PERT

Project Evaluation and Review Technique (PERT)

- **Developed in US Navy (1958) for the POLARIS missile program**
- **The emphasis was on completing the program in the shortest possible time.**
- **Many projects involve variability in activity durations due to factors such as lack of prior experience, equipment breakdown, unpredictable weather conditions, late delivery of supplies, and others.**

PERT

- PERT had the ability to cope with uncertain activity completion times (e.g. for a particular activity the most likely completion time is 4 weeks but it could be anywhere between 3 weeks and 8 weeks).
- Multiple task time estimates (probabilistic nature)
- PERT is based on the assumption that an activity's duration follows a probability distribution instead of being a single value.

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Three time estimates are required to compute the parameters of an activity's duration distribution:

- **pessimistic time (t_p) - the time the activity would take if things did not go well(worst case scenario)**
- **most likely time (t_m) - the consensus best estimate of the activity's duration**
- **optimistic time (t_o) - the time the activity would take if things did go well(best case scenario)**
- **Standard Deviation(σ)-shows how widely from the te that the actual values are spread.**

$$Expected\ Time(t_e) = \frac{t_o + 4t_m + t_p}{6}$$

$$\sigma = \frac{t_p - t_o}{6} \qquad v(variance) = \sigma^2$$

Probability Calculations

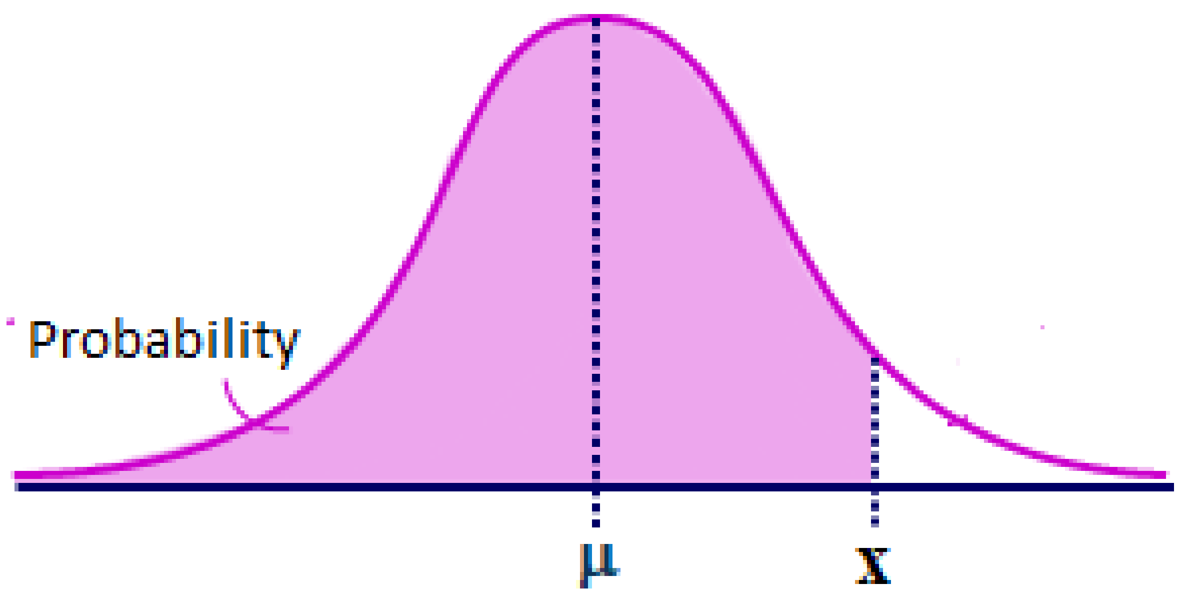
Determine probability that project is completed within specified time

$$Z = \frac{x - \mu}{\sigma_{CA}}$$

where μ = project mean(expected) time

σ_{CA} = project standard mean time

x = (proposed) specified time



PERT Analysis

- **Step 1:** Draw the network.
- **Step 2:** Analyze the paths through the network and find the critical path.
- **Step 3:** The length of the critical path is the mean of the project duration probability distribution which is assumed to be normal
- **Step 4:** The standard deviation of the project duration probability distribution is computed by adding the variances of the critical activities (all of the activities that make up the critical path) and taking the square root of that sum
- **Step 5:** Probability computations can now be made using the normal distribution table.

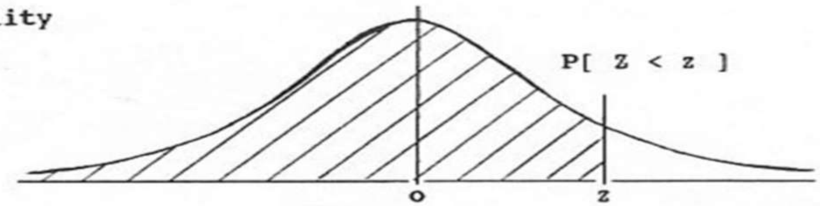
PERT

STANDARD STATISTICAL TABLES

1. Areas under the Normal Distribution

The table gives the cumulative probability up to the standardised normal value z i.e.

$$P[\ Z < z\ ] = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} \exp(-\tfrac{1}{2}Z^2)\ dZ$$



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5159	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7854
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8804	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9773	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9865	0.9868	0.9871	0.9874	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9924	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9980	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986

z	3.00	3.10	3.20	3.30	3.40	3.50	3.60	3.70	3.80	3.90
P	0.9986	0.9990	0.9993	0.9995	0.9997	0.9998	0.9998	0.9999	0.9999	1.0000

Problem on PERT

A company is launching a new product and has made estimates of the time for the various activities associated with the launch as follows:

Activity	Predecessor	Times (Days)		
		Optimistic	Most likely	Pessimistic
A	**None**	**1**	**3**	**5**
B	**None**	**3**	**4**	**5**
C	**A, B**	**1**	**3**	**11**
D	**B**	**3**	**3**	**9**
E	**A**	**1**	**2**	**3**
F	**C**	**2**	**5**	**14**
G	**E, F**	**2**	**3**	**4**
H	**D, F**	**2**	**2**	**2**
I	**G, H**	**10**	**10**	**10**

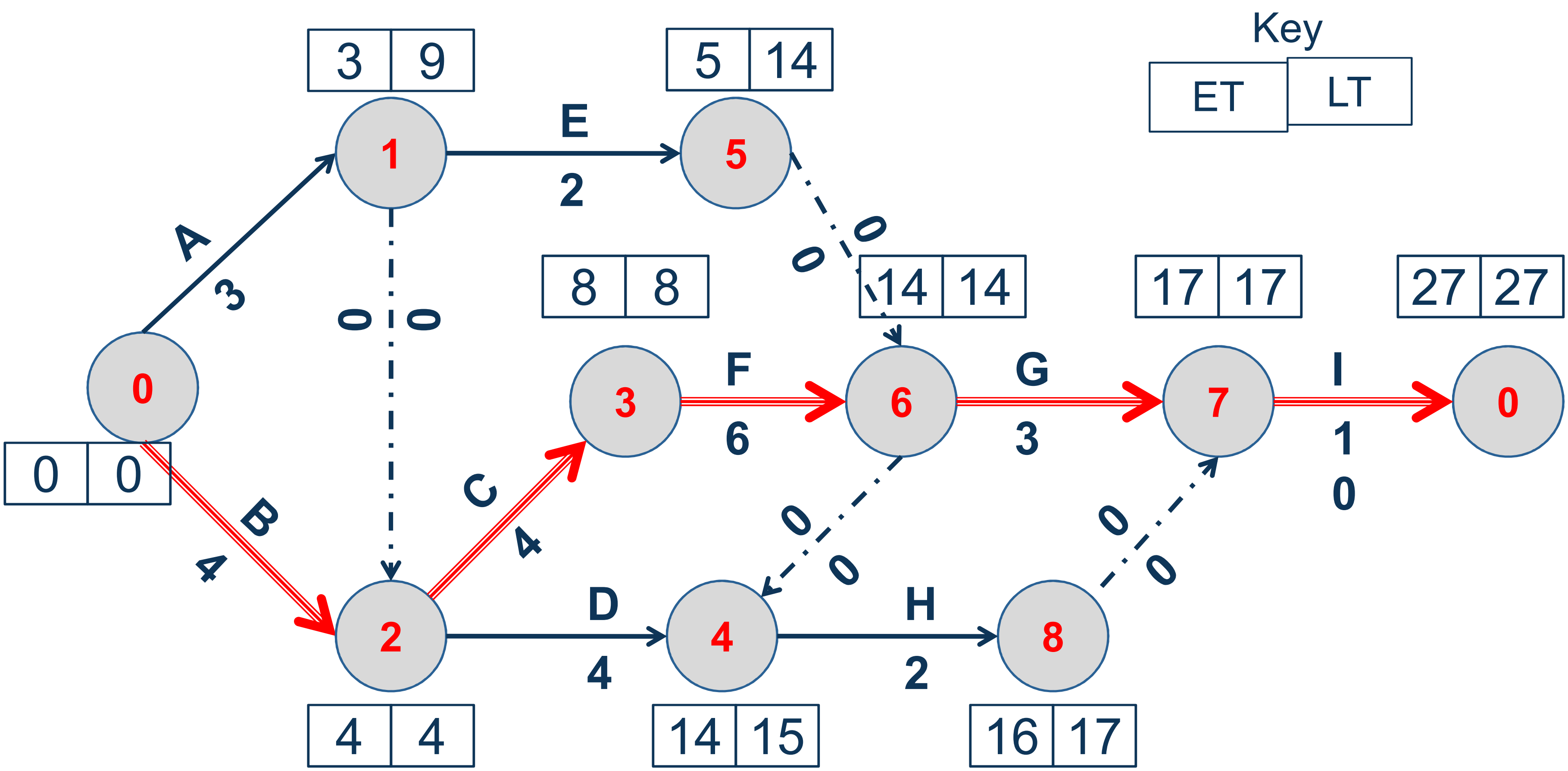
PERT

Required:

1. ***Draw the network diagram.***
2. ***Calculate the expected time and variance of each activity.***
3. ***Find out the expected length of critical path and its standard deviation.***
4. ***Find the probability that the launching will be completed in 29 days.***
5. ***Find the duration, which has 95% probability of completion.***

Answer

(i) Network Diagram



Critical Path B-C-F-G-I

(ii) Calculation of Expected Time , Standard Deviation and Variance of Activities

Activity	Pred.	Times (Days)					
		t_o	t_m	t_p	t_e	Standard dev. σ	Variance σ^2
A	-	1	3	5	(1+4*3+5)/6=3	(5-1)/6=0.67	0.44
B	-	3	4	5	(3+4*4+5)/6=4	(5-3)/6=0.33	0.11
C	A, B	1	3	11	(1+4*3+11)/6=4	(11-1)/6=1.67	2.78
D	B	3	3	9	(3+4*3+9)/6=6	(9-3)/6=1.00	1.00
E	A	1	2	3	(1+4*2+3)/6=2	(3-1)/6=0.33	0.11
F	C	2	5	14	(2+4*5+14)/6=6	(14-2)/6=2.00	4.00
G	E, F	2	3	4	(2+4*3+4)/6=3	(4-2)/6=0.33	0.11
H	D, F	2	2	2	(2+4*2+2)/6=2	(2-2)/6=0.00	0.00
I	G, H	10	10	10	(10+4*10+10)/6=10	(10-10)/6=0.00	0.00

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(iii) Standard Deviation of the Critical Path(B-C-F-G-I)

$$\sigma = \sqrt{(\text{total variance of critical activities})}$$

$$= \sqrt{(0.11+2.78+4+0.11+0)}$$

$$= 2.645$$

(iv) Probabilities of completion of job in 29 days. (from the table).

X = 29 days

Z = (29-27)/2.645 = 0.76

For Z = 0.76 the probability is 0.7764 from the table of area under normal curve or 77.64%.

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(v) For 95% of area the corresponding Z value is 1.64

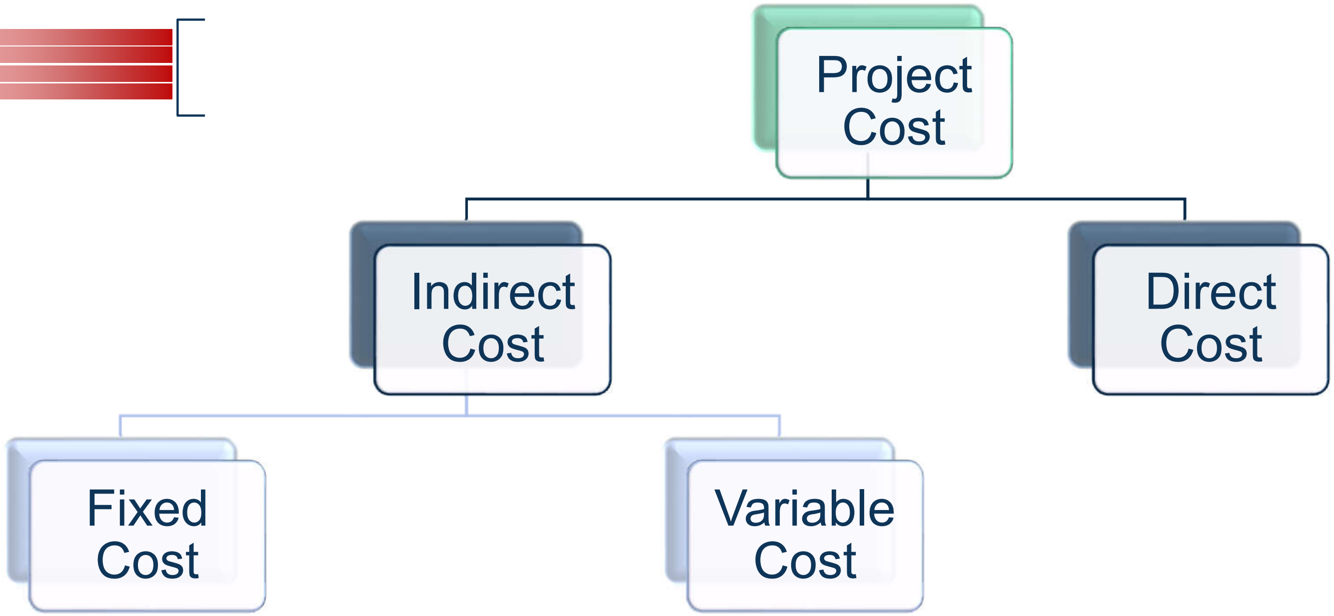
$$1.64 = \frac{x-27}{2.645}$$

Then x = 27+2.645*1.64

=31.33 days

Project Cost

- Project managers may have the option or requirement to crash the project, or accelerate the completion of the project.
- This is accomplished by reducing the length of the critical path(s).
- The length of the critical path is reduced by reducing the duration of the activities on the critical path.
- If each activity requires the expenditure of an amount of money to reduce its duration by one unit of time, then the project manager selects the least cost critical activity, reduces it by one time unit, and traces that change through the remainder of the network.
- Generally, project cost may be classified as follows:



- **Direct Cost:** It is the amount of cost, which is directly dependent on the amount of resources involved to complete activities. The resources include labor, materials, plants, equipment, machineries, payment to subcontractors, etc.
- **Fixed Indirect Cost:** These costs are independent of the progress of the project. For example, initial expenditure, license, taxes, etc.
- **Variable Indirect Cost:** These costs are directly proportional to the project time. For example, office OH, supervision costs, cost of providing utilities, etc.

Project Crushing

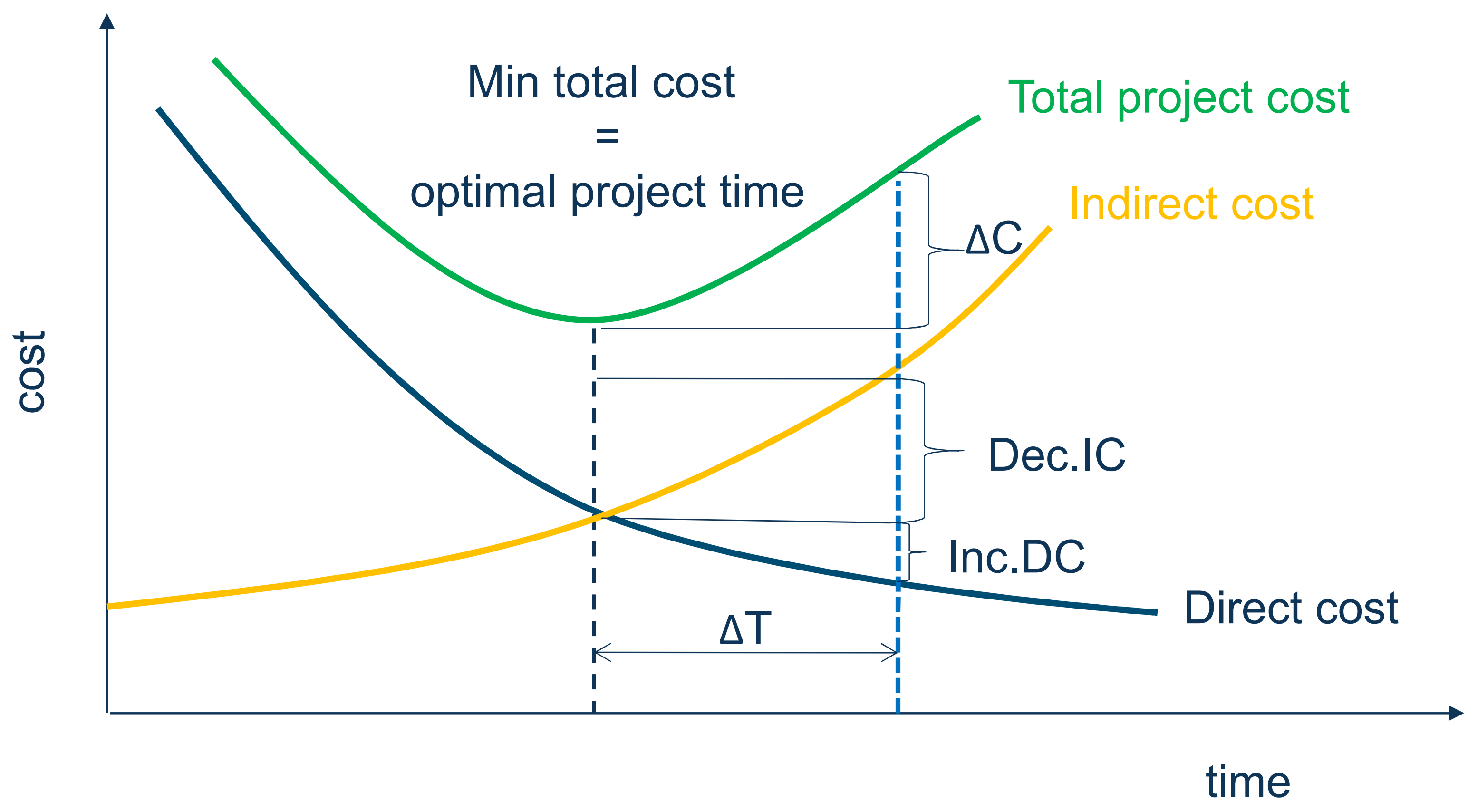
- ✓ In many instances, it is desirable to cut down the project duration which naturally leads to cost considerations.
- ✓ The search for optimum duration and minimum cost can be obtained by crushing techniques.
- ✓ Terminologies
  - Project Crushing: reducing project time by expending additional resources
  - Crushed Time: The minimum duration of activity after crushing.
  - Crashed cost: The total cost of activity if it is crushed to it's crushed time.
  - Crushable time: The maximum allowable amount of time an activity can be reduced.
  - Crashing cost: cost of reducing activity per unit time

Time-Cost Tradeoff

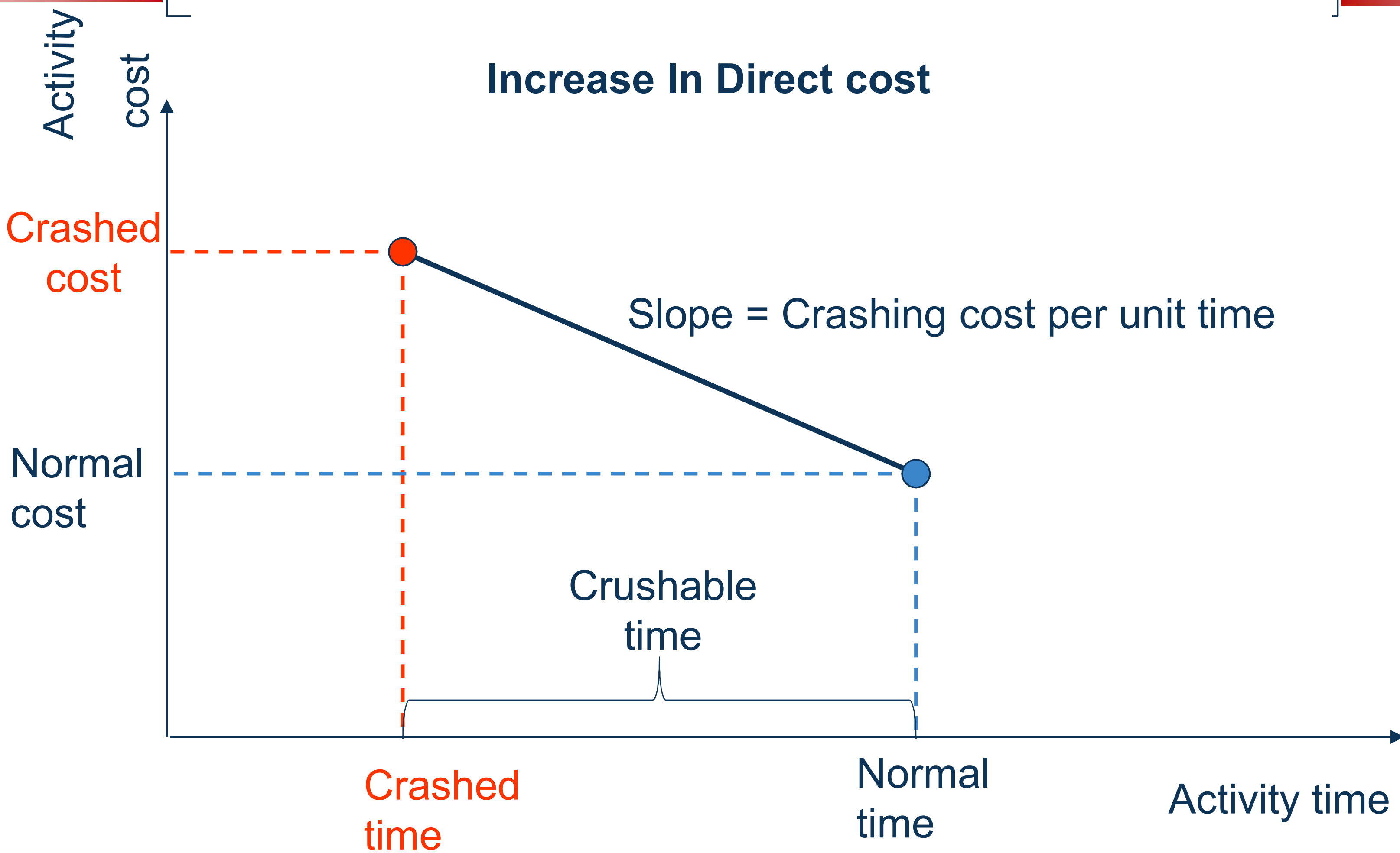
- **Goal**:
  - To reduce project duration at minimum cost
- Project completion times may need to be shortened because:
  - Different deadlines
  - Penalty clauses
  - Need to put resources on a new project
  - Promised completion dates
- Crashing a project needs to balance
  - Shorten a project duration
  - Cost to shorten the project duration

Time-Cost Tradeoff

- Crashing costs increase as project duration decreases
- Indirect costs decrease as project duration decreases
- Reduce project length as long as crashing costs are less than indirect costs



Activity Crashing



Steps on Project Crushing

- **Step 1:** Compute the crash cost per time period(Crushing cost)
- **Step 2:** Find the current critical path (CP)
- **Step 3:** Find the lowest cost way to crash the CP by 1 time period
- **Step 4:** Update all activity times. If further crashing is needed, go to step 2.

✓ **Note:**

- ✓ ***When there is more than one critical path, each of the critical paths must be reduced.***
- ✓ ***Total Project Cost*** $= \sum NDC_i + t^*IC + \sum CC_i$

  Where

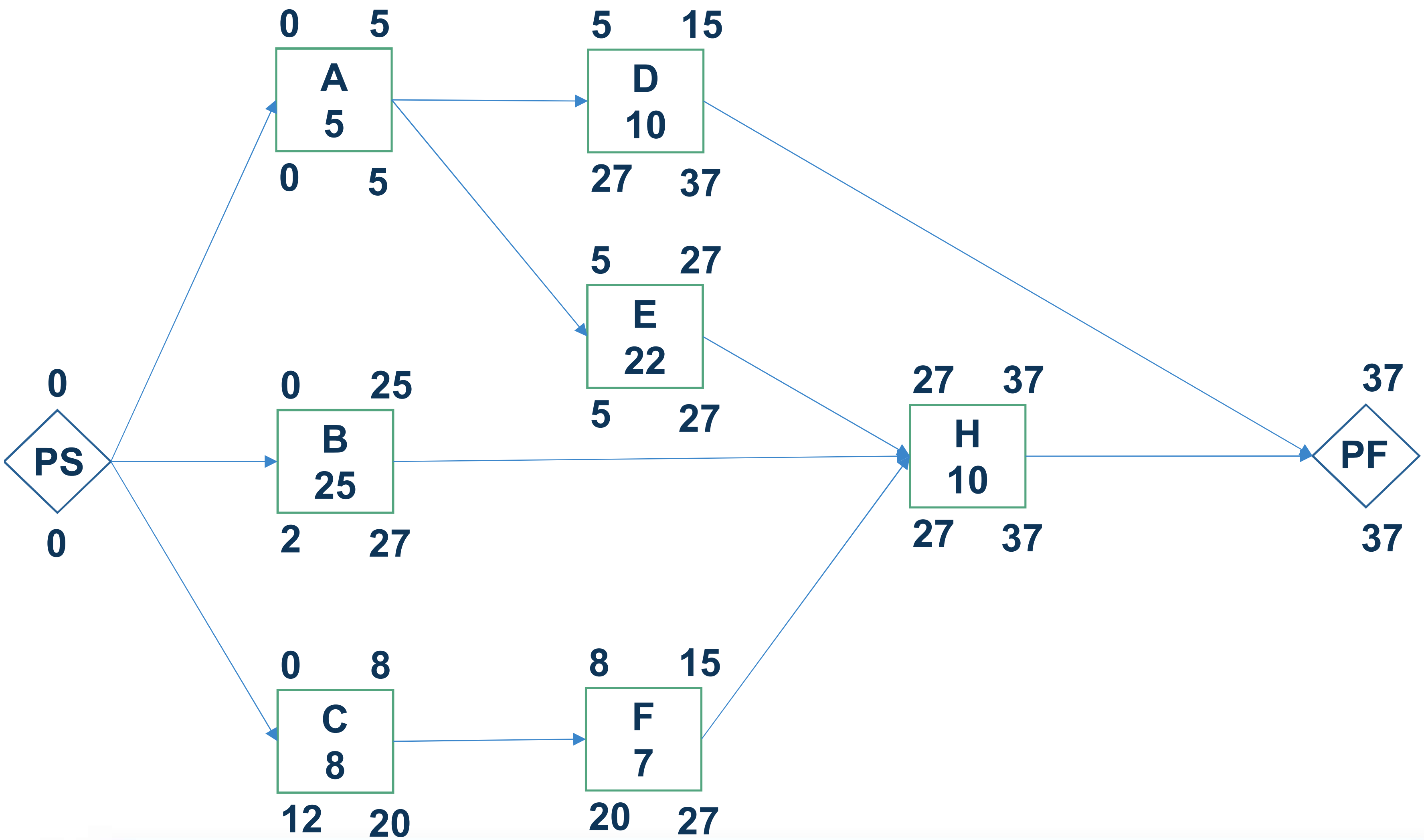
  - ✓ ***NDC is normal direct cost***
  - ✓ ***IC is indirect cost and***
  - ✓ ***CC is crushing cost***

Example 1

Calculate the optimum duration for the following project data. Indirect cost is 200ETB/day. Time in days & crushing cost is ETB per day.

Activity	Prede.	Normal Time	Crushed Time	Normal Cost	Crushing Cost
A	-	5	3	1,400	50
B	-	25	15	3,000	100
C	-	8	7	3,600	120
D	A	10	8	2,900	100
E	A	22	19	2,250	75
F	C	7	5	2,600	90
G	D	6	6	2,250	75
H	B, E, F	10	8	7,500	250

AON Diagram


0
5
A
5
0
5
5
15
D
10
27
37
5
27
E
22
5
27
0
PS
0
0
25
B
25
2
27
27
37
H
10
27
37
37
PF
37
0
8
C
8
12
20
8
15
F
7
20
27

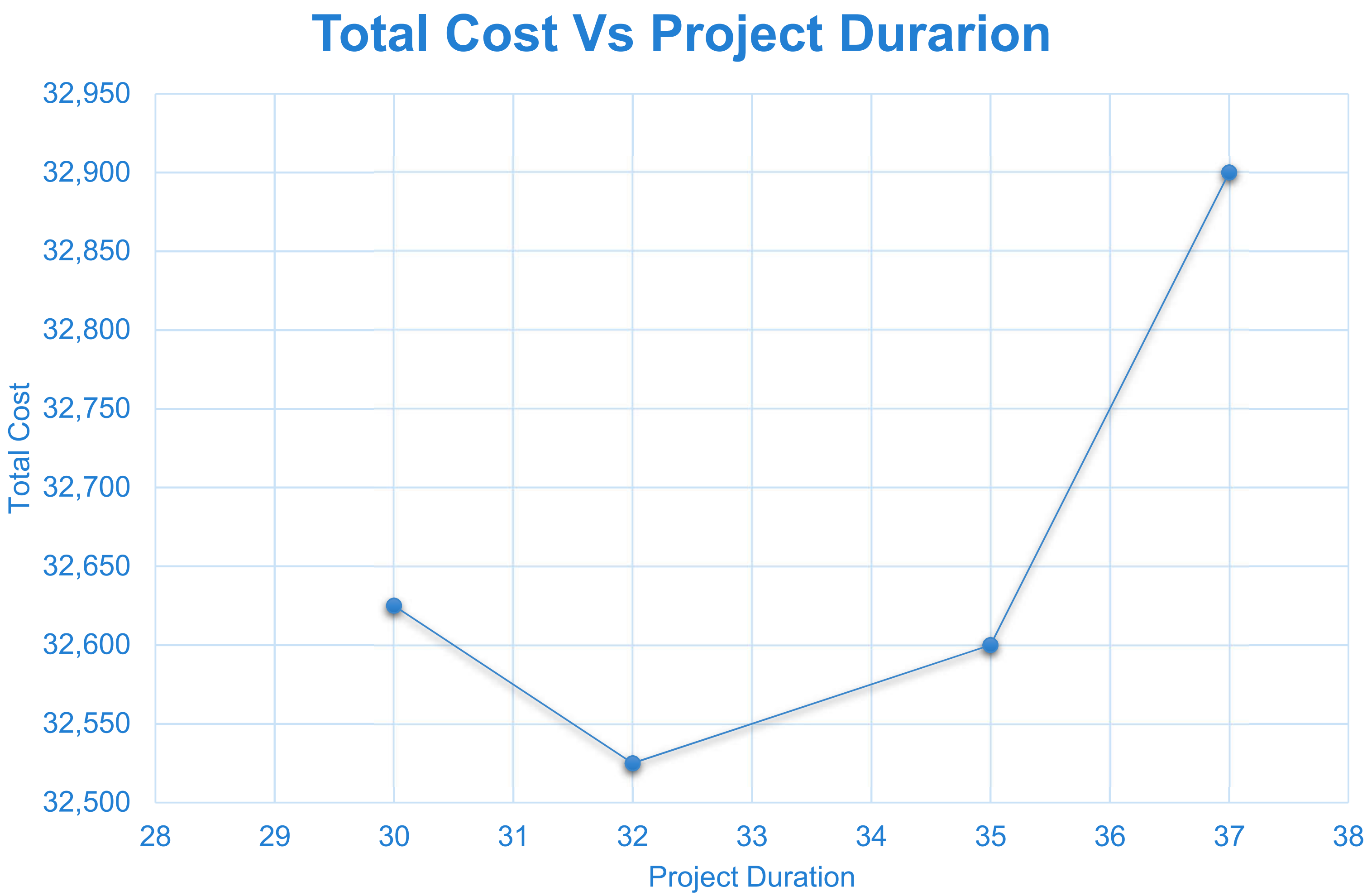
- **Stage 1(s1)**
  - **Network diagram is draw, critical path analyzed i.e. A-E-H with duration 37 days.**
- **Stage 2(s2)**
  - **Among activities A, E & H, activity A is cheapest to crash. Crash to full by 2days. Normally, 1 day at a time shall be crushed.Now, there are two critical paths i.e. A-E-H & B-H each with 35 days.**
- **Stage 3(s3)**
  - **Cost of H > cost of (E + B). So, crush E & B by 3 days each. Critical path remains the same but duration 32 days. Each path can also be separately crushed 1 day at a time.**
- **Stage 4(s4)**
  - **A, E are fully crushed. B has scope but cannot be selected for crushing. So the only one is H. Crash fully by 2 days. The duration will be 30 days.**

Result Tabulation

Stage	Description	Duration	Indirect Cost	Normal Direct cost	∑Crushing Cost	Total Cost	Remark
1	All normal	37	7,400	25,500	0	32,900	Normal
2	Crush ‘A ‘ by 2 @50ETB/day	35	7,000	25,500	100	32,600	
3	Crush ‘B’ & ‘E’ by 3 @(100+75)ETB/day	32	6,400	25,500	625	32,525	Optimum
4	Crush ‘H’ by 2 @250ETB/day	30	6,000	25,500	1125	32,625	minimum

Normal Direct Cost =
1,400+3,000+3,600+2,900+2,250+2,600+2,250+7,500 = 25,500

Result in graph



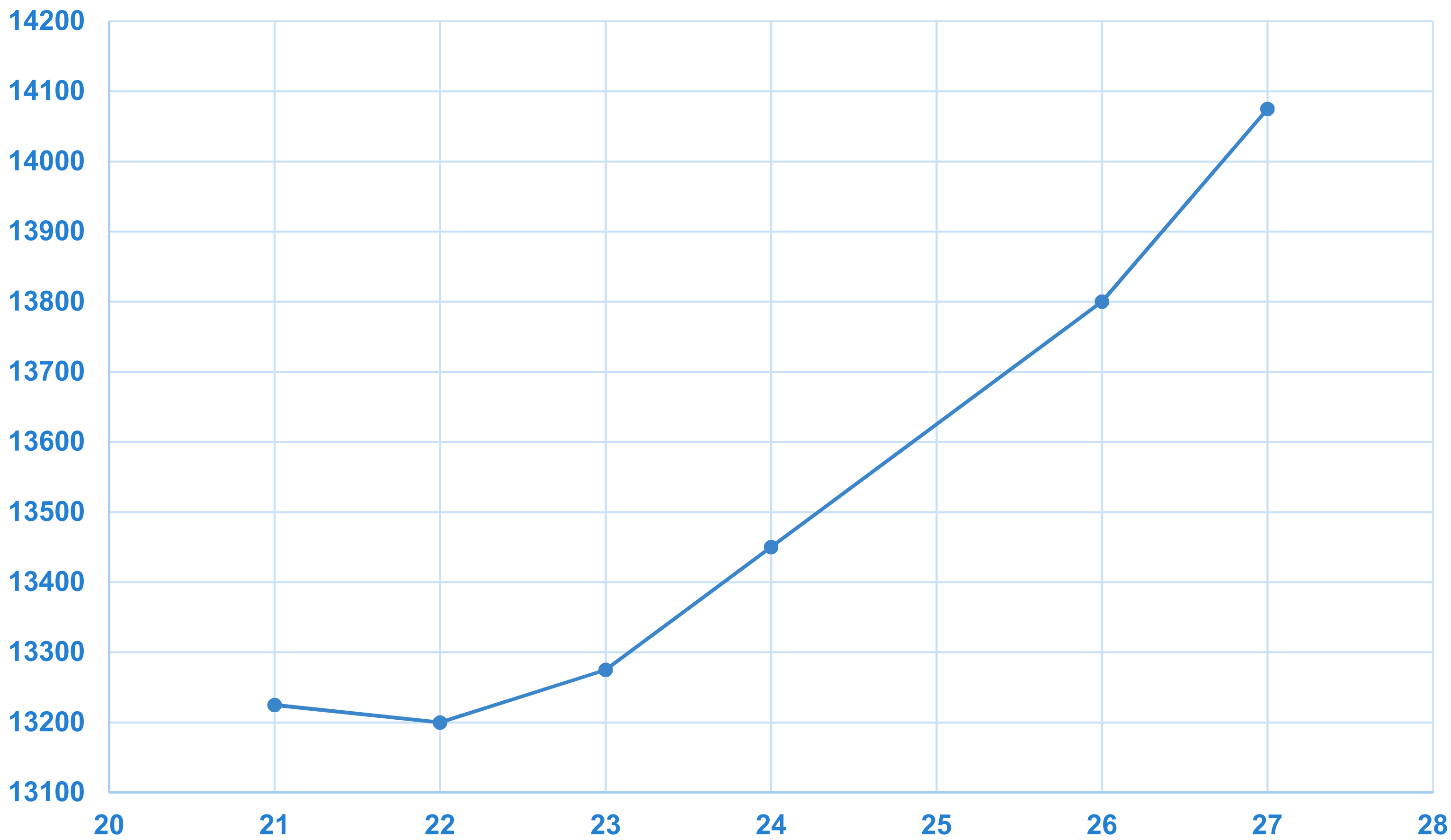
Example 2

Calculate the optimum duration for the following project data. Indirect cost is 325 ETB/day

Activity	IPA	Time(Days)		Cost(ETB)	
		Normal	Crashed	Normal	Crashed
A	-	1	1	800	800
B	A	7	4	1000	1600
C	A	6	4	300	500
D	A	3	2	400	800
E	B	3	1	100	200
F	B,C	7	5	500	800
G	D	8	4	200	1400
H	F,E	7	6	350	600
I	F	5	3	700	850
J	F,G	5	4	500	1000
K	H,I,J	5	4	450	800

Example 2

Activity	IPA	Time(Days)			Cost(ETB)		
		Normal	Crashed	Crushable	Normal	Crashed	Crushing
A	-	1	1	0	800	800	0
B	A	7	5	2	1000	1100	50
C	A	6	4	2	300	500	100
D	A	3	2	1	400	800	400
E	B	3	1	2	100	200	50
F	B,C	7	5	2	500	800	150
G	D	8	4	4	200	1400	300
H	F,E	7	6	1	350	600	250
I	F	5	3	2	700	850	75
J	F,G	5	4	1	500	1000	500
K	H,I,J	5	4	1	450	800	350
				∑	5300		



Optimum Duration: 22Days   Minimum Cost=13200

Comparison of CPM & PERT

	Factors	CPM	PERT
1	Field of application	Construction	Research & Development
2	Model emphasis	Activity oriented	Event oriented
3	Duration estimation	One-time method	Three-times method
4	Time-cost tradeoff	Feasible	Not feasible
5	Resources optimization	Feasible	Not feasible
6	Technique complexity	Simple	Comparatively difficult
7	Typical feature	Uses network, calculate float, identify critical path & activities, guides to monitor & controlling prj.	Same as CPM
8	Importance	Minimizing cost is more important	Meeting time target or estimating % completion is more important
9	Usage	Used where times can be estimated with confidence, familiar activities	Used where times cannot be estimated with confidence. Unfamiliar or new activities

Thank You !