FOUNDATION ENGINEERING I (CONTD…)

CEng 3204

CHAPTER THREE

Design of Shallow Foundations**: COMBINED FOOTINGS**

Example on Combined footings

- Design a rectangular base to support two columns carrying the following loads:

  Column 1: dead load = 310 kN, imposed load = 160 kN
  Column 2: dead load = 430 kN, imposed load = 220 kN

  The columns are each 350 mm square and are spaced at 2.5 m centers. The width of the base is not to exceed 2.0m. The safe bearing pressure on the ground is 160 kN/m^2. Take fck = 30 MPa concrete and fyk = 500 MPa.

Example on Combined footings

a. **Base arrangement and soil pressure**

- Assume the **weight of the base is 130 kN**.
- Various load conditions are examined.
- It is assumed here that the imposed loads on the columns are independent loads and therefore carry different load factors. *If this is not the case, then a single load factor should be applied for both the loads.*

Example on Combined footings

Case 1: Dead + imposed load on both columns.

Use SLS values

Axial load = (310 + 160) + (430 + 220) + 130 = 1250 kN

Area of base = 1250/160 = 7.81 m2

Length of base = 7.81/2.0 = 3.91 m

Choose **4.5 m × 2.0 m × 0.6 m** deep base.

The weight of the base is (4.5 × 2.0 × 0.6 × 25) = 135.0 kN ≈ 130 kN.

Example on Combined footings

Area = 4.5 × 2.0 = 9.0 m^2

Section modulus = 2.0 × 4.52/6 = 6.75 m^3

- The base is arranged so that the **center of gravity** of the loads coincides with the **center line of the base**, in which case the **base pressure will be uniform**. This arrangement will be made for the **maximum ultimate loads**.

- The ultimate loads are

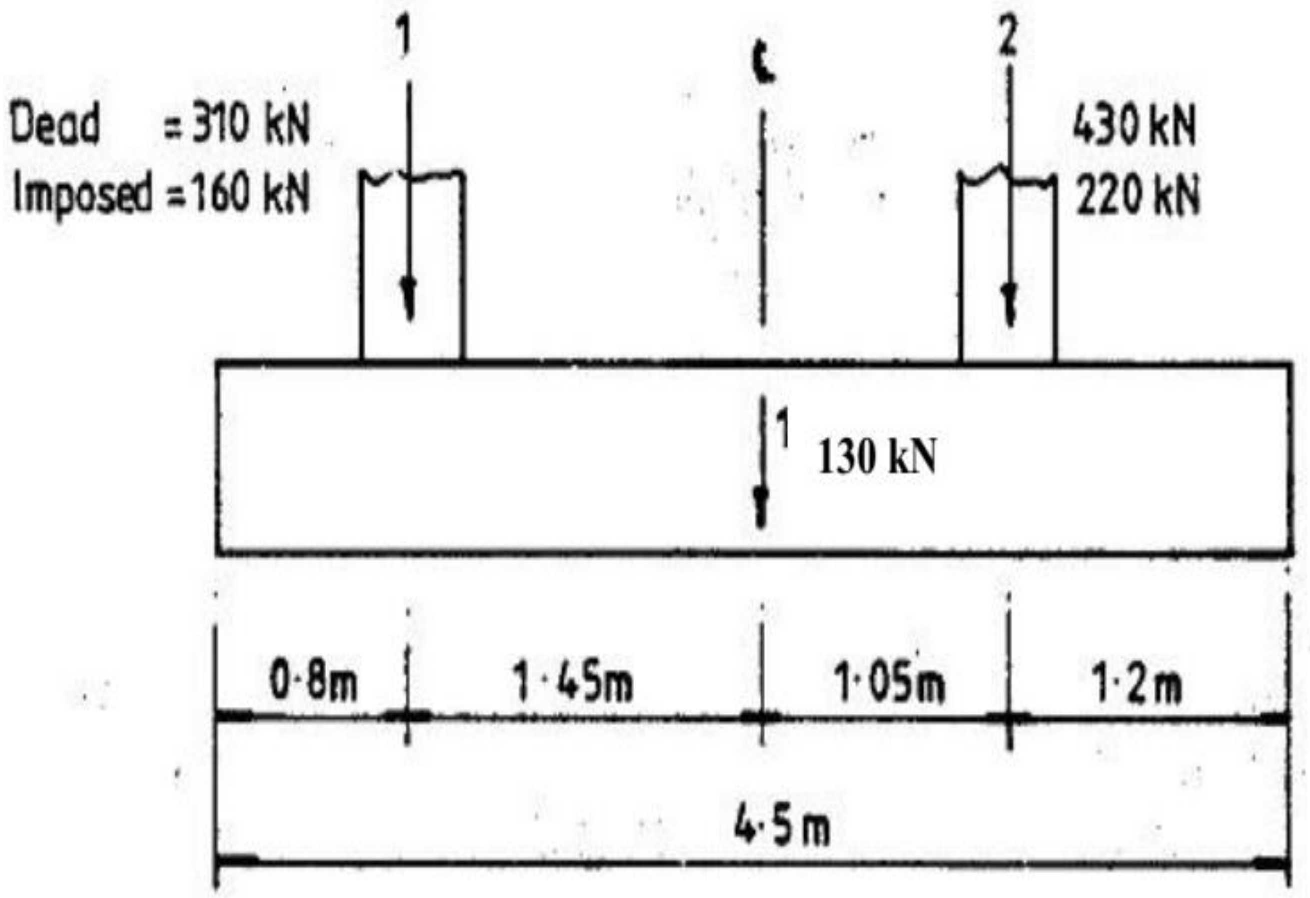
Column 1: load = 1.35 × 310 + 1.5 × 160 = 658.5 kN

Column 2: load = 1.35 × 430 + 1.5 × 220 = 910.5 kN

The distance of the center of gravity from column 1 is

x = (910.5 × 2.5)/ (658.5 + 910.5) = 1.45 m

Example on Combined footings



Base arrangement

Example on Combined footings

- The soil pressure is checked for service loads for case 1:

Direct vertical load = 310 + 160 + 430 + 220 + 130 = 1250 kN

- Since the centroid of the loads does not exactly coincide with the centroid of the base, check for maximum pressure which is non-uniform. The moment about the centre line of the base is

M = (430 + 220) × 1.05 – (310 + 160) × 1.45 = 1.0 kNm

The moment is very small and can be ignored. The base pressure is practically constant.

Base pressure = 1250/9.0 = 138.9 kN/m² < 160.0 kN/m²

Example on Combined footings

Case 2: Column 1, dead + imposed load; column 2, dead load only. Use SLS values

Axial load = (310+160) + (430 + 0) + 130.0 = 1030 kN

Moment = M = (430+0) × 1.05 – (310 + 160) × 1.45 = –230 kN m

Maximum pressure = 1030/9.0 + 230.0/6.75 = 148.5 kN/m^2 < 160.0 kN/m^2

Maximum base pressure occurs toward the column 1 side.

Example on Combined footings

Case 3: Column 1: dead load only; column 2: dead + imposed load. Use SLS values

Axial load = (310+0) + (430 + 220) + 130 = 1090 kN

Moment = M = (430+220) × 1.05 – (310 + 0) × 1.45 = 233 kN m

Maximum pressure = 1090/9.0 + 233.0/6.75 = 155.6 kN/m^2 < 160.0 kN/m^2

Maximum base pressure occurs toward the column 2 side.

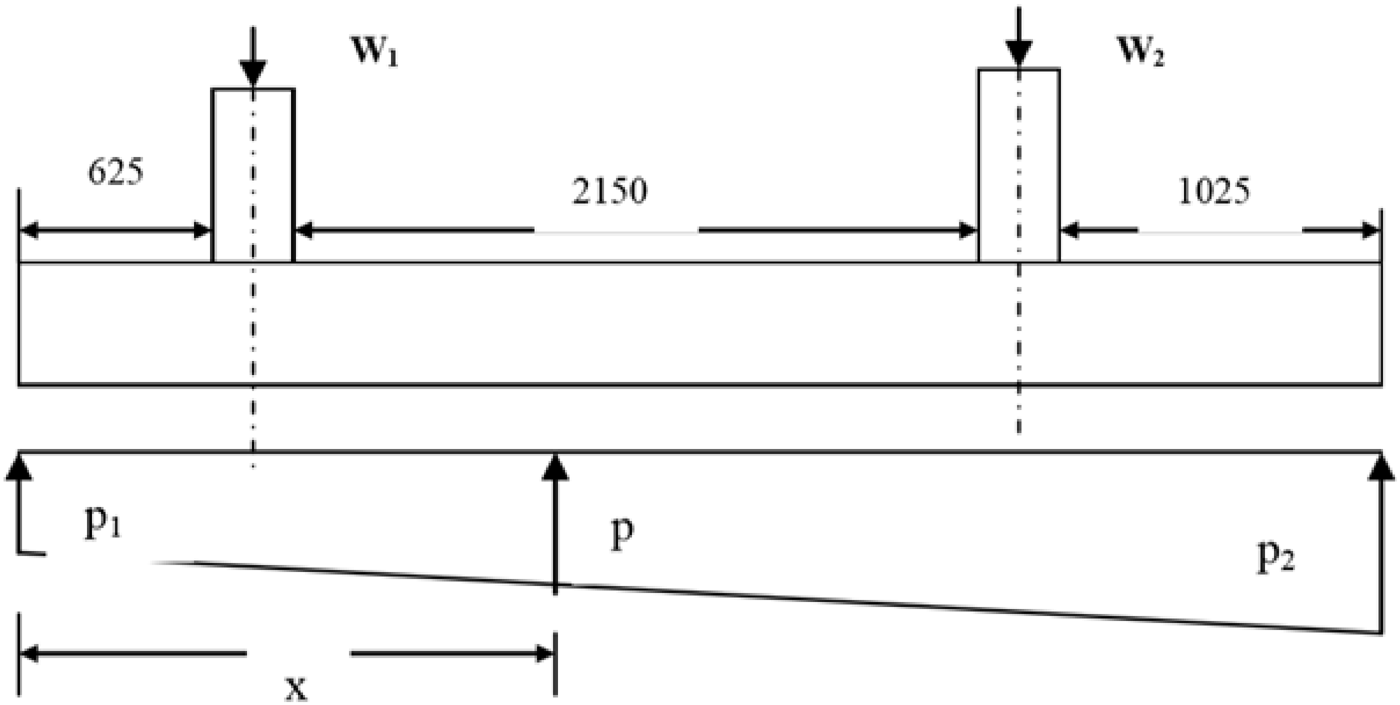
The base is satisfactory with respect to soil pressure.

Example on Combined footings

b. **Analysis for actions in longitudinal direction at ULS**

Take cover as 40 mm, and 20 mm diameter bars,

$$\mathbf{d_{eff} = 600 - 40 - 20/2 = 550\ mm.}$$



Example on Combined footings

- Using the ‘Macaulay bracket notation’, the shear force V and moment M in the longitudinal direction due to ultimate loads are calculated by statics.

$$\{x-a\}^n = \begin{cases} 0, & x < a \\ (x-a)^n, & x \geq a. \end{cases} \quad (n \geq 0)$$

$$\langle x-a\rangle^0 \equiv \{x-a\}^0 = \begin{cases} 0, & x < a \\ 1, & x > a. \end{cases}$$

$$p = p_1 + (p_2 - p_1)\frac{x}{4.5}$$

$$V = 2\{\frac{(p_1 + p)}{2}x\} - W_1\langle x - 0.8\rangle^0 - W_2\langle x - 3.3\rangle^0$$

$$M = 2\{p_1\frac{x^2}{2} + \frac{(p - p_1)}{2}\frac{x^2}{3}\} - W_1\langle x - 0.8\rangle - W_2\langle x - 3.3\rangle$$

Example on Combined footings

- The ***maximum design moments*** are at the **column face** and **between the columns**, and ***maximum shears*** are at ***d*** **from the column face**.
- The load cases are as follows. The weight of the base is ignored as the corresponding base pressure will cancel the pressure due to the weight of the base.

Example on Combined footings

- In the following load factors from Equation (6.10), Table A2.4 (B) from *BS EN 1990:2002, Eurocode-Basis of Structural Design* are used.

$$\gamma_{Gj,\,sup} = 1.35,\ \gamma Gj_{,\,inf} = 1.0,\ \gamma_{Q,\,1} = 1.5,\ \psi_{0,\,i} = 0.7$$

- Six loading cases are discussed in detail.

Example on Combined footings

- **Case 1A: Maximum load on both columns with column 1 carrying leading variable load.**
- **Case 1B: Maximum load on both columns with column 2 carrying leading variable load.**
- **Case 2A: Maximum load on column 1 and minimum load on column 2 with column 1 carrying leading variable load.**
- **Case 2B: Maximum load on column 1 and minimum load on column 2 with column 2 carrying leading variable load.**
- **Case 3A: Minimum load on column 1 and maximum load on column 2 with column 1 carrying leading variable load.**
- **Case 3B: Minimum load on column 1 and maximum load on column 2 with column 2 carrying leading variable load.**

Example on Combined footings

Case 1A: Maximum load on both columns with column 1 carrying leading variable load.

Treat G_k as unfavorable on both columns, Q_k on column 1 as leading variable action and Q_k on column 2 as accompanying variable action

W1: 1.35 × 310 + 1.5 × 160 = 658.5 kN

W2: 1.35 × 430 + 1.5 × 0.7× 220 = 811.5 kN

W1 + W2 = 658.5 + 811.5 = 1470.0 kN,

Moment M = 811.5 × 1.05 – 658.5 × 1.45 = -102.75 kN m

p1 = 1470.0/9.0 + 102.75/6.75 = 178.6 kN/m^2

p2 = 1470.0/9.0 – 102.75/6.75 = 148.1 kN/m^2

Example on Combined footings

x	V	M	Remarks
0.075	26.8	1.0	d from left face of column 1
0.625	220.6	69.2	Left face of column 1
0.975	–316.7	52.5	Right face of column 1
1.525	–129.5	–70.1	d from right face of column 1
1.89	0	***–95.1***	Maximum negative moment
2.575	**216.3**	–23.2	d from left face of column 2
3.125	391.6	144.2	Left face of column 2
3.475	–310.6	**158.4**	Right face of column 2
4.025	–142.1	34.1	d from right face of column 2

Design values: shear force = 216.3 kN, moment = 158.4 kNm and –95.1 kNm.

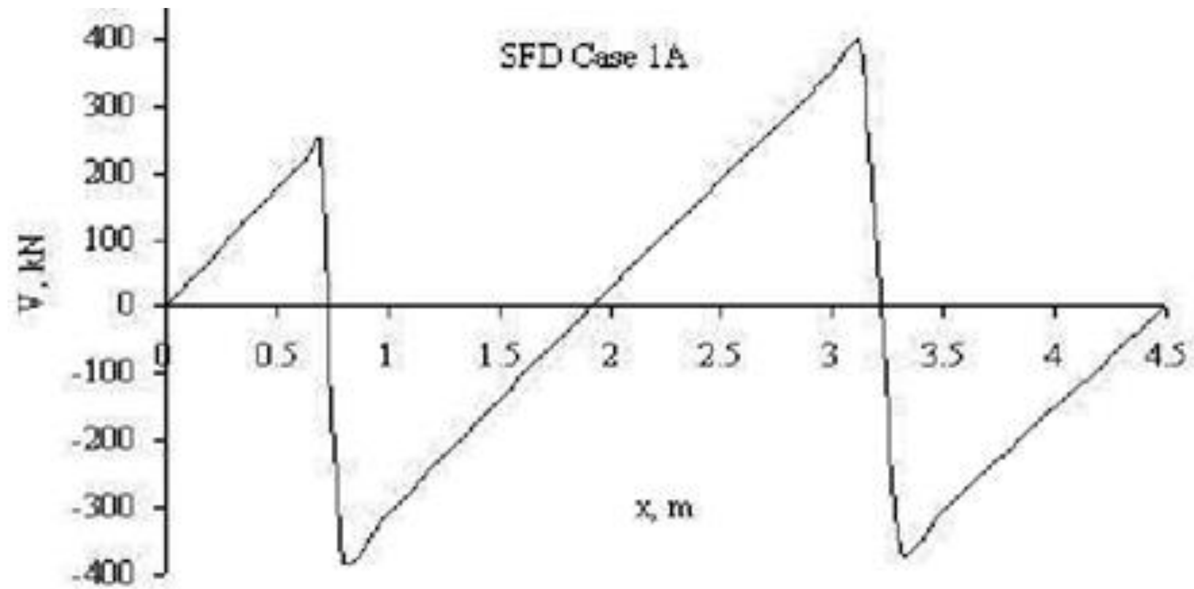
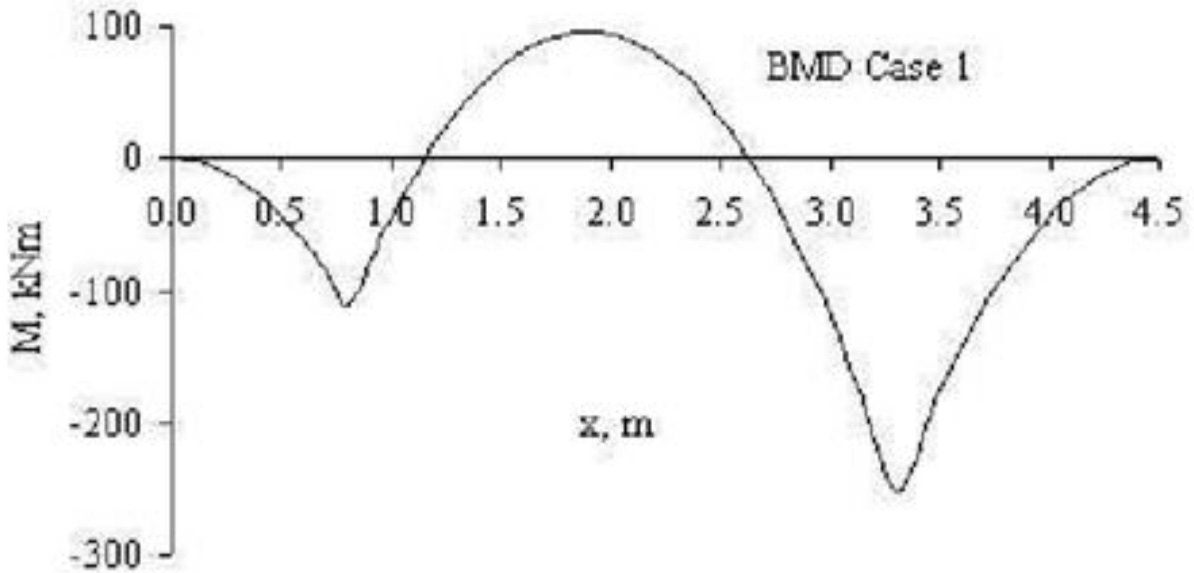


Fig. 11.22 Shear force diagram for case 1A.



Example on Combined footings

Case 1B: Maximum load on both columns with column 2 carrying leading variable load.

Treat G_k as unfavorable on both columns, Q_k on column 2 as leading variable action and Q_k on column 1 as accompanying variable action.

W1: $1.35 \times 310 + 1.5 \times 0.7 \times 160 = 586.5$ kN

W2: $1.35 \times 430 + 1.5 \times 220 = 910.5$ kN

W1 + W2 = 586.5 + 910.5 = 1497.0 kN,

Moment $M = 910.5 \times 1.05 - 586.5 \times 1.45 = 105.6$ kN m

$p1 = 1497.0/9.0 - 105.6/6.75 = 150.7$ kN/m^2

$p2 = 1497.0/9.0 + 105.6/6.75 = 181.96$ kN/m^2

Example on Combined footings

x	V	M	Remarks
0.075	22.6	0.8	d from left face of column 1
0.625	191.1	59.4	Left face of column 1
0.975	–286.0	42.8	Right face of column 1
1.525	–110.7	–66.5	d from right face of column 1
1.89	0	**–85.4**	Maximum negative moment
2.575	$\boxed{\textbf{235.7}}$	–2.3	d from left face of column 2
3.125	423.3	178.8	Left face of column 2
3.475	–365.7	**188.8**	Right face of column 2
4.025	–171.3	40.9	d from right face of column 2

Design values: shear force = 235.7 kN, moment = 188.8 kNm and –85.4 kNm.

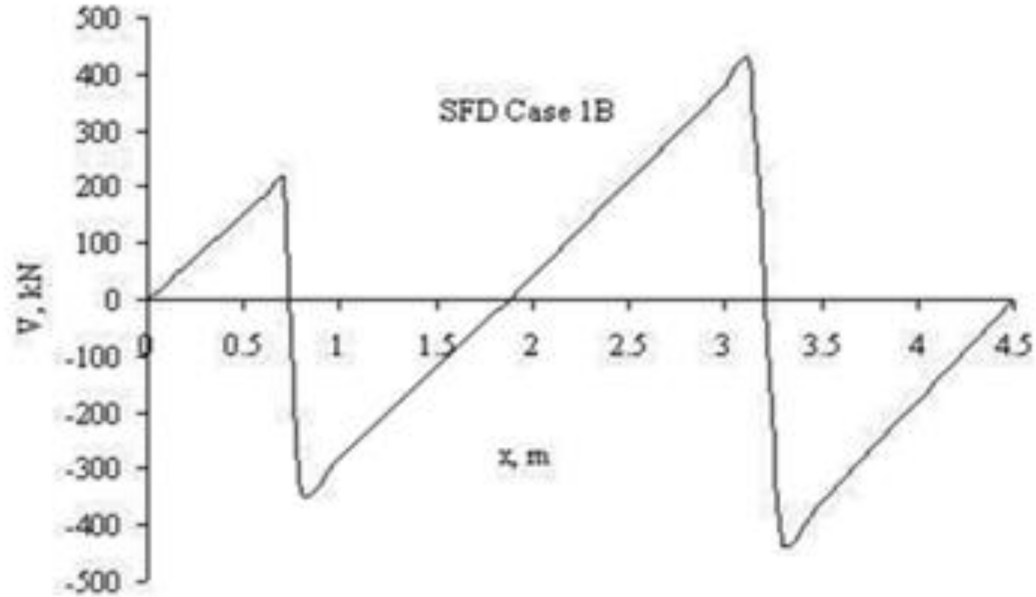
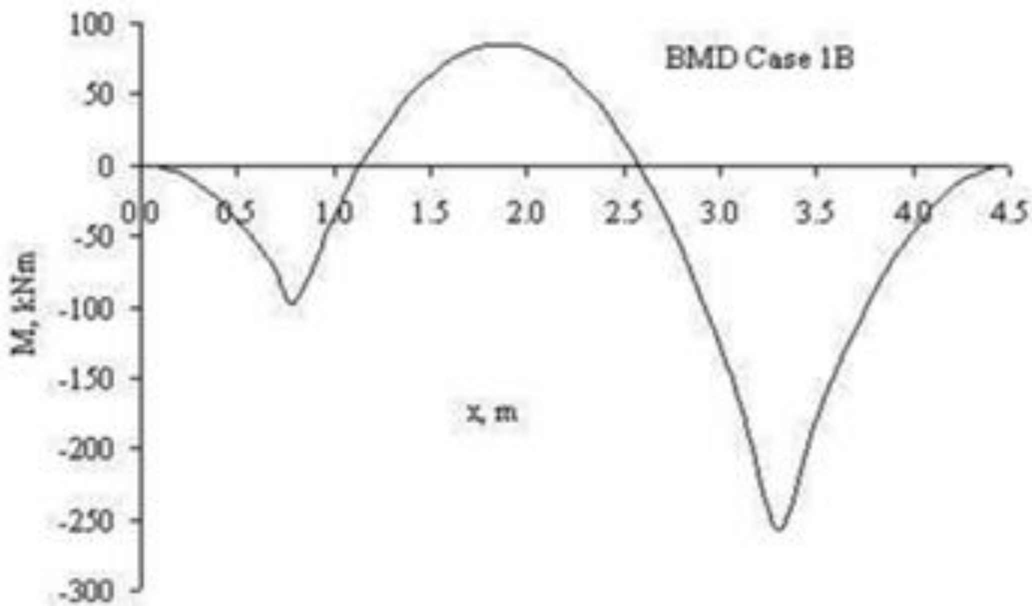


Fig. 11.24 Shear force diagram for case 1B.



Example on Combined footings

Case 2A: Maximum load on column 1 and minimum load on column 2 with column 1 carrying leading variable load.

Treat G_k as unfavorable on column 1 and as favorable on column 2, Q_k on column 1 as leading variable action and Q_k on column 2 as accompanying variable action..

W1: $1.35 \times 310 + 1.5 \times 160 = 658.5$ kN

W2: $1.0 \times 430 + 1.5 \times 0.7 \times 220 = 661.0$ kN

W1 + W2 = $658.5 + 661.0 = 1319.5$ kN,

Moment $M = 661.0 \times 1.05 - 658.5 \times 1.45 = -260.8$ kN m

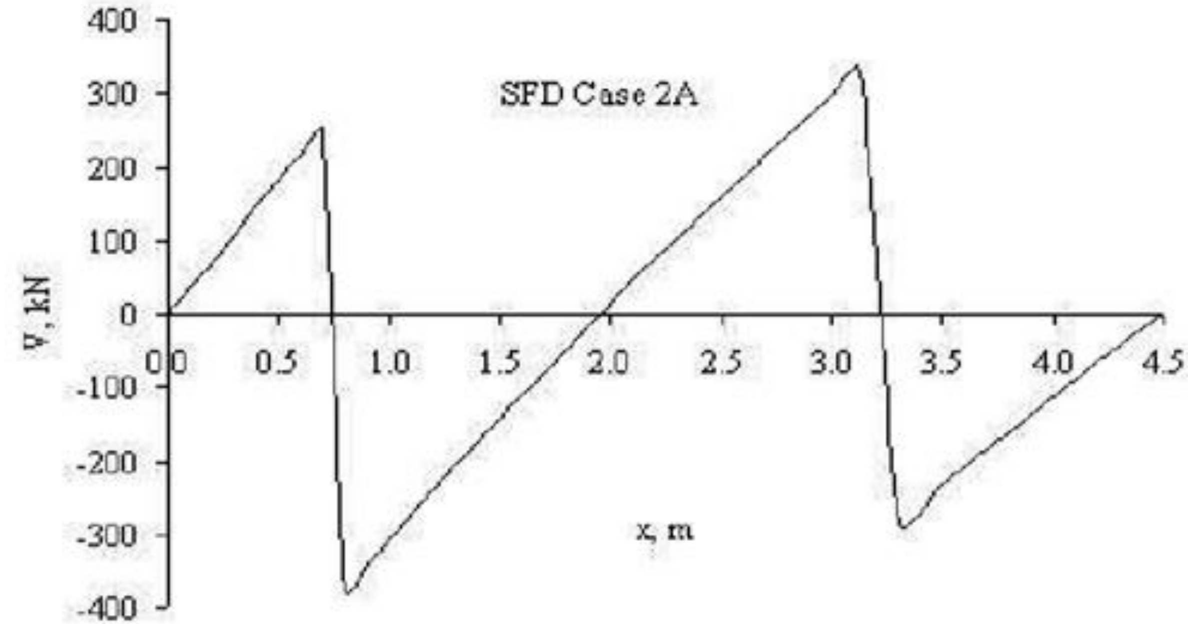
p1 = $1319.5 / 9.0 + 260.8 / 6.75 = 185.2$ kN/m2

p2 = $1319.5 / 9.0 - 260.8 / 6.75 = 108.0$ kN/m2

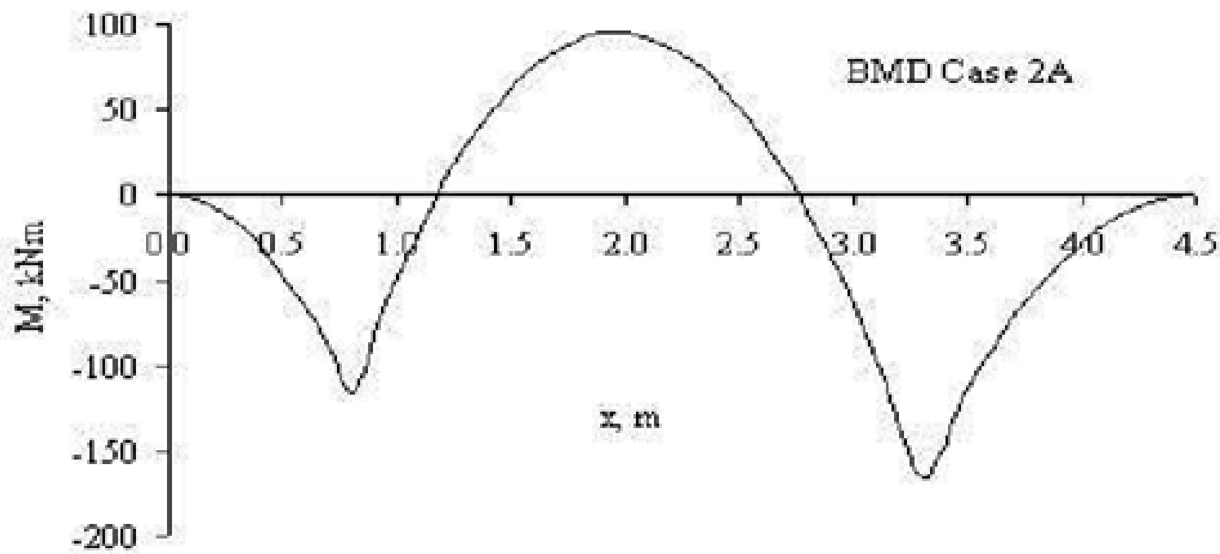
Example on Combined footings

x	V	M	Remarks
0.075	27.7	1.0	d from left face of column 1
0.625	224.9	71.0	Left face of column 1
0.975	–313.6	55.6	Right face of column 1
1.525	–133.4	–66.9	d from right face of column 1
2.04	0	***–95.3***	Maximum negative moment
2.575	**181.7**	–38.2	d from left face of column 2
3.125	331.7	103.4	Left face of column 2
3.475	–239.3	**119.7**	Right face of column 2
4.025	–106.3	25.2	d from right face of column 2

Design values: shear force = 181.7 kN, moment = 119.7 kNm and –95.3 kNm.



SFD Case 2A
400
300
200
100
0
-100
-200
-300
-400
V, kN
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
x, m



BMD Case 2A
100
50
0
-50
-100
-150
-200
M, kNm
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
x, m

Example on Combined footings

Case 2B: Maximum load on column 1 and minimum load on column 2 with column 2 carrying leading variable load.

Treat G_k as unfavorable on column 1 and as favourable on column 2, Q_k on column 2 as leading variable action and Q_k on column 1 as accompanying variable action.

W1: $1.35 \times 310 + 1.5 \times 0.7 \times 160 = 586.5$ kN

W2: $1.0 \times 430 + 1.5 \times 220 = 760.0$ kN

W1 + W2 = 586.5 + 760.0 = 1346.5 kN,

Moment $M = 760.0 \times 1.05 – 586.5 \times 1.45 = -52.4$ kN m

p1 = 1346.5 /9.0 + 52.4 /6.75 = 157.4 kN/m2

p2 = 1346.5 /9.0 - 52.4 /6.75 = 141.9 kN/m2

Example on Combined footings

x	V	M	Remarks
0.075	23.6	1.0	d from left face of column 1
0.625	195.4	61.2	Left face of column 1
0.975	–282.9	45.9	Right face of column 1
1.525	–114.5	–63.3	d from right face of column 1
2.04	0	**–84.9**	Maximum negative moment
2.575	**201.1**	–17.2	d from left face of column 2
3.125	363.4	138.1	Left face of column 2
3.475	–294.4	**150.2**	Right face of column 2
4.025	–135.5	32.1	d from right face of column 2

Design values: shear force = 201.1 kN, moment = 150.2 kNm and –84.9 kNm.

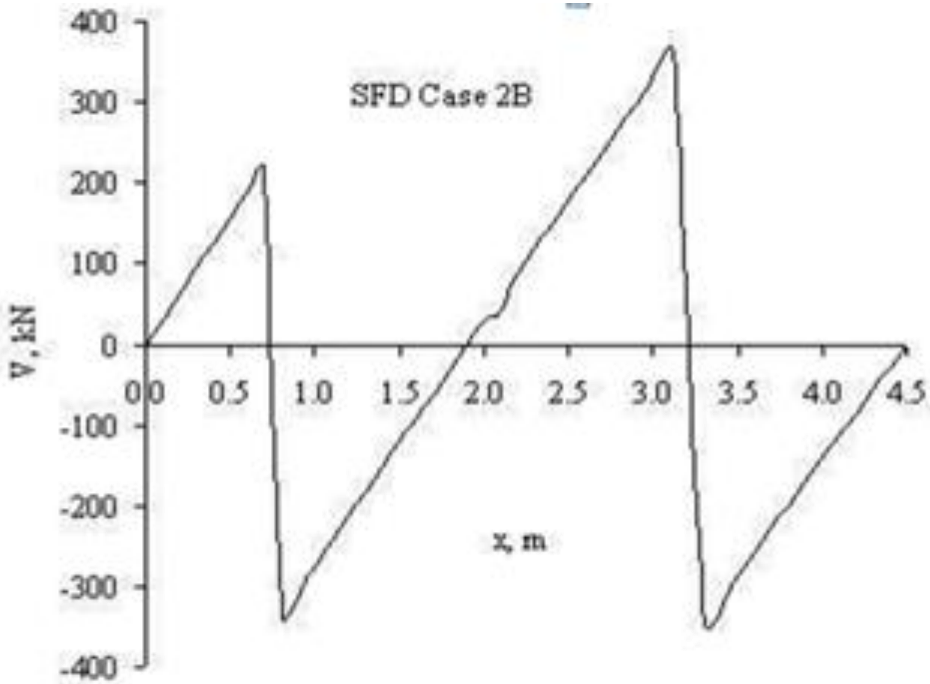
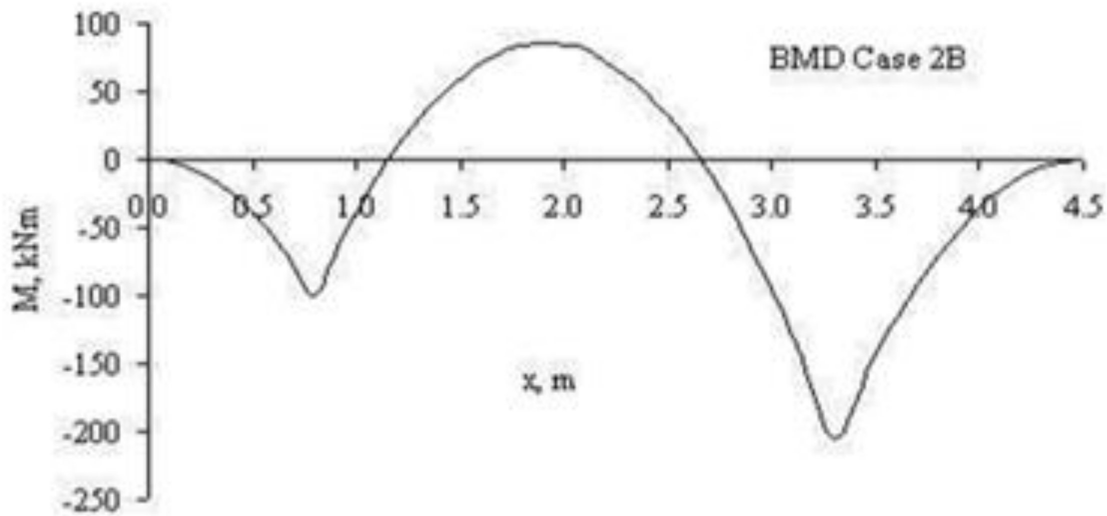


Fig. 11.28 Shear force diagram for case 2B.



Example on Combined footings

Case 3A: Minimum load on column 1 and maximum load on column 2 with column 1 carrying leading variable load.

Treat Gk as unfavorable on column 2 and as favorable on column 1, Qk on column 1 as leading variable action and Qk on column 2 as accompanying variable action.

$W1: 1.0 \times 310 + 1.5 \times 160 = 550.0$ kN
$W2: 1.35 \times 430 + 1.5 \times 0.7 \times 220 = 811.5$ kN
$W1 + W2 = 550.0 + 811.5 = 1361.5$ kN,
Moment $M = 811.5 \times 1.05 – 550.0 \times 1.45 = 54.6$ kN m

$p1 = 1361.5 /9.0 - 54.6 /6.75 = 143.2$ kN/m2
$p2 = 1361.5 /9.0 + 54.6 /6.75 = 159.4$ kN/m2

Example on Combined footings

x	V	M	Remarks
0.075	21.5	1.0	d from left face of column 1
0.625	180.4	56.2	Left face of column 1
0.975	–267.3	41.0	Right face of column 1
1.525	–104.9	–61.5	d from right face of column 1
2.04	0	**–79.9**	Maximum negative moment
2.575	**211.4**	–6.3	d from left face of column 2
3.125	380.2	156.3	Left face of column 2
3.475	–322.8	**166.3**	Right face of column 2
4.025	–150.4	36.1	d from right face of column 2

Design values: shear force = 211.4 kN, moment = 166.3 kNm and –79.9 kNm.

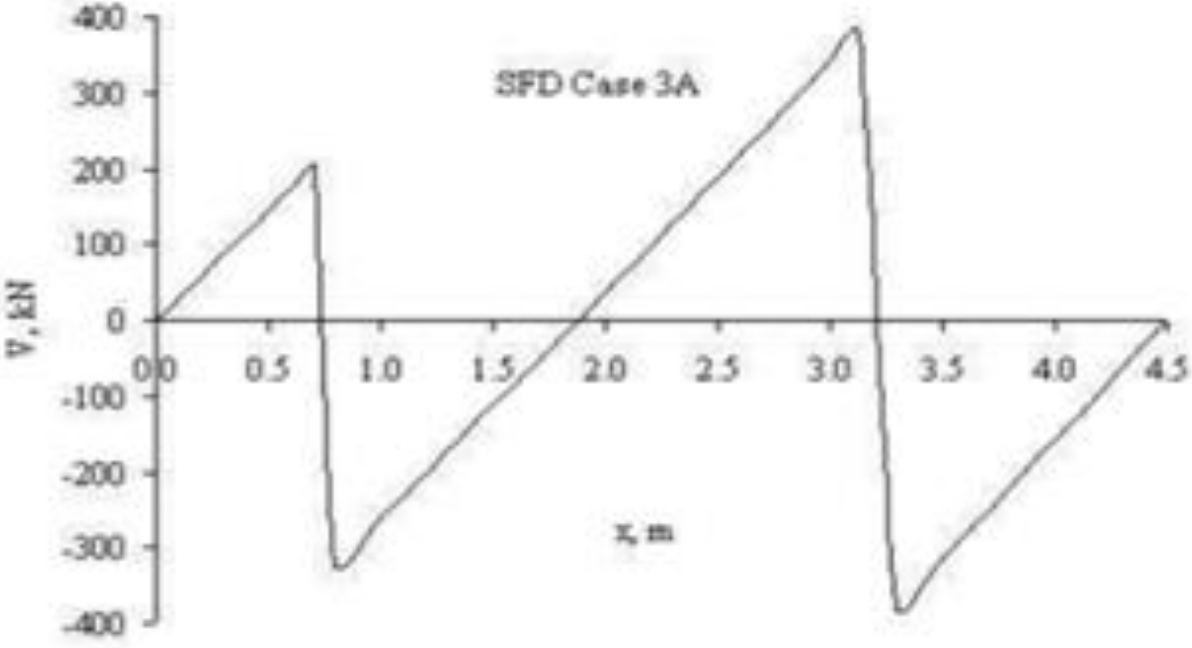
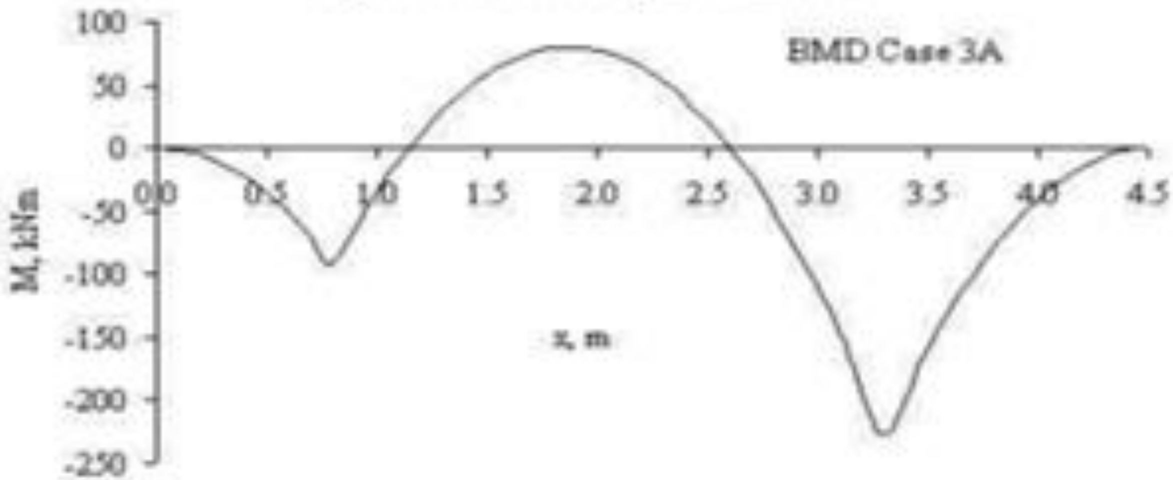


Fig. 11.30 Shear force diagram for case 3A.



Example on Combined footings

Case 3B: Minimum load on column 1 and maximum load on column 2 with column 2 carrying leading variable load.

Treat G_k as unfavorable on column 2 and as favorable on column 1, Q_k on column 2 as leading variable action and Q_k on column 1 as accompanying variable action.

W1: $1.0 \times 310 + 1.5 \times 0.7 \times 160 = 478.0$ kN

W2: $1.35 \times 430 + 1.5 \times 220 = 910.5$ kN

W1 + W2 = 478.0 + 910.5 = 1388.9 kN,

Moment $M = 910.5 \times 1.05 – 478.0 \times 1.45 = 262.9$ kN m

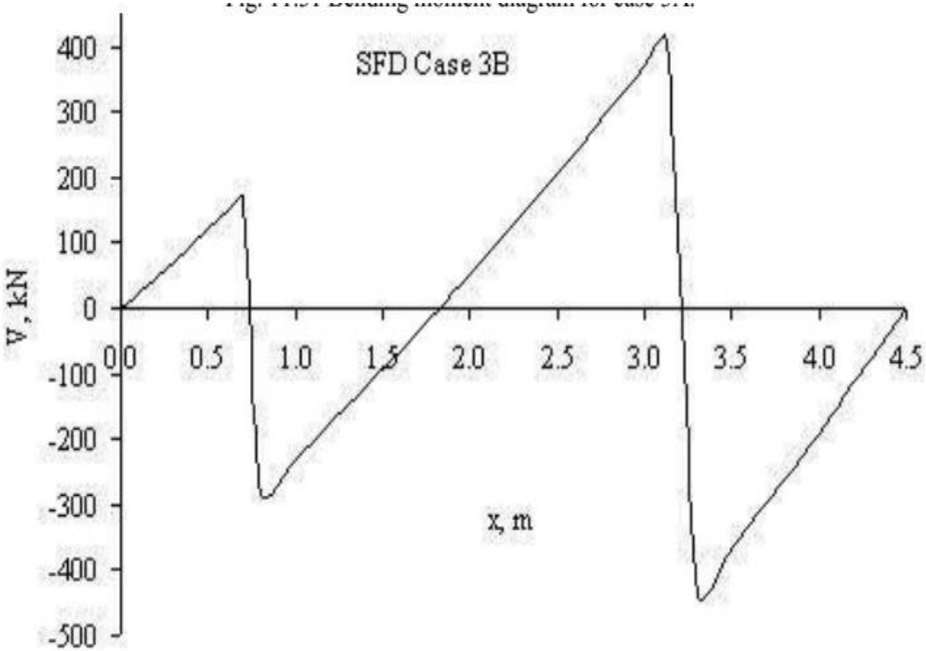
p1 = 1388.9 /9.0 - 262.9 /6.75 = 115.3 kN/m^2

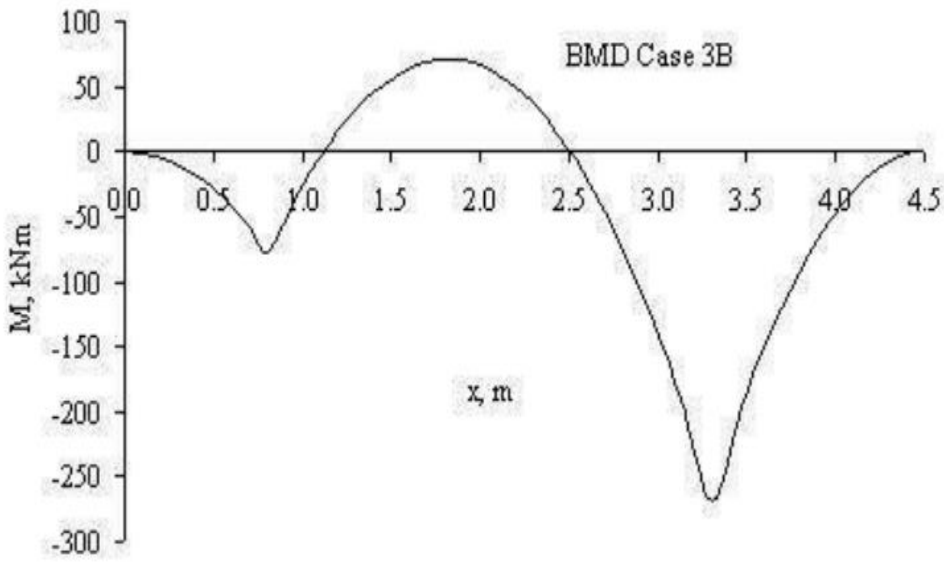
p2 = 1388.9 /9.0 + 262.9 /6.75 = 193.2 kN/m^2

Example on Combined footings

x	V	M	Remarks
0.075	17.4	1.0	d from left face of column 1
0.625	150.9	46.4	Left face of column 1
0.975	−236.7	31.3	Right face of column 1
1.525	–86.1	−57.9	d from right face of column 1
2.04	0	**−70.8**	Maximum negative moment
2.575	**230.6**	–14.6	d from left face of column 2
3.125	411.7	190.8	Left face of column 2
3.475	–378.0	**196.6**	Right face of column 2
4.025	–179.8	42.7	d from right face of column 2

Design values: shear force = 230.6 kN, moment = 196.6 kNm and –70.8 kNm.


SFD Case 3B
V , kN
400
300
200
100
0
-100
-200
-300
-400
-500
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
x, m


BMD Case 3B
M, kNm
100
50
0
-50
-100
-150
-200
-250
-300
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
x, m

Example on Combined footings

c. **Design of longitudinal reinforcement**

i. ***Bottom steel***

The maximum moment is from case 3B

M = 196.6 kNm

$k = M/ (bd^2 f_{ck}) = 196.6 \times 106/ (2000 \times 550^2 \times 30)$

$= 0.011 < 0.196$

$$\frac{z}{d} = 0.5[1.0 + \sqrt{(1 - 3\frac{k}{\eta})}]$$ z/d= 0.99

$f_{yk} = 500$, $f_{yd} = 500/1.15 = 435$ MPa

$As = 196.6 \times 106/ (435 \times 0.99 \times 550) = 830\ mm^2$

Example on Combined footings

Check minimum steel:

$A_s, min = 0.26\times (f_{ctm}/f_{yk}) \times bd \geq 0.0013\ bd$

$f_{ctm} = 0.3 \times f_{ck}\ 0.67 = 0.3 \times 30\ 0.67 = 2.9$ MPa, $f_{yk} = 500$ MPa

$b = 2000$ mm, $d = 550$ mm

$A_s, min = 0.26\times (2.9/500) \times 2000 \times 550 \geq 0.0013 \times 2000 \times 550$

$A_s, min = 1659\ mm^2 > 830\ mm^2$

Provide minimum reinforcement.

Provide 9ϕ16 at 240 mm centers to give a total area of 1809 mm².

$(\ell c = 1000mm) < \{0.75(c + 3d) = 0.75(350 + 3 \times 550) = 1500\ mm\}$

Reinforcement should be spread uniformly across the width.

Example on Combined footings

ii. ***Top steel***

The maximum moment from case 2A is ,M = 95.3 kNm

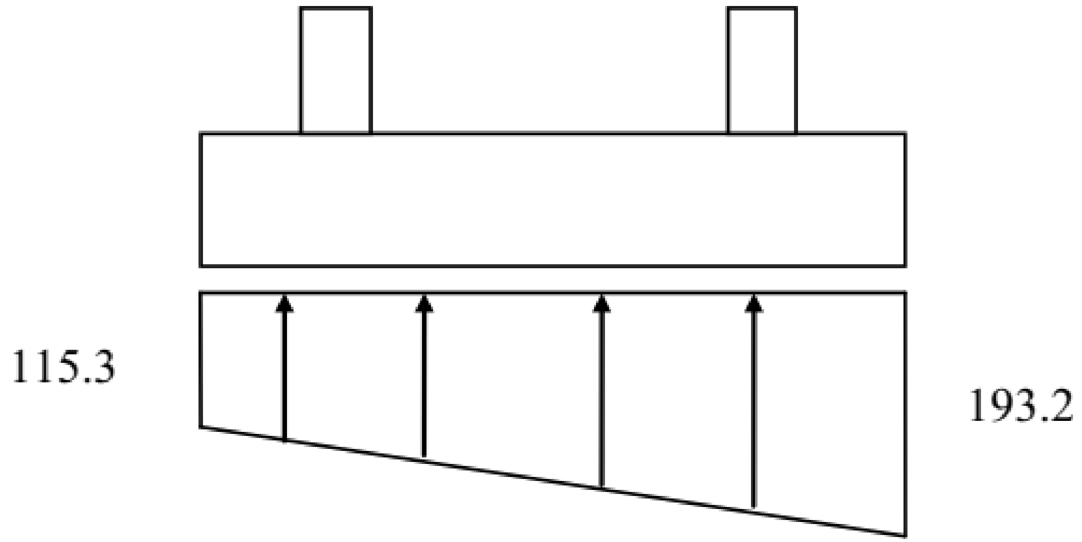
Provide minimum reinforcement as bottom steel.

Provide 9ϕ16 at 240 mm centers to give a total area of 1809 mm^2.

Example on Combined footings

d. **Transverse reinforcement**

At ULS, the base pressure distribution in kN/m^2 is shown below. The maximum pressure under the base is for case 3B.



Example on Combined footings

- The bending moment along the length of 4.5 m is variable. In order to calculate a moment which is reasonable, the average pressure over a width of 0.5 m of the footing length is calculated. The pressure at 0.5 m from the end is

  115.3 + (193.2 – 115.3) × 4.0/4.5 = 184.5 kN/m^2

  The average pressure on a 0.5 m length at the heavier end is

  (193.2 + 184.5) /2 = 188.9 kN/m^2

  (2000 – 350)/2 = 825 mm, d = 550 –16 = 534 mm

  The moment at the face of the columns on a 0.5 m length strip at the heaviest loaded end is

  M = {188.9 × (0.5 × 0.825) × 0.825/2 = 32.14 kNm

  k = M/ (bd^2 fck) = 32.14 × 106/ (500 × 5342 × 30) = 0.008 < 0.196

  z/d= 0.99

Example on Combined footings

fyk = 500, fyd = 500/1.15 = 435 MPa

As = 32.14 × 106/ (435 × 0.99 × 534) = 140 mm^2.

Check minimum steel:

As, min = 0.26× (fctm/fyk) × bd ≥ 0.0013 bd

fctm= 0.3 × fck 0.67 = 0.3 × 30 0.67 = 2.9 MPa,

fyk = 500 MPa, b = 500 mm, d = 534 mm

As, min = 0.26× (2.9/500) × 500 × 534 ≥ 0.0013 × 500 × 534

As, min = 402 mm^2 > 140 mm^2 Provide minimum reinforcement.

Total steel for a width of 4500 mm is 402 × 4500/500 = 3624 mm^2.

Provide 19ϕ16 at 245 mm centers to give a total area of 3820 mm².

Reinforcement should be spread uniformly across the length of the base.

Note that same reinforcement is provided at top and bottom faces.

Example on Combined footings

e. **Vertical shear**

The maximum vertical shear from case 1B is

V_{Ed} = 235.7 kN

v_{Ed} = 235.7 × 10^3/ (2000 × 550) = 0.21 MPa

A_{sl} = 9H16 = 1810 mm2

100 × ρ1 = 100 × 1810/ (2000 × 550) = 0.165 < 2.0

$C_{Rd,\,c}$ = 0.18/ (γc = 1.5) = 0.12, k = 1 + √ (200/550) = 1.60 ≤ 2.0,

$C_{Rd,\,c}$ × k × (100 × ρ1 × fck) 0.33 = 0.12 × 1.60 × (0.165 × 30)0.33 = 0.33

v_{min} = 0.035 × k1.5 × √fck= 0.035 ×1.60 1.5 × √30 = 0.39 > 0.33

$v_{Rd,\,c}$ = 0.39 MPa

(v_{Ed} = 0.21) < ($v_{Rd,c}$ = 0.39)

No shear reinforcement is required.

Example on Combined footings

f. **Punching shear**

Check punching shear **at column perimeter**.

V_{Ed} = column load – base pressure × column area.

The maximum value is 889.4 kN for column 2 from case 3B.

$u_0 = 2 \times (350 + 350) = 1400$ mm, d = 550 mm

$v_{Ed} = 889.4 \times 10^3 / (1400 \times 550) = 1.16$ MPa

$v_{Rd,max} = 0.3 \times (1 - fck/250) \times fcd = 0.3 \times (1 - 30/250) \times (30/1.5) = 5.28$ MPa

$v_{Ed} < v_{Rd}$, max.

The thickness of slab is adequate.

Example on Combined footings

Case	Column 1			Column 2		
	Load	Pressure	V_{Ed}	Load	Pressure	V_{Ed}
Case 1A	658.5	173.1	637.3	811.5	156.2	792.4
Case 1B	586.5	156.3	567.4	910.5	173.6	889.2
Case 2A	658.5	171.5	637.4	661,0	128.6	645.3
Case 2B	586.5	154.6	567.6	760.0	145.9	742.1
Case 3A	550.0	146.1	532.1	811.5	155.1	792.5
Case 3B	478.0	129.2	462.2	910.5	172.5	**889.4**

$$p = p_1 + (p_2 - p_1)\frac{x}{4.5}$$

X = 0.8m Column 1

X = 3.3m Column 2

V_{Ed} = column load – base pressure × column area.

Example on Combined footings

- Check punching shear is checked at perimeters at d to 2 d from the face of a column.

At d = 550 mm from the face of the column,

u = perimeter = 2 × (350 + 350) + 2 × π × 550 = 4856 mm

A = Area under perimeter = [4 ×350 × (350/2 + 550) + π × 550^2] × 10-6 = 1.965 m^2

Column load = 910.5 kN, base pressure at centre line of column= 172.5 kN/m^2

- $V_{ed,red}$ = 910.5 – (172.5 × 1.965) = 571.5 kN

v_{Ed} = 571.5 × 10^3/ (4856 × 550) = 0.21 MPa

Asl in x-direction = 9H16 = 1810 mm2

100 × ρ_x = 100 × 1810/ (2000 × 550) = 0.165

Asl in y-direction = 19H16 = 3992 mm2

100 × ρ_y = 100 × 3992/ (4500 × 550) = 0.16

100ρ1 = √ (0.165 × 0.16) = 0.162 < 2.0

Example on Combined footings

$C_{Rd,c}$ = 0.18/ (γ_c = 1.5) = 0.12,   k = 1 + √ (200/550) = 1.60 ≤ 2.0,

$C_{Rd,c}$ × k × (100 × ρ1 × fck) 0.33 = 0.12 × 1.60 × (0.162 × 30)0.33 = 0.33

v_{min} = 0.035 × k1.5 × √fck= 0.035 × 1.60 1.5 × √30 = 0.39 > 0.33

$v_{Rd,c}$ = 0.39 MPa

(v_{Ed} = 0.21) < (v_{Rd} = $v_{Rd,c}$ × {2d/ (a = d)} = 0.78)

The thickness of slab is adequate.

At **1.5 d from the face of the column**, the perimeter touches the edge of the slab on the width side. The punching shear is **less critical** than the vertical shear in this case. **The slab is safe against punching shear failure.**

Example on Combined footings

g. **Sketch of reinforcement**