

COSMIC RAY INTERACTIONS, PROPAGATION, AND ACCELERATION IN SPACE PLASMAS

LEV DORMAN



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COSMIC RAY INTERACTIONS, PROPAGATION, AND ACCELERATION IN SPACE PLASMAS

By

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Dedicated to the memory of my eldest brother Abraham Argov (1914-2003), whom I met for the first time in 1989 in Paris (see the picture above, he is on the left). In 1925 he emigrated with our great grand parents Globman (on our mother's side) from Ukraine to Palestine. Later he took part in the foundation and governing of the prominent kibbutz Beit-Hashita in the Yizreel Valley. As an officer, he took part in the War of Independence in 1948 (when his family name was changed to Hebrew, Argov). Abraham, together with my cousin Michal Govrin-Brezis, organized my first visit to Israel in 1990 and arranged my meeting with Prof. Yuval Ne'eman. Ne'eman, who soon became the Minister of Science, played an important role in the formation of the Israel Cosmic Ray and Space Weather Centre and the Emilio Segre' Observatory.

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PREFACE

As I mention in the Preface to the previous book (Dorman, M2004), after graduation in December 1950 Moscow Lomonosov State University (Nuclear and Elementary Particle Physics Division, the Team of Theoretical Physics), my supervisor Professor Dmitry Ivanovich Blokhintsev planned for me, as a winner of a Red Diploma, to continue my education as an aspirant (a graduate student) to prepare for Ph.D. in his very secret Object in the framework of what was in those time called the Atomic Problem. To my regret the KGB withheld permission, and I, together with other Jewish students who had graduated Nuclear Divisions of Moscow and Leningrad Universities and Institutes, were faced with a real prospect of being without any work. It was our good fortune that at that time there was being brought into being the new Cosmic Ray Project (what at that time was also very secret, but not as secret as the Atomic Problem), and after some time we were directed to work on this Project. It was organized and headed by Prof. Sergey Nikolaevich Vernov (President of All-Union Section of Cosmic Rays) and Prof. Nikolay Vasiljevich Pushkov (Director of IZMIRAN); Prof. Evgeny Lvovich Feinberg headed the theoretical part of the Project. Within the framework of this Project there was organized in former Soviet Union in 1951-1952 a wide network of CR stations equipped with a Compton type of large ASC-1 and ASC-2 ionization chambers developed in USSR (see Sections 1.2.7 and 4.2 in Dorman, M2004).

At that time many experimental results on CR time variations were obtained, but they were very considerably affected by meteorological effects and by meson-nuclear cascade in the atmosphere. Therefore it was not possible to make reasonable transformation from observed CR time variations in the atmosphere and underground to the variations expected in space. To solve this problem, it became necessary to develop a full theory of cosmic ray meteorological effects and a special method of coupling functions between primary and secondary CR variations (this work was finished at the end of 1951 and was described in the IZMIRAN's *Instructions on CR Data Processing*, see References to Chapter 1 of Dorman, M2004; Dorman, 1951a,b). Only from 1954 it becomes possible for our work on CR variations to appear in the open scientific literature, and from 1955 – to take part (by presentation of papers) in International Cosmic Ray Conferences. Mainly our results of that time were described in my first book (Dorman, M1957, which was translated very soon into English in the USA, thanks to the help of Professor John Simpson, at those time President of International CR Commission). Soon after this, in 1958, under the auspices of the International CR Commission the Committee of CR Meteorological Effects was organized, and I became its Chairman. Under the auspices of this Committee a special Instruction for CR Data Processing was developed which took into account corrections on meteorological effects.

In 1957 I was invited to work on special problems in Magnetic Laboratory of the Academy of Sciences of USSR as a Head of Department of Magnetic Hydrodynamics. In few years this Laboratory was transferred into the I.V. Kurchatov Institute of Atomic Energy, and I continued to work in this Institute up to 1965. In parallel I also worked at Moscow State University as Professor in the CR and Space Research Team. I also gave lectures in Irkutsk, Alma-Ata, Nalchik, Tbilisi, Erevan, Samarkand, and others places. Over about 40 years of teaching under my supervision more than hundred graduate students and scientists in USSR and some other countries gained their Ph.D. and several tenths became Doctors of Science. As my hobby I continued to work in CR research, and as Vice-President of All-Union Section of Cosmic Rays and Radiation Belts, took an active part in preparing the Soviet net of CR stations to the IGY (International Geophysical Year, 1957-1958): we equipped all soviet stations in USSR and in Antarctica with standard cubic and semi-cubic muon telescopes and with neutron monitors of IGY (or Simpson's) type. In connection with preparing for the IQSY (the International Quiet Sun Year, 1964-1965), the soviet net of CR stations was extended about two fold and they were equipped with neutron super-monitors of IQSY type (with an effective surface about 10 times bigger than the previous monitor of IGY type).

In 1965 I returned to IZMIRAN, and founded the Cosmic Ray Department (thanks to help of Professor N.V. Pushkov and Academicians M.D. Millionshchikov, L.A. Artsymovich, and V.I. Veksler). For the next 30 years, I was a Head of this Department, which became the center in the Soviet Union of scientific CR research in geophysical and astrophysical aspects. Our Department supported the work of all Soviet CR stations in the USSR and undertook the entire work of Soviet CR stations in Antarctica. We organized many CR expeditions inside USSR and in the Arctic Ocean, as well as in Pacific, Atlantic, Indian, and Southern Oceans on the ships "Academician Kurchatov", "Kislovodsk" and others (expeditions were equipped with a neutron super-monitor of IQSY type, with a multi-directional muon telescope, with radio-balloon CR measurements in the troposphere and stratosphere). Much very important data were obtained about coupling functions, integral multiplicities, and on the planetary distribution of cutoff rigidities.

From 1955 I took part in all International Cosmic Ray Conferences by presenting of original papers, as well as Invited Papers (in 1959 and 1965), Rapporteur Papers (in 1969 and 1987), Highlight Paper (in 1999), but I was able to go abroad only in 1966-1969 (thanks to N.V. Pushkov and M.D. Millionshchikov) and then from 1988, after "perestroika". The first country I traveled to was Bulgaria (the International School on Space Physics), then Yugoslavia (the International Symposium on Solar-Terrestrial Relations) in 1966. In 1967-1968 I headed the CR expedition to South America on the ship "Kislovodsk", went to Czechoslovakia in 1968, and to the International CR Conference in Budapest in 1969. Then up to 1988 I had no permission to go abroad. After 'perestroika', thanks to invitations: from K. Nagashima I went to Japan, from C.J. Cesarsky to France, from A.W. Wolfendale

and J.J. Quenby to England, from K. Otaola and J.F. Valdes-Galicia to Mexico, from D. Venkatesan to Canada, from J.A. Simpson and H. Ahluwalia to USA, from W.I. Axford and H.J. Völk to Germany, from A. Bishara to Egypt, from L.O'C. Drury to Ireland, from N. Iucci, G. Villoresi, and M. Parisi to Italy, from P.J. Tanskanen to Finland, from M. Duldig to Australia.

In October of 1989 in Paris I met for the first time with my eldest brother Abraham Argov. In 1925 with our great grand parents Globman (from the mother side) he went from Ukraine to Palestine, were in those time been Akiva, Pinhas, and Shlomo Globman (Govrin after 1948), three youngest brothers of my mother (they came to Palestine at the beginning of 1920-th). In Palestine Abraham took active part in the foundation and governing of the prominent kibbutz Beit-Hashita in the Yizreel Valley. As an officer, he took part in the War of Independence in 1948 (when his family name was changed to Hebrew, Argov). Abraham, together with my cousin Michal Govrin-Brezis, organized my first visit to Israel in 1990 and arranged my meeting with Prof. Yuval Ne'eman. Ne'eman, who soon became the Minister of Science, played an important role in the formation of the Israel Cosmic Ray and Space Weather Centre and the Emilio Segre' Observatory. In 1991 I was invited by Prof. Yuval Ne'eman, as Minister of Science, to visit Israel with family for one year (the Institute of Advance Study at Tel Aviv University) to give lectures and organize a Cosmic Ray Research Center. Step by step, thanks to great help of Prof. Yuval Ne'eman, Dr. Abraham Sternlieb, Mr. Aby Har-Even, Major of Qazrin Sami Bar Lev, and of three Italian colleagues, Prof. Nunzio Iucci, Dr. Giorgio Villoresi, and Prof. Mario Parisi, there was founded the Israel Cosmic Ray & Space Weather Center with the Israeli-Italian Emilio Segre' Observatory on Mt. Hermon (now 2055 m above sea level, cut-off rigidity 10.8 GV; see description in Dorman, M2004, Section 4.8), and I became a Head of this Center and Observatory (up to present I continue also to work as a volunteer in IZMIRAN as Chief Scientist of the Cosmic Ray Department, which has been headed since 1995 by my former student, Dr. Victor G. Yanke).

About four years ago I was invited by Dr. Harry Bloom to prepare monographs on geophysical and space aspects of CR research and possible applications of them. The monograph *Cosmic Rays in the Earth's Atmosphere and Underground* was published in 2004. Now is ready the monograph *Cosmic Ray Interactions, Propagation and Acceleration in Space Plasmas*, and the next *Cosmic Rays in the Magnetospheres of the Earth and other Planets* will be ready after about one year.

With the problems of CR interactions, propagation and acceleration (or deceleration) in space plasmas I meet at the first time at the beginning of 1950-th when considered the nature of solar and galactic CR variations in the interplanetary space (great solar flare events, 11-year CR variations connected with solar activity cycle, Forbush decreases of CR intensity caused by geomagnetic storms, CR solar diurnal and semi-diurnal variations caused by CR anisotropy and the Earth's rotation, 27-day variations caused by the sun rotation, and so on). These problems became especially actually when we start to understand the nature of giant solar CR

event at February 23, 1956: how particles were accelerated for very short time up to energies more than 10 GeV in conditions of solar chromosphere and corona; how these energetic particles propagate in the interplanetary space; why at the beginning of event was observed a big anisotropy, but after 20-30 minutes the distribution of solar CR became about isotropic? Many scientists tried to give answers on these questions.

The next great CR event was magnetic storm at 29 August 1957, when we at the first time observed CR particle acceleration by the interplanetary shock wave, so called pre-increase effect (this shock wave caused also geomagnetic storm and Forbush decrease in CR intensity). Investigation of the pre-increase effect stimulated developing of the drift mechanism of charged particle acceleration at single interaction with the shock wave. After about 20 years this mechanism was extended by taken into account scattering and multi-interaction of accelerated particles with shock wave (so called diffusion or regular mechanism of particle acceleration by shock waves).

The detailed Contents gave information on the problems considered and discussed in the monograph. The **Chapter 1** shortly describes main properties of space plasmas and main properties of primary CR, considered in details different types of CR interactions: with space plasmas matter with generation many secondary particles, ionization and other energy losses, interactions with photons and electromagnetic radiation, interactions with frozen in space plasmas stationary and moving magnetic fields of different configurations (including magnetic traps). In this Chapter we consider also the interaction of extremely high energy CR with relict 2.7° K and extremely high energy gamma-rays with magnetic fields of the Sun and planets. We consider here also gamma-ray generation in solar and stellar winds by interactions of galactic and flare energetic particle with space plasma matter.

In **Chapter 2** we consider the problem of CR propagation in space plasmas describing by kinetic equation and different types of diffusion approximation. Especially are considered the kinetics of CR in a large scale magnetic fields, diffusion in the momentum space and in pitch-angle space, anisotropic diffusion. In details are considered balance of CR energy in multiple scattering in expanding magnetic fields, anomaly CR diffusion and mean free path in the Alfvén wave heated space plasma, bulk speeds of CR resonant with parallel plasma waves, the CR cross-field diffusion in the presence of highly perturbed magnetic fields, dispersion relations for CR particle diffusive propagation and path integral solution to the stochastic differential equation of the Markov process for CR transport, the compound diffusion, the influence of magnetic clouds on the CR propagation, non-diffusive CR particle pulse transport, and so on.

Chapter 3 devoted to CR non-linear effects in space plasma caused by CR pressure CR kinetic stream instabilities. These effects are important in our Galaxy and other galaxies (galactic wind driven by CR and influence on CR propagation, chemical composition and energy spectrum formation), in the Heliosphere

(dynamics effects of galactic CR pressure on solar wind propagation, on the formation of terminal shock wave and the boundary of the Heliosphere, Alfvén turbulence generation by kinetic stream instability of non isotropic CR fluxes and its influence on CR propagation and modulation), in CR and gamma-ray sources (influence of CR pressure and CR stream instability of escaping energetic particles on acceleration efficiency and formation of energy spectrum and chemical composition of escaping particles, influence of nonlinear effects on gamma-ray emissivity distribution), in the processes of CR acceleration by shock waves and in other acceleration processes (inverse influence of pressure and stream instability of accelerated particles on the structure and propagation of shock waves, on processes of reconnection, on formation of accelerated particles energy spectrum and chemical composition).

In **Chapter 4** we consider different processes of CR acceleration in space plasma. In the first, we show that the particles acceleration in space plasma is an universal phenomenon in the Universe, realized in about all astrophysical objects. In details are considered the Fermi statistical mechanism of particle acceleration and its developing, formation of particle energy spectrum during statistical acceleration by taking into account the dependence of main parameters of mechanism (transport path, velocities of scatterers, escaping parameters) with increasing of particle energy, determining of injection energy and the portion of the accelerated particles in the statistical mechanism. Especially are considered statistical acceleration in the turbulent plasma and by electromagnetic radiation, by the Alfvén mechanism of magnetic pumping, and so on. Critically are considered possibility of induction acceleration mechanisms. We consider mechanisms of particle acceleration by shock waves and other moving magneto-hydrodynamic discontinuities during single interaction, particle acceleration in case of magnetic collapse and compression, the cumulative acceleration mechanism near the zero lines of magnetic field, particle acceleration in shear flows of space plasma, and additional regular particle acceleration in space plasma with two types of scatters moving with different velocities. In details we consider also quickly developed in the last 30 years mechanisms of shock-wave diffusion (regular) acceleration without and with accounting non-linear processes, particle acceleration by relativistic shock waves and by the electrostatic shock waves.

At the beginning of monograph, there is **Frequently used Abbreviations and Notations**. At the end of book, in the **Conclusion and Problems** we summarize main results and consider some unsolved key problems, important for development of the considered branch of CR Astrophysics and Geophysics. In the **References** there are separately references for Monographs and Books as well as for each Chapter. For the convenience of the reader, at the end of book we also put **Object Index** and **Author Index**.

We shall be grateful for any comments, suggestions, preprints and reprints which can be useful in our future research, and can make the next Edition of the book better and clearer; they may be sent directly to me by e-mail

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FREQUENTLY USED ABBREVIATIONS AND NOTATIONS

- A_{sid} — amplitude of sidereal CR anisotropy
 CME — coronal mass ejections
 CR — cosmic rays
 $D(E_k)$ — differential energy spectrum of primary CR
 $(dE/dt)_{\text{ion}}$ — energy losses of particles on ionization
 $(dE/dt)_{\text{brems}}$ — energy losses of electrons on generation of bremsstrahlung radiation
 E — total energy of particles
 E_k — kinetic energy of particles
 E_o — total energy of primary CR particle
 EAS — External Atmospheric Showers of CR
 EHE — extra high energy CR (particles, gamma-rays, and so on)
 ESA — European Space Agency
 ESO — Israel-Italian Emilio Segre' Observatory (Mt. Hermon, Israel)
 $F(E_k)$ — CR spectrum in sources
 FEP — Flare Energetic Particles
 G — Green's function
 GCR — galactic cosmic rays
 GLE — Ground Level Enhancement of solar CR
 $\mathbf{H}$ — vector of magnetic field
 $H = |\mathbf{H}|$ — strength of magnetic field
 H — altitude
 h — atmospheric pressure
 h_o — pressure on the level of observations
 HMF — Heliospheric magnetic field
 ICRC — International Cosmic Ray Conference
 ICRC — Israel Cosmic Ray Center (from 1992)
 ICRSWC — Israel Cosmic Ray and Space Weather Center (from 2003)
 ICRS — International Cosmic Ray Service (proposed in 1991)
 IGY — International Geophysical Year (July 1957-December 1958)
 IHY — International Heliospheric Year (2007-2008)
 IMF — interplanetary magnetic field
 IQSY — International Quiet Sun Year (1964-1965)
 IPY — International Polar Year (2007-2008)
 ISS — International Space Station
 IZMIRAN — Institut of Terrestrial Magnetism, Ionosphere, and Propagation of Radio-Waves Russian Academy of Sciences
 J_i — ionization potential
 L — mean path of particle interaction with magnetic inhomogeneities
 l — average distance between magnetic inhomogeneities

$m = 1, 2, 3, \dots$ — neutron multiplicities: number of pulses in NM from one neutron, proton, pion or muon in dependence of their energy during the time-gate ($\sim 10^{-3}$ sec)

m — mass of electron

m_{ac} — mass of accelerated particle

m w.e. — meters of water equivalent

$m_i(R, h)$ — integral multiplicity: number of secondary CR particles of type i on level h from one primary CR particle with rigidity R on the top of atmosphere

m_π, m_μ — rest mass of pions, muons

MT — muon or meson telescope

$N(R_c, h)$ or $I(R_c, h)$ — CR intensity

NM — neutron monitor or super-monitor

NM-64 or NM-IQSY — neutron super-monitor of IQSY type

NM-IGY — neutron monitor of IGY or Simpson's type

P_{ji} — fragmentation coefficient (average number of nuclei of type i formatted from one more heavy nuclei of type $j \geq i$).

QLT — quasy linear theory

$R = pc/Ze$ — particle rigidity

R_c — geomagnetic cutoff rigidity

$r_D = (kT/4\pi Ne^2)^{1/2}$ cm — Debye radius in space plasma

$r_g = cp/ZeH$ cm — Larmor radius

SA — solar activity

SCR — solar cosmic rays

SEP — solar energetic particles

SW — Space Weather

$T(h)$ — vertical air temperature distribution

$T_i = \lambda_i / \rho v_i$ — time life of CR particles relative to nuclear interactions in space plasma with density ρ (v_i is the velocity of CR particles of type i)

$t_m = L^2 / \nu_m = 2.5 \times 10^{-13} L^2 T^{3/2}$ — the magnetic field time dissipation in space plasma

u_1 — velocity of shock front

u_2 — velocity of matter after shock front

$u_1/u_2 = r$ — shock wave ratio

v — velocity of particle

$\langle v_e \rangle = \sqrt{3kT_e/m} \approx 6.8 \times 10^5 T_e^{1/2}$ cm/sec — mean velocities of electron motion in ionized hydrogen (T_e is the electron temperature in $^\circ\text{K}$).

$W(R, h)$ — coupling function

$X(E_k)$ — number of g/cm^2 of matter transferred by CR

$Y(R, h_o)$ or $Y(E, h_o)$ — yield function (characterized the dependence of CR detector counting rate per one primary proton from particle rigidity or energy)

Ze — charge of particle

Z_g — charge of atoms in background plasma

$Z^* \approx (v/\langle v_e \rangle) Z^{1/3}$ — effective charge of particle at $1 < v/\langle v_e \rangle < Z^{1/3}$ (v — velocity of particle, $\langle v_e \rangle$ — average velocity of electrons in atoms of background plasma).

θ — scattering angle

Θ — zenith angle

α — scattering angle

λ — latitude

λ — average size of magnetic inhomogeneities in space plasma

φ — longitude

Λ — CR transport path in space plasma

$\Lambda_G(E_k)$ — CR transport path in the Galaxy

$\Lambda_{\text{Gloc}}(E_k)$ — CR transport path in the local region near the Sun of the Galaxy

$\Lambda_i = 2 \times 10^4 T^2 N^{-1}$ — transport path of ions in space plasma

λ_i — transport paths (in g/cm^2) for absorption of different nuclei of CR

μ^+, μ^- — positive and negative muons

$\nu_m = 4 \times 10^{12} T^{-3/2}$ — magnetic viscosity in space plasma

π^+, π^-, π^0 — positive, negative and neutral pions

$\sigma = H_2/H_1 = u_1/u_2$ — degree of the compression of transverse magnetic field in the shock wave

$\sigma = 2 \times 10^7 T^{3/2}$ — conductivity in space plasma

σ_i — cross sections for nuclear interactions of different nuclei of CR (in 10^{-26} cm^2)

$\sigma_i = K \frac{\pi Z^2 e^4}{J_i} \frac{m_{ac}}{m E_k} \ln \left(\frac{m E_k}{m_{ac} J_i} \right)$ — ionization cross section of shell i with ionization potential J_i (K is the coefficient of the order of unity).

τ_π, τ_μ — life-time of rest charged pions and muons

$\omega_g = mc/ZeH \text{ sec}^{-1}$ — Larmor or gyro-frequency

Chapter 1

Cosmic Ray Interactions in Space Plasmas

1.1. Main properties of space plasma

1.1.1. *Neutrality of space plasma and Debye radius*

Space plasma is mostly very highly ionized gas where the main interactions between particles are Coulomb interactions. One of the main characteristics of space plasma (as in any plasma) is the value of the Debye radius:

$$r_D = \left(kT / 4\pi N e^2 \right)^{1/2} \approx 6.9 (T/N)^{1/2} \text{ cm}, \quad (1.1.1)$$

where the temperature of plasma T is in $^{\circ}\text{K}$, and concentration of plasma particles N is in cm^{-3} . It is important that the Debye radius for space plasma be much smaller than the dimension of the system. For example, in solar wind near the Earth's orbit ($T \leq 10^5 \text{ K}$, $N \geq 1 \text{ cm}^{-3}$) we obtain $r_D \leq 2 \times 10^3 \text{ cm}$; in the solar corona $r_D \leq 10 \text{ cm}$; in the interstellar space $r_D \leq 10^3 \text{ cm}$. In all space processes with characteristic dimensions bigger than r_D plasma must be considered as neutral: in any volume bigger than r_D the number of negative particles (mostly electrons) and positive particles (mostly ions) is equal.

1.1.2. *Conductivity and magnetic viscosity of space plasma*

Space plasma is mainly fully ionized hydrogen. In this case, the conductivity σ and magnetic viscosity ν_m are determined only by the plasma temperature:

$$\sigma = 2 \times 10^7 T^{3/2}, \quad (1.1.2)$$

$$\nu_m = c^2 / 4\pi\sigma = 4 \times 10^{12} T^{-3/2}. \quad (1.1.3)$$

1.1.3. *The time of magnetic fields dissipation; frozen magnetic fields*

The time (in sec) of ohm's dissipation of a magnetic field characterized by the dimension L will be

$$t_m = L^2 / \nu_m = 2.5 \times 10^{-13} L^2 T^{3/2}. \quad (1.1.4)$$

This gives: for magnetic inhomogeneities in the interplanetary space ($T \approx 10^5 K$, $L \geq 10^9$ cm) $t_m \geq 7 \times 10^{12}$ sec $\approx 2 \times 10^5$ years; for processes near solar spots ($T \approx 6 \times 10^3 K$, $L \approx 3 \times 10^9$ cm) $t_m \approx 10^{12}$ sec $\approx 3 \times 10^4$ years; for processes in interstellar space ($T \approx 10^3 K$, $L \geq 10^{15}$ cm) $t_m \geq 10^{20}$ sec $\approx 3 \times 10^{12}$ years; in supernova remnants ($T \geq 10^4 K$, $L \geq 10^{12}$ cm) $t_m \geq 3 \times 10^{17}$ sec $\approx 10^{10}$ years. These times are several orders bigger than characteristic times of processes in corresponding space conditions, and in some cases are bigger than the age of Universe. It means that magnetic fields in space plasmas can be considered as frozen in plasmas and moving together with moving matter.

1.1.4. Transport path of ions in space plasma and dissipative processes

Space plasma can be mainly considered as un-collisions. In actuality the transport path of ions Λ_i (in cm) in the fully ionized hydrogen plasma is

$$\Lambda_i = 2 \times 10^4 T^2 N^{-1}. \quad (1.1.5)$$

According to Eq. 1.1.5 we obtain for space plasma a very long transport path for collisions: for example, in solar wind ($T \approx 10^5 K$, $N \leq 10$ cm $^{-3}$) $\Lambda_i \geq 2 \times 10^{13}$ cm. It means that with all dissipative processes caused by plasma, particle collisions can be neglected.

1.1.5. Space plasma as excited magneto-turbulent plasma

Space plasma, as a rule, can be considered as highly excited magneto-turbulent plasma with intensive macroscopic, collective movements. Sources of space plasma excitation are the following: thermal convection leads to the generation of magnetic fields and their floating to the Sun's surface as sunspots (on the Sun); great discharging processes in solar flares; flowing of the inhomogeneous solar wind with frozen in magnetic field around the Earth's magnetic field and formation of the Earth's magnetosphere with radiation belts and very exiting plasma; explosions of Novae and Supernovae stars in our Galaxy; explosions of galactic nucleuses and collisions of galaxies in the Universe, etc..

Large scale movements in space plasma generate in plasma currents and electro-magnetic fields that lead finally to charged particle acceleration.

1.1.6. Main channels of energy transformation in space plasma

According to Syrovatsky (1968) the first main channel of the space plasma kinetic energy transformation is generation and amplification of a magnetic field up to the equilibrium value H determined by the relation

$$H^2/8\pi \approx \rho u^2/2, \quad (1.1.6)$$

where ρ is the plasma density, and u represents the characteristic velocities of macroscopic movements. Then the movements of magnetic fields frozen in space plasma lead to the generation of electric fields, and by them the generation of non-thermal particles, which give electromagnetic radiation, thermal heating of plasma and runaway accelerated particles (internal CR). Let us note that in fact the situation is much more complicated, because usually there are also external CR which together with internal CR influence space plasma through the non-linear processes: CR pressure and (in the case of existing of CR anisotropy) kinetic stream instability effects. The channels of energy transformation in space plasma, according to Syrovatsky (1968), taking into account non-linear CR processes (Section 1.7, and in more details Chapter 3) are shown in Fig. 1.1.1.

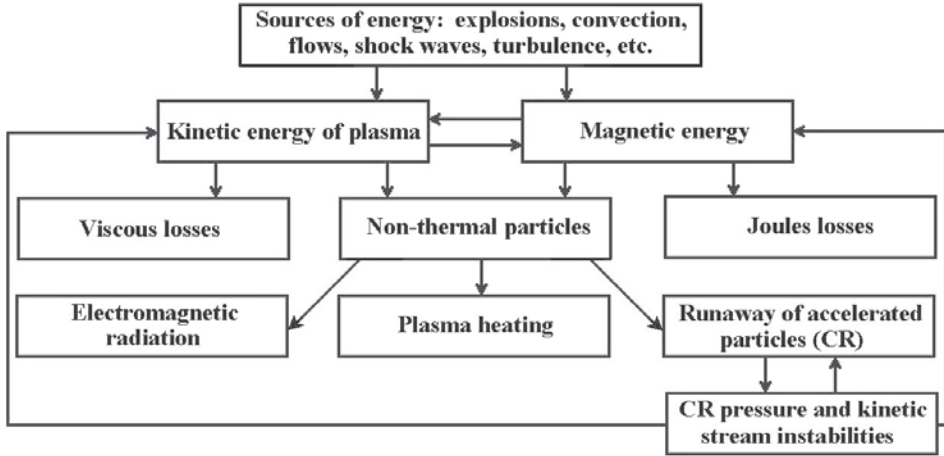


Fig. 1.1.1. Main channels of energy transformation in space plasma according to Syrovatsky (1968), expanded and taking into account non-linear CR processes (described in detail below in Chapter 3).

1.1.7. Particle acceleration in space plasma and the second fundamental law of thermodynamics

The phenomenon of particle acceleration in space plasma is, at first sight, in sharp contradiction with the second fundamental law of thermodynamics. Namely, by particle acceleration processes plasma transforms, one would think, into an evidently non-equilibrium state: thermal plasma + very small number of accelerated particles with energy density of the same order or much higher than energy density of the thermal plasma. However, as was emphasized by Syrovatsky (1968), there is no contradiction. The matter is that the particle acceleration proceeds during a time that is much smaller than the time of thermal relaxation of space plasma. In fact, the

system of space plasma is very far from the state of thermodynamically equilibrium. As it was shown in Dorman (M2004, Section 1.1.2), for space plasma it is statistically advantageous to have particle distribution with a raising ‘tile’ in the high energy range. So the acceleration processes transform the space plasma into a more advantageous state (i.e., in full agreement with the second fundamental law of thermodynamics), into a state of bigger entropy.

1.2. Main properties and origin of CR

1.2.1. *Internal and external CR of different origin*

As was considered in Dorman (M2004, Section 1.1.1), it is natural to define CR as particles and photons with energies at least several orders of magnitude higher than the average energy of thermal particles of background plasma. There are internal CR, generated inside the background plasma of some object, and external CR, generated in other objects and propagated into the object considered. For example, metagalactic (or extragalactic) CR of very high energy (up to 10^{21} eV), are generated in radio galaxies, quasars and other powerful objects in the Universe, and come through intergalactic space to our Galaxy, to the Heliosphere, and into the Earth’s atmosphere. Therefore they are internal CR relative to the Metagalaxy and external CR relative to the Galaxy. Galactic CR with energy at least up to $10^{15} \div 10^{16}$ eV, generated mainly in supernova explosions and supernova remnants, in magnetospheres of pulsars and double stars, by shock waves in the interstellar space and other objects in the Galaxy, are internal relative to the Galaxy and external for Heliosphere and the Earth’s magnetosphere. Solar CR with energy up to $15 \div 30$ GeV, generated in the solar corona in periods of powerful solar flares, are internal for the Sun’s corona and external for interplanetary space and the Earth’s magnetosphere. Interplanetary CR with energy up to $10 \div 100$ MeV, generated by terminal shock wave on the boundary of the Heliosphere and by powerful interplanetary shock waves, are internal for the Heliosphere and external for the Earth’s magnetosphere. Magnetospheric (or planetary) CR with energy up to 10 MeV for Jupiter and Saturn, and up to 0.030 MeV for the Earth, generated inside the magnetospheres of rotated magnetic planets, are internal in magnetospheres of planets and external in the interplanetary space.

1.2.2. *On the main properties of primary and secondary CR*

The main properties of primary CR, according to measurements by balloons in the upper atmosphere and by satellites outside the atmosphere and magnetosphere (protons and nuclei with different charge Ze , electrons and positrons, anti-protons and gamma rays) were considered in detail in Dorman (M2004, Section 1.4). Below in Sections 1.2.3–1.2.13 we will consider shortly main properties of observed energy spectrum, anisotropy, transport paths, and chemical composition of galactic

CR and show that they are in close relationships caused by peculiarities of CR interactions and propagation in space plasmas which will be considered in the remaining part of Chapter 1 and in Chapter 2.

Properties of secondary CR (neutrons, protons, pions, muons, electrons, positrons, gamma rays, neutrinos, etc.) generated in nuclear meson and electromagnetic cascades in the Earth's atmosphere as a result of interactions of primary CR with the nucleus of atmospheric atoms (and for neutrinos also as generated in the solar interior) were considered in detail in Chapter 2 of the book Dorman (M2004). Below, in Section 1.3 we will consider CR interactions with the matter of space plasma, nuclear reactions, fragmentations, and generation of secondary elementary particles and daughter nuclei in the space plasma.

1.2.3. Five intervals in the observed CR energy spectrum

According to Dorman (1977a,b,c), the observed CR spectrum near the Earth's orbit can be broken into five intervals (see Fig. 1.2.1): 1 – kinetic energy interval $10^{21} \text{ eV} \geq E_k \geq 3 \times 10^{15} \text{ eV}$, 2 – $3 \times 10^{15} \text{ eV} \geq E_k \geq 3 \times 10^{11} \text{ eV}$, 3 – $3 \times 10^{11} \text{ eV} \geq E_k \geq 30 \text{ MeV/nucleon}$, 4 – $30 \text{ MeV/nucleon} \geq E_k \geq 1 \text{ MeV/nucleon}$, 5 – $E_k \leq 1 \text{ MeV/nucleon}$. Such a division in Fig. 1.2.1 is based on some physical considerations and observation data.

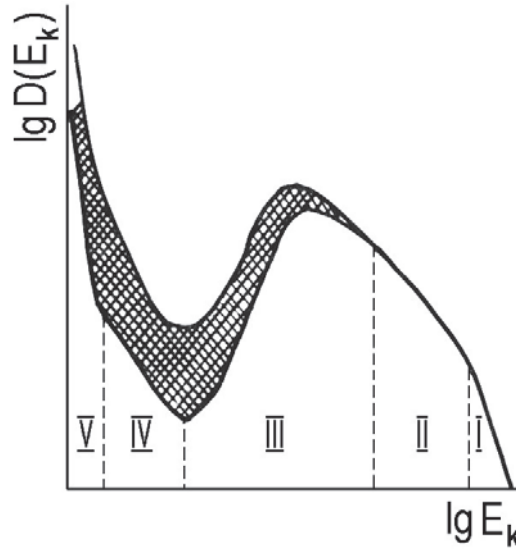


Fig. 1.2.1. The observed CR spectrum broken into five energy ranges. The shaded area shows the region subjected to solar modulation. According to Dorman (1977a).

The upper boundary of interval I should be determined by CR interactions with the 2.7 °K relict microwave radiation in case of meta-galactic origin of the highest

energy CR; the numerous available EAS experimental data are indicative of the existence of the particles with energies more than 10^{20} eV in the primary CR, but not more than $10^{21} - 10^{22}$ eV.

The boundary between intervals 1 and 2 is characterized by the jump change in the power exponent of the CR differential spectrum from 3.2–3.5 to 2.7 from interval 1 to interval 2 (this fact was first established on the basis of EAS measurements (e.g., Khristiansen, M1974).

The boundary between intervals 2 and 3 has particular meaning in the case of observations inside the solar system and corresponds to the upper energy boundary of CR modulation in the interplanetary space established on the basis of the data of many years of underground and ground based observations (Bishara and Dorman, 1973a,b,c, 1974a,b, 1975).

The chemical and isotopic composition and the regularities of the CR modulation by solar wind in the energy range 3 have been sufficiently studied and it is undoubted that interval 3 is completely of galactic origin.

The boundary between intervals 3 and 4 corresponds to the minimum of the CR spectrum in kinetic energy/nucleon and is probably somewhat variable with solar activity. This boundary separates the energy range of explicitly galactic origin (interval 3) from range 4 whose origin is being extensively discussed and has not become clear as yet. The problem of CR origin for interval 4 is discussed in detail in Dorman (1974, 1977d,e) where the following possible alternatives are treated: the solar (generated in solar flare acceleration processes and trapped for some time in the solar corona and in the Heliosphere), anomalous CR formed by ionization of interstellar atoms penetrating into interplanetary space and then accelerated in the vicinity of terminal shock wave, and galactic origin (small energy galactic CR so called sub-CR, penetrating from interstellar space into Heliosphere along the magnetic channels).

The boundary between energy ranges 4 and 5 is somewhat artificial, though it was assumed in Dorman (1977a) to be about 1 MeV/ nucleon. As the solar activity changes, this boundary may shift to both sides and the displacement may be from several tenths of an MeV/nucleon to several MeV/nucleon. The physical meaning of this boundary is that interval 5 is markedly different in the chemical composition, form of energy spectrum, and mode of temporal variations from interval 4. This fact is undoubtedly indicative of the different origin of CR in intervals 4 and 5. It is not excluded that the relative importance of various sources of interval 5 (low energy CR generation in solar corona in connection with chromospheric flares and during the quiet Sun; acceleration by the interplanetary shock waves and other disturbances in solar wind; generation and escaping from magnetospheres of rotating planets with a large magnetic field such as Jupiter, Saturn, and even the Earth; low energy particle generation in the transient layer between the solar wind and galactic magnetic field) varies markedly in time thereby resulting in the shift of the boundary between intervals 4 and 5.

The lower boundary of interval 5 extends, according to numerous works up to energies of ~ 0.01 MeV/nucleon and, perhaps, even lower, essentially coinciding with the upper energy boundary of the solar wind particles (let us note that these very low energy CR particles may have their origin from acceleration of background plasma particles in planetary magnetospheres and in the interplanetary space). Thus the observed CR spectrum is extended from $\sim 10^4$ eV/nucleon to $\sim 10^{21}$ eV (since the super-high energy particles are exclusively detected with EAS arrays, only the total energy can be determined), within ~ 17 orders.

1.2.4. Main CR properties and origin of CR in the interval 1

In various years, and up to recently, many researchers were of the opinion that the CR particles in the super-high energy interval 1 (10^{21} eV $\geq E_k \geq 3 \times 10^{15}$ eV according to the classification in Section 1.2.3) were mainly of metagalactic origin (Cocconi, 1960; Oda, 1961; Fichtel, 1963; Laster, 1964; Johnson, 1970; Berezhinsky et al., 1974; Hillas, 1975; Colgate, 1975a). This hypothesis was critically analyzed by Ginzburg and Syrovatsky (M1963). The following arguments favoring the metagalactic origin of the interval 1 (or its highest energy side) were considered: (i) the absence of the known sources of such high energies (up to about 10^{21} eV) in the Galaxy, (ii) the serious difficulties associated with the retention of the particles of very high energies in the Galaxy. The discovery and the study of the pulsars, however, have made it possible to suggest highly probable mechanisms of particle acceleration in the Galaxy up to $\sim 10^{20}$ eV (Ginzburg, 1969; Gunn and Ostriker, 1969; Silvestro, 1969; Colgate, 1975b,c). In particular, it was argued in Silvestro (1969) that the pulsars were capable of accelerating also the very heavy nuclei up to super-high energies. It is not excluded, either, that the particles of such high energies are generated in powerful processes taking place in the vicinities of the galactic center (Dorman, 1969). A serious argument favoring the galactic origin of the super-high energy CR is the absence of the spectrum cut-off at the high-energy side up to $\sim 10^{20}$ eV. Such cut-off should necessarily take place in the case of metagalactic (or extragalactic) origin owed to interactions with the 2.7 °K relict microwave radiation (Zatsepin and Kuzmin, 1966; Greisen, 1966; Hillas, 1968; Prilutsky and Rozental, 1969). Ginzburg (1968) presents a number of additional arguments against the hypothesis of the metagalactic origin of main part of observed CR, and Syrovatsky (1971) argues that the CR up to the highest observable energies may be of galactic origin.

1.2.5. The anisotropy in energy intervals 1 and 2

The anisotropy and mode of propagation in the Galaxy of super-high energy CR are of special interest in connection with the examined problem of their origin. The published data of the measurements of 84 largest size of EAS with four EAS arrays at Sydney, Volcano Ranch, Haverah Park, and Yakutsk have been used by Hillas and Ouldrige (1975) to study the distribution of the arrival of the $\geq$

2×10^{19} eV CR particles to the Earth. The search for sidereal anisotropy on the basis of the above data has given a value of $\sim 60\%$ for the amplitudes of the first and second harmonics. The possibility was analyzed in Hillas and Ouldrige (1975) that the obtained results were owed to the particles' arrival from the galactic clusters or super-clusters. The estimates of Hillas and Ouldrige (1975), show however, that, if the galactic super-clusters contain from 5×10^3 to 10^4 galaxies of the type of our Galaxy, the flux of the super-high energy particles expected from such super-clusters proves to be at least 400 times as small as the flux observed on the basis of EAS measurements. The data of measurements in the lower energy range also analyzed in Hillas and Ouldrige (1975) show that the amplitude of the sidereal anisotropy (see Fig. 1.2.2) in the $3 \times 10^{15} \text{ eV} \geq E_k \geq 10^{11} \text{ eV}$ energy range (interval 2 according to the classification given in Section 1.2.3) varies very little with energy and remains constant ($\sim 0.1\%$) with the peak near 19^{h} of sidereal time, which corresponds to an inconsiderable flux of CR along the force lines of the galactic spiral field.

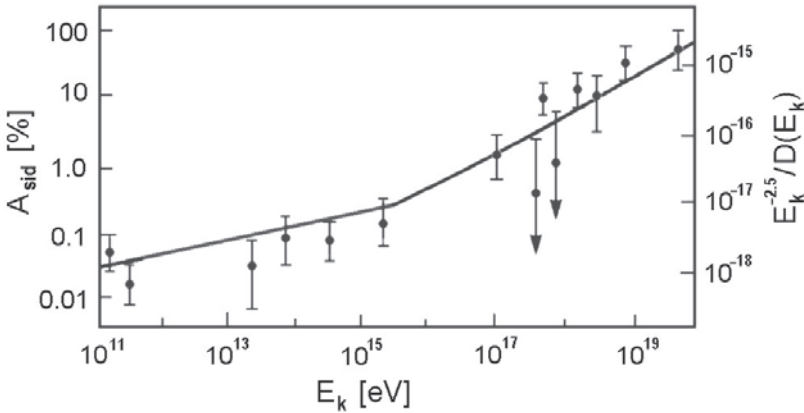


Fig. 1.2.2. The amplitude of the sidereal CR anisotropy A_{sid} as a function of energy E_k in the range $10^{14} - 10^{20}$ eV (the points with the vertical bars denoting the measurement errors). The solid line shows the dependence of A_{sid} from E_k according to Eq. 1.2.1 and the dependence $E_k^{-2.5}/D(E_k)$ by taking into account of Eq. 1.2.2.

The data of the various observations displayed in Fig. 1.2.2 show that the best agreement with the experimental data can be obtained on the assumption that the amplitude of the sidereal anisotropy A_{sid} in the energy range $3 \times 10^{15} \text{ eV} \geq E_k \geq 10^{11} \text{ eV}$ is not constant but increases approximately as $A_{\text{sid}} \propto E_k^{0.2}$. This corresponds to an increase of A_{sid} approximately an order (from $\sim 0.02\%$ to $\sim 0.2\%$) as energy increases from 10^{11} eV to $3 \times 10^{15} \text{ eV}$. Such regularity

in the variations of A_{sid} agrees with the mode of variations of the content of the daughter nuclei depending on the particle energies in the $10^9 \div 10^{11}$ eV (see below, Section 1.2.6). The time of the maximum is shifted with increasing the particle energies to 13^{h} of sidereal time, which corresponds to the appearance of the drift flux of CR with energies $\geq 3 \times 10^{15}$ eV from the Galaxy across the magnetic force lines. In this case the anisotropy amplitude increases rather rapidly with energy and reaches several tens of percent at $10^{19} \div 10^{20}$ eV. The results on the anisotropy discussed may be approximated by the expression

$$A_{\text{sid}}(E_k) \approx \begin{cases} 0.2 \times (E_k / 3 \times 10^{15} \text{ eV})^{0.2} \%, & \text{if } 10^{11} \text{ eV} \leq E_k \leq 3 \times 10^{15} \text{ eV}, \\ 0.2 \times (E_k / 3 \times 10^{15} \text{ eV})^{0.6} \%, & \text{if } 3 \times 10^{15} \text{ eV} \leq E_k \leq 10^{20} \text{ eV}, \end{cases} \quad (1.2.1)$$

which gives $A_{\text{sid}} \approx 0.03\%$ at $E_k = 3 \times 10^{11}$ eV ; $A_{\text{sid}} \approx 0.2\%$ at $E_k = 3 \times 10^{15}$ eV and $A_{\text{sid}} \approx 35\%$ at $E_k \approx 2 \times 10^{19}$ eV .

1.2.6. Relationships between the observed CR spectrum, the anisotropy, the relative content of the daughter nuclei, and the transport scattering path

It can be easily shown (see in Ginzburg and Syrovatsky, M1963; Dorman, 1969) that any diffusion model of CR propagation in the Galaxy involves a certain relationship between the observed CR spectrum $D(E_k)$ and the total spectrum of generation in all sources $F(E_k)$, the mean penetrable amount of the interstellar matter $X(E_k)$ (determining the relative content of the daughter nuclei of the type of Li, Be, B and some secondary isotopes which are explicitly absent from the sources), the anisotropy amplitude $A_{\text{sid}}(E_k)$, and the transport scattering path in the Galaxy $\Lambda_G(E_k)$:

$$D(E_k) \propto F(E_k) \Lambda_G^{-1}(E_k), \quad (1.2.2)$$

$$X(E_k) \propto \Lambda_G^{-1}(E_k), \quad (1.2.3)$$

$$A_{\text{sid}}(E_k) \propto \Lambda_{\text{Gloc}}(E_k), \quad (1.2.4)$$

where $\Lambda_G(E_k)$ is the transport path averaged over the entire region of particle propagation; $\Lambda_{\text{Gloc}}(E_k)$ is the local transport path in the region of the measurements of the anisotropy. It follows from the comparison between Eq. 1.2.1 and Eq. 1.2.4, that

$$\Lambda_{\text{Gloc}}(E_k) \propto \begin{cases} E_k^{0.2}, & \text{if } 10^{11} \text{ eV} \leq E_k \leq 3 \times 10^{15} \text{ eV}, \\ E_k^{0.6}, & \text{if } 3 \times 10^{15} \text{ eV} \leq E_k \leq 10^{20} \text{ eV}. \end{cases} \quad (1.2.5)$$

It should be expected that, although $\Lambda_G(E_k)$ and $\Lambda_{\text{Gloc}}(E_k)$ may be qualitatively different, the mode of their dependence on E_k is most probably the same. Then it follows from Eq. 1.2.2) that if the total spectrum of CR generation $F(E_k) \propto E_k^{-2.5}$ the observed spectrum $D(E_k) \propto E_k^{-2.7}$ in the interval $10^{11} \text{ eV} \leq E_k \leq 3 \times 10^{15} \text{ eV}$, and $D(E_k) \propto E_k^{-3.1}$ in the interval $3 \times 10^{15} \text{ eV} \leq E_k \leq 10^{20} \text{ eV}$. Thus the data presented above on the anisotropy are in agreement with the assumption of the unified spectrum of CR generation in the Galaxy of the form $F(E_k) \propto E_k^{-2.5}$ (the solid curve in Fig. 1.2.2) in the entire energy range of $10^{11} \text{ eV} \leq E_k \leq 10^{20} \text{ eV}$. This result is definitely indicative of the galactic origin of CR up to $\sim 10^{20} \text{ eV}$. In this case, according to the estimates of Hillas and Ouldrige (1975), if the CR of such high energies are protons or not too heavy nuclei, it is necessary for them to be retained and that the galactic magnetic field H_G of $\sim 2 \times 10^{-6} \text{ Gs}$ intensity would extend to the distances of at least 1 kps on either side of the galactic plane (this conclusion agrees with the measurements by Davies et al. (1974) of the Faraday rotation of the pulsar radiation, according to which $H_G = (2.2 \pm 0.4) \times 10^{-6} \text{ Gs}$ on the average in the said region). It should be noted that if $\Lambda_G(E_k)$ is described by the dependence of the type Eq. 1.2.5, the following important conclusion may be drawn from Eq. 1.2.3: the penetrable amount of interstellar medium $X(E_k)$ at $E_k \approx 3 \times 10^{15} \text{ eV}$ should be an order smaller than that at $E_k \approx 10^{11} \text{ eV}$. Such mode of variations in $X(E_k)$ depending on E_k is confirmed by the data of the direct measurements of the chemical composition of the CR in the energy range $E_k \leq 3 \times 10^{11} \text{ eV}$ (where it was found that $X \propto E_k^{-0.2}$). Of course, it would be extremely important to verify whether such trend also takes place at higher energies (the available data on the chemical composition in the high- and super-high energy ranges are not reliable yet). Thus if Eq. 1.2.5 is valid this means that the relative portion of the daughter nuclei and secondary isotopes should rapidly decrease with increasing energy and already between intervals 1 and 2 the observed CR composition should be close to the composition of the accelerated particles in the sources.

1.2.7. Chemical composition in the $10^9 \text{ eV/nucleon} \leq E_k \leq 3 \times 10^{11} \text{ eV/nucleon}$ range and the expected dependence of $\Lambda_G(E_k)$ and $A_{\text{sid}}(E_k)$ on E_k .

According to numerous experimental data on the chemical composition of CR, the relative content of the daughter nuclei in the range $10^9 \leq E_k \leq 3 \times 10^{11} \text{ eV/nucleon}$ decreases with increasing energy as $E_k^{-0.2}$. Therefore the penetrable amount of interstellar matter X_G will also similarly decrease with increasing E_k . Since, according to Eq. 1.2.3, $X_G \propto \Lambda_G^{-1}$ then $\Lambda_G \propto E_k^{0.2}$ in the above mentioned energy range. At the same time it follows from Eq. 1.2.4 that the sidereal anisotropy amplitude A_{sid} is (if other conditions being equal) $\propto \Lambda_G$, whence $A_{\text{sid}} \propto E_k^{0.2}$, as the particle energy decreases, A_{sid} should also decrease and reach $\sim 0.01\%$, which agrees with the measurement data (Jacklin, 1965; Dorman et al., 1967, 1969). Let us note that in the above mentioned energy range the ground based measurements of the sidereal anisotropy are difficult owing to the additional scattering of particles and distortion of their trajectories in the Heliosphere which give rise to the problem of correct interpretation of the observation results. Therefore, the available data on the sidereal anisotropy in the low energy range should be treated as rough estimates.

1.2.8. Chemical composition in the energy range $3 \times 10^7 \text{ eV/nucleon} \leq E_k \leq 10^9 \text{ eV/nucleon}$ and the nature of the scattering elements in the Galaxy

Since, the content of the daughter nuclei and secondary isotopes is practically invariants, as the particle energy decreases further down to the lower boundary of interval 3, it should be expected that $X_G \approx \text{const}$ in the range $3 \times 10^7 \text{ eV/nucleon} \leq E_k \leq 10^9 \text{ eV/nucleon}$ and hence, according to Eq. 1.2.4 $\Lambda_G \approx \text{const}$ too. Therefore, from the analysis of Fig. 1.9.4–1.9.8 in Section 1.9.7, when the particle scattering in the Galaxy is determined by the inhomogeneities with field structure of various complexities, the smallest scale of the inhomogeneities should be at least smaller than $R_{\text{min}}/300H_o$. Since $E_{k \text{ min}} = 3 \times 10^7 \text{ eV/nucleon}$ corresponds to $R_{\text{min}} = 2 \times 10^8 \text{ V}$, this gives $\lambda_1 \leq 3 \times 10^{11} \text{ cm}$ at $H_o \approx 2 \times 10^{-6} \text{ Gs}$ (otherwise, as follows from Fig. 1.9.4–1.9.8 in Section 1.9.7 Λ_G should abruptly increase with decreasing $E_{k \text{ min}}$, which would result in a pronounced decrease of the relative content of the daughter nuclei at very low energies).

If, however, the particles are scattered in the Galaxy by the magnetic clouds λ_1 may be many orders larger, since according to Fig. 1.9.1 from Section 1.9.3 in this case $\Lambda_G \approx \text{const}$ with decreasing E_k (even though $E_k \ll E_{k\min}$). Of course, the real situation may be more complex, namely, the variations in λ may be accompanied not only by the variations in the parameters α and β characterizing the spectrum of the scattering elements, but also by a change of the relative importance of the scattering of particles of various energies by the magnetic clouds and the inhomogeneities of the form $j = 1, 2, 5$ (see below in Section 1.9).

1.2.9. The nature of the energy boundary between intervals 3 and 2

According to the analysis of Bishara and Dorman (1973a,b,c, 1974a,b, 1975) carried out on the basis of many years underground measurements of the CR muon component, the energy boundary between intervals 3 and 2 (the upper energy boundary of CR modulation in the interplanetary space) is 150-300 GeV (this boundary varies markedly throughout the 11-year solar activity cycle). Let us note that the CR modulation in the Heliosphere is very significant in the energy range studied (the particle flux is modulated by a factor of from $\sim 2-5$ to several thousands). In accordance with the numerous theoretical and experimental data, however, the amplitude of the spectral modulation is determined only by the rigidity R and velocity v of the particles. Therefore the modulation amplitude for all particles with the same ratios A/Z and the same energy/nucleon (the same R and v) is also the same, i.e. CR relative chemical composition inside the Heliosphere will be the same as that in the interstellar space.

At higher energies the modulation spectrum falls with increasing particle rigidity R as R^{-2} or even more rapidly, whereas at lower energies the spectrum modulation depth $\delta D(R)/D_o(R) \propto R^{-(0.8 \pm 1.0)}$. The upper energy boundary of the CR heliospheric modulation is owed to the magnetic inhomogeneities spectrum in interplanetary space being limited at the side of λ by certain λ_2 , the largest scale of the inhomogeneities. According to Bishara and Dorman (1974b) the elements of the sector structure are most probably such largest scale of inhomogeneities. Therefore at a velocity of the solar wind ~ 400 km/sec, the complete revolution of a force line of the regular component of the interplanetary magnetic field (in the form of the Archimedes spiral) is in each about 6 A.U., for four sectors $\lambda_2 \approx 1.5$ A.U. whence the upper boundary $E_{k\min} \approx 270$ GeV at $H \approx 4 \times 10^{-5}$ Gs. When moving away from the Sun (at distances more than 6 A.U. from the Sun), λ_2 is almost invariant, but H decreases (approximately inversely to the distance r from the Sun) and, therefore, $E_{k\max}(r) \approx 270(r/r_e)^{-1}$ GeV, the size of the effective modulation region decreases with increasing the particle energy. According to Fig. 1.9.4–1.9.8 at $j = 1$ in Section 1.9.7 and Fig. 1.9.1 in Section 1.9.3, at $E_k \geq E_{k\max}$ the transport path in

interplanetary space $\Lambda \propto E_k^2$, and since the modulation depth $\propto \Lambda^{-1}$ at the ultrarelativistic energies it will be $\propto E_k^{-2}$, which explains the results of Bishara and Dorman (1973a,b,c, 1974a,b, 1975).

At $E_k < E_{k \max}$ the dependence of Λ on E_k is determined by the parameters λ_1/λ_2 , α and β , and (as can be seen from Fig. 1.9.4–1.9.8 at $j = 1$ in Section 1.9.7 and Fig. 1.9.1 in Section 1.9.3) the appropriate selection of these parameters may ensure that $\Lambda \propto E_k^{0.8}$ in a broad energy range from $E_{k \max}$ to ~ 3 GeV.

1.2.10. The mode of the dependence of Λ on particle rigidity R from solar modulation data of protons, electrons, and nuclei with various Z

A vast amount of experimental material has been accumulated at present for the spectrum modulation and the radial gradient of protons, electrons, and various nuclei during a period of about one solar activity cycle (and during almost four 11-year cycles for moderate energies from ground based observations). Analysis of these data on the basis of the modern theory of modulation including the diffusion, convection, and adiabatic cooling makes it possible to obtain fairly complete information on the mode of the dependence of Λ on R in the interplanetary space for a broad energy range. The results of the theoretical calculations (both analytical and numerical, see in Dorman, M1975a,b) for the spectrum modulation may be presented, as a first approximation, in the form

$$D_{p,\text{obs}} = D_{po} \exp\left(-\frac{a}{\Lambda v_p}\right), \quad D_{e,\text{obs}} = D_{eo} \exp\left(-\frac{a}{\Lambda v_e}\right), \quad D_{Z,\text{obs}} = D_{Zo} \exp\left(-\frac{a}{\Lambda v_Z}\right) \quad (1.2.6)$$

for the protons, electrons, and nuclei Z respectively. The parameter a in Eq. 1.2.6 is determined by the solar wind velocity, the size and geometry of the modulation region, whilst the particle velocities for CR particles are interrelated as

$$\frac{v_e}{v_p} = R^{-1} \left(R^2 + m_p^2 c^4 \right)^{1/2}, \quad \frac{v_e}{v_Z} = R^{-1} \left(R^2 + \left(A m_p c^2 / Z \right)^2 \right)^{1/2}. \quad (1.2.7)$$

Using the results of determination of the interstellar electron spectrum D_{eo} on the basis of the data on the nonthermal galactic radio emission, the interstellar spectra of protons and nuclei Z may be determined from Eq. 1.2.6 without knowing the modulation parameters:

$$D_{po} = D_{p,\text{obs}} \left(D_{eo} / D_{e,\text{obs}} \right)^{v_e/v_p}, \quad D_{Zo} = D_{Z,\text{obs}} \left(D_{eo} / D_{e,\text{obs}} \right)^{v_e/v_Z}, \quad (1.2.8)$$

where v_e/v_p and v_e/v_Z are determined by the Eq. 1.2.8. The parameter a/Λ may be determined from the comparison between the observed spectra on the basis of Eq. 1.2.6:

$$\frac{a}{\Lambda(R)} = -v_e(R) \ln(D_{e,\text{obs}}(R)/D_{eo}(R)) = -v_p(R) \ln(D_{p,\text{obs}}(R)/D_{po}(R)). \quad (1.2.9)$$

Such analysis was carried out in many works (see, for example, Dorman and Dorman, 1965, 1967; Earl, 1972; Cummings et al., 1973; Cechini et al., 1974; Lezniak and Webber, 1974). It was found by Lezniak and Webber (1974), in particular, that the coordination of numerous experiments can be achieved on the assumption that Λ should markedly increase with decreasing E_k (see Fig. 1.2.3) in the low energy range ($E_k < 100$ MeV for electrons and $E_k < 10$ MeV for protons).

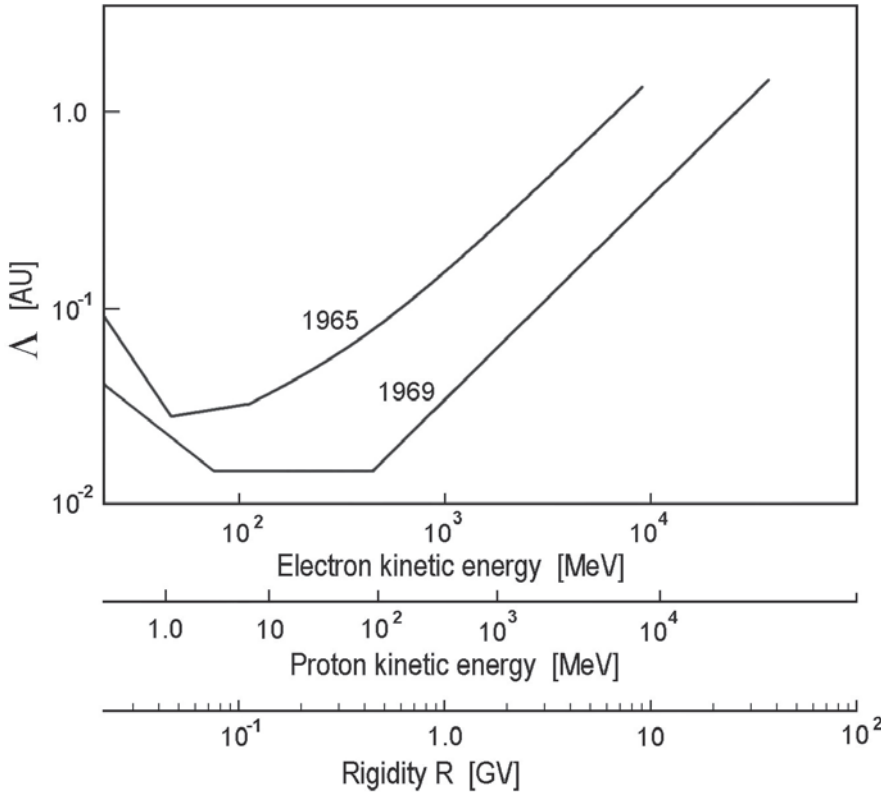


Fig. 1.2.3. The behavior of Λ as a function of rigidity and kinetic energy for galactic CR at low solar activity in 1965 and at high solar activity in 1969. According to Lezniak and Webber (1974).

Results shown in Fig. 1.2.3 are in accordance with Fig. 1.9.4-1.9.8 in Section 1.9.7. This means that the main role in the low energy particle scattering in the interplanetary space should be played not by the magnetic clouds but by the inhomogeneities with the field structure of various complexities. The maximum size of the inhomogeneities may be estimated on the basis of comparison between Fig. 1.2.3 with Fig. 1.9.4-1.9.8 in Section 1.9.7. Since it follows from this comparison that $R_{\min} \approx 3 \times 10^7$ V in 1965 and $R_{\min} \approx 7 \times 10^7$ V in 1969, we obtain $\lambda_1 \approx 2.5 \times 10^{10}$ cm in 1965 and $\lambda_1 \approx 6 \times 10^{10}$ cm in 1969 near the Earth orbit at $H_o \approx 4 \times 10^{-5}$ Gs.

1.2.11. The dependence of Λ on E_k from data of solar CR propagation

The numerous data on the time dependence of the solar CR (see in Dorman and Miroshnichenko, M1968; Dorman, M1978; Miroshnichenko, M2001) show that the transport scattering path $\Lambda \propto E_k^{0.8 \div 1.0}$ in the high energy range ($\geq$ several hundreds of MeV), whereas the dependence of Λ on E_k approaches the form $\Lambda \sim \text{const}$ with decreasing energy. The Gorchakov et al. (1975) analysis of the observations of the time dependence of the flux and anisotropy of solar CR from some chromospheric flares has shown that in this case the protons exhibit a trend of the increase in Λ with decreasing E_k at $E_k \leq 10 \div 30$ MeV (see Fig. 1.2.4) in an agreement with Fig. 1.2.3.

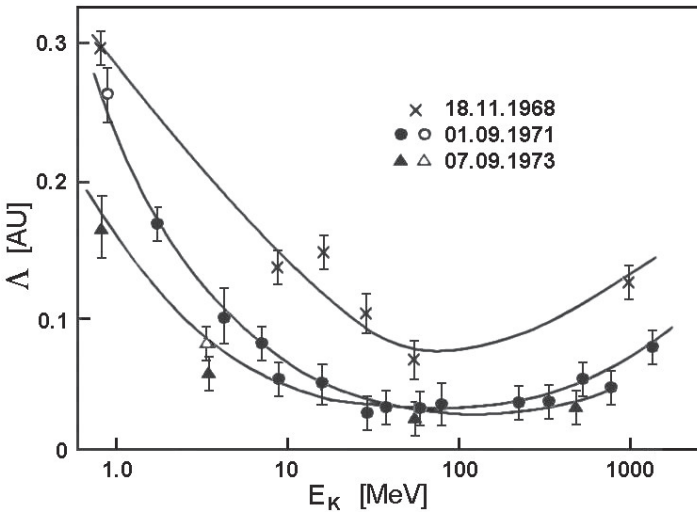


Fig. 1.2.4. The behaviour of Λ as a function of E_k for solar CR from the flares of November 18, 1968, September 1, 1971 and September 7, 1973. According to Gorchakov et al. (1975).

Some qualitative difference in Λ between Fig. 1.2.3 and Fig. 1.2.4 is because Λ in the first case is the effective value for galactic CR averaged over the entire modulation region, whereas Λ for solar CR reflects the local conditions in a near-Sun region of size of only several AU.

1.2.12. The features of the solar modulation of the CR spectrum and the measurements of the radial gradient

The numerous measurements of the radial CR gradient have given values several times smaller than those predicted by the modulation theory in terms of the spherically symmetrical model. This seems to place doubt on the correctness of the interpretation of the features of the solar modulation of the CR spectrum presented above. As was shown in Alania et al. (1977), however, the inclusion of the helio-latitude dependence of the solar wind parameters completely eliminates this difficulty (the CR penetrate through the high helio-latitude region and leak convectively near the helio-equator resulting in a pronounced decrease of the radial gradient compared with the value expected in terms of the spherically symmetrical model, which agrees with the experimental data). In this case, according to Alania et al. (1977), the spectrum modulation mode proves to be practically the same as in the spherically-symmetrical model.

1.2.13. The nature of the CR in energy intervals 3 - 5

It follows from the above analysis that the data on the chemical and isotopic composition, the energy spectrum, and the solar modulation prove without any doubt the galactic origin of interval 3 (30 MeV÷300 GeV). The problem is, however, much more complicated for intervals 4 and 5 and requires a special and comprehensive analysis including the diversity of the data on the chemical composition, the solar modulation, the temporal variations at different energies, the data on CR generation on the Sun, Jupiter, and in interplanetary space, and the indirect data on the possible existence of such particles in interstellar space (see in Dorman, 1977a,b). However, the result obtained above that Λ increases rapidly with decreasing E_k is indicative of the possibility of penetration of the particles of such low energies from the Galaxy to the inside of the Heliosphere.

1.3. Nuclear interactions of CR with space plasma matter

1.3.1. Cross sections, paths for absorption, and life time of CR particles relative to nuclear interactions in space plasma

Nuclear interactions of CR particles take place during processes of CR generation (in supernova remnants, stellar coronas, in the interstellar space and so on) and propagation in space plasma. These interactions lead to fragmentation of CR nuclei and absorption. Cross sections σ_i and transport paths λ_i for absorption

of different nuclei of CR in the interstellar plasma (93% of hydrogen and 7% of helium) according to Ginzburg and Syrovatsky (1966) is shown in Table 1.3.1.

Table 1.3.1. Cross sections σ_i and transport paths λ_i for absorption of different nuclei of CR.

Values	CR nuclei					
	p ($Z = 1$)	α ($Z = 2$)	L ($3 \leq Z \leq 5$)	M ($6 \leq Z \leq 9$)	H ($Z \geq 10$)	VH ($Z \geq 20$)
σ_i , 10^{-26} cm^2	2.8	11.2	20.7	29.7	50.5	74.3
λ_i , g/cm ²	72	18	9.8	8.0	6.0	2.7

From these data can be very easy determined the time life T_i of CR particles relative to nuclear interactions in space plasma with density ρ :

$$T_i = \lambda_i / \rho v_i, \quad (1.3.1)$$

where v_i is the velocity of CR particles of type i .

1.3.2. CR fragmentation in space plasma

Nuclear interactions of CR particles of type j with the matter of space plasma lead to their fragmentation and formation of nuclei of type i , where $i \leq j$ (we assume that i and j increase with increasing of Z). The contents n_i of CR in space plasma will be determined by following equation:

$$\frac{\partial n_i}{\partial t} - \text{div}(\kappa \nabla n_i) + \frac{n_i}{T_i} = Q_i(\mathbf{r}, t) + \sum_{j \geq i} P_{ji} \frac{n_j}{T_j}, \quad (1.3.2)$$

where κ is the diffusion coefficient of CR in space plasma (determined by the distribution of magnetic fields in space), $Q_i(\mathbf{r}, t)$ is the source function, T_i is the time life of CR particles relative to nuclear interactions (determined by Eq. 1.3.1), P_{ji} is the coefficient of fragmentation (i.e. average number of nuclei of type i formatted from one more heavy nuclei of type $j \geq i$). The approximate values of P_{ji} for α particles and groups of nuclei L, M, and H are shown in Table 1.3.2, and more detailed data for relativistic nuclei from Li to Fe in Table 1.3.3.

Table 1.3.2. Approximate values of P_{ji} for α particles and groups of nuclei L, M, and H. According to Fowler et al. (1957).

Primary nuclei of CR	Generated daughter nuclei			
	H ($Z \geq 10$)	M ($6 \leq Z \leq 9$)	L ($3 \leq Z \leq 5$)	α ($Z = 2$)
H ($Z \geq 10$)	0.31 ± 0.07	0.36 ± 0.07	0.12 ± 0.04	1.35 ± 0.18
M ($6 \leq Z \leq 9$)		0.11 ± 0.02	0.28 ± 0.04	1.22 ± 0.11
L ($3 \leq Z \leq 5$)			0.15 ± 0.05	1.09 ± 0.17
α ($Z = 2$)				0.41 ± 0.03

Table 1.3.3. Values of P_{ji} for relativistic nuclei from Li to Fe. According to Verschuur (1979).

Daughter nuclei	Primary nuclei of CR													
	Li	Be	B	C	N	O	F	Ne	Na	Mg	Al	Si	P-Cr	Fe
Li	.16	.13	.16	.10	.08	.07	.07	.07	.07	.07	.06	.06	.07	.09
Be		.05	.15	.07	.06	.05	.07	.09	.08	.07	.07	.07	.06	.05
B			.08	.07	.06	.05	.10	.11	.09	.08	.07	.07	.05	.04
C				.28	.16	.11	.12	.10	.09	.07	.05	.04	.03	.02
N					.29	.17	.19	.13	.10	.08	.07	.06	.04	.02
O						.29	.28	.21	.17	.15	.12	.10	.06	.03
F							.00	.14	.10	.05	.04	.04	.02	.01
Ne								.00	.23	.20	.16	.13	.08	.03
Na									.00	.14	.10	.05	.03	.01
Mg										.00	.23	.19	.11	.04
Al											.00	.14	.07	.03
Si												.00	.15	.07
P-Cr													.30	.64
Fe														.21

As we mentioned above, the Eq. 1.3.2 is valid for diffusion propagation of CR in space plasma (see Chapter 2 in more detail). In another case, when the propagation of CR is along magnetic field lines from some stationary source and there are no other CR sources in space, the contents of CR on the matter depth s from the stationary source (in units g/cm^2) whilst taking into account fragmentation of nuclei will be determined by the following equation

$$\frac{\partial n_i(s)}{\partial s} + \frac{n_i(s)}{\lambda_i} = \sum_{j \geq i} P_{ji} \frac{n_j(s)}{\lambda_j}, \quad (1.3.3)$$

with the boundary condition

$$n_i(s)\big|_{s=0} = n_{io}, \quad (1.3.4)$$

where n_{io} is the CR contents at the exit from the source.

1.3.3. Expected fluxes of secondary electrons, positrons, γ -quanta, and neutrinos

The nuclear interactions of CR with the space plasma matter in sources and during CR propagation also produce unstable secondary particles (K mesons, π^0 mesons, $\pi^\pm$ mesons, neutrons) whose decay eventually gives γ -quanta, electrons, and neutrinos. Since in this case the processes of generation and the kinematics of decay ($\pi^0 \rightarrow \gamma + \gamma$, $\pi^\pm \rightarrow \mu^\pm \rightarrow e^\pm$, $n \rightarrow p + e^-$, etc) are absolutely the same as those in the Earth's atmosphere, it is possible to use the result of (Greisen, 1960) and write, as a first approximation, in accordance with (Ginzburg and Syrovatsky, M1963) that the secondary particle generation by, for example, galactic CR will give the fluxes (in $\text{cm}^{-2}\text{ster}^{-1}\text{sec}^{-1}$):

$$I(E)dE \approx 10^{-4} E^{-2.64} K X dE, \quad (1.3.5)$$

where $K = 3.27$ for γ -quanta, 3.05 for neutrinos, 1.11 for electrons. In Eq. 1.3.5 the energy of secondary particles and gamma rays E is measured in GeV; X is the thickness of matter in g/cm^2 at the beam along the line of sight in the region filled with CR. In the Galaxy the > 1 GeV secondary particle fluxes prove to be inconsiderable; even the largest expected flux (from the galactic center at $X \sim 0.1 \text{ g/cm}^2$) is about $2 \times 10^{-5} \text{ cm}^{-2}\text{ster}^{-1}\text{sec}^{-1}$ for γ -quanta and neutrinos. According to Eq. 1.3.1, the estimate of generation rate for secondary electrons with $E \geq 1$ GeV in the halo (at a $\approx 10^{-2} \text{ atom/cm}^3$ concentration) gives an emissivity $\sim 2 \times 10^{-29} \text{ electron.cm}^{-3}\text{sec}^{-1}$, the value two orders smaller than that required to explain the observed flux of synchrotron radiation and the galactic electron flux observed on the Earth (Ginzburg and Syrovatsky, M1963). It can be seen below, however, that the inclusion of secondary particle generation is sometimes absolutely necessary in the problems of the study of CR generation and propagation. A lot of calculations of expected fluxes of secondary electrons, positrons, γ -quanta and neutrinos in space and in the atmosphere (including comparison with experimental data) were made by Daniel and Stephens (1974), Orth and Buffington (1974), Ling (1975), Verma (1977a,b), Badhwar and Stephens (1977).

In particular, Daniel and Stephens (1974) made calculations of the expected intensity of electrons, positrons and γ -quanta of various energies depending on the thickness of the atmosphere passed from 0 to 1000 g/cm^2 ; the data on nucleus-meson cascade of cosmic radiation in the Earth's atmosphere and energy

transmission from the hadron component to the electron-photon component were included in the calculations. The differential energy spectra of secondary electrons, positrons, γ -quanta and their variation with a depth of the atmosphere obtained are of primary importance in determining the respective corrections to the measurements of the fluxes of these particles from space. To estimate the effect of geomagnetic latitude the calculations were carried out for several values of geomagnetic cutoff rigidity: 0; 2; 4.5; 10 and 17 GV. To include approximately a chemical composition of primary CR for the free path to absorption, the following values were fixed: 120; 52; 33; 33, 32, 29 and 20 g/cm², respectively, for protons, α -particles, and nuclear groups L, M, H1, H2, H3. A comparison of calculated results with available experimental data made it possible to refine the general parameters of elementary acts, which are included in a theory. The results of calculations from (Daniel and Stephens, 1974) for space-energy distribution of electrons, positrons and γ -quanta in the Earth's atmosphere are presented in the form of three-dimensional diagrams, shown in Dorman, M2004 (Fig. 2.11.1–2.11.6 in Chapter 2).

Orth and Buffington (1974) computed by the Monte-Carlo method a meson-nucleus cascade of primary CR in the Earth's atmosphere and interstellar space at the depth ≤ 10 g/cm²; the expected fluxes and energy spectra of secondary electrons and positrons were found in the interval of energies 1÷100 GeV, arising as a result of the processes of decay of pions and muons: a development of subsequent electromagnetic cascades was also included. An approximate analytical expression was presented which makes it possible to determine easily the fluxes of secondary electrons and positrons with various energies, medium parameters and spectra of CR. The calculations were based on the following data: differential spectrum of primary protons of galactic CR depending on the proton energy E_p (in interstellar space: $2.0 \times 10^4 E_p^{-2.75} \text{ m}^{-2} \text{ ster}^{-1} \text{ sec}^{-1} \text{ GeV}^{-1}$; the spectrum at the boundary of the Earth's atmosphere, including solar modulation, was taken in the form $8.6 \times 10^3 E_p^{-2.55} \text{ m}^{-2} \text{ ster}^{-1} \text{ sec}^{-1} \text{ GeV}^{-1}$); the proton path with respect to inelastic interaction was taken to be 53.6; 57.3 and 100 g/cm² in hydrogen, interstellar medium and the air, respectively; the ratios of a number of electron and positrons, generated by all of CR nuclei, to their number, generated by the protons only: 1.36, 1.34 and 1.20 in hydrogen, interstellar medium, and in the air; the conversion path of γ -rays in the air, 48 g/cm²; the path lengths of $\mu^\pm$ and $\pi^\pm$ mesons to decay, $6.24 E_\mu \text{ km}$ and $0.0559 E_\pi \text{ km}$ (where E_μ and E_π are the energies of muons and pions, in GeV). It was shown that to account for the recent measurements of positron flux with the energy > 4 GeV, one has to assume that primary CR pass, on average, $(4.3^{+1.8}_{-1.2}) \text{ g/cm}^2$ of the interstellar medium from the moment of their generation.

It is of especial interest to study high energy neutrinos. Silberberg and Shapiro (1977) studied a diffusion background of neutrinos which is caused by the three sources: *a*) generated by CR in the Earth's atmosphere; *b*) generated in our Galaxy; *c*) generated out of the Galaxy. The flux of atmospheric neutrinos formed as a result of a decay of $\pi^\pm$ and $k^\pm$ -mesons generated in interaction of CR with the Earth's atmosphere, exceeds the flux of neutrinos of extra-terrestrial origin up to the energies $\sim 10^{14}$ eV. At $E > 10^{15}$ eV the direct generation of lepton pairs could be a prevailing source of atmospheric neutrinos. At higher energies ($\geq 10^{16}$ eV) a considerable uncertainty and a strong sensitivity to a model of neutrino source were observed in the value of diffusion flux. At these energies the following processes of neutrino generation take place: *a*) direct production of neutrino in the atmosphere; *d*) neutrinos from radio galaxies; *c*) neutrinos from galaxies; *d*) neutrinos from pulsars. To determine what of the mechanisms gives a prevailing contribution to neutrino fluxes, the direct observations are necessary. According to Margolis and Schramm (1977), a study of super high energy neutrinos gives important information about CR in the Universe, about physics of high energies, and about the physics of weak interactions. Measurements of neutrino flux from point sources make it possible to study acceleration of CR by such astrophysical objects as pulsars, Supernovas, and radio galaxies. In Margolis and Schramm (1977), an estimate was made of the flux value of neutrinos with the energies $E > 10^{11}$ eV from several types of astronomical sources. Berezhinsky (1977) considered some possible sources of neutrinos of super-high energies: Supernovae and interactions of CR protons with atomic nuclei and microwave photons in interstellar and inter galactic space. The author presented energy spectra of neutrinos calculated for various mechanisms of their generation. Beresinsky (1977) emphasized the following important aspects of astrophysics of neutrinos of super-high energies ($E \geq 10^{15}$ eV): a search for bursts of CR in remote cosmological epochs; a search for point sources of high-energy neutrinos; a study of interaction of neutrinos at the energies, unattainable for accelerators, a search for the W-bosons with a mass $30 \div 100$ GeV, and the measurement of a cross-section of neutrino interaction at the energy $E \geq 10^{15}$ eV. Berezhinsky and Zatsepin (1977) assumed that in the epoch of galactic and early-class stars formation, a burst took place with an energy output $\sim 5 \times 10^{60}$ erg in CR for our Galaxy. It is considered that the observed diffusion X- and γ -radiation in the range $1 \text{ keV} \div 30 \text{ MeV}$ is caused by high energy CR from this burst. Neutrinos with the energy $E \geq 3 \times 10^{15}$ eV should be produced as a result of interaction of high energy protons with microwave photons. The estimate was obtained for the flux value of these neutrinos. This estimate, as well as the assumption that neutrinos generated in early cosmological epoch did not undergo interactions in later stages of the Universe's expansion, makes it possible to register such neutrinos by means of a detector with the volume $\sim 10^9 \text{ m}^3$.

1.3.4. Expected fluxes of secondary protons and antiprotons

The problem of low energy secondary protons generated in nuclear reactions of CR with interstellar medium, was studied in the work of Wang (1973). The refined cross sections of (pH) and (pHe) interactions were used to obtain the intensities of secondary protons with an energy lower than 100 MeV, non-included processes were estimated. The rates of producing secondary protons were calculated and then a solution of the equation for the stationary density of secondary particles in the Galaxy was found. The intensity obtained of protons in interstellar medium for the energy range 10÷100 MeV appeared to be by 3÷5 times higher than the observed proton intensity near the Earth (this weakening is related to particle interaction with solar wind in interplanetary space. The value of intensity of protons obtained, born in nuclear interactions of high energy components of CR, is the lower limit of the actual intensity of protons near these energies in interstellar medium. Ganguli and Sreekantan (1976) calculated the expected fluxes of various secondary particles with the energy ≥ 10 GeV produced in nuclear interactions of CR with interstellar medium in the Galaxy, based on accelerator data on the effective cross-sections of the generation of $\pi^\pm$ mesons, antiprotons, and deuterons in proton-proton interactions up to the energies ~ 1500 GeV in nuclear interactions of CR with interstellar medium in the Galaxy. It was found that of γ -quanta with $E \geq 10$ GeV from the Galactic center on the boundary of the Earth's atmosphere should be $\approx 10^{-4}$ and the flux of antiprotons $(2 \div 3) \times 10^{-4}$ of the proton flux in CR; the expected deuterium flux appeared to be negligibly small compared to the flux arising as a result of fragmentation of α -particles in their interaction with interstellar hydrogen.

1.4. CR absorption by solid state matter (stars, planets, asteroids, meteorites, dust) and secondary CR albedo

Because, when passing through a grain of dust, a particle traverses $\leq 10^{-4} \text{ g.cm}^{-2}$ of matter (the mean size of grains of dust is $\approx 4 \times 10^{-5} \text{ cm}$), significantly below its interaction path, the dust grains will not in practice absorb CR and will give only some additional contribution to the fragmentation and production of secondary particles by the gaseous and ionized material in the space. In most cases, however, such a contribution is negligible (some 1% for interstellar space). The rest solids in the space (stars, planets, asteroids, meteorites), whose sizes (in g/cm^2) are as a rule much in excess of the nuclear interaction path, will be the CR absorbers and the generators of secondary albedo CR. If the bodies of type i with cross section $S_i = 4\pi r_i^2$ (where r_i is a radius of body) are spatially distributed with concentration $N_i = d_i^{-3}$ (where d_i is an average distance between bodies of type i), the mean lifetime T of a CR particle (with velocity v) relative to absorption by all the above mentioned types of celestial bodies will be

$$T = \sum_i (S_i N_i v)^{-1} = \sum_i \frac{d_i^3}{4\pi n r_i^2}. \quad (1.4.1)$$

In some cases, the importance of CR absorption by the bodies is negligible. According to (Ginzburg and Syrovatsky, M1963), for example, the life-time of a relativistic CR particles in the Galaxy before the particle arrives at any star ($S \approx 3 \times 10^{22} \text{ cm}^2$, $N \approx 10^{-54} \text{ cm}^{-3}$) is $\sim 3 \times 10^{13}$ years, the value which is many orders greater than the mean lifetime relative to nuclear interaction in the interstellar medium and CR diffusion from the Galaxy ($\sim 3 \times 10^7$ years). The absorption, however, should be taken into account in the problems of propagation of galactic and solar CR in vicinities of stars and planets (for example, in interplanetary space) the absorption is always of decisive importance to some modulation models (Dorman, M1963a)

1.5. CR interactions with electrons of space plasma and ionization losses

1.5.1. Ionization energy losses by CR nuclei during propagation in space

The ionization losses (energy losses for excitation and ionization of atoms of medium and for generation of Cherenkov radiation and δ -electron production) by a proton or nucleus with total energy E , rest mass M , and charge Z is in eV/sec (according to Ginzburg and Syrovatsky, M1963):

$$\begin{aligned} \left(\frac{dE}{dt} \right)_{\text{ion}} &= - \frac{4\pi e^4 Z^2 N_e}{mv} \ln \left[\frac{2mv^2}{J} \left(\frac{E}{Mc^2} \right)^2 - \frac{v^2}{c^2} \right] = -7.62 \times 10^{-9} Z^2 N_e \times \\ &\times \begin{cases} \left(11.8 + \ln(E_k / Mc^2) \right) \sqrt{(2Mc^2 / E_k)}, & \text{if } E_k \ll Mc^2, \\ \left(20.2 + 4 \ln(E / Mc^2) \right), & \text{if } (M/m)Mc^2 \gg E \gg Mc^2, \\ \left(19.5 + \ln(M/m) + 3 \ln(E / Mc^2) \right), & \text{if } E \gg (M/m)Mc^2, \end{cases} \end{aligned} \quad (1.5.1)$$

where v and E_k are the particle velocity and kinetic energy; N_e is the concentration of the medium electrons; m is the electron mass, J is the ionization potential. The Eq. 1.5.1 is also valid within a sufficient accuracy for a medium comprising, apart from hydrogen, helium and other light elements. In a completely ionized medium with electron concentration N_e the loss for production of δ -electrons (close collisions) and Cherenkov radiation of plasma waves (remote collisions) is significant. Such a loss has been totaled (in eV/sec):

$$\left(\frac{dE}{dt}\right)_{\text{ion}} = -7.62 \times 10^{-9} Z^2 N_e \times \begin{cases} \left(38.7 + \ln(E_k/Mc^2) - 0.5 \ln N_e\right) \sqrt{2Mc^2/E_k}, & \text{if } E_k \ll Mc^2, \\ \left(74.1 + 2 \ln(E/Mc^2) - \ln N_e\right), & \text{if } (M/m)Mc^2 \gg E \gg Mc^2, \\ \left(74.1 + \ln(E/Mc^2) - \ln N_e\right), & \text{if } E \gg (M/m)Mc^2. \end{cases} \quad (1.5.2)$$

The Eq. 1.5.1 and Eq. 1.5.2 are valid only for the particles whose velocities v are much in excess of the mean velocities $\langle v_e \rangle$ of electron motion in medium (in atomic hydrogen $\langle v_e \rangle = e^2/h \approx 2 \times 10^8$ cm/sec, in ionized hydrogen $\langle v_e \rangle = \sqrt{3kT_e/m} \approx 6.8 \times 10^5 T_e^{1/2}$ cm/sec, where T_e is the electron temperature in °K). However, the region $v \leq \langle v_e \rangle$ is of great interest to studying the mechanisms of CR acceleration in magnetized completely or incompletely ionized plasma (especially for the problem of injection into an acceleration process under the conditions of chromospheric flares, in active solar regions, in shock waves, in supernova explosions, etc.). In this energy range the loss (in eV/sec) is

$$\left(\frac{dE}{dt}\right)_{\text{ion}} = -2.34 \times 10^{-23} N (Z^* + Z_g) v^2 \quad (1.5.3)$$

in a gas with concentration of N atoms with atomic number Z_g . In Eq. 1.5.3 Z^* is the effective charge of particle, which is according to (Bohr, M1960) at $1 < v/\langle v_e \rangle < Z^{1/3}$ approximately

$$Z^* \approx (v/\langle v_e \rangle) Z^{1/3}. \quad (1.5.4)$$

This relation reflects the simple fact that the loss of a subsequent electron takes place when the velocity of particle equalizes the orbital velocity of this electron. At $v > \langle v_e \rangle Z$ and sufficiently frequent collisions, the ionization proves to be practically complete, $Z^* \approx Z$. It should be borne in mind when studying the particle motion in rarified gas that the ionization cross section σ_i of shell i with ionization potential J_i is maximum at in energy $E_{mi} \approx 3(M/m)J_i$ while the dependence of σ_i on E_k is determined by the relation

$$\sigma_i = K \frac{\pi e^4}{J_i} \frac{M}{mE_k} \ln \left(\frac{mE_k}{MJ_i} \right), \quad (1.5.5)$$

where K is the coefficient of the order of unity (see in Post, M1959). According to Eq. 1.5.5, the characteristic cross section of the loss of the first electron is of the order of gas-kinetic cross section ($\sim 10^{-16} \text{cm}^2$), whereas the cross section of the electron loss from inner shells is significantly smaller (for example, the cross section of the loss of the last electron from the K shell of Fe is only 10^{-21}cm^2).

In an ionized gas with concentration of N_e electrons and temperature T_e the energy loss (in eV/sec) is

$$\left(\frac{dE}{dt} \right)_{\text{ion}} = -1.8 \times 10^{-12} \frac{(Z^*)^2 N_e E_k}{A T_e^{3/2}}, \quad (1.5.6)$$

where A is the atomic number and Z^* is the effective charge of the particle.

1.5.2. Ionization and bremsstrahlung losses for CR electrons

The formulas for the ionization losses of relativistic electrons (in eV/sec) are somewhat different from Eq. 1.5.1 and Eq. 1.5.2 (the difference is mainly associated with the fact that the maximum energy transferred to electron in collisions with electron is $\sim E/2$):

$$\left(\frac{dE}{dt} \right)_{\text{ion}} = -7.62 \times 10^{-9} N_e \times \begin{cases} \left[18.8 + 3 \ln(E/mc^2) \right] & \text{in atomic hydrogen,} \\ \left[73.4 - \ln N_e + \ln(E/mc^2) \right] & \text{in plasma.} \end{cases} \quad (1.5.7)$$

Besides that, owing to their collisions with nuclei and electrons of medium, the electrons lose their energy by production of bremsstrahlung γ -quanta (Ginzburg and Syrovatsky, M1963):

$$\frac{1}{E} \left(\frac{dE}{dt} \right)_{\text{brems}} = -10^{-6} N_e \times \begin{cases} 8.0 & \text{in atomic hydrogen,} \\ \left[0.50 + 1.37 \times \ln(E/mc^2) \right] & \text{in plasma.} \end{cases} \quad (1.5.8)$$

Since the bremsstrahlung γ -quanta's energy is of the order of the electron energy, the continuous low-portion energy loss is substituted by discrete loss of electrons; it may be assumed in this case that the probability $P(s)$ for an electron to traverse a path s (in g/cm^2) is, according to Eq. 1.5.8:

$$P(s) = \begin{cases} \exp(-s/62) & \text{in atomic hydrogen,} \\ \exp(-s/52) & \text{in 90\% H + 10\% He mixture.} \end{cases} \quad (1.5.9)$$

1.6. CR interactions with photons in space

1.6.1. CR nuclei interactions with space photons

When propagating in space, the CR nuclei interact with photons of various energies, first of all with the universal microwave radiation at a temperature of about 2.7 °K (with very high density about 200 cm^{-3} in galactic and intergalactic space), and the infrared and light quanta of radiation from stars. In this case the photon energy E_{ph} in the center of mass system CR particle – background photon will be

$$E_{\text{ph}} = E_{\text{pho}} \left(\frac{E}{Mc^2} \right) (1 - \beta \cos \alpha), \quad (1.6.1)$$

where E_{pho} and E are the energies of photon and particle in the laboratory coordinate system, M is the rest mass of particle, $\beta = v/c$, and α is the angle between the directions of the photon and particle motion in laboratory coordinate system. These interactions are most significant for the high energy particles and give rise to the following two important effects. First, the interactions with the universal microwave radiation may be the main reason for constraining the primary CR spectrum at the high energy side; second, the interactions of high energy heavy nuclei with photon emission from the Sun will result in photodisintegration of nuclei (into a photo-nucleon and residual nucleus), in generation of correlated cascades when observed on the Earth (Gerasimova and Zatsepin, 1960). The energy loss in interactions with the microwave universal radiation at $T \sim 2.7 \text{ °K}$ becomes significant only at $E \geq 10^{19} \div 10^{20} \text{ eV}$.

Puget et al. (1976) studied the following effects of interactions of CR nuclei of high energy with photons: 1. Compton interaction; 2. Pair production in a field of a nucleus (generally, electron-positron pairs); 3. Photo-splitting of nuclei; 4. Photo-production of hadrons. In the system of the rest of a nuclei of CR the process 1 takes place at every energy E_{ph} ; the process 2 occurs at the minimum energy $E_{\text{ph}} = 2mc^2 \approx 1 \text{ MeV}$; for the process 3 the resonance increase of a cross section takes place at the photon energy E_{ph} from 15 to 25 MeV, and the process 4 occurs with the minimum energy $E_{\text{ph}} \sim 145 \text{ MeV}$. The data were presented for the effective cross sections for the listed processes in a relation to CR nuclei from deuterium to iron. The analysis and estimates of expected density of photons of

various energies E_{pho} in the Galaxy and in intergalactic space were made. Based on these data the calculations of the efficiency of these processes depending on the energy of various CR nuclei and their importance in forming the energy spectrum and chemical composition of primary CR in the range of super-high energies were carried out. Puget et al.(1976) draw the conclusion that as no cut-off of the spectrum up to the energies $(1 \div 2) \times 10^{20}$ eV was revealed, this undoubtedly gave the evidence against the model of uniform filling of Metagalaxy by CR of high energy (independently of a nature of high-energy CR, whether they are nuclei or protons). The obtained results give evidences in favor of local origin of super high-energy CR (in the Galaxy or in a local group of galaxies). New result was obtained concerning protons: it was shown that in the case of meta-galactic origin of CR, a cutoff must take place not only for nuclei but as well for protons (at the energy $\sim 5 \times 10^{19}$ eV owed to photo-producing mesons). The discussed effects are of substantial importance in a propagation of super-high energy CR, in forming their spectrum and chemical composition.

1.6.2. CR electron interactions with the photon field

In contrast to the loss of nucleus energy the electron energy loss (in eV/sec) in interactions with photon emission may be very significant (Ginzburg and Syrovatsky, M1963):

$$\left(\frac{dE}{dt} \right)_{\text{ph}} = -10^{-14} W_{\text{ph}} \begin{cases} 2(E/mc^2), & \text{if } E \ll mc^2 (mc^2/E_{\text{pho}}) \\ (mc^2/E_{\text{pho}})^2 \ln(2EE_{\text{pho}}/m^2c^4), & \text{if } E \gg mc^2 (mc^2/E_{\text{pho}}) \end{cases} \quad (1.6.2)$$

where $W_{\text{ph}} = N_{\text{ph}} E_{\text{pho}}$ is the mean density of the photon emission energy. The effects of the interaction of electrons with the photon field must be taken into account both in the processes of propagation of electrons of CR and in the problem of their origin. In particular, including of these effects and bremsstrahlung losses (see above, Section 1.8.2) results in the conclusion that high energy electrons, observed near the Earth, certainly cannot be of extragalactic origin and cannot come from such remote distances as the region of the center of the Galaxy.

1.7. Energy variations of CR particles in their interactions with magnetic fields

1.7.1. Synchrotron losses of energy by CR particles in magnetic fields

In the relativistic case the synchrotron loss (in eV/sec) by a particle with rest mass M and charge Z moving in a magnetic field of strength $H_{\perp}$ in a direction perpendicular to the particle velocity will be determined by the expression

$$\begin{aligned}
\left(\frac{dE}{dt}\right)_{\text{syn}} &= -\frac{2c}{3}\left(\frac{e^2}{mc^2}\right)\left(\frac{Z^2m}{M}\right)^2 H_{\perp}^2 \left(\frac{E}{Mc^2}\right)^2 = \\
&= -H_{\perp}^2 \times \begin{cases} 2.9 \times 10^{-10} \left(E/M_p c^2\right)^2 & \text{for protons,} \\ 1.8 \times 10^{-10} \left(E/M_p c^2\right)^2 & \text{for nuclei } (A/Z \approx 2), \\ 0.98 \times 10^{-3} \left(E/mc^2\right)^2 & \text{for electrons.} \end{cases} \quad (1.7.1)
\end{aligned}$$

It follows from Eq. 1.7.1 that the synchrotron loss for protons and nuclei is many orders (13÷14) lower than that for electrons at the same energies E . The electron synchrotron losses proves in many cases to be significant. A relativistic electron with energy E moving in a magnetic field with component $H_{\perp}$ (which is perpendicular to the electron's velocity) emits electromagnetic waves predominantly in the direction of its instantaneous velocity within a narrow cone of angle $\theta \approx mc^2/E \ll 1$. In each direction in the orbital plane within a time $2\pi E/ceH_{\perp}$ the radiation bursts will arise, each of duration

$$\tau \approx \frac{r_L \theta}{c} \left(\frac{mc^2}{E}\right)^2 \approx \frac{mc}{eH_{\perp} r_L} \left(\frac{mc^2}{E}\right)^2, \quad (1.7.2)$$

where the factor $(mc^2/E)^2$ arises owing to the Doppler effect. Therefore the frequencies

$$\omega_m = \frac{eH_{\perp}}{mc} \left(\frac{E}{mc^2}\right)^2 \quad (1.7.3)$$

will be presented most intensively. According to Ginzburg and Syrovatsky (1965) the energy emitted by the electron within 1 sec is

$$P(\nu, E) d\nu = 16 \frac{e^3 H_{\perp}}{mc^2} P\left(\frac{\omega}{\omega_m}\right) d\nu, \quad (1.7.4)$$

where the emission spectrum $P(\omega/\omega_m)$ is shown in Fig. 6 in (Ginzburg and Syrovatsky, 1965). If the CR electron concentration is

$$n_e(E)dE = K_e E^{-\gamma} dE, \quad (1.7.5)$$

the intensity I_ν extended along the sight line L will be

$$I_\nu = f(\gamma) K_e L H_\perp^{(\gamma+1)/2} \nu^{-(\gamma-1)/2}, \quad (1.7.6)$$

where $f(\gamma)$ is some function of γ (at $\gamma=3$, $f(\gamma) \approx 170$); I_ν is measured in $\text{erg.cm}^{-2}.\text{ster}^{-1}.\text{sec}^{-1}.\text{Hz}^{-1}$. In determining experimentally the radio emission spectrum we find the dependence on ν , hence the power exponent $(\gamma-1)/2$ and then γ in the electron spectrum (for example for the total radio emission from the Galaxy $(\gamma-1)/2 = 0.7$ whence $\gamma = 2.4$). As shown in (Dorman and Miroshnichenko, M1968) the above-mentioned processes of radio emission are also of great importance to the fast particle generation on the Sun. As a result of bremsstrahlung generation of relativistic electrons of CR, non-thermal radiation from various objects in Metagalaxy (radio galaxies, Seifert galaxies, quasars etc.) and the disc and halo of the Galaxy, of remnants of Supernovae and of the other objects in our Galaxy is formed.

1.7.2. Acceleration and deceleration of particles in their interactions with moving magnetic fields

The charged particle motion in plasma with magnetic fields is accompanied, apart from the various energy losses, by particle acceleration and deceleration by various mechanisms based on one or another mode of the transfer of the magnetic field energy and the energy of plasma kinetic motion to a comparatively small number of particles. In this case the particle acceleration up to comparatively low energies is owed to the first-order mechanisms, namely, the betatron acceleration in enhanced magnetic field, the acceleration in magnetic traps or, in general, in some region between two mutually approaching magnetic plasma formation, the particle acceleration owing to magnetic field dissipation during collapse of oppositely directed fields, etc. These mechanisms are probably realized under the conditions of chromospheric flares and, in general, in the active regions of the solar corona. They may also be the injectors during supernova explosions and explosions in quasars, with subsequent particle acceleration up to higher energies by the second-order mechanisms including the Fermi statistical acceleration mechanism (CR particle collisions with chaotically moving magnetic clouds). In the latter mechanism the particles gain energy in head on collisions and lose energy in overtaking collisions; since, however, the head on collisions are somewhat more probable, the particle energy, on the average, gradually increases. The statistical mechanisms also include the mechanisms of acceleration by various types of waves, namely, plasma waves, radio waves, Alfvén waves, magneto-sonic and shock waves. The energy gain is accompanied by energy loss owed to the various effects considered above. The

energy loss depends on particle energy, and in many cases, starting with some energy called the injection energy the loss becomes smaller than the gain due to some acceleration mechanism. Therefore, only a small portion of the plasma particles satisfying definite conditions, and not all of them, are accelerated. As a result the composition of the accelerated particles may be significantly different from the composition in the source. There may probably also exist injection-less mechanisms when practically all plasma particles are accelerated in a small region of the space within a comparatively short period (it is natural that in this case the chemical composition of the accelerated particles will resemble the composition in the source). An example of an injection-less mechanism may be the acceleration during dissipation of oppositely directed magnetic fields collapsing at a sufficiently high velocity. The energy spectrum constraint at the low energy side is most probably owed to the increase of the loss with decreasing of the particle energy. At the high energy side the spectrum is constrained by the finite time of acceleration owed to either a limited period of the acceleration mechanism action or to the ejection of fast particles from the acceleration zones. Since the probability of ejection increases with particle energy, the generated particle flux decreases pronouncedly with increase of the particle energy. We shall limit ourselves to the above brief pattern, the more that the problem of particle acceleration mechanisms and formation of nuclear composition and energy spectrum of CR in their sources is one of the fundamental problems in the astrophysical aspect of CR studies and will be quantitatively studied and analyzed in details in Chapter 4.

1.8. CR particle motion in magnetic fields; scattering by magnetic inhomogeneities

1.8.1. CR particle motion in the regular magnetic fields frozen into moving plasma formations

The magnetized plasma moving at high velocities is an important constituent of the space. The electric fields are known to be practically absent under the conditions of a high conductivity (the time of their neutralization $\propto \sigma^{-1}$, where $\sigma \geq 10^{12} \div 10^{14}$ CGSE units), so that only magnetic fields $\mathbf{H}$ may exist in a coordinate system related to the moving plasma (and, correspondingly, to the moving frozen magnetic field). The magnetic and induced electric fields in a coordinate system with respect to which a plasma formation moves at velocity $\mathbf{u}$ (here the case where $|\mathbf{u}| \ll c$ is of practical interest) are determined on the basis of Lorentz transformations by the relations

$$\mathbf{H}' = \mathbf{H}, \quad \mathbf{E}' = -\frac{1}{c}[\mathbf{u}\mathbf{H}]. \quad (1.8.1)$$

When affected by these fields the motion of a particle with momentum $\mathbf{p}$, velocity $\mathbf{v}$ and charge Ze will be defined by the equation

$$\frac{d\mathbf{p}}{dt} = \frac{Ze}{c}[\mathbf{v}\mathbf{H}] - \frac{Ze}{c}[\mathbf{u}\mathbf{H}]. \quad (1.8.2)$$

Since $v \gg u$ the motion in homogeneous field may be treated as that along a spiral line with the curvature (Larmor) radius

$$r_L = cp_{\perp}/ZeH. \quad (1.8.3)$$

In this case the energy of a particle traversing a region of size l in the direction perpendicular to $\mathbf{u}$ and $\mathbf{H}$ changes by

$$\Delta E = (Zel/c)uH. \quad (1.8.4)$$

If the size of the region l is such that $l \geq r_L$, such a region will significantly change the particle motion direction. If $r_L \ll l$ the particle will move practically along magnetic force lines and, if the force lines are closed such plasma formation may be a magnetic trap for particles with given r_L (see below, Section 1.10). Many publications have been devoted to the problems of charged particle motion in regular magnetic fields; we shall dwell on these problems below in connection with specific problems of CR propagation in space plasma (interplanetary space, the Earth's and other planets magnetospheres, interstellar and intergalactic space). Here we shall only refer the reader to the monographs (Boguslavsky, M1929; Alfvén, M1950; Störmer, M1955; Spitzer, M1956; Pikelner, M1961) and to the general works (Hayakawa and Obayashi, 1963a,b), in which these problems are formulated and solved to various approximations (analytical finding of trajectories, the drift approximation, numerical integration of the motion equations).

1.8.2. CR particle moving in essentially inhomogeneous magnetized plasma

Strictly speaking, the propagation of the CR particles in essentially inhomogeneous magnetized plasma should be studied by solving the Boltzmann kinetic equation for the function of CR distribution $f(t, \mathbf{r}, \mathbf{p})$:

$$\frac{\partial f}{\partial t} + \frac{\partial f}{\partial \mathbf{r}} \mathbf{v} + \frac{Ze}{c} \frac{\partial f}{\partial \mathbf{p}} [\mathbf{v} - \mathbf{u}, \mathbf{H}] = A_{\text{col}}, \quad (1.8.5)$$

where A_{col} is the term reflecting the role of elastic and inelastic collisions of CR particles with plasma particles (this term also reflects the fragmentation and energy

losses according to Sections 1.1–1.4). The structure of the real fields is, however, fairly complex, and therefore the Eq. 1.8.5 can be successfully solved as yet only for the simplest cases and by reducing to the diffusion approximation. However, to understand the basic features of CR interactions with magnetic fields the elementary approach is sufficient in many cases. In this approach at first the features of an isolated particle interaction with various types of magnetic field inhomogeneities is considered and then one or another statistical set of inhomogeneities is treated and the transport scattering path of particles and the diffusion coefficient are estimated.

1.8.3. Two-dimensional model of CR particle scattering by magnetic inhomogeneities of type $\mathbf{H} = (0, 0, H)$

Dorman and Nosov (1965) studied the scattering properties of the magnetic fields of the simplest configurations in a plane perpendicular to magnetic field $\mathbf{H} = (0, 0, H)$. For the two-dimensional case the differential effective cross section is

$$d\sigma = (dr/d\theta)d\theta, \quad (1.8.6)$$

where r is the impact parameter, θ is the scattering angle (in the two-dimensional case σ is of dimensionality of length).

1.8.4. Scattering by cylindrical fibers with a homogeneous field

If r_o is the radius of a cylinder inside which $H = \text{const}$, then

$$\theta = 2\text{arctg}\left(\sqrt{r_o^2 - r^2}/(r_L - r)\right), \quad (1.8.7)$$

where $r_L = cp/ZeH$ is the curvature radius of particle inside the cylinder (see Figs 1.8.1 and 1.8.2).

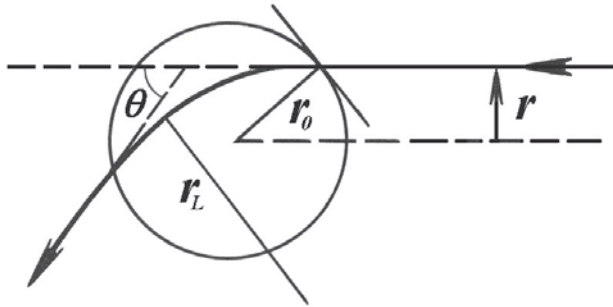


Fig 1.8.1. Geometry of scattering, and the notations

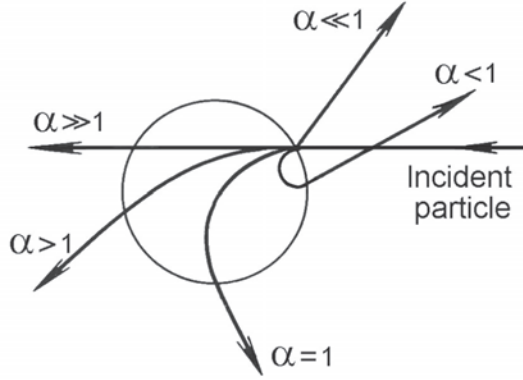


Fig. 1.8.2. The cases of scattering for various values of $\alpha = r_L/r_o$

After calculating $dr/d\theta$ by Eq. 1.8.7 and substituting in Eq. 1.8.6, we find that

$$d\sigma = \frac{r_o}{2} \left(\frac{r_L}{r_o} + a - \frac{1 - (r_L/r_o)^2}{2a} \left(1 + \operatorname{tg}^2 \frac{\theta}{2} \right) \right) \sin \theta d\theta, \quad (1.8.8)$$

where we have denoted:

$$a = \left[1 + \left(1 - (r_L/r_o)^2 \right) \operatorname{tg}^2 \frac{\theta}{2} \right]^{1/2}. \quad (1.8.9)$$

It follows from Eq. 1.8.8, in particular, that at $r_L/r_o \leq 1$ any scattering angle θ is possible (depending on the impact parameter r), and at $r_L/r_o > 1$ the angle θ is limited by the value

$$\theta_{\max} = 2 \operatorname{arctg} \left((r_L/r_o)^2 - 1 \right)^{1/2}. \quad (1.8.10)$$

It can be easily seen that at $r_L/r_o \ll 1$ the Eq. 1.8.8 gives the classical cross section of scattering by a solid sphere of radius r_o :

$$d\sigma = \frac{r_o}{2} \sin \frac{\theta}{2} d\theta. \quad (1.8.10a)$$

1.8.5. Scattering by cylindrical fibers with field of type $h = M/r^n$

The law of conservation of the angular momentum gives for $n \neq 2$ the following equation for the particle trajectory in the field $h = M/r^n$:

$$r^2 \frac{d\theta}{ds} + \frac{ZeM}{(2-n)r^{n-2}pc} - \frac{ZeC_1}{pc} = \frac{C_2}{p}, \quad (1.8.11)$$

where r, θ are the cylindrical coordinates of particle motion; ds is the element of the trajectory length; C_1 and C_2 are constants. Inserting the new unit length

$$l = (ZeM/pc)^{1/n-1} \quad (1.8.12)$$

we obtain

$$r_1^2 \frac{d\theta}{ds_1} + \frac{1}{(2-n)r_1^{n-2}pc} = \text{const}, \quad (1.8.13)$$

where $r_1 = r/l$, $s_1 = s/l$. At $n = 3$ the Eq. 1.8.12 determines the Störmer unit (see in Dorman et al., M1971). Fig. 1.8.3 shows the dependence of the scattering angle θ on the impact parameter r_1 (in units l) for dipolar field ($n = 3$).

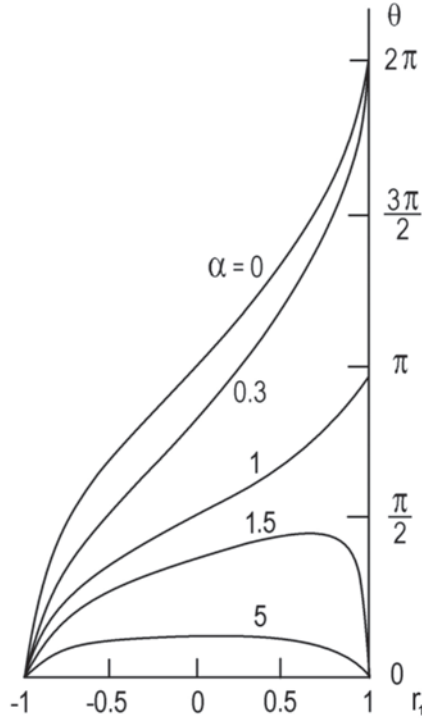


Fig.1.8.3. Dependence of the angle of scattering θ on the collision parameter r_1 (in terms of l) for a dipole field at various values of parameter α .

The dependence of θ on r_1 for a value of θ smaller than some small θ_o may be approximated by the expression $\theta = \theta_o (r_{1o}^2 / r_1^2)$, where r_{1o} is the value of the impact parameter at small deflection θ_o . From this we get for $r_1 \gg 1$:

$$d\sigma = \frac{1}{2} r_{1o} l \theta_o^{1/2} \theta^{-3/2} d\theta. \quad (1.8.14)$$

For other r_1 the value of $d\sigma$ is determined from Eq. 1.8.6 using the plot shown in Fig. 1.8.3.

1.8.6. Three-dimensional model of scattering by inhomogeneities of the type $\mathbf{h} = (0, h(x), 0)$ against the background of general field $\mathbf{H}_o = (H_o, 0, 0)$

Parker (1964) considered the scattering of a particle moving along the basic field $\mathbf{H}_o = (H_o, 0, 0)$ in the x direction from $-\infty$ to $+\infty$ by an inhomogeneity $\mathbf{h} = (0, h(x), 0)$, where $h(x)$ was set in the form $h(x) = dF/dx$ (i.e. $F(x)$ is the field flux from $-\infty$ to x). The equations of particle motion in such field will be written in the form

$$\frac{d^2x}{dt^2} = -\frac{dz}{dt} \omega_L \frac{dF}{dx}, \quad \frac{d^2y}{dt^2} = \frac{dz}{dt} \omega_L, \quad \frac{d^2z}{dt^2} = \frac{dx}{dt} \omega_L \frac{dF}{dx} - \frac{dy}{dt} \omega_L, \quad (1.8.15)$$

where $\omega_L = ZeH_o/Mc$ is the cyclotron frequency. Integration of Eq. 1.8.15 gives

$$\frac{dy}{dt} = \omega_L z, \quad \frac{dz}{dt} = \omega_L (F - y), \quad \left(\frac{dx}{dt} \right)^2 + \left(\frac{dy}{dt} \right)^2 + \left(\frac{dz}{dt} \right)^2 = v^2, \quad (1.8.16)$$

where v is the particle velocity. It follows from Eq. 1.8.15 and Eq. 1.8.16 that

$$\frac{d^2y}{dt^2} = \frac{dz}{dt} \omega_L = \omega_L^2 (F - y), \quad (1.8.17)$$

the solution of which is

$$y(t) = \omega_L \int_{-\infty}^t F(x) \sin(\omega_L(t - \tau)) d\tau. \quad (1.8.18)$$

Assuming that the scattering is inconsiderable, we may set $x \approx v_{||}t$. Then the scattering angle $\theta \approx v_{\perp}/v_{||}$ at $t \rightarrow \infty$, where $v_{\perp} = \left((dy/dt)^2 + (dz/dt)^2 \right)^{1/2}$ will be for the three types of inhomogeneities ($j = 1, 2, 3$ respectively; see Fig. 1.8.4)¹:

$$\theta \approx \exp\left(-\left(\frac{\lambda H_o}{2R}\right)^2\right) \times \begin{cases} \frac{\sqrt{\pi}\lambda h_o}{R} & \text{for } h(x) = h_o \exp(-x^2/\lambda^2), j=1, \\ \frac{\sqrt{\pi}\lambda^2 H_o h_o}{2R^2} & \text{for } h(x) = h_o \frac{x}{\lambda} \exp(-x^2/\lambda^2), j=2, \\ \frac{\sqrt{\pi}\lambda^3 H_o^2 h_o}{2R^3} & \text{for } h(x) = h_o \left(1 - \frac{2x^2}{\lambda^2}\right) \exp\left(-\frac{x^2}{\lambda^2}\right); j=3, \end{cases} \quad (1.8.19)$$

where $R = pc/Ze$ is the particle rigidity, λ and h_o are the effective size and the characteristic field strength of inhomogeneities, the values R/H_o and R/h_o in Eq. 1.8.19 are the Larmor radii of particles in the basic field H_o and inhomogeneities field h_o . Strictly speaking, the above presented results are valid when $v_{||} \gg v_{\perp}$, i.e. when $\theta \ll 1$. This is realized when $R/H_o \gg \lambda$. In this case the exponential multiplier approaches 1 and $\theta \propto R^{-1}$, R^{-2} , or R^{-3} , depending on the type of magnetic inhomogeneities. It should be noted that the obtained results are to some extent analogous to (Dorman and Nosov, 1965).

The first type of inhomogeneities ($j = 1$) corresponds to the pronouncedly limited field in the region $x \leq \lambda$ where approximately $h \approx h_o$, and beyond this region $h \approx 0$ (see Fig.1.8.4). In this case, as in (Dorman and Nosov, 1965), the angle $\theta \propto R^{-1}$ at great R .

The second type of inhomogeneities ($j = 2$) corresponds to the case where the field h is absent in the inhomogeneity center, increases gradually when moving away from the center and runs through its maximum but with opposite directions, so that the effective size of the inhomogeneity is as if it is dependent on particle rigidity according to the law $\lambda_{\text{eff}} = \lambda(\lambda/(R/H_o))$ (see Fig.1.8.4), decreases inversely to the rigidity.

For the third type of inhomogeneities ($j = 3$), the field is of even more complex nature and reaches 0 at $x = \pm \lambda/\sqrt{2}$; here $h < 0$ in the regions $x > \lambda/\sqrt{2}$ and $x < -\lambda/\sqrt{2}$, and $h > 0$ in the region $-\lambda/\sqrt{2} < x < \lambda/\sqrt{2}$. In this case $\lambda_{\text{eff}} = \lambda(\lambda/(R/H_o))^2$.

¹ These results are presented here in the form (Dorman, 1969b) which is somewhat different from (Parker, 1964) and seems to us to be more convenient for interpretation.

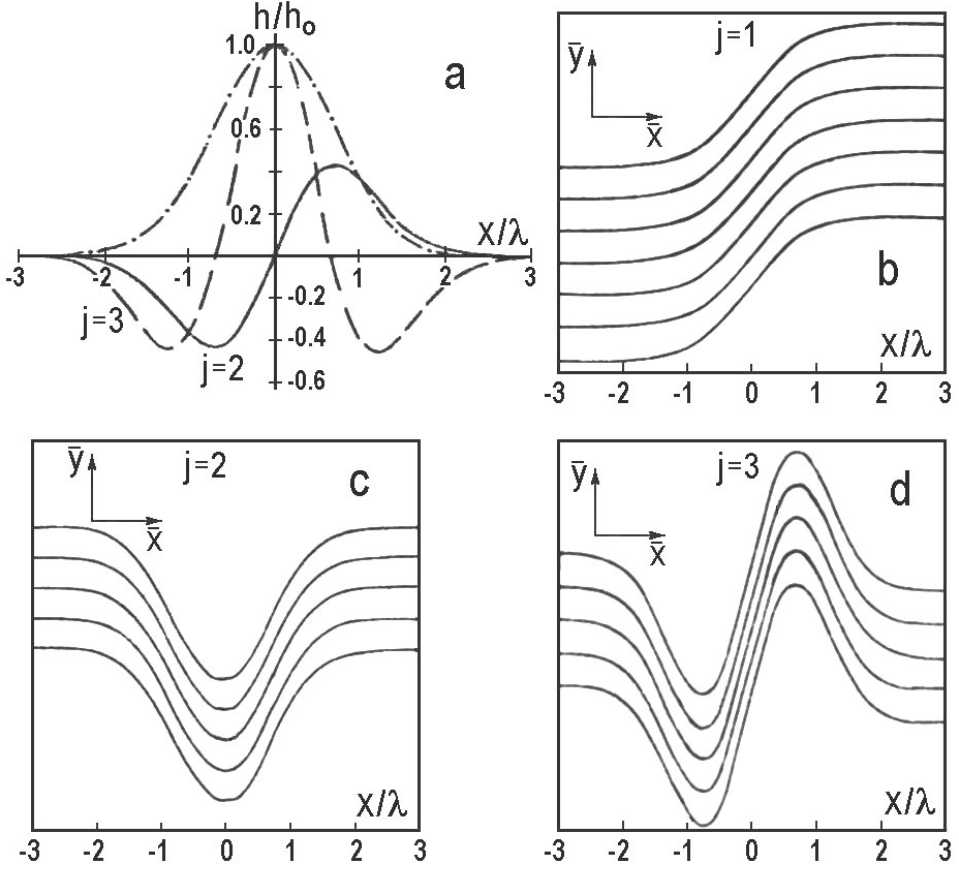


Fig.1.8.4. The character of a dependence of h/h_0 on x/λ (a) and the shape of magnetic lines of force in inhomogeneities of the types: $j = 1$ (b), $j = 2$ (c), $j = 3$ (d) according to Eq. 1.8.19.

Thus the main result of Eq. 1.8.19 for great R may be written in the form $\theta = \lambda_{\text{eff}}/(R/H_0)$, i.e. it is determined by the ratio of the effective size of magnetic inhomogeneity to the Larmor radius in the inhomogeneity field. At small R , when the Larmor radius is comparable with the inhomogeneity's size, the approximation used above is invalid since the condition $v_{\parallel} \gg v_{\perp}$ is violated (in this case $v_{\parallel} \sim v_{\perp}$). However, the qualitative conclusion following from Eq. 1.8.19 that the inhomogeneities are again transparent for the particles with very small R seems to be correct since the assumption of adiabatic invariant conservation is valid for such particles, and the particles, when winding up the force lines, will freely penetrate through the inhomogeneities. Thus it should be expected that the most considerable scattering will be observed for the particles whose Larmor radius is of the effective size of magnetic inhomogeneity (in accordance with Dorman, 1959; Dorman and

Nosov, 1965). Thus it can be probably considered that the Eq. 1.8.19 is approximately valid throughout the range of variations of R . The maximum values of θ for the various types of inhomogeneities can be reached at the following values of $R_{\max,j}$:

$$\theta_{\max,j} = \frac{h}{H_o} (2\pi j)^{1/2} \exp(-j/2) \quad \text{at} \quad R_{\max,j} = \lambda H_o (2j)^{1/2}, \quad (1.8.20)$$

where $j = 1, 2, 3$ is the type of inhomogeneity. Table 1.8.1 presents the variations of θ with R .

Table 1.8.1. Values of $\theta/\theta_{\max,j}$ for various $R/R_{\max,j}$

j	$R/R_{\max,j}$							
	0.2	0.4	0.6	1.0	1.5	3	10	30
1	3×10^{-5}	0.16	0.68	1.0	0.88	0.52	0.16	5.5×10^{-2}
2	9×10^{-10}	3.2×10^{-2}	0.47	1.0	0.78	0.27	2.7×10^{-2}	3.0×10^{-3}
3	9×10^{-14}	5.9×10^{-3}	0.32	1.0	0.69	0.14	4.4×10^{-3}	1.6×10^{-4}

It can be seen from Table 1.8.1. that the particles with $0.5R_{\max} \leq R \leq 3R_{\max}$ are scattered through the largest angles and that the scattering by inhomogeneities of the first type is in the largest rigidity interval, whilst that by the third type of inhomogeneities is in the smallest rigidity interval (Dorman, 1969). The inhomogeneities may also be in the form of condensations and rarefactions of magnetic force lines, the magneto-hydrodynamic and shock waves, the strengthens magnetic formation etc. We shall not dwell here on these cases and only note that the abovementioned formations may also be characterized by the size λ (for the condensations and rarefactions of the field and for the strengthens closed formations this is merely their effective size; for the magneto-hydrodynamic or magneto-sonic wave this is the wavelength; for the shock wave this is the width of the front) and field h . The scattering by such formations will be determined, as a first approximation, by the Eq. 1.8.8, 1.8.14, 1.8.19.

1.9. The transport path of CR particles in space magnetic fields

1.9.1. The transport path of scattering by magnetic inhomogeneities of the type of isolated magnetic clouds of the same scale

Let N be the concentration of magnetic inhomogeneities of size λ ; the mean path of particle interaction with inhomogeneity is then

$$L = N^{-1} \lambda^{-2} = l^3 \lambda^{-2}, \quad (1.9.1)$$

where $l = N^{-1/3}$ is the mean distance between inhomogeneities. If the scattering through an angle $\theta \leq \theta_o$ and $\theta_o \ll 1$ is on the average of equal probability in each interaction, the particle after n interactions will be scattered with equal probability through an angle $\theta \leq \theta_o \sqrt{n}$. Thus after $n = (2/\theta_o)^2$ collisions the scattering will be practically isotropic and the transport path will be

$$\Lambda \approx nl \approx (2/\theta_o)^2 l^3 \lambda^{-2}. \quad (1.9.2)$$

Examine first the inhomogeneities of the same scale of the type of isolated magnetic clouds. It was shown above that in such case $\theta_o \approx 2$ if $R/h = r_L \ll \lambda$ and $\theta_o \approx 2\lambda/r_L$ if $r_L \geq \lambda$. Taking account of Eq. 1.9.2 we shall obtain

$$\Lambda \approx l^3 \lambda^{-2} \times \begin{cases} 1 & \text{if } r_L \leq \lambda, \\ r_L^2 \lambda^{-2} & \text{if } r_L \geq \lambda. \end{cases} \quad (1.9.3)$$

The Eq. 1.9.3 may be rewritten, accurate to within a factor of ~ 2 , in the form

$$\Lambda \approx l^3 \lambda^{-4} (r_L^2 + \lambda^2). \quad (1.9.4)$$

1.9.2. Transport scattering path in case of several scales of magnetic inhomogeneities

Let it now be assumed that the space contains some spectrum of magnetic inhomogeneities filling the entire space, $l \sim \lambda$. If the field strength in inhomogeneities of all scales is approximately the same, the following simple observations (Dorman, 1959) will make it possible to estimate the dependence of Λ on R . Consider the motion of particles whose curvature radius in a field H is r_L . The scattering by inhomogeneities for which $r_L \sim \lambda$ will be determined by the path $\Lambda_1 \sim r_L \sim \lambda$. At the same time, the scattering by big inhomogeneities of size $\lambda_b \gg r_L$ will be determined by the path $\Lambda_2 \sim \lambda_b \gg \Lambda_1$, and that by small inhomogeneities of size $\lambda_s \ll r_L$ by the path $\Lambda_3 \sim (r_L/\lambda_s)^2 \lambda_s \gg \Lambda_1$, in both last cases the scattering proves to be much less effective and may be neglected as compared with the scattering by inhomogeneities with $\lambda \sim r_L$ (resonance scattering).

If we have a set of inhomogeneities within $\lambda_1 \div \lambda_2$, then, in compliance with the above simple observations, the diffusion path will be

$$\Lambda_{\text{eff}} \approx \begin{cases} \lambda_1 & \text{if } cp \leq Ze\lambda_1 H, \\ cp/(ZeH) & \text{if } \lambda_1 \leq cp/ZeH \leq \lambda_2, \\ c^2 p^2 / (Z^2 e^2 H^2 \lambda_2) & \text{if } cp \geq ZeH\lambda_2. \end{cases} \quad (1.9.5)$$

It follows from Eq. 1.9.5 that at $\lambda_1 < r_L < \lambda_2$ the particles are effectively scattered by the inhomogeneities whose sizes are of the order of Larmor radius. In the general case, however, at an arbitrary dependence of l and h on λ this may be not the fact. Indeed, let it be assumed that the space contains only three basic scales of inhomogeneities $\lambda_1, \lambda_2, \lambda_3$ and the intensities of their magnetic fields and the distance between them are the following functions of the size of the inhomogeneities:

$$h(\lambda) = h_2 (\lambda/\lambda_2)^\beta, \quad l(\lambda) = l_2 (\lambda/\lambda_2)^\alpha. \quad (1.9.6)$$

It follows from Eq. 1.9.6 that

$$r_L(\lambda) = r_{L2} (\lambda/\lambda_2)^{-\beta}; \quad r_{L2} = r_L(\lambda_2) = cp/Zeh_2; \quad r_{Lk} = r_{L2} (\lambda_k/\lambda_2)^{-\beta}, \quad (1.9.6a)$$

where $k = 1, 2, 3$. Since the times between collisions are the additive values, then

$$\Lambda^{-1} = \Lambda_1^{-1} + \Lambda_2^{-1} + \Lambda_3^{-1}, \quad (1.9.7)$$

where, according to Eq. 1.9.4 $\Lambda_k = l_k^3 \lambda_k^{-4} (r_{Lk}^2 + \lambda_k^2)$ is the transport path in inhomogeneities of the scale λ_k ($k = 1, 2, 3$). From this,

$$\Lambda = \frac{l_2^3}{\lambda_2^{3\alpha}} \left[\frac{\lambda_1^{4-3\alpha}}{r_{L2}^2 (\lambda_2/\lambda_1)^{2\beta} + \lambda_1^2} + \frac{\lambda_2^{4-3\alpha}}{r_{L2}^2 + \lambda_2^2} + \frac{\lambda_3^{4-3\alpha}}{r_{L2}^2 (\lambda_2/\lambda_3)^{2\beta} + \lambda_3^2} \right]^{-1}. \quad (1.9.8)$$

Examine the importance of individual terms (the scales of inhomogeneities 1, 2, 3) in the right part of Eq. 1.9.8 for the particles of rigidity $R = 300h_2\lambda_2$ (i.e. $r_{L2} = \lambda_2$) when the inhomogeneity scales differ by an order, $\lambda_1 : \lambda_2 : \lambda_3 = 0.1 : 1 : 10$. Table 1.9.1 presents the ratios $\Lambda_1 : \Lambda_2 : \Lambda_3$ for the various power exponents α and β in Eq. 1.9.6.

Table 1.9.1. Ratios $\Lambda_1 : \Lambda_2 : \Lambda_3$ for the various values of α and β at $\lambda_1 : \lambda_2 : \lambda_3 = 0.1:1:10$ for particles with $r_{L2} = \lambda_2$.

β	$\alpha = 1$	$\alpha = 4/3$	$\alpha = 5/3$	$\alpha = 2$
0	5:1:5	0.5:1:50	0.05:1:500	0.005:1:5000
0.5	50:1:5	5:1:50	0.5:1:500	0.05:1:5000
1	500:1:5	50:1:50	5:1:500	0.5:1:5000
1.5	5000:1:5	500:1:50	50:1:500	5:1:5000
2	50000:1:5	5000:1:50	500:1:500	50:1:5000

It can be seen from Table 1.9.1 that the conclusion of Dorman (1959) that the particles are effectively scattered mainly by inhomogeneities whose sizes are of the order of a Larmor radius is valid in mainly the cases for all β at $\alpha = 1$, for $\beta \geq 0.5$ at $\alpha = 4/3$, and so on, but at some values of β and α is not valid. In more probable case when $\beta = 0$ (if the field strength is the same in all the inhomogeneities scales), and $\alpha = 1$ (when the distances between the inhomogeneities are proportional to their sizes) the particles are most effectively scattered by the inhomogeneities whose sizes are of the order of Larmor radius (the transport path Λ_2 is five times smaller than Λ_1 and Λ_3). Thus in this case (probably realizable most frequently) the conclusion of Dorman (1959) is valid to a high precision. At the same $\beta = 0$ with increasing α to $4/3$, $5/3$ or 2 more importance became smaller scale of inhomogeneities. In the case of $\beta = 1$ (when the field strength in inhomogeneities is proportional to their size) the conclusion of Dorman (1959) is valid for $1 \leq \alpha \leq 5/3$, but at $\alpha = 2$ (when the distances between the inhomogeneities are proportional to the square of their sizes) the importance of inhomogeneities changes, namely the inhomogeneities with scale smaller than r_{L2} are of major importance.

1.9.3 The transport scattering path in the presence of a continuous spectrum of the cloud type of magnetic inhomogeneities

Realized most probably in the nature is some continuous spectrum of magnetic inhomogeneities with the density $N(\lambda) = l^{-3}(\lambda)$ in the interval of the scales $\lambda_1 \leq \lambda \leq \lambda_2$ and a definite dependences of l and h on λ according to Eq. 1.9.6 (it means that $l(\lambda) = l_2(\lambda/\lambda_2)^\alpha$ and $r_L(\lambda) = r_{L2}(\lambda/\lambda_2)^{-\beta}$, where $r_{L2} = cp/Zeh_2$ (see Eq. 1.9.6a). Using these denominations, we shall find according to Dorman (1969) that

$$\Lambda_{\text{eff}} = \frac{l_2^3(\lambda_2 - \lambda_1)}{\lambda_2^{3\alpha}} \left(\int_{\lambda_1}^{\lambda_2} \frac{\lambda^{4-3\alpha} d\lambda}{r_{L2}^2 (\lambda/\lambda_2)^{-2\beta} + \lambda^2} \right)^{-1} = \frac{l_2^3(\lambda_2 - \lambda_1)}{\lambda_2^{3\alpha} F(\alpha, \beta)}, \quad (1.9.9)$$

where $F(\alpha, \beta)$ are presented below for $\beta = -1, 0, 1/2$, and 1 at any values of α :

$$F(\alpha, \beta = -1) = \frac{1}{1 + (r_{L2}/\lambda_2)^2} \begin{cases} \ln(\lambda_2/\lambda_1) & \text{if } \alpha = 1, \\ \frac{\lambda_2^{3(1-\alpha)} - \lambda_1^{3(1-\alpha)}}{3(1-\alpha)} & \text{if } \alpha \neq 1, \end{cases} \quad (1.9.10)$$

$$F(\alpha, \beta = 0) = \begin{cases} \frac{1}{r_{L2}^2} \left(\frac{\lambda_2^{5-3\alpha} - \lambda_1^{5-3\alpha}}{5-3\alpha} - F\left(\alpha - \frac{2}{3}, \beta = 0\right) \right) & \text{if } \alpha \geq 2, \\ \frac{1}{r_{L2}^2} \ln \left(\frac{\lambda_2^2 (r_{L2}^2 + \lambda_1^2)}{\lambda_1^2 (r_{L2}^2 + \lambda_2^2)} \right) & \text{if } \alpha = 5/3, \\ r_{L2}^{-1} \left(\operatorname{arctg} \frac{\lambda_2}{r_{L2}} - \operatorname{arctg} \frac{\lambda_1}{r_{L2}} \right) & \text{if } \alpha = 4/3, \\ \frac{1}{2} \ln \left(\frac{r_{L2}^2 + \lambda_2^2}{r_{L2}^2 + \lambda_1^2} \right) & \text{if } \alpha = 1, \\ \frac{\lambda_2^{3(1-\alpha)} - \lambda_1^{3(1-\alpha)}}{3(1-\alpha)} - r_{L2}^2 F\left(\alpha + \frac{2}{3}, \beta = 0\right) & \text{if } \alpha \leq 2/3, \end{cases} \quad (1.9.11)$$

$$F\left(\alpha, \beta = \frac{1}{2}\right) = \begin{cases} \frac{r_{L2}^{-2} \lambda_2^{-1}}{3(\alpha-2)} \left(\lambda_1^{-3(\alpha-2)} - \lambda_2^{-3(\alpha-2)} - 3(\alpha-2) F\left(\alpha-1, \beta = \frac{1}{2}\right) \right) & \text{if } \alpha \geq 7/3, \\ \frac{r_{L2}^{-2} \lambda_2^{-1}}{3} \ln \left(\frac{\lambda_2^2 (r_{L2}^2 \lambda_2 / \lambda_1 + \lambda_1^2)}{\lambda_1^2 (r_{L2}^2 + \lambda_2^2)} \right) & \text{if } \alpha = 2, \\ \frac{(r_{L2} \lambda_2)^{-2/3}}{3} \left[\frac{\ln b_1}{2} + \sqrt{3} \left(\operatorname{arctg} \frac{\sqrt{3} \lambda_2}{2 r_{L2}^{2/3} \lambda_2^{1/3} - \lambda_2} - \right. \right. \\ \left. \left. - \operatorname{arctg} \frac{\sqrt{3} \lambda_1}{2 r_{L2}^{2/3} \lambda_2^{1/3} - \lambda_1} \right) \right] & \text{if } \alpha = 5/3, \\ \frac{1}{3} (r_{L2}^2 \lambda_2)^{-1/3} \left[-\frac{\ln b_1}{2} + \sqrt{3} \left(\operatorname{arctg} \frac{2 \lambda_2 - r_{L2}^{2/3} \lambda_2^{1/3}}{\sqrt{3} r_{L2}^{2/3} \lambda_2^{1/3}} - \right. \right. \\ \left. \left. - \operatorname{arctg} \frac{2 \lambda_1 - r_{L2}^{2/3} \lambda_2^{1/3}}{\sqrt{3} r_{L2}^{2/3} \lambda_2^{1/3}} \right) \right] & \text{if } \alpha = 4/3, \end{cases} \quad (1.9.12)$$

$$F\left(\alpha, \beta = \frac{1}{2}\right) = \begin{cases} \frac{1}{3} \ln \left(\frac{\lambda_2 (r_{L2}^2 + \lambda_2^2)}{\lambda_1 (r_{L2}^2 \lambda_2 / \lambda_1 + \lambda_1^2)} \right) & \text{if } \alpha = 1, \\ \frac{\lambda_2^{3(1-\alpha)} - \lambda_1^{3(1-\alpha)}}{3(1-\alpha)} - r_{L2}^2 \lambda_2 F\left(\alpha + 1, \beta = \frac{1}{2}\right) & \text{if } \alpha \leq 2/3 \end{cases} \quad (1.9.12a)$$

$$F(\alpha, \beta = 1) = \begin{cases} \frac{r_{L2}^{-2} \lambda_2^{-2}}{3\alpha - 7} \left(\lambda_1^{-(3\alpha-7)} - \lambda_2^{-(3\alpha-7)} - (3\alpha - 7) F(\alpha - 4/3, \beta = 1) \right) & \text{if } \alpha \geq 8/3, \\ r_{L2}^{-2} \lambda_2^{-2} \ln \left(\frac{\lambda_2^2 (r_{L2}^2 \lambda_2^2 / \lambda_1^2 + \lambda_1^2)}{\lambda_1^2 (r_{L2}^2 + \lambda_2^2)} \right) & \text{if } \alpha = 7/3, \\ (2r_{L2} \lambda_2)^{-3/2} \left[\frac{\ln b_2}{2} + \operatorname{arctg} \frac{(2r_{L2} \lambda_2)^{1/2}}{r_{L2} - \lambda_2} - \operatorname{arctg} \frac{(2r_{L2} \lambda_2)^{1/2}}{\lambda_2 r_{L2} / \lambda_1 - \lambda_1} \right] & \text{if } \alpha = 2, \\ (2r_{L2} \lambda_2)^{-1} \left(\operatorname{arctg} \frac{\lambda_2}{r_{L2}} - \operatorname{arctg} \frac{\lambda_1}{\lambda_2 r_{L2} / \lambda_1} \right) & \text{if } \alpha = 5/3, \\ \frac{(r_{L2}^2 \lambda_2)^{-1/2}}{2} \left[-\ln b_2 - \operatorname{arctg} \frac{(2r_{L2} \lambda_2)^{1/2}}{r_{L2} - \lambda_2} + \operatorname{arctg} \frac{(2r_{L2} \lambda_2)^{1/2}}{\lambda_2 r_{L2} / \lambda_1 - \lambda_1} \right] & \text{if } \alpha = 4/3, \\ \frac{1}{4} \ln \left(\frac{\lambda_2^2 (r_{L2}^2 + \lambda_2^2)}{\lambda_1^2 (r_{L2}^2 \lambda_2^2 / \lambda_1^2 + \lambda_1^2)} \right) & \text{if } \alpha = 1, \\ \frac{\lambda_2^{3(1-\alpha)} - \lambda_1^{3(1-\alpha)}}{3(1-\alpha)} - r_{L2}^2 \lambda_2^2 F\left(\alpha + \frac{4}{3}, \beta = 1\right) & \text{if } \alpha \leq 2/3, \end{cases} \quad (1.9.13)$$

where

$$b_1 = \frac{(\lambda_2 + r_{L2}^{2/3} \lambda_2^{1/3})^2 (\lambda_1^2 - r_{L2}^{2/3} \lambda_2^{1/3} \lambda_1 + r_{L2}^{4/3} \lambda_2^{2/3})}{(\lambda_1 + r_{L2}^{2/3} \lambda_2^{1/3})^2 (\lambda_2^2 - r_{L2}^{2/3} \lambda_2^{4/3} + r_{L2}^{4/3} \lambda_2^{2/3})}, \quad (1.9.14)$$

$$b_2 = \frac{(\lambda_2^2 + (2r_{L2}\lambda_2)^{1/2}\lambda_2 + r_{L2}\lambda_2)(\lambda_1^2 - (2r_{L2}\lambda_2)^{1/2}\lambda_1 + r_{L2}\lambda_2)}{(\lambda_2^2 - (2r_{L2}\lambda_2)^{1/2}\lambda_2 + r_{L2}\lambda_2)(\lambda_1^2 + (2r_{L2}\lambda_2)^{1/2}\lambda_1 + r_{L2}\lambda_2)}, \quad (1.9.15)$$

The results of calculations of Λ_{eff} performed by Dorman and Sergeev (1976) according to Eq. 1.9.9–Eq. 1.9.15, are presented in Fig. 1.9.1 for the values of the parameters $\alpha = 0.25, 0.5, 1$ and 1.5 ; $\beta = -0.5, 0, 0.5, 1$ and 1.5 ; $\lambda_1/\lambda_2 = 10^{-1}, 10^{-3}, 10^{-5}$. Along the ordinate axis Λ_{eff}/G is shown with $G = (l_2/\lambda_2)^3(\lambda_2 - \lambda_1)$.

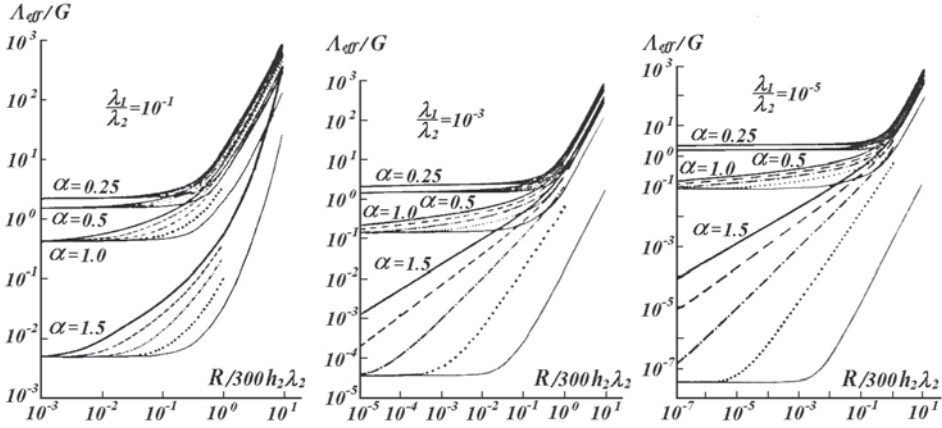


Fig. 1.9.1. Dependence of Λ_{eff}/G (where $G = (l_2/\lambda_2)^3(\lambda_2 - \lambda_1)$) on $R/300h_2\lambda_2$ (where rigidity R in V, h_2 in Gs, λ_2 in cm) for the model of inhomogeneities of a type of "magnetic clouds" for $\alpha = 0.25, 0.5, 1.0$ and 1.5 at $\lambda_1/\lambda_2 = 10^{-1}, 10^{-2}, 10^{-3}$. The thick, dashed, dotted, dash-dotted, and thin lines correspond to $\beta = 1.5, 1.0, 0.5, 0$, and -0.5 respectively.

Analysis of the above presented expressions (see also Fig. 1.9.1) shows that:

(1) If $r_{L2} \gg \lambda_2$ then $\Lambda_{\text{eff}} \propto r_{L2}^2$ irrespective of the character of the inhomogeneity spectrum (at any α and β). Thus if the particle's radius of curvature is in excess of the size of the largest inhomogeneity scale, then $\Lambda_{\text{eff}} \propto R^2$.

(2) If $r_{L1} = r_{L2}(\lambda_2/\lambda_1)^\beta \ll \lambda_1$, i.e. $r_{L2} \ll \lambda_1(\lambda_1/\lambda_2)^\beta$, then in all cases Λ_{eff} approaches a constant value independent of particle rigidity R .

(3) In the region $r_{L2} \leq \lambda_2$ (but $r_{L1} \geq \lambda_1$) the mode of the dependence of Λ_{eff} on R is determined by the parameters α and β . For example, at $\beta = -1$ for any α the dependence of Λ_{eff} on R is very weak (as r_{L2} varies from λ_2 to zero the path

Λ_{eff} decreases by a factor of only 2). At $\beta = 0$ (which is probably the most realistic case), $\Lambda_{\text{eff}} \propto R$ when $\alpha = 4/3$. If, however, $\alpha \geq 5/3$ then $\Lambda_{\text{eff}} \propto R^2$, and at $\alpha \leq 1$ the path Λ_{eff} has not in practice to be dependent on R . At $\beta = 0.5$ a dependence close to $\Lambda_{\text{eff}} \propto R$ will take place at $4/3 \leq \alpha \leq 5/3$ (approximately, at $\alpha = 4/3$ $\Lambda_{\text{eff}} \propto R^{2/3}$, and at $\alpha = 5/3$ $\Lambda_{\text{eff}} \propto R^{4/3}$). At $\beta = 1$ (when the field strength in inhomogeneities is proportional to the inhomogeneity size, which is probably a fairly realistic case) $\Lambda_{\text{eff}} \propto R^2$ for $\alpha \geq 7/3$, $\Lambda_{\text{eff}} \propto R^{3/2}$ for $\alpha = 2$, $\Lambda_{\text{eff}} \propto R$ for $\alpha = 5/3$, $\Lambda_{\text{eff}} \propto R^{1/2}$ for $\alpha = 4/3$, and, finally, is practically independent of R at $\alpha \leq 1$.

It should be again emphasized that the results obtained are valid for the isotropic scattering of particles by inhomogeneities which is determined by the Eq. 1.9.3 (or, approximately, by Eq. 1.9.4).

1.9.4. Transport path in a plane perpendicular to cylindrical fibers with a homogeneous field

Let us estimate the transport path for particle scattering by an ensemble of inhomogeneities of the type of the simplest two-dimensional and three-dimensional models considered above (see Section 1.8.4). In the general case for non-isotropic scattering

$$\Lambda_{\text{eff}} = [N\sigma(1 - \overline{\cos\theta})]^{-1}, \quad (1.9.16)$$

where N is the density of scatters, σ is the effective cross section of scatters, θ is the scattering angle. If the mean distance between the axes of the scattering cylinders is l , then $N \approx l^{-2}$. Let $\cos\theta$ and then $\overline{\cos\theta}$ be found. Considering that $r = r_o \cos\chi$ (where χ is the particle incidence angle varying from 0 to π) and after trigonometric transformations of the Eq. 1.8.7 we obtain

$$\cos\theta = 1 - 2\sin^2\chi \left[(r_L/r_o)^2 - 2(r_L/r_o)\cos\chi + 1 \right]^{-1}. \quad (1.9.17)$$

Integrating Eq. 1.9.17 over $r = r_o \cos\chi$ from $-r_o$ to $+r_o$, we shall find that

$$\overline{\cos\theta} = 1 + \frac{(1 - (r_L/r_o)^2)^2}{8(r_L/r_o)^3} \ln \left(\frac{r_o + r_L}{r_o - r_L} \right)^2 - \frac{r_o^2 + r_L^2}{2r_L^2}. \quad (1.9.18)$$

The Eq. 1.9.18 gives the following asymptotic representations:

$$1 - \overline{\cos \theta} = \begin{cases} (4/3)(r_L/r_o)^{-2} & \text{if } r_L/r_o \gg 1, \\ 4/3 & \text{if } r_L/r_o \ll 1. \end{cases} \quad (1.9.19)$$

Substituting Eq. 1.9.18 in Eq. 1.9.16 we find that

$$\Lambda = \frac{l^2 r_L^2}{r_o (r_o^2 + r_L^2) - \frac{(r_o^2 - r_L^2)^2}{4r_L} \ln \left(\frac{r_o + r_L}{r_o - r_L} \right)^2}. \quad (1.9.20)$$

Fig. 1.9.2 shows the dependence of Λ on r_L/r_o . Plotted as the ordinate in the Fig. 1.9.2 is $\Lambda/\Lambda_{\min}$, where $\Lambda_{\min} = (\Lambda)_{r_L/r_o \rightarrow 0} = 3l^2/8r_o$, and as the abscissa is $r_L/r_o = pc/(ZeHr_o)$. In the extreme cases the Eq. 1.9.19 gives

$$\Lambda = \frac{3l^2}{8r_o} \begin{cases} \left(\frac{pc}{ZeHr_o} \right)^2 & \text{if } pc \gg ZeHr_o, \\ 1 & \text{if } pc \ll ZeHr_o. \end{cases} \quad (1.9.21)$$

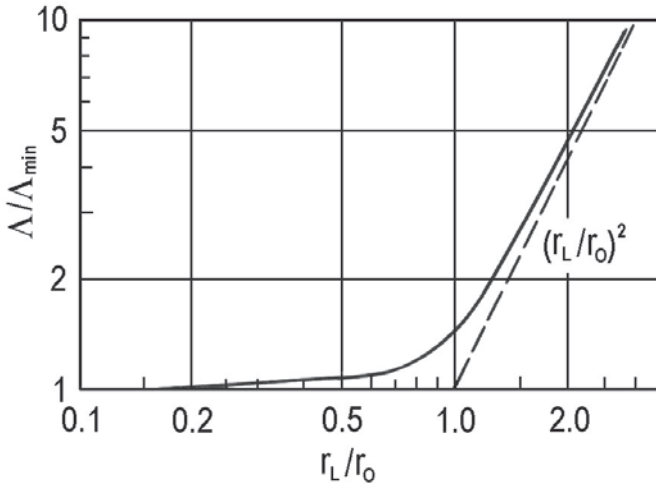


Fig. 1.9.2. The dependence of $\Lambda/\Lambda_{\min}$ on r_L/r_o according to Eq. 1.9.20.

It can be seen from Fig.1.9.2 that at $r_L/r_o \leq 0.3$, $\Lambda/\Lambda_{\min} \approx 1$; in the region $r_L/r_o \approx 1$, $\Lambda/\Lambda_{\min}$ increases with rigidity $\propto R$, whilst in the region $r_L/r_o \geq 2$, $\Lambda/\Lambda_{\min} \propto R^2$.

1.9.5. Transport path of scattering by cylindrical fibers with field $h = M/r^n$ in the two-dimensional case

Consider first the case $n = 3$ (scattering by magnetic dipoles). Then the Eq. 1.8.14 will be used for the impact parameters $r_1 \geq r_o$ (considering that $\theta_o \ll 1$, see Section 1.8.5) and numerical averaging will be carried out for $r_1 \leq r_o$ using the plot of Fig. 1.8.3. The resultant expression is

$$\Lambda \approx 0.36 l^2 (pc/ZeM)^{1/2}. \quad (1.9.22)$$

Similarly, it can be easily shown that at an arbitrary n

$$\Lambda \approx l^2 (pc/ZeM)^{1/(n-1)}. \quad (1.9.23)$$

This result follows physically from the simple observation. If the field $h = M/r^n$ the effective size of the scatter λ are as if dependent on pc . The value of λ may be estimated as the distance at which the Lorentz force equals the centrifugal force:

$$\frac{ZeMv}{c\lambda^n} = \frac{pv}{\lambda}, \quad (1.9.24)$$

whence

$$\lambda = (ZeM/pc)^{1/(n-1)}. \quad (1.9.25)$$

Since in the two-dimensional case $\Lambda \approx l^2/\lambda$, it is the Eq. 1.9.23 that follows from Eq. 1.9.25.

1.9.6. The transport path in the three-dimensional case of scattering by fields of the type $h = M/r^n$

Since in the three-dimensional case λ is also determined by the Eq. 1.9.25 and the mean path of collisions with inhomogeneities of effective size λ is determined by the Eq. 1.9.1, then approximately

$$\Lambda \approx l^3 (pc/ZeM)^{2/(n-1)}. \quad (1.9.26)$$

Thus it should be expected in the three-dimensional case of CR scattering by the dipole type of fields that Λ will be proportional to particle rigidity R . For the field of the type of quadruple ($n = 4$) $\Lambda \propto R^{2/3}$, at $n = 5$ $\Lambda \propto R^{1/2}$, etc., i.e. the dependence of Λ on R weakens as the field becomes more complicated.

1.9.7. Transport path of scattering by inhomogeneities of the type $h = (0, h(x), 0)$ against the background of the regular field $H_o = (H_o, 0, 0)$

Let us now find the transport path for the inhomogeneities the elementary scattering by which is set by the Eq. 1.8.19. According to Eq. 1.9.1 and considering Eq. 1.9.4, we shall obtain for inhomogeneities of type j ($j = 1, 2, 3$):

$$\Lambda_i = l^3 \lambda^{-2} \left(\frac{\theta_{\max}}{\theta} \right)^2 \left(\frac{2}{\theta_{\max}} \right)^2 = \frac{4el^3 H_o^2}{2\pi \hbar^2 \lambda^2} \left[\frac{R(2je)^{1/2}}{\lambda H_o} \exp \left(-\frac{(\lambda H_o)^2}{4jR^2} \right) \right]^{2i}. \quad (1.9.27)$$

Assuming that we have a spectrum of inhomogeneities in the range between λ_1 and λ_2 with the dependence of l and h on λ in the form described by Eq. 1.9.6. In this case we shall obtain similarly to Eq. 1.9.9:

$$\Lambda_{\text{eff},j} = \frac{2l_2^3 R^{2j} (2j)^j \exp(2j)(\lambda_2 - \lambda_1)}{\pi \hbar^2 \lambda_2^{3\alpha-2\beta} H_o^{2(j-1)} \Phi(k, R)}. \quad (1.9.28)$$

where

$$\Phi(k, R) = \int_{\lambda_1}^{\lambda_2} \lambda^k \exp \left(-\frac{\lambda^2 H_o^2}{2R^2} \right) d\lambda, \quad k = -3\alpha + 2(1 + j + \beta). \quad (1.9.29)$$

The functions $\Phi(k, R)$ forming part of Eq. 1.9.28 are presented below for the odd values k for which $\Phi(k, R)$ are expressed through the elementary functions

$$\Phi(k, R) = \frac{R^2}{H_o^2} \times \begin{cases} \left[\exp \left(-\frac{\lambda_1^2 H_o^2}{2R^2} \right) - \exp \left(-\frac{\lambda_2^2 H_o^2}{2R^2} \right) \right] & \text{if } k = 1, \\ \left[\left(\lambda_1^2 + \frac{2R^2}{H_o^2} \right) \exp \left(-\frac{\lambda_1^2 H_o^2}{2R^2} \right) - \left(\lambda_2^2 + \frac{2R^2}{H_o^2} \right) \exp \left(-\frac{\lambda_2^2 H_o^2}{2R^2} \right) \right] & \text{if } k = 3, \\ \left\{ \left(\lambda_1^4 + \frac{4\lambda_1^2 R^2}{H_o^2} + \frac{8R^4}{H_o^4} \right) \exp \left(-\frac{\lambda_1^2 H_o^2}{2R^2} \right) - \right. \\ \left. - \left(\lambda_2^4 + \frac{4\lambda_2^2 R^2}{H_o^2} + \frac{8R^4}{H_o^4} \right) \exp \left(-\frac{\lambda_2^2 H_o^2}{2R^2} \right) \right\} & \text{if } k = 5. \end{cases} \quad (1.9.30)$$

It can be easily seen that at $R \gg \lambda_2 H_o$ the asymptotic expression for function $\Phi(k, R)$ will be

$$\Phi(k, R) \Big|_{R \gg \lambda_2 H_o} = \frac{\lambda_2^{k+1} - \lambda_1^{k+1}}{k+1} - \frac{\lambda_2^{k+3} - \lambda_1^{k+3}}{k+3} \frac{H_o^2}{2R^2} \approx \frac{\lambda_2^{k+1} - \lambda_1^{k+1}}{k+1}, \quad (1.9.31)$$

so the asymptotic expression for $\Lambda_{\text{eff}, j}$ will take the form

$$(\Lambda_{\text{eff}, j})_{R \gg \lambda_2 H_o} \approx \frac{2kl_2^3 R^{2j} (2j)^j \exp(2j)(k+1)(\lambda_2 - \lambda_1)}{\pi j h_2^2 \lambda_2^{3\alpha-2\beta} H_o^{2(j-1)} (\lambda_2^{k+1} - \lambda_1^{k+1})}. \quad (1.9.32)$$

Analysis of the expressions presented above shows the following:

(1) If $R/H_o \gg \lambda_2$ then at $j = 1$, $\Lambda_{\text{eff}, 1} \propto R^2$ for any k (the dependence is the same as in the case of magnetic clouds treated above). At $j = 2$, $\Lambda_{\text{eff}, 2} \propto R^4$; and at $j = 3$, $\Lambda_{\text{eff}, 3} \propto R^6$.

(2) If $R/H_o \ll \lambda_1$ then in all cases $\Lambda_{\text{eff}, j} \rightarrow \infty$, i.e. the examined set of inhomogeneities proves, in contrast to magnetic clouds, to be transparent for low energy CR particles.

(3) A significant difference from the case of scattering by magnetic clouds will be also observed for the interval $\lambda_1 \leq R/H_o \leq \lambda_2$ when an appreciable scattering takes place only for a comparatively narrow interval of R near R_o which, for example for $j = 1$ and $k = 1$, is determined by the relation

$$R_o = \frac{H_o}{2} \left[\frac{\lambda_2^2 - \lambda_1^2}{2 \ln(\lambda_2/\lambda_1)} \right]. \quad (1.9.33)$$

Shown as an example in Fig. 1.9.3 is the dependence of $\Lambda_{\text{eff}, 1}$ on $R/H_o \lambda_2$ at $\alpha = 1$, $\beta = 0$ and $\lambda_1/\lambda_2 = 0.1$.

It can be seen from Fig. 1.9.3 that $\Lambda_{\text{eff}, 1}$ in the interval $\lambda_1 \leq R/H_o \leq \lambda_2$ varies comparatively little (by less than a factor of 2) and reaches its minimum (i.e. the scattering is most effective) at $R/H_o \lambda_2 \approx 0.3$ in accordance with Eq. 1.9.33. Beyond the above mentioned limits $\Lambda_{\text{eff}, 1}$ increases rapidly, namely in proportion to R^2 at large R and even more rapidly (exponentially) at small R .

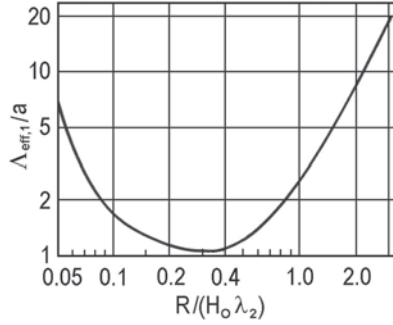


Fig. 1.9.3. Dependence of $\Lambda_{\text{eff},1}$ on $R/H_o\lambda_2$; the quantity $\Lambda_{\text{eff},1}/a$ (where $a = 3.6e^2H_o^2l_2^3/(\pi\lambda_2^2h_o^2)$) is counted along the ordinate axis.

The results of calculations of $\Lambda_{\text{eff},j}$, made by Dorman and Sergeev (1975, 1976), according to Eq. 1.9.30 including Eq. 1.9.31–1.9.33, are presented in Fig. 1.9.4–1.9.8 for $j = 1, 2, 3$, and the values of parameters $k = -3\alpha + 2(1 + j + \beta)$ which are equal to $-1, 0, 1, 2, 3, 4$ and $\lambda_1/\lambda_2 = 10^{-1}, 10^{-3}, 10^{-5}$. Here the values $\Lambda_{\text{eff},j}/G_j$ are put along the ordinate axis, where

$$G_j = \pi^{-1}(2j)^2 l_2^3 \lambda_2^{-2} (1 - \lambda_1/\lambda_2) (H_o/h_2)^2. \quad (1.9.34)$$

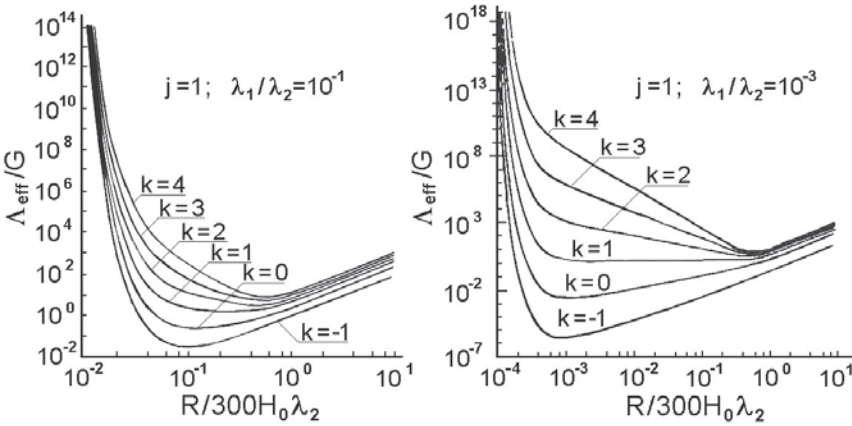


Fig. 1.9.4. Dependence of $\Lambda_{\text{eff},j}/G_j$ (where G_j is determined by Eq. 1.9.34) on $R/H_o\lambda_2$ (or on $R/300H_o\lambda_2$, if R in V, H_o in Gs, and λ_2 in cm) for a model of magnetic inhomogeneities of the type $\mathbf{h} = (0, h(x), 0)$ existing on a background of the general field $\mathbf{H}_0 = (H_o, 0, 0)$ for $k = -1, 0, 1, 2, 3$ and 4. Left panel for $j = 1$ and $\lambda_1/\lambda_2 = 10^{-1}$, right panel for $j = 1$ and $\lambda_1/\lambda_2 = 10^{-3}$.

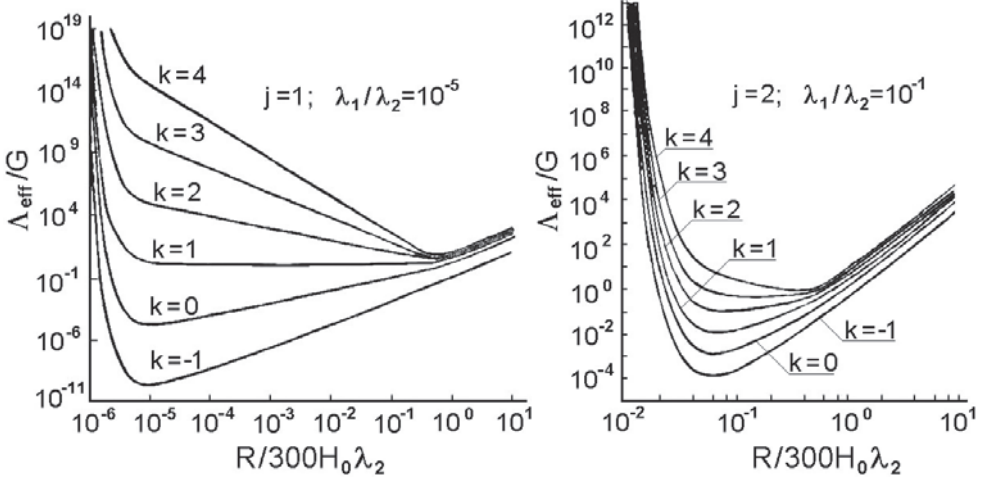


Fig. 1.9.5. The same as in Fig. 1.9.4, but for $j = 1$ and $\lambda_1/\lambda_2 = 10^{-5}$ (left panel), and for $j = 2$ and $\lambda_1/\lambda_2 = 10^{-1}$ (right panel).

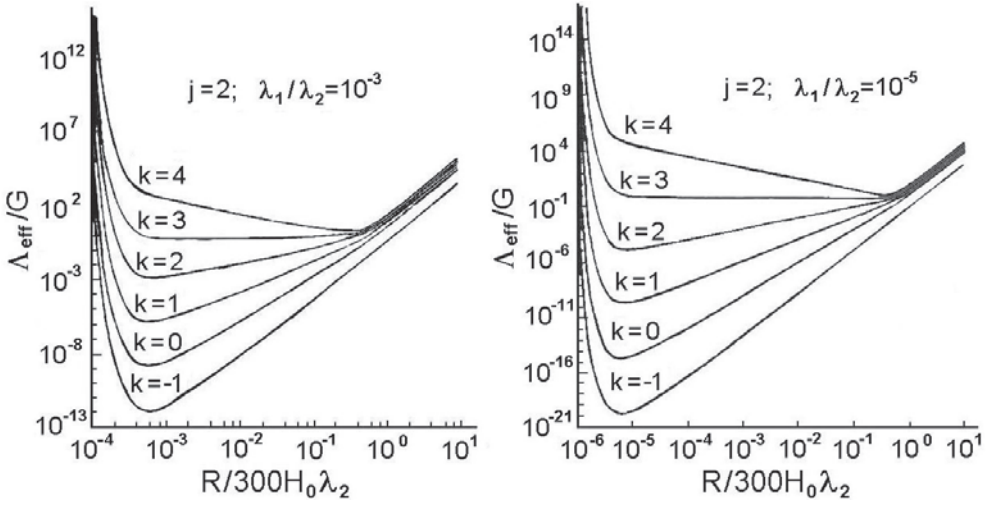


Fig. 1.9.6. The same as in Fig. 1.9.4, but for $j = 2$ and $\lambda_1/\lambda_2 = 10^{-3}$ (left panel), and for $j = 2$ and $\lambda_1/\lambda_2 = 10^{-5}$ (right panel).

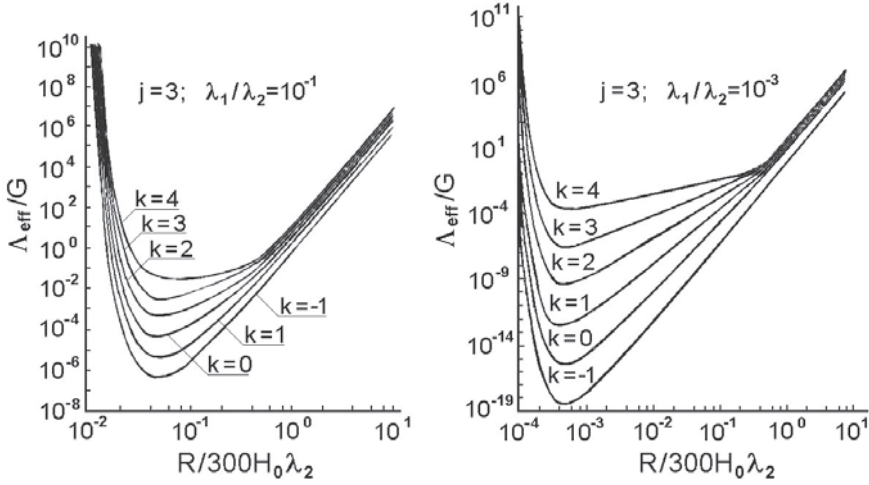


Fig. 1.9.7. The same as in Fig. 1.9.4, but for $j = 3$ and $\lambda_1/\lambda_2 = 10^{-1}$ (left panel), and for $j = 3$ and $\lambda_1/\lambda_2 = 10^{-3}$ (right panel).

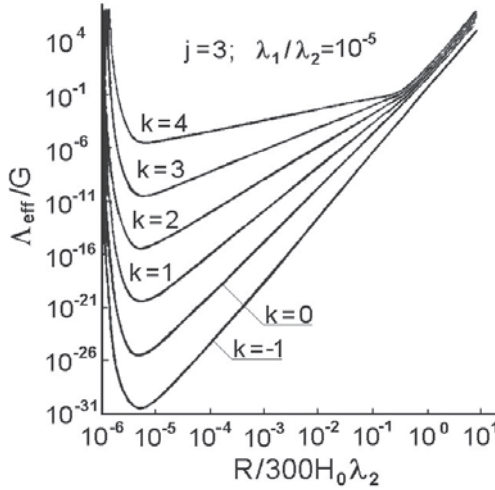


Fig. 1.9.8. The same as in Fig. 1.9.4, but for $j = 3$ and $\lambda_1/\lambda_2 = 10^{-5}$.

1.9.8. The transport scattering path including the drift in inhomogeneous fields

Consideration will be given now to the CR propagation as a result of drift in inhomogeneous magnetic fields. In this case a particle trajectory will be a trochoidal with curvature radius $r_L = cp_{\perp}/(ZeH)$ and the center of curvature will drift at the velocity

$$v_{dr} = (r_L v_{\perp} / 2H) \vec{\nabla}_{\perp} \mathbf{H}. \quad (1.9.35)$$

If the inhomogeneous sectors of the field fill the entire space at a certain distribution $N(\lambda)$, the particle propagation will be diffusive with the transport path Λ_{eff} determined by the Eq. 1.9.4 and Eq. 1.9.9; the diffusion coefficient, however, will contain not the particle velocity v , but the velocity of the drift in inhomogeneous sectors of the field v_{dr} :

$$\kappa = \Lambda_{\text{eff}} v_{dr} / 3. \quad (1.9.36)$$

Such diffusion may be treated as particle propagation at velocity $v_{\perp}$, but with transport path

$$\Lambda_{dr} = \Lambda_{\text{eff}} v_{dr} / v \approx (\Lambda_{\text{eff}} r_L v_{\perp} / 2Hv) \vec{\nabla}_{\perp} \mathbf{H}. \quad (1.9.37)$$

Obviously such modes of propagation will take place only for particles with $r_L \leq \lambda$ for which each interaction with inhomogeneity will be effective. If the field inhomogeneities fail to fill the entire space and are spaced apart on the average by a distance l , the particle will traverse the field inhomogeneities within time $\Delta t_1 = \lambda / v_{dr}$ and pass the space between the inhomogeneities within a time $\Delta t_2 = l^3 / (\lambda^2 v)$; hence the mean effective velocity of particle motion will be

$$v_{\text{eff}} = \frac{\lambda + l^3 / \lambda^2}{\lambda / v_{dr} + l^3 / \lambda^2 v} = \frac{v_{dr} v (\lambda^3 + l^3)}{\lambda^3 v + l^3 v_{dr}}, \quad (1.9.38)$$

and the diffusion coefficient will be

$$\kappa = \Lambda_{\text{eff}} v_{\text{eff}} / 3. \quad (1.9.39)$$

where v_{eff} is determined by the Eq. 1.9.38.

1.9.9. The transport scattering path in the presence of the regular background field

It should be noted that the particles are scattered in reality by magnetic inhomogeneities practically always against a background of some regular magnetic field (which may be the field of larger inhomogeneities). In this case the scattering, and hence the diffusion coefficient, along the field is practically the same while the diffusion across the field will be significantly hampered. Let a homogeneous magnetic field exist in the space. Then in the absence of inhomogeneities the

diffusion coefficient will be zero. An increase in the number of inhomogeneities will result in an increase of the diffusion coefficient, whereas, in the case of absence of the regular field, the same increase would, on the contrary, result in a decrease of the diffusion coefficient (see above the Eq. 1.9.4 and Eq. 1.9.9 which show that Λ_{eff} should decrease pronouncedly with decreasing l). Thus the regular field give qualitatively different results for the diffusion coefficient. In the general case of presence of the regular field H_o the particle motion will be an anisotropic diffusion with the diffusion coefficient along the field $\kappa_{//}$ being as a first approximation, the same as that in the absence of the field, whilst the diffusion coefficient across the field will be determined by the expression

$$\kappa_{\perp} = \kappa_{//} \left(1 + \omega_L^2 \tau^2 \right)^{-1}, \quad (1.9.40)$$

where the Larmor frequency is

$$\omega_L = ZeH_o v / cp, \quad (1.9.41)$$

and the time τ between the ‘effective’ particle collisions with field inhomogeneities (when the motion direction is significantly changed) is

$$\tau \approx \Lambda_{\text{eff}} / v_{\text{eff}}. \quad (1.9.42)$$

If $\omega_L^2 \tau^2 \ll 1$ the conventional isotropic diffusion takes place. If, however, $\omega_L^2 \tau^2 \gg 1$ the anisotropy in the particle’s motion should be taken into account.

Consider now at greater length the mechanism of particle scattering in a plane perpendicular to the regular field which will be assumed, for the sake of simplicity, to be homogeneous with intensity H_o (if the field is inhomogeneous this will additionally give rise to a systematic drift). Let the size of inhomogeneities be $\lambda \leq R_{\perp} / H_o$ and the field intensity in them be h . It can be easily seen that, during the time of the particle’s motion within an inhomogeneity, the center of curvature will shift by ξ determined from the relation

$$\xi / \lambda = \left| (R_{\perp} / H_o - R_{\perp} / h) \right| / (R_{\perp} / h), \quad (1.9.43)$$

i.e. by

$$\xi \approx \lambda \left| H_o - h \right| / H_o. \quad (1.9.44)$$

If the distance between inhomogeneities is l , the mean time between two collisions is $\tau = l^3 \lambda^{-2} v_{\perp}$, and the diffusion coefficient of the curvature center is

$$\kappa_{\perp} = \frac{\langle \xi^2 \rangle}{3\tau} = \frac{\lambda^4 v_{\perp}}{3l^3} \frac{\langle (H_o - h)^2 \rangle}{H_o^2}, \quad (1.9.45)$$

and the transport scattering path

$$\Lambda_{\perp} = \lambda^4 l^{-3} \langle (h/H_o - 1)^2 \rangle, \quad (1.9.46)$$

i.e. it should be practically independent of a particle's rigidity (it will be reminded that this result is valid subject that $R_{\perp} \geq \lambda H_o$). If, however, $R_{\perp} \leq \lambda H_o$, or $R_{\perp} \leq \lambda h$, then $\xi = R_{\perp}/H_o$ at $H_o \leq h$ and $\xi = R_{\perp}/h$ at $H_o \geq h$ which may approximately be written in the form

$$\xi \approx R_{\perp} (|h| + |H_o|) / |h H_o|, \quad (1.9.47)$$

whence

$$\kappa_{\perp} \approx \frac{R_{\perp}^2 \lambda^2 v_{\perp} (|h| + |H_o|)^2}{3l^3 (h H_o)^2}, \quad \Lambda_{\perp} \approx \frac{R_{\perp}^2 \lambda^2 (|h| + |H_o|)^2}{l^3 (h H_o)^2}, \quad (1.9.48)$$

i.e. $\kappa_{\perp} \propto R_{\perp}^2$ and $\Lambda_{\perp} \propto R_{\perp}^2$. The Eq. 1.9.46 and Eq. 1.9.48 for $\Lambda_{\perp}$ may be combined to within a factor of ~ 2 :

$$\Lambda_{\perp} \approx \frac{R_{\perp}^2 \lambda^4 l^{-3} \delta^2 (1 + \delta^2)}{R_{\perp}^2 (1 + \delta^2) + \delta^2 h^2 \lambda^2}, \quad (1.9.49)$$

where

$$\delta = \langle (h - H_o)^2 / H_o^2 \rangle^{1/2}. \quad (1.9.50)$$

Let $\delta \ll 1$ (the field in inhomogeneities are little different from the basic field); then

$$\Lambda_{\perp} \approx \frac{\lambda^4 l^{-3} \delta^2}{1 + \delta^2 H_o^2 \lambda^2 / R_{\perp}^2}, \quad (1.9.51)$$

and at $R_{\perp} \geq \delta H_o \lambda$ it will be $\Lambda_{\perp} \rightarrow \lambda^4 l^{-3} \delta^2$, whereas at $R_{\perp} < \delta H_o \lambda$ it will be $\Lambda_{\perp} \rightarrow R_{\perp}^2 \lambda^2 / (H_o^2 l^3)$.

If $\delta \approx 1$ then

$$\Lambda_{\perp} \approx \frac{R_{\perp}^2 \lambda^4 l^{-3}}{R_{\perp}^2 + h^2 \lambda^2}, \quad (1.9.52)$$

i.e. $\Lambda_{\perp} \approx \lambda^4 l^{-3}$ at $R_{\perp} \gg h \lambda$ and $\Lambda_{\perp} \approx R_{\perp}^2 \lambda^2 / (l^3 h^2)$ at $R_{\perp} \ll h \lambda$. Thus, the measurements of the transport scattering path on the basis of data on CR variations are of extreme interest since this parameter is a sensitive characteristic of the magnetic inhomogeneities in space.

1.9.10. The transport path for scattering with anisotropic distribution of magnetic inhomogeneities in space

Many of the diffusion models of a propagation of CR in interplanetary space are based on the concep of the scattering centers in solar wind. These scattering centers are magnetic inhomogeneities frozen in interplanetary plasma which radially move away from the Sun together with solar wind (Belov and Dorman, 1972). In a region remote from the Sun at the distance r , let the average distance between inhomogeneities along the radius is $l_r(r)$ and transverse to the radius (over θ and φ , where θ is polar, φ is azimuthal angles) is $l_{\theta}(r)$ and $l_{\varphi}(r)$. Suppose that at some distance r_o from the Sun $l_r(r_o) = l_{\theta}(r_o) = l_{\varphi}(r_o)$, i.e. the scattering centers are isotropically distributed. If a diffusion picture does not vary with time, then

$$l_r(r) = l_r(r_o) = l_o, \quad l_{\theta}(r) = l_{\varphi}(r) = l_o r / r_o. \quad (1.9.53)$$

Then at $r \neq r_o$ the isotropy in a distribution of inhomogeneities is conserved only in the plane, normal to the radius. The natural question arises of whether the anisotropic distribution of scattering centers will result in anisotropic diffusion. To verify this assumption let us consider the following spatial structure in the location of inhomogeneities: let the neighboring inhomogeneities be located at the same distances which are equal to a in radial direction, and in the directions, perpendicular (over θ and φ) to a radius, they equal b . Let us consider as well that every scattering occurs isotropically and all of inhomogeneities has the same dimension λ . A probability that a particle, moving initially along the radius, will be scattered at the distance a is λ^2 / b^2 ; the probability $(\lambda^2 / b^2) (1 - \lambda^2 / b^2)$ corresponds to the distance $2a$, etc. A probability of a free path ka in the radial direction will be $(\lambda^2 / b^2) (1 - \lambda^2 / b^2)^{k-1}$. Summing with the respective weight all possible free paths, we obtain the average free path

$$\Lambda_r = \sum_{k=1}^{\infty} k a \left(\lambda^2 / b^2 \right) \left(1 - \lambda^2 / b^2 \right)^{k-1} = a b^2 \lambda^{-2}. \quad (1.9.54)$$

Similarly

$$\Lambda_{\theta} = \Lambda_{\varphi} = \sum_{k=1}^{\infty} k b \left(\lambda^2 / (ab) \right) \left(1 - \lambda^2 / (ab) \right)^{k-1} = a b^2 \lambda^{-2}. \quad (1.9.55)$$

Therefore

$$\Lambda_r = \Lambda_{\theta} = \Lambda_{\varphi} = a b^2 \lambda^{-2} = 1 / (N \sigma), \quad (1.9.56)$$

where $N = 1 / (a b^2)$ is a density of inhomogeneities; $\sigma = \lambda^2$ is the differential cross-section of scattering. Thus in the absence of a regular magnetic field the diffusion remains isotropic, in spite of anisotropy in scattering centers.

Consider now a large-scale regular magnetic field. For particles with Larmor radius $r_L \gg \max\{a, b\}$ the isotropy of diffusion is not violated, but for particles with Larmor radius $r_L \ll \min\{a, b\}$ diffusion will be essentially anisotropic. These particles will be bound to the lines of force of a regular magnetic field, and a free path in the direction of the field will be $\Lambda_{\parallel} = d r^2 / \lambda^2$, where d is the average distance between inhomogeneities along the lines of force. In particular, if the regular field has a spiral structure then

$$d = a_o \left(\frac{1 + (r/r_o)^2 ((\Omega r/u) \sin \theta)^2}{1 + ((\Omega r/u) \sin \theta)^2} \right)^{1/2}, \quad (1.9.57)$$

where u is the solar wind velocity, and Ω is the angular velocity of solar rotation. We see that the distance d is essentially dependent on the distance from the Sun. Near the Sun $d \approx a_o$, and at great distances (near the ecliptic plane) $d \approx a_o r / r_o$. A free path Λ varies together with a distance r . Thus in the presence of a regular magnetic field an additional radial dependence of the diffusion coefficient arises in interplanetary space which is related to a divergent character of a motion of scattering centers (resulting in an anisotropic distribution in space). A similar situation may occur in expanding shells of Supernovae, in spiral branches of galaxies, in Metagalaxy.

1.10. Magnetic traps of CR in space

1.10.1. Types of CR magnetic traps and main properties

CR in space are essentially confined within magnetic traps of one or another scale and fail to propagate freely (excluding for CR γ -quanta and neutrinos which

cannot be trapped by magnetic fields). Enormous CR traps are very diverse in their properties and the charged particle behavior in such traps is essentially energy-dependent. The trap in the Earth's environment formed by the near-dipolar magnetic field exhibits a high degree of stability and considerable lifetime of particles in the trap. At the same time the traps in the vicinities of chromospheric flares or in solar corpuscular streams of magnetized plasma are much more transparent for particles and the mode of particle ejection from such traps resembles the diffusion in irregular magnetic fields. Traps of various kinds are also formed in the vicinities of normal stars, in particular in the solar system and supernova shells. On the other hand, the Galaxy (the galactic disc and halo) also forms a peculiar trap of dimensions of many thousand of parsecs which can safely (with a $\sim 10^7$ years life time) retain the particles of moderate and high energies and is very transparent for the super-high energy particles. It is quite possible that the clusters of galaxies form even more enormous traps of the super-high energy particles.

What formation in the space should be considered as a magnetic trap? Perhaps they are the formation with regular magnetic fields of peculiar configuration where the charged particle lifetime is very great and the accumulation effect is significant, or should the term be much extended? It seems to be expedient from the viewpoint of the study of the general regularities of the temporal variations of CR intensity to consider the CR traps as any magnetic formations in which the motion and time of residence of charged particles is substantially different from those in the free space of the same volume (it should be emphasized that the properties of the traps are essentially dependent on particle energy and that the same magnetic formation may be an excellent trap for particles with energies lower than some critical energy and, at the same time, may be practically transparent for particles of higher energies). The CR intensity inside a trap is determined by the powers of both internal and external source of particles, the absorption, nuclear conversions, loss owed interactions with magnetic fields (the latter is of importance to electrons) inside a trap, and the extent of the exchange with the outer space particles. The temporal variations of the above said factors will in their turn result in the temporal variations of the trapped radiation. Such an approach will permit the diverse types of CR to be uniquely considered and understood. The cosmic traps are characterized, first of all, by the structure of the magnetic fields that determine the charged particle's motion, the exchange with the outer space (ejection from a trap and the possibility of being trapped) and, to a great extent, by the particle absorption inside a trap. An extremely important characteristic of the traps is their dynamism; the traps can be static, moving, expanding, compressed, and, besides that, can exhibit their internal dynamics.

Many works devoted to the theoretical development and experimental study of traps with regular magnetic fields for containing hot plasma appeared in connection with the recent research into controllable thermonuclear reactions (see, for example, in Artsymovich, M1961). These problems have been also sufficiently elaborated in connection with the development of space electrodynamics (Pikelner, M1961;

Spitzer, M1956) and intensive exploration of the magnetic trap and the properties of trapped radiation in the Earth's environments.

1.10.2. Traps of cylindrical geometry with a homogeneous field

The simplest traps of cylindrical type with regular magnetic fields resembling homogeneous fields may be formed, for example, in the galactic arms, in solar corpuscular streams, and in extended magnetic formations in interplanetary space. In this case CR particles move along a spiral with radius of curvature $r_L = cp_{\perp}/(ZeH)$, where $p_{\perp} = p \sin \theta$; θ is the angle between the particle motion direction and a magnetic force line. The particles will move along the field at a velocity $v_{\parallel} = p_{\parallel}c^2/E = pc^2 \cos \theta/E$ where $E = (p^2c^2 + m_o^2c^4)^{1/2}$ is the total energy of particle. Thus if the width of the regular field region is L , the particles with $r_L \ll L$ cannot be ejected through the side walls across the field, excluding for the surface layer of thickness $2r_L$. If $r_L \approx L$ such a region will only scatter the particle and change the direction of its motion by an angle determined by Eq. 1.8.7 (see Section 1.8.4).

1.10.3. Traps with strength-less structure of the field

Willis (1966) has studied the motion of an individual charged particle in a relatively simple strength-less field $\mathbf{H} = (0, H_o \sin \alpha, H_o \cos \alpha)$, where H_o and α are constants. In such field the equations of particle motion can be exactly solved without assuming a leading center. One of the components of the equation of particle motion is formally reduced to the general equation of pendulum motion. It has been shown that under some conditions a particle may move predominantly across the lines of force and thus be ejected from magnetic trap. The criterion of realization of such possibility is the condition $L < r_L/2$, where L is the characteristic scale of the field; r_L is the maximum gyration radius of particle in homogeneous magnetic field H_o . It should be noted that examination of the traps of such kind is of great interest in analyzing the possibility of solar CR trapping by sunspot magnetic fields and in studying the propagation of galactic CR through the interstellar and interplanetary space.

1.10.4. The effect of magnetic field inhomogeneities

If the trap field comprises magnetic field irregularities which can scatter the particles, the diffusion across the field will also take place. Let the field inhomogeneities be characterized by size λ and the mean distance between them l ; let also the field in the inhomogeneities differ from the regular part by, on the average, $\pm \Delta H$ and the field direction in inhomogeneities be the same as in the main part of the trap. Within the time of particle motion through an inhomogeneity, the curvature center will shift by

$$\Lambda_{\perp} = \left(\frac{cp_{\perp}}{ZeH} - \frac{cp_{\perp}}{Ze(H + \Delta H)} \right) \frac{\lambda Ze(H + \Delta H)}{cp_{\perp}} = \lambda \frac{\Delta H}{H}. \quad (1.10.1)$$

Since within 1 sec a particle will encounter $v\lambda^2 l^{-3}$ inhomogeneities, the mean velocity of the displacement of the center of curvature will be

$$u_{\perp} = \Lambda_{\perp} v \lambda^2 l^{-3}, \quad (1.10.2)$$

whence, including Eq. 1.10.1, we shall obtain for the diffusion coefficient of the curvature center across the magnetic field:

$$\kappa_{\perp} = \frac{u_{\perp} \Lambda_{\perp}}{3} = \frac{\lambda^4 v l^{-3}}{3} \left(\frac{\Delta H}{H} \right)^2. \quad (1.10.3)$$

It follows from Eq. 1.10.3 that if the inhomogeneities fill the entire trap (if $\lambda \sim l$), then

$$\kappa_{\perp} = \frac{\lambda v}{3} \left(\frac{\Delta H}{H} \right)^2. \quad (1.10.4)$$

CR particle diffusion across the magnetic field is also possible owing to elastic and inelastic scattering by plasma particles, which is of significant importance for the low energy particle behavior in the high density matter traps.

1.10.5. Traps with an inhomogeneous regular field

Consider first an inhomogeneous field with parallel magnetic force lines. In this case a drift should take place since the radius of curvature decreases with increasing the field intensity; the drift will be perpendicular to the field gradient. The drift velocity may be found using the perturbation method if the field intensity varies little at a distance of the radius of curvature, if $(r_L \nabla)H \ll H$. The radius of curvature in the vector form (directed as $\mathbf{\omega}_L$) is

$$\mathbf{r}_L = \frac{c}{ZeH^2} [\mathbf{pH}], \quad (1.10.5)$$

and the shift of the center of curvature is

$$\mathbf{ds} = -\frac{cdH}{ZeH^3} [\mathbf{pH}], \quad (1.10.6)$$

whence, after averaging over the period of particle gyration, we shall find that

$$\mathbf{v}_{\text{dr}} = \frac{1}{T} \int_0^T \mathbf{ds} = -\frac{c}{ZeH^2} [\mathbf{f}_m \mathbf{H}]. \quad (1.10.7)$$

Here $\mathbf{f}_m$ is the force of interaction of elementary magnet $\mathbf{M}$ (formed by the current $\mathbf{I} = Ze\omega_L/2\pi$ as a result of particle gyration) with the field $\mathbf{H}$:

$$\mathbf{f}_m = M \nabla \mathbf{H}, \quad M = \frac{\pi I r_L^2}{c} = \frac{c^3 p_{\perp}^2}{2HE}, \quad (1.10.8)$$

where E is the total energy of particle; I is the current. Substituting Eq. 1.10.8 in Eq. 1.10.7 we obtain

$$\mathbf{v}_{\text{dr}} = -\frac{c^4 p_{\perp}^2}{2ZeH^3 E} [(\nabla \mathbf{H}) \mathbf{H}]. \quad (1.10.9)$$

In the ultra-relativistic case, we get

$$\mathbf{v}_{\text{dr}} = -\frac{c^3 p_{\perp}}{2ZeH^3} [(\nabla \mathbf{H}) \mathbf{H}], \quad (1.10.10)$$

and in the non-relativistic case we have

$$\mathbf{v}_{\text{dr}} = -\frac{v_{\perp} c^2 p_{\perp}}{2ZeH^3} [(\nabla \mathbf{H}) \mathbf{H}]. \quad (1.10.11)$$

1.10.6. Traps with a curved magnetic field

Examine now the particle motion in curved magnetic field. If a particle moves along a curved line or spirals around a force line, the centrifugal force

$$\mathbf{f}_c = \frac{p_{\parallel}^2 c^2}{EH} \mathbf{H} \nabla \frac{\mathbf{H}}{H} \quad (1.10.12)$$

arises. The force will result in a drift in the direction perpendicular to the plane comprising the given section of the force line:

$$\mathbf{v}_{\text{dr},c} = \frac{p_{\parallel}^2 c^3}{ZeH^3 E} \left[\mathbf{H} \left(\mathbf{H} \nabla \frac{\mathbf{H}}{H} \right) \right]. \quad (1.10.13)$$

1.10.7. Traps with a magnetic field varying along the force lines

The case is of great interest where the field intensity varies along the force lines (in this case the force lines are not parallel) and the Lorentz force component parallel to $v_{||}$ appears. In connection with this, the field-aligned particle energy varies and, since the total particle energy remains constant in stationary magnetic field, the transverse component of momentum should also vary. Since in the case of motion towards a stronger field the Lorentz force component is directed opposite to the motion, the longitudinal momentum decreases and may even vanish. In this case the particle will move with acceleration in opposite direction, reflection from the sector with stronger field will occur. It can be easily shown (see, for example, in Pikelner, M1961) that if the field inhomogeneity is weak, magnetic moment M is conserved. From this, it follows from Eq. 1.10.8 that the value

$$\sin^2 \theta / H = \text{const} \quad (1.10.14)$$

is conserved during the particle motion. A particle with angle θ to the field at point with intensity H will be reflected at a point where $\theta' = \pi/2$, and the field intensity $H' = H / \sin^2 \theta$. It can be seen from the above that the smaller the initial value of θ the more intense a field the particle will penetrate. Such reflection region is essentially a magnetic mirror. The particles, in moving between the magnetic mirrors, will be confined within the trap.

1.10.8. Traps with a magnetic field varying with time

It is of great interest to examine the case of particle motion in a magnetic field whose intensity varies with time. Consider a small region of the space where the field may be treated as homogeneous. Under the influence of the induced electric field and as the magnetic field intensity increases, the particles will drift towards the OZ axis at velocity

$$dr/dt = -(1/2) \mathbf{r} \mathbf{H} / H, \quad (1.10.15)$$

whence

$$r \propto H^{-1/2}. \quad (1.10.16)$$

In this case $r_L \propto H^{-1/2}$ and

$$cp \propto r_L H \propto H^{1/2}. \quad (1.10.17)$$

The existence of the regions with magnetic field inhomogeneities and the collisions of CR particles with plasma particles will result in particle scattering, diffusion across the field, and, ejection from the traps.

1.11. Cosmic ray interactions with electromagnetic radiation in space plasma

1.11.1. Effects of Compton scattering of photons by accelerated particles

The Compton effect on relativistic electrons (or, as it is called, the ‘inverse’ Compton effect) has been studied in many works. The first efforts in this direction were made by Feenberg and Primakoff (1948) in connection with the problem of whether the CR may have the electronic component. It has been shown in (Felten and Morrison, 1963; Ginzburg and Syrovatsky, M1963) that the Compton effect of thermal photons by electrons of CR of galactic origin may make certain contribution to the isotropic γ -ray background. Gordon (1960), Shklovsky (1964a), Zheleznyakov (1965), Korchak (1965a,b) studied the possible importance of the inverse Compton effect of thermal photons on the electron component of solar CR to generation of X-rays and γ -rays in solar flares. Ginzburg (1964) and Shklovsky (1964b) estimate the possible contribution from the inverse Compton effect to the generation of electromagnetic radiation from various radio objects. Korchak and Ponomarenko (1966) have calculated the expected spectrum of photon emission generated by the inverse Compton effect in interactions between accelerated electrons and isotropic background of thermal photons. In this case Korchak and Ponomarenko (1966) proceed from the formula for Compton cross section (Akhiezer and Berestetsky, M1959):

$$\sigma_k(E_{ph1}, E_{ph2}, E, \nu_1, \nu_2, \nu) = \frac{A}{2} r_e^2 \left(\frac{m_e c^2}{E} \right)^2 \left(\frac{E_{ph2}}{E_{ph1}} \right)^2 \nu_1^{-2}, \quad (1.11.1)$$

where

$$A = \frac{(m_e c^2/E)^2}{\nu_1 \nu_2} \left(\frac{\nu(m_e c^2/E)^2}{\nu_1 \nu_2} - 2 \right) + \frac{\nu_1 - \nu(E_{ph2}/E)}{\nu_1} + \frac{\nu_1}{\nu_1 - \nu(E_{ph2}/E)}. \quad (1.11.2)$$

Here E_{ph1} and E are the photon and electron energy before scattering;

$$E_{ph2} = \frac{\nu_1 E_{ph1}}{\nu_2 + \nu(E_{ph1}/E)} \quad (1.11.3)$$

is the photon energy after scattering; $r_e = e^2/m_e c^2$ is the classical radius of electron; $\nu_{1,2} = 1 - \beta \cos \theta_{1,2}$; $\nu = 1 - \beta \cos \theta$ ($\beta = v/c$ is the electron velocity relative to the velocity of light; θ is the angle between the photon momentum $\mathbf{k}_1$ and $\mathbf{k}_2$ before and after scattering; θ_1 and θ_2 are the angles between these momentums and the initial electron momentum $\mathbf{p}$). Assume that the thermal photons are isotropic in space with the Planck distribution of the photon energy at a temperature T_{ph} in energetic units):

$$n_{ph} = \frac{n_{pho}}{a_{ph} T_{ph}^3} \frac{E_{ph1}^2}{\exp(E_{ph1}/T_{ph}) - 1}, \quad (1.11.4)$$

where $a_{ph} = 2.404$. If the electrons are also isotropic, the effective cross section $\sigma(E_{ph2}, E)$ of generation of the photons with energy E during scattering of thermal photons with temperature T_{ph} by electrons with energy E will be obtained by integrating Eq. 1.11.1 and taking into account Eq. 1.11.4:

$$\sigma(E_{ph2}, E) = \frac{r_e^2}{4a_{ph}} \frac{(m_e c^2/E)^2}{T_{ph}} \sum_{k=1}^{\infty} k^{-2} I(T_{ph}/k), \quad (1.11.5)$$

where

$$I(T_{ph}/k) = \int_0^{2\pi} d\varphi \int d\theta_1 \int d\theta_2 \sin \theta_1 \sin \theta_2 A \nu_2^{-1} (T_{ph}/k)^{-2} \times \left(\frac{\nu_2 E_{ph2}}{\nu_1 - \nu(E_{ph2}/E)} \right) \exp \left(- \frac{\nu_2 E_{ph2}/(T_{ph}/k)}{\nu_1 - \nu(E_{ph2}/E)} \right). \quad (1.11.6)$$

Here the value A is determined from Eq. 1.11.2; the range of integration over θ_1 and θ_2 is $0 \leq \theta_{1,2} \leq 2\pi$ when the condition $\nu_2 E_{ph2}/(\nu_1 - \nu(E_{ph2}/E)) > 0$ which is equivalent to the condition $E_{ph2} < \nu_1 E/\nu$ is satisfied.

The Eq. 1.11.6 can be significantly simplified if the energy of the scattered photons is examined in the range

$$E(1 + \beta) \gg E_{ph2} \gg T_{ph}/\beta. \quad (1.11.7)$$

In this case

$$I(T_{\text{ph}}) = \frac{4\pi e^{-y}(1+\beta)}{\beta^4} \left\{ \frac{1+\beta^2}{2} + \frac{y(1+\beta^2)}{2} + e^4 \text{Ei}(-y)y(1+\beta) \left[1 + \frac{y(1+\beta^2)}{2} \right] \right\}, \quad (1.11.8)$$

where $y = E_{\text{ph}2}(1-\beta)/T_{\text{ph}}(1+\beta)$. For relativistic electrons at $E \gg m_e c^2$ (i.e. $\beta \approx 1$), we shall obtain by substituting Eq. 1.11.8 at $\beta=1$ in Eq. 1.11.5:

$$\sigma(E_{\text{ph}2}, E) = \frac{3\sigma_T}{4a_{\text{ph}}T_{\text{ph}}} \left(\frac{m_e c^2}{E} \right)^2 \sum_{k=1}^{\infty} k^{-2} \exp(-kz) G(kz), \quad (1.11.9)$$

where $\sigma_T = (8/3)\pi r_e^2 = 6.65 \times 10^{-25} \text{ cm}^2$ is the Thompson cross-section, and

$$G(z) = 1 + 2z + 2z(1+z)\exp(z)\text{Ei}(-z); \quad z = (E_{\text{ph}2}/4T_{\text{ph}})(m_e c^2/E)^2. \quad (1.11.10)$$

The function $G(z)$ depends little on z in the interval $(0, \infty)$: the values of this function are within 0.7-1.0 (see Fig. 1.11.1).

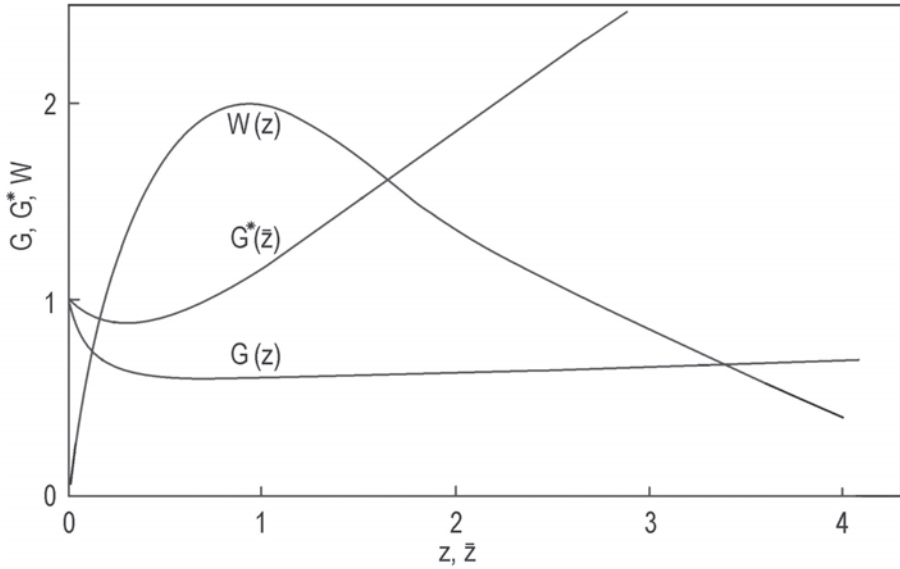


Fig. 1.11.1. Special functions functions W and G versus z , and G^* versus $\bar{z}$ for determining the expected spectrum of photon emission generated by the inverse Compton effect in interactions between accelerated electrons and isotropic background of thermal photons. The dimensionless variables z and $\bar{z}$ are determined by Eq. 1.11.10 and Eq. 1.11.22, respectively. According to Korchak and Ponomarenko (1966).

The spectral power of radiation per a single electron with energy E

$$W(E_{\text{ph2}}, E) = cn_{\text{pho}} E_{\text{ph2}} \sigma(E_{\text{ph2}}, E) \quad (1.11.11)$$

has a maximum value at $(E_{\text{ph2}}/4T_{\text{ph}})(m_e c^2/E) \approx 1$ (see Fig. 1.11.1).

If the differential energy spectrum of electrons is of the power form

$$n(E) = n_o (\gamma - 1) E_o^{\gamma-1} E^{-\gamma}, \quad (1.11.12)$$

(where E_o is the boundary of the spectrum on the low energy side, n_o is the total concentration of electrons with energy $E \geq E_o$), the total spectral power of radiation is

$$W(E_{\text{ph2}}) = \int_{E_o}^{\infty} n_o (\gamma - 1) E_o^{\gamma-1} E^{-\gamma} W(E_{\text{ph2}}, E) dE = \frac{3\sigma_T}{2a_{\text{ph}}} cn_o (\gamma - 1) n_{\text{pho}} z_o^{(1-\gamma)/2} I(z_o) \quad (1.11.13)$$

where

$$I(z_o) = \sum_{k=1}^{\infty} k^{-2} \int_0^{z_o} \exp(-kz) z^{(\gamma-1)/2} G(kz) dz; \quad z_o = \frac{E_{\text{ph2}}}{2T_{\text{ph}}} \left(\frac{m_e c^2}{E_o} \right)^2, \quad (1.11.14)$$

and the function G is determined by the Eq. 1.11.10. According to Korchak and Ponomarenko (1966), in the extreme case of low E_{ph2} when $z_o \ll 1$, then

$$I(z_o) \approx \frac{\pi^2}{3(\gamma+1)} z_o^{(\gamma+1)/2} \quad (1.11.15)$$

and

$$W(E_{\text{ph2}}) \approx \frac{\pi^2 \sigma_T}{8a_{\text{ph}}} cn_o n_{\text{pho}} \frac{\gamma-1}{\gamma+1} \left(\frac{m_e c^2}{E_o} \right)^2 \frac{E_{\text{ph2}}}{T_{\text{ph}}}, \quad (1.11.16)$$

i.e. at low E_{ph2} the spectral power $W(E_{\text{ph2}}) \propto E_{\text{ph2}}$. In the opposite extreme case of high E_{ph2} , when $z_o \gg 1$, then

$$W(E_{\text{ph2}}) \approx f(\gamma) \sigma_T cn_o n_{\text{pho}} \left(\frac{E_o}{m_e c^2} \right)^{\gamma-1} \left(\frac{E_{\text{ph2}}}{4T_{\text{ph}}} \right)^{(1-\gamma)/2}, \quad (1.11.17)$$

where

$$f(\gamma) = \frac{3}{2a_{\text{ph}}} \frac{(\gamma^2 + 4\gamma + 1)(\gamma - 1)}{(\gamma + 3)(\gamma + 5)} \Gamma\left(\frac{\gamma + 1}{2}\right) \xi\left(\frac{\gamma + 5}{2}\right), \quad (1.11.18)$$

and ξ is the Riemannian zeta-function. At $\gamma = 1.5, 2, 3, 4$, and 5 , the function $f(\gamma)$ takes the values $0.216, 0.410, 0.90, 1.79$, and 3.62 respectively. Thus at high $E_{\text{ph}2}$ the spectral power $W(E_{\text{ph}2}) \propto E_{\text{ph}2}^{(1-\gamma)/2}$ (similarly to the synchrotron radiation). The maximum of power is reached at

$$E_{\text{ph}2}^{\text{max}} = 4T_{\text{ph}}\phi(\gamma)\left(E_o/m_e c^2\right)^2, \quad (1.11.19)$$

where $\phi(\gamma) = 0.6, 0.71, 0.94, 1.16$, and 1.38 at $\gamma = 1.5, 2, 3, 4$ and 5 respectively.

It is of great interest to consider the case where all the thermal photons move isotropic (Korchak and Ponomarenko, 1966). In this case, the angle θ between the radially moving primary photons and the scattered photons (moving along the sight line) is fixed. The cross section $d\sigma$ of generation of photons with energy $E_{\text{ph}2}$ and direction of the momentum within the solid angle $d\Omega_{\text{ph}2}$ will then be

$$d\sigma = d\Omega_{\text{ph}2} \int \nu_1 \delta\left(E_{\text{ph}2} - \frac{\nu_1 E_{\text{ph}1}}{\nu_2 + \nu(E_{\text{ph}1}/E)}\right) \sigma_k \frac{n_{\text{ph}}}{n_{\text{ph}o}} dE_{\text{ph}1} \frac{d\Omega_E}{4\pi}, \quad (1.11.20)$$

where σ_k is determined from Eq. 1.11.1; $d\Omega_E$ is the solid angle characterized the direction of the initial electron momentum; the rest designations are as above. After integrating Eq. 1.11.20 over $E_{\text{ph}1}$ and θ_2 we shall obtain for relativistic electrons ($\beta \approx 1$):

$$d\sigma(E_{\text{ph}2}, E) = \frac{3\sigma_T d\Omega_{\text{ph}2}}{16\pi a_{\text{ph}} T_{\text{ph}}} \left(\frac{m_e c^2}{E}\right)^2 \sum_{k=1}^{\infty} \bar{k}^{-2} \exp(-k\bar{z}) G^*(k\bar{z}), \quad (1.11.21)$$

where

$$G^*(\bar{z}) = 1 - \bar{z} - 2\bar{z}^2 \exp(\bar{z}) \text{Ei}(-\bar{z}); \quad \bar{z} = \frac{E_{\text{ph}2} (m_e c^2 / E)^2}{2T_{\text{ph}} (1 - \cos \theta)}. \quad (1.11.22)$$

The plot of the function $G^*(\bar{z})$ is also shown in Fig. 1.11.1. The maximum value of $d\sigma$ is at $\theta = \pi/2$ and decreases down to 0 at $\theta = 0$; this is quite understandable

since in this case the observer receives only the photon from the Planck distribution, but not the scattered photons. If the electron spectrum is of the power form described by Eq. 1.11.12, we shall obtain taking account of Eq. 1.11.21:

$$dW(E_{\text{ph2}}) = \frac{3\sigma_T d\Omega_{\text{ph2}}}{16a_{\text{ph}}\pi} c n_o n_{\text{pho}} I(\bar{z}_o) (\gamma - 1) (1 - \cos\theta)^{(\gamma+1)/2} \\ \times \left(\frac{E_o}{m_e c^2} \right)^{(\gamma-1)} \left(\frac{E_{\text{ph2}}}{2T_{\text{ph}}} \right)^{(1-\gamma)/2}, \quad (1.11.23)$$

where

$$\bar{z}_o = E_{\text{ph2}} \left(m_e c^2 / E_o \right)^2 [2T_{\text{ph}} (1 - \cos\theta)]^{-1}. \quad (1.11.24)$$

It follows from Eq. 1.11.23, including Eq. 1.11.22, that at $\bar{z}_o \ll 1$ $dW \propto E_{\text{ph2}}/T_{\text{ph}}$ and at $\bar{z}_o \gg 1$ $dW \propto (1 - \cos\theta)^{(1+\gamma)/2} (E_{\text{ph2}}/T_{\text{ph}})^{(1-\gamma)/2}$. In this case the energy $E_{\text{ph2}}^{\text{max}}$, at which dW is a maximum, will be proportional to $(1 - \cos\theta) T_{\text{ph}} (E_o / m_e c^2)$. It also follows from Eq. 1.11.23 that in the high energy photon range (when $(E_{\text{ph2}}/4T_{\text{ph}})(m_e c^2 / E_o) \gg 1$) the emitted power depends on angle θ as $(\sin\theta/2)^{\gamma+1}$, where θ is the angular distance between the emitting region and the central meridian (for solar flares), i.e. the most considerable flux of hard X-rays generated by accelerated relativistic electrons will be from the flares at the edge of the solar limb. The probability of Compton scattering $W(E, \nu_{\text{ph1}}, \nu_{\text{ph2}})$ of isotropic radiation by relativistic electron with energy E is generalized in (Charugin and Ochelkov, 1974) for the case where the frequency ν_{ph2} of the scattered quantum is smaller than the frequency ν_{ph1} . It has been shown that the intensity of the scattered radiation $\propto \nu_{\text{ph2}}$ at $\nu_{\text{ph2}} > \nu_{\text{ph1}}$ and is proportional to ν_{ph2}^2 at $\nu_{\text{ph2}} < \nu_{\text{ph1}}$ for a mono-energetic electron beam. In case of induced Compton scattering the relativistic electrons always gains energy. If the brightness temperature of radiation T_{ph} is such that $kT_{\text{ph}} \gg m_e c^2$ in some range of frequencies, the rate of heating $\approx (kT_{\text{ph}}/m_e c^2) (m_e c^2 / E)^5 c \sigma_T \rho$. Since the cooling owed to spontaneous Compton loss $\propto E^2$, the electrons even in intense sources may be heated only up to energy $E \approx m_e c^2 (kT_{\text{ph}}/m_e c^2)^{1/7}$. Despite the fact that the effectiveness of the electron heating owing to induced Compton scattering in real sources is small, considerable distortions of the spectra are expected in quasars if the density of electrons with

energies $E \leq 10m_e c^2$ exceeds 10^6 cm^{-3} . Sweeney and Stewart (1974) have studied the nonlinear Compton radiative group deceleration. The equations of electron motion in a strong, plane, and circularly polarized electromagnetic wave have been numerically integrated by taking account of radiative deceleration. The principal attention was paid to the electron interactions with the electromagnetic wave whose phase velocity c/α (where $0 < \alpha \leq 1$) exceeds the velocity of light in vacuum. It has been shown that the radiative deceleration accounts for the rapid evolution of charged particles. Irrespectively of the initial energy, the evolution of charged particles is rapid and their energies approach the asymptotic value $E \approx m_e c^2 f / \sqrt{1-d^2}$ at which the energy loss for radiative deceleration and the energy gain under the effect of the electric field of the strong wave are equalized. For the pulsar in the Crab nebula, the parameter of wave force $f \approx 10^6$ and the electrons acquire their asymptotic energies already at distances of $\sim 2 \times 10^{13} \text{ cm}$ from neutron star.

Milton et al. (1974), Hari Dass et al. (1975) have studied the Compton scattering of photons by charged particles in the presence of homogeneous magnetic field whose value is comparable with critical (for electrons, $4 \times 10^{13} \text{ Gs}$). In this case the effect of such a field of all orders should be taken into account or else the charge should be considered as bound. The calculation scheme and the expression for the cross-section are presented. The extreme cases are examined. Ochelkov and Prilutsky (1974) study the effect of the energy loss for the Compton radiation on the electron spectrum in the 'plasma kettles', i.e. the regions with high density of electromagnetic radiations, which may probably exist in the galactic nuclei and quasars. It was shown earlier that a power spectrum of electrons with exponent $\gamma = 3$ which is universal, independent of specific size of a kettle, was generated in turbulent plasma in homogeneous magnetic field. It is emphasized, however, that the results obtained are inapplicable at sufficiently high densities of electromagnetic radiation in a kettle when the Compton scattering of the radiation by relativistic electrons becomes to be of importance. It has been shown that the power spectra of electrons with exponent $\gamma \neq 3$ are generated in the kettles with high density of radiation (of the order of the energy density of plasmons or magnetic field); the value of γ is already dependent on specific parameter of a kettle, for example the size, the relativistic electron density, etc. It is indicated that the value may be unambiguously determined from observations on the basis of the slope of the spectrum of X-rays from a kettle of Compton nature. For practical determination of γ , however, the spectrum of X-rays from galactic nuclei (supposedly the Compton X-rays) has been insufficiently studied as yet. Similar results in the study of the comptonization and the generation of the relativistic electron spectrum in the plasma kettles have been obtained in (Nikolaev and Tsyrovich, 1976; see Section 4.10.1) taking account of the Compton scattering of

reabsorbed radiation. The universality of the plasma turbulent kettle as a source of relativistic electrons with a power spectrum under the conditions close to the real situation in the space in the presence of magnetic fields and magnetic turbulent modes of pulsations has been demonstrated. The dependence of the spectrum exponent γ on the parameters characterizing the plasma of the turbulent reactor has been studied for various types of turbulence. The found $\gamma \leq 3$ correspond to the range of the most probable values obtained in the studies of cosmic radio sources.

1.11.2. The influence of nuclear photo effects on accelerated particles

Gerasimova and Rozentel (1961) have estimated the variations of the CR spectrum in case of a nuclear photo effect on stellar photons. The values have been obtained for iron nuclei which undergo a photo-effect on the photons whose spectrum is determined by the Planck function of black body radiation at $T = 5800$ °K. The nuclei of galactic origin fail to disintegrate completely; only their isotopic composition changes, since the (γ, n) and $(\gamma, 2n)$ photo-neutron reactions are predominant. The change of the exponent of the integral spectrum of galactic CR has been estimated. The nuclei of intergalactic origin produced more than 10^{10} years ago underwent the photo-effect completely. The nuclei that are being produced may enter the Galaxy from a region of 5×10^{25} cm size which fails to give any significant contribution from CR to the spectrum observed on the Earth.

Pollack and Shen (1969a,b) have noted that, because of the Doppler effect, the photons of moderate energy in the coordinate system of their sources are shifted to the γ -ray region in the coordinate system of highly energetic CR. Therefore such photons may knock out individual nucleons from compound nuclei and decrease the proton energy. The calculations show that the photon density during supernova explosions, in quasars, and in some pulsar models is sufficiently high to ensure a splitting of α -particles with total energies above 4×10^{15} eV when they are ejected from these potential sources of CR. The corresponding value of energy for nuclei of group VH is some 2×10^{17} eV (see Table 1.11.1).

Table 1.11.1. Critical energies (in eV) of various nuclei for an essential energy loss in photon field. According to Pollack and Shen (1969a,b).

Source	Total photodisintegration		Significant energy loss protons
	α -particles	very heavy nuclei, $A \geq 50$	
Supernova, type II	4×10^{15}	9×10^{16}	7×10^{16}
Quasars	2×10^{15}	8×10^{16}	8×10^{16}
Pulsars, $r = 10$ km	2×10^{15}	1.2×10^{17}	2×10^{17}
Pulsars, $r = 1000$ km	7×10^{18}	5×10^{20}	2×10^{22}

It follows from the results presented in Table 1.11.1 that the $10^{17} \div 10^{19}$ eV CR must be almost completely protons. Besides that, the order of magnitude of the photon field is sufficient to result in a significant energy loss at $E > 10^{16}$ eV. This is a probable reason for the decrease in the number of such high energy particles as observed in the spectrum.

Attention is paid in (Rengarajan, 1973) to the fact that during the early stage of a pulsar when an appreciable portion of the energy of neutron star rotation may be converted into high-energy CR and γ -quanta, the surface of the star is very hot (with $T \sim 10^7$ °K), and therefore it is necessary to take account of the CR and γ -quantum interactions with the photons of this radiation. Considering the photon emission to be black dark with $T = 10^7$ °K, the author has calculated the γ -quantum absorption as a result of γ - γ collisions and the heavy nucleus photo-disintegration. It has been found that during the initial stage of a pulsar, when $T \sim 10^7$ °K on its surface, all the γ -quanta with energy $10^8 - 10^{12}$ eV are completely absorbed and all the nuclei of the iron group with energy $10^{13} - 10^{15}$ eV/nucleon disintegrate almost completely. The calculations has been also carried out for $T = 2 \times 10^6$ °K and $T = 5 \times 10^6$ °K. At $T = 5 \times 10^6$ °K, the γ -quantum absorption is still significant, the optical thickness $\tau_{\gamma\gamma} \sim 10$ (for the $10^9 \div 10^{10}$ eV energy range), whereas at $T = 2 \times 10^6$ °K $\tau_{\gamma\gamma} \leq 1$. The iron nucleus photo-disintegration decreases pronouncedly with decreasing T, so that at already $T = 5 \times 10^6$ °K the optical thickness $\tau_{\gamma Fe} \sim 1$ only at $E \sim 5 \times 10^{13}$ eV/nucleon and decreases abruptly at either side of this value of energy.

1.11.3. Effect of the universal microwave radiation on accelerated particles

Daniel and Stephens (1966) studied the effect of the isotropic thermal radiations of the Universe at $T = 2.7$ °K on the energy spectrum of high energy electrons by measuring the differential energy spectrum of electrons with energies $E > 12$ GeV in primary CR. Analysis of 28 detected electrons has shown that the total flux of primary electrons with effective energy $E > 12$ GeV is 0.51 ± 0.10 (m²sec.ster)⁻¹. The differential energy spectrum of the 12÷350 GeV electrons is of the form $n(E)dE = 12.7E^{-\gamma}dE$, where E is the electron energy in GeV; $\gamma = 2.1 \pm 0.2$. The positron share in the total number of electrons and positrons is 0.70 ± 0.20 , which is indicative of a positron excess at $E > 12$ GeV in contrast to a negative excess of electrons at lower energies. The measured spectrum of the 12÷350 GeV electrons and the 1÷10 GeV electron spectrum obtained from other experiments were compared with the calculated spectrum obtained on the assumption of electron equilibrium in the galactic halo and including the Compton backscattering of electrons by the luminescence photons and by the photons of black-body radiation at $T = 2.7$ °K. The comparison has shown that at $E < 2$ GeV the experimental and theoretical spectra fail to be in agreement, which may be explained by the solar

modulation effect on the low-energy electron fluxes. On the other hand, at $E \leq 12$ GeV the observed appreciable excess of the experimental spectrum over the theoretical one is far beyond the possible errors. In this energy range the exponent of the theoretical spectrum $\gamma = 3.4$ in comparison with the experimental $\gamma = 2.1 \pm 0.2$. The observed discord indicates that either the black-body radiation at $T = 2.7$ °K does not exist in the Universe or the adopted model of galactic halo is invalid. However, the theoretical spectrum calculated in terms of the extragalactic model for electrons gives even larger disagreement with the experimental data. For the subsequent experiments have confirmed existence of the Planck radiation in the Universe, the extragalactic model have been completely rejected. In this case a dual explanation may be given to the available experimental data on the electron spectrum at $E > 12$ GeV, namely (1) the electrons are probably not in the equilibrium state in the halo and (2) the entire observed spectrum of the 1-350 GeV electrons consists of two different components in the galactic halo model. One of the components, which accounts for the existence of the electrons with energies of up to ~ 10 GeV and has the spectrum $n(E)dE = 50E^{-2.4}dE$, comprises the directly accelerated electrons and the secondary electrons produced in nuclear interactions of CR when traversing a 2.5 g/cm^2 path in the interstellar hydrogen. The second component accounts for the existence of the 10-350 GeV electrons and has the spectrum $n(E)dE = 0.54E^{-1.1}dE$. Because of the Planck radiation this spectrum begins to fall more steeply at $E \geq 20$ GeV and reaches the exponent $\gamma = 2.1$ at high energies.

Cowsik et al. (1966) emphasize that the universal radiation at $T = 2.7$ °K makes it possible to estimate the upper limit of the leakage lifetime of the primary CR electrons. For this purpose the equilibrium differential energy spectrum of electrons was calculated for various electron leakage lifetimes τ and the results were compared with the corresponding experimental spectrum in the 1-350 GeV energy range. The exponent of the power energy spectrum of electron injection calculated disregarding the energy loss was assumed to be $\gamma = 2.4$. The comparison showed that the spectrum calculated for $\tau = 10^7$ years was in a good agreement with experimental data if the possible errors are included, whereas the spectrum with $\tau \geq 10^8$ years gives too low an electron flux at energies of 100 GeV and higher. Thus, the leakage lifetime of electrons cannot exceed 10^7 years if the universal $T = 3$ °K radiation exists and if the electrons with energies of up to several hundreds of GeV are generated in but a single source.

1.11.4. Effect of infrared radiation on accelerated particles

Shen (1970) notes that if the recently discovered (Shivanandan et al. 1968), IR radiation exists actually in the Galaxy, Vela X is probably the sole source of the very high energy CR electrons measured on the Earth (Shivanandan et al., 1968).

An abrupt cutoff of the energy spectrum of CR electrons is predicted at $E \sim 2 \times 10^3$ GeV.

1.12. CR interaction with matter of space plasma as the main source of cosmic gamma radiation

1.12.1. *The matter of the problem*

The interaction of CR particles (protons, nuclei, and electrons) with matter determine the main processes of high energy gamma ray generation through neutral pions decay and bremsstrahlung emission. In Dorman (1996) there was estimated the expected gamma ray intensity generated by local and outer CR in different astrophysical objects for outer and inner observers. Any astrophysical object containing CR (of local and outer origin), magnetic fields and matter must generate gamma rays by neutral pion's decay (generated in interactions of CR protons and nuclei with matter), and by the generation of bremsstrahlung, synchrotron and curvature radiation of relativistic electrons, and by inverse Compton scattering of relativistic electrons on optical, infrared and relict 2.7 °K photons. The intensity and spectrum of gamma radiation depend on the CR spectrum, on the CR space-time distribution function, as well as on the spacial distribution of matter, magnetic fields and small energy background photons. Below we shall consider general formulas for gamma ray generation through neutral pion's decay (generated in nuclear interactions of proton-nuclear CR component with the matter of space plasma; see below Section 1.12.2), and gamma ray generation through interactions of CR electrons with matter and low energy photons in space plasmas (bremsstrahlung and inverse Compton radiation, respectively; see Section 1.12.3). On the basis of these formulas we shall make several estimations of expected gamma ray generation by flare CR from the Sun by their interactions with the matter of solar corona and solar wind as well as gamma ray generation by flare CR from other stars by interactions with the matter of stellar winds (Section 1.13). The same will be made for gamma ray generation by galactic CR interactions with matter of solar and stellar winds (Section 1.14).

1.12.2. *Gamma rays from neutral pions generated in nuclear interactions of CR with space plasma matter*

Let the distribution of space plasma matter in the spherical system of coordinates r, θ, ϕ be determined by $n(r, \theta, \phi)$ in units of atom.cm^{-3} . Let us suppose that $N_{pn}(E, r, \theta, \phi)$ is the space distribution of the differential intensity of the proton-nuclear component of CR, where E is the total CR particle energy in GeV/nucleon. The gamma ray intensity from some space plasma volume boundared by the surface $r_o(\theta, \phi)$ from neutral pions decay in this volume at the distance $r_{obs} \gg r_o(\theta, \phi)$ will then be

$$F_{\gamma, \text{pn}}(r_{\text{obs}}, E_{\gamma}) = r_{\text{obs}}^{-2} \int_{-\pi/2}^{\pi/2} \cos \theta d\theta \int_0^{r_o(\theta, \phi)} dr \int_0^{2\pi} d\phi \int_0^{\infty} dE \sigma_{\text{pn}}(E, E_{\gamma}) N_{\text{pn}}(E, r, \theta, \phi) n(r, \theta, \phi), \quad (1.12.1)$$

where, according to Stecker (M1971), Dermer (1986a,b),

$$\sigma_{\text{pn}}(E, E_{\gamma}) = 2 \int_{E_{\pi, \text{min}}(E_{\gamma})}^{\infty} dE_{\pi} \sigma_{\pi}(E) (E_{\pi}^2 - m_{\pi}^2 c^4)^{-1/2}. \quad (1.12.2)$$

In Eq. 1.12.2

$$E_{\pi, \text{min}}(E_{\gamma}) = E_{\gamma} + m_{\pi}^2 c^4 / E_{\gamma}, \quad (1.12.3)$$

and the cross-section of pion generation $\sigma_{\pi}(E)$ can be approximated by (the momentum p of protons is in GeV/c):

$$\sigma_{\pi}(E) = \begin{cases} 0 & \text{if } p < 0.78, \\ 0.032\eta^2 + 0.040\eta^6 + 0.047\eta^8 & \text{if } 0.78 \leq p \leq 0.96, \\ 32.6(p - 0.8)^{3.21} & \text{if } 0.96 \leq p \leq 1.27, \\ 5.40(p - 0.8)^{0.81} & \text{if } 1.27 \leq p \leq 8.00, \\ 32.0 \ln p + 48.5 p^{-1/2} - 59.5 & \text{if } p \geq 8.00. \end{cases} \quad (1.12.4)$$

In Eq. 1.12.4 there was used the notation

$$\eta = \left[\left(2m_p c^2 E_k - m_{\pi}^2 c^4 \right)^2 - 16m_{\pi}^2 m_p^2 c^8 \right]^{1/2} \left[2m_{\pi} c^2 \left(2m_p c^2 (E_k + 2m_p c^2) \right)^{1/2} \right]^{-1}, \quad (1.12.5)$$

and E_k is the kinetic energy of protons. The dependence of $\sigma_{\pi}(E)$ from kinetic energy of protons E_k (calculated according to Eq. 1.12.4) is shown in Fig. 1.12.1.

For some rough estimates let us use the demodulated differential CR proton spectrum in the interstellar space as in Dermer (1986a,b):

$$N_p(E) = 2.2 E^{-2.75} \text{ protons} / (\text{cm}^2 \cdot \text{sec} \cdot \text{GeV} \cdot \text{sr}). \quad (1.12.5a)$$

For α -particles the differential energy spectrum will be the same, but the coefficient will be 0.7 instead of 2.2 and E will denote the energy/nucleon. According to Dermer (1986a,b) the inclusion of additional channels of nuclear interactions p -He,

α -H, and α -He gives an increase in gamma ray emissivity of 28%, 9%, and about 2% relative to p -H channel considered above.

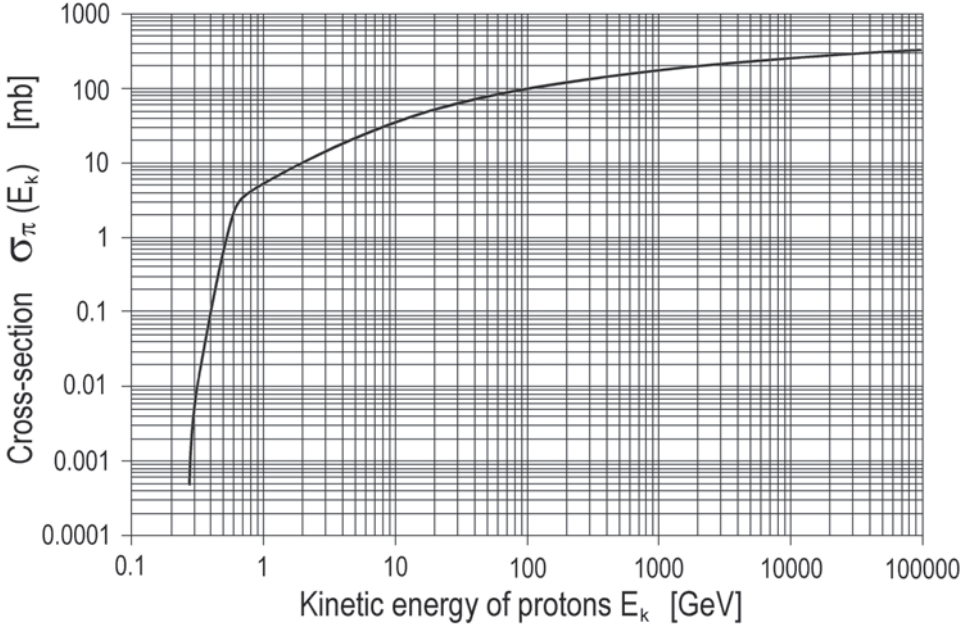


Fig. 1.12.1. The inclusive cross-section $\sigma_{\pi}(E_k)$ for reactions $p + p \rightarrow \pi^0 + \text{anything}$ as a dependence upon the kinetic energy of protons E_k . Calculated according to Eq. 1.12.4.

Therefore for rough estimations we can consider only the channel p -H and then multiply the result by a factor 1.39; if we also take into account heavier nuclei this factor will be 1.45. For the demodulated differential energy CR proton spectrum (Eq. 1.12.5) with factor 1.45, the expected gamma ray emissivity per 1 atom H in cm^3 was found by Dermer (1986a,b) as $Q_{pn}(E_{\gamma})$ in units photons/($\text{cm}^3 \text{ sec GeV}$), which can be approximated in the energy interval from 10^{-3} GeV up to 10^3 GeV by

$$\lg(Q_{pn}(E_{\gamma})) \approx -21.4 - \gamma \left[1 + (\lg(E_{\gamma}/E_{\gamma,\max}))^2 \right]^{1/2}, \quad (1.12.6)$$

where $\gamma = 2.75$ and $E_{\gamma,\max}$ denotes the position where $Q_{pn}(E_{\gamma})$ attains the maximum:

$$\lg(E_{\gamma,\max}) = -1.17. \quad (1.12.7)$$

1.12.3. Gamma ray generation by CR electrons in space plasma (bremsstrahlung and inverse Compton effect)

By using results of Cesarsky et al. (1978) on the bremsstrahlung gamma ray generation by electrons of galactic CR in the interstellar medium, we obtain for the expected bremsstrahlung gamma ray flux from some volume of space plasma at some distance r_{obs} from this volume the following formula:

$$F_{\gamma, \text{bs}}(r_{\text{obs}}, E_{\gamma}) = r_{\text{obs}}^{-2} \int_{-\pi/2}^{\pi/2} \cos \theta d\theta \int_0^{r_o(\theta, \phi)} dr \int_0^{2\pi} d\phi \int_{E_{\gamma}}^{\infty} dE \sigma_{\text{bs}}(E_e, E_{\gamma}) N_e(E_e, r, \theta, \phi) n(r, \theta, \phi), \quad (1.12.8)$$

where the definitions are the same as for Eq. 1.12.1, but E_e is the energy of electrons and $N_e(E_e, r, \theta, \phi)$ is the space distribution of the differential intensity of the electron component of CR. In Eq. 1.12.8 $\sigma_{\text{bs}}(E_e, E_{\gamma})$ is the cross-section of bremsstrahlung gamma ray generation with energy E_{γ} by electrons with energy E_e , which according to Cesarsky et al. (1978) can be approximated by the following equation:

$$\sigma_{\text{bs}}(E, E_{\gamma}) = \alpha_e^2 \left\{ (2E_e^2 - 2E_e E_{\gamma} + E_{\gamma}^2) E_{\gamma}^{-2} \varphi_1 - (E_e - E_{\gamma}) E_e^{-1} \varphi_2 \right\}, \quad (1.12.9)$$

where $\alpha \approx 1/137$ is the fine structure constant, r_e is the classical electron radius, φ_1 and φ_2 are functions from variable

$$\xi = \chi(Z) (m_e c^2 E_{\gamma}) / (E_e (E_e - E_{\gamma})), \quad (1.12.10)$$

$\chi(Z=1) = 34.259$, $\chi(Z=2) = 20.302$. The functions φ_1 and φ_2 are tabulated in Blumental and Gould (1970). According to Pohl (1994) for the standard He-to-H ratio of 0.1 for space plasma matter roughly $\varphi_1 \approx \varphi_2 \approx 58$ and

$$\sigma_{\text{bs}}(E_e, E_{\gamma}) \approx 0.42 r_e^2 \left((4/3) - (E_{\gamma}/E_e) - (E_{\gamma}/E_e)^2 \right), \quad (1.12.11)$$

For the demodulated differential energy spectrum of electrons in the interstellar space $N_e(E_e)$ by Cesarsky et al. (1978) was used spectrum of CR electrons measured on $r = r_1 = 1 \text{ AU}$ from the Sun in the minimum of solar activity:

$$N_e(E_e) = N_e(E_e, r_1) \approx 10^{-2} E_e^{-1.8} \text{ electron}/(\text{cm}^3 \cdot \text{sec} \cdot \text{GeV}). \quad (1.12.12)$$

The expected bremsstrahlung gamma ray emissivity per 1 atom H in cm^3 for this spectrum was found by Cesarsky et al. (1978) in the form

$$Q_{bs}(E_\gamma) \approx 10^{-26.4} E_\gamma^{-1.8} \text{ photons}/(\text{cm}^3 \cdot \text{sec} \cdot \text{GeV}). \quad (1.12.13)$$

For the total inverse Compton gamma ray emissivity (for starlight and 2.7 °K photons) was found

$$Q_{IC}(E_\gamma) \approx 10^{-27.3} E_\gamma^{-1.8} \text{ photons}/(\text{cm}^3 \cdot \text{sec} \cdot \text{GeV}), \quad (1.12.14)$$

about 10 times smaller than expected from bremsstrahlung gamma ray emissivity. Obtained in Cesarsky et al. (1978) gamma ray spectrum is too hard and contradicts to measurements by COMPTEL (Strong et al., 1994) and to theoretical models of Skibo and Ramaty (1993), Pohl (1993, 1994). On the basis of investigation of the CR hysteresis effect relative to solar activity (Dorman and Dorman 1967a,b), is possible to determine the modulation of CR in the interplanetary space in the minimum of solar activity and to restore the demodulated spectrum of galactic CR out of the Heliosphere (Dorman M1975a,b; Zusmanovich, M1986; Belov et al., 1990). For galactic CR electrons the demodulated differential spectrum $N_e(E_e)$ according to Webber (1987) can be described by power law $\propto E_e^{-\gamma}$ with gradually increasing γ with increasing of E_e from $\gamma \approx 2.3$ for $E_e \approx 1\div 2$ GeV to $\gamma \approx 3.2$ for $E_e \approx 30\div 100$ GeV:

$$N_e(E_e) \approx \begin{cases} 2 \times 10^{-2} E_e^{-2.3} & \text{for } E_e \leq 3 \text{ GeV}, \\ 3.3 \times 10^{-2} E_e^{-3.0} & \text{for } 3 \text{ GeV} < E_e < 10 \text{ GeV}, \\ 5.5 \times 10^{-2} E_e^{-3.2} & \text{for } 10 \text{ GeV} < E_e \leq 400 \text{ GeV}, \end{cases} \quad (1.12.15)$$

where $N_e(E_e)$ is in units of electron/(cm².sec.sr.GeV). The demodulated differential energy spectrum of CR electrons described by Eq. 1.12.15 will give the expected gamma ray emissivity in accordance with measurements by COMPTEL (Strong et al. 1994) and with theoretical models Skibo and Ramaty (1993); Pohl (1993, 1994).

1.13. Gamma ray generation in space plasma by interactions of flare energetic particles with solar and stellar winds

1.13.1. The matter of problem and the main three factors

The generation of gamma rays by interaction of flare energetic particles (FEP) with solar and stellar wind matter shortly was considered in Dorman (1996, 1997). In Dorman (2001a) was given a development of this research with much more details. As an example we consider the first the situation with gamma ray

generation in the interplanetary space by solar FEP in periods of great events, determined mainly by three factors:

The 1st factor – by the space-time distribution of solar FEP in the Heliosphere, their energetic spectrum and chemical composition (see review in Dorman, M1957, M1963a,b, M1978; Dorman and Miroshnichenko, M1968; Dorman and Venkatesan, 1993; Stoker, 1995; Miroshnichenko, M2001). For this distribution can be important nonlinear collective effects (especially for great events) of FEP pressure and kinetic stream instability (Berezinsky et al., M1990; Dorman, Ptuskin, and Zirakashvili, 1990, Zirakashvili et al., 1991; see in more details below, Chapter 3).

The 2nd factor – by the solar wind matter distribution in space and its change during solar activity cycle; nonlinear effects will also be important for this distribution: pressure and kinetic stream instability of galactic CR as well as of solar FEP (especially in periods of very great events) – see references above and in more details below, in Chapter 3.

The 3rd factor – by properties of solar FEP interaction with the solar corona and solar wind matter accompanied with gamma ray generation through decay of neutral pions (Stecker, M1971; Dermer, 1986a,b; see above Section 1.12.2).

After consideration of these 3 factors we calculate the expected space-time distribution of gamma ray emissivity, and expected fluxes of gamma rays for measurements on the Earth's orbit of a dependence upon time after the moment of FEP generation, for different directions of gamma ray observations. We calculate expected fluxes also for different distances from the Sun inside and outside the Heliosphere. We expect that the same 3 factors will be important for gamma ray generation by stellar FEP in stellar winds, but for some types of stars the total energy in FEP is several orders higher than in solar flares and the speed of lost matter is several orders higher than from the Sun (Gershberg and Shakhovskaya, 1983; Korotin and Krasnobaev, 1985; Gershberg et al., 1987; Kurochka, 1987).

According to Dorman (2001a), observations of gamma rays generated in interactions of solar FEP with solar wind matter can give during the periods of great events valuable information about the 3D-distribution of solar wind matter as well as about properties of solar FEP and its propagation parameters. Especially important will be observations of gamma rays generated in interactions of stellar FEP with stellar wind matter. In this case information can be obtained about total energy and energetic spectrum in stellar FEP, about the mode of FEP propagation, as well as information about stellar wind matter distribution.

1.13.2. The 1st factor: solar FEP space-time distribution

The problem of solar FEP generation and propagation through the solar corona and in the interplanetary space as well as its energetic spectrum and chemical and isotopic composition was reviewed in Dorman (M1957, M1963a,b, M1978), Dorman and Miroshnichenko (M1968), Dorman and Venkatesan (1993), Stoker (1995), Miroshnichenko (M2001). In the first approximation, according to numeral data from observations of many events for about 5 solar cycles the time change of

solar FEP and energy spectrum change can be described by the solution of isotropic diffusion (characterized by the diffusion coefficient $\kappa_i(E_k)$) from some pointing instantaneous source $Q_i(E_k, \mathbf{r}, t) = N_{oi}\delta(\mathbf{r})\delta(t)$ of solar FEP of type i (protons, α – particles and heavier particles, electrons) by

$$N_i(E_k, \mathbf{r}, t) = N_{oi}(E_k) \left[2\pi^{1/2} (\kappa_i(E_k)t)^{3/2} \right]^{-1} \times \exp(-\mathbf{r}^2 / (4\kappa_i(E_k)t)), \quad (1.13.1)$$

where $N_{oi}(E_k)$ is the energetic spectrum of total number of solar FEP in the source. At the distance $r = r_1$ the maximum of solar FEP density

$$N_{i\max}(r_1, E_k) / N_{oi}(E_k) = 2^{1/2} 3^{3/2} \pi^{-1/2} \exp(-3/2) r_1^{-3} = 0.925 r_1^{-3} \quad (1.13.2)$$

will be reached according to Eq. 1.13.1 at the moment

$$t_{\max}(r_1, E_k) = r_1^2 / 6\kappa(E_k), \quad (1.13.3)$$

and the space distribution of solar FEP density at this moment will be

$$\frac{N_i(r, E_k, t_{\max})}{N_{oi}(E_k)} = (54/\pi)^{1/2} r_1^{-3} \exp(-3r^2/2r_1^2) = 4.146 r_1^{-3} \exp(-3r^2/2r_1^2). \quad (1.13.4)$$

According to numerical experimental data the energetic spectrum of generated solar energetic particles in the source can be described approximately as (see the review in Dorman and Venkatesan, 1993):

$$N_{oi}(E_k) \approx N_{oi}(E_k / E_{k\max})^{-\gamma}, \quad (1.13.5)$$

where γ increases with increasing of energy from about $0 \div 1$ at $E_k \leq 1$ GeV/nucleon to about $6 \div 7$ at $E_k \approx 10 \div 15$ GeV/nucleon. Parameters N_{oi} and γ are changing sufficiently from one event to other: for example, for the greatest observed event of February 23, 1956 $N_{oi} \approx 10^{34} \div 10^{35}$, in the event of November 15, 1960 $N_{oi} \approx 3 \times 10^{32}$, in the event of July 18, 1961 $N_{oi} \approx 4 \times 10^{31}$, in the event of May 23, 1967 $N_{oi} \approx 10^{31}$. For the greatest observed event of February 23, 1956 parameter γ had values ≈ 1.2 at $E_k \approx 0.3$ GeV/nucleon, $\gamma \approx 2.2$ at $E_k \approx 1$ GeV/nucleon, $\gamma \approx 4$ at $E_k \approx 5 \div 7$ GeV/nucleon, and $\gamma \approx 6 \div 7$ at $E_k \approx 10 \div 15$ GeV/nucleon. This change of γ is typical for many great solar energetic particle events: see in Dorman (M1957,

M1963a,b) about event of February 23, 1956, and review about many events in Dorman (M1963a,b, M1978), Dorman and Miroshnichenko (M1968), Dorman and Venkatesan (1993), Stoker (1995), Miroshnichenko (M2001). Approximately the behavior of value γ in Eq. 1.13.5 can be described as

$$\gamma = \gamma_o + \ln(E_k/E_{ko}), \quad (1.13.6)$$

where parameters γ_o and E_{ko} are different for individual events, but typically they are in intervals $2 \leq \gamma_o \leq 5$ and $2 \leq E_{ko} \leq 10$ GeV/nucleon. The position of maximum in Eq. 1.13.5 taking into account Eq. 1.13.6 is determined by

$$E_{k \max} = E_{ko} \exp(-\gamma_o), \quad N_{oi}(E_{k \max}) = N_{oi}. \quad (1.13.7)$$

The total energy contained in FEP will be according to Eq. 1.13.5-1.13.7:

$$E_{\text{tot}} = N_{oi} \int_0^{\infty} E_k (E_k/E_{k \max})^{-\gamma_o - \ln(E_k/E_{ko})} d(E_k/E_{k \max}) = b N_{oi} E_{k \max}, \quad (1.13.8)$$

where

$$b = \int_0^{\infty} x^{1 - \ln x} dx = 4.82. \quad (1.13.9)$$

For great solar FEP events $E_{\text{tot}} \approx 10^{31} \div 10^{32}$ erg, and more (see in Dorman, M1957, M1963a,b, M1978; Dorman and Miroshnichenko, M1968; Dorman and Venkatesan, 1993; Stoker, 1995; Miroshnichenko, M2001), for great stellar FEP events $E_{\text{tot}} \approx 10^{35} \div 10^{37}$ erg (see in Gershberg and Shakhovskaya, 1983; Korotin and Krasnobaev, 1985; Gershberg et al., 1987; Kurochka, 1987).

In Eq. 1.13.1

$$\kappa_i(E_k) = \Lambda_i(E_k) v(E_k)/3 \quad (1.13.10)$$

is the diffusion coefficient, $\Lambda_i(E_k)$ is the transport path for particle scattering in the interplanetary space, $v(E_k)$ is the particle velocity as a dependence on the kinetic energy per nucleon E_k :

$$v(E_k) = c \left(1 - \left(1 + E_k/m_n c^2 \right)^{-2} \right)^{1/2}, \quad (1.13.11)$$

where $m_n c^2$ is the rest energy of the nucleon. According to numeral experimental data and theoretical investigations $\Lambda_i(E_k)$ have a bridge minimum in the region 0.1–0.5 GeV/nucleon and increases with energy decreasing lower than this region at about $\propto E_k^{-1}$ (caused by ‘tunnel’ effect for particles with curvature radius in the interplanetary magnetic field smaller than the smallest scale of hydromagnetic turbulence, see in Dorman, M1975a) as well as with energy increasing over this interval as $\propto E_k^\beta$, where β depends from the spectrum of turbulence and usually increases from 0 up to about 1 for high energy particles of few GeV/nucleon and then up to about 2 for very high energy particles with curvature radius in IMF bigger than biggest scale of magnetic inhomogeneities in IMF (according to investigations of galactic cosmic ray modulation in the Heliosphere it will be at $E_k \geq 20 \div 30$ GeV/nucleon). For calculations of expected space-time distribution of gamma ray emissivity we try to describe this dependence for the most part of spectrum what is important for gamma ray emission (from about 0.01 GeV/nucleon up to about 20 GeV/nucleon) approximately as

$$\Lambda_i(E_k) \approx \Lambda_i(W, r, t) \left(\frac{E_1}{E_k} + \frac{E_k}{E_2} + \left(\frac{E_k}{E_3} \right)^2 \right). \quad (1.13.12)$$

To determine the parameters E_1, E_2, E_3 we use observations of solar CR events as well as observations of galactic CR modulation in interplanetary space. The time dependences of galactic CR primary fluxes for effective rigidities $R=2, 5, 10$ and 25 GV were found in Belov et al. (1990) on the basis of ground measurements of muon and neutron components as well as measurements in stratosphere on balloons and in space on satellites and spacecrafts. The residual modulation (relative to the flux out of the Heliosphere) for $R \approx 10$ GV in the minimum and maximum of solar activity was determined as 6 and 24 % (what is in good agreement with results on CR-SA hysteresis effects according to Dorman and Dorman, 1967a,b, 1968; Dorman, M1975b; Dorman et al., 1997a,b). According to convection-diffusion model of CR solar cycle modulation (Parker, M1963; Dorman, 1959, M1975b), the slope of the residual spectrum $\Delta D(R)/D_0(R) \propto R^{-\beta}$ reflects the dependence $\Lambda(R) \propto (\Delta D(R)/D_0(R))^{-1} \propto R^\beta$. In Belov et al. (1990) the spectral index β was determined as $\beta \approx 0.4$ at $2-5$ GV, $\beta \approx 1.1$ at $5-10$ GV, and $\beta \approx 1.6$ at $10-25$ GV. Eq. 1.13.12 will be in agreement with these results and with data on FEP events in smaller energy region if we choose $E_1 \approx 0.05$ GeV/nucleon, $E_2 \approx 2$ GeV/nucleon, and $E_3 \approx 5$ GeV/nucleon.

The dependence of the transport path from the level of solar activity is characterized by $\Lambda_i(W)$. This parameter can be determined from investigations of

galactic CR modulation in the interplanetary space on the basis of observations by neutron monitors and muon telescopes for several solar cycles. According to Dorman and Dorman (1967a,b, 1968), Dorman (M1975b), Dorman et al. (1997a,b), $\Lambda_i(W) \propto W^{-1/3}$ for the period of high solar activity and $\Lambda_i(W) \propto W^{-1}$ for the period of low solar activity. According to Dorman et al. (1997a,b), the hysteresis phenomenon in the connection of long term CR intensity variation with solar activity cycle can be explained well by the analytical approximation of this dependence, taking into account the time lag of processes in the interplanetary space relative to caused processes on the Sun:

$$\Lambda_i(W, r, t) = \Lambda_i(W_{\max}) \times \left(W \left(t - \frac{r}{u} \right) / W_{\max} \right)^{-\frac{1}{3} - \frac{2}{3} \left(1 - W \left(t - \frac{r}{u} \right) / W_{\max} \right)}, \quad (1.13.13)$$

where $W_{\max}$ is the sunspot number in maximum of solar activity and $\Lambda_i(W_{\max}) \approx 10^{12} \text{ cm}^{-2}$.

1.13.3. The 2nd factor: space time distribution of solar wind matter

The detail information on the 2-nd factor for distances smaller than 5 AU from the Sun was obtained by the mission of Ulysses. Important information for bigger distances (up to about 100 AU) was obtained from missions Pioneer 10, 11, Voyager 3, 4, but only not far from the ecliptic plane. If we assume for the first approximation the model of Parker (M1963) of radial solar wind expanding into the interplanetary space which is in good accord with all available data of direct measurements in the Heliosphere, then the behavior of the matter density of solar wind will be described by the relation

$$n(r, \theta) = n_1(\theta) u_1(\theta) r_1^2 / (r^2 u(r, \theta)), \quad (1.13.14)$$

where $n_1(\theta)$ and $u_1(\theta)$ are the matter density and solar wind speed at the helio-latitude θ on the distance $r = r_1$ from the Sun ($r_1 = 1 \text{ AU}$). The dependence $u(r, \theta)$ is determined by the interaction of solar wind with galactic CR and anomaly component of CR, with interstellar matter and interstellar magnetic field, by interaction with neutral atoms penetrating from interstellar space inside the Heliosphere, by the nonlinear processes caused by these interactions (Dorman, 1995a,b; Le Roux and Fichtner, 1997; see also below, Chapter 3). According to calculations of Le Roux and Fichtner (1997) the change of solar wind velocity can be described approximately as

$$u(r) \approx u_1(1 - b(r/r_o)), \quad (1.13.15)$$

where the distance to the terminal shock wave $r_o \approx 74$ AU and parameter $b \approx 0.13 \div 0.45$ in dependence of subshock compression ratio (from 3.5 to 1.5) and from injection efficiency of pickup protons (from 0 to 0.9). From our investigations of CR-SA hysteresis phenomenon (Dorman and Dorman, 1967a,b, 1968; Dorman, M1975b; Dorman et al., 1997a,b), we estimate $r_o \approx 100$ AU .

1.13.4. The 3rd factor: gamma ray generation by FEP in the Heliosphere

Let us consider in the first generation of neutral pions. According to Stecker (M1971), Dermer (1986a,b), the neutral pion generation caused by nuclear interactions of energetic protons with hydrogen atoms through reaction $p + p \rightarrow \pi^0 + \text{anything}$ will be determined by

$$F_{pH}^{\pi}(E_{\pi}, r, \theta, t) = 4\pi n(r, \theta, t) \int_{E_{k \min}(E_{\pi})}^{\infty} dE_k N_p(E_k, r, t) \langle \zeta \sigma_{\pi}(E_k) \rangle (dN(E_k, E_{\pi})/dE_{\pi}), \quad (1.13.16)$$

where $n(r, \theta, t)$ is determined by Eq. 1.13.14, $E_{k \min}(E_{\pi})$ is the threshold energy for pion generation, $N_p(E_k, r, t)$ is determined by Eq. (1.13.1), $\langle \zeta \sigma_{\pi}(E_k) \rangle$ is the inclusive cross section for reactions $p + p \rightarrow \pi^0 + \text{anything}$, and

$$\int_0^{\infty} (dN(E_k, E_{\pi})/dE_{\pi}) dE_{\pi} = 1. \quad (1.13.16a)$$

Gamma ray emissivity caused by nuclear interactions of FEP protons with solar wind matter will be determined according to Stecker (M1971), Dermer (1986a,b), by

$$F_{pH}^{\gamma}(E_{\gamma}, r, \theta, t) = 2 \int_{E_{\pi \min}(E_{\gamma})}^{\infty} dE_{\pi} (E_{\pi}^2 - m_{\pi}^2 c^4)^{-1/2} F_{pH}^{\pi}(E_{\pi}, r, \theta, t), \quad (1.13.17)$$

where $E_{\pi \min}(E_{\gamma}) = E_{\gamma} + m_{\pi}^2 c^4 / 4E_{\gamma}$. Let us introduce Eq. 1.13.1 in Eq. 1.13.16 and Eq. 1.13.17 by taking into account Eq. 1.13.14:

$$F_{pH}^{\gamma}(E_{\gamma}, r, \theta, t) = B(r, \theta, t) \int_{E_{\pi \min}(E_{\gamma})}^{\infty} \left(E_{\pi}^2 - m_{\pi}^2 c^4 \right)^{-1/2} dE_{\pi} \\ \times \int_{E_{k \min}(E_{\pi})}^{\infty} N_{op}(E_k) \langle \sigma_{\pi}(E_k) \rangle (t/t_1)^{-3/2} \exp\left(-3r^2 t_1 / 2r_1^2 t\right) dE_k, \quad (1.13.18)$$

where

$$B(r, \theta, t) = 3^{3/2} 2^{7/2} \pi^{1/2} r_1^2 n_1(\theta, t) u_1(\theta, t) / r^2 u(r, \theta, t) \quad (1.13.19)$$

and

$$t_1 = r_1^2 / 6\kappa_p(E_k) \quad (1.13.20)$$

is the time in which the density of FEP at a distance of 1 AU reaches the maximum value. The space distribution of gamma ray emissivity for different t/t_1 will be determined mainly by function

$$f(t, t_1) = r^{-2} (t/t_1)^{-3/2} \exp\left(-3r^2 t_1 / 2r_1^2 t\right), \quad (1.13.21)$$

where t_1 , determined by Eq. 1.13.20, corresponds to some effective value of E_k in dependence of E_{γ} , according to Eq. 1.13.16 and Eq. 1.13.17. The biggest gamma ray emission is expected in the inner region

$$r \leq r_i = r_1 (2t/3t_1)^{1/2}, \quad (1.13.22)$$

where the level of emission $\propto r^{-2} (t/t_1)^{-3/2}$. Outside this region gamma ray emissivity decreases very quickly with r as $\propto r^{-2} \exp\left(-(r/r_i)^2\right)$. For an event with total energy 10^{32} ergs at $t = t_1 = 10^3$ sec, $r_i = 10^{13}$ cm, $n_1(\theta, t) \approx 5 \text{ cm}^{-3}$, $\kappa_p(E_k) \approx 4 \times 10^{22} \text{ cm}^2/\text{sec}$, we obtain for emissivity of gamma rays with energy > 100 MeV:

$$F_{pp}^{\gamma}(E_{\gamma} > 0.1 \text{ GeV}, r) \approx 10^8 r^{-2} \text{ photon.cm}^{-3} \text{ sec}^{-1}. \quad (1.13.23)$$

Let us note that at the distance of 5 solar radius it gives $10^{-15} \text{ photon.cm}^{-3} \text{ sec}^{-1}$. Eq. 1.13.18 describes the space-time variations of gamma ray emissivity

distribution from interaction of solar energetic protons with solar wind matter (see Fig. 1.13.1).

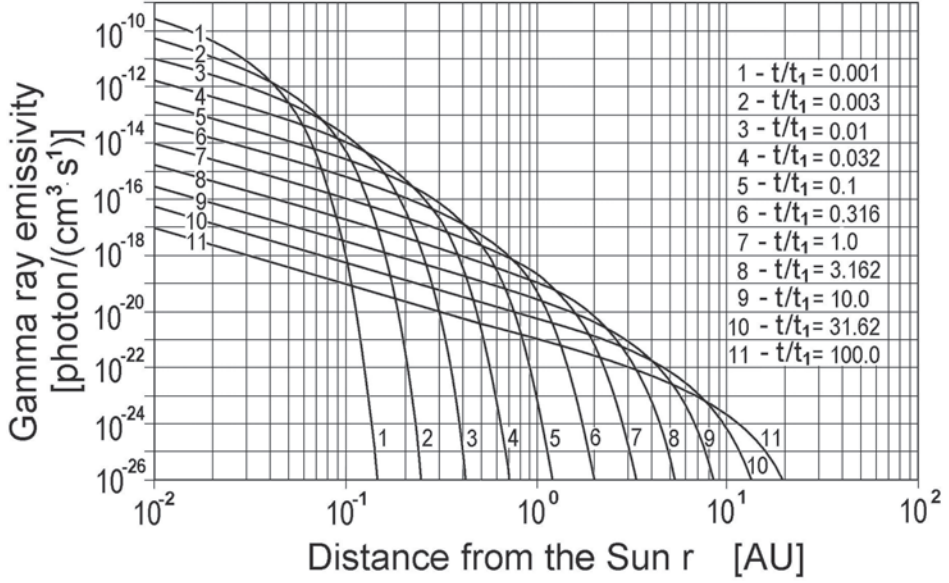


Fig. 1.13.1. Expected for the event with energy 10^{32} ergs space-time emissivity distribution of gamma rays with energy > 100 MeV for different time t after FEP generation in units of time maximum t_1 on 1 AU, determined by Eq. (1.13.20). The curves are from $t/t_1 = 0.001$ up to $t/t_1 = 100$. From Dorman (2001a).

1.13.5. Expected angle distribution and time variations of gamma ray fluxes for observations inside the Heliosphere during FEP events

Let us assume that the observer is inside the Heliosphere at the distance $r_{\text{obs}} \leq r_o$ from the Sun and helio-latitude θ_{obs} (here r_o is the radius of Heliosphere). The sight line of observation we can determine by the angle θ_{sl} , computed from the equatorial plane from direction to the Sun to the North. In this case the expected angle distribution and time variations of gamma ray fluxes will be

$$\Phi_{pH}^{\gamma}(E_{\gamma}, r_{\text{obs}}, \theta_{\text{sl}}, t) = \int_0^{L_{\text{max}}(\theta_{\text{sl}})} F_{pH}^{\gamma}(E_{\gamma}, L(r_{\text{obs}}, \theta_{\text{sl}}), t) dL. \quad (1.13.24)$$

In Eq. 1.13.24 gamma ray emissivity

$$F_{pH}^{\gamma}(E_{\gamma}, L(r_{\text{obs}}, \theta_{\text{sl}}), t) = F_{pH}^{\gamma}(E_{\gamma}, r, \theta, t) \quad (1.13.25)$$

is determined by Eq. 1.13.18 taking into account that

$$r = \left(r_{\text{obs}}^2 + L^2 + 2r_{\text{obs}}L\Delta\theta \right)^{1/2}, \quad \theta = \theta_{\text{obs}} + \arccos \left(\frac{r_{\text{obs}}^2 + r_{\text{obs}}L\Delta\theta}{r_{\text{obs}} \left(r_{\text{obs}}^2 + L^2 + 2r_{\text{obs}}L\Delta\theta \right)^{1/2}} \right), \quad (1.13.26)$$

where $\Delta\theta = \theta_{\text{sl}} - \theta_{\text{obs}}$. In Eq. 1.13.24

$$L_{\text{max}}(\theta_{\text{sl}}) = \frac{r_o}{\sin \Delta\theta} \sin \left[\Delta\theta - \arcsin \left(\frac{r_{\text{obs}}}{r_o} \sin \Delta\theta \right) \right]. \quad (1.13.27)$$

According to Eq. 1.13.18 and Eq. 1.13.24–1.13.27 the expected angle distribution and time variations of gamma ray fluxes for local observer ($r_{\text{obs}} \leq r_o$) from interaction of solar energetic protons with solar wind matter will be determined by the energy spectrum of proton generation on the Sun $N_{\text{op}}(E_k)$, by the diffusion coefficient $\kappa_p(E_k)$, and parameters of solar wind in the period of event near the Earth orbit $n_1(\theta, \tilde{t})$ and $u_1(\theta, \tilde{t})$.

In the case of spherical symmetry we obtain

$$\Phi_{pH}^\gamma(E_\gamma, r_{\text{obs}}, \varphi, t) \approx F_{pH}^\gamma(E_\gamma, r = r_{\text{obs}} \sin \varphi, t) (\theta_{\text{max}} - \theta_{\text{min}}) r_{\text{obs}} \sin \varphi, \quad (1.13.28)$$

where φ is the angle between direction on the Sun and direction of observation,

$$\theta_{\text{max}} = \arccos(r_{\text{obs}} \sin \varphi / r_i), \quad (1.13.29)$$

$$\theta_{\text{min}} = \begin{cases} -\arccos(r_{\text{obs}} \sin \varphi / r_i) & \text{if } r_{\text{obs}} > r_i, \\ \varphi - \pi/2 & \text{if } r_{\text{obs}} \leq r_i. \end{cases} \quad (1.13.30)$$

For the great solar FEP event with the total energy in FEP about 10^{32} ergs Eq. 1.13.28 for $r_{\text{obs}} = 1$ AU gives

$$\begin{aligned} & \Phi_{pH}^\gamma(E_\gamma > 0.1 \text{ GeV}, r_{\text{obs}} = 1 \text{ AU}, \varphi, t) \\ & \approx \frac{6.7 \times 10^{-6}}{\sin \varphi} \left(\frac{t}{t_1} \right)^{-\frac{3}{2}} \exp \left(-\frac{3t_1 \sin^2 \varphi}{2t} \right) \text{ photon.cm}^{-2}.\text{sr}^{-1}.\text{sec}^{-1}. \end{aligned} \quad (1.13.31)$$

Expected fluxes of gamma rays with energy $E_\gamma > 0.1$ GeV during a large FEP event with total energy 10^{32} ergs for different directions of observation characterized by an angle φ from 2° up to 179° as a dependence upon t/t_1 are shown in Fig. 1.13.2–1.13.5.

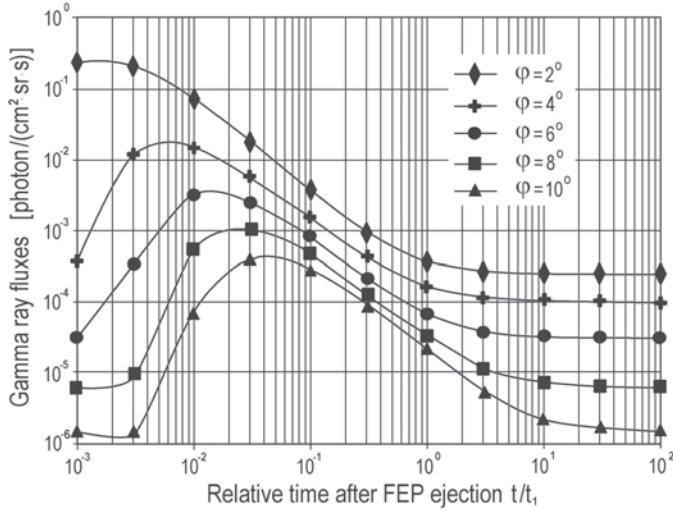


Fig. 1.13.2. Expected fluxes of gamma rays with energy more than 100 MeV during FEP event with total energy 10^{32} ergs for directions from $\varphi = 2^\circ$ to $\varphi = 10^\circ$ from the Sun as a dependence on t/t_1 , where t_1 was determined by Eq. (1.13.20).

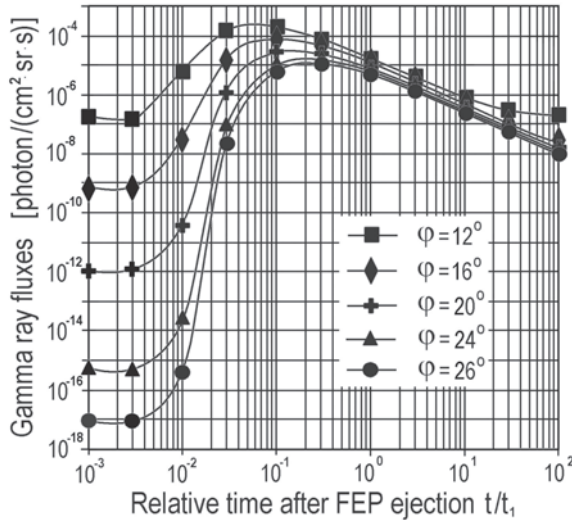


Fig. 1.13.3. The same as in Fig. 1.13.2, but for $\varphi = 12^\circ$ to $\varphi = 26^\circ$.

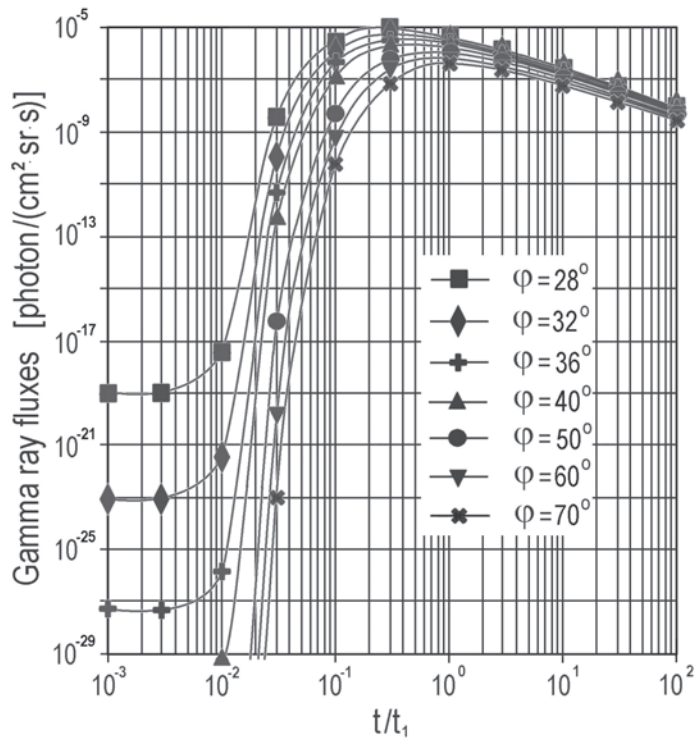


Fig. 1.13.4. The same as in Fig. 1.13.2, but for $\varphi = 28^\circ$ to $\varphi = 70^\circ$.

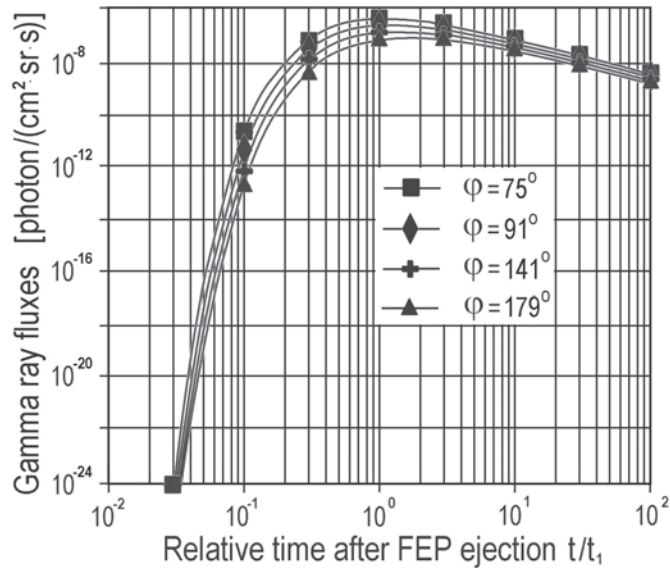


Fig. 1.13.5. The same as in Fig. 1.13.2, but for $\varphi = 75^\circ$ to $\varphi = 179^\circ$.

1.13.6. Gamma rays from interaction of FEP with stellar wind matter

Let us suppose that some observer is at the distance $r_{\text{obs}} \gg r_o$, where r_o is radius of a stellar-sphere. In this case

$$\Phi_{pH}^{\gamma}(E_{\gamma}, r_{\text{obs}}, t) = 2\pi r_{\text{obs}}^{-2} \int_{-\pi/2}^{\pi/2} \cos \theta d\theta \int_0^{r_o} r^2 dr F_{pH}^{\gamma}(E_{\gamma}, r, \theta, t), \quad (1.13.32)$$

where $F_{pH}^{\gamma}(E_{\gamma}, r, \theta, t)$ was determined by Eq. (1.13.18). For spherical symmetrical modes of FEP propagation and stellar wind matter distribution, we obtain

$$\Phi_{pH}^{\gamma}(E_{\gamma}, r_{\text{obs}}, t) = 4\pi r_{\text{obs}}^{-2} F_{pH}^{\gamma}(E_{\gamma}, r_1, t_1) (t/t_1)^{-1} r_1^3 \Phi\left(\frac{r_o}{r_1} (3t_1/t)^{1/2}\right) \text{ ph.cm}^{-2}\text{sec}^{-1}, \quad (1.13.33)$$

where $\Phi(x)$ is the probability function. For a flare star with total energy in an FEP event of 10^{36} ergs and $n_1(\theta, t) \approx 500 \text{ cm}^{-3}$ the expected emissivity at $t = t_1 = 10^3 \text{ sec}$ will be

$$F_{pp}^{\gamma}(E_{\gamma} > 0.1 \text{ GeV}, r) \approx 10^{14} r^{-2} \text{ photon.cm}^{-3}.\text{sec}^{-1}. \quad (1.13.34)$$

For this case Eq. (1.13.33) gives

$$\Phi_{pH}^{\gamma}(E_{\gamma} > 0.1 \text{ GeV}, r_{\text{obs}}, t) = 2 \times 10^{28} r_{\text{obs}}^{-2} (t/t_1)^{-1} \Phi\left(\frac{r_o}{r_1} (3t_1/t)^{1/2}\right) \text{ ph.cm}^{-2}\text{sec}^{-1}. \quad (1.13.35)$$

According to Eq. 1.13.35 for $t_1 = 10^3 \text{ sec}$ at $t = 10 \text{ sec}$ and 100 sec the value

$\frac{r_o}{r_1} (3t_1/t)^{1/2} \gg 1$ and $\Phi(x) \approx 1$, then at the distance $r_{\text{obs}} = 10^{19} \text{ cm}$ (about 3 pc) the expected gamma ray flux $\Phi_{pH}^{\gamma}(E_{\gamma} > 0.1 \text{ GeV}, r_{\text{obs}}, t)$ will be 2×10^{-8} and $2 \times 10^{-9} \text{ ph.cm}^{-2}\text{sec}^{-1}$, respectively. Eq. 1.13.35 shows that the total flux of gamma rays from stellar wind generated by FEP interaction with wind matter must fall inverse proportional with time and does not depend upon the details of the event. It is important for the separation of gamma ray generation in stellar wind from the direct generation in stellar flare.

1.13.7. Expected gamma ray fluxes from great FEP events

Estimates according to Eq. 1.13.31 show that in periods of great solar FEP events with total energy $\approx 10^{32}$ ergs the expected flux of gamma rays with energy $>$

100 MeV in the direction 2° from the Sun at $t/t_1=1/3$ reaches $\approx 10^{-3} \text{ photon.cm}^{-2}.\text{sr}^{-1}.\text{sec}^{-1}$. It means that according to Fig. 1.13.2 the expected flux in the same direction at $t/t_1 = 1/30$ reaches $\approx 2 \times 10^{-2} \text{ photon.cm}^{-2}.\text{sr}^{-1}.\text{sec}^{-1}$. In the direction 30° from the Sun (see Fig. 1.13.4) expected gamma ray fluxes are much smaller: the maximum will be at $t/t_1 = 1/3$ and reaches value only $\approx 10^{-5} \text{ photon.cm}^{-2}.\text{sr}^{-1}.\text{sec}^{-1}$. Expected gamma ray fluxes are characterized by great specific time variations, which depend from direction of observations relative to the Sun, total FEP flux from the source, parameters of FEP propagation (summarized in value of t_1), and properties of solar wind (see Fig. 1.13.1 for expected space time distribution of gamma ray emissivity and Fig. 1.13.2–1.13.5 for expected gamma ray fluxes).

1.13.8. On the possibility of monitoring gamma rays generated by FEP interactions with solar wind matter; using for forecasting of great radiation hazard

At energies above about 30 MeV pair production is the dominant photon interaction in most materials. In gamma ray **Pair Telescopes** this process is used to detect the arrival of a gamma ray photon through the electron-positron pair created in the detector. The space-telescopes **COS-B** and **EGRET** are well known (collection area of the latter about 1600 cm^2), which gave well energy and spatial resolution (see review in Weekes, 2000). These telescopes can detect objects with gamma ray fluxes of an energy bigger than 100 MeV at the detection limit of present day gamma ray telescopes of order 10^{-6} – $10^{-7} \text{ photon.cm}^{-2}.\text{sec}^{-1}$; these fluxes are several orders lower than expected from FEP interactions with solar wind matter (see Fig. 1.13.2–1.13.5). According to Gehrels and Michelson (1999) a further advance in the energy and spatial resolution is expected from the **Gamma ray Large Area Space Telescope (GLAST)**. In this telescope solid state detectors will be use as the tracking material instead of a gas filled chamber. It is planned to be launched in 2006. This telescope will allow for improved energy resolution (10% resolution) and spatial location (0.5-5.0 arc minutes). Figures 1.13.2–1.13.5 show that present gamma ray telescopes might measure expected gamma ray fluxes in periods of great FEP events. These observations of gamma rays generated in interactions of FEP with solar wind matter can give important information about 3D-distribution of the solar wind as well as about properties of solar FEP generation and propagation parameters. Let us note that the model of isotropic diffusion may be used only after about 15–30 minutes after an FEP event starting on the Earth, when the expected gamma ray fluxes are not so big, but measurable with present gamma ray telescopes and those available/planned in the near future. To obtain more exact information about solar wind properties as well as about the mode of FEP generation and propagation during the beginning stage, in the first several minutes after an FEP event starting on the Earth and starting even before

this (when it is expected the biggest fluxes of gamma rays in directions of a few degrees from the Sun), – it is necessary to recalculate the expected space-time distribution of gamma ray emissivity and gamma ray fluxes in the frame of more real and more complicated model of solar CR propagation based on the theory of anisotropic diffusion and kinetic equation. Let us note that these observations can be used for forecasting of great radiation hazard in the Earth's environment (Dorman, 2001f).

1.14. Gamma ray generation in space plasma by interactions of galactic CR with solar and stellar winds

1.14.1. The matter of problem and the main three factors

The generation of gamma rays by the interaction of galactic CR with solar and stellar winds matter was considered in Dorman (1996, 1997b, 2001b). These was considered the situation with gamma ray generation in the interplanetary space by galactic CR and expected time variations of gamma ray fluxes dependent on the direction of the observations and in connection with solar activity (SA) cycle (about 11 years from minimum to minimum of SA) and with the solar magnetic cycle (with a period of about 22 years including odd and even SA cycles, and periods of reversal general solar magnetic field near both maximums of SA). By data obtained from investigations of the hysteresis phenomenon in dependence of galactic CR intensity from solar activity level it was determined the change of CR density distribution in the Heliosphere during solar cycle as a dependence on particle energy. On the basis of observational data and investigations of CR nonlinear processes in the Heliosphere we also determined the space-time distribution of solar wind matter. Then we calculate the generation of gamma rays by the decay of neutral pions generated in the nuclear interactions of modulated galactic CR with solar wind matter and determine the expected space-time distribution of gamma-ray emissivity. On the basis of these results we calculate the expected time variation of the angle distribution and spectra of gamma ray fluxes generated by interaction of modulated galactic CR with solar wind matter for local (inside the Heliosphere) and distant observers (for stellar winds).

The space-time distribution of gamma ray emissivity will be determined mainly by 3 factors:

The 1st factor – space-time distribution of galactic CR in the Heliosphere, their energetic spectrum and chemical composition; for this distribution nonlinear collective effects of galactic CR pressure and kinetic stream instability can be important (Berezinsky et al., M1990; Dorman, Ptuskin, and Zirakashvili, 1990; Zirakashvili et al., 1991; Dorman, 1995b; Le Roux and Fichtner, 1997; see also below, Chapter 3).

The 2nd factor – the solar wind matter distribution in space and its change during the solar activity cycle; for this distribution pressure and kinetic stream instability

of galactic CR also be important will (Dorman, 1995b; Le Roux and Fichtner, 1997; see also below, Chapter 3).

The 3rd factor – properties of galactic CR interaction with solar wind matter accompanied with gamma ray generation through the decay of neutral pions (Stecker, M1971; Dermer, 1986a,b; see above, Section 1.12.2).

After consideration of these 3 factors we calculate the expected space-time distribution of gamma ray emissivity, and the expected fluxes of gamma rays for measurements on the Earth's orbit as a dependence on the level of solar activity for different directions of gamma ray observations. We also calculate the expected gamma ray fluxes for different distances from the Sun inside the Heliosphere (local observations) and outside (distant observations). We expect that the same 3 factors will play an important role for gamma ray generation by galactic CR in stellar winds, but for some types of stars the speed of lost matter is several orders higher than from the Sun.

Observations of gamma rays generated in interactions of galactic CR with solar wind matter can give valuable information about the 3D-distribution of solar wind matter as well as on properties of galactic CR global modulation and its propagation parameters. Especially important will be observations of gamma rays generated in interactions of galactic CR with stellar wind matter. It will be shown that in this case important information on galactic CR modulation in the stellar-sphere can be obtained as well as information about stellar activity and stellar wind.

1.14.2. The 1st factor: galactic CR space-time distribution in the Heliosphere

The problem of galactic CR propagation through interplanetary space as well as modulation of its intensity and energetic spectrum in the Heliosphere (the 1st factor) was reviewed in Dorman (M1957, M1963a,b, M1975b), and taking into account CR nonlinear processes in Dorman (1995b). According to this research the convection-diffusion modulation of energy spectra of proton component of galactic CR can be described in the quasy-stationary approximation of the spherical symmetrical geometry (Parker, 1958, M1963; Dorman, 1959) as

$$N_p(r, E_k) = N_p(E_k) \exp \left(- \frac{\gamma + 2}{3} \int_r^{r_0} \frac{u(r, t) dr}{\kappa_p(r, E_k, t)} \right), \quad (1.14.1)$$

where according to Simpson (1983)

$$N_p(E_k) = 2.2 \times (E_k + m_p c^2)^{-\gamma} \text{ proton} \cdot \text{sr}^{-1} \cdot \text{cm}^{-2} \cdot \text{s}^{-1} \cdot \text{GeV}^{-1} \quad (1.14.2)$$

is the differential energy spectrum of proton component of galactic CR outside the Heliosphere, the slope of the primary spectrum $\gamma = 2.75$, $u(r, t)$ is the solar wind velocity, and

$$\kappa_p(r, E_k, t) = \Lambda_p(r, E_k, t) v(E_k) / 3 \quad (1.14.3)$$

is the diffusion coefficient, $\Lambda_p(r, E_k, t)$ is the transport path for particle scattering, $v(E_k)$ is the particle velocity in dependence of kinetic energy per nucleon E_k :

$$v(E_k) = c \left(1 - \left(1 + E_k / m_n c^2 \right)^{-2} \right)^{1/2}. \quad (1.14.4)$$

According to numeral experimental data and theoretical investigations $\Lambda_i(E_k)$ has a wide minimum in the region 0.1–0.5 GeV/nucleon and increases with energy decreasing lower than this region as about $\propto E_k^{-1}$ (caused by the ‘tunnel’ effect for particles with curvature radius in the interplanetary magnetic field smaller than the smallest scale of hydromagnetic turbulence, see in Dorman, M1975a, and above in Section 1.9) as well as with energy increasing over this interval as $\propto E_k^\gamma$, where γ depends from the spectrum of the turbulence, and usually increases from 0 up to about 1 for high energy particles of a few GeV/nucleon and then up to about 2 for very high energy particles with radius of curvature in the IMF larger than the biggest scale of magnetic inhomogeneities in the IMF (according to investigations of galactic CR modulation in the Heliosphere it happens at $E_k \geq 15 \div 20$ GeV/nucleon). For calculations of expected space-time distribution of gamma ray emissivity we try to describe this dependence approximately as

$$\Lambda_i(E_k) \approx \Lambda_i(W, r, t) \left(\frac{E_1}{E_k} + \frac{E_k}{E_2} + \left(\frac{E_k}{E_3} \right)^2 \right). \quad (1.14.5)$$

To determine the parameters E_1, E_2, E_3 in Eq. 1.14.5 we used observations of solar CR events as well as observations of galactic CR modulation in interplanetary space. The time-dependence of galactic CR primary fluxes for effective rigidities $R=2, 5, 10$ and 25 GV were found in Belov et al. (1988, 1990) on the basis of ground measurements (muon and neutron components) as well as measurements in the stratosphere by balloons and in space by satellites and spacecrafts. The residual modulation (relative to the flux out of the Heliosphere) for $R \approx 10$ GV in the minimum and maximum of solar activity was determined as 6 and 24 % (what is in good agreement with results from hysteresis effect obtained on the basis of neutron monitor data (Dorman and Dorman, 1967a,b, 1968; Dorman, M1975b; Alania and Dorman, M1981; Alania et al., M1989; Dorman, Villorresi et al., 1997a,b). According to convection-diffusion model of CR cycle modulation, the slope of the residual spectrum $\Delta D(R) / D_0(R) \propto R^{-\gamma}$ reflects the dependence

$$\Lambda(R) \propto (\Delta D(R)/D_0(R))^{-1} \propto R^\gamma. \quad (1.14.6)$$

In Belov et al. (1988, 1990) the spectral index γ was determined as $\gamma \approx 0.4$ at 2–5 GV, $\gamma \approx 1.1$ at 5–10 GV, and $\gamma \approx 1.6$ at 10–25 GV. Eq. 1.14.5 will be in agreement with these results and with data on FEP events in smaller energy range if we choose $E_1 \approx 0.05$ GeV/nucleon, $E_2 \approx 2$ GeV/nucleon, and $E_3 \approx 5$ GeV/nucleon.

The dependence of the transport path from the level of solar activity is characterized by $\Lambda_i(W)$. This parameter can be determined from investigations of galactic CR modulation in interplanetary space on the basis of observations by neutron monitors and muon telescopes for several solar cycles. According to Dorman and Dorman (1967a,b, 1968) $\Lambda_i(W) \propto W^{-1/3}$ for the period of high solar activity and $\Lambda_i(W) \propto W^{-1}$ for the period of low solar activity. According to Dorman and Dorman (1967a,b, 1968), Dorman (M1975b), Dorman, Villaresi et al. (1997a,b), the hysteresis phenomenon in the connection of long-term CR intensity variation with solar activity cycle can be explained well by the analytical approximation of this dependence, taking into account the time lag r/u of electromagnetic processes in the interplanetary space relative to solar activity phenomena on the Sun caused these processes:

$$\Lambda_i(W, r, t) = \Lambda_i(W_{\max}) \times \left(W \left(t - \frac{r}{u} \right) / W_{\max} \right)^{-\frac{1}{3} - \frac{2}{3} \left(1 - W \left(t - \frac{r}{u} \right) / W_{\max} \right)}, \quad (1.14.7)$$

where $W_{\max} \approx 200$ and $\Lambda_i(W_{\max}) \approx 10^{12}$ cm.

In the some rough approximation the convection-diffusion global modulation described by Eq. 1.14.1 can be determined as

$$N_{gp}(r, E_k) \approx N_{gp}(E_k) \exp \left(- \frac{B(t)}{R^\gamma \beta} \left(1 - \frac{r}{r_o} \right) \right), \quad (1.14.8)$$

where r_o is the size of the modulation region, the parameter γ determines the dependence $\Lambda(R) \propto R^\gamma$,

$$R = cp/Ze = \left(E_k^2 + 2E_k m_p c^2 \right)^{1/2} / Ze \quad (1.14.9)$$

is the particle rigidity (in GV), and

$$\beta = v/c = \left(E_k^2 + 2E_k m_p c^2\right)^{1/2} / \left(E_k + m_p c^2\right) \quad (1.14.10)$$

is the particle velocity for protons in units of light speed c , and $B(t)$ is the parameter of modulation. According to Dorman and Dorman (1967a,b, 1968), Dorman (M1975b), Zusmanovich (M1986), Dorman, Villaresi et al. (1997a,b), the parameter $B(t)$ changes with solar activity in the first approximation inverse proportional to Λ_i . Near the minimum of solar activity $B_{\min} \approx (0.3 \div 0.4)$ GV. In the maximum of solar activity the modulation became higher and $B_{\max} \approx (1.6 \div 2.5)$ GV for different solar cycles in dependence of direction of solar general magnetic field and sign of CR particles charge (taking into account drift effects). For α -particles in galactic CR the space distribution of the modulated spectrum will be

$$N_{g\alpha}(r, E_k) \approx N_{g\alpha}(E_k) \exp\left(-\frac{B(t)}{R^\gamma \beta} \left(1 - \frac{r}{r_o}\right)\right), \quad (1.14.11)$$

where $B(t)$, R , γ , and β are the same as in Eq. 1.14.8 for protons, and $N_{g\alpha}(E_k)$ is the α -particle spectrum outside of the Heliosphere, which according to Simpson (1983) is

$$N_{g\alpha}(E_k) = 0.07 \left(E_k + m_p c^2\right)^{-2.75} \text{ particle.sr}^{-1}.\text{cm}^{-2}.\text{s}^{-1}(\text{GeV/nucleon})^{-1}, \quad (1.14.12)$$

and E_k is the kinetic energy of α -particles per nucleon. For heavier particles with $A \sim 2Z$ we have an equation, similar to Eq. 1.14.11, but

$$R = cp/Ze = (A/Ze) \left(E_k^2 + 2E_k m_p c^2\right)^{1/2}. \quad (1.14.13)$$

Described above is the modulation of galactic CR caused by convection-diffusion processes. To this modulation it is necessary to add modulation caused by drift effects which can be determined mainly by the value of the tilt angle between the neutral current sheet and the solar equatorial plane (Burger and Potgieter, 1999; Dorman, 2001c; Dorman, Dorman et al., 2001). This type of modulation changes the sign in periods of solar magnetic field reversal (near maxima of solar activity). The amplitude of drift modulation A_{dr} as well as dimension of the modulation region r_o where determined in Dorman (2001c) as average for even and odd cycles on the basis of data for four solar cycles 19–22 (see Fig. 1.14.1).

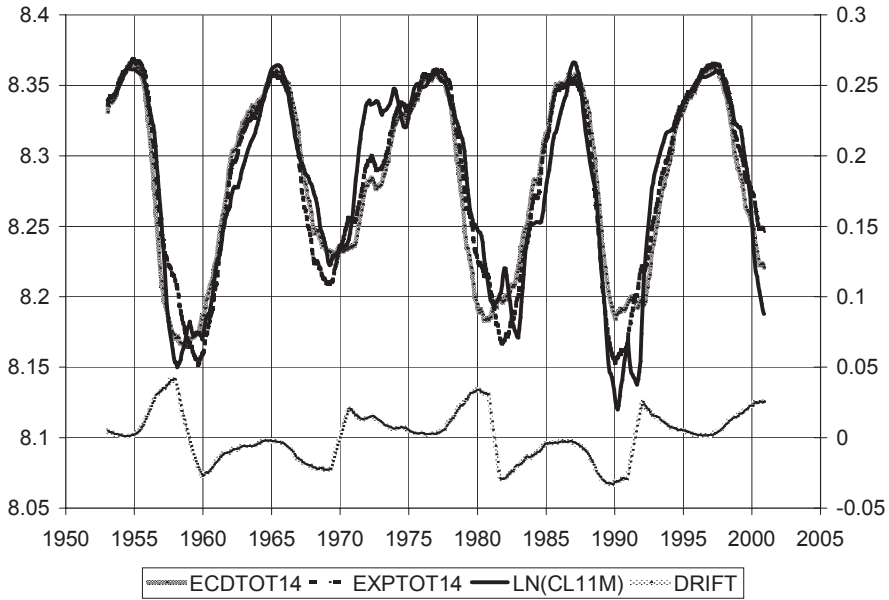


Fig. 1.14.1. Observed long term modulation of galactic CR in 1953–2000 according to Climax NM data (effective rigidity $R_{\text{ef}} \approx 10 \text{ GV}$, curve LN(CL11M)) in comparison with that expected (curve EXPTOT14) at $r_o = 14 \text{ av. month} \times u_{\text{av}} \approx 108 \text{ AU}$ (for this period $u_{\text{av}} \approx 7.7 \text{ AU/av. month}$). Convection-diffusion modulation (curve ECDTOT14) and drift modulation (curve DRIFT) are also shown. Left, Y-scale for natural logarithm of Climax NM counting rate; right, Y-scale for drift modulation. Interval between two horizontal lines corresponds to 5% variation. According to Dorman (2001c).

In Dorman, Dorman et al. (2001) the drift modulation was determined for cycle 22 as a dependence on effective particle rigidity R_{ef} . For galactic electrons the modulation in the Heliosphere will be determined also by Eq. 1.14.1 or Eq. 1.14.8, but the drift effects will be opposite in comparison with protons and α -particles.

1.14.3. The 2nd factor: space-time distribution of solar wind matter

The detail information on this factor we considered above, in Section 1.13.3 (see Eq. 1.13.14 and Eq. 1.13.15).

1.14.4. The 3rd factor: gamma ray generation by galactic CR in the Heliosphere

According to Stecker (M1971), Dermer (1986a,b) the neutral pion generation by nuclear interactions of energetic protons with hydrogen atoms (reactions $p + p \rightarrow \pi^0 + \text{anything}$) will be determined by

$$F_{pH}^{\pi}(E_{\pi}, r, \theta, t) = 4\pi m(r, \theta, t) \int_{E_{k \min}(E_{\pi})}^{\infty} dE_k N_p(E_k, r, t) \langle \zeta \sigma_{\pi}(E_k) \rangle (dN(E_k, E_{\pi})/dE_{\pi}), \quad (1.14.14)$$

where $n(r, \theta, t)$ is determined by Eq. 1.13.14, $E_{k \min}(E_{\pi})$ is the threshold energy for pion generation, $N_p(E_k, r, t)$ is determined by Eq. 1.14.1, $\langle \zeta \sigma_{\pi}(E_k) \rangle$ is the inclusive cross section for reactions $p + p \rightarrow \pi^0 + \text{anything}$ (see Section 1.12.2), and $\int_0^{\infty} (dN(E_k, E_{\pi})/dE_{\pi}) dE_{\pi} = 1$.

Gamma ray emissivity caused by nuclear interactions of galactic CR protons with solar wind matter will be determined according to Stecker (M1971), Dermer (1986a,b) by

$$F_{pH}^{\gamma}(E_{\gamma}, r, \theta, t) = 2 \int_{E_{\pi \min}(E_{\gamma})}^{\infty} dE_{\pi} (E_{\pi}^2 - m_{\pi}^2 c^4)^{-1/2} F_{pH}^{\pi}(E_{\pi}, r, \theta, t), \quad (1.14.15)$$

where $E_{\pi \min}(E_{\gamma}) = E_{\gamma} + m_{\pi}^2 c^4 / 4 E_{\gamma}$.

Let us introduce Eq. 1.14.2, Eq. 1.14.8-1.14.10 and Eq. 1.13.15 in Eq. 1.14.14 and then the result obtained in Eq. 1.14.15:

$$F_{pH}^{\gamma}(E_{\gamma}, r, \theta, t) = 2.2 \left(\frac{8\pi m_1(\theta, t) r_1^2}{r^2 (1 - b(r/r_o))} \right) \int_{E_{\pi \min}(E_{\gamma})}^{\infty} dE_{\pi} (E_{\pi}^2 - m_{\pi}^2 c^4)^{-1/2} \times \\ \int_{E_{k \min}(E_{\pi})}^{\infty} dE_k \langle \zeta \sigma_{\pi}(E_k) \rangle (E_k + m_p c^2)^{-2.75} \exp \left(- \frac{B(t) (E_k + m_p c^2) e}{(E_k^2 + 2E_k m_p c^2)^{(\gamma+1)/2}} \left(1 - \frac{r}{r_o} \right) \right). \quad (1.14.16)$$

The expected gamma ray emissivity distribution from the interaction of α -particles with solar wind matter will be determined by introducing Eq. 1.14.9-1.14.12, Eq. 1.13.15 in Eq. 1.14.14 and then in Eq. 1.14.15:

$$F_{\alpha H}^{\gamma}(E_{\gamma}, r, \theta, t) = 0.07 \left(\frac{8\pi m_1(\theta, t) r_1^2}{r^2 (1 - b(r/r_o))} \right) \int_{E_{\pi \min}(E_{\gamma})}^{\infty} dE_{\pi} (E_{\pi}^2 - m_{\pi}^2 c^4)^{-1/2} \times \\ \int_{E_{k \min}(E_{\pi})}^{\infty} dE_k \langle \zeta \sigma_{\pi}(E_k) \rangle (E_k + m_p c^2)^{-2.75} \exp \left(- \frac{B(t) (E_k + m_p c^2) (e/2)}{(E_k^2 + 2E_k m_p c^2)^{(\gamma+1)/2}} \left(1 - \frac{r}{r_o} \right) \right). \quad (1.14.17)$$

For the radial extended of solar or stellar wind the space-time distributions of gamma ray emissivity according to Eq. 1.14.16 will be mainly determined by the function

$$F_{pH}^\gamma(E_\gamma, r, t) = F_{pH}^\gamma(E_\gamma) \frac{n_1(\theta, t)}{n_o} \left(1 - \frac{br}{r_o}\right)^{-1} \left(\frac{r_1}{r}\right)^2 \exp\left(-A(E_\gamma, t) \left(1 - \frac{r}{r_o}\right)\right), \quad (1.14.18)$$

where $F_{pH}^\gamma(E_\gamma)$ is the emissivity spectrum from galactic CR protons in the interstellar space (as background emissivity from interstellar matter with density n_o according to Dermer, 1986a,b), and $n_1(\theta, t)$ is the density of solar or stellar wind on the latitude θ on the distance $r_1 = 1$ AU from the star. In Eq. 1.14.18

$$A(E_\gamma, t) = B(t) / (R^\gamma \beta)_{\text{ef}}(E_\gamma) \quad (1.14.19)$$

and $(R^\gamma \beta)_{\text{ef}}(E_\gamma)$ is some effective value of $R^\gamma \beta$ for particles responsible for gamma ray generation with energy E_γ . According to Dermer (1986a,b) the expected gamma ray emissivity from all particles in galactic CR $F^\gamma(E_\gamma, r, t)$ will increase in about 1.45 times if we take into account also α -particles and heavier particles in galactic CR:

$$F^\gamma(E_\gamma, r, t) = 1.45 F_{pH}^\gamma(E_\gamma) \frac{n_1(\theta, t)}{n_o} \left(1 - \frac{br}{r_o}\right)^{-1} \left(\frac{r_1}{r}\right)^2 \exp\left(-A(E_\gamma, t) \left(1 - \frac{r}{r_o}\right)\right). \quad (1.14.20)$$

1.14.5. Expected angular distribution of gamma ray fluxes from solar wind

Let us assume that the observer is inside the Heliosphere, at a distance $r_{\text{obs}} \leq r_o$ from the Sun and at a helio-latitude θ_{obs} . We can determine the line of sight of observation by the angle θ_{ls} , computed from the equatorial plane from the anti-Sun direction to the North. In this case the expected angular distribution and time variations of gamma ray fluxes for a local observer from interaction of galactic CR with solar wind matter will be

$$\Phi^\gamma(E_\gamma, r_{\text{obs}}, \theta_{\text{ls}}, t) = 1.45 \times \int_0^{L_{\text{max}}(\theta_{\text{ls}})} dL F_{pH}^\gamma(E_\gamma, L(r_{\text{obs}}, \theta_{\text{ls}}), t). \quad (1.14.21)$$

In Eq. 1.14.21

$$F_{pH}^{\gamma}(E_{\gamma}, L(r_{\text{obs}}, \theta_{\text{ls}}), t) = F_{pH}^{\gamma}(E_{\gamma}, r, \theta, t) \quad (1.14.22)$$

determined by Eq. 1.14.16, and

$$r = r(L) = (r_{\text{obs}}^2 + L^2 + 2r_{\text{obs}}L\Delta\theta)^{1/2}, \quad \Delta\theta = \theta_{\text{ls}} - \theta_{\text{obs}}, \quad (1.14.23)$$

$$\theta = \theta(L) = \theta_{\text{obs}} + \arccos\left(\left(r_{\text{obs}}^2 + r_{\text{obs}}L\Delta\theta\right) / \left(r_{\text{obs}}(r_{\text{obs}}^2 + L^2 + 2r_{\text{obs}}L\Delta\theta)^{1/2}\right)\right), \quad (1.14.24)$$

$$L_{\text{max}}(\theta_{\text{ls}}) = r_o \sin\left[\Delta\theta - \arcsin\left(\frac{r_{\text{obs}}}{r_o} \sin\Delta\theta\right)\right] / \sin\Delta\theta. \quad (1.14.25)$$

Let us consider the spherically symmetric case. For the spherically symmetric problem for observation at a distance r_{obs} in the direction determined by the angle φ between direction of observation and the direction to the Sun, on the basis of Eq. 1.14.21–1.14.25 we obtain

$$\Phi^{\gamma}(E_{\gamma}, r_{\text{obs}}, \varphi, t) = 1.45 F_{pH}^{\gamma}(E_{\gamma}) (n_1(t)/n_o) G(r_{\text{obs}}, \varphi, b, A(E_{\gamma}, t)), \quad (1.14.26)$$

where $A(E_{\gamma}, t)$ is determined by Eq. 1.14.19, and

$$G = r_1^2 (r_{\text{obs}} \sin\varphi)^{-1} \int_{\theta_{\text{min}}}^{\theta_{\text{max}}} \left(1 - \frac{kr_{\text{obs}} \sin\varphi}{r_o \cos\theta}\right)^{-1} \exp\left(-A(E_{\gamma}, t) \left(1 - \frac{kr_{\text{obs}} \sin\varphi}{r_o \cos\theta}\right)\right) d\theta, \quad (1.14.27)$$

where

$$\theta_{\text{max}} = \arccos(r_{\text{obs}} \sin\varphi / r_o), \quad \theta_{\text{min}} = -(\pi/2) + \varphi. \quad (1.14.28)$$

We calculate Eq. 1.14.27 numerically for $r_{\text{obs}} = r_1 = 1 \text{ AU} = 1.5 \times 10^{13} \text{ cm}$, $b = 0.13, 0.30, 0.45$, $\varphi = 2^\circ, 10^\circ, 45^\circ, 90^\circ$, and 178° , and $A(E_{\gamma}, t) = 0$ (no modulation), and $A(E_{\gamma}, t) = 0.2, 0.4, 0.8, 1.6, 3.2, 6.4$ and 12.8 . The case $E_{\gamma} \geq 100 \text{ MeV}$ corresponds to $R^{\gamma}\beta \geq 2 \text{ GV}$ which means $A(E_{\gamma}, t) \leq 0.2$ at the minimum of solar activity and $A(E_{\gamma}, t) \leq 1.2$ at the maximum of solar activity. The dependence on b is sufficient only for $A(E_{\gamma}, t) \geq 6.4$; in other cases G is about the same for $b = 0.13, 0.30$, and 0.45 . In Table 1.14.1 we show values $G(r_{\text{obs}}, \varphi, k, A)$, sufficient for cases

of generation gamma rays with $E_\gamma \geq 100$ MeV (corresponds to $A \leq 1.2$) and for $E_\gamma < 100$ MeV ($A > 1.2$).

Table 1.14.1. The parameter $G(r_{\text{obs}}, \varphi, b, A)$ (in cm) according to Eq. 1.14.27 for $r_{\text{obs}} = 1$ AU, $b = 0.3$ as a dependence on φ and of $A(E_\gamma, t)$ (characterize both the effective E_γ and the level of solar activity).

φ	$A = 0$	$A = 0.2$	$A = 0.4$	$A = 0.8$	$A = 1.6$	$A = 3.2$
2°	1.3E+15	1.1E+15	9.0E+14	6.0E+14	2.7E+14	5.5E+13
10°	2.6E+14	2.1E+14	1.7E+14	1.2E+14	5.3E+13	1.1E+13
45°	5.0E+13	4.1E+13	3.4E+13	2.3E+13	1.1E+13	2.3E+12
90°	2.4E+13	2.0E+13	1.6E+13	1.1E+13	5.3E+12	1.2E+12
178°	1.5E+13	1.2E+13	1.0E+13	7.2E+12	3.5E+12	8.7E+11

1.14.6. Gamma ray fluxes from stellar winds

In this case $r_{\text{obs}} \gg r_o$, and the expected gamma ray fluxes will be

$$\begin{aligned}
 \Phi^\gamma(E_\gamma, r_{\text{obs}}, t) &= 1.45 \times 2\pi r_{\text{obs}}^{-2} \int_{-\pi/2}^{\pi/2} \cos \theta d\theta \int_0^{r_o} r^2 F_{pH}^\gamma(E_\gamma, r, \theta, t) dr \\
 &\approx 2.9\pi r_{\text{obs}}^{-2} F_{pH}^\gamma(E_\gamma) \frac{n_1(t)}{n_o} r_1^2 r_o J(b, A(E_\gamma, t)), \quad (1.14.29)
 \end{aligned}$$

where

$$J(b, A) = \exp(-A) \times \left[\left(1 + \frac{A}{2} + \frac{A^2}{6} + \dots \right) + b \left(\frac{1}{2} + \frac{A}{3} + \dots \right) + b^2 \left(\frac{1}{3} + \frac{A}{4} + \dots \right) + \dots \right]. \quad (1.14.30)$$

Values of $J(b, A)$ for $E_\gamma \geq 100$ MeV are shown in Table 1.14.2.

Table 1.14.2. Parameter $J(b, A)$ in dependence of b and A .

b	$A = 0$	$A = 0.1$	$A = 0.2$	$A = 0.3$	$A = 0.4$	$A = 0.6$	$A = 0.8$
0.13	1.07	1.02	0.97	0.93	0.88	0.80	0.73
0.30	1.18	1.13	1.08	1.03	0.98	0.89	0.81
0.45	1.29	1.24	1.18	1.13	1.08	0.98	0.90

1.14.7. Summary of main results and discussion

The results obtained allowed the estimate of the expected distribution of gamma ray emissivity in the Heliosphere or in some stellar-sphere, to estimate the

expected fluxes of gamma rays and their time variations for observations of gamma rays from the solar wind or from the nearest stellar winds. According to Eq. 1.14.20 gamma ray emissivity in the interplanetary space will be bigger than in the interstellar space only in the inner part of the Heliosphere at $r < 3$ AU and with decreasing of r gamma ray emissivity will increase as about $\propto r^{-2}$. In the main part of the Heliosphere gamma ray emissivity will be many times smaller than in the interstellar space. This means that the Heliosphere, as well as many stellar-spheres, can be considered as holes in the galactic background gamma ray emissivity distribution.

According to Eq. 1.14.26 and Table 1.14.1 the biggest expected gamma ray flux from the solar wind ($n_1 \approx 5 \text{ cm}^{-3}$) in the direction 2° from the Sun and near the minimum of solar activity will be about the same as from interstellar matter ($n_o \approx 0.1 \text{ cm}^{-3}$) with a size $\approx 10^{17} \text{ cm}$ (at about two orders more than the size of the Heliosphere). This expected gamma ray flux decreases by time several times during maximum of solar activity and decreases by about two orders with increasing angle φ up to the opposite direction from the Sun (see Table 1.14.2). Let us compare this variable gamma ray flux with the background gamma ray flux from galactic CR interactions with interstellar matter. In the direction perpendicular to the disc plane background gamma ray flux is formatted on the distance about 200 pc $\approx 6 \times 10^{20} \text{ cm}$, therefore this background gamma ray flux will be about 6×10^3 times more than the largest expected from solar wind gamma ray flux in the direction 2° from the Sun. From this, it follows that it is now not possible to measure gamma rays from the solar wind generated by galactic CR modulated by the solar activity cycle. But in the future, with the increasing of accuracy of gamma ray telescopes and by using the big variability of this very weak gamma ray source, it will be possible to investigate this phenomenon and obtain some additional information about the solar wind's matter distribution and galactic CR modulation in the Heliosphere.

If measurements of gamma rays from the solar wind generated by galactic CR are made outside the Heliosphere, the following effect can be observed: in directions not far from the Sun this gamma ray flux will be about two orders larger than from interstellar matter of the same size as the Heliosphere, but in measurements at large angles from the Sun gamma ray flux much smaller than expected from interstellar matter of the same size as the Heliosphere will be observed. According to Eq. 1.14.29 the ratio of total gamma ray flux from the solar wind or from stellar wind Φ_{SW}^γ to the gamma ray flux from the same volume of interstellar medium Φ_{IM}^γ will be

$$\Phi_{SW}^\gamma / \Phi_{IM}^\gamma \approx 1.5 n_1 r_1^2 J(b, A(E_\gamma, t)) / (n_o r_o^2), \quad (1.14.31)$$

which will be changed in time according to Table 1.14.2 with the solar or stellar cycle and depends on E_γ . Table 1.14.2 shows that $J(b, A)$ for the Heliosphere increases from 0.73 to 1.29 with decreasing $A(E_\gamma, t)$ from 0.8 to 0, and with increasing b from 0.13 to 0.45, so that for the rough estimates we can put $J(b, A) \approx 1$. In this case Eq. 1.14.31 for the solar wind ($r_1 = 1 \text{ AU}$, $r_o \approx 100 \text{ AU}$, $n_1 \approx 5 \text{ cm}^{-3}$, $n_o \approx 0.1 \text{ cm}^{-3}$) gives $\approx 7.5 \times 10^{-3}$. It means that the Heliosphere can be considered as a deep variable hole in the background gamma ray emissivity distribution.

The value of ratio (Eq. 1.14.31), stellar wind density $n_{1\text{St}}$ at the distance 1 AU from the star and dimension of the hole $r_{o\text{St}}$ are determined mainly by the value of mass loosing rate by star $\dot{M}_{\text{St}}$ (for the Sun $\dot{M}_{\text{Sun}} \approx -10^{-14} M_{\text{Sun}}/\text{year}$) and speed of wind $u_{1\text{St}}$ on the distance 1 AU from the star:

$$n_{1\text{St}} = n_1 u_1 \dot{M}_{\text{St}} / (u_{1\text{St}} \dot{M}_{\text{Sun}}), \quad r_{o\text{St}} = r_o (u_{1\text{St}} \dot{M}_{\text{St}} / u_1 \dot{M}_{\text{Sun}})^{1/2}, \quad (1.14.32)$$

from which it follows that

$$\Phi_{\text{StW}}^\gamma / \Phi_{SW}^\gamma = u_1^2 / u_{1\text{St}}^2. \quad (1.14.33)$$

Here we assume that conditions around the star (CR and magnetic field pressure) are the same as near the Sun. Eq. 1.14.32 shows that the dimension of holes in gamma ray emissivity increases with increasing of stellar wind speed and the rate of mass loosing in degree $1/2$. The depths of the gamma ray hole according to Eq. 1.14.33 increases with stellar wind velocity in square. For example, for a star with $\dot{M}_{\text{St}} \approx 10^{-8} M_{\text{Sun}}/\text{year}$ and $u_{1\text{St}} \approx 10^8 \text{ cm/sec}$, we obtain $n_{1\text{St}} \approx 2 \times 10^6 \text{ cm}^{-3}$, $r_{o\text{St}} \approx 1.5 \times 10^5 \text{ AU}$, and $\Phi_{\text{StW}}^\gamma / \Phi_{SW}^\gamma \approx 1/6$. Let us note that in this case observation along a line near the star (see Table 1.14.1) will give a gamma ray flux corresponding to the background gamma rays from about $3 \times 10^{22} \text{ cm}$ of interstellar medium, which is much bigger than the gamma ray background in all directions from the galactic disc; this variable gamma ray flux can be measured by present day gamma ray telescopes (described in Gehrels and Michelson, 1999; Weekes, 2000).

1.15. On the interaction of extra high energy gamma rays with the magnetic fields of the Sun and the planets

1.15.1. The matter of the problem

In the last 10–15 years several CR events with energies above the predicted Greisen-Zatsepin-Kuzmin cutoff at $\sim 4 \times 10^{19}$ eV have been detected (Efimov et al., 1991, Bird et al., 1994, Hayashida et al. 1994). It is not excluded that some of these events are initiated by extra high energy (EHE) photons which content in CR above $\sim 10^{19}$ eV (McBreen and Lambert, 1981; Aharonian et al., 1991; Karakuła et al., 1994; Karakuła and Tubek, 1995; Karakuła and Bednarek, 1995; Karakuła, 1996; Stanev and Vankov, 1996; Kasahara, 1997; Bednarek, 1999); they also may be owed to the interaction of CR hadrons with the microwave background radiation (Wdowczyk and Wolfendale, 1990; Halzen et al., 1990), or by the decay of massive particles, such as Higgs and Gauge bosons predicted by some more exotic theories (Bhattacharjee et al., 1998). Gamma rays with energies above 10^{19} eV may develop magnetic $e^{\pm}$ pair cascades in the dipole magnetic field of compact objects in the Solar system: the Earth, the Sun, Jupiter, and others. The cascades initiated by EHE photons in the Earth's dipole magnetic field have been considered by McBreen and Lambert (1981) and Aharonian et al. (1991). The EHE CR events with energies $> 10^{20}$ eV were analyzed under the hypothesis of their photonic origin by Karakuła et al. (1994), Karakuła and Tubek (1995), Karakuła and Bednarek (1995), Karakuła (1996), Stanev and Vankov (1996), Kasahara (1997), Bednarek (1999). In Bednarek (1999) there are discussed the observational consequences of the cascading of EHE gamma rays in the magnetic field of the Sun. The magnetic field of the Sun is about an order of magnitude stronger than that of the Earth, for which photons have to have energies above $\sim 5 \times 10^{19}$ eV in order to cascade efficiently. Therefore detection of secondary photons from cascades initiated in the magnetic field of the Sun may allow investigation of the photon content in the EHE CR spectrum at energies about an order of magnitude lower, provided that a large enough detector of CR showers is available. According to Bednarek (1999) the content of EHE gamma rays in the highest energy CR can be investigated by observations of high energy CR showers from the direction of the Sun. If photons are numerous at the highest energies then a deficit of showers with energies $> 10^{19}$ eV, and multiple synchronous showers at lower energies might be detected from the certain circle around the Sun. Bednarek (1999) investigates these processes by performing Monte Carlo simulations of cascades initiated by EHE photons in the magnetic field of the Sun. Based on simulations he predict that the Auger array (see the short description in Section 4.5 in Dorman, M2004) may detect multiple, synchronous showers initiated by photons with energies above $\sim 10^{16}$ eV at a rate about one per year if photons are common above 10^{19} eV in CR.

1.15.2. Magnetic $e^\pm$ pair cascades in the magnetosphere of the Sun

According to Erber (1966), an EHE photon with energy E can convert into $e^\pm$ pair in the magnetic field B if the dimensionless parameter

$$\chi_\gamma = (E_\gamma / mc^2) / (B/B_{cr}) \quad (1.15.1)$$

is high enough ($\chi_\gamma \geq \chi_{\gamma,th} \approx 0.05$), where

$$B_{cr} = 4.414 \times 10^{13} \text{ Gs}, \quad (1.15.2)$$

and mc^2 is the electron's rest energy. The secondary $e^\pm$ pairs can then produce synchrotron photons in the magnetic field, which energies are high enough to produce the next generation of $e^\pm$ pairs. Bednarek (1999) simulates the development of such a type of cascade by using the Monte Carlo method and applying the rates of $e^\pm$ pair production by a gamma ray photon and synchrotron emission by secondary $e^\pm$ pairs are given by Baring (1988). Bednarek (1999) notes that except Kasahara (1997), all previous simulations of such a type of cascade based on the approximate rates of pair production and synchrotron emission given by Erber (1966). The magnetic field of the Sun during the minimum of solar activity can be well approximated as a dipole with a magnetic moment

$$M_s \approx 6.87 \times 10^{32} \text{ Gs.cm}^3. \quad (1.15.3)$$

In Bednarek (1999) the influence of the active regions on the Sun with strong magnetic fields was neglected, since they dominate only in the solar photosphere and chromosphere. CR EHE photons may initiate cascades in the dipole magnetic field of the Sun if their energies are

$$E_\gamma > E_{\gamma \min} = \chi_{\gamma,th} mc^2 B_{cr} r_s^3 / \left(M_s \sqrt{1 + 3 \cos^2 \varphi} \right) \text{ MeV}, \quad (1.15.4)$$

where $\chi_{\gamma,th} \approx 0.05$. Taking into account Eq. 1.15.1-1.15.3, the threshold for which photons have chance of cascading efficiently, will be

$$E_\gamma > E_{\gamma \min} \approx 1.13 \times 10^{12} / \left(B_{s,sur} \sqrt{1 + 3 \cos^2 \varphi} \right) \text{ MeV}. \quad (1.15.5)$$

In Eq. 1.15.5 $B_{s,sur}$ is the magnetic field at the surface of the Sun, and φ is the

zenith angle of the photon at the moment of its closest approach to the Sun.

It is assumed that photons are injected randomly within the circle with radius r_c around the Sun. The number of secondary photons from cascades initiated by primary photons with energy 10^{19} and 10^{20} eV, within a circle $r_c = 1.5; 2$ and $3 r_s$ around the Sun (where r_s is the radius of the Sun) are shown in Fig. 1.15.1.

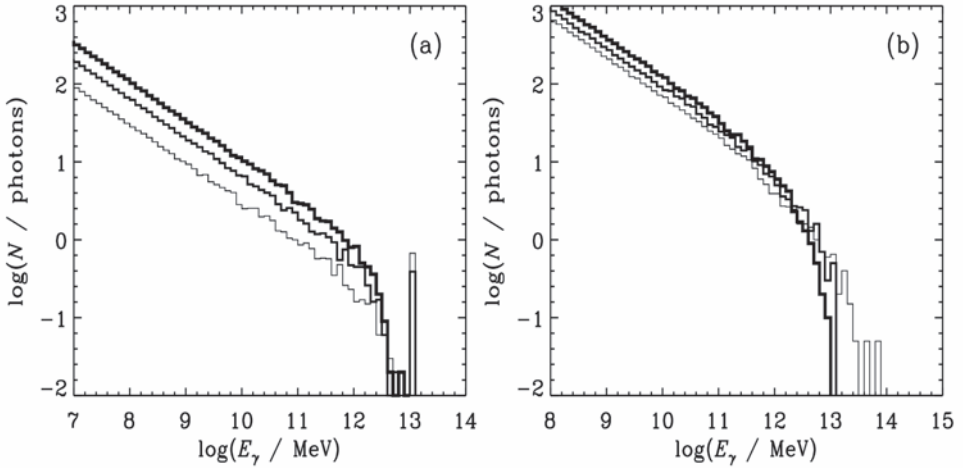


Fig. 1.15.1. The average number of secondary gamma rays (within $\Delta \log E_\gamma = 0.1$) from cascades initiated by one hundred primary EHE photons with energies 10^{19} eV (panel **a**), and ten photons with energy 10^{20} eV (panel **b**) which are injected within the circle $r_c = 1.5 r_s$ (the thickest full curve), $2 r_s$, and $3 r_s$ (the thinnest two full curves) around the Sun. According to Bednarek (1999).

In Fig. 1.15.1 the secondary photons are grouped into bins $\Delta(\log E_\gamma) = 0.1$, and the results are averaged over 100 simulations in the case of 10^{19} eV primary photons, and 10 simulations in the case of 10^{20} eV photons. Note that all primary photons with energy 10^{19} eV injected within the circle $r_c = 1.5 r_s$ from the Sun cascade, but only part of such photons interact if injected within a larger circle (61% for $r_c = 2 r_s$, and 33% for $r_c = 3 r_s$). All primary photons with energy 10^{20} eV cascade if injected within the range considered of parameter r_c .

1.15.3. The possibility that extra high energy CR spectrum at $> 10^{19}$ eV contains significant proportion of photons

Next, in Bednarek (1999) the possibility was considered that the extra high energy CR spectrum at $> 10^{19}$ eV contains significant proportion of photons. He computes the spectra of secondary cascade gamma rays assuming that the primary

photons, which enter the magnetosphere of the Sun at certain circle r_c , have a power law spectrum with the spectral index -2.7 and a cutoff at different energies. In Fig. 1.15.2 (panel **a**) are shown the spectra of secondary photons (multiplied by the photon energy square) from cascades initiated by primary extra high energy photons injected within $r_c = 1.5, 2$, and $3 r_s$. The primary photon spectrum extends up to $E_{\max} = 3 \times 10^{20}$ eV and is normalized to the observed CR spectrum at 10^{19} eV.

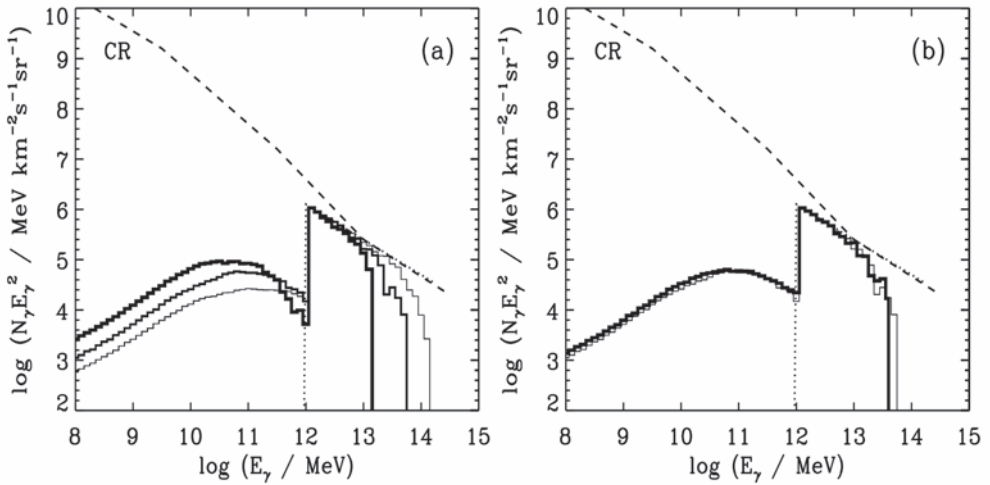


Fig. 1.15.2. Panel **a**: the spectra of secondary gamma rays (multiplied by the square of the photon energy) from the cascades initiated by the primary photons with the power law spectrum and spectral index -2.7 above 10^{18} eV and the cutoff at 3×10^{20} eV (marked by the dotted curve); the spectra emerging from the Sun's magnetosphere are shown for primary photons injected within the circle $r_c = 1.5 r_s$ around the Sun (the thickest full curve), $2 r_s$, and $3 r_s$ (the thinnest curves). Panel **b**: as in panel **a** but for the primary gamma ray spectrum injected within $r_c = 2 r_s$ and extending up to 3×10^{20} eV (thin curve) and 3×10^{21} eV (thick curve). The observed CR spectrum is schematically marked by the dashed curve. According to Bednarek (1999).

The observed CR spectrum is indicated in Fig. 1.15.2 schematically by the dashed curve. The primary photon spectrum which extends above 10^{18} eV is shown by the dotted curve. In Fig. 1.15.2 (panel **b**) are shown that the spectrum of secondary gamma rays, produced by the primary gamma ray spectrum with a cut off at 3×10^{21} eV, is almost the same as in the case of its cut off at 3×10^{20} eV.

1.15.4. Summering of main results and discussion

Simulations of Bednarek (1999) show that a significant part of extra high energy photons with energies above 10^{19} eV should cascade if injected within the circle of $r_c = 2 r_s$ around the Sun. However, the solid angle corresponding to such a circle on the sky is relatively small ($\sim 2.2 \times 10^{-5}$ sr). The Auger experiment is expected to detect about 50–100 particles with energy $> 10^{20}$ eV per year (Boratav, 1997; see also the short description of Auger experiment in Section 4.5 in Dorman, M2004) and about 2×10^4 particles with energy $> 10^{19}$ eV per year. Some showers initiated by particles with energy $> 10^{19}$ eV should be detected from the circle of $2 r_s$ around the Sun within a few years of operation. Let us assume that all these particles with an energy above 10^{19} eV are photons. Bednarek's (1999) simulations show that these photons should cascade in the Sun's magnetic field, and as a result of cascading about 12 secondary photons with energy $> 10^{17}$ eV and 50 secondary photons with energy $> 10^{16}$ eV should arrive at the Earth's surface synchronously with a rate corresponding to the number of events with energy $> 10^{19}$ eV expected from the direction of the Sun. Bednarek (1999) estimates the energy weighted perpendicular spread of secondary photons (its half thickness) based on the simulations described above. It is found that secondary photons from a cascade initiated by a primary photon with energy 10^{19} eV should fall on the Earth's surface within the circle with average radius ~ 19 km (an estimate based on ten simulations). If the primary photons have an energy 10^{20} eV, then the secondary photons should be contained within the circle with radius ~ 51 km.

If such synchronous multiple showers initiated by photons with energies above 10^{17} eV can be observed by the Auger experiment, then a bunch containing half of the number of these secondary photons should fall on the Auger array with a frequency of about one per year. Note that for such photon bunches the effective detection area of the Auger array becomes larger by a factor close to two for geometrical reasons. Observation of such multiple showers from the direction of the Sun should make possible the estimate of the content of the photons in CR with energy above 10^{19} eV at a level of 10 percent during several years of operation.

The spectra of secondary photons produced within the circle r_c around the Sun by primary photons with the spectrum observed in CR with energy above 10^{19} eV (and with normalization to the observed CR spectrum) are shown in Fig. 1.15.2 (panels **a** and **b**). Based on these computations Bednarek (1999) estimated the ratio of CR photons to CR particles at lower energies (see Table 1.15.1).

Table 1.15.1. The ratios of CR photons to CR particles at energy E from the direction of the Sun at primary CR energy 3×10^{20} eV. According to Bednarek (1999).

$E, \text{ eV}$	r_c/r_s		
	1.5	2	3
10^{15}	5×10^{-6}	$\sim 2 \times 10^{-6}$	$\sim 8 \times 10^{-7}$
10^{16}	1.5×10^{-3}	$\sim 6 \times 10^{-4}$	$\sim 2 \times 10^{-4}$
10^{17}	3×10^{-3}	$\sim 2 \times 10^{-3}$	$\sim 8 \times 10^{-4}$

From the Table 1.15.1 it can be seen that the expected ratios of CR photons to CR particles are of the order of $\sim 10^{-3}$ in the energy range 10^{16} – 10^{17} eV if primary photons are injected within the circle of less than $2 r_s$ around the Sun. However, in order to detect one shower initiated in the atmosphere by photon with energy 10^{16} eV from the direction of the Sun, the statistics of showers of the order of $\sim 10^8$ is needed because of small solid angle of the Sun.

Chapter 2

Cosmic Ray Propagation in Space Plasmas

2.1. The problem of CR propagation and a short review of a development of the basically concepts

Propagation of charged particles of CR in space plasmas (interstellar and interplanetary media, intergalactic space, various types of CR sources) is one of the actual problems of CR astrophysics and geophysics. The foundations of the theory of CR interaction with magnetized space plasma were developed more than fifty years ago: Fermi (1949) showed that charged particles of CR in a 'collision' with 'clouds' of magnetized plasma moving in the inverse direction should be accelerated, and in the opposite case - decelerated. Detailed studies of the processes of CR propagation in space began, however, after giant solar flare event at 23 February 1956 and International Geophysical Year (IGY) in 1957-1958. These studies were stimulated by extensive development of experimental methods connected with the formation of the world-wide network of ground based neutron monitors (NM) and muon telescopes (MT), and with direct observations in the magnetosphere and in space by satellites and spacecrafts.

The initial attempts at forming a statistical theory of CR propagation were based on a simple model of isotropic diffusion (Meyer et al., 1956; Dorman, M1957, 1958, M1963a,b; Parker, M1963; Ginzburg and Syrovatsky, M1963; Dorman and Miroshnichenko, M1968; Dorman, M1975a,b; Berezhinsky et al., M1990). This theory was complicated later on the basis of phenomenological considerations and experimental data. A detailed description of the theory of propagation of CR and a detailed analysis of various effects in the framework of the isotropic diffusion model has been presented in a series of monographs (Parker, M1963; Ginzburg and Syrovatsky, M1963; Dorman, M1963a,b). The theory of anisotropic diffusion, including the case of expanding space plasma, was developed for the first time in papers of Parker (1965), Dorman (1965, 1967). A treatment and development of the theory of CR propagation on the basis of kinetic approach were presented in (Shishov, 1966; Dolginov and Toptygin, 1966a,b, 1967, 1968a,b; Tverskoy, 1967a,b, 1969; Galperin et al., 1971; Toptygin, 1972, 1973a,b; Dorman and Katz, 1972a,b, 1973, 1974a,b, 1977a,b,c; Klimas and Sandri, 1973, 1975; Scudder and Klimas, 1975; Jokipii et al., 1995; Earl et al., 1995; Kóta, 1995, 2000; Otsuka and Hada, 2003). The corresponding reviews of the kinetic theory of CR propagation were presented in (Jokipii, 1971; Völk, 1975;

Dorman and Katz, 1977d; Dröge, 2000). In De Koning and Bieber (2001) there are analyzed the particle-field correlation in a flowing plasma. The case of small pitch-angle scattering was considered by Shakhov and Stehlik (2001), and in Dorman, Shakhov, and Stehlik (2003) the second order pitch-angle approximation for the CR Fokker-Planck kinetic equation was considered. Burgoa (2003) proposed a Lagrangian density for obtaining the Fokker-Planck CR transport equation and determining the energy-momentum tensor and CR currents.

In the work of Dolginov and Toptygin (1966a) a consistent theory of CR propagation in an inhomogeneous medium (where on a background of a regular magnetic field the stochastic inhomogeneities of magnetic field are present which are transferred by the cosmic plasma with a certain velocity) was developed basing on the collisionless Boltzman equation. The kinetic equation obtained, Dolginov and Toptygin (1966a,b) made a transformation to the diffusion approximation, and found the expression for the flux density of particles including the variations of particle energy, and considered some special problems of the theory of CR propagation.

A substation progress in a study of the processes of CR propagation is connected with the exploration of propagation of the solar CR. The analysis of the data on the tremendous flare of CR on February 23, 1956 has already shown that propagation of high energy particles from the Sun is well described by the diffusion theory (Meyer et al., 1956; Dorman, M1957, 1958). The interaction of galactic CR with the front of solar corpuscular streams (or as it is now called, with the front of the ‘coronal mass ejection’) leads to CR particles acceleration and increasing of CR intensity on the Earth (Dorman, 1959b). The analysis of the observational data of the flare on May 1959 has shown that a transfer of low energy solar CR in a trap formed in the frontal part of the solar corpuscular stream is possible (Dorman, 1959a).

Tverskoy (1967a,b, 1969) has formed the hypothesis of a transfer of fast particles behind the front of a shock wave where a developed hydromagnetic turbulence arises. It was assumed there that the Larmor radius of particles is far less than the main scale of turbulence. In this case the kinetic coefficients describing the process of particle propagation are determined by a detailed form of the spectrum function of a stochastic magnetic field. Note that a developed hydromagnetic turbulence (Alfvén waves) was recognized as a result of numerous direct measurements in interplanetary space (see, for example, Belcher and Davis, 1969, 1971).

In recent years, as a result of the progress in the experimental technology, the new class of CR variation, CR pulsations, i.e. more or less regular variations of CR intensity with the period of several minutes or less is studied. When studying CR pulsations the most informative is a comparison of the theory with experimental data on the variation of correlation function of fluctuations or the function of particle distribution. The theory of pulsation effects in CR was developed in (Shishov, 1968; Dorman and Katz, 1973, 1974a,b, 1977d; Jokipii

and Owens, 1976; Vasilyev and Toptygin, 1976a,b; Dorman, Katz, Stehlik, 1976, 1977; Dorman, Katz, Yukhimuk, 1977).

A substantial progress in the development of the theory of CR propagation in interplanetary space is undoubtedly connected with wide application of computers for a solution of problems of CR propagation in the conditions close to the actual ones (Urch and Gleeson, 1972; Dorman and Kobylinski, 1968, 1972a,b,c, 1973; Dorman, Kobylinski, and Khadakhanova, 1973; Fisk et al., 1973; Barnden, 1973 a,b; Cecchini and Quenby, 1975; Dorman and Milovidova, 1973, 1974, 1975a,b, 1976a,b,c,d, 1977, 1978; Alaniya and Dorman, 1977, 1978, 1979, M1981; Alania et al., 1976, 1977a,b, 1978, 1979, M1987). A sufficient success was achieved in the solution of the problems of CR propagation at the presence of moving hydromagnetic discontinuities in interplanetary space (Dorman, 1959c, M1963a,b, 1969, 1973a,b, 1975; Parker, M1963; Blokh et al., 1964; Bagdasariyan et al., 1971; Belov et al., 1973, 1975, 1976; Dorman and Shogenov, 1973a,b, 1974a,b, 1975 a,b, 1977, 1979; Dorman, Babayan et al., 1978a), in the problems of distortion of the external anisotropy during propagation of particles in interplanetary space (Parker, 1967; Belov and Dorman, 1969, 1972, 1977), in the self-consistent problems of propagation of CR including their non-linear interaction with the solar wind (Dorman and Dorman, 1968a,b, 1969; Babayan and Dorman, 1976; 1977a,b, 1979a,b,d; see below, Chapter 3).

2.2. The method of the characteristic functional and a deduction of kinetic equation for CR propagation in space in the presence of magnetic field fluctuations

CR moving in interplanetary space can be considered as a flow of non-interacting charged particles in a magnetic field

$$\mathbf{H}(\mathbf{r}, t) = \mathbf{H}_o(\mathbf{r}, t) + \mathbf{H}_1(\mathbf{r}, t), \quad (2.2.1)$$

which has a regular $\mathbf{H}_o(\mathbf{r}, t)$ and a random $\mathbf{H}_1(\mathbf{r}, t)$ components; in this case $\langle \mathbf{H} \rangle = \mathbf{H}_o$, $\langle \mathbf{H}_1 \rangle = 0$. The oblique brackets denote an averaging over random fields. Since a magnetic field in interplanetary space is transferred by the magnetized plasma of the solar wind, we should take into account the field motion with respect to an observer. The velocity which should be assigned to the field depends on a degree of freezing of the magnetic field in plasma. We intend to consider the case when the field is completely frozen in plasma and is involved by plasma into a motion with the velocity $\mathbf{u}_o(\mathbf{r})$. In the general case this velocity has various values in different points of a space and $u_o \ll c$. The most complete description of the propagation of CR in the interplanetary space with the magnetic field, which is determined by Eq. 2.2.1, is given by the collisionless kinetic

equation for a CR distribution function $f(\mathbf{r}, \mathbf{p}, t)$ (Dolginov and Toptygin, 1966a,b; Dorman and Katz, 1972a):

$$\frac{\partial f}{\partial t} + \mathbf{v} \frac{\partial f}{\partial \mathbf{r}} - \mathbf{H} \mathbf{D} f = 0, \quad (2.2.2)$$

where

$$\mathbf{D} = \frac{e}{c} \left(\mathbf{v} - \mathbf{u}_0, \frac{\partial}{\partial \mathbf{p}} \right) \quad (2.2.3)$$

is the operator of particle momentum, $\mathbf{p}$ is the momentum, $\mathbf{v} = c\mathbf{p}/E$ is the velocity and E is the total energy of a particle. The distribution function $f(\mathbf{r}, \mathbf{p}, t)$ varies fast following a random field variation. Averaged over a stochastic field the distribution function $F(\mathbf{r}, \mathbf{p}, t) = \langle f(\mathbf{r}, \mathbf{p}, t) \rangle$ is of interest. To obtain the equation for F we shall apply the method which has been developed in the quantum theory of fields (Schwinger, 1951; Fradkin, 1965) and statistical fluid mechanics (Hopf, 1952; see also Monin and Yaglom, M1965; M1967). This method was intensively developed in the problems of wave propagation in the medium with stochastic inhomogeneities (Tatarsky, M1967; Klyatskin and Tatarsky, 1973).

As is known (Hopf, 1952; Tatarsky, M1967), the statistical properties of a stochastic function $F_1(\mathbf{r}, \mathbf{p}, t) = F_1(\mathbf{x}, t)$ (in further consideration we shall frequently denote a set of variables $\{\mathbf{r}, \mathbf{p}\}$ by a single letter $\{\mathbf{x}\}$) are completely determined if its characteristic functional is settled:

$$\Phi[\boldsymbol{\eta}(\mathbf{x}, t)] = \langle \exp(i\boldsymbol{\eta} \mathbf{F}_1) \rangle, \quad (2.2.4)$$

where

$$\boldsymbol{\eta} \mathbf{F}_1 = \int d\mathbf{x} dt \boldsymbol{\eta}_\alpha(\mathbf{x}, t) F_{1\alpha}(\mathbf{x}, t) \quad (2.2.5)$$

is the ‘scalar product’ in the functional space and summation over the repeated indices is assumed here and below. All moments of a stochastic field can be derived from Eq. 2.2.4 as the functional derivatives at zero-valued functional argument $\boldsymbol{\eta}(\mathbf{x}, t)$:

$$\frac{1}{i} \frac{\delta \Phi[\boldsymbol{\eta}]}{\delta \eta_\alpha(\mathbf{x}, t)} \Big|_{\boldsymbol{\eta}=0} = \langle F_{1\alpha}(\mathbf{x}, t) \rangle = 0, \quad (2.2.6)$$

$$\frac{1}{i^2} \frac{\delta^2 \Phi[\boldsymbol{\eta}]}{\delta \eta_\alpha(\mathbf{x}, t) \delta \eta_\beta(\mathbf{x}, t)} \Big|_{\boldsymbol{\eta}=0} = \langle F_{1\alpha}(\mathbf{x}, t) F_{1\beta}(\mathbf{x}, t) \rangle = 0, \quad (2.2.7)$$

and so on. In particular, the statistical properties of a magnetic field are completely determined if its characteristic functional is set:

$$\Phi[\boldsymbol{\eta}(\mathbf{r}, t)] = \langle \exp i \int (\boldsymbol{\eta}(\mathbf{r}, t) \mathbf{H}_1(\mathbf{r}, t)) d\mathbf{r} dt \rangle. \quad (2.2.8)$$

According to the above considerations, all the moments of a stochastic magnetic field can be obtained from Eq. 2.2.8 as variation derivatives at the functional argument equal to zero. For instance, the value

$$\mathbf{B}_{\alpha\beta}(\mathbf{r}_1, t_1; \mathbf{r}_2, t_2) = \langle \mathbf{H}_{1\alpha}(\mathbf{r}_1, t_1) \mathbf{H}_{1\beta}(\mathbf{r}_2, t_2) \rangle = \frac{\partial^2 \Phi[\boldsymbol{\eta}]}{\partial \eta_\alpha(\mathbf{r}_1, t_1) \partial \eta_\beta(\mathbf{r}_2, t_2)} \Big|_{\boldsymbol{\eta}=0} \quad (2.2.9)$$

is the correlation tensor of a random magnetic field of the second rank which, in general, can be determined from experimental data.

Multiplying Eq. 2.2.2 by $\exp i \int (\boldsymbol{\eta}(\mathbf{r}, t) \mathbf{H}_1(\mathbf{r}, t)) d\mathbf{r} dt$ and averaging the equation obtained over possible realizations of a random magnetic field, we have the equation in the variance derivatives (Dorman and Katz, 1972a)

$$\left(\frac{\partial}{\partial t} + L_o \right) \Psi[\boldsymbol{\eta}; \mathbf{r}, \mathbf{p}, t] = \frac{ie}{c} D_\alpha \left\{ \frac{\delta \ln \Phi[\boldsymbol{\eta}]}{\delta \eta_\alpha(\mathbf{r}, t)} + \frac{\delta}{\delta \eta_\alpha(\mathbf{r}, t)} \right\} \Psi[\boldsymbol{\eta}; \mathbf{r}, \mathbf{p}, t], \quad (2.2.10)$$

where

$$L_o = \mathbf{v} \frac{\partial}{\partial \mathbf{r}} - \mathbf{H}_0 \mathbf{D}, \quad (2.2.11)$$

And

$$\Psi[\boldsymbol{\eta}; \mathbf{r}, \mathbf{p}, t] = \langle f(\mathbf{r}, \mathbf{p}, t) \exp i \int (\boldsymbol{\eta} \mathbf{H}_1) d\mathbf{r} dt \rangle \quad (2.2.12)$$

is the functional with the value at $\boldsymbol{\eta} = 0$ to be a distribution function averaged over a random field

$$\Psi[0; \mathbf{r}, \mathbf{p}, t] = \langle f(\mathbf{r}, \mathbf{p}, t) \rangle = F(\mathbf{r}, \mathbf{p}, t). \quad (2.2.13)$$

It is assumed that summing over repeated indices is carried out in Eq. 2.2.10. A common method of solving the Eq. 2.2.10 is to represent the functionals $\Psi[\boldsymbol{\eta}; \mathbf{r}, \mathbf{p}, t]$ and $\delta \ln \Phi[\boldsymbol{\eta}] / \delta \eta_\alpha(\mathbf{r}, t)$ as the functional power series

$$\begin{aligned} \Psi[\boldsymbol{\eta}; \mathbf{r}, \mathbf{p}, t] = & F(\mathbf{r}, \mathbf{p}, t) + \int \eta_\alpha(\mathbf{r}_1, t_1) F_{1\alpha}(\mathbf{r}, t; \mathbf{r}_1, t_1; \mathbf{p}) d\mathbf{r}_1 dt_1 \\ & + \int \eta_\alpha(\mathbf{r}_1, t_1) \eta_\beta(\mathbf{r}_2, t_2) F_{2\alpha\beta}(\mathbf{r}, t; \mathbf{r}_1, t_1; \mathbf{r}_2, t_2; \mathbf{p}) d\mathbf{r}_1 dt_1 d\mathbf{r}_2 dt_2 + \dots, \end{aligned} \quad (2.2.14)$$

$$\begin{aligned} \left. \frac{1}{i} \frac{\delta \Phi[\boldsymbol{\eta}]}{\delta \eta_\alpha(\mathbf{r}, t)} \right|_{\boldsymbol{\eta}=0} = & - \int B_{\alpha\beta}(\mathbf{r}, t; \mathbf{r}_1, t_1) \eta_\beta(\mathbf{r}_1, t_1) d\mathbf{r}_1 dt_1 \\ & + \int B_{\alpha\beta\gamma}(\mathbf{r}, t; \mathbf{r}_1, t_1; \mathbf{r}_2, t_2) \eta_\beta(\mathbf{r}_1, t_1) \eta_\gamma(\mathbf{r}_2, t_2) d\mathbf{r}_1 dt_1 d\mathbf{r}_2 dt_2 + \dots, \end{aligned} \quad (2.2.15)$$

where $F, F_{1\alpha}, F_{2\alpha\beta}$ are the functionals of the zero, the first and the second power, respectively; $B_{\alpha\beta}$ is the correlation tensor of a random magnetic field of the second rank which is determined by the Eq. 2.2.9, and $B_{\alpha\beta\gamma}$ is the correlation tensor of the third rank.

Substituting these expressions into Eq. 2.2.10 and equating the functionals of equal power on the left and right hand sides of Eq. 2.2.10, as a result we obtain an infinite set of related equations. The simplest method of solving of these equations is to equate one of the functionals F_n to zero. Let $F_2 = 0$, then

$$\begin{aligned} \left(\frac{\partial}{\partial t} + L_o \right) F(\mathbf{r}, \mathbf{p}, t) = & -i D_\alpha F_{1\alpha}(\mathbf{r}, t; \mathbf{r}, t; \mathbf{p}), \\ \left(\frac{\partial}{\partial t} + L_o \right) F_{1\alpha}(\mathbf{r}, t; \mathbf{r}_1, t_1; \mathbf{p}) = & i B_{\alpha\beta}(\mathbf{r}, t; \mathbf{r}_1, t_1) D_\beta F(\mathbf{r}, \mathbf{p}, t). \end{aligned} \quad (2.2.16)$$

To solve the set of Eq. 2.2.16 let us introduce the functions $\varphi_\alpha(\mathbf{r}, t; \mathbf{r}_1, t_1; \mathbf{p})$ according to the relation

$$F_{1\alpha}(\mathbf{r}, t; \mathbf{r}_1, t_1; \mathbf{p}) = \exp(-L_o t) \varphi_\alpha(\mathbf{r}, t; \mathbf{r}_1, t_1; \mathbf{p}). \quad (2.2.17)$$

The action of the operator $\exp(-L_o t)$ on an arbitrary function of the coordinates and moments is the replacement $\mathbf{r}$ by $\mathbf{r} - \Delta \mathbf{r}(t)$ and $\mathbf{p}$ by $\mathbf{p} - \Delta \mathbf{p}(t)$, where $\Delta \mathbf{r}(t)$ and $\Delta \mathbf{p}(t)$ is the variance of the radius-vector and the momentum of a particle in a regular field. Substituting the expression for $F_{1\alpha}$ into the second equation of the set Eq. 2.2.16, we obtain

$$F_{1\alpha}(\mathbf{r}, t; \mathbf{r}_1, t_1; \mathbf{p}) = -i \int_0^t \exp(-L_o(t-t')) B_{\alpha\beta}(\mathbf{r}, t'; \mathbf{r}_1, t_1) D_\beta F(\mathbf{r}, \mathbf{p}, t') dt'. \quad (2.2.18)$$

Substituting this relation into the first equation of the set Eq. 2.2.16 we find the equation for the function $F(\mathbf{r}, \mathbf{p}, t)$:

$$\left(\frac{\partial}{\partial t} + L_o \right) F(\mathbf{r}, \mathbf{p}, t) = D_\alpha \int_0^t dt' \left\{ \exp[-L_o(t-t')] B_{\alpha\beta}(\mathbf{r}_1, t; \mathbf{r}, t') D_\beta F(\mathbf{r}, \mathbf{p}, t') \right\}_{\mathbf{r}_1=\mathbf{r}}. \quad (2.2.19)$$

On the right hand side of this equation $t_1 = t$ was set, according to the first equation of the set Eq. 2.2.16, and it is necessary to keep in mind that we should put $\mathbf{r}_1 = \mathbf{r}$ after the action of the operator $\exp[-L_o(t-t')]$. For further analysis Eq. 2.2.19 it is necessary to concretize a dependence of the correlation tensor $B_{\alpha\beta}(\mathbf{r}, t; \mathbf{r}_1, t_1)$ on the coordinates and time. The most general form of the correlation tensor $B_{\alpha\beta}$ fitting the experiment and the Maxwell equation is

$$B_{\alpha\beta}(\mathbf{r}_1, t; \mathbf{r}, t') = B_{\alpha\beta}(\mathbf{r}_1, \mathbf{r}_1 - \mathbf{r} - u_o(t-t')). \quad (2.2.20)$$

Setting $t - t' = \tau$, we obtain from Eq. 2.2.19:

$$\left(\frac{\partial}{\partial t} + L_o \right) F(\mathbf{r}, \mathbf{p}, t) = D_\alpha \int_0^t d\tau \left\{ \exp(-L_o\tau) B_{\alpha\beta}(\mathbf{r}_1, \mathbf{r}_1 - \mathbf{r} - u_o\tau) D_\beta F(\mathbf{r}, \mathbf{p}, t - \tau) \right\}_{\mathbf{r}_1=\mathbf{r}} \quad (2.2.21)$$

The right hand side of the Eq. 2.2.21 differs from zero in time intervals of the order of the correlation time of a random field. If the correlation time of a random field is small compared to the characteristic time of the distribution function variation, i.e., $t \gg \tau$, then it is possible to write

$$\left(\frac{\partial}{\partial t} + L_o \right) F(\mathbf{r}, \mathbf{p}, t) = D_\alpha \int_0^t d\tau \left\{ \exp(-L_o\tau) B_{\alpha\beta}(\mathbf{r}_1, \mathbf{r}_1 - \mathbf{r} - u_o\tau) D_\beta F(\mathbf{r}, \mathbf{p}, t) \right\}_{\mathbf{r}_1=\mathbf{r}} \quad (2.2.22)$$

2.3. Kinetic equation in the case of weak regular and isotropic random fields

If the momentum of a particle is varied weakly at distances of the order of the correlation radius of a random field, one can set the variation of a particle momentum as $\Delta \mathbf{p}(\tau) = 0$ and the variation of its radius-vector as $\Delta \mathbf{r}(\tau) = \mathbf{v} \tau$ for the action of the operator $\exp(-L_o\tau)$. Then

$$\left(\frac{\partial}{\partial t} + L_o\right)F(\mathbf{r}, \mathbf{p}, t) = D_\alpha B_{\alpha\beta}(\mathbf{r}, \mathbf{v}) D_\beta F(\mathbf{r}, \mathbf{p}, t), \quad (2.3.1)$$

where

$$B_{\alpha\beta}(\mathbf{r}, \mathbf{v}) = \int_0^\infty B_{\alpha\beta}(\mathbf{r}, \mathbf{v}\tau - \mathbf{u}_o\tau) d\tau. \quad (2.3.2)$$

If a random magnetic field is statistically isotropic the correlation tensor $B_{\alpha\beta}(\mathbf{r}, \mathbf{x})$ has the following form:

$$B_{\alpha\beta}(\mathbf{r}, \mathbf{x}) = \frac{1}{3} \left\langle H_1^2(\mathbf{r}) \right\rangle \left\{ \Psi\left(\frac{x}{l_c}\right) \delta_{\alpha\beta} - \Psi_1\left(\frac{x}{l_c}\right) \frac{x_\alpha x_\beta}{x^2} \right\} \quad (2.3.3)$$

with

$$\Psi_1\left(\frac{x}{l_c}\right) = \Psi\left(\frac{x}{l_c}\right) - \frac{2l_c^2}{x^2} \int_0^{x/l_c} y \Psi(y) dy. \quad (2.3.4)$$

Here $\delta_{\alpha\beta}$ is the unit tensor and $\Psi(x/l_c)$ is a dimensionless function, which is assumed to be known from observations. Usually it is suitable to choose the function $\Psi(x/l_c)$ in the form

$$\Psi(x/l_c) = (x/l_c)^{(\nu-1)/2} K_{(\nu-1)/2}(x/l_c). \quad (2.3.5)$$

Here $K_\nu(x)$ is the McDonald function, ν is the index of the inhomogeneities spectrum of the interplanetary magnetic field (usually $\nu > 1$). Direct measurements of the magnetic field in the interplanetary space give for ν the values $1 \leq \nu \leq 3.8$. Fourier-image of the function $\Psi(x/l_c)$ corresponds to a power spectrum decreasing in the region of small scales of inhomogeneities

$$\Psi(k) = \frac{A_\nu}{(l_c^{-2} + k^2)^{\nu/2+1}}; \quad A_\nu = \frac{\nu \Gamma(\nu/2)}{4\pi^{3/2} l_c^{\nu-1} \Gamma(\nu/2 - 1/2)}. \quad (2.3.6)$$

Substituting Eq. 2.3.3 - 2.3.6 into Eq. 2.3.1, we obtain

$$\frac{\partial F}{\partial t} + \mathbf{v} \frac{\partial F}{\partial \mathbf{r}} = \mathbf{H}_o \mathbf{D} F + \gamma l_c \left\langle H_1^2(\mathbf{r}) \right\rangle \mathbf{D} |\mathbf{v} - \mathbf{u}_o|^{-1} \mathbf{D} F, \quad (2.3.7)$$

where

$$\gamma = \sqrt{\pi} \Gamma(\nu/2) / 4\nu \Gamma(\nu/2 - 1/2). \quad (2.3.8)$$

In the case in which the regular field is strong enough the approximation $\Delta \mathbf{p}(\tau) = 0$ is not valid, and one should include a spiral character of particle motion at the distances of the order of the correlation radius of a stochastic field.

2.4. Kinetic equation for CR propagation including fluctuations of plasma velocity

Fluctuations of plasma velocity

$$\mathbf{u}(\mathbf{r}, t) = \mathbf{u}_o(\mathbf{r}) + \mathbf{u}_1(\mathbf{r}, t); \quad \langle \mathbf{u}(\mathbf{r}, t) \rangle = \mathbf{u}_o(\mathbf{r}); \quad \langle \mathbf{u}_1(\mathbf{r}, t) \rangle = 0 \quad (2.4.1)$$

were taken into account in (Dorman and Katz, 1972a) besides the fluctuations of a magnetic field. In this case, one should take into account the action of the induced electric field

$$\mathbf{E}(\mathbf{r}, t) = -\frac{1}{c} [\mathbf{u}(\mathbf{r}, t) \times \mathbf{H}(\mathbf{r}, t)] \quad (2.4.2a)$$

on a particle. Because of the non-stationary character of the processes on the Sun, as well as a development of turbulence immediately in the interplanetary space, a widespread spectrum of turbulent pulsations (Alfvén, magneto-sonic waves, etc.) is generated in the solar wind plasma parallel with stochastic inhomogeneities frozen in it. The stochastic electromagnetic fields of these pulsations sufficiently affect the motion of charged particles. The distribution function $f(\mathbf{r}, \mathbf{p}, t)$ of non-interacting charged particles moving in the magnetic and electric fields which are determined by the Eq. 2.2.1 and Eq. 2.4.1 satisfy the collisionless kinetic equation

$$\frac{\partial f}{\partial t} + \mathbf{v} \frac{\partial f}{\partial \mathbf{r}} + F \frac{\partial f}{\partial \mathbf{p}} = 0, \quad (2.4.2b)$$

where $\mathbf{v} = c^2 \mathbf{p} / E$ is the velocity, $E = c(p^2 + m^2 c^2)^{1/2}$ is the total energy of a particle with the momentum $\mathbf{p}$ and rest mass m . The term $\mathbf{F}$ is the force acting on a particle:

$$\mathbf{F} = e \left(\mathbf{E} + \frac{1}{c} [\mathbf{v} \times \mathbf{H}] \right). \quad (2.4.3)$$

Let us present $\mathbf{F}$ in the form of the sum of regular

$$\mathbf{F}_o = e \left(\mathbf{E}_o + \frac{1}{c} [\mathbf{v} \times \mathbf{H}_o] \right) \quad (2.4.4)$$

and a stochastic

$$\mathbf{F}_1 = e \left(\mathbf{E}_1 + \frac{1}{c} [\mathbf{v} \times \mathbf{H}_1] \right) \quad (2.4.5)$$

component. If the magnetic field is completely frozen in plasma, the regular component of the electric field $\mathbf{E}_o$ has a form

$$\mathbf{E}_o = -\frac{1}{c} [\mathbf{u}_o \times \mathbf{H}_o], \quad (2.4.6)$$

and a stochastic component will be

$$\mathbf{E}_1 = -\frac{1}{c} ([\mathbf{u}_o \times \mathbf{H}_1] + [\mathbf{u}_1 \times \mathbf{H}_o]). \quad (2.4.7)$$

According to Eq. 2.4.3 - 2.4.7 we write the kinetic Eq. 2.4.2b in the form

$$\left(\frac{\partial}{\partial t} + L_o \right) f + \mathbf{F}_1 \mathbf{L} f = 0, \quad (2.4.8)$$

where

$$L_o = \mathbf{v} \frac{\partial}{\partial \mathbf{r}} + \mathbf{F}_o \mathbf{L} \quad (2.4.9)$$

is the operator related to the regular component $\mathbf{F}_o$ of the force $\mathbf{F}$ and $\mathbf{L} = \frac{\partial}{\partial \mathbf{p}}$. As for the case of fluctuations of a magnetic field alone, the distribution function $f(\mathbf{r}, \mathbf{p}, t)$ varies irregularly in space and time following the variations of a stochastic field, so that the actual meaning is proper to the distribution function $\langle f(\mathbf{r}, \mathbf{p}, t) \rangle$ which is averaged over a statistical ensemble corresponding to a stochastic field. To deduce the equation which the function $\langle f(\mathbf{r}, \mathbf{p}, t) \rangle$ satisfies, let us use the method of characteristic functional considered in Section 2.2. Then in a general case the correlation tensor of electromagnetic field (in contrast to Eq.

2.2.9 for pure magnetic fluctuations) will be determined by a more complicated relation

$$D_{\alpha\lambda} = e^2 \left\{ T_{\alpha\lambda} + \frac{1}{c} (\varepsilon_{\alpha\beta\gamma} V_\beta \Pi_{\gamma\lambda} + \varepsilon_{\lambda\mu\nu} V_\mu \Pi_{\nu\alpha}) + \varepsilon_{\alpha\beta\gamma} \varepsilon_{\lambda\mu\nu} V_\beta V_\mu B_{\gamma\nu} \right\}, \quad (2.4.10)$$

where $\varepsilon_{\alpha\beta\gamma}$ is the united anti-symmetrical tensor of the third rank; $T_{\alpha\lambda} = \langle E_{1\alpha} E_{1\lambda} \rangle$ and $B_{\gamma\nu} = \langle H_{1\gamma} H_{1\nu} \rangle$ are the correlation tensors of the electric and magnetic fields, respectively, $\Pi_{\alpha\beta} = \langle H_{1\alpha} E_{1\beta} \rangle$ is the crossed correlation tensor of electric and magnetic fields. If the magnetic field is completely frozen into the plasma then $D_{\alpha\lambda}$ will have a form

$$D_{\alpha\lambda} = \left(\frac{e}{c} \right)^2 \varepsilon_{\alpha\beta\gamma} \varepsilon_{\lambda\mu\nu} \{ W_\beta W_\mu B_{\gamma\nu} - W_\beta H_{\alpha\nu} S_{\mu\gamma} - W_\mu H_{\alpha\gamma} S_{\beta\nu} + H_{\alpha\gamma} H_{\alpha\nu} Q_{\beta\mu} \}, \quad (2.4.11)$$

where $\mathbf{W} = \mathbf{V} - \mathbf{u}_o$ and $S_{\mu\gamma} = \langle u_{1\mu} H_{1\gamma} \rangle$ is the crossed correlation tensor of the magnetic and velocity fields; $Q_{\beta\mu} = \langle u_{1\beta} u_{1\mu} \rangle$ is the correlation tensor of the velocity field. When writing Eq. 2.4.11 we neglected the term $(e/c)^2 \langle [\mathbf{u}_1 \times \mathbf{H}_1]_\alpha [\mathbf{u}_1 \times \mathbf{H}_1]_\lambda \rangle$ assuming them to be small.

Let us multiply Eq. 2.4.8 by $\exp(i(\boldsymbol{\eta} \mathbf{F}_1))$, where $\mathbf{F}_1$ is determined by Eq. 2.4.5 and average the derived equation over the statistical ensemble corresponding to a stochastic field. As a result, we obtain the equation in the functional derivatives

$$\left(\frac{\partial}{\partial t} + L_o \right) F(\boldsymbol{\eta}; \mathbf{x}, t) = iL_\alpha \left\{ \frac{\delta \ln \Phi[\boldsymbol{\eta}]}{\delta \eta_\alpha(\mathbf{x}, t)} + \frac{\delta}{\delta \eta_\alpha(\mathbf{x}, t)} \right\} F(\boldsymbol{\eta}; \mathbf{x}, t) \quad (2.4.12)$$

with respect to the functional

$$F[\boldsymbol{\eta}; \mathbf{x}, t] = \frac{\langle f(\mathbf{x}, t) \exp(i(\boldsymbol{\eta} \mathbf{F}_1)) \rangle}{\Phi[\boldsymbol{\eta}]}, \quad (2.4.13)$$

the value of which at the functional argument $\boldsymbol{\eta} = 0$ coincides with the distribution function averaged over a statistical ensemble corresponding to a stochastic field

$$F[0; \mathbf{x}, t] = \langle f(\mathbf{x}, t) \rangle. \quad (2.4.14)$$

To obtain the equation for the averaged distribution function (see below, Eq. 2.4.15) from Eq. 2.4.12, we present, similar to Eq. 2.4.14 the functional $F[\boldsymbol{\eta}(\mathbf{x}, t); \mathbf{x}, t]$ in the form of a functional power series:

$$\begin{aligned} F[\boldsymbol{\eta}; \mathbf{x}, t] = & F_o(\mathbf{x}, t) + \int d\mathbf{x}_1 dt_1 \eta_\alpha(\mathbf{x}_1, t_1) F_{1\alpha}(\mathbf{x}, t; \mathbf{x}_1, t_1) + \int d\mathbf{x}_1 dt_1 d\mathbf{x}_2 dt_2 \eta_\alpha(\mathbf{x}_1, t_1) \\ & \times \eta_\beta(\mathbf{x}_2, t_2) F_{2\alpha\beta}(\mathbf{x}, t; \mathbf{x}_1, t_1; \mathbf{x}_2, t_2) + \int d\mathbf{x}_1 dt_1 d\mathbf{x}_2 dt_2 d\mathbf{x}_3 dt_3 \eta_\alpha(\mathbf{x}_1, t_1) \eta_\beta(\mathbf{x}_2, t_2) \\ & \times \eta_\gamma(\mathbf{x}_3, t_3) F_{3\alpha\beta\gamma}(\mathbf{x}, t; \mathbf{x}_1, t_1; \mathbf{x}_2, t_2; \mathbf{x}_3, t_3), \end{aligned} \quad (2.4.15)$$

where $F_o, F_{1\alpha}, F_{2\alpha\beta}, F_{3\alpha\beta\gamma}, \dots$ are the power functionals of the zero, first, second, third, etc. powers, respectively.

The expansion $\delta \ln \Phi[\boldsymbol{\eta}] / \delta \eta_\alpha(\mathbf{x}, t)$ is the functional power series, the n -th term of which is determined by the form of the correlation tensor of $(n+1)$ -th rank:

$$\begin{aligned} \delta \ln \Phi[\boldsymbol{\eta}] / \delta \eta_\alpha(\mathbf{x}, t) = & - \int d\mathbf{x}_1 dt_1 \eta_\beta(\mathbf{x}_1, t_1) D_{\alpha\beta}(\mathbf{x}, t; \mathbf{x}_1, t_1) \\ & + \int d\mathbf{x}_1 dt_1 d\mathbf{x}_2 dt_2 \eta_\beta(\mathbf{x}_1, t_1) \eta_\gamma(\mathbf{x}_2, t_2) D_{\alpha\beta\gamma}(\mathbf{x}, t; \mathbf{x}_1, t_1; \mathbf{x}_2, t_2) + \dots \end{aligned} \quad (2.4.16)$$

Substituting Eq. 2.4.15 and Eq. 2.4.16 in Eq. 2.4.12 we equate the functionals of the same power in the left and right hand parts of Eq. 2.4.12 to each other. The resultant infinite chain of connected equations is

$$\left\{ \begin{aligned} \left(\frac{\partial}{\partial t} + L_o \right) F_o(\mathbf{x}, t) &= i L_\alpha F_{1\alpha}(\mathbf{x}, t; \mathbf{x}, t), \\ \left(\frac{\partial}{\partial t} + L_o \right) F_{1\alpha}(\mathbf{x}, t; \mathbf{x}_1, t_1) &= -i D_{\alpha\beta}(\mathbf{x}, t; \mathbf{x}_1, t_1) L_\beta F_o(\mathbf{x}, t) + 2i L_\beta F_{2\alpha\beta}(\mathbf{x}, t; \mathbf{x}, t; \mathbf{x}_1, t_1), \\ \left(\frac{\partial}{\partial t} + L_o \right) F_{2\alpha\beta}(\mathbf{x}, t; \mathbf{x}_1, t_1; \mathbf{x}_2, t_2) &= -D_{\alpha\beta\gamma}(\mathbf{x}, t; \mathbf{x}_1, t_1; \mathbf{x}_2, t_2) L_\gamma F_o(\mathbf{x}, t) \\ &\quad - i D_{\alpha\gamma}(\mathbf{x}, t; \mathbf{x}_1, t_1) L_\gamma F_{1\beta}(\mathbf{x}, t; \mathbf{x}_2, t_2) - i D_{\beta\gamma}(\mathbf{x}, t; \mathbf{x}_2, t_2) L_\gamma F_{1\alpha}(\mathbf{x}, t; \mathbf{x}_1, t_1) \\ &\quad + 3i L_\gamma F_{3\alpha\beta\gamma}(\mathbf{x}, t; \mathbf{x}_1, t_1; \mathbf{x}_2, t_2; \mathbf{x}_3, t_3), \\ &\vdots \end{aligned} \right. \quad (2.4.17)$$

In writing Eq. 2.4.16 we took into account that it was necessary to carry out a symmetrization over the arguments and indices of the factors $\eta_\alpha(\mathbf{x}, t)$ in the highest terms of the expansions described by Eq. 2.4.15 and Eq. 2.4.16. Assuming that one of the functionals F_n is equal to zero we shall obtain a closed set of

equations. In particular, assuming $F_2 = 0$ we obtain from Eq. 2.4.17 the following set of equations:

$$\left(\frac{\partial}{\partial t} + L_o\right)F_o(\mathbf{x}, t) = iL_\alpha F_{1\alpha}(\mathbf{x}, t; \mathbf{x}, t), \quad (2.4.18)$$

$$\left(\frac{\partial}{\partial t} + L_o\right)F_{1\alpha}(\mathbf{x}, t; \mathbf{x}_1, t_1) = -iD_{\alpha\beta}(\mathbf{x}, t; \mathbf{x}_1, t_1)L_\beta F_o(\mathbf{x}, t). \quad (2.4.19)$$

To solve the set of Eq. 2.4.18 and Eq. 2.4.19, we introduce the functions $\varphi_\alpha(\mathbf{x}, t; \mathbf{x}_1, t_1)$ according to the relation

$$F_{1\alpha}(\mathbf{x}, t; \mathbf{x}_1, t_1) = \exp(-L_o t)\varphi_\alpha(\mathbf{x}, t; \mathbf{x}_1, t_1). \quad (2.4.20)$$

The action of the operator $\exp(-L_o t)$ on an arbitrary function of coordinates and moments is, as it known, the substitution of $\mathbf{r} - \Delta\mathbf{r}(t)$ for $\mathbf{r}$ and $\mathbf{p} - \Delta\mathbf{p}(t)$ for $\mathbf{p}$, where $\Delta\mathbf{r}(t)$ and $\Delta\mathbf{p}(t)$ are the variations of the radius-vector and momentum of a particle in the regular field during the time t . Substituting Eq. 2.4.20 in Eq. 2.4.13, we shall obtain

$$F_{1\alpha}(\mathbf{x}, t; \mathbf{x}_1, t_1) = -i \int_0^t dt' \exp(-L_o(t-t')) D_{\alpha\lambda}(\mathbf{x}, t; \mathbf{x}_1, t_1) L_\lambda F_o(\mathbf{x}, t). \quad (2.4.21)$$

Using Eq. 2.4.21 we derive from Eq. 2.4.20 the equation for the function $F_o(\mathbf{x}, t)$:

$$\left(\frac{\partial}{\partial t} + L_o\right)F(\mathbf{r}, \mathbf{p}, t) = L_\alpha \int_0^t dt' \exp(-L_o(t-t')) D_{\alpha\lambda}(\mathbf{r}, \mathbf{p}, t; \mathbf{r}_1, \mathbf{p}_1, t) L_\lambda F(\mathbf{r}, \mathbf{p}, t). \quad (2.4.22)$$

We have returned here to the previous notations $\{\mathbf{x}\} \rightarrow \{\mathbf{r}, \mathbf{p}\}$ and have omitted the index o in F_o . On the right hand side of Eq. 2.4.22 according to Eq. 2.4.20 and Eq. 2.4.21 $t_1 = t$ was set and one should keep in mind that after the action of the operator $\exp(-L_o(t-t'))$ it is necessary to set $\mathbf{r}_1 = \mathbf{r}$ and $\mathbf{p}_1 = \mathbf{p}$.

For further analysis of Eq. 2.4.22 one should concretize a dependence of the correlation tensor on the coordinate axes and time. If a magnetic field is completely frozen in plasma the most general form of the correlation tensor $D_{\alpha\lambda}$ compatible with the experimental data and satisfying Maxwell's equations will be as follows

$$D_{\alpha\lambda}(\mathbf{r}_1, t_1; \mathbf{r}_2, t_2) = D_{\alpha\lambda}(\boldsymbol{\rho}, \mathbf{r} - \mathbf{u}_o T), \quad (2.4.23)$$

where $\boldsymbol{\rho} = (\mathbf{r}_1 + \mathbf{r}_2)/2$, $\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1$, $T = t_2 - t_1$.

A stochastic magnetic field which is described by the tensor Eq. 2.4.23 corresponds to the case in which turbulence presents an aggregate of regions with the scales of the order of the correlation radius of the stochastic field. Inside every one of these regions the turbulence is uniform; however, the total intensity of turbulent pulsations of magnetic field varies slowly at a transition from one to another of turbulent regions. Corresponding to this the first argument in the right hand side of Eq. 2.4.23 describes a smooth variation of the intensity of turbulent pulsations at a transition from one turbulent region to another and reflects the fact that the pulsation intensity varies considerably only with the variation of $\boldsymbol{\rho}$ by the value of the order of the correlation radius l_c of a stochastic field. The second argument describes a local structure of turbulence which is a universal parameter inside a region with the characteristic scales of the order of l_c . Notation of the second argument in the form of $\mathbf{r} - \mathbf{u}_o T$ implies that one can neglect the proper motion of magnetic field inhomogeneities and consider that all space time variations of a stochastic magnetic field are connected with a transfer of stochastic inhomogeneities with the velocity $\mathbf{u}_o$. If turbulence is not only uniform but also statistically isotropic, the correlation tensor of the second rank of a stochastic magnetic field will have the following form (Monin and Yaglom, M1965, M1967; Dolginov and Toptygin, 1968):

$$D_{\alpha\lambda}(\boldsymbol{\rho}, \mathbf{r}) = B_{\alpha\lambda}(\boldsymbol{\rho}, \mathbf{r}) = \frac{1}{3} \langle H_1^2(\boldsymbol{\rho}) \rangle \left\{ \Psi(r/l_c) \delta_{\alpha\lambda} - \Psi_1(r/l_c) \frac{r_\alpha r_\lambda}{r^2} \right\}, \quad (2.4.24)$$

where

$$\Psi_1(r/l_c) = \Psi(r/l_c) - \frac{2l_c^2}{r^2} \int_0^{r/l_c} y \Psi(y) dy, \quad (2.4.25)$$

and $\Psi(r/l_c)$ is a scalar function, which is assumed to be known from observation; $\langle H_1^2(\boldsymbol{\rho}) \rangle$ is the mean square of a stochastic magnetic field.

Including Eq. 2.4.23 and setting $t - t' = \tau$ we shall write Eq. 2.4.22 in the form

$$\left(\frac{\partial}{\partial t} + L_o\right)F(\mathbf{r}, \mathbf{p}, t) = L_\alpha \int_0^t dt' \exp(-L_o \tau) \times B_{\alpha\lambda}\left(\frac{\mathbf{r} + \mathbf{r}_1}{2}, \mathbf{p}_1; \mathbf{r}_1 - \mathbf{r} - \mathbf{u}_o \tau, \mathbf{p}\right) L_\lambda F(\mathbf{r}, \mathbf{p}, t - \tau). \quad (2.4.26)$$

The right hand side of Eq. 2.4.26 differs from zero in the time intervals of the order of the correlation periods of a stochastic field. Assuming that the correlation period is small as compared with the characteristic time of variation of the average distribution function F we can write

$$\left(\frac{\partial}{\partial t} + L_o\right)F(\mathbf{r}, \mathbf{p}, t) = L_\alpha \int_0^\infty d\tau \exp(-L_o \tau) B_{\alpha\lambda}\left(\frac{\mathbf{r} + \mathbf{r}_1}{2}, \mathbf{p}_1; \mathbf{r}_1 - \mathbf{r} - \mathbf{u}_o \tau, \mathbf{p}\right) L_\lambda F(\mathbf{r}, \mathbf{p}, t). \quad (2.4.27)$$

If the momentum of a particle in the regular magnetic field H_o varies weakly at distances of the order of the correlation radius of a stochastic field one will be able to set $\Delta\mathbf{r}(\tau) = \mathbf{v}(\tau)$ and $\Delta\mathbf{p}(\tau)$ at the action of operator $\exp(-L_o \tau)$. Then

$$\left(\frac{\partial}{\partial t} + L_o\right)F(\mathbf{r}, \mathbf{p}, t) = L_\alpha \overline{B_{\alpha\lambda}(\mathbf{r}, \mathbf{p})} L_\lambda F(\mathbf{r}, \mathbf{p}, t), \quad (2.4.28)$$

where

$$\overline{B_{\alpha\lambda}(\mathbf{r}, \mathbf{p})} = \int_0^\infty d\tau B_{\alpha\lambda}(\mathbf{r}, \mathbf{v} \tau - \mathbf{u}_o \tau; \mathbf{p}). \quad (2.4.29)$$

In Eq. 2.4.28 it was included that the first argument in $B_{\alpha\lambda}$ describes a slow variation of mean square of a stochastic field with a distance (see Eq. 2.4.23), and therefore one can neglect the action of the operator $\exp(-L_o \tau)$ on this argument. At stochastic velocities $\mathbf{u}_1 = 0$ Eq. 2.4.27–2.4.29 transform into the equations which have been derived for the first time by Dolginov and Toptygin (1966a) using the diagrams technique.

Let us elucidate the character of the approximations which were made in deducing the kinetic equation. Deducing Eq. 2.4.26–2.4.29, we have closed the chain of equations postulating $F_2 = 0$. This assumption holds true in the case in which the corrections to the distribution function F connected with including the subsequent terms of the functional series Eq. 2.4.15 are small. In the case under consideration, however, it is not necessary to calculate the subsequent vanishing approximation but is possible to use the known quantum mechanical analogy (see, for example, Bonch-Bruевич and Tyablikov, M1961) according to which the

approximation based on the assumption $F_2 = 0$ corresponds to Born's approximation for perturbation theory. If a magnetic field is stationary in time the conditions of applicability of Born's approximation in the case under consideration mean that variations of a particle momentum $\delta \mathbf{p}$ in a stochastic field $\mathbf{H}_1$ are small compared to the particle momentum $\mathbf{p}$. The ratio of these quantities

$$\frac{\delta p}{p} \approx \frac{el_c \sqrt{\langle H_1^2 \rangle}}{cp} = \frac{l_c}{r_{L1}} \ll 1, \quad (2.4.30)$$

determines the condition of applicability of the considered approximation. As is seen, the condition reduces to a small value of the ratio of the correlation radius of a stochastic field to a particle Larmor radius $r_{L1} = cp / \left(e \sqrt{\langle H_1^2 \rangle} \right)$ in the stochastic

field. This condition means that a particle is scattered at a small angle $\theta \approx l_c / r_{L1}$ by every inhomogeneity. Thus an application of the diagram technique with a Gaussian distribution of inhomogeneities (Dolginov and Toptygin, 1966a, 1968a,b) and approximation $F_2 = 0$ in the functional method (Dorman and Katz, 1972a,b; 1974a,b) resulted in the kinetic equation with a collisional term which is determined by the correlation tensor of the second rank of the stochastic magnetic field. As was noted, in this case particles are scattered at a small angle by every inhomogeneity. To consider the cases when particles are scattered at large angles, one should take into account the correlators of the higher ranks in the kinetic equation. According to (Katz, 1973) the functional method makes it possible to exceed the limits of Born's approximation and to take into account the triple correlation, i.e. to consider the cases when a particle in interaction with a separate inhomogeneity of magnetic field is scattered at large angles.

2.5. Kinetic equation for propagation of CR including electric fields in plasma

In actual conditions, a turbulent motion is not obliged to be a set of some separate inhomogeneities of a magnetic field. Very often it can be represented as a set of weakly interacting collective oscillations of a medium. These oscillations in the solar wind have the most frequently a form of Alfvén waves. One should take into account the presence of the random electric fields of oscillations in plasma in addition to the turbulent magnetic fields, considering the interaction of charged particles of cosmic radiation with the magnetized plasma of the solar wind (Dorman and Katz, 1972b). In this case the kinetic equation has the following form:

$$\left(\frac{\partial}{\partial t} + L_o\right)F(\mathbf{r}, \mathbf{p}, t) = \left(\frac{e}{c}\right)^2 \frac{\partial}{\partial p_\alpha} \int_0^\infty d\tau \left\{ \exp(-L_o\tau) T_{\alpha\lambda}(\mathbf{r}, \mathbf{p}; \mathbf{r}', \mathbf{p}', \tau) \frac{\partial}{\partial p_\lambda} F(\mathbf{r}, \mathbf{p}, t) \right\}_{\substack{\mathbf{r}'=\mathbf{r} \\ \mathbf{p}'=\mathbf{p}}}, \quad (2.5.1)$$

where

$$\begin{aligned} T_{\alpha\lambda} = & \varepsilon_{\alpha\beta\gamma} \varepsilon_{\lambda\mu\nu} v_\beta v'_\mu \langle H_{1\gamma} H'_{1\nu} \rangle + c \varepsilon_{\alpha\beta\gamma} v_\beta \langle H_{1\gamma} E'_\lambda \rangle \\ & + c \varepsilon_{\lambda\mu\nu} v'_\mu \langle E_\alpha H'_{1\nu} \rangle + c^2 \langle E_\alpha E'_\lambda \rangle, \end{aligned} \quad (2.5.2)$$

and $\mathbf{E}$ are the random electric fields of oscillations and the dashed symbols correspond to the field components depending on $\mathbf{r}'$. If the regular magnetic field can be considered as homogeneous in space at distances of the order of the correlation radius of a stochastic field, the variations of the radius-vector and of the momentum of a particle are determined by the expressions:

$$\Delta \mathbf{r}(\tau) = \mathbf{R}(0, \tau, \mathbf{r}; \mathbf{p}) = \mathbf{r} + (\mathbf{h}\mathbf{v})\mathbf{v}\tau + [\mathbf{h}[\mathbf{h}\mathbf{v}]] \frac{\sin \omega_L \tau}{\omega_L} - [\mathbf{h}\mathbf{v}] \frac{1 - \cos \omega_L \tau}{\omega_L}, \quad (2.5.3)$$

$$\Delta \mathbf{p}(\tau) = \mathbf{P}(0, \tau, \mathbf{p}) = (\mathbf{h}\mathbf{p})\mathbf{h} - [\mathbf{h}[\mathbf{h}\mathbf{p}]] \cos \omega_L \tau + [\mathbf{h}\mathbf{p}] \sin \omega_L \tau, \quad (2.5.4)$$

where $\mathbf{h} = \mathbf{H}_o/H_o$ and $\omega_L = ecH_o/E$ is the Larmor frequency of a particle with total energy E . In this case Eq. 2.5.1 takes a form

$$\left(\frac{\partial}{\partial t} + L_o\right)F(\mathbf{r}, \mathbf{p}, t) = e^2 \frac{\partial}{\partial p_\alpha} \int_0^\infty d\tau T_{\alpha\lambda}[\mathbf{r}, \mathbf{p}; \mathbf{R}(0, \tau, 0; \mathbf{p}), \mathbf{P}(0, \tau, \mathbf{p}); \tau] \left(\frac{\partial F}{\partial p_\lambda}\right)_{\substack{\mathbf{r} \rightarrow \mathbf{R} \\ \mathbf{p} \rightarrow \mathbf{P}}}. \quad (2.5.5)$$

The Eq. 2.5.1–2.5.5 at the given components of corresponding correlation tensors, completely describe the process of a propagation of CR in the random electromagnetic fields. The collisional terms of these equations include both elastic and inelastic particle interactions with magnetic field inhomogeneities (or with turbulent pulsations of electromagnetic fields of oscillations). These equations give the most complete description of the spatial and angular distribution and also of variation of energy spectrum of galactic and solar CR in their interaction with space plasma.

2.6. Kinetic equation for the propagation of CR in the presence of a strong regular field in low-turbulence magnetized plasma in which the Alfvén waves are excited

2.6.1. Formulation of the problem and deduction of the basic equation

Experimental studies of CR of low energies (1–10 MeV) showed that the free path of these particles in interplanetary space exceeds 1 AU (see, for example, Vernov et al., 1968a). As a result of this fact there was found a presence of the pronounced anisotropy in the angular distribution of particles moving from the Sun. To study a propagation of such particles one should directly use the kinetic equation. The first theoretical treatment of the processes of propagation of low energy CR were carried out by Tverskoy (1967b, 1969) who paid his greatest attention to an analysis of the effects of particle acceleration in the interplanetary space. The formulation of the problem proposed by B. A. Tverskoy was used, however, as the basis in a majority of the subsequent studies where a propagation of charged particles in cosmic conditions was investigated. The most detailed consideration of the process of multiple scattering of low energy charged particles by stochastic inhomogeneities of a magnetic field has been carried out by Galperin et al. (1971) and Toptygin (1973a,b). In this Section, basing on the kinetic equation we consider a motion of low energy charged particles through weakly turbulent magnetized solar wind plasma in which Alfvén oscillations are induced. Note that the presence of Alfvén waves in the solar wind plasma is confirmed by direct measurements (see, for example, Belcher and Davis, 1969, 1971). When analyzing the motion of charged particles, together with particle scattering on the turbulent pulsations of the magnetic field we consider also the energy exchange between turbulent pulsations and charged particles owed to particle interaction with stochastic electric fields of Alfvén waves.

A propagation of CR in low turbulent magnetized plasma in which the Alfvén waves are excited has been studied in Toptygin (1971), Dorman and Katz (1972b). The process was considered of a propagation of particles the Larmor radius of which is far less than the correlation radius of a random magnetic field.

The particles of relatively small energy satisfy this condition and the magnetic field should be strong enough. The particles with energies up to ~ 1000 MeV satisfy this condition ($cp/eH_0 \ll l_c$) when CR propagate in the interplanetary field. The collision term of the kinetic equation in the case under consideration is determined by the right hand side of Eq. 2.5.4:

$$StF = e^2 \frac{\partial}{\partial p_\alpha} \int_0^\infty d\tau T_{\alpha\lambda} [\mathbf{r}, \mathbf{p}; \mathbf{R}(0, \tau, 0; \mathbf{p}), \mathbf{P}(0, \tau, \mathbf{p}); \tau] \left(\frac{\partial F}{\partial p_\lambda} \right)_{\mathbf{p} \rightarrow \mathbf{P}}^{\mathbf{r} \rightarrow \mathbf{R}}, \quad (2.6.1)$$

where $\mathbf{R}$ and $\mathbf{P}$ are determined by the Eq. 2.5.3 and Eq. 2.5.4 and $T_{\alpha\lambda}$ is determined by Eq. 2.5.2. If the Alfvén waves are excited with the frequency

$$\omega(\mathbf{k}) = v_a k_z - i\gamma(\mathbf{k}), \quad (2.6.2)$$

where v_a is the Alfvén velocity, $\gamma(\mathbf{k})$ is the fading coefficient of the Alfvén waves with the wave vector $\mathbf{k}$, then the electric and magnetic fields of the oscillation in these wave are connected by the relation

$$\mathbf{E}(\mathbf{k}, \omega) = \frac{v_a^2}{c\omega} [\mathbf{h}[\mathbf{h}[\mathbf{kH}_1(\mathbf{k}, \omega)]]]. \quad (2.6.3)$$

If these waves are isotropically distributed over a space and are statistically independent, the correlation tensor of the second rank corresponding to a stochastic magnetic field, has the following form:

$$\begin{aligned} \langle H_\gamma(\mathbf{k}, \omega) H_\nu(\mathbf{k}', \omega') \rangle &= B_{\gamma\nu}(\mathbf{k}) \delta(\mathbf{k} + \mathbf{k}') \delta(\omega + \omega') \delta(\omega + \omega(\mathbf{k})), \\ B_{\gamma\nu}(\mathbf{k}) &= (\delta_{\gamma\nu} - k_\gamma k_\nu / k^2) B(\mathbf{k}). \end{aligned} \quad (2.6.4)$$

Substituting the Eq. 2.6.3 and Eq. 2.6.4 into Eq. 2.6.1, we obtain

$$StF = \frac{\partial}{\partial p_\alpha} D_{\alpha\lambda} \frac{\partial p_z}{\partial P_\lambda} \frac{\partial F}{\partial p_z}, \quad (2.6.5)$$

and in this case

$$\begin{aligned} D_{\alpha\lambda} &= e^2 \int_0^\infty d\tau \int \exp(-i\mathbf{kR}(0, \tau, 0; \mathbf{p}) - i\omega(\mathbf{k})\tau) B(\mathbf{k}) \\ &\times \left[\frac{c^2}{E^2} N_{\alpha\lambda} - \frac{c}{E} (Q_{\alpha\lambda} + \Pi_{\alpha\lambda}) + v_a^2 \bar{T}_{\alpha\lambda} \right] d\mathbf{k}, \end{aligned} \quad (2.6.6)$$

where

$$N_{\alpha\lambda} = \delta_{\alpha\lambda}(\mathbf{pP}) - p_\alpha P_\lambda - [\mathbf{kp}]_\alpha [\mathbf{kP}]_\lambda / k^2, \quad (2.6.7)$$

$$Q_{\alpha\lambda} = a_\alpha [\mathbf{Pv}_a]_\lambda - b(\delta_{\alpha\lambda}(\mathbf{v}_a \mathbf{P}) - P_\alpha v_{a\lambda}), \quad (2.6.8)$$

$$\Pi_{\alpha\lambda} = a_\lambda [\mathbf{p}\mathbf{v}_a]_\alpha - b(\delta_{\alpha\lambda}(\mathbf{v}_a\mathbf{p}) - v_{\alpha\alpha}p_\lambda), \quad (2.6.9)$$

$$\bar{T}_{\alpha\lambda} = a_\alpha a_\lambda + b^2(\delta_{\alpha\lambda} - h_\alpha h_\lambda), \quad (2.6.10)$$

$$\mathbf{a} = [\mathbf{v}_a\mathbf{k}]/c\omega, \quad b = (\mathbf{v}_a\mathbf{k})/c\omega. \quad (2.6.11)$$

In obtaining of the Eq. 2.6.5 the assumption was made that $f(\mathbf{p}) = f(p_\perp, p_z)$ and then $f(\mathbf{p}) = f(\mathbf{P})$. Including this circumstance, let us write the kinetic equation in cylindrical coordinates in the momentum space with the Z axis directed along the regular field:

$$\begin{aligned} \frac{\partial F}{\partial t} + v_z \frac{\partial F}{\partial z} - \frac{1}{2}(\text{div}\mathbf{h})v_\perp \left(\frac{p_z \partial}{\partial p_\perp} - \frac{p_\perp \partial}{\partial p_z} \right) F = \langle \text{St}F \rangle_\varphi = & \left(\frac{p_z}{p_\perp} \frac{\partial}{\partial p_\perp} p_\perp - \frac{p_\perp \partial}{\partial p_z} \right) \\ & \times D_1 \left(\frac{p_z \partial}{\partial p_\perp} - \frac{p_\perp \partial}{\partial p_z} \right) f + \frac{1}{p_\perp} \frac{\partial}{\partial p_\perp} p_\perp D_2 \left(\frac{p_\perp \partial}{\partial p_z} - \frac{p_z \partial}{\partial p_\perp} \right) \\ & + \left(\frac{p_\perp \partial}{\partial p_z} - \frac{p_z \partial}{\partial p_\perp} \right) D_2 \frac{\partial f}{\partial p_\perp} + \frac{1}{p_\perp} \frac{\partial}{\partial p_\perp} p_\perp D_3 \frac{\partial f}{\partial p_\perp} \end{aligned} \quad (2.6.12)$$

with the coefficients

$$\begin{aligned} \left. \begin{aligned} D_1 \\ D_2 \\ D_3 \end{aligned} \right\} = e^2 \int_0^\infty d\tau \exp\{i(k_z v_z - \omega)\tau + i\alpha[\sin(\varphi - \psi + \omega_L \tau)\sin(\varphi - \psi)]\} d\mathbf{k} B(\mathbf{k}) \\ \times \left\{ \begin{aligned} & \frac{c^2}{E} \left[\cos \omega_L \tau - \left(\frac{k_\perp}{k} \right)^2 \sin(\varphi - \psi) \sin(\varphi - \psi + \omega_L \tau) \right] \\ & \frac{v_a^2 k_z \cos \omega_L \tau}{E \omega} \\ & \frac{v_a^4}{c^2 \omega^2} \left[k_\perp^2 \sin(\varphi - \psi) \sin(\varphi - \psi + \omega_L \tau) + k_z^2 \cos \omega_L \tau \right] \end{aligned} \right\}, \quad (2.6.13)$$

where φ and ψ are the azimuthally angles of the vectors $\mathbf{p}$ and $\mathbf{k}$, respectively. For a further consideration it is necessary to set the form of the function $B(\mathbf{k})$. As was shown in Toptygin (1971), Galperin et al. (1971) $B(\mathbf{k})$ has a form corresponding to the power spectrum decreasing for large k :

$$B(\mathbf{k}) = \frac{A_\nu}{(k_o^2 + k^2)^{\nu/2+1}}, \quad A_\nu = \frac{\langle H_1^2(r) \rangle \nu \Gamma(\nu/2)}{4\pi^{3/2} \Gamma(\nu/2 - 1/2)} k_o^{\nu-1}, \quad (2.6.14)$$

where $k_o^{-1} = l$ is the external scale of the turbulence, and $\langle H_1^2(r) \rangle$ is the mean square of a stochastic field. According to the experimental data for the interplanetary field in the various regions the power index ν of the spectrum has the values from 1 to 3.8. Substituting Eq. 2.6.14 into Eq. 2.6.13 we obtain for the coefficients D_1, D_2, D_3 the following expressions

$$D_1 = \left(\frac{ec}{E} \right)^2 \pi \left\{ \frac{2A_\nu v_\perp^\nu}{(v_z - v_a) \omega_L^\nu} \sum_{n=0}^{\infty} n^2 \int_0^{\infty} \frac{\alpha J_n^2(\alpha)}{(\mu_n^2 + \alpha^2)^{\nu/2+2}} d\alpha \right. \\ \left. + \frac{A_\nu v_\perp^{\nu+2}}{(v_z - v_a)^3 \omega_L^\nu} \sum_{n=0}^{\infty} (n+1)^2 \int_0^{\infty} \frac{\alpha [J_n^2(\alpha) + J_{n+2}^2(\alpha)]}{(\mu_{n+1}^2 + \alpha^2)^{\nu/2+2}} d\alpha + \beta \right\}, \quad (2.6.15)$$

$$D_2 = \frac{e^2 v_a}{E} \pi \left\{ \frac{2A_\nu v_\perp^\nu}{(v_z - v_a) \omega_L^\nu} \int_0^{\infty} \frac{\alpha [J_n^2(\alpha) + J_{n+2}^2(\alpha)]}{(\mu_{n+1}^2 + \alpha^2)^{\nu/2+1}} d\alpha + \beta \right\} \quad (2.6.16)$$

$$D_3 = \left(\frac{ev_a}{c} \right)^2 \pi \left\{ \frac{A_\nu (v_z - v_a) v_\perp^{\nu-2}}{\omega_L^\nu} \sum_{n=0}^{\infty} \frac{1}{(n+1)^2} \int_0^{\infty} \frac{\alpha^3 [J_n^2(\alpha) + J_{n+2}^2(\alpha)]}{(\mu_{n+1}^2 + \alpha^2)^{\nu/2+1}} d\alpha \right. \\ \left. + \frac{A_\nu v_\perp^\nu}{(v_z - v_a) \omega_L^\nu} \sum_{n=0}^{\infty} \int_0^{\infty} \frac{\alpha [J_n^2(\alpha) + J_{n+2}^2(\alpha)]}{(\mu_{n+1}^2 + \alpha^2)^{\nu/2+1}} d\alpha - \frac{2A_\nu (v_z - v_a) v_\perp^{\nu-2}}{\omega_L} \right. \\ \left. \times \sum_{n=1}^{\infty} \int_0^{\infty} \frac{\alpha J_n^2(\alpha)}{(\mu_n^2 + \alpha^2)^{\nu/2+1}} d\alpha + \beta \right\}, \quad (2.6.17)$$

where $J_n(\alpha)$ are the Bessel functions, and

$$\alpha = \frac{k_\perp v_\perp}{\omega_L}, \quad \beta = \frac{1}{\pi} \int \frac{\gamma(\mathbf{k}) B(\mathbf{k}) J_n^2(\alpha)}{k_z^2 (v_z - v_a)^2 + \gamma^2(\mathbf{k})} d\mathbf{k}, \quad \mu_n = \frac{v_\perp n}{v_z - v_a}. \quad (2.6.18)$$

2.6.2. The case of large wave lengths

Since $\alpha \ll 1$ for the large wave lengths, it is possible to put $J_n(\alpha) \approx \alpha^n / 2^n n!$. In this case the expressions for the coefficients D_1, D_2, D_3 have the form:

$$D_1 = \left(\frac{ec}{E} \right)^2 \pi \left[\frac{2A_\nu (v_z - v_a)^{\nu-1}}{(\nu+2)\omega_L^\nu} + \beta \right], \quad (2.6.19)$$

$$D_2 = \frac{e^2 v_a}{E} \pi \left[\frac{A_\nu (v_z - v_a)^{\nu-1}}{\nu \omega_L^\nu} + \beta \right], \quad (2.6.20)$$

$$D_3 = \left(\frac{ev_a}{c} \right)^2 \pi \left[\frac{2A_\nu \Gamma(\nu/2 - 1) (v_z - v_a)^{\nu-1}}{2\omega_L^\nu \Gamma(\nu/2)} + \beta \right], \quad \nu \neq 2. \quad (2.6.21)$$

It is seen from Eq. 2.6.21 for D_3 that D_3 is divergent at the value $\nu = 2$ of the index of the spectral function of turbulence. This is connected with the circumstance that at $\nu = 2$ a particle interacts with the waves of vanishing amplitudes, and when there is a sufficiently weak dependence of the amplitude on the scale of pulsations, the effective time of a particle interaction with a wave tends to zero.

Actually integrating over $\mathbf{k}$, one should cut off the integral at the wave length corresponding to a particle Larmor radius. At $\nu = 2$ this results in a logarithmic dependence of the diffusion coefficient on the momentum in the momentum space (Tverskoy, 1967a).

2.6.3. The case of small wave lengths

In this case $\alpha \gg 1$, and the expressions for D_1, D_2, D_3 are

$$D_1 = \left(\frac{ec}{E} \right)^2 \left\{ \frac{A_\nu (\nu+1)}{2\sqrt{\pi}} \zeta(\nu+1) \left[1 + \frac{(v_z - v_a)^2}{v_\perp^2} \right] \frac{(v_z - v_a)^\nu}{v_\perp \omega_L^\nu} + \beta \right\}, \quad (2.6.22)$$

$$D_2 = \frac{e^2 v_a}{E} \left[\frac{A_\nu \Gamma\left(\frac{\nu+1}{2}\right)}{\nu \Gamma(\nu/2) \sqrt{\pi}} \zeta(\nu+1) \frac{(v_z - v_a)^\nu}{v_\perp \omega_L^\nu} + \beta \right], \quad (2.6.23)$$

$$D_3 = \left(\frac{ev_a}{c} \right)^2 \times \left[\frac{A_\nu \Gamma(\nu/2 - 1/2)}{8\Gamma(\nu/2)\sqrt{\pi}} \zeta(\nu+1) \frac{(v_z - v_a)^\nu}{v_\perp \omega_L^\nu} - \frac{2A_\nu \Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma(\nu/2)\sqrt{\pi}} \zeta(\nu+1) \frac{(v_z - v_a)^{\nu+2}}{v_\perp^3 \omega_L^\nu} + \beta \right], \quad (2.6.24)$$

where $\zeta(\nu+1)$ is the Riemann ζ -function. To obtain the Eq. 2.6.22–2.6.24, the asymptotic behavior of the Bessel function was used and $\cos^2 x$ replaced by its mean value $\overline{\cos^2 x} = 1/2$. The item β describes the particles which are in Cherenkov resonance with the waves (Galperin et al., 1971).

Generally speaking, as we consider the particles with a velocity far more than the Alfvén velocity ($v_a \approx 60$ km/sec in the solar wind plasma), it could appear that the Cherenkov resonance is of no importance but the interaction is caused by cyclotron resonances of all orders. However, the Cherenkov resonance must be included when evaluating the time of isotropization and acceleration of particles.

The limiting transition $\gamma \rightarrow 0$ was carried out in all of the items' cyclotron summands (in all items except for β) when calculating D_1, D_2, D_3 (Galperin et al., 1971). This approximation is not applicable for calculation of β as the limit condition $\gamma \rightarrow 0$ means that the effective time particle interaction with the waves appears to be infinitely large. In fact, the presence of an imaginary part of a frequency gives a finite width to the region of interaction of a separate Fourier harmonic of the wave with moving particles. This property results in the occurrence of a finite interaction time between a wave and moving particles. As is known (Braginsky, 1963), the Alfvén waves fade out with the decrement of fading

$$\gamma(\mathbf{k}) = c^2 k_z^2 / 4\pi\sigma_\perp, \quad (2.6.25)$$

where $\sigma_\perp$ is the coefficient of the transverse conductivity of plasma. Substituting the Eq. 2.6.25 for $\gamma(\mathbf{k})$ in Eq. 2.6.18 we obtain the following expression for β (at $(v_z - v_a)^2 \gg c^2 / 4\pi\sigma_\perp$):

$$\beta = \frac{c_\nu A_\nu v_\perp^{\nu-1}}{(v_z - v_a)^2 \omega_L^{\nu-1}}, \quad (2.6.26)$$

where

$$c_v = \frac{c^2}{2\pi\sigma_\perp} \int_0^\infty dx \int_0^\infty \frac{x J_1^2(x) dy}{(x^2 + y^2)^{v/2+1}}. \quad (2.6.27)$$

The importance of the Cherenkov resonance was emphasized by Galperin et al., 1971 (see also Vedenov et al., 1962). The Eq. 2.6.17–2.6.27 together with Eq. 2.6.12 describe completely a particle motion in low-turbulent magnetized plasma. The Eq. 2.6.12 at $D_2 = D_3 = 0$ represents the process of particle diffusion in the angular space with the energy conservation. This case was studied in detail by Tverskoy (1967b). The equation of the type of Eq. 2.6.12 at $\beta = 0$ was minutely investigated also in the works of Tverskoy (1967a,b). Another possible cause of broadening of the Cherenkov resonance is the scattering of particles which was studied in details by Galperin et al. (1971).

2.7. Green's function of the kinetic equation and the features of propagation of low energy particles

Let us write Eq. 2.6.12 in the spherical coordinates in the momentum space ($p_z = p \cos \theta$, $p_\perp = p \sin \theta$):

$$\begin{aligned} \frac{\partial F}{\partial t} + v \cos \theta \frac{\partial F}{\partial z} - \frac{1}{2} (\text{div} \mathbf{h}) v \sin \theta \frac{\partial F}{\partial \theta} &= \frac{1}{p^2 \sin \theta} \frac{\partial}{\partial \theta} D_{\theta\theta} \sin \theta \frac{\partial F}{\partial \theta} \\ &+ \frac{1}{p \sin \theta} \frac{\partial}{\partial \theta} D_{\theta p} \frac{\partial F}{\partial p} + \frac{1}{p^2} \frac{\partial}{\partial p} p D_{p\theta} \frac{\partial F}{\partial \theta} + \frac{1}{p^2} \frac{\partial}{\partial p} p D_{pp} \frac{\partial F}{\partial p}, \end{aligned} \quad (2.7.1)$$

where

$$\begin{aligned} D_{\theta\theta} &= p^2 D_1 + \cos \theta (D_3 \cos \theta - 2p D_2), \quad D_{pp} = D_3 \sin \theta, \\ D_{\theta p} &= D_{p\theta} = \sin \theta (D_3 \cos \theta - p D_2). \end{aligned} \quad (2.7.2)$$

The coefficients D_1, D_2, D_3 are determined by Eq. 2.6.13–2.6.24 with the corresponding substitution of variables. At $D_{\theta p} = D_{p\theta} = D_{pp} = 0$ the Eq. 2.7.1 describes a diffusion process in the angular space taking place with energy conservation. Let us consider the Eq. 2.7.1 for this case. In the general case with arbitrary values θ of a particle's pitch-angle it was not possible to obtain a solution of the Eq. 2.7.1; however for the angles $\theta \ll 1$ there is the analytical solution of the Eq. 2.7.1. At $\theta \ll 1$ the diffusion coefficient in the angular space is determined by the Eq. 2.6.22–2.6.24 where $v_z = v \cos \theta$. Using Eq. 2.6.22–2.6.24

and Eq. 2.7.2 we write the Eq. 2.7.1 for the stationary case including in the right-hand part of the equation a point source with the coordinates z_o and θ_o , i.e. consider the equation for the Green's function $G_{//}(z)$ of the kinetic equation:

$$\frac{\partial G_{//}}{\partial z} - \frac{1}{2}(\text{div} \mathbf{h})\theta \frac{\partial G_{//}}{\partial \theta} = \frac{1}{\Lambda_{//}(z)} \frac{1}{\theta} \frac{\partial}{\partial \theta} \theta \frac{\partial G_{//}}{\partial \theta} + \frac{1}{\theta} \delta(\theta - \theta_o) \delta(z - z_o), \quad (2.7.3)$$

where $G_{//} = G_{//}(z, \theta; z_o, \theta_o)$ is Green's function of the kinetic equation and the quantity $\Lambda_{//}(z)$ according to Galperin et al. (1971) is determined by the relation

$$\Lambda_{//}(z) = \frac{4(\nu + 2)\Gamma\left(\frac{\nu - 1}{2}\right)}{\nu\sqrt{\pi}\Gamma\left(\frac{\nu}{2}\right)} \frac{H_o^2}{\langle H_1^2 \rangle} \left(\frac{l_c}{r_L}\right)^{\nu-2} l_c \quad (2.7.4)$$

and presents a particle free path along the lines of force of the regular magnetic field. For solving the Eq. 2.7.3, it is convenient to make a substitution of variables:

$\rho' \rightarrow \frac{1}{2} \ln(H_o(z))$, $\theta \rightarrow \exp(\rho' - \xi')$. As a result we obtain the equation

$$\frac{\partial G_{//}}{\partial \rho'} = \exp[2(\xi' - \rho')] \varphi(\rho') \frac{\partial^2 G_{//}}{\partial \xi'^2} \delta(\xi' - \ln \theta_o) \delta(\rho'), \quad (2.7.5)$$

where

$$\varphi(\rho') = \left[\Lambda_{//}(z) \frac{\partial \rho'}{\partial z} \right]^{-1} \bigg|_{z=z(\rho')}. \quad (2.7.6)$$

The solution of the Eq. 2.7.5 has the following form (in a detail see Dorman and Katz, 1974a):

$$G_{//}(z, \theta; z_o, \theta_o) = \frac{H_o(z_o)}{\pi H_o(z) \theta^2(z)} \exp\left(-\frac{\theta_o^2 + \theta^2}{\theta^2(z)}\right) J_o\left(\frac{2\theta_o\theta}{\theta^2(z)}\right), \quad (2.7.7)$$

where

$$\overline{\theta^2(z)} = 4 \int_{z_o}^z dz' \frac{H_o(z)}{H_o(z') \Lambda_{//}(z')}. \quad (2.7.8)$$

Here $J_o(x)$ is a modified Bessel function. We returned to the variables z, θ in Eq. 2.7.7 and Eq. 2.7.8.

At $\theta_0 = 0$ the expression for the Green's function Eq. 2.7.7 transforms to the expression derived for the first time in (Galperin et al., 1971) where it was used to explain the cases of anisotropic propagation of particles with the energy 1–5 MeV observed by direct measurements in the interplanetary space (see below, Section 2.10).

The Green's function Eq. 2.7.7 describes a distribution of particles ejected by a point source. When considering the concrete cases one should know the actual source function (it is possible that the conditions in a process of particle propagation which are constrained by conservation of the adiabatic invariant $\sin^2 \theta / H$ result in an insignificant of the character of the angular distribution of particles in a source). We should note that it is necessary to know Green's function of the kinetic equation when we analyze the finer questions of a kinetic of CR pertaining to fluctuation effects arising in their motion in the interplanetary space (see below, Section 2.8).

The Green's function of the kinetic equation can be derived in a non-stationary case as well as in the case when the particles propagate in a diffusion way across the regular magnetic field. We now write the resulting expression

$$G = G_{//}(z, \theta; z_o, \theta_o) G_{\perp}(x, y, z; x_o, y_o, z_o), \quad (2.7.9)$$

where x_o, y_o, z_o are the source coordinates, $G_{//}$ is the Green's function of the field-aligned particle motion determined by Eq. 2.7.7, and

$$G_{\perp}(x, y, z; x_o, y_o, z_o) = \frac{3}{4\pi \int_{z_o}^z dz' \Lambda_{\perp}(z')} \exp \left[- \frac{3 \left((x - x_o)^2 + (y - y_o)^2 \right)}{4 \int_{z_o}^z dz' \Lambda_{\perp}(z')} \right] \quad (2.7.10)$$

is the Green's function of the transversal motion of particles, $\Lambda_{\perp}$ is the free path of a particle across the lines of force of regular field (Toptygin, 1973a,b).

Now we consider particle scattering in the range of angles in which the inequality $|\cos \theta| = x \ll 1$ is satisfied (Galperin et al., 1971). The coefficient of diffusion $D_{\theta\theta}$ in angular space as a function of θ is somewhat decreased as θ

grows of owing to a decreasing contribution of the cyclotron resonances of higher orders. At the same time the presence of the second summand β (see Eq. 2.7.2) including Eq. 2.6.22–2.6.23 related to Cherenkov resonance starting from some θ value in $D_{\theta\theta}$ results in the increase of $D_{\theta\theta}$. This behavior of the diffusion coefficient is repeated at $\theta > \pi/2$, so that $D_{\theta\theta}(\theta) = D_{\theta\theta}(\pi - \theta)$. Thus there are two maxima on the curve of the dependence of $D_{\theta\theta}(\theta)$ at $x = x_o = |\cos \theta_o|$ on the location and depth, depending on the relation between Cherenkov's and cyclotron summands. It can be shown (Toptygin, 1973a,b) that the value of x_o is determined by the expression

$$x_o = (\gamma_o / \omega_L)^{1/(\nu+2)}, \quad \gamma_o = \gamma(k_{||} = r_L^{-1}). \quad (2.7.11)$$

It appeared that the values of free path and isotropization time are substantially different for the cases in which $x_o \approx 1$ and when $x_o \ll 1$. In the former of these cases the minimum is not deep or is even absent and it does not affect considerably the scattering of particles. As in the region of Cherenkov resonance the scattering is rapid, the isotropization time is generally determined by the range of angles $0 < \theta \leq 1$, i.e. by the range where the expression for D_1 determined by Eq. 2.6.23 is applicable (or, in other words, the field-aligned free path is determined by the Eq. 2.7.4). In this case the isotropization time is determined by the relation $\tau = \Lambda_{||} / v$. We shall emphasize the characteristic dependence of a free path on the momentum of a particle (Galperin et al., 1971). At $\nu > 2$ the free path is decreased with the growth of a particle momentum. This is related to the fact that with a growth of Larmor radius, the particles will be scattered by inhomogeneities of a more scale of increasing number. At $\nu = 2$ the free path stops being dependent on the momentum. This circumstance was emphasized by Dorman and Miroshnichenko (1965) when analyzing the data on a propagation of CR from solar flares.

In the case of a narrow Cherenkov resonance, the pitch-angle scattering of particles in the range $\cos \theta \approx x_o$ is abruptly weakened; this results in a considerable increase of isotropization time and free path of particles. The analytical solution of the Eq. 2.7.1 in this case can be obtained with the conditions $x_o \leq x \ll 1$, $H_o = \text{const}$:

$$\frac{1}{v} \frac{\partial F}{\partial t} \pm x \frac{\partial F}{\partial z} = \frac{1}{l} \frac{\partial}{\partial x} x^\nu \frac{\partial F}{\partial x}. \quad (2.7.12)$$

The signs $\pm$ corresponds to $\cos \theta > 0$ or $\cos \theta < 0$, respectively, and the quantity

$$l = \frac{4(\nu+2)}{(\nu^2-1)\xi(\nu+1)} \frac{H_o^2}{\langle H_1^2 \rangle} \left(\frac{l_c}{r_L} \right)^{\nu-2} l_c \quad (2.7.13)$$

differs from $\Lambda_{//}$ in Eq. 2.7.4 by a factor of the order of unity. We shall consider the case $\nu=2$. We neglect a spatial inhomogeneity of the system ($\partial F/\partial z=0, l=\text{const}$) and follow a population of the range of angles between $x=x_o$ and $x=x_1$ ($x_o \ll x \ll 1$). The Eq. 2.7.13 takes the form

$$\frac{\partial F}{\partial \tau'} = x^2 \frac{\partial^2 F}{\partial x^2} + 2x \frac{\partial F}{\partial x}, \quad (2.7.14)$$

where a dimensionless time $\tau' = \nu t/l$. Constrain the distribution functions by the boundary conditions $F(x_1) = F_1$, $F(x_o) = 0$. The constant $F_1 = (2\pi)^{-1}$ if the region $x > x_1$ is occupied by particles and the distribution function is normalized to unit.

The second condition corresponds to the assumption that the particles coming onto the boundary $x = x_o$ are immediately removed into the backward hemisphere of the angular space. This approximation is sufficient for evaluation of the order of the isotropization time. Solution of the Eq. 2.7.14 with the boundary conditions described has the form

$$F(x, \tau') = F_1 \left(1 - \frac{x_o}{x} \right) + x^{-1/2} \sum_{n=1}^{\infty} A_n \exp \left[- \left(\frac{1}{4} + \lambda_n^2 \right) \tau' \right] \sin \left(\lambda_n \ln \frac{x}{x_o} \right), \quad (2.7.15)$$

where $\lambda_n = \pi n \ln(x_1/x_o)$; the factors λ_n and the coefficients A_n are determined by the initial condition. The dimensionless time of filling of the zone $x_o < x < x_1$ as it results from Eq. 2.7.15 is of the order of

$$\tau_1 = \frac{\nu t_1}{l} \approx \left[\frac{1}{4} + \left(\frac{\pi}{\ln(x_1/x_o)} \right)^2 \right]^{-1}. \quad (2.7.16)$$

This time is varied from zero in the case of a broad resonance $x_o \approx x_1 \approx 1$ to the value $\tau_1 = 4$ at $x_o \rightarrow 0$. The time τ_o for the particle to scatter through an angle $\theta \approx 1$ is of the order of unity according to the previous results. If the initial distribution function of particles is such that their number is approximately equal

at $\theta < \pi/2$ and at $\theta > \pi/2$, the isotropization period $\tau_s \approx \tau_o + \tau_1$. If, however, the distribution function is pronouncedly anisotropic in the initial instant of time, the time of isotropization will be considerably more. These result from the fact that the penetration of particles from the frontal to backward hemisphere is slower at small x_o . We shall obtain the velocity of the particle passing to a backward hemisphere is found by integrating Eq. 2.7.14 over x :

$$\frac{dN}{d\tau} = -x_o^2 \left(\frac{\partial F}{\partial x} \right) \Big|_{x=x_o}. \quad (2.7.17)$$

This results in the following order of the value:

$$\tau_s = - \left(2\pi \frac{dN}{d\tau'} \right)^{-1}. \quad (2.7.18)$$

If $\tau_s \gg \tau_o + 1$ then at $\tau_s \gg \tau \gg \tau_1$ a single term remains in the right-hand part of Eq. 2.7.15 which describes the quasi-stationary distribution of particles in the frontal hemisphere. Using this value of F from Eq. 2.7.18, we obtain

$$\tau_s = x_o^{-1} \gg 1. \quad (2.7.19)$$

As results from the estimates, during the time interval $t_s = l/vx_o$ a stream of CR has a specific structure: the frontal hemisphere is completely filled by particles and in the backward hemisphere there are few particles and abrupt gradient in the angular distribution exists near $x = x_o$.

At $\nu \neq 2$ qualitative features of the isotropization process remain the same as in the case $\nu = 2$. For the isotropization time the estimate is $\tau_s \approx x_o^{1-\nu}$, which holds true for $x_o^{1-\nu} \gg 1$. A path for scattering at the angle π according to this estimate and Eq. 2.7.11 and Eq. 2.7.13 has the order of value

$$\Lambda = l\tau_s \approx l(\omega_L/\gamma_o)^{(\nu-1)/(\nu+2)}. \quad (2.7.20)$$

An additional increase of the free path Λ occurs if the regular field H_o is inhomogeneous and particles move in the direction of its decrease. Focusing of particles arising as a result of the conservation of the adiabatic invariant $\sin^2 \theta/H_o$ prevents particle penetration into backward of the angular space. An estimate of

τ_s can be obtained in this case in a following way. In a weakly inhomogeneous field the Eq. 2.7.1 for a stationary case takes a form ($x_o < x \ll 1$):

$$x^\nu \frac{d^2 F}{dx^2} + (\nu x^{\nu-1} - \theta_1) \frac{dF}{dx} = 0, \quad (2.7.21)$$

where $\theta_1 = (l/2)\text{div}\mathbf{h} = \text{const}$; $\theta_1 > 0$ if the particles move towards the decrease in H_o . The solution of Eq. 2.7.21 with the same boundary conditions as for Eq. 2.7.12 at $\theta_1 \ll \nu x_o^{\nu-1}$ then gives the same result as in the case $H_o = \text{const}$, and at $\theta_1 \gg \nu x_o^{\nu-1}$ we obtain

$$F(x) = F_1 \left\{ 1 - \exp \left[\frac{\theta_1 (x^{\nu-1} - x_o^{\nu-1})}{(\nu-1)x_o^{\nu-1}x^{\nu-1}} \right] \right\}. \quad (2.7.22)$$

Estimating the isotropization time we obtain

$$\tau_s = \frac{x_o^{2-\nu}}{\theta_1} \exp \left[\frac{\theta_1}{(\nu-1)x_o^{\nu-1}} \right]. \quad (2.7.23)$$

Therefore an additional factor $(x_o/\theta_1)\exp(\nu/(\nu-1)x_o^{\nu-1})$ will appear in the expression for the free path Λ in Eq. 2.7.20. Basing on the obtained relations and using observational data on the spectrum of magnetic field inhomogeneities, one can estimate a free path of low energy particles in the interplanetary space. However, the experimental data obtained by various authors in different time, are considerably different. Using, for example, the data from (Jokipii and Coleman, 1968) and estimating the collision width of the Cherenkov resonance, we find $x_o = 0.9$ for protons with the energy ~ 1 MeV. This means that a weakening of scattering at $x \approx x_o$ is small. The estimate of a free path according to Eq. 2.7.4 gives the value of the order of 1 AU. This value of $\Lambda_{//}$ is in agreement with the observational data presented in (Vernov et al., 1968a). On the other hand, a free path calculated from the data of (Sari and Ness, 1969, 1970) appears to be more than 1 AU. It should be emphasized that if the main contribution in the observed spectrum of magnetic field inhomogeneities is caused by hydromagnetic discontinuities (as was assumed by Sari and Ness, 1969), the developed theory can appear to be in applicable since a particle can be scattered at a large angle immediately when passing through a discontinuity.

2.8. Kinetics of CR in a large scale magnetic field

2.8.1. The kinetic equation deriving on the basis of the functional method

The problem of propagation of CR in a large scale magnetic field has been discussed in Toptygin (1973a, M1983) on the basis of the drift kinetic equation. The quasi-linear approximation, which permits describing in unique way processes of CR scattering and their diffusion across the lines of force of a regular magnetic field, was used by Toptygin (1973a, M1983) to derive the kinetic equation for the average distribution function of CR. The problem of the diffusion of CR in a large scale field has been discussed by other methods in Ptuskin (1985) and Zybin and Istomin (1985). A kinetic equation describing the propagation of CR in a large scale field was derived by Dorman, Katz and Stehlik (1988) on the basis of the functional method (Klyatskin, M1975; Rytov et al., M1977; see also above, Section 2.2).

In order to describing the motion of CR particles in a strong magnetic field we shall use the equations of motion in the drift approximation (Sivukhin, 1963)

$$dR/dt = V_{||}h, \quad dP_{\perp}^2/dt = -V_{||}P_{\perp}^2\nabla h, \quad (2.8.1)$$

$$dP_{||}/dt = -(1/2)P_{||}^{-1}dP_{\perp}^2/dt = (1/2)V_{\perp}P_{\perp}\nabla h, \quad (2.8.2)$$

where $R(t)$ is the radius vector of the guiding center, $P_{\perp}$ and $P_{||}$ are the transverse and longitudinal components of the particle's momentum with respect to the direction of the magnetic field $\mathbf{H}(\mathbf{r}, t)$, $\mathbf{h} = \mathbf{H}/H$, and $V_{\perp}$ and $V_{||}$ are the components of the particle's velocity perpendicular and parallel to the magnetic field. The large scale field $\mathbf{H}(r, t)$ has regular $\mathbf{H}_o$ and random $\mathbf{H}_1(\mathbf{r}, t)$ components:

$$\mathbf{H} = \mathbf{H}_o + \mathbf{H}_1, \quad \langle \mathbf{H} \rangle = \mathbf{H}_o, \quad \langle \mathbf{H}_1 \rangle = 0. \quad (2.8.3)$$

If the random component is much larger than the regular one, then expanding Eq. 2.8.1 and Eq. 2.8.2 into series in powers of the random field to within the accuracy of second-order terms and averaging the equations obtained over the directions of the particle's momentum in the plane perpendicular to the regular field $\mathbf{H}_o$, we obtain:

$$dr_{\alpha}/dt = V_{||}(n_{\alpha} + H_{1\alpha\perp} + \xi_{\alpha\beta\gamma}H_{1\beta}H_{1\gamma}), \quad (2.8.4)$$

$$dp_{\perp}^2/dt = -V_{||}p_{\perp}^2\nabla_{r\alpha}(H_{1\alpha\perp} + \xi_{\alpha\beta\gamma}H_{1\beta}H_{1\gamma}), \quad (2.8.5)$$

$$dp_{//}/dt = (1/2)V_{\perp}p_{\perp}\nabla_{r\alpha}(H_{1\alpha\perp} + \xi_{\alpha\beta\gamma}H_{1\beta}H_{1\gamma}) = -(1/2)p_{//}^{-1}(dp_{\perp}^2/dt), \quad (2.8.6)$$

where

$$\begin{aligned} \xi_{\alpha\beta\gamma} &= -n_{\beta}\Delta_{\alpha\gamma}(\mathbf{n}) - (1/2)n_{\alpha}\Delta_{\beta\gamma}(\mathbf{n}), \quad \Delta_{\alpha\gamma}(\mathbf{n}) = \delta_{\alpha\gamma} - n_{\alpha}n_{\gamma}, \\ \mathbf{n} &= \mathbf{H}_o/|\mathbf{H}_o|, \quad H_{1\alpha\perp} = \Delta_{\alpha\beta}(\mathbf{n})H_{1\beta} = \Delta_{\alpha\gamma}(\mathbf{n})H_{1\gamma}, \end{aligned} \quad (2.8.7)$$

and $\mathbf{r}(t)$ is the radius vector of the guiding center of the particle averaged over the directions of the particle's momentum vector in the plane perpendicular to the field $\mathbf{H}_o$, which is assumed to be uniform in space and constant in time, and $p_{\perp}$ and $p_{//}$ are the components of the particle's momentum vector across and along the direction of the magnetic field $\mathbf{H}_o$. The random field $\mathbf{H}_1(\mathbf{r}, t)$ in Eq. 2.8.4–2.8.7 is measured below in units of H_o .

The system of Eq. 2.8.4–2.8.6 satisfies the condition (Sivukhin, 1963)

$$\nabla_r \frac{d\mathbf{r}}{dt} + \frac{\partial}{\partial p_{\perp}^2} \frac{dp_{\perp}^2}{dt} + \frac{\partial}{\partial p_{//}} \frac{dp_{//}}{dt} = 0 \quad (2.8.8)$$

a consequence of which is the Liouville theorem

$$\frac{df}{dt} = \frac{\partial f}{\partial t} \nabla_r \frac{d\mathbf{r}}{dt} f + \frac{\partial}{\partial p_{\perp}^2} \frac{dp_{\perp}^2}{dt} f + \frac{\partial}{\partial p_{//}} \frac{dp_{//}}{dt} f = 0, \quad (2.8.9)$$

i.e., the equality to zero of the total derivative of the distribution function $f(\mathbf{r}, p_{\perp}, p_{//}, t)$, calculated along the drift trajectory of motion of the particles. If particles moving in a large scale field interact with very small scale inhomogeneities of the magnetic field, it is necessary to supplement the Eq. 2.8.9 with a collision integral, which determines the variations of the distribution function as a consequence of the scattering of particles in the very small scale random magnetic field:

$$\frac{\partial f}{\partial t} \nabla_r \frac{d\mathbf{r}}{dt} f + \frac{\partial}{\partial p_{\perp}^2} \frac{dp_{\perp}^2}{dt} f + \frac{\partial}{\partial p_{//}} \frac{dp_{//}}{dt} f = \text{St } f. \quad (2.8.10)$$

It is necessary to average Eq. 2.8.10 over the ensemble of realizations of the large scale random field:

$$\frac{\partial F}{\partial t} + \nabla_r \left\langle \frac{d\mathbf{r}}{dt} f \right\rangle + \frac{\partial}{\partial p_{\perp}^2} \left\langle \frac{dp_{\perp}^2}{dt} f \right\rangle + \frac{\partial}{\partial p_{\parallel}} \left\langle \frac{dp_{\parallel}}{dt} f \right\rangle = \langle \text{St} f \rangle, \quad (2.8.11)$$

where $F = \langle f \rangle$ is the average distribution function. We shall make use of the functional method of Klyatskin (M1975) and Rytov et al. (M1977) to carry out the averaging. We shall write the average which figures in the second term in Eq. 2.8.11 in the form

$$\left\langle \frac{dr_{\alpha}}{dt} f \right\rangle = V_{\parallel} n_{\alpha} F + V_{\parallel} \langle H_{1\alpha\perp}(\mathbf{r}, t) f \rangle + V_{\parallel} \xi_{\alpha\beta\gamma} \langle H_{1\beta}(\mathbf{r}, t) H_{1\gamma}(\mathbf{r}, t) f \rangle. \quad (2.8.12)$$

For the averaging of the second and third terms on the right hand side of Eq. 2.8.12 we shall make use of the well known formula (Klyatskin, M1975; Rytov et al., M1977):

$$\langle H_{1\beta}(\mathbf{r}, t) f[\mathbf{H}_1] \rangle = \int_0^t dt_1 \int d\mathbf{r}_1 B_{\beta\mu}(\mathbf{r}, t; \mathbf{r}_1, t_1) \left\langle \frac{\delta f[\mathbf{H}_1]}{\delta H_{1\mu}(\mathbf{r}_1, t_1)} \right\rangle, \quad (2.8.13)$$

which is valid for any Gaussian field $\mathbf{H}_1(\mathbf{r}, t)$ with zero average value and correlation tensor

$$B_{\beta\mu}(\mathbf{r}, t; \mathbf{r}_1, t_1) = \langle H_{1\beta}(\mathbf{r}, t) H_{1\mu}(\mathbf{r}_1, t_1) \rangle. \quad (2.8.14)$$

It is necessary for performing the averaging to calculate the functional derivative $\delta f[\mathbf{H}_1] / \delta H_{1\mu}(\mathbf{r}_1, t_1)$. We shall make use for its calculation of Eq. 2.8.10

$$\begin{aligned} & \left(\frac{\partial}{\partial t} + V_{\parallel} \nabla - St \right) \frac{\delta f[\mathbf{H}_1]}{\delta H_{1\mu}(\mathbf{r}_1, t_1)} \\ & = -V_{\parallel} \delta(t - t_1) \left[\nabla_{\perp r\mu} \delta(\mathbf{r} - \mathbf{r}_1) - \left(\nabla_{\perp r\mu} \delta(\mathbf{r} - \mathbf{r}_1) \hat{O} p_{\perp}^2 \right) \right] f[\mathbf{H}_1], \end{aligned} \quad (2.8.15)$$

where $\hat{O} = \partial / \partial p_{\perp}^2 - \partial / \partial p_{\parallel}^2$.

The formal solution of Eq. 2.8.15 is of the form

$$\begin{aligned} & \frac{\delta f[\mathbf{H}_1]}{\delta H_{1\mu}(\mathbf{r}_1, t_1)} \\ & = - \int d\mathbf{p}_1 V_{\parallel} \left\{ G_{\mathbf{pp}_1}(\mathbf{r}, t; \mathbf{r}_1, t_1) \nabla_{\perp r\mu} - \left(\nabla_{\perp r\mu} G_{\mathbf{pp}_1}(\mathbf{r}, t; \mathbf{r}_1, t_1) \hat{O} p_{\perp}^2 \right) \right\} f[\mathbf{H}_1, \mathbf{r}_1, \mathbf{p}_1, t_1], \end{aligned} \quad (2.8.16)$$

where $G_{\mathbf{pp}_1}(\mathbf{r}, t; \mathbf{r}_1, t_1)$ is the Green's function of Eq. 2.8.15, $\mathbf{p} = \{p_\perp, p_\parallel\}$, and $d\mathbf{p} = p_\perp dp_\perp$. Taking Eq. 2.8.16 into account we obtain from Eq. 2.8.13:

$$\begin{aligned} \left\langle H_{1\beta}(\mathbf{r}, t) f[\mathbf{H}_1] \right\rangle = & - \int_0^t dt_1 \int d\mathbf{r}_1 d\mathbf{p}_1 V_{1//} G_{\mathbf{pp}_1}(\mathbf{r}, t; \mathbf{r}_1, t_1) \\ & \times \left\{ B_{\beta\mu}(\mathbf{r}, t; \mathbf{r}_1, t_1) \nabla_{\perp r\mu} - \left(\nabla_{\perp r\mu} B_{\beta\mu}(\mathbf{r}, t; \mathbf{r}_1, t_1) \hat{O}_1 p_\perp^2 \right) \right\} F(\mathbf{r}_1, \mathbf{p}_1, t_1), \end{aligned} \quad (2.8.17)$$

Averaging of the third term of the right hand side of Eq. 2.8.12 to within the accuracy of second-order terms in the random field leads to the relationship

$$\left\langle H_{1\beta}(\mathbf{r}, t) H_{1\gamma}(\mathbf{r}, t) f[\mathbf{H}_1] \right\rangle = B_{\beta\gamma}(\mathbf{r}, t; \mathbf{r}, t) F(\mathbf{r}, \mathbf{p}, t). \quad (2.8.18)$$

Taking Eq. 2.8.17 and Eq. 2.8.14 into account we obtain:

$$\begin{aligned} \left\langle \frac{dr_\alpha}{dt} f[\mathbf{H}_1] \right\rangle = & V_{//} \eta_\alpha F(\mathbf{r}, \mathbf{p}, t) - V_{//} \Delta_{\alpha\beta}(\mathbf{n}) \int_0^t dt_1 \int d\mathbf{r}_1 d\mathbf{p}_1 G_{\mathbf{pp}_1}(\mathbf{r}, t; \mathbf{r}_1, t_1) V_{1//} \\ & \times \left\{ B_{\beta\mu}(\mathbf{r}, t; \mathbf{r}_1, t_1) \nabla_{\perp \mathbf{r}_1 \mu} - \left(\nabla_{\perp \mathbf{r}_1 \mu} B_{\beta\mu}(\mathbf{r}, t; \mathbf{r}_1, t_1) \hat{O}_1 p_{1\perp}^2 \right) \right\} F(\mathbf{r}_1, \mathbf{p}_1, t_1), \end{aligned} \quad (2.8.19)$$

where

$$\eta_\alpha = n_\alpha + \xi_{\alpha\beta\gamma} B_{\beta\gamma}(\mathbf{r}, t; \mathbf{r}, t). \quad (2.8.19a)$$

Averaging of the third and fourth terms on the left side of Eq. 2.8.11 is performed similarly:

$$\begin{aligned} \left\langle \frac{dp_\perp^2}{dt} f[\mathbf{H}_1] \right\rangle = & -V_{//} p_\perp^2 \int_0^t dt_1 \int d\mathbf{r}_1 d\mathbf{p}_1 G_{\mathbf{pp}_1}(\mathbf{r}, t; \mathbf{r}_1, t_1) V_{1//} \\ & \times \left\{ \nabla_{\perp \mathbf{r}\beta} \nabla_{\perp \mathbf{r}\mu} B_{\beta\mu}(\mathbf{r}, t; \mathbf{r}_1, t_1) \hat{O}_1 p_{1\perp}^2 - \left(\nabla_{\perp \mathbf{r}\beta} B_{\beta\mu}(\mathbf{r}, t; \mathbf{r}_1, t_1) \nabla_{\perp \mathbf{r}\mu} \right) \right\} F(\mathbf{r}_1, \mathbf{p}_1, t_1); \\ \left\langle \frac{dp_\parallel}{dt} f[\mathbf{H}_1] \right\rangle = & -\frac{1}{2} \left\langle p_\parallel^{-1} \frac{dp_\perp^2}{dt} f[\mathbf{H}_1] \right\rangle. \end{aligned} \quad (2.8.20)$$

We shall make use of the simplest model of scattering of particles by statistical inhomogeneities of the magnetic field (Dolginov and Toptygin, 1966a), in which

one can neglect the influence of the large scale field on scattering of particles at scales of the order of the correlation radius L_c of the very small scale field, to average the collision integral $\text{St}f$. In this case

$$\langle \text{St}f \rangle = 2p_{//} \hat{O} \frac{V}{\Lambda} p_{\perp}^2 p_{//} \hat{O} F(\mathbf{r}, \mathbf{p}, t), \quad (2.8.21)$$

where $\Lambda(p)$ is the transport mean free path of a particle with respect to scattering by very small scale inhomogeneities of the magnetic field. Taking Eq. 2.8.19–2.8.21 into account, we write the kinetic Eq. 2.8.11 averaged over the large-scale random field:

$$\begin{aligned} \left(\frac{\partial}{\partial t} + \nabla_{\mathbf{r}} \eta V_{//} - St \right) F(\mathbf{r}, \mathbf{p}, t) = & \nabla_{\perp \mathbf{r} \alpha} \hat{\kappa}_{\perp \alpha \lambda} \nabla_{\perp \mathbf{r} \lambda} F(\mathbf{r} - \rho, \mathbf{p}_1, t - \tau) \\ & + V_{//} \hat{O} p_{\perp}^2 \hat{D} \hat{O} F(\mathbf{r} - \rho, \mathbf{p}_1, t - \tau), \end{aligned} \quad (2.8.22)$$

where the operators $\hat{\kappa}_{\perp \alpha \lambda}$ and $\hat{D}$ are defined by the relationships

$$\hat{\kappa}_{\perp \alpha \lambda} = V_{//} \int_0^t d\tau \int d\rho d\mathbf{p}_1 G_{\mathbf{p}\mathbf{p}_1}(\rho, \tau) V_{1//} B_{\alpha \lambda}(\rho, \tau), \quad (2.8.23)$$

$$\hat{D} = \int_0^t d\tau \int d\rho d\mathbf{p}_1 G_{\mathbf{p}\mathbf{p}_1}(\rho, \tau) \nabla_{\perp \rho \alpha} \nabla_{\perp \rho \lambda} B_{\alpha \lambda}(\rho, \tau). \quad (2.8.24)$$

If the random field described by the tensor $B_{\alpha \lambda}(\rho, \tau)$ is delta-correlated in time, Eq. 2.8.22 takes the form corresponding to the Fokker-Planck kinetic equation:

$$\left(\frac{\partial}{\partial t} + \nabla_{\mathbf{r}} \eta V_{//} - St \right) F(\mathbf{r}, \mathbf{p}, t) = \nabla_{\perp \mathbf{r} \alpha} \kappa_{\perp \alpha \lambda} \nabla_{\perp \mathbf{r} \lambda} F(\mathbf{r}, \mathbf{p}, t) + V_{//} \hat{O} p_{\perp}^2 D \hat{O} F(\mathbf{r}, \mathbf{p}, t), \quad (2.8.25)$$

where

$$\kappa_{\perp \alpha \lambda} = V_{//}^2 B_{\alpha \lambda}(0, 0), \quad D = \nabla_{\perp \rho \alpha} \nabla_{\perp \rho \lambda} B_{\alpha \lambda}(\rho, 0)_{\rho \rightarrow 0}. \quad (2.8.26)$$

In many cases the average distribution function and its derivatives with respect to the momentum vary slowly at the characteristic spatial and temporal scales of variation of the random field, one consequently can remove the distribution

function from under the integral sign on the right hand side of Eq. 2.8.22 and switch to the Fokker-Planck Eq. 2.8.25. In this case

$$\kappa_{\perp\alpha\lambda} = V_{//} \int_0^{\infty} d\tau \int d\rho d\mathbf{p}_1 G_{\mathbf{pp}_1}(\rho, \tau) \mathcal{V}_{1//} B_{\alpha\lambda}(\rho, \tau), \quad (2.8.27)$$

$$D = \int_0^{\infty} d\tau \int d\rho d\mathbf{p}_1 G_{\mathbf{pp}_1}(\rho, \tau) \nabla_{\perp\rho\alpha} \nabla_{\perp\rho\lambda} B_{\alpha\lambda}(\rho, \tau). \quad (2.8.28)$$

It is necessary for subsequent analysis of the kinetic equation to specify an explicit form of the tensor $B_{\alpha\lambda}(\rho, \tau)$. The experimental data of Matthaeus and Smith (1981) show that fluctuations of the magnetic field are statistically anisotropic; the spectrum of the fluctuations is axe-symmetric with respect to the direction $\mathbf{n}$ of the regular field, so that the spectral tensor

$$\begin{aligned} B_{\alpha\lambda}(\mathbf{k}, \tau) = & \frac{1}{3} \left\langle H_1^2 \right\rangle \left\{ B(\mathbf{k}, \tau) \Delta_{\alpha\lambda}(\mathbf{k}) + B_1(\mathbf{k}, \tau) \left(n_{\alpha} n_{\lambda} + x^2 \frac{k_{\alpha} k_{\lambda}}{k^2} - x \frac{n_{\alpha} k_{\lambda} + n_{\lambda} k_{\alpha}}{k} \right) \right. \\ & + k^{-1} B_2(\mathbf{k}, \tau) (n_{\alpha} [\mathbf{k}\mathbf{n}]_{\lambda} + n_{\lambda} [\mathbf{k}\mathbf{n}]_{\alpha}) + x k^{-1} B_2(\mathbf{k}, \tau) (k_{\alpha} [\mathbf{k}\mathbf{n}]_{\lambda} + k_{\lambda} [\mathbf{k}\mathbf{n}]_{\alpha}) \\ & \left. + i B_3(\mathbf{k}, \tau) \varepsilon_{\alpha\lambda\gamma} (k_{\gamma} / k) \right\}, \end{aligned} \quad (2.8.29)$$

where B , B_1 , and B_3 , are even and B_2 is an odd function of the wave vector $\mathbf{k}$, $\Delta_{\alpha\lambda}(\mathbf{k}) = \delta_{\alpha\lambda} - k_{\alpha} k_{\lambda} / k^2$, $x = \mathbf{n}\mathbf{k} / k$, and $\varepsilon_{\alpha\lambda\gamma}$ is a unit vector of the third rank.

Using Eq. 2.8.29 we obtain from Eq. 2.8.28:

$$D = \frac{1}{3} \left\langle H_1^2 \right\rangle \int_0^{\infty} d\tau \int d\mathbf{k} d\mathbf{p}_1 G_{\mathbf{pp}_1}(\mathbf{k}, \tau) \frac{k_{\perp}^2 k_{//}^2}{k^2} \left\{ B(\mathbf{k}, \tau) + \frac{k_{\perp}^2}{k^2} B_1(\mathbf{k}, \tau) \right\}, \quad (2.8.30)$$

where $G_{\mathbf{pp}_1}(\mathbf{k}, \tau)$ is the Fourier transform of the Green's function, and $k_{//} = \mathbf{n}\mathbf{k}$, $k_{\perp} = [\mathbf{n}[\mathbf{k}\mathbf{n}]]$. It is evident from Eq. 2.8.30 that $D = 0$ in cases of one-dimensional and two-dimensional turbulence. If the regular field is uniform in space then the Green's function (at $x \ll 1$) is

$$G_{\mathbf{pp}_1}(\mathbf{p}, \tau) = \frac{1}{p^2} \delta(p - p_1) \delta(\mathbf{p} - \mathbf{V}_{//}\tau) \sum_{l=0}^{\infty} \exp\left(-\frac{l(l+1)}{\tau_s} \tau\right) \left(l + \frac{1}{2}\right) P_l(\mathbf{n}\mathbf{g}) P_l(\mathbf{n}\mathbf{g}_1), \quad (2.8.31)$$

where $\tau_s = \Lambda / 4V$ is the scattering time of particles by very small scale inhomogeneities of the field, $\mathbf{n}\mathbf{g} = \mathbf{n}\mathbf{p} / p$, $\mathbf{n}\mathbf{g}_1 = \mathbf{n}\mathbf{p}_1 / p_1$, and $P_l(x)$ is the Legendre

polynomial. Taking account of Eq. 2.8.31, we obtain from Eq. 2.8.27 and Eq. 2.8.28:

$$\kappa_{\perp\alpha\lambda} = \pi V_{//}^2 \int_0^\infty d\tau \exp\left(-\frac{\tau}{\tau_s}\right) B_{\alpha\lambda}(V_{//}\tau, \tau), \quad D = \int_0^\infty d\tau \nabla_{\perp\rho\alpha} \nabla_{\perp\rho\lambda} B_{\alpha\lambda}(\rho, \tau) \Big|_{\rho=V_{//}\tau}. \quad (2.8.32)$$

If the spectral tensor $B_{\alpha\lambda}(\mathbf{k}, \tau)$ is defined by Eq. 2.8.29, then the coefficient

$$D = \frac{1}{3} \left\langle H_1^2 \right\rangle_0^\infty \int d\tau \int dk \exp(i\mathbf{k}\mathbf{V}_{//}\tau) \frac{k_\perp^2 k_{//}^2}{k^2} \left\{ B(\mathbf{k}, \tau) + \frac{k_\perp^2}{k^2} B_1(\mathbf{k}, \tau) \right\}, \quad (2.8.33)$$

and it vanishes in the case in which the tensor $B_{\alpha\lambda}(\mathbf{k}, \tau)$ does not explicitly depend on time or in the case of frozen turbulence.

2.8.2. Diffusion approximation

We shall write Eq. 2.8.22, in which we shall set $\hat{D} = 0$, in the form

$$\partial F / \partial t + \nabla \mathbf{J} = StF, \quad (2.8.34)$$

$$J_\alpha(\mathbf{r}, p, t) = V_{//} \eta_\alpha F(\mathbf{r}, \mathbf{p}, t) - \hat{\kappa}_{\perp\alpha\lambda} \nabla_{\perp\mathbf{r}\lambda} F(\mathbf{r}, \mathbf{p}, t), \quad (2.8.35)$$

where $J_\alpha(\mathbf{r}, p, t)$ is the particle flux in space. It is assumed in writing Eq. 2.8.34 and Eq. 2.8.35 that the distribution function varies weakly at spatial scales of the order of the correlation radius of the large scale field. The particle flux with the specified magnitude of the momentum is

$$\mathbf{J} = \eta J_{//} + \mathbf{J}_\perp, \quad J_{//} = \overline{V_x F(\mathbf{r}, \mathbf{p}, t)}, \quad (2.8.36)$$

$$J_{\perp\alpha} = -\overline{\hat{\kappa}_{\perp\alpha\lambda} \nabla_{\perp\mathbf{r}\lambda} F(\mathbf{r}, \mathbf{p}, t - \tau)}, \quad (2.8.37)$$

where $J_{//}$ is the particle flux along the direction of the regular field and $J_{\perp\alpha}$ is the particle flux across the direction of the regular field. The bar in Eq. 2.8.36 and Eq. 2.8.37 denotes averaging over pitch-angle, and $x = \mathbf{n}\mathbf{g} = \cos\theta$.

Representing the distribution function in the form of a series of Legendre polynomials and restricting ourselves to two terms of the expansion

$$F(\mathbf{r}, \mathbf{p}, t) = (1/2)(N + (3/V)xJ_{//}), \quad (2.8.38)$$

where $N = \overline{F(\mathbf{r}, \mathbf{p}, t)}$ is the particle density, we obtain from Eq. 2.8.34 a system of equations of the diffusion approximation

$$\frac{\partial N(\mathbf{r}, t)}{\partial t} + \nabla_{\mathbf{r}} \boldsymbol{\eta} J_{//}(\mathbf{r}, t) = \nabla_{\perp \alpha} \pi V_{//}^2 \int_0^t d\tau \exp(-\tau/\tau_s) \overline{x^2 B_{\alpha\lambda}(V_{//}\tau)} \nabla_{\perp \lambda} N(\mathbf{r}, t - \tau), \quad (2.8.39)$$

$$\frac{\partial J_{//}(\mathbf{r}, t)}{\partial t} = \frac{J_{//}(\mathbf{r}, t)}{\tau_s} = -\frac{1}{3} V_{//}^2 (\boldsymbol{\eta} \nabla_{\mathbf{r}}) N(\mathbf{r}, t), \quad (2.8.40)$$

whence

$$J_{//}(\mathbf{r}, t) = -\frac{1}{3} V_{//}^2 \int_0^t d\tau \exp(-\tau/\tau_s) (\boldsymbol{\eta} \nabla_{\mathbf{r}}) N(\mathbf{r}, t - \tau). \quad (2.8.41)$$

Substituting Eq. 2.8.41 into Eq. 2.8.39, we obtain the transport equation

$$\frac{\partial N(\mathbf{r}, t)}{\partial t} = \nabla_{\mathbf{r} \alpha} \int_0^t d\tau \exp(-\tau/\tau_s) \kappa_{\alpha\lambda}(\tau) \nabla_{\mathbf{r} \lambda} N(\mathbf{r}, t - \tau), \quad (2.8.42)$$

where

$$\kappa_{\alpha\lambda}(\tau) = V^2 \left\{ \frac{1}{3} \eta_{\alpha} \eta_{\lambda} + \pi x^2 \overline{B_{\alpha\lambda}(V_{//}\tau)} \right\}. \quad (2.8.43)$$

If the scattering time τ_s is small then small τ makes the main contribution to the integrated term in Eq. 2.8.42. In the region of small τ the tensor $B_{\alpha\lambda}$ can be replaced by its value at zero. As a result we obtain

$$\frac{\partial N(\mathbf{r}, t)}{\partial t} = \nabla_{\mathbf{r} \alpha} \kappa_{\alpha\lambda}(0) \int_0^t d\tau \exp(-\tau/\tau_s) \nabla_{\mathbf{r} \lambda} N(\mathbf{r}, t - \tau), \quad (2.8.44)$$

which reduces to the telegraph equation (for example, see Earl, 1976; Dorman, Fedorov et al., 1983):

$$\frac{\partial N(\mathbf{r}, t)}{\partial t} + \tau_s \frac{\partial^2 N(\mathbf{r}, t)}{\partial t^2} = \nabla_{\mathbf{r} \alpha} \kappa_{\alpha\lambda}(0) \tau_s \nabla_{\mathbf{r} \lambda} N(\mathbf{r}, t). \quad (2.8.45)$$

For large τ Eq. 2.8.45 changes into the diffusion equation (Toptygin, 1973, M1983). As follows from Eq. 2.8.45, the diffusion coefficient is

$$\kappa_{\alpha\lambda}(\tau) = \kappa_{\alpha\lambda}(0)\tau_s = \kappa_{//}\eta_\alpha\eta_\lambda + \kappa_{\perp\alpha\lambda}, \quad \kappa_{//} = \frac{1}{3}V\Lambda_{//}, \quad \Lambda_{//} = V\tau_s, \quad (2.8.46)$$

and

$$\kappa_{\perp\alpha\lambda} = \pi V^2 \overline{x^2 B_{\perp\alpha\lambda}(0) \tau_s}. \quad (2.8.47)$$

If the tensor $B_{\alpha\lambda}$ is defined by the Eq. 2.8.29, the diffusion coefficient across the lines of force of the regular field is

$$\kappa_{\perp\alpha\lambda} = \frac{1}{3}V\Lambda_{\perp}\Delta_{\alpha\lambda}(\mathbf{n}), \quad (2.8.48)$$

where the mean free path across the lines of force of the regular field is

$$\Lambda_{\perp} = \frac{\pi}{12} \langle H_1^2 \rangle \Lambda \int dk \left\{ \left(1 + \frac{k_{//}^2}{k^2} \right) B(\mathbf{k}) + \frac{k_{\perp}^2 k_{//}^2}{k^2} B_1(\mathbf{k}) \right\}. \quad (2.8.49)$$

The vector $\boldsymbol{\eta}$ which enters into the Eq. 2.8.46 coincides in this case with the vector $\mathbf{n}$ (see Eq. 2.8.19a). The Eq. 2.8.48 for the anisotropic random field changes into the expression for $\Lambda_{\perp}$ calculated in Toptygin (1973, M1983).

In the case in which the scattering time τ_s is large, Eq. 2.8.42 reduces to the diffusion equation (Toptygin, 1973, M1983):

$$\partial N(\mathbf{r}, t) / \partial t = \nabla_{\mathbf{r}\alpha} (\kappa_{//} \eta_\alpha \eta_\lambda + \kappa_{\perp\alpha\lambda}) \nabla_{\mathbf{r}\lambda} N(\mathbf{r}, t). \quad (2.8.50)$$

where $\kappa_{//}$ is defined by the Eq. 2.8.46 and

$$\kappa_{\perp\alpha\lambda} = \pi V^2 \int_0^\infty d\tau \exp(-\tau/\tau_s) \overline{x^2 B_{\perp\alpha\lambda}(V_{//}\tau)}. \quad (2.8.51)$$

Let us note again that in this Section we have used the model of particle scattering by very small scale inhomogeneities, in which the large scale field has no influence on the nature of the motion of particles at scales of the order of the correlation radius of the very small-scale field. A more general model which takes account of the spiral motion of particles at scales of the order of L_c has been discussed in Toptygin (1973, M1983). It follows from Toptygin (1973, M1983) that all the relationships obtained above keep their form when the influence of the large scale field at scales of the order of L_c is taken into account. It is only

necessary to use an expression for $\Lambda_{//}$ which takes account of particle motion within the confines of the correlation region of the very small-scale field when doing specific calculations.

2.8.3. Diffusion of CR in a large scale random field

If the scattering frequency of particles by very small scale inhomogeneities significantly exceeds the characteristic frequencies of fluctuations of the large scale field, then the process of propagation of CR is diffusion of CR along the direction of the large-scale field:

$$\partial N / \partial t = \nabla_{\mathbf{r}\alpha} \kappa_o h_\alpha h_\lambda \nabla_{\mathbf{r}\lambda} N. \quad (2.8.52)$$

Taking account of fluctuations of the large scale field and their influence on the propagation of CR on the basis of Eq. 2.8.52 was discussed by Ptuskin (1985) and by Zybin and Istomin (1985). Another approach to the calculation of the CR diffusion coefficient in a large scale field has been proposed by Berezhko (1985). This problem we discuss here using the functional method of averaging. Expanding the unit vector $\mathbf{h}(\mathbf{r}, t)$ into a series in powers of the random field $H_1(r, t)$ and restricting ourselves to second-order terms, we shall average the expression obtained over the ensemble of realizations of the random field. As a result we obtain

$$\left(\frac{\partial}{\partial t} + L_o \right) \langle N \rangle = \nabla_{\mathbf{r}\alpha} \Gamma_{\alpha\lambda\beta} \langle H_\beta \nabla_{\mathbf{r}\lambda} N \rangle + \nabla_{\mathbf{r}\lambda} \gamma_{\alpha\lambda\beta\gamma} \langle H_\beta H_\gamma \nabla_{\mathbf{r}\lambda} N \rangle, \quad (2.8.53)$$

where $\langle N \rangle$ is the average CR density, and

$$\begin{aligned} L_o &= \kappa - \nabla_{\mathbf{r}\alpha} \kappa_o n_\alpha n_\lambda \nabla_{\mathbf{r}\lambda}, \quad \Gamma_{\alpha\lambda\beta} = n_\alpha \Delta_{\lambda\beta}(\mathbf{n}) + n_\lambda \Delta_{\alpha\beta}(\mathbf{n}), \quad \gamma_{\alpha\lambda\beta\gamma} = \kappa_o \\ &\times \{ \Delta_{\alpha\beta}(\mathbf{n}) \Delta_{\lambda\gamma}(\mathbf{n}) + n_\alpha \Pi_{\lambda\beta\gamma} + n_\lambda \Pi_{\alpha\beta\gamma} \}, \quad \Pi_{\alpha\beta\gamma} = \frac{1}{2} n_\alpha (3n_\beta n_\gamma - \delta_{\beta\gamma}) - n_\beta \delta_{\alpha\gamma}. \end{aligned} \quad (2.8.54)$$

We shall make use of the Eq. 6.8.13 for averaging the terms on the right hand side of Eq. 2.8.53:

$$\langle H_{1\beta}(\mathbf{r}, t) \nabla_{\mathbf{r}\lambda} N[\mathbf{H}_1] \rangle = \int d\mathbf{r}_1 dt_1 B_{\beta\gamma}(\mathbf{r}, t; \mathbf{r}_1, t_1) \nabla_{\mathbf{r}\lambda} \left\langle \frac{\delta N[\mathbf{H}_1]}{\delta H_{1\gamma}(\mathbf{r}_1, t_1)} \right\rangle. \quad (2.8.55)$$

We shall determine the functional derivative, which enters into Eq. 2.8.55 based on perturbation theory:

$$\frac{\delta N[\mathbf{H}_1]}{\delta H_{1\gamma}(\mathbf{r}_1, t_1)} = -\Gamma_{\alpha\lambda\gamma} \left\{ \nabla_{\mathbf{r}_1\alpha} G(\mathbf{r}, t; \mathbf{r}_1, t_1) \right\} \nabla_{\mathbf{r}_1\lambda} N(\mathbf{r}_1, t_1). \quad (2.8.56)$$

where G is the Green's function of the operator $\partial/\partial t + L_o$. From Eq. 2.8.55 we have:

$$\langle H_{1\beta}(\mathbf{r}, t) \nabla_{\mathbf{r}\lambda} N[\mathbf{H}_1] \rangle = \Gamma_{\alpha\mu\gamma} \int_0^\infty d\tau \int d\rho G(\rho, \tau) \left\{ \nabla_{\rho\zeta} \nabla_{\rho\lambda} B_{\beta\gamma}(\rho, \tau) \right\} \nabla_{\mathbf{r}\mu} \langle N(\mathbf{r}, t) \rangle. \quad (2.8.57)$$

Averaging of the second term on the right hand side of Eq. 2.8.53 leads to the relationship

$$\langle H_{1\beta}(\mathbf{r}, t) H_{1\gamma}(\mathbf{r}, t) \nabla_{\mathbf{r}\lambda} N[\mathbf{H}_1] \rangle = B_{\beta\gamma}(\mathbf{r}, t; \mathbf{r}, t) \nabla_{\mathbf{r}\lambda} \langle N(\mathbf{r}, t) \rangle. \quad (2.8.58)$$

Substituting Eq. 2.8.57 and Eq. 2.8.58 into Eq. 2.8.53, we obtain the equation:

$$\partial \langle N \rangle / \partial t = \nabla_{\mathbf{r}\alpha} \kappa_{\alpha\lambda} \nabla_{\mathbf{r}\lambda} \langle N \rangle, \quad (2.8.59)$$

where

$$\kappa_{\alpha\lambda} = \kappa_o n_\alpha n_\lambda + \gamma_{\alpha\lambda\beta\gamma} B_{\beta\gamma}(\mathbf{r}, t; \mathbf{r}, t) + \Gamma_{\alpha\mu\beta} \Gamma_{\zeta\lambda\gamma} \int_0^\infty d\tau G(\rho, \tau) \nabla_{\rho\zeta} \nabla_{\rho\mu} B_{\beta\gamma}(\rho, \tau). \quad (2.8.60)$$

If the regular field is uniform in space, then having made use of the well known Green's function of the operator $\partial/\partial t + L_o$, we obtain from Eq. 2.8.60:

$$\kappa_{\alpha\lambda} = \left(1 - \langle H_1^2 \rangle \right) \kappa_o n_\alpha n_\lambda. \quad (2.8.61)$$

Consequently small fluctuations of a large scale field do not lead to diffusion of CR across the direction of the regular component of the large scale magnetic field, in accordance with Ptuskin (1985). As is evident from the results of Dorman, Katz, and Stehlik (1988) described in Sections 2.8.1–2.8.3, if the characteristic time of fluctuations of the large scale magnetic field significantly exceeds the scattering time of particles by very small scale inhomogeneities of the magnetic field, diffusion of the CR is described by the telegraph equation, in accordance with Earl (1976) and Dorman et al. (1983), which takes account of the presence of diffusion of CR across the lines of force of the regular field. The mean free path of particles across lines of force is related by the Eq. 2.8.49 to their mean free path with respect to scattering by very small scale inhomogeneities and is

determined in the case of anisotropic turbulence of the large scale field (see Eq. 2.8.29) by the two spectral functions B and B_1 .

2.8.4. CR transport in the random girotropic magnetic field

The problem of propagation of CR in the random girotropic magnetic field has been discussed by Fedorov et al. (1992) and Dolginov and Katz (1994). These examinations are a major preoccupation to the investigation of particles motion in the small-scale random girotropic magnetic field. Katz and Yacobi (1997) considered the effects are owed to existence of the large scale magnetic field. The influence of small-scale magnetic field provides the effective particles scattering whereas the nonzero helicity of the turbulence leads to the particles acceleration. Katz and Yacobi (1997) obtained the drift kinetic equation including these effects and derive the kinetic coefficients describing the particles propagation at these conditions.

The distribution function $f(\mathbf{r}, \mathbf{p}, t)$ of the ensemble of non-interacting particles moving in the small scale random girotropic magnetic field obeys the equation (Fedorov, et al., 1992; Dolginov and Katz, 1994):

$$\left(\frac{\partial}{\partial t} + \mathbf{V} \nabla_{\mathbf{r}} + \frac{\partial}{\partial \mathbf{p}} \mathbf{F} \right) f(\mathbf{p}, \mathbf{r}, t) = Stf(\mathbf{r}, \mathbf{p}, t), \quad (2.8.62)$$

where

$$\mathbf{F} = \frac{e}{c} (\mathbf{V} - \mathbf{u}) \times \mathbf{H} - \alpha \mathbf{H} \quad (2.8.63)$$

is the force acting on the particle with charge e and momentum p corresponding to the velocity $\mathbf{V} = c\mathbf{p}/E$ and energy E ; $\mathbf{u}$ is plasma velocity which transfers the large scale magnetic field $\mathbf{H}$ frozen into it. The second term in Eq. 2.8.63 is owed to particles acceleration in the small scale random girotropic magnetic field. The coefficient α is expressed in helicity terms of the turbulence. The collision integral $Stf(\mathbf{r}, \mathbf{p}, t)$ in Eq. 2.8.62 has Fokker-Planck form:

$$Stf(\mathbf{r}, \mathbf{p}, t) = \frac{\partial}{\partial p_{\alpha}} D_{\alpha\lambda} \frac{\partial f}{\partial p_{\lambda}}; \quad D_{\alpha\lambda} = \frac{p^2}{2\Lambda} V \left(\delta_{\alpha\lambda} - \frac{p_{\alpha} p_{\lambda}}{p^2} \right), \quad (2.8.64)$$

where Λ is particle transport path respectively its scattering on the random inhomogeneities of the small scale magnetic field. If the magnetic field $\mathbf{H}$ is sufficiently strong, it is conveniently to use the drift approximation (Toptygin, M1983). In the drift approximation Eq. 2.8.62 reads

$$\frac{\partial f}{\partial t} + \nabla_{\mathbf{r}} \dot{\mathbf{R}} f + \frac{\partial}{\partial p_{\perp}^2} \frac{dp_{\perp}^2}{dt} f + \frac{\partial}{\partial p_{\parallel}} \frac{dp_{\parallel}}{dt} f = \langle \text{St}f \rangle_{\mathbf{h}}, \quad (2.8.65)$$

where

$$\dot{\mathbf{R}} = V_{\parallel} \mathbf{h} + \mathbf{u}_{\perp}, \quad \mathbf{h} = \mathbf{H}/H, \quad \frac{dp_{\perp}^2}{dt} = -V_{\parallel} p_{\perp}^2 \nabla_{\mathbf{r}} \mathbf{h}, \quad \frac{dp_{\parallel}}{dt} = \frac{1}{2} V_{\perp}^2 p_{\perp}^2 \nabla_{\mathbf{r}} \mathbf{h} - \alpha(\mathbf{h}\mathbf{H}), \quad (2.8.66)$$

and where $f(\mathbf{r}, p_{\parallel}, p_{\perp}, t)$ is the drift distribution function and indices $\perp$ and $\parallel$ mark particle momentum components across and along the direction of the magnetic field $\mathbf{H}$. The symbol $\langle \dots \rangle_{\mathbf{h}}$ denotes averaging over directions of the momentum vector in the plane normal to vector $\mathbf{h}$. If the large scale magnetic field $\mathbf{H}$ is random function of the coordinates and time we have to average the Eq. 2.8.65 over the fluctuations of the large-scale magnetic field. This may be performed if the random component of the large scale magnetic field is small: $\mathbf{H} = \mathbf{H}_0 + \mathbf{H}_1$, where $\mathbf{H}_0$ is the regular component and $\mathbf{H}_1$ is the random component. Along with this we have to change the variables in Eq. 2.8.62 on the other that are related to the direction of the large scale regular magnetic field $\mathbf{H}_0$. In this case the Eq. 2.8.62 will have the following form

$$\frac{\partial f}{\partial t} + \nabla_{\mathbf{r}} \frac{d\mathbf{r}}{dt} f + \frac{\partial}{\partial p_{\perp}^2} \frac{dp_{\perp}^2}{dt} f + \frac{\partial}{\partial p_{\parallel}} \frac{dp_{\parallel}}{dt} f = \langle \text{St}f \rangle_{\mathbf{n}}, \quad (2.8.67)$$

where

$$\frac{d\mathbf{r}}{dt} = V_{\parallel} \left\{ [1 - H_{1\perp}^2] \mathbf{n} + (1 - H_{1\parallel}) H_{1\perp} \right\} + \mathbf{u}_{\perp} - (\mathbf{u} H_{1\perp}) \mathbf{n} - (\mathbf{u} \mathbf{n}) H_{1\perp}, \quad (2.8.68)$$

$$\frac{dp_{\perp}^2}{dt} = -V_{\parallel} p_{\perp}^2 \nabla_{\mathbf{r}} \left\{ (1 - H_{1\parallel}) H_{1\perp} - \frac{1}{2} H_{1\perp}^2 \mathbf{n} \right\}, \quad (2.8.69)$$

$$\frac{dp_{\parallel}}{dt} = \frac{1}{2} V_{\perp} p_{\perp}^2 \nabla_{\mathbf{r}} \left\{ (1 - H_{1\parallel}) H_{1\perp} - \frac{1}{2} H_{1\perp}^2 \mathbf{n} \right\} - \alpha. \quad (2.8.70)$$

In Eq. 2.8.67–2.8.70 the indices $\perp$ and $\parallel$ mark the components associated with the direction $\mathbf{n} = \mathbf{H}_0/H_0$ of the regular magnetic field $\mathbf{H}_0$. The Eq. 2.8.67 is described with accuracy of terms second order relative to the random magnetic field H_1 . The last is measured in the units of H_0 . The next step according to Katz

and Yacobi (1997) is averaging Eq. 2.8.67 over the ensemble of realization of the random magnetic field, and velocity $\mathbf{u}$: $\langle f \rangle = F$. Assuming the Gaussian distribution for random fields we obtain

$$\frac{\partial F}{\partial t} + \nabla_{\mathbf{r}} \mathbf{w} F - \frac{\partial}{\partial p_{\parallel}} \alpha F = \nabla_{\mathbf{r}} \alpha \kappa_{\perp \alpha \lambda} \nabla_{\mathbf{r}} \lambda F + \text{St} F, \quad (2.8.71)$$

where

$$\mathbf{w} = \left(1 - \frac{1}{3} \langle H_{\perp}^2 \rangle \right) \mathbf{n} V_{\parallel}, \quad (2.8.72)$$

$$\begin{aligned} \kappa_{\perp \alpha \lambda} = & \int_0^{\infty} d\tau \int d\mathbf{x} \left\{ V_{\parallel} G_{\mathbf{p}}(\mathbf{x}, \tau) B_{\alpha \lambda}(\mathbf{x}, \tau) + V_{\parallel} \Gamma_{\mathbf{p}}(\mathbf{x}, \tau) S_{\alpha \lambda}(\mathbf{x}, \tau) \right. \\ & \left. + G_{\mathbf{p}}(\mathbf{x}, \tau) S_{\alpha \lambda}(\mathbf{x}, \tau) + \Gamma_{\mathbf{p}}(\mathbf{x}, \tau) Q_{\alpha \lambda}(\mathbf{x}, \tau) \right\} \end{aligned} \quad (2.8.73)$$

$$\text{St} F = \frac{\partial}{\partial p_{\perp}^2} \left(D_{\perp \perp} \frac{\partial}{\partial p_{\perp}^2} + D_{\perp \parallel} \frac{\partial}{\partial p_{\parallel}} \right) F + \frac{\partial}{\partial p_{\parallel}} \left(D_{\parallel \perp} \frac{\partial}{\partial p_{\perp}^2} + D_{\parallel \parallel} \frac{\partial}{\partial p_{\parallel}} \right) F. \quad (2.8.74)$$

Components of tensor D are as following:

$$\begin{aligned} D_{\perp \perp} = & -2p_{\perp}^2 \left(\frac{V}{\Lambda} \beta + \frac{p_{\perp}^2}{2p_{\parallel}} V_{\parallel} D_1 \right), \quad D_{\perp \parallel} = D_{\parallel \perp} = -2p_{\perp}^2 p_{\parallel} \left(\frac{V}{\Lambda} \beta + \frac{p_{\perp}}{2p_{\parallel}} D_1 \right), \\ D_{\parallel \parallel} = & \frac{1}{2} p_{\perp}^2 \left(\frac{V}{\Lambda} \beta + \frac{1}{2} V_{\perp}^2 D_2 \right), \end{aligned} \quad (2.8.75)$$

where

$$\begin{aligned} \beta = & 1 - \frac{p^2}{2p_{\perp}^2} \langle H_{\perp}^2 \rangle, \quad D_1 = \int_0^{\infty} d\tau \int d\mathbf{x} G_{\mathbf{p}}(\mathbf{x}, \tau) \nabla_{\perp \mathbf{x} \alpha} \nabla_{\perp \mathbf{x} \mu} B_{\alpha \mu}(\mathbf{x}, \tau), \\ D_2 = & \int_0^{\infty} d\tau \int d\mathbf{x} \Gamma_{\mathbf{p}}(\mathbf{x}, \tau) \nabla_{\perp \mathbf{x} \alpha} \nabla_{\perp \mathbf{x} \mu} B_{\alpha \mu}(\mathbf{x}, \tau). \end{aligned} \quad (2.8.76)$$

In Eq. 2.8.73 and Eq. 2.8.76

$$G_{\mathbf{p}}(\mathbf{x}, \tau) = \int d\mathbf{p}_1 G_{\mathbf{p}\mathbf{p}_1}(\mathbf{x}, \tau), \quad \Gamma_{\mathbf{p}}(\mathbf{x}, \tau) = \int d\mathbf{p}_1 V_{1//} G_{\mathbf{p}\mathbf{p}_1}(\mathbf{x}, \tau), \quad (2.8.77)$$

and $G_{\mathbf{p}\mathbf{p}_1}(\mathbf{x}, \tau)$ is the Green's function of the equation describing particles scattering in the small scale random magnetic field, $B_{\alpha\lambda}(\mathbf{x}, \tau)$, $Q_{\alpha\lambda}(\mathbf{x}, \tau)$ and $S_{\alpha\lambda}(\mathbf{x}, \tau)$ are correlation tensors of the large scale magnetic field $\mathbf{H}_1$, of the velocity field $\mathbf{u}$, and their cross-correlation tensor, respectively.

Representing the distribution function in the form

$$F = N + \delta F, \quad (2.8.78)$$

Katz and Yacobi (1997) obtain from Eq. 2.8.71 the set of equations for the density $N(\mathbf{r}, p, t)$ and a small anisotropic addition $\delta F(\mathbf{r}, p, t)$:

$$\frac{\partial N}{\partial t} + (\nabla_{\mathbf{r}\mathbf{n}}) V \langle \mu \delta F \rangle_{\mu} - \frac{1}{p^2} \frac{\partial}{\partial p} p^2 \alpha \langle \mu \delta F \rangle_{\mu} = \nabla_{\mathbf{r}\alpha} \langle \kappa_{\perp\alpha\lambda} \rangle_{\mu} \nabla_{\mathbf{r}\lambda} N, \quad (2.8.79)$$

$$\frac{\partial}{\partial \mu} (1 - \mu^2) \left\{ D(\mu) \frac{\partial}{\partial \mu} + \frac{\alpha}{p} \right\} \delta F = V \mu \left\{ (\nabla_{\mathbf{r}\mathbf{n}}) N - \frac{\alpha}{V} \frac{\partial N}{\partial p} \right\}, \quad (2.8.80)$$

where symbol $\langle \rangle_{\mu}$ denotes the averaging over the particle pitch-angle $\mu = \mathbf{n}\mathbf{p}/p$, and $D(\mu)$ is the pitch-angle diffusion coefficient, determined by expression

$$p^2 D(\mu) = \mu^2 D_{\perp\perp} - 2\mu (1 - \mu^2)^{-1/2} D_{\perp//} + (1 - \mu^2) D_{////}. \quad (2.8.81)$$

The solution of Eq. 2.8.79–2.8.80 has the form

$$\delta F(\mu) = -\frac{1}{2} q(\mathbf{r}, p) \int_0^{\mu} \frac{d\mu'}{D(\mu')} \exp \left(-\frac{\alpha}{p} \int_{\mu'}^{\mu} \frac{d\mu_1}{D(\mu_1)} \right), \quad (2.8.82)$$

where

$$q(\mathbf{r}, p) = V \left\{ (\nabla_{\mathbf{r}\mathbf{n}}) N - \frac{\alpha}{V} \frac{\partial N}{\partial p} \right\}. \quad (2.8.83)$$

If the particles are highly scattered then

$$\langle \mu \delta F(\mu) \rangle_\mu = -\frac{1}{3} \Lambda_{//} q(\mathbf{r}, p), \quad (2.8.84)$$

where $q(\mathbf{r}, p)$ is determined by Eq. 2.8.83, and

$$\Lambda_{//} = \frac{3}{4} V \int_0^1 \frac{(1 - \mu^2) d\mu}{D(\mu)}. \quad (2.8.85)$$

Substituting Eq. 2.8.81 into Eq. 2.8.78 Katz and Yacobi (1997) obtain the transport equation

$$\begin{aligned} \frac{\partial N}{\partial t} - \nabla_{\mathbf{r}\alpha} \kappa_{\alpha\lambda} \nabla_{\mathbf{r}\lambda} N + (\nabla_{\mathbf{r}\mathbf{n}}) D_{\mathbf{r}p} \frac{\partial N}{\partial p} + \frac{1}{p^2} \frac{\partial}{\partial p} p^2 D_{p\mathbf{r}} (\nabla_{\mathbf{r}\mathbf{n}}) N \\ = \frac{1}{p^2} \frac{\partial}{\partial p} p^2 D(p) \frac{\partial N}{\partial p}, \end{aligned} \quad (2.8.86)$$

where

$$\kappa_{\alpha\lambda} = \kappa_{//} n_\alpha n_\lambda + \langle \kappa_{\perp\alpha\lambda} \rangle_\mu, \quad D_{\mathbf{r}p} = D_{p\mathbf{r}} = \frac{\alpha}{V} \kappa_{//}, \quad D(p) = \frac{\alpha^2}{V^2} \kappa_{//}, \quad \kappa_{//} = \frac{V \Lambda_{//}}{3}. \quad (2.8.87)$$

According to Katz and Yacobi (1997), Eq. 2.8.86 may be re-written in the form of the continuity equation in the coordinate space and in the space of the absolute values of the momentum (Fedorov et al., 1992; Dolginov and Katz, 1994):

$$\frac{\partial N}{\partial t} - \nabla_{\mathbf{r}} J_\alpha + \frac{1}{p^2} \frac{\partial}{\partial p} p^2 J_p = 0, \quad (2.8.88)$$

where

$$J_\alpha = -\kappa_{\alpha\lambda} \nabla_{\mathbf{r}\alpha} N + n_\alpha D_{\mathbf{r}p} \frac{\partial N}{\partial p} \quad (2.8.89)$$

is the vector of the particles flow in the coordinate space, and

$$J_p = D_{p\mathbf{r}} + (\nabla_{\mathbf{r}\mathbf{n}}) N - D(p) \frac{\partial N}{\partial p} \quad (2.8.90)$$

is the particles flow in the space of absolute values of the particles momentum.

2.9. CR diffusion in the momentum space

Consider the collision integral in the kinetic Eq. 2.7.1 written in the spherical coordinates in the momentum space ($p_z = p \cos \theta$, $p_\perp = p \sin \theta$):

$$\begin{aligned} \langle \text{Stf} \rangle_\phi = & \frac{1}{p^2 \sin \theta} \frac{\partial}{\partial \theta} \sin \theta D_{\theta\theta} \frac{\partial f}{\partial \theta} + \frac{1}{p \sin \theta} \frac{\partial}{\partial \theta} \sin \theta D_{\theta p} \frac{\partial f}{\partial p} \\ & + \frac{1}{p^2} \frac{\partial}{\partial p} p D_{p\theta} \frac{\partial f}{\partial \theta} + \frac{1}{p^2} \frac{\partial}{\partial p} p^2 D_{pp} \frac{\partial f}{\partial p}, \end{aligned} \quad (2.9.1)$$

where $D_{\theta\theta}, D_{\theta p}, D_{p\theta}, D_{pp}$ are determined by Eq. 2.7.2, and D_1, D_2, D_3 are determined by Eq. 2.6.13–2.6.24 with respective substitution for the variables. The terms of the collision integral including the components $D_{\theta\theta}$ of the tensor $D_{\alpha\lambda}$ describe an elastic scattering of charged particles on turbulent pulsations of the magnetic field and the rest terms include the energy interchange between turbulent pulsations and charged particles resulting in the acceleration of the latter. Note the following property. If we compare the value of various terms in the collision integral, we obtain that the term with $D_{\theta\theta}$ is the main term. The effective frequency of scattering of charged particles in the collisions with the magnetic field inhomogeneities $\nu = p^{-2} D_{\theta\theta}$ is far higher than the frequency of inelastic collisions, which is represented by the coefficients $D_{p\theta}$ and D_{pp} . Otherwise the particles are very quickly got into a chaotic state owing to scattering on turbulent pulsations of a magnetic field and their further diffusion in momentum space is described by an isotropic distribution function. According to this we shall search for the solution of Eq. 2.7.1 in the form (Ryutov, 1969):

$$f = \langle f \rangle_\theta + \delta f, \quad (2.9.2)$$

where the second term is far less than the first, and

$$\langle \dots \rangle_\theta = \frac{1}{2\pi} \int_0^\pi (\dots) \sin \theta d\theta \quad (2.9.3)$$

means averaging over θ . By averaging the kinetic Eq. 2.7.1 over θ angle we obtain

$$\frac{\partial}{\partial t} \langle f \rangle_\theta = \frac{1}{p^2} \frac{\partial}{\partial p} p^2 \left[\langle D_{pp} \rangle_\theta \frac{\partial}{\partial p} \langle f \rangle_\theta + \left\langle \frac{D_{p\theta}}{p} \frac{\partial \delta f}{\partial \theta} \right\rangle_\theta \right]. \quad (2.9.4)$$

The equation for a correction $\mathcal{J}$ to a distribution function has the following form:

$$\frac{1}{p \sin \theta} \frac{\partial}{\partial \theta} \sin \theta \left[D_{p\theta} \frac{\partial}{\partial p} \langle f \rangle_\theta + \frac{D_{\theta\theta}}{p} \frac{\partial \mathcal{J}}{\partial \theta} \right] = 0, \quad (2.9.5)$$

which results in

$$\frac{\partial \mathcal{J}}{\partial \theta} = -p \frac{D_{p\theta}}{D_{\theta\theta}} \frac{\partial}{\partial p} \langle f \rangle_\theta. \quad (2.9.6)$$

Substituting the latter equation in Eq. 2.9.4 we obtain

$$\frac{\partial f}{\partial t} = \frac{1}{p^2} \frac{\partial}{\partial p} p^2 D(p) \frac{\partial f}{\partial p}, \quad (2.9.7)$$

where

$$D(p) = \left\langle D_{pp} - \frac{D_{p\theta}^2}{D_{\theta\theta}} \right\rangle_\theta, \quad (2.9.8)$$

and the sign $\langle \rangle_\theta$ around the function $\langle f \rangle_\theta$ in Eq. 2.9.7 is omitted for brevity.

Substituting the expressions for D_{pp} , $D_{p\theta}$ and $D_{\theta\theta}$ in Eq. 2.9.8 we get the equation describing a particle diffusion in the momentum space (Tverskoy, 1967b):

$$\frac{\partial f}{\partial \tau} = \frac{1}{p^2} \frac{\partial}{\partial p} E p^{\nu+1} \frac{\partial f}{\partial p}, \quad (2.9.9)$$

where

$$\tau = \zeta(\nu+1) \Gamma\left(\frac{\nu+1}{2}\right) \frac{\nu(\nu+1) \Gamma\left(\frac{\nu}{2}\right) - (\nu+2) \Gamma\left(\frac{\nu+1}{2}\right)}{2\pi\nu(\nu+1)(\nu+2) \Gamma\left(\frac{\nu}{2}\right) \Gamma\left(\frac{\nu-1}{2}\right)} \left(\frac{mc^2}{eH_o l_c}\right)^{\nu-2} \frac{\langle H_1^2 \rangle}{H_o^2} \left(\frac{v_a}{c}\right)^2 \frac{ct}{l_c}. \quad (2.9.10)$$

The Eq. 2.9.9 is written in dimensionless variables: a momentum is measured in units of mc and energy in units of mc^2 . The solution of the Eq. 2.9.9 can be

obtained for non-relativistic ($E=1$) and for relativistic ($E=p$) cases. Applying the Fourier-Bessel transformation to the Eq. 2.9.9 we obtain for $\nu \neq 3$ as follows,

$$f(p, \tau) = \frac{1}{(3-\nu)p^{\nu/2}} \int_0^\infty q^{2-\nu/2} f_o(q) \exp\left(-\frac{q^{3-\nu} + p^{3-\nu}}{(3-\nu)^2}\right) I_{\frac{\nu}{3-\nu}}\left(\frac{2(qp)^{(3-\nu)/2}}{\tau(3-\nu)^2}\right) dq. \quad (2.9.11)$$

Here $f_o(p) \equiv f(p, \tau=0)$ and $I_\mu(x)$ is the modified Bessel function. If the initial distribution function $f_o(p)$ differs from zero in some region $p \leq p_o \ll 1$, the asymptotic behavior of $f(p, \tau)$ at large τ and $p \gg p_o$ has the following form:

$$f(p, \tau) = \frac{(3-\nu)^{2\nu/(\nu-3)}}{2\Gamma\left(\frac{3}{3-\nu}\right)} N_\nu \tau^{3/(\nu-3)} \exp\left(-\frac{p^{3-\nu}}{\tau(3-\nu)^2}\right), \quad (2.9.12)$$

where

$$N_\nu = \int_0^\infty q^\nu f_o(q) dq. \quad (2.9.12a)$$

As is seen from Eq. 2.9.12 the exponential spectrum of accelerated particles is asymptotically formed at $\nu=2$ (Tverskoy, 1967a,b).

The value N_ν has the meaning of the total particle number injected in 1 cm^3 and is not changed in the process. At $\nu=3$, the solution of Eq. 2.9.9 in the case of a point source has the following form

$$f(p, \tau) = \frac{1}{2\sqrt{\pi\tau}} p^{-3/2} \exp\left(-\frac{(\ln(p/p_o))^2}{4\tau}\right), \quad (2.9.13)$$

where p_o is the particle momentum in the source. As we have noted, the Eq. 2.9.11 describes the particle distribution in the super-relativistic case ($E=p$). For this purpose one should make the substitution $\nu \rightarrow \nu+1$. As consistent with this, Eq. 2.9.13 represents the distribution function in the super-relativistic case for $\nu=2$ (Tverskoy, 1967b).

Let us compare the cases considered of particle acceleration by a wave turbulence with a pure Fermi acceleration of particles (particle collisions with stochastically moving clouds). Fermi acceleration can be obtained from the Eq. 2.9.9 if we set in it formally $\nu=1$ (non-relativistic case) or $\nu=2$ (super-relativistic case). As is seen from the above Eq. 2.9.11–2.9.13 the shape of the

accelerated particles is sufficiently dependent on the index in the power spectrum of turbulent pulsations.

2.10. CR diffusion in the pitch-angle space

If we neglect the action of electric fields of oscillations on the particles in the initial kinetic Eq. 2.6.13 (or if we equate to zero the coefficients D_2 and D_3 in the collision integral of Eq. 2.6.13), we shall obtain the equation describing the process of particle diffusion in angular space which passes with energy conservation. In this case one should solve directly the kinetic Eq. 2.5.1 because the diffusion approximation with respect to coordinates certainly is not applicable. In the general case this problem presents serious mathematical complications; however, if the regular magnetic field is sufficiently strong so that a perturbation of particle movement by a stochastic field during the time of the order of cyclotron rotation is small, one can average the Eq. 2.5.1 over the angle of particle rotation, pass to the drift approximation (Sivukhin, 1963). The collisional term of the kinetic Eq. 2.5.1 is determined in this case by the Eq. 2.9.1 at $D_2 = D_3 = 0$ and the averaging of the right side of Eq. 2.5.1 is known from the drift theory (Sivukhin, 1963; Galperin et. al., 1971):

$$\frac{\partial F}{\partial t} + v \cos \theta \frac{\partial F}{\partial z} - \frac{v}{2} \sin \theta (\nabla \mathbf{h}) \frac{\partial F}{\partial \theta} = \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} D_1 \sin \theta \frac{\partial F}{\partial \theta}. \quad (2.10.1)$$

An analytical solution of Eq. 2.10.1 can be obtained for the angles fitting the condition $\theta \ll 1$. In this case Eq. 2.5.1 takes the form:

$$\frac{\partial F}{\partial z} - \frac{1}{2} (\nabla \mathbf{h}) \theta \frac{\partial F}{\partial \theta} = \frac{1}{\Lambda_{//}(z)} \frac{\partial}{\partial \theta} \theta \frac{\partial F}{\partial \theta} + \frac{1}{\theta \cos \theta_o} \delta(\theta - \theta_o) \delta(z - z_o). \quad (2.10.2)$$

The Eq. 2.10.2 is written for a stationary case and the point source (with the coordinates z_o, θ_o) is added in the right side. The value $\Lambda_{//}(z)$ is determined by means of Eq. 2.6.19–Eq. 2.6.21 and has the meaning of a particle transport path (Galperin et al., 1971):

$$\Lambda_{//}(z) = \frac{4(\nu + 2)\Gamma((\nu - 1)/2)}{\nu \sqrt{\pi} \Gamma(\nu/2)} \frac{H_o^2}{\langle H_1^2 \rangle} \left(\frac{l_c}{r_L} \right)^{\nu-2} l_c. \quad (2.10.3)$$

The transport path $\Lambda_{//}(z)$ depends on a particle momentum and on the field intensities $\langle H_1^2 \rangle$ and H_o^2 according to the law $\Lambda_{//}(z) \propto p^{2-\nu} H_o^2 / \langle H_1^2 \rangle$. At $\nu > 2$ the transport path decreases with the growth of particle energy. This is explained

by the fact that the particles are scattered by the inhomogeneities of a higher scale the greater is a particle Larmor radius, and the number of inhomogeneities in this case is increased with their scale. To solve the Eq. 2.10.2 let us introduce the new variable $\rho = (1/2)\ln H_o$. Then Eq. 2.10.2 takes the form

$$\frac{\partial F}{\partial \rho} + \theta \frac{\partial F}{\partial \theta} = \frac{\varphi(\rho)}{\theta} \frac{\partial}{\partial \theta} \theta \frac{\partial F}{\partial \theta} + \frac{1}{\theta \cos \theta_o} \delta(\theta - \theta_o) \delta(\rho), \quad (2.10.4)$$

where

$$\varphi(\rho) = \left(\Lambda_{//}(z) \frac{\partial \rho}{\partial z} \right)^{-1} \bigg|_{z=z(\rho)}. \quad (2.10.5)$$

It is suitable to make the substitution of variables $\rho \rightarrow \rho$, $\theta \rightarrow \exp(\rho - \xi)$. As a result we obtain the equation

$$\alpha(\rho) \frac{\partial F}{\partial \rho} = \exp(2\xi) \frac{\partial^2 \varphi}{\partial \xi^2} + \frac{\alpha(\rho)}{\cos \theta_o} \delta(\xi - \ln \theta_o) \delta(\varphi), \quad (2.10.6)$$

where

$$\alpha(\rho) = \exp(2\rho) (\varphi(\rho))^{-1}. \quad (2.10.7)$$

The Eq. 2.10.6 is equivalent to the equation

$$\frac{\partial F}{\partial \rho} = \exp(2\xi) \frac{\partial^2 \varphi}{\partial \rho^2} \quad (2.10.8)$$

with the additional condition

$$F|_{\rho=0} = \frac{\alpha(0)}{\cos \theta_o} \delta(\xi - \ln \theta_o). \quad (2.10.9)$$

The Eq. 2.10.8 has the special solution of the type

$$F_\lambda = \exp\left(-\lambda^2 \overline{\theta^2}(z)\right) J_o\left(\lambda \frac{\theta}{\sqrt{H_o}}\right), \quad (2.10.10)$$

where

$$\overline{\theta^2}(z) = 4 \int_{z_o}^z dz' \frac{H_o(z')}{H_o(z') \Lambda_{//}(z')}. \quad (2.10.11)$$

In Eq. 2.10.10 we have returned to the initial variables, J_o is the Bessel function. The general solution of Eq. 2.10.10 can be presented in the form

$$F = \int_0^\infty \sqrt{\lambda} \Psi(\lambda) \exp\left(-\lambda^2 \overline{\theta^2}(z)\right) J_o\left(\lambda \frac{\theta}{\sqrt{H_o}}\right) d\lambda, \quad (2.10.12)$$

which results in

$$F|_{z=z_o} = \frac{1}{\theta \cos \theta_o} \delta(\theta - \theta_o) = \int_0^\infty \sqrt{\lambda} \Psi(\lambda) J_o\left(\lambda \frac{\theta}{\sqrt{H_o}}\right) d\lambda. \quad (2.10.13)$$

Applying the Fourier-Bessel theorem we obtain

$$\Psi(\lambda) = \frac{\sqrt{\lambda}}{H_o(z_o) \cos \theta_o} J_o\left(\lambda \frac{\theta_o}{\sqrt{H_o(z_o)}}\right). \quad (2.10.14)$$

Using Eq. 2.10.14 we obtain finally (Galperin et al., 1971; Dorman and Katz, 1974b):

$$F(z, \theta; z_o, \theta_o) = \frac{H_o(z)}{\pi H_o(z_o) \overline{\theta^2}(z)} \exp\left(\frac{\theta^2 + \theta_o^2}{\overline{\theta^2}(z)}\right) I_o\left(\frac{\theta_o \theta}{\overline{\theta^2}(z)}\right), \quad (2.10.15)$$

where I_o is the modified Bessel function. A quantity $\overline{\theta^2}(z)$ has the meaning of a mean square angle of a particle's scattering (Galperin et al., 1971).

If we can neglect the dependence of H_o and $\Lambda_{//}$ on z , then a mean square angle of a scattering increases proportionally to a distance from the source. If H_o and H_1 vary at distance according to the same law

$$H_o \propto H_1 \propto (z_o/z)^\beta \quad (2.10.16)$$

and the spectrum shape (the values l_c and ν) remains unchanged, then at $z \gg z_o$

$$\overline{\theta^2}(z) = \frac{4z_o}{(\beta(\nu-1)+1)\Lambda_{//}(z_o)} \left(\frac{z}{z_o} \right)^{\beta(\nu-2)+1}. \quad (2.10.17)$$

Let us apply these results to a scattering of the low energy solar particles. According to the Parker's model in a region inside the Earth's orbit $\beta \approx 2$ for a regular field H_o . The spectrum index $\nu = 1.5 \pm 0.2$. If magnetic inhomogeneities are generated near the Sun and then transferred into interplanetary space by the solar wind, the intensity H_1 should be varied proportionally to H_o . This assumption is founded by measurements for a region outside the Earth's orbit. Thus according to the data of Jokipii and Coleman (1968) the value $\langle H_1^2 \rangle$ was decreased by 2.4 times and H_o^2 was decreased by 2.5 times when the distance from the Sun was increased from 1 AU to 1.44 AU. Substituting $\beta = 2$, $\nu = 1.5$ in Eq. 2.10.17 we find that $\overline{\theta^2}$ is independent of z . If in this case $\overline{\theta^2} < 1$, the particles generated on the Sun should come to the Earth in the form of a flux with a pronounced anisotropy. These anisotropic fluxes of protons with the energies of 1–10 MeV and a duration of several hours were recurrently registered in the experiments of Vernov et al. (1968a).

Earl (1976) also paid his attention to the effect of adiabatic focusing in the propagation of charged particles of CR in stochastic magnetic fields on the background of a regular field with divergent magnetic lines of force in the direction of the field weakening. In this paper the kinetic equation is obtained for which the proper functions of scattering are found (these functions appeared to be symmetric with respect to $\cos\theta$ where θ is a pitch-angle) and the proper functions of focusing (which appeared to be asymmetric with respect to $\cos\theta$). Numerical calculations which were carried out by means of these functions show that in the case of a weak divergent field there is obtained a diffusion approximation in the pitch-angle space where the effect of adiabatic focusing can be neglected; in the case of a strong field there appeared to be a mode of coherent propagation of particles which is completely determined by the effect of adiabatic focusing.

Morfill et al. (1976) presented the arguments against the model of a superposition of the averaged over extensive time intervals of a regular spiral magnetic field with small scale inhomogeneities causing a resonance scattering of particles with Larmor radius close to inhomogeneity's dimensions, which is generally applied to calculations of a transfer of galactic CR in interplanetary space. There is proposed a model of an irregular spiral field taking into account the middle-scale variations of the interplanetary field owed to the presence of sector structure, of tangential discontinuities, of jet streams in the solar wind etc. which are statistically described by a frequency distribution of the field direction.

The averaged Fokker-Plank equation is obtained for a pitch-angle diffusion of galactic CR (the effects of direct reflection of particles and their large scale drift are not included). Basing on this equation the spatial coefficients of diffusion were derived and the expected radial gradients were estimated. The obtained values appeared to be in good agreement with the results of radial gradient measurements from the data of synchronous observations from various spacecrafts. Calculations have also been carried out of the expected temporal variations of the diffusion coefficient during a solar activity cycle including the data of corresponding variations of the parameters characterizing the spectra of magnetic inhomogeneities in the range of small and moderate scales.

Alpers et al. (1975) carried out a study consisting of charged particles diffusion in the magnetic field which is a superposition of the regular constant magnetic field $\mathbf{H}_0$ and a rapidly varying in space stochastic field $\mathbf{H}_1(\mathbf{r})$. It was shown that a contribution to the diffusion coefficient in the form of a δ -function for particles with the pitch-angles θ (with respect to the force lines of the field $\mathbf{H}_0$) close to 90° is not caused neither by pitch-angle θ scattering nor peculiarities of particle propagation along the force lines of the average field $\mathbf{H}_0$. Alpers et al. (1975) have drawn the conclusion that abnormal behavior of the coefficient of pitch-angle diffusion at $\theta \approx 90^\circ$ is caused by the fact that generally used to determine coefficient of diffusion theory of a weak interaction of particles with magnetic inhomogeneities in the same point has a singularity. This singularity, however, has no physical sense. A detailed analysis shows that if in the initial state the ensemble of charged particles has $\theta = 90^\circ$, its broadening over pitch-angle occurs considerably longer than in the case when $\theta \neq 90^\circ$ is the initial state. The matter is that at $\theta = 90^\circ$ a particle is in the situation as if it was frozen in a magnetic inhomogeneity and a variation of its state is owed only to the extremely slow regular acceleration, whereas the particles with $\theta \neq 90^\circ$ very rapidly change their energy in a stochastic way due to the action of the statistic acceleration mechanism.

Lee and Völk (1975a,b) have solved in a quasi-linear approximation the equations of diffusion of CR particles in the space of pitch-angles and energies. There is considered an interaction of particles with a field of hydromagnetic waves in the presence of a regular field $\mathbf{H}_0$. The commonly used assumption of an isotropic tensor of the spectrum of power of magnetic inhomogeneities require in the given case the equality of spectra of Alfvén and magneto-sonic waves. For the solar wind plasma this assumption is not considerably realistic. It is shown that the coefficient of pitch-angle diffusion in the case under consideration, as well as in the other quasi-linear approximations, vanishes at the pitchangles $\sim 90^\circ$. Thus in quasi-linear theory there is no reflection of CR particles. This difficulty can be overcome by non-linear treatment of particle dynamics.

Basing on the kinetic Vlasov equation Goldstein (1976) has derived a diffusion approximation for the function of distribution over the pitch-angle

variable $\mu = \cos\theta$ (where θ is a pitch-angle) for a propagation of CR in strongly turbulent magneto-active plasma in which the bonds are considered to be weak, according to Kadomtsev's hypothesis. With the assumption that the correlation function of a stochastic magnetic field has the exponential character, the detailed calculations of the diffusion coefficient D_μ over the pitch-angle variable have been carried out. Special attention is paid to the behavior of D_μ at $\theta \rightarrow \pi/2$ (i.e. at $\mu \rightarrow 0$). It was shown that $D_\mu(\mu=0)$ has a finite value in a good agreement with the results of Monte Carlo numerical calculations, in contrast to the works of the other authors based on the linear theory when it was assumed that $D_\mu(\mu=0)=0$ or $D_\mu(\mu=0) \propto \delta(\mu)$, where $\delta(\mu)$ is the Dirac δ -function.

Goldstein (1977) has presented a critical analysis of theoretical models of pitch-angle scattering and spatial diffusion of particles based on a quasi-linear approximation of the kinetic theory which were developed in the literature. Goldstein (1977) gives also a generalization of the resonance theory of disturbed trajectories of particle pitch-angle diffusion in a model of magneto-static turbulence; the results obtained are used for numerical calculations of the spatial coefficient of the field-aligned diffusion. In this case it is possible to eliminate the all divergences which are proper to a quasi-linear formalism for a spectrum of magnetic field fluctuations of the type $k^{-\nu}$ at $\nu \geq 2$ (here k is the wave number). It was found that different methods give, in the first approximation, for $1 < \nu < 2$ close values for the spatial diffusion coefficient; the method of disturbed trajectories which is used in the paper only gives systematically slightly lower values of the diffusion coefficients than the models of quasi-linear theory.

In the works of Jones et al. (1973), Jones (1975) the expression for the coefficient of a diffusion over pitch-angles has been obtained on the basis of a kinetic equation for a distribution function averaged over fluctuations. It was shown that the developed theory gives the values of the diffusion coefficient coincident with those which are expected in quasi-linear theory at $0.6 \leq \mu \leq 1$ (where $\mu = \cos\theta$ and θ is a pitch-angle); at lower values of μ the new theory gives the values for the diffusion coefficient which are considerably higher than the values expected in the usual theory.

Jones et al. (1978) obtained the kinetic equation describing particle interaction with turbulent fluctuations of a magnetic field, using the non-linear theory which has been developed in Jones et al. (1973), Jones (1975), and made it possible to overcome correctly the difficulties proper to quasi-linear theory. The effect of fluctuations in the method developed by Jones et al. (1978) is determined from particle orbits which, in their turn, include a statistical averaging over a series of possible configuration of turbulence. In the method of a partially averaged field the averaging procedure is made from a sample of all realizations for which the field intensity takes a fixed value in a given point. Using the new

method, the calculations of the coefficient $D_{\mu\mu}$ of a diffusion over pitch-angles for particles interacting with a 'stratified' model of magnetic turbulence in which the fluctuations of magnetic field are linearly polarized transverse to the direction of the average magnetic field $\langle \mathbf{H} \rangle$. The results obtained are compared with the data of quasi-linear theory and of Monte Carlo a numerical model experiment. The conclusion was drawn that the main result of quasi-linear theory consists in determining $D_{\mu\mu}$ in the pitch-angle range near 90° where quasi-linear approximation is violated. Using $D_{\mu\mu}$ value, the coefficient $\kappa_{\parallel}$ of the spatial diffusion along the direction of magnetic field $\langle \mathbf{H} \rangle$ has been estimated. It was noted that the method of partially averaged field is not restricted by a criterion of smallness of the amplitudes of fluctuating fields, and therefore it is not a perturbation theory.

Developing this study, Kaiser (1975), Kaiser et al. (1978) presented the results of numerical Monte Carlo simulating the process of charged particles diffusion over the velocities in a stochastic turbulent magnetic field. The coefficient of diffusion over pitch-angles was determined by means of exact calculation of the orbits of particles moving in a great ensemble of realizations of a stochastic magnetic field with the statistic properties which are selected in a certain way. The calculations were carried out for a wide range of particle rigidities and of mean square intensities of a magnetic field. A comparison has been made of the results given by standard quasi-linear theory with the conclusions of non-linear theory which uses partially averaged fields.

Moussas et al. (1975), Moussas and Quenby (1977) have carried out numerical calculations of the diffusion coefficient $D(\mu)$ of CR in the pitch-angle space ($\mu = \mathbf{v}\mathbf{H}/vH$, where $\mathbf{v}$ is the particle velocity and $\mathbf{H}$ is the interplanetary magnetic field) basing on the data of the three-dimensional structure of interplanetary magnetic field. A comparison was made with $D(\mu)$ which is expected according to the usual quasi-linear kinetic theory of CR propagation. It was found that at $\mu \rightarrow 0$ the numerical calculations basing on the interplanetary magnetic field data of HEOS-2 give the values ≈ -1.5 for $\lg D(\mu)$, whereas quasi-linear theory gives $\lg D(\mu) < -3$, i.e. there is a discrepancy of almost two orders. At μ from 0.3 to 0.8 a discrepancy is also about 3-4 times. Analytical corrections for quasi-linear theory providing determination of correct results at $\mu \rightarrow 0$ in the case of very weak stochastic perturbations of a regular component of interplanetary field have been obtained. The problem of a rotational discontinuity effect on the pitch-angle distribution of solar CR has been solved numerically assuming in this case that the distribution in a stream of CR before passing through a discontinuity was axially-symmetric. The results were presented for numerical calculations of the expected particle distribution over the phases φ and over μ values depending on a distance from the discontinuity. It was predicted that there arises a two-

directed pitch-angle distribution after passing through a rotational discontinuity. It was observed that arising of this peculiarity in the pitch-angle distribution of solar CR is not connected with acceleration of particles.

2.11. Fokker-Planck CR transport equation for diffusion approximation

2.11.1. Diffusion approximation including the first spherical mode

At distances exceeding the free large angle scattering path of particles the distribution function is close to the isotropic distribution. In this case the CR propagation may be described by using the diffusion approximation equation (Dolginov and Toptygin, 1966a,b). After series expanding the function in spherical harmonics we obtain

$$F(\mathbf{r}, \mathbf{p}, t) = \frac{1}{4\pi} \left[n(\mathbf{r}, p, t) + \frac{3}{vp} \mathbf{p} \mathbf{J}(\mathbf{r}, p, t) \right], \quad (2.11.1)$$

where $n(\mathbf{r}, p, t)$ is the particle concentration; $\mathbf{J}(\mathbf{r}, p, t)$ is the density of the particle flux. Substituting Eq. 2.11.1 in Eq. 2.3.7 and multiplying Eq. 2.3.7 by 1 and by $\mathbf{p}$, we shall integrate the obtained expressions over vector angles $\mathbf{p}$ taking account of the first non-vanishing terms in powers u_o/v . The resultant set of equations for $n(\mathbf{r}, p, t)$ and $\mathbf{J}(\mathbf{r}, p, t)$ is

$$\begin{aligned} \frac{\partial n}{\partial t} + \text{div} \mathbf{J} = & \frac{u_o^2}{9\kappa_o} \left[p^2 \frac{\partial^2 n}{\partial p^2} + \left(1 + \frac{v^2}{c^2} \right) p \frac{\partial n}{\partial p} \right] + \frac{\mathbf{u}_o}{3\kappa_o} \left(p \frac{\partial \mathbf{J}}{\partial p} + \frac{v^2}{c^2} \mathbf{J} \right) \\ & - \frac{1}{vr_L} [\mathbf{u}_o \times \mathbf{h}] \left[p \frac{\partial \mathbf{J}}{\partial p} + \left(1 + \frac{v^2}{c^2} \right) \mathbf{J} \right], \end{aligned} \quad (2.11.2)$$

$$\frac{\Lambda}{v} \frac{\partial \mathbf{J}}{\partial t} + \mathbf{J} = -\kappa_o \frac{\partial n}{\partial \mathbf{r}} - \left(\mathbf{u}_o + \frac{\Lambda}{r_L} [\mathbf{h} \times \mathbf{u}_o] \right) \frac{p}{3} \frac{\partial n}{\partial p} - \frac{\Lambda}{r_L} [\mathbf{h} \times \mathbf{J}], \quad (2.11.3)$$

where

$$\Lambda = \frac{12c^2 p^2 v \Gamma((v-1)/2)}{\sqrt{\pi} \Gamma(v/2) l_c \langle H_1^2(\mathbf{r}) \rangle}, \quad \kappa_o = \frac{v\Lambda}{3}, \quad r_L = \frac{cp}{eH_o}. \quad (2.11.4)$$

When writing Eq. 2.11.2–2.11.4 it was included that the correlation tensor of a stochastic magnetic field $B_{\alpha\beta}$ has a form determined by Eq. 2.3.3 and that $\Psi(r/l_c)$ in Eq. 2.3.3 is determined by the Eq. 2.3.5. If the time for a marked change in the particle flux density $\mathbf{J}(\mathbf{r}, p, t)$ is long compared with the characteristic time of particle diffusion, the term $\frac{\Lambda}{v} \frac{\partial \mathbf{J}}{\partial t}$ in Eq. 2.11.3 may be neglected. After solving the equation obtained relative to $\mathbf{J}(\mathbf{r}, p, t)$ we obtain (Dolginov and Toptygin, 1966a,b):

$$J_\alpha = -\kappa_{\alpha\beta} \frac{\partial n}{\partial r_\beta} - u_{o\alpha} \frac{p}{3} \frac{\partial n}{\partial p}, \quad (2.11.5)$$

where the tensor diffusion coefficient is of the form

$$\kappa_{\alpha\beta} = \frac{\kappa_o r_L^2}{r_L^2 + \Lambda^2} \left(\delta_{\alpha\beta} + \frac{\Lambda^2}{r_L^2} h_\alpha h_\beta + \frac{\Lambda}{r_L} \varepsilon_{\alpha\beta\gamma} h_\gamma \right). \quad (2.11.6)$$

In the coordinate axes directed along the vectors

$$\mathbf{n}_1 = \frac{[\mathbf{u}_o \times \mathbf{H}_o]}{[\mathbf{u}_o \times \mathbf{H}_o]}, \quad \mathbf{n}_2 = \frac{[\mathbf{n}_1 \times \mathbf{H}_o]}{[\mathbf{n}_1 \times \mathbf{H}_o]}, \quad \mathbf{n}_3 = \mathbf{h} = \mathbf{H}_o/H_o, \quad (2.11.7)$$

the diffusion tensor is of the form

$$\kappa_{\alpha\beta} = \frac{\kappa_o r_L^2}{r_L^2 + \Lambda^2} \begin{pmatrix} 1 & -\frac{\Lambda}{r_L} & 0 \\ \frac{\Lambda}{r_L} & 1 & 0 \\ 0 & 0 & \frac{r_L^2 + \Lambda^2}{r_L^2} \end{pmatrix}, \quad (2.11.8)$$

similar to the form of the electro-conduction tensor for a collision plasma in a magnetic field.

The first term in Eq. 2.11.5 is the conventional diffusion flux proportional to the concentration gradient. In the absence of the regular magnetic field ($r_L \rightarrow \infty$)

$$\kappa_{\alpha\beta} = \kappa_o \delta_{\alpha\beta}, \quad (2.11.9)$$

which corresponds to the isotropic diffusion, and the expression Eq. 2.11.5 takes the form

$$\mathbf{J} = -\kappa_o \frac{\partial n}{\partial \mathbf{r}} - \mathbf{u}_o \frac{p}{3} \frac{\partial n}{\partial p}. \quad (2.11.10)$$

In this case Λ has the meaning of the transport free path of particles and κ_o is the scalar diffusion coefficient. The second term in Eq. 2.11.5 describes the convective flux owed to the motion of magnetic field inhomogeneities frozen into the solar wind plasma. Substituting Eq. 2.11.5 in Eq. 2.11.2 we obtain the equation of anisotropic diffusion for the particle concentration $n(\mathbf{r}, p, t)$:

$$\frac{\partial n}{\partial t} = \frac{\partial}{\partial r_\alpha} \left(\kappa_{\alpha\beta} \frac{\partial n}{\partial r_\beta} \right) - u_{o\alpha} \frac{\partial n}{\partial r_\alpha} + \frac{\partial u_{o\alpha}}{\partial r_\alpha} \frac{p}{3} \frac{\partial n}{\partial p}. \quad (2.11.11)$$

The last term in the right hand part of Eq. 2.11.11 describes the adiabatic cooling of charged particles associated with radial divergence of solar wind plasma with the frozen in magnetic field inhomogeneities. A consistent derivation of the Eq. 2.11.11, on the basis of the kinetic equation was first considered by Dolginov and Toptygin (1966a,b).

2.11.2. Including of magnetic inhomogeneities velocity fluctuations

The Eq. 2.11.11 is obtained with the assumption that a proper motion of magnetic inhomogeneities is neglected, i.e. $u_1 = 0$. Including a stochastic velocity of magnetic inhomogeneities is equivalent to appearance of stochastic electric fields resulting in acceleration of particles (Fermi mechanism of acceleration). Owing to general properties of the Fermi acceleration mechanism (see Chapter 4) the necessary condition for the efficiency of this mechanism is a high degree of isotropy of particle distribution in the momentum space. Therefore the acceleration of particles can be considered in a diffusion approximation. The procedure similar to that used in deducing equation Eq. 2.11.11 results in the equation of anisotropic diffusion including the effect of particle acceleration (Dolginov and Toptygin, 1967):

$$\frac{\partial n}{\partial t} = \frac{\partial}{\partial r_\alpha} \left(\kappa_{\alpha\beta}(\mathbf{r}, p) \frac{\partial n}{\partial r_\beta} \right) - u_{o\alpha} \frac{\partial n}{\partial r_\alpha} + \frac{\partial u_{o\alpha}}{\partial r_\alpha} \frac{p}{3} \frac{\partial n}{\partial p} + \frac{1}{p^2} \frac{\partial}{\partial p} p^2 D(\mathbf{r}, p) \frac{\partial n}{\partial p}, \quad (2.11.12)$$

where

$$D(\mathbf{r}, p) = \frac{\langle u_1^2 \rangle p^2}{3v\Lambda(\mathbf{r}, p)} \quad (2.11.13)$$

is the coefficient of particle diffusion in the momentum space; $\langle u_1^2 \rangle$ is the mean-square velocity fluctuation.

2.11.3. Diffusion approximation including the second spherical harmonic

Dorman, Katz and Fedorov (1977, 1978a,b), basing on the kinetic equation which includes an interaction of charged particles with stochastic magnetic fields in space, have obtained a set of equations for the diffusion approximation taking into account the second spherical harmonic. Let us start from the kinetic equation describing a propagation of CR in magnetized moving plasma

$$\left(\frac{\partial}{\partial t} + L_o \right) F(\mathbf{r}, \mathbf{p}, t) = D_\alpha \int_0^t d\tau \exp(-L_o \tau) \times B_{\alpha\lambda} \left(\frac{\mathbf{r} + \mathbf{r}_1}{2}, \mathbf{p}_1; \mathbf{r}_1 - \mathbf{r} - \mathbf{u}\tau \right) D_\lambda F(\mathbf{r}, \mathbf{p}, t) = \text{St}F, \quad (2.11.14)$$

where $F(\mathbf{r}, \mathbf{p}, t)$ is the distribution function of CR;

$$L_o = \mathbf{v} \frac{\partial}{\partial \mathbf{r}} - \mathbf{D} \mathbf{H}_o \quad (2.11.15)$$

is the operator describing a motion of charged particles in the regular magnetic field;

$$\mathbf{D} = \frac{e}{c} \left[\mathbf{v} - \mathbf{u}, \frac{\partial}{\partial \mathbf{p}} \right] \quad (2.11.16)$$

is the operator describing a variation of a particle momentum in magnetized plasma moving with the velocity $u \ll c$, $\mathbf{v}$ is the velocity of a particle with a momentum $\mathbf{p}$ and a charge e , c is the velocity of light;

$$B_{\alpha\lambda}(\mathbf{r}, t; \mathbf{r}', t') = \langle H_\alpha(\mathbf{r}, t) H_\lambda(\mathbf{r}', t') \rangle \quad (2.11.17)$$

is the correlation tensor of a stochastic field $\mathbf{H}(\mathbf{r}, t)$. The angular brackets in Eq. 2.11.17 mean the averaging over a statistic ensemble which corresponds to a stochastic magnetic field. The action of the operator $\exp(-L_o \tau)$ in the collision

integral $\text{St}F$ of the kinetic Eq. 2.11.14 reduces to a substitution of a radius-vector $\mathbf{r}$ of the particle by $\mathbf{r} - \delta\mathbf{r}(\tau)$ and its momentum $\mathbf{p}$ by $\mathbf{p} - \delta\mathbf{p}(\tau)$, where $\delta\mathbf{r}(\tau)$ and $\delta\mathbf{p}(\tau)$ are variations of the radius-vector and momentum of a particle in the regular magnetic field after a time interval τ .

Let us consider a stochastic magnetic field to be statistically isotropic. Then, according to Section 2.3 we shall have

$$B_{\alpha\lambda}(\mathbf{r}, \mathbf{x}) = \frac{1}{3} \langle H^2(\mathbf{r}) \rangle \left\{ \Psi\left(\frac{x}{l_c}\right) \delta_{\alpha\lambda} - \Psi_1\left(\frac{x}{l_c}\right) \frac{x_\alpha x_\lambda}{x^2} \right\}, \quad (2.11.18)$$

where $\Psi(x/l_c)$ and $\Psi_1(x/l_c)$ are some scalar functions and their interrelation is determined by the equation $\text{div}\mathbf{H}=0$ so that it is always possible to determine the function Ψ_1 from a given function Ψ ; l_c is the correlation radius of a stochastic magnetic field. Let us represent the distribution function $F(\mathbf{r}, \mathbf{p}, t)$ in the form of a series of expansion over spherical harmonics limiting it by the three terms of the expansion:

$$F(\mathbf{r}, \mathbf{p}, t) = \frac{1}{4\pi} \left[n(\mathbf{r}, p, t) + \frac{3}{vp} \mathbf{p} \mathbf{J}(\mathbf{r}, p, t) + \frac{p_\alpha p_\beta}{p^2} f_{\alpha\beta}(\mathbf{r}, \mathbf{p}, t) \right], \quad (2.11.19)$$

where $n(\mathbf{r}, p, t)$ and $\mathbf{J}(\mathbf{r}, p, t)$ are the density of particles and the flux density of particles with a given value of momentum, respectively, $f_{\alpha\beta}(\mathbf{r}, \mathbf{p}, t)$ is the symmetric tensor of the second rank, the components of which determine a contribution of the second spherical harmonic into a distribution of CR. The quantities $\mathbf{J}$ and $f_{\alpha\beta}$ characterize the anisotropy distribution of the CR. Observe that the trace of the tensor $f_{\alpha\beta}$ is equal to zero, i.e. $f_{\alpha\alpha}=0$ as a consequence of the identity $n_\alpha^2 = 1(\mathbf{n} = \mathbf{p}/p)$. Using Eq. 2.11.19 for the collision integral in Eq. 2.11.14, we obtain

$$\begin{aligned} \text{St}F = & \frac{p}{4\pi m \Lambda} (\delta_{\alpha\beta} - 3n_\alpha n_\beta) L_{\alpha\beta} + \frac{3m}{4\pi \Lambda} (\delta_{\alpha\beta} - 2n_\alpha n_\beta) \mu_\beta \frac{\partial I_\alpha}{\partial p} \\ & - \frac{3m}{4\pi p \Lambda} (\delta_{\alpha\beta} - 3n_\alpha n_\beta) \mu_\beta I_\alpha - \frac{3}{4\pi \Lambda} (\mathbf{n} \mathbf{I}), \end{aligned} \quad (2.11.20)$$

where m is mass of the particle and the quantity

$$\Lambda = \frac{3c^2 p^2}{2e^2 l_c \left\langle H^2(\mathbf{r}) \right\rangle \int_0^\infty dz \Psi(z)} \quad (2.11.21)$$

has the significance of the transport length of a particle free path;

$$\mathbf{I} = \mathbf{J} + \mathbf{u} \frac{p}{3} \frac{\partial n}{\partial p}, \quad (2.11.22)$$

$$L_{\alpha\beta} = f_{\alpha\beta} - \frac{m^2}{2} u_\alpha u_\beta \left(\frac{\partial^2 n}{\partial p^2} - \frac{1}{p} \frac{\partial n}{\partial p} \right). \quad (2.11.23)$$

Multiplying successively the kinetic Eq. 2.11.14 with the collision integral (Eq. 2.11.20) by 1, components of $\mathbf{n}$ vector, components of the tensor $n_i n_k - (1/3)\delta_{ik}$, and integrating over the angular variables in the momentum space, we obtain a set of equations for the quantities $n, \mathbf{J}, f_{\alpha\beta}$:

$$\frac{\partial n}{\partial t} + \text{div} \mathbf{I} = \mathbf{u} \frac{p}{3} \frac{\partial^2 n}{\partial p \partial \mathbf{r}} + \text{div} \mathbf{u} \frac{p}{3} \frac{\partial n}{\partial p} + \frac{m}{r_L} \left([\mathbf{u} \mathbf{h}] \frac{\partial \mathbf{I}}{\partial p} \right) + \frac{m}{\Lambda} \left(\mathbf{u} \frac{\partial \mathbf{I}}{\partial p} \right) + \frac{m}{pr_L} ([\mathbf{u} \mathbf{h}] \mathbf{I}), \quad (2.11.24)$$

$$\mathbf{I} + \frac{\Lambda}{r_L} [\mathbf{h} \times \mathbf{I}] = \kappa_o \frac{\partial n}{\partial \mathbf{r}}, \quad (2.11.25)$$

$$\begin{aligned} \left(T_{ik\alpha\beta}^{(3)} + 2 \frac{\Lambda}{r_L} T_{ik\alpha\beta}^{(7)} \right) L_{\alpha\beta} = & - \frac{m^2}{2} \left(\frac{\partial^2 n}{\partial p^2} - \frac{1}{p} \frac{\partial n}{\partial p} \right) u_\alpha u_\beta T_{ik\alpha\beta}^{(1)} \\ & - \frac{m\Lambda}{3} \left(T_{ik\alpha\beta}^{(1)} - 3T_{ik\alpha\beta}^{(2)} \right) \left(\frac{3}{p} \frac{\partial I_\alpha}{\partial r_\beta} - u_\beta \frac{\partial^2 n}{\partial r_\alpha \partial p} - \frac{\partial u_\alpha}{\partial r_\beta} \frac{\partial n}{\partial p} \right) \\ & + \frac{m^2}{2} u_\beta \frac{\partial I_\alpha}{\partial p} \left(2T_{ik\alpha\beta}^{(1)} - 6T_{ik\alpha\beta}^{(2)} - 3 \frac{\Lambda}{r_L} T_{ik\alpha\beta}^{(7)} - \frac{\Lambda}{r_L} A_{ik\alpha\beta}^{(1)} - 3 \frac{\Lambda}{r_L} A_{ik\alpha\beta}^{(4)} \right) \\ & + \frac{m^2}{p^2} \left(9T_{ik\alpha\beta}^{(2)} - 3T_{ik\alpha\beta}^{(1)} + 6 \frac{\Lambda}{r_L} T_{ik\alpha\beta}^{(7)} + 2 \frac{\Lambda}{r_L} A_{ik\alpha\beta}^{(1)} + 6 \frac{\Lambda}{r_L} A_{ik\alpha\beta}^{(4)} \right) u_\beta I_\alpha, \end{aligned} \quad (2.11.26)$$

where

$$\kappa_o = v\Lambda/3, \quad \mathbf{h} = \mathbf{H}_o/H_o \quad (2.11.27)$$

represent the scalar coefficient of diffusion and the unit vector in the direction of the regular component of magnetic field. In Eq. 2.11.26 $T_{ik\alpha\beta}^{(j)}$ ($j = 1, 2, 3 \dots 9$) are the tensors of the fourth rank which are symmetric with respect to the first and the second pairs of indices:

$$\begin{aligned} T_{ik\alpha\beta}^{(1)} &= \delta_{ik} \delta_{\alpha\beta}, \quad T_{ik\alpha\beta}^{(2)} = (1/2)(\delta_{i\alpha} \delta_{k\beta} + \delta_{i\beta} \delta_{k\alpha}), \quad T_{ik\alpha\beta}^{(3)} = \delta_{ik} h_{\alpha} h_{\beta}, \\ T_{ik\alpha\beta}^{(4)} &= h_i h_k \delta_{\alpha\beta}, \quad T_{ik\alpha\beta}^{(5)} = (1/4)(\delta_{ik} h_{\alpha} h_{\beta} + \delta_{i\beta} h_k h_{\alpha} + \delta_{k\alpha} h_i h_{\beta} + \delta_{k\beta} h_i h_{\alpha}), \\ T_{ik\alpha\beta}^{(6)} &= h_i h_k h_{\alpha} h_{\beta}, \quad T_{ik\alpha\beta}^{(7)} = (1/4)(\delta_{i\alpha} \varepsilon_{\beta\gamma k} + \delta_{i\beta} \varepsilon_{\alpha\gamma k} + \delta_{k\alpha} \varepsilon_{\beta i \gamma} + \delta_{k\beta} \varepsilon_{\alpha i \gamma}) h_{\gamma}, \\ T_{ik\alpha\beta}^{(8)} &= (1/4)(\varepsilon_{\beta\gamma k} h_i h_{\alpha} + \varepsilon_{\alpha\gamma k} h_i h_{\beta} + \varepsilon_{\beta i \gamma} h_k h_{\alpha} + \varepsilon_{\alpha i \gamma} h_k h_{\beta}) h_{\gamma}, \\ T_{ik\alpha\beta}^{(9)} &= (1/2)(\varepsilon_{\alpha i \gamma} \varepsilon_{\beta k \lambda} + \varepsilon_{\alpha k \gamma} \varepsilon_{\beta i \lambda}) h_{\gamma} h_{\lambda}, \end{aligned} \quad (2.11.28)$$

where $\varepsilon_{\alpha\beta\gamma}$ is the unit anti-symmetric tensor of the third rank. Tensors $A_{ik\alpha\beta}^{(j)}$ ($j = 1, 2, 3, 4$, and 5) in Eq. 2.11.27 are symmetric relative to the first pair of indices and anti-symmetric with respect to the second pair:

$$\begin{aligned} A_{ik\alpha\beta}^{(1)} &= \delta_{ik} \varepsilon_{\alpha\beta\gamma} h_{\gamma}, \quad A_{ik\alpha\beta}^{(2)} = h_i h_k \varepsilon_{\alpha\beta\gamma} h_{\gamma}, \\ A_{ik\alpha\beta}^{(3)} &= (1/4)(\delta_{i\alpha} h_k h_{\beta} + \delta_{k\alpha} h_i h_{\beta} - \delta_{i\beta} h_k h_{\alpha} - \delta_{k\beta} h_i h_{\alpha}), \\ A_{ik\alpha\beta}^{(4)} &= (1/4)(\delta_{i\alpha} \varepsilon_{\beta k \gamma} + \delta_{k\alpha} \varepsilon_{\beta i \gamma} - \delta_{i\beta} \varepsilon_{\alpha k \gamma} - \delta_{k\beta} \varepsilon_{\alpha i \gamma}) h_{\gamma}, \\ A_{ik\alpha\beta}^{(5)} &= (1/4)(\varepsilon_{\beta k \gamma} h_i h_{\alpha} + \varepsilon_{\beta i \gamma} h_k h_{\alpha} - \varepsilon_{\alpha k \gamma} h_i h_{\beta} - \varepsilon_{\alpha i \gamma} h_k h_{\beta}) h_{\gamma}. \end{aligned} \quad (2.11.29)$$

When deducing the set of Eq. 2.11.24–2.11.26, the terms of order of $(u^2/v^2)n$, $(u/v^2)\mathbf{J}$, and $f_{\alpha\beta}$ were hold and the terms of higher orders were omitted, taking into account that $u/v \ll 1$. After solving the Eq. 2.11.25 relative to $\mathbf{I}$, we have the relation obtained in Section 2.11.1:

$$J_{\alpha} = -\kappa_{\alpha\beta} \frac{\partial n}{\partial r_{\beta}} - u_{\alpha} \frac{p}{3} \frac{\partial n}{\partial p}, \quad (2.11.30)$$

where

$$\kappa_{\alpha\beta} = \frac{\kappa_0 r_L^2}{r_L^2 \Lambda^2} \left(\delta_{\alpha\beta} + \frac{\Lambda^2}{r_L^2} h_{\alpha} h_{\beta} + \frac{\Lambda}{r_L} \varepsilon_{\alpha\beta\gamma} h_{\gamma} \right) \quad (2.11.31)$$

is the tensor coefficient of diffusion. The first term on the right hand side of Eq. 2.11.31 corresponds to isotropic diffusion in absence of the regular magnetic field ($r_L \rightarrow \infty$). The second term is related to a diffusion stream of particles along the direction of the regular field. The third term determines the presence of a stream of particles normal to the regular magnetic field and the gradient of particle density. Using Eq. 2.11.30 we obtain from Eq. 2.11.24 the equation of anisotropic diffusion for particle density $n(\mathbf{r}, p, t)$:

$$\frac{\partial n}{\partial t} = \frac{\partial}{\partial r_\alpha} \kappa_{\alpha\beta} \frac{\partial n}{\partial r_\beta} - \mathbf{u} \frac{\partial n}{\partial \mathbf{r}} + \frac{p}{3} \frac{\partial n}{\partial p} \text{div} \mathbf{u}. \quad (2.11.32)$$

Let us solve the Eq. 2.11.26 relative to the tensor $L_{\alpha\beta}$. For this purpose determine the tensor $G_{\mu\nu ik}$ satisfying the relation

$$G_{\mu\nu ik} \left(3T_{ik\alpha\beta}^{(2)} + 2\frac{\Lambda}{r_L} T_{ik\alpha\beta}^{(7)} \right) = T_{\mu\nu\alpha\beta}^{(2)}. \quad (2.11.33)$$

From Eq. 2.11.33 we find

$$G_{\mu\nu ik} = \sigma_2 T_{\mu\nu ik}^{(2)} + \sigma_5 T_{\mu\nu ik}^{(5)} + \sigma_6 T_{\mu\nu ik}^{(6)} + \sigma_7 T_{\mu\nu ik}^{(7)} + \sigma_8 T_{\mu\nu ik}^{(8)} + \sigma_9 T_{\mu\nu ik}^{(9)}, \quad (2.11.34)$$

where

$$\begin{aligned} \sigma_2 &= \frac{1}{6} + \frac{3}{2}\eta_1, \quad \sigma_5 = -\frac{1}{3} - 3\eta_1 + 6\eta_o, \quad \sigma_6 = \frac{1}{2} + \frac{3}{2}\eta_1 - 6\eta_o, \quad \sigma_7 = -\frac{\Lambda}{r_L}\eta_1, \\ \sigma_8 &= 2\frac{\Lambda}{r_L}(\eta_1 - \eta_o), \quad \sigma_9 = \frac{1}{6} - \frac{3}{2}\eta_1, \quad \eta_o = \left(9 + \frac{\Lambda^2}{r_L^2}\right)^{-1}, \quad \eta_1 = \left(9 + 4\frac{\Lambda^2}{r_L^2}\right)^{-1}. \end{aligned} \quad (2.11.35)$$

Using the tensor $G_{\mu\nu ik}$ we obtain for the tensor $f_{\mu\nu}$ from Eq. 2.11.26:

$$\begin{aligned} f_{\mu\nu} &= \frac{m^2}{6} \left(3u_\mu u_\nu - u^2 \delta_{\mu\nu} \right) \left(\frac{\partial^2 n}{\partial p^2} - \frac{1}{p} \frac{\partial n}{\partial p} \right) - \frac{m\Lambda}{3} \eta_{\mu\nu\alpha\beta} \frac{\partial^2 n}{\partial r_\alpha \partial p} u_\beta \\ &\quad - \frac{m\Lambda}{p} \zeta_{\mu\nu\alpha\beta} \left(\frac{\partial}{\partial r_\beta} \kappa_{\alpha\gamma} \frac{\partial n}{\partial r_\gamma} - \frac{\partial u_\alpha}{\partial r_\beta} \frac{p}{3} \frac{\partial n}{\partial p} \right) - \frac{m\Lambda}{3p} \xi_{\mu\nu\alpha\beta} \frac{\partial n}{\partial r_\alpha} u_\beta, \end{aligned} \quad (2.11.36)$$

where

$$\begin{aligned}
\zeta_{\mu\nu\alpha\beta} &= \sum_{j=1}^9 a_j T_{\mu\nu\alpha\beta}^{(j)}; \quad \eta_{\mu\nu\alpha\beta} = \sum_{j=1}^9 c_j T_{\mu\nu\alpha\beta}^{(j)} + \sum_{j=1}^5 q_j A_{\mu\nu\alpha\beta}^{(j)}; \\
\xi_{\mu\nu\alpha\beta} &= \sum_{j=1}^9 \rho_j T_{\mu\nu\alpha\beta}^{(j)} + \sum_{j=1}^5 g_j A_{\mu\nu\alpha\beta}^{(j)}.
\end{aligned}
\tag{2.11.37}$$

In this case $A_{\mu\nu\alpha\beta}^{(j)}$ and $T_{\mu\nu\alpha\beta}^{(j)}$ are determined by Eq. 2.11.29 and Eq. 2.11.28, and the factors $a_j, c_j, q_j, \rho_j, g_j$ are determined by:

$$\begin{aligned}
a_1 &= 1/3, \quad a_2 = -3\sigma_2, \quad a_3 = a_4 = 0, \quad a_5 = -3\sigma_5, \quad a_6 = -3\sigma_6, \quad a_7 = -3\sigma_7, \\
a_8 &= -3\sigma_8, \quad a_9 = -3\sigma_9; \quad c_1 = r_L^2 / (r_L^2 + \Lambda^2), \quad c_2 = -3c_1, \quad c_3 = -(\Lambda^2 / r_L^2) c_1, \\
c_4 &= 0, \quad c_5 = -3c_3, \quad c_6 = 0, \quad c_7 = -(\Lambda / r_L) c_1, \quad c_8 = c_9 = 0; \quad q_1 = -(\Lambda / r_L) c_1, \\
q_2 &= 0, \quad q_3 = 3c_3, \quad q_4 = -c_7, \quad q_5 = 0; \quad \rho_1 = \left(1 - (\Lambda^2 / r_L^2)\right) c_1^2, \quad \rho_2 = -3\rho_1, \\
\rho_3 &= (\Lambda^2 / r_L^2) \left(3 + (\Lambda^2 / r_L^2)\right) c_1, \quad \rho_4 = 0, \quad \rho_5 = -\rho_3, \quad \rho_6 = 0, \quad \rho_7 = 6(\Lambda / r_L) c_1^2, \\
\rho_8 &= \rho_9 = 0; \quad g_1 = -2(\Lambda / r_L) c_1^2, \quad g_2 = 0, \quad g_3 = \rho_3, \quad g_4 = -\rho_7, \quad g_5 = 0.
\end{aligned}
\tag{2.11.38}$$

Consider first the case of a weak regular magnetic field when a particle's Larmor radius r_L is much large compared to the transport length Λ of a particle free path, i.e. $\Lambda / r_L \ll 1$. In this case the expression for $f_{\mu\nu}$ is considerably simplified:

$$\begin{aligned}
f_{\mu\nu} &= \frac{m^2}{6} \left(3u_\mu u_\nu - u^2 \delta_{\mu\nu} \right) \left(\frac{\partial^2 n}{\partial p^2} - \frac{1}{p} \frac{\partial n}{\partial p} \right) - \frac{m\Lambda}{3p} \left\{ \delta_{\mu\nu} \frac{\partial}{\partial r_\alpha} \kappa_o \frac{\partial n}{\partial r_\alpha} \right. \\
&\quad \left. - \frac{3}{2} \left(\frac{\partial}{\partial r_\nu} \kappa_o \frac{\partial n}{\partial r_\mu} + \frac{\partial}{\partial r_\mu} \kappa_o \frac{\partial n}{\partial r_\nu} \right) \right\} - \frac{m\Lambda}{9} \frac{\partial n}{\partial p} \left\{ \delta_{\mu\nu} \text{div} \mathbf{u} - \frac{3}{2} \left(\frac{\partial u_\mu}{\partial r_\nu} + \frac{\partial u_\nu}{\partial r_\mu} \right) \right\} \\
&\quad - \frac{m\Lambda}{3} \left\{ \delta_{\mu\nu} \left(\mathbf{u} \frac{\partial^2 n}{\partial \mathbf{r} \partial \mathbf{p}} \right) - \frac{3}{2} \left(u_\nu \frac{\partial^2 n}{\partial r_\mu \partial p} + u_\mu \frac{\partial^2 n}{\partial r_\nu \partial p} \right) \right\} \\
&\quad - \frac{m\Lambda}{3p} \left\{ \delta_{\mu\nu} \left(\mathbf{u} \frac{\partial n}{\partial \mathbf{r}} \right) - \frac{3}{2} \left(u_\nu \frac{\partial n}{\partial r_\mu} + u_\mu \frac{\partial n}{\partial r_\nu} \right) \right\}.
\end{aligned}
\tag{2.11.39}$$

In the opposite extreme case of a strong field, when $\Lambda / r_L \ll 1$, the expression for $f_{\mu\nu}$ takes the form

$$\begin{aligned}
f_{\mu\nu} = & \frac{m^2}{6} \left(3u_\mu u_\nu - u^2 \delta_{\mu\nu} \right) \left(\frac{\partial^2 n}{\partial p^2} - \frac{1}{p} \frac{\partial n}{\partial p} \right) + \frac{m\Lambda}{6p} \left\{ 2\delta_{\mu\nu} \text{div} \mathbf{I} - \frac{3}{2} \left(\frac{\partial I_\mu}{\partial r_\nu} + \frac{\partial I_\nu}{\partial r_\mu} \right) \right. \\
& + \frac{3}{2} \left[h_\nu \left(\mathbf{h} \frac{\partial I_\mu}{\partial \mathbf{r}} \right) + h_\mu \left(\mathbf{h} \frac{\partial I_\nu}{\partial \mathbf{r}} \right) + h_\nu \left(\mathbf{h} \frac{\partial \mathbf{I}}{\partial r_\mu} \right) + h_\mu \left(\mathbf{h} \frac{\partial \mathbf{I}}{\partial r_\nu} \right) \right] - \frac{3}{2} \varepsilon_{\mu\gamma\alpha} \varepsilon_{\nu\lambda\beta} h_\gamma h_\lambda \\
& \times \left(\frac{\partial I_\alpha}{\partial r_\beta} + \frac{\partial I_\beta}{\partial r_\alpha} \right) - 9h_\mu h_\nu h_\alpha h_\beta \frac{\partial I_\alpha}{\partial r_\beta} \left. \right\} - \frac{m\Lambda}{18} \frac{\partial n}{\partial p} \left\{ 2\delta_{\mu\nu} \text{div} \mathbf{u} - \frac{3}{2} \left(\frac{\partial u_\mu}{\partial r_\nu} + \frac{\partial u_\nu}{\partial r_\mu} \right) \right. \\
& + \frac{3}{2} \left[h_\nu \left(\mathbf{h} \frac{\partial u_\mu}{\partial \mathbf{r}} \right) + h_\mu \left(\mathbf{h} \frac{\partial u_\nu}{\partial \mathbf{r}} \right) + h_\nu \left(\mathbf{h} \frac{\partial \mathbf{u}}{\partial r_\mu} \right) + h_\mu \left(\mathbf{h} \frac{\partial \mathbf{u}}{\partial r_\nu} \right) \right] - \frac{3}{2} \varepsilon_{\mu\gamma\alpha} \varepsilon_{\nu\lambda\beta} h_\gamma h_\lambda \\
& \times \left(\frac{\partial u_\alpha}{\partial r_\beta} + \frac{\partial u_\beta}{\partial r_\alpha} \right) - 9h_\mu h_\nu h_\alpha h_\beta \frac{\partial u_\alpha}{\partial r_\beta} \left. \right\} - \frac{m\Lambda}{3} \left\{ \delta_{\mu\nu} (\mathbf{u}\mathbf{h}) - \frac{3}{2} (u_\mu h_\nu + u_\nu h_\mu) \right\} \\
& \times \left(\mathbf{h} \frac{\partial^2 n}{\partial \mathbf{r} \partial p} \right) - \frac{m\Lambda}{3p} \left\{ \delta_{\mu\nu} (\mathbf{u}\mathbf{h}) - \frac{3}{2} (u_\mu h_\nu + u_\nu h_\mu) \right\} \left(\mathbf{h} \frac{\partial n}{\partial \mathbf{r}} \right), \tag{2.11.40}
\end{aligned}$$

where

$$\mathbf{I} = -\kappa_o \mathbf{h} \left(\mathbf{h} \frac{\partial n}{\partial \mathbf{r}} \right). \tag{2.11.41}$$

The Eq. 2.11.26, or its extreme cases Eq. 2.11.39 and Eq. 2.11.40, together with the equation of anisotropic diffusion Eq. 2.11.32 and the expression for the vector of particle flux density Eq. 2.11.30 solve the problem of deducing the set of equations in the diffusion approximation including the second spherical harmonic. Observe that this equation set has a remarkable feature: the tensor $f_{\mu\nu}$ does not give a contribution either in the equation of anisotropic diffusion Eq. 2.11.32 or in the expression for the particle flux density Eq. 2.11.30. The circumstance mentioned simplifies substantially the analysis of propagation of CR particles when it is necessary to take into account the third spherical harmonic.

2.11.4. Drift effects in a diffusion propagation of CR

With the assumption that the regular magnetic field component, with lines of force in the form of Archimedean spirals, consists of several sectors with alternating field direction (anti-Sunward and Sunward), Barnden and Bercovitch (1975) have carried out Monte Carlo calculations of trajectories of test particles of cosmic radiation in their stochastic wandering in the solar system. The calculations were carried out including parallel with diffusion, a convective transfer of particles by the solar wind, as well as their energy losses owing to adiabatic deceleration caused by radially divergent inhomogeneities. It was shown

that for a transfer of CR in interplanetary space the latitude drift of a particle is of substantial significance, which arises owing to a curvature of lines of force of the regular component of the magnetic field and to the presence of the field gradients in a vicinity of the sector boundaries. It was found that particles coming to the Earth with energy lower than 100 GeV have a wide distribution of the energy losses and of the duration of their wandering in the solar system, and that they come from the Galaxy in a wide ranges of helio-latitudes.

Forman (1975) considered the general expression, which describes a formation of CR anisotropy in interplanetary space and includes four terms. The first term represents a convective transfer of CR in a radial direction from the Sun by the solar wind (the Compton-Getting effect). The second term reflects a diffusion along magnetic lines of force (inverse to the gradient of CR density). The third term is owed to diffusion across the field lines of force, and the fourth term is caused by a transverse gradient drift that is directed normally to the field lines of force and normally to the gradient of CR density. To account for the experimental data obtained by means of the neutron super-monitors, according to which the anisotropy vector of CR in interplanetary space in 20-35% of observations occurs to be directed at the angle above 30° to the direction of the magnetic field projection onto the ecliptic plane (averaged over 24 hours), the following two possibilities are analyzed: either the expression stated for CR anisotropy in interplanetary space is not valid, or CR diffusion across magnetic lines of force and (or) a transverse gradient drift are of substantially greater importance sometimes than is usually considered. Forman (1975) draws the conclusions that the second possibility is, rather, realized. In this case the important role must be played by the transverse gradient drift whereas the case $\kappa_{\perp}/\kappa_{\parallel} \approx 1$ (where $\kappa_{\perp}$ and $\kappa_{\parallel}$ are the components of the coefficient of diffusion across and along the field) is realized in rare days.

Jokipii and Levy (1977), Jokipii et al. (1977) have shown that a drift of CR particles in twisted in the Archimedean spirals interplanetary magnetic field which is related to the gradient of magnetic field and the curvature of the lines of force, affects considerably diffusion propagation and the effect of modulation of galactic CR by the solar wind. The case is that the drift velocities of CR particles (the rigidities $R > 0.3$ GV) appear to be higher than the solar wind velocity, and the value of the radial component of drift velocity is comparable to or higher than the wind's velocity. Preliminary results of Monte Carlo calculations are presented for CR modulation in a spherically symmetric solar wind carrying magnetic field in the form of Archimedean spiral. The calculations show that including particle drift can result in a considerable decrease of modulation, heliocentric gradient, and energy variation (for particles with $R \sim 1$ GV) inside the solar system. It was observed that, though the calculations have been carried out for a magnetic field of certain configuration, the drift effect should act as well in a more general case.

Therefore, always when the drift velocity is comparable to or higher than the velocity of the solar wind, it is easier for galactic CR to penetrate inside the solar system, and this results in a decrease of energy variation and in a decrease of the radial gradient of CR.

In the paper of Dorman, Dremukhina and Okulov (1977a,b) the component of CR current was considered which is caused by gradient drift of high energy charge particles of CR in a stationary non-uniform interplanetary magnetic field. As charge particles with the energy E in the field $\mathbf{H}$ have the magnetic moment

$$\mu = E \sin^2 \theta / H, \quad (2.11.42)$$

where θ is a particle pitch angle, there will arise in a non-uniform field the force

$$\mathbf{F} = -\vec{\nabla}(\mu \mathbf{H}), \quad (2.11.43)$$

under the action of which the particles will drift with the velocity

$$\mathbf{v}_{dr} \propto \mathbf{F} \times \mathbf{H}. \quad (2.11.44)$$

To determine from Liouville's theorem the CR current arising in the stationary case, the equation was obtained for the distribution function of particles in the six-dimensional phase space $\mathbf{r}, \mathbf{p}$ (where $\mathbf{r}$ are the spatial coordinates, $\mathbf{p}$ is the particle momentum) through Poissonian brackets with the Hamiltonian of the system in which the gradient drift is included. Parker's model of interplanetary field in the form of Archimedean spirals was considered as an example. It was shown that if the transverse coefficient of diffusion is $\approx 5 \times 10^{20} \text{ cm}^2/\text{sec}$, the radial gradient of CR density is $\approx 10\%/AU$, then with a solar wind velocity $\sim 400 \text{ km/sec}$ the anisotropy of CR owing to gradient drift will have the value of about 0.2% which is comparable with the components of solar anisotropy owing to the other causes. It is shown that in the presence of sector structure of the interplanetary field under the action of gradient drift there will also arise abruptly changing on the sector boundaries North-South asymmetry of CR which is comparable in amplitude with that observed by means of the global net of neutron monitors and muon telescopes.

It is necessary to take into account the gradient and centrifugal drifts when considering a process of CR transfer in interplanetary space was also proved by Isenberg and Jokipii (1978). If the radial diffusion coefficient κ_{rr} is independent of particle energy and is proportional to the radial distance from the Sun, it will be possible to obtain a solution of the Fokker-Plank equation including convective and drift transfer, anisotropic diffusion and adiabatic cooling. For this purpose a transformation was made to the pitch-angle variable $\mu = \cos \theta$, where θ is the pitch-angle of CR particles relative to a line of force of the regular component of

interplanetary magnetic field (which is chosen, according to Parker's model, in the form of spiral field, with a neutral sheet in the equatorial plane). The authors presented graphs of the expected modulation depth, radial gradient and radial flux of CR including the reversal of interplanetary field direction in a 22-year cycle of the solar activity as the functions of r and θ with the solar wind velocity 4×10^7 cm/sec, $\kappa_{rr} = 1.68 \times 10^{21}$ and 1.25×10^{20} cm²/sec (corresponding to protons with a rigidity of 1 GV and an energy 10 MeV). It was shown that including the drift affects substantially the results of calculations of the expected modulation of galactic CR (for example, for particles with a rigidity ~ 1 GV, including the drift results in a decrease of the relative radial gradient almost by 5 times).

2.11.5. General poloidal magnetic field effects in a diffusion propagation of CR

Gall et al. (1977) considered Störmer's theory for a model of interplanetary magnetic field in the form of an Archimedean spiral:

$$H_r = H_o (r_o/r)^2 [1 + (r_o/r) \sin \lambda], \quad (2.11.45)$$

$$H_\phi = H_o r_o^2 \Omega \cos \lambda / u_r, \quad (2.11.46)$$

where r is the distance from the solar center, $r_o = 2.5 \times r_s$ (r_s is the solar radius), λ is the helio-latitude, Ω is the angular velocity of solar rotation, $\mathbf{u}$ is the solar wind velocity. The field values were given in this case, according to Altschuler et al. (1974). Störmer's constant of motion γ_{St} is derived from a Lagrangian of the equation set of motion of a charged particle in a similar way as it is made for a dipole magnetic field (see Dorman et al., M1971). As a result, Störmer's cut-off rigidity of interplanetary CR was determined as

$$R_{ci} = 1.68 \times 10^4 \left(\frac{r_o}{r} \right)^2 \cos^2 \lambda \left[1 + \cos \lambda \pm \left((1 + \cos \lambda)^2 - \sin^2 \theta \cos^2 \lambda \right)^{1/2} \right]^{-2} \text{ GV}, \quad (2.11.47)$$

where $\theta = 0^\circ, 90^\circ$ and -90° correspond to a particle coming along a vertical, from the East and the West (with respect to the ecliptic plane). The results of the calculation are presented for the expected values of R_{ci} as a function of λ for $\theta = 0^\circ, 90^\circ$, and -90° for observations at $r = 0.25$ AU and 1 AU (see Fig. 2.11.1). At $\lambda = 90^\circ$ the value $R_{ci} = 0$; the maximum R_{ci} should be at $\lambda = 0^\circ$ (in the equatorial plane). It was found that $R_{ci} = 1.1$ GV and 0.07 GV in the case $r = 0.25$ and 1 AU

at $\lambda = 0^\circ$; the values of R_{ci} are 1.7 GV and 0.11 GV at $\theta = +90^\circ$; $R_{ci} = 6.6$ GV and 0.41 GV at $\theta = -90^\circ$ respective for $r = 0.25$ and 1 AU.

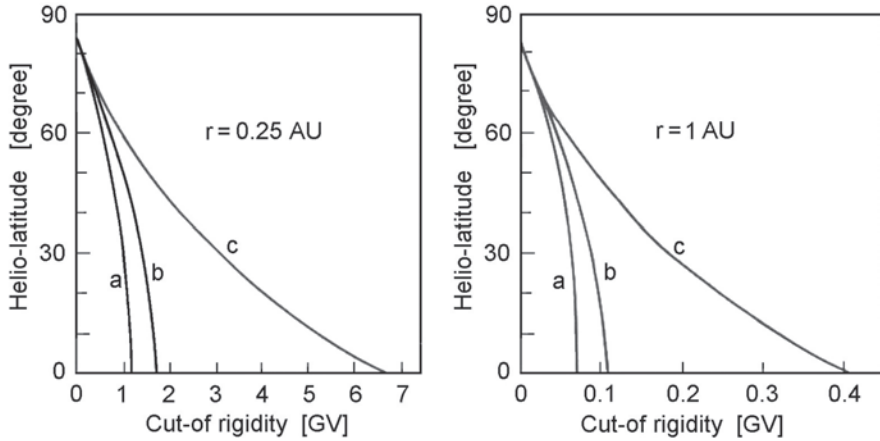


Fig. 2.11.1. Variation of interplanetary magnetic cut-off rigidity with latitude and directions of incidence at two distances from the Sun. Curves *a*, *b*, *c* correspond to the 0° , 90° and -90° respectively. According to Gall et al. (1977).

From Fig. 2.11.1 can be seen the presence of the East-West asymmetry in R_{ci} , which results in appearance of CR anisotropy with the direction to the maximum at 18 hours of the local solar time (i.e. with the same phase as it results from the convection-diffusion theory); in this case the amplitude of anisotropy should be increased approaching the Sun. As R_{ci} is substantially dependent on r ($\propto r^{-3}$ according to Eq. 2.11.47), this will result in the appearance of an additional positive gradient, i.e. again of the same sign as expected from diffusion theory. The importance was emphasized of the experimental test of the predicted effects the relative significance of which should pronouncedly increase approaching the Sun.

With the assumption that the sector structure of the interplanetary magnetic field is a separating boundary between the magnetic field of the opposite polarities in the northern and southern hemispheres of the Sun, Svalgaard and Wilcox (1976) studied a connection between the extent of these field and the 11-year variation of CR. The sector magnetic field in the photosphere with the intensity 0.5 Gauss near the minimum of the solar activity is extended in the latitude range $\pm 40^\circ$ and its extent at the distance of 1 AU is only $\pm 15^\circ$. This field compression may be caused of an excess magnetic pressure in the polar regions of the Sun. Near the maximum of solar activity when the sign of the general field changes, the field intensity in the polar region is decreased and a compression of the structure of the equatorial field will also be decreased. An increase of the volume occupied

by the sector field with a complicated structure should result in an increase of scattering of galactic CR in their diffusion into the solar system. This geometrical effect may be the main cause of the 11-year variation of CR. To test this hypothesis, Svalgaard and Wilcox (1976) determined the 11-year variation of helio-latitude extent of sector structure of magnetic field. It was shown that this parameter is in a good correlation with the inverse wave of the 11-year variation of CR intensity measured in the stratosphere above Murmansk and Mirny (Antarctica).

Humble and Pelechaty (1977) have made calculations of trajectories of CR with a rigidity from 150 to 9000 GV in interplanetary space, including the sector structure of the magnetic field at low helio-latitudes, and that the field is unidirectional in the high latitude region. The calculations were carried out for particles coming to Hobart (Australia) in various seasons of a year. A possibility of change the field polarity in the high latitude region owed to inversion of the solar general magnetic field has also been taken into account.

Kraiev and Stozhkov (1977) reported their theoretical model of a magnetic field in interplanetary space based on the data on large scale photospheric magnetic field which have a dipole character. The presence of general magnetic field of the Sun and their variations with the 22-year period will result in the corresponding variations of intensity and anisotropy of galactic CR. The model was developed in the paper (Kraiev, 1978) in which the equation of CR diffusion in interplanetary field was solved including a dipole character of the high latitude magnetic field of the Sun stretched out by the solar wind. To simplify the calculations, variations of particle energy in the process of CR propagation in interplanetary space were neglected. It was found that the depth of modulation depends substantially on a direction of the solar magnetic dipole $\mathbf{M}_S$ and on the sign of the charge of CR particles: near the ecliptic plane the depth of modulation is considerable (by 2–4 times) larger at sign $(H_{\perp}, q) < 0$ than at sign $(H_{\perp}, q) > 0$ (here q is a charge of particles, $H_{\perp}$ is the interplanetary magnetic field component normal to the ecliptic plane); when moving away from the ecliptic plane both the depth of modulation and the ratio value described are decreased. It is assumed that $\mathbf{M}_S$ changes its direction to the opposite near the epoch of the maximum of solar activity every ~ 11 years. The spectrum over total energy E per nucleon of the type of $\propto E^{-2.6}$ was chose as the non-modulated interstellar spectrum. The considered model results in the appearance of 22-year harmonic in the variation of intensity of galactic CR in interplanetary space which is superposed in the 11-year harmonics caused by the 11-year cycle of the solar activity.

2.11.6. Derivation of the Fokker-Planck CR transport equation from variational principle

Burgoa (2003) proposed a Lagrangian density for obtaining the Fokker-Planck CR transport equation and determining the energy-momentum tensor and

CR currents of a single CR source by applying the Noether's theorem (see in Sokolov et al., M1989).

According to Burgoa (2003), the Fokker-Planck CR transport field is possible define by $\psi = \psi(x^\mu)$ and the complex conjugate $\psi^+(x^\mu)$ with parameters $x^\mu = x^0, x^1, x^2, x^3, x^4$, where $x^0 = ct$; $x^1, x^2, x^3 = x, y, z$ and $x^4 = bp$, where p is the momentum modulus, b a dimensional constant and x, y, z the position coordinates. So here, the Latin index i, j from 1 to 3 and the Greek index μ, ν, ρ from 0 to 4 in analogy of the relativity theory. In this model the diffusion tensor $\kappa_\mu^\nu(x^i, x^4)$ is defined by:

$$\kappa_\mu^\nu = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \kappa_j^i & 0 \\ 0 & 0 & \kappa_4^4 \end{bmatrix}, \quad (2.11.48)$$

where $\kappa_j^i = \kappa_j^i(x, y, z)$ is the spatial diffusion coefficient, $\kappa_4^4 = \alpha K(p)$ with α a dimensional constant and $K(p)$ is the momentum-space diffusion coefficient and the product $\psi(x^\mu) \psi^+(x^\mu)$ is the density per unit of total particle momentum.

The Lagrangian L_{FP} of Fokker-Planck equation proposed is:

$$L_{FP} = L_{DIFF} + L_{SOURCE} + L_{CONV} \quad (2.11.49)$$

where

$$L_{DIFF} = \frac{i}{2} g \kappa_\mu^\nu \left(\frac{\psi}{\psi^+} \partial^\mu \psi^+ \partial_\nu \psi^+ - \frac{\psi^+}{\psi} \partial^\mu \psi \partial_\nu \psi \right), \quad (2.11.50)$$

$$L_{SOURCE} = \frac{i}{2} g Q_\mu \left(\frac{1}{\psi} \partial^\mu \psi - \frac{1}{\psi^+} \partial^\mu \psi^+ \right), \quad (2.11.51)$$

$$L_{CONV} = \frac{i}{2} g a_\mu \left(\psi^+ \partial^\mu \psi - \psi \partial^\mu \psi^+ \right), \quad (2.11.52)$$

In Eq. 2.11.50-2.11.52 g is a dimensional constant and i the imaginary unit,

$$a_\mu = \left[c, v_i, N x^4 + \frac{2}{x^4} \kappa_4^4 - \frac{x^4}{3} \frac{\partial v_i}{\partial x^i} \right], \quad (2.11.53)$$

where v is the convection velocity, and the source term:

$$Q_\mu = [Q(t), Q(x, y, z), Q(p)]. \quad (2.11.53a)$$

Now, using the Euler-Lagrange equations we obtain the motion equations:

$$\begin{aligned} \partial_\rho \left(a^\rho \psi - 2 \frac{\psi}{\psi^+} \kappa_\mu^\rho \partial^\mu \psi^+ \right) + a_\mu \partial^\mu \psi^+ \\ - \kappa_\mu^\rho \psi \left(\frac{\partial^\mu \psi^+}{\psi^+} \frac{\partial_\rho \psi^+}{\psi^+} + \frac{\partial^\mu \psi}{\psi} \frac{\partial_\rho \psi}{\psi} \right) = -\psi \partial_\rho Q^\rho, \end{aligned} \quad (2.11.54)$$

$$\begin{aligned} \partial_\rho \left(a^\rho \psi^+ - 2 \frac{\psi^+}{\psi} \kappa_\mu^\rho \partial^\mu \psi \right) + a_\mu \partial^\mu \psi \\ - \kappa_\mu^\rho \psi^+ \left(\frac{\partial^\mu \psi}{\psi} \frac{\partial_\rho \psi}{\psi} + \frac{\partial^\mu \psi^+}{\psi^+} \frac{\partial_\rho \psi^+}{\psi^+} \right) = -\psi^+ \partial_\rho Q^\rho, \end{aligned} \quad (2.11.55)$$

By introducing Eq. 2.11.49 into the expression of Noether's theorem for complex fields given by

$$J^\nu = -i \left(\frac{\partial L}{\partial u_{,\nu}} u - \frac{\partial L}{\partial \bar{u}_{,\nu}} \bar{u} \right), \quad \frac{\partial J^\rho}{\partial x^\rho} = 0, \quad (2.11.56)$$

with the momentum-energy spectrum

$$T_\alpha^\nu = -L \delta_\alpha^\nu + \frac{\partial L}{\partial u_{,\nu}^A} u_{,\alpha}^A, \quad (2.11.57)$$

Burgoa (2003) obtain:

$$\frac{\partial J^\rho}{\partial x^\rho} = \frac{\partial}{\partial x^\rho} \left(a^\rho W - \kappa_\mu^\rho \partial^\mu W - Q^\mu \right), \quad (2.11.58)$$

where $W = \psi^+ \psi$. By introducing Eq. 2.11.53 in Eq. 2.11.58, Burgoa (2003) find the Fokker-Planck equation:

$$\frac{\partial W}{\partial t} + \frac{\partial}{\partial x^i} \left(v_i W - \kappa_j^i \frac{\partial W}{\partial x^j} \right) - \frac{\partial}{\partial p} \left[p^2 D_{pp} \frac{\partial}{\partial p} \left(\frac{W}{p^2} \right) - \frac{p}{3} W \frac{\partial v_i}{\partial x^i} \right] + NW = q(r, p, t), \quad (2.11.59)$$

where

$$D_{pp} = \kappa_4^4, \quad N = \frac{1}{\tau_f} - \frac{1}{\tau_r}, \quad q(r, p, t) = \frac{\partial Q}{\partial t} + \frac{\partial Q}{\partial r} + \frac{\partial Q}{b \partial p}, \quad (2.11.60)$$

and τ_f is the time scale for fragmentation and τ_r is the time scale for the radioactivity decay. The Eq. 2.11.58 for index $\rho = 4$ gives:

$$\frac{\partial}{\partial p} \left[\left(Nbp + \frac{2\alpha K(p)}{bp} - \frac{bp}{3} \frac{\partial v_i}{\partial x^i} \right) (\psi^+ \psi) - \frac{\alpha K(p)}{bp} \frac{\partial (\psi^+ \psi)}{\partial p} - Q(p) \right] = 0, \quad (2.11.61)$$

which in the case when the convection velocity v is a constant and $N \rightarrow 0$, has solution

$$\psi^+ \psi = -bp^2 \int \frac{Q(p)}{\alpha K(p)} p^{-2} dp. \quad (2.11.62)$$

If the CR source has an exponential dependence of the form $Q(p) \propto \int p^{-\gamma} dp$, the solution for differential energy spectrum of CR density according to Eq. 2.11.62 will be

$$\frac{\partial (\psi^+ \psi)}{\partial E} = \frac{\gamma - 2}{\gamma(\gamma - 1)} \left(\frac{E}{c} \right)^{-(\gamma-1)}. \quad (2.11.63)$$

2.12. Phenomenological description of CR anisotropic diffusion

2.12.1. Deduction of general equation

In some cases, the approximation of anisotropic diffusion is sufficient for the study of CR propagation and their energy variations (CR propagation in the galactic arms, interaction of CR of moderate energy with solar wind etc.). The matter is that CR distribution is isotropic in the first approximation, a relative variance from isotropy (so called anisotropy) is very small; as a rule it is $\leq 1\%$, and in this case a distribution of CR in space can be described, with good accuracy, by a particle density $n(\mathbf{r}, p, t)$ instead of a distribution function $f(\mathbf{r}, \mathbf{p}, t)$ (Dorman, 1965, 1967). Thus a density of CR in space is $n(\mathbf{r}, R, Ze, t)$, where $\mathbf{r}$ is the spherical coordinates with the center in the Sun, R and Ze are the particle rigidity and charge, t is the time. Then in the approximation of anisotropic diffusion n will be determined by a continuity equation

$$\frac{\partial n}{\partial t} + \sum_{\alpha=1}^4 \frac{\partial J_{\alpha}}{\partial x_{\alpha}} = \Phi(\mathbf{r}, R, Ze, t) \quad (2.12.1)$$

including the initial and boundary conditions determined by a particular problem of CR propagation. In Eq. 2.12.1 $\Phi(\mathbf{r}, R, Ze, t)$ is the source function; J_{α} are the components of particle flux in terms of $\mathbf{r}$ and R . It will be taken into account that in the case of anisotropic diffusion the spatial flux is

$$J_i = - \sum_{k=1}^3 \kappa_{ik} \frac{\partial n}{\partial x_k} + u_{dr,i} n, \quad (2.12.2)$$

where κ_{ik} is the tensor diffusion coefficient and $u_{dr,i}$ is the drift velocity of CR in space arising from regular motions of magnetized plasma and from the presence of inhomogeneous magnetic fields and CR density gradients. Substituting Eq. 2.12.2 in Eq. 2.12.1 we obtain the general equation describing a propagation of CR in the approximation of anisotropic diffusion.

2.12.2. The case of propagation in a galactic arm

For the first approximation the regular magnetic field component in an arm of the Galaxy can be considered as uniform field $\mathbf{H}_0$. Let the x-axis is directed along $\mathbf{H}_0$. Then in a rectangular coordinate system x, y, z we obtain

$$\kappa_{ik} = \kappa_0 \times \begin{pmatrix} 1 & 0 & 0 \\ 0 & (1 + \omega_L^2 \tau^2)^{-1} & \omega_L \tau (1 + \omega_L^2 \tau^2)^{-1} \\ 0 & -\omega_L \tau (1 + \omega_L^2 \tau^2)^{-1} & (1 + \omega_L^2 \tau^2)^{-1} \end{pmatrix}, \quad (2.12.3)$$

where $\kappa_0 = v\Lambda/3$ is the coefficient of particle diffusion in the absence of a regular magnetic field (Λ is the transport path for scattering). Here $\tau = \Lambda/v$ is the mean time between collisions, $\omega_L = ZeH_0/Mc$ is the Larmor frequency of the motion of a particle with a charge Ze and a relativistic mass $M = Amc^2(1 - v^2/c^2)^{-1/2}$ in a large scale field $\mathbf{H}_0$. Including that Λ depends on a particle rigidity $R = cp/Ze$ and curvature-radius in the magnetic field $\mathbf{H}_0$ is $r_L = R/300H_0$ (if R is expressed in volts, H_0 is in gauss, then r_L is expressed in cm), and we obtain

$$\omega_L \tau = \Lambda/r_L = 300\Lambda(R)H_0/R, \quad (2.12.4)$$

i.e. $\omega_L \tau$ is a function only of R and is independent of a particle charge Ze and of its velocity v . Substituting Eq. 2.12.4 in Eq. 2.12.3 we have

$$\kappa_{ik} = \kappa_o \times \begin{pmatrix} 1 & 0 & 0 \\ 0 & \alpha_1 & \alpha_2 \\ 0 & -\alpha_2 & \alpha_1 \end{pmatrix}, \quad (2.12.5)$$

where

$$\alpha_1 = \frac{R^2}{R^2 + (300\Lambda H_o)^2}, \quad \alpha_2 = \frac{300\Lambda H_o R}{R^2 + (300\Lambda H_o)^2}. \quad (2.12.6)$$

If we consider that in a galactic arm the regular component of plasma motion velocity is $\mathbf{u} = 0$, then $u_{dr} = 0$, and $J_R = 0$. In this case

$$J_x = -\kappa_o \frac{\partial n}{\partial x}, \quad J_y = -\kappa_o \alpha_1 \frac{\partial n}{\partial y} - \kappa_o \alpha_2 \frac{\partial n}{\partial z}, \quad J_z = \kappa_o \alpha_2 \frac{\partial n}{\partial y} - \kappa_o \alpha_1 \frac{\partial n}{\partial z}. \quad (2.12.7)$$

Substituting Eq. 2.12.7 in Eq. 2.12.1 we find the searched equation of anisotropic diffusion for the case under consideration:

$$\begin{aligned} \frac{\partial n}{\partial t} - \frac{\partial}{\partial x} \left(\kappa_o \frac{\partial n}{\partial x} \right) - \frac{\partial}{\partial y} \left(\kappa_o \alpha_1 \frac{\partial n}{\partial y} + \kappa_o \alpha_2 \frac{\partial n}{\partial z} \right) - \frac{\partial}{\partial z} \left(\kappa_o \alpha_2 \frac{\partial n}{\partial y} - \kappa_o \alpha_1 \frac{\partial n}{\partial z} \right) \\ = \Phi(\mathbf{r}, R, Ze, t), \end{aligned} \quad (2.12.8)$$

where α_1 and α_2 in general are dependent of $\mathbf{r}$ and determined by Eq. 2.12.6.

2.12.3. The case of CR propagation in interplanetary space

Moving in this case to a spherical coordinate system centered in the Sun, we can rewrite Eq. 2.12.2 in the form

$$\begin{aligned} J_r = -\kappa_{rr} \frac{\partial n}{\partial r} - \frac{\kappa_{r\theta}}{r} \frac{\partial n}{\partial \theta} - \frac{\kappa_{r\varphi}}{r \sin \theta} \frac{\partial n}{\partial \varphi} + u_{dr,r} n, \quad J_\theta = -\kappa_{\theta r} \frac{\partial n}{\partial r} - \frac{\kappa_{\theta\theta}}{r} \frac{\partial n}{\partial \theta} \\ - \frac{\kappa_{\theta\varphi}}{r \sin \theta} \frac{\partial n}{\partial \varphi} + u_{dr,\theta} n, \quad J_\varphi = -\kappa_{\varphi r} \frac{\partial n}{\partial r} - \frac{\kappa_{\varphi\theta}}{r} \frac{\partial n}{\partial \theta} - \frac{\kappa_{\varphi\varphi}}{r \sin \theta} \frac{\partial n}{\partial \varphi} + u_{dr,\varphi} n. \end{aligned} \quad (2.12.9)$$

Let us now determine the flux $J_R = n dR/dt$ relative to the rigidity axis caused, by the fact that in the case of radial divergence of magnetic inhomogeneities from

the Sun, there is a systematic loss of particle energy owing to prevailing scattering at the overtaking collisions (in the directions θ and φ) (Ginzburg et al., 1955). Since the particle energy E is decreased in average by $\Delta E = -Ev\Delta u/c^2$ in an elementary collision act ($\Delta u = u\Lambda/r$), the time between two collision is $\Delta t = \Lambda/v$, and the energy decrease takes place only for the directions normal to $\mathbf{r}$ (in the directions θ and φ), we shall have

$$\frac{dE}{dt} = -\frac{2}{3} \frac{Ev^2}{c^2} \frac{u}{r}. \quad (2.12.10)$$

Since

$$E = Ze \left(R^2 + \left(Am_p c^2 / Ze \right)^2 \right)^{1/2}, \quad dE = Ze R dR \left(R^2 + \left(Am_p c^2 / Ze \right)^2 \right)^{-1/2},$$

$$\frac{v^2}{c^2} = R^2 \left(R^2 + \left(Am_p c^2 / Ze \right)^2 \right)^{-1}, \quad (2.12.11)$$

we obtain

$$\langle dR/dt \rangle = -2uR/3r. \quad (2.12.12)$$

Substituting Eq. 2.12.9 and Eq. 2.12.12 in Eq. 2.12.1, we find a general equation in anisotropic diffusion approximation for a study of modulation effects of galactic CR in interplanetary space and of solar CR propagation:

$$\begin{aligned} \frac{\partial n}{\partial t} - \frac{1}{r^2} \frac{\partial}{\partial r} r^2 \left(\kappa_{rr} \frac{\partial n}{\partial r} + \frac{\kappa_{r\theta}}{r} \frac{\partial n}{\partial \theta} + \frac{\kappa_{r\varphi}}{r \sin \theta} \frac{\partial n}{\partial \varphi} - u_{dr,r} n \right) - \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} \sin \theta \\ \times \left(\kappa_{\theta r} \frac{\partial n}{\partial r} + \frac{\kappa_{\theta\theta}}{r} \frac{\partial n}{\partial \theta} + \frac{\kappa_{\theta\varphi}}{r \sin \theta} \frac{\partial n}{\partial \varphi} - u_{dr,\theta} n \right) - \frac{1}{r \sin \theta} \frac{\partial}{\partial \varphi} \left(\kappa_{\varphi r} \frac{\partial n}{\partial r} + \frac{\kappa_{\varphi\theta}}{r} \frac{\partial n}{\partial \theta} \right. \\ \left. + \frac{\kappa_{\varphi\varphi}}{r \sin \theta} \frac{\partial n}{\partial \varphi} - u_{dr,\varphi} n \right) - \frac{2u}{3rR^2} \frac{\partial}{\partial R} R^2 (Rn) = \Phi(r, \theta, \varphi, R, Z, t). \end{aligned} \quad (2.12.13)$$

To determine κ_{ik} one should pay attention to the following circumstances:

1. Electromagnetic conditions in interplanetary space for CR propagation and modulation (which are eventually determined by anisotropic diffusion tensor κ_{ik} and by the velocity $\mathbf{u}$ of plasma's motion) do not remain uniform but vary in time. These variations are detailed relative to the processes on the Sun generating plasma streams, and this time delay is $t_d \approx r/u$. Let S be a factor controlling

plasma outbursts from the Sun (number of solar spots, chromospheres flares, calcium flocculates etc.), then at a distance r from the Sun κ_{ik} will be

$$\kappa_{ik} = \kappa_{ik}(r, S(t - r/u)). \quad (2.12.14)$$

2. The kinetic energy density of plasma released by the Sun is far higher than the magnetic field related to the plasma. This relation (which holds true, according to direct measurements near the Earth's orbit) should be valid also at the larger distances from the Sun (since the kinetic energy density decreases $\propto r^{-2}$, the magnetic energy density decreases in the same ways). Then the geometrical place of plasma portions, released from the same region of the Sun, is, according to Parker (M1963), an Archimedes spiral owing to the solar rotation with the angular frequency $\Omega = 2\pi/T$ ($T \approx 27$ days is weakly dependent on the polar angle θ):

$$r - r_s = -\frac{u(\theta_s, \varphi_s)}{\Omega(\theta_s)}(\varphi - \varphi_s), \quad (2.12.15)$$

where θ_s and φ_s are a polar angle and helio-longitude, respectively, of a plasma generation region on the solar surface. The large scale magnetic field will tend to be stretched along these spirals. Since, however, $u(\theta_s, \varphi_s)/\Omega(\theta_s)$ in Eq. 2.12.15 can vary considerably from region to region on the solar surface, it will result in a complicated interaction between solar plasma and magnetic field which can be a possible cause of generation of magnetic field inhomogeneities radial moving from the Sun.

3. The angle between magnetic force lines and a direction of the radial motion of magnetic inhomogeneities,

$$\Psi = \arctg\left(\frac{r\Omega(\theta_s)\sin\theta_s}{u(\theta_s, \varphi_s)}\right), \quad (2.12.16)$$

determines a transformation of the anisotropic diffusion tensor κ_{ik} .

4. In the coordinate system in which the direction of the force line of the interplanetary magnetic field is chosen as the x - axis in any point $\mathbf{r}$, the anisotropic diffusion tensor will have a form determined by Eq. 2.12.5. Note that the term $\Lambda^2 H^2 \propto r^{-2}$ decreases rapidly with the distance from the Sun at $\Lambda \approx \text{const}$ (i.e. at Λ , independent of r), since $H \approx H_E(r_E/r)$, for $r > r_E$ (here $r_E = 1$ AU is the radius of the Earth's orbit). Therefore it may be assumed that, from a certain r onwards, $300\Lambda H \ll R$, and, according to Eq. 2.12.5, $\alpha_1 \rightarrow 1, \alpha_2 \rightarrow 0$, i.e. the diffusion gets strong anisotropic. If, however, Λ increases directly

proportional to r , then $\Lambda H \approx \text{const}$ and the anisotropy will also be significant at large r .

5. One can show that the determinant of a transformation a_{ik} from the system related to magnetic lines of force to the system of r, θ, φ has the form (the upper sign corresponds to force lines coming out of the Sun, the lower sign corresponds to those coming into the Sun):

$$a_{ik} = \begin{pmatrix} \cos \Psi & 0 & \pm \sin \Psi \\ 0 & 1 & 0 \\ \pm \sin \Psi & 0 & \cos \Psi \end{pmatrix}. \quad (2.12.17)$$

In the system of coordinates (r, θ, φ) , the anisotropic diffusion tensor will be determined by

$$\kappa_{ik} = \sum_{m,n} a_{mi} a_{nk} \bar{\kappa}_{mn}, \quad (2.12.18)$$

where $\bar{\kappa}_{mn}$ is determined by Eq. 2.12.5. From Eq. 2.12.18 taking into account Eq. 2.12.17 there follows

$$\kappa_{ik} = \begin{pmatrix} \cos^2 \Psi + \alpha_1 \sin^2 \Psi & \mp \alpha_2 \sin \Psi & \pm (1 - \alpha_1) \sin \Psi \cos \Psi \\ \pm \alpha_2 \sin \Psi & \alpha_1 & \pm \alpha_2 \cos \Psi \\ \mp (1 - \alpha_1) \sin \Psi \cos \Psi & \mp \alpha_2 \cos \Psi & \sin^2 \Psi + \alpha_1 \cos^2 \Psi \end{pmatrix}, \quad (2.12.19)$$

where α_1 and α_2 are determined by Eq. 2.12.6. One should keep in mind that according to Eq. 2.12.16, Ψ is a function of r, θ_s, φ_s , and $\kappa_o = v\Lambda/3$ is a function of r, θ, φ , and of some parameter of solar activity with definite time delay $S(t - r/u)$.

Let us now determine the drift velocities $\mathbf{u}_{dr}$ of CR particles. The drift can be caused by many factors: electromagnetic drift caused by the moving of solar wind plasma with frozen in magnetic fields, density drift in magnetic field, curvature drift. Here we will consider electromagnetic drift caused the convection of CR particles. When plasma with magnetic field $\mathbf{H}$ moves with a velocity $\mathbf{u} = u\mathbf{r}/r$ an electric field $\mathbf{E}$ arises in the Sun-stars coordinate system. Charged particles will drift under action of $\mathbf{E}$ and $\mathbf{H}$ in this system with the velocity

$$\mathbf{u}_{dr} = \frac{[\mathbf{H} \times [\mathbf{u} \times \mathbf{H}]]}{H^2} = \frac{u\mathbf{r}}{r} - \frac{u(\mathbf{r}\mathbf{H})\mathbf{H}}{rH^2}. \quad (2.12.20)$$

Consider first the drift velocity under action of regular spiral interplanetary magnetic field. Since the angle between $\mathbf{r}$ and $\mathbf{H}$ is determined by Eq. 2.12.16, we shall obtain basing on Eq. 2.12.20 (irrespective of the magnetic field direction to the Sun or away from it):

$$(u_{dr,r})_{\text{reg}} = \frac{u\Omega^2 r^2 \sin^2 \theta}{u^2 + \Omega^2 r^2 \sin^2 \theta}, \quad (u_{dr,\theta})_{\text{reg}} = 0, \quad (u_{dr,\varphi})_{\text{reg}} = \frac{u^2 \Omega r \sin \theta}{u^2 + \Omega^2 r^2 \sin^2 \theta}. \quad (2.12.21)$$

Consider now the second extreme case. Let us assume that the interplanetary field is only a set of inhomogeneities in which the direction of the magnetic field $\mathbf{H}$ may be arbitrary with equal probability (a turbulent field). Let the angle between $\mathbf{r}$ and $\mathbf{H}$ be ξ . Then averaging Eq. 2.12.20 over all possible directions of $\mathbf{H}$ we obtain

$$(u_{dr,r})_{\text{turb}} = \frac{2}{3}u, \quad (u_{dr,\theta})_{\text{turb}} = (u_{dr,\varphi})_{\text{turb}} = 0. \quad (2.12.22)$$

In the presence of the field inhomogeneities on the background of a regular magnetic field, including Eq. 2.12.21 and Eq. 2.12.22 and designating a relative share of the large-scale field by

$$\beta = H_o / (H_o + H_1), \quad (2.12.23)$$

we have

$$u_{dr,r} = \frac{\beta u \Omega^2 r^2 \sin^2 \theta}{u^2 + \Omega^2 r^2 \sin^2 \theta} + \frac{2(1-\beta)u}{3}, \quad u_{dr,\theta} = 0, \quad u_{dr,\varphi} = \frac{\beta u^2 \Omega r \sin \theta}{u^2 + \Omega^2 r^2 \sin^2 \theta}. \quad (2.12.24)$$

It is easy to see that with $\beta \rightarrow 1$ Eq. 2.12.24 transforms to Eq. 2.12.21, and with $\beta \rightarrow 0$ it transforms to Eq. 2.12.22.

2.12.4. On rotation of CR gas in the interplanetary space

The assumption is of common use in the literature that a gas of CR co-rotates with the Sun with the same angular velocity Ω . In particular, a mean solar anisotropy of galactic CR is usually explained by this phenomenon. It should be emphasized that this concept holds, to a certain degree, true for a region near the Earth's orbit, whilst this concept is completely wrong with removal from the Sun. First of all, Eq. 2.12.21 and Eq. 2.12.22 show that the rotation of CR gas ($u_{dr,\varphi}$ component) is provided by a drift in a regular field with spiral force lines, whilst a turbulent field component gives a sufficient contribution to $u_{dr,r}$. Since the radial

component of drift velocity is balanced mainly by a diffusion flow, the real average motion of the CR gas is provided by $u_{dr,\varphi}$.

Let us suppose that $\beta \rightarrow 1$ (the field is purely regular). Consider the region near the plane of the helio-equator ($\theta \approx \pi/2$). Since the average solar wind velocity $u \sim 400$ km/sec, we have $u \approx \Omega r_E$, where $r_E = 1.5 \times 10^{13}$ cm is the distance from the Earth to the Sun. The Eq. 2.12.21 results in $(u_{dr,r})_{reg} \approx \Omega r(r/r_E) \ll u$ and $(u_{dr,\varphi})_{reg} \approx \Omega r$ at $r \ll r_E$. Near the Earth's orbit ($r \approx r_E$), we have $(u_{dr,r})_{reg} \approx u/2$, $(u_{dr,\varphi})_{reg} \approx \Omega r_E/2$ (about half of the Sun angle rotation). Far behind the Earth's orbit at $r \gg r_E$ we have $(u_{dr,r})_{reg} \approx u$, $(u_{dr,\varphi})_{reg} \approx \Omega r(r_E/r)^2 \ll \Omega r$ (much smaller than the Sun angle rotation).

Thus, the radial component of the drift velocity near the Sun where the field is radial, is sufficiently less than the wind's velocity; however, CR gas in this region must co-rotate synchronously together with the Sun. Near the Earth's orbit the radial drift increases to one half of the wind velocity and the rotation of CR gas is decelerated to half of the velocity of solar rotation. Finally, at distances more than several AU, where the field is practically azimuthally, the rotation of CR gas is $\propto (r/r_E)^{-2}$ from the solar rotation. The above considerations result in there being a differential rotation of CR gas in the interplanetary space and its angular velocity near the Sun coincides with the solar rotation, and this velocity is sharply decreased with the distance from the Sun.

2.12.5. Temporal variations and spatial anisotropy of CR in the interplanetary space

Substituting Eq. 2.12.19 and Eq. 2.12.24 in Eq. 2.12.13 we obtain the equation determining the spatial-temporal variation of CR density. Using the boundary and initial conditions it is possible to determine the sought function $n(r, \theta, \varphi, R, Z, t)$ by means of this equation. Substituting the $n(r, \theta, \varphi, R, Z, t)$ obtained and Eq. 2.12.19 and Eq. 2.12.24 in Eq. 2.12.9 we shall then find the spatial particle fluxes which determine CR anisotropy: in the radial direction it is the so called 12- and 24-hour anisotropy (J_r/nv); in the direction normal to the helio-equatorial plane it is the so called North-South asymmetry (J_θ/nv); in the direction along the Earth's orbit it is 6- and 18-hour anisotropy (J_φ/nv). When comparing theoretical results with experimental data it is necessary to keep in mind that Eq. 2.12.13 gives the results, which are related to the coordinate system connected with the helio-equator.

2.12.6. The region where the CR anisotropic diffusion approximation is applicable

Let us now discuss the question concerning the region where the Eq. 2.12.1 is applicable. It is known that the anisotropic diffusion approximation is the are better applicable to a description of a process the slower are the processes of variations of density and fluxes within the distances of the order of free path. For the estimates one can use the following criteria:

$$\frac{1}{n} \frac{\partial n}{\partial x_\alpha} \Lambda \ll 1, \quad \frac{1}{J} \frac{\partial J}{\partial x_\alpha} \Lambda \ll 1. \quad (2.12.25)$$

In all cases of galactic CR modulation, which are of practical interest, the criteria determined by Eq. 2.12.25 hold true. In fact, after the measurements Neher and Anderson (1964) the value of the relative radial intensity gradient of CR was in 1962 $\frac{1}{n} \frac{\partial n}{\partial r} \approx (12 \pm 4) \% / \text{AU}$ in the interplanetary space near the Earth's orbit for particles with the energy ≥ 10 MeV. However, at this distance from the Sun for such soft particles, according to the study of solar CR propagation (Dorman and Miroshnichenko, M1968), $\Lambda \approx 10^{12}$ cm; therefore in this case $\frac{1}{n} \frac{\partial n}{\partial r} \Lambda \leq 0.01$.

For the energetic particles Λ increases approximately $\propto R$ but (according to Dorman, M1963a) the relative gradient should decrease $\propto R^{-1}$, so that the above estimate should not in practice be strongly dependent on particle energy. In the maximum of solar activity, the value of relative gradient should be increased several times (this results from the experimental data on the 11-year CR variation) but in this case Λ slightly decreases so that the criterion 2.12.25 is still satisfied.

It is easy to show that the criterion 2.12.25 is equivalent to the condition $J/nv \ll 1$. But according to numerous investigations of diurnal variations, $J/nv \leq 0.01$ in the whole range of the studied rigidities except for some occasional periods of large Forbush effects when J/nv reaches 0.03-0.05.

However, in all cases when it is necessary to determine the distribution function of CR (numerous problems of solar CR propagation, the effects of CR interaction with interplanetary shock waves, a distortion of the external anisotropy of CR in the interplanetary space and so on), one carries out the study based on the kinetic equation.

2.13. On a relation between different forms of the equation of anisotropic diffusion of CR

In the theory of CR propagation different forms of the equation of anisotropic diffusion are used, depending on the choice of variables: the momentum p (Dolginov and Toptygin, 1966), rigidity R (Dorman, 1965), total energy E and

kinetic energy E_k (Parker, 1965; Jokipii, 1971). When applying these equations the necessity arises of inter-relating the quantities included in these equations. In the papers (Dorman, Katz and Shakhov, 1976, 1977) the various forms of the anisotropic diffusion equations were analyzed, their identity was proved and the relation was found between the phase density of particles and the flux density of particles, expressed in different variables.

We shall start from the anisotropic diffusion equation for the phase density $n(\mathbf{r}, p, t)$ and the expression for the flux density $\mathbf{J}(\mathbf{r}, p, t)$ of particles (which were obtained in Dolginov and Toptygin, 1966a,b):

$$\frac{\partial n(\mathbf{r}, p, t)}{\partial t} = \frac{\partial}{\partial r_\alpha} \kappa_{\alpha\lambda}(\mathbf{r}, p) \frac{\partial n(\mathbf{r}, p, t)}{\partial r_\lambda} - u_\alpha(\mathbf{r}) \frac{\partial n(\mathbf{r}, p, t)}{\partial r_\alpha} + \frac{\partial u_\alpha(\mathbf{r})}{\partial r_\alpha} \frac{p}{3} \frac{\partial n(\mathbf{r}, p, t)}{\partial p}, \quad (2.13.1)$$

$$J_\alpha(\mathbf{r}, p, t) = -\kappa_{\alpha\lambda}(\mathbf{r}, p) \frac{\partial n(\mathbf{r}, p, t)}{\partial r_\lambda} - u_\alpha(\mathbf{r}) \frac{p}{3} \frac{\partial n(\mathbf{r}, p, t)}{\partial p}, \quad (2.13.2)$$

where $\kappa_{\alpha\lambda}(\mathbf{r}, p)$ is the tensor diffusion coefficient, $\mathbf{u}(\mathbf{r})$ is the solar wind velocity, and summation over the repeated indices is assumed in Eq. 2.13.1 and Eq. 2.13.2. After obvious transformation, the Eq. 2.13.1 and Eq. 2.13.2 can be written in the form:

$$\frac{\partial n(\mathbf{r}, p, t)}{\partial t} = \frac{\partial}{\partial r_\alpha} \left\{ \kappa_{\alpha\lambda}(\mathbf{r}, p) \frac{\partial}{\partial r_\lambda} - u_\alpha(\mathbf{r}) \right\} n(\mathbf{r}, p, t) + \frac{1}{3} \frac{\partial u_\alpha(\mathbf{r})}{\partial r_\alpha} \frac{1}{p^2} \frac{\partial}{\partial p} p^3 n(\mathbf{r}, p, t), \quad (2.13.3)$$

$$J_\alpha(\mathbf{r}, p, t) = - \left\{ \kappa_{\alpha\lambda}(\mathbf{r}, p) \frac{\partial}{\partial r_\lambda} - u_\alpha(\mathbf{r}) \right\} n(\mathbf{r}, p, t) - \frac{u_\alpha(\mathbf{r})}{3} \frac{1}{p^2} \frac{\partial}{\partial p} p^3 n(\mathbf{r}, p, t). \quad (2.13.4)$$

If one selects the particle rigidity $R = cp/Ze$ as the variable, Eq. 2.13.3 will take the form (Dorman, 1965):

$$\frac{\partial n(\mathbf{r}, R, t)}{\partial t} = \frac{\partial}{\partial r_\alpha} \left\{ \kappa_{\alpha\lambda}(\mathbf{r}, R) \frac{\partial}{\partial r_\lambda} - u_\alpha(\mathbf{r}) \right\} n(\mathbf{r}, R, t) + \frac{1}{3} \frac{\partial u_\alpha(\mathbf{r})}{\partial r_\alpha} \frac{1}{R^2} \frac{\partial}{\partial R} R^3 n(\mathbf{r}, R, t), \quad (2.13.5)$$

and the expression for the flux density of particles will be

$$J_\alpha(\mathbf{r}, R, t) = - \left\{ \kappa_{\alpha\lambda}(\mathbf{r}, R) \frac{\partial}{\partial r_\lambda} - u_\alpha(\mathbf{r}) \right\} n(\mathbf{r}, R, t) - \frac{u_\alpha(\mathbf{r})}{3} \frac{1}{R^2} \frac{\partial}{\partial R} R^3 n(\mathbf{r}, R, t). \quad (2.13.6)$$

When writing Eq. 2.13.5 and Eq. 2.13.6 we used the relation, connecting the particle density $n(\mathbf{r}, t)$ with differential particle density $n(\mathbf{r}, p, t)$ in the phase space,

$$n(\mathbf{r}, t) = \int_{p_0}^{\infty} p^2 n(\mathbf{r}, p, t) dp, \quad (2.13.7)$$

which gives the relations

$$n(\mathbf{r}, t) = \int_{R_0}^{\infty} R^2 n(\mathbf{r}, R, t) dR, \quad (2.13.8)$$

$$n(\mathbf{r}, p, t) = \left(\frac{c}{Ze} \right)^3 n(\mathbf{r}, R, t), \quad (2.13.9)$$

Selecting the total energy $E = c(m_0^2 c^2 + p^2)^{1/2}$ (where m_0 is the rest mass of a particle) as the variable, one can write Eq. 2.13.1 and Eq. 2.13.2 in the form

$$\begin{aligned} \frac{\partial n(\mathbf{r}, E, t)}{\partial t} = & \frac{\partial}{\partial r_\alpha} \left\{ \kappa_{\alpha\lambda}(\mathbf{r}, E) \frac{\partial}{\partial r_\lambda} - u_\alpha(\mathbf{r}) \right\} n(\mathbf{r}, E, t) \\ & + \frac{1}{3} \frac{\partial u_\alpha(\mathbf{r})}{\partial r_\alpha} \frac{\partial}{\partial E} \frac{(E^2 - m_0^2 c^4)}{E} n(\mathbf{r}, E, t), \end{aligned} \quad (2.13.10)$$

$$J_\alpha(\mathbf{r}, E, t) = - \left\{ \kappa_{\alpha\lambda}(\mathbf{r}, E) \frac{\partial}{\partial r_\lambda} - u_\alpha(\mathbf{r}) \right\} n(\mathbf{r}, E, t) - \frac{u_\alpha(\mathbf{r})}{3} \frac{\partial}{\partial E} \frac{(E^2 - m_0^2 c^4)}{E} n(\mathbf{r}, E, t). \quad (2.13.11)$$

In this case the particle density $n(\mathbf{r}, t)$ is determined by the expression

$$n(\mathbf{r}, t) = \int_{E_0}^{\infty} n(\mathbf{r}, E, t) dE, \quad (2.13.12)$$

and the phase density is related by the expression

$$n(\mathbf{r}, p, t) = \frac{c^2}{E(E^2 - m_0^2 c^4)^{1/2}} n(\mathbf{r}, E, t) = \frac{c^2}{pE} n(\mathbf{r}, E, t). \quad (2.13.13)$$

Let us pass now to the kinetic energy

$$E_k = E - m_0 c^2. \quad (2.13.14)$$

Then the Eq. 2.13.1 and Eq. 2.13.2 will take the form which is often used in literature:

$$\begin{aligned} \frac{\partial n(\mathbf{r}, E_k, t)}{\partial t} = & \frac{\partial}{\partial r_\alpha} \left\{ \kappa_{\alpha\lambda}(\mathbf{r}, E_k) \frac{\partial}{\partial r_\lambda} - u_\alpha(\mathbf{r}) \right\} n(\mathbf{r}, E_k, t) \\ & + \frac{1}{3} \frac{\partial u_\alpha(\mathbf{r})}{\partial r_\alpha} \frac{\partial}{\partial E_k} \alpha(E_k) E_k n(\mathbf{r}, E_k, t), \end{aligned} \quad (2.13.15)$$

$$\begin{aligned} J_\alpha(\mathbf{r}, E_k, t) = & - \left\{ \kappa_{\alpha\lambda}(\mathbf{r}, E_k) \frac{\partial}{\partial r_\lambda} - u_\alpha(\mathbf{r}) \right\} n(\mathbf{r}, E_k, t) \\ & - \frac{u_\alpha(\mathbf{r})}{3} \frac{\partial}{\partial E_k} \alpha(E_k) E_k n(\mathbf{r}, E_k, t), \end{aligned} \quad (2.13.16)$$

where

$$\alpha(E_k) = \frac{E_k + 2m_0 c^2}{E_k + m_0 c^2}. \quad (2.13.17)$$

The expression for the flux density of particles (Eq. 2.13.16) can be written in the form (Parker, 1965; Jokipii, 1971):

$$J_\alpha(\mathbf{r}, E_k, t) = -\kappa_{\alpha\lambda}(\mathbf{r}, E_k) \frac{\partial n(\mathbf{r}, E_k, t)}{\partial r_\lambda} - u_\alpha(\mathbf{r}) C(E_k) n(\mathbf{r}, E_k, t), \quad (2.13.18)$$

where

$$C(E_k) = 1 - \frac{1}{n(\mathbf{r}, E_k, t)} \frac{\partial}{\partial E_k} \alpha(E_k) n(\mathbf{r}, E_k, t) \quad (2.13.19)$$

is the so called Compton-Getting's factor which was called after the paper (Compton and Getting, 1935).

2.14. Spectral representations of Green's function of non-stationary equation of CR diffusion

2.14.1. Formulation of the problem

In studying CR propagation in interplanetary space, the model of isotropic diffusion including convection and adiabatic deceleration of particles is successfully applied. In recent years a substantial result was achieved in this direction, and first of all in this Section we should notice the solution used above (see Section 2.7) of the equation for the Green's function of the stationary equation of isotropic transfer with arbitrary dependence of the diffusion coefficient on the particle momentum and with the power dependence on the distance (Toptygin, 1973a,b). As to solutions of non-stationary problems which are of primary importance in studying a great number of aspects of the propagation theory (propagation of the solar CR, Forbush effect, the 11-year variation including the hysteresis phenomena, etc), the situation is more complicated in this case. Mathematical difficulties arising in the solution of non-stationary problems are very substantial and it is extremely difficult to obtain closed expressions for a solution of non-stationary equations describing the actual physical situations. Having no claim on a complete solution of the problem described, Dorman and Katz (1977a,b,c) considered some simplest models of non-stationary propagation of CR in a medium with the constant diffusion coefficient. As will be shown below, it is possible in these models to find a spectral expression for the Green's function of the equation of a transfer of CR. The assumption of constancy of the diffusion coefficient is, of course, an idealization, but we hope that in future it will be possible to generalize the considered class of non-stationary solutions for the Green's function taking into account a dependence of the diffusion coefficient on a distance and from particle momentum.

2.14.2. Determining of the radial Green's function for a non-stationary diffusion including convection

Consider initially a non-stationary diffusion of CR including their convective transfer by radially expanding plasma of solar wind. If we neglect the process of adiabatic variation of particle energy, the particle density $n(\mathbf{r}, t)$ satisfies the equation of a non-stationary diffusion including convection:

$$\frac{\partial n(\mathbf{r}, t)}{\partial t} = (\kappa \Delta - \mathbf{u}_0 \vec{\nabla}) n(\mathbf{r}, t) + Q_o(\mathbf{r}, t), \quad (2.14.1)$$

where κ is the coefficient of particle diffusion, $\mathbf{u}_0$ is the solar wind velocity which is assumed to be directed along a radius away from the Sun, i.e. $\mathbf{u}_0 = u_0 \mathbf{r}/r$,

$Q_o(\mathbf{r}, t)$ is the source function. Including the radial dependence of solar wind velocity $\mathbf{u}_o$, we write the Eq. 2.14.1 in the spherical coordinates

$$\frac{\partial n}{\partial \tau} = \left(\Delta_r + \Delta_{\theta\varphi} - 2\mathbf{u} \frac{\partial}{\partial \mathbf{r}} \right) n + Q_1, \quad (2.14.2)$$

where the notations

$$\tau = \kappa t, \quad 2\mathbf{u} = \mathbf{u}_o / \kappa, \quad Q_1 = Q_o / \kappa, \quad (2.14.3)$$

are used. In Eq. 2.14.2

$$\Delta_r = \frac{1}{r^2} \frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r}; \quad \Delta_{\theta\varphi} = \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \sin \theta \frac{\partial}{\partial \theta} + \frac{\partial^2}{\partial \varphi^2} \frac{1}{r^2 \sin \theta}, \quad (2.14.4)$$

represents the radial and angular parts of Laplacian. Writing Eq. 2.14.2 in the form

$$\frac{\partial n}{\partial \tau} = \left(\frac{\partial^2}{\partial r^2} + 2q(r) \frac{\partial}{\partial r} + \Delta_{\theta\varphi} \right) n + Q_1, \quad (2.14.5)$$

where

$$q(r) = \frac{1}{r} - u \quad (2.14.6)$$

and making a substitution the unknown function according the relation¹

$$n(\mathbf{r}, \tau) = r \Psi(\mathbf{r}, \tau) \exp \left\{ - \int^r q(r) dr \right\} \quad (2.14.7)$$

we obtain the equation for $\Psi(\mathbf{r}, \tau)$ function

$$\frac{\partial \Psi}{\partial \tau} = \Delta \Psi + \left(\frac{2u}{r} - u^2 \right) \Psi + Q, \quad (2.14.8)$$

¹ In the integral in the exponent, the lower limit of integration is not shown because its concrete value is inessential; in the inverse transition from $\Psi(\mathbf{r}, \tau)$ to the function $n(\mathbf{r}, \tau)$ the lower limit of integration disappears (a similar method was used in Vasilyev and Toptygin, 1976).

where

$$Q = \frac{1}{r} \exp \left\{ - \int^r q(r) dr \right\} Q_1, \quad (2.14.9)$$

and the expression for Q_1 is given in Eq. 2.14.3. We shall start below from the equation for the Green's function of the Eq. 2.14.8:

$$\frac{\partial G(\mathbf{r}, \tau; \mathbf{r}_o, \tau_o)}{\partial \tau} = \Delta G(\mathbf{r}, \tau; \mathbf{r}_o, \tau_o) + \left(\frac{2u}{r} - u^2 \right) G(\mathbf{r}, \tau; \mathbf{r}_o, \tau_o) + \delta(\mathbf{r} - \mathbf{r}_o) \delta(\tau - \tau_o). \quad (2.14.10)$$

now consider a general method of composing the Green's function which satisfies the Eq. 2.14.10. For this purpose note that the function G depends only on the difference $\tau - \tau_o$ owing to the invariance of the Eq. 2.14.10 and of the initial conditions with respect to the onset of the time scale. Therefore it is possible to write a formal expansion of this function into the Fourier integral

$$G(\mathbf{r}, \tau; \mathbf{r}_o, \tau_o) = \frac{1}{2\pi} \int_{-\infty}^{\infty} G(\mathbf{r}, \mathbf{r}_o; \omega) \exp(-i\omega(\tau - \tau_o)) d\omega, \quad (2.14.11)$$

where the spectral representation of the Green's function $G(\mathbf{r}, \mathbf{r}_o; \omega)$ satisfies the equation

$$\Delta G(\mathbf{r}, \mathbf{r}_o; \omega) + \frac{2u}{r} G(\mathbf{r}, \mathbf{r}_o; \omega) + k^2 G(\mathbf{r}, \mathbf{r}_o; \omega) = \delta(\mathbf{r} - \mathbf{r}_o), \quad (2.14.12)$$

where

$$k^2 = i\omega - u^2. \quad (2.14.13)$$

As usual, in the integral Eq. 2.14.11, a substitution is assumed of the quantity ω in the argument of the function $G(\mathbf{r}, \mathbf{r}_o; \omega)$ by $\omega + i\varepsilon$ ($\varepsilon > 0$) with conservation of integration along the real axis. This is connected with the fact that the multi-leaved function $G(\mathbf{r}, \mathbf{r}_o; \omega)$ at a certain place of a complex variable ω at $\text{Im} \omega > 0$ being to have no discontinuities, according to the general theory (Morse and Feshbach, 1953). Including this property, let us start to compose the Green's function for the Eq. 2.14.12. Note first of all that Eq. 2.14.12 coincides formally with Schrödinger's equation for a particle in a Coulomb field. Therefore the methods which were developed for solving Schrödinger's equation with a Coulomb potential (Bahrah and Vetchinkin, 1971), can be applied to the problem under consideration. Using a similarity of the Eq. 2.14.12 to Schrödinger's equation, we

separate the variables in Eq. 2.14.12, presenting the Green's function $G(\mathbf{r}, \mathbf{r}_o; \omega)$ in the form of an expansion over proper functions of the angular part of the Laplacian

$$G(\mathbf{r}, \mathbf{r}_o; \omega) = \sum_{l=0}^{\infty} \frac{2l+1}{4\pi} P_l(\cos \theta) G_l(r, r_o; \omega), \quad (2.14.14)$$

where θ is the angle between $\mathbf{r}$ and $\mathbf{r}_o$; $l = 0, 1, 2, \dots$; $P_l(\cos \theta)$ are Legendre's polynomials; $G_l(r, r_o; \omega)$ are Green's function for the radial equation corresponding to Eq. 2.14.12:

$$\left[\frac{1}{r^2} \frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r} + \frac{l(l+1)}{r^2} + \frac{2u}{r} + k^2 \right] G_l(r, r_o; \omega) - \frac{1}{rr_o} \delta(r - r_o) = 0. \quad (2.14.15)$$

To obtain the Green's function $G_l(r, r_o; \omega)$ consider the solutions $f_l(r)$ and $\varphi_l(r)$ of the equation

$$\left[\frac{1}{r^2} \frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r} + \frac{l(l+1)}{r^2} + \frac{2u}{r} + k^2 \right] \begin{pmatrix} f_l(r) \\ \varphi_l(r) \end{pmatrix} = 0, \quad (2.14.16)$$

where $f_l(r)$ is regular at $r \rightarrow 0$ and the function $\varphi_l(r)$ is regular at $r \rightarrow \infty$. If the functions $f_l(r)$ and $\varphi_l(r)$ are known, the Green's function will be determined, according to Titchmarsh (M1958), by the relation

$$G_l(r, r_o; \omega) = \frac{\theta(r - r_o) f_l(r) \varphi_l(r_o) + \theta(r_o - r) f_l(r_o) \varphi_l(r)}{r^2 \Delta\{f_l(r), \varphi_l(r)\}}, \quad (2.14.17)$$

where

$$\theta(x) = \begin{cases} 1, & \text{if } x > 0, \\ 0, & \text{if } x < 0, \end{cases} \quad (2.14.18)$$

and

$$\Delta\{f_l(r), \varphi_l(r)\} = \varphi_l(r) \frac{\partial f_l(r)}{\partial r} - f_l(r) \frac{\partial \varphi_l(r)}{\partial r} \quad (2.14.19)$$

is Wronskian of the Eq. 2.14.16. Now, we start to solve the Eq. 2.14.16, i.e. to determine directly the functions $f_l(r)$ and $\varphi_l(r)$. The substitution of the independent variable $r = \xi/2k$ and the unknown function

$(f_l, \varphi_l) \Rightarrow (2k/\xi)(\chi_l^+(r), \chi_l^-(r))$ reduces the Eq. 2.14.16 to the canonic form of equation for Whittaker's function:

$$4\xi^2 \frac{\partial^2 \chi_l^\pm}{\partial \xi^2} = (\xi^2 - 4\mu\xi + 4\lambda^2) \chi_l^\pm, \quad (2.14.20)$$

where $\mu = u/k$, $\lambda^2 = l(l+1)$. The condition of regularity at a zero point is satisfied by the solution of the Eq. 2.14.20 in the form of the Whittaker's function $M_{\mu, l+1/2}(\xi)$ and the regularity at infinity is satisfied by the Whittaker's function $W_{\mu, l+1/2}(\xi)$. Using the corresponding asymptotic expression for these functions, we calculate the Wronskian of Eq. 2.14.19 and obtain the Green's function $G_l(r, r_o; \omega)$ by means of Eq. 2.14.17:

$$G_l(r, r_o; \omega) = \frac{\Gamma(1+l-\mu)}{\Gamma(2l+2)rr_o} [\theta(r-r_o)M_{\mu, l+1/2}(2kr_o)W_{\mu, l+1/2}(2kr) + \theta(r_o-r)M_{\mu, l+1/2}(2kr)W_{\mu, l+1/2}(2kr_o)] \quad (2.14.21)$$

where $\Gamma(x)$ is Euler's Γ - function. The Eq. 2.14.21 completely solves the formulated problem of determining the radial Green's function of the Eq. 2.14.12. It is convenient for further considerations to represent the Green's function (Eq. 2.14.21) in the form of contour integral (Hostleger, 1964). For this purpose we shall use the integral representation (Ryzhik and Gradstein, M1971) for the product of Whittaker's functions included in Eq. 2.14.21:

$$M_{\mu, \nu}(y)W_{\mu, \nu}(y_o) = \frac{\Gamma(1+2\mu)}{\Gamma(\mu-\nu+1/2)} \sqrt{yy_o} \int_0^\infty dx \exp\left(-\frac{y^2+y_o^2}{2} \text{ch} x\right) \times \left(\text{cth} \frac{x}{2}\right)^{2\nu} J_{2\mu}(\sqrt{yy_o} \text{sh} x), \quad (2.14.22)$$

where $J_{2\mu}$ is the modified Bessel's function and the conditions take place:

$$\text{Re}(\mu-\nu+1/2) > 0, \quad \text{Re} \mu > 0. \quad (2.14.23)$$

The latter condition may occur and be too restrictive in actual physical problems. To eliminate the restriction determined by Eq. 2.14.23 we use the analytical continuation to the complex plane by setting $\zeta = \text{ch} x$. As a result we have

$$W_{\mu, l+1/2}(2kr)M_{\mu, l+1/2}(2kr_o) = -\Gamma(2l+2)\Gamma(\mu-l)\exp(\pi i\Gamma(\mu-l))\frac{\sqrt{rr_o}}{\pi i} \\ \times \int_{+\infty}^{(1+)} d\zeta \exp(-k(r+r_o)\zeta) \frac{(\zeta+1)^{\mu-1/2}}{(\zeta-1)^{\mu+1/2}} J_{2l+1}\left(2k\sqrt{rr_o}(\zeta^2-1)\right), \quad (2.14.24)$$

where the contour of integration goes round the point $\zeta = +1$ in the positive direction and tends to infinity along the real semi-axis $\zeta > 0$.

Using Eq. 2.14.24 we obtain the expression for the Green's function

$$G_l(r, r_o; \omega) = \Gamma(\mu-l)\Gamma(1-\mu-l)\frac{\exp(\pi i\Gamma(\mu-l))}{\pi i\sqrt{rr_o}} \\ \times \int_{+\infty}^{(1+)} d\zeta \exp(-k(r+r_o)\zeta) \frac{(\zeta+1)^{\mu-1/2}}{(\zeta-1)^{\mu+1/2}} J_{2l+1}\left(2k\sqrt{rr_o}(\zeta^2-1)\right) \quad (2.14.25)$$

of the radial part of Eq. 2.14.12.

Notice that the Green's function determined by Eq. 2.14.25 is symmetric relative to its arguments. At $l = 0$, Eq. 2.14.21 and Eq. 2.14.25 represent the Green's functions of the equation for the spherically symmetric isotropic diffusion including convection.

2.14.3. Green's function of the three-dimensional transfer equation including convection

In the three-dimensional case the Green's function of the transfer equation is determined by Eq. 2.14.14. The problem is to find a closed expression for $G(\mathbf{r}, \mathbf{r}_o; \omega)$. Using the integral representation for the Green's function $G_l(r, r_o; \omega)$ (Eq. 2.14.25), we obtain

$$G(\mathbf{r}, \mathbf{r}_o; \omega) = \frac{1}{4\pi^2 i \sqrt{rr_o}} \int_{+\infty}^{(1+)} d\zeta \exp(-k(r+r_o)\zeta) \frac{(\zeta+1)^{\mu-1/2}}{(\zeta-1)^{\mu+1/2}} \Psi(\zeta, \beta, \theta), \quad (2.14.26)$$

where

$$\Psi(\zeta, \beta, \theta) = \sum_{l=0}^{\infty} (2l+1)\Gamma(\mu-l)\Gamma(1-\mu-l)\exp(\pi i\Gamma(\mu-l))P_l(\cos\theta)J_{2l+1}(\beta), \quad (2.14.27)$$

and the quantity

$$\beta = 2k\sqrt{rr_o}(\zeta^2 - 1). \quad (2.14.28)$$

Thus the problem reduces to a summation of the series in Eq. 2.14.27. To do this we use the following method from the paper (Hostleger, 1964). Representing the series in Eq. 2.14.27 in the form of the sum over odd and even indices and summarizing the corresponding terms, we obtain

$$\Psi(\zeta, \beta, \theta) = \frac{\pi \exp(i\pi\mu)}{\sin(\pi\mu)} \sum_{l=0}^{\infty} (2l+1) P_l(\cos \theta) J_{2l+1}(\beta). \quad (2.14.29)$$

Then using the expression for Legendre's polynomials in terms of the hypergeometric function (Bateman and Erdelyi, M1953)

$$P_l(\cos \theta) = (-1)^l F\left(-l, l+1; 1; \cos^2 \frac{\theta}{2}\right), \quad (2.14.30)$$

and representing the series in Eq. 2.14.29 in the form of Neumann's expansion (see Watson, M1949) we obtain the closed expression for the function $\Psi(\zeta, \beta, \theta)$:

$$\begin{aligned} \Psi(\zeta, \beta, \theta) &= \frac{\pi \exp(i\pi\mu)}{\sin(\pi\mu)} \sum_{l=0}^{\infty} \frac{(2l+1)}{l!} (-1)^l \Gamma(l+1) F\left(-l, l+1; 1; \cos^2 \frac{\theta}{2}\right) \\ &\times J_{2l+1}(\beta) = \frac{\pi \exp(i\pi\mu)}{2\sin(\pi\mu)} \beta J_0\left(\beta \cos \frac{\theta}{2}\right). \end{aligned} \quad (2.14.31)$$

Substituting Eq. 2.14.31 in Eq. 2.14.26 we derive the expression for the Green's function $G(\mathbf{r}, \mathbf{r}_o; \omega)$ in the form of contour integral

$$\begin{aligned} G(\mathbf{r}, \mathbf{r}_o; \omega) &= \frac{ik \exp(i\pi\mu)}{4\pi \sin(\pi\mu)} \\ &\times \int_{+\infty}^{(1+)} d\zeta \exp(-k(r+r_o)\zeta) \left(\frac{\zeta+1}{\zeta-1}\right)^{\mu} J_0\left(2k\sqrt{rr_o}(\zeta^2-1)\right) \cos\left(\frac{\theta}{2}\right). \end{aligned} \quad (2.14.32)$$

Then in the integral in Eq. 2.14.32 we make the substitution of variables according to the relations

$$\frac{1}{2}(\rho_1 + \rho_2) = \omega = r + r_o; \quad \sqrt{\rho_1 \rho_2} = v = 2\sqrt{rr_o} \cos \frac{\theta}{2}. \quad (2.14.33)$$

Using Eq. 2.14.33 and the known relation $J_0(z) = \frac{1}{z} \frac{\partial}{\partial z} J_1(z)$ we write the Eq. 2.14.32 in the form

$$G(\mathbf{r}, \mathbf{r}_o; \omega) = \frac{i \exp(i\pi\mu)}{4\pi \sin(\pi\mu)} \frac{1}{v} \frac{\partial}{\partial v} v \int_{+\infty}^{(1+)} d\zeta \exp(-k\omega\zeta) \frac{(\zeta+1)^{\mu-1/2}}{(\zeta-1)^{\mu+1/2}} J_1\left(kv\sqrt{\zeta^2-1}\right). \quad (2.14.34)$$

Comparing this expression with the integral representation of the product of Whittaker's functions (Eq. 2.14.24) we obtain the expression for the Green's function of the three-dimensional equation in the form which does not include the integration operation

$$G(\mathbf{r}, \mathbf{r}_o; \omega) = \frac{\Gamma(1-\mu)}{4\pi} \frac{1}{v} \frac{\partial}{\partial v} W_{\mu, 1/2}(k\rho_2) M_{\mu, 1/2}(k\rho_1). \quad (2.14.35)$$

The variables ρ_1 and ρ_2 are related with the variables r and r_o by the expressions

$$\rho_1 = r + r_o - |\mathbf{r}_o - \mathbf{r}|, \quad \rho_2 = r + r_o + |\mathbf{r}_o - \mathbf{r}|. \quad (2.14.36)$$

Writing the differential operator in Eq. 2.14.35 in terms of these variables, we finally obtain

$$G(\mathbf{r}, \mathbf{r}_o; \omega) = \frac{\Gamma(1-\mu)k}{4\pi|\mathbf{r}_o - \mathbf{r}|} \left(\frac{\partial}{\partial x} - \frac{\partial}{\partial y} \right) W_{\mu, 1/2}(y) M_{\mu, 1/2}(x), \quad (2.14.37)$$

where $x = k\rho_1$, $y = k\rho_2$. Let us write the solution of the transfer Eq. 2.14.1 through the Green's function described by Eq. 2.14.37. Taking into account Eq. 2.14.3, Eq. 2.14.6 and Eq. 2.14.9, we obtain for the particle density $N(\mathbf{r}, \omega)$:

$$N(\mathbf{r}, \omega) = \frac{\Gamma(1-\mu)}{4\pi\kappa} k \int d\mathbf{r}_o \frac{\exp(-u(r-r_o))}{|\mathbf{r}_o - \mathbf{r}|} \left(\frac{\partial}{\partial x} - \frac{\partial}{\partial y} \right) W_{\mu, 1/2}(y) M_{\mu, 1/2}(x) Q_o(\mathbf{r}_o). \quad (2.14.38)$$

As results from Eq. 2.14.38, the Green's function of Eq. 2.14.1 has the following form

$$G(\mathbf{r}, \mathbf{r}_o; \omega) = \frac{\Gamma(1-\mu)k \exp(-u(r-r_o))}{4\pi\kappa|\mathbf{r}_o - \mathbf{r}|} \left(\frac{\partial}{\partial x} - \frac{\partial}{\partial y} \right) W_{\mu, 1/2}(y) M_{\mu, 1/2}(x). \quad (2.14.39)$$

2.14.4. Possible inclusion of the variations of particle energy

Now we show the way in which it is possible to take into account the process of particle energy variations in the framework of the considered formalism. In this case, instead of the equation Eq. 2.14.1, one should consider the equation

$$\frac{\partial n(\mathbf{r}, p, t)}{\partial t} = (\kappa_o \Delta - \mathbf{u}_o \vec{\nabla}) n(\mathbf{r}, p, t) + \frac{2u_o}{3r} p \frac{\partial n(\mathbf{r}, p, t)}{\partial p} + Q_o(\mathbf{r}, p, t), \quad (2.14.40)$$

where p is the particle's momentum. Using the Mellin transform

$$n(\mathbf{r}, s, t) = \int dp p^{s-1} n(\mathbf{r}, p, t) \quad (2.14.41)$$

we write the Eq. 2.14.40 in the form

$$\frac{\partial n(\mathbf{r}, s, t)}{\partial t} = (\kappa_o \Delta - \mathbf{u}_o \vec{\nabla}) n(\mathbf{r}, s, t) + \frac{2u_o}{3r} s n(\mathbf{r}, s, t) + Q_o(\mathbf{r}, s, t). \quad (2.14.42)$$

The substitution of the unknown function according to the Eq. 2.14.7 transforms the Eq. 2.14.32 to the form

$$\frac{\partial \Psi(\mathbf{r}, s, t)}{\partial t} = \Delta \Psi(\mathbf{r}, s, t) + \left(\left(2 - \frac{s}{3} \right) \frac{u}{r} - u^2 \right) \Psi(\mathbf{r}, s, t) + Q_o(\mathbf{r}, s, t), \quad (2.14.43)$$

i.e. the Eq. 2.14.40 is actually transformed to Eq. 2.14.18 with inessential (in the framework of the considered method) variation of Coulomb's potential which is represented by the coefficient in the square brackets on the right-hand side of the Eq. 2.14.43. Thus, to determine $\Psi(\mathbf{r}, s, t)$, the developed above formalism can be completely applied (one should observe that making of the inverse Mellin transformation to determine the unknown Green's function may produce considerable mathematical difficulties).

2.14.5. The Green's function for the stationary isotropic diffusion in the case of power dependence of the diffusion coefficient on the distance

Basing on the non-stationary diffusive-convective transfer of CR in interplanetary space and taking into account adiabatic cooling of particles, Webb and Gleeson (1977) composed the equation with the source in the form of a five-dimensional δ -function (time, particle rigidity, and 3 spatial coordinates) to determine the Green's function. The further integration over five-dimensional volume made it possible to represent the Green's function in the form of solution

of a multi-integral equation. In the special case of the stationary spherically symmetric model of isotropic diffusion the Green's function is written in analytical form through Bessel's function of the first kind. If the coefficient of isotropic diffusion $\kappa = \kappa_o(p)r^b$, where p is a particle momentum, r is the distance from the Sun, $\kappa_o(p)$ is an arbitrary scalar function of p , then the Green's function

$$G(r_o, p_o; r, p) = \begin{cases} \frac{3r_o^2}{4pr^2|\beta+1|u} \left(\frac{x}{x_o}\right)^\beta \left(\frac{x}{y}\right)^2 \exp\left(-\frac{x^2+x_o^2}{4y^2}\right) I_{|\beta|}\left(\frac{xx_o}{2y^2}\right), & \text{if } p \leq p_o \\ 0, & \text{if } p > p_o, \end{cases} \quad (2.14.44)$$

where u is the radial velocity of the solar wind, $I_{|\beta|}$ is the modified Bessel's function of the first kind. To abbreviate writing, the following notations are used in Eq. 2.14.44:

$$\begin{aligned} x &\equiv \frac{2}{1-b} \left(rp^{3/2}\right)^{(1-b)/2}, & x_o &\equiv \frac{2}{1-b} \left(rp_o^{3/2}\right)^{(1-b)/2}, \\ y^2 &\equiv \frac{3}{2u} \int_p^{p_o} \kappa_o(z) z^{(1-b)/2} dz, & \beta &\equiv \frac{1+b}{1-b}. \end{aligned} \quad (2.14.45)$$

2.15. On a relation between the correlation function of particle velocities and pitch-angle and spatial coefficients of diffusion

2.15.1. Correlation function of particle velocities

Forman (1977a,b) developed the concept of the correlation function of particle velocities $\langle v_i(t)v_j(t') \rangle$ which was proposed by Kubo (1957). If μ_i is the corresponding cosine of the angle between the velocity direction and the i -axis of coordinates, then

$$\langle v_i(t)v_j(t') \rangle = \frac{v^2}{2} \int_{-1}^1 \mu_i d\mu_i \int_{-1}^1 \mu'_j d\mu'_j \Omega(\mu_i, \mu'_j; t-t'), \quad (2.15.1)$$

where $\Omega(\mu_i, \mu'_j; t-t') d\mu_i$ is a number of particles between μ_i and $\mu_i + d\mu_i$ in the time instant t which had in the instant t' the direction cosine μ'_j . The function Ω is a solution of the equation

$$\partial\Omega/\partial t + \hat{D}\Omega = 0, \quad (2.15.2)$$

where $\hat{D}$ is the operator of the equation of a transfer in the pitch-angle space with the initial condition

$$\Omega(\mu, \mu'; 0) = \delta(\mu - \mu') \quad (2.15.3)$$

and can be expanded in the series

$$\Omega(\mu, \mu'; t - t') = \sum_k R_k(\mu) R_k(\mu') \exp(-(t - t')/\tau_k) \left/ \int_{-1}^1 R_k^2(\mu) d\mu \right. . \quad (2.15.4)$$

Here $R_k(\mu)$ and τ_k are the eigen-functions and eigen-values of the operator $\hat{D}$ (i.e. $\hat{D}R_k = R_k/\tau_k$). In a special case of isotropic scattering and injection the functions $R_k(\mu)$ at $\mu=1$ transform into Legendre's polynomials $P_k(\mu)$ and τ_k transforms into $2\tau_1/k(k+1)$; in this case $v\tau_1$ is the transport path of particles for scattering. The method developed of the correlation function of particle velocities makes it possible to apply the theory of CR diffusion to the actual cases when a scattering is not isotropic, but, for example, takes place mainly along the field.

2.15.2. Connection between the correlation function of particle velocities, pitch-angle and spatial coefficients of diffusion

In general form, a connection between the correlation function of particle velocities $\langle v_i(t) v_j(t') \rangle$ and the spatial coefficient of anisotropic diffusion D_{ij} is determined by the relation

$$D_{ij} = \int_{-\infty}^t \langle v_i(t) v_j(t') \rangle dt' . \quad (2.15.5)$$

To determine a relation with the pitch-angle diffusion coefficient, Forman (1977b) starts from the diffusion equation in the pitch-angle space for the distribution function $f(\mu, v, z)$:

$$\frac{\partial f}{\partial t} + \mu v \frac{\partial f}{\partial z} = \frac{\partial}{\partial \mu} D_{\mu\mu} \frac{\partial f}{\partial \mu} , \quad (2.15.6)$$

where $\mu = \cos \theta$ (θ is a pitch-angle between a particle velocity $\mathbf{v}$ and the direction of a magnetic line of force); v is the absolute value of a particle velocity; z is a coordinate along of a line of force of the regular component of magnetic field; $D_{\mu\mu}$ is the diffusion coefficient in the pitch-angle space. In the case of the

scattering inhomogeneities being able to be represented in the form of solid spheres, we then have

$$D_{\mu\mu} = \nu (1 - \mu^2) / 2, \quad (2.15.7)$$

where ν is the collision frequency. The spatial field-aligned coefficient of diffusion D_{zz} is related to $D_{\mu\mu}$ by the expression

$$D_{zz} = \frac{\nu^2}{8} \int_{-1}^{+1} \frac{(1 - \mu^2)^2}{D_{\mu\mu}} d\mu. \quad (2.15.8)$$

On the other hand,

$$\kappa_{//} = D_{zz} = \int_0^\infty \langle v_z(0) v_z(t) \rangle dt, \quad (2.15.9)$$

where $\langle v_z(0) v_z(t) \rangle$ is the correlation function of the velocities along the field; this results in

$$\langle v_z(0) v_z(t) \rangle = \frac{1}{2} \int_{-1}^{+1} \mu d\mu \left(\mu + \int_0^t \langle d\mu/dt \rangle dt \right). \quad (2.15.10)$$

For the correlation function of transverse velocities $\langle v_x(0) v_x(t) \rangle$ the relation with the transverse spatial coefficient of diffusion is determined by the expression

$$\kappa_{\perp} = D_{xx} = \int_0^\infty \langle v_x(0) v_x(t) \rangle dt. \quad (2.15.11)$$

If the transport path for scattering Λ is far more than the correlation length l_c , then

$$\kappa_{\perp} = \frac{\nu}{u} W_{xx}(0) H_o^2, \quad (2.15.12)$$

where $W_{xx}(0)$ is the spectrum of the field fluctuation power at zero frequency, H_o is the intensity of the regular magnetic field component. If $\Lambda \ll l_c$, then

$$\kappa_{\perp}/\kappa_{\parallel} = \left\langle \delta H_x^2 \right\rangle / H_o^2 . \quad (2.15.13)$$

It was shown that including the effects of moderate-scale turbulence on the particle transfer results in a decrease of the parallel diffusion coefficient; the decrease is determined by the additional factor

$$\langle \cos(\delta\theta) \rangle^2 \approx 1 - \left\langle \delta H_x^2 + \delta H_y^2 \right\rangle / H_o^2 . \quad (2.15.14)$$

2.16. On a balance of CR energy in multiple scattering in expanding magnetic fields

The problem of the balance of CR energy in expanding magnetic fields is of greatest importance, because without its solution it is impossible to study the features of the propagation of CR of internal and external origin in expanding shells of a Supernova in galaxies in the presence of galactic wind, in the expanding Metagalaxy, in stellar winds, in particular, a propagation of CR of solar and galactic origin in the solar wind. The most carefully studied problem is the problem of propagation of CR in interplanetary space when there is energy exchange between charged particles and stochastic inhomogeneities of interplanetary field which are frozen in solar wind plasma. The prevailing concept when considering energy dissipation in the system of CR-solar wind, is the assumption of adiabatic deceleration of charged particles of cosmic radiation. This concept is related to the prevailing probability of overtaking collisions with radially moving inhomogeneities of magnetic field. Dorman, Katz, Fedorov and Shakhov (1978c, 1979) showed that these concepts are restricted owing to ignoring the concrete character of particle spatial distribution and, as a result, owing to ignoring the necessity of revising the notion of the character of CR propagation in interplanetary space. Furthermore, pronouncedly inhomogeneous character of expansion of the solar wind plasma results in the presence of a specific mechanism of CR acceleration caused by the spatial inhomogeneity of the distribution function of particles.

We start from the equation of CR transfer (Dolginov and Toptygin, 1966a):

$$\frac{\partial n(\mathbf{r}, p, t)}{\partial t} - \frac{\partial}{\partial r_{\alpha}} \kappa_{\alpha\lambda}(\mathbf{r}, p, t) \frac{\partial n(\mathbf{r}, p, t)}{\partial r_{\lambda}} + \mathbf{u} \frac{\partial n(\mathbf{r}, p, t)}{\partial \mathbf{r}} - \frac{p}{3} \frac{\partial n(\mathbf{r}, p, t)}{\partial p} \text{div} \mathbf{u} = 0 , \quad (2.16.1)$$

where $n(\mathbf{r}, p, t)$ is the density of particles with given value of momentum p , $\kappa_{\alpha\lambda}(\mathbf{r}, p, t)$ is the tensor of particle diffusion in space, $\mathbf{u}(\mathbf{r})$ is the solar wind velocity.

The energy density $W(\mathbf{r}, t)$ of CR is determined by the equation

$$W(\mathbf{r}, t) = \int_0^{\infty} dp p^2 E_k n(\mathbf{r}, p, t), \quad (2.16.2)$$

where E_k is the kinetic energy of particles. The continuity equation for $W(\mathbf{r}, t)$ results from Eq. 2.16.1 and has the following form:

$$\frac{\partial W(\mathbf{r}, t)}{\partial t} + \text{div} \mathbf{q}(\mathbf{r}, t) = \frac{1}{3} \int_0^{\infty} dp p^3 v \left(\mathbf{u} \frac{\partial n(\mathbf{r}, p, t)}{\partial \mathbf{r}} \right), \quad (2.16.3)$$

where

$$\mathbf{q}(\mathbf{r}, t) = \int_0^{\infty} dp p^2 E_k \mathbf{J}(\mathbf{r}, p, t) \quad (2.16.4)$$

is the flux density of CR energy and

$$J_{\alpha}(\mathbf{r}, p, t) = -\kappa_{\alpha\lambda}(\mathbf{r}, p, t) \frac{\partial n(\mathbf{r}, p, t)}{\partial r_{\lambda}} - u_{\alpha}(\mathbf{r}) \frac{p}{3} \frac{\partial n(\mathbf{r}, p, t)}{\partial p} \quad (2.16.5)$$

is the flux density of CR. On the other hand, the law of conservation of particle number corresponds to the Eq. 2.16.1:

$$\frac{\partial n(\mathbf{r}, t)}{\partial t} + \text{div} \mathbf{I}(\mathbf{r}, t) = 0 \quad (2.16.6)$$

where

$$n(\mathbf{r}, t) = \int_0^{\infty} dp p^2 n(\mathbf{r}, p, t) \quad (2.16.7)$$

is the density of particles, and

$$\mathbf{I}(\mathbf{r}, t) = \int_0^{\infty} dp p^2 \mathbf{J}(\mathbf{r}, p, t) \quad (2.16.8)$$

is the flux density of CR with all energies.

The Eq. 2.16.3 has the form of the continuity equation with a source on the right-hand side the sign of which determines just the character of the variations of energy density of CR. As results from Eq. 2.16.3, a sign of the term, corresponding to the source in the case of radial outflow of solar wind plasma, is determined of a direction of the radial gradient of CR and with the positive

gradient of CR (taking place for galactic CR) this term represents the amount of energy which is accumulated by particles in unit volume per unit time-interval in their interaction with moving inhomogeneities of the magnetic field. Thus, the total number of particles in this case is conserved, according to Eq. 2.16.6, and the energy density of particles is increased; it is the typical situation for the presence of a process of particle acceleration. If the radial gradient of CR is negative, the inverse process takes place, i.e. particles transfer their energy to inhomogeneities of the magnetic field and are decelerated. The same conclusion concerning the character of energy exchange between CR and moving inhomogeneities of magnetic field results from the Eq. 2.16.1. The Eq. 2.16.1 is an equation of Fokker-Plank type, and to estimate the physical meaning of the terms in this equation, one should write it in the canonical form, i.e. in the form of conservation of particle number in phase space:

$$\frac{\partial n(\mathbf{r}, p, t)}{\partial t} + \text{div} \mathbf{J}(\mathbf{r}, p, t) + \text{div}_p \mathbf{J}_p(\mathbf{r}, p, t) = 0, \quad (2.16.9)$$

where $\mathbf{J}_p(\mathbf{r}, p, t)$ is the flux density of particles in the momentum space and the subscript index p of the operator div_p implies that in the case under consideration one should take into account only the part of divergence operator in the momentum space which depends on the absolute value of the momentum. Including Eq. 2.16.9 we write Eq. 2.16.1 in the form

$$\begin{aligned} \frac{\partial n(\mathbf{r}, p, t)}{\partial t} + \frac{\partial}{\partial r_\alpha} \left\{ -\kappa_{\alpha\lambda}(\mathbf{r}, p, t) \frac{\partial}{\partial r_\lambda} + D_{\alpha p} \frac{\partial}{\partial p} \right\} n(\mathbf{r}, p, t) \\ + \frac{1}{p^2} \frac{\partial}{\partial p} p^2 D_{p\alpha} \frac{\partial n(\mathbf{r}, p, t)}{\partial r_\alpha} = 0, \end{aligned} \quad (2.16.10)$$

Where

$$D_{\alpha p} = -D_{p\alpha} = -pu_\alpha/3 \quad (2.16.11)$$

are the components of the crossed tensor of particle diffusion which describe the process of the energy exchange between CR and magnetic field inhomogeneities frozen in the solar wind plasma. In agreement with the general theory the quantities $D_{\alpha p}, D_{p\alpha}$ (as well as $\kappa_{\alpha\lambda}$) satisfy the principle of the symmetry of kinetic coefficients. As results from Eq. 2.16.10, the value of the vector of particle density in the momentum space is determined by the expression

$$J_p = D_{p\alpha} \frac{\partial n(\mathbf{r}, p, t)}{\partial r_\alpha}. \quad (2.16.12)$$

It should be noted that in the initial formulation of the problem of CR propagation (Krymsky, 1964; Dorman, 1965; Parker, 1965), a form of Fokker-Planck equation was postulated, basing on the concept of a systematic energy losses of particles in their interaction with the radially-divergent inhomogeneities of magnetic field (in contrast to the paper Dolginov and Toptygin (1966a,b) where a consistent deduction of this equation was carried out for the first time). In this case, the exact expression was used for the particle flux $\mathbf{J}(\mathbf{r}, p, t)$ in space and the particle flux in the momentum space was determined by the expression

$$J_p = \left\langle \frac{\partial p}{\partial t} \right\rangle n(\mathbf{r}, p, t), \quad (2.16.13)$$

where the kinetic coefficient $\langle \partial p / \partial t \rangle$ has the meaning of variation of particle momentum per unit time, and for calculation of this coefficient some intuitive considerations were involved using the assumption of systematic losses of particle energy. In spite of the fact that the equation (obtained from these not completely exact assumption) having been a quite correct equation of the transfer, a canonical form ascribed to it does not correspond to reality² and based on the interpretation of physical phenomena taking place in the CR propagation in interplanetary space appears to be incorrect. Moreover, in a consistent phenomenological treatment there does not arise the problem of calculation of the kinetic coefficient $\langle \partial p / \partial t \rangle$, but, as seen from Eq. 2.16.10, it is necessary to determine the crossed coefficient of diffusion $D_{p\alpha}$ characterizing the process of energy exchange between CR and magnetic inhomogeneities which is caused by spatial inhomogeneity of the particle distribution function, in accordance with the general conclusion resulting from the Eq. 2.16.3. Therefore, the concept of adiabatic deceleration of particles does not have a global character and the process of energy exchange in the system CR-solar wind is determined by a concrete form of the distribution function of particles. In this case, galactic CR when propagating in the solar wind, appear to be in an acceleration regime, accumulating energy in the process of scattering on the radially moving inhomogeneities of the magnetic field.

In the conclusion of this problem we present the relations resulting from Eq. 2.16.10, which determine a variation of momentum and energy of a particle per unit time:

² In a later publication (see, for example, Jokipii, 1971) the correct expression for the flux $\mathbf{J}$ of particles was used (see Eq. 2.16.5) corresponding to the expression obtained in (Dolginov and Toptygin, 1966a,b).

$$\frac{\partial p}{\partial t} = \frac{p}{3} \left(\mathbf{u} \frac{1}{n(\mathbf{r}, p, t)} \frac{\partial n(\mathbf{r}, p, t)}{\partial \mathbf{r}} \right), \quad \frac{\partial E_k}{\partial t} = \frac{a E_k}{3} \left(\mathbf{u} \frac{1}{n(\mathbf{r}, p, t)} \frac{\partial n(\mathbf{r}, p, t)}{\partial \mathbf{r}} \right), \quad (2.16.14)$$

where

$$a = (E_k + 2mc^2) / (E_k + mc^2) \quad (2.16.15)$$

and m is a particle rest mass.

These relations show as well that a variation of particle momentum and energy is determined by a sign of the radial gradient of CR, and with the positive radial gradient, an increase of particle energy takes place. The specific feature of the quoted relation consists in the fact that mean variation of particle energy being determined by the value of the relative gradient of CR, which is the parameter characterizing the collective properties of the particle ensemble under consideration. Similar results was obtained some later also by Gleeson and Webb (1978).

2.17. The second order pitch-angle approximation for the CR Fokker-Planck kinetic equation

2.17.1. The matter of the problem

The study of multiple charged particle scattering in magnetic field with random inhomogeneities as scattering centers is important in turbulent plasma theory (e.g., Shkarofsky et al., M1966), in problems of cosmic ray particle propagation through cosmic media (e.g., Jokipii, 1966; Dorman and Katz, 1977), and many other problems of particle transport (e.g., Case and Zweifel, M1967). If the magnetic field is sufficiently strong that the Larmor radius of particle $r_L \ll \lambda$ (λ being the particle mean free path with respect to its scattering by inhomogeneities of the magnetic field), the averaging over a particle's spiral motion around the magnetic field can be performed, and one can restrict oneself to a simple rectilinear system.

The paper of Dorman, Shakhov and Stehlik (2003) deals with solution of the equation for the particle distribution function in the second order approximation in the pitch-angle. The exact analytical solution is obtained in an integral form. The well known solution in the first order pitch-angle approximation can be restored by performing the small time limit in the result. Unlike the first order solution the solution obtained in the second approximation rightly shows that the pitch-angle diffusion is closely connected with the particle transport along the mean magnetic field.

The diffusive particle propagation and its angular scattering along the mean magnetic field is governed by kinetic equation of the Fokker - Planck form, and

the particle distribution function f depends only on location x , the pitch-angle, and the time t . Note that the Fokker-Planck scattering represents the scattering of particles in continuously fluctuating fields. Introducing the dimensionless variables, $y = x/\lambda$, $\tau = vt/\lambda$, (v is the particle velocity) and $\mu = \cos\theta$, the kinetic equation for $f(y, \tau, \mu)$ reduces to the well known form (Gleeson and Axford, 1967; Galperin et al., 1971):

$$\frac{\partial f}{\partial \tau} + \mu \frac{\partial f}{\partial y} = \frac{\partial}{\partial \mu} (1 - \mu^2) \frac{\partial f}{\partial \mu} + \frac{1}{\lambda} \delta(y) \delta(\tau) \delta(\mu - \mu_o). \quad (2.17.1)$$

Note that cross-field transport (i.e. perpendicular diffusion and drift, energy change, or adiabatic focusing) is not included into the model.

2.17.2. The first order approximation

Using the Fourier transform in the space variable y , and the Laplace transform in the time τ , Eq. 2.17.1 gives the ordinary differential equation that does not lead to known special functions (Komarov et al., M1976); therefore some approximation of Eq. 2.17.1 is necessary. The simplest approximation corresponds to very small pitch-angle θ , when one can put $\sin\theta \rightarrow \theta$ and $\cos\theta \rightarrow 1$. In this case the function $f_1(y, \tau, \theta, \theta_o)$ in the first order approximation is well known (e.g., Dorman and Katz, 1977d):

$$f_1(y, \tau, \theta, \theta_o) = \frac{1}{2\lambda\tau} \exp\left(-\frac{\theta^2 + \theta_o^2}{4\tau}\right) I_o\left(\frac{\theta\theta_o}{2\tau}\right) \delta(y - \tau), \quad ..(2.17.2)$$

where $I_o(x)$ is the zero order Bessel function with an imaginary argument.

2.17.3. The second order approximation

In the second order pitch-angle approximation one must also hold the term of $O(\theta^2)$. This means that $\sin\theta \rightarrow \theta$ and $\cos\theta \rightarrow 1 - \theta^2/2$ and Eq. 2.17.1 for f_2 in the second order approximation reads

$$\frac{\partial f_2}{\partial \tau} + \left(1 - \frac{\theta^2}{2}\right) \frac{\partial f_2}{\partial y} = \frac{1}{\theta} \frac{\partial}{\partial \theta} \theta \frac{\partial}{\partial \theta} f_2 + \frac{1}{\lambda} \delta(y) \delta(\tau) \frac{\delta(\theta - \theta_o)}{\theta_o}. \quad (2.17.3)$$

The resulting solution takes the form:

$$f_2(y, \tau, \eta, \eta_o) = \frac{1}{8\pi\lambda} \int_0^\infty \left\{ \exp \left[-ik(y - \tau) - (1+i)\sqrt{k}(\eta + \eta_o) \coth \left((1+i)\tau\sqrt{k} \right) \right] \times \right. \\ \left. \times I_o \left(\frac{(1+i)\sqrt{k\eta\eta_o}}{2 \sinh \left((1+i)\tau\sqrt{k} \right)} \right) \frac{2(1+i)\sqrt{k}}{\sinh \left((1+i)\tau\sqrt{k} \right)} + C.C. \right\} dk, \quad (2.17.4)$$

or in terms of the variables $\{x, t, \theta, \theta_o\}$,

$$\tilde{f}_2(y, t, \theta, \theta_o) = \frac{1}{8\pi\lambda} \int_0^\infty \left\{ \exp \left[-\frac{ik}{\lambda}(y - vt) - (1+i)\frac{\sqrt{k}}{4}(\theta^2 + \theta_o^2) \coth \left((1+i)\frac{vt}{\lambda}\sqrt{k} \right) \right] \times \right. \\ \left. \times I_o \left(\frac{(1+i)\theta\theta_o\sqrt{k}}{2 \sinh \left((1+i)\frac{vt}{\lambda}\sqrt{k} \right)} \right) \frac{2(1+i)\sqrt{k}}{\sinh \left((1+i)\frac{vt}{\lambda}\sqrt{k} \right)} + C.C. \right\} dk, \quad (2.17.5)$$

where $C.C.$ denotes the complex conjugate term. This function is shown in Fig. 2.17.1.

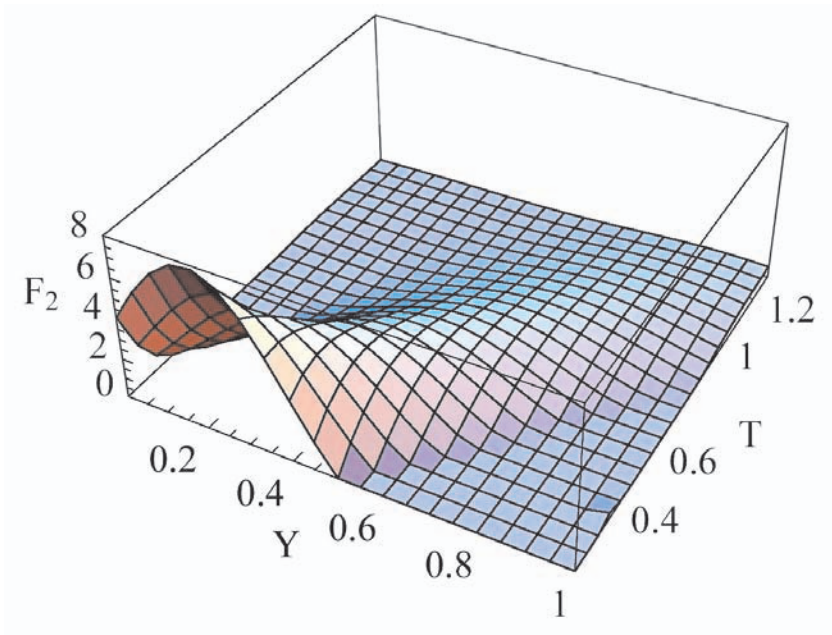


Fig. 2.17.1. The space distribution $f_2(y, \tau)$ in the second approximation in the interval $\tau = 0.3-1.3$ for $\theta = 0.1$, $\theta_o = 0$. According to Dorman, Shakhov and Stehlik (2003).

Let us consider the Eq. 2.17.5 in the non-zero but small time limit $t \rightarrow 0$. Then the expression for $\tilde{f}_2(x, t, \theta, \theta_0)$ acquires the form analogous to Eq. 2.17.2 obtained in the first order approximation:

$$\tilde{f}_2(x, t, \theta, \theta_0) = \frac{\lambda}{2vt} \exp\left(-\frac{\lambda(\theta^2 + \theta_0^2)}{4vt}\right) I_0\left(\frac{\lambda\theta\theta_0}{2vt}\right) \delta\left(x - vt\left(1 - \frac{\theta^2 + \theta_0^2}{6}\right)\right). \quad (2.17.6)$$

One can see from the Eq. 2.17.6 that both processes of the pitch-angle diffusion and the particle transport are connected to each other here. Unlike the first approximation f_1 there is not 'free' propagation in the small time limit solution as well as in the second approximation f_2 . So the first approximation is suitable only in a very small time interval, $\tau \ll 1$.

2.17.4. Peculiarities of the second pitch-angle approximation

Unlike the first approximation, the function $f_2(\theta)$ describes the initially anisotropic stream during a longer time after the particle injection, and one has a low level at $y \approx \tau$ especially for $y \gg 1$. The space distribution $f_2(y)$ has been depicted in Fig. 2.17.1, where the pitch-angle θ is fixed. The picture is similar for non-zero θ . The space distribution possess rather wide 'tail' behind the front of the first particles at $y = \tau$. Its width increase with increasing time, and the maximum decreases in amplitude and becomes later with increasing time. Temporal development of $f_2(y, \tau)$ is shown in Fig. 2.17.2. We conclude that unlike the first approximation in pitch-angle the derived expressions for the particle distribution function in the second approximation as well as the particle density gives a more realistic picture of the pitch-angle distribution after an immediately unidirectional particle injection.

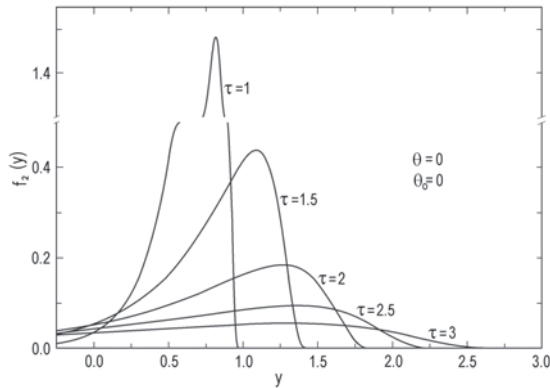


Fig. 2.17.2. The space distribution $f_2(y)$ at time $\tau = 1, 1.5, 2, 2.5$, and 3 for $\theta = \theta_0 = 0$. According to Dorman, Shakhov and Stehlik (2003)

2.18. Anomalous diffusion: modes of CR diffusion propagation

2.18.1. Three modes of particle propagation: classical diffusion, super-diffusion and sub-diffusion

According to Otsuka and Hada (2003), anomalous diffusion is observed in many branches of science, e.g., anomalous diffusion in rotating flows (Solomon et al., 1993), particle motion in nonlinear dynamical systems (Klafter et al., 1993), chaotic phase diffusion in Josephson junctions (Geisel et al., 1985), field line diffusion in solar wind magnetic turbulence (Pommois et al., 2001), and transport in turbulent plasmas (Balescu, 1995). The term 'anomalous' is used to emphasize deviation from classical (normal) diffusion, in which the mean squared displacement of particles increases proportional with time. Namely, if we define the diffusion coefficient κ as

$$\kappa = \left\langle (\Delta r)^2 \right\rangle / \tau \propto \tau^\beta, \quad (2.18.1)$$

where Δr is the particle displacement within the time scale τ , and the bracket denotes an ensemble average, then $\beta = 0$ for the **classical diffusion**. This is a consequence of the well-known central limit theorem, which states that in the long time limit the p.d.f. of Δr approaches a normal (Gaussian) distribution with its variance $\propto \tau$. On the other hand, when a particle can travel long distances ballistically, the so-called Levy flights or Levy walks can arise, and the resultant diffusion process of the ensemble of particles becomes **super-diffusive** ($\beta > 0$). When a particle can be trapped within a certain bounded region for a long time, then the **sub-diffusion** ($\beta < 0$) emerges.

2.18.2. Simulation of particle propagation in a two-dimensional static magnetic field turbulence

Otsuka and Hada (2003) compute numerically orbits of CR particles in a two-dimensional static magnetic field turbulence, and show that anomalous diffusion can appear in general. The result may have an important implication for plasma astrophysics, since, up to now, various diffusion processes (including the cross-field diffusion) are almost always discussed within the framework of the quasi-linear theory, which in principle is a combination of the classical diffusion equation for particles and an evolution equation for the energy of the turbulence, which in turn determines the diffusion coefficient. The spatial diffusion problem, in particular, is important for the shock acceleration of CR charged particles (see Chapter 4).

Although the cross-field diffusion in reality is a three-dimensional problem, Otsuka and Hada (2003) limit their discussion to the case where all the physical variables depend only on two spatial coordinates (x and y), and the magnetic field

lines are perpendicular to the x - y plane. By taking such geometry, the effective cross-field diffusion resulting from parallel motion along the twisted field lines (field-line random walk (Jokipii, 1966)) will be excluded. In general, in a model with only two spatial dimensions, particles are tied to the magnetic field lines since the canonical momentum associated with the ignorable coordinate becomes an invariant of the motion (Jokipii et al., 1993). Since the energetic particle velocities considered are much larger than the MHD velocities it may be assumed that the field turbulence is time stationary (fossil turbulence). The particle energy is then conserved, and the position $\mathbf{r} = (x, y)$ and the velocity $\mathbf{V} = (V_x, V_y)$ obey the equations of motion

$$\dot{\mathbf{V}} = \mathbf{V} \times (1 + b)\mathbf{z}; \quad \dot{\mathbf{r}} = \mathbf{V}, \quad (2.18.2)$$

where $\mathbf{z}$ is a unit vector in the z direction, b is the fluctuation part of the normalized magnetic field, and time is normalized to the reciprocal of the average particle gyro-frequency. The turbulence field is given by

$$b(x, y) = \sum_m \sum_n A(k) \cos(mx + ny + \phi(m, n)), \quad (2.18.3)$$

where $k = 2\pi/\lambda$ is the wave number, and

$$A(k) \propto \begin{cases} k^{-\gamma} & \text{for } k_{\min} \leq k \leq k_{\max}, \\ k_{\min}^{-\gamma} & \text{for } k_{\text{sys}} \leq k \leq k_{\min}. \end{cases} \quad (2.18.4)$$

In Eq. 2.18.3 and 2.18.4 there are

$$k = (m^2 + n^2)^{1/2}, \quad k_{\text{sys}} = 2\pi/L, \quad (2.18.5)$$

and L is the system size. Boundary conditions are periodic, and phases $\phi(m, n)$ are random. The magnetic field correlation length L_c was defined as

$$L_c^2 = \left\langle (2\pi/k)^2 A(k) \right\rangle / \left\langle A(k) \right\rangle. \quad (2.18.6)$$

There were chosen $\gamma = 1.5$, $L = 512$, and $\langle b^2 \rangle^{1/2} = 0.01$, then $L_c \approx 61$. By giving different velocities to the particles, it was running with different r_L/L_c , where r_L is the Larmor radius.

The results are summarized in Fig. 2.18.1. The upper panels show the expected diffusion coefficient κ defined in Eq. 2.18.1 versus τ in logarithmic scales for three different regimes of r_L/L_c , and the lower panels are the y component of the guiding center $\mathbf{r}_g = \mathbf{r} + \mathbf{V} \times \mathbf{z}$ versus time τ , plotted for some particles. The orbits look quite different for the three runs. When $r_L/L_c = 10$ the orbits look more or less similar to a Brownian motion, whilst for $r_L/L_c = 1$ and $r_L/L_c = 0.1$ they are composed of segments with different characters – sometimes almost ballistic, sometimes trapped at a certain locations, and sometimes like a Brownian motion. This diversity of the types of the orbits is a reflection of the presence of multi-scales in the magnetic field turbulence. Namely, if a particle is guided by a large scale inhomogeneity of the magnetic field for a longer time period than the ‘observation’ time scale τ , then its orbit will appear to be almost ballistic, while a particle trapped by a small scale inhomogeneity will appear as trapped if it makes many rotations around the inhomogeneity within τ .

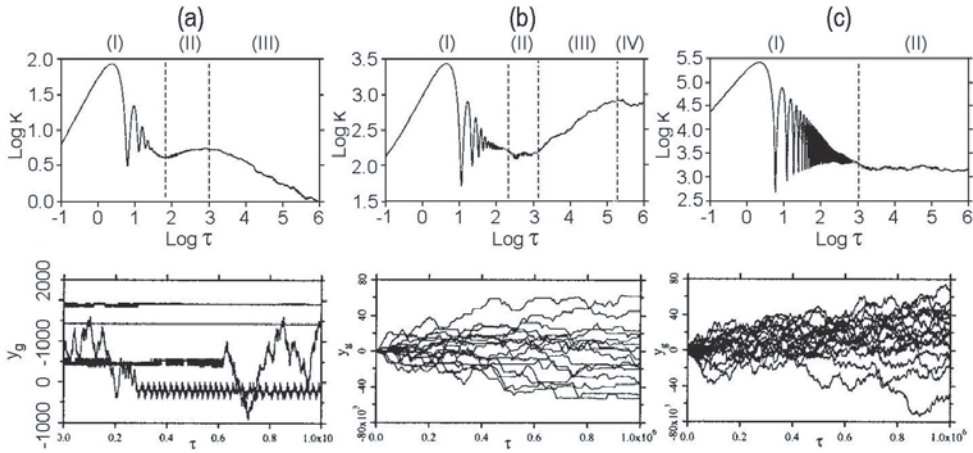


Fig. 2.18.1. Diffusion coefficients κ (upper panels) and typical guiding center trajectories (lower panels) for (a) $r_L/L_c = 0.1$, (b) $r_L/L_c = 1$, and (c) $r_L/L_c = 10$. According to Otsuka and Hada (2003).

The diffusion coefficients represent the different characteristics of the orbits. Let us first look at the **case c** in Fig. 2.18.1, $r_L/L_c = 10$. When (i) $\tau < 10^3$, the value of κ is still influenced by Larmor rotations (large amplitude oscillations in the figure), and so there is no sense in discussing statistics in this regime. For longer time scale (ii) $\tau > 10^3$, the value of κ became almost constant, suggesting that the diffusion is almost classical and the orbits are essentially Brownian. This is reasonable, since when $r_L/L_c = 10$, a particle traverses many inhomogeneities of

the magnetic field during one gyration, and the force acting on the particle, which will be a sum of many fluctuations, will be random and incoherent.

When $r_L/L_c = 0.1$ (**case a**), the gyration regime (i) is followed by two distinct regimes (ii) and (iii) as τ is increased. In (ii) the process is slightly super-diffusive ($\beta > 0$ in Eq. 2.18.1) since within this time scale the majority of the particles gradient-**H** drift around the field inhomogeneities without making a complete rotation. For longer time scales many particles are trapped (as seen in the lower panel), resulting in the sub-diffusion ($\beta < 0$ in Eq. 2.18.1). The values of β and the transition time scale which separates regimes (ii) and (iii) depend on the parameters for the turbulence.

The **case b**, $r_L/L_c = 1$, illustrates the possibility that even more distinct types of orbits can exist. In the super-diffusive regime (iii) some particles ‘percolate’ along infinitely long open paths, which result from the assumed periodicity of the simulation system. At longer time scales the percolation orbits start to mix (percolation random walk), and thus the diffusion becomes classical again.

Otsuka and Hada (2003) came to the conclusion that in two-dimensional static magnetic field turbulence different types of cross-field diffusion of energetic particles are observed for different regimes of r_L/L_c , and for a finite observation time scale τ . When $r_L/L_c > 1$ the diffusion is classical asymptotically ($\tau \rightarrow \infty$), whilst at super- and sub-diffusion can be realized when $r_L/L_c < 1$ and when $r_L/L_c \sim 1$.

2.19. Energetic particle mean free path in the Alfvén wave heated space plasma

2.19.1. Space plasma heated by Alfvén waves and how it influences particle propagation and acceleration

Vainio et al. (2003a) present a simple analytical expressions for the power spectrum of cascading Alfvén waves and the resulting CR energetic particle mean free path in the solar wind. The model can reproduce the short coronal mean free path required for efficient acceleration of charged particles in coronal shock waves (see Chapter 4, Section 4.20) as well as a longer interplanetary mean free path required for a rapid propagation of the accelerated particles to a distance 1 AU from the Sun. Recent observations of high and anisotropic ion temperatures in the solar corona (Kohl et al., 1998) give observational support to models employing the cyclotron heating mechanism to heat the plasma on open magnetic field lines. In these models the energy input for heating the plasma comes from Alfvén waves created at the solar surface. The waves propagate until their frequency is comparable to the local ion-cyclotron frequency, and the wave energy is absorbed by the plasma ions via the cyclotron-resonance. Energetic particles interact strongly with the waves responsible for heating the corona

(Vainio et al., 2003b). The same waves that heat the solar corona can help to rapidly accelerate charged particles in coronal shock waves, and thus explain particle acceleration in small SEP events, where self-generated waves (see Chapter 3) can not explain the rapid acceleration. On the other hand, observed parameters of SEP transport in the solar wind give constraints on the wave-heating models, limiting the magnitude and spatial extent of wave heating in the solar wind.

2.19.2. Determining of the Alfvén wave power spectrum

Vainio et al. (2003a) considered Alfvén waves propagating in the solar corona and solar wind in the framework of the model developed in Hu et al. (1999). An equation governing the power spectrum $P(f, r)$ of outward propagating Alfvén waves in the solar wind in the steady state, is given according to Tu et al. (1984) by

$$\frac{v_a}{AV} \frac{\partial}{\partial r} \left(\frac{AV^2}{v_a} P \right) = - \frac{\partial F}{\partial f}; \quad F = 2\pi\alpha\alpha_1 \frac{v_a}{V} \frac{f^{5/2} P^{3/2}}{B}, \quad (2.19.1)$$

where f is the wave frequency, $V = u + v_a$ is the inertial-frame speed (u is the solar wind plasma velocity, and v_a – the Alfvén velocity), r – the heliocentric distance. The given form of the spectral flux function $F(f, r)$ corresponds to the Kolmogorov cascade phenomenology. It is proportional to the cascade constant $\alpha = 1.25$ and to the square root of the ratio of the inward and outward wave intensity, α_1 , which is a model parameter taken to depend on r , only. The flux-tube cross-sectional area A is inversely proportional to the magnetic field B , which is taken to point in the radial direction.

According to Vainio et al. (2003a) the Alfvén wave power spectrum can be solved in an analytic form if α_1 is a given function of position. In this case the spectrum can be given as

$$P = \frac{v_a}{AV^2} \frac{A_s V_s^2}{v_{as}} \frac{f_o^{5/3} I(x(f), \tau(r))}{f^{5/3}} P(f_o, r_s), \quad (2.19.2)$$

where the dimensionless function $I(x, \tau)$ fulfils the equation

$$I(x, \tau) = I(x, 0) \left(x + \tau I^{1/2}(x, \tau) \right). \quad (2.19.3)$$

In Eq. 2.19.2 and Eq. 2.19.3

$$I(x,0) = \frac{P(f_o x^{-3/2}, r_s)}{x^{5/2} P(f_o, r_s)}; x = \left(\frac{f_o}{f}\right)^{5/3}; \tau = 2\pi f_o \varepsilon_p^{1/2} \alpha \int_{r_s}^r \frac{\alpha_1(r') V_s v_a(r')}{V^3(r')} \left(\frac{n_{es}}{n_e(r')}\right)^{\frac{1}{4}} dr'. \quad (2.19.4)$$

Here f_o is an arbitrary normalization frequency, n_e is the electron density, and $\varepsilon_p \equiv f_o P(f_o, r_s) / B_s^2 \ll 1$ is a dimensionless constant. All quantities indexed by s refer to the values at the solar surface. When deriving the spectrum, conservation of mass and quasi-neutrality in an electron–proton plasma are used, i.e., $An_e u = \text{const}$.

In the special case, considered by Vainio et al. (2003a), when $P(f, r_s) = \varepsilon_p B_s^2 / f$, the initial dimensionless spectrum becomes $I(x, 0) = x$. In this case the power spectrum of the Alfvén waves can approximate by

$$P(f, r) = \frac{v_a}{AV^2} \frac{A_s V_s^2}{v_{as}} \frac{\varepsilon_p B_s^2}{f \left(1 + (f/f_c)^{2/3}\right)}, \quad (2.19.5)$$

where the spectral break point frequency $f_c(r) = f_o / \tau(r)$ decreases with heliocentric distance (the spectrum is a broken power law with a spectral index of -1 and $-5/3$ below and above $f_c(r)$; such a form of the power spectrum is supported by observations in the solar wind according to Horbury, 1999).

2.19.3. Determining of the energetic particle mean free path

Vainio et al. (2003a) show that in the case of a wave-heated solar wind the power spectrum of the Alfvén waves determines the mean free path $\lambda(v, r)$ of energetic particles with velocity v (excluding electrons). Taking the Alfvén waves to be linearly polarized quasi-parallel propagating waves, the mean free path will be:

$$\lambda(v, r) = \frac{3v}{8} \int_{-1}^{+1} d\mu \frac{(1 - \mu^2)^2}{D_{\mu\mu}}, \quad (2.19.6)$$

where $D_{\mu\mu}$ is the pitch-angle diffusion coefficient over pitch-angle cosine μ :

$$D_{\mu\mu} = \frac{\pi}{4} \Omega (1 - \mu^2) \frac{f_r P(f_r, r)}{B^2}; \quad f_r = \frac{\Omega}{2\pi} \frac{V}{v|\mu|}. \quad (2.19.7)$$

Here Ω and f_r are the (angular) particle cyclotron and resonant wave frequency, respectively. Substituting the form of the power spectrum described by Eq. 2.19.5 gives

$$\lambda(v, r) = \frac{2v}{\pi \varepsilon_p \Omega} \left(1 + \frac{27}{7} \left(\frac{\Omega V}{2\pi f_c v} \right)^{2/3} \right) \left(\frac{n_e}{n_{es}} \right)^{1/2} \frac{V^2}{V_s^2}. \quad (2.19.8)$$

Assuming a constant value of $V = 400$ km/s, the magnetic field $B = 1.3(r_s/r)^2(1 + 1.9(r_s/r)^6)$ Gs, and the electron density of 10 cm^{-3} at 1 AU gives the density, flow speed, and Alfvén speed profiles depicted in Fig. 2.19.1.

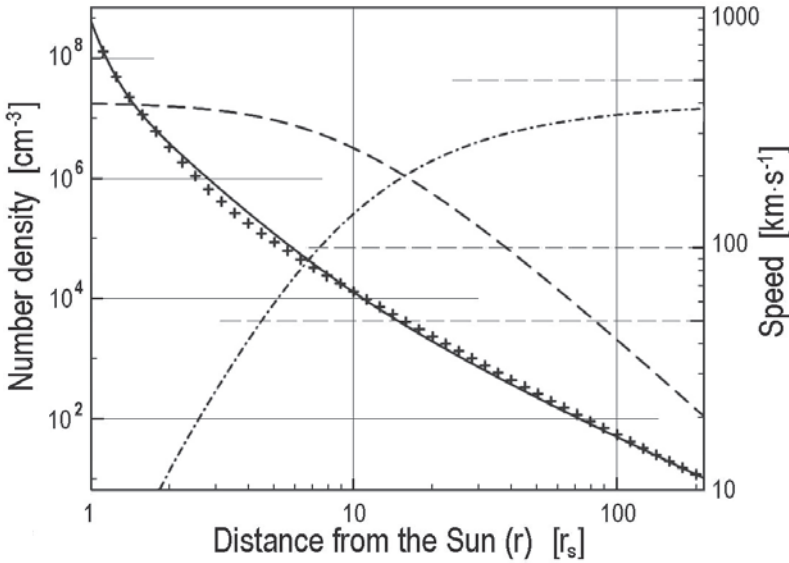


Fig. 2.19.1. Solar wind electron density (solid), Alfvén speed (dashed), and flow speed (dot-dashed). By crosses are shown results of Bird and Edenhofer (1990) of semi-empirical electron densities. From Vainio et al. (2003a).

In Fig. 2.19.2 the resulting 10 MeV proton mean free path is plotted for wave parameters $\varepsilon_p = 5 \times 10^{-5}$ and α_1 which has a constant value at $r > 10r_s$ and increases linearly from 0 to this value at $r_s < r < 10r_s$ (such values of ε_p are needed to produce a solar wind fulfilling observational criteria of mass flux and speed according to Laitinen et al., 2003; the values for α_1 are taken from papers modeling the solar wind expansion: Hu et al., 1999; Laitinen et al., 2003).

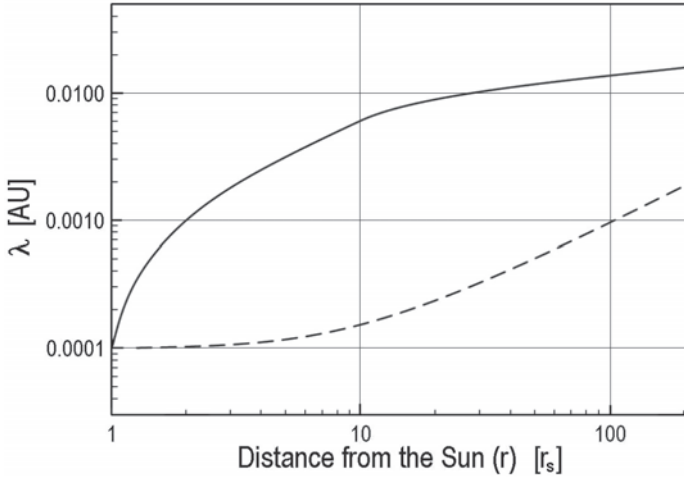


Fig. 2.19.2 10-MeV proton mean free path for the solar wind model depicted in Fig. 2.19.1. Results for $\alpha_1 = 0.05$ (solid) and $\alpha_1 = 0$ (dashed) at $r > 10 r_s$ are shown. According to Vainio et al. (2003a).

The solid curve in Fig. 2.19.2 representing cascading Alfvén waves connects a very short mean free path close to the solar surface ($r < 2r_s$) with a larger, spatially almost constant, value at larger distances from the Sun. Thus the model may offer a consistent explanation of both efficient SEP acceleration at coronal shocks (requiring small λ) in small SEP events, where efficient generation of the Alfvén waves by the energetic protons themselves is not possible, and of the subsequent rapid interplanetary propagation from the acceleration site to the observer. As was shown by Laitinen et al. (2003), the self-consistent modeling of the Alfvén wave propagation and the solar wind expansion in case of no cascade term in the wave transport equation ($\alpha_1 = 0$) produces too small SEP mean free paths in the solar wind. From the other hand, Vainio et al. (2003a) have demonstrated that cascading can dramatically increase the values of the mean free path in the solar wind (in accordance with observations).

2.20. Bulk speeds of CR resonant with parallel plasma waves

2.20.1. Formation of the bulk speeds that are dependent on CR charge/mass and momentum

According to Vainio and Schlickeiser (1999) the quasi-linear interaction of CR particles with transverse parallel propagating plasma waves occurs via gyro resonance. To interact efficiently with a circularly polarized wave the particle must gyrate around the mean magnetic field in the same sense and with the same frequency as the electric field of the wave when viewed in the rest frame of the

particle's guiding center (GC frame). Augmented with the dispersion relations of the relevant wave modes, this condition determines the wave numbers and frequencies of the waves resonant with particles of a given type (charge/mass) and velocity. The intensity of the waves at these wave numbers, in turn, determines how fast the given particle is diffusing in momentum space. If the particle's guiding center moves much faster than the waves relative to the plasma, one may neglect the plasma-frame wave frequency in the (Doppler-shifted) GC-frame wave frequency and make the so called magneto-static approximation (e.g., Jokipii, 1966). This approximation, however, does not give correct results for particles with pitch-angles close to 90° . Since the description of particle scattering in this region determines the fundamental CR transport parameter, the spatial diffusion coefficient (Schlickeiser and Miller, 1998), one has to abandon the magneto-static approximation at least when computing this parameter from the assumed/observed spectrum of magnetic fluctuations. Vainio and Schlickeiser (1999) studied the effect of finite phase speeds of the waves on another transport coefficient, the bulk speed of the CR, which is the effective speed of the waves that scatter the CR particles. Dispersive waves, therefore, can give rise to bulk speeds that are dependent on CR charge/mass and momentum. It was also studied how this affects the scattering-center compression ratio in low Mach number parallel shock waves.

2.20.2. Dispersion relation and resonance condition

The dispersion relations of parallel transverse waves in a cold electron-proton plasma can be described according to Vainio and Schlickeiser (1999) with the equation (e.g., Steinacker and Miller, 1992):

$$k^2 c^2 = \omega^2 \left(1 + \frac{c^2}{v_a^2} \frac{\Omega_e \Omega_p}{(\Omega_p - \omega)(\Omega_e - \omega)} \right), \quad (2.20.1)$$

where k is the wave number and ω is the wave frequency,

$$\Omega_e = \frac{q_e B}{m_e c}, \quad \Omega_p = \frac{q_p B}{m_p c} \quad (2.20.2)$$

are non-relativistic electron and proton gyro-frequencies (q_e and m_e are the electron charge and mass; q_p and m_p are the proton charge and mass; B is the background magnetic field magnitude), c is the speed of light, and

$$v_a = B(4\pi n_e(m_p + m_e))^{-1/2} \quad (2.20.3)$$

is the non-relativistic Alfvén speed, and n_e is the electron density of the plasma. Negative (positive) frequencies denote right (left) handed polarization and the sign of ω/k fixes the propagation direction of the wave relative to the background magnetic field direction. Assuming that $(v_a/c)^2 \ll 4\Omega_p/|\Omega_e| = 0.00218$, Vainio and Schlickeiser (1999a) write the dispersion relation described by Eq. 2.20.1 in the dimensionless form

$$\kappa \approx \pm f \sqrt{\frac{\Phi_p}{(\Phi_p - f)(1 + f)}}, \quad (2.20.4)$$

where

$$\kappa = kv_a/|\Omega_e|, \quad f = \omega/|\Omega_e|; \quad \Phi_p = \Omega_p/|\Omega_e| = 1/1836. \quad (2.20.5)$$

The wave frequency f takes values between $-1 \leq f \leq \Phi_p$; and the sign fixes the wave propagation direction relative to the background magnetic field. When $|f| \ll \Phi_p$ the dispersion relation Eq. 2.20.4 describes Alfvén waves. At positive frequencies the Alfvén waves are converted to proton-cyclotron waves as $f \rightarrow \Phi_p$. At negative frequencies they are first converted to whistlers at $f \approx -\Phi_p$ and finally to electron-cyclotron waves as $f \rightarrow -1$.

Finally, the gyro-resonance condition between the CR and the parallel/anti-parallel waves is

$$f' - \kappa' v_{||} / v_a = \Phi, \quad (2.20.6)$$

where κ' and f' are the dimensionless resonant wave number and wave frequency, v is the CR particle speed and $v_{||}$ is the particle velocity parallel to the background magnetic field, $\Phi = qB/(\gamma mc|\Omega_e|)$ is the (signed) dimensionless gyro-frequency, q is the charge, γ is the Lorentz factor, and m is the mass of the CR particle.

2.20.3. Effective wave speed

Vainio and Schlickeiser (1999) treat $\Phi_o = \Phi\gamma = m_e q/(m|q_e|)$ as constant. The combining Eq. 2.20.4 and Eq. 2.20.6 allows the writing down of an equation for the phase speed $w = v_a f' / \kappa'$ of the waves resonant with CR particles of fixed v as a function of $v_{||}$ in a parametric form

$$w(f') = \pm v_a \sqrt{\frac{(\Phi_p - f')(1 + f')}{\Phi_p}}, \quad v_{||}(f') = w(f') \left(1 - \frac{\Phi}{f'}\right), \quad (2.20.7)$$

from which an implicit form, i.e., $v_{||} = v_{||}(w)$, may be derived straightforwardly. In Fig. 2.20.1 and Fig. 2.20.2 are plotted the solutions of Eq. 2.20.7 for two values of Φ corresponding to non-relativistic protons and mildly relativistic electrons.

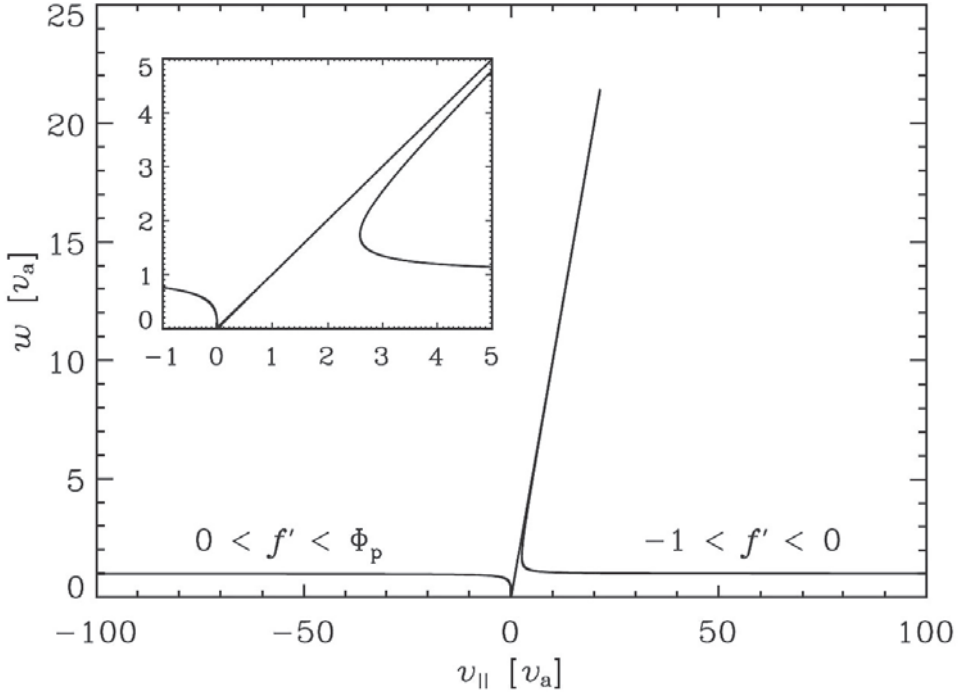


Fig. 2.20.1. Phase speed w , as a function of parallel particle velocity, of parallel-propagating transverse waves resonant with CR particles having constant dimensionless gyro-frequency of $\Phi = \Phi_p$ (left) and $\Phi = -1/3$ (right). According to Vainio and Schlickeiser (1999).

In Fig. 2.20.1 and Fig. 2.20.2 there are indicated what values the wave frequency f' takes in each branch of the curves. The curves are plotted for parallel-propagating waves; for anti-parallel waves, both $v_{||}$ and w change signs for constant f' , and a complete figures would include the negative w axes with curves obtained by rotating the plots in Fig. 2.20.1 and Fig. 2.20.2 about their origins by 180° .

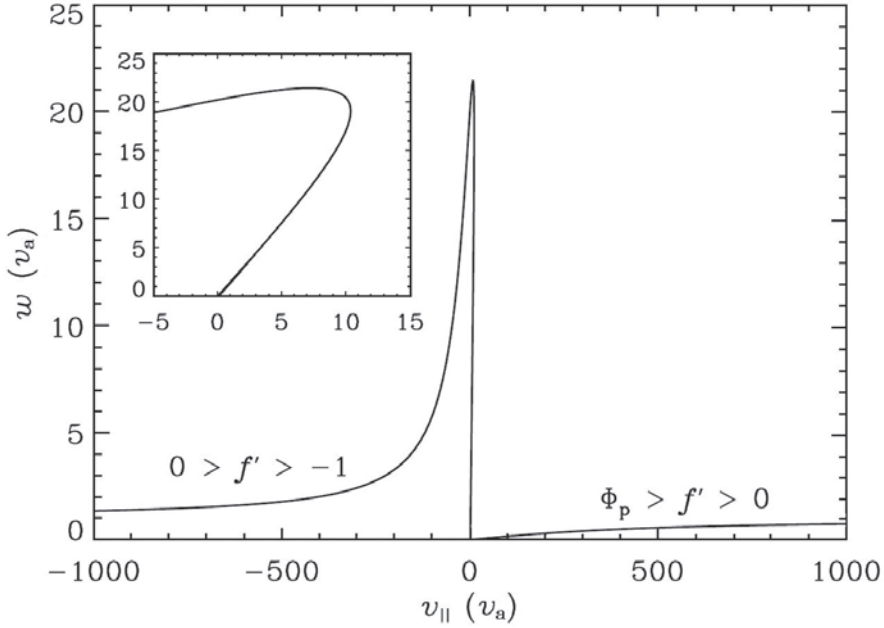


Fig. 2.20.2. The same as in Fig. 2.20.1 but for $\Phi = -1/3$. According to Vainio and Schlickeiser (1999).

2.20.4. Bulk motion of the CR in space plasma

The scattering by waves which all move with the same phase speed, e.g., parallel Alfvén waves, tends to make the CR particle distribution isotropic in the wave frame, i.e., the coordinate system moving with the phase speed of the waves relative to the plasma. This results in a plasma-frame bulk motion of the CR with the phase speed of the waves. If waves with several speeds are present the situation is a bit more involved, but the use of quasi-linear theory with the diffusion approximation for CR propagation gives the plasma-frame bulk speed of the particles in form (Schlickeiser, 1989)

$$V(p) = \frac{1}{3p^2} \frac{\partial}{\partial p} (p^3 D(p)), \quad D(p) = \frac{3v}{4p} \int_{-1}^{+1} (1 - \mu^2) \frac{D_{\mu p}}{D_{\mu\mu}} d\mu, \quad (2.20.8)$$

where p and $\mu = v_{||}/v$ are particle momentum and pitch-angle cosine, and $D_{\mu p} = (1/2)(\Delta\mu\Delta p)/\Delta t$ and $D_{\mu\mu} = (1/2)(\Delta\mu)^2/\Delta t$ are components of the momentum diffusion tensor in the CR kinetic equation. In general, the diffusion tensor components $D_{\mu\mu}(\mu, p)$ and $D_{\mu p}(\mu, p)$ are obtained by taking ensemble averages of the first-order corrections owed to wave fields to the helical particle orbit. For

parallel cold-plasma waves, they have been calculated by Steinacker and Miller (1992). Vainio et al. (2003a), however, show that the bulk speed $V(p)$ in Eq. 2.20.8 may be estimated without knowing the detailed form of these coefficients. They study the interaction of the CR particle with a single resonant wave mode with phase speed w . The interaction between the particle and the wave component can be viewed in the wave frame, where the wave's magnetic field is static making the scattering elastic. Thus we may write the equation $\Delta p' = 0$ for the wave-frame momentum,

$$p' = p \left(1 - 2\mu w/v + (w/v)^2 \right)^{1/2}, \quad (2.20.9)$$

which leads to

$$\Delta p = wp\Delta\mu/(v - \mu w) \quad (2.20.10)$$

if terms of the order (w/c) are neglected. Thus in this case we may write for the ratio of the momentum-diffusion-tensor components

$$\frac{D_{\mu p}}{D_{\mu\mu}} = \frac{\langle \Delta\mu\Delta p \rangle}{\langle (\Delta\mu)^2 \rangle} = \frac{wp}{v - \mu w}. \quad (2.20.11)$$

If several waves, numbered by α , are scattering the particle with given μ and p , we may write

$$\frac{D_{\mu p}}{D_{\mu\mu}} = \sum_{\alpha} a_{\alpha} \frac{w_{\alpha} p}{v - \mu w_{\alpha}}, \quad (2.20.12)$$

where

$$a_{\alpha} = D_{\mu\mu}^{\alpha} \left(\sum_{\alpha} D_{\mu\mu}^{\alpha} \right)^{-1} \quad (2.20.13)$$

and

$$D_{\mu\mu}^{\alpha} = A_{\alpha} (1 - \mu^2) (v - \mu w_{\alpha})^2 / |\mu v - w_{g,\alpha}| \quad (2.20.14)$$

is the pitch-angle diffusion coefficient related to the wave α ; w_{α} and $w_{g,\alpha}$ are the phase and group speeds of the wave α . In Eq. 2.20.14 coefficient $A_{\alpha}(p)$ is proportional to the power in the magnetic field fluctuations of the wave α . Vainio

and Schlickeiser (1999a) note that this result agrees with Steinacker and Miller (1992), where both coefficients were calculated using quasi-linear theory directly. The combining of Eq. 2.20.11 and Eq. 2.20.12 with Eq. 2.20.7 allows one to calculate the CR bulk speed $V(p)$ determined by Eq. 2.20.8, if the scattering frequencies as a function of wave frequency are specified. In particular, if the spectrum of waves as a function of wave number is steep enough, we may approximate a_α as unity for the resonant wave with the lowest wave number and zero for the others.

2.21. Non-resonant pitch-angle scattering and parallel mean-free-path

2.21.1. The problem of the non-resonant pitch-angle scattering

According to Ragot (1999) the scattering of charged particles through the zero pitch-angle cosine, μ , has long remained a challenging problem in the theory of CR particle transport, in a magnetic turbulence composed of plasma waves superimposed on a larger-scale regular magnetic field (see *e.g.*, Bieber et al. 1994; Ragot, 1999 and references therein). The quasilinear theory (Vedenov et al., 1962; Jokipii 1966), which usually describes this problem of particle transport only includes the resonant interactions between the waves and the particles and, when the real spectral shape of the turbulence is taken into account, can fail to produce any significant scattering through $\mu = 0$ at low particle's rigidities. More sophisticated, nonlinear theories (see references in Ragot, 1999) require enhanced levels of fluctuations to achieve this scattering through $\mu = 0$, which are not necessarily observed each time the particles are efficiently scattered. The occurrence of this problem of pitch-angle scattering at precisely $\mu = 0$ is, however, somewhat surprising. Indeed, why should the point where $\mu = 0$ always be the critical point, when μ is defined with respect to the main magnetic field $\mathbf{B}_0$, while locally the real field lines are not along $\mathbf{B}_0$? The answer to this problem according to Ragot (1999) can be formulated in a quite simple form. In order to correctly describe the pitch-angle scattering through $\mu = 0$, *one must take into account the lower-frequency waves even if they are not in resonance with the particles*, because they determine the local variations of the field line direction. The frequencies at which resonant interactions can take place between waves and particles depend on the particles' rigidity and the dispersion relation of the waves. It appears that the most dramatic effect – extremely weak scattering and resulting divergence of the parallel mean free path – occurs when the resonant frequencies fall in the dissipation range of the turbulence, leaving the waves of the inertial range out of the wave-particle interaction description, despite the fact that these later lower-frequency waves are responsible for the local variations of the field line direction. This is the case, in particular, for low-rigidity CR in the solar wind. They cannot gyro-resonate with MHD waves in the inertial range of the

turbulence, below a few hundreds of MV when their pitch-angle cosine μ approaches 0, and below ≈ 1 MV for any μ . Gyro-resonance between waves of frequency ω (parallel wavenumber $k_{\parallel}$) and particles of gyro-frequency

$$\Omega = |q|B/(mc\gamma) = (m_p/m)(\Omega_{po}/\gamma) \quad (2.21.1)$$

occurs when the condition

$$k_{\parallel}z - \omega_j t \pm n|\Omega|t = 0 \quad (2.21.2)$$

is satisfied for some integer $n \neq 0$, $j = \pm 1$ denoting forward and backward propagating waves. At small μ , less than the ratio Alfvén speed v_a over particle speed v , no transit-time damping (TTD) interaction ($n = 0$ resonance) is possible either, with the fast magneto-sonic component of the spectrum. As a consequence the quasi-linear theory, which only takes into account the resonant interactions (gyro-resonant and TTD), predicts a very low pitch-angle scattering and a very long mean-free-path along the direction of the magnetic field lines. The original quasi-linear prediction (Jokipii, 1966) gave a short parallel mean-free-path, but this was owing to the absence of cutoff in the turbulence spectrum. Latter measurements in the solar wind (Coroniti et al., 1982; Denskat et al., 1983) showed a strong steepening of the spectrum above the ion gyro-frequency $\Omega_{po} \approx 1$ Hz, with a spectral index γ going from $\gamma \approx -1.7$ to $\gamma \approx -2.9$, which is responsible for the ‘divergence’ of the mean-free-path below about 100 MV. Besides this problem of cutoff and divergence the original quasi-linear mean-free-path in fact never gave the right dependence of λ as a function of rigidity R : λ kept decreasing with decreasing R , whereas the observational data show little dependence of λ on R for rigidities between 10^{-1} and 10^3 MV, which is known as the ‘flatness problem’.

If it seems relatively clear that the problem of scattering through $\mu = 0$ arises from the exclusion of the lower-frequency waves, the precise, quantitative prediction of the parallel mean free path for solar CR or small energy galactic CR, and the solution of the ‘flatness problem’, require a more detailed description of the process of wave-particle interaction, and of the turbulence. We do *not* know for sure what the real composition of the turbulence is. However, it is likely that the wave turbulence is made of Alfvén and fast magneto-sonic waves, because in a magnetized, low but finite β plasma ($\beta = 2c_s^2/v_a^2$, c_s being the sound speed) like the solar wind, these two types of waves are the less heavily damped. As for the distributions of $\mathbf{k}$ -vectors for these waves, Ragot (1999a) make following assumptions: as in the papers by Schlickeiser and Miller (1998), Ragot and Schlickeiser (1998a,b) and Ragot (1999b), a *slab* Alfvén turbulence (parallel

propagating, with $\mathbf{k}$ along $\mathbf{B}_0$) and *isotropic* fast magneto-sonic waves. Ragot (1999a) presents the main lines of the derivation of the non-resonant pitch-angle scattering process with these waves, and shows that it very efficiently scatters the particles through $\mu = 0$. Detailed calculations for this process can be found in the paper by Ragot (1999b), and fits of the parallel mean free path as a function of the rigidity, deduced from measurements for solar CR, are presented in Ragot, 1999c (see below, Section 2.21.5). The case of oblique Alfvén waves is also briefly considered in Ragot (1999c). In Ragot (1999a) the slab Alfvén waves, for reasons of symmetry, do not contribute to the non-resonant interaction process. Ragot (1999a) thus ignores them in the evaluation of this effect.

2.21.2. Derivation of the non-resonant scattering process

According to Ragot (1999a) gyro-resonance is very inefficient at scattering low rigidity CR, because most of the energy is in the waves that have much *too low frequencies* to be in gyro-resonance with these particles. The limit between gyro-resonant and non gyro-resonant waves is given, for a linear dispersion relation of the waves $\omega_j = jk v_a$, valid well below $k_c = \Omega_{po}/v_a$, by:

$$\frac{k}{k_c} \approx \frac{v_a/c}{\max(|\eta\mu|, v_a/v)} \frac{m_p c^2}{ZeR} \quad (2.21.3)$$

which can be easily shown, for $\mu < v_a/v$, to be larger than 1 as soon as

$$R < 938 \left[1 + \left(m_p c^2 / ZeR \right)^2 \right]^{-1/2} \text{ MV}. \quad (2.21.4)$$

Consequently, the *equation of motion* for a particle of momentum $\mathbf{p}$, Lorentz factor γ , mass m , and charge q ,

$$\frac{d\mathbf{p}}{dt} = q\delta\mathbf{E} + \frac{q}{mc\gamma} \mathbf{p} \times (\mathbf{B}_0 + \delta\mathbf{B}) \quad (2.21.5)$$

can be averaged on the short timescale Ω^{-1} (Ragot, 1999b) and for fast magneto-sonic waves turbulence:

$$\delta\mathbf{E} = \sum_j \int d\mathbf{k} \frac{\omega_j}{kc} \delta B_{\mathbf{k}}^j \cos\psi_{\mathbf{k}} \begin{pmatrix} -\sin\phi_{\mathbf{k}} \\ \cos\phi_{\mathbf{k}} \\ 0 \end{pmatrix}, \quad \delta\mathbf{B} = \sum_j \int d\mathbf{k} \delta B_{\mathbf{k}}^j \cos\psi_{\mathbf{k}} \begin{pmatrix} -\eta \cos\phi_{\mathbf{k}} \\ -\eta \sin\phi_{\mathbf{k}} \\ \sqrt{1-\eta^2} \end{pmatrix} \quad (2.21.6)$$

with $\psi_{\mathbf{k}} = \mathbf{k}\mathbf{x} - \omega_{\mathbf{k}}t + \alpha_{\mathbf{k}}$, k the norm of the wave-vector, η the cosine of the angle between $\mathbf{k}$ and $\mathbf{B}_o$, and $\phi_{\mathbf{k}}$ the angle between $\mathbf{k}$ and the plane (x, z) , the z -axis being along $\mathbf{B}_o$. Once averaged, the equation for the pitch-angle cosine,

$$\dot{\mu} = \frac{\Omega_o}{\gamma} \sin \theta \sum_j \int d\mathbf{k} \frac{\delta B_{\mathbf{k}}^j}{B_o} \left(\eta - \frac{\mu m \gamma \omega^j}{pk} \right) \cos \psi_{\mathbf{k}}^j \sin(\varphi - \phi_{\mathbf{k}}), \quad (2.21.7)$$

becomes

$$\dot{\mu} = \frac{\Omega_o}{\gamma} \sin \theta \sum_j \int d\mathbf{k} \frac{\delta B_{\mathbf{k}}^j}{B_o} \left(\eta - \frac{\mu \omega^j}{vk} \right) J_1 \left(\frac{v\gamma \sqrt{1-\mu^2}}{|\Omega_o|} k_{\perp} \right) \sin(k\eta v \int \mu dt - \omega_{\mathbf{k}}^j t + \alpha_{\mathbf{k}}) \quad (2.21.8)$$

in terms of the ‘constant’ speed v and Lorentz factor γ (φ denotes the gyro-phase of the particle, and p and Ω_o , the norm of its momentum and non-relativistic gyro-frequency, respectively). Integrating the time-averaged Eq. 2.21.8 on a time-scale short enough to keep the particles rigidity constant, and in the range of pitch-angles $\mu < v_a/v$ where the scattering by resonant processes is known to be the most deficient, it can be shown that the particles are in fact linearly pushed out of the small μ range by the low-frequency waves. Indeed,

$$\mu - \mu_o \approx -\frac{\Omega_o}{\gamma} (t - t_o) \sum_j \int d\mathbf{k} \frac{\delta B_{\mathbf{k}}^j}{B_o} \eta J_1 \left(\frac{\gamma m k}{(v_a/v) m_p k_c} \sqrt{1-\eta^2} \right) \sin(\alpha'_{\mathbf{k}}^j) \quad (2.21.9)$$

with $\alpha'_{\mathbf{k}}^j = \alpha_{\mathbf{k}} + (\eta\mu/(v_a/v) - j)k v_a t_o$ constant on the time interval $[t_o, t]$, if we only keep in the spectrum the wave numbers smaller than $K_M = \min(K, \Delta k_F)$, K being the largest wave-number satisfying the condition: $t - t_o < (\pi/2)(k_c/K)(1/\Omega_{po})$, and Δk_F the actual width of the FMW spectrum. The contribution to the variation of μ from the wave-numbers larger than K is negligibly small, because it is given by the integral of an oscillating sine function of time and k . It was checked that there exists a time interval $t - t_o$ such that K is larger than the lower boundary of the spectrum, and $\mu - \mu_o \approx v_a/v$, *i.e.*, that the Eq. 2.21.9 holds until the particles leave the small- μ range. Once they have reached the boundary v_a/v , they are efficiently scattered away by the transit-time damping interaction with the fast magneto-sound waves (in accordance with Schlickeiser and Miller, 1998; Ragot, 1999b).

Averaging on many successive passages through this small- μ region, *i.e.*, over the phases $\alpha'_{\mathbf{k}}$, one can estimate the average exit time τ , and an equivalent

‘diffusion’ coefficient $D_{\mu\mu r} = (v_a/v)^2 / (2\tau)$. Assuming that the spectrum is a simple power law, i.e., $\propto k^{-q}$ above k_m , one can write:

$$\tau \approx \frac{\gamma(v_a/v)}{\delta b \Omega_o \sqrt{q-1}} \left(\frac{(v_a/v) m_p k_c}{\gamma m k_m} \right)^{(q-1)/2} (I_M - I_m)^{-1/2} \quad (2.21.10)$$

with

$$I_{m,M} = \int_0^1 d\eta (1-\eta^2)^{(q-1)/2} \eta^2 Z_{m,M} \int_0^{\sqrt{1-\eta^2}} dZ Z^{-q} J_1^2(Z), \quad (2.21.11)$$

where

$$Z_m = \frac{\gamma m k_m}{(v_a/v) m_p k_c} = \frac{RZe}{m_p c^2} \frac{k_m}{(v_a/c) k_c}; \quad Z_M = \frac{RZe}{m_p c^2} \frac{K_M}{(v_a/c) k_c}. \quad (2.21.12)$$

The corresponding parallel mean-free-path will be

$$\lambda_{nr} \approx \frac{3v_a}{4D_{\mu\mu r}(|\mu| < v_a/v)} \approx \frac{3}{2} \frac{v_a \tau(K_M(R))}{(v_a/v)^2} \approx \frac{3}{2k_c} \frac{\Omega_{po} \tau(K_M(R))}{(v_a/c)^2 \left[1 + (mc^2/RZe)^2 \right]}, \quad (2.21.13)$$

where it was assumed that the slowest scattering process still occurs at small μ . Ragot (1999) notes that despite the apparent dependence of the average exit time τ on the lowest wave number k_m , which is very badly known, there is no problem of lower cut-off value. As was shown in Fig. 2.21.1, the lowest wavenumbers only give a negligible contribution at low rigidities, and the scattering process is dominated by higher and higher frequencies as the rigidity of the particles decreases.

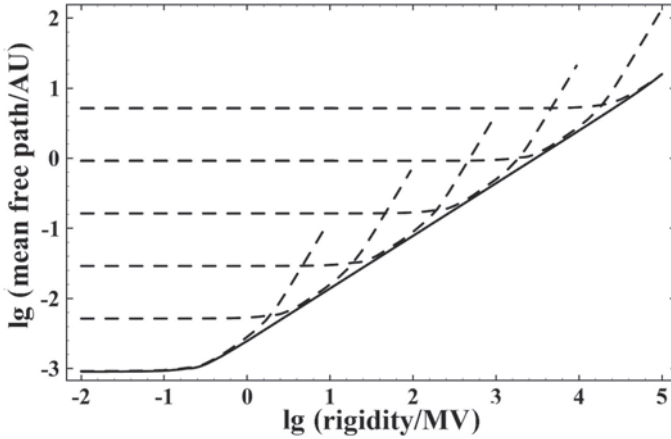


Fig. 2.21.1. Logarithm of the non-resonant mean free path λ_{nr} for $\Delta k_F = k_c$. The contribution from each 'decade' of the spectrum is given in dashed line starting, in the top of the figure, with the wavenumber interval $10^{-6}k_c$ to $10^{-5}k_c$. According to Ragot (1999a).

2.21.3. Resulting mean free path and comparison with gyro-resonant model

By the comparison of the found mean-free-path λ_{nr} with the one resulting from gyro-resonance with slab Alfvén waves (see Fig. 2.21.2), Ragot (1999a) shows that the non-resonant scattering process becomes much more efficient than the gyro-resonant one below about 100 MV, *even* in the absence of spectral cut-off.

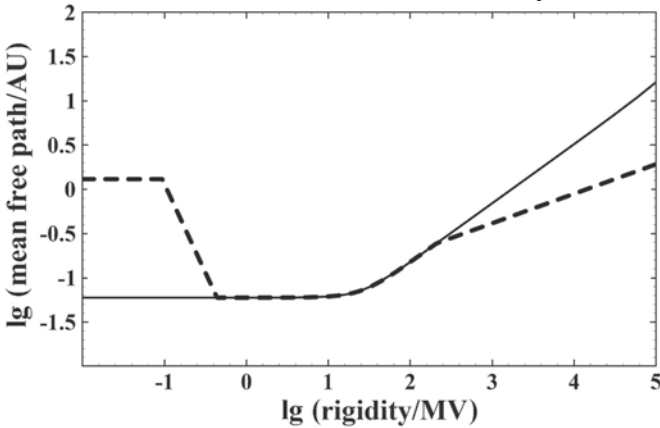


Fig. 2.21.2. Mean free path λ , for $\Delta k_F = k_c/100$ and $\Delta k_a \leq k_c$. Continuous line: λ_{nr} ; thick dashed line: effective λ . Above a few 100 MV, λ results from gyro-resonance at $\mu < v_a/v$ with Alfvén waves. Below 1 MV, for electrons, it is determined by TTD at $\mu \geq v_a/v$. From Ragot (1999a).

Further comparison with the mean-free path derived from the smaller pitch-angles indicates that the slowest scattering process does not occur around $\mu = 0$ any longer for rigidities less than ≈ 1 MV. This again results from the upper steepening of the spectrum. Indeed, when the particles' rigidity is really low, gyro-resonance also becomes impossible around $|\mu|=1$. As the transit-time damping interaction, which is owed to the compressive component of the magnetic field – along $\mathbf{B}_0$ – is very inefficient at these large $|\mu|$, it produces a relatively large mean-free-path. Note that this mean-free-path is constant below ≈ 0.1 MV. Ragot (1999a) notes also that the developed theory can reproduce the main features of the parallel mean free path as a function of the particles rigidity – for electrons, down to 1 keV, and for protons, down to 1 or 10 MV. The flatness of the curve between 1 MV and 10^3 MV seems to require a steepening of the fast mode component of the turbulence spectrum above about $10^{-2}k_c$. This is, in the presence of low-energy CR, plausible, given that the fast magneto-sonic waves in the range $[10^{-2}k_c, k_c]$ give the main contribution to the transit-time damping acceleration process.

2.21.4. Contribution from slab and oblique Alfvén waves to the non-resonant pitch-angle scattering

In Ragot (1999c) was shown that the slab Alfvén component of the spectrum, for reasons of symmetry, does not contribute to the non-resonant scattering and also oblique Alfvén waves do not produce any significant scattering by the non-resonant scattering process. In a turbulence of slab Alfvén waves the fluctuating fields consist of transversal left- and right-hand polarized waves propagating parallel and anti-parallel to the homogeneous magnetic field $\mathbf{B}_0$. The polarization of the waves is circular. It follows that the integral (over $\mathbf{k}$) describing the variations of μ has an oscillatory integrand in $\exp(\pm i(\psi_{\mathbf{k}} + \varphi))$, where $\psi_{\mathbf{k}} = \mathbf{k}\mathbf{x} - \omega_{\mathbf{k}}t + \alpha_{\mathbf{k}}$ and φ is the gyro-phase of the particle. As a consequence, averaging over the particle gyro-period – the shortest timescale of the problem, see Ragot (1999a,b) – will just reduce the μ variation to a negligible contribution. The case of oblique Alfvén waves is close to the one of the oblique fast magneto-sonic waves presented by Ragot (1999a,b). When $\mathbf{k}$ is not along the main magnetic field $\mathbf{B}_0$, the Alfvén waves are linearly polarized waves with, if the inertia of the electrons is neglected, an electric field $\delta\mathbf{E}$ normal to $\mathbf{B}_0$, in the plane of $\mathbf{k}$ and $\mathbf{B}_0$. The magnetic field $\delta\mathbf{B}$ is normal to $\delta\mathbf{E}$ and $\mathbf{B}_0$. The different configuration of the magnetic field perturbation results here in an equation for the pitch-angle cosine μ of a form similar to Eq. 2.21.7, but where $\cos\psi_{\mathbf{k}}^j \cos(\varphi - \varphi_{\mathbf{k}})$ substitutes for $\cos\psi_{\mathbf{k}}^j \sin(\varphi - \varphi_{\mathbf{k}})$, $\varphi_{\mathbf{k}}$ being the angle between $\mathbf{k}$ and the plane (x, z) , with a z -axis along $\mathbf{B}_0$. The averaging of this equation over

the particle gyro-period will not permit to extract any constant term of significant amplitude, on the expected timescale of pitch-angle variation. Indeed, an expansion of $\cos\psi_k^j \sin(\varphi - \varphi_k)$ in Bessel functions only displays oscillatory terms in $\cos(\varphi - \varphi_k)$ or $\sin(\varphi - \varphi_k)$ with n a strictly positive integer. This, according to Ragot (1999c), shows that the contribution from oblique Alfvén waves to the non-resonant pitch-angle scattering is also negligible.

2.21.5. Parallel mean free path: comparison of the theoretical predictions with the measurements

If the Alfvén waves, as argued in the Section 2.21.4, do not produce any significant contribution to the pitch-angle scattering by the non-resonant effect, it means that the result obtained by Ragot (1999a,b) might already provide with a reasonable description of the CR scattering in the solar wind. Ragot (1999c) tried to compare her theoretical predictions with the measurements. The sensitivity of the mean free path to the characteristics of the fast magneto-sonic waves spectrum (in particular, spectral index and cutoff wave number) and the fact that the data obtained from different solar events are often presented together, without reference to the distinct events, makes this comparison difficult (see Fig. 2.21.3).

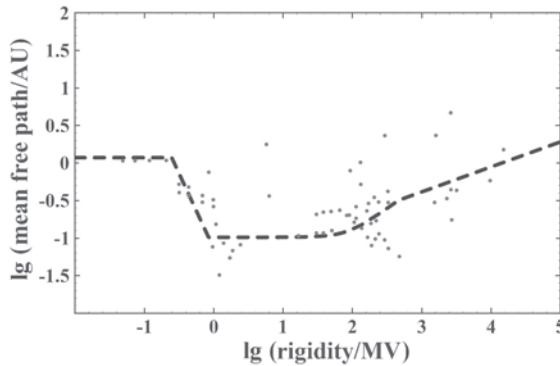


Fig. 2.21.3. Parallel mean free path versus particle rigidity, in logarithm, for various solar events. The dots represent parallel mean free paths derived above 10 MV from proton observations, and below from electron observations, as published by Bieber et al. (1994). The theoretical curve, in the dashed line, has been obtained with cutoff of the Alfvén and fast magneto-sonic waves spectra at $0.4k_c$ and $0.003k_c$, respectively; $\delta b_a = 0.1$, $\delta b_F = 0.13$, and $v_a = 10^{-4}c$. The extension of the plateau at very low rigidities is directly related to the cut-off wavenumber of the Alfvén spectrum. This cut-off value is observed in the solar wind at about k_c . A value of $0.4k_c$ to produce the best fit presented here is reasonable, since the precise characteristics of the turbulence spectra might vary during a solar event from those of the ‘quiet’ solar wind. From Ragot (1999c).

From Fig. 2.21.3 it can be seen that the theoretical curve globally fits the data points. The dispersion of the points around the theoretical curve presented on Fig.

2.21.3 should not be interpreted as uncertainty of the measurements, or inappropriateness of the theory to fit all the data. The data shown on Fig. 2.21.3 have been obtained from many different solar events. Their dispersion only indicates that the turbulence spectrum in the solar wind varies from one event to another. Ragot (1999c) has studied how the theoretical prediction is modified by variations of the turbulence spectrum, both fast magneto-sonic and Alfvén component. There was found a rather strong sensitivity of the theoretical prediction on the precise shape of the spectra. Even if the main features of the curve in Fig. 2.21.3 are preserved (e.g., separation in three domains where the transit-time damping, non-resonant and gyro-resonant interactions successively determine the parallel mean free path), it is always possible to find a curve which will fit one subset of data points, keeping reasonable turbulence spectra. Ragot (1999c) fits in Fig. 2.21.4 one particular event, namely Nov 22, 1977, which looks very similar to Dec 27, 1977, and Apr 11, 1978 (see Beeck et al., 1987; Valdes-Galicia et al., 1988; Dröge et al., 1993). All the measurements, for this particular event, appear to be in the range where the non-resonant interaction with the fast magneto-sonic waves dominates. It would be necessary, in order to validate the theory and obtain the whole information on the turbulence spectra, to have data for single events over a broader range of rigidities, spanning the intervals where the transit-time damping (below 1 MV) and gyro-resonant (above 10^3 MV) interactions determine the parallel mean free path.

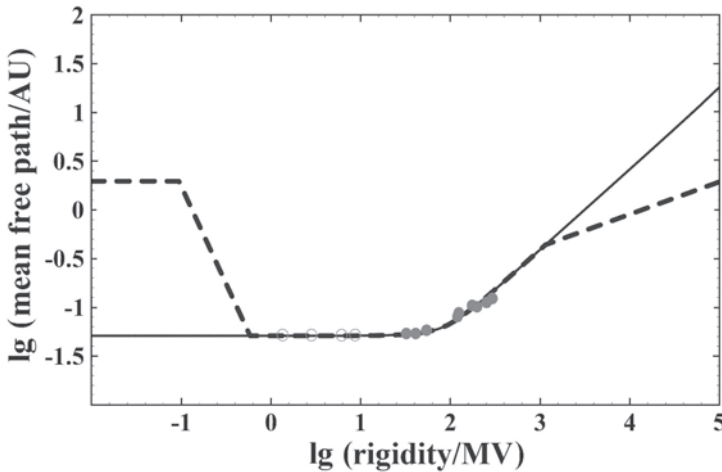


Fig. 2.21.4. Parallel mean free path versus particle rigidity for the solar event of Nov 22, 1977 measured by Helios-1 (Beeck et al., 1987; Valdes-Galicia et al., 1988; Dröge et al., 1993). From Ragot (1999c).

The circles represent measurements for electrons, and the disks for protons. The theoretical curve remains valid on the whole range of rigidities for electrons. It only holds above about 10 MV for protons, but all the data for protons are

obtained above 20 MV, so the theory is consistent with the observations presented in this figure and Fig. 2.21.3. The measurements for this particular event appear to be in the range where the non-resonant interaction with the fast magneto-sonic waves dominates. The theoretical curve in thick dashed line is calculated with an Alfvén spectrum of Kolmogorov type up to k_c , and a fast mode wave spectrum damped above $3.2 \times 10^{-3} k_c$, with a spectral index of 1.35 below. The continuous line plots the mean free path resulting from the non-resonant interaction alone, assuming that the slowest scattering process occurs at small μ . From Ragot (1999c).

2.22. On the cosmic ray cross-field diffusion in the presence of highly perturbed magnetic fields

2.22.1. *The matter of the problem*

According to Michałek and Ostrowski (1999), the investigation of CR transport in highly perturbed magnetic fields raises a number of issues which are poorly understood. In particular, an analytic theory enabling derivation of particle diffusion across the magnetic field is still not available. The quantitative analytical derivations of the cross-field diffusion coefficient $\kappa_{\perp}$ in turbulent magnetic fields are limited to small perturbation amplitudes, $\delta B \ll B_0$ (e.g. Jokipii, 1971; Achterberg and Ball, 1994). A significant result in this respect was achieved by Giacalone and Jokipii (1994), Jones et al. (1998). They provided a proof that the cross-field diffusion requires a three-dimensional nature of the turbulent field. A process of particle cross-field diffusion in high amplitude Alfvénic turbulence is considered in Michałek and Ostrowski (1997, 1998, 1999) using the Monte Carlo particle simulations. They derive the cross-field diffusion coefficient $\kappa_{\perp}$ in the presence of different 1-D, 2-D and 3-D turbulent wave field models. Vanishing of $\kappa_{\perp}$ in 1-D turbulence models is used as an accuracy check for the numerical computations. They found substantial differences in the cross-field diffusion efficiency at the same perturbation amplitude, depending on the detailed form of the turbulent field considered. Michałek and Ostrowski (1997, 1998, 1999) reproduced the expected increase of $\kappa_{\perp}$ with the growing power of waves propagating perpendicular to $\mathbf{B}_0$. Substantially larger values of $\kappa_{\perp}$ appear in the presence of long compressive fast-mode waves in comparison with the Alfvén waves. This result was interpreted in terms of particle drifts in non-uniform magnetic fields. In some cases an initial regime of sub-diffusive transport appears in the simulations.

2.22.2. *Description of Monte Carlo particle simulations*

Michałek and Ostrowski (1997, 1998, 1999) considered an infinite region of tenuous plasma with a uniform mean magnetic field along the z -axis. It is

perturbed by propagating MHD waves (described below in Section 2.22.3 for different turbulence models). Test particles are injected at random positions into this turbulent magnetized plasma and their trajectories are followed by integrating particle equations of motion in space and momentum. By averaging over a large number of trajectories one derives the required diffusion coefficients for turbulent wave fields. In the simulations 500 relativistic particles were used with the same initial velocity $v_{in} = 0.99c$ in an individual run.

2.22.3. Wave field models

For high amplitude waves there are no analytic models available reproducing the turbulent field structure. Because of that, in Michałek and Ostrowski (1997, 1998, 1999) approximate models representing such fields are considered, with turbulence represented as a superposition of Alfvén or fast-mode waves. The wave parameters (wave vectors $\mathbf{k}$, wave amplitudes δB_o and initial phases ϕ) are drawn in a random manner from the flat ($F(k) \propto k^{-1}$) or the Kolmogorov ($F(k) \propto k^{-5/3}$) wave spectra. The wave vectors are expressed in units of the ‘resonance’ wave vector

$$k_{res} = 2\pi / r_g \langle B \rangle / p_o \quad (2.22.1)$$

for the injected particle with momentum $p = p_o$ in the mean magnetic field

$$\langle B \rangle = (B_o^2 + \delta B^2)^{1/2}. \quad (2.22.2)$$

The wave vectors are selected from the range $0.08 k_{res} < k < 8.0 k_{res}$. Integration time is expressed in units of $\Omega_o = eB_o / \gamma mc$. The magnetic field fluctuation vector related to the wave ‘ i ’, $\delta \mathbf{B}^{(i)}$, is given in the form:

$$\delta \mathbf{B}^{(i)} = \delta \mathbf{B}_o^{(i)} \sin(k^{(i)} \mathbf{r} - \omega^{(i)} t - \phi^{(i)}). \quad (2.22.3)$$

In Michałek and Ostrowski (1997, 1998, 1999) are considered the following turbulence models:

(i) Linearly polarized plane waves (model A)

In this model are considered superposition of plane Alfvén waves propagating with the same intensity along the z -axis, in the positive (forward) and the negative (backward) direction.

(ii) ‘Wave-packets’ models (two models B1 and B2)

It was proposed a simple extension of the above model A to three dimensions by considering wave packets, involving wave modulation in one direction

perpendicular to the propagation direction by using Eq. 2.22.3 for $\delta\mathbf{B}^{(i)}$, where the phase parameter is subject to sinusoidal modulation. Two types of modulation (presented for the x -components in Eq. 2.22.3) are considered: **model B1** with the ‘smooth’ sinusoidal modulation characterized by

$$\phi_x^{(i)}(y) = \sin(k_y^{(i)}y), \quad (2.22.4)$$

and **model B2** with the ‘sharp-edged’ modulation characterized by

$$\phi_x^{(i)}(y) = y \bmod (1/k_y^{(i)}). \quad (2.22.5)$$

The y -components can be obtained from Eq. 2.22.4 and Eq. 2.22.5 by interchanging x and y . Vectors $k_x^{(i)}$ and $k_y^{(i)}$ are drawn in a random manner from the respective wave-vector range for $\mathbf{k}^{(i)}$.

(iii) Oblique MHD waves (four models C-AF, C-AK, C-MF, and C-MK)

There was considered a superposition of plane MHD waves propagating obliquely to the average magnetic $\mathbf{B}_o \equiv B_o \mathbf{e}_z$. The wave propagation angle with respect to $\mathbf{B}_o$ is randomly chosen from a uniform distribution within a cone (‘wave-cone’) along the mean field. For a given simulation two symmetric cones are considered centered along $\mathbf{B}_o$, with the opening angle 2α , directed parallel and anti-parallel to the mean field direction. The same number of waves is selected from each cone in order to model the symmetric wave field. For the model (iii) four different turbulent fields were considered characterized with parameters α and δB and labeled as follows:

- Alfven waves with the flat wave spectrum (**model C-AF**),
- Alfven waves with the Kolmogorov spectrum (**model C-AK**),
- Fast-mode magneto-sonic waves with the flat spectrum (**model C-MF**),
- Fast-mode magneto-sonic waves with the Kolmogorov spectrum (**model C-MK**).

2.22.4. Simulations for Alfvenic turbulence models A, B1, B2

Examples of the derived formal (e.i., the derived particle dispersion squared and divided by the integration time multiplied by two) cross-field diffusion coefficients versus the integration time are presented for the considered Alfvénic turbulence **models A, B1, and B2** in Fig. 2.22.1.

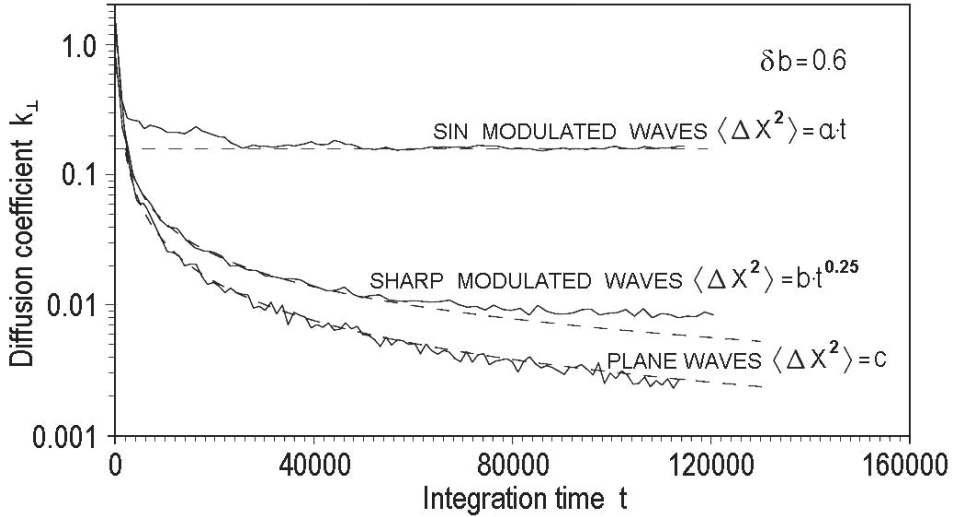


Fig. 2.22.1. Examples of simulated $\kappa_{\perp}$ versus the integration time t . The power-law fits of the cross-field particle dispersion $\langle \Delta x^2 \rangle$ are presented for the **model A** and for the initial part of the curve for the **model B2** (A , B and C are constants). A constant fit is provided for the **model B1**. From Michałek and Ostrowski (1999).

For the one dimensional plane wave **model A** one can note that (as required by Giacalone and Jokipii, 1994) the cross-field diffusion coefficient falls off as $\propto t^{-1}$, as expected for a particle dispersion constant in time. An important feature seen in Fig. 2.22.1 is that the value of $\kappa_{\perp}$ depends substantially on the assumed shape of the magnetic field perturbations. For the same amplitude and a ‘similar’ form of modulation applied in **models B1** and **B2** the diffusion coefficient values can differ by more than an order of magnitude. For the **model B2** (sharp-edge modulated waves), the regime of sub-diffusive transport across the mean magnetic field is discovered on a short time scale, with $\kappa_{\perp}$ slowly decreasing in the beginning and it approaches a constant value at large t . This time evolution of particle spatial cross-field dispersion differs from the one expected for the ordinary diffusion, with a short initial free-streaming followed by the phase with $\kappa_{\perp}$ fluctuating near some constant value. The observed behavior reflects the restraining influence of stochastic particle trapping by large amplitude magnetic waves. When inspecting the initial part of the curve for the **model B1**, one observes in a narrow time range an analogous sub-diffusive evolution of particle distribution. In this case the ordinary particle cross-field diffusion is much larger than in the case **B2** and particles decorelate from any given ‘trap’ much earlier. Numerical experiments performed by Michałek and Ostrowski (1997, 1998, 1999) proved that the phenomenon is caused by the long distance correlations introduced

by the longest waves. In Fig. 2.22.2 are presented the simulated values of $\kappa_{\perp}$ versus the wave amplitude δB .

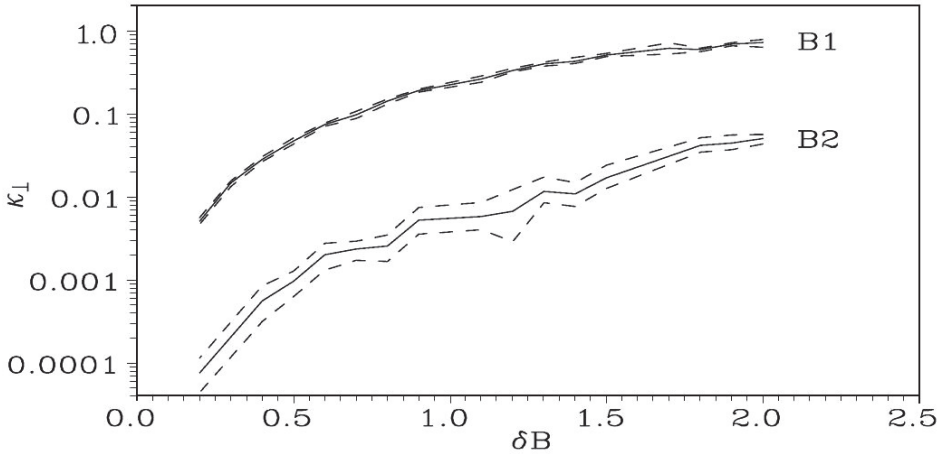


Fig. 2.22.2. The simulated values of $\kappa_{\perp}$ versus δB for the wave **models B1 and B2**. Solid lines join the results obtained using fitting procedure. The adjacent dashed lines provide information about errors as they join the maximum (or respectively minimum) values of the quantity measured within the range used for fitting. From Michałek and Ostrowski (1999).

For the 3-D turbulence models considered Michałek and Ostrowski (1997, 1998, 1999) proved the possibility of substantial (by more than one order of magnitude at the same δB) difference in $\kappa_{\perp}$ between at first glance similar turbulence models. Such a difference does not disappear for $\delta B \geq 1$. The reason for this difference is a more uniform modulation pattern in model **B2** with respect to **B1**. The value of $\kappa_{\perp}$ is closely related to the value of the magnetic field line diffusion coefficient D_m (Michałek and Ostrowski, 1997), but the growth of the wave amplitude is accompanied by a slight increase in the ratio of $\kappa_{\perp}/D_m$. This corresponds to the relative increase of the particle cross-field scattering owed to particle-wave interactions relative to the diffusion caused by magnetic field line wandering.

2.22.5. Simulations for oblique MHD waves models C-AF, C-AK, C-MF, and C-MK

The derived values of $\kappa_{\perp}$ for different wave-cone opening angles and for different turbulence amplitudes are presented in Fig. 2.22.3. For the flat spectrum turbulence a systematic increase of $\kappa_{\perp}$ with amplitude occurs and the rate of this increase roughly scales as δB^2 . The value of $\kappa_{\perp}$ at any given δB is a factor ~ 10

larger for the fast-mode waves in comparison to the Alfvén waves. It grows substantially with the increasing wave cone opening α , i.e. with increasing power of waves perpendicular to the mean magnetic field. For the Kolmogorov spectrum a dependence of $\kappa_{\perp}$ on the perturbation amplitude is flatter, the values of the cross-field diffusion coefficient at small δB are larger and there is a smaller difference between the fast-mode waves and the Alfvén waves.

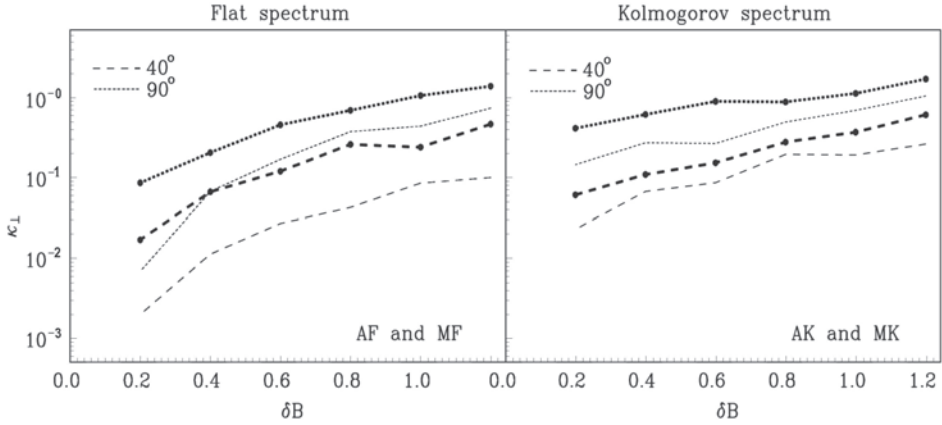


Fig. 2.22.3. Variation of the cross-field diffusion coefficient $\kappa_{\perp}$ versus the perturbation amplitude δB and the wave propagation anisotropy (angle α) for the flat spectrum and the Kolmogorov spectrum. Results for the Alfvén turbulence (thin lines) and the fast-mode turbulence (thick lines with indicated simulation points) are superimposed on the same panels. From Michałek and Ostrowski (1999).

The characteristic features seen in Fig. 2.22.3 can be qualitatively explained with the use of simple physical arguments discussed by Michałek and Ostrowski (1998). Results presented there show much larger increases of respective D_m than $\kappa_{\perp}$. It proves that in the range of δB considered here $\kappa_{\perp}$ is in a substantial degree controlled by the cross-field drifts and the resonance cyclotron scattering, and not by the field line diffusion. Michałek and Ostrowski (1999) stressed that the substantial cross-field shifts accompany wave particle interaction involving the so called ‘transit time damping resonance’, where for the effective cross-field drift, the particle velocity $v_{\parallel}$ and the wave phase velocity $V_{\parallel}$ along the mean field are approximately equal ($V_{\parallel} = v_a$ for the Alfvén waves and $V_{\parallel} = v_a(k/k_{\parallel})$ for the magneto-sonic fast-mode waves). For $v_a = 10^{-3}c$ and $v = 0.99c$ considered in the described simulations a noted difference between $\kappa_{\perp}$ for the Alfvén and the fast-mode waves occurs as a result of satisfying the resonance condition in a wider range of $v_{\parallel}$ by oblique fast-mode waves. Another difference arises from the linear compressive terms occur only in fast-mode waves. It enables for gradient drifts at

small perturbation amplitudes and enables particle cross-field transport when interacting with long waves.

2.23. Dispersion relations for CR particle diffusive propagation

2.23.1. The matter of the problem and denominations

Kóta (1999) introduced and evaluated the dispersion relations for CR particles diffusive propagation. He presented illustrative examples for cases including dominant helicity, focusing, and hemispherical scattering. It was shown that the dispersion relations can be quickly computed and can be a useful diagnostic tool for exploring the validity range of various approximations. The matter of the problem is as following.

The evolution of the distribution function $f(z, \mu, t)$ for CR particle diffusive propagation in time t , space z , and cosine of pitch-angle μ is governed in the simplest rectilinear geometry by the equation

$$\frac{\partial f}{\partial t} + v\mu \frac{\partial f}{\partial z} = \frac{\partial}{\partial \mu} D_{\mu\mu} \frac{\partial f}{\partial \mu}, \quad (2.23.1)$$

where $D_{\mu\mu}$ is the pitch-angle scattering coefficient. Eq. 2.23.1 is often approximated by the diffusion equation which operates with the omni-directional density $f_o(z, t) = \langle f(z, \mu, t) \rangle$ only ($\langle \rangle$ indicates average over μ). The diffusion model is inaccurate for short times and fails to describe the early phase of SEP events. Several efforts have been made to improve the diffusion model. Fisk and Axford (1969) introduced the telegrapher's equation. According to Kóta (1999), the modified equations can be written in the general form of

$$\alpha \frac{\partial^2 f_o}{\partial t^2} + \Lambda \frac{\partial^2 f_o}{\partial z \partial t} + \frac{1}{\tau} \frac{\partial f_o}{\partial t} = \frac{1}{A} \frac{\partial}{\partial z} \left(\frac{Av^2}{3} \frac{\partial f_o}{\partial z} \right) = \frac{v^2}{3} \left(\frac{\partial^2 f_o}{\partial z^2} + \frac{1}{L} \frac{\partial f_o}{\partial z} \right), \quad (2.23.2)$$

where $\alpha = 0$, $\Lambda = 0$ corresponds to the standard diffusion equation, $\alpha = 1$, $\Lambda = 0$ yields the telegrapher's equation. Pauls et al (1993) suggested a cross-derivative term ($\Lambda \neq 0$) to account for a possible dominant helicity in the random component of the Heliospheric magnetic field. Gombosi et al (1993) pointed out that a modified α , which depends on the actual form of $D_{\mu\mu}$ gives better approximation. The right hand side of Eq. 2.23.2 includes adiabatic focusing owed to the possible divergence of field lines ($A(z)$ is the area element, and L is the focusing length, with $1/L = \partial \ln A / \partial z$).

One way to explore these approximations is to look at the dispersion relations they yield. Kóta (1999) supposes to consider the solution for $f(z, \mu, t)$ as a sum of eigenfunctions $F(k, \mu) \exp(ikz - \nu t)$. Eq. 2.23.2 then transforms into

$$\alpha \nu^2 + i\Lambda k \nu - \nu/\tau = -\nu^2 (k^2 - ik/L)/3. \quad (2.23.3)$$

2.23.2. Dispersion relations for diffusion and telegrapher's equations

The resulting dispersion relations $\nu(k)$, evaluated in Kóta (1999) from Eq. 2.23.3, are shown in Fig. 2.23.1. Here there is covered only the half plane, obviously $\text{Re}(\nu)$ is an even and $\text{Im}(\nu)$ is an odd function of k . To use dimensionless quantities, Kóta (1999) takes the particle speed, v , and the scattering time, τ , to be unity ($v = \tau = 1$). These dispersion relations of the approximations can then be compared to those obtained from the full Eq. 2.23.1. In general, the full equation has an infinite number of eigenfunctions and eigenvalues (see Earl, 1974). In Kóta (1999) it was focused on the two lowest eigenvalues which are the most important in determining the evolution of the particle density and anisotropy. Clearly the value of α appears in $\nu_1(k=0)$, whilst a non-zero Λ would appear as a non-zero (imaginary) value of $d\nu_1/dk$ at $k=0$. The dispersion relations for the 'billiard-ball' scattering were first given by Fedorov and Shakhov (1993). 'Hemispherical' scattering was considered by Kóta (1994).

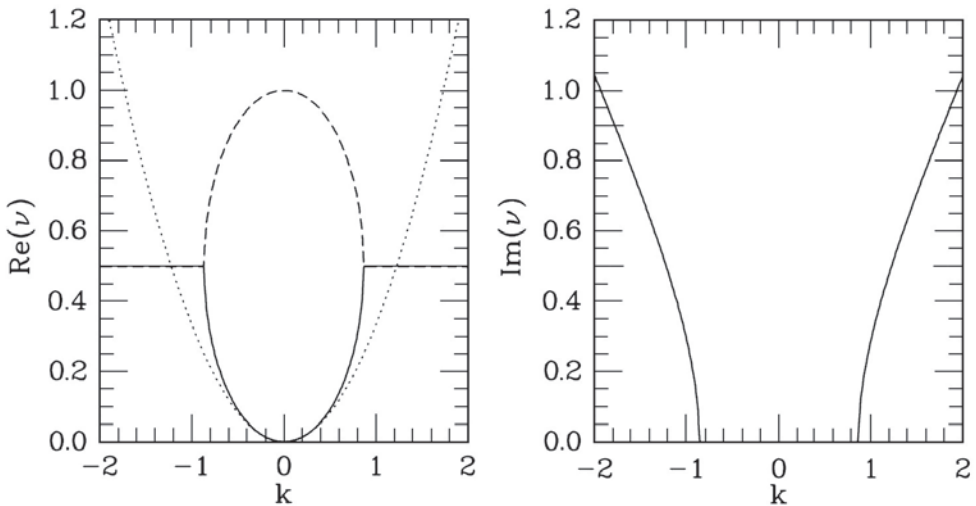


Fig. 2.23.1. Dispersion relations for the diffusion (dotted line) and the telegrapher's equation (solid lines). From Kóta (1999).

2.23.3. Dispersion relations in general case

For the sake of simplicity Kóta (1999) assume that both $D_{\mu\mu}$ and the focusing length L are independent of location, which corresponds to an exponentially diverging geometry (according to Earl, 1981). The pitch-angle scattering coefficient $D_{\mu\mu}$ is allowed to be arbitrary function of μ , so it was assumed that

$$D_{\mu\mu} = \frac{v(1-\mu^2)}{2\lambda(\mu)} = \frac{(1-\mu^2)\mu^q}{(1\pm\sigma)(1-q)(3-q)\tau}. \quad (2.23.4)$$

In this formulation σ accounts for helicity (Bieber et al., 1987) and τ represents the effective scattering time so that the resulting spatial diffusion coefficient along the magnetic field lines will be

$$\kappa_{//} = v^2\tau/3 \quad (2.23.5)$$

according to Hasselman and Wibberenz (1970). The Fokker-Planck equation including focusing can then be rewritten as

$$\frac{\partial f}{\partial t} + v\mu \frac{\partial f}{\partial z} = -\frac{v(1-\mu^2)}{2L} \frac{\partial f}{\partial \mu} + \frac{\partial}{\partial \mu} D_{\mu\mu} \frac{\partial f}{\partial \mu} = e^{G(\mu)} \frac{\partial}{\partial \mu} \left(e^{-G(\mu)} D_{\mu\mu} \frac{\partial f}{\partial \mu} \right), \quad (2.23.6)$$

where $\partial G/\partial \mu = \lambda(\mu)/L$ (Kunstmann, 1979). In terms of the eigenfunctions $F(k, \mu)$ reads as

$$-vF + ikv\mu F - e^{G(\mu)} \frac{\partial}{\partial \mu} \left(e^{-G(\mu)} D_{\mu\mu} \frac{\partial F}{\partial \mu} \right) = 0. \quad (2.23.7)$$

The eigenvalues $v = v_j(k)$ ($j = 0, 1, 2, \dots$) are complex in general. Slow spatial variation corresponds to $k \approx 0$. At $k = 0$ the lowest eigenvalue is always $v_0 = 0$, corresponding to the completely homogeneous and isotropic solution, and all the other eigenvalues are real and the eigenfunctions are identical with the eigenfunctions of the scattering operator (Earl, 1974). Moving from k to $k + \delta k$ the eigenfunctions and eigenvalues change to $F + \delta F$ and $v + \delta v$, yielding

$$-v\delta F + ikv\mu\delta F - e^{G(\mu)} \frac{\partial}{\partial \mu} \left(e^{-G(\mu)} D_{\mu\mu} \frac{\partial \delta F}{\partial \mu} \right) = \delta vF + i\delta k v\mu F. \quad (2.23.8)$$

Multiplying Eq. 2.23.8 by $F(k, \mu)$ and integrating over μ , the left hand side should vanish

$$\int_{-1}^{+1} \left(-\delta v F^2 + i \delta k v \mu F \right) e^{-G(\mu)} d\mu = 0, \quad (2.23.9)$$

hence the variation of $v(k)$ is given by

$$\frac{dv}{dk} = iv \frac{\left\langle \mu e^{-G} F^2 \right\rangle}{\left\langle e^{-G} F^2 \right\rangle}. \quad (2.23.10)$$

The derivative dv/dk is the group velocity, which can be associated with the coherent propagation speeds while the second derivative d^2v/dk^2 is characteristic of the dispersion and can be associated with the diffusion coefficients. It can be shown that for the rectilinear case d^2v/dk^2 at $k = 0$ exactly returns the diffusion coefficient derived by Hasselman and Wibberenz (1970). Kóta (1999) presents some examples to illustrate the method for various kinds of scattering. It was assumed that the dependence of $D_{\mu\mu}$ was as given in Eq. 2.23.6. Clearly, $q = 0$, $\sigma = 0$ describes isotropic scattering, $\sigma \neq 0$ implies dominant helicity, whilst $q \approx 1$ represents hemispherical scattering. Kóta (1999) consider both rectilinear and focusing geometries with a constant focusing length L .

2.23.4. Dispersion relations for isotropic pitch-angle scattering

The simplest scattering is isotropic pitch-angle scattering. Then the eigenfunctions at $k = 0$ are the spherical harmonics, whilst the eigenvalues are $v_j = j(j+1)/2\tau$ ($j = 0, 1, 2, \dots$). The variations of $v_o(k)$ and $v_1(k)$ as function of k are shown in Fig. 2.23.2.

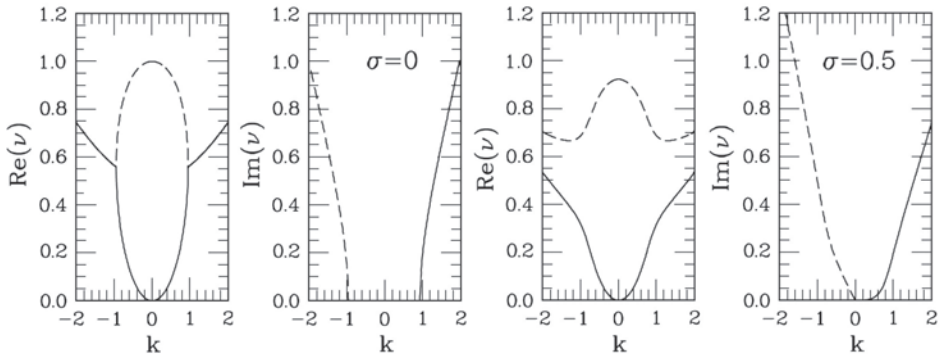


Fig. 2.23.2. $v_o(k)$ (solid line) and $v_1(k)$ (dashed line) for isotropic pitch-angle scattering without a dominant helicity ($\sigma = 0$, left) and with a dominant helicity ($\sigma = 0.5$, right). From Kóta (1999).

From Fig. 2.23.2 can be seen that for isotropic pitch-angle scattering the general pattern is similar to that of the telegrapher's equation (compare with Fig. 2.23.1), but there are noticeable differences at the same time.

2.23.5. Dispersion relations for the cases with dominant helicity

Bieber et al. (1987) called attention to the possible role of a dominant helicity, which introduces an asymmetry into $D_{\mu\mu}$. Fig. 2.23.3 shows the dispersion relations for a dominant helicity characterized by $\sigma = 0.5$.

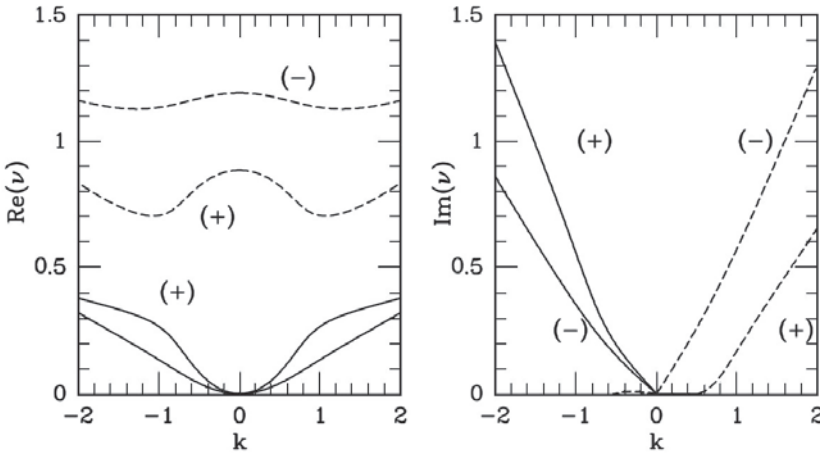


Fig. 2.23.3. Dispersion relations for a focusing geometry ($L = 1$), with $\sigma = +0.5$ (+) and $\sigma = -0.5$ (-) helicities. According to Kóta (1999).

Kóta (1999) notes that the imaginary part of the derivative dv_1/dk becomes finite at $k = 0$ in accord with the predictions of Eq. 2.23.2 for a non-zero value of Λ (Pauls et al., 1993).

2.23.6. Dispersion relations for focusing scattering

Adiabatic focusing becomes important when field lines diverge on a scale comparable with or smaller than the scattering mean free path. According to Kóta (1999) focusing appears in Eq. 2.23.8 through the function $G(\mu)$. In a focusing geometry, Eq. 2.23.3 suggests that κ can be obtained as $\kappa = -iLdv_o/dk$ at $k = 0$. Since the zeroes eigenfunction F_o is always constant at $k = 0$, Eq. 2.23.10 immediately leads to

$$\kappa = -vL \left\langle \mu e^{-G} \right\rangle \left/ \left\langle e^{-G} \right\rangle \right. . \quad (2.23.11)$$

which is identical to the expression inferred by Bieber and Burger (1990) using a Born approximation. Bieber et al. (1987) pointed out that the combined effect of

focusing and dominant helicity leads to charge dependence in κ . This effect is clearly demonstrated in the dispersion relations shown in Fig. 2.23.3 for a focusing length $L = 1$, and helicities $\sigma = 0.5$ and $\sigma = -0.5$. Both the curvature of $\text{Re}(v_o)$ and the slopes of $\text{Im}(v_o)$ at $k = 0$ indicate different effective diffusion coefficients for the two different signs of helicity.

2.23.7. Dispersion relations for hemispherical scattering

A case of particular importance is the hemispherical scattering when particles are strongly scattered both in the $\mu < 0$ and $\mu > 0$ hemispheres but scattering through $\mu = 0$ is restricted. Such a case is described, for instance, by $q \approx 1$ or, in another formulation, by the introduction of two distinct levels f_+ and f_- for the two hemispheres. The equations for $f_{\pm}$ have been developed and discussed in detail by Isenberg (1997) and Schwadron (1998). Fig. 2.23.4 shows the dispersion relations for $q = 0.9$ for a rectilinear case, without focusing (left panels), and for a focusing scenario ($L = 1$).

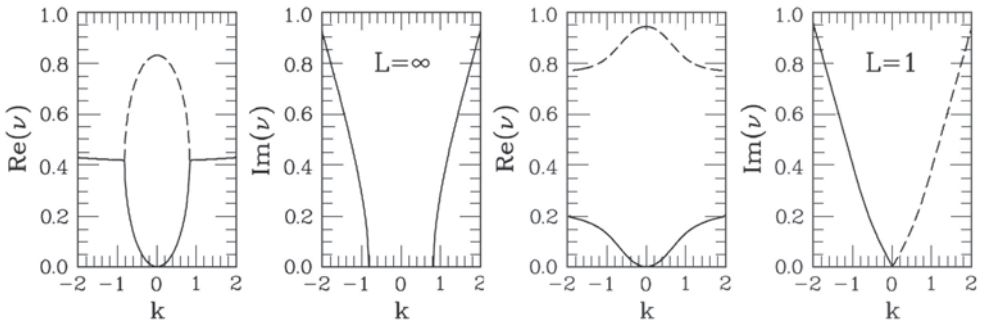


Fig. 2.23.4. Hemispherical scattering ($q = 0.9$) for rectilinear (left) and focusing ($L = 1$, right) geometries. According to Kóta (1999).

From Fig. 2.23.4 it can be seen that for the rectilinear case the dispersion relations are quite similar to those of the telegrapher's equation (see Fig. 2.23.1). Moreover, the higher eigenvalues v_j ($j = 2, 3, \dots$) are remarkably large. For instance, the second eigenvalue is already $v_2 \approx 23$, thus the contributions from the higher eigenfunctions vanish quickly and can be neglected. This also reaffirms that the use of the two distinct levels, f_- and f_+ is a good approximation. For the evolution of f_+ and f_- , Kóta (1999) suggest the coupled equations

$$\frac{\partial f_-}{\partial t} - \frac{v}{2} \frac{\partial f_-}{\partial z} = -\frac{f_- - f_+}{T}, \quad (2.23.12)$$

$$\frac{\partial f_+}{\partial t} + \frac{v}{2} \frac{\partial f_+}{\partial z} + \frac{v}{2L} (f_+ - f_-) = -\frac{f_+ - f_-}{T}, \quad (2.23.13)$$

which, in the focusing term, are somewhat different from the equations of Schwadron (1998) and Isenberg (1997). Combining Eq. 2.23.12 and Eq. 2.23.13 leads to

$$v^2 - \left(\frac{v}{2L} + \frac{2}{T} \right) v + \frac{v^2}{4} \left(k^2 - \frac{ik}{L} \right) = 0 \quad (2.23.14)$$

implying a modification of $v_o(k)$ owed to the effect of focusing. The right panels of Fig. 2.23.4 show that, as expected from Eq. 2.23.14, the dispersion d^2v/dk^2 does indeed decrease in the presence of focusing.

2.24. The dynamics of dissipation range fluctuations with application to CR propagation theory

2.24.1. The matter of the problem

According to Leamon et al. (1999a), relatively few studies of the dissipation range of interplanetary magnetic turbulence exist when compared to the inertial range at lower frequencies. Fig. 2.24.1 shows an example of a high-resolution spectrum taken by the WIND spacecraft in near-Earth orbit and its associated reduced magnetic helicity spectrum.

From Fig. 2.24.1 can be seen that the inertial range spectrum terminates at 0.44 Hz in a spectral break to a steeper spectral index. This break marks the onset of the dissipation range. The possible involvement of ion cyclotron activity in the observed onset of steepening has been discussed by Behannon (1976) and Denskat et al. (1983). In all events observed by Leamon et al. (1999a) this steepening of the dissipation range sets in at $f_{sc} > \Omega_p/2\pi$, where Ω_p is the proton cyclotron frequency, but the $\mathbf{k} \cdot \mathbf{V}_{SW}$ Doppler shift makes it likely that $\omega < \Omega_p$. In the spacecraft's frame it may be found that as a reasonable first approximation, the break frequency f_{sc} is about 4 times the gyro-frequency $\Omega_p/2\pi$.

Although the dissipation range contains very little energy, it is important because low rigidity particles and all particles at large pitch-angles become resonant with fluctuations at those scales. Magneto-static quasi-linear scattering by the 'slab' geometry which omits consideration of the dissipation range gives too much scattering, especially at low rigidity. To counter this Bieber et al. (1988) and Smith et al. (1990) argue that incorporation of a dissipation range in magneto-static scattering significantly alters the mean-free-paths of energetic particles. Bieber et al. (1994) employ the dissipation range, together with magneto-dynamic

effects, to produce mass-dependent mean-free-paths that are distinct from the usual rigidity-dependent forms. This leads to differing mean-free-paths for protons and electrons of equal rigidity, in general agreement with a large class of solar energetic particle observations. Understanding supra thermal particle scattering therefore requires better determination of the turbulence geometry, i.e., the direction of $\mathbf{k}$.

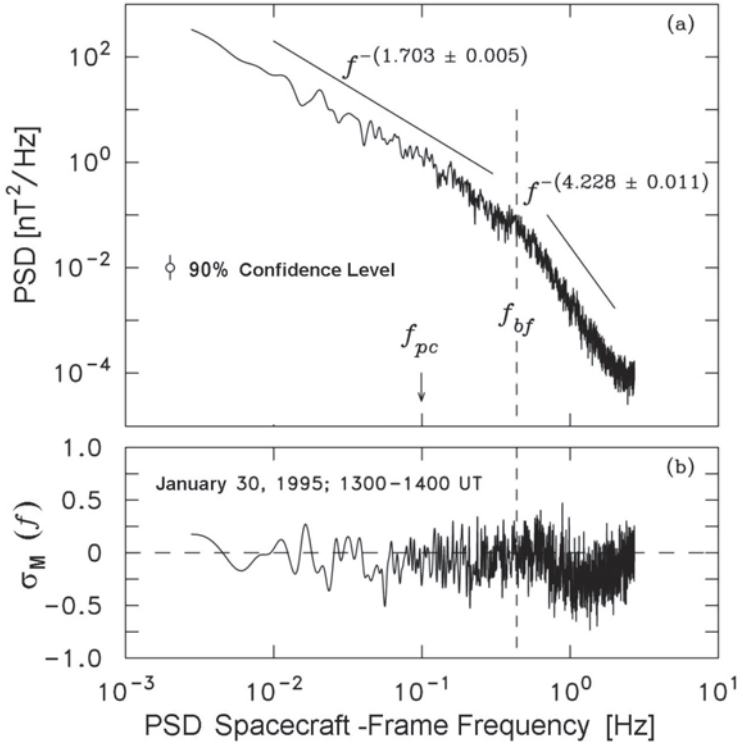


Fig. 2.24.1. Typical interplanetary power spectrum showing the inertial and dissipation ranges: (a) Trace of the spectral matrix with a break at ≈ 0.4 Hz where the dissipation range sets in; (b) The corresponding magnetic helicity spectrum. According to Leamon et al. (1999a).

Traditionally the reported observation of magnetic fluctuations perpendicular to the mean magnetic field (Belcher and Davis, 1971) has been used to motivate $\mathbf{k} \perp \mathbf{B}$. However, the possibility that an energetically significant fraction of the wave vectors could be nearly at $\mathbf{k} \parallel \mathbf{B}$ was shown by Matthaeus et al. (1990). Bieber et al. (1996) assumed a composite two-dimensional (2D)/slab model for the magnetic turbulence and determined that in the inertial range there is a dominant ($\approx 85\%$ by energy) 2D component. The 2D component does not contribute to resonant scattering of very energetic particles (CR) and can explain

the observed problem of ‘too small’ CR mean free paths (Bieber et al., 1994). Whereas Bieber et al. (1996) considered solar particle events, we shall extend their methods to the undisturbed solar wind and in frequency to the high-frequency end of the inertial range (~ 0.02 to ~ 0.2 Hz) and the low-frequency end of the dissipation range (~ 0.5 to ≤ 2 Hz).

2.24.2. Magnetic helicity according to WIND spacecraft measurements

The results presented by Leamon et al. (1999a) are based on the analysis of 33 one-hour intervals of quiet solar wind data from the magnetic field and thermal plasma instruments of the WIND spacecraft. This data set and the method of analysis is described in detail by Leamon et al. (1998). For the 33 quiet solar wind intervals the spectral indices of the inertial range were between -1.46 and -1.93 , with an average of -1.67 . The dissipation range indices range from -1.93 to -4.43 , with the average being -3.01 . No clear correlation between the fitted indices of the two ranges is observed. The panel **b** in Fig. 2.24.1 shows the reduced magnetic helicity spectrum for that interval. Note the negative signature at dissipation frequencies, averaging -0.275 over those frequencies used to compute the dissipation range spectral slope. If there is finite magnetic helicity, the sign of the particle's charge can enter into the rigidity dependence of the mean free path as a second order effect (Goldstein and Matthaeus, 1981; Bieber et al., 1987, 1994). This is accomplished by changing the amount of energy available for resonant scattering by adjusting the net polarization of the power spectrum within any given range. Perhaps more importantly, nonzero magnetic helicity can lead to resonant scattering dependent on the sign of the particle's charge. The apparent depletion of outward propagating Alfvén waves at frequencies comparable to the proton gyro-frequency naturally suggests resonant cyclotron damping of such Alfvén waves as the leading candidate for the formation of the dissipation range.

2.24.3. Anisotropy according to WIND spacecraft measurements

The classic study of inertial range magnetic fluctuations is that of Belcher and Davis (1971). They defined a coordinate system relative to the mean magnetic field direction $\hat{\mathbf{B}}$, and radial direction $\hat{\mathbf{R}}$, according to $\{\hat{\mathbf{B}} \times \hat{\mathbf{R}}, \hat{\mathbf{B}} \times (\hat{\mathbf{B}} \times \hat{\mathbf{R}}), \hat{\mathbf{B}}\}$ and showed that the average variances for these three components are in the ratio 5:4:1. Leamon et al. (1999) note that this implies a ratio for the total variance transverse to aligned with the mean field of 9 : 1. This high level of anisotropy is in accordance with the fluctuations consisting of Alfvén waves. Leamon et al. (1999a) define $P_{\parallel}$ to be the power in fluctuations parallel to $\hat{\mathbf{B}}$ and $P_{\perp}$ to be the total power in both components perpendicular to the mean field. For the high-frequency end of the inertial range it was found a mean $P_{\perp}/P_{\parallel}$ ratio of 14 : 1, with a range $3.0 \leq P_{\perp}/P_{\parallel} \leq 53.2$. For the dissipation range there was found a mean ratio of 5.4:1 with a range $2.36 \leq P_{\perp}/P_{\parallel} \leq 12.8$. The dissipation range ratios $P_{\perp}/P_{\parallel}$ are

consistently less than inertial range ratios, implying a decreased importance of transverse fluctuations in the dissipation range and an increase in the compression of the plasma at these scales.

2.24.4. Slab waves and 2D turbulence according to WIND spacecraft measurements

The Belcher and Davis (1971) anisotropy is usually taken as evidence of slab waves, even though it is consistent with 2D turbulence. By 2D turbulence is meant fluctuations which have wave-vectors that are nearly transverse to $\mathbf{B}$. Most people interpret Belcher and Davis (1971) 5:4:1 anisotropy as a $P_{\perp}/P_{\parallel} = 9:1$ ratio; the 5:4 part is considered physically unimportant. However, there is physical meaning to the ratio of the power in the two perpendicular directions (i.e., $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ in the mean-field coordinate system outlined in Section 2.24.3), and reason to expect that they should not be equal. Following Bieber et al. (1996), in a test based on the analysis of Oughton (1993) this ratio was used as a direct link to the percentage of slab waves and 2D modes in the fluctuations. Bieber et al. (1996) use a coordinate system that is a 90° right-handed rotation away from Belcher and Davis (1971) around the $\hat{\mathbf{z}}$ or $\hat{\mathbf{B}}$ axis). In the analysis that follows Bieber's conventions were used such that $\hat{\mathbf{y}} = \hat{\mathbf{B}} \times \hat{\mathbf{R}}$. It was assumed that the magnetic fluctuations consist of a mixture of slab and 2D geometries and compute their relative strengths from the ratio of transverse spectral powers: C_S and C_{2D} are the amplitudes of the slab and 2D components, respectively; i.e., the slab spectrum in the range of interest is parameterized by $C_S k^{-q}$ and the 2D spectrum by $C_{2D} k^{-q}$. It was further assumed that the two components obey the same power law (that is, they have the same spectral index $-q$). This is equivalent to the statement that P_{xx} and P_{yy} obey the same power law, which is not strictly obeyed, at least not within data set, but is approximately true. The 'slab fraction'

$$r = C_S / (C_S + C_{2D}), \quad (2.24.1)$$

is the contribution of the slab component to the energy spectrum, relative to the total energy. From Bieber et al. (1996) and the above definition leads to the following formula for the ratio of power between components:

$$\frac{P_{yy}(f)}{P_{xx}(f)} = \frac{\left(\frac{2\pi f}{V_{SW} \cos \theta} \right)^{1-q} + \frac{2q(C_{2D}/C_S)}{1+q} \left(\frac{2\pi f}{V_{SW} \sin \theta} \right)^{1-q}}{\left(\frac{2\pi f}{V_{SW} \cos \theta} \right)^{1-q} + \frac{2(C_{2D}/C_S)}{1+q} \left(\frac{2\pi f}{V_{SW} \sin \theta} \right)^{1-q}}. \quad (2.24.2)$$

The ratio P_{yy}/P_{xx} (which becomes independent of frequency in the relevant range) and the parameters V_{SW} , θ , the angle between the magnetic field and solar wind's velocity, and q are derivable from observations by a single spacecraft. Thus the only unknown in Eq. 2.24.2 is ratio C_{2D}/C_S , which, in turn, gives the slab fraction r determined by Eq. 2.24.1. For the 'middle' of the inertial range, Bieber et al. (1996) conclude that IMF geometry is $\sim 85\%$ 2D and only $\sim 15\%$ slab waves. Results of Leamon et al. (1999a,b) provide an essentially identical result for the high-frequency end of the inertial range, with $\sim 89\%$ of the energy in 2D fluctuations. In the dissipation range, on the other hand, the 2D component falls to $\sim 55\%$, which it may explain by preferential dissipation of 2D structures. In terms of application to scattering theory, the large 2D component reduces the overall scattering rate by the same percentage. Perpendicular wavevectors are inefficient scatterers of particles, essentially making their percentage of the total energy unavailable for particles. In Leamon et al. (1998, 1999a,b) have shown that there is both observational and theoretical evidence to support the claim that the dissipation range forms as the result of dissipating energy associated with wave vectors at large angles to the mean magnetic field. This is consistent with inertial range studies (Matthaeus et al., 1990; Bieber et al., 1996) that indicate the same geometry at these larger scales and CR mean-free-path analyses (Bieber et al., 1994). The results described above are expected to aid in the refinement of ongoing CR propagation analyses. Also important is to examine the possible role of magnetic helicity within the dissipation range in determining CR propagation. Since resonant scattering of large pitch-angle particles by the dissipation range is balanced against magneto-dynamic effects and other considerations, the possible role of helicity at the small scales is unclear.

2.25. A path integral solution to the stochastic differential equation of the Markov process for CR transport

2.25.1. The matter of the problem

CR transport in interplanetary or interstellar magnetic fields is often studied in the framework of diffusion models (e.g., Parker, 1965; Ginzburg and Ptuskin, 1975). For interplanetary transport a Fokker-Planck diffusion equation for the isotropic part of the CR distribution function can be derived from the collisionless Boltzmann equation with the help of observations of interplanetary magnetic fields (Skillington, 1976). The mechanism for motion of CR in the interstellar medium is not yet clear simply because of insufficient information on the interstellar medium and the galactic magnetic field. But on the overall scale size of the galaxy and on the time scale of CR life time ($\sim 10^7$ years) the diffusion approximation seems to be a suitable approach because it is consistent with the observation of small CR flux anisotropy and large amount of secondly produced

nuclei in CR relative to the interstellar medium composition. In addition, acceleration of CR by astrophysical shocks may also be studied in the framework of diffusion models (Drury, 1983). According to Zhang (1999a,b,c), CR transport in interplanetary or interstellar magnetic fields can be viewed as a Markov stochastic process and the transport equation has therefore been reformulated with a set of stochastic differential equations that describe the guiding center and the momentum of individual charged particles. The Fokker-Planck diffusion equation for the CR flux can be derived from these stochastic differential equations. Alternatively, the Fokker-Planck equation, like the Schrödinger equation in quantum mechanics, can be solved with a path integral method. Both new methods enable one to solve modulation, propagation and acceleration problems for CR spectra. In addition, both can reveal insights into the physical processes behind the solutions to these problems since they follow the trajectory and the momentum of individual particles. In papers of Zhang (1999a,b,c) stochastic differential equations were used that describe Markov stochastic processes to replace the diffusion equation as the fundamental transport equation of CR. From the stochastic differential equation, the stochastic process was discretized to obtain a path integral solution for the transition probability, which is consistent with the Green's function of the diffusion equation. A Lagrangian was found which, if minimized, describes the most probable trajectory of particles in diffusion process. The path integral derived from the Markov stochastic process is consistent with the path integral derived from the diffusion equation with quantum mechanics method (Zhang, 1999a). In Zhang (1999a,b,c) was shown that both the stochastic process method and path integral approach give excellent results for CR spectrum calculation (see below, Section 2.25.4).

2.25.2. Diffusion and Markov stochastic processes; using definitions

In diffusion models for CR studies, the distribution function or flux obeys a second-order d-dimensional partial differential equation, which can be in general written as:

$$\frac{\partial u}{\partial t} = \left[\sum_{\mu, \nu} \frac{\partial}{\partial q_{\mu}} \left(\frac{1}{2} a_{\mu, \nu} \frac{\partial}{\partial q_{\nu}} - b_{\mu} \right) + c \right] u + Q, \quad (2.25.1)$$

where the coordinate q_{μ} ($\mu = 1, 2, \dots, d$) is composed of spatial coordinates and particle momentum or energy, and Q is the source term. Table 2.25.1 lists the variables and parameters for studying solar modulation, interstellar propagation and diffusive particle acceleration.

In Section 2.25, and particularly in the Table 2.25.1, the following definitions are used: f – the isotropic distribution function; N_i – the flux per energy range for nuclei; N_e – the flux per energy range for the electrons; $\mathbf{x}$ – the guiding center

position; p – the momentum; E – the energy; $\kappa_{\mu\nu}$ – the diffusion coefficient tensor; κ – the diffusion coefficient scalar; D_p – the Fermi acceleration coefficient; $\mathbf{V}$ – the convection speed of plasma; $\mathbf{V}_d$ – the drift speed in magnetic fields; b_i – the ionization energy loss rate; b_e – the synchrotron energy loss rate; n – the interstellar medium density; v – the particle speed; σ_i – the total cross scalar section for specie i ; σ_{ik} – the cross section matrix from species k to i ; τ_i – the radioactive decay time for specie i ; τ_{ik} – the radioactive decay time matrix from species k to i .

Table 2.25.1. Parameters in the diffusion equation for applications to CR modulation, propagation and acceleration studies. According to Zhang (1999c).

Parameters	Heliospheric modulation	Interstellar propagation		Shock acceleration
		Nuclei	Electrons	
u	f	$N_i (i - \text{species})$	N_e	f
q-space	$\mathbf{x}$	$\mathbf{x}$	$\mathbf{x}$	$\mathbf{x}$
	p	E/amu	E	p
$(1/2)a_{\mu\nu}$	$\kappa_{\mu\nu}$	κ	κ	$\kappa_{\mu\nu}$
	D_p	D_p	D_p	D_p
b_μ	$\mathbf{V} + \mathbf{V}_d$	usually 0	0	$\mathbf{V} + \mathbf{V}_d$
	$-(1/3)p\nabla \cdot \mathbf{V}$	b_i	b_e	$-(1/3)p\nabla \cdot \mathbf{V}$
Q	boundary problem	continues	continues	injection at shock

The motion of individual particles in diffusion models has always been viewed as a random walk since the beginning of theoretical efforts (e.g. Parker, 1965). However, it is only a recent development that the CR transport equation can be reformulated with stochastic differential equations (Zhang, 1999b). In this approach the guiding center position of the particle and its momentum (energy) follow a set of Itô stochastic differential equations

$$dq_\mu = \beta_\mu dt + \sum_{\sigma} \alpha_{\mu,\sigma} dw_\sigma(t), \quad (2.25.2)$$

where $w_\sigma(t)$ is a Wiener process (see below) and the sum of σ runs over all required independent random noises. The probability density for the particle in the Markov process determined by Eq. 2.25.2 to appear in a unit volume at a

particular location in q -space at time t , $P(t, q)$, follows the same Fokker-Planck diffusion equation as Eq. 2.25.1 (Zhang, 1999b) if we let

$$a_{\mu\nu} = \sum_{\sigma} \alpha_{\mu,\sigma} \alpha_{\nu,\sigma}, \quad \beta_{\mu} = \beta_{\mu} + (1/2) \sum_{\nu} \partial a_{\mu\nu} / \partial q_{\nu}, \quad (2.25.3)$$

and let the process be created at an exponential rate of c as a function of time, i.e. $d(\ln P)/dt = c$. The probability density in q -space can be made proportional to the CR flux or distribution function. If the probability density starts with a δ -function initially, i.e., the stochastic process starts from a single location point, the solution is the Green's function to the diffusion Eq. 2.25.1 (thus, the Green's function is often called the transition probability density or propagator). Therefore the stochastic differential Eq. 2.25.2 with an additional creation term can be used to describe diffusion.

Zhang (1999b) applied the Itô stochastic differential equation to studies of modulation, and the results from Monte Carlo simulation of the stochastic process completely agree with those by directly solving the diffusion equation. One obvious advantage of using the stochastic process approach is that it can reveal the physics of particle diffusion in more detail. For example, we can investigate the trajectory of simulated particles traveling through heliospheric or interstellar magnetic fields and when an ensemble of particles is simulated, we can find the distributions of source particles in terms of entry location at the boundary, initial momentum and propagation time (which is approximately proportional to path length). The path length distribution is particularly useful for studies of nuclear fragmentation during interstellar propagation.

2.25.3. Path integral representation for the transition probability of Markov processes

For simplicity Zhang (1999a) considers only a 1-dimensional stochastic diffusion process governed by an Ito stochastic differential equation

$$dq = \beta(t, q)dt + \alpha(t, q)dw(t), \quad (2.25.4)$$

with a creation rate $c(t, q)$. The Wiener process has an associated probability for the process $w(t)$ to transit from w_o at time t_o to an interval $w_1 < w(t_1) < w_1 + dw_1$ at t_1 ($t_1 > t_o$):

$$P_1(t_o, w_o; t_1, w_1) = \frac{dw_1}{\sqrt{2\pi(t_1 - t_o)}} \exp\left(-\frac{(w_1 - w_o)^2}{2(t_1 - t_o)}\right). \quad (2.25.5)$$

The Wiener process can be understood as the simplest diffusion with a coefficient of $1/2$ and no convection. To calculate the transition probability density for the process described by Eq. 2.25.4 to get from q_o at time t_o to q at t , we normally divide the time interval $\{t_o, t\}$ into N small segments $\{t_o, t_1, t_2, \dots, t_{N-1}, t_N\}$, where $t_N = t$. This method is often called discretization. The probability for the process to go through a path

$$\{q_o, q_1 < q(t_1) < q_1 + dq_1, q_2 < q(t_2) < q_2 + dq_2, \dots, q_N < q(t_N) < q_N + dq_N\}, \quad (2.25.6)$$

during which the driving Wiener process goes through

$$\{w_o, w_1 < w(t_1) < w_1 + dw_1, w_2 < w(t_2) < w_2 + dw_2, \dots, w_N < w(t_N) < w_N + dw_N\}, \quad (2.25.7)$$

is

$$P_N(\{w_i\}) = \prod_{i=1}^N \frac{dw_i}{\sqrt{2\pi\Delta t_i}} \exp\left(-\left(\frac{(\Delta w_i)^2}{2\Delta t_i}\right) - c(t_{i-1}, q_{i-1})\Delta t_i\right). \quad (2.25.8)$$

where $\Delta t_i = t_i - t_{i-1}$ and $\Delta w_i = w_i - w_{i-1}$. When $\Delta t_i \rightarrow 0$, Δw_i must be $O(\sqrt{\Delta t_i})$ in order to have non-vanishing probability. The transition probability density from the initial point (t_o, q_o) and the end point (t, q) can be obtained by integrating all the intermediate points, $w_1, w_2, \dots, w_{N-1}$. However, the probability density, as obtained directly from Eq. 2.25.8, is for the w -space. To calculate the probability density in the q -space, we need to find the Jacobean for the transformation to q -coordinates, which can be obtained by finite expansion of the Ito stochastic differential Eq. 2.25.4 to the 6th order (Langouche et al., 1982). Replacing also the argument w_i in the exponential of Eq. 2.25.8, we obtain a path integral representation for the transition probability density:

$$G(t_o, q_o; t, q) = \frac{1}{\sqrt{2\pi a(t_o, q_o)\Delta t_o}} \int_{q_o=q_o}^{q_N=q} \prod_{i=1}^{N-1} \frac{dq_i}{\sqrt{2\pi a(t_{i-1}, q_{i-1})\Delta t_i}} \\ \times \exp\left\{-\sum_{i=1}^{N-1} \frac{1}{2a(t_{i-1}, q_{i-1})} \left(\frac{\Delta q_i}{\Delta t_i} - \beta\right)^2 - \frac{\sqrt{a(t_{i-1}, q_{i-1})}}{2} \frac{\partial}{\partial q} \frac{\beta}{\sqrt{a(t_{i-1}, q_{i-1})}} - c\right\}, \quad (2.25.9)$$

where $\beta \equiv \beta(t_{i-1}, q_{i-1})$, $c \equiv c(t_{i-1}, q_{i-1})$, $a = \alpha^2$. When we take the limit $N \rightarrow \infty$, Eq. 2.25.9 may be written in a continuous path integral:

$$G(t_o, q_o; t, q) = \int \hat{D}q \exp(-\int L(t, q, \dot{q}) dt), \quad (2.25.10)$$

where

$$\hat{D}q = \frac{1}{\sqrt{2\pi a(t_o, q_o)\Delta t_o}} \lim_{N \rightarrow \infty} \prod_{i=1}^{N-1} \frac{dq_i}{\sqrt{2\pi a(t_{i-1}, q_{i-1})\Delta t_i}}, \quad (2.25.11)$$

and the Lagrangian, $L(t, q, \dot{q})$, where $\dot{q} = \Delta q / \Delta t$, is:

$$L(t, q, \dot{q}) = \frac{1}{2a} (\dot{q} - \beta(t, q))^2 + \frac{\sqrt{a}}{2} \frac{\partial}{\partial q} \frac{\beta}{\sqrt{a}} - c. \quad (2.25.12)$$

The path integral in Eq. 2.25.10 is consistent with the path integral directly derived from the Fokker-Planck equation (Drozdov, 1993). For higher dimensions the derivation of the path integral from stochastic differential equations is much more complicated. Interested readers may find rigorous calculations by Langouche et al. (M1982). When the functional integral $\int L(t, q, \dot{q}) dt$ is minimized, it yields an Euler-Lagrange equation. Thus the Lagrangian in Eq. 2.25.12 may be used to find the most probable trajectory for particles.

2.25.4. Main results and method's of checking

Zhang (1999a,b,c) have presented a Markov stochastic process approach to the diffusion theory of CR modulation, propagation and acceleration. The CR transport equation is reformulated with the Itô stochastic differential equation. From the stochastic differential equation a Fokker-Planck equation can be derived for the probability density, which is proportional to the CR flux. The transition probability density of the Markov process is obtained as a path integral consistent with that derived in quantum mechanics. Fig. 2.25.1 shows an example of computer calculations of modulated CR spectra.

From Fig. 2.25.1 it can be seen that the three different methods – the path integral approach, stochastic process simulation and numerical method to solve the diffusion equation ('SolMod' according to Fisk, 1971) – all agree with each other. In addition to the ability to solve the CR diffusive transport equation, the two new methods provide the detailed physical processes behind their solutions (Zhang, 1999a,b).

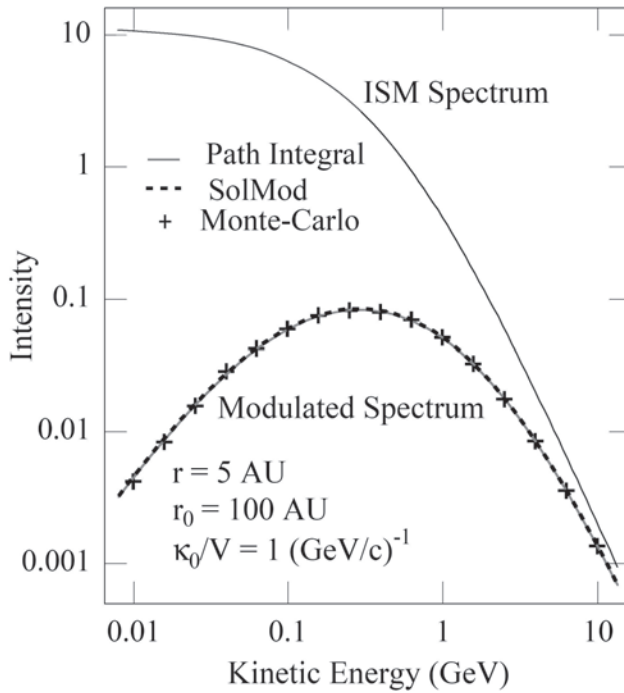


Fig. 2.25.1. Three different calculations of modulated CR spectra at 5 AU with an input ISM spectrum. According to Zhang (1999c).

2.26. Velocity correlation functions and CR transport (compound diffusion)

2.26.1. The matter of problem

The transport of energetic charged particles in a turbulent magnetic field is often diffusive, where the time evolution of the omnidirectional particle density $f_o(x_i, t)$ is described by a diffusion equation with diffusion tensor κ_{ij} . Jokipii and Kóta (1999) consider some consequences of a particular way of looking at the diffusion. For a random, diffusive motion, the spatial diffusion tensor κ_{ij} can be related, under very broad conditions, to the velocity correlation function Kubo (1957)

$$\kappa_{ij} = \int_0^\tau \langle v_j(t) v_i(t+t') \rangle dt' \quad (2.26.1)$$

in the limit that $\tau \rightarrow \infty$. Here $\Delta x_i = x_i(t + t') - x_i(t)$ is the spatial displacement of particle positions between times t and $t + t'$; the brackets $\langle \rangle$ denote averages over an ensemble. It was assumed that the fluctuating velocities are statistically homogeneous over the time and length scales of interest, so the velocity correlation function $\langle v_j(t) v_i(t + t') \rangle$ depends only on the time difference t' . For any physical random velocity $\langle v_j(0) v_i(t') \rangle$ must go to zero at large t' , and the integral in Eq. 2.26.1 approaches a constant value for $t' \rightarrow \infty$. The virtue of this method is that only the particle velocity needs to be considered. Forman (1977a) was the first to apply Kubo's formalism to the transport of CR. It has recently been invoked by Bieber and Matthaeus (1997), who postulated a simple exponential form for the correlation tensor $\langle v_j(0) v_i(t') \rangle$ to infer the corresponding perpendicular diffusion coefficient $\kappa_{\perp}$ and effective drift, which is related to the anti-symmetric component of κ_{ij} . Jokipii and Kóta (1999) consider the special case of the perpendicular diffusion of particles tied to the turbulent magnetic field lines, which is important in understanding the transport of energetic charged particles in the Heliosphere, and for which the application of Kubo's formalism is not obvious.

2.26.2. Compound CR diffusion

According to Jokipii and Kóta (1999) the most poorly-understood area of CR transport at present is the transport of particles perpendicular to the direction of the average magnetic field. This motion is owed to at least two distinct effects. Particles may scatter across field lines and the field lines may depart from the mean field owing to the random walk and mixing of field lines (Jokipii and Parker, 1969). This random walk of the field lines plays an important role in the perpendicular diffusion (see, e.g. Jokipii, 1966; Forman et al., 1974; Giacalone and Jokipii, 1999). Low rigidity particles in certain cases may be effectively tied to magnetic field lines, so it is useful to consider an idealized, but physically consistent, approximation, in which particles are assumed to be strictly tied to the field lines. The particles are assumed scatter back and forth along the field lines, in which case the particle perpendicular transports arises solely from the random walk of field lines. This can then serve as a starting point for understanding the more general problem of particle transport. This approximation has been termed compound diffusion, and has been used to discuss transport of CR in the Galaxy (e.g. Getmantsev, 1963; Lingenfelter et al., 1971; Allan, 1972).

Compound diffusion may be written as the convolution of two diffusive processes. Particles scatter back and forth and spread strictly along the field lines with a diffusion coefficient $\kappa_{\parallel}$, and the field lines, in turn, diffuse perpendicular to the mean field's in z-direction with a diffusion coefficient D_L . The mean square displacement in a perpendicular direction, say x , is then proportional to the

length traveled along the field line, which, from simple scaling properties, is proportional to $\sqrt{\Delta t}$. A quantitative calculation evaluating the convolution of the x and t motions yields

$$\langle \Delta x^2 \rangle = 2D_L \langle |\xi| \rangle = 4D_L \sqrt{\kappa_{\parallel} \Delta t / \pi} \quad (2.26.2)$$

which is slower than the standard diffusion, where $\langle \Delta x^2 \rangle = 2\kappa_{\perp} t$, and so is fundamentally non-Markovian.

2.26.3. The Kubo formulation applied to compound diffusion

Kubo's (1957) formalism states essentially that the mean square displacement $\langle \Delta x^2 \rangle$ in a time Δt can be obtained from very general principles as

$$\langle \Delta x^2 \rangle = \left\langle \left(\int_0^{\Delta t} v_x(t') dt' \right)^2 \right\rangle = 2 \int_0^{\Delta t} (\Delta t - t') \langle v_x(0) v_x(t') \rangle dt', \quad (2.26.3)$$

which for Δt large compared with the coherence time of $v_x(t)$ yields diffusive motion with a diffusion coefficient given by

$$\kappa_{xx} = \langle \Delta x^2 \rangle / \Delta t = \int_0^{\infty} \langle v_x(0) v_x(t') \rangle dt'. \quad (2.26.4)$$

The only requirement, in addition to statistically homogeneous conditions, is that the velocity correlation function $v_x(0)v_x(t)$ should vanish sufficiently fast as the time lag τ increases. Under these conditions the Kubo model would always give a diffusion $\propto \Delta t$, and could not yield compound diffusion, which results in a slower transport $\propto \Delta t^{1/2}$. There is clearly a problem with the application of Kubo's formulation to this problem. To explore this more deeply, to see where the problem lies, Jokipii and Kóta (1999) have considered a simple, transparent model in which the particles propagate either forward or backward along a magnetic field line, which executes random walk about the main field direction in z . The z axis points in the direction of the mean background field; ξ denotes the position, measured along the field line, and $x(\xi)$ stands for the departure of the field line from the mean field. They consider particles released, in random directions, at $\xi = 0$ ($x = 0$) at time $t = 0$. The variation of the number of forward (n_+) and backward

(n_-) moving particles as a function of time t , and position along the field line ξ is governed by the pair of equations:

$$\frac{\partial n_+}{\partial t} + v \frac{\partial n_+}{\partial \xi} = -\frac{n_+ - n_-}{2\tau} + q_+ \delta(t) \delta(\xi), \quad (2.26.5)$$

$$\frac{\partial n_-}{\partial t} + v \frac{\partial n_-}{\partial \xi} = -\frac{n_- - n_+}{2\tau} + q_- \delta(t) \delta(\xi), \quad (2.26.6)$$

where τ represents the average time of scattering, q_+ and q_- are the number of particles released in positive and negative directions, respectively.

Obviously, the velocity in the x direction is $\pm v(dx/d\xi)$ depending on whether the particle happens to move forward or backward along the field line. Thus to obtain the velocity correlation $\langle v_x(0)v_x(t) \rangle$, the mean velocity ($vn_+ - vn_-$) along the field is to be averaged over position, with the inclusion of the actual orientation of the field line ξ

$$\langle v_x(0)v_x(t) \rangle = v^2 \int_{-\infty}^{\infty} \left\langle \left(\frac{dx}{d\xi} \right) \left(\frac{dx}{d\xi} \right) \right\rangle_{|_0}^{\xi} (n_+ - n_-) d\xi, \quad (2.26.7)$$

where subscripts imply the position in ξ . Since the source q_+ accounts for positive initial speed, whilst q_- corresponds to negative initial speed, n_+ and n_- can be taken as the solutions for sources $q_+ = 1/2$ and $q_- = -1/2$. This ensures that the initial speed $v_x(0)$ is properly taken into account. Instead of considering the directly the velocity correlation $\langle v_x(0)v_x(t) \rangle$ Jokipii and Kóta (1999) consider its Laplace transform

$$L_{xx}(s) = \int_0^{\infty} \langle v_x(0)v_x(t) \rangle e^{-st} dt. \quad (2.26.8)$$

They note that $L_{xx}(s=0)$ yields exactly the corresponding perpendicular diffusion coefficient κ_{xx} , whilst the behavior of L_{xx} at small s values brings information on the behavior of $\langle v_x(0)v_x(t) \rangle$ for large times ($t \gg \tau$).

Jokipii and Kóta (1999) adopt the technique of Fourier and Laplace transforms (Fedorov and Shakhov, 1993; Kóta, 1994). First, taking the Fourier transform of Eq. 2.26.5 and Eq. 2.26.6, and Laplace transforming the resulting pair of equations yields a solution for $L_{xx}(s)$:

$$L_{xx}(s) = \frac{v}{2} \sqrt{\frac{s\tau}{s\tau+1}} \int_{-\infty}^{\infty} \left\langle \left(\frac{dx}{d\xi} \right) \left(\frac{dx}{d\xi} \right) \right\rangle \exp(-k_o \xi) d\xi. \quad (2.26.9)$$

Inspection of Eq. 2.26.9 shows that $L_{xx}(0)=0$, unless the integral over ξ , which is related to the random walk of field lines, is infinite (this would be the case only if the field had a nonzero regular component in the x direction). Since the Laplace transform at $s=0$ is identical to κ_{xx} , the derivation above demonstrates that Kubo's theorem yields precisely zero perpendicular diffusion coefficient κ_{xx} for compound diffusion. By the opinion of Jokipii and Kóta (1999) this result could intuitively be anticipated, since compound diffusion produces slower than x - t diffusion.

A further study of Eq. 2.26.9 reveals the character of $\langle v_x(0)v_x(t) \rangle$ in more detail. First Jokipii and Kóta (1999) notice that, for small values of s , $k_o \approx 0$, and the integral over ξ in Eq. 2.26.9 gives the power of field fluctuations at zero wavenumber, which is equivalent to $2D_L$, where D_L is the diffusion coefficient of field line random walk (Jokipii, 1966). For small values of s , $L_{xx} \approx vD_L(st)^{1/2}$, and Eq. 2.26.9 implies that for large values of t

$$\langle v_x(0)v_x(t) \rangle \approx -\frac{vD_L\sqrt{\tau}}{2\sqrt{\pi}} t^{-3/2}. \quad (2.26.10)$$

The velocity correlation function has a long negative tail to balance the positive values at smaller t , and to give an exactly vanishing integral in Eq. 2.26.4. This long term behavior could be obtained directly from considering the solutions for n_+ and n_- of Fisk and Axford (1969) in the $t \gg \tau$ limit, when the exact solutions can be approximated by diffusive time profiles.

Thus Jokipii and Kóta (1999) find that, in a broader sense, compound diffusion fits into Kubo's theory. The velocity correlation function $\langle v_x(0)v_x(t) \rangle$ exhibits a long-term anticorrelation, causing the diffusion coefficient κ_{xx} (i.e. the integral in Eq. 2.26.4) to vanish. This is connected with the non-Markov nature of the compound diffusion. At this point it is of interest to establish the connection between the present discussion and some current ideas in time-series analysis. The fact that the mean square displacement $\langle \Delta x^2 \rangle$ increases as Δt^5 means that the motion is non-Markov. According to opinion of Jokipii and Kóta (1999), the case in which $\langle \Delta x^2 \rangle \propto \Delta t^{2H}$ ($0 < H < 1$) has been given the name fractional Brownian motion, where H is the Hurst exponent (Mandelbrot and Van Ness, 1968). The case of compound diffusion corresponds to a Hurst exponent $H = 0.25$. It may be

shown (see Section 9.4 in Feder, M1988), that if $H > 0.5$ the process exhibits long-term positive correlation and conversely, if $H < 0.5$ corresponds to long-term anticorrelation of the process. Clearly, then, the case of no correlation requires $H = 0.5$. This agrees with the determination using Laplace transforms, described previously. Now, physically, it is expected that particles in a turbulent magnetic field will loose correlation and it will be retrieve the standard form $\langle \Delta x^2 \rangle \propto t$. But this cannot occur if particles are strictly tied to field lines.

2.26.4. Main results

Jokipii and Kóta (1999) considered compound diffusion, which is a non-Marko diffusion leading to $\Delta x^2 \propto \Delta t^{1/2}$. This idealized but valid motion is seemingly in contradiction with Kubo's theory, which yields $\Delta x^2 \propto \Delta t$. They have shown that compound diffusion fits into the general theory in a broader sense and determined the Laplace transform of the velocity correlation, and showed that the diffusion coefficient, as defined by Eq. 2.26.1, turns out to vanish. A study of the Laplace transform revealed, furthermore, that the velocity correlation has a negative non-exponential tail, $\langle v_x(0)v_x(t) \rangle \propto -t^{-3/2}$ indicating a long-term anticorrelation. The $\Delta x^2 \propto \Delta t^{1/2}$ behavior of the compound diffusion could also be recovered from Kubo's formalism. The compound diffusion may serve as a starting point for understanding the perpendicular transport of low rigidity particles. The question is how to proceed from this picture to a model including some scattering across field lines. A small amount of cross-field scattering can be amplified by the subsequent mixing of field lines; originally nearby field lines may separate to great distances. The time scales of these processes may be large for low rigidity particles. In this case the long non-exponential tail of the velocity correlation, which is a result of the long-term anticorrelation, may be of importance; the velocity correlation function may considerably differ from the simple exponential decay postulated by Bieber and Matthaeus, (1997). These questions, in the opinion of Jokipii and Kóta (1999), need further exploration. They also point out that consideration of temporally varying magnetic fields suggests that the conclusions derived here apply also to this situation.

2.27. The BGK Boltzmann equation and anisotropic diffusion

2.27.1. The matter of problem

Early work by Parker (1965) and Axford (1965) derived the form of the diffusion tensor for CR in a random magnetic field for the case of isotropic scattering. Forman et al. (1974) used quasi-linear theory in slab turbulence to determine the diffusion coefficients parallel $\kappa_{//}$ and perpendicular $\kappa_{\perp}$ to the mean magnetic field $\mathbf{B}_0$, as well as the anti-symmetric component of the diffusion

tensor κ_A , associated with particle drifts, for the case where the distribution function could be expanded in spherical harmonics. Jokipii (1971) and Hasselmann and Wibberenz (1970), pointed out that the detailed dependence of the pitch-angle diffusion coefficient $D_{\mu\mu}$ on μ is important in determining $\kappa_{//}$. Webb et al. (2001) study a model of CR diffusion based on a gyro-phase, and pitch-angle dependent BGK Boltzmann model, involving two collision time scales $\tau_{\perp}$ and $\tau_{//}$ associated with scattering perpendicular and parallel to the background magnetic field $\mathbf{B}_o$. The time scale $\tau_{//}$ describes the ironing out of gyro-phase anisotropies, and the relaxation of the full gyro-phase distribution f to the gyroaveraged distribution f_o . The time scale $\tau_{\perp}$ determines the diffusion coefficient $\kappa_{\perp}$, perpendicular to the mean magnetic field, and the corresponding anti-symmetric diffusion coefficient κ_A associated with particle drifts. The time scale $\tau_{//}$ describes the relaxation of the pitch-angle distribution f_o to the isotropic distribution F_o , and determines the parallel diffusion coefficient $\kappa_{//}$. The Green's function solution of the model equation is obtained, for the case of delta function initial data in position, pitch-angle, and gyro-phase, in terms of Fourier-Laplace transforms. The solutions are used to discuss non-diffusive and diffusive particle transport. The gyro-phase dependent solutions exhibit cyclotron resonant behavior, modified by resonance broadening due to $\tau_{\perp}$. Below, in Sections 2.27.2–2.27.7, the model of Webb et al. (2001) will be considered in details.

2.27.2. Description of the model

According to Webb et al. (2001) the BGK Boltzmann equation for the momentum-space distribution function $f(\mathbf{r}, \mathbf{p}, t)$, for particles with momentum $\mathbf{p}$, (or velocity $\mathbf{v}$), at position $\mathbf{r}$ at time t , in a uniform background magnetic field $\mathbf{B}_o = \{0, 0, B_o\}$ along the z -axis, may be written in the form:

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f - \Omega \frac{\partial f}{\partial \phi} = -\frac{f - f_o}{\tau_{\perp}} - \frac{f_o - F_o}{\tau_{//}}, \quad (2.27.1)$$

where

$$f_o = \frac{1}{2\pi} \int_0^{2\pi} f d\phi, \quad F_o = \frac{1}{2} \int_{-1}^1 f_o d\mu. \quad (2.27.2)$$

denote the gyro-phase averaged distribution function (f_o), and the isotropic component of the distribution function (F_o) in momentum space, and $\mu = \cos\theta$ is

the pitch-angle cosine. The gyro-phase derivative term $-\Omega(\partial f/\partial\phi)$, on the left hand side of Eq. 2.27.1, is the Lorentz force term, where $\Omega = qB_o/mc$ is the particle gyro-frequency, and m is the relativistic particle mass. Note that (v, θ, ϕ) are spherical polar coordinates for the velocity, where the polar axis is along $\mathbf{B}_o$. Kóta (1993) used a model similar to Eq. 2.27.1, except that he used a pitch-angle and gyro-phase diffusion term for the collision term.

2.27.3. The diffusion approximation

Following the approach of Kóta (1993), Webb et al. (2001) expand the distribution function in the series:

$$f = \sum_{n=-\infty}^{\infty} f_n \exp(in\phi), \quad (2.27.3)$$

where $f_{-n} = f_n^*$. Multiplying the Boltzmann Eq. 2.27.1 by $\exp(im\phi)$, and integrating over the gyro-phase ϕ from $\phi = 0$ to $\phi = 2\pi$, yields the moment equations:

$$\begin{aligned} \frac{\partial f_m}{\partial t} + \frac{v \sin \theta}{2} \left(\frac{\partial f_{m-1}}{\partial x} - i \frac{\partial f_{m-1}}{\partial y} \right) + \frac{v \sin \theta}{2} \left(\frac{\partial f_{m+1}}{\partial x} + i \frac{\partial f_{m+1}}{\partial y} \right) \\ + v \cos \theta \frac{\partial f_m}{\partial z} - im\Omega f_m = - \frac{f_m - f_o \delta_o^m}{\tau_+} - \frac{(f_o - F_o) \delta_o^m}{\tau_{//}}, \end{aligned} \quad (2.27.4)$$

where $m = 0, \pm 1, \pm 2, \dots$, and δ_i^j is the Kronecker delta symbol. In particular, for $m = 0$, Eq. 2.27.4 multiplied by $2\pi p^2 \sin \theta$, and integrated over θ from $\theta = 0$ to $\theta = \pi$, yields the number density conservation equation:

$$N + \vec{\nabla} \cdot \mathbf{S} = 0, \quad (2.27.5)$$

where $N = p^2 \int f d\Omega$ and $\mathbf{S} = p^2 \int \mathbf{v} f d\Omega$ are the particle number density and current, and the integrations over $d\Omega$ are over solid angle in momentum space. In the diffusion approximation one uses the approximate moment balance equations for $m = 0$ and 1:

$$2\pi p^2 v \int_0^\pi d\theta \sin \theta \cos \theta \left(v \cos \theta \frac{\partial f_o}{\partial z} + \frac{f_o - F_o}{\tau_{//}} \right) \approx 0, \quad (2.27.6)$$

$$2\pi p^2 v \int_0^\pi d\theta \sin\theta^2 \left(\frac{v \sin\theta}{2} \left(\frac{\partial f_o}{\partial x} - i \frac{\partial f_o}{\partial y} \right) - i\Omega f_1 + \frac{f_1}{\tau_\perp} \right) \approx 0 \quad (2.27.7)$$

to determine the diffusive current $\mathbf{S}$. The diffusion approximation assumes that the scattering is strong enough to drive the distribution function to a near isotropic state, and that the effective scattering time is much shorter than the time scale for the evolution of F_o . Using Eq. 2.27.6 and Eq. 2.27.7 it follows that the diffusive current has the form:

$$\mathbf{S} = -\kappa_{//} N_z \mathbf{e}_B - \kappa_\perp \nabla_\perp N - \kappa_A \nabla N \times \mathbf{e}_B, \quad (2.27.8)$$

where $\mathbf{e}_B \equiv \mathbf{e}_z$ is the unit vector along $\mathbf{B}_o$, and

$$\kappa_{//} = \frac{v^2 \tau_{//}}{3}, \quad \kappa_\perp = \frac{v^2 \tau_\perp}{3(1 + \Omega^2 \tau_\perp^2)}, \quad \kappa_A = \Omega \tau_\perp \kappa_\perp. \quad (2.27.9)$$

The expressions in Eq. 2.27.9 for $\kappa_{//}$, $\kappa_\perp$ and κ_A have the same form as in Forman et al. (1974).

2.27.4. Evaluation of the Green function

Introducing the Laplace-Fourier transform:

$$\tilde{f}(\mathbf{k}, \mathbf{p}, s) = \int_0^\infty dt \int_{-\infty}^\infty \frac{d^3 r}{(2\pi)^3} \exp(-st - i\mathbf{k} \cdot \mathbf{r}) f(\mathbf{r}, \mathbf{p}, t) \quad (2.27.10)$$

the BGK Boltzmann Eq. 2.27.1 reduces to the ordinary differential equation:

$$\Omega \tilde{f}_\phi - (s + i\mathbf{k} \cdot \mathbf{v} + v_\perp) \tilde{f} = \left[-\hat{f}(\mathbf{k}, \mathbf{p}, 0) + (v_{//} - v_\perp) \tilde{f}_o - v_{//} \tilde{F}_o \right], \quad (2.27.11)$$

where $v_{//} = 1/\tau_{//}$, $v_\perp = 1/\tau_\perp$ and $\hat{f}(\mathbf{k}, \mathbf{p}, 0)$ is the Fourier transform of the initial data $f(\mathbf{r}, \mathbf{p}, 0)$. For Dirac-delta function for initial data, with

$$f(\mathbf{r}, \mathbf{p}, 0) = A \delta(\mathbf{r} - \mathbf{r}_o) \delta(\mu - \mu_o) \delta(\phi - \phi_o) \quad (2.27.12)$$

it will be

$$\hat{f}(\mathbf{k}, \mathbf{p}, 0) = \frac{A}{(2\pi)^3} \exp(-i\mathbf{k} \cdot \mathbf{r}_o) \delta(\mu - \mu_o) \delta(\phi - \phi_o). \quad (2.27.13)$$

Using Eq. 2.27.13 as the source term in Eq. 2.27.11, and integrating Eq. 2.27.11 yields the solution:

$$\tilde{f} = (\Omega I(\phi, \theta))^{-1} \left[(v_{||} \tilde{F}_o - (v_{||} - v_{\perp}) \tilde{f}_o) (L(2\pi, \theta)/\zeta - L(\phi, \theta)) \right] + Q, \quad (2.27.14)$$

where

$$Q = \frac{AI(\phi_o, \theta_o) \exp(-i\mathbf{k} \cdot \mathbf{r}_o) \delta(\mu - \mu_o)}{(2\pi)^3 \Omega I(\phi, \theta)} \left(\frac{1}{\zeta} - H(\phi - \phi_o) \right) \quad (2.27.15)$$

is the source term associated with the initial data described by Eq. 2.27.13. In deriving Eq. 2.27.14 and Eq. 2.27.15, the angles ϕ and ϕ_o are restricted to the range $[0, 2\pi]$, and the condition $\tilde{f}(\phi = 0) = \tilde{f}(\phi = 2\pi)$ is used to determine the integration constant. In Eq. 2.27.14 and Eq. 2.27.15

$$I(\phi, \theta) = \exp\left(-(\bar{s} + \bar{v}_{\perp} + ik_{||}r_g \cos\theta)(\phi - \Phi) - ik_{\perp}r_g \sin\theta \sin(\phi - \Phi)\right) \quad (2.27.16)$$

is the integrating factor for Eq. 2.27.11 where used the notations $\bar{s} = s/\Omega$, $\bar{v}_{\perp} = v_{\perp}/\Omega$, $\bar{v}_{||} = v_{||}/\Omega$, and

$$L(\phi, \theta) = \int_0^{\phi} I(\phi', \theta) d\phi', \quad \zeta = 1 - \exp\left(-2\pi(\bar{s} + \bar{v}_{\perp} + ik_{||}r_g \cos\theta)\right). \quad (2.27.17)$$

In Eq. 2.27.15–2.27.17 (k, Θ, Φ) are spherical polar coordinates for $\mathbf{k}$, with polar axis along $\mathbf{B}_o$; $k_{||} = k \cos\Theta$, $k_{\perp} = k \sin\Theta$, and $H(x)$ is the Heaviside step function. Eq. 2.27.14 can be regarded as an integral equation for $\tilde{f}$, and is a central result in the analysis.

By using the standard generating function identity for Bessel functions (e.g. Abramowitz and Stegun, M1965, p. 361, formula 9.1.41), Webb et al. (2001) obtained

$$L(\phi, \theta) = \sum_{n=-\infty}^{\infty} \frac{J_n(k_{\perp} r_g \sin \theta)}{\bar{s} + \bar{v}_{\perp} + i(k_{\parallel} r_g \cos \theta + n)} \exp[(\bar{s} + \bar{v}_{\perp} + i k_{\parallel} r_g \cos \theta + in)\phi] \\ \times \left\{ 1 - \exp[-(\bar{s} + \bar{v}_{\perp} + i k_{\parallel} r_g \cos \theta + in)\phi] \right\} \quad (2.27.18)$$

where $r_g = pc/(qB_o)$ is the particle gyro-radius and $J_n(x)$ is the Bessel function of the first kind of order n and argument x . By noting that $\bar{s} = s/\Omega$, and setting $s = -i\omega$ one finds that the denominator of the n -th term in Eq. 2.27.18

$$\bar{s} + \bar{v}_{\perp} + i k_{\parallel} r_g \cos \theta + n = 0 \text{ when } \omega - k_{\parallel} v \mu = n\Omega - iv_{\perp} \text{ (} n \text{ integer),} \quad (2.27.19)$$

where $\mu = \cos \theta$. Thus the role for the term indexed by n in the series of Eq. 2.27.18 corresponds to the cyclotron resonance condition $\omega - k_{\parallel} v \mu = n\Omega$ broadened by scattering owed to $v_{\perp}$. Averaging Eq. 2.27.14 over gyro-phase ϕ yields the integral equation

$$\tilde{f}_o = (a\tilde{F}_o + \bar{Q})/[1 + (1 - \tau_{\parallel}/\tau_{\perp})a], \quad (2.27.20)$$

relating $\tilde{f}_o$ and $\tilde{F}_o$ where $\bar{Q} = \int_0^{2\pi} Q d\phi / (2\pi)$. The function a in Eq. 2.27.20 can be expressed in the form:

$$a = \bar{v}_{\parallel} \sum_{n=-\infty}^{\infty} \frac{J_n^2(k_{\perp} r_g \sin \theta)}{\bar{s} + \bar{v}_{\perp} + i(k_{\parallel} r_g \cos \theta + n)}. \quad (2.27.21)$$

The source term $\bar{Q}$ in Eq. 2.27.20 can be expressed in the form

$$\bar{Q} = \frac{AI(\phi_o, \theta_o) \exp(-i\mathbf{k} \cdot \mathbf{r}_o)}{(2\pi)^4 \Omega} \hat{Q}, \quad (2.27.22)$$

where

$$\hat{Q} = \exp(-\bar{s} - \bar{v}_{\perp} - i k_{\parallel} r_g \cos \theta) \sum_{n=-\infty}^{\infty} \frac{J_n(k_{\perp} r_g \sin \theta) \exp(-in\Phi)}{\bar{s} + \bar{v}_{\perp} + i(k_{\parallel} r_g \cos \theta + n)} \\ \times \left\{ 1 + \exp[2\pi(\bar{s} + \bar{v}_{\perp} + i k_{\parallel} r_g \cos \theta)] - \exp[\phi_o(\bar{s} + \bar{v}_{\perp} + i k_{\parallel} r_g \cos \theta)] \right\}. \quad (2.27.23)$$

Again note the singularities in Eq. 2.27.21 and Eq. 2.27.23 at the cyclotron resonances described by Eq. 2.27.19. By using the Newberger sum rule (Newberger, 1982)

$$\sum_{n=-\infty}^{\infty} \frac{J_n^2(z)}{n + \chi} = \frac{\pi J_{\chi}(z) J_{-\chi}(z)}{\sin(\pi\chi)} \quad (2.27.24)$$

in Eq. 2.27.21, one can obtain

$$a = i\bar{v}_{||} \frac{J_{\chi}(k_{\perp} r_g \sin \theta) J_{-\chi}(k_{\perp} r_g \sin \theta)}{\sin(\pi\chi)}, \quad (2.27.25)$$

as an alternative, more compact expression for a , where

$$\chi = (\omega - k_{||} v \mu + i v_{\perp}) / \Omega \quad (2.27.26)$$

is the normalized Doppler shifted frequency ω relative to the particle, taking into account perpendicular scattering.

Averaging of Eq. 2.27.20 over μ (the pitch-angle cosine), yields a simple algebraic equation for $\tilde{F}_o$ with solution

$$\tilde{F}_o = \frac{\langle \bar{Q} / [1 + (1 - \tau_{||} / \tau_{\perp}) a] \rangle}{1 - \langle a / [1 + (1 - \tau_{||} / \tau_{\perp}) a] \rangle}, \quad (2.27.27)$$

where the angular brackets in Eq. 2.27.27 denote an average over μ . For the case of isotropic scattering ($\tau_{||} = \tau_{\perp}$), Eq. 2.27.27 simplifies to

$$\tilde{F}_o = \langle \bar{Q} \rangle / (1 - \langle a \rangle). \quad (2.27.28)$$

2.27.5. Long-scale, large-time asymptotes

From Fedorov et al. (1995), Kota (1994) and Webb et al. (2000), the long time asymptotes for F_o can be obtained by investigating the dispersion equation

$$D(\mathbf{k}, s) = 1 - \langle a / [1 + (1 - \tau_{||} / \tau_{\perp}) a] \rangle, \quad (2.27.29)$$

associated with the singular eigen-solutions of Eq. 2.27.27. In particular, the diffusive behavior of the solution follows from the large space-scale ($k \rightarrow 0$) and long time ($s \rightarrow 0$) behavior of Eq. 2.27.29). For example, consider the case of

isotropic scattering ($\tau_{//} = \tau_{\perp}$) for which $D(\mathbf{k}, s) = 1 - \langle a \rangle = 0$ is the singular manifold. Using the expansion of the Bessel functions in Eq. 2.27.25 for $|k_{\perp} r_g \sin \theta| \ll 1$, Webb et al. (2001) obtain

$$a \approx \frac{i\bar{v}_{//}}{\chi} \left(1 + \frac{(k_{\perp} r_g \sin \theta)^2}{2(\chi^2 - 1)} + O[(k_{\perp} r_g)^4] \right), \quad (2.27.30)$$

Using Eq. 2.27.30 Webb et al. (2001) find

$$\langle a \rangle \approx -i\bar{v}_{//} b \left(I_1 + \frac{1}{2} (k_{\perp}^2 / k_{//}^2) I_2 \right) \quad (2.27.31)$$

for the approximate value of a over pitch-angle at long wavelengths, where

$$b = (k_{//} r_g)^{-1}, \quad I_1 = \frac{1}{2} \ln \left(\frac{\mu_c - 1}{\mu_c + 1} \right), \quad I_2 = \frac{1}{4b^2} \left\{ \ln \left(\frac{\mu_c - b - 1}{\mu_c - b + 1} \right) [1 - (\mu_c - b)^2] \right. \\ \left. + [1 - (\mu_c + b)^2] \ln \left(\frac{\mu_c + b - 1}{\mu_c + b + 1} \right) - 2(1 - \mu_c^2) \ln \left(\frac{\mu_c - 1}{\mu_c + 1} \right) \right\}, \quad \mu_c = ib(\bar{s} + \bar{v}_{\perp}). \quad (2.27.32)$$

From Eq. 2.27.31 and Eq. 2.27.32 the dispersion equation $D(\mathbf{k}, s) = 1 - \langle a \rangle = 0$ for $|s\tau| \ll 1$ and $kr_g \ll 1$ has the approximate solution

$$s = -(\kappa_{//} k_{//}^2 + \kappa_{\perp} k_{\perp}^2) + O(k^4), \quad (2.27.33)$$

where $\kappa_{//}$ and $\kappa_{\perp}$ are the parallel and perpendicular diffusion coefficients in Eq. 2.27.9 for $\tau_{//} = \tau_{\perp} = \tau$. Eq. 2.27.33 is the dispersion equation for the diffusion equation obtained from Eq-s 2.27.5, 2.27.8 and 2.27.9, but with no drift terms, since the background state is uniform.

On the other hand, if $k_{\perp}^2 / k_{//}^2 \ll 1$ the term I_2 can be dropped in Eq. 2.27.31, and the dispersion Eq. 2.27.29 has the approximate solution:

$$s \approx -\left(\kappa_{//} k_{//}^2 + \frac{1}{5} \kappa_{//}^2 k_{//}^4 \tau\right) + O(k_{//}^6). \quad (2.27.34)$$

The latter dispersion equation is equivalent to the equation:

$$\frac{1}{5} s^2 \tau + s + \kappa_{//} k_{//}^2 \approx 0. \quad (2.27.35)$$

In the space-time domain, Eq. 2.27.35 becomes the telegraph equation of Gombosi et al. (1993). Clearly, to obtain an equivalent telegraph equation including perpendicular diffusion, one needs to retain terms $O(k_{\perp}^4)$ in Eq. 2.27.30.

2.27.6. Pitch-angle evolution and perpendicular diffusion

It is instructive to consider the integral Eq. 2.27.20 under the assumption that $|k_{\perp} r_g| \ll 1$, so that the approximation described by Eq. 2.27.30 applies for a . Eq. 2.27.30 can be re-written in the form:

$$(v_{//}/a - v_{\perp}) \tilde{f}_o = v_{//} (\tilde{F}_o - \tilde{f}_o) + v_{//} \tilde{Q}/a. \quad (2.27.36)$$

Using the usual Fourier space map

$$\frac{\partial}{\partial t} \rightarrow -i\omega, \quad \nabla \rightarrow i\mathbf{k}, \quad (2.27.37)$$

and using the approximation described by Eq. 2.27.30 for a , Eq. 2.27.36 reduces to the approximate, integro-differential evolution equation

$$\frac{\partial f_o}{\partial t} + v\mu \frac{\partial f_o}{\partial z} - \frac{1}{2} \frac{(\partial_t + v\mu \partial_z + v_{\perp}) v^2 \sin^2 \theta}{(\partial_t + v\mu \partial_z + v_{\perp})^2 + \Omega^2} \nabla_{\perp}^2 = v_{//} (F_o - f_o) + v_{//} S^{-1} \left(\frac{\tilde{Q}}{a} \right), \quad (2.27.38)$$

where $\nabla_{\perp}^2 = \partial_x^2 + \partial_y^2$ is the Laplacian operator transverse to the magnetic field, which is assumed to lie along the z -axis, and S^{-1} is the inverse Laplace and Fourier transform operator. Assuming that f_o evolves on much longer time scales than $\tau_{\perp}, \tau_{//}$ and the gyro-period $2\pi/\Omega$ (i.e. $|f_{ot}/f_o| \ll v_{\perp}, v_{//}, \Omega$) and on space scales much larger than the mean free paths $v\tau_{//}$ and $v\tau_{\perp}$, then Eq. 2.27.38 can be approximated by the equation:

$$\frac{\partial f_o}{\partial t} + v\mu \frac{\partial f_o}{\partial z} - \frac{v^2 v_\perp (1 - \mu^2)}{2(v_\perp^2 + \Omega^2)} \nabla_\perp^2 f_o = v_\parallel (F_o - f_o) + v_\parallel S^{-1} \left(\frac{\tilde{Q}}{a} \right), \quad (2.27.39)$$

which is the pitch-angle evolution equation for f_o incorporating the effects of cross-field diffusion (the $\nabla_\perp^2 f_o$ term). Multiplying Eq. 2.27.39 by $2\pi p^2$ and integrating Eq. 2.27.39 over μ from $\mu = -1$ to $\mu = 1$, using the diffusion approximation, and neglecting the source, or initial value term in Eq. 2.27.39 results in the usual diffusion Eq. 2.27.5 in the form:

$$\frac{\partial N}{\partial t} + \frac{\partial}{\partial z} \left(-\kappa_\parallel \frac{\partial N}{\partial z} \right) + \nabla_\perp \cdot (-\kappa_\perp \nabla_\perp N) = 0, \quad (2.27.40)$$

where $\kappa_\perp$ and $\kappa_\parallel$ are given by Eq. 2.27.9. In the derivation of Eq. 2.27.40 it is also necessary to take the first moment of Eq. 2.27.39 (i.e. multiply Eq. 2.27.39 by $2\pi p^2 v\mu$ and integrate over μ from $\mu = -1$ to $\mu = 1$, and use the diffusion approximation to find the diffusive streaming parallel to the field). It is clear that accurate approximate solutions of Eq. 2.27.39 can be obtained by expanding the distribution function in terms of Legendre's polynomials, and taking moments of Eq. 2.27.39 (e.g., Gombosi et al. 1993; Lu et al., 2001).

2.27.7. Summary of main results

Summarizing the results discussed above, Webb et al. (2001) note that from the explicit solution for $\tilde{F}_o$ in Eq. 2.27.27, the complete solution for $f(\mathbf{r}, \mathbf{p}, t)$ for the case of Dirac-delta initial data in position, pitch-angle, and gyro-phase, can be constructed by Laplace-Fourier inversion. By the first determining $\tilde{F}_o$ from Eq. 2.27.27, and using the result to determine $\tilde{f}_o$ from Eq. 2.27.20, and then obtain $\tilde{f}$ from Eq. 2.27.14, followed by Laplace and Fourier inversion – to determine f . A multiple scattering analysis (e.g. Webb et al., 2000) and eigenfunction/moment equation methods should reveal further aspects of the solution. There are several outstanding issues raised by the above analysis. For example, in a non-uniform background magnetic field there is a non-zero contribution to the divergence of the particle current owed to curvature and gradient drifts associated with the anti-symmetric diffusion coefficient κ_A . It is of interest to determine whether the effects of these drifts can be included in a pitch-angle evolution equation analogous to Eq. 2.27.39 in this case. It is also of interest to investigate higher order transport effects in the model, e.g. the incorporation of CR inertial effects in telegraph type equations for CR transport including cross-field diffusion, that generalize the telegraph equation obtained by Gombosi et al. (1993). Other aspects of CR transport theory that are raised by the analysis, concern the form of

the pitch-angle evolution equation obtained by Skilling (1975) for particle transport in the solar wind, or its relativistic generalization (e.g. Webb, 1985) when cross field transport is included, and the role of cross-field transport effects on CR viscosity, and non-inertial acceleration effects.

2.28. Influence of magnetic clouds on the CR propagation

2.28.1. *The matter of the problem*

The propagation of energetic charged particles through interplanetary space is normally described by a transport equation which considers the effects of propagation parallel to the field, pitch-angle scattering at magnetic field irregularities, and focusing in the diverging interplanetary magnetic field (Roelof, 1969) or, in addition to the above effects, also convection with the solar wind and adiabatic deceleration (Ruffolo, 1995). Focusing is always considered for simple geometries, in general the Archimedean spiral field, although variations in the large scale magnetic field structure, in particular propagating magnetic flux ropes (ejects following coronal mass ejections, CMEs, also called magnetic clouds; for a review see e.g. Burlaga, M1995), modify the local focusing length and therefore also particle propagation.

In their detail investigation Kallenrode (2001a) takes into account that magnetic clouds modify the structure of the interplanetary magnetic field on spatial scales of tenth of AU. Their influence on the transport of energetic charged particles is studied with a numerical model that treats the magnetic cloud as an outward propagating modification of the focusing length. As a rule of thumb the influence of the magnetic cloud on particle intensity and anisotropy profiles increases with decreasing particle mean free path and decreasing particle speed. Special attention is paid to energetic particles running into a magnetic cloud released at an earlier time: here the cloud acts as a barrier storing the bulk of the particles in its downstream medium.

2.28.2. *The numerical model*

Since Kallenrode (2001a) is concerned with particles with energies in the MeV and tens of MeV range, solar wind effects such as convection and adiabatic deceleration are of minor importance (Ruffolo, 1995), in particular, if there are concerned with a long-lasting injection from a propagating interplanetary shock (Lario et al., 1998; Kallenrode, 2001b). For a first approach on the influence of a magnetic cloud, they started from the model of focused transport (Roelof, 1969):

$$\frac{\partial f}{\partial t} + \mu v_p \frac{\partial f}{\partial s} + \frac{1 - \mu^2}{2\zeta} v_p \frac{\partial f}{\partial \mu} - \frac{\partial}{\partial \mu} \left(\kappa \frac{\partial f}{\partial \mu} \right) = Q(t, s, \mu), \quad (2.28.1)$$

where $f(t, s, \mu)$ being the distribution function, t time, s distance along the Archimedian magnetic field spiral, v_p particle speed, μ pitch-cosine, $\kappa(s, \mu)$ pitch-angle diffusion coefficient, and

$$\zeta(s) = -B(s)/(\partial B/\partial s) \quad (2.28.2)$$

the focusing length. The terms in the transport Eq. 2.28.1 from left to right describe the field parallel propagation, focusing in a magnetic field with focusing length $\zeta(s)$ depending on distance, and pitch-angle scattering. The source term is allowed to propagate along the field line, simulating the long lasting injection of energetic particles from a shock as described in Kallenrode and Wibberenz (1997), the transport of energetic particles through the shock front is treated as described in Kallenrode (2001b). The magnetic cloud is assumed to be of spherical cross section with the interplanetary magnetic field draped symmetrically around it (see Fig. 2.28.1).

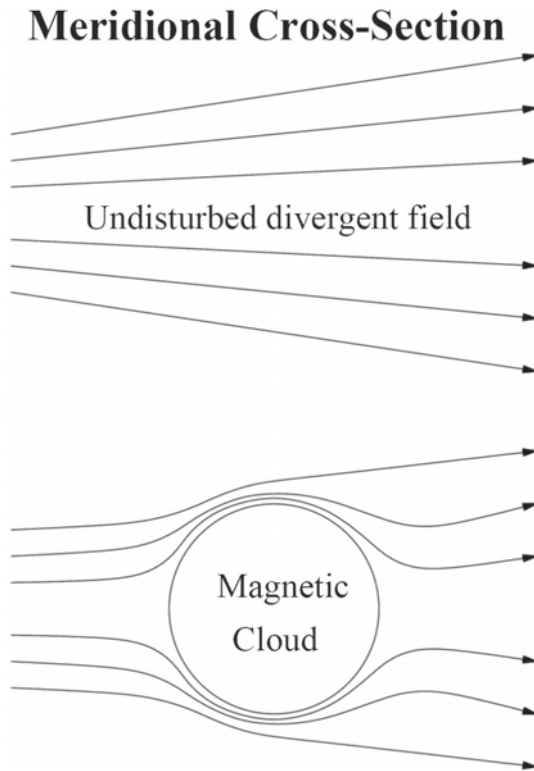


Fig. 2.28.1. Cross-section (perpendicular to the plane of ecliptic) for the undisturbed expanding magnetic field (top) and a field disturbed by a magnetic cloud (bottom). The field converges at the flanks of the cloud. According to Kallenrode (2001a).

Kallenrode (2001a) note that the main change is a compression of the interplanetary magnetic field at the flanks of the cloud. The magnetic cloud is characterized by its diameter d_c as a certain fraction of the distance r_s of the shock from the Sun, the distance r_{cs} of its leading edge from the shock, also expressed as a certain fraction of r_s , and its magnetic compression r_B at the flanks. For applications these data can be inferred from the observations; for the numerical study below Kallenrode (2001a) used $d_c = 0.2$ and $r_{cs} = 0.1$ (Bothmer, 1993). With

$$s_1 = s(r_s - (d_c + r_{cs})) \quad \text{and} \quad s_2 = s(r_s - r_{cs}) \quad (2.28.3)$$

this configuration than is translated into a sinusoidal variation of the focusing length

$$\zeta(s) = \begin{cases} \zeta_o(s) & \text{for } s \leq s_1, \\ \zeta_o(s) \pm \zeta_o(s) \sin\left(\frac{s-s_1}{s_2-s_1}\right) f(r_B) & \text{for } s_1 < s < s_2, \\ \zeta_o(s) & \text{for } s \geq s_2, \end{cases} \quad (2.28.4)$$

and a corresponding elongation of the interplanetary magnetic field line. The $\pm$ allows for the consideration of the magnetic cloud or a void in the field instead of the cloud. Asymmetric draping of field lines (Vandas et al., 1996) can be considered by assuming a stronger (or weaker) compression of the magnetic field with a more (or less) pronounced elongation of the field line. Kallenrode (2001a) note that this approach allows to describe the particle propagation in a flux tube draped around the magnetic cloud but not the features of energetic particles directly inside the cloud. It also does not consider the cross-field transport of energetic particles from the ambient medium into the magnetic cloud.

2.28.3. Numerical results

Fig. 2.28.2 shows intensity and anisotropy profiles for a solar energetic particle event (lower set of curves; observer at 1 AU, particle speed $v_p = 1$ AU/h corresponding to ~ 10 MeV protons, radial mean free path $\lambda_r = 0.1$ AU, δ -injection on the Sun) followed by a magnetic cloud with a constant speed of 800 km/s (no shock with particle acceleration considered here). The upper set of curves is for particles accelerated at a shock with constant speed of 800 km/s and constant acceleration efficiency, followed by a magnetic cloud. All other parameters are the same as for the solar event. The shock arrives at the drop in particle intensity around 50 h. The solid line gives the particle event without ejecta, the dotted lines are for a cloud geometry with a magnetic compressions at

its flank of 1.3 (lower amplitude) and 2. The latter value is in agreement the values inferred from numerical simulations (Vandas and Romashets, 2001).

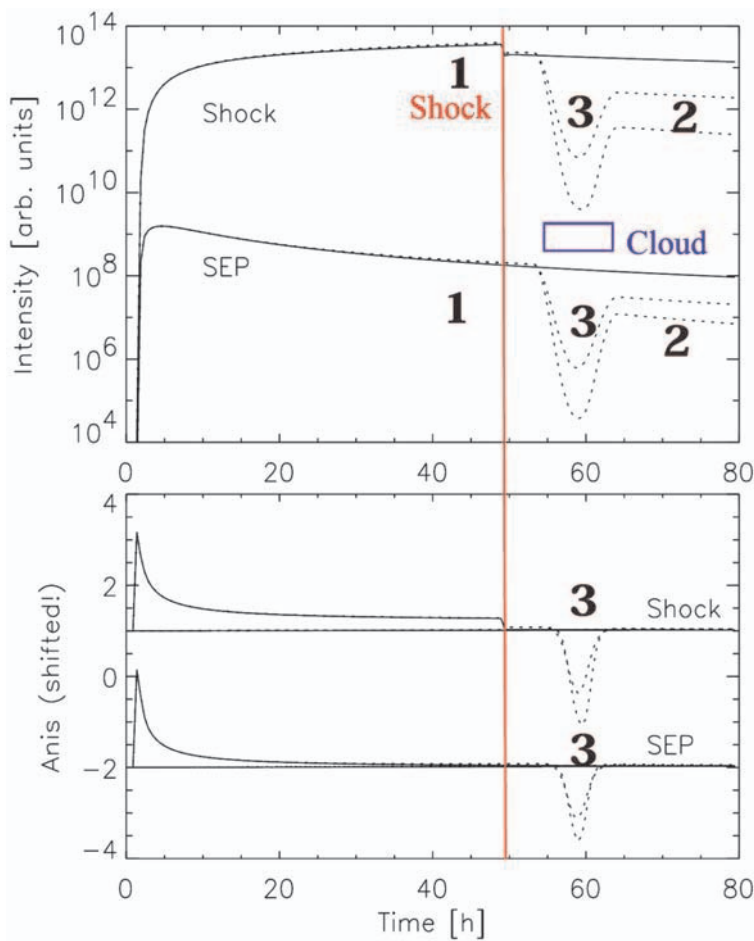


Fig. 2.28.2. Solar energetic particle event (lower set of curves) and shock accelerated particles (upper set of curves) followed by a magnetic cloud. The upper panel gives intensities for the two scenarios, the lower ones anisotropies (shifted with respect to each other). According to Kallenrode (2001a).

The presence of the magnetic cloud leads to: (1) a slight increase in intensities upstream of the cloud by a few percent, (2) a strong drop in intensities downstream of the cloud by about an order of magnitude, depending on the strength of the magnetic compression, and (3) a sharp drop of intensity at the time of cloud passage (remember, this is at the flanks not inside the cloud) combined with a strong anisotropy indicating a net-streaming of particles from the cloud's upstream medium (where intensities are high) into its downstream medium (where

intensities are low). Note that these effects are very similar for a simple solar injection as well as for the continuous particle injection from a propagating interplanetary shock. Quantitatively the influence of the magnetic cloud depends on particle speed and strength of the interplanetary scattering. With increasing scattering the increase in upstream intensities increases while the drop in downstream intensities decreases. The intensity drop at the time of cloud passage is independent of scattering while the anisotropy decreases with increasing scattering. With decreasing particle speed both upstream intensity increases and downstream intensity drops increase and the negative anisotropy inside the cloud becomes more pronounced. Thus faster particles are less influenced by the presence of the magnetic cloud than are slower ones. Fig. 2.28.3 gives the same set of curves as Fig. 2.28.2 except that the ejection has started 24 hours prior to the release of the energetic particles in a different solar event. In this case the ejection is running ahead of the particles and is at a radial distance of about 0.5 AU at the start of the particle injection. Again, solid lines are calculated without ejects, dotted ones with.

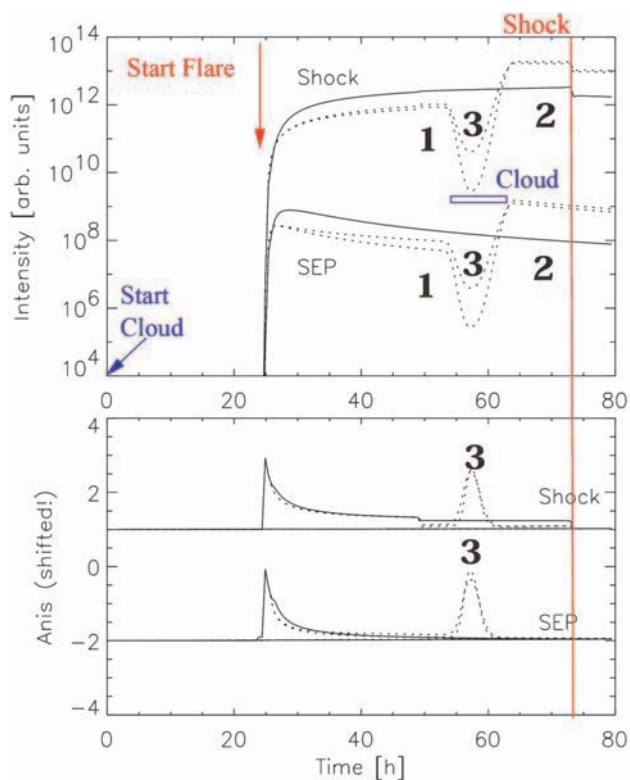


Fig. 2.28.3. The same as in Fig. 2.28.2, but the magnetic cloud is running ahead of a solar energetic particle event (lower set of curves) and a shock accelerated particle event (upper). According to Kallenrode (2001a).

According to Kallenrode (2001a), the most important results are (1) a pronounced decrease in intensities upstream of the magnetic cloud combined with (2) a pronounced increase in intensities downstream of the cloud, and (3) a strong drop in intensity at the time of passage of the cloud combined with a pronounced positive anisotropy, indicating a net-streaming of particles from the cloud's downstream into its upstream medium (again following the gradient in particle intensities). Again, effects are very similar for a solar injection and a continuous injection from a propagating shock. These results strongly point to a barrier effect of the magnetic cloud for the propagation of energetic particles.

2.28.4. Comparison with observations

Fig. 2.28.4 shows a comparison between a model run and the Helios observations in the 27 May 1981 event (for a detailed description see Kallenrode, 1997).

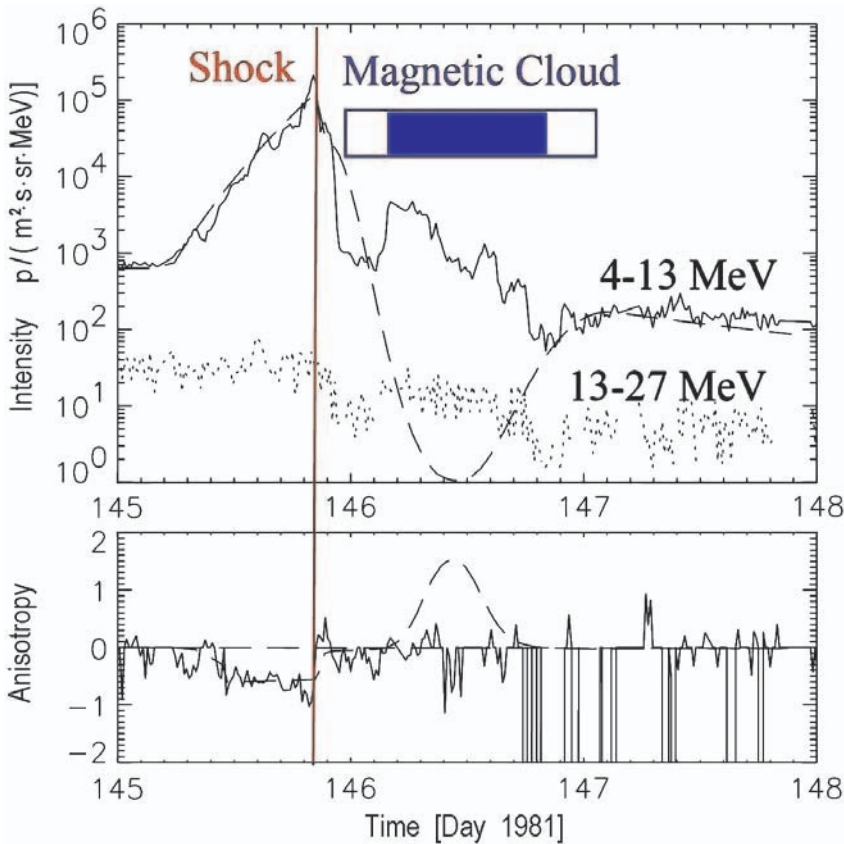


Fig. 2.28.4. Comparison between model (dashed) and observations, see text. According to Kallenrode (2001a).

In Fig. 2.28.4 the passage of the magnetic cloud is marked by a filled rectangle; interplanetary field lines draped around the cloud are represented by the adjacent open rectangles. These latter field lines are the ones which can be approximated in this model while the field lines inside the cloud are not considered. For modeling, this event is a challenge in so far as it shows a rather strong increase in intensity towards the shock combined with a drop in intensity short before the arrival of the magnetic cloud. This is impossible to model in a simple transport model because particles cannot be removed fast enough to get a significant decrease in intensity. If the magnetic cloud is considered, however, not only intensities upstream of the cloud can be fitted but also the fast decrease of intensity associated with the arrival of the field lines draped around the cloud and the reduced intensity in the cloud's downstream medium can be described properly. The description fails, by definition, right inside the cloud since the model only gives intensities along the field lines draped around the cloud but not inside the cloud; the satellite, on the other hand, cuts right through the cloud. Kallenrode (2001a) came to following conclusions:

(1) If the cloud follows the particle source, the upstream intensity is increased by a few percent for 10 MeV protons under average scattering conditions ($\lambda = 0.1$ AU);

(2) This increase increases with decreasing energy and increasing scattering;

(3) The downstream intensities are reduced by about an order of magnitude;

(4) If the cloud is ahead of the particle source, it is an effective barrier for particle propagation;

(5) The model allows the fitting of observations, although by definition intensities and anisotropies inside the cloud are not described correctly.

All these properties can be understood from the modified focusing: viewed from the outside, the bottleneck configuration shows a converging field and thus reflects part of the particles. Consequently, the cloud is a barrier that separates the upstream and downstream medium and allows for markedly different intensities in both of them. Intensities are higher on that side of the cloud where the source is located (upstream in case of a traveling shock, downstream in case of a magnetic cloud from an earlier event). At the bottleneck, intensities are reduced because only the relatively small number of particles just in transit can propagate through. In addition, anisotropies are relatively high because only particles with small pitch-angle can propagate into the bottleneck. Changes in intensity and anisotropy related to the presence of the cloud increase with decreasing energy and mean free path because in these cases particles stay longer in the vicinity of the cloud and thus can perform multiple interactions. For weak scattering and high energies, on the other hand, once a particle has passed the cloud it has only a small return probability. The enhancement of the barrier function of the cloud with increasing scattering also had been proposed by Lario et al. (1999). A relatively unexpected effect was the strong barrier action of a cloud ahead of the particle source. Since SOHO observations show a large number of CMEs during solar maximum (about

2 per day, see in St.Cyr et al., 2000), magnetic clouds in interplanetary space ahead of a particle source might be a relatively common feature. Fits of a transport equation on particle events neglecting the influence of a magnetic cloud might be faulty. This might explain part of the discrepancy between particle mean free paths determined from fits and particle mean free paths determined from the analysis of magnetic field fluctuations (Wanner and Wibberenz, 1993). In addition, the barrier properties of the magnetic cloud as demonstrated in Fig. 2.28.3 can be used to simulate rogue events where converging shocks lead to unusual high particle intensities as described for the August 1972 event by Levy et al. (1976). First examples are described in Kallenrode and Cliver (2001).

2.29. Non-diffusive CR particle pulse transport

2.29.1. *The matter of the problem*

In Fedorov et al. (2002) there are developed a theory of the transport of an anisotropic pulse of CR charged particles injected into moved space plasma with frozen in magnetic field (with applications to the anisotropic ground level solar CR events). For these events the kinetic regime is considered when the mean free path is comparable with the distance from particle source to detector. The problem is that in many cases the ground-level neutron monitors network detects complicated temporal solar CR intensity profiles, when the profile starts with narrow peak of ‘direct particles’ with a following diffusion tail of many times scattered particles (Fisk and Axford, 1969; Lupton and Stone, 1973; see reviews in Dorman and Miroshnichenko, M1968; Dorman, M1978; Dorman and Venkatesan, 1993, Miroshnichenko, M2001). In these cases a strong anisotropic pitch-angle distribution of particles in the interplanetary magnetic field is observed which implies a need to consider non-diffusive particle transport (Earl, 1994; Fedorov and Shakhov, 1994; Fedorov et al., 2002), because the mean free path determined by the collision integral is comparable with the distance from the source to the detector. Some of the ground level events are characterized by an impulse peak, which has been observed by the ground-based neutron monitors at Kerguelen and Apatity during the solar proton event on 7–8 December 1982, and at Deep River and Apatity during the event of 16 February 1984 (Borovkov et al., 1987; Smart et al., 1987; Smart and Shea, 1990; Perez-Pereza et al., 1992). Unlike these events, during the event on 22 October 1989 neutron monitors at the South Pole and Calgary have registered a short intensive peak after which a basic enhancement followed (Bieber et al., 1990; Flückiger and Köbel, 1993). A similar event, which was observed on 24 May 1990, is also characterized by a strong anisotropy of particle angular distribution as well as by very complicated temporal structure (Morishita et al., 1995; Torsti et al., 1996; Debrunner et al., 1992, 1997). Fedorov et al. (2002) attempt to give simplified model of these events on the basis of the kinetic theory approach of Fedorov and Shakhov (1994), Fedorov and Stehlik (1997), Fedorov et al. (1995). Their solution includes both the angle

distribution of injected particles and the angular response function of NM as well as a finite time of the particle injection in the source.

2.29.2. Kinetic equation

According to Fedorov et al. (2002), in the theoretical consideration the regular IMF is taken to be homogeneous. Nevertheless, the formation of an initial angle distribution of particles into narrow stream along the regular IMF (Lumme et al., 1986) owed to the magnetic focusing of force-lines near the Sun is included, see below. Thus, evolution of the particle distribution function $f(y, \tau, \mu)$ follows from the kinetic equation written in the drift approximation, in which the particle scattering on stable magnetic inhomogeneities is supposed to be isotropic (Fedorov et al., 1995):

$$\partial_\tau f + \mu \partial_y f + f - \frac{1}{2} \int_{-1}^1 f d\mu = \frac{\nu_s}{\nu} \delta(y) \delta(\tau) \varphi(\mu), \quad (2.29.1)$$

where y and τ are the coordinate along regular IMF and the time, respectively (in the dimensionless units $y = z\nu_s/\nu$, $\tau = \nu_s t$; ν_s is the collision frequency of particles with the magnetic clouds; z is the coordinate along the IMF), $\mu = \cos \theta$, θ is the particle pitch-angle. The right-hand side of Eq. 2.29.1 describes an instantaneous injection of CR particles with an initial angular distribution

$$\varphi(\mu) = \frac{a_\mu \Delta_\mu}{2(\Delta_\mu^2 + (\mu - \mu_o)^2)}, \quad (2.29.2)$$

where $\mu \in (-1, 1)$. The value of the constant a_μ , which depends on a maximal value direction of μ_o and a width Δ_μ , can be found from the normalization condition, $\int_{-1}^1 \varphi(\mu) d\mu = 1$, and it is equal to

$$a_\mu = 2 \left(\arctan \frac{1 - \mu_o}{\Delta_\mu} + \arctan \frac{1 + \mu_o}{\Delta_\mu} \right)^{-1}. \quad (2.29.3)$$

Owing to the focusing effect in the IMF mentioned, μ_o and Δ_μ have values equal to 1 and 0.01, respectively (see numerical estimations by Fedorov and Stehlik, 1997).

2.29.3. Pitch-angle response function for neutron monitors

Fedorov et al. (2002) used the pitch-angle response function for neutron monitors $\psi(\lambda)$ which is similar to $\varphi(\mu)$ in which μ has to be replaced by λ (λ is the pitch-angle of an asymptotic NM direction related to the regular IMF direction). The value of λ_o corresponds to the angle of a maximal sensitivity of detector, the parameter Δ_λ characterizes a width of directional diagram of the neutron monitors.

2.29.4. Time-finite injection

According to Fedorov et al. (2002), an intensity enhancement of the registered by neutron monitors solar energetic particles arises suddenly at $\tau = y$ for a δ -like particle injection, and a width of the impulse peak connected with arriving of the first particles is very short. Usually one needs to suppose, based on the description of measured temporal profiles of past solar proton events, that the injection of high energy particles into the interplanetary medium has a finite duration, which is caused mainly by the propagation of accelerated particles in the solar corona (Lumme et al., 1986; Borovkov et al., 1987).

The injection of accelerated particles from the source into the IMF during a finite time can be represented by the following time injection function:

$$\chi(\tau) = \nu_o^2 \tau \exp(-\nu_o \tau), \quad (2.29.4)$$

where the dimensionless quantity ν_o^{-1} is an unique parameter, which characterizes the mean duration of the injection as well as the instant of maximum at $\tau_m = \nu_o^{-1}$. It was assumed that τ is measured in the dimensionless quantity $\tau = t\nu_s = vt/\Lambda$, where Λ is the particle mean path. It is also reasonable to suppose that the duration of the emission by that ‘particle source’ of the lower energy particles is longer, so the quantity ν_o will be dependent on the particle rigidity. Building all these ‘weight’ functions above into the consideration, a detector will register

$$G(y, \tau) = \int_0^\tau d\xi \int_{-1}^1 d\mu \chi(\tau - \xi) f(y, \xi, \mu) \psi(\mu). \quad (2.29.5)$$

2.29.5. Three parts of resulting solution

The solution $G(y, \tau)$, described by Eq. 2.29.5, has been obtained by Fedorov and Stehlik (1997) using the method of the direct and inverse Fourier–Laplace transform and it consists of three terms:

$$G(y, \tau) = G_{us}(y, \tau) + G_s^o(y, \tau) + G_s^d(y, \tau). \quad (2.29.6)$$

The first component describes a contribution of the unscattered particles which exponentially decreases with time τ :

$$G_{us}(y, \tau) = \frac{\nu_s \nu_o^2 \exp(-\nu_o \tau)}{\nu} \int_0^\tau \frac{d\xi}{\xi} (\tau - \xi) \varphi\left(\frac{y}{\xi}\right) \psi\left(\frac{y}{\xi}\right) \exp(\xi(\nu_o - 1)). \quad (2.29.7)$$

A contribution of the scattered particles can be divided into two parts. One, the non-diffusive term $G_s^o(y, \tau)$, also exponentially decreases with time, and another term, $G_s^d(y, \tau)$, has a leading meaning in the diffusive limit of $\tau \gg 1$. Namely, the non-diffusive term reads

$$G_s^o(y, \tau) = \frac{\nu_s \nu_o^2 \exp(-\nu_o \tau)}{8\pi\nu} \times \left\{ \int_0^{y/\tau} d\eta \Psi(y, \tau, \eta) [S(\tau) - S(y)] + \int_{y/\tau}^1 d\eta \Psi(y, \tau, \eta) [S(y/\eta) - S(y)] \right\}, \quad (2.29.8)$$

where

$$\Psi(y, \tau, \eta) = \frac{\exp(y\Lambda(\eta)/2)}{((\mu_o - \eta)^2 + \Delta_\mu^2)((\lambda_o - \eta)^2 + \Delta_\lambda^2)}, \quad (2.29.9)$$

$$S(\xi) \equiv S(\xi; y, \tau, \eta) = \frac{\exp(A)}{A^2 + (\pi\eta/2)^2} \times \{(\beta_1 b_1 + \beta_2 b_2) \cos(\pi(y - \xi\eta)) + (\beta_1 b_2 - \beta_2 b_1) \sin(\pi(y - \xi\eta))\}, \quad (2.29.10)$$

In Eq. 2.29.9 and 2.29.10 the following denominations are used:

$$\begin{aligned} A &= \nu_o - 1 - \eta\Lambda(\eta), \quad \beta_1 = (\tau - \xi)A + \frac{A^2 - (\pi\eta/2)^2}{A^2 + (\pi\eta/2)^2}, \quad \beta_2 = (\tau - \xi)\frac{\pi\eta}{2} + \frac{\pi\eta A}{A^2 + (\pi\eta/2)^2}, \\ \Lambda(\eta) &= \ln(1 - \eta) - \ln(1 + \eta), \quad b_1 = \pi(\Delta_\mu a_\mu \Pi_\lambda(\eta) + \Delta_\lambda a_\lambda \Pi_\mu(\eta)), \\ b_2 &= \Pi_\mu(\eta) \Pi_\lambda(\eta) - \pi^2 \Delta_\mu \Delta_\lambda a_\mu a_\lambda, \quad \Pi_\mu(\eta) = 2(\mu_o - \eta) + \Delta_\mu(2\alpha_\mu + a_\mu \Lambda(\eta)), \\ \alpha_\mu &= \frac{a_\mu}{4} \left[\ln\left(\Delta_\mu^2 + (1 + \mu_o^2)^2\right) - \ln\left(\Delta_\mu^2 + (1 - \mu_o^2)^2\right) \right]. \end{aligned} \quad (2.29.11)$$

The last (diffusive non-vanishing) term in Eq. 2.29.6 has a sense only for $|y| < \tau$ and reads as

$$G_s^d(y, \tau) = \frac{\nu_s \nu_o^2}{4\pi\nu} \int_{-\pi/2}^{\pi/2} \frac{dk}{k^2} \Phi(y, k) \left\{ e^{\tau\kappa} - e^{(y-\tau)\nu_o + y\kappa} [1 + (\tau - y)(\nu_o + \kappa)] \right\}, \quad (2.29.12)$$

where $\kappa \equiv k \cot k - 1$, and

$$\Phi(y, k) = \frac{(B_\mu B_\lambda - \Gamma_\mu \Gamma_\lambda) \cos(ky) + (B_\mu \Gamma_\lambda + \Gamma_\mu B_\lambda) \sin(ky)}{D_\mu D_\lambda \cos^2 k}, \quad (2.29.13)$$

$$B_\mu(k) = \Delta_\mu a_\mu (\mu_o^2 + \Delta_\mu^2) k \tan^3 k + (\mu_o^2 - \Delta_\mu^2 + 2\mu_o \Delta_\mu \alpha_\mu) \tan^2 k - \Delta_\mu a_\mu k \tan k + 1 \quad (2.29.14)$$

$$\Gamma_\mu(k) = [(\mu_o + \Delta_\mu \alpha_\mu)(\mu_o^2 + \Delta_\mu^2) \tan^2 k - 2\mu_o \Delta_\mu a_\mu k \tan k + (\mu_o - \Delta_\mu \alpha_\mu)] \tan k, \quad (2.29.15)$$

$$D_\mu(k) = \left[1 - (\mu_o^2 + \Delta_\mu^2) \tan^2 k \right]^2 + 4\mu_o^2 \tan^2 k. \quad (2.29.16)$$

The expressions for similar quantities $B_\lambda, \Gamma_\lambda, D_\lambda, \Pi_\lambda, a_\lambda, \alpha_\lambda$ follow from the above expressions by substituting $\mu \rightarrow \lambda, \mu_o \rightarrow \lambda_o, \Delta_\mu \rightarrow \Delta_\lambda$. Fedorov et al. (2002) note that both terms $G_{us}(y, \tau)$ and $G_s^o(y, \tau)$ vanish in the diffusive limit owed to the remaining factor of $\exp(-\nu_o \tau)$; so only $G_s^d(y, \tau)$ gives the main contribution in this limit.

2.29.6. Expected temporal profiles for neutron monitors and comparison with observations

According to Fedorov et al. (2002), the main peculiarity of the solar CR events is connected with some neutron monitors (Hobart – HO, Mt. Wellington – WE, Lomnický Štít – LS) having the narrow peak of the anisotropic stream of the first fast particles, other neutron monitors (Oulu – OU, Apatity – AP, Thule – TH, Durham – DU, Mt. Washington – WA) show a diffusive tail with a wide maximum at a later time, or, show both – the first narrow peak with a second diffusion maximum (South Pole – SP). For example, some selected NM data for the 24 May 1990 are demonstrated in Fig. 2.29.1. The time (in min) is measured from the onset of particle injection taken as 20.50 UT of May 24, 1990.

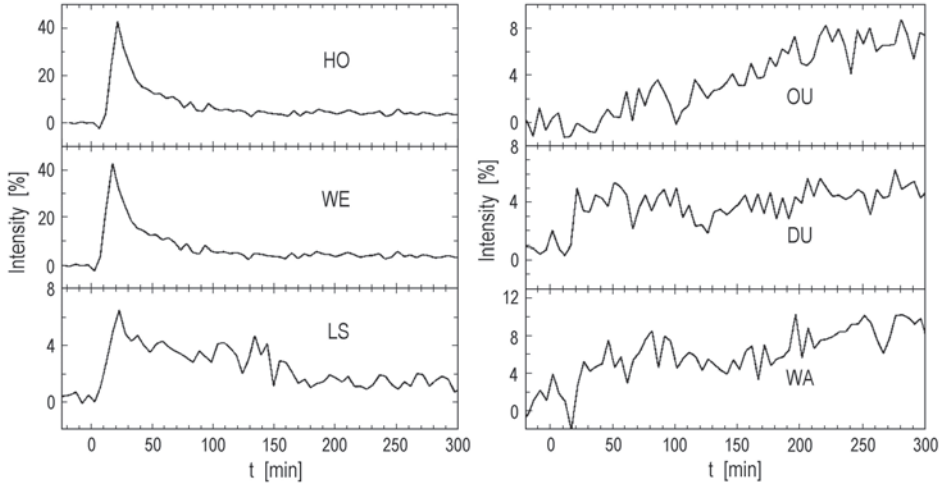


Fig. 2.29.1. Two groups of NM records of the event on 24 May 1990. Left - have the narrow peak of the anisotropic stream of the first fast particles (HO – Hobart, WE– Mt. Wellington, LS – Lomnický Štít); right - show a diffusive tail with a wide maximum at a later time (OU – Oulu, DU – Durham, WA– Mt. Washington). According to Fedorov et al. (2002).

Comparison of the NM data with the theoretical prediction based on the kinetic equation solution requires a choice of a ‘normalizing NM station’ and consequent rigidity-dependent re-calculation of input parameters entering into the theoretical profile calculation. The NM station HO was chosen to be that station because it allows to determine the starting parameters at its mean rigidity $\bar{R}(\text{HO}) = 2.3$ GV. This value, as well as the others, have been calculated by assumption of a particle rigidity spectrum roughly $\propto R^{-5}$ in the initial phase. For other NMs was taken the following calculated values of the mean rigidity: $\bar{R}_{WE} = 2.3$ GV, $\bar{R}_{LS} = 2.3$ GV, $\bar{R}_{OU} = 1.0$ GV, $\bar{R}_{DU} = 2.0$ GV, $\bar{R}_{WA} = 1.8$ GV, and $\bar{R}_{SP} = 0.8$ GV. The mean rigidity $\bar{R}$ was obtained from trajectory computations for $R < 10$ GV with a step of 0.01 GV by a technique owed to Kassovicova and Kudela (1998). Each allowed trajectory was assigned by the weight corresponding to the solar proton spectra $\propto R^{-5}$ and the coupling function according to Dorman (M1975a). The geomagnetic field model for trajectory calculations included the IGRF plus the Tsyganenko 89 model (Tsyganenko, 1989) for $Kp > 5$.

The asymptotic directions λ_o for NMs have been obtained by numerical integration of particle motion in the geomagnetic field (by the method described in Kassovicova and Kudela, 1998; see in details in the Chapter 3 of Dorman, M2006) for the given epoch at 21:00 hours, and then they were averaged over both the rigidity-dependent response function of NM and the particle rigidity spectrum. For each allowed trajectory the pitch-angle was assigned and the mean

value λ_o as well as dispersion $\Delta\lambda$ were obtained from the histogram of the expected pitch-angle distribution (for the computations are used vertically incident particles). This simplification is used because:

- (a) the contribution to NM count rate is in the geometric approach inversely proportional to cosine of zenith angle,
- (b) the main limitation for the trajectory computations is the magnetic field model (Smart et al., 2000),
- (c) these computed results (Kassovicova and Kudela, 1998) can be compared with the vertical cutoff rigidities obtained by other methods (Shea and Smart, 2001).

The mean transport path Λ in IMF is supposed to be independent of rigidity for the considered interval of NM sensitivity rigidity. Elementary calculation shows that

$$\nu_o = (10.8/t_m)(\Lambda/z) = 10.8/(yt_m), \quad (2.29.17)$$

where z and $y = z/\Lambda$ is the distance of the detector (the Earth) from the source (the Sun) and the dimensionless one, respectively. Therefore the fit of the theoretical curve to experimental data of HO determines $\nu_o = \nu_o(t_m)$ for given y_{HO} . The best fit gives $t_m = 12.4$ min for $y_{HO} = 0.6$. Values of ν_o for the other NMs are calculated assuming the rigidity dependence of $t_m \propto \bar{R}^{-\beta}$ using the given value y_{HO} . The spectrum index β characterizes shape of a low energy particle delay in the corona. Fedorov et al. (2002) have used the value of $\beta = 1$.

For comparison of theoretical dependences with the experimental data the dependence of GCR intensity on particle rigidity also has been taken into consideration. Let this dependence be $I_g \propto R^{-\gamma_g}$, where γ_g is GCR spectrum index, and the SCR rigidity dependence at the instant of its injection into IMF is $I_s \propto R^{-\gamma_s}$. All theoretical curves are standardized to a maximum relative to the mean rigidity of HO, i.e., the curve HO has the value 1 in maximum. The multipliers $(\bar{R}_{HO}/\bar{R}_i)^{\gamma_s - \gamma_g}$ (where $i = OU, WE, WA, \text{etc.}$) which take into account the rigidity spectra of GCR and SCR, must be used in calculation of the rigidity dependence of the i -th NM. The values of the maxima of the theoretical curves of the i -th NM are conditioned by the difference of $\Delta\gamma = \gamma_s - \gamma_g$.

The experimental records of temporal profiles can be divided roughly into two groups, one of NMs which in the initial phase (about the first hour after the particle onset) have asymptotic direction near the regular IMF direction (Fig. 2.29.1, left panel), and the others whose the asymptotic direction differs from it (Fig. 2.29.1, right panel). The theoretically predicted temporal profiles for the selected NMs in the model described in Section 2.29.5 are demonstrated in Fig.

2.29.2, left and right panels, respectively, using the calculated asymptotic direction for each NM station. This calculation shows that HO and WE have very similar characteristics, λ_o is 0.9 and 0.86, respectively, with $\Delta\lambda = -0.26$. Station LS has $\lambda_o = 0.34$, $\Delta\lambda = 0.4$. In the second group of NM's, OU, DU, and WA have $\lambda_o = -0.94, -0.9, -0.85$ and $\Delta\lambda = 0.06, 0.1, 0.3$, respectively. Note that Oulu and Apatity give absolutely the same theoretical curves resulting from their similar characteristics and very similar temporal profiles of the event. The last two NMs (DU and WA) experienced small increases at 1–2 hours after onset, as the theory predicts, see Fig. 2.29.2 (right panel), owing to smaller λ_o and larger $\Delta\lambda$ and larger mean $\bar{R}$.

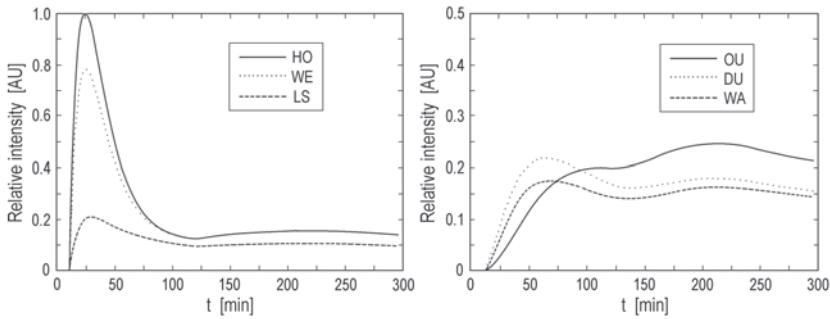


Fig. 2.29.2. Theoretical prediction of temporal profiles for the selected NMs using calculated parameters λ_o , $\Delta\lambda$ and mean $\bar{R}$ for each NM. On the ordinate axis are shown expected intensity relative to HO in maximum. According to Fedorov et al. (2002).

Especially interesting are data of the South Pole (SP) station (see Fig. 2.29.3).

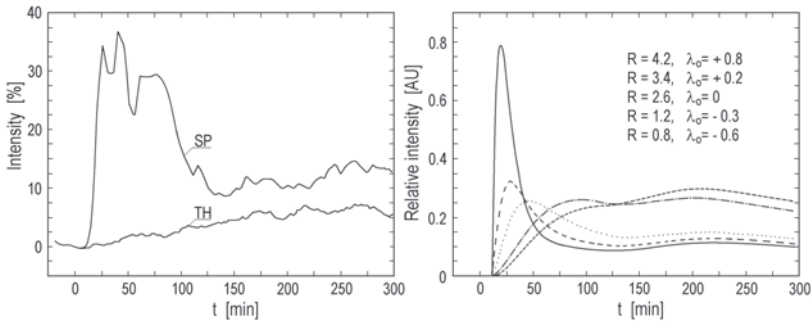


Fig. 2.29.3. The NM record and theoretical prediction for South Pole (SP) station, which detected both the high-energy stream and the diffusive tail. The northern Thule (TH) station detected only the diffusive tail. Left panel – observations; right panel – predicted temporal curves for different values of $\bar{R}$ (in GV) and λ_o . According to Fedorov et al. (2002).

From Fig. 2.29.3 (left panel) it can be seen that South Pole records indicate a more complicated structure being probably a mixture of the anisotropic stream of fast particles and a diffusive tail of lower energy particles. In the initial phase SP looked at the source with very narrow width $\Delta\lambda = 0.1$ and registered the anisotropic stream of particle rigidity up to 10 GV. The double peak structure is probably caused by very fast irregular changes in latitude as well as longitude of the apparent source direction (IMF direction). Thus the SP asymptotic direction could jump from one force-line to another and, therefore, it could repeatedly see the source. The model (right panel in Fig. 2.29.3) predicts the two types of time profile as a function of $\bar{R}$. In contrast to the South Pole the Thule station in the north (both with zero vertical cutoff rigidities and with the same receipt parameters) shows only the second, diffusive-like tail (see in Fig. 2.29.3, left panel). Probably the North–South anisotropy of lower energy particles which is not described in the present model and/or the latitudinal component of IMF direction can cause such difference. Therefore it is not possible to consider SP in the simple model with one characteristic (mean rigidity $\bar{R}$ of a NM station), especially in the initial phase of the event. Thus the real picture of registered profile on Fig. 2.29.3 (left panel) is some mixing of these theoretical curves (presented in the right panel of Fig. 2.29.3).

2.30. Pitch-angle diffusion of energetic particles by large amplitude MHD waves

2.30.1. *The matter of the problem*

Hada et al. (2003a) consider some fundamental properties of pitch-angle diffusion of charged particles by MHD waves by performing test particle simulations. Even at a moderate normalized turbulence level (turbulence magnetic field energy density normalized to the background field energy density ~ 0.1), both the mirroring and the resonance broadening effects become important, and the diffusion starts to deviate substantially from the standard quasi-linear diffusion model. Generally speaking, the transport of CR charged particles by MHD turbulence is one of the key issues in space and astro-plasma physics. Pitch-angle diffusion is fundamental to other transport processes such as the energy and the parallel diffusion (Jokipii, 1966; Terasawa, 1991; Michalek and Ostrovski, 1996; Tsurutani et al., 2002). For the discussion of the various transport processes of CR in space plasma the quasi-linear theory is frequently used, in which two assumptions are fundamental. First, the turbulence amplitude is sufficiently small, so that truncation at the second power of the turbulence is guaranteed. Second, the wave phases are random (random phase approximation), so that any effect of mode-mode coherence is destroyed by phase mixing. However, the MHD turbulence in space does not necessarily satisfy these assumptions: in particular, the waves excited near collisionless shocks have the wave magnetic field

amplitude comparable to or even larger than the background field. In addition, their waveforms show consequences of strong nonlinear evolution (e.g., the shocklets found in the Earth's foreshock region – according to Hoppe et al., 1981), suggesting the presence of the phase coherence (Hada et al., 2003b). It is a main cause why Hada et al. (2003a) investigate the pitch-angle diffusion of energetic particles by MHD waves, which are not necessarily small amplitude, and their phases not necessarily random, by numerically integrating in time the equations of motion of charged particles under influence of given MHD turbulence.

2.30.2. The model used

Hada et al. (2003a) employ the so called slab model for the MHD turbulence, although this is probably an over-simplification for the turbulence in reality (e.g., in the solar wind according to Matthaeus et al., 1990). Within this model the fluctuation electromagnetic field is given as a superposition of parallel propagating, circularly polarized finite amplitude Alfvén waves, with different wave numbers and different polarizations. Since the typical particle velocity far exceeds the Alfvén wave's speed, it was assumed that the waves to be non-propagating: within this system, particle energy is conserved. For both groups of waves with different polarizations it was assumed that the wave spectrum is given by a power law (with an index γ), and their phases be related by the iteration formula defined in Eq. 4 of Kuramitsu and Hada (2000).

2.30.3. Main results of simulation

Fig. 2.30.1 shows the time evolution of distribution of particle pitch-angle cosine, μ , defined as an inner product of the unit vectors parallel to the particle velocity and the local magnetic field.

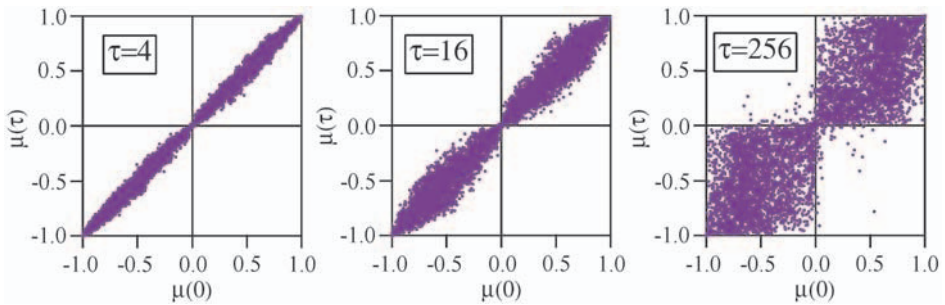


Fig. 2.30.1. Time evolution of μ for $\delta B = 0.01$. According to Hada et al. (2003a).

For each panel in Fig 2.30.1 the horizontal axis represents the initial distribution, $\mu(0)$, and the vertical axis denotes the distribution at some later times, $\mu(\tau)$. Each dot represents a single test particle. Important parameters used here

are: $\gamma = 1.5$, $c_\phi = 0$ (random phase), and the variance of the normalized perpendicular magnetic field fluctuations, $\delta B = 0.01$. At $\tau = 1$ the distribution of μ has not evolved much, and so the dots are almost aligned along the diagonal line. Later at $\tau = 16$ pitch-angle diffusion is more evident, but is still absent around $\mu \sim 0$ and $|\mu| \sim 1$. The former is owed to the lack of waves which resonate with near 90° pitch-angle, and the latter is simply owing to geometry. At an even later time at $\tau = 256$, substantially longer than the pitch-angle diffusion time scale, it is clear that the majority of particles stay within the hemisphere they belonged to initially. Three panels from the left in Fig. 2.30.2 show the same plots as before except that the turbulence level is increased to $\delta B = 0.1$, keeping other parameters unchanged.

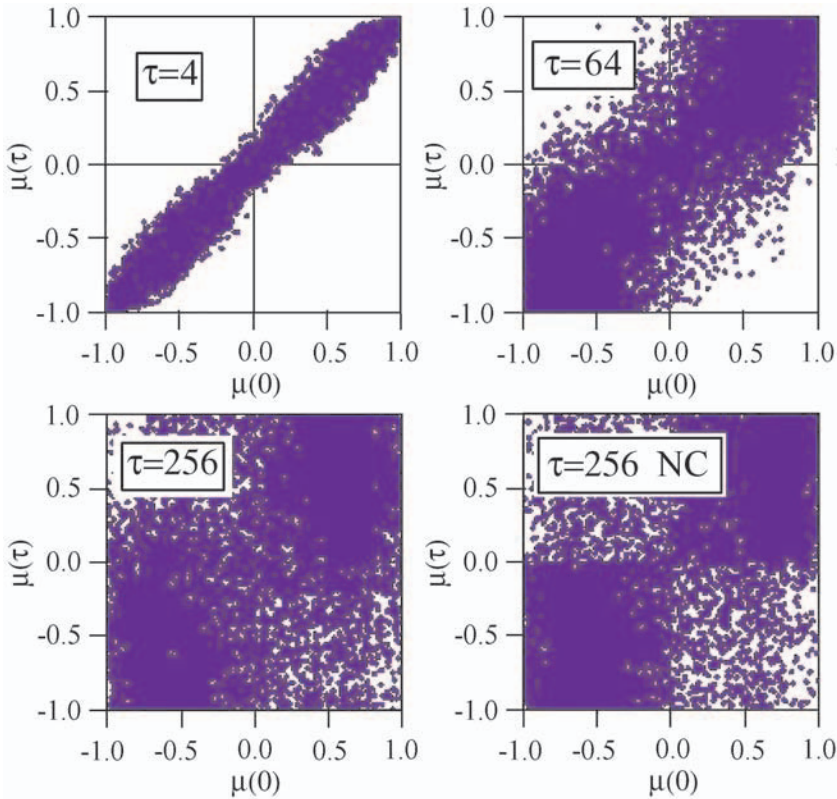


Fig. 2.30.2. Time evolution of μ for $\delta B = 0.1$. Non-compressional turbulence is used for the run shown in the right bottom panel. According to Hada et al. (2003a).

From the comparison of the two runs it is clear that not only the diffusion occurs on a faster time scale but also that many particles traverse the 90° pitch-angle. This is mainly owed to the mirroring and the resonance broadening, both of which are the consequences of finite amplitude waves. These two effects can be

separate by making the turbulence non-compression, $\mathbf{b}'(x) = \delta B \mathbf{b}(x)/|\mathbf{b}(x)|$, where $\mathbf{b}(x)$ is the given compression turbulence (the power spectrum and the phase distribution of $\mathbf{b}(x)$ and $\mathbf{b}'(x)$ are not exactly the same). The distribution of μ as diffused by such a non-compression turbulence is shown in the right bottom panel of Fig. 2.30.2. Although the number of particles crossing the 90° pitch-angle is less compared with the compression case, it is shown that the resonance broadening alone can mix the particles across $\mu = 0$. Fig. 2.30.3 as well as Fig. 2.30.4 summarizes the numerically evaluated pitch-angle diffusion coefficient D , compared with the value D_{QL} obtained from the quasi-linear theory,

$$D_{QL} = \frac{\pi e^2}{2m^2 c^2 v |\mu|} (1 - \mu^2) P(k_r), \quad (2.30.1)$$

where $k_r = -\Omega/v\mu$ is the resonance wave number, $P(k)$ is the wave power spectrum, Ω is the particle gyro-frequency, and other notations are standard (Gary and Feldman, 1978; Kennel and Engelmann, 1966; Lee, 1971).

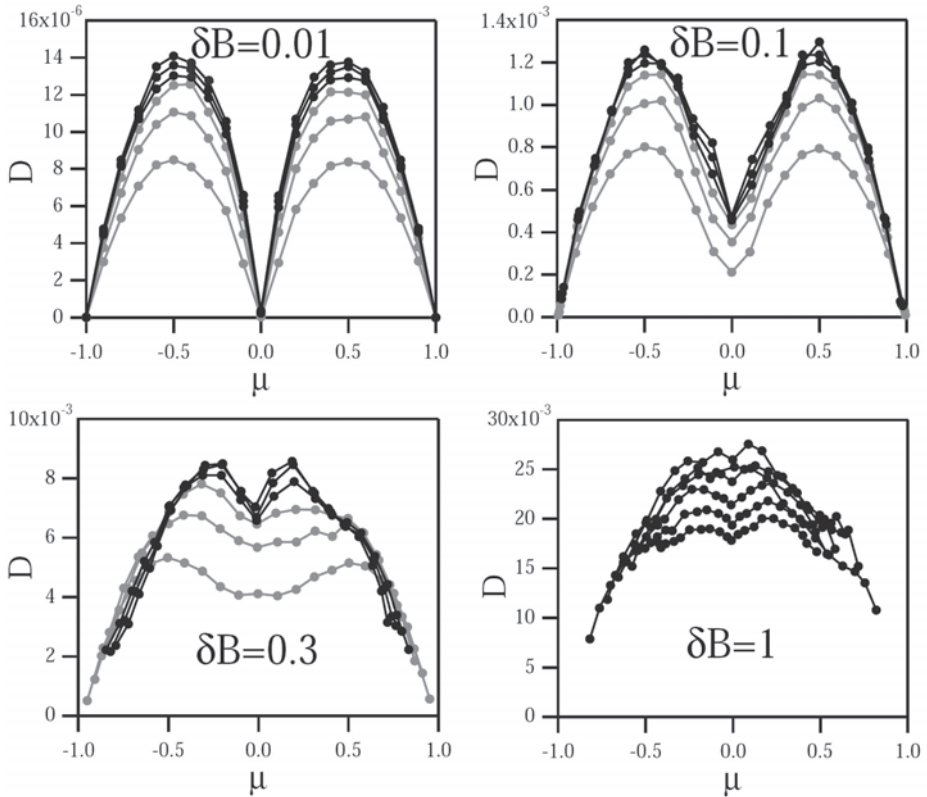


Fig. 2.30.3. D versus μ . According to Hada et al. (2003a).

Four panels of Fig. 2.30.3 show D versus μ for various values of δB . The turbulence is compression, and the wave phases are random. When δB is small, D is doubly peaked, as it vanishes at $\mu = 0$, -1 , and 1 . However, as the turbulence amplitude is increased, the diffusion at $\mu = 0$ becomes drastically enhanced. At $\delta B \sim 0.3$, D is of the same order with respect to μ . This is also apparent in Fig. 2.30.4, in which D is plotted against δB . When $0 < \mu < 1$, numerically computed D matches well with D_{QL} (thick broken line), whilst they start to deviate around $\delta B \sim 0.1$. From the numerical results discussed above, Hada et al. (2003a) are tempted to model the pitch-angle diffusion process by a simple equation,

$$\frac{\partial f(\mu)}{\partial t} = \frac{\partial}{\partial \mu} D^* \frac{\partial f(\mu)}{\partial \mu} - \frac{f(\mu) - f(-\mu)}{\tau(\mu)}, \quad (2.30.2)$$

where $f(\mu)$ is the distribution function, $D^*(\mu)$ is the modified pitch-angle diffusion coefficient including the resonance broadening effect (and thus $D^*(0) \neq 0$), and $\tau(\mu)$ is the time scale for the mirror reflection, which may be determined by statistics of compression magnetic field (one should note, however, that the mirror reflection is not always adiabatic as assumed in Eq. 2.30.2). If there is a finite coherence in the MHD turbulence, as evidenced by recent spacecraft data analysis (Hada et al., 2003b), it strongly influences $\tau(\mu)$, which in turn modifies the pitch-angle diffusion.

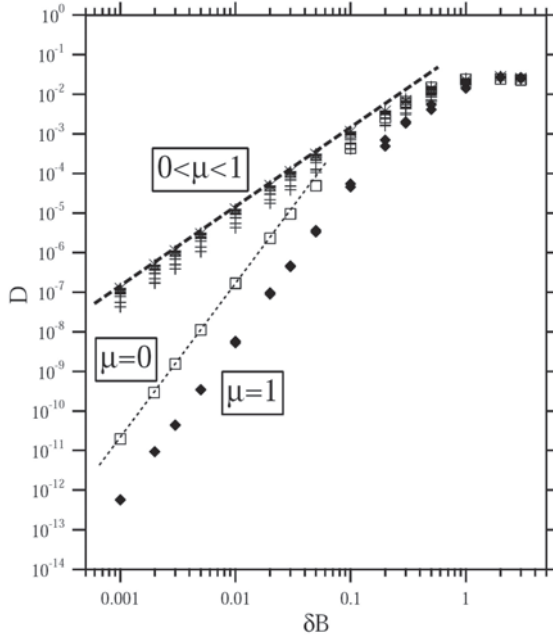


Fig. 2.30.4. D versus δB . According to Hada et al. (2003a).

2.31. Particle diffusion across the magnetic field and the anomalous transport of magnetic field lines

2.31.1. On the anomalous transport of magnetic field lines in the quasi-linear regime

The transport of CR particles across the regular component of the magnetic field in space is for a large part induced by the transport of the magnetic field lines themselves (so called compound diffusion, see in Jokipii, 1966; Schlickeiser, 1994). At the shock fronts of supernovae like SN1987A the observed acceleration time of GeV electrons suggests a transport also dominated by the wandering of the magnetic field lines, as the inferred diffusion coefficient of the electrons by far exceeds the Bohm value of this coefficient (Ball and Kirk, 1992; Ragot, 2001a,b). Understanding the behavior of magnetic field lines in a turbulence composed of random fluctuations $\delta\mathbf{B}$ superimposed on a regular magnetic field $\mathbf{B}_0$ is thus of prime importance to model the propagation of charged particles in space plasma. The case of small magnetic field perturbation is treated by the quasi-linear theory (Jokipii and Parker, 1968) for weak magnetic turbulence. This theory, which neglects the perpendicular displacement of the field lines in the derivation of their spreading (first order derivation in $\delta b \equiv \delta B/B_0$), predicts a diffusion of the field lines beyond the parallel correlation length, $L_{c//}$, defined as the characteristic scale of the two-point correlation function. There is a strong belief amongst astrophysicists and physicists in general that, as long as the quasi-linear approximation holds, *i.e.*, as long as the perpendicular displacement can be neglected, the quasi-linear theory does predict a diffusion of the magnetic field lines or, more accurately, their linear spreading across the direction of $\mathbf{B}_0$ with the distance Δz along $\mathbf{B}_0$. However, this diffusive result is conditioned by the existence of a finite correlation length, $L_{c//}$, small enough to consider the transport of the field lines on much longer scales. In the papers by Jokipii and Parker (1968), Jokipii and Coleman (1968), this correlation length was estimated as the inverse of the upper wave-number in the low, flat part of the turbulence spectrum. A power spectrum flat below $k = L_c^{-1}$ produces indeed a correlation function of the magnetic field perturbation with an exponential cutoff of characteristic scale L_c . Ragot (2001c) note that yet a flattening of the spectrum at sufficiently high frequency is not guaranteed. For instance, in the solar wind the early observations apparently indicating a flattening at 10^{-5} Hz, which would have given a quasi-linear correlation just short enough, have not been confirmed by more recent measurements which show power-law spectra down to lower frequencies (Goldstein et al., 1995). In general the presence of such extended, projected spectra, relatively smooth but not flat, is expected for an anisotropic turbulence (*e.g.*, Ragot, 1999a), and as the damping rates of many plasma waves

depend on the propagation angle of the waves, anisotropic turbulence is likely to be a quite common feature of plasmas. Clearly, in those cases of extended projected spectra a study of the transport of field lines is still needed even in the quasi-linear regime of magnetic field perturbation, as the spreading of the field lines on any relevant scale will be determined by a part of the spectrum that is not flat, hence neglected in the original quasi-linear theory.

In Ragot (2001c) is introduced the assumption concerning the existence of a short correlation length and express the spreading of the field lines along the axis x normal to the average magnetic field as a function of the projected power spectrum of turbulence. In the case when this projected spectrum can be described as a power law on an interval of wave-numbers around $1/\Delta z$, which is generally assumed in any study of turbulence, it can be then establish a new asymptotic expansion for the variance (Δx^2) . With this expansion it may be analytically proved that whenever the spectral index of the turbulence does not vanish exactly on an interval of wave-numbers at least two or three decades broad around $1/\Delta z$, the transport of the field lines is non-diffusive, or anomalous: (Δx^2) increases as $(\Delta z)^\alpha$ with α different from 1. This confirms the numerical result obtained by Ragot (1999a,b) for similar power-law spectra. Then can be established simple expressions for the transport exponent α , as well as the transport coefficient D_α , defined by $(\Delta x^2) = D_\alpha (\Delta z)^\alpha$. These expressions are particularly important for a quantitative comparison with the spreading predicted by the original quasi-linear theory.

Ragot (2001c) consider (as in the paper by Ragot, 1999a) a three-dimensional turbulence in quasi-linear regime with a *continuous* spectrum; hence unlike Pommois et al. (1999), he always keep the length scale Δz much shorter than the inverse of the minimum wave-number, which is an absolute requisite to model a continuous spectrum. Below by drawing by Ragot (2001c) the main lines of the classical quasi-linear derivation.

2.31.2. *Quasi-linear theory for magnetic lines diffusion*

In the quasi-linear approximation, *i.e.*, if the perpendicular deviation is neglected, the displacement along the axis x of the field line that goes through the point $\mathbf{r}_0 = (x_0, y_0, z_0)$ can be written as

$$\Delta x = x(\mathbf{r}_0, z) - x_0 = \int_{z_0}^z b_x(x_0, y_0, z') dz', \quad (2.31.1)$$

where b stands for $\delta \mathbf{B}/B_0$, and the variance $\langle \Delta x^2 \rangle$ can be expressed as:

$$\langle \Delta x^2 \rangle = \int_{z_o}^z dz' \int_{z_o}^z dz'' \langle b_x(x_o, y_o, z') b_x(x_o, y_o, z'') \rangle = 2\Delta z \int_0^{\Delta z} ds \left(1 - \frac{s}{\Delta z} \right) R_{xx}(s), \quad (2.31.2)$$

where $\Delta z = z - z_o$. The brackets $\langle \rangle$ denote an average over a statistical ensemble of systems and $R_{xx}(s) = \langle b_x(x_o, y_o, z') b_x(x_o, y_o, z'') \rangle$ stands for the two-point correlation function of the magnetic field along x . In the usual quasi-linear theory R_{xx} is assumed to cut off on the length scale $L_{c//}$, known as the parallel correlation length, and the limit $\Delta z \gg L_{c//}$ is taken so that

$$\frac{\langle \Delta x^2 \rangle}{2\Delta z} \approx \int_0^{+\infty} ds R_{xx}(s) \equiv D. \quad (2.31.3)$$

It shows that the magnetic field lines diffuse with the diffusion coefficient D on length scales much longer than $L_{c//}$. However, it does not prove that $L_{c//}$ exists and is very much smaller than the size of the system, which happens to be necessary to observe a diffusion *in* the system. In the following, Ragot (2001c) introduce the assumption concerning the existence of a finite correlation length and derive a general expression for the spreading of magnetic field lines in the quasi-linear regime of turbulence.

2.31.3. Quasi-linear spreading of magnetic field lines

If k_m and k_M denote the lowest and highest wave-numbers in the spectrum, the spreading of the field lines:

$$\langle \Delta x^2 \rangle = 2k_m^3 \int_{z_o}^z dz' \int_{z_o}^z dz'' \int d\mathbf{k} b_x^2(\mathbf{k}) \cos[k_{//}(z' - z'')], \quad (2.31.4)$$

can be deduced from

$$\frac{b_x(\mathbf{r})}{2} = \int d\mathbf{k}_\perp \int_0^{k_M} dk_{//} \tilde{b}_x(\mathbf{k}) \cos(\mathbf{k}_\perp \cdot \mathbf{r} + k_{//}z + \phi_{\mathbf{k}}), \quad (2.31.5)$$

where $\tilde{b}_x(\mathbf{k}) \exp(i\phi_{\mathbf{k}})$ is the Fourier transform of $\tilde{b}_x(\mathbf{r})$ with $\tilde{b}_x(\mathbf{k}) > 0$. The derivation of Eq. 2.31.4 assumes, as in the quasi-linear theory, that the phases $\phi_{\mathbf{k}}$ decorrelate on the scale k_m but this assumption of no spectral structuring could of course be given up by introducing a different phase-correlation scale and

substituting for the factor k_m^3 . Integrating now over z' and z'' , we obtain in the quasi-linear regime of magnetic field perturbation:

$$\langle \Delta x^2 \rangle = 4k_m^3 \int_0^{k_M} dk_{//} [1 - \cos(k_{//}z)] \frac{P_{x//}(k_{//})}{k_{//}^2}, \quad (2.31.6)$$

where

$$P_{x//}(k_{//}) = \int_0^{2\pi} d\phi \int_{k_{\min}(k_{//})}^{k_{\max}(k_{//})} dk_{\perp} k_{\perp} b_x^2(k_{//}, k_{\perp}, \phi) \quad (2.31.7)$$

is the x -component of the power spectrum projected along $\mathbf{B}_o$, and

$$k_{\min}(k_{//}) = [\max(0, k_m^2 - k_{//}^2)]^{1/2}; \quad k_{\max}(k_{//}) = (k_M^2 - k_{//}^2)^{1/2}. \quad (2.31.8)$$

When the spectrum is smooth enough to be represented as a series of power laws, the right-hand side of Eq. 2.31.6 can be integrated over the parallel wave-numbers to obtain an explicit form of the field lines spreading. For a power-law spectrum

$$P_{x//}(k_{//}) = P_x(k_1)(k_{//}/k_1)^{-a} \quad (2.31.9)$$

from k_1 to $+\infty$, it was found for the quasi-linear regime:

$$\begin{aligned} \langle \Delta x^2 \rangle = & 4k_m^3 P_{x//}(k_1) k_1^{-1} \left\{ \frac{1}{1+a} + |k_1 \Delta z|^{1+a} \Gamma(-1-a) \sin \frac{a\pi}{2} \right. \\ & \left. - \frac{1}{1+a} F_{P,Q} \left[\left\{ \frac{-1-a}{2} \right\}, \left\{ \frac{1}{2}, \frac{1-a}{2} \right\}; -\frac{(k_1 \Delta z)^2}{4} \right] \right\}, \end{aligned} \quad (2.31.10)$$

where $F_{P,Q}$ denotes the hyper-geometric function and $a > -1$. When $a > -1$ and $k_1 \Delta z \ll 1$, an expansion of the hyper-geometric function gives:

$$\langle \Delta x^2 \rangle = 4k_m^3 P_{x//}(k_1) k_1^{-1} \left\{ |k_1 \Delta z|^{1+a} \Gamma(-1-a) \sin \frac{a\pi}{2} - \frac{(k_1 \Delta z)^2}{2(1-a)} + O((k_1 \Delta z)^4) \right\}. \quad (2.31.11)$$

Eq. 2.31.11 shows that the spreading of the field lines is not linear unless $a = 0$. Moreover, since $\Gamma(-1-a)\sin(a\pi/2) \rightarrow \pi/2$ as $a \rightarrow 0$, the usual quasi-linear diffusion coefficient $2\pi k_m^3 P_{x//}(k_1)$ is recovered in this limit of a flat spectrum. For a finite upper wave-number k_2 one has to subtract $4k_m^3 P_{x//}(k_1) k_1^a \int_{k_2}^{+\infty} dk_{//} [1 - \cos(k_{//} z)] k_{//}^{-2-a}$ from the right-hand side of Eq. 2.31.10, which can be estimated in a way similar to that as the integral from k_1 to $+\infty$. However, the first term resulting from the integration of $k_{//}^{-2-a}$ is small compared to the part in k_1 as soon as $(k_1/k_2)^{1+a} \ll 1$. As for the other term, it is negligible for $k_2 \Delta z \gg 1$ and $a > -1$ because of the Riemann-Lebesgue lemma (Bender and Orszag, M1978), since $\int_{k_2}^{+\infty} dk_{//} |k_{//}|^{-2-a}$ exists. Consequently if $k_2^{-1} \ll \Delta z \ll k_1^{-1}$ and $a > -1$, the Eq. 2.31.10 and 2.31.11 still apply for a power-law spectrum on a finite interval $[k_1, k_2]$.

2.31.4. The transport exponent and transport coefficient for magnetic field lines

In accordance with Ragot (2001c) the transport exponent α and transport coefficient $D_{m\alpha}$, defined by

$$\langle \Delta x^2 \rangle \approx D_{m\alpha} (\Delta z)^\alpha \quad (2.31.12)$$

can be expressed in analytical form on the basis of results in Section 2.31.3. From $\alpha = d(\log \langle \Delta x^2 \rangle) / d(\log \Delta z)$ there follows for spectral indexes $a > -1$ or $a > -0.5$ (depending on how small $k_1 \Delta z$ is)

$$\alpha = 1 + \frac{aA_1(k_1 \Delta z)^{1+a} + A_2(k_1 \Delta z)^2}{A_1(k_1 \Delta z)^{1+a} + A_2(k_1 \Delta z)^2} \quad (2.31.13)$$

with

$$A_1 = \Gamma(-1-a)\sin(a\pi/2), \quad A_2 = -1/(2-2a). \quad (2.31.14)$$

In the limit of small $|a|$, namely, $|a| < 0.5$ for $k_1 \Delta z \approx 10^{-1}$ and $|a| < 1$ for $k_1 \Delta z \rightarrow 0$, it gives

$$D_{m\alpha} = 4k_m^3 P_{x//}(k_1) k_1^a A_1, \quad (2.31.15)$$

whereas in the limit of larger $|a|$ ($|a| > 2$ for $k_1 \Delta z \approx 10^{-1}$ and $|a| > 1$ for $k_1 \Delta z \rightarrow 0$),

$$D_{m\alpha} = 4k_m^3 P_{x//}(k_1) k_1^a A_2, \quad (2.31.16)$$

where coefficients A_1 and A_2 are determined by Eq. 2.31.14. Fig. 2.31.1 shows the transport exponent α as a function of the spectral index a .

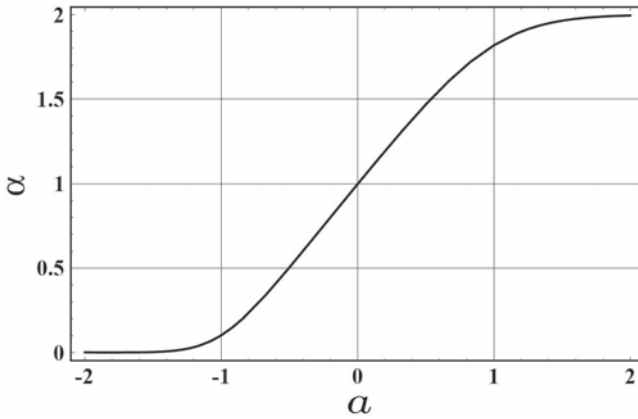


Fig. 2.31.1. Transport exponent α as a function of the spectral index a for $k_1 \Delta z = 10^{-2}$. From Ragot (2001c).

The spreading of magnetic field lines in the quasi-linear regime of turbulence is only linear for a flat spectrum. For all decreasing power laws ($a > 0$) the field lines supra-diffuse ($\alpha > 1$), whereas for inverted power laws the field lines sub-diffuse ($\alpha < 1$) as long as $a > -2$. For $a < -0.5$, α is averaged over a broad range of Δz . From Fig. 2.31.1 it can be seen that the transport of the magnetic field lines is supra-diffusive ($\alpha > 1$) for any positive spectral index of turbulence and sub-diffusive ($\alpha < 1$) for any inverted spectrum, which confirms the results of Ragot (1999a,b). As the spectral index a approaches 1 from below, the term in $-(k_1 \Delta z)^2 / (2 - 2a)$ has a growing weight in Eq. 2.31.11, owing to the factor $(2 - 2a)^{-1}$. It is dominant for $a > 1$, therefore its sum with $(k_1 \Delta z)^{1+a} \Gamma(-1 - a) \sin(a\pi/2)$, which is then negative, remains always positive. In the limit of very small $k_1 \Delta z$ the other term becomes completely negligible and α

converges to 2 as soon as $a > 1$. For $k_1 \Delta z = 10^{-2}$ as in Fig. 2.31.1, the first term still has a significant weight up to $a = 1.5 - 2.0$, but for all spectral indexes steeper than 2 the transport exponent α is practically equal to 2.

For $|a| < 0.5 - 0.8$, $\langle \Delta x^2 \rangle$ is accurately determined by the first term. In this range of spectral indexes the transport exponent simply reduces to $\alpha = 1 + a$. This case is of particular interest since it corresponds to a spectrum that would tend to flatten at low frequency, but not perfectly, as is observed for instance in the solar wind (Goldstein et al., 1995).

Ragot (2001c) note that $P_{x//}(k_1)k_1^a = P_{x//}(k_//)k_//^a$ for any $k_//$ in the interval $[k_1, k_2]$ so that the value of $D_{m\alpha}$ does not depend on the lower limit k_1 of the interval on which $P_{x//}$ is in $k_//^a$ but solely on the level of turbulence in this interval of parallel wave-numbers. The range of validity in a for $\alpha = 1 + a$, however, does depend on the value of $k_1 \Delta z$. If $k_1 \Delta z \rightarrow 0$, it extends from -1 to 1 .

The condition established by Ragot (1999a) to observe, in the quasi-linear regime of turbulence, a diffusive spreading of the field lines on at least one decade is confirmed; namely, the spectrum should be flat on at least three decades around $1/\Delta z$ (2 decades for $k_2^{-1} \ll \Delta z \ll k_1^{-1}$ plus 1 decade for the variation of Δz). This means that even a flattening at 10^{-5} Hz in the solar wind would not have guaranteed a diffusive spreading of the field lines on a scale of length shorter than the typical distance between strong inhomogeneities, since the sun rotates at a frequency of $(3.2 - 4.6) \times 10^{-7}$ Hz less than 100 times smaller. This conclusion of no quasi-linear diffusion of magnetic field lines in the inner Heliosphere is not contradicted by the observation of fluctuating field line directions and could account for the lack of mixing of charged particles propagating through the turbulent solar wind (Zurbuchen et al., 2000) recently observed with the SWICS instrument on ACE. A supra-diffusion is indeed characterized by a lower dispersion and ordered fields on many scales, as is considered below in Section 2.31.5.

2.31.5. Comparison with the original quasi-linear prediction

A quantitative comparison of Ragot (2001c) prediction for the magnetic field line spreading with the prediction of the original quasi-linear theory now is relatively straightforward. For a level of turbulence to be the same at the lower wave-number k_1 , the ratio of the two predicted variances can be written as

$$\frac{\langle \Delta x^2 \rangle}{D_{m1} \Delta z} = \frac{2}{\pi} A_1 (k_1 \Delta z)^a \left[1 + \frac{A_1}{A_2} (k_1 \Delta z)^{1-a} \right] + O((k_1 \Delta z)^3). \quad (2.31.17)$$

This ratio is plotted in Fig. 2.31.2 for various values of $k_1\Delta z$.

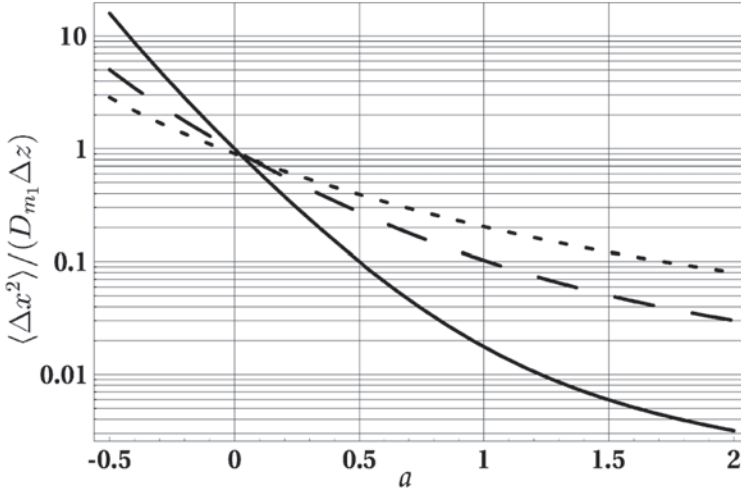


Fig. 2.31.2. Ratio of the quasi-linear field line spreading over the original diffusive quasi-linear prediction for a given value of $P_{x//}(k_1)$. Continuous line: $k_1\Delta z = 0.01$. Long-dashed line: $k_1\Delta z = 0.1$. Short-dashed line: $k_1\Delta z = 0.3$. From Ragot (2001c).

What strikes at once is that the supra-diffusion ($a > 0$) seems to give a much slower spreading of the field lines than would be expected for the diffusion of the original quasi-linear theory, whereas the sub-diffusion apparently gives a much faster transport. Whilst this might not be entirely accurate (one chooses a lower turbulence amplitude for larger a by taking the same value of $P_{x//}(k_1)$), it serves the purpose of Ragot (2001c) pretty well here. He emphasizes the following. A supra-diffusion does not necessarily mean that the transport is faster, nor does a sub-diffusion imply a slower transport. This very much depends on the value of the transport coefficient. Supra-diffusion is characterized by a lesser dispersion of the field lines which tend to behave in a more orderly manner. Whilst the small-scale irregularities still exist and might give the impression that the field lines are ‘diffusing’ in an erratic and uncorrelated way, the large-scale transport is significantly influenced by the lower part of the spectrum and ordered behavior occurs on all scales, even the largest ones. The propagator derived by Ragot and Kirk (1997) illustrates this property with a peaked shape shifted away from zero. For comparison the propagator of diffusion is the well known Gaussian centered around the origin. In the sub-diffusive case ($a < 0$) the propagator is more widespread and peaks at the origin. The transport is dominated by the small scales and long ordered ‘flights’ are extremely rare. A greater dispersion might still result, though, from some field lines being able to wander relatively quickly in

some part of the space while others (the majority) are trapped in smaller-scale domains on longer length scales.

2.31.6. Summary of main results and discussion

Ragot (2001c) have shown analytically that in the quasi-linear regime of turbulence the transport of magnetic field lines is anomalous on the length scale Δz whenever the projected spectrum of turbulence is not perfectly flat below the parallel wave-number $10/\Delta z$. The field line spreading $\langle \Delta x^2 \rangle$ varies as $(\Delta z)^\alpha$ with $\alpha \neq 1$, $0 \leq \alpha \leq 2$. A decreasing spectrum results in a supra-diffusion of the field lines ($\alpha > 1$), whereas an inverted spectrum implies a sub-diffusion ($\alpha < 1$). For a spectrum that takes the form of a power-law on an interval of parallel wave-numbers around $(\Delta z)^{-1}$, there were established new, simple expressions for the transport exponent and coefficient (Eq. 2.31.13–2.31.15). These expressions generalize the quasi-linear prediction for the spreading of magnetic field lines.

2.32. CR transport in the fractal-like medium

2.32.1. The matter of problem and main relations

In papers Lagutin et al. (2001b,d, 2005), Erlykin et al. (2003), Lagutin and Uchaikin (2003) a model of phenomenological anomalous diffusion, in which the high energy CR propagation in the galactic medium is simulated as fractal walks, has been developed. The anomalous diffusion results from large free paths ('Lévy flights') of particles between magnetic domains-traps of the returned type. These paths are distributed according to power law

$$P(r, R) \approx A(R, \alpha) r^{-\alpha-1}; \quad \alpha < 2 \quad \text{at } r \rightarrow \infty \quad (2.32.1)$$

being an intrinsic property of fractal structures. Here R is the particle's magnetic rigidity. It is also assumed that the particle can spend a long time in the trap. A long time means that the distribution of the particles staying in traps, $q(\tau, R)$, has a tail of power-law type

$$q(\tau, R) \approx B(R, \beta) \tau^{-\beta-1} \quad (2.32.2)$$

with $\beta < 1$ at $\tau \rightarrow \infty$ (Lévy trapping time).

Without energy losses and nuclear interactions, the propagator $G(\mathbf{r}, t, R; R_o)$, describing such a process, obeys the equation (Lagutin and Tyumentsev, 2004):

$$\frac{\partial G}{\partial t} = -\kappa(R, \alpha, \beta) D_{0+}^{1-\beta} (-\Delta)^{\alpha/2} G(\mathbf{r}, t, R; R_o) + \delta(\mathbf{r}) \delta(t) \delta(R - R_o). \quad (2.32.3)$$

Here, $\kappa(R, \alpha, \beta)$ is the anomaly diffusion coefficient, D_{0+}^{μ} denotes the Riemann-Liouville fractional derivative, and $(-\Delta)^{\alpha/2}$ is the fractional Laplacian, so called ‘Riss’ operator (see in Samko et al., M1987).

In the case of punctual impulse source of duration T with inverse power spectrum

$$S(\mathbf{r}, t, R) \approx S_o R^{-p} \delta(\mathbf{r}) \Theta(T - t) \Theta(t), \quad (2.32.4)$$

where $\Theta(t)$ is the Heviside function, CR concentration is

$$n(r, t, R) = \frac{S_o R^{-p}}{\kappa(R, \alpha, \beta)} \int_{\max[0, t-T]}^t \tau^{-3\beta/\alpha} \Psi_3^{(\alpha, \beta)} \left(|r| \left(\kappa(R, \alpha, \beta) \tau^{\beta} \right)^{-1/\alpha} \right) d\tau, \quad (2.32.5)$$

where the scaling function $\Psi_3^{(\alpha, \beta)}(r)$,

$$\Psi_3^{(\alpha, \beta)}(r) = \int_0^{\infty} q_3^{(\alpha)}(r \tau^{\beta}) q_1^{(\beta, 1)}(\tau) \tau^{3\beta/\alpha} d\tau, \quad (2.32.6)$$

is determined by three-dimensional spherically-symmetrical stable distribution $q_3^{(\alpha)}$ at $\alpha \leq 2$, and one-sided stable distribution $q_1^{(\beta, 1)}(t)$ with characteristic exponent β (Zolotarev, M1983; Uchaikin and Zolotarev, M1999). The diffusion coefficient $\kappa(R, \alpha, \beta)$ is determined by the positive constants $A(R, \alpha)$ and $B(R, \beta)$ (in the asymptotic behavior) for the ‘Lévy flight’ (A) and the ‘Lévy waiting time’ (B) distributions:

$$\kappa(R, \alpha, \beta) \propto A(R, \alpha) / B(R, \beta). \quad (2.32.7)$$

Taking into account that both the free path and the probability to stay in trap during the time interval τ for particle with charge Z and mass number A depend on particle magnetic rigidity R , we accept

$$\kappa(R, \alpha, \beta) = (v/c) \kappa_o(\alpha, \beta) R^{\delta}. \quad (2.32.8)$$

2.32.2. Formation of CR spectrum in the frame of anomaly diffusion in the fractal-like medium

It has been shown (Lagutin et al., 2001b,d; Lagutin and Uchaikin, 2003; Erlykin et al., 2003) that in the framework of anomaly diffusion model it is possible to explain the locally observed basic features of the CR in the wide energy range. This model was proposed with the aim to understand the nature of the knee in primary CR spectrum and explain why the spectral exponent of protons and other nuclei at $E \approx 10^2 - 10^5$ GeV/nucleon has different values.

The physical arguments and the calculations indicate that the bulk of observed CR with energy $10^8 - 10^{10}$ eV is formed by numerous distant sources. It means that the contribution of these sources to the observed flux may be evaluated in the framework of the steady-state approach. Using results of Lagutin et al. (2001a), in paper Lagutin et al. (2005), the total flux J_D^i of the particles of type i from all distant ($r > 1$ kpc) sources have been presented in the form

$$J_D^i(E, r > 1 \text{ kpc}) = v_i C_{0i} E^{-p-\delta/\beta}, \quad (2.32.9)$$

where v_i is a particle velocity, C_{0i} is a constant evaluated via fitting of experimental data.

The contribution $J_L^i(E, r \leq 1 \text{ kpc})$ of the nearby or local ($r \leq 1$ kpc) relatively young ($t \leq 10^5$ years) sources defines the spectrum in the high energy region and, as it was shown in papers Lagutin et al. (2001d), Lagutin and Uchaikin (2003), provides the knee in the spectrum of galactic CR observed on the Earth:

$$J_L^i(E, r \leq 1 \text{ kpc}) = \frac{v_i}{4\pi} \sum_j n_i(\mathbf{r}_j, t_j, E), \quad (2.32.10)$$

where $\mathbf{r}_j, t_j$ are the coordinate and the age of the source j , $n_i(\mathbf{r}_j, t_j, E)$ is the CR concentration from this source.

The similar separation of the flux into two components with significantly different properties is frequently used in the CR studies. However, the presence of the large free paths of the particles (the ‘Lévy flights’) in considered model leads to the introduction of the third component. This third component is formed by the particles, which pass a distance between an acceleration site of a source and solar system without scattering. The flux of non-scattered particles J_{NS}^i is determined by the injected flux $\propto S_{0i} E^{-p}$ and the ‘Lévy flight’ probability $P(> r, E)$. Taking into account that for the particle with energy E the probability

$$P(> r, E) \approx A(E, \alpha) r^\alpha \propto E^{\delta_L}. \quad (2.32.11)$$

we have

$$J_{NS}^i = C_{li}^0 E^{-p+\delta_L}. \quad (2.32.12)$$

Lagutin et al. (2005) assumed that this component defines the spectrum in the ultrahigh energy region $E \geq 10^{18}$ eV and provides the flattening of the spectrum. In other words, in this model the ‘ankle’ in primary CR spectrum is also due to the ‘Lévy flights’ of the CR particles.

Thus, the differential flux $J_i(E)$ of the particles of the type i from all Galactic sources may be presented in the form:

$$J_i(E) = J_D^i(E) + J_L^i(E) + J_{NS}^i(E). \quad (2.32.13)$$

2.32.3. Parameters of the model and numerical calculations

The first free path distribution of CR particle traveling through highly inhomogeneous medium of fractal type was investigated by Lagutin et al. (2005) numerically. It was obtained that first free path distribution in the medium with mass fractal dimension $0 < d_M < 2$ has power-law asymptotic $P(r) \propto r^{-\alpha-1}$. The index α dependence on fractal dimension of the medium under different assumptions on cross-section of particle interaction with elementary structures of the medium (parameter ρ/x_o) is shown in Fig. 2.32.1. In case of small cross-sections the relation, obtained in Isliker and Vlahos (2003),

$$\alpha + 1 = 3 - d_M, \quad (2.32.14)$$

is confirmed by calculations of Lagutin et al. (2005). The violation of linear dependence of $\alpha(d_M)$ appears with increasing cross-section as a consequence of finiteness of medium and also overlapping inhomogeneities of the medium. Thus, assuming fractal dimension of the Galaxy as $d_M \approx 1.7$ (according to Combes, 2000), from Eq. 2.32.14 follows $\alpha \approx 0.3$ (Lagutin et al., 2004). The other parameters of the model ($p, \delta, \beta, \kappa_o$) were evaluated from experimental data. There were found as follows (in accordance with Lagutin and Tyumentsev, 2004):

$$p \approx 2.85, \quad \delta \approx 0.27, \quad \kappa_o \approx 3 \times 10^{-6} \text{ pc}^{0.3} \text{ yr}^{-0.8}. \quad (2.32.15)$$

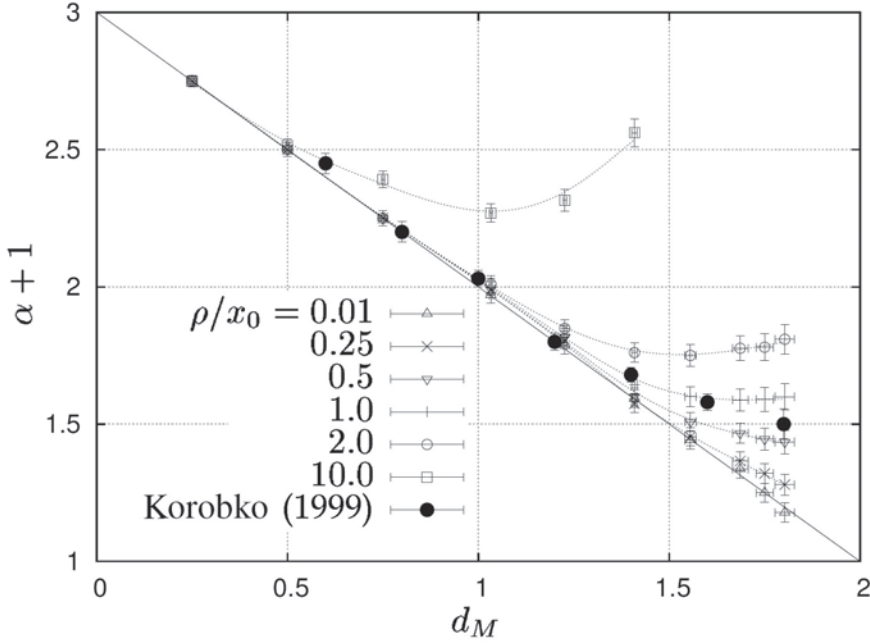


Fig. 2.32.1. Dependence of α on fractal dimension of the medium d_M under different assumptions on cross-section of CR particle interaction ρ/x_0 from 0.01 up to 10. According to Lagutin et al. (2005).

2.32.4. Application to the problem of galactic CR spectrum formation

Possible candidates of the CR sources, located within 1 kpc from the Sun, with ages less than 4×10^5 years, and the contribution of each source to total proton flux, are presented in Fig. 2.32.2. This Figure illustrates the contribution of each source to proton flux observed near the Sun, assuming that the output of protons from each supernova is the same and equal to

$$Q_p(E > 1 \text{ GeV}) = 4 \times 10^{50} \text{ erg/SN}. \quad (2.32.16)$$

From Fig. 2.32.2 follows, that only two SNRs give significant contribution to observed proton flux in the high energy region: Loop-I gives from 60% to 70% and Loop-II from 12% to 7% (in dependence of energy). Lagutin et al. (2005) note that this result contradicts with observed very small amplitude of CR anisotropy, what can be owed either to not correctness of assuming about equal output of SN in CR protons (Eq. 2.32.16) or there are some other main sources of high energy CR (see the discussion also in Ptuskin, 1997; Cronin, 2001; Olinto, 2001; Hoerandel, 2004).

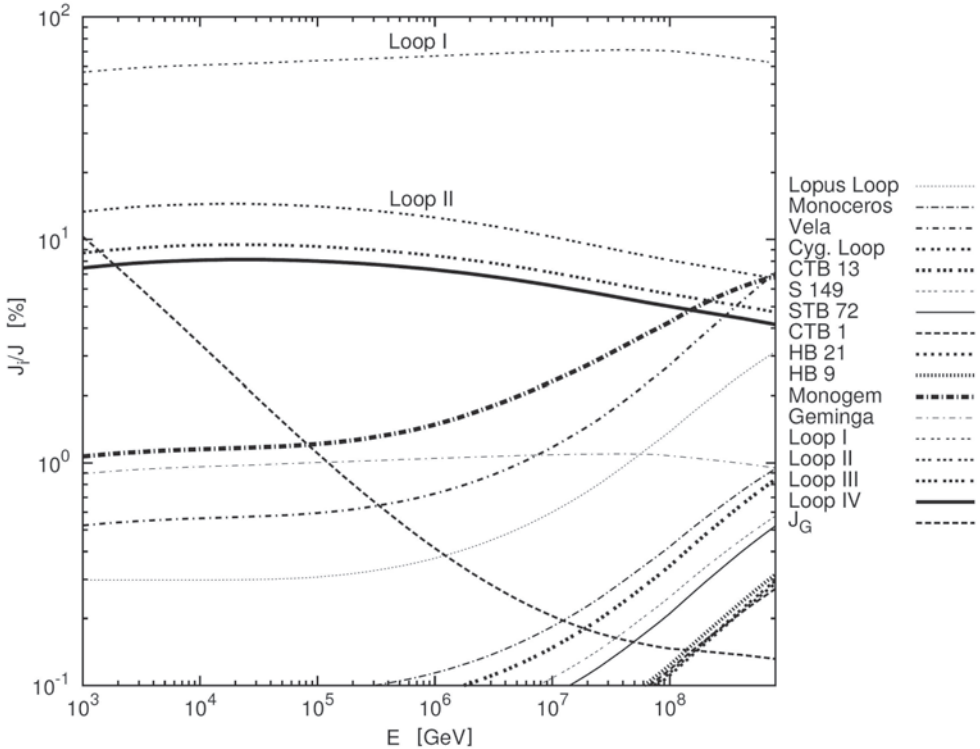


Fig. 2.32.2. Relative contribution of each nearby SNRs to proton flux near the solar system. According to Lagutin et al. (2005).

2.33. CR propagation in large-scale anisotropic random and regular magnetic fields

In series of papers of Mel'nikov (1996, 2000, 2005a,b,c) kinetic coefficients and parallel (to the mean field) mean free paths of CR particles in large-scale anisotropic random magnetic field are obtained with using nonlinear collision integral, i.e., by taking into account the strong random scattering.

2.33.1. The matter of problem

It follows from the analysis of experimental data performed by Matthaeus et al. (1990) and Bieber et al. (1996) that the distribution of interplanetary magnetic field fluctuations is anisotropic. In the weakly disturbed inner Heliosphere, the preferential direction of the magnetic field fluctuations is perpendicular to the regular magnetic field. The wave vectors of the fluctuations are also mainly perpendicular to the regular magnetic field, which gives rise to two-dimensional fluctuations. In the interplanetary medium the energy of the two-dimensional fluctuations can reach 85% of the energy of the random magnetic field. The

parallel transport mean free paths of high energy particles in the interplanetary magnetic field, including the anisotropy of random fluctuations, were calculated numerically by Bieber et al. (1994), Teufel and Schlickeiser (2002, 2003), Teufel et al. (2003), Shalchi and Schlickeiser (2004) and analytically by Dröge (2003). These authors used a quasi-linear random magnetic field approximation and introduced the cyclotron resonance broadening using a de-correlation in the correlation tensor of the random magnetic field. They showed that CR particles are scattered weakly by two-dimensional fluctuations. The calculated transport mean free paths of solar CR protons exceed their observed values by several tens or hundreds of times. Mel'nikov (2005a) shows that for nonlinear broadening of two-dimensional perturbations the random scattering frequency increases significantly, and the transport mean free path decreases.

2.33.2. Main equations and transforming of collision integral

Mel'nikov (2005a) has considered the kinetic coefficients and particle transport mean free paths over a wide energy range from 1 MeV to several GeV in the inner Heliosphere and at energies above 10 GeV in the outer Heliosphere, including those at the energies at which $r_g \approx L_{//}, L_{\perp}$ where r_g is the gyro-radius in the random magnetic field, and $L_{\perp}$ and $L_{//}$ are the perpendicular and parallel (relative to the regular magnetic field) correlation lengths, respectively. It was used the following kinetic equation for the average particle distribution function $F(\mathbf{r}, \mathbf{p}, t)$ with the nonlinear collision integral (Mel'nikov, 1996, 2000):

$$\left\{ \frac{\partial}{\partial t} + \mathbf{v} \frac{\partial}{\partial \mathbf{r}} - \mathbf{H}_0 \mathbf{D} \right\} F(\mathbf{r}, \mathbf{p}, t) = \text{St}F, \quad (2.33.1)$$

where

$$\text{St}F = D_{\alpha} \int dx_1 B_{\alpha\beta}(\mathbf{r}, t; \mathbf{r}_1, t_1) G_1(x, x_1) D_{1\beta} F(x_1, x_0), \quad \mathbf{D} = \frac{e}{c} \left[(\mathbf{v} - \mathbf{u}) \times \frac{\partial}{\partial \mathbf{p}} \right], \quad (2.33.2)$$

and $x \equiv \mathbf{r}, \mathbf{p}, t$; $\mathbf{r}$ is the coordinate, $\mathbf{p}$ is the momentum, $\mathbf{v}$ is the particle velocity; t is the time; $\mathbf{H}_0$ is the strength of the regular magnetic field, e is the particle charge, c is the speed of light, $\mathbf{u}$ is the velocity of the magnetic field, and $G_1(x, x_1)$ is the one-particle Green function that is the solution of the linear kinetic equation. Mel'nikov (2005a) choused the correlation tensor of the random anisotropic magnetic field $\mathbf{H}_1$ for a power-law spectrum in the form (Matthaeus et al. 1990; Toptygin 1985; Chuvilgin and Ptuskin 1993):

$$B_{\alpha\beta}(\mathbf{k}) = P(\mathbf{k}) \frac{[\mathbf{k} \times \mathbf{h}_o]_\alpha [\mathbf{k} \times \mathbf{h}_o]_\beta}{k_\perp^2}, \quad (2.33.3)$$

where

$$P(\mathbf{k}) = A_\nu (q_\perp q_{//})^{-1-\nu/2} \left(\frac{k_{//}^2}{q_{//}^2} + \frac{k_\perp^2}{q_\perp^2} \right) \left[1 + \frac{k_{//}^2}{q_{//}^2} + \frac{k_\perp^2}{q_\perp^2} \right]^{-2-\nu/2}, \quad (2.33.4)$$

and $\mathbf{k}$ is the wave vector, ν is the spectral index,

$$\mathbf{h}_o = \mathbf{H}_o / H_o, \quad \mathbf{k}_{//} = (\mathbf{k} \mathbf{h}_o) \mathbf{h}_o, \quad \mathbf{k}_\perp = \mathbf{k} - \mathbf{k}_{//}, \quad \mathbf{q}_{//} = (\mathbf{q} \mathbf{h}_o) \mathbf{h}_o, \quad \mathbf{q}_\perp = \mathbf{q} - \mathbf{q}_{//}, \quad (2.33.5)$$

$$q_\perp = 2\pi L_\perp^{-1}, \quad q_{//} = 2\pi L_{//}^{-1}, \quad A_\nu = 2\Gamma\left(2 + \frac{\nu}{2}\right) q_\perp^{\nu/2-1} q_{//}^{\nu/2} \langle H_1^2 \rangle \left(3\pi^{3/2} \Gamma\left(\frac{\nu-1}{2}\right) \right)^{-1}, \quad (2.33.6)$$

Γ is the Gamma function. Passing to the drift approximation, we obtain:

$$\left\{ \frac{\partial}{\partial t} + \nu \mu \frac{\partial}{\partial z} - \frac{1}{2} \nu \operatorname{div} \mathbf{h}_o \sin \theta \frac{\partial}{\partial \theta} \right\} \Phi = \langle \operatorname{St} F \rangle_\varphi, \quad (2.33.7)$$

where the coordinate z is along the vector $\mathbf{h}_o$, $\Phi = \langle F \rangle_\varphi$, θ is the angle between $\mathbf{p}$ and $\mathbf{h}_o$, $\mu = \cos \theta$, φ is the azimuthally angle between $\mathbf{p}$ and $\mathbf{h}_o$.

The nonlinear average collision integral is

$$\langle \operatorname{St} F \rangle_\varphi = \frac{\partial}{\partial \mu} (1 - \mu^2) b(\mu) \frac{\partial}{\partial \mu} \Phi(r, p, \mu, t), \quad (2.33.8)$$

where the kinetic coefficient is

$$b(\mu) = \frac{e^2}{2m^2 c^2} \int d\tau \int d\mathbf{k} P(\mathbf{k}) \cos(\varphi_k - \varphi) \cos(\varphi_k - \varphi - \Omega \tau) \Gamma_0(\omega) \exp(i\mathbf{k} \Delta \mathbf{r}(\tau)), \quad (2.33.9)$$

m is the particle mass, φ_k is the azimuthally angle of the vector $\mathbf{k}$, Ω is the gyro-frequency in the regular magnetic field, ω is the gyro-frequency in the random magnetic field, $\Delta \mathbf{r}(\tau)$ is the change in the radius vector of the particle in the regular magnetic field, $\Gamma_0(\omega)$ is a factor that is related to the additional Green

function of the particle in the nonlinear collision integral and that yields the damping of the resonant wave–particle interaction:

$$\Gamma_0(\omega) = \exp\left(-\frac{\omega_{\perp}^2}{16} v_{\parallel}^2 k_{\perp}^2 \tau^4 - \frac{\omega_{\perp}^2}{4\Omega^2} v_{\perp}^2 k_{\parallel}^2 \tau^2\right). \quad (2.33.10)$$

2.33.4. Kinetic coefficients and transport mean free paths

Let us first consider the limiting case of the absence of resonance broadening, $\omega = 0$ and $\Gamma_0(\omega) = 1$. In the kinetic coefficient (2.33.9), we expand the corresponding functions in terms of Bessel functions. We transform the series of Bessel functions and add the series using the addition formula for the Bessel functions

$$J_n(\rho) \exp\left(in \frac{\pi - \beta}{2}\right) = \sum_{k=-\infty}^{+\infty} J_{n+k}(z) J_k(z) \exp(ik\beta), \quad (2.33.11)$$

where $\rho = 2z \sin(\beta/2)$, $J_n(\rho)$, is the Bessel function of order n . The integrations in Eq. 2.33.9 yield a kinetic coefficient in the form

$$b_o(\mu) = C_o(\nu) \omega_{\perp} (\omega_{\perp}/\Omega) (q_{\parallel} \mu R)^{\nu-1}, \quad (2.33.12)$$

where

$$C_o(\nu) = \frac{\sqrt{\pi} \Gamma(\nu/2 + 2)}{3\nu \Gamma((\nu-1)/2)}, \quad R = |\mathbf{v}| \Omega^{-1}. \quad (2.33.13)$$

Let us now turn to the diffusion approximation using the formulae

$$\Lambda_0 = \frac{3|\mathbf{v}|}{4} \int_0^1 d\mu \frac{1-\mu^2}{b(\mu)}. \quad (2.33.14)$$

In this case

$$\Lambda_0 = 3(\Omega^2 \omega^{-2}) R (q_{\parallel} R)^{1-\nu} (4(2-\nu)(2-\nu/2) C_o(\nu))^{-1}. \quad (2.33.15)$$

In the case of strong random scattering at $v_{\parallel}^2 q_{\perp}^2 \gg v_{\perp}^2 q_{\parallel}^2$ following factor makes a major contribution to the resonance damping:

$$\Gamma_1(\omega, \tau) = \exp\left(-\frac{\omega_{\perp}^2}{16} v_{\parallel}^2 k_{\perp}^2 \tau^4\right). \quad (2.33.16)$$

The integrations in Eq. 2.33.9 yield a kinetic coefficient in the form

$$b_1(\mu) = C_1(\nu) \omega_{\perp} (\omega_{\perp}/\Omega)^{\nu} (q_{\perp} R)^{\nu-1}, \quad (2.33.17)$$

where

$$C_1(\nu) = \left(\frac{5}{4}\right)^{\nu-\frac{1}{2}} \frac{\sqrt{\pi}(\nu/2+1)\Gamma(1/4)\Gamma(\nu-1/2)}{3 \times 2^{\nu/2+1}\Gamma((\nu-1)/2)}. \quad (2.33.18)$$

In the opposite case $v_{\parallel}^2 q_{\perp}^2 \ll v_{\perp}^2 q_{\parallel}^2$, the following integrand factor makes a major contribution to the damping function :

$$\Gamma_2(\omega, \tau) = \exp\left(-\frac{\omega_{\perp}^2}{4\Omega^2} v_{\perp}^2 k_{\parallel}^2 \tau^2\right). \quad (2.33.19)$$

Substituting it into Eq. 2.33.9, we obtain after transformations and integrations

$$b_2(\mu) = C_2(\nu) \omega_{\perp} (\omega_{\perp}/\Omega) (q_{\parallel} R)^{\nu-1} \left(\frac{\sqrt{\pi}}{\Gamma(\nu/2)} \mu^{\nu-1} + (\omega_{\perp}/\Omega)^{\nu-1} \sqrt{1-\mu^2} \right), \quad (2.33.20)$$

where

$$C_2(\nu) = \frac{\Gamma(\nu/2)\Gamma(\nu/2+2)}{3\nu\Gamma((\nu-1)/2)}. \quad (2.33.21)$$

It is convenient to combine Eq. 2.33.12, Eq. 2.33.17, and Eq. 2.33.20 into a general interpolation formula for $b(\mu)$ that is valid at any pitch angle; as result, Mel'nikov (2005a) obtain

$$b(\mu) = \omega \left(\frac{\omega}{\Omega} \right) (q_{\parallel} R)^{\nu-1} \left[C_0 \mu^{\nu-1} + C_1 \mu^{\nu-1} \left(\frac{\omega q_{\perp}}{\Omega q_{\parallel}} \right)^{\nu-1} + C_2 \left(\frac{\omega}{\Omega} \right)^{\nu-1} \sqrt{1-\mu^2} \right]. \quad (2.33.22)$$

Using the interpolation formula for the integral in Eq. 2.33.14, he obtained the final formula for the parallel transport mean free path, including strong random scattering:

$$\Lambda_{//} = \Lambda_0 \sqrt{\pi} (2 - \nu) \left(2 - \frac{\nu}{2} \right) \left(\frac{\Omega}{\omega} \right)^{\nu-1} \Gamma^{-1} \left(\frac{\nu}{2} \right) \times \left[1.63 + \frac{\nu-1}{3} (f_1(\omega))^{0.7} + (2.13 - \nu) f_1(\omega) \right]^{-1}, \quad (2.33.23)$$

where

$$f_1(\omega) = C_0(\nu) C_2^{-1}(\nu) (\Omega \omega^{-1})^{\nu-1} + C_1(\nu) C_2^{-1}(\nu) (q_{\perp} q_{//}^{-1})^{\nu-1}. \quad (2.33.24)$$

In this case, $\Lambda_{//} \propto p^{2-\nu}$. The contribution of strong random scattering is significant at any strengths of the random magnetic field. The momentum dependence of $\Lambda_{//}$ in this case is similar to that numerically calculated by Teufel and Schlickeiser (2002, 2003) and Shalchi and Schlickeiser (2004).

2.33.5. Comparison with experimental data

For protons with energy of 200 MeV scattered in a weakly disturbed interplanetary medium, substituting $R = 4 \times 10^{10}$ cm, $L_{//} = L_{\perp} = 2 \times 10^{12}$ cm, $\nu = 1.67$, and $\Omega^2/\omega^2 = 8$ yields $\Lambda_{//} \approx 0.25$ AU. This value of $\Lambda_{//}$ is close to the mean experimental values from Palmer (1982). The values of $\Lambda_{//}$ are close to those from Shalchi and Schlickeiser (2004), in which, however, slab turbulence produces the main scattering. Thus, the weak momentum dependence of $\Lambda_{//}$ for solar CR over a wide energy range from several MeV to several GeV can be explained in terms of strong (moderate) random scattering by two-dimensional turbulence in the solar wind.

In the case of the very strong turbulence in co-rotating interaction region of the outer Heliosphere

$$\Lambda_{//} = \frac{3\nu \times 2^{3\nu/2-2} \Gamma\left(\frac{\nu-1}{2}\right)}{\pi \Gamma\left(\frac{\nu}{2} + 2\right)} \left(\sqrt{2} \frac{\Omega}{\omega} \right)^{\nu+1} \left(\frac{q_{//}}{q_{\perp}} \right)^{\frac{\nu}{2}-1}. \quad (2.33.25)$$

Calculations using Eq. 2.33.25 for $\omega \approx 0.7\Omega$ yield $\Lambda_{//} \approx 1$ AU for galactic CR with an energy of 10 GeV. The numerical value and rigidity dependence of $\Lambda_{//} \propto R$ is in agreement with experimental data on galactic CR modulation in the outer Heliosphere (Fujii and McDonald, 1995).

For galactic CR with energies above 4 GeV scattered in the interstellar medium, when the random magnetic field has a Kolmogorov spectrum with $\nu \approx 1.7$, we obtain the following order-of-magnitude estimate from Eq. 2.33.23:

$$\Lambda_{//} = 1.6q_{//}^{-1}(Rq_{//})^{2-\nu}(\Omega/\omega)^{\nu+1}. \quad (2.33.26)$$

Assuming that $\omega \approx 1.8\Omega$, $L_{//} \approx 100$ pc, and $H_1 \approx 0.3$ nT, we obtain for relativistic protons

$$\Lambda_{//} \approx 1.8 \times 10^{18} E^{2-\nu} \text{ cm}, \quad (2.33.27)$$

where E is the particle energy in GeV. Calculated value of $\Lambda_{//}$ is close to the experimental mean free path (Ptuskin, 2001).

2.34. CR perpendicular diffusion calculations on the basis of MHD transport models

2.34.1. The matter of problem

As it is mention in le Roux et al. (1999a), quasi-linear theory (QLT) for the parallel diffusion (diffusion coefficient $\kappa_{//}$) of CR appears to be understood reasonably well, unlike perpendicular diffusion ($\kappa_{\perp}$). This hampers our understanding of CR modulation in the context of well-established CR transport theory. le Roux et al. (1999a) present calculations of the radial cosmic ray diffusion coefficient in the ecliptic plane on the basis of three different theories for perpendicular diffusion assuming that large-scale field line random walk dominates resonant perpendicular diffusion. The radial dependence of κ_{rr} is determined completely theoretically using a promising recent model for the combined transport of a predominantly 2D component (80%), and a minor slab component (20%) of MHD turbulence in the solar wind.

2.34.2 Three models for perpendicular diffusion coefficient

On the basis of standard QLT for the cyclotron resonant interaction of CR with random Heliospheric magnetic field (HMF) slab fluctuations le Roux et al. (1999a) derived the CR parallel mean free path

$$\lambda_{//} = 2.433 \frac{r_g^{1/3} l_b^{1/3}}{A^2} \left[1 + 0.0972 \left(\frac{r_g}{l_b} \right)^{5/3} \right], \quad (2.34.1)$$

where r_g is the particle gyro-radius, l_b is the wavelength for slab turbulence at the break point in the power spectrum of HMF fluctuations, A is the normalized amplitude of the x -component of the slab fluctuations ($A = \delta B_x / B$, where B is the magnitude of the mean HMF; $(\delta B_x / B)^2 = 0.05$ and $B = 5$ nT at 1 AU).

The first model for $\kappa_{\perp}$ is given by

$$\kappa_{\perp} = (1/4) v l_c A^2, \quad (2.34.2)$$

where v is CR particle speed, l_c is the correlation length of slab turbulence, and the amplitude ($l_c = 0.79256 l_b$ where $l_b = 0.03$ AU at 1 AU), A is the sum of slab and 2D turbulence amplitudes. Eq. 2.34.2 corresponds to the QLT of Jokipii (1971) for slab turbulence and implies that CR are tied to and moving along a large-scale random-walking field line without experiencing resonant spatial diffusion. The only modification is that A denotes the sum of slab and 2D turbulence instead of just the slab component. This theory is tied to the condition $\lambda_{//} \gg l_c$ indicating applicability for rigidities $R \gg 2 \times 10^{-4}$ GV at 1 AU. Thereby, all R -values of relevance for CR modulation are covered. This model is referred to as the modified QLT (MQLT) model (see also Zank et al., 1998).

In the limit $\lambda_{//} \ll l_c$ or $R \ll 2 \times 10^{-4}$ GV at 1 AU, $\kappa_{\perp}$ is given by

$$\kappa_{\perp} = 0.5 A^2 \kappa_{//}, \quad (2.34.3)$$

where A is the sum of the amplitudes of slab and 2D turbulence. This expression is an outflow of the QLT by Chuvilgin and Ptuskin (1993) on anomalous perpendicular diffusion. It means that CRs are resonantly diffusing primarily along and weakly across large-scale random walking field lines, so allowing CR to change field lines. It implies that the rate at which large-scale neighboring field lines separate then plays a major role in determining the effective $\kappa_{\perp}$ of CR across B (see large ratio of $\kappa_{\perp} / \kappa_{//}$ in Eq. 2.34.3). Unfortunately, this model applies at R below that of interest for CR modulation. However, test particle simulations by Giacalone (1998) suggest that large-scale field line separation effects also occur at energies relevant for CR modulation, but at a reduced level. Using their work as a guide, le Roux et al. (1999) assume for **the 2nd model** of $\kappa_{\perp}$ that

$$\kappa_{\perp} = 0.02(A/A_o)^2 \kappa_{//}, \quad (2.34.4)$$

where A_o is the amplitude of the turbulence at 1 AU. This model is referred to as the modified anomalous diffusion (MAD) model.

The 3rd model makes use of the well-known basic expression for $\kappa_{\perp}$ given by

$$\kappa_{\perp} = \frac{1}{3} v r_g \frac{\omega \tau}{1 + \omega^2 \tau^2}, \quad (2.34.5)$$

where ω is the particle gyro-frequency, and τ is the scattering or relaxation time. Bieber and Matthaeus (1997) suggest that $\omega \tau$ can be expressed as

$$\omega \tau = \frac{2r_g}{3D_{B\perp}}, \quad (2.34.6)$$

where $D_{B\perp}$ describes large-scale field line wandering across $\mathbf{B}$. The expression for $D_{B\perp}$ is given by

$$D_{B\perp} = \frac{1}{2} D_{sl} + \frac{1}{2} \sqrt{D_{sl}^2 + 4D_{2D}^2}, \quad (2.34.7)$$

where

$$D_{sl} = \frac{1}{2} l_{sl} A_{sl}^2, \quad D_{2D} = l_{2D} A_{2D}. \quad (2.34.8)$$

In Eq. 2.34.8, D_{sl} describes the magnetic field wandering for slab turbulence, A_{sl} is the amplitude of this turbulence, and l_{sl} is its correlation length along B , while D_{2D} describes the magnetic field wandering for 2D turbulence, A_{2D} is the amplitude of this turbulence, and l_{2D} is its correlation length across $\mathbf{B}$ (Matthaeus et al., 1995). The advantage of this approach is that there is a clear distinction between l_c along and across B tied to the 2 components of solar wind turbulence. The MQLT model allows just for l_c parallel to $\mathbf{B}$. In addition, the approach is not limited to small amplitudes as QLT. Assuming $\omega \tau \gg 1$, and $D_{2D} = 0$, $\kappa_{\perp}$ in Eq. 2.34.5 is the same as $\kappa_{\perp}$ in Eq. 2.34.2 for slab turbulence. Although not well known, the expectation is that $l_{2D} \gg l_{sl}$ so that $\kappa_{\perp}$ is larger compared to the MQLT model in the limit $\omega \tau \gg 1$ and smaller when $\omega \tau \ll 1$. Le Roux et al.

(1999a) use $l_{2D} = 100 l_{sl}$ and for reference they call this model the nonperturbative (NP) model (see also Zank et al., 1998).

The dependence of the diffusion coefficients on radial distance r is theoretically determined with the MHD model for HMF turbulence transport in the solar wind according to Zank et al. (1996). The model gives a good reproduction of the observed r -dependence in the energy density of HMF fluctuations and also specifies the r -dependence of l_c . Key elements in its success are the generation of turbulence by corotating interaction regions close to the Sun, and by isotropizing pickup ion (PI) ring distributions beyond the ionization cavity ($r > 6$ AU). In an extended version of the model (le Roux et al., 1999b), it is shown that near isotropic PI distributions can also damp turbulence for $r > 30$ AU, but that turbulence generation by PI still dominates. The increase in the energy density of the turbulence and the decrease in l_c across the termination shock is estimated simply with the extended model.

2.34.3. The main results for diffusion coefficients

In Fig. 2.34.1 are shown theoretically calculated mean free paths for 930 MV CR He^+ with the MQLT model for $\kappa_{\perp}$ in the ecliptic plane as a function of increasing r from the Sun.

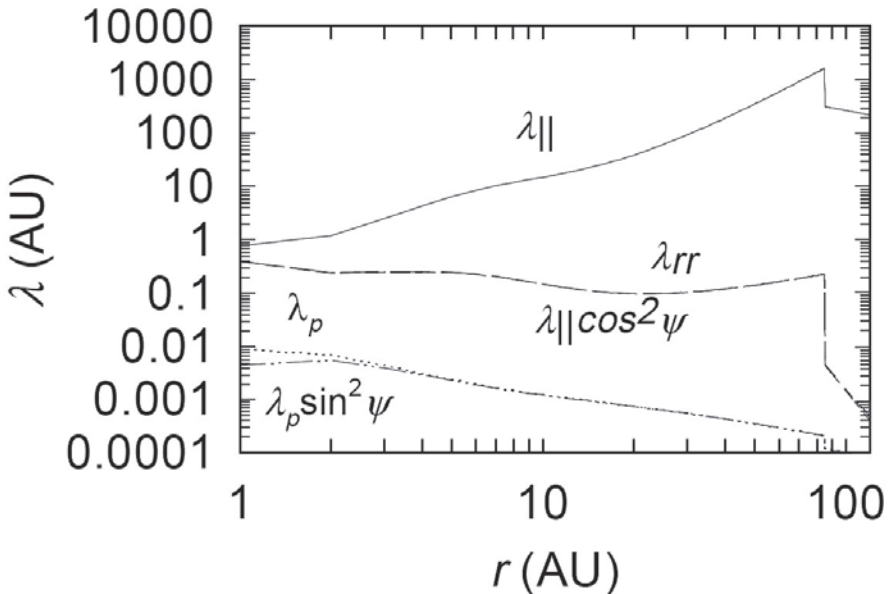


Fig. 2.34.1. Mean free paths in the ecliptic plane for the modified quasi-linear (MQLT) model of $\kappa_{\perp}$. The curves denote for anomalous CR He^+ with $R = 930$ MV ($E_k = 28$ MeV/nucleon). From le Roux et al. (1999a).

The radial mean free path λ_{rr} in Fig. 2.34.1 is calculated according to

$$\lambda_{rr} = \lambda_{||} \cos^2 \psi + \lambda_{\perp} \sin^2 \psi, \quad (2.34.9)$$

where ψ is the Parker field spiral angle, and $\lambda_{\perp} = 3\kappa_{\perp}/v$ is the perpendicular mean free path. The three important results to emerge from the MQLT model are as follows (see Fig. 2.34.1):

(1.1) λ_{rr} is determined solely by $\lambda_{||}$ without any contribution from $\lambda_{\perp}$ so that large negative radial gradients in λ_{rr} exist for $R \ll 1$ GV.

(1.2) There is a big decrease in the magnitude of $\lambda_{||}$ across the termination shock at 85 AU ($\cos^2 \psi \propto u^2$ where u is the solar wind speed) implying that a strong modulation barrier to galactic CR exists downstream.

(1.3) Close to 1 AU $\lambda_{rr} \propto \lambda_{||} \propto R^{1/3}$ because CR interact resonantly with the inertial range of the power spectra; at larger distances $\lambda_{rr} \propto \lambda_{||} \propto R^2$ because of resonant interaction of CR with the energy range.

In Figure 2.34.2 are shown calculations that consider the MAD model for $\kappa_{\perp}$.

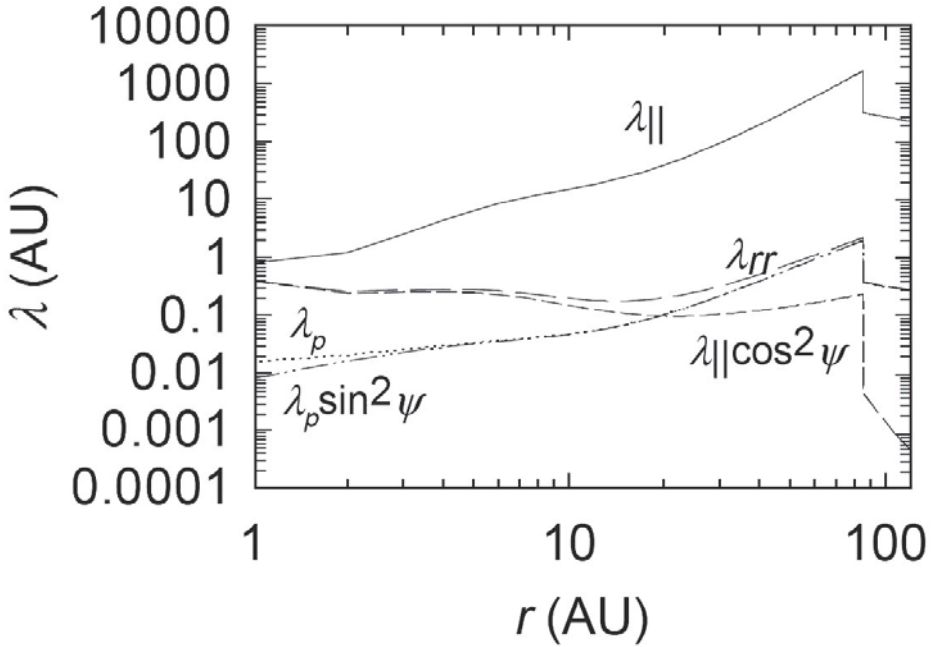


Fig. 2.34.2. The same as in Fig. 2.34.1, but for the modified anomalous diffusion (MAD) model of $\kappa_{\perp}$. From le Roux et al. (1999a).

The main results are (see Fig. 2.34.2):

- (2.1) $\lambda_{\perp} \propto r_g^{-2}$ contributes mainly to λ_{rr} beyond ~ 20 AU so that λ_{rr} has a strong r -dependence upstream beyond ~ 30 AU.
- (2.2) The drop in λ_{rr} across the termination shock is reduced ($\sin^2 \psi$ is less sensitive to the shock jump than $\cos^2 \psi$).
- (2.3) The R -dependence of λ_{rr} is determined by $\lambda_{\parallel}$, giving it the same dependence as for the MQLT model.
- (2.4) The negative r -dependence of λ_{rr} below $R < 1$ GV is weakened by the important contribution of $\lambda_{\perp}$ to λ_{rr} .

In Fig. 2.34.3 and Fig. 2.34.4 calculations of mean free paths are presented on the basis of the NP model.

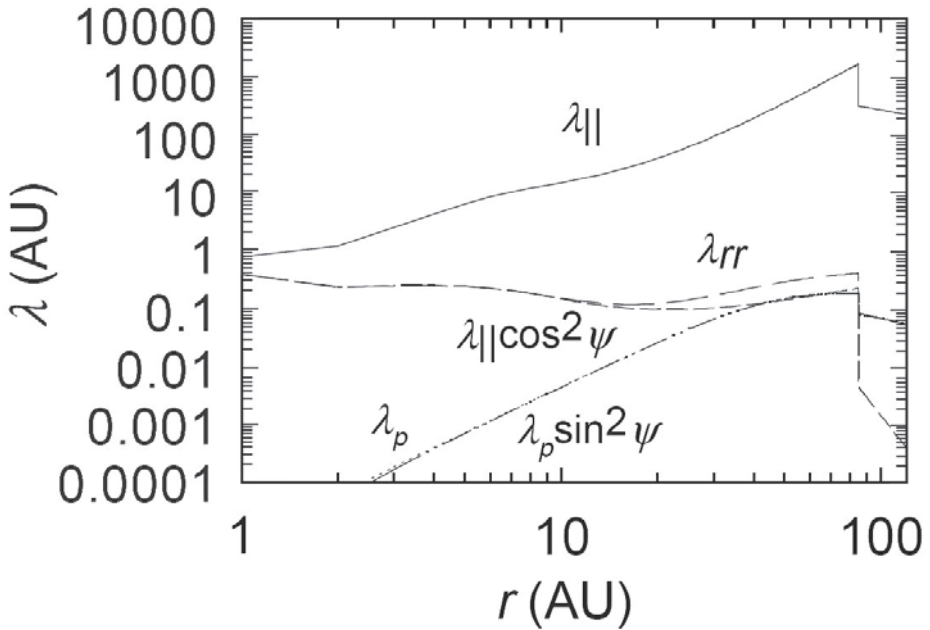


Fig. 2.34.3. As in Fig. 2.34.1, but for the non-perturbative (NP) model of $\kappa_{\perp}$. From le Roux et al. (1999a).

The key results produced by the NP model are the following:

- (3.1) $\lambda_{\perp}$ contributes significantly to λ_{rr} beyond ~ 20 AU from the Sun below ~ 3 GV so that the r -dependence of λ_{rr} upstream is reduced compared to the MAD case for intermediate R -values (Fig. 2.34.3). This is because $\lambda_{\perp}$ is independent of r_g ($\omega\tau \gg 1$ in Eq. 2.34.5).

(3.2) For $R \ll 3$ GV beyond ~ 20 AU, $\lambda_{\perp}$ and therefore λ_{rr} is strongly dependent on r ($\omega\tau \ll 1$ in Eq. 2.34.5).

(3.3) Above ~ 3 GV beyond ~ 20 AU $\lambda_{//}$ contributes the most to λ_{rr} .

(3.4) Consequently, λ_{rr} features a three interval R -dependence in the outer Heliosphere with the weakest dependence $\lambda_{rr} \propto R$ for intermediate values of $R < 3$ GV in the middle interval, and $\lambda_{rr} \propto R^2$ for $R > 3$ GV and $R \ll 3$ GV in the other two intervals (Fig. 2.34.4). It was proposed and demonstrated first by Moraal et al. (1999), in an empirical approach to the CR diffusion tensor, that a similar three interval R -dependence for λ_{rr} , with the weakest R -dependence in the center interval, is necessary for the simulation of both observed galactic and anomalous CR spectra.

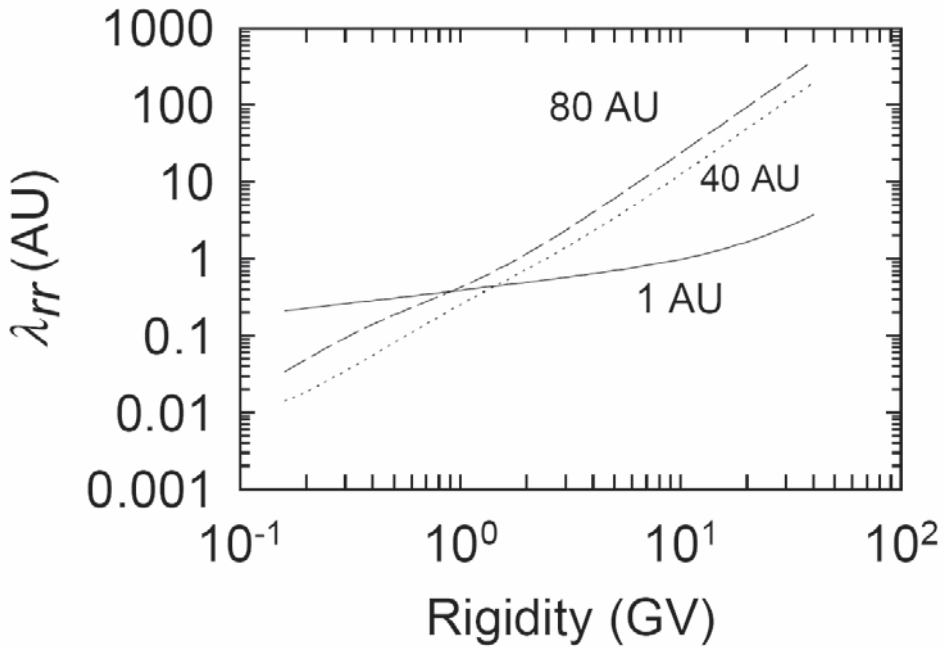


Fig. 2.34.4. The rigidity dependence of the radial mean free path λ_{rr} in the ecliptic plane for the NP model of $\kappa_{\perp}$. From le Roux et al. (1999a).

2.34.4. Summarizing and comparison of used three models

Le Roux et al. (1999a) summarized main results as following. The parallel, perpendicular and radial mean free paths for CR were determined theoretically on the basis of three plausible theories for $\kappa_{\perp}$ assuming that field line random walk is

more important than resonant perpendicular diffusion. A MHD model for field turbulence transport in the solar wind (Zank et al., 1996, 1998) was used to calculate the spatial dependence of the mean free paths. Concerning the MQLT model for $\lambda_{\perp}$, $\lambda_{\parallel}$ contributes solely to λ_{rr} , and consequently a big drop in λ_{rr} across the termination shock implying a strong galactic CR modulation barrier, is predicted. For the MAD model of $\lambda_{\perp}$, $\lambda_{\parallel}$ produces a strong contribution to λ_{rr} for $r > 20$ AU from the Sun resulting in a large r -dependence for λ_{rr} for $r > 30$ AU upstream. For both models, $\lambda_{rr} \propto R^2$ at large r due to the resonant interaction of CR with the energy range of the power spectra. Regarding the NP model for $\lambda_{\perp}$, $\lambda_{\parallel}$ contributes significantly to λ_{rr} for $R < 3$ GV at large r but $\lambda_{\parallel}$ dominates in λ_{rr} for $R > 3$ GV. This leads to a complex three interval R -dependence for λ_{rr} , with the weakest R -dependence $\lambda_{rr} \propto R$ given by the middle interval, and $\lambda_{rr} \propto R^2$ in the other two intervals. A similar R -dependence was first proposed empirically by Moraal et al. (1999) as a necessary condition for the simulation of both observed galactic and anomalous CR spectra. The NP model tentatively provides a theoretical basis for the work of Moraal et al. (1999).

2.35. On the role of drifts and perpendicular diffusion in CR propagation

2.35.1. Main equations for CR gradient and curvature drifts in the interplanetary magnetic field

Jokipii and Levy (1977) show that the CR gradient and curvature drifts in an Archimedean-spiral magnetic field produce a significant effect in the galactic CR propagation and modulation in the Heliosphere. The effects of drifts are due to the fact that CR small energy particles for which the drift velocity is comparable to the solar wind velocity have more rapid access (in case when the drift velocity directed to the Sun) to the inner Heliosphere than in the absence of drifts; in the opposite case, when the drift velocity directed from the Sun, the result could be inverse. Although drifts are explicitly contained in standard transport theories (e.g., Parker, 1965; Dorman, 1965; Axford, 1965a,b; Jokipii and Parker, 1970) they have been neglected in all models of galactic CR or SEP propagation in the interplanetary space. Jokipii and Levy (1977) note that Jokipii (1971), Levy (1975, 1976a,b), Barnden and Bercovitch (1975) pointed out some consequences of drifts, but did not construct complete models. Jokipii and Levy (1977) suggest using the term ‘drift’ to refer to gradient and curvature drifts, and not to the convection with the solar wind. Jokipii and Levy (1977), Jokipii et al. (1977) use the general formulation of CR transport written down by Jokipii and Parker (1970) and start

from decompose the CR diffusion tensor κ_{ij} into its symmetric and anti-symmetric parts $\kappa_{ij,S}$ and $\kappa_{ij,A}$. Then the average particle drifts may be written as

$$v_{dr,i} = \frac{\partial}{\partial x_j} (\kappa_{ij,A}). \quad (2.35.1)$$

Noting that according to Levy (1976a)

$$\text{div}(\mathbf{v}_{\mathbf{dr}}) = 0, \quad (2.35.2)$$

One may write the equation for the CR density n as a function of position $\mathbf{r}$, time t , and kinetic energy E_k as

$$\frac{\partial n}{\partial t} = \frac{\partial}{\partial x_i} \left(\kappa_{ij,S} \frac{\partial n}{\partial x_j} - (u_{sw,i} + v_{dr,i}) n \right) + \frac{1}{3} \frac{\partial u_{sw,i}}{\partial x_i} \frac{\partial}{\partial E_k} (\alpha E_k n), \quad (2.35.3)$$

where

$$\alpha = (2m_o c^2 + E_k) / (m_o c^2 + E_k). \quad (2.35.4)$$

For simplicity Jokipii and Levy (1977), Jokipii et al. (1977) assume that the electromagnetic conditions in the interplanetary space are symmetric about the Sun's rotation axis, and that $\kappa_{rr}, \kappa_{\theta\theta}$ are independent of r and θ , and define the new function

$$f = r^2 n \sin \theta. \quad (2.35.5)$$

The resulting equation for f is

$$\begin{aligned} \frac{\partial f}{\partial t} = & \frac{1}{2} \frac{\partial^2}{\partial r^2} (2\kappa_{rr} f) + \frac{1}{2} \frac{\partial^2}{\partial \theta^2} \left(\frac{2\kappa_{\theta\theta}}{r^2} f \right) - \frac{\partial}{\partial r} \left(\left(\frac{2\kappa_{rr}}{r} + v_{dr,r} + u_{sw} \right) f \right) \\ & - \frac{\partial}{\partial \theta} \left(\left(\frac{2\kappa_{\theta\theta}}{r} \cot \theta + \frac{v_{dr,\theta}}{r} \right) f \right) + \frac{\partial}{\partial E_k} \left(\frac{2u_{sw}}{3r} \alpha E_k f \right). \end{aligned} \quad (2.35.6)$$

Eq. 2.35.6 is a Fokker-plank equation (Chandrasekhar, 1943) with transition moments

$$\begin{aligned}\langle \Delta r^2 \rangle &= 2\kappa_{rr}\Delta t, \quad \langle \Delta \theta^2 \rangle = 2\kappa_{\theta\theta}\Delta t, \quad \langle \Delta r \rangle = (2\kappa_{rr}/r + u_{sw} + v_{dr,r})\Delta t, \\ \langle \Delta \theta \rangle &= (\kappa_{\theta\theta} \cot \theta / r^2 + v_{dr,\theta}/r)\Delta t, \quad \langle \Delta E_k \rangle = -2u_{sw}\alpha E_k/3r.\end{aligned}\quad (2.35.7)$$

Jokipii and Levy (1977) consider the steady-state solution to Eq. 2.35.7 with outer boundary condition $n = n_o(E_k)$ at $r = r_o$; it is presumed that the inner boundary at $r = r_a$ is an absorber of CR (because the inner absorbs boundary occupies relatively very small region of space, the its nature makes very little impact on the solution). The solution for f is obtained by introducing particles at $r = r_o$, distributed in θ as $\sin \theta$. Each particle random walks in r and θ according to the prescription

$$\begin{aligned}r_{i+1} &= r_i \pm \left(\langle \Delta r^2 \rangle \right)^{1/2} + \langle \Delta r \rangle, \quad \theta_{i+1} = \theta_i \pm \left(\langle \Delta \theta^2 \rangle \right)^{1/2} + \langle \Delta \theta \rangle, \\ E_{k,i+1} &= E_{k,i} + \langle \Delta E_k \rangle,\end{aligned}\quad (2.35.8)$$

where the time step Δt is chosen to be some convenient value, and the plus or minus signs are chosen randomly. Each particle is followed for successive time steps until r_{i+1} is either greater than r_o or less than r_a , at which point it is regarded as having escaped from the system and a new particle is introduced at $r = r_o$. The space in r, θ, E_k is divided into bins, and at each step the bin in which the particle is located is incremented by 1; the resulting r, θ, E_k histogram corresponds to the time independent solution for f . The corresponding solution for n is obtained by dividing on $r^2 \sin \theta$ (corresponding to Eq. 2.35.5).

2.35.2. The using of Archimedean-spiral model of interplanetary magnetic field

The used in Jokipii and Levy (1977) the model of interplanetary magnetic field corresponds to the classical Parker's (M1963) Archimedean-spiral magnetic field for constant radial solar wind velocity, but in which the field changes sign at the solar equator. The field may be written as

$$\mathbf{B}(r, \theta) = A(1 - 2S(\theta - \pi/2)) \times \left[\frac{\mathbf{e}_r}{r^2} - \mathbf{e}_\theta \frac{\Omega_o \sin \theta}{u_{sw} r} \right], \quad (2.35.9)$$

where $S(\theta)$ is the Heaviside step function and A is a constant; the sign of A changes with successive 11-year solar cycles and is positive for positive CR particles between general solar magnetic field reversals from even to odd cycle (as in 1950-1960, 1970-1980), and negative from odd to even cycle (as in 1960-1970, 1980-

1990, and so on). For negative CR particles the situation is inversely. On the opinion of Jokipii and Levy (1977) this is a very good approximation to that magnetic field which was shown by Levy (1975, 1976a,b) to provide a natural interpretation of interplanetary sector-structure observations. The corresponding particle velocity drift for positive CR particles is given by (Jokipii et al., 1977):

$$\mathbf{v}_{dr} = \left(1 - 2S\left(\theta - \frac{\pi}{2}\right)\right) \langle \mathbf{v}_{dr} \rangle_m + \delta\left(\theta - \frac{\pi}{2}\right) \frac{2vpc}{3qAu_{sw}} \times \frac{\Omega_o r}{1 + \Omega_o^2 r^2 \sin^2 \theta / u_{sw}^2} \left(\mathbf{e}_r + \mathbf{e}_\phi \frac{u_{sw}}{\Omega_o r} \right), \quad (2.35.10)$$

where

$$\langle \mathbf{v}_{dr} \rangle_m = \left(-\mathbf{e}_r \frac{\Omega_o r}{u_{sw}} \cos \theta + \mathbf{e}_\theta \left(2 + \frac{\Omega_o^2 r^2 \sin^2 \theta}{u_{sw}^2} \right) \frac{\Omega_o r \sin \theta}{u_{sw}} + \mathbf{e}_\phi \frac{\Omega_o^2 r^2 \cos \theta \sin \theta}{u_{sw}^2} \right) \times \frac{2vpcr}{3qA \left(1 + \Omega_o^2 r^2 \sin^2 \theta / u_{sw}^2 \right)^2}, \quad (2.35.11)$$

p is the momentum of CR particle, v is its speed, and q its charge; u_{sw} is the velocity of the solar wind, Ω_o is the angular velocity of the Sun; r , θ , and ϕ are the usual spherical polar coordinate centered in the Sun. To simplify the calculations Jokipii and Levy (1977) assume that κ_{rr} , $\kappa_{\theta\theta}$, v_{dr} are independent of energy. They ran simulations using 60,000 to 200,000 particles with: $u_{sw} = 4 \times 10^7 \text{ cm}^2 \text{s}^{-1}$, $\kappa_{rr} = 5 \times 10^{21} \text{ cm}^2 \text{s}^{-3}$, $\kappa_{\theta\theta} = 0.1 \kappa_{rr}$, $A = \pm 4.7 \times 10^{21} \text{ cm}^2 \text{Gs}$ (corresponding to $|\mathbf{B}| = 5 \times 10^{-5} \text{ Gs}$ at 1 AU), $pc = 1 \text{ GeV}$, $r_a = 10^{12} \text{ cm}$, $r_o = 10 \text{ AU}$, $n_o(E_k) \propto E_k^{-2.5}$. The solution is symmetric about $\theta = \pi/2$, so it was considered the range $0 \leq \theta \leq \pi/2$.

2.35.3. The illustration results on the nature of CR drift modulation

Results shown in Fig. 2.35.1 and Fig. 2.35.2 are histograms of the number density of CR particles within 27° of the equatorial plane, averaged over 0.5 AU, with the density normalized to 1 at the outer boundary $r_o = 10 \text{ AU}$ (really, as it is considered in Chapter 3, $r_o \approx 100 \text{ AU}$, but results shown in Fig. 2.35.1 and Fig. 2.35.2 are interested as illustration of drifts influence on CR propagation and modulation). In Fig. 2.35.1, A is positive, and in Fig. 2.35.2, A is negative. The

dashed lines in each case correspond to the modulation solution with the drifts set equal to zero.

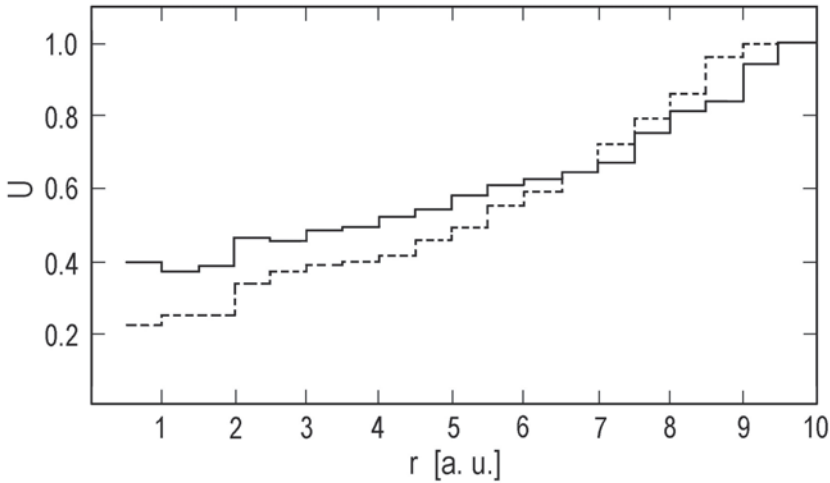


Fig. 2.35.1. Histograms of the number of CR particles within $\pm 27^\circ$ of the solar equatorial plane, as a function of heliocentric radius r , for positive A . The solid line is the solution with drifts; the dashed line is the solution in the absence of drifts. The statistical uncertainties scale as r^{-2} and are about $\pm 15\%$ for the innermost bin shown. From Jokipii and Levy (1977).

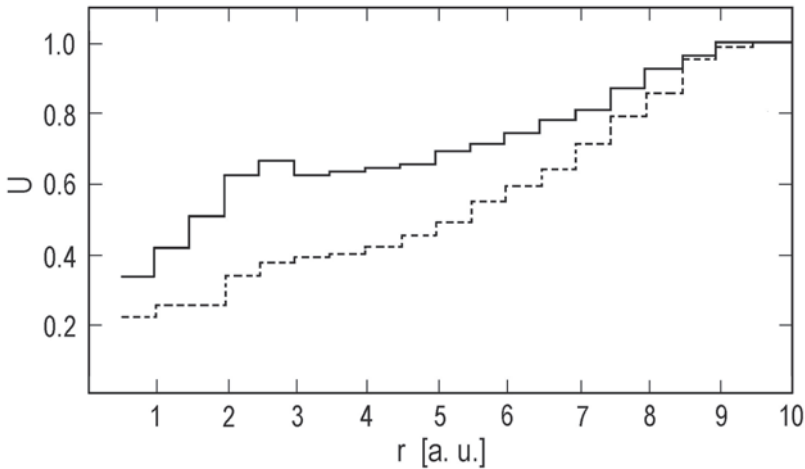


Fig. 2.35.2. Same as in Fig. 2.35.1, but for A negative. The bump at ~ 3 AU is a statistical deviation. From Jokipii and Levy (1977).

From Fig. 2.35.1 and Fig. 2.35.2 may be clear seen that for both positive and negative A , the drifts considerably change the modulated CR density. An item of

major interest is that the CR radial gradient may be substantially reduced by the inclusion of realistic drifts. According to Jokipii et al. (1977), since the divergence of the average drift velocity is zero, the drift by itself cannot cause CR modulation; however, in the presence of a CR particle gradient produced by the usual convection-diffusion modulation, the drifts can have a substantial effect.

2.36. Drifts, perpendicular diffusion, and rigidity dependence of near-Earth latitudinal proton density gradients

2.36.1. The matter of the problem

Burger et al. (1999) note that from September 1994 to July 1995, the Ulysses spacecraft executed a fast latitude scan by moving from 80° South to 80° North at solar distances between 1.3 and 2.2 AU. During this first comprehensive exploration of the latitudinal dependence of modulation, a number of discoveries were made (see Simpson, 1998 and McKibben, 1998 for recent overviews). It was observed the unexpected small latitudinal CR proton density gradients, and its rigidity dependence (Heber et al., 1996.) These authors also attempted to model the observed gradients. They found that the discrepancy between measurements and model results increased as rigidity is decreased. The magnitude problem was subsequently solved (Potgieter et al, 1997, 1999; Hattingh et al., 1997) by using anisotropic perpendicular diffusion (Jokipii and Kóta, 1995). Burger et al. (1999) show that it is the rigidity dependence of the perpendicular diffusion coefficient in the polar direction that controls that of the latitudinal gradient, and that this coefficient's rigidity dependence cannot be the same as that of parallel diffusion.

2.36.2 The propagation and modulation model, and diffusion tensor

According to Burger et al. (1999) the modulation of galactic CR is described by Parker's transport equation (Parker, 1965) for the omni-directional distribution function $f_o(\mathbf{r}, p)$ for particles with rigidity p at position $\mathbf{r}$, which can be written in the steady state as

$$\begin{aligned} & \left(\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \kappa_{rr}) \right) \frac{\partial f_o}{\partial r} + \left(\frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} (\kappa_{\theta\theta}) \right) \frac{\partial f_o}{\partial \theta} + \kappa_{rr} \frac{\partial^2 f_o}{\partial r^2} + \frac{\kappa_{\theta\theta}}{r^2} \frac{\partial^2 f_o}{\partial \theta^2} \\ & - (\langle v_{dr} \rangle_r) \frac{\partial f_o}{\partial r} - \left(\frac{1}{r} \langle v_{dr} \rangle_\theta \right) \frac{\partial f_o}{\partial \theta} - u_{sw} \frac{\partial f_o}{\partial r} + \left(\frac{1}{3r^2} \frac{\partial}{\partial r} (r^2 u_{sw}) \right) \frac{\partial f_o}{\partial \ln p} = 0. \end{aligned} \quad (2.36.1)$$

Here r and θ are heliocentric radial distance and colatitude (polar angle) respectively, u_{sw} is the solar wind speed, and v_{dr} is the drift velocity. The coefficient $\kappa_{\theta\theta}$ describes diffusion perpendicular to the mean magnetic field in the polar direction, while the radial coefficient is

$$\kappa_{rr} = \kappa_{//} \cos^2 \psi + \kappa_{\perp}^{r\phi} \sin^2 \psi, \quad (2.36.2)$$

where $\kappa_{//}$ is the diffusion coefficient parallel to the mean magnetic field, $\kappa_{\perp}^{r\phi}$ is the diffusion coefficient perpendicular to the field in the radial/azimuthal direction and ψ is the spiral angle. In the two-dimensional model this coefficient acts only in the radial direction. In Eq. 2.36.1 the first 4 terms described diffusion, 5th and 6th – drifts, 7th – convection, and the last, 8th – adiabatic energy loss. Burger et al. (1999) used a steady-state two-dimensional model that simulate the effect of a wavy current sheet (Burger and Hattingh, 1995, Hattingh and Burger 1995) by using for the three-dimensional drift pattern in the region swept out by the wavy current sheet, an averaged field with only an r - and a θ -component. The Heliospheric boundary is assumed at 100 AU while the solar wind speed is 400 km/s within $\sim 30^\circ$ of the ecliptic plane and increases within $\sim 10^\circ$ to 800 km/s in the polar regions. A modified Heliospheric magnetic field (HMF) is used (Jokipii and Kóta 1989). The tilt angle of the wavy current sheet is 15° . The diffusion tensor on which the current one is based, is described in detail in Burger and Hattingh (1998). For diffusion parallel to the magnetic field, Burger et al. (1999) used

$$\kappa_{//} = \frac{9vB_o^{5/3}l_s^{5/3}}{28\pi^2 sC_s} \left(\frac{R}{c}\right)^{1/3}, \quad (2.36.3)$$

if the quantity $D = (c/R)B_o l_s$ is greater than 1, while if it is less than one

$$\kappa_{//} = \begin{cases} \frac{v}{8\pi^2 sC_s l_s} \left(\frac{R}{c}\right)^2 \left[\frac{1}{4\delta} + \left(2 + \frac{b}{2}\right) D^2 - \frac{1}{12} \left(\frac{8}{7} + b^3\right) D^4 \right] & \text{if } bD \leq 1, \\ \frac{vB_o}{4\pi^2 sC_s} \left(\frac{R}{c}\right) \left[\frac{1}{3} + D - \frac{1}{21} D^3 \right] & \text{if } bD > 1. \end{cases}, \quad (2.36.4)$$

In Eq. 2.36.3 and Eq. 2.36.4 v is the particle speed, B_o is the magnitude of the background magnetic field, l_s is the correlation length of the magnetic field, s is the fraction of slab turbulence, C_s is the level of the turbulence, c is the speed of light, and R is the particle rigidity. The quantity b determines the transition from $\kappa_{//}/v \propto R$ to $\kappa_{//}/v \propto R^2$ for particles resonant with fluctuations in the energy range of the magnetic field power spectrum: if b is equal to 1, only $\kappa_{//}/v \propto R^2$ occurs, while if it is greater than one both occur. In Eq. 2.36.4 the terms in square brackets ensure a smooth transition from one rigidity dependence to the next. To

describe the anisotropic diffusion perpendicular to the field, and drift, Burger et al. (1999) used

$$\kappa_{\perp}^{r\phi} = 0.007 \kappa_{\parallel} R^{\gamma}, \quad \kappa_R = \frac{\nu R}{3cB_o} \frac{10R^2}{1+10R^2}, \quad \kappa_{\theta\theta} = \begin{cases} 0.007 \kappa_{\parallel} R^{\eta} & \text{ecliptic} \\ 0.1 \kappa_{\parallel} R^{\eta} & \text{polar} \end{cases} \quad (2.36.5)$$

The ecliptic region spans the solar equatorial plane with a half-angle of 35° . $\gamma = \eta = -0.4$. Fig. 2.36.1, panels (a) and (b) show the spatial dependence of the radial and polar mean free paths, and the drift scale, while panels (c) and (d) show their rigidity dependence at $\gamma = \eta = -0.4$. Burger et al. (1999) note that the spatial and the rigidity dependence of the diffusion coefficients cannot be separated and this leads to the different behavior of these quantities in different regions in space.

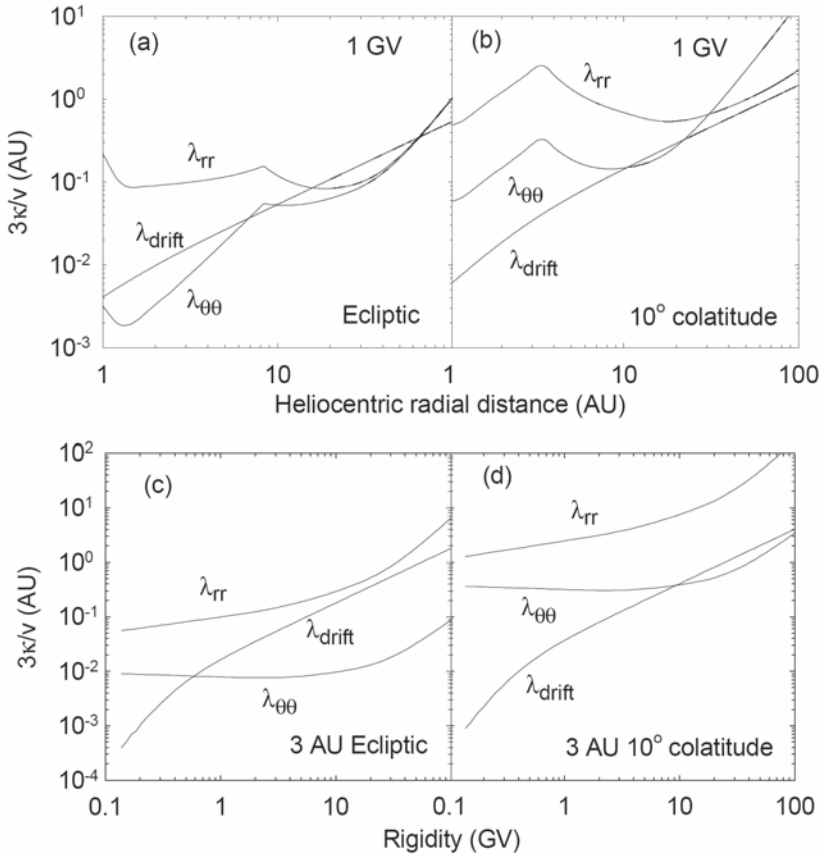


Fig.2.36.1. Radial dependence of the radial and polar mean free paths, and the drift scale for 1 GV protons in the ecliptic and at 10° colatitude (upper panels). The two lower panels show the rigidity dependence of the same variables at a radial distance of 3 AU. In all cases $\gamma = \eta = -0.4$. From Burger et al. (1999).

From the panel (a) in Fig. 2.36.1 can be seen that in the ecliptic region λ_{rr} approaches $\lambda_{\theta\theta}$ beyond 30 AU. Radial diffusion is dominated by $\kappa_{\perp}^{r\phi}$ at large radial distances where the field becomes azimuthal, and in the ecliptic region $\kappa_{\perp}^{r\phi} = \kappa_{\theta\theta}$. In the polar region (see panel (b) in Fig. 2.36.1), $\lambda_{\theta\theta}$ exceeds λ_{rr} at large radial distances where $\kappa_{\perp}^{r\phi}$ again begins to dominate radial diffusion; but here $\kappa_{\perp}^{r\phi} < \kappa_{\theta\theta}$. The panels (c) and (d) in Fig. 2.36.1 show that the polar mean free path has a flatter rigidity dependence than the radial mean free path. Drifts are slightly reduced (below about 1 GV) with respect to the weak scattering case which is proportional to R at all rigidities.

The parameters $\gamma = \eta = -0.4$ in Eq. 2.36.5 are chosen to fit solar minimum data at Earth for both solar polarity epochs, by changing *only* the sign of the magnetic field. Although optimized for protons, good fits to galactic helium and high energy electrons are also obtained.

2.36.3. Latitudinal gradients for CR protons

The central result of paper Burger et al. (1999) is Fig. 2.36.2, which shows a comparison of the latitudinal gradient for CR protons calculated at 2 AU between the ecliptic and 10° colatitude, and Ulysses data.

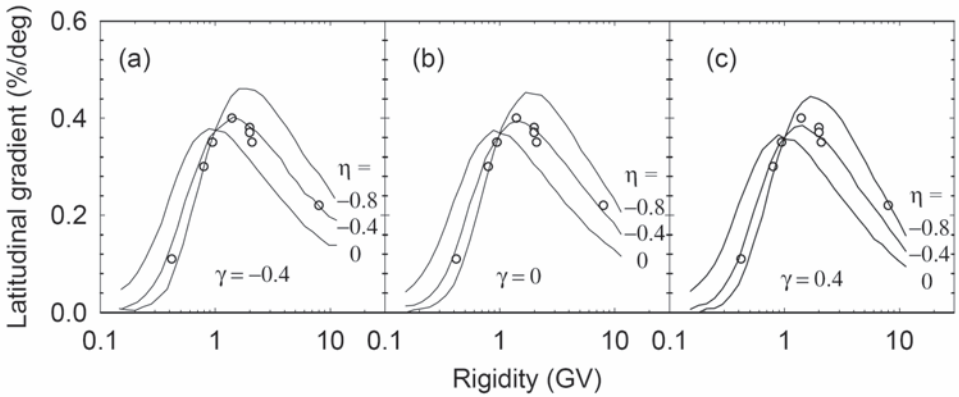


Fig. 2.36.2. The calculated latitudinal CR proton gradients at radial distance 2 AU from the Sun for different values γ and η in Eq. 2.36.5 for perpendicular diffusion, and comparison with Ulysses data (open circles, according to Heber et al., 1996). From Burger et al. (1999).

Comparing panels (a), (b), and (c) in Fig. 2.36.2 it is evident that changing the rigidity dependence of $\kappa_{\perp}^{r\phi}$ has little effect on the latitudinal gradient as γ changes from -0.4 to $+0.4$. It reduces the gradient somewhat at high rigidity, but it does not shift the maximum. In contrast, changing the rigidity dependence of $\kappa_{\theta\theta}$ changes

both the magnitude and the position of the maximum as η changes from -0.8 to 0 in each panel. The best values are $\gamma = \eta = -0.4$.

2.36.4. Discussion on the nature of CR latitudinal transport

Burger et al. (1999) came to conclusion that to obtain the correct magnitude of the observed near-Earth latitudinal gradient, enhanced latitudinal transport is required. In the described above model, this is accomplished by increasing the cross-field diffusion in the polar direction with respect to that in the other direction perpendicular to the HMF. To obtain the observed rigidity dependence of this gradient, the cross-field diffusion in the polar direction must have a flatter rigidity dependence than parallel diffusion; at rigidities below about 10 GV $\kappa_{\perp}/v$ should be almost independent of rigidity. Before coming to this conclusion, numerous other options were tried. However, looking at the transport Eq. 2.36.1, it is obvious that $\kappa_{\theta\theta}$, which appears as a coefficient of $\partial f_o / \partial t$, should play a dominant role in governing latitudinal transport. At least two other studies support this conclusion. Comparing Ulysses high-latitude data on the rate of change of integral CR intensities with IMP-8 data, Simpson (1998) concludes that if cross-field diffusion (as opposed to direct magnetic field "channeling"; see Fisk, 1996) occurs, it should be independent of rigidity. In an independent study, Potgieter et al. (1999) came to a similar conclusion studying CR electron modulation and using Ulysses electron data (see below, Section 2.40).

There are however other studies that at a first glance appear to contradict the above conclusion. A numerical simulation of Giacalone (1998) predicts $\kappa_{\perp}/v \propto R^{1/2}$ in the range $40 \text{ MV} < R < 2 \text{ GV}$. A second different conclusion follows from the interpretation of Voyager anomalous nuclear component data (Cummings and Stone, 1998 and references therein) which suggests that the perpendicular mean free path is proportional to R^2 below about 1 GV, in agreement with quasi-linear theory (e.g., Bieber et al., 1995).

Can all these different results be reconciled? The answer of Burger et al. (1999) is a guarded yes, but only if those that appear to have observational support are considered, i.e. if the numerical simulations reported by Giacalone (1998) is neglected for the moment. One possibility is that perpendicular transport in the ecliptic and in the polar region is different. In the ecliptic region, QLT may apply – this will explain the result reported in Cummings and Stone (1998). The relative insensitivity of the rigidity dependence of perpendicular gradient G_{θ} on $\kappa_{\perp}^{r\phi}$ near Earth, is an indication (albeit not a strong one) that the results of the discussed study will not necessarily be invalidated if $\kappa_{\perp}^{r\phi}$ from QLT is used. More problematic is $\kappa_{\theta\theta}$. The enhanced latitudinal transport may also be due to direct magnetic field "channeling" in the HMF model of Fisk (1996). The question in this case is, if the transport is parallel to the field, should this process not have the same rigidity

dependence as parallel diffusion? The first implementation of the Fisk field in a numerical modulation model (Kóta and Jokipii, 1997) unfortunately did not address this issue.

2.37. CR drifts in dependence of Heliospheric current sheet tilt angle

2.37.1. *The matter of the problem*

According to Burger and Potgieter (1999), the effect of particle drifts, and in particular drift along the wavy current sheet, has long been thought to be responsible for the characteristic shape of the CR intensity profile observed near Earth around times of minimum solar activity (e.g., Jokipii and Thomas, 1981). Positively charged particles, during a positive solar polarity cycle (when the field in the northern hemisphere of the Sun points outward; denoted by $A > 0$) exhibit a rather flat response to the changing tilt near solar minimum. During alternate cycles, denoted by $A < 0$, and for the same range of tilt angles, the intensity profile shows a peak around solar minimum. It is by now well-established that drift-dominated models can readily explain these different profiles (e.g., Jokipii and Thomas, 1981). While the role of drifts during periods of minimum solar activity appear to be well understood, the same cannot be said for periods when the Sun approaches maximum activity, and the tilt angle becomes large. Previous studies (Potgieter and Burger, 1990; Webber et al., 1990) with steady-state two-dimensional models that simulate the effect of a wavy current sheet, suggest that the flat response of positively charged particles during $A > 0$ cycles would persist for large values of the tilt angle. Using a newer version of such a two-dimensional simulated wavy current sheet model, Burger and Hattingh (1998) show that the intensity of CR protons during an $A > 0$ cycle does respond markedly when the tilt becomes larger than about 40° , approaching the intensity for an $A < 0$ cycle. In the described below study Burger and Potgieter (1999) extend the analysis of Burger and Hattingh (1998) to show what happens when the tilt angle approaches $\sim 90^\circ$ near maximums of solar activity.

2.37.2 *CR propagation and modulation model; solar minimum spectra*

The two-dimensional, steady-state numerical modulation model that is used in the study of Burger and Potgieter (1999) was described in detail by Burger and Hattingh (1995). A comparison of CR electron spectra from this model and those from a three-dimensional model was considered in Ferreira et al. (1999); it was found good agreement between the two models (see below, in Section 2.41). Therefore, as Burger and Potgieter (1999) note, from a modeling point of view there is no reason to doubt the validity of the results from the two-dimensional model.

In Fig. 2.37.1 are shown resulting solar minimum spectra (tilt angle is taken 15°) for CR electrons, protons and helium at Earth for the two polarity cycles of the

solar magnetic field: $A > 0$ (e.g., 1996) and $A < 0$ (e.g., 1987). The modulated CR spectra are shown in comparison with corresponding spectra out of the Heliosphere.

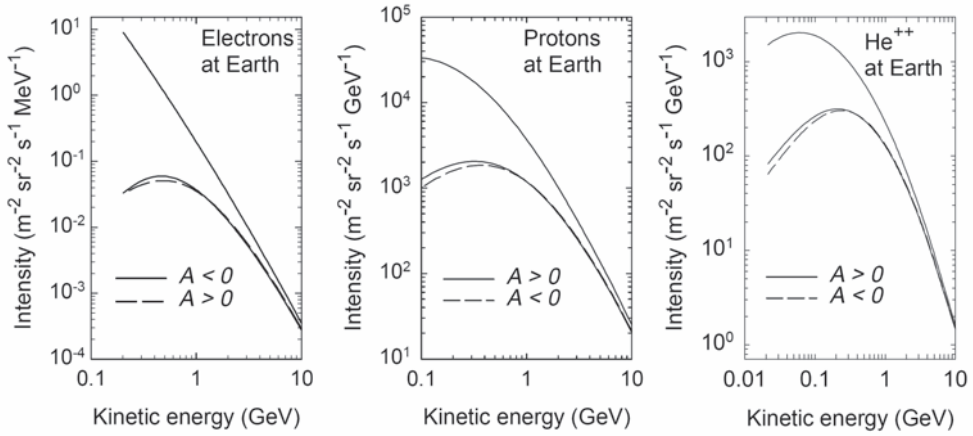


Fig. 2.37.1. Solar minimum spectra for CR electrons, protons and helium at Earth for the two polarity cycles of the solar magnetic field: $A > 0$ (e.g., 1996) and $A < 0$ (e.g., 1987). The tilt angle is 15° . By thick curves are shown corresponding spectra out of the Heliosphere. From Burger and Potgieter (1999).

2.37.3. Tilt angle dependence of CR protons at Earth

Fig. 2.37.2 shows how the intensity of CR protons, relative to the corresponding interstellar value, varies as function of tilt angle. From Fig. 2.37.2 can be seen that at all three energies the classic drift behavior, with the intensity-tilt profiles for an $A > 0$ cycle flatter than for an $A < 0$ cycle, is evident only for tilt angles up to about 45° . From 45° to about 60° , the intensity-tilt profiles for both cycles have similar slopes. Beyond about 60° , the $A > 0$ intensity-tilt profile drops, while the $A < 0$ intensity-tilt profile flattens, both approaching the no-drift value, indicated with a filled circle. The fact that this approach to the no-drift value becomes more evident as the energy decreases is due to numerical boundary effects, which in these cases diminishes as the particle's gyro-radius decreases. Note that Webber et al. (1990) used such intensity-tilt profiles to deduce that drift effects need to be reduced in a rigidity dependent manner, as is done in the study of Burger et al., 1999 (see above, Section 2.36, Eq. 2.36.5).

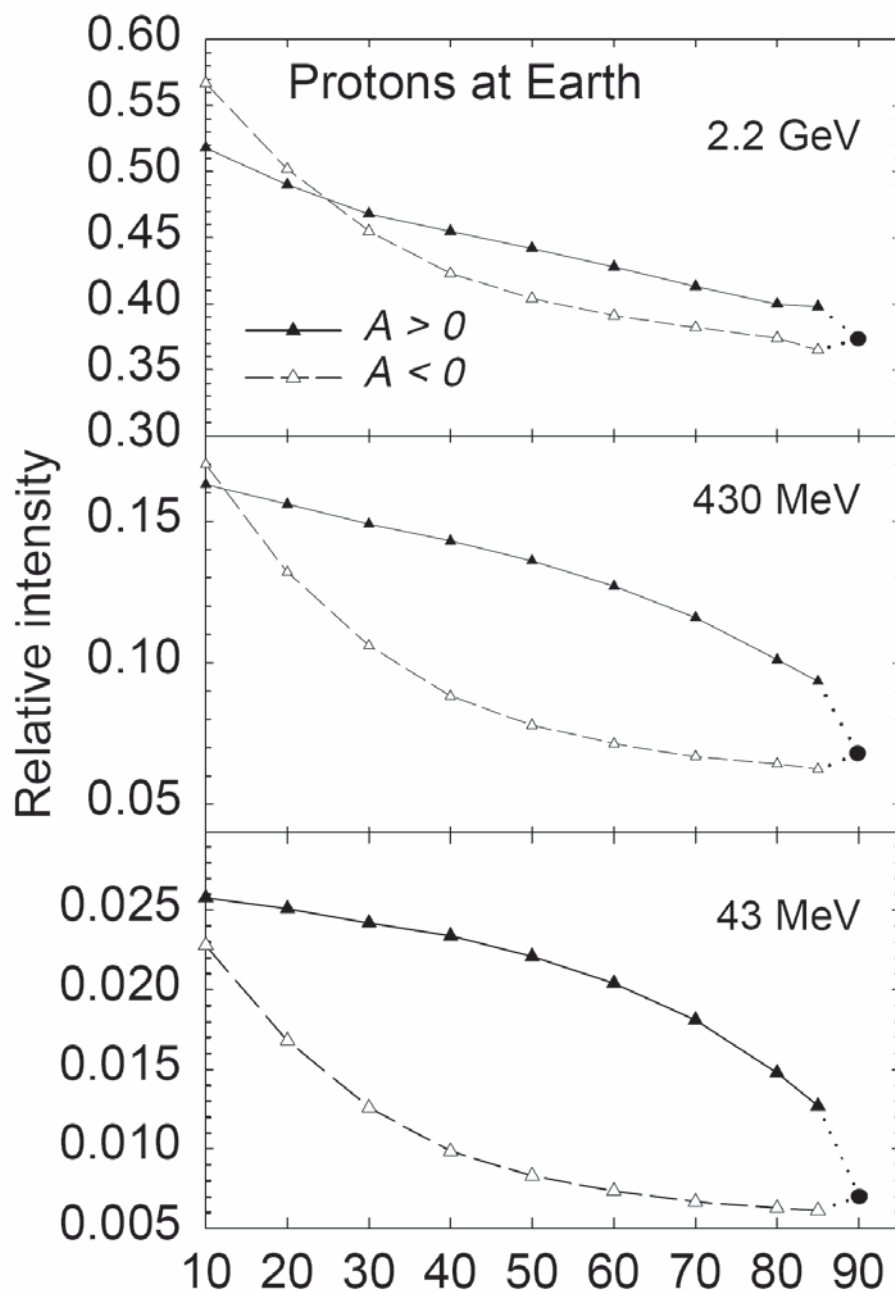


Fig. 2.37.2. Intensity-tilt profiles for CR protons at Earth relative to intensity out of the Heliosphere. The filled circles denote no-drift values, and the dotted lines are straight-line interpolations. From Burger and Potgieter (1999).

A comparison between the two-dimensional model and a three-dimensional model (Hattingh, 1998) is shown in Fig. 2.37.3

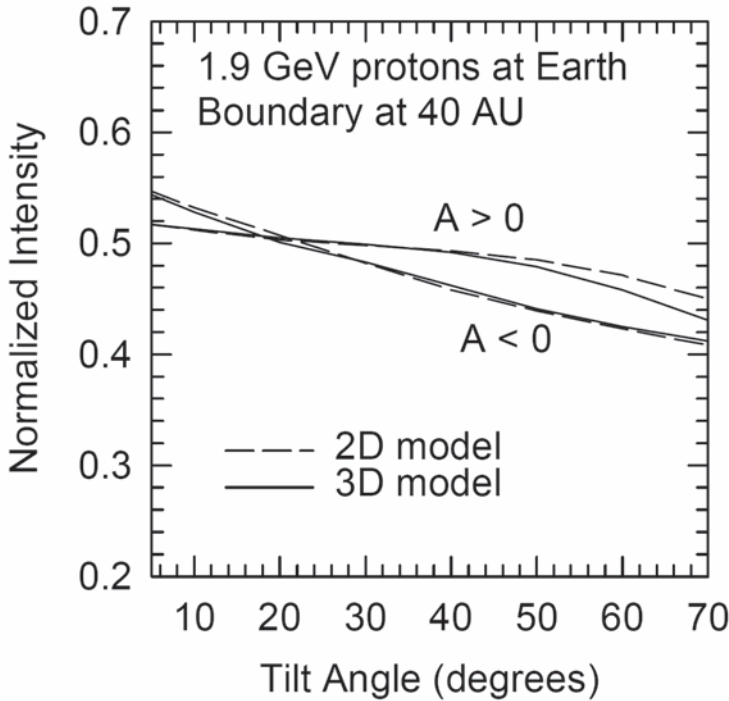


Fig. 2.37.3. Comparison of intensity-tilt profiles from a two-dimensional and a three-dimensional models for 1.9 GeV protons at the Earth's orbit. From Burger and Potgieter (1999).

In Fig 2.37.3 a different diffusion tensor is used, and the boundary is set at 40 AU. The convergence of the intensity-tilt profiles in the three-dimensional model to a common value is somewhat faster than in the two-dimensional model. Note, however, that the difference between the $A > 0$ intensities at a tilt angle of 70° is less than 5%. Clearly the two models show the same qualitative behavior, and to a large extent the same quantitative behavior, as the tilt angle increases (Ferreira et al., 1999a; see also below, Section 2.39).

2.37.4. Tilt angle dependence of CR intensity ratios at Earth orbit

The tilt angle and solar polarity-sign dependence of the ratios e^-/He^{++} and e^-/p are shown in Fig. 2.37.4, normalized with respect to the minimum value for each ratio. Although there are some quantitative differences between the two ratios, their qualitative behavior is the same. During an $A > 0$ cycle, the ratio has a 'w' shape,

and shows smaller changes when the tilt angle changes than during an $A < 0$ cycle, when the ratio has an 'm' shape.

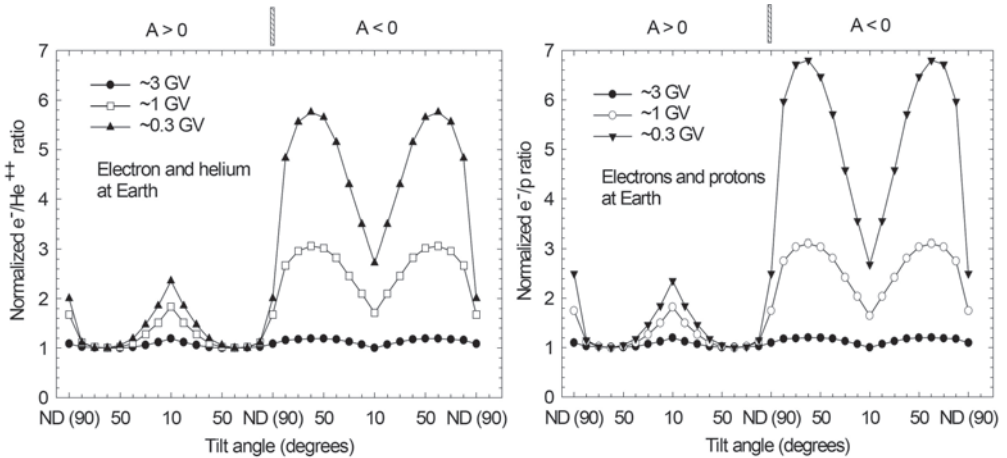


Fig. 2.37.4. The tilt angle and solar polarity-sign dependence of the ratios e^-/He^{++} and e^-/p , normalized with respect to the minimum value for each ratio. "ND (90)" denotes a no-drift value at a hypothetical tilt angle of 90° . From Burger and Potgieter (1999).

From Fig 2.37.4 can be seen that the changes in the ratio becomes larger as the rigidity becomes smaller. In earlier studies (e.g., Potgieter and Burger 1990; Webber et al., 1990) the smooth transition from one polarity cycle to the next does not occur. The reason for this is that at large tilt angles, predecessors of the current two-dimensional model predicted a much flatter intensity-tilt response of positively charged particles during an $A > 0$ cycle, and therefore of negatively charged particles during an $A < 0$ cycle.

2.37.5 Discussion of main results

Apart from magnetic polarity, only the tilt angle is changed to obtain the present results. Since the tilt angle is a proxy for solar activity, we therefore employ drifts to construct a simplified solar-activity cycle. In Burger and Potgieter (1999) model, intensity-tilt profiles (Fig. 2.37.2) show three distinct regimes. During periods when the tilt is small, the well-known peaked profile for protons occurs when $A < 0$, and the "flat" profile (actually only flatter than the peaked profile) when $A > 0$. For larger tilt angles, a second regime occurs when the two profiles more-or-less track each other. Cane et al. (1999) find observational evidence for both regimes at neutron-monitor energies, but conclude that the second is not due to drift effects, in contrast to the results presented here. The third regime is when the $A > 0$ profile drops while the $A < 0$ profile flattens to converge to the no-drift intensity. Clearly, drifts are phased out as the tilt angle increases for both polarities.

The ratio of differently charged particle intensities throughout hypothetical solar activity cycle, where only the tilt angle changes, shows that during $A > 0$ cycles, the largest changes occur around solar minimum modulation, and the local maximum occurs at solar minimum. At other times, little change in the ratio occurs, but there is an sharp increase in the ratio going from an $A > 0$ to an $A < 0$ cycle, which decreases as the rigidity of the particles decreases. During an $A < 0$ cycle, changes in the ratio is typically larger than during the alternate cycle, especially at lower rigidities. At solar minimum modulation the ratio is at a local minimum. Burger and Potgieter (1999) note that the qualitative features of the ratio-tilt angle profiles for the electron/helium ratio agrees remarkably well with observations (Bieber et al., 1999a, b).

According to Burger and Potgieter (1999), before attempting a detailed comparison of the described results with observations, one should bear in mind the following:

- (i) In a dynamical model, the symmetry with respect to solar minimum modulation is broken (le Roux and Potgieter, 1990).
- (ii) Modulation caused by ‘barriers’ cannot be neglected during non solar minimum modulation periods (e.g., Potgieter and le Roux, 1992a,b).
- (iii) The electron measurement may contain a sizable fraction of positrons (e.g., Evenson, 1998).
- (iv) The state of the Heliosphere during the approach to solar maximum, is certainly different from that in the considered model (e.g., review by Jokipii and Wibberenz, 1998).
- (v) The sign of the solar magnetic field does not change abruptly through solar maximum, and as a rule, this does not occur at a tilt angle of 90° .

2.38. CR drifts in a fluctuating magnetic fields

2.38.1. *The matter of problem*

Giacalone et al. (1999) examine the drifts of CR particles in a fluctuating magnetic fields using direct numerical simulation of particle trajectories. They superimpose a randomly fluctuating magnetic field upon a background uniform field. Particle drifts in a magnetic field which has a mean which varies with position are a basic aspect of the motion of CR energetic particles. In general, the motion of CR is composed of the diffusive motion caused by the scattering of the particles due to the fluctuating part of magnetic field and the drift motions resulting from large-scale gradient and curvature of the average magnetic field. The nature of the diffusive transport, and the relation of the diffusion coefficients to the turbulent structure of the magnetic field has been extensively studied over the years. In particular, the diffusion parallel to the average magnetic field seems to be fairly well understood, whereas the perpendicular diffusion with coefficient $\kappa_\perp$ is less so (Fisk et al., M1998). In addition to the perpendicular and parallel diffusion,

determined by the *symmetric* part of the diffusion tensor, a mean magnetic field produces in general an *anti-symmetric* part to the diffusion tensor, usually termed κ_A . In general the diffusion tensor may be write as

$$\kappa_{ij} = \kappa_{\perp} \delta_{ij} - (\kappa_{\perp} - \kappa_{\parallel}) \frac{B_i B_j}{B^2} + \kappa_A \varepsilon_{ijk} \frac{B_k}{|\mathbf{B}|}, \quad (2.38.1)$$

where ε_{ijk} is the unit totally anti-symmetric tensor. The drift velocity $\mathbf{v}_{dr}$ (averaged over the nearly-isotropic distribution) may be shown to be precisely the divergence of the anti-symmetric part of the diffusion tensor (Jokipii et al., 1977). Depending on the situation, one may work in terms of either the drift velocity itself or the anti-symmetric diffusion tensor. In the following Giacalone et al. (1999) will use the term drift velocity or anti-symmetric diffusion tensor inter-changeably.

Giacalone et al. (1999) examine the nature of the gradient and curvature drifts in the presence of turbulent fluctuations. The standard expression for the drift velocity of a charged particle of mass m , charge q , momentum p , and speed v in a magnetic field $\mathbf{B}$, in the limit that the scattering mean free path is much larger than the gyro-radius r_g , is

$$\mathbf{v}_{dr} = (pcv/3q)\nabla \times (\mathbf{B}/B^2), \quad (2.38.2)$$

where c is the speed of light. The corresponding $\kappa_A = vr_g/3$. This is the limit most-frequently used, since Giacalone et al. (1999) expect that the mean free path is generally somewhat larger than the gyro-radius. A finite amount of scattering should reduce this somewhat. A simple analysis based on the venerable billiard ball scattering picture suggests that scattering by fluctuating magnetic field might reduce the drifts by a noticeable amount for CR in the Heliosphere (Burger and Moraal, 1990; Jokipii, 1993). Similarly some analyses of the modulation of galactic CR by the solar wind suggest that the drift motions in the Heliospheric magnetic field are significantly reduced from the classical value given above (e.g., Potgieter et al., 1989). In this special case the expressions for $\kappa_{\perp}$ and κ_A become, in terms of the ratio η of the mean free path λ to the gyro-radius r_g ,

$$\kappa_{\perp} = \kappa_{\parallel} / (1 + \eta^2), \quad \kappa_A = \kappa_{\parallel} \eta / (1 + \eta^2), \quad (2.38.3)$$

where $\kappa_{\parallel}$ is the parallel diffusion coefficient. Again $\mathbf{v}_{dr}$ is the divergence of the anti-symmetric part of the diffusion tensor. Giacalone et al. (1999) utilize direct numerical simulations of particle motions in the turbulent magnetic field to analyze the effects of fluctuations on the drifts.

2.38.2. Analytical result and numerical simulations for CR particle drifts

Before proceeding to the results of the numerical simulations, Giacalone et al. (1999) first present an analytical result which enables to simplify and make more precise the numerical analysis. For simplicity in notation, Giacalone et al. (1999) assume without loss of generality that the average magnetic field, at least locally, is in the z direction, so that the perpendicular directions are x and y . In determining the transport coefficients from numerical simulations it is usual to work in terms of the Fokker-Planck transition moments $\langle \Delta x^2 \rangle / \Delta t$, etc (e.g., Giacalone and Jokipii, 1999). In this case the drift term appears in one or more of the first-order coefficients, for example $\langle \Delta x \rangle / \Delta t$. However, this will only be non-zero when the magnetic field has spatial variation, and this is more complicated to compute numerically. Hence it is usually more convenient to work with the anti-symmetric diffusion coefficient, which is non-zero even if there are no gradients, and whose divergence is the drift velocity. But the obvious Fokker-Planck coefficient $(\Delta x \Delta y) / \Delta t$ is obviously symmetric. The reason is that the divergence of the anti-symmetric tensor is zero if the field does not vary, and in this case the anti-symmetric coefficient does not appear in the diffusion equation. But it does appear in the equation for the streaming flux, or anisotropy. It must proceed differently.

It may be shown that, in general, the equation for the streaming flux in a simple system with no convection, may be written as

$$F_i = -\kappa_{ij} \frac{\partial f}{\partial x_j}, \quad (2.38.4)$$

where the diffusion tensor κ_{ij} can be written as $\kappa_{ij} = \langle v_i \Delta x_j \rangle$. It is easily seen that this also gives the anti-symmetric part of κ_{ij} . Furthermore, this form is much simpler to compute in a simulation.

According to Giacalone et al. (1999) the above result can be demonstrated as follows. At some time t , we may express the value of the distribution function $f(x_i, p_i, t)$ in terms of the values of the $p_i = p'_i$ and $x_i = x'_i$ corresponding to the x_i, p_i at some other time t' (following the actual particle trajectories) by the *exact* relation

$$f(x_i, p_i, t) = f(x'_i, p'_i, t), \quad (2.38.5)$$

which is simply a restatement of Liouville's theorem. Now consider the situation where the time $\Delta t = t' - t$ is many scattering times, but where the corresponding $\Delta x_i = x'_i - x_i$ is much smaller than the scale of spatial variation of f (this is

equivalent to the usual diffusion approximation). Then, since f is nearly isotropic and the momentum magnitude of a particle is constant, the spatial gradient $\partial f / \partial x_i$ is approximately the same for all directions (all particles at a given p) and it may be write

$$f(x'_i, p'_i, t) \approx f(x_i, p_i, t) + \Delta x_i (\partial f(x_i, p_i, t) / \partial x_i). \quad (2.38.6)$$

Therefore, since the p'_i at x'_i are scrambled relative to the p_i , so that

$$\int v f(x'_i, p'_i, t) d\Omega = 0, \quad (2.38.7)$$

the diffusive flux at x_i, t ,

$$F_i(p, x_i, t) = -\int v_i f d\Omega = -\int v_i \Delta x_j d\Omega \frac{\partial f}{\partial x_j} = -\langle v_i \Delta x_j \rangle \frac{\partial f}{\partial x_j} = -\kappa_{ij} \frac{\partial f}{\partial x_j}, \quad (2.38.8)$$

which is the desired result Eq. 2.38.4. Here it was used of the fact that the diffusion coefficient κ_{ij} depends on the magnitude of p as well as x_i and t , so the integral over Ω sums over all the particles at a given p and the $\partial f / \partial x_i$ properly weights the sum over particles. Below Giacalone et al. (1999) compute κ_A from the relationship $\kappa_{ij} = \langle v_i \Delta x_j \rangle$.

2.38.3. Numerical simulations by integration of particle trajectories

Giacalone et al. (1999) integrate the trajectories of particles moving under the influence of a time-independent magnetic field of the form

$$\mathbf{B}(\mathbf{r}) = B_o \hat{z} + \delta \mathbf{B}(\mathbf{r}). \quad (2.38.9)$$

The fluctuating component, $\delta \mathbf{B}(\mathbf{r})$, is determined in a manner similar to that which was described previously in Giacalone and Jokipii, 1994, 1996, 1999). They are characterized by a discrete sum of individual stationary plane waves with random wave vectors, phases, and polarizations. The amplitudes are given by a Kolmogorov-like power spectrum which is described mathematically in terms of three parameters: the total integrated power σ^2 , the correlation length l_c , and the spectral index γ (for all simulations considered in Giacalone et al. (1999) it was set $\gamma = 5/3$). It were considered fluctuations which are approximately spatially homogeneous and isotropic.

Particles are injected at a given energy (which remains constant since the field is time stationary) chosen in such a way that the particle gyro-radius is $0.1 l_c$. For

the interplanetary magnetic field with a typical correlation length of 0.01 AU and mean field strength at 1 AU of 5 nT, this would correspond to proton with an energy of 31.6 MeV. The particles are released isotropically in velocity space at a point in space which it was arbitrarily take to be the origin. They are followed for 1000 gyro-periods, which is larger than the scattering time for all runs that it was report here. The numerical scheme is described in detail in Giacalone and Jokipii (1996, 1999). The diffusion coefficients are compute in the following manner: the cross-field and parallel diffusion coefficients are determined by computing the averages over all particles of $\langle \Delta x^2 \rangle / (2\Delta t)$ and $\langle \Delta z^2 \rangle / (2\Delta t)$, respectively.

The anti-symmetric diffusion coefficients are determined from 2.38.8 as the average over all particles of $\kappa_{xy} = \langle \Delta x v_y \rangle$, and $\kappa_{yx} = \langle \Delta y v_x \rangle$, respectively.

In order to compare the numerical results with Eq. 2.38.3 it must vary the particle mean free path. To accomplish this, Giacalone et al. (1999) vary the power in the random fluctuations, σ^2 . According to the standard quasilinear theory (e.g. Jokipii, 1966) the mean free path varies as the inverse of σ^2 . Giacalone et al. (1999) emphasize, however, that here they compute the mean free path directly from the simulations from the relationship $\lambda_{||} = 3\kappa_{||}/v$ (which was divide by the particle gyro-radius to get η).

Shown in Fig. 2.38.1 are the ratios $\kappa_{\perp}/\kappa_{||}$ and $\kappa_A/\kappa_{||}$ as a function of η . The corresponding values of the turbulence variance range from $0.03 < \sigma^2/B_o^2 < 30$. The curves in Fig. 2.38.1 are from Eq. 2.38.3.

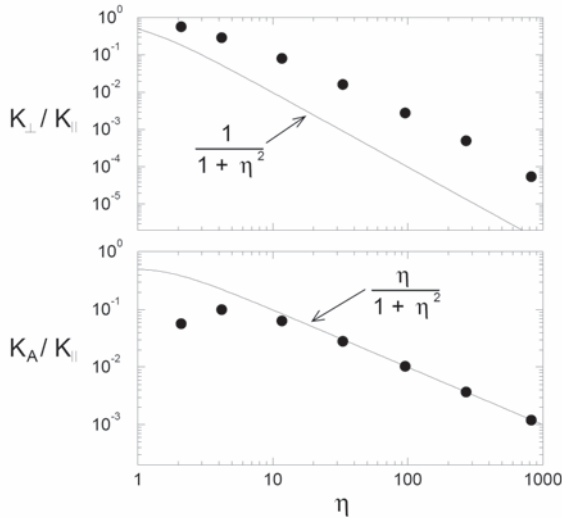


Fig. 2.38.1. Comparison of numerical simulations (solid circles), and analytic theory based on classical scattering (curves, determined by Eq. 2.38.3). From Giacalone et al. (1999).

Fig. 2.38.1 shows that the cross-field diffusion coefficient is considerably larger than the classical scattering result of Eq. 2.38.3. This is due to the fact the $\kappa_{\perp}$ is enhanced by the field-line random walk. This result is consistent with obtained in Giacalone and Jokipii (1999). On the other hand, the simulated values of κ_A agree nicely with the classical scattering result of Eq. 2.38.3 for large values of η . In order to obtain the smaller values of κ_A Giacalone et al. (1999) had to set the power in the random fluctuations considerably larger than the power in the mean field. Consequently, the field becomes almost completely random with no preferential direction. There should be no drifts under such a situation. This is the reason why the simulation results deviate noticeably from the curve. Giacalone et al. (1999) point out however that the statistics were very poor in determining these points and that additional simulations are needed to verify these findings.

2.38.4. Summary of main results

Giacalone et al. (1999) have performed numerical simulations of charged-particles moving in turbulent magnetic fields and compared these with analytic theory. They have concentrated primarily on the drifts associated with these motions and have derived expressions for determining the anti-symmetric diffusion coefficients. Giacalone et al. (1999) have found that the computed anti-symmetric diffusion coefficient agrees well with the classical theory when mean-free path largely exceeds the particle gyro-radius. On the other hand, κ_A is significantly smaller than the predicted value when the mean-free path is less than several particle gyro-radiuses, which occurs when the power in the random fluctuations exceeds that in the mean field. These conclusions regarding κ_A are restricted to a small range of parameters. Future work will extend this to a more comprehensive range of parameters. The small value of κ_A at $\eta < 5$ is potentially of significance for models of CR transport in the Heliosphere, where drifts play an important role.

2.39. Increased perpendicular diffusion and tilt angle dependence of CR electron propagation and modulation in the Heliosphere

2.39.1. The matter of the problem

It is well-known that the wavy Heliospheric current sheet (HCS) is a very important modulation parameter as were predicted by drift models (Jokipii and Thomas, 1981; le Roux and Potgieter, 1990). The computed effects of the HCS “tilt angle” α which represents the extend to which it is warped is however dependent of other modulation parameters, in particular the parallel $\kappa_{\parallel}$ and perpendicular $\kappa_{\perp}$ diffusion coefficients. Concerning $\kappa_{\perp}$ it was argued by Kóta and Jokipii (1995) that it is not isotropic but seems enhanced in the polar directions. This enhancement has been studied intensively in modulation models (e.g., Potgieter, 1997) and it was

illustrated that the enhancement is necessary to make these models compatible with the small latitude effects observed for protons onboard the Ulysses spacecraft.

Ferreira et al. (1999b) note that for numerical solutions of Parker's CR transport equation to be compatible with the small latitudinal gradients observed for protons by Ulysses, enhanced perpendicular diffusion seems needed in the polar regions of the Heliosphere. The role of enhanced perpendicular diffusion was further investigated by examining electron modulation as a function of the tilt angle α of the wavy current sheet, using a comprehensive modulation model including convection, diffusion, gradient, curvature and neutral sheet drifts. Ferreira et al. (1999b) found that by increasing perpendicular diffusion in the polar direction, a general reduction occurs between the modulation differences caused by drifts effects for galactic CR electrons as a function of α for the $A > 0$ (e.g. ~ 1990 to ~ 2000) and $A < 0$ (e.g. ~ 1980 to ~ 1990) solar magnetic polarity cycles. This aspect is also pursued in Potgieter, 1996 (detailed description of the importance of the various parameters in electron modulation) and in Ferreira and Potgieter, 1999 (where the effects are illustrated for spectra and differential intensities as a function of radial distance and polar angle).

2.39.2 The propagation and modulation model

The model for the study of Ferreira et al. (1999b) is based on the numerical solution of the Parker's (1965) transport equation:

$$\frac{\partial f}{\partial t} = -(\mathbf{u} + \langle \mathbf{v}_{dr} \rangle) \cdot \nabla f + \nabla \cdot (\hat{\kappa}_S \cdot \nabla f) + \frac{1}{3}(\nabla \cdot \mathbf{u}) \frac{\partial f}{\partial \ln R}, \quad (2.39.1)$$

where $f(\mathbf{r}, R, t)$ is the CR distribution function, R is rigidity, $\mathbf{r}$ is position, and t is time. Terms on the right-hand side represent convection, gradient and curvature drifts, diffusion and adiabatic energy changes respectively, with $\mathbf{u}$ the solar wind velocity. The symmetric part of the tensor $\hat{\kappa}_S$ consists of a parallel diffusion coefficient $\kappa_{\parallel}$ and a perpendicular diffusion coefficient $\kappa_{\perp}$. The anti-symmetric part $\hat{\kappa}_A$ describes gradient and curvature drifts in the large scale Heliospheric magnetic field (HMF) with the pitch angle averaged guiding center drift velocity for a near isotropic CR distribution is given by

$$\langle \mathbf{v}_{dr} \rangle = \nabla \times \hat{\kappa}_A \mathbf{e}_B, \quad (2.39.2)$$

where $\mathbf{e}_B = \mathbf{B}/B$, with B the magnitude of the background HMF. Eq. 2.39.1 was solved in a spherical coordinate system assuming azimuthal symmetry, and for a steady-state, that is $\partial f / \partial t = 0$.

The HMF was modified according to Jokipii and Kóta (1989). Qualitatively, this modification is supported by measurements made of the HMF in the polar

regions of the Heliosphere by Ulysses (Balogh et al., 1995). The solar wind speed u was assumed to change from 450 km/s in the equatorial plane ($\theta = 90^\circ$) to a maximum of 850 km/s when $\theta < 60^\circ$, with θ the polar angle. The outer boundary of the simulated Heliosphere was assumed at 100 AU which is a reasonable consensus value. The galactic electron spectrum published from the COMPTEL results (Strong et al., 1994) was used as the local interstellar spectrum; see also Potgieter (1996). Solutions for tilt angle α up to 70° were computed for both $A > 0$ and $A < 0$ polarity epochs. For the parallel and perpendicular diffusion coefficients, and the 'drift' coefficient, the following general forms were assumed respectively:

$$\kappa_{//} = \kappa_o \beta f_1(R) f_2(R), \quad \kappa_{\perp r} = a \kappa_{//}, \quad \kappa_{\perp \theta} = b \kappa_{//}, \quad \kappa_A = (\kappa_A)_o \frac{\beta R}{3B_m}. \quad (2.39.3)$$

Here β is the ratio of the speed of the CR particles to the speed of light; $f_1(R)$ gives the rigidity dependence in GV; κ_o is a constant in units of $6.0 \times 10^{20} \text{ cm}^2 \text{ s}^{-1}$ with $\kappa_o = 25$; $a = 0.05$ is a constant which determines the value of $\kappa_{\perp r}$ which contributes to perpendicular diffusion in the radial direction, and b is a constant determining the value of $\kappa_{\perp \theta}$ which contributes to perpendicular diffusion in the polar direction. Diffusion perpendicular to the HMF was therefore enhanced in the polar direction by assuming $\kappa_{\theta\theta} = \kappa_{\perp \theta} = b \kappa_{//}$ with $b = 0.05$ and 0.15 respectively. (see also Kóta and Jokipii, 1995; Potgieter, 1996). The coefficient $(\kappa_A)_o$ specifies the amount of drifts allowed, with $(\kappa_A)_o = 1.0$ a maximum. The effective radial diffusion coefficient is given by

$$\kappa_{rr} = \kappa_{//} \cos^2 \psi + \kappa_{\perp} \sin^2 \psi, \quad (2.39.4)$$

with ψ the angle between the radial direction and the averaged HMF direction. Note that $\psi \rightarrow 90^\circ$ when $r \geq 10$ AU with the polar angle $\theta \rightarrow 90^\circ$, and $\psi \rightarrow 0^\circ$ when $\theta \rightarrow 0^\circ$, which means that $\kappa_{//}$ dominates κ_{rr} in the inner and polar regions and $\kappa_{\perp}$ dominates in the outer equatorial regions of the Heliosphere. Differential intensities, $J \propto R^2 f$, are calculated as particles $\text{m}^{-2} \text{sr}^{-1} \text{s}^{-1} \text{MeV}^{-1}$.

Solutions were computed in Ferreira et al. (1999b) with a simple rigidity dependence for $\kappa_{//}$ and $\kappa_{\perp}$ (meaning both $\kappa_{\perp r}$ and $\kappa_{\perp \theta}$) given by

$$f_1(R) = \begin{cases} \beta R / R_o & \text{at } R > 0.4 \text{ GV}, \\ \beta (0.4 \text{ GV}) / R_o & \text{at } R \leq 0.4 \text{ GV}, \end{cases} \quad (2.39.5)$$

where $R_o = 1$ GV. This simple approach has proven to be most useful (Potgieter, 1996). For the spatial dependence,

$$f_2(\theta, r) = 1 + r/\eta_1 \quad (2.39.6)$$

was assumed, with $\eta_1 = 1$ AU. Note that $\kappa_{//}$ and $\kappa_{\perp}$ have the same rigidity dependence that becomes flat and constant below 0.4 GV. This feature causes the electron modulation at a given position in the Heliosphere to become almost constant at energies ≤ 50 MeV. At energies ≤ 10 MeV Jovian electrons may contribute to the computed spectra (Haasbroek et al., 1996) but is neglected in Ferreira et al. (1999b) study. Different assumptions for $f_1(R)$ may change the slope of the spectra at low energies as was illustrated in detail by Potgieter (1996) but this is not important for the results and conclusions of the Ferreira et al. (1999b) study.

Modeling the modulation of electrons in the Heliosphere, a 2D model with an emulated wavy current sheet was used as developed by Hattingh and Burger (1995a). Obviously, the 2D model differs from a 3D model in the way the HCS is handled. However, using a 2D model is well justified and for a comparison between this 2D model and the 3D model developed by Hattingh (1998) - see also Hattingh and Burger (1995b); it will be considered below in detail in Section 2.41 on the basis of paper Ferreira et al. (1999a). For an additional description of the ‘tilt angle’ dependence of the model see Section 2.37 on the basis of paper Burger and Potgieter (1999).

2.39.3. Main results and discussion

The electron differential intensities as a function of tilt angle α are shown in Fig. 2.39.1 for 1.94 GeV electrons at $\theta = 90^\circ$ (equatorial plane) for both polarity cycles. Solutions are shown at three radial distances and for two different values of $\kappa_{\perp\theta}$. Panels (a) and (b) show solutions at 1 AU; panels (c) and (d) for 5 AU and panels (e) and (f) for 80 AU for $b = 0.05$ and $b = 0.15$ respectively.

From Fig. 2.39.1 follows that at 1 AU the intensity for the $A < 0$ polarity cycle is higher than for the $A > 0$ cycle. As $\kappa_{\perp\theta}$ was enhanced by increasing b from 0.05 to 0.15, a reduction occurs in the differences between the two epochs. The intensities for both epochs are lower for the increased value of $\kappa_{\perp\theta}$ and do not have such a strong dependence as for a smaller $\kappa_{\perp\theta}$. For $A > 0$ this diminished dependence on α is especially evident for $\alpha < 40^\circ$. At 5 AU the intensities for the $A > 0$ and $A < 0$ cycles cross at $\alpha \sim 15^\circ$ with the $A < 0$ intensities lower than those for the $A > 0$ for $\alpha < 15^\circ$. As for 1 AU, the increase in $\kappa_{\perp\theta}$ led to a decrease of the α dependence, especially with $\alpha < 40^\circ$. The spectra shown no longer cross, but it still occurs at a slightly larger radial distance. For 80 AU, the intensities for the $A > 0$ are consistently higher than for the $A < 0$ epoch and the increase in $\kappa_{\perp\theta}$ had little or no effect on the differential intensities as a function of α . This indicates that the increase in $\kappa_{\perp\theta}$ is more important in the inner and middle Heliosphere (compare also Ferreira and Potgieter, 1999).

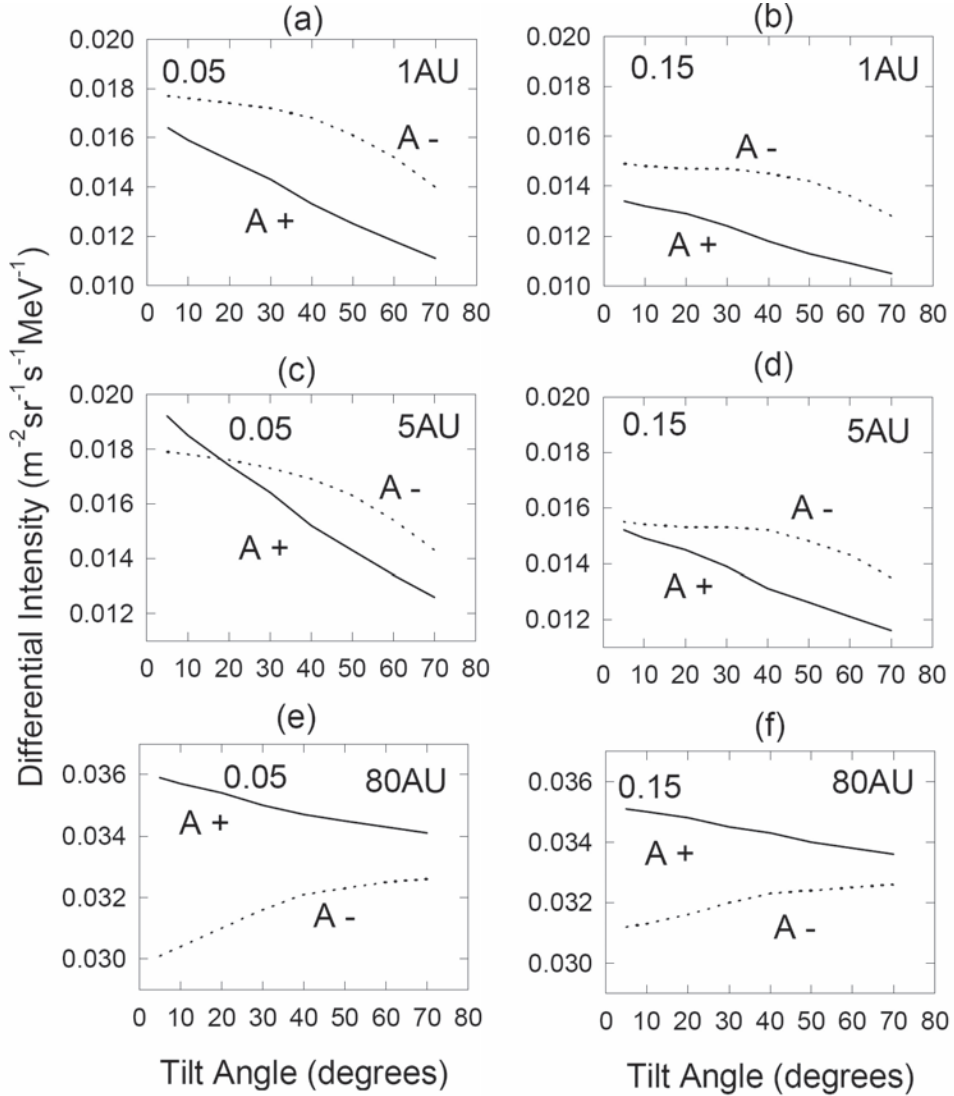


Fig. 2.39.1. Electron differential intensity at 1.94 GeV as a function of tilt angle α , shown in the equatorial plane ($\theta = 90^\circ$) for $A > 0$ and $A < 0$ polarity epochs. Solutions are shown at 1 AU, 5 AU and 80 AU for different values of $\kappa_{\perp\theta}$: panels (a), (c) and (e) with $b = 0.05$ and panels (b), (d) and (f) with $b = 0.15$ in Eq. 2.39.3. From Ferreira et al. (1999b).

The process was repeated for 0.30 GeV electrons and the results are shown in Fig. 2.39.2.

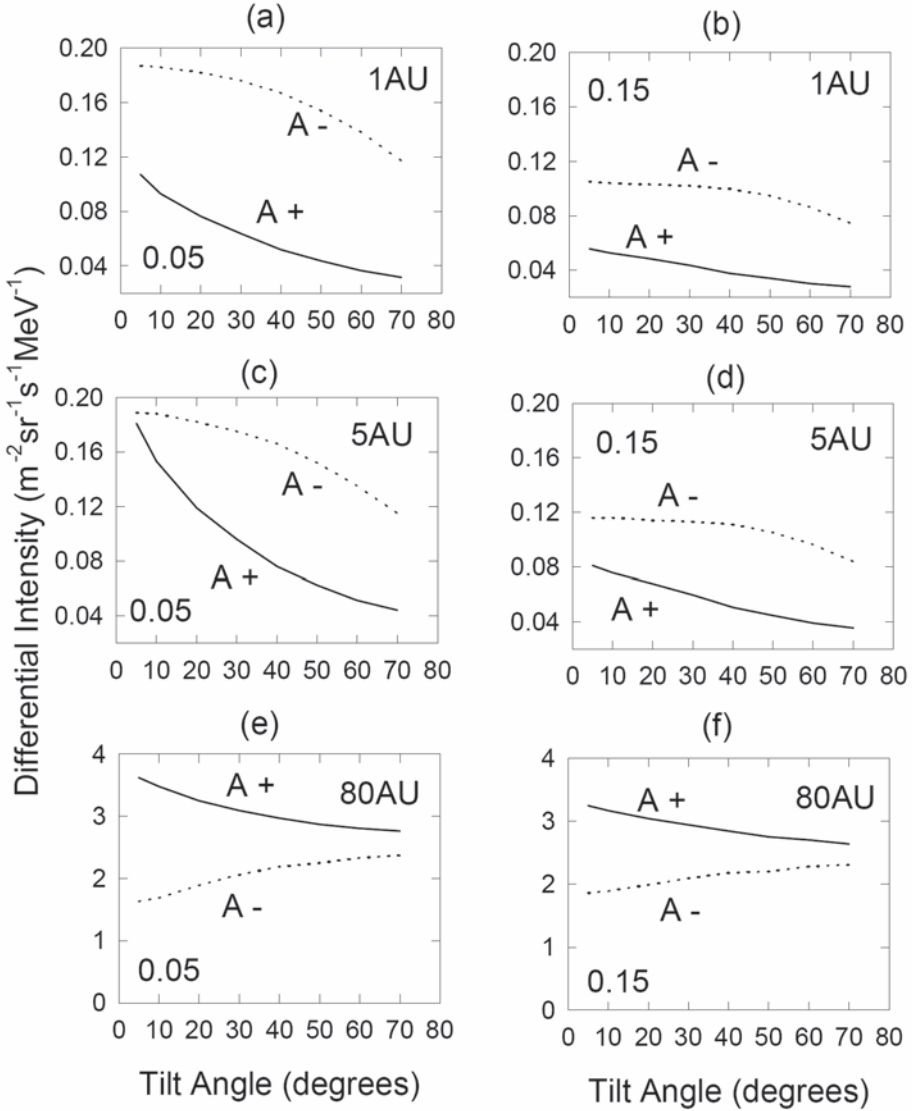


Fig. 2.39.2. As in Fig. 2.39.1, but for 0.30 GeV electrons. From Ferreira et al. (1999b).

Qualitatively, Fig. 2.39.2 shows a similar response to changes in $\kappa_{\perp\theta}$ and no significant deviations occur with the changing α compared to Fig. 2.39.1. The cross-over shown in Fig. 2.39.1 now occurs at $r > 5$ AU. Quantitatively, the α dependence of the $A > 0$ intensities is less linear and the differences between the two epoch solutions are evidently larger than for the higher electron energies shown in Fig. 2.39.1.

Fig. 2.39.3 shows solutions as in Fig. 2.39.2 except that they were obtained at $\theta = 5^\circ$ to illustrate what happens to the α dependence of the solutions in the polar regions of the Heliosphere.

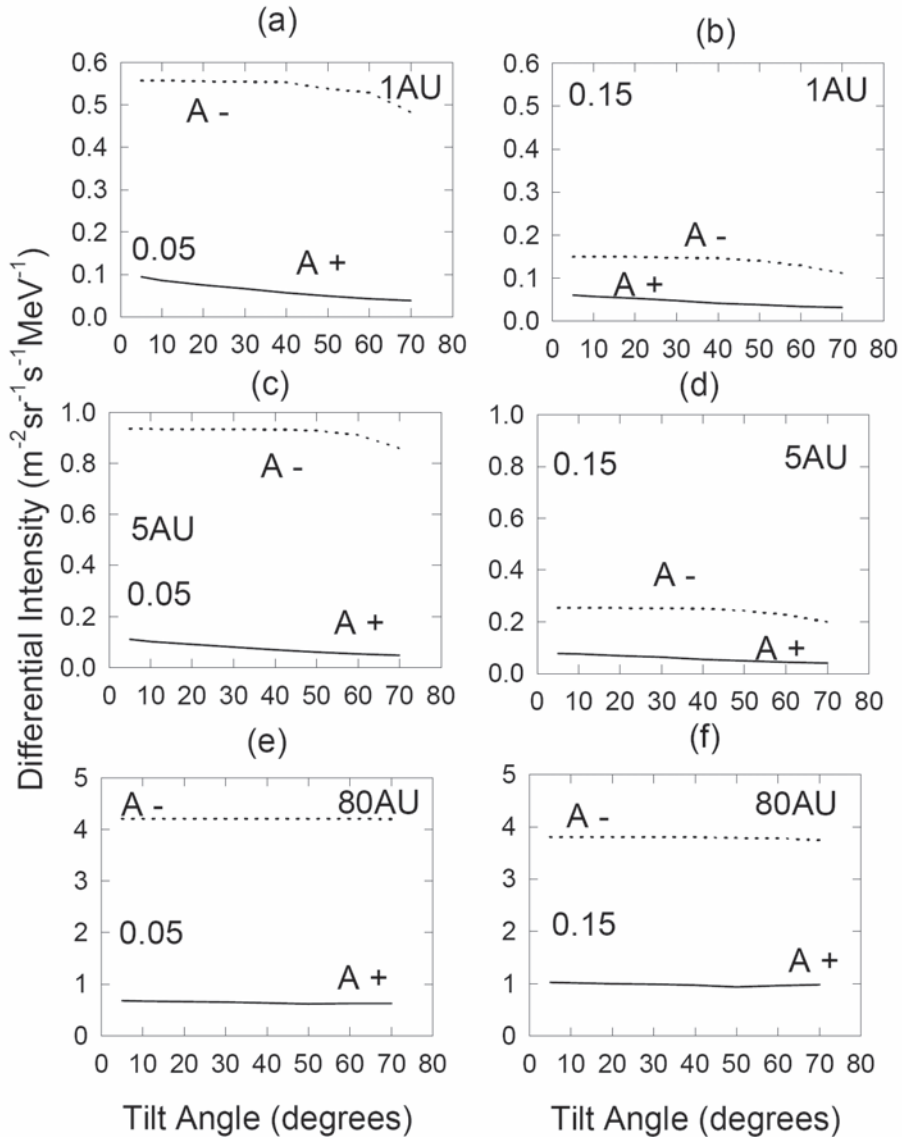


Fig. 2.39.3. As in Fig. 2.39.2, but at a polar angle of $\theta = 5^\circ$. From Ferreira et al. (1999b).

The α dependence is clearly negligible compared to the equatorial regions as follows from comparing panels (a) to (f) in Fig. 2.39.3 with those in Fig. 2.39.2. The only significant α dependence is when $\alpha > 60^\circ$. This is understandable, because

when the tilt angles become very large the modulation of intensities must respond to the presence of the wavy HCS in these high latitude regions of the simulated Heliosphere. The increase in $\kappa_{\perp\theta}$ from 5% to 15% of $\kappa_{//}$ gives the largest effect on the level of modulation in the polar regions as a comparison of panels (b), (d) and (f) in Fig. 2.39.3 with those in Fig. 2.39.2 illustrates. This is due to the fact that the enhancement of $\kappa_{\perp\theta}$ in the polar directions becomes more effective with decreasing polar angles. The significant reduction of the latitude dependence of the electron intensities due to the enhancement of $\kappa_{\perp\theta}$ also follows clearly from comparing panels (b),(d) and (f) in Fig. 2.39.3 with those in Fig. 2.39.2.

2.39.4. Summary and conclusions

According to Ferreira et al. (1999b) studying the effects on electron modulation of enhancing $\kappa_{\perp\theta}$ in the polar directions, from 5% to 15% of $\kappa_{//}$, it was found that this increase reduced the differences between the modulated intensities as a function of tilt angle α for the two magnetic polarity cycles. This is especially strong for the inner Heliosphere in the equatorial regions and most of the Heliosphere in the polar regions. The increase in $\kappa_{\perp\theta}$ also led to a decrease in the α dependence of the differential intensities for $\alpha < 40^\circ$ for the inner Heliosphere in the equatorial regions as shown in Fig. 2.39.1 and Fig. 2.39.2. For the polar regions, shown in Fig. 2.39.3, the increase in $\kappa_{\perp\theta}$ had little or no change in the α dependence of the intensities for $\alpha < 60^\circ$, but it caused a significant reduction in the global latitude dependence of electron modulation.

2.40. Rigidity dependence of the perpendicular diffusion coefficient and the Heliospheric modulation of CR electrons

2.40.1. The matter of problem

Potgieter et al. (1999) note that the diffusion perpendicular to the Heliospheric magnetic field (HMF) plays an important role in the modeling of the Heliospheric propagation and modulation of galactic CR. This followed directly from the simulation of latitude dependent modulation, first studied about 30 years ago with a two-dimensional model by Fisk (1976). Even with the introduction of global and neutral sheet drifts in models of increasing complexity (Kóta and Jokipii, 1983; Potgieter and Moraal, 1985; le Roux and Potgieter, 1991; Burger and Hattingh, 1998) the importance of the perpendicular diffusion coefficient has remained and is arguably the most important element of the diffusion tensor. But because no comprehensive theory exists for it, the best that can be done at this stage is to make reasonable assumptions about its value, spatial and rigidity dependence. Fortunately, the modulation of CR electrons in the Heliosphere provides a useful tool in understanding and in determining the diffusion coefficients. Computed

electron modulation responds directly to what is assumed for the energy dependence of the diffusion coefficients below 500 MeV, in contrast to protons which experience large adiabatic energy changes below this energy and which consequently obscure the effects of changing the energy dependence of any of the diffusion coefficients. Another aspect is that drifts become progressively less important with decreasing electron energy, to have almost no effect on electron modulation below 100–200 MeV. For the work of Potgieter et al. (1999), electron modulation was used to illustrate how important perpendicular diffusion is, in particular its rigidity dependence, to the Heliospheric modulation of CR electrons.

According to Potgieter et al. (1999), the modulation of CR electrons in the Heliosphere provides a useful tool in understanding and estimating the diffusion tensor applicable to Heliospheric modulation. Using a comprehensive modulation model including all major mechanisms to study electron modulation, especially at energies below 500 MeV, Potgieter et al. (1999) found that perpendicular diffusion is very important to electron modulation at these energies. Electrons respond directly to the energy dependence of the diffusion coefficients below 500 MeV, in contrast to protons which experience large adiabatic energy losses below this energy. As a result of this and because drifts become unimportant for electrons at these low energies, important conclusions can be made about the absolute values, spatial and especially the rigidity dependence of the diffusion coefficients.

2.40.2. The propagation and modulation model, main results, and discussion

The propagation and modulation model that was used in Potgieter et al. (1999) is the same which was used in Ferreira et al. (1999b) and was described above, in Section 2.39.2 (see Eq. 2.39.1 up to Eq. 2.39.6). The results are shown in Fig. 2.40.1 and illustrate in general that when the rigidity dependence of $\kappa_{\perp}$ (that is both $\kappa_{\perp r}$ and $\kappa_{\perp \theta}$ in Eq. 2.39.3) is taken independently from that for $\kappa_{\parallel}$ at energies below ~ 500 MeV, it clearly dominates modulation at these lower energies. Potgieter et al. (1999) assumed for these results that function $f_2(\theta, r)$ in Eq. 2.39.3 is described by Eq. 2.39.6, with $r_1 = 1$ AU, $\kappa_0 = 25$, $a = 0.05$ and $b = 0.15$. The HMF magnetic cycle was chosen to be $A > 0$ (e.g. the solar polarity cycle in 1999). The different rigidity dependencies for $\kappa_{\parallel}$ and $\kappa_{\perp}$ are shown in the inserted graph where $\kappa_{\perp}$ was multiplied by 10 for illustrative purposes. In this case the rigidity dependence for $\kappa_{\parallel}$ was according to the damping model - composite slab - 2D geometry of Bieber et al. (1994); see also Potgieter (1996). It is evident that the computed spectra in the inner Heliosphere are still compatible to data at higher energies but below ~ 100 MeV the modulation becomes unreasonably large which is apparently not supported by measurements. However, these results illustrate that although $\kappa_{\perp r}$ and $\kappa_{\perp \theta}$ is only 5% and 15% of the value of $\kappa_{\parallel}$ respectively, perpendicular diffusion, especially in the polar direction, dominates electron

modulation below ~ 100 MeV and that it is as such a very important parameter that should be studied in detail. If the increase in the low energy part of observed electron spectra with decreasing energy was taken as a characteristic of modulated electron spectra then $\kappa_{\perp} \propto \beta R$, as shown in Fig. 2.40.1, is not a workable option.

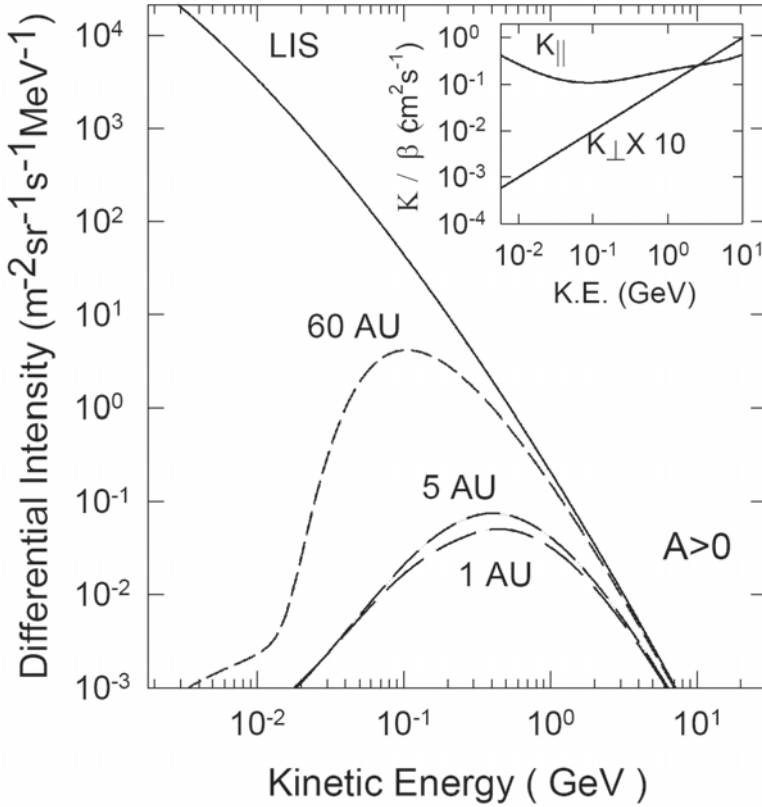


Fig. 2.40.1. Computed electron modulation at 1, 5, and 60 AU in the equatorial plane with the LIS at 100 AU for an $A > 0$ epoch. Insert shows the values of $\kappa_{||}$ and $\kappa_{\perp r}$ in units of $6 \times 10^{20} \text{ cm}^2 \text{ s}^{-1}$; note that $\kappa_{\perp \theta} = 3 \times \kappa_{\perp r}$. From Potgieter et al. (1999).

As it was argued by Kóta and Jokipii (1995) that $\kappa_{\perp \theta}$ plays a crucial role in CR modulation which is the assumption for work of Potgieter et al. (1999), because perpendicular diffusion enhanced in the polar direction seems a necessity for getting computed latitude dependencies compatible to the Ulysses observations. Because $\kappa_{\perp \theta}$ may be considerably larger than $\kappa_{\perp r}$ in the Heliospheric polar regions, Fig. 2.40.2 illustrates whether a further enhancement of $\kappa_{\perp \theta}$ may change the features shown in Fig. 2.40.1 by taking $b = 0.40$ instead of 0.15 in Eq. 2.39.3. This increase

caused additional low energy electrons to reach the equatorial plane, compared to Fig. 2.40.1. A further increase in b had little additional effect at these low energies, so that there is clearly a limit to what his approach can do.

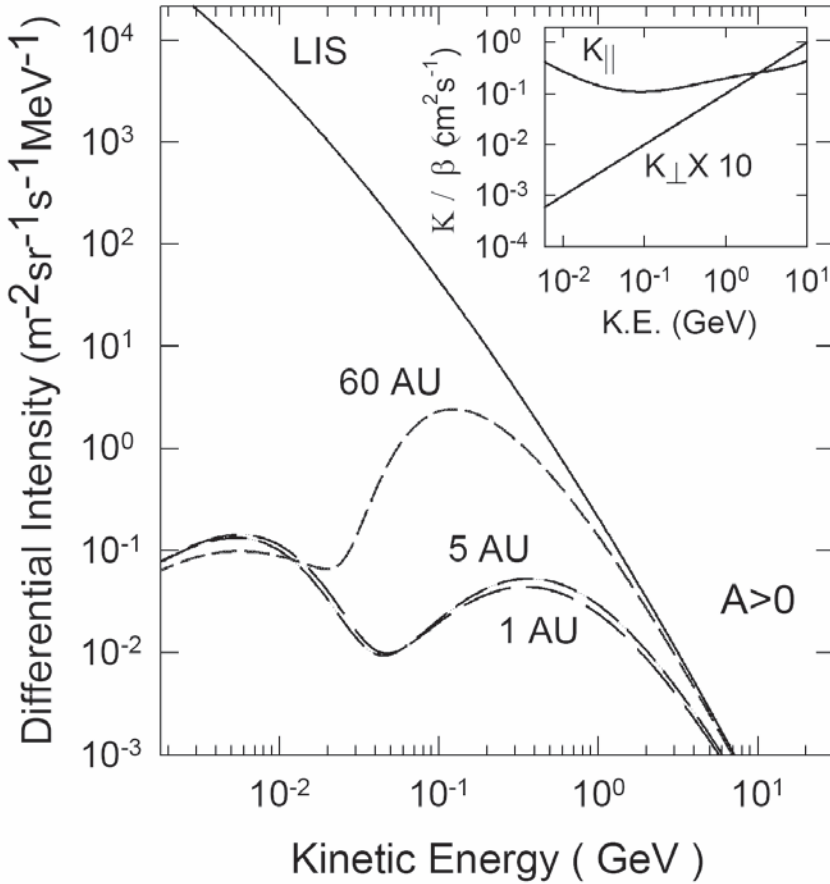


Fig. 2.40.2. Similar to Fig. 2.40.1 but with $\kappa_{\perp\theta} = 8 \times \kappa_{\perp r}$. Note how the situation changed at the low energies, especially in the inner Heliosphere. From Potgieter et al. (1999).

To extend the study on the modulation aspects shown in Fig. 2.40.1 and Fig. 2.40.2, Ferreira (1999) constructed an analytical expression for $\kappa_{||}$, applicable to electrons, using the theoretical work of Hattingh (1998) and Burger and Hattingh (1998) where they on their part used the formalism of Bieber et al. (1994), especially the random sweeping model for dynamical turbulence with pure slab geometry (see also Zank et al., 1998). This expression is depicted in Fig. 2.40.3 as a function of kinetic energy for 1 AU, 10 AU, 50 AU and 100 AU in the equatorial plane. No explicit latitude dependence was assumed.

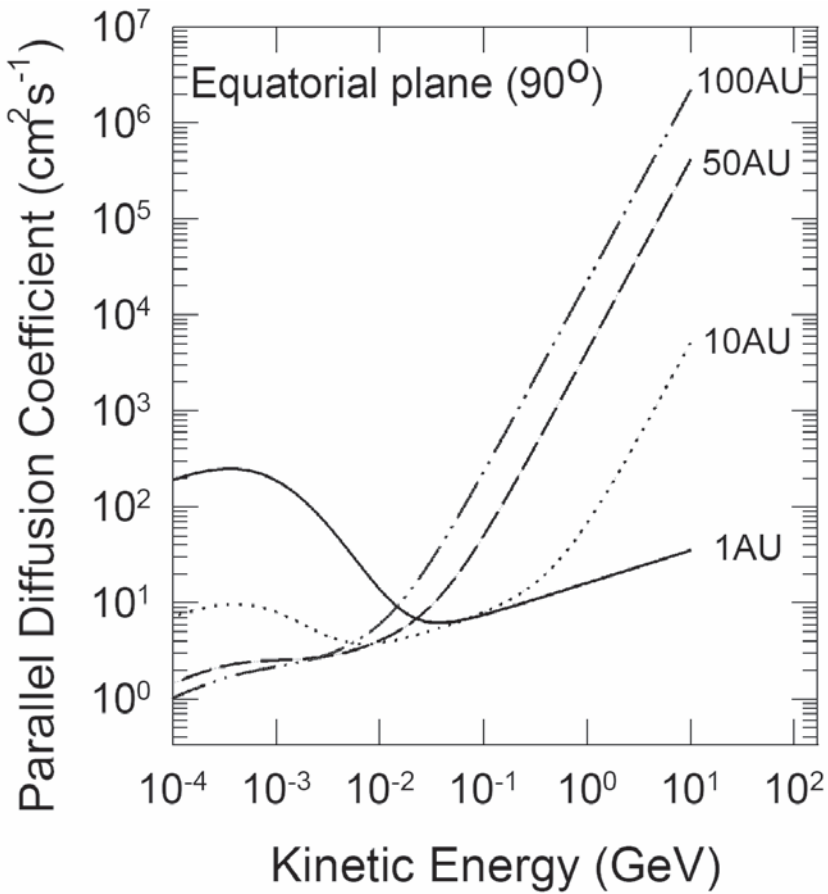


Fig. 2.40.3. The parallel diffusion coefficient $\kappa_{//}$ for CR electrons in units of $6 \times 10^{20} \text{ cm}^2 \text{ s}^{-1}$ at 1 AU, 10AU, 50AU and 100AU in the equatorial plane as a function of kinetic energy. Note the changes from 1 AU to 100 AU. From Potgieter et al. (1999).

It is evident from Fig. 2.40.3 that the radial dependence of $\kappa_{//}$ is much more sophisticated with main feature the changing slopes of the function and that $\kappa_{//}$ is much larger in the outer Heliosphere at high energies than at low energies, with the opposite at 1 AU.

The corresponding computed spectra are shown in Fig. 2.40.4. In this case $a = 0.05$ and $b = 0.15$ in Eq. 2.39.3. The electron data from the Ulysses/KET experiment for 1997 are shown to provide a reference for inner Heliospheric electron intensities during minimum modulation. The compatibility between the data and the model is reasonable at energies $> 400 \text{ MeV}$, but not at energies < 100

MeV. Although Potgieter et al. (1999) used a very sophisticated function for $\kappa_{\parallel}$ and $\kappa_{\perp}$ it does not give electron modulation compatible to Ulysses data at low energies. In this case the only way to assure compatibility is to make $\kappa_{\perp}$ almost independent of kinetic energy at low energies because it dominates electron modulation at these low energies – see also Ferreira (1999), Ferreira and Potgieter (1999). It should be kept in mind, however, that measured low energy electrons might contain a Jovian contribution.

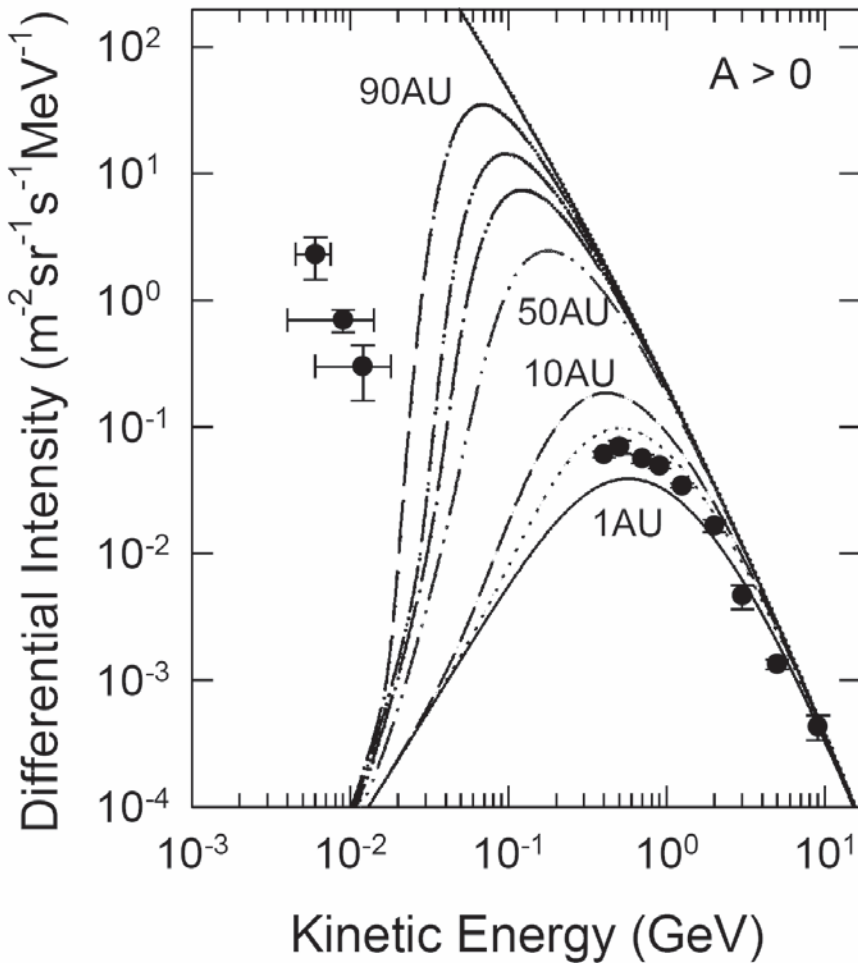


Fig. 2.40.4. Computed electron spectra in the equatorial plane at 1, 5, 10, 50, 70, 80, and 90 AU, corresponding to the function shown for $\kappa_{\parallel}$ in Fig. 2.40.3 and based on the assumption that $\kappa_{\perp} \propto \kappa_{\parallel}$. Data are at ~ 5 AU for 1997 from the Ulysses/KET experiment. From Potgieter et al. (1999).

Potgieter et al. (1999) came to conclusion that the analysis of electron modulation illustrates how important $\kappa_{\perp r}$, but especially $\kappa_{\perp \theta}$ and its rigidity dependence is to electron modulation below 100 - 300 MeV. It was illustrated that although $\kappa_{\perp r}$ and $\kappa_{\perp \theta}$ was only 5% and 15% of the value of $\kappa_{\parallel}$ respectively, perpendicular diffusion dominates electron modulation below ~ 100 MeV. It was argued that if the increasing intensity with decreasing energy below ~ 100 MeV in the observed electron spectra in the inner Heliosphere were taken as a characteristic of modulated CR electron spectra, as in Fig. 2.40.4, then, to assure reasonable compatibility with data below ~ 100 MeV, $\kappa_{\perp r}$ and certainly $\kappa_{\perp \theta}$ must be nearly independent of kinetic energy below ~ 100 MeV.

2.41. Comparison of 2D and 3D drift models for galactic CR propagation and modulation in the Heliosphere

2.41.1. The matter of problem

Ferreira et al. (1999a) note that the propagation and modulation of galactic CR in the Heliosphere is described successfully by Parker's (1965) transport equation (see Eq. 2.39.1 in Section 2.39). This equation has been solved with increasing complexity over the years. However, to solve it numerically for three spatial dimensions (3D), a rigidity and a time-dependence is rather complex and has not yet been done successfully. By assuming an axisymmetric CR distribution one can neglect the equation's azimuthal dependence which leads to 2D models which have been used widely for modulation studies (le Roux and Potgieter, 1990). The main difficulty in 2D models is how to emulate the effect of the wavy Heliospheric current sheet (HCS) because it cannot be done directly. This was done successfully for the first time by Potgieter and Moraal (1985). The technique was improved by Burger and Potgieter (1989). Hattingh (1993) developed a refined 2D model, which was called the WCS model, and after several years also a 3D model which includes an actual wavy HCS (Burger and Hattingh, 1995; Hattingh 1998). This 3D model was compared carefully to the first 3D model developed by Kóta and Jokipii (1983) with excellent results. For a review and detail of the different models, see le Roux and Potgieter (1990), Hattingh and Burger (1995a,b), Burger and Hattingh (1995) and for an application of the 3D model, see Burger and Hattingh (1998). An obvious next step was to compare the 2D and 3D models to establish how reliable the 2D models are, and to establish to what extent they can be used for modulation studies. This was done by Hattingh (1998) for CR protons and it was found that the agreement between the 3D and the 2D WCS model varied between good to excellent. At Earth, the largest variation of $\sim 16\%$ in the ratio of the two sets of solutions was found at low rigidities for a tilt angle $\alpha = 20^\circ$ during an $A < 0$ (e.g. ~ 1980 to ~ 1990) solar polarity cycle. At 60 AU, the largest variation was $\sim 26\%$ at low energies during the $A < 0$ cycle. This comparative study was continued by Ferreira (1999) who concentrated on the modulation of CR electrons in the

Heliosphere because electrons may have a different diffusion tensor than protons, experience less adiabatic energy losses than protons at energies of interest to modulation, and for which drifts become less significant with decreasing energy.

The paper of Ferreira et al. (1999a) reports on the comparative study of the 2D and 3D models using electron modulation, with emphasis on the tilt angle dependence because the computation of the wavy HCS and its effects on modulation are the important difference between the 2D and 3D numerical models. In Ferreira et al. (1999a) the modulation of galactic CR electrons in the Heliosphere was used to compare solutions of a 2D and 3D drift model, both developed by the Potchefstroom Modulation Group. These steady-state models are based on the numerical solution of Parker's transport equation and include the main modulation mechanisms: convection, diffusion, gradient, curvature and neutral sheet drifts. Examining computed electron spectra, with identical modulation parameters in both models, as a function of the Heliospheric neutral sheet tilt angle yielded no qualitative differences and insignificant quantitative differences between the solutions of the 2D and 3D models. Taking into account the large amount of resources needed for the 3D model, the use of a 2D model for modulation studies is well justified.

2.41.2. The propagation and modulation models

A short description of the 2D WCS model is given by Ferreira et al., 1999b (see Section 2.39) with detail given by Burger and Hattingh (1995). The 3D model is based on the numerical solution of Parker's (1965) equation:

$$\frac{\partial f}{\partial t} = -(\mathbf{u} + \langle \mathbf{v}_{dr} \rangle) \cdot \nabla f + \nabla \cdot (\kappa_S \cdot \nabla f) + \frac{1}{3}(\nabla \cdot \mathbf{u}) \frac{\partial f}{\partial \ln R}, \quad (2.41.1)$$

where $\mathbf{u}$ is the solar wind velocity and $f(\mathbf{r}, p, t)$ is the CR distribution function where p is rigidity, $\mathbf{r}$ is position, and t is time. The symmetric part of the diffusion tensor κ_S consists of a parallel diffusion coefficient $\kappa_{\parallel}$ and a perpendicular diffusion coefficient $\kappa_{\perp}$. The antisymmetric part κ_A describes gradient and curvature drifts in the large scale Heliospheric magnetic field (HMF). The pitch angle averaged guiding centre drift velocity for a near isotropic CR distribution is given by

$$\langle \mathbf{v}_{dr} \rangle = \nabla \times [h(r) \kappa_A \mathbf{e}_B] \quad (2.41.2)$$

with $\mathbf{e}_B = \mathbf{B}/B$, where B is the magnitude of the background HMF and $h(r)$ is a transition function which varies from 1 to -1 across the HCS and is zero in the HCS. This transition function modifies κ_A across the wavy HCS which is positioned at

$$\theta' = \frac{\pi}{2} + \alpha \sin\left(\phi + r \frac{\Omega}{u}\right) \quad (2.41.3)$$

with Ω the angular velocity of the Sun and θ , ϕ , and r the heliocentric spatial coordinates. The solar wind speed u was assumed to change from 450 km.s^{-1} in the equatorial plane ($\theta = 90^\circ$) to a maximum of 850 km.s^{-1} when $\theta \leq 60^\circ$. The HMF was modified according to Jokipii and Kóta (1989) and the outer boundary of the simulated Heliosphere was assumed at 100 AU. The galactic electron spectrum based on COMPTEL results (Strong et al., 1994) was assumed as the local interstellar spectrum. To produce spectra compatible to both Ulysses and Voyager 1 measurements (Potgieter et al., 1999), the following parallel diffusion coefficient was used:

$$\kappa_{//} = \frac{v}{3} \beta \eta_1 \left[\left(\frac{p}{p_o} \right)^2 D_1 D_2 + \frac{(p/p_o)^2}{10} D_2 + \left(1 + \frac{r}{10\eta_1} \right) \left(\left(\frac{\eta_1}{r} \right)^2 - \frac{1 - r/\eta_1}{(r/\eta_1)^3} \right) D_2 D_3 \right] \quad (2.41.4)$$

where $p_o = 1 \text{ GV}$ and $\eta_1 = 1 \text{ AU}$, and

$$D_1 = \frac{(r/\eta_1)^{0.8} - 1}{20}, \quad D_2 = 4 - \frac{r/\eta_1}{7 - \frac{r}{40\eta_1}} - \left(\frac{r}{\eta_1} \right)^{0.6}, \quad D_3 = \left[5 - \frac{5}{\exp\left(\frac{0.9}{p/p_o}\right)} \right]. \quad (2.41.5)$$

Assuming Eq. 2.41.4, and apart from the inherent azimuthal dependence of the HCS, no additional azimuthal dependence was incorporated in the 3D model. For the perpendicular diffusion and the ‘drift’ coefficient the following general forms were assumed respectively:

$$\kappa_{\perp r} = a \kappa_{//}, \quad \kappa_{\perp \theta} = b \kappa_{//}, \quad \kappa_A = (\kappa_A)_o \frac{\beta p}{3B_m}. \quad (2.41.6)$$

Here β is the ratio of the speed of the CR particles to the speed of light, B_m is the magnitude of the modified HMF, $a = 0.05$ is a constant determining the value of $\kappa_{\perp r}$ which contributes to perpendicular diffusion in the radial direction, $b = 0.15$ is a constant determining the value of $\kappa_{\perp \theta}$ which contributes to perpendicular diffusion in the polar direction. Diffusion perpendicular to the HMF was therefore enhanced in the polar direction by assuming $b > a$ (Kóta and Jokipii, 1995; Potgieter, 1996). The coefficient $(\kappa_A)_o$ in Eq. 2.41.6 specifies the amount of drifts

allowed. According to Ferreira et al. (1999a) it was necessary to take $(\kappa_A)_0 = 0.5$ which corresponds to medium drift effects.

The effective radial diffusion coefficient is given by

$$\kappa_{rr} = \kappa_{//} \cos^2 \psi + \kappa_{\perp} \sin^2 \psi, \quad (2.41.7)$$

with ψ the angle between the radial direction and the averaged HMF direction. Note that $\psi \rightarrow 90^\circ$ when $r \geq 10$ AU with the polar angle $\theta \rightarrow 90^\circ$, and $\psi \rightarrow 0^\circ$ when $\theta \rightarrow 0^\circ$, which means that $\kappa_{//}$ dominates κ_{rr} in the inner and polar regions and $\kappa_{\perp r}$ dominates in the outer equatorial regions of the Heliosphere. The differential intensity, $J \propto p^2 f$, is calculated in units of particles $\text{m}^{-2}\text{sr}^{-1}\text{s}^{-1}\text{MeV}^{-1}$.

2.41.3. Main results on comparison and discussion

To compare the results of the 3D model with the 2D WCS model, the average of the 3D solutions had to be calculated in Ferreira et al. (1999a) over one solar rotation, i.e. for azimuthal angles $\phi = 0 \rightarrow 2\pi$. As a first comparison the modulated electron spectra computed with both models are shown in Fig. 2.41.1 for the polar regions, $\theta = 30^\circ$ (panel a), and for the equatorial regions, $\theta = 90^\circ$ (panel b), at radial distances of 1 AU and 60 AU with tilt angle $\alpha = 20^\circ$.

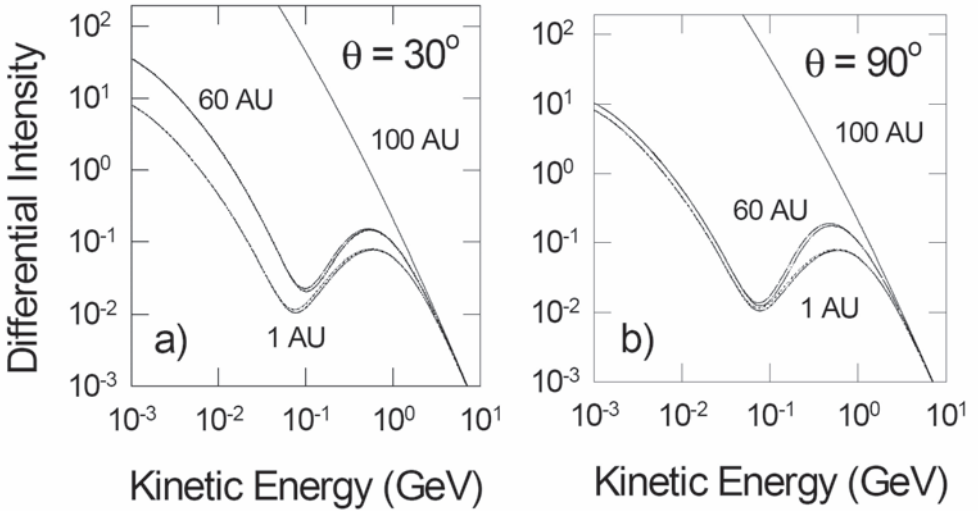


Fig. 2.41.1. Panel a: Computed electron spectra produced by the 2D and 3D drift model. Differential intensities are shown for 1 AU and 60 AU at a polar angle of $\theta = 30^\circ$ and a tilt angle $\alpha = 20^\circ$ in units of $\text{m}^{-2}\text{sr}^{-1}\text{s}^{-1}\text{MeV}^{-1}$ for the $A > 0$ polarity cycle. Panel b: Similar to panel a, but for a polar angle of $\theta = 90^\circ$. Note that the spectra for the two models essentially coincide. From Ferreira et al. (1999a).

Solutions in Fig. 2.41.1 are shown for the $A > 0$ polarity epoch (e.g. ~ 1990 to ~ 2000) only, because during this cycle when electrons are drifting in along the HCS the largest difference between the two models occurs (see discussion below). These spectra can be considered as typical for minimum modulation periods. Evidently, the electron spectra produced by the two models as shown in Fig. 2.41.1 essentially coincide despite the use of a rather complex rigidity dependence for $\kappa_{\parallel}$ and $\kappa_{\perp}$.

The two models obviously differ in the way the HCS is treated. Therefore, an appropriate way to compare the two models is by examining the α dependence of the differential intensities. In Fig. 2.41.2 the ratio of the computed 2D and 3D differential intensities is shown as a function of tilt angle α for both the $A > 0$ and $A < 0$ magnetic polarity cycles.

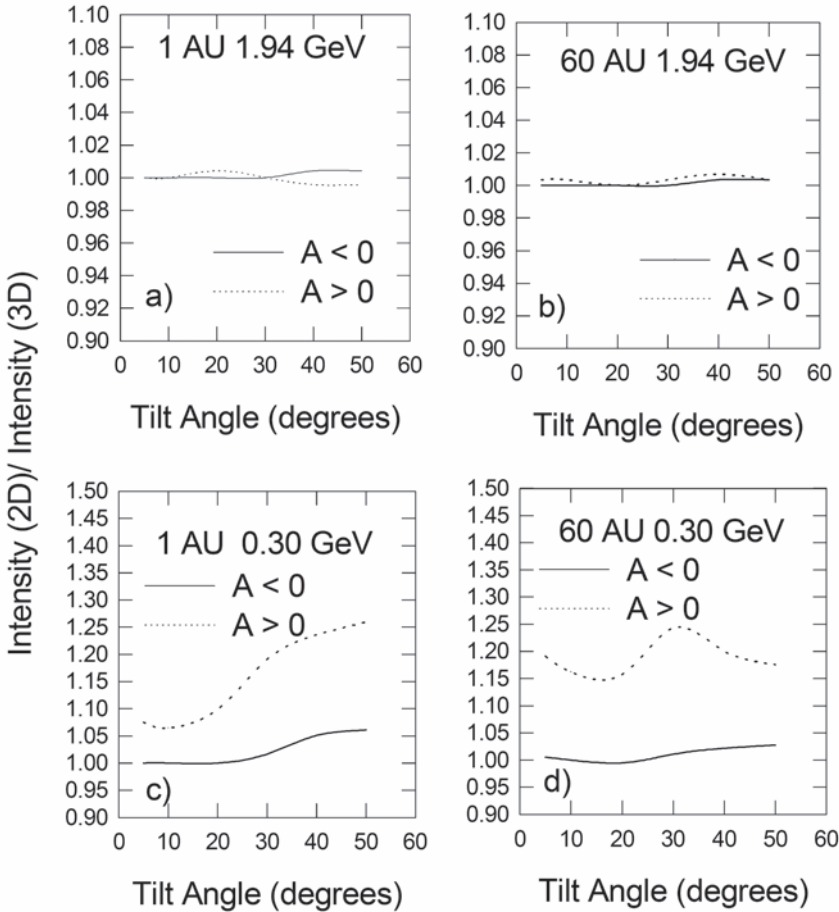


Fig. 2.41.2. The ratio of electron differential intensities computed with the 2D and 3D drift models as a function of tilt angle α . Panel **a** shows the ratio for 1.94 GeV electrons for both the $A > 0$ and $A < 0$ polarity cycles at 1 AU and panel **b** shows the situation at 60 AU. Panels **c** and **d** display the situation for 0.30 GeV electrons at 1 AU and 60 AU respectively. From Ferreira et al. (1999a).

The modulation parameters in Fig. 2.41.2 are the same as for Fig. 2.41.1. Values are shown for 1.94 GeV electrons at 1 AU in panel **a** and at 60 AU in panel **b**. Panels **c** and **d** show the same situation but for 0.30 GeV electrons. From Fig. 2.41.2 follows that for 1.94 GeV electrons the ratio varies with $\leq 1\%$ for both polarity cycles, at 1 AU and at 60 AU, for all tilt angles α . At 0.30 GeV, the $A < 0$ polarity cycle exhibits again a very small deviation from unity in the ratio, not more than $\sim 5\%$. For the $A > 0$ cycle, however, the ratio is > 1.0 at 1 AU, increasing with increasing α , with a maximum of 1.25. At 60 AU the ratio has a peculiar dependence varying between 1.15 and 1.25 which is larger than at 1 AU, especially for $\alpha \leq 30^\circ$. The differences between the model solutions are obviously the largest for the intermediate to lower energies during the $A > 0$ cycle when the electrons drift in along the HCS. At energies below ~ 0.05 GeV the differences between the intensities dissipate quickly because electrons experience less and less drift effects with decreasing energy. The largest variation in the ratio between the two sets of solutions as a function of energy occurs at ~ 0.2 GeV and varies between 12% at 1 AU and 24% at 60 AU, with no difference at kinetic energies ≥ 1 GeV. No qualitative differences were found between the solutions of the two models despite the difference in spatial dimensions and the different way the HCS was handled in the numerical schemes.

Hattingh (1998) indicated that the difference in the solutions of the 2D and 3D models using the same set of modulation parameters was somewhat dependent on the parameter values. The values used above correspond to solar minimum modulation conditions for which the steady-state models were developed. When more extreme variations were used the differences between the two models increased, indicating that some caution is required during periods of large modulation. Investigating this aspect further using electron modulation it was found that by increasing $\kappa_{\perp\theta}$, which has become a very important parameter in modulation models, a reduction in the differences between the 2D and 3D model solutions followed: see also Section 2.39 (Ferreira et al., 1999b) and Ferreira and Potgieter, 1999). This is expected because an increasing $\kappa_{\perp\theta}$ causes less pronounced drift effects. It is worthwhile to mention that when no-drifts were used the two models produce identical solutions under all circumstances. Reducing the azimuthal, radial, polar and rigidity grid intervals, that is increasing the number of total grid points in the numerical scheme, resulted in only a slight reduction in the difference between the two models while the runtime in computing one solution increased considerably for the 3D model (Ferreira, 1999).

According to Ferreira et al. (1999a), comparing the solutions produced by the 2D and 3D numerical modulation models that were both developed by the Modulation Group in Potchefstroom, it was found that when examining electron spectra as a function of the HCS tilt angle α , no qualitative differences occurred between the two sets of solutions when using identical parameters. Quantitatively, in the inner Heliosphere the ratio between the two sets of solutions increased with

increasing α , with 25% the largest difference at intermediate energies (~ 0.30 GeV) for $\alpha > 30^\circ$ during the $A > 0$ cycle; at 60 AU the ratio varied with 15% to 25% with no clear trend in the α dependence of the intensities. At energies below ~ 0.05 GeV the differences between the intensities dissipate quickly because electrons experience diminishing drift effects with decreasing energy. For the $A < 0$ cycle, the solutions were essentially identical. Thus, with no qualitative differences and insignificant quantitative differences between the solutions of the 2D and 3D models, and taking into account the amount of computing time and resources needed for the 3D model, the use of the 2D drift model for modulation studies is still well justified.

2.41.4. General comments to the Sections 2.34-2.41

In Sections 2.34-2.41 we considered in detail very important principal problems on galactic CR charged particles diffusion (especially enhanced perpendicular diffusion), convection, and drifts (gradient, curvature, and especially along Heliospheric current sheet) during their propagation and modulation in the Heliosphere as well as comparison with CR observation data (especially are important data near the Earth's orbit, on different distances from the Sun and Ulysses data on different helio-latitudes). In all these Sections it was not taken into account the time-lag of processes in the Heliosphere relative to corresponding causes processes on the Sun. The second what is also does not accounted in Sections 2.34-2.41 is the time lag caused by galactic CR particles penetrating into the inner Heliosphere. Below, in Sections 2.45 and 2.46 we will try to account these two points when we solve the inverse problems for CR propagation and modulation in the Heliosphere.

2.42. The inverse problem for solar CR propagation

2.42.1. Observation data and inverse problems for isotropic diffusion, for anisotropic diffusion, and for kinetic description of solar CR propagation

It is well known that Solar Energetic Particle (SEP) events in the beginning stage are very anisotropic, especially during great events as in February 1956, July 1959, August 1972, September-October 1989, July 2000, January 2005, and many others (Dorman, M1957, M1963a,b, M1978; Dorman and Miroshnichenko, M1968; Miroshnichenko, M2001). To determine on the basis of experimental data the properties of the SEP source and parameters of propagation, i.e. to solve the inverse problem, is very difficult, and it needs data from many CR stations. By the procedure developed in Dorman and Zukerman (2003), Dorman, Pustil'nik, Zukerman and Sternlieb, 2005; see review in Chapter 3 in Dorman, M2004), for each CR station the starting moment of SEP event can be automatically determined and then for different moments of time by the method of coupling functions to determine the energy spectrum of SEP out of the atmosphere above the individual

CR station. As result we may obtain the planetary distribution of SEP intensity out of the atmosphere and then by taking into account the influence of geomagnetic field on particles trajectories – the SEP angle distribution out of the Earth's magnetosphere. By this way by using of the planetary net of CR stations with on-line registration in real time scale can be organized the continue on-line monitoring of great ground observed SEP events (Dorman, Pustil'nik, Sternlieb et al., 2004; Mavromichalaki, Yanke, Dorman et al., 2004).

In paper Dorman (2005) we practically base on the two well established facts:

- (i) the time of particle acceleration on the Sun and injection into solar wind is very short in comparison with time of propagation, so it can be considered as delta-function from time;
- (ii) the very anisotropic distribution of SEP with developing of the event in time after few scattering of energetic particles became near isotropic (well known examples of February 1956, September 1989 and many others).

The paper of Dorman (2005), described below, is the first step for solution of inverse problem in the theory of solar CR propagation by using only one on-line detector on the ground for high energy particles and one on-line detector on satellite for small energies. Therefore we will base here on the simplest model of generation (delta function in time and in space) and on the simplest model of propagation (isotropic diffusion). The second step will be based on anisotropic diffusion, and the third – on kinetic description of SEP propagation in the interplanetary space.

The observed energy spectrum of SEP and its change with time are determined by the energy spectrum in the source, by the time of SEP ejection into the solar wind and by the parameters of SEP propagation in the interplanetary space in dependence of particle energy. Here we will try to solve the inverse problem on the basis of CR observations by the ground base detectors and detectors in the space to determine the energy spectrum of SEP in the source, the time of SEP ejection into the solar wind and the parameters of SEP propagation in the interplanetary space in dependence of particle energy. In general, this inverse problem is very complicated, and we suppose to solve it approximately step by step. In this Section we present the solution of the inverse problem in the frame of the simple model of isotropic diffusion of solar CR (the first step). We suppose that after start of SEP event, the energy spectrum of SEP at different moments in time is determined with good accuracy in a broad interval of energies by the method of coupling functions (see in detail in Chapter 3 in Dorman, M2004). We show then that after this the time of ejection, diffusion coefficient in the interplanetary space and energy spectrum in source of SEP can be determined. This information, obtained on line on the basis of real-time scale data, may be useful also for radiation hazard forecasting.

2.42.2. The inverse problem for the case when diffusion coefficient depends only from particle rigidity

In this case the solution of isotropic diffusion for the pointing instantaneous source described by function

$$Q(R, r, t) = N_o(R) \delta(r) \delta(t) \quad (2.42.1)$$

will be

$$N(R, r, t) = N_o(R) \times \left[2\pi^{1/2} (\kappa(R)t)^{3/2} \right]^{-1} \times \exp\left(-\frac{r^2}{4\kappa(R)t}\right), \quad (2.42.2)$$

where r is the distance from the Sun, t is the time after ejection, $N_o(R)$ is the rigidity spectrum of total number of SEP at the source, and $\kappa(R)$ is the diffusion coefficient in the interplanetary space during SEP event. Let us suppose that at distance from the Sun $r = r_1 = 1 \text{ AU}$ and at several moments of time t_i ($i = 1, 2, 3, \dots$) after SEP ejection into solar wind the observed rigidity spectrum out of the Earth's atmosphere $N(R, r_1, t_i) \equiv N_i(R)$ are determined in high energy range on the basis of ground CR measurements by neutron monitors and muon telescopes (by using method of coupling functions, spectrographic and global spectrographic methods, see review in Dorman, M2004)) as well as determined directly in low energy range on the basis of satellite CR measurements. Let us suppose also that the UT time of ejection T_e as well as the diffusion coefficient $\kappa(R)$ and the SEP rigidity spectrum in source $N_o(R)$ are unknown. To solve the inverse problem, i.e. to determine these three unknown parameters, we need information on SEP rigidity spectrum $N_i(R)$ at least at three different moments of time T_1 , T_2 and T_3 (in UT). In this case for these three moments of time after SEP ejection into solar wind we obtain:

$$t_1 = T_1 - T_e = x, \quad t_2 = T_2 - T_e = T_2 - T_1 + x, \quad t_3 = T_3 - T_e = T_3 - T_1 + x, \quad (2.42.3)$$

where $T_2 - T_1$ and $T_3 - T_1$ are known values and $x = T_1 - T_e$ is unknown value to be determined (because T_e is unknown). From three equations for t_1 , t_2 and t_3 of the type of Eq. 2.42.2 by taking into account Eq. 2.42.3 and dividing one equation on other for excluding unknown parameter $N_o(R)$, we obtain two equations for determining unknown two parameters x and $\kappa(R)$:

$$\frac{T_2 - T_1}{x(T_2 - T_1 + x)} = -\frac{4\kappa(R)}{r_1^2} \times \ln \left\{ \frac{N_1(R)}{N_2(R)} (x/(T_2 - T_1 + x))^{3/2} \right\}, \quad (2.42.4)$$

$$\frac{T_3 - T_1}{x(T_3 - T_1 + x)} = -\frac{4\kappa(R)}{r_1^2} \times \ln \left\{ \frac{N_1(R)}{N_3(R)} (x/(T_3 - T_1 + x))^{3/2} \right\}. \quad (2.42.5)$$

To exclude unknown parameter $\kappa(R)$ let us divide Eq. 2.42.4 by Eq. 2.42.5; in this case we obtain equation for determining unknown $x = T_1 - T_e$:

$$x = [(T_2 - T_1)\Psi - (T_3 - T_1)] / (1 - \Psi), \quad (2.42.6)$$

where

$$\Psi = [(T_3 - T_1) / (T_2 - T_1)] \times \frac{\ln \left[\frac{N_1(R)(x / (T_2 - T_1 + x))^{3/2} / N_2(R)}{N_1(R)(x / (T_3 - T_1 + x))^{3/2} / N_3(R)} \right]}{\ln \left[\frac{N_1(R)(x / (T_2 - T_1 + x))^{3/2} / N_2(R)}{N_1(R)(x / (T_3 - T_1 + x))^{3/2} / N_3(R)} \right]}. \quad (2.42.7)$$

Eq. 2.42.6 can be solved by the iteration method: as a first approximation, we can use $x_1 = T_1 - T_e \approx 500$ sec which is the minimum time propagation of relativistic particles from the Sun to the Earth's orbit. Then, by Eq. 2.42.7 we determine $\Psi(x_1)$ and by Eq. 2.42.6 we determine the second approximation x_2 . To put x_2 in Eq. 2.42.7 we compute $\Psi(x_2)$, and then by Eq. 2.42.6 we determine the third approximation x_3 , and so on. After solving Eq. 2.42.6 and determining the time of ejection, we can compute very easily diffusion coefficient from Eq. 2.42.4 or Eq. 2.42.5:

$$\kappa(R) = - \frac{r_1^2 (T_2 - T_1) / 4x (T_2 - T_1 + x)}{\ln \left\{ \frac{N_1(R)}{N_2(R)} (x / (T_2 - T_1 + x))^{3/2} \right\}} = - \frac{r_1^2 (T_3 - T_1) / 4x (T_3 - T_1 + x)}{\ln \left\{ \frac{N_1(R)}{N_3(R)} (x / (T_3 - T_1 + x))^{3/2} \right\}}. \quad (2.42.8)$$

After determining the time of ejection and diffusion coefficient, it is easy to determine the SEP source spectrum:

$$\begin{aligned} N_o(R) &= 2\pi^{1/2} N_1(R) \times (\kappa(R)x)^{3/2} \exp(r_1^2 / (4\kappa(R)x)) \\ &= 2\pi^{1/2} N_2(R) \times (\kappa(R)(T_2 - T_1 + x))^{3/2} \exp(r_1^2 / (4\kappa(R)(T_2 - T_1 + x))) \\ &= 2\pi^{1/2} N_3(R) \times (\kappa(R)(T_3 - T_1 + x))^{3/2} \exp(r_1^2 / (4\kappa(R)(T_3 - T_1 + x))) \end{aligned} \quad (2.42.9)$$

2.42.3. The inverse problem for the case when diffusion coefficient depends from particle rigidity and from the distance to the Sun

Let us suppose, according to Parker (M1963), that the diffusion coefficient

$$\kappa(R, r) = \kappa_1(R) \times (r / r_1)^\beta. \quad (2.42.10)$$

In this case the solution of diffusion equation will be

$$N(R, r, t) = \frac{N_o(R) \times r_1^{3\beta/(2-\beta)} (\kappa_1(R)t)^{-3/(2-\beta)}}{(2-\beta)^{(4+\beta)/(2-\beta)} \Gamma(3/(2-\beta))} \times \exp\left(-\frac{r_1^\beta r^{2-\beta}}{(2-\beta)^2 \kappa_1(R)t}\right), \quad (2.42.11)$$

where t is the time after SEP ejection into solar wind. So now we have four unknown parameters: time of SEP ejection into solar wind T_e , β , $\kappa_1(R)$, and $N_o(R)$. Let us assume that according to ground and satellite measurements at the distance $r = r_1 = 1$ AU from the Sun we know $N_1(R)$, $N_2(R)$, $N_3(R)$, $N_4(R)$ at UT times T_1 , T_2 , T_3 , T_4 . In this case

$$t_1 = T_1 - T_e = x, \quad t_2 = T_2 - T_1 + x, \quad t_3 = T_3 - T_1 + x, \quad t_4 = T_4 - T_1 + x, \quad (2.42.12)$$

For each $N_i(R, r = r_1, T_i)$ we obtain from Eq. 2.42.11 and Eq. 2.42.12:

$$N_i(R, r = r_1, T_i) = \frac{N_o(R) \times r_1^{3\beta/(2-\beta)} (\kappa_1(R)(T_i - T_1 + x))^{-3/(2-\beta)}}{(2-\beta)^{(4+\beta)/(2-\beta)} \Gamma(3/(2-\beta))} \times \exp\left(-\frac{r_1^2 (2-\beta)^{-2}}{\kappa_1(R)(T_i - T_1 + x)}\right), \quad (2.42.13)$$

where $i = 1, 2, 3$, and 4. To determine x let us step by step exclude unknown parameters $N_o(R)$, $\kappa_1(R)$, and then β . In the first we exclude $N_o(R)$ by forming from four Eq. 2.42.13 for different i three equations for ratios

$$\frac{N_1(R, r = r_1, T_1)}{N_i(R, r = r_1, T_i)} = \left(\frac{x}{T_i - T_1 + x}\right)^{-3/(2-\beta)} \times \exp\left(-\frac{r_1^2}{(2-\beta)^2 \kappa_1(R)} \left(\frac{1}{x} - \frac{1}{T_i - T_1 + x}\right)\right), \quad (2.42.14)$$

where $i = 2, 3$, and 4. To exclude $\kappa_1(R)$ let us take logarithm from both parts of Eq. 2.42.14 and then divide one equation on another; as result we obtain following two equations:

$$\frac{\ln(N_1/N_2) + (3/(2-\beta))\ln(x/(T_2 - T_1 + x))}{\ln(N_1/N_3) + (3/(2-\beta))\ln(x/(T_3 - T_1 + x))} = \frac{(1/x) - (1/(T_2 - T_1 + x))}{(1/x) - (1/(T_3 - T_1 + x))}, \quad (2.42.15)$$

$$\frac{\ln(N_1/N_2) + (3/(2-\beta))\ln(x/(T_2 - T_1 + x))}{\ln(N_1/N_4) + (3/(2-\beta))\ln(x/(T_4 - T_1 + x))} = \frac{(1/x) - (1/(T_2 - T_1 + x))}{(1/x) - (1/(T_4 - T_1 + x))}. \quad (2.42.16)$$

After excluding from Eq. 2.42.15 and Eq. 2.42.16 unknown parameter β , we obtain equation for determining x :

$$x^2(a_1a_2 - a_3a_4) + xd(a_1b_2 + b_1a_2 - a_3b_4 - b_3a_4) + d^2(b_1b_2 - b_3b_4) = 0, \quad (2.42.17)$$

where

$$d = (T_2 - T_1)(T_3 - T_1)(T_4 - T_1), \quad (2.42.18)$$

$$a_1 = (T_2 - T_1)(T_4 - T_1)\ln(N_1/N_3) - (T_3 - T_1)(T_4 - T_1)\ln(N_1/N_2), \quad (2.42.19)$$

$$a_2 = (T_3 - T_1)(T_4 - T_1)\ln(x/(T_2 - T_1 + x)) - (T_2 - T_1)(T_3 - T_1)\ln(x/(T_4 - T_1 + x)), \quad (2.42.20)$$

$$a_3 = (T_2 - T_1)(T_3 - T_1)\ln(N_1/N_4) - (T_3 - T_1)(T_4 - T_1)\ln(N_1/N_2), \quad (2.42.21)$$

$$a_4 = (T_3 - T_1)(T_4 - T_1)\ln(x/(T_2 - T_1 + x)) - (T_2 - T_1)(T_4 - T_1)\ln(x/(T_3 - T_1 + x)), \quad (2.42.22)$$

$$b_1 = \ln(N_1/N_3) - \ln(N_1/N_2), \quad b_2 = \ln(x/(T_2 - T_1 + x)) - \ln(x/(T_4 - T_1 + x)), \quad (2.42.23)$$

$$b_3 = \ln(N_1/N_4) - \ln(N_1/N_2), \quad b_4 = \ln(x/(T_2 - T_1 + x)) - \ln(x/(T_3 - T_1 + x)). \quad (2.42.24)$$

As it can be seen from Eq. 2.42.20 and Eq. 2.42.22-2.42.24, coefficients a_2, a_4, b_2, b_4 very weakly (as logarithm) depend from x . Therefore Eq. 2.42.17 we solve by iteration method, as above we solved Eq. 2.42.6: as a first approximation, we use $x_1 = T_1 - T_e \approx 500$ sec (which is the minimum time propagation of relativistic particles from the Sun to the Earth's orbit). Then by Eq. 2.42.20 and Eq. 2.42.22-2.42.2 we determine $a_2(x_1)$, $a_4(x_1)$, $b_2(x_1)$, $b_4(x_1)$ and by Eq. 2.42.17 we determine the second approximation x_2 , and so on. After determining x , i.e. according Eq. 2.42.12 determining t_1 , t_2 , t_3 , t_4 , the final solutions for β , $\kappa_1(R)$, and $N_o(R)$ can be found. Unknown parameter β in Eq. 2.42.10 we determine from Eq. 2.42.15 and Eq. 2.42.16:

$$\beta = 2 - 3 \left[\left(\ln(t_2/t_1) \right) - \frac{t_3(t_2 - t_1)}{t_2(t_3 - t_1)} \ln(t_3/t_1) \right] \times \left[\left(\ln(N_1/N_2) \right) - \frac{t_3(t_2 - t_1)}{t_2(t_3 - t_1)} \ln(N_1/N_3) \right]^{-1}. \quad (2.42.25)$$

Then we determine unknown parameter $\kappa_1(R)$ in Eq. 2.42.10 from Eq. 2.42.14:

$$\begin{aligned} \kappa_1(R) &= \frac{r_1^2 (t_1^{-1} - t_2^{-1})}{3(2 - \beta) \ln(t_2/t_1) - (2 - \beta)^2 \ln(N_1/N_2)} \\ &= \frac{r_1^2 (t_1^{-1} - t_3^{-1})}{3(2 - \beta) \ln(t_3/t_1) - (2 - \beta)^2 \ln(N_1/N_3)}. \end{aligned} \quad (2.34.26)$$

After determining parameters β and $\kappa_1(R)$ we can determine the last parameter $N_o(R)$ from Eq. 2.42.13:

$$\begin{aligned} N_o(R) &= N_i (2 - \beta)^{(4+\beta)/(2-\beta)} \Gamma(3/(2 - \beta)) r_1^{-3\beta/(2-\beta)} (\kappa_1(R) t_i)^{3/(2-\beta)} \\ &\times \exp \left(\frac{r_1^2}{(2 - \beta)^2 \kappa_1(R) t_i} \right), \end{aligned} \quad (2.42.27)$$

where index $i = 1, 2$ or 3 .

Above we show that for some simple model of SEP propagation is possible to solve inverse problem on the basis of ground and satellite measurements at the beginning of the event. Obtained results we used in the method of great radiation hazard forecasting based on on-line CR one-minute ground and satellite data (Dorman et al., 2005b).

Let us note that described solutions of inverse problem may be partly useful for solving more complicated inverse problems in case of SEP propagation described by anisotropic diffusion and by kinetic equation.

2.43. The checking of solution for SEP inverse problem by comparison of predictions with observations

2.43.1. The checking of the model when diffusion coefficient does not depend from the distance from the Sun

Let us in the first checking the model of SEP propagation in the interplanetary space, described in Section 2.42.2 (when the value of the diffusion coefficient does not depend from the distance from the Sun). We will use the data obtained during

the great SEP event in September 1989 by NM on the top of Gran-Sasso in Italy (Dorman et al., 2005a,b). This NM detects one-minute data not only of total neutron intensity, but also many of neutron multiplicities (≥ 1 , ≥ 2 , ≥ 3 , up to ≥ 8), what gave possibility by using method of coupling functions to determine the energy spectrum in high energy range (≥ 6 GV) for each minute. On the basis of these data we determine at first the values of diffusion coefficient $\kappa(R)$. These calculations have been done according to the procedure described above, by supposing that $K(R)$ does not depend on the distance to the Sun (see Eq. 2.42.8). Results are shown in Fig. 2.43.1.

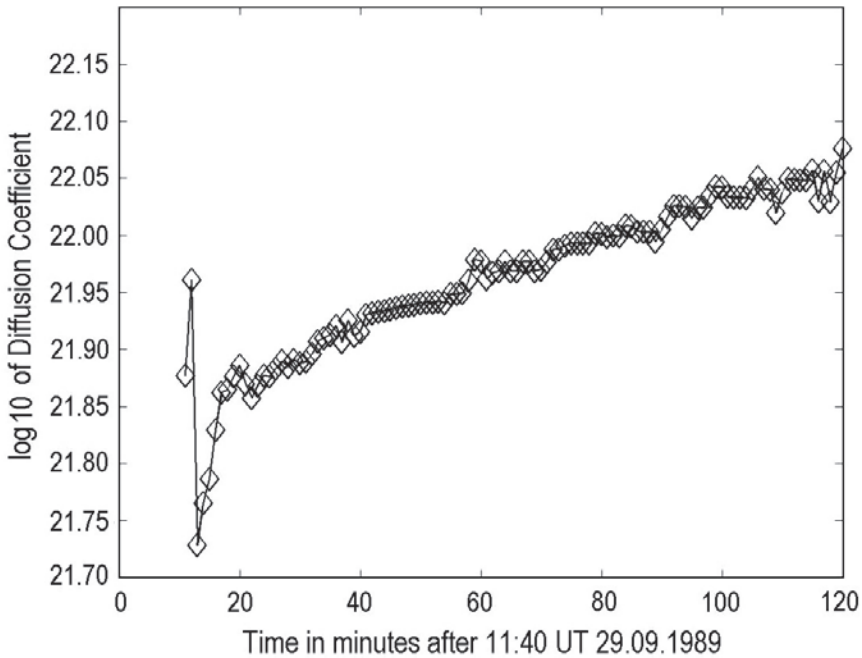


Fig. 2.43.1. The time behavior of $\kappa(R)$ for $R \sim 10$ GV for the SEP event 29 September, 1989. According to Dorman et al. (2005a,b).

From Fig. 2.43.1 can be seen that at the beginning of the event the obtained results are not stable, due to large relative statistical errors. After several minutes the amplitude of CR intensity increase becomes many times bigger than statistical error for one minute data σ (about 1%), and we can see a systematical increase of the diffusion coefficient $\kappa(R)$ with time. This result contradicts the conditions at which was solved the inverse problem in Section 2.42.2. Really the systematical increase of the diffusion coefficient with time reflects the increasing of $\kappa(R)$ with the diffusion propagation of solar CR from the Sun, i.e. reflects the increasing of

$\kappa(R)$ with the distance from the Sun. It means that for the considered SEP event we need to apply the inverse problem described in Section 2.42.3, where it was assumed increasing of diffusion coefficient with the distance from the Sun according to Eq. 2.42.10.

2.43.2. The checking of the model when diffusion coefficient depends from the distance to the Sun

On the basis of the inverse problem solution described in Section 2.42.3, by using the first few minutes NM data of the SEP event we can determine the effective parameters β by Eq. 2.42.25, $\kappa_1(R)$ by Eq. 2.42.26, and $N_o(R)$ by Eq. 2.42.27, corresponding to high rigidity, about 10 GV. In Fig. 2.43.2 the values of parameter $\kappa_1(R)$ are shown.

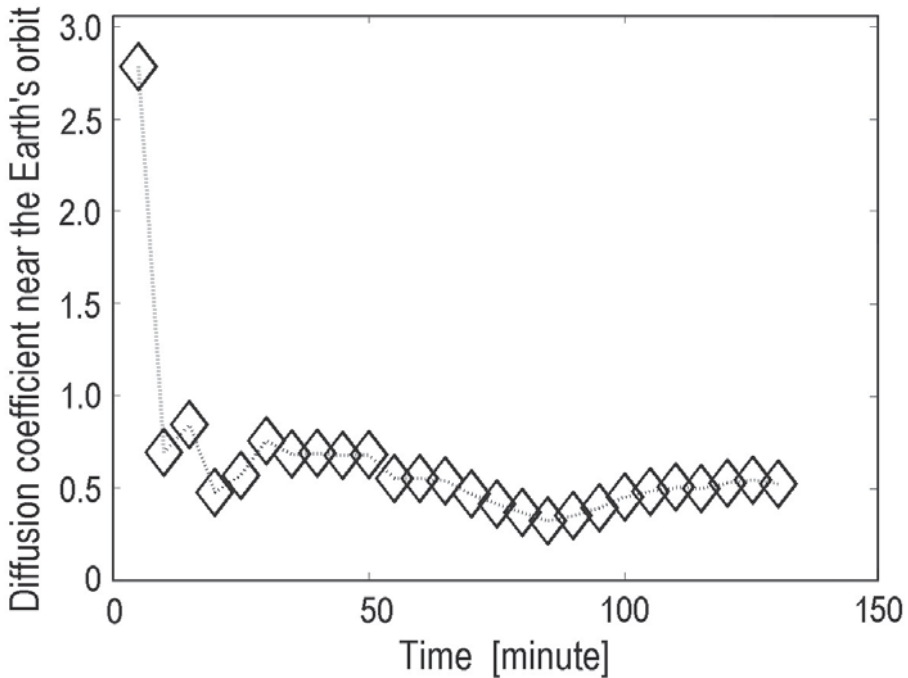


Fig. 2.43.2. Diffusion coefficient $\kappa_1(R)$ near the Earth's orbit (in units $10^{23} \text{ cm}^2 \text{ sec}^{-1}$) in dependence of time (in minutes after 11.40 UT of September 29, 1989).

From Fig. 2.43.2 it can be seen that at the very beginning of event (the first point) the result is unstable: in this period the amplitude of increase is relatively small, so the relative accuracy is too low, and we obtain very big diffusion coefficient. Let us note, that at the very beginning of the event the

diffusion model can be very hardly applied (more correct would be the application of kinetic model of SEP propagation). After the first point we have about stable result with accuracy $\pm 20\%$ (let us compare with Fig. 2.43.1, where the diffusion coefficient was found as effectively increasing with time). In Fig. 2.43.3 are shown values of parameter β .

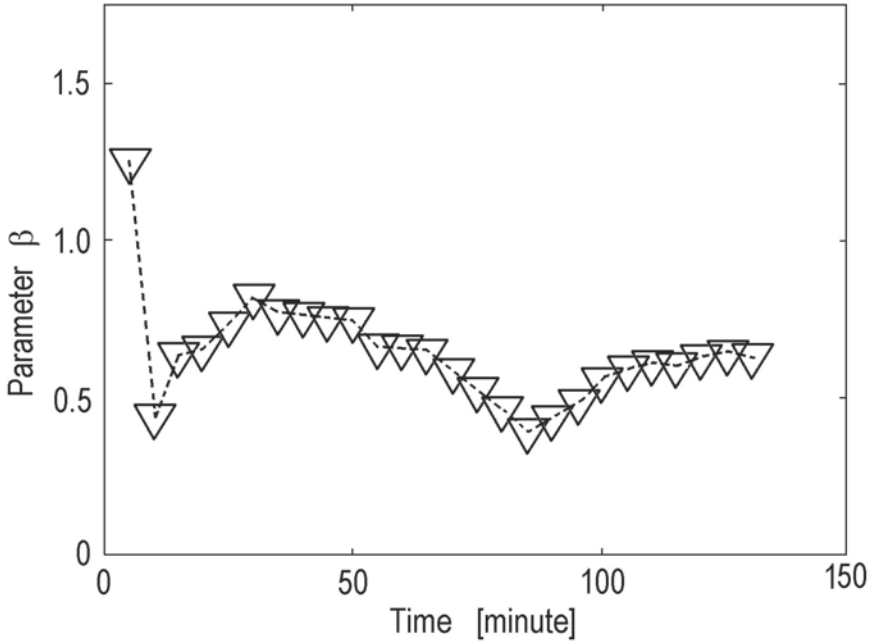


Fig. 2.43.3. Values of parameter β in dependence of time (in minutes after 11.40 UT of September 29, 1989).

It can be seen from Fig. 2.43.3 that again the first point is anomalously big, but after the first point the result become almost stable with average value $\beta \approx 0.6$ (with accuracy about $\pm 20\%$). Therefore, we can hope that the model of the inverse problem solution, described in Section 2.42.3 (the set of Eq. 2.42.10–2.42.27) reflects adequately SEP propagation in the interplanetary space.

2.43.3. The checking of the model by comparison of predicted SEP intensity time variation with NM observations

More accurate and exact checking of the solution of the inverse problem can be made by comparison of predicted SEP intensity time variation with NM observations. For this aim after determining of the effective parameters β , $\kappa_1(R)$, and $N_o(R)$ we may determine by Eq. 2.42.11 the forecasting curve of expected

SEP flux behavior for total neutron intensity. With each new minute of observations we can determine parameters β , $\kappa_1(R)$, and $N_o(R)$ more and more exactly. It means that with each new minute of observations we can determine more and more exactly the forecasting curve of expected SEP flux behavior. We compare this forecasting curve with time variation of observed total neutron intensity (see Fig. 2.43.4 which contains 8 panels for time moments $t = 10$ min up to $t = 120$ min after 11.40 UT of 29 September, 1989).

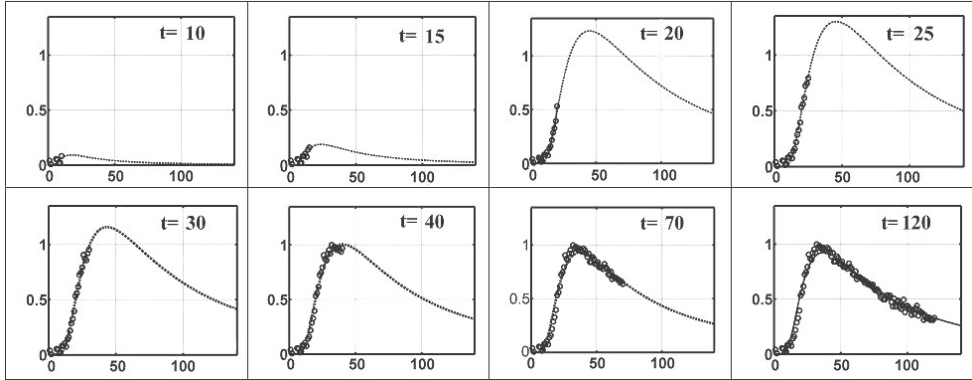


Fig. 2.43.4. Calculation for each new minute of SEP intensity observations parameters β , $\kappa_1(R)$, $N_o(R)$ and forecasting of total neutron intensity (time t is in minutes after 11.40 UT of September 29, 1989; curves – forecasting, circles – observed total neutron intensity). From Dorman et al. (2005a,b).

From Fig. 2.43.4 it can be seen that it is not enough to use only the first few minutes of NM data ($t = 10$ min): the obtained curve forecasts too low intensity. For $t = 15$ min the forecast shows some bigger intensity, but also not enough. Only for $t = 20$ min (15 minutes of increase after beginning) and later (up to $t = 40$ min and more) we obtain about stable forecast with good agreement with observed CR intensity.

2.43.4. The checking of the model by comparison of predicted SEP intensity time variation with NM and satellite observations

The results described above, based only on NM on data, reflect the situation in SEP behavior in the high energy (more than 6 GeV) region. For extrapolation of these results to the low energy interval (dangerous for space-probes and satellites), we use satellite on-line data available through the Internet. The problem is how to extrapolate the SEP energy spectrum from high NM energies to very low energies detected by GOES satellite. The main idea of this extrapolation is the following: 1) the time of ejection for high and small energy ranges (detected by NM and by satellite) is the same, so it can be determined by using only NM data; 2) the source

function relative to time is a δ -function, and relative to energy is a power function with an energy-dependent index $\gamma = \gamma_o + \ln(E_k/E_{ko})$ with maximum at $E_{k\max} = E_{ko} \exp(-\gamma_o)$:

$$N_o(R, T) = N_o \delta(T - T_e) R^{-(\gamma_o + \ln(E_k/E_{ko}))}. \quad (2.43.1)$$

Fig. 2.43.5 shows results based on the NM and satellite data of forecasting of expected SEP fluxes also in small energy intervals and comparison with observation satellite data.

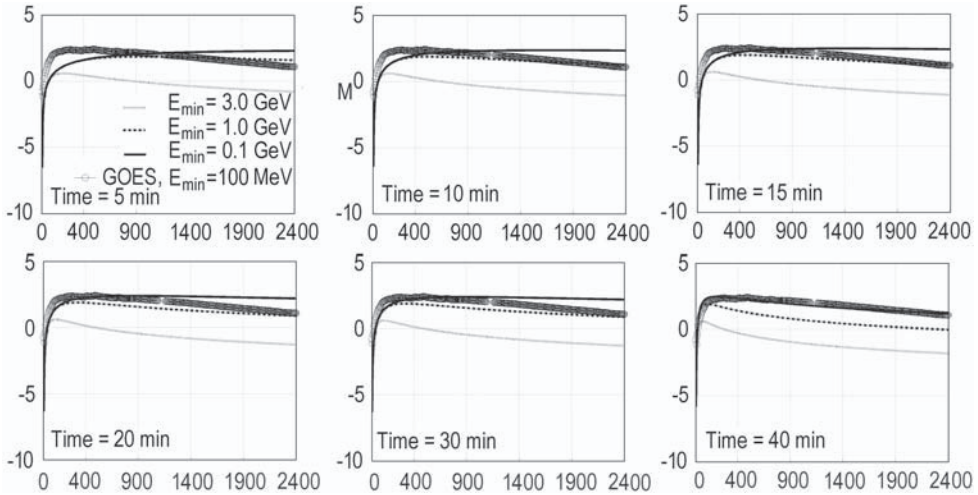


Fig. 2.43.5. Predicted SEP integral fluxes for $E_k \geq E_{\min} = 0.1, 1.0$, and 3.0 GeV. The forecasted integral flux for $E_k \geq E_{\min} = 0.1$ GeV is compared with the observed fluxes for $E_k \geq 100$ MeV on GOES satellite. The ordinate is $\log_{10}$ of the SEP integral flux (in $\text{cm}^{-2}\text{sec}^{-1}\text{sr}^{-1}$), and the abscissa is time in minutes from 11.40 UT of September 29, 1989. From Dorman et al. (2005a,b).

Results of comparison presented in Fig. 2.43.4 and Fig. 2.43.5 show that by using on-line data from ground NM in the high energy range and from satellite in the low energy range during the first 30-40 minutes after the start of the SEP event, it is possible by using only CR data to solve the inverse problem by formulas in Sections 2.42.2 and 2.42.3: to determine the properties of SEP source on the Sun (time of ejection into solar wind, source SEP energy spectrum, and total flux of accelerated particles) and parameters of SEP propagation in the interplanetary space (diffusion coefficient and its dependence from particle energy and from the distance from the Sun).

2.43.5. The inverse problems for great SEP events and space weather

Let us note that the solving of inverse problems for great SEP events has important practical sense: to predict the expected SEP differential energy spectrum on the Earth's orbit and integral fluxes for different threshold energies up to many hours (and even up to few days) ahead. The total (event-integrated) fluency of the SEP event, and the expected radiation hazards can also be estimated on the basis of the first 30-40 minutes after the start of the SEP event and corresponding Alerts to experts operating different objects in space, in magnetosphere, and in atmosphere at different altitudes and at different cut-off rigidities can be sent automatically. These experts should decide what to do operationally (for example, for space-probes in space and satellites in the magnetosphere to switch-off the electric power for few hours to save the memory of computers and high level electronics; for jets to decrease their altitudes from 10 km to 4-5 km to protect crew and passengers from great radiation hazard, and so on). From this point of view especially important are the solving of inverse problems for great SEP by using on-line data of many NM and several satellites in the frame of models in which CR propagation described by the theory of anisotropic diffusion or by kinetic theory. The solving of these inverse problems will made possible on the basis of world-wide CR Observatories and satellite data (in real scale time, applicable from Internet) to made forecasting on radiation hazard for much shorter time after SEP event beginning. These important problems are formulated below, in Section "Conclusions and Problems" at the end of monograph.

2.44. The inverse problems for CR propagation in the Galaxy

The main parameters of CR propagation in the Galaxy can be determined by the solving the inverse problem for the some model of CR propagation (boxes model, diffusion model in disc or/and in halo, model with galactic wind, model of rotating Galaxy with galactic wind driven by pressure of CR, and so on). Partly these models we consider in Chapter 3 with account nonlinear effects (which are sufficient in case of CR propagation in the Galaxy) and experimental data on relative content of radioactive nuclei in CR ^{10}Be and others (what determines the average time-life of CR in the Galaxy), contents of elements Li, Be, B in CR (determined the grammar of matter transferred by CR before they escape from the Galaxy), and data on gamma ray distribution (determined the distribution of CR sources). The paper of Bloemen et al. (1993) can be considered as a classical example of using these data for solving inverse problem of CR propagation in the disc and halo of Galaxy with account galactic wind (see in detail Chapter 3, Section 3.13). As result it was determined the diffusion coefficient and the velocity of galactic wind.

In Sections 3.14-3.16 we consider in detail other important inverse problems for CR propagation and distribution in the Galaxy with taking into account non-linear phenomena: CR pressure and kinetic stream instabilities; galactic wind driving by CR and generation of Alfvén turbulence by CR and its influence on CR

propagation; self-consistent problem for dynamic halo in rotating Galaxy for CR propagation and space-distribution, for formation of galactic wind and magnetic field; transport of random magnetic fields from the disc by galactic wind driven by CR and its influence on CR propagation; nonlinear Alfvén waves generated by CR streaming instability and their influence on CR propagation in the Galaxy; the balance of Alfvén wave generation by CR with damping mechanisms, and others.

2.45. The inverse problem for high energy galactic CR propagation and modulation in the Heliosphere on the basis of NM data

2.45.1. Hysteresis phenomenon and the inverse problem for galactic CR propagation and modulation in the Heliosphere

By the solving of the inverse problem for galactic CR propagation and modulation in the interplanetary space on the basis of observation data of CR-SA (solar activity) hysteresis phenomenon can be obtained important information on the main properties of the Heliosphere. The investigation of the hysteresis phenomenon in the connection between long-term variations in CR intensity observed at the Earth and SA, started about 50 years ago (Dorman, M1957; Forbush, 1958; Neher and Anderson, 1962; Simpson, 1963; Dorman, M1963a, M1963b). In the middle of 60-th many scientists came to conclusion that the dimension of modulation region (or Heliosphere) is about 5 AU, and not more than 10-15 AU (Quenby, 1965; Kudo and Wada, 1968; Charakhchyan and Charakhchyan, 1968, 1971; Stozhkov and Charakhchyan, 1969; Pathak and Sarabhai, 1970). It was found that the radius r_o of the CR modulation region is very small either by analysis of the intensity of coronal green line in some helio-latitude regions (as controlled solar activity factor; in this case was obtain $r_o \approx 5$ AU), or by investigation the CR modulation as caused by sudden jumps in solar activity ($r_o \approx 10-15$ AU). In Dorman and Dorman (1965, 1967a,b,c), Dorman (M1975b) the hysteresis phenomenon was investigated on the basis of neutron monitor (NM) data for about one solar cycle in the frame of convection-diffusion model of CR global modulation in the Heliosphere with taking into account time lag of processes in the interplanetary space relative to processes on the Sun. It was shown that the dimension of the modulation region should be about 100 AU (much bigger than accepted in those time in literature, 5–15 AU). These investigations were continued on the basis of CR and SA monthly average data for about four solar cycles in Dorman et al. (1997, 1999). Let us note that many authors worked on this problem, used sunspot numbers or other parameters of solar activity for investigations of CR long-term variations, but they did not take into account time lag of processes in the interplanetary space relative to processes on the Sun as integral action (see review in Belov, 2000). The method, described below, takes into

account that CR intensity observed on the Earth at moment t is caused by solar processes summarized for the long period started many months before t . In recent paper Dorman (2001) was considered again by this method CR and SA data for solar cycles 19-22, but with taking into account drift effects according to Burger and Potgieter (1999). It was shown that including in the consideration drift effects (as depending from the sign of solar polar magnetic field (sign of parameter A) and determined by difference of total CR modulation at $A > 0$ and $A < 0$, and with amplitude proportional to the value of tilt angle between interplanetary neutral current sheet and equatorial plane) is very important: it became possible to explain the great difference in time-lags between CR and SA in hysteresis phenomenon for even and odd solar cycles.

2.45.2. Hysteresis phenomenon and the model of CR global modulation in the frame of convection-diffusion mechanism

It was shown in Dorman and Dorman (1965) that the time of propagation through the Heliosphere of particles with rigidity bigger than 10 GV (to which NM are sensitive) is not longer than one month. This time is at least about one order of magnitude smaller than the observed time-lag in the hysteresis phenomenon. It means that the hysteresis phenomenon on the basis of NM data can be considered as quasi-stationary problem with parameters of CR propagation changing in time. In this case according to Parker (1958, M1963), Dorman (1959c):

$$n(R, r_{obs}, t) / n_o(R) \approx \exp \left(-a \int_{r_{obs}}^{r_o} \frac{u(r, t) dr}{\kappa_r(R, r, t)} \right), \quad (2.45.1)$$

where $n(R, r_{obs}, t)$ is the differential rigidity CR density, $n_o(R)$ is the differential rigidity density spectrum in the local interstellar medium out of the Heliosphere, $a \approx 1.5$, $u(r, t)$ is the effective solar wind velocity (taking into account also shock waves and high speed solar wind streams), and $\kappa_r(R, r, t)$ is the radial diffusion coefficient in dependence of the distance r from the Sun of particles with rigidity R at the time t . According to Dorman and Dorman (1967a,b), Dorman (M1975b) the connection between $\kappa_r(R, r, t)$ and solar activity can be described by the relation

$$\kappa_r(R, r, t) \propto r^\beta (W(t - r/u))^{-\alpha}, \quad (2.45.2)$$

where $W(t - r/u)$ is the sunspot number in the time $t - r/u$. By the comparison with observation data it was determined in Dorman and Dorman (1967a,b), Dorman (M1975b) that parameter $0 \leq \beta \leq 1$ and $\alpha \approx 1/3$ in the period of high solar activity ($W(t) \approx W_{\max}$) and $\alpha \approx 1$ near solar minimum ($W(t) \ll W_{\max}$). Here we suppose, in accordance with Dorman et al. (1997), that

$$\alpha(t) = 1/3 + (2/3)(1 - W(t)/W_{\max}), \quad (2.45.3)$$

where $W_{\max}$ is the sunspot number in the maximum of solar activity cycle.

According to Eq. 2.45.1 the value of the natural logarithm of observed CR intensity global modulation at the Earth's orbit, taking into account Eq. 2.45.2 and Eq. 2.45.3, will be

$$\ln(n(R, r_E, t)_{obs}) = A(R, X_o, \beta) - B(R, X_o, \beta) F\left(t, X_o, \beta, W(t - X)\right)_{X_E}^{X_o}, \quad (2.45.4)$$

where

$$F\left(t, X_o, \beta, W(t - X)\right)_{X_E}^{X_o} = \int_{X_E}^{X_o} \left(\frac{W(t - X)}{W_{\max}} \right)^{\frac{1}{3} + \frac{2}{3}(1 - W(t - X)/W_{\max})} X^{-\beta} dX, \quad (2.45.5)$$

$X = r/u$, $X_E = 1AU/u$, $X_o = r_o/u$ (X_E and X_o are in units of average month = (365.25/12) days = 2.628×10^6 sec). Let us note that the solving of Eq. 2.45.4 on the basis of experimental data will give solution also for the inverse problem because the regression coefficient $A(R, X_o, \beta)$ determines the CR intensity out of the Heliosphere, regression coefficient $B(R, X_o, \beta)$ characterized the effective diffusion coefficient of CR in the interplanetary space, and $X_o = r_o/u$ characterized the dimension of modulation region. These three coefficients can be determined by correlation between observed values $\ln(n(R, r_E, t)_{obs})$ and the values of F , calculated according to Eq. 2.45.5 for different values of X_o and β . In Dorman et al. (1997) three values of $\beta = 0; 0.5; 1$ have been considered; it was shown that $\beta = 1$ strongly contradicts CR and SA observation data, and that $\beta = 0$ is the most reliable value. Therefore, we will consider here only this value.

2.45.3. Even-odd cycle effect in CR and role of drifts for NM energies

To determine $X_{o\max}$, corresponding to the maximum value of the correlation coefficient for regression Eq. 2.45.4, we compare 11 months moving averages of the Climax NM ($H = 3400$ m, cut-off rigidity $R_c = 2.99$ GV) for solar cycles 19-22 and onset of cycle 23 (Dorman, 2001). For each time-lag, $X_o = r_o/u = 1, 2, 3, \dots, 60$ av. months, we determined the correlation between observed and expected CR intensities. The Climax NM data correspond to an effective rigidity of primary CR

of about 10-15 GV. For higher energy particles (about 30-40 GV) we used Huancayo ($R_c = 12.92$ GV, $H = 3400$ m)/Haleakala ($R_c = 12.91$ GV, $H = 3030$ m) NM data from January 1953 to August 2000. Results are summarized below in Table 2.45.1 in columns $A_{dr}=0\%$. It can be seen a big difference in $X_{o\max}$ for odd and even solar cycles.

We assume that observed long-term CR modulation is caused by two processes: the convection-diffusion mechanism (e.g. Parker, 1958, M1963; Dorman, 1959c, 1965), which is independent of the sign of the solar magnetic field, and the drift mechanism (e.g., Jokipii and Davila, 1981; Burger and Potgieter, 1999; Ferreira et al., 1999), what gave opposite effects with changing sign of solar magnetic field. For the convection-diffusion mechanism we use the model described in detail in Dorman (2001), shortly given above by Eqs. 2.45.1–2.45.5. For drift effects we use results of Burger and Potgieter, 1999 (see also above, Section 2.37), and assume that the drift effect is proportional to the value of the tilt angle α with negative sign at $A > 0$ and positive sign at $A < 0$, and in the period of reversal we again suppose linear transition through 0 from one polarity cycle to other (see Fig. 2.37.1-2.37.4 in Section 2.37; we assume that average of curves for $A > 0$ and $A < 0$ in these figures characterized convection-diffusion modulation, and difference of these curves – double drift modulation). Data on tilt-angles for solar cycles 19 and 20 are not available. We used relation between sunspot numbers W and α to made homogeneous analysis of the period 1953-2000. Based on data for 18 years (May 1976- September 1993), we found that there are very good relation between α and W ; for 11 months smoothed data

$$\alpha = 0.349W + 13.5^\circ \quad (2.45.6)$$

with correlation coefficient 0.955. An example for correction of observed CR intensity on the drift effects (to obtain only convection-diffusion modulation) is shown for period January 1953-November 2000 in Fig. 2.45.1.

We used 11 months smoothed data of W (shown in Fig. 2.45.1) and determined the amplitude A_{dr} of drift effects as drift modulation at $W11M = 75$ (average value of $W11M$ for 1953-1999). The reversal periods were determined as: August 1949 $\pm$ 9 months, December 1958 $\pm$ 12 months, December 1969 $\pm$ 8 months, March 1981 $\pm$ 5 months, and June 1991 $\pm$ 7 months. We determined correlation coefficients between the expected integrals F according to Eq. 2.45.5 for different values of $X_o = 1, 2, 3, \dots, 60$ av months with the observed LN(CL11M) and LN(HU/HAL11M), as well as with corrected for the drift effects according to A_{dr} from 0.15% up to 4%.

In Table 2.45.1 are shown results of the determination of $X_{o\max}$ for solar cycles 19, 20, 21, and 22 without corrections on drift effect, and with corrections owed to the drift effects in dependence of the value of A_{dr} . (from 0.5% to 4% for Climax NM and from 0.15% to 1.0% for Huancayo/Haleakala NM).

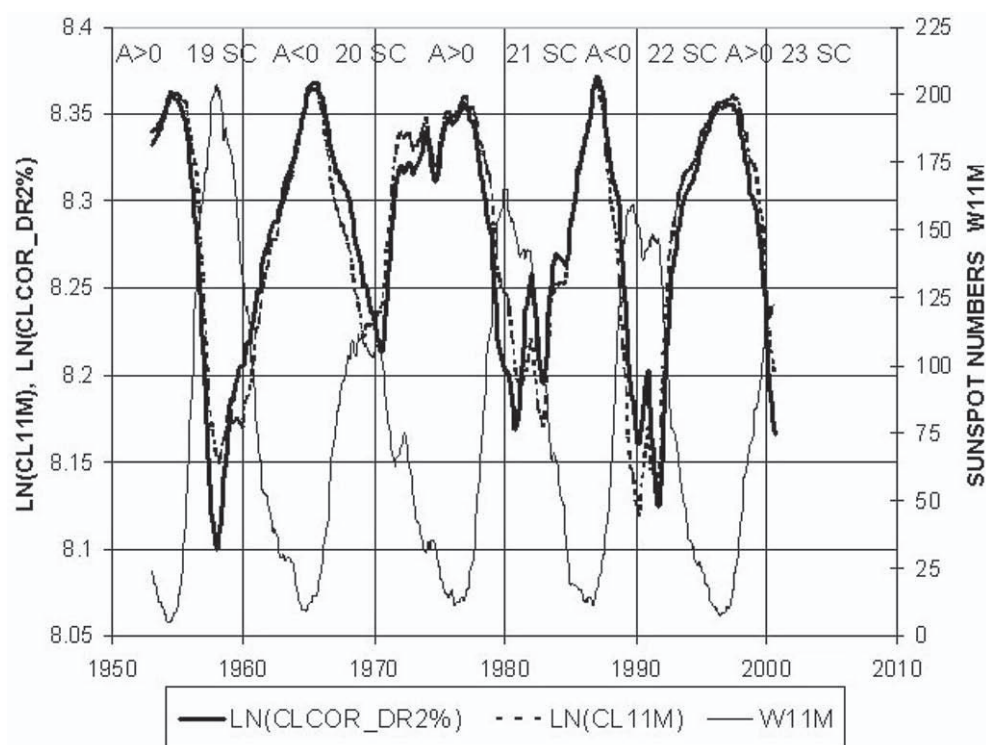


Fig. 2.45.1. An example of CR data correction on drift effects in 1953-2000 (19-22 cycles and onset of 23 cycle): LN(CL11M) – observed natural logarithm of Climax NM counting rate smoothed for 11 months, LN(CLCOR3_DR2%) – corrected on assumed $A_{dr}=2\%$ at W11M=75. Interval between two horizontal lines corresponds 5% of CR intensity variation.

Table 2.45.1. Values of $X_{o\max}$ (in av. months) for observed data ($A_{dr}=0\%$) and corrected on drift effects with different amplitudes A_{dr} .

CLIMAX NM, LN(CL11M)								
CY	0%	0.5%	1%	1.5%	2%	2.5%	3%	4%
19	21	18.5	16.5	14.5	12.5	11	9	6
20	6.5	8	9.5	12	16.5	20	27	34
21	31	27	23	20	16.5	15	12	9
22	8	10	11	12	14	16.5	18	24
HUANCAYO/HALEAKALA NM, LN(HU/HAL11M)								
CY	0%	0.15%	0.25%	0.35%	0.5%	0.75%	1.0%	
19	20	18	16.5	14	12	9	6	
20	10.5	15	18	25	31	39	46	
21	32	23	18	15	12	9	7	
22	9	12	11	12	14	16.5	22	

In Fig. 2.45.2 the dependences of $X_{o\max}$ on A_{dr} are shown for Climax NM. From Fig. 2.45.2 can be seen that the region of crossings of $X_{o\max}(A_{dr})$ for odd and even cycles is: $13 \leq X_{o\max} \leq 16.5$, $1.7\% \leq A_{dr} \leq 2.3\%$. For Huancayo/Haleakala NM this region is: $13 \leq X_{o\max} \leq 18$, $0.23\% \leq A_{dr} \leq 0.43\%$. Thus we came to conclusion that the amplitude of the drift effect is about 2.0% for Climax NM and about 0.33% for Huancayo/Haleakala NM. It means that for primary CR with rigidity 10-15 GV a relative contribution of drift effects is about 20-25%. For CR with rigidity 35-40 GV a relative role of drift effects is about 2-3 times smaller. For $X_{o\max}$ we obtained for both 10-15 and 35-40 GV about 15 av. months, what corresponds $r_o \approx 100$ AU (at average solar wind speed 400 km/s).

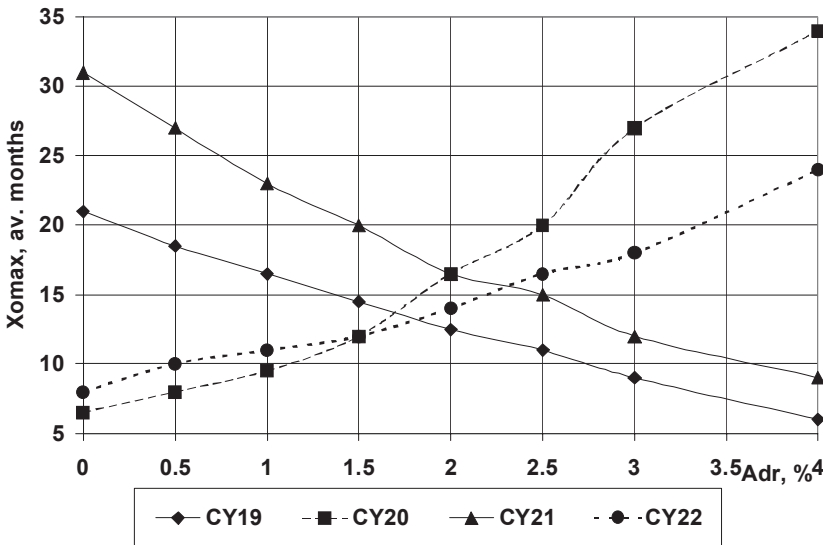


Fig. 2.45.2. Dependences $X_{o\max}(A_{dr})$ for Climax NM.

2.45.4. The inverse problem for CR propagation and modulation during solar cycle 22 on the basis of NM data

In the Section 2.45.4 was considered the relative role of convection-diffusion and drifts in the long-term CR modulation on the basis of a comparison of observations in odd and even cycles of SA: it was shown that the time-lag $X_{o\max}$ between CR and SA in the odd cycles 19, 21 decreases with increasing of the amplitude of the drift effect A_{dr} , but in the even cycles 20, 22, $X_{o\max}$ increases with increasing A_{dr} . To determine $X_{o\max}$ and A_{dr} separately, in the Section 2.45.4 was assumed that for a first approximation $X_{o\max}$ and A_{dr} are about the same in odd and even solar cycles. In this case the crossing of dependences $X_{o\max}(A_{dr})$ for

odd and even cycles determines the expected values of $X_{o\max}$ and A_{dr} . In this Section we try to solve the inverse problem of determining A_{dr} and $X_{o\max}$ only on the basis of data during solar cycle 22 (Dorman, 2003a,b). We will therefore correct the observed CR long-term variation in cycle 22 for drift effects with different values of the amplitude A_{dr} ; for each A_{dr} we determine the correlation coefficient $R(X_o, A_{dr})$ of corrected CR long-term variation according to a convection-diffusion model for different values of the time-lag X_o (from 0 to 60 av. months with monthly steps). Then we determine the value of $X_{o\max}(A_{dr})$ when $R(X_o, A_{dr})$ reaches the maximum value $R_{\max}(X_{o\max}, A_{dr})$. For each A_{dr} we will determine $R_{\max}$ and $X_{o\max}$. It is natural to assume that the most reliable value of A_{dr} will correspond to the biggest $R_{\max}(X_{o\max}, A_{dr})$ value, i.e. when the correction for drift effects is the best (in the frame of the model used for drift effects for long-term CR variations). By this way will be also possible to determine the most reliable value for $X_{o\max}$ characterizing the dimension of the CR modulation region in the Heliosphere. We will base on the convection-diffusion quasi-stationary model of CR-SA hysteresis phenomenon which was described in detail in Section 2.45.3 (Eqs. 2.45.1-2.45.5), and on drift model (both these models were used in Section 2.45.4). According to the main idea of the drift mechanism (see Jokipii and Davila, 1981; Jokipii and Thomas, 1981; Lee and Fisk, 1981; Kota and Jokipii, 1999; Burger and Potgieter, 1999; Ferreira et al., 1999) we assume that the drift's CR amplitude are proportional to the value of tilt angle T and changed sign during periods of the SMF polarity reversal. Important for the cycle 22 reversal periods are: March 1981 ± 5 months and June 1991 ± 7 months. The expected drift effect according to this model for the period January 1985-December 1996 is shown in Fig. 2.45.3 for the 11-month-smoothed data of W and $A_{dr}=1\%$ at $W=75$.

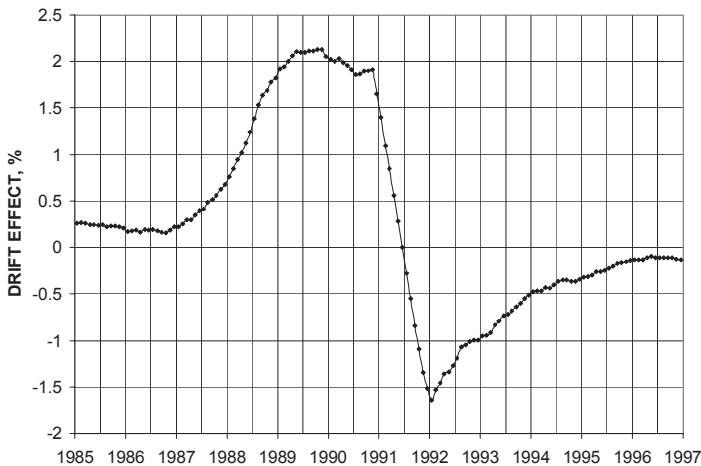


Fig. 2.45.3. An example of assumed drift modulation in cycle 22 for $A_{dr}=1\%$ at $W=75$.

Results for Climax NM data. According to the procedure described in Section 2.45.4 we correct the 11-month-smoothed data on the drift effect for different values of A_{dr} from 0% (no drift effect) up to 4% at $W=75$. The dependence of the correlation coefficient on the value of the expected time-lags is shown in Fig. 2.45.4. For each value of A_{dr} in Fig. 2.45.4 can be easily determined the value of $X_{o\max}(A_{dr})$ at which the correlation coefficient reaches a maximum value $R_{\max}$.

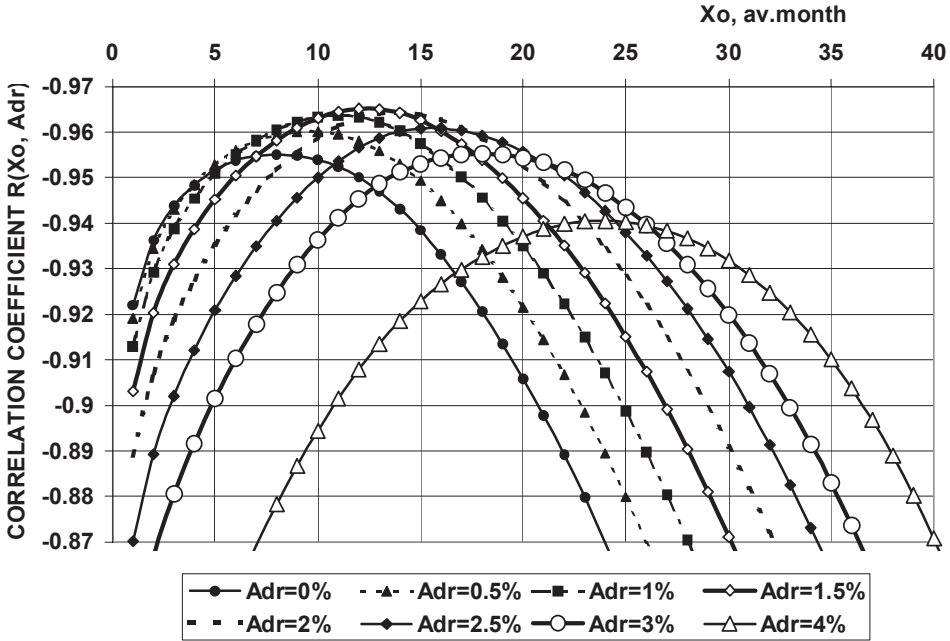


Fig. 2.45.4. Correlation coefficient $R(X_o, A_{dr})$ according to 11-month-smoothed data of Climax NM (N39,W106; height 3400 m, 2.99 GV) in Cycle 22 for different A_{dr} from 0% up to 4% at $W=75$.

The functions $R_{\max}(A_{dr})$ and $X_{o\max}(A_{dr})$ are shown in Fig. 2.45.5. The function $R_{\max}(A_{dr})$ can be approximated with correlation coefficient 0.9985 ± 0.0007 by parabola

$$R_{\max}(A_{dr}) = aA_{dr}^2 + bA_{dr} + c, \quad (2.45.7)$$

where $a = 0.004065 \pm 0.000079$, $b = -0.01253 \pm 0.00024$, and $c = -0.9551 \pm 0.0185$.

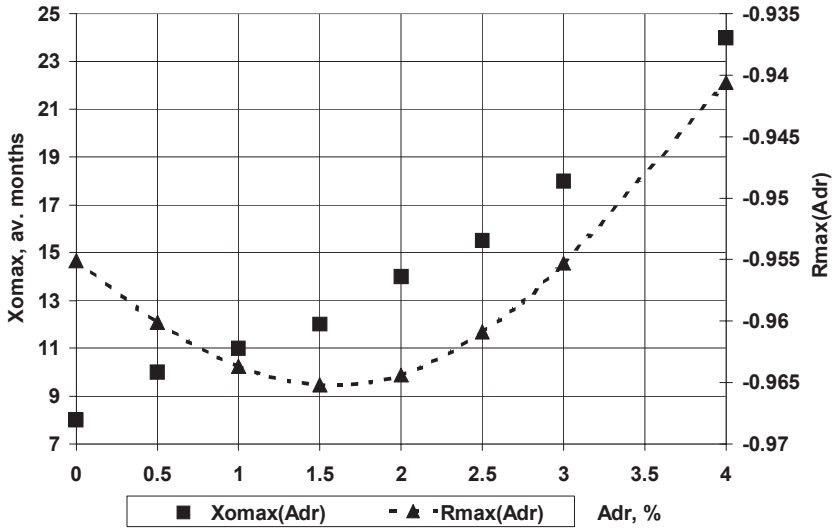


Fig. 2.45.5. Functions $R_{\max}(A_{dr})$ and $X_{o\max}(A_{dr})$ for Climax NM data in cycle 22.

From Eq. 2.45.7 we can determine $A_{dr\max}$ at which $R_{\max}$ reaches the biggest value:

$$A_{dr\max} = -b/2a, \quad (2.45.8)$$

what gives $A_{dr\max} = 1.54 \pm 0.04\%$. With this information, we can now correct the Climax NM data of cycle 22 for drifts, with the most reliable amplitude $A_{dr\max}$ according to Eq. 2.45.8 and the function $R(X_o, A_{dr\max})$ is shown in Fig. 2.45.6.

From Fig. 2.45.6 can be seen that the function $R(X_o, A_{dr\max})$ can be approximated with a correlation coefficient 0.99994 ± 0.00003 by a parabola:

$$R(X_o, A_{dr\max}) = dX_o^2 + eX_o + f, \quad (2.45.9)$$

where $d = 0.000377 \pm 0.000002$, $e = -0.00942 \pm 0.00004$, and $f = -0.906 \pm 0.004$. By Eq. 2.45.9 we can determine the most reliable value of $X_{o\max}$ corresponding to $A_{dr\max}$:

$$X_{o\max} = -e/2d, \quad (2.45.10)$$

what gives $X_{o\max} = 12.5 \pm 0.1$ av. month. At obtained values of $A_{dr\max}$ and $X_{o\max}$ the connection between expected and observed CR intensity is characterized by correlation coefficient $R_{\max}(X_{o\max}, A_{dr\max}) = 0.9652$ (see Fig. 2.45.6).

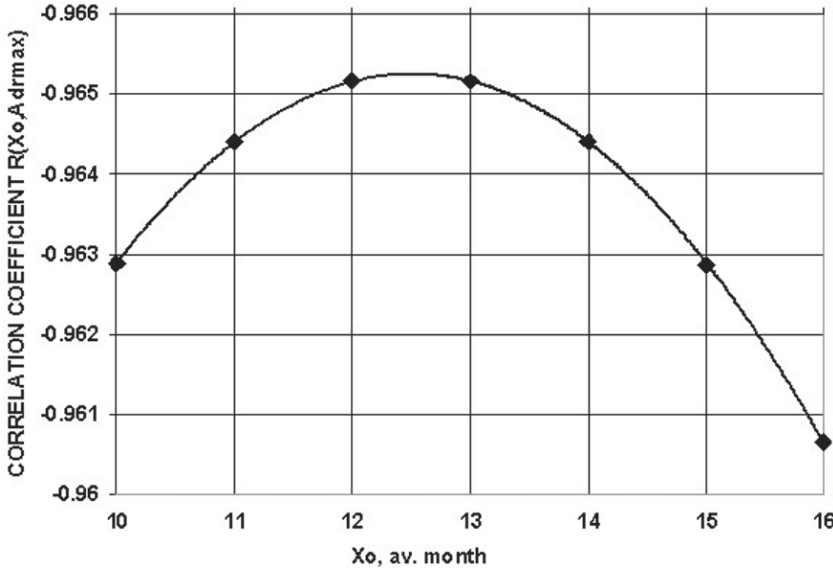


Fig. 2.45.6. The function $R(X_o, A_{dr\max})$ for Climax NM data in cycle 22.

Results for Kiel NM data. The function $R_{\max}(A_{dr})$ for Kiel NM data (sea level; $R_c = 2.32$ GV) can be approximated with a correlation coefficient 0.9992 ± 0.0004 by Eq. 2.45.7 with regression coefficients $a = 0.0095 \pm 0.0001$, $b = -0.0250 \pm 0.0004$, $c = -0.960 \pm 0.014$, what gives, according to Eq. 2.45.8, $A_{dr\max} = 1.32 \pm 0.04\%$. Next, we determine $R(X_o, A_{dr\max})$ that can be approximated with a correlation coefficient 0.99988 ± 0.00006 by Eq. 2.45.9 with a regression coefficients $d = 0.000466 \pm 0.000003$, $e = -0.01191 \pm 0.00007$, $f = -0.897 \pm 0.005$, that gives, according to Eq. 2.45.10, $X_{o\max} = 13.4 \pm 0.2$ av. months. The obtained values for $A_{dr\max}$ and for $X_{o\max}$ are about the same as for the Climax NM. In this case the correlation between the predicted and observed CR intensity is characterized by a coefficient of $R_{\max}(X_{o\max}, A_{dr\max}) = 0.977$.

Results for Tyan-Shan NM data. The Tyan-Shan NM (43N, 77E, near Alma-Ata; 3.34 km above sea level, $R_c = 6.72$ GV) is sensitive to more energetic particles than the Climax NM and the Kiel NM. For the Alma-Ata NM the function $R_{\max}(A_{dr})$ can be approximated with correlation coefficient of 0.9996 ± 0.0002 by Eq. 2.45.7, with regression coefficients $a = 0.0149 \pm 0.0015$, $b = -0.019 \pm 0.002$, $c = -0.957 \pm 0.009$, that gives $A_{dr\max} = 0.634 \pm 0.012\%$ according to Eq. 2.45.8. Next, we determined $R(X_o, A_{dr\max})$ that can be approximated with a correlation coefficient of 0.9997 ± 0.0002 by Eq. 2.45.9 with a regression coefficients $d = 0.000388 \pm 0.000004$, $e =$

-0.00845 ± 0.00005 , $f = -0.917 \pm 0.008$, that gives, according to Eq. 2.45.10, $X_{o\max} = 10.9 \pm 0.2$ av. months. In this case the correlation between the predicted and observed CR intensity is characterized by a coefficient of $R_{\max}(X_{o\max}, A_{dr\max}) = 0.963$.

Results for Huancayo/Haleakala NM data. The Huancayo NM (12S, 75W; 3.4 km above sea level, $R_c = 12.92$ GV)/ Haleakala NM (20N, 156W; 3.03 km above sea level, $R_c = 12.91$ GV) is sensitive to primary CR particles of 35–40 GV which is about 2–3 times larger than for the Climax and Kiel NM. For Huancayo/ Haleakala NM the function $R_{\max}(A_{dr})$ can be approximated with a correlation coefficient of 0.9998 ± 0.0001 by Eq. 2.45.7, with regression coefficients $a = 0.0621 \pm 0.0004$, $b = -0.0165 \pm 0.0001$, $c = -0.978 \pm 0.007$, which gives $A_{dr\max} = 0.133 \pm 0.002\%$ according to Eq. 2.45.8. Next, we determined $R(X_o, A_{dr\max})$ that can be approximated with a correlation coefficient 0.99998 ± 0.00001 by Eq. 2.45.9 with regression coefficients $d = 0.000406 \pm 0.000001$, $e = -0.00842 \pm 0.00002$, $f = -0.935 \pm 0.002$, that gives $X_{o\max} = 10.38 \pm 0.05$ av. months according to Eq. 2.45.10. In this case the correlation between the predicted and observed CR intensity is characterized by $R_{\max}(X_{o\max}, A_{dr\max}) = 0.979$.

Main results for the inverse problem for the solar cycle 22 on the basis of NM data. The taking into account drift effects (see Fig. 2.45.4) gives an important possibility, using data only for solar cycle 22, to determine the most reliable amplitude $A_{dr\max}$ (at $W=75$) and the time-lag $X_{o\max}$ (the effective time of the solar wind moving with frozen magnetic fields from the Sun to the boundary of the modulation region on the distance $r_o \approx uX_{o\max}$). We found that with an increasing effective CR primary particle rigidity from 10–15 GV (Climax NM and Kiel NM) up to 35–40 GV (Huancayo/Haleakala NM) are decreased both the amplitude of drift effect $A_{dr\max}$ (from about 1.5% to about 0.15%) and time-lag $X_{o\max}$ (from about 13 av. months to about 10 av. months). It means that in cycle 22, for the total long term modulation of CR with rigidity 10–15 GV, the relative role of the drift mechanism was $4 \times 1.5\% / 25\% \approx 1/4$ and the convection-diffusion mechanism about $3/4$ (we take into account that observed total 11-year variation in Climax and Kiel NM is 25%, and according to Fig. 2.45.3 the total change of CR intensity owed to drift effects is about 4 times more than the amplitude A_{dr}); for rigidity 35–40 GV these values were $4 \times 0.15\% / 7\% \approx 1/10$ for the drift mechanism, and about $9/10$ for the convection-diffusion mechanism. If we assume that the average velocity of the solar wind in the modulation region was about the same as the observed average velocity near the Earth's orbit in 1965–1990: $u = 4.41 \times 10^7$ cm/s = 7.73 AU/(average month), the estimated dimension of modulation region in cycle 22 will be ~ 100 AU

for CR with rigidity of 10–15 GV and about 80 AU for CR with rigidity of 35–40 GV. It means that at distances more than 80 AU the magnetic fields in solar wind and in inhomogeneities are too weak to influence intensity of 35–40 GV particles.

2.46. The inverse problem for small energy galactic CR propagation and modulation in the Heliosphere on the basis of satellite data

2.46.1. Diffusion time lag for small energy particles

As it was shown by Dorman and Dorman (1965), the time of diffusion propagation through the Heliosphere of particles with rigidity greater than 10 GV (to whom NM are sensitive) should be shorter than one month. This time is at least one order of magnitude smaller than the observed time-lag in the hysteresis phenomenon. It means that the CR long-term variation on the basis of NM data can be considered as a quasi-stationary problem with parameters of CR propagation changing with time. In this case, according to Parker (1958), Dorman (1959)

$$n(R, r_{obs}, t)/n_o(R) \approx \exp \left(-a \int_{r_{obs}}^{r_o} \frac{u(r, t) dr}{D_r(R, r, t)} \right), \quad (2.46.1)$$

where $n(R, r_{obs}, t)$ is the measured differential rigidity CR density at the time t , at the distance r_{obs} from the Sun; $n_o(R)$ is the differential rigidity density spectrum in the local interstellar medium out of the Heliosphere; $a \approx 1.5$; $u(r, t)$ is the effective solar wind velocity (taking into account also shock waves and high speed solar wind streams); and $D_r(R, r, t)$ is the radial diffusion coefficient, in dependence of the distance r from the Sun, of particles with rigidity R at the same time t of observations (if we neglect the time of diffusion through the Heliosphere). In Dorman (2003a,b) it was taken into account the time lag of processes in the interplanetary space relative to processes on the Sun, determined by the value r/u .

For small energy particles measured on satellites and balloons, it is necessary to take into account the additional time-lag $T_{dif}(R, r_{obs}, r, r_o)$ caused by the particle diffusion through the Heliosphere from r to r_{obs} . This diffusion time-lag can be approximately estimated. In Dorman et al. (1997) it was shown that in a first approximation the value u/D_r in Eq. 2.46.1 can be considered as not dependent from r , and some effective values of solar wind speed $u_{ef}(t)$ and of diffusion coefficient $D_{r,ef}(R, t)$ can be used. In this case, instead of Eq. 2.46.1, we obtain

$$n(R, r_{obs}, t)/n_o(R) \approx \exp\left(-\frac{au_{ef}(t)(r_o - r_{obs})}{D_{r,ef}(R, t)}\right). \quad (2.46.2)$$

The diffusion propagation time of CR particles with rigidity R from the distance r to the distance of observations r_{obs} can be approximately estimated as

$$T_{dif}(R, t, r_{obs}, r, r_o) \approx \frac{(r_o - r_{obs})^2 - (r_o - r)^2}{6D_{r,ef}(R, t)} \approx C(R, t) \times \frac{(r - r_{obs})(2r_o - r - r_{obs})}{u_{ef}(t)(r_o - r_{obs})}, \quad (2.46.3)$$

where

$$C(R, t) = -\frac{\ln(n(R, r_{obs}, t)/n_o(R))}{6a}. \quad (2.46.4)$$

and $D_{r,ef}(R, t)$ was determined by Eq. 2.46.2. Instead of the distances from the Sun it is possible to introduce the variables used by Dorman (2003a,b):

$$X_{obs} = r_{obs}/u_{ef}, \quad X = r/u_{ef}, \quad X_o = r_o/u_{ef}; \quad (2.46.5)$$

these variables and T_{dif} are in units of av. month = (365.25/12) days = 30.44 days = 2.628×10^6 sec. By combining Eq. 2.46.5 and Eq. 2.46.3 we obtain

$$T_{dif}(R, t, X_{obs}, X, X_o) \approx C(R, t) \times \frac{(X - X_{obs})(2X_o - X - X_{obs})}{X_o - X_{obs}}. \quad (2.46.6)$$

From Eq. (6) it follows that $T_{dif}(R, t, X_{obs}, X, X_o)$ reaches the maximum value at $X = X_o$, and the coefficient $C(R, t)$ reaches the maximum value, according to Eq. 2.46.4, at the minimum of CR intensity (near the maximum of solar activity; then

$$\frac{T_{dif}(R, t, X_{obs}, X, X_o)}{X_o - X_{obs}} \leq C(R, t). \quad (2.46.7)$$

For high and middle latitude NM data (effective particle rigidity 10-15 GV) the amplitude of 11-year modulation is about 25% and according to Eq. 2.46.4 we obtain for solar maximum $C(R, t) \approx 0.028$. It means that $T_{dif}(R, t, X_{obs}, X, X_o)/(X_o - X_{obs}) \leq 0.028$ according to Eq. 2.46.7, i.e. the diffusion time-lag is negligible in comparison with the time propagation of solar

wind from the Earth's orbit to the boundary of Heliosphere. On the basis of Burger and Potgieter (1999) we estimate $C(R, t)$ for smaller rigidities, observed on satellites. Results are shown in Table 2.46.1.

Table 2.46.1. Coefficient $C(R, t)$ for different rigidities, for periods of maximum and minimum solar activity

particle rigidity and kinetic energy	solar activity	
	MAX	MIN
3 GV (protons, 2.2 GeV)	0.107	0.067
1.0 GV (protons, 430 MeV)	0.30	0.20
0.3 GV (protons, 43 MeV)	0.55	0.41

2.46.2. Convection-diffusion modulation for small energy galactic CR particles

According to Eq. 2.46.7 and Table 2.46.1, for small energy galactic CR particles it is necessary to take into account the additional time-lag caused by the particle diffusion in the interplanetary space. In a first approximation we use the quasi-stationary model of convection-diffusion modulation described by Dorman (2003a), and here developed by taking into account the diffusion time-lag:

$$\ln(n(R, r_{obs}, t)) = A(X_o, \beta, t_1, t_2) - B(X_o, \beta, t_1, t_2) \times F\left(t, X_o, \beta, W(t - X^*)\right)_{X_{obs}}^{X_o}, \quad (2.46.8)$$

where

$$F\left(t, X_o, \beta, W(t - X^*)\right)_{X_{obs}}^{X_o} = \int_{X_{obs}}^{X_o} \left(\frac{W(t - X^*)}{W_{max}} \right)^{\frac{1}{3} + \frac{2}{3}(1 - W(t - X^*)/W_{max})} X^{-\beta} dX, \quad (2.46.9)$$

and X, X_{obs}, X_o are determined by Eq. 2.46.5, and

$$X^* = X + C(R, t) \times \frac{(X - X_{obs})(2X_o - X - X_{obs})}{X_o - X_{obs}}. \quad (2.46.10)$$

Different Approaches can be considered for small energy convection-diffusion modulation:

1-st Approach: $C(R, t) = 0$ - no diffusion time-lag. This Approach was used by Dorman (2003a,b) for NM energies; for small energies this Approach will be used for comparison.

2-nd Approach: $C(R, t) = C(R)_{av} \approx (C_{\max} + C_{\min})/2$, where $C_{\max}$ and $C_{\min}$ are listed in Table 1, and $C(R)_{av} \approx 0.087$ for 3 GV, 0.15 for 1 GV, and 0.48 for 0.3 GV are obtained.

3-rd Approach: $C(R, t)$ is determined by Eq. (2.46.4). In Fig. 2.46.1 the dependences of C from tilt angle α , calculated on the basis of results obtained in Burger and Potgieter, 1999; see also Section 2.37), are shown.

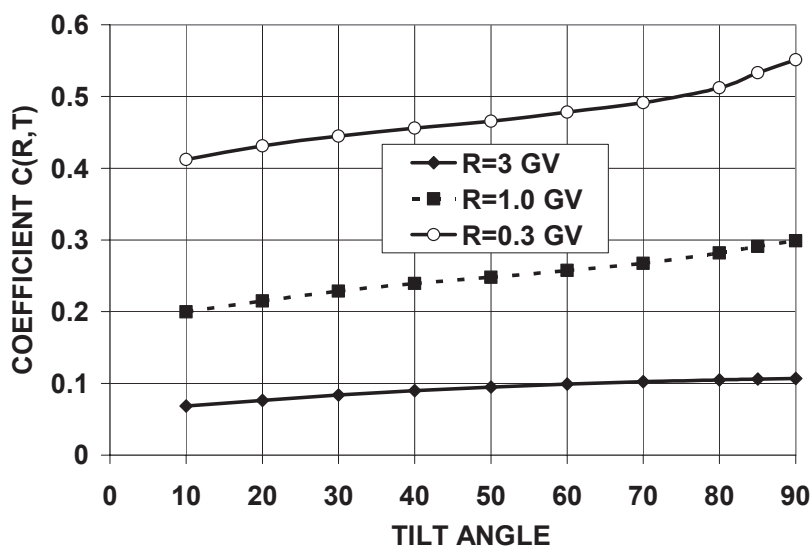


Fig. 2.46.1. The dependences of coefficient $C(R, \alpha)$ from tilt angle α for CR particles with rigidities 3, 1, and 0.3 GV. From Dorman et al. (2005c).

The dependences shown in Fig. 2.46.1 can be approximated for $R = 3$ GV by

$$C(3 \text{ GV}, T) = 0.000464\alpha + 0.0685 \quad (2.46.11a)$$

with correlation coefficient 0.978. On the basis of Eq. 2.45.6 (in Section 2.45.3), the Eq. 2.46.11 can be presented through sunspot number W as

$$C(3 \text{ GV}, W) = 0.000162 W + 0.0747. \quad (2.46.11b)$$

For 1 GV we obtain

$$C(1 \text{ GV}, \alpha) = 0.00116 \alpha + 0.190, C(1 \text{ GV}, W) = 0.000407 W + 0.206, \quad (2.46.12)$$

with correlation coefficient 0.997. For 0.3 GV it will be

$$C(0.3 \text{ GV}, \alpha) = 0.00156 \alpha + 0.394, C(0.3 \text{ GV}, W) = 0.000545 W + 0.415, \quad (2.46.13)$$

with correlation coefficient 0.980. Eq. 2.46.11a – 2.46.11b and Eq. 2.46.12 – 2.46.13 can be combined approximately as (here particles rigidity R is in GV)

$$C(R, \alpha) \approx (-3.94R + 1.63)10^{-3}\alpha - 0.142\ln(R) + 0.213, \quad (2.46.14)$$

$$C(R, W) \approx (-1.38R + 5.68)10^{-4}W - 0.148\ln(R) + 0.227. \quad (2.46.15)$$

2.46.3. Small energy CR long-term variation caused by drifts

According to the main idea of the drift mechanism (see Jokipii and Davila, 1981; Jokipii and Thomas, 1981; Lee and Fisk, 1981; Kóta and Jokipii, 1999; Burger and Potgieter, 1999; Ferreira et al., 1999), we assume that the drifts depend on the value of tilt angle α and change sign during periods of the SMF polarity reversal (see drift approach 3 according to Dorman, 2003a). We used data of tilt-angles for the period May 1976–September 1993. On the basis of these data we determined the correlation between α and W for 11 month-smoothed data as determined by Eq. 2.45.6 (in Section 2.45.3) with correlation coefficient 0.955 ± 0.013 . We assume that the drift effect is proportional to the theoretical value in dependence on tilt-angle α (or in dependence on the sunspot number W through Eq. 2.45.6) with negative sign for general solar magnetic field $A > 0$ and positive sign for $A < 0$, and in the period of reversal we suppose linear transition through 0 from one polarity cycle to another. The theoretical expected values of convection-diffusion modulation A_{cd} and drift modulation A_{dr} have been determined from Fig. 2.37.1 – 2.37.4 in Section 2.37 (from Burger and Potgieter, 1999); we assume that in these figures the average of curves for $A > 0$ and $A < 0$ characterizes the convection-diffusion modulation (which does not depend on the sign of general solar magnetic field), and the difference between these curves represents the double drift modulation (which depends on the sign of general solar magnetic field). Fig. 2.46.2 shows the drift modulation A_{dr} (relative to the intensity out of Heliosphere) for $R = 3, 1$, and 0.3 GV derived from theoretical results of Burger and Potgieter (1999) by taking into account Eq. 2.45.6 (in Section 2.45.3).

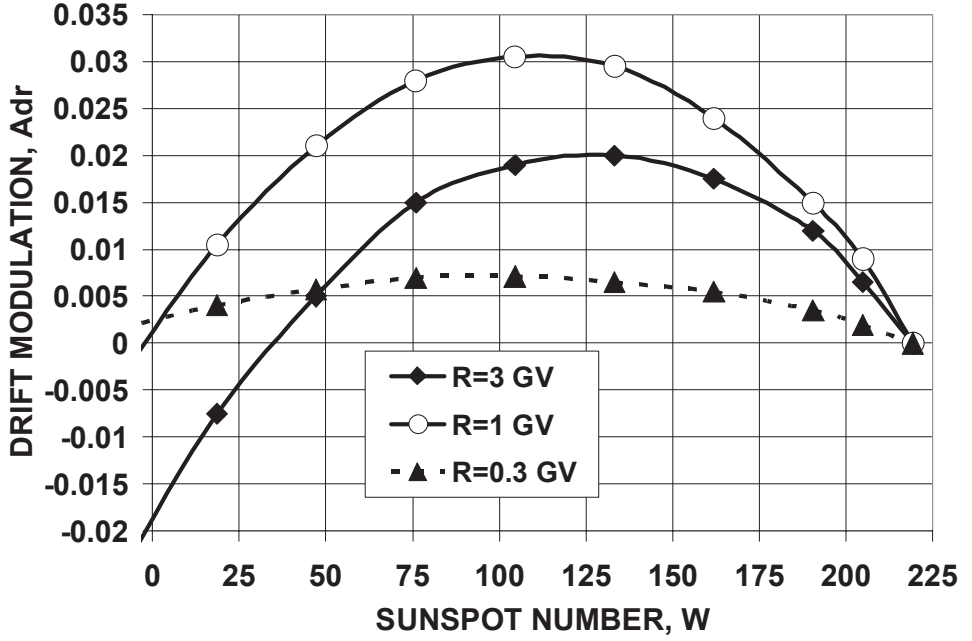


Fig. 2.46.2. Expected drift modulations for $R = 3$, 1, and 0.3 GV relative to the intensity out of the Heliosphere, in dependence of sunspot number W and derived from Burger and Potgieter (1999) by taking into account Eq. 2.45.6 (in Section 2.45.3). According to Dorman et al. (2005c).

The dependences shown in Fig. 2.46.2 can be approximated as:

$$A_{dr}(R = 3 \text{ GV}) = -2.399 \times 10^{-6} W^2 + 6.116 \times 10^{-4} W - 1.834 \times 10^{-2}, \quad (2.46.16)$$

with correlation coefficient 0.9993;

$$A_{dr}(R = 1.0 \text{ GV}) = -2.464 \times 10^{-6} W^2 + 5.379 \times 10^{-4} W + 1.414 \times 10^{-3}, \quad (2.46.17)$$

with correlation coefficient 0.9993;

$$A_{dr}(R = 0.3 \text{ GV}) = -4.826 \times 10^{-7} W^2 + 9.606 \times 10^{-5} W + 2.434 \times 10^{-3} \quad (2.46.18)$$

with correlation coefficient 0.9975. In Fig. 2.46.3 are shown ratios of A_{dr}/A_{cd} for $R = 3$, 1, and 0.3 GV, also derived from theoretical results Burger and Potgieter (1999) with account Eq. 2.45.6 (in Section 2.45.3).

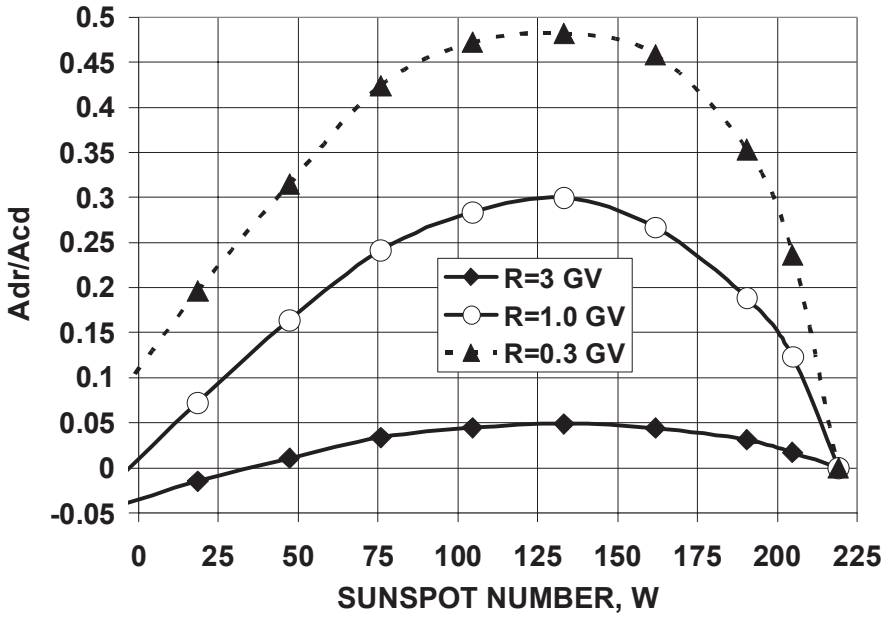


Fig. 2.46.3. The expected ratios A_{dr}/A_{cd} , for $R=3$, 1, and 0.3 GV in dependence of sunspot number W and derived from Burger and Potgieter (1999) by taking into account Eq. 2.45.6 (in Section 2.45.3). According to Dorman et al. (2005c).

The dependences shown in Fig. 2.46.3 can be approximated as (here A_{cd} is the amplitude of convection-diffusion modulation relative to the intensity out of the Heliosphere):

$$\frac{A_{dr}(R=3GV)}{A_{cd}(R=3GV)} = -5.161 \times 10^{-6} W^2 + 1.314 \times 10^{-3} W - 3.591 \times 10^{-2}, \quad (2.46.19)$$

with correlation coefficient 0.996;

$$\frac{A_{dr}(R=1.0GV)}{A_{cd}(R=1.0GV)} = -2.231 \times 10^{-5} W^2 + 5.104 \times 10^{-3} W - 2.602 \times 10^{-4}, \quad (2.46.20)$$

with correlation coefficient 0.975;

$$\frac{A_{dr}(R=0.3GV)}{A_{cd}(R=0.3GV)} = -3.278 \times 10^{-5} W^2 + 7.258 \times 10^{-3} W + 8.982 \times 10^{-2}, \quad (2.46.21)$$

with correlation coefficient 0.950.

2.46.4. The satellite proton data and their corrections on solar CR increases and jump in December 1995

We analyze the following data: IMP-8 monthly data of proton fluxes with kinetic energy $E_k \geq 106$ MeV ($R \geq 0.458$ GV) from October 1973 to December 1999 (http://data.ftcs.com/archive/imp_cpme/) and GOES daily data of proton fluxes from January 1986 to December 1999 (<http://spidr.ngdc.noaa.gov/spidr/>) with kinetic energies $E_k \geq 100$ MeV ($R \geq 0.444$ GV), $E_k \geq 60$ MeV ($R \geq 0.341$ GV), $E_k \geq 30$ MeV ($R \geq 0.239$ GV), $E_k \geq 10$ MeV ($R \geq 0.137$ GV), $E_k \geq 5$ MeV ($R \geq 0.097$ GV), as well as fluxes in intervals 60-100, 30-60, 10-30, and 5-10 MeV.

The first problem is that the original GOES data contain many increases caused by SEP events. To exclude these days we sorted daily data for each month and determined the averages from ten minimal, ten middle, and ten maximal daily values. In the present paper we used averages from ten minimal daily values for each month. Even by this method the influence of great solar energetic particle events was not totally eliminated (e.g., as in September 1989). These months have been excluded from our analysis. Then, we determined 11-months moving averages.

The second problem is that the original GOES data contain a jump in December 1995. To exclude this jump we compared GOES data for $E_k \geq 100$ MeV with IMP-8 monthly data for $E_k \geq 106$ MeV and estimated the value of jump as 0.006 proton.cm⁻².sec⁻¹.ster⁻¹. For $E_k \geq 60, \geq 30, \geq 10$, and ≥ 5 MeV, the value of the jump is 0.012, 0.025, 0.035, 0.040 proton.cm⁻².sec⁻¹.ster⁻¹, respectively.

In Fig. 2.46.4 the corrected data of IMP-8 data of proton intensities with energy $E_k \geq 106$ MeV are shown.

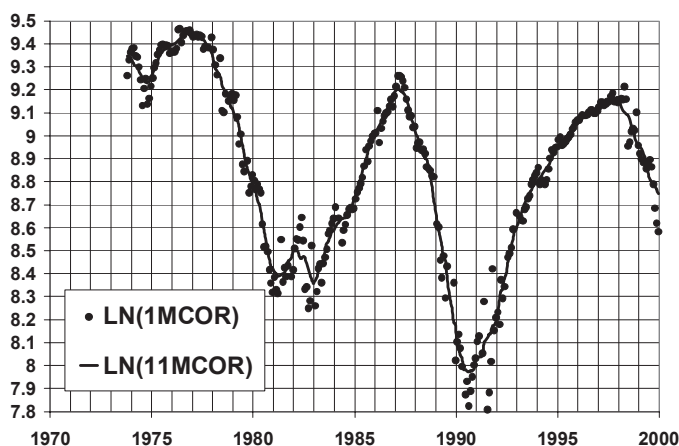


Fig. 2.46.4. Natural logarithm of monthly and 11-month moving averages IMP-8 data of proton intensities with energy $E_k \geq 106$ MeV, corrected by excluding days with increases mainly caused by SEP events. According to Dorman et al. (2005c).

In Fig. 2.46.5-2.46.7 results for GOES monthly data (averages from ten minimal daily values for each month) are shown for $E_k \geq 100$, ≥ 60 , and ≥ 30 MeV, respectively. Corrections have been applied by excluding few months with solar cosmic ray increases, and for the jump of December 1995; 11-month moving averages of obtained monthly data are also shown (to exclude big fluctuations in monthly data).

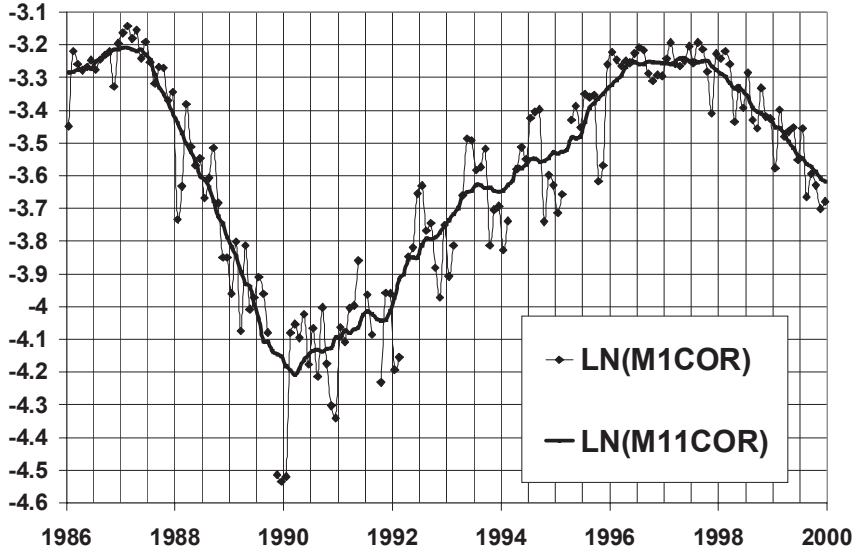


Fig. 2.46.5. Natural logarithm of monthly and 11-month moving averages obtained from GOES daily data of proton intensities with energy $E_k \geq 100$ MeV (ten minimal daily values for each month). Some months with solar CR increases are excluded. The jump of December 1995 is corrected. From Dorman et al. (2005c).

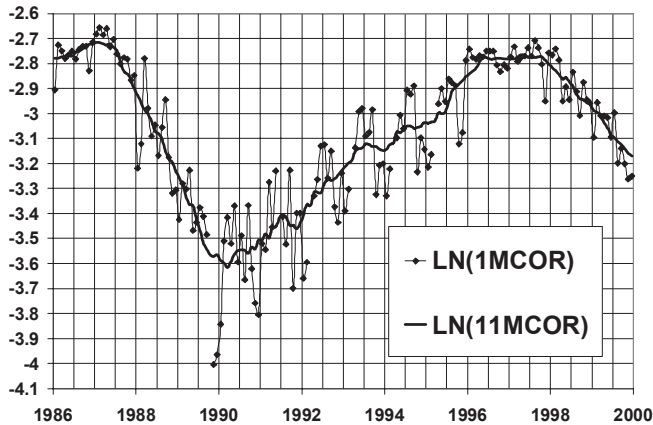


Fig. 2.46.6. The same as in Fig. 2.46.5, but for $E_k \geq 60$ MeV.

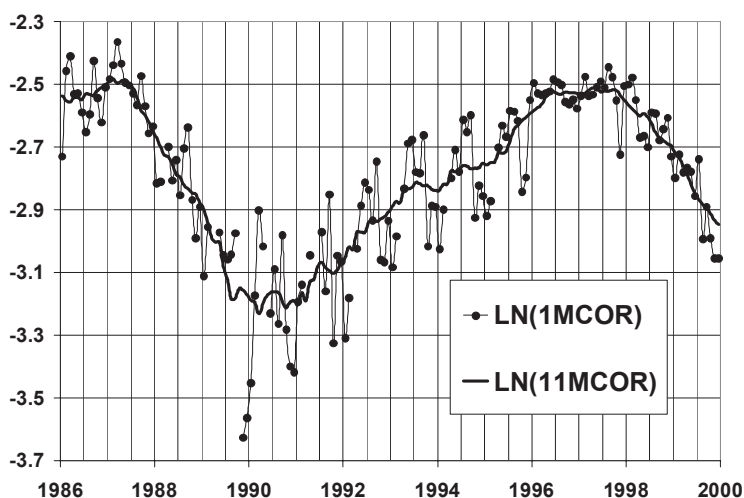


Fig. 2.46.7. The same as in Fig. 2.46.5, but for $E_k \geq 30$ MeV.

A more complicated situation is found for GOES data in narrow energy intervals: 60-100 MeV, 30-60 MeV, 10-30 MeV and 5-10 MeV. In these cases the number of months contaminated by solar CR increases is so large that it was necessary to do frequent data interpolation for excluding defect months. Moreover, after December 1995 all data have jumps, different for different energy intervals. We determined the values of jumps and applied correction on monthly data. Then, we computed 11-month moving averages and natural logarithms of monthly and 11-month moving averages of proton fluxes in different energy intervals. In Fig. 2.46.8 and 2.46.9 examples of this analysis for 60-100 MeV energy interval are shown.

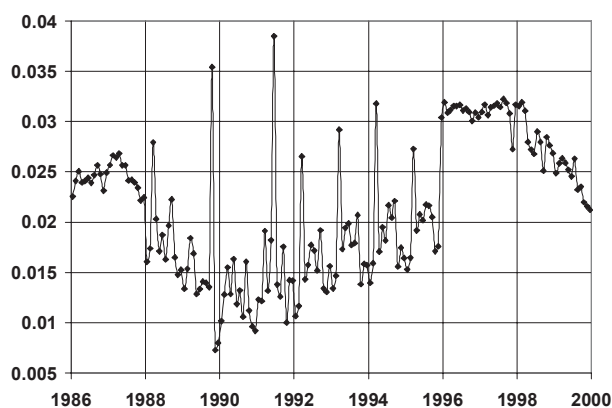


Fig. 2.46.8. Non-corrected GOES $60 \text{ MeV} \leq E_k \leq 100 \text{ MeV}$ monthly data (on the basis of 10 minimal daily values for each month). A great number of increases caused by solar CR, and a big jump in December 1995, are visible. From Dorman et al. (2005c).

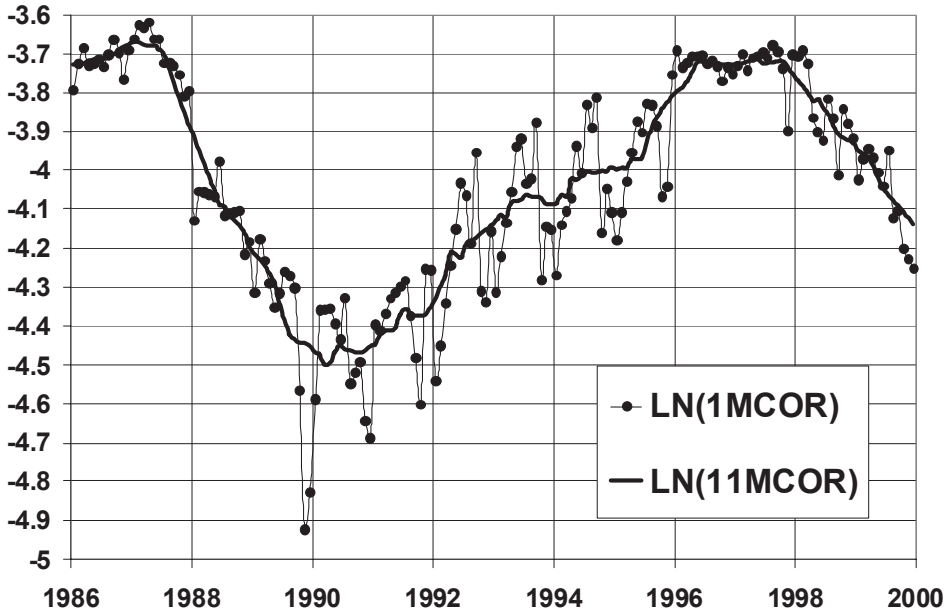


Fig. 2.46.9. Natural logarithms of corrected GOES monthly data for $60 \text{ MeV} \leq E_k \leq 100 \text{ MeV}$: full circles – LN(1MCOR). Months largely affected by solar CR increases are excluded; the missing values have been calculated by linear interpolation. Monthly data after December 1995 are corrected for the jump with amplitude 0.007. Also 11-month moving averages are shown: thick curve – LN(11MCOR). From Dorman et al. (2005c).

2.46.5. Convection-diffusion modulation and correction for drift modulation of the satellite proton data

For determining the diffusion time-lag T_{dif} we use the 2-nd Approach, in which all quantities are defined (see Section 2.46.2):

$$T_{dif} \approx C_{av}(R_{ef}) \times \frac{(X - X_{obs})(2X_o - X - X_{obs})}{X_o - X_{obs}}. \quad (2.46.22)$$

For observations near Earth's orbit ($r_{obs} \approx 1 \text{ AU}$) the value of $X_{obs} = r_{obs}/u_{ef} \ll 1 \text{ av. month}$, and in Eq. 2.46.22 we can neglect X_{obs} in comparison with $X_o = r_o/u_{dr}$ and with $X = r/u_{dr}$.

The dependence $C_{av}(R_{ef})$, estimated for several values of R_{ef} (0.3, 1, 3 and 12.5 GV), can be approximated as (with correlation coefficient 0.988):

$$C_{av}(R_{ef}) \approx 0.17 \times R_{ef}^{-0.9}. \quad (2.46.23)$$

The drift modulation expected for different values of A_{dr} can be estimated according to procedure described in Section 2.46.3.

2.46.6. Results for ≥ 106 and ≥ 100 MeV protons (IMP-8 and GOES data)

In Fig. 2.46.10 we show dependences of correlation coefficient $\Psi(X_o, A_{dr})$ between natural logarithm of 11-month moving averages IMP-8 data of proton intensities with energy $E_k \geq 106$ MeV (corrected by excluding months affected by SEP events (as shown in Fig. 2.46.4), and corrected also for drift effects with different amplitudes according to the procedure described in Section 2.46.5), with the value expected from convection-diffusion mechanism, by taking into account the diffusion time-lag (important for small energy particles observed on satellites).

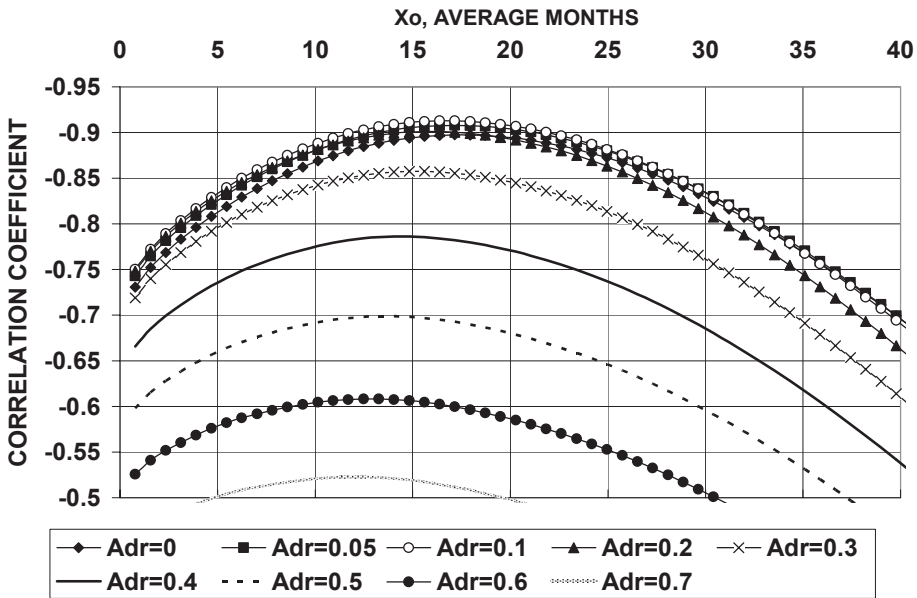


Fig. 2.46.10. Correlation coefficient $\Psi(X_o, A_{dr})$ for 11-month moving averages of IMP-8 data of proton intensities with energy $E_k \geq 106$ MeV from October 1973 to December 1999, corrected for drift modulation with different amplitudes A_{dr} from 0 to 0.7. From Dorman et al. (2005c).

From Fig. 2.46.10 it can be seen that $\Psi(X_o, A_{dr})$ reaches the greatest values for $A_{dr} \approx 0.1$ (i.e. 10%) with maximum value 0.9128 at $X_{o\max} \approx 17$ av. months, a

little bigger than the value obtained for NM data by Dorman (2001), and Dorman et al. (2001a,b). About the same result was obtained for monthly data, but with smaller values of correlation coefficient (maximum value 0.8993 at $X_{o\max} \approx 18$ av. months). In Fig. 2.46.11 is shown one result for GOES satellites. It can be seen that GOES data give about the same result as IMP-8 data, but with much bigger correlation coefficient.

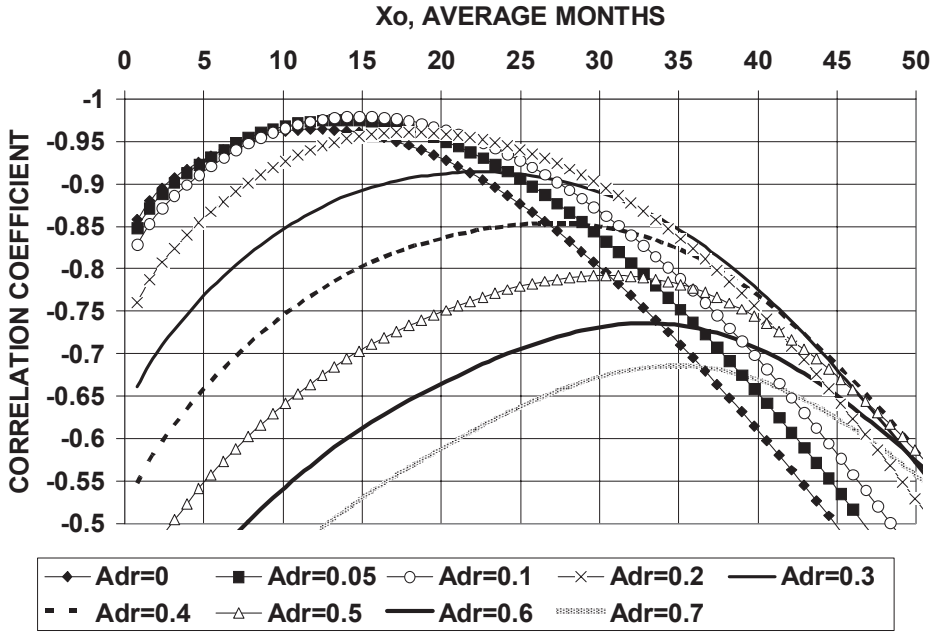


Figure 2.46.11. The same as in Fig. 2.46.10, but for GOES data of protons with energy $E_k \geq 100$ MeV from January 1986 to December 1999. From Dorman et al. (2005c).

The best correlation is found again at $A_{dr} \approx 0.1$, but with maximum value 0.9793 at $X_{o\max} \approx 15$ av. months, and regression equation

$$\ln(I_{cor}) = -3.226 - 0.0525 \times F, \quad (2.46.24)$$

where F is determined by Eq. 2.46.9. From Eq. 2.46.24 it follows that the intensity out of Heliosphere $\ln I_o = -3.226$ (for GOES data of protons with energy $E_k \geq 100$ MeV). From the obtained results, according to Le Roux and Fichtner (1997) and Dorman et al. (2001b), we can estimate the dimension of modulation region

$$r_o \approx X_{o \max} u_{ef} \approx 0.84 X_{o \max} u(1 AU) \approx 97.4 AU, \quad (2.46.25)$$

and the effective radial diffusion coefficient

$$D_r(R_{ef}) = \frac{au_{ef}^2}{0.0525} \frac{(1 AU)^2}{av.month} \approx 1.03 \times 10^{23} \frac{cm^2}{sec}. \quad (2.46.26)$$

2.46.7. The satellite alpha-particle data and their main properties

According to the procedure described in Sections 2.46.1–2.46.3 for the hysteresis analysis of small energy galactic CR fluxes, the following information is needed: kinetic energy interval, rigidity interval and effective rigidity R_{ef} . These parameters are necessary for determining the coefficient $C_{av}(R_{ef})$, according to Eq. 2.46.23, and diffusion time lag, T_{dif} according to Eq. 2.46.22, for evaluating v/c interval and effective value $(v/c)_{ef}$ (for estimating the effective transport path of particles in the Heliosphere after the determination of diffusion coefficient). We used 5-minute GOES data of small energy alpha-particle fluxes (in units $particles \cdot cm^{-2} \cdot sec^{-1} \cdot sr^{-1} \cdot MeV^{-1}$) from January 1986 to May 2000 in three energy intervals with parameters listed in Table 2.46.2 (<http://spidr.ngdc.noaa.gov/spidr/>).

Table 2.46.2. Parameters for three used alpha-particles energy intervals.

E_k interval, MeV	R interval, GV	R_{ef} , GV	$C_{av}(R_{ef})$	$(v/c)_{ef}$
60–160	0.337–0.554	0.45	0.347	0.23
160–260	0.554–0.710	0.63	0.257	0.32
330–500	0.804–1.000	0.90	0.186	0.43

2.46.8. Results for alpha-particles in the energy interval 330–500 MeV

This energy interval is expected to have the lower influence from solar CR events. Therefore, the corrected GOES data (by excluding sudden increases caused by solar energetic particle effects) reflect better the long-term modulation of galactic CR. In Fig. 2.46.12 the monthly and 11-month running averages of corrected data are shown.

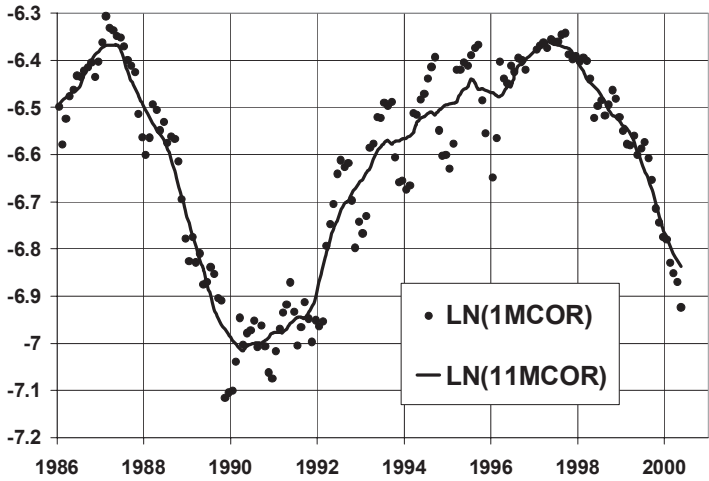


Fig. 2.46.12. Natural logarithm of monthly (as LN(1MCOR)) and 11-month moving averages (as LN(11MCOR)) of corrected GOES data of alpha-particle fluxes in the energy interval 330-500 MeV, during January 1986-May 2000. From Dorman et al. (2005c).

Then, we calculate the expected drift modulation for different A_{dr} from 0 (no drift modulation) up to 0.4 (i.e., 40%) on the basis of monthly data on tilt-angles T and sunspot numbers W , according to the procedure described by Dorman (2003a), and already used in Sections 2.46.4-2.46.6 for satellite proton data. In Fig. 2.46.13 the expected drift modulation for $A_{dr} = 0.2$ is shown as an example.

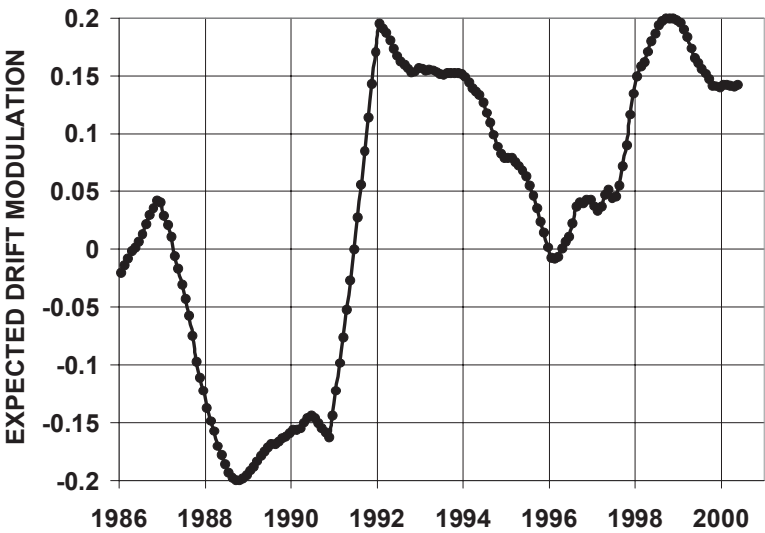


Fig. 2.46.13. Expected drift modulation for $A_{dr} = 0.2$. From Dorman et al. (2005c).

The next step is the correction of observed data for drift modulation at different values of A_{dr} (an example is shown in Fig. 2.46.14).

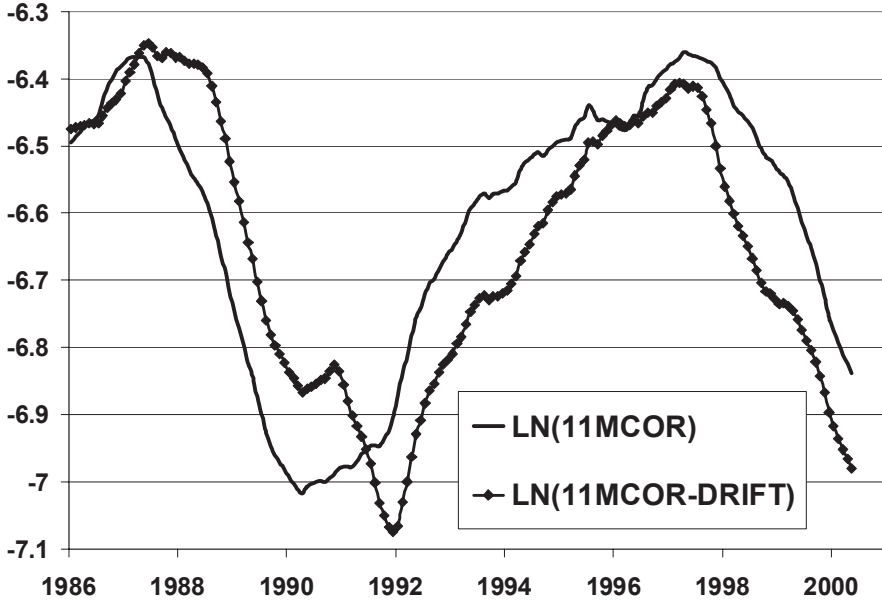


Fig. 2.46.14. Natural logarithm of 11-months running averages of alpha-particle fluxes for the energy interval 330-500 MeV (as LN(11MCOR), derived from Fig. 2.46.12) and corrected for drift at $A_{dr} = 0.2$ (expected clean convection-diffusion modulation, plotted as LN(11MCOR-DRIFT)). From Dorman et al. (2005c).

We compare the alpha-particle fluxes corrected for drift at different A_{dr} (expected clean convection-diffusion modulation) with values

$$F(t, X_o, R_{ef}) = \int_{X_{obs}}^{X_o} \left(\frac{W(t - X^*)}{W_{max}} \right)^{\frac{1}{3} + \frac{2}{3}(1 - W(t - X^*)/W_{max})} dX, \quad (2.46.27)$$

where X, X_{obs}, X_o are determined by Eq. 2.46.5, and, in the frame of the 2-nd Approach,

$$X^* = X + C_{av}(R_{ef})X(2 - X/X_o), \quad (2.46.28)$$

where coefficients $C_{av}(R_{ef})$ are given in Table 2.46.2. The Eq. 2.46.27 accounts for the time lag of interplanetary processes relative to processes on the Sun, and for

the diffusion time lag T_{dif} (which is especially important for small energy galactic alpha-particles detected on satellites). This comparison has been done for different values of X_o (characterizing the time propagation of solar wind from the Sun to the boundary of Heliosphere, in units av. month = $365.25/12 = 30.44$ days), in the frame of linear regression

$$\ln(I_{11MCOR}(t) - I_{dr}(A_{dr}, t)) = A(R_{ef}, A_{dr}, X_o) - B(R_{ef}, A_{dr}, X_o)F(t, X_o, R_{ef}), \quad (2.46.29)$$

and evaluating the correlation coefficients $\Psi(R_{ef}, A_{dr}, X_o)$, and corresponding regression coefficients $A(R_{ef}, A_{dr}, X_o)$ which determine the CR intensity out of the Heliosphere, and $B(R_{ef}, A_{dr}, X_o)$ characterizing the effective diffusion coefficient in the interplanetary space. The values of correlation coefficient $\Psi(R_{ef}, A_{dr}, X_o)$ are shown in Fig. 2.46.15 for different values of A_{dr} obtained for the period of solar cycle 22 which is totally covered by the used GOES data on alpha-particle fluxes.

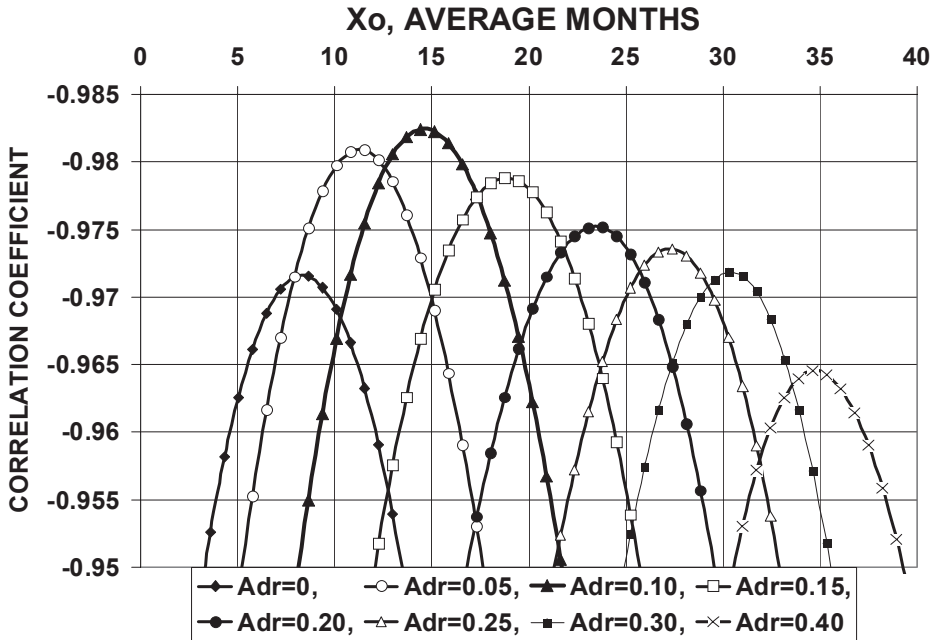


Fig. 2.46.15. Correlation coefficient $\Psi(R_{ef}, A_{dr}, X_o)$ in dependence from X_o at different values of drift modulation amplitude A_{dr} from 0 (no drift correction) to 0.40 for alpha-particle fluxes in energy interval 330-500 MeV during solar cycle 22. From Dorman et al. (2005c).

From Fig. 2.46.15 it can be seen that with increasing of A_{dr} from 0 to 0.40 the maximum of correlation coefficient changes from about 8 av. months to 35 av. months. The value of maximum of correlation coefficients $\Psi(R_{ef}, A_{dr}, X_o)$ increases with increasing A_{dr} up to $A_{dr} = 0.10$, then it decreases. The curves of Fig. 2.46.15 can be approximated by parabolas with correlation coefficients higher than 0.999:

$$\Psi(R_{ef}, A_{dr}, X_o) = a(R_{ef}, A_{dr})X_o^2 + b(R_{ef}, A_{dr})X_o + c(R_{ef}, A_{dr}), \quad (2.46.30)$$

where coefficients a , b , and c are given in Table 2.46.3 together with $X_{o\max} = -2b(R_{ef}, A_{dr})/a(R_{ef}, A_{dr})$ and $\Psi_{\max}$ determined on the basis of Eq. 2.46.30 at $X_o = X_{o\max}$:

$$\Psi_{\max}(R_{ef}, A_{dr}) = 2b^2(R_{ef}, A_{dr})/a(R_{ef}, A_{dr}) + c(R_{ef}, A_{dr}). \quad (2.46.31)$$

Table 2.46.3. Coefficients a , b , and c , and values of $X_{o\max}$ (in av. months) and maximal correlation coefficient $\Psi_{\max}$ for different values of A_{dr} from 0 (no drift corrections) up to 0.40. From Dorman et al. (2005c).

A_{dr}	a	b	c	$X_{o\max}$	$\Psi_{\max}$
0	0.000821	-0.0137	-0.914	8.35	-0.972
0.05	0.000828	-0.0187	-0.875	11.30	-0.981
0.10	0.000707	-0.0207	-0.830	14.66	-0.982
0.15	0.000604	-0.0228	-0.764	18.84	-0.979
0.20	0.000604	-0.0282	-0.645	23.38	-0.975
0.25	0.000710	-0.0387	-0.447	27.24	-0.973
0.30	0.000788	-0.0478	-0.247	30.33	-0.972
0.40	0.000758	-0.0528	-0.046	34.81	-0.964

In Fig. 2.46.16 the dependences of $X_{o\max}$ and $\Psi_{\max}$ from A_{dr} in the vicinity of the highest $\Psi_{\max}$ values are shown.

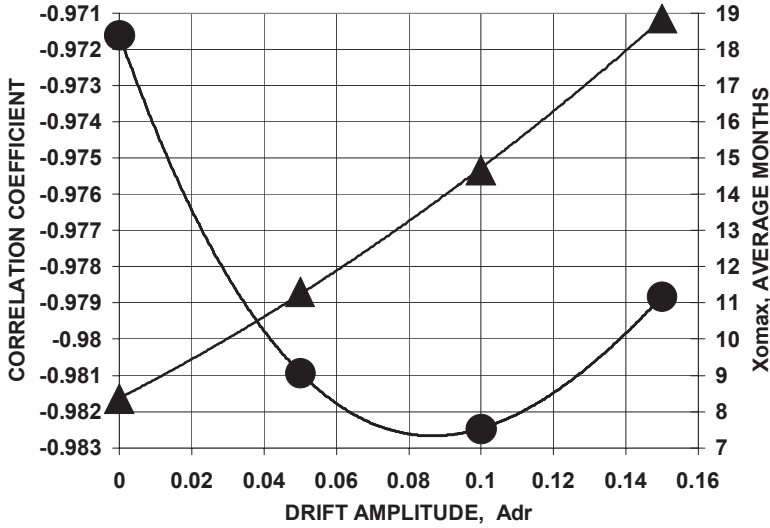


Fig. 2.46.16. The dependences of correlation coefficient $\Psi_{\max}$ (circles, left scale)) and $X_{o\max}$ (triangles, right scale) from drift amplitude A_{dr} . From Dorman et al. (2005c).

The dependences shown in Fig. 2.46.16 can be approximated as

$$\Psi_{\max} = -3.486A_{dr}^3 + 2.081A_{dr}^2 - 0.282A_{dr} - 0.972, \quad (2.46.32)$$

$$X_{o\max} = 123.13A_{dr}^2 + 51.23A_{dr} + 8.367. \quad (2.46.33)$$

From Eq. 2.46.32 and Eq. 2.46.33 we can determine the optimal values of A_{dr} and $X_{o\max}$ for which the correlation coefficient is the highest (-0.98275):

$$(A_{dr})_{opt} = 0.087, (X_{o\max})_{opt} = 13.76 \text{ av. months}. \quad (2.46.34)$$

2.46.9. Main results of the inverse problem solution for satellite alpha-particles

According to direct measurements on space probes the average solar wind speed for the period 1965-1990 near the Earth's orbit at $r=1\text{AU}$ was $u_1 = 4.41 \times 10^7 \text{ cm/sec} = 7.73 \text{ AU/av. month}$. The function $u(r)$ is determined by solar wind interactions with galactic CR and anomalous CR component, with neutral

atoms penetrating from interstellar space and others. According to calculations of Le Roux and Fichtner (1997), the change of solar wind speed with the distance r from the Sun can be described approximately as

$$u(r) \approx u_1(1 - b(r/r_{\text{tsw}})), \quad (2.46.35)$$

where r_{tsw} is the distance to the terminal shock wave and parameter $b \approx 0.13 \div 0.45$ depends on sub-shock compression ratio and on injection efficiency of pickup protons. On the basis of Eq. 2.46.35 we can determine the radius of CR modulation region r_{mod} from equation:

$$(X_{\text{omax}})_{\text{opt}} = \int_0^{r_{\text{mod}}} (u_1(1 - br/r_{\text{tsw}}))^{-1} dr = -r_{\text{tsw}} \ln(-b + r_{\text{mod}}/r_{\text{tsw}})/(bu_1), \quad (2.46.36)$$

which gives:

$$r_{\text{mod}} = r_{\text{tsw}} \left(b + \exp(-(X_{\text{omax}})_{\text{opt}} bu_1 / r_{\text{tsw}}) \right). \quad (2.46.37)$$

By assuming that the radius of modulation region r_{mod} for alpha-particle GOES data (effective rigidity 0.9 GV according to Table 2.46.2) is about the same as the radius of the Heliosphere r_{tsw} , we obtain from Eq. 2.46.37 for $r_{\text{mod}} = r_{\text{tsw}} = r_o$ the following result:

$$r_o = -bu_1(X_{\text{omax}})_{\text{opt}} / \ln(1 - b). \quad (2.46.38)$$

For the most reliable value of $b \approx 0.3$ we obtain from Eq. 2.46.34 and Eq. 2.46.38

$$r_o \approx 13.76 \times 7.73 \times 0.84 \approx 90 \text{ AU}. \quad (2.46.39)$$

We can compute regression coefficients A (indicating the natural logarithm of CR alpha-particle intensity out of the Heliosphere) and B (indicating the diffusion coefficient) in Eq. 2.46.29 for the values obtained by Eq. 2.46.34:

$$A = -6.3531, B = -0.03483(\text{av. month})^{-1}. \quad (2.46.40)$$

Since

$$B(X_{\text{omax}})_{\text{opt}} = au_{\text{av}} r_o / D_r(R_{\text{ef}}), \quad (2.46.41)$$

and according to Eq. 2.46.38

$$u_{av} = -bu_1/\ln(1-b) \approx 0.84u_1, \quad (2.46.42)$$

we obtain for the radial effective diffusion coefficient

$$\begin{aligned} D_r(R_{ef}) &= au_{av}^2/B \approx 0.707 au_1^2/B \approx 1530 AU^2/\text{av. month} \\ &\approx 1.31 \times 10^{23} \text{ cm}^2/\text{sec} \end{aligned} \quad (2.38.43)$$

and for the effective transport path (by using Table 2.46.2)

$$\Lambda_r(R_{ef}) \approx 3.05 \times 10^{13} \text{ cm}. \quad (2.46.44)$$

It should be noted that if the diffusion time lag is not taken into account, the result $(X_{o\max})_{opt} \approx 19$ av. months for alpha-particles in the energy interval 330-500 MeV will be obtained instead of the value described by Eq. 2.46.34, in contradiction with results based on satellite proton data and on NM data (Dorman, 2001; Dorman et al., 2001a,b).

Preliminary results, obtained for satellite alpha-particles data for other energy intervals 160-260 MeV and 60-160 MeV (values of $(A_{dr})_{opt}$ and $(X_{o\max})_{opt}$) show that in these cases the influence of SEP events was not totally excluded, and that there is a necessity of additional data cleaning.

2.46.10. Peculiarities in the solution of the inverse problem for small energy CR particles

The specific aspects in the solution of the inverse problem for small energy CR particles for long-term variations caused by propagation and modulation in the Heliosphere (convection-diffusion and drift processes) are the following:

- remarkable diffusion time-lag, increasing with decreasing particle energy, and
- remarkable drift modulation, whose relative role is also increasing with decreasing particle energy.

The obtained results for convection-diffusion and drift modulations have been used for the analysis of proton and alpha-particle satellite data. The results shown in Fig. 2.46.7 and Fig. 2.46.8 lead to the following conclusions:

- The procedure described here to obtain the expected convection-diffusion modulation, by taking into account the additional diffusion time lag in Heliosphere for small energy particles observed on satellites, and also by taking into account drift modulation based on Burger and Potgieter (1999),

can be used to describe the long-term variation of galactic CR intensity at small energies;

- The procedure described above for excluding solar energetic particle events from satellite data for particles with energies $E_k \geq 106$ MeV and $E_k \geq 100$ MeV – made it possible to obtain from satellite data information on real long-term variation of galactic small-energy CR intensity, which can be compared with theoretical expectations;
- The $X_{o\max}$ values obtained from this comparison, $A_{dr\max}$, the dimension of modulation region and the effective radial diffusion coefficient are in good agreement with those obtained by Dorman (2001), Dorman et al. (2001a,b) on the basis of NM data, and with those expected by Burger and Potgieter (1999). This means that the dimension of modulation region is very close to the dimension of the Heliosphere.
- GOES data for small energy intervals 60-100 MeV, 30-60 MeV, and others, have been also analyzed. A contradictory dependence of the determined $X_{o\max}$ and $A_{dr\max}$ on the energy has been obtained.

We think that the applied procedure for excluding events of CR increases for narrow small energy intervals is not enough accurate: these data still reflect an appreciable contribution of solar CR (without time-delay and with about opposite phase to variation of galactic CR) which leads to an appreciable decrease in the observed modulation and even to a change in phase. This could be the main reason for the contradictory determination of $X_{o\max}$ and A_{dr} . It is necessary to develop more effective procedure for excluding the local CR influence on small energy particle intensity variation observed by satellites. To do that it will be important to use also satellite data of isotopes which are not contaminated by small energy solar CR.

Chapter 3

Nonlinear Cosmic Ray Effects in Space Plasmas

3.1. The important role of nonlinear CR effects in many processes and objects in space

At the foundation of nonlinear CR effects in space plasma are two phenomena:

1) influence of CR pressure on plasma dynamics; this was first considered by Axford and Newman (1965) for solar wind propagation, and then by Dorman and Dorman (1968a,b, 1969), Dorman (M1975a,b), Babayan and Dorman (1977a,b, 1979a,b,c, 1981, 1990), Dorman (1995a,b, 1996);

2) kinetic stream instability of anisotropic CR and generation of Alfvén turbulence; this was first considered by Ginzburg (1965) for CR propagation in the interstellar medium, and then by Wentzel (1974), Cesarsky (1980), Babayan et al. (1987), Zirakashvili et al. (1991, 1993), Dorman (1995a,b, 1996).

The nonlinear CR effects are important (see the review in Dorman, 1995a,b; Dorman, 1996):

1. in our Galaxy and other galaxies (galactic wind driven by CR and its influence on CR propagation, chemical composition, and energy spectrum formation);

2. in the outer Heliosphere (dynamic effects of galactic CR pressure on solar wind and interplanetary shock waves propagation, on the formation of the terminal shock wave and the boundary of the Heliosphere, Alfvén turbulence generation by kinetic stream instability of non-isotropic CR fluxes and its influence on CR propagation and modulation);

3. in CR and gamma ray sources (influence of CR pressure and stream instability of escaping particles on acceleration efficiency and formation of energy spectrum and chemical composition of escaping particles, influence of nonlinear effects on gamma-ray emissivity distribution);

4. in the processes of CR acceleration by shock waves and in regions of magnetic field reconnection (inverse influence of pressure and stream instability of accelerated particles on the structure and propagation of shock waves, on processes of reconnection, on formation of accelerated particles energy spectrum and chemical composition).

3.2. Effects of CR pressure

The approximation of CR motion in electro-magnetic fields generally used is not valid in general. In many cases when CR density energy is comparable to the energy density of magnetic fields and kinetic energy of moving plasma, the inverse influence of CR on space plasma dynamics and electro-magnetic field structure are important. As was shown by Ptuskin (1984) on the basis of CR kinetic equation that in the diffusion approximation the ponderomotive force from CR particles on plasma with a stochastic magnetic field is determined by the pressure of these particles (Berezinsky et al., M1990):

$$P_c = \frac{4\pi}{3} \int v p^3 f_o(p) dp, \quad (3.2.1)$$

where v and p are the velocity and momentum of CR particles and $f_o(p)$ is the CR distribution function. The CR pressure P_c is connected with CR total energy density

$$W_c = 4\pi \int dp p^2 E(p) f_o(p), \quad (3.2.2)$$

where $E(p)$ is the kinetic energy of particle with momentum p . As was shown by Ptuskin (1984),

$$\frac{\partial P_c}{\partial W_c} = \frac{1}{3} \left\{ 1 - \frac{1}{3(W_c + P_c)} \sum_i m_i c^2 \int_{m_i c^2}^{\infty} dE \frac{2E^2 + m_i^2 c^4}{E^3} N_i(E) \right\}, \quad (3.2.3)$$

where

$$N(E) = (4\pi p^2 / v) f_o(p) \quad (3.2.4)$$

is the differential energy spectrum and index i shows the sort of CR particles. For

$$N(E) \propto E^{-\gamma}, \quad (3.2.5)$$

Eq. 3.2.1–3.2.3 give

$$P_c = \frac{2}{3\gamma} W_c; \quad \frac{\partial P_c}{\partial W_c} = \frac{2(5\gamma + 6)}{3(\gamma + 2)(3\gamma + 2)}. \quad (3.2.6)$$

For $\gamma = 2.7$ this gives $P_c \approx 0.25 W_c$ and $\partial P_c / \partial W_c \approx 0.27$. For ultra-relativistic gas ($E \gg m_i c^2$) it will be $P_c = (1/3) W_c$ and $\partial P_c / \partial W_c = 1/3$. Let us note that if the gradient of CR pressure is zero then the ponderomotive forces on the elemental volume of space plasma from all directions will be the same and therefore the resultant ponderomotive force acting on the elemental volume will be zero. CR pressure gradient influenced on the dynamic of space plasma according to the set of hydrodynamic equations:

$$\frac{\partial \rho}{\partial t} + \nabla(\rho \mathbf{u}) = 0, \quad (3.2.7)$$

$$\rho \left(\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \nabla) \mathbf{u} \right) = -\nabla P_g - \nabla P_c + \frac{1}{4\pi} [\nabla \times \mathbf{H}, \mathbf{H}], \quad (3.2.8)$$

$$\frac{\partial \mathbf{H}}{\partial t} = \nabla \times [\mathbf{u}, \mathbf{H}], \quad (3.2.9)$$

$$\nabla \cdot \mathbf{H} = 0, \quad (3.2.10)$$

where $\mathbf{u}$ is the velocity of matter ($|\mathbf{u}| \ll c$) and P_g is the gas-dynamical pressure of plasma.

3.3. Effects of CR kinetic stream instability

The importance of CR kinetic stream instability effects for CR acceleration and propagation was first noted by Ginzburg (1965), and then by Wentzel (1974) and Cesarsky (1980). This type of instability is well known in plasma physics (Tsytovich, M1971; Vedenov and Rjutov, 1972; Akhiezer et al., M1974; Artsimovich and Sagdeev, M1979). The CR kinetic stream instability generates a broad spectrum of waves in space plasma, but generation of high-frequency waves (Langmuir and whistler types) is not effective because of a big damping of these waves; moreover, these waves are not effective for CR particle scattering. Another situation is for magneto-hydrodynamic waves that are effective for CR scattering. The growth rate is the largest for waves propagated along magnetic field. The main interaction between CR particles and these waves is based on cyclotron resonant on the first harmonic; the growth rate $\Gamma_c(k)$ will be determined by equation (Berezinsky et al., M1990):

$$\Gamma_c(k) = \frac{Z^2 p^2 e^2 V_a^2}{2 c^2} \iint p^2 dp d\mu (1 - \mu^2) v \times \left\{ \frac{\partial f(p, \mu)}{\partial p} + \left(\frac{kv}{\omega_a(k)} - \mu \right) \frac{1}{p} \frac{\partial f(p, \mu)}{\partial p} \right\} \delta \left(kv\mu + \frac{ZeHc}{E} \right) \delta \left(kv\mu - \frac{ZeHc}{E} \right). \quad (3.3.1)$$

Here μ is the cosine of particle's pitch-angle, $\omega_a(k)$ is the frequency of an Alfvén wave with wave number $k = 2\pi/\lambda$. Let us suppose that the CR distribution function is characterized by isotropic part $f_o(p)$ and some small anisotropy with amplitude A which implies:

$$f(p, \mu) = f_o(p)(1 + A\mu) = f_o(p) - \frac{U_o}{v} p \frac{\partial f_o(p)}{\partial p} \mu. \quad (3.3.2)$$

In Eq. 3.3.2 the value

$$U_o = -(vf_o(p)A)/(p\partial f_o(p)/\partial p) \quad (3.3.3)$$

is the effective velocity of the observer relative to the system of coordinate with CR isotropic distribution function $f_o(p)$ to obtain anisotropy with amplitude A . If we take into account only resonance scattering with

$$p_{\text{res}} = \left| \frac{ZeH}{ck} \right| = \left| \frac{Zm_i \omega_{Hj}}{Z_i k} \right|, \quad (3.3.4)$$

where Z_i and ω_{Hi} are the charge and ion gyro-frequency of background plasma. Then according to Eq. 3.3.1 and Eq. 3.3.2 it will be

$$\Gamma_c(k) = \frac{\gamma - 1}{\gamma} \frac{\pi}{4} \left| \frac{Z\omega_{Hi}}{Z_i} \right| \frac{N_c(> p_{\text{res}})}{n_i} \left(\frac{U_o}{V_a} - 1 \right), \quad (3.3.5)$$

where

$$N_c(> p) = 4\pi \int_p^\infty p^2 dp f_0(p) \propto p^{-\gamma+1} \quad (3.3.6)$$

is integral CR momentum spectrum and n_i is the plasma density. Eq. 3.3.5 shows that $\Gamma_c(k) > 0$ for $U_o > V_a$. This condition is necessary for the development of CR stream instability and the generation of Alfvén turbulence.

For the spectrum described by Eq. 3.3.6 the increment $\Gamma_c(k) \propto k^{\gamma-1}$, but if the spectrum has a maximum at $p = p^*$ then for $p < p^*$ (i.e. for $k > k^*$) the increment $\Gamma_c(k) \propto k^{-1}$. Here it is assumed that $\Gamma_c(k) \ll \omega(k)$, i.e. that $AW_c \ll H_o^2/4\pi$. Usually this condition is valid because the amplitude of the anisotropy $A \leq 0.1$ and CR energy density $W_c \leq H_o^2/4\pi$ (in the Galaxy, in the main part of Heliosphere, in the processes of particle acceleration by shock waves, and in regions of magnetic field reconnection). Perhaps only in very powerful compact sources of CR can the accelerated particle anisotropy and energy density be so high that it becomes necessary to consider a stronger approximation.

3.4. On the structure and evolution of nonlinear CR-space plasma systems

3.4.1. Principles of hydrodynamic approach to the CR-space plasma nonlinear system

As was considered in Chapters 1 and 2, and in the previous Sections 3.2 and 3.3, CR interact with thermal plasma via magnetic clouds, hydromagnetic irregularities, and hydromagnetic waves in the plasma. Scattered by the magnetic irregularities (mostly by gyro-resonant scattering), CR propagate and diffuse through the plasma. CR acquire energy from the plasma if the plasma flow is systematically converging. This process is called the first order Fermi acceleration. Because CR are anisotropic their interaction with the plasma excite hydromagnetic waves via streaming instability. When waves of different phase velocities are present, CR diffuse in the momentum space also. This is called the second order Fermi acceleration, or stochastic acceleration. The system is self-consistent and is called a CR-plasma system (Ko, 1999).

As one can imagine solving the system in distribution function approach is very difficult (see e.g., Malkov, 1997a,b). On other hand, basing on the papers of Drury and Völk (1981), Axford, Leer, and McKenzie (1982), McKenzie and Völk (1982), Ko (1992), authors Jiang et al. (1996), Ko et al. (1997), Ko (1998, 1999) came to conclusion that the hydrodynamic approach is a fairly good approximation for studying the structure and evolution of any CR-plasma system. In this approach every component is considered as a fluid. For instance, CR and waves are treated as massless fluids but with significant energy density and pressure. For example, Ko (1999) consider a four-fluid model which comprises the thermal plasma, CR, and two oppositely propagating Alfvén waves. It was shown that in general there are three energy exchange mechanisms: 1) work done by plasma flow, 2) CR streaming instability, and 3) stochastic acceleration. In Ko

(1999) are presented several steady state profiles of the CR-plasma system which demonstrate the interplay between these three energy exchange mechanisms.

3.4.2. Four-fluid model for description CR-plasma system

Ko (1992) proposed a fairly comprehensive version of the hydrodynamic approach. That is a four-fluid model which comprises thermal plasma, CR and two oppositely propagating Alfvén waves. The governing equations are the total mass and momentum equations, and energy equations of various components (i.e., kinetic energy and thermal energy of plasma, CR energy and wave energies). In the one dimensional approximation with magnetic field parallel to the plasma flow, these equations will be

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} = 0, \quad (3.4.1)$$

$$\rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial x} = -\frac{\partial}{\partial x} (P_{th} + P_c + P_w^+ + P_w^-), \quad (3.4.2)$$

$$\frac{\partial E_k}{\partial t} + \frac{\partial E_k}{\partial x} = -u \frac{\partial}{\partial x} (P_{th} + P_c + P_w^+ + P_w^-), \quad (3.4.3)$$

$$\frac{\partial E_{th}}{\partial t} + \frac{\partial E_{th}}{\partial x} = u \frac{\partial P_{th}}{\partial x}, \quad (3.4.4)$$

$$\frac{\partial E_c}{\partial t} + \frac{\partial E_c}{\partial x} = (u + (e_+ - e_-)V_a) \frac{\partial P_c}{\partial x} + \frac{P_c}{\tau}, \quad (3.4.5)$$

$$\frac{\partial E_w^\pm}{\partial t} + \frac{\partial E_w^\pm}{\partial x} = u \frac{\partial P_w^\pm}{\partial x} \mp e_\pm V_a \frac{\partial P_c}{\partial x} - \frac{P_c}{2\tau}, \quad (3.4.6)$$

where ρ and u are the density and velocity of the plasma; the indexes k , th , c and w denote the kinetic part of the plasma, the thermal part of the plasma, CR and wave parts, respectively; and $\pm$ denote forward and backward propagating waves. The energy densities and energy fluxes are given by:

$$\begin{aligned} E_k &= (1/2)P_k = (1/2)\rho u^2, \quad E_{th} = P_{th}/(\gamma_g - 1), \quad E_c = P_c/(\gamma_c - 1), \quad E_w^\pm = 2P_w^\pm, \\ F_k &= uE_k, \quad F_{th} = u(E_{th} + P_{th}), \quad F_c = (u + V_a(e_+ - e_-))(E_c + P_c) - \kappa(\partial E_c/\partial x), \\ F_w^\pm &= E_w^\pm(u \pm V_a) + uP_w^\pm. \end{aligned} \quad (3.4.7)$$

The Alfvén speed is given by

$$V_a = B(u_o \rho)^{-1/2}, \quad (3.4.8)$$

and B is constant in the one dimensional problem.

Ko (1992) gave a simple model of the coupling between plasma, CR and waves. The diffusion coefficient κ , the stochastic acceleration rate $1/\tau$ and $e_{\pm}$ (which are related to streaming instability) are given by

$$k = \frac{c^2}{3\alpha(P_w^+ + P_w^-)}, \quad \frac{1}{\tau} = \frac{16\alpha V_a^2 P_w^+ P_w^-}{c^2(P_w^+ + P_w^-)}, \quad e_{\pm} = \frac{P_w^{\pm}}{(P_w^+ + P_w^-)}, \quad (3.4.9)$$

where c is the speed of light, and α indicates the strength of coupling.

3.4.3. Steady state profiles of the CR-plasma system

According to Ko (1999), in steady state there are six integration constants:

1) magnetic flux

$$\Phi = B, \quad (3.4.10)$$

2) mass flux

$$F_m = u\rho, \quad (3.4.11)$$

3) entropy constant

$$\Theta = P_{th} \rho^{-\gamma_g}, \quad (3.4.12)$$

4) total energy flux

$$F_{\text{tot}} = F_k + F_{th} + F_c + F_w^+ + F_w^-, \quad (3.4.13)$$

5) total momentum

$$P_{\text{tot}} = P_k + P_{th} + P_c + P_w^+ + P_w^-, \quad (3.4.14)$$

and 6) wave-action

$$W_A = \left(F_c + E_w^+ \frac{(u + V_A)^2}{V_A} - E_w^- \frac{(u - V_A)^2}{V_A} \right). \quad (3.4.15)$$

In CR-plasma systems without waves, or in systems where the thermal plasma is dominant (the so called nonlinear test particle picture) physical solutions can be classified completely (Drury and Völk, 1982; Axford, Leer, and McKenzie, 1982; Jiang, Chan, and Ko, 1996; Ko, Chan, and Webb, 1997; Ko, 1998). Unfortunately the mathematics of the full system is too complicated to sort out every physical solution.

Ko (1999) works out several typical solutions numerically. A solution of structure is deemed physical if its pressures are non-negative, and it approaches

uniform states both far upstream ($x \rightarrow -\infty$) and far downstream ($x \rightarrow +\infty$). Moreover, owing to stochastic acceleration at least one of the three pressures P_c, P_w^+, P_w^- must be zero as $x \rightarrow \pm\infty$ (see Eqs. 3.4.5 and 3.4.6).

Recall that in CR-plasma systems without waves there are two generic steady state structures. The flow profile is monotonically decreasing and it is either continuous or have one discontinuity (i.e., a sub-shock): Drury and Völk (1981), Axford, Leer and McKenzie (1982), Ko, Chan and Webb (1997). For systems with a unidirectional wave it is possible to consider only continuous flow, because a sub-shock generates both waves downstream. In this case the flow profile is also monotonically decreasing (McKenzie and Völk, 1982). It is necessary to point out that uniform states are physically allowable solutions in the simplified systems mentioned above but not in the full system. Ko (1999) concentrates only on the continuous flow profile of the full system (i.e., with both forward and backward waves). Furthermore, in paper of Ko (1999) has super-Alfvénic flows were considered only (i.e., $u/V_a > 1$ everywhere). In all these calculations the magnetic field, velocity, density, pressures, and length are nominated as following: $B_o, u_o, \rho_o, P_o, L_o$, where $B_o^2/\mu_o = \rho_o u_o^2 = P_o$ and $L_o = c^2(\alpha P_o u_o)^{-1}$. To integrate the set of equations, besides assigning values to γ_g and γ_c , eight constants are required, e.g., three integration constants $\Phi, F_m, F_{\text{tot}}$, and five initial values of $u, P_{th}, P_c, P_w^+, P_w^-$ at $x = 0$. Results of numerical calculations for $\gamma_g = 5/3, \gamma_c = 4/3$ are shown in Fig. 3.4.1–3.4.4.

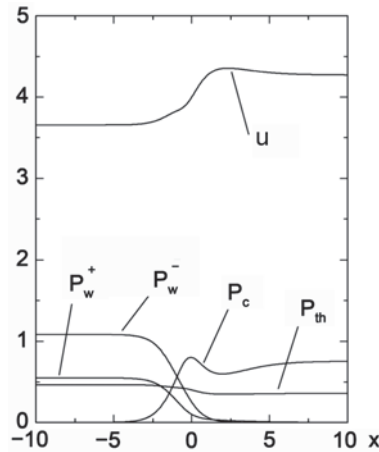


Fig. 3.4.1. Profiles of CR-plasma systems in the hydrodynamic approach. The parameters are the following: $\Phi = 1.0, F_m = 1.6, F_{\text{tot}} = 31.26$ and $u = 4.0, P_{th} = 0.4, P_c = 0.8, P_w^+ = 0.1, P_w^- = 0.25$ at $x = 0$; moreover $\Theta = 1.842, P_{\text{tot}} = 7.95, W_A = 12.82$. According to Ko (1999).

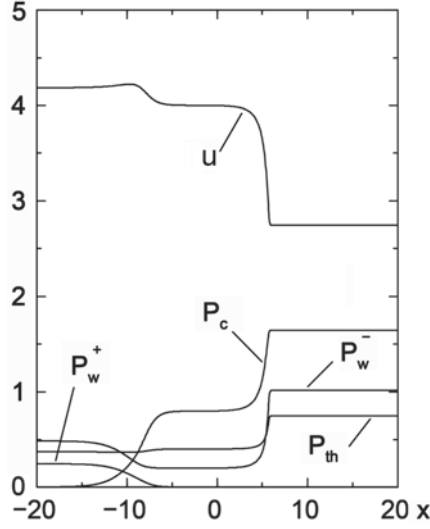


Fig. 3.4.2. Profiles of CR-plasma systems in the hydrodynamic approach. The parameters are the following: $\Phi = 1.0$, $F_m = 1.6$, $F_{\text{tot}} = 26.30$ and $u = 4.0$, $P_{th} = 0.4$, $P_c = 0.8$, $P_w^+ = 10^{-6}$, $P_w^- = 0.2$ at $x = 0$; moreover $\Theta = 1.842$, $P_{\text{tot}} = 7.80$, $W_A = 6.25$. According to Ko (1999).

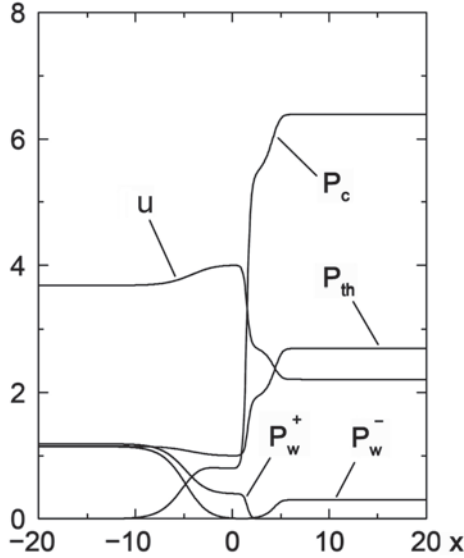


Fig. 3.4.3. Profiles of CR-plasma systems in the hydrodynamic approach. The parameters are the following: $\Phi = 1.0$, $F_m = 4.0$, $F_{\text{tot}} = 63.53$ and $u = 4.0$, $P_{th} = 1.0$, $P_c = 0.8$, $P_w^+ = 0.4$, $P_w^- = 0.01$ at $x = 0$; moreover $\Theta = 1.0$, $P_{\text{tot}} = 18.21$, $W_A = 35.65$. According to Ko (1999).

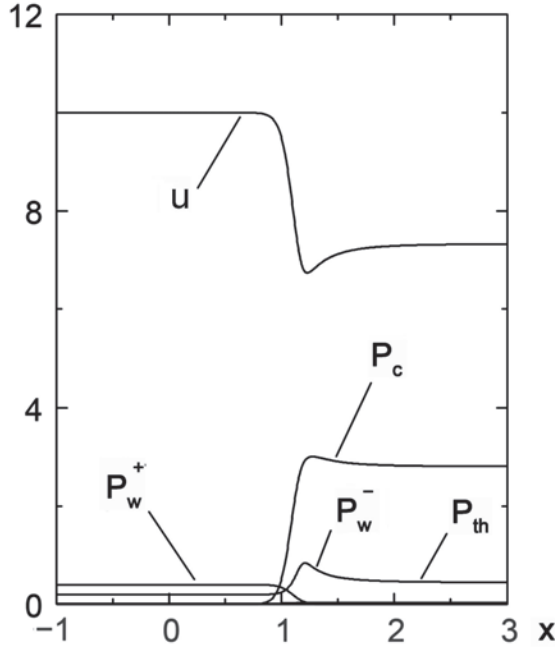


Fig. 3.4.4. Profiles of CR-plasma systems in the hydrodynamic approach. The parameters are the following: $\Phi = 1.5$, $F_m = 1.0$, $F_{\text{tot}} = 70.4$ and $u = 10.0$, $P_{th} = 0.02$, $P_c = 10^{-8}$, $P_w^+ = 0.4$, $P_w^- = 0.2$ at $x = 0$; moreover $\Theta = 0.9283$, $P_{\text{tot}} = 10.62$, $W_A = 34.33$. According to Ko (1999).

The most significant features in Fig. 3.4.1–3.4.4 are the flow and pressure profiles which can be non-monotonic, and are in sharp contrast with systems without waves or systems with a unidirectional wave. Fig. 3.4.1 is a reminiscence of the non-linear test particle picture of Jiang, Chan and Ko (1996), where the CR pressure can be increasing non-monotonically. In Fig. 3.4.2 the downstream state closely resembles a system without forward wave ($P_w^+ = 0$), but the upstream state is totally different. Fig. 3.4.3 shows a prominent peak in velocity and a valley in backward wave pressure, while Fig. 3.4.4 shows the opposite. In these examples the CR pressure far downstream is always larger than the CR pressure far upstream, i.e., CR are always accelerated.

According to Ko (1999) the rich morphology of structures is the result of the interplay between the three basic energy transfer mechanisms (see Eqs. 3.4.3–3.4.6): (i) work done by plasma flow; (ii) CR streaming instability; and (iii) stochastic acceleration. Thus (i) and (ii) are facilitated by pressure gradients, (ii) and (iii) involve energy exchange between CR and waves. Ko (1999) notes that (i) and (ii) can accelerate or decelerate CR, while (iii) can only accelerate. As shown in the non-linear test particle picture, work done by plasma flow is, in general, the

major accelerating mechanism for CR. The relative contributions of the three mechanisms along x produce the fine details of the profiles.

Let us note also that according to Ko (1999) the other class of CR-plasma systems with a sub-shock is rather complicated mathematically, but one thing is clear: the profile structure ought to be qualitatively different from the structure of systems without waves. Besides being non-monotonic, the downstream state will not be uniform (recall that the uniform state is the only physically allowable downstream state available to systems without waves). As far as the upstream state has a wave, both forward and backward waves are generated downstream by the shock. When CR and both waves are present, no uniform state is possible because of the stochastic acceleration.

3.5. Nonlinear Alfvén waves generated by CR streaming instability

3.5.1. Possible damping mechanisms for Alfvén turbulence generated by CR streaming instability

In Sections 3.1 and 3.3 there was mention that the CR streaming instability can play an important role in processes of CR particles' diffusive propagation through space plasma and in diffusive shock acceleration since it can supply Alfvén waves which scatter the particles on different pitch angles (see also Lerche, 1967; Kulsrud and Pearce, 1969; Wentzel, 1969). In order to balance Alfvén wave generation some damping mechanism is usually considered. As Alfvén waves are weakly linearly damped, various nonlinear effects are currently used. CR streaming generates waves in one hemisphere of wave vectors. It is well known that such waves are not subject to any damping in incompressible magneto-hydrodynamics. Using compressibility results in a ponder-motive force that gives a second order plasma velocities and electric field perturbations along the mean magnetic field. These perturbations can yield wave steepening as well as nonlinear damping, if kinetic effects of thermal particles are included. Those effects were taken into account in order to obtain nonlinear damping rates of parallel propagating Alfvén waves (Lee and Völk, 1973; Kulsrud, 1978; Achterberg, 1981). The importance of trapping of thermal particles for nonlinear dissipation of sufficiently strong waves that results in the saturation of wave damping was also understood many years ago (Kulsrud, 1978; Völk and Cesarsky, 1982). Corresponding saturated damping rates which take into account dispersive effects were calculated. Nevertheless dispersive effects can be rather small for Alfvén waves that are in resonance with galactic CR nuclei. Hence the effect of Coulomb collisions can be important. Zirakashvili et al. (1999) derived the nonlinear Alfvén wave damping rate in presence of thermal collisions.

3.5.2 Basic equations described the nonlinear Alfvén wave damping rate in presence of thermal collisions

Zirakashvili et al. (1999) consider Alfvén waves propagating in one direction along the ambient magnetic field. It is convenient to write the equations in the frame of coordinates moving with the waves. In such a frame there are only quasi-static magnetic and electric fields slowly varying in time owed to wave dispersion and nonlinear effects. The case of a high- β Maxwellian plasma is considered. Electric fields are negligible for nonlinear damping in such a plasma. Zirakashvili et al. (1999) investigated waves with wavelengths much greater thermal particles gyro-radii and used drift equations for distribution function of those particles (Chandrasekhar, M1960):

$$\frac{\partial F}{\partial t} + v\mu(\mathbf{b}\nabla)F + \frac{1-\mu^2}{2}v\frac{\partial F}{\partial\mu}\nabla\mathbf{b} = \text{St}F, \quad (3.5.1)$$

where F is the velocity distribution of thermal particles that is averaged on gyro-period, v is particle velocity, $\mathbf{b} = \mathbf{B}/B$ is the unit vector along the magnetic field $\mathbf{B}$, $\mu = \mathbf{p}\mathbf{B}/B$ is the cosine of the pitch angle of the particle. The right hand side of Eq. 3.5.1 describes collisions of particles. For the Maxwell equations it is necessary to know the flux of particles. It is given by drift theory (Chandrasekhar, M1960):

$$\mathbf{J}_\perp = \frac{1-\mu^2}{2}\frac{v^2}{\Omega}\mathbf{b} \times \left[\nabla F + \mu\frac{\partial F}{\partial\mu}(\mathbf{b}\nabla)\mathbf{b} \right], \quad (3.5.2)$$

where Ω is particle gyro-frequency in local field. The last term on the left hand side of Eq. 3.5.1 describes the mirroring of particles. Because the field is static in this frame, the particle energy is constant, and in a time asymptotic state wave dissipation is absent without collisions. In the presence of wave excitation it will only deal with the time asymptotic state in the following. It will be used for the collision operator a simplified form

$$\text{St}F = \Delta_v v^2 (F - F_M), \quad (3.5.3)$$

where F_M is the Maxwellian distribution function shifted by the Alfvén velocity V_a ; Δ_v is the Laplace operator in velocity space and v is the collision frequency. This operator tends to make the particle distribution function Maxwellian. Introducing the coordinate s along the magnetic field, and the distribution function $f = F - F_M$ one obtains the following equation for f :

$$v\mu \frac{\partial f}{\partial s} - \frac{1-\mu^2}{2} v \frac{\partial f}{\partial \mu} \frac{\partial \ln B(s)}{\partial s} - \Delta_v v w^2 f = \frac{1-\mu^2}{2} v \frac{\partial F_M}{\partial \mu} \frac{\partial \ln B(s)}{\partial s}. \quad (3.5.4)$$

For sufficiently small magnetic field perturbations (conditions for that case will be derived later) one can neglect the mirroring term on the left hand side of Eq. 3.5.4. Without collisions this leads to the well known nonlinear damping mentioned above. Zirakashvili et al. (1999) take into account the mirroring term here and use standard quasilinear theory (Galeev and Sagdeev, 1979). The function f can be written in the form $f = f_o + \delta f$, where $f_o = \langle f \rangle$ is the ensemble averaged distribution function f . They are interested in the case of a small magnetic field amplitude $A \ll 1$, where $\mathbf{A} = (\mathbf{B} - \mathbf{B}_o)/B_o$. Taking also into account that mirroring is sufficient for small $\mu \ll 1$ of particles they leave in the collision operator the second derivative on μ only and come to the equation:

$$v\mu \frac{\partial f}{\partial s} - \frac{v}{4} \frac{\partial}{\partial \mu} (f + F_M) \frac{\partial A^2(s)}{\partial s} - v \frac{\partial^2 f}{\partial \mu^2} = 0. \quad (3.5.5)$$

Taking into account that average distribution function is s independent one can obtain equation for Fourier transform $\delta f_k = \int ds \delta f(s) \exp(-isk)$:

$$ikv\mu \delta f_k - v \frac{\partial^2 \delta f_k}{\partial \mu^2} = \frac{1}{4} ikvA_k^2 \frac{\partial}{\partial \mu} (f_o + F_M). \quad (3.5.6)$$

The functions f_o and δf_k are peaked near $\mu = 0$. It is convenient to introduce the Fourier transform on

$$\mu \tilde{f}_o(\xi) = \int d\mu f_o(\mu) \exp(-i\xi\mu) \text{ and } \tilde{\delta f}_k(\xi) = \int d\mu \delta f_k(\mu) \exp(-i\xi\mu). \quad (3.5.6a)$$

Then Eq. 3.5.6 will transmit into

$$kv \frac{\partial \tilde{\delta f}_k}{\partial \xi} + v \xi^2 \tilde{\delta f}_k = \frac{v}{4} ikA_k^2 \left(2\pi \delta(\xi) \frac{\partial F_M}{\partial \mu} \Big|_{\mu=0} + i \xi \tilde{f}_o(\xi) \right). \quad (3.5.7)$$

This equation has a solution

$$\begin{aligned} \tilde{\mathcal{F}}_k = & i \frac{k}{4|k|} A_k^2 \int_{-\infty}^{+\infty} d\xi' \theta(k(\xi - \xi')) \\ & \times \left(2\pi\delta(\xi') \frac{\partial F_M}{\partial \mu} \Big|_{\mu=0} + i\xi' \tilde{f}_o(\xi') \right) \exp\left(-\frac{\nu(\xi^3 - \xi'^3)}{3k\nu}\right). \end{aligned} \quad (3.5.8)$$

After ensemble averaging of Eq. 3.5.5 and using Eq. 3.5.8 one obtains an equation for $\tilde{f}_o(\xi)$:

$$\begin{aligned} \nu\xi^2 \tilde{f}_o(\xi) - \frac{i\xi}{16} \int dk dk_1 \int_{-\infty}^{+\infty} d\xi' |k| \nu \theta(k(\xi - \xi')) I(k_1) I(k+k_1) \\ \times \left(2\pi\delta(\xi') \frac{\partial F_M}{\partial \mu} \Big|_{\mu=0} + i\xi' \tilde{f}_o(\xi') \right) \exp\left(-\frac{\nu}{3k\nu}(\xi^3 - \xi'^3)\right) = 0. \end{aligned} \quad (3.5.9)$$

Here $I(k)$ is the spectrum of Alfvén waves normalized to the magnetic energy of the mean field:

$$\langle \delta B^2 \rangle = B_o^2 \int dk I(k). \quad (3.5.9a)$$

Wave numbers k with $+$ ($-$) sign correspond to right(left) hand circularly polarized wave. The equation obtained describes the influence of waves on the mean distribution function of thermal particles, in particular, well known in plasma theory quasilinear ‘plateau’ formation breaking by thermal collisions (Galeev and Sagdeev, 1979). The solution of this equation should be substituted into Eq. 3.5.8. This expression, together with the Eq. 3.5.2 for the flux determines the nonlinear electric current density (the input of thermal protons is taken into account only):

$$\mathbf{J}_k = i\pi k \int_{-\infty}^{+\infty} dk_1 \int_0^{\infty} \nu^2 d\nu M c \frac{\nu^2}{B_o} [\mathbf{A}_{k-k_1} \times \mathbf{e}_z] \tilde{\mathcal{F}}_{k_1} \Big|_{\xi=0}. \quad (3.5.10)$$

Substituting this current into the Maxwell equations and ensemble averaging one can derive an equation for the Alfvén wave spectrum

$$dI(k)/dt = -2\Gamma_{NL} I(k) \quad (3.5.11)$$

with the nonlinear Alfvén wave damping rate:

$$\Gamma_{NL} = -\frac{\pi}{8} \frac{MV_a}{B_o^2} k \int_{-\infty}^{+\infty} dk_1 I(-k_1) \int_0^{\infty} 2\pi v^2 dv v^2 \int_{-\infty}^{+\infty} d\xi' \frac{k+k_1}{|k+k_1|} \theta(-\xi'(k+k_1)) \\ \times \left(2\pi \delta(\xi') \frac{\partial F_M}{\partial \mu} \Big|_{\mu=0} + i \xi' \tilde{f}_o(\xi') \right) \exp \left(\frac{v \xi'^3}{3(k+k_1)v} \right), \quad (3.5.12)$$

where M is the ion mass and

$$\frac{\partial F_M}{\partial \mu} \Big|_{\mu=0} = -\frac{n V_a v}{(2\pi)^{3/2} v_T^5} \exp \left(-\frac{v^2 + V_a^2}{2v_T^2} \right). \quad (3.5.13)$$

Here n is the plasma density and v_T is the thermal velocity.

It is useful to transform Eq. 3.5.9 to a form more convenient for applications. It is possible to invert the integral operator and obtain the following equation:

$$8\xi^2 \int_{-\infty}^{+\infty} d\xi' \xi'^3 \tilde{f}_o(\xi') \int_{-\infty}^{+\infty} \frac{d\eta}{2\pi} \frac{\exp \left(\frac{i\eta}{3} (\xi^3 - \xi'^3) \right)}{\int_{-\infty}^{+\infty} dk dk_1 \frac{I(k_1) I(k+k_1) k^2 v^2}{v^2 + \eta^2 k^2 v^2}} + \xi \tilde{f}_o(\xi) \\ = 2\pi i \delta(\xi) \frac{\partial F_M}{\partial \mu} \Big|_{\mu=0}. \quad (3.5.14)$$

One should solve Eq. 3.5.14 in order to use Eq. 3.5.12, except in the case in which the collision frequency is large enough and a ‘plateau’ is absent. In this case one can neglect $\tilde{f}_o$ in Eq. 3.5.12 and obtain the well known unsaturated nonlinear damping rate (Lee and Völkl, 1973; Kulsrud, 1978; Achterberg, 1981):

$$\Gamma_{NL}^{(0)} = -\frac{\sqrt{2\pi}}{8} v_T k \int_{-\infty}^{+\infty} dk I(k) \frac{k-k_1}{|k-k_1|}. \quad (3.5.15)$$

In the opposite case of small ν one should use Eq. 3.5.14 and put $\nu=0$:

$$\frac{\partial}{\partial \xi} \frac{1}{\xi^2} \frac{\partial}{\partial \xi} \tilde{\mathcal{F}}_o(\xi) - \frac{1}{8} \langle A^2 \rangle^2 \tilde{\mathcal{F}}_o(\xi) = -\frac{i\pi}{4} \delta(\xi) \langle A^2 \rangle^2 \frac{\partial F_M}{\partial \mu} \Big|_{\mu=0}, \quad (3.5.16)$$

where

$$\langle A^2 \rangle = \int_{-\infty}^{+\infty} dk I(k). \quad (3.5.17)$$

Substituting the solution of Eq. 3.5.16 into Eq. 3.5.12 and expanding the exponent one can obtain the saturated non-linear damping rate:

$$\Gamma_{NL}^{\text{sat}} = \nu_o \langle A^2 \rangle^{-3/2} k \int_{-\infty}^{+\infty} dk_1 \frac{I(k_1)}{k - k_1}, \quad (3.5.18)$$

where

$$\nu_o = \frac{2^{1/4} \pi}{8} \frac{\Gamma(3/4)}{\Gamma(1/4)} \int_0^\infty \frac{4\pi w^4 dv}{(2\pi)^{3/2} v_T^5} \exp\left(-\frac{v^2 + V_a^2}{2v_T^2}\right). \quad (3.5.19)$$

3.5.3. On the possible role of nonlinear damping saturation in the CR-plasma systems

According to Zirakashvili et al. (1999), the trapping of thermal particles is essential for the damping of Alfvén waves if the frequency of collisions is small enough. For trapped particles $|\mu| < \mu_*$, where $\mu_* \approx \delta B/B$ for Alfvén waves. Hence the escape time is

$$t_{\text{esc}} \approx \mu_*^2 / \nu. \quad (3.5.20)$$

It should be compared with the period of particle oscillations inside the trap,

$$T \approx (k v_T \mu_*)^{-1}. \quad (3.5.21)$$

This gives the condition for saturation of nonlinear damping:

$$\nu \ll k v_T (\delta B/B)^3. \quad (3.5.22)$$

The saturated damping rate can be estimated as the unsaturated damping rate multiplied by the ratio T/t_{esc} . It is easy to see that such an estimate is in accordance with Eq. 3.5.18.

In the self-consistent model of galactic wind flow developed by Zirakashvili et al. (1996), Ptuskin et al. (1997) where the unsaturated damping rate was used, $(\delta B/B) \approx 10^{-2}$ and is determined by the power of CR sources in the Galactic disk. For this case the critical value for the collision frequency is $10^{-12} \text{ sec}^{-1}$ for a

wavenumber $k \approx 10^{-13} \text{ cm}^{-1}$ that is in resonance with 1 GeV CR protons. This value is close to the value of the collision frequency of a hot rarefied plasma with number density 10^{-3} cm^{-3} and temperature 10^6 K . Therefore in the absence of other scattering processes, trapping effects might be relevant for Alfvén wave damping in our Galaxy (see in more details below Sections 3.13–3.14).

Another important feature of saturated damping according to Zirakashvili et al. (1999) is the possibility of not only damping but also energy transfer to smaller wavenumbers. This property is absent for unsaturated damping of unpolarized ($I(k) = I(-k)$) waves. Such energy transfer can be important for diffusive shock acceleration because it permits small energy particles to generate Alfvén waves that are in resonance with particles of greater energies and, hence determines the rate of acceleration.

3.6. Interplanetary CR modulation, possible structure of the Heliosphere and expected CR nonlinear effects

3.6.1. CR hysteresis effects and dimension of the modulation region; importance of CR nonlinear effects in the outer Heliosphere

The studies of the neutron component data have made it possible to find the hysteresis character of the relationships between the variations in solar activity (SA) and in CR intensity (Simpson, 1963; Dorman and Dorman, 1967a,b,c,d; Dorman, M1974, M1975a). This effect arises from the delay of the interplanetary processes (responsible for CR modulation) with respect to the initiating solar processes which correspond to some effective velocity of solar wind propagation (see Fig. 3.6.1 for $r_o/u = 10$ months and Fig. 3.6.2 for $r_o/u = 20$ months, where r_o is an effective radius of modulation region and u is an effective radial velocity of solar wind).

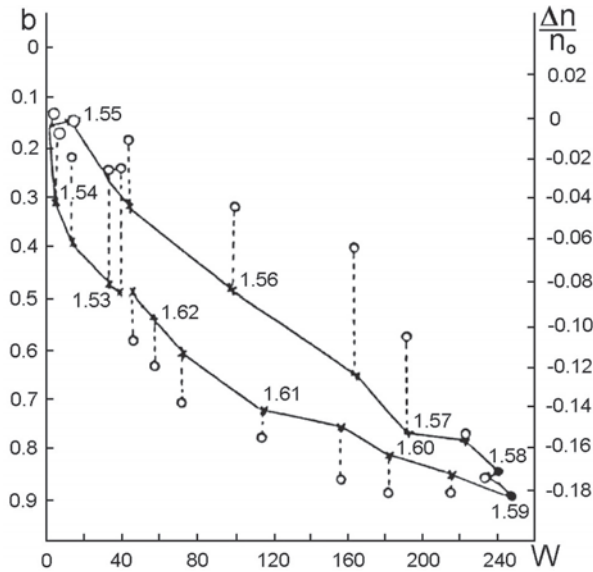


Fig. 3.6.1. Expected modulation in period July 1952 – July 1962 (curve, left scale) for $r_o/u = 10$ months (which corresponds to $r_o = 67$ AU at $u = 4 \times 10^7$ cm/s) as function of sunspot number W flattened over the 12 month period and comparison with observation data of the neutron component in Chicago, also flattened over the 12 month period (circles connected with the corresponding theoretical values by the dashed vertical lines, the right hand scale).

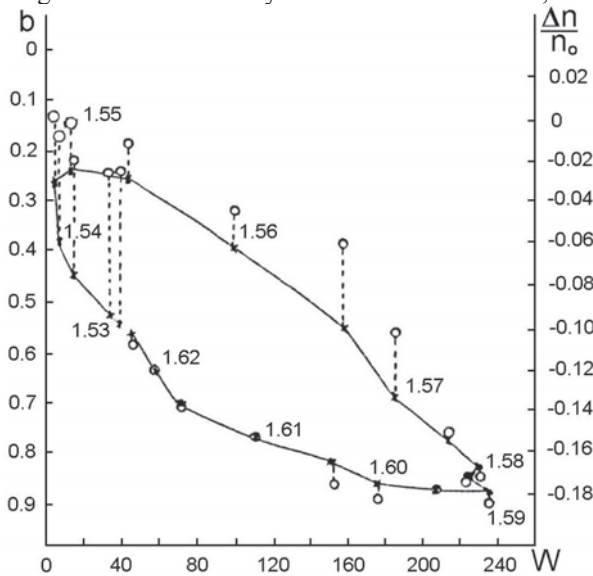


Fig. 3.6.2. The same as in Fig. 3.6.1 but for $r_o/u = 20$ months (which corresponds to $r_o = 133$ AU).

This hysteresis effect in Fig. 3.6.1 and 3.6.2 shows that the size of region of CR effective modulation for energy of few GeV is expected about 100 AU. On these great distances the energy density of IMF and moving solar wind became comparable with CR pressure, so the CR nonlinear effects are sufficient (see Sections 3.2 and 3.3)

3.6.2. Long - term CR spectrum modulation in the Heliosphere

The investigation of long-term CR modulation in the interplanetary space gives important information on dimensions of the Heliosphere (from hysteresis effect, see Section 3.6.1) as well as on the dependence of transport path Λ versus particle rigidity R (from long term CR rigidity spectrum of modulation). The time-dependence of the primary variations of CR with effective rigidities $R = 2, 5, 10$ and 25 GV was found by Belov, Dorman et al. (1988, 1990) on the basis of ground measurements (muon and neutron components) as well as measurements in stratosphere on balloons and in space on satellites and spacecrafts. Results are shown in Fig. 3.6.3.

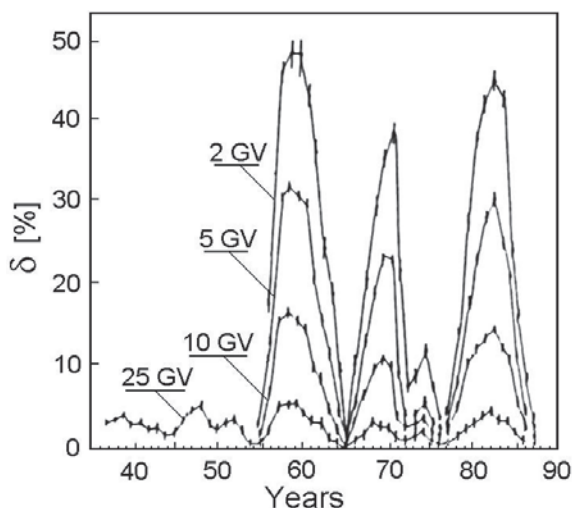


Fig. 3.6.3. Time dependence of primary CR with effective rigidity $R = 2, 5, 10$ and 25 GV (on the basis of ground level registration of muon and neutron components and measurements on balloons and satellites).

The smaller modulation in the 1965–1975 solar activity cycle reflects the influence of the reversal of the general magnetic field of the Sun (drift effects). Fig. 3.6.4 shows the observed rigidity spectrum of long-term modulation relative to the minimum of solar activity in 1965 as well as the residual modulation spectrum in 1965:

$$\begin{aligned}
\delta_{\text{obs}}(R) &= \Delta D(R)/D_{65}(R) = aR^{-\gamma}, \quad \Delta D(R) = D(R) - D_{65}(R), \\
\delta_o(R) &= \Delta D_o(R)/D_{65}(R) = (52 \pm 8)R^{-1.0 \pm 0.1}, \quad \Delta D_o(R) = D_o(R) - D_{65}(R), \\
\delta_{\text{tot}}(R) &= \Delta D_{\text{tot}}(R)/D_o(R) = aR^{-\gamma}, \quad \Delta D_{\text{tot}}(R) = D(R) - D_o(R),
\end{aligned} \tag{3.6.1}$$

where $D_{65}(R)$ is the observed spectrum in the minimum of SA in 1965, and $D_o(R)$ is the CR spectrum out of the Heliosphere, in the interstellar space.

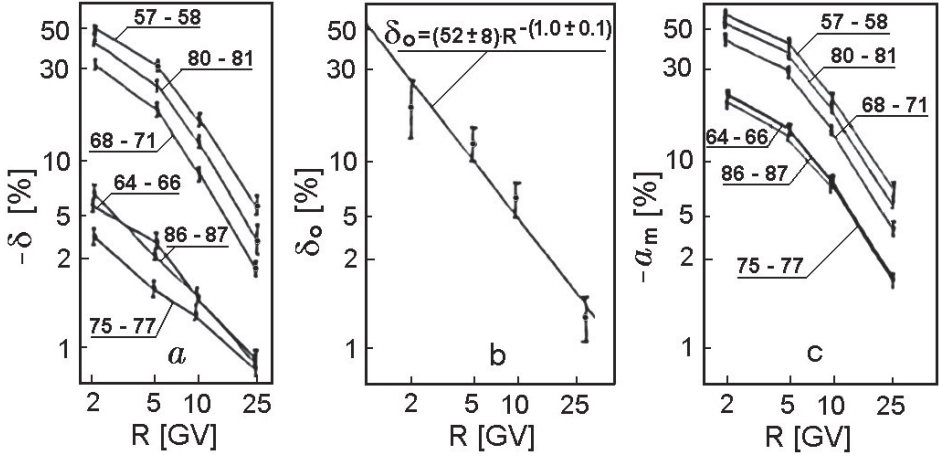


Fig.3.6.4. Rigidity spectra: of observed long term CR variations (a), of residual modulation (b) and of total CR modulation (c) during the various time intervals indicated on the curves.

The residual modulation was found $\delta_o(R) = 6.0 \pm 1.2 \%$ at an effective rigidity $R \approx 10$ GV (which is in good agreement with results in Fig. 3.6.2 for the hysteresis effect in Chicago neutron monitor data). The slope of the total spectrum modulation (the panel c in Fig. 3.6.4) gets steeper with increasing rigidity and the spectral index increases:

$$\begin{aligned}
\delta_{\text{tot}}(R) &\propto R^{-\gamma}; \quad \gamma \approx 0.4 \text{ at } R = 2 \div 5 \text{ GV}; \quad \gamma \approx 1.1 \text{ at } R = 5 \div 10 \text{ GV}; \\
\gamma &\approx 1.6 \text{ at } R = 10 \div 25 \text{ GV}.
\end{aligned} \tag{3.6.2}$$

In the first approximation the effective transport path in the interplanetary space will be

$$\Lambda \propto (\Delta D_{\text{tot}}(R)/D_o(R))^{-1} \propto R^{\gamma}, \tag{3.6.3}$$

where γ is determined by Eq. 3.6.2. The tendency of increasing γ with increasing rigidity R is seen also for CR propagation in the interstellar space for much higher energies 10^{14} – 10^{17} eV (Berezinsky et al., M1990) and for solar CR in the solar atmosphere and in interplanetary space for the much lower energy region 10^6 – 10^{10} eV (Dorman and Miroshnichenko, M1968; Dorman, M1978; Dorman and Venkatesan, 1993; Miroshnichenko, M2001). These intervals in the first approximation correspond to the product of magnetic field strength on the characteristic scale of turbulence in the space where CR propagate.

3.6.3. CR anisotropy in the Heliosphere

The information on possible types of galactic CR anisotropy in the interplanetary space and on their dependence on helio-latitude and radial distance as well as on the level of solar activity is very important for the problem of kinetic stream instability in the Heliosphere (in details see below Section 3.12). CR penetrate inside the Heliosphere and propagate in extended solar wind with frozen in regular spiral magnetic field with inhomogeneities. Fig. 3.6.5 (from Moraal, 1993) shows several mechanisms of CR anisotropy formation in the meridian plane: convection–diffusion, CR density gradient drift, terminal shock wave drift, polar drift and neutral current sheet drift.

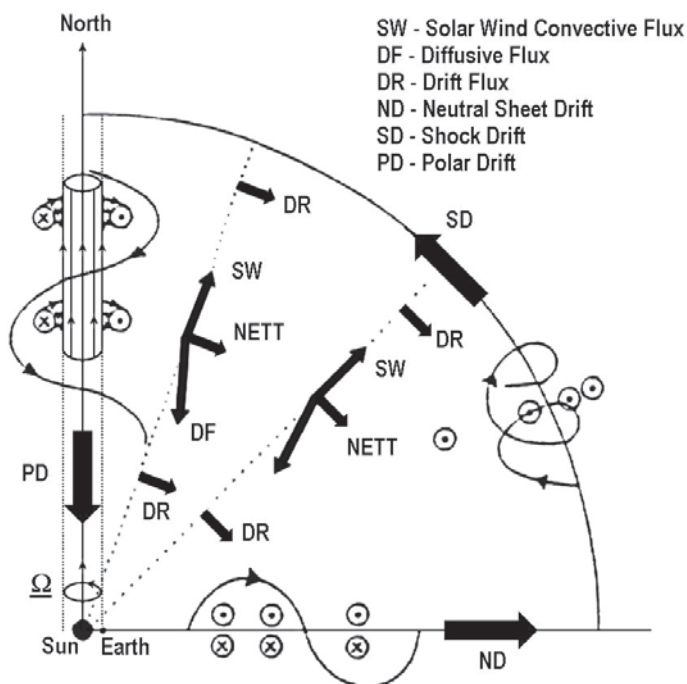


Fig. 3.6.5. Meridian projection of a quarter Heliosphere showing the major galactic CR transport processes and mechanisms of anisotropy formation (according to Moraal, 1993).

The convection anisotropy is determined by the velocity of the solar wind and directed radially from the Sun; diffusion anisotropy is mainly along spiral magnetic field and directed towards the Sun. Resulting anisotropy increased with the radial distance (proportionally in the first approximation) and does not depend on the direction of the spiral interplanetary magnetic field. The direction and the value of the gradient density drift anisotropy depends on the directions of gradient and magnetic field and is proportional to the product of their particle Larmor radius and CR density gradient. The drift along the terminal shock wave gives important particle acceleration (formation of so called anomaly CR in low energy range). The neutral current sheet drift is very important for CR long term modulation; its direction changed every 11 years with changing of the sign of the Sun's general magnetic field: it is a main cause of the 22-year CR variation. In the vicinity of equatorial plane there is also a very important density drift mechanism caused by CR density gradient perpendicular to the ecliptic plane that gives some average anisotropy perpendicular to IMF and CR gradient. There are also North-South CR anisotropy caused by the some asymmetry in latitudinal distribution of solar activity and IMF (see review in Dorman, 2000). Ahluwalia and Dorman (1995a,b), Dorman and Ahluwalia (1995) show that observed anisotropy is mixed, produced by several mechanisms with different properties and different rigidity spectra. The observed galactic CR anisotropies reflect real CR fluxes and are determined by complicate CR density distribution in space and energy balance caused by many processes: CR convection, anisotropy diffusion, neutral sheet drift, curvature drift, drift in inhomogeneous magnetic field, drift along shock wave front with energy change and so on. Let us note that the main galactic CR anisotropy caused by the convection-diffusion mechanism is expected to increase with increasing radial distance, which is important for CR kinetic stream instability.

3.6.4. Possible structure of the Heliosphere and expected nonlinear effects

A possible structure of the Heliosphere according to Dorman (1991) is shown on Fig. 3.6.6.

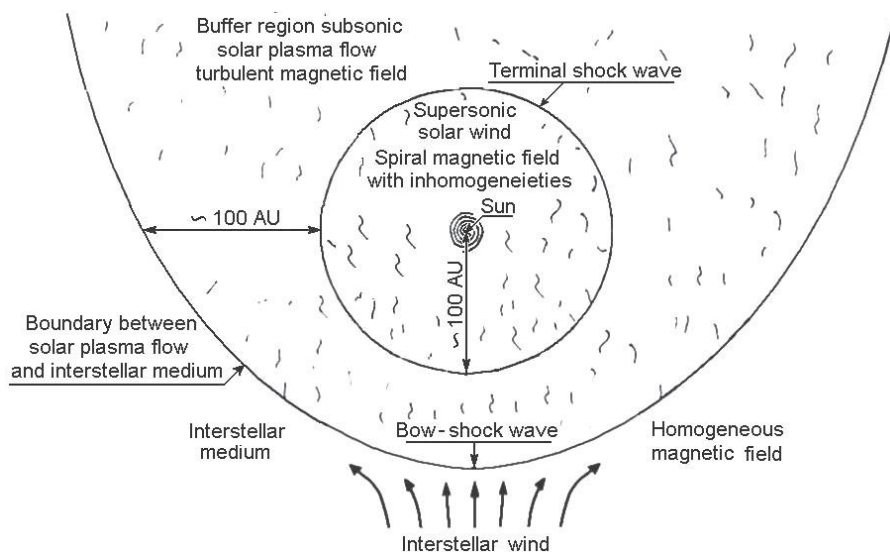


Fig. 3.6.6. Expected structure of the Heliosphere. According to Dorman (1991).

In the region of the inner planets the dynamic pressure of the solar wind is much larger than the CR pressure but at larger distances these pressures became of about the same order and the nonlinear effects then became important. The problem is what is the size of the Heliosphere. About 40 years ago many scientists came to the conclusion that the radius of the Heliosphere is not more than 10-15 AU, but from investigations of CR hysteresis phenomena we determine that this size must be not smaller than the size of effective modulation region for small energy particles, i. e. not smaller about 100 AU (see Section 3.6.1). If the size of the Heliosphere is as big as shown in Fig. 3.6.6, then the dynamical pressure of solar wind in the outer part of the Heliosphere becomes comparable with the CR pressure and it is necessary to take into account the influence of galactic CR pressure on the solar wind's propagation. This was first done by Axford and Newman (1965), and then by Dorman and Dorman (1968a,b,c, 1969), Babayan and Dorman (1977, 1979a,b, 1981, 1990). It was shown that the solar wind's radial deceleration by the pressure of galactic CR becomes important in the outer Heliosphere. The CR modulation in the interplanetary space is not spherically symmetric; the modulation is expected to be stronger in the low helio-latitude region. Therefore one expects that CR pressure in the high helio-latitude region will be higher than in the low helio-latitude region. If it is so, we shall expect the transverse compression of solar wind streams by CR pressure caused by the transverse CR density gradient (Dorman and Dorman, 1969; Babayan and Dorman, 1977).

3.6.5. Studies of the termination shock and heliosheath at > 92 AU: Voyager 1 magnetic field measurements

Now, about 40 years after our prediction (on the basis of investigation of the nature of CR-SA hysteresis effect) that the dimension of the Heliosphere is about 100 AU (Dorman and Dorman, 1967a-e), was obtained experimental evidence. According to Ness et al. (2005), the Heliospheric Magnetic Field (HMF) has been measured by twin Voyager spacecrafts, Voyager 1 and Voyager 2, which were launched in 1977. After encounters with the 4 giant outer planets, they have more or less continuously measured the Heliospheric Magnetic Field (HMF) from 1 to ~96 AU (at June 2005). Thus, magnetic field observations now cover well over a full 22 years long solar magnetic cycle. The temporal and spatial variations of the magnitude of the HMF have been found to be well described by Parker's Archimedean spiral structure (Parker, 1963) when due account is made for time variations of the source field strength and solar wind velocity. The HMF generally had the expected properties at these distances and epochs through several solar activity cycles until late in 2004 when Voyager 1 was at 94 AU and heliographic latitude of 35° N. The paper of Ness et al. (2005), summarizes HMF observations which demonstrate clearly that the theorized and long-sought Termination Shock (TS) associated with the interaction of the solar wind with the local interstellar medium was detected in mid-December 2004 by Voyager 1 at 94.0 AU at 35° N heliographic latitude: the magnitude of HMF increased by a factor of ~3-4 and fluctuations were enhanced significantly, it has been observing in-situ a new astrophysical plasma regime referred to as the Heliosheath (HS). It was noted that observations of the HMF in 2002–2003 did not provide evidence for any crossings of the termination shock near 85 AU as earlier was proposed (Burlaga et al., 2003; Krimigis et al., 2003). Main results of Ness et al. (2005) are shown in Fig. 3.6.7-3.6.9.

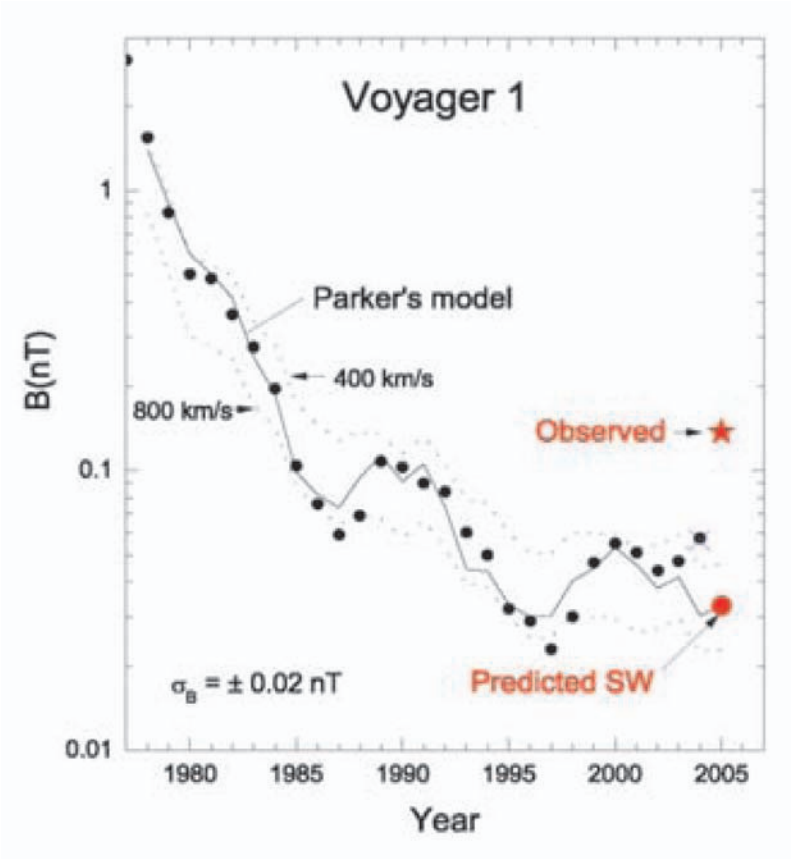


Fig. 3.6.7. Comparison of Voyager 1 annual averaged HMF magnitudes (black cyrcles) with estimated value (curve) according to Parker's model (Parker, M1963). From Ness et al. (2005).

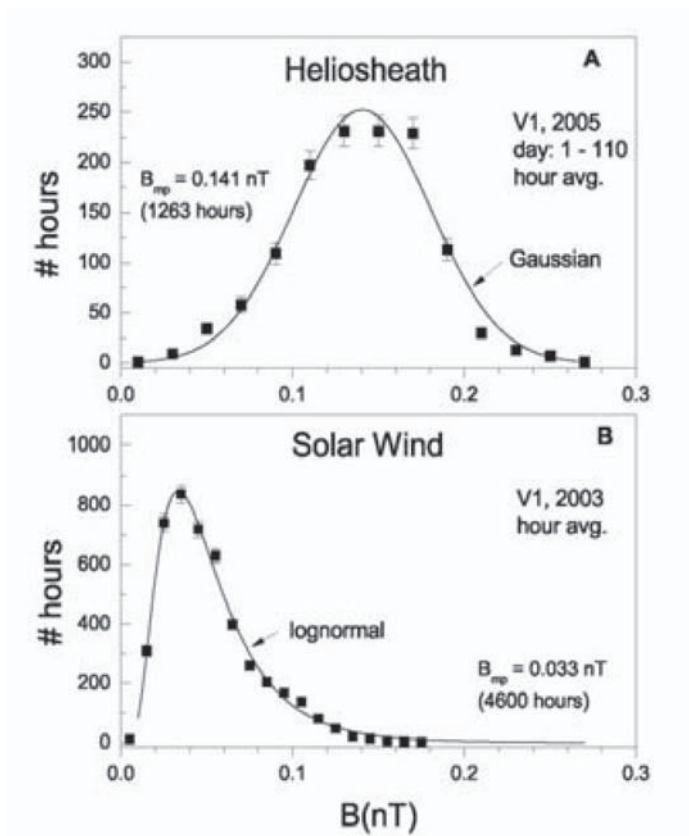


Fig. 3.6.8. Statistical distributions of observed by Voyager 1 (V1) HMF in Heliosheath (A) in 2005 and in Solar Wind (B) in 2003. From Ness et al. (2005).

The hourly averaged Heliospheric magnetic field observations by Voyager 1 during 2004-2005, 61 days before and 76 days after the Terminal shock crossing on day 351, 2004 are shown in Fig. 3.6.9.

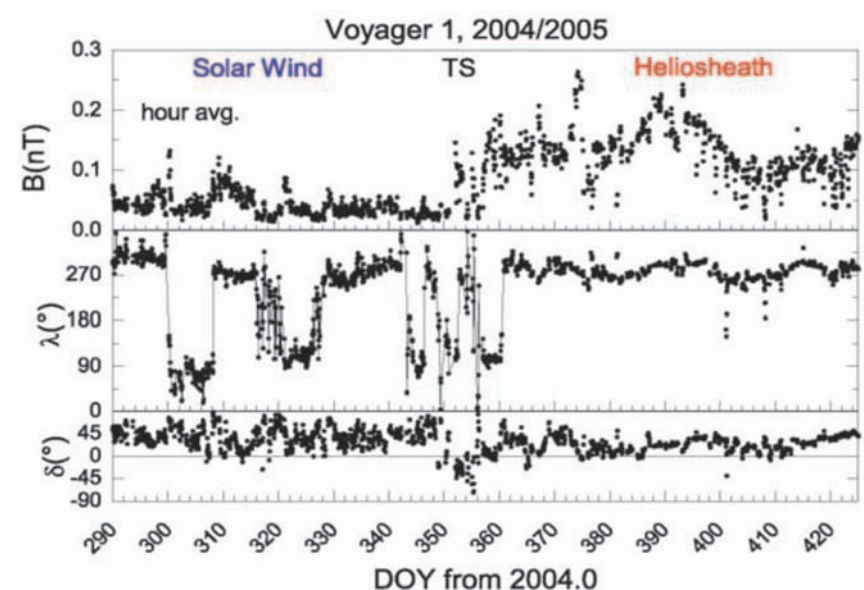


Fig. 3.6.9. V1 Hourly average Heliospheric Magnetic Field (HMF) in heliographic coordinates observed by Voyager 1. Crossing of Terminal Shock (TS) occurs at day 351 of 2004. Sector boundaries, crossings of the Heliospheric Current Sheet (HCS), readily evident in 2004 but not yet seen in 2005. According to Ness et al. (2005).

In Fig. 3.6.9 the field vector is represented by the magnitude, B , and the direction by the heliographic longitude and latitude angles λ and δ . Readily evident in the longitude angle plot is that it displays a well defined bi-modal distribution, characteristically near 270° or 90° . These correspond to Parker's Archimedean spiral angles at this distance for fields with a sense pointing outward from or inward toward the Sun. Sudden changes in λ correspond to crossings of the well known and long studied sector boundaries between Heliospheric magnetic field regions of uniform but opposite polarity in the solar wind: a manifestation of the Heliospheric current sheet. The large field values from day 352 onward are accompanied by large fluctuations of the magnitude while the longitude angle remains fixed near 270° . These field orientations and the sudden large increase in the average field magnitude indicate that the observed Terminal shock is, as expected, classified as a **perpendicular shock**.

There appear to be a few sector boundaries observed shortly after the Terminal shock crossing but for most of the time that Voyager 1 is in the Heliosheath, the polarity of the field remains the same. Since polarities of the fields across sector boundaries or the Heliospheric current sheet are expected to be transmitted through the Terminal shock without any field polarity reversal, this long duration of a fixed polarity is somewhat of a puzzle at the present time. One

suggested explanation is that the Terminal shock may be in motion relative to Voyager 1.

An expanded time scale of field magnitude and fluctuations across the Terminal shock crossing are shown in Fig. 3.6.10. The horizontal bars on either side of the Terminal shock represent the average values during the periods indicated and show a field jump by a factor of ~ 3 . The sudden large and sustained increase in this SD parameter, coincident with the field magnitude increase, suggests that the Heliosheath is a different astrophysical plasma regime than observed in earlier studies of the Heliospheric magnetic field. Throughout the many years prior to the Terminal shock crossing, the typical value of this SD parameter was 0.012-0.014 nT except during passage of a propagating Merged interaction regions (Burlaga et al., 2001).

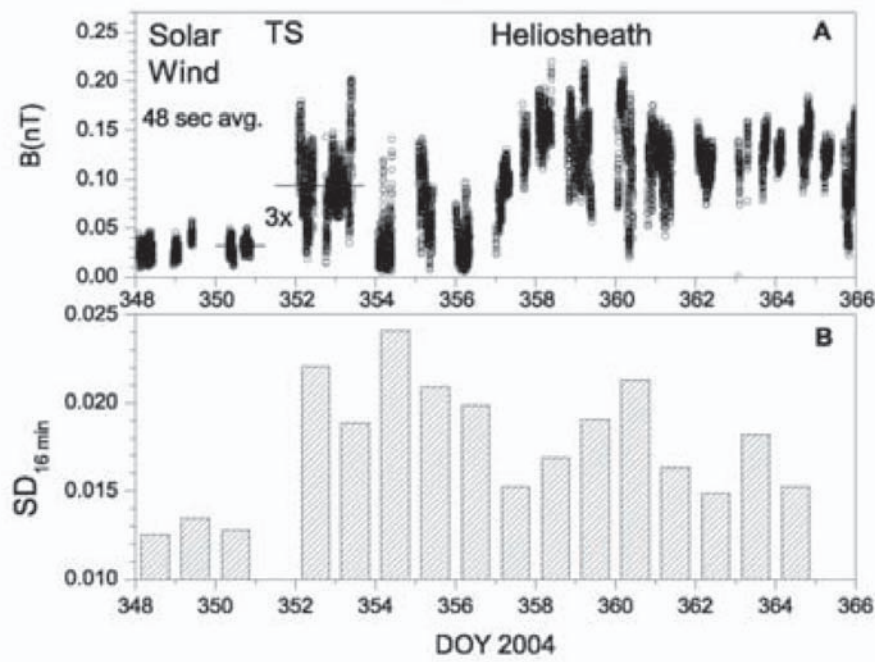


Fig. 3.6.10. 48 second averaged Heliospheric magnetic field magnitude observed by Voyager 1 (upper panel) near crossing of Termination shock on day 351 of 2004. Lower panel presents measurement of daily averaged higher frequency fluctuations up to 0.25 HZ over 16 min intervals. According to Ness et al. (2005).

Ness et al. (2005) came to conclusion that the Termination shock was observed by the Voyager 1 Magnetic Field Experiment in late 2004 when Voyager 1 crossed or was crossed by the Terminal shock at 94.0 AU and 35°N and entered the Heliosheath.

3.7. Radial CR pressure effects in the Heliosphere

3.7.1. On a necessity of including non-linear large-scale effects in studies of propagation of solar and galactic CR in interplanetary space.

In studies of a modulation of galactic CR and a propagation of solar CR, the solar wind is usually considered to be set, independent of intensity and gradients of CR whereas the effects of galactic CR is comparable, or even more than effects of the other factors also limiting solar wind propagation (re-charging, pressure of galactic magnetic field etc.). Therefore, it is necessary to take into account the inverse action of CR on the solar wind, i.e. to solve a self-consistent problem. Estimates of this action were obtained in Axford and Newman (1965), Dorman and Dorman (1968a,b,c, 1969, 1971), Sousk and Lenchek (1969), Dorman (1971b, 1972b), Holzer (1972), Belov et al. (1972), Dorman and Babayan (1975), Babayan and Dorman (1976, 1977a,b, 1979), Babayan et al. (1976), Dorman, Babayan et al. (1978a,b). In particular, in the work of Holzer (1972), there was considered the interaction of solar wind with neutral gas, galactic CR, thermal plasma and galactic magnetic field. The first two causes result in a volume force braking a supersonic flow. The second two causes provide the action of surface force and may result in arising of shock wave. The surface force has the normal and tangential components; therefore the shape of a heliospheric cavity, occupied by supersonic solar wind, will be stretched. A density of interstellar atomic hydrogen is, apparently, insufficient for the braking action to solar wind without forming a shock wave. It is expected that the penetration of interstellar neutral hydrogen into the Heliosphere result in a heating of solar wind at great distances from the Sun. This effect may however be at least partially, masked by dissipation processes and by a presence of jets in solar wind. It was noticed that penetration of interstellar gas into solar system can noticeably vary a state of ionization, for example, of helium.

The boundary of the Heliosphere, of the helio-pause, determined by the balance of pressure of radially outflowing solar wind plasma and interstellar magnetic field may be noticeably different from a spherical surface owing to anisotropy of a pressure of interstellar magnetic field along the surface. The Heliosphere has a tendency to be stretched along a local galactic magnetic field; the exact shapes and its dimension depend on the intensity of the interstellar magnetic field, the ion pressure in interstellar space, the density of interstellar hydrogen, and on the flux of the solar wind. Possible observable results of non-sphericity of Heliosphere are an additional contribution to anisotropy of galactic CR and anisotropic L_{α} distribution of a background radiation owed to scattering on 'hot' hydrogen atoms arising in the process of re-charging outside helio-pause (it is expected that the maximum intensity of this radiation will be observed along the direction of local galactic magnetic field).

3.7.2. Radial braking of solar wind and CR modulation: effect of galactic CR pressure

Babayan and Dorman (1976) obtained integer-differential equation describing non-linear interaction of galactic CR with solar wind for a spherically symmetric model of solar wind with assumption of isotropic diffusion. The equation was numerically solved by the Runge-Kutt method for interstellar spectrum of CR in the form of a power law on kinetic energy and the character of solar wind braking was determined in the minimum and maximum of solar activity depending on the value of $E_{k,\min}$ which determines the lower boundary of energy spectrum of primary CR in interstellar space. A solution was obtained for the case in which the transport path for scattering Λ is independent of the distance to the Sun. It was noticed that the same method could be applied to solve the integro-differential equation for the solar wind velocity also in the case in which Λ is presented in the form of a product of two functions, one of which depends only on a distance to the Sun and the second depends only on the energy of cosmic radiation particles.

For a spherically symmetric model of solar wind with assumption of isotropic diffusion, Babayan and Dorman (1977a) obtained a self-consistent integro-differential equation describing action of galactic CR on the solar wind. The equations, describing solar wind propagation and including the effect of CR in spherically symmetric case, have the following form:

$$d(r^2 \rho u)/dr = 0, \quad (3.7.1)$$

$$\rho u du/dr = -dP_c/dr, \quad (3.7.2)$$

where r is a distance to the Sun; ρ, u are a density and velocity of solar wind; P_c is a pressure of CR. Gravitation plays a substantial role only up to the distances at which a transition of solar wind from subsonic to supersonic flow takes place (Parker, M1963). As this occurs at the distances of several solar radii, and we consider distances greater than 1 AU, on the right-hand side of Eq. 3.7.2 the terms are absent which describe the gravitational effect and a gradient of solar wind. The pressure of CR with isotropic distribution is

$$P_c = \frac{1}{3} \int_0^\infty n(E_k, r) p v dE_k, \quad (3.7.3)$$

where $n(E_k, r)$ is the energy spectrum of CR density at the distance r from the Sun; p and v are a momentum and velocity of particles. Substituting Eq. 3.7.3 in Eq. 3.7.2 and using Eq. 3.7.1, we obtain

$$\frac{du}{dr} = -\frac{r^2}{3\eta_1^2 u_1 \rho_1} \int_0^\infty \frac{dn(E_k, r)}{dr} p v dE_k, \quad (3.7.4)$$

where $\eta_1 = 1$ AU; u_1, ρ_1 are respectively the velocity and the density of the solar wind on the Earth's orbit. The factor dn/dr , in its turn, is determined by the character of the modulation of galactic CR in interplanetary space, i.e. is determined, finally, by the solar wind's velocity $u(r)$ and by the transport path for particle scattering $\Lambda(r)$. As was shown in Dorman (1967), the exact analytical solution of the problem of modulation in the two simplest cases when $\Lambda = \text{const}$ and $\Lambda \propto r$ may be represented (with the relative accuracy to 10%) in the form:

$$n(E_k, r) = n_o(E_k) \exp\left(-\int_r^{r_o} \frac{3u(x)}{v\Lambda(E_k, x)} dx\right), \quad (3.7.5)$$

where $n_o(E_k)$ is the energy spectrum of CR density outside solar wind, r_o is the wind's dimension. Substituting Eq. 2.21.5 in Eq. 3.7.4 we have

$$\frac{du}{dr} = -\frac{r^2 u(r)}{\eta_1^2 u_1 \rho_1} \int_0^\infty n_o(E_k) \exp\left(-\int_r^\infty \frac{3u(r)}{v\Lambda(E_k)} dr\right) \frac{p}{\Lambda(E_k)} dE_k, \quad (3.7.6)$$

In the exponent in Eq. 3.7.6, integrating over r is carried out not from r to r_o , as in Eq. 3.7.5, but from r to ∞ since we consider here that a limiting of the solar wind is provided automatically by its non-linear interaction with CR. Therefore in the non-linear theory, in contrast to the linear theory, it is not necessary to introduce any assumptions about the dimension r_o of the region of solar wind propagation. This fact is one of the essential advances of non-linear theory as compared to the linear theory.

In the papers Dorman and Dorman (1968a,b), Dorman and Babayan (1975), Babayan et al. (1976), a similar equation with some simplifying assumptions was transformed to a non-linear differential equation of the second order, for which it appeared to be possible to obtain the analytical solution only for the regions not very distant from the Sun, i.e. for the regions where the effect of non-linear interaction is small.

In the work Babayan and Dorman (1977a) Eq. 3.7.6 was solved numerically by iteration method. The expression

$$n_o(E_k) = \begin{cases} aE_k^{-(\gamma+1)} & \text{if } E_k \geq E_{k,\min} \\ 0 & \text{if } E_k \leq E_{k,\min} \end{cases} \quad (3.7.7)$$

was taken as n_o , where $\gamma = 1.5$ and 2 , and $E_{k,\min} = 0.1, 0.01$ and 0.001 GeV. The calculations were also carried out for the spectrum described by Eq. 3.7.7 but over rigidity R with the index $\gamma + 1 = 2.5$. The coefficient a was determined from the condition that a density of kinetic energy of CR W_{CR} is 1 eV/cm^3 at $E_{k,\min} = 0.1$ GeV. A dependence of Λ on R was taken as in Dorman and Dorman (1968 a,b) and on a distance r , according to power law $\Lambda \propto r^\beta$ with the power index $\beta = 0$ and 1 , i.e.,

$$\Lambda = 0.63\Lambda_o \left(R^2 + 1.57\right)^{1/2} (r/r_1)^\beta, \quad (3.7.8)$$

where $\Lambda_o = 1.5 \times 10^{12} \text{ cm}$ is the transport path for scattering of particles with $R = 1$ GV at the distance from the Sun $r = r_1$, where $r_1 = 1 \text{ AU}$ is the radius of the Earth's orbit.

In Fig.3.7.1 the dependence is shown of the value of u/u_1 on the distance r/r_1 at $u_1 = 3 \times 10^7 \text{ cm/sec}$ and the density of solar wind particles $N_1 = 5 \text{ cm}^{-3}$ at the Earth's orbit for a spectrum described by Eq. 3.7.7 with $\gamma = 1.5$. Curves 1, 2, 3 correspond to the values of $E_{k,\min} = 0.1, 0.01$, and 0.001 GeV, and $\beta = 0$.

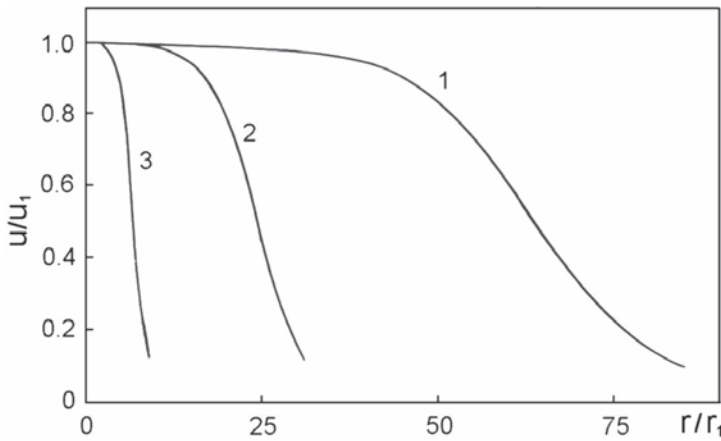


Fig. 3.7.1. Dependence of u/u_1 on r/r_1 for the case $u_1 = 3 \times 10^7 \text{ cm/sec}$, $N_1 = 5 \text{ cm}^{-3}$, $\beta = 0$, $\gamma = 1.5$, $n_o \propto E_k^{-2.5}$. Curve 1 for $E_{k,\min} = 0.1 \text{ GeV}$; 2 – for $E_{k,\min} = 0.01 \text{ GeV}$, 3 – for $E_{k,\min} = 0.001 \text{ GeV}$.

Fig. 3.7.2 presents the same as Fig. 3.7.1 but for $\beta = 1$.

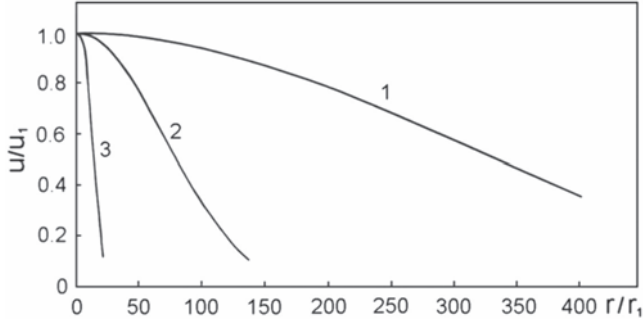


Fig. 3.7.2. The same , as in Fig. 3.7.1, but at $\beta = 1$.

A comparison of Fig. 3.7.1 and Fig. 3.7.2 shows that a variation of solar wind velocity is strongly dependent on how Λ depends on r/r_1 . In the first case the velocity falls by an order at the distances 85, 32, 9 AU, whereas in the second case this occurs at the distances 140 and 22 AU for $E_{k,\min} = 0.01$ and 0.001 GeV, and at $E_{k,\min} = 0.1$ GeV even at the distance 400 AU, the velocity falls down only by 3 times.

Fig. 3.7.3 presents the results of calculations for spectrum described by Eq. 3.7.7 with $\gamma = 2$ and $\beta = 0$. It easy to see that in this case a braking takes place at substantially less distances than with $\gamma = 1.5$. In particular, curve 3 shows that the wind velocity falls down by ten times yet at the distances ~ 5 AU at $E_{k,\min} = 0.001$ GeV. This is connected with that a density of kinetic energy of CR in this case is of the order of several ten eV/cm³, but it is not real.

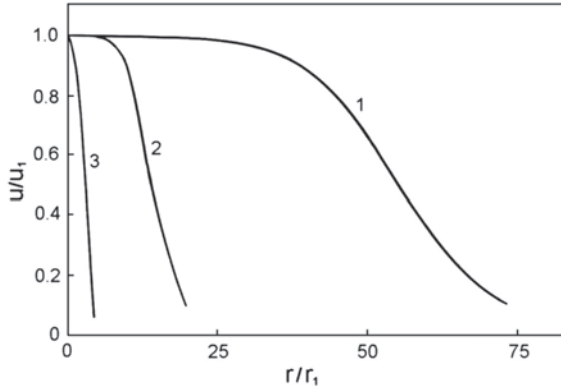


Fig. 3.7.3. Dependence of u/u_1 on r/r_1 for the case $u_1 = 3 \times 10^7$ cm/sec, $N_1 = 5$ cm⁻³, $\beta = 0$, $\gamma = 2$, $n_o \propto E_k^{-3.0}$. Curve 1 for $E_{k,\min} = 0.1$ GeV; 2 – for $E_{k,\min} = 0.01$ GeV, 3 – for $E_{k,\min} = 0.001$ GeV.

In Fig.3.7.4 the same is shown as in Fig. 3.7.1 but at $u_1 = 4 \times 10^7$ cm/sec and $N_1 = 10 \text{ cm}^{-3}$. In this case the wind's deceleration is substantially weaker than for the variants shown in Fig. 3.7.1. This means that a deceleration of the solar wind is essentially dependent on the parameter characterizing the solar activity (a velocity of the wind and its density near the Earth's orbit).

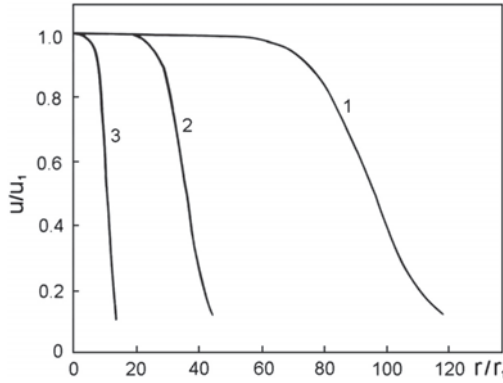


Fig. 3.7.4. Dependence of u/u_1 on r/r_1 for the case $u_1 = 4 \times 10^7$ cm/sec, $N_1 = 10 \text{ cm}^{-3}$, $\beta = 0$, $n_o \propto E_k^{-2.5}$. Curve 1 for $E_{k\min} = 0.1$ GeV; 2 – for $E_{k\min} = 0.01$ GeV, 3 – for $E_{k\min} = 0.001$ GeV.

The results of calculations for the rigidity spectrum are presented in Fig. 3.7.5. The dependence on $R_{\min}$ is weaker than for spectra over E_k . It is connected with that in the case of rigidity spectrum, W_{CR} is weakly dependent on $R_{\min}$.

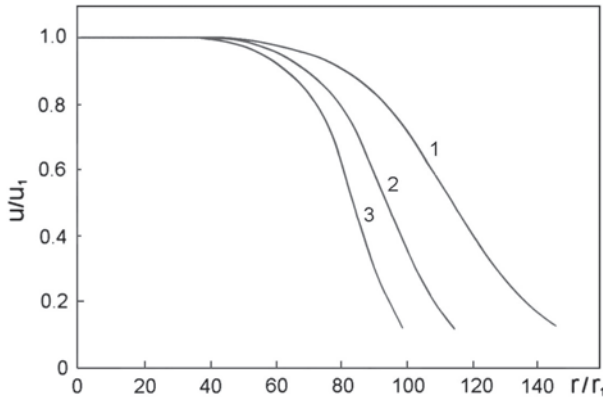


Fig. 3.7.5. Dependence of u/u_1 on r/r_1 for the case $u_1 = 3 \times 10^7$ cm/sec, $N_1 = 5 \text{ cm}^{-3}$, $\beta = 0$, $n_o \propto R^{-2.5}$. Curve 1 for $R_{\min} = 0.1$ GV; 2 – for $R_{\min} = 0.01$ GV, 3 – for $R_{\min} = 0.001$ GV.

The expected modulation of galactic CR including the solar wind braking by CR pressure was found in Babayan and Dorman (1977a). As an example, in Fig. 3.7.6 the results of calculations are shown for a relative modulation with the primary spectrum described by Eq. 3.7.7 at $E_{k,\min} = 0.01$ GeV, including non-linear interaction of solar wind with CR, i.e. taking into account the obtained dependence $u(r)$. Curves 1, 2, 3, 4, 5 were obtained for the rigidities 0.5, 1, 5, 10, and 100 GV, respectively.

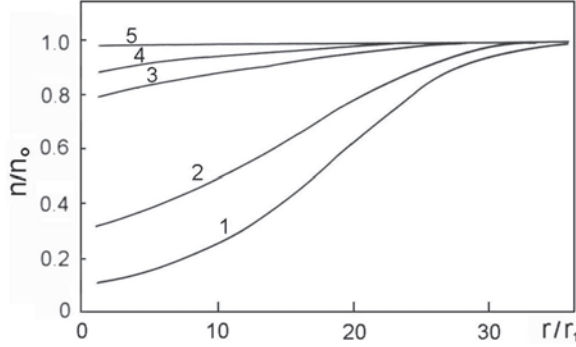


Fig. 3.7.6. Modulation depth n/n_0 at various distances from the Sun r/r_1 including non-linear interaction of CR with solar wind at $u_1 = 3 \times 10^7$ cm/sec, $N_1 = 5 \text{ cm}^{-3}$, $\beta = 0$, $n_0 \propto E_k^{-2.5}$, $E_{k,\min} = 0.01$ GeV. Curves 1 – 5 are respective to particles with the rigidities 0.5, 1.0, 5.0, 10, and 100 GV.

3.7.3. Radial braking of solar wind and CR modulation: effects of galactic CR pressure and re-exchange processes with interstellar neutral hydrogen atoms

The importance of including the effect of the re-charging of neutral hydrogen for a deceleration of solar wind was emphasized by Parker (M1963). A relative contribution of this effect depends on a number of factors and, first of all, on the density of neutral hydrogen in the vicinity of the solar system. The effect under consideration was taken into account in the papers Axford and Newman (1965), Sousk and Lenchek (1969), Semar (1970), etc. The complete equation set determining a propagation of solar wind in a spherically symmetric model, including a modulation of galactic CR and the phenomenon of re-charging of neutral hydrogen was obtained and solved by means of computer in the work Babayan and Dorman (1979).

The set of hydrodynamical equations that determines the solar wind propagation, taking into account the CR pressure P_c and re-exchange between solar wind protons and atoms of interstellar neutral hydrogen is given below:

$$\begin{aligned}\frac{d}{dr}(r^2 \rho u) &= 0, \quad \rho u \frac{du}{dr} = -\frac{dP_g}{dr} - \frac{dP_c}{dr} - Su, \\ \frac{d}{r^2 dr} \left(r^2 u \left(\frac{\rho u^2}{2} + \frac{\gamma P_g}{\gamma - 1} \right) \right) &= -u \left(\frac{dP_c}{dr} + Su \right),\end{aligned}\quad (3.7.9)$$

where ρ, u, P_g are solar wind density, velocity, and gas dynamic pressure, the gas constant $\gamma = 5/3$, and

$$S = Q_{ex} \rho u N_H(r), \quad P_c = (1/3) \int_0^\infty N(E_k, r) p v dE_k. \quad (3.7.10)$$

In Eq. 3.7.10 Q_{ex} is the cross-section for re-exchange processes of interstellar neutral hydrogen atoms and protons of the solar wind, $N_H(r)$ is the density distribution of neutral hydrogen in the interplanetary space which is proportional to the neutral hydrogen density N_{Ho} in the vicinity of the solar system, $N(E_k, r)$ is the CR density distribution inside the Heliosphere as a function of the particle kinetic energy E_k and distance to the Sun r . The modulation of galactic CR in interplanetary space in the convection-diffusion approximation can be described by equation:

$$r^{-2} \frac{\partial}{\partial r} \left(r^2 \left(u N - \kappa \frac{\partial N}{\partial r} \right) \right) - \frac{1}{3} \frac{\partial}{\partial E_k} \left(\alpha(E_k) E_k r^{-2} \frac{\partial}{\partial r} (r^2 u N) \right) = 0, \quad (3.7.11)$$

where

$$\alpha(E_k) = (E_k + 2m_o c^2) / (E_k + m_o c^2). \quad (3.7.12)$$

From Eq. 3.7.9–3.7.12 we obtain the integro–differential equation for determining the behavior of solar wind velocity

$$\frac{du}{dr} = -\frac{r^2}{3\mathfrak{I}} \int_0^\infty \frac{\partial N(E_k, r)}{\partial r} p v dE_k - Q_{ex} u N_H(r), \quad (3.7.13)$$

where

$$\mathfrak{I} = \rho_1 u_1 r_1^2, \quad (3.7.14)$$

and the subscript 1 denotes that these values are taken at $r = r_1 = 1$ AU.

In the work Babayan and Dorman (1979) it was assumed, according to the results of Fite et al. (1962), that a cross-section for the process of re-charging protons of solar wind is independent of energy and equal to

$$Q_{ex} = 3.04 \times 10^{-15} \text{ cm}^2. \quad (3.7.15)$$

As to the density N_{Ho} of interstellar neutral hydrogen in a vicinity of the solar system, there is still a great uncertainty: in various works, the values of N_{Ho} from 10 to 0.05 cm^{-3} are presented (Baranov and Kransnobaev, M1977; Holzer, 1972). Therefore, the calculations were made for the values $N_{Ho} = 0.1, 0.5$ and 1 cm^{-3} . The interstellar spectrum of galactic CR was taken as well as above in Section 3.7.2, in the form of a power spectrum on kinetic energy and rigidity of particles, and the transport path Λ for scattering of particles in interplanetary space was taken which respect to Eq. 3.7.8 at $\beta = 0, 0.5$ and 1 . Fig. 3.7.7 shows, as an example, the expected variation of solar wind velocity with a distance from the Sun r for $N_{Ho} = 0.1, 0.5$ and 1 cm^{-3} , $\beta = 0$, solar wind velocity $u_1 = 3 \times 10^7 \text{ cm/sec}$ and its density $N_1 = 5 \text{ cm}^{-3}$ near the Earth's orbit.

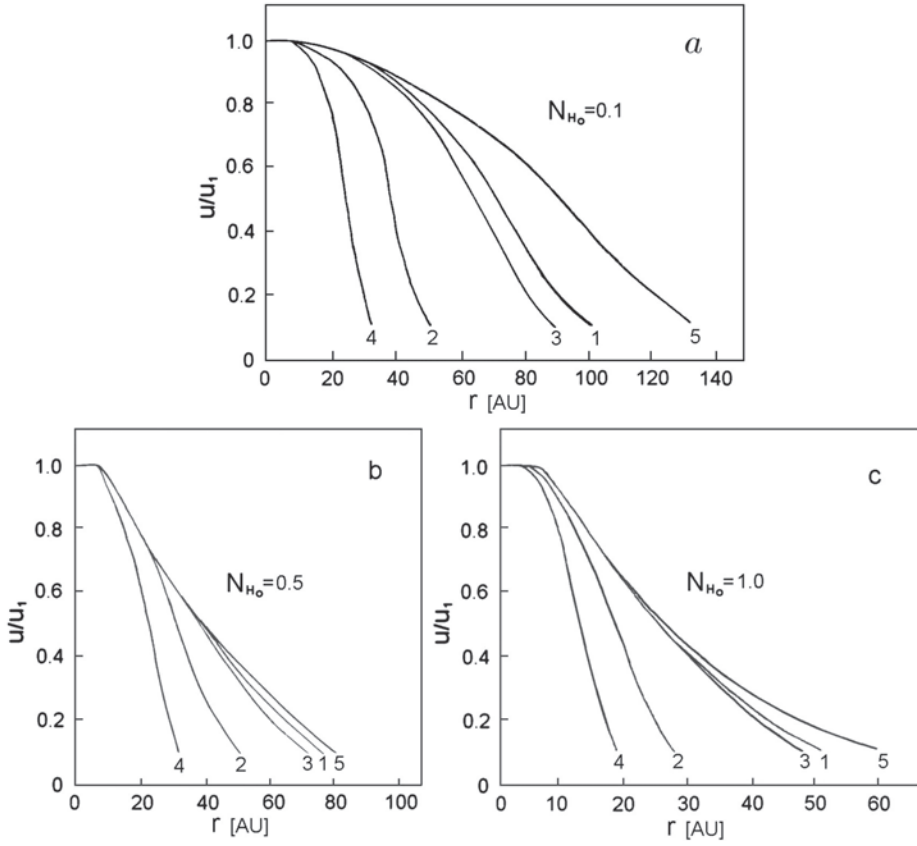


Fig. 3.7.7. The character of a decrease of solar wind velocity u with a distance from the Sun r including the process of re-charging at $u_1 = 3 \times 10^7$ cm/sec, $N_1 = 5 \text{ cm}^{-3}$, $\beta = 0$, $N_{Ho} = 0.1 \text{ cm}^{-3}$ (a), $N_{Ho} = 0.5 \text{ cm}^{-3}$ (b), and $N_{Ho} = 1.0 \text{ cm}^{-3}$ (c); curves: 1 – $n_o \propto E_k^{-2.5}$, $W_c = 1 \text{ eV/cm}^3$; 2 – $n_o \propto E_k^{-2.5}$, $W_c = 2.67 \text{ eV/cm}^3$; 3 – $n_o \propto E_k^{-3.0}$, $W_c = 1 \text{ eV/cm}^3$; 4 – $n_o \propto E_k^{-3.0}$, $W_c = 7.11 \text{ eV/cm}^3$; 5 – $n_o \propto R^{-2.5}$, $W_c = 1 \text{ eV/cm}^3$.

The calculations for Fig. 3.7.7 were made for various values of the interstellar density of energy of CR: curves 1, 3 and 5 correspond to $W_c = 1 \text{ eV/cm}^3$, and curves 2 and 4 are related to the values $W_c = 2.67$ and 7.11 eV/cm^3 . A character of dependence of u/u_1 on r is substantially dependent on W_c , N_{Ho} and on a form of interstellar energy spectrum. In Fig. 3.7.8 the dependences of u/u_1 on r are presented for $N_{Ho} = 0.5 \text{ cm}^{-3}$ at $W_c = 1 \text{ eV/cm}^3$ and the spectrum form $\propto E_k^{-3}$ for $\beta = 0, 0.5$ and 1 .

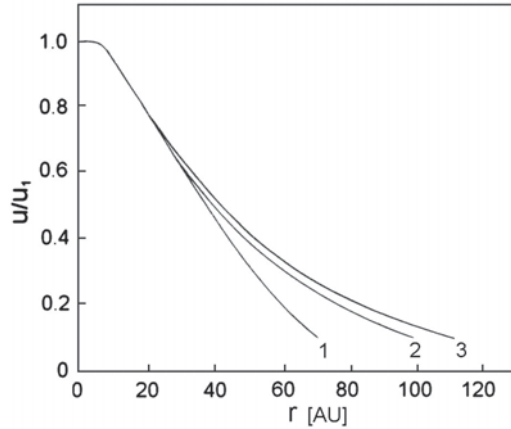


Fig.3.7.8. The character of a decrease of solar wind velocity u with distance from the Sun r , including re-charging process depending on parameters β at $u_1 = 3 \times 10^7$ cm/sec, $N_1 = 5 \text{ cm}^{-3}$, $N_{Ho} = 0.5 \text{ cm}^{-3}$, $n_o \propto E_k^{-3.0}$, $W_c = 1 \text{ eV/cm}^3$. Curves 1, 2 and 3 correspond to $\beta = 0, 0.5$ and 1.

It is seen from Fig. 3.7.8 that with a growth of parameter β in Eq. 3.7.8, the dimension of the region occupied by solar wind should be somehow increased: the distance where u/u_1 is decreased by an order, equals 72, 100, and 112 AU, respectively, at $\beta = 0, 0.5$, and 1.

The character of CR modulation at $N_{Ho} = 0.5 \text{ cm}^{-3}$ is seen from Fig. 3.7.9 for particles with the rigidity R from 0.5 to 100 GV.

A comparison with the results of Section 3.7.2 (where it was assumed $N_{Ho} = 0$) shows that including of the process of re-charging results in some weakening of the modulation depth; it is caused by more rapid deceleration of solar wind with a growth of N_{Ho} .

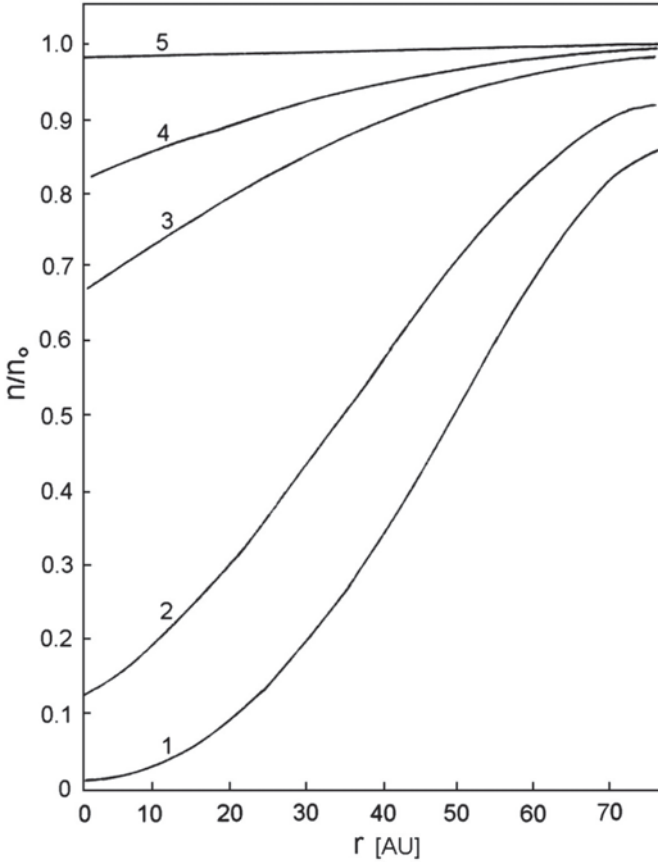


Fig. 3.7.9. Modulation depth of CR n/n_o depending on a distance to the Sun r for particles with the rigidity $R = 0.5, 1, 5, 10$, and 100 GV (Curves 1–5, respectively) at $N_H = 0.5 \text{ cm}^{-3}$, $\beta = 0$, $u_1 = 3 \times 10^7 \text{ cm/sec}$, $N_1 = 5 \text{ cm}^{-3}$, $n_o \propto E_k^{-2.5}$, $W_c = 1 \text{ eV/cm}^3$.

3.8. Expected change of solar wind Mach number accounting the effects of radial CR pressure and re-charging with neutral interstellar atoms

If we introduce the Mach number $M = u/V_s$, where

$$V_s = (\gamma P_g / \rho)^{1/2} \quad (3.8.1)$$

is the sound velocity, then on the basis of Eq. 3.7.13, taking into account the analytical solution of Eq. 3.7.11 for CR isotropic diffusion, we obtain:

$$M^{-2} \frac{dM}{dr^2} \left(8M^2 r^3 \left(\frac{dP_{CR}}{udr} + Q_{ex} \rho u N_H(r) \right) - 12 \mathfrak{S} \left(1 + \frac{M^2}{3} \right) \right) \left(3 \mathfrak{S} r (1 - M^2) \right)^{-1} = 0, \quad (3.8.2)$$

where $\mathfrak{S}$ is determined by Eq. 3.7.14. The behavior of $M(r)$ for several cases is shown in Fig. 3.8.1.

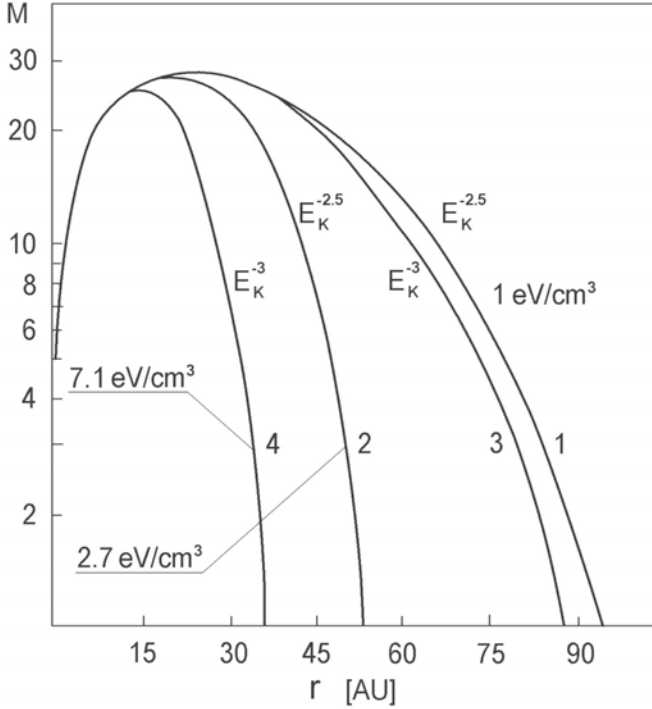


Fig. 3.8.1. Expected change of Mach number M versus radial distance r for $N_o \propto E_k^{-2.5}$ (curves 1 and 2) and for $N_o \propto E_k^{-3.0}$ (curves 3 and 4) for different CR energy density. According to Babayan and Dorman (1979a).

3.9. On the type of transition layer from supersonic to subsonic fluid of the solar wind

The especially important problem is the behavior of $M(r)$ and $u(r)$ near critical point $M^2 = 1$: what type of transition layer from supersonic regime to subsonic regime is realized – of gradual type or of shock wave type? To solve this problem Babayan and Dorman (1990) considered the behavior of the Eq. 3.8.2 near $M^2 = 1$. Let us suppose that near the critical point $r = r_c$ (where $M^2 = 1$)

$$\frac{dP_c}{u dr} \approx Kr^j, \quad (3.9.1)$$

where K and j are some constants. Then from Eq. 3.8.2 we obtain:

$$M^{-2} \frac{dM^2}{dr} = \left(\frac{8}{3} M^2 r^3 (Kr^j + \mathfrak{S}^{-2} L_{ex}^{-1} - \mathfrak{S} r^{-3} L L_{ex}^{-1}) - 4 \mathfrak{S} \left(1 + \frac{M^2}{3} \right) \right) \left(\mathfrak{S} r (1 - M^2) \right)^{-1}, \quad (3.9.2)$$

where j was determined by Eq. 3.9.1, and $\mathfrak{S}$ – by Eq. 3.7.14, $L_{ex} = Q_{ex} N_{Ho}$, and L is the distance at which $N_H(r)$ decreases by a factor of e . The behavior of integral curves of Eq. 3.9.2 near the critical point $r = r_c$ will be determined by the characteristic equation

$$X^2 - BX + C = 0, \quad (3.9.3)$$

where

$$B = \frac{16}{3} Kr_c^{3+j} + \mathfrak{S} \left(r_c L_{ex}^{-1} - L L_{ex}^{-1} - \frac{20}{3} \right); \quad C = \frac{8 \mathfrak{S} r_c}{3} (Kr_c^{2+j} (3+j) + \mathfrak{S} L_{ex}^{-1}). \quad (3.9.4)$$

The position of the critical point $r = r_c$ is determined by the equation which follows from Eq. 3.9.2:

$$Kr_c^{3+j} + \mathfrak{S} (r_c L_{ex}^{-1} - L L_{ex}^{-1} - 2) = 0. \quad (3.9.5)$$

According to the characteristic Eq. 3.9.3, the main Eq. 3.9.2 has critical point $r = r_c$ (which is determined by Eq. 3.9.5) of the type knot (gradual transition) if $B^2 - 4C \geq 0$ and of type focus (shock wave transition) if $B^2 - 4C < 0$. It means that if $j \leq j_c$ we obtain near the critical point $r = r_c$ gradual transition from supersonic flow to subsonic flow, but if $j > j_c$ we expect near the critical point $r = r_c$ shock wave transition from supersonic flow to subsonic flow. The value of j_c is determined from the condition $B^2 - 4C = 0$:

$$j_c = -2 - \frac{1 + 2 L L_{ex}^{-1}}{2(2 - r_c L_{ex}^{-1} + L L_{ex}^{-1})}. \quad (3.9.6)$$

According to Semar (1970) and McDonough and Brice (1971) the probable value of $L \approx 3 \div 5 \text{ AU} \approx (4.5 \div 7.5) \times 10^{13} \text{ cm}$ and $L_{ex} \approx 5 \times 10^{15} \text{ cm}$ (at $N_{Ho} = 0.1 \text{ cm}^{-3}$), so $2LL_{ex}^{-1} \approx (1.8 \div 3.0) \times 10^{-2}$. We expect that $r_c \approx 100 \text{ AU}$, so $r_c L_{ex}^{-1} \approx 0.3$, and $j_c \approx -(2.29-2.30)$. The value j_c depends very weakly on parameters L , L_{ex} and r_c . For example, if $r_c = 150 \text{ AU}$, then $j_c \approx -(2.32-2.33)$. From data on the radial CR gradient obtained for the inner Heliosphere by space probes Pioneer, Voyager, and others, it is expected that the value $j > -2$ (therefore we expect a shock wave transition), but what will be the situation near the critical point $r = r_c$, is not exactly clear. It needs a special investigation, including consideration of kinetic stream instability in the outer Heliosphere, which can change the diffusion coefficient and therefore the dependence determined by Eq. 3.9.1.

3.10. Non-linear influence of pickup ions, anomalous and galactic CR on the Heliosphere's termination shock structure

3.10.1. Why are investigations of the Heliosphere's termination shock important?

According to Le Roux and Fichtner (1997a,b) the shock transition terminating the supersonic solar wind, the so-called heliospheric shock, has received increasing attention for several reasons:

First, the deep space probes Pioneer and Voyager are entering the outer heliospheric region where the heliospheric shock is supposedly located, and it is of importance to have some expectation of how it might show up in the data (e.g., Barnes, 1993; Suess, 1993; Paularena et al., 1996). An indication that Pioneer 10 and Voyager 1, both located beyond a heliocentric distance of $\sim 60 \text{ AU}$, might, in fact, be relatively close to the heliospheric shock is given by the detection of anomalous hydrogen by these spacecrafts (Christian et al., 1995; McDonald et al., 1995; Stone et al., 1996).

Second, the heliospheric shock is a key element in structuring the global Heliosphere, which is currently the subject of extensive numerical modeling (e.g., Baranov and Malama 1993; Karmesin et al., 1995; Linde et al., 1996; Pauls and Zank, 1996; Ratkiewicz et al., 1996).

Third, the notion that beyond 120 AU the pressure of pickup ions might be much larger than the thermal pressure of the solar wind (Isenberg, 1986) or than the magnetic field pressure (Whang et al., 1995) drives some interest in its influence on the dynamics of the outer Heliosphere (e.g., Fahr and Fichtner, 1995) concerning the location and modification of the heliospheric shock (Zank et al., 1993; Lee, 1997).

Fourth, the properties of anomalous CR, probably produced at the heliospheric shock, give rise to the question of how the heliospheric shock structure, determining the diffusive shock acceleration process, looks in detail (e.g., Lee, 1997; Le Roux et al., 1996).

Previously the influence of pickup ions, anomalous CR, and galactic CR on the structure of the heliospheric shock have been studied separately (e.g., Ko et al., 1988; Lee and Axford 1988; le Roux and Ptuskin 1995a,b) or, in a simplified approach, for combinations of some or all of the energetic particle populations (Zank et al., 1993; Lee et al., 1996; Le Roux et al., 1996). Le Roux and Fichtner (1997a,b) studied the simultaneous influence of all energetic particle populations on the structure of the heliospheric shock and developed a self-consistent time-dependent model on the non-linear influence of pickup ions, anomalous and galactic CR on the Heliosphere's termination shock structure. They demonstrate that on the basis of the currently available data the heliospheric shock structure cannot be clarified unambiguously, but that there are at least two different alternatives consistent with observations obtained so far.

3.10.2. Description of the self-consistent model and main equations

Le Roux and Fichtner (1997a,b) developed a self-consistent time-dependent model of the non-linear influence of pickup ions, anomalous and galactic CR on the Heliosphere's termination shock structure. This model generalized earlier approaches of Ko et al. (1988) and Donohue and Zank (1993) by taking into account the self-consistent interaction of the thermal plasma of solar wind (including pickup ions) with anomalous and galactic CR which propagation is described by the transport equation (Parker, 1965):

$$\frac{\partial f}{\partial t} + u \frac{\partial f}{\partial r} - \frac{1}{r^2} \frac{\partial f}{\partial r} \left(r^2 \kappa \frac{\partial f}{\partial r} \right) - \frac{1}{3r^2} \frac{\partial f}{\partial r} (r^2 u) p \frac{\partial f}{\partial p} = \eta Q_{pi}. \quad (3.10.1)$$

In the transport Eq. 3.10.1 $u(r, t)$ is the solar wind velocity, $f(r, p, t)$ is the omnidirectional CR distribution function, and

$$\kappa = \kappa_{\parallel} \cos^2 \psi + \kappa_{\perp} \sin^2 \psi \quad (3.10.2)$$

is the radial diffusion coefficient ($\kappa_{\parallel}$ and $\kappa_{\perp}$ are the parallel and perpendicular components of CR diffusion coefficient relative to the regular component of interplanetary magnetic field $\mathbf{B}$, and $\psi = \tan^{-1}(\Omega r/u)$ is the winding angle between $\mathbf{B}$ and radial direction, Ω is the angular speed of the Sun). In numerical calculations of propagation of anomalous and galactic CR in Le Roux and Fichtner (1997a,b) were used

$$\kappa_{//} = \frac{v}{c} \frac{P}{1GV} \frac{5nT}{B} \times (3.8 \times 10^{22}) \text{ cm}^2 \text{ sec}^{-1}; \quad \kappa_{\perp} = \varepsilon \kappa_{//}; \quad \varepsilon = 0.015. \quad (3.10.3)$$

Let us note that in Le Roux and Fichtner (1997a,b) Eq. 3.10.1 was used also for describing the propagation of pickup ions with $p > mu$ as well as with $p < mu$ (for the last case in Eq. 3.10.1 was assumed diffusion coefficient $\kappa = 0$).

Let us consider the right hand part of Eq. 3.10.1. According to Le Roux and Fichtner (1997a,b) the calculation of Q_{pi} based on standard values for the interstellar neutral density of hydrogen H ($n_{\infty} = 0.077 \text{ cm}^{-3}$, where n_{∞} is the interstellar neutral density of H at large distances r from the Sun), and the ionisation frequency ($\nu_e = 5 \times 10^{-7} \text{ sec}^{-1}$, where ν_e is the ionisation frequency at the Earth's orbit). The injection efficiency η in the right hand of Eq. 3.10.1 represents the fraction of those pickup ions that, by adiabatic heating across the Heliosphere's terminal shock, attain momentum $p > mu$.

The solar wind speed is self-consistently calculated in Le Roux and Fichtner (1997a,b) from a system of time-dependent equations describing a spherically symmetric one-fluid solar wind propagation (including pickup ions) in the presence of anomalies and galactic CR:

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \rho u) = m Q_{ph}, \quad (3.10.4)$$

$$\frac{\partial \rho u}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \rho u^2) = - \frac{\partial (P + P_c)}{\partial r} - mu Q_{ce}, \quad (3.10.5)$$

$$\begin{aligned} \frac{\partial}{\partial t} \left(\frac{1}{2} \rho u^2 + \frac{P}{\gamma - 1} \right) + \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 u \left(\frac{1}{2} \rho u^2 + \frac{\lambda P}{\lambda - 1} \right) \right) = -u \frac{\partial P_c}{\partial r} \\ + \alpha_{pi} \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 u) - \frac{1}{2} m \left[u^2 H(r_{sh} - r) + v_{rms}^2 H(r - r_{sh}) \right] Q_{ce}, \end{aligned} \quad (3.10.6)$$

where ρ and P are the mass density and thermal pressure of the solar wind gas including pickup ions, $\gamma = 5/3$ is the polytropic index for the thermal gas, $P_c = (4\pi/3) \int p^3 v f dp$ is the CR pressure. In Eq. 3.10.4–3.10.6 the source and loss terms are determined by the production rates of pickup ions resulting from photoionisation Q_{ph} of and charge exchange Q_{ce} with, interstellar hydrogen. Following Lee (1997), the terms were derived under the assumption that upstream

of the heliospheric shock, the energy density loss of the solar wind, and Q_{ce} depend mainly on solar wind velocity u , but downstream of the heliospheric shock depends mainly on the root mean square velocity v_{rms} of solar wind protons. In Eq. 3.10.6 $H(r_{sh} - r)$ and $H(r - r_{sh})$ are the Heaviside function with r_{sh} the heliospheric shock radius. The coefficient α_{pi} in Eq. 3.10.6 results from the transfer of pickup ions with $p < mu$ across the threshold $p = mu$ from the thermal to the supra-thermal population by adiabatic heating (according to Zank et al, 1993).

3.10.3. Using methods of numerical calculations

According to Le Roux and Fichtner (1997a,b), the parabolic transport Eq. 3.10.1 describes both anomalous CR resulting from the injection and diffusive shock acceleration of pickup ions at the heliospheric shock and galactic CR incorporated by prescribing an interstellar spectrum (see, e.g., McDonald et al., 1995) at the outer boundary at 120 AU. This equation was solved by using a combination of the implicit Crank-Nicholson method for spatial diffusion and the explicit monotonic transport scheme for convection and adiabatic energy changes. For the system of hyperbolic fluid Eq. 3.10.4–3.10.6 describing the thermal gas mixture, solved with a Riemann algorithm (LeVeque, 1994), standard solar wind conditions at the inner boundary $r_i = 1$ AU are used ($u_i = 400 \text{ km s}^{-1}$, $\rho_i = 5 \text{ m}_p \text{ cm}^{-3}$, $T_i = 10^5 \text{ K}$). At the outer boundary $r_o = 120 \text{ AU}$, a constant downstream density (i.e. $\partial\rho/\partial r|_{r=r_o} = 0$), a mass flux decreasing proportional to $1/r^2$ (i.e. $\partial mr^2/\partial r|_{r=r_o} = 0$), and a thermal pressure equal to the local interstellar pressure (i.e. $p = p_{thLISM} = 1 \text{ eV/cm}^3$) are assumed.

3.10.4. Expected differential CR intensities on various heliocentric distances

For η , the injection efficiency of pickup protons into the process of diffusive acceleration at the heliospheric shock, Le Roux and Fichtner (1997a,b) found one high and one low value of the free parameter η resulting in CR flux levels consistent with *Pioneer* and *Voyager* observations during 1987. Fig. 3.10.1 shows the differential intensity $J(E_k) = p^2 f$ of combined pickup ions, anomalous and galactic CR as a function of kinetic energy E_k for various heliocentric distances for these solutions: (a) $\eta = 0.0003$ and (b) $\eta = 0.9$. Since the injection efficiency as defined above denotes only a fraction of those pickup ions with velocities $w > u$, the actual number of injected particles represents a smaller fraction of the total pickup ions population than is indicated by η . The percentage of pickup protons at the heliospheric shock having velocities greater than u as a consequence of

adiabatic heating is found to be 98% and 92% for (a) and (b), respectively. For $\eta = 0.0003$ and $\eta = 0.9$, Le Roux and Fichtner (1997a,b) found that 0.03% and 83% of all pickup ions are diffusively accelerated for (a) and (b), respectively. From an analytical estimate employing an upstream pickup ions distribution derived by Vasyliunas and Siscoe (1976) in combination with the self-consistently determined solar wind deceleration, one obtains (a) 0.02% and (b) 15% for the actual injection rate. These numbers demonstrate not only that the values obtained numerically represent a tendency of the algorithm to accelerate particles too efficiently (Hawley et al., 1984), but also that, in order to reproduce the observed spectrum with a high-injection case, the actual injection fraction has to be increased. Such an increase could be achieved by the inclusion of a pre-acceleration mechanism for pickup ions (e.g., Chalov and Fahr, 1996; Fichtner et al., 1996; Lee et al., 1996; Zank et al., 1996).

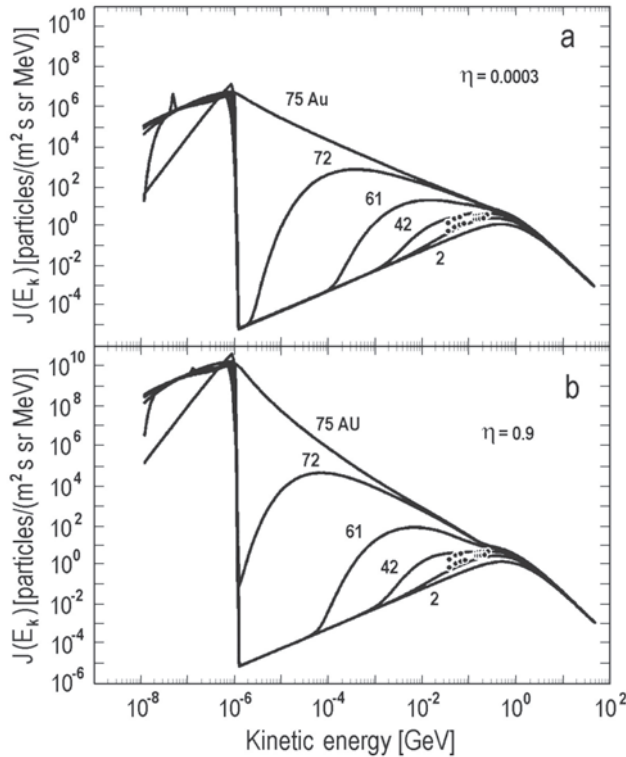


Fig. 3.10.1. The combined pickup ions, anomalous and galactic CR differential intensities in $\text{particles m}^{-2}\text{s}^{-1} \text{srad}^{-1}\text{MeV}^{-1}$ as a function of kinetic energy in GeV in the upwind direction. From bottom to top the spectra are shown at 2, 23, 42, 61, 72, and 75 AU, respectively, with 75 AU just downstream of the heliospheric shock. The top panel for $\eta = 0.0003$, the bottom panel for $\eta = 0.9$. The filled circles represent proton data in 1987 from Voyager 2 and Pioneer 10 at 23 and 42 AU, respectively (McDonald et al., 1996). According to Le Roux and Fichtner (1997a,b).

3.10.5. Different cases of heliospheric shock structure and solar wind expansion

From Fig. 3.10.1 it can be seen that the modulated spectra for distances smaller than 60 AU are basically identical; it means that there are differences farther out owing to the different heliospheric shock structure. The parameters describing this structure are listed in Table 3.10.1 for both injection cases (a) and (b) along with those for three non-injection cases (1–3) serving as reference solutions.

Table 3.10.1. Heliospheric shock parameters for different cases. According to Le Roux and Fichtner (1997b)

case	energetic particles	η	r_{sh} , AU	s	L , (AU)	$\Delta u/u_i$, %
1	no pickup ions or CR	0	80.8	4.0	0	0
2	pickup ions, but no CR	0	73.7	3.5	0	-12.8
3	pickup ions and galactic CR	0	71.8	3.5	37	-17.3
(a)	pickup ions, galactic and anomalous CR	0.0003	74.3	3.4	34	-19.8
(b)	pickup ions, galactic and anomalous CR	0.9	73.8	1.5	12	-45.0

Notes to Table 3.10.1: η is the pickup ions injection efficiency, r_{sh} is the heliocentric distance to the heliospheric shock, s is the sub-shock compression ratio, L – the extent of precursor, $\Delta u/u_i$ – the total solar wind deceleration ($u_i = 400$ km/s).

According to Le Roux and Fichtner (1997a,b), **case 1** in Table 3.10.1 corresponds to a solar wind expansion unaffected by the presence of pickup ions and CR. Consequently the heliospheric shock is strong, with a compression ratio $s = 4.0$, and there is no deceleration upstream. The mere presence of neutral particles and, subsequently, pickup ions, i.e., **case 2** (dashed lines in Fig. 3.10.2a and Fig. 3.10.2b), decelerates the solar wind on 12.8% according to Table 3.10.1. A higher temperature of the gas mixture in the outer Heliosphere causes its Mach number to decrease from 141 to 4.3; thus the compression ratio decreases to $s = 3.5$. Because of the lower solar wind ram pressure the heliospheric shock moves in from 80.8 to 73.7 AU. Reference **case 3** also includes galactic CR. The effect is twofold: besides further reducing the heliocentric distance of the heliospheric shock to 71.8 AU as a consequence of the increased external pressure ($p_{th,LISM} + p_{GCR}$), galactic CR enhance the solar wind deceleration. Compared with case 2 the deceleration is increased in a region of ~ 37 AU upstream by 14.5%, resulting in a total increase in deceleration of 17.3%. In the following, Le Roux and Fichtner (1997a,b) refer to this region of deceleration in excess of that owed to pickup ions

(case 2) as the ‘precursor’. If such a precursor exists, the shock transition is referred to as a sub-shock. The extended precursor of case 3 can be attributed to the high effective diffusion length (k/u) of galactic CR. The results obtained for these reference cases correspond to those reported by Ko et al. (1988), Lee and Axford (1988), Lee (1997).

Let us now turn to the situation where all three energetic particle populations are present simultaneously, i.e. cases (a) and (b) in Table 3.10.1. Fig. 3.10.2 shows the heliospheric shock structure resulting from (a) low and (b) high injection. According to Grzedzielski and Ziemkiewicz (1990) and Lee (1997), the anomalous CR should push the heliospheric shock away from the Sun. Results of Le Roux and Fichtner (1997a,b) confirm this finding (see Table 3.10.1). While the effects of galactic CR and anomalous CR on the sub-shock location nearly compensate each other, so that the location is close to that of the pure pickup ions (case 2 in Table 3.10.1), the other parameters defining the heliospheric shock structure are distinctly different. For the low injection (case (a) in Table 3.10.1) the compression ratio is slightly lower than for case 3, the extent of the precursor is somewhat shorter (~ 34 AU), and the deceleration of the solar wind is more pronounced (19.8%). Even so, the precursor is determined mainly by galactic CR and the sub-shock by pickup ions. For case (b) the solar wind deceleration is 45.0% and both the compression ratio ($s = 1.5$) and the precursor extent (~ 12 AU) are significantly reduced, indicating that anomalous CR with a relatively small effective diffusion length dominate the overall structure of the heliospheric shock.

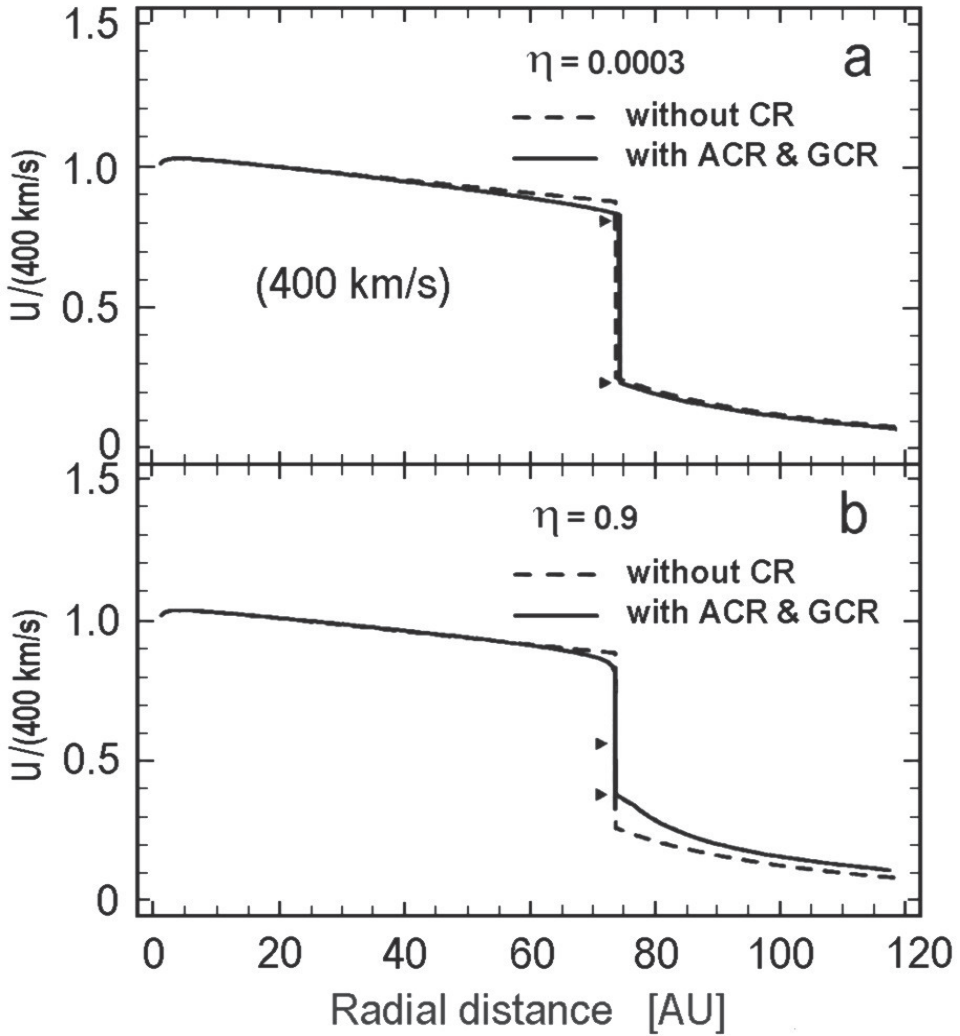


Fig. 3.10.2. Calculated solar wind speed normalised to 400 km/sec as a function of the radial distance to the Sun (in AU units) in the upwind direction. The dashed curve is the solution which includes the dynamic effects of pickup ions, but not anomalous and galactic CR. The solid curve inclines in addition the dynamic effects of anomalous and galactic CR. Top panel for $\eta = 0.0003$; bottom panel for $\eta = 0.9$. The solid triangles indicate the subshock. According to Le Roux and Fichtner (1997a,b).

The difference between the two cases (a) and (b) can be further illustrated with a comparison of the combined anomalous CR and galactic CR pressure with the solar wind ram and thermal pressure (Fig. 3.10.3).

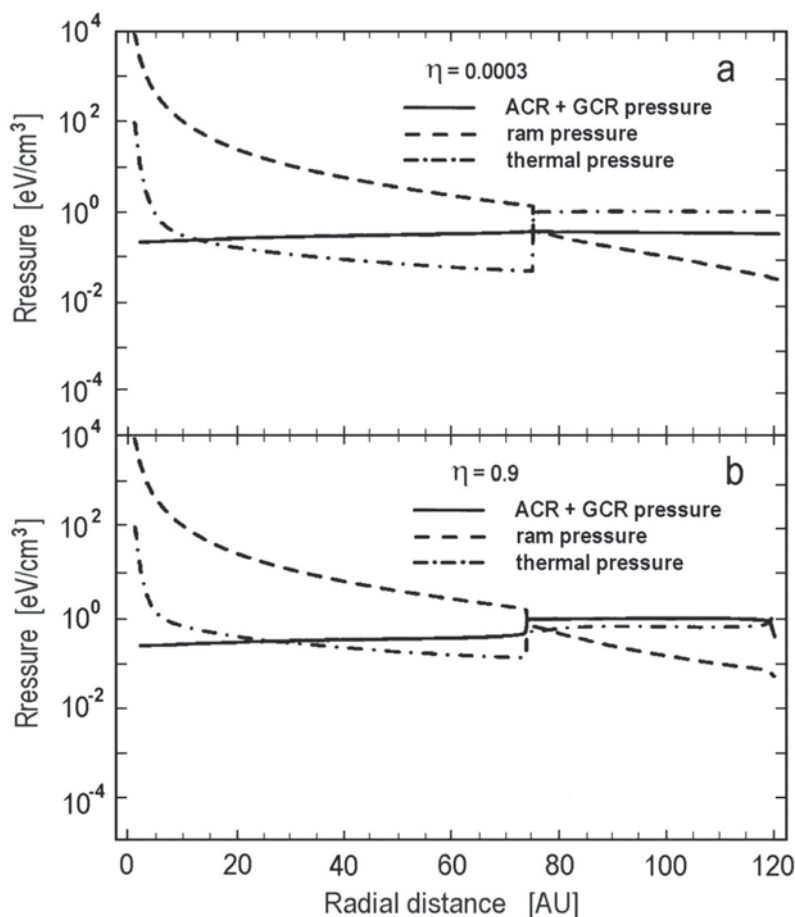


Fig. 3.10.3. CR pressure (solid line) compared to the ram (dashed line) and the thermal pressure (dash-dotted line) for (a) $\eta = 0.0003$ and (b) $\eta = 0.9$. All pressures are in eV cm^{-3} . According to Le Roux and Fichtner (1997b).

From Fig. 3.10.3 it can be seen that for $\eta = 0.0003$, the contribution from anomalous CR is negligible and the CR pressure profile is rather flat. Also the CR pressure is everywhere smaller than that of the thermal plasma; its ram pressure dominates upstream, its thermal pressure downstream. For $\eta = 0.9$, however, the acceleration of anomalous CR results in a significant CR pressure at and downstream of the heliospheric shock.

According to Le Roux and Fichtner (1997a,b) the strong modification of the heliospheric shock shown in Fig. 3.10.2b and given in Table 3.10.1 can be understood in view of the large CR pressure gradient close to the heliospheric shock seen in Fig. 3.10.3b. This gradient forces the solar wind to decelerate strongly. It is also evident why both the high- and the low-injection case result in

similar flux levels for distances smaller than ~ 60 AU: the production of anomalous CR leads to a pressure buildup close to, at, and beyond the heliospheric shock, but not very far upstream. If only a small fraction of the pickup ion population is injected, the sub-shock strength remains relatively high ($s = 3.4$) and the acceleration remains sufficiently efficient to produce the observed flux levels. If, on the other hand, a larger fraction of pickup ions becomes injected, the heliospheric shock is strongly modified ($s = 1.5$) and its acceleration is efficiency reduced, accompanied by stronger modulation, i.e., larger radial gradients $g_r = Cu/\kappa$, particularly beyond ~ 60 AU. These effects occur because the steeper spectral gradient $\partial J(E_k)/\partial E_k$ at the heliospheric shock (see Fig. 3.10.1) implies a larger Compton-Getting factor

$$C = \frac{1}{3} \left[2 - \frac{E_k + 2m_p c^2}{E_k + m_p c^2} \frac{\partial \ln J(E_k)}{\partial \ln E_k} \right]. \quad (3.10.7)$$

As a consequence of both effects, case (b) gives flux levels similar to those of case (a) inside ~ 60 AU. At distances smaller than ~ 60 AU it is difficult to distinguish between the low- and the high-injection case (see Fig. 3.10.1–3.10.3). Thus, an observational discrimination between the two alternatives (before a spacecraft encounters the heliospheric shock) can only be made in that part of the precursor that is close to the heliospheric shock.

3.10.6. The summary of obtained results

From Fig. 3.10.1–3.10.3 and Table 3.10.1 it can be seen how the heliospheric shock is modified by the simultaneous presence of pickup ions, anomalous and galactic CR for the low ($\eta = 0.0003$) and high ($\eta = 0.9$) injection efficiency cases. The presence of pickup ions in the solar wind according to Le Roux and Fichtner (1997a,b) results in:

1. The solar wind decelerates upstream of the heliospheric shock by about 12.8%, because of the charge exchange between solar wind protons and interstellar hydrogen; 2. Consequently the solar wind ram pressure is lower and the heliospheric shock moves inward from 80.8 AU (the initial position without pickup ions or CR) to 73.7 AU;
3. The heliospheric shock compression ratio is reduced from $s = 4$ to $s = 3.5$, because the mixture of solar wind protons with the hot pickup ions decreased the upstream Mach number from ≈ 14.1 to ≈ 4.3 .

According to Le Roux and Fichtner (1997a,b), **for very small injection efficiency ($\eta = 0.0003$)** the heliospheric shock moves outward from 73.7 to 74.3 AU. Because anomalous CR move the heliospheric shock outward and galactic CR move it inwards, it means that anomalous CR protons are more important than galactic CR protons in determining the heliospheric shock position. The outward

movement of the heliospheric shock is caused by the loss of internal energy of the solar wind (including pickup ions) across the heliospheric shock owed to the transfer of pickup ions across the threshold to the anomalous CR population by adiabatic heating. The inward movement of the heliospheric shock by galactic CR is owed to the positive galactic CR gradient which decelerates the solar wind and reduces the ram pressure. The compression ratio is reduced slightly by the anomalous CR to $s = 3.4$. Mainly owing to galactic CR, a precursor to the heliospheric shock's subshock is formed with a scale length of ≈ 34 AU which increases the total deceleration of the solar wind to $\approx 19.8\%$. Despite the combined presence of both anomalous CR and galactic CR, the heliospheric shock position and compression ratio are still dominated by pickup ions.

In the case of a big injection efficiency ($\eta = 0.9$) the heliospheric shock according to Le Roux and Fichtner (1997a,b) also moves outward, but to a lesser degree. The heliospheric shock is strongly modified by mainly anomalous CR protons from $s = 3.5$ to 1.5. Largely owing to anomalous CR, a precursor to the heliospheric shock's subshock is formed with a shorter scale length of ≈ 15 AU which increases the total solar wind deceleration dramatically to $\approx 45\%$. In this case the heliospheric shock is predominantly modified by anomalous CR protons, while its position is still mainly determined by pickup ions. Galactic CR protons contribute the least in modifying the heliospheric shock.

In Fig. 3.10.1 the combined spectra of pickup ions, anomalous and galactic CR protons are shown as solid curves at different radial distances r . The pickup ions spectra are noticeable at kinetic energies $E_k < 10^{-6}$ GeV, whilst for $E_k > 10^{-6}$ GeV, anomalous CR proton intensities dominate the galactic CR proton intensities, except for $r < 23$ AU, and $E_k > 200$ MeV where the reverse is true. Let us consider two cases:

1. For $\eta = 0.0003$ the spectrum at the heliospheric shock is a power law except at the highest energies where it rolls over because $\kappa/u > r_{sh}$. This implies that the galactic CR induced heliospheric shock precursor is too small so that anomalous CR at all energies see the same effective $s = 3.4$ value. The dynamical influence of CR on the heliospheric shock is small, and a test particle approach would have given a similar result.

2. For $\eta = 0.9$ the spectral slope of anomalous CR at the heliospheric shock is steeper overall and has a clear energy dependence. At low energies the spectrum has a steep slope because the anomalous CR see only the subshock with $s = 1.5$, while undergoing diffusive shock acceleration. At higher energies the particles have a larger effective diffusion length and see additionally the CR induced precursor in crossing the heliospheric shock (effectively seeing a larger s -value), leading to a decreasing slope. Despite these large differences in the spectral slopes of anomalous CR at the heliospheric shock, the roll over portion of the spectrum basically occurs at the same energies, because κ/u is the same for both η -values.

Consequently from Fig. 3.10.2 can be seen that the modulated intensity levels for energies close to the roll over energy are similar for the two injection efficiencies $\eta = 0.0003$ and 0.9 . They are also nearly the same at lower energies except close to the heliospheric shock beyond ≈ 60 AU and below ≈ 200 MeV, where the spectra have different radial gradients (this is owing to the Compton-Getting factor C because the CR radial gradient $\approx Cu/\kappa$ which is determined by the spectral slope at the heliospheric shock, see Eq. 3.10.7).

3.11. Expected CR pressure effects in transverse directions in Heliosphere

3.11.1. CR transverse gradients in the Heliosphere and its possible influence on solar wind moving

The data on annual and semi-annual variations of CR show a presence of transverse CR gradients in the Heliosphere. A presence of these gradients results also from an assumption that a part of the observed hysteresis of CR (Dorman and Dorman, 1967a,b,c) is caused by a shift of a zone of solar activity toward low latitudes during a cycle of solar activity (Stozhkov and Charakchyan, 1969). The existing of CR transverse gradients follows also from the analysis of NM data and muon telescopes underground data for about 25 years on CR drift and convection diffusion anisotropies (Ahluwalia and Dorman, 1995a,b; Dorman and Ahluwalia, 1995; see also Chapter 16). Whereas solar activity is concentrated generally in the latitude band $\pm 30^\circ$, one should expect that CR density be to decrease with approaching helio-equator. Therefore a compression of solar wind will take place. A focusing action of CR over helio-latitude is produced in this way. Approximate estimation of this focusing action was performed by Dorman and Dorman (1969) with assumption that CR density in interplanetary space is a power function depending on a distance r to the Sun. Babayan and Dorman (1977b) carried out more accurate calculations of a transverse interaction of CR with solar wind including a solution of the problem of non-linear interaction of CR with solar wind in the radial direction (Babayan and Dorman, 1977a) which made it possible to find actually the dependence of density and pressure of CR on r .

3.11.2. The simple model for estimation of upper limit of CR transverse effects on solar wind

Let us consider the following idealized simple model. Let solar wind be distributed uniformly over longitude and concentrated on the both sides from the helio-equator in the helio-latitude zone $-\theta_o \leq \theta \leq +\theta_o$. Outside this helio-latitude band let the density of CR be equal to its interstellar value $n_o(E_k)$ and the pressure of CR on the latitude boundaries

$$P_{co} = \frac{1}{3} \int_0^{\infty} n_o(E_k) p v dE_k \quad (3.11.1)$$

(here p and v are a momentum and velocity of particles). To simplify the problem we shall consider that a distribution of CR density $n(r, E_k)$ in the helio-equatorial plane is determined by a solution of the problem of non-linear modulation in a spherically symmetric case (see Sections 3.7–3.9). Respectively, the pressure of CR in the plane of the helio-equator will be

$$P_c(r) = \frac{1}{3} \int_0^{\infty} n(r, E_k) p v dE_k. \quad (3.11.2)$$

Let the CR pressure at $\theta = \pm\theta_0$ be equal to the pressure in the interstellar space P_{co} . In this case the average value of the gradient of CR across the plane of the helio-equator at a distance r from the Sun will be

$$G_{\perp}(r) \approx (P_{co} - P_c(r))/r\theta_0. \quad (3.11.3)$$

The energy spectrum of CR in the plane of the helio-equator at a distance r from the Sun, according to diffusion-convection theory, is determined by the expression

$$n(r, E_k) = n_o(E_k) \exp\left(-\int_r^{\infty} \frac{3u(r)dr}{\Lambda(r)v}\right), \quad (3.11.4)$$

where $n_o(E_k)$ is the spectrum outside of solar wind, $\Lambda(r)$ is transport path for scattering of particles, $u(r)$ is the radial velocity of solar wind, determined including an inverse action of CR onto solar wind, respective to Section 3.7.2.

Whereas the problem is symmetric relative to the plane of the helio-equator, consider then a motion of solar plasma above the helio-equator. For a motion of solar plasma under the action of CR across the plane of the helio-equator, we have the equation

$$\rho(\vec{u} \cdot \vec{\nabla}) u_{\perp}(r) \approx (P_{co} - P_c(r))/r\theta_0, \quad (3.11.5)$$

where ρ is solar wind density and $u_{\perp}(r)$ is the velocity of solar wind plasma in the transverse direction. In Eq. 3.11.5 the terms are absent which take into account the gas kinetic pressure and the pressure of magnetic field, because we can assume for the first approximation that the transverse gradients of these pressures are equal to zero. It should be noted that with a strong compression of solar wind by CR there must arise a difference in gas kinetic and magnetic pressures inside

and outside the modulation zone of CR; in this case the respective terms will appear in Eq. 3.11.5, which will result in a decrease of the compression effect. Moreover, in the case of strong compression of the solar wind one must take into account that there will occur a rapprochement of magnetic inhomogeneities, and therefore a decrease of transport path for scattering Λ , i.e. intensification of CR modulation which, in its turn, should result in an increase of the transverse gradient of CR pressure and, finally, in an increase of the effect of solar wind compression. A variation of CR modulation will result in a respective variation of the dependence $u(r)$. Therefore in the general case, we have a considerably complicated self-consistent non-linear problem, the solving of which is difficult. Therefore Babayan and Dorman (1977b) presented only estimates obtained on the basis of Eq. 3.7.12 including Eq. 3.7.8, Eq. 3.7.9 and Eq. 3.7.11 of a solution for $u(r)$ which was presented in Section 3.7.2, to determine what is the expected effect of solar wind compression by CR and when the effect of non-linear interaction of CR with solar wind in the transverse direction is substantial. Let us add to Eq. 3.7.12 the equation of continuity for the solar wind

$$\rho(r)r^2u(r)=\rho_1r_1^2u_1, \quad (3.11.6)$$

where ρ_1 and u_1 are, respectively, a density and velocity of the wind at the distance of the radius of the Earth's orbit from the Sun $r_1 = 1 \text{ a.u.}$ Substituting Eq. 3.11.6 in Eq. 3.11.5 we obtain

$$\frac{du_{\perp}(r)}{dr} = \frac{r(P_{co} - P_c(r))}{\rho_1 u_1 r_1^2 \theta_o}. \quad (3.11.7)$$

Let us follow an element of solar wind that had, in the instant of ejection from the Sun ($r = 0$) only the radial component of its velocity, i.e., $u_{\perp}(r = 0) = 0$, and find a transverse velocity at a distance r . Eq. 3.11.7 results in

$$u_{\perp}(r) = \left(\rho_1 u_1 r_1^2 \theta_o \right)^{-1} \int_0^r (P_{co} - P_c(r)) r dr. \quad (3.11.8)$$

The transverse displacement of an element of solar wind is

$$x_{\perp}(r) = \int_0^r (u_{\perp}(r)/u(r)) dr. \quad (3.11.9)$$

It is obvious that a focusing action of CR onto solar wind will be substantial if $x_{\perp}(r)$ is comparable to $r\theta_o$. The numerical calculations were made by Babayan and Dorman (1977b) with respect to Eq. 3.11.8 and Eq. 3.11.9 including Eq. 3.7.8, Eq. 3.7.9, and Eq. 3.7.11 for CR interstellar spectrum of the form described by Eq. 3.7.7 for the values of $\gamma = 1.5$ and 2.0; $E_{k,\min} = 0.1$ and 0.01 GeV. The values of Λ were taken as the same as in Section 3.7. The velocity and density of solar wind on the Earth's orbit were set to be $u_1 = 3 \times 10^7$ and 4×10^7 cm/sec, $N_1 = 5$ and 10 cm^{-3} . In Fig. 3.11.1 the expected values of $u_{\perp}(r)$ and $x_{\perp}(r)/r\theta_o$ are presented at $\gamma = 1.5$, $u_1 = 3 \times 10^7$ cm/sec and $N_1 = 5 \text{ cm}^{-3}$. It is seen that $u_{\perp}(r)$ is initially increased, then it starts to diminish with distance. It is connected with $P_c(r)$ tending to P_{co} at great distances. The calculations were made up to the distances where the radial velocity of solar wind falls by an order compared to its initial value, owing to non-linear interaction with CR in the radial direction, with respect to the results of Sect. 3.7.

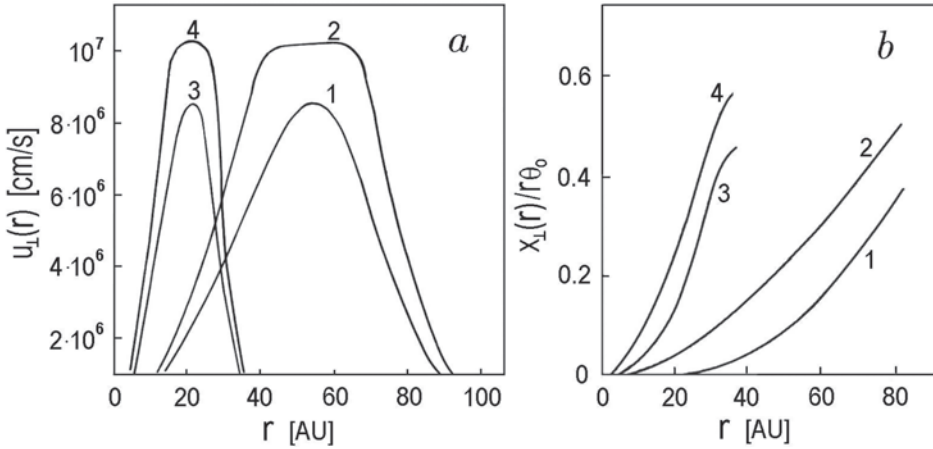


Fig. 3.11.1. Expected $u_{\perp}(r)$ and $x_{\perp}(r)/r\theta_o$ (a and b, respectively), for $\gamma = 1.5$; $u_1 = 3 \times 10^7$ cm/sec, $N_1 = 5 \text{ cm}^{-3}$. Curve 1 for $E_{k,\min} = 0.1$ GeV, $\theta_o = 30^\circ$; 2 - for $E_{k,\min} = 0.1$ GeV, $\theta_o = 25^\circ$; 3 - for $E_{k,\min} = 0.01$ GeV, $\theta_o = 30^\circ$, 4 - for $E_{k,\min} = 0.01$ GeV, $\theta_o = 25^\circ$.

In Fig. 3.11.2 the results of calculations are presented for $\gamma = 1.5$, $u_1 = 4 \times 10^7$ cm/sec and $N_1 = 10 \text{ cm}^{-3}$, and in Fig. 3.11.3 – the results are presented for $\gamma = 2.0$, $u_1 = 3 \times 10^7$ cm/sec and $N_1 = 5 \text{ cm}^{-3}$.

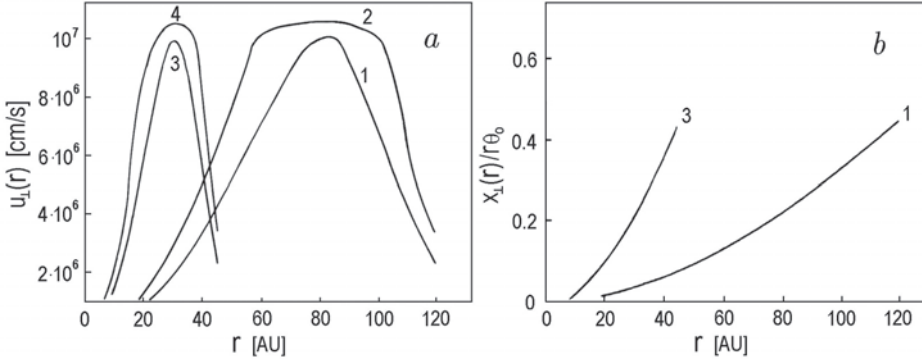


Fig. 3.11.2. Expected $u_{\perp}(r)$ and $x_{\perp}(r)/r\theta_0$ for $\gamma = 1.5$; $u_1 = 4 \times 10^7$ cm/sec, and $N_1 = 10 \text{ cm}^{-3}$. Notations are the same as in Fig. 3.11.1.

A comparison of Fig. 3.11.1 with Fig. 3.11.2 shows that with growth of u_1 and N_1 the effect of non-linear interaction of CR with solar wind is substantially decreased. This results from a comparison of Fig. 3.11.1 and Fig. 3.11.3 that with increasing γ the effect of non-linear interaction under consideration is increased.

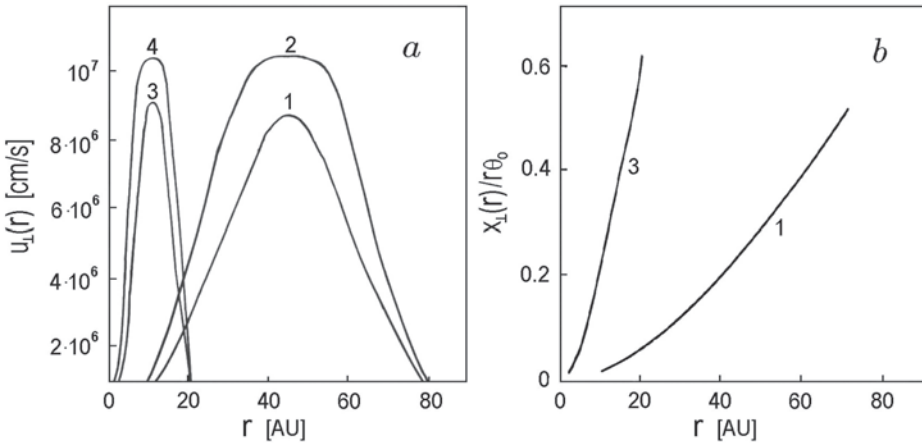


Fig. 3.11.3. Expected $u_{\perp}(r)$ and $x_{\perp}(r)/r\theta_0$ for $\gamma = 2.0$; $u_1 = 3 \times 10^7$ cm/sec, and $N_1 = 5 \text{ cm}^{-3}$. Notations are the same as in Fig. 3.11.1.

The obtained results show that the effect of non-linear interaction of CR with solar wind in the direction normal to ecliptic plane, becomes substantial (i.e., $x_{\perp}(r)/r\theta_0 \geq 0.1-0.2$) and it must be taken into account at the distances $r \geq 15-20$ AU with $E_{k,\min} = 0.01$ GeV and at $r \geq 30-40$ AU with $E_{k,\min} = 0.1$ GeV if γ

$=1.5$, $u_1 = 3 \times 10^7$ cm/sec, $N_1 = 5$ cm $^{-3}$. At the same time, if $u_1 = 4 \times 10^7$ cm/sec and $N_1 = 10$ cm $^{-3}$, the above distances are extended to 20–25 AU and 50–60 AU, respectively, for $E_{k,\min} = 0.01$ GeV and $E_{k,\min} = 0.1$ GeV. If, however, in the low energy range the power index in interstellar spectrum $\gamma = 2$, then at $u_1 = 3 \times 10^7$ cm/sec, $N_1 = 5$ cm $^{-3}$, the effect of non-linear transverse interaction becomes substantial at $r \geq 7$ –10 AU if $E_{k,\min} = 0.01$ GeV and at $r \geq 25$ –30 AU if $E_{k,\min} = 0.1$ GeV. In all of the mentioned cases, when non-linear effects of a transverse interaction of CR with solar wind become considerable, one must solve the self-consistent problem of CR modulation which was mentioned above when discussing Eq. 3.5.12.

3.11.3. The effect of the galactic CR gradients on propagation of solar wind in meridional plane

Babayán and Dorman (1981) considered a self-consistent set of equations describing the hydrodynamic flow of solar wind in the meridional plane including the pressure of galactic CR. The hydrodynamic equations are linearized assuming a small difference of the solar wind parameters from the spherically symmetric case. The differential equations have been obtained which describe the variations of the solar wind parameters in the meridional plane depending on the galactic cosmic ray gradients. We shall treat the stationary one-fluid polytropic model without magnetic field, i.e. we shall assume that the solar wind can be described by the following set of hydro-dynamic equations including equation of state:

$$\nabla(\rho \mathbf{u}) = 0; \quad \rho(\mathbf{u} \cdot \nabla)u = -\nabla P_g - \nabla P_c; \quad (3.11.10)$$

and that the equation of state is

$$P_g \rho^{-\gamma_g} = \text{const}. \quad (3.11.11)$$

Here ρ , $\mathbf{u}$, P_g are the density, velocity, and gas kinetic pressure of solar wind; r is the helio-centric distance; P_c is the pressure of galactic CR determined from the equation of anisotropic diffusion or by the Fokker-Planck equation including the diffusion, convection, drift, and energy change of CR particles. Gravitation is neglected since it is of significant importance only at the distances comparable with the distances at which the subsonic flow turns into supersonic flow. We are interested, however, in the distances much in excess of 1 AU.

Let the solution for the set determined by Eq. 3.11.10 be presented as

$$P_g = P_{go} + P'_g, \quad \rho = \rho_o + \rho', \quad u = u_o + u', \quad P_c = P_{co} + P'_c, \quad (3.11.12)$$

where the parameters with a dash will be assumed to be small compared with the parameters labelled by the index 'o', i.e. with non-disturbed parameters. Treated as the zero approximation will be the solution of the spherically symmetric model of solar wind including the effect of the radial gradient of galactic CR and the charge exchange of the solar wind protons with interstellar neutral hydrogen (Babayan and Dorman, 1979a,b; see also Section 3.7), i.e. we shall assume that

$$u_o = \{u_o(r), 0, 0\}, \quad \rho_o(r)u_o(r)r^2 = \text{const}, \quad P_{go}\rho_o^{-\gamma_g} = \text{const},$$

$$P_{co} = \frac{1}{3} \int_0^\infty n(E_k, r) p v dE_k, \quad (3.11.13)$$

where n , p , v , and E_k are the differential energy spectrum, momentum, velocity, and kinetic energy of CR particles. It will be noted that, in turn, the factor $n(E_k, r)$ is determined from the condition of equality between the diffusive and convective fluxes of galactic CR is a function of u_o .

After substituting Eq. 3.11.12 in Eq. 3.11.10 and linearizing the set of Eq. 3.11.10 for the solar wind propagation in meridional plane, the set of Eq. 3.11.10 will take the following forms: **for the r component**

$$\frac{\partial}{\partial r} \left(u_o u'_r + c_o^2 \frac{\rho'}{\rho_o} \right) - \frac{\rho'}{\rho_o^2} + \frac{1}{\rho_o} \frac{\partial P'_c}{\partial r} = 0; \quad (3.11.14)$$

for the θ component

$$\frac{\partial}{\partial r} \left(r u'_\theta \right) + \frac{c_o^2}{u_o} \frac{\partial}{\partial \theta} \left(\frac{\rho'}{\rho_o} \right) + \frac{1}{u_o \rho_o} \frac{\partial P'_c}{\partial \theta} = 0; \quad (3.11.15)$$

for the continuity condition

$$\frac{\partial}{\partial r} \left(\frac{u'_r}{u_o} + \frac{\rho'}{\rho_o} \right) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{u'_\theta}{u_o} \right) = 0, \quad (3.11.16)$$

where

$$P'_g = c_o^2 \rho', \quad c_o^2 = \frac{\gamma_g P_{go}}{\rho_o}. \quad (3.11.17)$$

Let us introduce the dimensionless variable $s = r/1\text{AU}$. The variables in the the set of Eq. 3.11.15–3.11.17 can be separated if the solution is presented as

$$\frac{u_r'}{u_o} = E_1(s)E_2(\theta), \quad \frac{u_\theta'}{u_o} = T_1(s)T_2(\theta), \quad \frac{\rho'}{\rho_o} = G_1(s)G_2(\theta), \quad P_c' = P_{c1}(s)P_{c2}(\theta), \quad (3.11.18)$$

and it is assumed that

$$E_2(\theta) = G_2(\theta) = P_{c2}(\theta). \quad (3.11.19)$$

Then the Eq. 3.11.14 takes the form

$$\frac{\partial}{\partial s} \left(u_o^2 E_1 + c_o^2 G_1 \right) - \frac{G_1}{\rho_o} \frac{\partial P_{co}}{\partial s} + \frac{1}{\rho_o} \frac{\partial P_{c1}}{\partial s} = 0, \quad (3.11.20)$$

and the Eq. 3.11.15, considering the condition described by Eq. 3.11.19, will be transformed into

$$\frac{u_o \rho_o}{c_o^2 G_1 \rho_o + P_{c1}} \frac{\partial}{\partial s} (s u_o T_1) = -\frac{1}{T_2} \frac{\partial P_{c2}}{\partial \theta} = K_1. \quad (3.11.21)$$

After separating the variables with due account of the condition described by Eq. 3.11.19, the continuity Eq. 3.11.16 will be

$$\frac{s}{T_1} \frac{\partial}{\partial s} (E_1 + G_1) = -\frac{1}{P_{c2} \sin \theta} \frac{\partial}{\partial \theta} (T_2 \sin \theta) = K_2. \quad (3.11.22)$$

Here K_1 in Eq. 3.11.21 and K_2 in Eq. 3.11.22 are separation constants. Thus, knowing the angular dependence of galactic CR pressure (for example, on the basis of the solution for the anisotropic diffusion equation) we may find the angular dependence of the θ -component of the velocity of inhomogeneous solar wind.

It will be noted that we may obtain for P_{c2} by combining the Eq. 3.11.21 and Eq. 3.11.22 the following equation:

$$\left(\cos^2 \theta - 1 \right) \frac{\partial^2 P_{c2}}{\partial (\cos \theta)^2} + 2 \cos \theta \frac{\partial P_{c2}}{\partial (\cos \theta)} + K_1 K_2 P_{c2} = 0 \quad (3.11.23)$$

the solution for which is the Legendre polynomials of the order of n at $K_1 K_2 = -n(n+1)$. The Eq. 3.11.20–3.11.23 may be used to obtain the differential equation for determining the radial dependence of the angular component of the solar wind velocity determining the meridional motion

$$\frac{\partial}{\partial s} \left[\left(\frac{u_o^2 K_2 T_1}{s} + \frac{\partial G_1}{\partial s} (c_o^2 - u_o^2) + G_1 \left(\frac{\partial c_o^2}{\partial s} - \frac{1}{\rho_o} \frac{\partial P_{co}}{\partial s} + \frac{\partial u^2}{\partial s} \right) + \frac{1}{\rho_o} \frac{\partial P_{cl}}{\partial s} \right) \frac{\partial u^2}{\partial s} \right] = -\frac{K_2 T_1}{s}, \quad (3.11.24)$$

where

$$G_1 = \frac{u_o}{K_1 c_o^2} \left(\frac{\partial}{\partial s} (\rho u_o T_1) - \frac{P_{cl} K_1}{\rho_o} \right). \quad (3.11.25)$$

3.12. Effects of CR kinetic stream instability in the Heliosphere

3.12.1. Rough estimation of stream instability effect at constant solar wind speed

First let us consider (Dorman et al., 1990) only the effects of CR kinetic stream instability in the outer Heliosphere without taking into account effects of CR pressure on the movement of the solar wind. Therefore we assume that the velocity of the solar wind $u = 400$ km/sec = const, the radius of the Heliosphere $r_o = 60$ AU, the spiral interplanetary magnetic field has components

$$H_r = (r/r_S)^{-2} H_S, \quad H_\phi = \left(ur/r_S^2 \Omega_S \right)^{-1} H_S \sin \theta, \quad H_\theta = 0, \quad (3.12.1)$$

where $r_S = 7 \times 10^{10}$ cm is the radius of the Sun, $H_S \approx 2$ Gs is the strength of the Sun's general magnetic field on the surface, $\Omega_S = 2.7 \times 10^{-6}$ sec⁻¹ is the angular velocity of solar rotation, θ is the polar angle ($\theta = \pi/2$ is the solar equator). At large distances from the Sun the full spiral interplanetary magnetic field H and the angle Ψ between magnetic field and radial direction will be

$$H \approx H_\phi \approx H_1 (r_1/r) \sin \theta, \quad \cos \Psi \approx r_1 (r \sin \theta)^{-1}, \quad (3.12.2)$$

where $r_1 = 1$ AU is the radius of the Earth's orbit, and $H_1 = H(r_1) \approx 5 \times 10^{-5}$ Gs is the strength of IMF at the distance of 1 AU from the Sun. The galactic CR

anisotropy generated in the interplanetary space is determined by the spiral magnetic field, particle scattering, and CR gradients and convection. Near the Earth's orbit the average anisotropy has an amplitude of about 0.5 % and is perpendicular to the radial direction. In the first approximation the amplitude of average anisotropy must be $\propto (\cos\Psi)^{-1} \propto r \sin\theta$, and at large distances from the Sun we expect a large amplitude of CR anisotropy. Therefore we expect that the effects of stream instability in the outer Heliosphere must be very important.

According to the Section 3.3 we consider the generation by CR stream instability of MHD waves propagated along the magnetic field (axis Z). the growth rate $\Gamma(k_z)$ determines the evolution of spectral energy density of MHD waves with wave number k_z in Z - direction and with accidental phases $W(k_z)$ will be determined by:

$$dW(k_z)/dt = 2\Gamma(k_z)W(k_z), \quad \langle (\delta H)^2 \rangle = 4\pi \int W(k_z) dk_z. \quad (3.12.3)$$

Resonance interaction of particles with momentum p and cosines of pitch angle $\mu = (\mathbf{pH})/(pH)$ will be with waves numbers $k_z = \pm ZeH/(pc\mu)$. We consider here only particles with velocity $v \approx c \gg V_a = H/(4\pi\rho)^{0.5} \approx 5 \times 10^6$ cm/sec for conditions in the interplanetary space. On the basis of Eq. 3.3.1 we obtain:

$$\begin{aligned} \Gamma(k_z) = & \frac{\pi^2 Z^2 e^2 V_a^2}{2c^2} \int d^3p \left(1 - \mu^2 \right) \left[\frac{\partial f(p, \mu)}{\partial p} + \left(\frac{k_z v}{\omega_A(k_z)} - \mu \right) \frac{\partial f(p, \mu)}{p \partial \mu} \right] \\ & \times \left[\delta \left(k_z \mu - \frac{ZeH}{pc} \right) \delta \left(k_z \mu + \frac{ZeH}{pc} \right) \right]. \end{aligned} \quad (3.12.4)$$

Here $\omega_A(k_z) = k_z V_a$. The connection between MHD waves and CR propagation is determined by the effective frequency of particle scattering on waves:

$$\nu^\pm = 2\pi^2 \omega_H k_{res} W^\pm(k_{res}), \text{ where } k_{res} = ZeH/(pc|\mu|)^{-1}, \quad \omega_H = ZeHv/(pc)^{-1}. \quad (3.12.5)$$

Let us consider the isotropic part of the CR distribution function $f_o = \int f(p, \mu) d\Omega / 4\pi$. For times $\Delta t \gg \nu^{-1}$ and distances $\Delta r \gg \nu \nu^{-1}$ the transport equation in the diffusion approximation will be

$$\frac{\partial f_o}{\partial t} - \nabla_i \kappa_{ij} \nabla_j f_o - (\nabla_i u_i) \frac{p \partial f_o}{3 \partial p} = 0, \quad (3.12.6)$$

where

$$\kappa_{ij} = \kappa_{//} h_i h_j, \quad \kappa_{//} = \frac{v^2}{2} \int_0^1 \frac{(1 - \mu^2) d\mu}{v^+ + v^-}, \quad h_i = H_i / H. \quad (3.12.7)$$

CR particles penetrating into the Heliosphere generate MHD waves along the spiral magnetic field in direction to the Sun ($k_z < 0$); therefore we will use sign ‘-’, i.e. v^- , W^- . Because we shall consider only this case, we do not use the index ‘z’ and the sign ‘-’ any further. Let us introduce the function $F(k) = kW(k)$. Then we obtain:

$$-r^{-2} \frac{\partial}{\partial r} \frac{r_1^4 c p v H_1^2}{4\pi^2 Z e r^2} \int_0^1 d\mu (1 - \mu^2) \left(F(ZeH/pc|\mu|) \right)^{-1} \frac{\partial f_o}{\partial r} + u \frac{\partial f_o}{\partial r} - \frac{2up}{3r} \frac{\partial f_o}{\partial p} = 0, \quad (3.12.8)$$

$$u \frac{\partial E}{\partial r} + \frac{3uF}{r} - \frac{2\pi V_A r_1^2 Z e H_1}{c k r^2} \int_{ZeH/ck}^{\infty} dp p^2 c \left(1 - \frac{ZeH}{pck} \right)^2 \frac{\partial f_o}{\partial r} = 0. \quad (3.12.9)$$

For approximate calculations we replace in the integral of Eq. 3.12.8 $F(ZeH/pc|\mu|)$ by $F(ZeH/pc)$ and introduce functions $F(p) = F(k = ZeH/pc)$ and $\Gamma(p) = \Gamma(k = ZeH/pc)$. Then from Eq. 3.12.8 and Eq. 3.12.9 we obtain

$$-r^{-2} \frac{\partial}{\partial r} \frac{r_1^3 c p v H_1}{6\pi^2 Z e r F(p) \sin \theta} \frac{\partial f_o}{\partial r} + u \frac{\partial f_o}{\partial r} - \frac{2up}{3r} \frac{\partial f_o}{\partial p} = 0, \quad (3.12.10)$$

$$u \frac{\partial F(p)}{\partial r} + \frac{3u}{r} F(p) - \frac{4\pi V_A v p^4 r_1}{3r \sin \theta} \frac{\partial f_o}{\partial r} = 0. \quad (3.12.11)$$

For the boundary condition $f_o(r = r_o, p) = f_e(p)$, where $f_e(p)$ is the CR distribution function out of the Heliosphere, the approximate solution of Eq. 3.12.10 and Eq. 3.12.11 will be

$$f_o(r, p) = \frac{f_e(p)}{1 + B(p)f_e(p)(r_o^3 - r^3)}; \quad F(p) = \frac{4\pi V_A}{3u} p^4 v f_o(r, p) \frac{r_1}{r \sin \theta}, \quad (3.12.12)$$

where

$$B(p) = \frac{8\pi^3 Z e V_A p^3}{3c r_1^2 H_1}. \quad (3.12.13)$$

The expected anisotropy $A(r, p)$ along interplanetary spiral magnetic field and radial diffusion coefficient $\kappa_{rr}(r, p)$ are as follows:

$$A(r, p) = \frac{3ur \sin \theta}{v r_1}, \quad (3.12.14)$$

$$\kappa_{rr}(r, p) = \kappa_{//} \cos^2 \Psi = \frac{ucH_1 r_1^2 \left[1 + B(p) f_e(p) (r_o^3 - r^3) \right]}{ZeV_A p^3 f_e(p) r_o^2} \left(\frac{r_o}{r} \right)^2. \quad (3.12.14a)$$

For $r = r_o = 60$ AU, $v = c$, $\theta = \pi/2$ we obtain $A \approx 20\%$. The expected dependence of κ_{rr} from kinetic energy of particles is shown in Fig. 3.12.1, the expected depth of modulation in the outer Heliosphere f_o/f_e in the interval 40–60 AU is in Fig. 3.12.2.

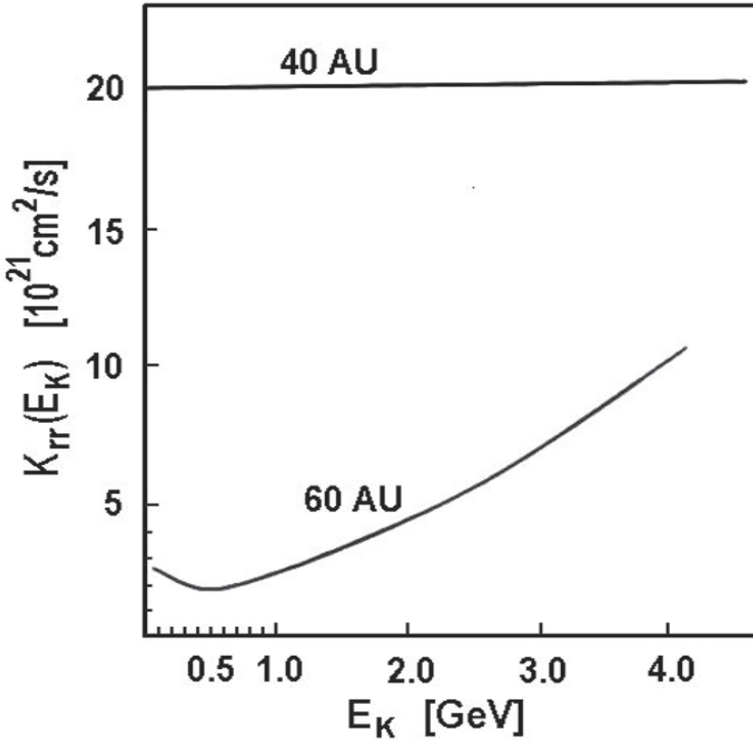


Fig. 3.12.1. CR radial diffusion coefficient $\kappa_{rr}(E_k)$ in units $10^{21} \text{ cm}^2/\text{sec}$ at heliocentric distances 40 AU (assumed according to spaceprobe measurements) and as expected at 60 AU (as developed from CR kinetic stream instability calculated in Dorman et al., 1990).

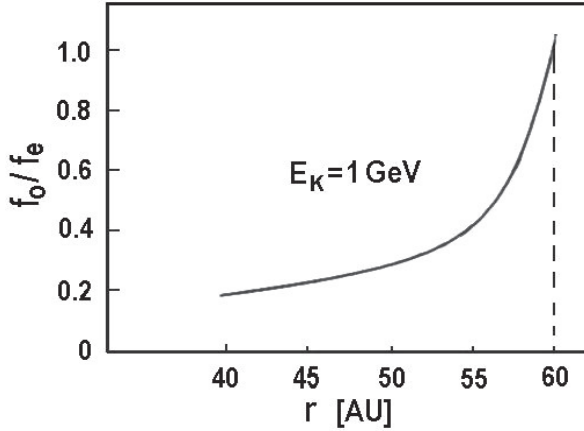


Fig. 3.12.2. CR modulation in the outer Heliosphere at distances 40–60 AU for particles with kinetic energy 1 GeV, expected according to Dorman et al. (1990).

3.12.2. Self-consistent problem including effects of CR pressure and kinetic stream instability in the Heliosphere

In Section 3.12.1 it was assumed that the solar wind velocity is constant up to the boundary of the Heliosphere and we did not take into account the influence of CR pressure on the solar wind moving, although it was shown in Sections 3.5–3.8 that this influence is very important in the outer Heliosphere. To take into account both effects (stream instability and CR pressure) Zirakashvili et al. (1991) considered self-consistent problem on the basis of the following set of equations in the hydrodynamic approximation (Drury and Völk, 1981; McKenzie and Webb, 1984):

$$\left\{ \begin{array}{l} \frac{\partial}{\partial r} (r^2 \rho u) = 0, \\ \rho u \frac{\partial u}{\partial r} = - \frac{\partial}{\partial r} (P_g + P_c + P_w + P_m), \\ \frac{\partial}{\partial r} \left[\rho u \left(\frac{u^2}{2} + \frac{\gamma_g}{\gamma_g - 1} \frac{P_g}{\rho} \right) + 3uP_w + 2uP_m + \frac{1}{\gamma_c - 1} \left(\gamma_c u P_c - \kappa \frac{\partial P_c}{\partial r} \right) \right] = 0, \\ \frac{\partial}{\partial r} (3ur^2 P_w) = u \frac{\partial P_w}{\partial r} \pm V_a \frac{\partial P_c}{\partial r} \cos \Psi, \\ u \frac{\partial P_c}{\partial r} = \frac{\partial}{\partial r} \left[\frac{r^2}{\gamma_c - 1} \left(\gamma_c u P_c - \kappa \frac{\partial P_c}{\partial r} \right) \right]. \end{array} \right. \quad (3.12.15)$$

Here the value $P_w = (1/2) \int W(\omega) d\omega$ is the pressure of MHD turbulence, and P_m is the magnetic pressure. We will consider an inner boundary problem at $r = r_o \gg 1 AU$, where $P_w = 0$ and the boundary of Heliosphere (the transition from supersonic to subsonic flow) will be determined on the basis of solution of the self-consistent problem described by the set of Eq. 3.12.15. To obtain some rough analytical solution of the problem Eq. 3.12.15 we assume that the dimension of layer between r_o and the transition layer is small relative to the radial distance r (i.e. $r - r_o \ll r$), so we can consider it as one-dimensional problem. In this case in the set of Eq. 3.12.15 we can consider $r^2 = r_o^2 + 2r_o(r - r_o) + (r - r_o)^2 \approx r_o^2(1 + 2(r - r_o)/r_o)$ to be a slow function of r , and take it outside the differentiation $\partial/\partial r$ and the set of Eq. 3.12.15 will transform to:

$$\begin{cases} \frac{\partial}{\partial r}(\rho u) = 0, \\ \rho u \frac{\partial u}{\partial r} = -\frac{\partial}{\partial r}(P_g + P_c + P_w + P_m), \\ \frac{\partial}{\partial r} \left[\rho u \left(\frac{u^2}{2} + \frac{\gamma_g}{\gamma_g - 1} \frac{P_g}{\rho} \right) + 3uP_w + 2uP_m + \frac{1}{\gamma_c - 1} \left(\gamma_c u P_c - \kappa \frac{\partial P_c}{\partial r} \right) \right] = 0, \\ \frac{\partial}{\partial r}(3uP_w) = u \frac{\partial P_w}{\partial r} \pm V_a \frac{\partial P_c}{\partial r} \cos \Psi, \\ u \frac{\partial P_c}{\partial r} = \frac{\partial}{\partial r} \left[\frac{1}{\gamma_c - 1} \left(\gamma_c u P_c - \kappa \frac{\partial P_c}{\partial r} \right) \right]. \end{cases} \quad (3.12.16)$$

Because the magnetic field is frozen in moving plasma, we obtain from the first equation of the set described by Eq. 3.12.16 that (subscript 'o' means the values at $r = r_o$) for the magnetic field

$$H = H_o(r_o u_o / ru), \quad (3.12.17)$$

for the magnetic pressure

$$P_m = H^2 / 8\pi = P_{mo} (u_o / u)^2, \quad (3.12.18)$$

for the Alfvén velocity

$$V_a = V_{ao} \sqrt{u_o/u} , \quad (3.12.19)$$

and for

$$\cos \Psi = (u/u_o) \cos \Psi_o . \quad (3.12.20)$$

Then on the basis of the set described by Eq. 3.12.16 we obtain

$$\rho u = \rho_o u_o, \quad P_c = P_{co} + \rho_o u_o (u_o - u) + P_{mo} \left(1 - (u_o/u)^2\right), \quad (3.12.21)$$

$$\kappa \left(1 - \frac{2P_{mo}}{\rho_o u_o^2} \left(\frac{u_o}{u}\right)^3\right) \frac{\partial u}{\partial r} = (u - u_o)(u - u_\infty) \left[\frac{\gamma_c + 1}{2} + \frac{P_{mo}(2 - \gamma_c)}{\rho_o u u_\infty} \right], \quad (3.12.22)$$

$$\begin{aligned} \frac{u_\infty}{u_o} &= \frac{\gamma_c - 1}{2(\gamma_c + 1)} + \frac{\gamma_c}{\gamma_c + 1} \frac{P_{co} + P_{mo}}{\rho_o u_o^2} \\ &+ \left[\left(\frac{\gamma_c - 1}{2(\gamma_c + 1)} + \frac{\gamma_c}{\gamma_c + 1} \frac{P_{co} + P_{mo}}{\rho_o u_o^2} \right)^2 + \frac{2P_{mo}}{\rho_o u_o^2} \frac{2 - \gamma_c}{\gamma_c + 1} \right]^{1/2}. \end{aligned} \quad (3.12.23)$$

We can find the pressure of Alfvén turbulence P_w from the equation

$$2u \frac{\partial P_w}{\partial u} + 3P_w = V_{ao} \rho_o u_o \cos \Psi_o \left(\frac{u}{u_o} \right)^2 \left(\frac{2P_{mo}}{\rho_o u_o^2} \left(\frac{u_o}{u} \right)^3 - 1 \right) \quad (3.12.24)$$

with the boundary condition $P_w(u = u_o) = 0$:

$$P_w = V_{Ao} \rho_o u_o \cos \Psi_o \left[\left(\frac{1}{4} + \frac{P_{mo}}{\rho_o u_o^2} \right) \left(\frac{u_o}{u} \right)^2 - \frac{P_{mo}}{\rho_o u_o^2} \left(\frac{u_o}{u} \right)^3 - \frac{1}{4} \right] \left(\frac{u}{u_o} \right)^2 . \quad (3.12.25)$$

Because $P_{mo}/\rho_o u_o^2 \approx 1/450$, and according to Eq. 3.12.23 $u_o/u < 7$ for $\gamma_c = 4/3$, we obtain approximately

$$P_w = (1/4) V_{Ao} \rho_o \cos \Psi_o \left((u_o/u)^2 - 1 \right) (u/u_o)^{1/2} . \quad (3.12.26)$$

Then CR diffusion coefficient will be determined by

$$\kappa(p) = \kappa_{\parallel}(p) \cos^2 \Psi + \kappa_{\perp}(p) \sin^2 \Psi; \quad \kappa_{\parallel}(p) = \frac{v^2}{3\nu(p)}; \quad \kappa_{\perp}(p) = \frac{v^2 \nu(p)}{3\omega_H^2}, \quad (3.12.27)$$

where

$$\omega_H = ZeHc/F(p); \quad \nu(p) = \pi\omega_H\omega_R W(\omega_R)/4P_m; \quad \omega_R = (V_A/\nu)\omega_H. \quad (3.12.28)$$

Approximately $\omega_R W(\omega_R) \approx 2P_w$, and for $E_k \approx 2$ GeV ($\nu \approx c$), $r_o = 60$ AU we find the expected dependence of $\kappa_{rr}(E_k)$ on r (see Fig. 3.12.3).

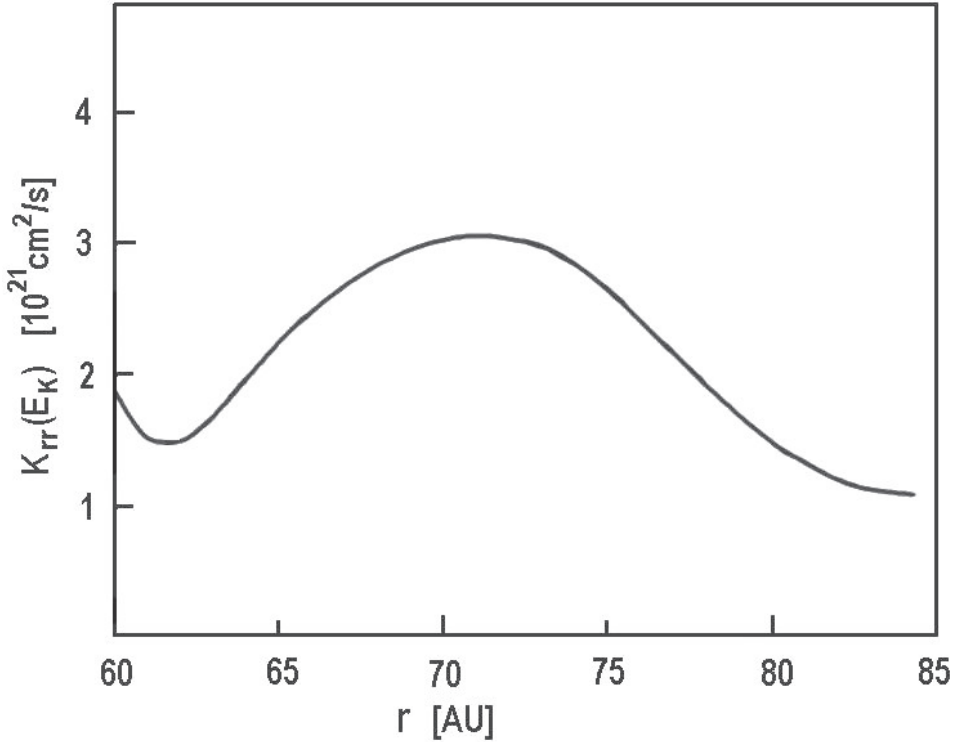


Fig. 3.12.3. Expected CR radial diffusion coefficient $\kappa_{rr}(E_k)$ for CR particles with kinetic energy 2 GeV in units $10^{21} \text{ cm}^2/\text{sec}$ as function of heliocentric distance r . According to Zirakashvili et al. (1991).

In Fig 3.12.4 is shown the expected dependence of $P_c/\rho_o u_o^2$ and u/u_o from r at $r \geq r_o = 60$ AU

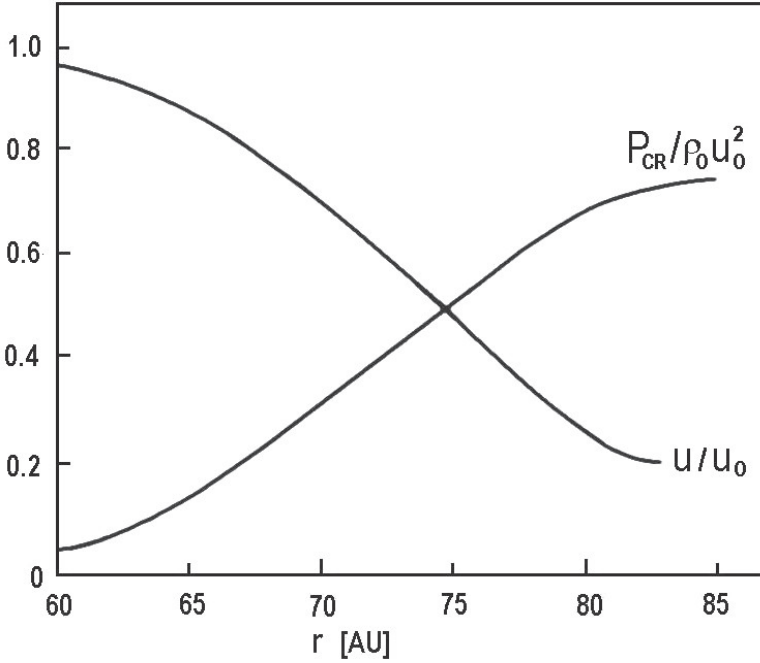


Fig. 3.12.4. Ratio of CR pressure to initial dynamic pressure of the solar wind and ratio of the solar wind speed u to initial speed $u_0 = 400$ km/s as functions of distance r . According to Zirakashvili et al. (1991).

It follows from Eq. 3.12.22 that the terminal transition will be of shock wave type only in the case when

$$1 - \frac{2P_{mo}}{\rho_0 u_0^2} \left(\frac{u_0}{u} \right)^3 = 0 \text{ at } u_\infty < u < u_0. \quad (3.12.29)$$

When Eq. 3.12.29 can not be satisfied, the terminal transition must be of a gradual type (without formation of terminal shock wave). For $P_{mo}/\rho_0 u_0^2 \approx 1/450$ the value

$1 - \frac{2P_{mo}}{\rho_0 u_0^2} \left(\frac{u_0}{u} \right)^3$ becomes equal to 0 at $u_0/u \approx 6.1$. Therefore if $u_0/u_\infty > 6.1$ we

obtain shock wave transition. The value u_0/u_∞ is determined by Eq. 3.12.23. If we consider CR as a relativistic gas ($\gamma_c = 4/3$) then $u_0/u_\infty \approx 7$ and we expect a shock wave transition. The problem is that non relativistic CR also contribute to CR pressure and stream instability and in this case the value u_0/u_∞ becomes smaller and for final solution we need some additional analyses.

3.12.3. Main results for Heliosphere; possible nonlinear effects for stellar winds

It is shown that CR nonlinear effects (pressure and stream instability) considered here play a vital role in the Heliosphere: influence on solar wind propagation; a role in the formation of terminal shock wave and a boundary between solar plasma and the interstellar medium; in the generation of MHD waves and formation of CR diffusion coefficient and anisotropy; significant influence on CR propagation and modulation in interplanetary space, especially in the outer Heliosphere. It is expected that these nonlinear effects can play an important role in dynamics of other stellar winds. CR nonlinear effects are expected to be especially important for stellar winds from quickly rotated stars with frozen in big magnetic fields (in this case both effects of CR pressure and kinetic stream instabilities effects will be important for limiting and formation of the stellar-sphere).

3.13. CR nonlinear effects in the dynamic Galaxy

3.13.1. CR propagation in the dynamic model of the Galaxy

CR can give important information on galactic wind. The extended dynamical halo (galactic wind) was first taken into account in CR propagation in the convection-diffusion model by Bulanov et al. (1972). Dogiel et al. (1980) extended this model, taking into account the adiabatic losses. Bloemen et al. (1993) investigated in detail CR propagation in the Galaxy, taking into account the galactic wind (diffusion - convection processes and adiabatic energy losses in extended halo). It was assumed that the velocity of galactic wind u increased proportionally to the distance z from the equatorial plane of the disk: $u = V_o z$. The problem is that we do not know exactly the coefficient V_o and effective diffusion coefficient in the halo κ_h . In the framework of this model there was calculated the expected depth X of CR crossing of matter (in g/cm^2) for the average time of CR living in the Galaxy, and relative contents f of radioactive isotope ^{10}Be . Fig. 3.13.1 shows the results of calculations of Bloemen et al. (1993) of expected relations $V_o - \kappa_h$ for $X = 6-8 \text{ g cm}^{-2}$ (which fits experimental data on CR chemical composition) and for $f = 0.2-0.3$ (that fits experimental data on relative contents of radioactive isotope ^{10}Be).

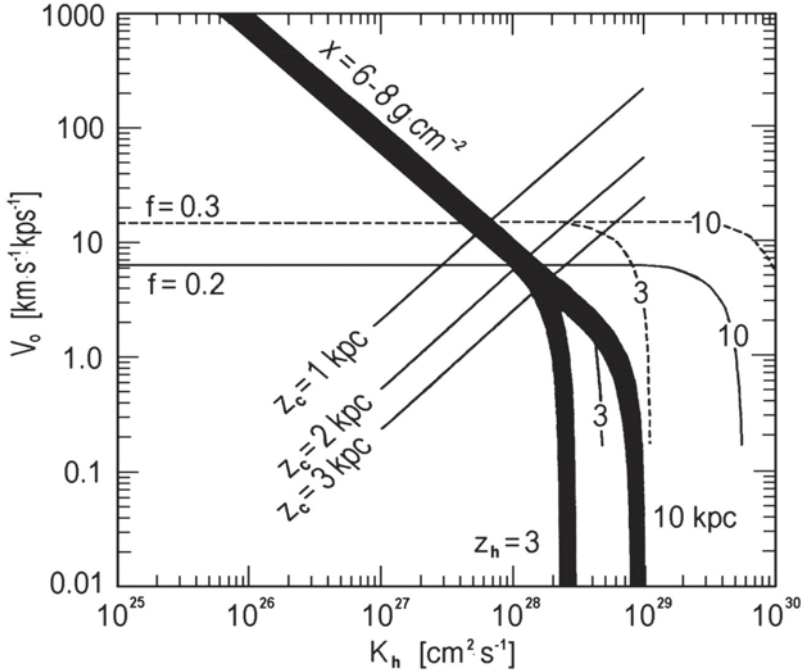


Fig. 3.13.1. Constraints on the relation between V_o and κ_h in a convection-diffusion model from the observed CR grammage X and the observed abundance f of radioactive isotope ^{10}Be . Results are shown for different extent of the halo z_h . According to Bloemen et al. (1993).

Fig. 3.13.1 shows that the best fit of V_o and κ_h that can explain simultaneously $X = 6-8 \text{ g cm}^{-2}$ and $f = 0.2-0.3$ are $\kappa_h \approx 10^{28} \text{ cm}^2 \text{ sec}^{-1}$ and $V_o \approx 10 \text{ km} \cdot \text{sec}^{-1} \text{ kpc}^{-1}$. At a distance 30 kpc from the disk the velocity of galactic wind is expected to be 300 km/sec. This result shows that the galactic wind plays an important role for CR propagation and formation of the chemical composition. Moreover, this result can be considered as some additional evidence of existence of galactic wind.

3.13.2. The geometry of galactic wind and possible role of CR

The model of galactic wind driven by CR was proposed by Ipavich (1975). Recently we have had some radio-astronomical evidence for the existence of galactic wind (Reich and Reich, 1988; Hummel et al., 1988; Hummel and Dettmar, 1990; Pohl et al., 1991). In Section 3.13.1 we considered the importance of the existence of galactic wind for CR propagation and formation of chemical composition. One-dimensional Cartesian geometry of galactic wind (which is valid near the center and for distances $z \leq 10 \text{ kpc}$ from the disk) was considered by Breitschwerdt et al. (1991) and Fichtner et al. (1991). On the other hand, one-

dimensional spherical symmetric geometry, considered by Ipavich (1975) and Zank (1989), is applicable only for very large galacto-centric distances >15 kpc. A multi-dimensional model for ellipsoidal geometry (contains planar and spherical regimes as asymptotic cases) was considered by Fichtner et al. (1991), Vormbrock and Fichtner (1993).

3.13.3. Expected distribution of galactic wind velocity and CR density in the halo (ellipsoidal geometry model)

Vormbrock and Fichtner (1993) considered ellipsoidal geometry for the model of galactic wind driving by CR for our Galaxy and for NGC 4631 with about the same mass $2.75 \times 10^{11} M_{\text{Sun}}$ and about the same dimensions of disk $r_d = 15$ kpc, $h_d = 1$ kpc. The basic set of hydrodynamic equations is:

$$\begin{aligned} \nabla(\rho \mathbf{u}) = 0, \quad \rho(\mathbf{u} \nabla) \mathbf{u} = \mathbf{f} - \nabla P_g - \nabla P_c, \quad \nabla \left(\rho \mathbf{u} \left(\frac{u^2}{2} + \frac{\gamma_g}{\gamma_g - 1} \frac{kT}{m} \right) \right) + \frac{1}{\gamma_c - 1} (\mathbf{u} \nabla) P_c \\ - \mathbf{u} \mathbf{f} - \Lambda_1 + \Lambda_2 = 0, \quad \nabla(\mathbf{u} \gamma_c P_c - \kappa \nabla P_c) - (\gamma_c - 1)(\mathbf{u} \nabla) P_c = 0, \end{aligned} \quad (3.13.1)$$

where $\mathbf{f}$ is the gravitational force, Λ_1 reflects the heating processes (Coulomb interaction, ionization by CR), Λ_2 takes into account cooling processes (bremsstrahlung, recombination, collision induced line emission). Fig. 3.13.2 shows the expected distribution of directions and values of galactic wind velocity in assuming that it has only one cause: driving by CR. The expected full mass loss rate is $\approx 0.8 M_{\text{Sun}}/\text{year}$, in good agreement with observations.

Let us note that the calculated values for galactic wind must be considered as a lower limit because there are at least several additional sources (supernova explosions, stellar winds etc.). In Fig. 3.13.3 the expected distribution of CR pressure P_c in the dynamical halo is shown (according to Vormbrock and Fichtner, 1993).

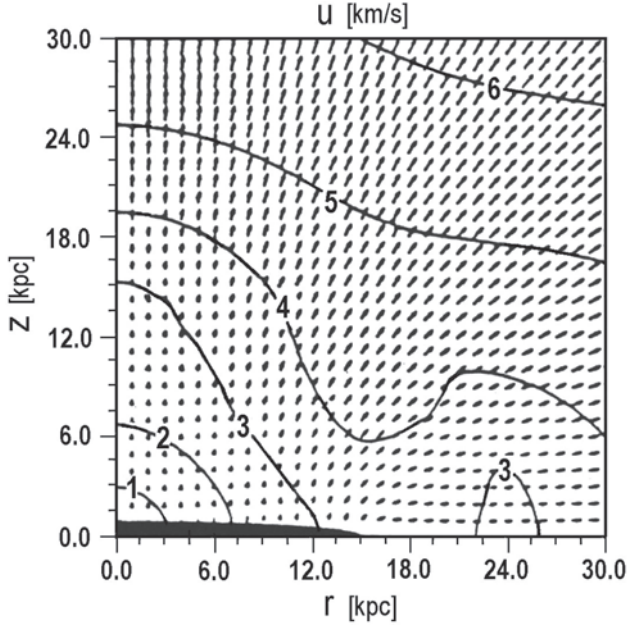


Fig. 3.13.2. Expected galactic wind velocity field: 1 - 25, 2 - 75, 3 - 150, 4 - 225, 5 - 300 and 6 - 350 km/s. According to Vormbrock and Fichtner (1993).

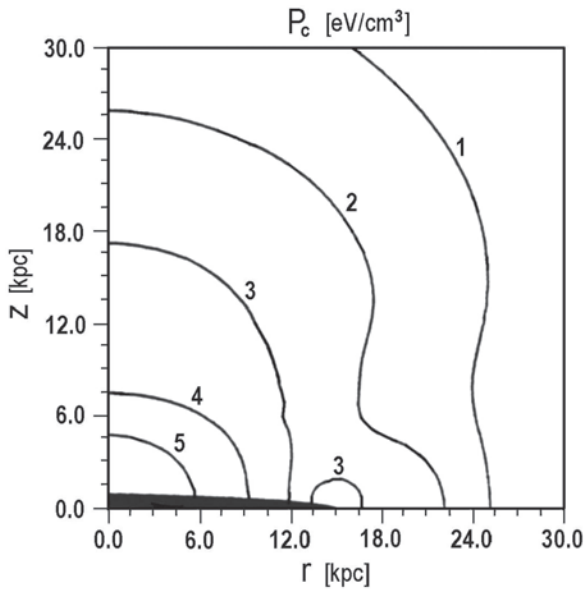


Fig. 3.13.3. Expected distribution of CR pressure: 1 - 0.001, 2 - 0.003, 3 - 0.015, 4 - 0.075, and 5 - 0.2 eV/cm³. According to Vormbrock and Fichtner (1993).

3.14. Self-consistent problem for dynamic halo in rotating Galaxy

3.14.1. Solution for galactic wind and magnetic field

The self-consistent problem of CR propagation in the expanded halo, taking into account the rotation of the Galaxy, was considered by Zirakashvili et al. (1993), Ptuskin and Zirakashvili (1993). In the frame of axisymmetrical model it was supposed that plasma moved along some surface of rotation S (according to Weber and Davis, 1967; Yeh, 1976). The vectors $\mathbf{u}$ (galactic wind velocity) and $\mathbf{H}$ (frozen magnetic field) are coplanar to S . It is assumed that $\partial/\partial s$ is the derivative in meridian direction (symbolized by $'$) and the cross section of the tube is $B(s, r_o)$. The basic steady state MHD equations which include CR pressure P_c (and under the assumption of the smooth transition of the solution through the possible critical points) will be:

$$B\rho u = \text{const}, \quad BH = \text{const}, \quad \rho(u' - u_r u_\Phi^2 / ur) = -(P_g + P_c)' - (r^2 H_\Phi^2)' / 8\pi r^2 + \rho f', \quad (3.14.1)$$

$$B^{-1} \left(B \left(\rho u \left(\frac{u^2}{2} + \frac{u_\Phi^2}{2} + \frac{\gamma_g P_g}{\rho(\gamma_g - 1)} - f \right) - \frac{\Omega r H H_\Phi}{4\pi} \right) \right)' + (u + V_a) P_c' = 0; \quad (3.14.2)$$

$$B^{-1} \left(B (\gamma_c (u + V_a) P_c - \kappa P_c') \right)' - (\gamma_c - 1) (u + V_a) P_c' = 0. \quad (3.14.3)$$

Here the gravitational potential $\mathbf{f}$ is taken as

$$\mathbf{f} = \mathbf{f}_o (1 + s/s_1)^{-1}, \quad (3.14.4)$$

where $\mathbf{f}_o = 1.9 \times 10^{15} \text{ cm}^2/\text{sec}^2$, $s_1 = 45 \text{ kpc}$. The shape of flux tube is taken in the form

$$B(s) = B_o \left(1 + (s/s_2)^2 \right), \quad r(s) = r_o \left(1 + (s/s_2)^2 \right)^{1/2}, \quad (3.14.5)$$

where $s_2 = 15 \text{ kpc}$. The calculations are made at $\gamma_g = 1.6$ for the boundary condition at $z_o = 3 \text{ kpc}$ as follows:

$$P_{co} = 2 \times 10^{-13} \text{ erg/cm}^3, \quad H_o = 10^{-6} \text{ Gs}, \quad T_o = 4 \times 10^5 \text{ K}, \\ n_o = 10^{-3} \text{ cm}^{-3}, \quad u_{\Phi o} = 250 \text{ km/sec}. \quad (3.14.6)$$

It was found that the meridional initial wind velocity $u_o = 28.4$ km/sec. The results of expected changes of the magnetic field H , temperature T , meridional u and azimuthal u_ϕ components of the galactic wind velocity with distance from the disco-equator z up to $z = 75$ kpc are shown on the Fig. 3.14.1 (the critical points with smooth transition are located at 5.3 kpc – the slow magnetosonic, at 8.1 kpc – the Alfvénic, and at 30 kpc – the fast magnetosonic; it was found also that generated by Galaxy rotation azimuthal magnetic field falls as s^{-1}).

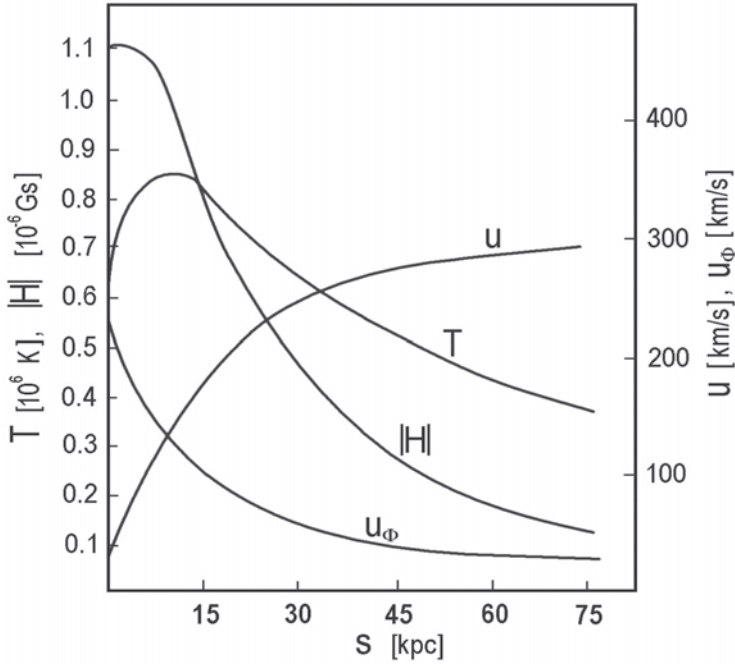


Fig. 3.14.1. Expected distribution of magnetic field $|H|$, meridional u and azimuthal u_ϕ galactic wind velocities and gas temperature T in the rotating Galaxy. According to Zirakashvili et al. (1993).

3.14.2. Solution for CR propagation in the rotating Galaxy

On the basis of the solution shown in Fig. 3.14.1 Ptuskin and Zirakashvili (1993) investigated CR propagation in the Galaxy governed by the diffusion-convection processes:

$$-B^{-1}(B\kappa f')' + (u + V_a)f' - B^{-1}(B(u + V_a))' \frac{p}{3} \frac{\partial f}{\partial p} = 2Q(p)\delta(s), \quad (3.14.7)$$

where $f(p, s)$ is the CR distribution function and CR sources concentrated in disk are assumed to have a power law spectrum $Q \propto p^{-\gamma_s}$. The diffusion coefficient is determined by the expression

$$\kappa = \kappa_{//} \cos^2 \alpha; \quad \kappa_{//} = v r_L H^2 / \left(6\pi^2 k_{res} W(k_{res}) \right), \quad (3.14.8)$$

where $k_{res} = 1/r_L$ and r_L is the Larmor radius. The growth rate of CR streaming instability for Alfvén waves generation

$$\Gamma_c = -6\pi^3 e^2 V_a \left(k_{res} c^2 \right)^{-1} \int_{p_{res}}^{\infty} dp \left(1 - p_{res}^2 / p^2 \right) p \kappa_{//} f' \cos \alpha, \quad (3.14.9)$$

where $p_{res} = e|\mathbf{H}|/ck_{res}$. The non-saturated Landau damping of Alfvén waves will be (according to Völk and Cesarsky, 1982; see also above, Section 3.5):

$$\Gamma_{NL} = 2\pi^2 \beta^{1/2} V_a k \int dk W(k) / H^2, \quad (3.14.10)$$

where

$$\beta = 4\pi m T / H^2. \quad (3.14.11)$$

The diffusion coefficient will be

$$\kappa_{//} = \frac{\gamma_s (\gamma_s - 2)}{9\pi^2 (\gamma_s - 4)} \beta^{1/2} c^3 \omega_{Hi}^{-1} H^2 E^{-1} (p/mc)^{\gamma_s - 3} B(s) \cos \alpha, \quad (3.14.12)$$

where E is the particle's full energy. This model is in a good agreement with data on mean matter thickness $\langle \mathbf{X} \rangle = 10(v/c) \text{ g/cm}^2$ for $pc/Ze < 5 \text{ GV}$ and $\langle \mathbf{X} \rangle \propto p^{-0.55}$ for $pc/Ze > 5 \text{ GV}$, but predicts too large stellar anisotropy

$$A_{st} = -3\kappa v^{-1} (\ln f)' \approx 10^{-4} (p/mc)^{0.55}, \quad (3.14.13)$$

which gives 5% at energy 10^{14} eV (measured amplitude is about 0.05 % at 10^{12} – 10^{14} eV and about 3% at 10^{17} eV). Ptuskin and Zirakashvili (1993) note that this discrepancy can be caused by peculiarities of local conditions near the Sun that do not affect the energy spectrum and chemical composition, but are very important for the observed CR anisotropy.

3.15. On the transport of random magnetic fields by a galactic wind driven by CR; influence on CR propagation

3.15.1. Random magnetic fields in the galactic disc and its expanding to the dynamic halo

Zirakashvili et al. (2001) considered the transport of random magnetic fields by galactic wind driven by CR and their influence on CR propagation in the Galaxy. As was considered in previous Sections 3.11 and 3.12, the galactic wind driven by CR is a prominent example of the dynamical importance of energetic particle nonlinear effects in our Galaxy (Ipavich, 1975; Breitschwerdt et al., 1987, 1991; Zirakashvili et al., 1996). The main matter is that CR sources in the galactic disk generate energetic particles which can not freely escape from the Galaxy but rather generate Alfvén waves through kinetic stream instability (see Section 3.3). In spite of strong nonlinear Landau damping (Livshits and Tsytovich, 1970; Lee and Völk, 1973; Kulsrud, 1978; Achterberg, 1981; Achterberg and Blandford, 1986; Fedorenko et al., 1990; Zirakashvili, 2000) such waves lead to an efficient coupling of thermal gas and energetic particles (Ptuskin et al., 1997) and CR drive galactic wind flow owing to their pressure gradient. In the simplest approximation one can assume that a frozen-in magnetic field is advected from near the galactic disk region to the galactic halo. Such a field and its tension were taken into account by Zirakashvili et al. (1996) for calculations of the galactic wind flow. An idealized regular magnetic field configuration was considered. Zirakashvili et al. (2001) develop these ideas further and take into account the random magnetic field component that exists in the galactic disk and dynamically dominates over the regular component.

3.15.2. Basic equations described the transport of the random magnetic fields

According to Zirakashvili et al. (2001), magneto-hydrodynamic (MHD) turbulence is created in the galactic disk mainly by the numerous supernovae; it seems that turbulent diffusion and helicity really provide dynamo action in the disk of our Galaxy (Parker, 1992). Those effects can be less important in the galactic halo, especially if galactic wind flow exists. The approximation used by Zirakashvili et al. (2001) is that at heights of several hundred pc the magnetic inhomogeneities created in the upper part of the disk are picked up by the wind flow and transported into the galactic halo. They neglect turbulent magnetic diffusion and reconnection here. The magnetic field $\mathbf{B}$ is frozen into the galactic wind gas and evolves according to equation

$$\frac{\partial \mathbf{B}}{\partial t} = [\nabla \times [\mathbf{u} \times \mathbf{B}]]. \quad (3.15.1)$$

It was assumed that gas velocity $\mathbf{u}$ and density ρ are non random quantities and are described by the steady state equations

$$\nabla(\rho\mathbf{u}) = 0, \quad (3.15.2)$$

$$\rho(\mathbf{u}\nabla)\mathbf{u} = -\nabla(P_g + P_c) + \rho\nabla\Phi - \frac{1}{4\pi}\langle[\mathbf{B} \times [\nabla \times \mathbf{B}]]\rangle. \quad (3.15.3)$$

Here P_g and P_c are the pressures of gas and CR respectively, Φ is the gravitational potential. Angular brackets mean averaging over volume. It is easy to see from Eq. 3.15.3 that dynamical effects of the magnetic field can be described if one knows the mean tensor $B_{ij} = \langle B_i B_j \rangle$. The equation for this tensor can be derived from Eq. 3.15.1:

$$\frac{\partial B_{ij}}{\partial t} = -u_k \nabla_k B_{ij} - 2B_{ij} \nabla_k u_k + B_{kj} \nabla_k u_i + B_{ik} \nabla_k u_j. \quad (3.15.4)$$

Then Zirakashvili et al. (2001) consider steady state solutions of Eq. 3.15.4 corresponding to the steady state Eq. 3.15.2 and Eq. 3.15.3. This is a development of previous results of Zirakashvili et al. (1996) (where the steady state Eq. 3.15.1 for the regular magnetic field was used) to the case including random magnetic fields.

3.15.3. The random magnetic field effects in the galactic wind flow with azimuthal symmetry

Assuming azimuthal symmetry of the galactic wind flow it is convenient to introduce the coordinate s in meridional direction and the azimuthal angle ϕ . The tensor components B_{ij} should be written in terms of those coordinates. The gas velocity has the meridional and azimuthal components u_s and u_ϕ , respectively. For the sake of simplicity it was assumed that the magnetic field is tangent to the surface S along which the galactic wind streams. Therefore there are only three independent components: B_{ss} , $B_{s\phi}$ and $B_{\phi\phi}$. Introducing the flux-tube cross-section $A(s)$ one can obtain:

$$B_{ss} A^2(s) = \text{const}, \quad (3.15.5)$$

$$\frac{u_\phi}{r} - \frac{u_s B_{s\phi}}{r B_{ss}} = \Omega, \quad (3.15.6)$$

$$\frac{u_s^2}{r^2} \left(\frac{B_{\phi\phi}}{B_{ss}} - \frac{B_{s\phi}^2}{B_{ss}^2} \right) = \text{const.} \quad (3.15.7)$$

Using the ϕ -component of Eq. (3.15.3) one finds angular momentum conservation along the surface S :

$$ru_\phi - \frac{rB_{s\phi}}{4\pi\varrho u_s} = C. \quad (3.15.8)$$

Here the quantities Ω and C are constant along the surface S , and r is the distance from the axis of rotation. Expressions for u_ϕ and $B_{s\phi}$ can be found using Eq. 3.15.6 and 3.15.7. They contain the denominator $1 - M_a^2$, where

$$M_a = \sqrt{4\pi\varrho u_s^2 / B_{ss}} \quad (3.15.9)$$

is the meridional Alfvén Mach number. Assuming acceleration of the wind flow from sub-Alfvénic to super-Alfvénic velocities one can find a relation between C and Ω which leads to finite values of u_ϕ and $B_{s\phi}$:

$$C = \Omega r_a^2, \quad (3.15.10)$$

where r_a is the distance from the Alfvénic point $M_a = 1$ to the axis of rotation. As a result

$$u_\phi = \frac{\Omega(r^2 - r_a^2 M_a^2)}{r(1 - M_a^2)}, \quad (3.15.11)$$

$$B_{s\phi} = B_{ss} \frac{\Omega M_a^2 (r^2 - r_a^2)}{ru_s (1 - M_a^2)}. \quad (3.15.12)$$

It is easy to see that these expressions are similar to those for the azimuthal components of gas velocity and magnetic field in the previous investigation of Zirakashvili et al. (1996) and in the theory of azimuthal symmetric MHD flows (Weber and Davis, 1969; Yeh, 1976; Sakurai, 1985). The only difference is the definition of the Alfvén Mach number. In Zirakashvili et al. (2001) it contains B_{ss} instead of the square of the meridional regular magnetic field component. Eq.

3.15.7 shows that $B_{\phi\phi}$ is not reduced to the square of the azimuthal field component in Zirakashvili et al. (1996) but rather contains an additional term. Nevertheless, this term is inversely proportional to the square of the meridional velocity and hence quickly drops with height over the disk. Therefore at large heights above the disk $B_{\phi\phi}$ is given by

$$B_{\phi\phi} = B_{ss} \frac{\Omega^2 M_a^4 (r^2 - r_a^2)^2}{r^2 u_s^2 (1 - M_a^2)^2}. \quad (3.15.13)$$

In the general case in which magnetic field components perpendicular to the surface S are present, Eq. 3.15.4 also describes the generation of a random magnetic field owed to differential rotation of neighboring surfaces. Nevertheless, all B_{ij} components except determined by Eq. 3.15.5, 3.15.12 and 3.15.13 tend to zero as the wind accelerates. Hence expressions determined by Eq. 3.15.5, 3.15.11, 3.15.12 and 3.15.13 are valid in the general case for large heights above the disk. It is easy to picture the magnetic field geometry in the galactic halo (see Fig. 3.15.1).

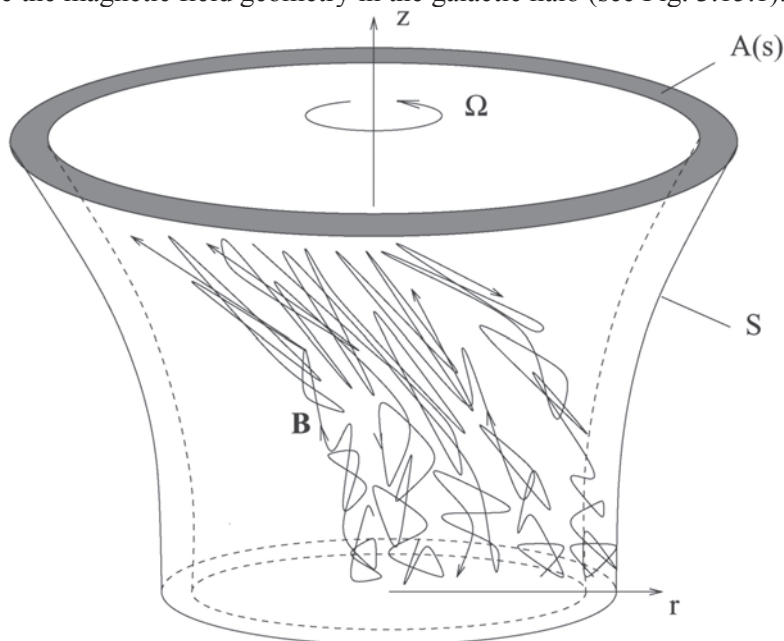


Fig. 3.15.1. Flux-tube geometry characterized by the surface S which contains the wind stream lines. The flux-tubes of cross section $A(s)$ have axial symmetry around z -axis. In the disk ($z = 0$) the gas rotates with angular velocity $\Omega(r)$. Magnetic field disturbances, near isotropic in the galactic disk, become strongly elongated in the galactic halo. According to Zirakashvili et al. (2001).

From Fig. 3.15.1 can be seen that magnetic field lines are strongly elongated in one direction owing to wind acceleration and bend away from the meridional direction because of the rotation of the Galaxy. This picture is similar to the one obtained in our previous investigation for the regular magnetic field. The presence of the magnetic field gives some properties of an elastic body to the surface S , which can now resist to velocity shear. This feature allows magnetic connection and corresponding transport of angular momentum along this surface even for the zero regular magnetic field case.

3.15.4. Results of numerical calculations

Galactic wind numerical calculations were performed by Zirakashvili et al. (2001) for the same parameters of our Galaxy as described in Zirakashvili et al. (1996). They include the gravitational potential of Miyamoto and Nagai (1975) and take into account a dark matter halo of the Galaxy (Innanen, 1973). The geometry of the flow is prescribed. The surface S is chosen to have a hyperbolic form

$$\frac{r^2}{r_o^2} - \frac{z^2}{z_o^2 - r_o^2} = 1, \quad (3.15.14)$$

where $z_o = 15$ kpc is the galactic halo radius, and r_o is that galacto-centric radius where the flux-tube under consideration originates. Energy conservation along the surface S was assumed (as in Zirakashvili et al., 1996):

$$\frac{u_s^2}{2} + \frac{u_g^2}{2} - \Omega r u_\phi + \frac{P_g \gamma_g}{\rho(\gamma_g - 1)} - \Phi + \frac{P_c \gamma_c (M_a + 1)}{\rho(\gamma_c - 1)} = \text{const}, \quad (3.15.15)$$

where γ_c and γ_g are the adiabatic indices of CR and gas respectively. The values $\gamma_c = 1.2$, and $\gamma_g = 1.6$ were used. The only difference in comparison with the previous consideration of Zirakashvili et al. (1996) is the substitution of the z -component of the regular magnetic field B_z by $B_{zz}^{1/2}$. These components coincide with the meridional component at small heights above the disk. Observations of the regular magnetic field in the galactic disk show a regular field of about 2 μ Gs which is parallel to the galactic disk (Rand and Kulkarni, 1989). This means that the vertical component of the regular field in the galactic halo is small and hardly exceeds 1 μ Gs. On the other hand, a random field of about 6 μ Gs exists in the disk.

Assuming it to be isotropic one finds $B_{zz}^{1/2} \sim 3.5$ μ Gs. The corresponding component can be smaller in the galactic halo, say ≈ 1.0 μ Gs. In that case the results of the calculations of Zirakashvili et al. (1996) with 1.0 μ G regular

magnetic field can be used. On the other hand, larger values of magnetic field strength are also possible. Zirakashvili et al. (2001) take the value $B_{zz}^{1/2} \sim 3.0 \mu\text{Gs}$ for the calculations described here. In addition it was used a smaller value of CR pressure $P_{co} = 1.0 \times 10^{-13} \text{ erg/cm}^3$ at the base level 3 kpc over the disk in order to maintain approximately the same cosmic ray energy flux in comparison with calculations of Zirakashvili et al. (1996). A gas number density $n_o = 10^{-3} \text{ cm}^{-3}$ at this base level was assumed. Radiative cooling losses are relevant for denser gas at smaller heights above the disk. Numerical results for the flux tube originating at galacto-centric distance $r_o = 8.5 \text{ kpc}$ (Sun's position) are shown in Fig. 3.15.2 and Fig. 3.15.3.

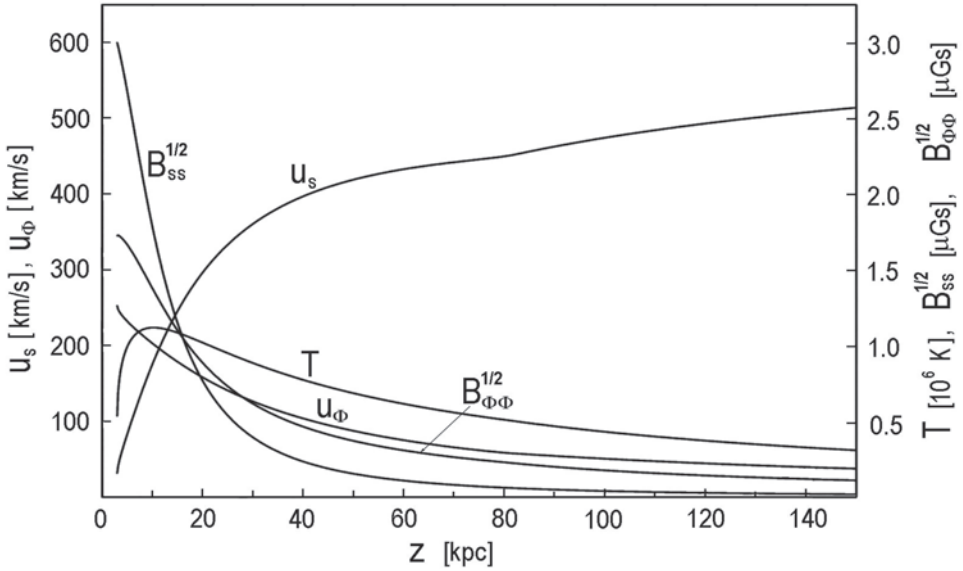


Fig. 3.15.2. Variation of the meridional and azimuthal velocities u_s and u_ϕ , azimuthal and meridional magnetic field strength $B_{\phi\phi}^{1/2}$ and $B_{ss}^{1/2}$, and gas temperature T , with distance from the disk. The resulting initial velocity is $u_o = 31.5 \text{ km/sec}$, and the critical points positions are $z_s = 7.1 \text{ kpc}$, $z_a = 21.6 \text{ kpc}$, and $z_f = 84.8 \text{ kpc}$. The terminal velocity is $u_f = 698 \text{ km/sec}$. According to Zirakashvili et al. (2001).

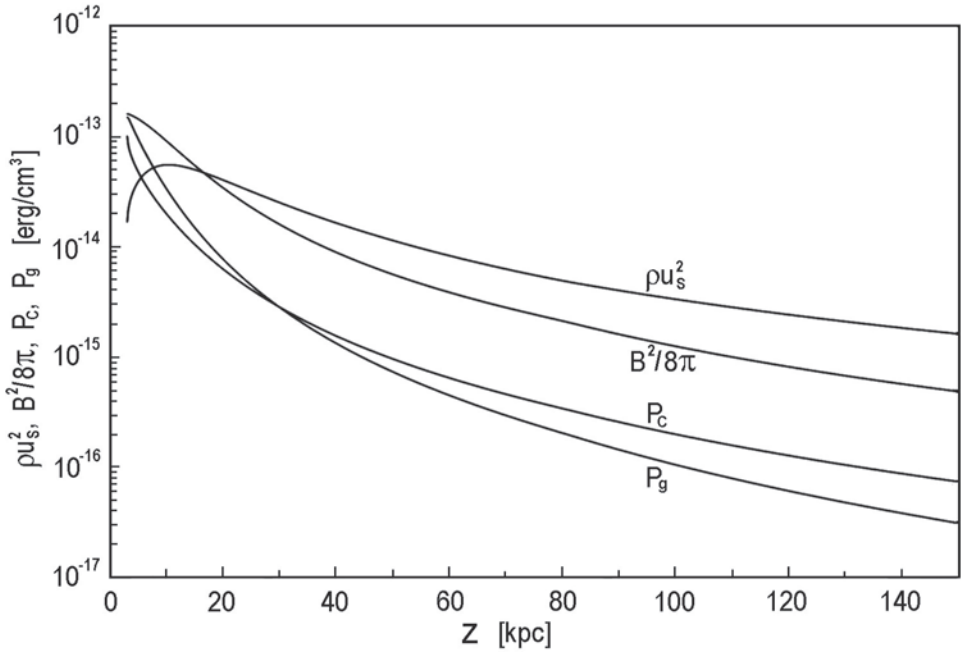


Fig. 3.15.3. Variation of dynamic pressure ρu_s^2 , CR pressure P_c , gas pressure P_g , and magnetic pressure $B^2/8\pi$ with distance z from the disk. According to Zirakashvili et al. (2001).

The height of the slow magneto-sonic point is practically the same $z_s = 7.1$ kpc above the disk. The Alfvén point $z_a = 21.6$ kpc and the fast magneto-sonic point $z_f = 84.8$ kpc move further out into the flow. The initial wind velocity at the base level is $u_o = 31.5$ km/sec, and the terminal velocity is $u_f = 698$ km/sec. The magnetic pressure dominates gas and CR pressures practically everywhere. Nevertheless, this is a CR driven wind because CR gives approximately half of the kinetic energy flux, the second half given by rotational effects (see Eq. 3.15.15).

On the basis of the results described, Zirakashvili et al. (2001) conclude that the inclusion of the random field component in the galactic wind model results in the possibility that our Galaxy is surrounded by a large wind halo with a rather strong magnetic field even though this field strength is probably an upper limit. The field geometry is rather simple. The magnetic field disturbances are nearly isotropic in the galactic disk and have a size of about 100 pc. They are strongly elongated in the galactic halo. The elongation estimated is 1:10–1:100. This magnetic field leads to an effective angular momentum transport and a correspondent additional centrifugal acceleration of the flow, which then results in larger terminal velocity of the wind. At large distances the field is practically

azimuthal and sign dependent with fluctuating direction. One can expect that CR diffusion in such a field is highly anisotropic, enhanced diffusion being in direction of the elongation. It can be also expected that high energy protons with energies larger than 3×10^{17} eV are hardly held by such a sign dependent field. The gas heating owed to damping of Alfvén waves generated by the CR streaming instability is rather effective, the wind halo being filled by a hot rarefied gas with a temperature of about one million degrees. The angular momentum loss rate of the Galaxy is mainly owing to magnetic torque and is about 50% in 10^{10} years.

3.16. Nonlinear Alfvén waves generated by CR streaming instability and their influence on CR propagation in the Galaxy

3.16.1. On the balance of Alfvén wave generation by CR streaming instability with damping mechanisms

Zirakashvili et al. (1999) consider Alfvén wave generation by CR streaming instability and nonlinear damping of parallel propagating Alfvén waves in high- β plasma. There was also taken into account trapping of thermal ions and Coulomb collisions, saturated damping rate be calculated, and applications was made for CR propagation in the Galaxy. As it was considered above, the CR streaming instability can play an important role in processes of diffusive shock acceleration and CR propagation in the Heliosphere and in Galaxy since it can supply Alfvén waves that scatter the particles on pitch angle (Lerche, 1967; Kulsrud and Pearce, 1969; Wentzel, 1969). In order to balance wave generation some damping mechanism is usually considered. As Alfvén waves are weakly linearly damped, various nonlinear effects are currently used. CR streaming generates waves in one hemisphere of wave-vectors. Such waves are not subject to any damping in incompressible magneto-hydrodynamics. The use of compressibility results in a pondermotive force gives a second order plasma velocity and electric field perturbations along the mean magnetic field. These perturbations can yield wave steepening as well as nonlinear damping, if kinetic effects of thermal particles are included. Those effects were taken into account in order to obtain nonlinear damping rates of parallel propagating Alfvén waves (Lee and Völk, 1973; Kulsrud, 1978; Achterberg, 1981). The importance of trapping of thermal particles for nonlinear dissipation of sufficiently strong waves that results in saturation of wave damping was also understood many years ago (Kulsrud, 1978; Völk and Cesarsky, 1982). Corresponding saturated damping rates that take into account dispersive effects were calculated. Nevertheless dispersive effects can be rather small for Alfvén waves that are in resonance with galactic CR nuclei. Hence the effect of Coulomb collisions can be important. Zirakashvili et al. (1999) derive the nonlinear Alfvén wave damping rate in the presence of thermal collisions.

3.16.2. Basic equations and their solutions

Zirakashvili et al. (1999) consider Alfvén waves propagating in one direction along the ambient magnetic field. It is convenient to write the equations in the frame moving with the waves. In such a frame there are only quasi-static magnetic and electric fields slowly varying in time owing to wave dispersion and nonlinear effects. The case of a high- β Maxwellian plasma was considered. Electric fields are negligible for nonlinear damping in such plasma. Zirakashvili et al. (1999) investigate waves with wavelengths much greater thermal particles gyro-radii and use drift equations for distribution function of those particles (Chandrasekhar, M1960):

$$\frac{\partial F}{\partial t} + v\mu(\mathbf{b}\nabla)F + \frac{1-\mu^2}{2}v\frac{\partial F}{\partial\mu}\nabla\mathbf{b} = \text{St}F. \quad (3.16.1)$$

Here F is the velocity distribution of thermal particles that is averaged over the gyro-period, $\mathbf{v}$ is the particle velocity, $\mathbf{b}=\mathbf{B}/B$ is the unit vector along the magnetic field $\mathbf{B}$, $\mu = \mathbf{p}\mathbf{B}/Bp$ is the cosine of the pitch angle of the particle. The right hand side of Eq. 3.16.1 describes collisions of particles. For Maxwell's equations it is necessary to know the flux of particles. It is given by drift theory (Chandrasekhar, M1960):

$$\mathbf{J}_\perp = \frac{1-\mu^2}{2}\frac{v^2}{\Omega}\mathbf{b}\times\left[\nabla F + \mu\frac{\partial F}{\partial\mu}(\mathbf{b}\nabla)\mathbf{b}\right], \quad (3.16.2)$$

where Ω is the particle gyro-frequency in the local field. The last term on the left hand side of Eq. 3.16.1 describes mirroring of particles. Because the field is static in this frame the particle energy is constant, and in a time asymptotic state wave dissipation is absent without collisions. In the presence of wave excitation we shall deal only with the time asymptotic state in the following. We shall use for the collision operator a simplified form

$$\text{St}F = \Delta_v v^2 (F - F_M), \quad (3.16.3)$$

where F_M is the Maxwellian distribution function shifted by the Alfvén velocity v_a ; Δ_v is the Laplace operator in velocity space, and v is the collision frequency. This operator tends to make the particle distribution function Maxwellian. Introducing the coordinate s along the magnetic field, and the distribution function $f = F - F_M$ one obtains the following equation for f :

$$v\mu \frac{\partial f}{\partial s} - \frac{1-\mu^2}{2} v \frac{\partial f}{\partial \mu} \frac{\partial \ln B(s)}{\partial s} - \Delta_v v^2 f = \frac{1-\mu^2}{2} v \frac{\partial F_M}{\partial \mu} \frac{\partial \ln B(s)}{\partial s}. \quad (3.16.4)$$

For sufficiently small magnetic field perturbations (conditions for that case will be derived later) one can neglect the mirroring term on the left hand side of Eq. 3.16.4. Without collisions this leads to the well known nonlinear damping mentioned above. Zirakashvili et al. (1999) take into account the mirroring term here and use the standard quasi-linear theory according to Galeev and Sagdeev (1979). The function f can be written in the form $f = f_o + \delta f$, where $f_o = \langle f \rangle$ is the ensemble averaged distribution function f . We are interested in the case of a small magnetic field amplitude $A \ll 1$, where $\mathbf{A} = (\mathbf{B} - \mathbf{B}_o)/B_o$. Taking also into account that mirroring is sufficient for small $\mu \ll 1$ particles we leave in the collision operator the second derivative on μ only and come to the equation:

$$v\mu \frac{\partial f}{\partial s} - \frac{v}{4} \frac{\partial}{\partial \mu} (f + F_M) \frac{\partial A^2(s)}{\partial s} - v \frac{\partial^2 f}{\partial \mu^2} = 0. \quad (3.16.5)$$

Taking into account that the average distribution function is independent of s one can obtain the equation for the Fourier transform $\delta f_k = \int ds \delta f(s) \exp(-isk)$:

$$ikv\mu \delta f_k - v \frac{\partial^2 \delta f_k}{\partial \mu^2} = \frac{ikv}{4} A_k^2 \frac{\partial}{\partial \mu} (F_M + f_o). \quad (3.16.6)$$

The functions f_o and δf_k are peaked near $\mu = 0$. It is convenient to introduce the Fourier transform on $\mu f_o(\xi) = \int d\mu f_o(\mu) \exp(-i\xi\mu)$ and $\delta f_k(\xi) = \int d\mu \delta f(\mu) \exp(-i\xi\mu)$. Then from Eq. 3.16.6 will be

$$kv \frac{\partial \delta f_k}{\partial \xi} + v \xi^2 \delta f_k = \frac{ikv}{4} A_k^2 \left(2\pi \delta(\xi) \frac{\partial F_M}{\partial \mu} \Big|_{\mu=0} + i \xi \tilde{f}_o(\xi) \right). \quad (3.16.7)$$

This equation has a solution

$$\begin{aligned} \delta f_k &= \frac{ik}{4|k|} A_k^2 \int_{-\infty}^{\infty} d\xi' \theta(k(\xi - \xi')) \\ &\times \left(2\pi \delta(\xi') \frac{\partial F_M}{\partial \mu} \Big|_{\mu=0} + i \xi' \tilde{f}_o(\xi') \right) \exp \left(-\frac{v}{3kv} (\xi^3 - \xi'^3) \right). \end{aligned} \quad (3.16.8)$$

After ensemble averaging of Eq. 3.16.5 and using Eq. 3.16.8 one obtains an equation for $\tilde{f}_o(\xi)$:

$$\begin{aligned} v\xi^2 \tilde{f}_o(\xi) - \frac{i\xi}{16} \int dk dk_1 \int_{-\infty}^{\infty} d\xi' |k| v \theta(k(\xi - \xi')) I(k_1) I(k + k_1) \\ \times \left(2\pi\delta(\xi') \frac{\partial F_M}{\partial \mu} \Big|_{\mu=0} + i\xi' \tilde{f}_o(\xi') \right) \exp\left(-\frac{v}{3kv} (\xi^3 - \xi'^3)\right) = 0. \end{aligned} \quad (3.16.9)$$

Here $I(k)$ is the spectrum of Alfvén waves normalized to the magnetic energy of the mean field: $\langle \delta B^2 \rangle = B_o^2 \int dk I(k)$. Wave-numbers with + or - sign correspond to right or left hand circularly polarized wave. Eq. 3.16.9 describes the influence of waves on the mean distribution function of thermal particles, in particular, well known in plasma theory quasi-linear ‘plateau’ formation breaking by thermal collisions (Galeev and Sagdeev, 1979). The solution of this equation should be substituted into Eq. 3.16.8. This equation, together with the Eq. 3.16.2 for the flux, determines the nonlinear electric current density (the input of thermal protons is taken into account only)

$$\mathbf{J}_k = i\pi k \int_{-\infty}^{\infty} dk_1 \int_0^{\infty} v^2 dv Mc \frac{v^2}{B_o} [\mathbf{A}_{k-k_1} \times \mathbf{e}_z] \tilde{g}_{k_1} \Big|_{\xi=0}. \quad (3.16.10)$$

Substituting this current into Maxwell’s equations and ensemble averaging one can derive an equation for the Alfvén wave spectrum

$$dI(k)/dt = -2\Gamma_{NL} I(k) \quad (3.16.11)$$

with the nonlinear Alfvén wave damping rate:

$$\begin{aligned} \Gamma_{NL} = -\frac{\pi}{8} \frac{Mv_a}{B_o^2} k \int_{-\infty}^{\infty} dk_1 I(-k_1) \int_0^{\infty} 2\pi v^2 dv v^2 \int_{-\infty}^{\infty} d\xi' \frac{k+k_1}{|k+k_1|} \theta(-\xi'(k+k_1)) \\ \times \left(2\pi\delta(\xi') \frac{\partial F_M}{\partial \mu} \Big|_{\mu=0} + i\xi' \tilde{f}_o(\xi') \right) \exp\left(-\frac{v\xi'^3}{3(k+k_1)v}\right), \end{aligned} \quad (3.16.12)$$

where M is the ion mass and

$$\frac{\partial F_M}{\partial \mu} \Big|_{\mu=0} = -\frac{nv_a v}{(2\pi)^{3/2} v_T^5} \exp\left(-\frac{v^2 + v_a^2}{2v_T^2}\right). \quad (3.16.13)$$

Here n is the plasma density and v_T is the thermal velocity. It is useful to transform Eq. 3.16.9 to a form more convenient for applications. It is possible to invert the integral operator and obtain the following equation:

$$\begin{aligned} 8\xi^2 \int_{-\infty}^{\infty} d\xi' \xi'^3 \tilde{f}_o(\xi') \int_{-\infty}^{\infty} \frac{d\eta}{2\pi} \frac{\exp\left(\frac{i\eta}{3}(\xi^3 - \xi'^3)\right)}{\int_{-\infty}^{\infty} dk dk_1 \frac{I(k_1)I(k+k_1)k^2 v^2}{v^2 + \eta^2 k^2 v^2}} + \xi \tilde{f}_o(\xi) \\ = 2\pi i \delta(\xi) \frac{\partial F_M}{\partial \mu} \Big|_{\mu=0}. \end{aligned} \quad (3.16.14)$$

One should solve Eq. 3.16.14 in order to use Eq. 3.16.12 except in the case when the collision frequency is large enough and a 'plateau' is absent. In this case one can neglect $\tilde{f}_o$ in Eq. 3.16.12 and obtain the well known unsaturated nonlinear damping rate (Lee and Völk, 1973; Kulsrud, 1978; Achterberg, 1981):

$$\Gamma_{NL}^{(0)} = \frac{1}{8} \sqrt{2\pi} v_T k \int_{-\infty}^{\infty} dk I(k) \frac{k - k_1}{|k - k_1|}. \quad (3.16.15)$$

In the opposite case of small v one should use Eq. 3.16.14 and put $v = 0$:

$$\frac{\partial}{\partial \xi} \frac{1}{\xi^2} \frac{\partial}{\partial \xi} \xi \tilde{f}_o(\xi) - \frac{1}{8} \langle A^2 \rangle^2 \xi \tilde{f}_o(\xi) = -\frac{i\pi}{4} \delta(\xi) \langle A^2 \rangle^2 \frac{\partial F_M}{\partial \mu} \Big|_{\mu=0}, \quad (3.16.16)$$

where $\langle A^2 \rangle^2 = \int_{-\infty}^{+\infty} dk I(k)$. Substituting the solution of Eq. 3.16.16 into Eq. 3.16.12 and expanding the exponent one can obtain the saturated damping rate:

$$\Gamma_{NL}^{sat} = v_o \langle A^2 \rangle^{-3/2} k \int_{-\infty}^{+\infty} dk_1 \frac{I(k_1)}{k - k_1}, \quad (3.16.17)$$

where

$$\nu_o = \frac{2^{1/4} \pi}{8} \frac{\Gamma(3/4)}{\Gamma(1/4)} \int_0^{+\infty} \frac{4\pi}{(2\pi)^{3/2} v_T^5} \exp\left(-\frac{v^2 + v_a^2}{2v_T^2}\right) dv. \quad (3.16.18)$$

3.16.3. Summary of main results

Zirakashvili et al. (1999) came to the conclusion that the trapping of thermal particles is essential for the damping of Alfvén waves if the frequency of collisions is small enough. For trapped particles $|\mu| < \mu_*$, where $\mu_* \approx \delta B/B$ for Alfvén waves. Hence the escape time is $t_{esc} \approx \mu_*^2/\nu$. It should be compared with the period of particle oscillations inside the trap $T \approx (kv_T \mu_*)^{-1}$. This gives the condition for saturation of nonlinear damping:

$$\nu \ll kv_T (\delta B/B)^3. \quad (3.16.19)$$

The saturated damping rate can be estimated as the unsaturated damping rate multiplied by the ratio T/t_{esc} . It is easy to see that such an estimate is in accordance with Eq. 3.16.17. In the self-consistent model of galactic wind flow (Zirakashvili et al., 1996; Ptuskin et al., 1997) where the unsaturated damping rate was used, $\delta B/B \approx 10^{-2}$ and is determined by the power of CR sources in the Galactic disk. For this case the critical value for the collision frequency is $10^{-12} \text{ sec}^{-1}$ for a wavenumber $k \approx 10^{-13} \text{ cm}^{-1}$ that is in resonance with 1 GeV CR protons. This value is close to the value of the collision frequency of a hot rarefied plasma with number density 10^{-3} cm^{-3} and temperature 10^6 K . Therefore in the absence of other scattering processes, trapping effects might be relevant for Alfvén wave damping in our Galaxy. According to Zirakashvili et al. (1999) another important feature of saturated damping is the possibility of not only damping but also energy transfer to smaller wavenumbers. This property is absent for unsaturated damping of unpolarized ($I(k) = I(-k)$) waves. Such energy transfer can be important for diffusive shock acceleration because it permits small energy particles to generate Alfvén waves that are in resonance with particles of greater energies and, hence determines the rate of acceleration.

Chapter 4

Cosmic Ray Acceleration in Space Plasmas

4.1. Acceleration particles in space plasmas as universal phenomenon in the Universe

Understanding the generation of CR (or acceleration of energetic charged particles) is one of the most fundamental goals of Astrophysics. As we note in Section 1.1 of Dorman (M2004), the basis of any mechanisms of charged particles acceleration in space plasma up to very high energies observed in CR is the interaction of individual particles with huge moving ensembles of particles through frozen in magnetic fields and induced electric fields. These moving ensembles (mass ejections, clouds, shock waves, magneto-hydrodynamic waves, etc.) have a huge kinetic energy many orders higher than energy of an individual particle. Therefore the energy of moving ensembles will lead to the gain of the energy of individual particles, and their energy become many orders higher than the energy of background plasma particles, so these individual particles became CR particles. The acceleration of an individual particle is possible only if the gain of energy per unit of time is bigger than the loss of energy. The loss of energy depends upon the mass m_{ac} , charge Ze , and total energy E (or kinetic energy E_k) of the accelerated particle, as well as upon properties of background plasma, magnetic field intensity, and electromagnetic radiation. The energy loss is especially important at small kinetic energies E_k of an accelerated particle (mostly ionization losses), and it become smaller than the energy gain only at $E_k \geq E_{ki}$, where E_{ki} is the minimal energy of ejection to the accelerated process. The value of E_{ki} which depends upon the mass m_{ac} and effective charge Z^*e of accelerated particles (where $Z^* \leq Z$), and properties of the background plasma will be determine the chemical and isotopic contents of accelerated particles. In the middle energy region the energy loss per unit of time becomes much smaller than the energy gain, and the energy spectrum of accelerated particles will be formatted mainly by the rate of energy gain and probability of accelerated particles escaping from the acceleration volume. In the very high, super-relativistic energy region, the loses of energy again becomes important (for CR electrons, – synchrotron radiation in the magnetic fields and interactions with photons; for CR protons and nuclei, – interactions with photons). These energy loses and CR escaping from the acceleration volume lead to sufficient

deformation of the power energy spectrum $\propto E^{-\gamma}$ with gradual increase of the power index γ with the particle energy increasing and to a sharp cutting of CR spectrum from the high energy side.

First we shall consider the statistical mechanism of charged particle acceleration when the energy of a particle increases and decreases in collisions with magnetic clouds, but increases are bigger and more often than decreases. This mechanism originally was proposed more than 50 years ago by Fermi (1949). According to Fermi (1949) the frequency of collisions with increasing energy is higher than the frequency of collisions with decreasing energy. This gives a gradual increase of particle energy with the time up to the moment when the acceleration mechanism finishes affecting particle's energy (e.g., the particle escaping from the acceleration volume). We shall show that there is also another cause of particle energy increasing: the energy increasing and decreasing in collisions are not equal, but systematically increasing energy is little bigger than decreasing. As result, we show that the statistical mechanism of acceleration is about two times more effective than was considered originally by Fermi (1949). We shall consider the description of this mechanism and its development including the problem of ejection and changing of effective parameters of the mechanism during particle acceleration in Sections 4.2–4.8.

Statistical acceleration by plasma turbulence and by electromagnetic radiation will be considered in Sections 4.9–4.10. We consider statistical acceleration of particles by the Alfvén mechanism of magnetic pumping and scattering in Section 4.11. The problem of the formation of accelerated particle flux escaped from CR source we consider in Section 4.12 (in general this flux is proportional to the CR intensity inside the source and to the probability of particles run away from the source, which depends from particle energy and other parameters).

The induction acceleration mechanisms, mostly by rotating magnetic stars we consider in Section 4.13, and particle acceleration by moving magnetic piston or magnetic cloud as result of single interaction and reflection we shortly consider in Section 4.14. Mechanisms of particle acceleration by shock waves and other moving magneto-hydrodynamic discontinuities during a single interaction are considered in Section 4.15. We consider the acceleration of particles in the case of magnetic collapse and the cumulative acceleration mechanism near the zero lines of magnetic field in Sections 4.16–4.17. The problem of tearing instability in neutral sheet region and triggering mechanisms of formatting fractals, percolation, and particle acceleration we consider in Section 4.18. Particle acceleration in sheer space plasma flows we consider in Section 4.19. Additional regular particle acceleration in space plasmas with two or more types of scatters moving with different velocities is considered in Section 4.20.

Very important universal shock-wave diffusion (regular) acceleration of charged particles, which is intensively developed during about last 30 years, we consider in details in Sections 4.21–4.31.

Different CR acceleration mechanisms in space plasmas were partly reviewed in the books Alfvén (M1950), Dorman (M1957, M1963a,b, M1972b, M1975a, M1978), Ginzburg and Syrovatsky (M1963), Parker (M1963), Pikelner (M1966), Rossi (M1966), Dorman and Miroshnichenko (M1968), Hayakawa (M1969), Tsytoich (M1971), Khristiansen (M1974), Arons et al., eds. (M1979), Melrose (M1980a,b), Priest (M1982), Toptygin (M1983), Berezhko et al. (M1988), Berezhinsky et al. (M1990), Zank and Gaisser, eds. (M1992), Benz (M1993), Sturrock (M1994), Ramaty et al., eds. (M1996), Priest and Forbes (M2000), Miroshnichenko (M2001), Schlickeiser (M2001), and in review papers Dorman and Katz (1977), Syrovatsky (1981), Axford (1987), Debrunner (1987), De Jager (1987), Galeev et al. (1987), Ginzburg (1987), Ramaty (1987), Völk (1987), Dorman and Venkatesan (1993), Biermann (1993), Mandzhavidze (1993), Berezhko (1997, 2001), Cane (1997), Baring (1999), Cliver (1999), Kirk and Duffy (1999), Akasofu (2001), Malkov and Drury (2001), Mazur (2001), Ostrowski (2001), Aschwanden (2002), Cohen (2003), Lin (2003), Moskalenko (2003), Ryan (2005), Kahler et al. (2005), Ptuskin (2005).

4.2. The Fermi mechanism of statistical acceleration

According to Fermi (1949) at each collision of a charged particle moving with velocity $\mathbf{v}$, with magnetic cloud moves with velocity $\mathbf{u}$, changes its energy according to the relation

$$(\Delta E/E)_{\pm} = \pm 2uv/c^2, \quad (4.2.1)$$

where the upper sign is for head-on collisions and bottom sign for overtaking collisions (see Fig. 4.2.1).

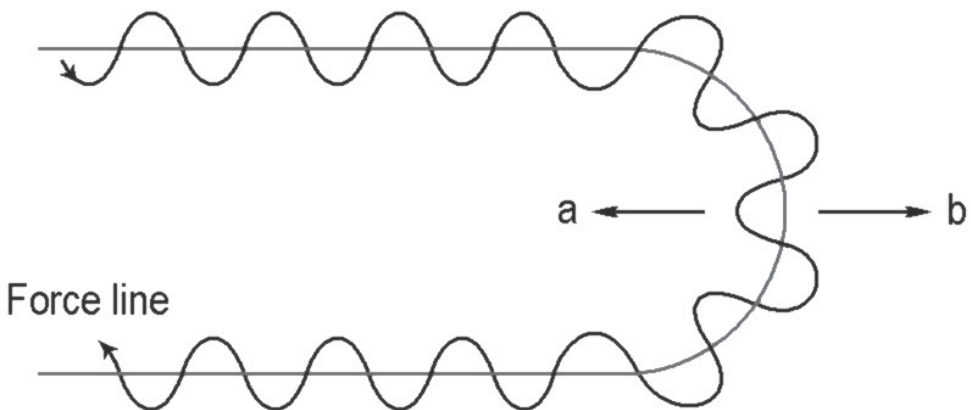


Fig. 4.2.1. Charged particle interaction with a moving magnetic cloud: **a** – the case in which the cloud moves against the particle (head-on collision), **b** – the case in which the cloud moves in the same direction as the particle (overtaking collision). According to Fermi (1949).

Therefore, according to Fermi (1949), in a head-on collision we shall have a relative gain energy of $2uv/c^2$, and in an overtaking collision the same relative loss energy of $2uv/c^2$. If λ is the mean free path for particle collisions with magnetic clouds, the corresponding frequencies $\nu_{\pm}$ for collisions will be

$$\nu_{\pm} = (v \pm u)/2\lambda; \quad \nu_+ + \nu_- = v/\lambda. \quad (4.2.2)$$

The average change energy per unit of time will be (including Eq. 4.2.1 and Eq. 4.2.2)

$$dE/dt = (\Delta E)_+ \nu_+ + (\Delta E)_- \nu_- = \alpha E, \quad (4.2.3)$$

where

$$\alpha = 2u^2 v / \lambda c^2 \quad (4.2.4)$$

is the so called parameter of acceleration. From Eq. 4.2.3 it follows that if particle start to accelerate at $t = 0$ from the initial energy E_i , their energy at moment t will be (if we neglect by energy losses on ionization and other processes):

$$E(t) = E_i \exp\left(\int_0^t \alpha dt\right), \quad (4.2.5)$$

and in case $\alpha = \text{const}$ it will be

$$E(t) = E_i \exp(\alpha t). \quad (4.2.6)$$

Let us suppose (following to Fermi, 1949), that the process of particle acceleration in some volume is stationary, at least for a time much larger than the average time of particles time life τ in the acceleration volume and that particles start to accelerate with the same probability in any time between 0 and τ . In this case the particle distribution over the total time t of acceleration will be

$$D(t)dt = \tau^{-1} \exp(-t/\tau)dt; \quad \int_0^{\tau} D(t)dt = 1. \quad (4.2.7)$$

In the case $\alpha = \text{const}$ it follows from Eq. 4.2.6 that

$$t = \alpha^{-1} \ln(E/E_i); \quad dt = dE/\alpha E. \quad (4.2.8)$$

Substituting Eq. 4.2.8 in Eq. 4.2.7 we arrive at a power distribution over the energy of accelerated particles in the accelerated volume, obtained by Fermi (1949):

$$D(E)dE \propto E^{-\gamma}dE; \quad \gamma = 1 + (\alpha\tau)^{-1}. \quad (4.2.9)$$

4.3. Development of the Fermi model: head-on and overtaking collisions

4.3.1. Non-relativistic case

In this case the particle's velocity is changed by $\pm 2u$ in each particle-cloud collision for the head-on and overtaking collisions, respectively. Therefore, the energy variation is

$$\Delta E_{k\pm} = \Delta E_{\pm} = m_{ac}(v \pm 2u)^2/2 - m_{ac}v^2/2 = \pm 2m_{ac}vu + 2m_{ac}u^2. \quad (4.3.1)$$

Considering that in this case $E \approx m_{ac}c^2$, we obtain

$$(\Delta E/E)_{\pm} = \pm 2uv/c^2 + 2u^2/c^2. \quad (4.3.2)$$

From Eq. 4.3.2 it follows that the energy gain and loss in the head-on and overtaking collisions are not equal: the relative energy gain is systematically bigger than the relative loss energy on $4u^2/c^2$. What is the physical sense of this difference? In Fig. 4.3.1 trajectories of particles inside moved with velocity $\mathbf{u}$ magnetic cloud for the head-on and overtaking collisions are shown in the laboratory system of coordinates.

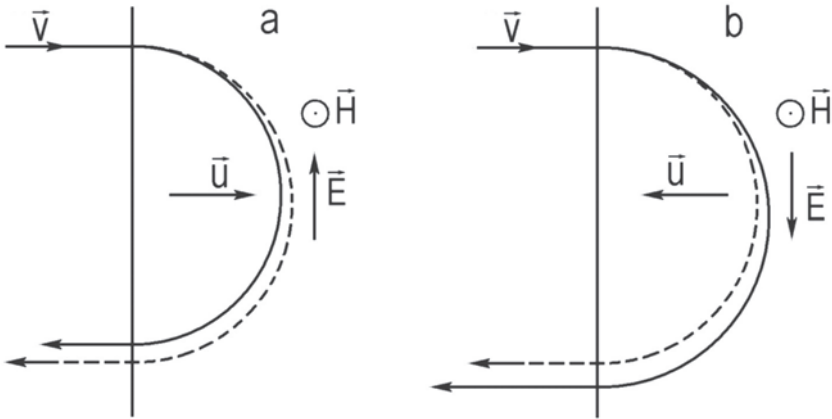


Fig. 4.3.1. Illustration of the derivation of the Eq. 4.3.2 in case of particle collision with a moving magnetic cloud in the laboratory coordinate system: **a** – overtaking collision, **b** – head-on collision. The magnetic field in the cloud $\mathbf{H}$ is perpendicular to the plane of the figure. The induced electric field $\mathbf{E}$ that changes the energy of particle during moving inside the cloud is also shown. The dashed curves show the trajectories suggested by Fermi (1949) and adopted in the scientific literature; the solid curves present the real trajectories.

Let us note, that the difference $4u^2/c^2$ which follows from Eq. 4.3.2 and Fig. 4.3.1 is very small in comparison with the total relative energy gain or loss $2uv/c^2$ (at $v \gg u$) and usually is neglected (starting from Fermi, 1949). However, after averaging, taking into account the frequencies of head-on and overtaking collisions, the second term in the right hand side of Eq. 4.3.2 gives two or more time bigger contribution to the total particle acceleration in the statistical mechanism than the first term usually used. If λ is the transport scattering path of particles before their collision with magnetic clouds, the frequency of the head-on and overtaking collisions will be in the non-relativistic case ($v \ll c, u \ll c$):

$$v_+ = (v + u)/2\lambda; \quad v_- = (v - u)/2\lambda, \quad (4.3.3)$$

which coincide with Eq. 4.2.2. The total variation of particle energy in unit time is

$$dE_k/dt = (\Delta E_k)_+ v_+ + (\Delta E_k)_- v_-, \quad (4.3.4)$$

whence, considering Eq. 4.3.1 and Eq. 4.3.3:

$$dE_k/dt = dE/dt = 4m_{ac} v u^2 / \lambda = \left(4u^2/\lambda\right) \sqrt{2m_{ac} E_k}. \quad (4.3.5)$$

It will be noted that the resultant dE/dt without including the second addend in the right hand side of Eq. 4.3.1 (as was done in Fermi, 1949; see Eq. 4.2.1) is twice as small (compare Eq. 4.2.3 and Eq. 4.3.5). By integrating of Eq. 4.3.5 for the initial condition $E_k = E_{ki}$ at $t = 0$, we obtain

$$E_k(t) = \left[E_{ki}^{1/2} + \int_0^t \frac{2u^2 \sqrt{2m_{ac}}}{\lambda} dt \right]^2 = \left[E_{ki}^{1/2} + \frac{2u^2 t \sqrt{2m_{ac}}}{\lambda} \right]^2, \quad (4.3.6)$$

where the last expression in Eq. 4.3.6 is valid at $u^2/\lambda = \text{const}$ (let us note that in general with increasing energy of accelerated particle, bigger magnetic clouds with bigger velocities became more effective for scattering, so this parameter can change during particle acceleration, even the conditions in the source are stationary; see in detail below Section 4.5).

4.3.2. Relativistic case

In this case, the particle velocity in the coordinate system related to the cloud will be

$$v' = (v \pm u) / \left(1 \pm uv/c^2\right), \quad (4.3.7)$$

where v is the velocity of particles before their collisions with the cloud; the $+$ and $-$ signs correspond to the head-on and overtaking collisions, respectively. In this coordinate system the particle velocity after reflection varies only in the direction having conservation of the modulus of the velocity. Returning to the laboratory coordinate system we find that the finite particle velocity after a head-on or overtaking collisions will be respectively

$$v_{\text{fin}} = (v' \pm u) / \left(1 \pm uv'/c^2\right), \quad (4.3.8)$$

or, considering Eq. 4.3.7 and Eq. 4.3.8, we obtain (taking into account that may be $v \sim c$, but $u \ll c$):

$$\left(\frac{\Delta E}{E}\right)_{\pm} = \left(\frac{m_{ac}}{\sqrt{1 - v_{\text{fin}}^2/c^2}} - \frac{m_{ac}}{\sqrt{1 - v^2/c^2}} \right) \frac{\sqrt{1 - v^2/c^2}}{m_{ac}} = \pm \frac{2uv}{c^2} + \frac{2u^2}{c^2}. \quad (4.3.9)$$

The result obtained coincides with Eq. 4.3.2 for non-relativistic case.

Let us determine now the frequencies of the head-on, ν_+ , and overtaking, ν_- , collisions in the relativistic case. We need to take into account the relativistic summation of velocities of particle and cloud (see Eq. 4.3.7 and Eq. 4.3.8), and the relativistic transformation of the transport path:

$$\lambda_+ = \lambda \left(1 + uv/c^2\right)^{-1}; \quad \lambda_- = \lambda \left(1 - uv/c^2\right)^{-1}. \quad (4.3.10)$$

Therefore the frequencies of the head-on, ν_+ , and overtaking, ν_- , collisions in the relativistic case will be

$$\nu_{\pm} = \frac{v \pm u}{2\lambda_{\pm} \left(1 \pm uv/c^2\right)} = \frac{v \pm u}{2\lambda}, \quad (4.3.11)$$

i.e. the same as was obtained for non-relativistic case (see Eq. 4.3.3). Considering Eq. 4.3.9 and Eq. 4.3.11, we obtain that the mean variation of energy with time in the relativistic case is

$$dE/dt = (\Delta E)_+ \nu_+ + (\Delta E)_- \nu_- = \alpha E, \quad (4.3.12)$$

where

$$\alpha = 4u^2v/\lambda c^2, \quad (4.3.13)$$

i.e. the parameter of acceleration α is two times larger than was obtained in Fermi (1949): compare with Eq. 4.2.4.

4.4. Development of the Fermi model: inclusion of oblique collisions

4.4.1. Non-relativistic case

In this case the particle velocity components perpendicular and parallel to the front of cloud will be $v_{\perp} = \mathbf{uv}/u$ and $v_{\parallel} = v\left(1 - (\mathbf{uv}/uv)^2\right)^{1/2}$. In the coordinate system related to the cloud, we shall obtain $v'_{\perp} = v_{\perp} - u$; $v'_{\parallel} = v_{\parallel}$. After reflection in the coordinate system of the cloud $v'_{\perp 2} = -v'_{\perp}$; $v'_{\parallel 2} = v'_{\parallel}$. When, after that, the laboratory coordinate system is again used, we shall obtain for the particle velocity after collision that

$$v_{\perp 2} = v'_{\perp 2} + u = -v_{\perp} + 2u = -\mathbf{uv}/u + 2u, \quad (4.4.1)$$

$$v_{\parallel 2} = v'_{\parallel 2} = v\left(1 - (\mathbf{uv}/uv)^2\right)^{1/2}, \quad (4.4.2)$$

whence

$$v_2^2 = v_{\perp 2}^2 + v_{\parallel 2}^2 = v^2 - 4\mathbf{uv} + 4u^2. \quad (4.4.3)$$

It follows from Eq. 4.4.3 that

$$E_{k2} = E_k - 2\mathbf{uv}m_{ac} + 2u^2m_{ac}, \quad (4.4.4)$$

whence the energy change in a single collision is

$$\Delta E_k = E_{k2} - E_k = -2\mathbf{uv}m_{ac} + 2u^2m_{ac}. \quad (4.4.5)$$

The relative change of energy in a single collision can be found to be

$$\frac{\Delta E_k}{E_k} = -\frac{4\mathbf{uv}}{uv} \frac{u}{v} + \frac{4u^2}{v^2} = -\frac{4\mathbf{uv}}{uv} \frac{u}{\sqrt{2E_k/m_{ac}}} + \frac{2u^2m_{ac}}{E_k}, \quad (4.4.6)$$

Since in the non-relativistic energy range the total particle energy $E \approx m_{ac}c^2$, we obtain for the relative change of the total energy:

$$\frac{\Delta E}{E} = -\frac{2\mathbf{u}\mathbf{v}}{c^2} + \frac{2u^2}{c^2}. \quad (4.4.7)$$

Thus the energy loss and gain also are not the same for the oblique head-on and overtaking collisions.

Assuming that the cloud velocity distribution is isotropic, we shall average the particle energy change over all the possible realizations of the relative velocity of the particle and cloud:

$$\mathbf{w} = \mathbf{v} - \mathbf{u}; \quad w(\varphi) = (v^2 - 2uv \cos \varphi + u^2)^{1/2}, \quad (4.4.8)$$

where φ is the angle between $\mathbf{u}$ and $\mathbf{v}$. If χ is the azimuthally angle of projection of $\mathbf{u}$ on a plane perpendicular to $\mathbf{v}$, then

$$\langle \Delta E_k \rangle = \frac{1}{4\pi} \int_0^{2\pi} d\chi \int_0^\pi \Delta E_k(\varphi) w(\varphi) \sin \varphi d\varphi / \langle w \rangle, \quad (4.4.9)$$

where $\Delta E_k(\varphi)$ is determined from Eq. 4.4.5 and

$$\langle w \rangle = \frac{1}{4\pi} \int_0^{2\pi} d\chi \int_0^\pi w(\varphi) \sin \varphi d\varphi = \begin{cases} u + v^2/3u & \text{if } v \leq u, \\ v + u^2/3v & \text{if } v \geq u. \end{cases} \quad (4.4.10)$$

In this way we find the average particle energy change per single collision with a cloud

$$\langle \Delta E_k \rangle = \langle \Delta E \rangle = 2u^2 m_{ac} + \begin{cases} (2/3)v^2 m_{ac} & \text{if } v \ll u, \\ \frac{2v^2(5u^2 - v^2)}{5(3u^2 + v^2)} m_{ac} & \text{if } v \leq u, \\ \frac{2u^2(5v^2 - u^2)}{5(3v^2 + u^2)} m_{ac} & \text{if } v \geq u, \\ (2/3)u^2 m_{ac} & \text{if } v \gg u. \end{cases} \quad (4.4.11)$$

The Eq. 4.4.11 implies an important conclusion which is at variance with the commonly accepted opinion that the energy gain in the statistical acceleration mechanism is owed to the difference between the frequencies of the head-on and overtaking collisions (the second term in Eq. 4.4.11). In the actuality the acceleration in the non-relativistic range is mainly (by more than 75%) owing to the difference between the energy loss and gain in the head-on and overtaking collisions (the first term in Eq. 4.4.11).

The mean relative change of the kinetic energy may be obtained as

$$\left\langle \frac{\Delta E_k}{E_k} \right\rangle = \frac{4E_{ko}}{E_k} \times \begin{cases} \left(1 + \frac{E_k(E_{ko} - E_k/5)}{E_{ko}(3E_{ko} + E_k)} \right) & \text{if } E_k \leq E_{ko}, \\ \left(1 + \frac{(E_k - E_{ko}/5)}{(3E_k + E_{ko})} \right) & \text{if } E_k \geq E_{ko}, \end{cases} \quad (4.4.12)$$

where $E_{ko} = m_{ac}u^2/2$. Since in the studied energy range $E \approx m_{ac}c^2$, then

$$\left\langle \frac{\Delta E}{E} \right\rangle = \frac{2u^2}{c^2} \times \begin{cases} \left(1 + \frac{E_k(E_{ko} - E_k/5)}{E_{ko}(3E_{ko} + E_k)} \right) & \text{if } E_k \leq E_{ko}, \\ \left(1 + \frac{(E_k - E_{ko}/5)}{(3E_k + E_{ko})} \right) & \text{if } E_k \geq E_{ko}. \end{cases} \quad (4.4.13)$$

It follows from Eq. 4.4.13 that in extreme cases

$$\left\langle \frac{\Delta E}{E} \right\rangle = \frac{2u^2}{c^2} \times \begin{cases} 1 & \text{if } E_k \ll E_{ko}, \\ 6/5 & \text{if } E_k = E_{ko}, \\ 4/3 & \text{if } E_k \gg E_{ko}. \end{cases} \quad (4.4.14)$$

The energy change in time averaged over all possible collisions will be determined by the relation

$$\left\langle \frac{dE_k}{dt} \right\rangle = \frac{1}{4\pi} \int_0^{2\pi} d\chi \int_0^\pi \Delta E_k(\varphi) \nu(\varphi) \sin \varphi d\varphi. \quad (4.4.15)$$

Here the frequency of particle collisions with magnetic clouds is

$$\nu(\varphi) = w(\varphi)/\lambda, \quad (4.4.16)$$

where λ is the particle transport path for collisions with clouds (e.g., if l is the characteristic size of clouds, $N_{cl} = d^{-3}$ is the clouds concentration, d is the mean distance between clouds, then $\lambda \approx d^3/l^2$). Taking into account Eq. 4.4.8 we obtain

$$\frac{dE_k}{dt} = \begin{cases} \frac{2u^2 m_{ac}}{\lambda} \left(u + \frac{v^2}{3u} \right) + \frac{2v^2 m_{ac}}{15\lambda u} (5u^2 - v^2) & \text{if } v \leq u, \\ \frac{2u^2 m_{ac}}{\lambda} \left(v + \frac{u^2}{3v} \right) + \frac{2u^2 m_{ac}}{15\lambda v} (5v^2 - u^2) & \text{if } v \geq u, \end{cases} \quad (4.4.17)$$

or, after expressing the velocities v and u in terms of E_k and E_{ko} , we find that

$$\frac{dE_k}{dt} = \frac{16E_{ko}}{3\lambda} \begin{cases} \frac{3}{4} \sqrt{2E_{ko}/m_{ac}} + \frac{E_k}{\sqrt{2E_{ko}m_{ac}}} - \frac{E_k^2}{10E_{ko}\sqrt{2E_{ko}m_{ac}}} & \text{if } E_k \leq E_{ko}, \\ \sqrt{2E_k/m_{ac}} + 2E_{ko} & \text{if } E_k \geq E_{ko}, \\ \sqrt{2E_k/m_{ac}} & \text{if } E_k \gg E_{ko}. \end{cases} \quad (4.4.18)$$

Let the particles be accelerated from some initial energy E_{ki} at the instant $t=0$. **Consider first the case in which $E_{ki} < E_{ko}$. In this case we obtain from Eq. 4.4.18 for the energy range $E_k < E_{ko}$, i.e. the time interval $0 \leq t \leq t_1$**

$$E_k(t) = E_{ko} \left(5 - \frac{2\sqrt{10}(b + \exp(at))}{b - \exp(at)} \right), \quad (4.4.19)$$

where

$$a = \frac{32\sqrt{2E_{ko}/m_{ac}}}{3\sqrt{10}\lambda}; \quad b = \frac{1 + 2\sqrt{2/5} - E_{ki}/5E_{ko}}{1 - 2\sqrt{2/5} - E_{ki}/5E_{ko}}; \quad t_1 = \frac{1}{a} \ln \frac{b(1 - 2\sqrt{2/5} - 1/5)}{1 + 2\sqrt{2/5} - 1/5}. \quad (4.4.20)$$

The following relation between E_k and t will be obtained from Eq. 4.4.18 in the energy range $E_k \geq E_{ko}$, i.e. for the time interval $t \geq t_1$

$$\frac{8\sqrt{2E_{ko}/m_{ac}}}{3\lambda} (t - t_1) = \sqrt{E_k/E_{ko}} - 1 - \sqrt{1/5} \arctg \frac{\sqrt{E_k/E_{ko}} - 1}{\sqrt{5E_k/E_{ko}} + \sqrt{1/5}}. \quad (4.4.21)$$

In the energy range $E_k \gg E_{ko}$ it follows from Eq. 4.4.21 that

$$\begin{aligned} E_k(t) &= E_{ko} \left(\left(1 + \sqrt{1/5} \arctg \sqrt{1/5} \right) + \frac{8\sqrt{2E_{ko}/m_{ac}}}{3\lambda} (t - t_1) \right)^2 \\ &= E_{ko} \left(1.187 + \frac{2.667u}{\lambda} (t - t_1) \right)^2. \end{aligned} \quad (4.4.22)$$

It should be noted that the Eq. 4.4.22 may be used in approximate calculations over the entire energy range since the term with $\arctg$ in Eq. 4.4.21 varies comparatively little, namely from 0 at $E_k = E_{ko}$ to 0.187 at $E_k \gg E_{ko}$.

If a particle starts being accelerated at the instant $t = 0$ from the initial energy $E_{ki} \geq E_{ko}$, the law of change in E_k with time t will be determined by the relation

$$\frac{8t\sqrt{2E_{ko}/m_{ac}}}{3\lambda} = \sqrt{E_k/E_{ko}} - \sqrt{E_{ki}/E_{ko}} - \sqrt{1/5} \arctan \frac{\sqrt{E_k/E_{ki}} - 1}{\sqrt{E_k/5E_{ki}} + \sqrt{5E_k/E_{ko}}} \quad (4.4.23)$$

or, for the energy range $E_k \gg E_{ko}$:

$$E_k(t) \approx E_{ko} \left(\sqrt{E_{ki}/E_{ko}} + \sqrt{1/5} \arctan \sqrt{E_{ko}/5E_{ki}} + \frac{2.667ut}{\lambda} \right)^2. \quad (4.4.24)$$

The Eq. 4.4.24 may be used over practically the entire energy range since the term with $\arctan$ in Eq. 4.4.21 varies comparatively little, namely from 0 at $E_k = E_{ko}$ to 0.187 at $E_k \gg E_{ko}$.

The accelerated particle spectrum will be determined using the particle acceleration time distribution (see Eq. 4.4.6); then it follows from Eq. 4.4.24 that

$$n(E_k)dE_k \propto \exp \left(- \frac{3\lambda\sqrt{E_k/E_{ko}}}{8\tau\sqrt{2E_{ko}/m_{ac}}} \right) \frac{dE_k}{\sqrt{E_k}} = \frac{\exp \left(- \frac{3v\lambda}{8u^2\tau} \right) dE_k}{\sqrt{E_k}}. \quad (4.4.25)$$

The spectrum described by Eq. 4.4.25 may be presented in the form

$$n(E_k)dE_k \propto E_k^{-\gamma} dE_k, \quad (4.4.25a)$$

where

$$\gamma = -\frac{E_k}{n(E_k)} \frac{dn(E_k)}{dE_k} = \frac{1}{2} \left(1 + \frac{3\lambda \sqrt{2E_k/m_{ac}}}{8u^2} \right) = \frac{1}{2} \left(1 + \frac{3v\lambda}{8u^2\tau} \right). \quad (4.4.26)$$

4.4.2. Relativistic case

Let us lift the limitation $v \ll c$ (but meet the condition $u \ll c$). If in the laboratory coordinate system the particle velocity prior to collision is $\mathbf{v}$ and the cloud velocity is $\mathbf{u}$, then in the coordinate system related to the cloud for the particle velocity prior to collision we obtain

$$\mathbf{v}' = (\mathbf{v} - \mathbf{u}) \left(1 - \mathbf{v}\mathbf{u}/c^2 \right)^{-1}. \quad (4.4.27)$$

In the same coordinate system, after reflection from the cloud, the longitudinal (along the cloud front) component of particle velocity will not change, whereas the sign of the transverse component will reverse. If, after that, the laboratory coordinate system is used, we obtain

$$\mathbf{v}_{\text{fin}} = \left(-\frac{\mathbf{v} - \mathbf{u}}{1 + \mathbf{v}\mathbf{u}/c^2} + \mathbf{u} \right) \left(1 - \frac{(\mathbf{v} - \mathbf{u})\mathbf{u}}{(1 - \mathbf{v}\mathbf{u}/c^2)c^2} \right)^{-1} \approx -\mathbf{v} \left(1 + 2\frac{\mathbf{v}\mathbf{u}}{c^2} \right) + 2\mathbf{u} \left(1 + \frac{3\mathbf{v}\mathbf{u}}{2c^2} \right), \quad (4.4.28)$$

where the terms of order higher than $\mathbf{v}\mathbf{u}/c^2$ are neglected. Let us find now the relative change of the particle energy in a single collision:

$$\frac{\Delta E}{E} = \left(\frac{m_{ac}}{\sqrt{1 - v_{\text{fin}}^2/c^2}} - \frac{m_{ac}}{\sqrt{1 - v^2/c^2}} \right) \frac{\sqrt{1 - v^2/c^2}}{m_{ac}} = \frac{\sqrt{1 - v^2/c^2}}{\sqrt{1 - v_{\text{fin}}^2/c^2}} - 1 = \frac{b}{1 - b}, \quad (4.4.29)$$

where it has been taken into account that

$$v_{\text{fin}}^2 = v^2 \left(1 + \frac{\mathbf{u}\mathbf{v}}{c^2} + 12 \left(\frac{\mathbf{u}\mathbf{v}}{c^2} \right)^2 - 2 \frac{u^2}{c^2} \right) + 4u^2 \left(1 + 3 \frac{\mathbf{u}\mathbf{v}}{c^2} \right) - 4\mathbf{u}\mathbf{v} \left(1 + \frac{2\mathbf{u}\mathbf{v}}{7c^2} \right); \quad (4.4.30)$$

$$b = \left(-4 \frac{\mathbf{u}\mathbf{v}}{c^2} - 12 \left(\frac{\mathbf{u}\mathbf{v}}{c^2} \right)^2 + 4 \frac{u^2}{c^2} \right), \quad (4.4.31)$$

and where the terms of order higher than u^2/c^2 are neglected. Substituting Eq. 4.4.31 in Eq. 4.4.29, we shall obtain, within the same accuracy, the expression

$$\frac{\Delta E}{E}(\varphi) = \frac{1}{2}b + \frac{3}{8}b^2 + \dots \approx -2\frac{\mathbf{u}\mathbf{v}}{c^2} + 2\frac{u^2}{c^2} = -2\frac{uv\cos\varphi}{c^2} + 2\frac{u^2}{c^2}, \quad (4.4.32)$$

where $\varphi = \arccos(\mathbf{u}\mathbf{v}/uv)$ is the angle between $\mathbf{v}$ and $\mathbf{u}$. Eq. 4.4.32 coincides with Eq. 4.4.18 obtained for non-relativistic case. Since in the relativistic case the relative velocity of particle and cloud is

$$\mathbf{w} = (\mathbf{v} - \mathbf{u}) / \left(1 - \mathbf{u}\mathbf{v}/c^2\right); w(\varphi) = \left(v^2 - 2uv\cos\varphi + u^2\right)^{1/2} / \left(1 - uv\cos\varphi/c^2\right), \quad (4.4.33)$$

the relative change in particle energy averaged over all possible angles between $\mathbf{v}$ and $\mathbf{u}$ will be

$$\left\langle \frac{\Delta E}{E} \right\rangle = \frac{1}{4\pi} \int_0^{2\pi} d\chi \int_0^\pi \frac{\Delta E}{E}(\varphi) v(\varphi) \sin\varphi d\varphi / \langle v(\varphi) \rangle, \quad (4.4.34)$$

where $\Delta E(\varphi)$ is determined by Eq. 4.4.32 and the distribution function of collision frequencies $\nu(\varphi)$ is

$$\nu(\varphi) = w(\varphi)/\lambda(\varphi); \quad \lambda(\varphi) = \lambda / \left(1 - uv\cos\varphi/c^2\right). \quad (4.4.35)$$

Since the non-relativistic case was analyzed in detail in Section 4.4.1, only the case $v \gg u$ will be considered below. From Eq. 4.4.35 follows that

$$\langle \nu(\varphi) \rangle = \frac{1}{4\pi} \int_0^{2\pi} d\chi \int_0^\pi \nu(\varphi) \sin\varphi d\varphi = \frac{2\pi/\lambda}{4\pi} \int_{-1}^{+1} \left(v^2 - 2uvx + u^2\right)^{1/2} dx \approx v/\lambda, \quad (4.4.36)$$

and from Eq. 4.4.34 we obtain (including Eq. 4.4.32, 4.4.33, 4.4.35, and 4.4.36):

$$\begin{aligned} \left\langle \frac{\Delta E}{E} \right\rangle &= \frac{1}{4\pi} \int_0^{2\pi} d\chi \int_0^\pi \left(-2\frac{uv\cos\varphi}{c^2} + 2\frac{u^2}{c^2} \right) \frac{\left(v^2 - 2uv\cos\varphi + u^2\right)^{1/2}}{\lambda} \sin\varphi d\varphi / \langle v/\lambda \rangle \\ &= \frac{uv}{c^2} \int_{-1}^{+1} \left(-x + \frac{u}{v} \right) \left(1 - \frac{2ux}{v} + \frac{u^2}{v^2} \right)^{1/2} dx \approx \frac{2}{3} \frac{u^2}{c^2} + 2\frac{u^2}{c^2}. \end{aligned} \quad (4.4.37)$$

This result coincides with that obtained for non-relativistic case (compare with Eq. 4.4.14 at $E_k \gg E_{ko} = m_{ac}u^2/2$). According to Eq. 4.4.37 only 25% of the energy gain is caused by the difference in the frequencies of particle collisions with clouds; 75% of energy gain is caused by the systematical small gain energy which does not depend from φ and which was neglected in original variant of statistical acceleration mechanism (Fermi, 1949) as well as in many subsequent papers of other authors.

Thus, the effect of a systematic small excess of energy gain over energy loss in each collision, when averaged over all possible angles between $\mathbf{u}$ and $\mathbf{v}$, is of main importance to the statistical energy gain by a particle - not only in the low-energy range (see Section 4.4.1) but also at relativistic energies.

Now let the particle energy change in time be found:

$$\frac{dE}{dt} = \frac{1}{4\pi} \int_0^{2\pi} d\chi \int_0^\pi \nu(\varphi) \Delta E(\varphi) \sin \varphi d\varphi = E \frac{2u^2 v}{c^2 \lambda} \left(\frac{1}{3} + 1 \right), \quad (4.4.38)$$

where the frequency of particle collisions with clouds $\nu(\varphi) = w(\varphi)/\lambda(\varphi)$ was determined by Eq. 4.4.33 and 4.4.35. Here $w(\varphi)$ is the velocity of particle relative to the cloud, and $\lambda(\varphi)$ is the particle transport path. In Eq. 4.4.38 we show separately two parts of the energy gain (the same 25% and 75%) caused correspondingly by differences in frequencies of collisions and by small but systematic energy gain. Integrating over Eq. 4.4.38 with the initial condition $E = E_i$ at $t = 0$, we obtain for relativistic particles ($v \approx c$):

$$E(t) \approx E_i \exp \left(\int_0^t \frac{8u^2}{3c\lambda} dt \right) \approx E_i \exp \left(\frac{8u^2 t}{3c\lambda} \right), \quad (4.4.39)$$

where the last expression is valid if the parameter of acceleration $\alpha = 8u^2/3c\lambda$ does not change with time during the particle's acceleration. Using the particle distribution determined by Eq. 1.1.6 over the age t from the acceleration onset and including the relations

$$t = \frac{3c\lambda}{8u^2} \ln(E/E_i), \quad dt = \frac{3c\lambda dE}{8u^2 E}, \quad (4.4.40)$$

we shall obtain for the accelerated particle spectrum

$$n(E)dE \propto E^{-\gamma} dE; \quad \gamma = 1 + 8u^2 \tau / 3c\lambda. \quad (4.4.41)$$

It is of importance to emphasize that in the relativistic case considered the particle energy gain and the generation of the spectrum are also mainly accounted for not by the difference in the frequency of the head-on and overtaking collisions (this concept is widely used in the literature) but by the effect of the systematic difference in the particle energy gain and loss in each particle collision with clouds, namely, the relative importance of the former phenomenon as compared with the latter is determined by a factor of 1:3.

4.5. Statistical acceleration of particles during the variations in the acceleration mechanism parameters as particles gain energy

4.5.1. The expected variations of the acceleration mechanism parameters as a particles gain energy

It was assumed above that the parameters λ , u , and τ are the constants independent of time. This is, however, is not the fact of reality, even in the stationary case. The fact is that, as the particle energy increases, the properties of the magnetic inhomogeneities (size, magnetic field intensity, velocity of motion) involved effectively in scattering and energy change of the particle vary; therefore, the effective values of λ and u will vary with changing of E . As it follows from the results presented in Chapter 1 (Section 1.9), it should be expected that over a wide energy range

$$\lambda = \lambda_i (E/E_i)^\beta, \quad (4.5.1)$$

where λ_i is the transport path at particle energy of injection E_i , and it is most probable that $0 \leq \beta \leq 1$. Only at very high E , when the Larmor radius of the accelerated particles exceeds the largest scale of inhomogeneities, $\beta \rightarrow 2$. Generally speaking, the specific value of the parameter β is a function of the magnetic inhomogeneity spectrum and the nature of the fields in the inhomogeneities (see Section 1.9). As to the inhomogeneity velocity u , it should increase with increasing λ (for example, in case of developed turbulence $u \propto \lambda^{2/3}$). Since according to Eq. 4.5.1 λ is a power function of E , we shall assume that

$$u = u_i (E/E_i)^\delta, \quad (4.5.2)$$

where u_i is the velocity of movement of the inhomogeneities ensuring the effective scattering of the particles with energy E_i (in the case of developed turbulence of Kolmogorov type $\delta = 2\beta/3$). If τ in Eq. 4.2.7 is determined by diffusive escaping from the acceleration region with effective size L , then

$$\tau \approx \frac{L^2}{2v\lambda} \approx \tau_i (E/E_i)^{-\beta} (v_i/v), \quad (4.5.3)$$

where $\tau_i = L^2/2v_i\lambda_i$ is the mean lifetime of particles with energy E_i and velocity v_i in their source. It follows from Eq. 4.5.3 that the effective time of particle acceleration in the source decreases with increasing the particle energy E .

4.5.2. The mode of particle energy change and formation of the spectrum in the non-relativistic range for the statistical acceleration mechanism including the dependence of λ and u on energy.

Let it be assumed that the dependences determined by Eq. 4.5.1, 4.5.2, and 4.5.3 are also valid for E_k , i.e. in the non-relativistic energy range where these dependences may be written in the form

$$\lambda = \lambda_i (E_k/E_{ki})^\beta, \quad u = u_i (E_k/E_{ki})^\delta, \quad \tau = \tau_i (E_k/E_{ki})^{-(\beta+1/2)}, \quad (4.5.4)$$

where λ_i , u_i , and τ_i are respectively the transport scattering path, the velocity of scattering inhomogeneities, and the mean time living of particles in the acceleration source at kinetic energy of ejection E_{ki} . Then instead of Eq. 4.4.18 we shall obtain at $E_k \gg m_{ac}u^2/2$:

$$\frac{dE_k}{dt} = \frac{8u_i^2 \sqrt{2m_{ac}}}{3\lambda_i E_{ki}^{2\delta-\beta}} \left(1 + \frac{u_i^2 m_{ac}}{5E_{ki}^{2\delta}} E_k^{2\delta-1} \right) E_k^{2\delta-\beta+1/2}. \quad (4.5.5)$$

Let us examine the following cases.

(1) The case $2\delta - 1 = 0$. Including the initial condition $E_k = E_{ki}$ at $t = 0$, and $\beta \neq 1/2$, we shall then obtain

$$E_k(t) = E_{ki} \left[1 + (\beta - 1/2) \frac{2u_i^2 t \sqrt{2m_{ac}/E_{ki}}}{3\lambda_i} \left(1 + \frac{u_i^2 m_{ac}}{5E_{ki}} \right) \right]^{\frac{1}{\beta-1/2}}. \quad (4.5.6)$$

If $\beta < 1/2$ then

$$E_k \rightarrow \infty \text{ at } t \rightarrow \frac{3\lambda_i}{2u_i^2} \left[\sqrt{2m_{ac}/E_{ki}} (1/2 - \beta) \left(1 + \frac{u_i^2 m_{ac}}{5E_{ki}} \right) \right]^{-1}. \quad (4.5.7)$$

It should be taken into account, however, that the result described by Eq. 4.5.7 is only formal and is not realistic, because Eq. 4.4.18 and hence Eq. 4.5.6 are valid for only non-relativistic energies and that one should use Eq. 4.4.38 for the range of sufficiently high energies. In order to find the spectrum of accelerated particles at $\beta < 1/2$ one should take into account the particle distribution over t according to Eq. 4.2.7, the dependence of τ on E_k according to Eq. 4.5.4, and that, according to Eq. 4.5.6,

$$t = \left[1 - (E_k/E_{ki})^{-(1/2-\beta)} \right]^{(1/2-\beta)} \left[\frac{16(1/2-\beta)u_i^2}{3\lambda_i \sqrt{2E_{ki}/m_{ac}}} \left(1 + \frac{u_i^2 m_{ac}}{5E_{ki}} \right) \right]^{-1}; \quad (4.5.8)$$

$$dt = (E_k/E_{ki})^{\beta-3/2} \left[\frac{16u_i^2 E_{ki}}{3\lambda_i \sqrt{2E_{ki}/m_{ac}}} \left(1 + \frac{u_i^2 m_{ac}}{5E_{ki}} \right) \right]^{-1} dE_k. \quad (4.5.9)$$

In such a way we shall obtain that

$$n(E_k) \propto E_k^{-(1-2\beta)} \times \exp \left\{ - \left(\frac{E_k}{E_{ki}} \right)^{\beta+1/2} \left[1 - \left(\frac{E_k}{E_{ki}} \right)^{-(1/2-\beta)} \right] \left[\frac{16(1/2-\beta)u_i^2 \tau_i}{3\lambda_i \sqrt{2E_{ki}/m_{ac}}} \left(1 + \frac{u_i^2 m_{ac}}{5E_{ki}} \right) \right]^{-1} \right\}. \quad (4.5.10)$$

If $\beta > 1/2$, then we shall obtain instead of Eq. 4.5.6:

$$E_k(t) = E_{ki} \left[1 + (\beta - 1/2) \frac{8u_i^2 t \sqrt{2m_{ac}/E_{ki}}}{3\lambda_i} \left(1 + \frac{u_i^2 m_{ac}}{5E_{ki}} \right) \right]^{1/(\beta-1/2)}. \quad (4.5.11)$$

It can be easily seen that in this case, at first at $t \ll t_1$, where

$$t_1 = 3\lambda_i \left[(\beta - 1/2) 8u_i^2 \sqrt{2m_{ac}/E_{ki}} \left(1 + \frac{u_i^2 m_{ac}}{5E_{ki}} \right) \right]^{-1}. \quad (4.5.12)$$

the particle energy increases very slowly:

$$E_k(t) = E_{ki} \left[1 + \frac{t}{(\beta - 1/2)t_1} \right]; \quad (4.5.13)$$

and, after that at $t \gg t_1$, it increases as

$$E_k(t) = E_{ki}(t - t_1)^{1/(\beta - 1/2)}. \quad (4.5.14)$$

The accelerated particle spectrum at $\beta > 1/2$ will be determined by the Eq. 4.5.10 found above.

If $\beta = 1/2$, then, after integrating Eq. 4.5.5 at $2\delta = 1$ and the initial condition $E_k|_{t=0} = E_{ki}$, we shall obtain

$$E_k(t) = E_{ki} \exp \left[\frac{8u_i^2 t \sqrt{2m_{ac}/E_{ki}}}{3\lambda_i} \left(1 + \frac{u_i^2 m_{ac}}{5E_{ki}} \right) \right]. \quad (4.5.15)$$

The accelerated particle spectrum at $\beta = 1/2$ will be found, using the distribution described by Eq. 4.2.7 over the times of particle t acceleration in the source and the dependence of the mean lifetime τ on E_k (see Eq. 4.5.4). The spectrum found of the accelerated particles may be presented in the form of Eq. 4.2.9, i.e. $n(E_k) \propto E_k^{-\gamma}$, but with a variable power exponent

$$\gamma = 2 + 3\lambda_i E_k / 8u_i^2 \tau_i \sqrt{2m_{ac}/E_{ki}} \left(1 + \frac{u_i^2 m_{ac}}{5E_{ki}} \right). \quad (4.5.16)$$

In the moderate-energy range

$$E_k \leq E_{kc} = 8u_i^2 \tau_i \sqrt{2m_{ac}/E_{ki}} \left(1 + \frac{u_i^2 m_{ac}}{5E_{ki}} \right) / 3\lambda_i, \quad (4.5.17)$$

the exponent $\gamma \approx 2$. At $E_k = E_{kc}$, $\gamma = 3$, where E_{kc} is determined by Eq. 4.5.17, and then γ increases rapidly with increasing E_k as $\gamma = 2 + E_k/E_{kc}$.

(2) The case $2\delta - 1 > 0$. In this case at $E_k \ll \left(5E_{ki}^{2\delta} / u_i^2 m_{ac} \right)^{1/(2\delta-1)}$ we shall obtain

$$E_k(t) = E_{ki} \left[1 - \frac{(2\delta - \beta - 1/2) 8 u_i^2 t \sqrt{2 m_{ac} / E_{ki}}}{3 \lambda_i} \right]^{-1/(2\delta - \beta - 1/2)} \quad (4.5.18)$$

The accelerated particle spectrum will be in the form

$$n(E_k) \propto E_k^{-2(\delta - \beta)} \times \exp \left\{ - \left(\frac{E_k}{E_{ki}} \right)^{\beta + 1/2} \left[1 + \left(\frac{E_k}{E_{ki}} \right)^{-(2\delta - \beta - 1/2)} \right] \left[\frac{8(2\delta - \beta - 1/2) u_i^2 \tau_i}{3 \lambda_i \sqrt{E_{ki} / 2 m_{ac}}} \right]^{-1} \right\} \quad (4.5.19)$$

If $E_k \gg (5 E_{ki}^{2\delta} / u_i^2 m_{ac})^{1/(2\delta - 1)}$, then

$$E_k(t) = E_{ki} \left[1 - \frac{(4\delta - \beta - 3/2) 8 \sqrt{2} u_i^4 m_{ac}^{3/2} t}{15 \lambda_i E_{ki}^{3/2}} \right]^{-1/(4\delta - \beta - 3/2)} ; \quad (4.5.20)$$

and for differential density spectrum of accelerated particles inside the source we obtain

$$n(E_k) \propto E_k^{-(4\delta - 2\beta - 1)} \times \exp \left\{ - \left(\frac{E_k}{E_{ki}} \right)^{\beta + 1/2} \left[1 - \left(\frac{E_k}{E_{ki}} \right)^{-(4\delta - \beta - 3/2)} \right] \left[\frac{8 \sqrt{2} (4\delta - \beta - 3/2) u_i^4 \tau_i}{15 \lambda_i E_{ki}^{3/2} m_{ac}^{-3/2}} \right]^{-1} \right\} \quad (4.5.21)$$

(3) The case $2\delta - 1 < 0$. Here, at $E_k \ll (5 E_{ki}^{2\delta} / u_i^2 m_{ac})^{1/(2\delta - 1)}$ we shall obtain for $E_k(t)$ and $n(E_k)$ the expressions coinciding with Eq. 4.5.20 and 4.5.21 respectively.

If, however $E_k \gg (5 E_{ki}^{2\delta} / u_i^2 m_{ac})^{1/(2\delta - 1)}$ we shall obtain for $E_k(t)$ and $n(E_k)$ the expressions coinciding with Eq. 4.5.18 and 4.5.19, respectively.

4.5.3. Particle acceleration and formation of the spectrum in relativistic energy range including the variations in the parameters λ and u with total particle energy E increasing

In the relativistic case we shall use Eq. 4.4.38. Substituting Eq. 4.5.1 and Eq. 4.5.2 in Eq. 4.4.38, we obtain

$$\frac{dE}{dt} = \frac{8u_i^2 v E^{2\delta-\beta+1}}{3c^2 \lambda_i E_i^{2\delta-\beta}}. \quad (4.5.22)$$

Since the non-relativistic energy range was examined above, we shall analyze here the ultra-relativistic case where it may be assumed that $v \approx c$. In this case it follows from Eq. 4.5.22 that

$$E(t) = E_i \times \begin{cases} \exp(8u_i^2 t / 3c\lambda_i) & \text{if } 2\delta - \beta = 0, \\ \left[1 - \frac{8(2\delta - \beta)u_i^2 t}{3c\lambda_i} \right]^{-1/(2\delta-\beta)} & \text{if } 2\delta - \beta \neq 0. \end{cases} \quad (4.5.23)$$

Examine separately the cases where $2\delta - \beta = 0$ and $2\delta - \beta \neq 0$.

(1) The case $2\delta - \beta = 0$. In this case, at $m_{ac}^2 c^4 / E^2 \ll 1$ it follows from Eq. 4.5.22 that the acceleration parameter

$$\alpha = \alpha_i = 8u_i^2 / 3c\lambda_i = \text{const}, \quad (4.5.24)$$

i.e. the relative rate $\Delta E/E$ of particle energy gain will be constant. Here, using Eq. 4.2.7 and taking account of Eq. 4.5.3, we shall obtain an expression of the form Eq. 4.2.9, i.e. $n(E) \propto E^{-\gamma}$, but with the power exponent being a function of E :

$$\gamma = 1 + \frac{(E/E_i)^\beta}{\alpha_i \tau_i}. \quad (4.5.25)$$

Thus, in this case the spectrum may be described by a power law with variable exponent γ increasing with particle energy (if $\beta > 0$). It can be easily seen that at $\beta \rightarrow 0$ the Eq. 4.5.25 for γ turns out to be Eq. 4.2.9 but with higher parameter of acceleration α .

(2) The case $2\delta - \beta \neq 0$. If $2\delta - \beta \neq 0$ then, according to Eq. 4.5.23 and using the notations of Eq. 4.5.24 we obtain

$$E(t) = E_i (1 - \alpha_i t (2\delta - \beta))^{-1/(2\delta-\beta)}. \quad (4.5.26)$$

Since in this case

$$t = \left[1 - (E/E_i)^{-(2\delta-\beta)} \right] / \alpha_i (2\delta - \beta), \quad dt = dE (E/E_i)^{-(2\delta-\beta+1)} / \alpha_i E_i, \quad (4.5.27)$$

we shall obtain for the accelerated particle density differential energy spectrum inside the source using Eq. 4.5.3:

$$n(E) \propto E^{-(1+2\delta-2\beta)} \exp \left[- (E/E_i)^\beta \left(1 - (E/E_i)^{-(2\delta-\beta)} \right) (\alpha_i \tau_i (2\delta - \beta))^{-1} \right]. \quad (4.5.28)$$

It can be easily seen that when $2\delta - \beta \rightarrow 0$, the Eq. 4.5.28 turns out to be a power spectrum of the form $n(E) \propto E^{-\gamma}$, where γ is determined by the Eq. 4.5.25.

If $2\delta - \beta > 0$, then in accordance with Eq. 4.5.26 and Eq. 4.5.28 the rate of the particle energy gain will be more rapid and the generated spectrum at $E \gg E_i$ will be the product of the power function $E^{-\gamma}$ with $\gamma = 1 - 2\beta + 2\delta$ by an exponential function of the form $\exp \left[- (E/E_i)^\beta (\alpha_i \tau_i (2\delta - \beta))^{-1} \right]$. The exponential factor will result in the spectrum cutoff on the high energy side at $E \geq E_i (\alpha_i \tau_i (2\delta - \beta))^{1/\beta}$.

If $2\delta - \beta < 0$, then in accordance with Eq. 4.5.26 and Eq. 4.5.28 the rate of the particle energy gain will be rather slower than exponential, while the spectrum at $E \gg E_i$ will be expressed as the product of a power function of the form $E^{-\gamma}$ with exponent $\gamma = 1 - 2\beta + 2\delta$ by the exponential factor $\exp \left[- (E/E_i)^{2\beta-2\delta} / \alpha_i \tau_i (\beta - 2\delta) \right]$. The spectrum cutoff on the high energy side is expected at $E \geq E_i (\alpha_i \tau_i (\beta - 2\delta))^{1/(2\beta-2\delta)}$.

4.5.4. The nature of the constraint of the accelerated particle's energy

It follows from the expressions presented above for the rate of particle energy gain that an infinite increase in particle energy with time should be expected in all cases. For example, in the cases described by the Eq. 4.2.9 and 4.5.25 at $\beta = 0$, $E \rightarrow \infty$ exponentially as $\exp(\alpha t)$ with time t . In the case $2\delta - \beta > 0$, $E \rightarrow \infty$ within a finite time $t = (\alpha_i (2\delta - \beta))^{-1}$. In the case $2\delta - \beta < 0$, $E \rightarrow \infty$ as $\propto t^{1/(\beta-2\delta)}$. Actually, however, these conclusions are erroneous.

The fact is that any source always comprises some maximum scale of inhomogeneities, the effective scattering by which corresponds to some particle energy E_{cr} . If $l_{\max}$ and $H(l_{\max})$ are the effective size and the mean intensity of the magnetic field in inhomogeneities of the largest scale, then

$$E_{cr} \approx Ze l_{\max} H(l_{\max}). \quad (4.5.29)$$

The value of E_{cr} can be reached in the case described in Section 4.2 at

$$t_{cr} \approx \ln(E_{cr}/E_i)/\alpha; \quad (4.5.30)$$

in the case described in Section 4.5.3 for $2\delta - \beta > 0$

$$t_{cr} \approx \left(1 - (E_{cr}/E_i)^{-(2\delta-\beta)}\right)/\alpha_i (2\delta - \beta); \quad (4.5.31)$$

and in the case described also in Section 4.5.3 but at $2\delta - \beta < 0$

$$t_{cr} \approx \left(1 + (E_{cr}/E_i)^{(\beta-2\delta)}\right)/\alpha_i (\beta - 2\delta). \quad (4.5.32)$$

Consider, for example, the acceleration up to high energies under the condition of constancy of the acceleration parameter, i.e. that $2\delta - \beta = 0$. It can be easily seen that the condition $2\delta - \beta = 0$ cannot be satisfied once $E \approx E_{cr}$ is reached. In fact, as was shown in Section 1.9, for any type of magnetic inhomogeneities, when the condition $E \geq E_{cr}$ is satisfied the transport scattering path should increase with energy as $\lambda \propto E^2$ or even more rapidly, i.e. it is explicit that $\beta_{cr} \geq 2$. On the other hand, since inhomogeneities exceeding the maximum scale are absent, it must be that $u_{cr} \approx \text{const}$, i.e. $\delta \approx 0$ for the energy range $E \geq E_{cr}$. Thus at $E \geq E_{cr}$ it is explicit that $2\delta - \beta < 0$; here, however, we obtain the case $2\delta - \beta \neq 0$ described above and the energy gain rate at $E \geq E_{cr}$ will be determined, according to Eq. 4.5.26, by

$$E \approx E_{cr} (1 + \alpha_{cr} \beta_{cr} (t - t_{cr}))^{1/\beta_{cr}}. \quad (4.5.33)$$

Here E_{cr} is determined by Eq. 4.5.29, t_{cr} is determined by Eq. 4.5.30 and

$$\alpha_{cr} = u_{cr}^2 / c \lambda_{cr}, \quad (4.5.34)$$

where λ_{cr} is transport path of particles with energy E_{cr} . In accordance with Eq. 4.5.28 the particle spectrum at $E \geq E_{cr}$ will be determined by the expression

$$n(E) \Big|_{E \geq E_{cr}} \propto (E/E_{cr})^{2\beta_{cr}-1} \exp \left\{ - (E/E_{cr})^{\beta_{cr}} \left((E/E_{cr})^{\beta_{cr}} - 1 \right) (\alpha_{cr} \tau_{cr} \beta_{cr})^{-1} \right\}, \quad (4.5.35)$$

where $\tau_{cr} \approx L^2 / 2c \lambda_{cr}$ is the mean lifetime of particles in their source with energy E_{cr} . It follows from Eq. 4.5.35 that an abrupt cutoff of the spectrum should take place at $E \geq E_{cr} (\alpha_{cr} \tau_{cr} \beta_{cr})^{1/2\beta_{cr}}$. The same will also take place at $2\delta - \beta \neq 0$. Once the particle energy reaches the value $E = E_{cr}$ determined by Eq. 4.5.29, the

energy gain rate and the accelerated particle spectrum will be expressed at $E > E_{cr}$ by the Eq. 4.5.33 and 4.5.35; in these relations, however, t_{cr} will be determined from Eq. 4.5.31 in case $2\delta - \beta > 0$ and by Eq. 4.5.32 in the case $2\delta - \beta < 0$.

4.6. Formation of the particle rigidity spectrum during statistical acceleration

4.6.1. General remarks and basic relations

The mode of particle motion in magnetic fields is determined by particle rigidity

$$R = cp/Ze, \quad (4.6.1)$$

where p is the momentum and Ze is the charge of particle. It is of interest, therefore, to determine how the particle rigidity varies in time during the acceleration processes and what is the generated spectrum of the accelerated particle rigidity. This can be done on the basis of the relations obtained in Sections 4.2 - 4.5 using the relativistic expressions determining the relationship between E , dE and v with R and dR :

$$\begin{aligned} E &= Ze \left(R^2 + \left(Am_p c^2 / Ze \right)^2 \right)^{1/2}; \quad dE = Ze R \left(R^2 + \left(Am_p c^2 / Ze \right)^2 \right)^{-1/2} dR; \\ v/c &= R \left(R^2 + \left(Am_p c^2 / Ze \right)^2 \right)^{-1/2}, \end{aligned} \quad (4.6.2)$$

where m_p is the proton mass; for nuclei A is the atomic weight ($m_{ac} = Am_p$); for electrons $A = 5.45 \times 10^{-4}$).

Let the spectrum and motion of inhomogeneities be such that the effective values of λ and u depend on R over a sufficiently wide range of rigidities as

$$\lambda = \lambda_i (R/R_i)^\beta; \quad u = u_i (R/R_i)^\delta, \quad (4.6.3)$$

where λ_i and u_i are respectively the transport scattering path and the chaotic velocity of inhomogeneity motion which are effective for particles with rigidity R_i . Then, taking into account Eq. 4.6.2 and 4.6.3 we shall obtain for the mean time of particle acceleration in the source:

$$\tau = \tau_i (R/R_i)^{-(\beta+1)} \left(R^2 + \left(Am_p c^2 / Ze \right)^2 \right)^{1/2} \left(R_i^2 + \left(Am_p c^2 / Ze \right)^2 \right)^{-1/2}, \quad (4.6.4)$$

where $\tau_i = L^2 / 2v_i \lambda_i$ is the mean time of acceleration in the source of particles with rigidity R_i ; here the particle's velocity $v_i = cR_i \left(R_i^2 + (Am_p c^2 / Ze)^2 \right)^{-1/2}$.

4.6.2. Non-relativistic range; λ and u are independent of R

If $R \ll Am_p c^2 / Ze$ it follows from Eq. 4.6.2 that

$$E_k = (ZeR)^2 / 2Am_p c^2; \quad dE_k = (Ze)^2 R dR / Am_p c^2; \quad v/c = ZeR / Am_p c^2. \quad (4.6.5)$$

Then for the non-relativistic range Eq. 4.6.3 will remain unchanged, and Eq. 4.6.4 will turn out to be

$$\tau = \tau_i (R/R_i)^{-1}. \quad (4.6.6)$$

Substituting Eq. 4.6.5 in Eq. 4.4.18 we shall obtain

$$\frac{d(R/R_o)}{dt} = \frac{8u}{3\lambda(R/R_o)} \begin{cases} 3/4 + (R/R_o)^2/2 - (R/R_o)^4/20 & \text{if } R/R_o \leq 1, \\ R/R_o + 1/5(R/R_o) & \text{if } R/R_o \geq 1, \\ R/R_o & \text{if } R/R_o \gg 1, \end{cases} \quad (4.6.7)$$

where

$$R_o = Am_p cu / Ze \quad (4.6.8)$$

is the rigidity of particles with velocity u .

If $R_i < R_o$ (i.e. $v_i < u$), then, after integrating Eq. 4.6.7 at the initial condition $R|_{t=0} = R_i$ we shall obtain

$$R = R_o \times \begin{cases} \left(\frac{a(2\sqrt{10} + 5) \exp(32ut/3\lambda\sqrt{10}) - 2\sqrt{10} + 5}{1 + a \exp(32ut/3\lambda\sqrt{10})} \right)^{1/2} & \text{if } t \leq t_1, \\ \left\{ \frac{4u(t-t_1)}{3\lambda} + \frac{3}{5} + \left[\frac{1}{4} \left(\frac{8u}{3\lambda} (t-t_1) + \frac{6}{5} \right)^2 - \frac{1}{5} \right]^{1/2} \right\} & \text{if } t \geq t_1, \end{cases} \quad (4.6.9)$$

where

$$a = \frac{2\sqrt{10} - 5 + (R_i/R_o)^2}{2\sqrt{10} + 5 - (R_i/R_o)^2}; \quad t_1 = \frac{3\lambda\sqrt{10}}{32u} \ln \left(\frac{2\sqrt{10} - 4}{a(2\sqrt{10} + 4)} \right). \quad (4.6.10)$$

The Eq. 4.6.9 seems to be rather cumbersome for further analysis. Within a relative error of less than 0.1, it follows from Eq. 4.6.9 that

$$R \approx R_o \times \begin{cases} \left(\left(3/2 + R_i^2/R_o^2 \right) \exp(8ut/3\lambda) \right)^{1/2} & \text{if } t < t_1, \\ [8u(t - t_1)/3\lambda + 1] & \text{if } t \geq t_1, \end{cases} \quad (4.6.11)$$

where

$$t_1 = \frac{3\lambda}{8u} \ln \left(\frac{5}{3 + 2R_i^2/R_o^2} \right). \quad (4.6.12)$$

It follows from Eq. 4.6.11 that the particle rigidity must rapidly (within time t_1) increase from R_i to R_o ; the time t_1 is a weak function of R_i , namely it varies from $0.192(\lambda/u)$ at $R_i/R_o = 0$ to $0.134(\lambda/u)$ at $R_i/R_o = 0.5$, and to $0.058(\lambda/u)$ at $R_i/R_o = 0.8$. Further, the particle's rigidity increase is directly proportional to t (see Eq. 4.6.11).

If, however, $R_i \geq R_o$ (i.e. $v_i \geq u$), then, after integrating Eq. 4.6.7, we find that

$$R - R_i - \frac{R_o}{\sqrt{5}} \arctg \left(\frac{\sqrt{5}(R - R_i)R_o}{R_i^2 + 5RR_i} \right) = \frac{8utR_o}{3\lambda}, \quad (4.6.13)$$

whence approximately

$$R \approx R_i + 8utR_o/3\lambda. \quad (4.6.14)$$

The accelerated particle spectrum will be found from Eq. 4.2.7 and Eq. 4.6.14 in the case where τ is independent of R in the form

$$n(R) \propto \exp(-3\lambda(R - R_i)/8uR_o\tau). \quad (4.6.15)$$

If, however, the expected variations of τ with R (even at constant λ and u) are taken into account according to Eq. 4.6.6, then

$$n(R) \propto \exp(-3\lambda R(R - R_i)/8uR_iR_o\tau). \quad (4.6.16)$$

The spectrum determined by Eq. 4.6.6 exhibits a peak maximum at

$$R_{\max} = \frac{R_i}{4} + \left(\frac{R_i^2}{16} + \frac{4uR_oR_i\tau_i}{3\lambda} \right)^{1/2} \quad (4.6.17)$$

and may be presented at high values of R in the power form $n(R) \propto R^{-\gamma}$ with the exponent γ increasing with R

$$\gamma = 3\lambda R(2R - R_i)/8uR_iR_o\tau_i - 1. \quad (4.6.18)$$

4.6.3. Non-relativistic case; λ and u are functions of R

In this case, according to Eq. 4.6.4, in the non-relativistic range we obtain at $R \ll Am_p c^2 / Ze$:

$$\tau = \tau_i (R/R_i)^{-(\beta+1)}. \quad (4.6.19)$$

Substituting Eq. 4.6.3 in Eq. 4.4.18 and considering Eq. 4.6.5 we shall find the following equation determining the variations in the particle rigidity R with time t of particle acceleration:

$$\frac{dx}{dt} = \frac{8u_i x^{2\delta-\beta-1}}{3\lambda_i x_i^{2\delta-\beta}} \times \begin{cases} \left[\frac{3}{4} x^\delta x_i^{-\delta} + \frac{1}{2} x^{2-\delta} x_i^\delta - \frac{1}{20} x^{4-3\delta} x_i^{3\delta} \right] & \text{if } x \leq x_i^{-\delta/(1-\delta)}, \\ \left[x + \frac{1}{5} x^{2\delta-1} x_i^{-2\delta} \right] & \text{if } x \geq x_i^{-\delta/(1-\delta)}, \end{cases} \quad (4.6.20)$$

where the designations

$$x = R/R_{oi}; \quad x_i = R_i/R_{oi}; \quad R_{oi} = Am_p c u_i / Ze \quad (4.6.21)$$

have been used. The physical meaning of R_{oi} and its relationship with R_i are as follows: R_{oi} is the rigidity of particles with the velocity u_i of the magnetic inhomogeneities which effectively scatter the particles with rigidity R_i . In turn,

$$x = v/u_i; \quad x_i = v_i/u_i. \quad (4.6.21a)$$

It can be easily seen that if $\delta = \beta = 0$, the Eq. 4.6.20 turns out to be Eq. 4.6.7. Consider the various possible cases.

(1). The case $\delta = 0$, $\beta \neq 0$. Here we shall obtain instead of Eq. 4.6.20:

$$\frac{dx}{dt} = \frac{8u_i x^{-\beta-1}}{3\lambda_i x_i^{-\beta}} \times \begin{cases} \left(\frac{3}{4} + \frac{1}{2}x^2 - \frac{1}{20}x^4 \right) & \text{if } x \leq 1, \\ \left(x + \frac{1}{5}x^{-1} \right) & \text{if } x \geq 1. \end{cases} \quad (4.6.22)$$

If $R_i < Am_p cu_i / Ze$ (i.e. if $x_i < 1$) the integration of Eq. 4.6.22 in the region $x < 1$ gives

$$\begin{aligned} & \frac{x^{\beta+2}}{\beta+2} \left[1 - \frac{2(\beta+2)}{3(\beta+4)}x^2 + \frac{23(\beta+2)}{45(\beta+6)}x^4 - \dots \right] \\ &= \frac{2u_i x_i^\beta t}{\lambda_i} + \frac{x_i^{\beta+2}}{\beta+2} \left[1 - \frac{2(\beta+2)}{3(\beta+4)}x_i^2 + \frac{23(\beta+2)}{45(\beta+6)}x_i^4 - \dots \right]. \end{aligned} \quad (4.6.23)$$

It follows from Eq. 4.6.23 that within a short time

$$t = t_1 = \frac{\lambda_i x_i^{-\beta}}{2u_i(\beta+2)} \left[1 - x_i^{\beta+2} - \frac{2(\beta+2)}{3(\beta+4)}(1 - x_i^{\beta+4}) + \frac{23(\beta+2)}{45(\beta+6)}(1 - x_i^{\beta+6}) - \dots \right]. \quad (4.6.24)$$

a particle gains rigidity $Am_p cu_i / Ze$, i.e. $x = 1$. We shall consider, therefore, the particle acceleration at $R_i \geq Am_p cu_i / Ze$ (i.e. when $x_i \geq 1$). It then follows approximately from Eq. 4.6.22:

$$R = R_i \left(\frac{8u_i t(\beta+1)}{3\lambda_i (R_i / R_{oi})} \right)^{1/(\beta+1)}. \quad (4.6.25)$$

Since according to Eq. 4.6.25

$$t = \frac{3\lambda_i R_i (R/R_i)^{(\beta+1)}}{8u_i (\beta+1) R_{oi}}; \quad dt \propto R^\beta dR, \quad (4.6.26)$$

then taking account of Eq. 4.2.7 and Eq. 4.6.19 we shall obtain for the particle rigidity spectrum

$$n(R)dR \propto R^{2\beta+1} \exp \left(- \frac{3\lambda_i R_i (R/R_i)^{2(\beta+1)}}{8u_i \tau_i (\beta+1) R} \right) dR. \quad (4.6.27)$$

The spectrum determined by Eq. 4.6.27 exhibits a peak maximum at

$$R_{\max} = R_i \left(\frac{4u_i \tau_i (2\beta + 1)}{3\lambda_i (R_i/R_{oi})} \right)^{1/2(\beta+1)}. \quad (4.6.28)$$

The Eq. 4.6.27 may be presented in the form of the power function $n(R) \propto R^{-\gamma}$ with the variable power exponent

$$\gamma = -\frac{d \ln n(R)}{d \ln R} = \frac{3\lambda_i R_i (R/R_i)^{2(\beta+1)}}{4u_i \tau_i R_{oi}} - (2\beta + 1). \quad (4.6.29)$$

(2). The case $\delta = 1$, β is arbitrary. Since in this case $x_i^{-\delta/(1-\delta)} = 0$ and x is always ≥ 0 , we shall obtain instead of Eq. 4.6.20:

$$\frac{dx}{dt} = \frac{8ux^{2-\beta}}{3\lambda_i x_i^{2-\beta}} \left(1 + \frac{1}{5} x_i^{-2} \right). \quad (4.6.30)$$

After integrating Eq. 4.6.30 from the initial value, we shall obtain

$$R = R_i \times \begin{cases} \left(1 - \frac{8(1-\beta)u_i t (1 + 0.2(R_{oi}/R_i)^2)}{3\lambda_i (R_i/R_{oi})} \right)^{-1/(1-\beta)} & \text{if } \beta < 1, \\ \exp \left(\frac{8u_i t (1 + 0.2(R_{oi}/R_i)^2)}{3\lambda_i (R_i/R_{oi})} \right) & \text{if } \beta = 1, \\ \left(1 - \frac{8(1+\beta)u_i t (1 + 0.2(R_{oi}/R_i)^2)}{3\lambda_i (R_i/R_{oi})} \right)^{1/(\beta-1)} & \text{if } \beta > 1. \end{cases} \quad (4.6.31)$$

If $\beta \neq 1$, we shall find from Eq. 4.6.31 that

$$t = \frac{3\lambda_i (R_i/R_{oi}) \left((R_{oi}/R_i)^{(\beta-1)} - 1 \right)}{8(\beta-1)u_i (1 + 0.2(R_{oi}/R_i)^2)}; \quad dt \propto R^{\beta-2} dR. \quad (4.6.32)$$

Whence, including Eq. 4.2.7 and Eq. 4.6.9, we obtain for the expected rigidity spectrum:

$$n(R) \propto R^{2\beta-1} \exp \left[-\frac{3\lambda_i (R_i/R_{oi}) (R/R_i)^{\beta+1} ((R/R_i)^{\beta-1} - 1)}{8(\beta-1)u_i\tau_i (1 + 0.2(R_{oi}/R_i)^2)} \right]. \quad (4.6.33)$$

It can be seen from Eq. 4.6.33 that if $\beta > 1$ then at $R \gg R_i$ the accelerated particle spectrum may be presented in the form

$$n(R) \propto R^{2\beta-1} \exp \left[-\frac{3\lambda_i (R_i/R_{oi}) (R/R_i)^{2\beta}}{8(\beta-1)u_i\tau_i (1 + 0.2(R_{oi}/R_i)^2)} \right], \quad (4.6.34)$$

i.e. the accelerated particle density in the source should exhibit a peak maximum at

$$R_{\max} = R_i \left(\frac{8(2\beta-1)(\beta-1)u_i\tau_i (1 + 0.2(R_{oi}/R_i)^2)}{3\lambda_i\beta(R_i/R_{oi})} \right)^{1/2\beta}. \quad (4.6.35)$$

When the spectrum described by Eq. 4.6.34 is presented in the power form $n(R) \propto R^{-\gamma}$, the variable power exponent γ will be determined by the expression

$$\gamma = -\frac{d \ln n(R)}{d \ln R} = \frac{3\lambda_i\beta(R_i/R_{oi}) (R/R_i)^{2\beta}}{4(\beta-1)u_i\tau_i (1 + 0.2(R_{oi}/R_i)^2)} - (2\beta-1). \quad (4.6.36)$$

If $\beta < 1$ then at $R \gg R_i$ the accelerated particle spectrum in the source will be

$$n(R) \propto R^{2\beta-1} \exp \left[-\frac{3\lambda_i (R_i/R_{oi}) (R/R_i)^{\beta+1}}{8(1-\beta)u_i\tau_i (1 + 0.2(R_{oi}/R_i)^2)} \right], \quad (4.6.37)$$

whence

$$\gamma = -\frac{d \ln n(R)}{d \ln R} = \frac{3\lambda_i(\beta+1)(R_i/R_{oi}) (R/R_i)^{\beta+1}}{4(1-\beta)u_i\tau_i (1 + 0.2(R_{oi}/R_i)^2)} - (2\beta-1). \quad (4.6.38)$$

When the condition

$$2\beta-1 > \frac{3\lambda_i(\beta+1)(R_i/R_{oi})}{8(1-\beta)u_i\tau_i (1 + 0.2(R_{oi}/R_i)^2)} \quad (4.6.39)$$

is satisfied, the spectrum described by Eq. 4.6.38 exhibits a peak maximum at

$$R_{\max} = R_i \left(\frac{8(2\beta - 1)(1 - \beta)u_i\tau_i(1 + 0.2(R_{oi}/R_i)^2)}{3\lambda_i(\beta + 1)(R_i/R_{oi})} \right)^{1/(\beta+1)}. \quad (4.6.40)$$

Let it now be assumed that $\beta = 1$; then it follows from Eq. 4.6.31 that

$$t = \frac{3\lambda_i(R_i/R_{oi})\ln(R/R_i)}{8u_i(1 + 0.2(R_{oi}/R_i)^2)}; \quad dt \propto R^{-1}dR, \quad (4.6.41)$$

whence, considering Eq. 4.2.7 and Eq. 4.6.19, we shall obtain the rigidity spectrum of accelerated particles in the power form $n(R) \propto R^{-\gamma}$, but with the variable exponent

$$\gamma = \frac{3\lambda_i(R_i/R_{oi})(R/R_i)^2}{8u_i\tau_i(1 + 0.2(R_{oi}/R_i)^2)} - 1. \quad (4.6.42)$$

It can be seen from Eq. 4.6.42 that when the condition

$$3\lambda_i(R_i/R_{oi}) < 8u_i\tau_i(1 + 0.2(R_{oi}/R_i)^2) \quad (4.6.43)$$

is satisfied the accelerated particle spectrum exhibits a maximum peak at

$$R_{\max} = R_i \left(\frac{8u_i\tau_i(1 + 0.2(R_{oi}/R_i)^2)}{3\lambda_i(R_i/R_{oi})} \right)^{1/2}. \quad (4.6.44)$$

If, however, the condition Eq. 4.6.43 is not satisfied, then already from $R = R_i$ the spectrum will descend with ever increasing power exponent γ .

(3) The case $0 < \delta < 1$; β is arbitrary. It will be assumed that $x_i < 1$ (i.e. $R_i < R_{oi}$). Since in the case examined $\delta/(1-\delta) > 0$ the value of $x^{-\delta/(1-\delta)} > 1$. Therefore the acceleration will take place first from $x = x_i$ to $x = x_i^{-\delta/(1-\delta)}$ according to Eq. 4.6.20 for $x \leq x_i^{-\delta/(1-\delta)}$, and then up to high values also according to Eq. 4.6.20 but for $x \geq x_i^{-\delta/(1-\delta)}$. The value of $x = x_i^{-\delta/(1-\delta)}$ is attained within a time

$$t_1 = \frac{3\lambda_i x_i^2}{8u_i} \int_1^{\frac{y^{1+\beta-2\delta}}{(3/4)y^\delta + (1/2)x_i^2 y^{2-\delta} - (1/20)x_i^4 y^{4-3\delta}}} dy. \quad (4.6.45)$$

Since $x_i < 1$ we shall, as a first approximation, neglect the second and third term in comparison with the first term in the denominator of the integrand in Eq. 4.6.45; the resultant expression is

$$t_1 \approx \frac{\lambda_i x_i^2}{2u_i} \times \begin{cases} \frac{x_i^{-(2+\beta-3\delta)/(1-\delta)} - 1}{2 + \beta - 3\delta} & \text{if } 2 + \beta - 3\delta > 0, \\ \ln \frac{1}{x_i(1-\delta)} & \text{if } 2 + \beta - 3\delta = 0, \\ \frac{1 - x_i^{(3\delta-\beta-2)/(1-\delta)}}{3\delta - \beta - 2} & \text{if } 2 + \beta - 3\delta < 0. \end{cases} \quad (4.6.46)$$

It follows from Eq. 4.6.46 that at $x_i \ll 1$ (when the assumption adopted is at its most accurate)

$$t_1 \approx \frac{\lambda_i}{2u_i} \times \begin{cases} \frac{x_i^{(\delta-\beta)/(1-\delta)} - 1}{2 + \beta - 3\delta} & \text{if } 2 + \beta - 3\delta > 0, \\ \frac{x_i^2 \ln(1/x_i)}{(1-\delta)} & \text{if } 2 + \beta - 3\delta = 0, \\ \frac{x_i^2}{3\delta - \beta - 2} & \text{if } 2 + \beta - 3\delta < 0. \end{cases} \quad (4.6.47)$$

The acceleration process in the examined time interval from 0 to t_1 (i.e. in the rigidity range of from R_i to $R_{oi}(R_i/R_{oi})^{-\delta/(1-\delta)}$) can be described by the relation

$$R \approx R_i \times \begin{cases} \left(1 + \frac{2(2 + \beta - 3\delta)u_i t}{\lambda_i (R_i/R_{oi})^2} \right)^{1/(2+\beta-3\delta)} & \text{if } 2 + \beta - 3\delta > 0, \\ \exp\left(\frac{2u_i t}{\lambda_i (R_i/R_{oi})^2} \right) & \text{if } 2 + \beta - 3\delta = 0, \\ \left(1 - \frac{2(3\delta - \beta - 2)u_i t}{\lambda_i (R_i/R_{oi})^2} \right)^{1/(3\delta-\beta-2)} & \text{if } 2 + \beta - 3\delta < 0. \end{cases} \quad (4.6.48)$$

In the abovementioned rigidity range from R_i to $R_{oi}(R_i/R_{oi})^{-\delta/(1-\delta)}$ the generated spectrum will, according to Eq. 4.2.7 and Eq. 4.6.19, be of the form

$$n(R) \propto \begin{cases} R^{2+2\beta-3\delta} \exp\left(-\frac{\lambda_i(R/R_i)^{\beta+1}\left((R/R_i)^{2+\beta-3\delta}-1\right)}{2(2+\beta-3\delta)u_i\tau_i(R_i/R_{oi})^{-2}}\right) & \text{if } 2+\beta-3\delta > 0, \\ R^{\beta-\frac{\lambda_i(R_i/R_{oi})^2(R/R_i)^{\beta+1}}{2u_i\tau_i}} & \text{if } 2+\beta-3\delta = 0, \\ R^{3\delta-2} \exp\left(-\frac{\lambda_i(R/R_i)^{\beta+1}\left((R/R_i)^{3\delta-\beta-2}-1\right)}{2(3\delta-\beta-2)u_i\tau_i(R_i/R_{oi})^{-2}}\right) & \text{if } 2+\beta-3\delta < 0. \end{cases} \quad (4.6.49)$$

At $t \geq t_1$ the acceleration will be determined by the Eq. 4.6.20 at $x = x_i^{-\delta/(1-\delta)}$ integration of which at initial condition $R|_{t=t_1} = R_{oi}(R_i/R_{oi})^{-\delta/(1-\delta)}$ gives

$$\frac{8u_i(t-t_1)}{3\lambda_i(R_i/R_{oi})} = \begin{cases} \sum_{k=0}^{\infty} \frac{(R/R_i)^{\beta-2\delta+1-2k(1-\delta)} - (R_{oi}/R_i)^{(\beta-2\delta+1/(1-\delta))-2k}}{(-1)^k 5^k (R_i/R_{oi})^{2k} (\beta-2\delta+1-2k(1-\delta))} & \text{if } 1+\beta-2\delta \neq 0, \\ \ln(R/R_i) + \frac{\ln(R_i/R_{oi})}{1-\delta} + \sum_{k=1}^{\infty} \frac{1-(R/R_i)^{-2k(1-\delta)}(R_i/R_{oi})^{-2k}}{(-1)^k 5^k 2k(1-\delta)} & \text{if } 1+\beta-2\delta = 0. \end{cases} \quad (4.6.50)$$

If $R \gg R_i$ it is possible in Eq. 4.6.50 to be limited to the first term; then the dependence $R(t)$ will be determined by the relations

$$R = R_i \times \begin{cases} \left((R_{oi}/R_i)^{(\beta-2\delta+1)/(1-\delta)} + \frac{8u_i(\beta-2\delta+1)(t-t_1)}{3\lambda_i(R_i/R_{oi})} \right)^{1/(\beta-2\delta+1)} & \text{if } 1+\beta-2\delta \neq 0, \\ (R_{oi}/R_i)^{1/(1-\delta)} \exp\left(\frac{8u_i(t-t_1)}{3\lambda_i(R_i/R_{oi})} \right) & \text{if } 1+\beta-2\delta = 0, \end{cases} \quad (4.6.51)$$

where t_1 is determined by Eq. 4.6.46 and Eq. 4.6.47. It follows from Eq. 4.6.51 based on Eq. 4.2.7 and including Eq. 4.6.19 that at $R \gg R_i$ the accelerated particle spectrum is

$$n(R) \propto \begin{cases} R^{2\beta-2\delta+1} \exp\left(-\frac{(R/R_i)^{\beta+1} b_1}{8u_i \tau_o}\right) & \text{if } 1 + \beta - 2\delta \neq 0, \\ R^\beta \exp\left(-\frac{(R/R_i)^{\beta+1} b_2}{8u_i \tau_i}\right) & \text{if } 1 + \beta - 2\delta = 0, \end{cases} \quad (4.6.52)$$

where

$$b_1 = 3\lambda_i (R_i/R_{oi}) \left[(R/R_i)^{\beta-2\delta+1} - (R_{oi}/R_i)^{(\beta-2\delta+1)/(1-\delta)} \right] + 8u_i t_1, \quad (4.6.52a)$$

$$b_2 = 3\lambda_i (R_i/R_{oi}) [\ln(R/R_i) + \ln(R_{oi}/R_i)/(1-\delta)] + 8u_i t_1. \quad (4.6.52b)$$

It will now be assumed that $x_i = 1$ (i.e. $R_i = R_{oi}$). Then $x_i^{-\delta/(1-\delta)}$ and the acceleration will be completely determined by the second expression in Eq. 4.6.20. If $x_i > 1$ (i.e. $R_i > R_{oi}$) then, since in this case $x_i^{-\delta/(1-\delta)} < 1$, the acceleration will also be completely determined by the second expression in Eq. 4.6.20. Thus at $x_i \geq 1$ and including the boundary condition $x|_{t=0} = x_i$ we obtain

$$\frac{8u_i t}{3\lambda_i x_i} = \int_1^{x/x_i} \frac{y^{\beta-2\delta} dy}{1 + y^{2\delta-2}/5x_i^2}. \quad (4.6.53)$$

Whence, similarly to Eq. 4.6.50, we have

$$\frac{8u_i t}{3\lambda_i (R_i/R_{oi})} = \begin{cases} \sum_{k=0}^{\infty} \frac{[(R/R_i)^{\beta-2\delta+1-2k(1-\delta)} - 1](R_i/R_{oi})^{-2k}}{(-1)^k 5^k (\beta-2\delta+1-2k(1-\delta))} & \text{if } \beta-2\delta+1 \neq 0, \\ \ln(R/R_i) + \sum_{k=1}^{\infty} \frac{1 - (R/R_i)^{-2k(1-\delta)}}{(-1)^k 5^k 2k(1-\delta)(R_i/R_{oi})^{2k}} & \text{if } \beta-2\delta+1 = 0. \end{cases} \quad (4.6.54)$$

It can be found from Eq. 4.6.54 at $R \gg R_i$ that

$$R = R_i \times \begin{cases} \left(1 + \frac{8u_i t(\beta-2\delta+1)}{3\lambda_i (R_i/R_{oi})}\right) & \text{if } \beta-2\delta+1 \neq 0, \\ \exp\left(\frac{8u_i t}{3\lambda_i (R_i/R_{oi})}\right) & \text{if } \beta-2\delta+1 = 0. \end{cases} \quad (4.6.55)$$

Whence, including Eq. 4.4.7 and Eq. 4.6.9,

$$n(R) \propto \begin{cases} R^{2\beta-2\delta+1} \exp\left(-\frac{3\lambda_i(R_i/R_{oi})(R/R_i)^{2\beta-2\delta+1}}{8u_i\tau_i}\right) & \text{if } \beta-2\delta+1 \neq 0, \\ R^\beta \exp\left(-\frac{3\lambda_i(R_i/R_{oi})(R/R_i)^{\beta+1}}{8u_i\tau_i}\right) & \text{if } \beta-2\delta+1 = 0. \end{cases} \quad (4.6.56)$$

It follows from (4.6.56) at $\beta-2\delta+1=0$ that the accelerated particle spectrum exhibits a peak maximum at

$$R_{\max} = R_i \left(\frac{8(2\beta-2\delta+1)u_i\tau_i}{3\lambda_i(2\beta-2\delta+2)(R_i/R_{oi})} \right)^{1/(2\beta-2\delta+2)}. \quad (4.6.57)$$

The spectrum may be presented in the power form $n(R) \propto R^{-\gamma}$, but with the variable exponent

$$\gamma = \frac{3\lambda_i(2\beta-2\delta+2)(R_i/R_{oi})(R/R_i)^{2\beta-2\delta+2}}{8u_i\tau_i} + 2\beta-2\delta+1. \quad (4.6.58)$$

If $2\beta-2\delta+1=0$ the spectrum exhibits a peak maximum, according to Eq. 4.6.56, at

$$R_{\max} = R_i \left(\frac{8\beta u_i\tau_i}{3\lambda_i(R_i/R_{oi})} \right)^{1/(\beta+1)}. \quad (4.6.59)$$

4.6.4. Relativistic range; λ and u are independent of R

Substituting Eq. 4.6.2 in Eq. 4.4.38, we obtain

$$\frac{dR}{dt} = \frac{2u^2}{c\lambda} (R^2 + R_1^2)^{1/2} \left(1 + \frac{R_1^2}{3(R^2 + R_1^2)} \right). \quad (4.6.60)$$

where the following denomination has been inserted for the sake of brevity:

$$R_1 = Am_p c^2 / Ze. \quad (4.6.61)$$

Integrating Eq. 4.6.60 from $R = R_i$ at $t = 0$, we find

$$\frac{2u^2 t}{c\lambda} = \int_{R_i/R_1}^{R/R_1} \frac{dy/\sqrt{y^2+1}}{1+1/3(y^2+1)} = \ln \left\{ \frac{R + \sqrt{R^2 + R_1^2}}{R_i + \sqrt{R_i^2 + R_1^2}} \left[\frac{3 + (1 + R_i^2/R_1^2)^{-1/2}}{3 + (1 + R^2/R_1^2)^{-1/2}} \right]^{3/4} \right\}. \quad (4.6.62)$$

It follows from Eq. 4.6.62 that

$$t = \frac{c\lambda}{2u^2} \ln \left\{ \frac{R + \sqrt{R^2 + R_1^2}}{R_o + \sqrt{R_o^2 + R_1^2}} \left[\frac{3 + (1 + R_i^2/R_1^2)^{-1/2}}{3 + (1 + R^2/R_1^2)^{-1/2}} \right]^{3/4} \right\}, \quad (4.6.63)$$

$$dt = \left\{ (R^2 + R_1^2)^{-1/2} + \frac{3R}{4R_1^2} \left[3 + (1 + R^2/R_1^2)^{-1/2} \right]^{-1} (1 + R^2/R_1^2)^{-3/2} \right\}, \quad (4.6.64)$$

whence including Eq. 4.2.7 and Eq. 4.6.4 at $\beta = 0$,

$$\begin{aligned} n(R) \propto & \left\{ \frac{R + \sqrt{R^2 + R_1^2}}{R_i + \sqrt{R_i^2 + R_1^2}} \left[\frac{3 + (1 + R_i^2/R_1^2)^{-1/2}}{3 + (1 + R^2/R_1^2)^{-1/2}} \right]^{3/4} \right\}^{-\frac{c\lambda R \sqrt{R_i^2 + R_1^2}}{2u^2 \tau_i R_i \sqrt{R^2 + R_1^2}}} \\ & \times \left\{ (R^2 + R_1^2)^{-1} + \frac{3R^2}{4R_1^3} \left[3 + (1 + R^2/R_1^2)^{-1/2} \right]^{-1} (1 + R^2/R_1^2)^{-2} \right\}. \end{aligned} \quad (4.6.65)$$

Consider individual cases.

(1). The case $R \ll R_1$ (and hence $R_i \ll R_1$). Here we obtain from Eq. 4.6.62:

$$R = R_i + 2u^2 R_1 t / c\lambda. \quad (4.6.66)$$

Taking account of Eq. 4.2.7 and Eq. 4.6.4 at $\beta = 0$, we shall find the following form of the accelerated particle spectrum in the source:

$$n(R) \propto \exp \left[-\frac{c\lambda(R - R_i)(R/R_i)}{2u^2 \tau_i R_1} \right] R. \quad (4.6.67)$$

The spectrum described by Eq. 4.6.67 may be presented in the power form $n(R) \propto R^{-\gamma}$ with the variable exponent

$$\gamma = -\frac{d \ln n(R)}{d \ln R} = \frac{c\lambda(2R - R_i)R}{2u^2\tau_i R_i R_1} - 1. \quad (4.6.68)$$

It can be seen from Eq. 4.6.67 that the described spectrum exhibits a peak maximum (when $\gamma = 0$) at

$$R_{\max} = \frac{R_i}{4} + \left(\frac{R_i^2}{16} + \frac{u^2\tau_i R_i R_1}{c\lambda} \right)^{1/2}. \quad (4.6.69)$$

(2) The case $R \gg R_1$, $R_i \gg R_1$. In this case we obtain from Eq. 4.6.62:

$$R = R_i \exp(2u^2 t / c\lambda), \quad (4.6.70)$$

whence it follows for the accelerated particle spectrum in the source taking account of Eq. 4.2.7 and Eq. 4.6.4 at $\beta = 0$ (in this case $\tau = \tau_i$) that $n(R) \propto R^{-\gamma}$, where

$$\gamma = \frac{c\lambda}{2u^2\tau} + 1. \quad (4.6.71)$$

(3) The case $R \gg R_1$, $R_i \ll R_1$. In this case, the particles are accelerated from non-relativistic up to ultra-relativistic energies. Here we obtain from Eq. 4.6.62:

$$R = \frac{R_1}{2} \left(\frac{3}{4} \right)^{3/4} \exp(2u^2 t / c\lambda), \quad (4.6.72)$$

In the case examined it follows from Eq. 4.6.4 at $\beta = 0$ that

$$\tau = \tau_i R_i / R_1 \quad (4.6.73)$$

whence we obtain for the accelerated particle spectrum in the source taking account of Eq. 4.2.7 that $n(R) \propto R^{-\gamma}$, where

$$\gamma = \frac{c\lambda R}{2u^2\tau_i R_i} + 1. \quad (4.6.74)$$

4.6.5. Relativistic range; λ and u are functions of R

Substituting Eq. 4.6.2 and Eq. 4.6.3 in Eq. 4.4.38, we find

$$\frac{dR}{dt} = \frac{2u_i^2 (R/R_i)^{2\delta-\beta}}{c\lambda_i} (R^2 + R_1^2)^{1/2} \left(1 + \frac{R_1^2}{3(R^2 + R_1^2)} \right), \quad (4.6.75)$$

where R_i is the rigidity of injected particles (at $t = 0$); u_i is the velocities of the inhomogeneities effectively scattering the particles with rigidity R_i ; λ_i is the transport scattering path of such particles; R_1 is determined by Eq. 4.6.61. Integration of Eq. 4.6.75 gives

$$\frac{2u_i^2 t}{c\lambda(R_i/R_1)^{2\delta-\beta}} = \int_{R_i/R_1}^{R/R_1} \frac{y^{\beta-2\delta} dy}{(1+y^2)^{1/2} [1 + 1/3(1+y^2)]}. \quad (4.6.76)$$

Consider individual cases.

(1) The case $R \ll R_1$ (and hence $R_i \ll R_1$). Here it follows from Eq. 4.6.76 that

$$R = R_i \times \begin{cases} \left(1 + \frac{8u_i^2 t (\beta - 2\delta + 1) R_1}{3c\lambda_i R_i} \right)^{1/(\beta-2\delta+1)} & \text{if } \beta - 2\delta + 1 \neq 0, \\ \exp\left(\frac{8u_i^2 t R_1}{3c\lambda_i R_i} \right) & \text{if } \beta - 2\delta + 1 = 0. \end{cases} \quad (4.6.77)$$

Since in this case

$$dR \propto \begin{cases} R^{\beta-2\delta} dR, & \text{if } \beta - 2\delta + 1 \neq 0; \\ R^{-1} dR, & \text{if } \beta - 2\delta + 1 = 0 \end{cases}; \quad \tau = \tau_i (R/R_i)^{-(\beta+1)}, \quad (4.6.78)$$

we shall obtain, in accordance with Eq. 4.4.7, for the accelerated particle spectrum:

$$n(R) \propto \begin{cases} R^{2\beta-2\delta+1} \exp\left(-\frac{3c\lambda_i R_i \left[(R/R_i)^{\beta-2\delta+1} - 1\right] (R/R_i)^{\beta+1}}{8u_i^2 \tau_i}\right) & \text{if } \beta - 2\delta + 1 \neq 0, \\ (R/R_i)^{\beta - \frac{3c\lambda_i R_i (R/R_i)^{\beta+1}}{8u_i^2 \tau_i R_i}} & \text{if } \beta - 2\delta + 1 = 0. \end{cases} \quad (4.6.79)$$

It follows from Eq. 4.6.79 that at $\beta - 2\delta + 1 = 0$ the spectrum will be of power form only if $\beta + 1 = 0$; $\delta = 0$. Here it follows from Eq. 4.6.76 that

$$R = R_i \times \begin{cases} \left(1 + 2u_i^2 t (\beta - 2\delta) / c\lambda_i\right)^{1/(\beta-2\delta)} & \text{if } \beta - 2\delta \neq 0, \\ \exp(2u_i^2 t / c\lambda_i) & \text{if } \beta - 2\delta = 0. \end{cases} \quad (4.6.80)$$

If $\beta - 2\delta \neq 0$ then since it follows from Eq. 4.6.80 and Eq. 4.6.4 that

$$t = \frac{c\lambda_i \left((R/R_i)^{\beta-2\delta} - 1\right)}{2u_i^2 (\beta - 2\delta)}; \quad dt \propto R^{\beta-2\delta-1} dR; \quad \tau = \tau_i (R/R_i)^{-\beta}, \quad (4.6.81)$$

and including Eq. 4.2.7, we shall obtain for the rigidity spectrum of accelerated particles in their sources:

$$n(R) \propto R^{2\beta-2\delta-1} \exp\left(-\frac{c\lambda_i \left((R/R_i)^{\beta-2\delta} - 1\right) (R/R_i)^{\beta}}{2u_i^2 \tau_i (\beta - 2\delta)}\right). \quad (4.6.82)$$

If, however, $\beta - 2\delta = 0$ we shall obtain $n(R) \propto R^{-\gamma}$, where

$$\gamma = \frac{c\lambda_i (R/R_i)^{\beta}}{2u_i^2 \tau_i} + 1 - \beta. \quad (4.6.83)$$

(2) The case $\beta - 2\delta = 0$. Since in this case the integral in the right hand side of Eq. 4.6.76 coincides with Eq. 4.6.62, the relationship between R and t will be determined by the Eq. 4.6.63, and we shall obtain for $n(R)$, including Eq. 4.6.4, that

$$n(R) \propto \left\{ \frac{R + \sqrt{R^2 + R_1^2}}{R_i + \sqrt{R_i^2 + R_1^2}} \left[\frac{3 + \left(1 + R_i^2/R_1^2\right)^{-1/2}}{3 + \left(1 + R^2/R_1^2\right)^{-1/2}} \right]^{3/4} \right\} \frac{c\lambda_i (R/R_i)^{1+\beta} \sqrt{R_i^2 + R_1^2}}{2u_i^2 \tau_i \sqrt{R^2 + R_1^2}} \\ \times \left\{ R^{1+\beta} (R^2 + R_1^2)^{-1} + \frac{3R^{2+\beta}}{4R_1^3} \left[3 + \left(1 + R^2/R_1^2\right)^{-1/2} \right]^{-1} \left(1 + R^2/R_1^2\right)^{-2} \right\}. \quad (4.6.84)$$

It follows from Eq. 4.6.84 that if $R_i \ll R_1$ (the acceleration starts from non-relativistic energies), then in the ultra-relativistic energy range at $R \gg R_1$ the spectrum takes the form $n(R) \propto R^{-\gamma}$, where

$$\gamma = \frac{c\lambda_i R_1 R^\beta}{2u_i^2 \tau_i R_i^{1+\beta}} + 1 - \beta. \quad (4.6.85)$$

In this case, if $\beta > 1$ the spectrum exhibits a peak maximum which reaches at

$$R_{\max} = \left(\frac{2u_i^2 \tau_i R_i^{1+\beta} (\beta - 1)}{c\lambda_i R_1} \right)^{1/\beta}. \quad (4.6.86)$$

(3) The case $\beta - 2\delta = 1$. In this case, after substituting $z = \cos(\text{arctg})$, we obtain from Eq. 4.6.6:

$$\frac{2u_i^2 R_i t}{c\lambda_i R_1} = \left(1 + R^2/R_1^2\right)^{1/2} - \left(1 + R_i^2/R_1^2\right)^{1/2} \\ - \frac{1}{\sqrt{3}} \text{arctg} \left[\frac{\left(1 + R^2/R_1^2\right)^{1/2} - \left(1 + R_i^2/R_1^2\right)^{1/2}}{\left(1/\sqrt{3}\right) + \sqrt{3} \left(1 + R^2/R_1^2\right)^{1/2} \left(1 + R_i^2/R_1^2\right)^{1/2}} \right]. \quad (4.6.87)$$

If $R_i \ll R_1$ (the acceleration starts from the non-relativistic energy) we shall obtain from Eq. 4.6.87 at $R \gg R_1$:

$$R \approx 2u_i^2 R_i t / c\lambda_i, \quad (4.6.88)$$

whence, on the basis of Eq. 4.2.7 and including Eq. 4.6.4, we have

$$n(R) \propto R^\beta \exp\left(-\frac{c\lambda_i R^{\beta+1} R_1}{2u_i^2 \tau_i R_i^{\beta+2}}\right). \quad (4.6.89)$$

The spectrum described by Eq. 4.6.89 may be presented in the power form $n(R) \propto R^{-\gamma}$, where

$$\gamma = \frac{(\beta+1)c\lambda_i R^{\beta+1} R_1}{2u_i^2 \tau_i R_i^{\beta+2}} - \beta \quad (4.6.90)$$

It follows from (4.6.90) that if $\beta > 0$ the spectrum described by Eq. 4.6.89 exhibits a peak maximum at

$$R_{\max} = \left(\frac{2u_i^2 \tau_i R_i^{\beta+2}}{(\beta+1)c\lambda_i R_1} \right)^{1/(\beta+1)}. \quad (4.6.91)$$

4.7. Statistical acceleration by scattering on small angles

In the above the statistical acceleration of particles in case of large-angle scatterings was considered. When treating the acceleration processes in the space plasmas, however, it is of great interest to consider also the small-angle scattering. First, we shall analyze the small-angle scattering (Section 4.7.1), then determine the energy change in an elementary scattering event (Sections 4.7.2–4.7.4), and finally, estimate the total energy change along the transport scattering path (Section 4.7.5).

4.7.1. Small-angle scattering

The particles are scattered through small angles by magnetic clouds when the particle Larmor radius in inhomogeneities $\rho \geq l$, where l is the characteristic scale of inhomogeneities. In this case the characteristic scattering angle of a particle with rigidity R is (see Section 1.8)

$$\theta \approx l/r_L = 300hl/R, \quad (4.7.1)$$

where h is the magnetic field intensity in inhomogeneities.

Let the energy change in an elementary scattering event be determined at first. The small-angle scattering also takes place during interactions of the particles with sufficiently high energies with inhomogeneities of types $j = 1, 2$, and 3 which are disturbances against the background of homogeneous field (considered in Chapter 1, Section 1.9). If in the Cartesian system x, y, z the homogeneous field $\mathbf{H}_0 = (H_0, 0, 0)$, then the disturbance $\mathbf{h} = (0, h(x), 0)$. In this case, according to Parker (1964),

$$h(x) = h_o \exp\left(-x^2/l^2\right) \left[\frac{1}{2} \left(1 + (-1)^j - j \left(-\frac{x}{l} \right)^j \right) \right], \quad (4.7.2)$$

where $j = 0, 1, 2$ is the type of inhomogeneity. The fields $h(x)$ in inhomogeneities of types $j = 1, 2$, and 3 were shown in Fig. 1.8.4 in Chapter 1. In this case, according to Parker (1964) the scattering angle is

$$\theta = \frac{2\sqrt{\pi}h_o}{H_o} \left(\frac{l}{2R/300H_o} \right)^{j+1} \exp\left(-\left(\frac{l}{2R/300H_o} \right)^2\right), \quad (4.7.3)$$

where l is in cm, R in V, and H_o in Gs. The mode of the dependence of θ on R for $j = 1, 2$, and 3 is shown in Table 1.8.1 (see Chapter 1, Section 1.8.6) and in Fig. 4.7.1-4.7.3.

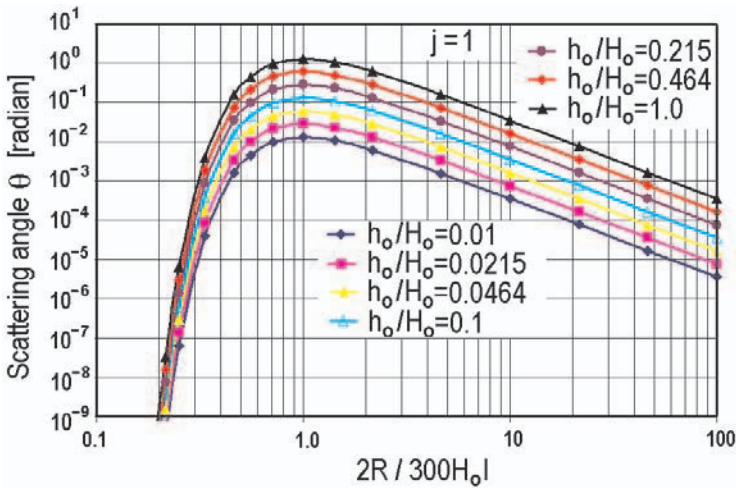


Fig. 4.7.1. The scattering angle θ for different values of ratio h_o/H_o (from 0.01 up to 1.0 from bottom to the top) in dependence of $2r_g/l = 2R/300H_o l$ for the scattering by inhomogeneities of type $j = 1$.

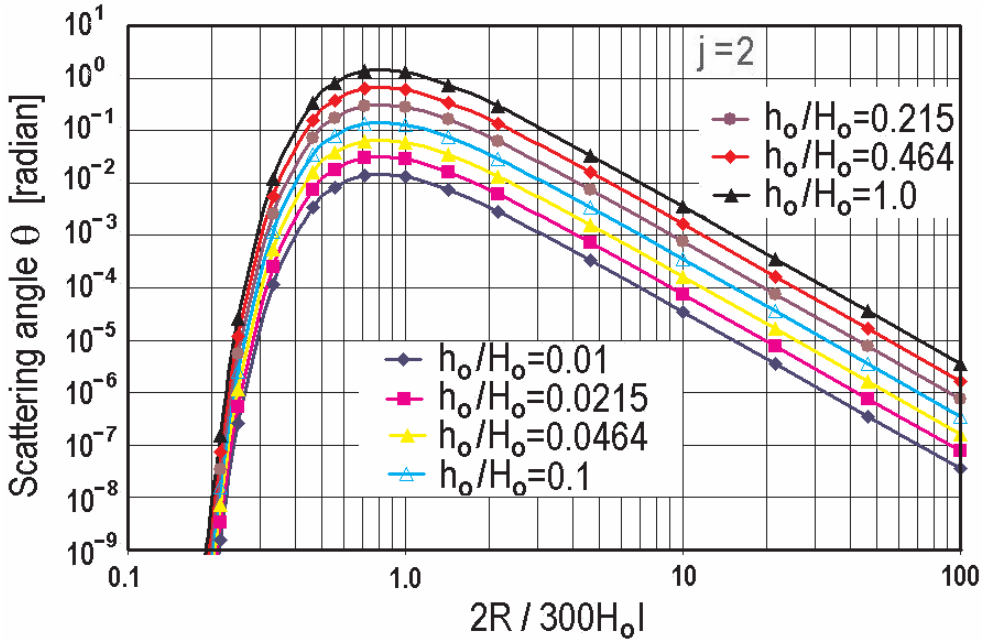


Fig. 4.7.2. The same as in Fig. 4.7.1 but for $j = 2$.

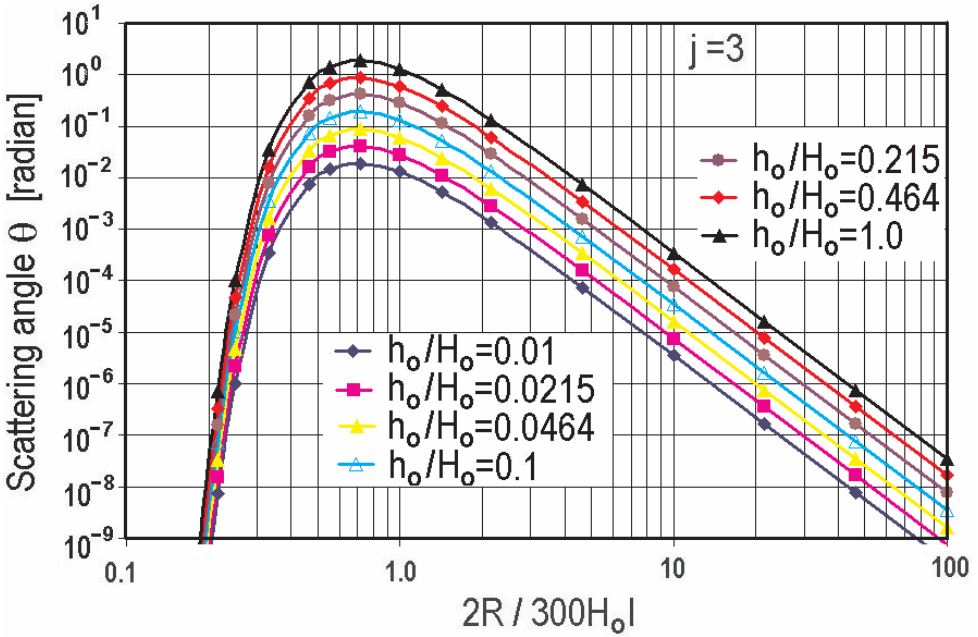


Fig. 4.7.3. The same as in Fig. 4.7.1 but for $j = 3$.

It can be seen from Fig. 4.7.1–4.7.3 that the scattering by inhomogeneities of types $j = 1, 2$, and 3 is characterized by a rapid decrease of θ both with increasing R (as $\propto R^{-1}$, $\propto R^{-2}$, $\propto R^{-3}$, respectively, for inhomogeneities of types $j = 1, 2$, and 3 , and with decreasing R when $2r_L/l = 2R/300H_o l < 1$ for $j = 1$, $2R/300H_o l < 0.8$ for $j = 2$, and $2R/300H_o l < 0.7$ for $j = 3$. The latter circumstance essentially differs the scattering by inhomogeneities of types $j = 1, 2$, and 3 from the scattering by magnetic clouds (in which at $R > 300h_o l$ $\theta \propto R^{-2}$, but at $R \leq 300h_o l$ the angle $\theta \geq 1$ and the value of θ is practically independent on R at $R \rightarrow 0$). It follows from Eq. 4.7.3 that for the inhomogeneities of types $j = 1, 2$, and 3 the scattering angle $\theta(R)$ exhibits a peak maximum at

$$2R_{\max}/300H_o l = \sqrt{2/(j+1)}; \quad R_{\max} = 150H_o l \sqrt{2/(j+1)}. \quad (4.7.4)$$

According to Eq. 4.7.4 we obtain $2R_{\max}/300H_o l = 1, 0.816, 0.707$ for $j = 1, 2$, and 3 , respectively. The small angle scatterings seem to be fairly frequent in the space; in particular, they will also take place during charged particle interactions with the plasma pulsations and the disturbances of various types (see in more detail in Chapters 1 and 2). It is obvious that the smaller the scattering angle θ is smaller the change of particle energy ΔE (in the extreme when $\theta \rightarrow 0$, it should be that $\Delta E \rightarrow 0$).

4.7.2. Energy gain in head-on and overtaking collisions in non-relativistic case for small angle scatterings

In order to estimate the mode of energy change for small angle scattering, we shall examine first the simplest example, namely a head-on collision of non-relativistic particle with magnetic cloud (see Fig. 4.7.4).

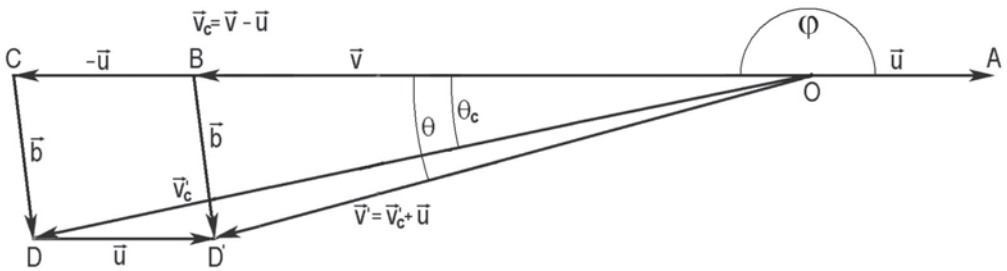


Fig. 4.7.4. A scheme of determination of the particle velocity change during non-mirror interactions for head-on collisions.

Let the cloud velocity be $\mathbf{u}$, the particle velocity $\mathbf{v}$, and $\mathbf{u} \uparrow \downarrow \mathbf{v}$. Fig. 4.7.4 shows the vectors $\mathbf{OA} = \mathbf{u}$, $\mathbf{OB} = \mathbf{v}$, and $\mathbf{BC} = -\mathbf{u}$. In a coordinate system related to the cloud the particle velocity will be

$$\mathbf{OC} = \mathbf{v}_c = \mathbf{v} - \mathbf{u}; \quad |\mathbf{v}_c| = (v^2 + u^2 - 2vu \cos \varphi)^{1/2}; \quad \varphi = \pi. \quad (4.7.5)$$

In this coordinate system the particle energy fails to vary, and as a result of scattering, the particle velocity vector will turn through angle θ so that

$$\mathbf{OD} = \mathbf{v}'_c = \mathbf{v} - \mathbf{u} + \mathbf{b}; \quad |\mathbf{v}'_c| = |\mathbf{v}_c| \quad (4.7.6)$$

is the particle velocity vector after scattering in the coordinate system related to the magnetic cloud. In Eq. 4.7.6

$$\mathbf{b} = \mathbf{CD}; \quad |\mathbf{b}| = 2 |\mathbf{v} - \mathbf{u}| \sin(\theta_c/2), \quad (4.7.7)$$

where $\theta_c = \angle COD$, i.e. the angle between vectors $\mathbf{v}_c$ and $\mathbf{v}'_c$ (scattering angle in the coordinate system related to the magnetic cloud). Since the magnetic field in the cloud may be arbitrarily oriented, the scattering vector $\mathbf{b}$ rotates around the OC axis and circumscribes a cone surface. After scattering in the coordinate system of the cloud the velocity vector $\mathbf{OD}$ will circumscribe a similar cone around the OC axis. Now we shall again use the laboratory coordinate system, i.e. we shall add the vector $\mathbf{OA}$ to the vector $\mathbf{OD}$; as a result we shall obtain for the particle velocity vector $\mathbf{OD}' = \mathbf{v}'$ after scattering in the laboratory system:

$$\mathbf{v}' = \mathbf{v} - \mathbf{u} + \mathbf{b} + \mathbf{u} = \mathbf{v} + \mathbf{b}. \quad (4.7.8)$$

It can be seen from the scheme in Fig. 4.7.4 that the angle between the vectors $\mathbf{v}$ and $\mathbf{b}$ is

$$\angle OBD' = \frac{\pi}{2} - \frac{\theta_c}{2}, \quad (4.7.9)$$

whence we find on taking into account of Eq. 4.7.7:

$$|\mathbf{v}'|^2 = v^2 + b^2 + 2\mathbf{v}\mathbf{b} = v^2 + 4(vu + u^2) \sin^2(\theta_c/2). \quad (4.7.10)$$

Since $E \approx m_{ac}c^2$ it follows from Eq. 4.7.10 that

$$(\Delta E/E)_+ = \frac{2(vu + u^2)}{c^2} \sin^2 \frac{\theta_c}{2}. \quad (4.7.11)$$

Similarly, for overtaking collision ($\mathbf{v} \uparrow \mathbf{u}$) we obtain (in this case $\varphi = 0$):

$$(\Delta E/E)_- = \frac{2(-vu + u^2)}{c^2} \sin^2 \frac{\theta_c}{2}. \quad (4.7.12)$$

It can be easily seen that if $\theta_c = \pi$ then Eq. 4.7.11 and Eq. 4.7.12 turn out to be the corresponding expressions from Sections 4.2 and 4.3 for mirror reflection.

From Fig. 4.7.4 can be seen that for small angle scattering the relation between scattering angle θ_c in the cloud coordinate system and scattering angle θ in the laboratory coordinate system is

$$\theta = \theta_c(v + u)/v; \quad \theta_c = \theta v/(v + u). \quad (4.7.13)$$

Substituting Eq. 4.7.13 in Eq. 4.7.11 and Eq. 4.7.12 we obtain

$$(\Delta E/E)_+ = \frac{2(vu + u^2)}{c^2} \sin^2 \frac{v\theta}{2(v + u)}; \quad (\Delta E/E)_- = \frac{2(-vu + u^2)}{c^2} \sin^2 \frac{v\theta}{2(v + u)}. \quad (4.7.14)$$

It is easy to see that at $v \gg u$ according to Eq. 4.7.13 $\theta_c \approx \theta$, and instead of Eq. 4.7.14 we obtain

$$(\Delta E/E)_+ = \frac{2(vu + u^2)}{c^2} \sin^2 \frac{\theta}{2}; \quad (\Delta E/E)_- = \frac{2(-vu + u^2)}{c^2} \sin^2 \frac{\theta}{2}. \quad (4.7.15)$$

4.7.3. Energy change in non-relativistic case for oblique collisions

Consider now an oblique collision (see Fig. 4.7.5). Here, as in Fig. 4.7.4, the cloud velocity $\mathbf{u} = \mathbf{OA}$, the particle velocity in the laboratory coordinate system $\mathbf{v} = \mathbf{OB}$, the angle between $\mathbf{u}$ and $\mathbf{v}$ is $\varphi = \angle AOB$.

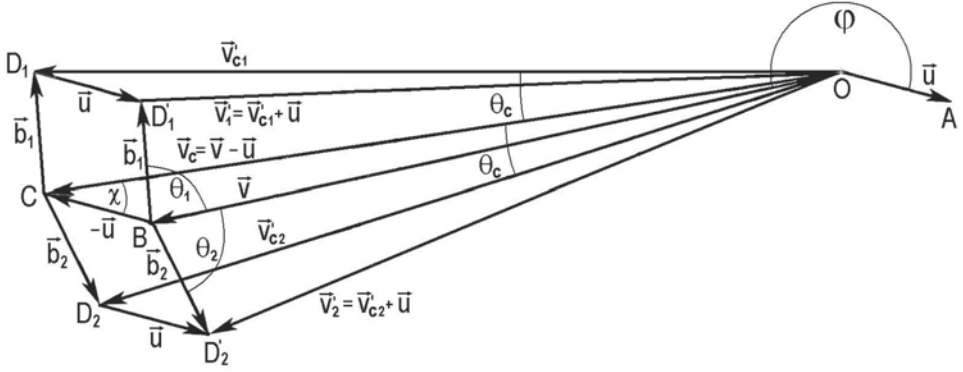


Fig. 4.7.5. A scheme of determination of the particle velocity change during non-mirror interactions for oblique collisions.

Using the coordinate system related to the magnetic cloud we find the velocity of particle

$$\mathbf{OC} = \mathbf{v}_c = \mathbf{v} - \mathbf{u}; \quad |\mathbf{v}_c| = \left(v^2 + u^2 - 2vu \cos \varphi \right)^{1/2}. \quad (4.7.16)$$

In this coordinate system, particles are scattered through angle θ_c and, as a result, the vector $\mathbf{OC}$ is transformed into

$$\mathbf{OD}_1 = \mathbf{v}'_{c1} = \mathbf{v}_c + \mathbf{b}_1; \quad |\mathbf{v}'_{c1}| = |\mathbf{v}_c|, \quad (4.7.17)$$

and

$$\mathbf{OD}_2 = \mathbf{v}'_{c2} = \mathbf{v}_c + \mathbf{b}_2; \quad |\mathbf{v}'_{c2}| = |\mathbf{v}_c|. \quad (4.7.18)$$

Let us note that actually, the resultant vector circumscribes a cone around axis $\mathbf{OC}$ with half-angle θ_c of cone apex O. The scattering velocity vector $\mathbf{b}$, which module determined by

$$|\mathbf{b}| = |\mathbf{b}_1| = |\mathbf{b}_2| = 2|\mathbf{v} - \mathbf{u}| \sin \frac{\theta_c}{2}, \quad (4.7.19)$$

circumscribes the cone D_1CD_2 with the half-angle $\pi/2 - \theta_c/2$ of cone apex C (in this case the section of the cone by the plane running through vectors $\mathbf{u}$ and $\mathbf{v}_c$ is determined by the vectors $\mathbf{CD}_1 = \mathbf{b}_1$ and $\mathbf{CD}_2 = \mathbf{b}_2$). Using again the laboratory

coordinate system we shall find the particle velocity vector after collision with the cloud:

$$\mathbf{OD}'_1 = \mathbf{v}'_1 = \mathbf{v}'_{c1} + \mathbf{u} = \mathbf{v} + \mathbf{b}_1; \quad |\mathbf{v}'_1| = \left(v^2 + b_1^2 + 2vb_1 \cos \theta_1 \right)^{1/2}, \quad (4.7.20)$$

$$\mathbf{OD}'_2 = \mathbf{v}'_2 = \mathbf{v}'_{c2} + \mathbf{u} = \mathbf{v} + \mathbf{b}_2; \quad |\mathbf{v}'_2| = \left(v^2 + b_2^2 + 2vb_2 \cos \theta_2 \right)^{1/2}. \quad (4.7.21)$$

where

$$\theta_1 = \angle D'_1 BO = \frac{3}{2}\pi - \varphi - \frac{\theta_c}{2} - \chi; \quad \theta_2 = \angle D'_2 BO = -\frac{\pi}{2} + \varphi - \frac{\theta_c}{2} + \chi. \quad (4.7.22)$$

In Eq. 4.7.22

$$\chi = \angle OCB = \arcsin \left(\frac{v \sin \varphi}{|\mathbf{v} - \mathbf{u}|} \right); \quad |\mathbf{v} - \mathbf{u}| = \left(v^2 + u^2 - 2vu \cos \varphi \right). \quad (4.7.23)$$

It may be assumed that approximately

$$\Delta E = \frac{m_{ac}}{2} \left(\frac{v_1'^2 + v_2'^2}{2} - v^2 \right). \quad (4.7.24)$$

Substituting Eq. 4.7.19 in Eq. 4.7.20–4.7.21 and then in Eq. 4.7.24 and considering that at non-relativistic energies $E \approx m_{ac}c^2$ we shall obtain, after tedious trigonometric transformations, the relation

$$\frac{\Delta E}{E} = \frac{2 \sin^2(\theta_c/2)}{c^2} (-uv \cos \varphi + u^2); \quad \left(\frac{\Delta E}{E} \right)_{\varphi=\pi,0} = \left(\pm \frac{2uv}{c^2} + \frac{2u^2}{c^2} \right) \sin^2(\theta_c/2), \quad (4.7.25)$$

which differ from the corresponding relation for mirror reflection only in the factor $\sin^2(\theta_c/2)$. As it was shown in Section 4.7.2 at $v \gg u$ we obtain $\theta_c \approx \theta$, and Eq. 4.7.23 will be transform into

$$\frac{\Delta E}{E} = \frac{2 \sin^2(\theta/2)}{c^2} (-uv \cos \varphi + u^2), \quad (4.7.26)$$

where θ is the scattering angle in the laboratory system of coordinates.

4.7.4. Energy change in relativistic case

In the relativistic case the mathematical operations of the previous Section 4.7.3 are also valid with the only exception that it becomes necessary to include relativistic composition of velocities. Considering that $u \ll c$ we shall obtain the following changes in Eq. 4.7.20, 4.7.21, 4.7.19, 4.7.23:

$$\mathbf{v}'_1 = (\mathbf{v} + \mathbf{b}_1) / (1 + \mathbf{v}\mathbf{b}_1/c^2), \quad \mathbf{v}'_2 = (\mathbf{v} + \mathbf{b}_2) / (1 + \mathbf{v}\mathbf{b}_2/c^2), \quad (4.7.27)$$

$$b_1 = b_2 = 2 \left| \mathbf{v} - \mathbf{u} - (\mathbf{v}\mathbf{u})\mathbf{v}/c^2 \right| \sin \frac{\theta}{2}, \quad (4.7.28)$$

$$\chi = \arcsin \left(\frac{v \sin \varphi}{\left| \mathbf{v} - \mathbf{u} - (\mathbf{v}\mathbf{u})\mathbf{v}/c^2 \right|} \right). \quad (4.7.29)$$

Taking into account that in the relativistic case

$$\Delta E = m_{ac} c^2 \left(\frac{1}{2\sqrt{1 - v_1^2/c^2}} + \frac{1}{2\sqrt{1 - v_2^2/c^2}} - \frac{1}{\sqrt{1 - v/c^2}} \right) \quad (4.7.30)$$

we shall obtain, after tedious transformations and neglecting the terms of higher orders than u^2/c^2 , the expression coinciding with Eq. 4.7.26.

4.7.5. The mode of particle energy change in time

The mean change in the particle energy with time dE/dt for scattering through some angles θ is determined not only by the value of $\Delta E/E$ in each collision event but also by the collision frequency ν . Since $\nu \propto (d^3/l^2)^{-1}$ then $dE/dt \propto (d^3/l^2)^{-1} \sin^2(\theta/2)$. On the other hand, in the models of magnetic clouds or inhomogeneities of type $j = 1, 2, 3$ the transport scattering path $\lambda \approx (d^3/l^2)/\theta^2$ at $\theta \ll 1$. Thus at small scattering angles we shall obtain the expressions of Sections 4.2–4.5 for dE/dt , in which λ should be meant not as the path before collisions with magnetic clouds but as the particle transport scattering path. Since at large scattering angles the path before scattering with magnetic clouds coincides with the transport scattering path, we can draw the following important conclusion: the results of Sections 4.2–4.5 (and hence of Section 4.6, the statistical acceleration in particle rigidities) are valid not only for mirror reflections but also for statistical scatterings

through any angles; it is necessary only to understand λ in the expressions of Sections 4.2–4.5 and 4.6 as the transport scattering path of particles.

4.8. Injection energy and the portion of the accelerated particles in the statistical mechanism

The initial acceleration process takes place most frequently in the non-relativistic energy range where the energy loss is of significance. It is the initial acceleration process that determines so called injection energy, i.e. the minimum energy from which the particle acceleration becomes possible. Detailed examination of the acceleration in the non-relativistic energy range makes it possible also to find the portion of the accelerated particles.

4.8.1. Injection energy in the statistical acceleration mechanism

Since the statistical acceleration mechanisms are characterized by a comparatively slow rate of energy gain, the various kinds of energy loss by the accelerated particles are very essential, especially in the first stages of acceleration. Let us consider the initial stage of acceleration (Dorman, 1959). The Eq. 4.3.5 in the non-relativistic energy range for the energy gain rate and analogues expressions in Sections 4.4–4.7, may be generalized in the form

$$\left(\frac{dE_k}{dt} \right)_{ac} = \alpha(E_k) \sqrt{2m_{ac}c^2 E_k}, \quad (4.8.1)$$

where m_{ac} and E_k are the rest mass and the kinetic energy of the accelerated particle; $\alpha(E_k)$ is the acceleration parameter including its dependence on E_k . It will be assumed in accordance with Section 4.5 that in the non-relativistic energy range

$$\alpha(E_k) = \alpha_T (E_k/kT)^{2\delta-\beta}, \quad (4.8.2)$$

where α_T is the acceleration parameter at $E_k = kT$ (where T is the plasma temperature). On the other hand, the energy loss for collisions (the ionization loss) is

$$\left(\frac{dE_k}{dt} \right)_{ion} = -4\pi Z^2 e^4 N_e L_1 \sqrt{m_{ac}/2E_k} / m_e, \quad (4.8.3)$$

where the logarithmic term $L_1 \approx 20$ for the characteristic space conditions, Ze is the charge of the accelerated particle, N_e is the electron concentration in the medium; m_e is the electron mass. It is obvious that the particle acceleration is possible only if

$$\left(\frac{dE_k}{dt}\right)_{ac} + \left(\frac{dE_k}{dt}\right)_{ion} \geq 0. \quad (4.8.4)$$

Substituting Eq. 4.8.2 in Eq. 4.8.1 and comparing with Eq. 4.8.3, we find that the condition described by Eq. 4.8.4 is satisfied at $E_k \geq E_{ki}$, where

$$E_{ki} = kT \left(\frac{2\pi Z^2 e^4 N_e L_1}{m_e \alpha_T c k T} \right)^{1/(2\delta - \beta + 1)}, \quad (4.8.5)$$

It can be easily seen that at $2\delta - \beta = 0$ and at a constant acceleration parameter we obtain the conventionally used expression for particle injection

$$E_{ki} = \frac{2\pi Z^2 e^4 N_e L_1}{m_e \alpha c} \approx 7 \times 10^{-9} \frac{Z^2 N_e}{\alpha c}. \quad (4.8.6)$$

If the initial energy of a particle $E_{ko} < E_{ki}$, such particle cannot be accelerated. The acceleration will occur only for the particles with $E_{ko} > E_{ki}$.

4.8.2. The injection from background plasma: conditions for acceleration of all particles

Let us consider different cases.

The case $2\delta - \beta = -1$. It can be seen from the comparison between Eq. 4.8.1 and Eq. 4.8.3 that in this case $(dE_k/dt)_{ac}$ and $(dE_k/dt)_{ion}$ vary with E_k in a similar manner. Therefore, if

$$\alpha_T > \frac{2\pi Z^2 e^4 N_e L_1}{m_e c k T} \approx 7 \times 10^{-9} \frac{Z^2 N_e}{c k T} \text{ sec}^{-1}, \quad (4.8.7)$$

then $E_{ki} \rightarrow 0$ and all the plasma particles should be accelerated. If, however, the Eq. 4.8.7 is not satisfied then $(dE_k/dt)_{ac} < (dE_k/dt)_{ion}$ over the entire range of nonrelativistic energies and the acceleration proves to be impossible at $2\delta - \beta = -1$.

The case $2\delta - \beta > -1$. If in this case Eq. 4.8.7 is satisfied then $E_{ki} \leq kT$, which is also equivalent to the acceleration of all the plasma particles since in this case the particle acceleration condition Eq. 4.8.4 is satisfied from thermal energies. Since the overall heating of the plasma takes place in this case, T increases abruptly and the

acceleration conditions change; such a process, which is essentially non-stationary, takes place at high values of α_T .

4.8.3. The injection from background plasma: quasi-stationary acceleration of a small part of the particles

If the parameter α_T (the acceleration parameter α at $E_k = kT$) is sufficiently small then $E_{ki} \gg kT$ and only small portion of plasma particles are accelerated. In this case the ion distribution function varies little in time and the acceleration is close to a quasi-stationary process. In this case, as was shown by Gurevich (1960), the rate of generation of run-away particles dn/dt from background plasma, including in the acceleration process, will be (in $\text{cm}^{-3} \text{sec}^{-1}$):

$$dn/dt = \sqrt{2/\pi} N \nu_o \exp\left(-(\pi/4)\sqrt{E_{ki}/kT}\right), \quad (4.8.8)$$

where N is the concentration of ions (cm^{-3}) and ν_o is the frequency of collisions (sec^{-1}) in the background plasma. The Eq. 4.8.8 is valid in the case when the distribution function of background plasma particles is close to the equilibrium (i.e. the part of accelerated particles for the average time of acceleration is much smaller than the total number of particles in the background plasma).

4.8.4. The problem of injection and acceleration of heavy nuclei from background plasma

From Eq. 4.8.6 follows that the injection energy $E_{ki} \propto Z^2$. It means that for heavy nuclei the injection energy will be sufficiently bigger than for protons, and in practice according to Eq. 4.8.8 the ratio of accelerated heavy nuclei number to contents in sources is expected to be much smaller than for protons. So the contribution of heavy nuclei to the total flux of accelerated particles expected to be negligible. However, the real situation is the opposite: in galactic CR the content of heavy nuclei relative to abundances in sources is much bigger than for protons (see review in Ginzburg and Syrovatsky, M1963; Dorman, M1963a, M1972, M1975a; Berezhinsky et al., M1990; Schlickeiser, M2001; see also in Section 1.4 in Dorman, M2004, and very short information above, in Section 1.2). This serious contradiction was widely discussed in the literature after the first paper of Fermi (1949) on the statistical acceleration mechanism, where this problem was for the first time noted.

To solve this problem Korchak and Syrovatsky (1959) take into account that if the temperature of background plasma is not too high, heavy ions are only one or two times ionized, so really the acceleration starts from the effective charge of heavy ion $Z^* = 1$ or 2, and in this case, according to Eq. 4.8.4 and Eq. 4.8.5, the energy of injection will be about the same as for protons. With energy increasing the effective

charge of particle will also be increased, but during all processes of acceleration the energy gain will be bigger than ionization losses.

Moreover, according to Korchak and Syrovatsky (1959), if the acceleration starts from velocities $v < v_e$, where v_e is the velocity of electrons of the background plasma, the value of the injection energy does not have a sense and practically all particles with these velocities will be involved in the acceleration process. Therefore from the condition

$$E_{kcr} = m_{ac} v_e^2 / 2 \quad (4.8.9)$$

may be found the critical value of the acceleration parameter α_{cr} , higher of which the injection threshold absent and particles are accelerated independence from their initial energy. In this case

$$\alpha_{cr}(A, Z) = \alpha_{cr}(p)(Z^*)^2 / A, \quad (4.8.10)$$

where $\alpha_{cr}(p)$ is the critical value of the acceleration parameter for protons. Let us account the loss of electrons (increasing Z^*) by ion with increasing energy during its acceleration (Bore's formula):

$$Z^* = (v/v_o) Z^{1/3}, \quad (4.8.11)$$

where v is the velocity of accelerated ion, and $v_o = e^2 / m_e r_o$ (here r_o is the classical radius of electron). After substituting Eq. 4.8.11 into Eq. 4.8.10 we obtain

$$\alpha_{cr}(A, Z) = \alpha_{cr}(p)(v_e/v_o)(Z^*)^{2/3} / A. \quad (4.8.12)$$

Because heavy ions of not very hot background plasma starts to accelerate from only single or double ionized ($Z^* = 1$ or 2), for them α_{cr} becomes smaller than for protons. Therefore, at the same initial ionization of background plasma is possible such parameter of acceleration α , that ions with big A will be accelerated independence from their initial energy, but the acceleration of protons will be depressed because of the high threshold of injection.

4.9. Statistical acceleration in the turbulent plasma confined within a constant magnetic field

As a rule the space plasma is turbulent and confined within a more or less regular magnetic field. Important studies of the turbulent acceleration mechanisms were

carried out in Lucke (1962), Tsytovich (1969), Schatzman (1969), Hall (1969) and others. The detail analysis of this problem was given by Tsytovich (1966c).

4.9.1. The magnetic field effect on plasma turbulence

The problem of the effect of large scale magnetic field on turbulent motion in an electro-conducting medium seems to be highly important to the problems of particle acceleration in such a medium and CR propagation. Without going into details of this complicated problem, we shall only note the work (Rädler, 1974) which considers the turbulent motions in homogeneous incompressible electro-conducting medium in the presence of a magnetic field which is on the average homogeneous and stationary. Adopting the model in which the turbulence is owed to stochastic volume force, and assuming weak interaction between the motion and the magnetic field, Rädler (1974) develops a method for calculating the paired correlation tensor of the velocity field. Calculated as an example is such a tensor for homogeneous stationary turbulence that is isotropic and mirror-symmetric at a beam magnetic field. It has been found that: first, the field suppresses the turbulence, namely, the component parallel to the field that is smaller than the perpendicular component; second, the correlation length parallel to the field tends to exceed the perpendicular length. The probability is considered of particular situations in which the turbulent velocities are enhanced by the field and the anisotropy of the velocity components and correlation lengths is opposite to that indicated above.

4.9.2. Particle acceleration by plasma fluctuations

The magnetic field gives rise to the change of the spectrum of the quasi-longitudinal plasma fluctuations determined by the equation

$$k_i k_j \xi_{ij} = 0, \quad k_{\perp}^2 \xi + k_z^2 \xi_z = 0, \quad (4.9.1)$$

where the dielectric constant

$$\xi = 1 - \omega_{oe}^2 / (\omega^2 - \omega_{Le}^2), \quad \xi_z = 1 - \omega_{oe}^2 / \omega^2 \quad (4.9.2)$$

at small kV_{Te}/ω . In the case of a weak spatial dispersion the effect of systematically change in the particle velocity for $v_z \gg v_{\perp}$ is

$$\begin{aligned} \frac{dE}{dt} = & \frac{e^2 \omega_{oe}^3 \hbar}{2m_{ac} v_z} \int u du \left\{ \left(\frac{m_e}{m_{ac}} h + \frac{m_{ac}}{m_e} \frac{u^2}{h} \right) \frac{|\xi_z|}{\xi^2} (N_- - N_+) \right. \\ & \left. - \frac{2u|\xi_z|}{\xi^2} (N_- + N_+) - \frac{4u}{|\xi|} N_o - \frac{2u^2 \omega_{oe}}{v_z |\xi|} N'_o \right\}, \end{aligned} \quad (4.9.3)$$

where the plasmons numbers

$$N'_o = \frac{dN_o}{dk_z}; N_\nu = N_{\omega, k_z} \text{ at } k_z = \frac{(\omega + \nu\omega_{Le})}{v_z}. \quad (4.9.4)$$

In Eq. 4.9.3 and 4.9.4 are used notations

$$u = \frac{\omega}{\omega_{oe}}; h = \frac{\omega_{Le}}{\omega_{oe}}; \text{index } \nu = 0, +, -. \quad (4.9.5)$$

Important application of CR charged particle's acceleration by plasma turbulence in the interstellar space pointed out by Jokipii (1977). It was shown that the age of CR particles in the Galaxy disk is enough to increase particle's energy several times from the turbulence energy of magnetized space plasma in the disk. The possibility of CR particle acceleration by frozen in magnetic turbulence of solar and stellar winds (in the case in which the CR density gradient is directed from the star outside) was shown in papers of Dorman et al. (1980, 1987).

4.9.3. Acceleration by magneto-sound and Alfvén waves

The phase velocities of the Alfvén and magneto-sound waves may be very high if $v_{ao}^2 = H^2/4\pi N_i m_i \gg 1$. In this case, the Alfvén velocity may approach the speed of light,

$$v_a^2 = c v_{ao}^2 / (1 + v_{ao}^2) \rightarrow c \text{ at } v_{ao} \rightarrow \infty. \quad (4.9.6)$$

This circumstance is of importance because the acceleration effect appears at $v_z < v_a$, where v_z is the field-aligned velocity of particle. According to Tsytovich (1966c) we obtain

$$\frac{dE_p}{dt} \approx \frac{2e^2 \omega_{Li}^2 v_a}{2\xi v_{ao}^2} \bar{N}_a, \quad (4.9.7)$$

where $\bar{N}_a$ is the mean number of Alfvén waves in unit of volume. The works of (Toptygin, 1972; 1973) treat the acceleration of fast particles by the Alfvén and magnetosonic waves of small amplitudes existing in a medium with a strong homogeneous magnetic field. The contribution from the small-scale (harmonics with $k > r_g^{-1}$, where r_g is the Larmor radius of particles) and large scale ($k < r_L^{-1}$) random field to the acceleration is included. The energy dependence of the particle diffusion coefficient in momentum space has been calculated. The effect of the

anisotropy of the particle distribution function on the acceleration has been considered. It has been shown that when the particles are accelerated by the Alfvén waves whose spectral function has the spectrum exponent $\nu > 2$ and the amplitude is sufficiently small, an energy dependence of the diffusion coefficient, which is stronger than that in the Fermi acceleration mechanism, may arise. In the case of magneto-sonic waves at $\nu \geq 2$ the energy dependence of the diffusion coefficient is the same as that in the acceleration by the Fermi mechanism, whilst at $\nu < 2$ the energy dependence is weaker.

4.9.4. Cyclotron acceleration of ions by plasma waves

The existence of the magnetic field facilitates the injection in the case of acceleration by high-frequency fluctuations since it is known that very slow waves of high frequencies may move perpendicularly to the magnetic field. The cyclotron acceleration of ions may be owed to the absorption and emission of the plasma waves with low phase velocities which propagate perpendicularly to the magnetic field. In this case (Tsytovich, 1963a):

$$\begin{aligned} \frac{dE_p}{dt} = \frac{2e^2}{m_e v_{\perp}} \sum_l \omega_L^3 l^4 \int_{-1}^{+1} n^2(\omega_L l, \mu) I_{\mu} \left(l v_{\perp} n(\omega_L l, \mu) \sqrt{1 - \mu^2} \right) N(\omega_L l, \mu) (1 - \mu^2)^{-1/2} \\ \times I'_l \left(l v_{\perp} n(\omega_L l, \mu) \sqrt{1 - \mu^2} \right) \left[\xi(\omega_L l, \mu) (1 - \mu^2) + \xi_z(\omega_L l, \mu) \mu^2 \right]^{-1} d\mu, \end{aligned} \quad (4.9.8)$$

where $\mu \equiv \cos \theta$, l is the order of harmonic, and $n(\omega_L l, \cos \theta)$ is the refractive index. If the fluctuation intensity is accumulated in the angular range of the order of $\mu_0 \approx \sqrt{1 - \xi_z/\xi} \ll 1$ and the mean ion velocity along the magnetic field is $\langle v_z \rangle$ we obtain the following estimate of the characteristic time of ion acceleration up to energy E_i by the first harmonic $l = 1$ (here W is the energy density of the waves):

$$\tau = \frac{1}{\omega_{oi}} \frac{E_i}{T_e} \langle v_z \rangle v_a \frac{n T_e}{W}; \quad v_a = \frac{H}{\sqrt{4\pi m_i m_i}}. \quad (4.9.9)$$

4.9.5. Cyclotron acceleration of ions by the combination frequency

Considered below will be the effects of ion acceleration in magnetic field in the events of induced scattering. The energy conservation law in the case of scattering in magnetic field is of the form

$$\omega - \omega' - (k_z v_z - k'_z v'_z) = \nu \omega_{Li}. \quad (4.9.10)$$

At $v = 0$ it is expedient to treat the scattering in the magnetic field; at $v = \pm 1$ the cyclotron acceleration by combination frequency $\omega - \omega'$ will be treated. The cyclotron acceleration by the combination frequency ($v = \pm 1$) in an intense magnetic field is much in excess of the acceleration owed to induced scattering ($v = 0$). In this case the ion velocity components perpendicular to the external magnetic field are predominantly increased. The estimate of the effect of acceleration by the plasma waves is of the form (Tsytovich, 1966c):

$$\frac{d}{dt} \langle E_{\perp} \rangle \approx \frac{\sqrt{2\pi}}{32} \frac{1}{m_i} \frac{k_{\perp}^2}{k^2} \frac{W_k^2}{n_o^2 v_{gr}}, \quad (4.9.11)$$

where v_{gr} is the wave group velocity, and W is the energy density of the waves.

4.9.6. Acceleration by electron plasma waves

The work (Smith, 1976) considers the acceleration of particles in their interactions with various plasma waves in a magnetic field $\mathbf{H}$ when the ratio of electron gyro-frequency to the electron plasma frequency $\omega_{Le}/\omega_{oe} \leq 1$. The kinetic equations are presented describing the evolution of the energy distribution of particles and plasmons including their forced scattering. Simplifying assumptions are made (the possibility of averaging over the non-relativistic and Maxwell distributions of ions and over the azimuth in a plane perpendicular to $\mathbf{H}$ and the possibility of limitation to the main terms of the expansion in $(\omega_{Le}/\omega_{oe})^2$, and others) and the approximate expressions for the scattering probability and the ratio of the electric energy of waves to their total energy have been obtained. The general mode of the behavior of the plasmons filling numbers has been studied and the relaxation time of the plasmons distribution has been estimated. It has been shown that the nonlinear process of the forced scattering by polarized ion clouds results in wave collapse and in an almost one-dimensional number spectrum extended along $\mathbf{H}$. The consecutive acceleration of relativistic and non-relativistic particles has been studied. It has been shown that such an acceleration is more effective for non-relativistic particles (protons); in this case, if the wave distribution is negatively sloped, the acceleration decreases for small velocities and increases for high velocities compared with the acceleration and isotropic distribution of the plasma waves in magnetic field. This should result in further changes of the wave spectrum and the value of the acceleration.

4.9.7. Acceleration by nonlinear waves

Gintzburg (1967, 1968) has obtained the solution for the equation of two-fluid magnetic hydrodynamics in the form of periodic ion waves of large amplitude with simultaneous rotation of the magnetic field vector and change of the magnetic field

modulus. The analytic expressions for the wave profile have also been found. The study of nonlinear ion waves with relativistic velocities shows that they may accelerate particles up to high energies. It is not excluded that this mechanism may be of definite importance in the case of CR generation in chromospheric flares on the Sun and other stars as well as in Supernova explosions. Particle acceleration by nonlinear waves was considered in details in Sagdeev et al. (M1988), and He (1998, 1999, 2000, 2002, 2003). He (2003) considered dimensionless Newtonian equations for charged particle in an electric field:

$$dx/dt = v, \quad dv/dt = -q\nabla\phi(x, t), \quad (4.9.12)$$

where x and v are particle position and velocity respectively, charge number $q = \pm 1$, and potential $\phi(x, t)$ is chosen as a solution of the driven/damped nonlinear drift-wave equation:

$$\frac{\partial\phi}{\partial t} + a \frac{\partial^3\phi}{\partial t\partial x^2} + c \frac{\partial\phi}{\partial x} + f\phi \frac{\partial\phi}{\partial x} = -\gamma\phi - \varepsilon \sin(x - \Omega t) \quad (4.9.13)$$

with a periodic system of length 2π and fixed constants of $a = -0.287$, $c = 1.0$, $f = -6.0$, $\gamma = 0.1$ (there was considered only the effect of nonlinear waves of Eq. 4.9.13 on the particle, while neglecting other effects related to the physical system from which the drift-wave equation is derived). A lot of simulations for different cases lead He (2003) to following conclusions. In all the tested non-steady wave fields a slow particle can be accelerated in the orientation of the steady waves. However, in the spatially regular field the particle is finally trapped by a wave trough and eventually acquires the group velocity of the steady waves, whilst in the chaotic both in time and in space field the particle experiences trapped and free phases randomly, depending on the charge sign the averaged velocity can be larger or smaller than the group velocity of the steady waves. It is shown that the virtual pattern of saddle steady waves plays the role of an asymmetric potential, which together with nonzero perturbation waves are necessary for the acceleration.

4.9.8. Acceleration by electrostatic waves

Bloomberg and Gary (1973) have considered the particle acceleration by electrostatic waves with a phase velocity increasing as such wave propagates in the space. The one-dimensional motion of a charged particle in an electrostatic wave propagating in inhomogeneous medium has been analyzed. It is assumed that the wave is of a fixed frequency and has the wave vector decreasing in space. The particle is accelerated in two stages. At first it is trapped between the wave crests. Then the particle is accelerated without oscillations in the well. The particle velocity at the ends of the first and second stages has been estimated. The estimate for the first stage has been made using the adiabatic invariant and gives a velocity $\propto E^{1/4}$,

where E is the electrostatic field in the wave. For the second stage the velocity $\propto E^{1/2}$. Numerical calculations have been carried out for a wave with wave vector depending on distance x according to the law $k(x) = k_o(1 + ax)^{-1}$. The calculations were carried out for 20 particles with the same initial velocities distributed uniformly along a segment equaling the period. The calculation results show that the particle may be considerably accelerated during the second stage, already after their oscillatory motion stops.

4.9.9. Stochastic Fermi acceleration by the turbulence with circularly polarized Alfvén waves

Ostrowski and Siemienieć-Oziębło (1997) demonstrated that the forward-backward asymmetry of particle scattering (as measured in the scattering center rest frame) at randomly moving scattering centers can lead to a first order regular acceleration term, in addition to the one resulting from the momentum diffusion. A physical example of such asymmetric scattering provides a finite amplitude circularly polarized Alfvén wave (Siemienieć-Oziębło et al., 1999). This research was continued in the paper of Michałek et al. (1999), in which were presented preliminary results of Monte Carlo modeling of the particle acceleration/diffusion process for protons interacting with finite amplitude circularly polarized Alfvén waves. It was shown that the scattering's forward-backward asymmetry occurring for such waves allows for the first order acceleration effects to occur in the stochastic acceleration process, enabling in favorable conditions for more effective acceleration in comparison to the linearly polarized Alfvén waves of the same amplitude.

4.10. Statistical acceleration of particles by electromagnetic radiation

4.10.1. Effectiveness of charged particle acceleration by electromagnetic radiation; comparison with the Fermi mechanism

Tsytoich (1963b,d), Nikolaev and Tsytoich (1976) studied the charged particle acceleration by electromagnetic radiation and compared the effectiveness of this mechanism with the Fermi mechanism. It has been shown that whilst the force affecting the charge in vacuum is determined by the light pressure proportional to the Thomson cross-section, the same force in the medium with the same radiation flux will increase by a factor of λ/r_o , where λ is the wavelength and r_o is the radius of a particle with charge Ze . The following form of relativistic expression has been obtained for the change of energy E of a particle with charge Ze in a medium with refractive index n in the field of isotropic radiation of density W with frequency ω , wavelength $\lambda = 2\pi u/\omega$ and propagation velocity $u = c/n$:

$$\frac{dE}{dt} = 2\pi p \frac{e^2 Z^2 \lambda u^2}{c^2 m_{ac} v} \left(1 - \frac{u^2}{v^2} \right) \left(\frac{E}{m_{ac} c^2} \right) \frac{1}{1 - \frac{\omega}{u} \frac{d\omega}{du}}, \quad (4.10.1)$$

where v and m_{ac} are respectively the velocity and mass of the accelerated particle. The Eq. 4.10.1 describes the influence of the radiation with chaotically phases.

4.10.2. On the injection in the particle acceleration by radiation

Detailed consideration is given in Tsytovich (1963b,d) to the problem of particle injection for the Fermi acceleration mechanism in the case of acceleration owed to radiation. With this purpose the curves of the energy gain are compared with the deceleration curves in the two mechanisms. For the same deceleration curves, the curves of acceleration gain prove to be significantly different, namely, the curves rise with increasing energy for the Fermi mechanism and fall for the mechanism of acceleration owed to radiation (in this case the decrease with increasing energy is always more rapid than that of the deceleration curve, so that the curves intersect at a certain value of energy). In connection with this the Fermi mechanism implies both injection and injection-less acceleration without visible limitation of the maximum energy of the accelerated particles (some limitation will be owed to only the inverse effect of the accelerated particles on the medium, the nuclear loss, and the escape from the acceleration region).

4.10.3. On the maximum energy and maximum density of accelerated particles in the case of particle acceleration by radiation

According to Tsytovich (1963b,d), in the case of the acceleration owed to radiation, there exists a maximum energy E_{cr} above which the deceleration is superior to the acceleration. If the radiation is characterized by the temperature T_{eff} (in eV) it appears that $E_{cr} \approx T_{eff}$. The total flux of the accelerated particles is determined by the condition that the density of their energy should not exceed the radiation energy density. This condition ensures an equilibrium energy distribution between the fast particles and the radiation, a fact which can be observed in the space. Tsytovich (1963b,d) notes that the state with $E \approx E_{cr}$ is unstable because the energy decrease makes the acceleration force superior to the deceleration force and the particle energy increases; in its turn, the energy increase results in the inverse effect. Tsytovich (1963b,d) also pays attention to the fact of the examined mechanism of particle acceleration owed to radiation is especially effective when the mean density of the radiation energy is much in excess of the mean kinetic energy of matter (for example, in the objects of the supernova type).

4.10.4. Cyclotron acceleration of relativistic electrons by lateral waves

In the magnetic field the particle energy may prove to change owed to the energy gain or loss in case of cyclotron radiation and wave absorption. This is clearly exemplified by the possibility of acceleration by the high-frequency lateral waves. For relativistic particles, the absorption and radiation of the frequencies multiple to the gyro-frequency $\omega \approx \mu \omega_L$ are of significance. According to Gailitis and Tsytovich (1963), we obtain in this case at $\omega \gg \omega_{oe}$ and $v_{\perp} \gg V_f$:

$$\frac{dE}{dt} = \frac{e^2 \omega_L^2}{\pi E \sqrt{3}} \int_0^{\infty} \chi^2 d\chi \{ 2(\mu + \eta) \xi K_{5/3}(\xi) - 3\eta \xi K_{1/3}(\xi) - 2\eta \int_0^{\infty} K_{1/3}(\xi') d\xi' \} N(\chi) \hbar, \quad (4.10.2)$$

where N is the number of waves and

$$\mu = 1 - v_{\perp}^2 / c^2, \quad \eta = 1 - \xi_z(\omega_L \mu), \quad \xi = \frac{2\chi}{3}(\mu + \eta). \quad (4.10.3)$$

When $\chi \gg \omega_{oe} m_e / eH(E/m_e)$ it may be assumed that

$$\frac{dE}{dt} = \frac{9\sqrt{3}e^2}{4\pi m_e^2} \left(\frac{eH}{m_e} \right)^2 \left(\frac{E}{m_e} \right)^3 \int_0^{\infty} \xi^2 N(\xi) K_{5/3}(\xi) \hbar d\xi. \quad (4.10.4)$$

For the 'temperature' radiation where $N(\omega) = T_{eff} / \hbar \omega$, the acceleration proves to be similar to the Fermi acceleration:

$$\frac{dE}{dt} = \alpha E; \quad \alpha = \frac{8}{3} \frac{e^4 H^2}{m_e^2} \frac{T_{eff}}{m_e^2}. \quad (4.10.5)$$

4.10.5. Electron acceleration by the radiation during their induced Compton scattering

Levich and Sunyaev (1971) showed that the induced Compton scattering of the radiation by the cold electrons brings about their effective heating to the relativistic temperatures. Of obvious interest is the further heating of already relativistic electrons with energies $E > m_e c^2$. The analysis of acceleration of such electrons owed to the induced Compton scattering was made in the papers Blandford (1973), Charugin and Ochelkov (1974, 1977), Kurlrud and Arons (1975).

Statistical acceleration of ultra-relativistic electrons by random electromagnetic waves is considered in the work (Blandford, 1973) in which the classical treatment of the induced Compton effect is generalized for relativistic case. The behavior of the

function of electron distribution in a stationary field of radiation has been studied. Kurlrud and Arons (1975) have studied the statistical acceleration of particles up to relativistic energies when affected by a set of spherical electromagnetic waves. They have shown that if a particle moves among a great number of antennas (for example, pulsars) emitting electromagnetic waves, the particle's energy and value of momentum increase in a stochastic manner. The essence of the mechanism is as follows: the trajectory of a particle which moves between the antennas (pulsars), each of which emits a strong electromagnetic wave, comprises the points at which the phase difference of the particle's oscillatory motions when affected by the two waves is constant in the coordinate system where the particle is at rest on the average. After arriving at such a point the particle will be accelerated until the drift motion or acceleration carries it away from the resonance region. Such a kind of the nonlinear resonant acceleration in which a particle interacts with two waves is known from Landau's theory of nonlinear attenuation, and it may be shown that such acceleration is a particular case of the Compton scattering. If the antennas (pulsars) are randomly located, the summation over all resonances and all pulsars will result in stochastic acceleration of the particles.

In the paper of Charugin and Ochelkov (1977) it was dwelt upon the discussion of the objects of relativistic electrons heating by the induced Compton scattering in the sources with relativistic brightness temperatures and isotropic distribution of the radiation.

4.10.6. Acceleration of charged particles by electromagnetic radiation pressure

Noerdlinger (1971) analyzes the motion of a particle ejected by a central body (star) when affected by radiation pressure. The expression for the accelerating force $\mathbf{F}$ affecting the particle which generalizes the corresponding results of (Chandrasekhar, 1934a,b) for the relativistic velocities of motion has been obtained. It was assumed when deriving the formula for $\mathbf{F}$ that the radiation field was purely radial and that the effective particle cross section σ was a function of the radiation field frequency. It has been shown that the final velocity of the ejected particle v_∞ depends on the value $F_o r_o / m_{ac} c^2$, where F_o is the force affecting the stationary particle at the initial moment $t = t_o$; r_o is the initial radial coordinate of the particle; m_{ac} is the accelerated particle mass. The plots of the dependences of the dimensionless velocity of particle motion $\beta = v_\infty / c$ on $F_o r_o / m_{ac} c^2$ for relativistic and non-relativistic cases were obtained. Noerdlinger (1974) has studied the effect of the finite size of the electromagnetic radiation source on charged particle acceleration by radiative pressure. The extreme value of the Lorentz-factor γ_L has been found (as a function of the distance from the source) above which any radiation field, whatever strong, cannot accelerate particles. The final energy of particles decreases significantly for very strong sources if a source ejects the particles within a large angle (at the same initial acceleration). In the asymptotic extreme, the final

energy of the particle at rest at the source surface is proportional to the fourth-power root of the source's luminescence for strong sources, whereas it is proportional to cube root of the luminescence for a point source.

Nakada (1973) examines the heavy ion acceleration in the case of resonant scattering of radiation near a bright source. The ion energy proves to be limited by the Doppler effect, aberration, photo-ionization, and ionization in the collision with the electrons of medium. It has been shown that the O and B stars may accelerate heavy ions up to 200 and 80 MeV/nucleon respectively, whilst the supernovae may accelerate them up to several hundreds of MeV/nucleon. Gordon (1975) has found the velocity distribution of fully ionized isolated atoms which are produced from partially ionized atoms and reside in the field of a point source of radiation with the power spectrum $I_\nu(\nu) = A\nu^\gamma/r^2$ (where ν is the frequency; r is the distance from the source; $I_\nu(\nu)$ is the radiation intensity). The radiation pressure onto the atoms in case of the radiation absorption in the resonance lines and the atomic photo-ionization have been taken into account. It has been found that the velocity distribution of the fully ionized atoms is independent of the radiation intensity and is a function of only the value of the exponent γ . It has been shown that for the exponent γ typical of the astrophysical objects the atoms cannot be accelerated by the radiation pressure up to relativistic velocities.

4.11. Statistical acceleration of particles by the Alfvén mechanism of magnetic pumping

4.11.1. *Alfvén's idea of particles acceleration by magnetic pumping*

Alfvén (1949, 1959) studied the acceleration of charged particles moving in an alternating magnetic field $\mathbf{H}$ and showed that a field enhancement would result in adiabatic acceleration of particles defined by the relation

$$p_\perp^2/H = \text{const}, \quad p_\parallel = \text{const}. \quad (4.11.1)$$

If in this case the particles are scattered by magnetic inhomogeneities the field will return to the initial value without decelerating the particles down to an energy corresponding to the initial state. The final result of a single cycle will be some gain in the particle energy. For the stationary state, if the rate of energy variation is independent of particle energy, the resultant spectra will be of the form $\propto p^{-\gamma}$, where $\gamma = 1$. It is assumed in order to obtain a higher γ in conformity to the experimental data that the spectrum is variable in space. Analysis of the assumption shows that the previously accelerated particles are injected in the vicinities of stars where they may be subsequently accelerated by the alternating magnetic field. The

particles of relatively low energies are confined within the regions close to the active stars, whereas the high energy particles are distributed over a much extended region. The inclusion of the absorption of the accelerated particles gives a power spectrum similar to the observed spectral form.

It is of importance to note that Alfvén's concept (1959) of a combination of the particle scattering and the betatron acceleration makes it possible to obtain a systematic increase of particle energy in a fluctuating magnetic field (even if the magnetic field intensity does not increase on the average). This phenomenon was subsequently called magnetic pumping. This mechanism, which is most probably of great importance to the acceleration processes in various objects, will be considered below in more details.

4.11.2. Relative change of the momentum, energy, and rigidity of particles in a single cycle of magnetic field variation in the presence of scattering

In accordance with the work (Alfvén, 1959), we shall examine the following simple model. Let us assume that the charged particles are confined within some volume comprising the magnetic field inhomogeneities against the background of homogeneous magnetic field. The frequency of collisions with inhomogeneities is $\nu = v/\lambda$, where λ is the transport scattering path of particles and v is the particle velocity. Let the field vary in time in the following manner:

(1) the field increases from H_0 to H_1 within a short time from t_1 to t_2 (here $t_2 - t_1 \ll \nu^{-1}$);

(2) then, within time from t_2 to t_3 (here $t_3 - t_2 \gg \nu^{-1}$), the field remains at the level H_1 ;

(3) after that the field intensity falls rapidly down to the initial value H_0 within time from t_3 to t_4 (here $t_4 - t_3 \ll \nu^{-1}$);

(4) during a period from t_4 to t_5 (here $t_5 - t_4 \gg \nu^{-1}$) the field remains if at the level H_0 .

Let us consider how the particle momentum and energy will vary during the above-mentioned cycle. Since, at the beginning of cycle (the same as period from t_4 to t_5), the field was equal to H_0 during a long period ($\gg \nu^{-1}$) and the equilibrium distribution of the energy degrees of freedom has set in owing to the collisions with inhomogeneities; so that the momentum components across and along the field were determined respectively as

$$p_{\perp 1}^2 = \frac{2}{3} p_1^2; \quad p_{//1}^2 = \frac{1}{3} p_1^2, \quad (4.11.2)$$

where p_1 is the particle momentum at the instant t_1 . During the first interval the collisions with inhomogeneities may be neglected and, according to Eq. 4.11.1:

$$p_{\perp 2}^2 = \frac{H_1}{H_o} p_{\perp 1}^2 = \frac{2}{3} \frac{H_1}{H_o} p_1^2; \quad p_{\parallel 2}^2 = p_{\parallel 1}^2 = \frac{1}{3} p_1^2. \quad (4.11.3)$$

From Eq. 4.11.3 follows that the total change will be

$$p_2^2 = p_{\perp 2}^2 + p_{\parallel 2}^2 = \frac{1}{3} \left(1 + \frac{2H_1}{H_o} \right) p_1^2. \quad (4.11.3a)$$

During the second interval from t_2 to t_3 an equilibrium distribution will set in owing to the collision with inhomogeneities and, since the field failed to vary in that period, then

$$p_3^2 = p_2^2; \quad p_{\perp 3}^2 = \frac{2}{3} p_3^2 = \frac{2}{9} \left(1 + \frac{2H_1}{H_o} \right) p_1^2; \quad p_{\parallel 3}^2 = \frac{1}{3} p_3^2 = \frac{1}{9} \left(1 + \frac{2H_1}{H_o} \right) p_1^2. \quad (4.11.4)$$

During the third interval from t_3 to t_4 the collisions with inhomogeneities may be neglected and we shall obtain from Eq. 4.11.1 as a result of the field decrease from H_1 to H_o :

$$p_{\perp 4}^2 = \frac{H_o}{H_1} p_{\perp 3}^2 = \frac{2}{9} \frac{H_o}{H_1} \left(1 + \frac{2H_1}{H_o} \right) p_1^2; \quad p_{\parallel 4}^2 = p_{\parallel 3}^2 = \frac{1}{9} \left(1 + \frac{2H_1}{H_o} \right) p_1^2;$$

$$p_4^2 = p_{\perp 4}^2 + p_{\parallel 4}^2 = \frac{1}{9} \left(5 + \frac{2H_1}{H_o} + \frac{2H_o}{H_1} \right) p_1^2. \quad (4.11.5)$$

During the fourth interval from t_4 to t_5 the equilibrium distribution will be restored owing to collisions with inhomogeneities, and since the field failed to vary during that period we shall obtain that

$$p_5^2 = p_4^2; \quad p_{\perp 5}^2 = \frac{2}{27} \left(5 + \frac{2H_1}{H_o} + \frac{2H_o}{H_1} \right) p_1^2; \quad p_{\parallel 5}^2 = \frac{1}{27} \left(5 + \frac{2H_1}{H_o} + \frac{2H_o}{H_1} \right) p_1^2. \quad (4.11.6)$$

Thus as a result of a single complete cycle of the field variation and owing to the particle scattering by magnetic inhomogeneities, the relative change of the momentum will be

$$\frac{\Delta p^2}{p^2} = \frac{1}{9} \left(\frac{2H_1}{H_o} + \frac{2H_o}{H_1} - 4 \right); \quad \frac{\Delta p}{p} = \frac{1}{3} \sqrt{5 + \frac{2H_1}{H_o} + \frac{2H_o}{H_1}} - 1. \quad (4.11.7)$$

and the relative change of the total energy will be

$$\frac{\Delta E}{E} = \left[1 + \frac{p^2 c^2}{9E^2} \left(\frac{2H_1}{H_o} + \frac{2H_o}{H_1} - 4 \right) \right]^{1/2} - 1. \quad (4.11.8)$$

It follows from Eq. 4.11.8 that

$$\frac{\Delta E_k}{E_k} = \frac{1}{9} \left(\frac{2H_1}{H_o} + \frac{2H_o}{H_1} - 4 \right), \quad (4.11.9)$$

in the non-relativistic energy range and

$$\frac{\Delta E}{E} = \left[1 + \frac{1}{9} \left(\frac{2H_1}{H_o} + \frac{2H_o}{H_1} - 4 \right) \right]^{1/2} - 1 \quad (4.11.10)$$

at ultra-relativistic energies. The relative change of particle rigidity $R = cp/Ze$ during a single cycle will be, according to Eq. 4.11.7:

$$\frac{\Delta R}{R} = \frac{1}{3} \left(5 + \frac{2H_1}{H_o} + \frac{2H_o}{H_1} \right)^{1/2} - 1. \quad (4.11.11)$$

It follows from Eq. 4.11.8, 4.11.9, and 4.11.11 at $H_1/H_o - 1 = \Delta H/H_o \ll 1$ that

$$\frac{\Delta E}{E} = \frac{p^2 c^2}{9E^2} \left(\frac{\Delta H}{H_o} \right)^2; \quad \frac{\Delta E_k}{E_k} = \frac{2}{9} \left(\frac{\Delta H}{H_o} \right)^2; \quad \frac{\Delta R}{R} = \frac{1}{9} \left(\frac{\Delta H}{H_o} \right)^2. \quad (4.11.12)$$

The mode of variations during single cycle in $\Delta E_k/E_k$ according to Eq. 4.11.9 and in $\Delta R/R$ according to Eq. 4.11.11 (depending on $\Delta H/H_o = H_1/H_o - 1$) can be seen from Table 4.11.1.

Table 4.11.1. Variations of $\Delta E_k/E_k$ in the non-relativistic case and $\Delta R/R$ depending on $\Delta H/H_o$.

$\Delta H/H_o$	0.1	0.2	0.5	1.0	2.0	3.0	4.0
$\Delta E_k/E_k$	0.0022	0.0072	0.037	0.111	0.296	0.500	0.711
$\Delta R/R$	0.0011	0.0036	0.018	0.054	0.138	0.225	0.308

The expected relative variations of $\Delta E/E$ depending on $p^2 c^2/E^2$ and $\Delta H/H_o$ are shown in Table 4.11.2.

Table 4.11.2. Variations of $\Delta E/E$ depending on $p^2 c^2/E^2$ and $\Delta H/H_o$.

$p^2 c^2/E^2$	$\Delta H/H_o$							
	0.1	0.2	0.5	1.0	2	3	4	9
0.10	0.00011	0.00036	0.00185	0.00553	0.0147	0.0247	0.0349	0.0863
0.20	0.00022	0.00072	0.00370	0.0110	0.0292	0.0488	0.0687	0.1662
0.40	0.00044	0.00144	0.00739	0.0220	0.0575	0.0954	0.1333	0.3115
0.60	0.00067	0.00216	0.0110	0.0328	0.0852	0.1402	0.1944	0.4422
0.80	0.00089	0.00288	0.0148	0.0435	0.1121	0.1832	0.2525	0.5620
0.90	0.00100	0.00324	0.0165	0.0488	0.1253	0.2042	0.2806	0.6186
0.95	0.00106	0.00342	0.0174	0.0514	0.1319	0.2145	0.2944	0.6462
0.98	0.00109	0.00353	0.0180	0.0530	0.1358	0.2207	0.3026	0.6625
0.99	0.00110	0.00356	0.0181	0.0536	0.1371	0.2227	0.3053	0.6679
1.00	0.00111	0.00360	0.0183	0.0541	0.1384	0.2247	0.3081	0.6733

4.11.3. The rate of the gain in energy and rigidity for the mechanism of acceleration by magnetic pumping

Let the duration of total cycle of magnetic field variations be

$$T = t_5 - t_1 = \text{const.} \quad (4.11.13)$$

The mean rate of the gain in the kinetic energy (in the non-relativistic case), rigidity, and total energy will then be, considering Eq. 4.11.9, 4.11.10, and 4.11.8 respectively:

$$\frac{dE_k}{dt} = \frac{E_k}{9T} \left(\frac{2H_1}{H_o} + \frac{2H_o}{H_1} - 4 \right); \quad \frac{dR}{dt} = \frac{R}{T} \left[\frac{1}{3} \left(5 + \frac{2H_1}{H_o} + \frac{2H_o}{H_1} \right)^{1/2} - 1 \right]; \quad (4.11.14)$$

$$\frac{dE}{dt} = \frac{E}{T} \left\{ \left[1 + \frac{p^2 c^2}{9E} \left(\frac{2H_1}{H_o} + \frac{2H_o}{H_1} - 4 \right) \right]^{1/2} - 1 \right\}. \quad (4.11.15)$$

If the onset of acceleration was at the instant $t = 0$ from $E_k = E_{ko}$ or from $R = R_o$, then

$$E_k(t) = E_{ko} \exp(\alpha_1 t/T); \quad R(t) = R_o \exp(\alpha_2 t/T), \quad (4.11.16)$$

where

$$\alpha_1 = \frac{1}{9} \left(\frac{2H_1}{H_o} + \frac{2H_o}{H_1} - 4 \right), \quad (4.11.17)$$

$$\alpha_2 = \frac{1}{3} \left(5 + \frac{2H_1}{H_o} + \frac{2H_o}{H_1} \right)^{1/2} - 1. \quad (4.11.18)$$

The values of α_1 and α_2 , as functions of $\Delta H/H_o = H_1/H_o - 1$ are the same values, which were presented in Table 4.11.1 (rows for $\Delta E_k/E_k$ and $\Delta R/R$, respectively).

The time variations in the particle total energy E will be determined on the basis of Eq. 4.11.14-4.11.15 and including the notation determined by Eq. 4.11.17 at the initial condition $E|_{t=0} = E_o$ by the relation

$$\int_{E_o}^E \frac{dE}{E} \left\{ \left[1 + \alpha_1 \left(1 - \frac{m_{ac}^2 c^4}{E^2} \right) \right]^{1/2} - 1 \right\}^{-1} = \frac{t}{T}. \quad (4.11.19)$$

After solving Eq. 4.11.19, we shall get

$$\frac{(1 + \alpha_1 - x_o)(x - 1)}{(1 + \alpha_1 - x)(x_o + 1)} = \exp(\alpha_1 t/T), \quad (4.11.20)$$

where

$$x_o = \left[1 + \alpha_1 \left(1 - \frac{m_{ac}^2 c^4}{E_o^2} \right) \right]^{1/2}; \quad x = \left[1 + \alpha_1 \left(1 - \frac{m_{ac}^2 c^4}{E^2} \right) \right]^{1/2}. \quad (4.11.21)$$

Resolving the Eq. 4.11.20 relative to E we shall find

$$E(t) = \sqrt{\alpha_1} m_{ac} c^2 \left\{ 1 + \alpha_1 - \left[\frac{(1 + \alpha_1)(x_o + 1) \exp(\alpha_1 t / T) + 1 + \alpha_1 - x_o}{(x_o + 1) \exp(\alpha_1 t / T) + 1 + \alpha_1 - x_o} \right]^2 \right\}^{-1/2} \quad (4.11.22)$$

It can be easily seen that in the non-relativistic energy range where $1 - m_{ac}^2 c^4 / E^2 \ll 1$ the Eq. 4.11.20 turns out to be

$$x_o \approx 1 + \frac{\alpha_1 E_{ko}}{2 m_{ac} c^2}; \quad x \approx 1 + \frac{\alpha_1 E_k}{2 m_{ac} c^2} \quad (4.11.23)$$

and Eq. 4.11.22 is transformed into Eq. 4.11.16 for dE_k / dt . In the ultra-relativistic case in which $m_{ac}^2 c^4 / E^2 \ll 1$ the Eq. 4.11.16 is transformed to

$$x_o = \sqrt{1 + \alpha_1} \left(1 - \frac{\alpha_1}{2(1 + \alpha_1)} \frac{m_{ac}^2 c^4}{E_o^2} \right); \quad x = \sqrt{1 + \alpha_1} \left(1 - \frac{\alpha_1}{2(1 + \alpha_1)} \frac{m_{ac}^2 c^4}{E^2} \right). \quad (4.11.24)$$

so that Eq. 4.11.22 will turn out to be

$$E(t) = E_o(t) \exp(\alpha_2 t / T). \quad (4.11.25)$$

4.11.4. Formation of the energy and rigidity spectra in the case of particle acceleration by magnetic pumping

Let the mean lifetime of particles in the acceleration region be determined by one of the following expressions:

$$\tau = \tau_o (E_k / E_{ko})^{-\beta}; \quad \tau = \tau_o (R / R_o)^{-\beta}; \quad \tau = \tau_o (E / E_o)^{-\beta}. \quad (4.11.26)$$

Taking into account the Eq. 4.2.7 describing the distribution of the accelerated particle number $n(t)$ over the particle lifetime t in the acceleration region and considering that, according to Eq. 4.11.16,

$$t = \frac{T}{\alpha_1} \ln \frac{E_k}{E_{ko}}, \quad dt = \frac{T dE_k}{\alpha_1 E_k}; \quad t = \frac{T}{\alpha_2} \ln \frac{R}{R_o}, \quad dt = \frac{T dR}{\alpha_2 R}; \quad (4.11.27)$$

we shall obtain for the kinetic energy and rigidity spectra of the particles respectively:

$$n(E_k) \propto E_k^{-\gamma}, \quad \gamma = \frac{T}{\alpha_1 \tau} \left(\frac{E_k}{E_{ko}} \right)^\beta + 1 - \beta; \quad (4.11.28)$$

$$n(R) \propto R^{-\gamma}, \quad \gamma = \frac{T}{\alpha_2 \tau} \left(\frac{R}{R_o} \right)^\beta + 1 - \beta. \quad (4.11.29)$$

Considering that, according to Eq. 4.11.20,

$$t = \frac{T}{\alpha_1} \ln \left(\frac{(1 + \alpha_1 - x_o)(x - 1)}{(1 + \alpha_1 - x)(x_o + 1)} \right); \quad dt = \frac{TdE}{E} \left\{ \left[1 + \left(1 - \alpha_1 \frac{m_{ac}^2 c^4}{E^2} \right) \right]^{1/2} \right\}^{-1}, \quad (4.11.30)$$

we shall obtain for the total energy spectrum of the accelerated particles:

$$n(E) \propto \frac{E^{\beta-1}}{x-1} \left[\frac{(1 + \alpha_1 - x_o)(x - 1)}{(1 + \alpha_1 - x)(x_o + 1)} \right]^{\frac{T(E/E_o)^\beta}{\alpha_1 \tau}}, \quad (4.11.31)$$

where x_o and x are determined from Eq. 4.11.21.

In the non-relativistic energy range, where it may be assumed that $1 - m_{ac}^2 c^4 / E^2 \ll 1$, we shall obtain, including Eq. 4.11.24, that Eq. 4.11.31 turns out to be a spectrum of the form described by Eq. 4.11.28. In the ultra-relativistic energy range where $m_{ac}^2 c^4 / E^2 \ll 1$ we shall find including Eq. 4.11.24 that Eq. 4.11.31 turns out to be a spectrum of the form

$$n(E) \propto E^{-\gamma}; \quad \gamma = 1 - \beta + \frac{T}{\alpha_2 \tau} (E/E_o)^\beta. \quad (4.11.32)$$

It can be seen from Eq. 4.11.28, Eq. 4.11.29, and Eq. 4.11.32 that at $\beta = 0$ the expected spectra of the accelerated particles are of power form. In the opposite case, at $\beta > 0$, the power exponent in the accelerated particle spectrum increases with increasing the energy or rigidity of the particles.

4.11.5. Formation of the particle spectrum in the magnetic pumping mechanism including absorption in the source

Fälthammer (1963) studied this acceleration mechanism in detail, examined the injection conditions, and derived the following form of the kinetic equation for the function of particle distribution in the space of coordinates and moments for a stationary case:

$$\operatorname{div}(\kappa \operatorname{grad} f) - \frac{\partial}{\partial p} \left(\frac{pf}{\tau} \right) - \frac{f}{\tau_{ab}} = 0, \quad (4.11.33)$$

where f is the sought distribution function, p is the particle momentum, κ is the diffusion coefficient, τ is the effective duration of acceleration, and τ_{ab} is the effective duration of absorption. Assuming that the particle injection is determined by the condition $pf/\tau = n_o \delta(r)$ at $p = p_o$, Fälthammer (1963) finds the solution of the equation presented above (subject to κ , τ , and τ_{ab} are independent of the spatial coordinates $\mathbf{r}$ of the leading center of particles but are functions of only momentum p) for three-dimensional case in the form:

$$f = \frac{n_o \tau}{p (\pi R_p^2)^{3/2}} \exp \left(-\frac{r^2}{R_p^2} - A(p) \right), \quad (4.11.34)$$

where

$$R_p^2 = 4 \int_{p_o}^p \kappa \tau \frac{dp}{p}, \quad A(p) = \int_{p_o}^p \frac{\tau}{\tau_{ab}} \frac{dp}{p}. \quad (4.11.35)$$

In the two-dimensional case the resultant solutions are the same; the only difference is that the denominator contains πR_p^2 instead of $(\pi R_p^2)^{3/2}$. Fälthammer (1963) has shown that Alfvén's partial solution (Alfvén, 1959) can be obtained from the reduced general solution if the following dependences on p for κ , τ and τ_{ab} are selected:

$$\kappa = \kappa_o (p/p_o)^{1-\varphi}, \quad \varphi = \text{const}; \quad \tau = \tau_o (p/p_o)^\alpha, \quad \alpha = \text{const}; \quad \tau_{ab} = \text{const}. \quad (4.11.36)$$

In this case the resultant spectrum in the range of large momentum is of a power form. In the small-momentum range at small distances from the injection region, the resultant spectrum is also of a power form, whereas at great distances the spectrum should have a significant fall at low energies. These results are qualitatively in a good agreement with the experimental data. In order to obtain the exponent $\gamma = 2.5$ in the differential spectrum at high energies, it is necessary to the two-dimensional

case that $\varphi = -0.5$ and in the three-dimensional case $\alpha = 3\varphi$ (for example, $\alpha = 0.3$ and $\varphi = 0.1$). It has been shown that this mechanism is reasonable to consider in interplanetary space only for those particles with energies not above 10^{12} eV and in interstellar space for those particles with energies not above 10^{15} eV.

4.11.6. The magnetic pumping mechanism in the case of field variations according to the power law

The mechanism of particle acceleration by magnetic pumping was initially proposed by Alfvén (1959) for the case of slow periodic change of a homogeneous magnetic field:

$$H = H_0(1 + \beta \cos(\omega t)) \quad (4.11.37)$$

(here $\beta < 1$ is the so called pumping parameter; ω is the frequency of the field variation) in turbulent plasma accompanied by conservation of the adiabatic invariants $p_{\parallel} = \text{const}$, $p_{\perp}^2/H = \text{const}$. In this case an exponential increase of the total momentum on hydro-magnetic turbulence in time and the betatron acceleration are possible. Because of the particle scattering by hydromagnetic turbulence, the portion of the momentum accumulated during the magnetic field enhancement (Eq. 4.11.37) owed to betatron acceleration ($p_{\perp}^2 \propto H$) is transferred to the parallel component of the momentum. As a result, if the scattering time is small (much smaller than the period of field variation), the particle momentum loss proves to be smaller than the momentum increase during the field enhancement; it is this circumstance that results in the acceleration. The mechanism for this case of magnetic pumping was further developed by Schluter (1957) who has shown that the mode of periodic variations of the field is of no importance in principle and that in the particular case described by Eq. 4.11.36 the acceleration effect for an ensemble of particles of the same energy is a maximum at $\nu_{\text{eff}} \approx \omega$, where ν_{eff} is the effective frequency of particle scattering.

4.11.7. Kinetic theory of particle acceleration by magnetic pumping

The most consistent theory of particle acceleration in variable magnetic fields was developed in the works by Bakhareva et al. (1970a,b). It is in these works that the particle scattering by hydromagnetic turbulence was proposed as the scattering mechanism and the problem of particle acceleration was formulated on the basis of the equations of quasi-linear kinetic which permitted both the accelerated particle spectrum and the spectrum of turbulent pulsations causing the particle scattering to be determined.

It may be expected that in the presence of a variable magnetic field the cyclotron instability associated with the anisotropy of the angular distribution of charged

particle velocities is the source of intense turbulence. In the initial isotropic plasma such anisotropy appears because of conservation of the adiabatic invariant $p_{\perp}^2/H = \text{const}$ and is owed to two-dimensional compression (expansion) of the Larmor orbits of particles owing to periodic variations of magnetic field. The above mentioned instability is owed to the cyclotron resonance between waves and particles on Larmor frequency including the Doppler effect and appears at very small anisotropy for sufficiently high particle velocities.

The equation set of quasi-linear approximation for the examined case may be written in the form

$$\frac{\partial f}{\partial t} + \frac{p \sin \theta}{2H} \frac{dH}{dt} \left(\sin \theta \frac{\partial}{\partial p} + \frac{\cos \theta}{p} \frac{\partial}{\partial \theta} \right) f = \left(\frac{Ze}{m_{ac}c} \right)^2 \frac{1}{E \sin \theta} \frac{\partial}{\partial \theta} \sin \theta |D_{\theta\theta}| \frac{\partial f}{\partial \theta}, \quad (4.11.38)$$

where p and E are the dimensionless momentum and energy (in units $m_{ac}c$ and $m_{ac}c^2$, respectively) and

$$D_{\theta\theta} = \frac{B(\omega_L/cp \cos \theta)}{cp \cos \theta}. \quad (4.11.39)$$

In this case

$$\gamma(\omega) = \frac{2}{m_{ac}} \left(\frac{\pi Ze}{ck} \right)^2 \omega \frac{d\omega}{dk} \int_0^{\infty} dp_{\perp}^2 \left[f \pm \frac{\omega_L}{\omega} \frac{p_{\perp}^2}{E} \left(\frac{\partial}{\partial p_{\perp}^2} - \frac{\partial}{\partial p_{\parallel}^2} \right) f \right], \quad (4.11.40)$$

where $p_{\parallel} \approx \omega_L/kc$. Here f is the particle distribution function; $\gamma(\omega)$ is the increment of the pulsations with frequency ω and wave vector $\mathbf{k}$; the upper and lower signs in Eq. 4.11.40 relate to the Alfvén and rapid magneto-sonic waves respectively. After averaging the Eq. 4.11.38 over the period $T = 2\pi/\omega$ of variation of the field (see Eq. 4.11.37) and over the angular variable θ , we obtain the equation

$$\frac{\partial f}{\partial t} = \frac{1}{p^2} \frac{\partial}{\partial p} p^2 D(p) \frac{\partial f}{\partial p}, \quad (4.11.41)$$

describing the particle acceleration process with diffusion coefficient

$$D(p) = \left(\frac{m_{ac}cp}{Ze} \right)^2 \frac{E}{6} \left(\frac{1}{2H} \frac{dH}{dt} \right)^2 \int_0^{\pi} d\theta \sin^3 \theta \left[\int \frac{d\theta \sin \theta \cos \theta}{|D_{\theta\theta}|} \right]. \quad (4.11.42)$$

In Eq. 4.11.42 the upper line denotes the averaging over the period $T = 2\pi/\omega$. Taking into account that at the instability boundary $\nu(\omega) = 0$ and that for resonance ($\cos\theta = \omega_L/ckp$) the function $B(\omega_L/cp \cos\theta) \equiv B(k)$, we shall obtain from Eq. 4.11.40:

$$B(k) = \mp \frac{1}{3} \frac{\omega_L}{k\omega} \frac{1}{2H} \frac{dH}{dt} \left[1 + \frac{1}{2} \lim \left(\frac{\omega_L^2}{c^2 k^2} - p^2 \right) \bar{f}(p) \int_{\omega_L/ck}^{\infty} p f(p) dp \right]. \quad (4.11.43)$$

The Eq. 4.11.41 with the diffusion coefficient determined by Eq. 4.11.42 and the Eq. 4.11.43 constitute a self-consistent set for determining the distribution function of accelerated particles $f(p)$ and the wave vector $B(k)$. Solution of this set by the method of successive approximation gives a spectral function $B(k)$ of the form

$$B(k) = \frac{B_1 H_o^2 \omega}{6\sqrt{2} \nu_a k^2}, \quad (4.11.44)$$

where B_1 is a constant. In accordance with Eq. 4.11.44, the Eq. 4.11.41 for the power exponent of magnetic inhomogeneity spectrum $\nu = 2$ gives an exponential spectrum of accelerated particles in the non-relativistic case. In fact, as was shown in (Dorman and Katz, 1977), if the initial function

$$f_o(p) \equiv f(p, t = 0) \quad (4.11.45)$$

differs from zero in some region $p \leq p_o \ll 1$ (in units $m_{ac}c$), the asymptotic $f(p, t)$ at great t and $p \gg p_o$ is of the following form:

$$f(p, t) = \frac{(3 - \nu)^{2\nu/(\nu-3)}}{2\Gamma\left(\frac{3}{3-\nu}\right)} N_\nu t^{3/(\nu-3)} \exp\left(-\frac{p^{3-\nu}}{(3-\nu)^2 t}\right), \quad (4.11.46)$$

where

$$N_\nu = \int_0^\infty q^\nu f_o(q) dq. \quad (4.11.47)$$

It can be seen from Eq. 4.11.46 that at $\nu = 2$ the exponential spectrum of accelerated particles is asymptotically generated. The value N_ν has the meaning of the total number of the particles injected in 1 cm^3 , and, as can be seen from Eq. 4.11.41, fails

to vary in the course of acceleration. At $\nu = 3$ the solution of the equation Eq. 4.11.41 for the case of point source is of the form

$$f(p, t) = \frac{1}{2\sqrt{\pi\tau}} p^{-3/2} \exp\left(-\frac{(\ln(p/p_o))^2}{4t}\right), \quad (4.11.48)$$

where p_o is the particle momentum during injection. The Eq. 4.11.46 also describes the particle distribution in the ultra-relativistic case. For this purpose the replacement $\nu \rightarrow \nu + 1$ should be made in this expression. In accordance with this the Eq. 4.11.46 describes the distribution function in the ultra-relativistic case for $\nu = 2$. The characteristic time of acceleration τ is determined in both cases by the relation:

$$\tau = \left(\frac{m_{ac} c \omega_L}{6Ze}\right)^2 \frac{6\sqrt{2}v_a}{B_1 H_o^2 \omega} \left(\frac{1}{2H} \frac{dH}{dt}\right)^2. \quad (4.11.49)$$

In the case of stationary acceleration we obtain the power spectra of charged particles. According to Bakhareva et al. (1970a), the magnetic pumping mechanism including the loss for synchrotron radiation may ensure the observed power of the synchrotron X-radiation from the Crab nebula. The kinetic theory of particle acceleration by magnetic pumping was further developed in the work (Bakhareva et al., 1973) which gives a more general derivation of the equation of particle diffusion in the momentum space for quasi-linear approximation. The equation describes the evolution of the averaged distribution function f in the variable external magnetic field (pumping field) subject to strong scattering by turbulent pulsations during the pumping period. To close the set of equations describing self-consistently the evolution of the particle and wave spectra (hydromagnetic turbulence), the exact equation of the quasi-linear theory for the rate of the increase in the spectral function of waves

$$d\Phi/dt = 2\gamma_k \Phi \quad (4.11.50)$$

is used here (where γ_k is the increment of the cyclotron instability owed to the distribution function deformation). The deformation is associated with particle acceleration by the inductive electric field of pumping. The increment γ_k is calculated as a function of the averaged distribution f described by the diffusion equation. The stationary solution of the set of equations so obtained has been studied for the case of ultra-relativistic electrons taking account of the synchrotron radiation forming a sink of the energy pumped by the variable magnetic field. The region of the momentum space where the acceleration is most effective has been found, which

makes it possible to construct simple formulas for estimating the electron energy density and the synchrotron radiation intensity.

4.12. Accelerated particle flux from sources

4.12.1. Particle flux from a source in stationary case

We found above the spectrum of accelerated particles in their source for different modes of statistical acceleration mechanism. Since the probability of particle ejection from the source may be energy-dependent, the spectrum of the outgoing flux may be appreciably different from the particle spectrum in the source. Consider a simple model. Let the source be a sphere of radius L , the transport scattering path inside the source be λ , the particle velocity be v ; then the diffusive particle flux from the source $I(E)$ will be determined by the expression

$$I(E) = -4\pi L^2 \frac{v\lambda}{3} (\text{grad}n)_L, \quad (4.12.1)$$

where

$$(\text{grad}n)_L = (dn/dr)_{r=L} \approx -a n/L, \quad (4.12.2)$$

and the parameter $a \sim 1$ is determined by the details of the diffusion model. Let the particle concentration and transport scattering path in the source be determined by the expressions

$$n(E) = n_o (E/E_o)^{-\gamma}; \quad \lambda = \lambda_o (E/E_o)^\beta. \quad (4.12.3)$$

Then, including Eq. 4.12.1, we obtain

$$I(E) \approx \frac{4\pi a}{3} L v n_o \lambda_o (E/E_o)^{-\gamma+\beta}. \quad (4.12.4)$$

In the non-relativistic energy range we get:

$$I(E_k) \propto (E_k/E_{ko})^{-\gamma+\beta+1/2}. \quad (4.12.5)$$

If n and λ are determined by the power functions from R with exponents γ and β (see Section 4.4), then

$$I(R) \approx \frac{4\pi a}{3} L n_o \lambda_o R \left[R^2 + (m_{ac} c^2 / Ze)^2 \right]^{-1/2} (R/R_o)^{-\gamma+\beta}. \quad (4.12.6)$$

4.12.2. Particle flux from the source in non-stationary case

Assume that the particles are accelerated in the source within time from t_0 to t_1 . At the instant t_1 the particle concentration in the source is

$$n(E, t_1) = n_1 (E/E_0)^{-\gamma}. \quad (4.12.7)$$

If $\lambda = \lambda_0 (E/E_0)^\beta$ is the transport scattering path the change of the particle number inside the source at $t \geq t_1$ will be determined, taking account of Eq. 4.12.3, by the equation

$$\frac{d}{dt} \left(\frac{4\pi}{3} L^3 n(E, t) \right) = - \frac{4\pi a}{3} L v n_0 \lambda_0 n(E, t) (E/E_0)^\beta, \quad (4.12.8)$$

the solution of which including the initial condition at $t = t_1$ according to Eq. 4.12.7 gives

$$n(E, t) = n_1 (E/E_0)^{-\gamma} \exp \left[- \frac{a(t-t_1) v \lambda_0 (E/E_0)^\beta}{L^2} \right]. \quad (4.12.9)$$

Taking account of Eq. 4.12.1, it is easy now to determine the time variations of the particle flux from the source at $t \geq t_1$:

$$I(E, t) \approx \frac{4\pi a}{3} L v \lambda_0 n_1 (E/E_0)^{-\gamma} \exp \left[- \frac{a(t-t_1) v \lambda_0 (E/E_0)^\beta}{L^2} \right]. \quad (4.12.10)$$

It can be seen from Eq. 4.12.10 that the energy spectrum of the particles ejected from the source varies significantly in time. If $\beta > 0$ then at

$$t - t_1 \geq L^2 a v \lambda_0 (E/E_0)^\beta \quad (4.12.11)$$

the exponential factor is already of significant importance and the spectrum becomes even softer in time.

4.12.3. Accelerated particles in the space beyond the stationary sources

Consider some volume of space in the form of a sphere of radius r_0 which contains stationary sources of accelerated particles. Let the total particle input from all sources to the considered volume per unit time be $F(E)$. If the transport scattering

path of particles within this volume is $\bar{\lambda}(E)$ the total diffusive particle flux from the space volume is

$$F_{out}(E) = -4\pi r_o^2 \frac{v\bar{\lambda}}{3} \text{grad}\bar{n}(E, r) \Big|_{r=r_o}, \quad (4.12.12)$$

where $\bar{n}(E, r)$ is the concentration of accelerated particles in the volume beyond the sources. If other losses of particles may be neglected we shall obtain from the balance equation

$$F(E) = -4\pi r_o^2 \frac{v\bar{\lambda}}{3} \text{grad}\bar{n}(E, r) \Big|_{r=r_o}. \quad (4.12.13)$$

Let us consider that approximately

$$\text{grad}\bar{n}(E, r) \Big|_{r=r_o} = -\frac{a_1 \bar{n}(E, r)}{r_o}, \quad (4.12.14)$$

where $a_1 \sim 1$ is the parameter determined by the details of the problem. Then the averaged spectrum (over the considered volume) of accelerated particles in the space beyond the sources will be

$$\langle \bar{n}(E, r) \rangle \approx 3F(E)/4\pi r_o v \bar{\lambda}. \quad (4.12.15)$$

It can be seen from Eq. 4.12.15 that the accelerated particle spectrum in the space beyond the sources may be significantly different both from the particle spectrum in the sources and from the spectrum of the particles ejected from the sources. If $\bar{\lambda}(E) \propto E^{\bar{\beta}}$ we obtain, considering that according to Eq. 4.12.3 $F(E) \propto E^{-\gamma+\beta}$ for relativistic particles with energy E and that according to Eq. 4.12.4 $F(E_k) \propto E_k^{-\gamma+\beta+1/2}$ for non-relativistic particles with energy E_k , that beyond the sources will be

$$\langle \bar{n}(E, r) \rangle \propto E^{-\gamma+\beta-\bar{\beta}}; \quad \langle \bar{n}(E_k, r) \rangle \propto E_k^{-\gamma+\beta-\bar{\beta}}. \quad (4.12.16)$$

4.12.4. The accelerated particle spectrum beyond non-stationary sources

Consider first the case where the flux of particles ejected from their sources may be presented in the form of product of δ -functions, i.e. $F(E)\delta(\mathbf{r} - \mathbf{r}_o)\delta(t - t_o)$. Then

the energy spectrum of particles beyond the source in case of isotropic diffusion will be

$$\bar{n}(E, r, t) = F(E) \left[\frac{4\pi}{3} v(t - t_o) \bar{\lambda}(E) \right]^{-3/2} \exp \left[-\frac{3(\mathbf{r} - \mathbf{r}_o)^2}{4v(t - t_o) \bar{\lambda}(E)} \right]. \quad (4.12.17)$$

For the particles with energy E , the peak at point $\mathbf{r}$ is reached at the moment

$$t_{\max}(\mathbf{r}, E) = t_o + (\mathbf{r} - \mathbf{r}_o)^2 / 4v\bar{\lambda}(E). \quad (4.12.18)$$

It follows from Eq. 4.12.17 that, at first, the accelerated particle spectrum increases rapidly, reaching the value

$$\bar{n}(E, \mathbf{r}, t_{\max}) = F(E) |\mathbf{r} - \mathbf{r}_o|^{-3} \left(\frac{1}{2} \sqrt{\frac{6}{\pi}} \right)^3 \exp(-3/2) \quad (4.12.19)$$

at $t = t_{\max}(\mathbf{r}, E)$, determined by Eq. 4.12.18, and then decreases according to the law $\propto (t - t_o)^{-3/2}$. It can be seen from Eq. 4.12.17 that if $\bar{\lambda}(E) \propto E^{\bar{\beta}}$ (where $\bar{\beta} > 0$), then the accelerated particle spectrum in the space first proves to be at $t < t_{\max}(\mathbf{r}, E)$ more harder than that ejected by the source and then becomes softer and softer. If $t \gg t_{\max}(\mathbf{r}, E)$, and $F(E) \propto E^{-\gamma+\beta}$ in the relativistic energy range, then

$$n(E, \mathbf{r}, t) \propto E^{-\gamma+\beta-(3/2)\bar{\beta}}. \quad (4.12.20)$$

In the non-relativistic energy range, where $F(E_k) \propto E_k^{-\gamma+\beta+1/2}$, we shall obtain at $t \gg t_{\max}(\mathbf{r}, E_k)$, that

$$n(E_k, \mathbf{r}, t) \propto E_k^{-\gamma+\beta-(3/2)\bar{\beta}-1/4}. \quad (4.12.21)$$

If the total particle flux from the source is variable in time, i.e. it may be presented in the form $F(E, t)\delta(\mathbf{r} - \mathbf{r}_o)$, then beyond the source

$$\bar{n}(E, r, t) = \left(\frac{4\pi}{3} v \bar{\lambda}(E) \right)^{-3/2} \int_0^t \frac{F(E, \tau)}{(t - \tau)^{3/2}} \exp \left[-\frac{3(\mathbf{r} - \mathbf{r}_o)^2}{4v(t - \tau) \bar{\lambda}(E)} \right] d\tau \quad (4.12.22)$$

in the case of isotropic diffusion.

4.13. Induction acceleration mechanisms

4.13.1. *The discussion on the problem of induction acceleration mechanisms*

The hypothesis that the CR particles can be accelerated up to very high energies by an electromagnetic mechanism of induction type was first set forth by Swann (1933) as long as more than 70 years ago. He proposed the betatron acceleration mechanism and qualitatively developed this model later in the work (Swann, 1960), in which he studied charged particle acceleration up to the CR energies; he treated particle acceleration as being owed to electromagnetic induction associated with alternating magnetic fields of the stars (the betatron mechanism). For the sake of brevity axial symmetry is considered, in which the magnetic field $\mathbf{H}$ and the relevant vector-potential $\mathbf{U}$ are generated by circular currents around the axis z . If U is independent of r (i.e. $dU/dr = 0$) and the magnetic field component H_z varies as $\propto r^{-1}$ the particle will move along a circular orbit. As the field increases for 10^6 sec from 0 to 2000 Gs inside a circle of radius $r_0 = 10^9$ cm, the particle energy increases up to 3×10^{14} eV. A decrease of the field, however, will result in a particle's deceleration. It has been shown that even if $\partial U / \partial z \neq 0$, a 'deep' trap of the curve $U(r)$ permits the existence of a stable circular orbit on which the particle may gain energy for a long period. Special selection of the function $U(r, t)$ will result in that the trap will shift with time outwards from the axis and disappear at a certain distance from the axis. At that moment the accelerated particles, without being decelerated, will be ejected from the trap along a rapidly unwinding spiral.

A similar concept was developed by Terletsky (1959) who examined the possibility of particle acceleration by the electromagnetic field generated in the vicinities of a rotating body when its rotation axis and the magnetic moment do not coincide as a result of unipolar induction. Swann (1960) noted in the discussion of paper Terletsky (1959) that he had considered this problem many years earlier and concluded that the results of the calculations were dubious owing to an uncertainty of the motion state of the medium surrounding such rotating astronomical body (star or planet). The fact is that if the medium co-rotates with a body, the electromotive forces will not be induced in such medium. The ions may have been accelerated at great distances from the body, but the magnetic field at those distances should be frozen into the plasma.

Therefore only a certain transient region is of interest in this case. It was noted, in numerous works that the induction mechanisms could not be effective because of the high conductivity of the cosmic plasma. In connection with that Swann (1960) noted, that the shielding electrical currents in rarified plasma could not be significant

and that in any case their density was smaller than Nec , which gives $\sim 5 \times 10^{-10} \text{ A/cm}^2$ at $N = 1 \text{ cm}^{-3}$, but such currents could not hamper the particle's energy gain in the induction acceleration mechanism.

4.13.2. Charged particle acceleration up to very high CR energies by rotating magnetized neutron star

Gunn and Ostriker (1969) suggested following induction mechanism of charged particle acceleration by fast rotating neutron star provided maximal energy of accelerated protons up to 10^{21} eV . For determinate it was considered pulsar in the Crab remnant. Its main parameters are supposed as following: the magnetic moment (perpendicular to the axes of rotation) $\mu = 4.17 \times 10^{30} \text{ Gs.cm}^3$, moment of inertia $I = 1.39 \times 10^{45} \text{ g.cm}^2$, quadruple inertia moment $I_q = 6.12 \times 10^{41} \text{ g.cm}^2$, the initial angle velocity $\Omega_o = 1.03 \times 10^4 \text{ sec}^{-1}$. Here the values for I and Ω_o are taken according to model of Hartle and Thorne (1968) for neutron star with the mass $1.4 M_S$ and with the strength of magnetic field on the star's surface $H_p \approx 10^{12} \text{ Gs}$. The value of I_q corresponds to ellipsoidality $\sim 10^{-4}$. Such star will emit quadruple gravitation radiation with frequency 2Ω and magnetic dipole radiation with frequency Ω . The equation of particle moving in this case can be integrated analytically with the solution

$$t = \tau_m \left[(x^2 - 1) - \frac{1}{\eta} \ln \left(\frac{1 + \eta x^2}{1 + \eta} \right) \right], \quad (4.13.1)$$

where

$$\begin{aligned} \tau_m &= \frac{3c^3 I}{\mu^2 \Omega_o^2} \approx 180 \text{ days}; \quad \tau_g = \frac{45c^5 I}{4GI_q^2 \Omega_o^4} \approx 0.16 \text{ days}; \\ x &= \Omega_o / \Omega; \quad \eta = 2\tau_g / \tau_m = 1.78 \times 10^{-3}. \end{aligned} \quad (4.13.2)$$

The gravitation radiation will be predominate during the time

$$t_g = (\tau_m^2 / 2\tau_g) (1 - \ln 2) \approx 81 \text{ years}. \quad (4.13.3)$$

In this period parameters will be change in the following way:

$$\begin{aligned}\Omega &\approx \Omega_o (1 + t/\tau_g)^{-1/4}; \quad H_r \approx H_{ro} (1 + t/\tau_g)^{-3/4}; \quad L_{md} \approx L_{mdo} (1 + t/\tau_g)^{-1}; \\ L_{gq} &\approx L_{gqo} (1 + t/\tau_g)^{-3/2},\end{aligned}\quad (4.13.4)$$

where

$$L_{mdo} = 4.8 \times 10^{45} \text{ erg/sec}; \quad L_{gqo} = 2.7 \times 10^{48} \text{ erg/sec}; \quad H_{ro} = 1.7 \times 10^{11} \text{ Gs}.$$

(4.13.5)

In Eq. 4.13.4 H_r is the strength of magnetic field in the radiation zone; the magnetic field on the surface H_p is supposed to be constant. At the moment of time $t = 915$ years after Supernova explosion the star radiated magnetic dipole and gravitation quadrupole radiation with the power

$$L_{md}(t = 915 \text{ years}) = 5.5 \times 10^{38} \text{ erg/sec}; \quad L_{gq}(t = 915 \text{ years}) = 1.0 \times 10^{38} \text{ erg/sec}.$$

(4.13.6)

For the all time of Crab remnant living it gives $\sim 5 \times 10^{50}$ erg total energy in the low-frequency electromagnetic radiation.

In paper of Gunn and Ostriker (1969) it was not considered more complicated problem on the interactions in the local wave zone $r \leq r_{lc}$, but investigated in details the region $r \geq r_{lc}$ where electromagnetic wave can be considered as spherical. The equation of moving of the test particle with charge Ze and mass m_{ac} in such wave will be

$$\frac{dv_\mu}{d\tau} = \frac{Ze}{m_{ac}c} F_{\mu\nu} v_\nu, \quad (4.13.7)$$

where $F_{\mu\nu}$ is the tensor of electro-magnetic field, and v_μ is the velocity vector of accelerated particle. Let us determine the wave in the point $(0, 0, r_o)$ where particle are injected; then

$$\begin{Bmatrix} E \\ H \end{Bmatrix} = \begin{Bmatrix} (1, 0, 0) \\ (0, 1, 0) \end{Bmatrix} \frac{m_{ac}c}{Ze} \omega_L \sin \Omega t, \quad (4.13.8)$$

where

$$r_{lc} = c/\Omega; \quad \omega_L = ZeH_r/m_{ac}c \quad (4.13.9)$$

are the radius of the wave zone and gyro-frequency, correspondingly. The equations of particle moving are as following:

$$\begin{aligned} \frac{dv_o}{d\tau} &= \frac{\omega_L r_{lc}}{rc} v_1 \sin \left[\Omega \left(t - \frac{r}{c} \right) \right]; \quad \frac{dv_1}{d\tau} = \frac{\omega_L r_{lc}}{r} (cv_o - v_3) \sin \left[\Omega \left(t - \frac{r}{c} \right) \right]; \\ \frac{dv_2}{d\tau} &= 0; \quad \frac{dv_3}{d\tau} = \frac{\omega_L r_{lc}}{r} v_1 \sin \left[\Omega \left(t - \frac{r}{c} \right) \right]. \end{aligned} \quad (4.13.10)$$

For particles which start from the rest the equations of particle moving will be

$$\frac{\sqrt{2}}{3} \frac{d(v_3/c)^{3/2}}{d(\ln r)} = \frac{\omega_L}{\Omega} \sin \left[\Omega \left(t - \frac{r}{c} \right) \right], \quad v_1 = (2v_3c)^{1/2}, \quad v_o = 1 + \frac{v_3}{c}. \quad (4.13.11)$$

Because the value $\omega_L/\Omega > 10^8$ for protons and electrons, the acceleration up to relativistic energies along the direction of the wave propagation will be realized for the time much smaller than the period of the wave. Therefore for the rough estimation can be supposed that $\varphi = \Omega(t - r/c) \approx \text{const} = \varphi_o$. By integrating of Eq. 4.13.11 from r_{lc} up to some cutting radius r_c , the final energy of accelerated particle will be obtained as

$$E_{\text{fin}} = m_{ac} c^2 \left[\frac{\omega_L}{\Omega} \frac{3}{\sqrt{2}} \ln \left(\frac{r_c}{r_{lc}} \right) \sin \varphi_o \right]^{2/3}. \quad (4.13.12)$$

At acceleration of the collective of particles, the energy of the wave will be decrease and their amplitude will $\rightarrow 0$ at $r \approx r_c$, so the final energy will be some smaller. Now it is easy to check the supposition on the constant of the phase. The expected change of the phase will be $\sim (r_c/r_{lc})(\Omega/\omega_L)^{4/3}(\sin \varphi_o)^{-4/3}$, i.e. really very small (excluding the point $\varphi_o = 0$). Therefore, charged particles are accelerated very fast and moved together with electromagnetic wave at constant phase. In the laboratory system of coordinates the test particle continuously gain energy in weekly inclined electric and magnetic fields which directions practically does not change in time. The mechanism of particle acceleration works effectively thanks very low frequency and very big amplitude of the field in the wave (the strength of magnetic field H_r at the basis of wave zone at present time is $\sim 10^6$ Gs, and at $t = 0$ it was $\sim 10^{10}$ Gs). Such non-linear interaction of charged particle with the very low frequency electromagnetic wave was discussed for another limit conditions in papers of Buchsbaum and Roberts (1964) and Jory and Trivelpiece (1968). In difference of these papers, in Gunn and

Ostriker (1969) the parameter ω_L/Ω is so very big that it ensures the moving of charged test particle at the practically constant phase.

Let us consider now the suggestion used in Gunn and Ostriker (1969) that the charged particles moved as in vacuum. This approximation is valid if

$$\gamma_o \omega_L \Omega > \omega_{oe}^2, \quad (4.13.13)$$

where $\gamma_o = E_o/m_{ac}c^2$ at injection, and $\omega_{oe} = (4\pi e^2 N_e/m_e)^{1/2}$ is the plasma frequency. If the Eq. 4.13.13 does not satisfied, the considered above mechanism of charged particles acceleration will work with smaller efficiency.

4.13.3. On the maximal energy of accelerated particles from fast rotated magnetic star

Eq. 4.13.12 for accelerated particles with the mass $m_{ac} = m_p A$ and the charge Ze gives

$$E_{\max} = 1.3 \times 10^{14} A^{1/3} Z^{2/3} \left(\frac{H_p}{10^{12}} \right)^{2/3} \left(\frac{\Omega}{200} \right)^{4/3} \left(\ln \frac{r_c}{r_g} \right)^{2/3} \text{ eV} \quad (4.13.14)$$

at $\sin \varphi_o = 1$. Using the parameters for pulsar in Crab (and suppose that $r_c \approx 0.01$ pc), the maximal energy for protons will be estimated as $E_{\max} \approx 3 \times 10^{15}$ eV and for iron nucleus as $E_{\max} \approx 5.5 \times 10^{16}$ eV. By the extrapolation to $t=0$ the corresponding maximal energies for protons and iron nucleus, will be $\sim 3 \times 10^{17}$ eV and $\sim 10^{19}$ eV.

What theoretical limit there is for the value of $E_{\max}$ in the frame of the considered mechanism? To answer on this question, Gunn and Ostriker (1969) consider some collapse object with the mass M_{col} and radius r_{col} near the Schwarzschild radius $r_g = 2GM_{col}/c^2$. The maximal angle velocity $\Omega_{\max} \approx c/r_g$, and maximal magnetic field on the surface $H_{p\max} \approx G^{1/2} M_{col}/r_g^2$. By using these values from Eq. 4.13.12 follows (without taking into account the logarithmic factor):

$$\frac{E_{\max}}{m_{ac}c^2} \approx \left(\frac{Z^3 e^3}{G m_p^2} \right) \approx 10^{12}, \quad (4.13.15)$$

i.e. does not depends from the mass of collapsing star. Eq. 4.13.15 gives for electrons $E_{\max} \sim 5 \times 10^{17}$ eV, for protons $\sim 10^{21}$ eV, and for iron nucleus $\sim 4 \times 10^{22}$ eV.

4.13.4. On the expected energy spectrum and total flux of accelerated particles from fast rotated magnetic star

If according to Goldreich (1969), let us suppose that the density in the magnetosphere of fast rotated magnetic star is so that the relation $\omega_{oe}^2 = \omega_o \Omega$ is satisfied. In this case the total flux of accelerated particles with charge e vs the time will be

$$\eta(t) = \frac{H_p \Omega^2 r_p^3}{ec} = 2.7 \times 10^{33} \left(\frac{\Omega}{200} \right)^2 \left(\frac{H_p}{10^{12}} \right) \left(\frac{r_p}{10^6} \right)^3 \text{ particles/sec}, \quad (4.13.16)$$

where r_p is the radius of the star. Eq. 4.13.16 gives about 3×10^{37} particles/sec at $\Omega = \Omega_o$ in the case of pulsar in Crab.

Let us estimate the energy spectrum of accelerated particles emitted from the rotated magnetic star for the all time with taking into account the braking of the star's rotation. If the rate of injection is constant at the phase ϕ_o , the mean energy of accelerated particles $\langle E \rangle$ will be about 80% from $E_{\max}$ (Goldreich, 1969). In this case the differential energetic spectrum of accelerated particles ejected into interstellar space will be

$$D(E)dE = \int_0^\infty \eta(t) \left(\frac{d\langle E \rangle}{dt} \right)^{-1} dt dE. \quad (4.13.17)$$

By using Eq. 4.13.4 for $\Omega(t)$ we obtain

$$D(E)dE = \frac{3}{4} \eta_i \tau_g \left(\frac{E_i}{E} \right)^{5/2} \frac{dE}{E_i}, \quad (4.13.18)$$

where τ_g is determined by Eq. 4.13.2, E_i is the initial average energy, and η_i is the initial rate of charged particles injection into acceleration process. The obtained slope 5/2 in the spectrum is in good agreement with observations of CR in high energy range (about 2.6 – 2.7).

Let us note that the interaction of charged particles with strong low-frequency electromagnetic field generated in case of rotation of inclined magnetic dipole has been studied analytically and numerically by Grewing and Heintzmann (1973a,b,c,d). It has been shown that, when reasonable initial conditions for rotating magnetized neutron star are properly selected, the particle may acquire an energy comparable with the highest observed CR energy.

4.14. Particle acceleration by moving magnetic piston

As a result of the ejection of magnetized plasma (for example, during the chromosphere flares, coronal ejections, during the explosions of Novae and Supernovae) the particles will be reflected from such a condensation (let us call it a magnetic piston) and the particle energy will increase or decrease depending on the type of an acting collision: head-on or overtaking collision.

4.14.1. Acceleration and deceleration at a single interaction of particles with magnetic piston

Let a magnetic piston have a thickness l , the intensity of a magnetic field in it be H , and let it move with a velocity u ($u \ll c$). If the angle between the velocity $\mathbf{v}$ of a particle's motion and the piston velocity $\mathbf{u}$ is φ , then the particle will be reflected from the piston having a Larmor radius inside the piston:

$$r_L \leq l / (1 - \sin \varphi). \quad (4.14.1)$$

Since a particle's velocity in the coordinate system related to the piston is $\mathbf{v} - \mathbf{u}$, then

$$r_L = \frac{cm_{ac}|\mathbf{v} - \mathbf{u}|}{ZeH} \left(1 - \frac{|\mathbf{v} - \mathbf{u}|^2}{c^2} \right)^{-1/2}. \quad (4.14.2)$$

The Eq. 4.14.1 and Eq. 4.14.2 result in that the piston not reflecting the particles which have the velocities $v > v_{cr}$, where

$$v_{cr} = u \cos \varphi + \left\{ \frac{(ZeH)^2}{m_{ac}^2 (1 - \sin \varphi)^2 \left(m_{ac}^2 c^4 (1 - \sin \varphi)^2 + (ZeH)^2 \right)} - u^2 \sin^2 \varphi \right\}^{1/2} \quad (4.14.3)$$

At a reflection when $v \leq v_{cr}$, a relative energy variation of a particle, according to Sections 2.2–2.5, will be

$$\frac{\Delta E}{E} = -\frac{2uv \cos \varphi}{c^2} + \frac{2u^2}{c^2}. \quad (4.14.4)$$

If $v \geq v_{cr}$, the particle will pass through a magnetic piston. In this case, it will be scattering at the angle θ which is determined by the equation:

$$\frac{l}{\sin(\varphi - \theta - \pi/2)} = 2r_L \sin \frac{\theta}{2}, \quad (4.14.5)$$

where r_L is determined according to Eq. 4.14.2. In the case of scattering at the small angles $\theta \ll 1$, the equation results in

$$\theta \approx 2 \arcsin \left(-\frac{l \cos \varphi}{2r_L} \right). \quad (4.14.6)$$

The energy variation in this case is determined according to Sections 4.3–4.5:

$$\frac{\Delta E}{E} = 2 \sin^2 \frac{\theta}{2} \left(-\frac{2uv \cos \varphi}{c^2} + \frac{2u^2}{c^2} \right) \approx \frac{l^2 \cos^2 \varphi}{r_L^2} \left(-\frac{2uv \cos \varphi}{c^2} + \frac{2u^2}{c^2} \right). \quad (4.14.7)$$

It should be noted that Eq. 4.14.4 and Eq. 4.14.6 hold true also at large values of u . For non-relativistic particle energies the relative variation of kinetic energy $\Delta E_k/E_k$ will be considerable even in a single reflection:

$$\frac{\Delta E_k}{E_k} = \begin{cases} \frac{4u}{v} \left(-\cos \varphi + \frac{u}{2v} \right) & \text{if } v \leq v_{cr}, \\ \frac{4u}{v} \frac{l^2 \cos^2 \varphi}{r_L^2} \left(-\cos \varphi + \frac{u}{2v} \right) & \text{if } v > v_{cr}. \end{cases} \quad (4.14.8)$$

The Eq. 4.14.7 results in $\Delta E_k/E_k$ being able to be very large at $v \sim u$. In the range of super-relativistic energies

$$\Delta E/E \approx -2u \cos \varphi / c \quad (4.14.9)$$

at the particle reflection from a magnetic piston and

$$\Delta E/E \approx -2ul^2 \cos^3 \varphi / c r_L. \quad (4.14.10)$$

when a particle crosses it. The Eq. 4.14.9 and Eq. 4.14.10 show that the relative energy variation is not large, of the order of u/c , in the case of relativistic energies.

4.14.2. Acceleration and deceleration of particles at the multiple interactions with magnetic piston

In the presence of scattering medium behind or/and before a magnetic piston, a multiple interaction of particles with a piston will take place and particle energy

variation will be very significant. In this case a share of particles interacting with a piston will be pronouncedly decreased with increase of the multiplicity of interaction. Therefore the differential energy spectrum of accelerated particles will fall down with a growth of particle energy; the detailed form of a spectrum will be determined by the probability dependence for a given multiplicity of particle interaction with a piston. The same conclusion can also be drawn for a dependence of the relative share of decelerated particles $\Delta n/n$ on the particle energy - it should also be decreased with a growth of particle energy. Some examples of particle acceleration and deceleration in the process of multiple interactions with a magnetic piston will be discussed below.

4.15. Mechanisms of particle acceleration by shock waves and other moving magneto-hydrodynamic discontinuities during a single interaction

The particle acceleration by the moving magneto-hydrodynamic discontinuities is probably one of the accelerated processes which are most frequent in the space (in the solar atmosphere, in interplanetary space and in the magnetospheres of the planets, in the Galaxy, etc.). The mechanism of acceleration by the transverse and oblique shock fronts for normal and oblique incidence of particles, including the scattering by magnetic inhomogeneities of medium has been most comprehensively developed (Dorman and Freidman, 1959; Shabansky, 1961, 1966; Schatzman, 1963; Korobeinikov and Lomnev, 1964; Alekseev and Kropotkin, 1970; Vasilyev et al., 1978; and others).

4.15.1. Acceleration for single passage of a laterally incident particle (the shock front is unlimited)

Consider a shock wave in a medium with a frozen magnetic field parallel to the shock front plane. Let the shock front move at velocity u_1 . In undisturbed space 1 the field intensity is H_1 , in disturbed space 2 moving at velocity $u_1 - u_2$ relative to the rest system, the field intensity is H_2 (see Fig. 4.15.1). A particle moving in undisturbed space 1 will collide with the magnetized shock front, be reflected from the front, and gain an additional momentum as in a head-on collision with mirror. Then the particle will again collide with the front, etc. After a while, however, the process will stop due to the particle drift to undisturbed space 2 behind the front. In addition to that, the drift along the front takes place owing to the difference between H_1 and H_2 . In this case if the front is limited, the acceleration may stop even earlier, before the particle is completely transferred to space 2. The calculations carried out by Dorman and Freidman (1959) for the case of an infinite front show that such a mechanism may give a considerable increase of the particle's energy. In Dorman and Freidman (1959) the particle acceleration was estimated, on the assumption of normal particle incidence onto the front.

In the coordinate system relative to the shock front (as in Fig. 4.15.1, in which the shock front is in plane yz , the magnetic field along z axis), the particle motion will be as follows. In space 1 a charged particle affected by magnetic field $\mathbf{H}_1$ and electric field $\mathbf{E}_1 = -\frac{1}{c}\mathbf{u}_1 \times \mathbf{H}_1$ will drift at velocity u_1 towards the front. Near the front the particle is affected by the difference in H_1 and H_2 , and will drift also along the front towards the Y axis (i.e. along the electric field $\mathbf{E}_1 = \mathbf{E}_2$); on traversing the shock front plane the particle will drift in disturbed space at a velocity u_2 .

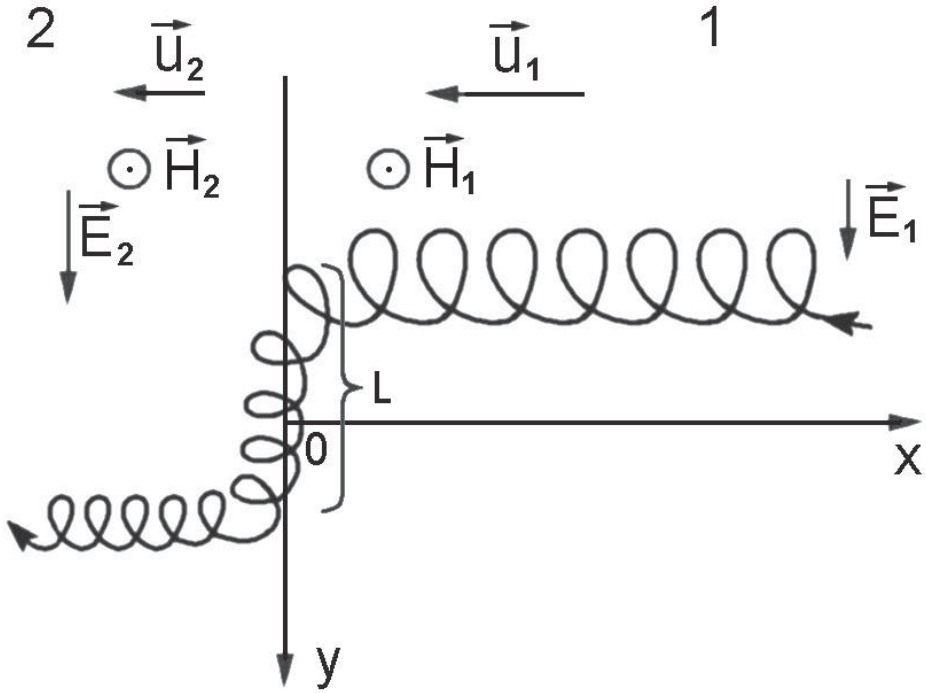


Fig. 4.15.1. Charged particle trajectory in shock wave (coordinate system related to the wave front). According to Dorman and Freidman (1959).

Let the particle move in undisturbed space at velocity v_0 perpendicular to the magnetic field. In a fixed coordinate system the particle will then move along a spiral with curvature radius $r_{L1} = cp_0 / ZeH_1$ and frequency $\omega_{L1} = ZecH_1 / E_0$. Here Ze is the particle charge, p_0 is the initial momentum, E_0 is the total initial energy of the particle. When the front approaches the particle, the drift will be toward the Y axis and the particle energy will change by

$$\Delta E = Ze l |\mathbf{E}| = Ze l u_1 H_1 / c, \quad (4.15.1)$$

where l is the drift along the y axis. Let us estimate l . The shift during a single cycle will be

$$\Delta l = 2(r_{g1} - r_{g2}) = \frac{2cp_o(\sigma - 1)}{ZeH_1\sigma}, \quad (4.15.2)$$

where $\sigma = H_2/H_1 = u_1/u_2$ is the degree of the compression of transverse magnetic field in the shock wave. We assume here that the particle energy is almost invariable during a single cycle. The time of a single cycle will be

$$\Delta t = \frac{\pi}{\omega_{g1}} + \frac{\pi}{\omega_{g2}} = \frac{\pi E_o}{Zec} \left(\frac{1}{H_1} + \frac{1}{H_2} \right) = \frac{\pi E_o(\sigma + 1)}{ZecH_1\sigma}, \quad (4.15.3)$$

whence the drift velocity in the direction of the Y axis is

$$u_y = \frac{\Delta l}{\Delta t} \approx \frac{2c^2 p_o(\sigma - 1)}{\pi E_o(\sigma + 1)}. \quad (4.15.4)$$

Considering that the particle moves across the shock front plane at velocity u_1 in space 1 and u_2 in space 2, we shall obtain that the time of particle drift along the y axis will be

$$\Delta t = \frac{r_{g1}}{u_1} + \frac{r_{g2}}{u_2} = \frac{cp_o}{ZeH_1u_1} + \frac{cp_o}{ZeH_2u_2} = \frac{2cp_o}{ZeH_1u_1}. \quad (4.15.5)$$

From this the shift along the Y axis is

$$l = u_y \Delta t = \frac{4c(cp_o)^2(\sigma - 1)}{\pi E_o ZeH_1u_1(\sigma + 1)}, \quad (4.15.6)$$

and according to Eq. 4.15.1, the energy change will be

$$\Delta E = \frac{4(cp_o)^2(\sigma - 1)}{\pi E_o(\sigma + 1)}. \quad (4.15.7)$$

It is of importance to emphasize that, according to Dorman and Freidman (1959), the energy increase is dependent on neither the particle's charge, nor the shade wave speed, nor the magnetic field intensity (this is associated with the fact that the increases of the intensity of field resulting in a more intensive drift and in a more rapid ejection of particles from the acceleration region, so that the total effect remains the same). The particle acceleration effect is eventually determined by the parameter $\sigma = H_2/H_1 = u_1/u_2$, i.e., the degree of enhancement of the transverse magnetic field and compression of medium during shock wave movement.

It follows from Eq. 4.15.7 in the non-relativistic case, when $E_o \approx m_{ac}c^2$, that

$$\Delta E_k / E_{ko} \approx 8(\sigma - 1) / \pi(\sigma + 1), \quad (4.15.8)$$

and in the ultra-relativistic case when it may be assumed that $cp_o \approx E_o$ we get

$$\Delta E / E_o \approx 4(\sigma - 1) / \pi(\sigma + 1). \quad (4.15.9)$$

4.15.2. Acceleration in a single passage of a transversely incident particle (the shock front is limited)

If the shock wave front is limited and its size is L , the particle may be ejected from the zone of shock wave acceleration even earlier than it can drift from region 1 to region 2 shown in Fig. 4.15.1. In this case the maximum energy that the particle may acquire will be

$$\Delta E_{\max} = \frac{Zeu}{c} HL, \quad (4.15.10)$$

where u is the movement velocity of the front, H is the field intensity in the front. Thus in case of a limited shock front the particle energy gain will be determined either according to Eq. 4.15.7 if $\Delta E \leq \Delta E_{\max}$, or otherwise by Eq. 4.15.10.

4.15.3. Exact integration of the particle motion equations for an oblique incidence of a non-relativistic particle onto a shock front

The above approximate estimates are concordant with the results obtained by Shabansky (1961) for trajectory calculations for a particle with the initial momentum p_o which is incident at some angle φ_o to a front normal. It was found that if $\varphi_o > 0$ the ratio p/p_o is not high (≤ 1.5); p/p_o increases rapidly and approaches the value ~ 4 at $\varphi_o = -\pi/4$. The maximum acceleration corresponding to $p/p_o \approx 5.23$ is realized at $\varphi_o = -\pi/2$.

4.15.4. Particle acceleration by a transverse shock wave at $v \gg u$ in general case (including oblique incidence of particles)

Such a problem was investigated by Shabansky (1966). Let us consider a plane hydromagnetic shock wave propagating in the direction normal to the magnetic field (a transverse shock wave). Let all the quantities with index '1' be related to those before the front, and after behind the front they have index '2'. An energetic particle with a velocity which is assumed to be far more than the front velocity passes from the medium 1 to the medium 2. During this transition a particle occurs alternatively in the regions 1 and 2 moving along the arcs of circles in the coordinate systems which are motionless relative to the media 1 and 2, respectively. Because in this case the velocity parallel to the magnetic field does not vary, we limit a priori our consideration by the motion of an energetic particle only in the plane normal to the field. In the general case this is equivalent to a consideration in the coordinate system moving with the velocity $v_{//}$ along the field.

At the successive passages of the regions 1 and 2, the arc of a particle's trajectory in the region 1 will be decreased and in the region 2 it will be increased until it will be equal to 2π and a particle will be always in the region 2 behind the wave front. If the angle between a particle velocity $\mathbf{v}$ and the normal to the front (directed from the medium 1 to 2) in the moment of transition is φ , the central angle θ of an arc of a particle motion in the medium 2 will be related to φ as follows: $\theta = \pi + 2\varphi$. In this case the angles φ and θ vary within the limits $-\pi/2 \leq \varphi \leq \pi/2$, $0 \leq \theta \leq 2\pi$. The front's displacement relative to the medium 2 during the time of a particle's motion in the medium 2 is, on the one hand, $\Delta x_2 = u_2 \Delta t_2 = u_2 (\theta_2 / \omega_{L2})$ and, on the other hand, $\Delta x_2 = r_{L2} \Delta \varphi_2 \cos \varphi_2$, where u_2 is the front velocity relative to the medium 2, $r_{Li} = pc / ZeH_i$ ($i = 1, 2$) is the Larmor radius, $\omega_{Li} = ZeH_i / m_{ac}c$ is the frequency of a particle Larmor rotation. Then the change of the angle φ and the front displacement relative to the medium 2 during a particle passage through the medium 2 is equal to

$$\Delta \varphi_2 = \frac{u_2}{v} \frac{\pi + 2\varphi}{\cos \varphi}, \quad \Delta x_2 = \frac{r_{g2} u_2}{v} (\pi + 2\varphi). \quad (4.15.11)$$

Similarly to this, during a particle's passage through the medium 1 the front displacement relative to the medium 1 $\Delta x'_1 = u_1 \Delta t_1 = u_1 (\theta_1 / \omega_{g1}) = u_1 (\pi + 2\varphi_1) / \omega_{g1}$ and on the other hand, $\Delta x'_1 = r_{g1} \Delta \varphi_1 \cos \varphi_1$. The front displacement relative to the medium 2 will be $\Delta x_1 = \Delta x'_1 (u_2 / u_1)$, where u_1 is the front velocity in the medium 1 and φ_1 is the angle between the front normal (directed from the medium 2 to medium 1) and a particle velocity. Since $\varphi_1 = -\varphi$, the angle variation $\Delta \varphi_1$ and the front

displacement relative to the medium 2 during a particle passage through the medium 1 are

$$\Delta\varphi_1 = \frac{u_1}{v} \frac{\pi - 2\varphi}{\cos\varphi}, \quad \Delta x_1 = \frac{r_{g1} u_1}{v} (\pi - 2\varphi). \quad (4.15.12)$$

Together with the angle variation described by Eq. 4.15.11 and Eq. 4.15.12 owed to finiteness of particle motion in the media 2 and 1, the angle will be increased by $\Delta\varphi'$ which is related to a change of a particle's momentum at the reflection from the medium 2. Normal to the front plane component of a momentum, $p_{\perp} = p \cos\varphi$, will be increased by $\Delta p_{\perp} = 2(u_2 - u_1)E/c^2$ and the longitudinal component $p_{\parallel} = p \sin\varphi$ will have $\Delta p_{\parallel} = 0$. Here E and p are the total energy and the momentum of a particle, c is the velocity of light. From the equations

$$\Delta p \sin\varphi + p \Delta\varphi_o \cos\varphi = 0, \quad \Delta p \cos\varphi - p \Delta\varphi_o \sin\varphi = 2(u_1 - u_2)E/c^2, \quad (4.15.13)$$

which have been obtained by differentiation of these expressions, we shall find $\Delta\varphi_o$ and the momentum variation Δp :

$$\Delta\varphi_o = -2(u_1 - u_2)E \sin\varphi / pc^2 = -2(u_1 - u_2) \sin\varphi / v, \quad (4.15.14)$$

$$\Delta p = 2(u_1 - u_2)E \cos\varphi / c^2. \quad (4.15.15)$$

The total increment of the angle during a circle is composed by the increments described by Eq. 4.15.11, 4.15.12 and 4.15.14 $\Delta\varphi = \Delta\varphi_1 + \Delta\varphi_2 + \Delta\varphi_o$ and is equal to

$$\Delta\varphi = \frac{u_1}{v \cos\varphi} \left[(\pi - 2\varphi) + (\pi - 2\varphi) \frac{u_2}{u_1} - 2 \left(1 - \frac{u_2}{u_1} \right) \sin\varphi \cos\varphi \right]. \quad (4.15.16)$$

Let us now determine a displacement of the instantaneous center of rotation during one revolution. As is seen from the expression for the Larmor radius $\mathbf{r}_g = (c/ZeH^2)[\mathbf{p}\mathbf{H}]$ a displacement in the direction of the wave propagation $\Delta x = (c/ZeH^2)[\Delta\mathbf{p}\mathbf{H}_2]_x$ is equal to zero at the transition through the front (since Δp is normal to the front).

One can neglect the angle variation $\Delta\varphi$ and a change of position of the front itself during a revolution when determining a displacement of the instantaneous

rotation center in the front plane in the direction normal to the field. A displacement of the instantaneous center is

$$\Delta\varphi = 2(r_1 - r_2)\cos\varphi = 2r_2\left(\frac{u_1}{u_2} - 1\right)\cos\varphi. \quad (4.15.17)$$

Dividing Eq. 4.15.15 by Eq. 4.15.16 and passing to the limit $\Delta p/\Delta\varphi \rightarrow dp/d\varphi$ we obtain the equation for a particle momentum

$$dp/d\varphi = pf(\varphi), \quad (4.15.18)$$

where

$$f(\varphi) = \frac{2\cos^2\varphi}{(2\pi/\alpha_2) + \pi - 2\varphi - 2\sin\varphi\cos\varphi}; \quad \alpha_2 = \frac{H_2}{H_1} - 1, \quad (4.15.19)$$

and where the evident equalities have been used:

$$\frac{r_{g1}}{r_{g2}} = \frac{H_2}{H_1} = \frac{u_1}{u_2} = \frac{\omega_{g2}}{\omega_{g1}} = \frac{\rho_2}{\rho_1} \quad (4.15.20)$$

including the condition of frozenness (ρ is a density). Integrating Eq. 4.15.18 we have

$$\frac{p}{p_1} = \exp\left(\int_{-\pi/2}^{\varphi} f(\varphi)d\varphi\right) = \left(\frac{H_2}{H_1}\right)^{1/2} \left[1 + \frac{\alpha}{2\pi}(\pi - 2\varphi - \sin 2\varphi)\right]^{-1/2}. \quad (4.15.21)$$

The lower limit of integration corresponds to the initial angle of entry into the region 2 behind the front in the first instant of particle contact with the front, i.e. $p = p(-\pi/2)$ is the momentum before the wave passage. Eq. 4.15.21 results in that after crossing the shock front ($\varphi = \pi/2$), the momentum $p_2 = p(\pi/2)$ is determined by the relation:

$$p_2/p_1 = (H_2/H_1)^{1/2}, \quad (4.15.22)$$

which is the expression for the law of magnetic moment conservation, i.e. $p^2/H = \text{const}$. Concrete properties of the medium parameter variations in the shock wave have not been used in the deduction of the Eq. 4.15.18–4.15.22. The only limitation for the field discontinuity have come to the condition of the conservation

of magnetic flux per unit volume at the crossing of a discontinuity surface: $u_1 H_1 = u_2 H_2$. Therefore the law of conservation of magnetic moment holds for any field discontinuity with this condition.

The law of magnetic moment conservation (see Eq. 4.15.22) having been deduced exactly is a result of the assumption that the front velocity u_1 is infinitely small compared with a particle's velocity v . Actually the Eq. 4.15.22 holds with the accuracy to the terms of the order of u_1/v as compared to unit. The matter of problem is that if a particle makes a finite number of revolutions during when it is on the front (u_1/v is not an infinitely small quantity), the magnetic moment is not conserved according to the general theory of a violation of adiabatic invariants in the presence of forces undergoing a discontinuity of any derivative. If a particle has time only for a single crossing of the shock front (u_1 is comparable with v), the momentum increment $\Delta p = (u_1 - u_2) E \cos \varphi / c^2$ is positive for head on ($-\pi/2 \leq \varphi \leq \pi/2$) and negative for overtaking collisions. In this case a magnetic moment is not conserved. It is evident that the less is a particle's velocity the less is the difference of its behavior on the front from the behaviors of thermal particles which are heated non-adiabatically on the shock front.

4.15.5. Particle acceleration by oblique shock waves

The acceleration of energetic particles on the front of an oblique shock wave have been analyzed by Alekseev and Kropotkin (1970). The particles with a Larmor radius r_L which is large compared to a discontinuity thickness of d were considered as energetic particles. It was assumed that the motion of such particles near the plane of discontinuity ($x = 0$) is determined by magnetic field which is uniform in each of semi-spaces (1 at $x > 0$ and 2 at $x < 0$). The collisions for discontinuities in collisional plasma are neglected; the latter assumption was argued for the particles with a free path $\lambda \gg r_g \gg d$. Since d is of the order of the free path length of the thermal plasma particles, and the free path of a given particle is increased with its energy, the above inequality will hold for the particles the energy of which is large compared to the thermal energy. It was assumed for collisionless discontinuities that an interaction of the particles under consideration with micro-fields (forming a jump of the magnetic field) is insignificant compared to the interaction with regular macroscopic magnetic field; this appears to be true if $r_g \gg d$. Particle interaction with an oblique shock front is essentially different from the interaction with a wave in purely transverse field (the normal field component $H_n = 0$) considered above. Note that there always exists the coordinate system in which an electric field $\mathbf{E} = 0$ (Landau and Lifshitz, M1957). In this coordinate system a particle trajectory consists from parts of spiral lines occurring by turns either in the region 1 with the magnetic field $\mathbf{H}_1$ (having the components $H_{x1} = H_n$, $H_{y1} = 0$, $H_{z1} = H_{t1}$) and in region 2

with the field $\mathbf{H}_2$ (its components are $H_{x2} = H_n$, $H_{y2} = 0$, $H_{z2} = H_{t2}$). A trajectory is determined by the following two parameters: pitch angle θ and the phase φ of Larmor precession in any point of crossing the front plane (the phase φ is counted from a normal to the force line which is directed from the space 1 to 2). Let $H_n/H_{t1} = \text{tg}\alpha$ and $H_n/H_{z2} = \text{tg}\beta$ be a tilt of the force lines to the front plane in the regions 1 and 2, respectively. Then θ_1 and φ_1 are related to θ_2 and φ_2 (the subscript indicates to which of the regions the variables θ and φ are related) by the condition of the continuity of the velocity at the crossing of a shock front

$$\begin{aligned}\cos\theta_2 &= \cos\gamma\cos\theta_1 + \sin\gamma\sin\theta_1\sin\varphi, \quad \sin\theta_2\cos\varphi_2 = \sin\theta_1\cos\varphi_1, \\ \sin\theta_2\sin\varphi_2 &= -\sin\gamma\cos\theta_1 + \cos\gamma\sin\theta_1\sin\varphi_1,\end{aligned}\quad (4.15.23)$$

where $\gamma = \alpha - \beta$. Furthermore, the phases of successive crossings (m and $m + 1$) of the front plane are connected by the relation:

$$\cos\varphi_{1,m+1} - \cos\varphi_{1,m} = \text{tg}\alpha\text{tg}\theta_1(\varphi_{1,m} - \varphi_{1,m+1}) \quad (4.15.24)$$

for the region 1 and by a similar relation for the region 2. For simplicity we shall assume below that the angles α and β are small. For an increments of a pitch angle and a phase per one revolution we then obtain

$$\Delta\theta = 2\gamma\sin\varphi, \quad \Delta\varphi\sin\varphi = \text{ctg}\theta[2\pi\alpha - \gamma(2\varphi - \sin 2\varphi)]. \quad (4.15.25)$$

The variations of parameters θ and φ along a particle trajectory determine a certain dependence of θ which is governed by the equation

$$\text{ctg}\theta d\theta = \frac{2\gamma\sin\varphi d\varphi}{2\gamma(\pi - \varphi) + \gamma\sin 2\varphi + 2\pi\beta}. \quad (4.15.26)$$

The solution of Eq. 4.15.26 is

$$\sin^2\theta = \frac{2\pi\alpha\sin^2\theta_o}{2\pi\alpha - 2\gamma\varphi + \gamma\sin 2\varphi}, \quad (4.15.27)$$

where θ_o is the pitch angle for the first crossing of the front. The phase φ varies from 0 to π at the transition from the region 1 to 2 and conversely otherwise. The Eq. 4.15.27 means that the magnetic flux enclosed by a particle during one revolution is constant. The particles moving from a region of weaker magnetic field with the pitch

angle $\theta_o > \theta_{cr}$ ($\sin^2 \theta_{cr} = \beta/\alpha$ at $\alpha > \beta$) are reflected from the plane of the shock front with conservation of the magnetic moment

$$\mu = m_{ac} v^2 \sin^2 \theta / 2H = m_{ac} v^2 \sin^2 \theta_k / 2H, \quad (4.15.28)$$

where the pitch angle after coming from the front is $\theta_k = \pi - \theta_o$. The rest particles pass through the front changing their pitch angles according to the conservation of the magnetic moment. It is possible to determine the trajectory in the front plane by using the integrals of motion

$$I_1 = p_y + \frac{Ze}{c} \left(\frac{|x|+x}{2} H_{t1} + \frac{x-|x|}{2} H_{t2} + H_n z \right), \quad (4.15.29)$$

$$I_2 = p_z + \frac{Ze}{c} H_n y, \quad (4.15.30)$$

where p_y and p_z are the momentum components of a particle. In the instant when a particle crosses the plane of the shock front $p_y = -p \sin \theta \cos \varphi$, $p_z = p \cos \theta$ (the terms $\sim \alpha$ are omitted). Putting $x = 0$ and neglecting the rapid oscillations of a particle in the direction normal to the front, we obtain from Eq. 4.15.29 and 4.15.30:

$$y - y_o = r_g (\cos \theta_o - \cos \theta), \quad z - z_o = r_g (\sin \theta_o \cos \varphi_o - \sin \theta \cos \varphi). \quad (4.15.31)$$

For definiteness we consider that a particle is positively charged. The quantity $r_g = pc/ZeH_n$ is the Larmor radius in the field H_n ; y_o and z_o are the particle's coordinates at the first crossing of the front; θ_o and φ_o are the initial pitch-angle and phase; θ and φ in Eq. 4.15.31 are determined by Eq. 4.15.27. The total displacement in the yz -plane (during the time when a particle is near the front) will be

$$\Delta y = r_L (\cos \theta_o - \cos \theta_k), \quad \Delta z = r_L (\sin \theta_o - \sin \theta_k). \quad (4.15.32)$$

For particle reflection Eq. 4.15.32 gives

$$\Delta y = 2r_L \cos \theta_o, \quad \Delta z = r_L \sin \theta_o. \quad (4.15.33)$$

In the case of small angles α and β Eq. 4.15.25 are complicated. The variations $\Delta \theta$ and $\Delta \varphi$ during one revolution are now not small. The pitch angle θ_k and φ_k of a

particle leaving the plane of the shock front are determined by the successive solutions of the Eq. 4.15.23 and Eq. 4.15.24. In this case θ_k also depends on the initial pitch angle θ_o and on the phase φ_o . A particle's magnetic moment is not conserved after passing through the front. However, the general character of the motion remains the same. The particles moving from the region of a weaker field will be reflected from the front if their pitch angle is more than θ_{cr} depending on the initial phase φ_o . The other particles will cross the front.

On the basis of the results considered, Alekseev and Kropotkin (1970) have calculated the expected particle acceleration. The matter of the problem is that in any coordinate system, except for the single one in which $[\mathbf{uH}] = 0$, acceleration will take place because there is a uniform electric field parallel to the plane of discontinuity. When crossing this plane the leading center of a particle's motion moves along the electric field so that there is a corresponding change of the particle's energy (see for comparison Section 4.15.1).

Let us pass to the coordinate system K' moving relative the initial frame with the velocity $u = cE/H_n$ along the z -axis (the z component of a magnetic field has a jump at the plane of discontinuity but the other two components are continuous). The electric field E' in the system K' is equal to zero; $H_{z1} = H_{zt}$ and $H'_{z2} = H_{z2}$; the magnetic field component parallel to the velocity $\mathbf{u}$ does not vary and is normal to the plane of discontinuity $H'_n = \left(H_n^2 - |\mathbf{E}|^2\right)^{1/2}$. At $|\mathbf{E}| \approx H_n$ the angle α' of force line tilt to the front plane will be small in the K' system even if the initial angle $\alpha \sim 1$. A particle trajectory in the K' system has been described above. The turn transition to the initial coordinate system K makes it possible to obtain the particle trajectory in the presence of an electric field.

The change of kinetic energy ΔE_k at the crossing of the front is given by the equation

$$\Delta E_k = Ze|\mathbf{E}|\Delta y = \frac{u\Delta p'_z}{\left(1 - u^2/c^2\right)^{1/2}}, \quad (4.15.34)$$

where Δy is determined by Eq. 4.15.32 in the case of oblique shock wave.

4.15.6. Particle acceleration by rotational discontinuities

Alekseev and Kropotkin (1970) have also considered the trajectories of motion and acceleration of energetic particles in the vicinity of moving rotational discontinuity. Near the plane of a rotational discontinuity a magnetic field has the components:

$$H_{x1} = H_n, \quad H_{y1} = H_t \cos \Psi / 2, \quad H_{z1} = H_t \sin \Psi / 2 \quad \text{for } x > 0; \quad (4.15.34a)$$

$$H_{x2} = H_n, \quad H_{y2} = H_t \cos \Psi / 2, \quad H_{z2} = H_t \sin \Psi / 2 \quad \text{for } x < 0; \quad (4.15.34b)$$

Here Ψ is the angle of the turn of the field's vector at the discontinuity. Similar to the case of an oblique shock wave, the boundary conditions (satisfied on the discontinuity surface) result in the existence of the coordinate system where $\mathbf{E} = 0$ (see, for example, Landau and Lifshits, M1957). In the case of a rotational discontinuity, assuming that the tilt angle of magnetic force lines is small, we obtain the following expressions (instead of Eq. 4.15.23):

$$\begin{aligned} \cos \theta_2 &= \cos \Psi \cos \theta_1 + \sin \Psi \sin \theta_1 \cos \varphi_1 + \alpha(1 - \cos \Psi) \sin \theta_1 \sin \varphi_1; \\ \sin \theta_2 \cos \varphi_2 &= -\sin \Psi \cos \theta_1 + \cos \Psi \sin \theta_1 \cos \varphi_1 - \alpha \sin \Psi \sin \theta_1 \sin \varphi_1; \\ \sin \theta_2 \sin \varphi_2 &= -\alpha(1 - \cos \Psi) \cos \theta_1 + \alpha \sin \Psi \cos \theta_1 + \sin \theta_1 \sin \varphi_1. \end{aligned} \quad (4.15.35)$$

Using Eq. 4.15.35 and Eq. 4.15.24) we obtain increments of pitch angle θ and phase φ during one revolution:

$$\begin{aligned} \Delta \theta &= 2\alpha[(1 - \cos \Psi) \sin \Psi - \tilde{\varphi} \sin \varphi (\sin \Psi \cos \varphi + \cos \Psi \operatorname{ctg} \theta)]; \\ \Delta \varphi \sin \varphi &= \alpha \left\{ \operatorname{ctg} \theta [2\alpha(\pi - \varphi) + (1 - \cos \Psi) \sin 2\varphi] - 2 \sin \Psi \sin \varphi \right. \\ &\quad \left. + \tilde{\varphi} [2 \operatorname{ctg} \theta (\cos^2 \Psi - \sin^2 \Psi \cos^2 \varphi) + \sin 2\Psi \cos \varphi (1 - \operatorname{ctg}^2 \theta)] \right\}, \end{aligned} \quad (4.15.36)$$

where

$$\tilde{\varphi} = \arctg \left(\cos \Psi \operatorname{ctg} \varphi - \sin \Psi \frac{\operatorname{ctg} \theta}{\sin \varphi} \right). \quad (4.15.37)$$

Applying Eq. 4.15.36 it is possible to obtain the differential equation which determines the relation of pitch angle and phase along a trajectory. The solution of this equation is

$$\begin{aligned} \sin^2 \theta [2(\pi - \tilde{\varphi} - \varphi) + (1 - \cos \Psi) \sin 2\varphi + \sin \Psi \sin \varphi \operatorname{ctg} \theta \\ + 2\tilde{\varphi} \sin^2 \Psi (\operatorname{ctg}^2 \theta - \cos^2 \varphi - 2 \operatorname{ctg} \Psi \cos \Psi \operatorname{ctg} \theta)] = C, \end{aligned} \quad (4.15.38)$$

where C is a constant of integration. Similarly to the Eq. 4.15.27, the Eq. 4.15.38 implies the conservation of the magnetic flux Φ which is enclosed by a particle during one revolution. A particle's behavior is essentially dependent on Ψ . At $\Psi = \pi/2$ the particles moving from the top ($x > 0$) cross the plane of discontinuity, and conservation of Φ means that θ_k (the pitch angle of the particles coming down

to $x < 0$) is equal to the initial pitch-angle. For moving particles upward, conservation of θ_k does not require any restriction of the final pitch angle. For $\pi/2 < \Psi < \pi$ a portion of the particles of the upper region with pitch angle $0 < \theta_o < \Psi - \pi/2$ will cross the discontinuity with conservation of their pitch angle, and another part of them will be reflected returning to the top region with the pitch angle $\theta_k = \Psi - \theta_o$. If $\Psi - \pi/2 < \theta_o < \pi/2$ a particle will cross the discontinuity plane and Eq. 4.15.38 results in θ_k be equal either to θ_o or to $\pi - \theta_o$. The particles moving from below with pitch-angle $\Psi < \theta_o < \pi$ will cross the discontinuity ($\theta_k = \theta_o$) but if $\pi/2 < \theta_o < \Psi$ a partial reflection of the particles into lower half space ($\theta_k = \Psi - \theta_o$) will occur and a fraction of particles will cross the discontinuity plane ($\theta_k = \theta_o$).

Using the integrals of motion in the field of a rotational discontinuity

$$I_1 = p_y + \frac{ZeH_n}{c} \left(z + |x| \operatorname{ctg} \alpha \sin \frac{\Psi}{2} \right); \quad I_2 = p_z + \frac{ZeH_n}{c} \left(y - x \operatorname{ctg} \alpha \cos \frac{\Psi}{2} \right), \quad (4.15.39)$$

and the fact that at the crossing of a discontinuity plane (at $x = 0$):

$$\begin{aligned} p_y &= p \left(\cos \frac{\Psi}{2} \cos \theta - \sin \frac{\Psi}{2} \sin \theta \cos \varphi \right); \\ p_z &= p \left(\sin \frac{\Psi}{2} \cos \theta + \cos \frac{\Psi}{2} \sin \theta \cos \varphi \right), \end{aligned} \quad (4.15.39a)$$

we can determine the trajectory in a discontinuity plane

$$\begin{aligned} y - y_o &= r_L \cos \frac{\Psi}{2} \left[\cos \theta_o - \cos \theta - \operatorname{ctg} \frac{\Psi}{2} (\sin \theta_o \cos \varphi_o - \sin \theta \cos \varphi) \right]; \\ z - z_o &= r_L \cos \frac{\Psi}{2} \left[\cos \theta_o - \cos \theta - \operatorname{tg} \frac{\Psi}{2} (\sin \theta_o \cos \varphi_o - \sin \theta \cos \varphi) \right], \end{aligned} \quad (4.15.40)$$

where θ and φ are co-related (see Eq. 4.15.38). The total displacement of a particle in a discontinuity plane is

$$\Delta y = r_L \left[\sin \left(\theta_o + \frac{\Psi}{2} \right) - \sin \left(\theta_k + \frac{\Psi}{2} \right) \right]; \quad \Delta z = r_L \left[\cos \left(\theta_o + \frac{\Psi}{2} \right) - \cos \left(\theta_k + \frac{\Psi}{2} \right) \right]. \quad (4.15.41)$$

Here the initial pitch angle θ_o and the final pitch angle θ_k are related to the region 1 at $x > 0$. If a particle comes from the region 2 then θ_o should be substituted for $\Psi - \theta_{2o}$ (θ_{2o} is the initial pitch-angle in the region 2). Similarly, for those particles

coming into the region 2, θ_k should be substituted for $\Psi - \theta_{2k}$. The final pitch angle is determined by Eq. 4.15.38. Particle acceleration in the interaction with a rotational discontinuity is determined by the Eq. 4.15.34 but Δy is defined in it by Eq. 4.15.41. In particular, for the particles reflected from the discontinuity plane $\Delta p'_z = 2p'_z$ and

$$\Delta E_k = \frac{2up'_z}{(1-u^2/c^2)^{1/2}} = \frac{2|\mathbf{E}|}{H_n^2 - |\mathbf{E}|^2} \left[cp_z H_n - |\mathbf{E}| (m_{ac} c^2 + E_k) \right], \quad (4.15.42)$$

where p'_z is the z component of a particle momentum in the system K' . For non-relativistic particles ($E_k \ll m_{ac} c^2$) we have

$$\Delta E_k = 2m_{ac} c^2 \frac{|\mathbf{E}|}{H_n^2 - |\mathbf{E}|^2} (u_z H_n / c - |\mathbf{E}|). \quad (4.15.43)$$

This shows that with $|\mathbf{E}| \sim H_n$ the particles will be accelerated to relativistic energies. For $|\mathbf{E}| \ll H_n$ the value $\Delta E_k = 2up'_z$, where $p'_z \approx p_{||}$ ($p_{||}$ is a particle's momentum along the magnetic field) for a small α' , and therefore a discontinuity with a tangential electric field $\mathbf{E}$ accelerates the particles similar to a magnetic mirror moving along the discontinuity with the velocity $u = c|\mathbf{E}|/H_n$.

4.15.7. Particle acceleration at a multiple reflection from a shock wave front

In the presence of inhomogeneities of the magnetic field ahead of the shock front there will be particle scattering both by undisturbed medium and by a shock wave front. As a result a certain share of particles will undergo a multiple acceleration on the shock wave front and their energy can be far increased. A realization of such a mechanism of acceleration with the certain concrete condition will be considered below (see Sections 4.21–4.24). Here we shall present the solution obtained by Vasilyev et al. (1978) for the problem of energetic particles acceleration by a shock wave propagating in a turbulent medium. The paper of Vasilyev et al. (1978) is founded upon the following assumptions: a) a particle's Larmor radius r_g is large compared to the shock front's thickness; b) the shock waves are propagated in the medium with a regular magnetic field H_o and a random field $\tilde{H}$, and $\tilde{H} \leq H_o$; c) the spectrum of the random field falls with the increase of the wave number k so that the large scale component of the field (the scales are more than a particle Larmor radius r_g) prevails over the small scale component; the main turbulence scale L_o

(correlation length) satisfies the condition $L_o \gg r_g$. If these conditions are satisfied, a particle's transport path will be large compared to the Larmor radius in the resultant field composed of a regular magnetic field H_o and by a large scale of a random field $\tilde{H}$. A small scale field within each of the Larmor revolutions will produce only a small distortion of a trajectory. Therefore near the front within several Larmor revolutions one can use the leading center approximation, neglecting a small scale component. In a system of coordinates with a shock front at rest let the plasma before the shock moves normally to it. The large-scale field before and behind the front is denoted by H_1 and H_2 , respectively, and the small random component is neglected. If the angles α_1, α_2 (related by the expression $u_1 \text{ctg} \alpha_1 = u_2 \text{ctg} \alpha_2$ owed to the boundary conditions) satisfy the inequality $u_i \text{ctg} \alpha_i < c$ ($i = 1, 2$), the coordinate system K' will exist in which $u' \parallel H'$ and an electric field $\mathbf{E}'$ is equal to zero in the whole space. The K' system moves opposite to the x -axis with the velocity $u' = u_i \text{ctg} \alpha_i$ (Landau and Lifshitz, M1957).

In this system the energy of particles does not change and their trajectories in the regions 1 and 2 have the form of spiral segments. At arbitrary values α the coefficients of particle reflection from the front $f(\mu)$ and those of particle's passage through the front $\phi(\mu)$ as functions of $\mu = \cos \theta$ (the pitch angle cosine), averaged over a period of cyclotron rotation, can be found by a numerical calculation. However, if the condition $\alpha_{1,2} \ll 1$ is valid the magnetic flux through the orbit (the transverse adiabatic invariant) will be conserved in the K' system when a particle crosses the wave front. Furthermore, we shall restrict our consideration to this case and for simplicity consider a non-relativistic case: $u/c \ll \alpha \ll 1$. The differences of α' from α and of H' from H as well as tangential velocity component behind the front will then be negligibly small. From a comparison of the transverse adiabatic invariant before and behind the front, $\sin^2(\theta'_1/H_1) = \sin^2(\theta'_2/H_2)$, we find the boundary value

$$\mu'_o = \sqrt{1 - H_1/H_2}, \quad (4.15.44)$$

separating the particles moving from the region 1 and reflecting from the front, from those passing through the front. Thus, the coefficient of passage is

$$\phi_{12}(\mu') = \begin{cases} 1 & \text{at } 1 \geq \mu' \geq \mu'_o, \\ 0 & \text{at } 0 \leq \mu' \leq \mu'_o; \end{cases} \quad (4.15.45)$$

and the reflection coefficient $f_{12}(\mu') = 1 - \phi_{12}(\mu')$. For the particles incident upon the front from the region 2,

$$\varphi_{21}(\mu') = 1, \quad f_{21}(\mu') = 0 \quad \text{at} \quad -1 \leq \mu' \leq 0. \quad (4.15.46)$$

A relation of pitch-angles in the stroked and initial systems is given by the expression

$$\cos \theta' = (\cos \theta + u_1/\alpha_1 v) \left[1 + (u_1/\alpha_1 v) \cos \theta + (u_1/\alpha_1 v)^2 \right]^{-1/2}, \quad (4.15.47)$$

which results in all values $\theta' - 1 \leq \cos \theta' \leq 1$ being possible with $u_1/\alpha_1 v < 1$. For $u_1/\alpha_1 v > 1$ only positive values $\sqrt{1 - (\alpha_1 v/u_1)^2} \leq \cos \theta \leq 1$ are possible. Since the reflection condition is $\cos \theta' \leq \sqrt{1 - H_1/H_2}$ both latter inequalities will be valid simultaneously, but having the condition $\alpha_1 \geq (u_1/v)(H_1/H_2)^{1/2}$. For $\alpha_1 < (u_1/v)(H_1/H_2)^{1/2}$ a particle with an arbitrary $\cos \theta$ coming to the front from the region 1 will pass through it.

The dependence of $\cos \theta$ on $\cos \theta'$ is double-valued:

$$\cos \theta = \pm \cos \theta' \sqrt{1 - (u_1/\alpha_1 v)^2 \sin^2 \theta'} - (u_1/\alpha_1 v) \sin^2 \theta'. \quad (4.15.48)$$

In the case $u_1/\alpha_1 v < 1$ one should take only the sign +. For $u_1/\alpha_1 v > 1$, one should take into account both of the signs. The Eq. 4.15.44–4.15.48 give the values of μ limiting the reflection region and the region of passage for the particles coming from the medium 1 into medium 2. They are different for three regions of values of α_1 :

region 1 $\alpha_1 > u_1/v$; reflection at $-u_1/\alpha_1 v \leq \mu \leq \mu_{o+}$, passage at $\mu_{o+} \leq \mu \leq 1$, where

$$\mu_{o\pm} = \pm \sqrt{(1 - H_1/H_2) \left(1 - (u_1/\alpha_1 v)^2 (H_1/H_2) \right)} - u_1 H_1 / \alpha_1 v H_2; \quad (4.15.49)$$

region 2 $(u_1/v)(H_1/H_2)^{1/2} < \alpha_1 < u_1/v$; reflection at $\mu_{o-} \leq \mu \leq \mu_{o+}$, passage at $\mu_{o+} \leq \mu \leq 1$ and $-1 \leq \mu \leq \mu_{o-}$;

region 3 $(u_1/c) < \alpha_1 < (H_1/H_2)^{1/2}$; only the passage through the front is possible.

For the particles in the medium 2 incidence onto the front is possible at $-1 \leq \mu \leq -u_1/\alpha_1 v$ providing by the condition $u_1/\alpha_1 v < 1$; all these particles will pass through the front; there is no reflection. At $u_1/\alpha_1 v > 1$ a hit onto the front from

the medium 2 is impossible. The coefficients of reflection f and of passage φ , which are considered as the functions of μ , are equal to 0 or 1 depending on in what range μ is located. When calculating an increment of energy of a particle reflecting from the shock front, let us include that in the system K' , the energy variation of a particle does not take place, and the longitudinal momentum is changed to the reversal momentum:

$$p''_{//} = -p'_{//} = -\left(p_{//} + \frac{m_{ac}u_1}{\alpha_1}\right), \quad E''_k = E'_k = E_k + \frac{p_{//}u_1}{\alpha_1} + \frac{m_{ac}}{2}\left(\frac{u_1}{\alpha_1}\right)^2. \quad (4.15.50)$$

Here we have neglected a difference between $p_{//} = p \cos \theta$ and p_x as a result of α_1 angle being small. The momentum along $\mathbf{H}_1$ and a particle energy in the initial system before a reflection are designated by $p_{//}$ and E_k , and $p''_{//}$ and E''_k correspond to the same quantities in the system K' after reflection. Furthermore, a particle was considered to be non-relativistic: $E_k = m_{ac}v^2/2$.

Passing again to the initial coordinate system, we obtain the energy variation at a reflection:

$$\Delta E_k = \frac{2m_{ac}vu_1}{\alpha_1} \left(\mu + \frac{vu_1}{\alpha_1} \right). \quad (4.15.51)$$

Remember that particle reflection is possible only with $\alpha_1 \geq (u_1/v)(H_1/H_2)^{1/2}$; substituting the boundary value $\alpha_1 = (u_1/v)(H_1/H_2)^{1/2}$ at which the single value $\cos \theta = \mu_0 = -(H_1/H_2)^{1/2}$ is possible, we shall obtain the maximum possible energy of the reflected particles (for a given ration H_1/H_2) :

$$\Delta E_{k \max} = 2m_{ac}v^2(H_2/H_1 - 1). \quad (4.15.52)$$

At the characteristic value $H_2/H_1 = 3$, typical for interplanetary shock waves, the energy of a particle increases no more than 9 times in a reflection.

In the opposite limiting case $u_1/\alpha_1 v \ll 1$, an energy increment at the reflection is a small portion of the initial particle energy:

$$\Delta E_k = 2m_{ac}vu_1\mu/\alpha_1 \ll m_{ac}v^2/2. \quad (4.15.53)$$

When passing through a shock front the energy and momentum of a particle varies in the coordinate system K' in a following way:

$$E''_k = E'_k, p''_{//} = p'_{//} \frac{\sqrt{1 - (H_2/H_1)(1 - \mu'^2)}}{\mu'}. \quad (4.15.54)$$

Returning to the initial coordinate system we find the increment of a particle energy ΔE_{k12} :

$$\Delta E_{k12} = \frac{m_{ac} v u_1}{\alpha_1} \left(\mu + \frac{u_1}{\alpha_1 v} - \sqrt{1 + \frac{2u_1 \mu}{\alpha_1 v} + \left(\frac{u_1}{\alpha_1 v} \right)^2 - \frac{H_2}{H_1} (1 - \mu^2)} \right). \quad (4.15.55)$$

Here $1 \geq \mu \geq \mu_o$ at $\alpha_1 > (u_1/v)(H_1/H_2)^{1/2}$ where μ_o is given by Eq. 4.15.49. The maximum energy increment of the passage's particle (as of those reflected) is reached at $\alpha_1 = (u_1/v)(H_1/H_2)^{1/2}$ and $\mu = \mu_o = -(H_1/H_2)^{1/2}$:

$$\Delta E_{k12 \max} = m_{ac} v^2 (H_2/H_1 - 1). \quad (4.15.56)$$

The increment is twice lower than for the reflected particles, since in the latter case particle interacts twice longer with a front (passing through it 'forward' and 'return'). At $\alpha_1 < (u_1/v)(H_1/H_2)^{1/2}$ (as has been noted) all particles pass through the front; but their maximum energy increment is decreased compared to that determined by Eq. 4.15.56; at $(\alpha_1 v/u_1)^2 (H_2/H_1) \ll 1$ the energy increment reaches the value:

$$\Delta E_{k12} = \frac{m_{ac} v^2}{2} (H_2/H_1 - 1) \quad (4.15.57)$$

irrespective of α_1 . This value is obtained for $\mu = \alpha_1 v/u_1$. Eq. 4.15.57 corresponds to a conservation of the adiabatic invariant of a non-relativistic particle at its passage through the front:

$$E_{k2}/E_{k1} = H_2/H_1. \quad (4.15.58)$$

This result is not a stochastic one: for a purely transverse shock wave ($\alpha_1 = \alpha_2 = 0$) for which it is impossible to introduce a coordinate system with $\mathbf{E}' = 0$, a conservation of the particle adiabatic invariant takes place for a particle crossing the front of a wave: $p_{1\perp}^2/H_1 = p_{2\perp}^2/H_2$. Eq. 4.15.58 is again a result of this relation for a non-relativistic particle.

Let us return to a general Eq. 4.15.55 and consider it at $\alpha_1 \gg u_1/v$. Neglecting the small terms we shall have:

$$\Delta E_{k12} = \frac{m_{ac} v u_1}{\alpha_1} \left(\mu - \sqrt{1 - \frac{H_2}{H_1} (1 - \mu^2)} \right). \quad (4.15.59)$$

for the condition $\mu \approx \mu'$ considered, so $1 \geq \mu \geq \sqrt{1 - H_2/H_1}$. At the transition from the region 2 into region 1 the corresponding increment is given by the equation:

$$\Delta E_{k21} = \frac{m_{ac} v u_1}{\alpha_1} \left(\mu + \frac{u_1}{\alpha_1 v} + \sqrt{1 + \frac{2u_1 \mu}{\alpha_1 v} + \left(\frac{u_1}{\alpha_1 v} \right)^2 - \frac{H_1}{H_2} (1 - \mu^2)} \right), \quad (4.15.60)$$

where $-1 \leq \mu \leq -u_1/\alpha_1 v$. A transition through the front is possible only with $\alpha_1 > u_1/v$. If $1 \gg \alpha_1 \gg u_1/v$, then

$$\Delta E_{k21} = \frac{m_{ac} v u_1}{\alpha_1} \left(\mu + \sqrt{1 - (H_1/H_2)(1 - \mu^2)} \right). \quad (4.15.61)$$

In all the cases the maximum increment of energy at a reflection or at a transition through the front is several times higher than the particle's initial energy, and is a small part of it at $\alpha_1 \gg u_1/v$.

Since a magnetic field has a random component the above formulae are valid if the angles α_1 and α_2 are weakly dependent on the distance which a particle covers during a time of acceleration. A simple estimate shows that this condition will be satisfied if the correlation length L_o fits the inequalities $L_o \gg r_g v/u_1 (H_2/H_1)$ at $\alpha_1 < u_1/v$ and $L_o \gg (r_g/\alpha_1)(H_2/H_1)$ at $\alpha_1 > u_1/v$. Since in interplanetary space the transport path $\Lambda_{//}$ in the direction of the magnetic field is usually larger than the correlation length L_o , the above inequalities also provide the small value of a particle pitch angle variation owed to scattering by a small scale field during the period of acceleration.

Let us now find, according to the paper (Vasilyev et al., 1978), the boundary conditions on the shock wave front. We shall consider the particle distribution function as sufficiently isotropic one both before a shock wave front and behind it in order to use the diffusion approximation. Furthermore suppose that $\alpha_1 \gg u_1/v$ which results in that ΔE_k is small as compared to a particle energy E_k . This condition makes it possible to expand the distribution function over the small

additions to the energy. Otherwise the boundary conditions will have the form of the equations in the finite differences over the energy. A deduction of the boundary conditions is based on a calculation of the particle fluxes through the wave front in the z -axis direction and in the opposite one.

The particle flux $J_2(E_k)$ through the plain, which is on the distance from the front of about one Larmor radius, in the direction of the z -axis (along the normal to the front) can be written by means of the distribution function $F_2(E_k, \mu)$:

$$J_2(E_k) = \int (\mathbf{n}\mathbf{v}) F_2(E_k, \mu) d\Omega, \quad (4.15.62)$$

where an integration is made within the limits from 0 to $\pi/2 + u_2/\alpha_2 v \approx \pi/2$ over a pitch angle and from 0 to 2π over the angle of cyclotron rotation of particles. Since a reflection of particles moving from the medium 2 with the condition under consideration is absent, the flux $J_2(E_k)$ can be formed only by the particles moving from the region 1 and crossing the shock wave front. Let us designate the flux of these particles as $J_{12}^\varphi(E_k)$:

$$J_{12}^\varphi(E_k) = \int (\mathbf{n}\mathbf{v}) \varphi_{12}(\mu) F_1(E_k - \Delta E_{k12}(\mu), \mu, t) d\Omega. \quad (4.15.63)$$

The argument of the distribution function includes the energy increment on the front; the factor $\varphi_{12}(\mu)$ (see Eq. 4.15.45) represents the probability of a particle transition through the front without reflection. The first boundary condition will have a form:

$$J_2(E_k) = J_{12}^\varphi(E_k) \quad \text{at } z = 0. \quad (4.15.64)$$

The second boundary condition is obtained from the consideration of the particles crossing the same plane but in the direction opposite to the z -axis. It has the form:

$$J_1(E_k) = J_{21}^\varphi(E_k) + J_{11}^f(E_k) \quad \text{at } z = 0, \quad (4.15.65)$$

where $J_1(E_k)$ is the flux through the plane expressed by the distribution function in the medium 1; $J_{21}^\varphi(E_k)$ is the flux of particles moving from the medium 2 and crossing the wave front; $J_{11}^f(E_k)$ is the flux of particles incident from the medium 1 and reflecting from the front. For all of these fluxes it is easy to write the expressions similar to Eq. 4.15.62 and 4.15.63.

The distribution function in the system of rest plasma in the diffusion approximation has the form

$$F(z', E', \mu', t) = \frac{1}{4\pi} \left(N(z', E'_k, t) - \frac{v'}{2} \frac{dN(z', E'_k, t)}{dz'} \int_0^{\mu'} \frac{d\mu}{D(\mu)} \right), \quad (4.15.66)$$

where $D(\mu)$ is the diffusion coefficient in angular space, $N(z', E'_k, t)$ is the isotropic part of the distribution function. The primed values correspond to the rest plasma system, z' is counted along magnetic lines of force. Since the distribution function is an invariant of the Lorentz transformation, it is sufficient to express the primed terms in the right hand part of Eq. 4.15.66 by those unprimed to the transition to the system of the rest shock front. The above assumptions $u \ll \alpha v_1$, $\alpha \ll 1$, make it possible to write $E'_k = E_k - \mathbf{pu}$, $|\mathbf{pu}| \ll E_k$. Expanding F over the small addition to the energy, we shall have

$$F(z, E, \mu, t) = \frac{1}{4\pi} \left(N(z, E, t) - \mathbf{pu} \frac{\partial N}{\partial E} - \frac{\alpha v}{2} \frac{\partial N}{\partial z} \int_0^{\mu} \frac{d\mu'}{D(\mu')} \right). \quad (4.15.67)$$

In the small anisotropic addition in Eq. 4.15.66 we have set $E'_k = E_k$, $\mu' = \mu$. Furthermore, we have included in one-dimensional case that the distribution function depends only on z so that $dz' = dz/\alpha$. Substituting Eq. 4.15.67 into Eq. 4.15.62 and integrating we have

$$J_2 = \frac{\alpha_2 v}{4} N_2 - \frac{1}{6} u_2 p v \frac{\partial N_2}{\partial E} - \frac{\alpha_2^2 v}{6} \Lambda_{//} \frac{\partial N_2}{\partial z}. \quad (4.15.68)$$

Here

$$\Lambda_{//} = \frac{3v}{4} \int_0^1 \frac{1 - \mu^2}{D(\mu)} d\mu \quad (4.15.69)$$

is the transport path along magnetic lines of force. We shall set below $D(\mu) = D = \text{const}$ and use the expression $\Lambda_{//} = 2D/v$.

The other fluxes are calculated in a similar way, in particular:

$$J_{12}^\varphi(E_k) = \frac{D_1}{4} \alpha_1 v N_1 - \frac{\alpha_1 v}{4} \overline{\Delta E_{k12}^\varphi} \frac{\partial N_1}{\partial E} - \frac{D'_1}{6} u_1 p v \frac{\partial N_1}{\partial E} - \frac{D''_1}{6} \alpha_1^2 v \Lambda_{1//} \frac{\partial N_1}{\partial z}, \quad (4.15.70)$$

where

$$\begin{aligned}
D_1 &= 2 \int_0^1 \mu \phi_{12}(\mu) d\mu = 1 - \mu_o^2; \quad D'_1 = 3 \int_0^1 \left[\mu^2 \sin^2 \alpha_1 + \frac{1 - \mu^2}{2} \cos^2 \alpha_1 \right] \phi_{12}(\mu) d\mu \\
&= 1 - \frac{3}{2} \mu_o + \frac{1}{2} \mu_o^3; \quad D''_1 = 3 \int_0^1 \mu^2 \phi_{12}(\mu) d\mu = 1 - \mu_o^3; \quad \overline{\Delta E_{k12}^\varphi} = \frac{2 p u_1}{3 \alpha_1} \mu_o^2 (1 - \mu_o); \\
\mu_o &\approx \mu'_o = \sqrt{1 - H_1/H_2}.
\end{aligned} \tag{4.15.71}$$

Substituting Eq. 4.15.68 and Eq. 4.15.70 into Eq. 4.15.64 and applying $\alpha_1/\alpha_2 = H_2/H_1$ and $\alpha_1 D_1/\alpha_2 = 1$ we reduce the Eq. 4.15.64 to the form

$$\begin{aligned}
N_1 - N_2 &= \frac{2 u_2 p}{3 \alpha_2} \frac{\partial N_2}{\partial E} + \frac{2 D'_1 u_1 p}{3 \alpha_2} \frac{\partial N_1}{\partial E} + \frac{\alpha_1}{\alpha_2} \overline{\Delta E_{k12}^\varphi} \frac{\partial N_1}{\partial E} - \frac{2}{3} \alpha_2 \Lambda_{2//} \frac{\partial N_2}{\partial z} \\
&\quad + \frac{2}{3} \frac{\alpha_1^2}{\alpha_2^2} D''_1 \Lambda_{1//} \frac{\partial N_1}{\partial z} \quad \text{at } z = 0.
\end{aligned} \tag{4.15.72}$$

In a similar way we shall obtain from the Eq. 4.15.65 the following relation:

$$\begin{aligned}
N_1 - N_2 &= -\frac{2 u_1 p}{3 \alpha_2} \frac{\partial N_1}{\partial E} - \frac{2}{3} \frac{\alpha_1^2}{\alpha_2^2} \Lambda_{1//} \frac{\partial N_1}{\partial z} - \frac{\alpha_1}{\alpha_2} \overline{\Delta E_{k11}^\varphi} \frac{\partial N_1}{\partial E} + \frac{2(1 - D'_1) u_1 p}{3 \alpha_2} \frac{\partial N_1}{\partial E} \\
&\quad - \frac{2}{3} \frac{\alpha_1^2}{\alpha_2^2} (1 - D''_1) \Lambda_{1//} \frac{\partial N_1}{\partial z} - \overline{\Delta E_{k21}^\varphi} \frac{\partial N_2}{\partial E} + \frac{2 u_2 p}{3 \alpha_2} \frac{\partial N_2}{\partial E} + \frac{2}{3} \alpha_2 \Lambda_{2//} \frac{\partial N_2}{\partial z}.
\end{aligned} \tag{4.15.73}$$

The left hand parts of the latter equations are equal. Therefore it is convenient to pass to the other two equations, which are obtained by subtracting corresponding term and summing of Eq. 4.15.72 and Eq. 4.15.73. The summing results in a relation in the right hand part of which the summands have the order of $(\Delta E_k/E_k)N \ll N$ and $\alpha \Lambda''(\partial N/\partial z) \ll N$. This means that the difference $N_1 - N_2$ is small compared to N_1, N_2 and it can be equated to zero: $N_1 \approx N_2$ at $z = 0$. As the result of subtraction, we have:

$$\kappa''_2 \frac{\partial N_2}{\partial z} - \kappa''_1 \left(\frac{H_2}{H_1} \right)^2 \frac{\partial N_1}{\partial z} = \frac{p}{3 \alpha_2^2} \frac{\partial N}{\partial p} u_1 \mu_o^2 (1 + \mu_o) \quad \text{at } z = 0. \tag{4.15.74}$$

The Eq. 4.15.74 can be written in a more compact form if we include that in the case under consideration (when a particle's motion across the magnetic field is not included), the quantities $\alpha_2^2 \kappa''_2 = \kappa_2$ and $\alpha_2^2 (H_2/H_1)^2 \kappa''_1 = \kappa_1$ represent the

diffusion coefficient in the normal to the front direction, and $u_1\mu_o^2 = u_1 - u_2$ is the jump of velocity on the front. As a result Eq. 4.15.74 comes to the form

$$\kappa_2 \frac{\partial N_2}{\partial z} - \kappa_1 \frac{\partial N_1}{\partial z} = \frac{p\Delta u}{3} (1 + \mu_o) \frac{\partial N}{\partial p}. \quad (4.15.75)$$

The boundary condition reflects the fact of particle acceleration at the crossing of the shock front. A part of this acceleration is caused by a difference of the magnetic inhomogeneity velocities before and behind the front and represents the Fermi acceleration. The value $1 + \mu_o = 1$ in the right hand of Eq. 4.15.75 should correspond to only this process. Another part of the acceleration is owed to a jump of the regular magnetic field on the wave front. The latter part is included by the summand $\mu_o(\Delta u/3)p(\partial N/\partial p)$ on the right-hand of Eq. 4.15.75. While the Fermi component of the acceleration can be written basing on a general structure of the transfer equation, the inclusion of an additional acceleration require a consideration of the elementary processes on the front and their including into the boundary conditions; this has been carried out in (Vasilyev et al., 1978).

4.16. Acceleration of particles in case of magnetic collapse and compression

It was shown above that in some cases the injection energy for the statistical acceleration mechanism could prove to be very high (see Section 4.8). In such a case an essential particle injector might be the first-order Fermi mechanism of particle acceleration in a magnetic trap between two mutually approaching magnetic mirrors considered in (Fermi, 1954; Spitzer, M1956) for the case in which the initial particle velocity $v_o \gg u$ (u is the speed of magnetic mirror motion to meet each other).

However, the acceleration stage when $v_o \leq u$ is of greatest interest from the viewpoint of the problem of injection. Such case was examined in (Dorman, 1959a) on the assumption of particle injection from the space of the magnetic mirrors. If, however, the particle's thermal velocity is much below the speed of mutual approach of the mirrors, such case is little effective. The case is of great interest where the particles are injected from the space between magnetic clouds (Dorman, 1959b).

4.16.1. Non-relativistic case of particle acceleration during magnetic collapse

Let the semi-space 1 (the left of plane A) and semi-space 2 (the right of plane B) contain homogeneous magnetic fields of intensity $\mathbf{H}$ (assumed, for the sake of simplicity, to be the same in either semi-space). The fields are parallel to the planes A and B, but, generally speaking, are not parallel to each other. The field between the planes is zero. In semi-space 1 a particle will be affected by the magnetic field $\mathbf{H}$

and the electric field $\mathbf{E} = -[\mathbf{uH}]/c$. The particle affected by these fields will move in a rest coordinate system along a trochoid and in a coordinate system relative to semi-space 1 along a circle at a velocity $v_o + u$ and frequency $\omega_L = ZeH/m_{ac}c$. In this case a particle, when affected by the electric field $\mathbf{E}$ during its motion in the magnetized semi-space, will gain velocity $2u$ if the particles are non-relativistic (see Section 4.3). Let a particle with velocity v_o be injected at the moment $t = -t_{in}/2 = -\pi m_{ac}c/ZeH$ to the semi-space 1 from the space between the planes A and B. At moment $t = 0$ the particle will be ejected from semi-space 1 at velocity $v_1 = v_o + 2u$ and, after that, from semi-space 2 at velocity $v_2 = v_o + 4u$. At that moment, the distance between the planes will be

$$l_2 = l_1 \frac{1 + \alpha_1}{3 + \alpha_1} - ut_{in}, \quad (4.16.1)$$

where $\alpha_1 = v_o/u$ and $t_{in} = 2\pi m_{ac}c/ZeH$.

Extending the above examination further, we shall find that at the k -th ejection of the particle from any semi-space the particle velocity will be

$$v_k = v_o + 2ku = u(2k + \alpha_1) \quad (4.16.2)$$

and the moment t_k/t_o of the k -th ejection will be

$$\tau_k \equiv \frac{t_k}{t_o} = \frac{2(k-1) + \beta_1(k-1)(k + \alpha_1 + 1)}{2k + \alpha_1 - 1}, \quad (4.16.3)$$

where

$$\begin{aligned} t_o &\equiv \frac{l_1}{2u}; \quad \beta_1 \equiv \frac{t_{in}}{2t_o} = \frac{2\pi m_{ac}c}{ZeHl_1} = \frac{\beta_o}{1 - \beta_o}; \quad \beta_o = \frac{2\pi m_{ac}c}{ZeHl_o}; \quad l_o = l_1 + t_{in}u; \\ 1 - \tau &= \frac{l}{l_1} = \frac{ZeHl}{ZeHl_o - 2\pi m_{ac}c}, \end{aligned} \quad (4.16.4)$$

where $0 \leq \tau \leq 1$. After expressing k from Eq. 4.16.3 in terms of τ_k , substitution in Eq. 4.16.2, and considering Eq. 4.16.4, we obtain

$$v = \left\{ v_o^2 + \frac{2ZeHl_o(u + v_o)}{\pi m_{ac}} \left[1 + \frac{lu}{l_o(u + v_o)} + \frac{2ZeHl^2}{2\pi m_{ac}l_o(u + v_o)} \right] \right\}^{1/2} - \frac{ZeHl}{\pi m_{ac}}. \quad (4.16.5)$$

At $l = 0$, we shall obtain the maximum velocity

$$v_{\max} = \left(v_o^2 + \frac{2ZeHl_o(u + v_o)}{\pi c m_{ac}} \right)^{1/2} \quad (4.16.6)$$

and the maximum kinetic energy of the particle

$$E_{k \max} = \frac{m_{ac} v_{\max}^2}{2} = \frac{m_{ac} v_o^2}{2} + \frac{ZeHl_o(u + v_o)}{\pi c}. \quad (4.16.7)$$

4.16.2. Relativistic case of particle acceleration during magnetic collapse

If the particle velocity is sufficiently high, $v \gg 2u$, the change of the distance between the planes during the collision time is relatively small. The energy increase will be

$$\Delta E_k = 2pcZeH / ZeHc = 2pu. \quad (4.16.8)$$

Since

$$\Delta E_k = p \Delta p \left(1 + p^2 / m_{ac}^2 c^2 \right)^{-1/2}, \quad (4.16.9)$$

we obtain

$$\Delta p = 2um_{ac} \left(1 + p^2 / m_{ac}^2 c^2 \right)^{1/2} = 2uE / c^2, \quad (4.16.10)$$

where E is the total energy of the particle. The distance between the planes during the period between the collisions

$$\Delta t = l / v + \pi E / ZeHc \quad (4.16.11)$$

will change by

$$\Delta l = -2u \Delta t = -2u(l / v + \pi E / ZeHc). \quad (4.16.12)$$

From this

$$dl / dp = -l / p - \pi c / ZeH, \quad (4.16.13)$$

and the solution will be found in the form

$$l = -\pi c p / 2ZeH + C_1 / p, \quad (4.16.14)$$

where the constant

$$C_1 = p_r(l_r + \pi c p_r / 2ZeH). \quad (4.16.15)$$

Here p_r and l_r are respectively the particle momentum and the distance between the planes at a selected moment when the condition $v \gg 2u$ is already satisfied. Thus we find

$$\frac{p}{p_r} = \sqrt{\left(\frac{ZeHl}{\pi c p_r}\right)^2 + \frac{2ZeHl_r}{\pi c p_r} + 1} - \frac{ZeHl}{\pi c p_r}. \quad (4.16.16)$$

The maximum value of the momentum will be obtained at $l = 0$:

$$p_{\max} = p_r \sqrt{1 + \frac{2ZeHl_r}{\pi c p_r}}. \quad (4.16.17)$$

4.16.3. The case of particle acceleration from very low energies up to relativistic energies

In this case the change of the velocity will be determined first by Eq. 4.16.5 and then, after satisfying the condition $v \gg 2u$, by Eq. 4.16.8. Since in almost all cases $u \leq 10^8$ cm/sec, it is expedient to take $v_r = 20u = 2 \times 10^9$ cm/sec as the boundary between the scopes of the effect of there expressions, for at $v \leq v_r$ particles are nonrelativistic and the Eq. 4.16.2 is valid, whilst at $v \geq v_r$ the condition $v \gg 2u$ is satisfied within a sufficient accuracy and the Eq. 4.16.8 is valid up to the highest energies.

According to Eq. 4.16.2, $v = v_r$ will be achieved if $k = 10$ is used (let us note that the condition $\beta_1 < (1 + \alpha_1)/9(11 + \alpha_1)$ or $\beta_o < (1 + \alpha_1)/10(10 + \alpha_1)$ should be satisfied in this case, otherwise the magnetic clouds will collide earlier than the particle velocity reaches $v = v_r = 20u$ and the Eq. 4.16.7 has to be used to estimate $E_{k \max}$. In this case $p_r = 20m_{ac}u + p_o$, where $p_o = m_{ac}v_o$. Thus we find that

$$p = (20m_{ac}u + p_o) \left\{ \left[1 + \frac{2(ZeHl_o - 2\pi c m_{ac}u)(1 + \alpha_1 - 9\beta_1(11 + \alpha_1))}{\pi c (20m_{ac}u + p_o)(19 + \alpha_1)} + \left(\frac{ZeHl}{\pi c (20m_{ac}u + p_o)} \right)^2 \right]^{1/2} - \frac{ZeHl}{\pi c (20m_{ac}u + p_o)} \right\}. \quad (4.16.18)$$

The maximum value of the particle momentum will be obtained at $l = 0$ using the denominations described by Eq. 4.16.4:

$$p_{\max} = \frac{2m_{ac}u}{\sqrt{\beta_o}} \sqrt{\frac{(1+\alpha_1)(1-\beta_o)-9\beta_o(1+\alpha_1)}{0.95(1+\alpha_1/20)(1+\alpha_1/19)} + 100\beta_o}$$

$$\approx \frac{2m_{ac}u}{\sqrt{\beta_o}} \sqrt{\frac{1-5\beta_o+\alpha_1(0.9-10\beta_o)}{0.95}}, \quad (4.16.19)$$

where the last expression contains an error smaller than 0.5% at $\alpha_1 \leq 1$. In this case $1 \leq \sqrt{[1-5\beta_o+\alpha_1(0.9-10\beta_o)]/0.95} \leq 1.3$ and, within a sufficient accuracy,

$$p_{\max} \approx \frac{2m_{ac}u}{\sqrt{\beta_o}} = \sqrt{\frac{2ZeHl_o m_{ac}u}{\pi c}}. \quad (4.16.20)$$

Considering that $E_{k \max} = m_{ac}c^2(\sqrt{1+\alpha_2}-1)$, where

$$\alpha_2 = \frac{p_{\max}^2}{m_{ac}^2 c^2} = \frac{2uZeHl_o}{\pi m_{ac} c^3} \quad (4.16.21)$$

we obtain that, if $\alpha_2 \gg 1$, the ultra-relativistic energies may be attained and, in this case,

$$E_{k \max} = \sqrt{\frac{2}{\pi} ZeHl_o m_{ac} u c} \quad (4.16.22)$$

If, however, $\alpha_2 \ll 1$ only non-relativistic energies may be achieved, and in this case $E_{k \max}$ will be determined by Eq. 4.16.17. If $\alpha_2 \gg 1$ ($p_o \gg 2m_{ac}u$), the Eq. 4.16.17 can be directly applied and in which $p_r = p_o$; $l_r = l_o$ should be set.

The conditions under which $u \approx 3 \times 10^7$ cm/sec, $l_o \approx 10^8 \div 10^9$ cm, $H \sim 10^2$ Gs may probably be realized in solar flares. From this, according Eq. 4.16.21, we shall obtain for protons $\alpha_2 = 2.2$ and 22 at $l_o = 10^8$ cm and $l_o = 10^9$ cm, respectively; for electrons $\alpha_2 = 4 \times 10^3$ and 4×10^4 cm at $l_o = 10^8$ cm and $l_o = 10^9$ cm, respectively. Thus, it may be expected, that $E_{k \max} \approx 0.8$ and 3.8 GeV for protons and $E_{k \max} \approx 32$ and 100 MeV for electrons at $l_o = 10^8$ cm and $l_o = 10^9$ cm,

respectively. However, it is necessary that the injection condition should also be satisfied for the above possibility of particle acceleration to be realized.

4.16.4. The particle injection conditions for acceleration in a magnetic trap

It is evident that the most crucial moment for the particle's acceleration in a magnetic trap is the initial stage of acceleration, i.e. the first collisions of particles with magnetized clouds. In a magnetized cloud the particle's velocity in the first collision will be $v = u + v_o$. According to Spitzer (M1956) the period between the collisions with the thermal particles of cloud plasma, i.e. the time during which our test particle is gradually deflected by 90° as a result of multiple small deflections, is determined as

$$t_d = \frac{m_{ac}^2 v^3}{8\pi N Z^2 n^4 \left[\Phi\left(\sqrt{m_i/2kT}\right) - G\left(v\sqrt{m_i/2kT}\right) \right] \ln \Lambda}, \quad (4.16.23)$$

where N is the concentration of the medium; m_i is the mass of the plasma particles; T is the temperature in the space plasma; $\Lambda = \left(3/2Ze^3\right)\left(k^3T^3/\pi N\right)^{1/2}$. The functions Φ and G and the values of $\ln \Lambda$ are presented in Chapter 5 of the monograph Spitzer (M1956). If $vt_d > \pi r_g$, i.e. exceeds the path in a magnetic cloud, the particle may be ejected from the cloud without colliding with the particles of cloud plasma. Later on the particle will move between the clouds, and if the condition $vt_d > l$ proves to be satisfied the particle can enter another cloud. Since the particle velocity increased as a result of the first and second collisions, the condition $vt_d > \pi r_g$ will be explicitly satisfied in the second cloud, and, moreover, this condition will be satisfied during the subsequent motion of the particle through the space between the clouds.

When the particle velocity exceeds the electron velocity in space, the energy loss for dynamical friction is more significant. In this case, according to Spitzer (M1956),

$$dv/dt = -v/t_s, \quad (4.16.24)$$

Where

$$t_s = \frac{m_{ac}^2 v}{8\pi N Z^2 e^4 (1 + m_{ac}/m_e) (m_e/2kT)^2 G\left(v\sqrt{m_i/2kT}\right) \ln \Lambda} \approx \frac{m_e m_{ac} v^3}{4\pi N Z^2 e^4 \ln \Lambda} \quad (4.16.25)$$

(the latter is valid at $v \gg \sqrt{2kT/m_e}$ and $m_{ac} \gg m_e$).

It can be seen from the comparison between Eq. 4.16.23 and Eq. 4.16.24 that under the above conditions $t_s/t_d \approx m_e/m_{ac}$, i.e. the validity of the conditions $vt_s > \pi r_g$ and $vt_s > l$ becomes more critical for heavy particles at high velocities. These conditions may be more accurately obtained by integrating Eq. 4.16.24 along the path of motion. The resultant path is

$$L = \frac{m_{ac} m_e (v_{\text{fin}}^4 - v_{\text{in}}^4)}{8\pi e^4 N Z^2 \ln \Lambda}, \quad (4.16.26)$$

where v_{in} and v_{fin} are the particle's initial and final velocities. The appropriate conditions will be $L > \pi r_g$ and $L > l$. When considering electron acceleration, $m_e = m_{ac}$ and the factors $m_e m_{ac}$ in Eq. 4.16.25 and Eq. 4.16.26 and m_{ac}^2 in Eq. 4.16.23 will turn out to be m_e^2 . In this case $t_d \approx t_s$ since $\Phi - G \approx 1$ in the cases of interest to us.

4.16.5. Diffusive compression acceleration of charged particles

Jokipii et al. (2003) consider the acceleration of fast charged particles by smooth compressions and expansions in a collisionless fluid by using the diffusion approximation. If the diffusion length κ/u is of the order of the fluid scale or larger, efficient acceleration occurs which has similarities with both 2nd-order Fermi acceleration and diffusive shock acceleration, but is different from both. A simple one-dimensional sinusoidal flow is analyzed in Jokipii et al. (2003). It was shown that the acceleration dominates, even with equal amounts of compression and expansion. The acceleration time is $\approx \kappa/u^2$. They suggest that this mechanism may be an important accelerator in regions where there are large-scale compressive disturbances, but few shocks. It may contribute to the acceleration of CR elsewhere in the Heliosphere and the Galaxy. It was suggested the name 'diffusive compression acceleration' for this mechanism.

Now a number of general acceleration mechanisms have been suggested. The most successful of these has been the acceleration by collisionless shocks (Krymsky, 1977; Axford et al., 1977; Bell, 1978; Blandford and Ostriker, 1978; Drury, 1983; Jones and Ellison, 1991; see also Sections 4.14, 4.15, 4.21–4.24). However, there are important situations in which energetic particles are accelerated with no shocks present. One recent and particularly clear example consists of the energetic ions observed in interplanetary co-rotating interaction regions near 1 AU, well inside the radius at which the associated co-rotating shocks form (Mason, 2000). Giacalone et al (2002) found that compression acceleration provided a natural and compelling interpretation of the observations.

Jokipii et al. (2003) consider the transport of CR in the diffusion approximation, in which the (nearly-isotropic) distribution function $f(\mathbf{r}, p, t)$ as a function of position $\mathbf{r}$, momentum magnitude p and time t satisfies the Parker equation (Parker, 1965):

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial x_i} \left(\kappa_{ij} \frac{\partial f}{\partial x_j} \right) - u_i \frac{\partial f}{\partial x_i} + \frac{1}{3} \frac{\partial u_i}{\partial x_i} \frac{\partial f}{\partial \ln(p)} + Q_s - Q_l, \quad (4.16.27)$$

where κ_{ij} is the diffusion tensor, u_i is the flow velocity of the background plasma, and Q_s and Q_l represent any additional sources and losses. This equation applies if there is enough scattering for the distribution function to remain nearly isotropic, even at discontinuities such as current sheets and shock waves. Particle acceleration is contained in the term $\partial u_i / \partial x_i$. Application of Eq. 4.16.27 to a one-dimensional system having a planar shock, where the flow velocity changes discontinuously, yields all of the results of diffusive shock acceleration. If the disturbance is not a discontinuity, but instead is a more-gradual compression having a characteristic length scale L_c , one can show that in the limit in which the ratio of the diffusive skin depth $L_d = \kappa_{xx} / u_x$ to the length scale L_c is large, or, equivalently, $\xi = \kappa_{xx} / (u_x L_c) \gg 1$, the solution for the CR distribution function f goes over to the standard diffusive shock solution. In the opposite limit $\xi \ll 1$ the CR are closely tied to the convecting fluid, and simply compress adiabatically.

Then consider the case $L_d \geq L_c$, but where the flow varies smoothly. Note that the scattering mean free path λ_{sc} does not appear explicitly in this inequality. So it is possible to have λ_{sc} small compared with the compression length scales L_c (so that the diffusion approximation applies) but where the diffusive skin depth L_d is of the order of L_c or larger. It was found that such non-shock compressions may be efficient accelerators even if there are associated expansions. The physical basis of the acceleration is the interplay between a) the energy change caused by the compression or expansion of the fluid and b) the diffusion into or away from the region of compression or expansion. Rapid diffusion leads to a particle being able to diffuse away from a region of compression or expansion before the compensating expansion or compression can occur. Hence statistically some few particles will be fortunate enough to gain energy in several compression regions. In this process, for large κ , the accelerations dominate the particle energy change, even in those cases where the compressions and expansions are equally present in the fluid flow. This is because statistically some particles can reach very high energies, but they cannot be decelerated to energies lower than zero. Note also that this acceleration can take place for any orientation of the magnetic field. Gradient and curvature drifts can in general significantly affect the particle trajectories as they are accelerated.

To illustrate this process, Jokipii et al. (2003) consider the simple, periodic one-dimensional velocity profile

$$u_x(x) = u_o(1 + a \sin(kx)), \quad (4.16.28)$$

and κ_{xx} independent of x or p . There are not been able to solve this analytically for general parameters, but it is simple to solve numerically, and the solutions depend only on the dimensionless parameters

$$\chi = (u_o/\kappa_{xx})x, \quad \tau = (u_o^2/\kappa_{xx})t, \quad \eta = (\kappa_{xx}/u_o)k, \quad (4.16.29)$$

and the amplitude a in Eq. 4.16.28. The solutions are clearly periodic in χ with a period $2\pi/\eta$. Illustrated in Fig. 4.16.1(a) is the initial rate of acceleration $d \ln(p)/d\tau$ (in units of $1/\tau$), averaged over x and plotted as a function of normalized wavenumber η , for the case in which the parameter $a = 0.6$, which corresponds to a ratio of maximum density (or velocity) to minimum density (or velocity) of 4.

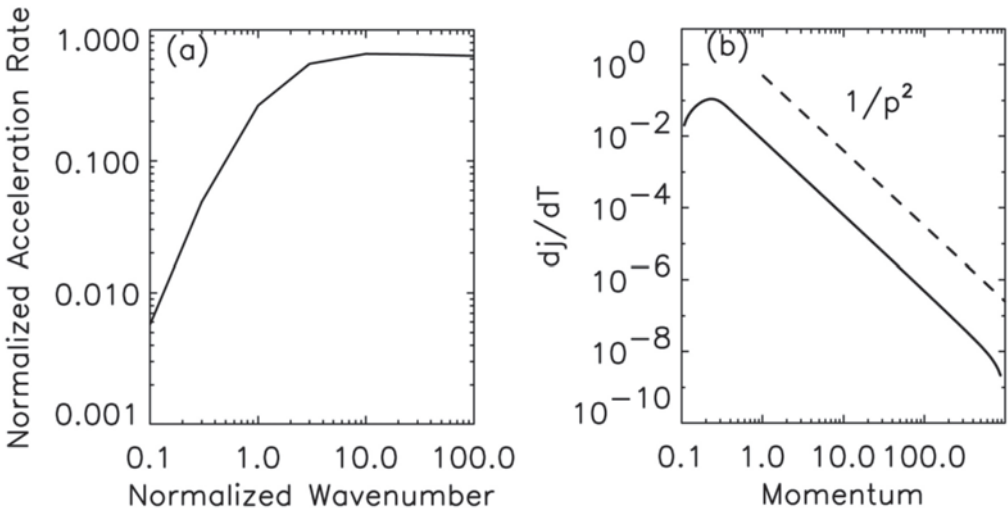


Fig. 4.16.1. Illustration of the acceleration times (a), and resulting energy spectrum (b). According to Jokipii et al. (2003).

It is apparent from Fig. 4.16.1(a) that the average acceleration rate decreases rapidly for a wavenumber much less than 1 (diffusion too slow), and asymptotically approaches a constant which is about unity for larger wavenumbers (when the diffusion becomes more important). A net acceleration occurs in spite of the balancing of compression and expansion.

To determine the energy spectrum for a simple confinement model Jokipii et al. (2003) consider next the solution to Eq. 4.16.27 for the case in which the system is not strictly periodic but that there are diffusive loss boundaries at $x = +15$ and for $\eta = 2$. The velocity is of the form

$$u(x, t) = a \sin(\eta x - t), \quad (4.16.30)$$

where $a = 0.6$, which is essentially the same as the periodic system used above, but is a propagating wave. The particles are injected continuously and uniformly in x at a momentum $p_o = 1$, so the source term in Eq. 4.16.27:

$$Q_s(x, p, t) = R_o \delta(p - p_o). \quad (4.16.31)$$

Figure 4.16.1(b) illustrates the energy spectrum obtained for this model system.

From Fig. 4.16.1 it can be seen that the significant acceleration by non-shock compressions, even in the presence of comparable expansions, is possible as long as the diffusion scale is comparable to the scale of the fluid variations, or larger. This diffusive compression acceleration has some similarities with 2nd-order Fermi acceleration and with shock acceleration, but is different from both. It appears to produce naturally a power law-like spectrum, similar to that in shock acceleration, over a broad range of parameters. According to Jokipii (2001), Jokipii et al. (2003), this acceleration can occur in a number of circumstances. For example, in the inner Heliosphere near 1 AU, for ≈ 1 MeV galactic CR where $\kappa \approx 10^{20}$ cm²/sec, and where compressive velocities should be of the order of Alfvén velocity $v_a \approx 50$ km/sec, we have $\zeta \geq 1$ for scales $L_c \leq 1$ AU. Observations suggest that there may be significant compressive fluctuations over these scales. In the interstellar medium, where the diffusion coefficient is typically $\geq 10^{26}$ cm²/sec, and typical fluid velocities are ≈ 100 km/sec or so, scales of several parsecs to tens of parsecs can correspond to $\zeta \geq 1$. Jokipii et al. (2003) conclude that compressive variations may contribute to the acceleration of energetic particles in many places in the Universe.

4.16.6. Acceleration at fluid compressions and comparison with shock acceleration

The study of particle acceleration at a shock discontinuity raises the question of acceleration at continuous fluid compressions that have not yet developed into shocks. This has previously been examined for a magnetic field parallel to the shock normal (Krulls and Achterberg, 1994). Particle acceleration was examined in a general, steady-state context in a preliminary report by Klappong et al. (2001), whilst in Jokipii (2001) and Giacalone et al. (2002) the problem for the situation of co-rotating interaction regions in the interplanetary space has been examined and they were able to explain observed time-intensity profiles. Malakit et al. (2003) examines

steady-state particle acceleration at continuous fluid compressions of varying width in comparison with that at a discontinuous shock for various shock-field angles. The configurations are shown in Fig. 4.16.2; in the compression case magnetic field lines have a hyperbolic shape and width (the semi-conjugate axis) b along the field.

In comparison with shocks the narrow compressions exhibit quantitatively similar particle acceleration, leading smoothly to the shock results as the width is reduced. However, compressions do not naturally yield a power law particle spectrum; rather, the resulting spectrum is sensitive to the velocity dependence of the mean free path of scattering. The study of acceleration at compression leads to better understanding of shock acceleration, especially regarding the effect of magnetic mirroring on the distribution function and hardening of the particle spectrum.

The transport and acceleration of energetic charged particles near a fluid compression in Malakit et al. (2003) is studied by numerically solving a time-dependent pitch angle transport equation for a general, static magnetic field. The numerical methods are based on those of Ruffolo (1999) and Nutaro et al. (2001). For a shock the transport equation is greatly simplified, but care is required when treating particles crossing the shock. The particle orbits are considered as they cross the shock, using a transfer matrix to assign the distribution function to the appropriate μ and z cells after the shock encounter. In a stringent test of the accuracy of the pitch-angle treatment, the simulations have been able to explain observed 'loss-cone' precursors to Forbush decreases (Leerunnavarat et al., 2003). Although the key results of the work of Malakit et al. (2003) are derived from a more fundamental treatment of pitch angle transport and diffusion-convection treatment. The diffusion-convection transport equation for the plane parallel configuration is an ordinary differential equation, which can be solved analytically for a shock, and can readily be solved numerically for a compression. In the paper Malakit et al. (2003) approximate diffusion-convection results are shown specifically to highlight the role of magnetic mirroring, which is neglected by diffusion-convection included in the full pitch angle treatment.

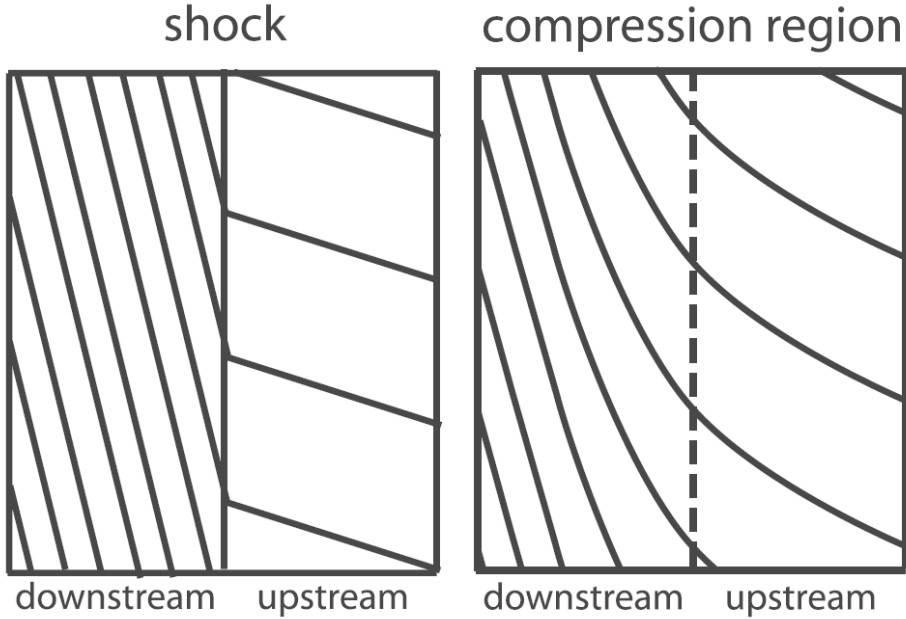


Fig. 4.16.2. Sample mean magnetic field configurations for a shock (left) and a compression region (right). According to Malakit et al. (2003).

In the results it was found that particle spectra from shocks, as predicted by pitch angle treatment, are not exactly power laws as predicted by diffusion-convection (Krymsky, 1977). The spectra are hardened at low energy, especially for the quasi-perpendicular (Q-Perp) case (upstream shock-field angle $\theta_1 = 75.96^\circ$). However, the particle spectrum in the case of a quasi-parallel (Q-Par) shock ($\theta_1 = 0.57^\circ$), predicted by pitch angle treatment, is still a power-law (see Fig. 4.16.3).

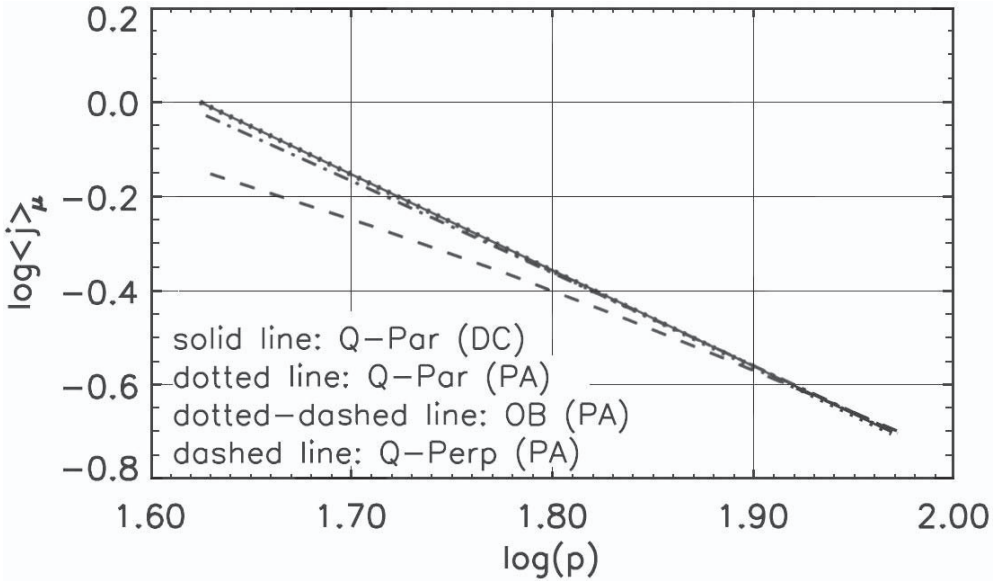


Fig. 4.16.3. Particle spectra for cases of shocks obtained by pitch angle treatment (PA) compared with a power-law spectrum obtained by diffusion-convection equation (DC). According to Malakit et al. (2003).

Similarly, the particle-density jump can be predicted by pitch angle treatment only. The jump is highest in the Q-Perp case, intermediate for the oblique (OB) case ($\theta_1 = 45^\circ$), and disappears for a Q-Par shock (see Fig. 4.16.4).

For compression regions, there is also a peak near the compression plane that is analogous with the jump in the case of shocks. This peak is not as high as the shock jump and the peak height decreases when the compression is wider (see Fig. 4.16.5).

In Fig. 4.16.5 compression width is expressed in terms of the ratio of b to the parallel mean free path. Malakit et al. (2003) conclude that the peak (or jump) should be owed to magnetic mirroring, which is neglected in the diffusion-convection approach. As further evidence, Fig. 4.16.6 shows equal-density contours in the μ - z plane, with a density peak near the compression plane for particles mirroring back upstream.

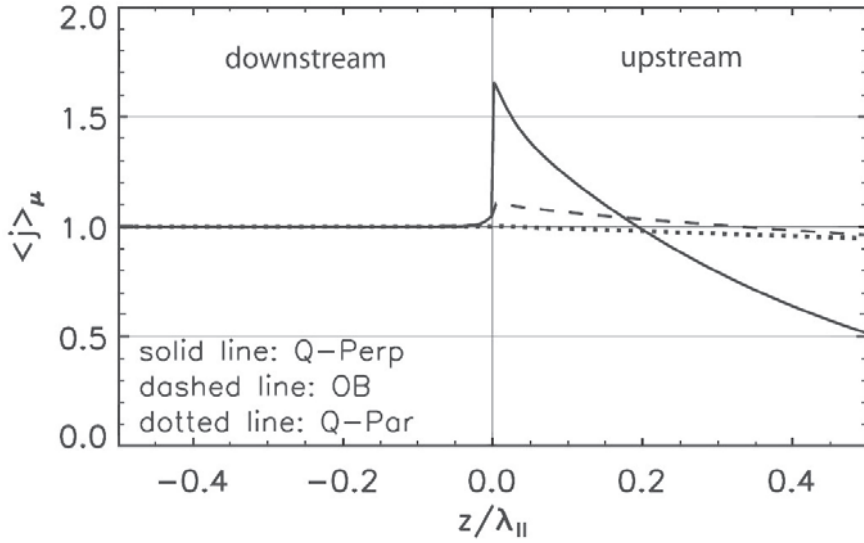


Fig. 4.16.4. Particle density vs. position, obtained by pitch angle treatment, with a higher jump for a more perpendicular shock-field angle. According to Malakit et al. (2003).

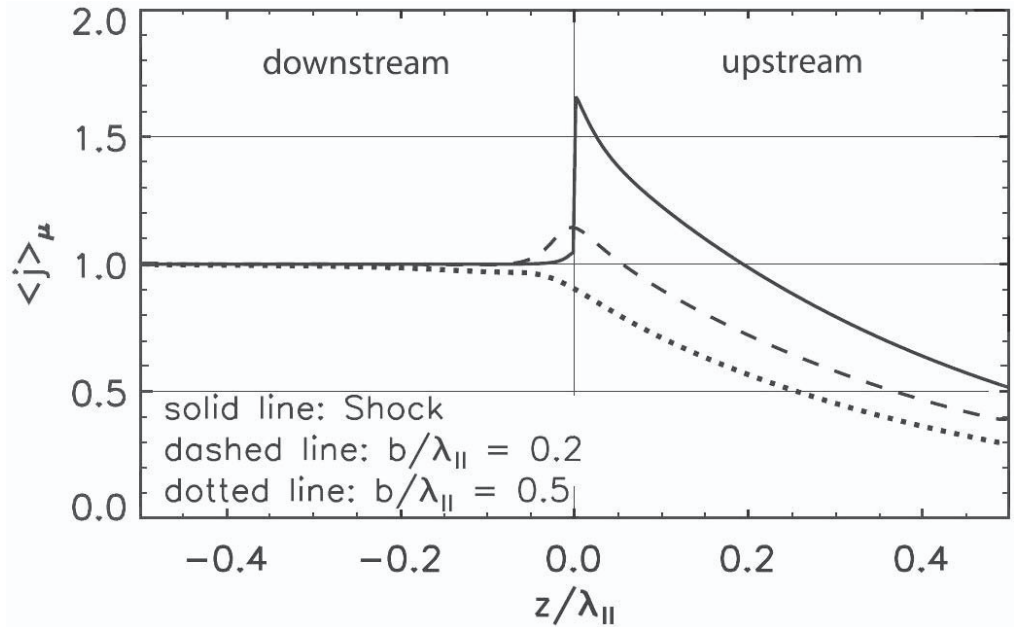


Fig. 4.16.5. Particle density vs. position, obtained by pitch angle treatment, for the Q-Perp case, with a higher peak (or jump) for a narrower compression. According to Malakit et al. (2003).

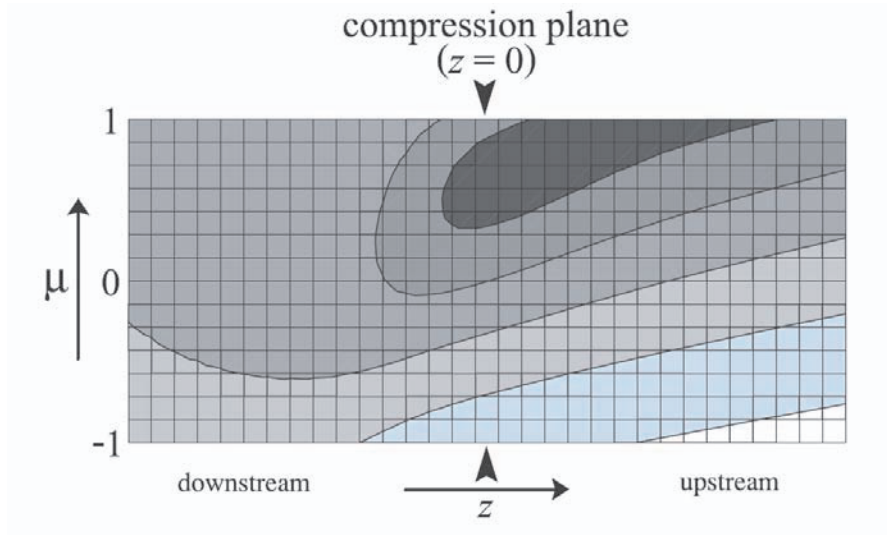


Fig. 4.16.6. Contour plot of the distribution function at a Q-Perp compression region with $b/\lambda_{\parallel} = 0.2$ (darker regions have a higher particle density). According to Malakit et al. (2003).

Malakit et al. (2003) conclude that the mirroring effect leads to more effective acceleration, especially at low energy, evidenced by the hardened spectrum. Another result is that spectra of particles accelerated by compression regions are generally not power laws but rather are hardened at high energy (see Fig. 4.16.7).

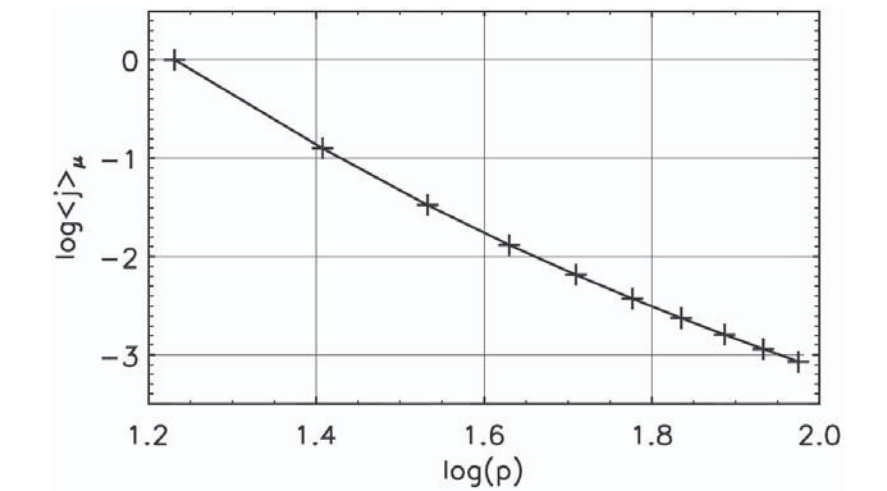


Fig. 4.16.7. Particle spectrum for a compression region with $b/\lambda_{\parallel} = 2.0$. According to Malakit et al. (2003).

Furthermore, Malakit et al. (2003) also found that the spectral index at a given particle energy increases approximately linearly with the compression width for wide compressions.

4.17. The cumulative acceleration mechanism near the zero lines of magnetic field

4.17.1. Injection-less acceleration of particles and the mechanism of magnetic field annihilation

It was shown above (see Sections 4.2–4.12) that the statistical acceleration mechanisms exhibited a high sensitivity to the type of the accelerated particles. At the same time the above mentioned features of the statistical acceleration are not observed for quite a number of events. Namely, the composition of nuclei accelerated in solar flares repeats the composition of the Sun's atmosphere (Dorman and Miroshnichenko, M1968; Dorman, M1978; Dorman and Venkatesan, 1993; Miroshnichenko, M2001) and the acceleration of the electrons observed directly and on the basis of their radio-emission and X-rays (Korchak, 1967) proves to be highly effective. A great number of the accelerated electrons are also present in the Earth's magnetosphere and magnetospheric tail, in the magnetospheres of Jupiter and other planets of the Solar system, in the galactic CR, in the supernova shells, in quasars and radio galaxies. The excess of heavy nuclei in the galactic CR, though observable, is not so high as may have been expected in the case of only statistical acceleration.

The above-mentioned data have to be explained based on some other mechanism of acceleration, or at least injection, which would not display the high sensitivity to the type of the accelerated particles inherent to the statistical mechanisms. It is of great interest from this viewpoint to consider the mechanism of annihilation of oppositely directed magnetic fields, because in this case the electric field induced in the vicinities of the zero line will accelerate all the particles of the medium in a region of a relatively small volume. It should be noted that amongst numerous theoretical mechanisms involving the magnetic field's annihilation; when interpreting the flare processes on the Sun and the particle acceleration, the theories considering the rapid rearrangement and dissipation of magnetic field in the class of two-dimensional streams seem to be most promising at present.

The first step in this direction was made by Sweet (1958), who examined the one-dimensional stationary compression of the plasma between two anti-parallel layers with due account of plasma streaming in the transverse direction along the layer. After qualitatively estimating Sweet's model, however, Parker (1963) showed that such model gave excessively long times to be necessary for the energy to be released. In order to obtain smaller times of magnetic field dissipation as compared with the values obtained by Sweet's model, Sweet's mechanism was examined by including the ambipolar diffusion. The essence of ambipolar diffusion is that the magnetic field moves with the electrically conducting ionized plasma component

whose motion relative to the neutral component is retarded by the friction owed to ion collisions with neutral atoms. Owing to this phenomenon, a decrease of the effective conductivity may be included in the estimate of the magnetic field's dissipation rate. Petchek (1964) reconsidered the Sweet-Parker mechanism and showed that even if the mechanism of ambipolar diffusion is included the time necessary for the magnetic energy to be converted into thermal energy still remains too great. In Petchek (1964) the Sweet-Parker model is also supplemented with magneto-hydrodynamic waves whose propagation will be of great importance to the stable plasma streams of high conductivity. According to Petchek's estimate for a compressible flux corresponding to solar flares, the energy necessary to a flare may be released within $\sim 10^2$ sec. Thus the model of Petchek (1964) can satisfactorily explain the rapid heating and ejections of plasma in the region of solar flares. In that theory, however, the problem of generation of acceleration particles remains to be solved.

4.17.2. Current sheets and rapid rearrangement of magnetic fields

The mechanism of particle acceleration in the case of magnetic field dissipation was further developed in the works of Syrovatsky (1966, 1967, 1968, 1969, 1971) which deal with the cumulative mechanism of acceleration near the zero lines of magnetic field, the mechanism that ensures acceleration in individual small regions of plasma for all charged particles irrespective of their properties. According to Syrovatsky (1966, 1967, 1968, 1969, 1971) such acceleration must take place in the vicinities of current sheets in the case of rapid rearrangement of magnetic field. According to the above-mentioned works, the magnetic field in the space is rapidly rearranged under the conditions of seeming ideal frozenness of the field. Such rearrangement is accompanied by particle acceleration, for example in chromospheric flares. Syrovatsky (1971) examined the rearrangement mechanism in detail. It is assumed in Syrovatsky (1971) for the sake of simplicity that the magnetic fields are plane and the plasma moves in a strong magnetic field, i.e.,

$$P/\rho u_A^2 \ll 1; \quad u^2/u_a^2 \ll 1, \quad (4.17.1)$$

where P and ρ is the pressure and density of the plasma; $u_a = H/\sqrt{4\pi\rho}$ is the Alfvén velocity; u is the velocity of plasma motion. In this case the equations of magneto-hydrodynamics for the perfectly conducting plasma will be written in the form

$$\frac{d\mathbf{A}}{dt} = \frac{\partial \mathbf{A}}{\partial t} + \mathbf{u} \nabla \mathbf{A} = 0; \quad \Delta \mathbf{A} = 0; \quad \frac{\partial \mathbf{u}}{\partial t} \times \nabla \mathbf{A} = 0; \quad \frac{\partial P}{\partial t} = -\rho \operatorname{div} \mathbf{u}. \quad (4.17.2)$$

Here $A(x, y, t)$ is the unique non-zero component of the vector-potential $\mathbf{A}$, so

$$H_x = \partial A / \partial y; \quad H_y = -\partial A / \partial x; \quad H_z = 0. \quad (4.17.3)$$

It has been shown in Syrovatsky (1971) that if the field of the external field sources in the volume studied has singular zero points then there exist regions of non-analytical solution of the equation system described by Eq. 4.17.2. By virtue of the assumption Eq. 4.17.1 the non-analyticity region may consist of only isolated points (linear currents) and cuts (plane currents). If the intensity of the linear current was zero at $t = 0$ (the simplest zero point of the type X exists at that moment, see Fig. 2.17.1a), and then increased gradually with time, the zero point should be 'doubled' and two points of the type X with a region of closed force lines between them should, appear (see Fig. 2.17.1b).

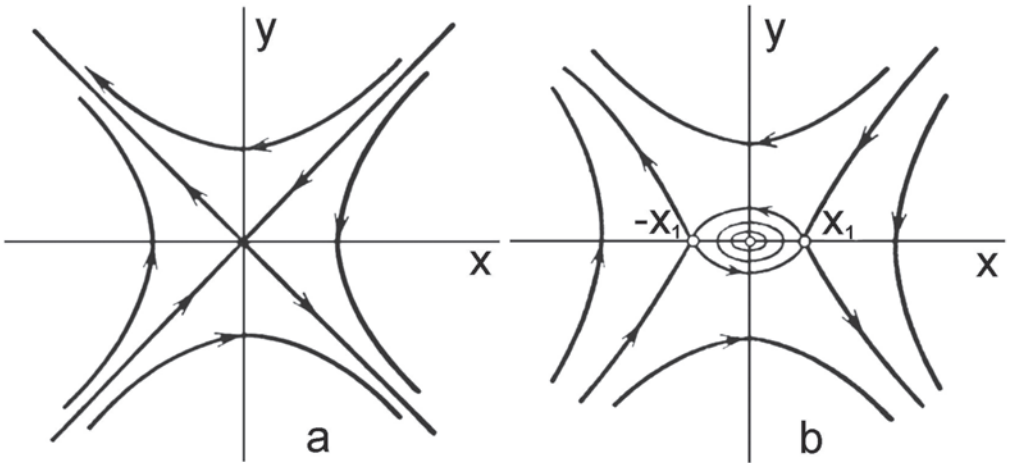


Fig. 4.17.1. When linear current I appears at the singular zero point (panel a), the zero point is 'doubled' (panel b). According to Syrovatsky (1971).

Under the condition of frozenness, however, the closed force lines cannot be obtained as a result of permanent deformation of the initial field which did not comprise such lines. Thus the isolated singular points cannot be used to construct a solution for Eq. 4.17.2 that will be continuous in the rest of the space. As a result only the solutions with cuts have to be accepted, and Syrovatsky (1971) asserts the following: if the external field comprises singular zero points the plane currents or (which is the same) the current sheets are formed in the plasma near such points. The location of the cuts corresponding to current sheets on the complex plane should be such that the boundary problem for Eq. 4.17.2 would have an infinite solution everywhere beyond the cuts. The following rule may be formulated to determine the location of the cuts. A cut must include the initial zero point and all zero points appearing if the initial zero point contain a linear current varying from zero to some finite value. In this case, the linear current direction must coincide with the electric

field's direction. The analysis carried out in Syrovatsky (1971) shows that a zero point is the place of development of a current sheet (see Fig. 4.17.2) in which the current direction coincides with the electric field direction excluding for small sectors at the edges of the cut where the current direction is opposite (the back currents in the region of zero line are generated and maintained not directly owing to the external electric field $\mathbf{E}$ but as a result of the plasma motion inhomogeneity caused by this field).

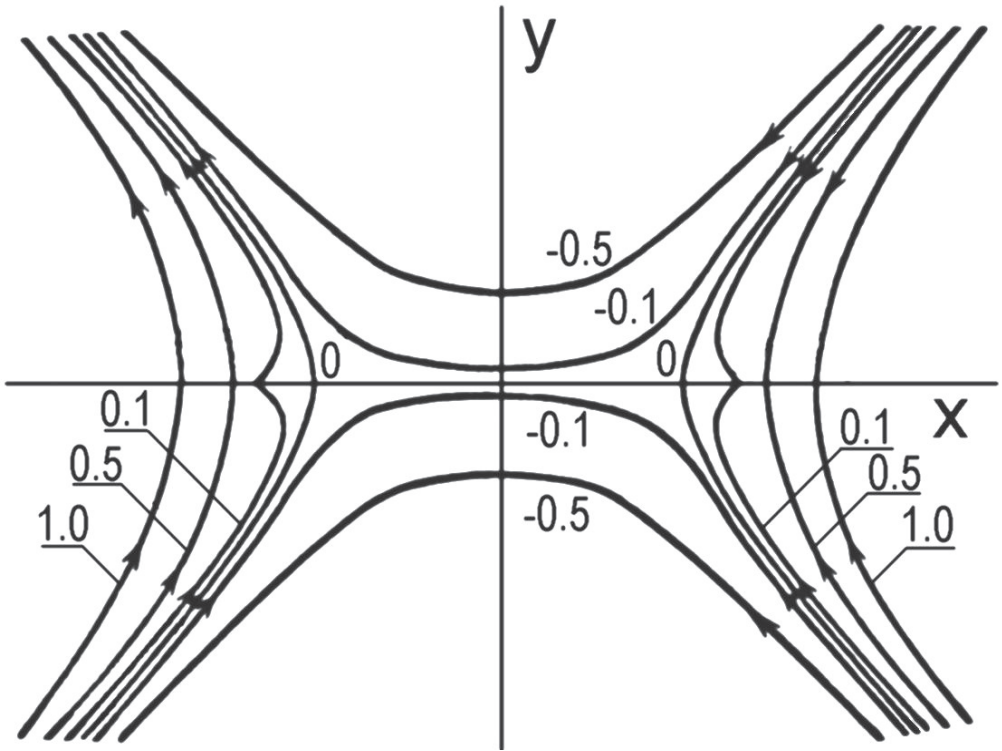


Fig. 4.17.2. The current sheet developing in the place of the zero point. The numerals show the value of the potential $\mathbf{A}$ on the corresponding force lines. According to Syrovatsky (1971).

Only in the special case in which the total current may be coordinated with the external field so that the inverse currents will be absent (this may be realized for sufficiently slow motions and at a finite, though high, conductivity of the plasma when the inverse currents are inconsiderable; in this case the width of the sheet is completely determined by the external field, see Fig. 4.17.3).

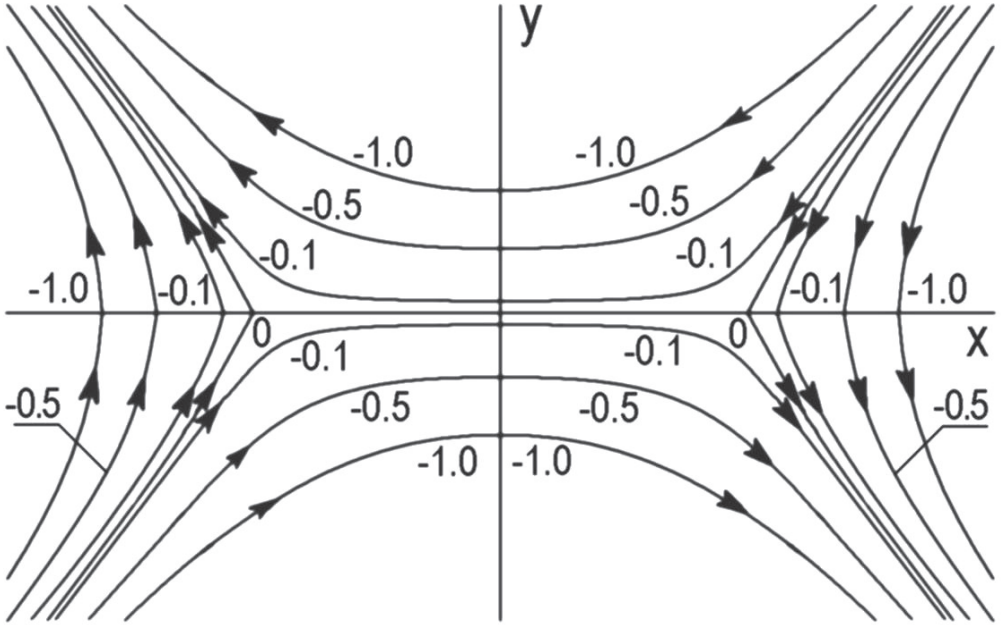


Fig. 4.17.3. The current sheet developing from the zero point in the absence of the inverse currents. According to Syrovatsky (1971).

In Syrovatsky (1966) the magneto-hydrodynamic equations have been solved for two-dimensional motions of the plasma in the regions where the plasma dynamics is completely determined by magnetic intensities, i.e. the condition

$$P \ll H^2/8\pi \quad (4.17.4)$$

is satisfied and the plasma motion equation turns out to be

$$\frac{\partial \mathbf{u}}{\partial t} = -\frac{1}{4\pi\rho} \Delta \mathbf{A} \nabla \mathbf{A}, \quad (4.17.5)$$

where $\mathbf{A}$ is the vector-potential of the magnetic field. It has been shown that in this case the disturbance of the initial equilibrium state results in a rapid (with the Alfvén velocity u_a) establishment of a certain new quasi-equilibrium state through a shift across the force lines; the system will run a number of quasi-equilibrium states determined by the Eq. 4.17.4.

The picture of the shift of the plasma and the frozen-in magnetic field shown in Fig. 4.17.4 may be obtained by treating the above mentioned problem for the region with a neutral point appearing between two parallel currents located on the x -axis

and shifted by a small distance $\sim \sqrt{\delta}$. It follows from the calculations of the Jacobean of transition from non-shifted to shifted coordinates that the regions with strong rarefaction and compression of the plasma are formed in the region $r \gg r_s$ (r_s is the distance at which the Alfvén velocity equals the sonic velocity) during the shift.

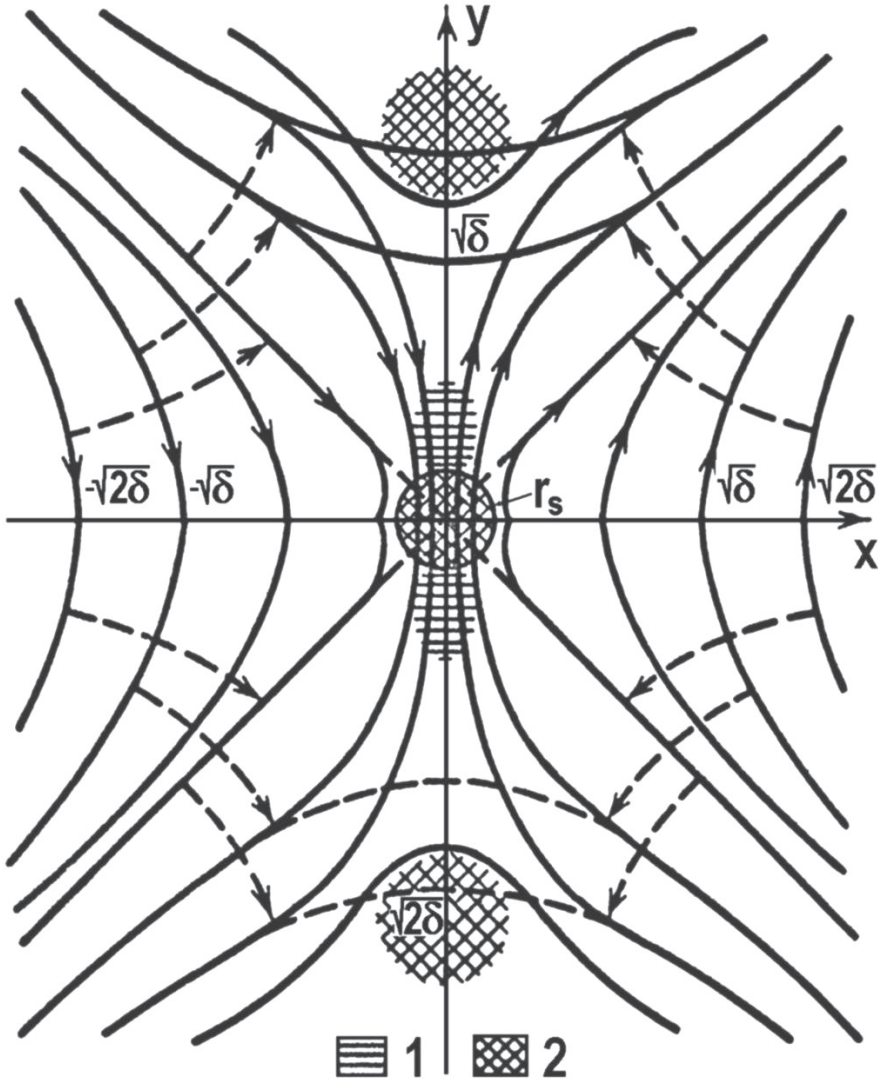


Fig. 4.17.4. Qualitative pattern of magnetic field deformation in the model of magnetic dissipation proposed by Syrovatsky (1966). 1 - rarefaction region; 2 - region of strong compression. The dashed arrows indicate the field deformation directions.

The nontrivial region $x \leq \sqrt{\delta}$ also comprises a strong-compression region where the characteristic ratio of the field gradient to plasma concentration is

$$\frac{h}{N} \approx \frac{h_o}{N_o} \frac{\delta^2}{r_s^4}, \quad (4.17.6)$$

where h and h_o are the magnetic field gradients.

The value determined by Eq. 4.17.6 defines the criterion of violation of the frozenness in this region since in virtue of the quasi-stationary equation

$$\text{rot} \mathbf{H} = \frac{4\pi}{c} \mathbf{J} = \frac{4\pi}{c} \sum_i N_i e \mathbf{V} \quad (4.17.7)$$

the ratio h/N cannot exceed the evident limit (since $V < c$):

$$h/N < 4\pi e, \quad (4.17.8)$$

where N is the concentration of the charges of both signs in the plasma.

Violation of Eq. 4.17.8 means that the charge concentration becomes insufficient to balance the increased gradient of the magnetic field. This gives rise to the induced electric field $\mathbf{E}$ which is directed along the current $\mathbf{J}$ and positively affects the particles, thereby increasing their energy. It is this process that provides the magnetic energy conversion into the particle energy, i.e. the dynamic dissipation of the magnetic field. It is characteristic that the process examined is not associated with the Joule dissipation, the current density is saturated when the frozenness condition is violated, and the field energy is lost to increase the total energy of the particles $E = \sqrt{c^2 p^2 + m_{ac}^2 c^4}$. It follows from the estimate of the energy concentrated in the magnetic compression region

$$W \approx (1/8) h_o^2 \delta^2 \quad (4.17.9)$$

that a fraction of the order of δ of the total released energy is concentrated in the compression region. The remaining energy will be lost for compression in the region $y \approx \sqrt{2\delta}$ and field deformation in the surrounding space.

The temporal characteristics of the processes are given by the relation between the pressure decrease in the examined region and the balancing gas-dynamic motion of the plasma. It has been shown in (Syrovatsky, 1966) that the plasma influx cannot balance the decreasing pressure if the basic currents are shifted at velocity $u \gg u_s \sqrt{\delta}$, where u_s is the sonic velocity in the plasma. Since the motions in the

region of the intense field take place at the magneto-hydrodynamic velocity $u_a \gg u_s$, the process considered above may be realized under various conditions in the space.

A further development of this mechanism was given in (Syrovatsky, 1975). It was determined the electric and magnetic fields arising with a discontinuity of the neutral current sheet in connection with the problem of particle acceleration. The neutral sheet is considered to be an infinitely thin and perfectly conducting formation. It has been shown that a weak zero line of a magnetic field was arising with the electric field directed along it.

Syrovatsky (1975) has considered the role of the neutral current sheets in the dynamics of magnetized plasma. The main parameters have been presented which determine the flow near the zero lines of a magnetic field, and their typical values for solar flares and for model experiments. The approximation of a strong magnetic field and the properties of a neutral current sheet in the stationary regime have been examined. It has been noted that there exist two modes of magnetic field dissipation: quasi-stationary, and explosion modes. It was shown that the main effect of a magnetic field's longitudinal component along the zero line comes to a decrease of the effective Mach number of the plasma flow. The interesting model study of plasma behavior in the vicinity of zero lines was carried out by Frank (1975). The results of model laboratory experiments for investigation of magnetic field structure and plasma dynamics in the vicinity of a magnetic zero line were presented in this paper. With the methods of phase location and holographic interferometry it has been shown that the plasma, similarly to a current, transforms into a sheet. Redistribution of electron density over a sheet thickness presents the evidence for a sheet to be broken into separate current filaments. A character of this discontinuity depends on the conditions of an experiment.

4.17.3. A development of magnetic field annihilation models and the model of magnetic force line reconnection; on the role of discharge phenomena in some astrophysical processes and particle acceleration

Priest (1972) has examined the mechanism proposed by Sweet (1958) for magnetic energy transformation into others forms of energy. The method of boundary layer theory was used to obtain the profiles (along a current sheet) of the quantities characterizing a flow. Priest and Sonnerup (1975) have presented a review of the stationary hydro-magnetic models of magnetic field annihilation. The exact three-dimensional solution of magneto-hydrodynamic equations has been described in which a magnetic field was parallel to the yz -plane but varied (depending on the coordinate x) in its value and direction. The application of the theory to the geomagnetic field near the sub-solar point of the magnetopause was considered in the case of slow reconnection of magnetic force lines. The intensity of the induced electric field was calculated depending on the angle between the interplanetary and geomagnetic fields with the condition that the interplanetary field was perpendicular to the Sun-Earth line and plasma density in the current sheet was constant. Yeh

(1976a) has investigated a diffusion hydro-magnetic flow in the vicinity of a neutral point. Yeh (1976a) has shown that the electrical resistance determined the time scale of approaching to the stationary state in the case of hydromagnetic interaction of conducting fluid at the merging of magnetic force lines near a neutral point. These proper scales differ from those which are ascribed to the flow when it is observed from outside. In the stationary state a transformation of magnetic energy into kinetic and thermal energy requires that the Alfvén number should be less than unit for the upward flow. The solutions in the vicinity of a neutral point show that Ohm's law for a simple resistance is applicable to a modeling of the magnetic field force line reconnection. Rüdiger (1975) has studied the interaction between a homogeneous turbulence and a non-uniform magnetic field in the vicinity of a neutral surface. It was shown that initially uniform and isotropic turbulence field becomes non-uniform and anisotropic in such a magnetic field. The finite correlation length results in the turbulence field being affected also on the neutral surface. The anisotropic decrease of motions in the vicinity of the neutral surface was determined for some special forms of one- and two-dimensional turbulence. Furthermore, the effects of the action of such a non-uniform field of turbulence onto an average magnetic field have been found. Using the Bochner's theorem on the spectral tensor of initially homogeneous turbulence, an additional decrease of an average magnetic field was obtained. Yang and Sonnerup (1976) have generalized a model of the field reconnection in non-compressive fluid for the case of compressive fluid. The properties of two plasma streams ejected from a reconnection region depending on plasma properties and on the inflow velocity were determined by means of numerical integration using the conditions on a shock wave front. The possibility was discussed of the existence of fast transverse shock waves in the outward streams. Fukao and Tsuda (1973) have solved a non-stationary problem of magnetic field line reconnection in the magnetic hydrodynamics of non-compressive fluid. A numerical model experiment has been carried out for a plane of non-compressive viscous conducting fluid in the vicinity of a stagnation line. The velocity and magnetic fields (the magnetic field of a plane current sheet which is orientated along one of the asymptotic directions of plasma flow from the stagnation line) were set as the initial conditions. The results of the solution have been presented; the dependence of the velocity and magnetic field components, the current density, etc, upon the coordinates and time. The magnetic field reconnects in the vicinity of the neutral line arising on the stagnation line of the stream. The velocity of reconnection is increased with the growth of the flow's initial velocity. The interpretation of the results is complicated by the fact that the procedure of calculation itself introduces disturbances which are equivalent to a certain effective diffusion of a magnetic field. The results obtained do not come out of the stationary regime during the time intervals for which the calculating scheme holds. However, some characteristic features (proper to the stationary solutions obtained earlier) can be as well recognized, in the opinion of the authors, in the non-stationary picture obtained.

Yeh (1976b) has studied reconnection of magnetic lines of force in a viscous conducting fluid. Two-dimensional solutions of the equations of non-compressible magnetic hydrodynamics including the terms of viscosity and conductivity have been considered. The solutions were obtained describing the flow which is typical for the case of the reconnection of magnetic force lines. These solutions do not contain any discontinuity. It has been shown that magneto-hydrodynamic flows in the problem of magnetic field line reconnection which were obtained in a non-dissipative approximation can be substantially different from the flows which were obtained in the case in which viscosity and conductivity are tending to zero in a dissipative flow. The properties of the solution obtained confirm the substantial role of boundary conditions far from the neutral point on the character of magnetic field line reconnection.

The paper of Bruce (1975), in which the results of more than hundred publications during the thirty years are summarized, is devoted to a study of the role of the process of electric discharge in various astrophysical phenomena. The phenomena have been described related to electric discharges: the propagation velocity, magnetic fields which are circular with respect to a discharge axis, pinch-effect; plasma jets; radiation of whistlers; and so on. An attempt was made to identify the electric discharges with a wide class of astrophysical phenomena. It is adopted that electric discharges give a certain contribution to the solar flares where a current of $\sim 10^4$ A is reached and the magnetic fields of the order of 10^4 – 10^5 Gs are required which are not observable with solar magnetometer owing to localization and absence of the neutral atoms. The variation of long period variable stars, a filamentary structure of the Crab nebula, the abrupt changes in the brightness curves of Novae, the giant scale eruptions in the radio galaxies and some other astrophysical processes following particle acceleration are explained basing on the electrical discharges.

4.17.4. Particle acceleration in the neutral current sheets

Bulanov and Syrovatsky (1972) have considered two models of charged particle acceleration by electric fields near the neutral current sheets. **For the first** example, particle acceleration by the electric field arising at the instantaneous decay of the sheet was considered. It was shown that a considerable share of magnetic field energy can be transferred to the particles energy. **In the second** case the interaction of a stepped electromagnetic impulse of finite amplitude with a neutral current sheet has been examined. If the impulse amplitude is sufficiently high, non-limited particle acceleration will take place in the sheet.

Levine (1974) has studied the behavior of thermal particles in the vicinity of a neutral sheet. It has been shown that the field compression towards the neutral sheet would produce particle acceleration. A mean energy increase was calculated both with no account of and including the Coulomb losses. Coulomb losses complicate the picture to a high degree. In particular, acceleration does not take place in certain directions of particle motion; the acceleration of electrons is far more complicated

problem than that of proton acceleration. The case is possible in which practically only the protons will be accelerated. The case of very rapid field compression was considered separately.

Particle acceleration in the vicinity of magnetic field neutral line has been studied as well by Bulanov and Sasorov (1974, 1975). Charged particle acceleration in a hyperbolic magnetic and uniform electric field has been investigated. The acceleration process consists from the direct acceleration by an electric field in a non-adiabatic region near the neutral line and from the betatron acceleration in a drift region. The characteristic energy of accelerated particles in non-relativistic and super-relativistic limits was determined. In the high energy region the energy spectra have an exponential form.

Bulanov and Syrovatsky (1976) have studied analytically and computed numerically the motion equations of charged particles in a uniform electric field which is directed along the zero of a hyperbolic magnetic field. The region of substantially non-adiabatic motion is located near the zero line, in which the particles are directly accelerated by electric field. Outside this region the particles are in a drift motion consisting of oscillation along the force lines and of a displacement across them; the oscillation amplitude is slowly increased (decreased) with moving away from (approaching to) the zero line. When approaching the zero line the adiabatic cooling of particles takes place and their heating takes place in the case of moving away.

4.17.5. Mechanism of magnetic field dissipation in a current sheet including non-anti-parallelism of the magnetic field, instabilities, and turbulence

Cowley (1976) has examined the phenomenon of reconnection of non anti-parallel magnetic fields. Some modifications of the existing models of magnetic field line reconnection in the vicinity of the x -line (where the effects of finite conductivity are substantial) have been considered. In the models of this kind (for non-compressible conducting fluid) the magnetic fields and the stream velocities in the 'convective region' are less restricted than was assumed earlier. In the general case it can be argued that:

- 1) the magnetic field components and stream velocities normal to the x -line should change their sign to the opposite of that in the current sheet and its vicinity, the absolute values of the field and velocity, however, can be different on both sides of the sheet;
- 2) the components parallel to the x -line are arbitrary and can undergo arbitrary jumps in the sheet;
- 3) the second statement results in that the current in the sheet is not obligatory parallel to the x -line.

The mechanism of fast magnetic field dissipation in the current sheets, which are formed by the velocity gradients in the solar wind and in the magnetosphere's tail was considered by Vainshtein and Tomozov (1975). Since plasma with these

conditions is collisionless, the dissipation processes are determined by the effective electric conductivities that are formed at a development of current sheets of ion-sound turbulence in plasma. The characteristic energies of electrons accelerated in the solar wind in the process of outflow of the magnetosphere have been estimated. Galeev and Zeleny (1976) have considered a development of the tearing mode of instability in plasma with a diffusive neutral sheet at the presence of a magnetic field normal to the sheet component with a small, but finite, value. The influence of this component on the electron orbits in the vicinity of the neutral sheet results in a stabilization of the electron-tearing mode even at very small amplitudes of the normal field. A development of the ion tearing mode of an instability of a given wave length is possible only in a 'slot'. This slot is a limited range of the normal magnetic field component for which it is on the ion orbits in the neutral sheet can be neglected but a stabilizing contribution to plasma dielectric properties (produced by the magnetized electrons) becomes small. It has been shown that the slot formation is possible only in the case of a neutral sheet with sufficient current. The states of plasma with the values of normal to the sheet of magnetic field component which are under the instability region appeared to be the metastable states with respect to generation of the main discontinuity mode.

Basing on the analysis of a tearing instability in a neutral sheet region, Pustil'nik (1973, 1976, 1977a,b, 1978, 1980, 1999a,b,c, 2001), Pustil'nik and Stasyuk (1973, 1974) have developed detailed models of solar flares (chromospheric and coronal) and the corresponding mechanisms of solar CR acceleration. This problem will be consider in detail in the next Section 4.18.

4.18. Tearing instability in neutral sheet region, triggering mechanisms of solar flares, turbulence, percolation, and particle acceleration

4.18.1. The problem of solar flare origin, particle acceleration and ejection into solar wind

The studies of solar flares (Kaplan et al., 1974; Syrovatsky, 1972; Pustil'nik, 1976) have shown that the flare occurs due to an anomalously rapid dissipation of the magnetic field in the solar atmosphere. Such dissipation takes place in the current sheet in the magnetic field shear zone between two magnetic tubes. The anomalous dissipation is triggered when the plasma of the initial 'thick' sheet become turbulent owing to a strong current corresponding to the electric field

$$E > E_{dr} = \frac{m_e v_{Te}}{e} v_{\text{eff}}, \quad (4.18.1)$$

where v_{T_e} is the velocity of thermal electrons and ν_{eff} is the effective frequency of collisions. According to Pustil'nik (1977a), the problem of flare origin is thereby reduced to the search for the initial disturbance resulting in the abrupt increase of the current and electric field in the sheet. Three types of hypotheses about the nature of such disturbances are considered: (1) an abrupt disturbance of the photospheric magnetic field (is at variance with the observations of Rust (1968) which show that the field is not disturbed before the flares); (2) transition of the field into an unstable state with the discontinuity modes of the tearing type (faces the difficulties of principle owed to stabilization of such instabilities by the longitudinal magnetic field and due to a slow mode of their occurrence (Fürth et al., 1962; Biskamp et al., 1970); (3) the instability of plasma formations hanging in the solar atmosphere near the initial sheet. Version (3), described in detail by Pustil'nik (1976), will be treated in detail here. Only two types of such formations exist in the solar atmosphere, namely the quiescent prominence and the coronal condensation.

4.18.2. The prominence channel of flares

The appearance of new magnetic fluxes in an active region should result, on the one hand, in a storage of the energy of the non-potential strength-less magnetic field, and, on the other hand, in a depression of the upper arc of the force lines (Kiepenhahn and Shindler, 1957; Syrovatsky, 1966). As it was shown by Pikelner (1971), such depression should leak to the quiescent prominence within a period $\tau_{\text{syphon}} \approx 10 \div 30$ hours. In other words, a situation takes place in this region which is sufficiently unstable relative to the balloon modes of the flute instability, i.e. a heavy prominence ($n_{\text{pr}} \geq 10^{10} \text{ cm}^{-3}$) above the light corona ($n_{\text{cor}} \approx 10^8 \text{ cm}^{-3}$). As it was shown by Pustil'nik (1973), when a prominence exceeds the critical value

$$d_{\text{pr}} \geq d_{\text{cr}} \approx 5 \times 10^8 \text{ cm}^{-3} \left(\frac{H}{50e} \right) \left(\frac{n_{\text{pr}}}{3 \times 10^{10} \text{ cm}^{-3}} \right)^{-1}, \quad (4.18.2)$$

it should be stabilized and eject downwards a flute with a field which, in its turn, should wobble with period $\tau_1 \approx 1 \div 3$ min and move downwards at a velocity

$$v_o \approx \sqrt{g_o n_{\text{pr}}} \approx 10^{6.5 \div 7.0} \text{ cm/sec}. \quad (4.18.3)$$

In this case, the amplitude of the wobbling is

$$\lambda \approx (\partial \ln n / \partial z)^{-1} \approx 10^{8.0 \div 8.5} \text{ cm}; \Delta v \approx 10^{6 \div 7} \text{ cm/sec}. \quad (4.18.4)$$

After colliding at velocity v_o with the low-lying shear zone or a singular point of type X, the flute should disturb the magnetic field in that region and induce an electric field sufficient to create turbulence in the sheet and trigger the flare with electric field

$$E_x \approx (v_o/c)H_x \gg E_{dr}. \quad (4.18.5)$$

Such a 'flare' version of the events requires a sufficiently high degree of the shear, $\theta_x \approx \pi \pm 0.1$, otherwise the prominence will appear in the quiet state with the damping of the excessive mass to the flute-column (Pustil'nik, 1973). The total number of wobbblings of the flute $N \approx n_{pr}/d_{pr} \approx 10 \div 20$, the time of the completed descent $\tau_{act} \approx N\tau_1 \approx (0.5 \div 1.0)$ hours.

The particles accelerated in the sheet will enter the chromosphere and corona and cause the observed flare displays in all bands (Kaplan et al., 1974; Syrovatsky, 1972). The detailed structure of the flare of such type is shown in Fig. 4.18.1.

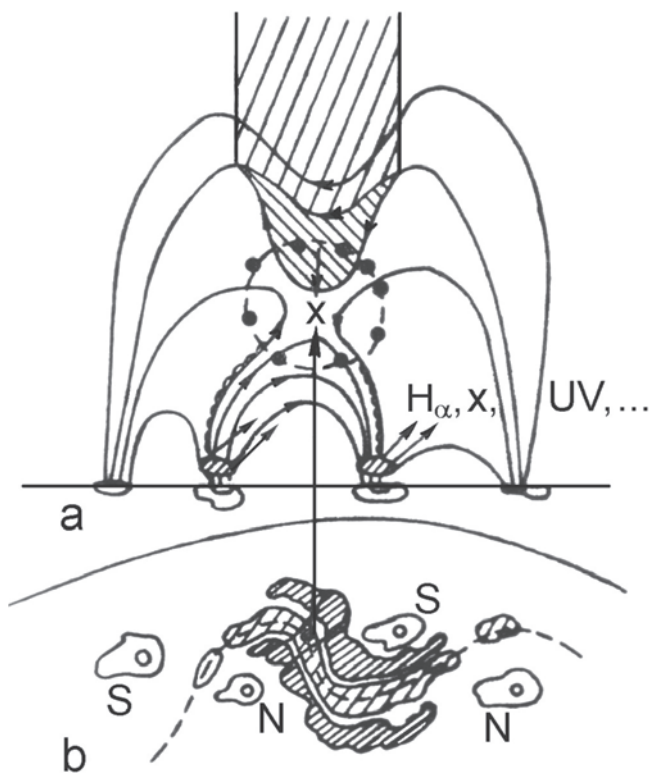


Fig. 4.18.1. The structure of the solar prominence flare (non-proton or proton-delayed). **a** – the structure in the vertical plane; **b** – the view on the photosphere-chromosphere level. According to Pustil'nik (1977a).

A very important property of the prominence of the flares will be noted. Since all the force lines involved in the merging through the current sheet are short-circuited to the photosphere, the direct ejection of accelerated particles from the flare to the solar wind is impossible, i.e. such flares are either of non-proton character or proton-delayed (when the particles leak from the trap due to the Bohm diffusion or an instability of the flute type within $\tau_{\text{del}} \approx 10^{3+4}$ sec after the flare).

The relationship between the solar flares of considered type and the ascent of new magnetic fluxes to the atmosphere has been confirmed by observations (Rust, 1976). The destabilization of the filaments (prominences) within 30÷60 min prior to a flare was observed for overwhelming majority of flares: $\geq 80\%$ according to Ramsey and Smith (1965) and $\approx 100\%$ according to Moreton (1965). The detailed pattern of filament activation also coincides with the predictions (Pustil'nik, 1976).

4.18.3. Non-evolutionary channels of triggering of the prominence type of flares

The development of the lower loop of the field should raise the X-type singular point and the shear zone, thereby decreasing the depth of the depression in the arc and, hence, increasing the thickness of the prominence. If in this case the thickness exceeds the critical value, the prominence will be automatically destabilized and a flare will be triggered within 50-60 min (in case of a favorable measure of the shear) according to Ramsey and Smith (1965). This pattern is probably observed in some cases with the pre-flare rising and thickening of filament (Martin and Ramsey, 1972). If a shock wave from a flare in one region catches up with a filament in a state near the stability threshold in another region, such filament will be excited and turn out to be abruptly unstable with subsequent triggering of a 'prominence' flare (Pustil'nik, 1976). When a fast particle flux is ejected from the outside to the force lines supporting the prominence, then such fast particles will move along the force line depression under the prominence and, creating a centrifugal acceleration $g = v^2/R \gg g_{\text{sun}}$, should make the prominence significantly 'heavier' thereby transferring it to the unstable state. This mechanism is of great importance to the proton flares.

4.18.4. The coronal channel of flares

The observations of Zhitnik and Lifshitz (1972) are indicative of the existence of dense and hot arc-like coronal condensations (cc) over the active region with the following properties:

$$n_{\text{cc}} \approx 10^{10} \text{ cm}^{-3}, \quad n_{\text{cor}} \approx 10^8 \text{ cm}^{-3}, \quad T_{\text{cc}} \approx (4 \div 6) \times 10^6 \text{ K} > T_{\text{cor}} \approx 2 \times 10^6 \text{ K}. \quad (4.18.6)$$

These condensations appear probably owed to overheating of the closed forced tubes by the waves from under the photosphere (Kaplan et al., 1974). Such condensation

hanging from the curved force lines cannot fly apart to the rarified corona due to the action of the force lines and is subjected to the balloon mode of flute instability. Such instability results in the flute soothing of the condensation surface with periods

$$\tau_b \approx 4D/V_a \approx 10^{3 \div 4} \text{ sec}, \quad (4.18.7)$$

and amplitudes

$$\lambda_{\perp} \approx 16\pi(nkT)_{cc} \times (d/H) \approx (3 \times 10^9 \div 10^{10}) \text{ cm}. \quad (4.18.8)$$

Since the condensation is a source of intense radio emission, such soothing should give rise to the fluctuations of the flux and other characteristics of the radio emission (Pustil'nik and Stasyuk, 1974) with a period

$$\tau_{fl} \approx \tau_b \approx 10^{3 \div 4} \text{ sec} \quad (4.18.9)$$

and an amplitude

$$A_o \approx (\Delta J/J_S)_o \approx 0.1 \div 0.3\%, \quad (4.18.10)$$

which was observed by Gelfreikh et al. (1969), Durasova et al. (1971). The permanent heating of the condensation increases the pressure in the condensation and, hence, the flute amplitude. Within the time

$$\tau_{crit} \approx (nkT)_{crit}/q_+S \approx 10^5 \text{ sec}, \quad (4.18.11)$$

such heating should wobble the flute soothing up to an amplitude

$$\lambda_{\perp} = \Delta h_Y = 10^{10.0 \div 10.5} \text{ cm}, \quad (4.18.12)$$

where $q_+ = 6 \times 10^6 \text{ erg/(cm sec)}$ is the power of the heating by the waves (Kaplan et al., 1974) and Δh_Y is the distance to the Y-type singular point. In this case the radio fluctuation amplitude should increase by

$$A_{crit}/A_o \approx 10 \div 30 \text{ times}. \quad (4.18.13)$$

This process will be accompanied by the disturbance of the Y-type singular point and the overlaying current sheet by the flutes from the condensation colliding with them at a velocity $V_o \approx 10^7 \text{ cm/sec}$. The electric field

$$E_Y \approx (V_o/c)H_Y \gg E_{dn} \quad (4.18.14)$$

and the currents

$$j > j_{crit} \geq nec_{si} \quad (4.18.15)$$

induced in the vicinities of the singular point will turbulently the plasma in that region and trigger a durable and low-power flare process (Ramsey and Smith, 1965; Pustil'nik and Stasyuk, 1974). It is of great importance that this type of flares should result in a direct injection of weak fluxes of the accelerated particles into the solar wind along the force lines open into the wind. In other words, these are the 'weak proton' flares (see Fig. 4.18.2).

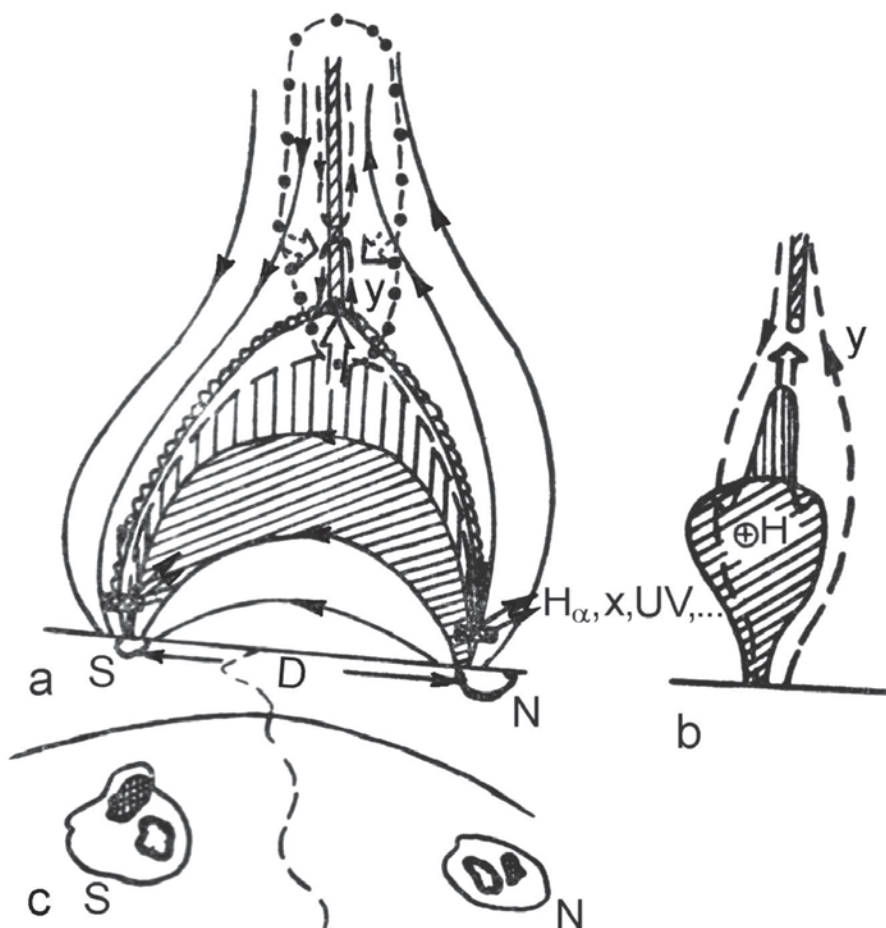


Fig. 4.8.2. The structure of weak-proton flare: **a** and **b** – in the vertical plane, **c** – the view on the photosphere-chromosphere level. According to Pustil'nik (1977a).

Comparison with observations shows that the realization of this type of flares is also fairly high, namely the weak flares with low-frequency bursts of type III are localized above the spots (Pustil'nik, 1976) and a considerable number of such flares give rise to faint bursts of solar CR in interplanetary space (Dorman and Miroshnichenko, M1968; Dorman, M1972, M1978; Miroshnichenko, M2001). Besides that, the kink-instability occurring in case of excessive twisting of the spots and the 'sporadic overheating' of the coronal condensation from under the low-lying prominence flare may be treated as the additional (non-evolutionary) ways of flare triggering.

4.18.5. Powerful proton flares

Examine the situation taking place when a large spot of a new loop of the field (photospheres field δ) appears in the penumbra. The X- type singular point with the depression of the upper open force lines over this point and the Y-type coronal singular point will appear in such region (see Fig. 4.18.3).

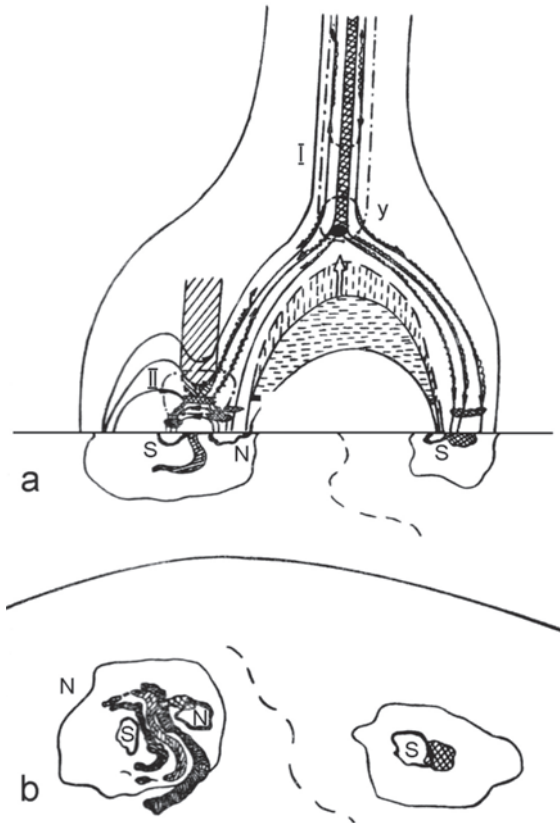


Fig. 4.18.3. The structure of powerful proton flare: **a** – in the vertical plane, **b** – the view on the photosphere-chromosphere level. According to Pustil'nik (1977a).

At the same time, the closed tubes of the upper field are also overheated, a coronal condensation with surface flute soothing is formed, and the chromosphere plasma leak into the force line depression. Similarly to the process described above, in Section 4.18.2, the overheating of the coronal plasma will result in the triggering of a flare in the upper corona. An accelerated particle flux will be ejected downwards from that region. When such flux reaches the field line depression above the X point, the centrifugal acceleration produced by the flux will make the plasma in the depression heavier and the system will turn out to be unstable with subsequent downward ejection of a flute with plasma, fast particles, and field. According to Pustil'nik (1977a), the total energetic of the coronal flares is quite sufficient for the above process to occur. As a result, a flare dissipation of the field will be triggered in the lower current sheet in the region of the strong field of the spot. Such flare is characterized by: (i) a high power owed to the strong field, (ii) the direct injection of intense fluxes of accelerated particles into the solar wind along the open force lines (see Fig. 4.18.3). In other words, this is the classical power proton flare.

All basic predictions bearing on this type of flares are actually realized in the observed proton flares; the proton flares occur in the region with magnetic geometry of type δ (Dodson and Hedeman, 1964), they are characterized by a pre-flare phase (pre-burst) within 20-60 min before the main flare (Lincoln, M1973), and, finally, the study of the radio fluctuations before the proton flares has shown that the overwhelming majority of the proton flares observed were preceded by a strong wobbling of hourly radio fluctuations which began a day before the flare and caused, a 10÷30-fold increase of the fluctuation amplitude by the moment of flare (Kobrin et al., 1975, 1978).

4.18.6. The problem of particle acceleration in the current layer of solar flares

According to Pustil'nik (1977b), the particle acceleration during anomalous dissipation of magnetic field is one of the most important processes in the problem of solar flares. The available observational data obtained by both direct measurements in the interplanetary space and indirect methods permit the following conclusions about the characteristics of accelerated particles to be drawn (Dorman and Miroshnichenko, M1968; Dorman, M1972): (i) the spectrum of the electron component is of the power form $D_e(E_e) \propto E_e^{-\gamma_e}$ with exponent $\gamma_e = 2.5 \div 4$ and the most frequently realization $\langle \gamma_e \rangle \approx 3$; (ii) the spectrum of the proton-nuclear component is also of a power form with exponent γ_p and an abrupt cut-off at energies $E_{p,cr}$ from few up to 10÷20 GeV; (iii) the chemical composition of the accelerated particles is similar to the chemical composition of the solar atmosphere.

According to the modern concepts, the flare energy is released in the turbulent current sheet where the high density of current $j \geq nec_{si}$ and the anomalously low conductivity result in a high power of the energy release in the course of magnetic

field dissipation $P \approx j^2/\sigma^*$. The theoretical estimates of the power are in a good agreement with the observation data. The thickness of the sheet may be estimated by equalizing the latest $P \approx 10^{28+29}$ erg/sec to the rate of the field processing in the sheet:

$$a \leq \frac{H^2 S v_d}{4\pi P^*} \leq \left(\frac{H}{300e} \right)^2 \left(\frac{\sigma}{10\omega_{oe}} \right)^{-1} \left(\frac{n}{10^8 \text{ cm}^{-3}} \right)^{-1/2} \left(\frac{P}{10^{28} \text{ erg/sec}} \right)^2 10^5 \text{ cm}, \quad (4.18.16)$$

Here $S = 10^{19} \text{ cm}^2$ is the area of the current sheet; $v_d = c^2/(4\pi\sigma a)$ is the rate of the field diffusion into the sheet. Since the particles accelerated in the field, are magnetized and move only along the force line, the maximum energy $E_{\max}$ which they may gain in the electric field E of the sheet, $E = j/\sigma$, is limited:

$$E_{\max} \approx ZeEa \approx Ze \frac{j}{\sigma} a \approx 100 \text{ keV} \times \left(\frac{H}{300 \text{ Gs}} \right) \left(\frac{v_{\text{eff}}}{0.1\omega_{oe}} \right) \left(\frac{n}{10^8 \text{ cm}^{-3}} \right)^{-1/2}, \quad (4.18.17)$$

which is much smaller than the observed energies. As it will be seen below (see Section 4.18.8), the visible paradox is associated with disregard of the three-dimensionality of the sheet.

The existing mechanism of acceleration may be broken into two following groups according to the inclusion of electric field and turbulence.

(1) Regular acceleration in the electric field of the sheet (Syrovatsky, 1966) (see the scheme in Fig. 4.18.4 along Z -axis).

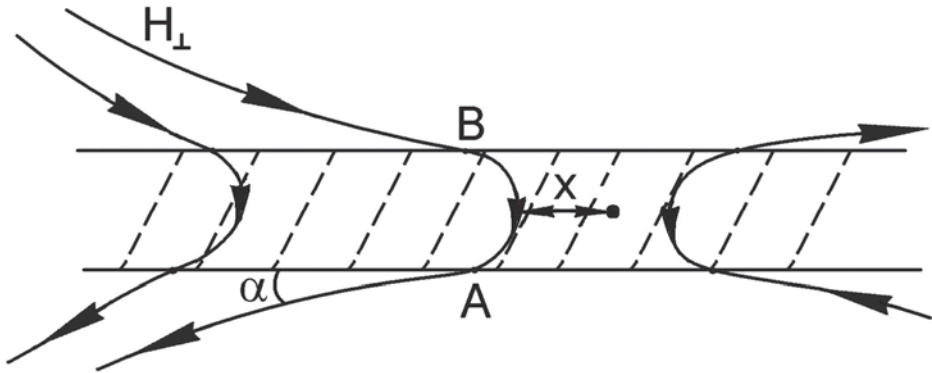


Fig. 4.18.4. The scheme of the sheet in the frame of two-dimensional geometry. According to Pustil'nik (1977b).

This approach gives rise to difficulties of principle when explaining the spectrum and chemical composition of the accelerated particles. It is of importance that the neglecting of the particle-plasmon elastic collisions involved in this approach is barely incorrect, since the sheet is opaque for them ($\nu_{eff}^p \gg l/\nu$).

(2) Stochastic acceleration in the turbulent pulsation field resulting in energy diffusion. In terms of this approach, the particles are accelerated in the particle-plasmon inelastic collisions (predominantly with the Longmire plasmons). Though this approach can more realistically include the conditions in the current sheet of a solar flare, it also faces difficulties due to a small thickness of the current sheet which makes the sheet "transparent" for inelastic scattering ($\nu < a/\nu$ for particles with energies $E \geq 10$ keV). Even along an acceleration path equaling the current sheet length $l = L = 10^{10}$ cm, the sheet is "transparent" for inelastic processes at $E \geq 10$ MeV, which is drastically at variance with observations (Pustil'nik, 1977c).

4.18.7. The spatial diffusion in the electric field of the sheet in the case of two-dimensional geometry with pure anti-parallel magnetic field

As it was noted above, the particles accelerated in the current sheet should suffer multiple elastic collisions with the ion-sound plasmons. The frequency of such collisions is (Kaplan and Tsytovich, M1972)

$$\nu_s^p \approx \frac{v^2}{c_s^2} \nu_s^E \approx \omega_{oe} \left(\frac{v}{v_{Te}} \right)^{-3} \left(\frac{W_s}{nT_e} \right) \left(\frac{m_e}{m_{ac}} \right)^2 Z. \quad (4.18.18)$$

As a result, the particle motion will take the form of one-dimensional diffusion to the walls of the sheet (one-dimensionality due to magnetization, $\nu_{eff}^p \ll \omega_H$). After reaching the walls, the particles will be ejected from the accelerated region along the force lines. The spatial diffusion coefficient is

$$\kappa_X(E) = \frac{v^2}{Y_{cs}^p} \approx \frac{c_s^2}{m_{ac}^2 v^2 D(E)}. \quad (4.18.19)$$

When diffusing along the electric field, a particle should gain energy at a rate

$$\dot{E} = Ze\dot{X} \approx (ZeE)^2 \frac{\kappa_X(E)}{E}. \quad (4.18.20)$$

Since in this pattern the particle lifetime in the sheet and the particle acceleration time are closely interrelated, it should be expected that the ejection energy spectrum

will be of a power form (Korchak, 1976). Let us consider this problem at greater length in according with Pustil'nik (1977b). The diffusive flux of the particles with energies E' arriving at point X' is

$$\begin{aligned} I_o &= I_+ - I_- = n(X' + \lambda/2, E' + \Delta E/2) \frac{v\lambda_+}{\lambda} - n(X' - \lambda/2, E' - \Delta E/2) \frac{v\lambda_-}{\lambda} \approx \\ &\approx ZeE\kappa_{X(E)} \left(\frac{\partial n}{\partial E} + \frac{2n}{E} \right), \end{aligned} \quad (4.18.21)$$

where the variations of the particle energies is the course of movement in the electric field is automatically included. The resultant equation involving the equilibrium distribution function (including $I_o = \text{const}$) is

$$0 = -\frac{\partial n(E)}{\partial E} - \frac{2n(E)}{E} + \frac{I_o}{ZeE\kappa_{X(E)}}, \quad (4.18.22)$$

With boundary conditions $n(E_{inj}) = n_{inj}$ and $I_o = n(E'_o) \left(E'_o / m_{ac} \right)^{1/2}$ (here E'_o is the energy of ejection from the sheet). The first term in Eq. 4.18.21 describes the conventional diffusion, the second term is identical with the thermo-diffusion (see §11 in Lifshitz and Pitaevsky, M1979) due to the particle energy dependence of the diffusion coefficient. The solution of Eq. 4.18.21 is (Pustil'nik, 1978):

$$n(E) = n_o \left(\frac{E}{E_T} \right)^{-2} \left(1 - \frac{E}{E^*} \right) \approx n_o \left(\frac{E}{E_T} \right)^{-2}, \quad (4.18.23)$$

where

$$E_T = \frac{m_{ac} v_T^2}{2}; \quad E^* = E_T \left(\frac{E}{E_{dr}} \right)^{-1} \approx (1 \div 10) E_T. \quad (4.18.24)$$

From Eq. 4.18.23 follows that the spectrum of accelerated and ejected particles is characterized by power index $\gamma = 2$, i.e. the resultant spectrum is more rigid than the observed spectrum. In this case the maximum particle energy is $E_{\max} = Zea \leq 50 \text{ keV}$ for $Z = 1$ what is much smaller than observed energies.

4.18.8. The spatial diffusion in the electric field of the sheet in the case of three-dimensional geometry

The observed discrepancy in previous Section 4.18.7 is associated with the application of the two-dimensional geometry with pure anti-parallel field assuming that $H_z = 0$. This pattern is explicitly unreal, which can be seen from the difficulties

arising due to the transverse electric and magnetic fields which, in case of the undoubted weaker electric field of the sheet, $E \leq E_{Dr} \approx 10^{-(3 \div 4)} H$, precludes the particle motion in the electric field everywhere in the sheet except for the narrow central zone (Pustil'nik, 1978): $\Delta a \approx 10^{-(3 \div 4)} a$. The situation will change if a quasi-homogeneous longitudinal magnetic field $H_z \neq 0$ is inserted in the system. The magnetic field in the sheet will exhibit a shear, the force line will be deflected to the Z-axis (see Fig. 4.18.5), the electric field will be projected onto force line, and the acceleration by the regular field will be possible. Another important consequence of the existence of the longitudinal magnetic field is the increase of the acceleration path depending on the distance to the center of the sheet. In fact, the merging force lines are injected into the sheet at an angle $\alpha = X/R$, where X is the distance to the center of the sheet, R is the curvature radius of the force lines, of the external field. The distance between the force lines in the sheet increases and the value of the transverse field decreases as $H_{\perp} \approx H_o \alpha \approx H_o (X/R)$. At the same time, the longitudinal field is invariable, which steepens the inclination of the force lines to the Z-axis as $\theta \approx H_{\perp}/H_{\parallel} \approx X/R$. In this case the acceleration path is effectively increased as $l = a/\theta = aR/X$ and reaches $l_{\max} = L \approx 10^{10}$ cm in the center of the sheet.

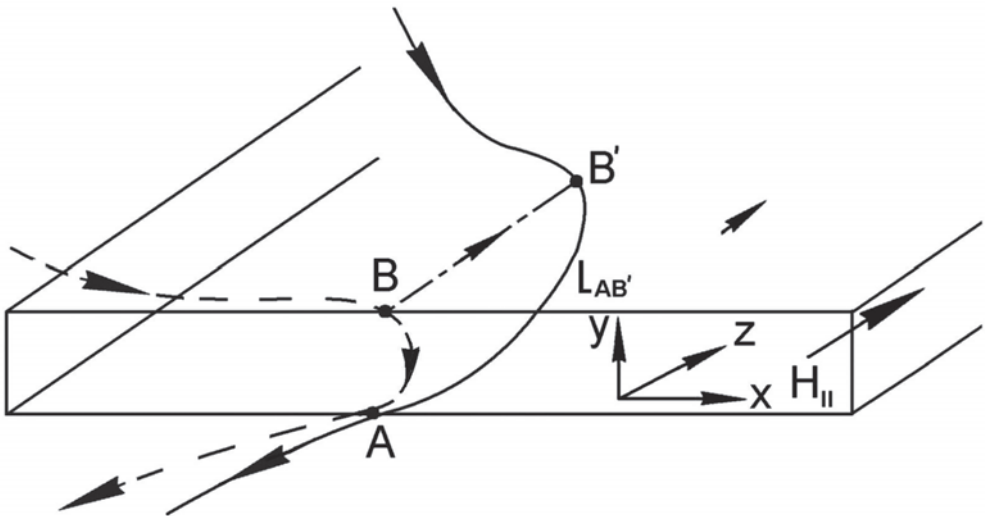


Fig. 4.18.5. The scheme of the sheet in the frame of three-dimensional geometry. By the thick curve B'A is shown the real 3-D magnetic force line, by broken curve BA is shown the 2-D projection of 3-D magnetic force line (usually used in the 2-D model of current sheet, see Fig. 4.18.4) According to Pustil'nik (1977b).

If the power spectrum $E_{\max} \propto 1/X$ with $D(E) \propto E^{-\gamma}$ is formed in each sector Δl , it can be easily shown (Pustil'nik, 1978) that the total spectrum $D^*(E)$ will also retain a power form but with $\gamma^* = \gamma + 1$ and the maximum particle energy $E_{\max}^* = ZeEL = E_o L/a \approx (1 \div 10)$ GeV. Thus, the following result is obtainable for the considered above case: the quasi-diffusion in the three-dimensional sheet generates the spectrum of ejection from the entire sheet $D^*(E) \propto E^{-3}$ with cut-off at high energies $E_{\max}^* \approx (1 \div 10)$ GeV, which is in a good agreement with observations.

4.18.9. Comparison of the quasi-diffusive acceleration and stochastic acceleration on the Langmuir plasmons

It can be seen from the comparison between the effectiveness of the quasi-diffusive acceleration (index *qd*) and stochastic acceleration (*st*) on the Langmuir plasmons (Pustil'nik, 1978):

$$\frac{\dot{E}_{qd}}{\dot{E}_{st}} = \left(\frac{A}{Z}\right)^2 \left(\frac{E/E_{Dr}}{W_s/0.1nT}\right)^2 \frac{W_s}{W_l} \left(\frac{\nu_{eff}^{\max}}{0.1\omega_{oe}}\right)^2 \frac{m_e}{M_i} \left(\frac{v}{c_s}\right)^6 \approx 10^{-3} \left(\frac{A}{Z}\right)^2 \left(\frac{v}{c_s}\right)^6 > 1, \quad (4.18.25)$$

that the quasi-diffusion acceleration completely defines the particle energy increase within the observed energy range. 2.

4.18.10. On the chemical composition of accelerated particles

The chemical composition of the particles accelerated by the quasi-diffusion is determined by the dependence of the particle injection parameters for the acceleration conditions on Z and A . In its turn, the injection is determined by the condition of the balance between the quasi-diffusive formation of the power spectrum and the relaxation of the distribution to the quasi-thermal form due to inelastic particle-ion sound plasmon collisions (the Longmuir plasmons fail to interact with particles in the injection region, since their velocity $v_{inj} < v_{Te}$). In this case according to Pustil'nik (1978):

$$\frac{\dot{E}_{qd}}{\dot{E}_{th}} = \left(\frac{A}{Z}\right)^2 \left(\frac{E/E_{Dr}}{W_s/0.1nT}\right)^2 \left(\frac{\nu_{eff}^{\max}}{0.1\omega_{oe}}\right)^2 \frac{4}{\pi^2} \left(\frac{v}{c_s}\right)^6 \approx \frac{1}{2} \left(\frac{A}{Z}\right)^2 \left(\frac{v}{c_s}\right)^6, \quad (4.18.26)$$

where $\dot{E}_{th}$ characterized energy losses by thermalization of accelerated particles as result of inelastic particle-ion sound plasmon collisions. It can be seen that the injection threshold for all particles is

$$v_{inj} \approx (0.9 \div 1.1)c_s. \quad (4.18.27)$$

On the other hand, at $v < c_s$ the particles are accelerated in the electric field without being scattered on the ion sound plasmons (in this case, the ionization losses are negligibly low). Each gain in the velocity is permanent up to where the particle energy is exchanged with the ion-sound plasmons. As it was shown just above, however, the particles in these regions are immediately affected by the quasi-diffusion acceleration. Thus, the chemical composition of the accelerated particles should be similar to that of the plasma in which the acceleration occurs, in agreement with the observation data (Dorman and Miroshnichenko, M1968; Dorman, M1972, M1978; Miroshnichenko, M2001). It can be seen when summarizing the above discussion that the inclusion of the particle-plasmon elastic collisions will unambiguously give the quasi-diffusive particle acceleration in the elastic field the main characteristics of which coincide with the observation data. This makes it possible to expect that this description of the processes resulting in particle acceleration in the current sheet of a flare is adequate.

4.18.11. Development of solar flare models and mechanisms of particle acceleration in the turbulent current sheet

In papers of Pustil'nik (1997, 1999a,b,c, 2001) given the development of discussed above solar flare models and mechanisms of particle acceleration in the turbulent current sheet. In more details are investigated the problem of the stability of a turbulent current sheet. Pustil'nik (1997) note that after successful progress in our understanding of the equilibrium state of a flare current sheet, it is natural to ask whether this equilibrium state is stable. This leads us to the unfortunate conclusion that the generally accepted equilibrium state of a turbulent current sheet is not stable, and this picture of a flare cannot be considered final. The instability of a turbulent current sheet forces us to drastically reconsider our approach to flare energy release, and to obtain a new, stable equilibrium state taking into account properties of dynamical instabilities. Let us first consider the main instabilities of a turbulent flare current sheet.

Tearing mode instability. These instabilities (Fürth et al., 1963) lead to the redistribution of the flat current in a plasma with finite conductivity into a set of parallel current strings (see Fig. 4.18.6).

The most unstable mode (tearing) has a development time

$$\tau_t \approx \tau_a (\text{Re}_H)^{1/2}, \quad (4.18.28)$$

where $\tau_a = a/v_a \approx 10^{-5} \div 10^{-3}$ sec is the Alfven time, $\text{Re}_H = \tau_d/\tau_a$ is the magnetic Reynolds number, and $\tau_d = a^2/(c^2/4\pi\sigma)$ is the diffusion time.

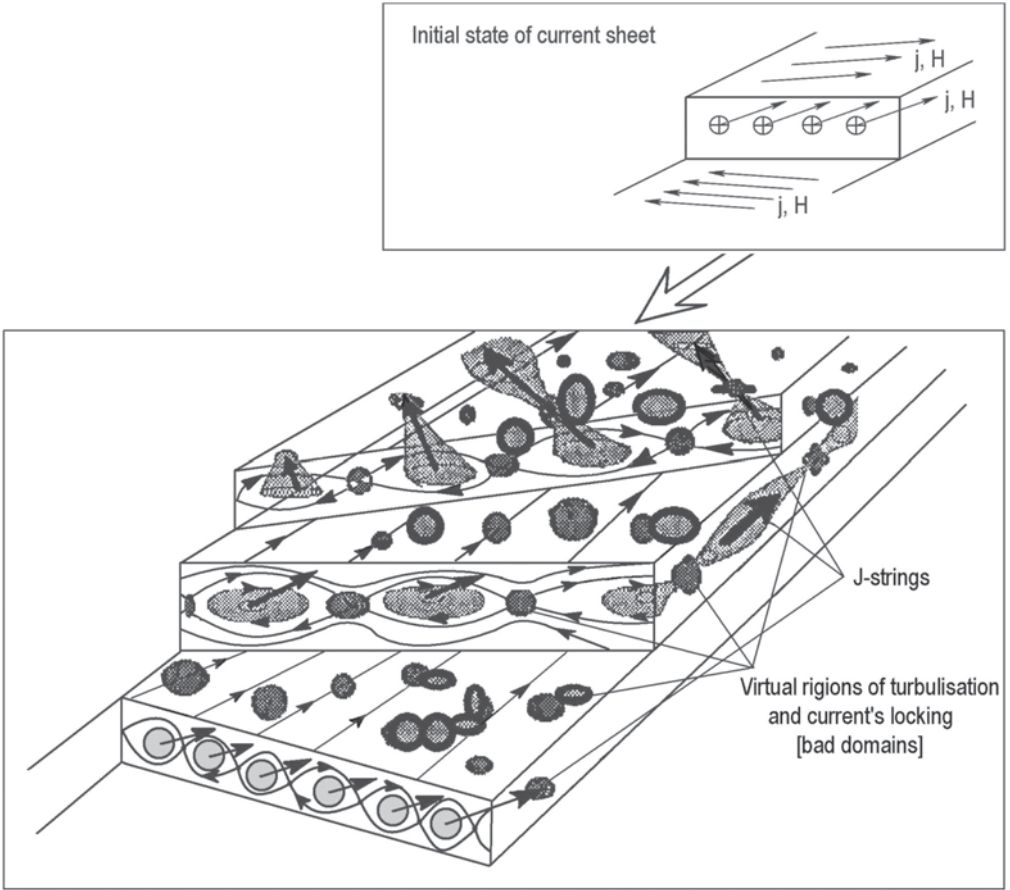


Fig. 4.18.6. Instabilities of the turbulent current sheet and 'domains' structure of the current sheet plasma. According to Pustil'nik (1997).

For standard current sheet parameters $H_{2.5} = 10^{2.5} \text{ Gs}$, $n_8 = 10^8 \text{ cm}^{-3}$, $T_6 = 10^6 \text{ K}$, this corresponds to a time for splitting of a flat current sheet into a system of strings of about 10-100 sec for Coulomb conductivity and 1-10 msec for turbulent conductivity. The tearing mode is very important, since it, has no threshold in the flare condition, and cannot be suppressed (stabilization by the rapid evacuation of plasma from the sheet, according to Bulanov and Sasorov (1978) does not act effectively for a thin current sheet). In the nonlinear state of the tearing mode, the opposite process of coalescence occurs. This leads to the joining of numerous generation strings, with additional energy release on the same tearing time scale. The final structure is determined by the competition between the coalescence of magnetic islands (current strings) and the ejection of plasma from the current sheet by magnetic tension.

Pinch type instabilities (sausage, kink, etc.). Current strings are in a Z pinch state, with the external pressure of azimuthally fields $H_\varphi^2/4\pi$ balanced by the internal pressure of the plasma nkT and the longitudinal field $H_{||}^2/4\pi$. This state is unstable to a set of MHD fast instabilities (Priest, 1982) (sausage, kink, and other more complicated instabilities), which result in the collapse of a local current strings and their disruption at numerous points, with the generation of an electrostatic double layers (in the sausage mode) and/or the formation of a current kink and straightening of its braided field lines (in the kink mode). There is some stabilization of the pinch mode by the influence of the longitudinal magnetic field on the length of the pinch

$$H_{||}/H_\varphi > \Lambda_{cr} = \alpha(l/a) \quad (4.18.29)$$

according to criteria of Kruskal and Schwarzschild (1954) and Shafranov (1957). However, this stabilization is not effective for the thin, long current strings with $l/a > 10^2 \div 10^3$ produced by the tearing mode in the turbulent current sheets of solar flares. The following two processes (c) and (d) disrupt the steady state of a current sheet under the action of a specific property of plasma turbulence, namely, the very narrow threshold for plasma turbulence generation: the directed velocity in the current must exceed the phase velocity of the excited waves. For the opposite sign of the ratio (even for $1 - u/V_{ph} \ll 1$), we have very rapid dissipation of the plasma waves in the same plasma. This can easily be seen from the example of the growth rate of the ion-acoustic wave instability in plasma with longitudinal current (Mikhailovsky, M1977):

$$\gamma_s = -\left(\frac{\pi}{8}\right)\omega_s \left\{ \frac{c_{si} - u}{c_{si}} + \left(\frac{T_e}{T_i}\right)^{3/2} \exp\left(-\frac{T_e}{2T_i}\right) \right\}, \quad (4.18.30)$$

where the first part is caused by the electron decrement/increment and the second is due to Landau damping of the thermal ions.

Overheating of turbulent regions in the current sheet. Turbulent plasma heating by anomalous current dissipation is another cause of the unstable state of current sheets (Pustil'nik, 1980). This phenomenon is due to the rapid heating of the plasma over an extremely short time

$$\tau_{Te} \approx (10^3 \div 10^4) \Omega_{oi}^{-1}. \quad (4.18.31)$$

It leads to changes of the current velocity/thermal velocity ratio over this time from $u/V_{Te} > 1$ (necessary for plasma wave generation) to $u/V_{Te} < 1$ with very rapid plasma wave dissipation by Landau absorption of the plasma waves by the thermal electrons (the time is of the order of Ω_{oi}^{-1}). This heating stops the local plasma turbulence and leads to a transition to a normal, high-conductive state. During the next stage, this region will be cooled by the thermal front of collision-less hot electrons, and will return into its initial turbulent state (the anomalous thermo-conductivity will restore the turbulent state after short cooling times $\tau_{th} = \xi(I_{||}/V_{Te})$). Subsequent anomalous heating in the turbulent state will repeat these local transitions in a pulsation regime, and create numerous normal and turbulent anomalous regions in the current sheet.

Splitting of current sheet at regions of discontinuous conductivity.

According to Pustil'nik (1997), the processes of rapid plasma wave generation in a turbulent regime lead to the rapid increase of the anomalous resistance and create a jump in the conductivity σ at the boundary between the normal and anomalous stage of magnetic field structure is caused by the slower diffusion process (then plasma instability), and at the first stage electric current is conserved, this will lead to a jump in the electric field at the boundary of the current sheet. This discontinuity in the electric field leads to a rapid redistribution of the current, with a decrease of the current density in the turbulent region to below u_{cr} , simultaneously, there is a rapid increase of the current density in the external region up to values exceeding u_{cr} near the boundary. In the inner region, with $u < u_{cr}$, plasma turbulence will disappear, and this local layer turns into the normal state. In the external region, we have the opposite result, with plasma turbulence and the generation of anomalous resistance for short times. New jumps of the conductivity arise at the new boundary between the normal and abnormal plasma, which will lead to a new splitting of the current sheet boundary. The resulting final state is a dynamic equilibrium of the current sheet, which contains numerous compact and short-lived turbulent and normal regions.

4.18.12. Unsteady state of turbulent current sheet and percolation

As it was demonstrated above, the standard steady state of a current sheet is unstable, and it must be disrupted into numerous local, short-lived, small-scale domains of "normal" and "abnormal" plasma (see Fig. 4.18.5). These domains form numerous virtual bond clusters from the "normal" and "turbulent" elements. Therefore, current propagation through a flare current sheet should be similar to percolation through a stochastic network of "good" and "bad" resistors with a constant source of electric current. This process has been studied both in experiments: in superconductive ceramic samples (Vedernikov et al., 1994) and in conductive graphite paper with random holes (Levinshstein et al., 1976; Last and

Thouless, 1971), in numerical simulations (Kirkpatrick, 1973), and in theoretical models (Render, 1983).

However, this important phenomenon was not taken into account in models of energy release and particle acceleration. There are some additional effects of current percolation in our situation - positive and negative feedback between elements, due to the dynamic redistribution of currents and thermo-conductive fluxes:

- (i) Current conservation leads to a redistribution of the currents in the sheet as a result of the permanent stochastic rebuilding of the network resistors. This will change the current density in the elements of the network and will lead to the generation of induced turbulence, with switch-on of plasma turbulence in some neighborhood and switch-off in the initial area. This will lead to constant transitions of the resistors in the network from the "bad" state to the "good" state and back.
- (ii) Thermo-conductive flux from the heated turbulent elements will escape into the surrounding cold plasma, heat this plasma, and thereby change the threshold current value $u_{cr} \propto V_{Te}$.

This very complex pattern, with intricate feedback between current propagation and the plasma turbulence state in local regions can be described using elementary transition probabilities in a stochastic resistor network, with the properties of the resistors dependent on the local current value, cross connections between resistors, and some delay effects. The best approximation to this process is percolation through a fractal network characterized by some cluster factuality, fractal dimension, and threshold of the percolation as infinite cluster disruption. Some general conclusions can be drawn from the first principles of percolation theory (Feder, M1988; see also Mogilevsky, M2004): 1) A fundamental property of a hold dependence of the global network conductivity on the density of bad elements, and, hence, on the current:

$$\sigma \propto (J - J_{cr})^{-\alpha}. \quad (4.18.32)$$

2) Another general property of a percolation process is the universal power-law dependence of the statistical properties in a global system (number versus amplitude, for example) on the characteristics of the domains (scaling):

$$N(x) \propto x^{-k}. \quad (4.18.33)$$

Here x is a parameter of the domain (size, amplitude, energy, etc.). The exponents α in Eq. 4.18.32 and k in Eq. 4.18.33 are determined by the fractal dimension of the clusters and the global dimensions of the system n , and are

$$\alpha = \begin{cases} 0.14 & \text{for } n = 2 \\ 0.40 & \text{for } n = 3 \end{cases} \quad k = \begin{cases} 1.6 \div 1.9 & \text{for } n = 2 \\ 2.5 & \text{for } n = 3 \end{cases} \quad (4.18.34)$$

Pustil'nik (1997) compared the general conclusions of the percolation approach with real flare observations in the solar atmosphere and flare stars, and obtained a remarkable correspondence: for all flare stars (UV Ceti-type red dwarfs; Gershberg and Shakhovskaya, 1983; Gershberg, 1989) and for various manifestations of solar flares in H α storms (Kurochka, 1987; Aschwanden et al., 1995; Merceier and Trotter, 1997) and hard X-ray solar bursts (Crosby et al., 1992), there is the same statistical dependence of the flare frequency and energy

$$f(W) \propto W^{-\beta}, \quad (4.18.35)$$

with the value of $\beta = 1.7 \div 1.8$, similar to that expected from percolation theories. The role of the percolation process and fractal formation in the formation of the frequency-energy spectrum was successfully considered by Wentzel et al. (1992) in a percolation model of active region formation from the convective zone, and by Vlahos et al. (1995) in a fractal model for the structure of magnetic elements over active regions.

4.18.13. Acceleration of particles in a fragmented turbulent current sheet

The fundamental property of solar flares is the acceleration of charged particles up to very high energies of several GeV/nucleon with energy spectrum of a power law

$$D(E) \approx E^{-\gamma}, \quad (4.18.36)$$

with the slope γ from 2 up to 7, but the mean value $\langle \gamma \rangle \approx 2 \div 3$ (Dorman, M1957, M1963a,b, M1978; Dorman and Miroshnichenko, M1968; Duggal, 1979; Dorman and Venkatesan, 1993; Stoker, 1994; Miroshnichenko, M2001). Pustil'nik (1997) considered two types of models for the particle acceleration during solar flares:

- (i) a turbulent boiler with a "particle-plasmon" energy exchange (Kaplan and Tsytovich, M1973),
- (ii) direct run-away of the particles in the DC electric field of a current sheet (Spicer, 1982).

The first model (i) is able to explain naturally the power-law energy spectrum as a consequence of turbulent diffusion in momentum space, but requires extreme assumptions about the turbulent energy. The second model (ii) can accelerate particles up to the maximum energies, but has difficulties in explaining the power-law dependence of the energy spectrum (the typical spectrum for run-away particles is an exponential). This approach does not take into account the fact that the motion of rapid particles in turbulent plasma is not a direct run-away, but rather a space diffusion, caused by effective elastic particle-plasmon scattering. Taking into account the cluster structure of a turbulent current sheet with numerous bad resistors (which play the role of plasma double layers and act as compact line accelerators in

the turbulent plasma of the current sheet) opens a new approach, where the elastic particle-plasmon interaction and direct energy change in the DC electric field are combined in a natural way. This approach leads both to the observed power-law energy spectrum and high energy limits (Pustil'nik, 1978).

According to Pustil'nik (1978), the physical reason for the formation of the spectrum in the model (ii) is the universal power-law dependence of the scattering probability on the particle energy: $P(E) \propto E^{-3}$. This leads to different free path lengths for particles with opposite velocity directions relative to the electric field. The result is that two diffusion fluxes are formed: the first is the standard flux caused by the density or potential gradient, and the second is the specific flux caused by variation of kinetic parameters in the medium, similar to thermo-diffusion. It was found for the second flux that the energy of particles $E = ZeEx$ and also their number $n(E)$ depends only on the distance x from the point of injection of the particles into the acceleration state to the space boundary where particles escape. This general connection leads to a direct "leakage-lifetime" relation with a power-law character for the energy spectrum. The resulting spectrum can be estimated from conservation of the diffusion flux (Pustil'nik, 1978):

$$F_x = \kappa_{\perp}(E) \left(\frac{\partial n(E)}{\partial E} + 2 \frac{n(E)}{v} f \right), \quad (4.18.37)$$

which, for a current sheet with the simplest geometry, leads naturally to a power-law energy spectrum for the ejected particles:

$$n(E) \propto \left(\frac{E}{E_T} \right)^{-2} \exp \left(- \frac{E}{E_*} \right), \quad (4.18.38)$$

where E_T is the thermal energy of the particles, $\kappa_{\perp}(E) = \kappa_o (E/E_T)^{5/2}$ is the space diffusion coefficient of a rapid particles with energy E in the field of the plasma turbulence, and $E_* = V_T \left(\frac{E_T^2}{ZeE\kappa_o} \right)$. Taking into account that the three-dimensional

distribution of the electric and magnetic fields in the turbulent current sheet will influence the power-law slope so that it can increase up to $\gamma \sim 3$. This means that fine cluster structure of a turbulent current sheet may provide a partial basis for understanding the threshold behavior of flare energy release, with its universal statistical properties, and also give a natural explanation for another fundamental property of the flare process - the universal power-law spectrum of the accelerated particles, with approximately the same slope over a wide class of flare objects. According to Pustil'nik (1997), for more progress in the percolation approach to flare

energy release, numerical simulations of a random resistive network are needed, taking into account lifetime effects, current feed-back influence, and the high self-induction of the magnetic fields in the normal domains in the sheet.

4.19. Particle acceleration in shear flows of space plasma

4.19.1. Space plasma's shear flows in different objects

In papers of Berezhko (1981, 1982), Berezhko and Krymsky (1981) there was assumed a particle acceleration mechanism in shear flows of space plasma. The shear flows of space plasma can be realized at boundaries of the magnetospheres of the Earth and other planets, at boundaries of the Heliosphere and stellar winds, and other astrophysical objects in regions of interactions of plasma flows with different velocities. In all these cases there are formatted shear flows of space plasma with regular changes of velocities of frozen in magnetic inhomogeneities which can be considered as scattering centers for energetic charged particles.

4.19.2. Particle acceleration in the two-dimensional shear flow of collisionless plasma

Berezhko (1981) investigates the idealized picture of the two-dimensional shear flow of collisionless plasma that is characterized by the presence of scattering centers whose role is played the magnetic field inhomogeneities. It was supposed that the hydrodynamic velocity $\mathbf{u}$ of the plasma is directed along the x axis and its magnitude varies as a function of the coordinate y (see Fig. 4.19.1).

In order to establish the energy variation law of a particle which, after scattering elastically, moves in the medium with a velocity $v \gg u$, it is convenient to represent this motion in the form of a population of vibrations between two scattering centers with an appropriate averaging over all possible pairs. Thus if the particle vibrates between the centers A and B and $u_A > u_B$ and $x_A < x_B$, as shown in Fig. 4.19.1, then its energy will increase since the centers A and B are brought together. However, for the center B there is a center B' that is placed symmetrically with respect to A ($y_{B'} = y_B$, $x_A - x_{B'} = x_B - x_A$) so that the particle moving between the centers A and B' with the same velocity loses energy if only the quantities of the order of u/v are taken into account. In other words, within the framework of the adiabatic approximation no changes occur in the particle energy: $\langle dE/dt \rangle = 0$ (the angular brackets denote averaging).

From other hand, if the terms $\sim (u/v)^2$ are taken into account, an analysis of the particle motion between the two scattering centers, which are moving with a relative velocity $dl/dt = w$, leads to the expression

$$\frac{dE}{dt} = -2 \frac{w}{l} \left(1 - \frac{w}{v} \right) E, \quad (4.19.1)$$

where l is the distance between the scattering centers. It can be seen from this that the total contribution of the centers A, B, and B' to dE/dt is positive and equal to $4Ew^2/(lv)$. An averaging of this expression over the locations of the B center with allowance for the fact that the probability of the particle traversing a path length l without scattering is $\exp(-l/\lambda)$ gives

$$\left\langle \frac{dE}{dt} \right\rangle = \alpha \frac{\lambda}{v} \left(\frac{du}{dy} \right)^2 E, \quad (4.19.2)$$

where λ is the free path length. The numerical coefficient α in this case is equal to $1/2$; a more systematic treatment can show that its value is somewhat different. An acceleration of charged particles, therefore, occurs in a shear flow of a collisionless plasma, and the rate of energy increase is proportional to the square of the curl of the hydrodynamic velocity of a plasma (in the analyzed case $(\text{rot } u)^2 \approx (du/dy)^2$).

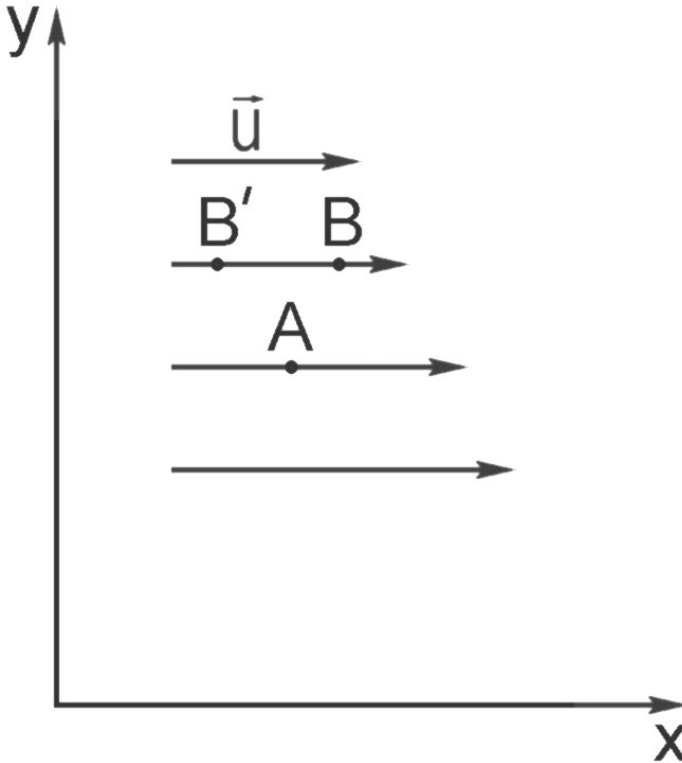


Fig. 4.19.1. The idealized picture of the two-dimensional shear flow of collisionless plasma. Horizontal rows show velocities of plasma $u(y)$; $B'(x,y)$, $B(x,y)$, and $A(x,y)$ are particle scattering inhomogeneities frozen in plasma and moved in x direction with velocity $u(y)$. According to Berezhko (1981).

4.19.3. Some examples of possible particle acceleration in shear flows

According to Sergeev and Tsyganenko (M1980) it has been established that a layer consisting of a shear flow of solar wind plasma exists at the boundary of the Earth's magnetosphere. The hydrodynamic velocity of the plasma in the layer varies from zero at the inside boundary to ~ 400 km/sec at the outside boundary. The characteristic thickness of the layer is $l \sim r_e$ and its longitudinal dimension is $L \geq 100r_e$, where r_e is the Earth's radius. Particles which have path lengths $\lambda \ll l$ are accelerated efficiently by means of the mechanism discussed in Section 4.19.2. The condition $\lambda \ll l$ can be satisfied only for electrons, since $\lambda \sim \rho \approx 100$ km for thermal protons, where ρ is the gyro-radius. It was assumed that the range of accelerated electrons is $E \leq 1$ MeV if $\lambda \sim \rho$ (the magnetic field at the boundary of the magnetosphere is $\sim 10^{-4}$ Gs). Since the electrons usually cannot penetrate the interior boundary of the acceleration region, which is formed by the regular magnetospheric magnetic field, a density gradient of accelerated electrons, which is directed toward the Earth, is formed. A corresponding diffusion flow is directed from the Earth, which accounts for the anisotropy of high energy electrons that is recorded in the experiments (Sergeev and Tsyganenko, M1980). The following can be said about the shape of their energy spectrum. Gnedin et al. (1972) have analyzed the formation of the spectrum of particles, which have been accelerated by means of a mechanism for which the characteristic acceleration time

$$\tau = E/(dE/dt) \propto E^{-\beta}. \quad (4.19.3)$$

They showed that at $\beta > 0$ a power spectrum $\propto E^\gamma$ is formed with an exponent

$$\gamma = -(1 + \beta) \quad (4.19.4)$$

at energies $E \gg E_0$ where E_0 is the initial energy of the particles. For the case considered in Section 4.19.2 the condition described by Eq. 4.19.3 means that $\lambda \propto E^\beta$. It can be assumed that $\lambda \sim \rho$ (i.e. $\lambda \propto E$) for most of the electrons energy range, and that $\lambda \propto E^2$ at high energies. Thus obtain a spectrum exponent γ in the limits from -3 to -1.5 , consistent with the experiment: $\gamma = -2$ to -1.5 for $E = 18-120$ keV and $\gamma = -4.5$ to -3 for $E \geq 100$ keV (Sergeev and Tsyganenko, M1980). The values of $\gamma < -2$ may be attributed to the influence of energy losses.

Universality of the spectrum produced is a characteristic feature of the discussed mechanism. An important feature of this mechanism is that it involves regular, large scale plasma motion, which sets it favorably apart from the other mechanisms, in particular, the mechanism of acceleration by turbulent pulsations (see Section 4.9).

A shear plasma flows can frequently occur under space conditions in different objects. A typical example for interplanetary space is the high velocity flows in the

solar wind. Because of this the acceleration process of charged particles in a shear flows can be important in the production CR in different energy ranges.

4.20. Additional regular particle acceleration in space plasma with two types of scatters moving with different velocities

4.20.1. Two types of scatters in space plasma as additional source of particle acceleration

The acceleration of particles to high energies is usually associated with chaotic motion of magnetized plasma (Fermi, 1949, 1954) and with shock waves (Dorman and Freidman, 1959; Shabansky, 1961; Alexeev and Kropotkin, 1970; Axford et al., 1977; Krymsky, 1977; Bell, 1978a,b; Blanford and Ostriker, 1978). It was shown that particle acceleration can be effective also by tangential discontinuities (Alexeev et al., 1970) and by macroscopic flows of magnetized plasma (Berezhko, 1981). Review of acceleration processes in space plasmas can be found in Dorman (M1972b, M1975a), Toptygin (M1983), Berezhinsky et al. (M1990). Here we shall consider some additional mechanism of particle acceleration in regions with two types of particle scatters according to Dorman and Shogenov (1985, 1999). The matter is that in many astrophysical objects there are background plasma moved with some speed $\mathbf{u}_{1o}$ and characterized by transport path $\lambda_1(R)$, where R is the CR particle rigidity, and some scatters moved with some different speed $\mathbf{u}_{2o}$ and characterized by the transport path $\lambda_2(R)$. As examples of these objects can be considered our Heliosphere (solar wind – as background plasma and interplanetary shock waves and magnetic clouds generated by coronal mass ejections as additional scatters of CR), stellar winds, our Galaxy, interacted galaxies.

4.20.2. General theory of CR propagation and acceleration in space plasma with two types of scatters moving with different velocities

According to Dorman and Shogenov (1985, 1990) the CR distribution function in the presence of two types of scatters will be described by the well known Boltzmann equation with collision term

$$\frac{\partial f(\mathbf{r}, \mathbf{p}, t)}{\partial t} + \mathbf{v} \frac{\partial f(\mathbf{r}, \mathbf{p}, t)}{\partial \mathbf{r}} + \frac{Ze}{c} [\mathbf{v} - \mathbf{u}_1, \mathbf{B}(\mathbf{r}, t)] \frac{\partial f(\mathbf{r}, \mathbf{p}, t)}{\partial \mathbf{p}} = \text{St}f, \quad (4.20.1)$$

where $\mathbf{v}$ and $\mathbf{p}$ are the velocity and momentum of the CR particle, $\mathbf{B}(\mathbf{r}, t)$ is the frozen magnetic field in the background plasma,

$$\text{St}f = n(\mathbf{r}, t) \int |\mathbf{v} - \mathbf{u}_2| \sigma(|\mathbf{v} - \mathbf{u}_2|, \alpha) \{f(\mathbf{r}, \mathbf{p}', t) - f(\mathbf{r}, \mathbf{p}, t)\} d\Omega', \quad (4.20.2)$$

and the distribution function of second type of scatters is

$$\varphi(\mathbf{r}, \mathbf{u}_2, t) = n(\mathbf{r}, t) \delta(\mathbf{u}_2 - \mathbf{u}_2(\mathbf{r}, t)). \quad (4.20.3)$$

Let us suppose that $\mathbf{B}(\mathbf{r}, t) = \mathbf{B}_0(\mathbf{r}, t) + \mathbf{B}_1(\mathbf{r}, t)$, where $\mathbf{B}_0(\mathbf{r}, t) = \langle \mathbf{B}(\mathbf{r}, t) \rangle$ and $\langle \mathbf{B}_1(\mathbf{r}, t) \rangle = 0$. Then $f(\mathbf{r}, \mathbf{p}, t) = F(\mathbf{r}, \mathbf{p}, t) + f_1(\mathbf{r}, \mathbf{p}, t)$, where $F(\mathbf{r}, \mathbf{p}, t) = \langle f(\mathbf{r}, \mathbf{p}, t) \rangle$ and $\langle f_1(\mathbf{r}, \mathbf{p}, t) \rangle = 0$. According to Eq. 4.20.1 and Eq. 4.20.2 we obtain

$$\frac{\partial F(\mathbf{r}, \mathbf{p}, t)}{\partial t} + \mathbf{v} \frac{\partial F(\mathbf{r}, \mathbf{p}, t)}{\partial \mathbf{r}} + \frac{Ze}{c} [\mathbf{v} - \mathbf{u}_1, \boldsymbol{\omega}(\mathbf{r}, t)] \frac{\partial F(\mathbf{r}, \mathbf{p}, t)}{\partial \mathbf{p}} = \text{St}F + \text{St}_1F, \quad (4.20.4)$$

where $\boldsymbol{\omega}(\mathbf{r}, t) = Zec\mathbf{B}_0(\mathbf{r}, t)/E$. Here Ze and E are electrical charge and total energy of CR particle and

$$\text{St}F = n(\mathbf{r}, t) \int |\mathbf{v} - \mathbf{u}_2| \sigma(|\mathbf{v} - \mathbf{u}_2|, \alpha) \{F(\mathbf{r}, \mathbf{p}', t) - F(\mathbf{r}, \mathbf{p}, t)\} d\Omega', \quad (4.20.5)$$

$$\text{St}_1F = (\partial/\partial p_i) D_{ik} (\partial F/\partial p_k), \quad (4.20.6)$$

where

$$D_{ik} = \frac{p^2 |\mathbf{v} - \mathbf{u}_1|}{\lambda_1} \left(\delta_{ik} - \frac{(\mathbf{v} - \mathbf{u}_1)_i (\mathbf{v} - \mathbf{u}_1)_k}{(\mathbf{v} - \mathbf{u}_1)^2} \right) \quad (4.20.6a)$$

is the diffusion coefficient in the momentum space and

$$\lambda_1 = 6c^2 p^2 / \sqrt{\pi} Ze l_c \langle \mathbf{B}_1^2 \rangle \quad (4.20.6b)$$

is the transport path in the background plasma and l_c is the correlation length of fluctuating field $\mathbf{B}_1(\mathbf{r}, t)$.

4.20.3. The diffusion approximation

Let us suppose that

$$F(\mathbf{r}, \mathbf{p}, t) = N(\mathbf{r}, p, t) + (\mathbf{p}/p) \mathbf{J}(\mathbf{r}, p, t), \quad (4.20.7)$$

and instead of Eq. 4.20.4 we obtain

$$\frac{\partial N}{\partial t} + \frac{v}{3} \frac{\partial J_k}{\partial x_k} = \frac{1}{p^2} \frac{\partial}{\partial p} \left\{ \frac{p^2 E}{3c^2} [\mathbf{u}_1 \boldsymbol{\omega}] \mathbf{J} + \frac{p^3}{3} \left(\frac{\mathbf{u}_1}{\lambda_1} + \frac{\mathbf{u}_2}{\lambda_2} \right) \mathbf{J} + \frac{p^4}{3} \left(\frac{u_1^2}{\lambda_1} + \frac{u_2^2}{\lambda_2} \right) \frac{\partial N}{\partial p} \right\}, \quad (4.20.8)$$

$$\frac{\partial J_k}{\partial t} + v \frac{\partial N}{\partial x_k} - [\mathbf{J} \boldsymbol{\omega}]_k - \frac{E}{c^2} [\mathbf{u}_1 \boldsymbol{\omega}]_k \frac{\partial N}{\partial p} = -v \left(\frac{1}{\lambda_1} + \frac{1}{\lambda_2} \right) J_k - p \left(\frac{u_{1k}}{\lambda_1} + \frac{u_{2k}}{\lambda_2} \right) \frac{\partial N}{\partial p}, \quad (4.20.9)$$

where $\lambda_2 = n(\mathbf{r}, t) \sigma_{tr}$ is the transport path in the second type of scatters and $\sigma_{tr} = \int \sigma (1 - \cos \theta) d\Omega$ is the transport cross-section of scattering, and θ is the angle between $\mathbf{p}$ and $\mathbf{p}'$. Here we neglect by members containing $u_1^3, u_1^2 \mathbf{J}, u_2^3, u_2^2 \mathbf{J}$, and by members higher order over $\mathbf{u}_1$ and $\mathbf{u}_2$. Let us suppose that $\mathbf{u}_1 = \mathbf{u}_{1o} + \mathbf{u}_{11}$, where $\mathbf{u}_{1o} = \langle \mathbf{u}_1 \rangle, \langle \mathbf{u}_{11} \rangle = 0$ and $\mathbf{u}_2 = \mathbf{u}_{2o} + \mathbf{u}_{21}$, where $\mathbf{u}_{2o} = \langle \mathbf{u}_2 \rangle, \langle \mathbf{u}_{21} \rangle = 0$. Then on the basis of Eq. 4.20.8 and Eq. 4.20.9 we obtain finally

$$\begin{aligned} \frac{\partial N}{\partial t} = & \frac{\partial}{\partial x_k} \left(\kappa_{kl} \frac{\partial N}{\partial x_l} - \frac{p}{3} \frac{q_k}{v^2 + \omega^2} \frac{\partial N}{\partial p} \right) \\ & + \frac{1}{p^2} \frac{\partial}{\partial p} \left\{ \frac{p^3}{3} \frac{\mathbf{q}_a \nabla N}{v^2 + \omega^2} + \frac{p^4}{3v^2} \left(v_1 \langle u_1^2 \rangle + v_2 \langle u_2^2 \rangle \right) \frac{\partial N}{\partial p} \right\}, \end{aligned} \quad (4.20.10)$$

where the frequencies of particle collisions with inhomogeneities

$$v = v_1 + v_2, \quad v_1 = v/\lambda_1, \quad v_2 = v/\lambda_2, \quad (4.20.11)$$

the space diffusion coefficient tensor

$$\kappa_{kl} = \frac{v^2 v}{3(v^2 + \omega^2)} \left(\delta_{kl} - v^{-2} \omega_k \omega_l - v^{-1} e_{kil} \omega_i \right), \quad (4.20.12)$$

and we have used the following nominations:

$$\mathbf{q} = v[\mathbf{u}_{1o} - \mathbf{W}_o, \boldsymbol{\omega}] - v^2 \mathbf{W}_o - \omega^2 \mathbf{u}_{1o} + \omega((\mathbf{u}_{1o} \boldsymbol{\omega}) - (\mathbf{W}_o v)), \quad (4.20.13)$$

$$\mathbf{q}_a = \mathbf{q} - 2v[\mathbf{u}_{1o} - \mathbf{W}_o, \boldsymbol{\omega}], \quad (4.20.14)$$

$$d = \nu(\nu^2 + \omega^2)^{-1} \times \left\{ \omega^2 u_{1o}^2 - (\mathbf{u}_{1o} \boldsymbol{\omega})^2 - 2\omega^2 (\mathbf{u}_{1o} \mathbf{W}_o) + 2(\mathbf{u}_{1o} \boldsymbol{\omega})(\mathbf{W}_o \boldsymbol{\omega}) - \nu^2 W_o^2 - (\mathbf{W}_o \boldsymbol{\omega})^2 \right\} \quad (4.20.15)$$

$$\mathbf{W}_o = \nu_1 \mathbf{u}_{1o} / \nu + \nu_2 \mathbf{u}_{2o} / \nu = (\mathbf{u}_{1o} \lambda_2 + \mathbf{u}_{2o} \lambda_1) / (\lambda_1 + \lambda_2). \quad (4.20.16)$$

The first member in the right hand side of Eq. 4.20.10 contains two terms: the first describes the anisotropic diffusion characterized by diffusion tensor Eq. 4.20.12 and the second describes the change of particle energy caused by convective moving of particles with effective speed determined by Eq. 4.20.16. The second term in the right hand side of Eq. 4.20.10 describes particle acceleration; to understand the meaning of this term let us consider the case when $\mathbf{B}_o = 0$.

4.20.4. The case $\mathbf{B}_o = 0$

In this case the Eq. 4.20.10 transforms into

$$\frac{\partial N}{\partial t} = \frac{\partial}{\partial x_k} \left(\kappa \frac{\partial N}{\partial x_k} - \frac{p}{3} W_{ok} \frac{\partial N}{\partial p} \right) + \frac{1}{p^2} \frac{\partial}{\partial p} \left(-\frac{p^3}{3} \mathbf{W}_o \bar{\nabla} N + p^2 D \frac{\partial N}{\partial p} \right), \quad (4.20.17)$$

where $D = D_1 + D_2 + D_{12}$ is the diffusion coefficient in the momentum space and

$$D_1 = p^2 \langle \Delta u_1^2 \rangle / 3\nu\lambda_1; \quad D_2 = p^2 \langle \Delta u_2^2 \rangle / 3\nu\lambda_2 \quad (4.20.18)$$

describe particle acceleration caused by usual Fermi mechanism (here $\langle \Delta u_1^2 \rangle = \langle u_1^2 \rangle - u_{1o}^2$, $\langle \Delta u_2^2 \rangle = \langle u_2^2 \rangle - u_{2o}^2$ and

$$D_{12} = p^2 \langle \mathbf{u}_{2o}^2 - \mathbf{u}_{1o}^2 \rangle / 3\nu(\lambda_1 + \lambda_2) \quad (4.20.19)$$

describes additional particle acceleration.

4.20.5. Space-homogeneous situation

Let us suppose that in this case $D_1 = D_2 = 0$. Then instead of Eq. 4.20.17 will be

$$\frac{\partial N}{\partial t} = \frac{1}{p^2} \frac{\partial}{\partial p} \left(p^2 D_{12} \frac{\partial N}{\partial p} \right). \quad (4.20.20)$$

Let us introduce

$$\tau_{ac} = 4p^2/D_{12} = 3v(\lambda_1 + \lambda_2)/4(\mathbf{u}_{2o}^2 - \mathbf{u}_{1o}^2). \quad (4.20.21)$$

If we multiply Eq. 4.20.20 on $p^3 dp$ and then integrate it, we obtain instead of Eq. 4.20.20

$$\frac{\partial \langle p \rangle}{\partial t} = \int p \frac{\partial}{\partial p} \left(\frac{4p^4}{\tau_{ac}} \frac{\partial N}{\partial p} \right) dp, \quad (4.20.22)$$

where $\langle p \rangle = \int p N p^2 dp$. From Eq. 4.20.22 follows that

$$d\langle p \rangle/dt = \langle p \rangle/\tau_{ac}, \text{ or } \langle p \rangle = p_o \exp(t/\tau_{ac}), \quad (4.20.23)$$

and τ_{ac} determined by Eq. 4.20.21 have a meaning of effective acceleration time.

4.20.6. Estimation of possible additional acceleration of CR particles in the Galaxy

According to data on the relative content of Be^{10} the life-time of CR particles in our Galaxy is (Berezinsky et al., M1990)

$$T_g = T_d + T_h \approx 3 \times 10^7 \text{ years}, \quad (4.20.24)$$

where T_d and T_h are the times of CR particles living in the disk and in the halo, respectively. On the other hand, from data on the chemical and isotopic composition of CR it follows that in average CR particles crossed some $X \text{ g/cm}^2$ of matter. If the density of matter in disk is ρ_d and in halo ρ_h we obtain

$$X = T_d v \rho_d + T_h v \rho_h, \quad (4.20.25)$$

where $v \approx c$ is the velocity of CR particles. From Eq. 4.20.24 and Eq. 4.20.25 follows that

$$T_h = (T_g v \rho_d - X) \times [v(\rho_d - \rho_h)]^{-1}; \quad T_d = T_g - T_h. \quad (4.20.26)$$

For particles with energy $E \sim 10 \text{ GeV/nucleon}$ the value $X \approx 7 \text{ g/cm}^2$. If $\rho_d \approx 10^{-24} \text{ g/cm}^3$ and $\rho_h \approx 10^{-26} \text{ g/cm}^3$ then according to Eq. 4.20.24 and Eq.

4.20.26 will be $T_d \approx 7 \times 10^6$ years and $T_h \approx 2.3 \times 10^7$ years. In the disk (the depth of which is $L_d \approx 1$ kpc) the expected transport path $\lambda_{1d} \approx L_d^2 / 2vT_d \approx 0.2$ pc. In the halo (the dimension $L_h \approx 15$ kpc) the expected transport path $\lambda_{1h} \approx L_h^2 / 2vT_h \approx 15$ pc.

Let us take into account that in many regions of the Galaxy's disk there are shock waves, extended supernova remnants and strong stellar winds (which we consider as scatters) moving relative to interstellar matter with a velocity $|\mathbf{u}_{2o} - \mathbf{u}_{1o}| \sim 10^8$ cm/s. We expect that the transport path λ_{2d} on these scatters is much bigger than λ_{1d} . Let us suppose that $\lambda_{2d} \sim 20 \lambda_{1d}$. In this case, according to Eq. 4.20.21, we obtain $\tau_{ac} \sim 10^6$ years for the time of acceleration. It means that in the disk according to Eq. 4.20.23 effective additional acceleration with increasing of energy up to $\exp(T_d / \tau_{ac}) \sim 10^3$ times can be realized.

Some part of shock waves, extended supernova remnants and strong stellar winds are also in the halo moving relative to the galactic wind with a velocity $|\mathbf{u}_{2o} - \mathbf{u}_{1o}| < 10^7$ cm/sec. We expect that the transport path λ_{2h} on these scatters is not smaller than λ_{1h} . In this case according to Eq. 4.20.21 we obtain for the time of acceleration $\tau_{ac} > 6 \times 10^8$ years. This means that in the halo according to Eq. 4.20.23 can not be realized additional acceleration by considered mechanism because $T_h / \tau_{ac} \ll 1$.

4.20.7. Estimation of possible additional acceleration of CR particles in the region of galaxies collision

Let us consider as an example the region of collision of the halos of two galaxies moving one against the other with a velocity $\sim 10^8$ cm/sec. Let us suppose that both galaxies to be characterized by about the same characteristics as considered in Section 4.20.6 for our Galaxy. Then we suppose that $\lambda_{1h} \sim \lambda_{2h} \sim 15$ pc, $T_h \approx 2.3 \times 10^7$ years. If $|\mathbf{u}_{2o} - \mathbf{u}_{1o}| \sim 2 \times 10^8$ cm/sec we obtain according to Eq. 4.20.21 that $\tau_{ac} \approx 3 \times 10^6$ years. This means that in the region of the halo's collision of two galaxies according to Eq. 4.20.23 can be realized effective additional acceleration with an increase of energy up to $\exp(T_h / \tau_{ac}) \sim 10^2 - 10^3$ times.

4.20.8. Estimation of possible additional acceleration of CR particles in the Heliosphere and in stellar winds

Let us consider first the situation with CR particles of small energy, about $E_k \sim 100$ MeV/nucleon. In our Heliosphere there is solar wind with average velocity $u_{1o} \sim 4 \times 10^7$ cm/sec (background plasma) and characterized in the

maximum of solar activity by a transport path $\lambda_1 \sim 3 \times 10^{12}$ cm for CR with energy $E_k \sim 100$ MeV/nucleon. The dimension of the Heliosphere $r_o \sim 100$ AU, so the life-time inside the Heliosphere of these particle will be $T \sim r_o^2 / 2\nu\lambda_1 \approx 3 \times 10^7$ sec ~ 1 year (Dorman and Dorman, 1967a,b; Dorman, 1991; Dorman et al., 1997a,b,c,d).

On the other hand, in the periods of high solar activity there are also shock waves, magnetic clouds, and high speed streams moving with an average velocity $u_{2o} \sim 10^8$ cm/sec. The CR particle scattering on these objects can be characterized in the periods of high solar activity by $\lambda_2 \sim 10^{13}$ cm. Then according to Eq. 4.20.21 the characteristic time of particle acceleration will be $\tau_{ac} \sim 3 \times 10^7$ sec ~ 1 year. According to Eq. 4.20.23 in this case there can be expected additional acceleration of small energy particles in $\exp(T/\tau_{ac}) \sim$ few times.

For low level of solar activity λ_1 increases and T decreases; there will also be a big increase of λ_2 , so τ_{ac} will increase. Therefore in periods of low solar activity we expect $T/\tau_{ac} \ll 1$ and the additional acceleration considered even for CR particles with energy $E_k \sim 100$ MeV/nucleon will be not effective.

For particles of higher energy with $E_k \geq 1$ GeV/nucleon the value of $\lambda_1 > 10^{13}$ cm in a maximum of solar activity, and are expect $T \sim r_o^2 / 2\nu\lambda_1 \leq 3 \times 10^6$ sec. Then in this case $T/\tau_{ac} \ll 1$ even in periods of high level of solar activity and additional acceleration is not effective. We came to the conclusion that the mechanism of CR particle acceleration considered can be effective in the Heliosphere only for small energy particles in periods of high solar activity.

In stellar winds the effectiveness of the mechanism of additional particle acceleration considered will depend on the ratio T/τ_{ac} which can be determined from the values of $\lambda_1, \lambda_2, r_o, \mathbf{u}_{2o} - \mathbf{u}_{1o}$ as

$$T/\tau_{ac} = 2r_o^2 (\mathbf{u}_{2o} - \mathbf{u}_{1o})^2 / 3\nu^2 \lambda_1 (\lambda_1 + \lambda_2) \quad (4.20.27)$$

and the increase of the particle's energy will be determined according to Eq. 4.20.27 as $\exp(T/\tau_{ac})$.

4.20.9. On the effectiveness of additional particle acceleration in the double star systems

We expect that this mechanism of additional particle acceleration will be especially effective for double star systems in regions of the collision of stellar winds where the situation will be about the same as in regions of galaxies collision (see above, Section 4.20.7). Let us suppose that both colliding stellar winds have the same characteristics as solar wind. In this case $\lambda_1 \sim \lambda_2$ and $(\mathbf{u}_{2o} - \mathbf{u}_{1o})^2$ will

be increased in 4 times, so T/τ_{ac} will increase in 4–8 times, which gives a particle energy increase according to Eq. 4.20.23 of 10^2 – 10^4 times. In a more detailed investigation it will be necessary to take into account that with increasing of particle energy there will also be an increase in λ_1 and λ_2 so according to Eq. 4.20.27 the parameter T/τ_{ac} will decrease and the increase of the total particle energy will be smaller.

4.20.10. Main results on the mechanism of CR particle additional acceleration and applications

The main results of the regular mechanism of CR particle additional acceleration considered above may be summarized as follows:

1. The considered mechanism of particle regular acceleration by scattering on different types of scatters moving with different regular velocities can give additional acceleration to well known mechanisms. The characteristic time τ_{ac} of particle acceleration is determined by Eq. 4.20.21. The total increasing of particle energy is determined by Eq. 4.20.23.

2. It is shown that the additional acceleration considered can be important in the disk of our Galaxy with an increase of CR particle energy up to about 10^3 times for the life time of particles in the disk. On the other hand, in the halo $T/\tau_{ac} \ll 1$ is expected and this mechanism of additional acceleration is not effective.

3. The mechanism considered of particle additional acceleration is expected to be effective in regions of galaxies collision with CR particle energy increasing by 10^2 – 10^3 times.

4. In the Heliosphere additional acceleration can be effective (with increase of the energy by about few times) only in periods of high solar activity and only for small energy particles of about few hundreds MeV/nucleon.

5. For stellar winds the effectiveness of additional particle acceleration depends upon ratio T/τ_{ac} determined by Eq. 4.20.27.

6. The mechanism of additional particle acceleration will be especially effective for double star systems in regions of stellar winds collision with particle energy increasing in 10^2 – 10^4 times.

7. In a more detailed investigation it is necessary to take into account the change of particle transport paths λ_1 and λ_2 with the increase of particle energy which leads to a change of the parameter T/τ_{ac} during the processes of acceleration.

4.21. Shock wave diffusion (regular) acceleration

4.21.1. Two types of particle interaction with a shock wave

In sections 4.15 we considered charged particle shock acceleration at the single crossing of a particle through the shock front (see Fig. 4.15.1). This happened when a shock wave propagated through space plasma with homogeneous frozen-in magnetic field. In other, much more frequent, cases, when the propagation of shock wave take place through excited, turbulence space plasma, a particle scatters on magnetic inhomogeneities behind and before the front and has some probability of interacting with the shock front several times (see the illustration of this diffusion process in Fig. 4.21.1).

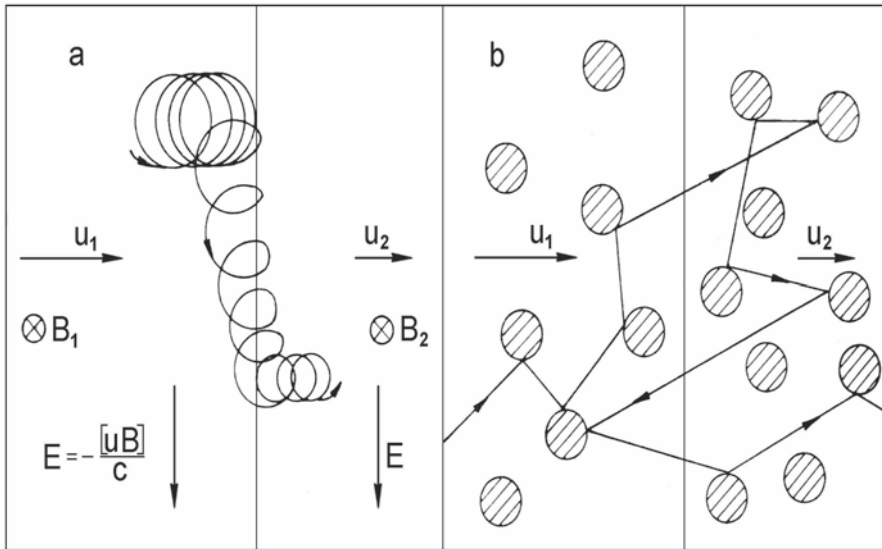


Fig. 4.21.1. The character of motion of a fast charged particle in the neighborhood of shock wave propagated through turbulent space plasma. According to Berezhko et al. (M1988).

This type of particle acceleration by shock wave in literature (Krymsky, 1977; Axford et al., 1977; Bell, 1978a,b) is called as regular shock acceleration (because at each crossing of shock front particle gain the energy) or diffusive shock acceleration (because the multi-crossing of shock front is caused by particle scattering and diffusion through magnetic inhomogeneities behind and before the shock front).

4.21.2. Elementary model of diffusive shock-wave acceleration

Let us consider the motion of a particle with velocity $v \gg u_1$ (where u_1 is the velocity of shock front) in neighborhood of the shock wave. Let us take into account that the relative velocities of magnetic irregularities in the space plasma are

very small in comparison with the velocity of a strong shock wave. So in the first approximation we may neglect the particle's change of energy during their interactions with magnetic irregularities. The change of particle momentum $\mathbf{p}$ as a result of scattering by the shock wave front will be

$$\Delta p = (\mathbf{p}_f - \mathbf{p}_i)\mathbf{u}_1/v, \quad (4.21.1)$$

where $\mathbf{p}_f$ and $\mathbf{p}_i$ are final and initial particle momentum. The change of particle momentum during double crossing of the shock wave front will be

$$\Delta p = (\mathbf{p}_k - \mathbf{p}_i)\mathbf{u}_1/v + (\mathbf{p}_f - \mathbf{p}_k)\mathbf{u}_2/v. \quad (4.21.2)$$

The flux of particles in the case of isotropic distribution after scattering by magnetic inhomogeneities per the unit of shock wave front, expressed through the accelerated particle density $n(p)$ and angle θ between momentum $\mathbf{p}$ and axis x , will be

$$J(\mathbf{p}) = n(p)v \cos \theta. \quad (4.21.3)$$

The averaging of Eq. 4.21.2 over the flux $J(\mathbf{p})$ in the angle intervals $\pi/2 \leq \theta_1 \leq \pi$, $\pi/2 \leq \theta_f \leq \pi$, and $0 \leq \theta_k \leq \pi/2$ gives the average gain of particle momentum for one cycle (a double crossing of the shock wave front):

$$\langle \Delta p \rangle = \frac{4(u_1 - u_2)p}{3v}. \quad (4.21.3a)$$

Because

$$n(p + \Delta p) = P_c n(p), \quad (4.21.4)$$

where P_c is the probability of realization of the next cycle of double crossing by particle of the shock wave front, we obtain the following equation for determining the integral spectrum of accelerated particles $N(> p)$:

$$\frac{dN(> p)}{dp} = \frac{P_c - 1}{\langle \Delta p \rangle} N(> p). \quad (4.21.5)$$

If P_{c1} is the probability that a particle from the region 1 (see Fig. 4.21.1) before the shock wave front will come back to the front, and P_{c2} is the same for a particle coming from the region 2 behind the shock wave front, the value of $P_c = P_{c1}P_{c2}$. It

is expected that $P_{c1} = 1$ because all particles from the region 1 by the convective motion came to the front. On the other hand, the probability

$$P_{c2} = (J_{12} - J_2)/J_{12}, \quad (4.21.6)$$

where J_{12} is the particle flux from the region 1 to region 2 (in the case of isotropic distribution of accelerated particles behind the front $J_{12} = nv/4$) and $J_2 = nu_2$ is the convection flux from the front to the region 2. Therefore by taking into account Eq. 4.21.6 we obtain

$$P_c = P_{c1}P_{c2} = 1 - 4u_2/v. \quad (4.21.7)$$

On the basis of Eq. 4.21.3a, 4.21.5, and 4.21.7 we obtain the following equation for the differential density of accelerated particles (differential energy spectrum) $n(p) = -dN(>p)/dp$:

$$\frac{d}{dp}(pn(p)) + \frac{3u_2}{u_1 - u_2}n(p) = 0 \quad (4.21.8)$$

with the solution

$$n(p) \propto p^{-\gamma}; \quad \gamma = \frac{u_1 + 2u_2}{u_1 - u_2} = \frac{\sigma + 2}{\sigma - 1}, \quad (4.21.9)$$

where

$$\sigma = u_1/u_2 = \rho_2/\rho_1 = H_2/H_1 \quad (4.21.10)$$

is the degree of plasma compressibility by the shock wave. On the other hand the gas-dynamical consideration (see Pikelner, M1966; Landau and Lifshitz, M1957; Zeldovich and Raizer, M1966; Longmair, M1966) shows that

$$\sigma = \frac{(\gamma_g + 1)M_1^2}{(\gamma_g - 1)M_1^2 + 2}, \quad (4.21.11)$$

where γ_g is the gas adiabatic index and $M_1 = u_1/u_{s1}$ is the Mach number, and the sound velocity is determined by $u_{s1} = (\gamma_g P_1/\rho_1)^{1/2}$. It is important to note that according to Eq. 4.21.9 with increasing σ from 2 to 4 the value of γ decreases from 4 to 2 in agreement with what is observed in galactic and solar CR.

4.21.3. Acceleration by the plane shock wave; diffusion approximation

Let us suppose, following to Berezhko et al. (M1988), that the plane shock wave propagates in the negative direction of the x -axis and the diffusion coefficient for particle scattering in the background plasma is κ . In this case the transport equation in the system of coordinates of the shock front will be

$$\frac{\partial f(x, p, t)}{\partial t} = \frac{\partial}{\partial x} \left(\kappa \frac{\partial f(x, p, t)}{\partial x} \right) - u \frac{\partial f(x, p, t)}{\partial x} - \frac{\Delta u}{3} \delta(x) p \frac{\partial f(x, p, t)}{\partial p} + Q(x, p), \quad (4.21.12)$$

where κ is the diffusion coefficient, $Q(x, p)$ is the source of particles, $\Delta u = u_1 - u_2$, and

$$u(x < 0) = u_1, \quad u(x > 0) = u_2. \quad (4.21.13)$$

Let us take into account that most probably the particle injection into the acceleration process is from the region of shock front the width of which l is supposed to be much smaller than the transport path λ ; in this case $Q(x, p) = Q_0(p) \delta(x)$. The other possibility which may be realized is the existence in the non-disturbed region 1 of fast particles with some spectrum $f_{1\infty}(p)$; in this case we shall have the boundary condition at $x \rightarrow -\infty$:

$$f_1(x = -\infty, p) = f_{1\infty}(p). \quad (4.21.14)$$

The boundary condition for $f(x, p, t)$ at $x = 0$ will be

$$f_1 = f_2; \quad \kappa_1 \frac{\partial f_1}{\partial x} + \frac{u_1}{3} p \frac{\partial f_1}{\partial p} = \kappa_2 \frac{\partial f_2}{\partial x} + \frac{u_2}{3} p \frac{\partial f_2}{\partial p} + Q_0(p). \quad (4.21.15)$$

The solution of Eq. 4.21.12 with boundary conditions described by Eq. 4.21.14 and Eq. 4.21.15 was found in Krymsky (1977), Berezhko et al. (M1988) as follows:

$$f_1(x, p) = f_2(p) \exp \left(\int_0^x (u_1 / \kappa_1) dx \right) + f_{\infty}(p) \left[1 - \exp \left(\int_0^x (u_1 / \kappa_1) dx \right) \right], \quad (4.21.16)$$

where

$$f_2(p) = \int_0^{\infty} G(p, p') [f_{\infty}(p') + Q_0(p') u_1] dp', \quad (4.21.17)$$

and

$$G(p, p') = \frac{\gamma}{p'} \left(\frac{p}{p'} \right)^{-\gamma} \theta(p - p'); \quad \gamma = 3u_1/(u_1 - u_2) \quad (4.21.18)$$

is the Green's function of the problem of particle acceleration by the plane shock wave (here $\theta(p - p')$ is the Heviside function).

4.21.4. The case of particle injection with mono-energetic spectrum

Let us suppose that the spectrum of injected particles is mono-energetic:

$$Q_o(p) = \frac{u_1 N_o}{4\pi p_o^2} \delta(p - p_o); \quad f_\infty(p) = \frac{N_\infty}{4\pi p_o^2} \delta(p - p_o). \quad (4.21.19)$$

In this case by substituting Eq. 4.9.19 in Eq. 4.21.17 we shall have

$$f_2(p) = \frac{N_o + N_\infty}{4\pi p_o^2} \gamma \left(\frac{p}{p_o} \right)^{-\gamma} \delta(p - p_o), \quad (4.21.20)$$

where the index of the power spectrum of accelerated particles

$$\gamma = 3\sigma/(\sigma - 1) \quad (4.21.21)$$

is determined only by the value of the compressibility $\sigma = u_1/u_2$ and does not depend upon the diffusion coefficient and other parameters of the concrete problem. This important result on the universality of the accelerated particle's spectrum was obtained for the first time about 30 years ago by Krymsky (1977) and Axford et al. (1977).

4.21.5. On the space distribution of accelerated particles

As follows from Eq. 4.21.17 and Eq. 4.21.20, in the region behind the shock wave's front the distribution of accelerated particles with momentum $p > p_o$ is homogeneous. Before the shock wave front, according to Eq. 4.21.16, the density of accelerated particles with increase of distance from the front falls exponentially (i.e., $\propto \exp(-x/L(p))$) with a characteristic length

$$L(p) = \kappa_1(p)/u_1. \quad (4.21.22)$$

4.21.6. The effect of finite width of shock wave front

According to Berezhko et al. (M1988) the effect of a finite width l of the shock wave's front can be accounted for approximately by changing the position of the

injection source: to put it behind the front at some distance $x = l$ instead of $x = 0$ as was assumed in the previous Sections 4.21.3–4.21.5. The physical meaning of this shifting is that for a real shock wave the heating of plasma occurs not at the point $x = 0$ but over the some length l , which may be considered as effective width of shock wave front. It means that the source function for Eq. 4.21.12 will now be

$$Q(x, p) = Q_o(p) \delta(x - l). \quad (4.21.23)$$

Let us suppose that this source is mono-energetic (i.e. $Q_o(p)$ is described by Eq. 4.21.19), and other sources are absent, i.e. $f_\infty(p) = 0$. In this case the solution before the front is described by the same Eq. 4.21.16, but for the region behind the front the solution will be

$$\begin{aligned} f_2(x, p) = & \frac{N_o}{4\pi p_o^3} \gamma \left(\frac{p}{p_o} \right)^{-\gamma} \theta(p - p_o) \exp\left(-\frac{u_2 l}{\kappa_2}\right) + \frac{\sigma N_o}{4\pi p_o^3} \delta(p - p_o) \\ & \times \left\{ \exp\left(\frac{u_2}{\kappa_2}(x - l)\right) \theta(l - x) - \exp\left(-\frac{u_2 l}{\kappa_2}\right) + \theta(x - l) \right\}. \end{aligned} \quad (4.21.24)$$

From Eq. 4.21.24 it can be seen that the taking into account of the finite width of the shock wave's front leads to the appearing of a modulating factor $\exp(-u_2 l / \kappa_2)$ which sufficiently decreases the flux of accelerated particles if $u_2 l \geq \kappa_2$. Berezhko et al. (M1988) came to the conclusion that the effective shock acceleration may be realized only for particles for which the diffusion length is bigger than the width l of the shock wave front, i.e.,

$$L(p) \geq l, \quad \text{or} \quad \kappa_1(p) / u_1 \geq l. \quad (4.21.25)$$

This means that for small energy particles with too small diffusion coefficient the Eq. 4.21.25 will not be satisfied and the acceleration of such particles will be not effective.

4.21.7. Effect of finite dimension of shock wave

In the Section 4.21.3 it was supposed that the plane shock wave front is infinite. In fact the dimension of the shock wave is limited. Berezhko et al. (M1988) show that the limiting of the shock wave's dimension can be roughly taken into account even in the simple one-dimensional approximation used in Section 4.21.3. Let us consider an accelerated particle which after interaction with the shock wave diffuses before the front. When a particle diffuses over the distance $x \geq R_s$, where R_s is the dimension of shock wave, it will have very little chance of coming back to the front

and being accelerated again (in comparison with the case of an infinite plane shock wave). Therefore we can consider such particles as running away from the acceleration process. It is equivalent to putting at the distance R_s before the shock wave front an absorber of accelerated particles, so the boundary condition described by Eq. 4.21.14 will be changed to

$$f_1(-R_s, p) = 0. \quad (4.21.26)$$

In this case the stationary solution of Eq. 4.21.12 will be transformed to

$$f_1(x, p) = f_2(p) \frac{\exp(xu_1/\kappa_1) - \exp(R_s u_1/\kappa_1)}{1 - \exp(R_s u_1/\kappa_1)}, \quad (4.21.27)$$

$$f_2(p) = \frac{N_o}{4\pi p_o^3} \gamma \left(\frac{p}{p_o} \right)^{-\gamma} \theta(p - p_o), \quad (4.21.28)$$

where the index of the power spectrum of accelerated particles is

$$\gamma = \frac{3\sigma}{\sigma - 1} [1 - \exp(-R_s u_1/\kappa_1)]^{-1}. \quad (4.21.29)$$

The Eq. 4.21.29 differs from Eq. 4.21.21 by the factor $[1 - \exp(-R_s u_1/\kappa_1)]^{-1} > 1$. It means that the limiting of the shock wave dimension leads to an increase of γ . As was shown in Chapters 1 and 2, with increasing particle momentum p usually increases diffusion coefficient. It means that at $p \geq p_{\max}$, where $p_{\max}$ is determined by the relation

$$\kappa_1(p_{\max}) = R_s u_1, \quad (4.21.30)$$

the value of γ quickly increases with increasing p . Therefore the value of $p_{\max}$ can be considered, according to Berezhko et al. (M1988), as the effective upper threshold of the momentum spectrum of particles accelerated by a shock wave with finite dimension R_s .

4.21.8. Effect of energy losses during particle shock acceleration

Above we do not take into account the energy lost by the particle during its regular acceleration by the shock wave. In real space plasma accelerated particles lose energy in different types of particle interactions with matter, magnetic fields and photons (see detailed description above, in Chapter 1). According to Bulanov

and Dogiel (1979) this account can be made on the basis of the Fokker-Planck equation which in the one-dimensional approximation will be (Berezhko et al., M1988):

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial x} \left(\kappa \frac{\partial f}{\partial x} - u f \right) + \frac{1}{p^2} \frac{\partial}{\partial p} \left[\left(\frac{\partial u}{\partial x} \frac{p}{3} - F \right) p^2 f + D p^2 \frac{\partial f}{\partial p} \right], \quad (4.21.31)$$

where the term $(\partial u / \partial x)(p/3)$ describes the adiabatic energy change caused by the change of plasma velocity, $F = \langle \Delta p / \Delta t \rangle$ is the averaged temp of momentum changing owing to energy lose, and $D = (1/2) \langle \Delta p^2 / \Delta t \rangle$ is the diffusion coefficient in momentum space. Berezhko et al. (M1988) for simplicity considered the case in which in the regions $i = 1, 2$ the energy lose can be presented in the form

$$F_i = -p / \tau_i, \quad (4.21.32)$$

where the characteristic times τ_i of energy lose do not depend on the momentum p . It is supposed also that the parameter $\alpha_i = 4\kappa_i / \tau_i u_i^2$ is the same in both regions, i.e., $\alpha_1 = \alpha_2 = \alpha$. In this case Eq. 4.21.31 can be solved and its solutions for regions $i = 1, 2$ are as follows:

$$\begin{aligned} f_i(x, p) = & \frac{N_o}{4\pi p_o^3} \frac{3\sigma}{\sigma-1} \left(1 - \frac{3\alpha(\sigma+1)}{4(\sigma-1)A} \right)^{-1} \exp \left(\frac{x u_i}{\kappa_i} \left(2 - i - (-1)^i \frac{A-1}{2} \right) \right) \\ & \times \left\{ \left(\frac{p}{p_o} \right)^{-\gamma} \theta(p - p_o) + \frac{1}{2} \left(\frac{p}{p_o} \right)^{-\gamma} \theta(p_o - p) \operatorname{erfc} \left(A \sqrt{\frac{\ln(p_o/p)}{\alpha}} - \frac{|x| u_i}{4\kappa_i} \sqrt{\frac{\alpha}{\ln(p_o/p)}} \right) \right. \\ & + \frac{1}{2} \left(1 - \frac{3\alpha(\sigma+1)}{2(\sigma-1)A} \right) \left(\frac{p}{p_o} \right)^{\gamma_1} \theta(p_o - p) \exp \left(- \frac{|x| u_i}{\kappa_i} \frac{3\alpha(\sigma+1)}{4(\sigma-1)} \right) \\ & \left. \times \operatorname{erfc} \left(\frac{|x| u_i}{4\kappa_i} \sqrt{\frac{\alpha}{\ln(p_o/p)}} + \left(A - \frac{3\alpha(\sigma+1)}{2(\sigma-1)} \right) \sqrt{\frac{\ln(p_o/p)}{\alpha}} \right) \right\}, \quad (4.21.33) \end{aligned}$$

where

$$A = \sqrt{\left(1 - \frac{3\alpha(\sigma+1)}{4(\sigma-1)} \right)^2 + \frac{3\alpha}{\sigma-1} + \frac{3\alpha(\sigma+1)}{4(\sigma-1)}}, \quad (4.21.34)$$

$$\gamma = \frac{3\sigma}{\sigma-1} \left(1 + \frac{(A-1)(\sigma+1)}{2\sigma} \right); \quad \gamma_1 = \frac{3}{\sigma-1} \left(1 + \frac{(A-1)(\sigma+1)}{2} - \frac{3\alpha(\sigma+1)^2}{4(\sigma-1)} \right). \quad (4.21.35)$$

Approximately at $p > p_o$ index γ in the power spectrum can be presented as

$$\gamma = \frac{3\sigma}{\sigma-1} \left(1 + \frac{3\alpha(\sigma+1)}{4\sigma(\sigma-1)} \right) + O(\alpha^2), \quad (4.21.36)$$

i.e., the index γ increases with increase of the parameter $\alpha = 4\kappa_1/\tau_1 u_1^2 = 4\kappa_2/\tau_2 u_2^2$. The account of energy lose leads to particles appearing with momentum smaller than that of injected particles. The relative contents of these particles depends on the value of the parameter α (e.g., at $\alpha \ll 1$ momentum spectrum at $p < p_o$ falls very quickly to zero as $\propto p^{1/\alpha}$).

The account of energy loses leads also to the sufficient change of space distribution of accelerated particles: behind the front it becomes non-homogeneous (the concentration falls with increasing of the distance from the front), and before the front the fall of the concentration of accelerated particles becomes more quick, approximately as $\propto \exp(-3|x|u_1\alpha/\kappa_1(\sigma-1))$.

4.21.9. *Simultaneously regular and statistical acceleration*

The propagation of a shock wave is accompanied by the excitation of space plasma, which leads to the development of different types of instabilities and the generation of magneto-hydrodynamic turbulence. Therefore, as was considered above in Sections 4.2–4.9, the particle scattering by moving magnetic inhomogeneities will give some additional statistical acceleration of particles simultaneously with the shock wave regular acceleration. This problem was considered approximately in the one-dimensional approximation in Berezhko et al. (1988). As it was shown in Section 4.9, the statistical acceleration by turbulent plasma can be considered as diffusion in the momentum space:

$$\frac{\partial f}{\partial t} = \frac{1}{p^2} \frac{\partial}{\partial p} \left(D p^2 \frac{\partial f}{\partial p} \right), \quad (4.21.37)$$

where D is the diffusion coefficient in momentum space. According to Tverskoy (1978), Vasilyev et al. (1980), Bulanov and Pukhov (1981), near the shock front conditions are realized suitable for turbulence developing. For the developing of turbulence may also be important accelerated by shock wave particles (mainly from streaming instability) generated Alfvén turbulence (Bell, 1978a,b; see also above, Chapter 3). In the case of Alfvén turbulence, according to Skilling (1975)

$$D = u_a^2 p^2 / 9\kappa_{//}, \quad (4.21.38)$$

where u_a is the Alfvén velocity, and $\kappa_{//}$ is the component of the space diffusion coefficient parallel to the magnetic field.

Let us multiply Eq. 4.21.37 by $4\pi p^3$, and then by integration over p from 0 to ∞ we obtain

$$\left\langle \frac{dp}{dt} \right\rangle = p/\tau_{st}; \quad \tau_{st} = p^3 \left(\frac{d}{dp} (Dp^2) \right), \quad (4.21.39)$$

where τ_{st} is the characteristic time of statistical acceleration.

According to Berezhko and Krymsky (1988), Berezhko et al. (M1988), the stationary equation described in the one-dimensional approximation simultaneously regular and statistical acceleration will be as following

$$\frac{\partial}{\partial x} \left(uf - \kappa \frac{\partial f}{\partial x} \right) - \frac{1}{p^2} \frac{\partial}{\partial p} \left(Dp^2 \frac{\partial f}{\partial p} \right) + \frac{\Delta u \delta(x)}{3p^2} \frac{\partial}{\partial p} (p^3 f) - \frac{N_o \delta(x)}{4\pi p_o^2} \delta(p - p_o) = 0. \quad (4.21.40)$$

For simplicity Berezhko et al. (M1988) considered the case when space diffusion coefficient does not depend from particle momentum; it means that

$$D(p) = D_o (p/p_o)^2, \quad (4.21.41)$$

where $D_o = D(p_o)$. In this case Eq. 4.21.40 can be solved by the substitution $f = \psi(x)p^{-\gamma}$. The equation for $\psi(x)$ will be

$$\frac{\partial}{\partial x} \left(u\psi - \kappa \frac{\partial \psi}{\partial x} \right) + D_o \gamma (3 - \gamma) \psi = 0, \quad (4.21.42)$$

the solution of which in both regions $i = 1, 2$ is

$$\psi_i = B \exp \left(\frac{|x| u_i}{\kappa_i} (i - 2 - A_i) \right), \quad (4.21.43)$$

where

$$A_i = \frac{1}{2} \left(\sqrt{1 + 4\alpha_i \gamma (3 - \gamma)} - 1 \right); \quad \alpha_i = D_o \kappa_i / u_i^2. \quad (4.21.44)$$

On the basis of sewing solutions in both regions at the shock wave front one can get algebraic equation for determining index γ in the power momentum spectrum of accelerated particles:

$$u_2 + (u_1 A_1 + u_2 A_2)/2 + (u_2 - u_1)(3 - \gamma)/3 = 0, \quad (4.21.45)$$

where A_1 and A_2 are determined by Eq. 3.19.44. According to the solution of Eq. 4.21.45 for the simplest case when $\alpha_1 = \alpha_2 = \alpha$, the index γ gradually changes in the limits from

$$\gamma = \frac{3\sigma}{\sigma - 1} \left(1 - \frac{9\alpha(\sigma + 1)}{2(\sigma - 1)^2} \right) + O(\alpha^2) \quad (4.21.46)$$

at $\alpha \ll 1$ up to

$$\gamma = 3 \left(1 - \frac{2(\sigma + 3)}{9\alpha(\sigma + 1)^2} \right) + O(\alpha^{-2}) \quad (4.21.47)$$

at $\alpha \gg 1$. The relative role of shock wave regular and statistical accelerations is mostly determined by the ratio $\tau/\tau_{st} \approx \alpha$. If $\alpha \ll 1$ the main role is played regular shock wave acceleration, and the accelerated particle distribution (see Eq. 4.21.43) and the index γ in the power spectrum (see Eq. 4.21.46) will be about the same as that considered in Section 4.21.5. Only in the region 2 (behind the shock front) on the big distances from the front $x \gg \kappa_2/u_2\alpha_2$ will be a sufficient role of statistical acceleration (according to Eq. 4.21.43): the density of accelerated particles will be increased. In the opposite case when $\alpha \gg 1$ statistical acceleration will be more important, and it will form the power spectrum with the index γ determined by Eq. 4.21.47.

In the case of **Alfvén turbulence** when the diffusion coefficient in the momentum space is determined by Eq. 4.21.38, we obtain $\alpha_{1,2} \approx (u_a/u_{1,2})^2$. For the **fast shock waves** $u_{1,2} \gg u_a$, then $\alpha \ll 1$, and the main role will be played by the regular acceleration. For **slow shock waves** $u_{1,2} \approx u_a$ can had and in this case the relative role of regular and statistical accelerations may be comparable.

4.21.10. Regular acceleration by spherical shock wave

According to Berezhko et al. (M1988), for the spherically symmetrical case for the shock front at some distance $r = r_o$, the main equation and boundary conditions will be as following:

$$\frac{\partial f}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \kappa \frac{\partial f}{\partial r} \right) - u \frac{\partial f}{\partial r} + \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 u \right) \frac{p}{3p} \frac{\partial f}{\partial p} + Q(r, p), \quad (4.21.48)$$

$$f_1(r_o, p) = f_2(r_o, p); \quad \left(\kappa_1 \frac{\partial f_1}{\partial r} + \frac{u_1 p}{3} \frac{\partial f_1}{\partial p} \right)_{r=r_o} = \left(\kappa_2 \frac{\partial f_2}{\partial r} + \frac{u_2 p}{3} \frac{\partial f_2}{\partial p} \right)_{r=r_o} + Q_o \times \text{sign}(u(r_o - 0) - u(r_o + 0)), \quad (4.21.49)$$

where $Q_o(p)\delta(r - r_o)$ is the part of the particle injection source located on the shock front, and the function $\text{sign}(u(r_o - 0) - u(r_o + 0))$ reflects that two different possibilities can be realized: when the region 1 (before the shock front) is at $r > r_o$ (as in solar or stellar wind), and when the region 1 is at $r < r_o$, as in case of accretion. For simplicity let us consider injection of mono-energetic particles from the undisturbed plasma before the shock front

$$f_\infty(p) = \left(N_\infty / 4\pi p_o^2 \right) \delta(p - p_o), \quad (4.21.50)$$

and injection from the front

$$Q_o(p) = \left(u_1 N_o / 4\pi p_o^2 \right) \delta(p - p_o). \quad (4.21.51)$$

Let us follow Berezhko et al. (M1988) by considering few important cases in which it is possible to obtain analytical solutions: a standing shock wave in the solar or stellar wind (as terminal shock wave on the boundary of Heliosphere; bow shocks in the magnetospheres of the Earth, Jupiter, Saturn, and so on), a standing shock wave in the case of accretion (as in double star system), a running shock wave (as in interplanetary space from solar flares and from CME – coronal mass ejections; as from supernova explosion).

4.21.11. Acceleration by spherical standing shock wave in the solar or stellar wind

According to Parker (M1963) and Baranov and Krasnobaev (M1977) the dependence of plasma velocity in its dependence on the distance to the star at the

presence of the standing shock wave at the distance $r = r_o$ can be described as follows:

$$u = \begin{cases} u_1 & \text{if } r < r_o, \\ u_2(r/r_o) & \text{if } r > r_o, \end{cases} \quad (4.21.52)$$

where $u_2 = u_1/\sigma$. As was shown in Webb et al. (1981), Forman et al. (1981), Webb et al. (1983), the analytical solution of Eq. 4.21.48 can be obtained if the diffusion coefficient has the form

$$K = \begin{cases} \kappa_1(r/r_o) & \text{if } r < r_o, \\ \kappa_2(r/r_o) & \text{if } r > r_o, \end{cases} \quad (4.21.53)$$

where κ_1 and κ_2 are constants.

By using Fourier's transformation, for the stationary case and injection sources described by Eq. 4.21.50 and Eq. 4.21.51 the following solution was obtained, following to Berezhko et al. (M1988):

$$f_1(r, p) = \frac{3(N_o + N_\infty \beta / (\sigma(1 - \beta)))}{8\pi p_o^3 \sqrt{\lambda}} \left[b_1 \left(\frac{p}{p_o} \right)^{-\gamma_1} \left(\frac{r}{r_o} \right)^{\varphi_1} \operatorname{erfc} \left(b_1 \eta - \frac{\ln(r_o/r)}{2\eta} \right) + b_2 \left(\frac{p}{p_o} \right)^{-\gamma_2} \right. \\ \left. \times \left(\frac{r}{r_o} \right)^{\varphi_2} \operatorname{erfc} \left(\frac{\ln(r_o/r)}{2\eta} - b_2 \eta \right) \right] \theta(p_o - p) + 2b_1 \left(\frac{p}{p_o} \right)^{-\gamma_1} \left(\frac{r}{r_o} \right)^{\varphi_1} \theta(p - p_o), \quad (4.21.54)$$

$$f_2(r, p) = f_1(r, p) \frac{1 - \exp\left(-\frac{g_2 r_o^2}{2r^2}\right)}{1 - \exp(-(g_2/2))} + \frac{N_\infty \delta(p - p_o)}{4\pi p_o^2} \frac{\exp\left(-\frac{g_2 r_o^2}{2r^2}\right) - \exp\left(-\frac{g_2}{2}\right)}{1 - \exp(-(g_2/2))}, \quad (4.21.55)$$

where

$$g_{1,2} = u_{1,2} r_o / \kappa_{1,2}, \quad \gamma_{1,2} = \frac{3}{2g_1} \left(b_{1,2}^2 - \left(1 - \frac{g_1}{2} \right)^2 \right), \quad \varphi_{1,2} = b_{1,2} - \left(1 - \frac{g_1}{2} \right), \quad \eta = \sqrt{\frac{3}{2g_1} \ln \left(\frac{p}{p_o} \right)}, \\ b_{1,2} = \frac{\sigma}{\sigma - 1} (1 \pm \sqrt{\lambda}), \quad \lambda = \left[1 - \left(\frac{\sigma - 1}{\sigma} \right) \left(1 - \frac{g_1}{2} \right) \right]^2 + \frac{2g_1 \beta (\sigma - 1)}{\sigma^2 (1 - \beta)}, \quad \beta = \exp\left(-\frac{g_2}{2}\right). \quad (4.21.56)$$

From Eq. 4.21.54 and Eq. 4.21.55 it follows that in the case considered there is important adiabatic deceleration of particles in the region 2 at $r < r_o$ and the finite dimension of the shock wave which leads to additional runaway particles in region 1 at $r > r_o$ from the neighborhood of the shock wave front compared with the case

of an infinite plane front (see Section 4.21.3). Both these processes sufficiently decrease the effectiveness of particle acceleration. The relative role of these processes is mostly determined by the values of the parameters $g_1 = u_1 r_o / \kappa_1$ and $g_2 = u_2 r_o / \kappa_2$ which are called modulation parameters: Eq. 4.21.54 and Eq. 4.21.55 show that they characterize the possibility of accelerated particles propagating against the plasma flux in regions 1 and 2, correspondingly. It is easy to see that if $g_1 \rightarrow 0$ or $g_2 \rightarrow 0$ we obtain

$$\gamma = \begin{cases} \frac{3(\sigma^2 + 1)(\sigma^2 + 2\sigma - 1)}{2g_1\sigma(\sigma - 1)} & \text{at } g_1 \ll \min(1, g_2), \\ \frac{6}{g_2(\sigma - 1)} & \text{at } g_2 \ll \min(1, g_1). \end{cases} \quad (4.21.57)$$

The bigger γ is means that in there cases particle acceleration to high energies becomes non-effective.

With increase of the modulation parameter g_1 the role of adiabatic particle energy decreasing falls sufficiently and at $g_1 \rightarrow \infty$ the value of the distribution function $f \propto (p/p_o)^{3g_1/8} \rightarrow 0$. This occurs because at $g_1 \gg 1$ the diffusion length of particles going inside region 2 with adiabatic cooling is very small: $L \approx \kappa_1/u_1 \ll r_o$. In Fig. 4.21.2 are shown results of calculations of expected momentum spectra at different distances from the standing shock front (for r/r_o from 0.01 to 100) at two values of modulation parameters: weak, $g_1 = g_2 = g = 0.1$, and very large, $g_1 = g_2 = g = 10$.

From Fig. 4.21.2 it can be seen that with increasing the modulation parameter g the relative role of particle deceleration ($p < p_o$) decreases sufficiently. At $g_1 \gg 1$ the spectral index of accelerated particles differs only little from the index $\gamma = 3\sigma/(\sigma - 1)$ for particle acceleration by the infinite plane shock wave (see Section 4.21.3); the difference is caused mainly by the finites of the standing spherical shock wave:

$$\gamma = \frac{3\sigma}{\sigma - 1} \left(\frac{1 + \beta(\sigma - 2)}{1 - \beta} \right) \quad \text{at } g_1 \gg 1. \quad (4.21.58).$$

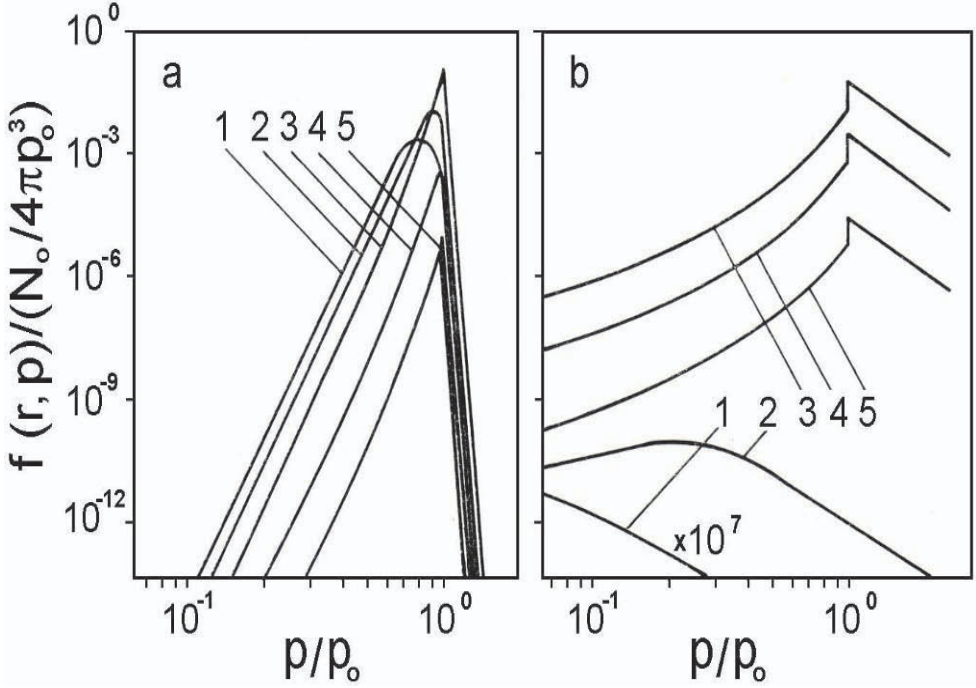


Fig. 4.21.2. Momentum spectra of particles accelerated by a standing spherical shock wave at different distances from the front: **a** – the case of weak modulation, $g_1 = g_2 = g = 0.1$; **b** – the case of strong modulation, $g_1 = g_2 = g = 10$. Curves from 1 to 5 correspond to $r/r_0 = 0.01, 0.1, 1, 10, 100$. According to Berezhko et al. (M1988).

4.21.12. Acceleration by spherical standing shock wave in the case of accretion

Let us consider, following Berezhko et al. (M1988), some elementary model of particle acceleration by a spherical standing shock wave in the case of accretion. The structure of accreting plasma in the approximation of spherical symmetry is characterized by the increase of plasma velocity u directed towards the gravitational center from zero on infinite distance to some value u_1 on the distance $r = r_0$ of shock wave transition and then have a constant value $u_2 = u_1/\sigma$ at $r < r_0$:

$$u(r) = \begin{cases} -u_1(r_0/r)^2 & \text{at } r > r_0, \\ -u_2 & \text{at } r < r_0, \end{cases} \quad (4.21.59)$$

where the sign $-$ means that the plasma flux is directed to the center of coordinates. In this case the solution of Eq. 4.21.48 with the boundary condition Eq. 4.21.49, and

with injection sources described by Eq. 4.21.50 and Eq. 4.21.51, will be for the outer region (before the shock front at $r > r_o$):

$$f_1(r, p)(\beta - 1) = f_{r_o}(p) \left(\exp \int_r^\infty \frac{u_1 r_o^2}{\kappa_1 r^2} dr - 1 \right) + \frac{N_\infty \delta(p - p_o)}{4\pi p_o^2} \left(\beta - \exp \int_r^\infty \frac{u_1 r_o^2}{\kappa_1 r^2} dr \right), \quad (4.21.60)$$

where

$$f_{r_o}(p) = f_1(r_o, p) = f_2(r_o, p), \quad \beta = \exp \left(\int_{r_o}^\infty \frac{u_1 r_o^2}{\kappa_1 r^2} dr \right). \quad (4.21.61)$$

For the region behind the shock front at $r < r_o$ the solution can be obtained very easily for a small diffusion coefficient, when $\kappa_2 \ll u_2 r_o$ or $g_2 \ll 1$ (it can be actually caused by high plasma excitation in this region). This means that in Eq. 4.21.48 can be neglected by the diffusion term in comparison with terms describing convection and adiabatic heating, and in the stationary case for the region $r < r_o$ Eq. 4.21.48 can be rewritten as

$$u \frac{\partial f_2}{\partial r} - \frac{\nabla \mathbf{u}}{3} p \frac{\partial f_2}{\partial p} = 0. \quad (4.21.62)$$

The solution of this equation with boundary condition $f_2(r_o, p) = f_{r_o}(p)$ is as follows

$$f_2(r, p) = f_{r_o} \left(p \exp \left(\frac{1}{3} \int_r^{r_o} \frac{\nabla \mathbf{u}}{u} dr \right) \right). \quad (4.21.63)$$

The factor $\exp \left(\frac{1}{3} \int_r^{r_o} \frac{\nabla \mathbf{u}}{u} dr \right)$ in Eq. 4.21.63 reflects the adiabatic increasing of particle momentum in compressing plasma ($\nabla \mathbf{u} < 0$).

The equation for the momentum spectrum on the shock front $f_{r_o}(p)$ can be found on the basis of the boundary condition described by Eq. 4.21.49 for the mono-energetic injection source (Eq. 4.21.50–4.21.51) and taking into account Eq. 4.21.60 and Eq. 4.21.63:

$$\frac{u_1 \beta}{1-\beta} f_{r_o}(p) + \frac{u_1 - u_2}{3} p \frac{\partial f_{r_o}(p)}{\partial p} \left(1 + \frac{2}{g_2} \right) = u_1 \frac{N_o + N_\infty \beta / (1-\beta)}{4\pi p_o^2} \delta(p - p_o). \quad (4.21.64)$$

The integration of this equation gives

$$f_{r_o}(p) = \frac{N_o + N_\infty \beta_o / (1-\beta_o)}{4\pi p_o^3} \theta(p - p_o) \exp \left(\frac{3u_1}{u_1 - u_2} \int_{p_o}^p \frac{dp'}{p'} \frac{\beta}{(\beta - 1)(1 + 2/g_2)} \right), \quad (4.21.65)$$

where $\beta_o = \beta(p_o)$.

4.21.13. Acceleration by spherical running shock wave

Let us consider, following Berezhko et al. (M1988), the particle acceleration by spherical running shock wave. The spherically symmetric running shock wave is determined by the law of its extending $r_o(t)$. The field of plasma velocities in the case in which the undisturbed plasma is in the rest will be

$$\mathbf{u}(r, t) = \begin{cases} \mathbf{r}u(r, t)/r & \text{if } r < r_o(t), \\ 0 & \text{if } r > r_o(t). \end{cases} \quad (4.21.66)$$

The plasma leaks on the shock front with the velocity $u_1 = dr_o/dt$ and flows behind the front with velocity $u_2 = u_1/\sigma$, where σ is the compressibility of plasma. The particle acceleration by this running shock wave is a sufficiently non-stationary problem, described by Eq. 4.21.48 for the field velocities of matter Eq. 4.21.66. But the main peculiarities of this acceleration investigated in Krymsky and Petukhov (1980), Prishchep and Ptuskin (1981), Berezhko and Krymsky (1982), Drury (1983) can be, according to Berezhko et al. (1984, M1988), analyzed on the basis of approximate solutions of Eq. 4.21.48. The matter of this method is that solutions before the shock front $f_1(r, p, t)$ and behind the front $f_2(r, p, t)$ are expressed through the momentum spectrum on the front $f_{r_o}(p, t) = f_1(r_o, p, t) = f_2(r_o, p, t)$ which then can be found from the boundary conditions described by Eq. 4.21.49–4.21.50 (as it was made in Section 4.21.12 for the problem of particle acceleration by the standing shock wave in the case of accretion). This method can be applied for any type of shock wave extending. The simple solutions can be obtained more easy for very small parameters of modulation $g_{1,2} \ll 1$ and very big parameters of modulation $g_{1,2} \gg 1$.

Let us consider first the problem for the region behind the front $r < r_o(t)$. If we suppose that the diffusion coefficient here is very small (as in Section 4.21.12), the transport Eq. 4.21.48 can be transformed into

$$\frac{\partial f_2}{\partial t} = \frac{\nabla \mathbf{u}}{3} p \frac{\partial f_2}{\partial r} - u \frac{\partial f_2}{\partial r}, \quad (4.21.67)$$

the solution of which satisfying the boundary condition $f_2(r_o, p, t) = f_{r_o}(p, t)$ will be

$$f_2(r, p, t) = f_{r_o} \left(p \exp \left(\int_{t_{r_o}}^t \frac{\nabla \mathbf{u}(s, t')}{3} dt' \right), t_{r_o} \right). \quad (4.21.68)$$

Here the factor $\exp \left(\int_{t_{r_o}}^t \frac{\nabla \mathbf{u}(s, t')}{3} dt' \right)$ reflects the adiabatic changing of the particle momentum $s(t)$ is the solution of the equation $ds/dt = u(s, t)$ with the boundary conditions $s(t_{r_o}) = r_o(t_{r_o})$, $s(t) = r$. This means that neglecting the particle diffusion leads to the situation in which accelerated particles in the region 2 are moving together with the plasma fluxes.

For the region 1 at $r > r_o$, where $u = 0$, let us suppose that the diffusion coefficient $\kappa_1(r) = \text{const}$ and injection of particles into the regime of acceleration is only at the shock front. In this case we obtain the transport equation

$$\frac{\partial f_1}{\partial t} = \frac{\kappa_1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial f_1}{\partial r} \right), \quad (4.21.69)$$

the solution of which for the boundary conditions

$$f_1(r_o, p, t) = f_{r_o}(p, t), \quad f_1(\infty, p, t) = 0 \quad (4.21.70)$$

will be

$$f_1(r, p, t) = \frac{1}{4r\sqrt{\pi}} \int_0^t \frac{r - r_o(t')}{(\kappa_1(t - t'))^{3/2}} \exp \left(-\frac{(r - r_o(t'))^2}{4\kappa_1(t - t')} \right) \mu(t') dt', \quad (4.21.71)$$

where the function $\mu(t)$ is the solution of the integral equation

$$\frac{1}{4\sqrt{\pi}} \int_0^t \frac{r_o(t) - r_o(t')}{(\kappa_1(t - t'))^{3/2}} \exp \left(-\frac{(r_o(t) - r_o(t'))^2}{4\kappa_1(t - t')} \right) \mu(t') dt' + \frac{\mu(t)}{2\kappa_1} = r_o(t) f_{r_o}(t). \quad (4.21.72)$$

For the case $g_1 \gg 1$ the approximate solution of Eq. 4.21.72 gives

$$\mu(t) = \kappa_1 r_o(t) f_{r_o}(p, t) \left[1 + (b+1)/g_1 + O(g_1^{-2}) \right], \quad (4.21.73)$$

where

$$b = d \ln f_{r_o}(p, t) / d \ln r_o(t). \quad (4.21.74)$$

By substituting Eq. 4.21.73 in Eq. 4.21.71 and integrating over t' we obtain

$$f_1(r, p, t) = f_{r_o}(p, t) \frac{r_o(t)}{r} \exp \left\{ -g_1 \frac{r - r_o(t)}{r_o(t)} \left[1 + \frac{b+1-(\nu-1)/\nu}{g_1} \right] \right\} + O(g_1^{-2}), \quad (4.21.75)$$

where the parameter ν determines the character of the expansion of the shock wave:

$$\nu = d \ln r_o(t) / d \ln t; \quad r_o(t) = r_{oo}(t/t_o)^\nu; \quad r_{oo} = r_o(t_o). \quad (4.21.76)$$

Eq. 4.21.75 shows that the width L of the region before the shock front occupied by accelerated particles at $g_1 \gg 1$ is very narrow in comparison with $r_o(t)$: $L \approx r_o/g_1 \approx \kappa_1/u_1$. With decreasing of modulation parameter g_1 the width L increases up to radius of shock wave r_o . The Eq. 4.21.68 and Eq. 4.21.75 described solutions for both regions behind and before shock front. The function $f_{r_o}(p, t)$ which enter into these equations can be determined by sewing together the solutions described by Eq. 4.21.68 and Eq. 4.21.75 on the basis of the boundary conditions on the shock front. Therefore we obtain

$$u_1 \left[1 + \frac{b+2-(\nu-1)/\nu}{g_1} + \frac{\sigma b}{g_2} \right] f_{r_o}(p, t) - \frac{\Delta u}{3} \left[1 - \frac{d}{g_2(\sigma-1)} \right] p \frac{\partial f_{r_o}(p, t)}{\partial p} = Q_o, \quad (4.21.77)$$

where

$$d = r_o \nabla \mathbf{u}(r = r_o - 0) / \mathbf{u}_2; \quad Q_o = u_1 \frac{N_o(t)}{4\pi p_o^2} \delta(p - p_o). \quad (4.21.78)$$

The solution of Eq. 4.21.77 with an accuracy about $1/g_1^2$ will according to Berezhko et al. (1984, M1988) be as follows:

$$f_{r_o}(p, t) = \frac{N_o(t)}{4\pi p_o^3} \gamma \left(\frac{p}{p_o} \right)^{-\gamma} \theta(p - p_o), \quad (4.21.79)$$

where the spectrum index

$$\gamma = \frac{3\sigma}{(\sigma - 1)} \left[1 + \frac{\delta + 2 - (\nu - 1)/\nu}{g_1} + \frac{\delta\nu + d/\Delta u}{g_2} \right], \quad (4.21.80)$$

and parameter

$$\delta = d \ln N_o(t) / d \ln r_o(p, t) \quad (4.21.81)$$

characterizes the particle injection rate with time. Let us suppose that the diffusion coefficients are increasing functions with momentum p , i.e.,

$$\kappa_1(p) = \kappa_{o1} (p/p_o)^\alpha; \quad \kappa_2(p) = \kappa_{o2} (p/p_o)^\alpha, \quad (4.21.82)$$

where $\alpha > 0$. In this case for the conditions $g_{1,2}(p, t) \geq 1$ and $g_2(p, t) \gg g_1(p, t)$ the requested region of the momentum $p_o \leq p \leq p_m(t)$ can be determined, where $p_m(t)$ is estimated from the equation $g_1(p_m, t) = 1$, i.e.,

$$p_m(t) = p_o \left(\nu r_{o2}^2 / \kappa_{o1} t_o \right)^{1/\alpha} (t/t_o)^{(2\nu-1)/\alpha}. \quad (4.21.83)$$

At $p \approx p_m(t)$ the momentum spectrum steepens sufficiently, so the value $p_m(t)$ can be considered as an effective upper limit of accelerated particles momentum spectrum.

4.21.14. Effects of finite duration shock acceleration

Ruffolo and Channok (2003) present analytic and numerical models of finite duration shock acceleration. For a given injection momentum p_o , after a very short time there is only a small boost in momentum, at intermediate times the spectrum is a power law with a hump and steep cutoff at a critical momentum, and at longer times the critical momentum increases and the spectrum approaches the steady-state power law. The composition dependence of the critical momentum is different from that obtained for other cutoff mechanisms. The spectral form of Ellison and Ramaty (1985), a power law in momentum with an exponential rollover in energy, has proved very useful in fitting spectra of solar energetic particles. The composition dependence of the rollover energy depends on the physical effect that causes the

rollover. For interplanetary shocks traveling well outside the solar corona, observations typically indicate a rollover at ~ 0.1 to 1 MeV/nucleon. Ruffolo and Channok (2003) consider what physical mechanism could explain this. If there is a cutoff for κ/u on the order of the shock thickness (Ellison and Ramaty, 1985), where κ is the parallel diffusion coefficient and u is the fluid velocity along the field, the observed long mean free paths for pickup ions (Gloeckler et al., 1995) would imply an extremely low cutoff energy. On the other hand, a cutoff owed to shock-drift acceleration across the entire width of a shock (such as that inferred for anomalous CR) is of the order of hundreds of MeV per unit charge. Ruffolo and Channok (2003) propose that the physical origin of such rollovers is the finite time available for shock acceleration. The typical acceleration timescale t_{ac} corresponding to observed mean free paths is of the order of several days, so the process of shock acceleration at an interplanetary shock near Earth should usually give only a mild increase in energy to an existing seed particle population. Indeed, the analyses of ACE observations argue for a seed population at substantially higher energies than the solar wind (Desai et al., 2003). On the other hand, finite duration shock acceleration should yield the standard power-law spectrum in the limit of a long duration t relative to the acceleration timescale. As a corollary of this idea, for an unusually strong shock (unusually short acceleration timescale) it is possible to obtain power-law spectra up to high energies (e.g., as observed by Reames et al., 1997). Therefore, Ruffolo and Channok (2003) derives a simple theory of finite duration shock acceleration and explores implications for the composition dependence of the spectrum. They consider a combinatorial model of finite duration shock acceleration assuming a constant acceleration rate r_a (i.e., the rate of a complete cycle returning upstream, or $1/\Delta t$ of Drury, 1983) and a constant escape rate r_e . After a time t the distribution of residence time T is

$$P(T) = r_e \exp(-r_e T) + \exp(-r_e t) \delta(T - t). \quad (4.21.84)$$

The Poisson distribution of the number of acceleration events n during T is

$$P(n, T) = \frac{(r_a T)^n}{n!} \exp(-r_a T). \quad (4.21.85)$$

The overall probability of n acceleration events is

$$\begin{aligned} P(n, t) = \int_0^t P(n, T) P(T) dT &= \frac{r_e}{r_a} \left(\frac{r_a}{r_a + r_e} \right)^{n+1} \exp(-t(r_a + r_e)) \sum_{k=n+1}^{\infty} \frac{(t(r_a + r_e))^k}{k!} \\ &+ \exp(-t(r_a + r_e)) \frac{(tr_a)^n}{n!}. \end{aligned} \quad (4.21.86)$$

The first term in Eq. 4.21.86 is an exponential in n times a Poisson probability of $> n$ acceleration events, and the second term, corresponding to a finite probability of residence time $T = t$, is a Poisson distribution at $\langle n \rangle = r_a t$. Usually $r_e \ll r_a$ so the result (in terms of momentum) is a power law spectrum with a hump and subsequent cutoff after $\sim r_a t$ acceleration events. A more complicated analytic expression can be derived for the more realistic case in which r_a and r_e depend on n (and particle momentum).

The following system of differential equations can be shown to be equivalent to the above approach, and is more convenient for computations. Ruffolo and Channok (2003) express $P(n, t)$ as the sum of $E(n, t)$ and $A(n, t)$, the fraction of particles escaping and remaining, respectively, after n acceleration events at time t . Then

$$\frac{dA_n}{dt} = -(r_{a,n} + r_{e,n})A_n + r_{a,n-1}A_{n-1}; \quad \frac{dE_n}{dt} = r_{e,n-1}A_{n-1} \quad (4.21.87)$$

with the initial condition $A(0,0) = 1$ and all other A, E equal to zero at $t = 0$.

For a general shock angle, we may use $r_{a,n}$ and $r_{e,n}$ which depend on the particle velocity v_n (following Drury, 1983):

$$r_{a,n} = \frac{v_n/4}{\frac{\kappa_1}{u_1} + \frac{\kappa_2}{u_2}}, \quad r_{e,n} = \frac{u_2 \cos \theta_2 / \cos \theta_1}{\left(\frac{\kappa_1}{u_1} + \frac{\kappa_2}{u_2} \right) \left(1 - \frac{4u_2 \cos \theta_2}{v_n \cos \theta_1} \right)}, \quad (4.21.88)$$

where θ is the field-shock normal angle, the subscript 1 refers, as usually, to upstream of the shock, and 2 refers to downstream. The particle momentum increases at each acceleration event according to

$$\frac{p_n}{p_{n-1}} = 1 + \frac{4}{3} \frac{u_1 \cos \theta_1 - u_2 \cos \theta_2}{v_{n-1} \cos \theta_1}. \quad (4.21.89)$$

The differential energy spectrum vs. kinetic energy E_k is calculated from

$$j(E_k, t) = P(n, t) / (T_{n+1} - T_n). \quad (4.21.90)$$

Fig. 4.21.3 shows results of Ruffolo and Channok (2003) for the time dependent energy spectrum of ^4He for an oblique shock wave with $u_1 = 540$ km/s, $u_2 = 140$ km/s, $\theta_1 = 45^\circ$, $\theta_2 = 75^\circ$, $\kappa = v\lambda/3$, and a parallel scattering mean free path $\lambda = 0.3$

AU (based on Rankine-Hugoniot conditions; see Ruffolo, 1999). Note that this corresponds to injection at 0.01 MeV/nucleon; for an interplanetary shock the resulting spectrum would be the convolution of such a ‘kernel’ with the seed particle spectrum. We see that after a short time the particles receive only a small boost in energy. At intermediate times, there is a power law at low energy and a hump at a certain critical energy, T_c , followed by a drastic decline. The power law and hump correspond to the two terms on the right hand side of Eq. 4.21.84; in particular, the hump corresponds to the fraction of particles that have not yet escaped and have a Poisson distribution of acceleration events n , with $\langle n \rangle \approx r_a t$. It is not clear whether a hump would be expected in observations, after convolution with the seed spectrum. The decline at high energy is qualitatively similar to that of Ellison and Ramaty (1985); however, Ruffolo and Channok (2003) obtain a different (Q/A) dependence, as shown below. At very long times the classic steady-state power law is recovered.

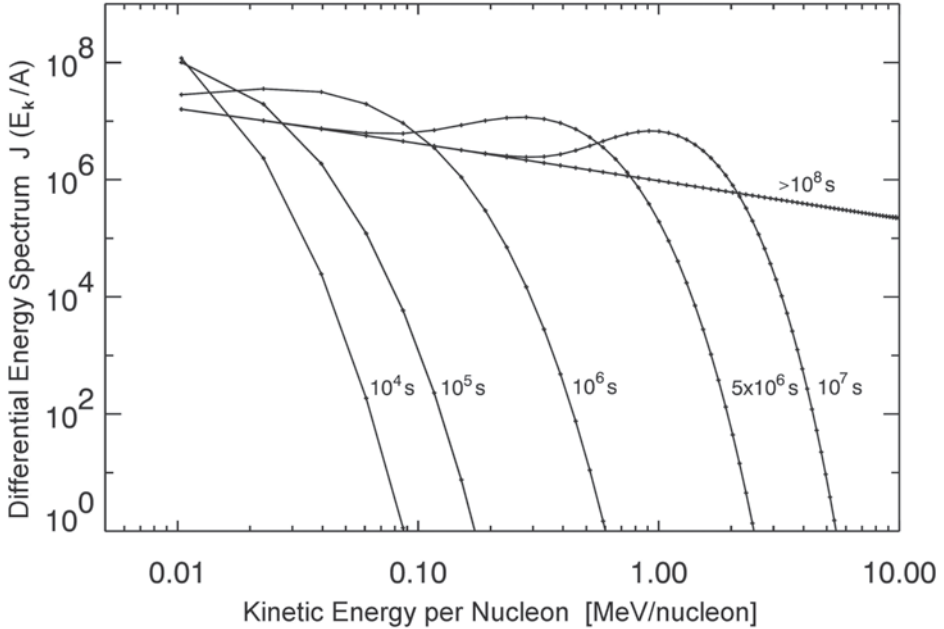


Fig. 4.21.3. Energy spectrum of ${}^4\text{He}$, injected at 0.01 MeV/nucleon, after shock acceleration for the indicated times. According to Ruffolo and Channok (2003).

For a constant acceleration rate r_a , i.e., a constant λ , there is expected the critical rigidity P_c to be approximately P_n for $n = r_a t$:

$$P_c \approx P_0 + \frac{4}{3} \frac{u_1 \cos \theta_1 - u_2 \cos \theta_2}{\cos \theta_1} \frac{mc}{q} r_a t \approx P_0 + \frac{A}{Q} \frac{m_{pc}}{e} \frac{u_1 \cos \theta_1 - u_2 \cos \theta_2}{\left(\frac{\lambda_1}{u_1} + \frac{\lambda_2}{u_2} \right) \cos \theta_1} t \quad (4.21.91)$$

(assuming non-relativistic particles). Thus for small P_0 or long times it is expected that the rollover rigidity will increase proportionally with time (with only a weak dependence on P_0), and the rollover energy to increase as t^2 . Note that for the above case of constant λ the rollover velocity (v_c) and kinetic energy per nucleon (E_{kc}/A) are independent of Q/A . For the more general case of $\lambda \propto P^\alpha$ it can be shown that

$$P_c \approx \left[P_0^{\alpha+1} + \frac{4}{3} (\alpha+1) \frac{A}{Q} \frac{m_{pc}}{e} \frac{u_1 \cos \theta_1 - u_2 \cos \theta_2}{\cos \theta_1} r_{ao} P_0^\alpha t \right]^{1/(\alpha+1)}, \quad (4.21.92)$$

or for late times,

$$E_{kc}/A \propto (Q/A)^{2\alpha/(\alpha+1)}. \quad (4.21.93)$$

For example, if $\lambda \propto P^{1/3}$ then $E_{kc}/A \propto (Q/A)^{1/2}$, a somewhat weaker dependence than the proportionality to (Q/A) that is sometimes assumed.

4.21.15. CR acceleration at quasi-parallel plane shocks (numerical simulations)

Kang and Jones (2003) studied the CR injection and acceleration efficiencies at cosmic shocks by performing numerical simulations of CR modified, quasi-parallel, shocks in 1D plane-parallel geometry for a wide range of shock Mach numbers and pre-shock conditions. According to the diffusive shock acceleration theory populations of CR particles can be injected and accelerated to very high energy by astrophysical shocks in tenuous plasmas (Malkov and Drury, 2001), and a significant fraction of the kinetic energy of the bulk flow associated with a strong shock can be converted into CR protons (Kang and Jones, 2002; Kang, 2003). Kang and Jones (2003) developed a numerical scheme that self-consistently incorporates a ‘thermal leakage’ injection model based on the analytic, nonlinear calculations of Malkov (1998a,b). This injection scheme was then implemented into the combined gas dynamics and the CR diffusion-convection code with subzone shock-tracking and multi-level adaptive mesh refinement techniques (Gieseler et al., 2000; Kang et al., 2002). Kang and Jones (2002), Kang (2003) applied this code to studying the CR acceleration at shocks by numerical simulations of CR modified, quasi-parallel shocks in 1D plane-parallel geometry with the physical parameters relevant for the

cosmic shocks emerging in the large scale structure formation of the Universe. In Kang and Jones (2003) are presented new simulations with a wide range of physical parameters. They calculated the CR acceleration at 1D quasi-parallel shocks that were driven by accretion flows in a plane-parallel geometry. Two sets of models are presented: 1) $T_o = 10^4$ K and $u_o = (15\text{km/s}) M_o$ and 2) $T_o = 10^6$ K and $u_o = (150\text{km/s}) M_o$, where T_o , u_o , and $M_o = 5-50$ are the temperature, accretion speed, and Mach number of the accretion flow, respectively. The Bohm diffusion model was adopted for the CR diffusion coefficient

$$\kappa(p) = \kappa_o p^2 / (p^2 + 1)^{1/2}, \quad (4.21.94)$$

where the particle momentum is expressed in units of $m_p c$. The choice of κ_o is arbitrary, since Kang and Jones (2003) present the results in terms of the diffusion time and length scales defined by $t_o = \kappa_o / u_o^2$ and $x_o = \kappa_o / u_o$. The gas density normalization constant, ρ_o , is arbitrary as well, but the pressure normalization constant depends on M_o as

$$P_o = \rho_o u_o^2 \propto M_o^2. \quad (4.21.95)$$

They adopted an injection scheme based on a ‘thermal leakage’ model that transfers a small proportion of the thermal proton flux through the shock into low energy CR (Malkov, 1998a,b; Gieseler et al., 2000). This model has a free parameter, $\varepsilon = B_o / B_\perp$, defined to measure the ratio of the amplitude of the post-shock MHD wave turbulence $B_\perp$ to the general magnetic field aligned with the shock normal, B_o (Malkov, 1998a,b). In Kang and Jones (2003) are presented models with $\varepsilon = 0.2$ only.

The injection efficiency in Kang and Jones (2003) are defined as the fraction of particles that have entered the shock from far upstream and then injected into the CR distribution:

$$\xi(t) = \int dx \int 4\pi f_{CR}(p, x, t) p^2 dp / (\int n_o u_s dt), \quad (4.21.96)$$

where $f_{CR}(p, x, t)$ is the CR distribution function, n_o is the particle number density far upstream, and u_s is the *instantaneous shock speed*. As a measure of acceleration efficiency they define the ‘CR energy ratio’, namely the ratio of the total CR energy within the simulation box to the kinetic energy in the *initial shock frame* that has entered the simulation box from far upstream,

$$\Phi(t) = \int dx E_{CR}(x, t) / (0.5 \rho_o u_{so}^3 t), \quad (4.21.97)$$

where u_{so} is the *initial shock speed* before any significant nonlinear CR feedback occurs. Although the sub-shock weakens as the CR pressure increases, the injection rate decreases accordingly and the sub-shock does not disappear. Kang and Jones (2003) found that the post-shock CR pressure reaches an approximately time-asymptotic value and the evolution of the CR shock becomes ‘self-similar’ owing to a balance between fresh injection/acceleration and advection/diffusion of the CR particles away from the shock. So the CR energy ratio Φ also changes asymptotically to a constant value, as shown in Fig. 4.21.4 (except in the model with $T_o = 10^6$ K and $M_o = 50$ which has not reached the time-asymptotic state up to $t/t_o = 40$). The time-asymptotic value of Φ increases with M_o , but it converges to $\Phi \approx 0.5$ – 0.6 for $M_o \geq 20$.

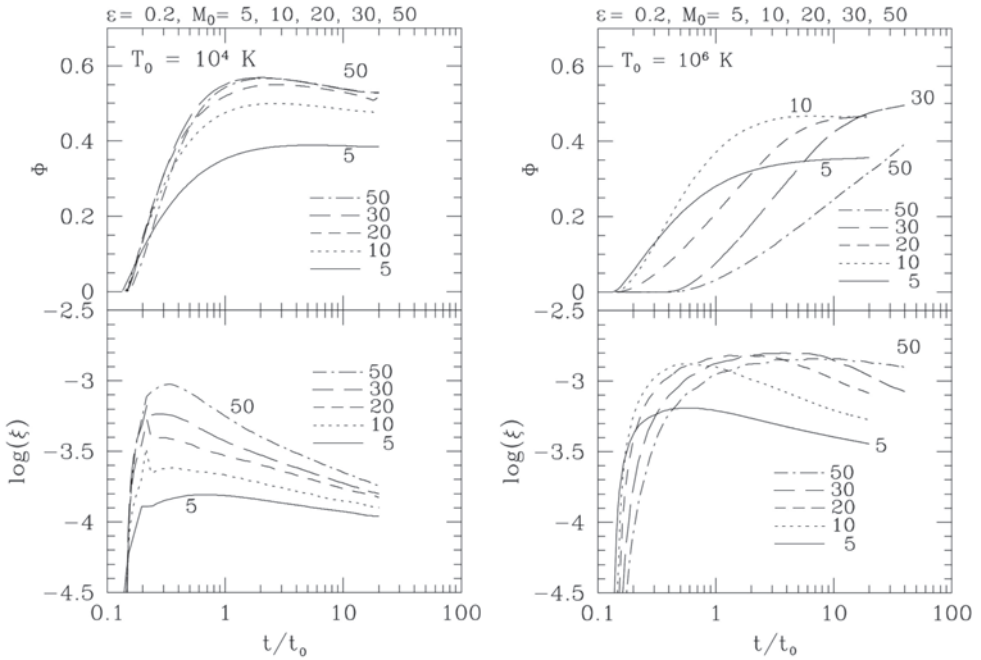


Fig. 4.21.4. The CR energy ratio, $\Phi(t)$, and time-averaged injection efficiency $\xi(t)$ for models with different accretion Mach number M_o . Left panels are for models with $T_o = 10^4$ K, while right panels for models with $T_o = 10^6$ K. Accretion speed of each model is given by $u_o = (15\text{km/s}) M_o$ for models with $T_o = 10^4$ K and by $u_o = (150\text{km/s}) M_o$ for models with $T_o = 10^6$ K. Time is given in terms of the diffusion time $t_o = \kappa_o / u_o^2 \propto M_o^{-2}$. According to Kang and Jones (2003).

The average injection rate varies in the interval $\xi \approx 10^{-4} \div 10^{-2.8}$, depending on M_o , u_o and ε . For two models with the same Mach number but different speeds (or different T_o) the injection rate is higher for models with higher speeds, but the CR energy increases more slowly in terms of the normalized time t/t_o .

Fig. 4.21.5 shows the total CR distribution within the simulation box,

$$G(p) = \int p^4 f_{cr}(p) dx \quad (4.21.98)$$

and its power law slope

$$q = -(\partial \ln G / \partial \ln p - 4). \quad (4.21.99)$$

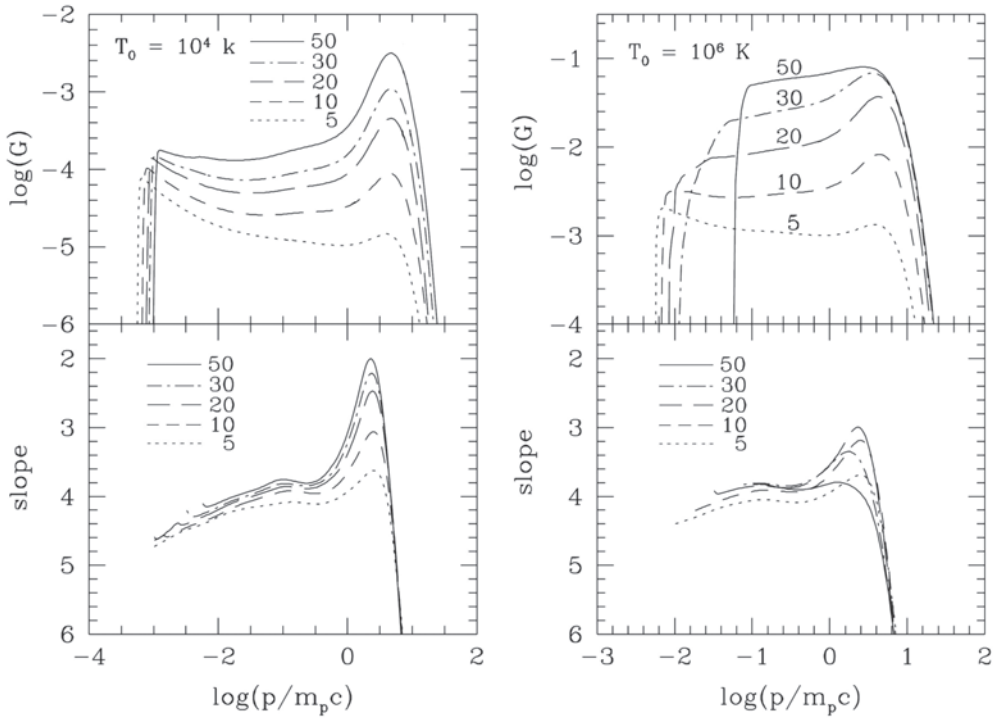


Fig. 4.21.5. CR distribution function integrated over the simulation box, $G(p)$, which is determined by Eq. 4.21.98, and its power law slope, q determined by Eq. 4.21.99, at $t/t_o = 20$. The curves are labeled with the accretion Mach number M_o . According to Kang and Jones (2003).

For all models shown in Fig. 4.21.5, $G(p)$ has an exponential cut-off at a similar momentum ($p_{\max} \approx 4$) regardless of values of u_o , since the results are shown at the same values of $t/t_o = 20$. The integrated distributions show the characteristic ‘concave upwards’ curves owed to nonlinear modification to the shock structures, and this ‘flattening’ trend is stronger for higher M_o models.

Kang and Jones (2003) came to the following conclusions:

1. The CR pressure seems to approach a steady-state value and the evolution of CR modified shocks becomes approximately ‘self-similar’.
2. Supra-thermal particles can be injected very efficiently into the CR population via the thermal leakage process, so that typically a fraction of $10^{-4} - 10^{-3}$ of the particles passed through the shock becomes CR.
3. For a given injection model, the acceleration efficiency increases with the shock Mach number, M_s , but it moves asymptotically to a limiting value of the CR energy ratio, $\Phi \approx 0.5 - 0.6$, for $M_s > 30$.

4.22. Simplified ‘box’ models of shock acceleration

4.22.1. Principles of ‘box’ models of shock acceleration

Many authors (Völk et al., 1981a,b; Bogdan and Völk, 1983; Moraal and Axford, 1983; Lagage and Cesarsky, 1983; Schlickeiser, 1984; Völk and Biermann, 1988; Ball and Kirk, 1992; Protheroe and Stanev, 1998) have used, under various guises, a simplified but physically intuitive treatment of shock acceleration, sometimes referred to as a ‘box’ model. Drury et al. (1999) present an alternative more physical interpretation of the ‘box’ model which can be significantly different when additional loss processes, such as synchrotron or inverse Compton losses, are included. The main features of the ‘box’ model, as presented in the literature (see references above) and exemplified by Protheroe and Stanev (1998), can be summarized as follows. The particles being accelerated (and thus ‘inside the box’) have differential energy spectrum $N(E)$ and are gaining energy at rate $r_{ac}E$ but simultaneously escape from the acceleration box at rate r_{esc} . Conservation of particles then requires

$$\frac{\partial N(E)}{\partial t} + \frac{\partial}{\partial E}(r_{ac}EN(E)) = Q(E) - r_{esc}N(E), \quad (4.22.1)$$

where $Q(E)$ is a source term combining advection of particles into the box and direct injection inside the box. In essence this approach tries to reduce the entire acceleration physics to a ‘black box’ characterized simply by just two rates, r_{ac} and r_{esc} . These rates have, of course, to be taken from more detailed theories of shock acceleration (e.g., above, and Drury, 1991).

4.22.2. Physical interpretation of the 'box' model

Drury et al. (1999) prefer a very similar, but more physical, picture of shock acceleration which has the advantage of being more closely linked to the conventional theory. For this reason they also choose to work in terms of particle momentum p and the distribution function $f(p)$ rather than E and $N(E)$. For an almost isotropic distribution $f(p)$ at the shock front where the frame velocity changes from u_1 to u_2 , then it is easy to calculate that there is a flux of particles upwards in momentum associated with the shock crossings of

$$\Phi(p, t) = \int p \frac{\mathbf{v}(\mathbf{u}_1 - \mathbf{u}_2)}{v^2} p^2 f(p, t) \mathbf{v} \cdot \mathbf{n} d\Omega = \frac{4\pi p^3}{3} f(p, t) \mathbf{n} \cdot (\mathbf{u}_1 - \mathbf{u}_2), \quad (4.22.2)$$

where $\mathbf{n}$ is the unit shock normal and the integration is over all directions of the velocity vector $\mathbf{v}$. This flux is localized in space at the shock front and is strictly positive for a compressive shock structure; in this description it replaces the acceleration rate r_{ac} . The other key element is the loss of particles from the shock by advection downstream. Drury et al. (1999) note that the particles interacting with the shock are those located within one diffusion length of the shock. Particles penetrate upstream a distance of order

$$L_1(p) = (\mathbf{n} \cdot \hat{\mathbf{K}}_1(p)) \cdot \mathbf{n} / (\mathbf{n} \cdot \mathbf{u}_1), \quad (4.22.3)$$

where $\hat{\mathbf{K}}_1(p)$ is the diffusion tensor and the probability of a downstream particle returning to the shock decreases exponentially with a scale length of

$$L_2(p) = (\mathbf{n} \cdot \hat{\mathbf{K}}_2(p)) \cdot \mathbf{n} / (\mathbf{n} \cdot \mathbf{u}_2), \quad (4.22.4)$$

Thus in this picture there are an energy dependent acceleration region extending a distance $L_1(p)$ upstream and $L_2(p)$ downstream. The total size of the box is then

$$L(p) = L_1(p) + L_2(p). \quad (4.22.5)$$

Particles are swept out of this region by the downstream flow at a bulk velocity $\mathbf{n} \cdot \mathbf{u}_2$. Conservation of particles then leads to the following approximate description of the acceleration,

$$\frac{\partial}{\partial t} (4\pi p^2 f(p, t) L(p)) + \frac{\partial \Phi}{\partial p} = Q(p, t) - 4\pi (\mathbf{n} \cdot \mathbf{u}_2) p^2 f(p, t). \quad (4.22.6)$$

4.22.3. Inclusion of additional loss processes

According to Drury et al. (2003), it is relatively straightforward to include losses of the synchrotron or inverse Compton type (Thomson regime) in the model. These generate a downward flux in momentum space, but one which is distributed throughout the acceleration region. Combined with the fact that the size of the ‘box’ or region normally increases with energy this also gives an *additional* loss process because particles can now ‘fall’ through the back of the ‘box’ as well as being advected out of it. Note that particles which ‘fall’ through the front of the box are advected back into the acceleration region and thus this process does not work upstream. This is shown schematically in Fig 4.22.1.

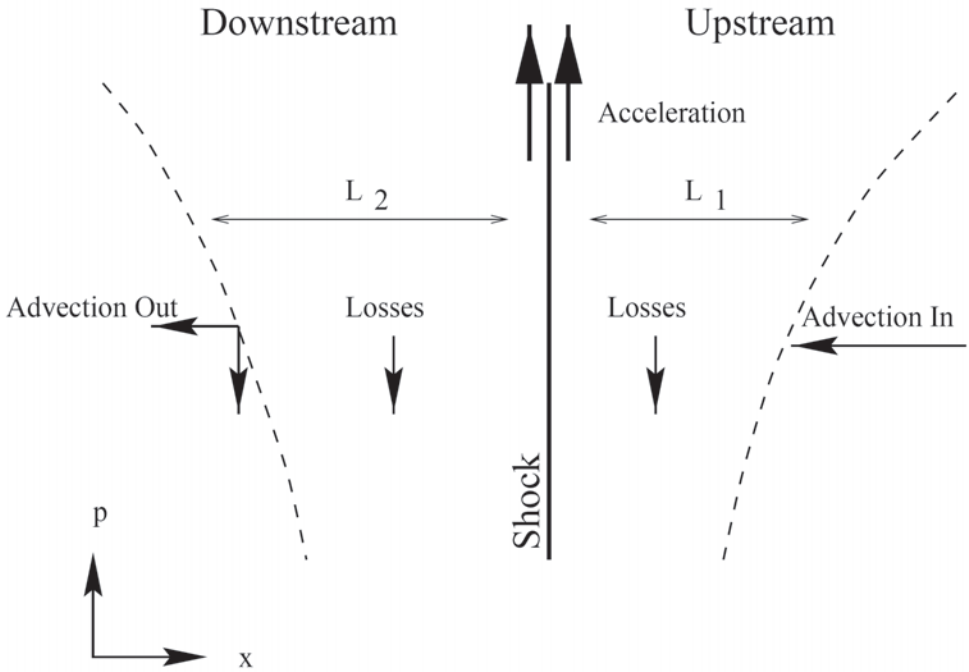


Fig. 4.22.1. A graphical representation in the x, p plane of the ‘box’ model of the shock wave particle acceleration. According to Drury et al. (1999).

If the loss rate is $\dot{p} = -\alpha p^2$ (the generalization to different loss rates upstream and downstream is trivial) the basic equation becomes

$$\frac{\partial}{\partial t} (4\pi p^2 L f) + \frac{\partial}{\partial p} \left(\Phi - 4\pi p^2 f(p) \alpha L \right) = Q - u_2 4\pi p^2 f(p) - 4\pi \alpha p^4 f(p) \frac{dL_2}{dp}. \quad (4.22.7)$$

In the steady state and away from the source region this gives immediately the remarkably simple result for the logarithmic slope of the spectrum,

$$\frac{\partial \ln f}{\partial \ln p} = - \frac{3(u_1 - 4\alpha p L - \alpha p^2 (dL_1/dp))}{u_1 - u_2 - 3\alpha p L}. \quad (4.22.8)$$

The denominator goes to zero at the critical momentum

$$p_{cr} = (u_1 - u_2)/(3\alpha L), \quad (4.22.9)$$

where the losses exactly balance the acceleration. If the numerator at this point is negative, the slope goes to $-\infty$ and there is no pile-up. However, the slope goes to $+\infty$ and a pile-up occurs if

$$u_1 - 4u_2 + 3\alpha p^2 (dL_1/dp) > 0 \quad \text{at} \quad p = p_{cr}. \quad (4.22.10)$$

In early analytic work (Webb et al., 1984; Bregman et al., 1981) the diffusion coefficient was taken to be constant, so $dL_1/dp = 0$ and this condition reduces to $u_1 > 4u_2$. However, if, as in the work of Protheroe and Stanev (1998), the diffusion coefficient is an increasing function of energy or momentum the condition becomes less restrictive. For a power-law dependence of the form $K \propto p^\beta$ the condition for a pile-up to occur reduces to

$$u_1 - 4u_2 + \beta(u_1 - u_2) \frac{L_1}{L_1 + L_2} > 0. \quad (4.22.11)$$

Drury et al. (1999) note that the equivalent criterion for the model used by Protheroe and Stanev (1998) is slightly different, namely,

$$u_1 - 4u_2 + \beta(u_1 - u_2) > 0 \quad (4.22.12)$$

because of their neglect of the additional loss process. For the case in which $L_1/L_2 = u_2/u_1$ and with $\beta = 1$ this condition predicts that shocks with compression ratios greater than about $r = 3.45$ will produce pile-ups whilst weaker shocks will not.

4.22.4. Including nonlinear effects in the ‘box’ model

Drury et al. (1999) note that at the phenomenological and simplified level of the ‘box’ models it is possible to allow for nonlinear effects by replacing the upstream velocity with an effective momentum-dependent velocity $u_1(p)$, reflecting the existence of an extended upstream shock precursor region sampled on different

length scales by particles of different energies. With a momentum-dependent $u_1(p)$ the logarithmic slope of the spectrum is

$$\frac{\partial \ln f}{\partial \ln p} = - \frac{3(u_1 - 4\alpha p L + (p/3)(du_1/dp) - \alpha p^2(dL_1/dp))}{u_1 - u_2 - 3\alpha p L} \quad (4.22.13)$$

with a pile-up criterion of

$$u_1(p) - 4u_2 - p(du_1/dp) + 3\alpha_1 p^2(dL_1/dp) > 0 \quad \text{at} \quad p = p_{cr}, \quad (4.22.14)$$

where p_{cr} is determined by Eq. 4.22.9. From Eq. 4.22.14 it can be seen that whether or not the nonlinear effects assist the formation of pile-ups depends critically on how fast they make the effective upstream velocity vary as a function of p . By making $u_1(p_{cr})$ larger they make it easier for pile-ups to occur. On the other hand, if the variation is more rapid than $u_1(p) \propto p$, the derivative term dominates and inhibits the formation of pile-ups. If the electrons are test-particles in a shock strongly modified by proton acceleration, and if the Malkov (1998a,b) scaling $u_1(p) \propto p^{1/2}$ holds even approximately, then a strong synchrotron pile-up appears inevitable (unless the maximum attainable momentum is limited by other effects to a value less than p_{cr}).

4.22.5. Main peculiarities of ‘box’ models

Drury et al. (1999) concluded that a major defect of all ‘box’ models is the basic assumption that all particles gain and lose energy at exactly the same rate. It is clear physically that there are very large fluctuations in the time particles spend in the upstream and downstream regions between shock crossings, and thus correspondingly large fluctuations for energy lost. The effect of these variations will be to smear out the artificially sharp pile-ups predicted by the simple ‘box’ models. However, the results described above are based simply on the scaling with energy of the various gain and loss processes together with the size of the acceleration region. Thus, they should be relatively robust, and Drury et al. (1999) expect that even if there is no sharp spike, the spectrum will show local enhancements over what it would have been in the absence of the synchrotron or inverse Compton losses in those cases in which this criterion is satisfied.

4.23. Diffusive shock wave acceleration in space plasma with accounting non-linear processes

4.23.1. Bulk CR transport in space plasma and diffusive shock wave acceleration

In Section 2.21 we reviewed the paper of Vainio and Schlickeiser (1999a) on the bulk transport of CR particles caused by the quasi-linear interactions with transverse, parallel-propagating plasma waves. Vainio and Schlickeiser (1999a) note that in cosmic shock waves particles can gain energy through first and second order Fermi mechanisms by multiple shock crossings and stochastic downstream acceleration, respectively. When first-order Fermi acceleration dominates, the spectral index of the shock accelerated particles is

$$\Gamma = (r_k + 2)/(r_k - 1) \quad (4.23.1)$$

and is thus determined by the scattering-center compression ratio of the shock,

$$r_k(p) = (u_1 + V_1(p))/(u_2 + V_2(p)), \quad (4.23.2)$$

where u_1 and u_2 are the upstream and downstream flow speeds of the plasma in the shock-frame, and V_1 and V_2 are the respective relative bulk speeds of particles owed to finite phase speed of the waves (see Section 2.21). Vainio and Schlickeiser (1999a) examine the effects of the non-zero wave speeds at the first-order Fermi acceleration of CR. In the upstream region of the shock they assume that all waves are propagating against the flow (backward waves, $w < 0$) if they are self-generated by the accelerated particles through the streaming instability. Then they assumed that backward waves are generated at all frequencies $-1 < f' < \Phi_p$. All upstream waves that have $-w < u_1$ will then be converted to the shock and become downstream waves. In the downstream region, owing to the interaction of the upstream waves with the shock, waves propagating in both directions will be present. For Alfvén waves Vainio and Schlickeiser (1999b,c) showed that the dominating downstream wave components are the backward ones. However, since it was assumed that the waves are generated in the upstream region it can not have downstream backward waves propagating faster than the shock relative to the downstream plasma. This implies that there are no backward waves at frequencies

$$\frac{1}{2}(-1 + \Phi_p - f_o) < f' < \frac{1}{2}(-1 + \Phi_p + f_o), \quad (4.23.3)$$

where

$$f_o^2 = (1 + \Phi_p)^2 - 4\Phi_p M^2 / r, \quad (4.23.4)$$

and M is the Alfvénic Mach number of the shock and r is the gas compression ratio of the shock. When the downstream Alfvénic Mach number

$$M/r^{1/2} > w_{\max}/v_a = (1 + \Phi_p)(4\Phi_p)^{-1/2} \approx 21.4 \quad (4.23.5)$$

all waves are able to propagate in the downstream region. For cold upstream plasma this means that $M > 42.8$ but since there are also considered the downstream modes in the cold plasma approximation, it must restrict ourselves too small gas compression ratios and shocks with $r^{1/2} < M < 2$. As an illustrative example, Vainio and Schlickeiser (1999a) consider downstream turbulence consisting of (i) Alfvén waves with $|f| \ll \Phi_p$ being dominated by the backward propagating waves, (ii) forward whistler waves with $\Phi_p \ll f \ll 1$, and (iii) equal intensities of forward and backward waves near the cyclotron frequencies. The latter assumption is made since it is not really known how waves with high frequencies and wave numbers interact with shocks and since other wave-generation processes may also be important at high frequencies. For upstream waves it was assumed that waves at low wave numbers dominate in intensity over the waves at high wave numbers. It was assumed also that all upstream waves are backward waves. Using these assumptions for turbulence near the shock and the results of Section 2.21 Vainio and Schlickeiser (1999a) conclude the following: (i) upstream and downstream bulk speed of the energetic ($v \gg v_a$) ions relative to the plasma is close to the local Alfvén speed, $V \geq v_a$; (ii) upstream bulk speed of energetic electrons is decreasing with momentum from $V_1 \approx -9v_a$ at $v \approx 2w_{\max}$ to $V_1 \geq -v_a$ at ultra-relativistic ($\gamma > 200$) energies; (iii) downstream bulk speed of energetic electrons is $V_2 > 0$ at non-relativistic energies and $V_2 \approx -v_a$ at ultra-relativistic energies. The study of Vainio and Schlickeiser (1999a) reveals that (i) for ions and ultra-relativistic ($E > 100$ MeV) electrons

$$r_k = r(M-1)/\left(M + H_{c,2}r^{1/2}\right), \quad (4.23.6)$$

where the downstream cross-helicity state is close to $H_{c,2} = -1$ and, thus, $\Gamma \sim 1$ (see Vainio and Schlickeiser, 1999b,c for a more details); (ii) for less energetic electrons, the first-order Fermi process will be less efficient and will, in fact, turn to deceleration at mildly relativistic or non-relativistic energies. Thus Vainio and Schlickeiser (1999a) expect stochastic acceleration in the downstream region to determine the spectrum of these electrons at the shock.

4.23.2. Simulating CR particle acceleration in shocks modified by CR non-linear effects

Jones and Kang (2003) developed a new, fast numerical scheme for the CR diffusion convection equation to be applied to the study of the nonlinear time evolution of CR modified shocks for arbitrary spatial diffusion properties. To reduce the effort required to solve the diffusion convection equation during diffusive shock acceleration, they take advantage of the expected smooth structure of the particle distribution function, $f(p)$, and used a finite volume approach in momentum space along with a simple model for the distribution inside individual volumes of momentum space. The method extends that in Jones (1999), and Jun and Jones (1999), and is similar to the scheme introduced by Miniati (2001), but computationally simpler. Those authors actually ignored spatial diffusion, so could not explicitly treat diffusive shock acceleration. They applied analytic test-particle solutions of the diffusion convection equation for $f(p)$ at shock jumps.

The method used in Jones and Kang (2003) is as follows. Ignoring momentum diffusion they write in 1D-geometry:

$$\frac{\partial f}{\partial t} + u \frac{\partial f}{\partial x} = \frac{\partial}{\partial x} \left(\kappa \frac{\partial f}{\partial x} \right) + \frac{1}{3} \left(\frac{\partial u}{\partial x} \right) p \frac{\partial f}{\partial p} + S, \quad (4.23.7)$$

where S is a convenient source term, and all the other symbols take their usual meanings. The number of CR particles in the momentum bin $\Delta p_i = [p_i, p_{i+1}]$ is

$$n_i = \int_{p_i}^{p_{i+1}} p^2 f(p) dp. \quad (4.23.8)$$

Integrating over the finite momentum volume bounded by Δp_i gives

$$\frac{\partial n_i}{\partial t} + \frac{\partial (un_i)}{\partial x} = F_{n_i} - F_{n_{i+1}} + \frac{\partial}{\partial x} \left(K_{n_i} \frac{\partial n_i}{\partial x} \right) + S_{n_i}, \quad (4.23.9)$$

where

$$F_{n_i} = \dot{p}_i p_i^2 f(p_i), \quad (4.23.10)$$

with the ‘momentum speed’, $\dot{p} = -\frac{p}{3} \frac{\partial u}{\partial x}$,

$$K_{n_i} = \int_{p_i}^{p_{i+1}} \kappa \frac{\partial f}{\partial x} p^2 dp \bigg/ \int_{p_i}^{p_{i+1}} \frac{\partial f}{\partial x} p^2 dp \Rightarrow \int_{p_i}^{p_{i+1}} \kappa \frac{\partial f}{\partial x} p^2 dp \bigg/ n_i, \quad (4.23.11)$$

and

$$S_{n_i} = \int_{p_i}^{p_{i+1}} p^2 S(p) dp. \quad (4.23.12)$$

The first pair of terms on the right hand side of Eq. 4.23.9 represents the net flux of CR across the boundaries of the individual momentum bins owed to adiabatic flow compression or expansion. Extension of this term to include fluxes owed to other energy loss or gain processes, such as momentum diffusion or radiative losses, is obvious. In addition there was defined

$$g_i = \int_{p_i}^{p_{i+1}} p^3 f(p) dp. \quad (4.23.13)$$

For relativistic particles g_i is proportional to the CR energy in the bin. With the above notations the diffusion convection equation is

$$\frac{\partial g_i}{\partial t} + \frac{\partial(ug_i)}{\partial x} = \frac{1}{3} \frac{\partial u}{\partial x} \left[\left(p^4 f(p) \right) \Big|_{p_i}^{p_{i+1}} - g_i \right] + \frac{\partial}{\partial x} \left(K_{g_i} \frac{\partial g_i}{\partial x} \right) + S_{g_i}, \quad (4.23.14)$$

where

$$K_{n_i} = \int_{p_i}^{p_{i+1}} \kappa \frac{\partial f}{\partial x} p^3 dp \Big/ \int_{p_i}^{p_{i+1}} \frac{\partial f}{\partial x} p^3 dp \Rightarrow \int_{p_i}^{p_{i+1}} \kappa f(p) p^3 dp \Big/ g_i. \quad (4.23.15)$$

Since the local slope

$$q(p) = -\partial \ln f(p) / \partial \ln p, \quad (4.23.16)$$

is a slowly varying function over much of momentum space, the simplest natural sub-grid model for $f(p)$ assumes q is constant inside Δp_i ; that is, we can apply a piecewise power-law model for $f(p)$ and use a logarithmic spacing in the momentum grid. Then, for example,

$$n_i = \frac{f_i p_i^3}{q_i - 3} \left(1 - d_i^{3-q_i} \right), \quad (4.23.17)$$

with obvious extension to g_i and the other terms in the diffusion convection equation moment equations, where

$$f_i = f(p_i) = (p_{i+1}/p_i)^{q_i} f_{i+1}; \quad d_i = p_{i+1}/p_i. \quad (4.23.18)$$

Using the ratio $g_i/(p_i n_i)$ one can derive the index q_i once n_i and g_i are known.

Fig 4.23.1. shows results from an initial test of the scheme inside a TVD hydrodynamics code and, for comparison, the equivalent simulation with a conventional finite difference scheme for solving the diffusion convection equation (Gieseler et al., 2000). The simulations involve evolution of an initial Mach 10 shock discontinuity with an initially uniform CR pressure equal to twice the upstream gas pressure with $f(p) \propto p^{-5}$.

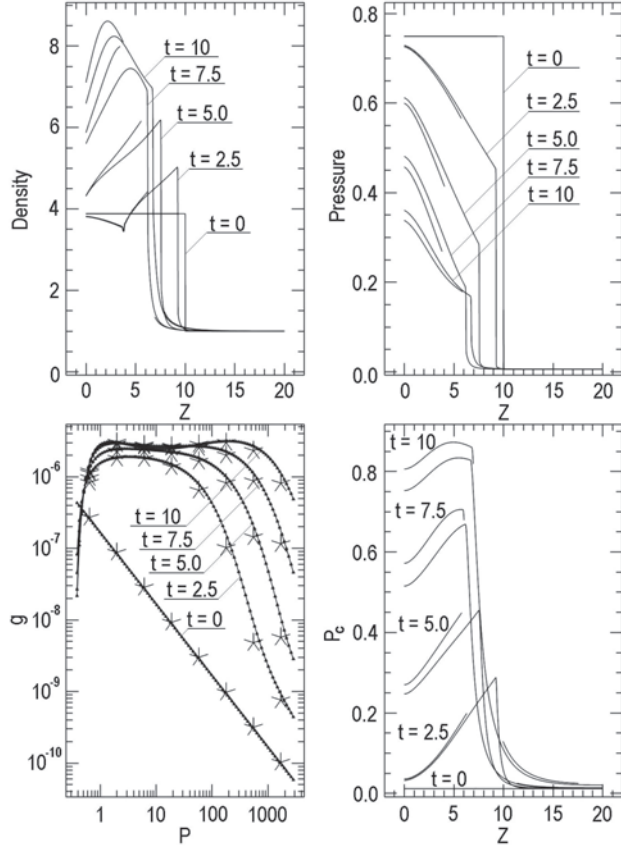


Fig. 4.23.1. Evolution of a CR modified Mach 10 plane shock using two different schemes for solving the diffusion convection equation. The simulations involve evolution of an initial Mach 10 shock discontinuity with an initially uniform CR pressure equal to twice the upstream gas pressure with $f(p) \propto p^{-5}$. The time is in diffusion time units, $t_d = \kappa(p=1)/u_s^2$, and momentum p in units $m_p c$. The shock structure and immediate post-shock particle distribution functions are shown at $t = 0, 2.5, 5, 7.5$, and 10. The solid lines (dynamical variables) and the large stars (distribution function) represent solutions based on 8 momentum bins and the new scheme described in the text. The dotted lines and small triangles come from a conventional finite difference scheme using 96 momentum points. The new code took about 80% less execution time. According to Jones and Kang (2003).

In Fig. 4.23.1 it was used the diffusion coefficient $\kappa(p) \propto p^{0.5}$. In diffusion time units, $t_d = \kappa(p=1)/u_s^2$, the shock structure and immediate-post-shock particle distribution function are shown at $t = 0, 2.5, 5, 7.5, 10$. The conventional diffusion convection equation solution used 96 momentum points spanning the momentum range of $\ln(p)$ from -1 to 8 . The coarse-grained solution used 9 logarithmically expanding momentum bins. It gave a factor 5 reduction in computation time. The code has been tested with simple CR injection models by currently implementing the thermal injection scheme of Gieseler et al. (2000) and porting the routine to CRASH of CR AMR code.

4.24. Thermal particle injection in nonlinear diffusive shock acceleration

4.24.1. Comparison semi-analytical and Monte Carlo models

Ellison et al. (2005) compare a recent semi-analytic model of non-linear diffusive shock acceleration (Blasi, 2002, 2004; Blasi et al., 2005) with a well-established Monte Carlo model (e.g., Ellison et al., 1990, 1999; Jones and Ellison, 1991). Both include a thermal leakage model for injection and the non-linear back reaction of accelerated particles on the shock structure, but they do these in very different ways. Also different is the way in which the particle diffusion in the background magnetic turbulence is modeled. The limited comparison of Ellison et al. (2005) shows that the important non-linear effects of compression ratios $\gg 4$ and concave spectra do not depend strongly on the injection model as long as injection is efficient. A fuller understanding of the complex plasma processes involved, particularly if injection is weak, will require particle-in-cell simulations (e.g., Giacalone and Ellison, 2000), but these simulations, which must be done fully in three-dimensions (Jones et al., 1998), cannot yet be run long enough, in large enough simulation spaces, to accelerate particles from thermal to relativistic energies in order to show strong non-linear effects. For now, approximate methods must be used. The Monte Carlo model is more general than the semi-analytic model. For instance, Ellison et al. (2005) note, that the Monte Carlo model can treat a specific momentum dependence for the scattering mean free path, particle acceleration in relativistic shocks (Ellison and Double, 2002, 2004), and non-linear effects in oblique shocks (Ellison et al., 1999), but it is considerably slower computationally. Since it is important in many applications, such as hydro models of supernova remnants, to include the dynamic effects of nonlinear diffusive shock acceleration in simulations which perform the calculation many times (e.g., Ellison et al., 2004), a rapid, approximate calculation, such as the semi-analytic model discussed here, is useful.

4.24.2. Injection models

In the semi-analytic model a free injection parameter, ξ_{SA} , determines the fraction of total particles injected into the acceleration mechanism and the injection momentum, p_{inj} . Specifically,

$$p_{inj} = \xi_{SA} p_{th}, \text{ where } p_{th} = \sqrt{2m_p k T_{DS}} \quad (4.24.1)$$

(T_{DS} is the downstream temperature). The fraction, η_{SA} , of un-shocked particles crossing the shock which become super-thermal in the semi-analytic model is

$$\eta_{SA} = \left(4/3\pi^{1/2}\right) (r_{sub} - 1) \xi_{SA}^3 \exp(-\xi_{SA}^2), \quad (4.24.2)$$

where r_{sub} is the sub-shock compression ratio. The fraction η_{SA} , which is approximately the number of particles in the Maxwellian defined by T_{DS} with momentum $p > p_{inj}$, is determined by requiring the continuity, at p_{inj} , of the Maxwellian and the super-thermal distribution (Blasi et al., 2005). Since p_{th} depends on the injected fraction, the solution must be obtained by iteration.

In the Monte Carlo model, the injection depends on the scattering assumptions. We assume that particles pitch-angle scatter elastically and isotropically in the local plasma frame and that the mean free path is proportional to the gyro-radius, i.e., $\lambda \propto r_g$, where $r_g = pc/qB$. With these assumptions, the injection is purely statistical with those ‘thermal’ particles which manage to diffuse back upstream gaining additional energy and becoming super-thermal. Note that in this scheme the viscous sub-shock is assumed to be transparent to all particles, even thermal ones, and that any downstream particle with $v \geq u_2$ has a chance to be injected (here u_2 is the downstream flow speed). For comparison with the semi-analytic model, we have included an additional parameter, v_{thres}^{MC} , to limit injection in the Monte Carlo simulation. Only downstream particles with $v \geq v_{thres}^{MC}$ are injected, i.e., allowed to re-cross the shock into the upstream region and become super-thermal. In our previous Monte Carlo results, with the sole exception of paper Ellison (1985), we have taken $v_{thres}^{MC} = 0$.

4.24.3. Models of momentum dependent diffusion

Diffusion is treated very differently in the two models. As just mentioned, the Monte Carlo simulation models pitch-angle diffusion by assuming a $\lambda(p)$ (in according with Ellison and Double, 2004). The semi-analytic model does not explicitly describe diffusion but assumes only that the diffusion is a strongly

increasing function of particle momentum p so that particles of different p interact with different spatial regions of the upstream precursor. Eichler (1984) used a similar procedure. With this assumption, particles of momentum p can be assumed to feel some average precursor fluid speed u_p and an average compression ratio $r_p \approx u_p/u_2$ (see Blasi et al., 2005). The different way diffusion is treated influences not only injection, but also the shape of the distribution function $f(p)$, where $f(p)$ is the momentum phase space density, i.e., particles/(cm³d³p)]. Both models give the characteristic concave $f(p)$ which hardens with increasing p and, since the overall shock compression ratio r can be greater than 4, this spectrum will be harder than p^{-4} at ultra-relativistic energies. In the results we show here, the acceleration is limited with a cutoff momentum so $f(p)$ cuts off abruptly at $p_{\max}$. More realistic models will show the effects of escape from some spatial boundary (e.g., finite shock size) or from a finite acceleration time. In either case, the spectrum will show a quasi-exponential turnover, e.g.,

$$f(p) \propto p^{-\sigma} \exp\left(-\alpha^{-1}(p/p_{\max})^{-\alpha}\right), \quad (4.24.3)$$

where α is included to emphasize that the detailed shape of the turnover depends on the momentum dependence of the diffusion coefficient near $p_{\max}$ (Ellison et al., 2000).

4.24.4. Thermalization

There is no thermalization process in the Monte Carlo simulation in the sense of particles exchanging energy between one another because particles scatter elastically in the local frame. However, a quasi-thermal low energy distribution is created as un-shocked particles cross the shock from upstream to downstream at different angles and receive different fractions of the speed difference, $u_o - u_2$. For the parameters used here, the low-energy peak is essentially a Maxwellian when $v_{thres}^{MC} = 0$.

In the semi-analytical model, the shocked thermal pressure and density are determined from the conservation relations and these are translated to a Maxwell-Boltzmann distribution.

4.24.5. Main results for both models and comparison

In Fig. 4.24.1 are shown distribution functions for a set of parameters and with $v_{thres}^{MC} = 0$ (left panel) and $v_{thres}^{MC} = 4u_2$ (right panel). In both panels, the solid curves are the Monte Carlo results and the dashed curves are the semi-analytical results where ξ_{SA} has been chosen to provide the best match to the Monte Carlo spectra.

In Fig. 4.24.2 is shown the low energy portions of the spectra along with the distributions of injected particles in the Monte Carlo model (dotted histograms). The vertical solid lines are at p_{inj} and they show the transition between the Maxwellian and the super-thermal population in the semi-analytical results.

Ellison et al. (2005) underlined the following important aspects of the plots in Fig. 4.24.1 and 4.24.2:

(i) The broad-band match between the two very different calculations is quite good, particularly for $v_{thres}^{MC} = 0$. Both models show the important characteristics of nonlinear diffusive shock acceleration, i.e., $r \gg 4$, concave spectra, and a sharp reduction in the shocked temperature from test-particle values.

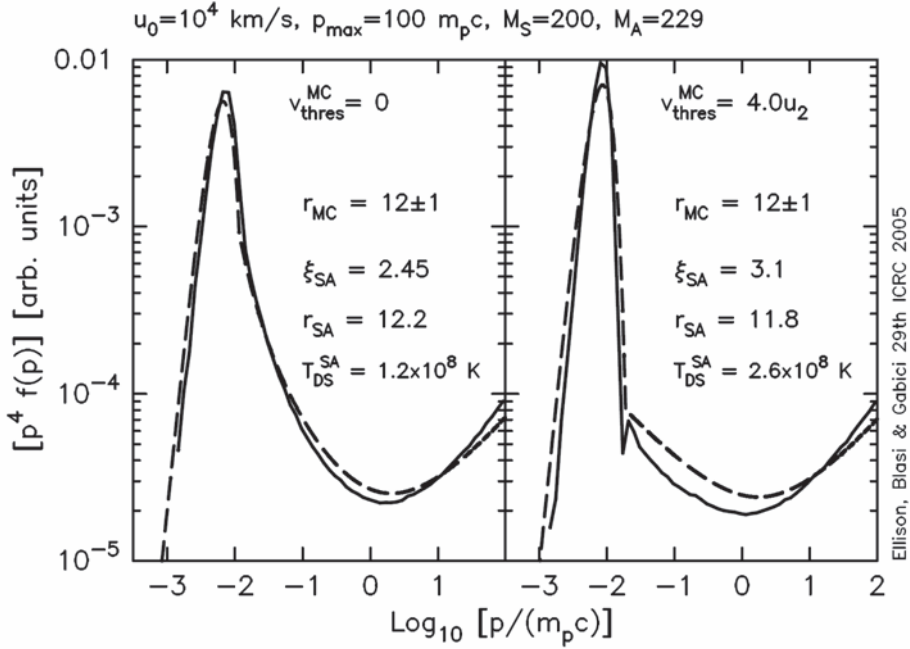


Fig. 4.24.1. Phase space distributions $f(p)$ multiplied on p^4 for a Monte Carlo model (solid curves) and a semi-analytical model (dashed curves). In the left panel $v_{thres}^{MC} = 0$, and in the right panel $v_{thres}^{MC} = 4u_2$ in the Monte Carlo model. Here, $u_2 = u_0/r$ is the downstream plasma speed in the shock frame, u_0 is the shock speed, and the Mach numbers, compression ratios, and shocked temperatures are indicated. In all cases, the shock is parallel and Alfvén heating is assumed in the shock precursor (Ellison et al., 2000). From Ellison et al., 2005).

(ii) The shocked temperature depends on v_{thres}^{MC} with weaker injection (i.e., larger v_{thres}^{MC}) giving a larger T_{DS} .

- (iii) The distribution $f(p)$ is harder near $p_{\max}$ in the Monte Carlo results than with the semi-analytical calculations. A common prediction of semi-analytical models is that the shape of the particle spectra at $p_{\max}$ has the form $\propto p^{-3.5}$ if the shock is strongly modified and the diffusion coefficient grows fast enough in momentum. This can be demonstrated by solving the equations in the extreme case of a maximally modified shock and approximating the spectrum with a power law at high momentum. The Monte Carlo model makes no such power-law assumption.
- (iv) The minimum in the $p^4 f(p)$ plot occurs at $p > m_p c$ in both models. The transition between $f(p)$ softer than p^{-4} , and $f(p)$ harder than p^{-4} varies with shock parameters and increases as $p_{\max}$ increases. This is an important difference from the algebraic model of Berezhko and Ellison (1999) where the minimum is fixed at $m_p c$.

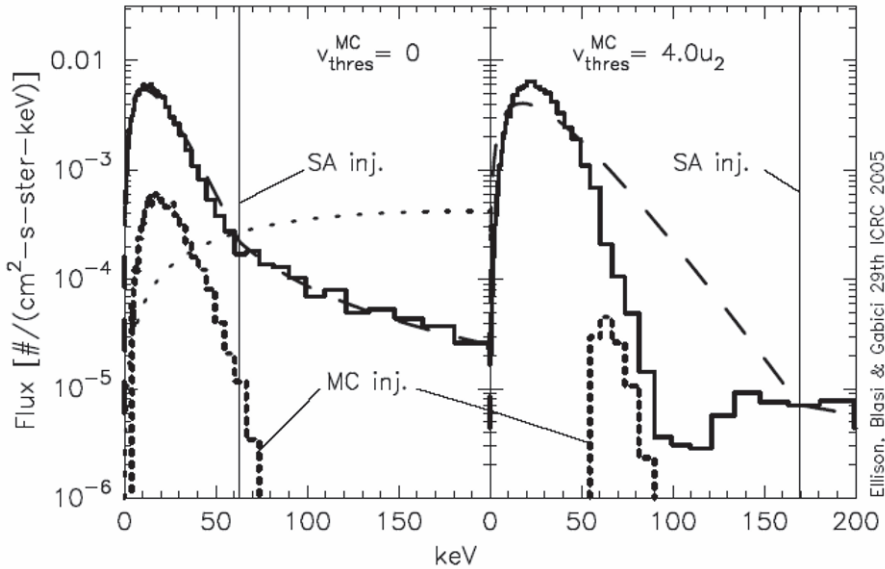


Fig. 4.24.2. Low energy portions of the spectra shown in Fig. 4.24.1 where the solid histograms are the Monte Carlo results and the dashed curves are the semi-analytical results. The heavy dotted histograms show the distribution of particles that were injected in the Monte Carlo model. The solid vertical lines are drawn at p_{inj} , the momentum at which particles are injected in the semi-analytical model. For $v_{\text{thres}}^{\text{MC}} = 0$, $\eta_{\text{MC}} = 5.7 \times 10^{-2}$ and $\eta_{\text{SA}} = 4.6 \times 10^{-2}$, while for $v_{\text{thres}}^{\text{MC}} = 4u_2$, $\eta_{\text{MC}} = 4.1 \times 10^{-3}$ and $\eta_{\text{SA}} = 2.5 \times 10^{-3}$. The light-weight dotted curve shows the Maxwellian ($T_{\text{DS}} = 2.2 \times 10^9$ K) that would have resulted if no diffusive shock acceleration occurred. According to Ellison et al., 2005.

(v) For the particular parameters used in these examples, the overall compression ratio is relatively insensitive to v_{thres}^{MC} , but shocks having other parameters may show a greater sensitivity. Note that Alfvén wave heating is assumed in all of the results presented here. If only adiabatic heating was assumed, the compression ratios would be much higher (in agreement with Ellison et al., 2000).

(vi) In all cases, the Monte Carlo model injects more particles than the semi-analytical model but the average energy of the injected particles is less, as indicated by the peak of the curve labeled 'MC inj' vs. the 'SA inj' energy in Fig. 4.24.2.

(vii) In contrast to $v_{thres}^{MC} = 0$, the 'thermal' part of $f(p)$ with $v_{thres}^{MC} = 4u_2$ (Fig. 4.24.2) shows large differences in the two models. While both conserve particle, momentum, and energy fluxes so that the broad-band $f(p)$ matches well for a wide range of parameters, the different treatments of the sub-shock lead to large differences in the critical energy range $2 \leq E/kT_{DS} \leq 5$. This offers a way to distinguish these models observationally.

Using two approximate acceleration models, Ellison et al. (2005) have shown that the most important features of non-linear diffusive shock acceleration, i.e., at $r \gg 4$ and concave spectra, are robust and do not strongly depend on the injection model as long as injection is efficient. If injection is weak, as might be the case in highly oblique shocks, accelerated spectra will depend more on the details of injection, at least in the transition range between thermal and super-thermal energies. Also, the relative efficiencies for injecting and accelerating electrons vs. protons or protons vs. heavier ions may require a more detailed description of injection, as may be provided by future PIC simulations.

4.25. Time evolution of CR modified MHD shocks

4.25.1. The matter of problem

Jones and Kang (2005a) present initial simulation results for the time evolution of CR modified plane parallel shocks in magneto-hydro-dynamical flows. The simulations utilize very efficient 'Coarse Grained finite Momentum Volume' (CGMV) transport scheme (Jones and Kang, 2005b). The simulations aim to explore nonlinear feedback among the particles, the wave turbulence, and the bulk flows. The calculations incorporate self-consistent treatment of the momentum-dependent CR diffusion- convection equation and, as it assumed by Jones and Kang (2005a), will soon be coupled with wave action equations for resonantly scattering Alfvén and fast mode waves, also treated through the CGMV scheme.

Jones and Kang (2005a) note that the physics of strong, CR modified shocks is complex and nonlinear; through diffusive shock acceleration, CR can capture a major portion of the energy flux through the shocks, greatly modifying the shock dynamics and structures in the process (e.g., Webb et al., 1986; Baring et al., 1993; Frank et al., 1995; Berezhko and Völkel, 2000; Bell and Lucek, 2001). CR

propagation in and around the shocks is mediated by the presence of the large-scale magnetic field and by resonant scattering on MHD waves, which are usually considered as Alfvén wave turbulence. Most, but not all, theoretical treatments of CR modified shocks assume a fixed CR scattering or diffusion law, and commonly the dynamical roles of the wave turbulence and the large scale magnetic field are ignored. On the other hand, a key feature of diffusive shock acceleration is that the relevant wave turbulence is strongly amplified by streaming CR near the shock (Bell, 1978). As a consequence, it can contribute a significant ponderomotive force on the bulk plasma flow, and its dissipation upstream of the classical gas sub-shock structure can preheat the upstream plasma, which also modifies the character of the shock transition. Lucek and Bell (2000), for example, have argued that the streaming instability can enhance the scattering waves so much that the scattering length is orders of magnitude less than expressed by Bohm diffusion using the upstream magnetic field. This has led several authors to argue that CR acceleration can be much more rapid than usually described (e.g., Bell and Lucek, 2001; Ptuskin and Zirakashvili, 2003). Furthermore, the scattering turbulence is likely to be anisotropic, which, among other things, can mean that the effective motion of the scattering centers will not match the motion of the bulk plasma, as usually assumed. For instance, Alfvén waves amplified by the streaming instability may be expected to propagate with respect to the bulk plasma along the large scale magnetic field at approximately the Alfvén speed. Since the Alfvén speed variations normal to the shock structure are not the same as the gas speed variations, this may modify the properties of diffusive shock acceleration through the shock (McKenzie and Völk, 1982; Jones, 1993). It is clearly important to carry out theoretical studies of the diffusive shock acceleration that can incorporate the physical processes just outlined. Jones and Kang (2005a) conclude that recent advances in MHD turbulence theory require a broadened outlook on scattering processes in CR modified shocks. The paper of Jones and Kang (2005a) reports initial steps in an effort to do just that.

4.25.2. Methods of calculations

For this study Jones and Kang (2005a) have incorporated new, efficient ‘Coarse Grained finite Momentum Volume’ (CGMV) scheme for solving the CR diffusion-convection equation (Jones and Kang, 2005b) into 1D TVD MHD code. This MHD code has been used effectively by in the past to study diffusive shock acceleration using conventional finite difference methods to evolve the diffusion-convection equation (Frank et al., 1995; Kang and Jones, 1997). The results presented in Jones and Kang (2005a) are based on a prescribed spatial diffusion coefficient, although they are in the process of incorporating a CGMV-based routine to evolve the wave action equation for Alfvénic turbulence, so that a self-consistent treatment of the full system can be carried out. The CGMV scheme for evolving the CR distribution utilizes the first two momentum moments of $f(t, x, p)$ over finite momentum bins; namely,

$$n_i = \int_{\Delta p_i} p^2 f(p) dp \text{ and } g_i = \int_{\Delta p_i} p^3 f(p) dp . \quad (4.25.1)$$

Assuming a piecewise power law momentum subgrid model, the first moment of $f(t, x, p)$ is, for example,

$$n_i = \left(f_i p_i^3 \right) \left(1 - d_i^{3-q_i} \right) / (q_i - 3), \quad (4.25.2)$$

where

$$f_i \equiv f(p_i) = (p_{i+1}/p_i)^{q_i} f_{i+1}, \quad d_i = p_{i+1}/p_i, \quad (4.25.3)$$

and q_i is the momentum index inside bin i . This leads to moments of the standard diffusion-convection equation (e.g., Skilling, 1975), such as

$$\frac{\partial n_i}{\partial t} + u \frac{\partial n_i}{\partial x} = F_{n_i} - F_{n_{i+1}} - n_i \frac{\partial u}{\partial x} + \frac{\partial}{\partial x} \left(K_{n_i} \frac{\partial n_i}{\partial x} \right) + S_{n_i}, \quad (4.25.4)$$

where $u = v + u_w$ is the net velocity of CR scattering centers, including the gas motion, v , and the mean wave motion, u_w (Jones and Kang, 2005b). In addition,

$$F_{n_i} = [\dot{p}_i + q(p_i) D(p_i) / p_i] p_i^2 f(p_i) \quad (4.25.5)$$

is a flux in momentum space, with $\dot{p} = -p(1/3)(\partial u / \partial x)$, K_{n_i} and S_{n_i} are the spatial diffusion coefficient, $\kappa(x, p)$, and a representative source term, S , averaged over the momentum interval. $D(p)$ is the momentum diffusion coefficient. We henceforth express particle momentum in units of mc , where m is the particle mass. In the preliminary study Jones and Kang (2005a) assume the convenient spatial diffusion form, $\kappa(p) = \kappa_o p^\alpha$, and ignore momentum diffusion ($D = 0$). Generally, spatial CR diffusion is not expected to be isotropic with respect to the direction of the local magnetic field. The degree of anisotropy has been shown to have important consequences, especially when the magnetic field is quasi-perpendicular to the shock normal (Baring et al., 1993; Frank et al., 1995; Jokipii, 1987). For these initial simulations, however, Jones and Kang (2005a) assume isotropic diffusion. They also assume a simple, common model for injection of low energy CR at the gas sub-shock; namely, that a fixed fraction, ε_{inj} , of the thermal proton flux through the sub-shock is able to escape upstream at momentum p_{inj} to join the diffusive CR population. This produces a source term in Eq. (4.25.4), for example,

$$S_{n_i} = (1/4\pi)(\rho_1/\mu m_p)v_s\phi(x), \quad (4.25.6)$$

where ρ_1 is the upstream plasma density, μ is the plasma molecular weight, v_s is the plasma flow speed across the shock and $\phi(x)$ is a normalized weighting function to distribute the injection across the numerical shock. These equations are coupled with the standard equations for compressible MHD by adding the pressure gradient of the CR, $\partial P_c/\partial x$, into the MHD Euler equation and by including in the MHD energy conservation equation the term $-u\partial P_c/\partial x$ to account for work done on the bulk fluid by the CR pressure gradient (this last contribution includes both the adiabatic compression of the plasma through the term $v\partial P_c/\partial x$, as well as a term representing dissipation of scattering wave energy generated through CR streaming, according to McKenzie and Völk, 1982; Jones, 1993). In addition, to account properly for thermal energy taken from the bulk plasma during the CR injection process, an energy sink term,

$$L = -(1/2)\epsilon_{inj}\phi(x)p_{inj}^2\rho_1v_s \quad (4.25.7)$$

must be, according to Jones and Kang (2005a), added to the MHD energy conservation equation.

4.25.3. Main results and discussion

According to Jones and Kang (2005a), CR-modified shocks are known to differ significantly from ordinary gas-dynamical or MHD shocks. CR diffusion upstream produces a pressure gradient, adding adiabatic compression to the gas. This preheats the gas and substantially increases compression through the transition. It also weakens the gas sub-shock. Typically, in fact, the compression through a strong CR shock precursor dominates the total shock compression, so that most of the CR acceleration actually takes place in the precursor, rather than in the thin, weakened sub-shock. Furthermore, energy extraction from the thermal plasma by the CR cools the bulk gas with respect to adiabatic gas shocks. Particularly if the CR escape upstream this allows the shock transition to resemble a radiatively cooled shock transition, further amplifying the total compression through the transition, while reducing the down stream temperature and pressure compared to gas-dynamical behaviors. Previous theoretical studies of CR-MHD shocks have emphasized major differences with respect to gas dynamical models for CR-modified shocks. For weak to moderate strength oblique shocks, magnetic field compression restricts the enhanced compression mentioned above. That becomes a relatively small effect in very strong MHD shocks, however, once the total downstream pressure is dominated by the CR (Webb et al., 1986; Frank et al., 1995). A much more significant influence in strong MHD shocks can be the difference between the motion of the bulk plasma and the motion of the CR scattering centers (Jones,

1993); i.e., the drift of the CR with respect to the fluid. If the upstream scattering is due primarily to Alfvén waves amplified by CR streaming away from the shock, then one expects the mean motion of the scattering centers to propagate upstream along the mean magnetic field at approximately the local Alfvén velocity with respect to the fluid flow. What matters is the component of this velocity along the shock normal, since it parallels the flow and CR gradients. This Alfvén velocity component will scale as $v_{ax} \propto B_x / \sqrt{\rho}$, where B_x is the mean magnetic field component along the shock normal. Since B_x does not change through the shock, $v_{ax} \propto 1/\sqrt{\rho}$, while the bulk flow speed varies as $1/\rho$, through mass conservation. Consequently, the effective rate of mirror convergence responsible for diffusive shock acceleration, $\partial u / \partial x$, is reduced compared to a pure gas flow, reducing the rate of CR acceleration. Moreover, the magnitude of u is increased by the upstream-facing drift, which increases the rate of precursor heating for a given CR pressure gradient. That reduces the strength of the shock transition, which also reduces the efficiency of diffusive shock acceleration. Jones and Kang (2005a) illustrate these effects by comparing in Fig. 4.25.1 three simulations involving a Mach 40 piston-generated shock.

In Fig. 4.25.1 each flow has an upstream magnetic field inclined 45 degrees from the shock normal with a magnetic pressure there equal to the gas pressure. The simple diffusion model assumed $\kappa_o = 0.1$ and $\alpha = 0.51$. CR injection was included with $\varepsilon_{inj} = 0.001$. Fourteen momentum bins were used in solving the diffusion-convection equation. The black, solid curves indicate behaviors when $u_w = 0$. This simulated shock is very similar to a gas-dynamical shock discussed in Jones and Kang (2005b) and illustrated in Fig. 4.25.2. The shock quickly becomes CR dominated, but only approaches dynamical equilibrium at the last time shown. The CR momentum distribution at the sub-shock shows the strongly concave form typical of such simulated CR shocks. By comparison the other plotted results include effects of CR drift. The red, dotted curves represent the behavior when $u_w = v_{ax}$, while the blue, dashed curves come from a simulation in which u_w was included in the MHD energy equation, but not the momentum equation. That approach has been used by some authors e.g., Berezhko and Völk (2000) in non-MHD models to approximate the influence of MHD by allowing for dissipation of wave energy. The differences between the gas-dynamical solutions and the MHD solutions are obvious. The magnetic, Maxwell stresses have had little impact on the MHD solutions. However, as anticipated from the above discussion, compression through the shock is much reduced through CR drift, while the efficiency of CR acceleration is reduced by about one third. These two effects have also mostly eliminated the strong concavity in the CR momentum distribution, since the shock precursor is a much less important contributor to diffusive shock acceleration. On the other hand, there is relatively little difference between the two CR-drift models.

In this one strong shock case, at least, modeling the MHD shock by estimating Alfvén wave heating of the bulk plasma would provide a reasonable approximation to the more complete model.

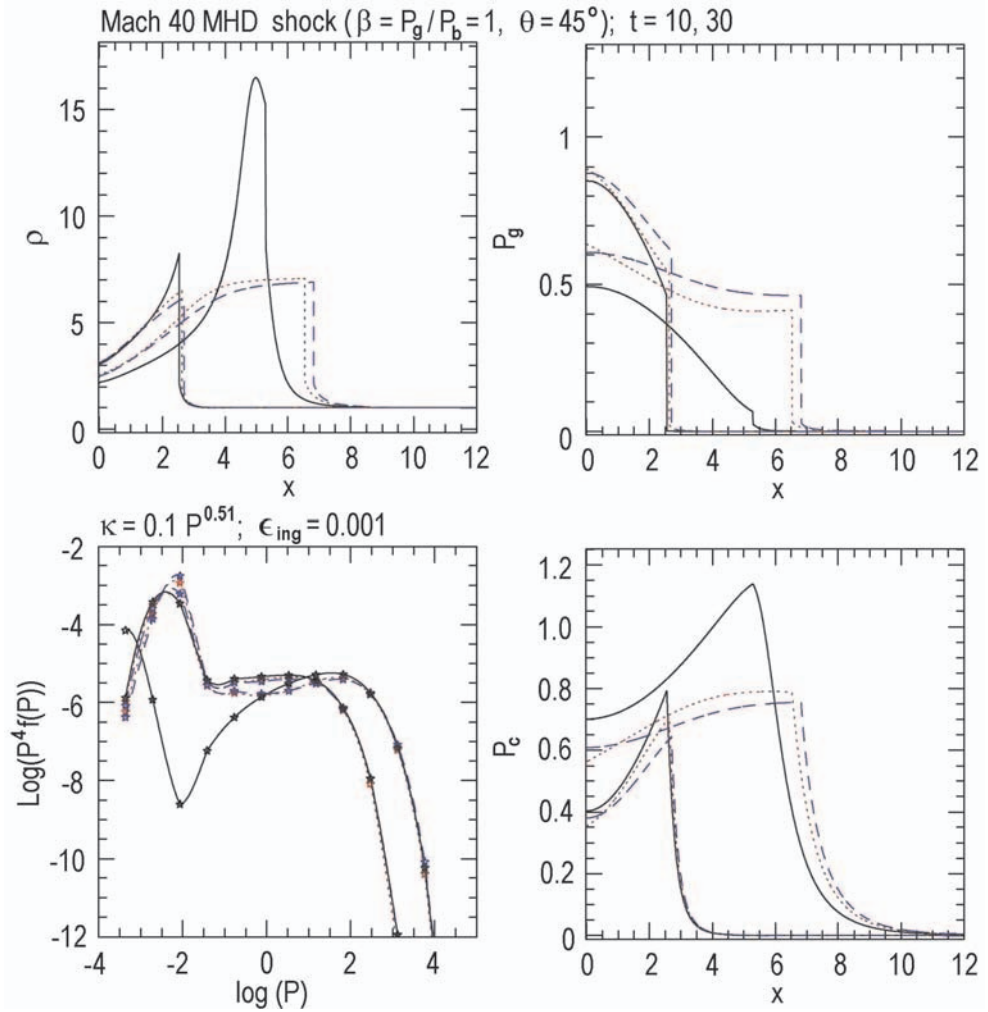


Figure 4.25.1. Three simulated MHD Mach 40 CR-modified shocks formed off a piston on the left boundary. The gas density ρ , gas pressure P_g , and CR pressure P_c spatial distributions are shown along with the CR momentum distribution $f(p)$ at the sub-shock. From Jones and Kang (2005a).

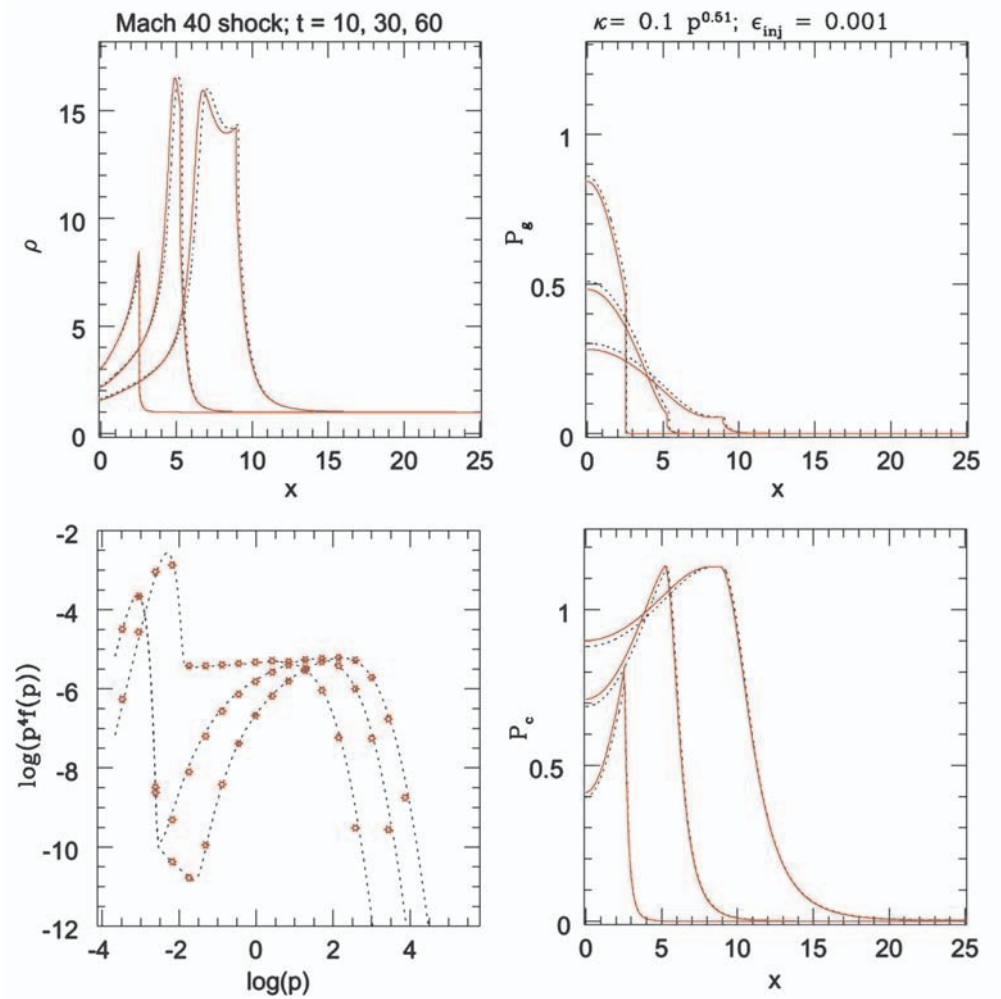


Fig 4.25.2. The same as in Fig. 4.25.1, but for gas-dynamical shock. According to Jones and Kang (2005b).

4.26. Particle injection and acceleration at non-parallel shocks

4.26.1. The matter of problem

Giagalone and Jokipii (2005) discuss new results in the physics of charged-particle acceleration by shock waves propagating at an arbitrary angle to the magnetic field. For the usually discussed case of a parallel shock acceleration by a supernova blast wave up to the knee in the CR spectrum requires very special assumptions such as a strong increase in the magnetic field, perhaps due to excitation from the streaming CR. They show that no such special circumstances

are required when one considers acceleration at nearly perpendicular shocks. The matter of problem is that the diffusive acceleration of charged particles at collisionless shocks, at which particles are accelerated by the converging flows and plasma compressions, naturally explains the observed universal power law of CR up to the knee in the spectrum at about 10^{15} eV (see, e.g., the reviews by Drury, 1983; Blandford and Eichler, 1987; Jones and Ellison, 1991). The acute angle between the shock-normal direction and the incident magnetic fields, θ_{Bn} , plays an important role in determining the resulting accelerated-particle spectrum. It was shown by Jokipii (1982, 1987) that the acceleration rate depends strongly on θ_{Bn} and is the highest when the shock is perpendicular ($\theta_{Bn} = 90^\circ$). Thus, given a particular time interval over which to accelerate particles, those with highest energy will originate from the perpendicular shock. An important issue in diffusive shock acceleration at nearly perpendicular shocks has been the well-known injection threshold problem. The problem arises because, until recently, it was assumed that particles move essentially along the lines of force which are convecting through the shock. Therefore, it was thought that there was no means by which low-energy particles could encounter the shock several times, which is required for efficient particle acceleration. Giacalone and Jokipii (2005) show that there is actually no such injection problem and, in fact, the injection does not depend strongly on the shock-normal angle. This can be understood in terms of the increased cross-field transport arising from so-called field-line random walk due to the large-scale (order of a parsec) turbulent interstellar magnetic field.

4.26.2. Analytical considerations

The main assumption in diffusive shock acceleration is that the pitch-angle distribution is nearly isotropic. By requiring the diffusive streaming anisotropy to be small, one can readily derive an expression for the injection velocity, w_{inj} (c.f., Giacalone and Jokipii, 1999). The most general expression is given by:

$$w_{inj} = 3u_1 \left(1 + \frac{\kappa_g^2 \sin^2 \theta_{Bn} + (\kappa_{//} - \kappa_{\perp})^2 \sin^2 \theta_{Bn} \cos^2 \theta_{Bn}}{(\kappa_{\perp}^2 \sin^2 \theta_{Bn} + \kappa_{//}^2 \cos^2 \theta_{Bn})^2} \right)^{1/2}, \quad (4.26.1)$$

where $\kappa_{\perp}$, and $\kappa_{//}$ are the components of the diffusion tensor perpendicular and parallel to the mean magnetic field, respectively, and the anti-symmetric component of the diffusion tensor is $\kappa_g = vr_g/3$ (r_g is the Larmor radius of accelerated particles in the mean magnetic field). For the case in which the correlation scale of the turbulent magnetic field is much larger than the gyro-radius of the particles of interest, it has been shown from numerical simulations that $\kappa_{\perp}/\kappa_{//}$ is independent

of energy (Giacalone and Jokipii, 1999). Thus, taking $\varepsilon = \kappa_{\perp}/\kappa_{\parallel} \ll 1$ and $\eta = \lambda_{\parallel} r_g/3$, where $\lambda_{\parallel}$ is the parallel mean-free path and r_g is the Larmor radius, Eq. 4.26.1 can be rewritten as:

$$w_{inj} = w_{inj,\parallel} \left(1 + \frac{(1/\eta)^2 \sin^2 \theta_{Bn} + \sin^2 \theta_{Bn} \cos^2 \theta_{Bn}}{(\varepsilon \sin^2 \theta_{Bn} + \cos^2 \theta_{Bn})^2} \right)^{1/2}, \quad (4.26.2)$$

where $w_{inj,\parallel} = 3u_1$ is the injection velocity for a parallel shock.

Shown in Fig. 4.26.1 is the solution to Eq. 4.26.2 for $\eta = 100$ and $\varepsilon = 0.02$. The dashed curve is $\sec \theta_{Bn}$, which is the scatter-free approximation which is clearly invalid for the case of a turbulent magnetic field. Note that at low-energies, the injection velocity at a perpendicular shock approaches $3u_1$, which is the same as that obtained for a parallel shock (Giacalone, 2003).

Thus, it can be conclude that enhanced motion normal to mean field by field-line random walk significantly decreases the injection velocity threshold for acceleration. Thus, the theory predicts that there should not be an injection problem at nearly perpendicular shocks. The acceleration rate, v_{ac} , in diffusive shock acceleration is given by

$$v_{ac} = \frac{1}{\tau_{ac}} \approx \frac{u_1^2}{\kappa_{\perp} \sin^2 \theta_{Bn} + \kappa_{\parallel} \cos^2 \theta_{Bn}}, \quad (4.26.3)$$

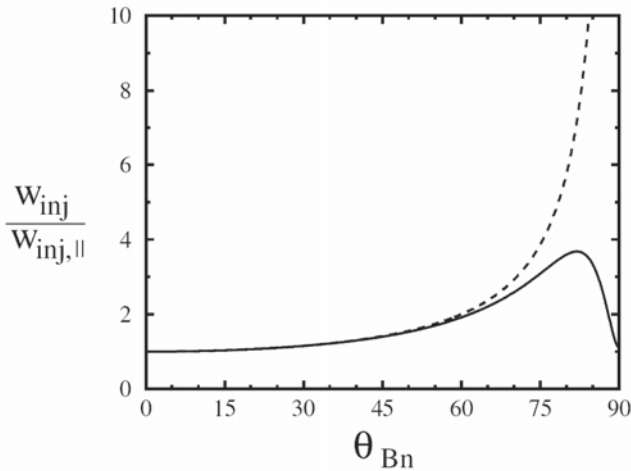


Fig. 4.26.1. The injection velocity derived from the diffusive streaming anisotropy for the case of field-line random walk (solid line) normalized to that at a parallel shock. The dashed curve assumes the scatter-free approximation. According to Giacalone and Jokipii (2005).

Thus, taking $v_{ac,||}$ as the acceleration rate at a parallel shock ($\theta_{Bn} = 0$), and $\varepsilon = \kappa_{\perp}/\kappa_{||}$ (as before), we obtain

$$\frac{v_{ac}}{v_{ac,||}} = \frac{1}{\varepsilon \sin^2 \theta_{Bn} + \cos^2 \theta_{Bn}}. \quad (4.26.4)$$

Eq. 4.26.4 is plotted as a function of θ_{Bn} for the case of $\varepsilon = 0.02$ in the Fig. 4.26.2. From Fig. 4.26.2 clearly can be seen that the acceleration rate is a maximum at perpendicular shocks. Therefore, it can be conclude that perpendicular shocks are both efficient and rapid accelerators of charged particles are most important in producing high-energy CR in a wide variety of astrophysical plasmas.

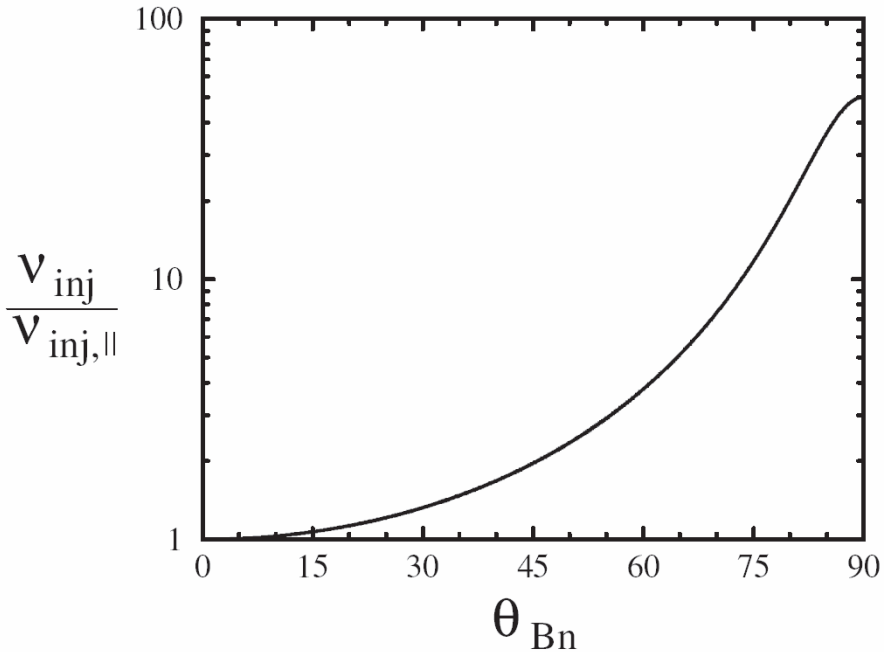


Fig. 4.26.2. The acceleration rate, normalized to that at a parallel shock, as a function of θ_{Bn} . According to Giacalone and Jokipii (2005).

4.26.3. Numerical calculations for test-particle simulations

Giacalone and Jokipii (2005), Giacalone (2005) consider then non-diffusive test-particle numerical simulations to better address the physics of acceleration at low energies. In these calculations, the trajectories of an ensemble of test particles are integrated by numerically solving the Lorentz force on each particle using pre-specified electric and magnetic fields. The mean magnetic field makes an angle

θ_{Bn} with respect to the shock-normal direction. Superimposed on this is a fluctuating component that is determined from a pre-specified power spectrum that resembles the usual Kolmogorov spectrum. The correlation scale of the turbulent magnetic field is taken to be $2000 u_1 / \Omega_i$, where u_1 is the upstream flow speed and Ω_i is the ion cyclotron frequency. Both components satisfy Maxwell's equations. Test particles (protons) are released with an energy of 3 times the plasma-ram energy in the local fluid frame just behind the shock front. Each particle's trajectory is integrated until it escapes downstream by convection (based on a probability of return criterion), or reaches an arbitrary high-energy cutoff (taken to be 2×10^5 times the plasma-ram energy). Fig. 4.26.3 shows the steady-state energy spectra downstream of the shock for 7 numerical simulations in which the only varying parameter is θ_{Bn} . Note that the spectra for the cases of $\theta_{Bn} = 0^\circ, 15^\circ, 30^\circ$ all lie on top of one another indicating that there is no dependence on this parameter at all for quasi-parallel shocks.

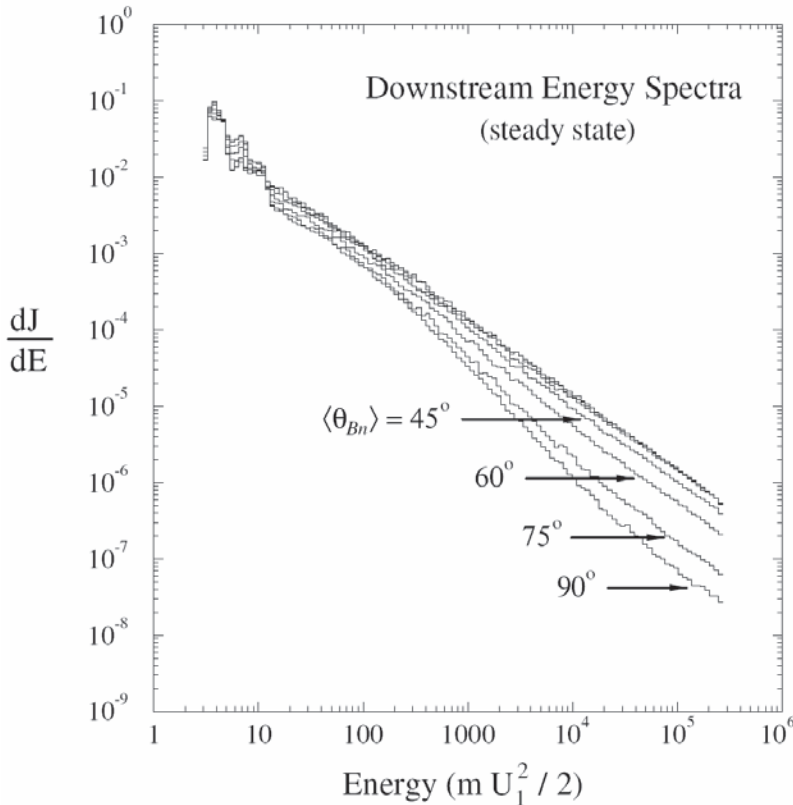


Fig. 4.26.3. Downstream energy spectra for test-particle numerical simulations for case of steady-state spectra obtained from simulations using different values of the shock-normal angle. According to Giacalone (2005a).

The Fig. 4.26.4 is for the case of a time-dependent acceleration process. Here, weaker turbulence was used and two different shock-normal angles are considered (as indicated).

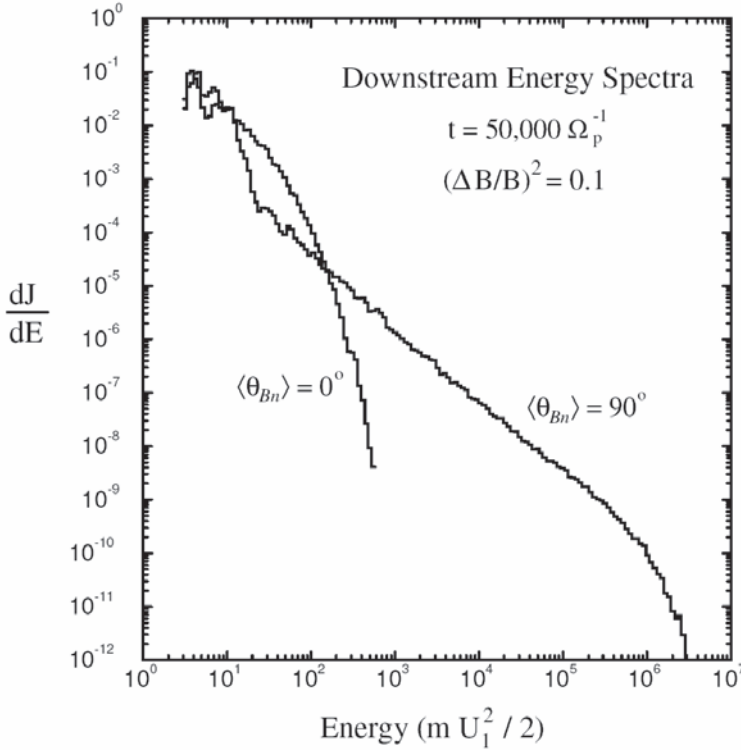


Fig. 4.26.4. Downstream energy spectra for test-particle numerical simulations for case of time-dependent spectra for two different shock-normal angles and weaker turbulence. According to Giacalone (2005a).

The results shown in Fig. 4.26.3 and Fig. 4.26.4 indicate that the injection energy, and therefore, the acceleration efficiency, does not have a strong dependence on the shock-normal angle. However, as shown in the Fig. 4.26.4, for any given time interval to accelerate the particles, perpendicular shocks produce the highest-energy particles. This is because, as it was discussed above, the acceleration rate is strongly dependent on the shock normal angle, provided $\kappa_{\perp}/\kappa_{\parallel} \ll 1$.

4.26.4. Numerical calculations for self-consistent hybrid simulations

Giacalone (2005b) performed massive-scale two-dimensional hybrid simulations of perpendicular shocks propagating into a turbulent upstream magnetic field. It was shown that a fraction of thermal particles encountering the shock are accelerated to high energies. The physics of this process is similar to that which we have already described above. However, the source of the high-energy particles

comes directly from the thermal population, which had not been seen in previous self-consistent plasma simulations. It has been long known that a fraction of thermal ions are assumed to be reflected by the shock and begin to gyrate within the shock ramp before becoming thermalized downstream. For the case in which the shock moves into an upstream region containing large-scale magnetic fluctuations, some of these ions can move upstream along these lines of force before returning to the shock. These ions can gain considerable energy because they can achieve multiple interactions with the shock. The efficiency for the acceleration in these large-scale hybrid simulations is difficult to estimate because the spatial domain is still rather limited by computation resources. However, it was estimated that the efficiency is probably comparable to that obtained for a parallel shock, or about 10-20% (Giacalone et al., 1997).

Giacalone and Jokipii (2005) have shown that the perpendicular shocks are as efficient as parallel shocks in accelerating particles to high energies using reasonable parameters. For these same parameters, perpendicular shocks are much more rapid accelerators. Thus, they conclude that perpendicular shocks are important sites of acceleration and can produce high-energy CR in a wide variety of astrophysical plasmas.

4.27. Numerical studies of diffusive shock acceleration at spherical shocks

4.27.1. *The matter of problem*

Collisionless shocks form ubiquitously in tenuous cosmic plasmas via collective, electromagnetic viscosities. The formation process of such shocks inevitably produces supra-thermal particles, which can be further accelerated to very high energies through the interactions with resonantly scattering Alfvén waves in the converging flows across a shock (Drury, 1983; Malkov and Drury, 2001). In the kinetic approach to study numerically the CR acceleration at shocks, the diffusion-convection equation for the particle momentum distribution, $f(p)$, is solved with suitably modified gas-dynamic equations. This numerical task is challenging, because the full CR shock transition includes a very wide range of length scales associated with the particle diffusion lengths, $\kappa(p)/u_s$, from CR injection scales near the shock to outer diffusion scales for the highest energy particles. To follow the acceleration of highly relativistic CR from supra-thermal energies, Kang and Jones (2005) have developed the CRASH (Cosmic-Ray Amr SHock) code in one dimensional (1D) plane-parallel geometry by combining a powerful Adaptive Mesh Refinement (AMR) technique and a shock tracking technique (Kang et al., 2001). Time-dependent nonlinear simulations of diffusive shock acceleration found that $10^{-4} - 10^{-3}$ of incoming thermal particles can be injected into the CR population via thermal leakage at quasi-parallel shocks, and that up to 50-60 % of the shock kinetic energy can be converted into CR at strong

shocks with $M_s > 10$ (Kang et al., 2002; Kang and Jones, 2005b). The presence of a preexisting CR population is equivalent to having efficient thermal leakage injection at the shock.

It is believed that the CR pressure is important in the evolution of the interstellar medium of our Galaxy and most of galactic CR protons with energies up to $\approx 10^{14}$ eV are accelerated by supernova remnant shocks (Blandford and Eichler, 1987). Simulations of diffusive shock acceleration in spherical supernova remnants also indicate that CR can absorb up to 50% of the initial blast energies (Berezhko and Völk, 1997, 2000). In paper of Kang and Jones (2005a) is described a new CRASH code in 1D spherical geometry. Kang and Jones (2005a) solved the flow equations in a frame comoving with the spherical shock, so the shock and refined region around it stay at the same grid locations. They present the numerical simulation results for a typical supernova remnant expanding into the uniform hot interstellar medium.

4.27.2. Comoving spherical grid

In order to ensure that the shock remains near the middle of the computational domain at all levels of refined grids, for a spherically expanding shock, it is necessary define a comoving frame which expands with the instantaneous shock speed. Following the conventional cosmological approach (Ryu et al., 1993), the **comoving** radial coordinate, $x = r/a$, is adopted, where a is the expansion factor and $a = 1$ at the start of simulations. The expansion rate, $\dot{a} = (u_s - v_s)/x_s$, is found from the condition that the shock speed is zero at the comoving frame. Here u_s and v_s are the shock radial velocities in the Eulerian frame and in the comoving frame, respectively. Then the comoving density and pressures are defined as

$$\tilde{\rho} = \rho a^3, \quad \tilde{P}_g = P_g a^3, \quad \tilde{P}_c = P_c a^3. \quad (4.27.1)$$

The gas-dynamic equation with CR pressure terms in the spherical comoving frame can be written as follows:

$$\frac{\partial \tilde{\rho}}{\partial t} + \frac{1}{a} \frac{\partial}{\partial x} (\tilde{\rho} v) = -\frac{2}{ax} \tilde{\rho} v, \quad (4.27.2)$$

$$\frac{\partial (\tilde{\rho} v)}{\partial t} + \frac{1}{a} \frac{\partial}{\partial x} (\tilde{\rho} v^2 + \tilde{P}_g + \tilde{P}_c) = -\frac{2}{ax} \tilde{\rho} v^2 - \frac{\dot{a}}{a} \tilde{\rho} v - \ddot{a} x \tilde{\rho}, \quad (4.27.3)$$

$$\begin{aligned}
& \frac{\partial(\tilde{\rho}\tilde{e}_g)}{\partial t} + \frac{1}{a} \frac{\partial}{\partial x} (\tilde{\rho}\tilde{e}_g v + \tilde{P}_g v + \tilde{P}_c v) \\
& = -\frac{v}{a} \frac{\partial \tilde{P}_c}{\partial x} - \frac{2}{ax} (\tilde{\rho}\tilde{e}_g v + \tilde{P}_g v) - \frac{2\dot{a}}{a} \tilde{\rho}\tilde{e}_g - \ddot{a}x\tilde{\rho}v - \tilde{L}(x, t).
\end{aligned} \tag{4.26.4}$$

The injection energy loss term, $L(r, t)$, accounts for the energy of the supra-thermal particles injected to the CR component at the sub-shock. The deceleration rate is calculated numerically by $\ddot{a} = (\dot{a}_n - \dot{a}_{n-1})/\Delta t_n$. The diffusion-convection equation for the function $\tilde{g} = p^4 \tilde{f}$, where $\tilde{f}(p, r, t)$ is the **comoving** CR distribution function, is given by

$$\frac{\partial \tilde{g}}{\partial t} + \frac{v}{a} \frac{\partial \tilde{g}}{\partial x} = \left(\frac{1}{3ax^2} \frac{\partial}{\partial x} (x^2 v) + \frac{\dot{a}}{a} \right) \left(\frac{\partial \tilde{g}}{\partial y} - 4\tilde{g} \right) + 3\frac{\dot{a}}{a} \tilde{g} + \frac{1}{a^2 x^2} \frac{\partial}{\partial x} \left(x^2 \kappa(x, y) \frac{\partial \tilde{g}}{\partial x} \right), \tag{4.27.5}$$

where $y = \ln(p)$ and $\kappa(x, p)$ is the diffusion coefficient.

4.27.3. Numerical models and results

Kang and Jones (2005a) considered a supernova explosion with $E_o = 10^{51}$ ergs and $M_{sn} = 10M_{Sun}$ in a uniform medium with $n_H = 3 \times 10^{-3} \text{ cm}^{-3}$. The physical quantities are normalized, both in the numerical code and in the plots below, by the following constants:

$$\begin{aligned}
\rho_o &= 7.0 \times 10^{-27} \text{ g.cm}^{-3}, \quad t_o = 6.1 \times 10^3 \text{ yr}, \quad r_o = 28.5 \text{ pc}, \quad u_o = 4.6 \times 10^3 \text{ km.sec}^{-1}, \\
P_o &= 1.5 \times 10^{-9} \text{ erg.cm}^{-3}, \quad \kappa_o = 4.0 \times 10^{28} \text{ cm}^2 \text{ sec}^{-1}.
\end{aligned} \tag{4.26.6}$$

It was assumed a Bohm type diffusion coefficient,

$$\kappa = \left(3 \times 10^{22} \text{ cm}^2 \text{ sec}^{-1} \right) p / B_\mu, \tag{4.27.7}$$

where $B_\mu = 5$ is the interstellar magnetic field strength in units of 10^{-6} Gs and p is the particle momentum in units of mc . The pressure of the background gas is set to be $P_{go} = 10^{-12} \text{ erg/cm}^3$ ($T_o \approx 10^6$ K), and the Mach number of the initial shock is 13. In the code units $\hat{P}_{go} = 1.67 \times 10^{-4}$ and $\hat{\kappa}(p) = 1.5 \times 10^{-7} p$. It is assumed that there exists a pre-existing CR population $f(p) \propto p^{-4.5}$, corresponding to an upstream CR pressure, $P_{co} = 0.5P_{go}$. The simulation is initialized at $t/t_o = 1$ by the Sedov-

Taylor similarity solutions which are characterized by the shock position $r_s/r_o = \xi_s(t/t_o)^{2/5}$ and speed, $u_s/u_o = (2/5)\xi_s(t/t_o)^{-3/5}$ with $\xi_s = 1.15167$. The spatial grid resolution in the code unit is $\Delta\hat{r}_o = 6.0 \times 10^{-4}$ at the base grid and $\Delta\hat{r}_s = 2.3 \times 10^{-6}$ at the 8-th refined grid, which is the finest refined grid for this simulation. When the simulation is repeated with 10 levels of refined grids, P_c increases less than 0.5 %, indicating true convergence in the simulation with 8 levels. This grid spacing is much larger than the diffusion length for $p_{\min} \approx 10^{-2}$, $\hat{l}_{dif} = 2.5 \times 10^{-9}$, which is contrary to what was found in previous simulations in Eulerian grid (Kang et al., 2001). The faster convergence at lower resolution seems to result from the fact that the shock stays in the same grid zone in the comoving frame. It was used 230 uniformly spaced logarithmic momentum zones in the interval

$$\log(p/mc) = [\log p_{\min}, \log p_{\max}] = [-3.0, +6.0]. \quad (4.27.8)$$

The CR modified shock structure and the CR momentum distribution inside the simulation box,

$$G(p) = 4\pi \int_{r_{\min}}^{r_{\max}} r^2 p^4 dr f(r, p), \quad (4.27.9)$$

are shown in Fig. 4.27.1 at $t/t_o = 1.0 - 1.5$.

The density in the precursor, ρ_1 , and in the post-shock region, ρ_2 , immediately before and after the sub-shock, respectively, are shown in the top panel of Fig. 4.27.2. In Fig. 4.27.2 the middle panel shows the Mach number of the sub-shock, while the bottom panel shows the CR pressure and gas pressure in units of the ram pressure of unmodified Sedov-Taylor similarity solution, $\rho_o u_{ST}^2 \propto t^{-1.2}$.

Kang and Jones (2005a) note the following important observations from Fig. 4.27.1 and Fig. 4.27.2:

1. The CR protons are accelerated to the proton knee energy in the spectrum of galactic CR (about $10^{14} - 10^{15}$ eV, $p/mc \approx 10^5 - 10^6$) in several thousand years, as expected from the standard estimate (Lagage and Cesarsky, 1983).
2. The ratios of both post-shock P_c and P_g relative the shock ram pressure approach to time-asymptotic values quickly. The post-shock P_c is about 50 % of the shock ram pressure, while the gas pressure takes only 20 %.

3. Both the CR momentum distribution at the shock, $g(r_s, p) = f(r_s, p)p^4$, and the integrated distribution, $G(p)$, exhibit characteristic concave curvature, reflecting the nonlinear velocity structure in the precursor.

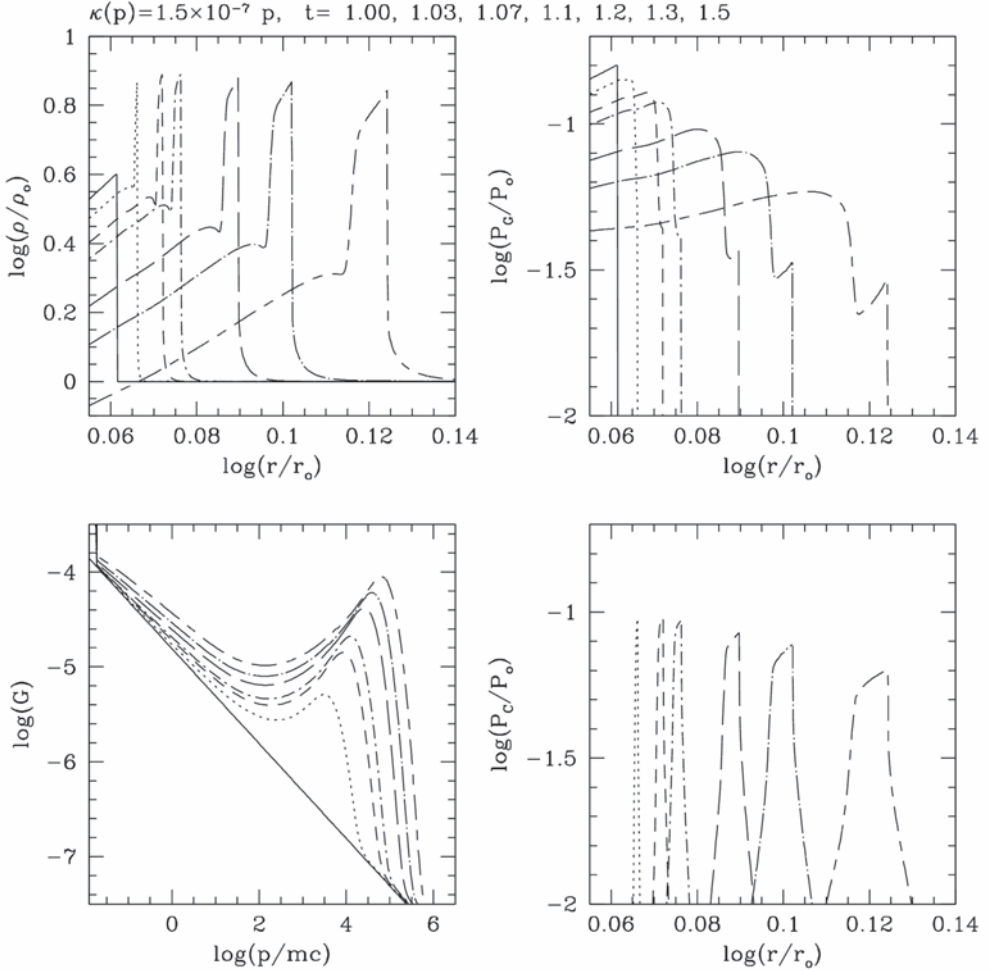


Fig. 4.27.1. Evolution a typical supernova remnant expanding into the uniform interstellar medium. The model parameters are $E_o = 10^{51}$ ergs, $M_{sn} = 10M_{Sun}$, $n_H = 3 \times 10^{-3} \text{ cm}^{-3}$, and $B_\mu = 5$. It assumes a preexisting CR population of $f(p) \propto p^{-4.5}$, with $P_{co} = 0.5P_{go}$, but thermal leakage is not included. The lower left panel shows the integrated particle spectrum according to Eq. 4.27.9. The time $t = 1$ corresponds to 6100 years. The initial condition at $t/t_o = 1.0$ (solid line) is set by the Sedov-Taylor similarity solution. According to Kang and Jones (2005a).

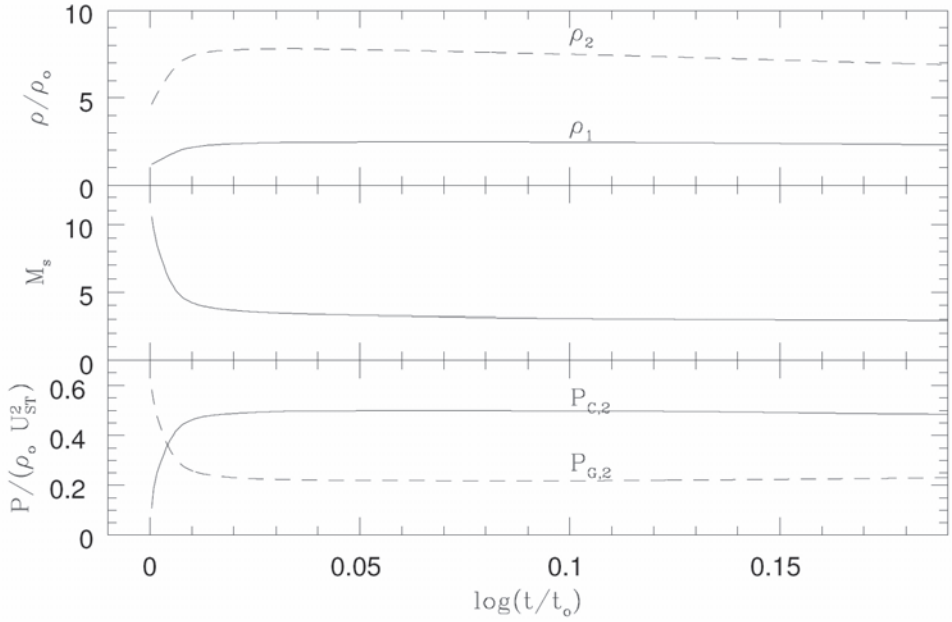


Fig. 4.27.2. Pre-shock density, ρ_1 , post-shock density, ρ_2 , the shock Mach number, M_s , the post-shock CR P_{c2} and gas pressure P_{g2} in units of the ram pressure of Sedov-Taylor solution, $\rho_o u_{ST}^2 \propto t^{-1.2}$. According to Kang and Jones (2005a).

4.28. Particle acceleration by the electrostatic shock waves

4.28.1. Formation of electrostatic shock waves in space plasma

Saito et al. (2003) note that even if there is charge neutrality and no magnetic field the plasma flows generate the intense electric and magnetic fields. This is referred to the counter-streaming instability that has the process similar to Weibel instability with anisotropic temperature (Weibel, 1959). Califano et al. (1997) investigated, both analytically and numerically, electron-electron counter-streaming instability in electron-ion plasma by using both two-fluid equations and Maxwell's equations. They derived the theoretical dispersion relation and applied it to beam-plasma instability in laser plasma. Kazimura et al. (1998) investigated electron-positron counter-streaming instability by using both linear theory and 'particle-in-cell' simulation based on Buneman (M1993). They derived the dispersion relation from four-fluid equations with Maxwell's equations, and compared the results derived from linear theory with the results of simulation. They noticed the linear stage of counter-streaming instability. In Haruki and Sakai (2003) there was investigated the non-linear stage of counter-streaming instability in pair plasma. They also derived the theoretical dispersion relation by using four-fluid equations

with Maxwell equations, and compared the growth rate derived from the dispersion relation with that of simulation result. Furthermore in non-linear stage they showed the generation of electrostatic shock waves which are caused by electrostatic counter-streaming instability. The generated electrostatic waves create some high energy particles. In the papers of Califano et al. (1997), Kazimura et al. (1998), Haruki and Sakai (2003) they do not take into account the possible role of the background magnetic field $\mathbf{B}_0$. Saito et al. (2003) developed this research and investigated the counter-streaming instability in pair plasma as well as particle acceleration taking into account the background magnetic field $\mathbf{B}_0$ parallel to the direction of counter-streaming.

4.28.2. The two-dimensional simulation model

Saito et al. (2003) used two-dimensional, fully electromagnetic, and relativistic ‘particle-in-cell’ code of Buneman (M1993). The lengths of the system are $L_x = 4000\Delta$ and $L_y = 64\Delta$, where Δ is the grid size. The periodic boundary conditions are imposed in both x and y directions. There are 60 electron-positron pairs in a cell uniformly in whole system. The background magnetic field and counter velocity are parallel, and their direction has been used to define the x -axis. To set the counter-streaming plasma Saito et al. (2003) divided all particles into two components. Both electrons and positrons in the left hand side have the velocity $0.5c$, and in the right hand side have the velocity $-0.5c$. The other parameters are as follows: the simulation time step is $\omega_{pe}t = 0.05$, where ω_{pe} is electron plasma frequency; the Debye length and skin depth are 1Δ and 10Δ , respectively; both the electron and positron thermal velocities are $0.1c$. The parameters associated with the background magnetic field $\mathbf{B}_0$ are listed in Table 4.28.1.

Table 4.28.1. The parameters associated with the background magnetic field (Saito et al., 2003).

The case No	Parameters			
	ω_{ce}/ω_{pe}	β	r_g	v_a
1	0	-	-	-
2	0.5	0.08	2.0Δ	$0.3c$
3	1.0	0.02	1.0Δ	$0.6c$
4	1.5	0.009	0.66Δ	$0.72c$
5	1.7	0.006	0.57Δ	$0.76c$
6	2.0	0.005	0.50Δ	$0.80c$

4.28.3. Generated electric and magnetic fields, and particle acceleration (results of simulation)

The simulation model described above gave the following main result: both the electric and magnetic fields are generated by counter-streaming instability. Fig. 4.28.1 shows the electric field E_x and magnetic field B_z configurations at $\omega_{pe}t = 25.0$ generated by this instability.

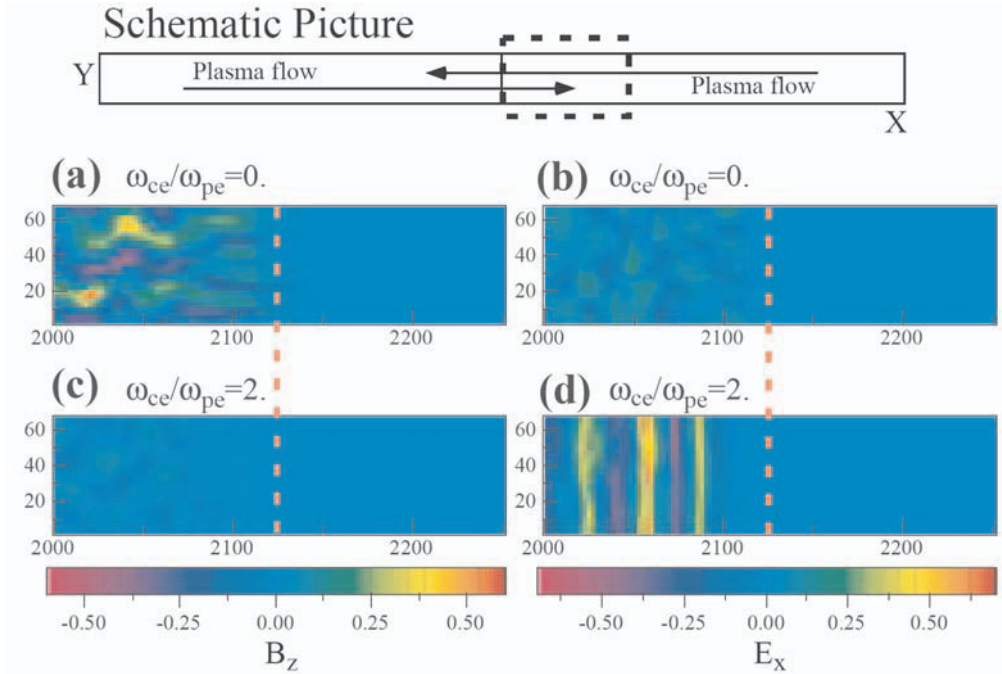


Fig. 4.28.1. The schematic picture in the top of the figure shows the whole system of this simulation. The dashed square indicates the plotting region for panels (a), (b), (c), and (d). Panels (a) and (b) show B_z and E_x configurations without a background magnetic field; panels (c) and (d) show them with the background magnetic field characterized by $\omega_{ce}/\omega_{pe} = 2$ at $\omega_{pe}t = 25.0$. The vertical dashed line indicates the front of the plasma flow. According to Saito et al. (2003).

The top panel in Fig. 4.28.1 shows the schematic picture which reflects the whole system of this simulation. Saito et al. (2003) plotted the magnetic field and electric field, generated in the region surrounded by dashed square of the schematic picture, in panels (a) and (b) of Fig. 4.28.1 which show B_z and E_x configuration without background magnetic field, and in panels (c) and (d) with the background magnetic field characterized by $\omega_{ce}/\omega_{pe} = 2$. The dashed red lines in panels (a), (b), (c) and (d) show the front of the plasma flow.

When ω_{ce}/ω_{pe} becomes large the generation of magnetic field is restrained by the background magnetic field, while the electric field becomes strong and turns into a longitudinal wave from a transverse wave. This means that the wave generated by counter-streaming instability is changed to the electrostatic mode from the electromagnetic mode by the strong background magnetic field.

Fig. 4.28.2 shows the ratio of the generated electric field energy E_{Electric} and the magnetic field energy E_{Magnetic} at the linear stage. In Fig. 4.28.2 the horizontal axis shows the ratio E_{B_0}/E_{flow} of background magnetic field energy E_{B_0} and plasma flow energy E_{flow} . The ratio $E_{\text{Electric}}/E_{\text{Magnetic}}$ of energies E_{Electric} and E_{Magnetic} suddenly increase when the ratio E_{B_0}/E_{flow} exceeds 3 or 4. This means that the generation of electric field is superior than the generation of magnetic field, which indicates that the nature of generated wave becomes electrostatic mode from electromagnetic mode. The generated electrostatic wave accelerates charged particles.

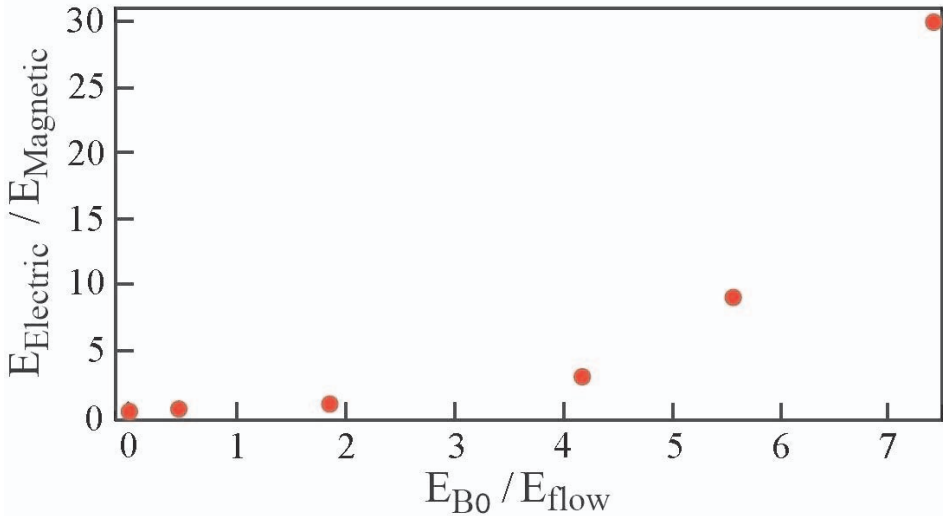


Fig. 4.28.2. The ratio $E_{\text{Electric}}/E_{\text{Magnetic}}$ of generated electric field and magnetic field energy in dependence on the ratio E_{B_0}/E_{flow} . According to Saito et al. (2003).

Fig. 4.28.3. shows the electron velocity distribution in the x - direction parallel to both the magnetic field and the counter-streaming. In Fig. 4.28.3 the red (1), green (2), and blue (3) lines indicate the electrons velocity distribution at the initial ($\omega_{pe}t = 0$), at $\omega_{pe}t = 200$ without the background magnetic field, and at $\omega_{pe}t = 200$ with the background magnetic field characterized by $\omega_{ce}/\omega_{pe} = 2$, respectively.

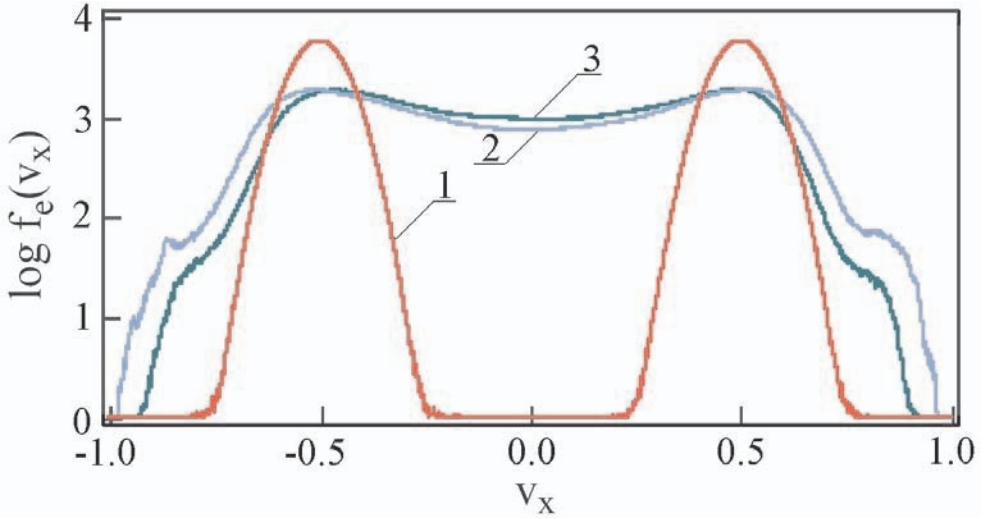


Fig. 4.28.3. The electron velocity distribution $f_e(v_x)$. The red (1), green (2), and blue (3) lines show $f_e(v_x)$ at $\omega_{pe}t = 0$, $\omega_{pe}t = 200$ without $\mathbf{B}_0$, and at $\omega_{pe}t = 200$ with $\mathbf{B}_0$ for $\omega_{ce}/\omega_{pe} = 2$, respectively. The electrons velocity v_x is normalized by the velocity of light. According to Saito et al. (2003).

From Fig. 4.28.3 it follows that with the condition $\omega_{ce}/\omega_{pe} = 2$, the electrostatic field can more effectively accelerate the electrons in the x -direction than the electromagnetic wave driven by the instability without the background magnetic field. In this condition the maximum Lorentz gamma factor of electrons is about 3, and the number of accelerated particles that exceeds $0.9c$ is about two hundreds times as the number of accelerated particles in the condition without background magnetic field.

Saito et al. (2003) came to conclusion that:

1. The background magnetic field restrains the generation of the electromagnetic wave driven by this instability, and consequently it generates the electrostatic wave that has the wave vector parallel to the both magnetic field and the counter-streaming. This means that the nature of counter-streaming instability becomes electrostatic mode from electromagnetic mode.
2. This electrostatic field effectively accelerates the particles in the direction parallel to both the magnetic field and the counter-streaming.
3. The positron velocity distributions are the almost same as the electron velocity distributions.

4.29. Particle acceleration by relativistic shock waves

4.29.1. Peculiarities of particle acceleration by relativistic shock waves

As Niemiec and Ostrowski (2003a) note, at a relativistic shock wave the bulk velocity of the flow is comparable to the particle velocity. This leads to anisotropy of particle angular distribution, which can substantially influence the process of particle acceleration. In contrast to the non-relativistic case the particle power-law spectral indices depend on the conditions at the shock, including the spectrum and amplitude of magnetic field perturbations and the mean field inclination to the shock's normal (Kirk and Schneider, 1987a,b; Heavens and Drury, 1988; Kirk and Heavens 1989; Ballard and Heavens, 1992; Ostrowski 1991, 1993; Bednarz and Ostrowski, 1996, 1998). In the case of a weakly perturbed magnetic field the acceleration process can be investigated via analytical methods (Kirk and Schneider, 1987a; Heavens and Drury, 1988; Kirk and Heavens, 1989). However, if finite-amplitude MHD waves are present in the medium these approaches are no longer valid and numerical methods have to be used. The particle acceleration studies so far have applied very simple models for numerical modeling of the perturbed magnetic field structure (Ostrowski, 1991, 1993; Ballard and Heavens, 1992; Bednarz and Ostrowski, 1996, 1998).

4.29.2. First-order Fermi particle acceleration at relativistic shock waves with a 'realistic' magnetic field turbulence model

Niemiec and Ostrowski (2003a) simulate the first order Fermi acceleration process at mildly relativistic shock waves propagating in more realistic perturbed magnetic fields, taking into account a wide wave vector range turbulence with the power-law spectrum and continuity of the magnetic field across the shock, involving the respective matching conditions at the shock: below the upstream (downstream) quantities are labeled, as usual, with the index '1' ('2'). In the simulations the trajectories of ultra-relativistic test particles are derived by integrating their equations of motion in the perturbed magnetic field. Niemiec and Ostrowski (2003a) consider a relativistic planar shock wave propagating in rarefied electron-proton plasma. Upstream of the shock the field consists of the uniform component, $B_{0,1}$, inclined at some angle ψ_1 to the shock's normal and finite amplitude perturbations imposed upon it. The perturbations are modeled as a superposition of 294 sinusoidal static waves of finite amplitudes (Ostrowski, 1993) which have either a flat

$$F(k) \propto k^{-1} \quad (4.29.1)$$

or a Kolmogorov

$$F(k) \propto k^{-5/3} \quad (4.29.2)$$

wave power spectrum in the wide wave vector range ($k_{\min}, k_{\max}$) and

$$k_{\max}/k_{\min} = 10^5. \quad (4.29.3)$$

The shock moves with the velocity u_1 with respect to the upstream plasma. The downstream flow velocity u_2 and the magnetic field structure are obtained from the hydrodynamic shock jump conditions, so that the field is continuous across the shock. Derivation of the shock compression ratio defined in the shock rest frame as $r = u_1/u_2$ is based on the approximate formulae derived in Heavens and Drury (1988). The acceleration process in Niemiec and Ostrowski (2003a) is considered in the particle energy range where radiative (or other) losses can be neglected.

In Fig. 4.29.1 there are presented particle spectra for the oblique sub-luminal

$$u_{B,1} \equiv u_1/\cos\psi_1 < c \quad (4.29.4)$$

shock wave with $u_1 = 0.5c$ and $\psi_1 = 45^\circ$ (the shock velocity along the mean magnetic field is then $u_{B,1} = 0.71c$, and the shock compression ratio is $r = 5.11$). The particle spectra are measured at the shock for three different magnetic field perturbation amplitudes and the flat wave power spectrum – panel (a) and the Kolmogorov spectrum – panel (b) in Fig 4.29.1.

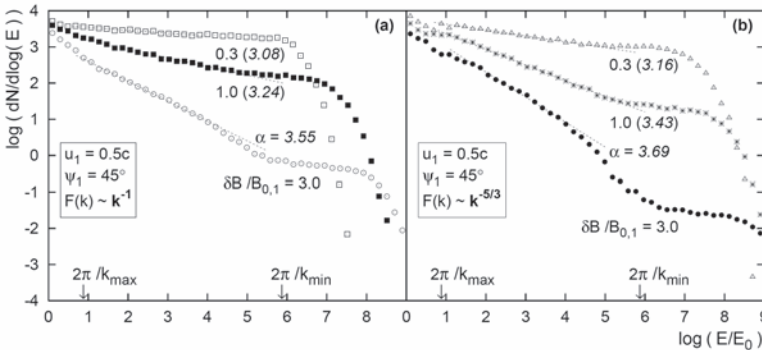


Fig. 4.29.1. Accelerated particle spectra at the sub-luminal shock wave ($u_1 = 0.5c$, $\psi_1 = 45^\circ$ and $u_{B,1} = 0.71c$) for (a) the flat (Eq. 4.29.1) and (b) the Kolmogorov (Eq. 4.29.2) wave spectrum of magnetic field perturbations. The upstream perturbation amplitude $\delta B/B_{0,1} = 0.3, 1.0$, and 3.0 are given near the respective results. Linear fits to the power-law parts of the spectra are presented and values of the phase space distribution function spectral indices $\alpha = 3.08, 3.24$, and 3.55 for (a) and $\alpha = 3.16, 3.43$, and 3.69 for (b) are given. Particles of energies in the range indicated by arrows can effectively interact with the magnetic field inhomogeneities ($k_{\min} < k_{res} < k_{\max}$). From Niemiec and Ostrowski (2003a).

The following features are visible in the spectra from Fig. 4.29.1:

1. the particle spectra diverge from a power-law in the full energy range;
2. before the spectrum cut-off a harder spectral component can appear;
3. the exact shape of the spectrum depends on both the amplitude of the magnetic field perturbations and the wave power spectrum.

One may note that a power-law part of the particle spectrum steeping with increasing amplitude of the field perturbations, more for the Kolmogorov perturbations, panel (b).

Niemiec and Ostrowski (2003a) note that the spectral indices obtained are consistent with previous numerical calculations of Ostrowski (1991, 1993) and with the analytic results obtained in the limit of small perturbations (Kirk and Heavens, 1989). The non power-law character of the obtained particle spectra results from the limited dynamic range of magnetic field perturbations. In the energy range within which the approximately power-law spectrum forms, particles can be effectively scattered by the magnetic field inhomogeneities. The character of the spectrum changes at the highest particle energies where $k_{res} \leq k_{min}$ and particles are only weakly scattered. Then the anisotropic distributed upstream particles can effectively reflect from the region of compressed magnetic field downstream of the shock, leading to the spectrum flattening (Ostrowski, 1991). The cut-off in the spectrum is formed mainly owing to very weakly scattered particles escaping from the shock to the introduced upstream free escape boundary.

In Fig. 4.29.2 are presented the spectra obtained for super-luminal shocks with $u_{B,1} = 1.93c$.

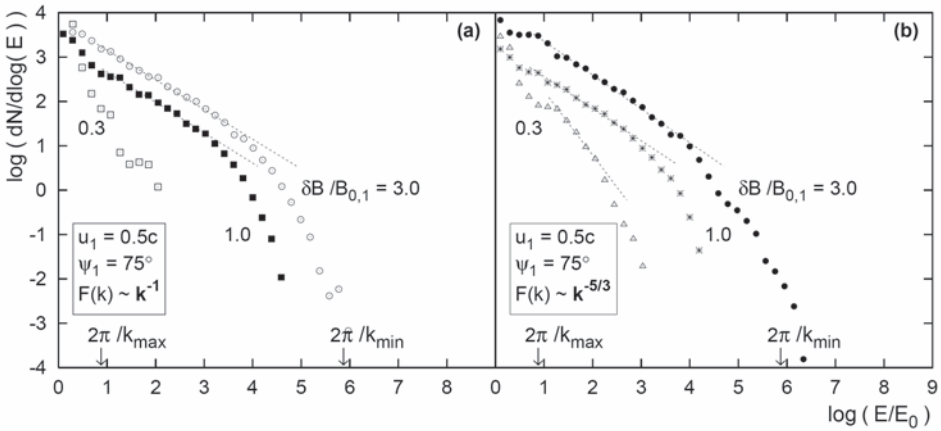


Fig. 4.29.2. Accelerated particle spectra at the super-luminal shock wave ($u_1 = 0.5c$, $\psi_1 = 75^\circ$ and $u_{B,1} = 1.93c$) for (a) the flat (Eq. 4.29.1) and (b) the Kolmogorov (Eq. 4.29.2) wave spectrum of magnetic field perturbations. The upstream perturbation amplitude $\delta B/B_{0,1} = 0.3, 1.0$, and 3.0 are given near the respective results. According to Niemiec and Ostrowski (2003a).

For the low amplitude turbulence ($\delta B/B_{0,1} = 0.3$) Niemiec and Ostrowski (2003a) approximately reproduce results of Begelman and Kirk (1990), with a ‘super-adiabatic’ compression of injected particles, but hardly any power-law spectral tail. At larger turbulence amplitudes power-law sections in the spectra are produced again, but the steepening and the cut-off occur at lower energies compared with the subluminal shocks (compare with Fig. 4.29.1).

4.29.3. Particle acceleration at parallel relativistic shocks in the presence of finite-amplitude magnetic field perturbations

As discussed by Ostrowski (1988b) for non-relativistic shocks, the presence of finite-amplitude magnetic field perturbations modifies the character of the diffusive particle acceleration at the shock wave with the mean field parallel to the shock’s normal. The effect arises owing to locally oblique field configurations formed by long-wave perturbations at the shock front and the respective magnetic field compressions. As a result the mean particle energy gains may increase and the particles reflected from the shock front may occur. The same phenomena should occur at relativistic shocks (Ostrowski, 1993).

In the simplified numerical simulations of the first-order Fermi acceleration at parallel mildly relativistic shocks the acceleration time scale reduces with increasing turbulence level, but no spectral index variation occurs (Bednarz and Ostrowski, 1996, 1998). However, the mentioned acceleration models apply very simple modeling of the perturbed magnetic field effects by introducing particle pitch-angle scattering. The purpose of the Niemiec and Ostrowski (2003b) work is to simulate the first order Fermi acceleration process at mildly relativistic shock waves propagating in more realistic perturbed magnetic fields, including a wide wave vector range of turbulence with the power-law spectrum. The magnetic field is continuous across the shock, according to the respective jump conditions. This feature leads to substantial modifications of the acceleration process at parallel shocks: as usually the upstream (downstream) quantities are labeled, as usual, with the index ‘1’ (‘2’).

In Niemiec and Ostrowski (2003b) the simulations trajectories of ultra-relativistic test particles are derived by integrating their equations of motion in the perturbed magnetic field. A relativistic shock wave is modeled as a plane discontinuity propagating in electron-proton plasma. The magnetic field is defined upstream of the shock. It consists of the uniform component, $B_{0,1}$, parallel to the shock normal and finite-amplitude perturbations imposed upon it. The perturbations are modeled as a superposition of 294 sinusoidal static waves of finite amplitudes (Ostrowski, 1993). They have either a flat (Eq. 4.29.1) or a Kolmogorov (Eq. 4.29.2) wave power spectrum in the wide wave vector range ($k_{\min}, k_{\max}$) with $k_{\max}/k_{\min} = 10^5$. The shock moves with the velocity u_1 with respect to the upstream plasma. The downstream flow velocity u_2 and the magnetic field

structure are obtained from the hydrodynamic shock jump conditions. Derivation of the shock compression ratio as measured in the shock rest frame, $R = u_1/u_2$, is based on the approximate formulae derived in Heavens and Drury (1988). In the analysis of the acceleration process the particle radiative (or other) losses are neglected. In Fig. 4.29.3 are presented particle spectra for the parallel shock wave with $u_1 = 0.5c$. The shock compression ratio is $r = 5.11$.

In Fig. 4.29.3 are reflected the particle spectra measured at the shock for three different magnetic field perturbation amplitudes and the flat, panel (a) or the Kolmogorov wave power spectrum, panel (b).

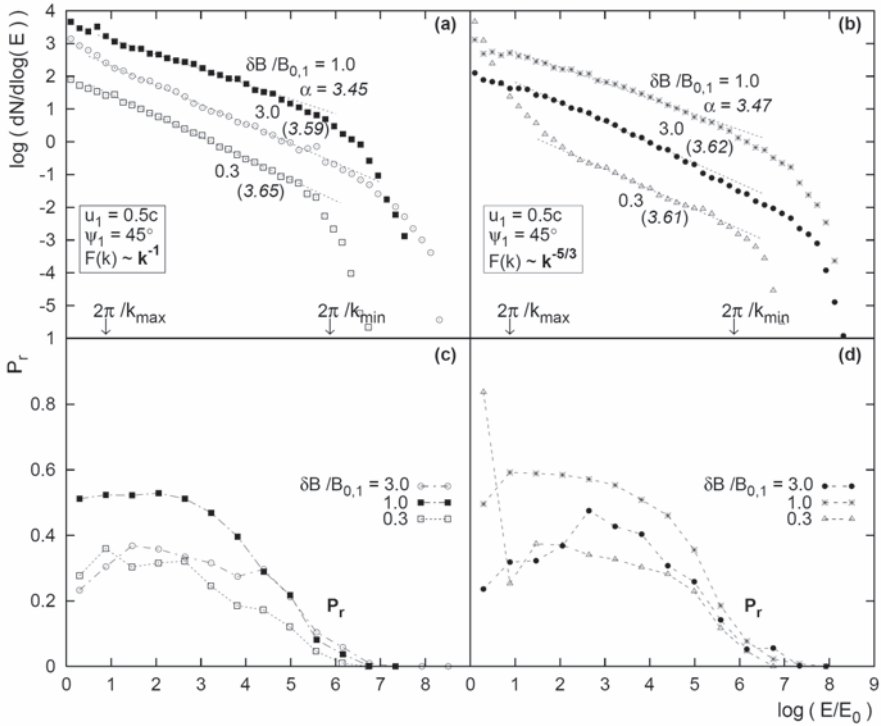


Fig. 4.29.3. Accelerated particle spectra at the parallel shock wave in the shock rest frame for (a) the flat (Eq. 4.29.1) and (b) the Kolmogorov (Eq. 4.29.2) wave spectrum of magnetic field perturbations. The upstream perturbation amplitude $\delta B/B_{0,1}$ is given near the respective results. Linear fits to the power-law parts of the spectra are presented and values of the phase space distribution function spectral indices α are given in parentheses. Particles in the energy range indicated by arrows can effectively interact with the magnetic field inhomogeneities ($k_{\min} < k_{\text{res}} < k_{\max}$). For upstream particles probabilities of reflection from the shock, P_r , are presented in the bottom panels as a function of particle energy for the respective particle spectra above (the transmission probability $P_{12} = 1 - P_r$). According to Niemiec and Ostrowski (2003b).

From Fig. 4.29.3 it can be seen that the particle spectral indices deviate from the small amplitude results of the pitch angle scattering model (Kirk and Schneider, 1987a,b; Heavens and Drury, 1988; Kirk and Heavens, 1989). In addition, the increasing magnetic field perturbations can produce non-monotonic changes of the particle spectral index – the feature which has not been discussed for parallel shocks so far. Analogously to oblique shock waves (Niemiec and Ostrowski, 2003a; see Section 4.29.2), the particle spectra obtained are non power-law in the full energy range and the shape of the spectrum varies with the amplitude of turbulence and the wave power index. The non-monotonic variation of the spectral index with the turbulence amplitude results from modifications of the particle acceleration process at the shock. The long-wave finite-amplitude perturbations produce locally oblique magnetic field configurations and lead to the occurrence of particles reflected from the compressed field downstream of the shock. The probability of reflection depends on the turbulence amplitude and the amount of field perturbations with wavelengths larger than the resonance wavelength for a given particle, as presented in the bottom panels (c) and (d) in Fig. 4.29.3. For $\delta B/B_{0,1} = 1.0$ the reflection probability is higher compared to the other perturbation amplitudes considered and the particle spectrum is flatter. For smaller ($\delta B/B_{0,1} = 0.3$) and larger ($\delta B/B_{0,1} = 3.0$) turbulence amplitudes the reflection and transmission probability do not differ considerably, which results in the similar values of the spectral indices.

From Fig. 4.29.3 can be seen also that the spectra obtained for the Kolmogorov case seem to exhibit a continuous slow change of inclinations. Thus the fitted power-laws depend to some extent on the energy range chosen for the fit. One can also note a steep part of the spectrum at low energies for $\delta B/B_{0,1} = 0.3$ in panel (b). The reflection (transmission) probabilities presented decrease (increase) at high particle energies owing to a limited dynamic range of the magnetic field turbulence. The locally oblique field configurations are mainly formed by long-wave perturbations ($k < k_{res}$) in accordance with Ostrowski (1988b). For high energy particles with $k_{res} < k_{min}$ there are no corresponding long waves and the upstream particles can be only transmitted downstream of the shock. In these conditions the acceleration process would converge to the ‘classic’ parallel shock acceleration model, but in the simulations considered particles move far to the introduced escape boundary forming a cut-off (Niemiec and Ostrowski, 2003b).

4.29.4. Electron acceleration in parallel relativistic shocks with finite thickness

Virtanen and Vainio (2003a) performed test-particle simulations of electron acceleration in parallel relativistic shock waves with finite width. The simulations trace individual electrons under the ‘guiding-center’ approximation in a homogeneous background magnetic field with superposed (magnetic) scattering centers frozen-in to the plasma flow. Scatterings off the irregularities are simulated

making small random displacements of the tip of the electron's momentum vector using a random generator (Vainio et al., 2000; Virtanen and Vainio, 2003b). The mean free path, λ , of all charged particles is taken to be a power-law function of particle rigidity, consistent with the assumed magnetic nature of scattering. They consider relativistic particles with speeds close to that of light and characterized by $\gamma = E/mc^2 \gg 1$. Such particles are efficiently scattered by Alfvén waves, and these wave-particle interactions can be, to the lowest approximation, described by quasi-linear theory. Thus, the scattering frequency of relativistic particles, $\nu = c/\lambda$, of species i is

$$\nu_i(\gamma') \approx \nu_o \left(\frac{m_e \Gamma_1}{m_i \gamma'} \right)^{2-q}, \quad (4.29.5)$$

where ν_o and q are parameters depending on the spectrum of magnetic fluctuations, and Γ_1 is the upstream bulk-speed Lorentz factor. The scatterings are performed in the local rest frame, denoted by primes, so the Lorentz factor is also measured in that frame. To simplify the numerical treatment there was neglected the dependence of ν on pitch angle. In standard quasilinear theory, q is the spectral index of the magnetic fluctuations causing the scattering. Two values for this parameter are considered:

Q1. $q = 2$ giving an energy-independent mean free path;

Q2. $q = 5/3$ corresponding to the Kolmogorov spectrum of turbulence.

Proton rigidity at constant Lorentz factor is m_p/m_e times the electron rigidity.

Thus the proton mean free path is $(m_p/m_e)^{2-q}$ times the electron mean free path.

Thus a shock wave having a thickness l_{sh} of about one thermal-proton mean free path,

$$l_{sh} \approx \lambda_{p,th} = c/\nu_p(\Gamma_1), \quad (4.29.6)$$

seems like a thick structure to all electrons with Lorentz factors less than

$$\gamma' \approx \Gamma_1 (m_p/m_e)^{2-q}, \quad (4.29.7)$$

and electron acceleration at these energies should be modest, resembling adiabatic compression.

Virtanen and Vainio (2003a) studied two velocity profiles, **U1** and **U2**.

U1. The tanh profile of Schneider and Kirk (1989):

$$u(x) = \frac{u_1 + u_2}{2} - \frac{u_1 - u_2}{2} \tanh\left(\frac{x}{W}\right), \quad (4.29.8)$$

where W is the width of the shock and x are axis points in the direction of the flow in the shock frame;

U2. A modified profile:

$$u(x) = u_1 - (u_1 - u_2)H(x) \tanh\left(\frac{2.4x}{\lambda_{p,th}}\right), \quad (4.29.9)$$

obtained by fitting the results of a self-consistent Monte Carlo simulation of shock structure (Vainio, 2003). Here $H(x)$ is the step function. To make the two models comparable, the shock width W was adjusted so that the transition from upstream (u_1) to downstream (u_2) values takes place over the same distance in both models. For this, it was used the width of the region where $u(x)$ is in the range

$$u_1 - \delta u < u < u_2 + \delta u \quad \text{with} \quad \delta u = 0.01 \, c. \quad (4.29.10)$$

This gives

$$W = \lambda_{p,th}/4.2. \quad (4.29.11)$$

Virtanen and Vainio (2003a) used the value of $\Gamma_1 = 10$ for the upstream bulk Lorentz factor, and $u_1 u_2 = c^2/3$. Electrons are injected into the acceleration process in the downstream region.

Two models are considered for the injection energy, **E1** and **E2**.

E1. A ‘kinematics’ injection energy:

$$\gamma = \Gamma \Delta = \Gamma_2 \Gamma_1 \left(1 - u_1 u_2 / c^2\right), \quad (4.29.12)$$

i.e., the energy of cold upstream electrons as seen from the downstream gas.

E2. A ‘thermalized’ injection energy:

$$\gamma = (1/2) \alpha \Gamma \Delta (m_p / m_e), \quad (4.29.13)$$

i.e., a fraction of proton thermal energy in the downstream region ($\alpha = 1$ corresponding to equal-partition). Virtanen and Vainio (2003a) used $\alpha = 0.2$.

The results of the simulated electron spectrum for all eight models (**Q1E1**, **Q1E2**, **Q2E1**, **Q2E2** for two velocity profiles, **U1** and **U2**) are plotted in Fig.

4.29.4, which shows that the two velocity profiles produce different results in the case **Q2**: for the injection **E1**, the speed profile **U1** produces a significantly harder spectrum than the speed profile **U2**, and the results are slightly different even for the injection model **E2**. The reason for the differences is probably that **U1** has a larger maximum value of the speed gradient than **U2**. The fact that the results are so similar in the case **Q1**, however, indicates that the adjustment of the shock width for **U1** is reasonable.

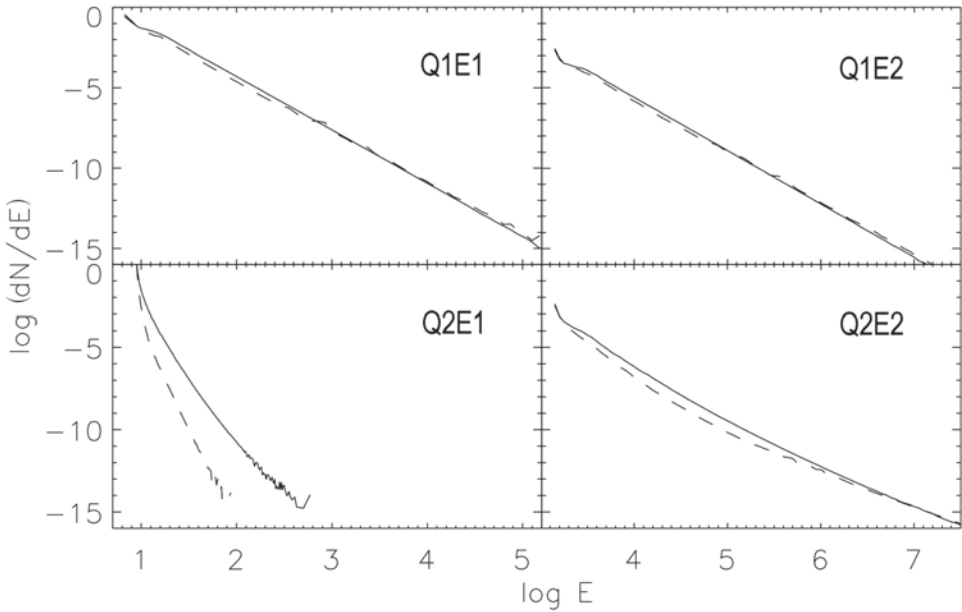


Fig. 4.29.4. The energy spectra of accelerated electrons in parallel relativistic shocks with finite thickness. Solid and dashed curves correspond to speed profile **U1** and **U2**, respectively. See text for a description of the different models. According to Virtanen and Vainio (2003a).

From Fig. 4.29.4 can be seen that the difference between the two turbulence models **Q1** and **Q2** is significant. The spectrum in the case **Q1** is a power law with a spectral index of ≈ 3.2 independent of the injection energy, as expected. The spectral shape in the case **Q2** is not a power law but hardens as a function of energy, because for a mean free path increasing with energy, the shock seems thinner for electrons at higher energies. In the case **E1** the Kolmogorov scattering law **Q2** produces accelerated particles much less efficiently than in the case of an energy-independent mean free path. The thermalized injection **E2** yields accelerated particle populations in both turbulence models. At the highest energies, the spectral index in the **Q2E2** model approaches the value of 2.2 obtained for a step-like shock at $\Gamma_1 \gg 1$ (Kirk and Duffy, 1999). Virtanen and Vainio (2003a) came to the

conclusion that electron acceleration in parallel relativistic shock waves with non-trivial internal structure is heavily dependent on the rigidity dependence of the particle's mean free path. For a shock thickness determined by ion dynamics and a mean free path increasing with energy the standard power-law electron spectra can be obtained only at very high energies, e.g., at $\gamma > 10^5$ for $\lambda \propto \gamma^{1/3}$.

4.29.5. *Small-angle scattering and diffusion: application to relativistic shock acceleration*

Protheroe et al. (2003) investigated ways of accurately simulating the propagation of energetic charged particles over small times where the standard Monte Carlo approximation to diffusive transport breaks down. Protheroe et al. (2003) find that a small-angle scattering procedure with appropriately chosen step-lengths and scattering angles gives accurate results, and they apply this to the simulation of propagation upstream in relativistic shock acceleration. The matter is that in diffusive shock acceleration at relativistic shocks problems arise when simulating particle motion upstream of the shock because the particle speeds, v , and the shock speed

$$v_{sh} = c(1 - \gamma_{sh}^{-2})^{1/2} \quad (4.29.14)$$

are both close to c , and so very small deflections are sufficient to cause a particle to re-cross the shock. Clearly, Monte Carlo simulation by a random walk with mean free path λ and large-angle scattering is inappropriate here, and in Monte Carlo simulations of relativistic shock acceleration at parallel shocks Achterberg et al. (2001) consider instead the diffusion of a particle's direction for a given angular diffusion coefficient D_θ ($\text{rad}^2 \text{ s}^{-1}$). Similarly, for a given spatial diffusion coefficient κ , Protheroe (2001) and Meli and Quenby (2001) adopted a random walk with a smaller mean free path, $\bar{l} \ll \lambda$, followed by scattering at each step by a small angle with mean deflection, $\bar{\theta} < 1/\gamma_{\text{shock}}$. (see Bednarz and Ostrowski, 2001 for a review of relativistic shock acceleration). Protheroe et al. (2003) consider propagation by small steps sampled from an exponential distribution with mean $\bar{l} \ll \lambda$, followed at each step by scattering through a small angle sampled from an exponential distribution with mean $\bar{\theta} \ll \pi$. The change in direction (θ_1, θ_2) may then be described as two-dimensional diffusion with angular diffusion coefficient $D_\theta = \bar{\theta}v_\theta/2$ ($\text{rad}^2 \text{ s}^{-1}$) where $v_\theta = \bar{\theta}/\bar{t}$, and $\bar{t} = \bar{l}/v$ such that $D_\theta = \bar{\theta}^2 v / (2\bar{l})$. The time t_{iso} , which gives rise to a deflection equivalent to a large angle (isotropic) scattering, is determined by the equation

$$(\sigma_{\theta_1}^2 + \sigma_{\theta_2}^2)^{1/2} = \sqrt{4D_\theta t_{\text{iso}}} \approx \pi/2, \quad (4.29.15)$$

giving

$$\lambda \approx vt_{iso} \propto \bar{l}/\bar{\theta}^2, \quad (4.29.16)$$

and a spatial diffusion coefficient

$$\kappa \propto \bar{l}v/\bar{\theta}^2 \propto v^2/D_{\theta}. \quad (4.29.17)$$

By using a Monte Carlo method it is straightforward to test this, determine the constant of proportionality, and thereby make the connection between diffusion and small angle scattering. The solution of the diffusion equation for a delta-function source in position and time $q(\mathbf{r}, t) = \delta(\mathbf{r})\delta(t)$ and an infinite diffusive medium is a three-dimensional Gaussian with standard deviation $\sigma = \sqrt{2\kappa t}$ (Chandrasekhar, 1943). The results from several Monte Carlo random walk simulations are shown in Fig. 4.29.5, from which it can be find that the expected dependence occurs for $\bar{\theta} < 5^\circ$ at times $t > 10^5 \bar{l}/v$.

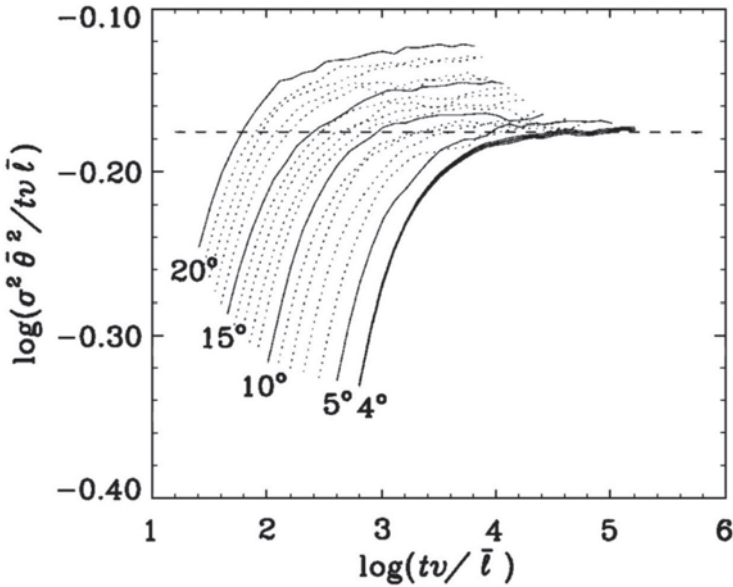


Fig.4.29.5. The value σ^2 vs. time for a 3D random walk with isotropic injection at the origin at $t = 0$. Step-lengths l were sampled from an exponential distribution with mean $\bar{l}$ followed by small-angle scattering with scattering angle θ sampled from an exponential distribution with mean $\bar{\theta}$ (the numbers attached to the curves). The dashed line is $\sigma^2 = 2tv\bar{l}/3\bar{\theta}^2$. Curves for 5° – 20° result from 10^4 simulations; 4° curve results from 8×10^4 simulations (width shows statistical error). According to Protheroe et al. (2003).

For this case it can be seen from Fig. 4.29.5 that $\sigma^2 \rightarrow 2tv\bar{l}/3\bar{\theta}^2$, and so it can be obtain the connection between small-angle scattering and diffusion theory, namely,

$$\kappa \approx \bar{l}v/(3\bar{\theta}^2) \approx v^2/(6D_\theta). \quad (4.29.18)$$

As viewed in the frame of reference of the upstream plasma, ultra-relativistic particles are only able to cross the shock from downstream to upstream if the angle θ between their direction and the shock normal pointing upstream is

$$\theta < \sin^{-1}(1/\gamma_{\text{sh}}), \quad (4.29.19)$$

where

$$\gamma_{\text{sh}} = (1 - \beta_{\text{sh}}^2)^{-1/2}; \quad \beta_{\text{sh}} = v_{\text{sh}}/c. \quad (4.29.20)$$

For highly relativistic shocks these particles cross the shock from downstream to upstream traveling almost parallel to the shock normal. Similarly, having crossed the shock, only a very slight angular deflection, by $\sim 1/\gamma_{\text{sh}}$ is sufficient to return them downstream of the shock. This change in particle direction gives rise to a change in particle energy E' and momentum p' , measured in the downstream plasma frame (primed coordinates), of

$$\frac{E'_{n+1}}{E'_n} \approx \frac{p'_{n+1}}{p'_n} = \frac{1 - \beta_{12} \cos \theta_{n+1}}{1 - \beta_{12} \cos \theta_n} \quad (4.29.21)$$

in an acceleration cycle (downstream $\rightarrow$ upstream $\rightarrow$ downstream), where β_{12} is the speed of the upstream plasma as viewed from the downstream frame. In 'parallel shocks' the magnetic field is parallel to the shock normal, and so the pitch angle ψ is the angle to the shock normal and $v \cos \psi$ gives the component of velocity parallel to the shock. Thus the small-angle scattering method described above is used here to simulate particle motion upstream of a parallel relativistic shock, including the effects of pitch-angle scattering, for a given diffusion coefficient. Ultra-relativistic particles were injected at the shock with downstream-frame energy E'_o travelling upstream parallel to the shock's normal, i.e., $\theta_o = 0$. Let us follow a particle's trajectory until the shock catches up with it, and it crosses from upstream to downstream with an upstream-frame angle θ_1 to the shock's normal and a downstream-frame energy E'_1 . The simulation was performed for $\gamma_{\text{sh}} = 10$ and five different mean scattering angles $\bar{\theta}$ to determine the maximum $\bar{\theta}$ -value that can safely be used for accurate simulation. The resulting distributions of $\cos \theta$ and $\log(E'_1/E'_o)$ are shown in Fig. 4.29.6.

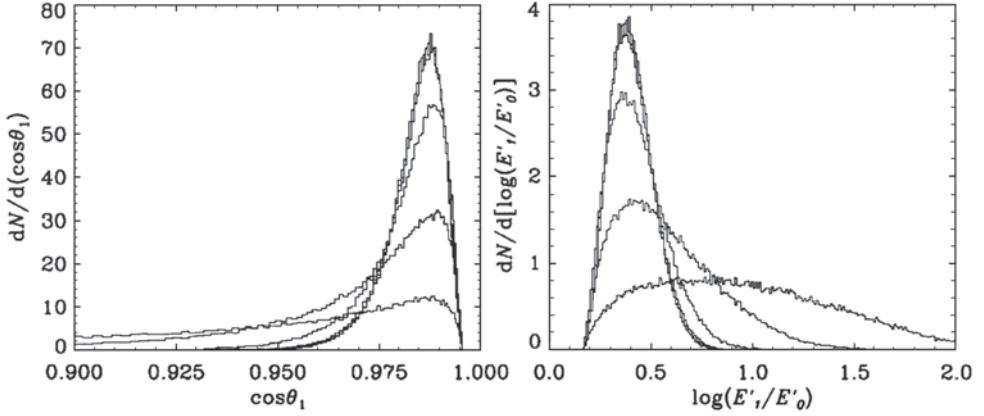


Fig. 4.29.6. Small-angle scattering simulation of excursion upstream in diffusive shock acceleration at a parallel relativistic shock with $\gamma_{\text{sh}} = 10$. Results are shown for 10^5 injected particles and $\bar{\theta} = 10^{-2}/\gamma_{\text{sh}}$ (top histogram), $3 \times 10^{-2}/\gamma_{\text{sh}}$, $10^{-1}/\gamma_{\text{sh}}$, $0.3/\gamma_{\text{sh}}$, $1/\gamma_{\text{sh}}$ and $3/\gamma_{\text{sh}}$ (bottom histogram). Note that the top three histograms are almost indistinguishable. According to Protheroe et al. (2003).

From Fig. 4.29.6 it can be seen that in this application one requires $\bar{\theta} < 0.1/\gamma_{\text{sh}}$. The results described are quite consistent with those of Achterberg et al. (2001), who used a diffusive angular step $\Delta\theta_{\text{st}} \leq 0.1/\gamma_{\text{sh}}$. Protheroe et al. (2003) came to the conclusion that the standard Monte Carlo random walk approach to the simulation of energetic charged particle propagation for a given spatial diffusion coefficient D can be extended to apply accurately to times much less than $\lambda/v = 3D/v^2$ by using a small-angle scattering procedure with steps sampled from an exponential distribution with mean free path $\bar{l} = \bar{\theta}^2 \lambda$ followed at each step by scattering with angular steps sampled from an exponential distribution with mean scattering angle $\bar{\theta} < 0.09$ rad (5°). The spatial and angular diffusion coefficients are then $D \approx \bar{l}v/(3\bar{\theta}^2)$ and $D_\theta \approx \bar{\theta}^2 v/(2\bar{l})$, and are related by $D \approx v^2/(6D_\theta)$. In the simulation of upstream propagation in relativistic shock acceleration one must use $\bar{\theta} < 0.1/\gamma_{\text{sh}}$ to obtain accurate results.

4.30. CR acceleration at super-luminal shocks

4.30.1. The matter of the problem

In paper Meli et al. (2005) the particle shock acceleration at super-luminal shocks is discussed and evaluated by performing Monte Carlo calculations. Relativistic beaming in many relativistic astrophysical sources suggests the

appearance of super-luminal shocks. The range of magnetic field orientations for which a shock is super-luminal increases as the upstream plasma flow speed increases. In order to study the sub-luminal case, it is possible to find a relativistic transformation to the frame of reference (so called De Hoffmann-Teller frame; introduced in paper De Hoffmann and Teller, 1950), in which the shock front is stationary and the electric field is zero ($\mathbf{E} = 0$) in both upstream and downstream regions. However, superluminal shock fronts do not admit a transformation to such a frame, the particle 'diffusive' approximation cannot be applied and the particles are more likely to gain energy as their gyro-center makes a single crossing of the shock front from upstream to downstream or else as doing 'drifts' parallel or anti-parallel to the present electric field. Meli et al. (2005) have established a numerical Monte Carlo method to study the CR acceleration properties (spectral shapes, energy gain, scattering model dependence, magnetic field dependence, etc) in highly relativistic super-luminal shocks by following the helix-trajectory of the particle in the region of a shock front.

4.30.2. Monte Carlo simulations

The aim of paper Meli et al. (2005) is to examine which is the role that different scattering models (large angle scattering or pitch angle diffusion) can play in reference to the spectral shape at very high gamma plasma flows, by considering superluminal shock configurations. It is necessary to note that the flow into and out of the shock discontinuity is not along the shock normal, but a transformation is possible into the normal shock frame to render the flows along the normal (Begelman and Kirk, 1990) and for simplicity it was assumed such transformation has already been made. For these simulation runs a Monte Carlo technique is applied by considering the motion of a particle of momentum p in a magnetic field $\mathbf{B}$. As it was mentioned above, in super-luminal conditions it is not possible to transform into a frame where $\mathbf{E} = 0$ (i.e., into De Hoffmann-Teller frame) a condition that is only possible for the sub-luminal case where $u < u_{sh} \tan \psi$. Thus, the frames to be used in this simulation will be the fluid frames, where still the electric field is zero $\mathbf{E} = 0$, and the shock frame, which it will be used only as a 'check frame' to test whether upstream or downstream conditions apply. Initially the particles are injected at 50λ , where λ is the particles' mean free path, from the shock and their guiding center is followed upstream, at the upstream frame until the particle reaches the shock at $x_{sh} = 0$ followed by an appropriate transformation to the shock frame. At injection the speed of the plasma upstream is highly relativistic and the values between $\Gamma = 10$ and 1000 are kept. For the pitch angle scattering Mali et al. (2005) follow a standard Monte Carlo random walk approach to simulate the energetic particle propagation by using a small angle scattering procedure with steps sampled from an exponential distribution with mean free path $L = \delta\theta^2 \lambda$ and keeping the angle $\delta\theta$ less than $0.1/\Gamma$ (Berezhko and Völk, 2000) while following the diffusion approximation of the scattering during and immediately before the

particle reaches the shock front. Since there is no easy approximation at this juncture to determine the probability of shock crossing or reflection, Meli et al. (2005) change the model following the helical trajectory of the particle, in the fluid frames upstream (index 1) or downstream (index 2) at $\mathbf{E} = 0$, respectively, where the velocity coordinates of the particle are calculated in a three-dimensional space. They assume that the tip of a particle's momentum vector undergoes randomly a small change θ_1 in its direction on the surface of a sphere and within a small range of polar angle (after a small increment of time). If the particle had an initial pitch angle θ_o , it was calculate its new pitch angle θ' by a trigonometric formula (Ryu et al., 1993). After they follow the trajectory in time, using $\phi_1 = \phi_o + \omega t$, and t is the time from first detecting the shock presence at x_{sh}, y_{sh}, z_{sh} and assuming that $\delta t = r_g / Hc$, where r_g is the Larmor radius, $H \geq 100$. After the suitable calculations it was checked whether the particle meets the shock again by transforming to the shock frame. If the particle meets the shock then the suitable transformations to the upstream frame are made again made again and they follow the particle's trajectory as described above. If the particle never meets the shock its guiding center is followed, the same way as mentioned earlier for the upstream side after the injection and it is left to leave the system if it reaches a well defined $E_{\max}$ momentum boundary or a spatial boundary of 100λ .

4.30.3. Main results

Main results are shown in Fig. 4.30.1-4.30.2.

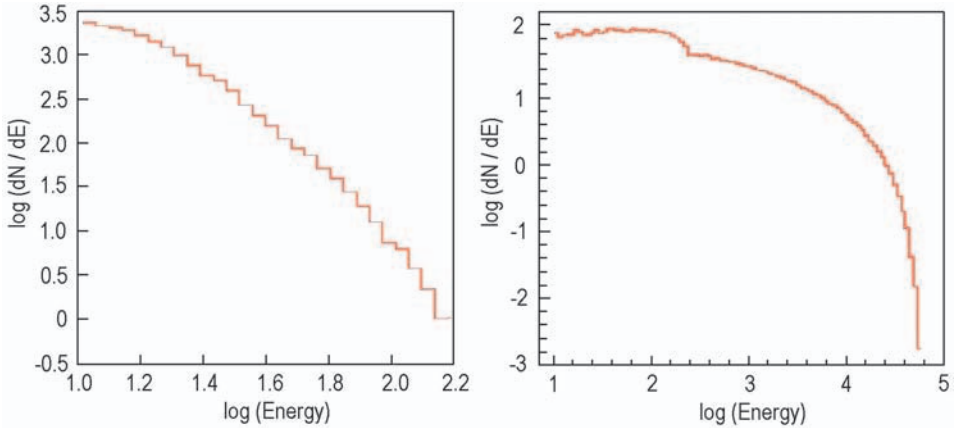


Fig. 4.30.1. Spectrum for the super-luminal pitch angle diffusion case, in the shock frame at the downstream side for $\Gamma = 10$ and $\psi = 89^\circ$. As an indication a gamma of 10^5 corresponds to ~ 100 GeV for protons. For the right panel it is note that the steep cutoff may suggest a connection to the spectra of relativistic electrons originating from observed hot spots (super-luminal shocks) in extragalactic radio sources. From Meli et al. (2005).

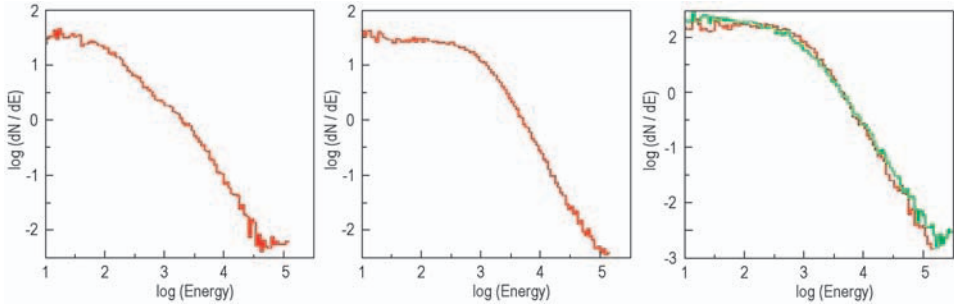


Fig. 4.30.2. Spectral shapes for $\Gamma = 500$ (left panel), $\Gamma = 1000$ (middle panel) for $\psi = 76^\circ$. In the right panel are shown two spectra for $\Gamma = 50$ and an inclination of $\psi = 50^\circ$ and $\psi = 89^\circ$, respectively. It may be seen no dependence of the spectrum with the angle of the magnetic field at the shock normal. This is tested for all gamma and different shock angles. The behavior is the same. From Meli et al. (2005).

From Fig. 4.30.1-4.30.2 one may understand that for pitch angle diffusion, the spectral shape of the accelerated particles follows a rather smooth power-law shape in comparison to the large angle scattering where the spectral shape gives a steep sudden cut-off. For both cases the simulations show that most of the particles are 'swept' downstream the shock after only a cycle. This condition limits the particle's ability to gain very high energies, contrary to the simulation findings in Meli and Quenby (2003a,b) for highly relativistic sub-luminal shocks, where plateau structured spectral shapes are seen, however.

4.30.4. Expected diffuse signal from sources with super-luminal shock fronts

The estimate above source spectra can be translated into an expected diffuse signal from certain astrophysical sources by folding the spectrum with the spatial distribution of the sources. In this ansatz, Meli et al. (2005) used Active Galactic Nuclei (AGN) and Gamma Ray Bursts (GRBs) as potential sources, since these are the sources with the highest observed boost factors. They assume that both source types follow Star Formation Rate (SFR) with a red-shift behavior as suggested in Hasinger et al. (2005). For AGN, $\Gamma = 10$ is assumed. For GRBs, the Γ dependence is considered by taking into account a range of boost factors of $100 < \Gamma < 1000$ with a maximum in the distribution at $\Gamma = 300$ (Guetta et al., 2004). As a simple model, it is assumed that 10% of all GRBs have $\Gamma = 100$, and further 10% are as powerful as $\Gamma = 1000$. For the remaining 80%, the average expected boost factor of $\Gamma = 300$ is assumed. The normalization of the expected signal is done using the most restrictive upper limit on the neutrino signal from extraterrestrial sources given by the AMANDA experiment (Münich and IceCube Collaboration, 2005):

$$E_\nu^2 \frac{dN_\nu}{dE_\nu} < 2.6 \times 10^{-7} \frac{\text{GeV}}{\text{s.sr.cm}^2}. \quad (4.30.1)$$

With an E^{-2} spectrum for both neutrinos and protons, the spectra are connected by assuming that the expected neutrino energy fluency is a fraction x of the proton spectrum,

$$\int \frac{dN_\nu}{dE_\nu} E_\nu dE_\nu = x \int \frac{dN_p}{dE_p} E_p dE_p \quad (4.30.2)$$

With $x = 1/40$, since only 20% of the proton flux goes into pion production via the delta resonance, 1/2 of the remaining flux goes into the charged pion component of which 1/4 goes into neutrinos. The resulting spectrum is shown in Fig. 4.30.3. It can be seen from Fig. 4.30.3 that the only possible contribution to the CR spectrum from the super-luminal shock sources as predicted in the paper of Meli et al. (2005) is around the knee of the measured CR spectrum. It is expected however, that the effective flux is actually even lower, since the normalization is based upon the assumption that the contribution cannot be more than the current neutrino flux limits omit.

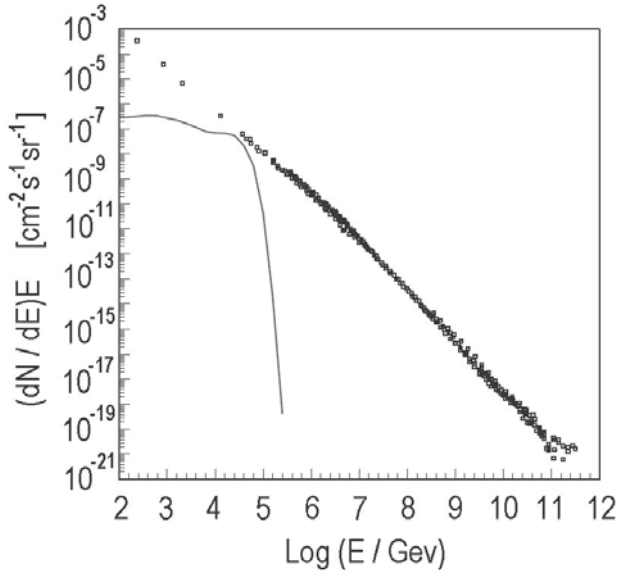


Fig. 4.30.3. The maximum predicted diffuse flux from GRBs and AGN with super-luminal shock fronts (the curve). The normalization is based on current neutrino flux limits (Münich and IceCube Collaboration, 2005). The flux is compared to the measured CR spectrum (small squares). From Meli et al. (2005).

Meli et al. (2005) conclude that described simulations are relevant to models of highly relativistic particle shock acceleration in sources as AGN jets and GRBs, and that:

- (i) Large angle scattering is unrealistic -as expected- (our test spectra gave a steep sudden cutoff) in such high plasma velocities and the pitch angle diffusion scheme resembling the high turbulence upstream the shock was simulated, keeping the scattering cone angle within $0.1/\Gamma$ at crossing the shock front (Protheroe et al., 2002).
- (ii) There is no decrease observed in the acceleration rate, comparing to results of diffusive relativistic shock acceleration (e.g. Lieu et al., 1994; Meli and Quenby, 2003a,b).
- (iii) In order to keep a power-law spectra the angular distribution at crossing the shock is highly anisotropic and 'beamed'.
- (iv) The energy gain of the CR in super-luminal shocks seems limited comparing to highly relativistic shocks.
- (v) The possible contribution to the CR spectrum from super-luminal shock sources is predicted around to the knee of the measured CR spectrum.

4.31. On the fraction of the kinetic energy of moving space plasma goes into energetic particles as result of diffusive shock acceleration

4.31.1. The problem of diffusive shock acceleration effectiveness

To solve this problem Mewaldt et al. (2005a) compare measurements of the energy content of large SEP events from 1997 to 2004 to the kinetic energy of the associated CME to study the efficiency of the diffusive shock acceleration. Mewaldt et al. (2005a) note that it is almost 30 years since the process of diffusive shock acceleration was described, and in the intervening period this process has successfully accounted for observations of several energetic particle components observed in the Heliosphere, including particles accelerated by CME driven shocks, planetary bow shocks, traveling and co-rotating interplanetary shocks, the solar wind termination shock, and supernova shocks. It is therefore of interest to measure the efficiency of this ubiquitous process. At the ACE-RHESSI-Wind workshop in October 2003, one working group had the objective of detailing the energy budget for two large solar events on 21 April, 2002 and 23 July, 2002. The first of these was a major SEP event while the second did not lead to an identified SEP event at 1 AU, possibly because it originated at solar longitude E72. One of the results of this exercise, as reported by Emslie et al. (2004), was that in the 21 April, 2002 event the SEP kinetic energy was a significant fraction ($\sim 15\%$) of the kinetic energy of the CME suggesting rather efficient acceleration. In a study of SEP events during the Halloween, 2003 period by Mewaldt et al. (2005b) five additional events were added to this comparison with SEP/CME energy ratios ranging from $\sim 1\%$ to $\sim 15\%$.

In these six events protons accounted for 69% to 82% of the SEP energy, He particles accounted for 10% to 19%, $Z \geq 6$ ions varied from 3% to 9%, and electrons accounted for ~1% to 18%. In paper of Mewaldt et al. (2005a) there are added preliminary results for 11 additional SEP events, test some of the assumptions of the simple model used to calculate SEP energies, and comment on how the precision of this comparison might be improved in the future.

4.31.2. Estimation of SEP and CME kinetic energies

According to Mewaldt et al. (2005a) several steps are involved in estimating the SEP kinetic energies (Emslie et al., 2004; Mewaldt et al., 2005b):

(i) Fitting spectra. The energy spectra were obtained by combining data from the ULEIS, EPAM and SIS instruments on ACE, the PET instrument on SAMPEX, and the GOES EPS sensor on GOES-8 and GOES-11. The spectra, extending from <0.1 to >100 MeV/nuc, were fit with one of two spectral forms: the double-power-law form of Band et al. (1993) or the model of Ellison and Ramaty (1985). The spectral fits were integrated from 0.01 to 1000 MeV/nuc to obtain the integrated fluencies at 1 AU.

(ii) Correcting for particles that cross 1 AU more than once. To obtain the energy/cm² escaping from 1 AU necessary to correct for the number of times that the average particle crosses 1 AU due to scattering on interplanetary turbulence. It was used the simulation by Giacalone (2005) shown in Fig. 4.31.1, which gives a logarithmic dependence on energy. On average, this reduces the estimated energy content of accelerated particles by a factor of ~3 to 4.

(iii) Correcting for longitude and latitude profiles. Studies of heavy ions >10 MeV/nuc show that the largest SEP events originate near central meridian. This is also seen in the longitude distribution of large proton events observed by GOES (see Fig. 4.31.1). From these data sets it was derived longitudinal e-folding longitudes of 45° for western events and 25° for eastern events. The e-folding latitude was chosen to be the average of these (about 35°). Using these dependences, it is possible to integrate the total particle energy escaping through 1 AU. In order to test whether the longitudinal profiles assumed here are reasonable, Helios 1 and 2 and IMP-8 data (Reames et al., 1996) were used to compare the estimated event fluencies from three separate vantage points, as shown in Fig. 4.31.2. The locations of the three spacecraft were spread over 158° in one event and 66° in the second event. The radial differences in the spacecraft locations were also corrected for by assuming that SEP fluencies scale as $\propto r^{-2}$, where r is the distance from the Sun (Reames and Ng, 1998). The uncertainties on the fluency estimates were taken to be the square-root of the sum of the correction factors for longitude, latitude, and multiple crossings (Mewaldt et al., 2005b). The agreement of the three independent estimates suggests that there are not significantly underestimated the uncertainties in this model.

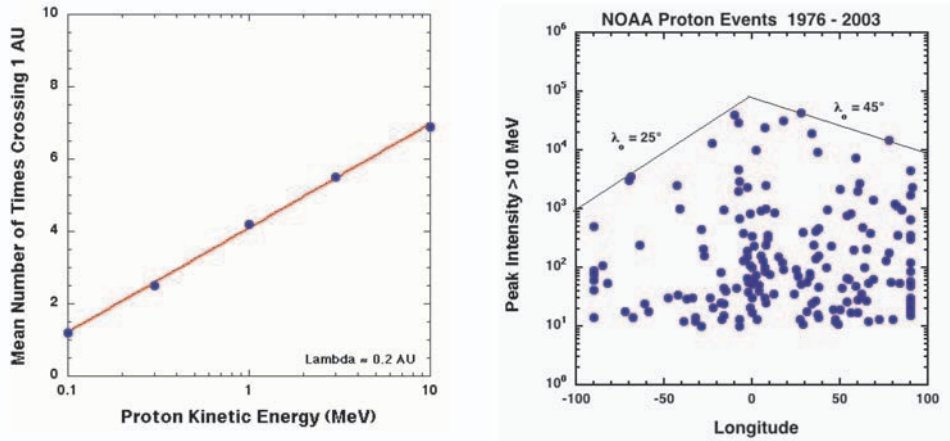


Fig. 4.31.1. The left panel shows the average number of times solar protons pass outward across 1 AU as a function of energy; based on a simulation by Giacalone (2005) assuming a mean free path of 0.2 AU. A logarithmic dependence was fit to these results and extrapolated to higher energy. The right panel shows the longitude distribution of large SEP events observed by the NOAA GOES satellites from 1976-2003. The e-folding longitudes used in this study are indicated. From Mewaldt et al. (2005a).

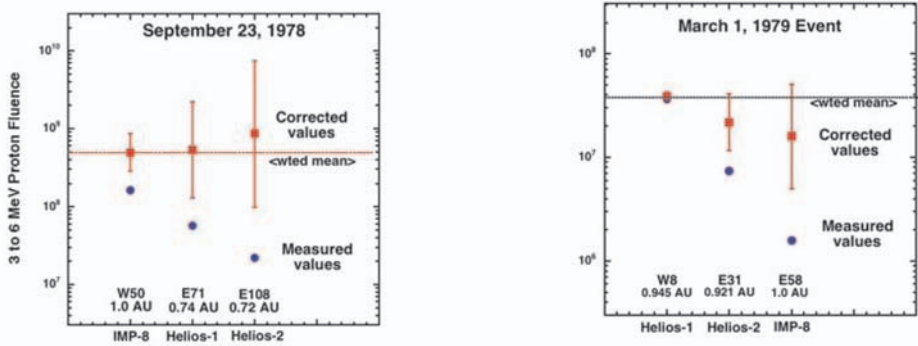


Fig. 4.31.2. A comparison of the fluence of 3 to 6 MeV protons measured by Helios-1 and 2, and IMP-8 for the September 23, 1978 and March 1, 1979 events (based on measurements by Reames et al., 1996). The locations of the three spacecraft relative to the flare site are indicated. Once corrected for latitude and radius, the three estimates are in reasonable agreement. From Mewaldt et al. (2005a).

Up to this point it was only fit the proton spectra for the 11 new events added in paper Mewaldt et al. (2005a). Based on the results from the first six events, where protons accounted for 69% to 82% of the total SEP kinetic energy, it was assumed that the protons make up 75% of the total kinetic energy. The uncertainty in the correction for other species is certainly small compared to the other uncertainties in these estimates.

The CME mass can be estimated from the total excess brightness and the velocity can be found from a fit to the radial profile (Vourlidas et al., 2000). It was used the results of Emslie et al. (2004) for the 21 April, 2002 event and those of Gopalswamy et al. (2005) for the Halloween events. For the eleven new events it was used tabulated CME masses and velocities from Gopalswamy et al. (2004). The CME masses are measured over a sector outlined by the measured angular width of the CME, its front, and the LASCO C2 or C3 occulter. Mass and energy estimates are more accurate for events on the limb than for halo CME events.

4.31.3. Main results of comparison

A comparison of CME and SEP kinetic energies for seventeen SEP events is shown in Fig. 4.31.3.

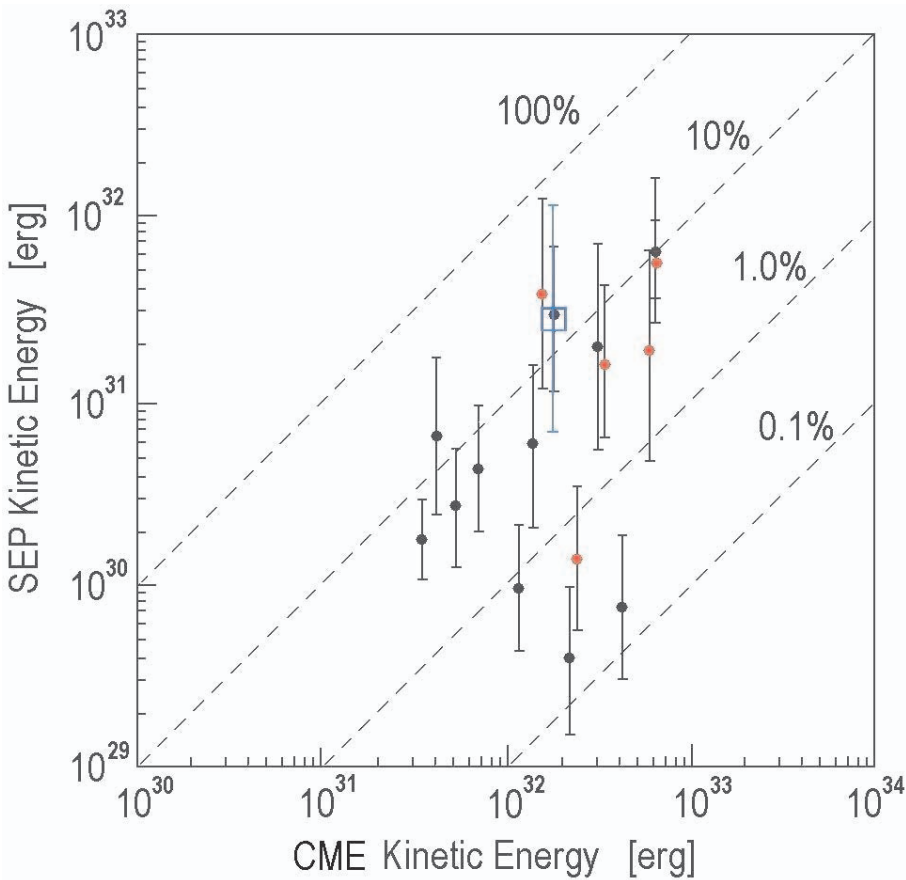


Fig. 4.31.3. A comparison SEP and CME kinetic energies for 17 SEP events including the April 21, 2002 event (open square), the five events from October-November 2003 (circles) and 11 other events observed from 1998-2003. From Mewaldt et al. (2005a).

Mewaldt et al. (2005a) note that the CME kinetic energies range from $\sim 3 \times 10^{31}$ ergs to $\sim 6 \times 10^{32}$ ergs, while the SEP kinetic energies range from $\sim 4 \times 10^{29}$ ergs to $\sim 7 \times 10^{31}$ ergs. Thus the spread in the SEP kinetic energies is about a factor of 10 greater. It is interesting that there is a group of eleven events where the SEP kinetic energy ranges from $\sim 3\%$ to $\sim 20\%$ of the CME kinetic energy. Thus, in spite of the sizable uncertainties, it appears that shock acceleration can often transform $\sim 10\%$ of the CME kinetic energy into energetic particles. There are also four events where the estimated efficiency is considerably lower (less than 1%). One of these events (February 20, 2002) is commonly regarded as an impulsive event (that also had a large CME) and a third (May 6, 1998) could also be impulsive (Von Rosenvinge et al., 2000). Of course, there are also CMEs for which no SEPs are observed at 1 AU. The events presented here were originally selected because the SEP intensities were sufficient for spectra to be measured, so they do not come from a representative sample of CMEs. Several considerations suggest that it was underestimated the SEP kinetic energies. It was not yet taken into account adiabatic energy losses, which may be as large as $\sim 50\%$ for particles accelerated near the Sun. In some events particle acceleration continues beyond 1 AU, and only particles that scatter back inside 1 AU are counted. In addition, CME kinetic energies derived from Gopalswamy et al., (2004) may be overestimated, since the tabulated maximum velocity was used. So, SEP/CME kinetic energy ratios may be even greater than indicated. According to Mewaldt et al. (2005a), it is interesting that galactic CR apparently extract a similar fraction of the kinetic energy from supernova shocks in order to sustain the energy density of CR in the Galaxy ($\sim 1 \text{ eV/cm}^3$) over the average CR lifetime of ~ 15 million years (Yanasak et al., 2001).

Mewaldt et al. (2005a) conclude that these preliminary comparisons indicate that particle acceleration at CME-driven shocks can be a surprisingly efficient process; particles frequently extract $\sim 10\%$ or more of the CME kinetic energy. It remains to be seen why some CME-driven shocks are more efficient accelerators than others. Further comparisons with Helios and Ulysses data can improve the corrections for longitude and latitude and they are also working to improve CME energy estimates. Finally, the combination of STEREO and 1-AU data will provide multipoint in-situ data and 3-point CME images that should greatly improve these comparisons and make it possible to correlate the acceleration efficiency with other SEP and CME characteristics.

Conclusion and Problems

In this monograph the main properties of space plasmas and main properties of primary CR as well as their interactions, propagation, non-linear effects, and acceleration were described. We show that space plasmas with frozen in magnetic fields is usually excited magneto-turbulent plasma with many channels of energy transformation and CR generation. The generation of CR in different objects of the Universe is the universal property of space plasmas owing mainly to energy transfer from macroscopic phenomena (kinetic energy of moving great ensembles of particles and energy of magnetic fields) to microscopic charged particles: protons, electrons, nuclei. These macroscopic phenomena are characterized by very high ‘effective temperature’, many orders higher than CR, so this process of energy transfer directly follows from the fundamental second law of thermodynamics.

In this energy transfer from space plasmas to CR the key role play interactions of fast charged particles with matter, magnetic fields and photons in moved, excited plasmas. It is a cause why we have considered in detail in Chapter 1 (Sections 1.3 – 1.15) different types of **CR interactions in space plasmas**, including nuclear and electromagnetic interactions of CR with nucleons and electrons of the space plasma matter (which determined the formation of elemental and isotopic composition of CR, and gamma ray generation as well as ionization energy losses), CR interactions with solid state matter (stars, planets, asteroids, meteorites, dust), interactions with electromagnetic radiation, with plasma waves and magnetic fields (moving and stationary), with magnetic traps. From other hand, all these types of CR interactions are in the basis of such fundamental CR phenomena as propagation, non-linear effects, and acceleration in space plasmas, considered in consequences Chapters.

The key problem of **CR propagation in space plasmas** we have considered in detail in Chapter 2 (Sections 2.1–2.46). The main results are as follows: the kinetic and diffusion descriptions of CR propagation in space plasmas have been intensively developed over the last about 50 years with many applications to different astrophysical objects. We have considered in detail following problems also: the balance of CR energy in multiple scatterings in expanding magnetic fields; the second order pitch-angle approximation for the CR Fokker-Planck kinetic equation; the anomalous CR diffusion; bulk speeds of CR resonant with parallel plasma waves; non-resonant pitch-angle scattering; CR cross-field diffusion in the presence of highly perturbed magnetic fields; dispersion relations for CR particle diffusive propagation; the dynamics of dissipation range fluctuations with application to CR propagation theory; a path integral solution of the stochastic differential equation of the Markov process for CR transport; velocity correlation functions and CR transport (compound diffusion); the influence of magnetic clouds on the CR propagation; non-diffusive CR particle

pulse transport; pitch angle diffusion of energetic particles by large amplitude MHD waves; particle diffusion across the magnetic field and the anomalous transport of magnetic field lines; CR transport in the fractal-like medium; CR propagation in large-scale anisotropic random and regular magnetic fields; CR perpendicular diffusion calculations on the basis of MHD transport models; on the role of drifts and perpendicular diffusion in CR propagation; drifts, perpendicular diffusion, and rigidity dependence of near-Earth latitudinal proton density gradients; CR drifts in dependence of Heliospheric current sheet tilt angle; CR drifts in a fluctuating magnetic fields; increased perpendicular diffusion and tilt angle dependence of CR electron propagation and modulation in the Heliosphere; rigidity dependence of the perpendicular diffusion coefficient and the Heliospheric modulation of CR electrons; comparison of 2D and 3D drift models for galactic CR propagation and modulation in the Heliosphere.

In Chapter 2 we consider also the first attempts to solve the inverse problems in CR propagation theory: inverse problems for solar CR propagation; the checking of solution for SEP inverse problem by comparison of predictions with observations; inverse problems for CR propagation in the Galaxy; inverse problem for high energy galactic CR propagation and modulation in the Heliosphere on the basis of NM data; inverse problem for small energy galactic CR propagation and modulation in the Heliosphere on the basis of satellite data.

The key problem of **CR nonlinear effects in space plasmas**, which is important for both CR propagation and acceleration, was discussed in Chapter 3 (Sections 3.1 - 3.16). We have considered in detail the effects of CR pressure and the effects of CR kinetic stream instability, the structure and evolution of CR-space plasma nonlinear systems and nonlinear Alfvén waves generated by CR streaming instability. As examples of applications we have considered: interplanetary CR modulation; the possible structure of the Heliosphere and expected CR nonlinear effects; radial CR pressure effects in the Heliosphere; the expected change of solar wind Mach number, accounting for the effects of radial CR pressure and re-charging with neutral interstellar atoms; the type of transition layer from supersonic to subsonic fluid of solar wind; the non-linear influence of pick-up ions, anomalous and galactic CR on the Heliosphere's termination shock structure; expected CR pressure effects in transverse directions in the Heliosphere; influence of CR kinetic stream instability effects in the Heliosphere on CR intensity and anisotropy distribution. We have also considered CR nonlinear effects in the dynamic Galaxy, the self-consistent problem for a dynamic halo in a rotating Galaxy, the transport of random magnetic fields by a galactic wind driven by CR, nonlinear Alfvén waves generated by CR streaming instability and their influence on CR propagation in the Galaxy.

The key problem of **CR acceleration in space plasmas** (as an universal phenomenon in the Universe), has been considered in detail in Chapter 4 (Sections 4.1 – 4.31). It was investigate in detail the Fermi mechanism of statistical

acceleration and its development with taking into account oblique collisions and variations of acceleration parameters (transport path and velocities of scatters) as particles gain energy. We also considered the formation of the particle energy spectrum during statistical acceleration and acceleration by scattering at small angles, the problem of injection energy and the portion of the accelerated particles in the statistical mechanism, statistical acceleration in the turbulent plasma confined within a constant magnetic field, and statistical acceleration of particles by electromagnetic radiation. We discussed in detail statistical acceleration of particles by the Alfvén mechanism of magnetic pumping, and estimated the accelerated particle flux from sources, and considered induction acceleration mechanisms and particle acceleration by a moving magnetic piston. Mechanisms have been considered in detail of particle acceleration by shock waves and other moving magneto-hydrodynamic discontinuities during single interaction, and acceleration of particles in the case of magnetic collapse and compression, the cumulative acceleration mechanism near the zero lines of a magnetic field, tearing instability in neutral sheet region, triggering mechanisms of solar flares, and particle acceleration in turbulent sheets accounting percolation processes. Particle acceleration in shear flows of space plasma and additional regular particle acceleration in space plasma with two types of scatters moving with different velocities have been also considered in detail.

Especially attention we pay when have considered very important mechanism of **shock wave diffusion (regular) acceleration** (Sections 4.21 – 4.31): elementary model of diffusive shock-wave acceleration, acceleration by the plane shock wave (diffusion approximation), particle injection into shock-wave acceleration, space distribution of accelerated particles, effects of finite width of shock wave front and finite dimension of shock wave, effect of energy losses during particle shock acceleration, the case of simultaneously regular and statistical acceleration, regular acceleration by spherical shock wave (standing wave in the solar or stellar wind, standing wave in the case of accretion, running shock wave), effects of finite-time shock acceleration, acceleration at quasi-parallel plane shocks (numerical simulations), accounting non-linear processes and bulk CR transport, acceleration by electrostatic and relativistic shock waves.

I think that any thoughtful reader according to his own scientific interest will be capable to formulate some actual Problems for any Chapter and Sections of this book, which needs to be solve and actually can be solved in correspondence with the current level of Science. The clear formulation of actual Problems is important not only for education (some Problems can be considered as a subject for Diploma Work in College or in University or as a subject for a Ph.D. Thesis), but also for acceleration of the progress in CR research and in connected branches of Science and Technology.

As example, let me formulate some problems, which are not so difficult and can be solved, from my opinion, in near future.

Problem 1. Let us assume that in the space plasma with frozen in homogeneous magnetic field $\mathbf{H}$ are propagate Alfvén wave, characterized with wave number $\mathbf{k}$ and velocity v_a . In this space move CR particle with mass m_a and charge Ze and momentum $\mathbf{p}$. Necessary to determine: the scattering angle and energy change of CR particle in dependence of $\mathbf{H}$, $\mathbf{k}$, v_a , m_a , Ze , and $\mathbf{p}$. How depend the final results from the angle between $\mathbf{H}$ and $\mathbf{k}$, between $\mathbf{H}$ and $\mathbf{p}$, between $\mathbf{p}$ and $\mathbf{k}$? What are dependences from other parameters of plasma and particle?

Problem 2. The same as in Problem 1, but there are spectrum of wave numbers $\mathbf{k}$. What will be in this case final result and average scattering angle in dependence of different parameters?

Problem 3. In Section 2.42 we consider the inverse problem for solar CR propagation in the frame of the simplest model of propagation: isotropic diffusion. Practically this model is valid only after some interval of time after injection into solar wind (it need few scatterings of solar CR, i.e. about 15-20 minutes). To use data from the beginning of GLE necessary to solve inverse problem for more complicated mode of propagation: for example, in the frame of the model of anisotropic diffusion. I think that in this case for solving the inverse problem will be possible to use data from about 5-10 minutes after the starting of GLE.

Problem 4. More complicated inverse problem for solar CR propagation is described by kinetic equation. In this case will be possible to use data directly from the starting of GLE. It is especially important for forecasting of radiation hazard with good accuracy basing on the several first minutes on-line one-minute data at the beginning of GLE observed on different ground based NM and satellites.

Problem 5. To solve the inverse problem for the interplanetary shock wave propagated from great solar flare or from CME on the basis of observation of pre-increase and pre-decrease effects in CR by neutron monitors and multidirectional muon telescopes. It is necessary to take into account that for galactic CR of energy from several GeV up to several tens GeV the interaction with shock wave is mostly single. The solving of this problem is important for exact forecasting for 10-20 hours ahead by on-line CR observations of expected effects in the Earth's magnetosphere and ionosphere (great geomagnetic storms and perturbations in radio wave propagation).

Problem 6. In Section 2.28 was considered the problem of the influence of magnetic clouds moving from the Sun on the CR propagation and modulation in the Heliosphere. This problem was considered as quasi-stationary. Necessary to make the second step and take into account: i) time-lag between magnetic clouds on different distances from the Sun and processes on the Sun caused these clouds; ii) time-evolution of magnetic clouds during their moving from the Sun; iii) for small energy galactic CR to take into account also diffusion and drifts time-lag of

energetic particles propagated from the boundary of Heliosphere (non-stationary problem).

Problem 7. In Sections 2.35 – 2.41 we consider the problem on the role of drifts and enhanced perpendicular diffusion in CR propagation and modulation. Up to present time in all described papers was considered quasi-stationary case. As the next step necessary to take into account the time-lag of processes in the Heliosphere relative to corresponding causing processes on the Sun in dependence from the radial distance. The second what is also necessary to take into account is the time lag caused by galactic CR particles penetrating into the interplanetary space from the boundary of Heliosphere (this is especially important for small energy particles measured on satellites and space-probes).

Problem 8. CR non-linear effects in the Heliosphere we considered in Sections 3.6 - 3.12. To try developing this consideration by taking into account that the solar wind velocity increased about two times from equator to the pole. It will lead to non-symmetrical Heliosphere. What will be CR modulation in dependence of particles direction arriving in this case?

Problem 9. Let us extend Problem 8 and take into account also the influence of the galactic magnetic field. What will be the form of Heliosphere? How it will be change during solar activity cycle and in periods of reversal of the Sun's general magnetic field? Will be any difference between odd and even cycles? What will be CR modulation?

Problem 10. What is the role in galactic CR modulation of the region with subsonic solar wind out of Heliospheric terminal shock up to bow shock and boundary with interstellar medium and interstellar magnetic field in dependence of particles energy? How it will be change during solar activity cycle? What will be change with distance and helio-latitude plasma velocity, magnetic fields, and level of turbulence out of Heliospheric terminal shock?

Problem 11. To extend Problem 10 and determine the expected CR intensity variation out of Heliospheric terminal shock with solar cycle, with distance from the terminal shock, and helio-latitude. How these results depend from CR particle rigidity

Problem 12. What is the role of CR nonlinear effects in solar wind plasma in formatting of CR anisotropy in dependence of particle rigidity, distance from the Sun, and helio-latitude; try to formulate system of equations for self-consistent problem taking into account galactic and anomaly CR pressure, kinetic stream instabilities, exchange of solar wind ions with neutral atoms from interstellar medium. What is expected dependence of CR anisotropy from the distance to the Sun, from helio-latitude, and particle rigidity? What is expected change of these results from the level of solar activity, for even and odd solar cycles?

Problem 13. On the basis of present day knowledge to develop mechanism of particle acceleration in variable magnetic fields of particles from background plasma (Alfvén mechanism of magnetic pumping, see Section 4.11). What is expected energy spectra, maximal energy, chemical composition of accelerated particles? What is the effectiveness of this acceleration mechanism?

Problem 14. The same as in Problem 13, but for mechanism of particle acceleration in the magnetosphere of fast rotated star (see Section 4.13).

Problem 15. The same as in Problem 13, but for particles acceleration by shock waves and other moving magneto-hydrodynamic discontinuities during single interaction (see Section 4.15).

Problem 16. The same as in Problem 13, but for particles acceleration in case of magnetic collapse and compression (see Section 4.16).

Problem 17. To develop the mechanism of acceleration particles from background plasma in the neutral sheet with account tearing instability, possible triggering mechanisms and percolation (see Sections 4.17 and 4.18). To determine expected energy spectrum, maximal energy, and chemical composition of accelerated particles. What part of energy of magnetic field dissipation will go to the accelerated particles (what is the expected effectiveness of the acceleration mechanism)?

Problem 18. The same as in Problem 13, but for particle acceleration in shear flows of space plasma (see Section 4.19).

Problem 19. The same as in Problem 13, but for particle acceleration in space plasma with two or more types of scatters moving with different velocities (see Section 4.20).

Problem 20. The same as in Problem 13, but for shock wave diffusion (regular) acceleration of charged particles in different cases (see Sections 4.21 – 4.30).

Problem 21. In Section 4.31 were considered results on experimental determination of the particle acceleration effectiveness; it was found that the effectiveness varies from case to case in broad interval from 0.1% to several tens percents. What is the cause of this big variation? What main parameters determined the particle acceleration effectiveness?

Problem 22. Try to consider the problems of particle acceleration as self-consistent problems (with taking into account nonlinear effects of back influence of accelerated particles on the background space plasma; see Chapter 3).

REFERENCES

References to Monographs and Books

- Abramowitz M. and I. Stegun, *Handbook of Mathematical Functions*, Dover, New York, M1965.
- Akasofu S.I. and S. Chapman, *Solar-Terrestrial Physics*, Clarendon Press, Oxford, M1972.
- Akhiezer A.I. and V.B. Berestetsky, *Quantum Electrodynamics*, Physmatgiz, Moscow, M1959 (in Russian).
- Akhiezer I.A., R.V. Polovin, A.G. Sitenko, and K.N. Stepanov, *Plasma Electrodynamics*, Nauka, Moscow, M1974 (in Russian).
- Alania M.V. and L.I. Dorman, *Cosmic Ray Distribution in the Interplanetary Space*. Metsniereba, Tbilisi, M1981 (in Russian).
- Alania M.V., L.I. Dorman, R.G. Aslamazashvili, R.T. Gushchina, and T.V. Dzhapiashvili, *Galactic Cosmic Ray Modulation by Solar Wind*. Metsniereba, Tbilisi, M1987 (in Russian).
- Aleksandrov A.F., L.S. Bogdankevich, and A.A. Rukhadze, *Principles of Plasma Electrodynamics*, Visshaya Shkola, Moscow, M1978 (in Russian).
- Alfvén H., *Cosmical Electrodynamics*, Oxford University Press, Oxford, M1950.
- Alfvén H. and C.-G. Fälthammer, *Cosmical Electrodynamics*, Oxford University Press, Oxford, M1962.
- Alpert Ya.L., A.V. Gurevich, L.P. Pitaevsky, *Artificial Earth Satellites in Rarified Plasma*, NAUKA, Moscow, M1964 (in Russian).
- Arons J., C. Max, and C. McKee (eds.), *Particle Acceleration Mechanisms in Astrophysics*, AIP Press, New York, M1979.
- Artsymovich L.A. and R.Z. Sagdeev, *Plasma Physics for Physicists*, Atomizdat, Moscow, M1979 (in Russian).
- Artsymovich L.A., *Controllable Thermonuclear Reactions*, Physmatgiz, Moscow, M1961 (in Russian).
- Baranov V.B. and K.V. Krasnobaev, *Hydromagnetic Theory of Cosmic Plasma*, Nauka, Moscow, M1977 (in Russian).
- Bateman H. and A. Erdelyi, *Higher Transcendental Functions*, McGraw-Hill Book, Co. Inc. N.Y.-Toronto-London, M1953 (In Russian: Inostrannaja Literatura, Moscow, Vol. 1 – M1965, Vol. 2 – M1966).
- Bender C.M. and S.A. Orszag, *Advanced Mathematical Methods for Scientists and Engineers*, McGraw-Hill, New York, M1978.
- Benz A.O., *Plasma Astrophysics: Kinetic Processes in Solar and Stellar Coronae*, Kluwer Academic Publishers, Dordrecht, M1993.
- Berezhko E.G., V.K. Elshin, G.F. Krymsky, and S.I. Petukhov, *Cosmic Ray Generation by Shock Waves*, Nauka, Novosibirsk, M1988 (in Russian).
- Berezin I.S. and I.P. Zhidkov, *Methods of Computations*, Moscow, Physmatgiz, M1959 (in Russian).
- Berezinsky V.S., S.V. Bulanov, V.L. Ginzburg, V.A. Dogiel, and V.S. Ptuskin (ed. V.L. Ginzburg), *Astrophysics of Cosmic Rays*, Physmatgiz, Moscow, M1990 (in Russian). English translation: North Holland, Amsterdam, M1990.

- Bleeker J.A.M., J. Geiss, and M.C.E. Huber (eds.) *The Century of Space Science*, Kluwer Academic Publishers, M2002.
- Boguslavsky S.A., *Paths of Electrons in the Electromagnetic Fields*, ONTI, Moscow-Leningrad, M1929 (in Russian).
- Bohr H., *Passage of Atomic Particles Through Matter*. Inostr. Literatura, Moscow, M1950 (in Russian).
- Bonch-Bruevich V.L. and S.V. Tyablikov, *Method of Green's Functions in Statistical Mechanics*, Fizmatgiz, Moscow, M1961 (in Russian).
- Buneman O., *Computer Space Plasma Physics: Simulation Techniques and Software*, Terra Scientific, New York, M1993.
- Burlaga L.F., *Interplanetary Magnetohydrodynamics*, Oxford University Press, M1995.
- Case K. and P. Zweifel, *Linear theory of transport*, Addison-Wesley Publ. Co., Massachusetts, M1967.
- Chandrasekhar S., *Plasma Physics*, Univ. Chicago Press, Chicago, M1960.
- Daglis I.A. (ed.), *Space Storms and Space Weather Hazards*, Kluwer Ac. Publ., Netherlands, M2001.
- Dorman Irena V., *Cosmic Rays (Historical Outlook)*, Nauka, Moscow, M1981 (in Russian).
- Dorman Irena V., *Cosmic Rays, Accelerators, and New Particles*, Nauka, Moscow, M1989 (in Russian).
- Dorman L.I., *Cosmic Ray Variations*. Gostekhteorizdat, Moscow, M1957 (in Russian). English translation: US Department of Defense, Ohio Air-Force Base, M1958.
- Dorman L.I., *Geophysical and Astrophysical Aspects of Cosmic Rays*. North-Holland, Amsterdam, in series "Progress in Physics of Cosmic Ray and Elementary Particles", ed. J.G. Wilson and S.A. Wouthuysen, Vol. 7, pp. 1-324, M1963a.
- Dorman L.I., *Cosmic Ray Variations and Space Research*. Nauka, Moscow, M1963b (in Russian).
- Dorman L.I., *Acceleration Processes in Space*, VINITI, Moscow, M1972b (in Russian).
- Dorman L.I., *Cosmic Rays: Variations and Space Exploration*. North-Holland, Amsterdam, M1974.
- Dorman L.I., *Experimental and Theoretical Principles of Cosmic Ray Astrophysics*. Physmatgiz, Moscow, M1975a (in Russian).
- Dorman L.I., *Variations of Galactic Cosmic Rays*. Moscow State University Press, Moscow, M1975b (in Russian).
- Dorman L.I., *Cosmic Rays of Solar Origin*, VINITI, Moscow (in series "Summary of Science", Space Investigations, Vol.12), M1978 (in Russian).
- Dorman L.I., *Cosmic Rays in the Earth's Atmosphere and Underground*, Kluwer Academic Publishers, Dordrecht/Boston/London, M2004.
- Dorman L.I. and L.I. Miroshnichenko, *Solar Cosmic Rays*, Physmatgiz, Moscow, M1968 (in Russian). English translation: NASA, Washington, DC, M1976.
- Dorman L.I., V.S. Smirnov and M.I. Tyasto, *Cosmic Rays in the Earth's Magnetic Field*. Physmatgiz, Moscow, M1971 (in Russian).
- Feder J., *Fractals*, Plenum Press, New York, M1988.
- Fisk L.A., J.R. Jokipii, G.M. Simnett, R. von Steiger, and K.-P. Wenzel (eds.), *Cosmic Rays in the Heliosphere*, Kluwer Academic Publishers, Dordrecht, M1998.
- Ginzburg V.L., *Propagation of Electromagnetic Waves in Plasma*, Fysmatgiz, Moscow, M1967 (in Russian).

- Ginzburg V.L. and S.I. Syrovatsky, *The Origin of Cosmic Rays*, Publ. Acad. Sci. USSR, Moscow, M1963 (in Russian). In English: Pergamon Press, M1964.
- Hayakawa Satio, *Cosmic Ray Physics: Nuclear and Astrophysical Aspects*, John Wiley & Sons, New York/London/Sydney/Toronto, M1969.
- Hundhausen R.I., *Coronal Expansion and Solar Wind*, Springer-Verlag, Berlin-Heidelberg-New York, M1972.
- Kadomtsev B.B., A.G. Sitenko, M.S. Rabinovich, V.P. Silin, and I.P. Yakimenko (eds), *The problems of Plasma Theory*, Naukova Dumka, Kiev, M1976 (in Russian).
- Kallenrode M.-B. *Space Physics: An Introduction to Plasmas and Particles in the Heliosphere and Magnetospheres*, Springer-Verlag, Berlin-New York, M1998.
- Kaplan S.A., S.B. Pikelner, and V.N. Tsytovich, *Plasma Physics of the Solar Atmosphere*, NAUKA, Moscow, M1977 (in Russian).
- Kaplan S.A. and V.N. Tsytovich, *Plasma Astrophysics*, Pergamon Press, M1973.
- Khristiansen G.B., *Cosmic Rays of Superhigh Energies*, Moscow State University Press, Moscow, M1974 (in Russian).
- Klyatskin V.I., *Statistical Description of Dynamical Systems with Fluctuating Parameters*, Nauka, Moscow, M1975 (in Russian).
- Komarov I.V., L.I. Ponomarev, and S.Yu. Slavyanov, *Spherical and Colon Spherical Functions*, Nauka, Moscow, M1976 (in Russian).
- Krymsky G.F., *Cosmic Ray Modulation in the Interplanetary Space*, Nauka, Moscow, M1969 (in Russian).
- Kuzmin A.I., *Variations of Cosmic Rays and Solar Activity*, Nauka, Moscow, M1968 (in Russian).
- Kuzmin A.I., *Variations of High Energy Cosmic Rays*, Nauka, Moscow, M1964 (in Russian).
- Landau L.D. and E.M. Lifshitz, *Electrodynamics of Continue Matters*, Gostekhizdat, Moscow, M1957 (in Russian).
- Langouche F., D. Roekaerts, and E. Tirapegui, *Functional Integration and Semi-Classical Expansions*, Reidel, Boston, M1982.
- Lans D.H., *Numerical Methods for Computers*, Foreign Literature Press (IL), Moscow, M1962 (in Russian).
- Lifshitz E.M. and L.P. Pitaevsky, *Physical Kinetics*, Physmatgiz, Moscow, M1979 (in Russian).
- Lincoln J.V. (ed.), *The Proton Flare Events of August 1972*, Report UAG, No. 28, M1973.
- Longmair K., *Plasma Physics*, Atomizdat, Moscow, M1966 (in Russian).
- Melrose D.B., *Plasma Astrophysics: Nonthermal Processes in Diffuse Magnetized Plasmas*, Volume 1, Gordon and Breach, New York, M1980a.
- Melrose D.B., *Plasma Astrophysics: Nonthermal Processes in Diffuse Magnetized Plasmas*, Volume 2, Gordon and Breach, New York, M1980b.
- Mikhailovsky A.B., *Theory of Plasma Instabilities*, Atomizdat, Moscow, M1977 (in Russian).
- Miroshnichenko L.I., *Solar Cosmic Rays*, Kluwer Ac. Publishers, Dordrecht/Boston/London, M2001.
- Miroshnichenko L.I., *Radiation Hazard in Space*, Kluwer Ac. Publishers, Dordrecht/Boston/London, M2003.
- Miroshnichenko L.I. and V.M. Petrov, *Dynamics of Radiation Conditions in Space*, Energoatomizdat, Moscow, M1985 (in Russian).

- Mogilevsky E.I., *Fractals on the Sun*, Moscow, M2004 (in Russian).
- Monin A.S. and A.M. Yaglom, *Statistical Hydromechanics*, Vol. 1, Nauka, Moscow, M1965 (in Russian).
- Monin A.S. and A.M. Yaglom, *Statistical Hydromechanics*, Vol. 2, Nauka, Moscow, M1967 (in Russian).
- Morse P.M. and H. Feshbach, *Methods of Theoretical Physics*, Vol. 1, 2, McGraw-Hill Book Co Inc., N.Y.-Toronto-London, M1953. In Russian: Inostrannaja Literatura, Moscow, M1958.
- Mysovskikh I.P., *Lectures on Computation Methods*, Moscow, Fizmatgiz, M1962 (in Russian).
- Parker E.N., *Interplanetary Dynamical Processes*, John Wiley and Sons, New York-London, M1963. In Russian (ed. L.I. Dorman; transl. L.I. Miroshnichenko): Inostrannaja Literatura, Moscow, M1965.
- Pikelner S.B., *Principles of Cosmic Electrodynamics*, Physmatgiz, Moscow, M1961 (1-st Ed.); M1966 (2-nd Ed.), in Russian.
- Post R.F., *High-Temperature Plasma Research and Controlled Fusion*, Ann. Rev. Inc., Palo Alto, California, M1959.
- Press W.H., S.A. Teulolsky, W.T. Wetterling, and B.P. Flannery, *Numerical Recipes in C, The Art of Scientific Computing*, Cambridge University Press, M1992
- Priest, E.R. (ed.), *Solar Flare Magnetohydrodynamics*, Gordon & Breach, New York, M1981.
- Priest E.R., *Solar Magnetohydrodynamics*, Reidel Publishing Company, Dordrecht, M1982.
- Priest E.R. and T. Forbes, *Magnetic Reconnection: MHD Theory and Applications*, Cambridge University Press, Cambridge, M2000.
- Ramaty R., N. Mandzhavidze, and X.-M. Hua (eds.), *High Energy Solar Physics*, AIP Press, New York, M1996.
- Rosen S. (ed.), *Selected Papers on Cosmic Ray Origin Theories*, New York, M1969.
- Rossi B., *Cosmic Rays*, Atomizdat, Moscow, M1966 (in Russian).
- Rytov S.M., Yu.A. Kravtsov, and V.I. Tatarsky, *Introduction to Statistical Radiophysics*, Nauka, Moscow, M1977 (in Russian).
- Ryzhik I.M. and S.M. Gradstein, *Tables of Integrals, Sums, Series and Products*, Nauka, Moscow, M1971 (in Russian).
- Sagdeev R.Z., D.A. Usikov, and G.M. Zaslavsky, *Nonlinear Physics*, Harwood Academic Publishers, London, M1988.
- Samko S.G., A.A. Kilbas, and O.I. Marichev, *Fractional Integrals and Derivations and some Applications*, Nauka, Minsk, M1987 (in Russian).
- Schlickeiser R., *Cosmic Ray Astrophysics*, Springer, Berlin, M2001.
- Sekido Y. and H Elliot (Eds.), *Early History of Cosmic Ray Studies*, Dordrecht, D. Reidel Publ. Co., M1985.
- Sergeev V.A. and N.A. Tsyganenko, *The Earth's Magnetosphere*, Nauka, Moscow, M1980.
- Shea M.A., D.F. Smart, and H. Carmichael, *Summary of Cutoff Rigidities Calculated with the International Geomagnetic Reference Field for Various Epochs*, Rep. AFGL-TR-76-0115, Air Force Geophys. Lab., Bedford, Massachusetts, M1976.
- Shkarofsky I., T. Johnston, and M. Bachynsky, *The Particle Kinetics of Plasmas*, Addison-Wesley Publ. Co., Massachusetts, M1966.

- Sokolov A.A., I.M. Ternov, V.Ch. Zhukovski, and A.V. Borisov, *Quantum Electrodynamics*, MIR, Moscow, M1989 (in Russian).
- Spitzer L., *The Physics of Fully Ionized Gasses*, Interscience, New York, M1956.
- Stecker F.W., *Cosmic Gamma Rays*, Mono Book Co, Baltimore, M1971.
- Stix M., *The Sun: an Introduction* (second edition), Springer-Verlag, M2002.
- Störmer C., *The Polar Aurora*, Oxford University Press, London and New York, M1955.
- Sturrock P.A., *Plasma Physics*, Cambridge University Press, Cambridge, M1994.
- Tatarsky V.I., *Theory of Fluctuation Events during Wave Propagation in Turbulent Atmosphere*, Acad. of Sci. USSR, Moscow, M1959 (in Russian).
- Tatarsky V.I., *Wave Propagation in Turbulent Atmosphere*, Nauka, Moscow, M1967 (in Russian).
- Titchmarsh E.C., *Eigenfunction Expansion of Differential Equations*, Vol. 1, 2, Clarendon Press, Oxford, M1958.
- Toptygin I.N., *Cosmic Rays in the Interplanetary Magnetic Fields*, Nauka, Moscow, M1983 (in Russian). English Translation: Reidel, Dordrecht, M1985.
- Tsytoich V.N., *Theory of Turbulent Plasma*, Atomizdat, Moscow, M1971 (in Russian).
- Uchaikin V.V. and V.M. Zolotarev, *Chance and Stability*, VSP, Utrecht, M1999.
- Walt M., *Introduction to Geomagnetically Trapped Radiation*, Cambridge Press, M1994
- Watson G.N., *A Treatise on the Theory of Bessel Functions*, Inostr. Literatura, Moscow, M1949 (in Russian).
- Zank G.P. and T.K. Gaisser (eds.), *Particle Acceleration in Cosmic Plasmas*, AIP Press, New York, M1992.
- Zeldovich Ya.B. and Yu.P. Raizer, *The Physics of Shock Waves and High Temperature Hydro-Dynamical Phenomena*, Nauka, Moscow, M1966 (in Russian).
- Zolotarev V.M., *One-Dimensional Stable Distribution*, Nauka, Moscow, M1983. Zombeck, M., *Handbook of Space Astronomy and Astrophysics*, Cambridge University Press, M1982.
- Zusmanovich A.G., *Galactic Cosmic Rays in the Interplanetary Space*, Nauka, Alma-Ata, M1986 (in Russian).

References for Chapter 1

- Aharonian F.A., B.L. Kanevsky, and V.A. Sahakian "On some peculiarities of EAS initiated by gamma rays of extremely high energies", *J. Phys. G, Nuclear and Particle Physics*, **17**, Issue 12, 1909-1924 (1991).
- Alania M.V., M.A. Alexidze, P.V. Dghandghagava, and L.I. Dorman "Galactic cosmic ray modulation at solar wind heliolatitude asymmetry with taking into account isotropy and anisotropy diffusion", *Geomagnetism and Aeronomy*, **17**, No. 2, 190-197 (1977).
- Badhwar G.D. and S.A. Stephens "Secondary positrons and electrons in the cosmic radiation", *Proc. 15th Intern. Cosmic Ray Conf.*, Plovdiv, **1**, 398-403 (1977).
- Baring M.G. "Synchrotron pair cascades in strong magnetic fields", *Astronomy and Astrophys.*, **225**, No. 1, 260-276 (1989).
- Bednarek W. "Interaction of EHE gamma-rays with the magnetic field of the Sun", *Proc. 26th Intern. Cosmic Ray Conf.*, Salt Lake City, **4**, 168-171 (1999).

- Belov A.V. and L.I. Dorman "The features of cosmic ray diffusion in radially divergent flux of magnetic inhomogeneities", *Geomagnetism and Aeronomy*, **12**, No.1, 113-114 (1972).
- Belov et al., 1988: Belov A.V., R.T. Gushchina, L.I. Dorman, I.V. Sirotina "Rigidity dependence of cosmic ray modulation parameter in the different epoch of solar activity cycle", *Izvestia Ac. of Sci. of USSR, Ser. Phys.*, **52**, No. 12, 2334-2337 (1988).
- Belov et al., 1990: Belov A.V., R.T. Gushchina, L.I. Dorman, I.V. Sirotina "Rigidity spectrum of cosmic ray modulation", *Proc. 21th Intern. Cosmic Ray Conf., Adelaide*, **6**, 52-55 (1990).
- Berezinsky V.S. "Extraterrestrial neutrinos and high energy neutrino astrophysics", *Proc. 15th Intern. Cosmic Ray Conf., Plovdiv*, **6**, 231-236 (1977).
- Berezinsky V.S. and G.T. Zatsepin "Dumand as a neutrino eye watching violent remote cosmological epochs", *Proc. 15th Intern. Cosmic Ray Conf., Plovdiv*, **6**, 248-252 (1977).
- Berezinsky V.S., S.I. Grigorieva, and G.T. Zatsepin "Metagalactic protons of super-high energy", *Izvestia Ac. of Science USSR, Ser. Phys.*, **38**, No.9, 1791-1795 (1974).
- Bhattacharjee P., Q. Shafi, and F.W. Stecker "TeV and Superheavy Particles from Supersymmetric Topological Defects, the Extragalactic gamma-ray Background, and the Highest Energy Cosmic Rays", *Phys. Rev. Lett.*, **80**, No. 17, 3698-3701 (1998).
- Bird D.J., S.C. Corbato, H.Y. Dai, B.R. Dawson, J.W. Elbert, B.L. Emerson, K.D. Green, M.A. Huang, D.B. Kieda, M. Luo, S. Ko, C.G. Larsen, E.C. Loh, M.H. Salamon, J.D. Smith, P. Sokolsky, P. Sommers, J.K.K. Tang, and S.B. Thomas "The cosmic-ray energy spectrum observed by the Fly's Eye", *Astrophys. J., Part 1*, **424**, No. 1, 491-502 (1994).
- Bishara A.A. and L.I. Dorman "Spectrum of 11-year cosmic ray variation in the high energy region and great-scale structure of solar wind", *Geomagnetism & Aeronomy*, **13**, No. 5, 782-787 (1973a).
- Bishara A.A. and L.I. Dorman "Estimation of cosmic ray variation spectrum in the high energy region by underground observations", *Izvestia Academy of Sciences USSR, Series Phys.*, **37**, No. 6, 1293-1297 (1973b).
- Bishara A.A. and L.I. Dorman "High-energy spectra of the 11-year, 27-day and solar-diurnal cosmic ray variations and Forbush-decreases according to the underground observation data", *Proc. 13th Intern. Cosmic Ray Conf., Denver*, **2**, 1218-1224 (1973c).
- Bishara A.A. and L.I. Dorman "Spectrum of cosmic ray Forbush-decreases in the high energy region", *Geomagnetism and Aeronomy*, **14**, No. 2, 357-360 (1974a).
- Bishara A.A. and L.I. Dorman "Spectrum of cosmic ray solar-diurnal variation in the high energy region", *Geomagnetism and Aeronomy*, **14**, No. 4, 573-579 (1974b).
- Bishara A.A. and L.I. Dorman "Some problems of cosmic ray variation investigation by underground observation data", *Cosmic Rays (Moscow, Nauka)*, **15**, 198-202 (1975).
- Blumenthal G.R. and R.J. Gould "Bremsstrahlung, Synchrotron Radiation, and Compton Scattering of High-Energy Electrons Traversing Dilute Gases", *Rev. Mod. Phys.*, **42**, No. 2, 237-271 (1970).
- Boratav M. (for the Auger Collaboration) "The Pierre Auger Observatory Project: An Overview", *Proc. 25th Intern. Cosmic Ray Conf., Durban*, **5**, 205-208 (1997).

- Breizman B.N. "Collective interactions of relativistic electron fluxes with plasma", In *Problems of Plasma Theory* (ed. B.B. Kadomtsev), Energatomizdat, Moscow, Vol. 15, 55-145 (1987).
- Burger R.A. and M.S. Potgieter "The effect of large heliospheric current sheet tilt angles in numerical modulation models: a theoretical assessment", *Proc. of 26th Intern. Cosmic Ray Conference*, Salt Lake City, 7, 13-16 (1999).
- Cechini S., I. Guidi, and N.J. Martinic, "Solar modulation of galactic protons, He nuclei and electrons during 1965-1968", *Nuovo Gimento*, **22B**, Ser. 2, No. 2, 237-248 (1974).
- Cesarsky C.J., J.A. Paul, and P.G. Shukla "Bremsstrahlung gamma radiation and interstellar electron spectrum in the local interstellar medium", *Astroph. Sp. Sci.*, **59**, No. 1, 73-83 (1978).
- Charugin V.M. and Yu.P. Ochelkov "Omnidirectional induced Compton scattering by relativistic electrons", *Astrophysics & Space Science*, **26**, No. 2, 337-344. (1974).
- Cocconi G. "The origin of the cosmic radiation", *Astrophys. J.*, Suppl. Ser., **4**, 417-422 (1960).
- Colgate S.A. "The origin of very high energy cosmic rays", *Proc. 14th Intern. Cosmic Ray Conf.*, Munchen, **2**, 723-727 (1975a).
- Colgate S.A. "Supernova and the origin of cosmic rays. I", In: *Origin of Cosmic Rays* (Proc. of the Advanced Study Institute, eds. J.L. Osborne and A.W. Wolfendale), Dordrecht, Netherlands, 425-445 (1975b).
- Colgate S.A. "Supernovae and the origin of cosmic rays. II. A model of cosmic ray production in supernovae", In: *Origin of Cosmic Rays* (Proc. of the Advanced Study Institute, eds. J.L. Osborne and A.W. Wolfendale), Dordrecht, Netherlands, 447-466 (1975c).
- Cowsik R., Y. Pal, S.N. Tandon, and R.P. Verma "3°K blackbody radiation and leakage lifetime of cosmic-ray electrons", *Phys. Rev. Lett.*, **17**, No. 26, 1298-1300 (1966).
- Cummings A.C., E.C. Stone, and R.E. Vogt "Analytic approximations in the study of the solar modulation of electrons", *Proc. 13th Intern. Cosmic Ray Conf.*, Denver, **2**, 765-770 (1973).
- Daniel R.R. and Stephens S.A. "Cosmic electrons above 10 Gev and the universal blackbody radiation at 3° K", *Phys. Rev. Letters*, **17**, No. 17, 935-939 (1966).
- Daniel R.R. and S.A. Stephens "Cosmic-ray-produced electrons and gamma rays in the atmosphere", *Revs. Geophys. and Space Phys.*, **12**, No. 2, 233-258 (1974).
- Davies R.D. "Propagation of cosmic rays in the Galaxy", *Observatory*, **94**, 112-113 (1974).
- Dermer C.D. "Secondary production of neutral pi-mesons and the diffuse galactic gamma radiation", *Astronomy and Astrophysics*, **157**, No. 2, 223-229 (1986a).
- Dermer C.D. "Binary collision rates of relativistic thermal plasmas. 2. Spectra", *Astrophys. J.*, Part 1, **307**, No. 1, 47-59 (1986b).
- Dorman I.V. and L.I. Dorman "Modulation of Protons and α -Particles of Small Rigidities and the Cosmic-Ray Spectrum in the Galaxy", *Geomagnetism and Aeronomy*, **5**, No. 4, 516-520 (1965), (English edition published by American Geophysical Union).
- Dorman I.V. and L.I. Dorman "Solar wind properties obtained from the study of the 11-year cosmic ray cycle", *J. Geophys. Res.*, **72**, No. 5, 1513-1520 (1967a).
- Dorman I.V. and L.I. Dorman "Propagation of energetic particles through interplanetary space according to the data of 11-year cosmic ray variations", *J. Atmosph. and Terr. Phys.*, **29**, No. 4, 429-449 (1967b).

- Dorman I.V. and L.I. Dorman "On the primary cosmic ray energetic spectrum in the very small energy region out of the solar system", *Izvestia Ac. Sci. USSR, Ser. Phys.*, **31**, No.8, 1239-1247 (1967c).
- Dorman I.V. and L.I. Dorman "Hysteresis phenomena in cosmic rays, properties of solar wind and energetic spectrum of different nuclei in the Galaxy", *Proc. 5th All-Union Winter School on Cosmophysics*, Apatity, 183-196 (1968).
- Dorman L.I. "On the theory of cosmic ray modulation by solar wind", *Proc. 6th Intern. CosmicRay Conf.*, Moscow, **4**, 328-334 (1959).
- Dorman L.I. "Subcosmic ray problems: modulation, spectrum, nuclear interactions, acceleration, role in the space", *Proc. 6th Leningrad Intern. Seminar on Cosmophysics*, Leningrad, 169-176 (1974).
- Dorman L.I. "The nature of the observed cosmic ray spectrum, I. Classification and the features of particle propagation in space", *Proc. 15th Intern. Cosmic Ray Conf., Plovdiv*, **1**, 405-410 (1977a).
- Dorman L.I. "The nature of the observed cosmic ray spectrum, II. Intervals 1 and 2 ($3 \times 10^{11} - 10^{20}$ eV). ", *Proc. 15th Intern. Cosmic Ray Conf., Plovdiv*, **1**, 411-415 (1977b).
- Dorman L.I. "The nature of the observed cosmic ray spectrum, III. Intervals 3-5 ($10^6 - 3 \times 10^{11}$ eV). ", *Proc. 15th Intern. Cosmic Ray Conf., Plovdiv*, **1**, 416-420 (1977c).
- Dorman L.I. "Subcosmic rays and their role in the space, I. Spectrum and chemical composition; possible origin", *Proc. 15th Intern. Cosmic Ray Conf., Plovdiv*, **2**, 364-368 (1977d).
- Dorman L.I. "Subcosmic rays and their role in the space, II. Three possible sources and temporal variations; role in the space", *Proc. 15th Intern. Cosmic Ray Conf., Plovdiv*, **2**, 369-374 (1977e).
- Dorman L.I. "On the possibility of concordance of cosmic ray measurements on satellite "Proton" with data on sidereal-diurnal variation and with various models of cosmic ray origin", *Space Research* (Moscow), Vol. 7, No. 3, pp. 402-414 (1969a).
- Dorman L.I. "Cosmic ray variations", *Izvestia Academy of Sci. of USSR, Series Phys.*, **33**, No. 11, 1832-1857 (1969b).
- Dorman L.I. "Cosmic ray nonlinear effects in space plasma, 2. Dynamic Heliosphere", In *Currents in High Energy Astrophysics*, ed. M.M. Shapiro, R. Silberberg, and J.P. Wefel, Kluwer Academic Publishers., Dordrecht/Boston /London, NATO ASI Serie, Vol. **458**, 193-208 (1995).
- Dorman L.I. "Cosmic ray nonlinear processes in gamma-ray sources", *Astronomy and Astrophysics*, Suppl. Ser., **120**, No. 4, 427-435 (1996).
- Dorman L.I. "Angle distribution and time variation of gamma ray flux from solar and stellar winds, 1. Generation by flare energetic particles", *Proc. 4th Compton Symposium* (eds. C.D. Dermer, M.S. Strickman, and J.D. Kurfess), Williamsburg, 1178-1182 (1997).
- Dorman Lev I. "Variable gamma ray sources, 1. Interactions of flare energetic particles with solar and stellar winds". In *Astrophysical Sources of High Energy Particles and Radiation*, ed. M.M. Shapiro et al., Kluwer Academic Publishers, Netherlands, 219-230 (2001a).
- Dorman Lev I. "Variable gamma ray sources, 2. Interactions of galactic cosmic rays with solar and stellar winds". In *Astrophysical Sources of High Energy Particles and*

- Radiation*, ed. M.M. Shapiro et al., Kluwer Academic Publishers, Netherlands, 231-243 (2001b).
- Dorman Lev I. "Cosmic ray long-term variation: even-odd cycle effect, role of drifts, and the onset of cycle 23", *Adv. Space Res.*, **27**, No. 3, 601-606 (2001c).
- Dorman Lev I. "Expected Gamma-Ray Fluxes from Interactions of Flare Energetic Particles with Solar Wind Matter", In *Gamma-Ray Astrophysics*, ed. S. Ritz et al., AIP Conf. Proc., Vol. **587**, 628-632 (2001d).
- Dorman Lev I. "Interactions of Flare Energetic Particles with Stellar Wind Matter: Expected Gamma-Ray Fluxes from Local Stars", In *Gamma-Ray Astrophysics*, ed. S. Ritz et al., AIP Conf. Proc., Vol. **587**, 633-637 (2001e).
- Dorman L.I. "Possible Using of Gamma Ray Measurements for Monitoring and Prediction of Radiation Hazard", In *Proc. 15th ESA Symposium on European Rocket and Balloon Programmes and Related Research*, Biarritz 28-31 May 2001, 457-462 (2001f).
- Dorman et al., 2001: Dorman L.I., I.V. Dorman, N. Iucci, M. Parisi, and G. Villoresi "Hysteresis between solar activity and cosmic rays during cycle 22: the role of drifts, and the modulation region", *Adv. Space Res.*, **27**, No. 3, 589-594 (2001).
- Dorman et al., 1967: Dorman L.I., O.I. Inozemtseva, S.F. Ilgach, and E.A. Masaryuk "On the sidereal-diurnal cosmic ray variation", *Izvestia Academy of Sciences of USSR, Series Phys.*, **31**, No. 8, 1357-1360 (1967).
- Dorman et al., 1969: Dorman L.I., O.I. Inozemtseva, S.F. Ilgach, and E.A. Masaryuk "Distortion in interplanetary space of cosmic ray anisotropy of galactic origin", *Cosmic Rays* (NAUKA, Moscow), **11**, 40-45 (1969).
- Dorman et al., 2001b: Dorman L.I., N. Iucci, and G. Villoresi "Time lag between cosmic rays and solar activity; solar minimum of 1994-1996 and residual modulation", *Adv. Space Res.*, **27**, No. 3, 595-600 (2001b).
- Dorman L.I. and Yu.G. Nosov "On the theory of charged-particle scattering by cosmic magnetic fields of the simplest types", *Geomagnetism and Aeronomy*, **5**, No. 1, 155-159 (1965).
- Dorman et al., 1990: Dorman L.I., V.S. Ptuskin, and V.N. Zirakashvili "Outer Heliosphere: pulsations, cosmic rays and stream kinetic instability", In *Physics of the Outer Heliosphere*, ed. S. Grzedzielski and D.E. Page, Pergamon Press, 205-209 (1990).
- Dorman L.I. and A.V. Sergeev "Cosmic ray transport path as a function of the type and the scale and field-intensity spectra of inhomogeneities", *Proc. 14th Intern. Cosmic Ray Conf.*, Munchen, **3**, 928 (1975).
- Dorman L.I. and A.V. Sergeev "Cosmic ray transport path in dependence of inhomogeneities types and its spectrums on scale and field intensity", *Izvestia Academy of Sciences of USSR, Series Phys.*, **40**, No. 3, 616-619 (1976).
- Dorman L.I. and D. Venkatesan "Solar cosmic rays, *Space Sci. Rev.*, **64**, 183-362 (1993).
- Dorman et al., 1997a: Dorman L.I., G. Villoresi, I.V. Dorman, N. Iucci, and M. Parisi "High rigidity CR-SA hysteresis phenomenon and dimension on modulation region in the Heliosphere in dependence of particle rigidity", *Proc. 25- th Intern. Cosmic Ray Conference*, Durban (South Africa), **2**, 69-72 (1997a).
- Dorman et al., 1997b: Dorman L.I., G. Villoresi, I.V. Dorman, N. Iucci, and M. Parisi "Low rigidity CR-SA hysteresis phenomenon and average dimension of the modulation region and Heliosphere", *Proc. 25th Intern. Cosmic Ray Conference*, Durban (South Africa), **2**, 73-76 (1997b).

- Earl J.A. "Modulation of cosmic-ray electrons", *Astrophys. J.*, **178**, No. 3, Part 1, 857-862 (1972).
- Efimov N.N., T.A. Egorov, A.V. Glushkov, M.I. Pravdin, and I.Ye. Sleptsov "The energy spectrum and anisotropy of primary cosmic rays at energy $E_0 > 10^{17}$ eV observed in Yakutsk", *Astrophysical Aspects on the Most Energetic Cosmic Rays, Proc. of the ICRR International Symposium* (ed. M. Nagano and F. Takahara), Singapore: World Scientific Publishing, 20 (1991).
- Erber T. "High-energy electromagnetic conversion processes in intense magnetic fields", *Rev. Mod. Phys.*, **38**, No. 4, 626-659 (1966).
- Erokhin N.S. and S.S. Moiseev "Wave processes in non-homogeneous plasma", In *Problems of Plasma Theory* (ed. M.A. Leontovich), Atomizdat, Moscow, Vol. 7, 146-204 (1973).
- Feenberg E. and H. Primakoff "Interaction of cosmic-ray primaries with sunlight and starlight", *Phys. Rev.*, **73**, No. 5, 449-469 (1948).
- Felten J.E. and P. Morrison "Recoil photons from scattering of starlight by relativistic electrons", *Phys. Rev. Lett.*, **10**, No. 10, 453-457 (1963).
- Fichtel C.E. "Very high-energy cosmic rays", *Phys. Rev. Lett.*, **11**, No. 4, 172-173 (1963).
- Fowler P.H., C.J. Waddington, P.S. Freier, J. Naugle, and E.P. Ney "The low energy end of the cosmic ray spectrum of alpha-particles", *Phil. Mag.*, **2**, Ser. 8, No. 14, 157-175 (1957).
- Galeev A.A. and R.Z. Sagdeev "Non-linear plasma theory", In *Problems of Plasma Theory*, M.A. Leontovich (ed.), Atomizdat, Moscow, **7**, 3-145 (1973).
- Ganguli S.N. and B.V. Sreekantan "Fluxes of γ -rays, antiprotons and deuterons in cosmic rays", *J. Phys. A: Math. and Gen.*, **9**, No. 2, 311-323 (1976).
- Gehrels N. and P. Michelson "GLAST: the next generation high energy gamma-ray astronomy mission", *Astropart. Phys.*, **11**, No. 1-2, 277-282 (1999).
- Gerasimova N.M. and I.L. Rozental "The influence of nuclear photo-effect on the spectrum of primary cosmic radiation", *JETP*, **41**, No. 2, 488-490 (1961).
- Gerasimova N.M. and G.T. Zatsepin "Splitting of cosmic ray nuclei by solar photons", *ZhETF*, **38**, No. 4, 1245-1252 (1960).
- Gershberg R.E., E.I. Mogilevsky, and V.N. Obridko "The energetics of the activity of flare stars and the Sun - A synergetic approach", *Kinem. and Phys. of Celestial Bodies*, **3**, No. 5, 3-17 (1987).
- Gershberg R.E. and N.I. Shakhovskaya "Characteristics of activity energetics of the UV C-type flare stars", *Astrophys. Space Science*, **95**, No. 2, 235-253 (1983).
- Ginzburg V.L. "Some results of radio-astronomical investigations", *Vestnik Academy of Sciences USSR*, No. 2, 17-21 (1964).
- Ginzburg V.L. "Cosmic rays and plasma phenomena in the Galaxy and Metagalaxy", *Astron. J. (Moscow)*, **42**, No. 6, 1129-1134 (1965).
- Ginzburg V.L. "On Metagalactic Cosmic Rays", *Astrophys. and Space Sci.*, **1**, No. 1, 125-128 (1968).
- Ginzburg V.L. "Pulsars and the origin of cosmic rays", *Comments Astrophys. and Space Phys.*, **1**, No. 6, 207-214 (1969).
- Ginzburg V.L. and S.I. Syrovatsky "The main problems of cosmic ray astrophysics", *Izvestia Academy of Sci. of USSR, Series Phys.*, **29**, No. 10, 1819-1824 (1965).
- Gordon I.M. "The nature of gamma-radiation of solar flares and cosmic ray particles generation in active regions", *Astron. J. (Moscow)*, **37**, No. 5, 934-937 (1960).
- Greisen K. "Cosmic ray showers", *Ann. Rev. Nucl. Sci.*, **10**, No. 1, 63-108 (1960).

- Greisen K. "End to the cosmic-ray spectrum?", *Phys. Rev. Lett.*, **16**, No. 17, 748-750 (1966).
- Gunn J.E. and J.P. Ostriker "Acceleration of high-energy cosmic rays by pulsars", *Phys. Rev. Lett.*, **22**, No.14, 728-731 (1969).
- Halzen F., R.J. Protheroe, T. Stanev, and H.P. Vankov, "Cosmology with 100-TeV gamma-ray telescopes", *Phys. Rev. D., Particles and Fields*, **41**, No. 2, 342-346 (1990).
- Hari Dass N.D., L.L. DeRaad, Jr, K.A. Milton, and W.-Y Tsai "Compton scattering in strong external electromagnetic fields", *Proc. Conf. on the Role of Magnetic Fields in Physics and Astrophysics* (NORDITA and Niels Bohr Institute, Copenhagen, Denmark, June 5-7, 1974), *New York Academy of Sciences Annals*, **257**, 72-75 (1975).
- Hayakawa S. and H. Obayashi "Canonical formalism of the motion of a charge particle in a magnetic field", *Redniconti della Scuola Internazionale di Fisica*, E. Fermi, Corso XXIV, 23-59 (1963a).
- Hayakawa S. and H. Obayashi "An effect of nonadiabativity on the structure of radiation belts", *J. Geophys. Res.*, **68**, No. 10, 3311-3313 (1963b).
- Hayashida N., K. Honda, M. Honda, S. Imaizumi, N. Inoue, K. Kadota, F. Kakimoto, K. Kamata, S. Kawaguchi, N. Kawasumi, Y. Matsubara, K. Murakami, M. Nagano, H. Ohoka, M. Takeda, M. Teshima, I. Tsushima, S. Yoshida, and H. Yoshii "Observation of a very energetic cosmic ray well beyond the predicted 2.7 K cutoff in the primary energy spectrum", *Phys. Rev. Lett.*, **73**, No. 26, 3491-3494 (1994).
- Hillas A.M. "Cosmic ray in an evolving universe", *Can. J. Phys.*, **46**, No. 10, Part 3, S623-S626 (1968).
- Hillas A.M. "The effect of intergalactic propagation on the energy spectrum of cosmic ray nuclei above 10^{15} eV", *Proc. 14th Intern. Cosmic Ray Conf.*, Munchen, **2**, 717-722 (1975).
- Hillas A.M. and M. Ouldrige "Cosmic rays and the Galaxy", *Nature*, **253**, No.5493, 609-610 (1975).
- Jacklyn R.M. "The Apparent Sidereal Daily Variation of Cosmic Ray Intensity at 40 m w. e. at Hobart, Tasmania", *Nuovo Cimento*, **36**, Ser. 10, No. 4, 1135-1148 (1965).
- Johnson M. "Trapping Regions for Cosmic Rays of the Highest Energies", *Observatory*, **90**, No. 974, 31-33 (1970).
- Kadomtsev B.B. "Plasma turbulence", In *Problems of Plasma Theory* (ed. M.A. Leontovich), Atomizdat, Moscow, Vol. 4, 188-339 (1964).
- Karakula S. "Can EHE photons be responsible for the highest energy cosmic ray showers?", *Proc. Vulcano Workshop 1996* (ed. F. Giovannelli and G. Mannocchi), Italian Physical Society, Bologna, Italy, **57**, 355 (1997).
- Karakula S. and W. Bednarek "Can photos with energies $> 10^{20}$ eV pass the Earth's magnetosphere? Analysis of the obseved highest energy cosmic rays", *Proc. 24th Intern. Cosmic Ray Conf.*, Rome, **1**, 266-269 (1995).
- Karakula S., T. Mlyńczyk, and W. Tubek "Interaction of EHE gamma-rays with the magnetic field of the Sun" *Proc. Frascati Workshop 1995* (ed. F. Giovannelli and L. Sabau- Graziati), Italian Physical Society, Bologna, Italy, **47**, 547 (1996).
- Karakula S. and W. Tubek "The influence of EHE photon interaction with Earth's magnetic field and LPM effect on the cascade development in the atmosphere", *Multifrequency behaviour of high energy cosmic sources*, *Vulcano*, 22-27 May

- 1995 (ed. Giovannelli F. and L. Sabau-Graziati), *Memorie della Societa Astronomica Italiana*, **67**, No. 1-2, 65-72 (1995).
- Kasahara K., *Proc. Intern. Symp. on Extremely High Energy Cosmic Rays: Astrophysics and Future Observatories* (ed. M. Nagano), University of Tokyo, 221 (1997).
- Korchak A.A. "On the origin of hard X- and radio-radiation during solar flare at September 28, 1961", *Geomagnetism and Aeronomia*, **5**, No. 1, 32-39 (1965a).
- Korchak A.A. "On the origin of of continue electromagnetic radiation during solar flares", *Izvestia AN SSSR, Ser. Phys.*, **29**, No. 10, 1813-1818 (1965b)
- Korchak A.A. and Yu.B. Ponomarenko "On the Compton effect on relativistic electrons in the Sun's atmosphere", *Geomagnetism and Aeronomia*, **6**, No. 3, 417-423 (1966).
- Korotin S.A. and V.I. Krasnobaev "Distribution of flares of UV Cet-type stars in open clusters and in the solar vicinity over optical radiation energies", *Izvestia Krimea Astrophysical Observatory*, **73**, 131-143 (1985).
- Kurochka L.N. "The energy distribution of 15 thousand solar flares", *Astronomy J. (Moscow)*, **64**, No. 2, 443-446 (1987).
- Laster H. "Galactic and extragalactic propagation of cosmic rays", *Phys. Rev.*, **135**, No. 5B, 1274-1279 (1964).
- le Roux J.A. and H. Fichtner "The influence of pickup, anomalous, and galactic cosmic-ray protons on the structure of the heliospheric shock: a self-consistent approach", *Astrophys. J.*, **477**, L115-L118 (1997).
- Lezniak J.A. and W.R. Webber "Implications of the reported low energy electron gradients", *Solar Phys.*, **34**, No. 2, 477-489 (1974).
- Ling J.C. "Semiempirical model for atmospheric gamma rays from 0.3 to 10 MeV at $\lambda = 40^\circ$ ", *J. Geophys. Res.*, **80**, No. 22, 3241-3252 (1975).
- Margolis S.H. and D.N. Schramm "Galactic and extragalactic ultra-high energy neutrinos", *Proc. 15th Intern. Cosmic Ray Conf.*, Plovdiv, **6**, 244-247 (1977).
- McBreen B. and C.J. Lambert "Interactions of high-energy ($E > 5 \times 10^{19}$ eV) particles in the Earth's magnetic field", *Phys. Rev. D, Particles and Fields*, **24**, No. 9, 2536-2538 (1981).
- Milton K.A., Tsai Wu-yang, L.L. DeRaad, Jr, and N.D. Hari Dass "Compton scattering in external magnetic fields. II. Spin-1/2 charged particles", *Phys. Rev. D*, **10**, No. 4, 1299-1309 (1974).
- Oda M. "A note on the role of colliding galaxies as a source of cosmic radiation", *Inst. Nucl. Study, Univ. Tokyo*, Technical Report, 1-10 (1961).
- Orth C.D. and A. Buffington "Secondary cosmic ray $e^\pm$ from 1 to 100 GeV in the upper atmosphere and interstellar space and interpretation of a recent $e^\pm$ flux measurement", *Astrophys. J.*, **206**, No. 1, Part 1, 312-332 (1976).
- Parker E.N. "The scattering of charged particles by magnetic irregularities", *J. Geophys. Res.*, **69**, No. 9, 1755-1758 (1964).
- Pohl M. "On the predictive power of the minimum energy condition. I - Isotropic steady-state configurations", *Astronomy and Astrophysics*, **270**, No. 1-2, 91-101 (1993).
- Pohl M. "On the predictive power of the minimum energy condition. 2: Fractional calorimeter behaviour in the diffuse high energy gamma emission of spiral galaxies", *Astronomy and Astrophysics*, **287**, No. 2, 453-462 (1994).
- Pollack J.B. and B.S. Shen "Disintegration and energy degradation of very high-energy cosmic rays in intense photon fields", *Phys. Rev. Lett.*, **23**, No. 23, 1358-1361 (1969a).

- Pollack J.B. and B.S. Shen "Decomposition and energy degradation of very high-energy cosmic rays by Doppler-shifted photons", *Bulletin of the American Astronomical Society*, **1**, 360 (1969b).
- Prilutsky O.F. and I.L. Rozental "Cosmic rays in the extended Universe", *Izvestia Ac. of Sci. USSR, Ser. Phys.*, **33**, No. 11, 1776-1786 (1969).
- Ptuskin V.S. "Pressure of a gas of fast charged particles that diffuse in a medium with a stochastic magnetic field", *JETP*, **86**, No. 2, 483-486 (1984).
- Puget J.L., F.W. Stecker, and J.H. Bredekamp "Photonuclear interactions of ultrahigh energy cosmic rays and their astrophysical consequences", *Astrophys. J.*, Part 1, **205**, No.2, 633-654 (1976).
- Rengarajan T.N. "Radio emission from pulsars and surface temperature of neutron stars", *Nature (Physical Science)*, **242**, No. 120, 102-104 (1973a).
- Rengarajan T.N. "Absorption of High Energy Heavy Nuclei and Gamma Rays at the Surface of Hot Neutron Stars", *Proc. 13th Intern. Cosmic Ray Conf.*, Denver, **1**, 627-632 (1973b).
- Sagdeev R.Z. "Collective processes and shock waves in rare plasma", In *Problems of Plasma Theory* (ed. M.A. Leontovich), Atomizdat, Moscow, Vol. 4, 20-80 (1964).
- Shafranov V.D. "Electromagnetic waves in plasma", In *Problems of Plasma Theory* (ed. M.A. Leontovich), Atomizdat, Moscow, Vol. 3, 3-140 (1963).
- Shen C.S. "Pulsars and very high-energy cosmic-ray electrons", *Astrophys. J.*, **162**, No. 3, Part 2, L181-L186 (1970).
- Shivanandan K., J.R. Houck, and M.O. Harwit "Preliminary observations of the far-infrared night-sky background radiation", *Phys. Rev. Lett.*, **21**, No. 20, 1460-1462 (1968).
- Shklovsky I.S. "Nature of solar X-ray emission", *Astronomicheskii Zhurnal*, **41**, No. 4, 676-683 (1964a).
- Shklovsky I.S. "Nature of jets in radio galaxies", *Soviet Astron. AJ.*, **7**, No. 6, 748-754 (1964b), (English Transl.).
- Silberberg R. and M.M. Shapiro "Diffuse background of cosmic neutrinos at high energies", *Proc. 15th Intern. Cosmic Ray Conf.*, Plovdiv, **6**, 237-242 (1977).
- Silvestro G. "Pulsars as possible sources of superheavy nuclei in the primary cosmic radiation", *Lett. Nuovo Cimento*, **2**, No. 16, 771-772 (1969).
- Simpson J.A. "Elemental and isotopic composition of the galactic cosmic rays", *Annual Review of Nuclear and Particle Science*, **33**, Annual Reviews Inc., Palo Alto, CA, USA, 323-381 (1983).
- Sivukhin D.V. "Drift theory of charged particle moving in electro-magnetic fields" In *Problems of Plasma Theory* (ed. M.A. Leontovich), Atomizdat, Moscow, Vol. 1, 7-97 (1963).
- Skibo J.G. and R. Ramaty "Diffuse Galactic low energy gamma ray continuum emission", *Astronomy and Astrophysics, Supplement Series*, **97**, No. 1, 145-148 (1993).
- Stanev T. and H.P. Vankov "Nature of the highest energy cosmic rays", *Phys. Rev. D, Particles and Fields*, **55**, No. 3, 1365-1371 (1997).
- Stoker P.H. "Relativistic solar proton events", *Space Sci. Rev.*, **73**, No. 1-4, 327-385 (1994).
- Strong A.W., K. Bennett, H. Bloemen, R. Diehl, W. Hermsen, D. Morris, V. Schoenfelder, J.G. Stacy, C. de Vries, M. Varendorff, C. Winkler, and G. Youssefi "Diffuse continuum gamma rays from the Galaxy observed by COMPTEL", *Astronomy and Astrophysics*, **292**, No. 1, 82-91 (1994).

- Sweeney G.S.S. and P Stewart "Non-linear Compton radiative group locking", *Astronomy and Astrophysics*, **37**, No. 2, 201-207 (1974)
- Syrovatsky S.I. "Particle acceleration in the space", *Proc. 5th All-Union Winter School on Cosmophysics*, Apatity, 58-72 (1968).
- Syrovatsky S.I. "Cosmic rays of ultra-high energy", *Comments on Modern Phys., Part C: Comments on Astrophys. and Space Phys.*, **3**, No. 5, 155-162 (1971).
- Vedenov A.A. "Introduction in the theory of weak-turbulent plasma", In *Problems of Plasma Theory* (ed. M.A. Leontovich), Atomizdat, Moscow, Vol. 3, 203-244 (1963).
- Vedenov A.A. and L.D. Rjutov "Quasi-linear effects in stream instabilities", In *"Problems of Plasma Theory"* (ed. M.A. Leontovich), Atomizdat, Moscow, **6**, 3-68 (1972).
- Verma S.D. "Energy spectra of splash and re-entrant albedo electrons of cosmic radiation", *Indian J. Radio and Space Phys.*, **6**, No. 3, 171-173 (1977a).
- Verma S.D. "Energy spectra of splash and re-entrant albedo electrons of cosmic radiations. Proc. 15th Intern. Cosmic Ray Conf., Plovdiv, 4, 197-202 (1977b).
- Verschuur G.L. "Observations of the galactic magnetic field", *Fundamentals of Cosmic Physics*, **5**, No. 1-2, 113-191 (1979).
- Völk H.J. and C.J. Cesarsky "Nonlinear Landau damping of Alfvén waves in a high beta plasma", *Z. Naturforsch.*, **37A**, No. 8, 809-815 (1982).
- Volkov T.F. "Hydro-dynamical description of very rare plasma", In *Problems of Plasma Theory* (ed. M.A. Leontovich), Atomizdat, Moscow, Vol. 4, 3-19 (1964).
- Wang H.T. "Low-energy cosmic ray protons from nuclear interactions of cosmic rays with the interstellar medium", *J. Geophys. Res.*, **78**, No. 25, 5693-5697 (1973).
- Wdowczyk J. and A.W. Wolfendale "Implications of an extragalactic origin for the highest energy cosmic rays", *Astrophys. J.*, Part 1, **349**, No. 1, 35-40 (1990)
- Webber W.R. "Cosmic ray antiprotons, electrons, positrons, deuterium and ^3He and new measurements of nuclear fragmentation and their role in understanding cosmic ray propagation in the Galaxy", *Proc. 20th Intern. Cosmic Ray Conf.*, Moscow, **8**, 65-86 (1987).
- Weekes T. "Gamma-Ray Telescopes", in *Encyclopedia of Astronomy and Astrophysics* (ed. P. Murdin), DOI: 10.1888/0333750888/1953 (2000).
- Wentzel D.G. "Cosmic-ray propagation in the Galaxy: collective effects", *Ann. Rev. of Astron. and Astrophys.*, **12**, 71-96 (1974).
- Willis D.M. "The motion of an isolated charged particle in a simple force-free magnetic field", *Planet. and Space Sci.*, **14**, No. 6, 483-507 (1966).
- Zatsepin G.T. and V.A. Kusmin "On the upper boundary of cosmic ray spectrum", *Lett. JETP*, **4**, No. 3, 114-117 (1966).
- Zheleznyakov V.V. "On the mechanism of gamma-ray emission by solar flares", *Astronomicheskii Zhurnal*, **42**, No. 1, 96-101(1965a).
- Zheleznyakov V.V. "On the origin of solar radio bursts in the meter-wavelength range", *Astronomicheskii Zhurnal*, **42**, No. 2, 244-252(1965b).
- Zirakashvili V.N., L.I. Dorman, V.S. Ptuskin, and V.Kh. Babayan "Cosmic ray nonlinear modulation in the outer Heliosphere", *Proc. 22th Intern. Cosmic Ray Conf.*, Dublin, **3**, 585-588 (1991).

References for Chapter 2

- Achterberg A. and L. Ball "Particle acceleration at superluminal quasi-perpendicular shocks. Application to SN 1978K and SN 1987A", *Astronomy and Astrophysics*, **285**, No. 2, 687-704 (1994)
- Akopyan S.K., Asatryan G.A., Babayan Kh.P., Bagdasaryan M.B. and Dorman L.I. "Automodel solutions of the theory of cosmic ray nonstationary modulation and problems of cosmic ray experimental research on the mountain Aragaz", *Izvestia Academy of Sciences USSR, Series Phys.*, **42**, No. 5, 1098-1101 (1978)
- Alania et al., 1976: Alania M.V., M.A. Aleksidze, E.R. Bagdavadze, P.A. Menteshashvili, and L.I. Dorman "Anisotropic and isotropic diffusion of galactic cosmic rays in interplanetary space", *Proc. Symp. on Solar-Terrestrial Phys.*, Tbilisi-Moscow, Part 1, 64-66 (1976).
- Alania M.V. and L.I. Dorman "Propagation of galactic cosmic rays in interplanetary space in case of helio-latitudinal asymmetry of solar wind", *Proc. 15th Intern. Cosmic Ray Conf.*, Plovdiv, **3**, 40-45 (1977).
- Alania M.V. and L.I. Dorman "Modulation of cosmic ray intensity in interplanetary space with account of "royal zones" in solar activity distribution", *Izvestia Academy of Sci. of USSR, Series Phys.*, **42**, No.5, 1038-1042 (1978).
- Alania M.V. and L.I. Dorman "Interplanetary modulation of galactic cosmic rays including the generalized tensor of anisotropic diffusion", *Izvestia Academy of Sci. of USSR, Series Phys.*, **43**, No. 4, 769-777 (1979).
- Alania et al., 1977a: Alania M.V., M.A. Aleksidze, P.V. Janjagava, and L.I. Dorman "Modulation of galactic cosmic rays by helio-latitudinal asymmetry of solar wind with account of isotropic and of anisotropic diffusion", *Geomagn. and Aeron.*, **17**, No. 2, 190-197 (1977a).
- Alania et al., 1977b: Alania M.V., M.A. Aleksidze, and L.I. Dorman "Choice of boundary conditions in approximately solving the boundary problems describing the diffusion of galactic cosmic rays". *Rep. Acad. Sci. Georgian SSR*, **88**, No.1, 69-72 (1977b).
- Alania et al., 1978: Alania M.V., E.R. Bagdavadze, A.S. Gabuniya, Ts.A. Gabuniya, R.T. Gushchina, and L.I. Dorman "Propagation of galactic cosmic rays in interplanetary space including the real distribution of solar activity", In *Cosmic Ray Modulation*, Metzniereba, Tbilisi, 15-36 (1978).
- Alania et al., 1979: Alania M.V., A.S. Gabunia, L.I. Dorman, and R.T. Gushchina "Propagation of galactic cosmic rays in interplanetary space including the real distribution of solar activity", *Proc. of 16th Intern. Cosmic Ray Conf.*, Kyoto, **3**, 63-68 (1979).
- Alania et al., 1980: Alania M.V., L.I. Dorman, A.S. Gabunia, and R.T. Gushchina "Anisotropic Diffusion of Galactic Cosmic Rays with Allowance for the Real Solar Activity Distribution", *Geomagnetism and Aeronomy*, **20**, No. 3, 281-284 (1980). English edition published by American Geophysical Union.
- Allan H.R. "Interpretation of the cosmic ray anisotropy", *Astrophys. Lett.*, **12**, No. 4, 237-241 (1972).
- Alpers W., K. Hasselmann, and J. Kunstmann "On the behaviour of the pitch angle diffusion coefficients near pitch angles of 90°", *Proc. 14th Intern. Cosmic Ray Conf.*, Munchen, **3**, 888-892 (1975).
- Altschuler M.D., D.E. Trotter, and G.J. Newkirk "The large-scale solar magnetic field", *Solar Phys.*, **39**, No. 1, 3-17 (1974).

- Axford W.I. "The modulation of galactic cosmic rays in the interplanetary medium", *Planet. Space Sci.*, **13**, No. 2, 115-130 (1965a).
- Axford W.I. "Anisotropic diffusion of solar cosmic rays", *Planet. Space Sci.*, **13**, No. 12, 1301-1309 (1965b).
- Ayubasheva S.I. and E.Ya. Gidalevich "Determination of the cosmic ray diffusion coefficient in interplanetary space on the basis of the study of Forbush-decreases", In *Cosmophysics Research*, Sverdlovsk, 12-19 (1977).
- Babayan V.Kh. and L.I. Dorman "The nonlinear theory of cosmic ray modulation by solar wind, 2. The focusing effect in the asymmetric model", *Proc. 15th Intern. Cosmic Ray Conf.*, Plovdiv, **3**, 113-118 (1977b).
- Babayan V.Kh. and L.I. Dorman "Nonlinear interactions of cosmic rays with solar wind", *Proc. Symp. on Solar-Terrestrial Phys.*, Tbilisi-Moscow, Part I, 69-70 (1976).
- Babayan V.Kh. and L.I. Dorman "Nonlinear interactions of galactic cosmic rays with solar wind", *Izvestia Academy of Sci. of USSR, Series Phys.*, **43**, No. 4, 793-799 (1979).
- Babayan V.Kh. and L.I. Dorman "The nonlinear theory of cosmic ray modulation by solar wind, 1. Spherically-symmetrical model", *Proc. 15th Intern. Cosmic Ray Conf.*, Plovdiv, **3**, 107-112 (1977a).
- Bagdasaryan M.B. and L.I. Dorman "Superposition of the increases and Forbush-decreases in galactic cosmic rays in the presence of a sequence of shock waves", *Proc. 16th Intern. Cosmic Ray Conf.*, Kyoto, **12**, 165-169 (1979).
- Bagdasaryan M.B. and L.I. Dorman "Cosmic-ray modulation by a shock wave with allowance for adiabatic cooling and transverse diffusion (self-similar solution)", *Geomagn. and Aeron.*, **20**, No. 6, 698-701 (1980). English edition published by American Geophysical Union.
- Bagdasaryan et al., 1971: Bagdasaryan M., A.V. Belov, L.I. Dorman, and B.A. Shakhov "Some problems in the theory of cosmic ray modulation effects". *Izvestia Academy of Sci. of USSR, Series Phys.*, **35**, No. 12, 2492-2498 (1971).
- Bagdasaryan et al., 1974: Bagdasaryan M.B., R.T. Gushchina, L.I. Dorman, I.A. Pimenov, and V.G. Yanke "Some problems of the theory of cosmic ray variations", *Cosmic Rays*, Nauka, Moscow, No. 14, 57-85 (1974).
- Bahrah B.L. and S.I. Vetchinkin "Green's function of the Schrödinger equation for the simplest systems", *Theor. Mat. Phys.*, **6**, No. 3, 392-402 (1971).
- Balescu R. "Anomalous transport in turbulent plasmas and continuous time random walks", *Phys. Rev. E*, **51**, No. 5, 4807-4822 (1995).
- Balogh A., E.J. Smith, B.T. Tsurutani, D.J. Southwood, R.J. Forsyth, and T.S. Horbury "Heliospheric magnetic field over the south polar region of the sun", *Science*, **268**, No. 5213, 1007 (1995).
- Barnden L.R. "The origin-of-scatter model for cosmic ray diffusion and the anisotropy predicted at Earth", *Proc. 13th Intern. Cosmic Ray Conf.*, Denver, **2**, 680-685 (1973a).
- Barnden L.R. "The diurnal and semidiurnal variations from the origin-of-scatter model for different values of k and its r -dependence", *Proc. 13th Intern. Cosmic Ray Conf.*, Denver, **2**, 686-691 (1973b).
- Barnden L.R. and M. Bercovitch "Field gradient and curvature drifts and cosmic ray transport into the solar system", *Proc. 14th Intern. Cosmic Ray Conf.*, Munchen, **3**, 875-880 (1975).

- Beeck J., G.M. Mason, D.C. Hamilton, G. Wibberenz, H. Kunow, D. Hovestadt, and B. Klecker "A multispacecraft study of the injection and transport of solar energetic particles", *Astrophys. J.*, Part 1, **322**, No. 2, 1052-1072 (1987).
- Behannon Kenneth Wayne "Observations of the interplanetary magnetic field between 0.46 and 1 angstrom by the Mariner 10 spacecraft", *Ph.D. Thesis*, Catholic Univ. of America, 1-234 (1976).
- Belcher J. and L. Davis "Large-amplitude Alfvén waves in the interplanetary medium: Mariner 5", *J. Geophys. Res.*, **74**, No. 9, 2302-2308 (1969).
- Belcher J. and L. Davis "Large-amplitude Alfvén waves in the interplanetary Medium, 2", *J. Geophys. Res.*, **76**, No. 16, 3534-3563 (1971).
- Belov A. "Large Scale Modulation: View From the Earth", *Space Sci. Reviews*, **93**, Issue 1/2, 79-105 (2000).
- Belov A.V. and L.I. Dorman "Distortion of the external anisotropic flux of galactic cosmic rays in solar system", *Geomagnetism and Aeronomy*, **9**, No. 4, 613-616 (1969).
- Belov A.V. and L.I. Dorman "Distortion of stellar anisotropy of galactic cosmic rays by solar wind", *Geomagnetism and Aeronomy*, **12**, No. 1, 111-112 (1972).
- Belov A.V. and L.I. Dorman "The modulating effect of solar wind on the outer anisotropy of cosmic rays", *Proc. 15th Intern. Cosmic Ray Conf.*, Plovdiv, 128-131 (1977).
- Belov et al., 1973: Belov A.V., L.I. Dorman, and B.A. Shakhov "The model of a moving magnetic mirror and Forbush-effect in cosmic rays", *Geomagnetism and Aeronomy*, **13**, No. 3, 399-405 (1973).
- Belov et al., 1975: Belov A.V., L.I. Dorman, and B.A. Shakhov "Asymmetric model of the Forbush-effect recovery phase", *Proc. 14th Intern. Cosmic Ray Conf.*, Munchen, **3**, 1076-1081 (1975).
- Belov et al., 1976: Belov A.V., L.I. Dorman, E.A. Eroshenko, K.G. Ivanov, O.I. Ionozemtseva, G.I. Kulanina, and N.V. Mikerina "Structure of Forbush-decreases", *Geomagnetism and Aeronomy*, **16**, No. 5, 761-764 (1976).
- Berezhko E.G. "Global cosmic-ray diffusion in the galaxy", *Sov. Astron. Lett.*, **11**, No. 4, 250-251 (1985).
- Bieber J.W. and R.A. Burger "Cosmic-ray streaming in the Born approximation", *Astrophys. J.*, Part 1, **348**, No. 1, Jan. 10, 597-607 (1990).
- Bieber et al., 1987: Bieber J.W., P. Evenson, and W.H. Matthaeus "Magnetic helicity of the IMF and the solar modulation of cosmic rays", *Geophys. Res. Letters*, **14**, 864-867 (1987).
- Bieber et al., 1988: Bieber J.W., C.W. Smith, and W.H. Matthaeus "Cosmic-ray pitch-angle scattering in isotropic turbulence", *Astrophys. J.*, Part 1, **334**, No. 1, 470-475 (1988).
- Bieber et al., 1990: Bieber J.W., P. Evenson, and M.A. Pomerantz "A barrage of relativistic solar particle events", *EOS Transactions*, Amer. Geophys. Union, **71**, 1027-1035 (1990).
- Bieber et al., 1994: Bieber J.W., W.H. Matthaeus, C.W. Smith, W. Wanner, M.-B. Kallenrode, and G. Wibberenz "Proton and electron mean free paths: The Palmer consensus revisited", *Astrophys. J.*, Part 1, **420**, No. 1, 294-306 (1994).
- Bieber et al., 1995: Bieber John W., R.A. Burger, and William H. Matthaeus "The Diffusion Tensor throughout the Heliosphere", *Proc. 24-th Intern. Cosmic Ray Conf.*, Rome, **4**, 694-697 (1995).

- Bieber et al., 1996: Bieber J.W., W. Wanner, and W.H. Matthaeus "Dominant two-dimensional solar wind turbulence with implications for cosmic ray transport", *J. Geophys. Res.*, **101**, No. A2, 2511-2522 (1996).
- Bieber et al., 1999a: Bieber J.W., R.A. Burger, R. Engel, T.K. Gaisser, S. Roesler, T. Stanev "Antiprotons at Solar Maximum", *Phys. Rev. Lett.*, **83**, 674-677 (1999a).
- Bieber et al., 1999b: Bieber J.W., R.A. Burger, R. Engel, T.K. Gaisser, and T. Stanev "Antiprotons as Probes of Solar Modulation", *Proc. 26-th Intern. Cosmic Ray Conf.*, Salt Lake City, 1999, **7**, 17-20 (1999b).
- Bieber J.W. and W.H. Matthaeus "Perpendicular diffusion and drift at intermediate cosmic-ray energies", *Astrophys. J.*, Part 1, **485**, No. 2, 655-659 (1997).
- Bird M.K. and P. Edenhofer "Remote sensing observations of the solar corona", in *Physics of the Inner Heliosphere* (Eds. R. Schewenn and E. Marsch), Springer-Verlag, Berlin-Heidelberg, Vol. 1, 13 (1990).
- Blokh et al., 1959a: Blokh Ya.L., L.I. Dorman, and N.S. Kaminer "Cosmic ray increase effect prior to magnetic storm", *Proc. 6th Intern. Cosmic Ray Conf.*, Moscow, **4**, 178-191 (1959a).
- Blokh et al., 1959b: Blokh Ya.L., E.S. Glokova, and L.I. Dorman "Study of the nature of the cosmic ray effect during the storm of August 29, 1957 on the basis of the data from the worldwide network of IGY stations", *Cosmic Rays*, No. 1, Akad. Sci. USSR, Moscow, 7-36 (1959b).
- Blokh et al., 1959c: Blokh Ya.L., E.S. Glokova, L.I. Dorman, and O.I. Inozemtseva "Electromagnetic conditions in interplanetary space in the period from August 29 to September 10, 1957 determined by cosmic ray variation data", *Proc. 6th Intern. Cosmic Ray Conf.*, Moscow, **4**, 172-177 (1959c).
- Blokh et al., 1964: Blokh G.M., Ya.L. Blokh, and L.I. Dorman "Some results of the calculations of the expected spectrum of variations in terms of the dynamic model of Forbush-effect", *Izvestia Academy of Sci. of USSR, Series Phys.*, **28**, No. 12, 1985-1988 (1964).
- Bloemen J.B.G.M., V.A. Dogiel, V.L. Dorman, and V.S. Ptuskin "Galactic diffusion and wind models of cosmic-ray transport .1. Insight from CR composition studies and gamma-ray observations", *Astronomy and Astrophysics*, **267**, No.2, 372-387 (1993).
- Borovkov L.P., L.L. Lazutin, O.I. Shumilov, and E.V. Vashenyuk "Injectin characteristics of energetic particles on the Sun during GLE", *Proc. 20th Intern. Cosmic Ray Conf.*, Moscow, **3**, 124-127 (1987).
- Bothmer V. "Die Struktur magnetischer Wolken im Sonnenwind", Ph.D. Thesis, Univ. Göttingen (1993).
- Braginsky S.I. "Transfer events in plasma", *Problems of Plasma Theory*, **1**, Atomizdat, Moscow, 183-273 (1963).
- Burger R.A. and M. Hattingh "Steady-state drift-dominated modulation models for Galactic cosmic rays", *Astrophys. and Space Sci.*, **230**, No. 1-2, 375-382 (1995).
- Burger R.A. and M. Hattingh "Toward a realistic diffusion tensor for Galactic cosmic rays", *Astrophys. J.*, **505**, No. 1, Part 1, 244-251 (1998).
- Burger R.A. and H. Moraal "Effect of small and medium scale magnetic field irregularities on particle drift", *Proc. 21-st Intern. Cosmic Ray Conf.*, Adelaide, 1990, **5**, 272 (1990).
- Burger R.A. and M.S. Potgieter "The calculation of neutral sheet drift in two-dimensional cosmic-ray modulation models", *Astrophys. J.*, **339**, No. 1, Part 1, 501-511 (1989).

- Burger R.A. and M.S. Potgieter “The Effect of Large Heliospheric Current Sheet Tilt Angles in Numerical Modulation Models: A Theoretical Assessment”, *Proc. 26-th Intern. Cosmic Ray Conf.*, **7**, 13-16 (1999).
- Burger R.A., M.S. Potgieter, and B. Heber “Rigidity dependence of near-Earth latitudinal proton density gradients”, *Proc. 26-th Intern. Cosmic Ray Conf.*, Salt Lake City, 1999, **7**, 242-245 (1999).
- Burgoa O. “Variational principle for Fokker-Planck cosmic rays transport equation”, *Proc. 28th Intern. Cosmic Ray Conf.*, Tsukuba (Japan), **4**, 2011-2014 (2003).
- Bykov A.M. and I.N. Toptygin “Particle kinetics in highly turbulent plasmas (renormalization and self-consistent field methods)”, *UFN (Uspekhi Fizicheskikh Nauk)*, **163**, No. 11, 19-56 (1993).
- Carbone V., F. Malara, and P. Veltri “A model for the three-dimensional magnetic field correlation spectra of low-frequency solar wind fluctuations during Alfvénic periods”, *J. Geophys. Res.*, **100**, No. A2, 1763-1778 (1995).
- Cecchini S. and J.J. Quenby “Three-dimensional models of galactic cosmic ray modulation”, *Proc. 14th Intern. Cosmic Ray Conf.*, Munchen, **3**, 911-916 (1975).
- Chandrasekhar S. “Stochastic Problems in Physics and Astronomy”, *Rev. Mod. Phys.*, **15**, Issue 1, 1-89 (1943).
- Charakhchyan A.N. and T.N. Charakhchyan “Secular cosmic ray modulation in interplanetary space”, *Izvestia Academy of Sci. of USSR, Series Phys.*, **31**, No. 8, 1313-1318 (1967).
- Charakhchyan A.N. and T.N. Charakhchyan “Eleven-year modulation of galactic cosmic rays in interplanetary space”, *Canad. Journ. of Phys.*, **46**, No. 10, part 4, S879-S882 (1968).
- Chuvilgin L.G. and V.S. Ptuskin “Anomalous diffusion of cosmic rays across the magnetic field”, *Astronomy & Astrophysics*, **279**, No. 1, 278-297 (1993).
- Combes F. “Astrophysical Fractals: Interstellar Medium and Galaxies”, *Advanced Series in Astrophysics and Cosmology* (Proceedings of the Second ICRA Network Workshop, Edited by V.G. Gurzadyan and R. Ruffini, World Scientific), **10**, 143 (2000).
- Compton A. and R. Getting “An apparent effect of Galactic rotation on the intensity of cosmic rays”, *Phys. Rev.*, **47**, No. 11, 817-827 (1935).
- Coroniti F.V., C.F. Kennel, F.L. Scarf, and E.J. Smith “Whistler mode turbulence in the disturbed solar wind”, *J. Geophys. Res.*, **87**, No. A8, 6029-6044 (1982).
- Cronin J.W. “Ultra high energy cosmic rays”, *Nuclear Physics B, Proceedings Supplements*, **97**, Issue 1-3, 3-9 (2001).
- Cummings A.C. and E.C. Stone “Anomalous Cosmic Rays and Solar Modulation”, *Space Sci. Rev.*, **83**, No. 1-2, 51-62 (1998).
- Danju M. and V. Sarabhai “Short-period variations of cosmic ray intensity”, *Phys. Rev. Lett.*, **19**, No. 5, 252-254 (1967).
- Debrunner et al., 1992: Debrunner H., J.A. Lockwood, and J.M. Ryan “The solar flare event on 1990 May 24 - Evidence for two separate particle accelerations”, *Astrophys. J.*, **387**, No. 1, L51-L54 (1992).
- Debrunner et al., 1997: Debrunner H., J.A. Lockwood, C. Barat, R. Buttikofer, J.P. Dezalay, E. Fluckiger, A. Kuznetsov, J.M. Ryan, R. Sunyaev, O.V. Terekhov, G. Trotter, and N. Vilmer “Energetic neutrons, protons, and gamma rays during the 1990 May 24 solar cosmic-ray event”, *Astrophys. J.*, Part 1, **479**, No. 2, 997-1011 (1997).

- De Koning C.A. and J.W. Bieber "Calculating the particle-field correlation in a flowing plasma", *Proc. 27th Intern. Cosmic Ray Conf.*, Hamburg, **8**, 3345-3348 (2001).
- Denskat K.U., H.J. Beinroth, and F.M. Neubauer "Interplanetary magnetic field power spectra with frequencies from 2.4 to 470 Hz from HELIOS-observations during solar minimum conditions", *J. of Geophysics - Zeitschrift fuer Geophysik.*, **54**, No. 1, 60-67 (1983).
- Dolginov A.Z., and M.E. Katz "Interaction of cosmic-rays with magneto-hydrodynamic turbulence in cosmic media", *Phys. Rep.*, **239**, No. 5-6, 285-364 (1994).
- Dolginov A.Z. and I.N. Toptygin "Multiple scattering of particles in a magnetic field with random inhomogeneities", *ZhETF*, **51**, No. 6, 1771-1783 (1966a).
- Dolginov A.Z. and I.N. Toptygin "Cosmic particle motions in random magnetic field", *Izvestia Academy of Sci. of USSR, Series Phys.*, **30**, No. 11, 1780-1783 (1966b).
- Dolginov A.Z. and I.N. Toptygin "Cosmic ray diffusion in interplanetary space", *Geomagnetism and Aeronomy*, **7**, No. 6, 967-973 (1967).
- Dolginov A.Z. and I.H. Toptygin "Cosmic rays in the interplanetary magnetic fields", *Icarus*, **8**, 54-60 (1968a).
- Dolginov A.Z. and I.N. Toptygin "The dependence of the cosmic ray diffusion coefficient on the spectrum of interplanetary magnetic field inhomogeneities and on the distance from the Sun", *Izvestia Academy of Sci. of USSR, Series Phys.*, **32**, No. 11, 1821-1828 (1968b).
- Dorman L.I. "The generation and propagation of solar cosmic rays", *Nuovo Cim.*, **8**, Suppl. No. 2, 391-402 (1958).
- Dorman L.I. "To the problem on the nature of soft radiation in the upper atmosphere", *Proc. 6th Intern. Cosmic Ray Conf.*, Moscow, **3**, 87-94 (1959a).
- Dorman L.I. "The energy spectrum and ground-based duration of cosmic ray intensity increase produced by shock wave and albedo from the magnetized front of corpuscular stream", *Proc. 6th Intern. Cosmic Ray Conf.*, Moscow, **4**, 132-139 (1959b).
- Dorman L.I. "To the theory of cosmic ray modulation by solar wind", *Proc. 6-th Intern. Cosmic Ray Conf.*, Moscow, **4**, 328-334 (1959c).
- Dorman L.I. "Galactic and solar cosmic rays in interplanetary space", *Proc. 9th Intern. Cosmic Ray Conf.*, London, **1**, 292-295 (1965).
- Dorman L.I. "Cosmic ray modulation in interplanetary space", *Cosmic Rays*, No. 8, Nauka, Moscow, 305-320 (1967).
- Dorman L.I. "Experimental and theoretical implications of Forbush-decreases in cosmic rays", *Izvestia Academy of Sci. of USSR, Series Phys.*, **33**, No. 11, 1890-1894 (1969).
- Dorman L.I. "Interplanetary shock waves and cosmic rays", *Comments on Astrophys. and Space Phys.*, **5**, No. 3, 67-74 (1973a).
- Dorman L.I. "On interplanetary shock wave generation", *Comments on Astrophys. and Space Phys.*, **5**, No. 4, 87-91 (1973b).
- Dorman L.I. "Studies of strong fluxes of solar plasma by means of cosmic rays", *Proc. 7th Leningrad Intern. Symp. on Corpuscular Streams from the Sun and the Radiation Belts of the Earth and Jupiter*, Leningrad, 193-211 (1975).
- Dorman L.I. "Cosmic ray long-term variation: even-odd cycle effect, role of drifts, and the onset of cycle 23", *Adv. Space Res.*, **27**, No. 3, 601-606 (2001).

- Dorman L.I. "Using galactic cosmic ray observations for determination of the Heliosphere structure during different solar cycles", *Solar Wind Ten* (ed. M. Velli et al.), AIP Conf. Proc., Melville, New York, 148-151 (2003a).
- Dorman L.I. "Changing of the modulation region structure with particle rigidities according to data of hysteresis phenomena", *Solar Wind Ten* (ed. M. Velli et al.), AIP Conf. Proc., Melville, New York, 164-167 (2003b).
- Dorman L.I. "The inverse problem: determining of energy spectrum and total flux of SEP in the source, time of ejection into solar wind, and propagation parameters in the interplanetary space on the basis of ground and satellite CR measurements", *Proc. 29th Intern. Cosmic Ray Conf.*, Pune, **1**, 281-284 (2005a).
- Dorman Lev I. "Using cosmic rays for monitoring and forecasting dangerous solar flare events", *Neutrinos and Explosive Events in the Universe* (Ed. M.M. Shapiro, T. Stanev, J.P. Wefel), NATO Science Series, Vol. 209, 131-142 (2005b).
- Dorman Lev I. "Combined ground-based and satellite cosmic ray measurements for forecasting of great radiation hazards", *Proc. 17-th ESA Symposium on European Rocket and Balloon Programmes and Related Research* (Ed. B. Warmbein), Sanefjord, Norway, 219-224 (2005c).
- Dorman L.I. "Monitoring and forecasting of radiation hazard from great SEP events for aircrafts by using on-line one-min cosmic ray data", *Proc. 3-rd Ankara Intern. Aerospace Conf.*, Ankara, Paper Track # 1119, 1-11, in DVD (2005d).
- Dorman Lev I. "Space Weather and Cosmic Rays: Applications to Aerospace (Invited Lecture)", *Proc. 3-rd Ankara Intern. Aerospace Conf.*, Ankara, pp 38, in DVD (2005e).
- Dorman I.V. and L.I. Dorman "Investigation of 11-year cosmic ray variations (on the base of sea-level observations)", *Cosmic Rays*, NAUKA, Moscow, **7**, 5-17 (1965).
- Dorman I.V. and L.I. Dorman "Solar wind properties obtained from the study of the 11-year cosmic ray cycle. 1", *J. Geophys. Res.*, **72**, No. 5, 1513-1520 (1967a).
- Dorman I.V. and L.I. Dorman "Propagation of energetic particles through interplanetary space according to the data of 11-year cosmic ray variations", *J. Atmosph. and Terr. Phys.*, **29**, No. 4, 429-449 (1967b).
- Dorman I.V. and L.I. Dorman "Cosmic rays and solar wind dynamics, I.", *Geomagnetism and Aeronomy*, **8**, No. 5, 817-821 (1968a).
- Dorman I.V. and L.I. Dorman "Cosmic rays and solar wind dynamics, II.", *Geomagnetism and Aeronomy*, **8**, No. 6, 1008-1013 (1968b).
- Dorman I.V. and L.I. Dorman "Inverse effect of cosmic rays on solar wind", *Izvestia Academy of Sci. of USSR, Series Phys.*, **33**, No.11, 1908-1917 (1969).
- Dorman et al., 1973a: Dorman L.I., M.E. Katz, and V.Kh. Shogenov "Kinetic description of cosmic ray interactions with shock waves", *Izvestia Academy of Sci. of USSR, Series Phys.*, **37**, No. 6, 1254-1258 (1973a).
- Dorman et al., 1973b: Dorman L.I., Z. Kobylinski, and T.S. Khadakhanova "Galactic cosmic ray modulation by the heliolatitude-asymmetric solar wind", *Geomagn. and Aeron.*, **13**, No. 1, 20-25 (1973b).
- Dorman et al., 1976a: Dorman L.I., M.E. Katz, and M. Stehlik "Cosmic ray fluctuations in interplanetary space relevant to the spectrum of magnetic field inhomogeneities", *Proc. Intern. Symp. on Solar-Terrestrial Phys.*, Part I, Tbilisi-Moscow, 50-52 (1976a).

- Dorman et al., 1976b: Dorman L.I., M.E. Katz, and M. Stehlik "Kinetics of Cosmic Rays in a Large-Scale Magnetic Field", *Geomagnetism and Aeronomy*, **28**, No. 2, in English Edition pages 173-177 (1976b).
- Dorman et al., 1976c: Dorman L.I., M.E. Katz, and A.K. Yukhimuk "The fluctuation effects in cosmic rays", *Trans. Resp. Geophys. Com. Acad. Sci. Ukr.SSR*, Kiev, No. 15, 43-48 (1976c).
- Dorman et al., 1976d: Dorman L.I., M.E. Katz, and B.A. Shakhov "About a connection between different forms of the equation of anisotropic diffusion of cosmic rays", *Geomagnetism and Aeronomy*, **16**, No. 5, 919-920 (1976d).
- Dorman et al., 1977a: Dorman L.I., L.A. Dremukhina, and Yu.I. Okulov "Calculations of three-dimensional distribution of the interplanetary magnetic field and the problems of cosmic ray propagation. I. Calculations of the field distribution", *Proc. 15th Intern. Cosmic Ray Conf.*, Plovdiv, **3**, 119-124 (1977a).
- Dorman et al., 1977b: Dorman L.I., L.A. Dremukhina, and Yu.I. Okulov "Calculations of three-dimensional distribution of the interplanetary magnetic field and the problems of cosmic ray propagation. II. Inclusion of the gradient drift", *Proc. 15th Intern. Cosmic Ray Conf.*, Plovdiv, **3**, 125-129 (1977b).
- Dorman et al., 1977c: Dorman L.I., M.E. Katz, and B.A. Shakhov "Various forms of the equation for the cosmic ray anisotropic diffusion in the radially expanding solar wind and the relationship between them", *Proc. 15th Intern. Cosmic Ray Conf.*, Plovdiv, **3**, 63-66 (1977c).
- Dorman et al., 1977d: Dorman L.I., M.E. Katz, and M. Stehlik "On the kinetic theory of cosmic ray fluctuations", *Proc. 15th Intern. Cosmic Ray Conf.*, Plovdiv, **3**, 76-83 (1977d).
- Dorman et al., 1977e: Dorman L.I., M.E. Katz, and Yu.I. Fedorov "The diffusion approximation in the theory of cosmic ray propagation in interplanetary space", *Proc. 15th Intern. Cosmic Ray Conf.*, Plovdiv, **3**, 67-71 (1977e).
- Dorman et al., 1978a: Dorman L.I., M.E. Katz, and Yu.I. Fedorov "The theory of cosmic ray diffusion in the interplanetary space with account of the second spherical harmonic", *Izvestia Academy of Sci. of USSR, Series Phys.*, **42**, No. 5, 978-983 (1978a).
- Dorman et al., 1978b: Dorman L.I., M.E. Katz, and Yu.I. Fedorov "Diffusion approach in the solar cosmic ray propagation theory including the second spherical harmonic", *Proc. IX Leningrad Intern. Seminar on Cosmophysics*, Leningrad, 338-346 (1978b).
- Dorman et al., 1978c: Dorman L.I., M.E. Katz, Yu.I. Fedorov, and B.A. Shakhov "On the cosmic ray energy balance under multiple scattering in a random-inhomogeneous magnetic field", *Pis'ma ZhETF*, **27**, No. 6, 374-378 (1978c).
- Dorman et al., 1978d: Dorman L.I., V.Kh. Babayan, A.V. Belov, O.N. Gulinsky, I.V. Dorman, A.M. Samir-Debish, and B.A. Shakhov "Development of the theory of cosmic ray modulation by solar wind and interplanetary shock waves", *Cosmic Rays*, Nauka, Moscow, 26-45 (1978d).
- Dorman et al., 1979: Dorman L.I., M.E. Katz, Yu.I. Fedorov, and B.A. Shakhov "The mode of energy variations of the galactic and solar cosmic rays in interplanetary space", *Izvestia Academy of Sci. of USSR, Series Phys.*, **43**, No. 4, 778-792 (1979).
- Dorman et al., 1983: Dorman L.I., Yu.I. Fedorov, M.E. Katz, F.S. Nosov, and B.A. Shakhov "Anisotropic stage of solar cosmic ray propagation", *Proc. 18th Intern. Cosmic Ray Conf.*, Bangalore, **10**, 63-66 (1983).

- Dorman et al., 1997: Dorman L.I., G. Villoresi, I.V. Dorman, N. Iucci, and M. Parisi "High rigidity CR-SA hysteresis phenomenon and dimension of modulation region in the Heliosphere in dependence of particle rigidity", *Proc. 25th Intern. Cosmic Ray Conf.*, Durban (South Africa), **2**, 69-72 (1997).
- Dorman et al., 1999: Dorman L.I., I.V. Dorman, N. Iucci, M. Parisi, and G. Villoresi "Hysteresis phenomenon: the dimension of modulation region in dependence of cosmic ray energy", *Proc. 26-th Intern. Cosmic Ray Conference*, Salt Lake City, **7**, 194-197 (1999).
- Dorman et al., 2001a: Dorman L.I., I.V. Dorman, N. Iucci, M. Parisi and G. Villoresi "Hysteresis between solar activity and cosmic rays during cycle 22: the role of drifts, and the modulation region", *Adv. Space Res.*, **27**, No. 3, 589-594 (2001a).
- Dorman et al., 2001b: Dorman L.I., N. Iucci and G. Villoresi "Time lag between cosmic rays and solar activity; solar minimum of 1994-1996 and residual modulation", *Adv. Space Res.*, **27**, No. 3, 595-600 (2001b).
- Dorman et al., 2003: Dorman L.I., B.A. Shakhov, and M. Stehlik "The second order pitch-angle approximation for the cosmic ray Fokker-Planck kinetic equations", *Proc. 28th Intern. Cosmic Ray Conf.*, Tsukuba (Japan), **6**, 3535-3538 (2003).
- Dorman et al., 2004: Dorman L.I., L.A. Pustil'nik, A. Sternlieb, I.G. Zukerman, A.V. Belov, E.A. Eroshenko, V.G. Yanke, H. Mavromichalaki, C. Sarlanis, G. Souvatzoglou, S. Tatsis, N. Iucci, G. Villoresi, Yu. Fedorov, B.A. Shakhov, and M. Murat "Monitoring and Forecasting of Great Solar Proton Evnts Using the Neutron Monitor Network in Real Time", *IEEE Transactions on Plasma Science*, 0093-3813, pp. 1-11 (2004).
- Dorman et al., 2005a: Dorman L.I., L.A. Pustil'nik, A. Sternlieb, and I.G. Zukerman "Forecasting of Radiation Hazard: 1. Alerts on Great FEP Events Beginning; Probabilities of False and Missed Alerts; on-Line Determination of Solar Energetic Particle Spectrum by using Spectrographic Method", *Adv. Space Res.*, **35**, (2005a), in press.
- Dorman et al., 2005b: Dorman L.I., N. Iucci, M. Murat, M. Parisi, L.A. Pustil'nik, A. Sternlieb, G. Villoresi, and I.G. Zukerman "Forecasting of Radiation Hazard: 2. On-Line Determination of Diffusion Coefficient in the Interplanetary Space, Time of Ejection and Energy Spectrum at the Source; on-Line Using of Neutron Monitor and Satellite Data", *Adv. Space Res.*, **35**, (2005b), in press.
- Dorman et al., 2005c: Dorman L.I., N. Iucci, M. Parisi, and G. Villoresi "The role of drift and convection-diffusion mechanisms in small energy global cosmic ray modulation: appliction to the hysteresis phenomenon of proton and alpha particle satellite data", *Adv. Space Res.*, **35**, 569-578 (2005c).
- Dorman L.I. and M.E. Katz "Study of cosmic ray propagation in interplanetary magnetic field on the basis of kinetic equation", *Izvestia Academy of Sci. of USSR, Series Phys.*, **36**, No.11, 2271-2277 (1972a).
- Dorman L.I. and M.E. Katz "Motion of charged particles in random magnetic fields of interplanetary space", *Proc. 3rd Leningrad Intern. Seminar on Similarity of the particle acceleration mechanisms in the space*, Leningrad, 237-246 (1972b).
- Dorman L.I. and M.E. Katz. "On charged particle intensity fluctuation in interplanetary magnetic field", *Proc. 13th Intern. Cosmic Ray Conf.*, Denver, **2**, 706-712 (1973).
- Dorman L.I. and M.E. Katz "Fluctuations in the intensity of solar cosmic rays", *Proc. 5th Leningrad Intern. Seminar*, Leningrad, 311-322 (1974a).

- Dorman L.I. and M.E. Katz "The fluctuation events in cosmic ray propagation in interplanetary space", *Izvestia Academy of Sci. of USSR, Series Phys.*, **38**, No.9, 1961-1965 (1974b).
- Dorman L.I. and M.E. Katz "The small-scale structure of solar corpuscular streams and fluctuation effects in cosmic rays", *Izvestia Academy of Sci. of USSR, Series Phys.*, **40**, No. 3, 510-513 (1976).
- Dorman L.I. and M.E. Katz "Green's function of the transfer equation for the simplest models of cosmic ray propagation", *Geomagnetism and Aeronomy*, **17**, No. 4, 615-621 (1977a).
- Dorman L.I. and M.E. Katz "Green's function of the transfer equation for the simplest models of cosmic ray propagation. I. Green's radial function", *Proc. 15th Intern. Cosmic Ray Conf.*, Plovdiv, **3**, 53-58 (1977b).
- Dorman L.I. and M.E. Katz "Green's function of the transfer equation for the simplest models of cosmic ray propagation. II. Green's function of three dimensional transfer equation", *Proc. 15th Intern. Cosmic Ray Conf.*, Plovdiv, **3**, 59-62 (1977c).
- Dorman L.I. and M.E. Katz "Cosmic ray kinetics in space", *Space Sci. Rev.*, **20**, No. 5, 529-575 (1977).
- Dorman L.I. and Z. Kobylinski "Cosmic ray modulation by the solar wind whose characteristics depend on the angle with helioequator plane", *Izvestia Academy of Sci. of USSR, Series Phys.*, **32**, No. 11, 1928-1932 (1968).
- Dorman L.I. and Z. Kobylinski "Calculation of cosmic ray modulation in cosmic space taking into account possible dependence of transport way for particle scatter and of solar wind velocity on angle with helioequator plane (isotropic diffusion case)", *Cosmic Rays*, Nauka, Moscow, No. 13, 108-134 (1972a).
- Dorman L.I. and Z. Kobylinski "Galactic cosmic ray modulation with non-spherically symmetrical solar wind taking into account anisotropic conditions near the Sun", *Izvestia Academy of Sci. of USSR, Series Phys.*, **36**, No. 11, 2332-2345 (1972b).
- Dorman L.I. and Z. Kobylinski "Expected gradients of cosmic radiation density for various models of galactic cosmic ray propagation in the interplanetary space", *Izvestia Academy of Sci. of USSR, Series Phys.*, **36**, No. 11, 2346-2353 (1972c).
- Dorman L.I. and Z. Kobylinski "Modulation of the spectrum, the radial and transverse gradients of galactic cosmic rays in a combined model of anisotropic and isotropic diffusion including convection and adiabatic cooling of particles in interplanetary space", *Proc. 13th Intern. Cosmic Ray Conf.*, Denver, **2**, 721-731 (1973).
- Dorman L.I. and N.P. Milovidova "Galactic cosmic ray density distribution in interplanetary space expected on the basis of anisotropic diffusion theory including the actual distribution of solar activity over solar disc", *Proc. 13th Intern. Cosmic Ray Conf.*, Denver, **2**, 713-720 (1973).
- Dorman L.I. and N.P. Milovidova "Annual cosmic ray variations in 1958-1968", *Izvestia Academy of Sci. of USSR, Series Phys.*, **38**, No. 9, 1928-1931 (1974).
- Dorman L.I. and N.P. Milovidova "Expected spatial distribution of cosmic rays in interplanetary space including the real heliolatitude distribution of solar activity", *Proc. 14th Intern. Cosmic Ray Conf.*, Munchen, **3**, 917-922 (1975a).
- Dorman L.I. and N.P. Milovidova "Expected 11-year and annual variations in cosmic rays of various rigidities in terms of the anisotropic diffusion model on the basis of the data on heliolatitude distribution of solar activity", *Proc. 14th Intern. Cosmic Ray Conf.*, Munchen, **3**, 923-927 (1975b).

- Dorman L.I. and N.P. Milovidova "Anisotropic diffusion of cosmic rays in the interplanetary space, I. Density distribution", *Geomagnetism and Aeronomy*, **16**, No. 1, 30-36 (1976a).
- Dorman L.I. and N.P. Milovidova "Anisotropic diffusion of cosmic rays in the interplanetary space, II. 11-years variation", *Geomagnetism and Aeronomy*, **16**, No. 2, 221-224 (1976b).
- Dorman L.I. and N.P. Milovidova "Anisotropic diffusion of cosmic rays in the interplanetary space, III. Radial and transversal gradients", *Geomagnetism and Aeronomy*, **16**, No. 3, 401-406 (1976c).
- Dorman L.I. and N.P. Milovidova "Anisotropic diffusion of cosmic rays in the interplanetary space, IV. Annual variations", *Geomagnetism and Aeronomy*, **16**, No. 4, 587-591 (1976d).
- Dorman L.I. and N.P. Milovidova "The expected solar-diurnal cosmic ray variation at various distances from the Sun and from the helioequator plane in 1959-1968 including the real heliolatitude distribution of solar activity", *Proc. 15th Intern. Cosmic Ray Conf.*, Plovdiv, **4**, 122-127 (1977).
- Dorman L.I. and N.P. Milovidova "Anisotropic diffusion of cosmic rays in interplanetary space. V. Solar-diurnal variations", *Geomagnetism and Aeronomy*, **18**, No. 3, 406-409 (1978).
- Dorman L.I. and L.I. Miroshnichenko "Burst of solar cosmic rays on September 28, 1961", *Geomagnetism and Aeronomy*, **5**, No. 3, 377-383 (1965).
- Dorman L.I. and V.Kh. Shogenov "Cosmic ray kinetics in interplanetary space before the moving magnetic mirror", *Geomagnetism and Aeronomy*, **13**, No. 6, 1006-1010 (1973a).
- Dorman L.I. and V.Kh. Shogenov "Galactic cosmic ray propagation in interplanetary space in the presence of a strong shock wave", *Proc. 13th Intern. Cosmic Ray Conf.*, Denver, **2**, 1233-1239 (1973b).
- Dorman L.I. and V.Kh. Shogenov "Shock-wave effects in cosmic ray kinetics", *Izvestia Academy of Sci. of USSR, Series Phys.*, **38**, No. 9, 1888-1894 (1974a).
- Dorman L.I. and V.Kh. Shogenov "Cosmic ray kinetics before the moving magnetic mirror", *Geomagnetism and Aeronomy*, **14**, No. 2, 355-257 (1974b).
- Dorman L.I. and V.Kh. Shogenov "Cosmic ray scattering by a statistically rough surface", *Geomagnetism and Aeronomy*, **15**, No. 1, 10-12 (1975a).
- Dorman L.I. and V.Kh. Shogenov "Kinetic theory of cosmic ray interaction with powerful interplanetary shock waves", *Proc. 14th Intern. Cosmic Ray Conf.*, Munchen, **3**, 1070 (1975b).
- Dorman L.I. and V.Kh. Shogenov "The function of cosmic ray distribution during Forbush-decrease", *Proc. 15th Intern. Cosmic Ray Conf.*, Plovdiv, **3**, 352-356 (1977).
- Dorman L.I. and V.Kh. Shogenov "Kinetic theory of galactic cosmic ray modulation by interplanetary magnetic piston". *Geomagnetism and Aeronomy*, **20**, No. 4, 588-594 (1980).
- Dorman L. and I. Zukerman "Initial Concept for Forecasting the Flux and Energy Spectrum of Energetic Particles Using Ground-Level Cosmic Ray Observations", *Adv. Space Res.*, **31**, No. 4, 925-932 (2003).
- Dröge W. "Transport of solar energetic particles", *Astrophys. J. Suppl. Ser.*, **90**, No. 2, 567-576 (1994).
- Dröge W. "Particle scattering by magnetic fields", *Space Sci. Rev.*, **93**, 121-151 (2000).

- Dröge W. "Solar particle transport in a dynamical quasi-linear theory", *Astrophys. J.*, **589**, Issue 2, 1027-1039 (2003).
- Drozhdov A.N. "Exponential power series expansion for the propagator of general diffusion processes", *Physica A*, **196**, No. 2, 283-312 (1993).
- Drury L. O'C. "An introduction to the theory of diffusive shock acceleration of energetic particles in tenuous plasmas", *Rep. Progr. Phys.*, **46**, No. 8, 973-1027 (1983).
- Dryer M., M.A. Shea, D.F. Smart, H.R. Collard, J.D. Mihalov, J.H. Wolfe, and J.W. Warwick "On the observation of a Flare-generated shock wave at 9.7 AU by Pioneer 10", *J. Geophys. Res.*, **A83**, No. 3, 1165-1168 (1978).
- Earl J.A. "Coherent propagation of charged-particle bunches in random magnetic fields", *Astrophys. J.*, **188**, No.2, 379-397 (1974).
- Earl J.A. "Analytical description of charged particle transport along arbitrary guiding field configurations", *Astrophys. J.*, Part 1, **251**, No.2, 739-755 (1981).
- Earl J.A. "The effect of adiabatic focusing upon charged-particle propagation in random magnetic fields", *Astrophys. J.*, **205**, No. 3, 900-919 (1976).
- Earl J.A. "New description of charged particle propagation in random magnetic fields", *Astrophys. J.*, **425**, No. 1, 331-342 (1994).
- Earl J.A., D. Ruffolo, H.L. Pauls, and J.W. Bieber "Comparison of three numerical treatments of charged particle transport", *Proc. 24th Intern. Cosmic Ray Conf.*, Rome, **4**, 341-344 (1995).
- Elliot H. "Time variations of cosmic radiation intensity", in *Progress in Cosmic Ray Physics* (ed. J.G. Wilson), Amsterdam, Vol. 1, Ch. VIII (1952). In Russian: Inostrannaja Literatura, Moscow, 379-430 (1954).
- Erlykin A.D., A.A. Lagutin, and A.W. Wolfendale "Properties of the interstellar medium and the propagation of cosmic rays in the Galaxy", *Astropart. Phys.*, **19**, Issue 3, 351-362 (2003).
- Evenson P. "Cosmic Ray Electrons", *Space Sci. Rev.*, **83**, No. 1-2, 63-73 (1998).
- Fedorov et al., 1992: Fedorov Yu.L, M.E. Katz, L.L. Kitchatinov, and M. Stehlik "Cosmic-ray kinetics in a random anisotropic reflective non-invariant magnetic-field", *Astronomy and Astrophysics*, **260**, No. 1-2, 499-509 (1992).
- Fedorov et al., 1995: Fedorov Yu I, B.A. Shakhov, and M. Stehlik "Non-diffusive transport of cosmic rays in homogeneous. regular magnetic fields", *Astron. Astrophys.*, **302**, No. 2, 623-634 (1995).
- Fedorov et al., 2002: Fedorov Yu., M. Stehlik, K. Kudela, and J. Kassovicova "Non-diffusive particle pulse transport: Application to an anisotropic solar GLE", *Solar Physics*, **208**, No. 2, 325-334 (2002).
- Fedorov Yu.I. and B.A. Shakov "Solar cosmic rays in homogeneous regular magnetic-field", *Proc. 23-rd Intern. Cosmic Ray Conf.*, Calgary, **3**, 215-218 (1993).
- Fedorov Yu.I. and B.A. Shakov "Propagation of solar cosmic-rays in a homogeneous regular magnetic-field", *Geomagnetism and Aeronomia*, **34**, No. 1, 19-29 (1994).
- Fedorov Yu.I. and M. Stehlik "Description of anisotropic particle pulse transport based on the kinetic equation", *Astrophys. Space Sci.*, **253**, No. 1, 55-72 (1997).
- Ferreira S.E.S., Master's Dissertation (M.Sc.), Potchefstroom University for CHE, South Africa (1999).
- Ferreira S.E.S. and M.S. Potgieter "Effects of anisotropic perpendicular diffusion on the energy and spatial dependence of galactic electron modulation in the Heliosphere", *Proc. 26-th Intern. Cosmic Ray Conf.*, Salt Lake City, **7**, 25-28 (1999).

- Ferreira S.E.S., M.S. Potgieter and R.A. Burger "Comparison of a Two- and Three-Dimensional Drift Model", *Proc. 26-th Intern. Cosmic Ray Conf.*, **7**, 77-80 (1999a)
- Ferreira S.E.S., M.S. Potgieter and R.A. Burger "Implications of increased perpendicular diffusion on the tilt angle dependence of electron modulation in the Heliosphere", *Proc. 26-th Intern. Cosmic Ray Conf.*, Salt Lake City, **7**, 53-56 (1999b).
- Fisk L.A. "Solar modulation of galactic cosmic rays", *J. Geophys. Res.*, **76**, No.1, 221-226 (1971).
- Fisk L.A. "Solar modulation of galactic cosmic rays. 4. Latitude-dependent modulation", *J. Geophys. Res.*, **81**, No. 25, 4646-4650 (1976).
- Fisk L.A. "Motion of the footpoints of heliospheric magnetic field lines at the Sun: implications for recurrent energetic particle events at high heliographic latitudes", *J. Geophys. Res.*, **101**, No. A7, 15547-15553 (1996).
- Fisk L.A. and W.I. Axford "Anisotropies of solar cosmic rays", *Solar Phys.*, **7**, No. 3, 486-498 (1969).
- Fisk L.A., M.A. Forman, and W.I. Axford "Solar modulation of galactic cosmic rays. 3. Implications of the Compton-Getting coefficient", *J. Geophys. Res.*, **78**, No. 7, 995-1006 (1973).
- Flückiger E.O. and E. Köbel "The ground level enhancement on 22 October 1989 as a test of the Tsyganenko (1989) magnetospheric magnetic field model", *Proc. 23-rd Intern. Cosmic Ray Conf.*, Calgary, **3**, 793-796 (1993).
- Forbush S.E. "World-wide changes in cosmic ray intensity", *Rev. Mod. Phys.*, **11**, No. 3-4, 168-172 (1939).
- Forbush S.E. "Cosmic-Ray Intensity Variations during Two Solar Cycles", *J. Geophys. Res.*, **63**, No. , 651- (1958)
- Forman M.A. "Possible theoretical explanations for occasional days of non-field-aligned, diffusion at neutron monitor energies", *Proc. 14th Intern. Cosmic Ray Conf.*, Munchen, **3**, 899-903 (1975).
- Forman M.A. "The velocity correlation function in cosmic ray diffusion theory", *Astrophys. Space Sci.*, **49**, No. 1, 83-97 (1977a).
- Forman M.A. "The velocity correlation function in cosmic ray diffusion theory", *Proc. 15th Intern. Cosmic Ray Conf.*, Plovdiv, **3**, 1-4 (1977b).
- Forman M.A., J.R. Jokipii, and A.J. Owens "Cosmic-ray streaming perpendicular to the mean magnetic field", *Astrophys. J.*, **192**, No. 2, 535-540 (1974)
- Fradkin E.S. "The method of Green's functions in quantized-field theory and in quantum statistics", *Trans. FIAN*, **29**, 3-139 (1965).
- Fujii Z. and F.B. McDonald "Study of the properties of the step decreases in galactic and anomalous cosmic rays over solar cycle 21", *J. Geophys. Res.*, **100**, Issue A9, 17043-17052 (1995).
- Gall R., R. Perez-Enriquez, and J. Otaola "In search of interplanetary magnetic cutoffs", *Proc. 15th Intern. Cosmic Ray Conf.*, Plovdiv, **3**, 130-132 (1977).
- Galperin B.A., I.N. Toptygin, and A.A. Fradkin "Particle scattering by magnetic inhomogeneities in an intense magnetic field", *ZhETF*, **60**, No. 3, 972-981 (1971).
- Gary S.P. and W.C. Feldman "A second-order theory for $\mathbf{k}$ parallel $\mathbf{B}_0$ electromagnetic instabilities", *Phys. Fluids*, **21**, No. 1, 72-80 (1978).
- Geisel T., J. Nierwetberg, and A. Zacherl "Accelerated diffusion in Josephson junctions and related chaotic systems", *Phys. Rev. Lett.*, **54**, No. 7, 616-619 (1985).
- Getmantsev G.G. "On the isotropy of primary cosmic rays", *Soviet Astron., A. J.*, **6**, No. 4, 477-479 (1963).

- Giacalone J. "Cosmic-Ray Transport Coefficients", *Space Sci. Rev.*, **83**, No. 1-2, 351-363 (1998).
- Giacalone J. and J.R. Jokipii "Charged-particle motion in multidimensional magnetic-field turbulence", *Astrophys. J.*, **430**, No. 2, L137-L140 (1994).
- Giacalone J. and J.R. Jokipii "Perpendicular transport in shock acceleration", *J. Geophys. Res.*, **101**, No. A5, 11095-11105 (1996).
- Giacalone J. and J.R. Jokipii "The transport of cosmic rays across a turbulent magnetic field", *Astrophys. J.*, **520**, No. 1, 204-214 (1999).
- Giacalone J., J.R. Jokipii, and J. Kóta "Particle drifts in a fluctuating magnetic field", *Proc. 26-th Intern. Cosmic Ray Conf.*, Salt Lake City, **7**, 37-40 (1999).
- Ginzburg V.L., S.B. Pikelner, and I.S. Shklovsky "On the problem of the mechanism of particle acceleration in nova and supernova shells", *Astronom. Zh.*, **32**, No. 6, 503-513 (1955).
- Ginzburg V.L. and V.S. Ptuskin "On the origin of cosmic rays: Some problems in high-energy astrophysics", *Rev. Mod. Phys.*, **48**, No. 2, 161-189 (1976).
- Gleeson L.J. and W.I. Axford "Cosmic rays in the interplanetary medium", *Astrophys. J.*, **149**, No. 3, L115-L118 (1967).
- Goldstein M.L. "A nonlinear theory of cosmic ray pitch-angle diffusion in homogeneous magnetostatic turbulence", *Astrophys. J.*, **204**, No. 3, 900-919 (1976).
- Goldstein M.L. "Consequences of using nonlinear particle trajectories to compute spatial diffusion coefficients", *J. Geophys. Res.*, **82**, No. 7, 1071-1076 (1977).
- Goldstein M.L. and W.H. Matthaeus "The role of magnetic helicity in cosmic ray transport theory", *Proc. 17th Intern. Cosmic Ray Conf.*, Paris, **3**, 294-297 (1981).
- Gombosi T.I., J.R. Jokipii, J. Kóta, K. Lorencz, and L.L. Williams "The telegraph equation in charged particle transport", *Astrophys. J.*, **403**, No. 1, 377-384 (1993).
- Haasbroek L.J., M.S. Potgieter, and J.A. le Roux "Some Modulation Effects of Termination Shock Accelerated Galactic and Jovian Electrons", *Proc. 25th Intern. Cosmic Ray Conf.*, Durban, **2**, 29-32 (1997).
- Hada et al., 2003a: Hada T., F. Otsuka, Y. Kuramitsu, and B.T. Tsurutani "Pitch angle diffusion of energetic particles by large amplitude MHD waves", *Proc. 28th Intern. Cosmic Ray Conf.*, Tsukuba, **6**, 3709-3712 (2003a).
- Hada et al., 2003b: Hada T., D. Koga, and E. Yamamoto, "Phase coherence of MHD waves in the solar wind", *Space Sci. Rev.*, **107**, No. 1, 463-466 (2003b).
- Hasselmann K. and G. Wibberenz "A note on the parallel diffusion coefficient", *Astrophys. J.*, **162**, No. 3, 1049-1051 (1970).
- Hattingh M., *M.Sc.-Dissertation*, Potchefstroom University for CHE, Potchefstroom. South Africa (1993).
- Hattingh M., *Ph.D.-Thesis*, Potchefstroom University for Christian Higher Education, Potchefstroom, South Africa (1998).
- Hattingh M. and R.A. Burger "A new simulated wavy neutral sheet drift model", *Adv. Space Res.*, **16**, No. 9, 213-216 (1995a).
- Hattingh M. and R.A. Burger "Some properties of a fully three-dimensional drift model for the modulation of galactic cosmic rays", *Proc. 24-th Intern. Cosmic Ray Conf.*, Rome, 1995, **4**, 337-340 (1995).
- Hattingh M., R.A. Burger, G. Wibberenz, and B. Heber "A 3D Simulation of Intensities along Ulysses' Trajectory", *Proc. 25-th Intern. Cosmic Ray Conf.*, Durban, **2**, 17-20 (1997).

- Heber B., W. Dröge, P. Ferrando, L.J. Haasbroek, H. Kunow, R. Müller-Mellin, C. Paizis, M.S. Potgieter, A. Raviart, and G. Wibberenz "Spatial variation of >40 MeV/n nuclei fluxes observed during the Ulysses rapid latitude scan", *Astronomy and Astrophysics*, **316**, No. 2, 538-546 (1996).
- Hoerandel J.R. "Models of the knee in the energy spectrum of cosmic rays", *Astropart. Phys.*, **21**, Issue 3, 241-265 (2004).
- Hopf E. "Statistical hydrodynamics and functional calculus", *J. Rat. Mech. Anal.*, **1**, No. 1, 87-123 (1952).
- Hoppe M.M., C.T. Russell, L.A. Frank, T.E. Eastman, and E.W. Greenstadt, "Upstream hydromagnetic waves and their association with backstreaming ion populations - ISEE 1 and 2 observations", *J. Geophys. Res.*, **86**, No. A6, 4471-4492 (1981).
- Horbury T.S. "Waves and turbulence in the solar wind - an overview", in *Plasma Turbulence and Energetic Particles in Astrophysics, Proc. Intern. Conf.* (eds. M. Ostrowski and R. Schlickeiser), Cracow (Poland), 5-10 September 1999, Obserwatorium Astronomiczne, Uniwersytet Jagielloński, Kraków, 115-134 (1999).
- Hostler L. "Coulomb Green's functions and the Furry approximation", *J. Math. Phys.*, **5**, No. 5, 591-611 (1964).
- Hu Y.Q., S.R. Habbal, and X. Li "On the cascade processes of Alfvén waves in the fast solar wind", *J. Geophys. Res.*, **104**, No. A11, 24819-24834 (1999).
- Humble J.E. and P.R. Pelechaty "Interplanetary cosmic-ray trajectories", *Proc. 15th Intern. Cosmic Ray Conf.*, Plovdiv, **3**, 5-5 (1977).
- Isenberg P.A. "A hemispherical model of anisotropic interstellar pickup ions", *J. Geophys. Res.*, **102**, No. A3, 4719-4724 (1997).
- Isenberg P.A. and J.R. Jokipii "Effects of particle drift on cosmic ray transport, II. Analytical solution to the modulation problem with no latitudinal diffusion", *Astrophys. J.*, **219**, No. 2, 740-749 (1978).
- Islaker H. and L. Vlahos "Random walk through fractal environments", *Phys. Rev.*, **E67**, No. 2, 026413: 1-21 (2003).
- Jokipii J.R. "Cosmic-ray propagation. I. Charged particles in a random magnetic field", *Astrophys. J.*, **146**, No. 2, 480-487 (1966).
- Jokipii J.R. "Propagation of cosmic rays in the solar wind", *Rev. Geophys. Space Phys.*, **9**, No. 1, 27-87 (1971).
- Jokipii J.R. "Particle drifts for a finite scattering rate", *Proc 22-th Intern. Cosmic Ray Conf.*, Calgary, **3**, 497-500 (1993).
- Jokipii J.R. and T. Barry "Effects of drift on the transport of cosmic rays. IV. Modulation by a wavy interplanetary current sheet", *Astrophys. J.*, Part 1, **243**, 1115-1122 (1981).
- Jokipii J.R. and P. Coleman "Cosmic ray diffusion tensor and its variation observed with Mariner 4", *J. Geophys. Res.*, **73**, No. 17, 5495-5503 (1968).
- Jokipii J.R. and J.M. Davila "Effects of Particle Drift on the Transport of Cosmic Rays. IV. More Realistic Diffusion Coefficients", *Astrophys. J.*, Part 1, **248**, 1156-1161 (1981).
- Jokipii et al., 1977: Jokipii J.R., E.H. Levy, and W.B. Hubbard "Effects of particle drift on cosmic ray transport, 1. General properties application to solar modulation", *Astrophys. J.*, **213**, No. 2, 861-868 (1977).

- Jokipii et al., 1993: Jokipii J.R., J. Kóta, and J. Giacalone "Prependicular transport in 1- and 2-dimensional shock simulations", *J. Geophys. Res. Lett.*, **20**, No. 17, 1759-1761 (1993).
- Jokipii et al., 1995: Jokipii J.R., J. Giacalone, F.C. Jones, and J. Kóta "Numerical simulations and analytic theory of cross-field transport", *Proc. 24th Intern. Cosmic Ray Conf.*, Rome, **4**, 329-332 (1995).
- Jokipii J.R. and J. Kóta "The polar heliospheric magnetic field", *Geophys. Res. Lett.*, **16**, No. 1, 1-4 (1989).
- Jokipii J.R. and J. Kóta "The Maximum Energy of Anomalous Cosmic Rays", *Proc. 24-th Intern. Cosmic Ray Conf.*, Rome, **4**, 718-721 (1995).
- Jokipii J.R. and J. Kóta "Velocity correlation functions and cosmic-ray transport", *Proc. 26th Intern. Cosmic Ray Conf.*, Salt Lake City, **7**, 29-32 (1999).
- Jokipii J.R. and E.H. Levy "Effects of particle drifts on the solar modulation of galactic cosmic rays", *Astrophys. J.*, **213**, No. 2, L85-L88 (1977).
- Jokipii J.R. and A.J. Owens "Cosmic ray scintillations. 4. The effects of non-field-aligned diffusion", *J. Geophys. Res.*, **81**, No. 13, 2094-2096 (1976).
- Jokipii J.R. and Parker E.N. "Cosmic-ray life and the stochastic nature of the galactic magnetic field", *Astrophys. J.*, **155**, No. 3, 799-806 (1969).
- Jokipii J.R. and Parker E.N. "On the convection, diffusion, and adiabatic deceleration of cosmic rays in the solar wind", *Astrophys. J.*, **160**, No. 2, 735-744 (1970).
- Jokipii J.R. and B. Thomas "Effects of drift on the transport of cosmic rays, IV. Modulation by a wavy interplanetary current sheet", *Astrophys. J.*, Part 1, **243**, 1115-1122 (1981).
- Jokipii J.R. and G. Wibberenz "Epilogue: Cosmic Rays in the Active Heliosphere", *Space Sci. Rev.*, **83**, No. 1-2, 365-368 (1998).
- Jones F.C. "Charged particle propagation in strong disordered magnetic fields", *Proc. 14th Intern. Cosmic Ray Conf.*, Munchen, **3**, 856-860 (1975).
- Jones et al., 1973: Jones F.C., T.B. Kaiser, and T.J. Birmingham "A new approach to cosmic ray diffusion theory", *Proc. 13th Intern. Cosmic Ray Conf.*, Denver, **2**, 669-674 (1973).
- Jones et al., 1978: Jones F.C., T.J. Birmingham, and T.B. Kaiser "Partially averaged field approach to cosmic ray diffusion", *Phys. Fluids*, **21**, No. 3, 347-360 (1978).
- Jones et al., 1998: Jones F.C., J.R. Jokipii, and M.G. Baring "Charged-particle motion in electromagnetic fields having at least one ignorable spatial coordinate", *Astrophys. J.*, **509**, No. 1, 238-243 (1998).
- Kaiser T.B. "Simulation of the velocity diffusion of charged particles in turbulent magnetic fields", *Proc. 14th Intern. Cosmic Ray Conf.*, Munchen, **3**, 861-865 (1975).
- Kaiser T.B., T.J. Birmingham, and F.C. Jones "Computer simulation of the velocity diffusion of cosmic rays", *Phys. Fluids*, **21**, No. 3, 361-373 (1978).
- Kallenrode M.-B. "The temporal and spatial development of MeV proton acceleration at interplanetary shocks", *J. Geophys. Res.*, **102**, No. A10, 22347-22364 (1997).
- Kallenrode M.-B. "The influence of magnetic clouds on the propagation of energetic charged particles in interplanetary space", *Proc. 27th Intern. Cosmic Ray Conf.*, Hamburg, **8**, 3273-3276 (2001a).
- Kallenrode M.-B. "Shock as a black box II: Effects of adiabatic deceleration and convection included", *J. Geophys. Res.*, in press (2001b).

- Kallenrode M.-B., and E.W. Cliver "Rogue SEP events: Modeling", *Proc. 27th Intern. Cosmic Ray Conf.*, Hamburg, **8**, 3318-3321 (2001).
- Kallenrode M.-B. and G. Wibberenz "Propagation of particles injected from interplanetary shocks: A black box model and its consequences for acceleration theory and data interpretation", *J. Geophys. Res.*, **102**, No. A10, 22311-22334 (1997).
- Kassovicova J. and K. Kudela "On the computations of cosmic ray trajectories in the geomagnetic field, *Preprint IEP SAS*, Kosice, 1-12 (1998).
- Katz M.E. "Study of cosmic ray propagation in interplanetary space on the basis of kinetic equation", *Ph. D. Thesis*, IZMIRAN-Moscow State University, Moscow (1973).
- Katz M.E. and V.M. Yacobi "Cosmic ray transport in the large-scale random magnetic field", *Proc. 25th Intern. Cosmic Ray Conf.*, Durban, **1**, 225-228 (1997).
- Kennel C.F. and F. Engelmann "Velocity space diffusion from weak plasma turbulence in a magnetic field", *Phys.Fluids*, **9**, No. 12, 2377-2388 (1966).
- Klafter J., G. Zumofen, and M.F. Shlesinger "Levy walks in dynamical-systems", *Physica A.*, **200**, No. 1-4, 222-230 (1993).
- Klimas A. and G. Sandri "The parallel diffusion coefficient for cosmic rays in a turbulent magnetic field", *Proc. 13th Intern. Cosmic Ray Conf.*, Denver, **2**, 659-662 (1973).
- Klimas A.J. and G. Sandri "A time-averaged cosmic ray propagation theory", *Proc. 14th Intern. Cosmic Ray Conf.*, Munchen, **3**, 844-849 (1975).
- Klyatskin V.I. and V.I. Tatarsky "Approximation of the diffusive random process in some nonstationary problems of physics", *Uspekhi Fiz. Nauk*, **110**, No. 4, 499-536 (1973).
- Kohl J.L., G. Noci, E. Antonucci, G. Tondello, M.C.E. Huber, S.R. Cranmer, L. Strachan, M.V. Panasyuk, L.D. Gardner, M. Romoli, S. Fineschi, D. Dobrzycka, J.C. Raymond, P. Nicolosi, O.H.W. Siegmund, D. Spadaro, C. Benna, A. Ciaravella, S. Giordano, S.R. Habbal, M. Karovska, X. Li, R. Martin, J.G. Michels, A. Modigliani, G. Naletto, R.H. O'Neal, C. Pernechele, G. Poletto, P.L. Smith, and R.M. Suleiman "UVCS/SOHO empirical determinations of anisotropic velocity distributions in the solar corona", *Astrophys. J.*, **501**, L127-L131 (1998).
- Korobko D. "Modelling of multiple scattering in fractal-like medium", *Ph.D. Thesis*, Ulyanovsk (1999).
- Kóta J. "A note on the perpendicular diffusion of cosmic rays", *Proc. 23-rd Intern. Cosmic Ray Conf.*, Calgary, **3**, 290-293 (1993).
- Kóta J. "Coherent pulses in the diffusive transport of charged particles", *Astrophys. J.*, **427**, No. 2, 1035-1041 (1994).
- Kóta J. "Transport coefficients in stochastic magnetic fields", *Proc. 24th Intern. Cosmic Ray Conf.*, Rome, **4**, 333-336 (1995).
- Kóta J. "Dispersion relations for diffusive motion", *Proc. 26th Intern. Cosmic Ray Conf.*, Salt Lake City, **6**, 362-365 (1999).
- Kóta J. "Diffusion of energetic particles in focusing fields", *J. Geophys. Res.*, **105**, No. A2, 2403-2411 (2000).
- Kóta J. and J.R. Jokipii "Effects of drift on the transport of cosmic rays", *Astrophys. J.*, **265**, No. 1, Part 1, 573-581 (1983).
- Kóta J. and J.R. Jokipii "3-D distribution of cosmic rays in the outer Heliosphere", *Proc. 24-th Intern. Cosmic Ray Conf.*, Rome, **4**, 680-683 (1995).
- Kóta J. and J.R. Jokipii "3-D simulations of Heliospheric transport: A comparison of models", *Proc. 25th Intern. Cosmic Ray Conf.*, Durban, **2**, 25-28 (1997).

- Kóta J. and J.R. Jokipii "Cosmic-ray modulation and the structure of the Heliospheric magnetic field", *Proc. 26th Intern. Cosmic Ray Conf.*, Salt Lake City, 7, 9-12 (1999).
- Krainev M.B. and Yu.I. Stozhkov "On the solar magnetic cycle in the isotropic and anisotropic components of the galactic cosmic rays", *Proc. 15th Intern. Cosmic Ray Conf.*, Plovdiv, 3, 204 (1977).
- Krymsky G.F. "Diffusive mechanism of the diurnal cosmic ray variation", *Geomagnetism and Aeronomy*, 4, No. 6, 977-987 (1964).
- Krymsky et al., 1975: Krymsky G.F., I.A. Transky, G.V. Shafer, V.P. Mamrukova, and V.K. Elshin "Shock models and observed properties of Forbush effects", In *Results into Cosmophysics and Aeronomy*, Yakutsk, 58-68 (1975).
- Krymsky et al., 1976: Krymsky G.F., S.I. Petukhov, I.A. Transky, V.P. Mamrukova, and G.V. Shafer "Cosmic ray distribution in the vicinity of the front of a non-stationary shock wave", *Geomagnetism and Aeronomy*, 16, No. 1, 25-29 (1976).
- Krymsky G.F. and I.A. Transky "Convective shock waves in interplanetary space", In *Results into Cosmophysics and Aeronomy*, Yakutsk, 36-57 (1975).
- Krymsky G.F. and I.A. Transky "The Forbush-decrease profile and convective shock waves in the interplanetary medium", *Proc. 15th Intern. Cosmic Ray Conf.*, Plovdiv, 3, 300-304 (1977).
- Kubo R. "Statistical-Mechanical theory of irreversible processes, 1. General theory and simple applications to magnetic and conduction problems", *J. Phys. Soc. Japan*, 2, No. 6, 570-586 (1957).
- Kudo S. and M. Wada "Eleven-year variation of cosmic ray intensity and heliolatitudes of sunspots", *Report of Ionosp. and Space Res. in Japan*, 22, No. 3, 137-147 (1968).
- Kunstmann J.E. "A new transport mode for energetic charged particles in magnetic fluctuations superposed on a diverging mean field", *Astrophys. J.*, Part 1, 229, No. 2, 812-820 (1979).
- Kuramitsu Y and T. Hada "Acceleration of charged particles by large amplitude MHD waves: effect of wave spatial correlation", *Geophys. Res. Lett.*, 27, No. 5, 629-632 (2000).
- Lagutin et al., 2001a: Lagutin A.A., V.V. Makarov, and A.G. Tyumentsev "Anomalous diffusion of the cosmic ray: steady-state solution", *Proc. 27th Intern. Cosmic Ray Conf.*, Hamburg, 5, 1889-1891 (2001a)
- Lagutin et al., 2001b: Lagutin A.A., Yu.A. Nikulin, and V.V. Uchaikin, "The "knee" in the primary cosmic ray spectrum as consequence of the anomalous diffusion of the particles in the fractal interstellar medium", *Nucl. Phys. B, Proc. Suppl.*, 97B, Issue 1-3, 267-270 (2001b).
- Lagutin et al., 2001c: Lagutin A.A., K.I. Osadchiy, and D.V. Strelnikov "Propagation of cosmic ray electrons in the Galaxy", *Proc. 27th Intern. Cosmic Ray Conf.*, Hamburg, 5, 1852-1855 (2001c).
- Lagutin et al., 2001d: Lagutin A.A., D.V. Strelnikov, and A.G. Tyumentsev "Mass composition of cosmic rays in anomalous diffusion model: comparison with experiment", *Proc. 27th Intern. Cosmic Ray Conf.*, Hamburg, 5, 1896-1899 (2001d).
- Lagutin et al., 2004: Lagutin A.A., R.I. Raikin, and A.G. Tyumentsev " ", *Izv. ASU*, 5, 27- (2004)

- Lagutin et al., 2005: Lagutin A.A., R.I. Raikin, A.G. Tyumentsev, and A.V. Yushkov "Cosmic rays transport in the fractal-like galactic medium", *Proc. 29-th Intern. Cosmic Ray Conf.*, Pune, **3**, 197-200 (2005).
- Lagutin A.A. and A.G. Tyumentsev " ", *Izv. ASU*, **5**, 4- (2004).
- Lagutin A.A. and V.V. Uchaikin "Anomalous diffusion equation: Application to cosmic ray transport", *Nucl. Instrum. Meth. Phys. Res.*, **201B**, Issue 1, 212-216 (2003).
- Laitinen T., H. Fichtner, and R. Vainio "Toward a self-consistent treatment of the cyclotron wave heating and acceleration of the solar wind plasma", *J. Geophys. Res.*, **108**, No. A2, SSH 9-1, CiteID 1081, DOI 10.1029/2002JA009479 (2003).
- Lario et al., 1998: Lario D., B. Sanahuja, and A.M. Heras "Energetic particle events: efficiency of interplanetary shocks as 50 keV < E < 100 MeV protons accelerators", *Astrophys. J.*, **509**, No. 1, 415-434. (1998).
- Lario et al., 1999: Lario D., M. Vandas, and B. Sanahuja "Energetic particle propagation in the downstream region of transient interplanetary shocks", in *Solar Wind 9, Proceedings of the Ninth International Solar Wind Conference*, Nantucket, MA, October 1998, (eds. S.R. Habbal, R. Esser, J.V. Hollweg, and P.A. Isenberg), AIP Conference Proceedings, **471**, 741-744 (1999).
- Leamon et al., 1998: Leamon R.J., C.W. Smith, N.F. Ness, W.H. Matthaeus, and H.K.Wong, "Observational constraints on the dynamics of the interplanetary magnetic field dissipation range", *J. Geophys. Res.*, **103**, No A3, 4775-4787 (1998).
- Leamon et al., 1999a: Leamon R.J., N.F. Ness, and C.W. Smith "The dynamics of dissipation range fluctuations with application to cosmic ray propagation theory", *Proc. 26th Intern. Cosmic Ray Conf.*, Salt Lake City, **6**, 366-369 (1999a).
- Leamon et al., 1999b: Leamon R.J., C.W. Smith, N.F. Ness, and H.K.Wong, "Dissipation range dynamics: Kinetic Alfvén waves and the importance of β_e ", *J. Geophys. Res.*, **104**, No. A10, 22331-22344 (1999b).
- Lee M.A. "Self-consistent kinetic equations and the evolution of a relativistic plasma in an ambient magnetic field", *Plasma Physics*, **13**, No. 12, 1079-1098 (1971).
- Lee M.A. and L.A. Fisk "The Role of Particle Drifts in Solar Modulation", *Astrophys. J.*, Part 1, **248**, 836-844 (1981).
- Lee M.A. and H.J. Völk "Hydromagnetic waves and cosmic ray diffusion theory", *Astrophys. J.*, **198**, No. 2, 485-492 (1975a).
- Lee M.A. and H.J. Völk "Hydromagnetic waves and cosmic ray diffusion theory", *Proc. 14th Intern. Cosmic Ray Conf.*, Munchen, **3**, 838-843 (1975b).
- le Roux et al., 1999a: le Roux J.A., G.P. Zank, V.S. Ptuskin, and H. Moraal "Cosmic ray diffusion coefficients determined for different models of perpendicular diffusion on the basis of a MHD transport model for solar wind turbulence", *Proc. 26-th Intern. Cosmic Ray Conf.*, Salt Lake City, **7**, 65-68 (1999a).
- le Roux et al., 1999b: le Roux J.A., G.P. Zank, and V.S. Ptuskin, "An evaluation of perpendicular diffusion models regarding cosmic ray modulation on the basis of a hydromagnetic description for solar wind turbulence", *J. Geophys. Res.*, **104**, Issue A11, 24845-24862 (1999b).
- le Roux J.A. and H. Fichtner, The influence of pickup, anomalous, and galactic cosmic ray protons on the structure of the heliospheric shock: a self consistent approach, *Astrophys. J.*, **477**, L115-L118, 1997.
- le Roux J.A. and M.S. Potgieter "A time-dependent drift model for the long-term modulation of cosmic rays with special reference to asymmetries with respect to the solar minimum of 1987", *Astrophys. J.*, **361**, No. 1, Part 1, 275-282 (1990).

- le Roux J.A. and M.S. Potgieter "The simulation of Forbush decreases with time-dependent cosmic-ray modulation models of varying complexity", *Astron. and Astrophys.*, **243**, No. 2, 531-545 (1991).
- Levy E.H. "Origin of the twenty year wave in the diurnal variation", *Proc. 14-th Intern. Cosmic Ray Conf.*, Munich, **4**, 1215-1220 (1975).
- Levy E.H. "Theory of solar magnetic cycle wave in diurnal-variation of energetic cosmic-rays - physical basis of anisotropy", *J. Geophys. Res.*, **81**, No. 13, 2082-2088 (1976a).
- Levy E.H. "The interplanetary magnetic field structure, *Nature*, **261**, No. 5559, 394-395 (1976b).
- Levy E.H., S.P. Duggal, and M.A. Pomerantz "Adiabatic Fermi acceleration of energetic particles between converging interplanetary shock waves", *J. Geophys. Res.*, **81**, No. 1, 51-59 (1976).
- Lingenfelter R.E., R. Ramaty, and L.A. Fisk "Compound diffusion of cosmic rays", *Astrophys. Lett.*, **8**, No.2, 93-97 (1971).
- Lu J.Y., G.P. Zank, and G.M. Webb "Numerical solution of the time-dependent kinetic equation for anisotropic pitch-angle scattering", *Astrophys. J.*, **550**, No. 1, 34-51 (2001).
- Lumme M., M. Nieminen, J.J. Torsti, E. Vainikka, and J. Peltonen "Interplanetary propagation of relativistic solar protons", *Solar Phys.*, **107**, No. 1, 183-194 (1986).
- Lupton J.E. and E.C. Stone "Solar flare particle propagation: Comparison of a new analytic solution with spacecraft measurements", *J. Geophys. Res.*, **78**, No. 7, 1007-1018 (1973).
- Mandelbrot B.B. and J.W. Van Ness, "Fractional Brownian motion, fractional noises and applications", *SIAM Review*, **10**, No. 4, 422-437 (1968).
- Matthaeus W.H., M.L. Goldstein, and D.A. Roberts "Evidence for the presence of quasi-two-dimensional nearly incompressible fluctuations in the solar wind", *J. Geophys. Res.*, **95**, No. , Dec. 1, 20673-20683 (1990).
- Matthaeus W.H., P.C. Gray, J.R. Pontius, Jr., and J.W. Bieber "Spatial Structure and Field-Line Diffusion in Transverse Magnetic Turbulence", *Phys. Rev. Lett.*, **75**, Issue 11, 2136-2139 (1995).
- Matthaeus W.W. and C. Smith "Structure of correlation tensors in homogeneous anisotropic turbulence", *Phys. Rev. A*, **24**, No. 4, 2135-2144 (1981).
- Mavromichalaki H., V. Yanke, L. Dorman, N. Iucci, A. Chilingaryan, and O. Kryakunova, "Neutron Monitor Network in Real Time and Space Weather", *Effects of Space Weather on Technology Infrastructure*, ed. by I.A. Daglis, NATO Science Series II, Mathematics, Physics and Chemistry, 176, Kluwer Academic Publishers, Dordrecht, 301-317 (2004).
- Mccay J. and H. Andersen "Analysis of time variations in the interplanetary radiation intensity observed by Mariner 4", *J. Geophys. Res.*, **73**, No. 9, 2911-2917 (1968).
- McKibben R.B. "Three-Dimensional Solar Modulation of Cosmic Rays and Anomalous Components in the Inner Heliosphere", *Space Sci. Rev.*, **83**, No. 1-2, 21-32 (1998).
- Mel'nikov Yu.P. "A theory of particle diffusion in a magnetic field with strong small-scale random scattering", *JETP*, **82**, Issue 5, 860-874 (1996).
- Mel'nikov Yu.P. "Diffusion of Charged Particles in Strong Large-Scale Random and Regular Magnetic Fields", *JETP*, **90**, Issue 3, 460-469 (2000).

- Mel'nikov Yu.P. "Strong cosmic-ray scattering in large-scale anisotropic random and regular magnetic fields", *Proc. 29-th Intern. Cosmic Ray Conf.*, Pune, Paper rus-melnikov-Y-abs1-sh25-poster (2005a)
- Mel'nikov Yu.P. "The basic correlation lengths of the interplanetary magnetic field fluctuations", *Proc. 29-th Intern. Cosmic Ray Conf.*, Pune, Paper rus-melnikov-Y-abs2-sh25-poster (2005b)
- Mel'nikov Yu.P. "Cosmic ray diffusion with an allowance for anisotropy the interplanetary magnetic field", *Proc. 29-th Intern. Cosmic Ray Conf.*, Pune, Paper rus-melnikov-Y-abs3-sh25-poster (2005c)
- Messerschmidt W. "Über Schwankungsmessungen der Ultrastrahlung, 2", *Z. Phys.*, **85**, No. 3, 332-335 (1933).
- Meyer P., E.N. Parker, and J.A. Simpson "Solar cosmic rays of February, 1956 and their propagation through interplanetary space", *Phys. Rev.*, **104**, No. 3, 768-783 (1956).
- Michalek G. and M. Ostrowski "Cosmic ray momentum diffusion in the presence of nonlinear Alfvén waves", *Nonlinear Processes in Geophys.*, **3**, No. 1, 66-76 (1996).
- Michalek G. and M. Ostrowski "Simulations of cosmic ray cross field diffusion in highly perturbed magnetic fields", *Astronomy and Astrophysics*, **326**, No. 2, 793-800 (1997).
- Michalek G. and M. Ostrowski "On the cosmic ray cross field diffusion in the presence of oblique MHD waves", *Astronomy and Astrophysics*, **337**, No. 2, 558-564 (1998).
- Michalek G. and M. Ostrowski "On the cosmic ray cross-field diffusion in the presence of highly perturbed magnetic fields", *Proc. 26th Intern. Cosmic Ray Conf.*, Salt Lake City, **6**, 340-343 (1999).
- Minter A.H. and S.R. Spangler "Observation of Turbulent Fluctuations in the Interstellar Plasma Density and Magnetic Field on Spatial Scales of 0.01 to 100 Parsecs", *Astrophys. J.*, **458**, No. 1, 194-214 (1996).
- Moraal H., C.D. Steenberg, and G.P. Zank "Simulations of Galactic and anomalous cosmic ray transport in the Heliosphere", *Adv. Space Res.*, **23**, No. 3, 425-436 (1999).
- Morfill G.E., H.J. Völk, and M.A. Lee "On the effect of directional medium-scale interplanetary variations on the diffusion of galactic cosmic rays and their solar cycle variation", *J. Geophys. Res.*, **81**, No. 34, 5841-5852 (1976).
- Morishita I., S. Sakakibara, Z. Fujii, Y. Muraki, K. Koi, and K. Takahashi "Anisotropic space distribution of solar particles of the GLE observed on 24 May 1990", *Proc. 24th Intern. Cosmic Ray Conf.*, Rome, **4**, 220-223 (1995).
- Moussas X. and J.J. Quenby "Numerical simulation of energetic particle-propagation employing interplanetary magnetic field data", *Proc. 15th Intern. Cosmic Ray Conf.*, Plovdiv, **3**, 34-39 (1977).
- Moussas X., J.J. Quenby, and S. Webb "Numerical experiments on energetic particle scattering in pitch angle", *Proc. 14th Intern. Cosmic Ray Conf.*, Munchen, **3**, 866-871 (1975).
- Neher H.V. and Anderson H.R. "Cosmic Rays at Balloon Altitudes and the Solar Cycle", *J. Geophys. Res.*, **67**, 1309-1315 (1962).
- Neher H.V. and H.R. Anderson "Change of cosmic ray intensity with distance from the Sun", *J. Geophys. Res.*, **69**, No. 9, 1911-1913 (1964).
- Newberger B.S. "New sum rule for products of Bessel functions with application to plasma physics", *J. Math. Phys.*, **23**, No. 7, 1278-1281 (1982).

- Olinto A.V. "The origin of ultra-high energy cosmic ray: new physics and astrophysics", *Nucl. Phys. B, Proc. Suppl.*, **97B**, Issue 1-3, 66-77 (2001).
- Otsuka F. and Hada T. "Cross field diffusion of cosmic rays in a two-dimensional magnetic field turbulence", *Space Sci. Rev.*, **107**, No. 1-2, 499-502 (2003).
- Oughton Sean "Transport of solar wind fluctuations: A turbulence approach", *Ph.D. Thesis*, Univ. of Delaware, 1-224 (1993).
- Palmer I.D. "Transport coefficients of low-energy cosmic rays in interplanetary space", *Rev. Geophys. Space Phys.*, **20**, No. 2, 335-351 (1982).
- Parker E.N. "Cosmic ray modulation by solar wind", *Phys. Rev.*, **110**, 1445-1449 (1958).
- Parker E.N. "The passage of energetic charged particles through interplanetary space", *Planet. Space Sci.*, **13**, No. 1, 9-49 (1965).
- Parker E.N. "The effect of adiabatic deceleration on the cosmic ray spectrum in the solar system", *Planet. Space Sci.*, **14**, No. 4, 371-380 (1966).
- Parker E.N. "Cosmic ray diffusion energy loss, and the diurnal variation", *Planet. Space Sci.*, **15**, No. 11, 1723-1746 (1967).
- Pathak P.N. and V. Sarabhai "A study of the long-term modulation of galactic cosmic ray intensity", *Planet. Space Sci.*, **18**, Issue 1, 81-94 (1970).
- Pauls H.L., R.A. Burger, and J.W. Bieber "The Born approximation: A new telegrapher's equation for helicity-modified solar particle transport", *Proc. 23-rd Intern. Cosmic Ray Conf.*, Calgary, **3**, 183-186 (1993).
- Perez-Pereza J., A. Gallegos-Cruz, E.V. Vashenyuk, and L.I. Miroshnichenko "Spectrum of accelerated particles in solar proton events with a fast component", *Geomagnetism and Aeronomy*, **32**, No. 2, 1-12 (1992).
- Pinter S. "Propagation pattern of flare-generated IP shock waves deduced from multiple spacecraft observations", *Proc. 15th Intern. Cosmic Ray Conf.*, Plovdiv., **5**, 191-196 (1977).
- Pommois P., P. Veltri, and G. Zimbardo "Field line diffusion in solar wind magnetic turbulence and energetic particle propagation across heliographic latitudes", *J. Geophys. Res.*, **106**, No. A11, 24965-24978 (2001).
- Potgieter M.S. "Heliospheric modulation of Galactic electrons: consequences of new calculations for the mean free path of electrons between 1 MeV and ~10 GeV", *J. Geophys. Res.*, **101**, No. A11, 24411-24422 (1996).
- Potgieter M.S. "Implications of Enhanced Perpendicular Diffusion in the Heliospheric Modulation of Cosmic Rays", *Proc. 25th Intern. Cosmic Ray Conf.*, Durban, **2**, 1-4 (1997).
- Potgieter M.S. and R.A. Burger "The modulation of cosmic-ray electrons, positrons and helium nuclei as predicted by a drift model with a simulated wavy neutral sheet", *Astronomy and Astrophysics*, **233**, No. 2, 598-604 (1990).
- Potgieter et al., 1989: Potgieter M.S., J.A. le Roux, and R.A. Burger "Interplanetary cosmic ray radial gradients with steady state modulation models", *J. Geophys. Res.*, **94**, No. A3, 2323-2332 (1989).
- Potgieter et al., 1997: Potgieter M.S., L.J. Haasbroek, P. Ferrando, and B. Heber "The modelling of the latitude dependence of cosmic ray protons and electrons in the inner heliosphere", *Adv. Space Res.*, **19**, No. 6, 917-920 (1997).
- Potgieter et al., 1999: Potgieter M.S., S.E.S. Ferreira, and B. Heber "The Heliospheric modulation of cosmic ray electrons: rigidity dependence of the perpendicular diffusion coefficient", *Proc. 26-th Intern. Cosmic Ray Conf.*, Salt Lake City, 1999, **7**, 57-60 (1999).

- Potgieter M.S. and J.A. le Roux "The simulated features of heliospheric cosmic-ray modulation with a time-dependent drift model. I. General effects of the changing neutral sheet over the period 1985-1990", *Astrophys. J.*, **386**, No. 1, Part1, 336-346 (1992a).
- Potgieter M.S. and J.A. le Roux "The simulated features of heliospheric cosmic-ray modulation with a time-dependent drift model. III. General energy dependence", *Astrophys. J.*, **392**, No. 1, Part1, 300-309 (1992b).
- Potgieter M.S. and H. Moraal "A drift model for the modulation of galactic cosmic rays", *Astrophys. J.*, **294**, No. 2, Part 1, 425-440 (1985).
- Ptuskin V.S. "Diffusion of strongly magnetized cosmic ray particles in a turbulent medium", *Proc. 19th Intern. Cosmic Ray Conf.*, La Jolla, **3**, 75-78 (1985).
- Ptuskin V.S. "Transport of high energy cosmic rays", *Advances in Space Research*, **19**, Issue 5, 697-705 (1997).
- Ptuskin V.S. "Propagation, Confinement Models, and Large-Scale Dynamical Effects of Galactic Cosmic Rays", *Space Sci. Rev.*, **99**, No. 1, 281-293 (2001).
- Quenby J.J. "Cosmic rays and the interplanetary field", *Proc. 9th Intern. Cosmic Ray Conf.*, London, **1**, 3-13 (1965).
- Ragot B.R. "Non-resonant pitch-angle scattering, and parallel mean-free-path", *Proc. 26th Intern. Cosmic Ray Conf.*, Salt Lake City, **4**, 322-325 (1999a).
- Ragot B.R. "Nongyroresonant Pitch Angle Scattering", *Astrophys. J.*, **518**, No. 2, 974-984 (1999b).
- Ragot B.R. "Non-resonant pitch-angle scattering: comparison with the measurements for solar cosmic rays", *Proc. 26-th Intern. Cosmic Ray Conf.*, Salt Lake City, **7**, 83-86 (1999c).
- Ragot B.R. "On the quasi-linear transport of magnetic field lines", *Astrophys. J.*, **525**, No. 1, 524-532 (1999d).
- Ragot B.R. "Anomalous transport of magnetic field lines in quasilinear regime: Analytical expressions", *Proc. 27-th Intern. Cosmic Ray Conf.*, Hamburg, **8**, 3293-3296 (2001).
- Ragot B.R. and R. Schlickeiser "Cosmic ray acceleration by fast magnetosonic waves", *Astronomy and Astrophysics*, **331**, No. 3, 1066-1069 (1998a).
- Ragot B.R. and R. Schlickeiser "The acceleration of energetic particles by transit-time damping", *Astroparticle Phys.*, **9**, No. 1, 79-95 (1998b).
- Rao U.R. "Solar modulation of galactic cosmic radiation", *Space Sci. Rev.*, **12**, No. 6, 719-809 (1972).
- Roelof E.C. "Propagation of solar cosmic rays in the interplanetary magnetic field", in *Lectures in High Energy Astrophysics* (eds. H. Ögelmann and J.R. Wayland), NASA SP-199, 111-135 (1969).
- Romashets E., and M. Vandas "Dynamics of a toroidal magnetic cloud in the solar wind", *J. Geophys. Res.*, **106**, No. A6, 10615-10624 (2001).
- Ruffolo D. "Effect of adiabatic deceleration on the focused transport of solar cosmic rays", *Astroph. J.*, **442**, No. 2, 861-874 (1995).
- Ryutov D.D. "Contribution to the theory of beam heating of a plasma in an open trap", *Nucl. Fus.*, **9**, No. 4, 297-306 (1969).
- Sari J.W. and H.F. Ness "Power spectra of the interplanetary magnetic field", *Solar Phys.*, **8**, No. 1, 155-165 (1969).
- Sari J.W. and H.F. Ness "Power spectral studies of the interplanetary magnetic field", *Acta Acad. Sci., Hungaricae*, **29**, Suppl. 2, 373-378 (1970).

- Schlickeiser R. "Cosmic-ray transport and acceleration. I - Derivation of the kinetic equation and application to cosmic rays in static cold media. II - Cosmic rays in moving cold media with application to diffusive shock wave acceleration", *Astrophys. J.*, **336**, No. 1, 243-293 (1989).
- Schlickeiser R. and J.A. Miller "Quasi-linear theory of cosmic-ray transport and acceleration: The role of oblique magnetohydrodynamic waves and transit-time damping", *Astrophys. J.*, **492**, No. 1, 352-378 (1998).
- Schwadron N.A. "A model for pickup ion transport in the Heliosphere in the limit of uniform hemispheric distributions", *J. Geophys. Res.*, **103**, No. A9, 20643-20650 (1998).
- Schwinger J. "On the Green's functions of quantized fields", *Proc. Nat. Acad. Sci.*, **37**, 452-459 (1951).
- Scudder J. and A.J. Klimas "An examination of the adiabatic approximation in cosmic ray propagation theory", *Proc. 14th Intern. Cosmic Ray Conf.*, Munchen, **3**, 850-855 (1975).
- Shakhov B. and M. Stehlik "The small pitch angle scattering", *Proc. 27-th Intern. Cosmic Ray Conf.*, Hamburg, **8**, 3352-3355 (2001).
- Shalchi A. and R. Schlickeiser "The Parallel Mean Free Path of Heliospheric Cosmic Rays in Composite Slab/Two-dimensional Geometry. I. The Damping Model of Dynamical Turbulence", *Astrophys. J.*, **604**, Issue 2, 861-873 (2004).
- Shea M.A. and D.F. Smart "Vertical cutoff rigidities for cosmic ray stations since 1955", *Proc. 27th Intern. Cosmic Ray Conf.*, Hamburg, **3**, 4063-4066 (2001).
- Shishov V.I. "Intensity fluctuations of charged particles moving in random magnetic field", *ZhETF*, **55**, No. 4, 1449-1455 (1968).
- Shishov V.I. "Propagation of high-energy solar protons in interplanetary magnetic field", *Geomagnetism and Aeronomy*, **6**, No. 2, 223-230 (1966).
- Simpson J.A. "Recent Investigations of the Low Energy Cosmic and Solar Radiation", *Semaine d'Etude sur le Probleme du rayonnement cosmique dans l'espace interplanetaire*, Pontifical Academy of Sciences, Vatican City, 323-352 (1963).
- Simpson J.A. "Recurrent Solar Modulation of the Galactic Cosmic Rays and the Anomalous Nuclear Component in Three Dimensions of the Heliosphere", *Space Sci. Rev.*, **83**, No. 1-2, 7-19 (1998).
- Sivukhin D.V. "Drift theory of charged particle moving in electro-magnetic fields" In *Problems of Plasma Theory* (ed. M.A. Leontovich), Atomizdat, Moscow, Vol. 1, 7-97 (1963).
- Skilling J. "Cosmic ray streaming -I. Effect of Alfvén waves on particles", *Mon. Notic. Roy. Astron. Soc.*, **172**, No.3, 557-566 (1975).
- Smart D.F. and M.A. Shea "The concept of using the Deep River and Kerguelen neutron monitors as "flagship" stations for ground-level solar cosmic ray events", *Proc. 21-st Intern. Cosmic Ray Conf.*, Adelaide, **5**, 144-147 (1990).
- Smart D.F., M.A. Shea, and E.O. Flückiger "Unusual aspects of the ground-level cosmic ray event of 7/8 December 1982", *Proc. 20th Intern. Cosmic Ray Conf.*, Moscow, **3**, 135-138 (1987).
- Smart D.F., M.A. Shea, and E.O. Flückiger "Magnetospheric models and trajectory computations", *Space Sci. Rev.*, **93**, No. 1-2, 305-333 (2000).
- Smith C.W., J.W. Bieber, and W.H. Matthaeus "Cosmic-ray pitch angle scattering in isotropic turbulence. II - Sensitive dependence on the dissipation range spectrum", *Astrophys. J.*, **363**, No. 1, 283-291. (1990).

- Solomon T.H., E.R. Weeks, and H.L. Swinney "Observation of anomalous diffusion and Levy flights in a 2-dimensional rotating flow", *Phys. Rev. Lett.*, **71**, 3975–3978 (1993).
- Spangler S.R. "Multi-Scale Plasma Turbulence in the Diffuse Interstellar Medium", *Space Sci. Rev.*, **99**, No. 1, 261-270 (2001)
- Steinacker J. and J.A. Miller "Stochastic gyroresonant electron acceleration in a low-beta plasma. I - Interaction with parallel transverse cold plasma waves", *Astrophys. J.*, **393**, No. 2, 764-781 (1992).
- St.Cyr O.C., R.A. Howard, N.R. Sheeley, Jr., S.P. Plunkett, D.J. Michels, S.E. Paswaters, M.J. Koomen, G.M. Simnett, B.J. Thompson, J.B. Gurman, R. Schwenn, D.F. Webb, E. Hildner, and P.L. Lamy "Properties of coronal mass ejections: SOHO LASCO observations from January 1996 to June 1998", *J. Geophys. Res.*, **105**, No. A8, 18169-18186 (2000).
- Steinmaurer R. and H.T. Graziadei "Ergebnisse der Registrierung der Kosmischen Ultrastrahlung auf dem Hafelekar (2300 m) bei Innsbruck, II Teil: Meteorologische und Solare einflüsse auf die ultrastrahlung", *Berl. Ber. Zitz. Bericht. Acad. Wiss (Wien)*, **22**, No. 21/22, 672-685 (1933).
- Stozhkov Yu.I. and T.N. Gharakhchyan "11-year modulation of cosmic ray intensity relevant to heliolatitudinal distribution of sunspots", *Geomagn. and Aeron.*, **9**, No. 5, 803-808 (1969).
- Strong A.W., K. Bennett, H. Bloemen, R. Diehl, W. Hermsen, D. Morris, V. Schoenfelder, J.G. Stacy, C. de Vries, M. Varendorff, C. Winkler, and G. Youssefi "Diffuse continuum gamma rays from the Galaxy observed by COMPTEL", *Astronomy and Astrophysics*, **292**, No. 1, 82-91 (1994).
- Svalgaard L. and J.M. Wilcox "Structure of the extended solar magnetic field and the sunspot cycle variation in cosmic ray intensity", *Nature*, **262**, No. 5571, 766-768 (1976).
- Terasawa T. "Acceleration mechanisms for cometary ions", in *Cometary plasma processes* (ed. A.D. Johnstone), *Geophysical monograph*, **61**, American Geophysical Union, 277-286 (1991).
- Teufel A., L. Lerche, and R. Schlickeiser "Cosmic ray transport in anisotropic magneto-hydrodynamic turbulence. II. Shear Alfvén waves", *Astron. Astrophys.*, **397**, 777-788 (2003)
- Teufel A. and R. Schlickeiser "Analytic calculation of the parallel mean free path of heliospheric cosmic rays. I. Dynamical magnetic slab turbulence and random sweeping slab turbulence", *Astron. Astrophys.*, **393**, 703-715 (2002).
- Teufel A. and R. Schlickeiser "Analytic calculation of the parallel mean free path of heliospheric cosmic rays. II. Dynamical magnetic slab turbulence and random sweeping slab turbulence with finite wave power at small wavenumbers", *Astron. Astrophys.*, **397**, 15-25 (2003).
- Toptygin I.N. "The effect of a large-scale random field on transverse diffusion of particles", *Proc. 3-rd Leningrad. Intern. Seminar*, Leningrad, 184-190 (1971).
- Toptygin I.N. "Particle scattering in interplanetary space and the properties of solar corpuscular fluxes", *Izvestia Academy of Sci. of USSR, Series Phys.*, **36**, No. 11, 2258-2264 (1972).
- Toptygin I.N. "Interaction of fast particles with magneto-hydrodynamical turbulence", *Astrophys. and Space Sci.*, **20**, No. 2, 329-350 (1973a).

- Toptygin I.N. "Direct and inverse problems of cosmic ray propagation in interplanetary space", *Geomagnetism and Aeronomy*, **13**, No. 2, 212-218 (1973b).
- Toptygin I.N. and V.N. Vasilyev "On the interplanetary cosmic-ray scintillations", *Astrophys. and Space Sci.*, **48**, No. 2, 267-281 (1977).
- Torsti J., L.G. Kocharov, R. Vainio, A. Anttila, and G.A. Kovaltsov "The 1990 May 24 solar cosmic-ray event", *Solar Phys.*, **166**, No. 1, 135-158 (1996).
- Transky I.A., G.F. Krymsky, V.K. Elshin, and V.P. Mamrukova "Distribution of cosmic rays in inhomogeneous solar wind", In *Results into Cosmophysics and Aeronomy*, Yakutsk, 3-21 (1975).
- Tsurutani B.T., L.D. Zhang, G.L. Mason, G.S. Lakhina, T. Hada, J.K. Arballo, and R.D. Zwickl "Particle transport in 3 He-rich events: wave-particle interactions and particle anisotropy measurements", *Annales Geophys.*, **20**, No. 4, 427-444 (2002).
- Tsyganenko N.A. "A magnetospheric magnetic field model with a warped tail current sheet", *Planet. Space Sci.*, **37**, No. 1, 5-20 (1989).
- Tu C.-Y., Z.-Y. Pu, and F.-S. Wei "The power spectrum of interplanetary Alfvénic fluctuations. Derivation of the governing equation and its solution", *J. Geophys. Res.*, **89**, No. A11, 9695-9702 (1984).
- Tverskoy B.A. "On the theory of statistical Fermi acceleration", *ZhETF*, **52**, No. 2, 483-497 (1967a).
- Tverskoy B.A. "On the theory of turbulent acceleration of charged particles in plasma", *ZhETF*, **53**, No. 4, 1417-1430 (1967b).
- Tverskoy B.A. "Theory of fast-particle interactions with hydromagnetic turbulence", *Proc. 1-st Leningrad Intern. Seminar on Study of Interpl. Space by Means of Cosmic Rays*, Leningrad, 159-168 (1969).
- Urch I.H. and L.J. Gleeson "Galactic cosmic ray modulation from 1965-1970", *Astrophys. and Space Sci.*, **17**, No. 2, 426-446 (1972).
- Urch I.H. and L.J. Gleeson "Energy losses of galactic cosmic rays in the interplanetary medium", *Astrophys. and Space Sci.*, **20**, No. 1, 177-185 (1973).
- Vainio R., T. Laitinen, and H. Fichtner "Energetic Particle Mean Free Path in the Wave Heated Solar Wind", *Proc. 28th Intern. Cosmic Ray Conf.*, Tsukuba, **6**, 3539-3542 (2003a).
- Vainio R., T. Laitinen, and H. Fichtner "A simple analytical expression for the power spectrum of cascading Alfvén waves in the solar wind", *Astronomy and Astrophysics*, **407**, No. 2, 713-723 (2003b).
- Vainio R. and R. Schlickeiser "Bulk speeds of cosmic rays resonant with parallel plasma waves", *Proc. 26th Intern. Cosmic Ray Conf.*, Salt Lake City, **4**, 310-313 (1999).
- Valdes-Galicia J.F., G. Wibberenz, J.J. Quenby, X. Moussas, G. Green, and F.M. Neubauer "Pitch angle scattering of solar particles - Comparison of "particle" and "field" approach. I - Strong scattering", *Solar Phys.*, **117**, No. 1, 135-156 (1988).
- Vandas M., S. Fischer, M. Dryer, Z. Smith, and T. Detman "Simulation of magnetic cloud propagation in the inner heliosphere in two-dimensions, 2, A loop parallel to the ecliptic plane and the role of helicity", *J. Geophys. Res.*, **101**, No. A2, 2505-2510 (1996).
- Vasilyev V.N. and I.N. Toptygin "Two-particle distribution function and the fluctuations in cosmic ray intensity", *Geomagnetism and Aeronomy*, **16**, No. 6, 953-959 (1976a).
- Vasilyev V.N. and I.N. Toptygin "A relation of interplanetary magnetic field inhomogeneities with the intensity fluctuation spectrum of cosmic rays", *Izvestia Acad. Sci. USSR, Series Phys.*, **40**, No. 3, 627-629 (1976b).

- Vedenov A.A., E.P. Velikhov, and R.Z. Sagdeev "Quasilinear theory of plasma oscillations", *Nucl. Fus.*, **2**, Suppl., Part 2, 465-475 (1962).
- Vernov S.N., A.E. Chudakov, P.V. Vakulov, E.V. Gorchakov, N.N. Kontor, S.N. Kuznetsov, Yu.I. Logachev, G.P. Lyubimov, A.G. Nikolaev, N.V. Pereslegina, and B.A. Tverskoy "Cosmic ray study during the flights of automatic space probes", *Proc. Vth Winter School of Cosmophysics*, Kola Branch, Acad. Sci. USSR, Apatity, 5-23 (1968a).
- Vernov S.N., L.I. Dorman, and B.A. Tverskoy "Magnetic inhomogeneity spectrum in the interplanetary space and cosmic ray variations", *Izvestia Acad. Sci. USSR, Series Phys.*, **32**, No. 11, 1834-1837 (1968b).
- Völk H.J. "Cosmic ray propagation in interplanetary space", *Rev. Geophys. and Space Phys.*, **13**, No. 4, 547-566 (1975).
- Wanner W. and G. Wibberenz "A study of the propagation of solar energetic protons in the inner Heliosphere", *J. Geophys. Res.*, **98**, No. A3, 3513-3528 (1993).
- Webb G.M. "Relativistic transport theory for cosmic rays", *Astrophys. J.*, **296**, No. 2, 319-330 (1985).
- Webb G.M., J. Kóta, G.P. Zank, and J.Y. Lu "The BGK Boltzmann equation and anisotropic diffusion", *Proc. 27th Intern. Cosmic Ray Conf.*, Hamburg, **8**, 3326-3329 (2001).
- Webb G.M., M. Pantazopoulou, and G.P. Zank "Multiple scattering and the BGK Boltzmann equation", *J. Phys. A: Math. and Gen.*, **33**, No. 16, 3137-3160 (2000).
- Webber W.R., M.S. Potgieter, and R.A. Burger "A comparison of predictions of a wavy neutral sheet drift model with cosmic-ray data over a whole modulation cycle: 1976-1987", *Astrophys. J.*, **349**, No. 2 Part 1, 634-640 (1990).
- Zank G.P., W.H. Matthaeus, and C.W. Smith "Evolution of turbulent magnetic fluctuation power with heliospheric distance", *J. Geophys. Res.*, **101**, No. A8, 17093-17107 (1996).
- Zank G.P., W.H. Matthaeus, J.W. Bieber, and H. Moraal "The radial and latitudinal dependence of the cosmic ray diffusion tensor in the heliosphere", *J. Geophys. Res.*, **103**, No. A2, 2085-2097 (1998).
- Zastenker G.N., V.V. Temni, C. Uston, and J.M. Bosquerd "The form and energy of the shock waves from the solar flares of August 2, 4, and 7, 1972", *J. Geophys. Res.*, **A83**, No. 3, 1035-1041 (1978).
- Zhang M. "A path integral approach to the theory of Heliospheric cosmic-ray modulation", *Astrophys. J.*, **510**, No. 2, 715-725 (1999a).
- Zhang M. "A Markov stochastic process theory of cosmic-ray modulation", *Astrophys. J.*, **513**, No. 1, 409-420 (1999b).
- Zhang M. "A path integral solution to the stochastic differential equation of the Markov process for cosmic ray transport", *Proc. 26th Intern. Cosmic Ray Conf.*, Salt Lake City, **7**, 1-4 (1999c).
- Zybin K.P. and Ya.N. Istomin "Diffusional motion of charged-particles in a random magnetic-field", *Zh. Eksp. Teor. Fiz.*, **89**, No. 3, 836-841 (1985).

References for Chapter 3

- Achterberg A. "On the Propagation of Relativistic Particles in a High β Plasma", *Astronomy and Astrophysics*, **98**, No. 1, 161-172 (1981)

- Achterberg A. and R.D. Blandford “Transmission and damping of hydromagnetic waves behind a strong shock front: implications for cosmic ray acceleration”, *Monthly Notices of the Royal Astronomical Society*, **218**, No. 3, 551–575 (1986).
- Ahluwalia H.S. and L.I. Dorman “On cosmic ray convection-diffusion and drift anisotropies”, *Nuclear Physics B*, **49A**, 121-124 (1995a).
- Ahluwalia H.S. and L.I. Dorman “The cosmic ray convection-diffusion anisotropy according to ground and underground observations in 1965-1990”, *Proc. 24th Intern. Cosmic Ray Conf.*, Rome, **4**, 660-663 (1995b).
- Axford W.I. and R.C. Newman “The effect of cosmic ray friction on the solar wind”, *Proc. 9th Intern. Cosmic Ray Conf.*, London, **1**, 173-175 (1965).
- Axford W.I., E. Leer, and J.F. McKenzie “The structure of cosmic ray shocks”, *Astronomy and Astrophysics*, **111**, No. 2, Part 2, 317-325 (1982).
- Babayan V.Kh. and L.I. Dorman “The nonlinear theory of cosmic ray modulation by solar wind. I. Spherically-symmetrical model”, *Proc. 15th Intern. Cosmic Ray Conf.*, Plovdiv, **3**, 107-112 (1977a).
- Babayan V.Kh. and L.I. Dorman “The nonlinear theory of cosmic ray modulation by solar wind. II. The focusing effect in the asymmetric model”, *Proc. 15th Intern. Cosmic Ray Conf.*, Plovdiv, **3**, 113-118 (1977b).
- Babayan V.Kh. and L.I. Dorman “Mach number of solar wind in nonlinear theory of cosmic ray modulation”, *Proc. 16th Intern. Cosmic Ray Conf.*, Kyoto, **3**, 117-121 (1979a).
- Babayan V.Kh. and L.I. Dorman “Nonlinear effects of the interactions between the galactic cosmic ray and solar wind including the charge exchange with interstellar neutral hydrogen”, *Proc. 16th Intern. Cosmic Ray Conf.*, Kyoto, **3**, 123-128 (1979b).
- Babayan V.Kh. and L.I. Dorman “On the nonlinear interaction of galactic cosmic rays with solar wind”, *Izvestia Ac. of Sci. of USSR, Ser. Phys.*, **43**, No. 4, 793-799 (1979c).
- Babayan V.Kh. and L.I. Dorman “The effect of the galactic cosmic ray gradients on solar wind propagation in meridional plane”, *Proc. 17th Intern. Cosmic Ray Conf.*, Paris, **3**, 373-376 (1981).
- Babayan V.Kh. and L.I. Dorman “Nonlinear effects of cosmic ray interaction with solar wind in the outer Heliosphere” In *Physics of the Outer Heliosphere*, ed. S. Grzedzielski and D.E. Page, Pergamon Press, 204-205 (1990).
- Babayan V.Kh., L.I. Dorman, and V.S. Ptuskin “Cosmic ray propagation in the outer Heliosphere: nonlinear effects”, *Proc Intern. Conf. on Plasma Physics*, Kiev, **4**, 228-232 (1987).
- Baranov V.B. and Yu.G. Malama “Model of the solar wind interaction with the local interstellar medium - Numerical solution of self-consistent problem”, *J. Geophys. Res.*, **98**, No. A9, 15157-15163 (1993).
- Barnes A. “Motion of the heliospheric termination shock - A gas dynamic model”, *J. Geophys. Res.*, **98**, No. A9, 15137-15146 (1993).
- Belov A.V., R.T. Gushchina, L.I. Dorman, and I.V. Sirotina “The rigidity spectrum of long-periodic cosmic-ray variations”, *Geomagnetism and Aeronomy*, **28**, No. 4, 550-559 (1988).
- Belov A.V., R.T. Gushchina, L.I. Dorman, and I.V. Sirotina “Rigidity spectrum of cosmic ray modulation”, *Proc. 21th Intern. Cosmic Ray Conf.*, Adelaide, **6**, 52-55 (1990).

- Bloemen J.B.G.M., V.A. Dogiel, V.L. Dorman, and V.S. Ptuskin "Galactic diffusion and wind models of cosmic-ray transport .I. Insight from CR composition studies and gamma-ray observations", *Astronomy and Astrophysics*, **267**, No.2, 372-387 (1993).
- Breitschwerdt D., J.F. McKenzie, and H.J. Völk "Cosmic Ray and Wave Driven Galactic Wind Solution", *Proc. 20th Intern. Cosmic Ray Conf.*, Moscow, 2, 115-118 (1987).
- Breitschwerdt D., J.F. McKenzie, and H.J. Völk "Galactic Winds .I. Cosmic-Ray and Wave-Driven Winds from the Galaxy", *Astronomy and Astrophysics*, **245**, No. 1, 79-98 (1991).
- Bulanov S.V., V.A. Dogiel, and S.I. Syrovatsky "The electron component of cosmic rays. I. Spatial distribution and energy spectrum", *Kosmicheske Issledovaniya*, USSR, **10**, No. 4, 532-544 (1972a).
- Bulanov S.V., V.A. Dogiel, and S.I. Syrovatsky "Electron component of cosmic rays. II. Radio-frequency emission of relativistic electrons in the Galaxy", *Kosmicheske Issledovaniya*, USSR, **19**, No.5, 721-731(1972b).
- Burlaga L.F., N.F. Ness, and F.B. McDonald "Voyagers 1 and 2 Observe a GMIR and Associated Cosmic Ray Decreases at 61 and 78 AU", *Proc. 27th Intern. Cosmic Ray Conf.*, Hamburg, Germany, **9**, 3641-3643 (2001).
- Burlaga L.F., N.F. Ness, E.C. Stone, F.B. McDonald, M.H. Acuna, R.P. Lepping and J.E.P. Connerney "Search for the heliosheath with Voyager 1 magnetic field measurements", *Geophys. Res. Lett.*, **30**, No. 20, 2072, DOI: 10.1029/2003GL018291, SSC9-1-4 (2003).
- Cesarsky C.J. "Cosmic-ray confinement in the Galaxy", *Ann. Rev. Astron. Astrophys.*, **18**, No. 1, 289-319 (1980).
- Chalov S.V. and H.J. Fahr "Reflection of pre-accelerated pick-up ions at the solar wind termination shock: The seed for anomalous cosmic rays", *Solar Phys.*, **168**, No. 2, 389-411 (1996).
- Christian E.R., A.C. Cummings, and E.C. Stone "Observations of anomalous cosmic-ray hydrogen from the Voyager spacecraft", *Astrophys. J.*, **446**, No. 2, L105-L108 (1995).
- Dogiel V.A., V.M. Kovalenko, and V.L. Prischep "Cosmic-ray electrons in the diffusion-convection model of particles propagation", *Lett. in J. Technical Phys.*, USSR, **6**, No. 11, 696-699 (1980).
- Donohue D.J. and G.P. Zank "Steady state and dynamical structure of a cosmic-ray-modified termination shock", *J. Geophys. Res.*, **98**, No. A11, 19005-19025 (1993).
- Dorman L.I. "Cosmic ray modulation", *Nuclear Physics B*, Suppl., **22**, No. 2, 21-45 (1991).
- Dorman L.I. "Cosmic ray nonlinear processes in gamma-ray sources", *Astronomy and Astrophysics*, Suppl. Ser., **120**, 427-435 (1996).
- Dorman Lev I. "Cosmic ray anisotropies in space". In *New Vistas in Astrophysics*, ed. M.M. Shapiro, R. Silberberg, T.S. Stanev, and J.P. Wefel, World Scientific, Singapore/New Jersey/London/Hong Kong, 343-369 (2000).
- Dorman L.I. and H.S. Ahluwalia "Drift anisotropy and transverse gradients of cosmic rays in the interplanetary space", *Proc. 24th Intern. Cosmic Ray Conf.*, Rome, **4**, 664-667 (1995).
- Dorman I.V. and L.I. Dorman "Solar wind properties obtained from the study of the 11-year cosmic-ray cycle, 1", *J. Geophys. Res.*, **72**, No. 5, 1513-1520 (1967a).

- Dorman I.V. and L.I. Dorman "Propagation of energetic particles through interplanetary space according to the data of 11-year cosmic ray variations", *J. Atmosph. and Terr. Phys.*, **29**, No. 4, 429-449 (1967b).
- Dorman I.V. and L.I. Dorman "11-year cosmic ray variation experimental data and mechanisms of galactic cosmic ray integral modulation in the interplanetary space", *Cosmic Rays*, Moscow, **8**, 65-73 (1967c).
- Dorman I.V. and L.I. Dorman "On the nature of cosmic ray intensity changes lag relative to change of solar activity", *Cosmic Rays*, Moscow, **8**, 100-110 (1967d).
- Dorman I.V. and L.I. Dorman "The nature of 11-year cosmic ray variations and properties of solar wind". *Izvestia Academy of Sciences of USSR, Series Phys.*, **31**, No. 8, 1273-1279 (1967e).
- Dorman I.V. and L.I. Dorman "Cosmic rays and the dynamics of solar wind, I", *Geomagnetism and Aeronomy*, **8**, No. 5, 817-821 (1968a).
- Dorman I.V. and L.I. Dorman "Cosmic rays and the dynamics of solar wind, II", *Geomagnetism and Aeronomy*, **8**, No. 6, 1008-1013 (1968b).
- Dorman I.V. and L.I. Dorman "The properties of the boundary between solar wind and interstellar space", *Izvestia Academy of Sci. of USSR, Series Phys.*, **32**, No. 11, 1838-1840 (1968c).
- Dorman I.V. and L.I. Dorman "Inverse effect of cosmic rays on solar wind", *Izvestia Academy of Sci. of USSR, Series Phys.*, **33**, No. 11, 1908-1917 (1969).
- Dorman I.V. and L.I. Dorman "Nonlinear interaction of galactic cosmic rays with solar wind", *Acta Phys. Ac. Sci. Hungaricae*, **29**, Suppl. 2, 17-19 (1970).
- Dorman L.I. and D. Venkatesan "Solar cosmic rays", *Space Sci. Rev.*, **64**, No. 3-4, 183-362 (1993).
- Dorman et al., 1990: Dorman L.I., V.S. Ptuskin, and V.N. Zirakashvili "Outer Heliosphere: pulsations, cosmic rays and stream kinetic instability", In *Physics of the Outer Heliosphere*, ed. S. Grzedzielski and D.E. Page, Pergamon Press, 205-209 (1990).
- Drury L.O'C. and H.J. Völk "Hydromagnetic shock structure in the presence of cosmic rays", *Astrophys. J.*, **248**, No. 1, Part 1, 344-351 (1981).
- Fahr H.J. and H. Fichtner "The influence of pick-up ion-induced wave pressures on the dynamics of the mass-loaded solar wind", *Solar Phys.*, **158**, No. 2, 353-363 (1995).
- Fedorenko V.N., V.M. Ostryakov, A.N. Polyudov, and V.D. Shapiro "Induced scattering in two-wave absorption of Alfvén waves in a plasma of arbitrary beta", *Fizika Plazmy, USSR*, **16**, No. 4, 443-451 (1990).
- Fichtner et al.: 1991a: Fichtner H., H.J. Fahr, W. Neutsch, R. Schlickeiser, A. Crusiuswatzel, and H. Lesch "Cosmic-Ray Driven Galactic Wind", *Nuovo Cimento*, **106**, No. 8, 909-925 (1991a).
- Fichtner et al.: 1991b: Fichtner H., W. Neutsch, H.J. Fahr, and R. Schlickeiser "3-Dimensional Models of a Galactic Wind Expansion with Ellipsoidal Geometry .1. The Hydrodynamical Test Case", *Astrophys. J.*, **371**, No. 1, 98-110 (1991b).
- Fichtner et al.: 1996: Fichtner H., J.A. Le Roux, U. Mall, and D. Rucinski "On the transport of pick-up ions in the Heliosphere", *Astron. and Astrophys.*, **314**, No. 2, 650-662 (1996).
- Fite W.L., A.C.H. Smith, and R.F. Steblings "Charge transfer in collisions involving symmetric and asymmetric resonance", *Proc. Roy. Soc., London*, **A268**, No. 1335 527-536 (1962).

- Galeev A.A. and R.Z. Sagdeev "Non-linear plasma theory", In *Problems of Plasma Theory*, M.A.Leontovich (ed.), Atomizdat, Moscow, **7**, 3-145 (1973).
- Ginzburg V.L. "Cosmic rays and plasma phenomena in the Galaxy and Metagalaxy", *Astron. J. (Moscow)*, **42**, No. 6, 1129-1134 (1965).
- Grzedzielski S. and J. Ziemkiewicz "LISM-heliosphere interaction mediated by suprathermal particles", *Physics of the Outer Heliosphere, Proc. of the 1st COSPAR Colloquium, Warsaw, Poland, Sept. 19-22, 1989*, ed. S. Grzedzielski and D.E. Page, New York, Pergamon Press, 363-366 (1990).
- Hawley J.F., L.L. Smarr, and J.R. Wilson "A numerical study of black hole accretion: II. Finite differencing and code calibration", *Astrophys. J. Suppl.*, **55**, 211 (1984).
- Holzer T.E. "Interaction of the solar wind with the neutral component of the interstellar gas", *J. Geophys. Res.*, **77**, 5407 (1972).
- Hummel E. and R.J. Dettmar "Radio Observations and Optical Photometry of the Edge-On Spiral Galaxy NGC4631", *Astron. and Astrophys.*, **236**, No. 1, 33-46 (1990).
- Hummel E., H. Lesch, R. Wielebinski, and R. Schlickeiser "The Radio Halo of NGC4631 - Ordered Magnetic-Fields Far Above the Plane", *Astronomy and Astrophysics*, **197**, No. 1-2, L29-L31 (1988).
- Innanen K.A. "Models of galactic mass distribution", *Astrophys. Space Sci.*, **22**, No. 2, 393-411 (1973).
- Ipavich F.M. "Galactic winds driven by cosmic rays", *Astrophys. J.*, **196**, No. 1, Part 1, 107-120 (1975).
- Isenberg P.A. "Interaction of the solar wind with interstellar neutral hydrogen - Three-fluid model", *J. Geophys. Res.*, **91**, No. A9, 9965-9972 (1986).
- Jiang I.G., K.W. Chan, and C.M. Ko "Hydrodynamic approach to cosmic ray propagation, 1. Nonlinear test particle picture", *Astronomy and Astrophysics*, **307**, No. 3, 903-914 (1996).
- Karmesin S.R., P.C. Liewer, and J.U. Brackbill "Motion of the termination shock in response to an 11 year variation in the solar wind", *Geophys. Res. Lett.*, **22**, No. 9, 1153-1156 (1995).
- Ko C.M. "A note on the hydrodynamical description of cosmic-ray propagation", *Astronomy and Astrophysics*, **259**, No. 1, 377-381 (1992).
- Ko C.M. "Hydrodynamic approach to cosmic ray propagation - II. Nonlinear test particle picture in a shocked background", *Astronomy and Astrophysics*, **340**, No. 2, 605-616 (1998).
- Ko C.M. "The structure of a cosmic-ray-plasma system", *Proc. 26th Intern. Cosmic Ray Conf., Salt Lake City*, **4**, 447-450 (1999).
- Ko et al., 1997: Ko C.M., K.W. Chan, and G.M. Webb "Cosmic-ray-modified shocks with injection in the hydrodynamic approach .1. Injection linear in the thermal pressure", *J. Plasma Phys.*, **57**, Part 3, 677-694 (1997).
- Ko et al., 1988: Ko C.M., J.R. Jokipii, and G.M. Webb "Cosmic-ray-modified stellar winds. III - A numerical iterative approach", *Astrophys. J.*, **326**, No. 2, 761-768 (1988).
- Krimigis S.M., R.B. Decker, M.E. Hill, T.P. Armstrong, G. Gloeckler, D.C. Hamilton, L.J. Lanzerotti, and E.C. Roelof "Voyager 1 exited the solar wind at a distance of ~85 AU from the Sun", *Nature*, **425**, No. 6962, 45-48 (2003).
- Kulsrud R.M. "Propagation of cosmic rays through a plasma", *Astronomical papers dedicated to Bengt Stromgren, Proceedings of the Symposium*, (eds. A. Reiz and T. Anderson), Copenhagen Univ. Obs., 317-326 (1978).

- Kulsrud R.M. and W.P. Pearce "The effect of wave-particle interactions on the propagation of cosmic rays", *Astrophys. J.*, **156**, No. 2, Part 1, 445-469 (1969).
- Lee M.A. "Effects of cosmic rays and the interstellar gas on the dynamics of the solar wind", *Cosmic Winds and the Heliosphere* (ed. J.R. Jokipii, C.P. Sonnett, and M.S. Giampapa), Univ. Arizona Press, Tucson, 833-856 (1997).
- Lee M.A. and W.I. Axford "Model structure of a cosmic-ray mediated stellar or solar wind", *Astron. and Astrophys.*, **194**, No. 1-2, 297-303 (1988).
- Lee M.A., V. Shapiro, and R.Z. Sagdeev "Pickup ion energization by shock surfing", *J. Geophys. Res.*, **101**, No. A3, 4777-4790 (1996).
- Lee M.A. and H.J. Völk "Damping and nonlinear wave-particle interactions of Alfvén-waves in the Solar wind", *Astrophys. Space Sci.*, **24**, No. 1, 31-49 (1973).
- Lerche I. "Unstable magnetosonic waves in a relativistic plasma", *Astrophys. J.*, **147**, No. 2, 689-696 (1967).
- Le Roux J.A. and H. Fichtner "The influence of pickup ions, anomalous and galactic cosmic rays on the structure of the heliospheric termination shock: a self-consistent approach", *Proc. 25th Intern. Cosmic Ray Conf.*, Durban, 2, 221-224 (1997a).
- Le Roux J.A. and H. Fichtner "The influence of pickup, anomalous, and galactic cosmic-ray protons on the structure of the heliospheric shock: a self-consistent approach", *Astrophys. J.*, **477**, L115-L118 (1997b).
- Le Roux J.A., M.S. Potgieter, and V.S. Ptuskin "A transport model for the diffusive shock acceleration and modulation of anomalous cosmic rays in the Heliosphere", *J. Geophys. Res.*, **101**, No. A3, 4791-4804 (1996).
- Le Roux J.A. and V.S. Ptuskin "Galactic cosmic-ray mediation of a spherical solar wind flow. I: The steady state cold gas hydrodynamical approximation", *Astrophys. J.*, **438**, No. 1, 427-433 (1995a).
- Le Roux J.A. and V.S. Ptuskin "Galactic cosmic-ray mediation of a spherical solar wind flow. II. The steady state hydromagnetic approximation", *Astrophys. J.*, **452**, No. 1, 423-433 (1995).
- LeVeque R.J. "CLAWPACK – A software package for solving multi-dimensional conservation laws", *Proc. 5th International Conference on Hyperbolic Problems*, Stony Brook, June, 1994, (J. Glimm et. al., eds.), World Scientific Press, 188-197 (1996).
- Linde T.J., T.I. Gombosi, L.A. Fisk, P.L. Roe, D.L. De Zeeuw, and K.G. Powell "Three-dimensional modelling of the outer Heliosphere: Implications on the geometry of the Heliosphere and the acceleration of energetic particles", *1996 Spring Meeting, AGU, Suppl. EOS*, **77**, S212-S212 (1996).
- Livshits M.A. and V.N. Tsytovich "The spectra of magnetohydrodynamic turbulence in collisionless plasma", *Nuclear Fusion*, **10**, No. 3, 241-250 (1970).
- Malkov M.A. "Analytic solution for nonlinear shock acceleration in the Bohm limit", *Astrophys. J.*, **485**, No. 2, Part 1, 638-654 (1997a).
- Malkov M.A. "Bifurcation, efficiency, and the role of injection in shock acceleration with the Bohm diffusion", *Astrophys. J.*, **491**, No. 2, 584-595 (1997b).
- McDonald F.B., A. Lukasiak, and W.R. Webber "Pioneer 10 and Voyager 1 observations of anomalous cosmic-ray hydrogen in the outer Heliosphere", *Astrophys. J.*, **446**, L101-L104 (1995).
- McDonough T.R. and N.M. Brice "The termination of the solar wind", *Icarus*, **15**, No. 3, 505-510 (1971).

- McKenzie J.F. and H.J. Völk "Non-linear theory of cosmic ray shocks including self-generated Alfvén waves", *Astronomy and Astrophysics*, **116**, No. 2, 191-200 (1982).
- McKenzie J.F. and G.M. Webb "Magnetohydrodynamic plasma instability driven by Alfvén waves excited by cosmic-rays", *J. Plasma Physics*, **31**, 275-299 (1984).
- Miyamoto M. and R. Nagai "Three-dimensional models for the distribution of mass in galaxies", *Publ. Astron. Soc. Japan*, **27**, No. 4, 533-543 (1975).
- Moraal H. "Cosmic ray modulation studies in the outer Heliosphere", *Nuclear Physics B, Suppl.*, **33**, No.1-2, 161-178 (1993).
- Ness N.F., L.F. Burlaga, M.H. Acuna, R.P. Lepping, and J.E.P. Connerney "Studies of the Termination Shock and Heliosheath at > 92 AU: Voyager 1 Magnetic Field Measurements", *Proc. 29-th Intern. Cosmic Ray Conf.*, Pune, **2**, 39-42 (2005)
- Parker E.N. "Fast dynamos, cosmic-rays, and the galactic magnetic-field", *Astrophys. J.*, **401**, No. 1, 137-145 (1992).
- Parker E.N. "The passage of energetic charged particles through interplanetary space", *Planet. Space Sci.*, **13**, No. 1, 9-49 (1965).
- Paularena K.I., J.W. Belcher, J.D. Richardson, G.S. Gordon, and A.J. Lazarus, "Voyager 2 fine-scale velocity oscillations at 48 AU", *Geophys. Res. Lett.*, **23**, No. 13, 1685-1688 (1996).
- Pauls H.L. and G.P. Zank "Interaction of a nonuniform solar wind with the local interstellar medium", *J. Geophys. Res.*, **101**, No. A8, 17081-17092 (1996).
- Pohl M., R. Schlickeiser, and E. Hummel "A statistical-analysis of cosmic-ray propagation effects on the low-frequency radio spectrum of spiral galaxies", *Astronomy and Astrophysics*, **250**, No. 2, 302-311 (1991).
- Ptuskin V.S. "Pressure of a gas of fast charged-particles diffusing in a medium with a stochastic magnetic-field", *JETP*, **86**, No. 2, 483-486 (1984)
- Ptuskin V.S., H.J. Völk, V.N. Zirakashvili, and D. Breitschwerdt "Transport of relativistic nucleons in a galactic wind driven by cosmic rays", *Astronomy and Astrophysics*, **321**, No. 2, 434-443 (1997).
- Ptuskin V.S. and V.N. Zirakashvili "Wind driven by cosmic rays in a rotating Galaxy. II. Propagation of cosmic rays", *Proc 23th Intern. Cosmic Ray Conf.*, Calgary, **2**, 290-292 (1993).
- Rand R.J. and S.R. Kulkarni "The local galactic magnetic-field", *Astrophys. J.*, **343**, No. 2, 760-772 (1989)
- Ratkiewicz R., A. Barnes, and J.R. Spreiter "Heliospheric termination shock motion in response to LISM variations: Spherically symmetric model", *Geophys. Res. Lett.*, **24**, No. 13, 1659-1662 (1997).
- Reich P. and W. Reich "Spectral index variations of the galactic radio-continuum emission - Evidence for a galactic wind", *Astronomy and Astrophysics*, **196**, No. 1-2, 211-226 (1988).
- Sakurai T. "Magnetic stellar winds - A 2-D generalization of the Weber-Davis model", *Astronomy and Astrophysics*, **152**, No. 1, 121-129 (1985).
- Semar C.L. "Effect of interstellar neutral hydrogen on the termination of the solar wind", *J. Geophys. Res.*, **75**, No. 34, 6892-6898 (1970).
- Simpson J.A. "The primary cosmic ray spectrum and the transition region between interplanetary and interstellar space", *Proc. 8th Intern. Cosmic Ray Conf.*, Jaipur, **2**, 155-168 (1963).

- Sousk S.F. and A.M. Lenchek "The Effect of Galactic Cosmic Rays upon the Dynamics of the Solar Wind", *Astrophys. J.*, **158**, No. 2, 781-795 (1969).
- Stone E.C., A.C. Cummings, and W.R. Webber "The distance to the solar wind termination shock in 1993 and 1994 from observations of anomalous cosmic rays", *J. Geophys. Res.*, **101**, No. A5, 11017-11026 (1996).
- Suess S.T. "Temporal variations in the termination shock distance", *J. Geophys. Res.*, **98**, No. A9, 15147-15155 (1993).
- Vasyliunas V.M. and G.L. Siscoe "On the flux and the energy spectrum of interstellar ions in the solar system", *J. Geophys. Res.*, **81**, 1247-1252 (1976).
- Vedenov A.A. and L.D. Rjutov "Quasi-linear effects in stream instabilities", In "*Problems of Plasma Theory*" (ed. M.A. Leontovich), Atomizdat, Moscow, **6**, 3-68 (1972).
- Völk H.J. and C.J. Cesarsky "Nonlinear Landau damping of Alfvén waves in a high beta plasma", *Z. Naturforsch.*, **37A**, No. 8, 809-815 (1982).
- Vormbrock N. and H. Fichtner "Moredimensional Modeling of Cosmic Ray Driven Galactic Winds", *Proc. 23th Intern. Cosmic Ray Conf.*, Calgary, **2**, 283-286 (1993).
- Weber E.J. and L. Davis, Jr. "The Angular Momentum of the Solar Wind", *Astrophys. J.*, **148**, No. 1, Part 1, 217-227 (1967).
- Wentzel D.G. "The propagation and anisotropy of cosmic rays 1. Theory for steady streaming", *Astrophys. J.*, **156**, No. 1, Part 1, 303-314 (1969).
- Wentzel D.G. "Cosmic-ray propagation in the Galaxy: collective effects", *Ann. Rev. of Astron. and Astrophys.*, **12**, 71-96 (1974).
- Whang Y.C., L.F. Burlaga, and N.F. Ness "Locations of the termination shock and the heliopause", *J. Geophys. Res.*, **100**, No. A9, 17015-17024 (1995).
- Yeh T. "Mass and angular momentum of fluxes of stellar winds", *Astrophys. J.*, **206**, No. 3, Part 1, 768-776 (1976).
- Zank G.P. "Solution Topologies for Cosmic-Ray Modified Galactic Winds, 1. Spherical-Symmetry", *Astronomy and Astrophysics*, **225**, No. 1, 37-47 (1989).
- Zank et al., 1993: Zank G.P., G.M. Webb, and D.J. Donohue "Particle injection and the structure of energetic-particle-modified shocks", *Astrophys. J.*, **406**, No. 1, 67-91 (1993).
- Zank et al., 1996: Zank G.P., H.L. Pauls, I.H. Cairns, and G.M. Webb "Interstellar pickup ions and quasi-perpendicular shocks: Implications for the termination shock and interplanetary shocks", *J. Geophys. Res.*, **101**, No. A1, 457-478 (1996).
- Zirakashvili V.N. "Induced scattering and two-photon absorption of Alfvén waves with arbitrary propagation angles", *JETP*, **90**, No. 5, 810-816 (2000).
- Zirakashvili et al., 1991: Zirakashvili V.N., L.I. Dorman, V.S. Ptuskin, and V.Kh. Babayan "Cosmic ray nonlinear modulation in the outer heliosphere", *Proc. 22th Intern. Cosmic Ray Conf.*, Dublin, **3**, 585-588 (1991).
- Zirakashvili et al., 1993: Zirakashvili V.N., D. Breitschwerdt, V.S. Ptuskin, and H.J. Völk "Wind Driven by Cosmic Rays in a Rotating Galaxy. I. Wind structure", *Proc. 23th Intern. Cosmic Ray Conf.*, Calgary, **2**, 287-289 (1993).
- Zirakashvili et al., 1996: Zirakashvili V.N., D. Breitschwerdt, V.S. Ptuskin, and H.J. Völk "Magnetohydrodynamic wind driven by cosmic rays in a rotating galaxy", *Astronomy and Astrophysics*, **311**, No. 1, 113-126 (1996).
- Zirakashvili et al., 1999: Zirakashvili V.N., V.S. Ptuskin, and H.J. Völk "On nonlinear Alfvén waves generated by cosmic ray streaming instability", *Proc. 26th Intern. Cosmic Ray Conf.*, Salt Lake City, **4**, 233-236 (1999).

Zirakashvili et al., 2001: Zirakashvili V.N., V.S. Ptuskin, and H.J. Völk “Random magnetic fields in the galactic wind flow”, *Proc. 27th Intern. Cosmic Ray Conf.*, Hamburg, **4**, 1827-1830 (2001).

References for Chapter 4

- Achterberg A., Y.A. Gallant, J.G. Kirk, and A.W. Guthmann “Particle acceleration by ultrarelativistic shocks: theory and simulations”, *Monthly Notices of the Royal Astronomical Society*, **328**, No. 2, 393-408 (2001).
- Akasofu S.-I. “Energy supply processes for solar flares and magnetospheric substorms”, *Space Sci. Rev.*, **95**, 613-621 (2001).
- Alekseev I.I. and A.P. Kropotkin “Passage of energetic particles through hydromagnetic discontinuity surface”, *Geomagnetism and Aeronomy*, **10**, No. 6, 953-957 (1970).
- Alekseev I.I., A.P. Kropotkin, and V.P. Shabansky “Cosmic ray acceleration by a magnetohydrodynamic discontinuity”, *Izv.Akad.Uauk USSR, Ser. Phys.*, **34**, No. 11, 2318-2321 (1970).
- Alfvén H. “On the origin of cosmic radiation”, *Phys. Rev.*, **75**, No. 11, 1732-1735 (1949).
- Alfvén H. “On the origin of cosmic radiation”, *Tellus*, **6**, No. 3, 232-253 (1954).
- Alfvén H. “Momentum spectrum of cosmic radiation”, *Tellus*, **11**, No. 1, 106-115 (1959).
- Aschwanden M.J. “Particle acceleration and kinematics in solar flares: A synthesis of recent observations and theoretical concepts (invited review)”, *Space Sci. Rev.*, **101**, 1-227 (2002).
- Aschwanden M.J., R.A. Schwartz, and D.M. Alt “Electron time-of-flight differences in solar flares”, *Astrophys. J.*, **447**, No. 2, 923-935 (1995).
- Axford W.I. “Cosmic ray acceleration”, *Proc. 20-th Intern. Cosmic Rays Conf.*, Moscow, **8**, 120-126 (1987).
- Axford W.I., E. Leer, and G. Skadron “The acceleration of cosmic rays by shock waves”, *Proc. 15th Intern. Cosmic Rays Conf.*, Plovdiv, **11**, 132-137 (1977).
- Bakhareva et al., 1970a: Bakhareva M.F., V.N. Lomonosov, and B.A. Tverskoy “Charged-particle acceleration in variable magnetic field”, *Zh. Eksp. Theor. Fiz.*, **59**, No. 6, 2003-2015 (1970a).
- Bakhareva et al., 1970b: Bakhareva M.F., V.N. Lomonosov, and B.A. Tverskoy “The acceleration of charged particles in a variable magnetic field” *Izvestiya Acad. Nauk USSR, Series Phys.*, **34**, No. 11, 2313-2317 (1970b).
- Bakhareva et al., 1973: Bakhareva M.F., V.N. Lomonosov, and B.A. Tverskoy “To the theory of charged particle acceleration in non-stationary magnetic fields”, *Geomagnetism and Aeronomy*, **13**, No. 5, 769-776 (1973).
- Ball L.T. and J.G. Kirk “Diffusive acceleration of electrons in SN 1987A”, *Astrophys. J.*, **396**, No. 1, L39-L42 (1992).
- Ballard K.R. and A.F. Heavens “Shock acceleration and steep-spectrum synchrotron sources”, *Monthly Notices of the Royal Astronomical Society*, **259**, No. 1, 89-94 (1992).
- Band D., J. Matteson, L. Ford, B. Schaefer, D. Palmer, B. Teegarden, T. Cline, M. Briggs, W. Paciesas, G. Pendelton, G. Fishman, C. Kouveliotou, C. Meegan, R. Wilson, and P. Lestrade “BATSE observations of gamma-ray burst spectra. I. Spectral diversity”, *Astrophys. J.*, **413**, No.1, Part 1, 281-192 (1993).

- Baring M.G. "Cosmic ray origin, acceleration and propagation", *Proc. 26th Intern. Cosmic Ray Conf.*, Salt Lake City, Vol. Invited, Rapporteur and Highlight Papers, 153-168 (1999)
- Baring M.G., D.C. Ellison, and F.C. Jones "The injection and acceleration of particles in oblique shocks: a unified Monte Carlo description", *Astrophys. J.*, **409**, No. 1, Part 1, 327-332 (1993).
- Bednarz J. and M. Ostrowski "The acceleration time-scale for first-order Fermi acceleration in relativistic shock waves", *Monthly Notices of the Royal Astronomical Society*, **283**, No. 2, 447-456 (1996).
- Bednarz J. and M. Ostrowski "Energy spectra of cosmic rays accelerated at ultrarelativistic shock waves", *Phys. Rev.Lett.*, **80**, No. 18, 3911-3914 (1998).
- Bednarz J. and M. Ostrowski "Efficiency of cosmic ray reflections from an ultrarelativistic shock wave", *Monthly Notices of the Royal Astronomical Society*, **310**, No. 1, L11-L13 (1999).
- Begelman M.C. and J.G. Kirk "Shock-drift particle acceleration in superluminal shocks - A model for hot spots in extragalactic radio sources", *Astrophys. J.*, **353**, No. 1, 66-80 (1990).
- Bell A.R. "The acceleration of cosmic ray in shock fronts. I.", *Monthly Notices of the Royal Astronomical Society*, **182**, No.1, 147-156 (1978a).
- Bell A.R. "The acceleration of cosmic rays in shock fronts. II.", *Monthly Notices of the Royal Astronomical Society*, **182**, No.2, 443-455 (1978b).
- Bell A.R. and S.G. Lucek "Cosmic ray acceleration to very high energy through the non-linear amplification by cosmic rays of the seed magnetic field", *Monthly Notices of the Royal Astronomical Society*, **321**, No .3, 433-438 (2001).
- Berezhko E.G. "Acceleration of charged particles in a cosmic-phase shear flow", *Pis'ma Zh.Eksp.Teor.Fiz.*, **33**, No. 8, 416-419 (1981).
- Berezhko E.G. "Friction mechanism of particle acceleration in conditions of interplanetary space", *Letters in Astron. J.*, **8**, No. 12, 747-750 (1982a).
- Berezhko E.G. "Acceleration of charged particles in condition of tangential discontinuity of hydro-dynamical velocity of space plasma", *Geomagnetism and Aeronomy*, **22**, No. 3, 353-361 (1982b).
- Berezhko E.G. "Generation of super-high energy cosmic rays in the vicinities of pulsars", *Letters in Astron. J.*, **20**, No. 2, 93-98 (1994).
- Berezhko E.G. "Cosmic ray sources", *Proc. 25th Intern. Cosmic Ray Conf.*, Durban, **8**, 281-290 (1997).
- Berezhko E.G. "Particle acceleration in supernova remnants", *Proc. 27th Intern. Cosmic Ray Conf.*, Hamburg, Vol. Invited, Rapporteur and Highlight Papers, 226-233 (2001).
- Berezhko E.G. "Influence of non-linear interactions of Alfvén waves on the regular particle acceleration; The near Earth bow shock wave", *Letters in Astron. J.*, **28**, No. 9, 701-710 (1994).
- Berezhko E.G. and D.C. Ellison "A Simple Model of Nonlinear Diffusive Shock Acceleration", *Astrophys. J.*, **526**, Issue 1, 385-399 (1999).
- Berezhko et al., 1984: Berezhko E.G., V.K. Elshin, G.F. Krymsky, and Yu.A. Romashchenko "Self-consistent particles spectrum at the shock wave front", *Izv. Ac. of Sci. USSR, Ser. Phys.*, **48**, No. 11, 2221-2224 (1984).

- Berezhko et al., 1991: Berezhko E.G., G.P. Krymsky, and A.A. Turpanov "Shock wave modification at different regimes of cosmic ray acceleration", *Izv. Acad. of Sci. USSR, Ser. Phys.*, **55**, No. 10, 2063-2065 (1991).
- Berezhko et al., 1996: Berezhko E.G., V.K. Elshin, and L.T. Ksenofonov "Numerical investigation of cosmic ray acceleration in supernova remnants", *Astron. J.*, **73**, No. 2, 176-188 (1996).
- Berezhko et al., 1997: Berezhko E.G., S.N. Taneev, and S.I. Petukhov "Regular ion acceleration at bow shock wave", *Izv. Russian Ac. of Sci., Ser. Phys.*, **61**, No. 6, 1137-1146 (1997).
- Berezhko et al., 1998: Berezhko E.G., S.I. Petukhov, and S.N. Taneev "Regular particle acceleration at the fronts of interplanetary shock waves", *Letters in Astron. J.*, **24**, No. 2, 151-160 (1998).
- Berezhko et al., 2001: Berezhko E.G., S.I. Petukhov, and S.N. Taneev "Solar cosmic ray acceleration by shock waves in solar corona", *Izv. Russian Ac. of Sci., Ser. Phys.*, **65**, No. 3, 339-342 (2001).
- Berezhko E.G. and G.F. Krymsky "Kinetic consideration of charged particle acceleration process in shearing flow of collisionless plasma", *Letters in Astron. J.*, **7**, No. 10, 636-640 (1981).
- Berezhko E.G. and G.P. Krymsky "Formation of universal energy spectrum of charged particles in the shear flow of cosmic plasma", *Izv. Acad. of Sci. USSR, Ser. Phys.*, **46**, No. 9, 1656-1658 (1982).
- Berezhko E.G. and G.P. Krymsky "Cosmic ray acceleration by shock waves", *UFN*, **154**, No. 1, 49-91 (1988).
- Berezhko E.G. and L.T. Ksenofonov "Contents of cosmic rays accelerated in supernova remnants", *Zh. Exp. Theor. Phys. (JETP)*, **116**, No. 3(9), 737-759 (1999).
- Berezhko E.G. and S.N. Taneev "Particle acceleration at the front of bow shock wave", *Space Research, Moscow*, **29**, No. 4, 582-592 (1991).
- Berezhko E.G. and S.N. Taneev "Solar cosmic ray acceleration by shock waves", *Letters in Astron. J.*, **29**, No. 8, 601-615 (2003).
- Berezhko E.G. and H.J. Völk "Kinetic theory of cosmic rays and gamma rays in supernova remnants. I. Uniform interstellar medium", *Astroparticle Physics*, **7**, No. 3, 183-202 (1997).
- Berezhko E.G. and H.J. Völk "Kinetic theory of cosmic ray and gamma-ray production in supernova remnants expanding into wind bubbles", *Astronomy and Astrophysics*, **357**, No. 1, 283-300 (2000).
- Biermann P.L. "Cosmic rays, origin and acceleration: what we can learn from radio-astronomy", *Proc. 23th Intern. Cosmic Ray Conf.*, Calgary, Vol. Invited, Rapporteur and Highlight Papers, 45-83 (1993).
- Biskamp D, R.Z. Sagdeev, and K. Schindler "Nonlinear evolution of the tearing instability in the geomagnetic tail", *Cosmic Electrodynamics*, **1**, No. 3, 297-310 (1970).
- Blandford R.D. "Statistical acceleration of ultrarelativistic electrons by random electromagnetic waves", *Astron. and Astrophys.*, **26**, No. 2, 161-170 (1973).
- Blandford R. and D. Eichler "Particle Acceleration at Astrophysical Shocks - a Theory of Cosmic-Ray Origin", *Phys. Rep.*, **154**, No. 1, 1-75 (1987).
- Blandford R.D. and J.K. Ostriker "Particle acceleration by astrophysical shocks", *Astrophys. J.*, **221**, L29-L32 (1978).
- Blasi P. "A semi-analytical approach to non-linear shock acceleration", *Astropart. Phys.*, **16**, Issue 4, 429-439 (2002).

- Blasi P. "Nonlinear shock acceleration in the presence of seed particles", *Astropart. Phys.*, **21**, Issue 1, 45-57 (2004)
- Blasi P., S. Gabici, and G. Vannoni "On the role of injection in kinetic approaches to non-linear particle acceleration at non-relativistic shock waves", *MNRAS*, **361**, Issue 3, 907-918 (2005).
- Bloomberg H.W. and S.P. Gary "Particle acceleration by an electrostatic wave and spatially increasing phase velocity", *J. Geophys. Res.*, **78**, No. 31, 7531-7535 (1973).
- Bogdan T.J. and H.J. Völk "Onion-shell model of cosmic ray acceleration in supernova remnants", *Astronomy and Astrophysics*, **122**, No. 1-2, 129-136 (1983).
- Bregman J.N., A.E. Glassgold, P.J. Huggins, M.J. Lebofsky, G.H. Rieke, M.F. Aller, H.D. Aller M.J. Lebofsky, M.F. Aller, G.H. Rieke, H.D. Aller, and P.E. Hodge "Multifrequency observations of the red QSO 1413 + 135", *Nature*, **293**, No. 5835, 714-717 (1981).
- Bruce C.E.R. "The role of electrical discharges in astrophysical phenomena" *Observatory*, **95**, No. 1008, 204-210 (1975).
- Buchsbaum S. and C. Roberts "Motion of a charged particle in a constant magnetic field: a transverse electromagnetic wave propagating along the field", *Phys. Rev.*, **135**, No. 2A, 381-389 (1964).
- Bulanov S.V. and V.A. Dogiel "The influence of energy losses on the acceleration of cosmic-ray particles at a shock front", *Sov. Astron. Letters*, **5**, 278-281 (1979), (English translation from Russian).
- Bulanov S.V. and A.A. Pukhov "On the joint action of the regular and the stochastic mechanisms of acceleration at the shock fronts", *Proc. 17th Intern. Cosmic Ray Conf.*, Paris, **2**, 322 (1981).
- Bulanov S.V. and P.V. Sazorov "Particle acceleration near the magnetic field zero line", *Proc. 6th Leningrad Intern. Seminar on Particle Acceleration and Nuclear Reactions in the Space*, Leningrad, 231-232 (1974).
- Bulanov S.V. and P.V. Sazorov "Energy spectrum of particles accelerated in the vicinities of magnetic field zero line", *Astronom. Zh.*, **52**, No. 4, 763-771 (1975).
- Bulanov S.V. and P.V. Sazorov "Stabilization of tearing instability in stationary flow of a plasma", *JETP Letters*, **27**, No. 10, 521-523 (1978)
- Bulanov S.V. and S.I. Syrovatsky "Simple models of charged particle acceleration in neutral current sheets", *Proc. 4th Leningrad Intern. Seminar on Uniformity of Particle Acceleration on Various Scales in the Space*, Leningrad, 101-108 (1972).
- Bulanov S.V. and S.I. Syrovatsky "On charged particle motion in the vicinities of magnetic field zero line", *Trans. FIAN*, **88**, 114-126 (1976).
- Burlaga L.F., N.F. Ness, and F.B. McDonald "Voyagers 1 and 2 Observe a GMIR and Associated Cosmic Ray Decreases at 61 and 78 AU", *Proc. 27th Intern. Cosmic Ray Conf.*, Hamburg, Germany, **9**, 3641-3643 (2001).
- Burlaga L.F., N.F. Ness, E.C. Stone, F.B. McDonald, M.H. Acuna, R.P. Lepping and J.E.P. Connerney "Search for the heliosheath with Voyager 1 magnetic field measurements", *Geophys. Res. Lett.*, **30**, No. 20, 2072, DOI: 10.1029/2003GL018291, SSC9-1-4 (2003).
- Califano F., F. Pegoraro, and S.V. Bulanov "Spatial structure and time evolution of the Weibel instability in collisionless inhomogeneous plasmas", *Phys. Rev. E*, **56**, No. 1, 963-969 (1997).
- Cane H.V. "Energetic particles in the solar wind: propagation, acceleration, and modulation", *Proc. 25th Intern. Cosmic Ray Conf.*, Durban, **8**, 135-150 (1997).

- Chandrasekhar S. "The Radiative Equilibrium of Extended Stellar Atmospheres", *Observatory* **57**, 225-227 (1934a).
- Chandrasekhar S. "The Radiative Equilibrium of Extended Stellar Atmosphere", *Monthly Notices of the Royal Astronomical Society*, **94**, 444-458 (1934b).
- Chandrasekhar S. "Stochastic Problems in Physics and Astronomy", *Rev. Mod. Phys.*, **15**, No. 1, 1-89 (1943).
- Charugin V.M. and Yu.P. Ochelkov "Systematic and stochastic acceleration of cosmic rays by radiation", *Proc. 15th Intern. Cosmic Ray Conf.*, Plovdiv, **1**, 422-427 (1977).
- Cliver E.W. "Solar energetic particles: acceleration and transport", *Proc. 26-th Intern. Cosmic Ray Conf.*, Salt Lake City, Vol. Invited, Rapporteur and Highlight Papers, 103-119 (1999).
- Cohen C.M.S. "Solar Energetic Particle Acceleration and Interplanetary Transport", *Proc. 28-th Intern. Cosmic Ray Conf.*, Tsukuba, **8**, 113-133 (2003).
- Cohen C.M.S., E.C. Stone, R.A. Mewaldt, R.A. Leske, A.C. Cummings, G.M. Mason, M.I. Desai, T.T. von Rosenvinge, and M.E. Wiedenbeck "Heavy ion abundances and spectra from the large solar energetic particle events of October–November 2003", *J. Geophys. Res.*, **110**, No. A9, A09S16, DOI: 10.1029/2005JA011004 (2005).
- Cowley S.W.H. "Comments on the merging of nonantiparallel magnetic fields", *J. Geophys. Res.*, **81**, No. 19, 3455-3458 (1976).
- Crosby N.B., M.J. Aschwanden, and B.R. Dennis "Frequency distributions and correlations of solar X-ray flare parameters", *Solar Phys.*, **143**, No. 2, 275-299 (1992).
- Debrunner H. "Coronal and interplanetary propagation of solar cosmic rays. Particle acceleration in interplanetary space", *Proc. 20-th Intern. Cosmic Ray Conf.*, Moscow, **8**, 141-164 (1987).
- De Hoffmann F. and E. Teller "Magneto-Hydrodynamic Shocks", *Phys. Rev.*, **80**, Issue 4, 692-703 (1950).
- De Jager C. "Energetic phenomena in impulsive solar flares", *Proc. 20-th Intern. Cosmic Ray Conf.*, Moscow, **7**, 66-76 (1987).
- Desai M.I., G.M. Mason, J.R. Dwyer, J.E. Mazur, R.E. Gold, S.M. Krimigis, C.W. Smith, and R.M. Skoug "Evidence for a suprathermal seed population of heavy ions accelerated by interplanetary shocks near 1 AU", *Astrophys. J.*, No. 2, 1149-1162 (2003).
- Dodson H.W. and E.R. Hedeman "Problems of differentiation of flares with respect to geophysical effects", *Planet. Space Sci.*, **12**, Issue 5, 393-418 (1964).
- Dorman L.I. "On the beginning stage of charge particles acceleration", *Proc. 6th Intern. Cosmic Ray Conf.*, Moscow, **3**, 245-252 (1959a).
- Dorman L.I. "On charge particle acceleration at powerful impulse discharges and at collision magnetized clouds", *Proc. All-Union Conf. on Magneto-hydrodynamics and Plasma Physics*, Latvia SSR Academy of Sciences Press, Riga, 83-88 (1959b).
- Dorman L.I. "Cosmic ray modulation", *Nuclear Physics B*, **228**, 21-45 (1991).
- Dorman I.V. and L.I. Dorman "Solar wind properties obtained from the study of the 11-year cosmic ray cycle", *J. Geophys. Res.*, **72**, No. 5, 1513-1520 (1967a).
- Dorman I.V. and L.I. Dorman "Propagation of energetic particles through interplanetary space according to the data of 11-year cosmic ray variations", *J. Atmosph. and Terr. Phys.*, **29**, No. 4, 429-449 (1967b).
- Dorman et al., 1980: Dorman L.I., M.E. Katz, Yu.I. Fedorov, and B.A. Shakhov "On the energy balance of charged particles at multi scattering in the stochastic

- inhomogeneous magnetic field”, *J. Exper. and Theor. Physics (JETP)*, Moscow, **79**, No. 4 (10), 1267-1281 (1980).
- Dorman et al., 1987: Dorman L.I., M.E. Katz M.E. and M. Stehlik “A particle energy change in stochastic magnetic fields”, *Proc. 20th Intern. Cosmic Ray Conf.*, Moscow, **4**, 21-23 (1987).
- Dorman et al., 1997a: Dorman L.I., G. Villoresi, I.V. Dorman, N. Iucci, and M. Parisi “High rigidity CR-SA hysteresis phenomenon and dimension of modulation region in the Heliosphere in dependence of particle rigidity”, *Proc. 25th Intern. Cosmic Ray Conf.*, Durban (South Africa), **2**, 69-72 (1997a).
- Dorman et al., 1997b: Dorman L.I., G. Villoresi, I.V. Dorman, N. Iucci, and M. Parisi “Low rigidity CR-SA hysteresis phenomenon and average dimension of the modulation region and Heliosphere”, *Proc. 25th Intern. Cosmic Ray Conf.*, Durban (South Africa), **2**, 73-76 (1997b).
- Dorman et al., 1997c: Dorman L.I., G. Villoresi, I.V. Dorman, N. Iucci, and M. Parisi “Solar-cycle changes of cosmic ray propagation parameters and heliospheric terminal shock wave”, *Proc. 25th Intern. Cosmic Ray Conf.*, Durban (South Africa), **7**, 341-344 (1997c).
- Dorman et al., 1997d: Dorman L.I., G. Villoresi, I.V. Dorman, N. Iucci, and M. Parisi “On the expected CR intensity global modulation in the Heliosphere in the last several hundred years”, *Proc. 25th Intern. Cosmic Ray Conf.*, Durban (South Africa), **7**, 345-348 (1997d).
- Dorman L.I. and G.I. Freidman “On the possibility of charged particle acceleration by shock waves in magnetized plasma”, *Problems of Magnetic Hydrodynamics and Plasma Dynamics*, **3**, Akad. Sci. Lat. SSR, Riga, 77-81 (1959).
- Dorman L.I. and M.E. Katz “Cosmic ray kinetics in space”, *Space Sci. Rev.*, **20**, No. 5, 529-575 (1977).
- Dorman L.I. and V.Kh. Shogenov “On a possibility of charged particle additional acceleration in the moved magnetized plasma”, *J. Exper. and Theor. Physics (JETP)*, Moscow, **89**, No. 5 (11), 1624-1631 (1985).
- Dorman L.I. and V.Kh. Shogenov “Additional particle acceleration in the heliosphere caused by interactions with scatterers moving with different speeds”. In *Solar Wind Nine* (eds. S.R. Habbal, R. Esser, J.W. Holweg, and P.A. Isenberg), Nantucket, October 1998, AIP Conf. Proc., **471**, 617-620 (1999).
- Dorman L.I. and D. Venkatesan “Solar cosmic rays”, *Space Sci. Rev.*, **64**, No. 3-4, 183-362 (1993).
- Drury L.O’C. “An introduction to the theory of diffusive shock acceleration of energetic particles in tenuous plasmas”, *Rep. Progr. Phys.*, **46**, No. 8, 973-1027 (1983).
- Drury L.O’C. “Time-dependent diffusive acceleration of test particles at shocks”, *Monthly Notices of the Royal Astronomical Society*, **251**, No. 2, 340-350 (1991).
- Drury L.O’C. “Acceleration and transport theory”, *Proc. 23th Intern. Cosmic Ray Conf.*, Calgary, Vol. Invited, Rapporteur and Highlight Papers, 307-320 (1993).
- Drury L.O’C., P. Duffy, D. Eichler, and A. Mastichiadis “On “box” models of shock acceleration and electron synchrotron spectra”, *Proc. 26th Intern. Cosmic Ray Conf.*, Salt Lake City, **4**, 394-397 (1999).
- Duggal S.P. “Relativistic solar cosmic rays”, *Rev. Geophys. Space Phys.*, **17**, 1021-1058 (1979).

- Durasova M.S., M.M. Kobrin, and O.I. Yudin "Solar physics-Evidence of quasi-periodic movements in the solar chromosphere and corona", *Nature Phys. Sci.*, **229**, No. 3, 82-84 (1971).
- Eichler D. "On the theory of cosmic-ray-mediated shocks with variable compression ratio", *Astrophys. J.*, Part 1, **277**, No. 1, 429-434 (1984).
- Ellison D.C. "Shock acceleration of diffuse ions at the earth's bow shock: Acceleration efficiency and A/Z enhancement", *J. Geophys. Res.*, **90**, No. A1, 29-38 (1985).
- Ellison D.C. and G.P. Double "Nonlinear particle acceleration in relativistic shocks", *Astropart. Phys.*, **18**, No. 3, 213-218 (2002).
- Ellison D.C. and G.P. Double "Diffusive shock acceleration in unmodified relativistic, oblique shocks", *Astropart. Phys.*, **22**, No. 3-4, 323-338 (2004).
- Ellison et al., 1990a: Ellison D.C., E. Moebius, and G. Paschmann "Particle injection and acceleration at Earth's bow shock: comparison of upstream and downstream events", *Astrophys. J.*, **352**, Part 1, No. 1, 376-394 (1990a).
- Ellison et al., 1990b: Ellison D.C., Jones F.C., Reynolds S.P. "First-order fermi particle acceleration by relativistic shocks", *Astrophys. J.*, **360**, Part 1, No. 2, 702-714 (1990b).
- Ellison et al., 1999: Ellison D.C., F.C. Jones, and M.G. Baring, "Direct Acceleration of Pickup Ions at the Solar Wind Termination Shock: The Production of Anomalous Cosmic Rays", *Astrophys. J.*, Part 1, 512, Issue 1, 403-416 (1999).
- Ellison et al., 2000: Ellison D.C., E.G. Berezhko, and M.G. Baring "Nonlinear shock acceleration and photon emission in supernova remnants", *Astrophys. J.*, Part 1, **540**, No. 1, 292-307 (2000).
- Ellison et al., 2004: Ellison D.C., A. Decourchelle, and J. Ballet "Hydrodynamic simulation of supernova remnants including efficient particle acceleration", *Astronomy and Astrophysics*, **413**, No. 1, 189-201 (2004)
- Ellison et al., 2005: Ellison D.C., P. Blasi, and S. Gabici "Thermal particle injection in nonlinear diffusive shock acceleration", *Proc. 29-th Intern. Cosmic Ray Conf.*, Pune, **3**, 261-264 (2005)
- Ellison D.C. and R. Ramaty "Shock acceleration of electrons and ions in solar flares", *Astrophys. J.*, Part 1, **298**, No. 1, 400-408 (1985).
- Emslie A.G., H. Kucharek, B.R. Dennis, N. Gopalswamy, G.D. Holman, G.H. Share, A. Vourlidas, T.G. Forbes, P.T. Gallagher, G.M. Mason, T.R. Metcalf, R.A. Mewaldt, R.J. Murphy, R.A. Schwartz, and T.H. Zurbuchen "Energy partition in two solar flare/CME events", *J. Geophys. Res.*, **109**, No. A10, A10104: 1-15, DOI: 10.1029/2004JA010571 (2004)
- Fälthammer C.-G. "Acceleration of cosmic ray particles by magnetic pumping", *Rend. Scuola internaz. fis. Enrico Fermi, 1961*, New York-London, 1963, Vol 19, 99-112 (1963).
- Fermi E. "On the origin of the cosmic radiation", *Phys. Rev.*, **75**, No. 8, 1169-1174 (1949).
- Fermi E. "Galactic magnetic fields and the origin of cosmic radiation", *Astrophys. J.*, **119**, No. 1, 1-6 (1954).
- Forbush S.E. "Cosmic-Ray Intensity Variations during Two Solar Cycles", *J. Geophys. Res.*, **63**, No. 4, 651-669 (1958)
- Forman M.A., G.M. Webb, and W.I. Axford "Cosmic-ray acceleration by stellar winds II. The spectrum of accelerated particles", *Proc. 17th Intern. Cosmic Ray Conf.*, Paris, **2**, 313 (1981).

- Frank A.G. "Experimental study of plasma behavior in the vicinities of magnetic zero lines", *Proc. 7th Leningrad Intern. Seminar on Solar Corpuscular Streams and the Radiation Belts of the Earth and Jupiter*, Leningrad, 81-91 (1975).
- Frank A., T.W. Jones, and D. Ryu "Time-dependent simulation of oblique MHD cosmic-ray shocks using the two-fluid model", *Astrophys. J.*, **441**, No. 2, Part 1, 629-643 (1995).
- Fukao S. and T. Tsuda "Reconnection of magnetic lines of force: evolution in incompressible MHD fluids", *Planet. and Space Sci.*, **21**, No. 7, 1151-1178 (1973).
- Fürth H.P., J. Killen, and M.N. Rosenbluth "Finite-Resistivity Instabilities of Sheet Pinch", *Phys. Fluids*, **6**, No. 4, 459-484 (1963).
- Gailitis A.K. and V.H. Tsytoich "Effect of environment on synchrotron radiation of relativistic particles", *Izv. Vuzov*, **6**, No. 6, 1103-1114 (1963).
- Galeev A.A. and L.M. Zeleny "Discontinuity instability in plasma configurations" *Zh. Eksp. Teor. Fiz.*, **70**, No. 6, 2133-2151 (1976).
- Galeev A.A., R.Z. Sagdeev, V.D. Shapiro, and V.I. Shevchenko "Solar wind interaction with comets as a model for the cosmic ray acceleration by shocks", *Proc. 20-th Intern. Cosmic Ray Conf.*, Moscow, **9**, 103-120 (1987).
- Gelfreikh G.B., S.G. Derevyanko, A.N. Korzhavin, and N.P. Stasyuk, *Solar Data*, **9**, 88 (1969).
- Gershberg R.E. "Time scales and energy of flares on red dwarf stars - A review", *Societa Astronomica Italiana, Memorie*, **60**, No. 1-2, 263-287 (1989).
- Gershberg R.E. and N.I. Shakhovskaya "Characteristics of activity energetics of the UV Cetype flare stars", *Astrophys. Space Science*, **95**, No. 2, 235-253 (1983).
- Giacalone J. "The physics of particle acceleration by collisionless shocks", *Planet. and Space Sci.*, **51**, Issue 11, 659-664 (2003).
- Giacalone J. "Particle acceleration at shocks moving through an irregular magnetic field", *Astrophys. J.*, **624**, No. 2, Part 1, 765-772 (2005a).
- Giacalone J. "The Efficient Acceleration of Thermal Protons by Perpendicular Shocks", *Astrophys. J. Lett.*, **628**, Issue 1, L37-L40 (2005b).
- Giacalone J. and D.C. Ellison "Three-dimensional numerical simulations of particle injection and acceleration at quasi-perpendicular shocks", *J. Geophys. Res.*, **105**, No. A6, 12541-12556 (2000).
- Giacalone et al., 1997: Giacalone J., D. Burgess, S.J. Schwartz, D.C. Ellison, and L. Bennett "Injection and acceleration of thermal protons at quasi-parallel shocks: a hybrid simulation parameter survey", *J. Geophys. Res.*, **102**, No. A9, 19789-19804 (1997).
- Giacalone J. and J.R. Jokipii "The Transport of Cosmic Rays across a Turbulent Magnetic Field", *Astrophys. J.* **520**, Issue 1, 204-214 (1999).
- Giacalone J. and J.R. Jokipii "Injection and Acceleration at Non-Parallel Shocks", *Proc. 29-th Intern. Cosmic Ray Conf.*, Pune, **3**, 265-268 (2005).
- Gieseler U.D.J., T.W. Jones, and H. Kang "Time dependent cosmic-ray shock acceleration with self-consistent injection", *Astronomy and Astrophys.*, **364**, No. 2, 911-922 (2000).
- Ginzburg M.A. "Electron and ion acceleration by nonlinear waves near the Earth", *J. Geophys. Res.*, **72**, No. 11, 2749-2764 (1967).
- Ginzburg M.A. "Nonlinear waves with magnetic field in space plasma", *Astron. J. (Moscow)*, **45**, No. 3, 610-615 (1967).

- Ginzburg V.L. "Astrophysical aspects of cosmic ray research(first 75 years and prospects for the future", *Proc. 20-th Intern. Cosmic Ray Conf.*, Moscow, **8**, 7-54 (1987).
- Gloeckler G., N.A. Schwadron, L.A. Fisk, and J. Geiss "Weak pitch angle scattering of few MV rigidity ions from measurements of anisotropies in the distribution function of interstellar pickup H^+ ", *Geophys. Res. Letters*, **22**, No. 19, 2665-2668 (1995).
- Gnedin Yu.N., A.Z. Dolginov, and V.N. Fedorenko "Statistical mechanism of acceleration under space conditions", *Proc. 5th Intern. Leningrad Seminar*, Leningrad, 27 (1972).
- Goldreich P. "The physics of rotating magnetic neutron stars", *Proc. Astron. Soc. Australia*, **1**, No. 5, 227-228 (1969).
- Gopalswamy N., S. Yashiro, S. Krucker, G. Stenborg, and R.A. Howard "Intensity variation of large solar energetic particle events associated with coronal mass ejections", *J. Geophys. Res.*, **109**, No. A12, A12105: 1-18 DOI: 10.1029/2004JA010602 (2004).
- Gopalswamy N., S. Yashiro, Y.Liu, G. Michalek, A. Vourlidas, M.L. Kaiser, and R.A. Howard "Coronal mass ejections and other extreme characteristics of the 2003 October–November solar eruptions", *J. Geophys. Res.*, **110**, No. A9, Cite ID A09S15, DOI: 10.1029/2004JA010958. (2005).
- Gordon R.H. "Acceleration of isolated atoms by radiation pressure", *Astrophysics and Space Science*, **35**, June, 197-202 (1975).
- Grewing M. and H. Heintzmann "The generation of the highest cosmic ray energies", *Phys. Lett. A*, **42**, No. 5, 345-346 (1973a).
- Grewing M. and H. Heintzmann "Charged particle acceleration in strong dipole fields", *Mitteilungen der Astronomischen Gesellschaft*, **32**, 214-218 (1973b).
- Grewing M. and H. Heintzmann "Rotating neutron stars: a model for pulsars", *Zeitschrift fur Naturforschung, Section A-A, Journal of Physical Sciences*, **28a**, No. 3-4, 377-382 (1973c).
- Grewing M. and H. Heintzmann "The origin of cosmic rays-new interest in an old question", *Zeitschrift fur Naturforschung, Section A-A, Journal of Physical Sciences*, **28a**, No. 3-4, 369-376 (1973d)..
- Guetta D., D. Hooper, J. Alvarez-Muniz, F. Halzen, and E. Reuveni "Neutrinos from individual gamma-ray bursts in the BATSE catalog", *Astroparticle Physics*, **20**, No. 4, 429-455 (2004).
- Gunn J.E. and J.P. Ostriker "Acceleration of high-energy cosmic rays by pulsars", *Phys. Rev. Letters*, **22**, No. 14, 728-731 (1969).
- Gurevich A.V. "On the problem of the number of accelerated particles in ionized gas for various acceleration mechanisms", *Zh. Eksp. Teor. Fiz.*, **38**, No. 5, 1597-1607 (1960).
- Hall D.E. "Stochastic acceleration in the presence of streaming", *129th Amer. Astron. Soc. Meet.*, Honolulu, **1**, 98 (1969).
- Hartle J.B. and K.S. Thorne "Slowly rotating relativistic stars and supermassive stars", *Astrophys. J.*, **153**, No. 3, Part 1, 807-834 (1968).
- Haruki T. and J.I. Sakai "Magnetic field generation and electrostatic shock wave formation driven by counter-streaming pair plasmas", *Plasma Physics*, 11th Intern. Congress on Plasma Physics, AIP Conference Proceedings, **669**, 762-765 (2003).

- Hasinger G., T. Miyaji, and M. Schmidt "Luminosity-dependent evolution of soft X-ray selected AGN: New Chandra and XMM-Newton surveys", *Astronomy and Astrophysics*, **441**, Issue 2, 417-434 astro-ph/0506118 (2005).
- He K. "Crisis-induced transition to spatiotemporally chaotic motions", *Phys. Rev. Lett.*, **80**, No. 4, 696-699 (1998).
- He K. "Trapped to free: A mechanism to spatiotemporal chaos", *Phys. Rev. E.*, **59**, No. 5, 5278-5284 (1999).
- He K. "Saddle pattern resonance and onset of crisis to spatiotemporal chaos", *Phys. Rev. Lett.*, **84**, No. 15, 3290-3293 (2000).
- He K. "Critical behavior of crisis-induced transition to spatiotemporal chaos in parameter space", *Phys. Rev. E.*, **63**, No. 1, 016218, 1-6 (2001).
- He K. "Stochastic acceleration of charged particle in nonlinear wave field", *Space Sci. Rev.*, **107**, No. 1, 467-474 (2003).
- Heavens A.F. and L'O.C. Drury "Relativistic shocks and particle acceleration", *Monthly Notices of the Royal Astronomical Society*, **235**, No. 4, 997-1009. (1988).
- Howard R. and H.W. Babcock "Magnetic Fields Associated with the Solar Flare of July 16, 1959", *Astrophys. J.*, **132**, 218-220 (1960).
- Jokipii J.R. "Fermi acceleration and the structure of interstellar turbulence", *Proc. 15th Intern. Cosmic Ray Conf.*, Plovdiv, **1**, 429-434 (1977).
- Jokipii J.R. "Particle drift, diffusion, and acceleration at shocks", *Astrophys. J.*, **255**, No. 2, Part 1, 716-720 (1982).
- Jokipii J.R. "Rate of energy gain and maximum energy in diffusive shock acceleration", *Astrophys. J.*, **313**, No. 2, Part 1, 842-846 (1987).
- Jokipii J.R., J. Kóta, and J. Giacalone "Perpendicular transport in 1- and 2-dimensional shock simulations", *J. Geophys. Res. Lett.*, **20**, No. 17, 1759-1761 (1993).
- Jones F.C. and D.C. Ellison "The plasma physics of shock acceleration", *Space Sci. Rev.*, **58**, No. 3-4, 1991, 259-346 (1991).
- Jones et al., 1998: Jones F.C., J.R. Jokipii, and M.G. Baring "Charged-particle motion in electromagnetic fields having at least one ignorable spatial coordinate", *Astrophys. J.*, Part 1, **509**, No. 1, 238-243 (1998).
- Jones T.W. "Alfvén wave transport effects in the time evolution of parallel cosmic-ray-modified shocks", *Astrophys. J.*, **413**, No. 2, Part 1, 619-632 (1993).
- Jones et al., 1999: Jones T.W., D. Ryu, and A. Engel "Simulating electron transport and synchrotron emission in radio galaxies: Shock acceleration and synchrotron aging in axisymmetric flows", *Astrophys. J.*, **512**, No. 1, 105-124 (1999).
- Jones T.W. and H. Kang "Simulating particle acceleration in modified shocks using a new coarse-grained finite momentum-volume scheme", *Proc. 28-th Intern. Cosmic Ray Conf.*, Tsukuba (Japan), **4**, 2035-2038 (2003).
- Jones T.W. and H. Kang "Time Evolution of Cosmic-Ray Modified MHD Shocks", *Proc. 29-th Intern. Cosmic Ray Conf.*, Pune, **3**, 269-272 (2005a).
- Jones T.W. and H. Kang "An efficient numerical scheme for simulating particle acceleration in evolving cosmic-ray modified shocks", *Astroparticle Physics*, **24**, Issue 1-2, 75-91 (2005b).
- Jory H.R. and A.W. Trivelpiece "Charged-particle motion in large amplitude electromagnetic fields", *J. Appl. Phys.*, **39**, No. 7, 3053-3060 (1968).
- Jun B.-I. and T.W. Jones "Radio emission from a young supernova remnant interacting with an interstellar cloud: Magnetohydrodynamic simulation with relativistic electrons", *Astrophys. J.*, **511**, No. 2, 774-791 (1999).

- Kahler et al., 2005: Kahler S.W., K. Kecskemety, and P. Kiraly "Acceleration and propagation in the heliosphere (SH-2)", *Proc. 29-th Intern. Cosmic Ray Conf.*, Pune, **10**, 367-376 (2005).
- Kang H. "Acceleration of cosmic rays at cosmic shocks", *J. of Korean Astron. Soc.*, **36**, No. 1, 1-12 (2003).
- Kang H. and T.W. Jones "Diffusive Shock Acceleration in Oblique Magnetohydrodynamic Shocks: Comparison with Monte Carlo Methods and Observations", *Astrophys. J.*, **476**, No. 2, Part 1, 875-888 (1997).
- Kang H. and T.W. Jones "Acceleration of Cosmic Rays at Large Scale Cosmic Shocks in the Universe", *J. of Korean Astron. Soc.*, **35**, No. 4, 159-174 (2002).
- Kang H. and T.W. Jones "Cosmic ray acceleration at quasi-parallel plane shocks", *Proc. 28th Intern. Cosmic Ray Conf.*, Tsukuba (Japan), **4**, 2039-2042 (2003).
- Kang H. and T.W. Jones "Numerical Studies of Diffusive Shock Acceleration at Spherical Shocks", *Proc. 29-th Intern. Cosmic Ray Conf.*, Pune, **3**, 257-260 (2005a).
- Kang H. and T.W. Jones "Efficiency of Nonlinear Particle Acceleration at Cosmic Structure Shocks", *Astrophys. J.*, **620**, Issue 1, 44-58 (2005b).
- Kang et al., 2001: Kang H., T.W. Jones, R.J. LeVeque, and K.M. Shyue "Time Evolution of Cosmic-Ray Modified Plane Shocks", *Astrophys. J.*, **550**, N 2, 737-751 (2001).
- Kang et al., 2002: Kang H., T.W. Jones, and U.D.J. Gieseler "Numerical studies of cosmic-ray injection and acceleration", *Astrophys. J.*, **579**, No. 1, 337-358 (2002).
- Kaplan S.A., S.B. Pikelner, and V.N. Tsytovich "Plasma physics of the solar atmosphere", *Physics Reports, Physics Letters Section C*, **15C**, No. 1, 1-82 (1974).
- Kazimura Y., J.I. Sakai, T. Neubert, and S.V. Bulanov "Generation of a small-scale quasi-static magnetic field and fast particles during the collision of electron-positron plasma clouds", *Astrophys. J. Lett.*, **498**, No. 2, L183- L186 (1998).
- Kiepenhahn R. and A. Shindler, *Zs. Astrophys.*, **43**, 36 (1957).
- Kirk J.G. and P. Duffy "Topical review: Particle acceleration and relativistic shocks", *J. Phys. G: Nucl. Part. Phys.*, **25**, No. 8, R163-R194 (1999).
- Kirk J.G. and A.F. Heavens "Particle acceleration at oblique shock fronts", *Monthly Notices of the Royal Astronomical Society*, **239**, No. 4, 995-1011. (1989).
- Kirk J.G. and P. Schneider "On the acceleration of charged particles at relativistic shock fronts", *Astrophys. J.*, Part 1, **315**, No. 2, 425-433 (1987a).
- Kirk J.G. and P. Schneider "Particle acceleration at shocks - A Monte Carlo method", *Astrophys. J.*, Part 1, **322**, No. 1, 256-265 (1987b).
- Kirkpatrick S. "Percolation and Conduction", *Rev. Modern Phys.*, **45**, No. 4, 574-588 (1973).
- Klappong K., K. Leerunnavarat, P. Chuychai, D. Ruffolo "Particle acceleration and pitch angle transport near a thin shock, a compression region, and a structured shock", *Proc. 27-th Intern. Cosmic Ray Conf.*, Hamburg, **8**, 3461-3464 (2001).
- Kobrin M.M., A.I. Korshunov, S.I. Arbutov, V.V. Pakhomov, and V.M. Fridman "About the existence of connection between quasi-periodic oscillations with periods more than 20 min in solar radio emission at 3 cm and appearance of proton flares", Preprint NRFI, No. 83, Gorky, 1-20 (1975).
- Kobrin M.M., A.I. Korshunov, S.I. Arbutov, V.V. Pakhomov, V.M. Fridman, and Yu.V. Tikhomirov "Manifestation of pulsation instability in solar radio emission preceding proton flares", *Solar Physics*, **56**, 359-373 (1978).
- Korchak A.A. "On possible mechanisms of rigid X-ray generation in solar flares", *Astronom. Zh.*, **44**, No. 2, 328-335 (1967).

- Korchak A.A. "On the formation of a power-law spectrum for particles that are accelerated in solar flares", *Izvestia Ac. of Sci. USSR, Series Phys.*, **41**, No. 2, 288-292 (1977).
- Korchak A.A. and S.I. Syrovatsky "On the composition of primary cosmic rays", *Proc. 6th Intern. Cosmic Ray Conf.*, Moscow, **3**, 211-219 (1959).
- Korobeinikov V.P. and S.P. Lomnev "Charged particle motion in plasma in the presence of magneto-hydrodynamic wave", *Zh. Prikl. Mekh. Tekhn. Fiz.*, No. 6, 89-92 (1964).
- Krimigis S.M., R.B. Decker, M.E. Hill, T.P. Armstrong, G. Gloeckler, D.C. Hamilton, L.J. Lanzerotti, and E.C. Roelof "Voyager 1 exited the solar wind at a distance of ~85 AU from the Sun", *Nature*, **425**, No. 6962, 45-48 (2003).
- Krülls W.M. and A. Achterberg "Computation of cosmic-ray acceleration by Ito's stochastic differential equations", *Astron. and Astrophys.*, **286**, 314-327 (1994).
- Kruskal M.D. and M. Schwarzschild "Some instabilities of a completely ionized plasma", *Proc. Roy. Soc. of London, Ser. A (Mathematical and Phys. Sciences)*, **223A**, No. 1154, 348-360 (1954).
- Krymsky G.F. "Regular mechanism of charged, particle acceleration at shock front", *Dokl. Akad. Nauk. SSSR*, **234**, No. 6, 1306-1308 (1977). Translation from Russian to English: *Sov. Phys. Doklady*, **22**, 327-328 (1977).
- Krymsky G.F. and S.I. Petukhov "Acceleration of particles by regular mechanism in the presence of a spherical shock wave", *Letters in Astron. J.*, Moscow, **6**, No. 4, 227-231 (1980).
- Kulsrud R. and J. Arons "Statistical acceleration of relativistic particles in and assembly of spherical electro-magnetic waves", *Astrophys. J.*, **198**, No. 3, 709-715 (1975).
- Kurochka L.N. "Energy Distribution of 15000 Solar Flares", *Soviet Astron.*, **31**, No. 2, 231-233 (1987).
- Lagage P.O. and C.J. Cesarsky "Cosmic ray shock acceleration in the presence of self-exited waves", *Astron. and Astrophys.*, **118**, No. 2, 223-228 (1983a).
- Lagage P.O. and C.J. Cesarsky "The maximum energy of cosmic rays accelerated by supernova shocks", *Astron. and Astrophys.*, **125**, No. 2, 249-257. (1983b).
- Last B.J. and D.J. Thouless "Evidence for power law localization in disordered systems", *Phys. Rev. Letters*, **27**, Issue 25, 1719-1721 (1971).
- Levich E.V. and R.A. Sunyaev "Heating of gas near quasars, Seyfert-galaxy nuclei, and pulsars by low-frequency radiation", *Astron. Zh.*, **48**, No. 3, 461-471 (1971).
- Levine R.H. "Acceleration of thermal particles in collapsing magnetic regions", *Astrophys. J.*, Part 1, **190**, No. 2, 447-456 (1974).
- Levinshstein M.E., M.S. Shur, and A.L. Efros "Galvanomagnetic phenomena in disordered systems. Theory and model experiments", *JETP*, **69**, No. 6, 2203-2211 (1975).
- Lieu R., J.J. Quenby, B. Drollas, and K. Naidu "Enhanced cosmic-ray acceleration rates in highly inclined astrophysical shocks", *Astrophys. J.*, **421**, No. 1, Part 1, 211-218 (1994).
- Lin R.P. "New RHESSI results on particle acceleration and energy release in solar flares", *Proc. 28-th Intern. Cosmic Ray Conf.*, Tsukuba, **8**, 335-345 (2003).
- Lucek S.G. and A.R. Bell "Non-linear amplification of a magnetic field driven by cosmic ray streaming", *Monthly Notices of the Royal Astronomical Society*, **314**, No. 1, 65-74 (2000).
- Lucke O. "Ober die Entstehung der suprathermischen Teilchen", *Abhandl. Geomagn. Inst. Potsdam*, No. 29, 125-127 (1962).
- Malakit K., K. Klappong, K. Leerunnavarat, P. Chuychai, N. Sanguansak, and D. Ruffolo "Particle acceleration at fluid compressions and what that teaches us about shock

- acceleration", *Proc. 28-th Intern. Cosmic Ray Conf.*, Tsukuba, **6**, 3677-3680 (2003).
- Malkov M.A. "Ion leakage from quasi-parallel collisionless shocks: Implications for injection and shock dissipation", *Phys. Rev. E*, **58**, No. 4, 4911-4928 (1998a).
- Malkov M.A. "Spectral universality of strong shocks accelerating charged particles", *Astro-Particle Phys.*, astro-ph/9807097 (1998b).
- Malkov M.A. and L.O'C. Drury "Nonlinear theory of diffusive acceleration of particles by shock waves", *Rep. Progr. Phys.*, **64**, No. 4, 429-481 (2001).
- Mandzhavidze N. "Solar particles and processes", *Proc. 23th Intern. Cosmic Ray Conf.*, Calgary, Vol. Invited, Rapporteur and Highlight Papers, 157-184 (1993)
- Martin S.F. and H.E. Ramsey "Early Recognition of Major Solar Flares in H-alpha", *Solar activity observations and predictions, Progress in Astronautics and Aeronautics*, **30**, (ed. P.S. McIntosh and M. Dryer), M.I.T. Press, Cambridge, 371 (1972)
- Mazur J.E. "Solar activity: minor ion enrichment, particle acceleration and transport, and extreme events", *Proc. 27th Intern. Cosmic Ray Conf.*, Hamburg, Vol. Invited, Rapporteur, and Highlight Papers, 103-108 (2001).
- McKenzie J.F. and H.J. Völk "Non-linear theory of cosmic ray shocks including self-generated Alfvén waves", *Astronomy and Astrophysics*, **116**, No. 2, Part 2, 191-200 (1982).
- Meli A. and J. Quenby "Spectra and time scales for particle acceleration in ultra-relativistic flows applicable to gamma-ray bursters", *Proc. 27th Intern. Cosmic Ray Conf.*, Hamburg, **7**, 2742-2745 (2001).
- Meli A and J.J. Quenby "Particle acceleration in ultra-relativistic parallel shock waves", *Astroparticle Physics*, **19**, No. 5, 637-648 (2003a).
- Meli A and J.J. Quenby "Particle acceleration in ultra-relativistic oblique shock waves", *Astroparticle Physics*, **19**, No. 5, 649-666 (2003b).
- Meli A., J.J. Quenby, and J.K. Becker "On the cosmic rays acceleration at super-luminal shocks", *Proc. 29-th Intern. Cosmic Ray Conf.*, Pune, **3**, 229-232 (2005).
- Mercier C. and G. Trotter "Coronal Radio Bursts: A Signature of Nanoflares?", *Astrophys. J. Letters*, **474**, No. 1, L65-L68 (1997).
- Mewaldt et al., 2005a: Mewaldt R.A., C.M.S. Cohen, G.M. Mason, D.K. Haggerty, M.D. Looper, A. Vourlidas, M.I. Desai, A.W. Labrador, R.A. Leske, and J.E. Mazur "What Fraction of the Kinetic Energy of Coronal Mass Ejections goes into Accelerating Solar Energetic Particles?", *Proc. 29-th Intern. Cosmic Ray Conf.*, Pune, **1**, 129-132 (2005a).
- Mewaldt et al., 2005b: Mewaldt R.A., C.M.S. Cohen, A.W. Labrador, R.A. Leske, G.M. Mason, M.I. Desai, M.D. Looper, J.E. Mazur, R.S. Selesnick, and D.K. Haggerty "Proton, helium, and electron spectra during the large solar particle events of October–November 2003", *J. Geophys. Res.*, **110**, No. A9, A09S18, DOI: 10.1029/2005 JA011038 (2005b).
- Michalek G., M. Ostrowski, and G. Siemieniec-Oziebło "Stochastic Fermi acceleration in turbulent fields with non-vanishing wave helicities", *Proc. 26th Intern. Cosmic Ray Conf.*, Salt Lake City, **6**, 284-287 (1999).
- Miniati F. "COSMOCR: A numerical code for cosmic ray studies in computational cosmology", *Comp. Phys. Comm.*, **141**, No. 1, 17-38 (2001).
- Moraal H. and W.I. Axford "Cosmic ray acceleration in supernova blast waves", *Astronomy and Astrophysics*, **125**, No. 2, 204-216 (1983).

- Moreton, G.E., in IAU Symp. 22, *Stellar and Solar Magnetic Fields*, ed. R. Lust, North-Holland, Amsterdam, 371 (1965).
- Moskalenko I.V. "Cosmic Ray Propagation and Acceleration", *Proc. 28-th Intern. Cosmic Ray Conf.*, Tsukuba, **8**, 183-204 (2003)
- Münich K. and IceCube Collaboration "Search for a Diffuse Flux of Non-Terrestrial Muon Neutrinos with the AMANDA Detector", *Proc. 29-th Intern. Cosmic Ray Conf.*, Pune, **5**, 17-20 (2005)
- Nakada M.P. "Acceleration of heavy ions by radiation pressure", *Astrophys. J.*, **184**, No. 2, 551-570 (1973).
- Neher H.V. and H.R. Anderson "Cosmic rays at balloon altitudes and the solar cycle", *J. Geophys. Res.*, **67**, No. 4, 1309-1315 (1962).
- Niemiec J. and M. Ostrowski "First-order Fermi particle acceleration at relativistic shock waves with a 'realistic' magnetic field turbulence model", *Proc. 28th Intern. Cosmic Ray Conf.*, Tsukuba, Japan, **4** 2015-2018 (2003a).
- Niemiec J. and M. Ostrowski "Cosmic ray acceleration at parallel relativistic shocks in the presence of finite-amplitude magnetic field perturbations", *Proc. 28th Intern. Cosmic Ray Conf.*, Tsukuba, Japan, **4**, 2019-2022 (2003b).
- Nikolaev Yu.A. and V.N. Tsytovich "The processes of Compton ionization and the spectra of relativistic electrons in a plasma turbulent reactor", *Astrofizika*, **12**, No. 3, 543-553 (1976).
- Noerdlinger P.D. "On the relativistic ejection of a particle by radiation pressure", *Astrophys. and Space Science.*, **13**, No. 1, 70-73 (1971).
- Noerdlinger P.D. "Relativistic ejection of a particle by radiation pressure. II", *Astrophys. J.*, **192**, No. 2, 529-533 (1974).
- Nutaro T., S. Riyavong, and D. Ruffolo "Application of a generalized total variation diminishing algorithm to cosmic ray transport and acceleration", *Comp. Phys. Comm.*, **134**, Issue 2, 209-222 (2001).
- Ostrowski M. "Acceleration of relativistic particles in shocks with oblique magnetic fields", *Monthly Notices of the Royal Astron. Society*, **233**, No. 2, 257-264 (1988a).
- Ostrowski M. "Particle acceleration at shock waves in the presence of finite amplitude perturbations of magnetic field. I - Parallel shock", *Astronomy and Astrophysics*, **206**, No. 1, 169-174. (1988b).
- Ostrowski M. "Monte Carlo simulations of energetic particle transport in weakly inhomogeneous magnetic fields. I - Particle acceleration in relativistic shock waves with oblique magnetic fields", *Monthly Notices of the Royal Astronomical Society*, **249**, No. 3, 551-559 (1991).
- Ostrowski M. "Cosmic-ray acceleration at relativistic shock waves in the presence of oblique magnetic fields with finite-amplitude perturbations", *Monthly Notices of the Royal Astronomical Society*, **264**, No. 1, 248-256 (1993).
- Ostrowski M. "Cosmic ray propagation and acceleration", *Proc. 27th Intern. Cosmic Ray Conf.*, Hamburg, Vol. Invited, Rapporteur, and Highlight Papers, 162-169 (2001).
- Ostrowski M. and G. Siemienieć-Oziebło "Diffusion in momentum space as a picture of second-order Fermi acceleration", *Astropart. Phys.*, **6**, No. 3-4, 271-277 (1997).
- Parker E.N. "The solar-flare phenomenon and the theory of reconnection and annihilation of magnetic fields", *Astrophys. J. Suppl.*, **8**, 177-211 (1963).
- Parker E.N. "The Dynamical State of the Interstellar Gas and Field", *Astrophys. J.*, **145**, No. 3, 811-833 (1966).

- Petchek H.E. "Magnetic field annihilation", In *Proc. AAS-NASA Sympos. on Physics of Solar Flares*, NASA SP-50, Ed. by W.N. Hess, Washington, 425-439 (1964).
- Pikelner S.B. "Nature of the Fine Structure of the Middle Chromosphere", *Solar Phys.*, **20**, No. 2, 286-294 (1971).
- Priest E.R. "Sweet's mechanism for the destruction of magnetic flux", *Quart. J. Mech. and Appl. Math.*, **25**, No. 3, 319-332 (1972).
- Priest E.R., in *Solar Magnetohydrodynamics* (ed. E.R. Priest), Reidel, Dordrecht, 315 (1982).
- Priest E.R., B.U.Ö. Sonnerup "Theories of magnetic field annihilation", *Geophys. J. Roy. Astron. Soc.*, **41**, No. 3, 405-413 (1975).
- Prishchep V.L. and V.S. Ptuskin "On the acceleration of fast particles at the spherical shock wave front", *Astronomy J.*, Moscow, **58**, No.4, 779-789 (1981).
- Protheroe R.J. "An efficient Monte Carlo scheme for relativistic shock acceleration", *Proc. 27th Intern. Cosmic Ray Conf.*, Hamburg, **6**, 2006-2009 (2001a).
- Protheroe R.J. "Energy spectrum, and acceleration time in relativistic shock acceleration", *Proc. 27th Intern. Cosmic Ray Conf.*, Hamburg, **6**, 2014-2017 (2001b).
- Protheroe R.J., A. Meli, and A.-C. Donea "Small-angle scattering and diffusion: application to relativistic shock acceleration", *Space Sci. Rev.*, **107**, Issue 1, 369-372 (2003).
- Protheroe R.J. and T. Stanev "Cut-offs and pile-ups in shock acceleration spectra", *Astropart. Phys.*, **10**, No. 2-3, 185-196 (1998).
- Ptuskin V.S. "Origin of galactic cosmic rays: sources, acceleration and propagation (OG-2)", *Proc. 29-th Intern. Cosmic Ray Conf.*, Pune, **10**, 317-328 (2005).
- Ptuskin V.S. and V.N. Zirakashvili "Limits on diffusive shock acceleration in supernova remnants in the presence of cosmic-ray streaming instability and wave dissipation", *Astronomy and Astrophysics*, **403**, No. 1, 1-10 (2003).
- Pustil'nik L.A. "Instability of quiet prominences and solar flare origin", *Astronomy J.*, Moscow, **50**, No.6, 1211-1219 (1973).
- Pustil'nik L.A. "Trigger mechanisms of the solar flares", in *Space Investigations of the Solar Active Regions*, Academy Press (Nauka), Moscow, 108-125 (1976).
- Pustil'nik L.A. "Triggering mechanisms of solar flares", *Proc. 15th Intern. Cosmic Ray Conf.*, Plovdiv, **5**, 18-22 (1977a).
- Pustil'nik L.A. "Particle acceleration in the current layer of solar flares", *Proc. 15th Intern. Cosmic Ray Conf.*, Plovdiv, **5**, 13-17 (1977b).
- Pustil'nik L.A. "Particle acceleration in the current sheet of a solar flare", *Astronomich. Zh.*, **55**, No. 3, 607-616 (1978). Translation in English: *Soviet Astronomy*, **22**, 350-355, May-June 1978.
- Pustil'nik L.A. "On the Energetics of the Current Sheet of a Flare", *Soviet Astron.*, **24**, No. 3, 347-352 (1980).
- Pustil'nik L.A. "Unsteady state of the turbulent current sheet of a flare", *Astrophys. and Space Sci.*, **252**, No. , 325-334 (1997).
- Pustil'nik L.A. "Solar flare phenomena as a phase transition caused by frustration of current percolation", *Astrophys. and Space Sci.*, **264**, Issue 1/4, 171-182 (1999a).
- Pustil'nik L.A. "Solar cosmic ray acceleration as space diffusion in the regular electric fields of double-layers clusters in random resistors network of the turbulent current sheet", *Proc. 26th Intern Cosmic Ray Conf.*, Salt Lake City, **6**, 228-231 (1999b).

- Pustil'nik L.A. "Critical state of current percolation, solar flare energy release, acceleration and energy spectrum", *Proc. 26th Intern Cosmic Ray Conf.*, Salt Lake City, **6**, 300-303 (1999c)
- Pustil'nik L.A. "Trigger process in solar flare as disruption of percolated current network by external disturbance", *Proc. 27-th Intern Cosmic Ray Conf.*, Hamburg, **8**, 3250-3253 (2001)
- Pustil'nik L.A. and N.P. Stasyuk "Instability of the coronal condensations", *Astronomical Circular USSR*, No. 778, 3-5 (1973).
- Pustil'nik L.A. and N.P. Stasyuk "Periodical fluctuations of a flux of solar local sources: 1. Origin of the periodical components", *Astrophysicheskiye Issledovaniya, Proceed. Special Astroph. Obs.*, **6**, 81-91 (1974)
- Rädler K.-H. "On the influence of a large-scale magnetic field on turbulent motions in an electrically conducting medium", *Astronomische Nachrichten*, **295**, No. 6, 265-273 (1974).
- Ramaty R. "Neutral radiations and particle acceleration at the Sun", *Proc. 20-th Intern Cosmic Ray Conf.*, Moscow, **8**, 127-140 (1987)
- Ramsey H. and S. Smith "Flare-Initiated Filament Oscillations", *Astron. J.*, **70**, No. 9, 688-688 (1965).
- Reames D.V., Barbier L.M., and Ng C.K. "The spatial distribution of particles accelerated by coronal mass ejection-driven shocks", *Astrophys. J.*, **466**, No. 1, Part 1, 473-486 (1996)
- Reames D.V., L.M. Barbier, T.T. von Rosenvinge, G.M. Mason, J.E. Mazur, and J.R. Dwyer "Energy spectra of ions accelerated in impulsive and gradual solar events", *Astrophys. J.*, **483**, 515-522 (1997).
- Reames D.V. and C.K. Ng "Streaming-limited intensities of solar energetic particles", *Astrophys. J.*, **504**, No. 2, Part 1, 1002-1005 (1998).
- Render S. "Percolation and conduction in random resistor-diode networks", *Annals Israel Phys. Soc.*, **5**, *Percolation structures and processes*, Eds. G Deutscher, R Zallen and J. Adler, Part D, 447-475 (1983).
- Rengarajan T.N. "Comments on Fermi acceleration mechanism for high-energy cosmic rays", *Lettere Al Nuovo Cimento*, **8**, ser.2, No. 14, 861-865 (1973).
- Rüdiger G "Die Wechselwirkung zwischen homogener Turbulenz und inhomogenem magnetischen Feld in der Umgebung neutraler Flächen", *Astron. Nachr.*, **296**, No. 3, 133-141 (1975)
- Ruffolo D. "Transport and acceleration of energetic charged particles near an oblique shock", *Astrophys. J.*, **515**, No. 2, 787-800 (1999).
- Ruffolo D. and C. Channok "Finite-time shock acceleration", *Proc. 28th Intern. Cosmic Ray Conf.*, Tsukuba (Japan), **6**, 3681-3684 (2003).
- Rust D.H. "Chromospheric Explosions and Satellite Sunspots", in *Structure and Development of Solar Active Region*, ed. H.O. Kiepenheuer, Dordrecht, D. Reidel, IAU Symp. No. 35, 77-84 (1968).
- Rust D.H. "An active role for magnetic fields in solar flares", *Solar Phys.*, **47**, No. 1, 21-40 (1976).
- Ryan J.M. "Solar emissions (SH1)", *Proc. 29-th Intern. Cosmic Ray Conf.*, Pune, **10**, 357-366 (2005).
- Ryu D, J.P. Ostriker, H. Kang, and R. Cen "A cosmological hydrodynamic code based on the total variation diminishing scheme", *Astrophys. J.*, **414**, No.1, Part 1, 1-19 (1993).

- Saito S., J.I. Sakai, and T. Haruki "Particle acceleration due to electrostatic shock wave driven by counterstreaming pair plasmas", *Proc. 28th Intern. Cosmic Ray Conf.*, Tsukuba (Japan), **1**, 311-314 (2003).
- Schatzman E. "On the acceleration of particles in shock fronts", *Annales d'Astrophysique*, **26**, No. 3, 234-249 (1963).
- Schatzman E. "Processus deceleration", *J. Phys.*, **30**, No. 11-12, Suppl., 8-13 (1969).
- Schlickeiser R. "An explanation of abrupt cutoffs in the optical-infrared spectra of nonthermal sources - A new pile-up mechanism for relativistic electron spectra", *Astronomy and Astrophysics*, **136**, No. 2, 227-236 (1984).
- Schneider P. and J.G. Kirk "Particle acceleration at modified shock fronts. I the power-law spectrum for relativistic flows", *Astronomy and Astrophysics*, **217**, No. 1-2, 344-350 (1989).
- Schluter A. "Solar radio emission and the acceleration of magnetic-storm particles", In *Radio Astronomy, Proceedings from 4th IAU Symposium* (Ed. H.C. Van de Hulst). Cambridge University Press, 356-357 (1957).
- Shabansky V.P. "Particle acceleration during passage of hydromagnetic shock front", *Zh. Eksp. Teor. Fiz.*, **41**, No.4 (10), 1107-1111 (1961).
- Shabansky V.P. "Interaction of energetic charged particles with hydromagnetic waves", *Geomagnetism and Aeronomy*, **6**, No.3, 472-478 (1966).
- Shafranov V.D., *J. Nucl. Energy*, **111**, No. 5, 86 (1957).
- Siemienieć-Oziebło G., G. Michalek, and M. Ostrowski "Asymmetric scattering of energetic particles by finite-amplitude circularly polarized Alfvén waves", *Astropart. Phys.*, **10**, No. 1, 121-128 (1999).
- Skilling J. "Cosmic ray streaming -I. Effect of Alfvén waves on particles", *Mon. Notic. Roy. Astron. Soc.*, **172**, No.3, 557-566 (1975).
- Smith D.F. "Acceleration of particles by electron plasma waves in moderate magnetic field", *Astrophys. and Space Sci.*, **42**, No. 2, 261-283 (1976).
- Spicer D.S. "An unstable arch model of a solar flare", *Solar Phys.*, **53**, No. 2, 305-345 (1977).
- Spicer D.S., "Magnetic energy storage and conversion in the solar atmosphere", *Space Sci. Rev.*, **31**, No. 4, 351-435 (1982).
- Stoker P.H. "Relativistic Solar Proton Events", *Space Sci. Rev.* **73**, 327-385 (1994).
- Swann W.F.G. "A mechanism of acquirement of cosmic-ray energies by electrons", *Phys. Rev.*, **43**, No. 4, 217-220 (1933).
- Swann W.F.G. "Processes involved in electromagnetic acceleration of particles to cosmic ray energies", *Proc. 6th Intern. Cosmic Ray Conf.*, 1959, Moscow, **3**, 167-170 (1960).
- Sweet A. "The production of high energy particles in solar flares", *Nuovo Cimento*, **8**, Suppl. 2, 188-196 (1958).
- Sweet P.A. "Mechanisms of Solar Flares", *Ann. Rev. Astron. Astrophys.*, **7**, 149-176 (1969).
- Syrovaty S.I. "Dynamic dissipation of magnetic field and particle acceleration", *Astronom. J.*, Moscow, **43**, No.2, 340-355 (1966).
- Syrovaty S.I. "Fast particle generation in the space", *Izv. Acad. of Science USSR, Ser. Phys.*, **31**, No.8, 1303-1306 (1967).
- Syrovaty S.I. "Particle acceleration in the space", *Proc. 5th All-Union Summer School of Cosmophysics*, Apatity, 58-72 (1968).

- Syrovatsky S.I. "Cosmic ray bursts and acceleration on the Sun", *Proc. Intern. Seminar on Study of Interplanetary Space Physics by Means of Cosmic Rays*, Leningrad, 7-9, (1969).
- Syrovatsky S.I. "Current sheets and particle acceleration in the space", *Proc. Intern. Seminar on Cosmic Ray Generation on the Sun*, Moscow, 15-36 (1971).
- Syrovatsky S.I., *Comments in Astrophys. and Space Physics*, **4**, 65 (1972).
- Syrovatsky S.I. "Process of magnetic force line merging and its role in cosmic plasma", *Proc. VIIth Leningrad Intern. Seminar on Solar Corpuscular Streams and the Radiation Belts of the Earth and Jupiter*, Leningrad, 63-80 (1975).
- Syrovatsky S.I. "Pinch sheets and reconnection in astrophysics", *Ann. Rev. Astron. Astrophys.*, **19**, 163-229 (1981).
- Terletsky Ya.P. "Possible acceleration of charges by electromagnetic field of the Earth's magnetic dipole", *Proc. Int. Conf. Cosmic Rays*, Moscow, **3**, 239-244 (1959).
- Toptygin I.N. "Particle acceleration by small-amplitude magnetohydrodynamic waves", *Proc. Intern. Seminar on Particle Acceleration in the Space (Earth Vicinities and Interplanetary Space)*, Leningrad, - (1972).
- Toptygin I.N. "Interaction of fast particles with magneto-hydrodynamical turbulence", *Astrophys. Space Sci.*, **20**, No. 2, 351-371 (1973).
- Tsytoich V.N. "On charged particle interaction with low-frequency oscillations of plasma", *Zh. Eksp. Teor. Fiz.*, Moscow, **44**, No. 3, 946-956 (1963a).
- Tsytoich V.N. "Acceleration by radiation and problems of fast particle generation in the space", *Astronomy J.*, Moscow, **40**, No. 4, 612-624 (1963b).
- Tsytoich V.N. "Effect of radiation on relativistic particles moving in magnetic field", *Izv. Vuzov, Radiofizika*, Moscow, **6**, No. 5, 918-927 (1963c).
- Tsytoich V.N. "Electron acceleration in the Earth's radiation belts", *Geomagnetism and Aeronomy*, Moscow, **3**, No. 4, 616-625 (1963d).
- Tsytoich V.N. "Statistical acceleration of particles in plasma", *Trans. FIAN*, Nauka, Moscow, **32**, 130-164 (1966a).
- Tsytoich V.N. "Statistical acceleration of particles in turbulent plasma", *Uspekhi Fiz. Nauk*, Moscow, **89**, No. 1, 89-146 (1966b).
- Tsytoich V.N. "Cosmic rays and plasma turbulence", *Izv. Acad. Sci. USSR, Ser. Phys.*, **34**, No. 11, 1800-1816 (1969).
- Tverskoy B.A. "Charged particle acceleration in the interplanetary medium", in *Nuclear Space Physics*, Leningrad, 137-146 (1978).
- Vainio R. "Conversion of blast wave energy into radiation: Particle transport simulations", In *High Energy Blazar Astronomy* (eds. L.O. Takalo and E. Valtaoja), ASP Conf. Proc., **299**, 143-148 (2003).
- Vainio R., L. Kocharov, and T. Laitinen "Conversion of blast wave energy into radiation: Particle transport simulations", *Astrophys. J.*, **528**, No. 2, 1015-1025 (2000).
- Vainio R. and R. Schlickeiser "Bulk speeds of cosmic rays resonant with parallel plasma waves", *Proc. 26th Intern. Cosmic Ray Conf.*, Salt Lake City, **4**, 310-313 (1999a).
- Vainio R. and R. Schlickeiser "Self-consistent generation of flat power-law particle momentum spectra by diffusive shock acceleration", *Proc. 26th Intern. Cosmic Ray Conf.*, Salt Lake City, **4**, 407-410 (1999b).
- Vainio R. and R. Schlickeiser "Self-consistent Alfvén-wave transmission and test-particle acceleration at parallel shocks", *Astronomy and Astrophysics*, **343**, 303-311 (1999c).

- Vainshtein S.I. and V.M. Tomozov "On the mechanism of rapid dissipation of magnetic field in collisionless plasma", *Research into Geomagnetism, Aeronomy and Solar Physics*, Nauka, Moscow, **37**, 100-105 (1975).
- Vasilyev et al., 1978: Vasilyev V.N., I.N. Toptygin, and A.A. Chirkov "Interaction of high-energy particles with a shock front in turbulent medium", *Geomagnetism and Aeronomy*, Moscow, **18**, No.3, 415-422 (1978).
- Vasilyev et al., 1980: Vasilyev M.V., A.L. Gyulameryan, A.V. Mamaev, V.V. Ragulsky, P.M. Semenov, and V.G. Sidorovich "Recording of phase fluctuations of stimulated scattered light", *JETP Letters*, **31**, No. 11, 634-638 (1980).
- Vedernikov N.F., K.M. Mukimov, G.P. Sigal and B.Yu. Sokolov "Conductivity of HTSC $\text{YBa}_2\text{Cu}_3\text{O}_{7.8}$ ceramics in the vicinity of the transition to the superconducting state", *Sverkhprovodimost': Fizika, Khimiya, Tekhnika*, **7**, No. 2, 316-321 (1994).
- Virtanen J. and R. Vainio "Monte Carlo simulations of electron acceleration in parallel relativistic shocks", *Proc. 28th Intern. Cosmic Ray Conf.*, Tsukuba (Japan), **4**, 2023-2026 (2003a).
- Virtanen J. and R. Vainio "Simulations on the effect of internal structure of shock fronts on particle acceleration", In *High Energy Blazar Astronomy* (eds. L.O. Takalo and E. Valtaoja), ASP Conf. Proc., **299**, 157-160 (2003b).
- Vlahos L., M. Georgoulis, R. Kluiving, and P. Paschos "The statistical flare", *Astronomy and Astrophysics*, **299**, No. 3, 897-911 (1995).
- Völk H.J. "Particle acceleration in astrophysical shock waves", *Proc. 20-th Intern. Cosmic Ray Conf.*, Moscow, **7**, 157-200 (1987).
- Völk H.J. and P.L. Biermann "Maximum energy of cosmic-ray particles accelerated by supernova remnant shocks in stellar wind cavities", *Astrophys. J.*, **333**, No. 2, L65-L68 (1988).
- Völk et al., 1981a: Völk H.J., G.E. Morfill, and M.A. Forman "The effect of losses on acceleration of energetic particles by diffusive scattering through shock waves", *Astrophys. J.*, **249**, No. 1, 161-175 (1981a).
- Völk et al., 1981b: Völk H.J., G.E. Morfill, and M. Forman "Cosmic-ray acceleration in the presence of losses", In *Origin of cosmic rays* (Eds. G. Setti, G. Spada, and A.W. Wolfendale), IAU Symp. 94, Bologna 1981, 359-360 (1981b).
- Von Rosenvinge T.T., C.M.S. Cohen, E.R. Christian, A.C. Cummings, R.A. Leske, R.A. Mewaldt, P.L. Slocum, E.C. Stone, and M.E. Wiedenbeck "The Solar Energetic Particle Event of 6 May 1998", In *Acceleration and Transport of Energetic Particles Observed in the Heliosphere. Proc. ACE-2000 Symposium*, Indian Wells, CA. AIP, **528**, Edited by Richard A. Mewaldt, J.R. Jokipii, Martin A. Lee, Eberhard Möbius, and Thomas H. Zurbuchen, Melville, NY, 111-114 (2000).
- Vourlidas A., P. Subramanian, K.P. Dere, and R.A. Howard "Large-Angle Spectrometric Coronagraph measurements of the energetics of coronal mass ejections", *Astrophys. J.*, **534**, No. 1, Part 1, 456-467 (2000).
- Webb G.M. "Boundary conditions for energetic particle transport at shocks", *Astron. and Astrophys.*, **124**, No.2, 163-171 (1983a).
- Webb G.M. "Oblique MHD shock waves modified by cosmic rays", *Proc. 16th Intern. Cosmic Ray Conf.*, Bangalore, **2**, 251-254 (1983b).
- Webb G.M. "The structure of oblique MHD cosmic ray shocks", *Astron. and Astrophys.*, **127**, No.1, 97-112 (1983c).

- Webb et al., 1981: Webb G.M., W.I. Axford, and M.A. Forman "Cosmic ray acceleration by stellar winds. I - Total density, pressure and energy flux", *Proc. 17th Inter. Cosmic Ray Conference*, Paris, **2**, 309-312 (1981).
- Webb et al., 1983: Webb G.M., W.I. Axford, and T. Terasawa "On the drift mechanism for energetic charged particles at shocks", *Astrophys. J.*, **270**, No. 2, Part 1, 537-553 (1983).
- Webb et al., 1984: Webb G.M., L.O'C. Drury, and P.L. Biermann "Diffusive shock acceleration of energetic electrons subject to synchrotron losses", *Astronomy and Astrophysics*, **137**, No. 2, 185-201 (1984).
- Webb et al., 1986: Webb G.M., L.O'C. Drury, and H.J. Völk "Cosmic-ray shock acceleration in oblique MHD shocks", *Astronomy and Astrophysics*, **160**, No. 2, Part 2, 335-346 (1986).
- Weibel E.S. "Spontaneously growing transverse waves in a plasma due to an anisotropic velocity distribution", *Phys. Rev. Lett.*, **2**, No. 3, 83-84 (1959).
- Wentzel D.G. and P.E. Seiden "Solar active regions as a percolation phenomenon", *Astrophys. J.*, Part 1, **390**, No. 1, 280-289 (1992).
- Yanasak N.E., M.E. Wiedenbeck, R.A. Mewaldt et al. "Measurement of the Secondary Radionuclides ^{10}Be , ^{26}Al , ^{36}Cl , ^{54}Mn , and ^{14}C and Implications for the Galactic Cosmic-Ray Age", *Astrophys. J.*, **563**, No 2, Part 1, 768-792 (2001)
- Yang C.-K. and B.U.Ö. Sonnerup, "Compressible magnetic field reconnection - A slow wave model", *Astrophys. J.*, **206**, No. 2, 570-582 (1976).
- Yeh T. "Diffusive hydromagnetic flow in the vicinity of a neutral point", *Astrophysical J.*, **207**, No. 3, 837-847 (1976a).
- Yeh T. "Reconnection of magnetic field lines in viscous conducting fluids", *J. Geophys. Res.*, **81**, No. 25, 4524-4530 (1976b).
- Zenitani S. and M. Hoshino "A plasma sheet as a source of nonthermal particles - relativistic magnetic reconnection and relativistic drift kink instability in $e^\pm$ plasmas", *Proc. 28th Intern. Cosmic Ray Conf.*, Tsukuba, **4**, 2043-2046 (2003).
- Zhitnik I.A. and M. Lifshitz "X Rays from Coronal Condensations and the Development of Active Regions", *Astronomich. Zh.*, **49**, No. 1, 137-147 (1972).

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