LATERAL EARTH PRESSURE

7 Introduction

A retaining wall is a structure that is used to support a vertical or near vertical slopes of soil. The resulting horizontal stress from the soil on the wall is called **lateral earth pressure**. To determine the magnitude of the lateral earth pressure, a geotechnical engineer must know the basic soil parameters – that is, unit weight γ, angle of friction ϕ, and cohesion c – for the soil retained behind the wall. In the evaluation of the magnitude of this lateral earth pressure, it is assumed that the soil behind the wall (called backfill soil) is on the verge of failure and obeys some failure criterion, for example, the Mohr-Coulomb failure criterion.

When you complete this chapter, you should be able to:

 • Understand lateral earth pressure.
 • Determine lateral earth pressure.

7.1 Definitions of Key Terms

At rest earth pressure coefficient (k_0) is the ratio between the lateral and vertical principal effective stresses when an earth retaining structure is at rest (or is not allowed to move at all).

Active earth pressure coefficient (k_a) is the ratio between the lateral and vertical principal effective stresses when an earth retaining structure moves away from the retained soil.

Passive earth pressure coefficient (k_p) is the ratio between the lateral and vertical principal effective stresses when an earth retaining structure is forced to move against a soil mass.

7.2 Lateral Earth Pressure at Rest

Consider a vertical wall of height H, as shown in Fig. 7.1, retaining a soil having a unit weight of γ. At any depth z below the ground surface the vertical effective stress is:

$$\sigma_z' = \gamma z - u \tag{7.1}$$

If the wall *is not allowed to move at all* either away from the soil mass or to the soil mass (or in other words if there is no lateral expansion or compression in the backfill soil), the lateral pressure is called **at rest earth pressure**. In this case, the lateral earth pressure σ_x' at a depth z is:

$$\sigma_x' = k_0 \sigma_z' \tag{7.2}$$

Where, k_0 is coefficient of at rest earth pressure. You must remember that k_0 applies only to effective stresses not to total stresses. The magnitude of k_0 depends on the type of the soil, its degree of compaction, plasticity characteristics, and degree of

disturbance (Bishop, 1958).
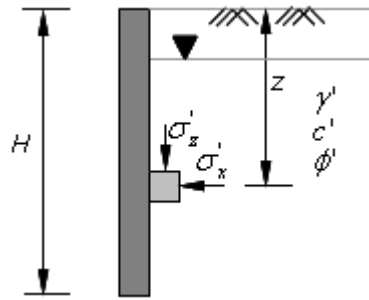
Figure 7.1: At rest earth pressure.

For truly normally consolidated soil that exhibits zero cohesion, a value for k_0 may be calculated from the following generally accepted empirical equation (Jaky, 1948):

$$k_0 = 1 - \sin\phi'$$ (7.3)

For overconsolidated soils the value of k_0 is higher than that given by Eq. (4.3). Alpan (1967) suggested the following relationship:

$$\frac{k_{0,OCR}}{k_{0,NC}} = (OCR)^n$$ (7.4)

Where, $k_{0,OCR}$ and $k_{0,NC}$ are the coefficient of at rest earth pressure for overconsolidated and normally consolidated soil, respectively, OCR is the overconsolidation ratio, and n is a number depending on the plasticity characteristics of the soil. Based on statistical analysis of several laboratory test results, Mayne & Kulhawy (1982) proposed that, $n = \sin\phi'$, thus:

$$k_{0,OCR} = (1 - \sin\phi')(OCR)^{\sin\phi'}$$ (7.5)

EXAMPLE 7.1

For the retaining wall shown in Fig. E7.1, determine the total resultant lateral earth force at rest per unit length of the wall. Also determine the location of the resultant earth pressure. Assume that the soil is a normally consolidated soil.
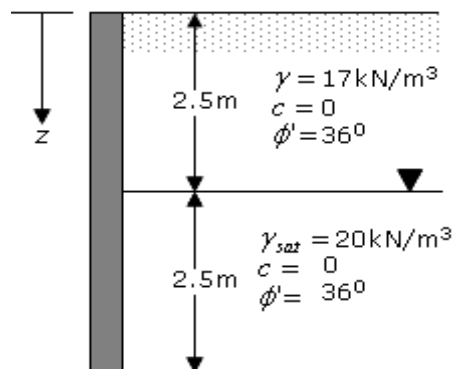
FIGURE E7.1

## 7.3 Active and Passive Lateral Earth Pressures

The lateral earth pressure condition described in section 7.2 involves walls that do not yield at all. In this section, we will discuss other conditions that involve movement of the wall and consequently failure of the backfill soil. Failure of the backfill soil occurs by two mechanisms depending on the direction of wall displacement. If the displacement of the wall is away from the backfill soil the resulting failure is called *active* and the lateral pressure exerted on the wall by the backfill soil is called *active lateral earth pressure* or simply *active earth pressure*. A *passive* failure occurs if the wall is displaced towards the backfill soil until the limiting displacement is achieved. In this case, the wall exerts a pressure on the backfill soil, and the passive resistance provided by the backfill soil against the wall displacement is called *passive earth pressure*.

In the next sections, we will deal with two active and passive earth pressure theories: one proposed by Rankine (1857) and the other by Coulomb (1776).

7.3 Rankine Active and Passive Earth Pressures

Consider a vertical frictionless (smooth) wall retaining a soil mass in both front and back of the wall as shown in Fig. 7.2a. If the wall remains rigid and no movement occurs, then the vertical and horizontal (lateral) effective stresses at rest on element *A*, at the back of the wall, and *B*, at the front of the wall are given by Eqns. (7.1 and 7.2) in section 7.2. Mohr's circle for at rest state is shown by circle ① in Fig. 7.2b.

Let us now assume a rotation about the bottom of the wall sufficient to produce slip planes in the soil mass behind and in front of the wall (Fig. 7.3). The rotation required, and consequently the lateral displacement or strain, to produce slip planes in front of the wall is much larger than that required for the back of the wall, as shown in Fig. 7.4. The soil mass at the back of the wall is assisting in producing failure, thus it is in the *active pressure state* while the soil mass at the front of the wall is resisting failure, thus in the *passive pressure state*.
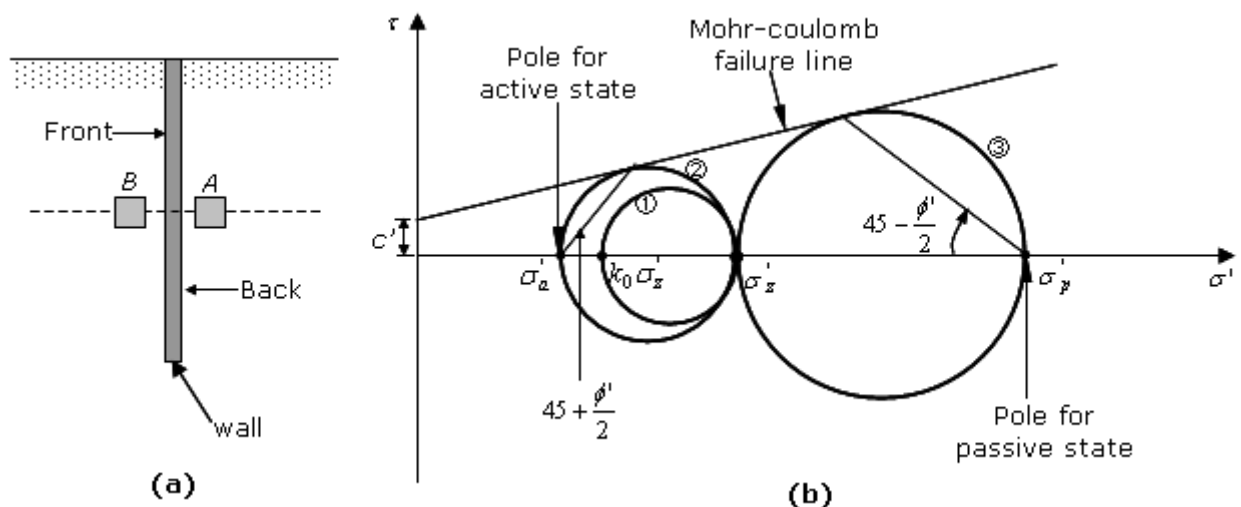
Figure 7.2: **a)** A smooth retaining wall, **b)** Mohr's circles for at rest, active and passive states.
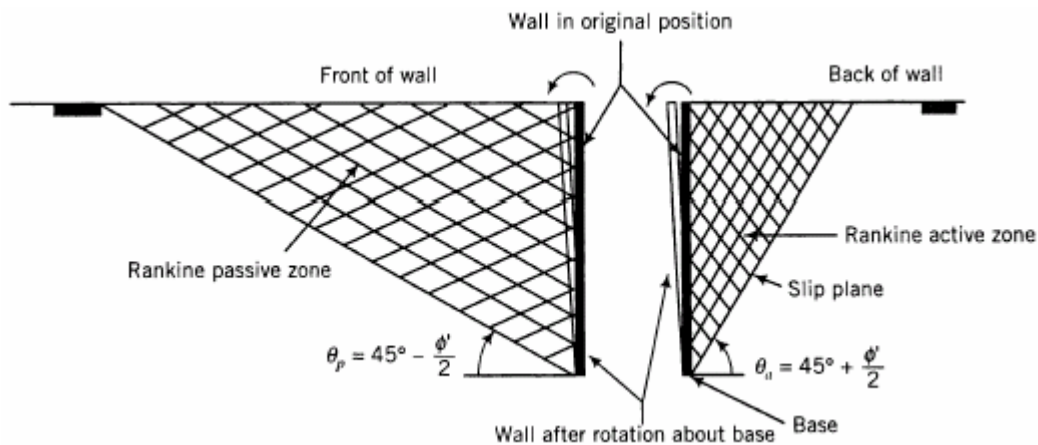
Figure 7.3: Failure planes within a soil mass near a retaining wall.
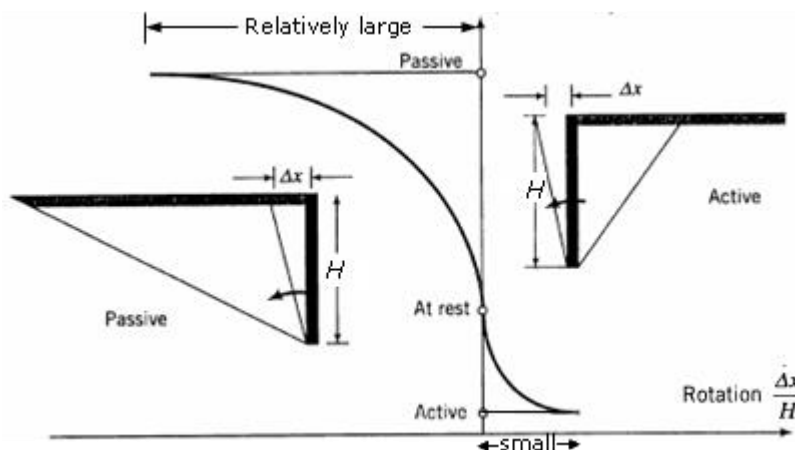
Figure 7.4: Rotation required mobilizing active and passive pressures.

What happens to the lateral effective stresses on elements *A* and *B* when the wall is rotated? The vertical stress will not change on either element but the lateral effective stress on element *A will be reduced* while that for element *B will be increased*. We can now plot two additional Mohr's circles: to represent the stress states of element *A* (circle ②) and the other to represent the stress state of element *B* (circle ③). Both circles are drawn such that the decrease (element *A*) or increase (element *B*) in lateral effective stresses is sufficient to bring the soil to Mohr-coulomb failure state. In other words, both circles ② and ③ will touch the Mohr-coulomb failure line as shown in Fig. 3.3 b. For element *B* to reach the failure state, the lateral effective stress must be greater than the vertical effective stress, as shown in Fig. 7.3 b.

The stress states of soil elements *A* and *B* are called the Rankine active state and the Rankine passive state, respectively (Named after the first developer of this theory Rankine, 1857). Each of these Rankine states is associated with a family of failure planes. For the Rankine active state, the failure planes are oriented at:

$$\theta_a = 45^0 + \frac{\phi'}{2} \tag{7.6}$$

to the horizontal, as illustrated in Fig. 7.3 b and proved in Chapter 1 (Eqn. 1.12). For

the Rankine passive state, the failure planes are oriented at:

$$\theta_p = 45^0 - \frac{\phi'}{2} \tag{7.7}$$

to the horizontal as illustrated in Fig. 4.3 a.

For the active state, the lateral effective stress σ_3' is equal to σ_a' as shown in Fig. 7.2 b (Mohr's circle ②). Substituting σ_a' into Eqn. (1.1.7) in Chapter 1, the Rankine active lateral effective stress is:

$$\begin{aligned}\sigma_a' &= \sigma_z' \frac{1-\sin\phi'}{1+\sin\phi'} - 2c'\sqrt{\frac{1-\sin\phi'}{1+\sin\phi'}} \\ &= \sigma_z' k_a - 2c'\sqrt{k_a}\end{aligned} \tag{7.8}$$

Where,

$$k_a = \frac{1-\sin\phi'}{1+\sin\phi'} = \tan^2(45 - \frac{\phi'}{2}) \tag{7.9}$$

K_a is the active earth pressure coefficient.

For the passive state, the lateral effective stress σ_p' becomes the major principal stress σ_1' and the vertical effective stress σ_z' becomes the minor principal stress σ_3' as shown in Fig. 7.2 b (Mohr's circle ③). Therefore, using Eqn. (1.16) in Chapter 1, the Rankine passive lateral effective stress is:

$$\begin{aligned}\sigma_p' &= \sigma_z' \frac{1+\sin\phi'}{1-\sin\phi'} + 2c'\sqrt{\frac{1+\sin\phi'}{1-\sin\phi'}} \\ &= \sigma_z' k_p + 2c'\sqrt{k_p}\end{aligned} \tag{7.10}$$

Where,

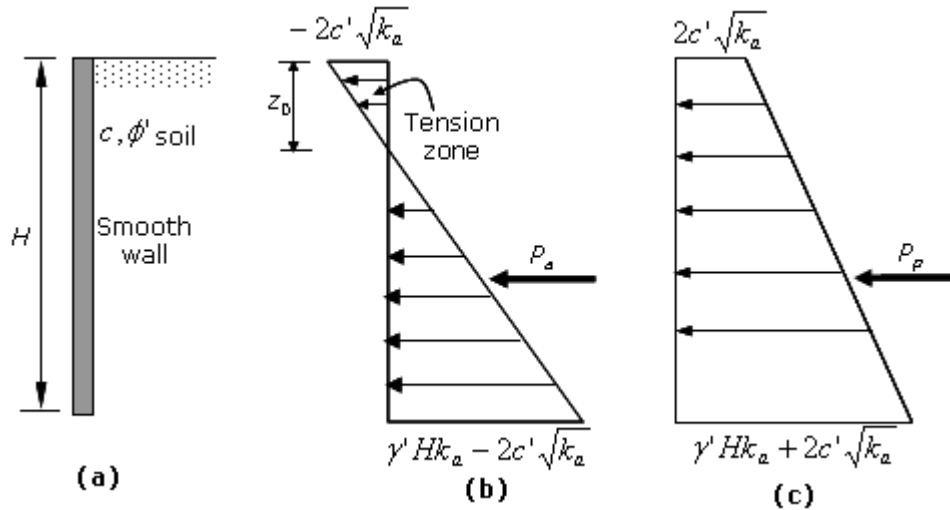$$k_p = \frac{1+\sin\phi'}{1-\sin\phi'} = \tan^2(45 + \frac{\phi'}{2}) \tag{7.11}$$

k_p is the passive earth pressure coefficient.

Based on Eqns. (7.9 and 7.11), we can easily get the following relation for the active and passive earth pressure coefficients:

$$k_p = \frac{1}{k_a} \tag{7.12}$$

Equations (7.8) and (7.10) indicate that, for a homogeneous soil layer, the lateral earth pressure varies linearly with depth z.

Figure 7.5: pressure distribution in c-ϕ' soil: a) c-ϕ' soil, b) active, c) passive state.

Figure 7.5 shows the active and passive lateral stress distribution for a smooth wall retaining a c-ϕ' soil. In the active state case, the soil at depth $z = 0$ is subjected to a tensile stress as shown in Fig. 7.5 b. Soils do not have tensile strength, as a result **tension cracks** will occur down to a depth z_0, where the tensile stress becomes zero. At depth z_0 (known as **depth of tension crack**), the stress is zero, thus,

$$0 = \gamma' z_0 k_a - 2c'\sqrt{k_a} \implies z_0 = \frac{2c'}{\gamma'\sqrt{k_a}} \tag{7.13}$$

The lateral earth force is the area of the lateral stress diagram (Fig. 7.5), which for the Rankine active state is:

$$P_a = \int_0^H (\gamma' z k_a - 2c'\sqrt{k_a}) = \tfrac{1}{2}k_a\gamma' H^2 - 2c'H\sqrt{k_a} \tag{7.14}$$

and, for the Rankine passive state, is

$$P_p = \int_0^H (\gamma' z k_p + 2c'\sqrt{k_p}) = \tfrac{1}{2}k_p\gamma' H^2 + 2c'H\sqrt{k_p} \tag{7.15}$$

For most retaining wall construction, a granular backfill is used and $c' = 0$, therefore, for granular soils Eqns. (7.13) and (7.14) can be rewritten as:

$$P_a = \tfrac{1}{2}k_a\gamma' H^2 \tag{7.16}$$

and

$$P_p = \tfrac{1}{2}k_p\gamma' H^2 \tag{7.17}$$

EXAMPLE 7.2

Draw the active pressure diagram per meter length of an 8 m high smooth vertical retaining wall. Also, calculate a) tension crack depth and b) the resultant (total) active force and its location. The properties of the backfill soil are c' = 20 kPa, $\phi'=25^0$ and γ =17.5 kN/m³. Note that the tension zone is usually ignored for finding the magnitude and location of the resultant force.

7.5   Lateral Earth Pressure due to Surcharge

Surfaces stresses (due to surcharge) also impose lateral pressure on retaining walls as illustrated in Fig. 7.6 d. A uniform surface stress, q_s, will transmit a uniform active lateral earth pressure of $k_a q_s$ and a uniform passive lateral earth pressure of $k_p q_s$. The active and passive lateral stresses due to the soil (i.e. c', ϕ' soil), and the uniform surfaces stresses are then:

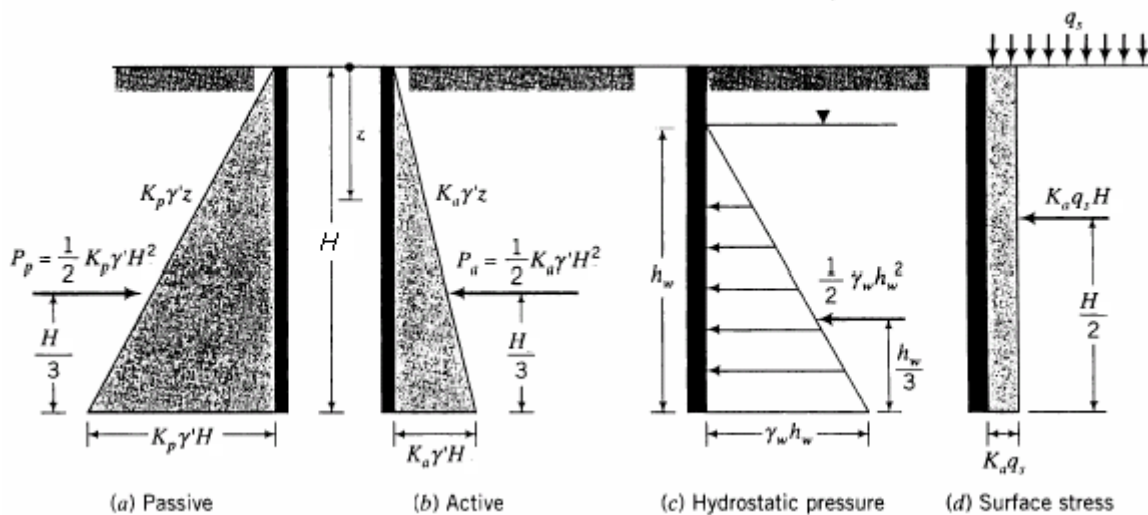$$\sigma_a^{'} = k_a \gamma' z + k_a q_s \tag{7.18}$$

and $$\sigma_p^{'} = k_p \gamma' z + k_p q_s \tag{7.19}$$

The corresponding active and passive lateral forces are also given by:

$$P_a = \tfrac{1}{2} k_a \gamma' H^2 + k_a q_s H \tag{7.20}$$

and

$$P_p = \tfrac{1}{2} k_p \gamma' H^2 + k_p q_s H \tag{7.21}$$

Figure 7.6: Variation of active and passive lateral earth pressures, hydrostatic pressure, and a uniform surface stress with depth. (Note: backfill soil is granular).

For a c', ϕ' backfill, Eqns. (3.18) and (3.19) will become:

$$\sigma_a^{'} = k_a \gamma' z + k_a q_s - 2c' \sqrt{k_a} \tag{7.22}$$

and
$$\sigma'_p = k_p \gamma' z + k_p q_s + 2c'\sqrt{k_p} \tag{7.23}$$

The corresponding active and passive lateral forces are also given by:

$$P_a = \tfrac{1}{2} k_a \gamma' H^2 + k_a q_s H - 2c'H\sqrt{k_a} \tag{7.24}$$

and

$$P_p = \tfrac{1}{2} k_p \gamma' H^2 + k_p q_s H + 2c'H\sqrt{k_p} \tag{7.25}$$

Note that for a purely cohesive saturated clay with undrained shear strength parameter of c_u and $\phi'_u = 0$, $k_a = k_p = 1$.

7.6   Lateral Earth Pressure When Groundwater is Present

If groundwater is present, you need to add the hydrostatic pressure (pore water pressure) to the lateral earth pressure. For example, if the groundwater level is at a distance h_w from the base of the wall as shown in Fig. 3.6, the hydrostatic pressure is,

$$u = \gamma_w h_w \tag{7.26}$$

and the hydrostatic force is:

$$P_w = \tfrac{1}{2} \gamma_w h_w^2 \tag{7.27}$$

7.7   Summary of Rankine Lateral Earth Pressure Theory

1. The lateral earth pressures on retaining walls are related directly to the vertical effective stress through two coefficients k_a and k_p.
2. Substantially more movement is required to mobilize the full passive earth pressure than the full active earth pressure.
3. A family of slip planes occurs in the Rankine active and passive states. In the active state, the slip planes are oriented at $(45^0 + \phi'/2)$ to the horizontal, and while for the passive case they are oriented at $(45^0 - \phi'/2)$ to the horizontal.
4. The lateral earth pressure coefficients, developed so far are only valid for a smooth, vertical wall supporting a soil mass with a horizontal surface; and must be applied to effective stresses only.

EXAMPLE 7.3

For the frictionless wall retaining a stratified soil and shown in Fig. E7.2, determine:
   (a) The active lateral earth pressure distribution with depth.
   (b) The passive lateral earth pressure distribution with depth.
   (c) The magnitude and location of the active and passive forces.
   (d) The resultant force.
   (e) The ratio of passive moment to active moment.
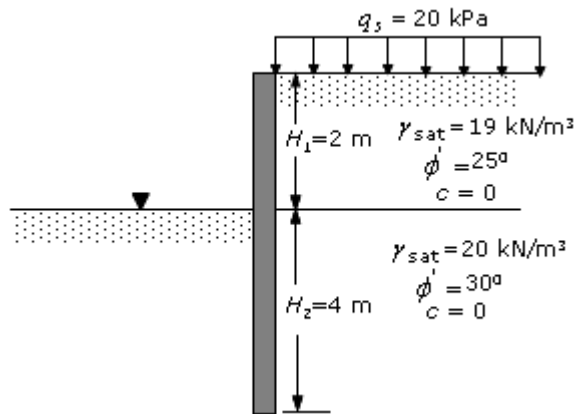
FIGURE E7.2

Strategy There are two layers. It is best to treat the layers separately. Neither k_a nor k_p should be applied to the pore water pressure. You do not need k_p for the top soil layer. Since the water level on both sides of the wall is the same, the resultant hydrostatic force is zero. However, you are asked to determine the forces on each side of the wall; therefore, you have to consider the hydrostatic force. A table is helpful to solve this type of problem.

7.8 Rankine Active & Passive Earth Pressure for Inclined Granular Backfill

If the backfill of a frictionless retaining wall is a granular soil ($c = 0, \phi^{'}$) and rises at an angle β ($\beta \leq \phi^{'}$) with respect to the horizontal (Figure 3.7), the Rankine active earth pressure coefficient k_a is expressed in the form:

$$k_a = \cos\beta \frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi^{'}}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi^{'}}} \tag{7.28}$$
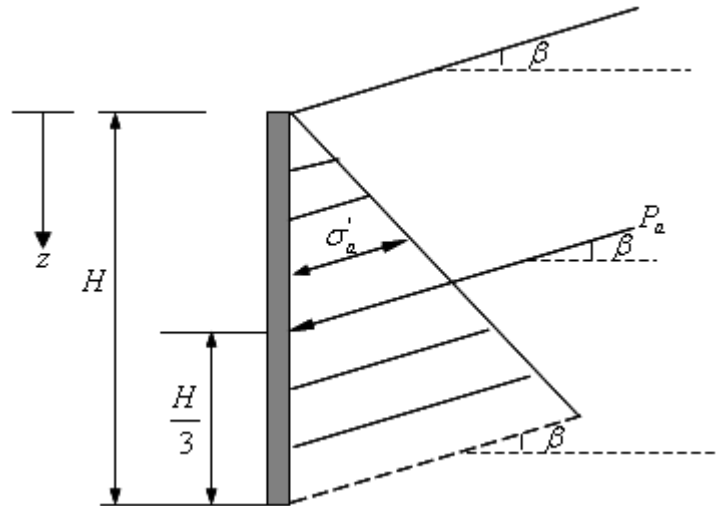
The Rankine active stress on the wall is:

$$\sigma_a^{'} = \gamma^{'} z k_a \tag{7.29}$$

and the Rankine active lateral force is:

$$P_a = \tfrac{1}{2} k_a \gamma^{'} H^2 \tag{7.30}$$

Note that, the direction of the lateral force P_a is inclined at an angle β to the horizontal and intersects the wall at a distance of $H/3$ from the base of the wall.

Figure 7.7: Rankine Active Earth Pressure for Inclined Granular Backfill

The Rankine passive pressure coefficient k_p for a wall with a granular sloping backfill is:

$$k_p = \cos\beta \frac{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi'}}{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi'}} \qquad (7.31)$$

The Rankine passive stress and passive lateral force are calculated using equations similar to Eqns. (7.24 and 7.25) in which k_a is replaced by k_p. As in the case of the active force, the resultant force P_p is inclined at angle β with the horizontal and intersects the wall at a distance of $H/3$ from the bottom of the wall.

EXAMPLE 7.4

A retaining wall with a vertical back is 5 m high and retains a sloping soil with $\beta = 20^0$. Determine the magnitude of the active pressure.

7.9   Coulomb's Earth Pressure Theory

As described in the previous sections, the Rankine earth pressure theory:

(1) Assumes the retaining wall is frictionless (or smooth), and

(2) Considers stress states and uses such tools as the Mohr's circle of stress.

Coulomb (1776) proposed a theory to determine the lateral earth pressure on a retaining wall by assuming a granular backfill ($c = 0$) and a plane sliding surface. He did this in order to simplify somewhat the mathematically complex problem introduced when cohesion and nonplane sliding surfaces are considered. He, however, account for the effects of friction (usually expressed by angle δ) between the backfill and the wall. Besides, he considered the more general case of the sloped face of a retaining wall, and in this respect, Coulomb's theory is a more general approach than the Rankine theory described earlier.
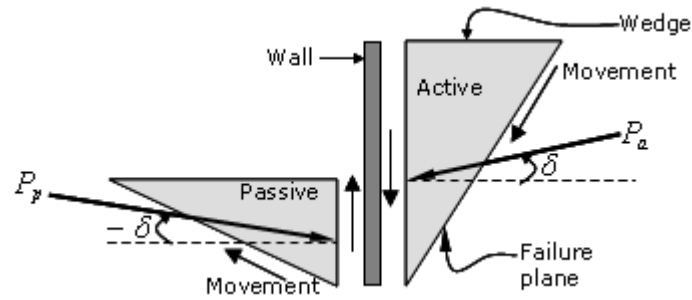
Figure 7.8: Direction of active and passive forces when wall friction is present.

Coulomb assumed a wedge shape collapse mechanism which is bounded by the face of the retaining wall, a horizontal or inclined ground surface and a linear failure plane. The wedge slides downwards on the failure plane in the active state or upwards in the passive state. Figure 3.8 illustrates direction of active and passive forces when wall friction is present.
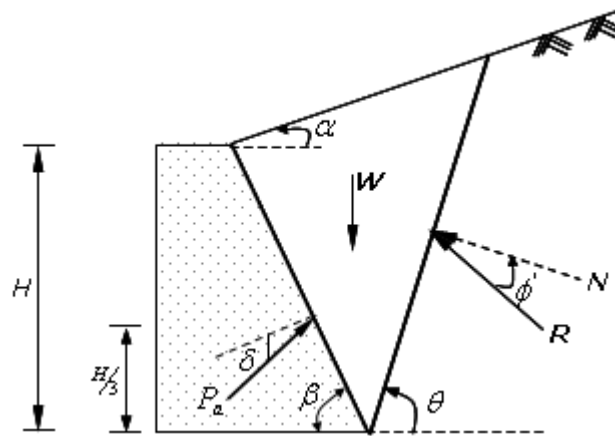
Figure 7.9: Retaining wall with slopping back, wall friction, and sloping soil surface for use with Coulomb's method for active state.

Based on Coulomb's theory, a condition of limit equilibrium exists through which a wedge of a soil mass behind a retaining wall will slip along a plane inclined at an angle θ to the horizontal. Figure 7.9 illustrates a retaining wall with slopping back, wall friction, and sloping soil surface for use with Coulomb's method for active state. Based on the equilibrium of the forces acting on the wedge (Fig. 7.9), Coulomb proposed the following equation to determine the active lateral force,

$$P_a = \tfrac{1}{2}k_{ac}\gamma'H^2 \tag{7.32}$$

Where, k_{ac} is Coulomb's active pressure coefficient, which is determined by the following expression.

$$k_{ac} = \frac{\sin^2(\beta + \phi')}{\sin^2\beta\sin(\beta - \delta)\left[1 + \sqrt{\dfrac{\sin(\phi' + \delta)\sin(\phi' - \alpha)}{\sin(\beta - \delta)\sin(\alpha + \beta)}}\right]^2} \tag{7.33}$$

Note that the line of action of the active force P_a will act at a distance *H/3* above the base of the wall and will be inclined at angle δ to the normal drawn to the back of the wall. In the actual design of retaining walls, the value of the wall friction angle, δ is assumed to be between $\phi'\!/2$ & $\frac{2}{3}\phi'$. Retaining walls are generally constructed of masonry or mass concrete. Table 7.1 shows the general range of the values of δ for various backfill materials.

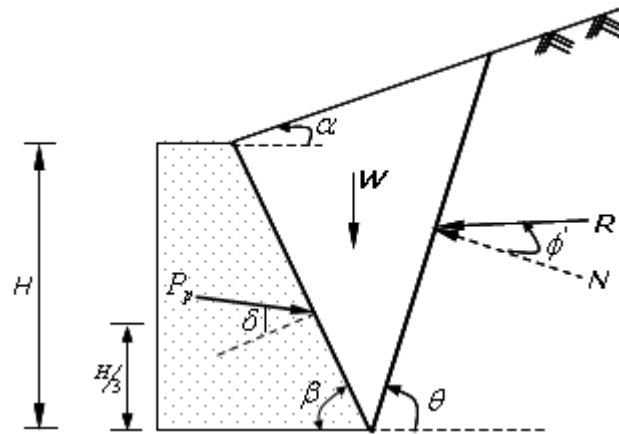**Table 7.1**: General range of wall friction angle for masonry or mass concrete walls

Backfill material	Range of δ in degrees
Gravel	27 – 30
Course sand	20 – 28
Fine sand	15 – 25
Stiff clay	15 – 20
Silty clay	12 – 16

Coulomb's passive earth pressure is determined similarly, except that passive pressure inclination at the wall and direction of the forces acting on the wedge will be as shown in Fig. 3.10. Coulomb's passive earth pressure is given by:

$$P_p = \tfrac{1}{2}k_{pc}\gamma'H^2 \tag{7.34}$$

Where, k_{pc} is Coulomb's passive pressure coefficient, which is determined by the following equation.

$$k_{pc} = \frac{\sin^2(\beta - \phi')}{\sin^2\beta\sin(\beta + \delta)\left[1 - \sqrt{\dfrac{\sin(\phi' + \delta)\sin(\phi' + \alpha)}{\sin(\beta + \delta)\sin(\alpha + \beta)}}\right]^2} \tag{7.35}$$
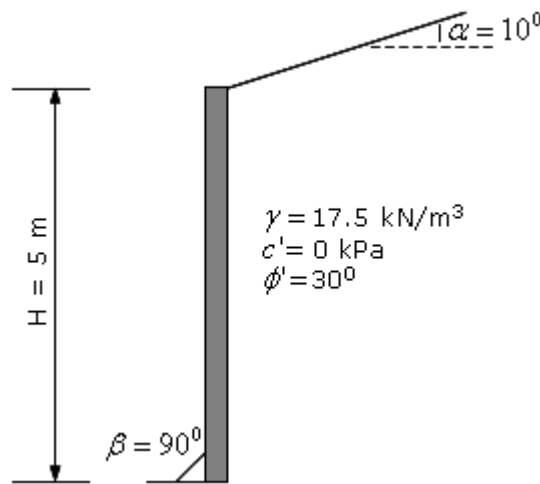
Figure 7.10: Retaining wall with slopping back, wall friction, and sloping soil surface for use with Coulomb's method for passive state.

EXAMPLE 7.5

Using the Coulomb equation, what is the total active force per meter of wall for the soil-wall system shown in Figure E7.4?

And where does P_a act?

Figure: E7.4