CHAPTER 4: EARTHWORK AND QUANTITIES

The term earthwork includes all clearing, grubbing, roadway and drainage excavation, excavation for structures, embankments, borrow, overhaul, machine grading, subgrade scarifying, rock fill, and all the operations of preparing the subgrade foundation for highway or runway pavement.

The quantity and cost of earthwork are calculated in terms of cubic meters of excavation in its original position on the basis of cross-section notes from field measurements.

Modern grading operations are carried on by power equipment including power shovels, scrapers, bulldozers, blade graders, rollers, dragline excavators, motor trucks, tractors, etc.

Classification of Excavated Material

Excavated material is usually classified as (1) common excavation, (2) loose rock, or (3) solid rock. Common excavation is largely earth, or earth with detached boulders less than **½ cu yd.** Loose rock usually refers to rock which can be removed with pick and bar, although the use of power shovels or blasting may be advantageous. Solid rock comprises hard rock in place and boulders that can be removed only by the use of drilling and blasting equipment.

Shrinkage and Swell Factors

When earth is excavated and hauled to form an embankment, the freshly excavated material generally increases in volume. However, during the process of building the embankment it is compacted, so that the final volume is less than when in its original condition. This difference in volume is usually defined as “shrinkage”. In estimating earthwork quantities, it is necessary to make allowance for this factor. The amount of shrinkage varies with the soil type and the depth of the fill. An allowance of 10 to 15 percent is frequently made for high fills and 20 to 25 percent for shallow fills. The shrinkage may be as high as 40 or 50 percent for some soils. This generally also allows for shrinkage due to loss of material in the hauling process.

When rock is excavated and placed in the embankment, the material will occupy a larger volume. This increase is called " swell" and may amount to 30 percent or more.

Percent shrinkage = (1 – (wt. bank measure / wt. compacted)) *100

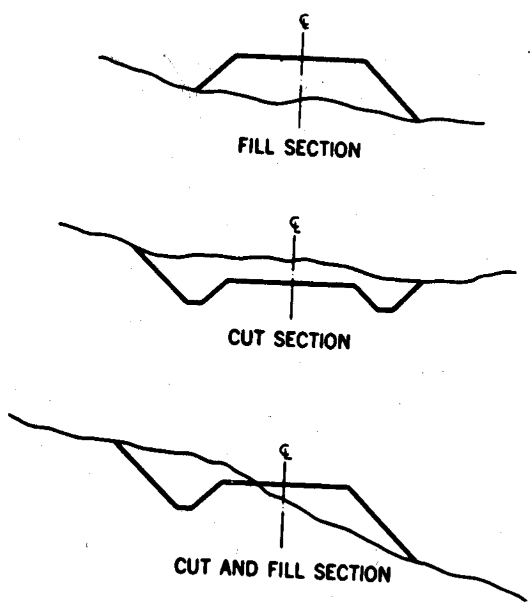
% sh. = $(1 - (\gamma_B / \gamma_C)) * 100$ …………………………………………..(1)

Percent swell = ((wt. bank measure / wt. loose measure) – 1) * 100

% sw. = $((\gamma_B / \gamma_L) - 1) * 100$ …………………………………………..(2)

Cross-Sections and Templates

In order to determine earth excavation and embankment requirements by manual means, a section outline of the proposed highway, commonly referred to as a template section, is placed on the original ground cross-section; the areas in cut and the areas in fill are determined; and the volumes between the sections are computed. Figure 4.1 shows various conditions that may be encountered when plotting these template sections. "Cut" and "fill" are the terms that are usually used for the areas of the sections, and the terms "excavation" and "embankment" generally refer to volumes.



Each cross-section should show the location or station of the original ground section and template section, the elevation of the proposed grade at the station, and the areas of cut and fill for each section. The computed volume of excavation and embankment may also be placed on the cross-section sheet between two successive cross-sections.

Figure 4.1 Original ground line and template sections

Slope Stakes

The final grade line having been established, slope stakes are set at points where the side slopes of the graded road will intersect the ground surface; they mark the limits of the excavation and embankment. The slope stakes are driven at points of zero cut or fill, but the numbers written on them give the vertical distance with reference to the finished grade of the subgrade. On the inner side of the stakes is marked in meters the "cut" or "fill" as the case may be. Thus, "C1.2" indicates that the centerline elevation of the roadbed is to be cut 1.2m below the ground at the slope stake; and "F2.3" indicates that the fill is to be 2.3m above the slope stake.

The stakes are driven with the tops slanting outward and with the sides upon which the cuts or fills are marked facing the roadway. The station number is marked on the outside of the stake. The usual equipment for setting slopes consists of a level, rod, tape, notebook, stakes, and keel.

If a profile map has been established from previous surveys and the final grade line drawn thereon, the centerline cut or fill may be found from the map by subtracting the elevation of grade from the elevation of ground.
Also, if a cross-section of the ground and finished shape of roadway have been plotted to scale, the position of the slope stakes could be scaled from the map.

Aside from the elevation of grade, the most satisfactory procedure is to determine all distances and elevations in the field. The method must necessarily be a "measure and try" process, but the work can be done rather easily.

General Method of Procedure: Let ABEPD in Figs. 4.2 and 4.3 represent the cross-sectional area of highway in fill or cut, for which we have the following general notation applying to both excavation and embankment:

b = AB = AC + CB = width of roadbed

s = "slope ratio" for the banks AD and BE = ratio of horizontal to vertical (plus for cut, minus for fill)

d = PC = depth of fill or cut at the center

x_1 = horizontal distance from P to slope stake at D

x_2 = horizontal distance from P to slope stake at E

y = vertical distance from P to slope stake

$h_1 = d + y_1$ = FD = vertical distance from C to D

$h_2 = d + y_2$ = GE = vertical distance from C to E

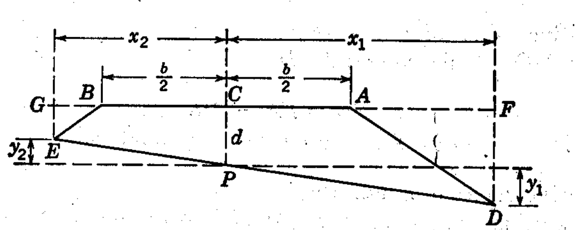


Figure 4.2 Cross section on fill (slope stakes at E and D)

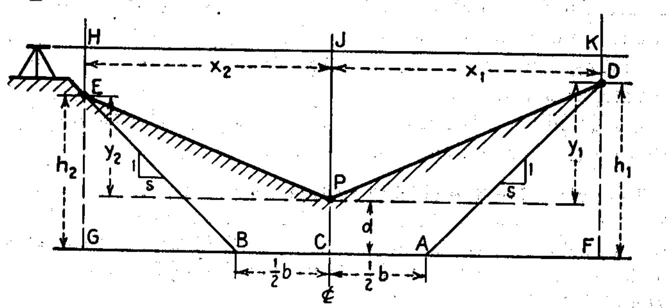


Figure 4.3 Cross section in cut (slope stakes at E and D)

The slope stake at point D on the right is correctly established if

$$x_1 = ½\, b + sh_1 = ½\, b + sd + sy_1 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Likewise, point E on the left is correctly established if

$$x_2 = ½\, b + sh_2 = ½\, b + sd + sy_2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

In the foregoing equations, b, d, and s are known, while x and h (or y) are measured and remeasured in the field until the equations are satisfied (trial-and-error method).

If the ground is level, $y_1 = y_2 = 0$; then $x_1 = x_2 = ½\, b + sd$.

In fig. 4.3, it is assumed that rod readings to all points within the cross profile under consideration can be taken from a single position of the level (which of course is not always possible). By means of previous differential leveling from the nearest benchmark, the HI (height of instrument or elevation of line HJK) is established.

If we imagine the bottom of the road to be at grade at point C, the rod reading would be CJ, which is called "grade rod". That is

$$\text{Grade rod} = \text{HI} - \text{grade elevation}$$

Negative values of grade rod would occur when the HI is below "grade".

Since the actual reading at the centerline stake (fig. 4.3) is PJ (not CJ), the depth of cut is CP = CJ – PJ; that is,

$$d = \text{grade rod} - \text{ground rod}$$

Negative values of d indicate fill.

Similarly at the slope stake D,

$$FD = FK - DK$$

That is, h_1 (or h_2) = grade rod – ground rod

Negative values of h indicate fill.

The values of x_1 and h_1 (or x_2 and h_2) corresponding to the intersection of two slopes are found easily after two or three field measurements.

Area of Cross-Section

From the data supplied by slope stake or cross-section notes, the area of cross-section may be calculated. If the ground is level or regular, simple geometry may be applied; for irregular ground, two general methods are used; (1) the graphical method and (2) the coordinate method.

- **Area for Level Ground.** For level ground, the area of cross-section in cut (or fill) is merely that of a trapezoid. In figure 4.4:

  b = width of base AB

  d = center cut (or fill)

  s = slope of banks = MD / AM = NE / BN

Hence,

Area = d(b + sd) ……………………………………..(5)

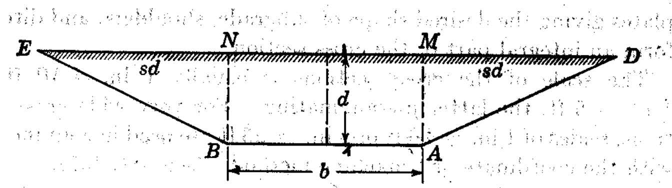


Figure 4.4

- **Area for Three-Level Section.** With three readings taken directly from slope stake notes, one at the center and one at each slope stake, the area of cross section may be obtained. For regular ground, this is an accurate and very satisfactory method. Such a section is known as a three-level section, and the area may be calculated readily from field notes without plotting.

  Imagine the area ABED (fig. 4.5) to be divided into four triangles, two having the common base d and altitudes x_1 and x_2, and two having bases = ½ b and altitudes h_1 and h_2. Hence the area of section is

$$A = \frac{1}{2}\left[d(x_1 + x_2) + \frac{1}{2}b(h_1 + h_2)\right] \quad \text{………………………….(6)}$$

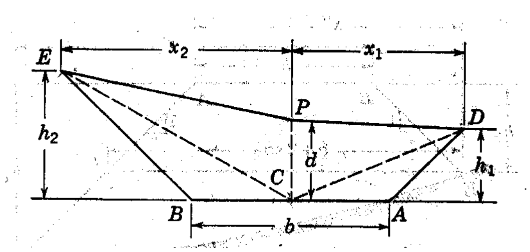


Figure 4.5

If the slope stake notes are not available and the center fill is known, the end area A may be found conveniently from the four field measurements indicated in fig. 4.6. Assuming uniform slope of the original ground underneath the fill,

$$A = \frac{1}{2}(h_1x'' + h_2x') \quad \text{………………………………….(7)}$$

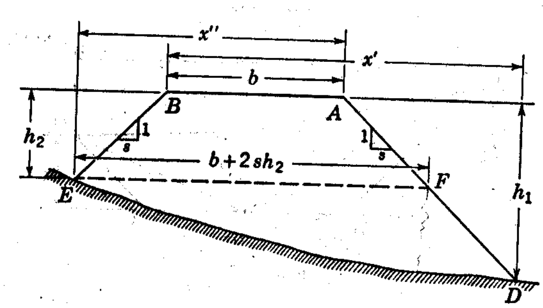


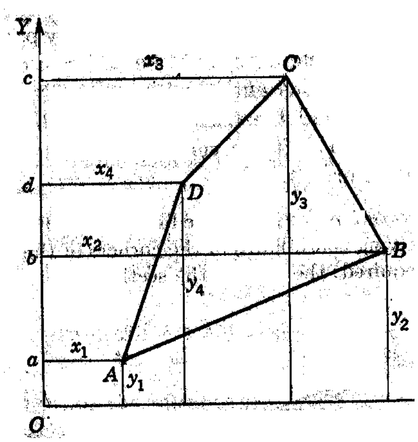
Figure 4.6

- **Area by Coordinate Method.** With the coordinates of all the corners of a cross-section known, the end area may be computed by means of the coordinate method.

Let the corners A, B, C, and D of the area ABCD (fig. 4.7) be located by the coordinates (x_1,y_1), (x_2,y_2), (x_3,y_3), and (x_4,y_4). Then the area is given by the algebraic sum of four trapezoids. Thus,

Area = ABba + BCcb – ADda - DCcd

$$=1/2\ [y_1(x_4 - x_2) + y_2(x_1 - x_3) + y_3(x_2 - x_4) + y_4(x_3 - x_1)] \quad \ldots\ldots\ldots\ldots\ldots(8)$$



From eqn.(8) we may state the following rule for area:

Multiply each ordinate by the algebraic difference between the adjacent abscissas, find the algebraic sum of the products, and then take half of this

Figure 4.7 Area by coordinate method

A simpler rule for area follows if we arrange in counterclockwise order the coordinates (fig. 4.7) in the form of fractions, the initial fraction (beginning at any corner) being repeated to give a closed boundary. Thus, we have

$$\frac{y_1}{x_1} \diagup \frac{y_2}{x_2} \diagup \frac{y_3}{x_3} \diagup \frac{y_4}{x_4} \diagup \frac{y_1}{x_1}$$

Multiply along the marked diagonals and add the products (all positive); multiply along the unmarked diagonals and add the products (all negative). The difference gives the double area.

Volume of Earthwork

The volume of earthwork may be found by means of either the **average end area** or the **prismoidal** formula. Although the former is less exact than the latter, it is generally accepted as the standard earthwork formula, on account of its simplicity.

- **Average End Area Formula.** The volume of a right prism equals the average area multiplied by the length. Assuming the average area to be the same as the average end area,

  Volume = V = ½ $(A_1 + A_2)L$

  In which: A_1 and A_2 = area of end sections (m^2)
  L = length of solid (m)

  This formula is applied to areas of any shape, but the results are slightly too large. The error is small if the sections do not change rapidly.

- **Prismoidal Formula.** A prismoid is a solid whose ends are parallel and whose sides are plane or wrapped surfaces. Fig. 4.8 represents a typical prismoid.

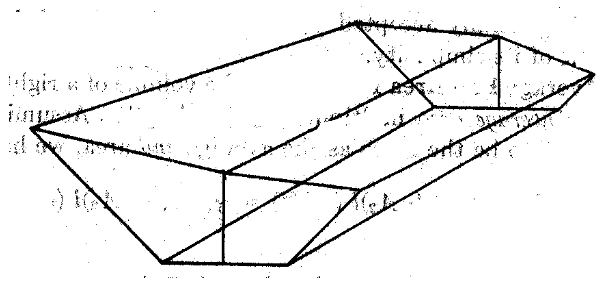


Figure 4.8

The volume of a prismoid is:

$$V = L/6\ (A_1 + 4A_m + A_2)$$

In which L is the distance between the two parallel bases A_1 and A_2 and A_m is a section midway between the two end bases and parallel to them. A_m is not an average of A_1 and A_2, but each of its linear dimensions is an average of the corresponding dimensions of A_1 and A_2.

Haul and Overhaul

In grading contracts for roads it is usually stipulated that the contractor shall be paid a certain price per cubic meter for excavating, hauling, and dumping the material, regardless of distance hauled, provided it does not exceed a specified limit called **free haul**. The free haul distance may be as low as 150m and as high as 900m or more.

If there is an **overhaul** on some of the material, that is, if the distance from excavation to embankment is beyond the free haul limit, then an extra charge may be allowed.

A mass diagram is helpful in determining the amount of overhaul and the most economical distribution of the excavated material.

Limit of Economic Haul

When there are long hauls, it may be more economical to waste and borrow materials rather than pay for the cost of overhauling. Equating the cost of excavation plus overhaul to the cost of excavation from both the roadway and borrow pit, one can estimate the limit of economic haul for making the embankment. Thus, let

c = cost of roadway excavation per cubic meter

b = cost of borrow per cubic meter

h = cost of overhaul, on the bases of $1m^3$ per station

x = economical length of overhaul

Cost to excavate and move $1m^3$ material from cut to fill

$$= c + hx \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(a)}$$

Cost to excavate from cut, waste, borrow, and place $1m^3$ material in fill

$$= b + c \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(b)}$$

Equating equations (a) and (b) and solving for x, we have

$$x = b/h\ (s + a) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(c)}$$

adding the free haul distance to equation (c), we get the limit of economical haul.

Mass Diagram

A mass diagram is a graphical representation of the amount of earthwork and embankment involved in a project and the manner in which the earth is to be moved. Its horizontal or x-axis represents distance and is usually expressed in meters or stations. It is drawn to the same horizontal scale as the profile. The vertical or y-axis represents the cumulative quantity of earthwork in cubic meters. The quantity of excavation on the mass diagram is considered positive, and embankment as negative. Preliminary to drawing the mass curve it is convenient to tabulate the cumulative volumes of cuts and fills at each station.

The mass diagram allows a highway engineer to determine direction of haul and the quantity of earth taken from or hauled to any location. It shows "balance points", the stations between which the volume of excavation (after adjustment for "shrinkage" or "swell") and embankment are equal.

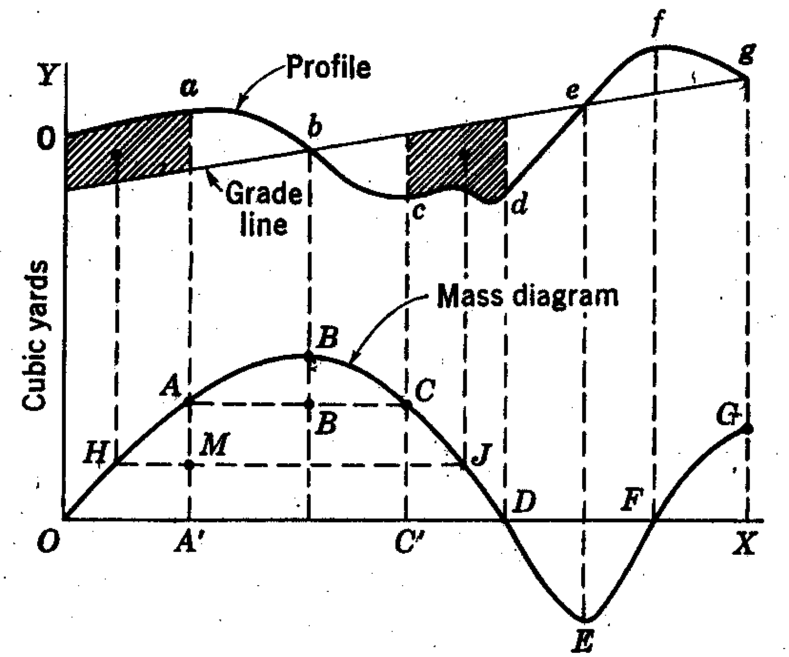


Figure 4.9: Profile and mass diagram. AC = free-haul distance; HJ – AC = overhaul distance; BB = free-haul volume; A'A = C'C = overhaul volume; OD = length of balance.

A study of the mass diagram (or curve) shown in figure 4.9 will verify the following statements:

- The ordinate at any point on the mass curve represents the cumulative volume to that point on the profile.
- Within the limits of a single cut, the curve rises from left to right; within the limits of a single fill, it falls from left to right.
- Sections where the volume changes from cut to fill correspond to a maximum; sections where the volume changes from fill to cut correspond to a minimum. Evidently the maximum and minimum points on the diagram occur at, or near, grade points on the profile.

- Any horizontal line, as AC, cutting off a loop of the mass curve, intersects the curve at two points between which the cut is equal to the fill (adjusted for shrinkage). Such a line is called a balance line.
- The loops convex upward indicate that the haul from cut to fill is to be in one direction (to the right in this case); loops concave upward indicate a reverse direction of haul.
- The final point on a mass diagram for a given project gives the overall net amount of earthwork for the entire project. This amount, if positive, would indicate a surplus of excavation material and a need to waste that quantity of material. If the final point on the mass diagram is a negative amount, it indicates a net shortage of earthwork for the project and a need to borrow that quantity of earthwork material.

Determination Of Overhaul From The Mass Diagram: One of the important uses of the mass diagram, aside from balancing cuts and fills and indicating the most advantageous distribution of the same, is to establish definitely the overhaul distance and the portion of the total volume which is to be regarded as hauled beyond the specified free-haul limit.

Referring to figure 4.9, proceed as follows:

i. Assuming the free-haul distance to be 150m, find by trial a horizontal line intersecting the curve at points A and C, such that AC = 150m. Then the material above line AC will be hauled at no extra cost. The amount of this material is given by the ordinate from line AC to point B and is a measure of the volume in cut from a to b, which makes the fill from b to c.

ii. Consider now the volume above the balance line OD. A study of the mass curve and the corresponding profile shows that the cut from o to b will make the fill from b to d. But since part of this solidity, the part above the balance line AC, is included in the free-haul limit, the other part between lines OD and AC – which is measured by the ordinate A’A – is subject to overhaul unless wastage and borrow take place. That is, some or all of the volume from o to a may be “overhauled” to make the fill from c to d.

The average length of haul of the solidity from o to a to make the fill from c to d is the distance between the centers of gravity of cut o to a and fill c to d. The gravity lines are found as follows: Bisect AA’ at M and draw a horizontal line intersecting the mass curve at H and J. These points H and J are assumed to be vertically below the desired centers of gravity. Therefore the average haul is given by the length of line HJ, and the overhaul is this distance HJ less the free haul distance AC. The overhaul distance (in stations) multiplied by the net volume gives the station-volumes of overhaul.

It should be note that the foregoing graphical method of determining the center of gravity of the masses in cut and fill is inaccurate when there is abruptness in the mass curve. In such cases, a more accurate method is to divide the volume in parts and take moments about a vertical line of reference just as is done in finding the center of gravity of a system of forces.

The mass diagram may be used to indicate the most economical procedure for disposing of excavated material, what part of it should be moved forward or backward, and whether borrowing and wasting are advisable. Thus if the balance line OD is continued horizontally to point X, it will be seen that the cuts and fills from o to f are balanced, but the solidity represented by the ordinate at G is excess cut (from f to g) which may be carried forward, backward, or wasted. If the project ends at point g or if there are no fills immediately ahead, then this excavated material should be carried backward to help make the fill from b to c (it being downhill and within the free-haul limit), while an equivalent amount of volume from the cut o to a would be wasted, thus reducing the station-volume of overhaul.

EXAMPLES: <u>Computing Fill and Cut Volumes Using the Average End-Area Method</u>
A roadway section is 1400 m long (14 stations). The cut and fill volumes are to be computed between each station. The following Table lists the station numbers (column 1) and lists the end area values (m^2) between each station that are in cut (column 2) and that are in fill (column 3). Material in a fill section will consolidate (known as shrinkage), and for this road

section, is 10 %. Determine the net volume of cut and fill, Mass Diagram Ordinate and plot the results.

Solution: For instance calculate the cut and fill volume between 0 and 1.

$$V_{cut} = L/2(A_{0C} + A_{1C}) = 100/2(3+2) = 250\ m^3$$

$$V_{fill} = L/2(A_{0F} + A_{1F}) = 100/2(18+50) = 3400\ m^3$$

Since, the materials always consolidate in fill state. So,

$$\text{Shrinkage} = V_{fill} * 10\% = 3400*0.1 = 340\ m^3$$

Therefore, Total fill volume required b/n 0 and 1 = 3400+340 = 3740 m^3 and

Total cut volume b/n 0 and 1 = 250 m^3

Net volume between stations 0-1 = Total cut - Total fill = 250-3740 = -3490 m^3

Note**:** Net fill volumes are negative (-) (column 8) and net cut volumes are positive (+) (column 9). Similar calculations are performed between all other stations, to obtain the remaining cut or fill values shown in columns 2 through 9.

Computing Ordinates (Volumes) of the Mass Diagram

The mass diagram is a series of connected lines that depicts the *net* accumulation of cut or fill between any two stations. The ordinate of the mass diagram is the net accumulation in cubic meters (m^3) from an arbitrary starting point. Thus, the difference inordinate between any two stations represents the net accumulation of cut or fills between these stations. If the first station of the roadway is considered to be the starting point, then the net accumulation at this station is zero. Use the tabulated data in the table to determine the net accumulation of cut or fill beginning with station 0+00, and Plot the results.

Hence, Columns 8 and 9 show the net cut and fill between each station. To compute the mass diagram ordinate between station *X* and *X* +1, add the *net* accumulation from Station *X* (the first station) to the net cut or fill volume (columns 8 or 9) between stations *X* and *X*+1. Enter this value in column 10.

Station 0 + 00 mass diagram ordinate = 0

Station 1 + 00 mass diagram ordinate = 0 – 3490 = - 3490 m^3

Station 2 + 00 mass diagram ordinate = -3490 - 7885 = - 11,375 m^3

Station 3 + 00 mass diagram ordinate = -11375 - 12185 = - 23,560 m^3 and Continue the calculation process for the remaining 11 stations (4+00 to 14+00).

A plot of the results is shown in a simple sketched figure below.

Interpretation of the Mass Diagram

1. When the mass diagram slopes downward (negative), the preceding section is in fill, and when the slope is upward (positive), the preceding section is in cut.
2. The difference in mass diagram ordinates between any two stations represents the *net accumulation* between the two stations (cut or fill).
3. A horizontal line on the mass diagram defines the locations where the net accumulation between these two points is zero. These are referred to as "balance points," because there is a balance in cut and fill volumes between these points. In Figure below, the "*x*" axis represents a balance between points *A* and *B* and a balance between points *B* and *C*. Beyond point *C*, the mass diagram indicates a Cut condition for which there is no compensating Fill. For this section, imported material (called borrow) will not be purchased from an off-site location and almost the overall Excavation cost will be safe.

END AREA (m^2)			**VOLUME (m^3)**				**NET VOLUME** 4-7		10 **Mass Diagram Ordinate**
1 Stations	**2 Cut**	**3 Fill**	4 Total Cut	5 Fill	6 Shrinkage 10%	7 Total Fill (5+6)	8 Fill (-)	9 Cut (+)	
0	**3**	**18**	250	3400	340	3740	3490	-	0
1	**2**	**50**	200	7350	735	8085	7885	-	- 3490
2	**2**	**97**	300	11350	1135	12485	12185	-	- 11,375
3	**4**	**130**	600	9050	905	9955	9355	-	- 23,560
4	**8**	**51**	2400	4800	480	5280	2880	-	-32,915
5	**40**	**45**	4250	3250	325	3575	-	675	-35,795
6	**45**	**20**	6250	1250	125	1375	-	4875	-35,120
7	**80**	**5**	10100	350	35	385	-	9715	-30,245
8	**122**	**2**	12600	100	10	110	-	12490	-20,530
9	**130**	**0**	13500	0	0	0	-	13500	-8,040
10	**140**	**0**	12000	150	15	165	-	11835	5,460
11	**100**	**3**	9000	1650	165	1815	-	7185	17,295
12	**80**	**30**	4000	7500	750	8250	4250	-	24,480
13	**0**	**120**	150	12000	1200	13200	13050	-	20,230
14	**3**	**120**	-	-	-	-	-	-	7180

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- A roadway section is 1 KM long between stations 351+00 to 352+00. The following Table shows the tabulated end areas from the earthwork site. If the material shrinks 12%,

    a. Determine the net volume of cut and fill,

    b. Mass Diagram Ordinate and

    c. Plot the results.

STATION	END AREAS(m^2)	
	CUT	FILL
351+000		57.93
351+050		52.28
351+125		23.58
351+250	8.4	3.73
351+375	13.8	
351+500	33.34	
351+650	39.45	
351+900	47.50	
352+000	55.75	