CHAPTER 1

1. General Introduction

1.1. Static and Kinematic Indeterminacy of Structures

The aim of structural analysis is to evaluate the external reactions, the deformed shape and internal stresses in the structure. If this can be determined strictly from equations of equilibrium, then such structures are known as determinate structures. However, in many structures, it is not possible to determine either reactions or internal stresses or both using equilibrium equations alone, because the structures having more unknown forces than available equilibrium equations such structures are known as the statically indeterminate structures. Static indeterminacy may be internal or external (or both), depending on the redundancy. The total number of releases required to make a structure statically determinate is called the degree of statical indeterminacy.

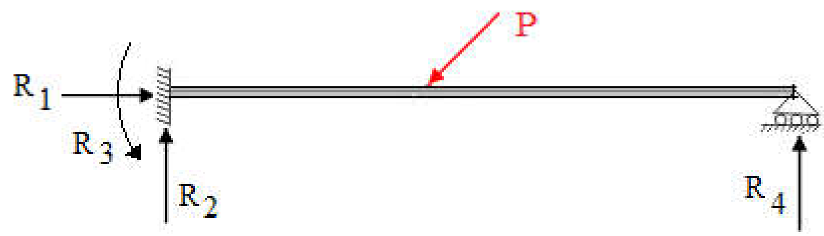


Fig 1.1 Statically indeterminate structure

For instance, the beam shown in Fig.1.1 has four reaction components, whereas we have only 3 equations of equilibrium. Hence the beam is externally indeterminate to the first degree.

Advantages and disadvantages of indeterminate structures

The advantages of statically indeterminate structures over determinate structures include the following.

- ***Smaller Stresses***- the maximum stresses in statically indeterminate structures are generally lower than those in comparable determinate structures.
- ***Greater Stiffnesses***- Statically indeterminate structures generally have higher stiffness's (i.e., smaller deformations), than those of comparable determinate structures.
- ***Redundancies***- Statically indeterminate structures, if properly designed, have the capacity for redistributing loads when certain structural portions become overstressed or collapse in cases of overloads due to earthquakes, impact (e.g. vehicle impacts), and other such events.

The following are some of the main disadvantages of statically indeterminate structures, over determinate structures.

- ***Stresses Due to Support Settlements*** - Support settlements do not cause any stresses in determinate structures; they may, however, induce significant stresses in indeterminate structures, which should be taken into account when designing indeterminate structures.
- ***Stresses Due to Temperature Changes and Fabrication Errors***- Like support settlements, these effects do not cause stresses in determinate structures but may induce significant stresses in indeterminate ones.

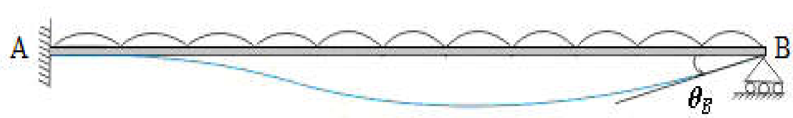
Kinematic Indeterminacy of structures

When the structure is loaded, the joints undergo displacements in the form of translations and rotations. In the displacement-based analysis, these joint displacements are treated as unknown quantities.

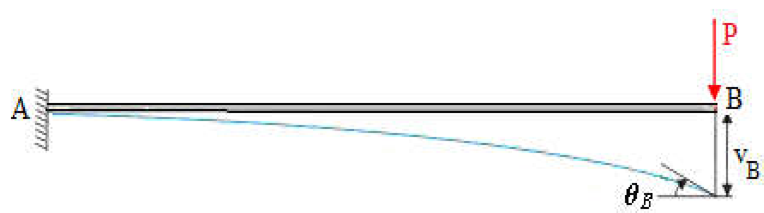
The joint displacements in a structure is treated as independent if each displacement (translation and rotation) can be varied arbitrarily and independently of all other displacements. The number of independent joint displacement in a structure is known as the degree of kinematic indeterminacy or the number of degrees of freedom.

Consider a propped cantilever beam shown in Fig. 1.2 (a). The displacements at a fixed support are zero. Hence, for a propped cantilever beam, we have to evaluate only rotation at B and this is known as the kinematic indeterminacy of the structure. A fixed-fixed beam is kinematically determinate but statically indeterminate to the 3rd degree. A simply supported beam and a cantilever beam shown in Fig. 1.2 (a) & (b) are kinematically indeterminate to 2nd degree.

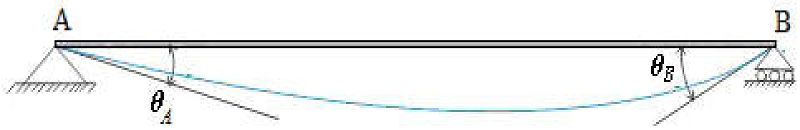
Remark: - Usually, the axial rigidity of the beam is so high that the change in its length along axial direction may be neglected.



(a) Propped Cantilever Beam



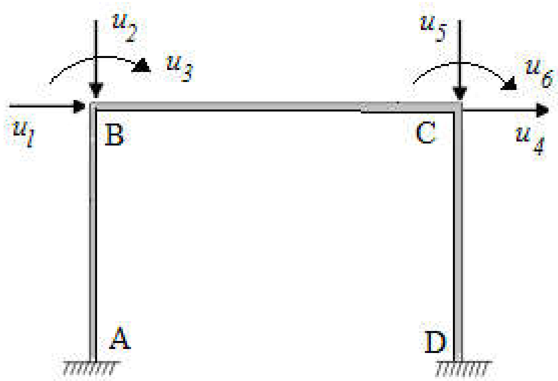
(b) Cantilever Beam



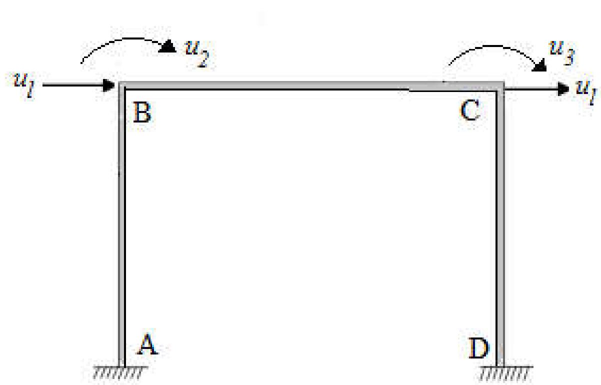
(c) Simply Supported Beam

Fig. 1.2 Kinematically indeterminate structures

In the plane frame shown in Fig.1.3 (a), the joints B and C have 3 degrees of freedom as shown in the figure. However if axial deformations of the members are neglected then $u_1 = u_4$ and u_2 and u_4 can be neglected. Hence, we have 3 independent joint displacement as shown in Fig. 1.3(b) i.e. rotations at B and C and one translation.



(a)



(b)

Fig. 1.3 Rigid frames

1.2. Analysis of Indeterminate Structures

In the analysis of statically determinate structures, the equations of equilibrium are first used to obtain the reactions and the internal forces of the structure; then the member force-deformation relations and the compatibility conditions are employed to determine the structure's displacements.

However, in the analysis of statically indeterminate structures, the equilibrium equations alone are not sufficient for determining the reactions and internal forces. Therefore, it becomes necessary to solve the equilibrium equations in conjunction with the compatibility conditions of the structure to determine its response. Because the equilibrium equations contain the unknown forces, whereas the compatibility conditions involve displacements as the unknowns, the member force-deformation relations are utilized to express the unknown forces either in terms of the unknown displacements or vice versa. The resulting system of equations containing only one type of unknowns is then solved for the unknown forces or displacements, which are then substituted into the fundamental relationships to determine the remaining response characteristics of the structure.

For analyzing statically indeterminate structures, many methods have been developed. These methods can be broadly classified into two categories, namely, the force (flexibility) methods and the displacement(stiffness) methods, depending on the type of unknowns (forces or displacements, respectively), involved in the solution of the governing equations.

Thus, some of these methods are:

- The consistent deformation method (force/ flexible method)
- Slope-displacement method
- Cross Moment distribution method
- Kani Method of Moment Distribution
- The stiffness method

Analysis of indeterminate structures using consistent deformation and slope deflection methods involve solutions of simultaneous equations. On the other hand, Cross and Kani moment distribution methods involve successive cycles of computation.

1.3. Revision on Consistent Deformation Method

The method of consistent deformations, or sometimes referred to as the force or flexibility method, is one of the several techniques available to analyze indeterminate structures. The following is the procedure that describes the concept of this method for analyzing externally indeterminate structures with single or double degrees of indeterminacy.

Principle: - Given a set of forces on a structure, the reactions must assume such a value as are not only in static equilibrium with the applied forces but also satisfy the conditions of geometry at the supports as well as the indeterminate points of the structure.

This method involves with the replacement of redundant supports or restrains by unknown actions in such a way that one obtain a basic determinate structure under the action of the applied loading and these unknown reactions or redundant. Then, the derived basic determinate structure must still satisfy the physical requirements at the location of the excess supports now replace by redundant reactions.

1.3.1. Beams by Consistent Deformation

The basic procedures to solve intermediate beams by the method of consistent deformation method are as follows:

- determine the degree of indeterminacy
- select redundant and remove restraint
- determine reactions and draw moment diagram for the primary structure
- calculate deformation at redundant
- write consistent deformation equation
- solve consistent deformation equation
- determine support reactions
- draw moment, shear, and axial load diagrams

For illustration, consider the beam loaded as shown in Fig. 1.4 (a).

(a) (b)

Fig. 1.4

The basic determinate beam under applied loading and redundant reaction, $R_B = 1$, is shown in Fig. 1.4 (b).

The condition of geometry dictates that: $\Delta_B - R_B\delta_B = 0 \quad \Rightarrow R_B = \dfrac{\Delta_B}{\delta_B} = 0$

The deflection of end B due to the applied load P and the redundant reaction R_B become.

$$\Delta_B = \frac{5}{48}\frac{PL^3}{EI} \quad \text{and} \quad \delta_B = \frac{R_B L^3}{3EI}$$

From statics, $R_B = \dfrac{11}{16}P$ and $M_A = \dfrac{3}{16}PL$

1.3.2. Trusses by Consistent Deformation

The method essentially consists of choosing a basic determinate truss (structure) on which the applied loading and the redundant force act and the applying conditions of geometry requiring the deflection in the direction of the redundant force must be zero or specified value. Once the redundant are determined, the member forces and other desired reaction components can be determined by the principle of super position.

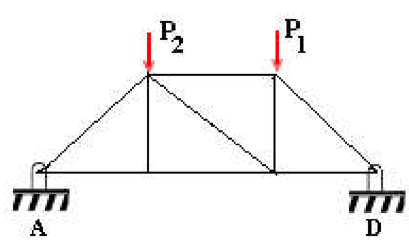


Fig 1.5

The given truss is indeterminate to the 1st degree externally. A basic determinate structure shown below is selected with external redundant H_D (the horizontal reaction).

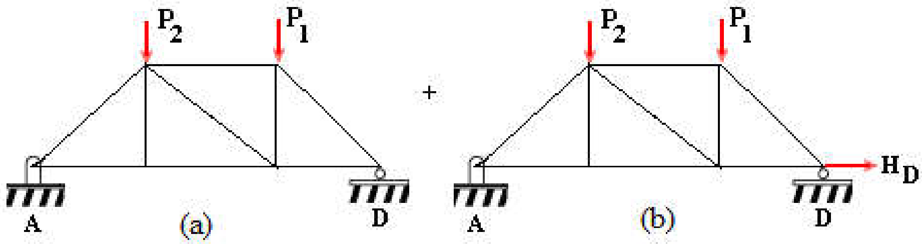


Fig. 1.6

From the geometry of the original structure shown in Fig.1.6(a), the horizontal displacement of support D due to the applied load becomes:

$$\Delta_0 = \Sigma u_1 \frac{SL}{AE}$$

Similarly, for Fig. 1.6 (b), the horizontal displacement of support D due to the fictitious load u_1=1kN, becomes:

$$\delta_1 = \Sigma \frac{u_1^2 L}{AE}$$

The horizontal displacement of support D of the actual structure is zero. Thus the following equation holds true.

$$\Delta_0 + H_D \delta_1 = 0 \text{ , } \Rightarrow H_D = -\frac{\Delta_0}{\delta_1}$$