THEORY OF STRUCTURE II

CHAPTER 1

1. ANALYSIS OF INDETERMINATE STRUCTURES

1.2. APPROXIMATE METHOD ANALYSIS

Introduction

In module 2, force method of analysis is applied to solve indeterminate beams, trusses and frames. In modules 3 and 4, displacement based methods are discussed for the analysis of indeterminate structures. These methods satisfy both equation of compatibility and equilibrium. Hence they are commonly referred as exact methods. It is observed that prior to analysis of indeterminate structures either by stiffness method or force method; one must have information regarding their relative stiffnesses and member material properties. This information is not available prior to preliminary design of structures. Hence in such cases, one can not perform indeterminate structural analysis by exact methods. Hence, usually in such cases, based on few approximations (which are justified on the structural behaviour under the applied loads) the indeterminate structures are reduced into determinate structures. The determinate structure is then solved by equations of statics. The above procedure of reducing indeterminate structures into determinate and solving them using equations of statics is known as approximate method of analysis as the results obtained from this procedure are approximate when compared to those obtained by exact methods. Also, approximate methods are used by design engineers to detect any gross error in the exact analysis of the complex structures. Depending upon the validity of assumptions, the results of approximate methods compare favourably with exact methods of structural analysis.

In some way, all structural methods of analysis are approximate as the exact loading on the structure, geometry; the material behaviour and joint resistance at beam column joints and soil-structure interaction are never known exactly. However, this is not a good enough reason for using approximate methods of analysis for the final design. After preliminary design, it is important to analyse the indeterminate structure by exact method of analysis. Based on these results, final design must be done. In this module both indeterminate industrial frames and building frames are analysed by approximate methods for both vertical and wind loads.

35.2 Indeterminate Trusses: Parallel-chord trusses with two diagonals in each panel.

Consider an indeterminate truss, which has two diagonals in each panel as shown in Fig. 35.1. This truss is commonly used for lateral bracing of building frames and as top and bottom chords of bridge truss.

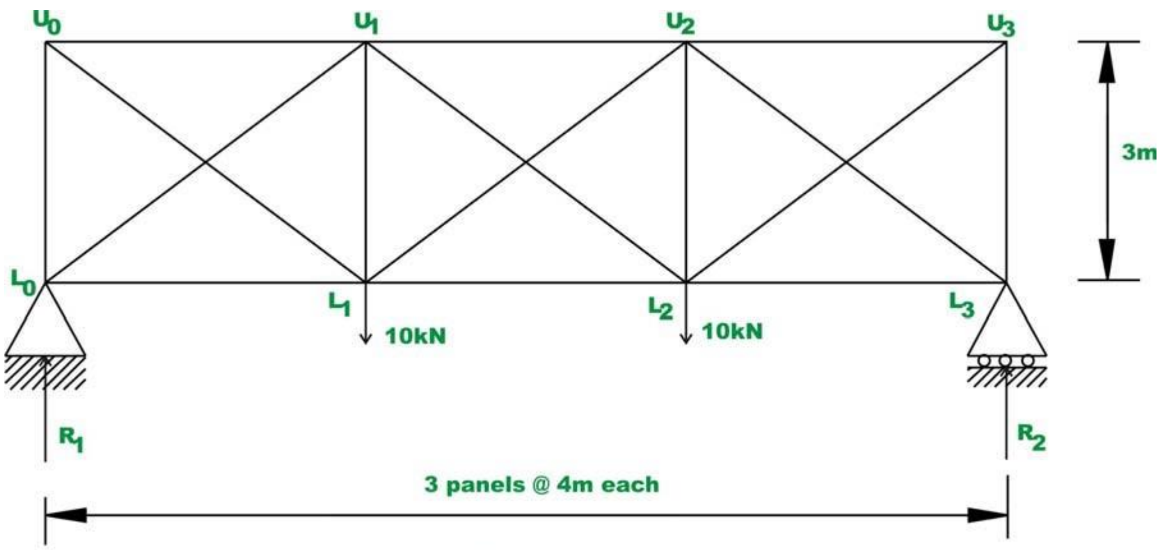


Fig. 35.1

This truss is externally determinate and internally statically indeterminate to 3^{rd} degree. As discussed in lesson 10, module 2, the degree of static indeterminacy of the indeterminate planar truss is evaluated by

$$i = (m + r) - 2j \qquad \text{(reproduced here for convenience)}$$

Where *m*, *j* and *r* respectively are number of members, joints and unknown reaction components. Since the given truss is indeterminate to 3^{rd} degree, it is required to make three assumptions to reduce this frame into a statically determinate truss. For the above type of trusses, two types of analysis are possible.

1. If the diagonals are going to be designed in such a way that they are equally capable of carrying either tensile or compressive forces. In such a situation, it is reasonable to assume, the shear in each panel is equally divided by two diagonals. In the context of above truss, this amounts to 3 independent assumptions (one in each panel) and hence now the structure can be solved by equations of static equilibrium alone.
2. In some cases, both the diagonals are going to be designed as long and slender. In such a case, it is reasonable to assume that panel shear is resisted by only one of its diagonals, as the compressive force carried/resisted by the other diagonal member is very small or

negligible. This may be justified as the compressive diagonal buckles at very small load. Again, this leads to three independent assumptions and the truss may be solved by equations of static alone.

Generalizing the above method, it is observed that one need to make n independent assumptions to solve n^{th} order statically indeterminate structures by equations of statics alone. The above procedure is illustrated by the following examples.

Example 35.1

Evaluate approximately forces in the truss members shown in Fig. 35.2a, assuming that the diagonals are to be designed such that they are equally capable of carrying compressive and tensile forces.

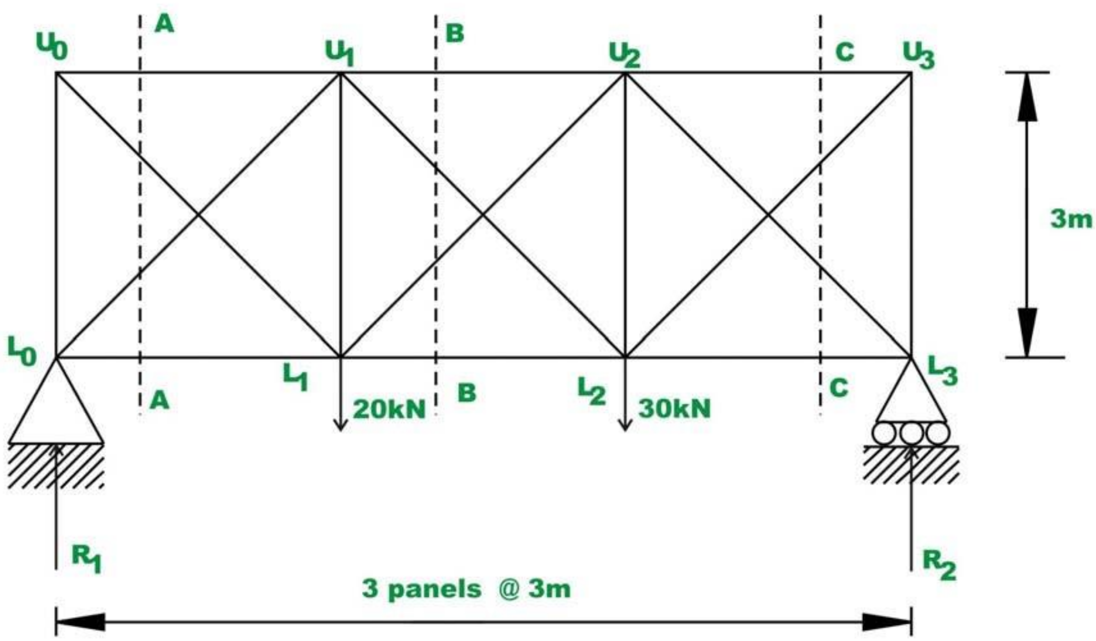


Fig. 35.2a

Solution:

The given frame is externally determinate and internally indeterminate to order 3. Hence reactions can be evaluated by equations of statics only. Thus,

$$R_1 = 23.33 \text{ kN} \quad (\uparrow)$$

$$R_2 = 26.67 \text{ kN} \quad (\uparrow) \tag{1}$$

Now it is required to make three independent assumptions to evaluate all bar forces. Based on the given information, it is assumed that, panel shear is equally resisted by both the diagonals. Hence, compressive and tensile forces in diagonals of each panel are numerically equal. Now consider the equilibrium of free body diagram of the truss shown left of $A-A$. This is shown in Fig. 35.2b.

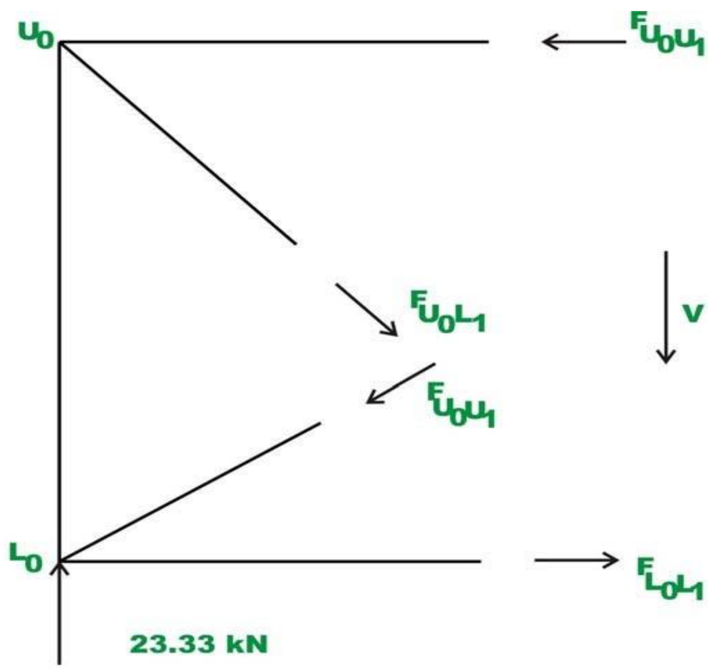


Fig.35.2b

For the first panel, the panel shear is 23.33 kN. Now in this panel, we have

$$F_{U_0L_1} = F_{L_0U_1} = F \tag{2}$$

Considering the vertical equilibrium of forces, yields

$$-F_{L_0U_1}\sin\theta - F_{L_0U_1}\sin\theta + 23.33 = 0 \tag{3}$$

$$2F\sin\theta = 23.33 \qquad \sin\theta = \frac{1}{\sqrt{2}}$$

$$F = \frac{23.33}{\sqrt{2}} \cong 16.50 \text{ kN} \tag{4}$$

Thus,

$$F_{U_1L_0} = 16.50 \quad \text{kN } (Tension)$$

$$F_{L_0U_1} = 16.50 \quad \text{kN } (Compression)$$

Considering the joint L_0,

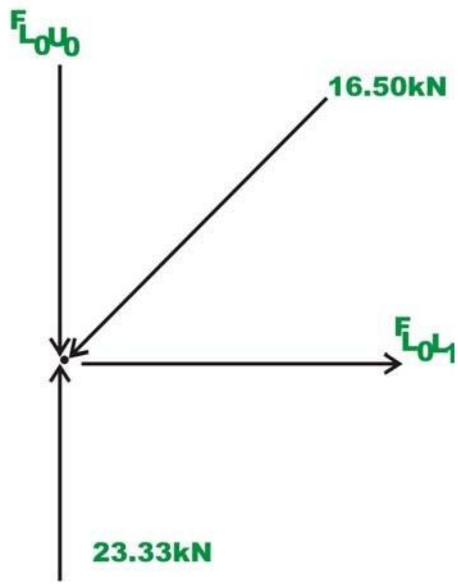


Fig.35.2c

$$\sum F_y = 0 \Rightarrow \qquad -F_{L_0U_0} - 16.50\sin 45 + 23.33 = 0$$

$$F_{L_0U_0} = 11.67 \text{ kN}(Comp.) \qquad (5)$$

$$\sum F_x = 0 \Rightarrow \qquad -16.50\cos 45 + F_{L_0L_1} = 0$$

$$F_{L_0L_1} = 11.67 \text{ kN}(Tension) \qquad (6)$$

Similarly, $F_{UU01} = 11.67$ kN$(comp.)$

Now consider equilibrium of truss left of section $C\ C-$ (ref. Fig. 35.2d)

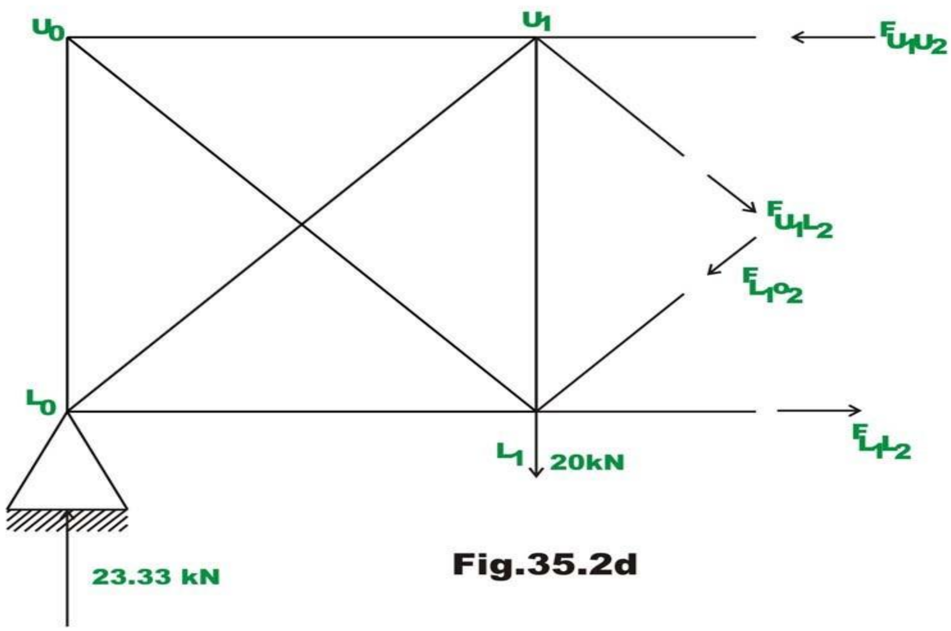


Fig.35.2d

In this panel, the shear is3.33 kN . Considering the vertical equilibrium of the free body diagram

$$\sum F_y = 0 \Rightarrow \quad -F_{L_1U_2}\sin 45 - F_{U_1L_2}\sin 45 + 23.33 - 20 = 0 \qquad (7)$$

It is given that $F_{L_1U_2} = F_{U_1L_2} = F$

$$2F\sin\theta = 3.33$$

$$F = \frac{3.33}{\sqrt{2}} \cong 2.36 \quad \text{kN}$$

Thus,

$$F_{U_1L_2} = 2.36 \quad \text{kN} \quad (Tension)$$
$$F_{L_1U_2} = 2.36 \quad \text{kN} \quad (Compression)$$

Taking moment about U_1 of all the forces,

$$-F_{L_1L_2}\times 3+2.36\left(\frac{1}{\sqrt{2}}\right)\times 3+23.33\times 3=0$$

$F_{L_1L_2} = 25$ kN$(Tension)$ (8)

Taking moment about L1of all the forces,

$$F_{U_1U_2} = 25 \text{ kN}(Comp.) \quad (9)$$

Considering the joint equilibrium of L_1(ref. Fig. 35.2e),

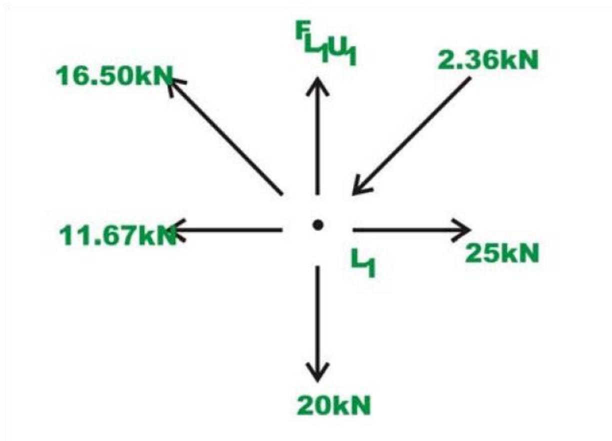


Fig.35.2e

$$\sum F_y = 0 \Rightarrow \quad F_{L_1U_1}+16.50\sin 45-2.36\sin 45-20=0$$

$F_{L_1U_1}=10$ kN$(Tension)$ (10)

Consider the equilibrium of right side of the section $B-B$(ref. Fig. 35.2f) the forces in the 3^{rd} panel are evaluated.

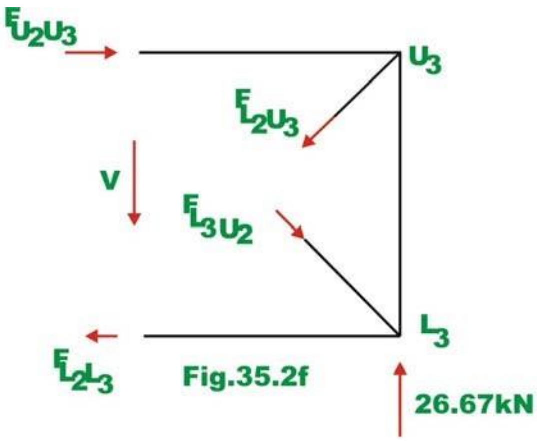


Fig.35.2f

We know that, $F_{L_3U_2} = F_{L_2U_3} = F$

$$\sum F_y = 0 \Rightarrow \quad -F_{L_3U_2}\sin 45 + F_{L_2U_3}\sin 45 + 26.67 = 0 \qquad (11)$$

$$F = \frac{26.67}{\sqrt{2}} \cong 18.86 \text{ kN}$$

$$F_{L_3U_2} = 18.86 \text{ kN} \quad (Comp.)$$

$$F_{L_2U_3} = 18.86 \text{ kN} \quad (Tension) \qquad (12)$$

Considering the joint equilibrium of L_3(ref. Fig. 35.2g), yields

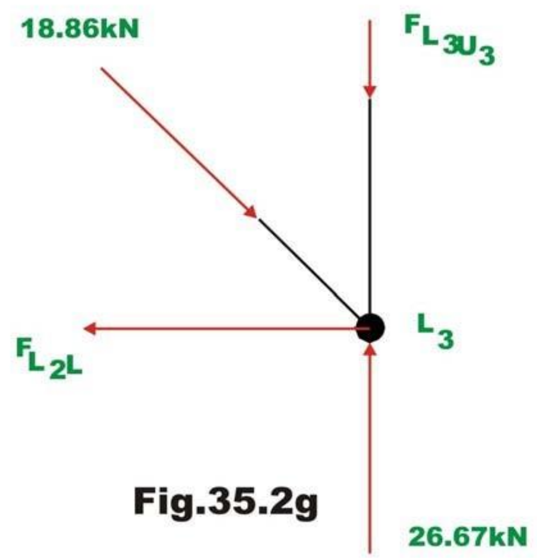


Fig.35.2g

$$\sum F_x = 0 \Rightarrow \qquad -18.86\cos 45 + F_{L_2L_3} = 0$$

$$F_{L_2L_3} = 13.34 \text{ kN}(Tension)$$

$$\sum F_y = 0 \Rightarrow \qquad F_{L_3U_3} = 13.33 \text{ kN}(Comp.)$$

The bar forces in all the members of the truss are shown in Fig. 35.2h. Also in the diagram, bar forces obtained by exact method are shown in brackets.

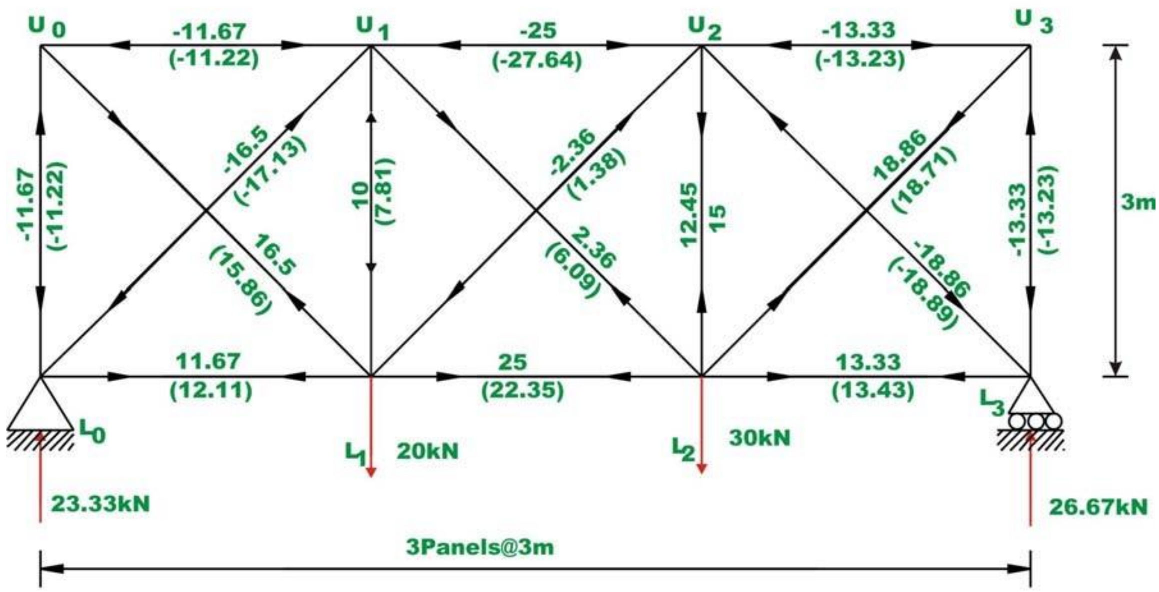


Fig.35.2h

Example 35.2

Determine bar forces in the 3-panel truss of the previous example (shown in Fig. 35.2a) assuming that the diagonals can carry only tensile forces.

Solution:

In this case, the load carried by the compressive diagonal member is zero. Hence the panel shear is completely resisted by the tension diagonal. Reactions of the truss are the same as in the previous example and is given by,

$$R_1 = 23.33 \text{ kN} \quad (\uparrow)$$

$$R_2 = 26.67 \text{ kN} \quad (\uparrow) \qquad (1)$$

Consider again the equilibrium of free body diagram of the truss shown left of $A-A$. This is shown in Fig. 35.3a.

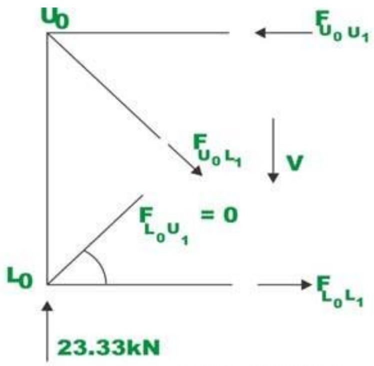


Fig.35.3a

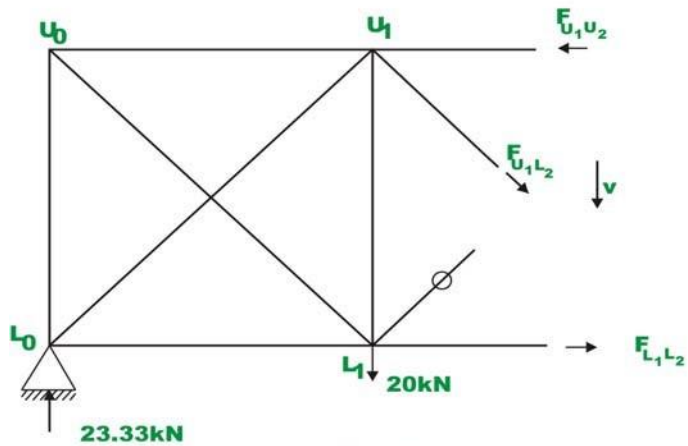


Fig.35.3b

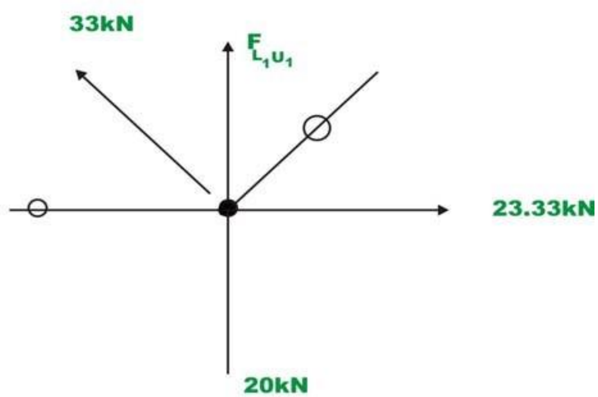


Fig.35.3c

Applying $\sum F_y = 0$,

$$-F_{UL_{01}} \sin 45 + 23.33 = 0$$

$$F_{UL_{01}} = 23.33\sqrt{2} \cong 33 \text{ kN}$$

$$F_{LU_{01}} = 0 \qquad (2)$$

It is easily seen that, $F_{LL_{01}} = 0$ and $F_{UU_{01}} = 23.33$ kN

Considering the vertical equilibrium of joint L_0, we get

$$F_{LU_{00}} = 23.33 \text{ kN } (Comp.) \qquad (3)$$

Since diagonals are inclined at 45° to the horizontal, the vertical and horizontal components of forces are equal in any panel.

Now consider equilibrium of truss left of section $C-C$ (ref. Fig. 35.3b)

In this panel, the shear is 3.33 kN . Considering the vertical equilibrium of the free body diagram,

$$\sum F_y = 0 \Rightarrow \quad -F_{UL_{12}} \sin 45 + 23.33 - 20 = 0 \qquad (4)$$

$$F_{UL_{12}} = 3.33\sqrt{2} \cong 4.71 kN$$

$$F_{LU_{12}} = 0 \qquad (5)$$

Taking moment of all forces about U_1,

$$-F_{LL_{12}} \times 3 + 23.33 \times 3 = 0$$

$$F_{LL_{12}} = 23.33 \text{ kN} (Tension) \qquad (6)$$

Taking moment about L_1 of all the forces,

$$-F_{U_1U_2} \times 3 + 4.71\left(\frac{1}{\sqrt{2}}\right) \times 3 + 23.33 \times 3 = 0$$
$$F_{U_1U_2} = 26.67 \text{ kN}(comp)$$

Considering the joint equilibrium of L_1(ref. Fig. 35.3c), yields

$$\sum F_y = 0 \Rightarrow \qquad F_{LU_{1\,1}} + 33\sin 45 - = 20\ 0$$
$$F_{LU_{1\,1}} = 3.33 \text{ kN}(comp) \qquad (7)$$

Considering the equilibrium of right side of the section $B-B$(ref. Fig. 35.3d) the forces in the 3^{rd} panel are evaluated.

$$\sum F_y = 0 \Rightarrow \qquad -F_{L\,U2\,3}\sin 45 + 26.67 = 0 \qquad (11)$$
$$F_{LU_{3\,2}} = 0$$
$$F_{LU_{2\,3}} = 37.71 \quad \text{kN}\ (Tension) \qquad (12)$$

Considering the joint equilibrium of L_3(ref. Fig. 35.3e), yields

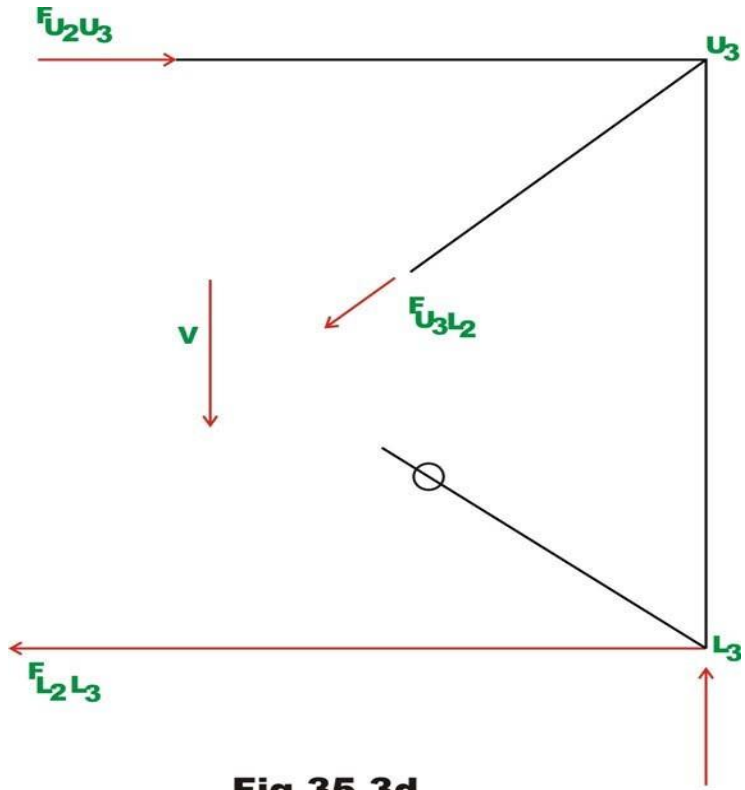


Fig.35.3d

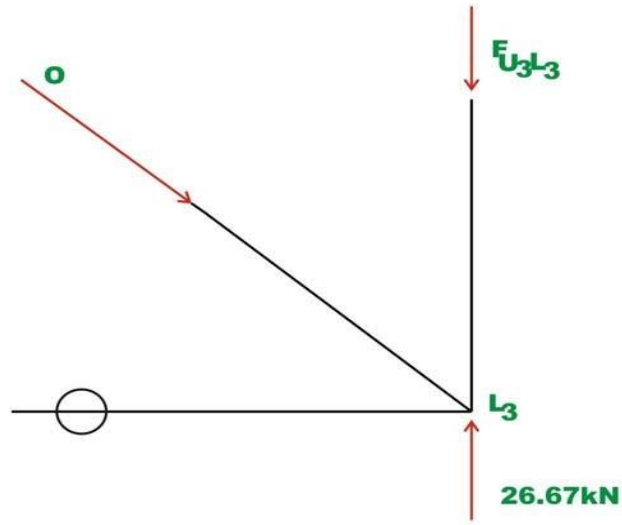


Fig. 35.3e

$\sum F_x = 0 \Rightarrow \qquad F_{L L2 3} = 0$

$\sum F_y = 0 \Rightarrow \qquad F_{LU_{33}} = 26.66 \text{ kN}(Comp.)$

The complete solution is shown in Fig. 35.3f. Also in the diagram, bar forces obtained by exact method are shown in brackets.

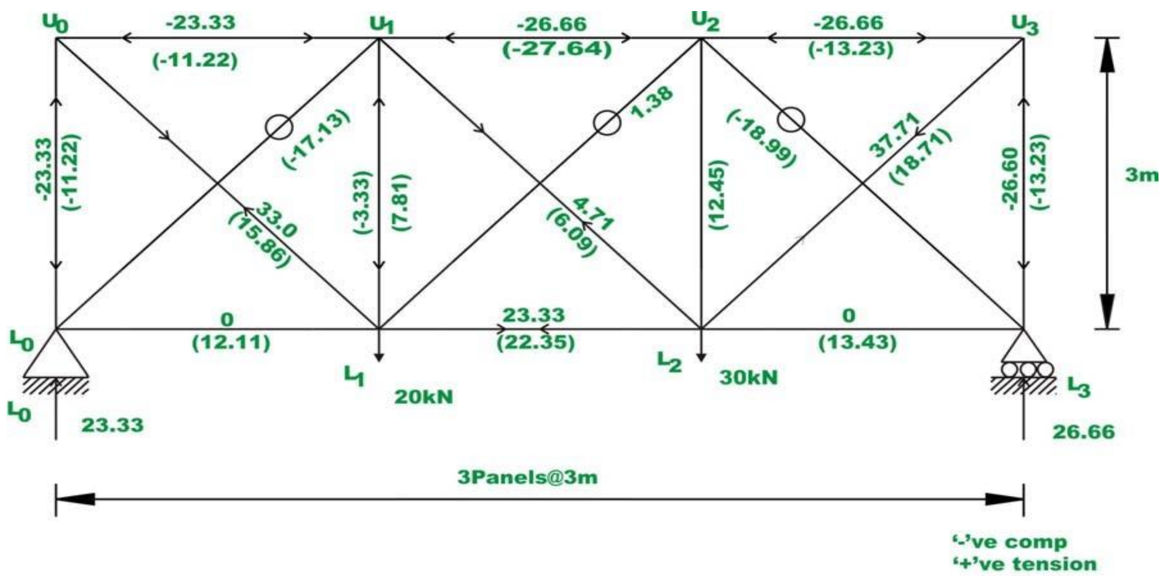
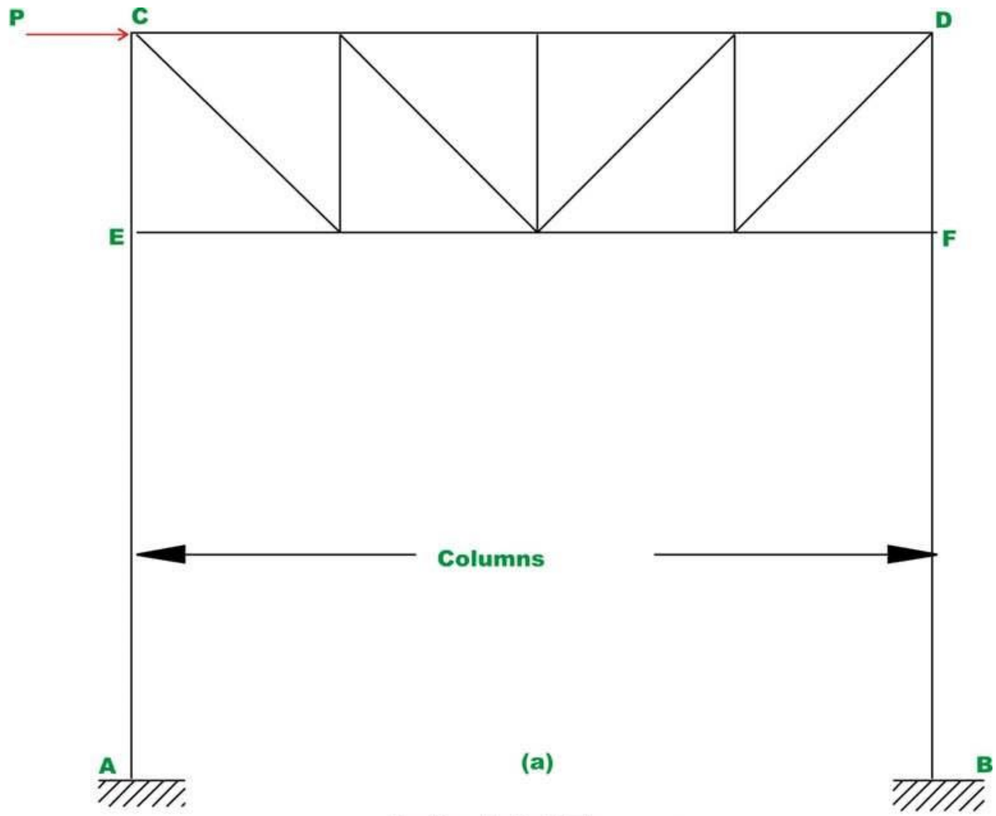


Fig.35.3f Final bar forces

35.3 Industrial frames and portals

Common types of industrial frames are shown in Fig. 35.4a and 35.4b. They consist of two columns and a truss placed over the columns. They may be subjected to vertical loads and wind loads (horizontal loads). While analyzing for the gravity loads, it is assumed that the truss is simply supported on columns. However, while analyzing the frame for horizontal loads it is assumed that, the truss is rigidly connected to columns. The base of the column are either hinged or fixed depending on the column foundation. When the concrete footing at the column base is small, then it is reasonable to assume that the columns are hinged at the base. However if the column are built into massive foundation, then the column ends are considered as fixed for the analysis purposes.



Industrial Frames

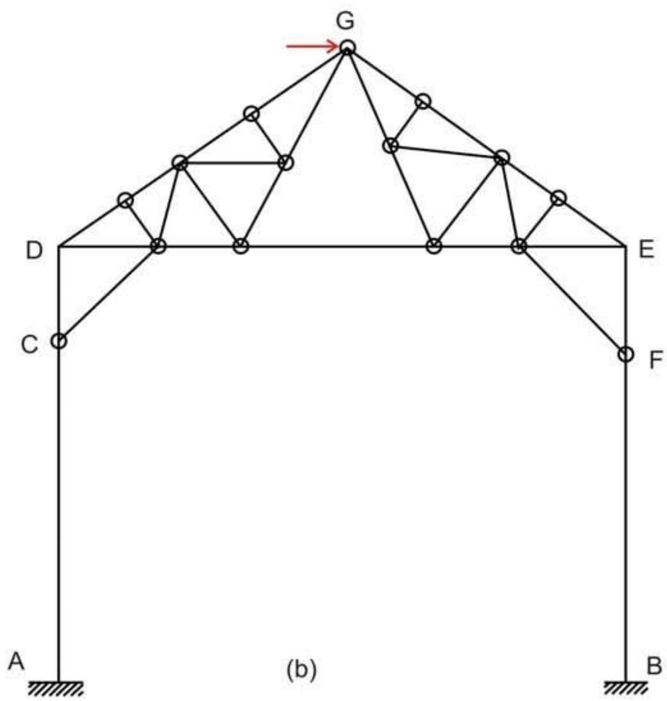


Fig. 35.4

Before considering the analysis of structures to wind load (horizontal load) consider the portals which are also used as the end portals of bridge structure (see Fig. 35.5). Their behaviour is similar to industrial trusses. The portals are also assumed to be fixed or hinged at the base depending on the type of foundation.

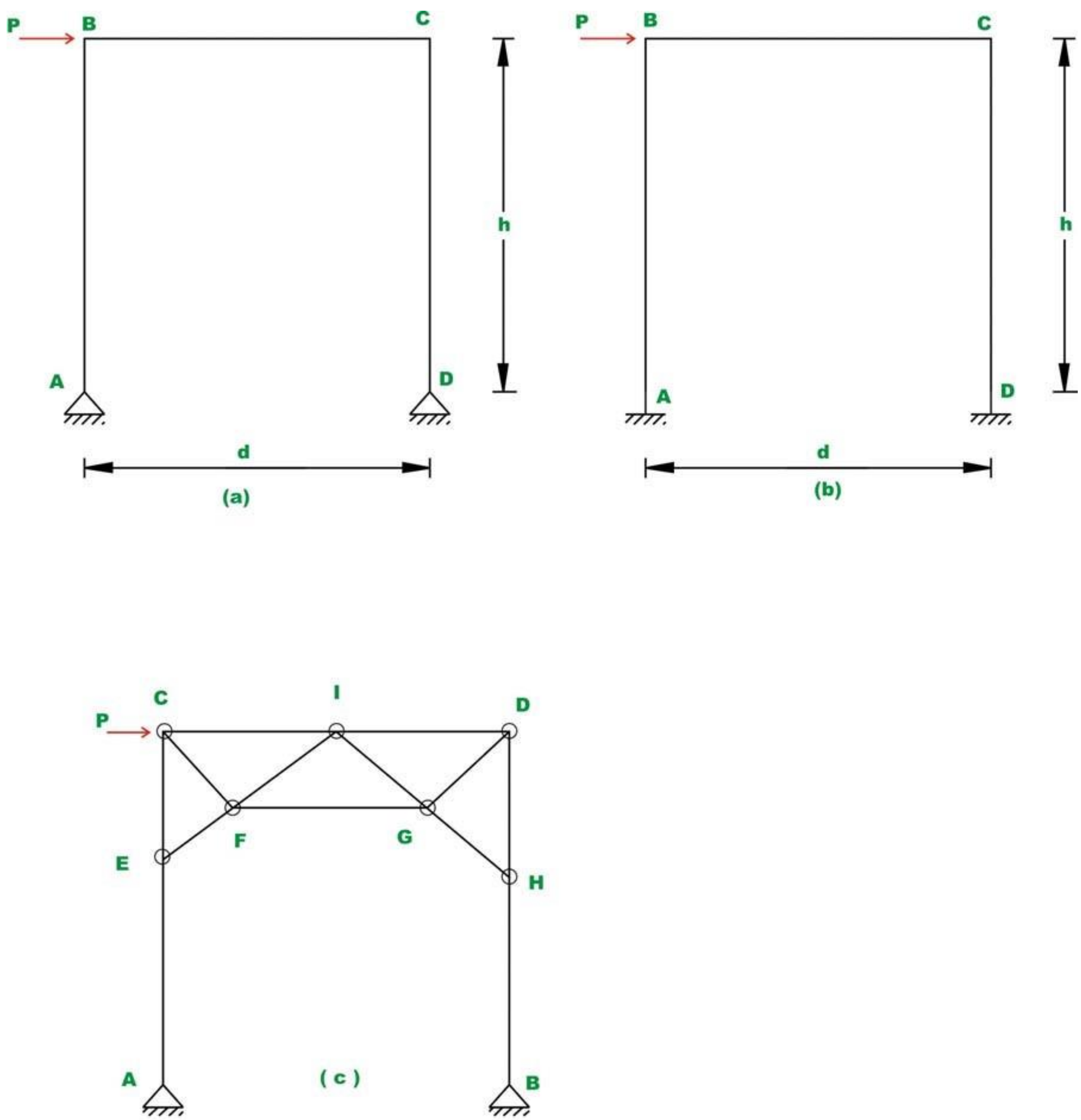


Fig.35.5 Portal Frames

Consider a portal which is hinged at the base, as shown in Fig. 35.5a. This structure is statically indeterminate to degree one. To analyse this frame when subjected to wind loads by only equations of statics, it is required to make one assumption. When stiffness of columns is nearly equal then it is assumed that the shear at the base of each column is equal. If stiffness of columns is unequal then it is assumed that the shear at the base of a column is proportional to its stiffness.

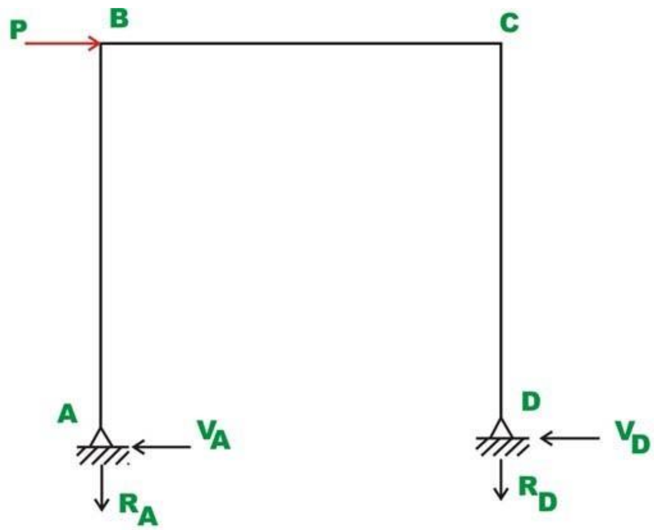


Fig.35.6

Reactions and Bending moments:

As per the assumption, shear at the base of columns is given by (vide Fig. 35.6)

Now $V_A = V_D = \dfrac{P}{2}$

Taking moment about hinge D,

$\sum M_D = 0 \qquad \Rightarrow R_A \times d = P \times h$

$$\Rightarrow R_A = \frac{Ph}{d}(\downarrow)$$

And $\qquad \Rightarrow R_D = \dfrac{Ph}{d}(\uparrow)$

The bending moment diagram is shown in Fig. 35.7.

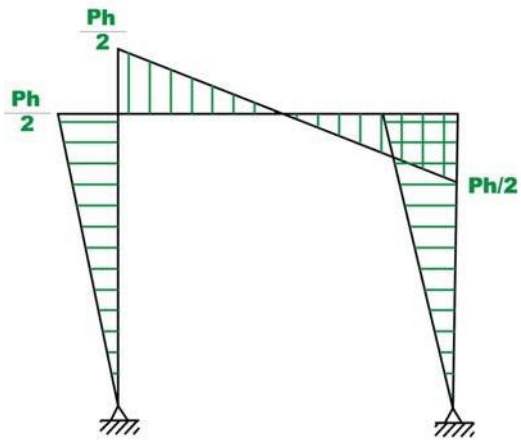


Fig 35.7a Bending Moment Diagram

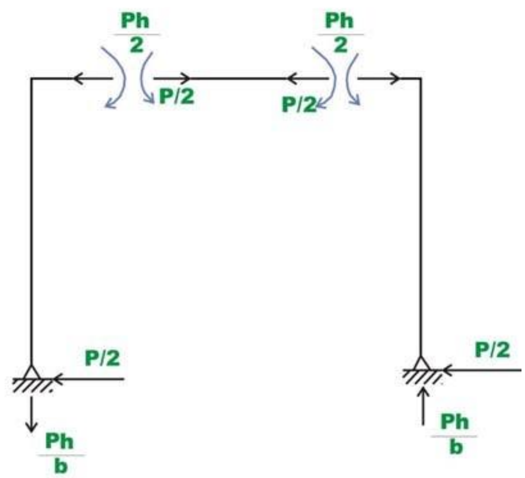


Fig.35.7(b) Reactions

It is clear from the moment diagram, an imaginary hinge forms at the mid point of the girders. Thus instead of making assumption that the shear is equal at the column base, one could say that a hinge forms at the mid point of the girder. Both the assumptions are one and the same.

Now consider a portal frame which is fixed at the base as shown in Fig. 35.5b. This is statically indeterminate to third degree and one needs to make three independent assumptions to solve this problem by equations of static equilibrium alone. Again it is assumed that the shear at the base of each column is equal provided their stiffnesses are equal. The deformed shape of the portal is shown in Fig. 35.8a and the deformed shape of the industrial frame is shown in Fig.35.8b.

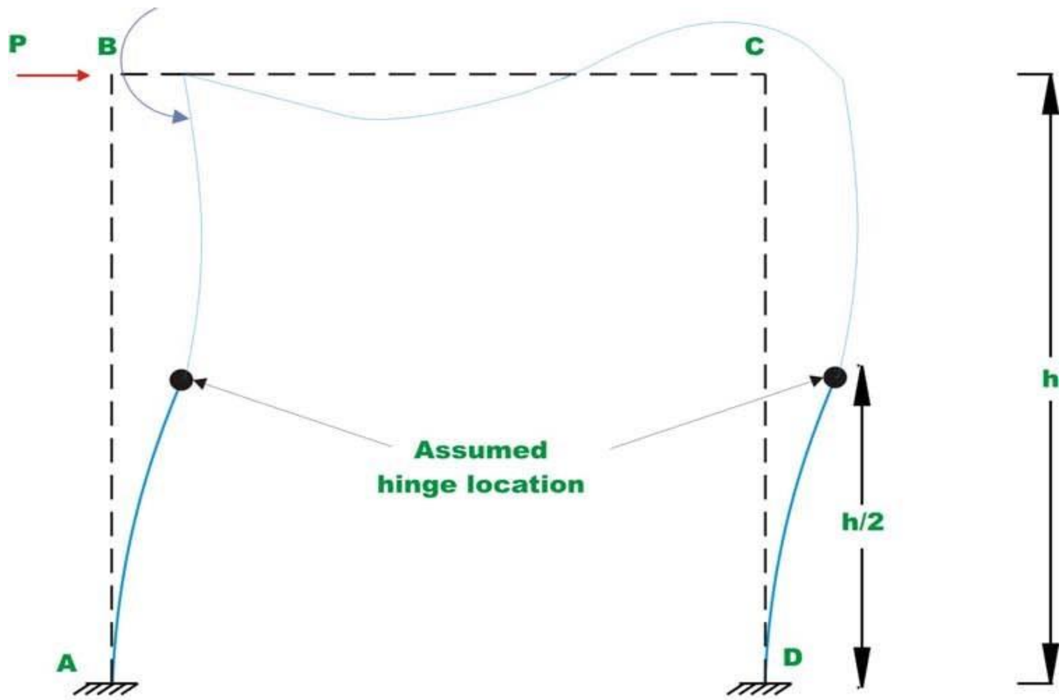


Fig.35.8a

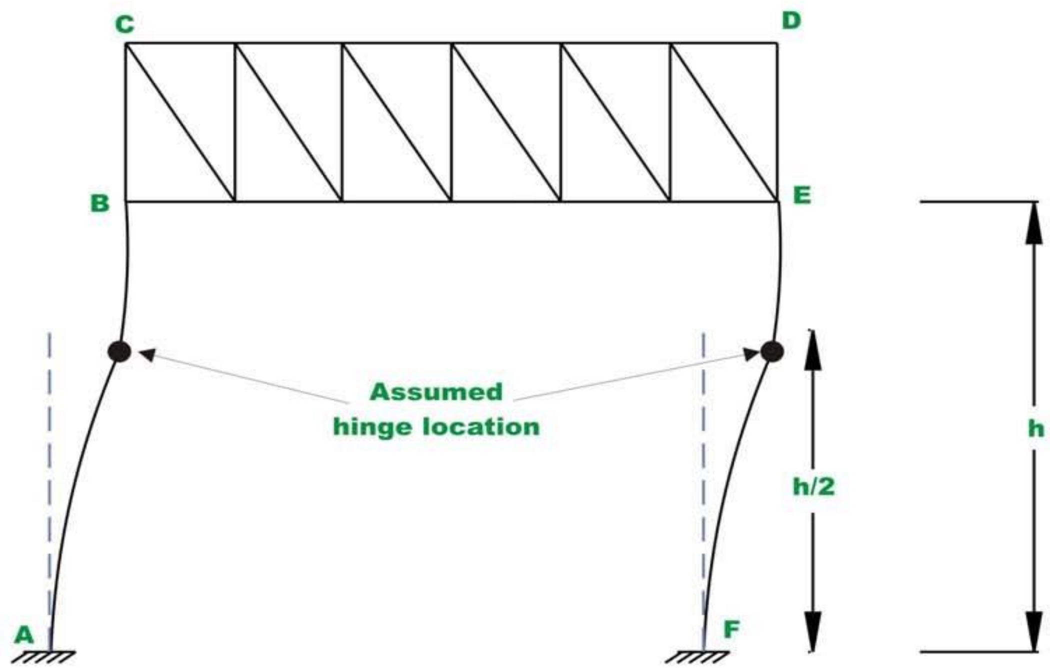


Fig.35.8b

In such a case, the bending moment at the base of the column (at A) produces tension on outside fibres of column cross section. The bending moment at top of column produces tension on inside fibres of column. Hence bending moment changes its sign between column base and top. Thus bending moment must be zero somewhere along the height of the portal. Approximately the inflexion point occurs at the mid height of columns. Now we have three independent assumptions and using them, we could evaluate reactions and moments. In the case of industrial frames, the inflexion points are assumed to occur at mid height between A and B .

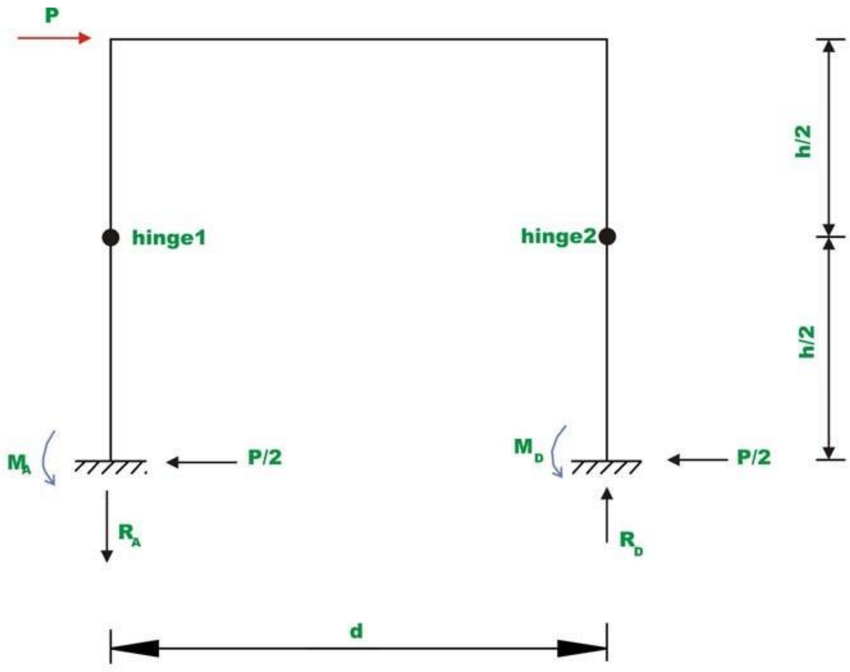


Fig.35.9a

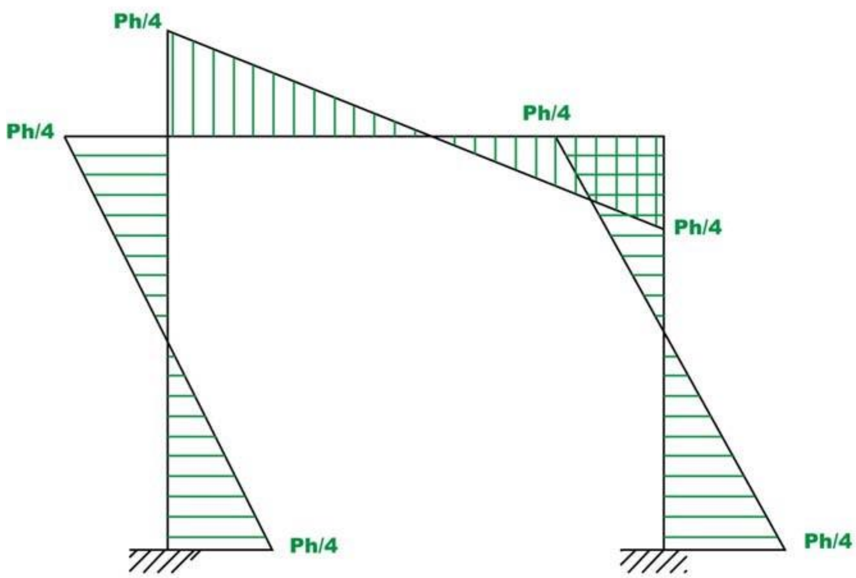


Figure 35.9b

Taking moment of all forces left of hinge 1 about hinge 1 (vide Fig. 35.9a), yields

$$\frac{Ph}{2\times 2} - M_A = 0 \qquad \Rightarrow \qquad M_A = \frac{Ph}{4}$$

Similarly taking moment of all forces left of hinge 2 about hinge 2,

$$\frac{Ph}{2\times 2} - M_D = 0 \qquad \Rightarrow \qquad M_D = \frac{Ph}{4}$$

Taking moment of all forces right of hinge 1 about hinge 1 gives,

$$R_D d + M_D - \frac{P}{2}\frac{h}{2} - \frac{Ph}{2} = 0 \qquad \Rightarrow \qquad R_D = \frac{Ph}{2d}(\uparrow)$$

Similarly

$$R_A = \frac{Ph}{2d}(\downarrow)$$

The bending moment diagram is shown in Fig. 35.9b.

If the base of the column is partially fixed then hinge is assumed at a height of $1/3^{rd}$ from the base. Note that when it is hinged at the base of the column, the inflexion point occurs at the support and when it is fixed, the inflexion point occurs at mid-height.

Example 35.3

Determine approximately forces in the member of a truss portal shown in Fig.

35.10a.

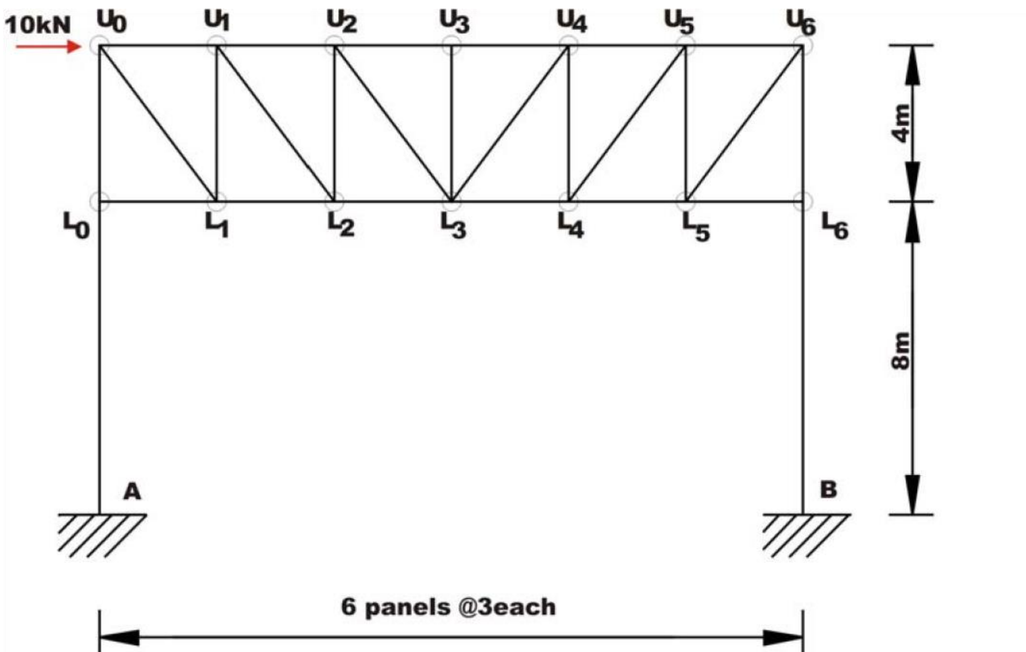


Fig. 35.10a

In this case, as per the first assumption, the shear at the base of each column is the same and is given by (ref. 35.10b)

$$V_A = V_D = \frac{10}{2} = 5kN \qquad (1)$$

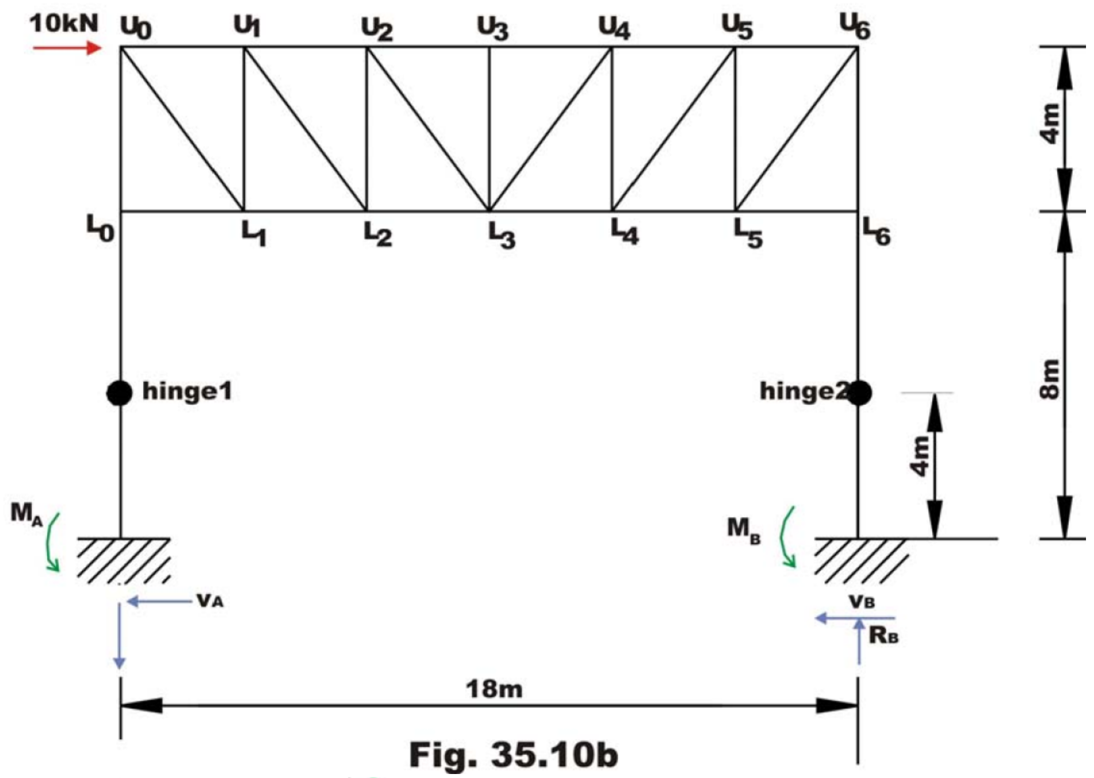


Fig. 35.10b

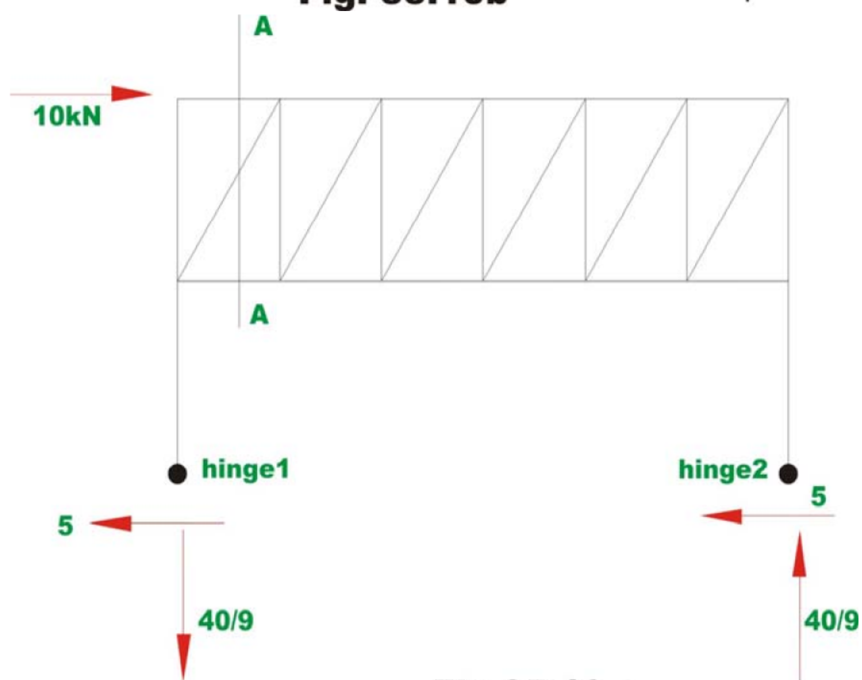


Fig 35.10.c

Taking moment of all forces right of hinge 2 about hinge 2, results

$$M_B = \frac{P}{2} \times 4 \quad \Rightarrow \quad M_B = 20\,\text{kN.m} \tag{2}$$

Similarly $M_A = 20$ kN.m (3)

Taking moment of all forces right of hinge 1 about hinge 1 gives,

$$R_B \times 18 - V_B \times 4 + 20 - 10(4+4) = 0 \quad \Rightarrow \quad R_B = \frac{80}{18} = \frac{40}{9}\,kN(\uparrow)$$

Similarly,

$$R_A = \frac{40}{9}\,kN(\downarrow) \tag{4}$$

Forces in the truss member can be calculated either by method of sections or by method of joints. For example, consider the equilibrium of truss left of $A-A$as shown in Fig. 35.10d.

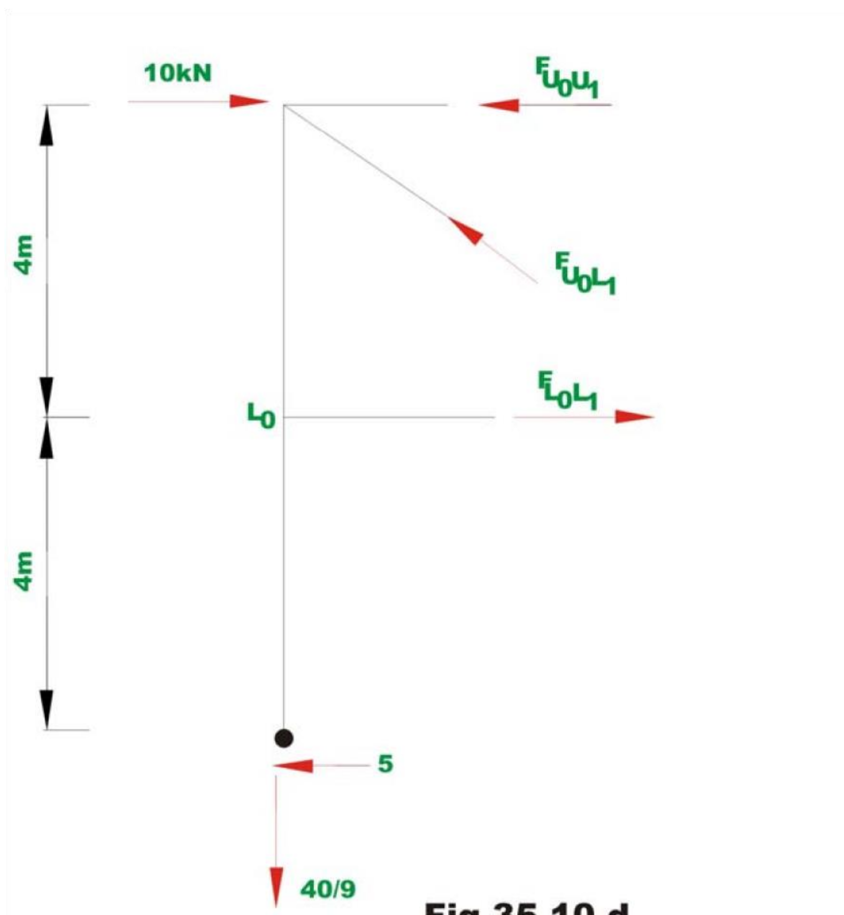


Fig 35.10.d

$$\sum F_y = 0 \Rightarrow \quad -\frac{40}{9} + F_{U_0L_1} \times \frac{4}{5} = 0$$

$$\Rightarrow F_{U_0L_1} = 5.55\,\text{kN}(Comp.) \qquad (5)$$

Taking moment about U_0,

$$5 \times 8 - F_{L_0L_1} \times 4 = 0$$

$$F_{L_0L_1} = 10\,\text{kN}(Tension) \qquad (6)$$

Taking moment about L_1,

$$10 \times 4 + 5 \times 4 - \frac{40}{9} \times 3 - F_{U_0U_1} \times 4 = 0$$

$$F_{U_0U_1} = 11.66\,\text{kN}(Comp) \qquad (7)$$

Summary

It is observed that prior to analysis of indeterminate structures either by stiffness method or force method; one must have information regarding their relative stiffnesses and member material properties. This information is not available prior to preliminary design of structures. Hence in such cases, one can not perform indeterminate structural analysis by exact methods. Hence, usually in such cases, based on few approximations (which are justified on the structural behaviour under the applied loads) the indeterminate structures are reduced into determinate structures. The determinate structure is then solved by equations of statics. This methodology has been adopted in this lesson to solve indeterminate trusses and industrial frames. Depending upon the validity of assumptions, the results of approximate methods compare favourably with exact methods of structural analysis as seen from the numerical examples.