# CHAPTER 3

## 3. Moment Distribution Method

### *3.1 Introduction*

The moment-distribution method can be used to analyze all types of statically indeterminate beams or rigid frames. Essentially, it consists in solving the linear simultaneous equations that were obtained in the slope-deflection method by successive approximations or moment distribution. Increased number of cycles would result in more accuracy. The iteration is stopped when, at all joints, the out of balance moment is a negligible value.

In moment distribution method; fixed end moment, stiffness, distribution and carry-over factors are of great importance and used frequently.

*Fixed-end moment:* The moment induced at member ends due to applied load (in-span load).

*Stiffness factor:* It is defined as the moment required to be applied at a joint to produce unit rotation at that joint.

*Distribution factor*: It is the ratio of the stiffness factor of the member being considered to the sum of the stiffnesses of all the members meeting at that particular continuous joint.

*Carry-over factor:* The constant (1/2) obtained in the slope-deflection equation is carry-over factor.

*Calculation of distribution factors*

Consider a simple structure shown below.

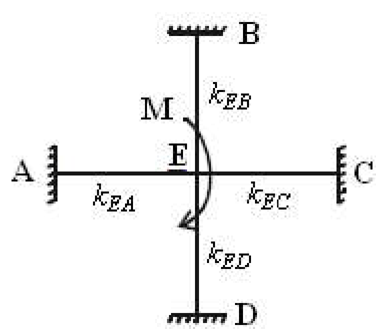


Fig. 3.1

where: $k_{EA}$, $k_{EC}$, $k_{EB}$, $k_{ED}$ are stiffness factors of members EA, EC, EB and ED respectively.

$$k_{ij} = \frac{4EI_{ij}}{L_{ij}} \quad \text{if the far end is fixed}$$

$$k_{ij} = \frac{3EI_{ij}}{L_{ij}} \quad \text{if the far end is pinned}$$

For the equilibrium requirements at the joint, it is obvious that the summation of moments ($\Sigma M$) should be zero at the joint. This means that the applied moment '$M$' will be distributed in all the members meeting at that joint in proportion to their stiffness factor. (This called stiffness – concept)

The distribution factor $DF_i$ of a member connected to any joint $j$ is:

$$DF_i = \frac{k_i}{\sum_{j=1}^{n} k_j} \leq 1 \qquad (3.1)$$

Note that $DF_i$=0 for fixed end and $DF_i$=1 for pinned end.

**where** $n$ :total number of members at joint $j$

$k_i$ :the stiffness of member $i$ connected to joint $j$

### *3.2 Moment distribution method for beams and non-sidesway frames*

Application of the moment distribution method for beams and frames having no sidesway follow the same procedure. Steps involved in moment distribution method:

- Calculate fixed end moments due to applied loads
- Calculate relative stiffness.
- Determine the distribution factors for various members framing into a particular joint.
- Distribute the net fixed end moments at the joints to various members by multiplying the net moment by their respective distribution factors in the first cycle.
- In the second and subsequent cycles, carry-over moments from the far ends of the same member (carry-over moment will be half of the distributed moment). Distribute these moments just "carried over"; Each cycle consists of two steps:
  - → distribution of out of balance moments,
  - → calculation of the carry over moment at the far end of each member
- Consider this carry-over moment as a fixed end moment and determine the balancing moment. This procedure is repeated from second cycle onwards till convergence
- Add all moments - fixed-end moments, distributed moments, moments carried over - at each end of each member to obtain the true moment at the end

### *3.3 Moment distribution method for sidesway frames*

In the case of sidesway frames, the analysis consists of two cases. First, the moment distribution is carried out without considering sway of the frame. This is done by fixing the joint to obtain non-sway moments, $M_o$. Second; the frame is allowed to sway by releasing the previously restrained joint. For a certain assumed value of δ (the members to be displaced) obtain the corresponding sway moments, $M_s$.

For illustration purpose, consider the frame shown below.

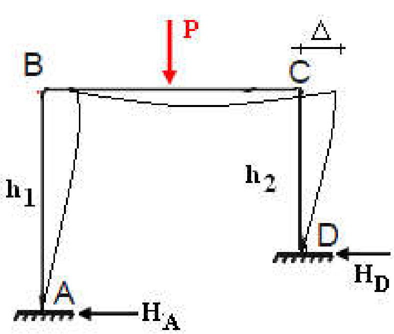


Fig. 3.2 (a) Actual frame

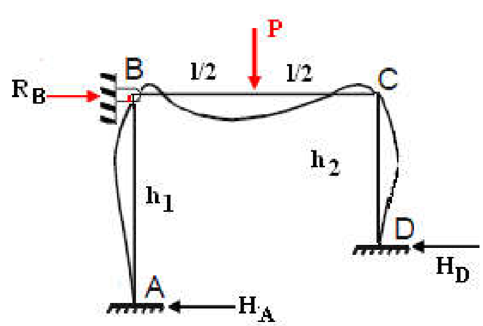


Fig. 3.2 (b) Joint B is restrained

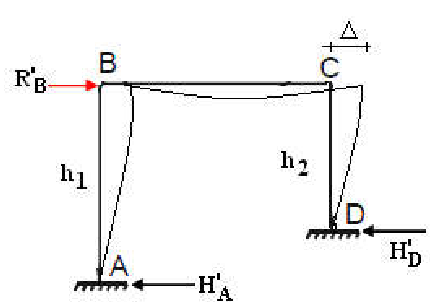


Fig. 3.2 (c) Joint B is released

**Case 1** (Fig. 3.2 (b)): The frame is a non sway frame since joint B is restrained. The moments, $M_o$, is obtained by following the same procedure as discussed for non-sway frames.

$$\begin{aligned} &\Sigma M_B = 0 \rightarrow M_{BA} + M_{BC} = 0 \\ &\Sigma M_C = 0 \rightarrow M_{CB} + M_{CD} = 0 \quad \text{and} \\ &\Sigma F_x = 0 \rightarrow H_A + H_D = R_B \end{aligned} \tag{3.2}$$

From FBD of member AB, $\Sigma M_B = 0 \rightarrow H_A = \dfrac{M_{BA} + M_{BC}}{h_1}$ and

From FBD of member CD, $\Sigma M_c = 0 \rightarrow H_D = \dfrac{M_{CD} + M_{CB}}{h_2}$

**Case 2** (Fig. 3.2 (c)): Release joint B and compute the corresponding sway moment, $M_s$. In this case, the FEM are computed for a certain assumed value of δ.

For member AB, $M^F_{BA} = M^F_{AB} = \dfrac{6EI\delta}{L^2}$; $L = h_1$

$$\begin{aligned} &\Sigma M_B = 0 \rightarrow M_{BA} + M_{BC} = 0 \\ &\Sigma M_C = 0 \rightarrow M_{CB} + M_{CD} = 0 \quad \text{and} \\ &\Sigma F_x = 0 \rightarrow H'_A + H'_D = R'_B \end{aligned} \tag{3.3}$$

From FBD of member AB, $\Sigma M_B = 0 \rightarrow H'_A = \dfrac{M_{BA} + M_{BC}}{h_1}$ and

From FBD of member CD, $\Sigma M_c = 0 \rightarrow H'_D = \dfrac{M_{CD} + M_{CB}}{h_2}$

The actual lateral displacement of the frame is $x$ times the assumed value, $\Delta = x\delta$, where $x$ is the sway correction factor. For the actual frame shown in Fig. 3.2 (a), summation of horizontal forces at joint B is zero.

$$\Sigma F_x = 0 \rightarrow R_B + xR'_B = 0 \quad \rightarrow x = -R_B / R'_B \tag{3.4}$$

The final moment for various members will be the sum of non-sway and sway moments, and it is given by:

$$M_{ij} = (M_{ij})_o + x(M_{ij})_s \tag{3.5}$$

### *3.4 Moment distribution for multistory frames*

Multistory frames may have more joint displacements and require more computations. To analyze such structures, the same procedure as that of single framed structures is used.