Addis Ababa University
Addis Ababa Institute of Technology
School of Civil and Environmental Engineering

Fundamentals of Geotechnical Engineering II (CEng2142)
Test 3 - Solution Set

Name	
ID No.	
Signature	
Section	
Exam Date:	08.06.2019

Instruction:

1) This examination is closed book and constitutes 5% of your final grade.
2) The time allowed for this exam is 1 hour.
3) Please read the questions carefully and make sure you understand the facts before you begin answering. Write as legibly and concisely as possible.
4) Use the provided space properly to present you answer.

Question #	**Weight** [marks]	**Score** [marks]
1	15	
2	30	
3	30	
4	25	

QUESTION 1: On Fundamentals of Stress and Strain [15%]

1.1 Define the following terms briefly and adequately? (4%)

Stress	(intensity of loading) - the load per unit area. (**1 *marks***)
Strain	(intensity of deformation) - the ratio of the change in a dimension to the original dimension or the ratio of change in length to the original length. (**1 *mark***)
Stiffness	resistance to deformation (**1 *mark***)
Strength	ability to withstand an applied load without failure or plastic deformation (**1 *mark***)

1.2 What exactly do we mean when we refer to soil as homogeneous and isotropic? (2%)

Homogeneous soil	Isotropic soil
Homogeneous means material properties are the same at every point within a soil mass. (**1 *mark***)	*Isotropic* means the material properties are the same in all directions, and the loadings are the same in all directions. (**1 *mark***)

1.3 A cylindrical soil, 75 mm in diameter and 150 mm long, is axially compressed. The length decreases to 147 mm and the radius increases by 0.3 mm.
Calculate: a) axial and radial strains; b) volumetric strains; c) Poisson's ratio (4%)

a)

$$\Delta\varepsilon_z = \frac{\Delta z}{H_o} = \frac{(150-147)}{150} = 0.02 \ (1\ mark)$$

$$\Delta\varepsilon_r = \frac{\Delta r}{r_o} = \frac{0.6}{75} = 0.008 (1\ mark)$$

b)

$$\varepsilon_p = \varepsilon_z + \varepsilon_r = 0.02 + 0.004 = 0.028 (1\ mark)$$

c)

$$\upsilon = \frac{-\Delta\varepsilon_r}{\Delta\varepsilon_z} = \frac{0.008}{0.02} = 0.4 (1\ mark)$$

1.4 Soils are not homogeneous, elastic, rigid bodies. Why do we then use elastic methods of analysis to calculate the stress distribution within a soil mass? (5%)

An elastic analysis of an isotropic material involves only two constants—Young's modulus and Poisson's ratio—and thus if we assume that soils are isotropic elastic materials, then we have a powerful, but simple, analytical tool to predict a soil's response under loading. We will have to determine only the two elastic constants from our laboratory or field tests. (**5 *marks***)

QUESTION 2: On Geostatic and Additional Stresses [30%]

2.1 At what depth would the vertical effective stress in a deep deposit of clay be 100 kPa, if e = 1.1? The groundwater level is at 1 m below ground surface and S = 95% above the groundwater level. Neglect pore air pressure. (10%)

$$n = \frac{e}{1+e} = \frac{1.1}{1+1.1} = 0.524 \ (2\ marks)$$

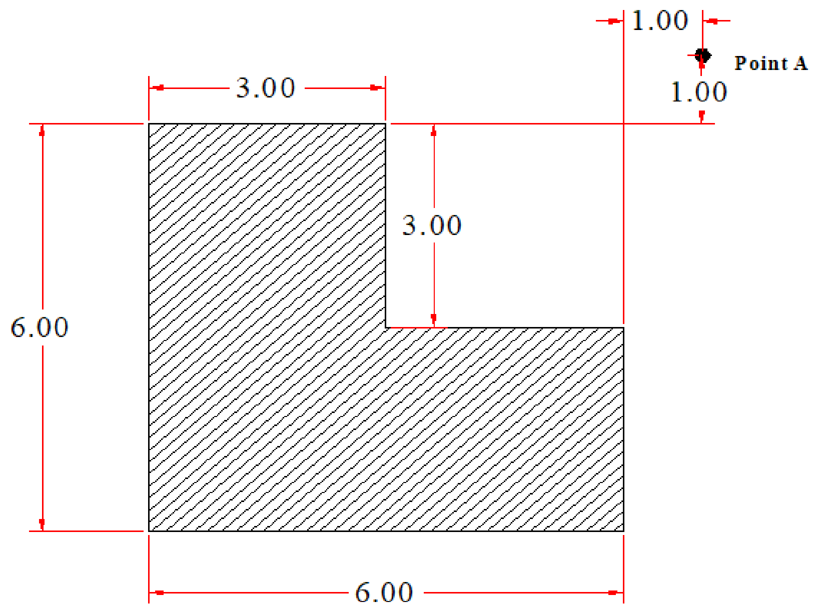
$$\gamma = \gamma_w[G_s(1-n) + Sn] = 9.81[2.7(1-0.524) + 1 \times 0.524] = 17.75\ \frac{kN}{m^3} \ (4\ marks)$$

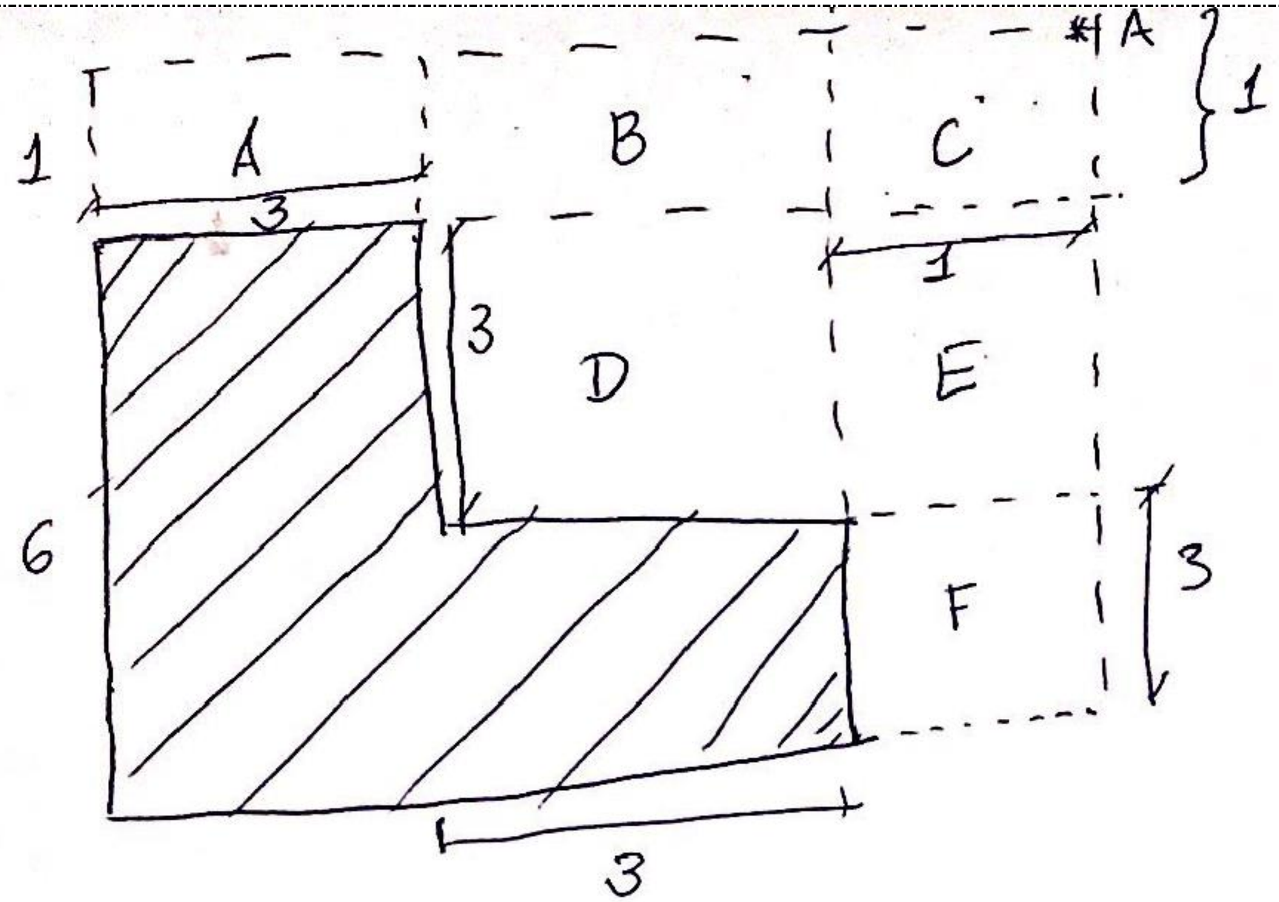
$$\sigma' = \sigma - u = \gamma \times z - \gamma_w \times z = 100\ kPa \ (2\ marks)$$

$$z = \frac{100\ kPa}{(17.75 - 9.81)\ kN/m^3} = 12.6\ m \ (2\ marks)$$

2.2 As shown in figure below, An L shape structure exerts a contact pressure of 100 kN/m². Find the stress at a point 5 m depth below point A due to this load. (20%)

$$I_z = \frac{1}{4\pi}\left[\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2+-m^2n^2+1}\left(\frac{m^2+n^2+2}{m^2+n^2+1}\right) + \tan^{-1}\left(\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2-m^2n^2+1}\right)\right]$$





(6 *marks*)

Area of the entire area = A_{7X7}

$m=B/z$; $n=L/z$

Total stress at A = $A_{7X7} - A_{CEF} - A_{ABC} - A_{BCDE} + A_{BC} + A_{CE}$

Area	B	L	m	n	I_z	$\Delta\sigma_z = I_z * q_s$
	(3 *marks*)		(3 *marks*)		(3 *marks*)	(3 *marks*)
A_{7X7}	*7*	*7*	*1.4*	*1.4*	*0.215*	*21.5*
A_{CEF}	*1*	*7*	*0.2*	*1.4*	*0.058*	*5.8*
A_{ABC}	*1*	*7*	*0.2*	*1.4*	*0.058*	*5.8*
A_{BCDE}	*4*	*4*	*0.8*	*0.8*	*0.15*	*15*
A_{BC}	*1*	*4*	*0.2*	*0.8*	*0.05*	*5*
A_{CE}	*1*	*4*	*0.2*	*0.8*	*0.05*	*5*

$\Delta\sigma_A = 21.5 - 5.8 - 5.8 - 15 + 5 + 5 = 4.9$ kN/m^2 (2 *marks*)

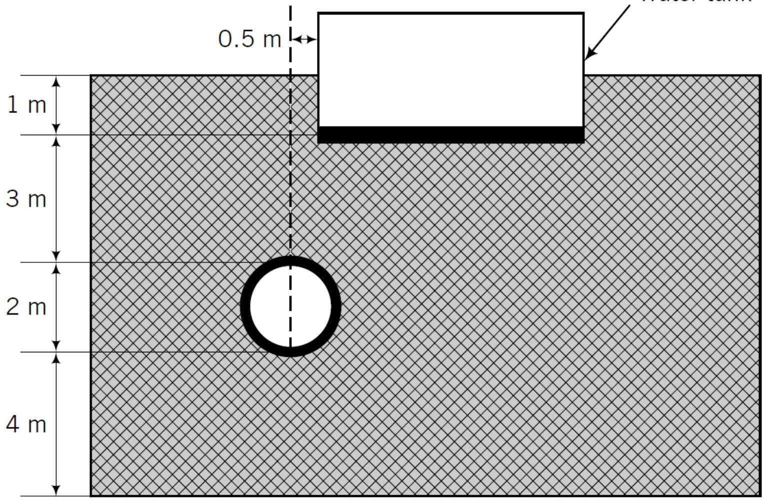
QUESTION 3: On Isobars and Contact Pressure [30%]

3.1 A water tank, 15 m in diameter and 10 m high, is proposed for a site where there is an existing pipeline. Plot the distribution of vertical and lateral stress increases imposed by the water tank on the pipeline along one-half the circumference nearest to the tank.

The empty tank's weight (deal load) is 350 kN. Assume the water tank is filled to its capacity.

Construct an isobar for 20% of the contact pressure. (30%)

$$\Delta\sigma_z = q_s\left[1-\left(\frac{1}{1+(r_o/z)^2}\right)^{3/2}\right] = q_s I_c;\ \Delta\sigma_r = \Delta\sigma_\theta = \frac{q_s}{2}\left[(1+2v) - \frac{4(1+v)}{[1+(r_o/z)^2]^{1/2}} + \frac{1}{[1+(r_o/z)^2]^{3/2}}\right]$$



Step 1 Calculate the stress increase at the base of the tank

Base area, $A = \pi\frac{D^2}{4} = \pi\frac{15^2}{4} = 176.7\ m^2$ (1 mark)

The empty tank's weight, DL=350 kN

Weight of water at full capacity, $LL = \pi\frac{15^2}{4}\times 10\times 9.81 = 17,336\ kN$

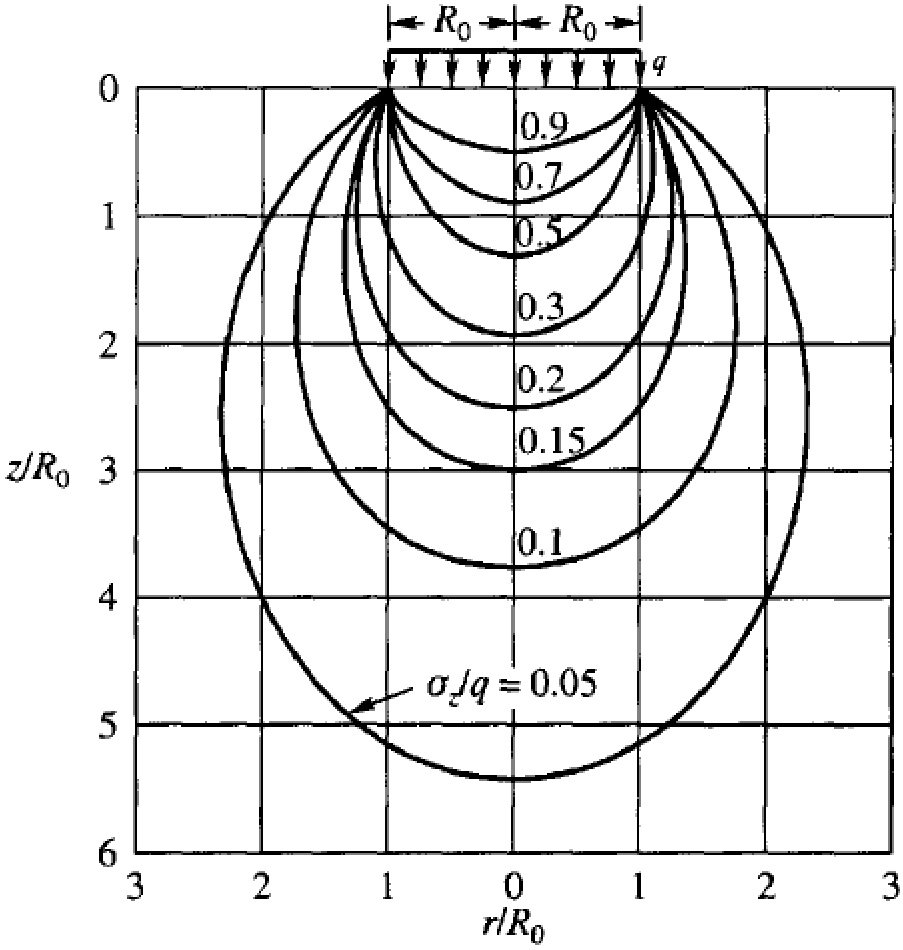
$$\Delta\sigma = \frac{DL+LL}{A} = \frac{350+17336}{176.7} \approx 100\ kPa = q_s\ (2\ marks)$$

$$r_o = 7.5\ m;\ v = 0.3\ (assumed)(2\ marks)$$

Location	Z (m)	$\Delta\sigma_z = q_s\left[1-\left(\frac{1}{1+(r_o/z)^2}\right)^{3/2}\right]$ (5 marks)	$\Delta\sigma_r = \frac{q_s}{2}\left[(1+2v) - \frac{4(1+v)}{[1+(r_o/z)^2]^{1/2}} + \frac{1}{[1+(r_o/z)^2]^{3/2}}\right]$ (5 marks)
Top	3	80.04	-14.00
	3.5	74.41	-26.17
Middle	4	68.70	-37.14
	4.5	63.05	-46.96
Bottom	5	57.60	-55.69

To construct an isobar for 20% of the contact pressure, $q_s = 20\ kPa$

z/Ro	*r/Ro*	$\Delta\sigma_z/q_s$
0	*0*	*0.2*
2	*1*	*0.2*
2.5	*0*	*0.2*



QUESTION 4: On Stress State and Stress Path [15%]

4.1 The initial principal stresses at a certain depth in a clay soil are 100 kPa on the horizontal plane and 50 kPa on the vertical plane. Construction of a surface foundation induces additional stresses consisting of a vertical stress of 45 kPa, a lateral stress of 20 kPa, and a counterclockwise (with respect to the horizontal plane) shear stress of 40 kPa.

Plot Mohr's circle

(1) for the initial state of the soil and (2) after construction of the foundation.

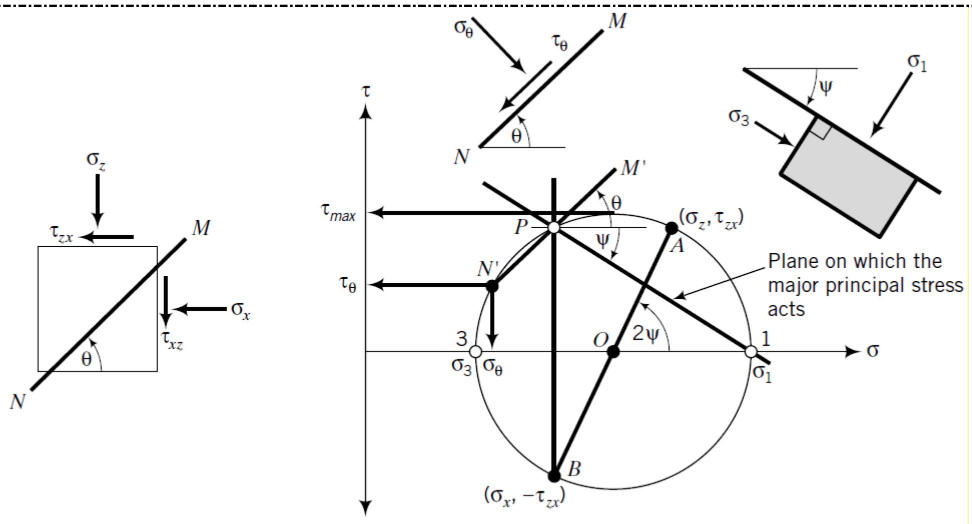
Determine

(a) the change in magnitude of the principal stresses,

(b) the change in maximum shear stress, and

(c) the change in orientation of the principal stress plane resulting from the construction of the foundation.

(25%)



Initial state

$$\sigma_1 = \sigma_z = 100\ kPa$$

$$\sigma_3 = \sigma_x = 50\ kPa$$

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{100 - 50}{2} = 25\ kPa$$

Due to the construction of the foundation

$$\sigma_z = 100 + 45 = 145\ kPa$$

$$\sigma_x = 50 + 20 = 70\ kPa$$

$$\tau_{zx} = 40\ kPa$$

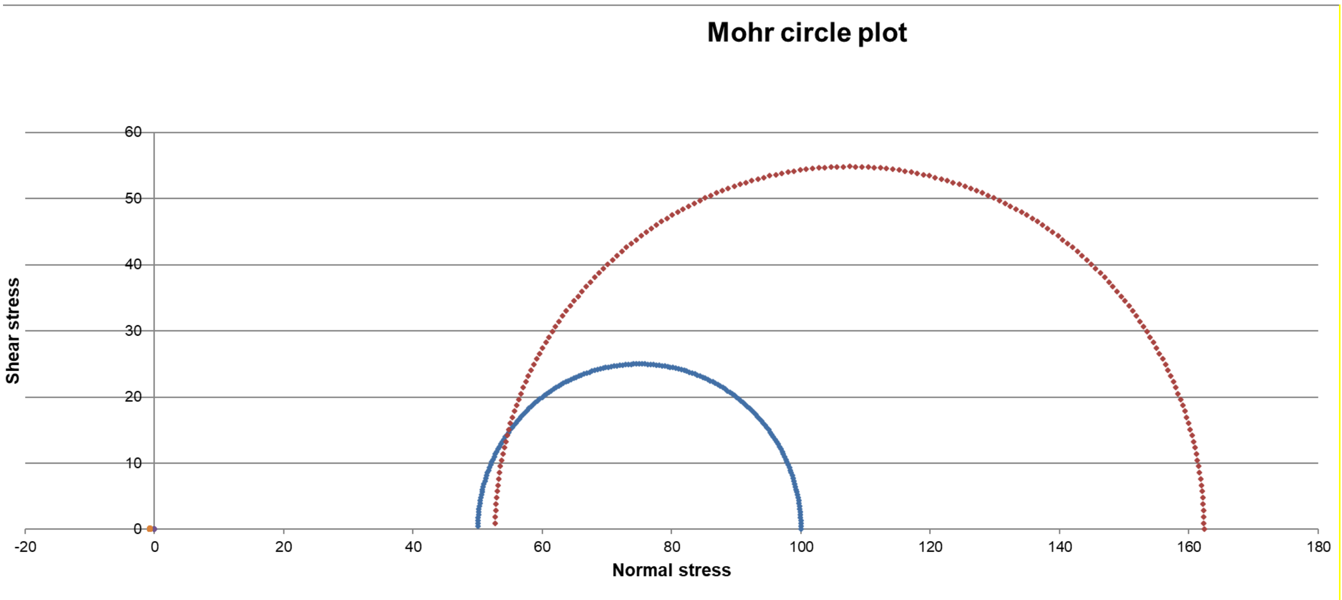
$$\sigma_1 = \frac{\sigma_z + \sigma_x}{2} + \sqrt{\left(\frac{\sigma_z - \sigma_x}{2}\right)^2 + \tau_{zx}^2} = \frac{145 + 70}{2} + \sqrt{\left(\frac{145 - 70}{2}\right)^2 + 40^2}$$

$$= 162.33\ kPa$$

$$\sigma_3 = \frac{\sigma_z + \sigma_x}{2} - \sqrt{\left(\frac{\sigma_z - \sigma_x}{2}\right)^2 + \tau_{zx}^2} = \frac{145 + 70}{2} - \sqrt{\left(\frac{145 - 70}{2}\right)^2 + 40^2}$$

$$= 52.67\ kPa$$

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{162.33 - 52.67}{2} = 54.83\ kPa$$



Change in principal stress

$$\Delta\sigma_1 = 162.33 - 100 = 63.33\ kPa$$

$$\Delta\sigma_3 = 52.67 - 50 = 2.67\ kPa$$

Change in maximum shear stress

$$\Delta\tau_{max} = 54.83 - 25 = 29.83\ kPa$$

The angle between the major principal stress plane and the horizontal plane (ψ) is after the construction of the foundation

$$\tan\psi = \frac{\tau_{zx}}{\sigma_1 - \sigma_x} = \frac{40}{162.33 - 70} = 0.43$$

$$\psi = \tan^{-1} 0.43 = 23.42°$$