Addis Ababa University
Addis Ababa Institute of Technology
School of Civil and Environmental Engineering

Fundamentals of Geotechnical Engineering III (CEng3143)
Mid-term Examination Solution Key

Name	
ID No.	
Signature	
Section	
Exam Date:	27.05.2019

Instruction:

1) This examination is closed book and constitutes 40% of your final grade.
2) The time allowed for this exam is 3 hours.
3) Please read the questions carefully and make sure you understand the facts before you begin answering. Write as legibly and concisely as possible.
4) Use the provided space properly to present you answer.

Question #	Weight [marks]	Score [marks]
1	60	
[redacted]		
2	40	

Examination paper set checked by: Henok Fikre (Dr.-Ing.)

Signature:

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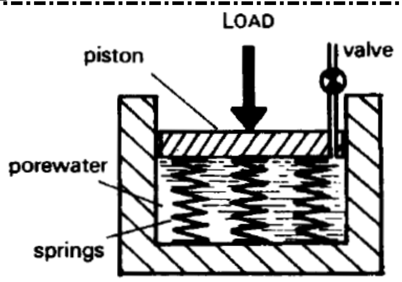
QUESTION 1: On Soil Compressibility & Settlement Analysis [60%]

1.1 Theoretical Background

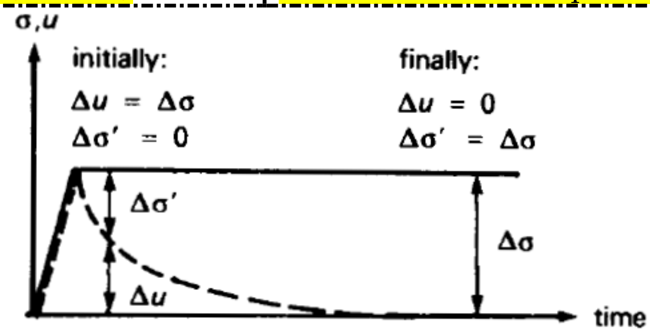
1.1.1 List out the components of settlement and their corresponding causes. (3 marks)

Settlement component	Corresponding cause
Immediate settlement (Elastic Settlement) (0.5 marks)	caused by the elastic deformation of dry soil and of moist and saturated soils without any change in the moisture content. (0.5 marks)
Primary consolidation settlement (Consolidation) (0.5 marks)	result of a volume change in saturated cohesive soil because of expulsion of the water that occupies the void spaces. (0.5 marks)
Secondary consolidation settlement (Creep) (0.5 marks)	observed in saturated cohesive soils and is the result of the plastic adjustment of soil fabrics. (0.5 marks)

1.1.2 With regard to the spring model put forward to simulate one-dimensional consolidation, draw a principal sketch and highlight the basic components and what they represent in actual soil.
Also explain each of the experiment's steps and results making parallel reference with that of actual soil consolidation phenomena.
Plot a rough sketch of stress vs time graph to complement your explanation. (12 marks)

Principal sketch	Components
 (1 mark)	Steel springs - represent the soil solid. (1 mark)
	The frictionless piston - supported by springs. (1 mark)
	The cylinder - filled with water. (1 mark)

Spring model	Actual soil
The initial increase in total stress (upon loading) is fully attributed to an increase in porewater pressure. (1 mark)	Load applied to piston with valves closed, length of springs remain unchanged, induced increase in total stress taken wholly by an equal increase in the water. (1 mark)
As time progresses, the porewater seeps out of the soil, the increase in porewater pressure is dissipated. When the whole excess pore water pressure has been dissipated the soil is fully consolidated. (1 mark)	Valve opened; excess water pressure causes the water to flow out; water pressure decreases & piston sinks as springs are compressed; load gradually transferred to the springs causing them to shorten until all the load is carried by the springs. (1 mark)
The rate of compression depends on the permeability of the soil. (1 mark)	The rate of compression depends on the extent to which the valve is opened. (1 mark)



1.1.3 Lay out the assumptions, indicate their implications and derive the Terzaghi-Froelich 1D consolidation equation for time rate of settlement using an element of the soil sample of thickness *dz* and cross-sectional area *dA*=*dx dy*. (10 marks)

Assumption	Implication
• Saturated (1 mark)	
• Isotropic (1 mark)	
• Homogeneous (1 mark)	
• Darcy's law is valid. (1 mark)	
• Flow only occurs vertically. (1 mark)	
• The strains are small. (1 mark)	

- The flow rate is the product of the velocity and the cross-sectional area normal to its direction. The change in flow is then $(\partial v/\partial z)dzdA$.
- The rate of change in volume of water expelled, which is equal to the rate of change of volume of the soil, must equal the change in flow.

That is, $\frac{\partial V}{\partial t} = \frac{\partial v}{\partial z}dzdA$

- The volumetric strain is $\varepsilon_p = \partial V/V = \partial e/(1+e_o)$

$$\partial V = \frac{\partial e}{1+e_o}dzdA = m_v \partial \sigma'_z dzdA = m_v \partial u dzdA$$

$$\frac{\partial V}{\partial z} = \frac{\partial u}{\partial t}m_v$$

From Darcy's law, the 1D flow of water is

$$v = k_z t = k_z \frac{\partial h}{\partial z}$$

where k_z is the hydraulic conductivity in the vertical direction.

- Partial differential equation wrt z

$$\frac{\partial v}{\partial z} = k_z \frac{\partial^2 h}{\partial z^2}$$

The pore water pressure at any time in our experiment is

$$u = h\gamma_w$$

where h is the height of water in the burette.

- Partial differential equation wrt z

$$\frac{\partial^2 h}{\partial z^2} = \frac{1}{\gamma_w}\frac{\partial^2 u}{\partial z^2}$$

- Through substitution

$$\frac{\partial v}{\partial z} = \frac{k_z}{\gamma_w}\frac{\partial^2 u}{\partial z^2}$$

Equating

$$\frac{\partial u}{\partial z} = \frac{k_z}{m_v\gamma_w}\frac{\partial^2 u}{\partial z^2}$$

Let $\frac{k_z}{m_v\gamma_w} = C_v$ with units like cm^2/min

$$\frac{\partial u}{\partial z} = C_v \frac{\partial^2 u}{\partial z^2}$$

(4 marks)

1.2 Oedometer Testing & Interpretation

A specimen of a fine-grained soil, 75 mm in diameter and 20 mm thick, was tested in an oedometer in a laboratory. The initial water content was 62% and Gs=2.7. The vertical stresses were applied incrementally—each increment remaining on the specimen until the porewater pressure change was negligible. The cumulative settlement values at the end of each loading step are as follows:

Vertical stress (kPa)	15	30	60	120	240	480
Settlement (mm)	0.10	0.11	0.21	1.13	2.17	3.15

The time–settlement data when the vertical stress was 200 kPa are:

Time (min)	0	0.25	1	4	9	16	36	64	100
Settlement (mm)	0	0.22	0.42	0.6	0.71	0.79	0.86	0.91	0.93

1.2.1 Generate the appropriate graphs required to determine different parameters of settlement computation. (15 marks)

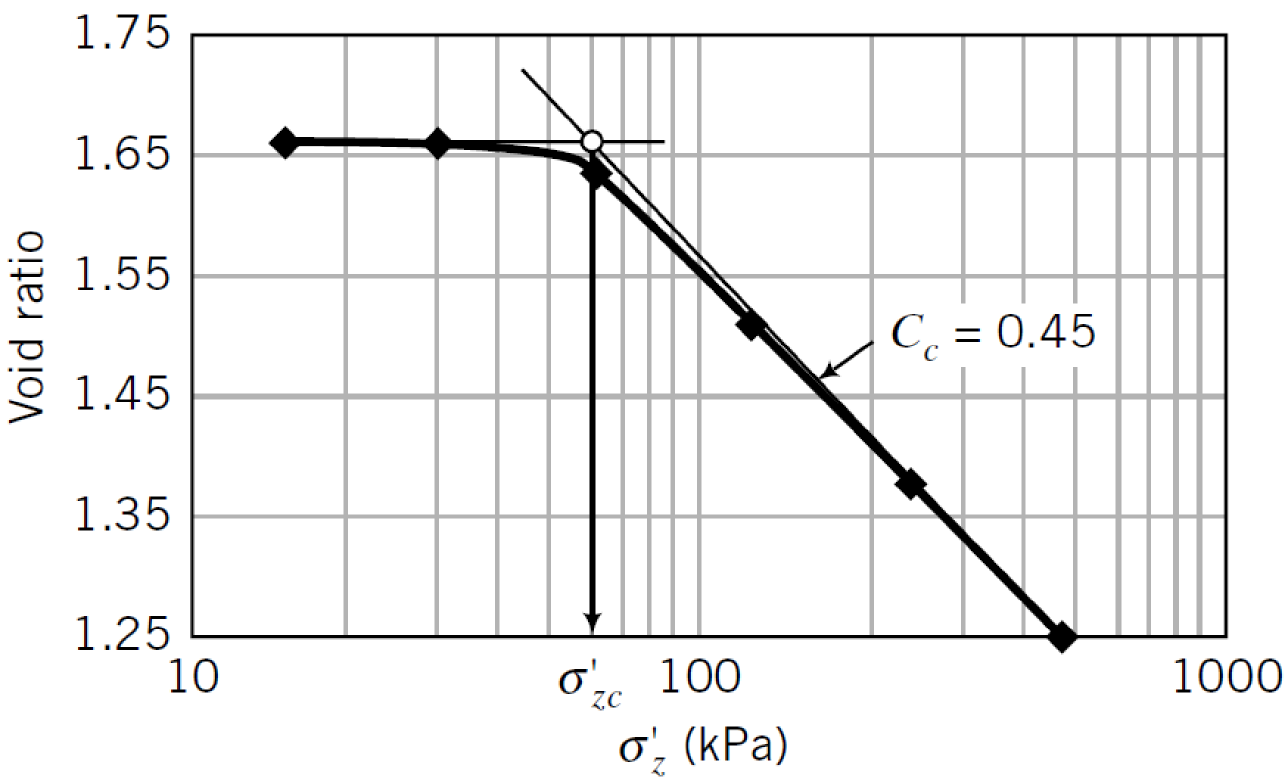
Initial void ratio: $e_o = \omega G_s = 0.62 \times 2.7 = 1.67$ **(1 mark)**

Void ratio: $e = e_o - \frac{\Delta z}{H_o}(1 + e_o) = 1.67 - \frac{\Delta z}{20}(1 + 1.67) = 1.67 - 13.35 \times 10^{-2}\Delta z$ **(1 mark)**

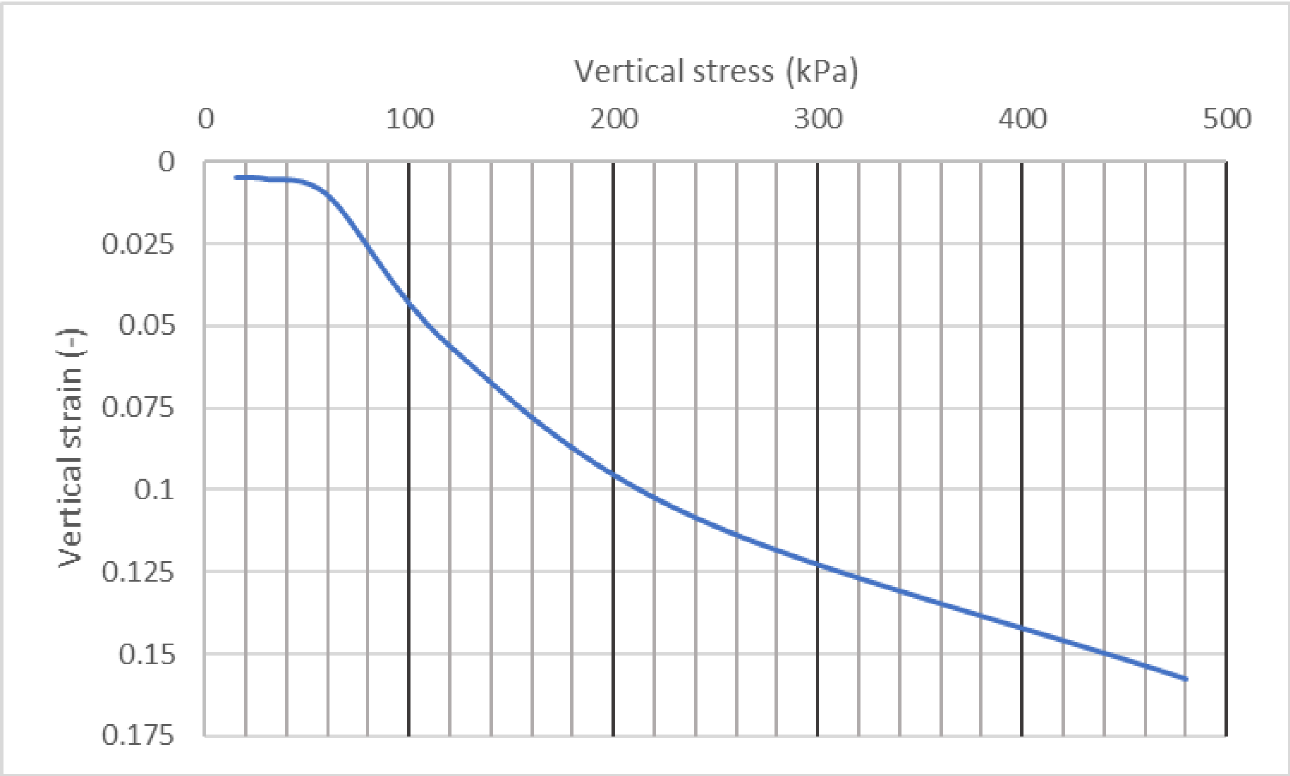
Vertical strain: $\varepsilon_z = \frac{\Delta z}{H_o} = \frac{\Delta z}{20}$ **(1 mark)**

Vertical stress (kPa)	15	30	60	120	240	480
Void ratio (-)	*1.66*	*1.65*	*1.64*	*1.52*	*1.38*	*1.25*
Vertical strain (-)	*0.005*	*0.0055*	*0.0105*	*0.0565*	*0.1085*	*0.1575*

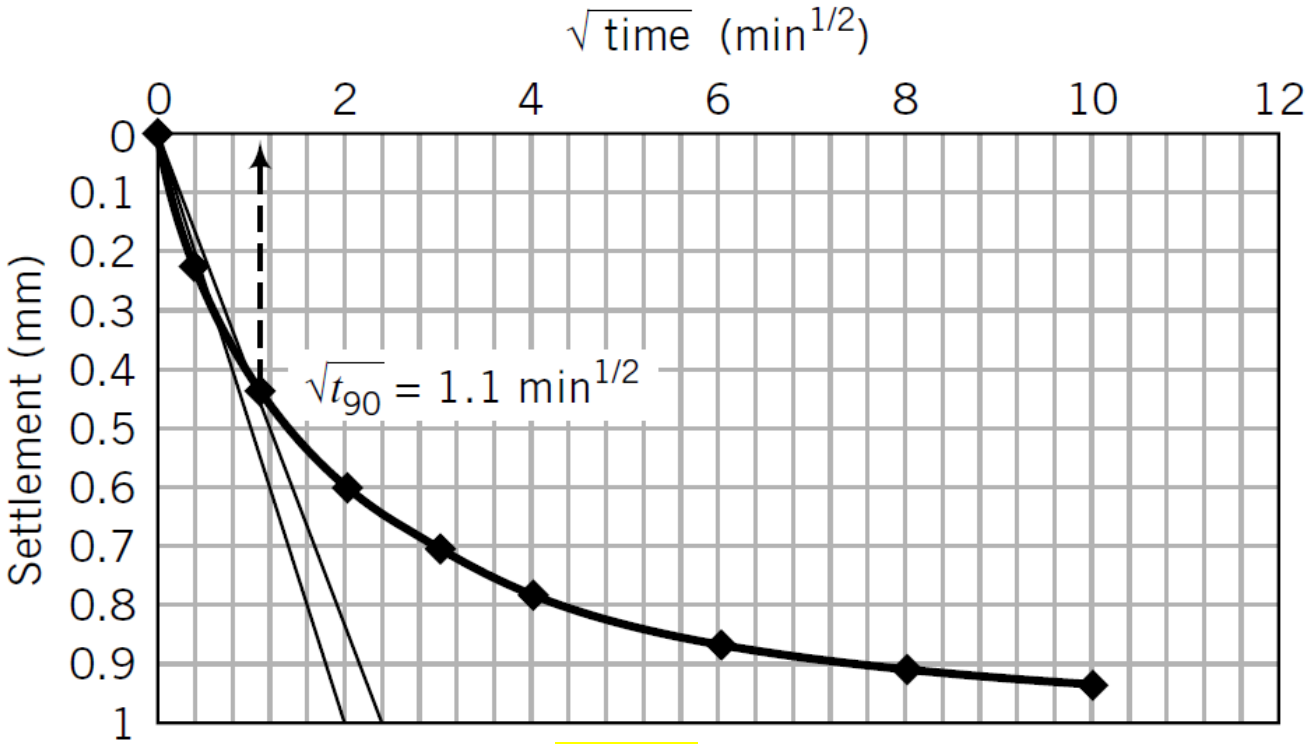
(6 marks)



(2 marks)



(2 marks)



(2 marks)

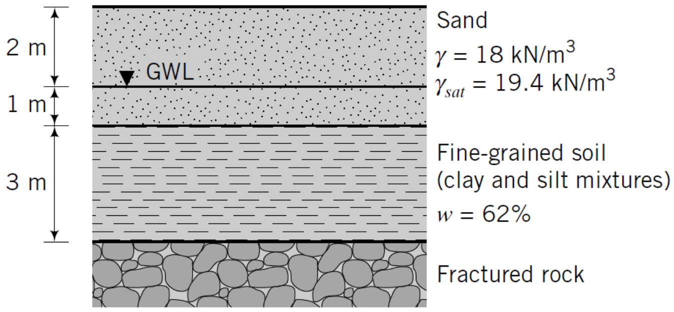
1.2.2 Determine the parameters required for calculation of elastic compression, primary consolidation and secondary consolidation (secondary compression). (10 marks)

Elastic modulus	$E' = \frac{\Delta\sigma}{\Delta\varepsilon} = \frac{480-120}{0.1575-0.0565} = 3570\ kPa$ (1 mark)
Poisson's ratio	$v' = 0.3\ (medium\ stiff\ clay\ assumed)$ (1 mark)
Pre-consolidation pressure	$\sigma'_{zc} = 60\ kPa$ (1 mark)
Over-consolidation ratio	$\gamma_{sat} = \frac{G_s + e_o}{1 + e_o}\gamma_w = \left(\frac{2.7 + 1.67}{1 + 1.67}\right) \times 9.81 = 16\ kN/m^3$ $\sigma'_{zo} = (18 \times 2) + (19.4 - 9.81) * 1 + (16 - 9.81) * 1.5 = 54.9\ kPa$ $OCR = \frac{\sigma'_{zc}}{\sigma'_{zo}} = \frac{60}{54.9} = 1.1$ (1 mark) For practical purposes, the OCR is very close to 1; that is, $\sigma'_{zo} \approx \sigma'_{zc}$. Therefore, the soil is normally consolidated.
Coefficient of consolidation	Use the data from the 240 kPa load step to plot a settlement versus time curve and to find Cv. From the curve, t_{90}=1.2 min. Height of sample at beginning of loading = 20 - 1.2 = 18.8 mm Height of sample at end of loading = 20 - 2.17 = 17.83 mm $H_{dr} = \frac{H_o + H_f}{4} = \frac{18.8 + 17.83}{4} = 9.16mm$ $C_v = \frac{T_v H_{dr}^2}{t_{90}} = \frac{0.848 \times 9.16^2}{1.21} = 59.3\ mm^2/min$ (1 mark)
Compression index	$C_c = \frac{1.52-1.25}{\log\left(\frac{480}{120}\right)} = 0.45$ (1 mark)
Modulus of volume Compressibility	$m_v = \frac{0.0565-0.1575}{480-120} = 0.00028\ m^2/kN$ (1 mark)
Recompression index	Data from unloading part of the test is not provided but inspection of the e versus log σ'_{zc} curve shows that Cr is approximately zero. (1 mark) $C_r = 0.15 * (e_o + 0.007) = 0.15 * (1.67 + 0.007) = 0.25155$ $C_r = 0.003 * (\omega + 7) = 0.003 * (62 + 7) = 0.207$ Azzouz et al., 1976
Modulus of volume recompressibility	Data from unloading part of the test is not provided (1 mark)
Secondary compression index	$\boldsymbol{C_\alpha / C_c = 0.03\ to\ 0.08}$ $\boldsymbol{C_\alpha = (0.03\ to\ 0.08) * 0.45 = 0.0135\ to\ 0.036}$ (1 mark)

1.3 Settlement Calculation

A foundation for circular, oil tank with diameter of 10m is proposed for a site with a soil profile as shown below.

The tank, when full, will impose vertical stresses of 90 kPa and 75 kPa at the top and bottom of the fine-grained soil layer, respectively. You may assume that the vertical stress is linearly distributed in this layer.



Use the parameters you determined in previous question (question 1.2) to perform the following tasks.

1.3.1 Calculate the immediate (elastic) settlement, primary consolidation, secondary compression (consolidation) and total settlement in the middle of the clay layer under the center of the footing. (6 marks)

Elastic compression

$p = \frac{90+75}{2} = 82.5\ kpa$ (1 mark)

$B = 10\ m;\ v = 0.30\ (medium\ stiff\ clay\ assumed);\ E = 3570\ kPa\ ;\ I_p = 0.73$

$S_i = \frac{pB(1-v^2)}{E} I_p = \frac{82.5*10*(1-0.30^2)}{3570} * 0.73 = 0.210\ m\ i.e\ around\ 21\ cm$ (1 mark)

Primary Consolidation

$S_c = C_c \frac{H_o}{1+e_o} log \frac{\sigma'_{zo}+\Delta\sigma_z}{\sigma'_{zo}} = 0.45 * \frac{1500}{1+1.67} log \frac{54.9+82.5}{54.9} = 201mm\ i.e.\ around\ 20cm$ (2 marks)

Secondary Compression

$S_s = \frac{H_o}{1+e_o} C_\alpha \log\left(\frac{t}{t_p}\right)$ No sufficient data (1 mark)

Total settlement

$S_{total} = S_i + S_c + S_s$ (1 mark)

1.3.2 Determine the time required for 90% consolidation to take place in the field.

(4 marks)

Double drainage condition (1 mark)

(H[redacted]$_{dr})_{field} = \frac{H_{clay\ layer}}{2} = \frac{3}{2} = 1.5m$ (1 mark)

$\frac{t_{field}}{t_{lab}} = \frac{(H^2_{dr})field}{(H^2_{dr})lab}$ (1 mark)

$t_{field} = \frac{(H^2_{dr})field}{(H^2_{dr})lab} * t_{lab} = \frac{1500^2}{9.16^2} * 1.21 = 32518.14\ minutes = 22.58\ days$ (1 mark)

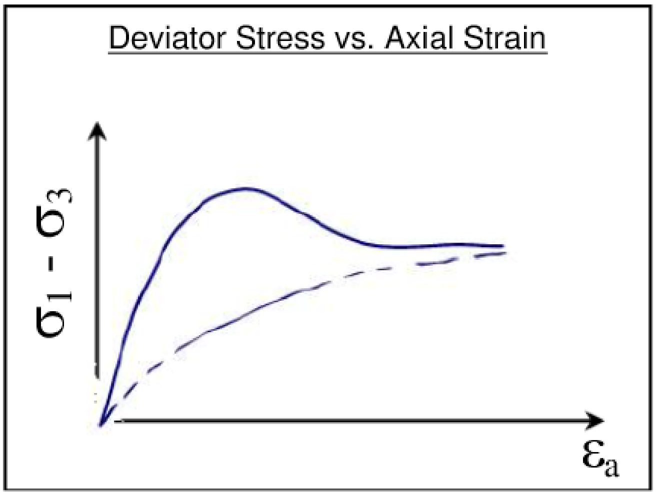
QUESTION 2: On Shear Strength of Soils [40%]

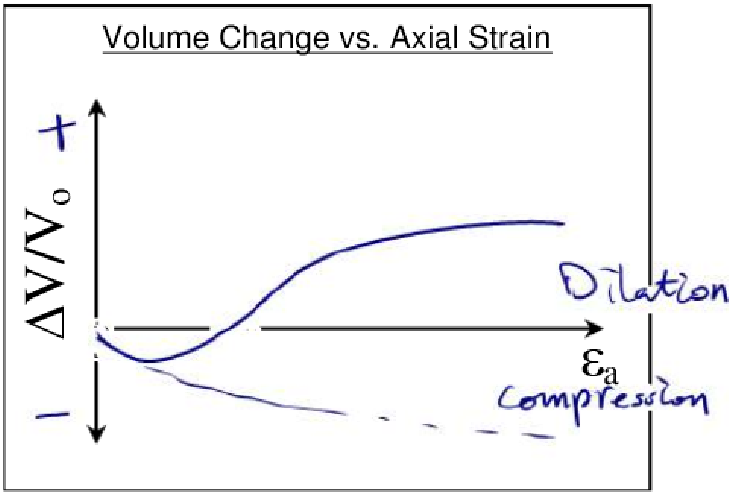
2.1 Theoretical Background

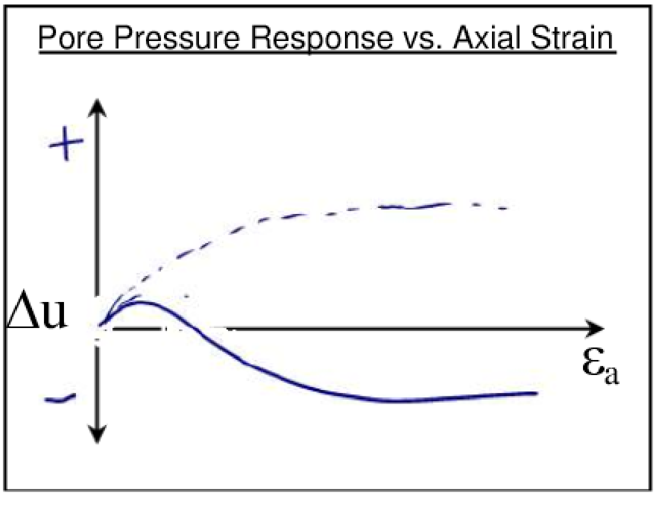
2.1.1 Mention three factors that control the strength of a mass of sand? Briefly outline the influence of each factor. (3 marks)

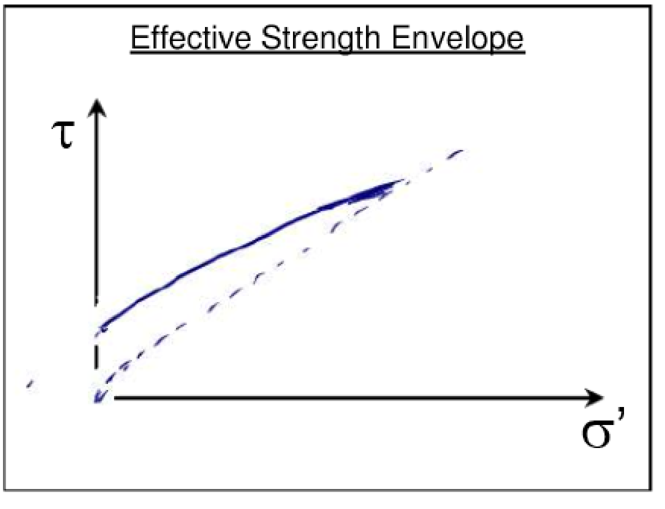
Void ratio	ϕ of low e > ϕ of high e
Grain size distribution	ϕ of SW > ϕ of SP
Grain shape	ϕ of angular > ϕ of rounded

2.1.2 Indicate the behavior of normally consolidated and over-consolidated clays by showing on the following typical diagrams. (4 marks)

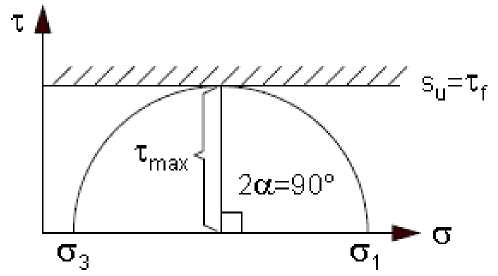
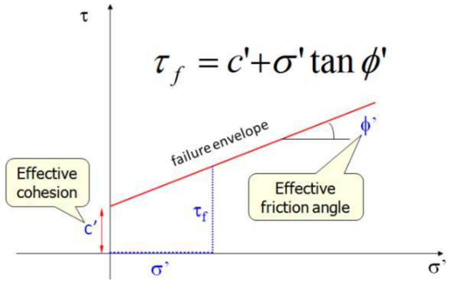








2.1.3 Compare and contrast Tresca and Mohr-Coulomb failure criteria (by means of equations and diagrams if need be). (6 marks)

Tresca Failure Criterion	Mohr-Coulomb Failure Criterion
relevant for loading on cohesive soils (clays and fine silts) when the condition may be assessed as undrained. (0.5 marks)	used in drained conditions where a change in stress also leads to a change in effective stresses. (0.5 marks)
defined by the undrained shear strength, su, being equal to the maximum shear stress at failure. $\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} = \tau_f = S_u$ (0.5 marks)	states that a material fails because of a critical combination of normal stress and shear stress, and not from their either maximum normal or shear stress alone. $\tau_f = C' + \sigma' \tan\phi$ (0.5 marks)
	

2.1.4 What do undrained and drained loading conditions mean? How does each arise in a soil mass? How do we simulate these in triaxial tests? (5 marks)

Undrained loading condition	Drained loading condition
Occur when excess pore water pressure can't drain, at least quickly from the soil. (0.5 marks)	Occur when the load case is a long term condition, allowing pore pressure changes to dissipate. (0.5 marks)
Volume of the soil remains constant. (0.5 marks)	Volume change in the soil imminent. (0.5 marks)
Mostly relevant for loading on cohesive soils (clays and fine silts) (0.5 marks)	Relevant for all granular materials and cohesive soils under extended loading (0.5 marks)
Associated failure criterion is TRESCA (0.5 marks)	Associated failure criterion is MC (0.5 marks)
By leaving the drainage valve open during shearing phase (0.5 marks)	By closing the drainage valve during shearing phase. (0.5 marks)

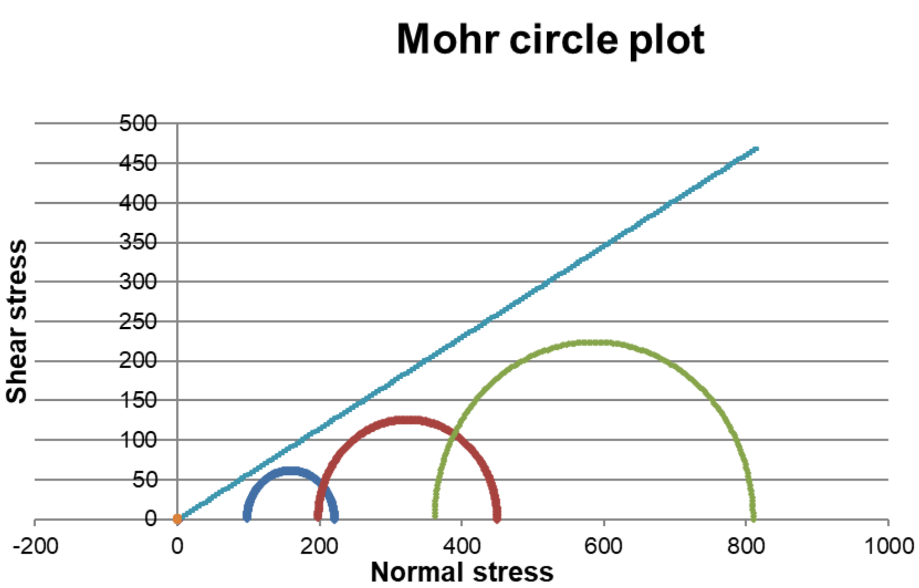
2.2 Triaxial Testing and Interpretation

The failure stresses and excess porewater pressures for three samples of a loose sand in CU tests are given below.

Sample no.	$(\sigma_3)_f$ (kPa)	$(\sigma_1-\sigma_3)_f$ (kPa)	$(\Delta u)_f$ (kPa)	$(\sigma_1)_f$ (kPa)	$(\sigma_1)_f{}'$ (kPa)	$(\sigma_3)_f{}'$ (kPa)	$\phi = \sin^{-1}\dfrac{(\sigma_1)'_f-(\sigma_3)_f{}'}{(\sigma_1)'_f+(\sigma_3)_f{}'}$
1	210	123	112	333	221	98	22.69125169
2	360	252	162	612	450	198	22.89698827
3	685	448	323	1133	810	362	22.48468014

(6 marks)

2.2.1 Perform the necessary calculation and plot Mohr's circle of effective stress from the data. Also determine the friction angle for each test. (12 marks)



(6 marks)

2.2.3 Determine the inclination of (a) the failure plane and (b) the plane of maximum shear stress to the horizontal plane for Test 2. (10 marks)

a)

$$\theta = 45° + \frac{\phi}{2} = 45° + \frac{22.89698827}{2} = 56.45°$$

(5 marks)

b)

(5 marks)