, with $x^\mu = (ct, \vec{x})$ the usual space-time four-vector, $dx^\mu/ds = \dot{x}^\mu$ the relativistic four-velocity, and $\mathcal{A}$ the maximal proper acceleration. (Conventions: $A, B = 0, \ldots, 7$, $\mu, \nu = 0, \ldots, 3$.) In the absence of gravity, $g_{AB} = \eta \otimes \eta$ (where $\eta = diag(+, -, -, -)$ is the Minkowski metric). The causality constraint $d\tau^2 \geq 0$ implies that proper accelerations must be limited, $|\ddot{x}| \leq \mathcal{A}$, in quantum geometry [3].

In 1984 Caianiello gave a proof [4–6] that an upper limit on the proper accelerations, fixed by the rest mass of the particle itself, $\mathcal{A} = 2mc^3/\hbar$, stems directly from the Heisenberg uncertainty relations of quantum mechanics. A detailed discussion of this point is given in the paper by G. Papini in this volume.

The maximal proper acceleration has been introduced with different motivations and from quite independent approaches [7–15]; in some of these works the maximal acceleration is a universal constant: $\mathcal{A} = 2m_P\frac{c^3}{\hbar}$; its value is fixed by the Planck mass $m_P = \sqrt{\hbar c/G}$, with G the gravitational constant.

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The introduction of an invariant interval in the eight-dimensional space-time tangent fiber bundle TM, and the consequent maximal proper acceleration, may also be interpreted as a regularization procedure of the field equations [16], alternative to that in which space-time is quantized by means of a fundamental length, as in a spin foam [17], or as in [18], where it is shown that, at the one loop level, the two-point and four-point Green's functions of a massless scalar field theory are explicitly devoid of ultraviolet divergences. *The advantage of Caianiello's quantum geometry theory is to preserve the continuum structure of space-time.*

An upper limit on the acceleration has been previously introduced, although not as a fundamental physical property, in the context of the physics of massive extended objects, where the critical acceleration is determined from the extension of the particles and from the causal structure of the space-time manifold.

For instance, in classical relativity [19] an object of proper length λ, in which one extreme is moving with acceleration a with respect to the other, develops a Rindler horizon at a proper distance a^{-1} from the accelerated extremity, so that all parts of the object can be causally connected only if $\lambda < a^{-1}$; this implies a proper critical acceleration $a_c \simeq \lambda^{-1}$ which depends on λ and diverges in the limit at which the object reduces to a point-like particle.

In the quantum relativistic context, the analysis of string propagation in cosmological backgrounds reveals that accelerations higher than the critical one give rise to the onset of Jeans-like instabilities [20, 21] in which the string oscillating modes develop imaginary frequencies and the string's proper length diverges. A kinematic interpretation of this string instability [22] shows that it occurs when the acceleration induced by the background gravitational field is large enough to render the two string extremities to be causally disconnected because of the Rindler horizon associated to their relative acceleration. This critical acceleration a_c is determined by the string size λ and is given by $a_c = \lambda^{-1} = (m\alpha)^{-1}$ where m is the string mass and α^{-1} the usual string tension. Frolov and Sanchez [23] derive a universal critical acceleration $a_c \simeq \lambda^{-1}$ as a general property of the strings.

In previous cases the critical acceleration arises as a dynamical effect due to the interplay of the Rindler horizon with the finite extension of the string; in Caianiello's proposal the maximal proper acceleration is a basic physical property, an inescapable consequence of quantum mechanics, and it must be included from the outset in the physical laws. This requires a modification of the metric structure of space-time. It leads, in the case of Rindler space, to a manifold with a nonvanishing scalar curvature and a shift in the horizon [24, 25]. The cut-off on the acceleration is the same as that required in an *ad hoc* fashion by Sanchez in order to regularize the entropy and the free energy of quantum strings [26] and introduced by McGuigan in the calculation of black hole entropy [27]. The dynamics of accelerated strings in the context of quantum geometry theory has been analyzed in [28].

In the cosmological framework, the maximal proper acceleration is a universal constant depending on the Planck mass: $\mathcal{A} = 2\sqrt{\frac{c^7}{\hbar G}}$. In order to analyze the consequences in cosmology of the quantum geometry theory, we have to study the dynamical laws defined through a suitable action in the eight-dimensional space-time tangent bundle TM, or by means of an embedding procedure [24, 25, 29], the first step of a process of successive approximations, by which one reduces to an effective four-dimensional space-time geometry. The embedding procedure constructs an effective new space-time metric $\tilde{g}_{\mu\nu}(\xi)$, through the eight parametric equations $x^A = x^A(\xi^\mu)$ that correlate the coordinates x^A of TM to the coordinates ξ^μ chosen to parametrize the four-dimensional space-time manifold M_4:

$$\tilde{g}_{\mu\nu}(\xi) = g_{AB}\frac{\partial x^A}{\partial \xi^\mu}\frac{\partial x^B}{\partial \xi^\nu} = g_{\alpha\beta}\left(\frac{\partial x^\alpha}{\partial \xi^\mu}\frac{\partial x^\beta}{\partial \xi^\nu} + \frac{1}{\mathcal{A}^2}\frac{\partial \dot{x}^\alpha}{\partial \xi^\mu}\frac{\partial \dot{x}^\beta}{\partial \xi^\nu}\right) \tag{1}$$

The first-order approximation introduced by this procedure consists in defining the eight parametric equations $\{x^\mu = x^\mu(\xi^\alpha); \dot{x}^\mu = \dot{x}^\mu(\xi^\alpha)\}$ by the solutions to the ordinary relativistic equations of motion. For instance, in a pure gravitational case, the velocity field obeys the classical geodesic equation of motion

$$d\dot{x}^\mu = -\Gamma^\mu_{\alpha\beta}\dot{x}^\alpha dx^\beta \tag{2}$$

and the "geodesic embedding" induces the following effective space-time metric $\tilde{g}_{\mu\nu}$:

$$d\tau^2 = \tilde{g}_{\mu\nu}dx^\mu dx^\nu = \left[g_{\mu\nu} + \frac{1}{\mathcal{A}^2}\Gamma_{\alpha\mu\beta}\Gamma^\alpha_{\nu\gamma}\dot{x}^\beta\dot{x}^\gamma\right]dx^\mu dx^\nu \tag{3}$$

This effective metric depends on the space-time x^μ, on the four-velocity field $\dot{x}^\mu$ and on the connection Γ and thus represents a generalization of the metric of Finsler spaces. The embedding procedure produces corrections to the given four-dimensional geometry, which disappear in the classical limit $\mathcal{A} \to \infty$; however these corrections induce in general a nonvanishing curvature, even starting from an eight-dimensional space-time tangent bundle TM with a flat metric.

The maximal proper acceleration implies interesting consequences in cosmology: in particular it avoids the divergence of curvatures and densities [30, 31]; this fact depends on the observation that the problem of limiting the curvature of a given background is equivalent to the problem of limiting the relative acceleration between two points of an elementary extended object.

In a curved space-time, in the absence of forces other than gravity, different points of a free falling particle will fall along different geodesics; the relative acceleration between these two points, separated by a spacelike distance λ, is given by the equation of geodesic deviation

$$a^\mu = -R^\mu{}_{\alpha\nu\beta}z^\nu u^\alpha u^\beta \tag{4}$$

where z^μ is a space-like vector connecting the two ends, $z^2 < 0$, $|z| = \lambda$, orthogonal to the velocity vector, $z^\mu u_\mu = 0$. The proper strength of this relative acceleration depends only on the components of the background curvature tensor, $R_{\mu\nu\alpha\beta}$, and then it diverges if and only if the curvature diverges.

We consider the case of a spatially homogeneous and isotropic background, described by the standard Friedmann–Robertson–Walker (FRW) scale factor $R(t)$.

In the local free falling frame of one of the two ends, supposed at rest in the comoving frame, the other end of the object has an acceleration, according to Equation (4), given by $|a| = \lambda|\ddot{R}/R|$ (where a dot denotes derivative with respect to the cosmic time t). The presence, in this geometry, of an extended object of minimal size λ is thus compatible with the causal bound on the accelerations, provided the curvature satisfies the Rindler constraint $|\lambda^2\ddot{R}/R| < 1$.

This constraint fixes a maximum allowed curvature, which cannot be satisfied by the standard FRW cosmology, in which the dominant source of gravity is a perfect fluid, and the curvature diverges like the energy density as the universe approaches the initial singularity at $t = 0$. We can say that singular cosmological models are not compatible with an early dominance of extended particles [10]. The maximal proper acceleration, on the contrary, modifies the geometry of the given singular FRW background, so as to satisfy the Rindler bound for objects of given size λ.

In Section 2, the geometric interpretation is given in terms of geodesic embedding of the given four-dimensional space-time into the larger eight-dimensional space-time tangent bundle TM, whose coordinates are positions and velocities. The interesting feature of this mechanism is that, as shown in Section 3, the modified geometry always includes, *whatever the given geometry may be*, an initial phase of exponential inflation, with expansion rate fixed by λ; at sufficiently late cosmic time, moreover, the modified geometry coincides with the original one. The proposed mechanism may also be regarded, therefore, as a standard procedure to generate deflationary cosmological models [32], i.e. models characterized by a metric which evolves smoothly from an initial de Sitter phase to a final state of decelerating FRW expansion.

2 Geodesic embedding and maximal curvature

A causal structure in which proper accelerations cannot exceed a given value $\mathcal{A} = \lambda^{-1}$ can be imposed over a space-time M_4, in the context of a geometric scheme [24, 25, 29] in which the given background M_4, with coordinates ξ^μ and metric $g_{\mu\nu}(\xi)$, is regarded as a four-dimensional hypersurface embedded in an eight-dimensional space-time tangent bundle TM, with metric $g_{AB} = g_{\mu\nu} \otimes g_{\mu\nu}$ and coordinates $x^A = (x^\mu, \lambda u^\mu)$, where $u^\mu = dx^\mu/ds$ is the usual velocity vector and $ds = (dx^\mu dx_\mu)^{\frac{1}{2}}$ (we shall use natural units in which $\hbar = c = 1$). The embedding of M_4 into TM, determined by the eight parametric equations $x^\mu = x^\mu(\xi^\alpha)$ and $u^\mu = u^\mu(\xi^\alpha)$, defines in fact a new space-time metric $\tilde{g}_{\mu\nu}(\xi)$, locally induced by the TM invariant interval, i.e.

$$d\tilde{s}^2 = g_{AB}dx^A\, dx^B = g_{\mu\nu}(dx^\mu dx^\nu + \lambda^2 du^\mu du^\nu) \equiv \tilde{g}_{\mu\nu}d\xi^\mu d\xi^\nu \tag{5}$$

where $\tilde{g}_{\mu\nu}$ is given by Equation (1) $e\lambda = A^{-1}$.

$$\tilde{g}_{\mu\nu} = g_{\alpha\beta}\left(\frac{\partial x^\alpha}{\partial\xi^\mu}\frac{\partial x^\beta}{\partial\xi^\nu} + \lambda^2\frac{\partial u^\alpha}{\partial\xi^\mu}\frac{\partial u^\beta}{\partial\xi^\nu}\right)$$

Along any given path, $x^\mu(s)$ and $u^\mu(s)$, the generalized proper-time interval (5) becomes $d\tilde{s}^2 = ds^2(1 - \lambda^2|a|^2)$, where $|a|^2 = |(du^\mu/ds)(du_\mu/ds)|$, so that the causality requirement $d\tilde{s}^2 > 0$ implies $|a| < \lambda^{-1}$.

This scenario implies that, at some fundamental level, space-time must loose the privileged role characteristic of the classical, macroscopic context, and only eight-dimensional space-time tangent bundle TM becomes physically meaningful. Special and general relativity, in particular, are to be generalized in such a way that the world geometry is determined by eight-dimensional gravitational equations.

The new metric $\tilde{g}_{\mu\nu}$ depends both on the coordinates ξ_μ chosen to parametrize M_4, *and on the velocity field*, $u^\mu(\xi)$, defined on it [24, 25, 29]. Assume to start with a given geometry described by a classical metric $g_{\mu\nu}$ on M_4. A particle coupled to this metric, in the absence of forces other than gravity, would tend to evolve in time according to the geodesic of $g_{\mu\nu}$. The corresponding velocity field $u^\mu(\xi)$, solution of the geodesic equations, defines an embedding of M_4 into TM which we shall call "geodesic embedding". The geodesic embedding of a singular FRW metric $g_{\mu\nu}$, in particular, leads to a modified metric $\tilde{g}_{\mu\nu}$ which satisfies the Rindler bound $\lambda|a| < 1$ on the relative accelerations. In order to illustrate this effect let us consider, for simplicity, a starting geometry parametrized by the conformal coordinates $\xi^\mu = (\eta, \vec{x})$, described by the metric

$$g_{\mu\nu}(\xi) = \mathrm{diag}\, R^2(\eta)(1, -1, -1, -1) \tag{6}$$

where η is the usual conformal time, defined by $R = dt/d\eta$, and R is the FRW scale factor. For an extended particle co-moving in this background, we choose the velocity field $u^\mu(\xi) = (R^{-1}, 0, 0, 0)$ which satisfies the geodesic equations for the metric (6). The geodesics embedding defined by this field leads, according to (5), to the generalized line-element

$$d\tilde{s}^2 = R^2(d\eta^2 - d\vec{x}^2) + \lambda^2 R^2 \left(\frac{dR^{-1}}{d\eta}\right)^2 d\eta^2$$

so that the corresponding generalized metric, written in terms of the original scale factor $R(\eta)$, becomes

$$\tilde{g}_{\mu\nu}(\xi) = \mathrm{diag}\, R^2 \left(1 + \lambda^2 \frac{R'^2}{R^4}, -1, -1, -1\right) \tag{7}$$

(a prime denotes differentiation with respect to η).

As a consequence, even starting from a FRW background in which the relative accelerations $|a|$ between different points of an extended object are unbounded, the curvature of the modified metric (7) turns out to be regularized, in such a way that the condition $\lambda|a| < 1$ is automatically satisfied.

Consider in fact the relative acceleration between two ends of an extended object of minimal size λ, which for the conformally flat metric (6) can be written, according to Equation (4),

$$|a| = \lambda \left| \frac{R''}{R^3} - \frac{R'^2}{R^4} \right| \tag{8}$$

For the generalized metric (7), instead, the acceleration (for the same separation λ) is

$$|\tilde{a}| = \frac{\lambda}{\left(1+\lambda^2\frac{R'^2}{R^4}\right)^2}\left|\frac{R''}{R^3} - \frac{R'^2}{R^4} + \lambda^2\frac{R'^4}{R^8}\right| \tag{9}$$

In the original FRW model, with flat spatial sections, the cosmological constant is vanishing and the gravitational sources are represented as a perfect fluid. Using the Einstein field equations one can show that, in the limit in which the cosmic time approaches zero, the acceleration $|a|$ of (8) diverges, for finite λ, like $H^2 = R'^2/R^4$ (remember that $R'/R^2 = \dot{R}/R \equiv H$ defines the standard Hubble expansion rate).

In the same limit, however, the corrections to the metric induced by the geodesic embedding diverge as H^4, so that their contributions to the acceleration, in (9), become dominant. We get then, in particular, that $|\tilde{a}| \to \lambda^{-1}$ whereas $|a|$ was divergent for the old metric, and that, for the new metric, $\lambda|\tilde{a}| \leq 1$.

The regularized geometry has a maximum curvature determined by the minimal size of the extended particles, so preventing the occurrence of relative accelerations larger than the Rindler limiting value λ^{-1}.

3 Deflationary behavior of the modified metrics

An interesting aspect of the new geometry, characterized by an initial phase of maximal finite curvature, is that it unavoidably includes a period of exponential inflation, since the initial phase of maximal curvature is always described by a de Sitter geometry, with limiting value of the Hubble parameter ($H = \lambda^{-1}$), *quite independently from the particular form of the geometry* (i.e. of the scale factor) *before the embedding*.

This property of the modified metric (7) can easily be verified in the new comoving frame, in which the modified geometry assumes the standard FRW form, by introducing a cosmic time t such that

$$t = \int d\eta \left(R^2 + \lambda^2\frac{R'^2}{R^2}\right)^{\frac{1}{2}} \tag{10}$$

In fact, in the regime in which the corrections become dominant, $\lambda R'/R^2 >> 1$, we have from (10) $\lambda\dot{R}/R = 1$, whose integration gives $R(t) = \exp(t/\lambda)$, i.e. the scale factor for a de Sitter manifold of minimal size $H^{-1} = \lambda$.

It should be noted that the maximum curvature of this phase cannot be exceeded, in this context, even if the starting metric is already a de Sitter one, with arbitrary value of the given cosmological constant $\Lambda = 3H^2$. In fact the modified metric, after the embedding, is still a de Sitter metric corresponding, however, to a new cosmological constant $\tilde{\Lambda} = 3\Lambda/(3+\lambda^2\Lambda)$, which is bounded, $\tilde{\Lambda} \leq 3/\lambda^2$, even if $\Lambda \to \infty$. The embedding procedure defines thus a maximum allowed value for the cosmological constant, $\Lambda_M = 3/\lambda^2$, compatible with the presence of extended particles of minimal size λ.

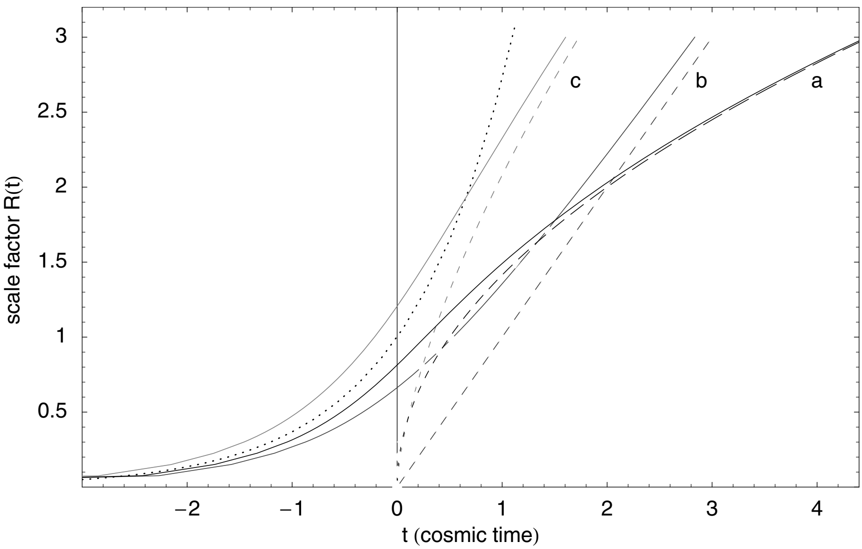

Fig. 1. *Dashed lines* describe cosmological models before embedding: (a) $R \propto t^{1/2}$, radiation-dominated expansion; (b) $R \propto t$, linear expansion; (c) $R \propto t^{2/3}$, matter-dominated expansion. *Solid lines* correspond to geometries after the embedding. Note that the singularity at $t = 0$ is avoided ($R(0) \neq 0$ for solid lines), and that all the modified geometries converge to an initial de Sitter phase of exponential expansion, $R \propto e^t$, represented in this plot by a *dotted line* (we have used units in which $\lambda = 1$)

The modified metrics (7) which, as we have seen, are of the de Sitter type at sufficiently early time, tend to coincide, on the other hand, with the original metric (6) when the background curvature becomes much smaller than the scale fixed by λ, as can easily be verified from (10). (These models provide a possible solution to the "graceful exit" problems that beset many inflationary scenarios.) The deflationary behavior of these metrics is illustrated in Fig. 1, for the three important cases in which the original metrics correspond to the radiation-dominated, matter-dominated, and linearly expanding models of FRW cosmology (the scale factors for the modified metrics are obtained from (10), by inserting the scale factors of the original metrics which, in terms of the conformal time, are given respectively by $R(\eta) \propto \eta,\ R(\eta) \propto \eta^2$ and $R(\eta) \propto \exp \eta$).

4 Discussion

The quantum geometry scheme, with its maximal proper acceleration constraint, introduced by Eduardo R. Caianiello in the early 1980s, revealed a very fruitful idea also in cosmology. Deflationary models in general, and the case of string-driven inflation in particular, can be automatically reproduced by modification of the space-time geometry induced by the maximal proper acceleration, which prevents the occurrence of accelerations larger than the Rindler limit. According to quantum geometry, the dominance of extended particles in the early universe prevents the big-bang singularity, by limiting the allowed curvature to a maximum value reached during an initial de Sitter phase. The corresponding curvature radius is fixed by the size of the object, $\lambda = H^{-1}$.

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Imagination and Rigor: Their Interaction Along the Way to Measuring Fuzziness and Doing Other Strange Things

Settimo Termini

1 Meandering around the problem

The development of information sciences, in their widest sense, has pointed out the crucial importance of the complementary role that innovative ideas and the power mathematical and technical tools play in this process showing how new emerging fields establish themselves as valid. This is also evident in the methodological attitude of a scientist like Eduardo Caianiello who, as a physicist, thought that the tradition of physics could provide some breakthroughs to the development of the new disciplines related to the mental and the investigation of intelligence. In the present paper some topics will be briefly discussed trying to point out the importance of the connection between intuition and rigor and, in particular, the process underlying the passage from a new intuitive idea to its formal realization. This process which has been outlined by Rudolph Carnap in his analysis of the notions of *explicandum* and *explicatum* is a form of what Gregory Bateson has superbly described as the tension existing between *imagination* and *rigor*, the couple of terms we chose as title of this volume of essays in honor of Eduardo Caianiello. The chosen topics all concern Eduardo's teachings and the multiplicity of his scientific interests, although in different ways and in various degrees, as will be clear in the following.

It has become more and more evident that the history and epistemology of science is strictly intertwined with science proper (what could, perhaps, be called its *technical* content) and in many cases the level of deepening (and formal elaboration) of the logical and conceptual analyses of the central notions of a specific discipline or theory done in a certain moment, determines the shape it assumes. So when we are trying to analyze the development of a certain field of investigation, one good strategy could be to look for the equilibrium that the used concepts and the necessary mathematical or formal tools – in simpler words, the *imagination* and the *rigor* – are reaching. This is particularly evident in the case of disciplines having to do with the "immaterial" (information, feedback, computation, cognition etc.), in a word (in an old word that today is not very fashionable), with cybernetics that, according to Eduardo, belongs to the sciences of intelligence (and which – according to his Linnaean classification – come after the sciences of the matter and the sciences of living). The reason for this fact is simple to understand. First, we are dealing with very difficult problems and our knowledge of these fields is relatively poor, when compared to our knowledge of physics (although their advancement is noteworthy) and, what is more, due to their subject matter, these disciplines come nearer to the necessity of analyzing what we are doing when we are reflecting on what we are doing. Feedback can be overreached by a sort of true short circuit. We know that things at this point become tremendously complicated. We can face the emergence of unpleasant paradoxes if we are not careful enough; but we also know that there are ways of facing the presence of self reference without producing short circuits. Another good strategy is that circumscribing domains of investigation that are relevant and not too close to the line where dangerous things can happen. Anyway, no simple solutions are graciously granted by some benevolent god. We must earn our salary as scientists and for doing this we must also understand what is possible and what is not possible at a certain moment, understanding the nature of the limitations that we must face and which may belong to mathematics, to the available technologies and, in some cases, also to the social reception of certain results as well as with the social acceptance of science in general.

2 Modeling and measuring fuzziness

I was introduced to fuzzy sets by Eduardo just after my arrival at the Laboratorio di Cibernetica (as then the Institute was called) in 1969. As was typical of Eduardo's he never told that he had read the paper and had found the *paper* interesting. He told me that Lotfi Zadeh had spoken to him of a new challenging and intriguing *idea*. I am quite sure that he really did not read the paper, maybe he skimmed it, although he was able to discuss its content on the basis of the conversation with Zadeh and his capacity of extracting the essential things. My first reflections on the subject resulted in a disappointment since the attempt of developing a quantum version of Zadeh's idea as well as the first results in this direction was cancelled by the appearance of Watanabe's paper [1] a few months later. It is interesting to recall Eduardo's reaction to that: if you were looking for something that senior people found out, well it means that you were headed in the right direction. The following step was the "definition of a non-probabilistic entropy in the setting of fuzzy sets theory" proposed and developed together with Aldo de Luca (a side effect of these investigations was the origin of my interest in the analysis of the presence of vague predicates in scientific theories[1]). This is not the place for providing details of the theory developed starting from this initial idea; the reader is referred to the references (and for a birds eye view to the Appendix). What these pages aim at instead is to briefly analyze the development of these ideas in light of the process of the interaction between *explicandum* and *explicatum* in this particular case.

2.1 As a sort of *introduction*

The concept of "fuzzy set", as is well known, was introduced by Lotfi Zadeh in 1965 [9]. He had already outlined the general idea in some previous papers under some different names. For a survey of this early history one can read the paper by Brian Gaines [10], which is also recommended for many other interesting discus-

[1] For ways of tackling the problem of the formalization of vague predicates different from Zadeh's, see, for instance, [2–5], and for the author's view [6–8].

sions. In a different setting and with a different terminology, an anticipation of the basic idea – although constrained by a probabilistic, however undogmatic, vision – can be traced in some brief notes by Karl Menger (see, for instance, [11, 12]) as well as in some remarks by Thoralf Skolem [13]. Finally, the proposal of Dieter Klaua shortly followed the appearence of Zadeh's seminal paper. A brief history of the work by Klaua and his group was outlined by Sigfried Gottwald in [14].

Why among all these attempts (and many others not mentioned here) is the one of Zadeh the unique one having a sudden impact from a sociological point of view coming to involve in a short period of time greater and greater numbers of researchers. Joseph Goguen has connected the general philosophy underlying Zadeh's proposal of "the social nature of truth". Another interesting phenomenon to be analyzed by historians of ideas is the apparently easier acceptance in eastern countries (China, Japan and India) of his idea. One main reason may be that among all these attempts, Zadeh's proposal is the only one bravely departing from the classical traditional use of mathematics without bothering too much about possible criticism and consequences; at the same time, it is also the only approach motivated by applications and, namely, by the inadequacy of classical languages to satisfactorily represent novel situations in applicative fields.

One of the most challenging points of Zadeh's proposal was that in the case of very complex systems, one could reduce the complexity by using a less idealized language from the start. It is clear that if one accepts the existence of fuzzy theories as acceptable models (i.e., as mathematically meaningful descriptions) of concrete systems of a high level of complexity, the question arises whether it is possible to control (and measure) the level of fuzziness present in the considered description. In discussions between Aldo de Luca and myself the idea of attempting to tackle this problem in a very general way emerged [15]. We were aware that many preliminary problems were present. One of these had to do with the fact that it is not clear how general the notion of fuzzy sets is as well as with the problem of the relationship between fuzziness and probability. We decided to move, strictly inside Zadeh's proposal. Possible changes, extensions and conceptual problems, would be taken into account at another time. The second point has to do with the choice of the most fruitful path to be followed in order to obtain a theory of the control of fuzziness that could be used in real, concrete applications. The simple idea followed was the one of proceeding by strictly connecting requirements to be imposed and measures satisfying these requirements, keeping in mind, however, the fact that not every requirement one could abstractly envisage could always be imposed in each specific situation. In other words, the axioms should not be put all on the same level. One should pick up some basic properties and requirements, necessary to characterize something that could be called *"entropy measure"* or *"measure of fuzziness"* of a fuzzy set. Other requirements could be imposed depending on the particular situation under study. By proceeding in this way, one could have available a wide class of measures from which one could choose the most adequate for the specific problem under study.

2.2 Fuzzy sets and measures of fuzziness a few decades later

Fuzzy sets are now 40 years old (more, if we take into account the preliminary intuitions of the same Zadeh or other related ideas from other authors recollected in the previous paragraph. Also, the theory of the measures of fuzziness is now more than 30 years old. So it is useful to draw some connections between the "general" contributions provided by way of tackling problems proposed by Zadeh (and, inside this setting, by the theory of measures of fuzziness).

Let me preliminarily observe that the development of fuzzy logic, triggered by the problems posed by the theory of fuzzy sets, has contributed to a rediscovery of Łukasiewicz infinite-valued logics also enriching them with new suggestions and contributions coming from different fields and which helped the introduction of "generalized" connectives. All this work then is an a posteriori support to the conviction of von Neumann[2] that escaping the constraints of all-or-none concepts would allow one to introduce (and use) results and techniques of mathematical analysis in the field of logic, making possible both major flexibility of logical tools and its wider application to different fields.[3]

As far as a general evaluation of fuzzy sets is concerned the global positive impact of Zadeh's approach is clear: his idea has triggered very strong conceptual innovations and *sperimentations*.

I shall limit myself to add a few unorthodox comments. Paradoxically, his general acceptance can be, in a sense, attributed to two not-so-well-known results, which apparently clashes with the original ambitions of the theory. First, due to a representation theorem by Pultr [16] all the machinery of fuzzy sets can be interpreted and translated into purely traditional mathematical terms by using only the classical notion of distance. Secondly, a long-standing controversy centreed about similarities and differences of fuzziness and probability, is solved in the setting of coherent conditional probabilities (see Sect. 4.1, below).

At the end of its long journey we can say that the notion of fuzzy set has affirmed itself – as all true innovations – as a very classical idea.

For what concerns the measures of fuzziness, in my view, the basic kernel of the theory may be considered fairly complete now, after the general classification of the various families of measures provided by Ebanks [17]. New proposals have appeared,

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[2] Let us remember that already in 1949 he had maintained that: "There exists today a very elaborate system of formal logic, and, specifically, of logic as applied to mathematics. This is a discipline with many good sides, but also with certain serious weaknesses. ... About the inadequacies ... this may be said: Everybody who has worked in formal logic will confirm that it is one of the most refractory parts of mathematics. The reason for this is that it deals with rigid, all-or-none concepts, and has very little contact with the continuous concept of the real or of the complex number, that is, with mathematical analysis. Yet analysis is the technically most successful and best-elaborated part of mathematics. Thus formal logic is, by the nature of its approach, cut off from the best cultivated portions of mathematics, and forced onto the most difficult part of the mathematical terrain, into combinatorics." [62, p. 303]

[3] In the following, another suggestion of von Neumann, namely the one of treating error thermodynamically, will be recollected in relation to the theory of measures of fuzziness. It is interesting to observe that fuzzy sets and the measures of fuzziness are strictly intertwined with two challenging observations of von Neumann and that the intertwining of imagination and rigor in this field has produced small steps in the direction envisaged by his remarks.

but many of these seems to ignore Ebanks' general classification. A notable exception is provided by Yager who proposed a new challenging idea, the one of measuring the fuzziness of a fuzzy set by means of its distance from the complement. However, it can be shown that every time a Yager measure is definable a measure of fuzziness can be also defined (see [18, 19] and a few brief remarks in the Appendix). So, we can conclude that this theory is robust and general enough to subsume new suggestions arising from different conceptual starting points without being substantially changed. The discovery of new general requirements to be imposed under the form of new axioms does not appear very likely as long as we move inside the basic scheme of the theory of fuzzy sets with the standard connectives. What is open to future work is the adaptation of the proposed axiomatic scheme to some variants of it. We may change the connectives (see, e. g., [20]) or also the range of the generalized characteristic functions.[4] Two developments in this direction can be found in [22] and in [25, Sect. 3.4]. The problem of *measuring* vagueness in general remains completely open, i.e., of constructing a *general* theory of measures of vagueness, a goal which presupposes the existence of a general formal theory of vague predicates.

Let me briefly comment on the connection between measures of fuzziness and von Neumann's remarks on the role of error in logics. The program of constructing a calculus of thermodynamical type which could be considered a development of von Neumann's idea of treating error thermodynamically[5] was explicitly mentioned in [23]. Measures of fuzziness are indeed an element which could contribute, inside the general framework of the theory of fuzzy sets, to the construction of a sort of *"thermodynamical logic"*.

They can, in fact, be viewed as a particular way of studying the levels of precision of a description. From this point of view they can already represent a treatment of error of a "thermodynamical" type in some definite – albeit still vague – sense. They, moreover, can be inserted in logical inference schemes in which approximation, vagueness, partial or revisable information play a role either at the level of the reliability of the premises or of the inference rules (or both). A satisfactory and fairly complete integration of all these aspects remains, however, to be done.[6] We shall briefly revisit this problem in an enlarged setting in the next subsection.

[4]The proposal has been recently made, for instance, to take the complex instead of the real numbers. In this, as in similar cases, one could take as a starting point the scheme outlined in [21], where the general case of an L-fuzzy set is considered for defining a measure of fuzziness.

[5]In [63, p. 329], von Neumann writes: "The subject matter ... is the role of error in logics, or in the physical implementation of logics – in automata synthesis. Error is viewed, therefore, not as an extraneous and misdirected or misdirecting accident, but as an essential part of the process under consideration ... ."

[6]Some preliminary results (see, for instance, [24]) obtained inside approaches making use of the measures of fuzziness (instead of the standard probabilistic ones) for measuring the level of reliability of an inference done under uncertainty conditions induce one to believe that this integration can produce interesting results.

2.3 Remarks on *"information dynamics"*

The informal notion of "information" (the *explicandum*) is very rich and multifaceted and so it is not strange that the formal theories that have been proposed do not capture all the nuances of the informal notion.

One could consider isolating some meaningful and coherent subsets of the properties and features of the explicandum and look for satisfying formalizations of these aspects. Since they are different aspects of one unique general concept anyway we must also pick up and study the way in which these subaspects interact. The process suggested above points then not to a very general but static theory of information in which a unique formal quantity is able to take the burden of a multifaceted informal notion, but instead pinpoints an *information dynamic* in which what the theory controls is a whole process (along with – under the pressure of changes in the boundary conditions – the relative changes of the main central (sub)notions involved in the theory itself and their mutual interactions). In this way we pass from a situation in which there is only one central notion on the stage to another one in which a report of what is happening in a process (in which information is transmitted, exchanged and the like) is provided by many actors on the stage, each of which represents one partial aspect of what the informal use of the word information carries with it. This scenario resembles the one of thermodynamics: no single notion suffices for determining what is happening in the system and the knowledge of the value assumed by one of the thermodynamical quantities can be obtained only as a function of (some of) the others, by knowing (and working out) the quantitative relationships existing among them. That is what an *informationdynamic* must look for: its principles, laws which quantitatively state the connections existing among some of the central quantities of the theory.

In [28], I tried to outline how a program of this type could be looked for in the setting of the theory of fuzzy sets. I shall briefly summarize now the general ideas. It is well known that many quantities have been introduced to provide a global (one could say, "macroscopic") control of the information conveyed by a fuzzy set; for instance, *measures of fuzziness* (see [15, 17, 23, 25]) or *measures of specificity* (see [26, 27]).

Measures of fuzziness want to provide an indication of how far a certain fuzzy set departs from a classical characteristic function; measures of specificity, instead, want to provide an indication of how much a fuzzy set approaches a singleton. These two classes of measures certainly control *different* aspects of the information conveyed by a fuzzy set; they are not, however, conceptually unrelated. If the measure of fuzziness is maximum (i.e., all the elements have a "degree of belonging" equal to 0.5) then we indirectly know something about specificity. Also, if the measure of specificity informs us that we are dealing exactly with a singleton, we can immediately calculate the corresponding measure of fuzziness. But a relationship between these measures also exists in not such extreme cases; in order to refine the way in which this kind of knowledge can be exchanged one could think to introduce other measures. An important role in this sense can be played by (generalized) cardinality of the fuzzy set and (if not all the elements are on an equal footing) also by a weighted cardinality, for instance, the so-called “energy” of a fuzzy set [23]. It would then be very interesting to have some explicit quantitative relations among these measures, since it would provide a way of transforming our knowledge regarding one or two of these quantities in a (more or less approximate) knowledge of the remaining one(s). All this stuff – this was the suggestion given in [28] – should be organized in a way similar to the structure of thermodynamics, listing principles and equations connecting the various central quantities. The final goal of the project being to obtain ways of calculating the values of one of these quantities, once the values of other ones are known, or to reconstruct the fuzzy set given the values of appropriate quantities.

3 Doing other strange things

In this Section I shall briefly touch upon two topics which allow reflection on a few epistemological themes in relation to some of Eduardo’s ideas. The first topic is non-monotonic logic. It was not of any interest to Eduardo; however, his most important contribution to cybernetics, his seminal paper [29], but see also [30], suggests a possible way for facing this problem although not in logical terms. This will be briefly discussed in the next subsection. The other topic, briefly outlined in the last subsection, has to do with the problem of the technological interaction between artefacts and pieces of biological tissuee.

3.1 How to formalize the process of “jumping to conclusions” and the corresponding withdrawal when we jumped too early?

It is typical of “everyday reasoning” to jump to conclusions in the light of the information available at a certain moment. We do not wait until all the conceivable and complete information is available for drawing a conclusion and then make a decision. We know what to do when additional information convinces us that we jumped too early to conclusions and then must withdraw (some of) them. “We” know what to do, but this knowledge – up to now, at least – is difficult to formalize, to constrain into a few, formal and mechanical rules. But what about machines? Machines behave wonderfully well and in a spectacular way in those cases in which formal and mechanical rules can be given (and are actually provided to them). So if we wish that machines also deal correctly with situations in which conclusions must be drawn in a context of partial and incomplete information, we have to formalize this process. The machine will then follow mechanically new rules representing this kind of situation. Considerations of this type are at the basis of the so-called “non-monotonic logic”.

Let us, incidentally, observe that a purely technological question (how to instruct machines to withdraw conclusions in light of new information in a way similar to the one followed by human beings – a very cybernetic question, indeed) poses very interesting, challenging and extremely difficult *theoretical* (mathematical and logical) and *conceptual* problems.

We face, then, an important new problem as well as a urgent "need", the need of finding a formal way of representing this new informal and innovative situation. Commonsense reasoning shows the importance of studying the unusual case of withdrawing conclusions in light of new information (unusual from the point of view of "classical" logic, better, of any logic which aims at preserving the essential features of what in the tradition, was considered as a *logic*). After a few isolated criticisms of the problem (non-monotonic logic is a contradiction in terms, is an oxymoron, since a monotonicity property – of the set of conclusions with respect to the set of premises – is a basic, key feature of anything that wants to call itself a *logic*), the academic response has been massive. For example, one can consult the handbook edited by Dov Gabbay [31], which contains extensive lists of references. Perhaps we can say that there are too many papers, too many, in particular, if we look at the very meagre concrete applications of all this huge activity. One could, perhaps, conclude that most of this work was done with rigor and without imagination. The various formalisms and results are unobjectable (the good ones, of course) from the point of views of the formal correctness and academic respectability. However, in many cases, they are not imaginative enough to provide an answer to the initial question[7] usable in a simple way in concrete cases. Moreover, like Tolomeus epicycles, it seems that some of these formalisms do not manifest that simplicity, which is a key note of the true turns in scientific discovery. The initial question, however, is a very good one and to find a good answer to it remains an interesting challenge.

But what does all this stuff have to do with Eduardo? Was he interested in non-monotonic logic? The relationship of all this with Eduardo is too complex to be clarified by a simple yes/no answer. The following few comments are aimed to throw some light on the complexity of the problem.

First remark. Eduardo was not very interested in logic, one could also say that he did not "appreciate" logic in itself, although he often had a logician working in his group.

Second remark. He liked to "manipulate" mathematics until it complied to his purposes, and did not appreciate very much those purely formal developments that seemed to go around in search of a justification of their existence. I remember that he used to suggest not to fly off at a "mathematical" tangent. I am not sure that I fully understood the general meaning of his suggestion since, he often used it *instrumentally* to induce people to follow lines of investigation he judged important at a certain moment and discourage investigations along lines that would eventually diverge from what he considered important. However, I think that the interpretation of his suggestion in the case of non-monotonic logic would be that too much work exists to justify the paucity of direct results (perhaps, a lot has not centreed on the target but has instead flown off "at mathematical tangent").

Third remark. His pioneering paper on neural networks can also be used in this context since his networks are able – due to the inhibition property – to do something which resembles a withdrawal of a conclusion previously obtained.

[7]There are some exceptions. One is provided by Gabbay's attempt to provide a definition of non-monotonic consequence relation at a very abstract level (see [32]).

Recall that his nets have been applied to problems of revisable reasoning (see [33]) and that this same problem is inserted in other general approaches in which neural nets play a central role. See, for instance, Valiant's approach [34].

Let me also observe that a "non-monotonic " behavior is "normal" not only in the setting of neural nets with inhibition properties but also in a probabilistic setting. Problems and questions become conceptually intriguing (and not only technically more or less difficult inside a setting that is not put into question) only for cases in which we want to use a *logical-like* framework.

So, the potential use of neural networks in revisable reasoning is certainly nothing new. It is mixture of the second and third remarks that opens an unusual perspective if combined with the simple observation that at the root of non-monotonic logic there is just one concept, the one of "incompatibility". It is, in fact, the incompatibility between one of the new premises added to impede that some of the previous conclusions can still be drawn. So, we should formally characterize the notion of *incompatibility* among statements and, afterwards, embed this characterization into a logical-like structure.

Some early attempts to analyze the questions asked by "commonsense reasoning",[8] according to the lines previously outlined, have shown the difficulty of solving the problem by a simple "application" and "use" of the available mathematical tools [35, 36]. Perhaps, a necessary passage will be to envisage an algebraic representation of a neural network with inhibition properties similar to the representation of a Boolean network through a Boolean algebra. But this – apparently – more limited problem also looks very hard.

The question: "Can we translate all the machinery grown around the problem of *commonsense reasoning* in clear algebraic structures that would unify all the scattered approaches existing in this interesting topic of artificial intelligence?" remains unanswered. However, the future cannot be forecast. It would be the irony of the historical development of science if one such innovative logical problem would eventually obtain some contribution to its *general* solution by using more or less direct suggestions from the work of a scientist like Eduardo, who did not like too much logic.

3.2 The chicken's neurons computer

The present interest for the interplay – especially at a technological level – of biological tissues with artefacts is well known. This can happen at very different levels. While, for instance, a very recent paper in Scientific American [37] showed the importance of the last achievements on "neuromorphic chips", which (besides a useful possible use in such artefacts as smart sensors and robotic eyes) could yield implantable silicon retinas to restore vision, there is a huge number of papers (for instance on "biochips") stressing a sort of situation inverse of the one described above. A piece of biological tissue is "integrated" with a technological artefact.

[8] Or, equivalently, to analyze the innovative conceptual points introduced by the same definition of a "non-monotonic" logic.

Eduardo liked *integrations* of this kind very much. In fact, he made a strong effort (or better said, forced some people to make an incredible effort) to realize what – at the time – at the Laboratorio di Cibernetica was ironically called "the chicken's neurons computer", that is, a machine whose working units should be made of live neurons cultured in vitro. Recall that we are speaking of knowledge and, more importantly, techniques and technologies of the late sixties. I quote, Paola Pierobon [38] who was in charge of the project at the time: "The problem was not at all easy to solve for at least two reasons: (1) these nerve cells not only had to be viable, but should also be prone to learn performing logical (mathematical) tasks in response to non-biological instructions, and therefore had to be trained to the purpose and (2) studies and methods for keeping cells alive outside their native organs/tissues had been developed with success but only for dividing cells. Neurons are adult cells that differentiate from embryonic precursors, the neuroblasts, and stop dividing upon attainment of the differentiated state (the discovery of tissue-specific stem cells, including nerve cells, is present knowledge). The first successful reports of cultured neuroblasts appeared in the early sixties. The protocol required: collagen extracted from rat tails, fetal calf serum (alternatively, serum from human umbilical cords), chicken homogenates and, for good measure, NGF extracted from snake salivary gland venom. All these ingredients were not yet commercially available and had to be collected, prepared and sterilized in the lab, as well as the salt and nutrient solutions. Needless to say, chick embryos also had to be collected in the form of fertilized eggs from astonished local farmers and scrupulously nursed in the lab till ready for use . . . "

Although the project did not fulfill its aims, it nevertheless must be remembered not only since it exhibits the incredible visionary capacity of Eduardo, who looked ahead almost forty years, but also since it is an interesting case-study from a methodological point of view for its side effects. First, it forced people to be explicitly aware of all the difficulties required by a *truly interdisciplinary* work done at the frontiers of the knowledge and possibilities of the time. One should really go into the working of the methodology of another discipline (besides deepening the working of his own discipline) before being able to appreciate what can and cannot be done by an interdisciplinary dialogue. However different the specificities of different disciplines are, their common basic core (at least, they all share a "scientific" methodology) guarantees that a dialogue *will* eventually be established. Secondly, it showed that the results of very innovative ideas cannot be obtained in the *short term* if they are too much ahead of time. They should be cultivated without rushing to obtain results. But this can be done only in the presence of stable scientific institutions as well as a scientific policy for the country.

4 On the interplay between imagination and rigor

The interplay between imagination and rigor is a crucial aspect of the development of scientific thought. It is also a privileged point for observing the birth and development of new theories and disciplines. Although in a superficial sense everyone is well aware of the role played by these notions and of their strict interaction, as far as I know, the only scholar who has looked from this vantage point at the development of science is Rudolf Carnap, who introduced the terms of *explicandum* and *explicatum* to characterize the different epistemological role the informal and formalized notions play in the development of a scientific discipline. They refer to a way of approaching and analyzing the problem of the development of scientific theories which, in my view, is very sympathetic to the logic underneath Bateson's vision.

In this section I shall go around this theme indicating a few topics in which a more detailed investigation along these lines could be done. I am firmly convinced that complete and deep historical and epistemological analyses of information sciences would provide a great help for the same future scientific development of these disciplines, by implicitly indicating the most promising paths to be followed for obtaining new innovative technical results. An analysis of the connections existing between the ideas of Bateson and Carnap – besides its interest in itself – could be a good starting point for deepening our knowledge of the innovative epistemological significance of information sciences

4.1 The dialectical interchange between the dynamics of concepts and the "resistance to change" of quantitative theories

The (implicit) epistemological thesis which I wish to discuss and defend here is the following: one of the driving forces in the development of science (by referring both to new results and to the adaptation of scientific method to new situations) especially in those emerging fields which strongly make use of new notions and concepts is the internal struggle between the informal requirements and the possible quantitative (formal, mathematical) versions of these same notions. In other words, the dynamics proper to the informal innovative concepts is different from the one proper to the development of formal theories. Needless to say, this tension changes over time, the dominant cultural background, and so on. But the process of interaction among different notions and their formalizations can appear as an interesting dynamical aspect of the development of the same construction of a scientific vision of the world: a conception of the knowledge which in every moment is ready to give up all the results previously obtained.

The problem of the formalization of uncertainty is a good example. Three centuries ago the discussions on probability concerned not only what is presently considered the domain of the *theory of probability*, but also had to do with many other aspects which today are either neglected or are described by means of different names. An interesting paper by Hannu Nurmi (see [39]) convincingly argues in favor of the fact that the *natural languages* of different European countries have grasped different nuances and aspects of a sort of general informal idea of uncertainty or vagueness. In other words, the different basic etymology of the words that today are used to denote probability, fuzziness, truth-likeness, etc., in different (European) countries, witnesses the fact that different languages pinpoint different facets and nuances of an originally very general informal notion. This does not mean that the developed theories are intended in different ways according to the country and the words that are used to denote them. This kind of curiosity is a sort of a fossil of the originally unique but very vague general concept. Today, when speaking about uncertainty in general we are confronted, on one side, with a sort of "residual" part of this general informal notion (which, from a conceptual point of view, is the part of the notion which is the most elusive and hard to grasp) and, on the other side, with different technical *explicata* which work marvellously well in specific domains and base their strength on the power of a complex and developed formal machinery.

A new reading – along these lines – of the debates concerning the relationship between fuzziness and probability would show the different roles played by the formal and informal notions (each with its own dynamics). It is also interesting to observe that recent new technical results in the field of coherent conditional probabilities (which extends De Finetti's approach) obtained by Coletti and Scozzafava (see, for instance, [40, 41]) seem to fill the gap between the two *informal* notions of fuzziness and probability, by providing a new, more general *explicatum* which, in principle, solves all the interpretative clashes between them.

Similar analyses could be made for other notions as, e. g., the ones of *information*, as partly done in Sect. 2 above, *complexity* and *computation*. This last one is particularly intriguing since it is the unique example of an informal notion whose *explicata* are all demonstrably (extensionally) equivalent (Church–Turing Thesis).

4.2 Thinking about cybernetics

Eduardo used to say that he was engaged in doing research both in physics and cybernetics: in the latter discipline with his left hand. OK, he could do this; for normal people it may be difficult to do so. They could be interested, instead, in asking the question: what is cybernetics or, at least, what is cybernetics today? It has been always clear that it cannot be considered a traditional, "normal" discipline although it always aspired to be considered – from a methodological point of view – a *classical* science. It could have been considered a *normal* discipline only if all the results which sprang out in a tumultuous way in the forties and in the fifties could be presented in a unitary way inside the general scheme outlined by Wiener. But in order to do this it would have been necessary to show non-trivial connections existing between such different things and results as mathematical biology and automated theorem proving or as chess strategies and pattern analysis. At least, it should have pursued an explicit scientific policy affirming that the strong connections would be found in a second moment; to follow a common path – due to some general connection – was in the mutual interest of all the subsectors involved whose further developments should have shown the deep reasons why these (apparently scattered) results were parts of a single whole. Eduardo's convinction was that the unifying staff of all this research was their having to do with "intelligence", with a scientific approach to modeling aspects of *intelligence* as he wrote in his preface to Aldo and Luigi's book "Introduzione alla Cibernetica" [42].

So we can certainly affirm that in the forties and fifties of the past century, cybernetics acted as a sort of catalyst indicating that a lot of interesting and new ideas, concepts and formalisms which were moving in a very creative as well as disordered way, breaking the boundaries of traditional disciplines, could be seen as parts of a unique, new scientific discipline and not only as scattered (albeit very interesting) results. This new scientific discipline, i.e., cybernetics (according to the definition and scientific work of Norbert Wiener as well as on the basis of the social acceptance of the scientific community of the time), moved along the way of the "classical" science; in a sense it could also be seen as part of physics although devoted to the investigation of new domains with their very peculiar features. The situation of apparent unity, however, was very unstable and did not last long. Among the reasons for this evolution one can also take into account the discrepancy between the general aims and ambitions of this new discipline and both the strength of the available formal tools and the obtained results. When there was a strong divarication between aims and (general) results, cybernetics as a unifying paradigm went through a critical period. Which are the consequences of Wiener's challenge? In a first moment, they were extremely positive since many interesting but scattered results could be seen as parts of a unitary effort. All the different results converge in reinforcing each other and all the interdisciplinary work can be seen as part of a big and important effort, not parasitic on the "goodwill" of one of the traditional disciplines which were so "liberal" as to allow that such unusual work could be done. However, in a second moment – due, perhaps, to questions having nothing to do with scientific development proper – some (natural) weaknesses of the new approach are used against the affirmation of cybernetics as the unique – and unitary – repository of the interdisciplinary work done in those years. And so, starting from the early sixties, the name cybernetics began to be not so fashionable as it had been before.

This was the path taken in those years by the scientific activities done under the general heading of cybernetics. The various subfields look for an autonomy jealously defended. They stress their mutual differences pointing out the similarities with traditional disciplines. For instance, the biologically oriented investigations prefer to use the name "biological" cybernetics instead of the simple one of cybernetics; people working on the theoretical aspects of automata tend to present their work as purely mathematical forgetting the interdisciplinary roots of the field; the community following the "symbolic approach" to what began to be called AI struggles against the "neural nets" community, and so on. Interdisciplinarity is no longer seen as a virtue. However, the path followed in the first years (chaotic but creative, unsystematic but full of innovative insights) deserves to be studied as a model of investigation, since it can provide useful suggestions for the future.

4.3 *Scientific* disciplines, *academic* disciplines, and interdisciplinarity

We have already seen that one cannot speak of cybernetics without dealing with interdisciplinarity. What is the *real* central role that interdisciplinarity plays in scientific development? Why does it play such an important role in cybernetics while it seems to play a negligible role in other (older) scientific disciplines?

Scientific disciplines spring out of problems and are related to the rational reconstruction of the connection of answers provided to the questions posed by them. Academic disciplines are the stabilization of the results obtained by studying important *old* problems. Interdisciplinarity often arises to fill the gap between (potentially important) *new* problems and questions the uneasiness to face them manifest in the academic disciplines present at a certain historical moment.

Undoubtedly, science goes forward by solving the problems that human kind encounters. Problems arise from nature (from a *natura naturata*, of course, that is not from a "naked" nature but from a nature seen and examined through the glasses of culture, of the specific tradition in which the *knowing person* is immersed, a cultural environment which induces one to consider something as a problem and other things as irrelevant or as no problem at all. The problems do not belong to a single and specific discipline, unless we assume an *essentialistic* attitude involving a classification of the phenomena according to their true nature, their essence: an attitude of Aristotelian type unfamiliar to modern scientific thought. Scientific disciplines *are* different and separate, but their difference and separation is a function of their development and of the historical moment. Think, for instance, to optics and electricity and magnetism *unified* by Maxwell theory.

Academic disciplines, on the contrary, are rigid; they obey a socially induced division of labor and have interests different from the ones of the pure development of scientific ideas. Interdisciplinarity plays a role just at this point. First, it points to the fact that a new problem and question can be tackled only if we escape the boundary of the "corral" of established disciplines. Secondly, interdisciplinarity induces an updating of scientific disciplines as they are organized at a certain time, in a specific phase of the evolution of human knowledge. It helps, moreover, to go over the rigid constraints imposed by "academic" disciplines, whose rigidity can impede the full explication of results the "scientific" disciplines of a certain period of time would allow anyway.

A look at the development of cybernetics from this observation point shows its positive role played for the renewal of all the disciplines which interacted with it not only at the height of its splendor but every time a complex problem was seriously approached through its undogmatic methodology.

4.4 Provisional conclusions

"Rigor alone is paralytic death, imagination alone is insanity", once observed Gregory Bateson. The scientific life of Eduardo can really be seen as a difficult and mostly successful "path" which always avoided clashing with these two poles, which can be considered the Scilla and Caribdis of our times, not only in the field of scientific investigation and which display all their creative strength only when the influence of one of them is tempered by the presence of the other. The capacity of preserving the equidistance between them is well documented in Eduardo's scientific life; for instance, on one side by his frequent advice of not pursuing mathematical

generalizations for their own sake, always having in mind the questions asked by the problem under study and, on the other hand, by his care in looking for the kind of mathematics most suitable for dealing with new ideas, and his capacity for inventing new formalisms and notations when the available ones looked unsatisfactory for his purposes. An historical reconstruction of his work will pinpoint when and where he succeeded spendidly and when the attempts remained attempts. These last lines will not be therefore devoted to commenting more on these aspects but will discuss one feature which I have considered for many years his biggest fault or, worse than a fault, his main methodological error. This error – in my view – was his desire (better: his strong will) of obtaining *"tutto e subito"* (everything and immediately) as a political slogan of the 1970s affirmed.[9] If something cannot be obtained immediately then it is better to give it up and concentrate on other promising intellectual adventures. A respectable promethean attitude. However, history often follows very tortuous paths and not speedy highways for obtaining innovative results. The example of the *chicken's neurons computer* was previously recollected – although I had not been personally involved in the project – just for providing a concrete example of Eduardo's impatience. Planning a complex strategy for the subsequent decades in light of the insurmountable problems encountered (*really* insurmountable by using the technologies of the time) would have given a primacy to the Institute in this (now) central topic. And the same could be said of many other (too) innovative ideas which were circulating in the *Laboratorio* thirty-five years ago. Planning a strategy, however, requires the presence of structures and institutions which guarantee long-term projects. Structures and institutions of this kind, unfortunately, are commodities which were not (and are not) easily available in Italy.

So, I am forced to sadly conclude, that Eduardo was right in requiring *"tutto e subito"* since in a precarious general context what is not fully obtained on the spot is lost forever.

Appendix: On the Measures of Fuzziness

The aim of this appendix is to present a brief overview of the essential points of the Measures of Fuzziness Theory, without providing proofs and details, for which the reader is referred to the literature. Within this presentation some comments will point out where the interaction between *informal ideas* and *technicalities* seems to be stronger.

We recall some elementary definitions before stating the axioms in order to fix the notation. Let X be an arbitrary set and L a partially ordered set. An *L-fuzzy set* [43] is any mapping $f : X \to L$ from X to L. In the case in which $L = [0, 1] = I$ one obtains the usual definition of *fuzzy subset* of X, (or, simply, fuzzy set). The *power* (or *generalized cardinality*) of a fuzzy set is given by [23]: $P(f) = \sum_{x \in X} f(x)$.

Let us denote by $\mathcal{L}(X)$ the class of all the applications from X to I. Let us, finally, introduce the operation of direct product $f \times g$ of two fuzzy sets $f \in \mathcal{L}(X)$ and $g \in \mathcal{L}(Y)$, defined for any $(x, y) \in X \times Y$, as:

$$(f \times g)(x) = f(x)g(y)$$

[9] Needless to say, he was in no way influenced either by the slogan or from the political climate. I am using this image just to radically stress the point.

It is possible to introduce a structure of (distributive) lattice in $\mathcal{L}(X)$ by means of the binary operations $\vee$ and $\wedge$ which associate to every pair of elements f and g of $\mathcal{L}(X)$ the elements $f \vee g$ and $f \wedge g$ of $\mathcal{L}(X)$ defined as follows: for any $x \in X$

$$(f \vee g)(x) = \max\{f(x), g(x)\}$$
$$(f \wedge g)(x) = \min\{f(x), g(x)\}$$

Furthermore, for any $f \in \mathcal{L}(X)$, one can introduce the so-called negation f' defined as: for any $x \in X$

$$f'(x) = 1 - f(x)$$

Let us now introduce in the interval $I = [0, 1]$ the partial order relation $\leq$ defined, for any $x, y \in I$, as

$$x \leq y \Leftrightarrow x \leq y \leq \tfrac{1}{2} \quad \text{or} \quad x \geq y \geq \tfrac{1}{2}$$

This relation can be extended, point by point, to $\mathcal{L}(X)$ as follows: for any f and $g \in \mathcal{L}(X)$,

$$f \leq' g \Leftrightarrow \forall x \in X[f(x) \leq g(x)]$$

If $f \leq g$ and $f \neq g$ we shall say that f is *sharper* than g.

A *measure of fuzziness* or *entropy measure* h is simply a functional $h : \mathcal{L}(X) \to \Re^+$, where $\Re^+$ denotes non-negative reals, satisfying some conditions which depend on the system under consideration.

The basic axioms that any measures of fuzziness must necessarily satisfy are:
(a) $h(f) = 0$ if and only if f is a classic characteristic function.
(b) $h(f)$ attains its maximum value if and only if $f = f'$.
(c) h is isotone with respect to the order $\leq$, that is, if $f \leq g$ then $h(f) \leq h(g)$

These conditions show that one requires that the concept of fuzziness disappears in the classical case, that the fuzziness be maximum when it is impossible to distinguish between a fuzzy set and its negation and, finally, that this measure must certify quantitatively the sharpness of a fuzzy set, described starting from the order relation $\leq$.

Other axioms that can be imposed in suitable situations are:
(d) $h(f) = h(f')$ for any $f \in \mathcal{L}(X)$
One imposes, then, a sort of symmetry between a fuzzy set and its negation for what regards the degree of fuzziness.
(e) h is a *valuation* of the lattice $\mathcal{L}(X)$, that is,

$$h(f \wedge g) + h(f \vee g) = h(f) + h(g)$$

The previous assumption (e) seems to be very natural; however, it is not satisfied by a very "reasonable" measure (see below).

Examples of simple measures of fuzziness (besides the well known measure of Shannon) are:

$$\sigma(f) = \sum_{x \in X} f(x)(1 - f(x))$$
$$u(f) = \sum_{x \in X} min\{f(x), 1 - f(x)\}$$

Measure $\sigma(f)$ is formally identical to the sum of the variance of the random variables $\xi(x)$ assuming the values 1 and 0 with probability $f(x)$ and $1 - f(x)$, respectively. Measure $\sigma(f)$ has been used in statistical pattern recognition [44] for representing the information content of a given image.

The measures of fuzziness $\sigma(f)$ and $u(f)$ as well as the logarithmic one satisfy all five axioms listed before, while an example of a measure which does not satisfy the valuation property (axiom e) is given by $[\sigma(f)]^2$.

Comment. All these simple examples are small stones useful for constructing small bridges with other theories (for investigating "in vivo" the interactions between "(informal) concepts" and "(possible) formalizations").

The proposal of developing a theory on the measures of fuzziness on an axiomatic basis has had many developments both on the theoretical and the application sides. Among the latter we just recall those pertaining to pattern analysis and recognition. Among the first theoretical contributions – soon after the theory was proposed – let us recall the papers by Alsina, Loo, Riera, Trillas [45–47]. In [21] the definition of entropy measure is generalized to the case of $L - \textit{fuzzy set}$.

Comment. It is interesting to note that in all these papers purely mathematical developments were looked for; however, these were near enough to the original informal ideas as to pose no essential interpretative problem. A different situation is presented by the following papers.

In [48] some convergence properties of specific measures in the case of a denumerable support are discussed and studied. Knopfmacher [49] presents an extension to the case in which the support X is non-denumerable under the hypothesis that in a σ-algebra of subsets of X a totally finite positive measure is defined, by suitably modifying the axioms we proposed.

In order to characterize families of measures of fuzziness, Ebanks [17] has introduced a further axiom (*generalized additivity*):

(f) There exist applications $\delta, \tau : [0, \infty[\to [0, \infty[$ such that

$$h(f \times g) = h(f)\tau(P(g)) + \delta(P(f))h(g)$$

for each $f \in \mathcal{L}(X)$ and $g \in \mathcal{L}(X)$, X and Y being finite.

This axiom requires, that the measure of fuzziness of the direct product of two fuzzy sets f and g be equal to the weighted sum of the measures of fuzziness of f and g, the weights depending on their generalized cardinalities.

Comment. In all these cases we could say that mathematics imposes its presence – in different degrees – forcing one to go along paths that are not suggested by the imagination sustaining the original proposal. This is true in particular for Ebank's work and his axiom (f). However, it is only with the help of (f) that Ebanks is able to characterize in a general way all the measures of fuzziness, providing explicitly all the measures of fuzziness which satisfy either all six axioms of the list or only five axioms (a, b, c, e, f). In this latter case the only condition which is not imposed is the one requiring that the measures of fuzziness of f and of its negation be equal (a strange situation, apparently, the one which is excluded). If one imposes all six proposed axioms the unique measure of fuzziness of a fuzzy set f is provided by the already met measure $\sigma(f)$.

In the previous lines only the general scheme of the theory has been briefly summarized. Among many others meaningful results, a new way of facing the problem of measuring how far a fuzzy set is from a classic characteristic function is due to Yager [18]. His proposal allows one to look at the problem of intuitive ideas versus formal results from another point of view. His challenging idea is that of measuring the "distance" or the "distinction" between a fuzzy set and its negation and the technical tool to do so is provided by the lattice theoretical notion of "betweeness".

Comment. It is possible to establish a formal connection between Yager's approach and the one presented here. In fact, it can be shown that in all the cases in which it is possible to define Yager's measure it is also possible to define a measure of fuzziness in the sense discussed above. The point of view of Yager, then, provides a new very interesting visualization but technically it does not allow one to extend the class of measures, as one would expect, due to the conceptual difference of the starting point. We refer to [19] for the proof of this connection and for a specification of the condition under which Yager's definition is applicable.

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Rational vs Reasonable

Giuliano Toraldo di Francia

1 Reasonableness

In one of the many witty sentences written by Eduardo Caianiello, we may find the expression *illuminata razionalità* (or "enlightened rationality"). Why enlightened? Could rationality be otherwise? Yes, it certainly could. As a matter of fact, everybody may have known a lot of perfectly "rational" people, who are absolutely ... "unreasonable". Let me try to explain this delicate point by taking as an example the struggle between old-fashioned "creationists" and "evolutionists".

Honestly speaking, not all the steps followed by biological evolution are perfectly clear. Sundry details (especially concerning the major transformations of genera and species) still escape careful analysis, and are waiting for a better understanding. Nevertheless, those difficulties are not sufficient to induce the great majority of scholars to revert to the old standpoint. Was that standpoint *irrational*? No, it was only *unreasonable*. For, suppose that a die-hard creationist should maintain that the whole world – including *homo sapiens* – was created *in its present state* only a few thousand years ago. How could we discover the trick, if that world showed exactly those features (such as geological layers, gradual succession of fossil remains, and so on) that, in due time, were to lead scholars to formulate the theory of evolution? We must acknowledge that purely rational thought could by no means falsify a creational hypothesis of that kind.

Nevertheless, it seems to me that that choice of integral creationism, though not downright irrational, would be unreasonable. For, why would a hypothetical omnipotent being (let us call it God) have created the world in such a devious and deceitful way as to induce us to imagine a bizarre and completely false history? (Incidentally, remember that a few philosophers have asserted that God cannot want to deceive us). In any case, and more generally, Occam's razor is not a merely rational choice, but a reasonable choice! Because, the assessment of what is reasonable, ostensibly involves, besides rational thought, also a number of pragmatic, ethical, social and economical factors. Ironically, ignoring such factors would be somewhat *irrational*.

For the sake of simplicity, we will stick, from now on, to the reasonable choice of Darwinian evolution (not ignoring, of course, the recent advances of genetics, psychology, as well as of biology in general). Stated in a few words, we will assume that our intelligence – in the same way as all other traits of *homo sapiens* – was gradually acquired by a process of *mutations plus natural selection*. As a consequence, we give up the Platonistic attitude, and recognize that our present way of reasoning may represent but one stage in the evolution of rational thought.

The modern evolutionary attitude is likely to encounter a stumbling block, right at the outset. It will be remembered that O. Neurath once asserted – in the context of logical empiricism – that we are like sailors striving to repair their ship from inside, while at sea, and not sitting in a dry dock. Now, the theory of biological evolution was arrived at by using the *present* (non-evolutionary) rules of human thought. How can we trust such rules, when trying to elaborate a *diachronic* and evolutionary theory of our mind?

Besides, in this case – unlike in the case of merely biological evolution – we have no observational basis to rely on, owing to the obvious lack of any fossil record of human thought. As a consequence, all we can offer is merely a number of hypotheses. Let us hope that they are good hypotheses. Be that as it may, we have *no choice*, and – though not believing human reason to have already attained its final or *absolute* stage – we cannot avoid using a sort of what may be called present-centric view. Most probably, our descendants will apply different criteria in judging what is reasonable.

2 Exaptation

Nature, proceeding by trial and error (i.e. via mutation and selection), in order to ensure survival and propagation of the species, has provided living beings, in particular humans, with a number of structures and capabilities, suitable for coping with the needs of life on the Earth. It is usual to speak of "adaptation" to the environment.

Today the new term *exaptation* has been coined too. I. Tattersall gives the definition: "This is a useful name for characteristics that arise in one context before being exploited in another". A very good example of exaptation may be offered by the *mouth*, which, originally meant for ingesting food, was then found useful for defense and attack, for emitting sounds of threat or alarm, and later employed by humans for *speaking* (as well as for a lot of minor purposes, like that of applying stamps on an envelope). Exaptation may, in turn, induce further natural adaptations of the body, like for instance a proper lengthening and adjusting of the vocal tract in *homo sapiens*, in order to produce a well-articuated language.

3 Philogenic behaviorism

It will be expedient to take a *behaviorist* standpoint and conceive man as a "trained" animal, like those performing in a circus. First, let us – extremely briefly – summarize a number of successive steps that seem to have taken place in evolution.

The primitive living organisms on Earth, say protozoa, acquired by selection a stimulus–response type of behavior. For a long time, that pattern was followed by all living creatures. The information coming from outside, plus the reaction elicited from the organism, formed a *fixed and finite* set of stimuli–responses. To stimulus A the organism had invariably to respond with reaction B.

If a living being, passing by chance close to the den of a wild animal, receives a painful bite, it will not think: "every time I pass there, I shall be bitten". No; next time, it will simply take a different route. As the trip now turns out safe, the animal's behavior willl thereby be reinforced. But, if the animal "thinks", it will not conceive that universal notion of *induction*, which was devised much later by our philosophers (in other words, by those who still believe that "all crows are black", and are prone to kindling endless discussions, in order to prove that the conclusion is sound).

The fundamental acquisition of the animal – and consequently of the species – is the capability of forming a network of *subjective probabilities*, and acting accordingly. By selection, such capability ends up being incorporated in the genome of the animal, simply because otherwise the species could not survive in the struggle for life (against both natural factors and other animals), and, as a consequnce, would become extinct. In this way, we have replaced the usual reward–punishment procedure of ontogenic behaviorism by the survival–extinction operation of philogenesis.

Let us again refer to human beings and their language. As stated above, they will always act by relying on an acquired (if partially innate) network of subjective probabilities. But there is a very important exception. When the probability of an event occurring takes such an extremely small value that, in order to have a chance to observe it, one would have to wait for an immense time, much longer than a human life, or the life of the universe, we, very conveniently, and *reasonably* at that, say that the probability is *zero*. We then talk of "certainty". It is never a true zero probability for the event to be observed. Nevertheless, we take that certainty for granted; and in our mind, the conclusion seems to be *evident*.

4 Reasoning and choosing

For a long time, (1) the information coming from outside, plus (2) the reaction elicited from the organism, formed a finite and fixed set of stimuli–responses. However, the number of different cases to be envisaged soon became countless. The more so as the surroundings of any species changes every time that other species vary ever so slightly. The fixed set of stimuli–responses turns out to be insufficient.

Nature made an enormous step forward when there arose a central nervous system, capable of coping with a virtually infinite number of different cases, by *reasoning* (thanks to induction) on all the collected mass of stimuli received, and thereafter *choosing* the (probably) most suitable response to give. At this point, "thought", and all its strategies were born. Then, the strategies started, in turn, to evolve by trial and error.

When modern humans appeared, with their symbolic thought and language, they started (once again by trial and selection) to set up a *syntax*, as well as a set of *logical rules*, such as non-contradiction, excluded middle, *modus ponens*, etc.; namely, all the equipment of *deduction*.

Please, don't ask me *why* there are such rules! They are once again the product of adaptation to a given environment. In fact, they work splendidly, and help us to survive in our surroundings. A species that does not respect them risks elimination, simply because our world is made like that. That's all there is to it.

As a consequence, we must aknowledge that those rules are not written in golden characters in a heavenly table above us. To be convinced, remember, for example, that they work rather poorly (or are downright wrong) when applied to the microphysics of subatomic particles.

Our intellectual history has led us (in the West) to imaging deduction as a kind of untouchable and sacred monster, much more powerfu and reliable than induction. However, the time has come to recognize that deductive reasoning is by no means more beautiful and sacred than the inductive procedure. On the contrary, they both share the same origin, as well as the same adaptive and selective justification. Ironically, one can maintain that deduction has an inductive justification! In fact, since the conclusion reached with "correct" deduction, proves always to be verified, it seems to us to be certain, or evident, or necessary. In other words our deductive procedure becomes, so to speak, incorporated in our DNA, and its results may appear apodictical.

That is what prompted the ancient Greek philosophers to assert that those were the eternal and right rules of thought.

5 The prostheses of the brain

The brain is certainly a marvellous structure. But, by itself, it is absolutely incapable of "doing" anything, except abstract thinking. In order to be able to operate outside itself, it needs to be provided with some suitable "prostheses". Our prostheses are arms, legs, hands, sense organs, and so on. As soon as humans reached the stage of *homo faber*, they started to build additional prostheses (i.e. instruments) to help their brains. For centuries, or millennia, we have used and adapted natural materials found in nature (like stone, wood, bone). Later, when we could master fire, we added metals and ceramics to the list.

However, note that our notion of prosthesis need not be limited to artificial instruments. In a broad sense, the horse, the ox, and all other domesticated animals, are prostheses. Agriculture is a prosthesis, too; the same can be said of the hut and clothes. All of these improvements allowed humans to reach the *explosion* of the neolithic age. After discovering the tremendous advantage of people living closely in a society, we passed from the cylindrical to the rectangular hut, that facilitates the assembly of buildings and regular road networks. Thus, the *city* was born with all its institutions. Progress continued for a long time at a comparatively moderate pace. But our modern improvements are simply *spectacular*. Suffice it to mention that today we are able to see objects billions of light years away, inspect the interior of the Earth, monitor the organs of a living body, including the brain, and safely operate on them. Further, we can communicate in virtually real time with all the inhabitants of the planet. But these are only a few (and perhaps even the less sophisticated) items of modern technology.

The new acquisitions will be termed "neostheses". I take the responsibility for that neologism. Neostheses enable us to do incredible things, like for instance EVA (extravehicular activity) about a space ship. Our entire *representation of reality* is thereby changing, so we can dare to speak of *augmented reality*.

6 Language

Let us now recall some of Noam Chomsky's views about laguage. According to Chomsky, language is an innate faculty, inscribed in our DNA. He distinguishes:

(1) FLB (or faculty of language in a broad sense), arisen during evolution, in order to communicate the presence of danger, or the discovery of a source of food, etc. These types of messages are but a limited set of stereotypes, characteristic of each species.

About 6 million years ago, the genus *homo* diverged from the other primates and acquired the faculty of spatial reasoning and of computing. Then, by means of an *exaptation*, there arose from FLB:

(2) FLN (or faculty of language in a narrow sense), i.e. a computational system that, starting from a *finite* set of elements (words), can build by *recursion* a virtually infinite string of discrete expressions (phrases, sentences, speeches).

This generative syntax, as it is called, first produces internal representations, then translates them through a sensory-motor interface, which activates both the conceptual system and the phonetic apparatus.

7 The sign language of deaf-mutes

As is well known, deaf-mutes can communicate between themselves, as well as with normal people, by means of a "sign language". This they generally learn from adults. But it can also spontaneously arise in an isolated community of people that are deaf from birth. One would expect that, when one individual of that community is "talking" with the others, he or she would activate that occipital area of the left hemisphere of the brain, which governs visual pattern recognition. However, NMR and Pet observations have revaled a quite different situation.

As a matter of fact, the regions of the cortex that are stimulated during the communication turn out to be the prefrontal and frontal areas that govern computation, syntax, logical reasoning, and the like. This seems to be a powerful support to Chomsky's notion af an innate and universal grammar of the language of present human beings.

8 Trouble from augmented reality

Everybody seems today to be complaining and bemoaning about the almost unlivable condition of the present world. We feel ovewhelmed by a number of modern acquisitions, that complicate *beyond sustainability* our existence, when they were allegedly intended to help.

Among the many things that are tormenting us, one cannot forget to mention the *tremendous quantity of information, that is assailing us from everywhere, be it required by us or not.*

The reason I offer for this situation is that our brain, originally constructed and selected to deal with common reality, is absolutely incapable of coping with an *augmented reality*. Providing the present neostheses to our brain is like offering a mechanical loom to a silkworm. The worm can already operate by itself in a marvellous way; but not with that loom!

What can we do in this situation? Perhaps, the way out is to *resume again the route pointed out by nature: evolution.*

Developing our brain further may appear as a crazy and desperate enterprise. But who knows? Are we not seeing every day the incredible successes of genetic engineering?

9 A rational, but unreasonable quest

Are we alone in the Universe? I have a personal view on the subject, and am well aware that I may be wrong. Anyway, I will set out and support my opinion.

The most common answer you are likely to hear to the above question is this. There are many billions of stars in our Galaxy and there are many billions of galaxies in the Universe. However small the probability may be of finding a planet similar to the Earth, that probability is not zero. Consequently, we can be pretty sure that there is such a planet somewhere in the Universe. The planet has most probably given birth to a life similar to ours. By evolution, it may very well have attained a stage similar to ours, and perhaps is now sending rational messages to us. Today, a lot of people are looking with powerful means to discover those signals.

Now, the argument is certainly rational, but I am afraid that the search is unreasonable! For, even if you admit that evolution on that planet has started with identical conditions as those existing on the primitive Earth, the probability of ending up with an *identical* product and finally with *homo sapiens*, is *virtually zero*.

This is because in the course of its development, natural evolution is a stochastic process, encountering on its course an endless number of *nonlinearities* and *bifurcations*, that reduce the allegedly billionth of a billionth probabilty to a trillionth of a trillionth of a trillionth. . . . , which practically amounts to zero.

However, as stated, that opinion may be proved any day to be wrong by experimental evidence.

Eduardo

Carla Persico Caianiello

It is obvious and quite natural that being members of the family, we know something more about Eduardo's personality, about his inner nature, which may be set within what you have respectfully and kindly recalled for the occasion. There is no need to underline our full approval and gratitude regarding this commemoration. Ten years after his death, it is for my daughters and I a great pleasure and honour to see how his life is recalled from both those who met him for a short time, and retained a stimulating impression, and those whose fellowship lasted a long time and was very productive and prestigious.

But I would like to go back to that particular aspect of his personality which will take us quite rationally to the subject I would like to express. It is well known that a great part of his thoughts, his way of seeing the outside world, and to a certain extent, the way he faced it, his recurrent detachment, are due (were due, unfortunately) to his deep fondness for the Oriental mentality, which started when he was very young.

For the occasion, we have had to fumble among his dearest books. He did not read them every day for lack of time, but they WERE THERE; and at certain times he read and alluded to them more often, even joking about them, saying that he wanted to retire *Under the Banyan Tree* after the fashion of Narayan!

Among his inspiring readings we find: harmonious fundaments of Buddhism from the "Light of Asia", "The Vedanta", the "Bhagavad-Gita", "The Upanishads". The theories he deeply interiorized, such as the pregnant wisdom of Confucian aphorisms and The Book of Mencio, or accurately applied to the body and spirit through yoga and the appealing paradoxes of Zen, show us his inner inclination towards a world of meditation. Moreover, a great number of manuals and dictionaries testify to how much he wanted to learn Chinese, Japanese, Tamil, Armenian, Samarqand Russian, only in part realized. But the truth is that he only wanted to know the deep essence of those people whose history and thoughts he loved.

An example is his close friendship with the Brahman scientist – I think R Vesudevan – with whom he spent a "tiring" but intense period in Madras, and many other Indians, well known in the world of Physics or Cybernetics.

But now I would like to talk about his Japanese "brainwave". He loved reading "No", "Kabuki" and "Haiku", the latter are brilliant short poems, poor in words but surprising for their meaning and scenarios hidden among the very concise and selected expressions. He even tried to find a philosophical, if not mathematical, explanation for his *passion*, lingering over "The Stone Garden" of Ryoanji Temple.

Trying to understand the Japanese, he, on many occasions visited the country: first of all a stay in Tokyo, as guest of the Fuji-tsu, where he was entrusted with prominent assignments and had the opportunity to meet interesting and important people (you have certainly seen the commemorative volume promoted by Professor Ishihara); and in 1976, after a very difficult time both professionally by and with health problems, when the Honda Fujsawa Memorial Foundation invited him to take part in an up-to-date Symposium.

The story of Discoveries began with Mr Honda's speech, from which we shall cite some short statements. Mr Honda was a factory worker, an engineer, a manager, whose motorbikes were admitted (in 1961 I think) to the prestigious race on the Isle of Man ... His personality was quite complex: after a period of happiness having achieved his aim (he came to dinner at our house a few times and was always smiling and cordial, even if he did not understand a single word of what was said around him), and after a life of hard work and sacrifice, his dissatisfaction grew into what he called a "crisis". But his inner crises were certainly positive, since every time he realized an idea, new questions arose, still without answers, and needing to be satisfied. Due to all this, undoubtedly the Honda factory was a triumph.

But at the same time the "patron" realized that something was missing: in himself, in Japanese corporations and in the civilization of the whole world (Honda's statement). Thus, with the intellectual support and, later, the operational help of Dr Segerstedt, Rector of Uppsala University (Sweden), centred on technological ideas based on experience, the Memorial became an International Symposium, quite unique for the times: Discoveries.

Why the word *Discoveries?* Every single letter of this word stands for a specific term. We can spare you for now. The interdisciplinary subject is Prediction, "all prediction must be about the future situation of man in society" (said Segerstedt). The project (as Honda stated, and repeated the year after in Rome) was to promote "interdisciplinary and international technology, knowledge and public welfare"; "humanize technological civilization" as a final aim, and last but not least "transfer the idea of art to technology" (as the well known Japanese scientist Shuei Aida summarized).

At the first symposium in 1976 there were about 40 Japanese scientists and many from other countries, all experts in various branches of knowledge: the only Italian was Eduardo, who was also a member of the Organizing Committee. In his talk he first of all considered the necessity to "correct the chaotic modern civilization" and then discussed "the most dramatic examples of cultural changes, such as the passage from the Middle Ages to modern science ... open to free research and centred on the Galilean method ... carried out in less than two centuries ... ".

He added: "Cybernetics is based on the fact that both evolution and revolution are essential paradigms in the development of a complex system like human society ... A new revolution of equal importance is in progress, and is an everyday challenge: the study of the past, from a new scientific point of view, may help us to overcome the problem as winners, not slaves".

In the beginning, Soichiro Honda only played the role of the perfect Japanese host, honoured to guest so many "enthusiastic scientists", but soon he called off all his commitments and as he personally said: "I ... spent three days, feeling as I was playing around the roots of the knowledge". And from then on he never missed one single day with *Discoveries*.

We are now in the year 2003 and if we consider what is happening around us, this unbelievable technological development, clonation, robotics, the new use of space and energy, biotech in competition with computers and so on, in complete chaos, the great expectations of the time may seem utopian, abstract and somewhat ingenuous.

But it was 1976 and *Discoveries* was not the only initiative of the scientific, industrial and political world of the time, worried about the impending crises of society. Professor Vincenzo Caglioti, president of the National Research Council, and sponsor of the Symposium in Rome in 1977, discussed the problem.

"Society is restlessly trying to face its many afflicting problems ... and is compelled ... to control the diffusion of alienations, frustrations and threats to fundamental human values".

In June of the same year, Mr Honda was visiting Europe and Italy, and on that occasion, during a press conference in the presence of the Minister of Research and Technology, Eduardo more or less stated:

– The whole development of western thought aimed at "conquering nature", with dramatic consequences on man himself.

Are scientific methods, enlightenment and speech the means or the aim? (Aldous Huxley had already foreseen that they would have show up as closely related.) In Tokyo a new perspective opened up, due to a characteristic form of Oriental mentality, the Japanese one.

Japanese people think that every animated or unanimated thing is part "divine", (in an animistic way, unknown to us, which only an anthropologist may understand); the use and study of every natural event, with every means, has always been a religious action for them. The Japanese consider nature and man as a single entity, something we have only just started, perhaps, to understand! We call it ecology, it would be better to call it harmony –.

All this may seem old-fashioned, but we are in 1977, not so long ago, but so many momentous events have occurred since then.

I would like to link these true thoughts of Eduardo's to what I was talking about at the beginning, that is his emotional inclination towards Oriental mentalities – and only God knows how complicated they are. But not even once did he under evaluate the pragmatism of Japanese people (to say it in a coherent way, they dress as businessmen in the morning working in local or foreign stock exchanges, but once at home they wear their traditional wooden sandals and kimono, drink hot rice wine and listen to the traditional music of Kotò and Shamisen ...), since he had well understood that their harmony was always pursued with purely pragmatic means.

In any case the Roman Symposium was a success (*The Human Use of Human Ideas*): there were 15 foreign scientists, just to mention a few, the Americans Bob Marshak and Linstone, the Balkan Damjanovic, Rusmunson from Stockholm, the Nobel Prize winner Prigogine, Dr Atsumi from Biomedical Engineering and President of the Japanese firm which produced artificial internal organs, Professor Piero Caldirola from Milan, Chiarelli from Turin, Mendia from Naples, Eng. Di Giulio from the Italian Montedison, and Siniscalco who was one of our eminent scientists.

Professor Jean Claude Simon of the University of Paris "Pierre et Marie Curie" organized the third Symposium in 1978 (51 experts); in 1979 in Stockholm, the Honda Symposium was inaugurated by the King of Sweden thanks to Professor Hambraeus, of the Swedish Royal Academy of Sciences (39 specialist in various fields, with Eduardo entrusted to present the results up to then achieved); 1982, Columbus, Ohio, with President Regan's blessing and the participation of 150 Japanese, American and European scientists – *Social Effects of Advanced Technologies*; 1983, in London the Symposium was opened by the Duke of Edinburgh – *Social and Cultural Challenges of Modern Technology*, with scientists, managers and ministers, about 150, from both Eastern and Western countries, among them, Umberto Agnelli ... In some way Mr Honda had to extend the market of his motorbikes, considering how much money he had spent!

Meanwhile, in 1980, Eduardo and some other eminent scientists established Discoveries Italia, which had the same aims as Discoveries International, and in 1981 an interesting Conference "*The Role of Science in the Post-Industrial Society*" was held, supported by Sperry Univac.

Just to mention a few lecturers: Professor Adriano Buzzati Traverso and the scientists Marchetti and Margulies from Vienna. Chiarelli and Arecchi from Florence. "How should men of science and experts behave when facing the violent impact of new technologies on western social structure?".

An International Symposium was held in Melbourne in 1984, on the use of resources and technology for human interest (the first multi-ethnical symposium in Australia).

In Brussels in 1985, professor Prigogine introduced Dr Salk and Alwin Toffler, among many others (94 participants from 20 countries); in 1986 in a solemn conference in Tokyo, which marked a pause in the Symposia (but in the meantime and later, more industrial ones followed) where the seniors were awarded a prize in a typically Japanese ceremony. ... On that occasion Eduardo made a short speech in Japanese, which touched Mrs Honda.

The last two European meetings, from what I can remember, were in Vienna in 1987 with 52 scientists from 13 countries (*The Complexity of Human Environmental Conditions*) and Bonn, with many professors and scholars from the Max Planck Institute and co-sponsored by the Alexander von Humboldt Foundation (*Basic Life Sciences and Human Society*).

The concrete results of all this were certainly not up to what had been expected, in some way too ingenuous, but accomplished by hard work and a strong will to fulfil difficult and valid projects. But I am still convinced that Eduardo's Japanese experience had to be known, even if it took me ten years to make up my mind. After all, Eduardo was the first to guide Mr Honda into Europe (he was fascinated by the non-material world, not only the material one). I am also convinced that a dose of witty, balanced, and brave idealism does no harm, more than ever today in this atmosphere of great but obscure progress.

I would like to thank once again Gerardo Marotta for hosting this commemoration and many others in this prestigious philosophical "hortus". His deep friendship with Eduardo, often supported by interesting Sunday chats, will always be in my dearest thoughts.