CHAPTER 5

FLUID DYNAMICS

5.1 Introduction

In discussing about hydrostatics we were concerned with forces (pressure forces) which are acting an object for a liquid at rest and when we deal with kinematics of fluid flow phenomena related with space time variation (velocity and acceleration) with out considering the effect of force. But in dealing with dynamics of fluid flow, all forces that affect the phenomenon are considered.

The dynamics of fluid motion deals with kinetics, which relates the kinematics with the forces responsible for causing the motion. This relation ship of fluid motion is established by the use of laws of nature.

i) The principle of conservation of mass (the continuity relation ship)
ii) Newton's laws of motion
iii) The 1st and second lows of thermodynamics

5.2 Equations of motion

The dynamic behavior of fluid motion is governed by a set of equations, known as equations of motion. These equations are obtained by using the Newton's second low, which may be written as $F_x = m.a_x$

Where, F_x is the net force acting in the x-direction upon a fluid element of mass m producing an acceleration a_x in the x-direction.

The forces which may be present in fluid flow problems are gravity forces F_g, pressure force F_p, force due to viscosity F_v, force due to turbulence F_t, Surface tension F_s, and force due to compressibility of fluid F_c.

Gravity forces (F_g.) is due to the weight of the fluid. Its component in flow direction results in acceleration.

Pressure force (F_p): It is equal to the product of pressure intensity and cross sectional area of the flowing fluid. Acts normal to the surface under consideration and produces acceleration in the given direction.

Viscous forces (F_v): - Exists in real fluids. It is the shearing resistance generated when there is relative motion between two layers of fluids. It acts opposite to the direction of motion, and retards flow.

Surface tension (F_s): This force is important when the depth of flow is extremely small.

Force due to compressibility (F_c): for incompressible fluids, this becomes significant in problems of unsteady flow like water hammer. In most of flow problems F_s and F_c are neglected.

Force due to turbulence (F_t): the continuous momentum transfer between layers in highly turbulent flow results in normal and shear stresses known as Reynolds's stress.

If the changes for the change in forces are small the forces can be taken negligible.

$$ma_x=(F_g)_x+(F_p)_x+(F_v)_x+(F_t)_x$$

The presence of such a complex system of forces in real fluid flow problems makes the analysis very complicated. Therefore, mathematical analysis of problems is generally possible only if certain simplifying assumptions are made.

5.2.1 Energy and Head

A liquid in motion may possess three forms of energy.

1. *Potential energy /elevation /positional energy/* because of its elevation above datum level. If a weight w of liquid is at a height of z above datum
   Potential energy = Wz
   Potential energy per unit weight = z (meters) = Potential head

2. *Pressure energy*: When a fluid flows in a continuous stream under pressure it can do work. If the area of cross – section of the stream of fluid is a, then force due to pressure p on cross- section is Pa.

   If a weight w of liquid passes the cross section.

   Volume passing in cross section = W/ρg

   Distance moved by liquid $= \dfrac{W}{\rho g a}$

   $W = F^* S = Pa \dfrac{W}{\rho g a} = W\ P / pg$

   Pressure energy per unit weight = P/ρg = pressure head.

3. *Kinetic energy*
   If a mass of fluid (m) moves at some velocity (v),
   Kinetic energy = ½ mV^2 = ½ W/g v^2

   Kinetic energy pr unit weight = $\dfrac{v^2}{2g}$ = kinetic head

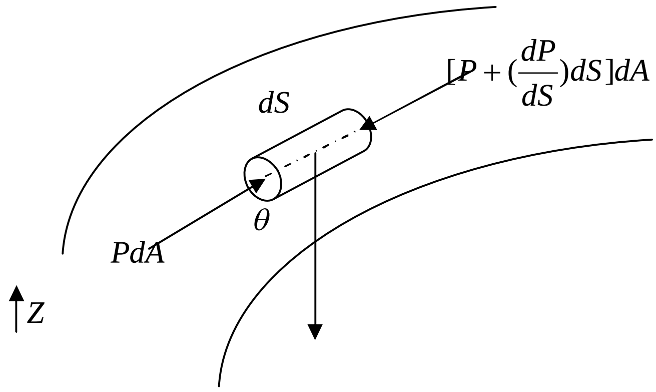
Total head = potential head + pressure head + velocity head = Z + $\dfrac{P}{\gamma}$ + $\dfrac{V^2}{2g}$

Bernoulli's theorem states that the total energy of each particle of a body of fluid is the same provided that no energy enters or leaves the system at any point. The division of this energy between potential, pressure and kinetic energy may vary, but the total remains constant. In symbols

$$Z + \frac{P}{\gamma} + \frac{V^2}{2g} = constant$$

5.3 Bernoulli's Equation

Consider a cylindrical element of stream tube having cross-sectional area d_A length ds unit weight γ as shown in motion along a streamline.



Bernoulli's assumptions:

- The flow is Steady an incompressible type of flow,
- Frictionless and non viscous flow,
- No shaft work - no pump or turbines on the streamline,
- No transfer of heat - either added or removed.

Fig: 5.1 Pressure and gravity forces on a cylindrical element along a streamline.

The normal forces on the side faces are in equilibrium and as the fluid is assumed non-viscous, there is no shear stress. The velocity varies along the streamline and there is acceleration. It is necessary to take into account force due to acceleration when considering the longitudinal balance of force.

But in the ease of steady flow the velocity doesn't vary at a point so that local acceleration will be zero $\left(\frac{\partial v}{\partial t} = 0\right)$ but for velocity variation with position convective acceleration will be different from zero $\left(V\frac{\partial v}{\partial s} \neq 0\right)$. The forces tending to accelerate the fluid mass are pressure force on the two ends of the element,

$[\sum Fs = ma_s]$ Summation of force in the arbitrary'sʼ direction.

$$PdA - \left(p + \frac{dp}{ds} ds \right) dA = -dp\ dA$$

Weight in the direction of motion

$$-\rho g ds dA \cos\theta = -\rho g ds dA \frac{dz}{ds} = -\rho g dA dz$$

Applying Newton's 2nd Law of motion F=m*a*

$$-dPdA - \rho g\ dA\ dz = \rho ds\ dAV \frac{dv}{ds} \qquad as \quad a_s = \frac{\partial v}{\partial t} + \frac{\partial v}{\partial s} V$$

$$\frac{dp}{\rho} + gdz + Vdv = 0 \qquad One\ dimensional\ Euler\ Equation$$

It can be applied for both compressible and incompressible flow.

$$\frac{dp}{\gamma} + dz + \frac{V}{g} dv = 0$$

For the case of an incompressible fluid γ may be treated as constant, the integration gives

$$\int \frac{dp}{\gamma} + \int dz + \int V \frac{dV}{g} = Constant$$

$$\frac{P}{\gamma} + Z + \frac{V^2}{2g} = Constant \text{ [Bernoulli's Equation]}$$

Under special conditions the assumption underlying Bernoulli's equations can be waived.

1. When streamlines originate from a reservoir,
2. For unsteady flow with gradually changing conditions (E.g. Emptying a reservoir) the equation may be applied without appreciable error,
3. It may be used for real fluids, by modifying the result experimentally.

The Bernoulli equation is the basis for the solution of a wide range of hydraulics problems. For two points along a streamline, the Bernoulli equation may be expressed in the form of

$$y_1 + p_1/\gamma + v_1^2/2g = y_2 + p_2/\gamma + v_2^2/2g$$

5.3.1 Bernoulli's Equation for real fluid

The Bernoulli's equation expressed by $\frac{p}{\gamma} + Z + \frac{v^2}{2g}$ is determined for an incompressible ideal fluid without taking in to account the effects of some other forces as viscous, etc.
In case of real fluid these forces should be introduced so that the equation needs some modification. A real fluid does possess viscosity and consequently it offers resistance to

flow. In order to overcome this viscous resistance and other resistances due to surface roughness and turbulence, some part of the total energy of the flow is lost. [Energy is neither created nor destroyed but may be changed to heat energy increasing temp of the fluid]

The increase in temperature of the fluid causes an increase in the internal energy. The increase in internal energy and the heat transfer from the fluid represent a loss of useful energy. The total loss per unit mass of fluid is (u_2-u_1-q).

Energy loss per unit weight in over coming resistance $h_l = \dfrac{u_2 - u_1 - q}{g}$ (head loss)

The total energy of flow decreases in the flow direction, and consequently the energy line has a down ward slope.

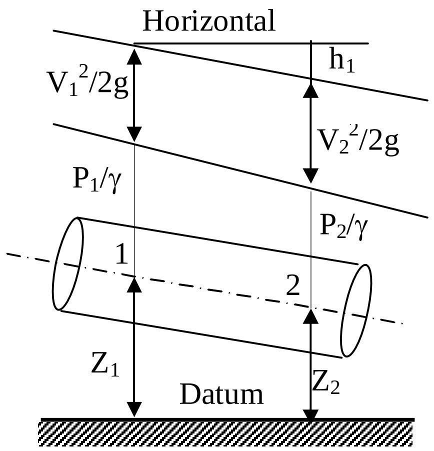


Fig. 5.2

The modified Bernoulli's equation for up stream section (1) and downstream section (2)

$$\frac{V_1^2}{2g} + \frac{P_1}{\gamma} + Z_1 = \frac{V_2^2}{2g} + \frac{p_2}{\gamma} + Z_2 + h_l$$

h_l=head loss between the two sections.

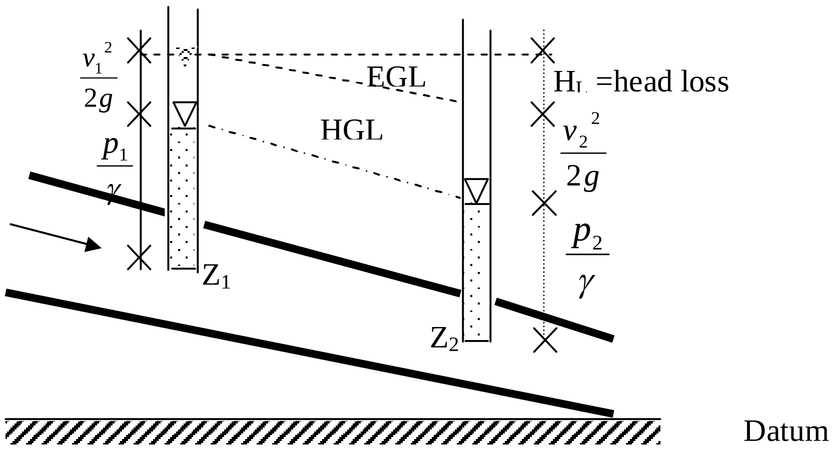


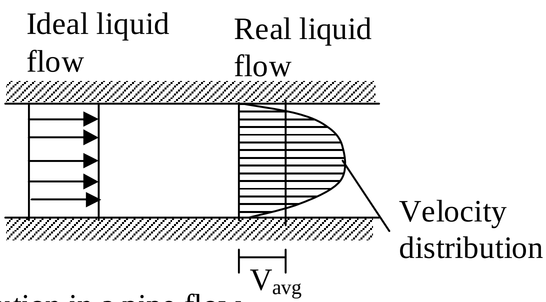
Fig.5.3 Energy & hydraulic grade lines

5.3.2 Energy correction factor

The analysis of flow problems is usually based on the one-dimensional approach. The entire flow is considered to be taking through stream tube with average velocity V at center of each cross-section. The velocity distribution at any x-section in real fluid flow is non uniform, on account of the boundary resistance and consequently the kinetic energy per unit weight given by $V^2/2g$ doesn't represent the kinetic energy across the section.

In order to compensate for the discrepancy a coefficient known as energy correction factor denoted by α is used. The multiplication of α with $V^2/2g$ yields the kinetic energy actually passing a section.

For the figure given,



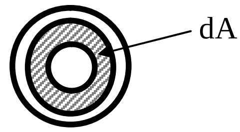


Fig.5.4. Velocity distribution in a pipe flow.

The kinetic energy per unit time passing through on elemental area *dA* is ½ ($\rho dA u$)u^2

u-velocity at that point

Total kinetic energy passing the section

$\int_A \frac{1}{2}\rho u^3 dA$ And the actual kinetic energy passed on average velocity V passing the section is equal to $\alpha \frac{1}{2}\rho V^3 A$

From the two equation

$$\alpha = \frac{1}{A}\int\left(\frac{u}{v}\right)^3 dA$$

Kinetic energy correction factor α is a measure of viscous resistance generated in a given flow, the effect of which is reflected uniform nature of velocity distribution. For a given pipe, it can be shown that its magnitude is a function of the type of flow and its turbulent characteristics.

Laminar flow is purely a viscous flow; the value of α is maximum and equals 2.0. But in case of fully developed turbulent flow in pipes, α is independent of Reynolds number and may be considered to have almost constant value (1.01 to 1.15) depending on surface roughness and Reynolds number. Lower value is appreciable for velocity rough surface and high Reynolds number.

$$\alpha_1\frac{V_1^2}{2g} + \frac{p_1}{\gamma} + Z_1 = \alpha_2\frac{V_2^2}{2g} + \frac{p_2}{\gamma} + Z_2 + h_\ell$$

For an identical velocity distribution at two sections, $\alpha_1=\alpha_2$,and if accuracy is not required in non uniform velocity distribution α =1.

5.3.3 Practical Applications of Bernoulli's Equation

Bernoulli's equation is applicable in all problems of incompressible fluid flow where energy considerations are involved. And it is practically applied for flow measurement using the following measuring devices.

1. Venturimeter (tube) (G.B Venturi (1746 –1822) Italian Eng)

It is a device used for measuring rate of flow in a pipeline and it consists of three components:

i) A converging entrance cone of angle of about 20^0.
ii) A cylindrical portion of short length called the "throat"
iii) A diverging section known as diffuser, of cone angle 5^0 to 7^0 to ensure a minimum loss of energy, but where this is unimportant the angle may be as large as 14^0.

The entrance tube and exit tube diameter are the same as that of the pipe line in to which it is inserted and the length of throat is equal to the throat diameter.

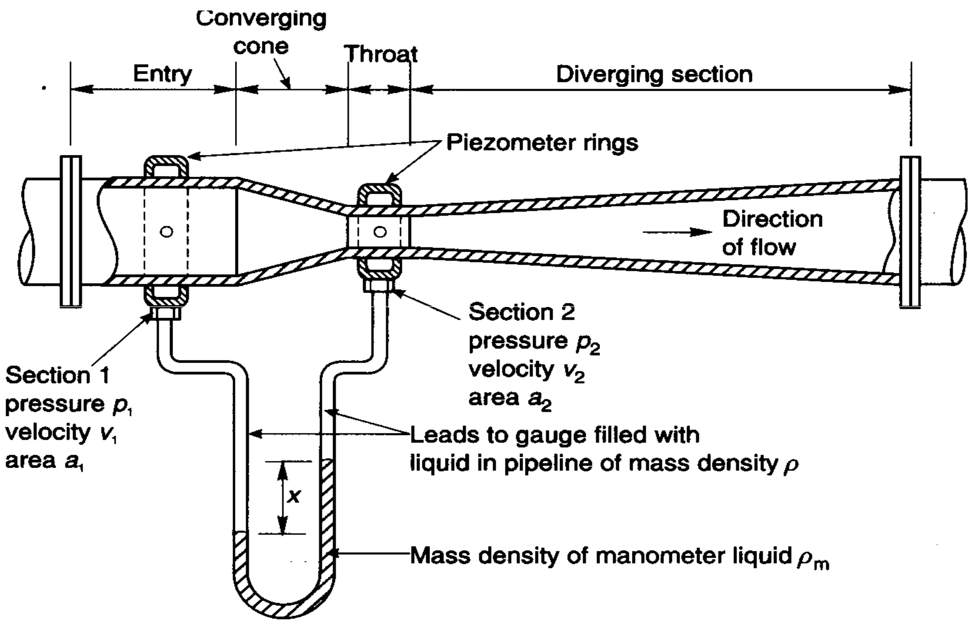


Fig.5.5 Horizontal Venturimeter

Assuming that the fluid is ideal (So that energy is not dissipated in overcoming frictional resistance) and that the velocities V_1 and V_2 at the inlet and throat respectively, are uniformly distributed over the cross section (so that the energy correction factor α is 1). Applying Bernoulli's equation between points (1) and (2) on a central stream line and assuming no frictional resistance,

$$\frac{V_1^2}{2g}+\frac{P_1}{\gamma}+Z_1=\frac{V_2^2}{2g}+\frac{P_2}{\gamma}+Z_2$$

For an incompressible fluid, continuity of flow at section 1 and 2 is,

$$Q = A_1 V_1 = A_2 V_2$$

$$V_2=\frac{A_1}{A_2}V_1 \quad \ldots\ldots(a)$$

If $A_1 > A_2$, $V_1 < V_2$ i.e. KE at section 2 (throat) > KE at section 1(the entrance)

P at throat < P at entrance

For a horizontal Venturi meter,

$$\frac{V_2{}^2-V_1^2}{2g}=\frac{P_1-P_2}{\rho g}\ \ldots\ldots(b)$$

Substituting equ (a) in to eqn(b)

$$\frac{V_1^2}{2g}\left[\left(\frac{A_1}{A_2}\right)^2-1\right]=\frac{P_1-P_2}{\rho\ g}$$

$$V_1=\sqrt{\frac{1}{\left(\frac{A_1}{A_2}\right)^2-1}*2g*\left(\frac{P_1-P_2}{\rho g}\right)}$$

$$V_1=\frac{A_2}{\sqrt{A^2{}_1-A_2^2}}\sqrt{2g\frac{P_1-P_2}{\rho g}}$$

Theoretical Discharge

$$Q_t = A_1 V_{1t} = \frac{A_1 A_2}{\sqrt{A^2{}_1 - A_2{}^2}}\sqrt{2g\ \frac{P_1-P_2}{\rho\ g}}$$

The theoretical discharge can be converted to actual discharge by multiplying

$$\textbf{Acutual discharge} = Cd * Q_t = Cd\ \frac{A_1 A_2}{\sqrt{A_1{}^2 - A_2{}^2}}\sqrt{2g\ \frac{P_1-P_2}{\rho\ g}}$$

The differential pressure is evaluated from manometry, and for figure.5.5

$$\frac{P_1}{\gamma}s + ys - xs - s_m(y-x) = \frac{P_2}{\gamma}s$$

$$\frac{P_1 - P_2}{\gamma} = x(\frac{s_m}{s} - 1)$$

For vertical or inclined Venturi meter, the actual discharge can be computed similarly.

2. Pitot tube (Total head tube) / (Henri Pitot)

Pitot tube is a device used for measuring velocity of flow at any point in a pipe or a channel. In its elementary form a Pitot tube consists of an L-shaped tube with open ends. Its may be aligned in open channel or pipe flow measurement as indicated below.

Stagnation pressure

For a figure below it has been seen that the central streamline terminates at B the entrance to the Pitot tube. This is on account of the inability of the streamline to take a sudden turn. The fluid flowing along the central streamline, therefore, stops moving as it reaches the point B. Hence the velocity of flow at this point is zero. This point is known as stagnation point.

Applying the Bernoulli's equation at points A and B, we obtain $P_B = P_O = P_A + \rho V_A^2/2$. *The pressure at the stagnation point is known as stagnation pressure composed of static pressure* P_A *and dynamic pressure* $\rho V_A^2/2$.

If the measurement is made on an open channel flow the surface will be exposed to the air and there is no static head from the surface, and if measurement is made on pipe flow there will be static head at A.

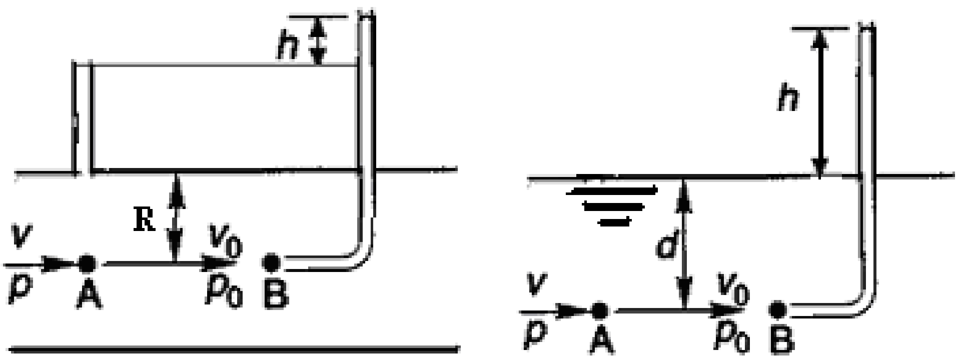


Fig.5.6 Simple Pitot tube in pipe flow and open channel flow.

Applying Bernoulli's equation to point A in the undisturbed flow region and the stagnation point B we have

$$\frac{V_O^{\ 2}}{2g}+\frac{P_O}{\gamma}+Z_O=\frac{V_A^{\ 2}}{2g}+\frac{P_A}{\gamma}+Z_A$$

$$\Rightarrow \frac{P_O}{\gamma}=\frac{V_A^{\ 2}}{2g}+\frac{P_A}{\gamma}$$

$$\frac{V_A^{\ 2}}{2g}=\frac{P_O-P_A}{\gamma}$$

$$V_A^{\ 2}=\frac{(P_O-P_A)}{\gamma}2g \Rightarrow V_A=\sqrt{2g\,(\frac{P_O-P_A}{\gamma})}=\sqrt{2gh} \quad \sin ce\frac{P_O-P_A}{\gamma}=h$$

A perfect Pitot tube should obey this equation exactly, but all actual instruments must be calibrated and a correction factor applied to make allowance for the small effects of nose shape and other characteristics.

Practically it is difficult to read h from a free surface. To overcome this difficulty, the static tube and the Pitot tube may combine in one instrument (differential U-tube).

For finding the velocity at any point in a pipe by Pitot tube, the following arrangements are adopted.

1. Pitot tube along with a vertical piezometer tube as shown in Fig.5.7.
2. Pitot tube connected with piezometer tube as shown in Fig.5.8
3. Pitot tube and vertical piezometer tube connected with a differential U-tube manometer as shown in Fig.5.9
4. Pitot static tube, which consists of two circular concentric tubes one inside the other with some annular space in between as shown in Fig.5.10. The outlets of these two tubes are connected to the differential manometer where the difference of pressure head is measured by knowing the difference of the levels of the manometer liquid h_m.

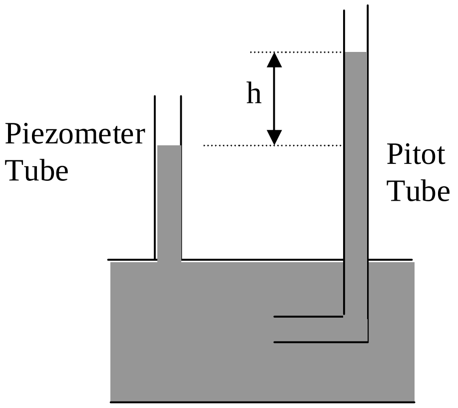


Fig.5.7

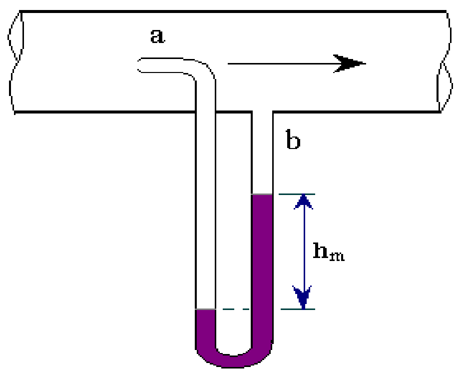


Fig.5.8

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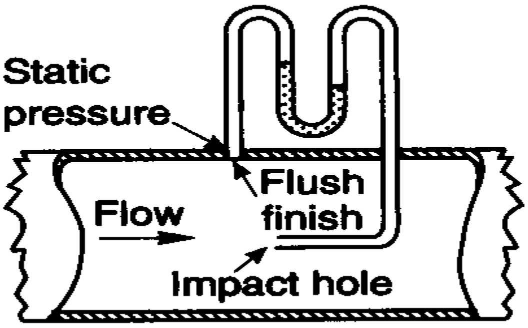

Fig.5.9

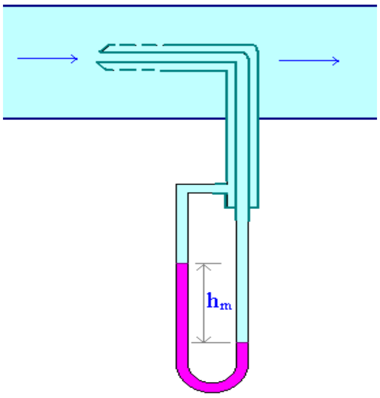

Fig.5.10

The Pitot tube measures the velocity of only a filament of liquid, and hence it can be used for exploring the velocity distribution across the pipe cross-section. If, however, it is desired to measure the total flow of fluid through the pipe, the velocity must be measured at various distances from the walls and the results integrated. The total flow rate can be calculated from a single reading only if the velocity distribution across the cross-section is already known.

The static tube measures the static pressure, since there is no velocity component perpendicular to its opening and the impact tube measures both the static pressure and impact pressure (due to kinetic energy). Impact tube head =pressure head + velocity head.

3. Orifices

An orifice is an opening (usually circular) in the wall of a tank or in a plate normal to the axis of a pipe, the plate being either at the end of the pipe or in some intermediate location and used for measuring rate of flow out of a reservoir (tank) or through a pipe.

a. Orifice flow in pipes, Orifice meter or orifice plate

The Venturi meter described earlier is a reliable flow-measuring device. Furthermore, it causes little pressure loss. For these reasons it is widely used, particularly for large-volume liquid and gas flows. However, this meter is relatively complex to construct and hence expensive. Especially for small pipelines, its cost seems prohibitive, so simpler devices such as orifice meters are used.

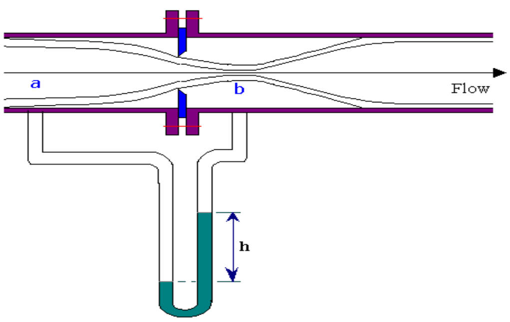

Fig.5.11 Orifice meter

The orifice meter consists of a flat orifice plate with a circular hole drilled in it. There is a pressure tap upstream from the orifice plate and another just downstream.

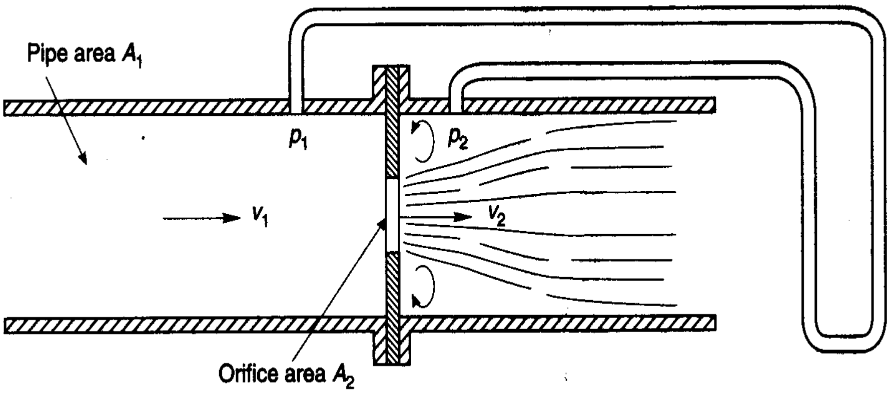

Fig.5.12 Orifice plate in a pipe

Applying the Bernoulli equation between at 1 (upstream of plate) and 2 (at the orifice)

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} = \frac{P_2}{\gamma} + \frac{v_2^2}{2g} \quad \textit{Since the orfice is horizontal } Z_1 = Z_2$$

$$\frac{V_1^2 - V_2^2}{2g} = \frac{P_2 - P_1}{\gamma}$$

$$\textit{Using continuity equation} \quad A_1 v_1 = A_2 V_2 \Rightarrow V_2 = \left(\frac{A_1}{A_2}\right) V_1$$

$$\frac{v_1^2 - (A_1/A_2)^2 v_1^2}{2g} = \frac{P_2 - P_1}{\gamma}$$

$$v_1^2 \left(\frac{1-(A_1/A_2)^2}{2g}\right) = \frac{p_2 - p_1}{\gamma} \Rightarrow V_1 = \sqrt{\left(\frac{p_2 - p_1}{\gamma}\right) * \frac{2g}{1-(A_1/A_2)^2}}$$

The theoretical discharge Q will therefore be,

$$Q = A_1 V_1 = A_1 \sqrt{\frac{(p_2 - p_1)}{p} * \frac{2}{1-(A_1/A_2)^2}}$$

The actual discharge will be less than the theoretical since the effective flow area near P_2 tapping will be less than A_2, the fluid forming a neck or vena contracta. In addition there will be some loss of energy between 1 and 2.

The actual discharge can be determined by determining coefficient of discharge.

b. Flow through a reservoir opening (orifice flow)

For a reservoir at water level h above orifice opening shown in the following figure. The reservoir is assumed to be very large as compared to size of the opening, so that the velocities of all points in the reservoir are negligibly small.

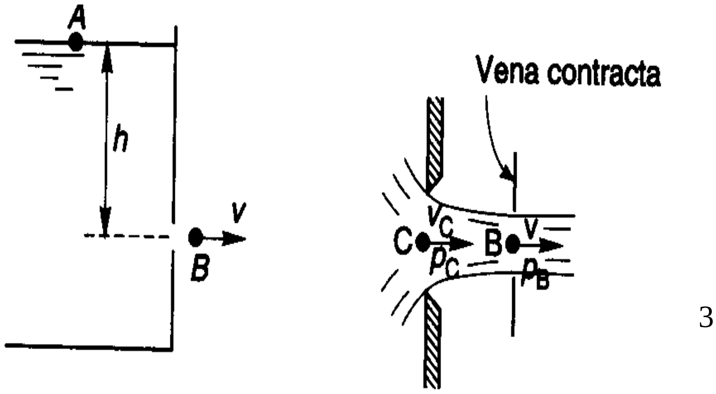

Fig. 5.13 Small orifice in the side of a large reservoir

Applying Bernoulli's equation neglecting losses between A and B (taking the datum level at the center of the orifice).

$$0 + 0 + h = \frac{V^2}{2g} + 0 + 0$$

$$V = \sqrt{2gh} \quad \text{(Theoretical velocity).}$$

Theoretical discharge, $Q_t = A_o \sqrt{2gh}$

Hydraulic Coefficients for flow through orifices

1) Coefficient of contraction, $C_c = \dfrac{Area of jet\ at\ vena\ contracta}{Area\ of\ orifice}$

$$A_0 = Area\ of\ orifice (Area\ of\ jet\ at\ C)$$

2) Coefficient of velocity, $C_v = \dfrac{Actual\ velocity\ at\ venacontracta}{Theoretical\ velocity}$

3) Coefficient of discharge, $C_d = \dfrac{Actual\ discharge}{Theoretical\ discharge}$

$$\text{Actual velocity } V_a = C_v \sqrt{2gh}$$

As shown in the figure, the paths of the particles of the liquid converge on the orifice so that the area of the issuing jet is less than the area of the orifice. In the plane of the orifice the particles have a component of velocity towards the center so that at C the pressure is greater than atmospheric pressure. It is only at B a little outside the orifice that the paths of the particles become parallel. The section through B is called the venacontracta.

Area of vena contracta $\quad A_B = C_c A_0$

Therefore

Actual discharge Q_a = Actual area * Actual velocity

$$= C_c A_0 C_v \sqrt{2gh}$$

It is customary to combine the two coefficients into a discharge coefficient C_d. ($C_d = C_v C_c$)

$$Qa = C_d A_0 \sqrt{2gh}$$

Determination of hydraulic coefficients of orifice.

<u>Determination of C_d</u>

By measuring area A_0, the head h and the discharge Q_a (by gravimetric or volumetric means), C_d is obtained using the above equation. Determination of either C_v or C_c then permits determination of the other by the equation $C_d = C_v C_c$.

<u>Determination of C_c and C_v</u>

1) Trajectory method

The equation for the trajectory may be obtained by applying Newton's equation of uniformly accelerated motion to a particle of the liquid passing from the nozzle to point P, whose coordinates are (x,y) in time t. Then $x= V_{xo}t$ and $z= V_{zo}t-1/_2gt^2$. Evaluating t from the first equation and substituting in the second gives

$$z = \frac{V_{z0}}{V_{x0}} x - \frac{g}{2V_{x0}^{\;2}} x^2$$

If the jet is initially horizontal, as in the flow from a vertical orifice, $V_{x0}=V_0$ and $V_{z0}=0$, the above equation is reduced in to;

$$V_0 = x\sqrt{\frac{g}{2z}}$$

$$\text{Then } C_v = \frac{V_0}{V_t} = \frac{V_0}{\sqrt{2gH}} = \frac{x}{\sqrt{2z/g}\sqrt{2gh}} = \frac{x}{\sqrt{4zh}}$$

2) Pitot tube method

Pitot tube can be set at the vena contracta so that actual velocity V_a is determined.

3) Calipers method

The diameter of the jet at the vena contracta can be approximately measured using an outside caliper. But this method is not precise and is less satisfactory than other methods.

Types of orifice

The following figure shows common types of orifice with their coefficient of discharge.

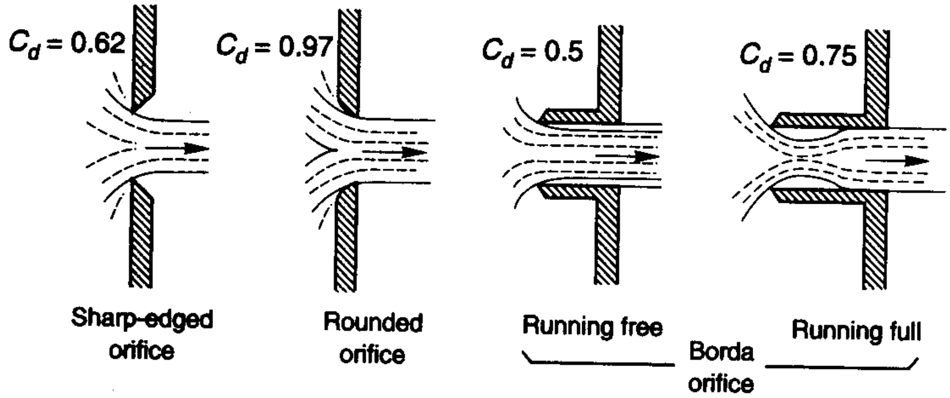

Fig.5.14 Types of orifice

The orifices are classified based on of their size, shape, nature of discharge and shape of the upstream edge.

1. *Depending upon their size*: small orifice and large orifice. If the head of liquid from the center of the orifice is more than five times the depth of the orifice, the orifice is called small orifice. If it less than five times it is known as large orifice.
2. *Depending upon shape*: as circular, triangular, rectangular and trapezoidal.
3. *Depending upon shape of edge:* as sharp edged and round or bell mouthed orifice. (Fig.5.14)
4. *Depending up on the nature of discharge*: as free discharging & drowned or submerged orifice. The submerged orifices are further classified as fully submerged and partially submerged orifice.

3. Unsteady orifice flow from reservoirs

The volume discharged from the orifice in time δt is $Q\delta t$, which must just equal the reduction in volume in the reservoir in the same time increment. AR ($-\delta y$), in which A_R is the surface area of the reservoir at level y above the orifice.

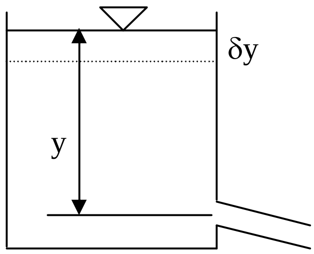

Fig 5.15 Un steady flow from reservoir

Equating the two expressions

$$Q\delta t = A_R\,(-\delta y)$$

Solving for δt and integrating yields

$$\delta t = \frac{A_R(-\delta y)}{Q}$$

$$\int_o^t dt = -\int_{y_1}^{y_2} \frac{A_R\,\delta y}{Q}$$

but $Q = C_d\, Ao\, \sqrt{2gy}$

$$\therefore\ t = \frac{-A_R}{C_d\, Ao\sqrt{2g}} \int_{y_1}^{y_2} y^{-1/2}\delta y$$

$$t = \frac{2A_R}{C_d\, A_0\, \sqrt{2g}}\, y_1^{1/2} - y_2^{1/2}$$ *This is the time for the liquid to fall from y_1 to y_2.*

4.Weirs

Open channel flow may be measured by a weir on obstruction in the channel that causes the liquid to back up behind it and flow over it or through it. There may be sharp crested or broad crested based on their length along the channel section.

a) Rectangular weir

The following figure shows rectangular notch of crest length (L) and working under a head H.

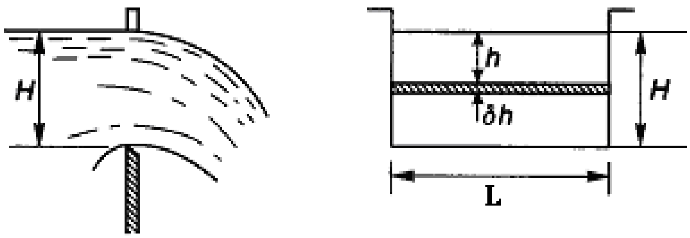

Fig. 5.16 Rectangular notch

By Torricelli's theorem the velocity of a particle discharged at any level h is $\sqrt{2gh}$ and will therefore vary from top to bottom of notch. Considering a horizontal strip at depth h and of thickness δh.

Discharge through strip = $\sqrt{2gh}$ * Lδh

Total discharge $Q_t = L\sqrt{2g}\int_0^H h^{1/2} d_h \quad = \frac{2}{3} L\sqrt{2g}H^{3/2}$

This is the theoretical discharge through a rectangular notch.

The value of Q given by the above equation is too high because no account has been taken of energy lost and also because, as shown below there will be a substantial reduction in the width and depth of the notch cross section because of the curved path lines of the liquid.

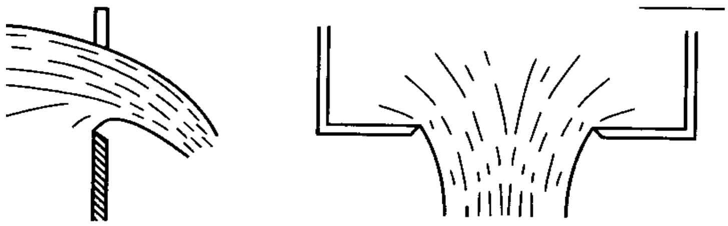

Fig 5.17 Effect of vena contracta in rectangular weir

Therefore, Actual discharge = $= \frac{2}{3} C_d L\sqrt{2g}H^{3/2}$

From experiment C_d=0.62 , Q = $1.831LH^{3/2}$

b) Triangular weir (V-notch)

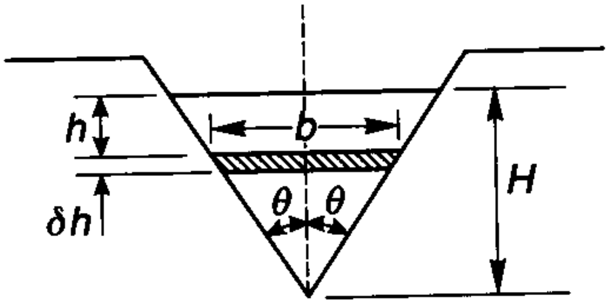

Fig. 5.18 Triangular weir

Since the velocity of flow through the notch varies from top to bottom, consider a strip of thickness δh at a depth h below the surface. If the velocity of approach is small:

Head producing flow =h

Velocity through strip = V = $\sqrt{2gh}$

If width of strip = b, Area of strip = $b\delta h$

Discharge through strip = δQ = V $b\delta h$

The width b depends on h and is given by b= 2(H-h) $\tan\theta$

$$\delta Q = \sqrt{(2g)}h^{1/2}x2(H-h)\tan\theta * \delta h$$

Thus

$$= 2\sqrt{2g}\tan\theta(Hh^{1/2} - h^{3/2})\delta h$$

Integrating between the limits h = 0 and h=H

$$Q = 2\sqrt{2g}\tan\theta\int_0^H (Hh^{1/2} - h^{3/2})dh$$

$$= 2\sqrt{2g}\tan\theta\left[\frac{2}{3}Hh^{3/2} - \frac{2}{5}h^{5/2}\right]_0^H$$

$$= \frac{8}{15}\sqrt{2g}\tan\theta H^{5/2}$$

This is the theoretical discharge

Actual discharge = $C_{d*}Q = C_d \frac{8}{15}\sqrt{2g}\tan\theta H^{5/2}$

From experiment $C_d = 0.58$, $Q_a = 1.37 \tan\theta\, H^{5/2}$,
For 90^0 V-notch, $Q_a = 1.37\, H^{5/2}$

5.4.Impulse_ momentum theorem

It is often important to determine the force produced on a solid body by fluid flowing steadily over it. For example, the force on a pipe bend caused by the fluid flowing through it; the force exerted by jet of fluid striking against a solid surface; thrust on a propeller. All these forces are ***hydrodynamic forces*** and they are associated with a change in the momentum of the fluid.

The magnitude of such a force is determined by ***Newton's second law*** of motion, by modifying the law to suit particularly to the steady flow of a fluid called the steady flow momentum equation. Only the forces acting at the boundaries of this space concern us, and use of momentum equation doesn't require the knowledge of the flow pattern in detail. Moreover, the fluid may be compressible or incompressible and the flow with or without friction.

Consider a stream tube shown below with the following assumptions

- The c/s of stream tube is sufficiently small so that the velocity may be considered uniformly distributed
- The flow is steady i.e. the stream tube remains stationary with respect to the fixed coordinate axis.

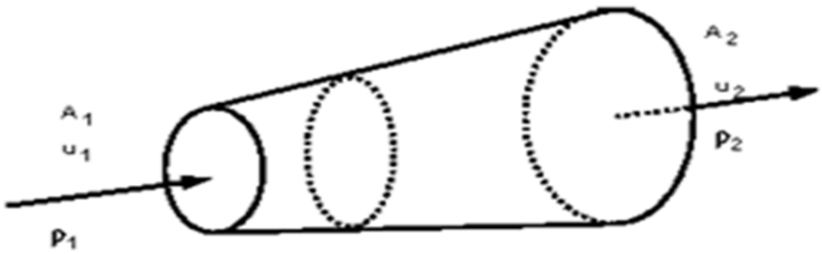

Fig Stream tube

Newton's second law

$F = m * a$

$F = m \frac{dv}{dt}$

$F * dt = m * dv$

- Momentum principle expresses that the rate of change of momentum is equal to the net force acting on the fluid mass.

Momentum of fluid entering section 1 –1 in a time Δt in the x –direction

$= \rho * dQ * \Delta t * V_1(x)$

Momentum leaving section 2- 2 in time Δt

$= \rho * dQ * \Delta t * v_2 (x)$

From momentum principle

$$dFx = \frac{P\ dQ\ \Delta t\ [v_2(x) - v_1(x)]}{\Delta t}$$

$dFx = \rho dQ\ [v_2 (x) – v_1 (x)]$

dFx = net force exerted on the fluid in the x –direction.

The total force in the x –direction is given by

$$Fx = \int_A dFx = \int_A \rho dQ v_2(x) - \int_A \rho dQ v_1(x)$$

$$= \int_A \rho(v_2 dA_2) v_2(x) - \int_A \rho(v_1 dA_1) v_1(x)$$

Assuming the fluid is incompressible

$Fx = \rho V_2 A_2 (V_2 (x) - PV_1 A_1 V_1 (x)$

$Fx = \rho Q [V_2(x) – V_1(x)]$

Similar equations for y and z directions may be written

$Fy = \rho Q [V_2(y) – V_1(y)]$

$Fz = \rho Q [V_2(z) – V_1(z)]$

Applications of momentum equations

1) The force caused by a jet striking a surface

A. Impact on a flat surface

i) Stationary plate A = area of jet
V_2 = velocity of jet
ρ = mass density

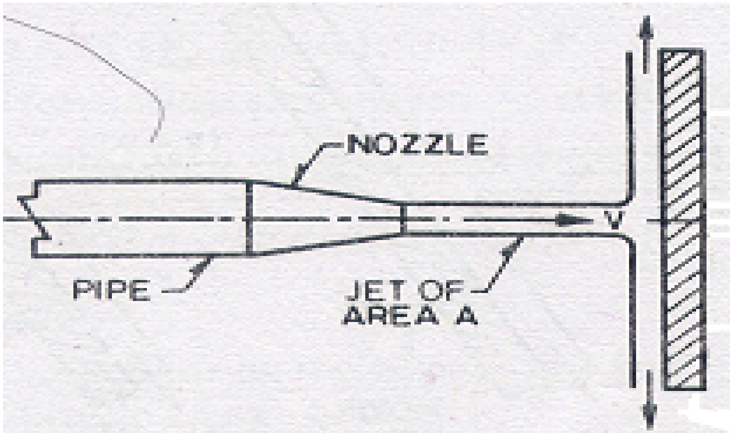

Fig. A jet hitting a vertical plate

Fx $= \rho Q\,(v_2(x) - v_1(x))$
$= \rho Q\,(0 - v_1)$ ➔ momentum normal to the plate is destroyed.

Fx $= -\rho\, A\, V^2$ (force in the jet)

Force exerted on plate $= \rho A V^2$

ii) Moving plate

Initial velocity of jet $v_1x = v_1$

Find vel of jet = velocity of $= v_2\,x = U$ plate

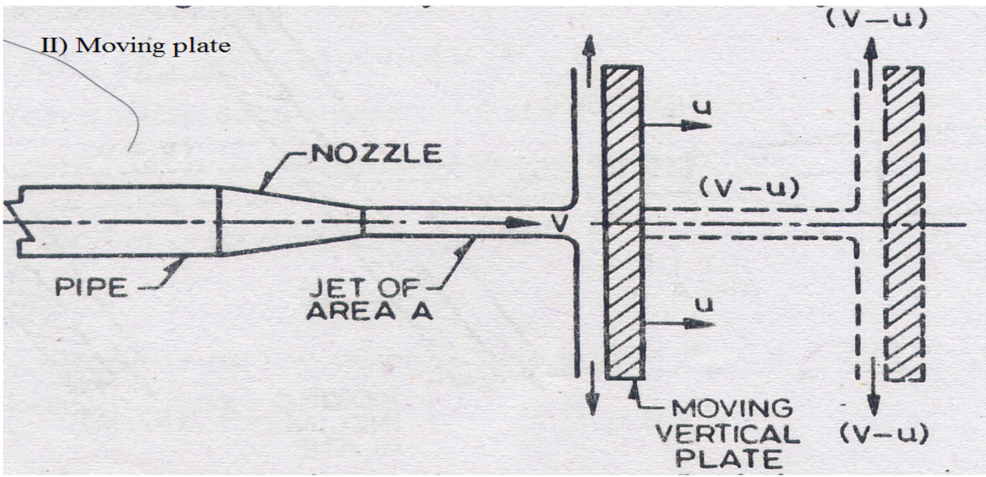

Fig. Ajet hitting a moving vertical plate

The velocity with which jet strikes the plate $= V - U$

Mass of fluid striking plate/sec $= \rho A (V - U) = \rho Q$

Force on plate $= \rho A (V - U) (V - U)$

$= \rho A (V - U)^2$

iii) Stationary inclined plate

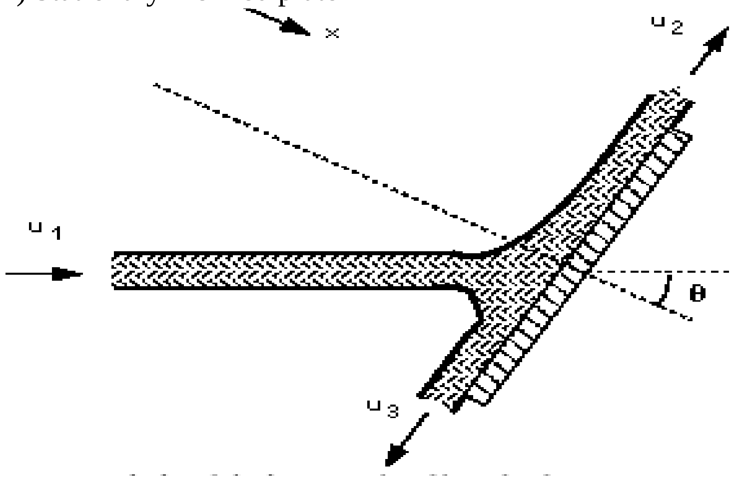

Fig. A jet hitting an inclined plate

$v_1 (x) = v \cos (90 - \theta) = v \sin \theta$

$v_2 (x) = 0$

Mass striking stationary plate $= \rho AV$

Normal force on the plate $= \rho AV (V \sin\theta) = \rho AV^2 \sin \theta$

Since the plate surface is smooth, there can be no force exerted by the plate on the fluid jet in the tangential direction.

$Ft = \rho Q_1 V - \rho Q_2 V - PQ\, V \cos\theta = 0$

$Q_1 - Q_2 - Q \cos\theta = 0$ (*)

Continuity equation $Q = Q_1 + Q_2 \Rightarrow Q_2 = Q - Q_1$

Substitute in (*) $\qquad Q_1 = Q - Q_2$

$Q_1 - Q + Q_1 - Q \cos\theta = 0$

$2Q_1 - Q (1 + \cos\theta) = 0 \qquad Q - Q_2 - Q_2 - Q \cos \theta = 0$

$Q_1 = Q/2 \ (1 + \cos\theta) \qquad Q (1 - \cos\theta) - 2 Q_2 = 0$

$Q_2 = Q/2 (1 - \cos \theta)$

For a vertical plate, $\theta = 90^0$

$Q_1 = Q/2 = Q_2$

iv) Moving inclined plate

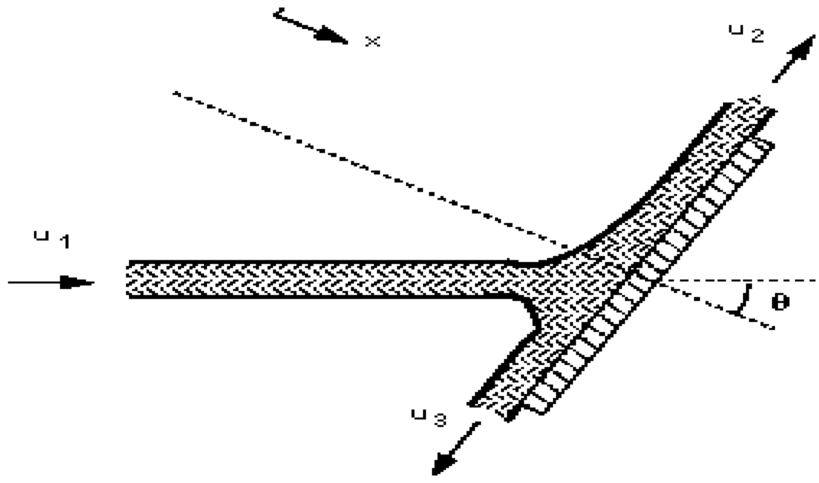

Fig. A jet hitting a moving inclined plate

V_1 (x) = v sin θ
V_2 (x) = U sin θ

Mass striking the moving plate per second = ρQ = ρA (v- u)
Normal force on the plate = ρA (V- U) (V sinθ- U sinθ)
= $\rho A (V - U)^2 \sin\theta$

Work done by this force = Fx * U = $\overline{F}n \sin\theta * u$

V) Series of flat plates
The force exerted by the impact of jet can be fruitfully utilized if the flat plates are mounted on the periphery of a wheel as shown below. The force exerted by the jet causes the rotation of the wheel. The flat plates thus occupy the bottom most position according to their turn.

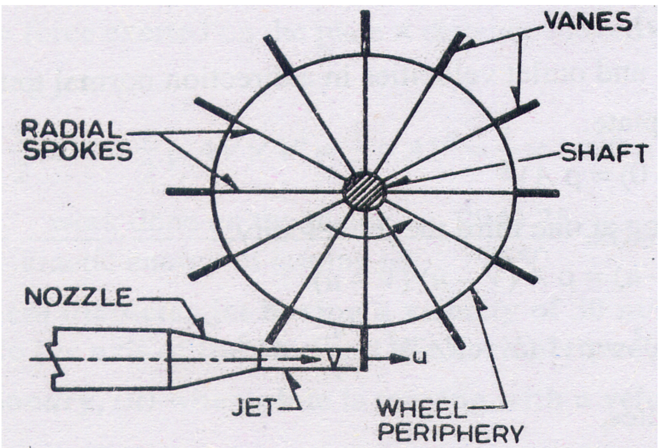

Fig Flat plates mounted a wheel

The number and location of the plates is so arranged that no portion of jet goes waste without doing work on the plate.

Initial velocity of jet = V
Final velocity = velocity of plate = U
Fluid mass striking the plate per second = ρQ = ρAV
Force exerted on the plate by jet = ρAV (V – U)

Work done per second = Force * distance moved per second
= ρAV (V –U) *U

KE of jet per second = ½ (ρAV) V^2

$$\text{Efficiency of jet} = \frac{Work\ done\ on\ plate}{KE\ of\ \ jet} = \frac{\rho\, AV(v-u)u}{1/2\,\rho\, AVV^2} = \frac{2(v-u)u}{v^2}$$

B. Force on curved vane

Consider a symmetrical curved vane having smooth surface. The jet strikes at the center and after impact it is deflected equally along the vane surface.

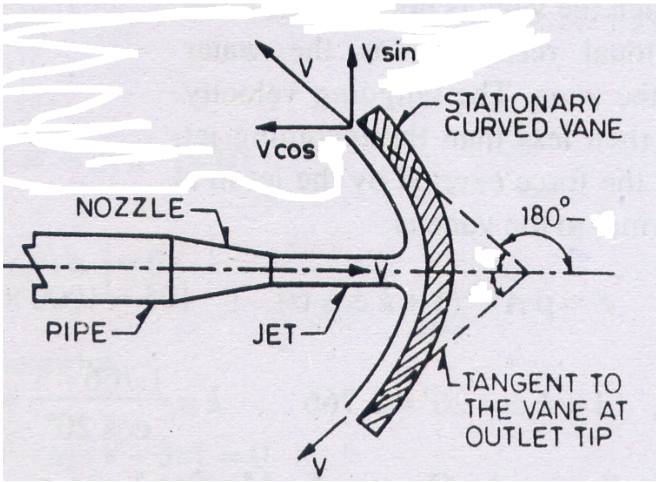

Fig. A jet hitting a curved vane

M_{in} = $\rho AV * V = \rho AV^2$

$M_{out} = -\ \rho \dfrac{Av}{2},\ V\cos\theta\ -\ \rho\ \dfrac{Av}{2} v\cos\theta$

$= -\ \rho AV^2 \cos\theta$

Force exerted by curved vane on the jet

Fx = $M_{out} - M_{in}$

$= -\ \rho AV^2 \cos\theta - \rho AV^2 = -\ \rho AV^2 (1+\cos\theta)$

Force on curve vane by jet = $\rho AV^2 (1+\cos\theta)$

For $\theta = 90^0$ – Flat plate at right angle to jet $F = \rho AV^2$
For $\theta = 0^0$ - semi –circular plate $F = 2\rho AV^2$

Curved vane moving in translation

Mass of fluid striking the vane = $\rho A(v-u)$

$M_{in} = \rho A(v-u)^2$
$M_{out} = -\rho A(v-u)^2 \cos\theta$

Force exerted by jet on the vane = Fx $= \rho A(v-u)^2 (1+\cos\theta)$

Work done by jet = $F_x U$

KE of jet /sec = $\frac{1}{2}(\rho AV) V^2$

Efficiency, $\eta = \dfrac{\text{workdone by jet}}{\text{KE of jet}}$

$$\eta = \frac{2(1+\cos\theta)(v-u)^2 u}{v^3}$$

Curved vane mounted on a wheel

Let the wheel rotate with a tangential velocity U and a jet moving with velocity v strikes the wheel.

Fluid mass striking the wheel per second = PAV

$M_{in} = \rho AV(v-u)$ (Before impact)

$M_{out} = \rho AV(v-u)\cos\theta$ (After impact)

Force exerted by jet on the van = $M_{in} - M_{out}$

$$F = \rho AV(v-u)(1+\cos\theta)$$

W = F * U

KE = $\frac{1}{2}\rho AV^2$ ➔ $\eta = \dfrac{2u(v-u)(1+\cos\theta)}{v^2}$

2. Force exerted on a reducing pipe bend.

As a general case a reducer bend has been selected which changes the magnitude and direction of velocity.

For simplicity assume the bend is in a horizontal plane.

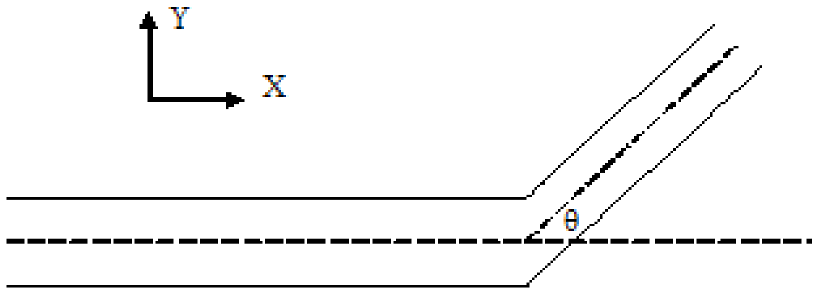

Fig. Force on a horizontal reducing bend

Let F_x and F_y be the components of the force exerted on the fluid by the pipe bend.

Then momentum equation in the x –direction can be written as

$$\rho_1 A_1 - \rho_2 A_2 \cos\theta - F_x = \rho Q (V_1 – V_2 \text{cost}\theta)$$

Momentum equation may be expressed as

$$P_1 A_1 - P_2 A_2 - R_z = pQ (V_2 - V_1)$$

As the discharge is to the atmosphere, $P_2 = 0$, and thus

$$R_z = P_1 A_1 - pQ(V_2 - V_1)$$

Further simple cases where the momentum equation may be profitably applied include the determination of the force acting on a pipeline at a contraction and that on a sluice.

$$R_z = 44.6\ x\ \frac{\pi}{4}\ x\ \frac{64^2}{10^0} - 8.5\,(30-2.65)/1000 = 1.21 kN$$

Change in velocity and direction

A reducing bend with deviation in the vertical plane is shown in Fig.below. Due to the hydrostatic and dynamic pressures a force is exerted by the fluid on the bend which has to be resisted by a thrust block or other suitable means. This force could be evaluated by plotting the stream lines and thus determining the pressure distribution. However, by a simple application of the momentum equation, and quite independently of any energy losses associated with turbulent eddying (real fluid), we obtain.

$$P_1 A_1 - P_2 A_2 \cos\theta - R_z = pQ(v_z \cos\theta - v_1)$$

And for the z direction

$$R_z - W - D_o A_o \sin\theta = pQv, \sin\theta$$

where W is the weight of fluid between the reference sections.

From these equations R_z and R_z may be determined and hence the resultant R =

$$R = \sqrt{R_z^{\ 2} + R_z^{\ 2}}$$

It is to be noted that the momentum equation gives no information concerning the location of the resultant, which necessitates an analysis involving forces and moments.

Applications of Momentum Equations

The momentum equation finds an application in many hydraulic problems. Generally, it is employed in conjunction with the continuity equation, and often additionally with the Bernoulli equation. Problems which involve a marked change in flow velocity or direction are particularly appropriate. The analysis of the hydraulic jump and the determination of the force exerted by a jet of water impinging on a

The correction factor β is obtained by integrating the momentum of the elemental stream rubes over the entire section and dividing by the momentum based on the mean velocity.

Thus, rotating vane are both in this category and are dealt with in later chapters. Our present consideration will be limited to two simple cases. When applying the equation to a real fluid it is important to remember that there are longitudinal frictional stresses present which can only be neglected when the distance between the reference sections is relatively short.

Change in velocity

A nozzle, attached to a pipeline, and discharging to the atmosphere provides a good example of a rapid change in velocity. The fluid exerts a force on the nozzle and in accordance with Newton's third law there is a similar force, of opposite sign, exerted by the nozzle on the fluid. This is the force which the tension bolts must be designated to withstand. The force could be evaluated by an analysis of the pressures acting on the surface of the nozzle but this procedure would, to say the least, be extremely tedious. By contrast, a simple application of the momentum equation between upstream and downstream reference sections will yield a direct solution. In Fig 4.16 the component forces are the hydrostatic forces P_1A_1 and P_2A_2 and the force R_1 exerted by the nozzle on the fluid. The rate of change of momentum is ρQ (V2 - V1) so that the momentum equation may be expressed as:

$$Fx = P_1A_1 - \rho Q(V_2 - V_1)$$