ADDIS ABABA INSTITUTE OF TECHNOLOGY
አዲስ አበባ ቴክኖሎጂ ኢንስቲትዩት
ADDIS ABABA UNIVERSITY
አዲስ አበባ ዩኒቨርሲቲ

SCEE, Geotechnical Engineering Chair

Student ID: ....

Academic contact during examination: Tewodros Gemechu

Phone: +251911394427

Solution set for CEng3143 Fundamentals of Geotechnical Engineering - III

Examination date: 26th June 2019

Examination time (from-to): 15:00 - 18:00

Permitted exam support material: No printed or handwritten material,

Number of pages enclosed: **8 including cover page**

Instruction:
-Write concise answers for theoretical questions.
-Show your steps clearly for problems involving calculations.
-The examination set shall be delivered with the answer sheet.

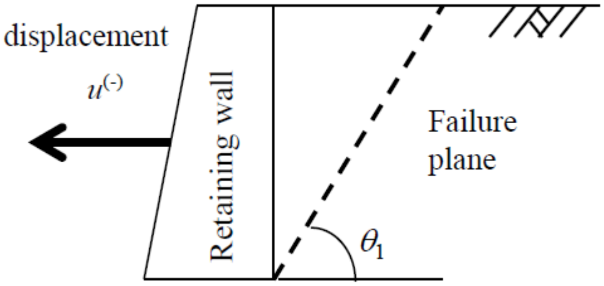
Examination paper set checked by: **Asrat Worku Setegn (Dr.-Ing.)**

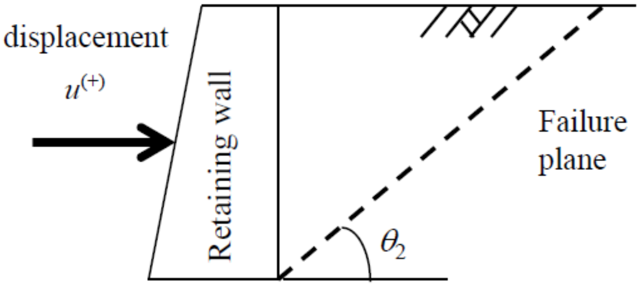
Date ______________________ Signature

Question #	Weight [%]	Score [%]
1	30	
2	40	
3	30	

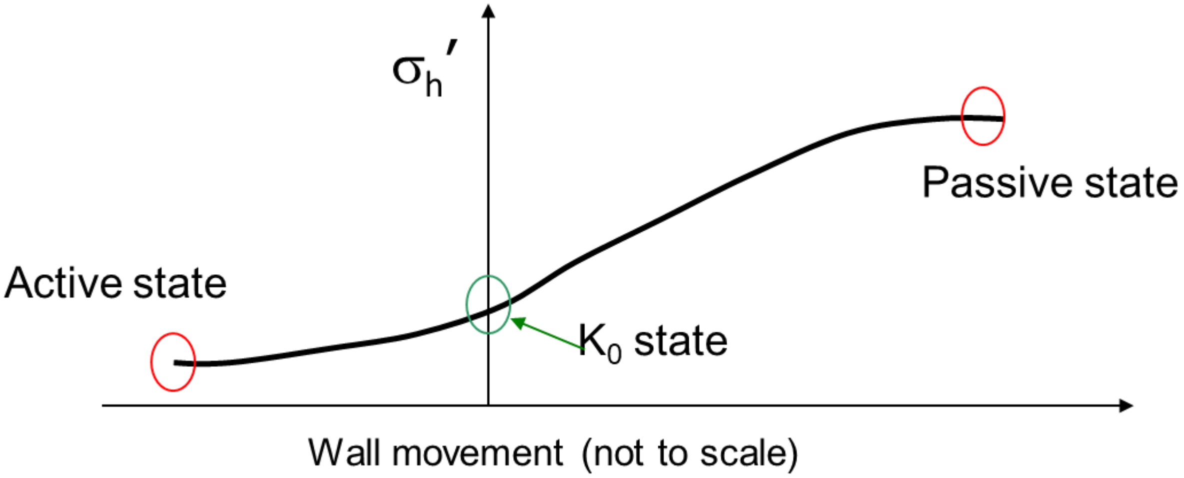
Question 1: On Lateral Earth Pressure [30%]

1.1 In reference to the figures below, indicate which kind of earth pressure each of the figures represent. Draw a graph showing the relationship between the displacement {u$^{(-)}$ and u$^{(+)}$} of the retaining wall and the earth pressure, p that acts on that wall. Also, within the graph, indicate the earth pressures which represent equivalent active and passive earth pressures. (6%)





…………***ACTIVE STATE***…… **(1 mark)** …. ……………***PASSIVE STATE***…… **(1 mark)** ….



(4 marks)

1.2 Explain the conditions at which Rankine's earth pressure theory and Coulomb's earth pressure theory become equal. (4%)

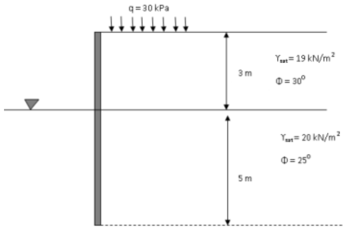
Upon fulfillment of the following conditions / assumptions

- ☐ ***Soil mass is semi-infinite, homogeneous, dry, cohessionless.*** **(2 marks)**
- ☐ ***The face of the wall in contact with the backfill is vertical and smooth.*** **(2 marks)**

1.3 For the frictionless wall shown, determine the following:

a. The active and passive lateral earth pressure distribution with depth.
b. The magnitude and location of the active and passive force.
c. The resultant force and its location.

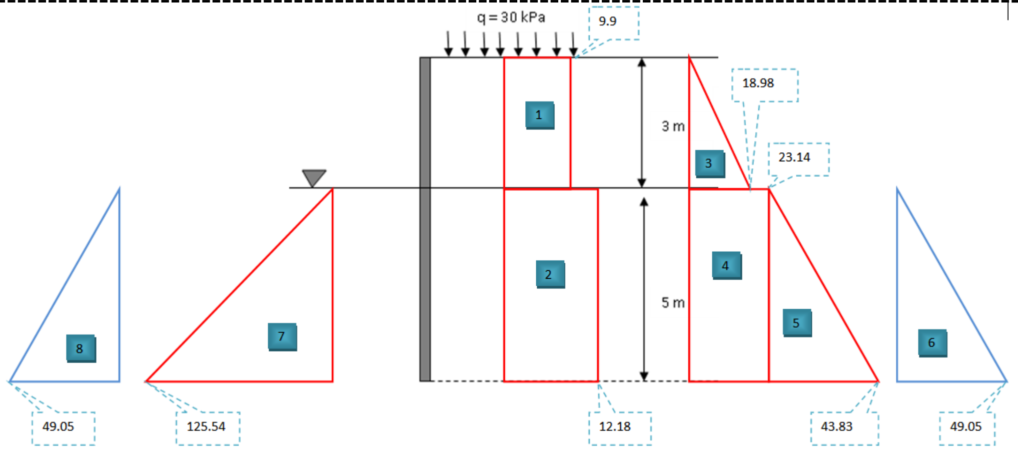
(20%)



Depth [m]	Active LEP Coefficient [-](*2 marks*)	Passive LEP Coefficient [-] (*1 mark*)
0 - 3	$K_a = \frac{1-\sin\phi}{1+\sin\phi} = \tan^2(45 - \frac{\phi}{2}) = 0.333$	
3 - 8	$K_a = \frac{1-\sin\phi}{1+\sin\phi} = \tan^2(45 - \frac{\phi}{2}) = 0.406$	$K_p = \frac{1}{K_a} = 2.464$

Depth, Z [m]	Surcharge, q [kPa]	$\sigma_z = \gamma * Z$ [kPa]	$U = \gamma_w * Z$ [kPa]	$\sigma'_z = \sigma_z - U$ [kPa]	$(\sigma_h)_a = Ka * \sigma v$	
					Ka*q (*2 marks*)	$Ka * \sigma'_z$ (*2 marks*)
0	30	0	0	0	9.9	0
3^-	30	57	0	57	9.9	18.98
3^+	30	57	0	57	12.18	23.14
8	30	157	49.05	107.95	12.18	43.83

Depth, z [m]	$\sigma_z = \gamma * z$ [kPa]	$U = \gamma_w * Z$ [kPa]	$\sigma'_z = \sigma_z - U$ [kPa]	$(\sigma_h)_p = Kp * \sigma v$ (*2 marks*)
0	0	0	0	0
5	100	49.05	50.95	125.54



Active Area	Depth [m]	Force [kN]	Moment Arm [m]	Moment [kN-m]
1	0-3	=9.9*3=29.7	=5+3/2=6.5	193.05
2	3-8	=12.18*5=60.9	=5/2=2.5	152.25
3	0-3	=0.5*18.98*3=28.47	=5+3/3=6	170.82
4	3-8	=23.14*5=115.7	=5/2=2.5	289.25
5	3-8	=0.5*(43.83-23.14)*5=51.73	=5/3=1.67	86.39
6	3-8	=0.5*49.05*5=122.63	=5/3=1.67	204.79
		P_A = 409.13 kN (*1 mark*)	M_A = 1096.55 kN-m (*1 mark*)	

Passive Area	Depth [m]	Force [kN]	Moment Arm [m]	Moment [kN-m]
7	3-8	=0.5*125.54*5=313.85	=5/3=1.67	524.13
8	3-8	=0.5*49.05*5=122.62	=5/3=1.67	204.78
		P_P=436.47 kN (*1 mark*)	M_P=728.91 kN-m (*1 mark*)	

Location of Active Force $Z_A = \frac{M_A}{P_A} = \frac{1096.55}{409.13} = 2.68m$ (*1 mark*)	Location of Passive Force $Z_P = \frac{M_P}{P_P} = \frac{728.91}{436.47} = 1.67m$ (*1 mark*)
Resultant Force R= P_P - P_A = 436.47 - 409.13 =27.54 kN (*1 mark*)	Location of resultant force $Z_R = \frac{M_A - M_P}{R} = \frac{1096.55-728.91}{27.54} = 13.35m$ (*1 mark*)

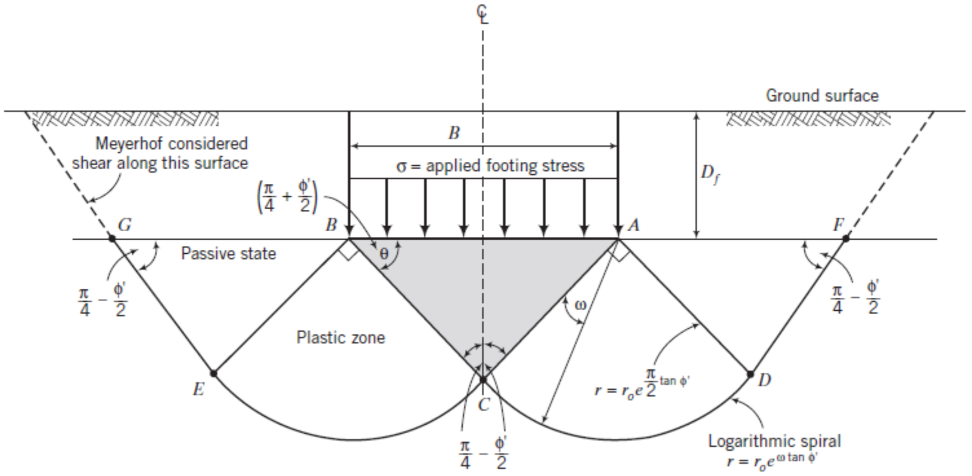
Question 2: On Bearing Capacity of Soils [40%]

2.1 Let B=foundation width, D_f=depth of foundation from the ground surface for a strip foundation.

(i) Draw the failure pattern of the foundation soil that is assumed by Terzaghi to derive his general bearing capacity equation. (3%)

(ii) List down at least 4 other assumptions of Terzaghi in deriving his equation. (4%)

(ii) Terzaghi's bearing capacity equation consists of three terms. Write down the equation and explain the meaning of each of those terms. (3%)



(3 marks)

Terzaghi assumed the following: **(4 marks)**

1. The soil is a semi-infinite, homogeneous, isotropic, weightless, rigid–plastic material.

2. The embedment depth is not greater than the width of the footing ($D_f < B$).

3. General shear failure occurs.

4. The angle θ in the wedge is ϕ. Later, it was found (Vesic, 1973) that $\theta = 45° + \phi'/2$.

5. The shear strength of the soil above the footing base is negligible. Later, Meyerhof (1951) considered the shear resistance above the footing base.

6. The soil above the footing base can be replaced by a surcharge stress ($= \gamma D_f$).

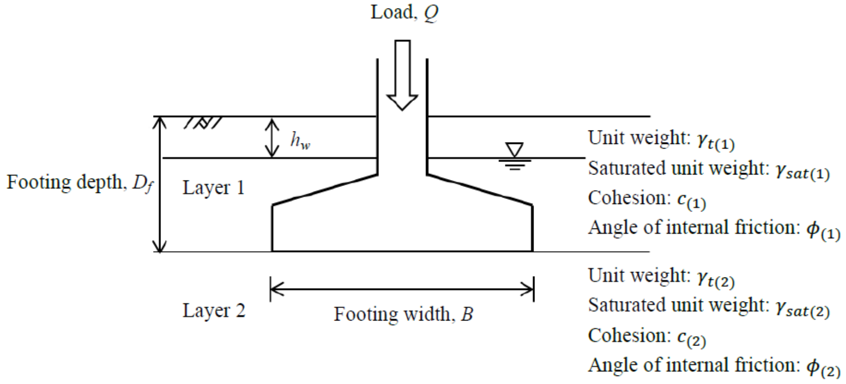
7. The base of the footing is rough.

Terzaghi's solution for strip footing is

$$\boldsymbol{q_u = cN_c + \gamma z N_q + 0.5\gamma B N_\gamma}$$

Contribution from the cohesion, surcharge and self-weight of the soil respectively. **(3 marks)**

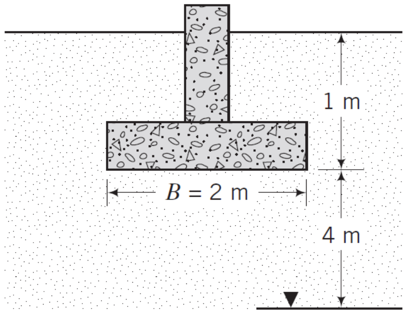
2.2 For the condition depicted in the figure below, re-write the ultimate bearing capacity equation considering the soil layering and the location of the ground water. (5%)



$$\boldsymbol{q_u = c_{(2)}N_c + [(\gamma_{t(1)} * h_w) + (\gamma_{sat(1)} - \gamma_w)(D_f - h_w)]N_q + 0.5(\gamma_{sat(2)} - \gamma_w)BN_\gamma}$$

(5 marks)

2.3 A footing 2 m square, subjected to a centric vertical load, is located at a depth of 1.0 m below the ground surface in a deep deposit of compacted sand $\phi'=35^{\circ}$ and $\gamma_{sat}= 18\text{kN/m}^3$. The groundwater level is 5 m below the ground surface, but you should assume that the soil above the groundwater is saturated. The friction angle is obtained from plain strain tests.
Determine the allowable bearing capacity for a factor of safety of 3 for both short-term and long-term conditions.
Also recalculate the allowable bearing capacity for a factor of safety of 3 if the same footing is subjected to a load inclined at 20° to the vertical.
Make a summarized comparison of the values you calculated. (25%)

	Ultimate bearing capacity equation for total and effective stress analyses Vertical Centric Load only on a Horizontal Footing Resting on a Horizontal Surface TSA: $q_u = 5.14 s_u s_c d_c$ ESA: $q_u = \gamma D_f (N_q - 1) s_q d_q + 0.5\gamma B N_\gamma s_\gamma d_\gamma$ Inclined Load only on a Horizontal Footing Resting on a Horizontal Surface TSA: $q_u = 5.14 s_u i_c$ ESA: $q_u = \gamma D_f (N_q - 1) i_q + 0.5\gamma B' N_\gamma i_\gamma$	
$N_q = e^{\pi \tan\phi'} \tan^2\left(45 + \frac{\phi'}{2}\right)$	$s_q = 1 + \frac{B'}{L'} \tan\phi'$	$d_q = 1 + 2\tan\phi' (1 - \sin\phi')^2 \tan^{-1}\left(\frac{D_f}{B'}\right)$
$N_\gamma = 0.1054 \exp(9.6\phi')$	$s_\gamma = 1 - 0.4\frac{B'}{L'}$	$d_\gamma = 1.0$
$n = n_B = \left(2 + \frac{B'}{L'}\right) / \left(1 + \frac{B'}{L'}\right)$	$i_q = \left(1 - \frac{H}{V_n}\right)^n = (1 - \tan\omega)^n$	$i_\gamma = \left(1 - \frac{H}{V_n}\right)^{n-1} = (1 - \tan\omega)^{n+1}$

Assume rough footing. Use $\phi' = 35^{\circ}$. The eccentricity is zero, so B'=B and L'=L

$$N_q = e^{\pi \tan\phi'} \tan^2\left(45 + \frac{\phi'}{2}\right) = e^{\pi \tan 35^{\circ}} \tan^2\left(45 + \frac{35^{\circ}}{2}\right) = 33.3 \ (1\ mark)$$

$$N_\gamma = 0.1054\, exp(9.6\phi') = 0.1054\, exp\left(9.6\frac{35^{\circ}\times\pi}{180^{\circ}}\right) = 37.1 \ (1\ mark)$$

$$s_q = 1 + \frac{B'}{L'} \tan\phi' = 1 + \frac{2}{2} \tan 35^{\circ} = 1.70 \ (1\ mark)$$

$$s_\gamma = 1 - 0.4\frac{B'}{L'} = 1 - 0.4\frac{2}{2} = 0.6 \ (1\ mark)$$

$$d_q = 1 + 2\tan\phi' (1 - \sin\phi')^2 \tan^{-1}\left(\frac{D_f}{B'}\right) = 1 + 2\tan 35^{\circ} (1 - \sin 35^{\circ})^2 \left[\tan^{-1}\left(\frac{1}{2}\right) \times \frac{\pi}{180^{\circ}}\right] = 1.13$$

(1 mark)

$$d_\gamma = 1.0 \ (1\ mark)$$

$$q_u = \gamma D_f (N_q - 1) s_q d_q + 0.5\gamma B N_\gamma s_\gamma d_\gamma$$
$$= 18 \times 1 \times (33.3 - 1) \times 1.7 \times 1.13 + 0.5 \times 18 \times 2 \times 37.1 \times 0.6 \times 1.0$$
$$= 1515\ kPa \ (2\ marks)$$

$$q_a = \frac{q_u}{FS} + \gamma D_f = \frac{1515}{3} + 18 \times 1 = 523\ kPa\ (2\ marks)$$

For the inclined load

B'=B; $\omega = 20°$

$$n = n_B = \left(2 + \frac{B'}{L'}\right) \Big/ \left(1 + \frac{B'}{L'}\right) = \left(2 + \frac{2}{2}\right) \Big/ \left(1 + \frac{2}{2}\right) = 1.5\ (1\ mark)$$

$$i_q = \left(1 - \frac{H}{V_n}\right)^n = (1 - \tan\omega)^n = (1 - \tan 20°)^{1.5} = 0.51\ (1\ mark)$$

$$i_\gamma = \left(1 - \frac{H}{V_n}\right)^{n-1} = (1 - \tan\omega)^{n+1} = (1 - \tan 20°)^{1.5+1} = 0.32\ (1\ mark)$$

$$q_u = \gamma D_f (N_q - 1) i_q + 0.5 \gamma B' N_\gamma i_\gamma = 18 \times 1 \times (33.3 - 1) \times 0.51 + 0.5 \times 18 \times 2 \times 37.1 \times 0.32 = 510\ kPa\ (2\ marks)$$

$$q_a = \frac{q_u}{FS} + \gamma D_f = \frac{510}{3} + 18 \times 1 = 188\ kPa\ (2\ marks)$$

The allowable bearing capacity for a vertical centric load is 523 kPa.

➔ ***Reduction in allowable bearing capacity is*** $\frac{523-188}{523} = 64\%\ (3\ marks)$

For short-term condition (i.e. total stress analysis), sufficient data has not been provided in both loading condition. (5 marks)

Question 3: On Stability of Soil Slopes [30%]

3.1 Fellenius's method is one of the slope stability analysis methods where failure surface is assumed to be a circular arc and the failing soil mass is divided into several slices. In this method the following three conditions are considered; a) the condition of equilibrium of forces along the direction perpendicular to the failure plane, b) the condition of moment equilibrium around the center of the circular arc, and (c) the conditional factor of safety (FS) with respect to Mohr-Coulomb failure condition.
Now, for the condition where pore water pressure is developed within the slope due to rain fall, write down the conditional equations (a)~(c) mentioned above. From these conditional equations, derive following equation for the safety factor.

$$FS = \frac{\sum(W_i \cos\alpha_i - U_i)\tan\phi_i}{W_i \sin\alpha_i}$$

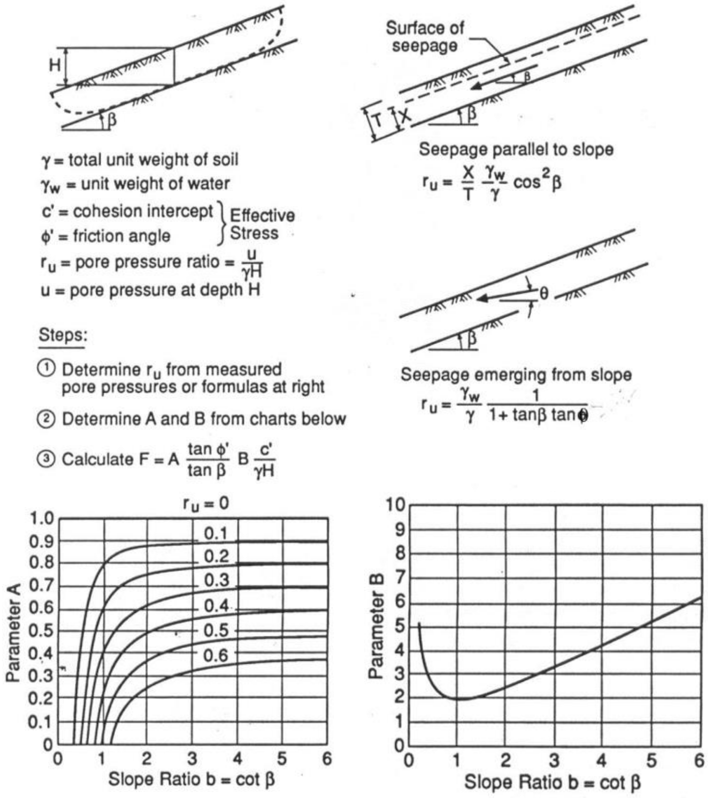
where ϕ_i = internal friction angle of soil; cohesion=0, α_i =sloping angle of the failure plane; W_i =self-weight of the slice, U_i=pore water pressure (15%)

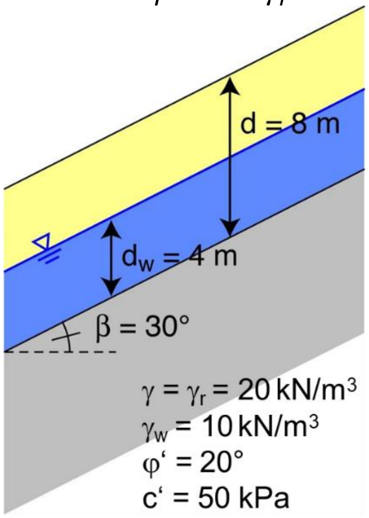
3.2 Consider the equation given in 3, explain why landslides are more likely to occur during/after rainfall. (5%)

3.3 Consider the slope represented in the figure on the right-hand side. Determine the factor of safety using the equation

$$FS = A \cdot \frac{\tan\phi'}{\tan\beta} + B \cdot \frac{c'}{\gamma_r \cdot d}$$

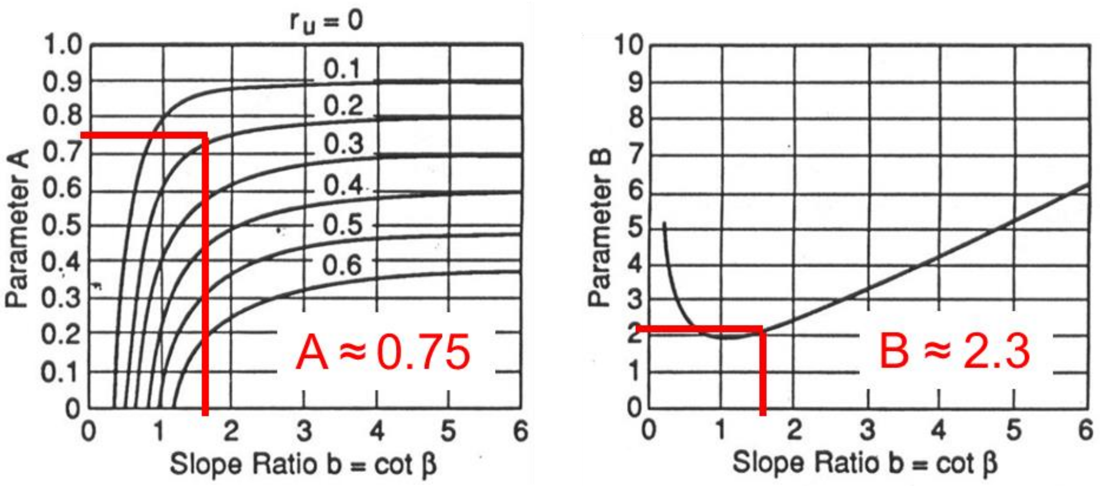
Use the modified Duncan stability chart given below to determine parameters A and B. (10%)





$$b = \cot\beta = 1.73\,;\quad r_u = \frac{X}{T}\frac{\gamma_w}{\gamma}\cos^2\beta = \frac{4}{8}\frac{10}{20}\cos^2 30 = 0.19$$

(4 marks)



(4 marks)

$$FS = 0.75 \cdot \frac{\tan 20}{\tan 30} + 2.3 \cdot \frac{50}{20 \cdot 8} = 1.19$$

(2 marks)