

PARTICLE ACCELERATION
AND DETECTION

S. Khan

Collective Phenomena in Synchrotron Radiation Sources

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Collective Phenomena in Synchrotron Radiation Sources

Prediction, Diagnostics, Countermeasures

With 92 Figures

 Springer

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Preface

This book is intended to dispel the notion that collective phenomena are an obscure and inaccessible topic. Having become increasingly important in modern storage rings, collective effects deserve to become part of the general education in accelerator physics.

I tried to write the book I was looking for when I started to study collective instabilities: an introduction that can be read from cover to cover with moderate effort and familiarizes the reader with the basic concepts. Self-contained appendices contain what I considered worth discussing but might impair the readability of the main text, such as a reminder of basic relations and some background information on selected topics. Finally, an extensive list of references was compiled (with some emphasis on accessibility of the source) in order to facilitate the search for more detailed information.

A previous version of the present text was submitted in German language as *Habilitationsschrift* – a postdoctoral thesis, which is a peculiarity of the German academic system – to the Humboldt University of Berlin. It grew from my occupation with collective phenomena over ten years at the third-generation synchrotron light source BESSY II. This has some consequences regarding the character of this text:

- There is a clear emphasis on synchrotron light sources and electron storage rings. However, many concepts (wake field, impedance, beam spectrum, beam lifetime, etc.) are quite general.
- The selection of material reflects my personal experience and some readers may want to skip over details that I considered important, while others may find that their favorite topic was not covered in sufficient detail.
- As suggested by the subtitle of the book, its content follows the logical sequence of prediction (in the design phase of a new facility), diagnostics (during commissioning, initial operation, and accelerator physics studies) and countermeasures (required for optimum routine operation).
- Most data and examples are taken from BESSY II, which makes the treatment monolithic and avoids confusion with varying machine parameters. In addition, references are given to direct the reader's attention to the work done and results obtained at other laboratories.

Synchrotron radiation sources *are* collective phenomena, since they are created by the coherent effort of many people. Therefore, I would like to thank all my colleagues who contributed to the successful operation of BESSY II. Many of them were directly involved in the work described in this book. In constructing and commissioning the multi-bunch feedback systems, I am particularly indebted to T. Knuth, who as a doctorate student did most of the work. Hardware and software contributions came from W. Anders, K. Bürkmann, V. Dürr, F. Falkenstein, R. Lange, J. Rahn, G. Schindhelm and others. I would also like to thank D. Krämer and P. Kuske for their continuous support and many helpful discussions. Last, but not least, I am grateful to the BESSY management, particularly to Prof. E. Jaeschke, for providing excellent working conditions with a considerable amount of freedom which made writing this book possible. All work at BESSY was funded by the Bundesministerium für Bildung und Forschung and by the Land Berlin.

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Hamburg, April 2006

Shaukat Khan

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Introduction

In the 1990s, progress in particle accelerators and storage rings resulted in ever higher beam intensities. Several particle “factories” came into existence – the B-meson factories at SLAC (Stanford, USA) and KEK (Tsukuba, Japan) as well as the Φ factory DAΦNE¹ (Frascati, Italy). Their design goal was the production of mesons and leptons at a high rate. Efforts to increase the “luminosity”, defined as the particle production rate divided by the cross section, were also made at existing facilities. Furthermore, several synchrotron radiation sources of the “third generation” were designed and built in order to meet the ever-increasing demand on intense radiation in the vacuum-ultraviolet (VUV) and X-ray regime.

Traditionally, the design of a particle accelerator or storage ring starts with the layout of the external electromagnetic fields that act on each individual particle in the same way. These include the static magnetic fields to guide and focus the beam as well as radio-frequency (rf) fields for acceleration. However, in view of the present tendencies towards higher beam current and smaller beam cross section, effects caused by the coaction of many particles become more and more important. These “collective effects” can easily prevent the prospective intensity gain of a new facility if efficient countermeasures are not considered at an early stage of the machine design.

This book is meant as an introduction to collective phenomena and how to avoid or counteract them.

1.1 Definition of the Topic

Collective phenomena in particle accelerators and storage rings are those phenomena that are caused by the interaction between all beam particles, or at least a large number of them. Apart from beam particles, other particles may

¹ Addresses of all facilities and institutes mentioned in this book are given in Appendix H.

exist inside the vacuum vessel of a machine and contribute in some way to collective effects: atoms and molecules of the residual gas, photons, photo electrons, or dust particles. However, the origin of the phenomena considered here is the joint action of the beam particles themselves.

Synchrotron radiation sources are – presently – storage rings for ultrarelativistic electrons or positrons emitting spontaneous electromagnetic radiation when the beam is transversely deflected by magnetic fields. These machines are optimized for ultraviolet, VUV and X-ray radiation, but their potential in other regimes such as THz radiation has been demonstrated as well. Several “third-generation” synchrotron light sources were built during the 1990s, others are under construction or planned. As an instructive example, BESSY II (Berlin, Germany) is described in Appendix B. Future synchrotron radiation sources are briefly discussed in Chap. 5. Restricting the discussion of collective phenomena to this class of electron² storage rings has several implications:

- The beam energy is constant. Effects occurring during acceleration in linear accelerators or circular machines are not considered.
- In a synchrotron radiation source, only one particle beam is present. Beam–beam interaction, an important topic for colliding-beam machines as used in elementary particle research, is not covered by this book.
- The particle speed is very close (and for practical purposes considered equal) to the speed of light.
- Electrons lose energy due to the emission of radiation. This energy is recovered when traversing a radio-frequency (rf) field at a particular phase. The phase restriction causes the electron beam in a storage ring to be a sequence of bunches much shorter than their spacing. The properties of a continuous (“coasting”) beam will not be discussed.

The definition of the topic is not as restrictive as it may appear at first glance. Due to the large number of existing and planned facilities and their wide range of applications, synchrotron radiation sources play a dominant role in accelerator physics. Furthermore, they are very well suited for the discussion of collective phenomena, which strongly influence the quality of synchrotron radiation delivered to the users. Much of what will be said applies equally to other storage rings. There is, in particular, a similarity between synchrotron radiation sources and modern meson factories as far as beam energy (1–8 GeV), electron density (typically 1 nC per electron bunch) and bunch spacing (≥ 2 ns) are concerned.

² If no explicit distinction is made between electrons and positrons, everything said about “electrons” applies to both.

1.2 Outline

The collective phenomena occurring in synchrotron radiation sources include:

1. *Beam instabilities.* Beam particles interacting with their surroundings create electromagnetic fields that act on subsequent beam particles and can cause them to oscillate about their equilibrium position. One can distinguish between
 - short-range fields acting within an electron bunch and long-range fields acting between different bunches.
 - longitudinal oscillations, i.e. motion along the beam axis accompanied by periodic energy changes, and transverse oscillations, i.e. motion perpendicular to the beam axis.
2. *Lifetime-limiting effects.* The lifetime of an electron beam in a storage ring is governed by effects that may be classified as “collective” due to their dependence on the beam current
 - Elastic (Coulomb-) scattering of electrons on residual gas nuclei or shell electrons causes an angular deflection. On the other hand, the emission of bremsstrahlung photons leads to a loss of energy. Both may cause the interacting beam electron to be lost. The residual gas density depends strongly on the number of synchrotron radiation photons hitting the walls of the vacuum vessel, which in turn depends on the beam current.
 - Møller scattering between two electrons within the same bunch (intra-beam scattering) causes an exchange of energy that may lead to the loss of both electrons. This is the “Touschek effect”, which is the most severe lifetime-limiting factor in third-generation synchrotron radiation sources.
3. *Ion effects.* If the residual gas density is high, e.g. during the commissioning phase of a new storage ring or after the vacuum vessel was vented, positive ions are created and trapped by stored electrons (not by positrons!) and may lead to beam instabilities. Because of their minor importance under normal operation conditions, ion effects will be discussed in the appendix.

These collective phenomena will be discussed in some detail, often using the synchrotron radiation source BESSY II as an example. Chapter 2 is dedicated to beam instabilities and Chap. 3 to lifetime issues. In Chap. 4, countermeasures employed at BESSY II and other facilities will be described. Finally, in Chap. 5, some conclusions for present-day and future synchrotron radiation sources will be drawn.

This text is complemented by an extensive appendix where more detailed information can be found. Throughout the book, SI units [1] will be used. Without further mention, c is always the speed of light ($2.998 \times 10^8 \text{ m s}^{-1}$), e is the elementary charge ($1.602 \times 10^{-19} \text{ C}$), ε_0 is the permittivity of free space ($8.859 \times 10^{-12} \text{ F m}^{-1}$), μ_0 is the permeability of free space ($1.257 \times 10^{-6} \text{ N A}^{-2}$), and i symbolizes $\sqrt{-1}$.

Collective Instabilities

2.1 Overview

Charged beam particles in a storage ring interact with the surrounding walls of the vacuum chamber and create electromagnetic fields that can act back on the particles themselves. These fields disturb their intended longitudinal and transverse position given by the magnetic guide fields and the electromagnetic radio-frequency (rf) fields. If, in turn, the disturbed particles create fields that tend to enhance the disturbed motion, a “collective instability” occurs. This must not be confused with disturbed particle trajectories due to optical resonances or nonlinear components of the guide field, which are not of a collective nature.

Collective instabilities show up as longitudinal or transverse beam excitations, where individual bunches or parts of bunches oscillate against each other. These oscillations may remain limited in amplitude, they may rise and collapse irregularly or periodically (sometimes in a sawtooth fashion), or they may increase in amplitude until the beam is lost.

Beam instabilities impair the quality of synchrotron radiation. One important figure of merit is the brilliance or spectral brightness¹ B :

$$B = \frac{\dot{N}_\gamma}{4\pi^2 \sigma_x \sigma_y \sigma_{x'} \sigma_{y'} dE/E_\gamma} \left(\frac{\text{photons s}^{-1}}{\text{mm}^2 \text{ mrad}^2 \text{ 0.1\% bandwidth}} \right), \quad (2.1)$$

where σ_x , σ_y , $\sigma_{x'}$ and $\sigma_{y'}$ are standard deviations of the horizontal and vertical spatial and angular distributions, respectively. By convention, the photon flux $\dot{N}_\gamma$ is integrated over a photon energy range (“bandwidth”) of $dE/E_\gamma = 0.001$ or 0.1%. Instabilities limit the brilliance in different ways:

¹ The term “brilliance” tends to be more popular in Europe. Depending on the author, “brightness” may denote the photon flux per bandwidth and solid angle, or the photon flux per area and solid angle; whereas “spectral brightness” is synonymous with brilliance. To increase the confusion, these quantities may or may not be normalized to the electron beam current.

- A transverse oscillation of electron bunches increases the time-averaged size and angular divergence of the radiation source.
- An oscillation in longitudinal phase space implies an oscillation of the electron energy E , which increases the time-averaged spread of photon energies in the undulator line spectrum, since the photon energy of undulator radiation is proportional to E^2 .
- Instabilities can be a serious limitation of the storable beam current I and consequently of the photon flux $\dot{N}_\gamma \sim I$.

It is instructive and for many applications sufficient to describe the longitudinal or transverse motion as a harmonic oscillation

$$A(t) = A_0 \exp(-i[\Omega + \Delta\Omega]t) . \quad (2.2)$$

The frequency shift $\Delta\Omega$ is a complex number. Its real part describes a modification of the oscillation frequency, whereas its imaginary part denotes the rate of change of the oscillation amplitude. The signature of an instability is an increasing amplitude, the imaginary part of $\Delta\Omega$ being positive.

For electrons travelling nearly at the speed of light, the electromagnetic fields created by the interaction with the surrounding walls act almost exclusively on trailing electrons, and they are figuratively called *wake* fields. In a storage ring, however, an electron can be influenced by the wake field of trailing particles from previous revolutions.

Instead of fully characterizing the wake field created by a given charge at each point in space and time, it is usually sufficient to specify the voltage experienced by a trailing “test” charge as function of its distance to the leading charge, integrated over a certain period of time (e.g. the time it takes to traverse significant structures like rf cavities). The Fourier transform of this (real) wake function is a complex function of frequency. This function is called *impedance*. In principle, impedance and wake function are equivalent descriptions of the same reality, one given in the frequency domain, the other in the time domain. Their usage, however, depends on the respective application. The wake function, for example, is better suited for time-dependent simulations, whereas analytical calculations are often simpler in the frequency domain since the beam motion is periodic and certain parameters (like the skin depth in the wall of a vacuum chamber) depend on frequency.

Impedance, wake function and the beam instabilities caused by them can be classified according to the following criteria:

- *Longitudinal* wake fields influence the energy of beam particles and their spatial distribution along the beam axis. *Transverse* wake fields deflect beam particles perpendicular to the beam axis in horizontal or vertical direction. Consequently, there are longitudinal and transverse wake functions, impedances and instabilities.

- *Long-range* wake fields decay slowly enough to act on trailing particle bunches. They correspond to *narrow-band* impedances in the frequency domain. They may decay over many revolution times of the storage ring causing *multi-bunch* instabilities where all bunches act like an ensemble of coupled pendula.
Short-range wake fields correspond to *broad-band* impedances and are, by definition, only significant within a particle bunch. Oscillations within a bunch are called *single-bunch* instabilities.
- There are distinct differences between beam instabilities in *linear accelerators* and *storage rings* as well as between oscillations of a *continuous beam* (also called *coasting beam*) and a *bunched beam*.

Concentrating on synchrotron radiation sources, this book is restricted to instabilities of bunched beams in storage rings. Under ordinary circumstances, only two types of instabilities are significant:

- “*Robinson*”-*type instabilities* are predominantly multi-bunch oscillations. The forces resulting from long-range wake fields create an imaginary frequency shift that is proportional to the beam current. This kind of instability exists at all values of beam current without any threshold. In practice, however, a threshold is given by the current above which the rise time of the instability is shorter than the damping time of stabilizing mechanisms (see Sect. 2.5). This instability type can essentially be understood using a one-particle model where each bunch is represented by a “macro particles”.
- *Mode-coupling instabilities* are single-bunch phenomena. Their model description requires to split a bunch into two or more macro particles, or to treat it as a continuous charge distribution. At low bunch current, the wake fields cause a current-dependent shift of the real frequency of each mode of oscillation. A non-zero imaginary frequency shift, that describes an instability, occurs only above a certain threshold current at which the real frequencies of two modes merge.

An example of a phenomenon that does not quite fit into these two categories is the “fast” ion instability, which is addressed in Appendix G. So far, it was only unambiguously observed in experiments where the residual gas pressure was intentionally increased.

Viewed in the frequency domain, instabilities can occur when the impedance is non-zero at a frequency that corresponds to a possible oscillation mode of the electron beam. Therefore, the frequency spectrum of a beam is described next. What follows is an introduction to wake functions and impedances, and how to predict them. Subsequently, different instability models with increasing degree of complexity are discussed, followed by a description of “natural” damping mechanisms. Finally, methods to measure the impedance at storage rings are addressed. The active control of collective instabilities using feedback systems is the topic of a later chapter.

2.2 The Frequency Spectrum of a Stored Beam

The beam spectrum, i.e. the beam current as function of frequency is deduced from the current as function of time, detected at a fixed position of the storage ring. This is analogous to the impedance, which is usually linked to geometrical or electrical properties at a fixed position of the vacuum vessel. Oscillations of the electron bunches create additional lines in the beam spectrum. Their frequencies depend not only on the oscillation frequency but also on the phase differences between subsequent bunches. Excitation of a particular oscillation requires an impedance component at its frequency to be present.

The beam spectrum $J(\omega)$ and the beam current as function of time $j(t)$ are connected via Fourier transform:

$$\begin{aligned} J(\omega) &= \int_{-\infty}^{\infty} dt j(t) \exp(-i\omega t), \\ j(t) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega J(\omega) \exp(i\omega t). \end{aligned} \quad (2.3)$$

The notion of a negative frequency² should not create any conceptual difficulty considering that only the real part of the oscillation is measurable:

$$\operatorname{Re} \exp(-i\omega t) = \operatorname{Re} \exp(i\omega t) = \cos(\omega t). \quad (2.4)$$

Thus, a measuring apparatus (e.g. a spectrum analyzer) does not distinguish between negative and positive frequencies. The negative part of a spectrum may be thought of as being “flipped over” to the positive side of the frequency axis.

A measured spectrum also depends on the frequency characteristics of the detector and electronic circuitry. In the following description of idealized spectra, technical issues of this kind are ignored. A more detailed discussion of beam spectra is given e.g. in [2], instruments like spectrum analyzers and network analyzers are explained in [3].

2.2.1 Pointlike Electron Bunches

As a starting point, consider a beam of pointlike electron bunches without any longitudinal or transverse oscillation (see Fig. 2.1).

Case 1a. Let a pointlike electron bunch make a single passage past a (likewise pointlike) detector. The current signal (in arbitrary units) can be described using Dirac’s δ -function

$$j(t) = \delta(t). \quad (2.5)$$

² Throughout this book, f denotes the frequency in terms of cycles per time unit, whereas $\omega = 2\pi f$ is the angular frequency, i.e. phase angle per time unit. For simplicity, the word “frequency” is used for both.

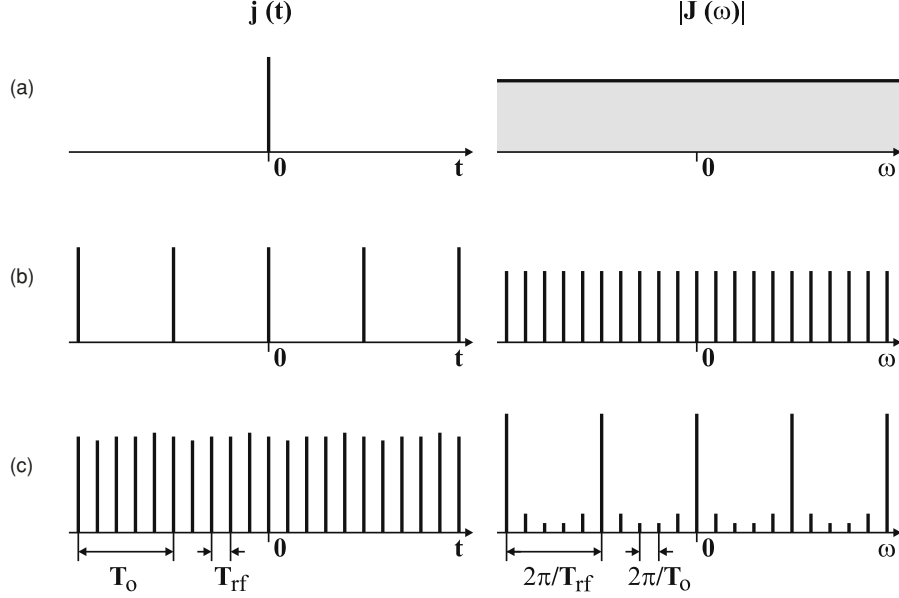


Fig. 2.1. Detected current signal caused by pointlike electron bunches as function of time (*left*) and frequency (*right*). (a) Single passage and (b) multiple passages of a single bunch with revolution time T_o , (c) multiple passages of T_o/T_{rf} equidistant bunches. Slight differences in bunch charge cause spurious revolution harmonics between the dominant rf harmonics

Fourier transformation results in an infinitely broad spectrum:

$$J(\omega) = \int_{-\infty}^{\infty} dt \delta(t) \exp(-i\omega t) = 1. \quad (2.6)$$

Case 1b. Let a pointlike electron bunch circulate in a storage ring with revolution time T_o and revolution frequency $\omega_o = 2\pi/T_o$. The beam current as function of time corresponds to a sequence of equidistant δ -functions:

$$j(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_o). \quad (2.7)$$

Its Fourier transform is

$$J(\omega) = \sum_{n=-\infty}^{\infty} \exp(-i\omega nT_o) = \omega_o \sum_{p=-\infty}^{\infty} \delta(\omega - p\omega_o), \quad (2.8)$$

where Poisson's sum rule for Fourier pairs $[f(t), F(\omega)]$ (e.g. [4])

$$\sum_{n=-\infty}^{\infty} f(an) = \frac{1}{a} \sum_{p=-\infty}^{\infty} F\left(\frac{2\pi p}{a}\right) \quad (2.9)$$

was applied with $f = \exp(-i\omega n T_o)$ and $a = T_o$. The electron bunch in the storage ring creates a spectrum of lines at integer multiples of the revolution frequency. These lines may be called “revolution harmonics”. In order to understand this result qualitatively, consider (2.7) to represent periodic samples of a sinewave. These samples are consistent with an oscillation of frequency ω_o , but any integer multiple frequency would yield the same sample values. This frequency ambiguity is called “aliasing”.

Case 1c. With $\omega_{\text{rf}} = h\omega_o$ being the rf frequency, h electron bunches correspond to a complete fill of the storage ring in the sense that each rf potential well (also called “bucket”) is occupied, and h is called the harmonic number of the storage ring. The current given by pointlike bunches with a temporal spacing of $T_{\text{rf}} = T_o/h$ is

$$j(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_{\text{rf}}) . \quad (2.10)$$

In analogy to (2.8), the beam spectrum is given by

$$J(\omega) = h\omega_o \sum_{p=-\infty}^{\infty} \delta(\omega - p\omega_{\text{rf}}) , \quad (2.11)$$

In this case, the lines of the spectrum are at integer multiples of the rf frequency and may be called “rf harmonics”. In real storage rings, however, small inhomogeneities in the fill pattern create spurious lines at multiples of the revolution frequency ω_o as well.

2.2.2 Extended Electron Bunches

The unphysical result of a spectrum extending to infinite frequency is a consequence of the assumption of pointlike bunches. As a more realistic approach, let the longitudinal charge density of each electron bunch be a Gaussian with standard deviation σ_τ in units of time (see Fig. 2.2).

Case 2a. A bunch with Gaussian charge distribution that passes the detector only once creates a current signal (in arbitrary units) of

$$j(t) = \frac{1}{\sqrt{2\pi}\sigma_\tau} \exp(-t^2/2\sigma_\tau^2) \quad (2.12)$$

and its spectrum is given by

$$J(\omega) = \exp(-\omega^2\sigma_\tau^2/2) . \quad (2.13)$$

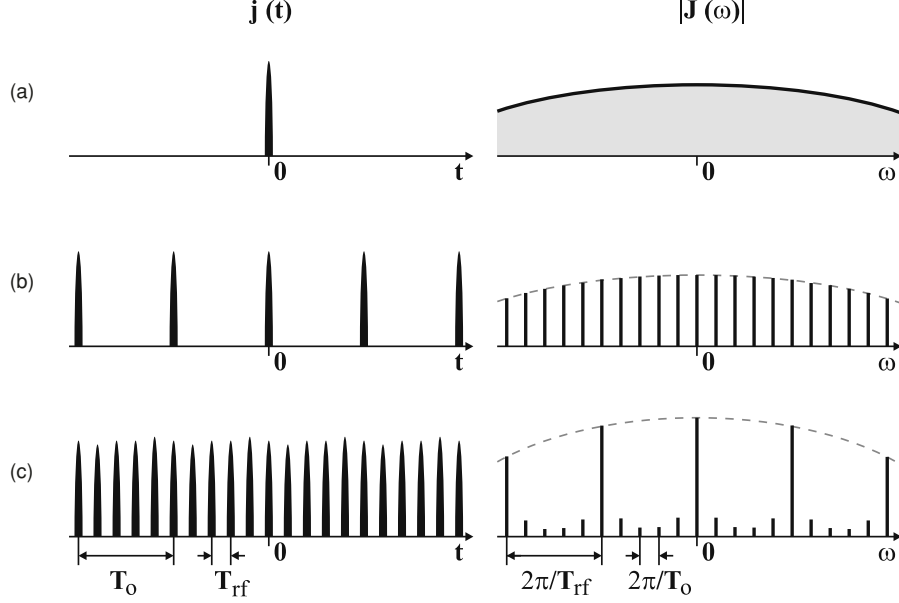


Fig. 2.2. Detected current signal caused by longitudinally extended bunches as function of time (*left*) and frequency (*right*). (a) Single passage and (b) multiple passages of a single bunch with revolution time T_0 , (c) multiple passages of T_0/T_{rf} equidistant bunches

Case 2b. For a Gaussian bunch in a storage ring with revolution time T_0 and revolution frequency $\omega_0 = 2\pi/T_0$, the beam current is a convolution of the previously assumed sequence of δ -functions and the charge density distribution:

$$\begin{aligned}
 j(t) &= \frac{1}{\sqrt{2\pi}\sigma_\tau} \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} dt' \delta(t' - nT_0) \exp(-[t - t']^2/2\sigma_\tau^2) \\
 &= \frac{1}{\sqrt{2\pi}\sigma_\tau} \sum_{n=-\infty}^{\infty} \exp(-[t - nT_0]^2/2\sigma_\tau^2). \quad (2.14)
 \end{aligned}$$

According to the convolution theorem, its Fourier transform is simply given by the product of the Fourier transforms of the convoluted functions:

$$J(\omega) = \omega_0 \exp(-\omega^2\sigma_\tau^2/2) \sum_{p=-\infty}^{\infty} \delta(\omega - p\omega_0). \quad (2.15)$$

The finite extension of the electron bunches defines the envelope of the line spectrum: the shorter the bunches, the broader the spectrum. The line width in the spectrum, on the other hand, is determined by the envelope of the distribution in time: the longer the measurement time (here assumed to be infinite), the smaller the line width.

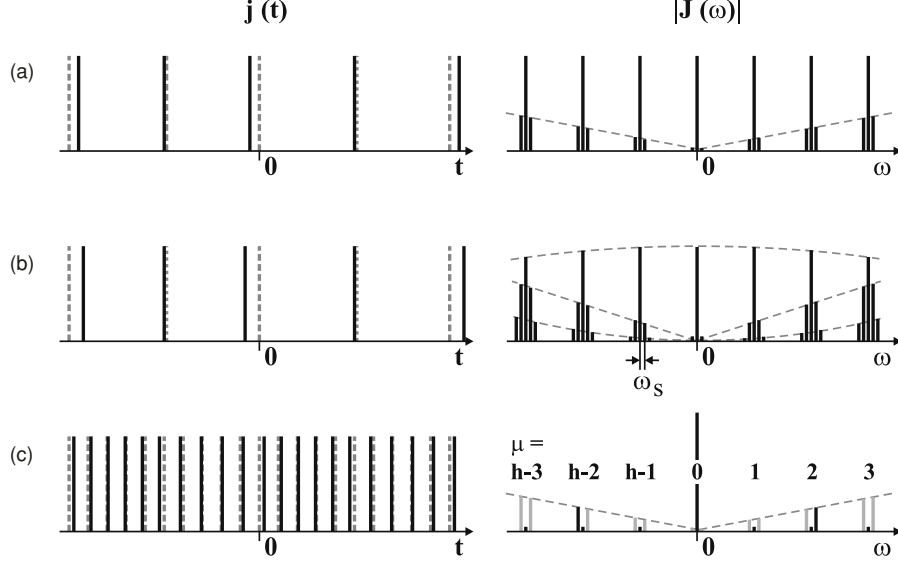


Fig. 2.3. Detected current signal as function of time (*left*) and frequency (*right*) caused by pointlike bunches that oscillate longitudinally with a synchrotron frequency ω_s . (a) Single bunch with small oscillation amplitude and (b) with larger oscillation amplitude, (c) $h = T_o/T_{rf}$ equidistant bunches oscillating with small amplitude. The symbol μ labels a mode with an upper sideband at $\mu\omega_o + \omega_s$

Case 2c. Filling the storage ring with h Gaussian bunches in analogy to case 1c creates a line spectrum consisting of rf harmonics with an envelope given by the Fourier transform of the charge density distribution of the individual bunches. Again, slight variations of the bunch charge cause spurious revolution harmonics, as shown in Fig. 2.2.

2.2.3 Longitudinal Bunch Oscillations

So far, the spacing between electron bunch signals was considered to be equal and time independent. Longitudinal oscillations (“synchrotron oscillations”) modulate the arrival time of the bunches at the detector (see Fig. 2.3).

Case 3a. Consider a pointlike electron bunch circulating in a storage ring with mean revolution frequency ω_o . In the case of a longitudinal motion with frequency ω_s and amplitude $\hat{\tau}$, the current signal is (in arbitrary units)

$$j(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_o - \hat{\tau} \cos[\omega_s nT_o]) , \quad (2.16)$$

where an arbitrary initial phase of the oscillation was suppressed. For small amplitudes, the spectrum can be expanded:

$$\begin{aligned}
J(\omega) &= \sum_{p=-\infty}^{\infty} \exp(-i\omega\{pT_o + \hat{\tau} \cos[\omega_s pT_o]\}) \\
&\approx \sum_{p=-\infty}^{\infty} (1 - i\omega\hat{\tau} \cos[\omega_s pT_o]) \exp(-i\omega pT_o) \\
&= \sum_{p=-\infty}^{\infty} \exp(-i\omega pT_o) - \frac{i\omega\hat{\tau}}{2} \left[e^{-ipT_o(\omega+\omega_s)} + e^{-ipT_o(\omega-\omega_s)} \right] \\
&= \omega_o \sum_{p=-\infty}^{\infty} \left\{ \delta(\omega - p\omega_o) - \frac{i\hat{\tau}}{2} (p\omega_o - \omega_s) \delta(\omega - p\omega_o + \omega_s) \right. \\
&\quad \left. - \frac{i\hat{\tau}}{2} (p\omega_o + \omega_s) \delta(\omega - p\omega_o - \omega_s) \right\}. \quad (2.17)
\end{aligned}$$

In this approximation, each line at integer multiples of the revolution frequency has a lower and an upper sideband at $p\omega_o \pm \omega_s$. Including one more term into the expansion creates additional small sidebands at $p\omega_o \pm 2\omega_s$.

Case 3b. Without restriction to small oscillation amplitudes, the spectrum is given by

$$J(\omega) = \omega_o \sum_{p=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} i^{-l} \mathcal{J}_l(\hat{\tau}\{p\omega_o + l\omega_s\}) \delta(\omega - p\omega_o - l\omega_s), \quad (2.18)$$

where $\mathcal{J}_l$ are Bessel functions of order l . This result can be obtained from (2.16) using (2.9) and the relation (e.g. [5])

$$\exp(ix \cos \alpha) = \sum_{l=-\infty}^{\infty} i^l \mathcal{J}_l(x) \exp(il\alpha). \quad (2.19)$$

Each line at $p\omega_o$ is now accompanied by a series of sidebands at distance $l\omega_s$ with $l = \pm 1, \pm 2$, and so on. The Bessel function of zeroth order constitutes a finite envelope of the revolution harmonics even if the bunches are assumed to be pointlike. The Bessel functions of order l act as envelopes for the respective sidebands. For small oscillation amplitudes $\hat{\tau}$, expression (2.17) can be recovered from (2.18).

Case 3c. A system of $h = T_o/T_{rf}$ equidistant electron bunches has h eigenmodes $\mu = 0 \dots h-1$ characterized by a phase difference $\Delta\varphi$ between the oscillation of successive bunches³. With just two oscillators, the eigenmodes with $\Delta\varphi = 0$ and π are well known from many examples in physics, e.g. two coupled pendula. For the general case of h oscillators, the phase difference for eigenmode μ is

³ As an intuitive definition, an eigenmode of a system of coupled oscillators is a mode of oscillation that, once excited, continues without beating, i.e. without energy exchange between the oscillators [6].

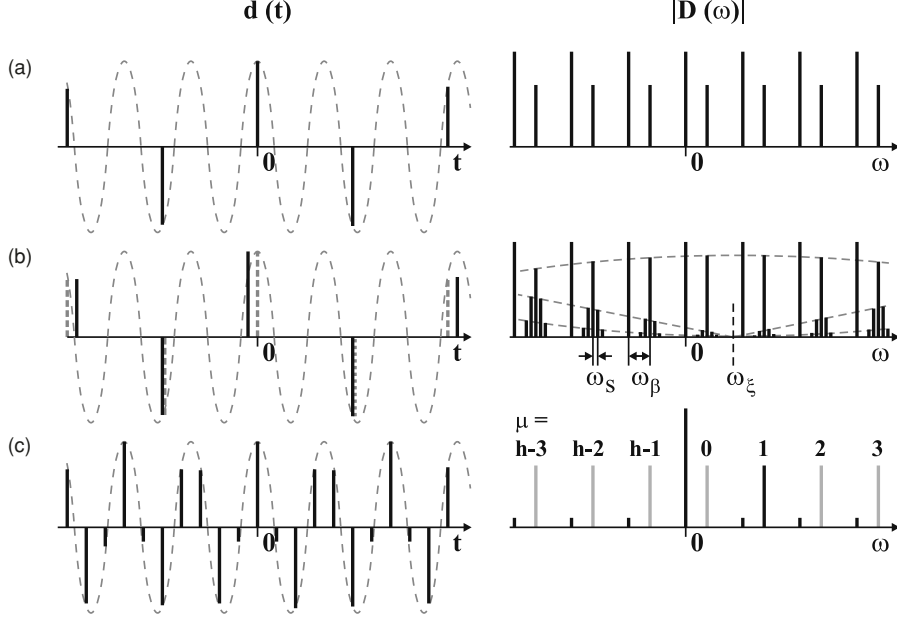


Fig. 2.4. Detected dipole moment as function of time (*left*) and frequency (*right*) for pointlike electron bunches. (a) Single bunch performing transverse oscillations with a betatron frequency ω_β , (b) with additional longitudinal oscillations, (c) $h = T_o/T_{rf}$ equidistant bunches performing transverse oscillations

$$\Delta\varphi = \frac{2\pi\mu}{h} . \quad (2.20)$$

Thus, the revolution harmonic at which the sidebands occur is shifted by the frequency

$$\frac{\Delta\varphi}{\Delta t} = \frac{2\pi\mu}{h} \frac{h}{T_o} = \mu\omega_o , \quad (2.21)$$

since the time difference from bunch to bunch is $\Delta t = T_o/h$. The sideband frequencies are $p\omega_{rf} \pm (\mu\omega_o + \omega_s)$. All h modes are found within a frequency range of $\omega_{rf}/2$, provided the frequency interval starts or ends at an integer multiple of ω_{rf} .

2.2.4 Transverse Bunch Oscillations

In the following discussion of transverse oscillations, $u(t)$ is either the horizontal or the vertical deviation of a bunch from its equilibrium orbit, and quantities like tune, dispersion or chromaticity are understood to carry a subscript x or y .

A detector, which is sensitive to transverse beam motion (see Fig.2.4), senses a dipole moment given by

$$m(t) = j(t) u(t) , \quad (2.22)$$

where $j(t)$ is the bunch current. Such a detector would also sense the longitudinal motion of bunches due to the variation of their arrival times, whereas a longitudinal detector is not necessarily sensitive to transverse motion.

Case 4a. Let a pointlike bunch in a storage ring perform transverse oscillations (“betatron oscillations”) with amplitude u_β and frequency

$$\omega_\beta = \nu_\beta \omega_\circ = (n_\beta + q_\beta) \omega_\circ , \quad (2.23)$$

where ν_β is the betatron tune, i.e. the number of oscillation periods during one revolution. The integer part of the tune is n_β , the fractional part is q_β . Adding a time-independent deviation u_c by which the beam deviates from the nominal orbit due to small errors of the guiding magnetic fields, the dipole moment (in arbitrary units) as function of time is

$$m(t) = \{u_c + u_\beta \exp(i\omega_\beta t)\} \sum_{n=-\infty}^{\infty} \delta(t - nT_\circ) . \quad (2.24)$$

In analogy to the previous examples, the Fourier spectrum is

$$M(\omega) = u_c \omega_\circ \sum_{p=-\infty}^{\infty} \delta(\omega - p\omega_\circ) + u_\beta \omega_\circ \sum_{p=-\infty}^{\infty} \delta(\omega - p\omega_\circ - \omega_\beta) . \quad (2.25)$$

The time-independent deviation results in a spectrum of revolution harmonics $p\omega_\circ$, whereas the betatron motion creates new lines displaced by $+\omega_\beta$ with respect to the revolution harmonics. In the measurable spectrum, i.e. the combination of positive and negative frequencies, there are two betatron lines between each pair of adjacent revolution harmonics. Their distance to the closest revolution harmonic is $\pm q'_\beta \omega_\circ$ with $q'_\beta < 0.5$. This observation is ambiguous in two ways:

- The integer part of the tune cannot be deduced since the detector installed at one position of the storage ring is insensitive to the integer number of cycles between two observations (alias effect).
- The fractional part of the tune could be $q_\beta = q'_\beta$ or $q_\beta = 1 - q'_\beta$. This ambiguity can be experimentally removed by increasing the field of a focussing quadrupole magnet, which causes the tune to rise. If the distance q'_β increases, then $q_\beta = q'_\beta$.

Case 4b. Let a pointlike, transversely oscillating bunch perform additional longitudinal oscillations with synchrotron frequency ω_s . This modifies the spectrum in several ways:

1. The arrival time of the bunch at the detector is modulated.
2. A longitudinal oscillation is also an oscillation in energy. If $\delta_E(t)$ denotes the relative deviation from the nominal beam energy, then
 - the transverse bunch position changes according to $D \delta_E(t)$ in a region with dispersion $D \neq 0$.

- the betatron tune is modulated by the energy oscillation if the chromaticity $\xi = d\nu_\beta/d\delta_E$ is non-zero.

Quantitatively, (A.32) and (A.33) show that an oscillation of the bunch arrival time $\tau = \hat{\tau} \cos(\omega_s t)$ corresponds to an energy oscillation $\delta_E(t) = \dot{\tau}/\eta = -(\omega_s \hat{\tau}/\eta) \sin(\omega_s t)$ with η being the momentum compaction factor. The dipole moment is then

$$m(t) = \left\{ u_c - D \frac{\omega_s \hat{\tau}}{\eta} \sin(\omega_s t) + u_\beta \exp(i\phi_\beta(t)) \right\} \times \sum_{n=-\infty}^{\infty} \delta(t - nT_o - \hat{\tau} \cos[\omega_s nT_o]) . \quad (2.26)$$

Here, the betatron motion is expressed by its phase advance $\phi_\beta(t)$ with

$$\dot{\phi}_\beta(t) = \omega_\beta + \Delta\omega_\beta(t) = \omega_\beta + \omega_o \xi \delta_E(t) = \omega_\beta + \frac{\omega_o \xi \dot{\tau}(t)}{\eta} . \quad (2.27)$$

Integration yields

$$\phi_\beta(t) = \phi_\beta(0) + \omega_\beta t + \frac{\omega_o \xi \tau(t)}{\eta} = \phi_\beta(0) + \omega_\beta t + \omega_\xi \hat{\tau} \cos(\omega_s t) , \quad (2.28)$$

where the so-called chromatic frequency $\omega_\xi = \omega_o \xi/\eta$ is introduced. The first term of (2.26) leads to synchrotron sidebands as in (2.18). The second term, which vanishes in dispersion-free regions, also creates sidebands at a distance $\pm l\omega_s$ from the revolution harmonics, but their envelopes differ from those of the first term. The third term gives rise to new lines in the spectrum. Its Fourier transform is given by [2]

$$M(\omega) = \omega_o u_\beta \sum_{p=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} i^{-l} \mathcal{J}_l(\hat{\tau}\{p\omega_o + \omega_\beta + l\omega_s - \omega_\xi\}) \times \delta(\omega - p\omega_o - \omega_\beta - l\omega_s) . \quad (2.29)$$

Thus, the energy dependence of the betatron frequency due to chromaticity creates two new phenomena:

- Not only the revolution harmonics but also the betatron lines are accompanied by synchrotron sidebands.
- The envelope of these sidebands is shifted by the chromatic frequency $\omega_\xi = \omega_o \xi/\eta$.

Case 4c. For $h = T_o/T_{rf}$ equidistant electron bunches executing transverse oscillations, there are h eigenmodes labeled $\mu = 0 \dots h-1$. As in the longitudinal case, they are characterized by a phase difference

$$\Delta\varphi = \frac{2\pi\mu}{h} , \quad (2.30)$$

between successive bunches and the frequencies of the betatron sidebands are $\omega = p\omega_{\text{rf}} + \mu\omega_o + \omega_\beta$. As in the longitudinal case, a measuring apparatus that does not distinguish between positive and negative frequencies, and all h transverse modes are found within a frequency interval of $\omega_{\text{rf}}/2$, provided it starts or ends at an integer multiple of ω_{rf} .

Since bunches were treated as pointlike or at least rigid objects without inner degrees of freedom, the results obtained here reappear in the context of one-particle models for multi-bunch instabilities.

Figure 2.5 shows examples of beam spectra recorded at the storage ring BESSY II using a spectrum analyzer. The central frequency was kept fixed at twice the rf frequency while the frequency span was reduced by a zoom factor of 250 from one spectrum to the next.

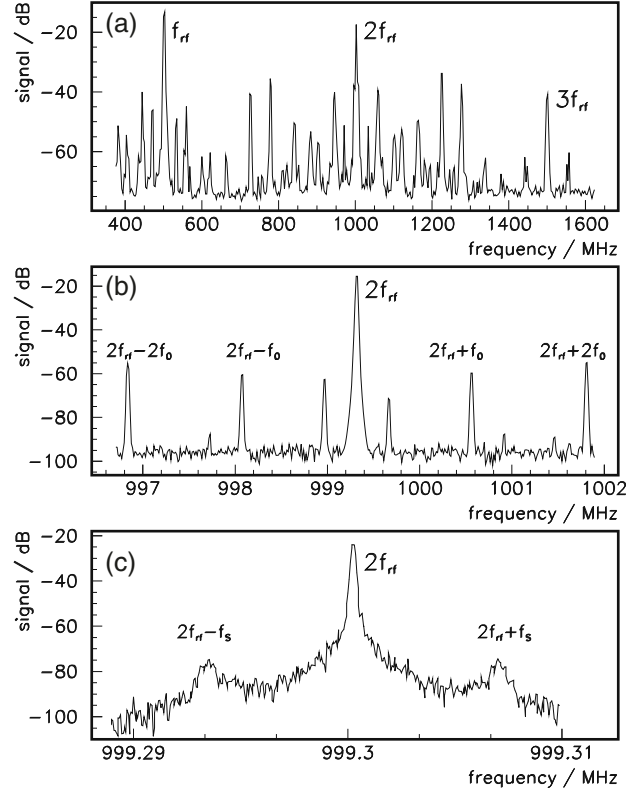


Fig. 2.5. Spectra of an unstable electron beam in BESSY II. In each case, the central frequency coincides with 999.3 MHz, which is twice the rf frequency f_{rf} . At a GHz span (a) groups of lines between the integer multiples of f_{rf} indicate the presence of instabilities; on a MHz scale (b) betatron sidebands between integer multiples of the revolution frequency f_o are visible, on the kHz scale (c) synchrotron sidebands are resolved

In order to decide whether the beam is stable or not, the span of the spectrum analyzer must cover all possible mode frequencies, i.e. at least half the rf frequency. Whether a line in the spectrum corresponds to a longitudinal or transverse oscillation is determined by measuring its frequency difference to an adjacent revolution harmonic, which either matches the synchrotron frequency or the product of fractional betatron tune and revolution frequency. For a storage ring filled with many bunches, the revolution harmonics may not even be visible (spurious revolution harmonics are more pronounced in the vicinity of rf harmonics) and its frequency must be carefully calculated from the known rf frequency – for kHz accuracy of the result, even the 1-Hz digit of the rf frequency should be included.

2.3 Wake Field and Impedance

In agreement with the majority of the literature (e.g. [7–11]), the following terms are introduced:

- *Wake field* is the electromagnetic field caused by an arbitrary charge distribution interacting with its surroundings.
- *Wake function* is the integral over time of the force caused by a pointlike unit charge and acting via wake fields on a pointlike trailing unit charge.
- *Wake potential* is the integral over time of the force caused by an extended unit charge and acting via wake fields on a pointlike trailing unit charge.
- *Impedance* is the Fourier transform of the wake function.

2.3.1 Longitudinal and Transverse Wake Function

Consider the situation displayed in Fig. 2.6: Two charges travel at almost the speed of light $\mathbf{v} \approx (0, 0, c)$ in the vacuum chamber of a storage ring. Let the coordinates of the leading charge q_1 and the trailing charge q_2 at a given time t be $(\mathbf{r}_1, s_1)$ and $(\mathbf{r}_2, s_2)$, respectively. The interaction of the leading charge

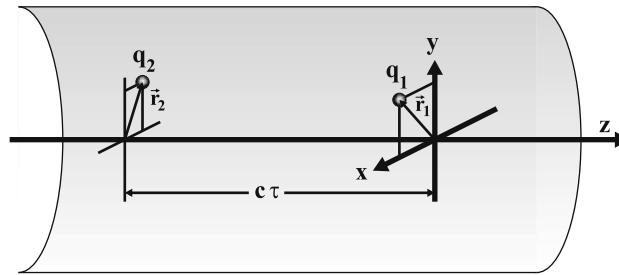


Fig. 2.6. Two charges q_1 and q_2 with transverse position vectors $\mathbf{r}_1$ and $\mathbf{r}_2$, respectively, travelling in a vacuum chamber. The distance between leading and trailing charge, measured along the z -axis, is $c\tau$

with the vacuum chamber causes an electromagnetic wake field that results in a force

$$\mathbf{F}(\mathbf{r}_1, s_1, \mathbf{r}_2, s_2, t) = q_2 \{ \mathbf{E}(\mathbf{r}_1, s_1, \mathbf{r}_2, s_2, t) + \mathbf{v} \times \mathbf{B}(\mathbf{r}_1, s_1, \mathbf{r}_2, s_2, t) \} \quad (2.31)$$

on the trailing charge. Assuming the force to be sufficiently weak, both charges maintain a nearly uniform motion characterized by a constant distance $c\tau = s_1(t) - s_2(t)$. In order to study the effect of a given structure (an rf cavity, for example) on a particle beam, it is sufficient to specify the time integral of the force, normalized to the values of both charges, as a function of the distance between them:

$$\mathbf{W}(\mathbf{r}_1, \mathbf{r}_2, \tau) = -\frac{c}{q_1} \int dt \{ \mathbf{E}(\mathbf{r}_1, \mathbf{r}_2, \tau, t) + \mathbf{v} \times \mathbf{B}(\mathbf{r}_1, \mathbf{r}_2, \tau, t) \} . \quad (2.32)$$

This is the wake function, the longitudinal part of which corresponds to the energy change ΔU experienced by the trailing charge under the influence of the wake field, normalized to the charges:

$$W_{\parallel}(\mathbf{r}_1, \mathbf{r}_2, \tau) = -\frac{\Delta U}{q_1 q_2} = -\frac{c}{q_1} \int dt E_z(\mathbf{r}_1, \mathbf{r}_2, \tau, t). \quad (2.33)$$

The unit of $W_{\parallel}$ is V/C. By convention, the minus sign makes the wake function positive for negative energy changes, i.e. energy loss. In the longitudinal case, there is no magnetic term, because $\mathbf{v} \times \mathbf{B}$ has no longitudinal component. The transverse wake function is

$$\mathbf{W}_{\perp}(\mathbf{r}_1, \mathbf{r}_2, \tau) = -\frac{c}{q_1} \int dt \{ \mathbf{E}_{\perp}(\mathbf{r}_1, \mathbf{r}_2, \tau, t) + [\mathbf{v} \times \mathbf{B}(\mathbf{r}_1, \mathbf{r}_2, \tau, t)]_{\perp} \} \quad (2.34)$$

and with this particular definition, the unit of $\mathbf{W}_{\perp}$ is J C⁻² or V C⁻¹. In the ultrarelativistic case considered here, the wake function is zero for negative values of τ due to causality. In the literature, $\mathbf{W}$ may also be defined as function of the distance d with $\mathbf{W}(d < 0) = 0$ or as function of the coordinate z in flight direction with the leading charge at $z = 0$, which implies $\mathbf{W}(z > 0) = 0$.

Sometimes, the transverse wake function is not normalized to the charge q_1 but to the dipole moment for a given horizontal or vertical offset of the leading charge, while $\mathbf{r}_2 = 0$. In this case, its unit is V C⁻¹ m⁻¹:

$$W_{uv}(\tau) = -\frac{c}{q_1 u} \int \{ \mathbf{E}_v(u, 0, \tau, t) + [\mathbf{v} \times \mathbf{B}(u, 0, \tau, t)]_v \} , \quad (2.35)$$

where $u = x$ or $u = y$ is the offset of q_1 and the subscript $v = x$ or $v = y$ indicates the field component under consideration. Mixed terms with $u = x$ and $v = y$ or $u = y$ and $v = x$ vanish if the vacuum chamber has rotational symmetry about the beam axis. If the vacuum chamber has no obvious symmetry, the dipole moments are not meaningful and (2.34) should be used.

In the case of rotational symmetry, a multipole expansion in cylindrical coordinates with unit vectors $\hat{r}, \hat{\theta}$ is advantageous. Defining $\theta \equiv \theta_2 - \theta_1$, the longitudinal and transverse wake functions are

$$\begin{aligned} W_{\parallel}(r_1, r_2, \theta, \tau) &= \sum_{m=0}^{\infty} \frac{2}{1 + \delta_{m0}} r_1^m r_2^m \cos m\theta W_{\parallel}^{(m)}(\tau) \\ W_{\perp}(r_1, r_2, \theta, \tau) &= \sum_{m=1}^{\infty} r_1^{m-1} r_2^m \left[\hat{r} \cos m\theta - \hat{\theta} \sin m\theta \right] W_{\perp}^{(m)}(\tau) . \end{aligned} \quad (2.36)$$

In this definition, the units of the longitudinal multipole components are $\text{V C}^{-1} \text{m}^{-2m}$ and the units of the transverse multipole components are $\text{V C}^{-1} \text{m}^{-2m+1}$. The transverse displacement r of a particle is often assumed to be small compared to the chamber radius R , and, according to dimensionality arguments, the higher orders decrease roughly like $(r/R)^{2m}$. Thus, for practical purposes, only the lowest-order terms are of interest:

- Longitudinal wake function $W_{\parallel} \approx W_{\parallel}^{(0)} (\text{V C}^{-1})$
- Transverse wake function $W_{\perp} \approx W_{\perp}^{(1)} (\text{V C}^{-1} \text{m}^{-1})$

In the ultrarelativistic case, the longitudinal wake function is independent of the transverse position r_2 of a hypothetical test charge. This fact is used to simplify the numerical determination of the wake potential (see Sect. 2.3.5).

2.3.2 Extended Charge Distribution: Wake Potential

The consideration of extended charge distributions is motivated by the fact that the size of real particle bunches usually cannot be neglected. Furthermore, the time-domain simulation of wake fields on a spatial and temporal grid requires the bunch charge to be distributed over several grid points. Such a simulation cannot directly determine the wake function, which is defined for point charges.

Starting from the longitudinal wake function $W_{\parallel}(t)$, the superposition principle yields the wake potential for a longitudinally extended charge distribution $j(\tau)$:

$$V_{\parallel}(\tau) = \int_{-\infty}^{\infty} dt W_{\parallel}(t) j(\tau - t) . \quad (2.37)$$

Assuming a transverse displacement $u(\tau)$ with mean value $\langle u \rangle$, the transverse wake potential is defined as

$$V_{\perp}(\tau) = \frac{1}{\langle u \rangle} \int_{-\infty}^{\infty} dt W_{\perp}(t) u(\tau - t) j(\tau - t) . \quad (2.38)$$

If the transverse displacement is constant along the bunch, the previous expression reduces to

$$V_{\perp}(\tau) = \int_{-\infty}^{\infty} dt W_{\perp}(t) j(\tau - t) . \quad (2.39)$$

Generally, a wake potential is obtained by a convolution of wake function and charge distribution. Its Fourier transform is just the product of the Fourier transforms of the convoluted functions. This can be used to recover the wake function from the wake potential obtained by numerical simulations (see Sect. 2.3.5).

2.3.3 Longitudinal and Transverse Impedance

The Fourier transform of the respective wake function

$$Z_{\parallel}(\omega) = \int_{-\infty}^{\infty} d\tau W_{\parallel}(\tau) \exp(-i\omega\tau), \quad (2.40)$$

$$Z_{\perp}(\omega) = i \int_{-\infty}^{\infty} d\tau W_{\perp}(\tau) \exp(-i\omega\tau) \quad (2.41)$$

is the longitudinal or transverse impedance (or “coupling impedance”). Following the usual convention, the definition of the transverse impedance includes a phase factor $i = \exp(i\pi/2)$, because the betatron oscillation transforms the kick from a transverse force into a transverse displacement after a phase advance of $\pi/2$. The notion of impedance was introduced to accelerator physics in the 1960s [12, 13]. To illustrate the analogy to the impedance used in electrical engineering, one may insert

$$j(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega J(\omega) \exp(i\omega\tau) \quad (2.42)$$

into (2.37) expressing the longitudinal wake potential as

$$\begin{aligned} V_{\parallel}(\tau) &= \int_{-\infty}^{\infty} dt W_{\parallel}(t) \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega J(\omega) \exp(i\omega\tau - i\omega t) \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega Z_{\parallel}(\omega) J(\omega) \exp(i\omega\tau) . \end{aligned} \quad (2.43)$$

On the other hand, the wake potential is the Fourier transform of $\tilde{V}_{\parallel}(\omega)$

$$V_{\parallel}(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \tilde{V}_{\parallel}(\omega) \exp(i\omega\tau) . \quad (2.44)$$

Comparison with (2.43) yields an expression that is analogous to Ohm’s law:

$$\tilde{V}_{\parallel}(\omega) = Z_{\parallel}(\omega) J(\omega) . \quad (2.45)$$

Thus, in the frequency domain the impedance connects a current distribution $J(\omega)$ to the voltage $\tilde{V}(\omega)$ induced by its wake field. On the other hand,

one may use this as a definition of the impedance and show that it is the Fourier transform of the wake function. Analogous impedance concepts exist for the description of mechanical and acoustic waves [4]. In either case, the impedance is a complex function of frequency and its real part describes dissipative effects:

- electrically through resistance according to Ohm’s law
- for mechanical systems through friction.

Its imaginary part is responsible for retarding effects or phase shifts:

- between an electric current and voltage via inductance and capacitance
- between a mechanical generator and an oscillator due to inertia.

2.3.4 Properties of Wake Function and Impedance

The Panofsky–Wenzel Theorem

There is a simple relationship between longitudinal and transverse wake function, known as the Panofsky–Wenzel theorem [14]:

$$\frac{1}{c} \frac{\partial}{\partial \tau} \mathbf{W}_{\perp}(\mathbf{r}_1, \tau) = -\nabla_{\perp} W_{\parallel}(\mathbf{r}_1, \tau) . \quad (2.46)$$

The time-derivative of the transverse wake function can be calculated from the transverse variation of the longitudinal wake function. In the frequency domain, the theorem also states that the transverse impedance can be calculated from the longitudinal impedance:

$$\mathbf{Z}_{\perp}(\mathbf{r}_1, \omega) = \frac{c}{\omega} \nabla_{\perp} Z_{\parallel}(\mathbf{r}_1, \omega) . \quad (2.47)$$

Originally formulated for geometrical structures with rotational symmetry, the Panofsky–Wenzel theorem is also valid for other geometries [15]. While the Panofsky–Wenzel theorem applies to each multipole order m separately, there is no simple connection between different multipole components. However, the multipole orders of the impedance for a vacuum chamber of radius R can be assumed to decrease roughly like

$$Z_{\parallel}^{(m)} \approx \frac{2}{R^{2m}} Z_{\parallel}^0 \quad (2.48)$$

and applying the Panofsky–Wenzel theorem

$$Z_{\perp}^{(m)} \approx \frac{2c}{\omega R^{2m}} Z_{\parallel}^0 . \quad (2.49)$$

Here, a factor of two was inserted arbitrarily to be consistent with (2.57) and (2.58) that hold for the resistive-wall impedance (see Sect. 2.3.5).

Narrow-Band Resonators

In analogy to an LRC resonator circuit, the impedance of cavity-like structures is often assumed to be of the form

$$Z_{\parallel}(\omega) = \frac{R_s}{1 + iQ(\omega_r/\omega - \omega/\omega_r)}, \quad (2.50)$$

$$Z_{\perp}(\omega) = \frac{c}{\omega} \frac{R_s}{1 + iQ(\omega_r/\omega - \omega/\omega_r)}. \quad (2.51)$$

In this case, the impedance is determined by the resonant frequency ω_r , the shunt impedance R_s and the quality factor Q . Fourier transform yields the wake functions, given here in the approximation of a weakly damped resonator ($Q \gg 1$):

$$W_{\parallel}(\tau) \approx \frac{\omega_r R_s}{Q} e^{-\omega_r \tau/2Q} \cos(\omega_r \tau) \quad (\tau > 0), \quad (2.52)$$

$$W_{\perp}(\tau) \approx -\frac{c R_s}{Q} e^{-\omega_r \tau/2Q} \sin(\omega_r \tau) \quad (\tau \geq 0). \quad (2.53)$$

Apparently, the derivative of the transverse wake function is proportional to the longitudinal wake function. This can be shown to be generally true by analyzing Maxwell's equations for rotationally symmetric boundary conditions [7].

Properties of the Wake Function

Some general properties of longitudinal wake functions $W_{\parallel}(\tau)$ and transverse wake functions $W_{\perp}(\tau)$ are summarized in the following and illustrated by Fig. 2.7. See e.g. [7,8] for detailed derivations. The symbols $\tau = 0^+$ and $\tau = 0^-$ represent infinitesimally small positive and negative time values.

- (a) $W_{\parallel}(0^-) = 0$ and $W_{\perp}(0^-) = 0$ for ultrarelativistic particles. This property, also referred to as “causality”, is due to the fact that electromagnetic fields cannot propagate faster than a charge travelling approximately at the speed of light.

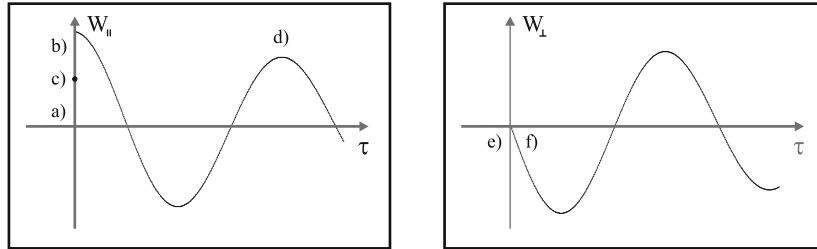


Fig. 2.7. Schematic illustration of longitudinal (*left*) and transverse (*right*) wake function versus time τ indicating the properties (a)–(f) as discussed in the text

- (b) $W_{\parallel}(0^+) \geq 0$. According to the definition (2.33), a positive value corresponds to energy loss. A charged particle cannot gain energy from its own field by traversing a passive structure.
- (c) $W_{\parallel}(0) = \frac{1}{2}W_{\parallel}(0^+)$, often called the “fundamental theorem of beam loading”, implies that a charge is retarded by its own wake field, but sees only half of it (see also Appendix D).
- (d) $W_{\parallel}(0^+) \geq |W_{\parallel}(\tau)|$. In the limit $W_{\parallel}(0^+) = |W_{\parallel}(\tau)|$, the maximum repeats itself with a period τ , corresponding to the wake caused by a resonator with infinitely large quality factor.
- (e) $W_{\perp}(0) = 0$, a charge is not transversely displaced by its own wake field. This and the following property can be shown from the longitudinal wake function being proportional to the derivative of the transverse wake function.
- (f) $W_{\perp}(0^+) < 0$. Furthermore, maxima of $W_{\perp}$ coincide with zero crossings of $W_{\parallel}$. In summary, one may say that the transverse wake function is sine-like and the longitudinal wake function is cosine-like.

Properties of the Impedance

The fact that a wake function is real and vanishes for $\tau < 0$ imposes some restrictions on its Fourier transform, the impedance (see Fig. 2.8).

- (a) $Z_{\parallel}^*(\omega) = Z_{\parallel}(-\omega)$ and $Z_{\perp}^*(\omega) = -Z_{\perp}(-\omega)$, where the asterisk indicates the complex conjugate value. Furthermore, $Z_{\parallel}(0) = 0$ and $\text{Re}Z_{\perp}(0) = 0$.
- (b) $\text{Re}Z_{\parallel}(\omega) \geq 0$ for all ω , whereas $\text{Re}Z_{\perp}(\omega) \geq 0$ for $\omega > 0$ and $\text{Re}Z_{\perp}(\omega) \leq 0$ for $\omega < 0$.
- (c) The real and imaginary parts of the impedance are not independent from each other, but are connected by a Hilbert transform [4]. For the resonator impedance at $\omega \neq 0$, the peaks of the real part coincide with zero crossings of the imaginary part.
- (d) To calculate the wake function from the impedance, it is sufficient to know its real or imaginary part, e.g.:

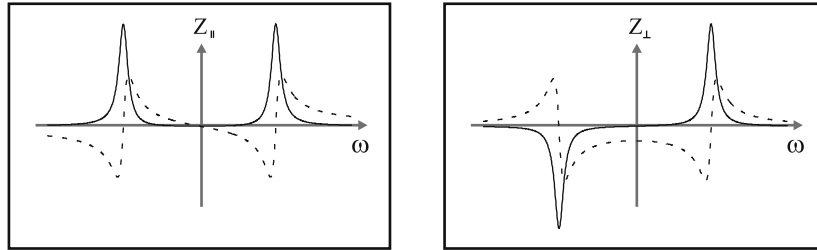


Fig. 2.8. Schematic illustration of longitudinal (*left*) and transverse (*right*) impedance versus frequency ω , where the *solid line* represents the real part and the *dashed line* is the imaginary part of the complex impedance

$$\begin{aligned}
W_{\parallel}(\tau) &= \frac{2}{\pi} \int_0^{\infty} d\omega \operatorname{Re} Z_{\parallel}(\omega) \cos(\omega\tau) \\
W_{\perp}(\tau) &= \frac{2}{\pi} \int_0^{\infty} d\omega \operatorname{Re} Z_{\perp}(\omega) \sin(\omega\tau) .
\end{aligned} \tag{2.54}$$

2.3.5 Prediction of Wake Potential and Impedance

There are ways to calculate and measure the impedance or the wake potential corresponding to a particular component of a storage ring. The impedance of that particular component can then be minimized under variation of the respective boundary conditions, i.e. its geometry and material.

In principle, the prediction of threshold currents or growth rates of collective instabilities requires the knowledge of the impedance of the whole storage ring rather than of individual components. Thus, additional assumptions must be made:

- Multi-bunch instabilities are driven by narrow-band impedances, which can be attributed to resonant structures like rf cavities, and the finite resistivity of the vacuum chamber (resistive-wall effect).
- The broad-band contributions to the total impedance from all storage ring components other than the cavities are combined to an “impedance budget”, often described by a simplified broad-band model or an equivalent circuit representing the whole storage ring.
- The measurement of beam properties at existing storage rings can give an indication of the total impedance while its assignment to particular storage ring components is often ambiguous.

The methods to calculate the impedance or wake function can be classified considering several criteria:

- Analytical or numerical treatment
- Time domain or frequency domain
- Longitudinal or transverse case
- 2-dimensional or 3-dimensional geometry
- Geometrical model or equivalent circuit

As for the experimental methods, there are bench measurements of individual components and measurements of beam properties. The latter are discussed in the last section of this chapter because they require some knowledge of instability models.

Analytical Methods

Calculating the impedance by directly solving Maxwell’s equations is only possible in a few cases. One example is the resistive-wall impedance or the respective wake function for a cylindrical beam pipe of radius R , length L and

electrical conductivity σ . For practical purposes, i.e. macroscopic distances τ and frequencies $\omega < 10^{12} \text{ s}^{-1}$, the following relations apply [7]⁴

$$\frac{W_{\parallel}^{\text{rw}}(\tau)}{L} = \frac{1}{4\pi c R} \sqrt{\frac{1}{\pi \varepsilon_0 \sigma}} \frac{1}{\sqrt{\tau^3}} \quad (2.55)$$

$$\frac{W_{\perp}^{\text{rw}}(\tau)}{L} = -\frac{1}{\pi R^3} \sqrt{\frac{1}{\pi \varepsilon_0 \sigma}} \frac{1}{\sqrt{\tau}} \quad (2.56)$$

$$\frac{Z_{\parallel}^{\text{rw}}(\omega)}{L} = \frac{1 - i \operatorname{sgn}(\omega)}{\sqrt{2 \varepsilon_0 \sigma} 2\pi c R} \sqrt{|\omega|} \quad (2.57)$$

$$\frac{Z_{\perp}^{\text{rw}}(\omega)}{L} = \frac{1 - i \operatorname{sgn}(\omega)}{\sqrt{2 \varepsilon_0 \sigma} \pi R^3} \frac{1}{\sqrt{|\omega|}}. \quad (2.58)$$

There are a number of analytical models that predict the approximate impedance for certain geometrical objects as described, for example, in [8, 16, 17]. The evaluation of these analytical expressions sometimes requires numerical techniques such as the inversion of large matrices. In this case, it is more appropriate to call them semi-analytical. Here are some examples:

- *Field matching.* If analytical solutions for electromagnetic fields excited in parts of a geometrical structure are known, a solution for the whole structure may be found under the condition of a smooth transition between the parts [18]. This applies e.g. to a cylindrical vacuum chamber with piecewise constant radius.
- *Perturbation methods.* “Small” deviations from a cylindrical vacuum chamber with infinite conductivity can be treated as perturbation of the electromagnetic field that accompanies a relativistic charged particle. Examples for such structures are single slits and holes or arrays of them, or step-like changes of the chamber radius [19, 20].
- *Diffraction methods.* The impedance at high frequency ($\omega \gg c/R$ with R being the chamber radius) can be deduced from the diffraction of the electromagnetic field when a charged particle passes an object like an iris or a cavity [21, 22].

The advantages of analytical expressions include smaller numerical complexity and insight into the relationship between geometric parameters and the resulting impedance.

On the other hand, the geometry of a real vacuum chamber often deviates considerably from the structures that can be described by analytical models. In the storage ring BESSY II, for example, hardly any component has rotational symmetry. The vacuum chambers are either approximately octagonal or elliptical, and most parts of the vacuum system show more than only small deviations from these basic shapes (see Fig.B.4). Only a few components can be approximated by a 2-dimensional (radial-longitudinal) geometry. The

⁴ Note that this book uses SI units while [7] adopted cgs units.

cylindrical symmetry of rf cavities and third-harmonic cavities, for example, is broken by plungers and ports for damping antennas. In all these cases, the impedance can only be estimated by numerical methods.

Numerical Methods

For a numerical treatment of electromagnetic fields excited by particle bunches in a storage ring, two basic tasks can be distinguished:

- Eigenmodes of resonant structures like rf cavities, for example, can be computed in the frequency domain. The goal of such a calculation is to obtain the parameters ω_r , R_s and Q that describe a narrow-band impedance according to (2.50) or (2.51).
- The more general task is to simulate the electromagnetic fields in discrete time steps for the case of an extended charge distribution traversing some geometrical structure. The result is a wake potential in the time domain, i.e. the convolution of wake function and charge distribution. Its Fourier transform is the product of impedance and current $J(\omega)$ in the frequency domain. The impedance itself, which does not depend on the charge distribution, can be computed by dividing the Fourier transform of wake potential by the current. For example, in the case of the longitudinal wake potential $V_{\parallel}(\tau)$ and its Fourier transform $\tilde{V}_{\parallel}(\omega)$, the impedance is given by

$$Z_{\parallel}(\omega) = \frac{\tilde{V}(\omega)}{J(\omega)} = \frac{\int_{-\infty}^{\infty} d\tau V_{\parallel}(\tau) \exp(-i\omega\tau)}{J(\omega)} . \quad (2.59)$$

This procedure only works to a certain degree. The frequency resolution improves with the total length of the simulated time interval. The smaller the time steps in the simulation and the shorter the assumed charge distribution, the larger is the valid frequency range.

Program packages for a numerical solution of Maxwell's equations use the finite integration technique (FIT), a modern version of the method of finite differences [23–27], or – less common – they may employ the finite element method [28–30].

A widely used program package is MAFIA [25,31], which – among other options – computes eigenmodes as well as wake potentials. In the 3-dimensional case, space is discretized by a cubic grid. Electric and magnetic field components are attributed to the grid points as sketched in Fig. 2.9. To give an idea of the numerical treatment of Maxwell's equations, the integral expression of the induction law

$$\oint \mathbf{E} \, d\mathbf{s} = - \int \dot{\mathbf{B}} \, d\mathbf{a} \quad (2.60)$$

is discussed as an example. Equation (2.60) is approximated on a rectangular grid at a point with index n by

$$\Delta x_n E_{x,n} - \Delta x_l E_{x,l} + \Delta y_n E_{y,n} - \Delta y_m E_{y,m} \approx -(\Delta x \Delta y)_n \dot{B}_{z,n} , \quad (2.61)$$

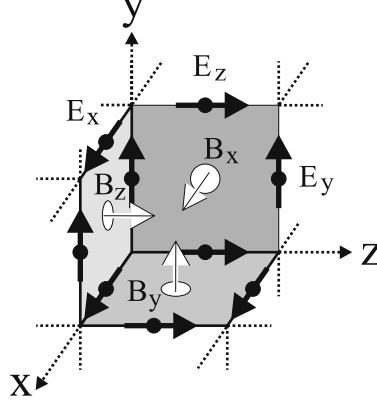


Fig. 2.9. Geometric assignment of electric and magnetic field components within a cubic grid for numerical simulations [23, 24]

where the indices l and m label the next adjacent point in x - and y -direction, respectively. These indices depend on the sequence in which the grid points are counted, and for a rectangular grid, the distances Δx_n and Δx_l are equal, likewise $\Delta y_n = \Delta y_m$, and $(\Delta x \Delta y)_n = \Delta x_n \Delta y_n$ is the area of the rectangle assigned to grid point n . In total, there are N grid points, given by the product of the number of points in either direction. Equation (2.61) can be thought of as one element of an equation of matrices and vectors of order $3N$

$$\mathbf{C} \mathbf{D}_s \mathbf{e} = -\mathbf{D}_a \dot{\mathbf{b}}, \quad (2.62)$$

where $\mathbf{e}$ and $\dot{\mathbf{b}}$ are vectors containing linearly labeled field components $E_{u,n}$ and $\dot{B}_{u,n}$, respectively, with $u = x, y, z$ and $n = 1, \dots, N$. The diagonal matrix

$$\mathbf{D}_s = \text{diag} [\Delta x_1, \dots, \Delta x_N, \Delta y_1, \dots, \Delta y_N, \Delta z_1, \dots, \Delta z_N] \quad (2.63)$$

contains the distances from a grid point to the next adjacent points in x -, y - and z -direction, and the corresponding area values are stored in the diagonal matrix $\mathbf{D}_a$

$$\begin{aligned} \mathbf{D}_a = \text{diag} [& (\Delta y \Delta z)_1, \dots, (\Delta y \Delta z)_N, \\ & (\Delta x \Delta z)_1, \dots, (\Delta x \Delta z)_N, \\ & (\Delta x \Delta y)_1, \dots, (\Delta x \Delta y)_N] . \end{aligned} \quad (2.64)$$

As sketched in Fig. 2.10, each row of the matrix $\mathbf{C}$ contains four non-zero elements ± 1 to select four elements of the column vector $\mathbf{D}_s \mathbf{e}$, creating the four terms on the left-hand side of (2.61). The remaining Maxwell's equations are treated in a similar fashion. In a geometrical model of a storage ring component, each grid point is associated with material properties like conductivity, dielectric constant and permeability. At the boundaries of the

$$\begin{pmatrix} \mathbf{C} \end{pmatrix} \begin{pmatrix} \mathbf{D}_s \end{pmatrix} \begin{pmatrix} \mathbf{e} \end{pmatrix} = - \begin{pmatrix} \mathbf{D}_a \end{pmatrix} \begin{pmatrix} \dot{\mathbf{b}} \end{pmatrix}$$

Fig. 2.10. Schematic representation of the matrix equation (2.62). The black and white squares in $\mathbf{C}$ represent 1 and -1 , respectively. The *black, grey and white dots* are the respective components in x, y and z . The *dashed lines* indicate the elements involved in the evaluation of one particular grid point n , leading to (2.61)

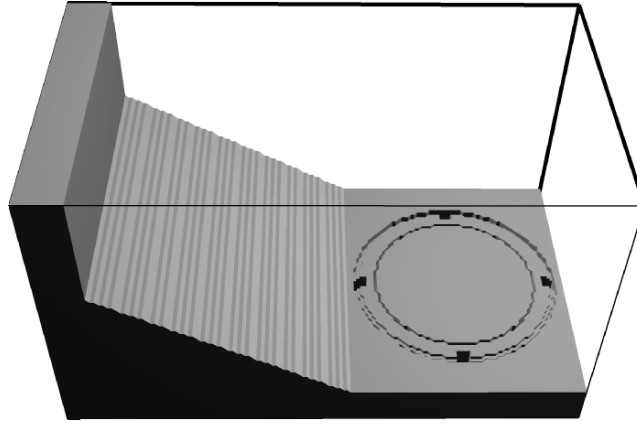


Fig. 2.11. Model of a beam position monitor in the BESSY II storage ring for a numerical determination of the wake potential using the MAFIA software package [31]. There are 112 beam position monitors installed in the ring, each comprising four button-type electrodes with 7.4 mm outer radius. Exploiting the geometrical symmetry to reduce the computation time, only one quarter of the whole structure is modelled

model, certain conditions may be imposed on the field components. In the model shown in Fig. 2.11, for example, the beam moves along the upper right edge, requiring the electric field to be always tangential to the upper and right boundary. “Open” boundary conditions are more complicated. Each storage ring component has at least two openings for the beam to enter and exit, which may e.g. be modelled as infinitely long waveguides.

As described in [9, 32], this approach allows to treat a variety of static and dynamical problems. In order to determine resonant modes of frequency ω , an eigenvalue equation with eigenvalues ω^2 can be found by considering $\dot{\mathbf{e}} = i\omega\mathbf{e}$ and $\dot{\mathbf{b}} = i\omega\mathbf{b}$.

Next, the approach for a time-dependent field simulation in equidistant time steps Δt is sketched using (2.62) as an example. Inserting

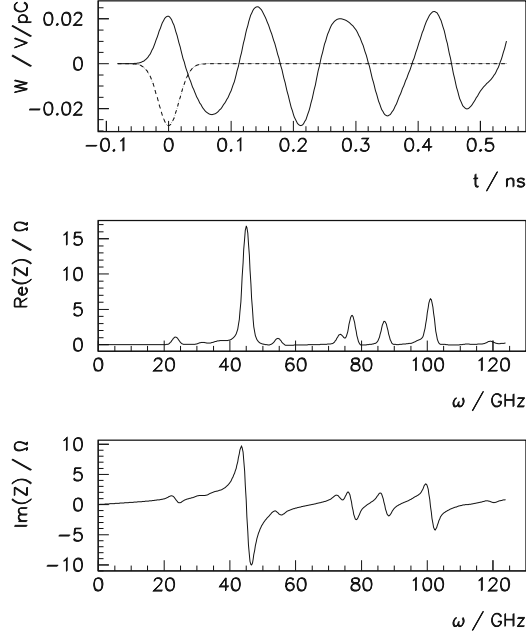


Fig. 2.12. (*Top*) Longitudinal charge distribution (*dashed line*) of an electron bunch and numerically determined wake potential (*solid line*) for the geometry shown in Fig. 2.11. (*Center*) Real part of the longitudinal impedance. (*Bottom*) Imaginary part of the longitudinal impedance

$$\dot{\mathbf{b}}(t - \Delta t/2) \approx \frac{\mathbf{b}(t) - \mathbf{b}(t - \Delta t)}{\Delta t} \quad (2.65)$$

into (2.62) yields an equation, where the calculated electric and magnetic field components are displaced by $\Delta t/2$ with respect to each other:

$$\mathbf{b}(t) = \mathbf{b}(t - \Delta t) - \mathbf{D}_a^{-1} \mathbf{C} \mathbf{D}_s \mathbf{e}(t - \Delta t/2) \Delta t. \quad (2.66)$$

Assuming that the longitudinal wake potential is independent of the transverse position of the test charge (see Sect. 2.3.1), the wake potential is generally not computed along the beam axis (“direct method”) but along a path parallel to the beam axis (“indirect method”), which partly coincides with the chamber wall [24]. This way, numerical “noise” is greatly reduced, while the direct method can yield very inaccurate results. The indirect method usually demands the beam entry and exit to be the smallest apertures within the model. Under this condition, protruding structures (e.g. scrapers) cannot be treated by the indirect method.

With the exception of the rf cavities, most storage ring components are candidates for a broad-band impedance, which has to be evaluated in the time domain. One standard deviation of the simulated electron bunch should

Table 2.1. Longitudinal loss factors from MAFIA [31] simulations for components of the BESSY II storage ring [33]

component	number n	$n k_{\parallel}$ (V pC ⁻¹)
rf cavities	4	4.57
undulator chambers	13	1.30
beam position monitors	112	0.88
bellows (short version)	80	0.17
bellows (long version)	48	0.14
flanges NW 100	192	0.13
flanges NW 250	32	0.02
depolarisation kicker	1	0.14
stripline electrodes	4	0.11
current monitors	2	0.05
dipole chambers with crotches	32	0.05
vacuum pumping ports	96	0.03
septum chamber	1	0.02

extend over typically ≥ 5 grid points. In view of a finite data storage capacity and computation time, this imposes a severe upper limit on the size of the modelled object and a lower limit on the bunch length. Figure 2.11 shows a 3-dimensional model of a BESSY II beam position monitor. The wake potential found for a bunch length of $\sigma = 5$ mm is shown in Fig. 2.12 together with the real and imaginary part of the impedance according to (2.59). As a figure of merit for impedance minimization, the longitudinal loss factor:

$$k_{\parallel} = \frac{1}{q} \int_{-\infty}^{\infty} W_{\parallel}(t) j(t) dt \quad (2.67)$$

was computed from the numerically evaluated wake function and the bunch current:

$$j(t) = \frac{q}{\sqrt{2\pi} \sigma_{\tau}} e^{-t^2/2\sigma_{\tau}^2}, \quad (2.68)$$

where q is the total bunch charge and σ_{τ} is the standard deviation of the charge distribution in units of time. Table 2.1 lists the loss factors for all storage ring components that were thought to be relevant [33]. The sum of all these impedance contributions and the longitudinal resistive-wall impedance are plotted in Fig. 2.13 as a function of frequency ω . As a worst-case scenario, partial cancellation of the contributions was excluded by adding the absolute impedance values. The bottom part of the figure shows the function $|Z|(\omega)/n$ with $n = \omega/\omega_{\circ}$, where $\omega_{\circ}$ is the revolution frequency. Analytical estimates concerning the beam stability sometimes use a broad-band impedance described by a quality factor $Q = 1$ and a global parameter $|Z|/n$, which is assumed to

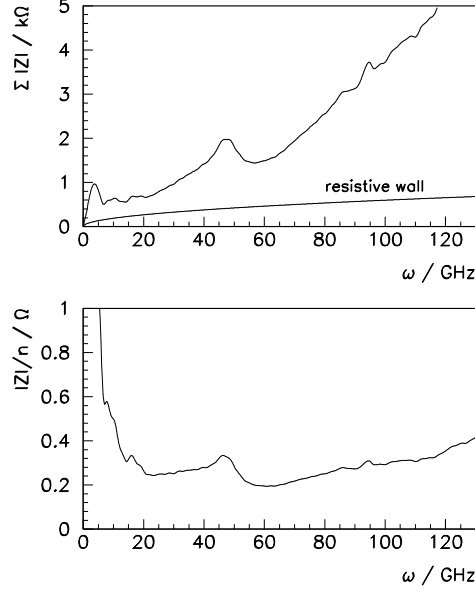


Fig. 2.13. (*Top*) Sum of the impedance contributions listed in Table 2.1 and resistive-wall impedance according to (2.57). (*Bottom*) The function $|Z|(\omega)/n$ with $n = \omega/\omega_0$

be frequency-independent. Indeed, the function $|Z|(\omega)/n$ in Fig. 2.13 changes only little over a wide frequency range, and for a coarse characterization of the total impedance a value of $|Z|/n \approx 0.3 \Omega$ may be concluded.

2.3.6 Bench Measurements of Impedance

The primary tool for bench measurements of the impedance of individual storage ring components is the network analyzer, which determines transmission and reflection of sinusoidal signals as function of frequency (see e.g. [3]). In the context of particle accelerator equipment, the following methods are usually applied:

- Coupling a signal into a cavity-like object and measuring the transmission parameter S_{21} between two electrodes yields a peak for each resonant mode from which the resonant frequency ω_r and the quality factor Q can be directly obtained [34]. The quality factor is proportional to the inverse width of the resonance.
- The measurement of the shunt impedance R_s of a narrow-band resonator involves the separate evaluation of Q (from the width of the resonance) and of the ratio R_s/Q determined by perturbing the field using a small metallic or ceramic object [35]. Usually, a sphere, ellipsoid or needle attached to a thread is pulled through the resonator while a resonant mode is excited by an external source. Each object has a set of characteristic form factors

K_i ($i = 1, \dots, 4$) to describe the contribution of the longitudinal and transverse electric and magnetic field to the frequency shift

$$\Delta\omega(z)/\omega = \frac{K_1\varepsilon_0 E_{\parallel}^2(z) + K_2\varepsilon_0 E_{\perp}^2(z) + K_3\mu_0 H_{\parallel}^2(z) + K_4\mu_0 H_{\perp}^2(z)}{4U} \quad (2.69)$$

measured as function of the longitudinal position z for a given energy U stored in the mode. The frequency shift obtained for perturbing objects of different shape allows to obtain different field components as function of position. Integration of the longitudinal E -field component yields:

$$\frac{R_s}{Q} = \frac{|\int E_{\parallel} \exp(i\omega z/c) dz|^2}{2\omega U}. \quad (2.70)$$

Using a network analyzer, one might as well measure the phase shift $\Delta\varphi$ rather than the frequency shift. The two are related by $\Delta\omega(z)/\omega = \tan \Delta\varphi(z)/2Q$.

- For objects that do not exhibit pronounced resonances, an impedance measurement can be performed using a wire along the chamber axis and simulating the field of a relativistic particle bunch by a short pulse traveling along the wire, see e.g. [36]. The longitudinal impedance is then given by

$$Z_{\parallel} = 2Z_c \frac{1 - S_{21}}{S_{21}}, \quad (2.71)$$

where Z_c is the characteristic impedance of the coaxial input and the signal source, and S_{21} is the transmission parameter determined by the network analyzer, which is usually measured relative to the value obtained for a smooth chamber. By using two parallel wires, the transverse impedance can be determined.

For the BESSY II storage ring, bench measurements were performed on the DORIS-type cavities [37] and several other components [38]. For many objects, their small impedance contribution was hard to measure but not negligible due to the large number of them installed in the ring (e.g. 112 beam position monitors). In these cases, it is better to rely on numerical simulations.

As another type of “bench measurement” one might consider to determine the wake potential of a geometrical structure by using two particle bunches under variation of their longitudinal distance and transverse offset. According to the definition of wake potential, the first bunch would be the charge exciting the wake field, while the second bunch would act as test charge. Such a measurement requires considerable effort and is not routinely performed when designing a new machine. An example is given in [39].

2.4 Models of Instabilities

Once an estimate of the impedance budget of a storage ring is obtained, its effect on the stored beam can be studied using various models:

- One-particle models, where a bunch of N electrons is represented by a single “macro particle” of charge $-e N$, are suitable for studying oscillations of bunches against each other, i.e. multi-bunch instabilities of the Robinson type.
- Models representing the bunch by a few macro particles shed light on its inner degrees of freedom. These models can describe multi-bunch instabilities as well as single-bunch modes quite precisely. They range from two-particle models (one representing the head, the other the tail of a bunch) to simulations using 10^3 and more macro particles.
- With 10^{10} and more electrons per bunch, the assumption of a continuous charge distribution is well justified. Such a description covers multi-bunch as well as single-bunch instabilities.

2.4.1 One-Particle Models

In the simplest case, a bunch containing N electrons is described as a harmonic oscillator subjected to a forced oscillation. Its equation of motion is

$$\ddot{u}(t) + \omega^2 u(t) = \frac{F(t)}{\gamma N m_e} . \quad (2.72)$$

In the longitudinal case, u is replaced by z and ω by the synchrotron frequency ω_s . For transverse oscillations, u stands for x or y and ω is the horizontal or vertical betatron frequency ω_β . The force $F(t)$ acting on the relativistic mass $\gamma N m_e$ is given

- in the time domain by the wake potential, or
- in the frequency domain by a frequency-dependent voltage, which is linked to the impedance and current by

$$\begin{aligned} V_{\parallel}(\omega) &= Z_{\parallel}(\omega) J_{\parallel}(\omega) \\ V_{\perp}(\omega) &= i Z_{\perp}(\omega) J_{\perp}(\omega). \end{aligned} \quad (2.73)$$

Assuming that the frequency of the forced motion deviates only little from the eigenfrequency ω , i.e.

$$u(t) = u_0 e^{-i\Omega t} \quad \text{with} \quad \Omega \approx \omega , \quad (2.74)$$

the left hand side of (2.72) can be rewritten as

$$(-\Omega^2 + \omega^2) e^{-i\Omega t} = (\omega + \Omega) (\omega - \Omega) e^{-i\Omega t} \approx 2\omega (\omega - \Omega) e^{-i\Omega t} . \quad (2.75)$$

The resulting frequency shift

$$\Delta\omega \equiv \Omega - \omega \approx -\frac{F(t)}{2\omega \gamma N m_e u_0} e^{i\Omega t} \quad (2.76)$$

is a complex number. Its real part corresponds to a shift in (real) frequency, whereas its imaginary part describes an exponential increase or decrease of the oscillation amplitude, depending on its sign.

For numerical simulations, the time-domain picture is advantageous, while the transition to the frequency domain simplifies the analytical treatment considerably. For the following description of multi-bunch oscillations, see also [7] and [40–42].

Longitudinal Case, 1 Bunch

The longitudinal force term for a single pointlike bunch is related to the longitudinal wake function:

$$\begin{aligned} \frac{F(t)}{z_0} &= \frac{N^2 e^2 \eta}{z_0 c T_o} \sum_{k=-\infty}^{\infty} W_{\parallel} [k T_o + z(t - k T_o)/c - z(t)/c] \\ &\approx \frac{N^2 e^2 \eta}{z_0 c T_o} \sum_{k=-\infty}^{\infty} W_{\parallel} (k T_o) + (z(t - k T_o) - z(t)) W'_{\parallel} (k T_o). \end{aligned} \quad (2.77)$$

Here, η is the momentum compaction factor, T_o is the revolution time, and $W'_{\parallel} = c^{-1} dW_{\parallel}/d\tau$ is the derivative of the wake function with respect to the longitudinal coordinate. Since $W_{\parallel}(\tau < 0) = 0$ due to causality, only the present and previous revolutions with $k \leq 0$ contribute to the sum. The expansion (second line) is based on the assumption that the displacement z is small compared to the period length of the wake function. The first term describes the static beam-loading effect (see Appendix D) and is ignored from now on. Under the assumption $z(t) = z_0 \exp(-i\Omega t)$ and with (2.76), the frequency shift reads

$$\Delta\omega = -\frac{N e^2 \eta}{2\omega_s \gamma m_e c T_o} \sum_{k=-\infty}^{\infty} (e^{i\Omega k T_o} - 1) W'_{\parallel} (k T_o). \quad (2.78)$$

The transition to the frequency domain requires the definition of the longitudinal impedance (2.40) and application of Poisson's sum rule (2.9). The result is

$$\Delta\omega = -i \frac{N e^2 \eta}{2\omega_s \gamma m_e c^2 T_o^2} \sum_{p=-\infty}^{\infty} \left(p\omega_o Z_{\parallel}(p\omega_o) - (p\omega_o + \Omega) Z_{\parallel}(p\omega_o + \Omega) \right) \quad (2.79)$$

with $\omega_o = 2\pi/T_o$ being the revolution frequency. Using the approximation $\Omega \approx \omega_s$ and $E = \gamma m_e c^2$, the real frequency shift is given by

$$\text{Re}(\Delta\omega) = \frac{N e^2 \eta}{2\omega_s E T_o^2} \sum_{p=-\infty}^{\infty} \left(p\omega_o \text{Im} Z_{\parallel}(p\omega_o) - (p\omega_o + \omega_s) \text{Im} Z_{\parallel}(p\omega_o + \omega_s) \right) \quad (2.80)$$

and the growth rate of the oscillation amplitude is

$$\text{Im}(\Delta\omega) = \frac{1}{\tau} = \frac{N e^2 \eta}{2\omega_s E T_o^2} \sum_{p=-\infty}^{\infty} (p\omega_o + \omega_s) \text{Re} Z_{\parallel}(p\omega_o + \omega_s). \quad (2.81)$$

Here, the first term of (2.79) vanishes due to $Z_{\parallel}^*(\omega) = Z_{\parallel}(-\omega)$. This term is independent of the bunch motion and represents a force periodic with the revolution frequency. This “potential-well distortion” modifies the shape of the rf potential and influences the synchrotron frequency but not the oscillation amplitude.

There is a potential source of confusion: in contrast to the spectra sketched in Fig. 2.3 that were based on a longitudinal motion $\sim \cos \omega_s t$, the present treatment using (2.74) yields only upper sidebands ($+\omega_s$ instead of $\pm \omega_s$). The difference between the two approaches vanishes when only positive (observable) frequencies are considered.

It is evident from the example shown in Fig. 2.14, that in the case of a narrow-band impedance (width $\ll \omega_o$) only two lines of the spectrum contribute to the sum:

$$\begin{aligned} \frac{1}{\tau} &= \frac{Ne^2\eta}{2\omega_s E T_o^2} \left(-|p|\omega_o \operatorname{Re} Z_{\parallel}(-|p|\omega_o + \omega_s) + |p|\omega_o \operatorname{Re} Z_{\parallel}(|p|\omega_o + \omega_s) \right) \\ &= \frac{Ne^2\eta}{2\omega_s E T_o^2} \left(-|p|\omega_o \operatorname{Re} Z_{\parallel}(|p|\omega_o - \omega_s) + |p|\omega_o \operatorname{Re} Z_{\parallel}(|p|\omega_o + \omega_s) \right) \end{aligned} \quad (2.82)$$

The fundamental mode of the rf cavities represents a narrow-band impedance that is always present in electron storage rings. It was in this context that this type of instability was discussed first and became known as the “Robinson instability” [44]. If the fundamental frequency of the rf cavities ω_r is above its closest revolution harmonic $h\omega_o$ (harmonic number times revolution frequency), the beam will be unstable.

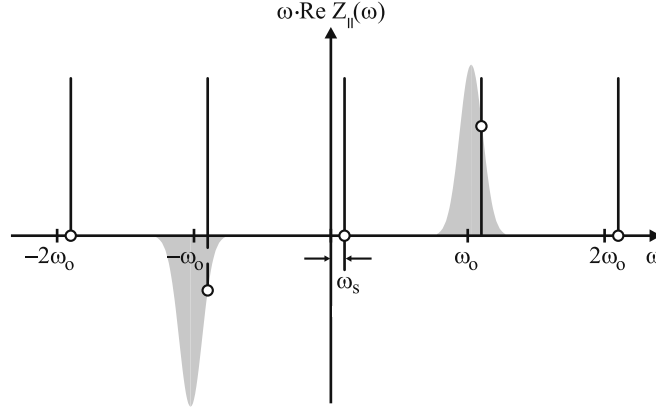


Fig. 2.14. Illustration of (2.81) applied to a narrow-band impedance (*grey*). Each *dot* corresponds to a term $(p\omega_o + \omega_s) \operatorname{Re} Z_{\parallel}(p\omega_o + \omega_s)$. In this example, the sum of these terms is positive, and so is the growth rate

Longitudinal Case, h Bunches

A system of h bunches has h independent eigenmodes, labeled by $\mu = 0, 1, \dots, h-1$ such that the phase difference in the oscillation of successive bunches is $2\pi\mu/h$. The longitudinal growth rate has to be calculated for each of these multi-bunch modes separately. In analogy to (2.81), the growth rate is (see e.g. [7]):

$$\text{Im}(\Delta\omega_\mu) = \frac{1}{\tau_\mu} = \frac{hNe^2\eta}{2\omega_s ET_o^2} \sum_{p=-\infty}^{\infty} (ph\omega_o + \mu\omega_o + \omega_s) \text{Re}Z_{\parallel}(ph\omega_o + \mu\omega_o + \omega_s), \quad (2.83)$$

where N is the number of electrons in each bunch. As an example, Fig. 2.15 shows growth rates computed for higher-order modes of the DORIS-type rf cavities used in the BESSY II storage ring. Since the exact mode frequencies are not predictable, two limiting cases are considered. The “worst case” is defined by maximum overlap of impedance and a synchrotron sideband $p\omega_o + \omega_s$. In the “best case”, each higher-order mode is placed half way between adjacent lines of the beam spectrum.

Transverse Case, 1 Bunch

The transverse wake function for a full revolution in a storage ring yields a transverse force acting on a pointlike bunch. In the case of a vertical motion, for example, the force term is given by

$$\frac{F(t)}{y_0} = \frac{N^2 e^2}{y_0 c T_o} \sum_{k=-\infty}^{\infty} y(t - k T_o) W_{\perp}(k T_o) \quad (2.84)$$

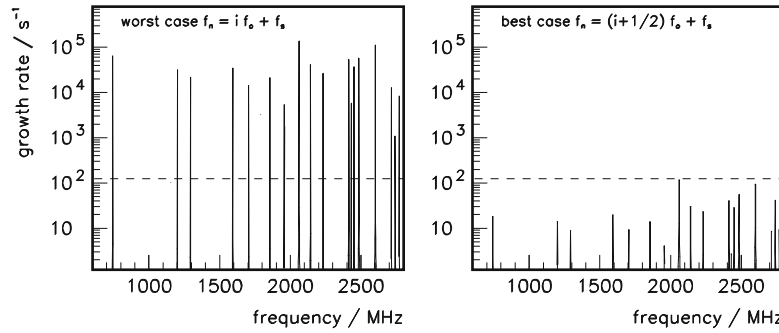


Fig. 2.15. Growth rates obtained in a one-particle model for longitudinal instabilities due to higher-order modes of DORIS-type rf cavities as function of their resonant frequency. The figure shows the largest (*left*) and lowest (*right*) possible growth rates as limiting cases [43]. The dashed line represents the radiation damping rate

with $W_{\perp}(\tau < 0) = 0$ due to causality. Assuming that the frequency of the bunch motion is close to the vertical betatron frequency, i.e.

$$y(t) = y_0 e^{-i\Omega t} \quad \text{with} \quad \Omega \approx \omega_{\beta}, \quad (2.85)$$

the complex frequency shift is

$$\Delta\omega = -\frac{Ne^2}{2\omega_{\beta} \gamma m_e c T_o} \sum_{k=-\infty}^{\infty} e^{i\Omega k T_o} W_{\perp}(k T_o). \quad (2.86)$$

The definition of the transverse impedance (2.41) together with Poisson's sum rule (2.9) yields

$$\Delta\omega = -i \frac{Ne^2}{2\omega_{\beta} \gamma m_e c T_o^2} \sum_{p=-\infty}^{\infty} Z_{\perp}(p\omega_o + \Omega). \quad (2.87)$$

With $\Omega \approx \omega_{\beta}$ and $E = \gamma m_e c^2$, the real frequency shift is

$$\text{Re}(\Delta\omega) = \frac{Ne^2 c}{2\omega_{\beta} E T_o^2} \sum_{p=-\infty}^{\infty} \text{Im} Z_{\perp}(p\omega_o + \omega_{\beta}) \quad (2.88)$$

and the growth rate is

$$\text{Im}(\Delta\omega) = \frac{1}{\tau} = -\frac{Ne^2 c}{2\omega_{\beta} E T_o^2} \sum_{p=-\infty}^{\infty} \text{Re} Z_{\perp}(p\omega_o + \omega_{\beta}). \quad (2.89)$$

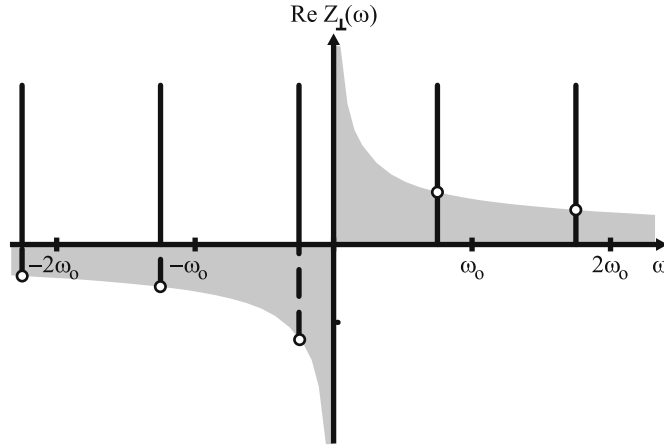


Fig. 2.16. Illustration of (2.89) assuming a transverse resistive-wall impedance (*grey*). Each *dot* corresponds to a term $\text{Re} Z_{\perp}(p\omega_o + \omega_{\beta})$. In this example, the sum is negative and the growth rate is positive

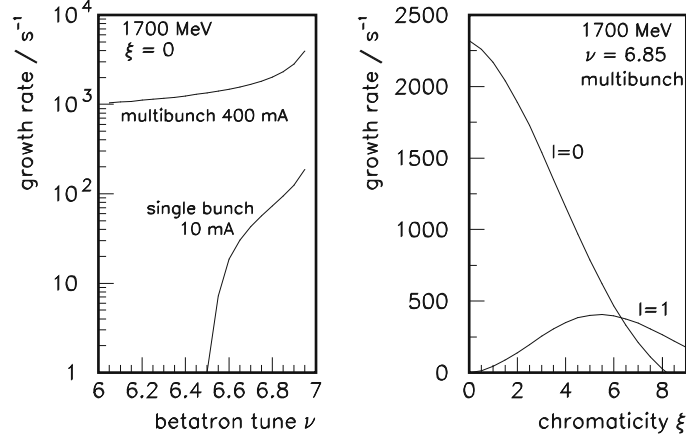


Fig. 2.17. Growth rates obtained in a one-particle model for vertical oscillations due to the resistive-wall impedance, plotted as a function of the vertical betatron tune ν for zero chromaticity, and as a function of chromaticity ξ for $\nu = 6.85$ [45], where l is the longitudinal mode number. The dependence on chromaticity is the result of an extension to the one-particle model that will be discussed in Sect. 2.4.3

In the case of a narrow-band transverse impedance, only the neighbouring lines of the beam spectrum contribute, which is the transverse version of the Robinson instability. Another important transverse impedance is due to the finite electrical conductivity of the vacuum chamber: the resistive-wall impedance, which is narrow-banded in the sense that it causes multi-bunch instabilities, but nevertheless covers a fairly broad frequency range as sketched in Fig. 2.16. According to (2.58), this impedance drops with increasing frequency $\sim 1/\sqrt{|\omega|}$, which requires the inclusion of many terms in the sum of (2.89) to accurately obtain the growth rate. Starting from zero and comparing successive terms of positive and negative frequency pairwise shows that a single-bunch beam is generally stable, if the fractional part of the betatron tune is below 0.5, since the resulting growth rate is always negative (cf. the single-bunch curve in Fig. 2.17).

Transverse Case, h Bunches

In the case of h equidistant electron bunches, the transverse growth rate has to be calculated for each multi-bunch mode separately. Analogously to (2.89), the growth rate is (see e.g. [7]):

$$\text{Im}(\Delta\omega_\mu) = \frac{1}{\tau_\mu} = \frac{hNe^2c}{2\omega_\beta ET_o^2} \sum_{p=-\infty}^{\infty} \text{Re}Z_\perp(p h\omega_o + \mu\omega_o + \omega_\beta). \quad (2.90)$$

In the case of the resistive-wall impedance, about half of the modes are unstable. The multi-bunch curve in Fig. 2.17 shows the growth rate of the most unstable mode as function of vertical betatron tune. In contrast to the

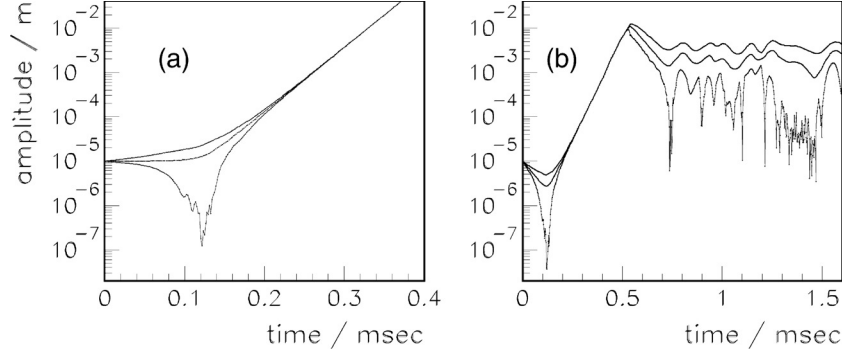


Fig. 2.18. Simulated vertical oscillation amplitudes for 400 electron bunches in the BESSY II storage ring: (a) without damping and (b) with radiation damping and amplitude-dependent betatron frequency. The three curves indicate the smallest, largest and the mean amplitude of all bunches

single-bunch case, there is no tune setting at which the resistive-wall instability would not occur.

A direct application of (2.72) together with (2.84) is the numerical simulation of bunch motion under the influence of the wake potential taken from (2.56), starting with small transverse random offsets for each bunch. Within a short period of time, the phases and amplitudes of the bunch oscillations rearrange themselves according to the mode with the largest growth rate. The growth of the amplitudes shown in the left part of Fig. 2.18 is consistent with the analytical result using (2.90) with (2.58), whereas the irregular motion in the right part of the figure results from the interplay of the instability, the damping effect of synchrotron radiation and an amplitude-dependent betatron frequency [46]. Figure 2.19 compares the oscillation of 400 electron bunches, occupying all rf buckets of the storage ring, with 320 bunches plus a gap of 80 empty buckets. In accordance with the theoretical expectation [47], the growth rate is the same, but in the presence of a gap the bunches differ in oscillation amplitude and the beam spectrum is more complex. The simulation also allows to demonstrate the difference between the usually assumed harmonic oscillation $\exp(-i\omega t)$ and the more irregular betatron motion in a real storage ring. For the vertical resistive-wall instability in BESSY II, introduction of the irregular betatron oscillation reduces the growth rate by about 20% [45].

Strictly speaking, the results discussed here in the context of a resistive-wall instability hold for a vacuum chamber with rotational symmetry and a given radius R . They may, however, serve as an estimate for a chamber in which R is the smallest distance to the beam. For rectangular and elliptical chambers, the impedance can be expressed analytically [48, 49]. For a general cross section, numerical methods exist [50].

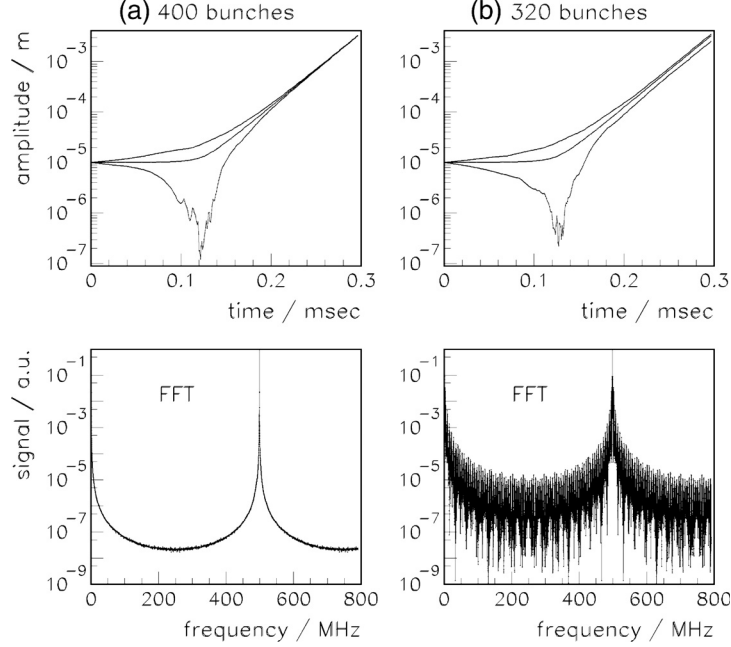


Fig. 2.19. Vertical oscillation amplitude of (a) 400 bunches (complete fill) and (b) 320 bunches in the BESSY II storage ring. The three curves in the upper figures correspond to the smallest, largest and the mean amplitude of all bunches. The lower figures show a numerical Fourier transform (FFT) of the beam motion as it would be sensed by a detector at a fixed location in the ring

2.4.2 Few-Particle Models

Models representing each electron bunch by two or more macro particles can describe effects that include the inner degrees of freedom of the bunches. This is the realm of short-range wake fields that do not act between different bunches.

Even a model that subdivides a bunch into only two longitudinal parts (“head” and “tail”) allows to identify two instabilities of practical importance: the head–tail effect and transverse mode coupling. The following equations describe the transverse (e.g. vertical) positions y_1 and y_2 of two macro particles, where the leading particle 1 is allowed to oscillate freely. The trailing particle 2 is subjected to a transverse force due to the wake field and transverse displacement of particle 1. For simplicity, the wake function $W_{\perp}$ is assumed to be constant over the length of the bunch.

$$\ddot{y}_1(t) + \omega_{\beta}^2(\delta_E) y_1(t) = 0 \quad (2.91)$$

$$\ddot{y}_2(t) + \omega_{\beta}^2(\delta_E) y_2(t) = \frac{Ne^2 W_{\perp}}{2\gamma c T_0 m_e} y_1 \quad (2.92)$$

After half a synchrotron period, the macro particles swap their longitudinal position and the equations hold with interchanged indices. For non-zero chromaticity ξ , the betatron frequency ω_β is a function of the relative energy deviation $\delta_E \equiv \delta E/E$:

$$\omega_\beta(\delta_E) = \omega_\beta(0) + \omega_o \xi \delta_E = \omega_\beta(0) + \omega_o \xi \frac{\omega_s \hat{z}}{c \eta} \cos \omega_s t, \quad (2.93)$$

where ω_o is the revolution frequency, ω_s is the synchrotron frequency, η is the momentum compaction factor, and the relation

$$\dot{z} = c \eta \delta_E \quad \text{with} \quad z(t) = \hat{z} \sin \omega_s t. \quad (2.94)$$

(see Appendix A) was used. Written in terms of complex numbers, the solution of the equation of motion for particle 1 is

$$\tilde{y}_1(t) = \{y_1(0) + i\dot{y}_1(0)/\omega_\beta\} \exp(-i\omega_\beta t). \quad (2.95)$$

Inserting this expression into (2.92) yields the following matrix formulation for the position of both particles after half a synchrotron period:

$$\begin{pmatrix} \tilde{y}_1 \\ \tilde{y}_2 \end{pmatrix}_{t=T_s/2} = \exp(-i\omega_\beta T_s/2) \begin{pmatrix} 1 & 0 \\ i a & 1 \end{pmatrix} \begin{pmatrix} \tilde{y}_1 \\ \tilde{y}_2 \end{pmatrix}_{t=0} \quad (2.96)$$

using the abbreviation

$$a \equiv \frac{\pi N e^2 W_\perp}{4 \gamma c T_o m_e \omega_\beta \omega_s} \left(1 + i \frac{4 \xi \omega_o \hat{z}}{\pi c \eta} \right). \quad (2.97)$$

The combination of two matrices with exchanged indices to describe a full synchrotron period leads to

$$\begin{aligned} \begin{pmatrix} \tilde{y}_1 \\ \tilde{y}_2 \end{pmatrix}_{t=T_s} &= \exp(-i\omega_\beta T_s) \begin{pmatrix} 1 & i a \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ i a & 1 \end{pmatrix} \begin{pmatrix} \tilde{y}_1 \\ \tilde{y}_2 \end{pmatrix}_{t=0} \\ &= \exp(-i\omega_\beta T_s) \begin{pmatrix} 1 - a^2 & i a \\ i a & 1 \end{pmatrix} \begin{pmatrix} \tilde{y}_1 \\ \tilde{y}_2 \end{pmatrix}_{t=0}. \end{aligned} \quad (2.98)$$

For zero chromaticity, a is real. The eigenvalues of the matrix are then given by the characteristic equation

$$\lambda^2 + \lambda(a^2 - 2) + 1 = 0 \quad \rightarrow \quad \lambda = 1 - \frac{a^2}{2} \pm \sqrt{\left(1 - \frac{a^2}{2}\right)^2 - 1}, \quad (2.99)$$

$$\lambda = e^{\pm i\Phi} \quad \text{with} \quad \cos \Phi \equiv 1 - \frac{a^2}{2}, \quad \text{if } a \leq 2, \quad (2.100)$$

$$\lambda = e^{\pm \mu} \quad \text{with} \quad \cosh \mu \equiv \frac{a^2}{2} - 1, \quad \text{if } a > 2. \quad (2.101)$$

This has two important consequences, described next in historic order.

Low Current – the Head–Tail Effect

In the case of $|a| \ll 1$, which is the case of a typical synchrotron radiation source in multi-bunch mode with $Ne \sim 1$ nC, the approximation $\Phi \approx a$ holds. The eigenvectors describe two oscillation modes of the macro particles with relative phase 0 or π , where a corresponds to a complex frequency shift with respect to the free betatron oscillation. In electron storage rings, η is usually positive and the chromaticity is negative in the absence of sextupole magnets. In this case, the oscillation amplitude of the 0 mode grows at a rate $\text{Im}(a)$, while the π mode is damped with the same rate. Therefore, sextupole magnets are used to set the chromaticity to a small positive value in order to damp the 0 mode. In this case, the π mode is excited, but the simple two-particle model overestimates its growth rate, and it is not excited as much as the 0 mode for negative chromaticity.

This head–tail effect occurs without current threshold and has been known for a long time [51–53]. By applying a larger positive chromaticity, the head–tail effect can be employed to damp multi-bunch oscillations, but this is usually done at the expense of a reduced dynamic aperture and beam lifetime.

High Current – Transverse Mode Coupling

Once the restriction $|a| \ll 1$ has been lifted, the motion can no longer be described by two isolated oscillation modes. The amplitude remains confined for $\xi = 0$ and a below 2. An instability occurs above $a = 2$, even at zero chromaticity. Immediately after exceeding this threshold, the real exponent of the eigenvalue leads to growth rates of the order of the synchrotron frequency.

This instability was first observed at the PETRA storage ring (DESY, Hamburg) and explained in 1980 [54]. It is therefore sometimes called the PETRA instability. Other names are head–tail turbulence, transverse microwave instability, transverse mode coupling or strong head–tail instability. In contrast to the Robinson mechanism and the “weak” head–tail instability, it does not occur below a certain threshold current. Above threshold, it constitutes a severe limitation of the single-bunch current in electron storage rings.

In the BESSY II storage ring, the onset of transverse mode coupling occurs at a current of ~ 6 mA in single-bunch operation and leads immediately to partial beam loss. With the transverse feedback system in operation, this threshold can be overcome and a single-bunch current of 20 mA can be stored (see Sect. 4.4).

The term “mode coupling” alludes to the two signals of the center-of-mass motion observed with a spectrum analyzer. At low current, they are found at frequencies ω_β and $\omega_\beta - \omega_s$, respectively. As the current increases, they approach each other and merge at the threshold of the instability. This fact is qualitatively reproduced by the simple two-particle model, but a better understanding requires a description with more macro particles or with a continuous charge distribution.

Models with More than Two Macro Particles

The description of more complex motion within a bunch requires models with more than two macro particles, and there are several time-domain simulation codes serving this purpose. These models may employ 10^2 – 10^5 macro particles, and are not restricted to a constant wake function, a particular charge distribution or other simplifications required to make the analytical models mathematically manageable. To some extent, they may offer better insight into the underlying mechanisms than experiments since they allow perfect control of the relevant parameters and a precise analysis of the results. As instructive examples, a simulation of transverse mode coupling and of the head–tail effect is described in [55, 56], respectively.

2.4.3 Models with Continuous Charge Distribution

A continuous charge distribution is an excellent approximation for bunches containing typically 10^9 – 10^{11} electrons. The properties of collective instabilities result from solutions of the Vlasov equation

$$\frac{\partial \psi}{\partial t} + \dot{q} \frac{\partial \psi}{\partial q} + \dot{p} \frac{\partial \psi}{\partial p} = 0, \quad (2.102)$$

where $\psi(q, p, t)$ is the density of particles that move in phase space (q, p) under the influence of external forces. Here, the “external” field is the collective wake field of all electrons. Dissipative effects (due to synchrotron radiation, for example) are negligible as long as the respective damping time is long compared to the growth times of the instabilities under consideration. The explicit inclusion of dissipative effects requires a solution of the Fokker–Planck equation as described e.g. in [57].

Longitudinal Case

As described in more detail in Appendix C, the usual approach is to consider a charge distribution ψ_0 in longitudinal phase space as function of polar coordinates (r, ϕ) , and to expand a small density perturbation ψ_1 oscillating with frequency Ω into longitudinal modes l :

$$\psi_1(r, \phi) = \sum_{l=-\infty}^{\infty} \alpha_l R_l(r) e^{il\phi}. \quad (2.103)$$

Since the longitudinal mode number l corresponds to the number of azimuthal nodes in longitudinal phase space, it is also called azimuthal mode number. The modes $l = 0, 1, 2$ are shown schematically in Fig. 2.20. The result of the calculation depends on the choice of ψ_0 . In the simplest case, the so-called water-bag model

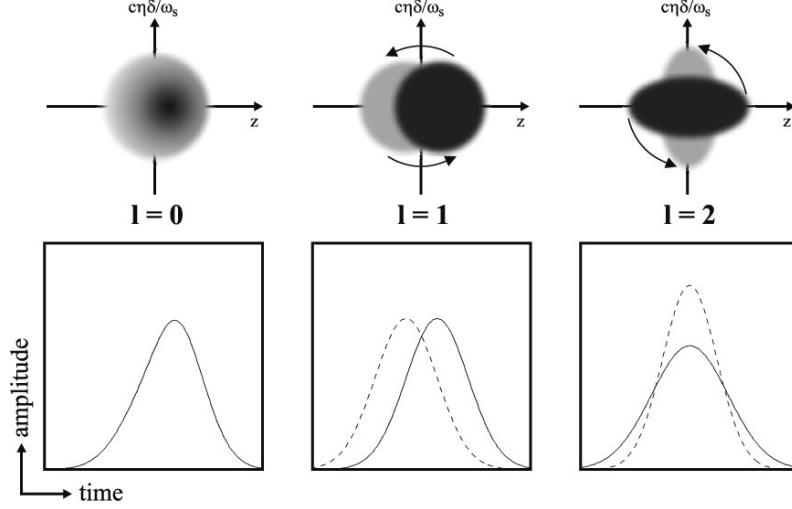


Fig. 2.20. Longitudinal modes $l = 0, 1, 2$ of a particle bunch, shown in phase space (*top*) and as signals from a detector at fixed position in a storage ring. The mode $l = 0$ is a static distortion (potential-well effect), while the distributions shown for $l > 0$ rotate in phase space with the synchrotron frequency

$$\begin{aligned} \psi_o &= \text{const.} \quad (r \leq \bar{z}) \\ \psi_o &= 0 \quad (r > \bar{z}) \end{aligned} \quad (2.104)$$

the perturbation is confined to the edge of the distribution: $R_l(r) \sim \delta(r - \bar{z})$. This simplification suppresses the occurrence of additional modes in radial direction and leads to the following system of equations:

$$(\Omega - l\omega_s) \alpha_l = i l \frac{2Ne^2c^2\eta}{E T_o^2 \omega_s \bar{z}^2} \sum_{l'=-\infty}^{\infty} \alpha_{l'} i^{l-l'} \sum_{p=-\infty}^{\infty} \frac{Z_{||}(\omega')}{\omega'} \mathcal{J}_l\left(\frac{\omega' \bar{z}}{c}\right) \mathcal{J}_{l'}\left(\frac{\omega' \bar{z}}{c}\right). \quad (2.105)$$

Here, N is the number of electrons, η is the momentum compaction factor, E is the beam energy, ω_s is the synchrotron frequency, $Z_{||}$ is the longitudinal impedance, T_o is the revolution time, $\mathcal{J}_l$ is the Bessel function of the order l and $\omega' \equiv p\omega_o + \Omega$ with $\omega_o = 2\pi/T_o$. This expression holds for a single bunch in a storage ring. Analogously to (2.83), the case of h equidistant bunches is described by the multi-bunch modes $\mu = 0 \dots h-1$, by replacing $N \rightarrow hN$ and with $\omega' \equiv ph\omega_o + \mu\omega_o + \Omega$. The system of equations can be written as a matrix equation

$$(\Omega - l\omega_s) \mathbf{I} \boldsymbol{\alpha} = \mathbf{M}_{||} \boldsymbol{\alpha} \quad (2.106)$$

with the vector $\boldsymbol{\alpha}$ containing the coefficients α_l , the unity matrix $\mathbf{I}$, and the interaction matrix $\mathbf{M}_{||}$, where the rows are labeled by l and the columns by l' . This model can describe both instability mechanisms mentioned above:

- *Robinson-type instabilities.* Neglecting the interaction, i.e. assuming a small number of electrons or a small impedance, the eigenvalues are given by

$$\Omega - l\omega_s = 0 \quad (2.107)$$

and $\alpha_l = 1$ for $l = l'$. Inserting this into (2.105) yields as an approximate solution

$$(\Omega - l\omega_s) = i l \frac{2Ne^2 c^2 \eta}{E T^2 \omega_s \bar{z}^2} \sum_{p=-\infty}^{\infty} \frac{Z_{\parallel}(\omega')}{\omega'} \mathcal{J}_l^2\left(\frac{\omega' \bar{z}}{c}\right). \quad (2.108)$$

For the dipole mode $l = 1$ and $J_1(x) \approx x/2$ (if $x \ll 1$) the results of the one-particle model (2.80) and (2.81) are recovered.

- *Mode coupling.* Lifting the restriction of a weak interaction, the matrix must be truncated to a finite number of modes l to allow for a numerical solution. The results are similar to the transverse mode coupling already described by the two-particle model:

All longitudinal modes with the exception of $l = 0$ experience a (real) frequency shift that increases with N and eventually causes the modes $l = 1$ and $l = 2$ to merge. At this point, an imaginary frequency shift occurs that also increases with N .

Thus, the longitudinal Robinson instability as well as the phenomenon of mode coupling are explained by one model. In addition, the model includes modes, which do not occur in a one-particle or few-particle description. However, some restrictions of this model should be kept in mind:

- Since only small perturbations of the charge distribution were assumed, the calculated growth rates are only valid at the threshold of the respective instability.
- More realistic distributions than the one given by the water-bag model result in greater mathematical complexity and allow only in few cases for a closed-form solution. The water-bag model ignores that each longitudinal mode l can split into a series of modes given by the nodes along the radial phase space coordinate.

Longitudinal mode coupling was proposed to be the mechanism responsible for the regularly observed current dependence of the bunch length, also dubbed “turbulent bunch lengthening” [58]. The frequency shift decreases as the typical size $\bar{z}$ of a bunch increases, while the explicit relationship depends on the assumed charge distribution. In this picture, a bunch excited by mode coupling is restabilized at a certain longitudinal amplitude at which it falls below the mode coupling threshold. This leads to an effective bunch length depending on the bunch current.

Transverse Case

A treatment of the charge density in transverse phase space alone is not useful, since many transverse phenomena – like the head–tail instability – result from an interplay of transverse and longitudinal motion. The longitudinal motion shown in Fig. 2.20 corresponds to a transverse mode number $m = 0$, whereas Fig. 2.21 shows the respective mode pictures for a transverse dipole motion with $m = 1$. When expanding the model sketched above by including a transverse dipole oscillation, the Vlasov formalism yields a system of equations

$$(\Omega - \omega_\beta - l\omega_s) \mathbf{I} \boldsymbol{\alpha} = \mathbf{M}_\perp \boldsymbol{\alpha} \quad (2.109)$$

with ω_β being the betatron frequency. The interaction matrix $\mathbf{M}_\perp$ now contains a sum over transverse impedance values $Z_\perp(\tilde{\omega})$ at frequencies $\tilde{\omega} = p\omega_o + \omega_\beta + \Omega$ for the case of a single bunch, or at $\tilde{\omega} = p h \omega_o + \mu \omega_o + \omega_\beta + \Omega$ for h equidistant bunches oscillating in multi-bunch mode μ . Again, the rows and columns of the matrix are labeled by longitudinal mode numbers l and l' , respectively. As in the longitudinal case, the model leads to different instability mechanisms:

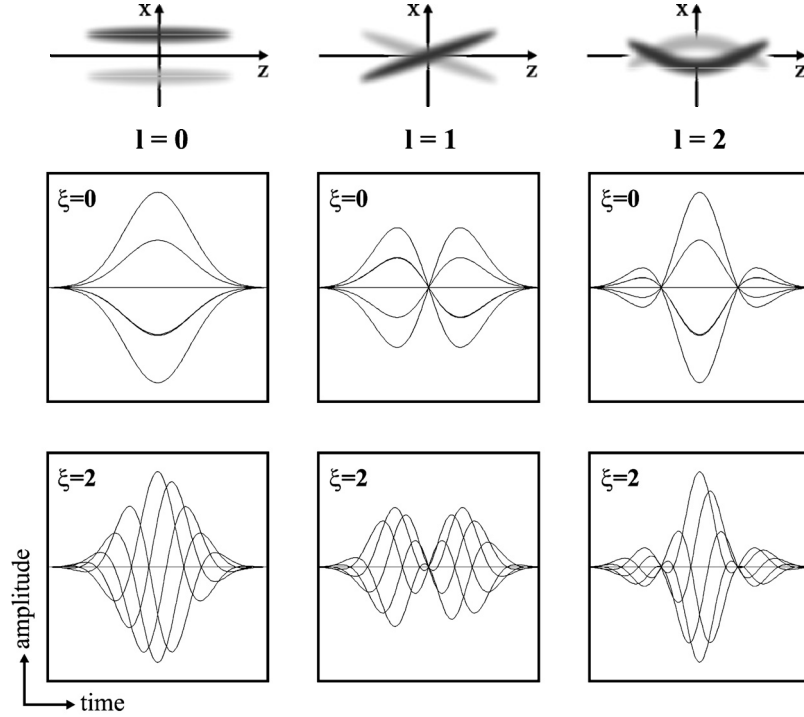


Fig. 2.21. Transverse $m = 1$ modes with the longitudinal mode numbers $l = 0, 1, 2$ in side view (*top*) and as signals sensed by a detector at fixed position for chromaticity $\xi = 0$ and $\xi = 2$

- *Robinson-type instabilities and head-tail instability.* For negligible interaction, i.e. for a small number of electrons or small impedance, the eigenvalues are

$$\Omega - \omega_\beta - l\omega_s = 0. \quad (2.110)$$

The approximate solution corresponding to (2.108) reads

$$(\Omega - \omega_\beta - l\omega_s) = -i \frac{Ne^2 c}{2ET_\omega^2 \omega_\beta} \sum_{p=-\infty}^{\infty} Z_\perp(\tilde{\omega}) \mathcal{J}_l^2 \left(\frac{\tilde{\omega} \tilde{z}}{c} - \chi \right). \quad (2.111)$$

An important difference to the purely longitudinal result is the head-tail phase in the argument

$$\chi = \frac{\xi \omega_o \hat{z}}{c \eta} \quad (2.112)$$

of the Bessel function, which shifts the beam spectrum for chromaticity $\xi = \Delta\nu_\beta/\delta_E \neq 0$. Again, the special case of $l = 0$, $\xi = 0$ and $\mathcal{J}_0(x) \approx 1$ ($x \ll 1$) reproduces the transverse Robinson-type instability obtained in the one-particle picture and given by (2.88) and (2.89). The head-tail effect shows up for chromaticity $\xi \neq 0$. In the right part of Fig. 2.22 with $\xi > 0$, the sum in (2.111) is positive and the growth rate is negative. Thus, the interaction between head and tail of a bunch has a damping effect.

- *Mode coupling.* A numerical treatment without the restriction of weak interaction yields a real frequency shift of all modes. The modes $l = 0$ and $l = -1$ finally merge and give rise to an instability, while the other modes do not change their frequency very much. This is displayed schematically together with longitudinal mode coupling in Fig. 2.23.

This model describes transverse multi-bunch instabilities of the Robinson type as well as single-bunch phenomena that first showed up in the two-particle model: the head-tail effect and transverse mode coupling.

As in the longitudinal case, (2.109) is based on a simple shape of the charge distribution. In the transverse case, the radial modes vanish only in the so-called air-bag model where the charge is concentrated on an ellipse in phase space [7].

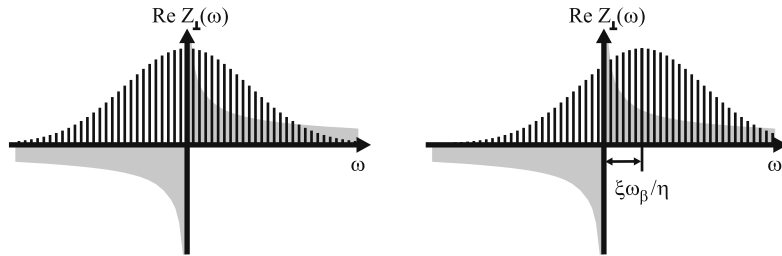


Fig. 2.22. Resistive-wall impedance as function of frequency (*grey*) and the distribution of betatron sidebands for chromaticity $\xi = 0$ (*left*) and $\xi > 0$ (*right*)

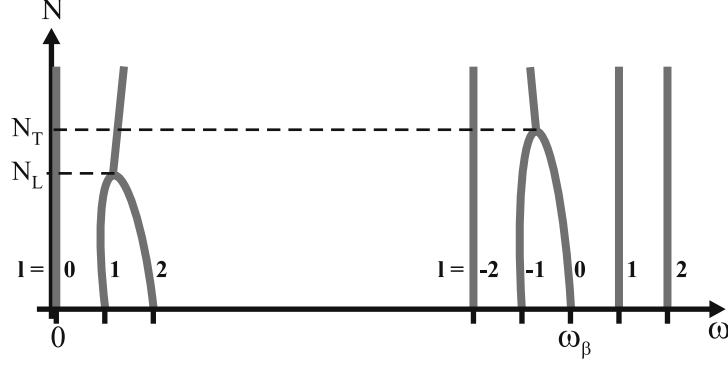


Fig. 2.23. Schematic view of sidebands $l\omega_s$ (transverse mode number $m = 0$) and $\omega_\beta \pm l\omega_s$ (transverse mode number $m = 1$) shifting in frequency with increasing number of electrons N . Above the longitudinal threshold N_L , a current-dependent increase of bunch length and energy spread is observed, whereas exceeding the transverse threshold N_T usually results in beam loss

2.5 Natural Damping Mechanisms

In contrast to mode coupling, there is no current threshold for instabilities of the Robinson type, below which they would not exist. On the other hand, everyday experience shows that the beam in an electron storage ring is usually stable at moderate current. This is due to “natural” damping mechanisms described in this section.

2.5.1 Robinson Damping

Equations (2.81), (2.83), (2.89) and (2.90) are used to calculate growth or damping rates of instabilities of the Robinson type, depending on the sign of the result. For narrow-band resonant impedances where only two terms contribute to the sum – see (2.82) – damping requires the resonant frequency to be below the closest revolution harmonic. In particular for the fundamental mode of rf cavities, this is referred to as the “Robinson-damped” side of the mode, and rf cavities are tuned accordingly. Otherwise, it would not be possible to store an electron beam.

However, other narrow-band impedances will – intentionally or not – be above their closest revolution harmonic. This is, for example, always the case for the fundamental mode of a higher-harmonic resonator used for bunch lengthening (see Sect. 4.6).

2.5.2 Damping via Synchrotron Radiation Emission

A detailed description of the damping effect due to synchrotron radiation can be found in almost any textbook of particle accelerator physics, e.g. [59, 60]:

- Longitudinal damping is due to the fact that the energy dissipated per turn V_S in a storage ring increases with electron energy E . For synchrotron oscillations, this leads to a damping effect that is analogous to Stokes' friction of a pendulum in a viscous medium (see Sect. A.3).
- Transverse radiation damping occurs because the momentum lost through photon emission has a transverse component, if the emitting electron executes a betatron oscillation. Since the rf cavities restore only the longitudinal component, the transverse momentum gradually decreases.

The longitudinal, horizontal and vertical damping rates are given by

$$\frac{1}{\tau_s} = \frac{V^S f_o}{2E} (2 + \mathcal{D}) \quad \frac{1}{\tau_x} = \frac{V^S f_o}{2E} (1 - \mathcal{D}) \quad \frac{1}{\tau_y} = \frac{V^S f_o}{2E} , \quad (2.113)$$

where f_o is the revolution frequency. The quantity $\mathcal{D}$ accounts for the influence of dispersion in the longitudinal and horizontal case and is usually negligible in present-day storage rings with separate dipole and quadrupole magnets.

2.5.3 Landau Damping

The Landau damping mechanism was originally studied in the context of plasma physics [61] and was applied to the motion of particles in a storage ring in the 1960s, see e.g. [62]. The following description assumes a transverse oscillation of a single bunch in a storage ring, where Landau damping arises from the fact that the oscillation frequency of particles within a bunch is not strictly uniform but is spread over a certain frequency range. This can limit the center-of-mass motion of the bunch even if the ensemble continuously absorbs energy from an external source. Thus, it is not truly a damping effect in the sense that it would counteract the force that drives the oscillation.

The forced transverse oscillation of an electron in the vertical coordinate, for example, can be described by

$$\ddot{y}(t) + \omega^2 y(t) = -\Delta\omega 2\omega \langle y \rangle(t) . \quad (2.114)$$

Here, the driving force is not expressed in terms of a wake function or impedance but – from (2.76) – in terms of the complex frequency shift $\Delta\omega$ from the single-particle model multiplied by the center-of-mass position of the bunch $\langle y \rangle$. The center-of-mass motion is assumed to follow

$$\langle y \rangle(t) = y_o e^{-i\Omega t} \quad (2.115)$$

with a frequency $\Omega \approx \omega$. The motion of a single electron as a solution of (2.114) is

$$y(t) = -y_o \frac{\Delta\omega 2\omega}{\omega^2 - \Omega^2} e^{-i\Omega t} \approx -y_o \Delta\omega e^{-i\Omega t} \frac{1}{\omega - \Omega} , \quad (2.116)$$

as can be seen by inserting (2.116) into (2.114). Let the frequency distribution $\varrho(\omega)$ of the electrons be characterized by a central frequency ω_c and width σ_ω and be normalized to N , the total number of electrons in the bunch:

$$\int_{-\infty}^{\infty} \varrho(\omega) d\omega = N . \quad (2.117)$$

The center-of-mass motion is given by integration over the whole distribution

$$\langle y \rangle(t) = -y_o \Delta\omega e^{-i\Omega t} \int_{-\infty}^{\infty} d\omega \frac{\varrho(\omega)}{\omega - \Omega - i\epsilon} \quad (2.118)$$

$$= -y_o \Delta\omega e^{-i\Omega t} \left\{ \int_0^{\infty} da \frac{\varrho(\Omega + a) - \varrho(\Omega - a)}{a} + i\pi\varrho(\Omega) \right\}. \quad (2.119)$$

In order to avoid a singularity at $\omega = \Omega$, an infinitesimally small quantity $i\epsilon$ was introduced. In (2.119), $a \equiv \omega - \Omega$ was defined and the real part of the integral was rewritten such that no divergence occurs at $\omega = \Omega$, while the imaginary part follows from the theorem of residues. The abbreviation

$$u \equiv \frac{\omega_c - \Omega}{\sigma_\omega} \quad (2.120)$$

replaces Ω by $\omega_c - \sigma_\omega u$. Defining the functions

$$f(u) \equiv \sigma_\omega \int_0^{\infty} da \frac{\varrho(\omega_c - \sigma_\omega u + a) - \varrho(\omega_c - \sigma_\omega u - a)}{a}, \quad (2.121)$$

$$g(u) \equiv \pi \sigma_\omega \varrho(\omega_c - \sigma_\omega u), \quad (2.122)$$

the result can be written as

$$\langle y \rangle(t) = -\frac{y_o \Delta\omega}{\sigma_\omega} e^{-i\Omega t} \{f(u) + ig(u)\}. \quad (2.123)$$

Comparison with the assumption (2.115) of a center-of-mass oscillation with constant amplitude yields the dispersion relation

$$-\frac{\Delta\omega}{\sigma_\omega} = \frac{1}{f(u) + ig(u)}. \quad (2.124)$$

The functions $f(u)$ and $g(u)$ depend on the shape of $\varrho(\omega)$ and are plotted in the left part of Fig. 2.24 for a Gaussian distribution of the electron frequencies. The right part of the figure shows the rhs of (2.124) in the complex plane, from which the following stability condition can be concluded:

If the value of $-\Delta\omega/\sigma_\omega$ lies on this graph, then (2.124) is satisfied, the frequency Ω of the center-of-mass oscillation is real and its amplitude does not change. For a value of $-\Delta\omega/\sigma_\omega$ above the graph, the amplitude decreases (damping). If $-\Delta\omega/\sigma_\omega$ is below the graph, the amplitude increases (instability). As stated above, $\Delta\omega$ is the complex frequency shift taken from the one-particle model, not including Landau damping. The minimum of the graph is at $-i\sqrt{2/\pi} \approx -i0.8$. Thus, Landau damping suppresses transverse oscillations with a growth rate below $0.8\sigma_\omega$. In the transverse case, the frequency width

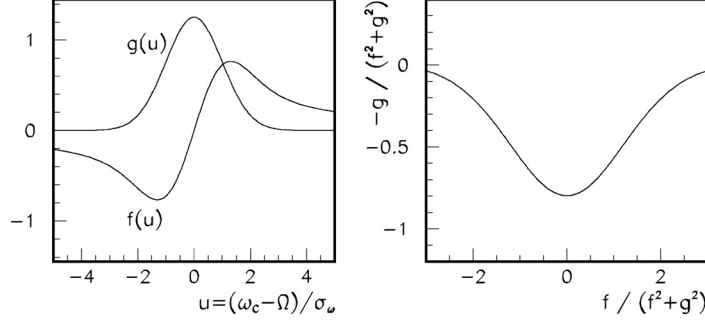


Fig. 2.24. Functions $f(u)$ and $g(u)$ (left) and representation of the rhs of (2.124) in the complex plane (right)

σ_ω is predominantly a consequence of the nonlinear magnetic guide fields due to sextupole magnets and depends on their setting.

There is an analogous stability condition for longitudinal oscillations of electron bunches, where the synchrotron frequency depends on the amplitude z according to

$$\omega_s(z) \approx \omega_s(0) \left(1 - \frac{\omega_{\text{rf}}^2 z^2}{16c^2} \right). \quad (2.125)$$

With an rf frequency $\omega_{\text{rf}} = 2\pi \cdot 500$ MHz, for example, and a synchrotron frequency $\omega_s(0) = 2\pi \cdot 7$ kHz, the frequency spread is only 30 Hz for an amplitude of 1 cm. Here, Landau damping appears to be quite inefficient. Higher-harmonic cavities can increase the frequency spread, and – if used for this purpose – are sometimes called “Landau cavities”.

2.6 Impedance Measurements Using the Beam

The total impedance of a storage ring or the corresponding wake potential can be estimated from its effect on the stored beam. The results are in two ways more general than those of the impedance calculations and bench measurements discussed in Sect. 2.3:

- The effect on the beam results from an “effective” impedance [58,63], i.e. a weighted sum of impedance components that coincide with the frequencies ω' and $\tilde{\omega}$ present in the beam spectrum for longitudinal and transverse oscillations, respectively:

$$(Z_{\parallel})_{\text{eff}} = \frac{\sum (\omega')^{-1} Z_{\parallel}(\omega') h_l(\omega')}{\sum h_l(\omega')}, \quad (2.126)$$

$$(Z_{\perp})_{\text{eff}} = \frac{\sum Z_{\perp}(\tilde{\omega}) h_l(\tilde{\omega} + \omega_{\xi})}{\sum h_l(\tilde{\omega} + \omega_{\xi})}, \quad (2.127)$$

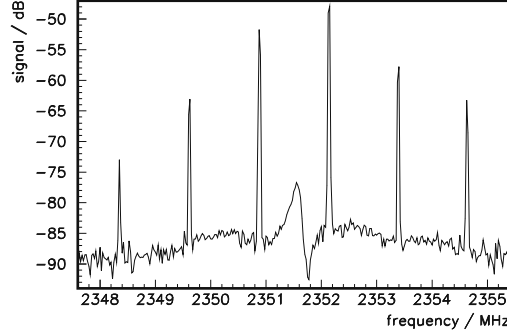


Fig. 2.25. Higher-order mode of a third-harmonic cavity and revolution harmonics observed in the BESSY II storage ring. A downshift of the mode frequency would excite the longitudinal multi-bunch mode $\mu = 2350.9 \text{ MHz}/f_o - 4h = 282$, where $f_o = 1.2491 \text{ MHz}$ is the revolution frequency and $h = 400$ is the harmonic number

where h_l is the spectral power density (i.e. the “shape” of the spectrum). The effective impedance defined this way corresponds to the sum contained in the previous equations (2.108) and (2.111).

- The effect on the beam comes from the total impedance, whereas the predictions always refer to individual components of the storage ring. Inferring the actual cause of an instability from beam properties is often difficult. Here are some general considerations:
 - There are well-known effects common to virtually all electron storage rings. One example is the transverse oscillation due to the resistive-wall effect, as discussed in Sect. 4.4.
 - Some sources of instabilities can be identified by variation of the geometric boundary conditions, e.g. by tuning a cavity.
 - The impedance may be identified by simultaneous observation of electromagnetic fields with an antenna. As an example, the signal picked up from a third-harmonic cavity is shown in Fig. 2.25.
 - Position-resolved measurements of energy loss and of frequency or phase of betatron oscillations can reveal the location of the disturbing impedance.

The effects of the impedance on several beam parameters described in the following paragraphs can be directly inferred from the impedance models discussed above.

2.6.1 Frequency Shift

A (real) frequency shift of longitudinal and transverse oscillations under variation of the beam current is mostly due to the imaginary (“reactive”) part of the impedance.

The measurement is usually performed using a spectrum analyzer where the betatron or synchrotron sidebands either show up spontaneously or are excited externally using a stripline electrode. Another possibility is digital sampling of the bunch motion as described later in the context of feedback systems (see Sects. 4.3 and 4.4). Transverse oscillations can be recorded by beam position monitors capable of operating in a fast “turn-by-turn” mode.

A position-resolved measurement of the betatron frequency and phase was performed at the LEP storage ring at CERN [64].

A new and rather sensitive method that can also sense quadrupole excitations (an oscillation without center-of-mass displacement) is to excite the beam with varying frequency while observing particle losses using scintillation detectors or pin diodes [65].

2.6.2 Growth Rates

An imaginary frequency shift, i.e. damping or growth of an instability, is caused by the real (“resistive”) part of the impedance.

In principle, an observed current threshold at which the growth rate equals the damping rate, allows to estimate the impedance, since the radiation damping rate is known. Landau damping, however, makes the issue more complex.

A direct observation of growth rates at small amplitudes (in order to avoid nonlinear effects) requires a fast transition from a stable state to a situation where the oscillation is allowed to grow freely. A digital feedback system that allows to sample the oscillation is ideally suited for this purpose (see Sects. 4.3 and 4.4).

2.6.3 Current Thresholds

Mode coupling instabilities are characterized by current thresholds below which there is no instability at all, but only a (real) frequency shift.

Assuming that longitudinal mode coupling is responsible for the “turbulent” bunch lengthening phenomenon (see Sect. 2.4.3), its current threshold can be determined by measuring the bunch length as function of current (see Sect. 2.6.4).

The transverse mode coupling threshold is evident from a rather violent instability that limits the storable bunch current.

2.6.4 Bunch Length and Shape

The measurement of bunch length σ_τ with increasing bunch current shows initially a moderate lengthening effect, which is known as potential-well distortion and results from the imaginary (inductive) broad-band impedance, while a real (resistive) impedance component influences the bunch shape. Above the threshold of longitudinal mode coupling, the bunch length increases roughly

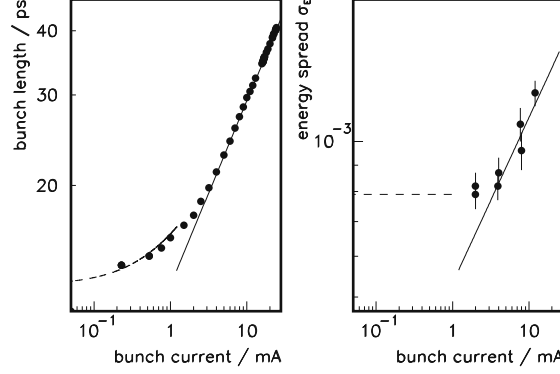


Fig. 2.26. Bunch length σ_τ in time units (*left*), and energy spread σ_E (*right*) of a single bunch in BESSY II as functions of current. The *dashed lines* correspond to (2.129) and to the natural energy spread, respectively. The *solid lines* show the I^x -dependence of bunch length and energy spread with $x \approx 1/3$ (courtesy P. Kuske, BESSY, and R. Klein, PTB, Berlin)

according to $\sigma_\tau \sim I^{1/3}$, as shown in Figs. 2.26 and 2.27 for BESSY II. Different power laws were found at other storage rings. When the bunch length was found to increase $\sim I^{1/1.32}$ at the SPEAR II storage ring at SLAC,

$$\sigma_\tau \sim I^{1/(2+a)} \quad (2.128)$$

was proposed [73] as a universal scaling law, where the parameter a describes the power-law behaviour of the impedance $Z(\omega) \sim \omega^a$.

A measurement of the longitudinal electron distribution to determine the real and imaginary part of the broad-band impedance is not a trivial task. The typical bunch length in electron storage rings of 3–30 mm (10–100 ps) requires the use of a streak camera (see e.g. [66]). Longer bunches can also be measured by analyzing stripline signals with a high-resolution oscilloscope [67]. The analysis of streak camera images of bunches in BESSY II [68] is shown in Fig. 2.26. For low current, the bunch length is in qualitative agreement with a potential-well calculation assuming a purely imaginary impedance with $|Z|/n = 0.3 \, \Omega$, as estimated from numerical wake field simulations (see Sect. 2.3.5). Haïssinski's equation [69] that follows from the Vlasov formalism and further simplifications regarding the bunch shape lead to a condition for the bunch length of

$$\left(\frac{\sigma_\tau(I)}{\sigma_\tau(0)} \right)^3 - \frac{\sigma_\tau(I)}{\sigma_\tau(0)} + D(I) = 0, \quad (2.129)$$

where $D(I)$ is a parameter proportional to the bunch current and the impedance [7]. The bunch shape can be inferred from streak camera images where current-dependent changes hint the presence of a real impedance component.

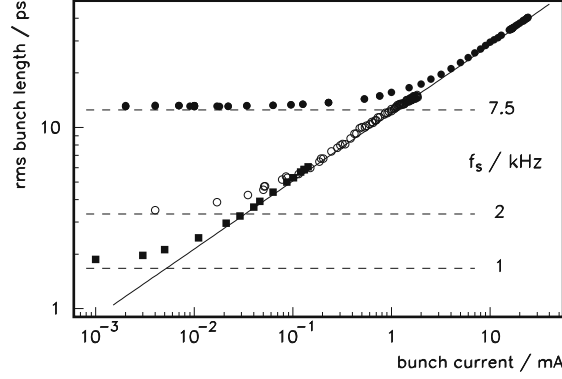


Fig. 2.27. Bunch length in time units as functions of current for different values of the synchrotron frequency f_s [71]. While $f_s = 7.5$ kHz corresponds to normal operation conditions of the BESSY II storage ring, the lower values are obtained in “low- α ” operation, i.e. with a reduced momentum compaction factor. The dashed lines indicate the respective natural bunch lengths, while the dots are data from streak camera measurements. The solid line corresponds to $\sigma_\tau \sim I^{0.37}$ (courtesy P. Kuske and G. Wüstefeld, BESSY)

The data of the left part of Fig. 2.26 suggest a longitudinal mode coupling threshold close to 4 mA above which the bunch length follows $\sigma_\tau \sim I^{0.36 \pm 0.01}$.

For a given storage ring, the power law appears to be the same under different operation conditions, e.g. under variation of the beam energy as in [73], or by changing the momentum compaction factor α , as shown in Fig. 2.27 for BESSY II. Here, the momentum compaction factor was reduced by almost two orders of magnitude to reduce the bunch length and to generate coherent far-infrared radiation in the THz regime [71, 72]. Because of the instability threshold, shortening the bunches by a factor of 10 requires to reduce their current by almost three orders of magnitude.

2.6.5 Energy Spread and Energy Loss

The natural energy spread is given by synchrotron radiation emission according to (A.42) and is not influenced by the potential-well effect. For longitudinal mode coupling, the energy spread should increase with current starting at the same threshold as bunch lengthening. The energy spread σ_E can be estimated from the horizontal rms beam size σ_x in the presence of non-zero dispersion D_x using

$$\sigma_x = \sqrt{\varepsilon_x \beta_x + \sigma_E^2 D_x^2}, \quad (2.130)$$

if the value of the beta function β_x is known. More precise measurements are elaborate. One example is the analysis of the Compton edge in the spectrum

of backscattered laser photons. If the detector resolution is known, the energy spread can be extracted from the shape of the Compton edge. The few measurements that were made at BESSY II [70] are shown in the right part of Fig. 2.26. Here, the energy spread rises with current as $\sigma_E \sim I^{0.32 \pm 0.04}$ in accordance with the bunch length behavior. The horizontal dashed line represents the natural energy spread of $7.9 \cdot 10^{-4}$ and suggests again a mode coupling threshold close to 4 mA.

The real part of the longitudinal impedance causes an energy loss as expressed by the longitudinal loss factor $k_{\parallel}$:

$$\Delta E/e = k_{\parallel} q = k_{\parallel} I/f_{\circ} . \quad (2.131)$$

Here, q is the charge, I is the current of the bunch and $f_{\circ}$ is the revolution frequency. This energy loss can be established by measuring a current-dependent shift of the synchronous phase angle in the rf cavity field by, for example, observing a stripline signal and the storage ring rf voltage simultaneously (e.g. [67]) or by streak camera measurements.

Observing the current-dependent change of the horizontal position $\Delta x/\Delta I$ under the condition of non-zero dispersion $D_x = \Delta x/(\Delta E/E)$ allows to determine the loss factor for different positions in the storage ring:

$$k_{\parallel} = \frac{E}{e} \frac{f_{\circ}}{D_x} \frac{\Delta x}{\Delta I} . \quad (2.132)$$

The effect is small but has been seen at very large storage rings with correspondingly small revolution frequency [64].

2.6.6 Transverse Displacement

In analogy to the energy loss in the longitudinal case, the imaginary part of the transverse impedance causes a kick described by the transverse kick factor

$$k_{\perp} = \frac{1}{\pi} \int_0^{\infty} d\omega J(\omega) \text{Im} Z_{\perp}(\omega), \quad (2.133)$$

where $J(\omega)$ represents the frequency spectrum of the beam. At the APS storage ring (Argonne, USA), very small displacements due to the resistive-wall impedance in a small-aperture undulator chamber were detected as deviations from a closed orbit bump [74].

The Lifetime of a Stored Electron Beam

3.1 Overview

In addition to the brilliance of the emitted radiation, the beam lifetime is another important quality criterion for a synchrotron radiation source. In order to prepare and perform an experiment, constant beam position and intensity without interruption may sometimes be more important than high brilliance or photon flux.

At third-generation synchrotron radiation sources, the beam lifetime is of the order of 10 hours and depends in detail on a large number of parameters, as described below. Typically, the beam current is allowed to decay by a factor of two before a new injection of electrons brings it back to its initial value (see Fig. 3.1).

The finite beam lifetime causes a varying thermal load on vacuum chambers, optical beamline elements and at the experiment, and often implies variations of the beam position. The most drastic change happens during injection, when the beam current changes significantly within a few minutes. A poor beam lifetime requires frequent injection during which the delivery of synchrotron radiation is usually interrupted for various reasons:

- The beam position is disturbed by pulsed magnetic elements (kicker and septa) used to inject the beam.
- Beamline shutters are usually closed during injection to ensure radiation safety at the experimental stations.
- In order to minimize radiation-induced demagnetization of permanent magnets in wigglers and undulators, these magnets are usually retracted during injection.

Topping up the beam in short time intervals or even a continuous injection in order to keep the beam current constant within 10^{-3} or better is a topic that is under discussion at many facilities. Routine operation of this kind, however, is to-date only practiced at the APS (Argonne, USA) [75] and more recently at the SLS in Switzerland [76] and at Spring-8 in Japan [77]. Top-up

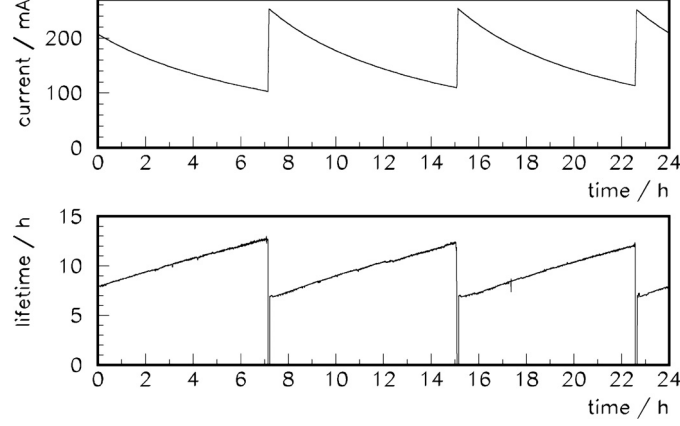


Fig. 3.1. Beam current (*top*) and beam lifetime (*bottom*) of the BESSY II storage ring during one day of operation

operation requires the beam to be injected at its final energy, which is not the case at every synchrotron radiation facility. The experiments must either be able to tolerate transient orbit distortions or must be interruptible during injection. A particular problem is to prevent an increased radiation level at the experiments and to ensure safe operation under all circumstances [78, 79].

In the following, the notion of a beam lifetime shall be defined more precisely. Then, different loss mechanisms limiting the lifetime in synchrotron radiation sources will be described.

3.1.1 Definition of the Beam Lifetime

The lifetime τ of a beam containing N particles is defined through its relative loss rate at a given time:

$$\frac{1}{\tau} \equiv -\frac{\dot{N}}{N}. \quad (3.1)$$

What sometimes causes confusion is that the beam lifetime is not constant. Generally, the lifetime is not the time interval after which the beam has reached $1/e$ of its initial current. This would only be true for a purely exponential decay given by

$$\begin{aligned} \dot{N}(t) &= -aN(t) \\ N(t) &= N(0) \exp(-at) \\ \tau_{1/e}(0) &= -\frac{N(0)}{\dot{N}(0)} = \frac{1}{a}, \end{aligned} \quad (3.2)$$

as would be the case for residual gas scattering under always constant conditions.

In reality, the residual gas pressure depends on the stored beam current, and the observed decay of current does not follow an exponential law. For electron–electron scattering (Touschek effect, see Sect. 3.4) the following relations hold:

$$\begin{aligned}\dot{N}(t) &= -aN^2(t) \\ N(t) &= \frac{N(0)}{1 + N(0)at} \\ \tau_{1/2}(0) &= -\frac{N(0)}{\dot{N}(0)} = \frac{1}{N(0)a} .\end{aligned}\tag{3.3}$$

Here, the inverse of the relative loss rate corresponds to a “half-life” time, but its value is by no means constant. Furthermore, the condition $\dot{N} \sim N^2$ is violated if the bunch volume depends on current, as it usually does. The bunch length changes due to potential-well distortion and turbulent bunch lengthening (cf. Fig. 2.26), and for electron energies below 1 GeV the transverse bunch dimensions increase with current due to small-angle multiple scattering between electrons.

It is certainly not meaningful to relate the lifetime values of (3.2) and (3.3) by a factor of $\ln 2$. The only precise statement is the loss rate $-\dot{N}/N$ at a given time. The inverse of a loss rate of $2 \times 10^{-5} \text{ s}^{-1}$, for example, would be a lifetime of 13.9 h, which is easier to conceive, but does not mean that 13.9 h is the time after which the beam current has decayed to a particular value.

If several processes contribute, the resulting beam lifetime is found by adding the respective loss rates:

$$\frac{1}{\tau} = \sum_i \frac{1}{\tau_i} .\tag{3.4}$$

3.1.2 Classification of Loss Mechanisms

Mechanisms leading to particle loss may be characterized by the phase space coordinate that changes during a scattering process. This is always a

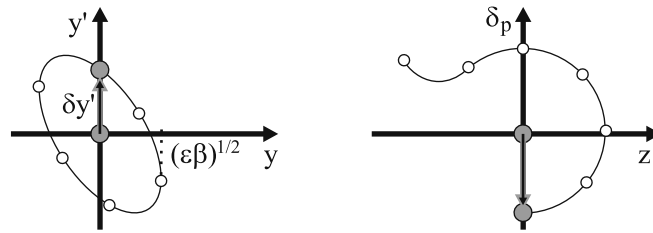


Fig. 3.2. Sketch of transverse phase space (*left*) with betatron oscillations excited by an angular kick, and of longitudinal phase space (*right*) with an electron losing momentum and exceeding the momentum acceptance. The *gray dots* and the *arrows* between them indicate the momentum change, the *open dots* mark subsequent positions of the electron

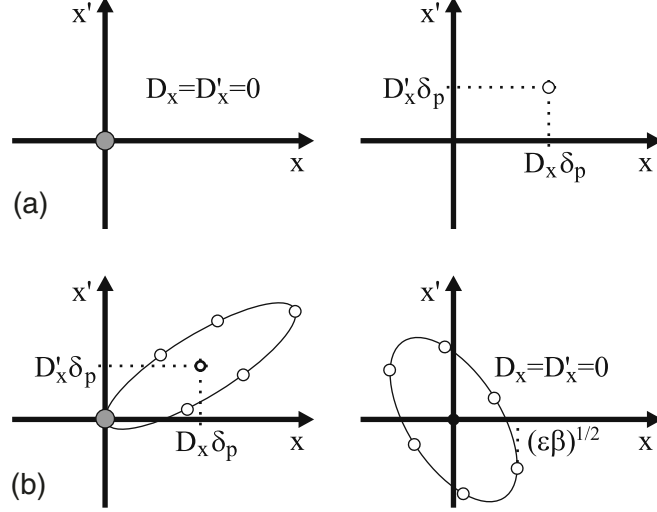


Fig. 3.3. Position of an electron in horizontal phase space after a change of longitudinal momentum. The *gray dot* marks the position at which the momentum change occurs, the *open dots* indicate later positions. (a) Momentum change in a dispersion-free region (*left*) and later position with dispersion (*right*). (b) Momentum change in a dispersive region (*left*) and later positions in a dispersion-free region (*right*)

momentum coordinate, since scattering cannot instantly change the particle position (see Figs. 3.2 and 3.3):

- *Change of longitudinal momentum* $\delta_p \equiv \delta p/p$ causes a particle to be lost, if it exceeds the momentum acceptance of the rf system given by (A.41). Even if this is not the case, a transverse displacement (see Appendix A)

$$\delta x = D_x \delta_p + \sqrt{\varepsilon_x \beta_x} \cos \phi \quad (3.5)$$

may cause particle loss if δx exceeds the physical aperture (the vacuum chamber wall or any other obstacle) or the dynamic aperture (the region that encloses stable orbits). The first term is the transverse displacement due to horizontal dispersion D_x , the second term describes a betatron oscillation where an unfavorable oscillation phase (with $\cos \phi \approx 1$, if $D_x \delta_p > 0$) occurs within a few turns. Normally, the betatron oscillation amplitude is much smaller than the aperture (Fig. 3.3a). However, if the particle momentum is instantly changed at a position with non-zero dispersion $\tilde{D}_x \neq 0$ and/or $\tilde{D}'_x \neq 0$, the quantity ε_x in (3.5) becomes

$$\varepsilon_x = \left(\tilde{\gamma}_x \tilde{D}_x^2 + 2\tilde{\alpha}_x \tilde{D}_x \tilde{D}'_x + \tilde{\beta}_x \tilde{D}_x'^2 \right) \delta_p^2, \quad (3.6)$$

where the tilde ($\sim$) marks the optical functions at the position of the momentum change. Figure 3.3b shows a betatron motion excited this way. If

- both, the scattering process and particle loss occur at positions with similar dispersion $\tilde{D}_x \approx D_x$ and beta function $\tilde{\beta}_x \approx \beta_x$, and assuming $\tilde{D}' = 0$ as well as $\tilde{\alpha}_x = 0$ for simplicity, the two terms in (3.5) would be equal.
- *Change of transverse momentum $\delta y'$* – assuming a vertical kick, for example – causes a displacement

$$\delta y = \sqrt{\varepsilon_y \beta_y} \cos \phi \quad (3.7)$$

with

$$\varepsilon_y = \tilde{\beta}_y \delta y'^2 \quad (3.8)$$

according to (A.9). Again, the tilde labels the beta function at the scattering position, and a betatron phase leading to maximum excursion ($\cos \phi \approx \pm 1$) is reached after a few turns. At third-generation synchrotron radiation sources, the vacuum chamber is usually “flat”, particularly in the region of wigglers and undulators, which makes the vertical aperture much more restrictive than the horizontal aperture.

In the following, the mechanisms leading to particle loss are classified according to the respective type of interaction. The primarily affected phase space coordinate is given in brackets:

- Quantum lifetime: emission of synchrotron radiation ($\delta p/p$).
- Coulomb scattering: elastic scattering on residual gas atoms ($\delta x'$, $\delta y'$).
- Bremsstrahlung: photon emission induced by residual gas atoms ($\delta p/p$).
- Touschek effect: electron–electron scattering ($\delta p/p$).

Other scattering processes, that are of no concern for synchrotron radiation sources such as the scattering on thermal photons (e.g. [80]), will not be discussed here.

In the context of collective phenomena, only electron–electron scattering is usually mentioned. In this book, however, the other processes shall be included for two reasons:

- An isolated treatment of the Touschek effect is not helpful, since the beam quality in a synchrotron radiation source depends on the total beam lifetime no matter what the dominant loss mechanism is.
- Secondly, residual gas scattering can be understood as a collective effect as well, since the gas pressure depends on the beam current. The quantum lifetime can also depend on beam current and thus on the presence of all beam particles.

3.2 Quantum Lifetime

The loss of momentum due to synchrotron photon emission is responsible for the electron distribution in all six phase space coordinates. These distributions can be assumed to be Gaussian to a certain degree of accuracy. Due to the

random nature of synchrotron radiation, each electron finds itself at some time in the tails of the distribution and will, if not lost by a faster process, eventually hit an obstacle at distance R from the beam center, such as the vacuum chamber wall. This constitutes the quantum lifetime τ_q given by (e.g. [81])

$$\tau_q = \frac{1}{2} \tau_d \frac{e^a}{a} \quad \text{with} \quad a = \frac{R^2}{2\sigma^2}, \quad (3.9)$$

where τ_d is the damping time and σ the rms beam size in the respective coordinate. The quantum lifetime is negligible for obstacles beyond 6–7 σ . In the BESSY II storage ring, for example, the horizontal aperture corresponds to about 100 σ_x , the vertical aperture to 300 σ_y (not because the vertical aperture is wider but because $\sigma_y \ll \sigma_x$). The energy acceptance, which depends on the rf voltage according to (A.41), is usually beyond 30 σ_E , where σ_E depends on the bunch current due to longitudinal mode coupling with (A.42) being a lower limit. The distribution of betatron frequencies can lead to lifetime limitations if the betatron tune is too close to an optical resonance. Lifetime restrictions of this nature can be observed and are eliminated by moving the tune away from the respective resonance.

3.3 Residual Gas Scattering

Once the total cross section σ_t of all scattering processes causing particle loss is known, the beam lifetime is given by (e.g. [82])

$$\frac{1}{\tau} = -\frac{\dot{N}}{N} = \sigma_t c n \rho, \quad (3.10)$$

where the electron velocity is approximated by c and n is the number of atoms per molecule. The molecule density ρ is given by the residual gas pressure p . At a temperature of $T = 300$ K, for example, it is

$$\rho [\text{m}^{-3}] = 2.45 \times 10^{22} p [\text{hPa}]. \quad (3.11)$$

3.3.1 Scattering Cross Sections

A twofold distinction has to be made when determining cross sections relevant to residual gas scattering:

- Scattering occurs primarily on the nuclei, but also on the electrons of residual gas atoms.
- The electric field of residual gas nuclei or electrons causes elastic scattering of beam electrons as well as inelastic scattering, where energy is carried away by the emission of bremsstrahlung quanta.

In this context, spin and screening effects can be neglected and for elastic (Coulomb) scattering, the classical Rutherford cross section is applicable. A clear-cut distinction between elastic scattering (with negligible energy loss) and bremsstrahlung (with negligible angular deflection) appears to be somewhat arbitrary, since both are just extreme cases of the general electromagnetic interaction between a beam electron and a residual gas atom. However, these cross sections are very small for the angles and energy losses relevant here, and the combined process of angular kick *and* energy loss can be safely ignored.

For elastic scattering, the total cross section is given by integrating the differential cross section from 180° to the minimum angle that causes an electron to reach the limiting aperture. For the bremsstrahlung case, the integration is carried out from the total electron energy to the minimum relevant energy loss. The total cross sections for scattering on nuclei and on electrons (labeled “N” and “e”, respectively) are given by [82]:

$$\sigma_{\text{elast}}^{\text{N}} = \frac{2\pi r_e^2 Z^2 \langle\beta\rangle \beta_a}{\gamma^2 R^2} \quad (3.12)$$

$$\sigma_{\text{inel}}^{\text{N}} = \frac{4 r_e^2 Z^2}{137} \frac{4}{3} \left(\ln \frac{183}{Z^{1/3}} \right) \left(\ln \frac{1}{\Delta p/p} - \frac{5}{8} \right) \quad (3.13)$$

$$\sigma_{\text{elast}}^{\text{e}} = \frac{2\pi r_e^2 Z}{\gamma} \frac{1}{\Delta p/p} \quad (3.14)$$

$$\sigma_{\text{inel}}^{\text{e}} = \frac{4 r_e^2 Z}{137} \frac{4}{3} \left(\ln \frac{2.5 \gamma}{\Delta p/p} - 1.4 \right) \left(\ln \frac{1}{\Delta p/p} - \frac{5}{8} \right). \quad (3.15)$$

Here, $r_e = 2.82 \times 10^{-15}$ m is the classical electron radius, Z is the charge number of the residual gas nucleus, R is the aperture radius, β_a is the beta function at the limiting aperture, $\langle\beta\rangle$ is the beta function averaged over the storage ring, and γ is the Lorentz factor. The momentum acceptance $\Delta p/p$ may be given by the rf system or by an aperture limitation together with non-zero dispersion as discussed in Sect. 3.1.2.

When trying to predict the beam lifetime, there is a considerable uncertainty regarding the composition and pressure of the residual gas. For a rough estimate, it is an established tradition to assume the residual gas to be equivalent to N_2 at 2×10^{-9} Torr (2.6×10^{-9} hPa). By experience, this yields lifetime values of the right order of magnitude. From there, one can study relative changes of the lifetime systematically under variation of relevant parameters such as geometric apertures or momentum acceptance.

3.3.2 Composition of the Residual Gas

Typical constituents of the residual gas in a stainless steel vessel under good vacuum conditions are hydrogen, carbon monoxide, carbon dioxide, methane, water, and noble gases. The respective partial pressure of these and possibly other components depends on a large variety of circumstances [83, 84]:

Material of the vacuum vessel. Commonly used materials are stainless steel, copper, and aluminium. At some parts of a storage ring, other materials may be employed, such as beryllium to minimize the scattering of particles to be detected, glass for visual inspection, or metal-coated ceramics at pulses magnets to avoid eddy currents.

Treatment of the material. Prior to installation, cleaning procedures and proper storage of vacuum components are important. Baking stainless steel chambers at 200–300°C predominantly removes H₂O from the walls.

Vacuum pumps. Usually, ion getter pumps in different varieties are employed, such as sputter-ion pumps, titanium sublimations pumps, or NEG (nonevaporative getter) pumps. The pumping speed depends on the respective molecule or atom and is particularly low for CH₄ and noble gases.

Operation time and beam current. Synchrotron radiation “scrubs” the vacuum chamber surfaces, and the received dose is commonly measured in Ampère times hours. After about 1000 A h, good vacuum conditions are usually achieved and the residual gas is dominated by H₂.

Contaminations. Traces of chemical cleaning agents, outgassing material, or hydrocarbons from vacuum pumps may contaminate the storage ring vacuum. Occasional venting by accident or to install new components contaminates the chamber walls, but there is a memory effect, i.e., the vacuum generally improves much faster after venting than after the initial installation of a vacuum chamber.

3.3.3 Residual Gas Pressure

Outgassing in electron storage rings is dominated by two processes:

- Thermal desorption, i.e., the release of weakly bound particles from the chamber walls via thermal excitation at a typical rate of 10^{-11} hPa m s⁻¹ (the total gas rate in hPa m³s⁻¹ per area in m²) [84].
- Beam-induced desorption, for electron storage rings mainly due to photo electrons produced by synchrotron radiation. It requires roughly 10^6 photons above 10 eV to release one gas particle [85].

The desorption rates mentioned above depend strongly on the circumstances that also influence the gas composition (Sect. 3.3.2). The thermal desorption rate may be higher by a factor of 10^3 , if the chamber was not baked before operation. At a typical beam current of a few 100 mA, beam-induced desorption usually dominates over thermal desorption.

It should be noted that the beam lifetime is not just determined by the average pressure in the storage ring but also depends on the optical functions, i.e., the beta function and dispersion. With (3.6) and (3.8) in mind, the loss rate can in principle be minimized by designing a machine such that a concurrence of large outgassing gas rate and large dispersion or beta function is avoided.

A simple model [84] of a pressure profile $p(s)$ – pressure as a function of position s along the ring – can be obtained by considering a vacuum vessel with constant cross section and an equidistant arrangement of pumps with equal pumping speed S (in $\text{m}^3 \text{s}^{-1}$). Let the distance between adjacent pumps be L and the outgassing rate per length unit q (in $\text{hPa m}^2 \text{s}^{-1}$) be constant. The gas flow Q (in $\text{hPa m}^3 \text{s}^{-1}$) is then given by

$$Q(s) = -w \frac{dp}{ds} \quad \text{and} \quad \frac{dQ}{ds} = q, \quad (3.16)$$

where w is the specific molecular conductance (in $\text{m}^4 \text{s}^{-1}$) given by the chamber cross section. Differentiation with respect to s yields

$$w \frac{d^2 p}{ds^2} = -q. \quad (3.17)$$

With each pump removing the total amount of gas from a section of length L , i.e., $p(nL) = qL/S$ (n being an integer), and $dp/ds(nL + L/2) = 0$ from symmetry considerations, the pressure profile is

$$p(s) = q \left(\frac{Ls - s^2}{2w} + \frac{L}{S} \right) \quad (0 \leq s \leq L). \quad (3.18)$$

This instructive model produces a parabolic pressure profile between adjacent pumps, as shown in Fig. 3.4. Even for infinite pumping speed, the mean pressure is finite, being limited by the molecular conductance:

$$\langle p \rangle = \frac{1}{L} \int_0^L p(s, S \rightarrow \infty) ds = \frac{qL^2}{12w}. \quad (3.19)$$

Using numerical methods, this model can easily be extended to more realistic cases. Important modifications are:

- the pumps are not necessarily equidistant,
- each pump has its own pumping speed,
- the pumping speed depends on the gas pressure,
- the desorption rate depends on the incident photon flux,
- the molecular conductance depends on the chamber cross section.

The local desorption rate follows from calculating the expected synchrotron radiation power hitting the chamber surface at a given position. Approximate expressions exist for the specific molecular conductance of simple chamber geometries. As an example, the conductance of a long tube with elliptical cross section is [84]

$$w[\text{m}^4 \text{s}^{-1}] \approx 431 \frac{a^2 b^2}{\sqrt{a^2 + b^2}} \sqrt{\frac{T}{M}}, \quad (3.20)$$

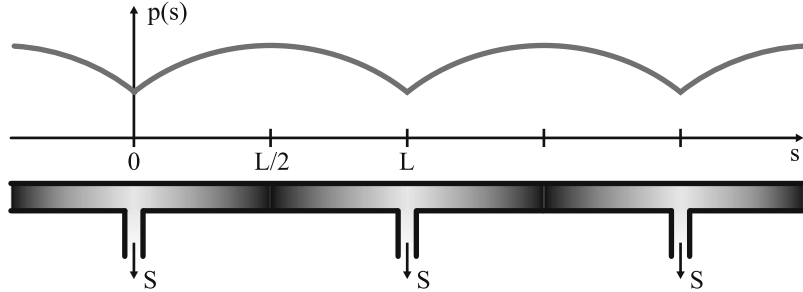


Fig. 3.4. Model of a vacuum chamber with constant cross section and equidistant pumps of equal pumping speed S , leading to a parabolic pressure profile (pressure as a function of position s) between adjacent pumps

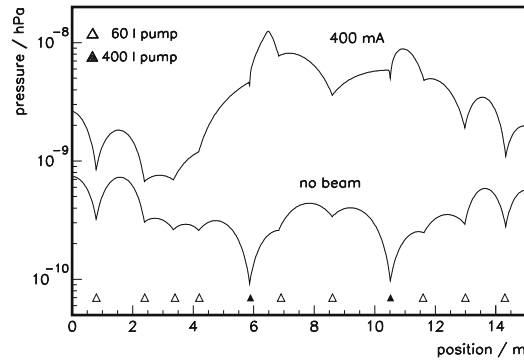


Fig. 3.5. Calculated pressure profile (residual gas pressure as a function of position) for 1/16 of the BESSY II storage ring without beam and with a stored current of 400 mA. The symbols mark the positions of pumps with 60 l s^{-1} and 400 l s^{-1} nominal pumping speed, respectively. With beam, pressure maxima occur where most of the radiation from dipole magnets is locally absorbed

where a and b are the semiaxes in m, T is the temperature in K, and M is the molecular weight. Data on the pumping speed as function of pressure – provided by the manufacturer of the respective pump – can be incorporated in an iterative procedure, repeating the numerical calculation of the pressure profile with updated pumping speed values according to the previously calculated pressure.

Figure 3.5 shows an example of a calculation for BESSY II that was made before the storage ring was actually built. The resulting gas pressure is plotted versus position along a 15 m long section of the ring. The calculation was repeated for different beam currents from zero (purely thermal desorption) up to 400 mA. With beam, pressure maxima occur at the locations of radiation absorbers. It may not be possible to exactly predict the composition and pressure profile of the residual gas, but the calculation is consistent with

typically measured pressure values and demonstrates their strong dependence on the beam current which is also observed during operation of the ring.

3.4 The Touschek Effect

3.4.1 Electron–Electron Scattering

Third-generation synchrotron radiation sources are characterized by a small emittance and thus by bunches of high electron density. Therefore, scattering between electrons within a bunch is highly relevant. The same is true for meson factories where the so-called Touschek effect also dominates the beam lifetime. This effect was for the first time observed at the storage ring *A dA* in Frascati (the world's first e^+e^- storage ring) and was correctly interpreted by Touschek and others as electron–electron scattering [86,87]. The loss mechanism can be understood as follows (see also Fig. 3.6):

In a reference frame moving with the electron bunch in a storage ring, individual electrons oscillate predominantly in horizontal direction due to their betatron motion. The average vertical momentum is typically lower by an order of magnitude according to the square root of the emittance ratio $\sqrt{\varepsilon_y/\varepsilon_x}$, and the longitudinal motion is negligible. In the BESSY II storage ring, for example, the momentum of the horizontal motion is of the order of 100 keV c^{-1} , depending locally on the beta function. Occasionally, scattering between two electrons occurs where the polar distribution of scattering angles in their center-of-momentum system is described by the differential Møller cross section (which holds for identical spin-1/2 particles). The scattering process leads to vertical and longitudinal components of the initially horizontal momenta. The transformation of the longitudinal component into the laboratory system corresponds approximately to a multiplication by the Lorentz factor $\gamma = E/m_e c^2$. At a beam energy of $E = 1700 \text{ MeV}$, for example, γ is 3327. Thus, a longitudinal momentum of, say, 20 keV c^{-1} in the center-of-momentum system, transforms into 67 MeV c^{-1} in the laboratory frame. This is a relative momentum deviation of $\delta_p \approx 0.04$, which clearly exceeds the momentum acceptance of most electron storage rings.

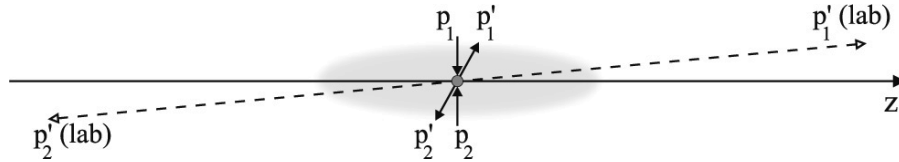


Fig. 3.6. Illustration of the Touschek effect. The initial momenta p_1, p_2 of two electrons in their center-of-momentum system are in good approximation purely transverse, whereas the momenta p'_1, p'_2 after scattering have a longitudinal component which is boosted by the Lorentz factor γ when transformed into the laboratory frame

3.4.2 Analytical Approaches

For an estimate of the “Touschek lifetime”, it is usually sufficient to consider a purely horizontal betatron oscillation in nonrelativistic approximation. The calculation is reproduced in Appendix F and yields an inverse lifetime (relative loss rate) of

$$\frac{1}{\tau} = -\frac{Nr_e^2 c}{8\pi\sigma_x\sigma_y\sigma_z\gamma^2(\Delta p/p)^3}\mathcal{C}\left(\frac{(\Delta p/p)^2}{\gamma^2\sigma_{x'}^2}\right), \quad (3.21)$$

where N is the number of electrons in a bunch, r_e is the classical electron radius, and σ_k is the standard deviation of the electron distribution in the coordinate labeled by $k = x, y, z$, or x' . The momentum acceptance $\Delta p/p$ may be given by the rf system or by a transverse aperture limitation. The function $\mathcal{C}$ is evaluated by numerical methods. For third-generation synchrotron radiation sources, its value is in the region of a broad maximum with $\mathcal{C} \approx 0.3$. It may be smaller for future facilities with significantly lower emittance.

Since the width of the electron distribution in the coordinates x , x' , and y varies with the beta function, it is more accurate to compute the total loss rate by summing the loss rates obtained at many points along the ring circumference, rather than to assume average values. This task is greatly facilitated by computer codes like ZAP [88].

The original analytical description given in [87] was later refined in many ways, sometimes accompanied by simplifications in other aspects:

Dispersion is explicitly accounted for in [89] and [90]. The vertical betatron motion for round and elliptical beams is included in [91] and [92].

It is usually assumed that only the rf bucket and the horizontal aperture define the momentum acceptance. However, in synchrotron radiation sources with extremely narrow undulator chambers, the vertical aperture can also be relevant, as shown by [93] and [94].

All calculations mentioned above treat the betatron motion in nonrelativistic approximation. An ultrarelativistic approximation is given in [95], a generalization to arbitrary kinetic energy in [96]. In [97], a general expression is derived that can be applied to particular cases like flat and round beams or the nonrelativistic and ultra-relativistic limit.

So far, a random orientation of the electron polarization was assumed. If, however, electron spins become aligned due to synchrotron radiation emission, known as the Sokolow–Ternow effect [98], the scattering cross section is reduced and the beam lifetime increases. The time to build up polarization depends on the bending radius ρ and the beam energy E and is proportional to ρ^3/E^5 . In BESSY II, for example, it amounts to about 20 minutes. The lifetime difference due to polarization is of practical relevance since resonant depolarization of the beam by an external rf field of known frequency can be used to precisely determine the beam energy. This method was established for metrological applications at the BESSY (I) ring [100] and is now routinely employed at BESSY II and other facilities (e.g., [101]). It is important to know whether the lifetime difference is significant enough for direct observation or

whether more elaborate methods (e.g., observing electromagnetic showers at particular locations) are required to detect small changes of the loss rate. Analytical expressions for the Touschek lifetime of a polarized beam are given in [99] for the nonrelativistic and ultrarelativistic approximation.

3.4.3 Monte Carlo Simulation

A completely different approach to calculate the Touschek lifetime is a Monte Carlo simulation of individual scattering events [102]. This way, effects that cannot be accessed analytically can be accounted for and the accuracy of the resulting loss rate depends mainly on the number of computed events. Thus, this approach allows to test the approximations made in analytical calculations. In the simulation, the following 12 parameters are chosen randomly on an event-by-event basis:

- the longitudinal scattering position s along the circumference
- three spatial coordinates (x, y, z) , common to both interacting electrons
- three momentum coordinates (x', y', z') for each of the two electrons
- the polar scattering angle Θ and the azimuthal angle Φ .

With n events and a total number of N electrons per bunch, the relative loss rate is given by

$$\frac{\dot{N}}{N} = -\frac{\Delta V}{n} \frac{N}{\gamma C} \sum_{k=1}^n 2 v_k \sigma_k \rho_k^{(1)} \rho_k^{(2)} \sin \Theta_k \Big|_{\substack{\delta p_i > \Delta p/p \\ \delta x_i > \Delta x}} . \quad (3.22)$$

Here, ΔV is the 12-dimensional volume of the parameter space from which the random variables are chosen, γ is the Lorentz factor, and C is the circumference of the storage ring. All quantities labeled by k are different for each event: the relative velocity v_k of the electrons, the scattering angle Θ_k , the angle-dependent cross section σ_k , and the densities $\rho_k^{(1)}$ and $\rho_k^{(2)}$ at the phase space locations of the two electrons. Due to the large number of parameters, a Monte Carlo calculation is much faster and more precise than other numerical integration methods. For each electron, the momentum deviation δp in the laboratory frame and the maximum horizontal displacement δx are computed. According to (3.5), the displacement δx can be a combination of the dispersive orbit and a betatron oscillation excited by a momentum change in a dispersive region. If the momentum acceptance $\Delta p/p$ or the horizontal aperture Δx are exceeded, the event contributes to the sum in (3.22). Calculations of this kind were performed during the design phase of BESSY II and are described in [103]. Here, only a few results are cited that apply to BESSY II and similar synchrotron radiation sources (see also Fig. 3.7):

- The relativistic treatment of Møller scattering leads to lifetime values that are 10% shorter compared to nonrelativistic calculations.

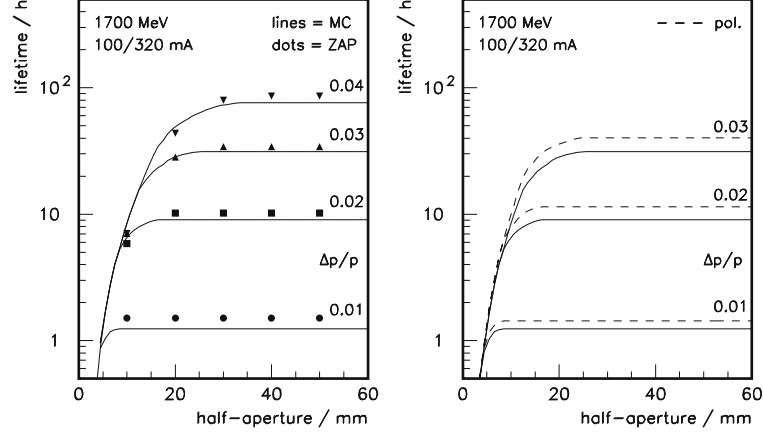


Fig. 3.7. Beam lifetime in BESSY II for 100 mA in 320 bunches due to Touschek scattering. The lifetime is plotted as function of the horizontal aperture and for different values of the rf momentum acceptance $\Delta p/p$. *Left:* Results of the relativistic Monte Carlo simulation (*solid lines*) and nonrelativistic calculations (*dots*) using the analytical code ZAP [88]. *Right:* Monte Carlo calculation for unpolarized (*solid lines*) and for polarized electrons (*dashed lines*)

- Nonlinearities in the betatron motion reduce the beam lifetime on the order of 10% if $\Delta p/p$ is given by horizontal aperture limitations, rather than by the rf momentum acceptance.
- Of all scattering events contributing to the loss rate, 80% take place in a dispersion-free region. The effect of additional betatron oscillations due to non-zero dispersion is less important in the case of BESSY II, since the electron density in dispersive regions is small.
- The lifetime of a polarized electron beam is 20% larger than that of an unpolarized beam. This difference is easily detectable and can be used to measure the beam energy via resonant depolarization.

As the simulation reveals, the approximate expression (3.21) is sufficiently accurate for many applications. However, it tends to overestimate the beam lifetime and provides no clue on polarized beams.

3.5 The Total Beam Lifetime

The inverse of the total beam lifetime is given by the sum of all relative loss rates according to (3.4). As a rule, all contributions are current-dependent, since the electron density as well as the residual gas pressure, increases with current.

When designing a new facility, only a rough estimate of the expected beam lifetime and its respective contributions can be given, since many parameters

contribute to the result. Some of these parameters depend on how carefully a storage ring is designed and constructed:

- Pressure and composition of the residual gas depend on the pretreatment of the vacuum components.
- The electron density within a bunch depends on its size.
 - The vertical emittance is mostly given by field errors and misalignment of the magnetic components.
 - The bunch length depends on the bunch current and the occurrence of longitudinal mode coupling (see Sect. 2.4.3) which is a consequence of the broad-band impedance of the storage ring.
- The dynamic aperture can be limited by nonlinear contributions to the magnetic guide field.

Ironically, some of the efforts made to improve a machine – minimization of field errors and impedance – tend to make the beam lifetime worse. Other parameters can be improved by investing more money:

- Vacuum chambers with a larger cross section represent a larger physical aperture and have a better molecular conductance. The vacuum chamber size is, among other factors, limited by the surrounding magnetic gaps.
- More pumps with larger pumping speed improve the vacuum. However, the molecular conductance always remains a limiting factor.
- A vacuum system comprising an antechamber can be advantageous for the vacuum at the position of the beam (see, e.g., the undulator chamber in Fig. B.4).
- Increasing the rf voltage improves the momentum acceptance but also reduces the bunch length (this is only true below the longitudinal mode coupling threshold). Consequently, once the transverse aperture limits the momentum acceptance, increasing the rf voltage actually reduces the beam lifetime.

In addition, the beam lifetime depends on the mode of operation:

- Synchrotron radiation desorption is a function of the stored beam current and of the integrated radiation dose (expressed as current $\times$ operation time), whereas the electron–electron scattering rate depends on the current per bunch.
- Elastic scattering on the residual gas and Touschek scattering are functions of the beam energy (see Fig. 3.8).
- The magnetic fields of undulators may influence the dynamic aperture and shift the betatron tune which could impair the beam lifetime. Furthermore, in-vacuum undulators or undulators with a flexible vacuum chamber change the physical aperture.

In view of the multitude of parameters and dependencies, an estimate of the expected beam lifetime can only be based on experience and plausible assumptions. The left part of Fig. 3.8 shows an estimate for BESSY II, made

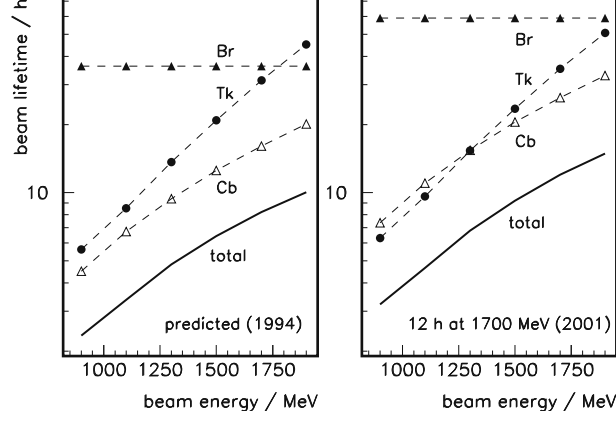


Fig. 3.8. Contributions to the beam lifetime in the BESSY II storage ring (Cb = Coulomb scattering, Br = Bremsstrahlung, Tk = Touschek effect) and total lifetime as a function of beam energy for a beam current of 100 mA. *Left:* prediction from 1994, with 8.2 h at a beam energy of 1,700 MeV. *Right:* Touschek lifetime corrected for 360 bunches and gas scattering contributions adjusted to match the observed lifetime of 12 h

prior to the construction of the facility. The total lifetime and its contributions are plotted versus beam energy in the range of 900–1900 MeV, for which the machine was designed. After 3 years of operation, the achieved beam lifetime at 1,700 MeV was by a factor 1.5 larger than predicted (right part of Fig. 3.8).

3.6 Experimental Determination of Loss Rates

When a new storage ring is being commissioned, it is of more than just academic interest to compare the observed beam lifetime to the predictions and to determine its various contributions:

- Is the beam lifetime unexpectedly low? This may hint a problem, e.g. an obstacle in the vacuum chamber.
- Which lifetime can be expected once the vacuum has been improved?
- How will the lifetime change under modification of the physical aperture e.g., by installing a smaller undulator chamber?
- Can more rf power improve the beam lifetime?

The beam current is continuously monitored e.g. using a current transformer, and the beam lifetime is determined by differential current measurements:

$$\tau = -\frac{I(t)}{\dot{I}(t)} \approx \frac{I(0) + I(\Delta t)}{2} \frac{\Delta t}{I(0) - I(\Delta t)}. \quad (3.23)$$

Due to random fluctuations in the current reading, the determination of a lifetime of the order of 10 h with 5% accuracy typically requires to average over a time interval of 1 minute.

The contributions of different loss mechanisms can be extracted by variation of the parameters which influence the respective loss rates in a characteristic way. The beam current is not a good choice for this purpose, since gas scattering losses show a roughly linear dependence on the beam current (due to synchrotron radiation desorption), and so does the Touschek effect (due to its dependence on the electron density).

In principle, a variation of the bunch current while keeping the stored beam current constant – i.e., a comparison of different fill patterns – allows to disentangle gas scattering from the Touschek effect. More reliable and reproducible are measurements under variation of one parameter in a short time interval to prevent other parameters from drifting away. Variation in many small steps reveals the functional dependence of the lifetime on the respective parameter. Well suited for this purpose is a variation of the momentum acceptance by changing the rf voltage, or a reduction of the aperture by using a scraper. Measurements of this kind were performed at different third-generation synchrotron radiation sources, see [104–106].

The measurements discussed in the following [107] were performed at BESSY II in its first year of operation, when the residual gas pressure was relatively high and the beam lifetime was dominated by gas scattering. It was nevertheless possible to identify the Touschek effect and to predict the lifetime for better vacuum conditions.

3.6.1 Variation of the Momentum Acceptance

The relationship between beam lifetime and momentum acceptance is very different for the Touschek effect and for gas scattering. The momentum acceptance $\Delta p/p$ is controlled by changing the rf voltage U_o . According to (A.41) it is $\sim \sqrt{U_o}$. Furthermore, the lifetime is influenced by the bunch length $\sigma_\tau \sim \sqrt{1/U_o}$, provided the bunch current is below the longitudinal mode coupling threshold. For the gas scattering contribution, additional assumptions must be made regarding the residual gas composition and the pressure profile in order to disentangle bremsstrahlung from elastic scattering, the latter being independent of the momentum acceptance.

In Fig. 3.9, the relative lifetime (normalized to the lifetime at the maximum rf voltage) is plotted against the momentum acceptance given by the rf voltage. The data for various values of beam current (top) and for two different fill patterns (bottom) follow the trend shown by the lines drawn for different ratios of the contributing processes, thus confirming the assumptions made in the model. Since the lifetime measurement is not very accurate at low beam current (i.e., large beam lifetime), the function must be sampled over a sufficiently wide range of $\Delta p/p$. Apparently, the momentum acceptance is determined by the rf bucket size and not by a transverse aperture, since the maximum rf voltage yields the maximum lifetime. If the aperture were the

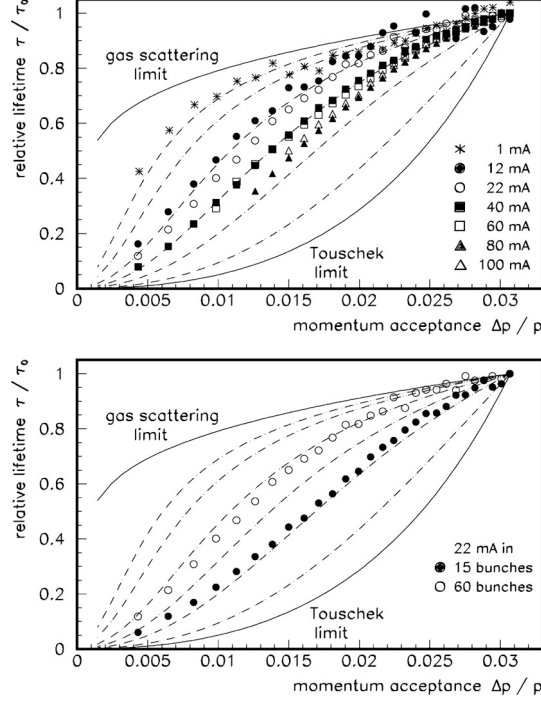


Fig. 3.9. Relative beam lifetime of the BESSY II storage ring as a function of momentum acceptance $\Delta p/p$ for beam currents between 1 and 100 mA (*top*) and for 22 mA with 15 and 60 bunches, respectively (*bottom*). In this experiment, the rf voltage was varied, and the momentum acceptance was calculated using (A.41)

limiting factor, the lifetime would decrease above a certain rf voltage due to bunch length reduction, which increases the electron scattering rate.

The resulting loss rates for gas scattering and Touschek effect are plotted in Fig. 3.10 as function of the beam current and show the expected behavior. The Touschek effect vanishes at low current while the gas scattering rate approaches a finite value. At 100 mA, the Touschek rate was presumably influenced by a multi-bunch instability – at the time of the experiment, the feedback systems (see Sect. 4.2) were not yet installed. This measurement demonstrates the sensitivity of the method to extract even small contributions of the Touschek effect to the beam lifetime.

3.6.2 Variation of the Vertical Aperture

A vertical obstacle – here, a scraper that can be precisely moved towards the beam – acts predominantly on electrons that were elastically scattered

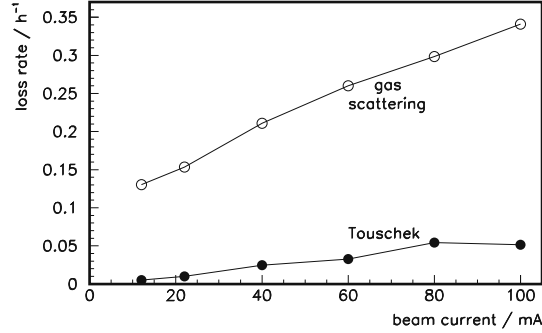


Fig. 3.10. Loss rates for gas scattering and Touschek effect as function of the beam current, based on the data shown in Fig. 3.9

on residual gas nuclei. Quantum lifetime effects and losses of off-momentum particles due to spurious vertical dispersion could not be identified in the measurements discussed below, since these effects occur at a scraper position where the beam is immediately lost. Figure 3.11 shows the relative lifetime as function of the distance between a vertical scraper and the beam. Elastic scattering alone would lead to a quadratic dependence on the scraper position (dashed line) until the scraper enters the shadow of another obstacle, e.g., the vacuum chamber itself. Deviations from the parabolic curve reveal the existence of other loss mechanisms. The dependence of the data on the beam current (top part of the figure) is due to the Touschek effect, since the ratio of bremsstrahlung and elastic scattering does not depend on the current. Even more pronounced is the dependence on momentum acceptance (bottom part of Fig. 3.11) which accounts for the Touschek effect as well as the bremsstrahlung contribution.

While fitting the curves to the data, the ratio between bremsstrahlung and elastic scattering was kept constant, while the Touschek contribution was varied. The satisfactory correspondence to the data confirms again the assumptions made on the gas composition and pressure profile.

These measurements were performed at an early stage of BESSY II operation. After 3 years of operation, the main contributor to the beam lifetime at typical beam currents of 150–250 mA was the Touschek effect. This is in accordance with the experience at other third-generation synchrotron radiation sources, e.g., [105,106]. On the other hand, the tendency towards ever smaller undulator chambers makes elastic (Coulomb) scattering relevant again. It should be kept in mind, that the residual gas contribution improves with operation time, while the Touschek effect constitutes a fundamental lifetime limitation for a given electron density.

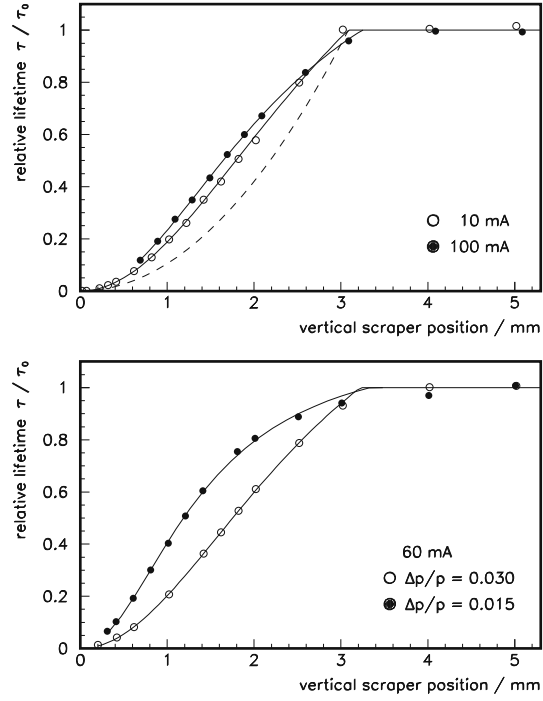


Fig. 3.11. Relative beam lifetime of the BESSY II storage ring as a function of the position of a vertical scraper, shown for a beam current of 10 and 100 mA (*top*) and for 60 mA with two values of momentum acceptance (*bottom*). The *dashed line* represents the expectation for purely elastic residual gas scattering

Measures Against Collective Effects

4.1 Overview

Synchrotron radiation sources of the third generation are extremely sensitive to collective effects. Due to their small horizontal beam emittance of a few 10^{-9} rad m, transverse instabilities of even small amplitude cause a noticeable increase of the average beam size and divergence. Furthermore, a small beam emittance implies a high electron density and the beam lifetime is usually dominated by electron–electron scattering. Another feature of these facilities is the extensive use of undulators in order to generate intense radiation within a small photon energy interval. Longitudinal instabilities increase the linewidths in the undulator spectrum and reduce the peak intensity.

The meson factories face similar problems. Their beam emittance may be somewhat larger – of the order of 10^{-8} rad m – but the beta function at the e^+e^- interaction point can be very small (~ 1 cm) and transverse oscillations or the longitudinal displacement of bunches would seriously impair the luminosity. The electron density in the bunches is comparable to that of synchrotron radiation sources.

It is thus not surprising that some measures against collective effects were originally developed in view of a future meson factory, and synchrotron radiation sources offered the opportunity to test them. This is particularly true for feedback systems designed to detect and damp multi-bunch instabilities, but also for design issues like impedance minimization strategies and the optimization of rf cavities.

4.1.1 Measures Against Collective Instabilities

Minimization of the Vacuum Chamber Impedance

The impedance of the surroundings of an electron beam is determined by the geometric boundary conditions and by the electric conductivity of the vacuum chamber wall. In a storage ring, there are many places where the vacuum

chamber is interrupted or changes its cross section. Generally, the impedance introduced by these discontinuities of the chamber can be minimized using numerical methods to evaluate the wake function or impedance for alternative designs. Intuitive criteria may provide some guidance:

- Interruptions of a smooth chamber wall, e.g., at flanges or bellows must be bridged with metallic material.
- Unavoidable changes of the chamber cross section, e.g., tapers at either end of an undulator, should be designed with a gentle slope (typically, 10° or less).
- Holes or slits in the chamber, e.g. vacuum pumping ports, synchrotron radiation ports, or the beam injection channel, should be designed such that a smooth wall is closer to the beam than the interruption.

When objects like rf cavities, diagnostics devices, or kickers are explicitly meant to interact with the beam, the impedance is necessarily large to make these devices efficient. The largest of these unavoidable disturbances comes from the accelerating rf cavities which have numerous higher-order modes (HOMs) in addition to the desired fundamental mode. The DORIS-type cavities currently employed at BESSY II are equipped with a combination of broadband absorbers and frequency-selective antennas to extract particular modes [303]. Damping antennas may also be useful in other devices such as the transverse kicker described in Sect. 4.4 or in higher-harmonic cavities (Sect. 4.6).

New developments of rf cavities aim at minimizing HOMs and maximizing the fundamental mode by an appropriate design. Two generic cavity shapes are shown in Fig. 4.1. The HOM content of a cavity with a curved (“bell-shaped”) longitudinal cross section, e.g., [108, 109], is generally lower than that of a cylindrical (“pillbox”) design. However, as discussed in [110], the difference becomes negligible once the unwanted modes are extracted by waveguides. Figure 4.2 shows calculated and measured HOMs with moderate impedance for a new 500-MHz pillbox cavity designed at BESSY [111]. While the HOM content is slightly larger than that of existing bell-shaped

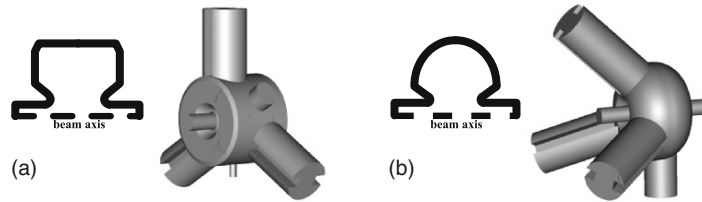


Fig. 4.1. Example of a pillbox cavity (a) and of a bell-shaped cavity (b), both with “nose cones” near the beam axis and equipped with damping waveguides. For better coupling to TM_{0mn} modes, the waveguides are longitudinally displaced in the pillbox case and inclined for bell-shaped cavities (courtesy F. Marhauser and E. Weihreter)

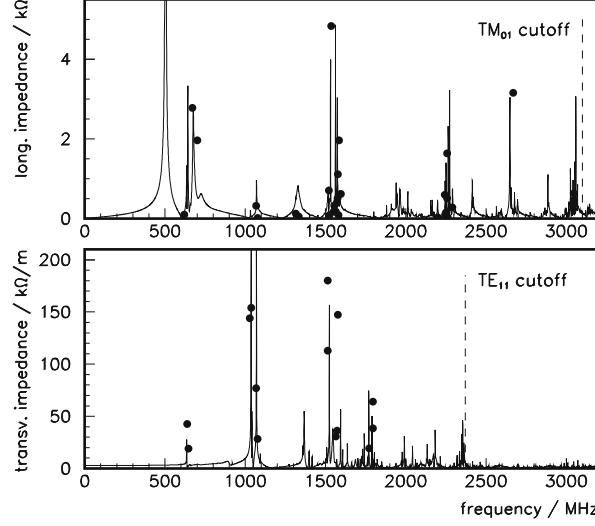


Fig. 4.2. Longitudinal (*top*) and transverse (*bottom*) HOM impedance of a HOM-damped cavity [111]. The fundamental mode is at 500 MHz, longitudinal and transverse cutoff frequencies of a beam pipe with 74 mm diameter are indicated by *dashed lines*. The *solid line* is derived from a 3d time-domain calculation using the MAFIA package [31], the *dots* are values measured at a high-power prototype (courtesy F. Marhauser and E. Weihreter)

Table 4.1. Electrical conductivity σ and specific resistance $\varrho = 1/\sigma$ at room temperature of common vacuum vessel materials

material	σ ($\Omega^{-1} \text{ m}^{-1}$)	ϱ ($\Omega \text{ m}$)
stainless steel	1.4×10^6	7.1×10^{-7}
aluminium	3.6×10^7	2.8×10^{-8}
copper	5.9×10^7	1.7×10^{-8}

cavities, the cylindrical cavity with three damping waveguides has a simpler and cost-saving mechanical design and a smaller insertion length.

The minimization of the resistive-wall impedance requires a large chamber cross section and good electrical conductivity σ . If the cross section has to be small, e.g., in undulators to produce a large magnetic field on the beam axis, the conductivity of the material is of prime importance. The resistive-wall impedance is proportional to $1/\sqrt{\sigma}$ – see (2.57) and (2.58). For the aluminium undulator chambers used in the straight sections of BESSY II, this means a reduction by a factor 5 compared to stainless steel used elsewhere in the ring (see Table 4.1).

Minimization of Instability Growth Rates

For a given chamber impedance, there are limited possibilities to improve on instability growth rates. Parameters like circumference, beam energy, synchrotron radiation losses, synchrotron frequency, and betatron tune influence the growth rates but their values are usually dictated by other necessities. Generally, operation at higher beam energy tends to reduce instabilities, and in particular cases such as the resistive-wall effect, the choice of the tune can play a role (see the left part of Fig. 2.17). An efficient way to suppress transverse instabilities is to shift the beam spectrum by increasing the chromaticity (see Fig. 2.22). However, since this is accomplished by increasing sextupole fields in dispersive regions – see (A.21) – the additional nonlinearity tends to reduce the dynamic aperture and thus the beam lifetime.

The effect of narrow-band impedances can be minimized by tuning their frequency as to minimize their overlap with the beam spectrum. For rf cavities, this may be done by using a plunger or by mechanically deforming the whole structure. Another way of deforming a cavity is to change its temperature, e.g., by controlling the flow of cooling water [112]. In each case, it is important to have control over more than one degree of freedom in order to shift HOMs while retaining the frequency of the fundamental mode [113].

Damping of Instabilities

The energy radiated per turn in dipole magnets, wigglers and undulators determines the synchrotron radiation damping rate (see Sect. 2.5.2). Generally, it is of the order of 100 s^{-1} and is often not sufficient to suppress instabilities.

Landau damping is provided by the nonlinear components of the potentials defined by the magnetic guide fields and by the rf voltage (see Sect. 2.5.3). Transversely, they are introduced by sextupole magnets, longitudinally by the sinusoidal shape of the rf voltage. In order to increase the damping effect, additional nonlinearity can be created, particularly in the longitudinal case by adding an rf voltage with an integer multiple of the fundamental rf frequency (“Landau cavities”, see, e.g., [114]).

Damping can also be provided by deliberate excitation of the beam. Allowing a longitudinal instability with small amplitude, for example, can prevent transverse oscillation modes. If such an instability cannot be controlled reliably, external excitation is another possibility, e.g., by phase modulation of the rf voltage [115]. Also, the fill pattern, i.e., the distribution of electrons among the rf buckets, has been found to influence the growth rate of instabilities [116–118].

The methods and procedures mentioned so far can help to increase the current thresholds for the occurrence of instabilities. However, most synchrotron radiation sources show unwanted oscillations at typical beam currents of 100–400 mA [119]. An initially stable machine can develop instabilities through later modifications or by raising the routinely stored current. Therefore, active damping via feedback must be included in the catalog of measures against instabilities.

Investing into feedback systems does not make the measures discussed so far less important. Quite the contrary: Feedback systems are only useful in a narrow range of growth rates. For a growth rate below the radiation damping rate, a feedback system would not be necessary. But if the growth rate is only two orders of magnitude larger, a feedback system may not be able to do its job. The availability of suitable amplifiers defines a limit on the kicker voltage that acts on the beam. This voltage depends as $\sim \sqrt{N_K \cdot P}$ on the number of installed kickers N_K and on the power P which is approximately proportional to the amplifier costs. It is therefore important to keep growth rates on a low level, even if the use of active feedback is intended. Feedback systems will be discussed in Sect. 4.2.

4.1.2 Measures to Improve the Beam Lifetime

Optimization of Aperture and Electron Optics

Residual gas scattering and the Touschek effect are determined either by a transverse aperture limitation or by the momentum acceptance of the rf system. It makes sense to adjust these limits to each other. A physical aperture that by far exceeds the dynamic aperture offers no advantage. On the other hand, it is not useful to increase the rf voltage if particles within the momentum acceptance are lost at a transverse aperture.

Increasing the momentum acceptance or aperture does not reduce the scattering rate itself but the total cross-section integrated over scattering angles or momenta that lead to particle loss. In principle, optimization of the electron optics can reduce both the total cross section and the scattering rate. Although the optical functions are usually determined by other criteria, the following points may be considered in view of the beam lifetime:

- By combining (3.7) and (3.8) for the case of elastic scattering, the amplitude of a betatron oscillation caused by a given scattering angle $\delta y'$ is given by

$$\delta y = \delta y' \sqrt{\tilde{\beta}_y \beta_y}, \quad (4.1)$$

where the tilde indicates the beta function at the scattering position. Thus, elastic scattering is more harmful where the beta function is large.

- From (4.1) follows that decreasing the half-aperture R of the vacuum chamber at a position with beta function $\beta(s)$ reduces the lifetime, if the ratio $R^2/\beta(s)$ is smaller than at other locations of the storage ring.
- Equations (3.5) and (3.6) show the influence of optical functions on the horizontal displacement due to electron–electron scattering or bremsstrahlung. A large betatron amplitude is caused by large dispersion D combined with a small beta function at the location of a momentum change (or by a large value of $D' = dD/ds$ combined with a large beta function).

- The function $\mathcal{C}(\xi)$ in the description of the Touschek scattering rate (3.21) depends on the horizontal emittance and on the beta function, as shown in Appendix F. For future synchrotron radiation sources with smaller emittance ε_x , the parameter

$$\xi \equiv \frac{(\Delta p/p)^2 \beta_x}{\gamma^2 \varepsilon_x} \quad (4.2)$$

may be larger and the Touschek rate may be smaller compared to present-day facilities with similar momentum acceptance $\Delta p/p$ and Lorentz factor γ .

Optimization of the Vacuum

Measures to improve the storage ring vacuum were already discussed in Sect. 3.3.2. At an electron storage ring, the vacuum without beam should reach its optimum level after a radiation dose (in terms of current $\times$ operation time) of the order of 1000 A h. The pressure rise due to synchrotron radiation desorption also decreases with the delivered dose.

Minimization of Losses Due to Electron–Electron Scattering

Apart from the spin-dependence of Møller scattering (see Sect. 3.4.3), the scattering rate of electrons within a bunch can only be reduced by decreasing the bunch density. It is useful to distribute the stored electrons evenly over many rf buckets and to minimize the gap in the fill pattern. Since this gap is meant to counteract ion effects (see Appendix G), its minimization requires good vacuum.

Occasionally, the electron density is reduced at the expense of a transverse increase of the beam size, e.g., by increasing the vertical emittance with tilted (“skew”) quadrupole magnets or by deliberate beam excitation. Moderate excitation of the beam, e.g., by phase modulation of the rf voltage [115], reduces the electron density. These measures, of course, reduce the brilliance of synchrotron radiation and lessen the merits of a third-generation source, but good beam lifetime may be more important in a particular case.

Another approach that does not reduce the brilliance is to increase the bunch size longitudinally using higher-harmonic cavities. As the name implies, their resonant frequency is close to an integer multiple of the fundamental rf frequency and their effect is to flatten the rf potential, thus increasing the bunch length without increasing the energy spread significantly – contrary to “turbulent bunch lengthening”, where the energy spread is increased as well. What makes this approach attractive is that most experiments at synchrotron radiation sources are not very sensitive to the bunch length. Higher-harmonic cavities are discussed in Sect. 4.6.

4.2 Active Damping of Instabilities

4.2.1 Feedback Control

Active damping of multi-bunch instabilities requires the use of feedback control. A “controller” is any arrangement of physical components (like mechanical devices, analog circuitry, or computers) interacting in such a way that a value of a system parameter equals a reference value. Feedback causes the action of a controller to depend on its output (see, e.g., [120, 121]).

The oscillation of a bunch with position $x(t)$ can be modeled as a forced oscillation of a damped harmonic oscillator:

$$\ddot{x}(t) + 2\alpha\dot{x}(t) + \omega^2x = F(t)/m, \quad (4.3)$$

where ω is the resonant frequency and α is the damping constant. The force term $F(t)/m$ is assumed to excite oscillations at a frequency close to ω . The measured quantity is usually the position x (although, in principle, a measurement of $\dot{x}$ is also possible). As can be seen from (4.3), feeding the quantity x back onto the beam would change the oscillation frequency but would not influence the damping term. This is called *reactive feedback* and can be useful for single-bunch instabilities in order to avoid mode coupling. The case of multi-bunch instabilities, however, requires a *resistive feedback* acting on the damping term $\sim \dot{x}$. Instead of measuring $\dot{x}$ directly, it can be deduced from measurements of

$$x(t) = A \cos(\omega t) \quad (4.4)$$

and, provided the frequency is known, by introducing a delay of $\Delta t = \pi/2\omega$ that corresponds to a 90° phase shift:

$$\dot{x}(t) = -A\omega \sin(\omega t) = -\omega x(t - \Delta t). \quad (4.5)$$

Alternatively, two subsequent measurements of x yield

$$\dot{x}(t) \approx \frac{x(t) - x(t - T_0)}{T_0}, \quad (4.6)$$

where T_0 is the time between the measurements. If one detector is used, T_0 is the revolution time or an integer multiple thereof.

4.2.2 Characteristics of Feedback Systems

A feedback system generally comprises a detector, a signal processing unit, an rf amplifier, and a kicker. Taking a closer look at these ingredients, there are some significant differences (e.g., [122, 123]):

Longitudinal and Transverse Feedback Systems

Longitudinal and transverse bunch motion requires different detectors and kickers. The bunch position is usually measured using electrodes that couple to the electric field of the beam. Transversely, the difference signal between opposite electrodes (top/bottom or left/right) is required. Longitudinally, the signal of opposite electrodes is summed, whereas the signal of a single electrode would be disturbed by transverse motion. Stripline structures are well established as transverse kickers, while longitudinal kickers resemble accelerating structures like cavities or drift tubes (see Appendix E).

In both cases, broad-band power amplifiers in solid-state technology are suitable. They are relatively robust against reflected power or power generated in the kicker by the beam. Travelling-wave tubes are also in use.

The very different frequency regimes of synchrotron and betatron motion imply differences in signal processing. The required delays, for example, are in the transverse case of the order of one revolution time, and for longitudinal motion of the order of 100 turns.

“Bunch-by-Bunch” and “Mode-by-Mode” Systems

A feedback system that detects and corrects all bunches independently is called a “bunch-by-bunch” system. The oscillation mode that emerges from the pattern given by all bunches is not considered explicitly. Naturally, such a feedback system is very flexible, but involves considerable electronic complexity due to the fact that bunches in a third-generation radiation source arrive at the rate of the rf frequency f_{rf} , which is typically 500 MHz. The required bandwidth to be covered by each component between detector and kicker corresponds to half the rf frequency (typically 250 MHz) since the highest occurring frequency is given by the opposite-phase oscillation of adjacent bunches. In the frequency domain, this corresponds to the statement made in Sect. 2.2 that all modes are contained in a frequency interval of width $f_{\text{rf}}/2$ which starts or ends at an integer multiple of f_{rf} .

A “mode-by-mode” system, in contrast, detects only oscillations at particular mode frequencies and acts only with these frequencies on the beam. This can be useful if only particular modes have to be damped with moderate effort. An example is the resistive-wall effect that shows up only around the lowest revolution harmonics. A mode-by-mode system that has a channel for each possible mode of oscillation is equivalent to a bunch-by-bunch system.

Analog and Digital Signal Processing

Signal processing involves tasks like filtering, delaying and amplifying a signal, or mixing several signals in a particular way. A feedback system may process analog input signals directly, or it may digitize them, then process and store the digital data, and finally convert them back to analog signals. Analog

systems are robust and can be built with moderate effort. On the other hand, digital systems offer a couple of advantages:

- Processing algorithms and parameters can be defined by software, which implies a large degree of flexibility.
- Digitized data can be stored and analyzed off-line which makes a digital system a valuable diagnostics tool. It allows measurements that are not easily performed by other means.
- Large delay times – as required in the longitudinal case due to the low synchrotron oscillation frequency – can be realized easily with digital data, whereas analog techniques, e.g., using coaxial delay lines or acoustic couplers are prone to losses and unwanted dispersion effects.

A digitization rate of 500 MHz is still close to the limits of commercially available technology. Care must be taken to keep the processing time within reasonable limits, e.g., by rejecting redundant information (downsampling) and by parallel use of multiple processors.

The distinction between digital and analog systems generally refers to the implementation of the processing algorithm. On the other hand, the transverse feedback system (TFB) of the B-meson factory PEP II at SLAC (Stanford, USA) [124] is an example of an analog system that digitizes signals just to solve the problem of long delay times.

Algorithms for Feedback Control

No matter what the architecture of a bunch-by-bunch feedback system is, the signal flow is always discontinuous. The position of a bunch is not sampled continuously but at discrete time steps. If the correction signal y_j is formed using a finite number M of past measurements x_{j-k} , the algorithm is called an M-tap-FIR (*finite impulse response*) filter:

$$y_j = \sum_{k=0}^{M-1} c_k x_{j-k} . \quad (4.7)$$

Each term of the sum contains a coefficient c_k and these coefficients determine the properties of the filter. After a single pulse at the input, the output will be zero after a finite number of time steps – hence its name. In contrast, the correction signal of an IIR (*infinite impulse response*) filter is formed by M past measurements and N past output signals:

$$y_j = \sum_{k=0}^{M-1} c_k x_{j-k} + \sum_{k=1}^N d_k y_{j-k} . \quad (4.8)$$

In this case, the response to a single input pulse can remain non-zero for ever. In comparison to FIR filters, IIR filters can perform a given task with less coefficients, but their design is more complex and they tend to be less stable. For both classes of filters, the required operations of delay, multiplication and summation can be implemented in digital form or as analog circuitry.

Another way to classify processing algorithms (more common in analog control theory) is to distinguish between controllers that produce a correction signal proportional to the input (P), proportional to the integral of the input (I), or proportional to the time derivative of the input (D). Usually, combined PID-controllers are employed and the respective functions contribute with different weight, depending on the task. For a discontinuous signal flow, the basic PID operations can be approximated by

$$\begin{aligned} \text{P: } y_j &= c x_j \\ \text{I: } y_j &= c x_j + c x_{j-1} + c x_{j-2} + \cdots = c x_j + y_{j-1} \\ \text{D: } y_j &= c x_j - c x_{j-1}. \end{aligned} \tag{4.9}$$

In order to damp multi-bunch oscillations where the measured signal corresponds to the bunch position, a D-controller is required to increase the damping term of (4.3). However, the control algorithms actually in use not always fit into the PID concept.

4.3 Example of a Longitudinal Feedback System

In this section, a longitudinal bunch-by-bunch feedback system based on digital signal processors (DSPs) will be introduced. This system was designed for application at the meson factories PEP II (Stanford, USA) [125,126] and DAΦNE (Frascati, Italy) [127]. A “quick prototype” [128] was installed in 1994 at the synchrotron radiation source ALS (Berkeley, USA) and later replaced by a more mature version. Its commissioning at the meson factories took place during 1997 and 1998. Presently, additional installations exist at BESSY II [129,130], at the PLS (Pohang, Korea) [131], and at SPEAR (Stanford, USA) [132]. These installations differ in some details. In the following, the system commissioned in 1999 at BESSY II will be described.

4.3.1 System Overview

Figure 4.3 shows a schematic overview of the longitudinal feedback system (LFB) at the BESSY II storage ring. Most of the electronic components are contained in a VXI crate, whereas the DSPs are mounted on VME boards comprising four processors each (electronic standards like VXI and VME are explained, e.g., in [133]).

Detection of the Longitudinal Bunch Position

The sum signal of four button-shaped electrodes of a beam position monitor passes a comb filter which produces a four-period pulse train for each bunch passing the electrodes. The frequency of the pulse train is 3 GHz, given by the geometry of the comb filter, and the amplitude is proportional to the current

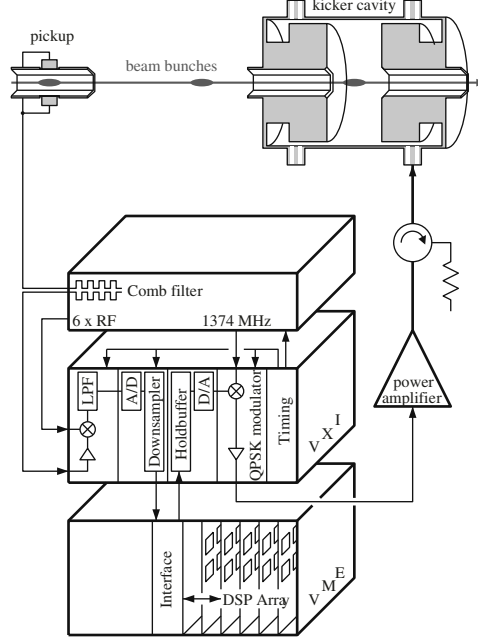


Fig. 4.3. Schematic view of the LFB at the BESSY II storage ring comprising a pickup, analog and digital electronics as described in Sect. 4.3, a power amplifier, and a kicker cavity. A circulator protects the amplifier against reflected or beam-induced power from the kicker

I of the bunch. Multiplication with a 3-GHz-reference signal, derived from the sixth harmonic of the storage ring rf voltage, yields

$$I \cos(\omega t + \varphi) \cos(\omega t + \varphi_{\text{ref}}) = \frac{I}{2} [\cos(\varphi - \varphi_{\text{ref}}) + \cos(2\omega t + \varphi + \varphi_{\text{ref}})]. \quad (4.10)$$

With the phase φ_{ref} of the reference signal set to $\pi/2$ and a low pass filter suppressing the second term, the resulting signal is proportional to the phase φ of the detected signal for small amplitudes since $\cos(\varphi - \pi/2) = \sin \varphi \approx \varphi$. For $|\varphi| > \pi/2$, the phase signal is ambiguous and for $|\varphi| > \pi$ it even changes its sign. The phase in this context refers to a 3 GHz sine wave. Its relation to the phase at $f_{\text{rf}} = 500$ MHz and to the longitudinal displacement z of the bunch is

$$\varphi(3 \text{ GHz}) = 6 \varphi(500 \text{ MHz}) = 6 \frac{2\pi f_{\text{rf}}}{c} z. \quad (4.11)$$

Signal Processing

The measured phase is digitized at the bunch rate of 500 MHz by an 8-bit A/D converter. In order to avoid processing redundant data, a large fraction of the

digitized values are ignored (downsampling). If the synchrotron oscillation period corresponds to 160 turns, for example, the Nyquist theorem (see, e.g., [134]) requires to sample the sinusoidal position signal only at every 80th turn, i.e., downsampling by a factor $D = 80$. For a FIR filter, downsampling reduces the number of arithmetic operations per unit time by D^2 since the number of output values in (4.7) as well as the number of newly computed terms is reduced [125]. For operation at BESSY II, it proved beneficial to process every 30th input value, i.e., $D = 30$. A serial data link sends the digitized phase values together with a bunch tag to a farm of 40 DSPs (AT&T 1610) where an interface module distributes the data to the processors. Each processor performs the same digital filter algorithm, usually a 5- or 6-tap FIR-filter. The digital output values are stored in the holdbuffer module and are converted to analog voltage values at the bunch frequency.

A simple M-tap-FIR-filter would be given by the coefficients

$$c_k = c'_k - \frac{1}{M} \sum_{l=0}^{M-1} c'_l \quad \text{with} \quad c'_k = \sin \left(2\pi k \frac{f_s}{f_0/D} + \delta \right) \quad (4.12)$$

where f_s is the synchrotron frequency and f_0/D is the sampling frequency. The values of c'_k give rise to a 90° phase shift between filter input and output, while the second term of (4.12) makes the output of (4.7) zero for constant input, i.e., for a DC (zero frequency) input signal, the system does not waste amplifier power on trying to shift the DC bunch position. The processing time and other delays are accounted for by an additional phase shift δ . As shown in Fig. 4.4, the response of this filter has a maximum at the synchrotron

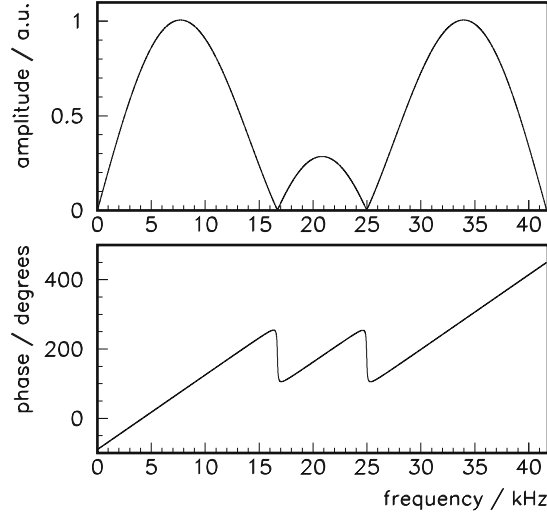


Fig. 4.4. Amplitude and phase as functions of frequency (known as Bode plot) of a 5-tap-FIR-Filter optimized for $f_s = 8.3$ kHz with a sampling frequency of 41.7 kHz and $\delta = 0$

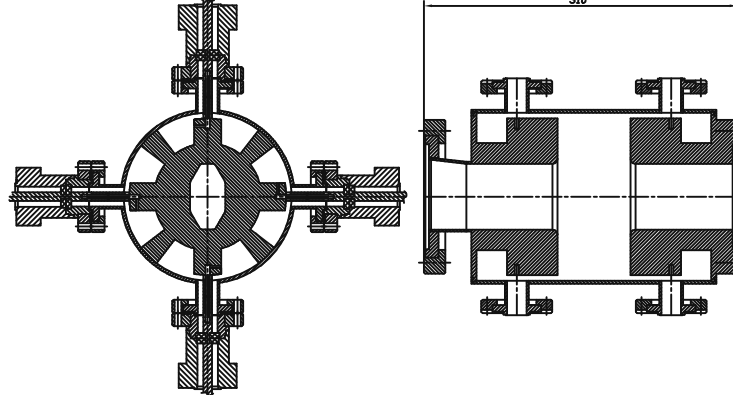


Fig. 4.5. Longitudinal feedback kicker in the BESSY II storage ring. *Left:* Cross section in the plane of the ceramic rf feedthroughs. *Right:* Longitudinal cross section in the horizontal midplane

frequency and there is a mirror maximum at $f_0/D - f_s$, since this frequency is indistinguishable from f_s . At $2f_s$, the phase shift is 180° and the filter response has a notch. The appropriate tool for a more profound filter analysis is the z -transform of a discrete sequence y_n defined by

$$Y(z) = \sum_{n=-\infty}^{\infty} y_n z^{-n}, \quad (4.13)$$

where z is a complex variable. The z -transform is the discrete-time counterpart of the continuous Laplace transform, and is used to analyze discrete signals or sampled data of continuous signals (see, e.g., [121, 134]).

Kicker and Power Amplifier

The task of the kicker is to accelerate or decelerate each bunch individually in order to damp each multi-bunch mode with similar efficiency, which demands a bandwidth of 250 MHz ($f_{rf}/2$). The longitudinal feedback kicker used at BESSY II (Fig. 4.5) is a cavity with a central frequency $f_c = 1374$ MHz (11/4th of f_{rf}) avoiding excitations at multiples of f_{rf} . In contrast to the usual accelerating structures, it is strongly damped by eight waveguides. This way, a quality factor of $Q \approx 5.5$ is obtained, yielding the required large bandwidth $\Delta f = f_c/Q$. This type of kicker cavity was designed for the LFB of the Φ -factory DAΦNE [135] and was modified for BESSY II in terms of central frequency, bandwidth and geometrical boundary conditions [136]. Compared to a drift-tube structure [137], this kicker offers a higher shunt impedance ($R_s = 960 \Omega$, see Sect. E.1.1) and has a more compact design (the total

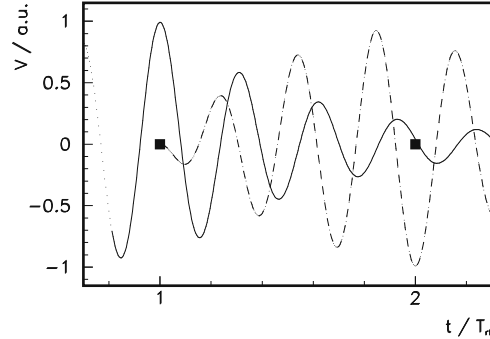


Fig. 4.6. Illustration of the effect of QPSK modulation. At time $t/T_{rf} = 1$, a bunch receives the kick voltage +1 (in arbitrary units). As the voltage decays, a new signal is superimposed. At $t/T_{rf} = 2$, this signal reaches the voltage required for a second bunch (-1, in this example). A phase difference of 90° between both signals causes the decaying kick voltage of the first bunch to cross zero at $t/T_{rf} = 2$, which minimizes its influence on the second bunch

length is 310 mm). In order to deliver an optimum kick voltage at the bunch frequency, the phase of each oscillation period is shifted by 90° . The advantage of this scheme, known as quad phase shift keyed (QPSK) modulation in communication theory, is illustrated by Fig. 4.6. The carrier signal is amplitude-modulated by the output of the digital filter and feeds the kicker through a broad-band power amplifier. Even though this solid-state amplifier is relatively robust against reflected power, it must be protected by circulators against beam-induced power.

Software

According to its nature, a digital system requires software on several levels of its operation [138]:

- A graphical user interface under UNIX allows to communicate with the system, to modify parameters, and to receive status messages.
- The actual control of the system takes place on several CPUs running under VxWORKS [139]. A VXI or VME bus links these CPUs directly to the respective electronic boards.
- When the system is initialized, the DSPs load the selected filter algorithm and execute it continuously.
- For the off-line analysis of recorded data, there are numerous routines available [118] using MATLAB [140].

The communication between the user interface and the VXI and VME CPUs is performed over a computer network and is based on Experimental Physics

and Industrial Control System (EPICS) [141], a software package widely used for accelerator control applications.

The Longitudinal Damping Rate

When considering the use of a LFB, the damping rate requirements may not be very clear. A particular narrow-band impedance of, say, an rf cavity can be accurately measured on the bench or calculated using numerical models, but its exact frequency during operation in a storage ring cannot be predicted. In the beam spectrum of BESSY II, the possible multi-bunch modes have a spacing of 1.25 MHz, i.e., the revolution frequency. Comparing the cases of minimum and maximum overlap of cavity HOMs and lines in the beam spectrum (see Fig. 2.15), the most optimistic assumption yields growth rates of 10^2 s^{-1} , which is of the order of the radiation damping rate. In the worst case, growth rates of 10^5 s^{-1} occur, about an order of magnitude higher than what can be handled by the feedback system. The achievable damping rate is given by

$$\frac{1}{\tau} = \frac{1}{2} f_0 \frac{\Delta U}{\Delta E/e} = \frac{1}{2} f_0 \frac{\eta h}{\nu_s E/e} \frac{\Delta U}{\Delta \phi} = \frac{1}{2} f_0 \frac{\eta h}{\nu_s E/e} G, \quad (4.14)$$

where ΔU is the kick voltage per turn, ΔE is the deviation from the nominal energy E , and f_0 is the revolution frequency, η is the momentum compaction factor, h is the harmonic number and ν_s is the synchrotron tune. The factor $1/2$ accounts for the fact that the detected deviation is sinusoidal [142]. The energy offset was converted to a phase deviation $\Delta \phi$ using (A.43) and (A.32), and the gain factor G was introduced to characterize the feedback system. This factor cannot be made arbitrarily large since noise is amplified as much as the position signal. What is more, the kick voltage for a given shunt impedance R_s of the kicker is limited by the amplifier power P according to $U \leq \sqrt{2R_s P}$. If only 50% of the amplifier power installed at BESSY II (220 W) act on the beam, which is a conservative assumption for damping losses and reflections, the maximum voltage is 470 V. Figure 4.7 shows the damping rate as a function of $\Delta \phi$. The point at which saturation of the amplifier causes the damping rate to decrease can be varied by changing the gain G and should be significantly above the resolution of the phase detector. Here, a damping rate of a few 10^3 s^{-1} appears to be realistic. Since it rises only with the square root of P , increasing the amplifier power soon reaches economic limits. If there is no space to install multiple kickers, longitudinal instabilities with growth rates exceeding 10^3 s^{-1} should be avoided.

4.3.2 Results

In order to verify the beneficial effect of a feedback system, the respective instabilities must be detected. Longitudinal bunch motion and oscillation of the electron energy are the characteristics of a synchrotron oscillation in the

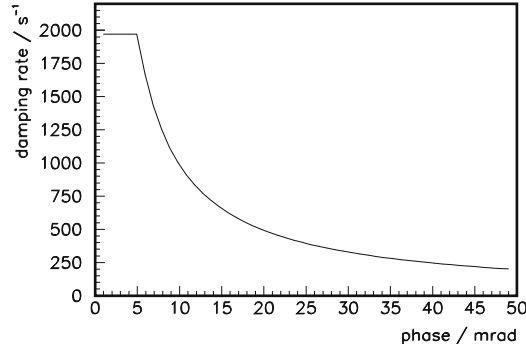


Fig. 4.7. Longitudinal damping rate as a function of oscillation amplitude (in units of the rf phase). In this example, the amplifier is saturated above an amplitude of 5 mrad

two longitudinal phase space coordinates. The longitudinal motion of electron bunches is detected by

- observing synchrotron sidebands in the beam spectrum (using a spectrum analyzer),
- measuring the variation of the bunch arrival time at a pickup electrode (using a fast oscilloscope or analyzing the data of the feedback system),
- measuring the variation of the arrival time of synchrotron radiation (using a streak camera).

Indications of an energy oscillation are:

- distortion of the synchrotron radiation spot in a dispersive region (observed with a CCD camera behind a pinhole),
- an increased energy spread of Compton-backscattered laser light (measured with a high-resolution gamma ray detector [307]),
- increased linewidths in undulator spectra (scanned by a monochromator).

The beam spectrum is the most easily accessible indicator of stability or instability of the beam (Fig. 4.8). Observation of the undulator linewidth, on the other hand, reflects the actual purpose of a longitudinal damping system. Its main task is to keep the linewidth small and the peak intensity large. Figure 4.9 shows an example.

As a further indication of the effectivity of the feedback system, Fig. 4.10 shows streak camera images of synchrotron radiation. In a streak camera, incident light on a photocathode is converted to electrons, which are accelerated and deflected horizontally and vertically by electric fields of different frequencies. The resulting two-dimensional pattern, where both axes represent time, is amplified by a microchannel plate and recorded using a CCD

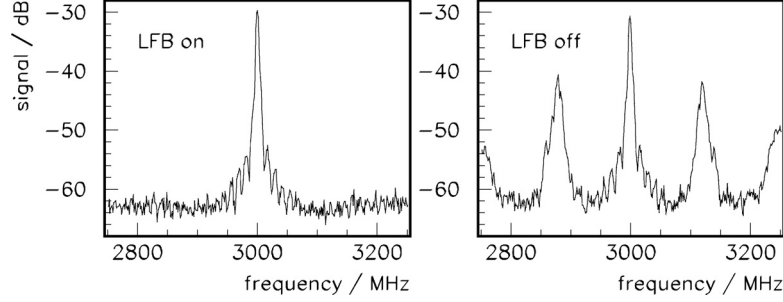


Fig. 4.8. Beam spectra in the range of (3000 ± 250) MHz. The longitudinal excitation seen in the right figure is successfully damped by the LFB

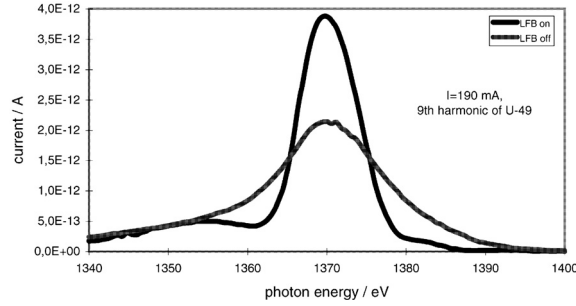


Fig. 4.9. Ninth harmonic in the photon spectrum of a planar undulator (U-49) in the BESSY II storage ring with and without damping of longitudinal instabilities [143]

camera. In Fig. 4.10, the horizontal axis covers 100 ns and shows a train of light pulses (since the camera was synchronized to 250 MHz, only every other bunch is visible). The vertical axis has a time resolution of the order of 1 ps and shows the arrival time and temporal profile of each pulse. For the multi-bunch mode $\mu = 278$ excited here, the phase difference between adjacent bunches is $2\pi \times 278/400 = 4.37$ rad, which is 2.45 rad (2×4.37 rad modulo 2π) between every other bunch. The damping effect of the feedback system is evident.

As Fig. 4.10 also demonstrates, the same information can be obtained by using the feedback system itself as diagnostics tool. Further examples shall be shown in the next section.

The present discussion was implicitly restricted to longitudinal dipole oscillations, i.e., $l = 1$ modes as depicted in Fig. 2.20. At the Φ factory DAΦNE (Frascati, Italy), however, the LFB was unexpectedly found to cure a longitudinal quadrupole instability as well [145]. In agreement with theoretical estimates [146], this instability limited the maximum stored current of the e^- storage ring. It could be suppressed by delaying the kicker pulse such that only the bunch tail was kicked (which only works if the bunch length is comparable to a quarter of the kicker wavelength).

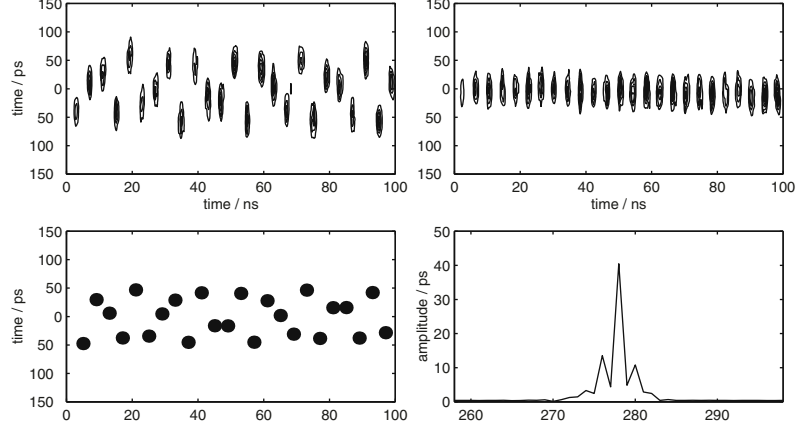


Fig. 4.10. *Top:* Contour plot of streak camera images at a beam current of 95 mA with the longitudinal feedback system (LFB) switched off (*left*) and on (*right*). The vertical scale represents deviations from the nominal arrival time of light pulses at the camera. *Bottom left:* Data recorded with the LFB, exhibiting almost the same pattern as the streak camera image. *Bottom right:* Mode spectrum extracted from the LFB data

4.3.3 Using the Digital Feedback System as Diagnostics Tool

The LFB allows to store digitized data over a period of typically 30–50 ms, depending on the downsampling factor, and to transfer these data over the VME bus without interrupting the system operation. As sketched in Fig. 4.11, the system switches at a predefined time t_1 prior to data acquisition from a digital filter #0 (the standard configuration) to a different filter #1. Data acquisition starts at t_2 and terminates at t_4 . In-between, at an adjustable time t_3 , the system switches back to filter #0. Some variations of this pattern,

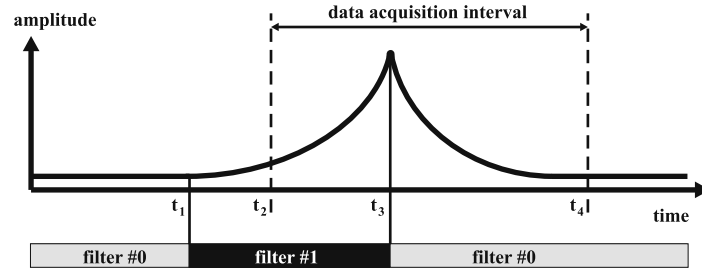


Fig. 4.11. Temporal scheme of data acquisition using the LFB. The system switches to a second digital filter function (filter #1) at an adjustable time t_1 prior to data acquisition and back to filter #0 at t_3 . In this example, filter #0 damps the instability, while filter #1 allows its amplitude to grow

Table 4.2. Gain settings for filter #0 and #1 of the LFB during data acquisition. The gain factor is defined such that a filter with $G > 0$ has a damping effect

$G(0)$	$G(1)$	
> 0	$= 0$	grow-damp measurement of unstable modes
$= 0$	< 0	grow-damp measurement of stable modes via anti-damping
$= 0$	$= 0$	unstable beam, e.g., measurement of mode frequencies
> 0	> 0	stable beam, e.g., measurement of synchronous phase angles

characterized by the gain factors $G(0)$ and $G(1)$ of filter #0 and #1, respectively, will be discussed in the following and are summarized in Table 4.2, where the sign of the gain factor is defined such that a filter with $G > 0$ has a damping effect.

$G(0) > 0, G(1) = 0$

If the gain of filter #1 is zero, instabilities are only inhibited by synchrotron radiation damping during the first part of data acquisition. After switching back to filter #0 at t_1 , these instabilities are damped by the feedback system. This pattern, known as grow-damp measurement, is used to determine growth rates of instabilities and the damping rate of the feedback system. The left part of Fig. 4.12 shows the envelopes of longitudinal oscillations as function of time and bunch index. Evidently, there were two bunch trains in the storage ring. A discrete Fourier transform (DFT, see, e.g., [144]) yields the amplitude as function of time and mode index, as shown in the right part of the figure. Here, several multi-bunch modes grow and are subsequently damped. Independently of the filter setting, there is a mode 0 oscillation, i.e., an in-phase motion of all bunches. This mode pulsates predominantly at a rate of 300 Hz, a harmonic of the 50 Hz mains frequency. Being imposed on the beam by the accelerating cavities, it cannot be counteracted by the relatively low feedback kicker voltage.

As an example, the amplitude of one particular mode is plotted versus time in Fig. 4.13. Since the measured growth rate (0.18 ms^{-1}) and damping rate (-0.85 ms^{-1}) includes radiation damping with $1/\tau_s = -0.13 \text{ ms}^{-1}$, the actual growth rate $1/\tau_g$ and damping rate $1/\tau_d$ is

$$1/\tau_g = 0.18 \text{ ms}^{-1} - 1/\tau_s = 0.31 \text{ ms}^{-1}$$

$$1/\tau_d = -0.85 \text{ ms}^{-1} - 1/\tau_g - 1/\tau_s = -1.03 \text{ ms}^{-1}.$$

Exponential damping at a constant rate indicates that the amplifier was not saturated. Otherwise, the damping rate would depend on the oscillation amplitude. According to (4.14), the damping rate is a function of the chosen gain value, which is not set higher than necessary.

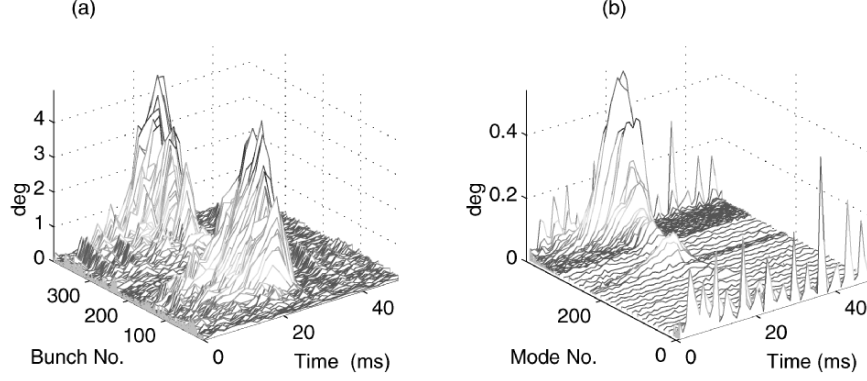


Fig. 4.12. Example of a grow-damp measurement of the LFB at BESSY II, where damping was activated after 20 ms. The longitudinal oscillation amplitude is plotted as function of time and bunch index (*left*) and as function of time and mode number (*right*)

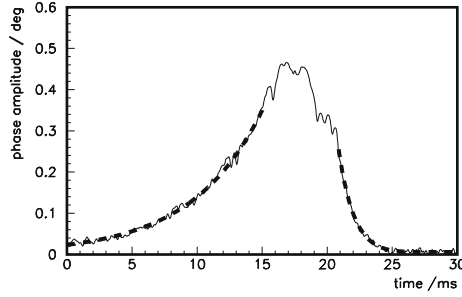


Fig. 4.13. Growth and damping of the longitudinal multi-bunch mode 240 at a beam current of 145 mA. The fitted exponential functions (*dashed*) correspond to a growth rate of 0.18 ms^{-1} and a damping rate of -0.85 ms^{-1}

Assuming an instability driven by a single narrow-band impedance at frequency

$$\omega' = p h \omega_0 + \mu \omega_0 + \omega_s, \quad (4.15)$$

and inserting the beam current $I = h N e / T_0$ and beam energy $E = \gamma m_e c^2$ into (2.83), the complex frequency shift is given by

$$\Delta\omega = \frac{h N e^2 \eta}{2 \omega_s E T_0^2} \omega' Z_{\parallel}(\omega') = \frac{I e \eta}{2 \omega_s E T_0} \omega' Z_{\parallel}(\omega') \quad (4.16)$$

with the symbols defined as in (2.83). Since a mode with index μ repeats itself after $h \omega_0 = \omega_{\text{rf}}$, the measurement does not determine the factor p and the actual frequency of the impedance. Figure 4.14 shows the measured growth rate $\text{Im } \Delta\omega + 1/\tau_s$ and frequency $(\text{Re } \Delta\omega + \omega_s)/2\pi$ as functions of the beam current for mode $\mu = 240$ caused by a HOM of one of the higher-harmonic

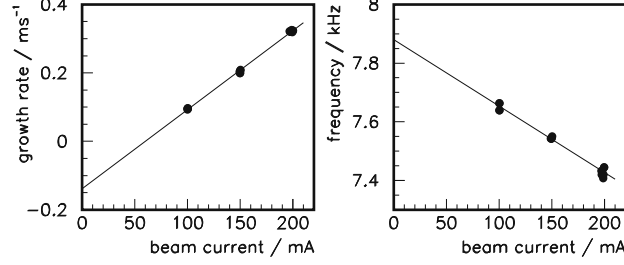


Fig. 4.14. Growth rate and frequency shift of multi-bunch mode 240 from grow-damp measurements as a function of beam current in BESSY II. The current dependence allows to determine the real and imaginary part of the impedance that causes this instability. Extrapolation to 0 mA yields the radiation damping rate (*left*) and the synchrotron frequency (*right*)

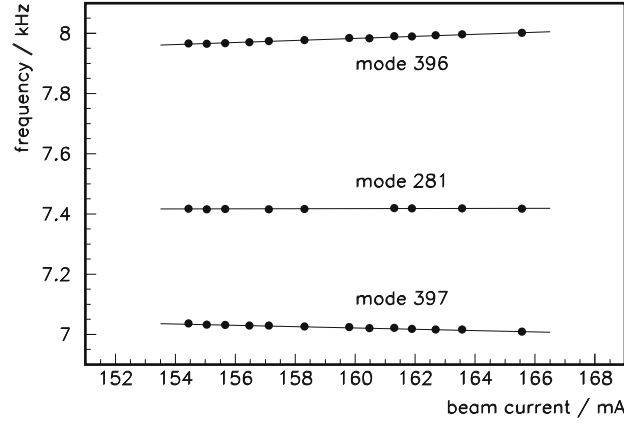


Fig. 4.15. Oscillation frequencies of multi-bunch modes 281, 396, and 397 from grow-damp measurements as a function of beam current in BESSY II. Within error margins, the fitted lines converge at zero current to 7.4 kHz (data from [147], courtesy D. Teytelman, SLAC)

cavities. With a known HOM frequency of $f \approx 2.3$ GHz, $p = 4$ can be assumed. The slopes of the two straight lines in Fig. 4.14 yield the impedance:

$$Z_{\parallel}(4 \cdot 400 \omega_0 + 240 \omega_0 + \omega_s) = (121 \pm 8) - i(166 \pm 11) \text{ k}\Omega, \quad (4.17)$$

In principle, the impedance might have changed in the course of the measurements, e.g., due to changes of the heat load on the higher-harmonic cavities, but some consistency checks can be made. The extrapolation of the observed growth rate to 0 mA, for example, matches the expected radiation damping rate of -0.13 ms^{-1} . When observing more than one mode, the extrapolation of the oscillation frequencies to 0 mA should lead to the same value of the synchrotron frequency, as has been shown in [147] (see also Fig. 4.15).

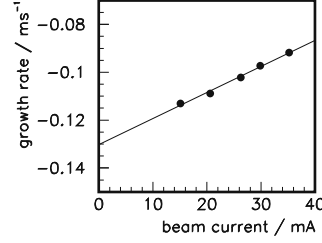


Fig. 4.16. Current dependence of the damping rate (negative growth rate) of a mode excited by the LFB

$$G(0) = 0, G(1) < 0$$

At low beam current, multi-bunch modes can be excited by negative feedback and subsequently damped by synchrotron radiation damping after switching to filter #0 with zero gain. The extrapolation to zero current of the values shown in Fig. 4.16 yields a radiation damping rate of $(-0.130 \pm 0.015) \text{ ms}^{-1}$. Using (2.113) with a beam energy of 1.70 GeV and other parameters from Table B.1, the theoretical radiation damping rate would be 0.125 ms^{-1} .

$$G(0) = 0, G(1) = 0$$

Observing the unstable beam with both filter gains set to zero can be instructive as well. In contrast to using a spectrum analyzer, the synchrotron sidebands of all revolution harmonics (i.e., all multi-bunch modes) are recorded simultaneously and as function of time.

The Fourier analysis projects the observed instabilities onto modes that are defined for a complete and homogeneous fill of the storage ring (even-fill eigenmodes, or EFEMs). Any other fill pattern has eigenmodes that are combinations of EFEMs and show up in the spectrum as a broad distribution of synchrotron sidebands. On the other hand, a single synchrotron sideband may be driven by a combination of several modes, and a unique assignment of sideband and mode can be difficult. In the projection onto EFEMs, different oscillation frequencies hint the presence of more than one eigenmode.

The left part of Fig. 4.17 is assembled from 400 spectra recorded sequentially using a spectrum analyzer. Each line of the picture is a spectrum with 25 kHz span and a central frequency coinciding with a revolution harmonic at $(6h + \mu)f_0$, where $h = 400$ is the harmonic number. The abscissa is the frequency relative to the respective revolution harmonic, the vertical axis is the EFEM index μ . Synchrotron sidebands are visible over a wide range of μ and vary in frequency between 6 and 7 kHz. This is shown more clearly by the LFB data in the right part of the figure, where oscillation frequency and mode index result from a Fourier transform of the recorded bunch oscillations and several discrete groups of EFEMs with identical frequency are visible. It

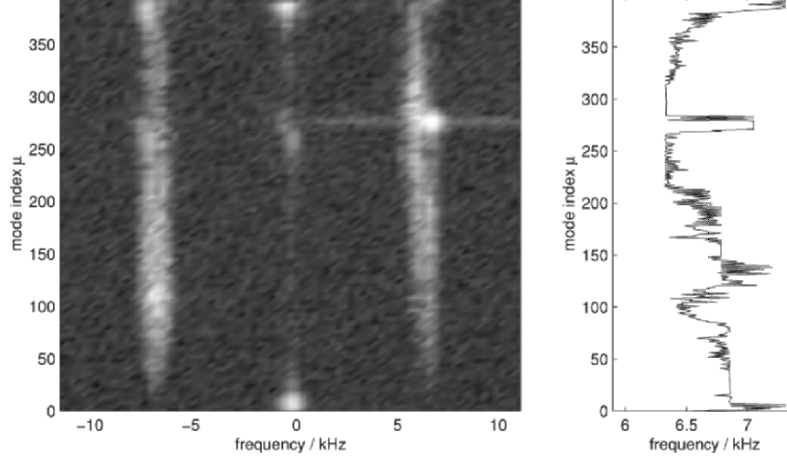


Fig. 4.17. Different frequencies of synchrotron sidebands. *Left:* 400 spectra recorded with a spectrum analyzer. The frequency is given relative to the revolution harmonics at $(6h + \mu)f_0$, where h is the harmonic number and f_0 is the revolution frequency. *Right:* Oscillation frequency from the analysis of longitudinal feedback data as function of mode number μ

should be noted that the LFB data were recorded within 30 ms, whereas the spectra in the left part of the figure were acquired over 10 minutes.

So far, very few investigations are devoted to the dynamics of instabilities in an unevenly filled storage ring. Besides numerical calculations (e.g., [148]) and analytical estimates (e.g., [47]), it is empirically known that gaps in the fill pattern can stabilize the beam. Apart from Landau damping via beam loading [116, 117], this may be due to the coupling of EFEMs (*modulation coupling*) as described in [118] and experimentally verified at PEP II, SPEAR and at the ALS.

$$G(0) > 0, G(1) > 0$$

If both filters, #0 and #1, have positive gain, the beam remains stable during data acquisition – except for some mode 0 excitation from the rf system. In this case, the LFB allows to measure the synchronous phase angle of each bunch. The detected signal is proportional to the product of phase and bunch current, and the bunch current can be estimated under the assumption that the mode 0 motion displaces all bunches equally. As an example, Fig. 4.18 shows simultaneous measurements of synchronous phase angles using a streak camera (top figure) and the LFB (center). The bottom part of the figure shows a simulation that reproduces the main features of the observed pattern. The phase angle $\phi_{s,k}$ of bunch k satisfies the condition that the energy loss per revolution is compensated by the rf voltage with amplitude U_0 :

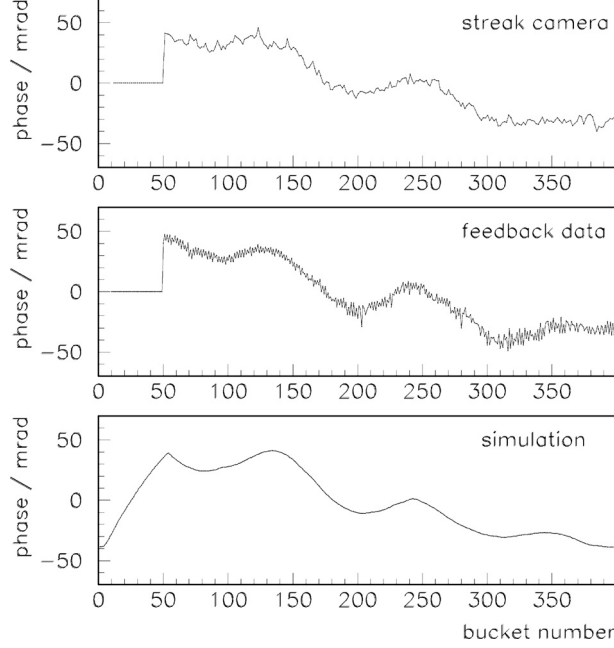


Fig. 4.18. Synchronous phase angles for 400 rf buckets in BESSY II with a gap of 50 buckets. *Top:* analysis of a streak camera image. *Center:* data of the LFB. *Bottom:* simulation (see Sect. D.2.2 in the appendix)

$$\sin \phi_{s,k} = \frac{V^S + V_k^W}{U_0} \quad \text{with} \quad V_k^W \approx \sum_{m=0}^{\infty} q_{k-m} W_{\parallel}(m T_{\text{rf}}). \quad (4.18)$$

The energy loss V^S is due to synchrotron radiation and is the same for all bunches while V_k^W is the energy loss of bunch k due to wake fields from all charges q_{k-m} preceding bunch k at distances which are (approximately) multiples of the rf period T_{rf} . Small deviations from ϕ_s , which is the phase angle in the absence of wake fields, are given by

$$\sin \phi_{s,k} - \sin \phi_s \approx (\phi_{s,k} - \phi_s) \cos \phi_s = \frac{V_k^W}{U_0}. \quad (4.19)$$

Finally, the phase difference $\phi_k = \phi_{s,k} - \phi_s$ is [118]

$$\phi_k = \frac{V_k^W}{U_0 \cos \phi_s}. \quad (4.20)$$

If the storage ring is filled with bunches of equal charge, the phase difference is the same for every bunch, and its current dependence shows only a summary effect of the wake fields. In the case of an uneven fill pattern, however, V_k^W is

different for every bunch and the distribution of measured phase angles contains detailed information on the wake function and impedance. The pattern shown in Fig. 4.18, for example, results from two contributions:

- The usual beam loading effect of the rf cavities changes the phase angles along the bunch train almost linearly (see Sect. D.2). The presence of higher-harmonic cavities enhances this effect.
- A sinusoidal modulation with a multiple of the revolution frequency (5 MHz, period length 200 ns or 100 buckets) is caused by two harmonic cavities “parked” 5 MHz away from their working frequency.

The modulation of phase angles corresponds to the frequency of the dominant impedance modulo, the rf frequency (499.7 MHz) at which the electron bunches “sample” the impedance. The impedance of the cavities “parked” at 1504 MHz causes a modulation of $1504 \text{ MHz} - 3 \times 499.7 \text{ MHz} \approx 5 \text{ MHz}$. An equation equivalent to (4.20) allows to determine the impedance [150] from

$$\Phi_n = \frac{h J_n}{U_0 \cos \phi_s} \sum_{p=-\infty}^{\infty} Z_{\parallel}(p h \omega_0 + n \omega_0) \quad (n = 0 \dots h - 1), \quad (4.21)$$

where Φ_n is the discrete-time Fourier transform of ϕ_k and J_n is the transform of bunch current $j_k = q_k \omega_0 / 2\pi$. Combining the complex frequency shift from a grow-damp measurement with the analysis of phase angles allows to remove the ambiguity regarding the factor p for narrow-band impedances. Equation (4.16) contains the product of impedance and frequency, while (4.21) contains the impedance alone. This can be traced to the fact that frequency shift and instability growth rate depend on the derivative $W'_{\parallel}$ – see (2.78) – while the synchronous phase angle depends directly on $W_{\parallel}$. An example with data taken at BESSY II can be found in [147].

Many investigations at the BESSY II storage ring and other facilities [118, 149–151] demonstrate that the data of a digital feedback system contain valuable information on impedances and other properties of a storage ring. Much of this information is not accessible by traditional diagnostics tools. Time-resolved observations can be performed with spectrum analyzers in *zero-span* mode, but only at one frequency at a time. The data acquisition capabilities of the LFB, on the other hand, allow to record time- and frequency-resolved data simultaneously. So-called “FFT signal analyzers” are similar in functionality, but their time resolution is typically orders of magnitude lower. Another option is to record data with a modern digital oscilloscope. Sampling the position of each bunch over several milliseconds, however, requires a large memory and some kind of downsampling mechanism.

4.4 Example of a Transverse Feedback System

The feedback system against transverse multi-bunch instabilities at BESSY II is an example of a purely analog system. Similar feedback systems are

described in [152, 153] for the ALS (Berkeley, USA) and in [154] for SRRC (Hsinchu, Taiwan). Arguments in favor of an analog system are low cost, relatively low complexity and a short development time compared to digital systems. For transverse oscillations in the MHz region, analog circuitry suggests itself provided the required signal delays are of the order of $1\text{ }\mu\text{s}$. For larger storage rings, e.g., PEP II at SLAC with a circumference of 2200 m, this is not the case [124].

4.4.1 System Overview

Figure 4.19 shows the transverse feedback system (TFB), which is in operation at BESSY II since 1999 [155]. The transverse beam position is detected by two different beam position monitors with four button-type electrodes each. The kicker, on the other hand, corrects the angle of the bunch trajectory rather than its position. Consequently, a satisfactory correction signal requires the phase advance of the betatron oscillation between detector and kicker to be close to $n \cdot 2\pi \pm \pi/2$ (with n being an integer). Since horizontal and vertical phase advance are generally different, and in order to maintain optimum performance if the betatron tune is changed, the bunch position is measured at two positions of the storage ring. Two measured values can be linearly combined to the equivalent of one measurement at the optimum phase advance, provided the phase advance between the detectors is not an integer multiple of π (a phase advance of $\pi/2$ would be ideal, but this is neither mandatory nor feasible in practice). Since the betatron amplitude does not change much from one revolution to the next, a linear combination of several measurements at a single detector would yield the same result, but would require long delay lines or digital processing.

Detection of the Transverse Position

In the receiver modules, the difference between signals from opposite button electrodes is formed. These signals are proportional to the respective dipole moment, i.e., the displacement x or y times the current I . Since this type of electrode is not very efficient in the baseband of multi-bunch oscillations (0–250 MHz), the detection is performed in a 250 MHz wide frequency band at 3.0–3.25 GHz. Mixing with a 3-GHz reference signal produces baseband signals as a lower sideband, while the upper sideband is eliminated by a low pass filter:

$$I y \cos(\omega t + \varphi) \cos(\omega t + \varphi_{\text{ref}}) = \frac{I y}{2} [\cos(\varphi - \varphi_{\text{ref}}) + \cos(2\omega t + \varphi + \varphi_{\text{ref}})] . \quad (4.22)$$

In contrast to the phase detector described by (4.10), distortions of the dipole moment measurement due to deviations of the phase φ from zero should be minimized, which is the case for $\varphi_{\text{ref}} = 0$.

Signal Processing

The mixer unit linearly combines the horizontal and vertical receiver signals $X_{1,2}$ and $Y_{1,2}$, respectively, to

$$X = a_1 X_1 + a_2 X_2, \quad Y = b_1 Y_1 + b_2 Y_2, \quad (4.23)$$

where the multiplication with the coefficients $a_{1,2}$ and $b_{1,2}$ is performed by mixing the signals with DC control voltages. The resulting products are summed using a “magic tee” hybrid (rf elements are described, e.g., in [156]). In practice, suitable coefficients are found empirically by varying the control voltages while observing betatron sidebands or just by looking at the synchrotron radiation image of the beam (see Fig.4.23). A correlation filter (notch filter) suppresses the revolution harmonics in the spectrum of the position signals without inhibiting the betatron sidebands. The occurrence revolution harmonics in the spectrum corresponds to static deviations from an arbitrary central position defined by the electrodes. The correlation filter is realized by the loops shown in Fig.4.19, delaying a position signal by one revolution time (800 ns) and subtracting it from the next signal of the same bunch. This corresponds to a differentiation which suppresses static signals and is an example of a 2-tap-filter implemented in an analog fashion. Here, a limitation of analog signal processing becomes apparent: due to dispersion in the delay lines (coaxial cables), the filter cannot be perfect over the whole baseband. In expectation of resistive-wall instabilities, the filter was optimized for multi-bunch modes at low frequency [130].

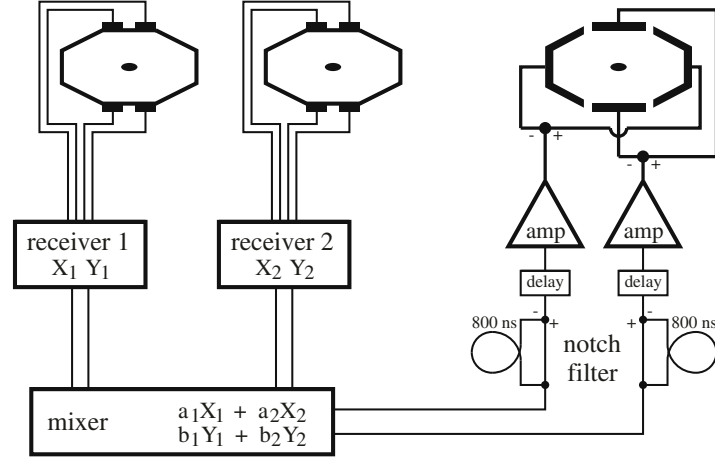


Fig. 4.19. The TFB at BESSY II comprising two beam position monitors, analog signal processing (receiver and mixer), correlation filtering (notch filter), amplifiers and kicker

Kicker and Power Amplifiers

A cross section of the stripline kicker developed for BESSY II [157] is shown in Fig. 4.20. The design is based on the following considerations:

- The shunt impedance of the kicker with electrodes of 30 cm length has notches at multiples of 500 MHz (rf harmonics).
- A structure with four electrodes that deflect the beam horizontally and vertically minimizes the required length.
- Small distances between the electrodes maximize the shunt impedance.
- Since tapering would create additional impedance, the four electrodes form a shape that matches the cross section of the adjacent vacuum chamber.
- Matching the amplifier requires the line impedance to be close to $50\ \Omega$ everywhere. At the electrodes, this was ensured by 2-dimensional field calculations [158]. In order to optimize the transitions between stripline and coaxial feedthroughs, a 1:1 low-power model was built and the reflected power was studied for different geometries by time-domain reflectometry (TDR) measurements.
- Wake fields building up in the cavity between the electrodes and the surrounding vacuum vessel are extracted by two antennas. The modes of this structure were determined numerically using the MAFIA eigenmode solver [31].
- Ohmic losses and wake fields may heat the electrodes, while the thermal conductivity of the rf feedthroughs is only marginal. In order to improve radiative heat dissipation, the electrodes and the inner walls of the chamber

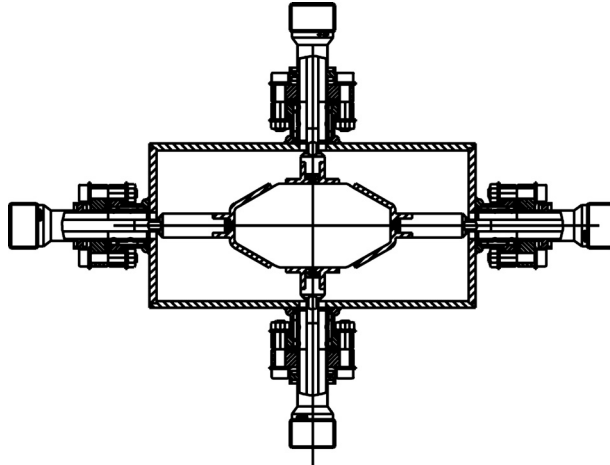


Fig. 4.20. Cross-section of the transverse feedback kicker used at BESSY II. The shape formed by the electrodes resembles the cross section of the adjacent vacuum chamber with 65 mm horizontal and 35 mm vertical aperture

- were blackened by a TiAlN layer [159]. The electrode temperature is monitored through a ZnSe window using a pyrometer.
- Small vanes perpendicular to the electrodes add mechanical stability, improve heat dissipation, and help to achieve $50\ \Omega$ matching.

The shunt impedance of the kicker depends on frequency. It varies between $10\ \text{k}\Omega$ at $1\ \text{MHz}$ and $4\ \text{k}\Omega$ at $250\ \text{MHz}$ horizontally and between $20\ \text{k}\Omega$ at $1\ \text{MHz}$ and $8\ \text{k}\Omega$ at $250\ \text{MHz}$ vertically, as shown in Fig. E.4 in the appendix. The larger vertical shunt impedance is advantageous in view of resistive-wall instabilities, which should be more pronounced in vertical direction.

The power amplifier is operated in the baseband from the smallest mode frequency, given by the fractional betatron tune q_β or $1 - q_\beta$ (whichever is smaller), up to $250\ \text{MHz}$. Its rise time must be shorter than $2\ \text{ns}$, the time between two consecutive bunches. At BESSY II, one horizontal and one vertical kicker electrode is driven by a solid-state $150\ \text{W}$ power amplifier, while keeping the opposite electrode floating. Both electrodes may be driven with signals of opposite phase, which requires a high-power 180° phase shifter and proved not to be necessary. Contrary to a cavity kicker, a stripline is sensitive to the beam direction. The amplifier must be connected to the downstream end of the electrode, while beam-induced power occurs only at the upstream end, which is equipped with a load resistor. Thus, a circulator to protect the amplifier is not required.

Software

Being an analog feedback system, the only digital part is the communication with the control system (see Sect. B.2.6), which is required to adjust the coefficients in (4.23) and the overall gain, and to monitor the status of the components. The software used here is derived from the control of other storage ring components and is based on EPICS [141].

Transverse Damping Rate

The resistive-wall effect was predicted to cause a vertical multi-bunch instability with a growth rate by far exceeding $1\ \text{ms}^{-1}$ at a beam current of $400\ \text{mA}$ (see Sect. 2.4.1), whereas the transverse radiation damping rate is only $0.06\ \text{ms}^{-1}$.

Similar to the longitudinal case, the damping rate of a TFB is given by

$$\frac{1}{\tau} = \frac{1}{2} f_0 \frac{\sqrt{\beta_P \beta_K}}{E/e} G, \quad (4.24)$$

where β_P and β_K are the beta functions at the pickup and at the kicker, respectively. The gain factor is the ratio of kick voltage and transverse displacement $G = \Delta U / \Delta x$ or $\Delta U / \Delta y$. As in the longitudinal case, the maximum kick voltage $U = \sqrt{2R_s P}$ depends on the shunt impedance R_s of the kicker

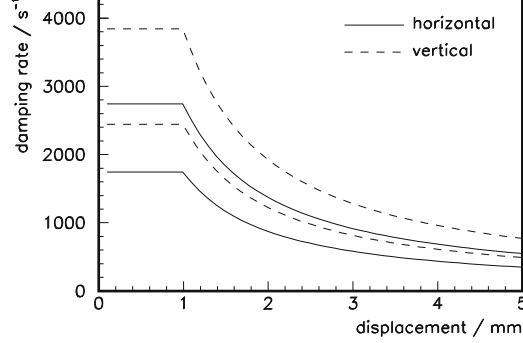


Fig. 4.21. Horizontal and vertical damping rate as functions of the oscillation amplitude. For each coordinate, the upper curve is valid at a frequency of 1 MHz, and the lower curve at 250 MHz

and on the amplifier power P . Assuming a power loss of 50%, Fig. 4.21 shows the resulting damping rates as function of transverse displacement for two frequencies at the lower and upper end of the baseband. The difference is due to the lower shunt impedance at higher frequency. The maximum damping rate depends on the setting of the gain factor G and is limited by noise exciting the beam, if the gain is too high.

4.4.2 Results

A signature for transverse multi-bunch instabilities is the occurrence of betatron sidebands in the beam spectrum. In the case of the resistive-wall effect, they only show up in the vicinity of multiples of the rf frequency. Inhomogeneities of the fill pattern cause the distribution of sidebands to be spread over several even-fill eigenmodes. Figure 4.22 shows snapshots from a spectrum analyzer documenting the effectiveness of the TFB.

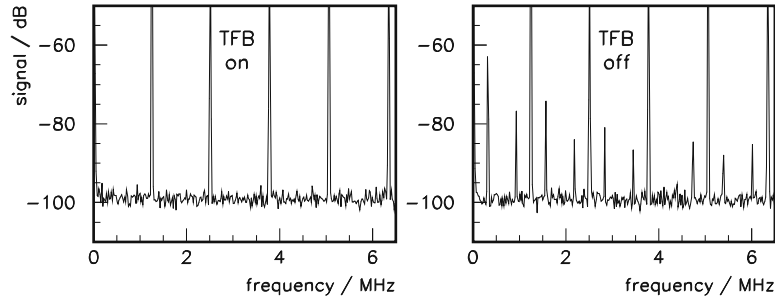


Fig. 4.22. Beam spectrum in the region of the first revolution harmonics. The excitation, which is visible as a series of betatron sidebands (*right*), is successfully suppressed by the feedback system (*left*)

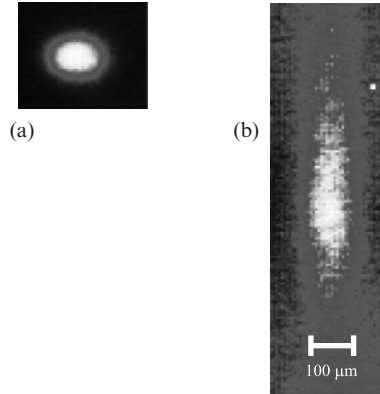


Fig. 4.23. Images of the synchrotron radiation source spot from a pinhole [310] behind a dipole magnet with the TFB (a) in operation and (b) switched off (courtesy K. Holldack, BESSY)

Transverse oscillations significantly enlarge the synchrotron radiation source spot, which reflects the time-averaged size of the electron beam. Figure 4.23 shows the source at a beam current of 150 mA with undamped instabilities (right) and damped in both planes (left) [160]. The enlargement of the spot depends on several parameters of the storage ring, particularly on the chromaticity and on nonlinearities of the magnetic guide field. In the case shown here, the feedback system reduces the spot size by almost an order of magnitude.

The TFB also helps to overcome the threshold of transverse mode coupling at about 6 mA (see Sect. 2.4.2), and single bunches with a current of more than 20 mA can be stored. The mechanism is yet unclear, but similar observations were made at the ALS [161]. It is possible that a reactive component in the feedback loop delays the coupling of modes $l = 0$ and $l = -1$ by causing a (real) frequency shift (see also [162]).

With its purely analog design, the TFB has no data acquisition capabilities by itself, but analog signals can be digitized and stored by other means, e.g., by employing the LFB as described in the next paragraph.

Digital Acquisition of Transverse Oscillations

Several experiments were performed at BESSY II digitizing the transverse correction signals at a sampling rate of 43 kHz using the LFB. As shown in Fig. 4.24, a pulse generator initiates data taking at time 1, and opens the TFB loop with a GaAs switch (switching time ≤ 3 ns) between time 2 and 3. The data taking period, typically 33 ms, is limited by the LFB memory size and ends at time 4. At the time of these experiments, the beam current was

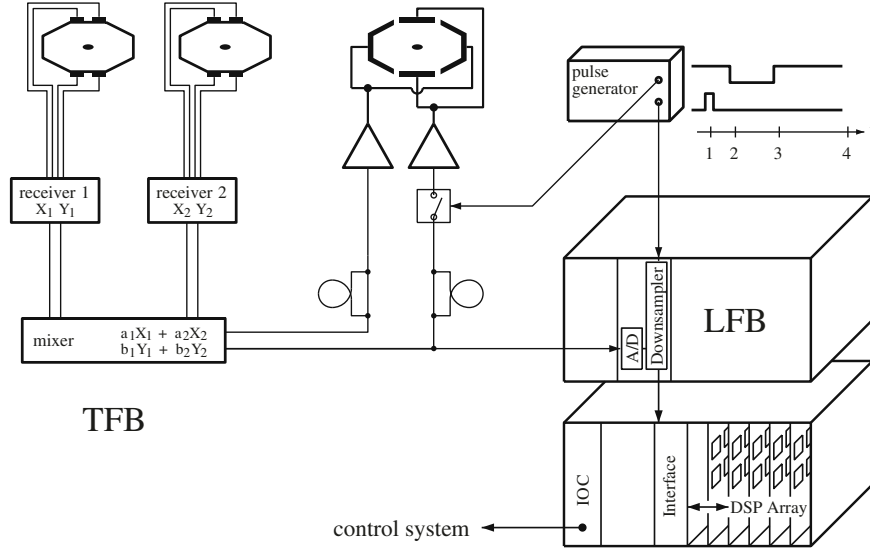


Fig. 4.24. Schematic of the digital acquisition of transverse instability data using the LFB. A pulse generator initiates data acquisition at time 1 and opens the TFB loop between time 2 and 3. Data recording is terminated at time 4. See Figs. 4.3 and 4.19 for more details of the respective feedback system

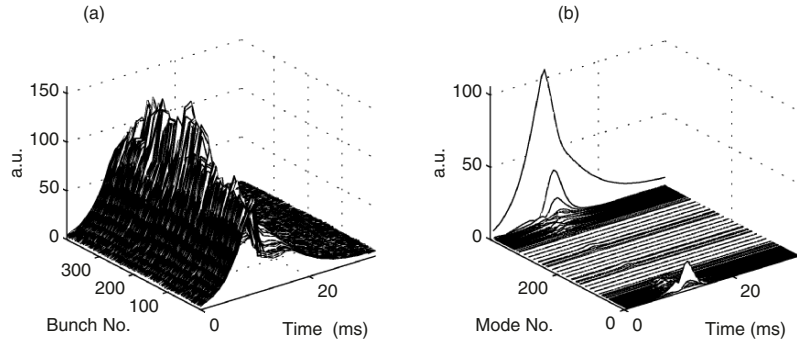


Fig. 4.25. Example of a grow-damp measurement of horizontal instabilities by combining the TFB with the longitudinal feedback electronics at BESSY II. The oscillation amplitude (in arbitrary units) is shown as function of time and bunch index (*left*) and as function of time and mode number (*right*)

restricted to 70 mA, where the beam was longitudinally stable without using the LFB, whereas transverse instabilities occurred already at 10–20 mA. At beam currents above 70 mA, longitudinal instabilities would suppress transverse oscillations.

Figure 4.25 shows horizontal oscillation amplitudes of 400 electron bunches (*left*) and the Fourier transformed signals as function of time and mode

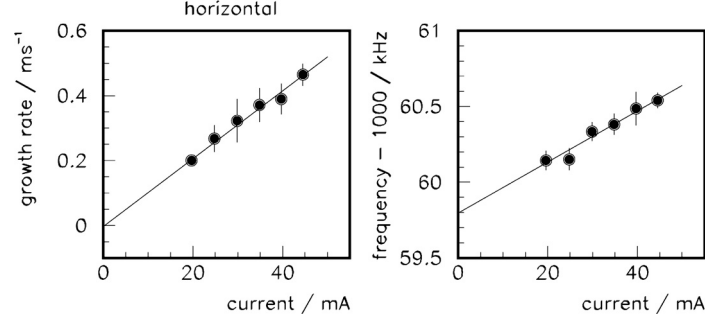


Fig. 4.26. Growth rate (*left*) and frequency minus 1000 kHz (*right*) of the horizontal multi-bunch mode labeled $\mu = 399$ as functions of beam current

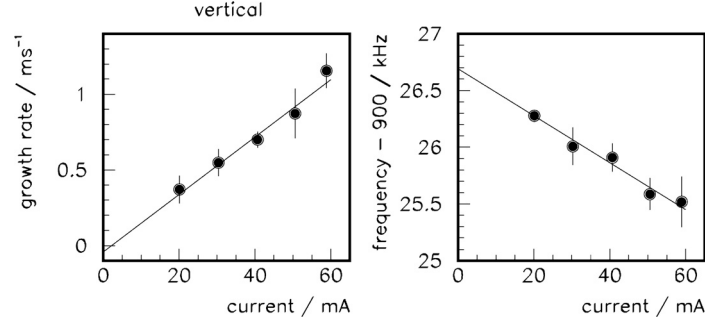


Fig. 4.27. Growth rate (*left*) and frequency minus 900 kHz (*right*) of the vertical multi-bunch mode labeled $\mu = 399$ as functions of beam current

number (right). Multi-bunch mode number $\mu = 399$ turns out to be dominant in both planes. Note that the integer part of the betatron tune is not detected and is therefore ignored in the assignment of mode numbers.

Horizontal and vertical growth rates and frequency shifts are shown in Figs. 4.26 and 4.27, respectively, as functions of beam current. For each value of the beam current, 2–8 measurements were performed and their rms variation is shown by the error bars. The vertical growth rate values are particularly prone to systematic errors since the growing amplitudes deviate considerably from the assumed exponential behavior.

The presumed resistive-wall impedance cannot be directly determined in analogy to (4.16) because many lines of the beam spectrum contribute to the complex frequency shift. The predictions of Sect. 2.4.3 may be too inaccurate for a quantitative comparison due to various simplifications in the model. Qualitatively, however, the results confirm the expectation of a predominantly vertical resistive-wall effect for a flat vacuum chamber:

- Multi-bunch mode $\mu = 399$ is dominant as expected for a fractional tune being > 0.5 and ignoring the integer part of the betatron tune.
- Due to the geometry of the vacuum chamber, vertical growth rates are larger than horizontal growth rates.
- The horizontal oscillation frequency increases, and the vertical frequency decreases with beam current. This is consistent with observations at other storage rings, e.g., at PEP II (Stanford, USA) [163] and at the ESRF (Grenoble, France) [164], and with numerical calculations, which can be performed for arbitrary chamber cross sections [50].

At the BESSY II storage ring under normal vacuum conditions, there were no clear indications of transverse instabilities other than those caused by the finite wall conductivity. However, after part of the vacuum vessel had been vented for new installations and particularly with all rf buckets occupied (i.e., omitting an ion-clearing gap), strong vertical oscillations with non-exponential growth and beating of different modes were observed and qualitatively reproduced by numerical simulations of a beam-ion instability [165].

4.5 Other Feedback Systems and New Developments

In the preceding paragraphs, the feedback systems implemented at BESSY II were described in some detail. Both systems were designed for storage rings with extremely short bunch spacing, i.e., meson factories and synchrotron light sources. In the case of the longitudinal system, the digital architecture has demonstrated a high degree of flexibility with six installations at storage rings of different size and purpose [132].

Apart from the systems described here, other feedback systems exist and the following tendencies are apparent:

- At large storage rings, it is difficult to realize the required delay times just by using cables. Apart from a digital architecture, the solution may be a hybrid system that adds and multiplies signals in an analog fashion but stores data digitally [124, 166].
- Due to the short period of the betatron oscillation, downsampling should be avoided for TFB. It has been shown for the B-meson factory KEKB (Tsukuba, Japan), that a digital 2-tap FIR filter with coefficients $c_0 = 1$ and $c_1 = -1$ is sufficient, if the delay between the measurements is adapted to the respective betatron tune [167]. Such a simple filter can be built using fast digital components without employing microprocessors. A prototype was built and tested at the TRISTAN accumulator ring [168]. In KEKB, the same electronics is employed for damping systems against longitudinal as well as transverse instabilities [169]. Similar to BESSY II, the longitudinal kicker of KEKB is derived from the DAΦNE kicker cavity [135] and the transverse kicker is a stripline structure.

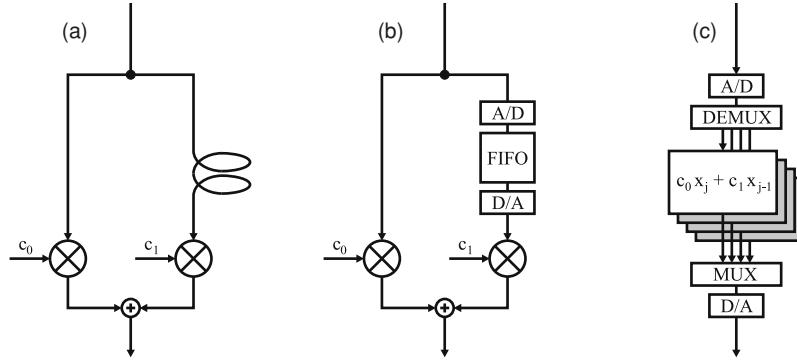


Fig. 4.28. Different implementations of the function $c_0 x_j + c_1 x_{j-1}$ of a 2-tap filter: (a) purely analog, (b) analog multiplication and addition combined with a digital delay in a first-in/first-out (FIFO) register, (c) purely digital system, where the data are typically distributed over an array of processors (de-multiplexed) and subsequently recombined (multiplexed)

- Field-programmable gate arrays (FPGAs, see e.g., [176]) are used in the subtraction stage of the KEKB feedback system, and may be considered to perform more complex filter functions. FPGAs are integrated circuits consisting of typically $10^3 - 10^4$ interconnected logical gates that can be programmed to perform complex functions. Potential advantages over DSPs are processing speed and ease of programming.
- While the electronics of the feedback system described in Sect. 4.3 was developed by several collaborating laboratories, DSP boards that can handle bunch rates of 500 MHz are nowadays commercially available. An example is the feedback system of the TLS in Taiwan, where longitudinal data are processed with a downsampling factor of 20 [170, 171]. Here, an unusual kicker structure was developed, combining a cavity with an arrangement of striplines [172].
- In a collaboration between the synchrotron radiation laboratories ELETTRA (Trieste, Italy) and SLS (Villigen, Switzerland), a feedback system was developed that can handle the full bunch rate of 500 MHz with a 5-tap filter running on commercially available DSP boards [173]. The system is designed to damp transverse and longitudinal instabilities at both synchrotron light sources. In order to make full use of the ADC range, the transverse system requires analog circuitry to reject a static position offset before digitization (common mode rejection) [174]. Here, the transverse kickers are separate stripline pairs for horizontal and vertical deflection, and the generic design of the longitudinal kicker is again that of the DAΦNE kicker cavity [175].

Finally, Fig. 4.28 shows different variations (analog, hybrid, digital) of a 2-tap filter. Faster microprocessors and commercially available user-friendly DSP

boards establish a clear tendency towards digital signal processing for future feedback systems.

4.6 Higher-Harmonic Cavities

4.6.1 Overview

Electron–electron scattering within a bunch is the dominant loss mechanism in third-generation synchrotron radiation sources (Touschek effect, see Sects. 3.4 and Appendix F). The only way to reduce the scattering rate without enlarging the beam transversely or in energy, i.e., without decreasing the brilliance of the emitted synchrotron radiation, is to lengthen the electron bunches using higher-harmonic cavities. Longitudinal single-bunch instabilities increase the bunch length *and* the energy width. For a sufficiently low bunch charge, at which this type of instability can be ignored, the bunch length is given by the shape of the rf potential. Making the rf potential more shallow increases the bunch length for a given energy width. The energy width itself is determined by the properties of the emitted radiation, i.e., by the electron energy and the magnetic guide fields (see Sect. A.3.2).

The rf potential can be modified by adding a voltage which is in phase with the accelerating rf voltage and has a frequency of $n f_{\text{rf}}$, i.e., an integer multiple of the rf frequency. This “harmonic” voltage can be provided in an “active” cavity by an rf transmitter or in a “passive” cavity by beam loading, where the energy to generate this voltage is drawn from the beam and eventually from the accelerating rf system (see Sect. D).

Higher-harmonic cavities can lengthen particle bunches in order to improve the beam lifetime but they can also shorten bunches if desired. Since the nonlinear shape of the modified rf potential causes additional Landau damping (Sect. 2.5.3), higher-harmonic cavities are also called Landau cavities, if used for this purpose. On the other hand, bunch lengthening is associated with a decrease of the synchrotron frequency, which has a destabilizing tendency. In synchrotron radiation sources, the main purpose of higher-harmonic cavities is to improve the beam lifetime.

In recent years, higher-harmonic cavities with $n = 5$ [177], $n = 4$ [178, 179] and $n = 3$ [180–184] were implemented or considered [185, 186] at synchrotron radiation sources. In [185], even higher harmonics were studied but the choice of n is practically limited by the cut-off frequency of the vacuum chamber. The cavities considered here are usually passive systems and mostly normal-conducting devices. The quality factor of superconducting cavities [187] is about 10^4 times higher. At the BESSY II storage ring, four normal-conducting passive cavities ($n = 3$, third-harmonic cavities) are in use since 1999 [183]. Later, HOM dampers were added [188], and work to replace the normal-conducting cavities by one superconducting device is underway [189].

4.6.2 Operation of Passive Cavities

The combination of accelerating rf cavities of frequency ω_{rf} and higher-harmonic cavities of the order n yields a total voltage of

$$U(z) = U_0 \sin(k_{\text{rf}}z + \phi_s) + r U_0 \sin(n k_{\text{rf}}z + n \phi_h), \quad (4.25)$$

where $k_{\text{rf}} = \omega_{\text{rf}}/c$ is the wave number and z is the longitudinal position in a coordinate system moving with the bunch. U_0 and $r U_0$ are the amplitudes of the accelerating voltage and harmonic voltage, respectively. Their phases are given by ϕ_s and ϕ_h , where ϕ_s is the usual synchronous phase angle. Figure 4.29 shows an example with $n = 3$, $r = 0.25$, and $\phi_h = 0$. The rf potential is given by (see e.g., [190])

$$\begin{aligned} \Phi(z) &= -\frac{\eta}{C E} \int_0^z [eU(z') - V^S] dz' \\ &= -\frac{\eta c U_0}{C E \omega_{\text{rf}}} \{\cos \phi_s - \cos(k_{\text{rf}}z + \phi_s)\} \\ &\quad - \frac{\eta c U_0}{C E \omega_{\text{rf}}} \frac{r}{n} \{\cos n \phi_h - \cos(n k_{\text{rf}}z + n \phi_h)\} + \frac{\eta V^S}{C E} z, \end{aligned} \quad (4.26)$$

where η is the momentum compaction factor, C is the storage ring circumference, and E is the beam energy. The energy loss per revolution V^S is mainly caused by the emission of synchrotron radiation. The distribution of the electron density in the bunch is [191, 192]

$$\rho(z) = \rho_0 \exp\left(\frac{-\Phi(z)}{\eta^2 \sigma_E^2}\right), \quad (4.27)$$

where σ_E is the relative energy width and ρ_0 is a normalization constant. The beam lifetime is proportional to the rms value of the distribution, ignoring

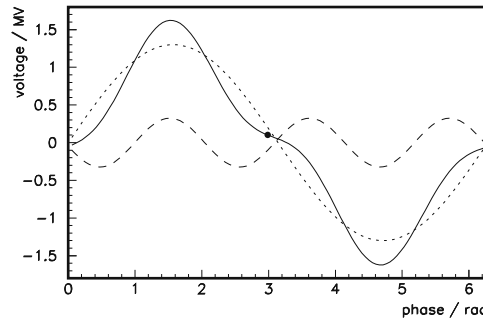


Fig. 4.29. Voltage of accelerating cavities (*dotted line*), harmonic cavities (*dashed line*), and their sum (*solid line*) as function of the rf phase. In this example, the amplitude ratio is $r = 0.25$. A dot marks the bunch position

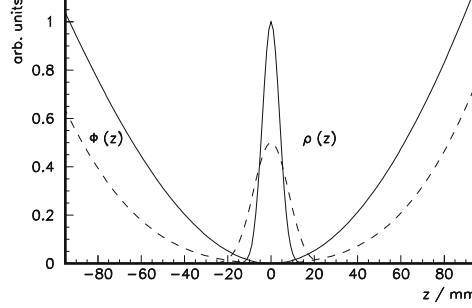


Fig. 4.30. Potential Φ and charge distribution ρ in arbitrary units as function of the longitudinal coordinate z without harmonic cavity (*solid line*), and for a voltage ratio $r = 0.25$ (*dashed line*)

a small decrease of the momentum acceptance of the modified rf potential. As shown in Fig. 4.30, the parameters used in the example of Fig. 4.29 nearly double the bunch length.

In the case of passive cavities, amplitude $U_h = r U_0$ and phase $\Psi_h = \pi/2 + n \phi_h$ of the harmonic voltage cannot be tuned independently. For an even fill of the storage ring, the following relations hold [191]:

$$U_h(z) = 2 I F R_h \cos \Psi_h \cos (n k_{rf} z - \Psi_h) . \quad (4.28)$$

Here, R_h is the shunt impedance and the formfactor

$$F = e^{-(n \omega_{rf} \sigma_\tau)^2} \quad (4.29)$$

accounts for the finite bunch length σ_τ (in units of time). The phase angle Ψ_h is a function of the eigenfrequency ω_r of the harmonic cavity that can be tuned by a mechanic plunger:

$$\tan \Psi_h = \frac{2Q_h}{\omega_r} (\omega_r - n \omega_{rf}) , \quad (4.30)$$

where Q_h is the quality factor of the harmonic cavity.

4.6.3 Harmonic Cavities and Feedback Systems

The experience of operating a storage ring with higher-harmonic cavities and bunch-by-bunch feedback systems shows that individual measures against collective phenomena cannot be considered independently, but are linked to each other in a complex way. Consequences of higher-harmonic cavities on the LFB are:

- Harmonic cavities have HOMs that can excite the beam considerably. These modes must be shifted by plungers or by controlling the cavity temperature such that their overlap with the beam spectrum is small. HOMs can be partly removed by implementing damping antennas.

- Making the rf potential more shallow reduces the synchrotron frequency. The filter algorithm of a LFB must be adapted accordingly. In the case of passive cavities, the reduction of the synchrotron frequency depends on the beam current.
- The nonlinearity of the modified rf potential creates a broad distribution of synchrotron frequencies that is another challenge for the filter algorithm. A possible remedy is to employ a more broad-banded IIR filter rather than the usual FIR filter [193].
- In the presence of a bunch gap which is used for ion clearing (see Appendix G), the synchronous phase varies along the bunch train due to beam loading (Sect. D.2.2). The accelerating cavities at BESSY II cause a phase variation of roughly 50 mrad at a beam current of 200 mA, while higher-harmonic cavities can cause variations of several 100 mrad [129]. This causes several problems for the operation of a LFB:
 - At the pickup, the different arrival time of different bunches is detected as additional DC phases. Detection at 3 GHz, for example, is not unique at deviations above 260 mrad (with respect to $f_{\text{rf}} = 500$ MHz) and even changes sign above 520 mrad.

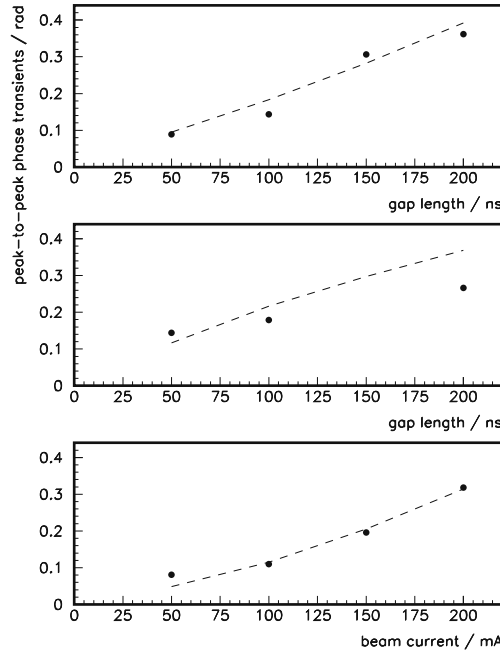


Fig. 4.31. Peak-to-peak variation of synchronous phase angles as function of the duration of one gap (*top*) and two opposite gaps (*center*), and as function of beam current (*bottom*). The *dots* are data measured with a streak camera [194] the *lines* are simulation results

- For a digital system, the range of the A/D converter may be exceeded.
- At the kicker, the variation of arrival time causes a variation of the kick voltage. With a carrier frequency of 1375 MHz in the BESSY II case, the kick voltage would be zero for a synchronous phase deviation of 570 mrad.

It may help to change the detection frequency to a smaller value which, however, would decrease the signal-to-noise ratio. The problem of synchronous phase variation is illustrated by Fig. 4.31 where calculated and observed variations are plotted as function of the width (in ns) of one or two bunch gaps and as function of the beam current. Clearly, the gap should be as short as feasible. Introducing a second gap does not significantly reduce the phase variations.

In the case of the BESSY II ring, the ion-clearing gap was reduced to 10% of the circumference, i.e. to 80 ns, as the vacuum steadily improved. Furthermore, the higher-harmonic cavities are operated at a moderate voltage which yields a significant improvement in lifetime while keeping the longitudinal feedback operational.

The variation of the synchronous phase amplified by the presence of higher-harmonic cavities also disturbs the position measurement of the TFB at 3 GHz. Here, it is possible to change the detection scheme from a fixed reference signal (heterodyne detection) to a reference signal that is variable in time (homodyne detection), which was successfully tested at the ALS [195] and at BESSY II [204]. The variation of the arrival time at a transverse stripline kicker is less critical than in the longitudinal case, since the correction signal is applied in the baseband between zero and $f_{\text{rf}}/2$.

Conclusions and Outlook

5.1 Summary

Novel storage rings such as third-generation synchrotron radiation sources and meson factories were built during the 1990s. The tendency toward ever higher intensity – radiation brilliance in one case, luminosity in the other – made active measures against collective effects indispensable. At the time of writing (2005), further storage-ring based synchrotron light sources with beam energies around 3 GeV have been completed [196], are under construction [197–199] or in the design phase [200–203].

In addition to a careful design of the storage ring, elaborate feedback systems are required to counteract multi-bunch instabilities. Two examples of feedback systems – one acting on longitudinal, the other on transverse instabilities – are described in Sects. 4.3 and 4.4. Beam stability and lifetime can be conflicting requirements. Particularly when the lifetime is dominated by the Touschek effect, an excited beam may have a longer lifetime than a stable one. Without loss of beam quality, the Touschek loss rate can only be reduced by artificially lengthening the bunches using higher-harmonic cavities. However, harmonic cavities may have adverse effects on the operation of feedback systems as discussed in Sect. 4.6.3, and the two must be reconciled.

Table 5.1 gives an overview of third-generation synchrotron radiation sources and meson factories at which feedback systems and harmonic cavities are currently employed or under development. Most of the synchrotron light sources mentioned here are facilities of moderate beam energy around 2 GeV, since storage rings operating at higher energy are less prone to collective effects as long as their beam current is only a few 100 mA.

In view of the high costs of constructing a synchrotron radiation facility, a reduction of the beam quality due to collective effects is not acceptable. Even so, compromises like increasing the transverse beam size in order to improve the lifetime are common practice. The countermeasures discussed in Chap. 4 significantly improve the performance, and the investment is small compared to the total costs of a facility. The actual improvement depends in detail on

Table 5.1. Overview of installed or planned (in brackets) longitudinal or transverse feedback systems (FB) against collective instabilities, as well as harmonic cavities (NC=normal, SC=superconducting) at third-generation synchrotron light sources and meson factories

facility	longitudinal FB	transverse FB	harm. cavities
ALS	digital [128]	analog [153]	NC [205]
BESSY II	digital [129]	analog [155]	NC (SC) [183, 189]
DAΦNE	digital [206]	digital [207]	(NC) [208]
ELETTRA	digital [209]	digital [209]	SC [210, 211]
ESRF	–	analog [212]	–
KEKB	digital [169]	digital [213]	–
MAX II	–	–	NC [180]
PEP II	digital [214]	hybrid [215]	–
PLS	digital [131]	analog [216]	(SC) [217]
SLS	(digital) [218]	digital [218]	SC [210]
SRRC	(digital) [171]	analog [154]	(NC) [219]

numerous parameters such as beam current, chromaticity, the impedance budget, and the insertion device properties. Typically, higher-harmonic cavities can double the beam lifetime, feedback systems improve the brilliance by an even larger factor.

5.2 Future Synchrotron Radiation Facilities

The large number of synchrotron radiation sources being constructed or planned shows that there is a huge demand for vacuum-ultraviolet (VUV) and X-ray radiation which can be met by the present-day technology of storage-ring-based sources. On the other hand, there is also a strong motivation to improve the quality of synchrotron radiation sources and to add features which may open up new frontiers of physics. Depending on the particular application, the catalog of quantities to improve includes:

- Average and peak brilliance
- Beam lifetime
- Short- and long-term orbit stability
- Duration of the radiation pulses
- Longitudinal and transverse coherence
- Compactness and cost-effectiveness

While third-generation synchrotron radiation sources have matured, the development of novel sources evolves at breathtaking speed, and the brief overview given in Sect. 5.2.1 may soon be outdated by new ideas and experimental achievements.

5.2.1 Storage Rings

It is envisaged that the brilliance of storage-ring based light sources can still be increased by two orders of magnitude [220]. Much of this increase is due to a reduction of the horizontal beam emittance which scales approximately according to $\varepsilon_x \sim E^2 \Theta^3$, where E is the electron energy and Θ is the bending angle of a single dipole magnet (e.g., [59]). Looking more closely, the emittance depends also on the adopted lattice and on the radiation from insertion devices, but the simple rule-of-thumb demonstrates the necessity to keep the beam energy moderate and to reduce Θ , i.e., increase the number of dipole magnets. Synchrotron radiation sources are usually designed as a sequence of achromatic or nearly-achromatic sections comprising two or three dipole magnets each, and the number of dipoles is increased by adding achromats. In essence, a lower emittance implies a larger storage ring, which also has the potential benefit of serving more beamlines simultaneously.

Beyond the diffraction limit $\varepsilon \approx \lambda/4\pi$ for radiation of wavelength λ (e.g., [221]), a further reduction of emittance yields no significant improvement. Assuming a circumference of 2 km as a practical limit, the horizontal emittance would be of the order of 10^{-10} rad m, and the diffraction limit for radiation of $\lambda \geq 0.1$ nm suggests to aim at a vertical emittance of 10^{-11} rad m.

Another way to increase the brilliance is to raise the beam current. While synchrotron light sources typically operate at a few 100 mA, the meson factories have demonstrated that a beam current of 1 A or more is feasible and the beam stability can be controlled using feedback systems.

Finally, the brilliance depends on the radiating device. For a wiggler, it is proportional to the number of periods, for an undulator even to the number of periods squared. Yet another option is the free-electron laser (FEL), where the interaction of the electron beam with undulator radiation gives rise to stimulated emission [222, 223]. Contemporary storage ring FELs are “oscillators,” where a radiation field in an optical cavity, i.e., between two mirrors, builds up as electron bunches interact repeatedly with the field. Several storage-ring FELs were built since the 1980s, operating in the infrared, visible, and UV regime – an overview is given in [224]. The use of mirrors at normal incidence limits the photon energy to about 6 eV (or a wavelength above ~ 200 nm). Another serious limitation is the effect of the FEL itself on the electrons by increasing their energy spread, which in turn reduces the FEL gain. Eventually, the process stops and restarts only after radiation damping has reduced the energy spread sufficiently.

Decreasing the emittance and raising the beam current implies a reduced Touschek lifetime which triggers the demand for a continuous – or very frequent – injection to keep the beam current constant. This so-called top-up operation is already practiced at a few synchrotron light sources [75–77], and is under study at others, e.g., [225]. Keeping the beam current and thus the heat load at a constant level is also a prerequisite to achieve sub-micrometer stability in the transverse coordinates [226].

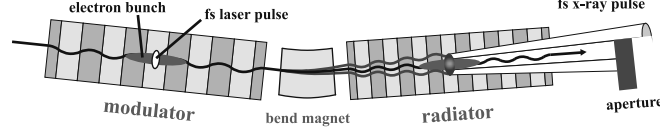


Fig. 5.1. Generation of ultrashort X-ray pulses via “femtosing” [231]: A femtosecond laser pulse, co-propagating with an electron bunch in an undulator (modulator) causes a periodic energy modulation within a short “slice” of the bunch. After passing a bend magnet, these electrons are transversely displaced and their radiation from a second undulator (radiator) can be selected by an aperture

There is growing interest in ultrashort VUV- or X-ray pulses. Visible laser pulses with a duration below 100 fs (fwhm) were available since the mid-1980s [227], while the bunch length in electron storage rings is almost three orders of magnitude larger and limited by the “natural” bunch length (A.43) as well as longitudinal mode coupling [228]. Reducing the bunch length by lowering the momentum compaction factor according to $\sigma_z \sim \sqrt{\eta}$ – insert (A.43) into (A.35) – only works when the bunch current is reduced below the mode-coupling threshold, as shown in Fig. 2.27. At BESSY, a bunch length below 1 ps (rms) has been demonstrated with a bunch current of 3×10^{-4} mA at a repetition rate of 500 MHz [229], and stable user operation with a bunch length of a few ps is offered on a regular basis [230].

The time resolution can also be improved by correlating time and transverse coordinate when tilting an electron bunch in the dipole field of an rf cavity [235]. In principle, the full bunch current and repetition frequency can be employed. The application of this technique to several synchrotron radiation sources is under study [236, 237], aiming at 1 ps time resolution.

The generation of ~ 100 -fs pulses by energy modulation of electrons with femtosecond laser pulses and their subsequent transverse displacement [231] (see Fig. 5.1) was demonstrated experimentally at the ALS (Berkeley, USA) [232], producing ultrashort radiation from a bend magnet. A first undulator-based facility was constructed at BESSY [233, 234]. Using this “femtosing” technique, the photon rate is limited by the fact that only $\sim 10^{-4}$ of the electron bunch contributes to the radiation and by the kHz repetition rate of the laser system. On the other hand, no other storage-ring-based technique presently offers a pulse duration on the 100 fs scale.

Another R&D activity is dedicated to the development of compact and low-cost facilities for industry, hospitals, or universities. Several compact storage rings were already built [238]. A light source using a miniature electron storage ring and a “laser undulator” – where laser photons are Compton-scattered from counter-propagating electrons [239] – is under development [240]. Extremely compact linear accelerators with rf frequencies around 100 GHz were also proposed (e.g., [241]).

5.2.2 Linear Accelerators

While the beam emittance increases quadratically with the Lorentz factor γ in a storage ring, it reduces during acceleration in a linear accelerator (“linac”) according to $\varepsilon = \varepsilon_n/\gamma$, where ε_n is the “normalized” emittance given by the electron source. The development of rf photoinjectors [242], where a cold photocathode inside an rf cavity emits electrons when illuminated by short laser pulses, has made electron bunches of high charge ($\sim$ nC) and a normalized emittance ε_n of the order of 10^{-6} rad m possible. Injectors with room-temperature cavities can operate at kHz rates with a duty cycle of a few percent, whereas continuous operation will be achievable with superconducting rf photoinjectors, which are currently under development (e.g., [243]).

Accelerating electron bunches off the crest of the linac voltage creates a correlation between energy and longitudinal coordinate. In a magnetic chicane, in which the path length varies with energy (“bunch compressor”), the bunch length can be reduced below 100 fs. Since emittance, bunch length and energy spread do not reach equilibrium values as in a storage ring, they are entirely given by the electron source and the processes during acceleration, and are insensitive to instabilities with rise times exceeding the few μ s spent between gun and beam dump. Thus, smaller bunch dimensions can be achieved in all phase space coordinates.

Several types of linac-based radiation sources are under consideration:

Short Pulses of Spontaneous Undulator Radiation

A combination of linac, bunch compressors and an undulator can be employed to produce ultrashort X-ray pulses. The Sub-Picosecond Photon Source (SPPS) at SLAC (Sunnyvale, USA) [244] makes temporarily use of the existing linac to lay groundwork for ultrafast science at FEL-based sources (see below). One issue, for example, is to determine the pulse arrival time by correlation techniques such as electro-optical sampling [245]. The SPPS produces X-ray pulses at a wavelength of 0.15 nm with a duration of 80 fs (fwhm), 10^7 photons per pulse and a repetition rate of 1–10 Hz.

Energy-Recovery Linacs

With a charge of a few nC and kHz bunch rates, the average beam current would be in the μ A regime. A beam current comparable to that of a storage ring (several 100 mA) could be achieved in continuous-wave operation of a linac, where the maximum bunch rate would be equal to the rf frequency as in storage rings. This, however, would imply klystrons delivering a power in the GW range to the beam, unless the beam energy is recovered after the electrons have served their purpose. In energy-recovery linacs (ERLs), the bunches travel through the accelerator once more after X-ray emission and – being shifted by 1/2 rf period and thus decelerated – return their energy

to the rf field. This is essentially the beam loading mechanism described in Appendix D: If bunches are accelerated in an rf cavity, they induce a decelerating voltage which requires additional energy, but if the bunches are decelerated, the induced voltage is in phase with the rf voltage and energy is returned to the cavity. This way, the power requirements as well as the radiation hazard at the beam dump can be greatly reduced. Energy recovery in combination with infrared FEL oscillators has been demonstrated at the Jefferson Laboratory (Newport News, USA) [246], at JAERI (Tokai, Japan) [247], and at BINP (Novosibirsk, Russia) [248], and there are several proposals for ERLs to produce high-brilliance X-ray radiation [249–251]. It can be cost-efficient to accelerate the beam more than once in the same linac. An example of a scheme combining recirculation and energy recovery is given in [252].

Single-Pass Free-Electron Lasers

Instead of (or in addition to) raising the average beam current, coherent radiation allows to increase the photon output from a given number of electrons by many orders of magnitude. In single-pass FELs, the quasicontinuous electron distribution within a bunch is converted to a sequence of “microbunches,” i.e., slices separated by one radiation wavelength, without employing an optical cavity. While the intensity of conventional synchrotron radiation is proportional to the number of contributing electrons, coherent radiation from microbunches is proportional to the number of electrons squared. It is expected that FELs will exceed the peak brilliance of present-day synchrotron radiation sources by 9–10 orders of magnitude and the average brilliance by about 4 orders of magnitude, e.g., [253, 271]. In addition, they offer a pulse duration in the femtosecond regime (particularly when seeded by femtosecond laser pulses, see below) and nearly full coherence, which will enable e.g., holographic imaging [254].

The FEL process is either initiated by spontaneous radiation from the first part of a long undulator – this is the SASE (self-amplified spontaneous emission) principle [223] – or by a seed pulse. The SASE process has been demonstrated at infrared and visible wavelengths at UCLA (Los Angeles, USA) [255], ANL (Argonne, USA) [256], and BNL (Brookhaven, USA) [257] and down to 30 nm wavelength at DESY (Hamburg, Germany) [258, 259]. Due to the random nature of this process, the spectrum and time structure of the radiation is spiky and varies from shot to shot. Seeding, on the other hand, leads to a smoother spectrum and allows to control the shape and duration of the radiation pulse. The question is (1) how to produce X-ray seed pulses or (2) how to initiate radiation at X-ray wavelength with laser pulses in the visible regime. Several schemes to this end have been invented, see e.g., [260] for case (1) and [261–263] for case (2). Among them, high-gain harmonic generation (HGHG) [261], has been demonstrated experimentally at the BNL [264]. A list of existing and proposed FELs can be found in [265],

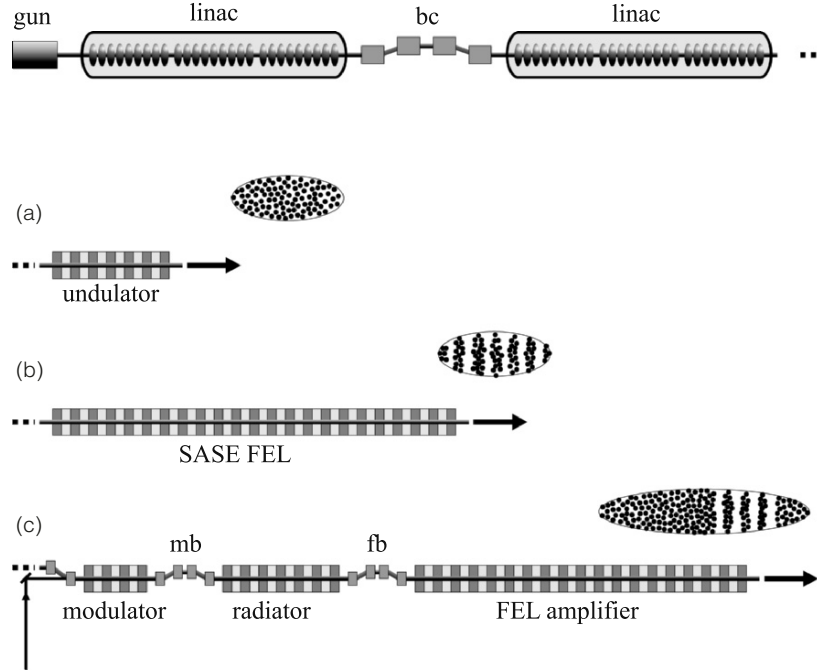


Fig. 5.2. Schematic view of radiation sources using short electron bunches from a combination of linear accelerator modules and one or more bunch compressors (bc): (a) Conventional undulator. (b) FEL based on self-amplified spontaneous emission (SASE), where spontaneous radiation at the beginning of a long undulator leads to microbunching which gives rise to coherent radiation. (c) Seeded FEL using high-gain harmonic emission. Interaction with a laser in an undulator (modulator) causes a periodic energy modulation which is converted into microbunching by a magnetic chicane (mb). Radiation from a subsequent undulator (radiator) at a higher harmonic of the laser wavelength seeds microbunching. The length of the seeded region is defined by the laser pulse duration and its position by a “fresh-bunch” chicane (fb). The FEL amplifier may be preceded by more than one modulator–radiator stage

comprising several SASE-FELs in the wavelength regime of 1 nm [266] and below [267–270] (partly considering a seeding option), as well as HGHG-FELs, also in the 1 nm regime [271, 272] and below [273].

Options of linac-based radiation sources (conventional undulators, SASE-FELs, and seeded FELs) are schematically sketched in Fig. 5.2.

5.2.3 Collective Effects

Collective effects occurring in future storage-ring-based synchrotron light sources are essentially those discussed in this book, albeit aggravated by higher beam current, reduced bunch dimensions or smaller undulator gaps. As for

collective instabilities, the development of future feedback systems profits from the experience gained at present-day light sources and meson factories.

Beam lifetime reductions due to higher electron densities and smaller undulator gaps are apparently overcome by frequent injection (top-up operation), but there are limits to the tolerable loss rate, e.g.,

- Acceptable injection frequency and duration for a given injection rate
- Legal limits for the radiation level in the experimental area
- Radiation hardness of insertion device magnets

Radiation issues can be improved by suitable collimators designed to concentrate the particle losses in particularly shielded regions.

The discussion of collective effects in linac-based synchrotron radiation sources is beyond the scope of this book, but some differences to storage rings shall be outlined:

- Single-pass FELs constitute additional demands on beam emittance and peak current.
- The beam exists only for one or a few μs . Thus, the lifetime issues and multi-bunch instabilities discussed in this book are not relevant. There is a transverse distortion of a bunch travelling in a linac called beam breakup (e.g., [7]), which resembles the head–tail interaction described in Sect. 2.4.2, except that head and tail never swap their position – there is no synchrotron oscillation.
- The beam energy is not constant, but starts almost at zero, where the beam emittance is strongly influenced by space charge effects.
- Using bunch compressors, the electron bunches are much shorter than in storage rings. For a bunch length of the order of $100\,\mu\text{m}$ and below, the surface roughness of the vacuum chamber [274–276] and even the effect of thin oxide layers in the case of aluminium chambers can be relevant [277]. Furthermore, coherent synchrotron radiation (CSR) in the bunch compressors starting from small electron density variations gives rise to a microbunching instability, which has been observed [278] and studied by simulation and analytically [279, 280].

A

Basic Relations in Accelerator Physics

A.1 General Conventions

The following brief review of accelerator physics relations defines the conventions and nomenclature used in this book. For a more profound introduction to accelerator physics, the reader is referred to textbooks, e.g., [59, 60, 81] or to review articles [190, 281].

For small deviations from the ideal orbit in an accelerator or storage ring, the motion of particles is conveniently described as depicted in Fig. A.1 by the following coordinates:

- s is the coordinate along the orbit, starting at a fixed position.
- z is the coordinate tangential to the orbit relative to an idealized particle.
- $\delta_p \equiv \Delta p/p$ is the relative deviation from the design momentum $|p| \equiv p$.
- x is the horizontal coordinate perpendicular to the orbit.
- $x' \equiv dx/ds \approx p_x/p$ is the horizontal momentum coordinate.
- y is the vertical coordinate perpendicular to the orbit.
- $y' \equiv dy/ds \approx p_y/p$ is the vertical momentum coordinate.

The coordinates x and y are called “transverse.” The “longitudinal” coordinate z is often expressed as deviation $\Delta\phi = \omega_{\text{rf}}z/c$ from the design (or “synchronous”) phase of the rf voltage with frequency ω_{rf} . Instead of δ_p , the relative energy deviation $\delta_E \equiv \Delta E/E$ may be used.

The origin of s is chosen arbitrarily at some point along the orbit. The origin of x , y , and z coincides with the position of an ideal particle defined by the fact that all deviating particles perform oscillations about this position: by tradition, the transverse motion is called *betatron* oscillation, the longitudinal motion (which is also a periodic variation of kinetic energy) is called *synchrotron* oscillation.

The present discussion is restricted to “flat” machines, i.e. the ideal orbit lies in the horizontal plane. A particle is completely determined by the six phase space coordinates shown in Fig. A.2. In the linear approximation, there is a transport matrix (or transfer matrix) that describes how the vector formed

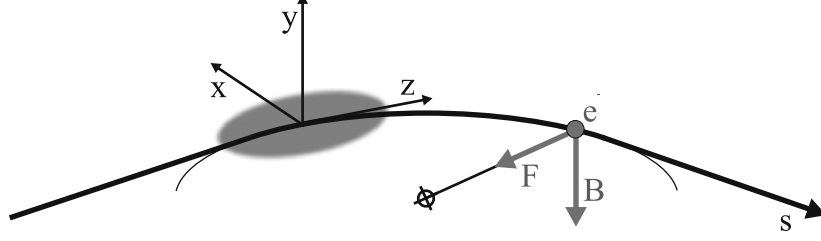


Fig. A.1. Coordinate system (x, y, z) copropagating with a particle bunch along the orbit. The z -coordinate is always tangential to the ideal orbit. In the presence of a magnetic field B , acting with a force F on each beam particle, the orbit is bent and the coordinate system rotates accordingly

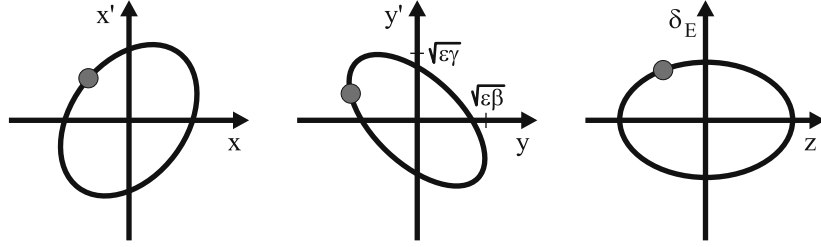


Fig. A.2. Beam particles move on phase space ellipses, here shown for the 2-dimensional phase space of the horizontal (*left*), vertical (*center*), and longitudinal (*right*) coordinates. A transverse ellipse is characterized by a beta function β , its derivative, usually expressed as $\alpha \equiv -\beta'/2$, and by $\gamma \equiv (1 + \alpha^2)/\beta$. It is tilted for $\beta' \neq 0$, whereas the longitudinal phase space ellipse is always upright

by these coordinates evolves when the particle moves from position $s = s_0$ to s_1 along the orbit:

$$\begin{pmatrix} x \\ x' \\ y \\ y' \\ z \\ \delta_E \end{pmatrix}_{s_1} = \begin{pmatrix} C_x & S_x & 0 & 0 & 0 & d_x \\ C'_x & S'_x & 0 & 0 & 0 & d'_x \\ 0 & 0 & C_y & S_y & 0 & 0 \\ 0 & 0 & C'_y & S'_y & 0 & 0 \\ r_c & r_s & 0 & 0 & 1 & r_d \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ x' \\ y \\ y' \\ z \\ \delta_E \end{pmatrix}_{s_0}. \quad (\text{A.1})$$

For a vanishing distance $s_1 - s_0$, the “cosine-like” matrix elements C and S' are 1, and the “sine-like” elements S and C' are 0. The 2×2 submatrices indicate that horizontal and vertical and phase space (Fig. A.2) are usually decoupled, provided there are no tilted (“skew”) quadrupole fields. The presence of horizontal arcs with radius $R(s)$, however, links horizontal and longitudinal phase space, making the horizontal position energy-dependent via d_x and d'_x – see (A.14) – and causing deviations in z described by

$$r_c = \int_{s_0}^{s_1} \frac{C_x(s)}{R(s)} ds \quad r_s = \int_{s_0}^{s_1} \frac{S_x(s)}{R(s)} ds \quad r_d = \int_{s_0}^{s_1} \frac{d_x(s)}{R(s)} ds. \quad (\text{A.2})$$

Equation (A.1) ignores changes of δ_E and is only valid for one or a few turns. To describe a full synchrotron oscillation over typically 10^2 turns, not only the magnetic properties of a machine as expressed by transport matrices but also the rf voltage must be taken into account (see Sect. A.3).

A.2 Transverse Phase Space

A.2.1 Solutions of the Linear Equations of Motion

The vertical component B_y of the transverse magnetic guide fields may be expanded as

$$\frac{e}{p} B_y(x) = \frac{e}{p} B_{y0} + \frac{e}{p} \frac{dB_y}{dx} x + \frac{1}{2} \frac{e}{p} \frac{d^2 B_y}{dx^2} x^2 + \dots = \frac{1}{R} + k \cdot x + \frac{1}{2} m \cdot x^2 + \dots \quad (\text{A.3})$$

The first term is the inverse bending radius for a particle with charge e and momentum p in a dipole field. For “flat” machines, there is no such term in the horizontal field components. The second term describes a quadrupole field with focusing strength k , the third term corresponds to a sextupole field of strength m . Linear particle optics is restricted to the first two terms, leading to the transverse equations of motion:

$$x''(s) + \left(\frac{1}{R^2(s)} - k(s) \right) x(s) = \frac{\delta_E}{R(s)} \quad (\text{A.4})$$

$$y''(s) + k(s) y(s) = 0. \quad (\text{A.5})$$

A dash denotes the derivative with respect to s . The sign of k is a matter of convention. Here, $k < 0$ denotes a horizontally focusing and vertically defocusing quadrupole magnet. The solution of the equations of motion can be expressed in several ways. The general solution is a transverse oscillation about the ideal orbit, e.g., in the horizontal plane:

$$x(s) = \sqrt{\varepsilon_x \beta_x(s)} \cos \mu_x(s) + D_x(s) \delta_E \quad (\text{A.6})$$

Here, ε_x is the Courant–Snyder invariant or “single-particle” emittance, $\beta_x(s)$ is the amplitude function or beta function, $\mu_x(s)$ is the betatron phase, and $D_x(s)$ is the dispersion function.

For particles with an energy deviation $\delta_E \neq 0$, the ideal orbit shifts horizontally by $D_x(s) \delta_E$, where the dispersion function $D_x(s)$ is the solution of (A.4) divided by δ_E . The beta function and the functions

$$\alpha_x(s) \equiv -\frac{\beta'_x(s)}{2}, \quad (\text{A.7})$$

$$\gamma_x(s) \equiv \frac{1 + \alpha_x^2(s)}{\beta_x(s)}, \quad (\text{A.8})$$

are called optical functions. At a location s , where their values are sometimes referred to as Twiss parameters, they define an ellipse

$$\varepsilon_x = \gamma_x(s) x^2(s) + 2\alpha_x(s) x(s) x'(s) + \beta_x(s) x'^2(s), \quad (\text{A.9})$$

in phase space. If $\alpha_x \neq 0$, the ellipse is tilted as shown in Fig. A.2. In the framework of this formalism [282, 283], each particle moves in horizontal and vertical phase space along an ellipse. The shape of these two ellipses changes with s , but their respective enclosed area $\pi\varepsilon_{x,y}$ is invariant. The phases $\mu_{x,y}$ denote the advance of the particle position along the horizontal or vertical ellipse and depend on the respective beta function, e.g., horizontally

$$\mu_x(s) = \int_0^s \frac{d\tilde{s}}{\beta_x(\tilde{s})}. \quad (\text{A.10})$$

An energy deviation δ_E changes the length L of the orbit, mainly due to different arc lengths in dipole fields. The quantity

$$\alpha = \frac{\Delta L/L}{\delta_E} = \frac{1}{L} \oint \frac{D_x(s)}{R(s)} ds \quad (\text{A.11})$$

is called *momentum compaction factor*. The change in revolution time is described by the slippage factor

$$\eta = \frac{\Delta T/T}{\delta_E} = \left(\alpha - \frac{1}{\gamma^2} \right), \quad (\text{A.12})$$

where $\gamma = E/m$ is the Lorentz factor, i.e., the total energy of the particle divided by its rest mass. Typical values for synchrotron radiation sources are $\alpha \sim 10^{-3}$ and $1/\gamma^2 \sim 10^{-6} \dots 10^{-8}$. In this case, the difference between α and η is negligible.

Another formulation of the solution of (A.4) and (A.5) is given by a specific transfer matrix for each magnetic element. For a path involving several magnets (e.g., the whole orbit around storage ring), the transfer matrix is calculated by multiplying the matrices of the individual elements. The horizontal and vertical 2×2 submatrices of (A.1) are:

$$\begin{array}{cc} \text{horizontal} & \text{vertical} \\ \begin{pmatrix} 1 & \Delta s \\ 0 & 1 \end{pmatrix} & \text{drift space} \\ & \text{i.e., no field} \end{array} \begin{pmatrix} 1 & \Delta s \\ 0 & 1 \end{pmatrix},$$

$$\begin{pmatrix} \cos \Omega & \frac{1}{\sqrt{|k|}} \sin \Omega \\ -\sqrt{|k|} \sin \Omega & \cos \Omega \end{pmatrix} \begin{array}{c} \text{quadrupole} \\ \text{(here : } k < 0 \text{)} \end{array} \begin{pmatrix} \cosh \Omega & \frac{1}{\sqrt{|k|}} \sinh \Omega \\ \sqrt{|k|} \sinh \Omega & \cosh \Omega \end{pmatrix}$$

$$\begin{pmatrix} \cos \frac{\Delta s}{R} & R \sin \frac{\Delta s}{R} \\ -\frac{1}{R} \sin \frac{\Delta s}{R} & \cos \frac{\Delta s}{R} \end{pmatrix} \quad \begin{array}{l} \text{"weak"} \\ \text{focusing} \end{array} \quad \begin{pmatrix} 1 & \Delta s \\ 0 & 1 \end{pmatrix} \\
\begin{pmatrix} 1 & 0 \\ \frac{\tan \Psi}{R} & 1 \end{pmatrix} \quad \begin{array}{l} \text{edge} \\ \text{focusing} \end{array} \quad \begin{pmatrix} 1 & 0 \\ -\frac{\tan \Psi}{R} & 1 \end{pmatrix}. \quad (\text{A.13})$$

Here, Δs is the length of the respective element and $\Omega \equiv \sqrt{|k|}\Delta s$. Edge focusing and weak focusing ("weak" compared to quadrupole magnets) are properties of a dipole magnet with bending radius R , while Ψ is the angle between the front edge of the magnet and a plane perpendicular to the ideal orbit. For $\delta_E \neq 0$, weak focusing also includes the matrix elements

$$d_x = R \left(1 - \cos \frac{\Delta s}{R}\right) \quad d'_x = \sin \frac{\Delta s}{R}. \quad (\text{A.14})$$

In the following relations (A.15)–(A.18), indices x or y denoting the horizontal or vertical coordinate, respectively, are omitted for brevity. Once the transfer matrix elements between two locations s_0 and s_1 are known, they can be used to transform the optical functions according to

$$\begin{pmatrix} \beta_1 \\ \alpha_1 \\ \gamma_1 \end{pmatrix}_{s_1} = \begin{pmatrix} C^2 & -2CS & S^2 \\ -CC' & CS' + SC' & -SS' \\ C'^2 & -2S'C' & S'^2 \end{pmatrix} \cdot \begin{pmatrix} \beta_0 \\ \alpha_0 \\ \gamma_0 \end{pmatrix}_{s_0}. \quad (\text{A.15})$$

On the other hand, if the optical functions at the locations s_0 and s_1 and the phase advance $\Delta\mu$ between them are known, the transfer matrix can be constructed:

$$M = \begin{pmatrix} \sqrt{\frac{\beta_1}{\beta_0}}(\cos \Delta\mu + \alpha_0 \sin \Delta\mu) & \sqrt{\beta_0\beta_1} \sin \Delta\mu \\ \frac{(\alpha_0 - \alpha_1) \cos \Delta\mu - (1 + \alpha_0\alpha_1) \sin \Delta\mu}{\sqrt{\beta_0\beta_1}} & \sqrt{\frac{\beta_0}{\beta_1}}(\cos \Delta\mu - \alpha_1 \sin \Delta\mu) \end{pmatrix}. \quad (\text{A.16})$$

Two general properties of transfer matrices are:

- The determinant $\det(M)$ of a transfer matrix is 1
- Stable particle trajectories require $|\text{Trace}(M)| \leq 2$

In a storage ring of circumference L , a periodicity condition holds for the beta function

$$\beta_{\circ} \equiv \beta(s_{\circ}) = \beta(s_{\circ} + L) \quad (\text{A.17})$$

and likewise for $\alpha_{\circ}$ and $\gamma_{\circ}$. For a given location s_0 , (A.15) yields

$$\begin{aligned} \beta(s_0) &= \frac{2\hat{S}}{\sqrt{2 - \hat{C}^2 - 2\hat{C}'\hat{S} - \hat{S}'^2}}, \\ \alpha(s_0) &= \frac{\hat{C} - \hat{S}'}{2\hat{S}}\beta(s_0), \\ \gamma(s_0) &= \frac{1 + \alpha^2(s_0)}{\beta(s_0)}, \end{aligned} \quad (\text{A.18})$$

where $\hat{S}$, $\hat{S}'$, $\hat{C}$, and $\hat{C}'$ are the matrix elements for a full revolution. The periodicity condition for the dispersion $D_x(s_0)$ and its derivative is

$$\begin{pmatrix} D_x \\ D'_x \\ 1 \end{pmatrix}_{s_0} = \begin{pmatrix} \hat{C}_x & \hat{S}_x & \hat{d}_x \\ \hat{C}'_x & \hat{S}'_x & \hat{d}'_x \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} D_x \\ D'_x \\ 1 \end{pmatrix}_{s_0}. \quad (\text{A.19})$$

With (A.18), the optical functions can be obtained at a location s_0 and via (A.15) for the whole storage ring. The dispersion at s_0 is given by (A.19) and is transformed to any other location by applying (A.1). Another quantity of interest is the phase advance for a full revolution $2\pi\nu_x$ or $2\pi\nu_y$, where ν_x and ν_y are the betatron *tunes*, i.e., the number of horizontal and vertical betatron oscillations per revolution, respectively. Optical resonances, at which small perturbations due to magnetic fields errors add up and cause particle trajectories to become unstable, are given by the condition

$$m\nu_x + n\nu_y = p \quad (m, n, p \text{ integer}), \quad (\text{A.20})$$

where $|m| + |n|$ is the order of the resonance and betatron tunes near low-order resonances must be avoided.

A.2.2 Chromaticity and Sextupole Magnets

Particles of different kinetic energy are focused differently by quadrupole magnets, leading to slightly different betatron tunes. This is expressed by the chromaticity

$$\xi_{x,z} \equiv \frac{\Delta\nu_{x,z}}{\delta_E} = \frac{1}{4\pi} \oint \{m(s)D_x(s) - |k|(s)\} \beta_{x,y}(s) ds. \quad (\text{A.21})$$

As the equation shows, the chromaticity can be controlled by placing sextupole magnets of strength m at locations with dispersion $D_x(s) \neq 0$. Fields that increase quadratically with x or y are beyond the regime of linear optics. Thus, there is no transfer matrix for a sextupole magnet. In order to track a particle through the magnetic lattice, the effect of a sextupole can be approximated by a kick (i.e., a small change in angle)

$$\Delta x' = \frac{1}{2} m l (x^2 - y^2) \quad \text{and} \quad \Delta y' = m l x y, \quad (\text{A.22})$$

where l is the effective length of the magnet. Due to nonlinearities, betatron oscillations change their frequency with amplitude and become unstable if the amplitude is too large. The borderline between stable and unstable trajectories is called the dynamic aperture. The effects of nonlinearities can be minimized by a suitable distribution of the “chromatic” sextupoles (those that control the chromaticity) and by placing additional “harmonic” sextupoles at dispersion-free locations.

A.2.3 Shape and Size of Particle Beams

In good approximation, the particle density of the beam in an accelerator or storage ring follows a normal distribution in all phase space coordinates e.g., horizontally

$$\rho(x, x') = \frac{N}{2\pi \sigma_x \sigma_{x'}} \exp\left(-\frac{x^2}{2\sigma_x^2} - \frac{x'^2}{2\sigma_{x'}^2}\right). \quad (\text{A.23})$$

N is the number of particles in a bunch, σ_x and $\sigma_{x'}$ are rms values of x and x' . The Courant–Snyder invariant ε_x of a particle with a phase space ellipse of extension σ_x and $\sigma_{x'}$ defines the “horizontal emittance of the beam” (and analogously for the vertical coordinate). Size and divergence of the beam scale with the square root of the emittance. The rms beam size $\sigma_x(s)$ as a function of s is called the “envelope” of the beam. At a location with $\alpha(s) = 0$, i.e., $\beta'(s) = 0$, the beam size has a minimum (a beam waist) or a maximum.

A.2.4 Synchrotron Radiation Damping

An electron emitting synchrotron radiation experiences a recoil, which is on average opposite to the direction of its trajectory. Since the longitudinal momentum is restored in rf cavities while transverse momentum of the betatron motion remains lost, there is a transverse damping effect. A quantitative treatment leads to the following damping rates (inverse damping times) for horizontal and vertical betatron oscillations:

$$a_x = \frac{1}{\tau_x} = \frac{V^S}{2E T_o} (1 - \mathcal{D}), \quad (\text{A.24})$$

$$a_y = \frac{1}{\tau_y} = \frac{V^S}{2E T_o}, \quad (\text{A.25})$$

where V^S is the energy dissipated due to radiation during one revolution time T_o . For an electron energy E and bending radius R , it is given by

$$V^S [\text{keV}] = 88.5 \frac{E^4 [\text{GeV}^4]}{R [\text{m}]}. \quad (\text{A.26})$$

The quantity $\mathcal{D}$ accounts for the fact that energy loss also changes the electron trajectory due to dispersion. However, for most present-day electron storage rings with separate magnets for dipole and quadrupole fields, $\mathcal{D} \ll 1$ can be assumed.

Despite this damping effect, electron bunches in a storage ring have a finite transverse size because photon emission also causes an excitation. In a region with non-zero dispersion D_x , a loss of energy δ_E shifts the orbit by $D_x \delta_E$ and $D'_x \delta_E$ in horizontal phase space, and the electron – no longer on its ideal orbit – starts to perform a betatron oscillation. The equilibrium between excitation and damping leads to a “natural” emittance in the horizontal coordinate given by

$$\varepsilon_x = \frac{C_\gamma \gamma^2}{1 - \mathcal{D}} \cdot \frac{\langle \mathcal{H}(s)/R^3 \rangle}{\langle 1/R^2 \rangle} \quad (\text{A.27})$$

with

$$C_\gamma = \frac{55\hbar}{32\sqrt{3}m_e c} = 3.83 \times 10^{-13} \text{ m} \quad (\text{A.28})$$

and

$$\mathcal{H}(s) = \gamma_x(s) D_x^2(s) + 2\alpha_x(s) D_x(s) D'_x(s) + \beta_x(s) D'^2_x(s), \quad (\text{A.29})$$

where the average $\langle \dots \rangle$ is calculated including all dipole magnets.

The vertical emittance ε_y is mainly given by coupling horizontal oscillations into the vertical plane via field errors and slight displacements of the magnets. The emittance ratio $\varepsilon_y/\varepsilon_x$ is often referred to as “coupling” and is of the order of 0.01.

A.3 Longitudinal Phase Space

A.3.1 Solution of the Longitudinal Equation of Motion

An electron in a storage ring loses energy V^S by synchrotron radiation emission, which is subsequently restored in the oscillatory electric field of an rf cavity. In the longitudinal equilibrium position

$$V^S = U_o \sin \phi_s \quad (\pi > \phi_s > \pi/2), \quad (\text{A.30})$$

where U_o is the amplitude of the rf voltage and ϕ_s is called the synchronous phase angle. The additional condition $\phi_s > \pi/2$ holds for storage rings with $\alpha > 1/\gamma^2$. Electrons at the synchronous phase and with nominal energy E maintain their position, while electrons with a phase offset $\Delta\phi$ undergo energy changes. For one revolution time T_o , the change of δ_E is given by

$$\dot{\delta}_E \approx \frac{\delta_E}{T_o} = \frac{1}{T_o} \frac{eU_o}{E} [\sin(\phi_s + \Delta\phi) - \sin \phi_s] - \frac{1}{T_o} \frac{dV^S}{dE} \delta_E. \quad (\text{A.31})$$

For small amplitudes, expression $[\dots]$ can be replaced by $\Delta\phi \cos \phi_s$, A phase offset $\Delta\phi$ corresponds to a longitudinal displacement z via

$$\Delta\phi = \frac{2\pi h}{c T_o} z, \quad (\text{A.32})$$

where h is the harmonic number, i.e., the number of rf periods per revolution time. The slippage factor η links the temporal variation of the longitudinal coordinate z according to (A.12) to the energy variation

$$\dot{z} \approx \frac{\Delta z}{T_o} = c \frac{\Delta T}{T_o} = c \eta \delta_E. \quad (\text{A.33})$$

Inserting this into the derivative of (A.31) with respect to time yields an equation for the variation of energy

$$\ddot{\delta}_E + 2a_s \dot{\delta}_E + \omega_s^2 \delta_E = 0 \quad (\text{A.34})$$

using the abbreviations

$$\omega_s = \sqrt{-\frac{2\pi e U_o h \eta \cos \phi_s}{E T_o^2}} \quad (\text{A.35})$$

and

$$a_s = \frac{1}{2 T_o} \frac{dV^S}{dE}. \quad (\text{A.36})$$

The solution of (A.34) is a damped harmonic oscillation, i.e., the synchrotron oscillation

$$\delta_E(t) = \delta_E(0) e^{-a_s t} e^{i\sqrt{\omega_s^2 - a_s^2} t}, \quad (\text{A.37})$$

with synchrotron frequency ω_s and damping rate a_s . Evaluating dV^S/dE yields

$$a_s = \frac{V^S}{2 E T_o} (2 + \mathcal{D}), \quad (\text{A.38})$$

where, as stated above, $\mathcal{D} \ll 1$ can be assumed. Note that, regardless of $\mathcal{D}$, the sum of the longitudinal and transverse damping rates (A.24) and (A.25) is

$$a_x + a_y + a_s = \frac{2V^S}{E T_o}, \quad (\text{A.39})$$

which is referred to as Robinson's Theorem.

An equation analogous to (A.34) describes the variation of the longitudinal coordinate z or phase offset $\Delta\phi$.

For larger oscillation amplitudes, the sinusoidal shape of the rf voltage leads to an anharmonic oscillation. Its amplitude-dependent frequency (see Fig. A.3) is approximately given by

$$\omega_s(z) = \omega_s(0) \left(1 - \frac{\pi^2 h^2}{4 c^2 T_o^2} z^2 \right). \quad (\text{A.40})$$

The potential well for stable oscillations is limited by the so-called separatrix. The harmonic number h is the number of such potential wells (also called buckets) along the storage ring circumference. The energy range of stable trajectories in longitudinal phase space, is given by

$$\Delta E/E = \pm \sqrt{\frac{2 e U_o}{\pi h E \eta} |\cos \phi_s + (\phi_s - \pi/2) \sin \phi_s|}. \quad (\text{A.41})$$

This energy acceptance (or the equivalent momentum acceptance $\Delta p/p$) is relevant for the lifetime of a beam in a storage ring.

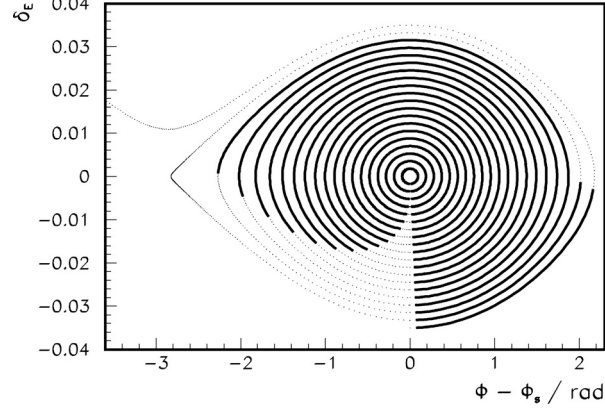


Fig. A.3. Calculated trajectories in longitudinal phase space (one of them is beyond the separatrix). Each dot corresponds to one revolution in the storage ring, the *solid lines* cover one synchrotron oscillation period

A.3.2 Energy Spread and Bunch Length

The natural energy distribution of a stored beam and the horizontal emittance are of similar origin. Both constitute an equilibrium between radiation damping and random energy losses due to photon emission. The variance of relative energy deviation is

$$\sigma_E^2 = \frac{C_\gamma \gamma^2}{2 + \mathcal{D}} \cdot \frac{\langle 1/R^3 \rangle}{\langle 1/R^2 \rangle} \quad (\text{A.42})$$

with C_γ as in (A.28). The energy width σ_E determines the natural bunch length via

$$\sigma_z = \frac{c \eta}{\omega_s} \sigma_E. \quad (\text{A.43})$$

B

Third-Generation Synchrotron Radiation Sources

B.1 Overview

The purpose of synchrotron radiation sources is to provide intense electromagnetic radiation, mainly in the vacuum-UV and X-ray regime, which has become indispensable for a multitude of scientific applications [284, 285]. Until recently, these facilities were solely based on electron or positron storage rings. The following classification is generally accepted:

1. *First generation:* Starting in the 1960s, the radiation from storage rings for elementary particle research was used parasitically. The horizontal emittance of these machines was typically 10^{-6} – 10^{-7} rad m. Some of them were later converted into dedicated synchrotron radiation sources, such as DORIS at DESY (Hamburg, Germany) and SPEAR at SLAC (Stanford, USA).
2. *Second generation:* In the 1970s, storage rings dedicated to synchrotron radiation were built. Their horizontal emittance was of the order of 10^{-8} rad m and their design included a few straight sections for wigglers and undulators. Examples are SUPER-ACO (Orsay, France) and BESSY (Berlin, Germany).
3. *Third generation:* In the 1990s, dedicated storage rings with a horizontal emittance of a few 10^{-9} rad m and up to 40 straight sections were constructed. At the time of writing (2005), more machines of this kind are still planned or under construction. There are two distinct groups:
 - Most third-generation light sources have a beam energy around 2–3 GeV and produce predominantly “soft” x-ray radiation like the ALS (Berkeley, USA), ELETTRA (Trieste, Italy), or BESSY II (Berlin, Germany).
 - There are three facilities with a beam energy of 6–8 GeV to generate “hard” x-rays: ESRF (Grenoble, France), APS (Argonne, USA), and Spring8 (Mikazuki, Japan).

Future synchrotron radiation sources may include storage rings with even smaller emittance (and consequently larger circumference) or linear accelerators in combination with conventional undulators or with free-electron lasers (FELs). For FELs producing short-wavelength radiation, the process of self-amplified spontaneous emission (SASE) [255, 256, 258] as well as “seeding” FELs [262, 264] are very active research topics (see also Sect. 5.2.2).

In the following, BESSY II in Berlin will be described in some detail as an example of a third-generation synchrotron radiation facility.

B.2 The Synchrotron Radiation Source BESSY II

The construction of BESSY II started in 1994 [286–289], and the operation of the storage ring commenced in 1998 [290–292]. The facility, shown in Fig. B.1, comprises:

- An electron gun and a 50 MeV microtron as preaccelerator
- A synchrotron of 96 m circumference, accelerating electrons to a final energy between 900 MeV and 1900 MeV [293, 294]
- A storage ring of 240 m circumference with 16 straight sections, usually operated at a beam energy of 1700 MeV
- Numerous X-ray beamlines with monochromators, providing synchrotron light for experiments

The availability of the storage ring – another important figure of merit for synchrotron radiation facilities – was already 95% in the first two years of operation [295, 296].

The following discussion of subcomponents concentrates on the storage ring. Table B.1 at the end of this section contains a list of parameters that are relevant in the context of this book.

B.2.1 Acceleration and Injection

In order to achieve a short injection time, electron bunches must be injected efficiently with a high repetition rate and at the final beam energy. The synchrotron combines high repetition rate with low power consumption by employing White circuits, where the magnets are part of a 10 Hz LRC oscillator [297]. At this frequency, a train of 160 electron bunches is injected, while the orbit in the storage ring is moved close to the pulsed septum for a few μs [298]. With 3 pC per injected bunch, an injection rate of 6 mA s^{-1} is achievable. The storage ring is filled evenly except for an ion-clearing gap of typically 10% of the circumference. Using a pulsed triode as electron gun permits the injection of a single bunch, which is required by specific experiments, e.g., time-of-flight spectroscopy.

B.2.2 The Radiofrequency (rf) System

The rf frequency is 3 GHz for the microtron and 500 MHz for all other components. In the synchrotron, the beam is accelerated by a five-cell PETRA-type

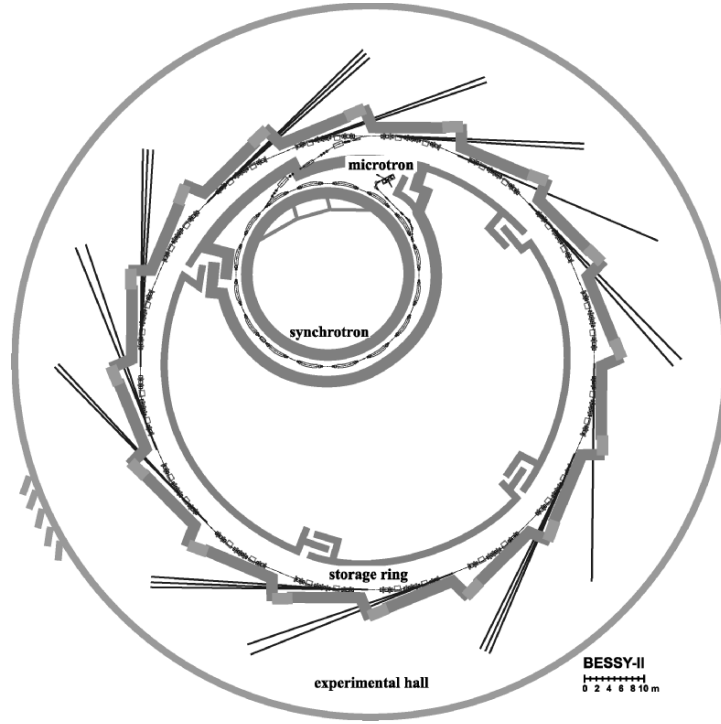


Fig. B.1. Layout of BESSY II, a third-generation synchrotron radiation source, shown without office and workshop buildings. The accelerator complex comprising a microtron, a synchrotron, and a storage ring, is separated from the experimental area by a radiation shielding wall (*gray*)

cavity, driven by a 35 kW klystron. The rf system of the storage ring comprises four pillbox cavities originally designed for the DORIS storage ring. Each cavity is driven by a 75 kW klystron. Damping antennas [303] suppress higher-order modes of these cavities. With synchrotron radiation losses around 200 keV per turn (depending on the settings of undulators and other insertion devices) and an accelerating voltage amplitude of typically 1.3 (max. 1.6) MV, the synchronous phase angle is $(\pi - 0.15)$ rad and the momentum acceptance is 3%. Phase and amplitude of the rf voltage are controlled by feedback loops in order to compensate current-dependent beam loading effects.

By the end of 1999, four passive 1.5 GHz cavities for bunch lengthening were installed [183] (see also Sect. 4.6).

B.2.3 Electron Optics and Magnets

The magnetic lattice of the storage ring [299] is derived from the Chasman–Green lattice [300] forming an achromat with two dipole magnets (double-bend achromat, DBA), i.e., the regions not enclosed by dipole pairs are

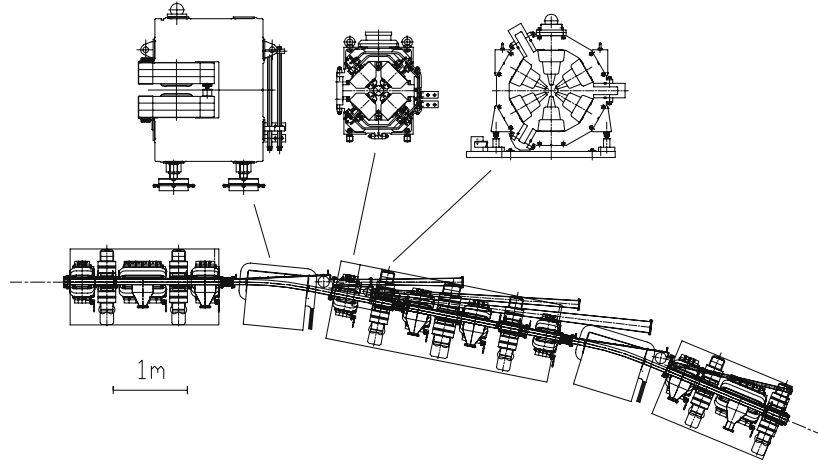


Fig. B.2. Top view of one achromat section of the BESSY II storage ring and front views ($2\times$ enlarged) of a dipole, quadrupole, and sextupole magnet

(ideally) free of dispersion. The magnets of one achromat are shown in Fig. B.2. The main objectives of the magnetic lattice design include:

- Minimum emittance, hence maximum brilliance of the emitted radiation
- Dispersion-free straight sections for wigglers and undulators
- Large dynamic aperture and hence optimum beam lifetime
- Flexibility to explore different configurations, e.g., operation with a reduced momentum compaction factor

Figure B.3 shows the magnetic lattice schematically together with the optical functions for a 30 m long section, i.e., one-eighth of the circumference. Straight sections with low horizontal beta function (< 1 m) alternate with high-beta (17 m) straight sections. The low-beta sections are required to minimize the effect of superconducting wigglers on the emittance [325]. The nonlinearities introduced by sextupole magnets within the achromats are reduced by additional sextupoles in dispersion-free regions. These “harmonic” sextupoles improve the dynamic aperture significantly, as shown by simulations [301] and by operational experience.

B.2.4 Vacuum System

The horizontal aperture of the stainless-steel vacuum chamber (± 32.5 mm) was chosen as to match a momentum acceptance of 3%, while the vertical aperture (± 17.5 mm) was limited by the gap within the dipole magnets. Indirectly cooled copper blocks inside the chambers absorb synchrotron radiation emitted by the dipole magnets. In the undulator regions, extruded aluminium

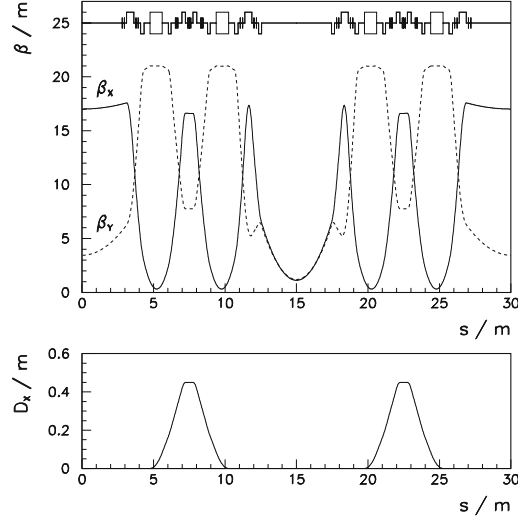


Fig. B.3. Beta functions β_x (top, solid line) and β_y (top, dashed line) and dispersion D_x (bottom) for a 30 m long section (one-eighth) of the BESSY II storage ring. The magnetic lattice is shown schematically: closed box = dipole, open box = quadrupole, 2 lines = sextupole

chambers [302] with a vertical aperture of ± 5.5 mm are employed. Cross sections are shown in Fig. B.4. Approximately 200 ion getter pumps are in use with a combined pumping speed of $> 20 \text{ m}^3 \text{ s}^{-1}$.

B.2.5 Beam Diagnostics

Numerous devices are used to monitor the electron beam in the storage ring (and their respective purpose is briefly stated in brackets):

- Eight fluorescence screens (qualitative determination of the beam position)
- 112 beam position monitors comprising four button-type electrodes coupling to the electromagnetic field of the beam [304, 305] (measuring the beam position with μm resolution)
- Two current transformers (measuring beam current and lifetime)
- Several stripline electrodes (beam spectrum, betatron tune, synchrotron frequency, identification of instabilities)
- Resonant beam depolarization [306] and Compton backscattering of laser photons [307] (to determine the beam energy with $< 10^{-4}$ resolution)
- Scintillation detectors and pin diodes (to detect local particle losses and small changes of beam lifetime)
- Horizontal and vertical scrapers (variable apertures, allowing controlled particle loss down to single-electron operation for metrology purposes)

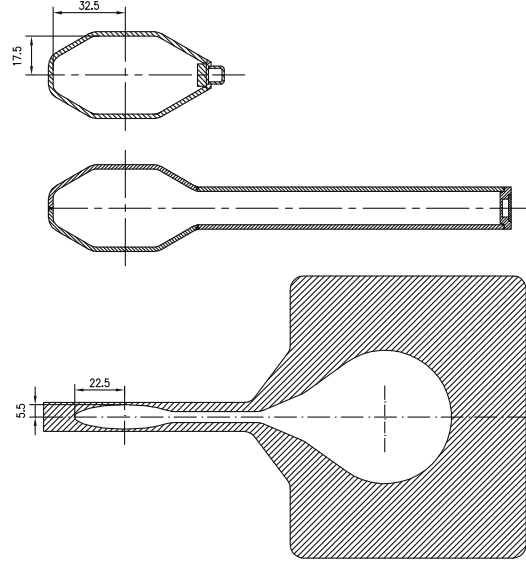


Fig. B.4. Cross sections of vacuum chambers in the BESSY II storage ring. *Top:* octagonal standard chamber, made of stainless steel. *Center:* dipole chamber with synchrotron radiation channel, also made of stainless steel. *Bottom:* extruded aluminium chamber with antechamber for wigglers and undulators

- Fast horizontal and vertical kicker [308] (exciting transverse oscillations for diagnostics purposes)
- Digital feedback system (recording longitudinal bunch positions)

Further instrumentation allows to deduce electron beam properties by observing synchrotron radiation:

- Streak camera (longitudinal electron distribution with 1 ps resolution)
- Photodiodes [309] (detection of extremely small beam currents)
- Pinhole arrays [310] (the beam cross section for emittance measurement)
- Bragg–Fresnel telescope [311] (imaging the beam cross section)
- X-ray beam position monitors using tungsten or copper blades [312] (time-resolved measurement of the beam position)
- Beamline monochromators (recording undulator spectra)

The stability of the electron beam against low-frequency disturbances ≤ 0.1 Hz is ensured by periodic orbit correction. A local orbit feedback in the vicinity of an undulator [313] with 100 Hz bandwidth against ground vibration or 50 Hz electric noise has been demonstrated [314]. Multi-bunch instabilities are counteracted by a digital longitudinal feedback system (see Sect. 4.3) and by an analog transverse feedback system (see Sect. 4.4).

B.2.6 The Control System

There are more than 20,000 input and output channels that require efficient control [315]. The tasks of the control system include:

- Setting and reading digital and analog control signals
- Performance of control algorithms e.g. orbit correction
- Collection of reference parameters in a data base
- Display of actual parameter values through a graphical user interface and their periodic archiving
- Alarm activation to bring malfunctions to the operator's attention
- Providing flexible tools for routine operation as well as for accelerator physics studies

Besides general demands like precision, reliability, and user friendliness [316], it is important that the control system can be easily extended to new applications and devices. The distributed architecture briefly described in the following paragraph helps to fulfill this requirement.

Hardware

The hardware for the BESSY II control system comprises several layers [317]:

- Console computers under UNIX providing a graphical user interface and alphanumerical data for the operator
- CPUs operating under VxWORKS [139] in distributed VME crates, acting as input/output controllers (IOCs) for specific devices (such as power supplies for the magnets)
- Embedded controllers, i.e., local processors inside or near certain devices which can be controlled or read out via A/D and D/A converters or by digital control signals

Console computers and IOCs are connected by a common network. The IOCs, in turn, communicate with several local processors using a field bus (Controller Area Network, CAN [318]).

Software

The communication between console computers and IOCs is based on the software package EPICS (Experimental Physics and Industrial Control System) [141]. The graphical user interface comprises EPICS applications, TCL/Tk scripts, and a commercial graphics package. Geometrical and electrical parameters of all components are collected in a relational data base. These data and the actual parameters of magnet power supplies are combined to construct a beam optics model of the storage ring using GOEMON [319]. Archiving and data acquisition is based on SDDS (Self-describing data sets) [320].

B.2.7 Emission of Synchrotron Radiation

In standard operation at a beam energy of 1.7 GeV, the critical energy of radiation from the dipole magnets is 2.5 keV. At a beam current of 250 mA, for example, each dipole magnet generates a power of 1.3 kW. There are up to four beamlines per achromat, one of which is aligned to the wiggler or undulator of the preceding straight section.

Out of 16 straight sections, one is occupied by the rf cavities, one is used for injection, and 14 are available for insertion devices (IDs), such as:

- Planar undulators (W/U-125, U-49, U-41) emitting linearly polarized radiation. Their names indicate the period length in mm. The design of these undulators combines permanent magnets and iron poles.
- Undulators of the Apple-II-type [321,322] (UE-56, UE-52) with permanent magnet poles, generating linearly, elliptically, or circularly polarized light. A modular UE-56 undulator can produce light of both helicities simultaneously.
- An electromagnetic undulator (U-180) [323].
- Superconducting 1-periodic wigglers, also dubbed wavelength shifters (WLS) with a peak field of 6–7 T.
- A superconducting multipole wiggler with 7 T peak field [324].

The average brilliance of some of these IDs as defined by (2.1) is plotted in Fig. B.5 as function of the photon energy.

The influence of IDs on the dynamic aperture and the storage ring emittance was the topic of detailed studies [325–327] indicating the necessity of straight sections with low horizontal beta function.

Examples of IDs *not* in use at BESSY II are in-vacuum undulators (e.g., [328]), superconducting undulators (e.g., [329]) and storage ring FELs (free-electron laser e.g., [330]).

B.2.8 Beamlines

At the beginning of 2001, 19 beamlines for wigglers and undulators and 12 beamlines for radiation from dipole magnets were in operation [296] and their number has steadily increased ever since. The purpose of a beamline is to image the radiation source on a sample and to select the wavelength required for the respective experiment. The optical elements of a beamline are mirrors, sometimes also Fresnel lenses. Dispersive elements are usually gratings in reflection. An overview of beamline physics and technology is given in [331]. The beamlines at BESSY are named after the type of monochromator in use, which in turn is classified by the geometry of the dispersive elements, e.g.,

- Plane grating monochromators (PGM)
- Spherical grating monochromators (SGM)
- Toroidal grating monochromators (TGM)

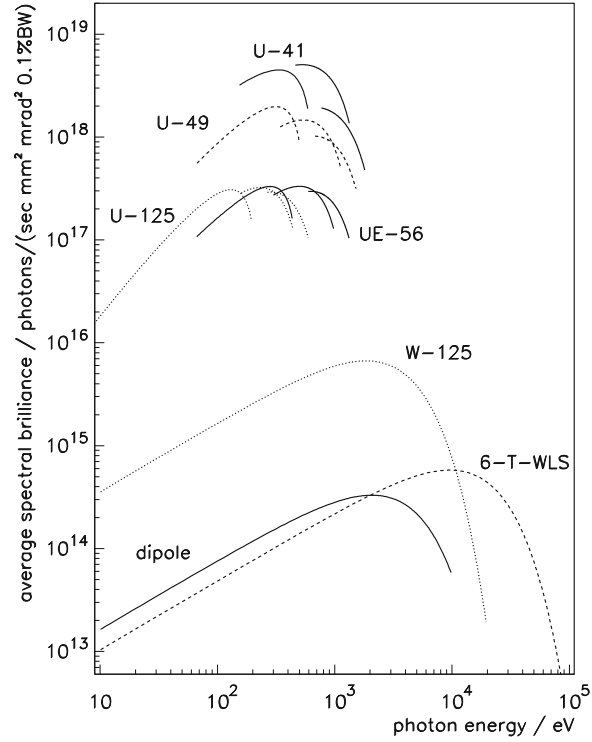


Fig. B.5. Brilliance as function of the photon energy, calculated for a beam current of 100 mA in BESSY II. The curves correspond to radiation from dipole magnets, from a 1-periodic superconducting wiggler (6-T-WLS), from a W/U-125 device in wiggler and undulator mode, and from the undulators UE-56, U-49, and U-41. For undulators, the first, third, and fifth harmonic is shown

or they are named after a specific focusing technique, e.g.,

- Rowland circle monochromators [332]
- Normal-incidence monochromators
- Petersen-type PGM [333]

Several of the beamlines operated at BESSY II are described in [334, 335]. At a photon energy of $E_\gamma = 64$ eV, a spectral resolution exceeding $\Delta E_\gamma / E_\gamma = 10^{-5}$ was achieved [336].

Table B.1. Parameters of the storage ring BESSY II

parameter	symbol	value	formula
circumference	C	240 m	
revolution time	T_o	800 ns	C/c
revolution frequency	$f_o = \omega_o/2\pi$	$1.25 \times 10^6 \text{ s}^{-1}$	$1/T_o$
beam energy	E	1.7 (0.9–1.9) GeV	
Lorentz factor	γ	3327	$E/m_e c^2$
dipole bending radius	R	4.35 m	
energy loss (dipoles only)	V^S	170 keV	(A.26)
longitudinal damping time	τ_s	8 ms	(A.38)
transverse damping time	$\tau_{x,y}$	16 ms	(A.24)
rf voltage	U_o	1300 (max. 1600) kV	
rf frequency	f_{rf}	$499.65 \times 10^6 \text{ s}^{-1}$	
harmonic number	h	400	$f_{\text{rf}} T_o$
synchronous phase angle	ϕ_s	171° ($\pi - 0.15$) rad	(A.30)
synchrotron frequency	$f_s = \omega_s/2\pi$	7800 s^{-1}	(A.35)
synchrotron tune	ν_s	6.2×10^{-3}	ω_s/ω_o
rf momentum acceptance	$\Delta p/p$	0.03	(A.41)
momentum compaction factor	η	8×10^{-4}	(A.11)
chromaticity	$\xi_{x,y}$	1–2	(A.21)
dispersion	D_x	max. 0.45 m	
horizontal beta function	β_x	0.5–18 m	
vertical beta function	β_y	1–21 m	
horizontal beam size	σ_x	55–330 μm	$\sqrt{\varepsilon_x \beta_x}$
vertical beam size	σ_y	8–35 μm	$\sqrt{\varepsilon_y \beta_y}$
horizontal divergence	$\sigma_{x'}$	18–110 μrad	$\sqrt{\varepsilon_x \gamma_x}$
vertical divergence	$\sigma_{y'}$	1.7–8 μrad	$\sqrt{\varepsilon_y \gamma_y}$
relative energy spread	σ_E	8×10^{-4}	(A.42)
natural bunch length	σ_z	3.9 mm	(A.43)
horizontal emittance	ε_x	$6 \times 10^{-9} \text{ rad m}$	(A.27)
vertical emittance	ε_y	$6 \times 10^{-11} \text{ rad m}$	
aperture (achromat)	$R_{x,y}$	32.5 mm, 17.5 mm	
aperture (undulators)	$R_{x,y}$	22.5 mm, 5.5 mm	
betatron tune	$\nu_{x,y}$	17.85, 6.74	
beam current (360 bunches)	I	250 (max. 400) mA	
electrons per bunch	N	3.5×10^9	$I/(360 e f_o)$
single-bunch current	$I_{(\text{sb})}$	20 mA	
electrons in single bunch	$N_{(\text{sb})}$	1.0×10^{11}	$I_{(\text{sb})}/(e f_o)$

C

Longitudinal Instabilities: Vlasov Formalism

C.1 The Vlasov Equation

The area of a phase space element $(\Delta q, \Delta p)$ in the presence of conservative forces is invariant under canonical transformations and consequently invariant with respect to time [337]. Let q be the spatial and p the momentum coordinate. The edge of a phase space element can be defined by a continuum of particle coordinates. In a deterministic system, a particle within such an element can never leave it: in order to do so it would have to coincide with a particle defining the edge at some time, which means that the two have the same trajectory at all times. For the particle density $\psi(q, p, t)$ at location (q, p) and time t , the invariance of the phase space area and of the number of enclosed particles implies

$$\frac{\partial \psi}{\partial t} + \dot{q} \frac{\partial \psi}{\partial q} + \dot{p} \frac{\partial \psi}{\partial p} = 0. \quad (\text{C.1})$$

This is the Vlasov equation, originally formulated for the case of electromagnetic forces acting on a collision-free plasma [338]. As for its application to a stored electron beam, a few remarks are in order:

- The Vlasov equation describes the effect of external forces on a system of particles. The wake fields acting on a particular electron are actually produced by other electrons within the system, but the interaction between individual electrons is negligible and the field generated collectively by all electrons can be treated like an “external” field.
- Since dissipative effects, such as the emission of synchrotron radiation, are ignored, the application of the Vlasov equation is only meaningful for instabilities growing on a timescale which is short compared to typical radiation damping times.

The following discussion is based on the more detailed treatments given in [7] and [339]. For the case of longitudinal instabilities, the phase space coordinates are z and δ_E , i.e., the longitudinal displacement from the ideal position, and

the energy deviation divided by the nominal energy. For a given synchrotron frequency ω_s , momentum compaction factor η , beam energy E , and energy loss per turn $eV(z, s)$ given by the collective wake field, the following relations hold for small amplitudes:

$$\begin{aligned} z' &= -\eta \delta_E \\ \delta_E' &= \frac{\omega_s^2 z}{\eta c^2} - \frac{e V(z, s)}{E c T_0} \end{aligned} \quad (\text{C.2})$$

by combination of (A.31)–(A.33) and (A.35), and differentiation of z and δE with respect to s (rather than time), as indicated by a dash. The Vlasov equation then reads

$$\frac{\partial \psi}{\partial s} - \eta \delta_E \frac{\partial \psi}{\partial z} + \frac{\omega_s^2 z}{\eta c^2} \frac{\partial \psi}{\partial \delta_E} - \frac{e V(z, s)}{E c T_0} \frac{\partial \psi}{\partial \delta_E} = 0 \quad (\text{C.3})$$

or

$$\frac{\partial \psi}{\partial s} + \frac{\omega_s}{c} \frac{\partial \psi}{\partial \phi} - \frac{e V(z, s)}{E c T_0} \frac{\partial \psi}{\partial \delta_E} = 0 \quad (\text{C.4})$$

after introducing polar coordinates given by¹

$$z = r \cos \phi \quad (\text{C.5})$$

and

$$\frac{\eta c}{\omega_s} \delta_E = r \sin \phi. \quad (\text{C.6})$$

C.2 The Instability Model

The instabilities are considered to be small deviations from an initial density distribution ψ_0 that depends only on the radial coordinate, while the perturbation ψ_1 also has an azimuthal dependence and oscillates with a particular frequency Ω , as sketched in Fig. C.1:

$$\psi(r, \phi, s) = \psi_0(r) + \psi_1(r, \phi) e^{-i\Omega s/c}. \quad (\text{C.7})$$

The projection of the perturbation onto the z -axis is

$$\rho_1(z) e^{-i\Omega s/c} = \int_{-\infty}^{\infty} d\delta_E \psi_1(r, \phi) e^{-i\Omega s/c}. \quad (\text{C.8})$$

The wake field of the initial distribution causes a static deformation (potential-well effect) which can be calculated numerically using Haïssinski's equation

$$\text{with } \frac{\partial r}{\partial z} = \cos \phi, \quad \frac{\partial r}{\partial \delta_E} = \frac{\eta c}{\omega_s} \sin \phi, \quad \frac{\partial \phi}{\partial z} = -\frac{1}{r} \sin \phi, \quad \frac{\partial \phi}{\partial \delta_E} = \frac{\eta c}{\omega_s r} \cos \phi.$$

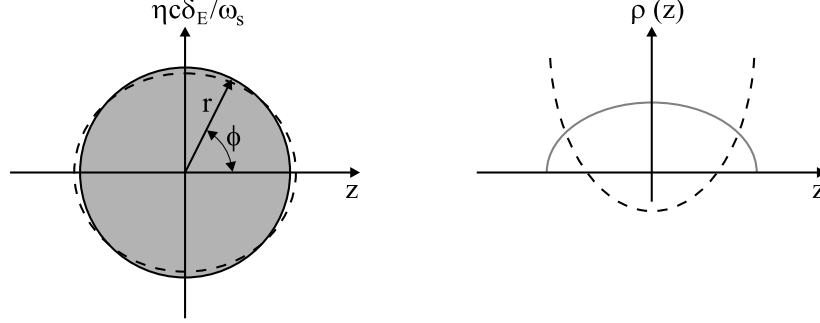


Fig. C.1. Electron density distribution in longitudinal phase space (*left*) and its projection onto the z -axis (*right*) for the so-called water-bag model. The figures show the unperturbed distribution (*solid lines*) and a perturbation of the longitudinal (“azimuthal”) mode $l = 2$ (*dashed lines*)

[69], but will be ignored here. The wake field of the perturbation $\rho_1(z)$, on the other hand, causes a time-dependent energy change $eV(z, s)$ of an electron at position z . Using the definition of the longitudinal wake function (2.33) and impedance (2.40), the energy change is given by

$$\begin{aligned} eV(z, s) &= e^2 \int_{-\infty}^{\infty} dz' \sum_{k=-\infty}^{\infty} \rho_1(z') e^{-i\Omega(s/c - kT_0)} W_{\parallel} \left(kT_0 - \frac{z}{c} + \frac{z'}{c} \right) \\ &= \frac{e^2}{T_0} e^{-i\Omega s/c} \sum_{p=-\infty}^{\infty} \tilde{\rho}_1(\omega') e^{i\omega' z/c} Z_{\parallel}(\omega') \end{aligned} \quad (\text{C.9})$$

with $\omega' \equiv p\omega_0 + \Omega$ and $\tilde{\rho}_1(\omega)$ denoting the Fourier transform of $\rho_1(z)$. The index k equals the number of revolutions in the storage ring. Due to causality, $W_{\parallel}(\tau) = 0$ for $\tau < 0$, i.e., an electron is only influenced by preceding electrons or by the wake fields of all electrons from previous revolutions.

Inserting the expressions (C.7) and (C.9) into the Vlasov equation (C.3) yields

$$-i\Omega\psi_1 + \omega_s \frac{\partial\psi_1}{\partial\phi} - \frac{\eta e^2 \sin\phi}{\gamma m_e c T_0^2 \omega_s} \frac{\partial\psi_0}{\partial r} \sum_{p=-\infty}^{\infty} \tilde{\rho}_1(\omega') e^{i\omega' z/c} Z_{\parallel}(\omega') = 0. \quad (\text{C.10})$$

Here, only terms are kept in which the (small) perturbation ψ_1 appears linearly. While ψ_0 depends only on r , the perturbation can be expanded into azimuthal modes:

$$\psi_1(r, \phi) = \sum_{l=-\infty}^{\infty} \alpha_l R_l(r) e^{il\phi}. \quad (\text{C.11})$$

The Fourier transform of its projection onto the z -axis is²

$$\begin{aligned}\tilde{\rho}_1(\omega') &= \int_{-\infty}^{\infty} dz \int_{-\infty}^{\infty} d\delta_E e^{-i\omega' z/c} \psi_1(r, \phi) \\ &= \frac{2\pi\omega_s}{\eta c} \sum_{l'=-\infty}^{\infty} \int_0^{\infty} r dr \alpha_{l'} R_{l'}(r) i^{-l'} \mathcal{J}_{l'}\left(\frac{\omega' r}{c}\right). \quad (\text{C.12})\end{aligned}$$

Inserting into the Vlasov equation and integrating over ϕ from 0 to 2π yields a system of equations

$$\begin{aligned}(\Omega - l\omega_s) \alpha_l R_l(r) &= -il \frac{2\pi e^2 c}{E T_0^2} \frac{\partial \psi_0 / \partial r}{r} \sum_{l'=-\infty}^{\infty} \int_0^{\infty} r' dr' \alpha_{l'} R_{l'}(r') i^{l-l'} \\ &\times \sum_{p=-\infty}^{\infty} \frac{Z_{\parallel}(\omega')}{\omega'} \mathcal{J}_l\left(\frac{\omega' r}{c}\right) \mathcal{J}_{l'}\left(\frac{\omega' r'}{c}\right). \quad (\text{C.13})\end{aligned}$$

For a given impedance $Z_{\parallel}$ and a choice of the initial distribution ψ_0 , the task is to find functions $R_l(r)$ and coefficients α_l that match (C.13). Of course, the result is only valid for the chosen distribution. In the most simple case, the distribution is constant, i.e.,

$$\psi_0(r) = \frac{N \eta c}{\pi \bar{z}^2 \omega_s} \text{ if } r \leq \bar{z}, \text{ otherwise } \psi_0(r) = 0, \quad (\text{C.14})$$

and the perturbation is limited to the edge of the distribution, as sketched in Fig. C.1. Consequently, this so-called water-bag model describes no radial but only azimuthal modes specified by index l . Even with this simple charge distribution, some important characteristics of the instabilities under consideration become apparent. Another model for ψ_0 that suggests itself is a normal distribution which, however, does not lead to a closed-form solution. With $R_l \sim \delta(r - \bar{z})$ according to the water-bag model, (C.13) reduces to

$$\begin{aligned}(\Omega - l\omega_s) \alpha_l &= il \frac{2N e^2 c^2 \eta}{E T_0^2 \omega_s \bar{z}^2} \\ &\times \sum_{l'=-\infty}^{\infty} \alpha_{l'} i^{l-l'} \sum_{p=-\infty}^{\infty} \frac{Z_{\parallel}(\omega')}{\omega'} \mathcal{J}_l\left(\frac{\omega' \bar{z}}{c}\right) \mathcal{J}_{l'}\left(\frac{\omega' \bar{z}}{c}\right). \quad (\text{C.15})\end{aligned}$$

This expression appears as (2.105) in Sect. 2.4.3 where its further interpretation is discussed. The analogous derivation of the transverse expression (2.111) is described, e.g., in [7], and shall not be repeated here.

² Here and in the next step, the following relations [340] with $\mathcal{J}_l(x)$ being the Bessel function are used:

$$\begin{aligned}\frac{1}{2\pi} \int_0^{2\pi} d\phi e^{il\phi - ix \cos \phi} &= i^{-l} \mathcal{J}_l(x) \\ \frac{1}{2\pi} \int_0^{2\pi} d\phi \sin \phi e^{ix \cos \phi - il\phi} &= -i^l (l/x) \mathcal{J}_l(x).\end{aligned}$$

D

Beam Loading

In rf cavities, electron bunches lose energy due to wake fields, and the voltage of the excited modes is opposite to the accelerating voltage. This is called *beam loading*. Due to the large quality factors of cavity modes, they keep ringing for a long time and act on subsequent bunches passing the cavity. Among the excited resonator modes is also the fundamental mode. The consequences are:

- The usable accelerating voltage is reduced.
- For an uneven fill of a storage ring, the beam loading effect is time-dependent and different bunches adopt different synchronous phases of the rf field.
- Multi-bunch instabilities can be excited.
- A cavity can be passively excited, i.e., it can be driven without an external energy source by the beam alone. This is often the case in higher-harmonic cavities employed for bunch lengthening (see Sect. 4.6).

Even though beam loading is usually discussed in the context of the fundamental cavity mode (e.g., [341]), the description applies to any Robinson-type instability caused by a narrow-band longitudinal impedance.

D.1 Basic Relations

The energy loss due to wake fields of a charge q passing a cavity is

$$\Delta E = q V_{\text{ind}} \quad (\text{D.1})$$

with $V_{\text{ind}} = q k_{\parallel}$ and thus

$$\Delta E = q^2 k_{\parallel} . \quad (\text{D.2})$$

The factor $k_{\parallel}$ that links q and the induced voltage V_{ind} is the longitudinal loss factor (see Sect. 2.3). However, V_{ind} is not equal to the voltage V_0 left behind in the cavity, as the following argument shows (e.g., [11]): A second charge

$q_2 = q$ trailing the first at infinitesimal distance loses additional energy due to the voltage V_0 left behind by the first charge. The total energy loss reads

$$\Delta E = \Delta E_1 + \Delta E_2 = \overbrace{q^2 k_{\parallel}}^{\text{Charge 1}} + \overbrace{q^2 k_{\parallel} + q V_0}^{\text{Charge 2}}. \quad (\text{D.3})$$

On the other hand, viewing the two charges as one bunch of charge $2q$ yields

$$\Delta E = (2q)^2 k_{\parallel} \quad (\text{D.4})$$

and thus

$$V_0 = 2q k_{\parallel}. \quad (\text{D.5})$$

The fact that $V_0 = 2V_{\text{ind}}$ is known as the “fundamental theorem of beam loading”. For a resonator with eigenfrequency ω_r , shunt impedance R_s and quality factor Q , the following relations hold [341]:

- Apart from a form factor ≈ 1 , the loss factor is given by

$$k_{\parallel} = \frac{\omega_r}{2} \frac{R_s}{Q}. \quad (\text{D.6})$$

- The so-called filling time of the cavity, i.e., the time for the rf field to build up or decay, is

$$T_f = \frac{2Q_1}{\omega_r} = \frac{2Q}{\omega_r(1+\beta)}. \quad (\text{D.7})$$

Here, Q_1 is the “loaded” quality factor, which depends on how the rf power couples to the cavity. In the DORIS-type cavities of BESSY II, for example, the coupling factor $\beta \approx 2.4$ is controlled by the rotation angle of a coupling loop.

- Bunches pass the cavity at a rate corresponding to the frequency ω of the driving rf field, which is generally not identical to the resonance frequency ω_r of the cavity.

Figure D.1 shows the voltage induced by five bunches acting on the fifth bunch (left) and the voltage in the cavity immediately after the passage of the fifth bunch (right) as phasors in the complex plane. Within a time interval Δt between bunches in consecutive rf buckets, the phasors rotate by an angle of $2\pi + \delta$ and shrink by a factor $\exp(-\tau)$, where

$$\delta = 2\pi \frac{\omega_r - \omega}{\omega} \quad \tau = \frac{\Delta t}{T_f}. \quad (\text{D.8})$$

The added phasors form a logarithmic spiral which converges after the passage of many bunches with equal charge q to the voltage

$$\begin{aligned} V_b &= V_0 [1 + \exp(i\delta - \tau) + \exp(i2\delta - 2\tau) + \dots] \\ &= \frac{V_0}{1 - \exp(i\delta - \tau)} \approx \frac{V_0}{1 - 1 - i\delta + \tau} = V_0 \frac{i\delta + \tau}{\delta^2 + \tau^2}. \end{aligned} \quad (\text{D.9})$$

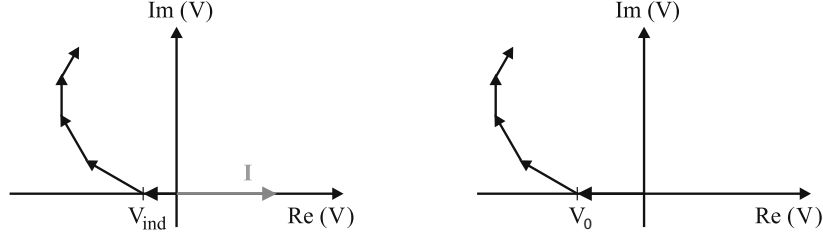


Fig. D.1. Phasor diagram of the voltage from five successive bunches passing an rf cavity. *Left:* Voltage acting on the fifth bunch, represented by a current I . *Right:* Voltage in the cavity immediately after the fifth bunch has passed, where $V_0 = 2 V_{\text{ind}}$

The approximation holds for small angles δ and long filling times $T_f \gg \Delta t$.

The detuning angle Ψ between the induced voltage and the (negative) real axis is defined as

$$\tan \Psi = \frac{\delta}{\tau} = T_f (\omega_r - \omega) = \frac{2Q}{\omega_r(1+\beta)} (\omega_r - \omega). \quad (\text{D.10})$$

With (D.6)–(D.9) and current $I = q/\Delta t$, the voltage reads

$$V_b = 2k_{\parallel} q \frac{\cos^2 \psi}{\tau} (1 + i \tan \Psi) = \frac{2I R_s}{1 + \beta} \cos \Psi \exp(i\Psi). \quad (\text{D.11})$$

In close analogy, the “generator” voltage in the cavity is

$$V_g = \frac{I_g R_s}{1 + \beta} \cos \Psi \exp(i\Psi), \quad (\text{D.12})$$

under the assumption that the rf power originates from a source with constant current I_g [342].

D.2 Applications

D.2.1 Frequency Detuning

Recovering the energy loss V^S due to synchrotron radiation requires an accelerating voltage

$$V_a = -V^S = V_c \cos \hat{\phi}_s \quad (\text{D.13})$$

determined by the synchronous phase angle¹ $\hat{\phi}_s$ of the rf voltage with amplitude V_c .

As sketched in Fig.D.2, the rf voltage is found by adding the phasors of the generator voltage V_g and the beam induced voltage V_b , both given by a

¹ Note that $\hat{\phi}_s$ as depicted in the phasor diagram of Fig.D.2 differs from the definition of ϕ_s given in Appendix A. Here, $\hat{\phi}_s = \phi_s - \pi/2$.

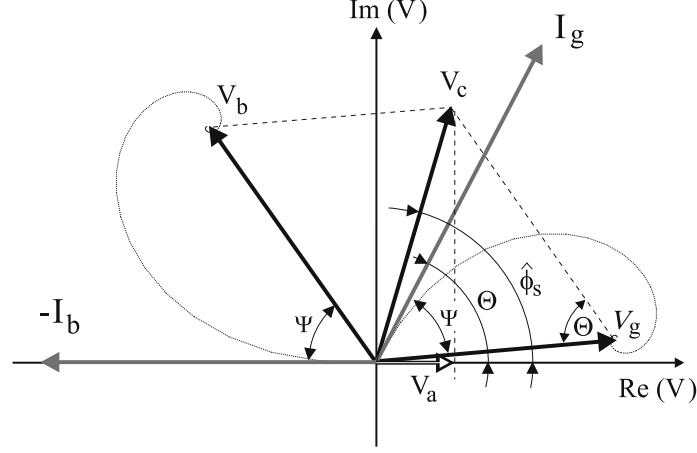


Fig. D.2. Phasor diagram of the voltages V_g and V_b , caused by the generator current I_g and by the beam current I_b , respectively. The respective phasors are given by the ends of logarithmic spirals based on the same detuning angle Ψ . The effective voltage V_c in the cavity is the sum of the phasors of V_g and V_b . This determines the synchronous phase angle $\hat{\phi}_s$ and the accelerating voltage V_a

logarithmic spiral according to (D.12) and (D.11), respectively, with the same detuning angle Ψ .

The power required by the rf transmitter is minimized by variation of the detuning angle and the coupling factor β . The minimum value is characterized by the following conditions [342]:

- The phasors of rf voltage V_c and generator current I_g are collinear, i.e., $\Theta = \hat{\phi}_s$. Applying the law of sines yields the relation

$$\sin \Psi = -\frac{V_b}{V_c} \sin \hat{\phi}_s. \quad (\text{D.14})$$

- The rf power is given by the sum of the beam power $P_b = V_a I_b$ and the power losses $P_c = V_c/2R_s$, and there should be no reflected power. This is true if $\beta = 1 + P_b/P_c$.

In practice, the coupling factor is tuned for a typical value of beam current. As the stored current decreases due to the finite beam lifetime, amplitude and phase are measured continuously and kept constant by controlling the generator voltage and the detuning angle by feedback loops.

D.2.2 Determination of Synchronous Phase Angles

Until now, it was assumed that all rf buckets are occupied by electron bunches of equal charge. However, in order to avoid ion effects (see Appendix G),

a number of consecutive buckets are left empty over typically 10–20% of the circumference. Furthermore, unintentional variations of the bunch charge can occur. In both cases, the rf voltages seen by different bunches vary and so do their synchronous phase angles, which may be disadvantageous. In e^+e^- colliders, for example, the luminosity suffers if the bunches do not collide at the point of minimum beta function. In order to determine the synchronous phase angles for an arbitrary fill pattern numerically, the following strategy may be pursued [343–345]:

- (a) Assume that the quantities R_s , Q , ω_r , and β are known for all rf cavities in the storage ring. Furthermore, the electron energy and energy loss per turn V^S is given, and a charge is assigned to each bunch.
- (b) Use the detuning angle for a complete fill according to (D.14) to compute starting values for the induced voltage (D.11) and the generator voltage (D.12).
- (c) Compute the rf voltage V_c in the complex plane for each bunch after equal time intervals by performing the following steps:
 1. Rotate and shrink V_b according to (D.8),
 2. Add the voltage $V_{\text{ind}} = V_0/2$ to V_b and compute V_c ,
 3. Add $V_0/2$ once again.
- (d) Deduce the motion of the bunch in longitudinal phase space under the influence of V_c using, e.g., (A.31) and (A.33).
- (e) At the end of a loop over all bunches, compare the average values of V_c and ϕ_s with their nominal values. In the case of deviations, adjust V_g and the detuning angle ψ to simulate the effect of amplitude and phase feedback loops in the rf system.
- (f) Repeat steps (c) through (e) until the synchronous phase angles have settled to equilibrium values.

As an example, Fig. D.3 shows the variation of synchronous phase angles for 320 bunches of equal charge (1 nC) in the BESSY II storage ring with four DORIS-type cavities. Due to beam loading acting differently on each bunch, the phase values show a variation over 38 mrad. Adding passive harmonic

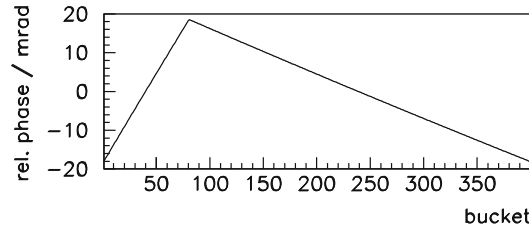


Fig. D.3. Computed modulation of synchronous phase angles for 400 rf buckets in BESSY II with an ion-clearing gap of 80 buckets and 320 buckets filled with electron bunches of equal charge

cavities causes the phase angles to vary over a much wider range, as shown in Fig. 4.18. The consequences of the variation for bunch-by-bunch feedback systems are discussed in Sect. 4.6.3.

D.2.3 Simulation of Longitudinal Multi-Bunch Instabilities

The computation strategy described in the previous paragraph is not only valid for the fundamental cavity mode but for higher-order modes as well. This way, Robinson-type longitudinal instabilities from any narrow-band impedance in the storage ring can be simulated. Only step (e), the modification of generator voltage and detuning angle, does not apply since the rf feedback loops respond only to the fundamental mode of the accelerating cavities. In principle, this time-dependent modeling is equivalent to the frequency-domain description of the one-particle model described in Sect. 2.4.1, but it allows to consider circumstances that are not easily accessible by analytic methods such as

- Uneven fill patterns of the storage ring,
- Properties of a longitudinal feedback system,
- Transient processes, e.g., during injection.

As an example, Fig. D.4 shows the effect of a feedback system on longitudinal oscillations under different premises. Simulations of this kind were helpful for a better understanding of operation and limitations of the longitudinal feedback implemented at BESSY II.

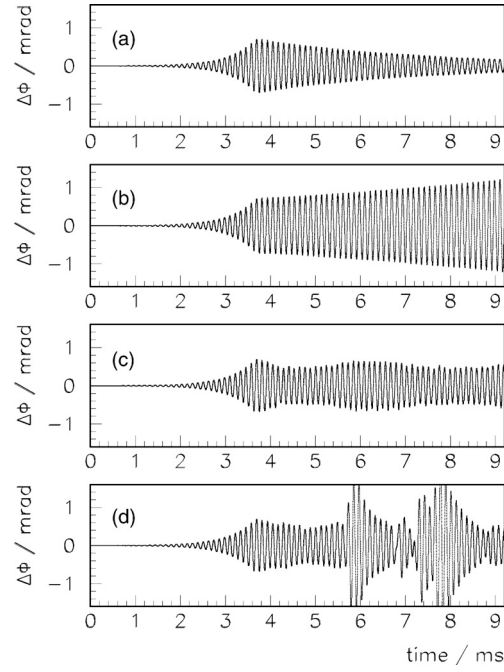


Fig. D.4. Simulation of longitudinal oscillations of an electron bunch as function of time under the influence of a feedback system, activated at time $t = 3.6$ ms: (a) idealized feedback system, (b) without QPSK modulation (see Sect. 4.3), (c) including the 8-bit resolution of the phase detector, (d) adding random noise to the phase detection (1 mrad rms)

E

Longitudinal and Transverse Kickers

Feedback systems to damp multi-bunch instabilities require kicker structures that act longitudinally or transversely on the beam with a bandwidth of at least half the bunch repetition frequency. For synchrotron radiation sources, the required bandwidth is typically 250 MHz. Under the assumption that the costs of power amplifiers scale linearly with the output power, the effective shunt impedance $R_s T^2$ is a reasonable figure of merit for a kicker:

$$R_s T^2 = \frac{V_0^2}{2P} T^2 = \frac{V^2}{2P} . \quad (\text{E.1})$$

Here, V_0 is the amplitude of a sinusoidal voltage, while V is the voltage averaged over the transition time of a particle. The so-called transit-time factor T accounts for the finite transition time which causes a reduction of the effective voltage as compared to the maximum voltage.

The discussion given here essentially follows [346] where more details and examples regarding the following statements can be found:

- There is a reciprocal relationship between kickers and pickup structures, i.e., a kicker optimized for a certain frequency can also be used as a pickup for signals at that frequency.
- Many structures can – in principle – be used as longitudinal kicker as well as transverse kicker (see below).
- According to the Panofsky–Wenzel theorem (see Sect. 2.3.4), a transverse voltage component follows from a gradient of the longitudinal kick voltage. This implies in particular that a purely transverse electromagnetic wave in a kicker has no effect – the effects of its electric and magnetic components cancel.

For longitudinal feedback systems, either cavities [135] or drift tubes [137] are employed (Fig. E.1). Transverse feedback systems use striplines in different variations. Examples are shown schematically in Fig. E.2.

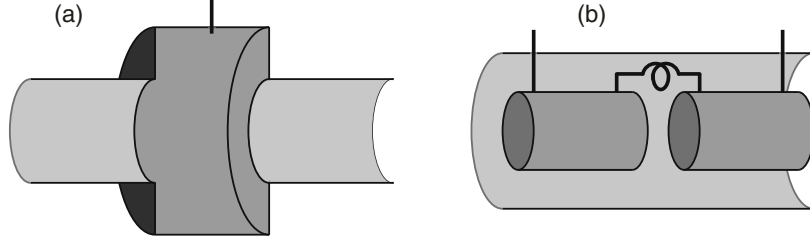


Fig. E.1. Cavity (a) and drift tube structure (b) as possible kickers for a longitudinal feedback system

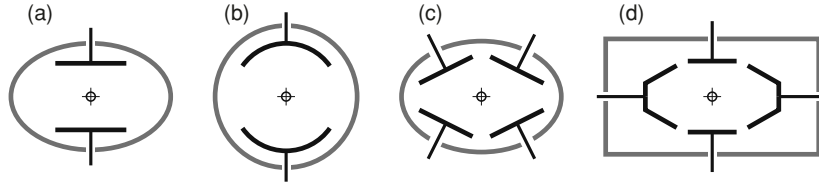


Fig. E.2. Schematic cross sections of different stripline structures: (a) and (b) act only in one transverse plane (here: vertically), while (c) and (d) act in both planes. The layout of the BESSY II electrodes (d) resembles the shape of the adjacent vacuum chamber

In the following, the longitudinal and transverse effects of cavities and striplines are discussed. Both are in use at the BESSY II storage ring. As usual, the transverse coordinates are x and y , and the longitudinal coordinate is z .

E.1 Kicker Cavities

For simplicity, a closed resonant cavity of length L is assumed. Let its cross section be quadratic with edge length b . Analogous results can be found for cylindrical (“pillbox”) cavities. A real cavity has ports for the beam, for the rf power, for plungers and damping antennas – all of this is neglected here. Accurate results for realistic models require numerical methods using program packages like MAFIA [31], SUPERFISH [347] or HFSS [348].

E.1.1 Longitudinal Case

The longitudinal shunt impedance is

$$R_{\parallel} T^2 = \frac{V^2}{2P} = \frac{V^2 Q}{2U\omega} \quad (\text{E.2})$$

using the definition of the quality factor $Q = U\omega/P$ of a resonator. Here, U is the stored energy and ω is the angular frequency of a mode, the most simple case being a TM_{110} mode with a purely longitudinal electric field:

$$E_z(x, y) = E_0 \cos \frac{\pi x}{b} \cos \frac{\pi y}{b}. \quad (\text{E.3})$$

With $k = \omega/c$ being the wave number, the voltage V integrated along the beam axis ($x = 0$ and $y = 0$) is

$$\begin{aligned} V &= \int_{-L/2}^{L/2} e^{ikz} E_z(0, 0) dz = \frac{E_0}{ik} \left(e^{ikL/2} - e^{-ikL/2} \right) \\ &= E_0 L \frac{\sin \Theta}{\Theta} = E_0 L T. \end{aligned} \quad (\text{E.4})$$

Here, the abbreviation $\Theta \equiv kL/2$ and the transit-time factor $T = \sin \Theta / \Theta$ are introduced. The stored energy is found by integrating (E.3) over the cavity volume:

$$U = \frac{\varepsilon_0}{2} L \iint E_z(x, y) dx dy = \frac{\varepsilon_0}{8} E_0^2 L b^2. \quad (\text{E.5})$$

The wavelength of a TM_{mnl} mode in a waveguide with square cross section is

$$\lambda = \frac{2}{\sqrt{(m/b)^2 + (n/b)^2 + (l/L)^2}} \quad \text{for } \text{TM}_{110} : \lambda = \frac{\sqrt{2}}{b}. \quad (\text{E.6})$$

Inserting (E.4)–(E.6) into (E.2) and using the definition of the vacuum impedance $Z_0 = (\varepsilon_0 c)^{-1} = 377 \text{ Ohm}$ yields the effective shunt impedance

$$R_{\parallel} T^2 = \frac{4}{\pi} Z_0 \frac{L}{\lambda} Q T^2 \approx 480 \frac{L}{\lambda} Q T^2 \text{ Ohm}. \quad (\text{E.7})$$

For a TM_{010} mode of equal wavelength in a cylindrical cavity, the shunt impedance is nearly the same. Variation of the length of a cylindrical cavity shows a broad maximum of the effective shunt impedance for $L = 0.44 \lambda$ with $T^2 = 0.51$.

The resonance curve of the strongly damped cavity ($Q \approx 5.5$) used as longitudinal kicker at BESSY II is shown in Fig. E.3. The frequency dependence of the transit-time factor causes a slight asymmetry of the numerical values with respect to the central frequency. The curve fitted to the points has a maximum at 1170 Ohm, while experimental excitation of the resonator using a wire on the cavity axis suggested a value of 960 Ohm [130, 136].

E.1.2 Transverse Case

The transverse (e.g., vertical) shunt impedance of a kicker cavity is

$$R_{\perp} T^2 = \frac{(\Delta p_y c / e)^2}{2P} = \left(\frac{1}{k} \frac{\partial V}{\partial y} \right)^2 \frac{Q}{2U\omega}, \quad (\text{E.8})$$

where the change of momentum Δp_y is described by the Panofsky–Wenzel theorem as

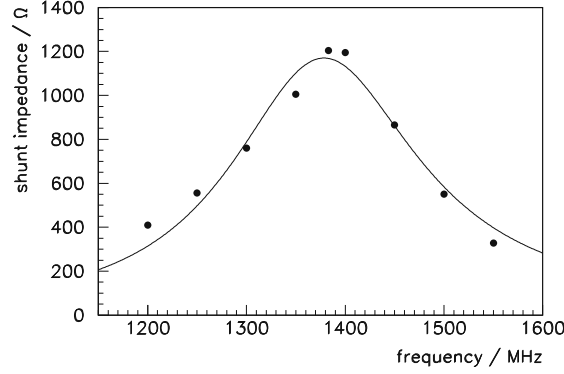


Fig. E.3. Shunt impedance as a function of frequency for the kicker cavity of the longitudinal feedback system at BESSY II (data from [130]). *Dots*: numerical simulation with HFSS [348]. *Line*: fit to the points using the impedance of a LRC resonator circuit (2.50)

$$\Delta p_y c/e = \frac{1}{k} \frac{\partial V}{\partial y} \quad (\text{E.9})$$

for highly relativistic particles and a sinusoidal longitudinal voltage of wave number k . Now, the stored energy U is calculated, assuming a TM_{120} mode with purely longitudinal electric field

$$E_z(x, y) = E_0 \cos \frac{\pi x}{b} \sin \frac{2\pi y}{b}. \quad (\text{E.10})$$

Integration of $\partial E_z / \partial y$ along the beam axis leads to

$$\frac{\partial V}{\partial y} = \int_{-L/2}^{L/2} e^{ikz} \frac{\partial E_z(0, 0)}{\partial y} dz = \frac{2\pi}{b} E_0 L \frac{\sin \Theta}{\Theta} = \frac{2\pi}{b} E_0 L T. \quad (\text{E.11})$$

Integration of (E.10) over the cavity volume yields the stored energy

$$U = \frac{\varepsilon_0}{2} L \iint E_z(x, y) dx dy = \frac{\varepsilon_0}{8} E_0^2 L b^2. \quad (\text{E.12})$$

With this and the wavelength of a TM_{120} mode given by (E.6)

$$\lambda = \frac{2\pi c}{\omega} = \frac{2}{\sqrt{5}} b, \quad (\text{E.13})$$

the transverse shunt impedance is

$$R_{\perp} T^2 = \frac{32}{25\pi} Z_0 \frac{L}{\lambda} Q T^2 \approx 154 \frac{L}{\lambda} Q T^2 \text{ Ohm}, \quad (\text{E.14})$$

using again $Z_0 = (\varepsilon_0 c)^{-1} = 377 \text{ Ohm}$. For the required bandwidth and a consequently low quality factor Q , only a few 100 Ohm can be achieved. As shown below, stripline kickers are much better suited for transverse deflection.

E.2 Stripline Kickers

In the most simple case, a stripline kicker consists of two striplines placed symmetrically with respect to the beam axis as in part (a) of Fig. E.2. Let L be the length of a stripline electrode and b the distance to the beam axis. The dependence of the shunt impedance on the detailed shape of the stripline can be expressed by a geometry factor which is generally determined by numerical methods.

For highly relativistic particles, it can be shown that the longitudinal voltage between striplines satisfies the 2-dimensional Laplace equation

$$\nabla_{\perp}^2 V(x, y) = 0. \quad (\text{E.15})$$

Thus, the computation of V is reduced to an electrostatic problem in a plane perpendicular to the beam axis for which numerical methods are readily available [349], e.g., the Gauss–Seidel algorithm. For this purpose, a unit potential $V_0 = 1$ V is assumed for both striplines

- With equal sign for the longitudinal case:
geometry factor $g_{\parallel} = V(0, 0)/V_0$;
- With opposite sign for the transverse case:
geometry factor $g_{\perp} = (b/\delta y)V(0, \delta y)/V_0$,

where $V(0, 0)$ and $V(0, \delta y)$ are results of the numerical calculations and δy is a small vertical offset. The distance b was absorbed in the definition of the transverse geometry factor to make $g_{\perp}$ dimensionless.

In order to avoid reflected power, the stripline impedance Z_L must match the output impedance of the amplifier that drives the kicker (usually 50 Ohm). The stripline impedance is determined by solving the 2-dimensional electrostatic problem and integrating the electric field component perpendicular to the electrode surface over the whole surface [158]:

$$Z_L = \frac{Z_0 V_0}{\oint \mathbf{E} d\mathbf{a}}. \quad (\text{E.16})$$

The boundary conditions are varied until Z_L is sufficiently close (say, ± 1 Ohm) to the desired value.

At both ends, there is a gap between the stripline electrode and the adjacent vacuum chamber. Assuming this gap to be small, its transit-time factor is $T \approx 1$.

E.2.1 Longitudinal Case

In the longitudinal case, both stripline electrodes have equal voltage of amplitude V_L . The longitudinal shunt impedance is

$$R_{\parallel} T^2 = \frac{V^2}{2P} = \frac{V^2 Z_L}{2V_L^2}, \quad (\text{E.17})$$

where Z_L is again the stripline impedance and P is the amplifier power. The actual acceleration takes place only in the gaps at either end of the stripline ($z = 0$ and $z = L$). Let the voltage be V_L when a bunch passes $z = 0$ at time $t = 0$. If an electromagnetic wave moves with velocity c in positive z direction and the bunch moves with velocity c in negative z direction, the bunch has passed $z = L$ at an earlier time $t = -L/c$. At $y = b$, the longitudinal voltage sum is

$$V(0, b) = V_L (1 - e^{-i2kL}) . \quad (\text{E.18})$$

The sign of the second term can easily be understood in the limit of zero frequency, where the accelerating voltages of the two gaps must be equal and of opposite sign. For signal and bunch moving in opposite direction, the factor e^{-ikL} occurs twice, for a signal copropagating with the bunch the voltage sum would be zero.

On the beam axis ($y = 0$), the voltage is

$$V(0, 0) = g_{\parallel} V_L (1 - e^{-i2kL}) = 2 g_{\parallel} V_L e^{i(\pi/2 - \Theta)} \sin \Theta \quad (\text{E.19})$$

with $\Theta \equiv kL$. Inserting this into (E.17) yields the shunt impedance

$$R_{\parallel} T^2 = 2 Z_L g_{\parallel}^2 \sin^2 \Theta . \quad (\text{E.20})$$

Thus, the stripline kicker is quite broadbanded, without additional damping being introduced. With $Z_L = 50 \text{ Ohm}$ and a geometry factor $g_{\parallel} \approx 1$, the longitudinal shunt impedance is 100 Ohm at best. In this respect, a stripline used as a longitudinal kicker is inferior to a cavity.

E.2.2 Transverse Case

With a voltage V_L of opposite phase and amplitude for the two stripline electrodes, the transverse shunt impedance can be written as

$$R_{\perp} T^2 = \frac{\Delta p_y c / e}{2P} = \left(\frac{1}{k} \frac{\partial V}{\partial y} \right)^2 \frac{Z_L}{2V_L^2} \quad (\text{E.21})$$

using the Panofsky–Wenzel theorem. Here, Δp_y is the vertical momentum change and Z_L is again the stripline impedance. Analogously to (E.19), the longitudinal voltage sum on the beam axis is

$$V(0, 0) = \frac{2y}{b} g_{\perp} V_L e^{i(\pi/2 - \Theta)} \sin \Theta \quad (\text{E.22})$$

with $\Theta \equiv kL$. Differentiating with respect to y and inserting the result into (E.21) yields the effective shunt impedance

$$R_{\perp} T^2 = 2 Z_L \frac{g_{\perp}^2}{b^2 k^2} \sin^2 \Theta , \quad (\text{E.23})$$

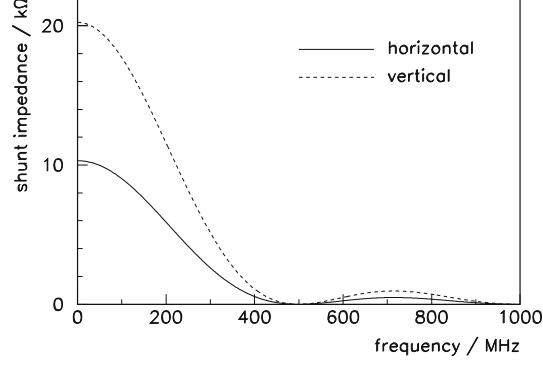


Fig. E.4. Horizontal and vertical shunt impedance of the transverse feedback kicker at BESSY II as a function of frequency

which reaches a maximum in the limit of zero frequency ($k \rightarrow 0$) given by

$$R_{\perp} T^2 \rightarrow 2 Z_L \frac{g_{\perp}^2}{b^2} L^2 \quad (\text{E.24})$$

and has notches at multiples of the frequency $c/2L$. The maximum value of the shunt impedance is typically of the order of 10^4 Ohm. As an example, Fig.E.4 shows the shunt impedance computed for the stripline kicker of the transverse feedback system at BESSY II.

F

The Touschek Effect

The Touschek effect¹ is a loss mechanism for electrons in a storage ring. Here, two electrons significantly change their longitudinal momentum in a single electron–electron scattering event when the initially transverse momentum from their betatron motion acquires a longitudinal component. Transformed into the laboratory frame, this longitudinal momentum component can exceed the momentum acceptance of the storage ring and both electrons are lost.

The following derivation of the relative loss rate, i.e., the inverse lifetime

$$\frac{1}{\tau} = -\frac{\dot{N}}{N} \quad (\text{F.1})$$

is essentially the same as in [87]. A description that includes the increase of the beam cross section in dispersive regions is given in [90]. The following simplifications are made (see also Sect. 3.4):

- The vertical and the longitudinal velocity is negligible in the center-of-momentum system.
- The velocity of the horizontal betatron motion is low enough to permit a nonrelativistic treatment.
- The orientation of the electron spins is not considered.
- The momentum acceptance $\Delta p/p$ is a fixed number, either given by the rf bucket height in longitudinal phase space or by a limiting transverse aperture.

F.1 Scattering in the Center-of-Momentum System

In the center-of-momentum of two electrons with relative velocity v , the Møller cross section, valid for unpolarized scattering of two identical spin-1/2 particles, can be written in nonrelativistic approximation as (see, e.g., [350])

¹ Also called *single* Touschek effect as opposed to emittance blow-up by multiple electron–electron scattering.

$$\frac{d\sigma}{d\Omega} = \frac{4r_e^2}{(v/c)^4} \left[\frac{4}{\sin^4 \Theta} - \frac{3}{\sin^2 \Theta} \right], \quad (\text{F.2})$$

where $r_e = 2.82 \times 10^{-15}$ m is the classical electron radius and Θ is the polar scattering angle. The total cross section describing the loss of electrons is given by integration over all scattering angles that lead to a longitudinal momentum component p_z/p in the laboratory system exceeding the momentum acceptance $\Delta p/p$. As shown in Fig. F.1, χ is the angle between the momentum vector after scattering and the z axis. For a Lorentz factor $\gamma \gg 1$, transforming longitudinal momentum into the laboratory frame corresponds in good approximation to a multiplication with γ . With $p \approx \gamma m_e c$ and m_e being the electron rest mass

$$p_z/p \approx \frac{\gamma m_e v/2}{\gamma m_e c} \cos \chi = \frac{v}{2c} \cos \chi. \quad (\text{F.3})$$

The condition $p_z/p > \Delta p/p$ implies

$$\cos \chi > 2 \frac{\Delta p/p}{v/c} \equiv \mu. \quad (\text{F.4})$$

Figure F.1 also illustrates the relations

$$d\Omega = \sin \chi \, d\chi \, d\phi \quad \text{and} \quad \cos \Theta = \sin \chi \cos \phi. \quad (\text{F.5})$$

Thus, the total cross-section is given by

$$\begin{aligned} \sigma_t &= \frac{4r_e^2}{(v/c)^4} \int_0^{\cos^{-1} \mu} \sin \chi \, d\chi \\ &\times \int_{-\pi}^{\pi} d\phi \left[\frac{4}{(1 - \sin^2 \chi \cos^2 \chi)^2} - \frac{3}{(1 - \sin^2 \chi \cos^2 \chi)} \right]. \end{aligned} \quad (\text{F.6})$$

Evaluation of the integral yields [351]

$$\sigma_t = \frac{8\pi r_e^2}{(v/c)^4} \left[\frac{1}{\mu^2} - 1 - \frac{1}{2} \ln \frac{1}{\mu^2} \right] \quad (\mu \leq 1) \quad (\text{F.7})$$

with the definition of μ according to (F.4). The condition $\mu \leq 1$ is required to avoid negative values of the cross section.

F.2 Loss Rate in the Laboratory System

A small volume dV with electron density ρ contains $dN = \rho dV$ electrons. Scattering between them results in a loss rate of

$$d\dot{N} = \frac{2}{\gamma^2} \cdot \overline{v \sigma_t} \cdot \rho dN = \frac{2}{\gamma^2} \cdot \overline{v \sigma_t} \cdot \rho^2 dV. \quad (\text{F.8})$$

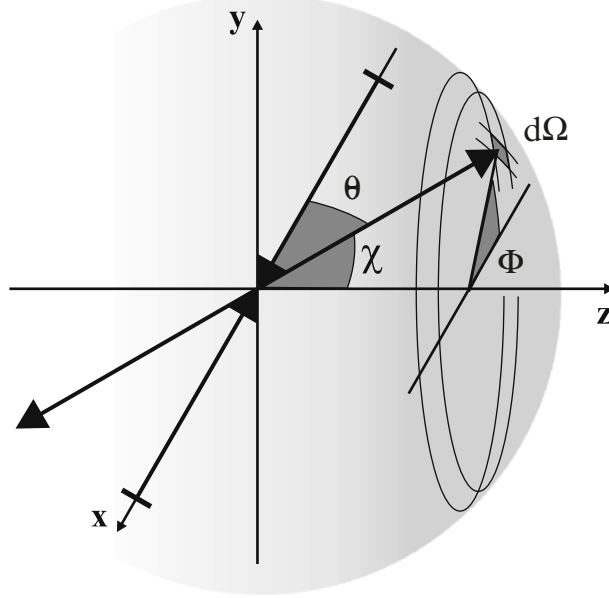


Fig. F.1. Scattering geometry of two electrons within a bunch, where Θ is the scattering polar angle with respect to the incident electron axis in the center-of-momentum system, Φ is the azimuthal angle and χ is the polar angle with respect to the z axis

The factor 2 accounts for the fact that two electrons are lost per scattering event, and γ^{-2} transforms the mean value $\overline{v\sigma_t}$ from the center-of-momentum frame into the laboratory system. Integrating over the electron distribution, the loss rate is

$$\begin{aligned}
 \frac{1}{\tau} &= -\frac{\dot{N}}{N} = -\frac{2}{N\gamma^2} \overline{v\sigma_t} \int_V \rho^2 dV \\
 &= -\frac{2}{N\gamma^2} \overline{v\sigma_t} \int_V \frac{N^2}{(2\pi)^3 \sigma_x^2 \sigma_y^2 \sigma_z^2} \exp \left\{ -\frac{x^2}{\sigma_x^2} - \frac{y^2}{\sigma_y^2} - \frac{z^2}{\sigma_z^2} \right\} dV \\
 &= -\frac{N}{\gamma^2} \frac{1}{4\pi^{3/2} \sigma_x \sigma_y \sigma_z} \overline{v\sigma_t}, \tag{F.9}
 \end{aligned}$$

where $\overline{v\sigma_t}$ depends only on the distribution of horizontal momentum differences

$$\Delta \equiv x'_1 - x'_2 \approx \frac{1}{\gamma c} v. \tag{F.10}$$

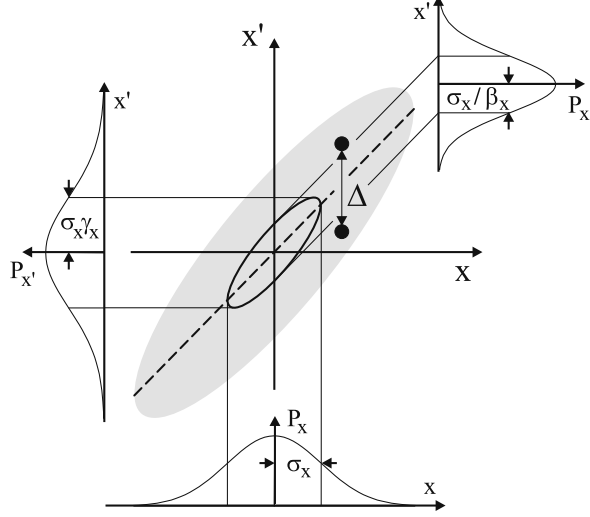


Fig. F.2. Electron distribution in horizontal phase space and different projections of the distribution. The *dots* indicate two electrons differing by Δ in momentum. The dashed line is $x' = -(\alpha_x/\beta_x)x$

Generally, the standard deviation of the momentum coordinate is $\sigma_{x'} = \sigma_x \sqrt{1 + \alpha_x^2/\beta_x}$ with $\alpha_x \equiv -\beta'_x/2$. Since both electrons are at equal position x , it reduces to $\sigma_{x'} = \sigma_x/\beta_x$, the standard deviation for a distribution relative to the axis $x' = -(\alpha_x/\beta_x)x$ as illustrated by Fig. F.2. With two electrons involved, the distribution P_Δ of momentum differences Δ is broader by a factor of $\sqrt{2}$. Thus

$$\begin{aligned} \overline{v\sigma_t} &= \int v(\Delta) \sigma_t(\Delta) P_\Delta d\Delta \\ &= \int \frac{1}{2\sqrt{\pi}} \frac{\beta_x}{\sigma_x} v(\Delta) \sigma_t(\Delta) \exp\left\{-\frac{\beta_x^2 \Delta^2}{\sigma_x^2 4}\right\} d\Delta. \end{aligned} \quad (\text{F.11})$$

This shortcut argument yields the same result as explicit integration over the coordinates x and x'_1 [90]. Inserting (F.7), (F.10) and (F.11) into (F.9) leads to

$$\begin{aligned} \frac{1}{\tau} = -\frac{\dot{N}}{N} &= -\frac{N}{\gamma^5} \frac{r_e^2 c}{\pi \sigma_x \sigma_y \sigma_z} \left(\frac{\beta_x}{\sigma_x}\right) \\ &\times \int_{\frac{2\Delta_P}{\gamma}}^{\infty} \frac{1}{\Delta^3} \left[\frac{1}{\mu^2} - 1 - \frac{1}{2} \ln \frac{1}{\mu^2} \right] \exp\left\{-\frac{\beta_x^2 \Delta^2}{\sigma_x^2 4}\right\} d\Delta \end{aligned} \quad (\text{F.12})$$

$$= -\frac{N}{\gamma^5} \frac{r_e^2 c}{8\pi \sigma_x \sigma_y \sigma_z} \left(\frac{\beta_x}{\sigma_x}\right)^3 \int_{\xi}^{\infty} \frac{1}{u^2} \left[\frac{u}{\xi} - 1 - \frac{1}{2} \ln \frac{u}{\xi} \right] \exp\{-u\} du \quad (\text{F.13})$$

using the definitions

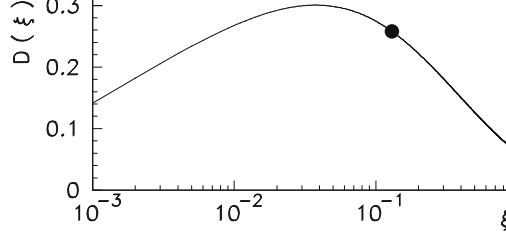


Fig. F.3. The function $\mathcal{C}(\xi)$ according to (F.17). A *dot* marks the value for the parameters of the BESSY II storage ring

$$u \equiv \left(\frac{\beta_x}{\sigma_x} \frac{\Delta}{2} \right)^2 \quad \text{and} \quad \xi \equiv \left(\frac{\Delta p/p}{\gamma} \frac{\beta_x}{\sigma_x} \right)^2 \quad (\text{F.14})$$

and thus

$$\frac{1}{\mu^2} = \frac{u}{\xi}. \quad (\text{F.15})$$

The lower integration limit corresponds to the restriction $\mu \leq 1$ according to (F.7). Equation (F.13) is equivalent to the expression found in the literature

$$\frac{1}{\tau} = -\frac{N}{\gamma^2} \frac{r_e^2 c}{8\pi\sigma_x\sigma_y\sigma_z} \frac{1}{(\Delta p/p)^3} \cdot \mathcal{C}(\xi) \quad (\text{F.16})$$

$$\text{with } \mathcal{C}(\xi) = \xi^{3/2} \int_{\xi}^{\infty} \frac{1}{u^2} \left[\frac{u}{\xi} - 1 - \frac{1}{2} \ln \frac{u}{\xi} \right] \exp\{-u\} du. \quad (\text{F.17})$$

The integral may be written in a more compact form, but must in any case be evaluated numerically. As shown in Fig. F.3, the function $\mathcal{C}(\xi)$ varies only slowly with ξ and the essential dependencies can be read from (F.16). For third-generation synchrotron radiation sources, ξ is typically 10^{-1} in the vicinity of a broad maximum with $\mathcal{C} \approx 0.3$. Note that ξ is proportional to $1/\varepsilon_x$ according to (F.14), and $\mathcal{C}$ may be significantly lower for future synchrotron radiation sources with smaller emittance.

G

Ion Effects in Electron Storage Rings

Due to their secondary importance for third-generation synchrotron radiation sources, ion effects were not discussed in the main part of this book. However, under adverse vacuum conditions, e.g., during commissioning of a storage ring or when the vacuum vessel had been recently vented, ion effects can be significant and some basic facts shall be presented in this section. Two distinctly different effects will be discussed:

- Ion trapping [352], building up over many revolutions, causes ions to gather in the potential well caused by the negative electron charge, where they lead to instabilities and reduction of the beam lifetime.
- An ion instability building up over a single revolution was postulated some years ago [353, 354] and was experimentally verified [355].

Ion effects do not occur if positrons are stored instead of electrons, as was temporarily the case, e.g., at the APS (Argonne, USA) [356] and other storage rings. However, analogous effects caused by photoelectrons and secondary electrons produced by them may show up [357]. A related problem is the capture of “macro particles”, i.e., small dust particles with a size of a few μm [358].

G.1 Ionization and Ion Trapping

A stored electron beam ionizes the residual gas mainly by direct collisions. The cross section is [359]

$$\sigma [\text{Mbarn}] = 1.87 \times 10^{-2} \{M^2(2 \ln \gamma - 1) + C\} \quad (\text{G.1})$$

with the dimensionless quantities M^2 and C depending on the gas molecule (e.g., $M^2 = 3.7$, $C = 35.1$ for carbon monoxide). Typical cross sections amount to several Mbarn. In the case of ionization due to synchrotron radiation and photoelectrons, the ions occupy a large volume and are less relevant. The ionization rate per unit length is

$$\dot{\lambda} = \sigma N_e f_0 \rho \approx 10^9 \dots 10^{10} \text{ m}^{-1} \text{ s}^{-1}, \quad (\text{G.2})$$

where N_e is the total number of electrons, f_0 is the revolution frequency, and ρ is the gas density according to (3.11). There are limits to the line density λ :

- Once the ion density reaches the electron density, the electron charge is neutralized and loses its effect.
- The ion density is limited by the gas density.

These limits are reached after roughly one second [360], provided ion trapping occurs at all, which requires stable ion trajectories in the potential of the electron charge. In linear approximation, the effect of an electron bunch with transverse dimensions σ_x and σ_y on ions with mass A and charge e can be described by a kick towards the beam axis with an angle $-a_u u$ proportional to the transverse offset $u = x$ or y , where [352]

$$a_u = \frac{N_e}{n_b} \frac{2 r_p}{\sigma_u (\sigma_x + \sigma_y)} \frac{1}{A} \quad (\text{G.3})$$

and $r_p = 1.535 \times 10^{-18} \text{ m}$ is the classical proton radius. For a train of n_b successive electron bunches and a harmonic number h , the ion position is described by one transport matrix for each bunch and matrices for the drift spaces in-between:

$$\begin{pmatrix} u \\ u' \end{pmatrix}_{s=h \lambda_{\text{rf}}} = \left[\begin{pmatrix} 1 & \lambda_{\text{rf}} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -a_u & 1 \end{pmatrix} \right]^{n_b} \begin{pmatrix} 1 & \lambda_{\text{rf}} \\ 0 & 1 \end{pmatrix}^{h-n_b} \begin{pmatrix} u \\ u' \end{pmatrix}_{s=0}. \quad (\text{G.4})$$

The length of each drift space corresponds to the rf period λ_{rf} . Transport matrices describe stable particle trajectories if the absolute value of their trace is ≤ 2 . For an even fill of the storage ring ($n_b = h$), the mass values, for which stable trajectories exist, are

$$A \geq \frac{N_e}{n_b^2} \frac{r_p h \lambda_{\text{rf}}}{2 \sigma_u (\sigma_x + \sigma_y)}. \quad (\text{G.5})$$

For a fill pattern with $n_b < h$, more elaborate computation is needed. Figure G.1 shows the result of a calculation for a beam current of 100 mA in the BESSY II storage ring. Stable combinations of ion mass and gap size are indicated by dots. According to this result, it would seem doubtful whether typical ion effects like

- tune shifts accompanied by particle loss due to optical resonances,
- instabilities with transverse oscillations of ions against electrons with frequencies in the MHz range,

can be eliminated by an ion-clearing gap of, say, $h/10$ empty buckets. This, however, is indeed the case, as the experience at BESSY II and many other

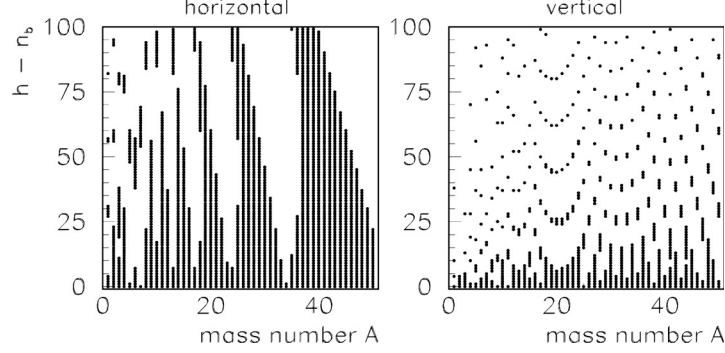


Fig. G.1. Combinations of mass number and $h - n_b$ (number of empty buckets), for which the linear theory predicts stable horizontal (*left*) and vertical (*right*) ion trajectories in the BESSY II storage ring

storage rings shows. The reason is the nonlinearity neglected in (G.3). Contrary to the approximation made before, a_u depends on the ion position relative to the electron beam axis. Simulations that take this into account yield significantly less stable ion trajectories than the linear theory [361].

For third-generation light sources, further countermeasures against ions such as static or oscillating electric fields (so-called clearing fields) are generally found to be dispensable.

G.2 The “Fast” Beam–Ion Instability

For an electron beam with small emittance, the ions produced during a single revolution may also lead to instabilities. Such a “fast” beam-ion instability was predicted in 1995 [353, 354] and is based on the idea that ions produced by a transversely displaced electron bunch can excite an oscillation of a trailing bunch. Starting from a small random displacement of a bunch, the oscillation amplitude increases exponentially with longitudinal distance z (measured against the direction of flight). For small excursions and $s = z - ct$, the oscillation amplitudes of the bunches are given in first approximation by

$$y(z, s) \sim y_0 \exp \left\{ \frac{z}{n_b \lambda_{rf}} \sqrt{\frac{t}{\tau_c}} \right\} \sin(\omega_i z/c - \omega_\beta s/c), \quad (\text{G.6})$$

where ω_i is the oscillation frequency of the ions, ω_β is the betatron frequency, and $1/\tau_c$ is the growth rate. Taking a spread of frequencies ω_i due to nonlinearities into account, y tends to grow at a reduced rate. At synchrotron radiation sources with an achromatic lattice, the frequency spread $\Delta\omega_i$ is more pronounced than at machines with a FODO structure.

Once the oscillation amplitude reaches the size of the electron bunches, the instability does not grow further. The oscillation frequency of ions with

charge eQ is given by [354]

$$\omega_i = c \sqrt{\frac{4Q N_e r_p}{3A n_b \lambda_{\text{rf}} \sigma_u (\sigma_x + \sigma_y)}} \quad (u = x \text{ or } y) \quad (\text{G.7})$$

and amounts to a few MHz. Experimentally, betatron sidebands appear in the beam spectrum, concentrated around a frequency that increases with the number of electrons N_e , i.e., with the beam current. The first observation of this instability at the ALS (Berkeley, USA) was made by artificially increasing the gas pressure in the vacuum vessel to $\sim 10^{-7}$ hPa [355]. Meanwhile, observations consistent with a fast beam-ion instability were also made at the B-meson factories PEP II (Stanford, USA) [362] and KEKB (Tsukuba, Japan) [363] and at the ESRF (Grenoble, USA) [364].

For present-day facilities, this instability is not a problem, but it should be kept in mind for future synchrotron light sources with smaller emittance. Possible remedies are an increased chromaticity or the introduction of additional empty rf buckets.

H

Address List

Here, the accelerators and storage rings explicitly mentioned in this book are listed alphabetically, and the addresses of their respective institutes are given below. The name given in brackets may deviate from the respective acronym.

H.1 Accelerators and Storage Rings

AdA (Anello di Accumulatione e^+e^-) see LNF
ALS (Advanced Light Source) see LBNL
APS (Advanced Photon Source) see ANL
BESSY I see BESSY
BESSY II see BESSY
DAΦNE (Φ -Factory) see LNF
DORIS see DESY
ELETTRA see Sincrotrone Trieste
ESRF see ESRF
KEKB (KEK B-Factory) see KEK
LEP (Large Electron-Positron Collider) see CERN
MAX II see MAX-laboratory
PETRA see DESY
PEP II (Asymmetric B-Factory) see SLAC
PLS (Pohang Light Source) see PAL
SLS (Swiss Light Source) see PSI
SPPS (Sub-picosecond Photon Source) see SLAC
SPring8 (Large-scale Synchrotron Radiation Facility) see JASRI
SRRC (Taiwan Light Source) see SRRC
SPEAR II or
SSRL (Stanford Synchrotron Radiation Laboratory) see SLAC
SUPER-ACO see LURE
TESLA (TeV-Energy Superconducting Linear Accelerator) see DESY
TRISTAN see KEK

H.2 Institutions

ANL (Argonne National Laboratory)
9700 S. Cass Avenue, Argonne, IL 60439, USA
(www.anl.gov)

BESSY (Berliner Gesellschaft für Synchrotronstrahlung mbH)
Albert-Einstein-Str. 15, 12489 Berlin, Germany
(www.bessy.de)

BINP (Budker Institute of Nuclear Physics)
Acad. Lavrentiev prospect 11, 630090 Novosibirsk, Russia
(www.inp.nsk.su)

BNL (Brookhaven National Laboratory)
P.O. Box 5000, Upton, NY 11973, USA
(www.bnl.gov)

CERN (European Organization of Nuclear Research)
1211 Geneva 23, Switzerland
(www.cern.ch)

DESY (Deutsches Elektronensynchrotron)
Notkestr. 85, 22607 Hamburg, Germany
(www.desy.de)

ESRF (European Synchrotron Radiation Facility)
BP 220, 38043 Grenoble Cedex, France
(www.esrf.fr)

JAERI (Japan Atomic Energy Research Institute)
2-4 Shirakata shirane, Tokai, Ibaraki 319-1195, Japan
(www.jaeri.go.jp)

JASRI (Japan Synchrotron Radiation Research Institute)
1-1-1 Kouto, Mikazuki-cho, Hyogo 679-5198, Japan
(www.spring8.or.jp)

Jefferson Lab (Thomas Jefferson National Accelerator Facility)
12000 Jefferson Avenue, Newport News, VA 23606, USA
(www.jlab.org)

KEK (High Energy Accelerator Research Organization)

1-1 Oho, Tsukuba, Ibaraki 305-0801, Japan

www.kek.jp**LBNL** (Ernest Orlando Lawrence Berkeley National Laboratory)

1 Cyclotron Rd, Berkeley, CA 94720, USA

www.lbl.gov**LNF** (Laboratori Nazionali di Frascati)

Via E. Fermi 40, 00044 Frascati, Italy

www.lnf.infn.it**LURE** (Laboratoire pour l'Utilisation du Rayonnement Electromagnétique)

Bât 209D Centre Universitaire, B.P. 34, 91898 Orsay Cedex, France

www.lure.u-psud.fr**MAX-Laboratory**

Box 118, 22100 Lund, Sweden

www.maxlab.lu.se**PAL** (Pohang Accelerator Laboratory)

San-31 Hyoja-dong, Pohang Kyungbuk, 790-784, South Korea

www.pal.postech.ac.kr**PSI** (Paul-Scherrer-Institut)

5232 Villigen, Schweiz

www.psi.ch**Sincrotrone Trieste S.C.p.A.** di interesse nazionale

Strada Statale 14 km 163.5, 34012 Basovizza, Trieste, Italy

www.elettra.trieste.it**SLAC** (Stanford Linear Accelerator Center)

2575 Sand Hill Road, Menlo Park, CA 94025, USA

www.slac.stanford.edu**SRRC** (Synchrotron Radiation Research Center)

No.1 R&D Road VI, Industrial Park, Hsinchu 300, Taiwan

www.srrc.gov.tw**UCLA** (University of California, Los Angeles)

405 Hilgard Avenue, Los Angeles, CA 90095, USA

www.ucla.edu

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