Darko Dujmović / Boris Androić / Ivan Lukačević

Composite Structures according to Eurocode 4

Worked Examples

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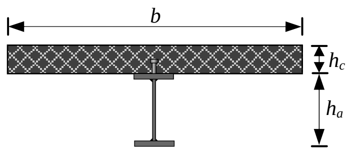
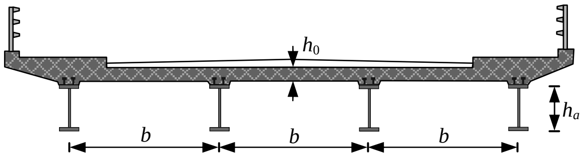
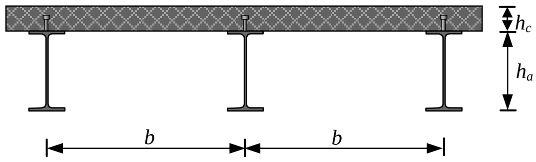
A1		3
A2		15
A3		27

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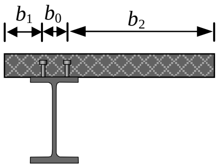
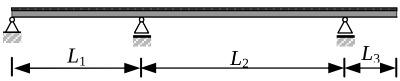
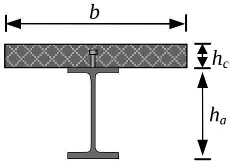
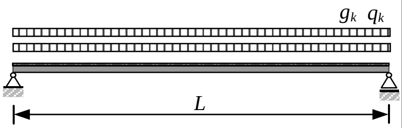
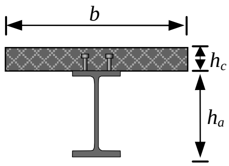
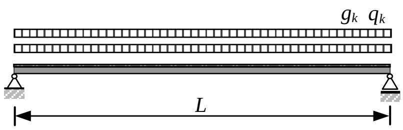
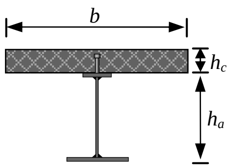
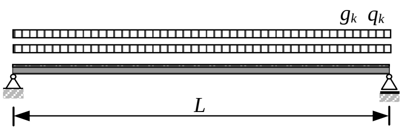
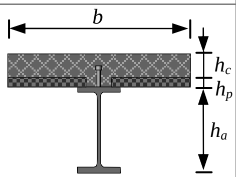
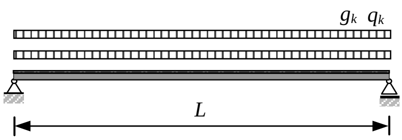
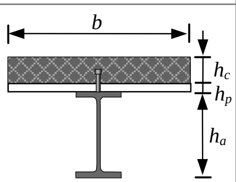
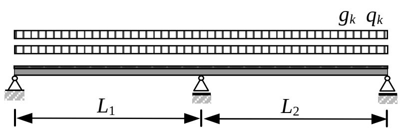
B1			47
B2			53
B3			67
B4			111
B5			151
B6			177

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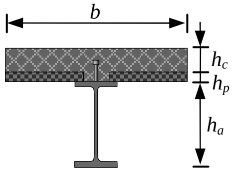
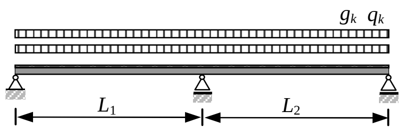
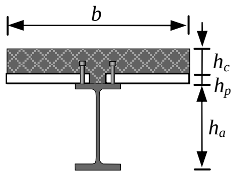
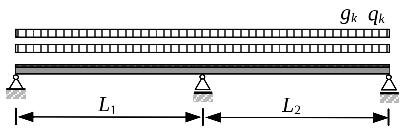
B7			243
B8			297

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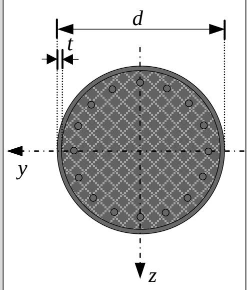
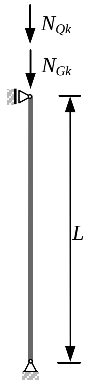
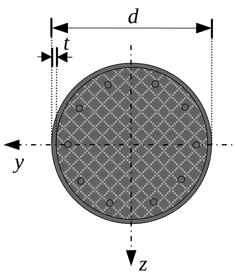
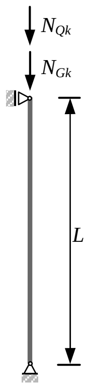
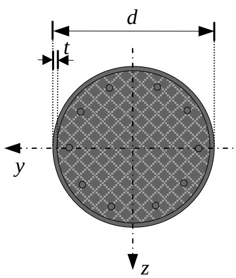
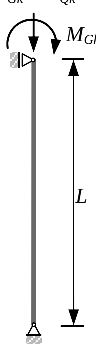
C1			399
C2			419
C3			445

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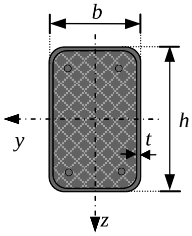
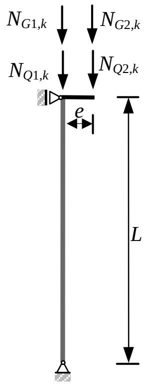
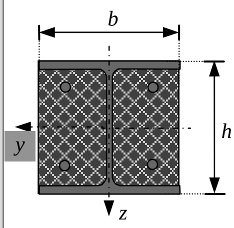
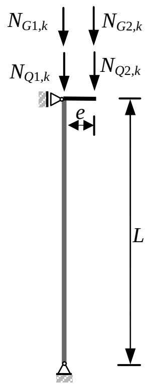
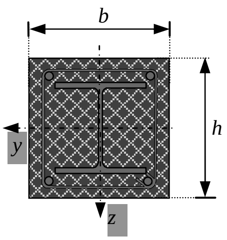
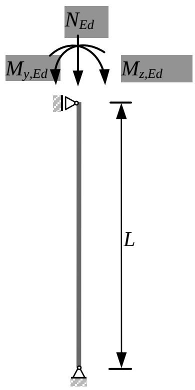
C4			487
C5			545
C6			615

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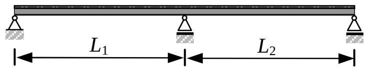
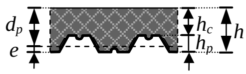
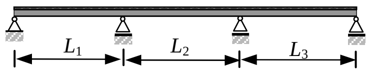
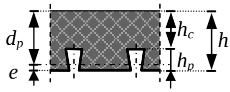
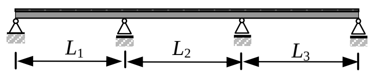
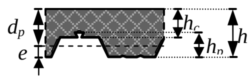
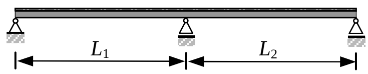
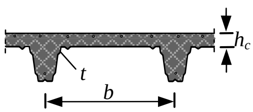
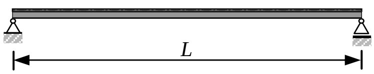
D1			673
D2			697
D3			723
D4			765
D5			797

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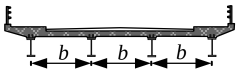
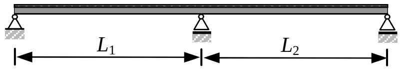
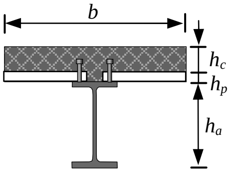
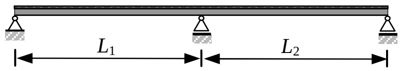
E1			827
E2			857

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Introduction

Between the development and the implementation of the Eurocodes that are currently published and in effect in many countries across Europe a number of years have passed. Given the long time of initial adoption of the Eurocodes some of the tests and methods of verification used in the current standards originate from the 1980s or 1990s. As is inherent in any standard, the Eurocodes have no educational character; their purpose is not to explain how they originated or developed. For Eurocode 4, this actually means that EN 1994-2004 no longer represents the state of the art for composite construction in Europe. The current level of scientific knowledge is not represented in the codes and they obviously do not take account of any forms of interaction between steel and concrete that now exist in the European construction markets but where the techniques were developed after adoption of the code. Only the next generation of Eurocode 4 due to come into force in 2018 as EN 1994-2018 will be based on the reactions and comments of the construction industry to the current standard.

In the meantime, we thought it highly necessary for practicing engineers to know well the details of calculation in accordance with the standard EN 1994-2004 that is currently published. This is exactly why we have compiled these fully worked out numerical examples in this book. The examples provided herein are intended for anyone involved in the detailed design of a composite structure of steel and concrete.

The examples listed in Chapter A represent the calculation of the values of the time-dependent concrete deformations due to creep and shrinkage. These values are included in EN 1992 (Design of concrete structures) but are used in EN 1994 as well. The final values of the creep coefficient are determined by means of nomograms in EN 1992. However, EN 1994 does not provide any nomograms for the determination of the final values of shrinkage deformations. For that reason, the nomograms that can be found in literature have not been used in these examples. The values of the time-dependent concrete deformations are given in the examples so as to enable the structural engineers to use them in practice.

The examples given in Chapter B refer to beams composed of steel profiles and concrete flanges. Although these structural elements have been thoroughly discussed in EN 1994, there are still some dilemmas about the calculation of the serviceability limit state. Those dilemmas are pointed out and commented at the end of the examples. It should be expected that they will be solved or better substantiated in the next edition of the Eurocode. Current practice utilises more and more often beams composed of structural steel and concrete with increased strength, but they are still not adequately represented in the current standard. Similarly, although frequently used, pre-stressed elements are not covered by the rules of EN 1994 at all. The examples given in Chapter B show in a detailed way a set of problems associated with the calculation of the bearing capacity of the shear connectors whose resistance is determined by a push-out test. The present test does not give sufficiently accurate data on ductility, so it will be necessary to present a more accurate, but also a more expensive, test in the future. Steel girders with openings, connected with concrete flanges represent a modern technical solution frequently applied, but there are no corresponding guidelines in EN 1994.

Chapter C provides examples for the calculation of composite columns consisting of structural steel and concrete. The recommendation of EN 1994-1-1 is that the calculation should be performed according to a simplified method. But when it comes to columns with non-uniform or asymmetric cross-sections, the verification can be produced only by a general method. Such a method is not convenient for practical purposes, so the standard does not contain any more detailed guidelines for its application. Even if a computer (software) support is used, it is necessary to know in advance the rules of the simplified method. For that reason, the articles associated with the simplified method are discussed in detail in Chapter C. For columns subjected only to axial pressure, the produced verification is the same for both structural steel and composite columns so the "χ" procedure can be used. However, for columns subjected to axial compression and bending, the verification is produced according to the second-order theory – through the introduction of equivalent imperfection. The imperfection is added only in the plane where a failure is to be expected. If it is not obvious which plane is in question, the verification should be produced for both planes. So, for instance, if the column is subjected to axial compression and uniaxial bending, the verification is frequently produced for axial force and biaxial bending. The modification of the new EN 1994-2018 will comprise the amendment to or correction of some informative Annexes because they have not been accepted by some countries. This refers specifically to the fire resistance of columns of concrete-filled tubes covered by the Annex H, EN 1994-1-1.

In the numerical examples given in Chapter D, the composite slabs consisting of steel profiled sheets and concrete are discussed. They highlight the complexities involved in their calculation, and also some dilemmas, which probably need to be resolved in the future. For the next generation EN 1994-2018 currently being developed, one special interest represents the introduction into the standard of new guidelines for some new types of composite slabs. These new types adhere to the principle that it is desirable to have more "hollow space" within the slab cross-section which reduces the amount of concrete as well as the slab's weight but still results in an effective flexural stiffness.

Fatigue problems are discussed in the examples included in Chapter E. A complete estimation of the fatigue of composite elements consisting of structural steel and concrete is given by EN 1994 only for headed studs. The fatigue estimation is produced for structural steel (EN 1993), concrete (EN 1992) and reinforcement (EN 1992), from which it can be concluded that such estimation represents a very complex problem.

The final chapter, F, deals with structural solutions for the joints applied most frequently in practice.

In recent years, composite elements consisting of steel and concrete have not, for various reasons, had the chance to be applied to any great extent – certainly not to the extent that they deserve. However, bearing in mind all the previously stated facts and also some of the dilemmas about the further development of the new generation of the Eurocode, we can say that there is now a new opportunity for the application of composite elements.

Zagreb, October 2014 The authors

A Creep and shrinkage

A1 Determination of creep and shrinkage values

1. Purpose of example

It is necessary to determine the values of creep and shrinkage of concrete in a composite beam with cross-section shown in Figure A1.1 as follows:

- The values of the creep coefficient at $t = \infty$, the final creep coefficient $\varphi(\infty, t_0)$, and at $t = 90$ days which is denoted with $\varphi(90, t_0)$,
- The values of the total shrinkage strain at $t = \infty$ which is denoted with $\varepsilon_{cs}(\infty)$ (the final value) and at $t = 90$ days which is denoted with $\varepsilon_{cs}(90)$.

2. Cross-section

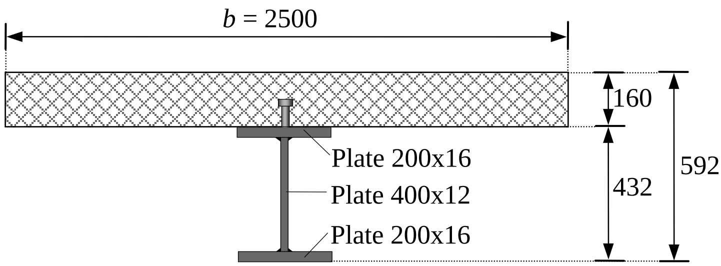

Figure A1.1 *Cross-section*

3. Input data

Concrete strength class: C 20/25 $f_{ck} = 20,0\ \text{N/mm}^2$

$$f_{cd} = \frac{f_{ck}}{\gamma_c} = \frac{20,0}{1,5} = 13,3\ \text{N/mm}^2$$

$$E_{cm} = 30000\ \text{N/mm}^2$$

Type of cement: *N, strength class according to* EN 197-1, 32,5 R $\alpha = 0$

$$\alpha_{ds1} = 4$$

$$\alpha_{ds2} = 0,12$$

Relative humidity: *inside conditions* RH 50%

First loading $t_0 = 28$ days

Beginning of drying $t_s = 3$ days

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4. Creep coefficients

4.1 Determination of final creep coefficient

For the calculation of the final creep coefficient $\varphi(\infty, t_0)$ the following is valid:

- the perimeter of that part which is exposed to drying, u

$$u = 2 \cdot b$$

$$u = 2 \cdot 2500 = 5000 \text{ mm}$$

- the notional size of the cross-section, h_0

$$h_0 = \frac{2 \cdot A_c}{u} = \frac{2 \cdot 2500 \cdot 160}{5000} = 160 \text{ mm } = 16 \text{ cm}$$

- t_0 = 28 days,
- inside conditions, the ambient relative humidity RH 50 %,
- the concrete strength class C 20/25,
- the type of cement – cement class N, strength class 32,5 R.

The final value of creep coefficient $\varphi(\infty, t_0)$ is determined using the nomogram shown in Figure 3.1, EN 1992-1-1. The process of determining the final value of the creep coefficient, taking into account these assumptions, is given in Figure A1.2:

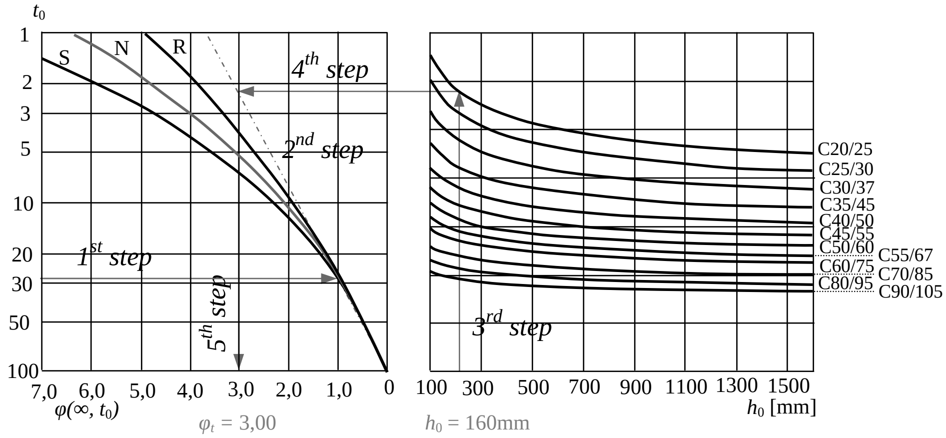

Figure A1.2 *Method for determining the creep coefficient*

The value of the final creep coefficient found from Figure A1.2 is:

$$\varphi_t = \varphi(\infty, t_0) = 3,00$$

4.2 Determination of creep coefficient at time t = 90 days

The value of creep coefficient $\varphi(t, t_0)$ for some arbitrary time t can be calculated from:

$$\varphi(t,t_0) = \varphi_0 \cdot \beta_c(t,t_0)$$

where:

φ_0 is the notional creep coefficient,

$\beta_c(t, t_0)$ is a coefficient to describe the development of creep with time after loading (at $t_0 = 0$, $\beta_c(t, t_0) = 0$, and at $t = \infty$, $\beta_c(t, t_0) = 1$),

The value of φ_0 is obtained as:

$$\varphi_0 = \varphi_{RH} \cdot \beta(f_{cm}) \cdot \beta(t_0)$$

where:

φ_{RH} is a factor to allow for the effect of relative humidity on the notional creep coefficient and is calculated as follows:

$$\varphi_{RH} = 1 + \frac{1 - RH/100}{0{,}1 \cdot \sqrt[3]{h_0}} \quad \text{for } f_{cm} \leq 35 \text{ N/mm}^2$$

$$\varphi_{RH} = [1 + \frac{1 - RH/100}{0{,}1 \cdot \sqrt[3]{h_0}} \cdot \alpha_1] \cdot \alpha_2 \quad \text{for } f_{cm} > 35 \text{ N/mm}^2$$

RH is the relative humidity of the ambient environment (in %),

h_0 is the notional size of the cross-section of the member (h_0 in mm), $h_0 = 2A_c / u$

A_c is the cross-sectional area of concrete (mm^2),

u is the perimeter of the member in contact with the atmosphere (mm),

$\beta(f_{cm})$ is a factor to allow for the effect of concrete strength on the notional creep coefficient and is determined as follows:

$$\beta(f_{cm}) = \frac{16{,}8}{\sqrt{f_{cm}}}$$

where:

f_{cm} is the mean compressive cylinder strength of concrete at the age of 28 days (N/mm^2, $f_{cm} = f_{ck} + 8$ N/mm^2),

$\beta(t_0)$ is a factor to allow for the effect of concrete age at loading on the notional creep coefficient and is determined as follows:

$$\beta(t_0) = \frac{1}{(0{,}1 + t_0^{0{,}20})}$$

The effect of the type of cement on the creep coefficient of concrete can be taken into account by modifying the age of loading t_0 according to the following expression:

$$t_0 = t_{0,T} \cdot [\frac{9}{2 + t_{0,T}{}^{1{,}2}} + 1]^{\alpha} \geq 0{,}5 \text{ days}$$

where:

α is the factor that takes into account the development of concrete strength as a function of type of cement,
$t_{0,T}$ is the temperature-adjusted age of concrete at loading in days.

The effect of elevated or reduced temperatures within the range 0–80°C on the maturity of concrete can be taken into account by adjusting the concrete age according to the following expression:

$$t_T = \sum_{i=1}^{n} e^{-(4000/[273+T(\Delta t_i)-13{,}65])} \cdot \Delta t_i$$

where:

t_T is the temperature-adjusted concrete age which replaces t in the corresponding expressions,
$T(\Delta t_i)$ is the temperature in °C during the time period Δt_i,
Δt_i is the number of days where a temperature T prevails.

$$\beta_c(t,t_0) = [\frac{(t-t_0)}{\beta_H + (t-t_0)}]^{0{,}3}$$

where:

t is the age of concrete in days at the time considered (in days),
t_0 is the age of concrete at first loading (in days),
$t - t_0$ is the non-adjusted duration of loading in days,
β_H is the coefficient depending on the relative humidity (RH in %) and the notional member size (h_0 in mm), and is estimated according to expressions:

$$\beta_H = 1{,}5 \cdot [1 + (0{,}012 \cdot RH)^{18}] \cdot h_0 + 250 \leq 1500 \quad \text{for } f_{cm} \leq 35 \text{ N/mm}^2$$

$$\beta_H = 1{,}5 \cdot [1 + (0{,}012 \cdot RH)^{18}] \cdot h_0 + 250 \cdot \alpha_3 \leq 1500 \cdot \alpha_3 \quad \text{for } f_{cm} > 35 \text{ N/mm}^2$$

α_i are correction factors which take into account the influence of the concrete strength according to the following expressions:

$$\alpha_1 = [35 / f_{cm}]^{0,7}$$

$$\alpha_2 = [35 / f_{cm}]^{0,2}$$

$$\alpha_3 = [35 / f_{cm}]^{0,5}$$

Thus, the mean compressive cylinder strength of concrete from Table 3.1, EN 1992-1-1 is:

$$f_{cm} = f_{ck} + 8 \text{ N/mm}^2 = 20 + 8 = 28 \text{ N/mm}^2$$

$\alpha = 0$ (type of cement N)

Correction factors which taken into account the influence of the concrete strength are:

$$\alpha_1 = [35 / f_{cm}]^{0,7} = [35 / 28]^{0,7} = 1{,}17$$

$$\alpha_2 = [35 / f_{cm}]^{0,2} = [35 / 28]^{0,2} = 1{,}05$$

$$\alpha_3 = [35 / f_{cm}]^{0,5} = [35 / 28]^{0,5} = 1{,}12$$

The factor to allow for the effect of relative humidity on the notional creep coefficient φ_0 for $f_{cm} \leq 35$ N/mm^2 is:

$$\varphi_{RH} = 1 + \frac{1 - RH / 100}{0{,}1 \cdot \sqrt[3]{h_0}} = 1 + \frac{1 - 50 / 100}{0{,}1 \cdot \sqrt[3]{160}} = 1{,}92$$

The factor to allow for the effect of concrete strength on the notional creep coefficient φ_0 is:

$$\beta(f_{cm}) = \frac{16{,}8}{\sqrt{f_{cm}}} = \frac{16{,}8}{\sqrt{28}} = 3{,}18$$

The effect of the type of cement on the creep coefficient of concrete can be taken into account by modifying the age of loading t_0 according to the following expression, where $t_{0,T} = t_0 = 28$ days:

$$t_0 = t_{0,T} \cdot [\frac{9}{2 + t_{0,T}^{\ 1,2}} + 1]^{\alpha} = 28 \cdot [\frac{9}{2 + 28^{1,2}} + 1]^{0}$$

$$t_0 = 28 \text{ days} \geq 0{,}5 \text{ days}$$

The factor to allow for the effect of concrete age at loading on the notional creep coefficient φ_0 is:

$$\beta(t_0) = \frac{1}{(0{,}1 + t_0^{0{,}20})} = \frac{1}{(0{,}1 + 28^{0{,}2})} = 0{,}49$$

The coefficient depending on the relative humidity (RH in %) and the notional member size h_0 for $f_{cm} \leq 35$ N/mm^2 is:

$$\beta_H = 1{,}5 \cdot [1 + (0{,}012 \cdot RH)^{18}] \cdot h_0 + 250 =$$
$$= 1{,}5 \cdot [1 + (0{,}012 \cdot 50)^{18}] \cdot 160 + 250 = 490 \leq 1500$$

The coefficient to describe the development of creep with time after loading is:

$$\beta_c(t, t_0) = [\frac{(t - t_0)}{\beta_H + (t - t_0)}]^{0{,}3} = [\frac{(90 - 28)}{490 + (90 - 28)}]^{0{,}3} = 0{,}52$$

The notional creep coefficient φ_0 is:

$$\varphi_0 = \varphi_{RH} \cdot \beta(f_{cm}) \cdot \beta(t_0) = 1{,}92 \cdot 3{,}18 \cdot 0{,}49 = 2{,}99 \approx 3{,}0$$

The value of notional creep coefficient represents the value of the final creep coefficient $\varphi(\infty, t_0)$ found from Figure A1.2. Thus, this result confirms the accuracy of the results obtained from the Figure A1.2 − see Section 4.1.

At t = 90 days the creep coefficient $\varphi(t, t_0)$ is:

$$\varphi(t, t_0) = \varphi_0 \cdot \beta_c(t, t_0) = 2{,}99 \cdot 0{,}52 = 1{,}55$$

5. Shrinkage strains

5.1 Determination of final value of shrinkage strain

The total shrinkage strain of concrete, ε_{cs}, is composed of two components:

$\varepsilon_{cs}(\infty) = \varepsilon_{cd}(\infty) + \varepsilon_{ca}(\infty)$

where:

$\varepsilon_{cd}(\infty)$ is the drying shrinkage strain,
ε_{ca} is the autogenous shrinkage strain (this develops during hardening of the concrete).

The final value of the drying shrinkage strain $\varepsilon_{cd}(\infty)$ is:

$$\varepsilon_{cd}(\infty) = k_h \cdot \varepsilon_{cd,0}$$

where:

k_h is a coefficient depending on the notional size of the member h_0, Table A1.1,
$\varepsilon_{cd,0}$ is the nominal unrestrained drying shrinkage value, which can be taken from Table A1.2 or can be calculated by means of the following expression:

$$\varepsilon_{cd,0} = 0{,}85 \cdot [(220 + 110 \cdot \alpha_{ds1}) \cdot \exp(-\alpha_{ds2} \cdot \frac{f_{cm}}{10})] \cdot 10^{-6} \cdot \beta_{RH}$$

$$\beta_{RH}(RH) = 1{,}55 \cdot [1 - (\frac{RH}{100})^3]$$

where:

f_{cm} is the mean compressive cylinder strength of concrete at the age of 28 days (N/mm^2, $f_{cm} = f_{ck} + 8$ N/mm^2),
α_{dsi} are factors which depend on the type of cement,
RH is the ambient relative humidity (%).

Table A1.1 *Values for factor k_h for calculation of final value of drying shrinkage strain*

h_0 [mm]	k_h
100	1,00
200	0,85
300	0,75
≥ 500	0,70

(1) h_0 – notional size of member (mm)

(2) h_0 = 2 x (cross-sectional area of concrete A_c)/(perimeter of member in contact with atmosphere)

Table A1.2 *Nominal unrestrained drying shrinkage values of $\varepsilon_{cd,0}$ (in ‰) for concrete with cement class N*

	Relative Humidity	
$f_{ck,cy}/f_{ck,cube}$ (N/mm^2)	Inside conditions, 50%	Outside conditions, 80%
20/25	0,54	0,30
40/50	0,42	0,24
60/75	0,33	0,19
80/95	0,26	0,15
90/105	0,23	0,13

The final value of the drying shrinkage strain $\varepsilon_{cd}(\infty)$ are determined using Tables A1.1 and A1.2.

The nominal unrestrained drying shrinkage value $\varepsilon_{cd,0}$ according to Table A1.2 for concrete strength class C 20/25 and RH 50% is 0,54‰.

The factor k_h depending on the notional size of the member h_0 according to Table A1.1 is:

For h_0 = 100 mm → k_h = 1,0
For h_0 = 200 mm → k_h = 0,85

Linear interpolation:

For h_0 = 160 mm → $k_h = 0,85 + \dfrac{200-160}{200-100} \cdot (1,0-0,85)$

k_h = 0,91

The final value of the drying shrinkage strain is:

$$\varepsilon_{cd}(\infty) = k_h \cdot \varepsilon_{cd,0} = 0,91 \cdot 5,4 \cdot 10^{-4} = 4,91 \cdot 10^{-4}$$

The final value of the autogenous shrinkage strain is:

$$\varepsilon_{ca}(\infty) = 2,5 \cdot (f_{ck} - 10) \cdot 10^{-6} = 2,5 \cdot (25 - 10) \cdot 10^{-6} = 3,75 \cdot 10^{-5}$$

The total shrinkage strain $\varepsilon_{cs}(\infty)$ is:

$$\varepsilon_{cs}(\infty) = \varepsilon_{cd}(\infty) + \varepsilon_{ca}(\infty) = 4,91 \cdot 10^{-4} + 3,75 \cdot 10^{-5} = 5,29 \cdot 10^{-4}$$

$\varepsilon_{cs}(\infty)$ = 0,529 ‰

5.2 Determination of shrinkage strain at time *t* = 90 days

The total shrinkage strain at time *t* is calculated as:

$\varepsilon_{cs}(t) = \varepsilon_{cd}(t) + \varepsilon_{ca}(t)$

The value of the drying shrinkage strain ε_{cd} at time *t* is:

$\varepsilon_{cd}(t) = \beta_{ds}(t,t_s) \cdot k_h \cdot \varepsilon_{cd,0}$

where:

$$\beta_{ds}(t,t_s) = \frac{(t-t_s)}{(t-t_s)+0,04\sqrt{h_0^3}}$$

t is the age of the concrete at the time considered, in days,

t_s is the age of the concrete in days at the beginning of drying shrinkage; normally this is at the end of the curing of the concrete.

The value of the autogenous shrinkage strain ε_{ca} at the age of concrete *t*, is given with the following expression:

$\varepsilon_{ca}(t) = \beta_{as}(t) \cdot \varepsilon_{ca}(\infty)$

where:

$\beta_{as}(t) = 1 - \exp(-0,2\sqrt{t})$, *t* in days

$\varepsilon_{ca}(\infty) = 2,5 \cdot (f_{ck} - 10) \cdot 10^{-6}$

The drying shrinkage strain is:

$\varepsilon_{cd}(t) = \beta_{ds}(t,t_s) \cdot k_h \cdot \varepsilon_{cd,0}$

$$\beta_{ds}(t,t_s) = \frac{(t-t_s)}{(t-t_s)+0,04\sqrt{h_0^3}} = \frac{(90-3)}{(90-3)+0,04\sqrt{160^3}} = 0,52$$

From Section 5.1 k_h = 0,91.

$\alpha_{ds1} = 4 \quad \alpha_{ds2} = 0,12$

$$\varepsilon_{cd,0} = 0,85[(220 + 110 \cdot \alpha_{ds1}) \cdot \exp(-\alpha_{ds2} \cdot f_{cm} / 10)] \cdot 10^{-6} \cdot \beta_{RH}$$
$$= 0,85[(220 + 110 \cdot 4) \cdot \exp(-0,12 \cdot 28 / 10)] \cdot 10^{-6} \cdot 1,36 = 5,45 \cdot 10^{-4}$$

$$\beta_{RH}(RH) = 1,55 \cdot [1 - (RH / 100)^3] = 1,55 \cdot [1 - (50 / 100)^3] = 1,36$$

$$\varepsilon_{cd}(t) = \beta_{ds}(t, t_s) \cdot k_h \cdot \varepsilon_{cd,0} = 0,52 \cdot 0,91 \cdot 5,45 \cdot 10^{-4} = 2,58 \cdot 10^{-4}$$

The autogenous shrinkage strain is:

$$\varepsilon_{ca}(t) = \beta_{as}(t) \cdot \varepsilon_{ca}(\infty)$$

$$\varepsilon_{ca}(\infty) = 2,5 \cdot (f_{ck} - 10) \cdot 10^{-6} = 2,5 \cdot (25 - 10) \cdot 10^{-6} = 3,75 \cdot 10^{-5}$$

$$\beta_{as}(t) = 1 - \exp(-0,2\sqrt{t}) = 1 - \exp(-0,2\sqrt{90}) = 0,85$$

$$\varepsilon_{ca}(t) = \beta_{as}(t) \cdot \varepsilon_{ca}(\infty) = 0,85 \cdot 3,75 \cdot 10^{-5} = 3,19 \cdot 10^{-5}$$

The total shrinkage strain is:

$$\varepsilon_{cs}(t) = \varepsilon_{cd}(t) + \varepsilon_{ca}(t)$$

$$\varepsilon_{cs}(90) = 2,58 \cdot 10^{-4} + 3,19 \cdot 10^{-5} = 2,89 \cdot 10^{-4} = 0,289\ ‰$$

6. Commentary

The effects of time-dependent strains of concrete (creep and shrinkage) in the analysis of structural elements of composite structures are different according to whether they are observed in *the level of cross-section or static system.*

The effects of creep and shrinkage of concrete produce internal forces and moments in cross-sections, and curvatures and longitudinal strains in members. The effects that occur in statically determinate systems are classified as primary effects. In statically indeterminate system, the primary effects of creep and shrinkage are associated with additional action effects, such that the total effects are compatible. These are classified as secondary effects and are considered as indirect actions which are sets of imposed deformations.

Computational methods, principles and basic equations for estimating the time-dependent strains of concrete are given in EN 1992-1-1. The determination of the final value of the creep coefficient $\varphi(\infty, t_0)$ for concrete under normal environmental conditions is possible using nomograms. However, for the determination of the values of shrinkage strains we need to use the extensive numerical procedure.

A2 Determination of creep and shrinkage values on an example composite highway bridge

1. Purpose of example

The values of creep and shrinkage of the concrete deck slab of a composite highway bridge are calculated at different time periods in the age of the concrete. For the corresponding values of creep and shrinkage and types of loading, the corresponding values of modular ratio, n_L, and the shrinkage strains are determined. Also, the primary effects of shrinkage are calculated. In the example, the following cases are considered:

- calculation of the modular ratio n_L for permanent action not changing with time. Two ages are considered: the first loading, which is applied at the age of t_0 = 28 days, and the end of the design life, for which it is assumed that $t = \infty$.
- calculation of the modular ratio n_L for permanent action not changing with time at bridge opening. Two ages are considered: the age at which the permanent actions are imposed, which is 28 days (t_0 = 28 days), and the age at opening to traffic, which is 63 days (t = 63 days).
- calculation of the modular ratio n_L for shrinkage at $t = \infty$ and the value of the total shrinkage deformation at $t = \infty$ denoted as $\varepsilon_{cs}(\infty)$. The age of concrete at the first loading is t_0 = 1 day (the beginning of drying shrinkage).
- calculation of the modular ratio n_L for shrinkage at bridge opening, for which the age is 63 days (t = 63 days) and the value of the total shrinkage deformation at t = 63 denoted as $\varepsilon_{cs}(63)$. The age of concrete at the first loading is t_0 = 1 day (the beginning of drying shrinkage).
- calculation of the primary effects of shrinkage only for the long-term situation.

2. Cross-section

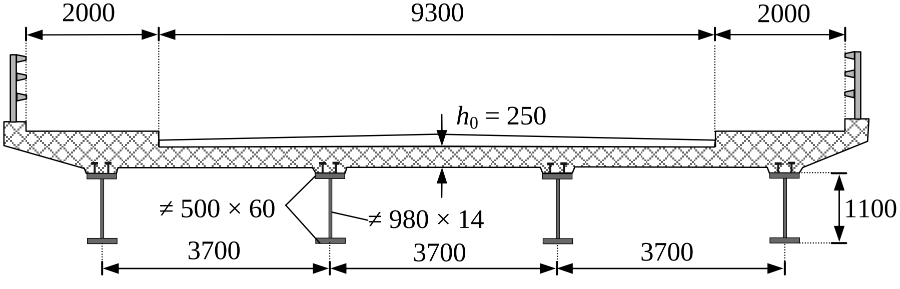

Figure A2.1 *Cross-section of composite bridge*

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3. Input data

Concrete strength class: C 40/50 $f_{ck} = 40,0\ \text{N/mm}^2$

$$f_{cd} = \frac{f_{ck}}{\gamma_c} = \frac{40,0}{1,5} = 26,7\ \text{N/mm}^2$$

$$E_{cm} = 35000\ \text{N/mm}^2$$

Type of cement: *N, strength class according to EN 197-1, 32,5 R* $\alpha = 0$

$\alpha_{ds1} = 4$

$\alpha_{ds2} = 0,12$

Relative humidity: RH 70%

First loading t_0 = 28 days

Beginning of drying t_s = 1 day

4. Calculation of modular ratio n_L for permanent action constant in time

4.1 Calculation of modular ratio n_L for permanent action constant in time at time $t = \infty$

For the age of concrete $t \to \infty$, $\varphi(t, t_0) = \varphi_0$, where φ_0 is the notional creep coefficient which may be estimated from:

$$\varphi_0 = \varphi_{RH} \cdot \beta(f_{cm}) \cdot \beta(t_0)$$

For the mean compressive cylinder strength of concrete $f_{cm} > 35$ N/mm², calculated according to Table 3.1, EN 1992-1-1, factor φ_{RH}, which allows for the effect of relative humidity on the notional creep coefficient, is determined as follows:

$$\varphi_{RH} = [1 + \frac{1 - RH/100}{0,1 \cdot \sqrt[3]{h_0}} \cdot \alpha_1] \cdot \alpha_2$$

The mean compressive cylinder strength of concrete is:

$$f_{cm} = f_{ck} + 8\ \text{N/mm}^2 = 40 + 8 = 48\ \text{N/mm}^2$$

$$\alpha_1 = [\frac{35}{f_{cm}}]^{0,7} = [\frac{35}{48}]^{0,7} = 0,802$$

$$\alpha_2 = [\frac{35}{f_{cm}}]^{0,2} = [\frac{35}{48}]^{0,2} = 0,939$$

For a 250 mm thick slab (h_0 = 250), factor φ_{RH} is:

$$\varphi_{RH} = [1 + \frac{1 - 70/100}{0,1 \cdot \sqrt[3]{250}} \cdot 0,802] \cdot 0,939 = 1,298$$

The factor to allow for the effect of concrete strength on the notional creep coefficient φ_0 is:

$$\beta(f_{cm}) = \frac{16,8}{\sqrt{f_{cm}}} = \frac{16,8}{\sqrt{48}} = 2,42$$

The effect of the type of cement on the creep coefficient of concrete can be taken into account by modifying the age of loading t_0 according to the following expression, where $t_{0,T}$ = t_0 = 28 days:

$$t_0 = t_{0,T} \cdot [\frac{9}{2 + t_{0,T}^{\ 1,2}} + 1]^0 = 21 \cdot [\frac{9}{2 + 28^{1,2}} + 1]^0$$

$$t_0 = 28 \text{ days} \geq 0,5 \text{ day}$$

The factor to allow for the effect of concrete age at loading on the notional creep coefficient φ_0 is:

$$\beta(t_0) = \frac{1}{(0,1 + t_0^{0,20})} = \frac{1}{(0,1 + 28^{0,20})} = 0,488$$

Thus, the notional creep coefficient is:

$$\varphi_0 = \varphi_{RH} \cdot \beta(f_{cm}) \cdot \beta(t_0)$$

$$\varphi_0 = 1,298 \cdot 2,42 \cdot 0,488 = 1,533$$

The value of the modulus of elasticity of concrete at the age of 28 days will be used for the determination of all the short-term effects and resistances, and therefore the modular ratio n_0 is:

$$n_0 = \frac{E_s}{E_{cm}} = \frac{210000}{35000} = 6,0$$

For the permanent action constant in time, ψ_L = 1,1 and therefore:

$$n_L = n_0 \cdot (1 + \psi_L \cdot \varphi_t)$$

$$\varphi_t = \varphi(t, t_0) = \varphi_0$$

$$n_L = 6,0 \cdot (1 + 1,1 \cdot 1,533) = 16,1$$

We can take into account the influence of concrete creep for permanent action constant in time and $t = \infty$ for permanent and constant actions by reducing the modulus of elasticity:

$$E_L = \frac{E_a}{n_L} = \frac{210000}{16,1} = 13043 \text{ N/mm}^2$$

4.2 Calculation of modular ratio n_L for permanent action constant in time at opening to traffic t = 63 days

If the design effects need to be determined at the time of opening the bridge to traffic (the age of concrete at opening to traffic is 63 days), the creep coefficient should be modified to reflect the short duration of loading (short-term loading). The effect of the age of the concrete at the time of opening on the creep coefficient was taken into account in this way. In this example, the average age of concrete at which the permanent actions are imposed is 28 days (t_0 = 28 days) and the age of concrete at opening to traffic is 63 days (t = 63 days). The creep coefficient is modified by the parameter $\beta_c(t, t_0)$ and in this case, with $t - t_0$ = 35 days, the value of the parameter $\beta_c(t, t_0)$ is calculated as follows:

$$\beta_c(t, t_0) = [\frac{(t - t_0)}{\beta_H + (t - t_0)}]^{0,3}$$

$$\alpha_3 = [\frac{35}{f_{cm}}]^{0,5} = [\frac{35}{48}]^{0,5} = 0,854$$

$$\beta_H = 1,5 \cdot [1 + (0,012 \cdot RH)^{18}] \cdot h_0 + 250 \cdot \alpha_3 \leq 1500 \cdot \alpha_3 \quad \text{for } f_{cm} > 35 \text{ N/mm}^2$$

$$\beta_H = 1,5 \cdot [1 + (0,012 \cdot 70)^{18}] \cdot 250 + 250 \cdot 0,854 \leq 1500 \cdot 0,854$$

$$\beta_H = 604,76 < 1500 \cdot 0,854 = 1281$$

$$\beta_c(t,t_0) = [\frac{(t-t_0)}{(\beta_H + t - t_0)}]^{0,3} = [\frac{(63-28)}{(604,76+63-28)}]^{0,3} = 0,418$$

At t = 63 days, the creep coefficient $\varphi(t, t_0)$ is:

$$\varphi(t,t_0) = \varphi_0 \cdot \beta_c(t,t_0) = 1,533 \cdot 0,418 = 0,641$$

The value of the modulus of elasticity of concrete at the age of 28 days will be used for the determination of all short-term effects and resistances, and therefore the modular ratio n_0 is:

$$n_0 = \frac{E_s}{E_{cm}} = \frac{210000}{35000} = 6,0$$

Thus, for permanent loads at opening to traffic:

$$n_L = n_0 \cdot (1 + \psi_L \cdot \varphi(t,t_0))$$

$$n_L = 6,0 \cdot (1 + 1,1 \cdot 0,641) = 10,2$$

The influence of concrete creep for permanent action constant in time and t = 63 days can be taken into account directly by the reduced modulus of elasticity:

$$E_L = \frac{E_a}{n_L} = \frac{210000}{10,2} = 20588 \text{ N/mm}^2$$

5. Calculation of modular ratio n_L for shrinkage and shrinkage strains

5.1 Calculation of modular ratio n_L for shrinkage and shrinkage strains at time $t = \infty$

The autogenous shrinkage strain at $t = \infty$ is:

$$\varepsilon_{ca}(\infty) = 2,5 \cdot (f_{ck} - 10) \cdot 10^{-6}$$

$$\varepsilon_{ca}(\infty) = 2,5 \cdot (40 - 10) \cdot 10^{-6} = 7,5 \cdot 10^{-5}$$

The drying shrinkage depends on the nominal unrestrained drying shrinkage, given by expression (B.11) in B.2, EN 1992-1-1 (or by interpolation in Table 3.2, EN 1992-1-1):

$$\varepsilon_{cd,0} = 0,85 \cdot [(220 + 110 \cdot \alpha_{ds1}) \cdot \exp(-\alpha_{ds2} \cdot \frac{f_{cm}}{10})] \cdot 10^{-6} \cdot \beta_{RH}$$

$$\beta_{RH}(RH) = 1,55 \cdot [1 - (\frac{RH}{100})^3]$$

For 70% relative humidity, f_{ck} = 40 N/mm^2 and class N cement:

$$\alpha_{ds1} = 4$$

$$\alpha_{ds2} = 0,12$$

$$\beta_{RH} = 1,55 \cdot [1 - (RH / 100)^3]$$

$$\beta_{RH} = 1,55 \cdot [1 - (70 / 100)^3] = 1,018$$

$$\varepsilon_{cd,0} = 0,85 \cdot [(220 + 110 \cdot \alpha_{ds1}) \cdot \exp(-\alpha_{ds2} \cdot \frac{f_{cm}}{10})] \cdot 10^{-6} \cdot \beta_{RH}$$

$$\varepsilon_{cd,0} = 0,85 \cdot [(220 + 110 \cdot 4) \cdot \exp(-0,12 \cdot \frac{40}{10})] \cdot 10^{-6} \cdot 1,018$$

$$\varepsilon_{cd,0} = 3,53 \cdot 10^{-4}$$

The final value of the drying shrinkage strain $\varepsilon_{cd}(\infty)$ is given by:

$$\varepsilon_{cd}(\infty) = k_h \cdot \varepsilon_{cd,0}$$

k_h = 0,80 (from Table 3.3, EN 1992-1-1, with h_0 = 250).

Thus, the drying shrinkage at $t = \infty$ is:

$$\varepsilon_{cd} = 0,80 \cdot 3,53 \cdot 10^{-4} = 28,2 \cdot 10^{-5}$$

The total shrinkage strain is:

$$\varepsilon_{cs} = \varepsilon_{ca}(\infty) + \varepsilon_{cd}(\infty)$$

$$\varepsilon_{cs} = 7{,}5 \cdot 10^{-5} + 28{,}2 \cdot 10^{-5} = 35{,}7 \cdot 10^{-5}$$

For the modular ratio, the creep factor is calculated as for long-term loading but the age at first loading is assumed to be 1 day and thus:

$$\beta(t_0) = \frac{1}{(0{,}1 + t_0^{0{,}20})} = \frac{1}{(0{,}1 + 1^{0{,}20})} = 0{,}91$$

The final creep coefficient is calculated as above for long-term effects but with $\beta(t_0)$ = 0,91, and therefore:

$$\varphi_0 = \varphi_{RH} \cdot \beta(f_{cm}) \cdot \beta(t_0)$$

$$\varphi_0 = 1{,}298 \cdot 2{,}42 \cdot 0{,}91 = 2{,}86$$

The value of the modulus of elasticity of concrete at the age of 28 days will be used for the determination of all short-term effects and resistances, and therefore the modular ratio n_0 is:

$$n_0 = \frac{E_s}{E_{cm}} = \frac{210000}{35000} = 6{,}0$$

For shrinkage, ψ_L = 0,55 and thus:

$$n_L = n_0 \cdot (1 + \psi_L \cdot \varphi(t, t_0))$$

$$n_L = 6{,}0 \cdot (1 + 0{,}55 \cdot 2{,}86) = 15{,}4$$

5.2 Calculation of modular ratio n_L for shrinkage and shrinkage strains at opening to traffic t = 63 days

The autogenous shrinkage strain at t = 63 days is:

$$\varepsilon_{ca}(t) = \beta_{as}(t) \cdot \varepsilon_{ca}(\infty)$$

where:

$$\beta_{as}(t) = 1 - \exp(-0{,}2\sqrt{t})$$

$$\beta_{as}(t) = 1 - \exp(-0{,}2\sqrt{63}) = 0{,}796$$

Thus:

$\varepsilon_{ca}(t) = \beta_{as}(t) \cdot \varepsilon_{ca}(\infty)$

$$\varepsilon_{ca}(63) = 0{,}796 \cdot 7{,}5 \cdot 10^{-5} = 5{,}97 \cdot 10^{-5}$$

The drying shrinkage strain at t = 63 days is:

$$\varepsilon_{cd}(t) = \beta_{ds}(t,t_s) \cdot k_h \cdot \varepsilon_{cd,0}$$

$$\beta_{ds}(t,t_s) = \frac{(t-t_s)}{(t-t_s) + 0{,}04\sqrt{h_0^3}}$$

For t = 63 and t_s = 1 (see clause 5.4.2.2(4), EN 1994-2):

$$\beta_{ds} = \frac{(63-1)}{(63-1) + 0{,}04\sqrt{250^3}} = 0{,}282$$

Thus the drying shrinkage at t = 63 days is:

$$\varepsilon_{cd} = 0{,}282 \cdot 0{,}80 \cdot 3{,}53 \cdot 10^{-4} = 7{,}96 \cdot 10^{-5}$$

The total shrinkage strain is:

$$\varepsilon_{cs} = \varepsilon_{ca}(63) + \varepsilon_{cd}(63)$$

$$\varepsilon_{cs} = 5{,}97 \cdot 10^{-5} + 7{,}96 \cdot 10^{-5} = 13{,}93 \cdot 10^{-5}$$

At opening to traffic (t = 63 days) the creep coefficient is modified by parameter $\beta_c(t, t_0)$ and in this case is:

$$\beta_c(t,t_0) = [\frac{(t-t_0)}{(\beta_H + t - t_0)}]^{0,3} = [\frac{(63-1)}{(604{,}76 + 63 - 1)}]^{0,3} = 0{,}490$$

At t = 63 days the creep coefficient $\varphi(t, t_0)$ is:

$$\varphi(t,t_0) = \varphi_0 \cdot \beta_c(t,t_0) = 2{,}86 \cdot 0{,}490 = 1{,}40$$

The value of the modulus of elasticity of concrete at the age of 28 days will be used for the determination of all short-term effects and resistances, and therefore the modular ratio n_0 is:

$$n_0 = \frac{E_s}{E_{cm}} = \frac{210000}{35000} = 6,0$$

For shrinkage, ψ_L = 0,55 and thus:

$$n_L = n_0 \cdot (1 + \psi_L \cdot \varphi(t, t_0))$$

$$n_L = 6,0 \cdot (1 + 0,55 \cdot 1,40) = 10,6$$

6. Primary effects of shrinkage

It is assumed that, for the majority of the shrinkage, the length of continuous concrete slab is such that shear lag effects are negligible. The effective area of the concrete flange is taken as the actual area, for both the shrinkage and its primary effects.

From Section 5.1, the modular ratio for shrinkage is n_L = 15,4 and the total shrinkage strain is $\varepsilon_{cs} = 35,7 \cdot 10^{-5}$.

From clause 2.4.2.1, EN 1992-1-1, the recommended partial factor for shrinkage is γ_{SH} = 1,0, so the design shrinkage strain, for both serviceability and ultimate limit states, is:

$$\varepsilon_{cs} = 1,0 \cdot 35,7 \cdot 10^{-5} = 35,7 \cdot 10^{-5}$$

To restore the slab to its length before shrinkage, the tensile force N_{cs} applies to the concrete a tensile stress of:

$$\varepsilon_{cs}(\infty) \cdot \frac{E_a}{n_L} = 35,7 \cdot 10^{-5} \cdot \frac{210000}{15,4} = 4,87 \text{ N/mm}^2$$

The area of the concrete cross-section is:

$$A_c = 3700 \cdot 250 + 500 \cdot 50 = 950000 \text{ mm}^2$$

The tensile force N_{cs} to restore the slab to its length before shrinkage is:

$$N_{cs} = \frac{\varepsilon_{cs} \cdot A_c \cdot E_a}{n_L} = \frac{35{,}7 \cdot 10^{-5} \cdot 950000 \cdot 210000}{15{,}4} \cdot 10^{-3} = 4625 \ \text{kN}$$

For n_L = 15,4, the location of the neutral axis of the uncracked unreinforced section is as shown in Figure A2.2.

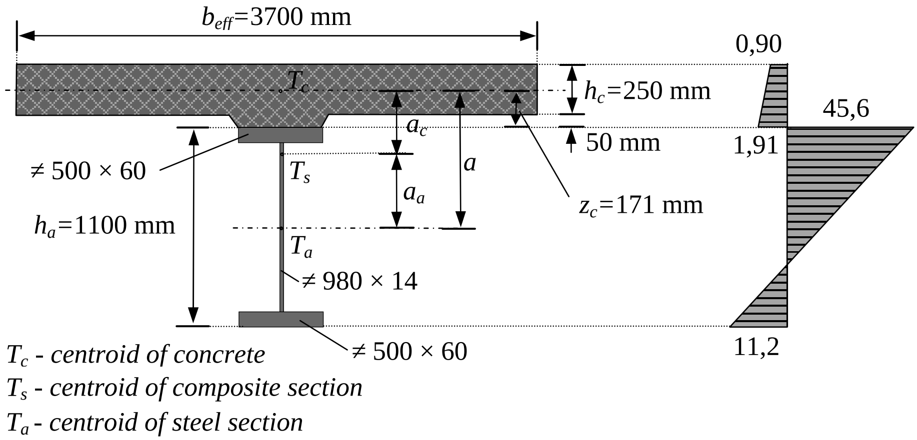

Figure A2.2 *Cross-section of composite beam and primary shrinkage stresses with n_L = 15,4*

The area of the steel cross-section is:

$$A_a = (500 \cdot 60) \cdot 2 + 980 \cdot 14 = 73720 \ \text{mm}^2$$

The ideal cross-sectional area of the composite section (the cross-section is considered as the transformed section), the sum of the area of steel section and the area of the concrete slab in “steel” units, is:

$$A_{id} = A_a + A_c / n_L = 73720 + 950000 / 15{,}4 = 135408 \ \text{mm}^2$$

The second moment of area of the steel section is:

$$I_a = 17340057333 \ \text{mm}^4$$

The second moment of area of the concrete section is:

$$I_c = 5370614035 \ \text{mm}^4$$

The distance between the centroidal axes of the concrete and the steel section a is:

$$a = \frac{h_a}{2} + z_c = \frac{1100}{2} + 171 = 721 \text{ mm}$$

The distance between the centroidal axes of the steel section and the composite section a_a is:

$$a_a = a \cdot \frac{A_c}{A_{id} \cdot n_L} = 721 \cdot \frac{950000}{135408 \cdot 15,4} = 328,5 \text{ mm}$$

The distance between the centroidal axes of the concrete and the composite section a_c is:

$$a_c = a \cdot \frac{A_a}{A_{id}} = 721 \cdot \frac{73720}{135408} = 392,5 \text{ mm}$$

The ideal second moment of area of the composite section is:

$$I_{id} = I_a + A_a \cdot a_a^2 + (I_c + A_c \cdot a_c^2) / n_L$$

$$I_{id} = 17340057333 + 73720 \cdot 328,5^2 + (5370614035 + 950000 \cdot 392,5^2) / 15,4$$

$$I_{id} = 3,51475 \cdot 10^{10} \text{ mm}^4$$

The location of the neutral axis of the uncracked unreinforced section is 392,5 mm below the centroid of the concrete area. The total external force is zero, so force N_{cs} is balanced by applying a compressive force of 4625 kN and a sagging moment of 4625·0,3925 = 1815 kNm to the composite section.

The long-term primary shrinkage stresses in the cross-section are as follows, with compression positive:

At the top of the slab

$$\sigma = -4,87 + \left(\frac{4625 \cdot 10^3}{135408} + \frac{4625 \cdot 10^3 \cdot 392,5 \cdot 521,5}{3,51475 \cdot 10^{10}}\right)\frac{1}{15,4} = -0,90 \text{ N/mm}^2$$

At the interface, in concrete

$$\sigma = -4,87 + \left(\frac{4625 \cdot 10^3}{135408} + \frac{4625 \cdot 10^3 \cdot 392,5 \cdot 221,5}{3,51475 \cdot 10^{10}}\right)\frac{1}{15,4} = -1,91 \text{ N/mm}^2$$

At the interface, in steel

$$\sigma = \frac{4625 \cdot 10^3}{135408} + \frac{4625 \cdot 10^3 \cdot 392{,}5 \cdot 221{,}5}{3{,}51475 \cdot 10^{10}} = 45{,}6 \ \text{N/mm}^2$$

At the bottom of the steel beam

$$\sigma = \frac{4625 \cdot 10^3}{135408} - \frac{4625 \cdot 10^3 \cdot 392{,}5 \cdot 878{,}5}{3{,}51475 \cdot 10^{10}} = -11{,}2 \ \text{N/mm}^2$$

7. Commentary

For different time periods the values of creep and shrinkage of concrete are variable. These values are used for taking into account the effects of creep and shrinkage in determining the modular ratios n_L for concrete, which depends on the type of loading (index L). The modular ratio n_L is determined according to clause 5.4.2.2(2), EN 1994-1-1. It is still used for the calculation of the bending stiffness of the cross-section composite beams.

The primary effects of shrinkage are calculated for uncracked cross-sections. After cracking, these effects remain in the concrete between cracks, but have negligible influence on stresses at the cracked cross-sections, at which stresses are verified.

A3 Determination of creep and shrinkage values and their effects at calculation of bending moments

1. Purpose of example

The purpose of the example is to show the calculation of the value of time-dependent strains of concrete due to creep and shrinkage. These values influence the determination of the effects of actions. In this example, the effect of action is related to the calculation of the design bending moment at the internal support of the continuous composite beam. This moment is obtained by summing the bending moment due to the actions (permanent and variable) and the bending moment due to shrinkage.

Primary and secondary effects due to creep and shrinkage of the concrete flange must be taken into account appropriately. These effects can be neglected in the analysis for verifications of ultimate limit states other than fatigue for composite elements with cross-sections of classes 1 and 2 and which do not need to take into account the lateral torsional buckling. For the serviceability limit state, these effects must be taken into account for the cross-sections of classes 1 and 2.

In this example, all the cross-sections are in class 1, and the cross-section and the static system are shown in Figure A3.1. Thus, the analysis of the influence of shrinkage for the ultimate limit state is not necessary. Despite the fact that the cross-section is in class 1, the method that takes into account the effects of shrinkage on the internal forces and bending moments is shown.

For this continuous beam, the global analysis is performed with defined regions of cracking of concrete, clause 5.4.2.3(2), EN 1994-1-1. The section is assumed to crack if the extreme-fibre tensile stress in concrete exceeds twice the mean value of the axial tensile strength, $2 \cdot f_{cm}$, given by EN 1992-1-1. The simplified method of allowing for cracking, clause 5.4.2.3(3), EN 1994-1-1, is used in this example.

This means that the effects of cracking of concrete are taken into account using the „cracked“ flexural stiffness $E_a I_2$, over 15% of the span on each side of each internal support, and as the „uncracked“ flexural stiffness $E_a I_1$ elsewhere.

Composite Structures according to Eurocode 4. Worked Examples.
First Edition. Darko Dujmović, Boris Androić, Ivan Lukačević.

2. Static system, cross-section and actions

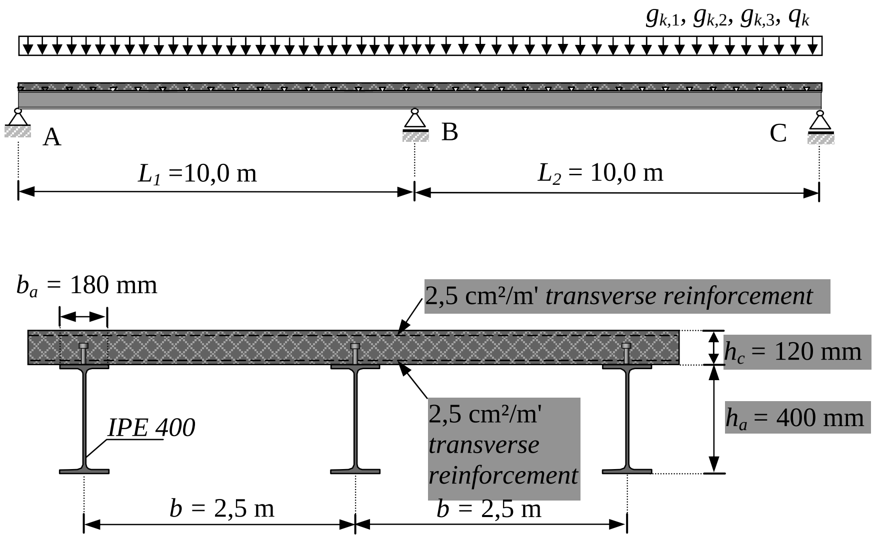

Figure A3.1 *Static system and cross-section of beam*

Actions

a) Permanent action

Remark:

According to EN 1991-1-1 the density of normal weight concrete is 24 kN/m^3, increased by 1 kN/m^3 for normal percentage reinforcement.

Concrete slab area per m width:

$$A_c = 1000 \cdot h_c$$

$$A_c = 1000 \cdot 120 = 120000 \ \text{mm}^2 = 1200 \ \text{cm}^2$$

concrete slab (the dry density) and reinforcement:

$$A_c \cdot 25 = 0{,}120 \cdot 25 = 3{,}00 \ \text{kN/m}^2$$

-concrete slab $g_{k,1} = 2{,}5 \cdot 3{,}00 = 7{,}50 \ \text{kN/m}$

-steel beam IPE 400 $g_{k,2} = 0{,}663 \ \text{kN/m}$

-floor finishes $g_{k,3} = 1,5 \cdot 2,5 = 3,75$ kN/m

b) Variable action

-imposed floor load, category of use C1 $q_k = 3,0 \cdot 2,5 = 7,50$ kN/m

3. Input data

Concrete strength class: C 25/30 $f_{ck} = 25$ N/mm^2

$$f_{cd} = \frac{f_{ck}}{\gamma_c} = \frac{25}{1,5} = 16,7 \text{ N/mm}^2$$

$$E_{cm} = 31000 \text{ N/mm}^2$$

Type of cement: *N, strength class according to EN 197-1, 32,5 R* $\alpha = 0$

$$\alpha_{ds1} = 4$$

$$\alpha_{ds2} = 0,12$$

Relative humidity RH 70%

First loading t_0 = 1 day

4. Creep and shrinkage

4.1 Determination of final creep coefficient

For the calculation of the final creep coefficient $\varphi(\infty, t_0)$ the following is valid:

- the perimeter of that part which is exposed to drying, u

$u = 2 \cdot b - b_a$ (b_a is not usually taken into account in the calculation)

$u = 2 \cdot 2500 - 180 = 4820$ mm

- the notional size of the cross-section, h_0

$$h_0 = \frac{2 \cdot A_c}{u} = \frac{2 \cdot 2500 \cdot 120}{4820} = 125 \text{ mm} = 12,5 \text{ cm}$$

- $t_0 = 1$ day,
- the ambient relative humidity, RH 70%,
- the concrete strength class C 25/30,
- the type of cement – cement class N, strength class 32,5 R.

Since the relative humidity RH = 70% is given, the final value of creep coefficient $\varphi(\infty, t_0)$ will be determined using the nomogram shown in Figure 3.1, EN 1992-1-1 by means of the linear interpolation for the values RH = 50% and RH = 80% . The process of determining the final value of creep coefficient for these assumptions is given in Figures A3.2 and A3.3:

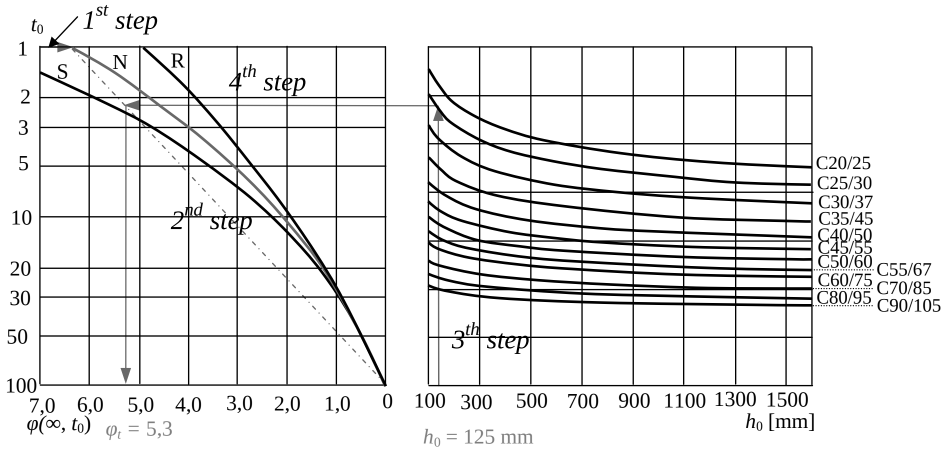

Figure A3.2 *Determination of the final value of creep coefficient (RH = 50%)*

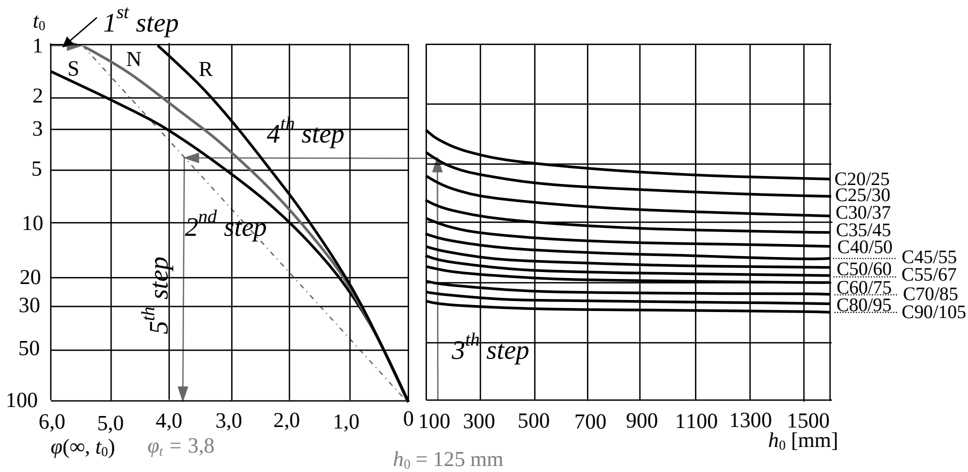

Figure A3.3 *Determination of the final value of creep coefficient (RH = 80%)*

From the diagram of Figure A3.2 for RH 50% we can find $\varphi_t^{RH=50\%} = 5,3$, and from the diagram of Figure A3.3 for RH = 80% we can find $\varphi_t^{RH=80\%} = 3,8$.

By means of linear interpolation, for the relative humidity RH = 70% we get:

$$\varphi_t^{RH=70\%} = \varphi_t^{RH=50\%} - \frac{RH^{70\%} - RH^{50\%}}{RH^{80\%} - RH^{50\%}}(\varphi_t^{RH=50\%} - \varphi_t^{RH=80\%})$$

$$\varphi_t^{RH=70\%} = 5,3 - \frac{70-50}{80-50}(5,3-3,8) = 4,3$$

If it is necessary to take non-linear creep into account, the creep coefficient can be calculated in accordance with clause B.1, EN 1992-1-1. Figure A3.4 shows the temporal evolution of creep coefficient. The final value is obtained by the method given in clause B.1, EN 1992-1-1, is 4,3.

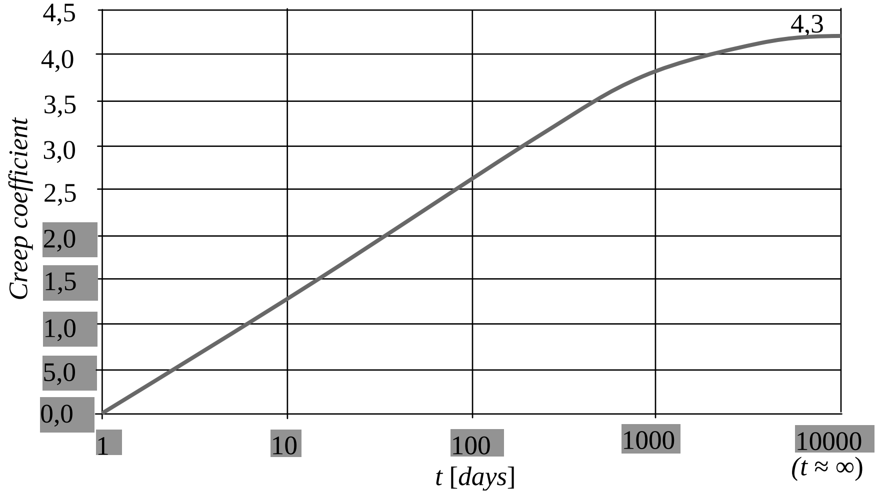

Figure A3.4 *Creep coefficient as a function of time t*

The modular ratio n_L is calculated according to the following:

$$n_L = n_0(1+\psi_L\varphi_t)$$

$$n_0 = \frac{E_a}{E_{cm}} = \frac{21000}{3100} = 6,77$$

$\psi_L = 0,55$ for shrinkage in accordance with clause 5.4.2.2(6), EN 1994-1-1,

$$n_L = n_0(1+\psi_L\varphi_t) = 6,77\cdot(1+0,55\cdot 4,3) = 22,78$$

4.2 Determination of shrinkage strain

The total shrinkage strain of concrete $\varepsilon_{cs}(\infty)$ at $t = \infty$ is calculated as:

$\varepsilon_{cs}(\infty) = \varepsilon_{cd}(\infty) + \varepsilon_{ca}(\infty)$

where:

$\varepsilon_{cd}(\infty)$ is the drying shrinkage strain,
$\varepsilon_{ca}(\infty)$ is the autogenous shrinkage strain (it develops during hardening of the concrete).

The autogenous shrinkage strain at $t = \infty$ is:

$$\varepsilon_{ca}(\infty) = 2{,}5 \cdot (f_{ck} - 10) \cdot 10^{-6}$$

$$\varepsilon_{ca}(\infty) = 2{,}5 \cdot (25 - 10) \cdot 10^{-6} = 3{,}75 \cdot 10^{-5}$$

The drying shrinkage depends on the nominal unrestrained drying shrinkage, given by expression (B.11) in B.2, EN 1992-1-1, (or by interpolation in Table 3.2, EN 1992-1-1). The basic drying shrinkage strain, $\varepsilon_{cd,0}$, is calculated using the following expression:

$$\varepsilon_{cd,0} = 0{,}85 \cdot [(220 + 110 \cdot \alpha_{ds1}) \cdot \exp(-\alpha_{ds2} \cdot \frac{f_{cm}}{10})] \cdot 10^{-6} \cdot \beta_{RH}$$

For the relative humidity RH = 70%, f_{ck} = 25 N/mm^2 and class N cement:

$$f_{cm} = f_{ck} + 8 = 25 + 8 = 33 \text{ N/mm}^2$$

$$\alpha_{ds1} = 4$$

$$\alpha_{ds2} = 0{,}12$$

$$\beta_{RH} = 1{,}55 \cdot [1 - (RH/100)^3]$$

$$\beta_{RH} = 1{,}55 \cdot [1 - (70/100)^3] = 1{,}018$$

$$\varepsilon_{cd,0} = 0{,}85 \cdot [(220 + 110 \cdot \alpha_{ds1}) \cdot \exp(-\alpha_{ds2} \cdot \frac{f_{cm}}{10})] \cdot 10^{-6} \cdot \beta_{RH}$$

$$\varepsilon_{cd,0} = 0{,}85 \cdot [(220 + 110 \cdot 4) \cdot \exp(-0{,}12 \cdot \frac{33}{10})] \cdot 10^{-6} \cdot 1{,}018$$

$$\varepsilon_{cd,0} = 3{,}84 \cdot 10^{-4}$$

The drying shrinkage strain at time t is given by:

$$\varepsilon_{cd}(t) = \beta_{ds}(t,t_s)\cdot k_h \cdot \varepsilon_{cd,0}$$

According to Table 3.3, EN 1992-1-1, and for h_0 = 125, k_h is calculated by means of linear interpolation:

- from Table 3.3, EN 1992-1-1, for h_0 = 100, it is found that k_h = 1,0,
- from Table 3.3, EN 1992-1-1, for h_0 = 200, it is found that k_h = 0,85,
- by means of linear interpolation for h_0 = 125:

$$k_h = 0,85 + \frac{200-125}{200-100}\cdot(1,0-0,85) = 0,96$$

At $t = \infty$, it holds that $\beta_{ds} = 1$. Thus, the drying shrinkage strain at $t = \infty$ is:

$$\varepsilon_{cd}(\infty) = k_h \cdot \varepsilon_{cd,0}$$

$$\varepsilon_{cd}(\infty) = 0,96\cdot 3,84\cdot 10^{-4} = 3,69\cdot 10^{-4}$$

The total shrinkage strain of concrete at $t = \infty$ is:

$$\varepsilon_{cs}(\infty) = \varepsilon_{cd}(\infty) + \varepsilon_{ca}(\infty)$$

$$\varepsilon_{cs}(\infty) = 3,69\cdot 10^{-4} + 3,75\cdot 10^{-5} = 4,07\cdot 10^{-4}$$

The dependence of shrinkage strain over time is shown in Figure A3.5.

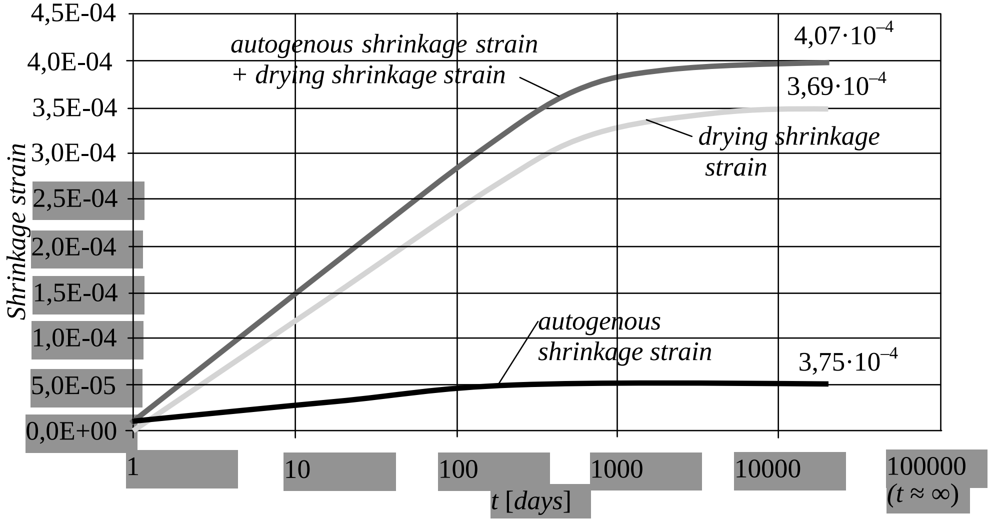

Figure A3.5 *The dependence of shrinkage strain over time*

5. Effective width of the concrete flange

5.1 Cross-section at mid-span

$$L_e = 0,85 \cdot L = 0,85 \cdot 1000 = 850 \text{ cm}$$

$$b_{e1} = L_e / 8 = 850 / 8 = 106 \text{ cm}$$

The effective width of the concrete flange at mid-span is:

$$b_{eff} = 2 \cdot b_{e1} = 2 \cdot 106 = 212 \text{ cm} < 250 \text{ cm}$$

5.2 Cross-section at support

$$L_e = 0,25 \cdot (L_1 + L_2) = 0,25 \cdot (1000 + 1000) = 500 \text{ cm}$$

$$b_{e1} = L_e / 8 = 500 / 8 = 62,5 \text{ cm}$$

The effective width of the concrete flange at support is:

$$b_{eff} = 2 \cdot b_{e1} = 2 \cdot 62,5 = 125 \text{ cm} < 250 \text{ cm}$$

6. Geometrical properties of composite cross-section at mid-span

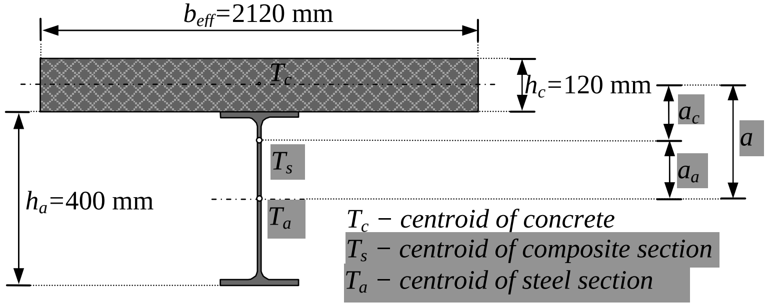

Figure A3.6 *Cross-section of composite beam at mid-span*

The distance between the centroidal axes of the concrete and the steel section a is:

$$a = \frac{h_a}{2} + \frac{h_c}{2} = \frac{40}{2} + \frac{12}{2} = 26 \text{ cm}$$

Steel section IPE400

$$A_a = 84{,}5 \text{ cm}^2$$

$$I_a = 23130 \text{ cm}^4$$

Concrete flange area

$$A_c = b_{eff} \cdot h_c = 212 \cdot 12 = 2544 \text{ cm}^2$$

$$I_c = \frac{b_{eff} \cdot h_c^3}{12} = \frac{212 \cdot 12^3}{12} = 30528 \text{ cm}^4$$

For *t*=∞, for the short-term loading

The modular ratio for the short-term loading is:

$$n_0 = E_a / E_{cm} = 21000 / 3100 = 6{,}77$$

From clause 5.4.2.2(11), EN 1994-1-1, the effects of creep may be allowed for by using the modular ratio $n_L = 2 \cdot n_0$ for both short-term and long-term loadings. Thus, the modular ratio is:

$$n_L = E_a / (0{,}5 \cdot E_{cm}) = 21000 / (0{,}5 \cdot 3100) = 13{,}55$$

The ideal cross-sectional area of the composite section, the sum of the area of steel section and the area of the concrete slab in “steel” units, is:

$$A_{id} = A_a + A_c / n_L = 84{,}5 + 2544 / 13{,}55 = 272{,}2 \text{ cm}^2$$

The distance between the centroidal axes of the steel section and the composite section is:

$$a_a = a \cdot \frac{A_c}{A_{id} \cdot n_L} = 26 \cdot \frac{2544}{272{,}2 \cdot 13{,}55} = 17{,}9 \text{ cm}$$

The distance between the centroidal axes of the concrete and the composite section is:

$$a_c = a \cdot \frac{A_a}{A_{id}} = 26 \cdot \frac{84{,}5}{272{,}2} = 8{,}1 \text{ cm}$$

The ideal second moment of area of the composite section is:

$$I_{id} = I_a + A_a \cdot a_a^{\,2} + (I_c + A_c \cdot a_c^{\,2}) / n_L$$

$$I_{id} = 23130 + 84{,}5 \cdot 17{,}9^2 + (30528 + 2544 \cdot 8{,}1^2) / 13{,}55$$

$$I_{id} = 64776 \ \text{cm}^4$$

For *t*=∞, taking into account the effects of creep

The modular ratio n_L is calculated as:

$$n_L = n_0 \cdot (1 + \psi_L \varphi_t)$$

The modular ratio for the short-term loading is:

$$n_0 = E_a / E_{cm} = 21000 / 3100 = 6{,}77$$

The creep multiplier n_L in clause 5.4.2.2(2), EN 1994-1-1, takes account of the shape of the stress – time curve for the effect considered, and is 0,55 for shrinkage.

The modular ratio for shrinkage effects is:

$$n_L = n_0 \cdot (1 + \psi_L \varphi_t) = 6{,}77 \cdot (1 + 0{,}55 \cdot 4{,}3)$$

$$n_L = 22{,}78$$

The ideal cross-sectional area of the composite section, the sum of the area of steel section and the area of the concrete slab in “steel” units, is:

$$A_{id} = A_a + A_c / n_L = 84{,}5 + 2544 / 22{,}78 = 196{,}2 \ \text{cm}^2$$

The distance between the centroidal axes of the steel section and the composite section is:

$$a_a = a \cdot \frac{A_c}{A_{id} \cdot n_L} = 26 \cdot \frac{2544}{196{,}2 \cdot 22{,}78} = 14{,}8 \ \text{cm}$$

The distance between the centroidal axes of the concrete and the composite section is:

$$a_c = a \cdot \frac{A_a}{A_{id}} = 26 \cdot \frac{84{,}5}{196{,}2} = 11{,}2 \ \text{cm}$$

The ideal second moment of area of the composite section is:

$$I_{id} = I_a + A_a \cdot a_a^{\,2} + (I_c + A_c \cdot a_c^{\,2}) / n_L$$

$$I_{id} = 23130 + 84{,}5 \cdot 14{,}8^2 + (30528 + 2544 \cdot 11{,}2^2) / 22{,}78$$

$$I_{id} = 56988 \ \text{cm}^4$$

7. Geometrical properties of composite cross-section at support

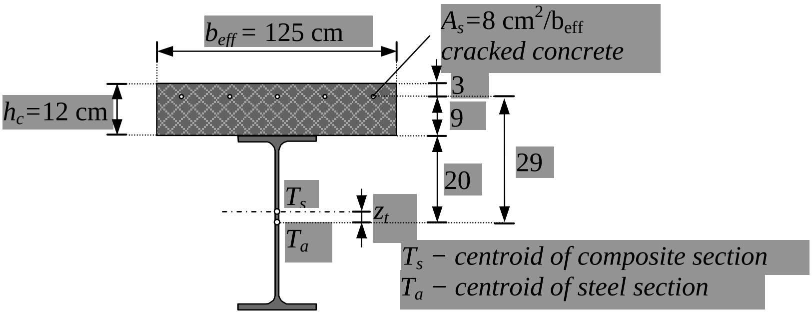

Figure A3.7 *Cross-section of composite beam at support*

In the slab at the internal support, the reinforcement is assumed to be A_s = 8 cm^2/b$_{eff}$.

The area of the cracked composite section:

$$A = A_s + A_a = 8 + 84{,}5 = 92{,}5 \ \text{cm}^2$$

The distance between the neutral axis of the cracked composite section and the centroidal axis of the steel section is:

$$z_t = A_s \cdot 29 / A = 8 \cdot 29 / 92{,}5 = 2{,}5 \ \text{cm}$$

Second moment of area:

$$I = I_a + A_a \cdot z_t^{\,2} + A_s \cdot (29 - z_t)^2 = 23130 + 84{,}5 \cdot 2{,}5^2 + 8 \cdot (29 - 2{,}5)^2 = 29276 \ \text{cm}^4$$

8. Effects of creep and shrinkage

In accordance with Section 5.4, EN 1994-1-1, the effects of actions may be calculated by elastic global analysis, even when the cross-sectional resistance is based on its plastic or non-linear resistance.

Generally, when the elastic global analysis is applied, it is necessary to take into account the effects of the concrete cracking, creep and shrinkage, the sequence of construction and the pre-stressing.

In accordance with EN 1994-1-1, for the calculation of the action effects of continuous composite beams, the simplified (approximate) methods based on the theory of elasticity and plasticity are applied.

For continuous composite beams with the concrete flanges above the steel section and not prestressed, including beams in frames that resist horizontal forces by bracing, the following simplified method may be applied for calculation of action effects:

- All the ratios of the lengths of adjacent continuous spans (shorter/longer) between supports are at least 0,6.
- The effect of cracking can be taken into account by using the flexural stiffness $E_a I_2$ over 15% of the span on each side of each internal support, and as the uncracked values $E_a I_1$ elsewhere.

The method, [35], is simple to implement and can be applied for the ultimate limit state and for the serviceability limit state.

8.1 Design bending moment for internal support

The design bending moment for the internal support is calculated for the static system and the design load, as shown in Figure A3.8.

Combination of actions:

$$e_d = 1,35 \cdot (g_{k,1} + g_{k,2} + g_{k,3}) + 1,5 \cdot q_k$$

The design value of actions in both spans is:

$$e_d = 1,35 \cdot (7,50 + 0,663 + 3,75) + 1,5 \cdot 7,50 = 27,3 \quad \text{kN/m}$$

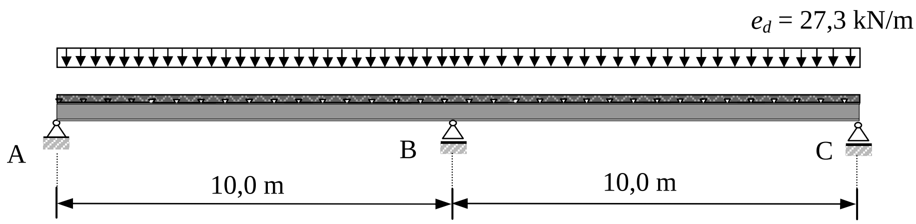

Figure A3.8 *Static system and design actions*

The calculation of the bending moment is carried out "by hand" according to the method given in [35].

Taking into account the symmetry of the structure and the load, the following statically indeterminate system is adopted for the calculation of the bending moment at point B, see Figure A3.9:

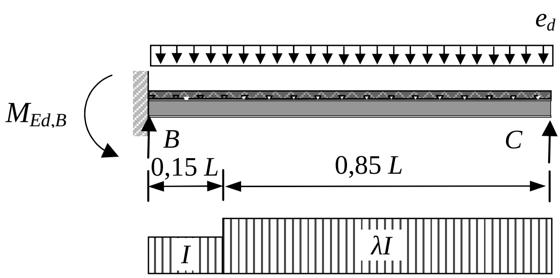

Figure A3.9 *Adopted static system – elastic propped cantilever with change of section at 0,15L*

The second moment of area of the cracked cross-section at the support, Figure A3.7, is:

$$I = 29276 \text{ cm}^4$$

The second moment of area of the uncracked cross-section at mid-span is:

$$I_{id} = 64776 \text{ cm}^4$$

The ratio of the second moments of area λ is:

$$\lambda = (I_{id}(t = \infty, n_L = 13{,}55))/I = \frac{64776}{29276} = 2{,}21$$

The design bending moment for the internal support, point B, excluding shrinkage effects, is calculated as follows [35]:

$$M_{Ed,B}^{g+q} = (\frac{e_d \cdot L^2}{4}) \cdot \frac{(0,110 \cdot \lambda + 0,890)}{(0,772 \cdot \lambda + 1,228)}$$

$$M_{Ed,B}^{g+q} = (\frac{27,30 \cdot 10,00^2}{4}) \cdot \frac{(0,110 \cdot 2,21 + 0,890)}{(0,772 \cdot 2,21 + 1,228)} = 263,6 \ \text{kNm}$$

8.2 Secondary effects of shrinkage

Shrinkage of the concrete flange causes sagging curvature and shortening of the composite beam. These are the primary effects of shrinkage. However, in a continuous beam, the curvature causes bending moments and shear forces. These are then the secondary effects. Both the curvature and the stresses from the primary effects are neglected in regions assumed to be cracked, clauses 5.4.2.2(8) and 6.2.1.5(5), EN 1994-1-1.

The hogging bending moment at the internal support is the important secondary effect for the considered beam. It is calculated as follows. Shrinkage is a permanent action, and so is not reduced by a combination factor ψ_0.

It is assumed that the concrete flange is separated from the steel beam. The concrete flange is shrunk due to shrinkage. The force that would cause the opposite effect – to extend the flange to its original length – is given by:

$$N_{cs} = \varepsilon_{cs}(\infty) \cdot E_S \cdot A_c$$

where:

$$E_S = \frac{E_a}{n_L}$$

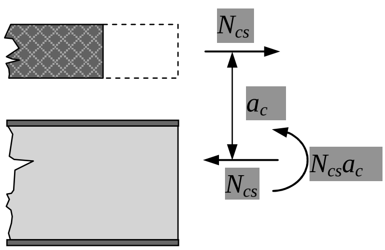

Figure A3.10 *Assumption – concrete flange is separated from the steel beam*

The total shrinkage strain, in accordance with EN 1992-1-1, is:

$\varepsilon_{cs}(\infty) = 4{,}07 \cdot 10^{-4}$

The force that would cause the opposite effect, to extend the flange to its original length, is:

$$E_S = \frac{E_a}{n_L} = \frac{21000}{22{,}78} = 922 \text{ kN/cm}^2$$

$$N_{cs} = \varepsilon_{cs}(\infty) \cdot E_S \cdot A_c = 4{,}07 \cdot 10^{-4} \cdot 922 \cdot 250 \cdot 12 = 1126 \text{ kN}$$

The force N_{cs} acts at the centre of the concrete flange, at a distance a_c above the centroid of the composite section:

$$a_c = a \cdot \frac{A_a}{A_{id}} = 26 \cdot \frac{84{,}5}{196{,}2} = 11{,}2 \text{ cm}$$

The parts of the beam are reconnected. To re-establish equilibrium, an opposite force N_{cs} and a bending moment $N_{cs} \cdot a_c$ (Figure A3.10) are applied to the composite section.

The radius of curvature of the uncracked part of the beam is given by:

$$R = \frac{E_a \cdot \lambda \cdot I_y}{N_{cs} \cdot a_c}$$

The second moment of area of the cracked cross-section is:

$$I = 29276 \text{ cm}^4$$

The ratio of the second moments of area λ is:

$$\lambda = (I_{id}(t = \infty, n_L = 22{,}78))/I = \frac{56988}{29276} = 1{,}95$$

The radius of curvature of the uncracked part of the beam is:

$$R = \frac{E_a \cdot \lambda \cdot I_y}{N_{cs} \cdot a_c} = \frac{21000 \cdot 1{,}95 \cdot 29276}{1126 \cdot 11{,}2} = 95063 \text{ cm} = 950{,}6 \text{ m}$$

If the centre support is removed, from the geometry of the circle, the deflection δ at that point is given by (Figure A3.11):

$$\delta = \frac{(0{,}85 \cdot L)^2}{2 \cdot R} = \frac{(0{,}85 \cdot 1000{,}0)^2}{2 \cdot 95063} = 3{,}8 \;\; \text{cm}$$

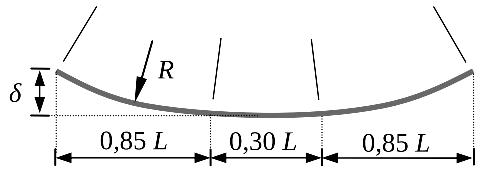

Figure A3.11 *Model for calculation of deflection δ*

The actual deflection at the middle is equal to zero. It is necessary to calculate the force *P*, applied at the point B, to reduce the deflection to zero, so the internal support can be replaced (Figure A3.12).

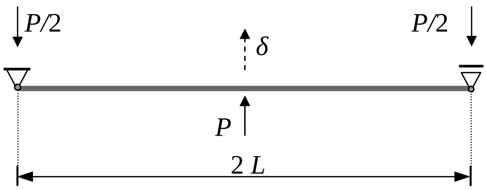

Figure A3.12 *Model for calculation force P*

The deflection due to the force *P* is:

$$\delta = \frac{P \cdot L^3 \cdot (0{,}13 \cdot \lambda + 0{,}20)}{2 \cdot E_a \cdot I \cdot \lambda} = 3{,}8 \;\; \text{cm}$$

From the above expression the force *P* is:

$$P = \frac{2 \cdot E_a \cdot I \cdot \lambda \cdot \delta}{L^3 \cdot (0{,}13 \cdot \lambda + 0{,}20)} = \frac{2 \cdot 21000 \cdot 29276 \cdot 1{,}95 \cdot 3{,}8}{1000^3 \cdot (0{,}13 \cdot 1{,}95 + 0{,}20)} = 20{,}1 \;\; \text{kN}$$

The secondary hogging bending moment at B is:

$$M_{Ed,sh,B} = \frac{P \cdot L}{2} = \frac{20{,}1 \cdot 10}{2} = 100{,}5 \;\; \text{kNm}$$

The design bending moment due to the loads and shrinkage is:

$$M_{Ed,B} = M_{Ed,B}^{g+q} + M_{Ed,sh,B} = 263{,}6 + 100{,}5 = 364{,}1 \;\; \text{kNm}$$

By taking into account shrinkage, the ultimate design bending moment at the support B is increased by 38%. In accordance with clause 5.4.2.2(7), EN 1994-1-1, the shrinkage effects can be neglected for the ultimate limit state if resistance is not influenced by lateral-torsional buckling. When the resistance moments of the sections are determined by plastic theory, the elastic deformations – such as those from shrinkage − are negligible compared to the total deformations.

However, if the resistance at the internal support is governed by lateral-torsional buckling, and if this resistance is significantly lower than the plastic resistance moment of the section, the inelastic behaviour due to the effect of shrinkage is not negligible.

9. Commentary

The previously used method of calculating of shrinkage effects is relatively complex for practical use.

In the appropriate software, which allows analysis of elements taking into account the concrete and steel section, the shrinkage effects may be included as an alternate load by temperature as:

$$\Delta T_{sh} = \frac{N_{cs} \cdot a_c \cdot h}{E_a \cdot \mathrm{I}_{id} \cdot \alpha_T}$$

where:

N_{cs} is the axial force due to shrinkage,
a_c is the distance between the centroidal axes of the concrete flange and the composite section,
h is the overall depth of the composite section,
E_a is the modulus of elasticity of structural steel,
I_{id} is the ideal second moment of area of the composite section,
α_T is the coefficient of thermal expansion, $12 \cdot 10^{-6}$ K.

This case of thermal action is taken into account only in the range of uncracked cross-sections of the beam over 85% of the span (0,85 L).

However, in practice we can also use the procedure that was implemented in examples B6 and B7.

B Composite beams

B1 Effective width of concrete flange

1. Purpose of example

This example illustrates the method of calculation of the effective width of the concrete flange due to shear lag. The method is recommended in clause 5.4.1.2, EN 1994-1-1.

It is considered that a continuous composite beam consists of two spans and a cantilever, as shown in Figure B1.1. We need to calculate the value of the effective width of the concrete flange b_{eff} for the mid-span regions AB and CD, for the support regions BC and DE and for the support at A. The recapitulation of the obtained results is shown in Table B1.1.

2. Static system and cross-section

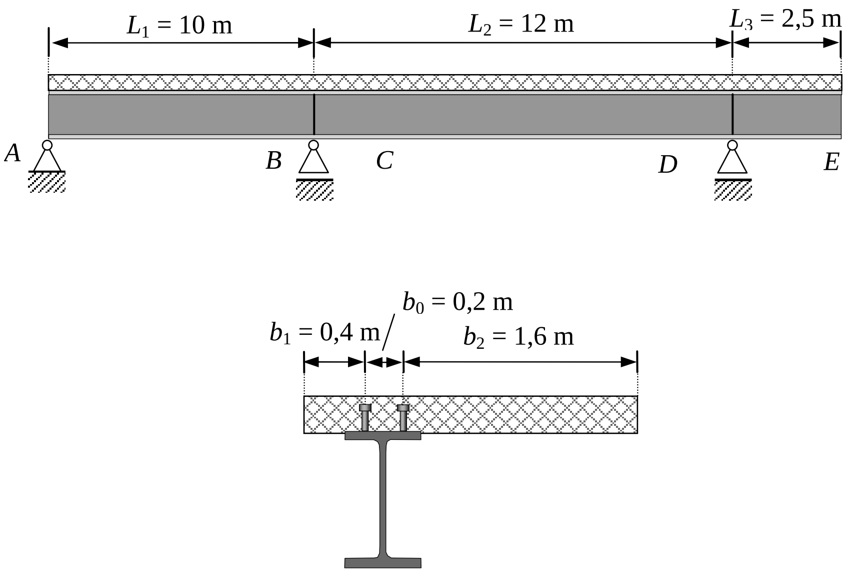

Figure B1.1 *Two-spans continuous beam and cross-section*

3. Calculation of effective width of the concrete flange

The calculation of the effective width for the required regions is carried out in accordance with clause 5.4.1.2, EN 1994-1-1, using the expressions (5.3), (5.4) and (5.5).

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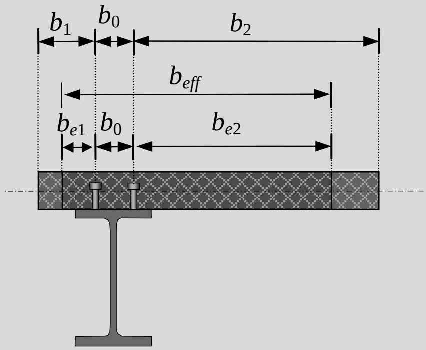

Figure B1.2 *Effective width dimensions*

3.1 Support A

$b_{eff} = b_0 + \Sigma\beta_i \cdot b_{ei}$ according to expression (5.4), EN 1994-1-1

$\beta_i = (0{,}55 + 0{,}025 \cdot \dfrac{L_e}{b_{ei}}) \leq 1{,}0$ according to expression (5.5), EN 1994-1-1

$L_e = 0{,}85 \cdot L_1$ for $b_{eff,1}$ according to Figure 5.1, EN 1994-1-1

$b_0 = 0{,}20$ m

$L_1 = 10{,}0$ m

$L_e = 0{,}85 \cdot 10{,}0 = 8{,}5$ m

$$b_{ei} = \frac{L_e}{8} \leq b_i$$

$$b_{e1} = \frac{L_e}{8} = \frac{8{,}5}{8} = 1{,}063 \text{ m} > b_1 = 0{,}4 \text{ m}$$

Adopted: $b_{e1} = 0{,}4$ m

$$b_{e2} = \frac{L_e}{8} = \frac{8{,}5}{8} = 1{,}063 \text{ m} < b_2 = 1{,}6 \text{ m}$$

Adopted: $b_{e2} = 1{,}063$ m

$\beta_1 = (0,55 + 0,025 \cdot L_e / b_{e1}) \leq 1,0$ according to expression (5.5), EN 1994-1-1

$\beta_1 = (0,55 + 0,025 \cdot 8,5 / 0,40) = 1,081 > 1,0$

$\beta_2 = (0,55 + 0,025 \cdot L_e / b_{e2}) \leq 1,0$ according to expression (5.5), EN 1994-1-1

$\beta_2 = (0,55 + 0,025 \cdot 8,5 / 1,063) = 0,750 < 1,0$

$b_{eff} = b_0 + \beta_1 \cdot b_{e1} + \beta_2 \cdot b_{e2} = 0,20 + 1,0 \cdot 0,40 + 0,750 \cdot 1,063 = 1,40 \text{ m}$

3.2 Mid-region AB

$b_{eff} = b_0 + \Sigma b_{ei}$ according to expression (5.3), EN 1994-1-1

$b_0 = 0,20$ m

$$b_{ei} = \frac{L_e}{8} \leq b_i$$

$L_e = 0,85 \cdot L_1$ for $b_{eff,1}$ according to Figure 5.1, EN 1994-1-1

$L_1 = 10,0$ m

$L_e = 0,85 \cdot 10,0 = 8,5 \text{ m}$

$$b_{e1} = \frac{L_e}{8} = \frac{8,5}{8} = 1,063 \text{ m} > b_1 = 0,4 \text{ m}$$

Adopted: $b_{e1} = 0,4$ m

$$b_{e2} = \frac{L_e}{8} = \frac{8,5}{8} = 1,063 \text{ m} < b_2 = 1,6 \text{ m}$$

Adopted: $b_{e2} = 1,063$ m

$b_{eff} = b_0 + b_{e1} + b_{e2} = 0,20 + 0,40 + 1,063 = 1,663 \text{ m}$

3.3 Support region BC

$b_{eff} = b_0 + \Sigma b_{ei}$ according to expression (5.3), EN 1994-1-1

b_0 = 0,20 m

$$b_{ei} = \frac{L_e}{8} \leq b_i$$

$$L_e = 0{,}25 \cdot (L_1 + L_2)$$

For $b_{eff,2}$ according to Figure 5.1, EN 1994-1-1:

L_1 = 10,0 m

L_2 = 12,0 m

$$L_e = 0{,}25 \cdot (10{,}0 + 12{,}0) = 5{,}50 \text{ m}$$

$$b_{e1} = \frac{L_e}{8} = \frac{5{,}50}{8} = 0{,}688 \text{ m} > b_1 = 0{,}4 \text{ m}$$

Adopted: $b_{e1} = 0{,}4$ m

$$b_{e2} = \frac{L_e}{8} = \frac{5{,}50}{8} = 0{,}688 \text{ m} < b_2 = 1{,}6 \text{ m}$$

Adopted: $b_{e2} = 0{,}688$ m

$$b_{eff} = b_0 + b_{e1} + b_{e2} = 0{,}20 + 0{,}40 + 0{,}688 = 1{,}288 \text{ m}$$

3.4 Mid-span region CD

$b_{eff} = b_0 + \Sigma b_{ei}$ according to expression (5.3), EN 1994-1-1

b_0 = 0,20 m

$$b_{ei} = \frac{L_e}{8} \leq b_i$$

$L_e = 0{,}70 \cdot L_2$ for $b_{eff,1}$ according to Figure 5.1, EN 1994-1-1

L_2 = 12,0 m

$L_e = 0{,}70 \cdot 12 = 8{,}4\ \text{m}$

$$b_{e1} = \frac{L_e}{8} = \frac{8{,}4}{8} = 1{,}050\ \text{m} > b_1 = 0{,}4\ \text{m}$$

Adopted: $b_{e1} = 0{,}4\ \text{m}$

$$b_{e2} = \frac{L_e}{8} = \frac{8{,}4}{8} = 1{,}050\ \text{m} > b_2 = 1{,}6\ \text{m}$$

Adopted: $b_{e2} = 1{,}050\ \text{m}$

$$b_{eff} = b_0 + b_{e1} + b_{e2} = 0{,}20 + 0{,}40 + 1{,}050 = 1{,}650\ \text{m}$$

3.5 Support region DE

$b_{eff} = b_0 + \Sigma b_{ei}$ according to expression (5.3), EN 1994-1-1

b_0 = 0,20 m

$$b_{ei} = \frac{L_e}{8} \le b_i$$

$L_e = 2 \cdot L_3$ for $b_{eff,2}$ according to Figure 5.1, EN 1994-1-1

L_3 = 2,5 m

$L_e = 2 \cdot 2{,}5 = 5{,}0\ \text{m}$

$$b_{e1} = \frac{L_e}{8} = \frac{5{,}0}{8} = 0{,}625\ \text{m} > b_1 = 0{,}4\ \text{m}$$

Adopted: $b_{e1} = 0{,}4\ \text{m}$

$$b_{e2} = \frac{L_e}{8} = \frac{5{,}0}{8} = 0{,}625\ \text{m} < b_2 = 1{,}6\ \text{m}$$

Adopted: $b_{e2} = 0{,}625$ m

$$b_{eff} = b_0 + b_{e1} + b_{e2} = 0{,}20 + 0{,}40 + 0{,}625 = 1{,}225 \text{ m}$$

4. Recapitulation of results

Table B1.1 *Effective width of the concrete flange b_{eff} of a continuous composite beam*

	Region				
	Support A	AB	BC	CD	DE
L_e (m)	8,50	8,50	5,50	8,40	5,00
$L_e/8$ (m)	1,063	1,063	0,688	1,050	0,625
b_{e1} (m)	0,40	0,40	0,40	0,40	0,40
b_{e2} (m)	1,063	1,063	0,688	1,050	0,625
b_{eff} (m)	1,40	1,663	1,288	1,650	1,225

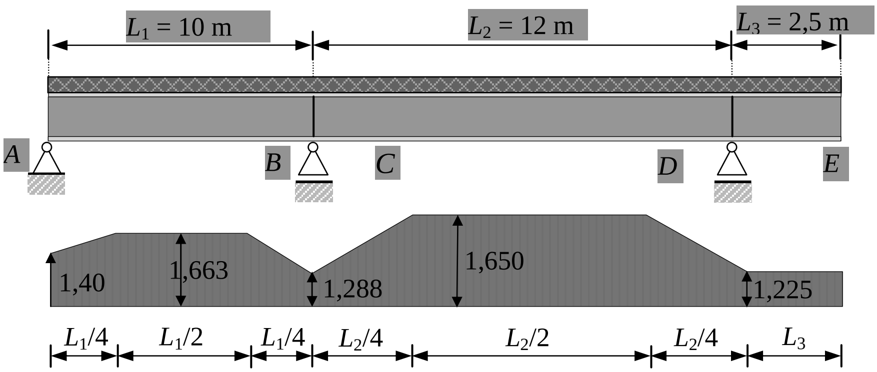

Figure B1.3 *Effective width of the concrete flange of a continuous composite beam*

5. Commentary

Elastic global analysis may be based on the stiffness calculated using the results for AB and CD. However, the differences between these values are so small that the structural element ABCDE can be analysed as a beam of uniform section with the effective width of the concrete flange b_{eff} = 1,650 m.

B2 Composite beam – arrangement of shear connectors in solid slab

1. Purpose of example

The purpose of the example is to show the calculation and the arrangement of the headed stud connectors in the case of the simple supported composite beam. The class of cross-section is such that we can calculate the resistance moment of the cross-section of the beam by means of rigid plastic theory. The thickness of the concrete slab is h_c = 14 cm.

The essence of the example is the calculation and the arrangement of the headed stud connectors in the solid slab. Therefore, the design of the beam is conducted more simply without considering creep and shrinkage of the concrete. Spacing and arrangement of the studs is determined according to the diagram of shear forces or by the shear flow, which is calculated according to the theory of elasticity which assumes a linear relationship between the shear force and the longitudinal shear force. The compressive normal force in the concrete flange N_c is transmitted to the composite beam by means of the headed studs.

2. Static system, cross-section and actions

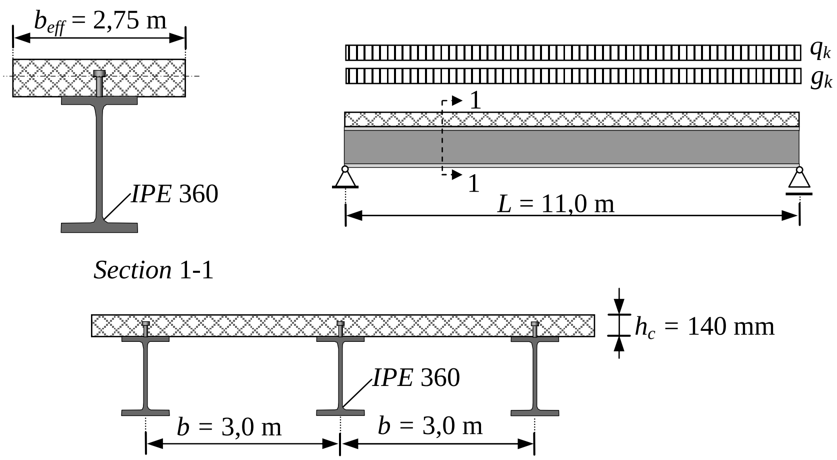

Figure B2.1 *Static system and cross-section*

Actions

Permanent action

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- *concrete slab and steel beam* $g_k = 11{,}1\ \text{kN/m}$

Variable action (including partitions)
- *variable and partitions* $q_k = 15{,}0\ \text{kN/m}$

3. Properties of materials

Concrete strength class: C 25/30 $f_{ck} = 25\ \text{N/mm}^2$

$$f_{cd} = \frac{f_{ck}}{\gamma_c} = \frac{25}{1{,}5} = 16{,}7\ \text{N/mm}^2$$

Structural steel: S355 $f_{yk} = 355\ \text{N/mm}^2$

$$f_{yd} = \frac{f_{yk}}{\gamma_M} = \frac{355}{1{,}0} = 355\ \text{N/mm}^2$$

Shear connectors: ductile headed studs $f_u = 450\ \text{N/mm}^2$

$d = 19\ \text{mm}$

$h_{sc} = 100\ \text{mm}$

$$\frac{h_{sc}}{d} = \frac{100}{19} = 5{,}3 > 4{,}0\ \ \rightarrow \alpha = 1{,}0$$

$P_{Rd} =$ 73,7 kN

4. Ultimate limit state

4.1 Design values of combined actions and design values of effects of actions

The design load of governed combination of actions is:

$$e_d = 1{,}35 \cdot g_k + 1{,}50 \cdot q_k$$

$$e_d = 1{,}35 \cdot 11{,}1 + 1{,}50 \cdot 15{,}0 = 37{,}5\ \text{kN/m}$$

The calculation of effects of actions is:

$$M_{Ed} = \frac{e_d \cdot L^2}{8} = \frac{37{,}5 \cdot 11{,}0^2}{8} = 567\ \text{kNm}$$

$$V_{Ed} = e_d \cdot \frac{L}{2} = 37{,}5 \cdot \frac{11{,}0}{2} = 206 \text{ kN}$$

4.2 Effective width of concrete flange

$$b_{eff} = b_0 + b_{e1} + b_{e2}$$

$$b_0 = 0$$

The distance between the centres of the rows of shear connectors is $b_0 = 0$ because there is only one row of shear connectors.

$$b_{e1} = b_{e2} = \frac{L_0}{8} = \frac{11{,}0}{8} = 1{,}375 \text{ m}$$

$$b_{eff} = 2 \cdot 1{,}375 = 2{,}75 \text{ m} < b = 3{,}0 \text{ m}$$

The effective cross-section of the concrete flange is shown in Figure B2.2.

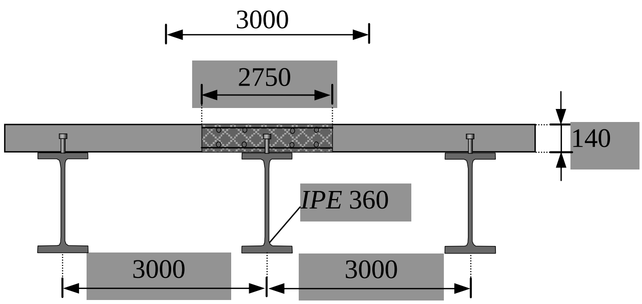


Figure B2.2 *Effective cross-section of the concrete flange*

4.3 Plastic resistance moment of composite cross-section

Since the main purpose of the example is to show the calculation and the arrangement of the headed stud connectors, a simplified verification for the ultimate limit state contains: *a) Check of the resistance of the composite cross-section to bending* and *b) Check of the resistance of the composite cross-section to vertical shear*.

The calculation of the plastic resistance moment $M_{pl,Rd}$ can be performed in two ways: (a) by calculating the position of the plastic neutral axis, x_{pl}, (b) directly without calculating x_{pl}.

a) By calculating of the position of the plastic neutral axis x_{pl}

If $N_{c,f} > N_{pl,a}$, the plastic neutral axis is located in the thickness h_c of the concrete of the slab:

$$N_{c,f} > N_{pl,a}$$

$$b_{eff} \cdot h_c \cdot 0{,}85 \cdot f_{cd} > A_a \cdot f_{yd}$$

$$2{,}75 \cdot 14 \cdot 0{,}85 \cdot 1{,}67 \geq 72{,}73 \cdot 35{,}5$$

$5465 > 2582$ $\rightarrow$ the plastic neutral axis is located in the slab depth.

The effective cross-section of the concrete flange, with dimensions, is shown in Figure B2.3.

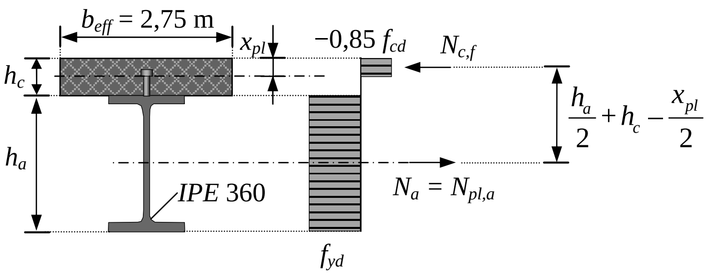


Figure B2.3 *Effective cross-section of the concrete flange with dimensions*

The plastic neutral axis lies a distance x_{pl} below the top of the concrete flange:

$$x_{pl} = \frac{A_a \cdot f_{yd}}{b_{eff} \cdot 0{,}85 \cdot f_{cd}} = \frac{72{,}73 \cdot 35{,}5}{275 \cdot 0{,}85 \cdot 1{,}67} = 6{,}61 \text{ cm} < h_c = 14{,}0 \text{ cm}$$

When the plastic neutral axis lies within the concrete slab, the plastic resistance moment $M_{pl,Rd}$ may be determined from:

$$M_{pl,Rd} = A_a \cdot f_{yd} \cdot \left(\frac{h_a}{2} + h_c - \frac{x_{pl}}{2}\right)$$

$$M_{pl,Rd} = 72{,}73 \cdot 35{,}5 \cdot \left(\frac{36}{2} + 14 - \frac{6{,}61}{2}\right) \cdot \frac{1}{100} = 741 \text{ kNm}$$

b) Directly without calculating x_{pl}

The design value of the plastic resistance of the structural steel section to normal force is:

$$N_{pl,a} = A_a \cdot f_{yd} = 72{,}73 \cdot 35{,}5 = 2582 \text{ kN}$$

The resistance of the effective area of the concrete flange acting compositely with the steel section is:

$$N_{c,f} = A_c \cdot 0{,}85 \cdot f_{cd} = 275 \cdot 14 \cdot 0{,}85 \cdot 1{,}67 = 5465 \text{ kN}$$

Since $N_{pl,a} < N_{c,f}$ the plastic neutral axis lies within the concrete slab. The lesser of the two values, $N_{pl,a}$ and $N_{c,f}$, is governed so that $N_a = N_{c,f} = N_{pl,a}$. The bending moment M_c, which takes the concrete flange, is:

$$M_c = N_c \cdot \frac{h_c}{2}(1 - \frac{N_a}{N_{c,f}}) = 2582 \cdot \frac{14{,}0}{2}(1 - \frac{2582}{5465}) \cdot \frac{1}{100} = 95 \text{ kNm}$$

The plastic resistance moment of the composite section $M_{pl,Rd}$ is:

$$M_{pl,Rd} = M_c + N_a \cdot z = 95 + 2582 \cdot (\frac{h_a}{2} + \frac{h_c}{2}) \cdot \frac{1}{100} = 741 \text{ kNm}$$

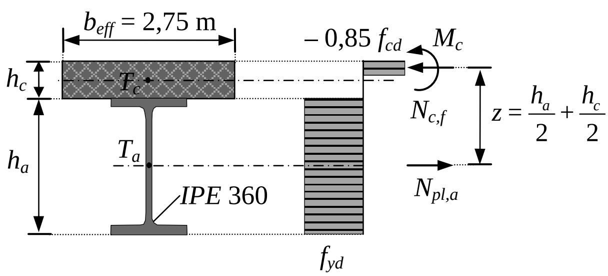


Figure B2.4 *Stress blocks for calculating the resistance of the composite cross-section*

Criterion:

$$\frac{M_{Ed}}{M_{pl,Rd}} \leq 1{,}0$$

$$\frac{567}{741} = 0{,}77$$

$0{,}77 < 1{,}0$, so the resistance moment of the composite cross-section is verified.

4.4 Vertical shear resistance

The shear buckling resistance of the web should be verified, for the unstiffened web when:

$$\frac{h_w}{t} > \frac{72}{\eta}\varepsilon$$

where:

$$\varepsilon = \sqrt{\frac{235}{f_y}} = \sqrt{\frac{235}{355}} = 0,81$$

$\eta = 1,2$, the factor defined in EN 1993-1-5

$$h_w = h_a - 2 \cdot t_f = 360 - 2 \cdot 12,7 = 335 \text{ mm}$$

$$\frac{72}{\eta}\varepsilon = \frac{72}{1,2} \cdot 0,81 = 48,6$$

$$\frac{h_w}{t} = \frac{h_w}{t_w} = \frac{335}{8,0} = 41,9$$

Since 41,9 < 48,6, the condition is satisfied. The shear buckling resistance of the web need not be verified.

Remark

The resistance of the composite beam to vertical shear is normally taken as the shear resistance of the steel section according to clause 6.2.6, EN 1993-1-1, which gives:

$$V_{pl,Rd} = V_{pl,a,Rd} = \frac{A_V (f_y / \sqrt{3})}{\gamma_{M0}}$$

For rolled I- and H-sections, if the load is applied parallel to the web, the shear area is calculated as:

$A_V = A_a - 2 \cdot b_a \cdot t_f + t_f \cdot (t_w + 2 \cdot r)$, but not less than $\eta \cdot h_w \cdot t_w$

The governed shear area A_V is shown in Figure B2.5, and it is:

$$A_V = 72,73 - 2 \cdot 17 \cdot 1,27 + 1,27 \cdot (0,8 + 2 \cdot 1,8)$$

$$A_V = 35,1 \ \text{cm}^2$$

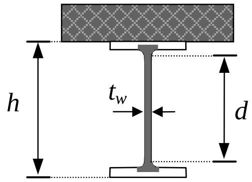


Figure B2.5 *Governed shear area*

$$\eta = 1,2$$

$$\eta \cdot h_w \cdot t_w = 1,2 \cdot 33,5 \cdot 0,8 = 32,2 \ \text{cm}^2$$

35,1 cm^2 > 32,2 cm^2

Thus, A_V = 35,1 cm^2

The design plastic shear resistance of the steel section is:

$$V_{pl,Rd} = V_{pl,a,Rd} = \frac{A_V (f_y / \sqrt{3})}{\gamma_{M0}}$$

$$V_{pl,Rd} = V_{pl,a,Rd} = 35,1 \frac{35,5}{\sqrt{3} \cdot 1,0} = 719 \ \text{kN}$$

Criterion:

$$\frac{V_{Ed}}{V_{pl,Rd}} \leq 1,0$$

$$\frac{206}{719} = 0,29 < 1,0$$

Since $0,29 < 1,0$, the condition is satisfied.

4.5 Check of resistance of headed stud connectors

The design value of the compressive normal force in the concrete flange N_c is transferred to the composite beam by means of the headed studs, as shown in Figure B2.6.

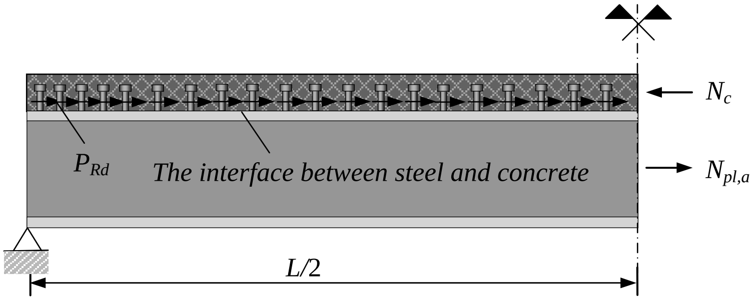


Figure B2.6 *Transfer of compressive normal force in concrete flange N_c by means of studs*

Calculating the plastic resistance moment of the composite section $M_{pl,Rd}$ is conducted so that the minimum force of N_c and $N_{pl,a}$ is multiplied by the lever arm z. In this example, the lesser force is $N_{pl,a}$ so that:

$$N_c = N_{pl,a} = 2582 \text{ kN}$$

The design value of the compressive normal force in the concrete flange N_c can be reduced in the ratio of the degree of utilization ($M_{Ed}/M_{pl,Rd}$):

$$red\ N_c = N_c \frac{M_{Ed}}{M_{pl,Rd}} = 2582 \cdot \frac{567}{741} = 1976 \text{ kN}$$

where $red\ N_c$ is the reduced compresive normal force in the concrete flange.

Since the design resistance of each stud is $P_{Rd} = 73{,}7$ kN, the required number of studs can be determined for half span $L/2$ as:

$$n = \frac{red\ N_c}{P_{Rd}} = \frac{1976}{73{,}7} \approx 27 \text{ studs}$$

In the elastic design, the shear connectors are spaced in accordance with the shear flow with a triangular distribution. If the span of the beam is divided into the several ranges where the studs are distributed at equal intervals, this leads to the idea that is shown in Figure B2.7. The total number of studs, n, is shared between lengths of the ranges in proportion to the areas of the design shear force diagram of the considered ranges. This option results in uniform resistance in each length. The longitudinal shear resistance is made to match the peak value of shear flow over that length and the following assumptions are valid:

- the peak shear flow within each length does not exceed the design longitudinal shear resistance per unit length by more than 10%,
- the total design longitudinal shear over the length does not exceed the total design resistance for this length.

This simplified model is shown in Figure B2.7.

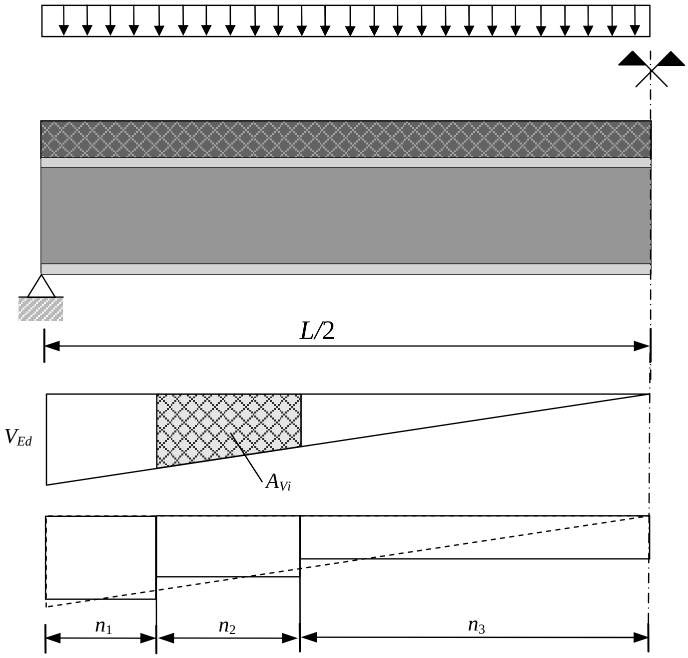


Figure B2.7 *Stepped form of shear flow*

The ranges in proportion to the areas of the shear force diagram are calculated as follows:

$$A_V = \frac{1}{2} V_{Ed} \cdot L$$

$$n_i = \frac{A_{Vi}}{A_V} \cdot n$$

$$\Sigma\, n_i \geq n$$

When determining the spacing of the studs, it is necessary to comply with the following conditions:

- the minimum spacing of studs in the direction of the shear force $e_L \geq 5d$, but not greater than six times the thickness of the concrete slab or 800 mm,

- the minimum spacing of studs in the direction transverse to the shear force $e_q \geq 2{,}5d$.

The arrangement of studs and the spacing of the studs are determined according to the diagram of the longitudinal shear flow. The half span of the beam is divided into three regions with the same areas of the shear flow. The lengths of these regions are 18%, 24% and 58% of the half span, $L/2$. The procedure is carried out as follows.

The reduced compressive normal force in the concrete flange is:

$$N_c = 1976 \text{ kN}$$

Clause 6.6.1.3(5), EN 1994-1-1, refers to the calculation of the longitudinal shear flow according to elastic theory. This assumes the use of the expression:

$$v_{L,Ed} = V_{Ed} \cdot \frac{S}{I}$$

where:

I is the ideal second moment of area of the cross-section,
S is the ideal first moment of area of the concrete slab or the steel section about the elastic neutral axis.

Thus, the distribution of the shear flow is obtained as shown in Figure B2.8.

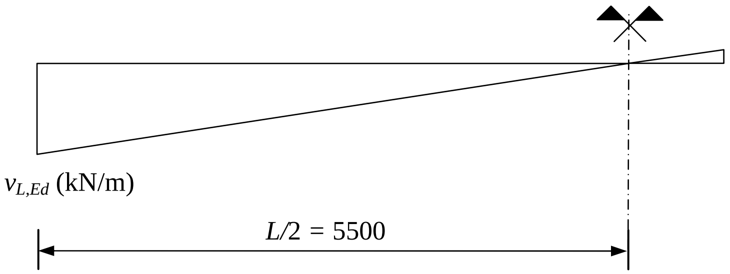


Figure B2.8 *Distribution of shear flow*

The diagram of the elastic shear flow is triangular, and the design value of the shear flow at the first support is:

$$v_{L,Ed} = 2 \cdot N_c / (L/2)$$

$$v_{L,Ed} = 2 \cdot 1976 / 5{,}5 = 719 \text{ kN/m}$$

Since we are dealing with the half span of the beam divided into three regions with the same shear flows, the model for calculating the lengths of the regions is formed as shown in Figure B2.9.

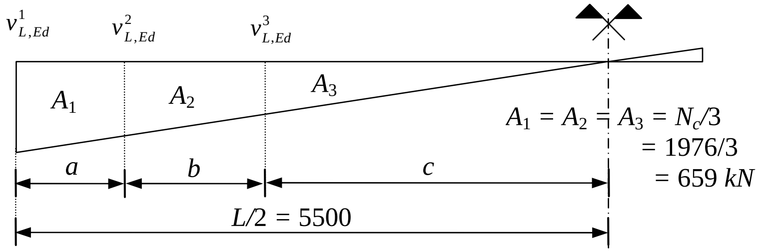


Figure B2.9 *Model for calculating the lengths of the regions*

In the first step, we calculate the length of the first region, *a*, and $v^2_{L,Ed}$ according to Figure B2.10.

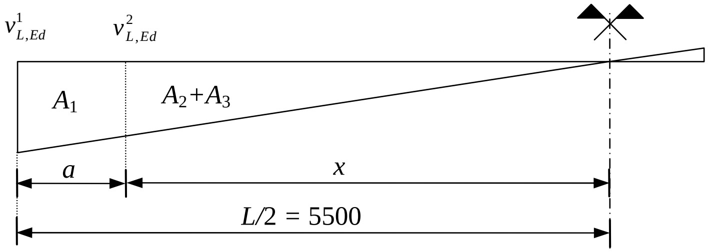


Figure B2.10 *Model for calculating the length of the first region*

The first equation is obtained from the calculation of the area of the triangle of the length *x*:

$$\frac{v^2_{L,Ed} \cdot x}{2} = A_2 + A_3$$

$$v^2_{L,Ed} \cdot x = 2 \cdot (659 + 659)$$

$$v_{L,Ed}^{2}\cdot x = 2636$$

The second equation is obtained from the ratio of the sides of triangles according to Figure B2.10:

$$v_{L,Ed}^{1} : v_{L,Ed}^{2} = (a + x) : x$$

$$(a + x) = 5{,}5$$

$$v_{L,Ed}^{1}\cdot x = 5{,}5\cdot v_{L,Ed}^{2}$$

Substituting the expression $v_{L,Ed}^{1}\cdot x = 5{,}5\cdot v_{L,Ed}^{2}$ into $v_{L,Ed}^{2}\cdot x = 2636$, and rearranging gives:

$$v_{L,Ed}^{2} = 587 \text{ kN/m}$$

The area of the first region from Figure B2.10 can be given in the following form:

$$(v_{L,Ed}^{1} - v_{L,Ed}^{2})\cdot\frac{a}{2} + v_{L,Ed}^{2}\cdot a = A_1$$

Substituting the previously calculated values in the above expression and the rearranging gives:

$$a = 1010 \text{ mm}$$

In relation to the half span of the beam, the length of the first region *a* is:

$$a / (L/2) = 1010 / 5500 = 0{,}18 \ (18\ \%)$$

Analogously, the value of $v_{L,Ed}^{3}$ is calculated, and then the other percentages of the lengths of the regions 2 and 3: 24% and 58% (Figure B2.11).

In each region, 10 studs are installed which are spaced uniformly over the length of the regions.

In regions 1 and 2 there are five pairs of studs, while in region 3 there are 10 studs in a row. The arrangement of the studs is shown in Figure B2.11.

Another option is that the half span of the beam is divided into two regions. If each region took 50% of the longitudinal shear force, the lengths of the regions should be divided in a ratio of 30% : 70% of the half span, *L*/*2*. The calculation procedure is the same as in the previous case.

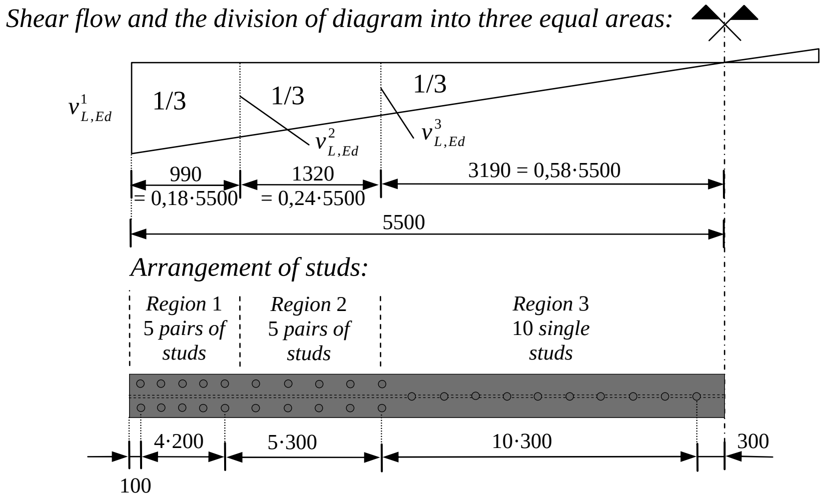


Figure B2.11 *Arrangement of stud connectors with regard to the shear flow diagram*

4.6 Check of the longitudinal shear resistance of the concrete flange

The transverse reinforcement in the slab is designed for the ultimate limit state so that premature longitudinal shear failure or longitudinal splitting is prevented. Figure B2.12 shows the critical sections for the failure of the concrete flange due to longitudinal shear. For these sections it is necessary to implement the verification.

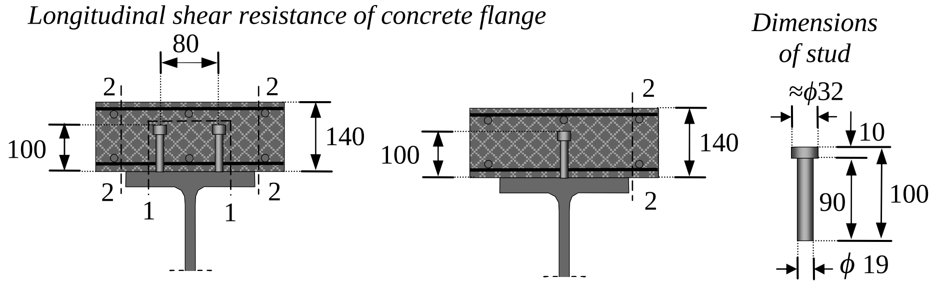


Figure B2.12 *Critical sections for failure of the concrete flange due to longitudinal shear*

5. Commentary

The example illustrates the arrangement of the studs according to clause 6.6.1.3(5), EN 1994-1-1, which relates to “longitudinal shear calculated by elastic theory”. Clause 6.6.1.3(1)P and to a lesser extent 6.6.1.1(2)P, refer to the “spacing” of the studs and the “appropriate distribution” of the longitudinal shear. The interpretation of “appropriate” depends on the applied method of analysis and the ductility of the studs.

The spacing of the studs in accordance with clause 6.6.1.3(5), i.e. the studs being spaced “elastically”, can be applied generally. The more appropriate use of uniform spacing requires the studs to satisfy clause 6.6.1.3 (3), which implies, but does not require, the use of plastic resistance moment. The studs must be “ductile”, as defined in clauses 6.6.1.1(4)P and 6.6.1.1(5)P. This is normally achieved by satisfying clause 6.6.1.2.

B3 Simply supported secondary composite beam supporting composite slab with profiled sheeting

1. Purpose of example

This example deals with a simply supported secondary composite beam under uniformly distributed loads. The composite slab is 130 mm deep with the profiled steel sheeting running perpendicular to the steel beam. Analysis of the composite beam is performed for the construction stage and for the composite stage. The design checks are conducted for both ultimate limit state and serviceability limit state. It is assumed initially that unpropped construction is used, and that the whole length of concrete flange is cast before composite action is developed. However, the check has shown that the steel beam does not have sufficient resistance to lateral-torsional buckling. Instead of selecting the beam with the greater lateral-torsional resistance, in this example it is assumed that the steel beam is fully propped. Also, the purpose of this example is to give the basis for some of the problems to be treated in more detail in the following examples. Therefore, this example is a "reference" for considering other examples.

2. Static system, cross-section and actions

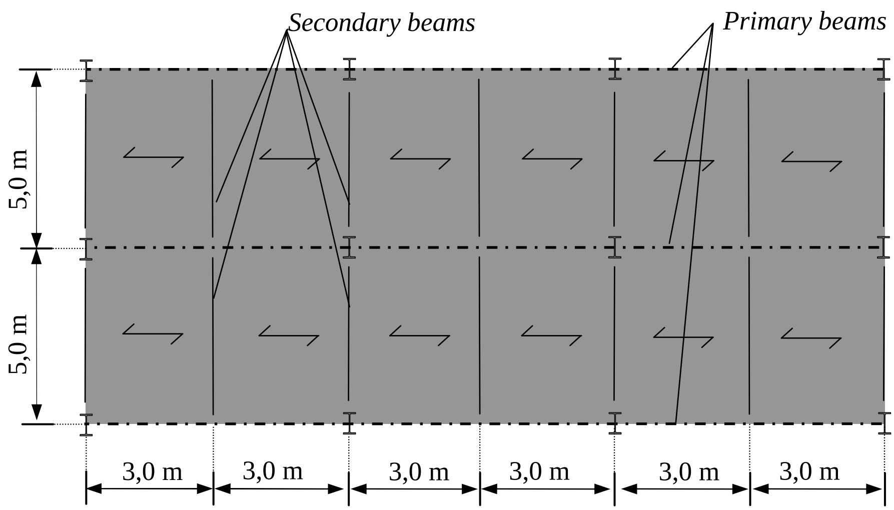
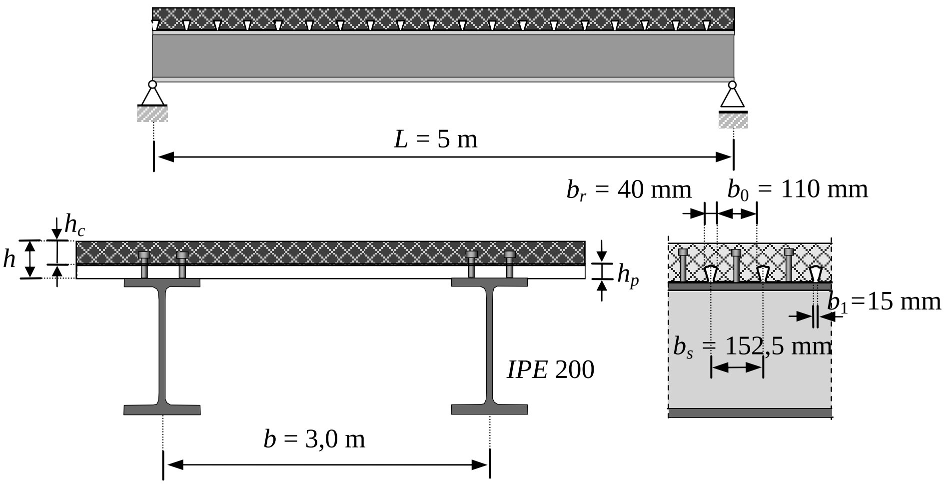


Figure B3.1 *Floor Layout*

Composite Structures according to Eurocode 4. Worked Examples.
First Edition. Darko Dujmović, Boris Androić, Ivan Lukačević.



$$h = 130 \text{ mm}, \ h_c = 79 \text{ mm}, \ h_p = 51 \text{ mm}$$

Figure B3.2 *Static system and section of secondary composite beam*

Actions

a) Permanent action

Remark:

According to EN 1991-1-1 the density of the normal weight concrete is 24 kN/m^3, increased by 1 kN/m^3 for normal percentage reinforcement, and increased for the wet concrete by another 1 kN/m^3.

The concrete slab area per m width is:

$$A_c = 1000 \cdot h - \left(\frac{1000}{b_s} \cdot \frac{b_1 + b_r}{2} \cdot h_p\right)$$

$$A_c = 1000 \cdot 130 - \left(\frac{1000}{152{,}5} \cdot \frac{15 + 40}{2} \cdot 51\right) = 120803 \text{ mm}^2 \approx 1200 \text{ cm}^2$$

- *concrete slab and reinforcement (wet concrete)*

$$A_c \cdot 26 = 0{,}120 \cdot 26 = 3{,}12 \text{ kN/m}^2$$

- *concrete slab and reinforcement (dry concrete)*

$$A_c \cdot 25 = 0{,}120 \cdot 25 = 3{,}00 \text{ kN/m}^2$$

Construction stage

- *concrete slab* $3{,}12 \text{ kN/m}^2$

- *profiled steel sheeting* $0{,}13 \text{ kN/m}^2$

- *steel beam* $0{,}20 \text{ kN/m}^2$

Total $g_{k,1} = 3{,}45 \text{ kN/m}^2$

Composite stage

- *concrete slab* $3{,}00 \text{ kN/m}^2$

- *profiled steel sheeting* $0{,}13 \text{ kN/m}^2$

- *steel beam* $0{,}20 \text{ kN/m}^2$

Total $g_{k,2} = 3{,}33 \text{ kN/m}^2$

Floor finishes $g_{k,3} = 0{,}15 \text{ kN/m}^2$

b) Variable action

Construction stage

-*construction loads* $q_{k,1} = 0{,}50 \text{ kN/m}^2$

Composite stage

- *imposed floor load, category of use C1* $q_k = 3{,}00 \text{ kN/m}^2$

-*movable partitions* $q_k = 0{,}80 \text{ kN/m}^2$

Total $q_{k,2} = 3{,}80 \text{ kN/m}^2$

Remark:

The variable actions q_k = 3,00 kN/m^2 and q_k = 0,80 kN/m^2 are mutually independent.

3. Properties of materials

Concrete strength class: C 25/30 $f_{ck} = 25 \text{ N/mm}^2$

$$f_{cd} = \frac{f_{ck}}{\gamma_c} = \frac{25}{1,5} = 16,7 \text{ N/mm}^2$$

$$0,85 f_{cd} = 0,85 \cdot 16,7 = 14,19 \text{ N/mm}^2$$

$$E_{cm} = 31\,000 \text{ N/mm}^2$$

Reinforcement: *ductility class B or C (Table C.1, EN 1992-1-1)* $f_{sk} = 500 \text{ N/mm}^2$

$$f_{sd} = \frac{f_{sk}}{\gamma_s} = \frac{500}{1,15} = 435 \text{ N/mm}^2$$

$$E_s = 210\,000 \text{ N/mm}^2$$

Structural steel: S275 $f_{yk} = 275 \text{ N/mm}^2$

$$f_{yd} = \frac{f_{yk}}{\gamma_M} = \frac{275}{1,0} = 275 \text{ N/mm}^2$$

$$E_a = 210\,000 \text{ N/mm}^2$$

Shear connectors: *ductile headed studs* $f_u = 450 \text{ N/mm}^2$

$$d = 19 \text{ mm}$$

$$h_{sc} = 100 \text{ mm}$$

$$\frac{h_{sc}}{d} = \frac{100}{19} = 5,26 > 4,0 \quad \rightarrow \alpha = 1,0$$

$$P_{Rd} = 73,7 \text{ kN}$$

4. Ultimate limit state

4.1 Design values of combined actions and of the effects of actions for the construction stage

$$e_d = b \cdot (\gamma_G \cdot g_{k,1} + \gamma_Q \cdot q_{k,1})$$

$b = 3,0$ m beam spacing

$$e_d = 3,0 \cdot (1,35 \cdot 3,45 + 1,5 \cdot 0,5) = 3,0 \cdot (4,65 + 0,75) = 16,2 \text{ kN/m}$$

$$M_{y,Ed} = \frac{e_d \cdot L^2}{8} = \frac{16,2 \cdot 5,0^2}{8} = 50,6 \text{ kNm}$$

$$V_{Ed} = \frac{e_d \cdot L}{2} = \frac{16,2 \cdot 5,0}{2} = 40,5 \text{ kN}$$

4.2 Design values of combined actions and of the effects of actions for the composite stage

$$e_d = b \cdot (\gamma_G \cdot g_{k,2} + \gamma_G \cdot g_{k,3} + \gamma_Q \cdot q_{k,1})$$

$b = 3,0$ m beam spacing

$$e_d = 3,0 \cdot (1,35 \cdot 3,33 + 1,35 \cdot 0,15 + 1,5 \cdot 3,8) = 3,0 \cdot (4,70 + 5,70) = 31,2 \text{ kN/m}$$

$$M_{y,Ed} = \frac{e_d \cdot L^2}{8} = \frac{31,2 \cdot 5^2}{8} = 97,5 \text{ kNm}$$

$$V_{Ed} = \frac{e_d \cdot L}{2} = \frac{31,2 \cdot 5,0}{2} = 78,0 \text{ kN}$$

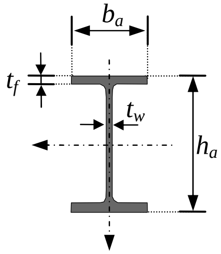
4.3 Check for the construction stage

4.3.1 Selection of steel cross-section

When the steel beam is unpropped at the construction stage, the steel beam carries the total load for the construction stage e_d = 16,2 kN/m. The plastic section modulus that is required to resist the construction stage maximum design bending moment is determined as:

$$W_{pl,y} = \frac{M_{y,Ed} \cdot \gamma_{M0}}{f_y} = \frac{50,6 \cdot 10^3 \cdot 1,0}{275} = 184 \text{ cm}^3$$

Section IPE 200 is selected with the cross-section and the dimensions shown in Figure B3.3.



$$W_{pl,y} = 220,6 \text{ cm}^3$$
$$A = 28,48 \text{ cm}^2$$
$$h_a = 200 \text{ mm}$$
$$b_a = 100 \text{ mm}$$
$$t_w = 5,6 \text{ mm}$$
$$t_f = 8,5 \text{ mm}$$
$$r = 12 \text{ mm}$$
$$I_{y,a} = 1943 \text{ cm}^4$$
$$I_{z,a} = 142,4 \text{ cm}^4$$
$$I_{w,a} = 12990 \text{ cm}^6$$
$$I_{t,a} = 6,98 \text{ cm}^4$$

Figure B3.3 *Cross-section and dimensions of IPE* 200

4.3.2 Classification of the steel cross-section

For t_f = 8,5 mm the yield strength is f_y = 275 N/mm^2.

$$\varepsilon = \sqrt{\frac{235}{f_y}} = \sqrt{\frac{235}{275}} = 0,92$$

For the execution stage, the neutral axis is located in the half depth of the web of the steel section.

The classification of steel cross-section is conducted according to Table 5.2, EN 1993-1-1.

Flange:

The outstand of compression flange is:

$$c = \frac{b_a - t_w - 2 \cdot r}{2} = \frac{100 - 5,6 - 2 \cdot 12}{2} = 35,2 \text{ mm}$$

$$\frac{c}{t_f} = \frac{35,2}{8,5} = 4,14$$

The limiting value for class 1 is:

$$\frac{c}{t_f} \leq 9\varepsilon = 9 \cdot 0,92 = 8,28$$

$4,14 < 8,28$ → Therefore, the flange in compression is class 1.

Web:

Web subject to bending

$$c = d = h_a - 2 \cdot t_f - 2 \cdot r = 200 - 2 \cdot 8,5 - 2 \cdot 12 = 159 \text{ mm}$$

$$\frac{c}{t_w} = \frac{159}{5,6} = 28,39$$

The limiting value for class 1 is:

$$\frac{c}{t_w} \leq 72\varepsilon = 72 \cdot 0,92 = 66,24$$

$28,39 < 66,24$ → Therefore the web in bending is class 1.

Therefore the cross-section in bending at the construction stage is class 1. At the composite stage, the cross-section will also be class 1.

4.3.3 Plastic resistance moment of the steel cross-section

The design resistance moment for class 1 sections is:

$$M_{c,Rd} = M_{pl,a,Rd} = \frac{W_{pl,y} \cdot f_{yd}}{\gamma_{M0}}$$

$$M_{c,Rd} = M_{pl,Rd} = \frac{220,6 \cdot 27,5}{1,0} = 6066,5 \text{ kNcm} = 60,7 \text{ kNm}$$

Verify that:

$$\frac{M_{y,Ed}}{M_{c,Rd}} \leq 1,0$$

$$\frac{50,6}{60,7} = 0,83 < 1,0$$

Therefore the resistance moment is adequate.

4.3.4 Shear resistance of the steel cross-section

The shear buckling resistance of web should be verified, for an unstiffened web when:

$$\frac{h_w}{t} > \frac{72}{\eta}\varepsilon$$

where:

$$\varepsilon = \sqrt{\frac{235}{f_y}} = \sqrt{\frac{235}{275}} = 0,92$$

$\eta = 1,2$, the factor defined in EN 1993-1-5

$$h_w = h_a - 2 \cdot t_f = 200 - 2 \cdot 8,5 = 183 \text{ mm}$$

$$\frac{72}{\eta}\varepsilon = \frac{72}{1,2} \cdot 0,92 = 55,2$$

$$\frac{h_w}{t} = \frac{h_w}{t_w} = \frac{183}{5,6} = 32,7$$

Since 32,7 < 55,2, the condition is satisfied. The shear buckling resistance of web should not be verified.

Remark:

The resistance of the composite beam to vertical shear is normally taken as the shear resistance of the steel section according to clause 6.2.6, EN 1993-1-1, which gives:

$$V_{pl,Rd} = V_{pl,a,Rd} = \frac{A_V (f_y / \sqrt{3})}{\gamma_{M0}}$$

For rolled I- and H-sections and the load applied parallel to the web, the shear area is calculated as:

$A_V = A_a - 2 \cdot b_a \cdot t_f + t_f \cdot (t_w + 2 \cdot r)$, but not less than $\eta \cdot h_w \cdot t_w$

The shear area A_V is:

$$A_V = 28,48 - 2 \cdot 10 \cdot 0,85 + 0,85 \cdot (0,56 + 2 \cdot 1,2)$$

$$A_V = 13,99 \ \text{cm}^2$$

$$\eta = 1,2$$

$$\eta \cdot h_w \cdot t_w = 1,2 \cdot 18,3 \cdot 0,56 = 12,29 \ \text{cm}^2$$

13,99 cm^2 > 12,29 cm^2

Therefore A_V = 13,99 cm^2.

The design plastic shear resistance of the steel section is:

$$V_{pl,Rd} = V_{pl,a,Rd} = \frac{A_V (f_y / \sqrt{3})}{\gamma_{M0}}$$

$$V_{pl,Rd} = V_{pl,a,Rd} = 13,99 \frac{27,5}{\sqrt{3} \cdot 1,0} = 222 \ \text{kN}$$

Verify that:

$$\frac{V_{Ed}}{V_{pl,Rd}} \leq 1,0$$

$$\frac{40,5}{222} = 0,18 < 1,0$$

Therefore the shear resistance of the cross-section is adequate.

4.3.5 Interaction of M-V (bending and shear force)

The reduction of the design resistance moment of the cross-section for the influence of shear force is defined in EN 1993-1-1. Because there is no shear force at the point of maximum bending moment (mid-span), no reduction (due to shear) in resistance moment is required. However, in this example, for pedagogic reasons, the check is carried out.

$$0{,}5 \cdot V_{pl,Rd} = 0{,}5 \cdot 222 = 111 \text{ kN}$$

$V_{Ed} = 40{,}5 \text{ kN} < 0{,}5\, V_{pl,Rd} = 111 \text{ kN}$, no reduction in the resistance moment

$$M_{y,V,Rd} = M_{c,Rd} = 60{,}7 \text{ kNm}$$

Verify that:

$$\frac{M_{Ed}}{M_{y,V,Rd}} \leq 1{,}0$$

$\dfrac{50{,}6}{60{,}7} = 0{,}83 < 1{,}0$, the resistance to combined shear and bending is satisfactory.

Remark:

Since there is no reduction of the plastic resistance moment of the steel section due to the shear force, the conducted verification is the same as in Section 4.3.3.

4.3.6 Lateral-torsional buckling if the steel beam

The resistance to the lateral-torsional buckling of the steel beam is carried out according to EN 1993-1-1. The elastic critical moment of lateral-torsional buckling is calculated using:

$$M_{cr} = C_1 \cdot \frac{\pi^2 \cdot E \cdot I_z}{(k \cdot L)^2} \cdot \left[\sqrt{\left(\frac{k}{k_w}\right)^2 \cdot \frac{I_w}{I_z} + \frac{(k \cdot L)^2 \cdot G \cdot I_t}{\pi^2 \cdot E \cdot I_z} + (C_2 \cdot z_g)^2} - C_2 \cdot z_g \right]$$

L = 500 cm, the length between points at which the compression flange is laterally restrained

$z_g = \frac{h}{2} = \frac{20,0}{2} = 10,0$ cm, the distance of the shear centre from the point application of the load

$$G = \frac{E}{2\,(1+\nu)} = \frac{21000}{2 \cdot (1+0,3)} = 8077 \ \text{kN/cm}^2$$

$k = 1,0 \quad k_w = 1,0$ the effective length factors that depend on the support conditions at the end sections,

$C_1 = 1,127$ the coefficient that takes into account the shape of the moment diagram,

$C_2 = 0,454$ the coefficient that takes into account the destabilizing or stabilizing effect of the position of the load,

$$M_{cr} = 1,127 \cdot \frac{\pi^2 \cdot 21000 \cdot 142,4}{(1 \cdot 500)^2} \cdot$$

$$[\sqrt{(\frac{1}{1})^2 \cdot \frac{12990}{142,4} + \frac{(1 \cdot 500)^2 \cdot 8077 \cdot 6,98}{\pi^2 \cdot 21000 \cdot 142,4} + (0,454 \cdot 10)^2} - 0,454 \cdot 10]$$

M_{cr} = 2626 kNcm = 26,3 kNm

Non-dimensional slenderness:

$$\bar{\lambda}_{LT} = \sqrt{\frac{W_y \cdot f_y}{M_{cr}}}$$

for classes 1 and 2 $W_y = W_{pl,y}$

$$\bar{\lambda}_{LT} = \sqrt{\frac{220,6 \cdot 27,5}{2626}} = 1,52 > \bar{\lambda}_{LT,0} = 0,4$$

The reduction factor for lateral-torsional buckling – General method:

$$\chi_{LT} = \frac{1}{\Phi_{LT} + \sqrt{\Phi_{LT}^2 - \bar{\lambda}_{LT}^2}} \ \text{but} \ \chi_{LT} \le 1,0$$

$$\Phi_{LT} = 0,5[1 + \alpha_{LT}\,(\bar{\lambda}_{LT} - 0,2) + \bar{\lambda}_{LT}^2]$$

$\frac{h}{b} = \frac{200}{100} = 2{,}0$ ≤ 2 rolled I section $\rightarrow$ the buckling curve *a* is governed

For the buckling curve a $\rightarrow$ $\alpha_{LT} = 0{,}21$, (α_{LT} is the imperfection factor)

$$\Phi_{LT} = 0{,}5\,[1 + 0{,}21 \cdot (1{,}52 - 0{,}2) + 1{,}52^2] = 1{,}794$$

$$\chi_{LT} = \frac{1}{1{,}794 + \sqrt{1{,}794^2 - 1{,}52^2}} = 0{,}36$$

The design buckling resistance moment is:

$$M_{b,Rd} = \chi_{LT} \cdot \frac{W_y \cdot f_y}{\gamma_{M1}}$$

for classes 1 and 2 $W_y = W_{pl,y}$

$$M_{b,Rd} = 0{,}36 \cdot \frac{220{,}6 \cdot 27{,}5}{1{,}0} = 2184 \text{ kNcm} = 21{,}8 \text{ kNm}$$

Verify that:

$$\frac{M_{Ed}}{M_{b,Rd}} \leq 1{,}0$$

$$\frac{50{,}6}{21{,}8} = 2{,}3$$

Therefore the buckling resistance moment of the steel beam is *not* adequate.

Remark:

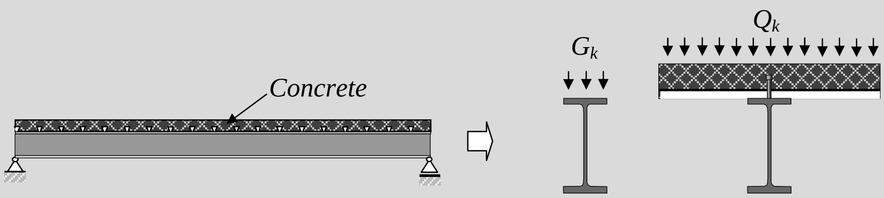
Because the verification of the lateral-torsional buckling resistance of the steel beam in the execution stage when the steel beam is unpropped is not met, there are two options:

Choose a steel section that has the greater resistance to lateral-torsional buckling or use propped construction. The solution with propped construction is selected in this case.

During execution, the steel beam can be fully propped or unpropped. If the steel beam is fully propped, the verification of the lateral-torsional buckling resistance of the steel beam at the execution stage is not necessary. However, if the steel beam is unpropped at the execution stage, it is possible to choose the solution in which the steel beam is laterally restrained along its length or it may be laterally restrained only at certain points of the span with the verification of the lateral-torsional buckling resistance between points of lateral restraint.

Considering the methods of construction, propped and unpropped, it is necessary to take into account the division of the loads from this point of view. In the case of unpropped construction, only the steel beam resists the permanent and variable loads during execution. When the steel beam is continuously propped, all loads take the composite beam. The methods of construction are as shown:

Case a) The steel beam is unpropped during concreting



G_k acts on the steel beam, and Q_k acts on the composite beam

Figure B3.4 *Steel beam is unpropped during concreting*

Permanent loads G_k on the steel beam:

- self-weight of the steel beam
- concrete beam
- construction loading
- shuttering

Variable loads Q_k on the composite beam:

- imposed load
- floor finishes, movable partitions etc.

Case b) The steel beam is continuously propped during concreting

If the steel beam is supported at intervals along its length, with one or more supports, and these supports are removed, it is necessary that the reactions of supports “give back” to the composite beam at the locations of the props.

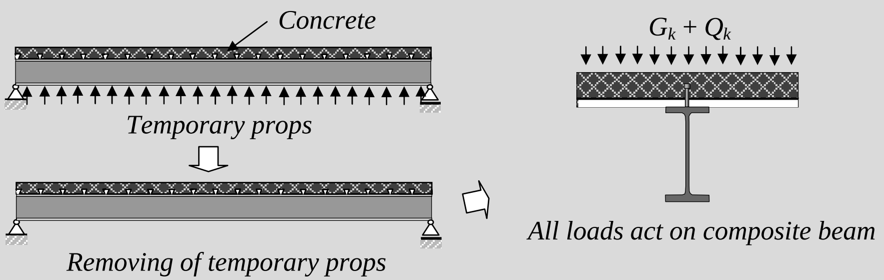


Figure B3.5 *Steel beam is continuously propped during concreting*

4.4 Check for the composite stage

4.4.1 Effective width of the concrete flange

The effective width of the concrete flange is calculated according to the expression (5.3), EN 1994-1-1:

$$b_{eff} = b_0 + \Sigma b_{ei}$$

$b_0 = 0$ (there is only one row of shear connectors)

$$b_{ei} = \frac{L_e}{8} = \frac{L}{8} = \frac{5}{8} = 0,625 \text{ m}$$

$b_{eff} = 0 + (2 \cdot 0,625) = 1,25 \text{ m} < 3,0 \text{ m}$ (beam spacing)

4.4.2 Check of shear connection

The design resistance of a single headed shear connector in a solid concrete slab, automatically welded in accordance with EN 14555, is determined as the smaller of:

$P_{Rd}^{(1)} = \dfrac{0,8 \cdot f_u (\pi d^2 / 4)}{\gamma_V}$ (shank failure of the stud)

$P_{Rd}^{(2)} = \dfrac{0,29 \cdot \alpha \cdot d^2 \sqrt{f_{ck} \cdot E_{cm}}}{\gamma_V}$ (concrete failure)

where:

$$\alpha = 0,2\,(\frac{h_{sc}}{d} + 1) \quad \text{for} \quad 3 \leq \frac{h_{sc}}{d} \leq 4$$

$$\alpha = 1,0 \quad \text{for} \quad \frac{h_{sc}}{d} > 4$$

$$\alpha = 1,0 \quad \text{for} \quad \frac{h_{sc}}{d} = \frac{100}{19} > 4$$

$$P_{Rd}^{(1)} = \frac{0,29 \cdot 1,0 \cdot 19^2 \sqrt{25 \cdot 31 \cdot 10^3}}{1,25} \cdot 10^{-3} = 73,7 \ \text{kN} \rightarrow \text{This resistance is governed}$$

$$P_{Rd}^{(2)} = \frac{0,8 \cdot 450 \cdot \pi \cdot 19^2 / 4}{1,25} \cdot 10^{-3} = 81,7 \ \text{kN}$$

The influence of profiled sheeting to the resistance of the headed stud:

For profiled sheeting with ribs running transverse to the supporting beam, the reduction factor k_t is given by:

$$k_t = (\frac{0,7}{\sqrt{n_r}})(\frac{b_0}{h_p})(\frac{h_{sc}}{h_p} - 1) \leq 1,0$$

(n_r = 1,0 for a single stud per trough of profiled sheeting)

$$k_t = (\frac{0,7}{\sqrt{1,0}})(\frac{112,5}{51})(\frac{100}{51} - 1) = 1,48 \ \leq 1,0$$

Adopted $k_t = 1,0$

Hence, the design resistance per stud in a rib where there is one stud per rib is (no reduction):

$$P_{Rd} = P_{Rd}^{(1)} = 73,7 \ \text{kN}$$

Since there is one stud per rib of the profiled sheeting, the spacing of the studs is 152,5 mm.

The number of studs for half span $L/2$ is:

$$n = \frac{\frac{L}{2} - \frac{h_a}{2}}{152{,}5} = \frac{\frac{5000}{2} - \frac{200}{2}}{152{,}5} \approx 16 \text{ studs}$$

The design longitudinal force in the steel-concrete interface is:

$$V_{L,Ed} = n \cdot P_{Rd} = 16 \cdot 73{,}7 = 1179 \text{ kN}$$

The degree of shear connection η is:

$$\eta = \frac{V_{L,Ed}}{N_{pl,a}} = \frac{1179}{783} = 1{,}5 > 1{,}0$$

Accordingly, the full shear connection is provided and no reduction in resistance moment is required.

4.4.3 Plastic resistance moment of the composite cross-section

According to clause 6.2.1.3, EN 1994-1-1, for the calculation of the resistance moment of the composite section at the mid-span sagging bending region, it is acceptable to use partial shear connection. However, in this case the full shear connection is applied.

The design value of the compressive normal force in the concrete flange with full shear connection is:

$$N_{c,f} = h_c \cdot b_{eff} \cdot 0{,}85 f_{cd}$$

$$N_{c,f} = 79 \cdot 1250 \cdot 14{,}19 \cdot 10^{-3} = 1401 \text{ kN}$$

The design value of the plastic resistance of the structural steel section to normal force is:

$$N_{pl,a} = A_a \cdot f_{yd}$$

$$N_{pl,a} = 28{,}48 \cdot 10^2 \cdot 275 \cdot 10^{-3} = 783 \text{ kN}$$

The position of the plastic neutral axis x_{pl}:

The plastic neutral axis lies within the concrete slab in compression if the following condition is satisfied:

$$N_{c,f} > N_{pl,a}$$

$1401 > 783$ $\rightarrow$ the plastic neutral axis lies within the concrete slab

$$x_{pl} = \frac{N_{pl,a}}{b_{eff} \cdot 0{,}85 \cdot f_{cd}}$$

$$x_{pl} = \frac{783}{1250 \cdot 14{,}19 \cdot 10^{-3}} = 44 \text{ mm} < h_c = 79 \text{ mm}$$

The design value of the plastic resistance moment of the composite section with full shear connection $M_{pl,Rd}$ is:

$$M_{pl,Rd} = \min(N_{c,f}\,; N_{pl,a}) \cdot z$$

$$M_{pl,Rd} = N_{pl,a}(0{,}5h_a + h_c + h_p - 0{,}5x_{pl})$$

$$M_{pl,Rd} = 783(0{,}5 \cdot 200 + 79 + 51 - 0{,}5 \cdot 44) \cdot 10^{-3} = 783 \cdot 208 \cdot 10^{-3} = 163 \text{ kNm}$$

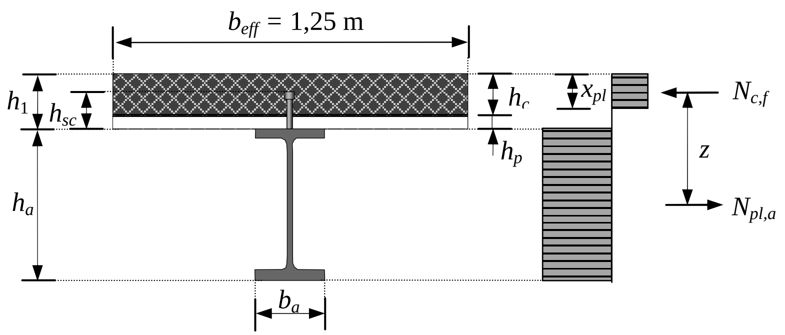


Figure B3.6 *Plastic stress distribution for a composite beam with profiled sheeting and full shear connection in sagging bending*

Check:

$$\frac{M_{y,Ed}}{M_{pl,Rd}} \leq 1{,}0$$

$$\frac{97{,}5}{163} = 0{,}60 < 1{,}0$$

4.4.4 Lateral-torsional buckling of the composite beam

Remark:

The composite slab provides continuous restraint to the top flange of the steel beam, so the beam is not susceptible to lateral torsional buckling.

4.4.5 Check of longitudinal shear resistance of the concrete flange

4.4.5.1 Check of transverse reinforcement

In practice it is usual to neglect the contribution of the steel sheeting, and the cross-sectional area of transverse reinforcement A_{sf} at spacing s_f should satisfied:

$$\frac{A_{sf}}{s_f} \cdot f_{sd} \geq v_{L,Ed} \cdot \frac{h_f}{\cot\theta}$$

where:

A_{sf}/s_f is the transverse reinforcement expressed in mm^2/m,
h_f is the depth of concrete above the profiled sheeting, see Figures B3.7 and B3.8,
θ is the angle between the diagonal strut and the axis of the beam (strut-and-tie model),
$v_{L,Ed}$ is the design longitudinal shear flow in the concrete slab.

Remark:

The profiled steel sheeting with ribs transverse to the beam, continuous across the top flange of the steel beam and with mechanical interlocking contributes to the transverse reinforcement. Its contribution to the transverse reinforcement for the shear surface can be allowed for by replacing above expression by:

$$\left(\frac{A_{sf}}{s_f} \cdot f_{sd}\right) + A_{pe} \cdot f_{yp,d} \geq v_{L,Ed} \cdot \frac{h_f}{\cot\theta}$$

where:

A_{pe} is the effective cross-sectional area of the profiled steel sheeting per unit length of the beam; for sheeting with holes, the net area should be used,
$f_{yp,d}$ is the design yield of strength of the profiled steel sheeting.

The contribution of the profiled steel sheeting is neglected in this example.

The transverse reinforcement (A_{sf}/s_f) expressed in mm^2/m can be denoted as A_t, the cross-sectional area of the top transverse reinforcement, for failure due to shear in the failure plane, shown in Figure B3.7 as section *a-a*.

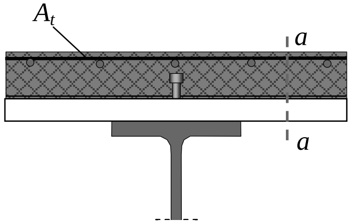


Figure B3.7 *Surface of potential failure in longitudinal shear*

It is necessary to ensure that the concrete flange can resist the longitudinal shear force transmitted to it by the shear connectors. At the steel-concrete interface, the distribution of longitudinal shear is influenced by yielding, by the spacing of the shear connectors, their load-slip properties, and shrinkage and creep of the concrete. The design resistance to longitudinal shear for the relevant shear failure surfaces is given in clause 6.2.4, EN 1992-1-1. The model is based on considering the flange to act like a system of compressive struts combined with a system of ties in the form of the transverse reinforcement.

Concrete flange in compression:

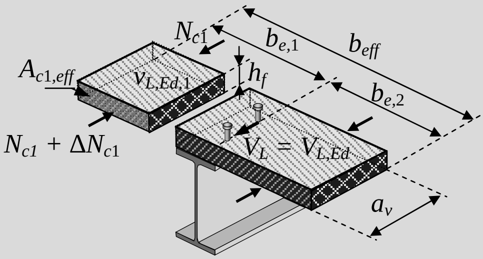


Figure B3.8 *Determination of longitudinal shear forces in the concrete flange*

When the concrete flange is in compression, longitudinal shear flow $v_{L,Ed}$ can be defined as:

$$v_{L,Ed,1} = \frac{\Delta N_{c1}}{a_v} = \frac{V_{L,Ed}}{a_v} \frac{A_{c1,eff}}{A_{c,eff}}$$

where:

a_v is the critical length (the distance between two given sections, Figure B3.8),

ΔN_{c1} is the change in the longitudinal compressive forces in the slab over the critical length a_v, see Figure B3.8.

$V_{L,Ed}$	is the design longitudinal shear force in the steel-concrete interface or in the concrete flange, $V_{L,Ed} = \min(N_{pl,a}, N_c, \Sigma P_{Rd})$

The length a_v is $L/2$ that is the distance between the section where the moment is maximal and the support.

The design longitudinal shear force is determined from the minimum resistance of the steel section, concrete and shear connectors.

$$V_{L,Ed} = \min(N_{pl,a}, N_c, \Sigma P_{Rd})$$

$$N_{pl,a} = 783 \text{ kN}$$

$$\Sigma P_{Rd} = n \cdot P_{Rd} = 16 \cdot 73,7 = 1179 \text{ kN}$$

$$N_{c,f} = 1401 \text{ kN}$$

In this example, with full shear connection, the maximum force that can be transferred is limited by the resistance of the steel section over half of the span, and is given by

$$V_{L,Ed} = 783 \text{ kN}$$

This force must be transferred over each half-span. As there are two shear planes (one on either side of the beam, running parallel to it), the design longitudinal shear stress is:

$$v_{L,Ed} = \frac{\Delta N_{c1}}{h_f \cdot a_v} = \frac{V_{L,Ed}}{2 \cdot h_f \cdot a_v}$$

With

$$f_{sd} = 435 \text{ N/mm}^2$$

$$h_f = h_c = 79 \text{ mm}$$

and

$$a_v = L / 2 = 5000 / 2 = 2500 \text{ mm}$$

the design longitudinal shear stress is:

$$v_{L,Ed} = \frac{783 \cdot 10^3}{2 \cdot 79 \cdot 2500} = 1{,}98 \ \text{N/mm}^2$$

Remark:
In order to prevent splitting of the concrete flange, for the adopted "truss model", according to clause 6.2.4(4) EN 1992-1-1, the angle θ between the concrete diagonals and the longitudinal direction is limited to the values:

$26{,}5^o \leq \theta \leq 45^o$ concrete flange in compression,

$38{,}6^o \leq \theta \leq 45^o$ concrete flange in tension.

In order to minimize the cross-sectional area of transverse reinforcement, the minimum angle θ is selected. For the concrete flange in compression, the minimum angle θ is:

$$\theta = 26{,}5^o$$

$$\frac{A_{sf}}{s_f} \cdot f_{sd} \geq v_{L,Ed} \cdot \frac{h_f}{\cot\theta}$$

$$\frac{A_{sf}}{s_f} \geq \frac{v_{L,Ed}}{f_{sd}} \cdot \frac{h_f}{\cot\theta} = \frac{1{,}98}{435} \frac{79}{\cot 26{,}5} \cdot 10^3 = 179 \ \text{mm}^2\text{/m}$$

The reinforcement provided is 8 mm bars at 150 mm, for which:

$$A_t = \frac{\pi \cdot 8^2}{4} \cdot \frac{1000}{150} = 335 \ \text{mm}^2\text{/m} > 179 \ \text{mm}^2\text{/m}$$

According to EN 1994-1-1, clause 6.6.6.3, the minimum area of transverse reinforcement is determined in accordance with EN 1992-1-1, clause 9.2.2(5), which gives the minimum area of reinforcement as a proportion of the concrete area. The ratio is:

$$\rho_{w,\min} = \frac{0{,}08\sqrt{f_{ck}}}{f_{yr,k}}$$

where:

f_{ck}	is the characteristic compressive cylinder strength of the concrete at 28 days in N/mm^2,
$f_{yr,k} = f_{sk}$	is the characteristic yield strength of the reinforcement in N/mm^2.

The minimum area of transverse reinforcement is:

$$\rho_{w,\min} = \frac{0,08\sqrt{f_{ck}}}{f_{yr,k}} = \frac{0,08\sqrt{25}}{500} = 0,0008$$

$$A_c = h_c \cdot b = 79 \cdot 1000 = 79000 \text{ mm}^2$$

$$A_{s,\min} = \rho_{w,\min} \cdot A_c = 0,0008 \cdot 79000 = 63 \text{ mm}^2\text{/m}$$

Since $A_t = 335$ mm^2/m $> A_{s,\min} = 63$ mm^2/m, the requirement of minimum transverse reinforcement is satisfied.

4.4.5.2 Crushing of the concrete flange

To prevent crushing of the compression struts in the flange, the following condition should be satisfied according to EN 1992-1-1, expression 6.22:

$$v_{L,Ed} \le v_{Rd}$$

$$v_{L,Ed} \le v \cdot f_{cd} \cdot \sin\theta \cdot \cos\theta$$

where:

$$v = 0,6 \cdot (1 - \frac{f_{ck}}{250})$$

θ is the angle between the concrete diagonals and the longitudinal direction.

In order to minimize the resistance of the concrete compression strut, we select the minimum angle θ. For concrete flange in compression, the minimum angle θ is:

$$\theta = 26,5°$$

$$v_{Rd} = v \cdot f_{cd} \cdot \sin\theta \cdot \cos\theta = 0,6 \cdot (1 - \frac{25}{250}) \cdot 16,7 \cdot \sin 26,5° \cdot \cos 26,5° = 3,60 \text{ N/mm}^2$$

Check:

$v_{L,Ed} = 1{,}98\ \text{N/mm}^2 < v_{Rd} = 3{,}60\ \text{N/mm}^2$

Therefore the crushing resistance of the concrete compression strut is adequate.

5. Serviceability limit state

5.1 General

Chapter 7, EN 1994-1-1, is limited to provisions relating to serviceability that are specific to composite structures. Serviceability verifications for composite structures generally include checks of stress, deflection and vibration, as well control of crack width.

For buildings, stress limitation is not required for beams if, in the ultimate limit state, no verification of fatigue is required and no pre-stressing by tendons and/or by controlled imposed deformations is provided. However, if stress limitation is required, clause 7.2, EN 1992-1-1, gives stress limits which may be applicable for buildings that have pre-stressing or fatigue loading.

Since the deflection is one of the most important verifications of the serviceability limit state, it is necessary to explain in detail the problems associated with deflection calculation. Deflections due to loads applied to the composite member are calculated using elastic theory, taking into account the following effects:

a) cracking of concrete,
b) creep and shrinkage of concrete,
c) sequence of construction,
d) influence of local yielding of structural steel at internal support (for continuous beams),
e) influence of incomplete interaction.

For a more detailed explanation of these effects, see example B6.

The expression for calculation of the maximum deflection of the beam due to uniformly distributed loads has the following general form:

$$\delta = \frac{5}{384} \cdot \frac{e_d \cdot L^4}{EI_L}$$

where:

e_d is the design value of load from governed combination of actions,

L is the span of beam,
EI_L is the effective flexural stiffness of composite sections, which depends on the type of loadings; the different types of loading are distinguished by a subscript L,
E_L is the effective modulus of elasticity of concrete, which depends on the same type of loadings as for the flexural stiffness EI_L. The effective modulus of elasticity of concrete E_L is denoted by $E_{c,eff}$ in EN 1994-1-1.

When calculating the deformation, i.e. deflection, the effect of creep is taken into account by using the effective flexural stiffness of the composite section EI_L.

The practical calculation can be carried out according to:

- the procedure with the effective flexural stiffness of the composite section EI_L,
- the procedure with the ideal second moment of area $I_{i,L}$, where the concrete flange is transformed in “steel” units by means of the appropriate modular ratio n_L, see example B4.

Thus, the effect of creep and shrinkage may be taken into account by using the effective modulus of elasticity of concrete $E_{c,eff}$, or by using the modular ratio n_L which depends on the type of loadings. The expression for the effective modulus of elasticity of concrete $E_{c,eff}$ is:

$$E_{c,eff} = \frac{E_{cm}}{1 + \psi_L \cdot \varphi(t, t_0)}$$

The expression for the modular ratio n_L is:

$$n_L = n_0(1 + \psi_L \cdot \varphi_t) = n_0 \cdot n_c$$

where:

n_L is the modular ratio appropriate to the type of loading,
n_0 is the modular ratio for short-term loading,
φ_t is the creep coefficient,
t is the age of the concrete in days at the time considered in the design,
t_0 is the age of the concrete in days at loading,
ψ_L is the creep multiplier, which depends on the type of loading.

For composite beams in structures for buildings, where the first-order elastic global analysis can be used, clause 5.4.2.2(11), EN 1994-1-1, allows for the nominal modular ratio to take the value $2 \cdot n_0$ for both short-term and long-term loading. The exceptions are:

- structures for which second-order global analysis is required according to clause 5.2, EN 1994-1-1,
- structures for buildings mainly intended for storage,
- structures pre-stressed by controlled imposed deformations – this would apply, for example, to the bending of steel beams by jacking before concrete is cast around one of the flanges.

When the conditions according to clause 5.4.2.2(11) do not apply, the modular ratio for use in the analysis of the effects of long-term loading, n_L, depends on the type of loadings and the creep coefficient φ_t. The creep coefficient depends on the age of the concrete at first loading, t_0, and the age of the concrete at the time observed in the analysis, which is usually taken as "infinite".

Figure B3.9 illustrates two possible methods taking into account the creep for composite structural members.

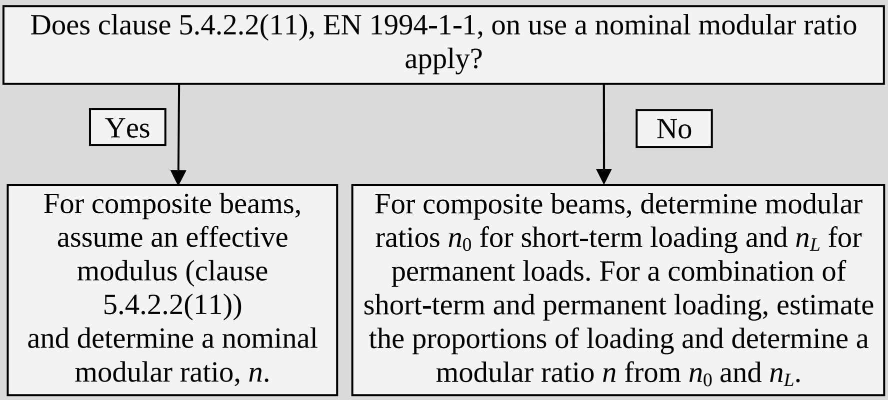


Figure B3.9 *Two possible methods taking into account the creep for composite structural members*

Clause 5.4.2.2(4), EN 1994-1-1 states that the age of concrete at the time of loading by the effects of shrinkage should generally be assumed to be one day. However, in clause 5.4.2.2(3), EN 1994-1-1, for permanent loads on composite structures cast in several phases, one mean value t_0 can be used for the calculation of the creep coefficient φ_t.

For the time-dependent strains of the concrete, it is always important to define the time of the beginning of the effect of certain phenomena. For example, the following values are common for t_0 (the age of the concrete on first loading) and t (the age at the time considered in the analysis):

t_0 = 1 day, the age of loading by the effects of shrinkage,
t_0 = 28 days, hardened concrete, i.e. the age of concrete on the first loading,
$t = \infty$, the age at the time considered in the analysis, which is normally taken as "infinity".

For structural elements propped during construction, the age at first loading can be considered to be when the props are removed.

Types of loadings and the corresponding flexural stiffness EI_L

In this example, the following values are used:

- Short-term loading

$$\psi_L = 0 \qquad E_0 = \frac{E_{cm}}{n_c} = \frac{E_{cm}}{1 + 0 \cdot \varphi(t, t_0)} = E_{cm} \qquad EI_0 \text{ flexural stiffness}$$

- Permanent loading constant in time

$$\psi_L = \psi_P = 1{,}10 \qquad E_P = \frac{E_{cm}}{n_c} = \frac{E_{cm}}{1 + 1{,}10 \cdot \varphi(t, t_0)} \qquad EI_P \text{ flexural stiffness}$$

- Action effects caused by shrinkage of concrete

$$\psi_L = \psi_S = 0{,}55 \qquad E_S = \frac{E_{cm}}{n_c} = \frac{E_{cm}}{1 + 0{,}55 \cdot \varphi(t, t_0)} \qquad EI_S \text{ flexural stiffness}$$

Effective flexural stiffness of composite sections

The flexural stiffness of the composite section of the beam with full shear connection is shown in the general format:

$$EI = E_a \cdot I_a + E_c \cdot I_c + \frac{E_a \cdot A_a \cdot E_c \cdot A_c}{E_a \cdot A_a + E_c \cdot I_c} \cdot a^2$$

If the influence of creep on the flexural stiffness of composite section is taken into account, in the above expression the effective modulus of elasticity of concrete E_L is introduced. After rearranging, the effective flexural stiffness EI_L is obtained. In this case the expression is:

$$EI_L = E_a \cdot I_a + E_L \cdot I_c + \frac{E_a \cdot A_a \cdot E_L \cdot A_c}{E_a \cdot A_a + E_L \cdot I_c} \cdot a^2$$

For example, for short-term loading the creep multiplier is $\psi_L = 0$ and the modulus of elasticity is $E_0 = E_{cm}$. In this case the expression for the flexural stiffness EI_0 is:

$$EI_0 = E_a \cdot I_a + E_0 \cdot I_c + \frac{E_a \cdot A_a \cdot E_0 \cdot A_c}{E_a \cdot A_a + E_0 \cdot I_c} \cdot a^2$$

The same procedure is used for calculating the flexural stiffness of the EI_P and EI_S with the corresponding effective modulus of elasticity of concrete E_P and E_S.

Combination of actions for serviceability limit state

The expressions for the combinations of actions for serviceability limit state design are given in EN 1990 for three possible combinations:

● Characteristic combination

$G_k + Q_{k,1} + \psi_{0,2} \cdot Q_{k,2}$ and $G_k + \psi_{0,1} \cdot Q_{k,1} + Q_{k,2}$

This is the least favourable combination and it is normally used for verifying irreversible limit states. An example of such a limit state is deformations that result from the yielding of steel. Also, this combination is applied when deformation causes cracking of a brittle floor finish or damage to fragile partitions. In this case, $\psi_{0,1}$ and $\psi_{0,2}$ denote the combination factors for the characteristic value of variable action, and the leading variable action is denoted with Q_1.

● Frequent combination

$G_k + \psi_{1,1} \cdot Q_{k,1} + \psi_{1,2} \cdot Q_{k,2}$

This combination is used in the case of reversible limit states, such as the elastic deflection of composite beams due to variable actions. However, if the deformation causes cracking of brittle finish floor structures or damage to fragile partitions then the limit state is not reversible. In this case, the verification must be carried out for the higher less probable loading of the characteristic combination. Then $\psi_{1,1}$ and $\psi_{1,2}$ denote the combination factors for the frequent value of variable action.

● Quasi-permanent combination

$G_k + \psi_{2,1} \cdot Q_{k,1} + \psi_{2,2} \cdot Q_{k,2}$

This combination is used for the estimation of deformation for the reversible limit states or the long-term effects such as creep and shrinkage of concrete, as well as for the appearance of the structure. Then $\psi_{2,1}$ and $\psi_{2,2}$ denote the combination factors for the quasi-permanent value of variable action.

According to EN 1990, for serviceability verification (deflections and vibrations) for permanent and variable actions the *characteristic* combination or the *frequent* combination apply. To determine the deflection due to creep we use the quasi-permanent combination. However, which combination will be chosen depends on the National Annex.

In the case of only one variable action Q_1, the combinations are:

characteristic: $G_k + Q_1$

frequent: $G_k + \psi_1 \cdot Q_1$

quasi-permanent: $G_k + \psi_2 \cdot Q_1$

In this example, for a building with floors in category B, office areas, the following combination factors ψ are selected:

$\psi_1 = 0,5$

$\psi_2 = 0,3$

Deflections of composite beam

If the deformation does not cause cracking of the floor structure or damage to partition walls then the serviceability limit state is reversible. In this case, the frequent combination is used. In this example, such a situation is assumed. Therefore, the frequent combination of permanent and variable actions G and Q should be used to determine the deflection of the composite beam:

$$G_k + \psi_1 \cdot Q_1 = G_k + 0,5 \cdot Q_1$$

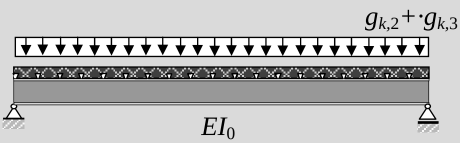
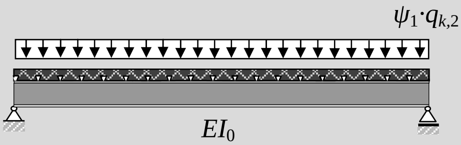
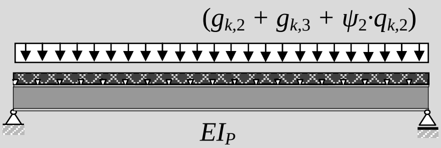
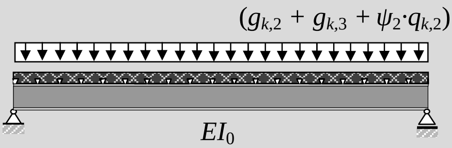
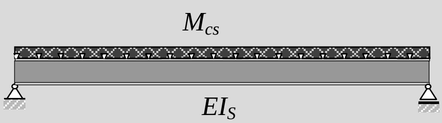
The quasi-permanent combination should be used to determine the deflection due to creep:

$$G_k + \psi_2 \cdot Q_1 = G_k + 0,3 \cdot Q_1$$

The effects of shrinkage are independent of the action, so that the deflection is calculated based on the moment M_{cs}, where M_{cs} is the bending moment due to shrinkage.

If the steel beam is fully propped, the total deflection of the composite beams is obtained by summing the following deflections:

Composite stage (verifications according to clause 7.3.1, EN 1994-1-1)

● Permanent action		
	*t = 0 → short-term cross section*	δ_1
● Frequent value of variable action (frequent combination)		
	*t = 0 → short-term cross section*	$\delta_{2,1}$
● Creep, deflection δ_{22} = *a) - b)*		
a) 	*$t = \infty$ → long-term cross section* (t_0 = 28 days)	$\delta_{2,2}$
b) 	*t = 0 → short-term cross section*	
● Shrinkage		
	*$t = \infty$ → long-term cross section* (t_0 = 1 day)	$\delta_{2,3}$

The total deflection is obtained from:

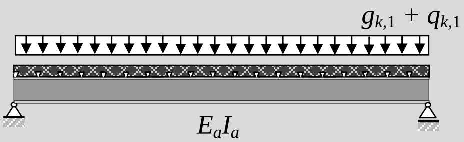
$\delta = \delta_1 + \delta_{2,1} + \delta_{2,2} + \delta_{2,3}$

where:

δ_1 is the deflection due to the permanent actions (the first loading is applied at age t_0 = 28 days),
$\delta_{2,1}$ is the deflection due to the frequent value of the variable action at the time of first loading,
$\delta_{2,2}$ is the deflection due to creep under the quasi-permanent value of variable action at time $t = \infty$,
$\delta_{2,3}$ is the deflection due to shrinkage.

If the steel beam at construction stage is unpropped, the calculation of deflection is performed separately for the construction stage and for the composite stage.

Construction stage (verifications according to clause 7.3.1, EN 1993-1-1)

● Permanent and variable action (characteristic combination)		
	*steel cross-section*	δ_0

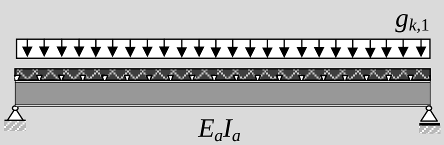
The total deflection is obtained from:

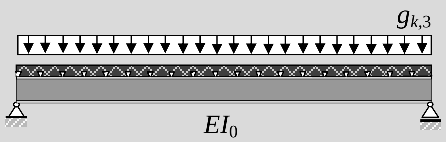
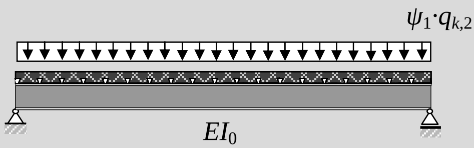
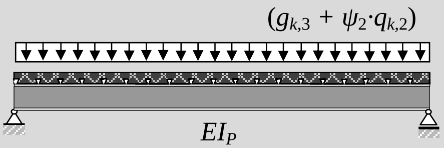
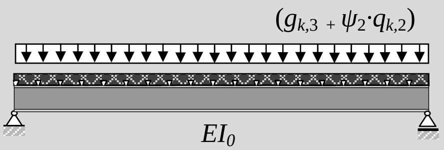
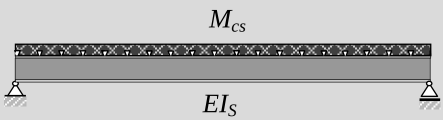
$\delta = \delta_0$ (verification for the construction stage)

where:

δ_0 is the deflection of the steel beam at the execution stage.

Composite stage (verifications according to clause 7.3.1, EN 1994-1-1)

● Permanent action		
	*Immediately after casting concrete (no shear connection)*	$\delta_{1,1}$

	*t = 0 → short-term cross section*	$\delta_{1,2}$
● Frequent value of variable action (frequent combination)		
	*t = 0 → short-term cross section*	$\delta_{2,1}$
● Creep (quasi-permanent combination), deflection δ_{22} = *a) – b)*		
a) 	*$t = \infty$ → long-term cross section* (t_0 = 28 days)	$\delta_{2,2}$
b) 	*t = 0 → short-term cross section*	
● Shrinkage		
	*$t = \infty$ → long-term cross section* (t_0 = 1 day)	$\delta_{2,3}$

The total deflection is obtained from:

$\delta = \delta_{1,1} + \delta_{1,2} + \delta_{2,1} + \delta_{2,2} + \delta_{2,3}$ (verification for the composite stage)

where:

$\delta_{1,1}$ is the deflection due to permanent action immediately after casting concrete (no shear connection),
$\delta_{1,2}$ is the deflection due to loads of floor finishes, partitions on the composite beam (first loading),
$\delta_{2,1}$ is the deflection due to the frequent value of variable action at the time of first loading,
$\delta_{2,2}$ is the deflection due to creep under the quasi-permanent value of variable action at time $t = \infty$,
$\delta_{2,3}$ is the deflection due to shrinkage.

5.2 Calculation of deflections

5.2.1 Construction stage deflection

Since that the verification has shown that the steel beam does not have sufficient resistance to lateral-torsional buckling, the steel beam is fully propped at the construction stage. In this case, there is no deflection of the steel beam.

5.2.2 Composite stage deflection

The total deflection of the composite beam has to be determined for the composite stage, i.e. for the stage when the concrete has hardened, and temporary props of steel beam have been removed.

Determination of creep coefficient and shrinkage

For the calculation of the creep coefficient $\varphi(t, t_0)$ the following is valid:

- the perimeter of that part which is exposed to drying, u

$$u = b$$

- the notional size of the cross-section, h_0

$$h_0 = \frac{2 \cdot A_c}{u} = \frac{2 \cdot b \cdot h_c}{b} = 2 \cdot h_c = 2 \cdot 79 = 158 \text{ mm}$$

- $t_0 = 1$ day, $t_0 = 28$ days,
- the ambient relative humidity, RH 50%,
- the concrete strength class C 25/30,
- the type of cement – cement class S, strength class 32,5 N.

The final value of creep coefficient $\varphi(\infty, t_0)$ will be determined using the nomogram shown in Figure 3.1, EN 1992-1-1. The process of determining the final value of creep coefficient for these assumptions is given in Figures B3.10 and B3.11:

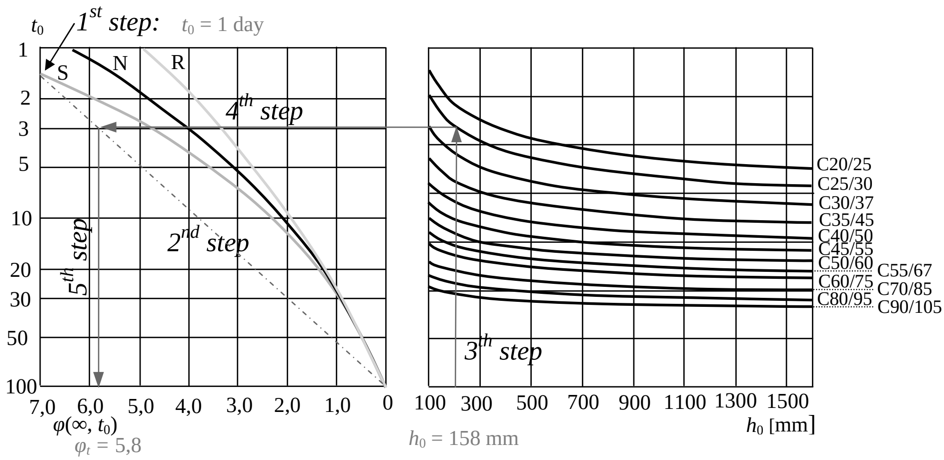


Figure B3.10 *Determination of the final value of creep coefficient*, t_0 = 1 *day*

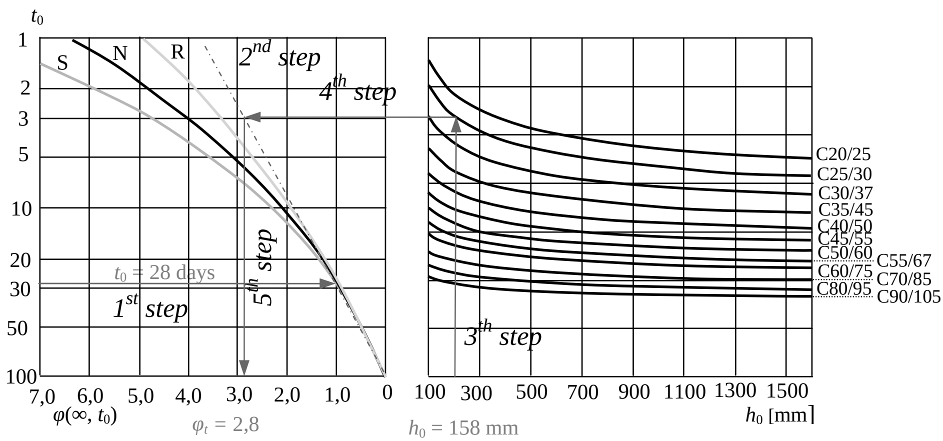


Figure B3.11 *Determination of the final value of creep coefficient*, t_0 = 28 *days*

The following creep coefficients are obtained:

$\varphi_t = \varphi(\infty, t_0 = 1 \text{ day}) = 5{,}8$
$\varphi_t = \varphi(\infty, t_0 = 28 \text{ days}) = 2{,}8$

The total shrinkage strain, according to clause 3.1.4, EN 1992-1-1, at the age of concrete at the beginning of drying shrinkage t_s = 3 days and the age at the time considered in the analysis $t = \infty$, is:

$\varepsilon_{cs}(\infty) = 4{,}14 \cdot 10^{-4}$
$\varepsilon_{cs}(\infty) = 0{,}414$ ‰

Effective flexural stiffness of composite section

The effective flexural stiffness of composite section EI_L as explained previously is:

$$EI_L = E_a \cdot I_a + E_L \cdot I_c + \frac{E_a \cdot A_a \cdot E_L \cdot A_c}{E_a \cdot A_a + E_L \cdot A_c} \cdot a^2$$

a) Short-term loading

$$E_a = 21000 \text{ kN/cm}^2 \quad I_a = 1943 \text{ cm}^4 \quad A_a = 28{,}48 \text{ cm}^2$$

$$I_c = \frac{b_{eff} \cdot h_c^3}{12} = \frac{125 \cdot 7{,}9^3}{12} = 5136 \text{ cm}^4$$

$$A_c = b_{eff} \cdot h_c = 125 \cdot 7{,}9 = 987{,}5 \text{ cm}^2$$

The distance between the centroidal axes of the concrete flange and the steel section is:

$$a = \frac{h_a}{2} + h_p + \frac{h_c}{2} = \frac{20}{2} + 5{,}1 + \frac{7{,}9}{2} = 19{,}05 \text{ cm}$$

$$n_c = 1$$

$$E_0 = \frac{E_{cm}}{n_c} = \frac{3100}{1{,}0} = 3100 \text{ kN/cm}^2 \quad E_L = E_0$$

$$EI_0 = 21000 \cdot 1943 + 3100 \cdot 5136 + \frac{21000 \cdot 28{,}48 \cdot 3100 \cdot 987{,}5}{21000 \cdot 28{,}48 + 3100 \cdot 987{,}5} \cdot 19{,}05^2$$

$$EI_0 = 238295590 \text{ kNcm}^2 = 23829 \text{ kNm}^2$$

b) Permanent loading constant in time

$$n_c = 1 + 1{,}10 \cdot \varphi\,(\infty, t_0) = 1 + 1{,}10 \cdot 2{,}8 = 4{,}08$$

$$E_P = \frac{E_{cm}}{n_c} = \frac{3100}{4,08} = 760 \text{ kN/cm}^2, \; E_L = E_P$$

$$EI_P = 21000 \cdot 1943 + 760 \cdot 5136 + \frac{21000 \cdot 28,48 \cdot 760 \cdot 987,5}{21000 \cdot 28,48 + 760 \cdot 987,5} \cdot 19,05^2$$

$$EI_P = 165494202 \text{ kNcm}^2 = 16549 \text{ kNm}^2$$

c) Action effects caused by shrinkage-primary effects

$$n_c = 1 + 0,55 \cdot \varphi(\infty, t_0) = 1 + 0,55 \cdot 5,8 = 4,19$$

$$E_S = \frac{E_{cm}}{n_c} = \frac{3100}{4,19} = 740 \text{ kN/cm}^2 \quad E_L = E_S$$

$$EI_S = 21000 \cdot 1943 + 740 \cdot 5136 + \frac{21000 \cdot 28,48 \cdot 740 \cdot 987,5}{21000 \cdot 28,48 + 740 \cdot 987,5} \cdot 19,05^2$$

$$EI_S = 163960845 \text{ kNcm}^2 = 16396 \text{ kNm}^2$$

Calculation of deflections

- Deflection due to permanent action, the first loading is applied at age t_0 = 28 days

$$e_d = b \cdot (g_{k,2} + g_{k,3}) = 3,00 \cdot (3,33 + 0,15) = 10,44 \text{ kN/m}$$

$EI_L = EI_0 = 23829$ kNm^2, for short-term loading

$$\delta_1 = \frac{5}{384} \cdot \frac{e_d \cdot L^4}{EI_0} = \frac{5}{384} \cdot \frac{10,44 \cdot 5^4}{23829} \cdot 1000 = 3,6 \text{ mm}$$

- Deflection due to the frequent value of the variable action at the time of the first loading t_0 = 28 days

For a building with floors in category B, office areas, the combination factor ψ is: ψ_1 = 0,5.

$$e_d = b \cdot \psi_1 \cdot q_{k,2} = 3,00 \cdot 0,5 \cdot 3,8 = 5,7 \text{ kN/m}$$

$EI_L = EI_0 = 23829$ kNm^2, for short-term loading

$$\delta_{2,1} = \frac{5}{384} \cdot \frac{e_d \cdot L^4}{EI_0} = \frac{5}{384} \cdot \frac{5{,}70 \cdot 5^4}{23829} \cdot 1000 = 1{,}9 \text{ mm}$$

- Deflection due to creep under the quasi-permanent value of the variable action at time $t = \infty$.

This deflection is the difference of the deflections at time $t = \infty$ and at the time of the first loading $t_0 = 28$ days.

$$e_d = b \cdot (g_{k,2} + g_{k,3} + \psi_2 \cdot q_{k,2}) = 3{,}00 \cdot (3{,}33 + 0{,}15 + 0{,}3 \cdot 3{,}8) = 13{,}9 \text{ kN/m}$$

$EI_0 = 23829$ kNm^2, for short-term loading

$EI_P = 16549$ kNm^2, permanent action constant in time

$$\delta_{2,2} = \frac{5}{384} \cdot \frac{e_d \cdot L^4}{EI_P} - \frac{5}{384} \cdot \frac{e_d \cdot L^4}{EI_0}$$

$$\delta_{2,2} = \frac{5}{384} \cdot \frac{13{,}9 \cdot 5^4}{16549} \cdot 1000 - \frac{5}{384} \cdot \frac{13{,}9 \cdot 5^4}{23829} \cdot 1000 = 6{,}8 - 4{,}7 = 2{,}1 \text{ mm}$$

- Deflection due to shrinkage

$$N_{cs} = \varepsilon_{cs}(\infty) \cdot E_S \cdot A_c = 4{,}14 \cdot 10^{-4} \cdot 740 \cdot 125 \cdot 7{,}9 = 303 \text{ kN}$$

$$a_c = \frac{E_a \cdot A_a}{E_a \cdot A_a + E_S \cdot A_c} \cdot a = \frac{21000 \cdot 28{,}48}{21000 \cdot 28{,}48 + 740 \cdot 987{,}5} \cdot 19{,}05 = 8{,}6 \text{ cm}$$

$$M_{cs} = N_{cs} \cdot a_c = 303 \cdot \frac{8{,}6}{100} = 26{,}1 \text{ kNm}$$

$$\delta_{2,3} = \frac{1}{8} \cdot \frac{M_{cs} \cdot L^2}{EI_S} = \frac{1}{8} \cdot \frac{26{,}1 \cdot 5^2}{16396} \cdot 1000 = 5{,}0 \text{ mm}$$

The effects of shear connection on the deflection of the beam can be neglected because there was full shear connection.

Deflection limits for composite beams are the same as for steel beams, and are determined by the National Annex.

Recommended limiting values for deflection of composite beams are:

$\delta_{tot} \leq \dfrac{L}{250}$, the deflection due to total load

$\delta_{\mathrm{var}} \leq \dfrac{L}{360}$, the deflection due to the variable load

The deflection due to the permanent action is:

$$\delta_1 = 3,6 \text{ mm}$$

The deflection due to the variable load, creep and shrinkage is:

$$\delta_2 = \Sigma\delta_{2,i} = 1,9 + 2,1 + 5,0 = 9,0 \text{ mm}$$

The total deflection due to permanent and variable loads, creep and shrinkage is:

$$\delta_{tot} = \delta_1 + \delta_2 = 3,6 + 9,0 = 12,6 \text{ mm} \leq \frac{L}{250} = \frac{5000}{250} = 20,0 \text{ mm}$$

The total deflection meets the criterion $L/250$.

The deflection due to variable load, creep and shrinkage is:

$$\delta_{\mathrm{var}} = \delta_2 = 9,0 \text{ mm} \leq \frac{L}{360} = \frac{5000}{360} = 13,9 \text{ mm}$$

The deflection due to variable load, creep and shrinkage meets the criterion $L/360$.

5.3 Simplified calculation of deflections

Remark:

As a simplification for the structures of buildings according to clause 5.4.2.2(11), EN 1994-1-1, the effects of creep may alternatively be taken into account by replacing the concrete area, A_c, by the effective steel area A_c/n for both short-term and long-term loading, where n is the nominal modular ratio corresponding to the effective modulus of elasticity of concrete $E_{c,eff}$ adopted as $E_{cm}/2$.

$$n = \frac{2 \cdot E_a}{E_{cm}} = 2n_0$$

Thus the nominal modular ratio *n* is:

$$n = 2n_0 = 2 \cdot 6,8 = 13,6$$

In this example, the deflection is calculated with effective flexural stiffness EI_L. The same result would be obtained if the procedure is applied with the ideal second moment of area $I_{i,L}$ so that the values of the concrete are reduced by the corresponding modular ratio n_L.

In this case, the simplified flexural stiffness values are taken into account for creep:

$$E_{c,eff} = \frac{E_{cm}}{2} = \frac{3100}{2} = 1550 \ \text{kN/cm}^2 \quad E_L = E_S$$

$$EI_L = E_a \cdot I_a + E_{c,eff} \cdot I_c + \frac{E_a \cdot A_a \cdot E_{c,eff} \cdot A_c}{E_a \cdot A_a + E_{c,eff} \cdot A_c} \cdot a^2$$

$$EI_P = 21000 \cdot 1943 + 1550 \cdot 5136 + \frac{21000 \cdot 28,48 \cdot 1550 \cdot 987,5}{21000 \cdot 28,48 + 1550 \cdot 987,5} \cdot 19,05^2$$

$$EI_P = EI_L = 204827738 \ \text{kNcm}^2 = 20483 \ \text{kNm}^2$$

Remark:

In contrast to the previous procedure for calculating deflection, a different simplification is used here. Deflections due to permanent action, variable action and creep are not calculated separately (using frequent and quasi-permanent combinations).
These deflections are calculated in a single step using the characteristic combination.

The following deflections are obtained:

a) Deflection due to the permanent action on the composite section (permanent action and the associated value of creep)

$$e_d = b \cdot (g_{k,2} + g_{k,3}) = 3,00 \cdot (3,33 + 0,15) = 10,44 \ \text{kN/m}$$

$$\delta_1 = \frac{5}{384} \cdot \frac{e_d \cdot L^4}{EI_P}$$

$$\delta_1 = \frac{5}{384} \cdot \frac{10,44 \cdot 5,0^4}{20843} \cdot 1000 = 4,1 \text{ mm}$$

b) Deflection due to the variable action on the composite section (variable action and the associated value of creep)

$e_d = b \cdot q_{k,2} = 3,00 \cdot 3,8 = 11,4$ kN/m

$$\delta_{2,1} = \frac{5}{384} \cdot \frac{e_d \cdot L^4}{EI_P}$$

$$\delta_{2,1} = \frac{5}{384} \cdot \frac{11,40 \cdot 5,0^4}{20843} \cdot 1000 = 4,5 \text{ mm}$$

c) Deflection due to shrinkage that was previously calculated is:

$$\delta_{2,3} = 5,0 \text{ mm}$$

The total deflection due to permanent and variable action, creep and shrinkage is:

$$\delta_{tot} = \delta_1 + \delta_{2,1} + \delta_{2,3} = 4,1 + 4,5 + 5,0 = 13,6 \text{ mm}$$

5.4 Pre-cambering of the steel beam

In high-rise buildings and industrial buildings it may be necessary to use a steel beam with pre-cambering. This is normally only adopted for beams longer than 10 m. For composite beams that are executed unpropped their own weight must be taken by the steel beam. This can result in excessive deflections. In this situation, where permanent load deflections are excessive, pre-cambering may be appropriate. However, in the case of propped execution, pre-cambering can be appropriate for beams with greater spans. The components of the pre-camber of the steel beam are represented schematically in Figure B.3.12. For the deflection due to variable load, the steel beam can be cambered to compensate this deflection completely or only partially. Usually, the deflection due to the imposed load is compensated only partially – see the upper part of the deflection $\delta_{2,1}$ in Figure B3.12.

A general recommendation is to design any pre-cambering to eliminate the dead load deflection and the deflection due to creep and shrinkage. Whether to take into account the camber (or the pre-camber) due to variable action is determined separately for each case.

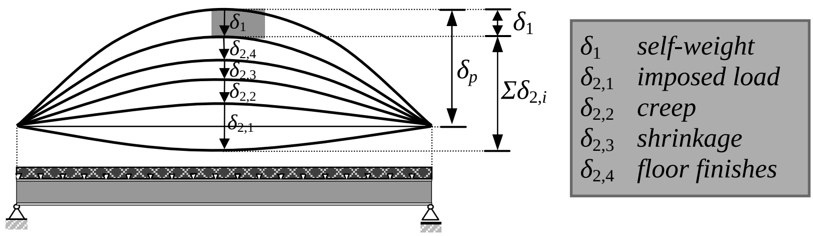


Figure B3.12 *Components of pre-camber of steel beam*

In the case of the composite beam with full shear connection, it is not necessary to take into account the influence of the degree of shear connection for calculating deflection. This also applies to the composite beam with partial shear connection, if the beam is unpropped, and the degree of shear connection is greater than $\eta > 0{,}5$.

In this example, the deflections are:

$\delta_1 = 3{,}6$ mm — deflection due to the permanent action, the first loading is applied at age $t_0 = 28$ days,

$\delta_{2,1} = 1{,}9$ mm — deflection due to the frequent value of the variable action at the time of the first loading $t_0 = 28$ days,

$\delta_{2,2} = 2{,}1$ mm — deflection due to creep under the quasi-permanent value of variable action at time $t = \infty$,

$\delta_{2,3} = 5{,}0$ mm — deflection due to shrinkage,

$\delta_{2,4} = 0$ mm — (the deflection due to floor finishes is not taken into account because its deflection is already included in the deflection δ_1).

Therefore, the pre-cambering of steel beam is:

$$\delta_p = 3{,}6 + 1{,}9 + 2{,}1 + 5{,}0 = 12{,}6 \text{ mm}$$

5.5 Check of vibration of the beam

The dynamic properties of composite floor systems should meet the criteria specified in clause A1.4.4, EN1990. This article states that the natural frequency of vibration of the structure or structural element should be kept above appropriate values, which depend on the function of the building and the source of the vibration. No values are given for either limiting frequencies or damping coefficients. However, it is proposed that criteria are agreed in consultation with the client and/or the relevant authority.

If the natural frequencies are found to be greater than 4 Hz, the floor is considered acceptable for normal use, such as offices.

Remark:

The limitation of the natural frequency to 4 Hz is a fairly generally accepted industry standard for vibration. However, the satisfying of this criterion does not guarantee that the behaviour of structural element or structure in general will be appropriate.

For the calculation of the natural frequency, the characteristic values of the permanent load for the composite stage, e_d, are taken into account, as is the effective flexural stiffness of the composite section for short-term loading EI_0.

This total load is:

$$e_d = b \cdot (g_{k,2} + g_{k,3}) = 3,00 \cdot (3,33 + 0,15) = 10,44 \text{ kN/m}$$

The deflection under this load is:

$$\delta = \frac{5}{384} \cdot \frac{e_d \cdot L^4}{EI_0} = \frac{5}{384} \cdot \frac{10,44 \cdot 5^4}{23829} \cdot 1000 = 3,6 \text{ mm}$$

The natural frequency of the beam is therefore:

$$f = \frac{18}{\sqrt{\delta}} = \frac{18}{\sqrt{3,6}} = 9,49 \text{ Hz} \geq 4 \text{ Hz} \quad \text{with } \delta \text{ in mm}$$

The criterion is satisfied. However, the improved estimation of the natural frequency is carried out taking into account the dynamic modulus of elasticity of concrete E_d that can be 10% higher than the modulus of elasticity of concrete E_{cm}, and so the dynamic modulus of elasticity of concrete is:

$$E_d = 1{,}1 \cdot E_{cm} = 1{,}1 \cdot 3100 = 3410 \text{ kN/cm}^2$$

The effective flexural stiffness of composite section is:

$$EI_d = E_a \cdot I_a + E_d \cdot I_c + \frac{E_a \cdot A_a \cdot E_d \cdot A_c}{E_a \cdot A_a + E_d \cdot A_c} \cdot a^2$$

$$EI_d = 21000 \cdot 1943 + 3410 \cdot 5136 + \frac{21000 \cdot 28{,}48 \cdot 3410 \cdot 987{,}5}{21000 \cdot 28{,}48 + 3410 \cdot 987{,}5} \cdot 19{,}05^2$$

$$EI_d = 242626250 \text{ kNcm}^2 = 24263 \text{ kNm}^2$$

For the calculation of the natural frequency, the characteristic values of the permanent load for the composite stage, $e_d = g_{k,2} + g_{k,3}$, and 10% of the variable actions are taken into account:

$$e_d = b \cdot (g_{k,2} + g_{k,3} + 0{,}10 \cdot q_{k,2}) = 3{,}00 \cdot (3{,}33 + 0{,}15 + 0{,}10 \cdot 3{,}8) = 11{,}58 \text{ kN/m}$$

The deflection under this load is:

$$\delta_d = \frac{5}{384} \cdot \frac{e_d \cdot L^4}{EI_d}$$

$$\delta_d = \frac{5}{384} \cdot \frac{11{,}58 \cdot 5^4}{24263} \cdot 1000 = 3{,}9 \text{ mm}$$

The natural frequency of the beam is therefore:

$$f = \frac{18}{\sqrt{\delta_d}} = \frac{18}{\sqrt{3{,}9}} = 9{,}11 \text{ Hz} \geq 4 \text{ Hz} \quad \text{with } \delta \text{ in mm}$$

The criterion is satisfied for initial calculation purposes. However, the dynamic performance of the entire floor is carried out using a method such as the one in [51].

5.6 Control of crack width

The appropriate theoretical model for cracking caused by restraint of the imposed deformation is different from that for cracking caused by applied loading. For this reason, design rules for control of cracking as two distinct procedures are introduced in EN 1994-1-1:

- for minimum reinforcement, clause 7.4.2, EN 1994-1-1,
- for reinforcement to control cracking due to direct loading, clause 7.4.3, EN 1994-1-1.

Clause 7.3.1(4), EN 1992-1-1 permits cracking of uncontrolled width in some circumstances. For example, this is true for a simply supported beam with a continuous concrete slab on top of a steel beam.

Clause 7.4.1(3), EN 1994-1-1 recommends the minimum reinforcement that will limit crack width to what is “acceptable”. For unpropped construction, the requirement is 0,2% of the area of concrete, and for propped construction this is doubled to 0,4%. The reinforcement area is calculated as a percentage of the concrete area above the profiled steel sheeting, using the depth used in beam design.

The area of the concrete is:

$$A_c = h_c \cdot b = 7{,}9 \cdot 100 = 790 \ \text{cm}^2$$

The criterion of minimum reinforcement is:

$$A_s \geq 0{,}004 \cdot A_c = 0{,}004 \cdot 790 \cdot 10^2 = 316 \ \text{mm}^2/\text{m}$$

The reinforcement provided is 10 mm bars at 200 mm, for which:

$$A_{s,\min} = \frac{\pi \cdot 10^2}{4} \cdot \frac{1000}{200} = 393 \ \text{mm}^2/\text{m} > 316 \ \text{mm}^2/\text{m}$$

For the adopted reinforcement, the maximum allowable thickness of slab h_c is:

$$h_c = \frac{A_{s,\min}}{0{,}004 \cdot 1000} = \frac{393}{0{,}004 \cdot 1000} = 98 \ \text{mm}$$

The maximum allowable thickness of slab h_c is higher than the existing thickness 79 mm, so this criterion is met.

6. Commentary

This example deals with the design of a secondary composite floor beam. Verifications for the ultimate limit state and serviceability limit state were conducted in accordance with EN 1994-1-1. The verification of deflections is explained in great detail, because deflections may govern the design, especially where beams are designed as simply supported and unpropped at the construction stage. The explanations are given for the application of a combination of actions from the aspects of ways of choosing a particular combination. According to EN 1990, there are three combinations of actions for the serviceability limit state: characteristic, frequent and quasi-permanent. The selection of an appropriate combination depends on whether the limit state is reversible or irreversible, that is whether or not the composite floor beam deflections cause cracking of a brittle floor finish or damage to fragile partitions.

It should be noted that EN 1994-1-1 does not specify any limits for serviceability criteria, but limits may be given in the National Annex. In addition, the National Annex may give references to publications that contain non-contradictory complementary information (NCCI) that will assist the designer.

B4 Calculation of simply supported composite beam according to the elastic resistance of the cross-section

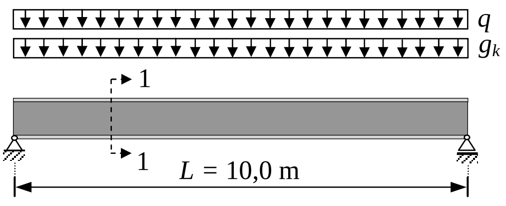
1. Purpose of example

It is necessary to design the simply supported composite beam according to the elastic-elastic procedure. Action effects are calculated by elastic global analysis and the resistance to bending is based on an elastic model. In the case of the elastic-elastic procedure the cross-section can be in class 3. The composite beam is a structural member of building category B, with office areas. In this case the combination factors ψ are:

$\psi_1 = 0,5$ – for the frequent value of variable action,
$\psi_2 = 0,3$ – for the quasi-permanent value of variable action.

During execution, the steel beam is fully propped. For the simply supported beam, according to clause 7.3.1(4), EN 1992-1-1, cracking of uncontrolled width is permitted, and clause 7.4.1(3), EN1994-1-1, recommends the minimum reinforcement. Since it is using the elastic-elastic procedure, the design value of concrete strength will not be reduced by the value 0,85. One of the essential features of this example is to illustrate the determination of the effective flexural stiffness EI_L by two procedures that are used in practice. The first procedure is the direct determination of the effective flexural stiffness, while the second is based on the modular ratio n_L. The procedure for determining the resistance of the composite section due to the vertical shear force, taking into account the verification of stress in the concrete slab, is also shown in the example. This problem is not covered in EN 1994-1-1, clause 6.2.2.

2. Static system, cross-section and actions



Section 1-1 *is shown in Figure B4.1b*

Figure B4.1a *Static system of composite beam*

Composite Structures according to Eurocode 4. Worked Examples.
First Edition. Darko Dujmović, Boris Androić, Ivan Lukačević.

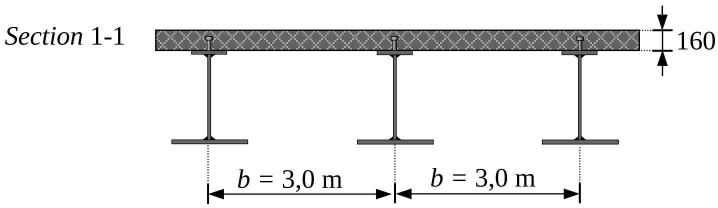


Figure B4.1b *Section 1-1 from* Figure B4.1a

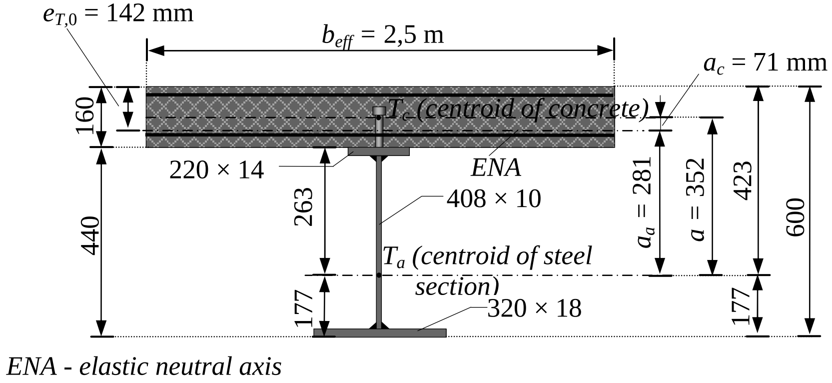


Figure B4.2 *Cross-section of the composite beam for short-term loading*

Actions

a) Permanent action

Remark:

According to EN 1991-1-1, the density of normal weight concrete is 24 kN/m^3, increased by 1 kN/m^3 for normal percentage reinforcement, and increased for wet concrete by another 1 kN/m^3.

Concrete slab area per m width:

$$A_c = 100 \cdot h = 100 \cdot 16{,}0 = 1600 \text{ cm}^2$$

- *concrete slab and reinforcement (dry concrete)* $A_c \cdot 25 = 0{,}16 \cdot 25 = 4{,}0 \text{ kN/m}^2$

- *steel beam* $= 0{,}33 \text{ kN/m}^2$

- *floor finishes* $= 2{,}0 \text{ kN/m}^2$

Total $g_k = 6{,}33 \text{ kN/m}^2$

b) Variable action

$$q_k = 5,0 \ \text{kN/m}^2$$

3. Properties of materials

Concrete strength class: C 20/25

$$f_{ck} = 20 \ \text{N/mm}^2$$

$$f_{cd} = \frac{f_{ck}}{\gamma_c} = \frac{20}{1,5} = 13,3 \ \text{N/mm}^2$$

$$E_{cm} = 30\,000 \ \text{N/mm}^2$$

Reinforcement: *ductility class B or C (Table C.1, EN 1992-1-1)*

$$f_{sk} = 500 \ \text{N/mm}^2$$

$$f_{sd} = \frac{f_{sk}}{\gamma_s} = \frac{500}{1,15} = 435 \ \text{N/mm}^2$$

$$E_s = 210\,000 \ \text{N/mm}^2$$

Structural steel: S235

$$f_{yk} = 235 \ \text{N/mm}^2$$

$$f_{yd} = \frac{f_{yk}}{\gamma_M} = \frac{235}{1,0} = 235 \ \text{N/mm}^2$$

$$\tau_{a,Rd} = \frac{f_{yd}}{\sqrt{3}} = \frac{235}{\sqrt{3}} = 136 \ \text{N/mm}^2$$

$$E_a = 210\,000 \ \text{N/mm}^2$$

Shear connectors: *ductile headed studs*

$$f_u = 450 \ \text{N/mm}^2$$

$$d = 22 \ \text{mm}$$

$$h_{sc} = 125 \ \text{mm}$$

$$\frac{h_{sc}}{d} = \frac{125}{22} = 5,7 > 4,0 \ \rightarrow \ \alpha = 1,0$$

$$P_{Rd} = 87,0 \ \text{kN}$$

4. Ultimate limit state

4.1 Design values of the combined actions and of the effects of actions

The design load for ultimate limit state is:

$$e_d = \text{b} \cdot (1,35 \cdot g_k + 1,50 \cdot q_k)$$

$$e_d = 3{,}0 \cdot (1{,}35 \cdot 6{,}33 + 1{,}50 \cdot 5{,}0) = 48{,}1 \text{ kN/m}$$

The maximum design bending moment is:

$$M_{Ed} = \frac{e_d \cdot L^2}{8} = \frac{48{,}1 \cdot 10{,}0^2}{8} = 601 \text{ kNm}$$

The maximum design shear force is:

$$V_{Ed} = e_d \cdot \frac{L}{2} = 48{,}1 \cdot \frac{10{,}0}{2} = 241 \text{ kN}$$

4.2 Effective width of the concrete flange

The effective width of the concrete flange is:

$$b_{eff} = b_0 + \Sigma b_{ei}$$

$$b_{ei} = \frac{L_e}{8} = \frac{L}{8} = \frac{10}{8} = 1{,}25 \text{ m}$$

$b_0 = 0$ (there is only one row of shear connectors)

$b_{eff} = 0 + (2 \cdot 1{,}25) = 2{,}5 \text{ m} < 3{,}0 \text{ m}$ (beam spacing)

The effective cross-section of the concrete slab is shown in Figure B4.3.

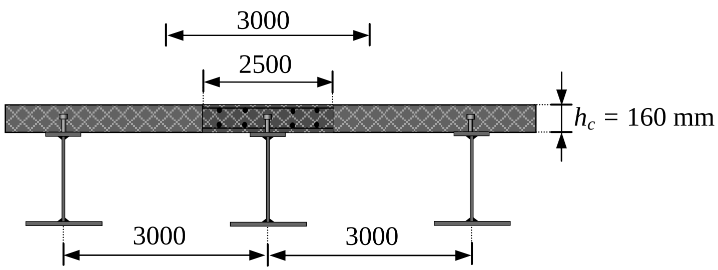


Figure B4.3 *Effective cross-section of the concrete slab*

4.3 Elastic resistance moment of the composite cross-section

4.3.1 Calculation of the centroid of the steel cross-section

The area of steel cross-section is:

$$A_a = 22,0 \cdot 1,4 + 1,0 \cdot 40,8 + 32,0 \cdot 1,8 = 30,8 + 40,8 + 57,6 = 129,2 \text{ cm}^2$$

The distance between the centroidal axis of the steel section and the extreme fibre of the steel section is:

$$e_a = \frac{30,8 \cdot 0,7 + 40,8 \cdot 21,8 + 57,6 \cdot 43,1}{129,2} = 26,3 \text{ cm}$$

4.3.2 Second moment of area of the steel cross-section

The second moment of area I_a for the top fibre of steel section is:

$$I_a = 22,0 \cdot \frac{1,4^3}{12} + 1,0 \cdot \frac{40,8^3}{12} + 32,0 \cdot \frac{1,8^3}{12} +$$
$$+30,8 \cdot 0,7^2 + 40,8 \cdot 21,8^2 + 57,6 \cdot 43,1^2 - 129,2 \cdot 26,3^2 = 42717 \text{ cm}^4$$

4.3.3 Flexural stiffness of the composite cross-section

The effective flexural stiffness of uncracked composite section is:

$$EI_L = E_a \cdot I_a + E_L \cdot I_c + \frac{E_a \cdot A_a \cdot E_L \cdot A_c}{E_a \cdot A_a + E_L \cdot A_c} \cdot a^2$$

Determination of the creep coefficient and shrinkage

For the calculation of the creep coefficient $\varphi(t, t_0)$ the following is valid:

- the perimeter of that part which is exposed to drying, u

$$u = 2 \cdot b$$

- the notional size of the cross-section, h_0

$$h_0 = \frac{2 \cdot A_c}{u} = \frac{b \cdot h_c}{b} = h_c = 160 \text{ mm}$$

- $t_0 = 1$ day, $t_0 = 28$ days,
- the ambient relative humidity, RH 50%,
- the concrete strength class C 20/25,
- the type of cement – cement class S, strength class 32,5 N.

The final value of creep coefficient $\varphi(\infty, t_0)$ is determined using the nomogram shown in Figure 3.1, EN 1992-1-1. Example A3, shows the detailed procedure for the determination of creep coefficients.

The following creep coefficients are obtained:

$\varphi_t = \varphi(\infty, t_0 = 1 \text{ day}) = 6{,}3$
$\varphi_t = \varphi(\infty, t_0 = 28 \text{ days}) = 3{,}1$

The total shrinkage strain, according to clause 3.1.4, EN 1992-1-1, at the age of concrete at the beginning of drying shrinkage t_s = 3 days and at the time considered in the analysis $t = \infty$, is:

$$\varepsilon_{cs}(\infty) = 4{,}26 \cdot 10^{-4}$$
$$\varepsilon_{cs}(\infty) = 0{,}426 \text{ ‰}$$

The effective flexural stiffness of composite section EI_L depends on the type of loadings, creep and shrinkage. This stiffness can be determined using the effective modulus of elasticity of concrete $E_{c,eff}$ or using the modular ratio n_L:

$$E_{c,eff} = \frac{E_{cm}}{1 + \psi_L \cdot \varphi_t} \qquad n_L = n_0 (1 + \psi_L \cdot \varphi_t)$$

$$E_{c,eff} = \frac{E_{cm}}{n_c} \qquad n_c = 1 + \psi_L \cdot \varphi_t$$

In practice, two numerical procedures are usually applied.

a) The first numerical procedure
The corresponding reduced flexural stiffness EI_L can be calculated directly. For example, in the case of short-term loading, the creep multiplier is $\psi_L = 0$. Therefore, $E_{c,eff} = E_{cm} = E_0$. This means that the effective flexural stiffness is $EI_L = EI_0$. The effective flexural stiffness EI_P (permanent action constant in time) and EI_S (shrinkage) is determined in a similar manner.

b) The second numerical procedure
For the corresponding type of loading, the modular ratio is calculated according to the expression $n_L = n_0 (1 + \psi_L \cdot \varphi_t)$

In the next step the "ideal" values of the composite section (I_{id}, A_{id}, etc.) are calculated in a way that takes into account the cross-sectional values of the steel section and the cross-sectional values of concrete divided by the corresponding modular ratio n_L (transformed cross-sectional values of concrete in “steel” units). For example, in the case of short-term loading, the creep multiplier is $\psi_L = 0$ and the modular ratio is $n_L = n_0 = \dfrac{E_a}{E_{cm}}$. In this case, the ideal area of cross-section is:

$$A_{id} = A_a(steel) + A_c(concrete) \cdot \frac{1}{n_0}$$

The other ideal values of composite section are calculated in a similar manner.

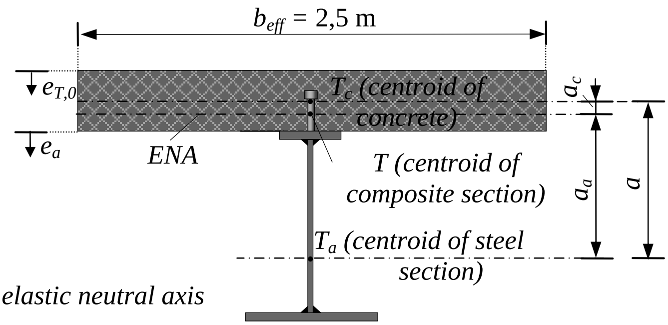
In this example, the first numerical procedure is used, while the second method will be illustrated for educational purposes only in (a) below.

Determination of the effective flexural stiffness of composite section EI_L for:

a) short-term loading, $n_c = 1$ → stiffness EI_0

b) permanent loading constant in time, $t = \infty$, $n_c = 1 + 1{,}10 \cdot \varphi(\infty, t_0)$ → stiffness EI_P

c) primary effects due to shrinkage $n_c = 1 + 0{,}55 \cdot \varphi(\infty, t_0)$ → stiffness EI_S



ENA - elastic neutral axis

- a *distance between the centroidal axes of the concrete slab and the steel section*
- a_c *distance between the centroidal axis of the concrete slab and the elastic neutral axis*
- a_a *distance between the centroidal axis of the steel section and the elastic neutral axis*
- e_a *distance between the centroidal axis of the steel section and the extreme fibre of the top flange of the steel section*
- $e_{T,0}$ *distance between the top surface of the concrete slab and the elastic neutral axis*

values depending on E_0, E_P, E_S or depending on modular ratio n_c

Figure B4.4 *Cross-section of composite beam and dimensions*

a) Short-term loading, $n_c = 1$

The first numerical procedure

$$E_0 = \frac{E_{cm}}{n_c} = \frac{3000}{1{,}0} = 3000 \text{ kN/cm}^2$$

$$A_c = b_c \cdot h_c = 250 \cdot 16 = 4000 \ \text{cm}^2 \ (b_c = b_{eff})$$

The distance between the centroidal axis of the concrete and the steel section is:

$$a = e_a + \frac{h_c}{2} = 26{,}3 + \frac{16{,}0}{2} = 34{,}3 \ \text{cm}$$

The elastic neutral axis (the centroid of composite section) lies within the concrete slab, and the distance between the centroidal axis of the steel section and the elastic neutral axis is:

$$a_a = \frac{E_0 \cdot A_c}{E_a \cdot A_a + E_0 \cdot A_c} \cdot a = \frac{3000 \cdot 4000}{21000 \cdot 129{,}2 + 3000 \cdot 4000} \cdot 34{,}3 = 28{,}0 \ \text{cm}$$

The distance between the top surface of the concrete slab, and the elastic neutral axis is:

$$e_{T,0} = a + \frac{h_c}{2} - a_a = 34{,}3 + \frac{16{,}0}{2} - 28{,}0 = 14{,}3 \ \text{cm} \le \ h_c = 16{,}0 \ \text{cm}$$

The elastic neutral axis lies within the concrete slab, and the area of the concrete slab in compression A_c is:

$$A_c = b_{eff} \cdot e_{T,0} = 250 \cdot 14{,}3 = 3575 \ \text{cm}^2$$

$$a = e_a + h_c - \frac{e_{T,0}}{2} = 26{,}3 + 16 - \frac{14{,}3}{2} = 35{,}2 \ \text{cm}$$

$$a_a = \frac{E_0 \cdot A_c}{E_a \cdot A_a + E_0 \cdot A_c} \cdot a = \frac{3000 \cdot 3575}{21000 \cdot 129{,}2 + 3000 \cdot 3575} \cdot 35{,}2 = 28{,}1 \ \text{cm}$$

Therefore, the distance between the top surface of the concrete slab and the elastic neutral axis is:

$$e_{T,0} = e_a + h_c - a_a = 26{,}3 + 16 - 28{,}1 = 14{,}2 \ \text{cm}$$

$$I_c = \frac{b_{eff} \cdot e_{T,0}^3}{12} = \frac{250 \cdot 14{,}2^3}{12} = 59652 \ \text{cm}^4$$

The effective flexural stiffness of the composite section EI_0 for short-term loading is:

$$EI_L = EI_0 = E_a \cdot I_a + E_0 \cdot I_c + \frac{E_a \cdot A_a \cdot E_0 \cdot A_c}{E_a \cdot A_a + E_0 \cdot A_c} \cdot a^2$$

$$EI_0 = 21000 \cdot 42717 + 3000 \cdot 59652 + \frac{21000 \cdot 129,2 \cdot 3000 \cdot 3575}{21000 \cdot 129,2 + 3000 \cdot 3575} \cdot 35,2^2$$

$$EI_0 = 3759029453 \ \text{kNcm}^2 = 375903 \ \text{kNm}^2$$

The second numerical procedure

$$n_L = n_0 \cdot n_c$$

$$n_0 = \frac{E_a}{E_{cm}} = \frac{21000}{3000} = 7,0$$

$$n_L = n_0 \cdot n_c = 7,0 \cdot 1,0 = 7,0$$

The ideal cross-sectional area of the concrete flange is:

$$A_c = b_{eff} \cdot h_c = 250 \cdot 16 = 4000 \ \text{cm}^2$$

$$A_{c,L} = \frac{A_c}{n_L} = \frac{4000}{7} = 571,4 \ \text{cm}^2$$

The cross-sectional area of the steel section is:

$$A_a = 129,2 \ \text{cm}^2$$

The ideal area of the composite section is:

$$A_{i,L} = A_a + A_{c,L} = 129,2 + 571,4 = 701 \ \text{cm}^2$$

$$a_a = e_a + \frac{h_c}{2} = 26,3 + \frac{16,0}{2} = 34,3 \ \text{cm}$$

The distance between the centroidal axes of the concrete and the composite section a_c is:

$$a_c^1 = \frac{A_a \cdot a_a}{A_{i,L}} = \frac{129{,}2 \cdot 34{,}3}{701} = 6{,}3 \text{ cm}$$

The centroid of the composite section lies within the concrete slab. Therefore, the procedure should be repeated in order to take into account the reduction of the area of the concrete flange.

The concrete flange portion which is in compression is:

$$e_{T,0}^1 = h_c - (\frac{h_c}{2} - a_c^1) = 16 - (\frac{16}{2} - 6{,}3) = 14{,}3 \text{ cm}$$

The ideal cross-sectional area of concrete flange is:

$$A_c = b_{eff} \cdot e_{T,0}^1 = 250 \cdot 14{,}3 = 3575 \text{ cm}^2$$

$$A_{c,L} = \frac{A_c}{n_L} = \frac{3575}{7} = 510{,}7 \text{ cm}^2$$

The cross-sectional area of the steel section is:

$$A_a = 129{,}2 \text{ cm}^2$$

The ideal area of the composite section is:

$$A_{i,L} = A_a + A_{c,L} = 129{,}2 + 510{,}7 = 640 \text{ cm}^2$$

The distance between the centroidal axes of the concrete and the composite section a_c is:

$$a_a = e_a + h_c - \frac{e_{T,0}^1}{2} = 26{,}3 + 16 - \frac{14{,}3}{2} = 35{,}2 \text{ cm}$$

$$a_c = \frac{A_a \cdot a_a}{A_{i,L}} = \frac{129{,}2 \cdot 35{,}2}{640} = 7{,}1 \text{ cm}$$

The concrete flange portion which is in compression is:

$$e_{T,0} = e_{T,0}^1 - (\frac{e_{T,0}^1}{2} - a_c) = 14{,}3 - (\frac{14{,}3}{2} - 7{,}1) = 14{,}2 \text{ cm}$$

The ideal second moment of area of the concrete flange is:

$$I_c = \frac{b_{eff} \cdot e_{T,0}^3}{12} = \frac{250 \cdot 14{,}2^3}{12} = 59652 \text{ cm}^4$$

$$I_{c,L} = \frac{I_c}{n_L} = \frac{59652}{7} = 8522 \text{ cm}^4$$

The ideal second moment of area of the composite section is:

$$I_{i,L} = I_a + I_{c,L} + \frac{A_a \cdot A_{c,L} \cdot a_a^2}{A_{i,L}} = 42717 + 8522 + \frac{129{,}2 \cdot 510{,}7 \cdot 35{,}2^2}{640}$$

$$I_{i,L} = 42717 + 8522 + 127761 = 179000 \text{ cm}^4$$

The effective flexural stiffness is calculated by multiplying the ideal second moment of area of the composite section by the modulus of elasticity of steel:

$$E_a = 21000 \text{ kN/cm}^2$$

$$I_{i,L} = 179000 \text{ cm}^4$$

$$EI_0 = E_a \cdot I_{i,L} = 21000 \cdot 179000 = 375900000 \text{ kNcm}^2$$

The effective flexural stiffness EI_0 for short-term loading is:

$$EI_0 = 375900 \text{ kNm}^2$$

Remark:

The value of effective flexural stiffness obtained by the second numerical procedure is equal to the value of the effective flexural stiffness obtained by the first numerical method.

b) Permanent loading constant in time

$$n_c = 1 + 1{,}10 \cdot \varphi(\infty, t_0 = 28 \text{ days}) = 1 + 1{,}10 \cdot 3{,}1 = 4{,}41$$

$$E_P = \frac{E_{cm}}{n_c} = \frac{3000}{4{,}41} = 680 \text{ kN/cm}^2$$

$$A_c = b_{eff} \cdot h_c = 250 \cdot 16 = 4000 \text{ cm}^2$$

$$a = e_a + \frac{h_c}{2} = 26{,}3 + \frac{16}{2} = 34{,}3 \text{ cm}$$

$$a_a = \frac{E_P \cdot A_c}{E_a \cdot A_a + E_P \cdot A_c} \cdot a = \frac{680 \cdot 4000}{21000 \cdot 129{,}2 + 680 \cdot 4000} \cdot 34{,}3 = 17{,}2 \text{ cm}$$

$$e_{T,P} = 42{,}3 - a_a = 42{,}3 - 17{,}2 = 25{,}1 \text{ cm} \geq h_c = 16{,}0 \text{ cm}$$

The elastic neutral axis (the centroid of composite section) lies within the steel section:

$$I_c = \frac{b_{eff} \cdot h_c^3}{12} = \frac{250 \cdot 16{,}0^3}{12} = 85333 \text{ cm}^4$$

$$EI_L = EI_P = E_a \cdot I_a + E_P \cdot I_c + \frac{E_a \cdot A_a \cdot E_P \cdot A_c}{E_a \cdot A_a + E_P \cdot A_c} \cdot a^2$$

$$EI_P = 21000 \cdot 42717 + 680 \cdot 85333 + \frac{21000 \cdot 129{,}2 \cdot 680 \cdot 4000}{21000 \cdot 129{,}2 + 680 \cdot 4000} \cdot 34{,}3^2$$

$$EI_P = 2553107304 \text{ kNcm}^2 = 255311 \text{ kNm}^2$$

c) Primary effects due to shrinkage

$$n_c = 1 + 0{,}55 \cdot \varphi(\infty, t_0 = 1 \text{ day}) = 1 + 0{,}55 \cdot 6{,}3 = 4{,}47$$

$$E_S = \frac{E_{cm}}{n_c} = \frac{3000}{4{,}47} = 671 \text{ kN/cm}^2$$

$$A_c = b_{eff} \cdot h_c = 250 \cdot 16 = 4000 \text{ cm}^2$$

$$a = e_a + \frac{h_c}{2} = 26{,}3 + \frac{16}{2} = 34{,}3 \text{ cm}$$

$$EA_s = E_a \cdot A_a + E_S \cdot A_c = 21000 \cdot 129{,}2 + 671 \cdot 4000 = 5397200 \text{ kN}$$

$$a_a = \frac{E_S \cdot A_c}{E_a \cdot A_a + E_S \cdot A_c} \cdot a = \frac{671 \cdot 4000}{21000 \cdot 129{,}2 + 671 \cdot 4000} \cdot 34{,}3 = 17{,}1 \text{ cm}$$

$$e_{T,S} = 42{,}3 - a_a = 42{,}3 - 17{,}1 = 25{,}2 \text{ cm} \geq h_c = 16{,}0 \text{ cm}$$

The elastic neutral axis (the centroid of composite section) lies within the steel section:

$$I_c = \frac{b_{eff} \cdot h_c^3}{12} = \frac{250 \cdot 16{,}0^3}{12} = 85333 \text{ cm}^4$$

$$EI_L = EI_S = E_a \cdot I_a + E_S \cdot I_c + \frac{E_a \cdot A_a \cdot E_S \cdot A_c}{E_a \cdot A_a + E_S \cdot A_c} \cdot a^2$$

$$EI_S = 21000 \cdot 42717 + 671 \cdot 85333 + \frac{21000 \cdot 129{,}2 \cdot 671 \cdot 4000}{21000 \cdot 129{,}2 + 671 \cdot 4000} \cdot 34{,}3^2$$

$$EI_S = 2541706935 \text{ kNcm}^2 = 254171 \text{ kNm}^2$$

4.3.4 Check of the resistance moment of the composite cross-section

The check for the bending moment will be conducted for the following criteria:

a) The age of concrete on the first loading with the corresponding flexural stiffness EI_0

b) At time $t = \infty$ with the corresponding flexural stiffness EI_P

c) For shrinkage with the corresponding flexural stiffness EI_S and the axial stiffness EA_S.

The composite beam is simply supported, statically determinate and the time-dependent secondary effects due to creep have no influence on bending moments.

However, since $EI_P < EI_0$, the deflections are increased from the age of the concrete on first loading to time $t = \infty$.

For the check, we need to determine the stresses in the steel section σ_a and in the concrete σ_c:

$$\sigma_a = E_a \cdot \frac{M}{EI} \cdot z_a$$

$$\sigma_c = E_c \cdot \frac{M}{EI} \cdot z_c$$

In the case of the composite beam with full shear connection, the bending moment and the normal force act on the cross-section, and the stresses in the steel section σ_a and in the concrete σ_c are:

$$\sigma_a = E_a \cdot \frac{N}{EA} + E_a \cdot \frac{M}{EI} \cdot z_a$$

$$\sigma_c = E_c \cdot \frac{N}{EA} + E_c \cdot \frac{M}{EI} \cdot z_c$$

The corresponding denotations are shown in Figure B4.5.

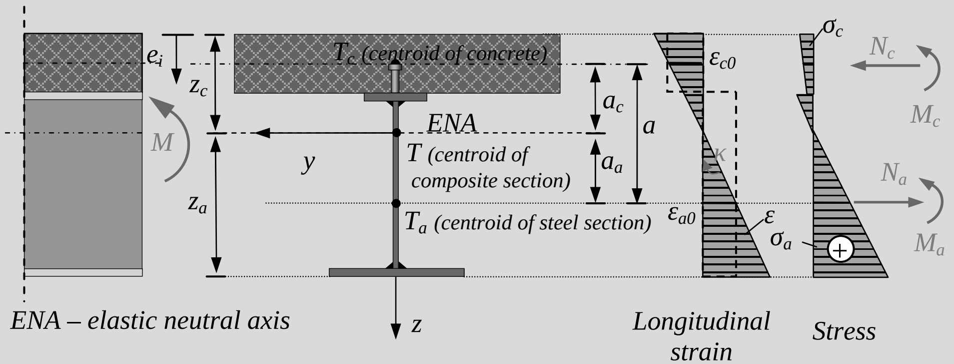


Figure B4.5 *Bending of the composite beam with full shear connection*

a) Check of bending moment, the age of concrete on the first loading with the corresponding flexural stiffness EI_0

Stresses due to sagging bending moment M_{Ed} are calculated at the following points of composite section 1, 2, 3, 4, 5 and 6, see Figure B4.6.

Point 1 – stress in concrete flange at level 1

$M_{Ed} = 601 \text{ kNm}$

$V_{Ed} = 0$ (corresponding design value of shear force)

$$\sigma_c = E_0 \frac{M_{Ed}}{EI_0} \cdot z_c$$

$$z_c = 0 - e_{T,0} = 0 - 14,2 = -14,2 \ \text{cm}$$

$$E_0 = E_{cm} = 3000 \ \text{kN/cm}^2$$

$$EI_0 = 375903 \cdot 10^4 \ \text{kNcm}^2$$

$$\sigma_c = 3000 \frac{60100}{375903 \cdot 10^4} \cdot (-14,2) = -0,681 \ \text{kN/cm}^2$$

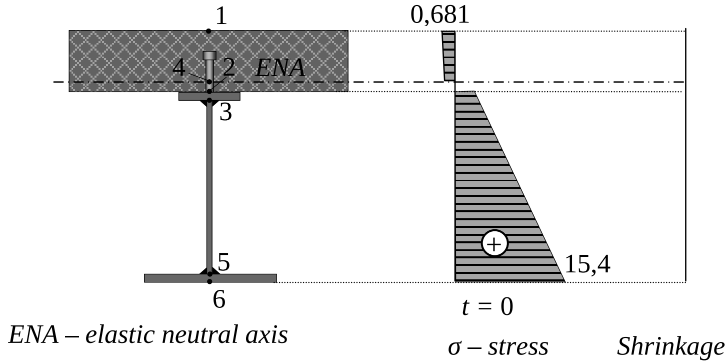


Figure B4.6 *Points of the composite section and the distribution of normal stresses (tensile stress positive)*

Point 6 - stress in steel section at level 6

$$M_{Ed} = 601 \ \text{kNm}$$

$V_{Ed} = 0$ (corresponding design value of shear force)

$$\sigma_a = E_a \frac{M_{Ed}}{EI_0} \cdot z_a$$

$$z_a = 60,0 - e_{T,0} = 60,0 - 14,2 = 45,8 \ \text{cm}$$

$$E_a = 21000 \ \text{kN/cm}^2$$

$$M_{Ed} = 60100 \ \text{kNcm}$$

$$EI_0 = 375903 \cdot 10^4 \ \text{kNcm}^2$$

$$\sigma_a = 21000 \cdot \frac{60100}{375903 \cdot 10^4} \cdot 45,8 = 15,4 \ \text{kN/cm}^2$$

Check for concrete flange:

$\dfrac{\sigma_c}{f_{cd}} = \dfrac{0,681}{1,33} = 0,51 \leq 1,0$, the condition is satisfied

Check for steel section:

$\dfrac{\sigma_a}{f_{yd}} = \dfrac{15,4}{23,5} = 0,66 \leq 1,0$, the condition is satisfied

b) Check of the bending moment at time $t = \infty$ with the corresponding flexural stiffness EI_P

Stresses due to sagging bending moment M_{Ed} are calculated in the following points of the composite section 1, 2, 3, 4, 5 and 6, see Figure B4.7.

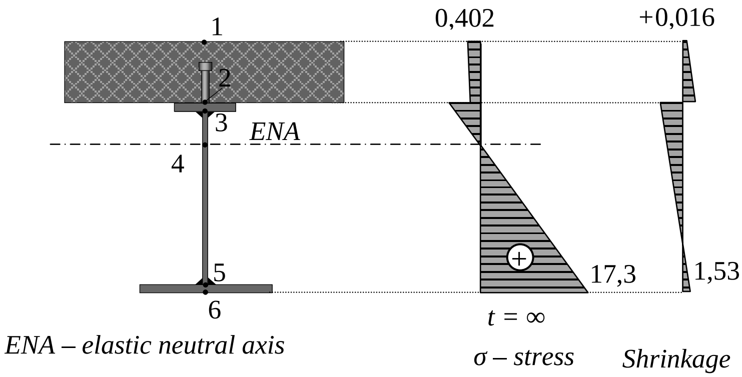


Figure B4.7 *Points of the composite section and the distribution of normal stresses at time t = ∞ (tensile stress positive)*

Point 1 – stress in concrete flange at level 1

$$M_{Ed} = 601 \ \text{kNm}$$

$V_{Ed} = 0$ (corresponding design value of shear force)

$$\sigma_c = E_P \frac{M_{Ed}}{EI_P} \cdot z_c$$

$$z_c = 0 - e_{T,P} = 0 - 25{,}1 = -25{,}1 \text{ cm}$$

$$E_P = 680 \text{ kN/cm}^2$$

$$EI_P = 255311 \cdot 10^4 \text{ kNcm}^2$$

$$\sigma_c = 680 \frac{60100}{255311 \cdot 10^4} \cdot (-25{,}1) = -0{,}402 \text{ kN/cm}^2$$

Point 6 - stress in the steel section at level 6

$$M_{Ed} = 601 \text{ kNm}$$

$$V_{Ed} = 0 \text{ (corresponding design value of shear force)}$$

$$\sigma_a = E_a \frac{M_{Ed}}{EI_P} \cdot z_a$$

$$z_a = 60{,}0 - e_{T,P} = 60{,}0 - 25{,}1 = 34{,}9 \text{ cm}$$

$$E_a = 21000 \text{ kN/cm}^2$$

$$EI_P = 255311 \cdot 10^4 \text{ kNcm}^2$$

$$\sigma_a = 21000 \cdot \frac{60100}{255311 \cdot 10^4} \cdot 34{,}9 = 17{,}3 \text{ kN/cm}^2$$

c) Check of the bending moment, or shrinkage with the corresponding flexural stiffness EI_S and the axial stiffness EA_S.

Point 1 – stress in the concrete flange due to shrinkage, which does not depend on the load

The width of the concrete flange b = 3,0 m, beam spacing, is taken into account, so that the area of the concrete is $A_c = b \cdot h_c$.

The compressive force in the concrete flange N_{cs} is:

$$N_{cs} = \varepsilon_{cs}(\infty) \cdot E_S \cdot A_c = 4{,}26 \cdot 10^{-4} \cdot 671 \cdot 300 \cdot 16 = 1372 \text{ kN}$$

$$N = -N_{cs} = -1372 \text{ kN}$$

$$a_c = a - a_a = 34{,}3 - 17{,}1 = 17{,}2 \text{ cm}$$

The bending moment due to the compressive force N_{cs} is:

$$M = M_{cs} = N_{cs} \cdot a_c = 1372 \cdot \frac{17{,}2}{100} = 236 \text{ kNm}$$

$$\sigma_c = E_S \frac{N_{cs}}{EA_s} + E_S \frac{M_{cs}}{EI_S} \cdot z_c + \frac{N_{cs}}{A_c}$$

$$z_c = 0{,}0 - e_{T,S} = 0{,}0 - 25{,}2 = -25{,}2 \text{ cm}$$

$$\sigma_c = -671 \frac{1372}{5397200} + 671 \frac{23600}{254171 \cdot 10^4} \cdot (-25{,}2) + \frac{1372}{4000} = +0{,}016 \text{ kN/cm}^2$$

Point 6 – stress in the steel section due to shrinkage, which does not depend on the load

The width of the concrete flange b = 3,0 m, beam spacing, is taken into account, so that the area of the concrete is $A_c = b \cdot h_c$.

The compressive force in the concrete flange N_{cs} is:

$$N_{cs} = \varepsilon_{cs}(\infty) \cdot E_S \cdot A_c = 4{,}26 \cdot 10^{-4} \cdot 671 \cdot 300 \cdot 16 = 1372 \text{ kN}$$

$$N = -N_{cs} = -1372 \text{ kN}$$

$$a_c = a - a_a = 34{,}3 - 17{,}1 = 17{,}2 \text{ cm}$$

$$\sigma_a = E_a \frac{N_{cs}}{EA_S} + E_a \frac{M_{cs}}{EI_S} \cdot z_a$$

$$z_a = 60{,}0 - e_{T,S} = 60{,}0 - 25{,}2 = 34{,}8 \text{ cm}$$

$$\sigma_a = -21000 \cdot \frac{1372}{5397200} + 21000 \cdot \frac{23900}{254171 \cdot 10^4} \cdot 34,8 = 1,53 \ \text{kN/cm}^2$$

The maximum stress in the steel section is obtained as the sum of the stress at time $t = \infty$ and the stress due to shrinkage:

$$\sigma_a = 17,3 + 1,53 = 18,83 \ \text{kN/cm}^2$$

Check for steel section:

$$\frac{\sigma_a}{f_{yd}} = \frac{18,83}{23,5} = 0,80 \leq 1,0,$$ the condition is satisfied

The maximum stress in the concrete is obtained as the sum of the stress at time $t = \infty$ and the stress due to shrinkage:

$$\sigma_c = -0,402 + 0,016 = -0,386 \ \text{kN/cm}^2$$

Check for concrete:

$$\frac{\sigma_c}{f_{cd}} = \frac{0,386}{1,33} = 0,29 \leq 1,0,$$ the condition is satisfied

4.4 Vertical shear resistance of the composite cross-section

Bending stresses near a support are within the elastic range in the case of simply supported steel beams. They are within the elastic range even when the design ultimate load is applied (for example, $1,35 \cdot G_k + 1,5 \cdot Q_k$). However, in a composite beam, maximum slip occurs at the end supports. Due to this behaviour, estimation of the bending stresses by simple elastic theory, based on plain sections remaining plane, may be unreliable.

From rates of change ($d\sigma/dx$) of bending stresses σ, the vertical shear stresses can be calculated. However, the estimation of this stresses near an end of the composite beam cannot easily be found. In tests conducted on composite beams, it has been shown that some of the vertical shear is resisted by the concrete slab. However, for this kind of behaviour, there is no simple design model. It has been found that the contribution of the concrete slab is influenced by whether the slab is continuous across the end support, by how much the slab is cracked and by local details of the shear connection.

Therefore, in practice the resistance to vertical shear can be taken as the resistance of the structural steel section unless the contribution from the concrete slab has been established.

According to clause 6.2.2, EN 1994-1-1, this problem is not treated in detail.

The distribution of shear stress depends on the time *t*. For this reason, the check for shear force is carried out for:

a) The age of the concrete on the first loading with the corresponding flexural stiffness EI_0

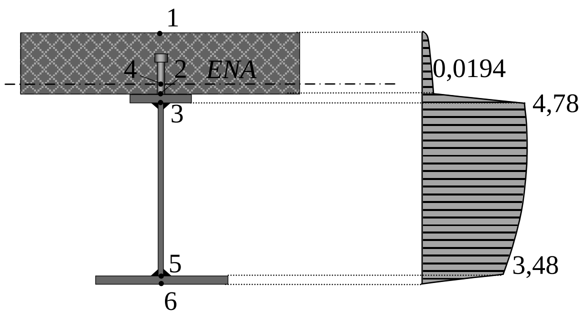
- The centroid of the composite section (the elastic neutral axis) lies within the concrete slab
- The first moment of area is $S_i = A_i \cdot e_i$
- The distance between the centroid of the composite section and the top surface of the concrete slab is $e_{T,0} = 14{,}2$ cm

b) At time $t = \infty$:

- The centroid of the composite section (the elastic neutral axis) lies within the steel section
- The first moment of area is $S_i = A_i \cdot e_i$
- The distance between the centroid of composite section and the top surface of concrete slab is $e_{T,P} = 25{,}1$ cm

ad a) The age of the concrete on first loading, the centroid of the composite section lies within the concrete slab ($e_{T,0}$ = 14,2 cm)

Stresses due to vertical force V_{Ed} are calculated at the following points of the composite section 1, 2, 3, 4, 5 and 6, see Figure B4.8.



ENA – elastic neutral axis *τ – stress*

Figure B4.8 *Points of the composite section and the distribution of shear stresses*

Point 2 – shear stress in the concrete flange

$$V_{Ed} = 241 \text{ kN}$$

$$\Sigma E_i \cdot S_i = E_0 \cdot (e_{T,0} \cdot b_{eff}) \cdot \frac{e_{T,0}}{2} = 3000 \cdot (14,2 \cdot 250) \cdot 14,2 / 2$$

$$\Sigma E_i \cdot S_i = 7562 \cdot 10^4 \text{ kNcm}$$

$$\tau_c = V_{Ed} \cdot \frac{\Sigma E_i \cdot S_i}{EI_0 \cdot b_{eff}} = 241 \cdot \frac{7562 \cdot 10^4}{375860 \cdot 10^4 \cdot 250} = 0,0194 \text{ kN/cm}^2$$

Point 3 – shear stress in the steel section

$$V_{Ed} = 241 \text{ kN}$$

$$\Sigma E_i \cdot S_i = E_a (40,8 \cdot 1,0) \cdot (20,4 + 1,4 + 1,8) + E_a (32,0 \cdot 1,8) \cdot (17,7 + 28,1 - 0,9)$$

$$\Sigma E_i \cdot S_i = 7453 \cdot 10^4 \text{ kNcm}$$

$$\tau_a = V_{Ed} \cdot \frac{\Sigma E_i \cdot S_i}{EI_0 \cdot t_w} = 241 \cdot \frac{7453 \cdot 10^4}{375860 \cdot 10^4 \cdot 1,0} = 4,78 \text{ kN/cm}^2$$

Point 5 – shear stress in the steel section

$$V_{Ed} = 241 \text{ kN}$$

$$\Sigma E_i \cdot S_i = E_a \cdot (32,0 \cdot 1,8) \cdot (17,7 + 28,1 - 0,9) = 5431 \cdot 10^4 \text{ kNcm}$$

$$\tau_a = V_{Ed} \cdot \frac{\Sigma E_i \cdot S_i}{EI_0 \cdot t_w} = 241 \cdot \frac{5431 \cdot 10^4}{375860 \cdot 10^4 \cdot 1,0} = 3,48 \text{ kN/cm}^2$$

The basic shear strength of the concrete according to [45] (there is no data about the basic shear strength of the concrete in EN 1992-1-1) is:

$$\tau_{c,Rd} = 0,389 \cdot f_{ctm} \leq 0,15 \text{ kN/cm}^2$$

This basic shear strength of the concrete is based on the partial factor for unreinforced concrete γ_c = 1,8. National Annexes could be recommend a different value of partial factor.

For concrete strength class C 20/25: $f_{ctm} = 0{,}22$ kN/cm^2

$$\tau_{c,Rd} = 0{,}389 \cdot f_{ctm} = 0{,}389 \cdot 0{,}22 = 0{,}0856 \text{ kN/cm}^2$$

Check for the concrete:

$$\frac{\tau_c}{\tau_{c,Rd}} = \frac{0{,}0194}{0{,}0856} = 0{,}23 \leq 1{,}0\text{, the condition is satisfied}$$

The basic shear strength of the structural steel is:

$$\tau_{a,Rd} = \frac{f_y}{\sqrt{3}}$$

For structural steel S 235: $f_y = 23{,}5$ kN/cm^2

$$\tau_{a,Rd} = \frac{f_y}{\sqrt{3}} = \frac{23{,}5}{\sqrt{3}} = 13{,}6 \text{ kN/cm}^2$$

Check for the steel section:

$$\frac{\tau_a}{\tau_{a,Rd}} = \frac{4{,}78}{13{,}6} = 0{,}35 \leq 1{,}0\text{, the condition is satisfied}$$

Generally we can assume that only the web of the steel section resists the shear force. In this case the maximum shear stress is at the centroid of the steel cross-section:

$$V_{Ed} = 241 \text{ kN}$$

$$\Sigma E_i \cdot S_i = E_a(32{,}0 \cdot 1{,}8)(17{,}7 - 0{,}9) + E_a \cdot \frac{1{,}0 \cdot (17{,}7 - 1{,}8)^2}{2}$$

$$\Sigma E_i \cdot S_i = 2298 \cdot 10^4 \text{ kNcm}$$

$$\tau_a = V_{Ed} \cdot \frac{\Sigma E_i \cdot S_i}{EI_a \cdot t_w} = 241 \cdot \frac{2298 \cdot 10^4}{21000 \cdot 42717 \cdot 1{,}0} = 6{,}17 \text{ kN/cm}^2$$

Check:

$\dfrac{\tau_a}{\tau_{a,Rd}} = \dfrac{6{,}17}{13{,}6} = 0{,}45 \leq 1{,}0$, the condition is satisfied

Based on the calculated shear stresses it can be concluded that the concrete flange participates at approximately 22% in the transmission of the transverse force.

b) At time $t = \infty$, the centroid of the composite section lies within the web of the steel section ($e_{T,P}$ = 25,1 cm)

Stresses due to vertical force V_{Ed} are calculated at the following points of the composite section 1, 2, 3, 4, 5 and 6, see Figure B4.9.

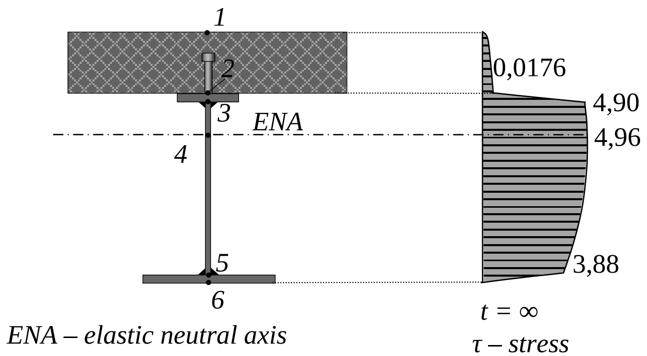


Figure B4.9 *Points of the composite section and the distribution of shear stresses at time t = ∞*

Point 2 - shear stress in the concrete flange

$$V_{Ed} = 241 \text{ kN}$$

$$\Sigma E_i \cdot S_i = E_P \cdot A_c \cdot (25{,}1 - 8{,}0) = 680 \cdot 4000 \cdot (25{,}1 - 8{,}0) = 4651 \cdot 10^4 \text{ kNcm}$$

$$\tau_c = V_{Ed} \cdot \frac{\Sigma E_i \cdot S_i}{EI_P \cdot b} = 241 \cdot \frac{4651 \cdot 10^4}{255311 \cdot 10^4 \cdot 250} = 0{,}0176 \text{ kN/cm}^2$$

Point 3 - shear stress in the steel section

$$V_{Ed} = 241 \text{ kN}$$

$$\Sigma E_i \cdot S_i = 4651 \cdot 10^4 + 21000 \cdot 30{,}8 \cdot 8{,}4 = 5194 \cdot 10^4 \text{ kNcm}$$

$$\tau_a = V \cdot \frac{\Sigma E_i \cdot S_i}{EI_P \cdot t_w} = 241 \cdot \frac{5194 \cdot 10^4}{255311 \cdot 10^4 \cdot 1{,}0} = 4{,}90 \text{ kN/cm}^2$$

Point 4 - shear stress in the steel section

$$V_{Ed} = 241 \text{ kN}$$

$$\Sigma E_i \cdot S_i = 5194 \cdot 10^4 + 21000 \cdot 1{,}0 \cdot \frac{7{,}7^2}{2} = 5256 \cdot 10^4 \text{ kNcm}$$

$$\tau_a = V_{Ed} \cdot \frac{\Sigma E_i \cdot S_i}{EI_P \cdot b} = 241 \cdot \frac{5256 \cdot 10^4}{255311 \cdot 10^4 \cdot 1{,}0} = 4{,}96 \text{ kN/cm}^2$$

Point 5 - shear stress in the steel section

$$V_{Ed} = 241 \text{ kN}$$

$$\Sigma E_i \cdot S_i = 21000 \cdot 57{,}6 \cdot 34{,}0 = 4113 \cdot 10^4 \text{ kNcm}$$

$$\tau_a = V_{Ed} \cdot \frac{\Sigma E_i \cdot S_i}{EI_0 \cdot t_w} = 241 \cdot \frac{4113 \cdot 10^4}{255311 \cdot 10^4 \cdot 1{,}0} = 3{,}88 \text{ kN/cm}^2$$

The basic shear strength of the concrete according to [45] (there is no data about the basic shear strength of the concrete in EN 1992-1-1) is:

$$\tau_{c,Rd} = 0{,}389 \cdot f_{ctm} \le 0{,}15 \text{ kN/cm}^2$$

This basic shear strength of the concrete is based on the partial factor for unreinforced concrete γ_c = 1,8. National Annexes may recommended a different value of partial factor.

For concrete strength class C 20/25: $f_{ctm} = 0{,}22 \text{ kN/cm}^2$

$$\tau_{c,Rd} = 0{,}389 \cdot f_{ctm} = 0{,}389 \cdot 0{,}22 = 0{,}0856 \text{ kN/cm}^2$$

Check for the concrete:

$$\frac{\tau_c}{\tau_{c,Rd}} = \frac{0,0176}{0,0856} = 0,20 \leq 1,0, \text{ the condition is satisfied}$$

The basic shear strength of the structural steel is:

$$\tau_{a,Rd} = \frac{f_y}{\sqrt{3}}$$

For structural steel S 235: $f_y = 23,5\ \text{kN/cm}^2$

$$\tau_{a,Rd} = \frac{f_y}{\sqrt{3}} = \frac{23,5}{\sqrt{3}} = 13,6\ \text{kN/cm}^2$$

Check for the steel section:

$$\frac{\tau_a}{\tau_{a,Rd}} = \frac{4,96}{13,6} = 0,36 \leq 1,0, \text{ the condition is satisfied}$$

The maximum shear stress in the steel section max τ_a occurs at time $t = \infty$.

Verification of the shear buckling resistance for the web:

$$\frac{h_w}{t_w} > 72 \cdot \frac{\varepsilon}{\eta} \qquad \eta = 1,2$$

$$\frac{h_w}{t_w} = \frac{40,8}{1,0} = 40,8 < 72 \cdot \varepsilon = 72 \cdot \frac{1,0}{1,2} = 60$$

The verification of the shear buckling resistance for the web is not necessary.

4.5 Calculation of shear connection

Based on the design vertical shear force V_{Ed}, we can calculate the longitudinal shear force per unit length of the beam or the longitudinal shear flow, v_b. Longitudinal shear flow v_b also depends on time t. In this case the age of the concrete on the first loading is taken into consideration.

The design longitudinal shear force per unit length, in the observed cross-sections, which is governed for the longitudinal shear failure, is denoted as $v_{L,Ed}$. The design longitudinal shear flow is:

$$v_b = v_{L,Ed} = V_{Ed} \cdot \frac{\Sigma E_i \cdot S_i}{EI_0}$$

$$V_{Ed} = 241 \text{ kN}$$

$$E_i \cdot S_i = E_0 \cdot (e_{T,0} \cdot b_{eff}) \frac{e_{T,0}}{2} = 3000 \cdot (14,2 \cdot 250) \frac{14,2}{2} = 7562 \cdot 10^4 \text{ kNcm}$$

$$EI_0 = 375903 \cdot 10^4 \text{ kNcm}^2$$

$$v_{L,Ed} = 241 \cdot \frac{7562 \cdot 10^4}{375903 \cdot 10^4} = 4,85 \text{ kN/cm}$$

The shear connectors are spaced in accordance with the shear flow with a triangular distribution. The span of the beam is divided into several ranges where the studs are distributed at equal intervals. The total number of studs, *n*, is shared between lengths of the ranges in proportion to the areas of the design shear force diagram of the considered ranges. In this case, the cross-section class 3, the peak shear flow within each length does not exceed the design longitudinal shear resistance per unit length.

The following spacings of studs are selected:

Range 1

$$e_L = 170 \text{ mm} \qquad v_{Rd} = \frac{P_{Rd}}{e_L} = \frac{87}{17,0} = 5,11 \text{ kN/cm} \quad \text{with 12 studs}$$

Range 2

$$e_L = 250 \text{ mm} \qquad v_{Rd} = \frac{P_{Rd}}{e_L} = \frac{87}{25,0} = 3,48 \text{ kN/cm} \quad \text{with 10 studs}$$

The design longitudinal shear force per unit length $v_{L,Ed}$ and the design longitudinal shear resistance per unit length $v_{L,Rd}$ are shown in Figure B4.10.

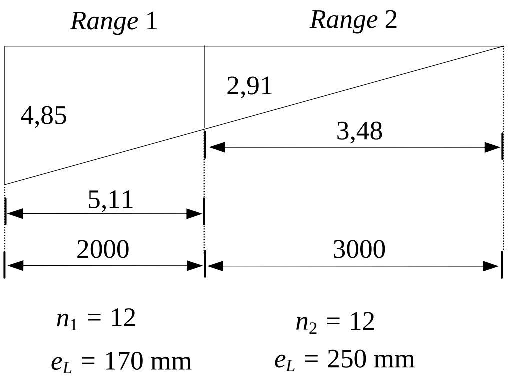


Figure B4.10 *Design longitudinal shear force per unit length $v_{L,Ed}$ and design longitudinal shear resistance per unit length $v_{L,Rd}$*

The total design longitudinal shear force on half span of the beam is:

$$V_{L,Ed} = \frac{1}{2} \cdot v_{L,Ed} \cdot \frac{L}{2} = \frac{1}{2} \cdot 4,85 \cdot \frac{1000}{2} = 1213 \text{ kN}$$

The total design longitudinal shear resistance on half span of the beam is:

$$V_{Rd} = n \cdot P_{Rd} = 24 \cdot 87 = 2088 \text{ kN}$$

Check:

$V_{L,Ed} \leq V_{Rd}$

$1213 \leq 2088$, the condition is satisfied

The verification of the selected spacings of studs:

$e_L = 170$ mm and $e_L = 250$ mm

$e_L > 5 \cdot d = 5 \cdot 22 = 110$ mm

$e_L \leq 800$ mm

$e_L \leq 6 \cdot h_c = 6 \cdot 160 = 960$ mm

4.6 Check of longitudinal shear resistance of the concrete flange

4.6.1 Check of transverse reinforcement

The cross-sectional area of transverse reinforcement is calculated according to the expression:

$$\frac{A_{sf}}{s_f} \cdot f_{sd} \geq v_{L,Ed} \cdot \frac{h_f}{\cot \theta}$$

where:

A_{sf}/s_f is the transverse reinforcement expressed in mm^2/m,
h_f is the thickness of concrete flange, see Figure B4.11,
θ is the angle between the diagonal strut and the axis of the beam (strut-and-tie model),
$v_{L,Ed}$ is the design longitudinal shear flow in the concrete slab.

The transverse reinforcement (A_{sf}/s_f) expressed in mm^2/m can be denoted as A_t for the top transverse reinforcement and A_b for the bottom transverse reinforcement.
It is necessary to verify the failure due to shear in the failure plane shown in Figure B4.11 as section *a-a* and section *c-c*.

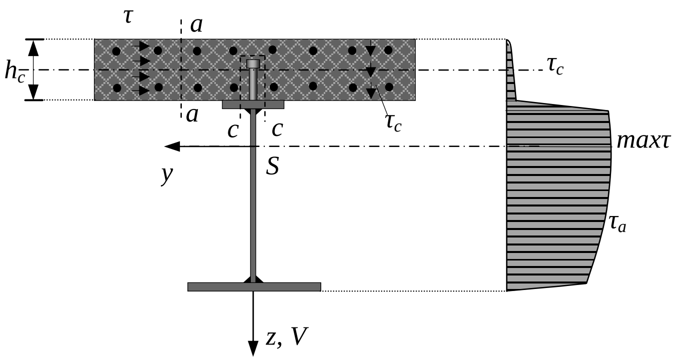


Figure B4.11 *Surfaces of potential failure in longitudinal shear*

a) Check for shear stress τ in section *a-a* of slab

The transverse reinforcement is selected for the transverse bending of the slab.

Adopted:

10 mm bars at 200 mm

$$d_s = 10 \text{ mm} \quad e_L = 200 \text{ mm} \quad A_t = A_b = 393 \text{ mm}^2 \quad A_s = A_t + A_b = 786 \text{ mm}^2$$

From the design shear force V_{Ed} at section *a-a* of slab, it is possible to calculate the design longitudinal shear flow v_a:

$$v_{L,Ed} = v_a = V_{Ed} \cdot \frac{\Sigma E_i \cdot S_i}{EI_0} \cdot \frac{b_i}{b_{eff}} = 241 \cdot \frac{7562 \cdot 10^4}{375860 \cdot 10^4} \cdot \frac{125}{250} = 2,42 \text{ kN/cm}$$

$$v_{L,Ed} = 242 \text{ N/mm}$$

The longitudinal shear stress is calculated if the longitudinal shear flow is divided by the thickness of concrete flange h_f:

$$v_{L,Ed} = \frac{V_{Ed}}{h_f} \cdot \frac{\Sigma E_i \cdot S_i}{EI_0} \cdot \frac{b_i}{b_{eff}} = \frac{241}{16} \cdot \frac{7562 \cdot 10^4}{375860 \cdot 10^4} \cdot \frac{125}{250} = 0,15 \text{ kN/cm}^2$$

$$v_{L,Ed} = 1,5 \text{ N/mm}^2$$

Remark:

In order to prevent splitting of the concrete flange, for the adopted “truss model”, according to clause 6.2.4(4) EN 1992-1-1, the angle θ between the concrete diagonals and the longitudinal direction is limited to the value:

$26,5° \le \theta \le 45°$ concrete flange in compression

$38,6° \le \theta \le 45°$ concrete flange in tension

In order to minimize the cross-sectional area of the transverse reinforcement, the minimum angle θ is selected. For the concrete flange in compression, the minimum angle θ is:

$$\theta = 26,5°$$

$$\frac{A_{sf}}{s_f} \cdot f_{sd} \ge v_{L,Ed} \cdot \frac{h_f}{\cot\theta}$$

$$\frac{A_{sf}}{s_f} \ge \frac{v_{L,Ed}}{f_{sd}} \cdot \frac{h_f}{\cot\theta} = \frac{1,5}{435} \frac{160}{\cot 26,5} \cdot 10^3 = 275 \text{ mm}^2\text{/m} < A_s = 786 \text{ mm}^2\text{/m}$$

The required reinforcement has to be distributed so that one half is placed in the upper zone, the top reinforcement, and the other half in the lower zone, the bottom reinforcement.

b) Check for shear stress τ for shear surface *c-c* in the slab

The local transmission of force through the shear connector in the concrete slab is carried out through the „surface surrounding the shear connector“. This is the shear surface *c-c* passing round the studs as shown in Figure B4.11. By means of appropriate transverse reinforcement, local failure of the concrete flange can be avoided.

From the design shear force V_{Ed}, it is possible to calculate the design longitudinal shear flow v_b. Longitudinal shear flow v_b also depends on time *t*. In this case the age of the concrete on first loading is taken into consideration.

The design longitudinal shear flow is:

$$v_{L,Ed} = v_b = V_{Ed} \cdot \frac{\Sigma E_i \cdot S_i}{EI_0} = 241 \cdot \frac{7562 \cdot 10^4}{375860 \cdot 10^4} = 4{,}84 \text{ kN/cm (range 1, Figure B4.10)}$$

$$v_{L,Ed} = 48{,}4 \text{ N/mm}$$

In this case the length of the shear surface *b-b* is:

$$h_f = 2 \cdot h_{sc} + 1{,}5 \cdot d = 2 \cdot 12{,}5 + 1{,}5 \cdot 2{,}2 = 28{,}3 \text{ cm}$$

The longitudinal shear stress is calculated if the longitudinal shear flow is divided by the thickness of the concrete flange h_f:

$$v_{L,Ed} = \frac{V_{Ed}}{h_f} \cdot \frac{\Sigma E_i \cdot S_i}{EI_0} = \frac{241}{28{,}3} \cdot \frac{7562 \cdot 10^4}{375860 \cdot 10^4} = 0{,}17 \text{ kN/cm}^2$$

$$v_{L,Ed} = 1{,}7 \text{ N/mm}^2$$

In order to minimize the cross-sectional area of the transverse reinforcement, the minimum angle θ is selected. For the concrete flange in compression, the minimum angle θ is:

$$\theta = 26{,}5°$$

$$\frac{A_{sf}}{s_f} \cdot f_{sd} \geq v_{L,Ed} \cdot \frac{h_f}{\cot\theta}$$

$$\frac{A_{sf}}{s_f} \geq \frac{v_{L,Ed}}{f_{sd}} \cdot \frac{h_f}{\cot\theta} = \frac{1,7}{435}\frac{283}{\cot 26,5} \cdot 10^3 = 551 \ \text{mm}^2/\text{m} < 2A_b = 2 \cdot 393 = 786 \ \text{mm}^2/\text{m}$$

According to EN 1994-1-1, clause 6.6.6.3, the minimum area of transverse reinforcement is determined in accordance with EN 1992-1-1, clause 9.2.2(5), which gives the minimum area of reinforcement as a proportion of the concrete area. The ratio is:

$$\rho_{w,\min} = \frac{0,08\sqrt{f_{ck}}}{f_{yr,k}}$$

where:

f_{ck} is the characteristic compressive cylinder strength of the concrete at 28 days in N/mm^2,

$f_{yr,k} = f_{sk}$ is the characteristic yield strength of the reinforcement in N/mm^2.

The minimum area of transverse reinforcement is:

$$\rho_{w,\min} = \frac{0,08\sqrt{f_{ck}}}{f_{yr,k}} = \frac{0,08\sqrt{20}}{500} = 0,0007$$

$$A_c = h_c \cdot b = 160,0 \cdot 1000,0 = 160000 \ \text{mm}^2$$

$$A_{s,\min} = \rho_{w,\min} \cdot A_c = 0,0007 \cdot 160000 = 112 \ \text{mm}^2/\text{m}$$

Since $A_t + A_b = 786 \ \text{mm}^2/\text{m} > A_{s,\min} = 112 \ \text{mm}^2/\text{m}$, the requirement of minimum transverse reinforcement is satisfied.

4.6.2 Crushing of the concrete flange

To prevent crushing of the compression struts in the flange, the following condition needs to be satisfied according to EN 1992-1-1, expression 6.22:

$$v_{L,Ed} \leq V_{Rd}$$

$$v_{L,Ed} \leq v \cdot f_{cd} \cdot \sin\theta \cdot \cos\theta$$

where:

$$v = 0{,}6 \cdot (1 - \frac{f_{ck}}{250})$$

θ is the angle between the concrete diagonals and the longitudinal direction.

In order to minimize the resistance of the concrete compression strut, the minimum angle θ is selected. For the concrete flange in compression, the minimum angle θ is:

$$\theta = 26{,}5°$$

$$v_{Rd} = v \cdot f_{cd} \cdot \sin\theta \cdot \cos\theta = 0{,}6 \cdot (1 - \frac{20}{250}) \cdot 1{,}33 \cdot \sin 26{,}5° \cdot \cos 26{,}5° = 0{,}29 \ \text{kN/cm}^2$$

$$v_{Rd} = 2{,}9 \ \text{N/mm}^2$$

Check:

$$v_{L,Ed} = 1{,}7 \ \text{N/mm}^2 < v_{Rd} = 2{,}9 \ \text{N/mm}^2$$

Therefore the crushing resistance of the concrete compression strut is adequate.

c) Concentrated longitudinal shear at the end of the concrete slab

The primary effects of shrinkage cause the design concentrated longitudinal shear at the end of the concrete slab. The concentrated longitudinal shear force is equal to the longitudinal force in the steel section or in the concrete flange. However, it is necessary to take into consideration the direction (signed) of these forces. This force is in the concrete flange with a positive sign while in the steel section it has a negative sign. Thus, this force acts opposite to the longitudinal shear forces due to self-weight and variable load.

This longitudinal shear force is obtained when the normal stress in the centroid of the steel section is multiplied by the area of steel section. The design shear force $V_{L,Ed}$ transferred across the steel-concrete interface can be triangular distributed over the length equal to b_{eff}. Where the ductile stud shear connectors are used, the design longitudinal shear flow can be uniformly distributed over the length b_{eff}.

The normal stress is given as:

$$\sigma_a = E_a \cdot \frac{N}{EA_S} + E_a \cdot \frac{M}{EI_S} \cdot z_T$$

A calculation of primary shrinkage stress is given in Section 4.3.4 (c). The force N and the moment M are found from these, and the normal stress is:

$$\sigma_{a,0} = -21000 \cdot \frac{1372}{5397200} + 21000 \cdot \frac{23900}{254171 \cdot 10^4} \cdot 17{,}1 = -1{,}96 \ \text{kN/cm}^2$$

The shear force $V_{L,Ed}$ transferred across the steel-concrete interface is:

$$N_{a,S} = V_{L,Ed} = \sigma_{a,0} \cdot A_a = -1{,}96 \cdot 129{,}2 = -253 \ \text{kN}$$

The design longitudinal shear flow is:

$$v_{L,Ed} = \frac{2 \cdot V_{L,Ed}}{b_{eff}} = \frac{2 \cdot 253}{250} = 2{,}02 \ \text{kN/cm}$$

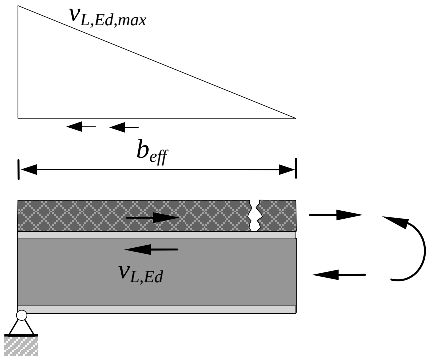


Figure B4.12 *Distribution of longitudinal shear force along the steel-concrete interface*

The design forces at the end of the concrete slab are reduced on the following value:

$$P_{d,s} = v_{L,Ed} \cdot e_L = 2{,}02 \cdot 17{,}0 = 34{,}3 \ \text{kN}$$

Further verification is not necessary.

5. Serviceability limit state

5.1 General

Chapter 7, EN 1994-1-1, is limited to provisions relating to serviceability that are specific to composite structures. Serviceability verifications in the case of the composite structures generally include checks of stress, deflection and vibration as well control of crack width.

For buildings, stress limitation is not required for beams if, in the ultimate limit state, no verification of fatigue is required and no pre-stressing by tendons and/or by controlled imposed deformations is provided. However, if the stress limitation is required, clause 7.2, EN 1992-1-1 gives stress limits which may be applicable for buildings that have pre-stressing or fatigue loading.

Since the deflection is one of the most important verifications of the serviceability limit state, it is necessary to explain in detail the problems associated with the deflection calculation. Deflections due to loads applied to the composite member are calculated using elastic theory, taking into account the following effects:

a) cracking of concrete,
b) creep and shrinkage of concrete,
c) sequence of construction,
d) influence of local yielding of structural steel at internal support (for continuous beams),
e) influence of incomplete interaction.

For a more detailed explanation of these effects, see example B6.

If the steel beam is fully propped, the total deflection of composite beams is obtained by summing the following deflections:

$$\delta = \delta_1 + \delta_{2,1} + \delta_{2,2} + \delta_{2,3}$$

where:

δ_1 is the deflection due to the permanent actions (the first loading is applied at the age of t_0 = 28 days),
$\delta_{2,1}$ is the deflection due to the frequent value of the variable action at time of first loading,
$\delta_{2,2}$ is the deflection due to creep under the quasi-permanent value of variable action at time $t = \infty$,

$\delta_{2,3}$ is the deflection due to shrinkage.

5.2 Calculation of deflections

5.2.1 Construction stage deflection

In this example, the steel beam is fully propped at the construction stage, and the deflection of the steel beam at the construction stage is:

$$\delta_0 = 0$$

5.2.2 Composite stage deflection

The expression for calculation of the maximum deflection of the beam due to uniformly distributed loads has the following general form:

$$\delta = \frac{5}{384} \cdot \frac{e_d \cdot L^4}{EI_L}$$

where:

e_d is the design value of load from the governed combination of action,
L is the span of the beam,
EI_L is the effective flexural stiffness of the composite sections which depends on the type of loadings; the different types of loading are distinguished by a subscript L,
E_L is the effective modulus of elasticity of the concrete which depends on the same type of loadings as for the flexural stiffness EI_L. The effective modulus of elasticity of concrete E_L is denoted by $E_{c,eff}$ in EN 1994-1-1.

Calculation of deflections

- Deflection due to permanent action, the first loading is applied at age t_0 = 28 days

$$e_d = b \cdot g_k = 3{,}0 \cdot 6{,}33 = 19{,}0 \text{ kN/m}$$

$$EI_L = EI_0 = 375903 \text{ kNm}^2$$

$$\delta_1 = \frac{5}{384} \cdot \frac{e_d \cdot L^4}{EI_0} = \frac{5}{384} \frac{19{,}0 \cdot 10^4}{375903} \cdot 100 = 0{,}66 \text{ cm}$$

- Deflection due to the frequent value of the variable action at the time of the first loading t_0 = 28 days

For a building with floors in category B, office areas, the combination factor is: ψ_1 = 0,5.

$$e_d = b \cdot \psi_2 \cdot q_k = 3{,}0 \cdot 0{,}5 \cdot 5{,}0 = 7{,}5 \text{ kN/m}$$

$$EI_L = EI_0 = 375903 \text{ kNm}^2$$

$$\delta_{2,1} = \frac{5}{384} \cdot \frac{e_d \cdot L^4}{EI_0} = \frac{5}{384} \frac{7{,}5 \cdot 10^4}{375903} \cdot 100 = 0{,}26 \text{ cm}$$

- Deflection due to creep under the quasi-permanent value of variable action at time $t = \infty$.

This deflection is the difference of deflections at time $t = \infty$ and at the time of the first loading t_0 = 28 days.

$$e_d = b \cdot (g_k + \psi_2 \cdot q_k) = 3{,}0 \cdot (6{,}33 + 0{,}5 \cdot 5{,}0) = 26{,}5 \text{ kN/m}$$

$EI_0 = 375903 \text{ kNm}^2$, for short-term loading

$EI_P = 255311 \text{ kNm}^2$, permanent action constant in time

$$\delta_{2,2} = \frac{5}{384} \cdot \frac{e_d \cdot L^4}{EI_P} - \frac{5}{384} \cdot \frac{e_d \cdot L^4}{EI_0}$$

$$\delta_{2,2} = \frac{5}{384} \cdot \frac{26{,}5 \cdot 10^4}{255311} \cdot 100 - \frac{5}{384} \cdot \frac{26{,}5 \cdot 10^4}{375903} \cdot 100 = 1{,}35 - 0{,}92 = 0{,}43 \text{ cm}$$

- Deflection due to shrinkage

$$\delta_{2,3} = \frac{1}{8} \cdot \frac{M_{cs} \cdot L^2}{EI_S} = \frac{1}{8} \cdot \frac{236 \cdot 10^2}{254171} \cdot 100 = 1{,}16 \text{ cm}$$

The effects of shear connection on the deflection of the beam can be neglected because there was full shear connection.

Deflection limits for composite beams are the same as for steel beams, and are determined by the National Annex.

Recommended limiting values for deflection of composite beams are:

$\delta_{tot} \le \dfrac{L}{250}$, the deflection due to the total load

$\delta_{tot} \le \dfrac{L}{360}$, the deflection due to the variable load

The deflection due to the permanent action is:

$$\delta_1 = 0{,}66 \text{ cm}$$

The deflection due to variable load, creep and shrinkage is:

$$\delta_2 = \Sigma\delta_{2,i} = 0{,}26 + 0{,}43 + 1{,}16 = 1{,}85 \text{ mm}$$

The total deflection due to the permanent and variable loads, creep and shrinkage is:

$$\delta_{tot} = \delta_1 + \delta_2 = 0{,}66 + 1{,}85 = 2{,}51 \text{ mm} \le \frac{L}{250} = \frac{1000}{250} = 4{,}0 \text{ mm}$$

The total deflection meets the criterion $L/250$.

The steel beam is fully propped at the construction stage and the deflection of the steel beam at the construction stage is $\delta_0 = 0$.

The deflection due to variable load, creep and shrinkage is:

$$\delta_{\text{var}} = \delta_2 = 1{,}85 \text{ mm} \le \frac{L}{360} = \frac{1000}{360} = 2{,}78 \text{ mm}$$

The deflection due to the variable load, creep and shrinkage meets the criterion $L/360$.

5.3 Pre-cambering of steel beam

For the estimation of pre-cambering of the steel beam, the deflections due to the permanent load, creep and shrinkage are taken into account:

$$\delta_p = \delta_1 + \delta_{2,2} + \delta_{2,3}$$

$$\delta_p = 0,66 + 0,43 + 1,16$$

$$\delta_p = 2,25 \ \text{cm}$$

5.4 Check of vibration of the beam

For the calculation of the natural frequency, the characteristic values of the permanent load for the composite stage $e_d = b \cdot g_k$ is taken into account, and the effective flexural stiffness of composite section for short-term loading EI_0.

This load is:

e_d = 19,0 kN/m

The deflection under this load is:

$$\delta = \frac{5}{384} \cdot \frac{e_d \cdot L^4}{EI_0} = \frac{5}{384} \cdot \frac{19,0 \cdot 10^4}{375903} \cdot 100 = 0,66 \ \text{cm} \ = 6,6 \ \text{mm}$$

The natural frequency of the beam is therefore:

$$f = \frac{18}{\sqrt{\delta}} = \frac{18}{\sqrt{6,6}} = 7,01 \ \text{Hz} \geq 4 \ \text{Hz} \quad \text{with} \quad \delta \text{ in mm}$$

The criterion is satisfied. However, the improved estimation of the natural frequency is illustrated in example B3.

5.5 Cracks

If the composite beam is simply supported with continuous concrete slab on the top of the steel beam, the control of crack width is not required. However, the longitudinal reinforcement provided within the effective width of the concrete slab should be not less than 0,4% of the area of concrete.

The area of concrete is:

$$A_c = h_c \cdot b = 16,0 \cdot 100 = 1600 \ \text{cm}^2$$

Criterion of minimum reinforcement:

$$A_s \geq 0{,}004 \cdot A_c = 0{,}004 \cdot 1600 \cdot 10^2 = 640 \text{ mm}^2/\text{m}$$

The reinforcement provided is 12 mm bars at 150 mm, for which:

$$A_{s,\min} = \frac{\pi \cdot 12^2}{4} \cdot \frac{1000}{150} = 754 \text{ mm}^2/\text{m} > 640 \text{ mm}^2/\text{m}$$

For the adopted reinforcement, the maximum allowable thickness of slab h_c is:

$$h_c = \frac{A_{s,\min}}{0{,}004 \cdot 1000} = \frac{754}{0{,}004 \cdot 1000} = 189 \text{ mm}$$

The maximum allowable thickness of slab h_c is higher than the existing thickness of 160 mm, so this criterion is met.

5.6 Stresses at the serviceability limit state

For buildings the stress limitation is not required for beams if in the ultimate limit state the verification of fatigue is not required and the pre-stressing by tendons and/or by controlled imposed deformations is not provided.

6. Commentary

In the case of the cross-section class 3, the resistance of cross-section should be calculated by elastic resistance using the effective width of the concrete flange. For cross-sections of class 4, the effective structural steel section, determined according to EN 1993-1-5, must be taken into account. The stress limitations for calculating the elastic resistance to bending using the effective cross-section are recommended in clause 6.2.1.5, EN 1994-1-1. In the verification procedure, the stresses due to design effects of actions (M_{Ed}, N_{Ed}, V_{Ed}) need to be limited, in accordance with clause 6.2.1.5, EN 1994-1-1, as follows:

$\sigma_c \leq f_{cd}$ (concrete in compression)

$\sigma_a \leq f_{yd}$ (structural steel in compression or tension)

$\sigma_s \leq f_{sd}$ (reinforcement in compression or tension)

The calculation should consider the method of execution (propped or unpropped), cracking of concrete and influences of creep and shrinkage.

Since EN 1994-1-1 does not give guidelines for resistance of composite cross-section relating to the vertical shear, the stresses due to the shear forces are calculated for the two cases:

a) The age of concrete on first loading with the corresponding flexural stiffness EI_0
b) At time $t = \infty$

B5 Calculation of simply supported composite beam according to the plastic resistance of the cross-section

1. Purpose of example

The example shows the design of a simply supported composite beam according to the elastic–plastic procedure. Action effects are calculated by elastic global analysis and the resistance to bending is based on a plastic model. For the elastic–plastic procedure, the cross-section should be at least in class 2. An important feature of this example is to explain the concept of the “partial shear connector”. The concepts of “full shear connection” and “partial shear connection” are applicable to composite beams for which plastic theory is used for calculating plastic resistance. Sufficient shear connection between the slab and the steel beam is needed to develop the full plastic resistance of the section. With composite beams, where sufficient connection exists, it is referred to as full shear connection. When the resistance moment of section increases with the number of shear connectors, the shear connection is partial. The shear connectors must be sufficiently ductile that the resistance moment with partial shear connection can be developed.

The concrete slab was made with prefabricated elements where the concreting has been done on the site. The steel beam is fully propped at the construction stage and the verification of lateral-torsional buckling is not necessary. The ductile connectors used are spaced uniformly and it is necessary to verify that is $M_{pl,Rd}/M_{pl,a,Rd} \leq 2{,}5$, clause 6.6.1.3(3), EN 1994-1-1.

2. Static system, cross-section and actions

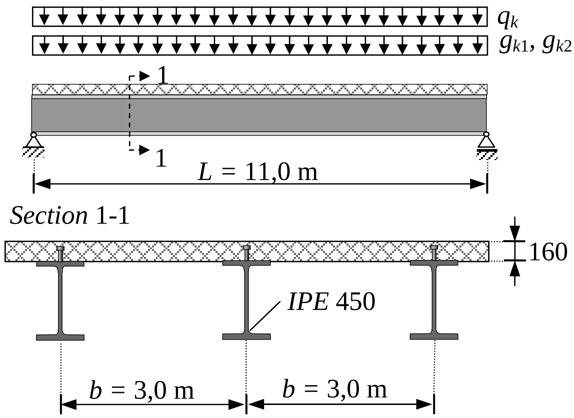


Figure B5.1 *Static system and the cross-section of the composite beam*

Composite Structures according to Eurocode 4. Worked Examples.
First Edition. Darko Dujmović, Boris Androić, Ivan Lukačević.

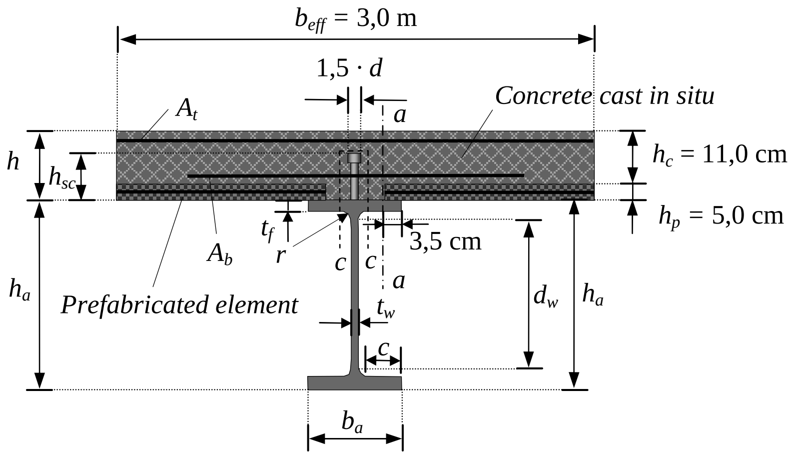


Figure B5.2 *Cross-section of composite beam*

Actions

Permanent action

- *concrete slab and reinforcement (dry concrete)* $25 \cdot 0{,}16 = 4{,}0 \ \text{kN/m}^2$

- *steel beam* $= 0{,}27 \ \text{kN/m}^2$

- *floor finishes* $= 2{,}5 \ \text{kN/m}^2$

Total $g_k = 6{,}77 \ \text{kN/m}^2$

Variable action

- *imposed floor load, category of use C3* $q_k = 5{,}00 \ \text{kN/m}^2$

3. Properties of materials

Concrete strength class: C 25/30 $f_{ck} = 25 \ \text{N/mm}^2$

$$f_{cd} = \frac{f_{ck}}{\gamma_c} = \frac{25}{1{,}5} = 16{,}7 \ \text{N/mm}^2$$

$$0{,}85 f_{cd} = 0{,}85 \cdot 16{,}7 = 14{,}17 \ \text{N/mm}^2$$

$$E_{cm} = 31000 \ \text{N/mm}^2$$

Reinforcement: *ductility class B or C* $f_{sk} = 500 \ \text{N/mm}^2$

$$f_{sd} = \frac{f_{sk}}{\gamma_s} = \frac{500}{1{,}15} = 435 \text{ N/mm}^2$$

$$E_s = 210\,000 \text{ N/mm}^2$$

Structural steel: S355

$$f_{yk} = 355 \text{ N/mm}^2$$

$$f_{yd} = \frac{f_{yk}}{\gamma_M} = \frac{355}{1{,}0} = 355 \text{ N/mm}^2$$

$$\tau_{a,Rd} = \frac{f_{yd}}{\sqrt{3}} = 204{,}9 \text{ kN/cm}^2$$

$$E_a = 210\,000 \text{ N/mm}^2$$

Shear connectors: *ductile headed studs*

$$f_u = 450 \text{ N/mm}^2$$

$$d = 22 \text{ mm}$$

$$h_{sc} = 125 \text{ mm}$$

$$\frac{h_{sc}}{d} = \frac{125}{22} = 5{,}68 > 4{,}0 \rightarrow \alpha = 1{,}0$$

$$P_{Rd} = 98{,}9 \text{ kN}$$

4. Ultimate limit state

4.1 Design values of combined actions and of the effects of actions

The design load for the ultimate limit state is:

$$e_d = b \cdot (\gamma_G \cdot g_k + \gamma_Q \cdot q_k)$$

$b = 3{,}0$ m beam spacing

$$e_d = 3{,}0 \cdot (1{,}35 \cdot 6{,}77 + 1{,}5 \cdot 5{,}00) = 3{,}0 \cdot (9{,}14 + 7{,}5) = 49{,}9 \text{ kN/m}$$

The maximum design bending moment is:

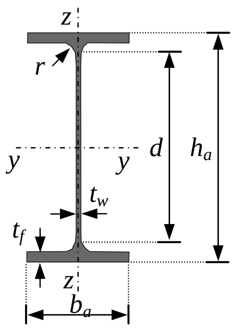
$$M_{Ed} = \frac{e_d \cdot L^2}{8} = \frac{49{,}9 \cdot 11{,}0^2}{8} = 755 \text{ kNm}$$

The maximum design shear force is:

$$V_{Ed} = e_d \cdot \frac{L}{2} = 49{,}9 \cdot \frac{11{,}0}{2} = 274 \text{ kN}$$

4.2 Selection of cross-section

Section IPE 450, grade 355, is selected with the cross-section and the dimensions shown in Figure B5.3.



$h_a = 450$ mm
$b_a = 190$ mm
$d = 379$ mm
$t_f = 14{,}6$ mm
$t_w = 9{,}4$ mm
$r = 21$ mm
$A_a = 98{,}8$ cm^2
$I_a = 33740$ cm^4
$W_{pl,y} = 1702$ cm^3

Figure B5.3 *Cross-section and dimensions of IPE* 450

4.3 Effective width of concrete flange

The effective width of the concrete flange is:

$$b_{eff} = b_0 + \Sigma b_{ei}$$

$$b_{ei} = \frac{L_e}{8} = \frac{L}{8} = \frac{11}{8} = 1{,}375 \text{ m}$$

$b_0 = 0$ (there is only one row of shear connectors)

$b_{eff} = 0 + (2 \cdot 1{,}375) = 2{,}75$ m $< 3{,}0$ m (beam spacing)

The effective cross-section of the concrete slab is shown in Figure B5.4.

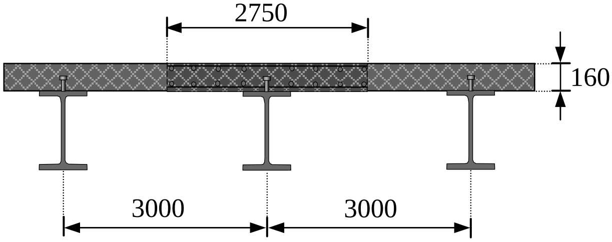


Figure B5.4 *Effective cross-section of the concrete slab*

4.4 Classification of the steel cross-section

For t_f = 14,6 mm the yield strength is f_y = 355 N/mm^2.

$$\varepsilon = \sqrt{\frac{235}{f_y}} = \sqrt{\frac{235}{355}} = 0,81$$

For the execution stage, the neutral axis is located in the half depth of the web of the steel section.

The classification of the steel cross-section is conducted according to Table 5.2, EN 1993-1-1.

Flange:

The outstand of the compression flange is:

$$c = \frac{b_a - t_w - 2 \cdot r}{2} = \frac{190 - 9,4 - 2 \cdot 21}{2} = 69,3$$

$$\frac{c}{t_f} = \frac{69,3}{14,6} = 4,75$$

The limiting value for class 1 is:

$$\frac{c}{t_f} \leq 9 \cdot \varepsilon = 9 \cdot 0,81 = 7,29$$

$4,75 < 7,29$ $\rightarrow$ therefore the flange in compression is class 1

Web:

Web subject to bending

$$c = h_a - 2 \cdot t_f - 2 \cdot r = 450 - 2 \cdot 14,6 - 2 \cdot 21 = 378,8 \text{ mm}$$

$$\frac{c}{t_w} = \frac{378,8}{9,4} = 40,3$$

The limiting value for class 1 is:

$$\frac{c}{t_w} \leq 72 \cdot \varepsilon = 72 \cdot 0{,}81 = 58{,}3$$

$40{,}3 < 58{,}3 \rightarrow$ therefore the web in bending is class 1.

Therefore the cross-section in bending at the construction stage is class 1. At the composite stage, the cross-section will also be class 1.

4.5 Check of shear connection

For full shear connection, the lesser value of N_{cf} and $N_{pl,a}$ is governed:

$$V_{L,Ed} = \min(N_{cf}; N_{pl,a})$$

The design value of the compressive normal force in the concrete flange with full shear connection is:

$$N_{cf} = b_{eff} \cdot h_c \cdot 0{,}85 \cdot f_{cd} = 275 \cdot 11 \cdot 0{,}85 \cdot 1{,}67 = 4294 \text{ kN}$$

The design value of the plastic resistance of the structural steel section to normal force is:

$$N_{pl,a} = A_a \cdot f_{yd} = 98{,}8 \cdot 35{,}5 = 3507 \text{ kN}$$

Thus, the design longitudinal force in the steel-concrete interface is equal to $N_{pl,a}$:

$$V_{L,Ed} = N_{pl,a} = 3507 \text{ kN}$$

The required number of studs for full shear connection is:

$$n_f = \frac{V_{L,Ed}}{P_{Rd}} = \frac{N_{pl,a}}{P_{Rd}} = \frac{3507}{98{,}9} = 35{,}5$$

The required minimum degree of shear connection η depending on the span of beam and uniform spacing of studs is:

$$L < 25 \text{ m}$$

$$\eta = 1 - \left(\frac{355}{f_{yk}}\right) \cdot (0{,}75 - 0{,}03 \cdot L_e) \qquad \eta \geq 0{,}4$$

$$\eta = 1 - (0,75 - 0,03 \cdot 11) = 0,58$$

$$0,58 > 0,4$$

With respect to the minimum degree of shear connection, the required number of studs is:

$$n = \eta \cdot n_f = 0,58 \cdot 35,5 = 20,6$$

The adopted number of studs is $n = 30$. For this number of studs, the degree of shear connection is:

$$\eta = \frac{30}{35,5} = 0,85 \geq 0,58$$

The design longitudinal force in the steel-concrete interface is:

$$V_{L,Ed} = n \cdot P_{Rd} = 30 \cdot 98,9 = 2967 \text{ kN}$$

The following spacing of studs in the longitudinal direction is selected:

$$e_L = \frac{L}{2n} = \frac{11000}{2 \cdot 30} = 183 \text{ mm}$$

Verification of the criteria for the spacing of studs:

$$e_L = 183 \text{ mm} > 5 \cdot d = 5 \cdot 22 = 110 \text{ mm}$$

$$e_L = 183 \text{ mm} \leq 800 \text{ mm}$$

$$e_L = 183 \text{ mm} \leq 6 \cdot h = 6 \cdot 160 = 960 \text{ mm}$$

4.6 Plastic resistance moment of the composite cross-section

According to clause 6.2.1.3, EN 1994-1-1, for the determination of the resistance moment at mid-span, or the region of sagging bending, we can use partial shear connection. In this case the force transferred to the concrete is limited by the resistance of the shear connectors.

If ductile shear connectors are used, the resistance moment can be calculated by means of rigid plastic theory according to 6.2.1.2, EN 1994-1-1. However, the reduced value of the compressive force in the concrete flange N_c should be taken into account in place of the compressive normal force in the concrete flange with full shear connection $N_{c,f}$.

The plastic neutral axis lies within the flange of the steel beam if:

$$N_c < N_{pl,a}$$

$$N_{pl,a} = \frac{A_a \cdot f_{yd}}{\gamma_{M,0}} = \frac{98{,}8 \cdot 35{,}5}{1{,}0} = 3507 \text{ kN}$$

$$N_c = \eta \cdot N_{c,f} = \Sigma P_{Rd} = 30 \cdot 98{,}9 = 2967 \text{ kN}$$

$2967 < 3507$ the plastic neutral axis lies within the flange of the steel beam

$$N_{pl,a} = (A_a - \rho \cdot A_V) f_{yd}$$

$$N_{pl,f} = \frac{N_{pl,a} - N_c}{2}$$

For $\rho = 0$, ρ is the parameter related to reduced design resistance moment, accounting for vertical shear:

$$N_{pl,a} = A_a \cdot f_{yd} = 98{,}8 \cdot 35{,}5 = 3507 \text{ kN}$$

$$N_{pl,f} = \frac{N_{pl,a} - N_c}{2} = \frac{3507 - 2967}{2} = 270 \text{ kN}$$

$$x_1 = \frac{N_c}{b_{eff} \cdot f_{cd}} = \frac{2967}{275 \cdot 0{,}85 \cdot 1{,}67} = 7{,}6 \text{ cm} < h_c = 11 \text{ cm}$$

$$x_2 = \frac{N_{pl,f}}{b_f \cdot f_{yd}} = \frac{270}{30 \cdot 35{,}5} = 0{,}25 \text{ cm} < t_f = 1{,}46 \text{ cm}$$

$$z = h_c + h_p + \frac{h_a}{2} - \frac{x_1}{2}$$

$$z = 11{,}0 + 5{,}0 + \frac{45{,}0}{2} - \frac{7{,}6}{2} = 11 + 5{,}0 + 22{,}5 - 3{,}8 = 34{,}7 \text{ cm} = 0{,}347 \text{ m}$$

The design resistance moment of the composite section is:

$$M_{Rd} = N_c \cdot z + N_{pl,f}(h_a - x_2)$$

$$M_{Rd} = 2967 \cdot 0,347 + 270(0,45 - 0,00025)$$

$$M_{Rd} = 1150 \text{ kNm}$$

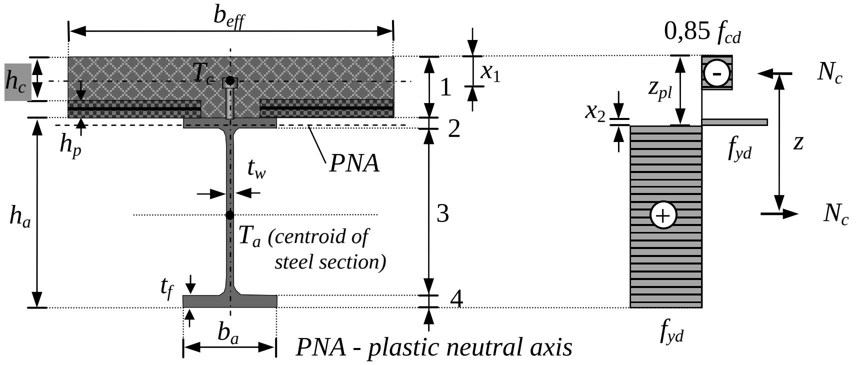


Figure B5.5 *Cross-section of the composite beam*

Since the design bending moment is M_{Ed} = 864 kNm, the check of the resistance of the composite section to bending is:

$$\frac{M_{Ed}}{M_{Rd}} \leq 1,0$$

$$\frac{755}{1150} = 0,66 < 1,0 \text{ the criterion is satisfied}$$

The ductile headed studs can be spaced uniformly according to 6.6.1.3(3), EN 1994-1-1, if the following conditions are satisfied:

- all critical sections in the span of the beam are in classes 1 or 2,
- the values of shear connection η meet the conditions of clause 6.6.1.2, EN 1994-1-1.

- $\dfrac{M_{pl,Rd}}{M_{pl,a,Rd}} \leq 2,5$

The first two conditions are met, so that should prove the third condition. Therefore, it is necessary to determine $M_{pl,a,Rd}$ and $M_{pl,Rd}$.

The design value of the plastic resistance moment of the structural steel section is:

$$M_{pl,a,Rd} = W_{pl,y} \cdot f_{yd} = 1702 \cdot \frac{35,5}{100} = 604 \text{ kNm}$$

The design value of the plastic resistance moment of the composite section with full shear connection $M_{pl,Rd}$ is calculated as follows.

The plastic neutral axis lies within the thickness of the concrete flange if:

$$N_{c,f} > N_{pl,a}$$

$$N_{pl,a} = \frac{A_a \cdot f_{yd}}{\gamma_{M,0}} = \frac{98,8 \cdot 35,5}{1,0} = 3507 \text{ kN}$$

$$N_{c,f} = h_c \cdot b_{eff} \cdot 0,85 f_{cd} = 11 \cdot 275 \cdot 0,85 \cdot 1,67 = 4294 \text{ kN}$$

$4294 > 3507$, the plastic neutral axis lies within the thickness of concrete flange.

In the case of full shear connection, the plastic neutral axis lies within the concrete flange, so that:

$$x_{pl} = \frac{N_{pl,a}}{b_{eff} \cdot 0,85 \cdot f_{cd}} = \frac{3507}{275 \cdot 0,85 \cdot 1,67} = 8,98 \text{ cm} \le h_c = 11,0 \text{ cm}$$

The design resistance moment of the composite section with the full shear connection $M_{pl,Rd}$ is:

$$M_{pl,Rd} = \min(N_{c,f}; N_{pl,a}) \cdot z_1$$

$$M_{pl,Rd} = N_{pl,a}(0,5h_a + h_c + h_p - 0,5x_{pl})$$

$$M_{pl,Rd} = 3507(0,5 \cdot 450 + 110 + 50 - 0,5 \cdot 89,8) \cdot 10^{-3} = 3507 \cdot 340,1 \cdot 10^{-3}$$

$$M_{pl,Rd} = 1193 \text{ kNm}$$

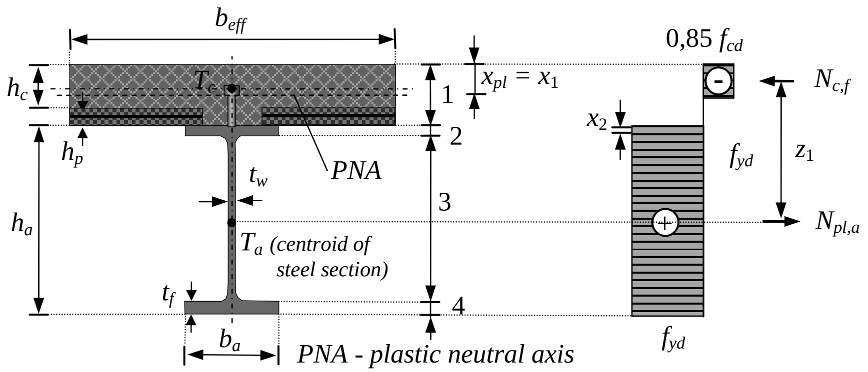


Figure B5.6 *Cross-section for determination of the design plastic resistance to bending*

Check:

$$\frac{M_{pl,Rd}}{M_{pl,a,Rd}} = \frac{1193}{604} = 1,98 \le 2,5$$

4.7 Vertical shear resistance of the composite cross-section

The shear buckling resistance of the web should be verified, for an unstiffened web, when:

$$\frac{h_w}{t} > \frac{72}{\eta}\varepsilon$$

where:

$$\varepsilon = \sqrt{\frac{235}{f_y}} = \sqrt{\frac{235}{355}} = 0,81$$

$\eta = 1,2$, the factor defined in EN 1993-1-5

$$h_w = h_a - 2t_f = 450 - 2 \cdot 14,6 = 420,8 \text{ mm}$$

$$72 \cdot \frac{\varepsilon}{\eta} = 72 \cdot \frac{0,81}{1,2} = 48,60$$

$$\frac{h_w}{t} = \frac{h_w}{t_w} = \frac{420,8}{9,4} = 44,77$$

Since 44,77 < 48,60, the condition is satisfied. The shear buckling resistance of the web need not be verified.

Remark:

The resistance of the composite beam to vertical shear is normally taken as the shear resistance of the steel section according to clause 6.2.6, EN 1993-1-1, which gives:

$$V_{pl,Rd} = V_{pl,a,Rd} = \frac{A_V(f_y / \sqrt{3})}{\gamma_{M0}}$$

For rolled I- and H-sections, if the load is applied parallel to the web, the shear area is calculated as:

$A_V = A - 2 \cdot b_a \cdot t_f + t_f \cdot (t_w + 2 \cdot r)$, but not less than $\eta \cdot h_w \cdot t_w$

The shear area A_V is:

$$A_V = 98,8 - 2 \cdot 19 \cdot 1,46 + (0,94 + 2 \cdot 2,1) \cdot 1,46$$

$$A_V = 50,8 \ \text{cm}^2$$

$$\eta = 1,2$$

$$\eta \cdot h_w \cdot t_w = 1,2 \cdot 42,08 \cdot 0,94 = 47,46 \ \text{cm}^2$$

50,8 cm^2 > 47,46 cm^2

Therefore A_V = 50,8 cm^2.

The design plastic shear resistance of the steel section is:

$$V_{pl,Rd} = V_{pl,a,Rd} = \frac{A_V(f_y / \sqrt{3})}{\gamma_{M0}}$$

$$V_{pl,Rd} = 50,8 \frac{35,5}{\sqrt{3} \cdot 1,0} = 1041 \text{ kN}$$

Verify that:

$$\frac{V_{Ed}}{V_{pl,Rd}} \leq 1,0$$

$$\frac{274}{1041} = 0,26 < 1,0$$

Therefore the shear resistance of the cross-section is adequate.

As there is no shear force at the point of maximum bending moment (mid-span), no reduction (due to shear) in resistance moment is required.

4.8 Check of longitudinal shear resistance of the concrete flange

4.8.1 Check of transverse reinforcement

The cross-sectional area of the transverse reinforcement is calculated according to the expression:

$$\frac{A_{sf}}{s_f} \cdot f_{sd} \geq v_{L,Ed} \cdot \frac{h_f}{\cot \theta}$$

where:

A_{sf}/s_f is the transverse reinforcement expressed in mm²/m,
h_f is the thickness of the concrete flange, see Figures B5.7 and B5.8,
θ is the angle between the diagonal strut and the axis of the beam (strut-and-tie model),
$v_{L,Ed}$ is the design longitudinal shear flow in the concrete slab.

The transverse reinforcement (A_{sf}/s_f) expressed in mm²/m can be denoted as A_t for the top transverse reinforcement and as A_b for the bottom transverse reinforcement. It is necessary to verify the failure due to shear in the failure plane shown in Figure B5.7 as section *a-a* and section *c-c*.

The transverse reinforcement provided is 8 mm bars at 150 mm, for which:

$$A_t = A_b = \frac{\pi \cdot 8^2}{4} \cdot \frac{1000}{150} = 335 \text{ mm}^2/\text{m}$$

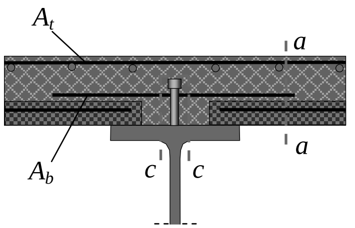


Figure B5.7 *Surfaces of potential failure in longitudinal shear*

It is necessary to ensure that the concrete flange can resist the longitudinal shear force transmitted to it by the shear connectors. At the steel-concrete interface, the distribution of longitudinal shear is influenced by yielding, by the spacing of the shear connectors, their load-slip properties and shrinkage and creep of the concrete. The design resistance to longitudinal shear for the relevant shear failure surfaces is given in clause 6.2.4, EN 1992-1-1. The model is based on considering the flange to act like a system of compressive struts combined with a system of ties in the form of transverse reinforcement.

Concrete flange in compression:

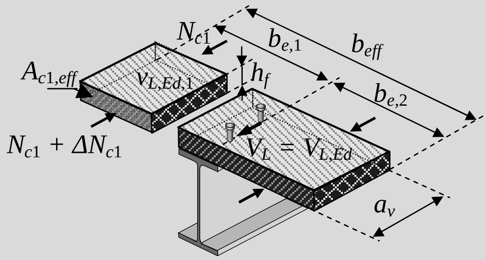


Figure B5.8 *Determination of longitudinal shear forces in the concrete flange*

When the concrete flange is in compression, longitudinal shear flow $v_{L,Ed}$ can be defined as:

$$v_{L,Ed,1} = \frac{\Delta N_{c1}}{a_v} = \frac{V_{L,Ed}}{a_v} \frac{A_{c1,eff}}{A_{c,eff}}$$

where:

a_v is the critical length (the distance between two given sections, Figure B5.8),

ΔN_{c1} is the change of the longitudinal compressive forces in the slab over the

	critical length a_v, see Figure B5.8.
$V_{L,Ed}$	is the design longitudinal shear force in the steel-concrete interface or in the concrete flange,
$V_{L,Ed} = \min(N_{pl,a}, N_c, \Sigma P_{Rd})$	

The length a_v is $L/2$, which is the distance between the section where the moment is maximum and the support.

The design longitudinal shear force is determined from the minimum resistance of the steel section, concrete and shear connectors.

$$V_{L,Ed} = \min(N_{pl,a}, N_c, \Sigma P_{Rd})$$

$$N_{pl,a} = 3507 \text{ kN}$$

$$\Sigma P_{Rd} = 2967 \text{ kN}$$

$$N_{c,f} = 4294 \text{ kN}$$

In this case, with partial shear connection, the maximum force that can be transferred is limited by the sum of the resistance of the studs, and is given by:

$$V_{L,Ed} = \Sigma P_{Rd} = 2967 \text{ kN}$$

This force must be transferred over each half-span.

Section *a-a*

$$V_{L,Ed} = 2967 \text{ kN}$$

$$f_{sd} = 435 \text{ N/mm}^2$$

As there are two shear planes, see Figure B5.7, one on either side of the beam, running parallel to it, and with $h_f = h_c = 110$ mm (the prefabricated element is neglected), the design longitudinal shear stress is:

$$h_f = h_c = 110 \text{ mm}$$

$$v_{L,Ed} = \frac{\Delta N_{c1}}{h_f \cdot a_v} = \frac{V_{L,Ed}}{2h_f a_v} = \frac{2967 \cdot 10^3}{2 \cdot 110 \cdot 5500} = 2,45 \text{ N/mm}^2$$

Remark:

In order to prevent splitting of the concrete flange, for the adopted "truss model", according to clause 6.2.4(4) EN 1992-1-1, the angle θ between the concrete diagonals and the longitudinal direction is limited to the value:

$26{,}5^\circ \le \theta \le 45^\circ$ concrete flange in compression

$38{,}6^\circ \le \theta \le 45^\circ$ concrete flange in tension

In order to minimize the cross-sectional area of the transverse reinforcement, the minimum angle θ is selected. For the concrete flange in compression, the minimum angle θ is:

$$\theta = 26{,}5^\circ$$

$$\frac{A_{sf}}{s_f} \cdot f_{sd} \ \ge \ v_{L,Ed} \cdot \frac{h_f}{\cot\theta}$$

$$\frac{A_{sf}}{s_f} \ge \frac{v_{L,Ed}}{f_{sd}} \cdot \frac{h_f}{\cot\theta} = \frac{2{,}45}{435} \cdot \frac{110}{\cot 26{,}5} \cdot 10^3 = 309 \ \text{mm}^2/\text{m}$$

The reinforcement is obtained that is less than the selected reinforcement $A_t + A_b = 2 \cdot 335 = 670 \ \text{mm}^2/\text{m}$.

Section *c-c*

$$V_{L,Ed} = 2967 \ \text{kN}$$

$$f_{sd} = 435 \ \text{N/mm}^2$$

The length h_f of the shear surface *c-c* passing round the studs as shown in Figure B5.7 is:

$$h_f = 2 \cdot h_{sc} + 1{,}5 \cdot d = 2 \cdot 125 + 1{,}5 \cdot 22 = 283 \ \text{mm}$$

The design longitudinal shear stress is:

$$v_{L,Ed} = \frac{\Delta N_{c1}}{h_f \cdot a_v} = \frac{V_{L,Ed}}{h_f \cdot a_v} = \frac{2967 \cdot 10^3}{283 \cdot 5500} = 1{,}91 \ \text{N/mm}^2$$

In order to minimize the cross-sectional area of the transverse reinforcement, the minimum angle θ is selected. For the concrete flange in compression, the minimum angle θ is:

$$\theta = 26{,}5°$$

$$\frac{A_{sf}}{s_f} \cdot f_{sd} \geq v_{L,Ed} \cdot \frac{h_f}{\cot\theta}$$

$$\frac{A_{sf}}{s_f} \geq \frac{v_{L,Ed}}{f_{sd}} \cdot \frac{h_f}{\cot\theta} = \frac{1{,}91}{435} \cdot \frac{283}{\cot 26{,}5} \cdot 10^3 = 620 \ \text{mm}^2/\text{m}$$

The reinforcement is obtained that is less than the selected reinforcement $A_t + A_b = 2 \cdot 335 = 670 \ \text{mm}^2$/m.

According to EN 1994-1-1, clause 6.6.6.3, the minimum area of transverse reinforcement is determined in accordance with EN 1992-1-1, clause 9.2.2(5), which gives the minimum area of reinforcement as a proportion of the concrete area. The ratio is:

$$\rho_{w,\min} = \frac{0{,}08\sqrt{f_{ck}}}{f_{yr,k}}$$

where:

f_{ck} is the characteristic compressive cylinder strength of the concrete at 28 days in N/mm^2,

$f_{yr,k} = f_{sk}$ is the characteristic yield strength of the reinforcement in N/mm^2.

The minimum area of transverse reinforcement is:

$$\rho_{w,\min} = \frac{0{,}08\sqrt{f_{ck}}}{f_{yr,k}} = \frac{0{,}08\sqrt{25}}{500} = 0{,}0008$$

$$A_c = h_c \cdot b = 110 \cdot 1000 = 110000 \ \text{mm}^2$$

$$A_{t,\min} = \rho_{w,\min} \cdot A_c = 0{,}0008 \cdot 110000 = 88 \ \text{mm}^2/\text{m}$$

Since $A_t + A_b = 670\ \text{mm}^2/\text{m} > A_{t,\min} = 88\ \text{mm}^2/\text{m}$, the requirement for minimum transverse reinforcement is satisfied.

4.8.2 Crushing of the concrete flange

To prevent crushing of the compression struts in the flange, the following condition should be satisfied according to EN 1992-1-1, expression 6.22:

$$v_{L,Ed} \leq v_{Rd}$$

$$v_{L,Ed} \leq v \cdot f_{cd} \cdot \sin\theta \cdot \cos\theta$$

where:

$$v = 0{,}6 \cdot \left(1 - \frac{f_{ck}}{250}\right)$$

θ is the angle between the concrete diagonals and the longitudinal direction.

In order to minimize the resistance of the concrete compression strut, the minimum angle θ is selected. For the concrete flange in compression, the minimum angle θ is:

$$\theta = 26{,}5°$$

$$v_{Rd} = v \cdot f_{cd} \cdot \sin\theta \cdot \cos\theta = 0{,}6 \cdot \left(1 - \frac{25}{250}\right) \cdot 16{,}7 \cdot \sin 26{,}5° \cdot \cos 26{,}5° = 3{,}60\ \text{N/mm}^2$$

Check:

$$v_{Ed} = 2{,}45\ \text{N/mm}^2 < v_{Rd} = 3{,}60\ \text{N/mm}^2$$

Therefore the crushing resistance of the concrete compression strut is adequate.

5. Serviceability limit state

5.1 General

Chapter 7, EN 1994-1-1 is limited to provisions relating to serviceability that are specific to composite structures. Serviceability verifications in the case of the composite structures generally include a check of stress, deflection and vibration as well as control of the crack width.

For buildings, stress limitation is not required for beams if, in the ultimate limit state, no verification of fatigue is required and no pre-stressing by tendons and/or by controlled imposed deformations is provided. However, if the stress limitation is required, clause 7.2, EN 1992-1-1, gives stress limits which may be applicable for buildings that have pre-stressing or fatigue loading.

Since the deflection is one of the most important verifications of the serviceability limit state, it is necessary to explain in detail the problems associated with the deflection calculation. Deflections due to loads applied to the composite member are calculated using elastic theory, taking into account the following effects:

a) cracking of the concrete,
b) creep and shrinkage of the concrete,
c) sequence of construction,
d) influence of local yielding of the structural steel at the internal support (in case of continuous beams),
e) influence of incomplete interaction.

For a more detailed explanation of these effects, see example B6.

If the steel beam is fully propped, the total deflection of composite beams is obtained by summing the following deflections:

$$\delta = \delta_1 + \delta_{2,1} + \delta_{2,2} + \delta_{2,3}$$

where:

δ_1 is the deflection due to the permanent actions (the first loading is applied at age t_0 = 28 days),
$\delta_{2,1}$ is the deflection due to the frequent value of the variable action at the time of first loading,
$\delta_{2,2}$ is the deflection due to creep under the quasi-permanent value of the variable action at time $t = \infty$,
$\delta_{2,3}$ is the deflection due to shrinkage.

5.2 Calculation of deflections

5.2.1 Construction stage deflection

In this example, the steel beam is fully propped at the construction stage and the deflection of the steel beam at the construction stage is:

$$\delta_0 = 0$$

5.2.2 Composite stage deflection

The expression for calculation of the maximum deflection of the beam due to uniformly distributed loads has the following general form:

$$\delta = \frac{5}{384} \cdot \frac{e_d \cdot L^4}{EI_L}$$

where:

e_d is the design value of the load from a governed combination of action,
L is the span of the beam,
EI_L is the effective flexural stiffness of the composite sections which depends on the type of loadings; the different types of loading are distinguished by a subscript L,
E_L is the effective modulus of elasticity of the concrete, which depends on the same type of loadings as for the flexural stiffness EI_L. The effective modulus of elasticity of the concrete E_L is denoted by $E_{c,eff}$ in EN 1994-1-1.

The total deflection of the composite beam will be determined for the composite stage, i.e. the concrete slab has hardened and any propps have been removed.

Determination of creep coefficient and shrinkage

For the calculation of the creep coefficient $\varphi(t, t_0)$ the following is valid:

- the perimeter of that part which is exposed to drying, u

$$u = b$$

- the notional size of the cross-section, h_0

$$h_0 = \frac{2 \cdot A_c}{u} = \frac{b \cdot h_c}{b} = h_c = 160 \text{ mm}$$

- $t_0 = 1$ day, $t_0 = 28$ days,
- the ambient relative humidity, RH 50%,
- the concrete strength class C 25/30,
- the type of cement – cement class S, strength class 32,5 N.

The final value of creep coefficient $\varphi(\infty,\ t_0)$ is determined using the nomogram shown in Figure 3.1, EN 1992-1-1. Example A3 shows the detailed procedure for determination of the creep coefficients.

The following creep coefficients are obtained:

$\varphi_t = \varphi(\infty,\ t_0\ =\ 1\ \text{day}) = 5{,}8$
$\varphi_t = \varphi(\infty,\ t_0\ =\ 28\ \text{days}) = 2{,}8$

The total shrinkage strain, according to clause 3.1.4, EN 1992-1-1, at the age of the concrete at the beginning of drying shrinkage t_s = 3 days and the age at the time considered in the analysis $t = \infty$, is:

$$\varepsilon_{cs}(\infty) = 4{,}14 \cdot 10^{-4}$$
$$\varepsilon_{cs}(\infty) = 0{,}414\ ‰$$

Effective flexural stiffness of the composite section

The effective flexural stiffness of the composite section EI_L (in this case approximately $h_c = h$) is:

$$EI = E_a \cdot I_a + E_L \cdot I_c + \frac{E_a \cdot A_a \cdot E_L \cdot A_c}{E_a \cdot A_a + E_L \cdot A_c} \cdot a^2$$

a) Short-term loading

$$E_a = 21000\ \text{kN/cm}^2 \quad I_a = 33740\ \text{cm}^4 \quad A_a = 98{,}8\ \text{cm}^2$$

$$I_c = \frac{b_{eff} \cdot h^3}{12} = \frac{275 \cdot 16^3}{12} = 93867\ \text{cm}^4$$

$$A_c = b_{eff} \cdot h = 275 \cdot 16 = 4400\ \text{cm}^2$$

The distance between the centroidal axes of the concrete flange and the steel section is:

$$a = 0{,}5 \cdot (h + h_a) = 0{,}5 \cdot (16 + 45) = 30{,}5\ \text{cm}$$

$$n_c = 1$$

$$E_0 = \frac{E_{cm}}{n_c} = \frac{3100}{1{,}0} = 3100 \text{ kN/cm}^2 \quad E_L = E_0$$

$$EI_0 = 21000 \cdot 33740 + 3100 \cdot 93867 + \frac{21000 \cdot 98{,}8 \cdot 3100 \cdot 4400}{21000 \cdot 98{,}8 + 3100 \cdot 4400} \cdot 30{,}5^2$$

$$EI_0 = 2674784657 \text{ Ncm}^2 = 267478 \text{ kNm}^2$$

b) Permanent loading constant in time

$$n_c = 1 + 1{,}10 \cdot \varphi(\infty, t_0) = 1 + 1{,}10 \cdot 2{,}8 = 4{,}08$$

$$E_P = \frac{E_{cm}}{n_c} = \frac{3100}{4{,}08} = 760 \text{ kN/cm}^2 \quad E_L = E_P$$

$$EI_P = 21000 \cdot 33740 + 760 \cdot 93867 + \frac{21000 \cdot 98{,}8 \cdot 760 \cdot 4400}{21000 \cdot 98{,}8 + 760 \cdot 4400} \cdot 30{,}5^2$$

$$EI_P = 1970953798 \text{ Ncm}^2 = 197095 \text{ kNm}^2$$

c) Primary effects due to shrinkage

$$n_c = 1 + 0{,}55 \cdot \varphi(\infty, t_0) = 1 + 0{,}55 \cdot 5{,}8 = 4{,}19$$

$$E_S = \frac{E_{cm}}{n_c} = \frac{3100}{4{,}19} = 740 \text{ kN/cm}^2 \quad E_L = E_S$$

$$EI_S = 21000 \cdot 33740 + 740 \cdot 93867 + \frac{21000 \cdot 98{,}8 \cdot 740 \cdot 4400}{21000 \cdot 98{,}8 + 740 \cdot 4400} \cdot 30{,}5^2$$

$$EI_S = 1956877034 \text{ Ncm}^2 = 195688 \text{ kNm}^2$$

Calculation of deflections

- Deflection due to permanent action, the first loading is applied at age t_0 = 28 days

$$e_d = b \cdot g_k = 3{,}0 \cdot 6{,}77 = 20{,}3 \text{ kN/m} \quad \text{and} \quad EI_L = EI_0$$

$$\delta_1 = \frac{5}{384} \cdot \frac{e_d \cdot L^4}{EI_0} = \frac{5}{384} \cdot \frac{20{,}3 \cdot 11^4}{267478} \cdot 100 = 1{,}45 \text{ cm}$$

- Deflection due to the frequent value of the variable action at the time of the first loading t_0 = 28 days

For a building with floors in category B, office areas, the combination factor ψ is: ψ_1 = 0,5.

$$e_d = b \cdot \psi_1 \cdot q_k = 3{,}0 \cdot 0{,}5 \cdot 5{,}0 = 7{,}5 \text{ kN/m}$$

$EI_L = EI_0 = 264310 \text{ kNm}^2$, for short-term loading

$$\delta_{2,1} = \frac{5}{384} \cdot \frac{e_d \cdot L^4}{EI_0} = \frac{5}{384} \cdot \frac{7{,}5 \cdot 11^4}{267478} \cdot 100 = 0{,}53 \text{ cm}$$

- Deflection due to creep under the quasi-permanent value of variable action at time $t = \infty$

This deflection is the difference of deflections at time $t = \infty$ and at the time of the first loading t_0 = 28 days.

$$e_d = b \cdot (g_k + \psi_2 \cdot q_k) = 3{,}0 \cdot (6{,}77 + 0{,}3 \cdot 5{,}0) = 24{,}8 \text{ kN/m}$$

$EI_0 = 267478 \text{ kNm}^2$, for short-term loading

$EI_P = 197095 \text{ kNm}^2$, permanent action constant in time

$$\delta_{2,2} = \frac{5}{384} \cdot \frac{e_d \cdot L^4}{EI_P} - \frac{5}{384} \cdot \frac{e_d \cdot L^4}{EI_0}$$

$$\delta_{2,2} = \frac{5}{384} \cdot \frac{24{,}8 \cdot 11^4}{197095} \cdot 100 - \frac{5}{384} \cdot \frac{24{,}8 \cdot 11^4}{267478} \cdot 100 = 2{,}40 - 1{,}77 = 0{,}63 \text{ cm}$$

- Deflection due to shrinkage

$$N_{cs} = \varepsilon_{cs}(\infty) \cdot E_S \cdot A_c = 4{,}14 \cdot 10^{-4} \cdot 740 \cdot 275 \cdot 16 = 1348 \text{ kN}$$

$$a_c = \frac{E_a \cdot A_a}{E_a \cdot A_a + E_S \cdot A_c} \cdot a = \frac{21000 \cdot 98,8}{21000 \cdot 98,8 + 740 \cdot 4400} \cdot 30,5 = 11,87 \text{ cm}$$

$$M_{cs} = N_{cs} \cdot a_c = 1348 \cdot \frac{11,87}{100} = 160 \text{ kNm}$$

$$\delta_{2,3} = \frac{1}{8} \cdot \frac{M_{cs} \nu L^2}{EI_S} = \frac{1}{8} \cdot \frac{160 \cdot 11^2}{195688} \cdot 100 = 1,24 \text{ cm}$$

The effects of shear connection on the deflection of the beam can be neglected because the condition $n/n_f \geq 0,5$ is satisfied.

Deflection limits for composite beams are the same as for steel beams, and are determined by the National Annex.

Recommended limiting values for deflection of composite beams are:

$\delta_{tot} \leq \dfrac{L}{250}$, the deflection due to the total load

$\delta_{\mathrm{var}} \leq \dfrac{L}{360}$, the deflection due to the variable load

The deflection due to permanent action is:

$$\delta_1 = 1,45 \text{ cm}$$

The deflection due to variable load, creep and shrinkage is:

$$\delta_2 = \Sigma\delta_{2,i} = 0,53 + 0,63 + 1,24 = 2,40 \text{ cm}$$

The total deflection due to permanent and variable loads, creep and shrinkage is:

$$\delta_{tot} = \delta_1 + \delta_2 = 1,45 + 2,40 = 3,85 \text{ cm} > \frac{L}{250} = \frac{1100}{250} = 4,4 \text{ cm}$$

The total deflection does not meet the criterion L/250. However, taking into account the pre-cambering calculated in Section 5.3 the criterion is satisfied.

The steel beam is fully propped and its deflection at the construction stage is zero, $\delta_0 = 0$.

The deflection due to variable load, creep and shrinkage is:

$$\delta_{\mathrm{var}} = \delta_2 = 2,40 \text{ cm} \leq \frac{L}{360} = \frac{1100}{360} = 3,05 \text{ cm}$$

The deflection due to variable load, creep and shrinkage meets the criterion $L/360$.

5.3 Pre-cambering of steel beam

In this example, the pre-cambering of the steel beam involves deflections due to permanent loads, creep and shrinkage:

$$\delta_p = \delta_1 + \delta_{2,2} + \delta_{2,3}$$

$$\delta_p = 1,45 + 0,63 + 1,24$$

$$\delta_p = 3,32 \text{ cm}$$

5.4 Check of vibration of the beam

For the calculation of the natural frequency, the characteristic value of the permanent load for the composite stage, $e_d = b \cdot g_k$, is taken into account and the effective flexural stiffness of composite section for short-term loading EI_0.

$$e_d = b \cdot g_k = 3,0 \cdot 6,77 = 20,3 \text{ kN/m}$$

The deflection under this load is:

$$\delta = \frac{5}{384} \cdot \frac{g_k \cdot L^4}{EI_0} = \frac{5}{384} \cdot \frac{20,3 \cdot 11^4}{267478} \cdot 100 = 1,45 \text{ cm}$$

The natural frequency of the beam is therefore:

$$f = \frac{18}{\sqrt{\delta}} = \frac{18}{\sqrt{14,5}} = 4,7 \text{ Hz} \geq 4 \text{ Hz} \quad \text{with} \quad \delta \text{ in mm}$$

The criterion is satisfied for initial calculation purposes. However, the dynamic performance of the entire floor is carried out using a method such as the one in [51].

5.5 Control of crack width

The control of crack width for the simply supported beam with continuous concrete slab on the top of steel beam is not necessary. Clause 7.4.1(3), EN 1994-1-1 recommends the minimum reinforcement which will limit crack width to what is "acceptable", see example B3.

6. Commentary

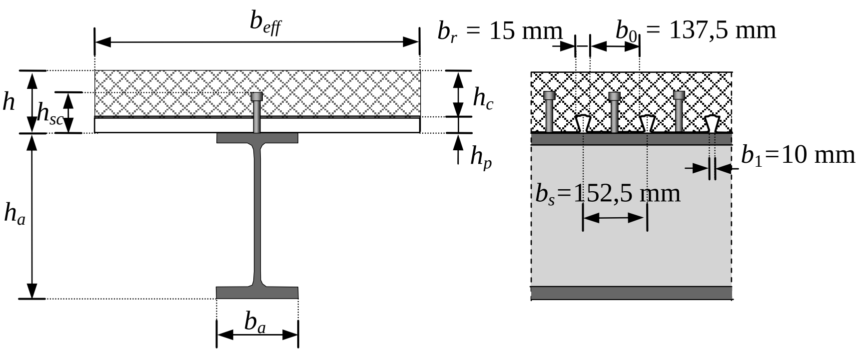
For the design of composite beams in structures for buildings it is normal to select beams with steel sections such that the composite sections are in classes 1 or 2. In such cases it is possible to use rigid plastic analysis provided that the cross-sections at locations of plastic hinges are in class 1. Further, the resistance moments of beams can be calculated using plastic theory and the limits of moment redistribution are more favourable than for classes 3 and 4. Where all beam cross-sections are in classes 1 or 2, it is possible to use partial shear connection. Therefore, as a rule, the cross-section of resistance is based on rigid plastic theory, and only in exceptional cases is it based on elastic analysis.

B6 Calculation of continuous beam over two spans by means of elastic–plastic procedure

1. Purpose of example

The example shows the design of a two-span composite beam using the elastic–plastic procedure. Action effects are calculated by elastic global analysis and the resistance to bending is based on a plastic model. To use the elastic–plastic procedure, the cross-section should be at least in class 2. Analyses for ultimate limit state and serviceability limit state are also conducted by elastic global analysis. This example illustrates the method based on uncracked analysis with limited redistribution. In accordance with clause A1.4.2, EN 1990, serviceability limit states in buildings should take into account criteria related to floor stiffness. These criteria are expressed in terms of limits for vertical deflections and for vibrations. The serviceability criteria required in this example are: $\delta_{req} = L/250$ for the total load and $\delta_{req} = L/360$ for variable load. The natural frequency should not be less than 4 Hz. Generally, the serviceability criteria should be specified for each project and agreed with the client. However, in accordance with EN 1990, the relevant National Annexes could give recommended values for the serviceability criteria. The continuous beam is unpropped at the construction stage. The composite slab is cast in situ on profiled steel sheeting with profile height 51 mm, creating the overall slab thickness 150 mm.

2. Static system, cross-section and actions



h = 150 mm, h_c = 99 mm, h_p = 51 mm

Figure B6.1 *Cross-section of composite beam*

Composite Structures according to Eurocode 4. Worked Examples.
First Edition. Darko Dujmović, Boris Androić, Ivan Lukačević.

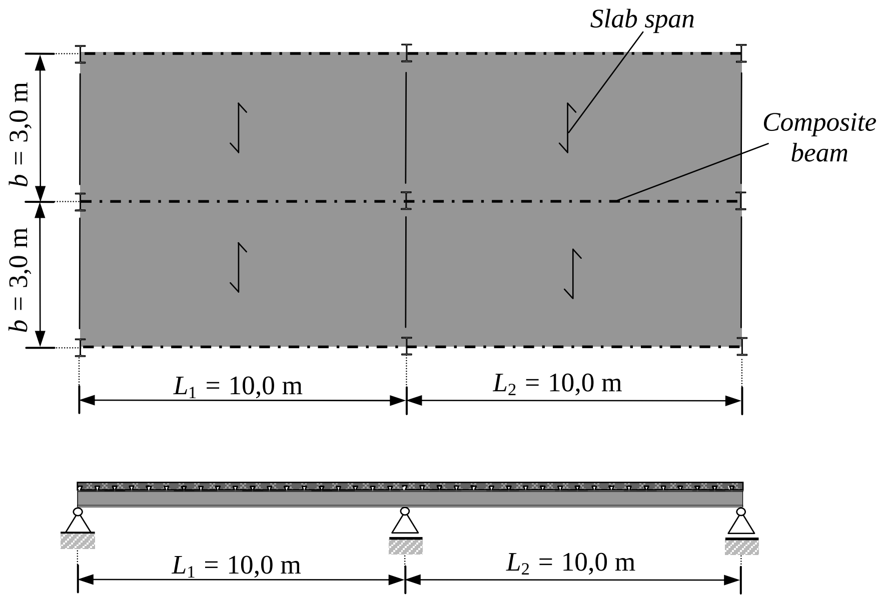


Figure B6.2 *Floor layout and static system*

Actions

a) Permanent action

Remark:

According to EN 1991-1-1 the density of normal weight concrete is 24 kN/m^3, increased by 1 kN/m^3 for normal percentage reinforcement, and increased for the wet concrete by another 1 kN/m^3.

The concrete slab area per m width is:

$$A_c = 1000 \cdot h - \left(\frac{1000}{b_s} \cdot \frac{b_1 + b_r}{2} \cdot h_p\right)$$

$$A_c = 1000 \cdot 150 - \left(\frac{1000}{152{,}5} \cdot \frac{10 + 15}{2} \cdot 51\right) = 145820 \ \text{mm}^2 = 1458 \ \text{cm}^2$$

- *concrete slab and reinforcement (wet concrete):*

$$A_c \cdot 26 = 0{,}1458 \cdot 26 = 3{,}79 \ \text{kN/m}^2$$

- *concrete slab and reinforcement (dry concrete):*

$$A_c \cdot 25 = 0{,}1458 \cdot 25 = 3{,}64 \ \text{kN/m}^2$$

Construction stage

- *concrete slab* $= 3{,}79\ \text{kN/m}^2$
- *profiled steel sheeting* $= 0{,}17\ \text{kN/m}^2$
- *steel beam* $= 0{,}30\ \text{kN/m}^2$

Total $g_{k,1} = 4{,}26\ \text{kN/m}^2$

Composite stage

- *concrete slab* $= 3{,}64\ \text{kN/m}^2$
- *profiled steel sheeting* $= 0{,}17\ \text{kN/m}^2$
- *steel beam* $= 0{,}30\ \text{kN/m}^2$

Total $g_{k,2} = 4{,}11\ \text{kN/m}^2$

Floor finishes $g_{k,3} = 0{,}50\ \text{kN/m}^2$

b) Variable action

Construction stage

- *construction loads* $q_{k,1} = 0{,}50\ kN/m^2$

Composite stage

- *imposed floor load, category of use C2* $= 4{,}00\ \text{kN/m}^2$
- *movable partitions* $= 0{,}50\ \text{kN/m}^2$

Total $q_{k,2} = 4{,}50\ \text{kN/m}^2$

Remark:

The variable actions q_k = 4,00 kN/m^2 and q_k = 0,50 kN/m^2 are mutually independent.

3. Properties of materials

Concrete strength class: C 40/50 $f_{ck} = 40\ \text{N/mm}^2$

$$f_{cd} = \frac{f_{ck}}{\gamma_c} = \frac{40}{1{,}5} = 26{,}7\ \text{N/mm}^2$$

$$0,85 f_{cd} = 0,85 \cdot 26,7 = 22,7 \text{ N/mm}^2$$

$$E_{cm} = 35\,000 \text{ N/mm}^2$$

$$f_{ctm} = 3,5 \text{ N/mm}^2$$

Reinforcement: *ductility class B or C (Table C.1, EN 1992-1-1)* $f_{sk} = 460 \text{ N/mm}^2$

$$f_{sd} = \frac{f_{sk}}{\gamma_s} = \frac{460}{1,15} = 400 \text{ N/mm}^2$$

$$E_s = 210\,000 \text{ N/mm}^2$$

Structural steel: S275 $f_{yk} = 275 \text{ N/mm}^2$

$$f_{yd} = \frac{f_{yk}}{\gamma_M} = \frac{275}{1,0} = 275 \text{ N/mm}^2$$

$$E_a = 210\,000 \text{ N/mm}^2$$

Shear connectors: *ductile headed studs* $f_u = 450 \text{ N/mm}^2$

$$d = 19 \text{ mm}$$

$$h_{sc} = 95 \text{ mm}$$

$$\frac{h_{sc}}{d} = \frac{95}{19} = 5,0 > 4,0 \rightarrow \alpha = 1,0$$

$$P_{Rd} = 82,0 \text{ kN}$$

4. Ultimate limit state

4.1 Design values of combined actions and of the effects of actions for the construction stage

Remark:

Calculation of internal forces and bending moments is carried out using commercial software.

The design load determined by governed combination of actions is:

$$e_d = b \cdot (\gamma_{G,\text{sup}} \cdot g_{k,1} + \gamma_Q \cdot q_{k,1})$$

$b = 3,0$ m beam spacing

$$e_d = 3,00 \cdot (1,35 \cdot 4,26 + 1,50 \cdot 0,50) = 19,5 \text{ kN/m}$$

a) Maximum design moment at the internal support

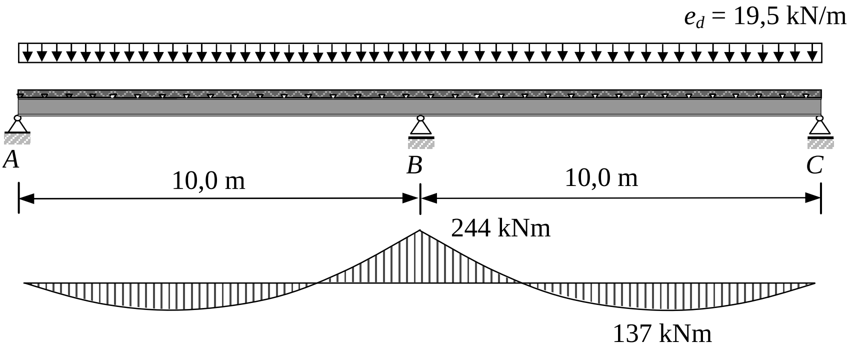


Figure B6.3 *Design load for maximum design moment at support with corresponding bending moment distribution*

$V_{Ed,A} = 73{,}1\ \text{kN}$

$V_{Ed,B} = 122\ \text{kN}$

b) Maximum design moment at mid-span

Remark:

For simplicity, in the determination of the maximum bending moment at mid-span, the self-weight of the steel beam is neglected. This assumption produces a conservative result.

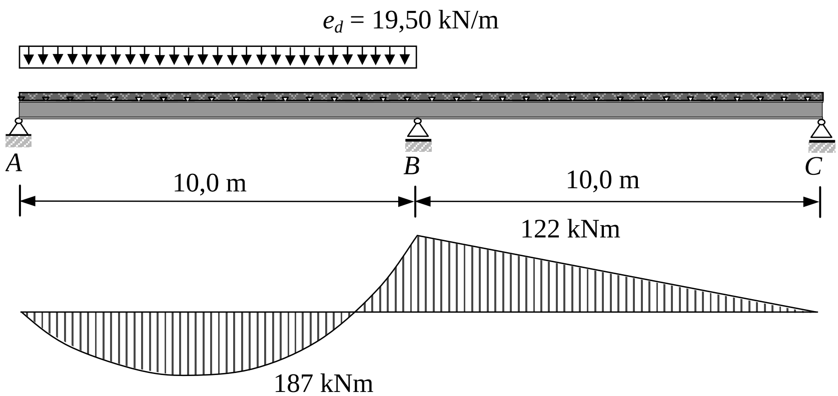


Figure B6.4 *Design load for maximum design moment at mid-span with corresponding bending moment distribution*

$V_{Ed,A} = 85{,}3\ \text{kN}$

$V_{Ed,B} = 110\ \text{kN}$

Considering both load cases, the maximum design moments on the steel section during the execution stage are:

- design negative moment $M_{Ed,B} = 244$ kNm
- design positive moment $M_{Ed,1} = 187$ kNm

The maximum design shear force on the steel section is: $V_{Ed,B} = 122$ kN.

4.2 Design values of combined actions and of the effects of actions for the composite stage

Remark:

Calculation of internal forces and bending moments is carried out using commercial software.

The design load determined by governed combination of actions is:

$$e_d = b \cdot (\gamma_{G,\sup} \cdot g_{k,2} + \gamma_{G,\sup} \cdot g_{k,3} + \gamma_Q \cdot q_{k,2})$$

$$e_d = 3{,}00 \cdot (1{,}35 \cdot 4{,}11 + 1{,}35 \cdot 0{,}5 + 1{,}50 \cdot 4{,}50) = 38{,}9 \text{ kN/m}$$

a) Maximum design moment at the internal support (load case 1)

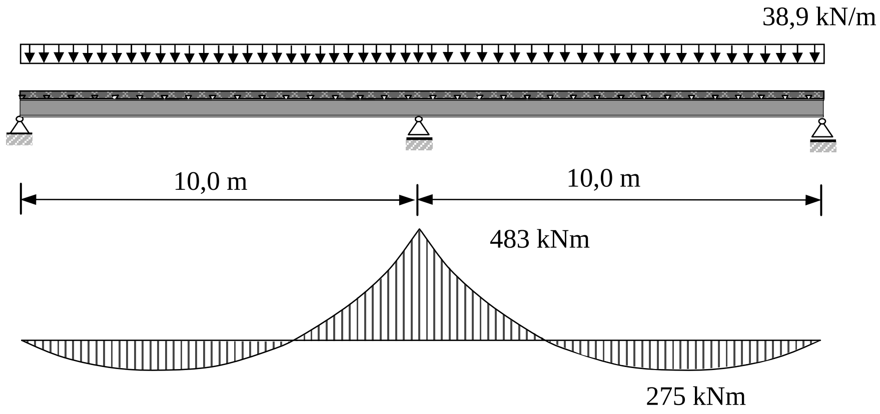


Figure B6.5 *Design load for maximum design moment at the support with corresponding bending moment distribution (without redistribution)*

$V_{Ed,A} = 146$ kN

$V_{Ed,B} = 243 \text{ kN}$

b) Maximum design moment at mid-span (load case 2)

$$e_d = b \cdot (\gamma_{G,\text{sup}} \cdot g_{k,2} + \gamma_{G,\text{sup}} \cdot g_{k,3} + \gamma_Q \cdot q_{k,2})$$

$$e_d = 3{,}00 \cdot (1{,}35 \cdot 4{,}11 + 1{,}35 \cdot 0{,}5 + 1{,}50 \cdot 4{,}50) = 38{,}9 \text{ kN/m}$$

$$e_{d,\min} = b \cdot (\gamma_{G,\inf} \cdot g_{k,2} + \gamma_{G,\inf} \cdot g_{k,3})$$

$$e_{d,\min} = 3{,}00 \cdot (1{,}00 \cdot 4{,}11 + 1{,}00 \cdot 0{,}5) = 13{,}8 \text{ kN/m}$$

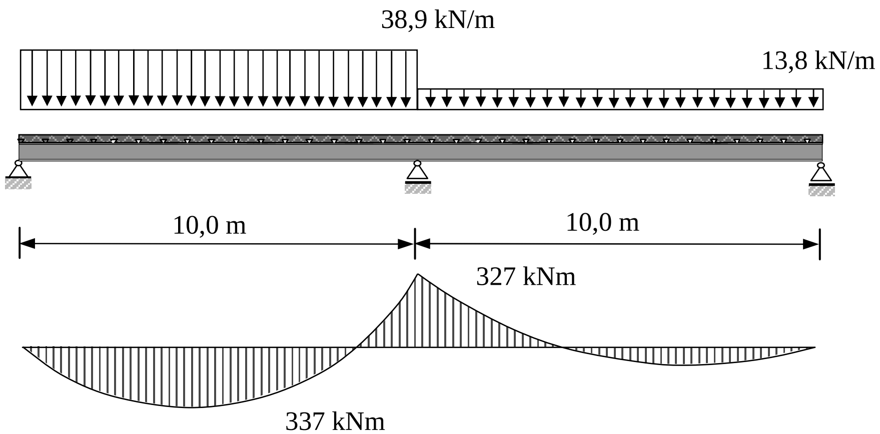


Figure B6.6 *Design load for maximum design moment at mid-span with corresponding bending moment distribution (without redistribution)*

$V_{Ed,A} = 162 \text{ kN}$
$V_{Ed,B} = 227 \text{ kN}$

Considering both load cases, the maximum design moments on the composite section without redistribution are:

- design negative moment $M_{Ed,B} = 483 \text{ kNm}$
- design positive moment $M_{Ed,1} = 327 \text{ kNm}$

The maximum design shear force on the composite section is:

$V_{Ed,B} = 243 \text{ kN}$

For the design of continuous composite beams, the effects of cracking of concrete should be taken into account. Cracking of concrete reduces the flexural stiffness at the internal support, at regions of hogging bending moments, but not in sagging regions. The reduction of flexural stiffness should be taken into account in elastic global analysis. In EN 1994-1-1, several different methods are proposed for allowing for cracking in beams. The two methods for calculation of action effects based on elastic theory are shown in Figure B6.7.

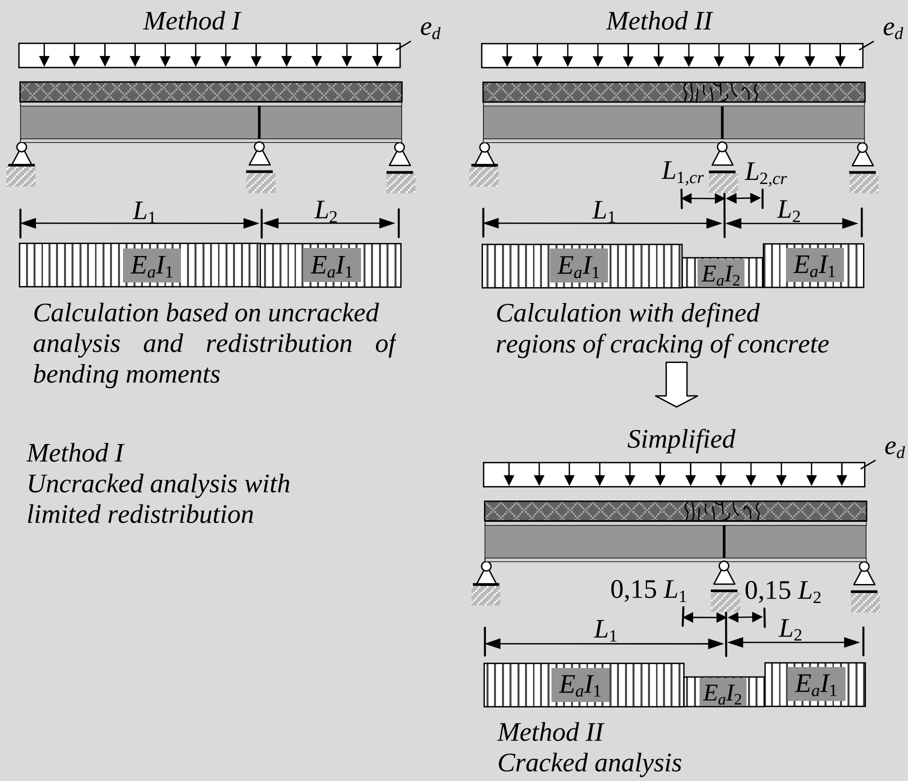


Figure B6.7 *Simplified methods for taking into account the effects of cracking based on elastic theory*

In Method I, Figure B6.7, the internal forces and bending moments are determined for the characteristic combination of actions with uncracked flexural stiffness E_aI_1. According to clause 5.4.4, EN 1994-1-1, the bending moments can be redistributed if the required conditions are satisfied. In Method II, effects of cracking in composite beam are taken into account as follows. The first step is to determine the regions of beam, L_{cr}, where the extreme fibre concrete tensile stress, σ_c, exceeds the limit value. For the ultimate limit state, the criterion is $\sigma_c > 2\,f_{ctm}$ (the extreme-fibre tensile stress in concrete exceeds twice the mean value of the axial tensile strength) and for the serviceability limit state $\sigma_c > 1{,}5\,f_{ctm}$, Figure B6.7. In the second step, the cracked stiffness is then adopted for such sections in the cracked regions and the structure is re-analysed. In this analysis, the beams with cracked regions are treated as beams of non-uniform section. However, for continuous beams, we can use the simplification as follows. Where all the ratios of the length of adjacent continuous spans (shorter/longer) between supports are at least 0,6, the effects of cracking can be taken into account by using the flexural stiffness E_aI_2 over 15% of the span on each side of each internal support, and as the un-cracked values E_aI_1 elsewhere. The simplified Method II is shown in Figure B6.7.

In this example, Method I is used. Accordingly, it is assumed that cross-sections in the hogging region are not cracked.

Remark:

The bending moments and internal forces are calculated using linear elastic global analysis so that, according to Table 5.1, clause 5.4.4, EN 1994-1-1, the appropriate redistribution of moments can be applied.

It is assumed that the cross-section at the support is class 1, such that the bending moment at the support, determined by linear global analysis with uncracked sections, can be reduced to 40%. This is the maximum amount of the redistribution for cross-sections class 1 according to Table 5.1, clause 5.4.4, EN 1994-1-1.

Remark:

For a beam with all cross-sections in class 1 or class 2 only, we can increase the maximum moment at the support by amounts not exceeding 10% for uncracked analysis or 20% for cracked analysis. This increase would lead to a solution closer to rigid plastic global analysis.

Adopting the maximum 40% reduction in support moment allowed, the maximum moments on composite section are:

Design moment at the support (design hogging moment):

$$M_{Ed,B} = 483 \cdot 0,6 = 290 \text{ kNm (load case 1)}$$

Design moment at mid-span (design sagging moment):

$$M_{Ed,1} = 327 + 0,5 \cdot (0,4 \cdot 325) = 392 \text{ kNm (load case 2)}$$

The redistributed shear force is:

$$V_{Ed} = e_{d,\max} \cdot \frac{L}{2} + \frac{M_{Ed,B}}{L} = 38{,}9 \cdot \frac{10}{2} + \frac{290}{10} = 224 \text{ kN}$$

Remark:

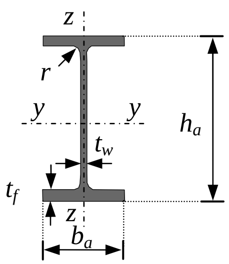
The redistributed design positive moment is an approximate value, assuming that the positive moment at mid-span increases by approximately 1/2 of the decrease in negative moment over the support.

4.3 Check for the construction stage

4.3.1 Selection of steel cross-section

The approximate ratio of span to depth of the steel beam for a continuous composite secondary beam is:

$$\frac{L}{h_a} \approx 25$$



$W_{pl,y} = 2194$ cm^3
$W_{el,y} = 1928$ cm^3
$A_a = 115{,}5$ cm^2
$h_a = 500$ mm
$b_a = 200$ mm
$t_w = 10{,}2$ mm
$t_f = 16$ mm
$r = 21$ mm
$I_{y,a} = 48200$ cm^4
$I_{z,a} = 2142$ cm^4
$I_{w,a} = 1249000$ cm^6
$I_{t,a} = 89{,}29$ cm^4
$g = 90{,}7$ kg/m

Figure B6.8 *Cross-section of steel beam IPE* 500

For span $L = 10$ m, the minimum depth of steel beam is:

$$h_a = \frac{L}{25} = \frac{10 \cdot 10^3}{25} = 400 \text{ mm}$$

Section IPE 500 is selected with the cross-section and the dimensions shown in Figure B6.8.

4.3.2 *Classification of the steel cross-section*

For t_f = 16 mm, the yield strength is f_y = 275 N/mm^2.

$$\varepsilon = \sqrt{\frac{235}{f_y}} = \sqrt{\frac{235}{275}} = 0{,}92$$

For the execution stage, the neutral axis is located in the half depth of the web of the steel section.

The classification of the steel cross-section is conducted according to Table 5.2, EN 1993-1-1.

Flange:

The outstand of the compression flange is shown in Figure B6.9.

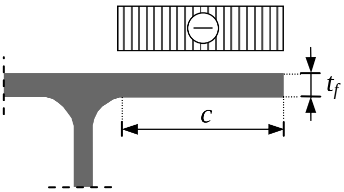


Figure B6.9 *Classification of the flange (compressive stress is negative)*

$$c = \frac{b_a - t_w - 2 \cdot r}{2} = \frac{200 - 10{,}2 - 2 \cdot 21}{2} = 73{,}9 \text{ mm}$$

$$\frac{c}{t_f} = \frac{73{,}9}{16} = 4{,}62$$

The limiting value for class 1 is:

$$\frac{c}{t_f} \leq 9 \cdot \varepsilon = 9 \cdot 0{,}92 = 8{,}28$$

$4{,}62 < 8{,}28$ → Therefore, the flange in compression is class 1

Web:

The web subject to bending is shown in Figure B6.10.

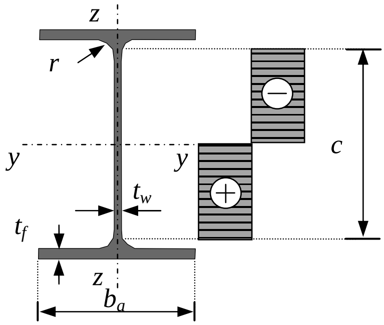


Figure B6.10 *Classification of the web (compressive stress is negative)*

$$c = d = h_a - 2 \cdot t_f - 2 \cdot r = 500 - 2 \cdot 16 - 2 \cdot 21 = 426 \text{ mm}$$

$$\frac{c}{t_w} = \frac{426}{10,2} = 41,8$$

The limiting value for class 1 is:

$$\frac{c}{t_w} \leq 72 \cdot \varepsilon = 72 \cdot 0,92 = 66,2$$

$41,8 < 66,2 \rightarrow$ Therefore the web in bending is class 1

The cross-section is class 1.

4.3.3 Plastic resistance moment of the steel cross-section

The design resistance moment for classes 1 and 2 cross sections is:

$$M_{c,Rd} = M_{pl,a,Rd} = \frac{W_{pl,y} \cdot f_{yd}}{\gamma_{M0}}$$

$$M_{c,Rd} = M_{pl,a,Rd} = \frac{2194 \cdot 27,5 \cdot 10^{-2}}{1,0} = 603 \text{ kNm}$$

Verify that:

$$\frac{M_{y,Ed}}{M_{c,Rd}} \leq 1,0$$

$$\frac{244}{603} = 0,40 < 1,0$$

Therefore the resistance moment is adequate.

4.3.4 Shear resistance of the steel cross-section

According to 6.2.2.3, EN 1994-1-1, the shear buckling resistance of an un-encased web should be verified using Section 5, EN 1993-1-5, if:

$$\frac{h_w}{t} > \frac{72}{\eta}\varepsilon$$

where:

$$\varepsilon = \sqrt{\frac{235}{f_y}} = \sqrt{\frac{235}{275}} = 0,92$$

$\eta = 1,2$, the factor defined in EN 1993-1-5

$$h_w = h_a - 2 \cdot t_f = 500 - 2 \cdot 16 = 468 \text{ mm}$$

$$\frac{72}{\eta} \cdot \varepsilon = \frac{72}{1,2} \cdot 0,92 = 55,2$$

$$\frac{h_w}{t} = \frac{h_w}{t_w} = \frac{468}{10,2} = 45,9$$

Since 45,9 < 55,2, the condition is satisfied. The shear buckling resistance of the web need not be verified.

Remark:

The resistance of the composite beam to vertical shear is normally taken as the shear resistance of the steel section according to clause 6.2.6, EN 1993-1-1, which gives:

$$V_{pl,Rd} = V_{pl,a,Rd} = \frac{A_V (f_y / \sqrt{3})}{\gamma_{M0}}$$

For rolled I- and H-sections where the load is applied parallel to the web, the shear area is calculated as:

$A_V = A_a - 2 \cdot b_a \cdot t_f + t_f \cdot (t_w + 2 \cdot r)$, but not less than $\eta \cdot h_w \cdot t_w$

The shear area A_V is:

$$A_V = 115{,}5 - 2 \cdot 20 \cdot 1{,}6 + 1{,}6 \cdot (1{,}02 + 2 \cdot 2{,}1)$$

$$A_V = 59{,}9 \ \text{cm}^2$$

$$\eta = 1{,}2$$

$$\eta \cdot h_w \cdot t_w = 1{,}2 \cdot 46{,}8 \cdot 1{,}02 = 57{,}3 \ \text{cm}^2$$

59,9 cm^2 > 57,3 cm^2

Therefore, A_V = 59,9 cm^2.

The design plastic shear resistance of the steel section is:

$$V_{pl,Rd} = V_{pl,a,Rd} = \frac{A_V (f_y / \sqrt{3})}{\gamma_{M0}}$$

$$V_{pl,Rd} = V_{pl,a,Rd} = \frac{59{,}9}{1{,}0} \cdot \frac{27{,}5}{\sqrt{3}} = 951 \ \text{kN}$$

Verify that:

$$\frac{V_{Ed}}{V_{pl,Rd}} \leq 1{,}0$$

$$\frac{122}{951} = 0{,}13 < 1{,}0$$

Therefore, the shear resistance of the cross-section is adequate.

4.3.5 Interaction of M-V (bending and shear force)

Where the shear force is less than half the plastic shear resistance its effect on the resistance moment can be neglected.

$$0{,}5 \cdot V_{pl,Rd} = 0{,}5 \cdot 951 = 476 \text{ kN}$$

$V_{Ed,B} = 122 \text{ kN} < 0{,}5\, V_{pl,Rd} = 476 \text{ kN}$ no reduction in the resistance moment

$$M_{y,V,Rd} = M_{c,Rd} = 603 \text{ kNm}$$

Verify that:

$$\frac{M_{y,Ed}}{M_{c,Rd}} \leq 1{,}0$$

$\dfrac{244}{603} = 0{,}40 < 1{,}0$, the resistance to combined shear and bending is satisfactory.

4.3.6 Lateral-torsional buckling of the steel beam

The continuous beam is unpropped at the construction stage. It is necessary to verify the resistance to lateral-torsional buckling of the steel beam according to EN 1993-1-1.

The elastic critical moment of lateral-torsional buckling is calculated as:

$$M_{cr} = C_1 \cdot \frac{\pi^2 \cdot E \cdot I_z}{(k \cdot L)^2} \cdot [\sqrt{(\frac{k}{k_w})^2 \frac{k}{k_w} \cdot \frac{I_w}{I_z} + \frac{(k \cdot L)^2 \cdot G \cdot I_t}{\pi^2 \cdot E \cdot I_z} + (C_2 \cdot z_g)^2} - C_2 \cdot z_g\,]$$

$L = 1000$ cm the length between points at which the compression flange is laterally restrained,

$z_g = \dfrac{h}{2} = \dfrac{50{,}0}{2} = 25{,}0$ cm the distance of the shear centre from the point of application of the load,

$$G = \frac{E}{2(1+v)} = \frac{21000}{2 \cdot (1+0{,}3)} = 8077 \text{ kN/cm}^2$$

The shape of the moment diagram and the load from Figure B6.3 give:

- the effective length factors that depend on the support conditions at the end sections $k = 1{,}0$ and $k_w = 1{,}0$,

- the coefficient which takes into account the shape of the moment diagram C_1 and the coefficient which takes into account the destabilizing or stabilizing effect of the position of the load C_2 are found according to [3].

$$\mu = \frac{qL^2}{8M} = -\frac{19{,}5 \cdot 10{,}0^2}{8 \cdot 243} = -1{,}00$$

$$\psi = \frac{0}{243} = 0$$

$$C_1 = 2{,}22$$

$$C_2 = 0{,}88$$

$$M_{cr} = 2{,}22 \cdot \frac{\pi^2 \cdot 21000 \cdot 2142}{(1 \cdot 1000)^2} \cdot \left[\sqrt{\left(\frac{1}{1}\right)^2 \cdot \frac{1249000}{2142} + \frac{(1 \cdot 1000)^2 \cdot 8077 \cdot 89{,}29}{\pi^2 \cdot 21000 \cdot 2142} + (0{,}88 \cdot 25)^2} - 0{,}88 \cdot 25\right]$$

M_{cr} = 29450 kNcm = 295 kNm

Non-dimensional slenderness:

$$\bar{\lambda}_{LT} = \sqrt{\frac{W_y \cdot f_y}{M_{cr}}}$$

for classes 1 and 2 $W_y = W_{pl,y}$

$$\bar{\lambda}_{LT} = \sqrt{\frac{2194 \cdot 27{,}5}{29450}} = 1{,}43 > \bar{\lambda}_{LT,0} = 0{,}4$$

The reduction factor for lateral-torsional buckling – General method:

$$\chi_{LT} = \frac{1}{\Phi_{LT} + \sqrt{\Phi_{LT}^2 - \bar{\lambda}_{LT}^2}} \quad \text{but} \quad \chi_{LT} \leq 1{,}0$$

$$\Phi_{LT} = 0{,}5\,[1 + \alpha_{LT}\,(\bar{\lambda}_{LT} - 0{,}2) + \bar{\lambda}_{LT}^2]$$

$\dfrac{h}{b} = \dfrac{500}{200} = 2{,}5 > 2$, rolled I-section → the buckling curve *b* is governed

For the buckling curve *b* → $\alpha_{LT} = 0{,}34$, (α_{LT} is the imperfection factor).

$$\Phi_{LT} = 0{,}5\,[1 + 0{,}34 \cdot (1{,}43 - 0{,}2) + 1{,}43^2] = 1{,}73$$

$$\chi_{LT} = \frac{1}{1{,}73 + \sqrt{1{,}73^2 - 1{,}43^2}} = 0{,}37$$

The design buckling resistance moment is:

$$M_{b,Rd} = \chi_{LT} \cdot \frac{W_y \cdot f_y}{\gamma_{M1}}$$

for classes 1 and 2 $W_y = W_{pl,y}$

$$M_{b,Rd} = 0{,}37 \cdot \frac{2194 \cdot 27{,}5}{1{,}0} = 22324 \text{ kNcm} = 223 \text{ kNm}$$

Verify that:

$$\frac{M_{Ed}}{M_{b,Rd}} \leq 1{,}0$$

$$\frac{244}{223} = 1{,}09 > 1{,}00$$

Therefore, the buckling resistance moment of the steel beam is *not* adequate and the steel beam must be laterally restrained at the construction stage.

Remark:

Where the profiled steel sheeting spans perpendicularly to the beam and is attached to its top flange, the beam can be considered as restrained along its length. For this case, the verification is conducted according to EN 1993-1-1.

4.4 Check for the composite stage

4.4.1 Effective width of the concrete flange

The effective width of the concrete flange is calculated according to the expression (5.3), EN 1994-1-1:

$$b_{eff} = b_0 + \Sigma b_{ei}$$

$b_0 = 0$ (there is only one row of shear connectors)

$$b_{ei} = \frac{L_e}{8} \le b_i$$

The equivalent span of the beam in the mid-span region (L_1 = L_2 = 10,0 m) in accordance with Figure 5.1, EN 1994-1-1 is:

$$L_{e,1} = 0,85 \cdot L_1 = 0,85 \cdot 10,0 = 8,5 \text{ m}$$

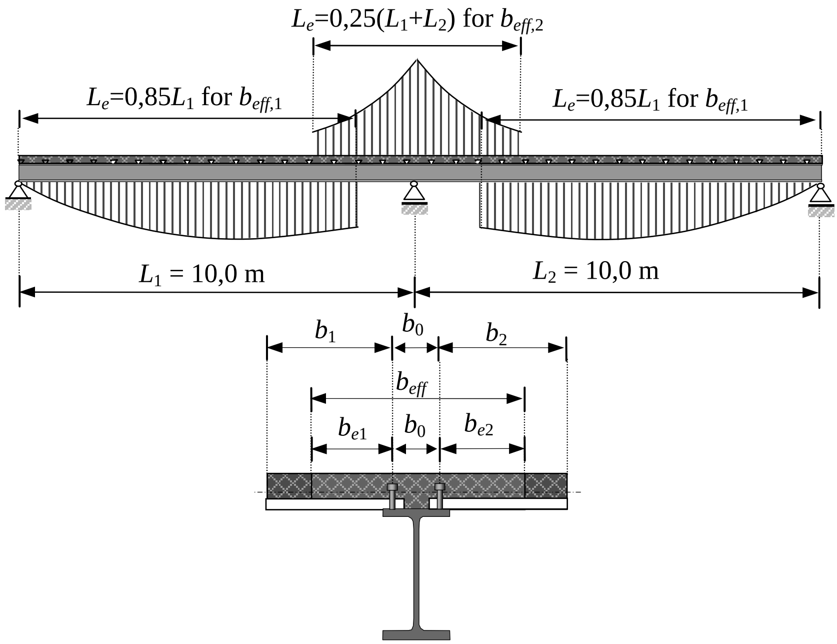


Figure B6.11 *Equivalent spans for effective width of the concrete flange and effective width dimensions*

$$b_1 = \frac{b}{2} = \frac{3,0}{2} = 1,5 \text{ m}$$

$$b_2 = \frac{b}{2} = \frac{3,0}{2} = 1,5 \text{ m}$$

$$b_{e1} = \frac{L_{e1}}{8} \le b_1$$

$$b_{e1} = \frac{8,5}{8} = 1,063 \text{ m} < b_1 = 1,5 \text{ m}$$

Adopted: $b_{e1} = 1,063 \text{ m}$

$$b_{e2} = \frac{L_{e,2}}{8} \le b_2$$

$$b_{e2} = \frac{8,5}{8} = 1,063 \text{ m} < b_2 = 1,5 \text{ m}$$

Adopted: $b_{e2} = 1,063 \text{ m}$

$$b_{eff,1} = b_0 + b_{e1} + b_{e2} = 0 + 1,063 + 1,063 = 2,125 \text{ m}$$

The equivalent span of the beam for the region at internal support ($L_1 = L_2 = 10,0$ m) in accordance with Figure 5.1, EN 1994-1-1 is:

$$L_{e,2} = 0,25 \cdot (L_1 + L_2) = 0,25 \cdot (10,0 + 10,0) = 5,0 \text{ m}$$

$$b_1 = \frac{b}{2} = \frac{3,0}{2} = 1,5 \text{ m}$$

$$b_2 = \frac{b}{2} = \frac{3,0}{2} = 1,5 \text{ m}$$

$$b_{e1} = \frac{L_{e,2}}{8} \le b_1$$

$$b_{e1} = \frac{5}{8} = 0,625 \text{ m} < b_1 = 1,5 \text{ m}$$

Adopted: $b_{e1} = 0{,}625$ m

$$b_{e2} = \frac{L_{e,2}}{8} \le b_2$$

$$b_{e2} = \frac{5}{8} = 0{,}625 \text{ m} < b_2 = 1{,}5 \text{ m}$$

Adopted: $b_{e2} = 0{,}625$ m

$$b_{eff,2} = b_0 + b_{e1} + b_{e2} = 0 + 0{,}625 + 0{,}625 = 1{,}25 \text{ m}$$

4.4.2 Classification of the composite cross-section

The local buckling of cross-sections affects the resistance and rotation capacity of sections. Therefore, local buckling should be considered in design. The classification of cross-sections of composite beams depends on the local slenderness of the flange (b/t) and the web (c/t) of steel beams, the position of the plastic neutral axis and the area of longitudinal reinforcement in the slab at the internal support.

The section classification in EN 1993-1-1 is adopted for composite sections. Table 5.2 of EN 1993-1-1 gives limits for the width-to-thickness ratios for the compression parts of a section for each classification. In addition to the limitations of local slenderness of the flange and the web of steel beams, requirements for ductility of reinforcement in tension are given for class 1 and class 2. The reinforcement should have the ductility class B or C, Table C.1, EN 1992-1-1, and according to clause 5.5.1(5), EN 1994-1-1, the minimum area of reinforcement A_s should satisfy the following condition:

$$A_s \ge \rho_s \cdot A_c$$

with,

$$\rho_s = \delta \frac{f_y}{235} \frac{f_{ctm}}{f_{sk}} \sqrt{k_c}$$

where:

A_c is the effective area of the concrete flange,

f_y is the nominal (characteristic) value of the yield strength of the structural steel in N/mm^2,

f_{sk}	is the characteristic yield strength of the reinforcement N/mm^2,
f_{ctm}	is the mean tensile strength of the concrete,
δ	is a factor which is:
	δ = 1,1 for the procedure plastic–plastic (cross-section class 1),
	δ = 1,0 for the procedure elastic–plastic (cross-section class 2),
k_c	is a coefficient which takes into account the stress distribution within the section immediately prior to cracking and is $k_c = \dfrac{1}{1 + h_c / (2z_0)} + 0,3 \le 1,0$
h_c	is the thickness of the concrete flange, excluding any haunch or ribs,
z_0	is the vertical distance between the centroids of the uncracked concrete flange and the uncracked composite section, calculated using the modular ratio $n_0 = E_a/E_{cm}$ for short-term loading, i.e. at time of the first loading t_0.

4.4.2.1 Cross-section at mid-span

The cross-section in bending at the construction stage is class 1 and therefore, at the composite stage, the cross-section is also class 1.

4.4.2.2 Cross-section at the internal support

Classification of flange

At the internal support, a region of hogging bending, the bottom flange of steel section is in compression. According to the classification of the flange from section 4.3.2, the flange satisfies the condition for class 1.

Classification of web

In composite beams subjected to hogging bending, addition of longitudinal reinforcement in the concrete flange increases the depth of the steel web in compression, α·*c*, in Figure B6.12.

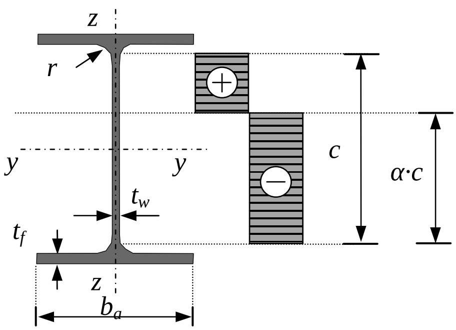


Figure B6.12 *Classification of the web (compression is negative)*

For $\alpha > 0{,}5$, the limiting value for class 1 is:

$$\frac{c}{t} \leq \frac{396 \cdot \varepsilon}{13\alpha - 1}$$

$$c = d = 500 - 2 \cdot 16 - 2 \cdot 21 = 426 \text{ mm}$$

To classify the web, the position of the plastic neutral axis should be determined.

The design resistance moment at internal support, M_{Rd}, can be calculated as:

$$M_{Rd} = \Sigma N_{si} \cdot z_i + M_{a,V,Rd} - 0{,}25 t_w \cdot d_0^2 (1 - \rho) \cdot f_{yd}$$

The area of longitudinal reinforcement in the concrete flange at the internal support rather affects the class of the web. It is necessary to choose a value for the area of longitudinal reinforcement. The reinforcement bars are assumed to be 16 mm at 200 mm with 25 mm of the concrete cover, because larger diameter bars may not give the required control of crack widths. Therefore:

$$d_{bar} = 16 \text{ mm}, \rightarrow A_{bar} = \pi \cdot d_{bar}^2 / 4 = 201 \text{ mm}^2, \rightarrow 201 \cdot 1000 / 200 = 1005 \text{ mm}^2\text{/m}$$

$$A_s = b_{eff,2} \cdot 1005 / 1000 = 1250 \cdot 1005 / 1000 = 1256 \text{ mm}^2 = 12{,}56 \text{ cm}^2$$

The force in these bars is:

$$N_s = f_{sd} \cdot A_s$$

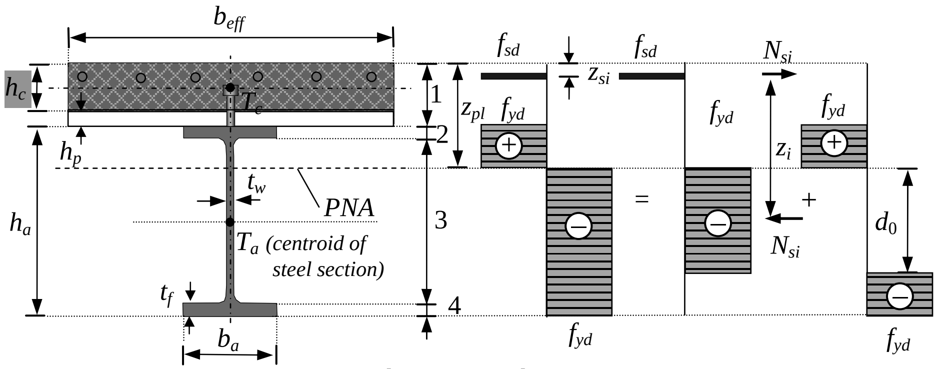
$$N_s = 400 \cdot 1256 \cdot 10^{-3} = 502 \text{ kN}$$

In the case of the hogging bending (moment at internal support), the plastic neutral axis lies within the web of the steel beam, as shown in Figure B6.13:

According to 6.2.2.4, EN 1994-1-1, where the shear force exceeds half the shear resistance its effect on the resistance moment should be taken into account. The reduction factor for the design yield strength of the web is $(1 - \rho)$, where:

$$\rho = \left(\frac{2V_{Ed}}{V_{Rd}} - 1\right)^2$$

and V_{Rd} is the design shear resistance.



PNA – plastic neutral axis

Figure B6.13 *Determination of the design resistance moment in the hogging region*

For the design shear force $V_{Ed} = 243$ kN and the design shear resistance, determined in Section 4.3.4 for the steel section only, $V_{pl,Rd} = V_{pl,a,Rd} = 951$ kN is:

$$\frac{V_{Ed}}{V_{Rd}} = \frac{243}{951} = 0,26 < 0,50$$

Therefore, there is no reduction in the resistance moment.

From equilibrium, the design tensile force in reinforcement $N_{si} = N_s$ and the design compressive force in the web $N_{si} = d_0 \cdot f_{yd} \cdot t_w$, which gives:

$$d_0 = \frac{N_s}{t_w \cdot f_{yd}} = \frac{502}{1,02 \cdot 27,5} = 17,9 \text{ cm}$$

The distance between the plastic neutral axis and the top of the slab z_{pl} is:

$$z_{pl} = h_c + h_p + \frac{h_a}{2} - \frac{d_0}{2} = 9,9 + 5,1 + \frac{50}{2} - \frac{17,9}{2} = 31,1 \text{ cm}$$

The distance between the centroid of the steel section and the centroid of the reinforcement is:

$$z_i = h_c + h_p + \frac{h_a}{2} - z_{si} = 9,9 + 5,1 + \frac{50}{2} - 2,5 = 37,5 \text{ cm}$$

Where the shear force reduces the resistance moment of the steel section, the reduced design resistance moment is:

$$M_{a,V,Rd} = M_{pl,f,Rd} + (M_{pl,a,Rd} - M_{p,f,Rd})(1-\rho)$$

In this case, there is no reduction in the resistance moment. Therefore, the design resistance moment of the steel section $M_{pl,a,Rd}$ is taken into account instead of the reduced design resistance moment $M_{a,V,Rd}$.

The design value of the plastic resistance moment of the steel section is:

$$M_{pl,a,Rd} = 60300 \text{ kNcm} = 603 \text{ kNm}$$

Therefore, the design value of the plastic resistance moment of the composite section at internal support is:

$$M_{pl,Rd} = N_s \cdot z_i + M_{pl,a,Rd} - \frac{t_w \cdot d_0^2 \cdot f_{yd}}{4}$$

$$M_{pl,Rd} = 502 \cdot 37{,}5 + 60300 - \frac{1{,}02 \cdot 17{,}9^2 \cdot 27{,}5}{4} = 76878 \text{ kNcm} = 769 \text{ kNm}$$

For I-sections subject to major-axis bending and axial force with the neutral axis in the web, the parameter α_c can be calculated as:

$$\alpha_c = \frac{1}{c}\left(\frac{h_a}{2} + \frac{1}{2}\frac{N_{Ed}}{t_w \cdot f_{yd}} - (t_f + r)\right)$$

In this case, the design axial compressive force N_{Ed} is equal to the design tensile force in reinforcement. The design axial compressive force N_{Ed} is:

$$N_{Ed} = N_s = 502 \text{ kN}$$

The parameter α_c is:

$$\alpha_c = \frac{1}{42{,}6}\left(\frac{50}{2} + \frac{1}{2}\frac{502}{1{,}02 \cdot 27{,}5} - (1{,}6 + 2{,}1)\right) = 0{,}71$$

For $\alpha > 0{,}5$, the limiting value for class 1 is:

$$\frac{c}{t} = \frac{426}{10,2} = 41,8 \leq \frac{396 \cdot \varepsilon}{13\alpha - 1} = \frac{396 \cdot 0,92}{13 \cdot 0,71 - 1} = 44,3$$

$41,8 < 44,3$ $\rightarrow$ the web is class 1

Therefore, the cross-section is class 1.

All sections within the span of the composite beam, for both sagging and hogging regions, are in class 1.

Minimum reinforcement area A_s

Within the effective width of the composite section, the ductile reinforcement is selected. According to clause 5.5.1(5), EN 1994-1-1, the minimum area of reinforcement A_s should satisfy the following condition:

$$A_s \geq \rho_s \cdot A_c$$

with,

$$\rho_s = \delta \frac{f_y}{235} \frac{f_{ctm}}{f_{sk}} \sqrt{k_c}$$

The area of reinforcement is:

$A_s = 12,56$ cm^2, (bars 16 mm at 200 mm)

The effective area of the concrete slab at the internal support is:

$$A_c = b_{eff,2} \cdot h_c = 125 \cdot 9,9 = 1238 \text{ cm}^2$$

For the elastic–plastic procedure, the factor δ is 1,0.

The coefficient k_c takes into account the stress distribution within the section immediately prior to cracking and is:

$$k_c = \frac{1}{1 + h_c / (2z_0)} \leq 1,0$$

The thickness of the concrete flange h_c is 9,9 cm. The vertical distance between the centroids of the uncracked concrete flange and the uncracked composite section is denoted by z_0. It is calculated using the modular ratio $n_0 = E_a/E_{cm}$ for short-term loading, i.e. at the time of the first loading t_0.

The modular ratio n_0 is:

$$n_0 = \frac{E_a}{E_{cm}} = \frac{210}{35} = 6$$

The effective width of the concrete flange at the internal support is:

$$b_{eff,2} = 125 \text{ cm}$$

Transformed to the ideal steel section, the effective width is:

$$\frac{b_{eff,2}}{n_0} = \frac{125}{6} = 20,8 \text{ cm}$$

The area of the ideal steel cross-section is:

$$A = A_a + \frac{b_{eff,2}}{n_0} \cdot h_c = 115,5 + 20,8 \cdot 9,9 = 321,4 \text{ cm}$$

The distance between the neutral axis and the centroid of the steel section is:

$$z_{n0} = \frac{\frac{b_{eff,2}}{n_0} \cdot h_c \cdot (\frac{h_a}{2} + \frac{h_c}{2} + h_p)}{A} = \frac{20,8 \cdot 9,9 \cdot (\frac{50}{2} + \frac{9,9}{2} + 5,1)}{321,4} = 22,5 \text{ cm}$$

Thus, the vertical distance between the centroids of the uncracked concrete flange and the uncracked composite section is:

$$z_0 = (\frac{h_a}{2} + \frac{h_c}{2} + h_p) - z_{n0} = (\frac{50}{2} + \frac{9,9}{2} + 5,1) - 22,5 = 35,1 - 22,5 = 12,6 \text{ cm}$$

Therefore:

$$k_c = \frac{1}{1 + h_c / (2z_0)} + 0,3 = \frac{1}{1 + 9,9 / (2 \cdot 12,6)} + 0,3 = 1,02 \leq 1,0$$

$$k_c = 1,00$$

$$\rho_s = \delta \frac{f_y}{235} \frac{f_{ctm}}{f_{sk}} \sqrt{k_c} = 1,0 \frac{275}{235} \frac{3,5}{460} \sqrt{1,0} = 0,890\ \%$$

The final verification of the minimum reinforcement is:

$$A_s = 12,56\ \text{cm}^2 \geq \rho_s \cdot A_c = 0,00890 \cdot 1238 = 11,02\ \text{cm}^2$$

The condition is satisfied.

4.4.3 Check of shear connection

4.4.3.1 Resistance of the headed stud connectors

The design resistance of a single-headed shear connector in a solid concrete slab is determined by shank failure of the stud ($P_{Rd}^{(1)}$) or by concrete failure ($P_{Rd}^{(2)}$). The design resistance of a single-headed shear connector in a solid concrete slab, automatically welded in accordance with EN 14555, should be determined as the smaller of:

$$P_{Rd} = \min(P_{Rd}^{(1)}, P_{Rd}^{(2)})$$

$$P_{Rd}^{(1)} = \frac{0,8 \cdot f_u \cdot (\pi \cdot d^2)/4}{\gamma_V}$$

$$P_{Rd}^{(2)} = \frac{0,29 \cdot \alpha \cdot d^2 \cdot \sqrt{f_{ck} \cdot E_{cm}}}{\gamma_V}$$

where:

- d is the diameter of the shank of the stud (16 mm $\leq d \leq$ 25 mm),
- h_{sc} is the overall nominal height of the stud,
- f_u is the specified ultimate tensile strength of the material of the stud but not greater than 500 N/mm^2,
- f_{ck} is the characteristic cylinder compressive strength of the concrete,
- E_{cm} is the mean value of the secant modulus of elasticity of the concrete,
- γ_V is the partial factor for the stud (γ_V = 1,25),
- α is the correction factor which takes into account the ratio of the height of the stud to the diameter of its shank.

The correction factor α is calculated as:

$$\alpha = 0,2[(\frac{h_{sc}}{d}) + 1] \quad \text{for} \quad 3 \le \frac{h_{sc}}{d} \le 4$$

$$\alpha = 1,0 \quad \text{for} \quad \frac{h_{sc}}{d} > 4$$

Since h_{sc}/d = 95/19 = 5,00, then α = 1.

Therefore:

$$P_{Rd}^{(1)} = \frac{0,8 \cdot f_u \cdot (\pi \cdot d_{stud}^2)/4}{\gamma_V}$$

$$P_{Rd}^{(1)} = \frac{0,8 \cdot 450 \cdot (\pi \cdot 19^2)/4}{1,25} \cdot 10^{-3} = 82 \ \text{kN}$$

$$P_{Rd}^{(2)} = \frac{0,29 \cdot \alpha \cdot d^2 \cdot \sqrt{f_{ck} \cdot E_{cm}}}{\gamma_V}$$

$$P_{Rd}^{(2)} = \frac{0,29 \cdot 1 \cdot 19^2 \cdot \sqrt{40 \cdot 35 \cdot 10^3}}{1,25} \cdot 10^{-3} = 99 \ \text{kN}$$

Remark:

The resistance of headed studs used as shear connectors with profiled steel sheeting is less than the design resistance of headed studs used as shear connectors in a solid concrete slab.

The resistance of a headed stud within profiled sheeting is determined by multiplying the design resistance for a headed stud connector in a solid concrete slab (P_{Rd}) by a reduction factor k_l for profiled steel sheeting spanning parallel to the supporting beam, and k_t for profiled steel sheeting spanning transverse to the supporting beam.

For profiled steel sheeting spanning parallel to the supporting beam, the design resistance of a single-headed shear connector is:

$$P_{Rd} = k_l \cdot \min(P_{Rd}^{(1)}, P_{Rd}^{(2)})$$

The reduction factor k_l is calculated according to clause 6.6.4.1(2), EN 1994-1-1:

$$k_l = 0,6(\frac{b_0}{h_p})(\frac{h_{sc}}{h_p} - 1) \leq 1,0$$

where h_{sc} is the overall height of the stud, but not greater than h_p + 75 mm.

For profiled steel sheeting spanning transverse to the supporting beam, the design resistance of a single-headed shear connector is:

$$P_{Rd} = k_t \cdot \min(P_{Rd}^{(1)}, P_{Rd}^{(2)})$$

The reduction factor k_t is calculated according to clause 6.6.4.2(1), EN 1994-1-1:

$$k_t = \frac{0,70}{\sqrt{n_r}} \frac{b_0}{h_p} (\frac{h_{sc}}{h_p} - 1) \leq k_{t,max}$$

In the above expression n_r is the number of studs in one rib at a beam intersection, not to exceed 2 in the calculations, and $k_{t,max}$ is the maximum value of the reduction factor k_t which is given in Table 6.2, EN 1994-1-1.

Since in this example the profiled steel sheeting has ribs running transverse to the supporting beam, we need to check the effect of a reduction factor, k_t, on the shear connector resistance.

The reduction factor depends on the overall height of the stud, h_{sc}, the dimensions of the trough in the profiled sheeting (Figure B6.2), the thickness of the profiled sheeting (assumed to be b_e > 1,0 mm) and the number of studs per trough, n_r.

For a single stud per trough the number of studs per trough is n_r = 1.

The reduction factor k_t is:

$$k_t = \frac{0,7}{\sqrt{n_r}} \cdot \frac{b_0}{h_p} \cdot (\frac{h_{sc}}{h_p} - 1)$$

$$k_t = \frac{0,7}{\sqrt{1}} \cdot \frac{140}{51} \cdot (\frac{95}{51} - 1) = 1,66 \text{ but} \leq 1,00 \text{ according to Table 6.2, EN 1994-1-1}$$

Since 1,66 > 1,00, there is no reduction in the shear connector resistance.

Therefore, the design shear connector resistance is:

$$P_{Rd} = P_{Rd}^{(1)} = 82 \text{ kN}$$

4.4.3.2 Arrangement of the headed studs and the degree of shear connection

For the span with loading 38,9 kN/m, the end reaction is $R_{Ed,A}$ = 146 kN. The point of maximum positive moment is at a distance 146/38,9 = 3,75 m from the support. Therefore, for the positioning of the shear stud connectors between this support and the point of maximum positive moment there are 24 available troughs. There are 41 shear stud positions between the internal support and the point of maximum negative moment.

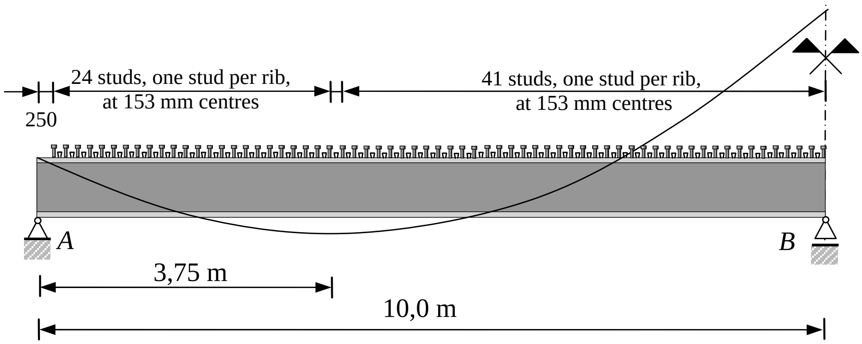


Figure B6.14 *Arrangement of the stud connectors for load case 1*

For load case 2 the end reaction is $R_{Ed,A}$ = 162 kN. The point of maximum positive moment is at a distance 162/38,9 = 4,16 m ≈ 4,00 from the support. Therefore, for the positioning of the shear stud connectors between this support and the point of maximum positive moment there are 26 available troughs. There are 39 shear stud positions between the internal support and the point of maximum negative moment.

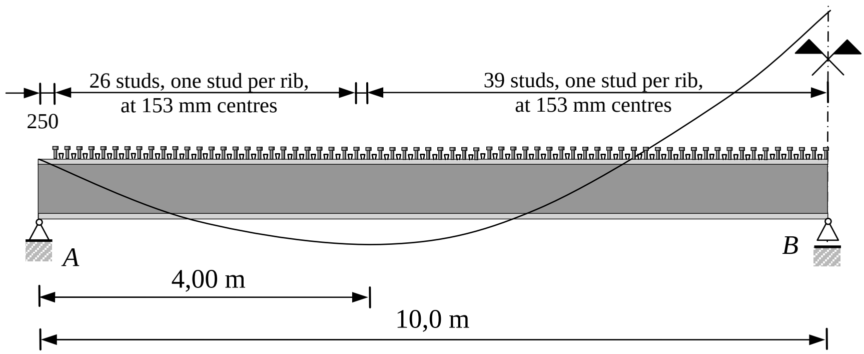


Figure B6.15 *Arrangement of the stud connectors for load case 2*

Remark:

Partial shear connection is permitted in sagging regions of composite beams.
Full shear connection is required in hogging regions of composite beams.

The longitudinal shear force transfer, $V_{L,Ed}$, between the support A and the point of maximum positive moment is:

$$V_{L,Ed} = 24 \cdot P_{Rd}^{(1)} = 24 \cdot 82 = 1968 \text{ kN (load case 1)}$$

$$V_{L,Ed} = 26 \cdot P_{Rd}^{(1)} = 26 \cdot 82 = 2132 \text{ kN (load case 2)}$$

Remark:

The assumed critical area is between the support A and the point of maximum positive moment because, between the point of maximum positive moment and the internal support there are significantly more studs, even if the studs within the hogging region of the beam are neglected. This assumption should be verified when the number of studs needed to yield the slab reinforcement in the hogging region is determined (see Section 4.4.4.2).

The equivalent span of the beam at mid-span (L_1 = L_2 = 10,0 m) according to Figure 5.1, EN 1994-1-1, is:

$$L_{e,1} = 0,85 \cdot L_1 = 0,85 \cdot 10,0 = 8,5 \text{ m}$$

For $L_e \leq$ 25 m, according to clause 6.6.1.2 (3), EN 1994-1-1, the limit for the degree of shear connection is:

$$\eta_{\min} = 1 - (\frac{355}{f_y}) \cdot (1,00 - 0,04 \cdot L_e), \qquad \eta \geq 0,4$$

$$\eta_{\min} = 1 - (\frac{355}{275}) \cdot (1,00 - 0,04 \cdot 8,5) = 0,15, \qquad \eta \geq 0,4$$

A larger value is adopted:

$$\eta_{\min} = 0,40$$

The actual degree of shear connection, η, is given by:

$$\eta = \frac{N_c}{N_{c,f}} = \frac{V_{L,Ed}}{N_{pl,a}} = \frac{1968}{3176} = 0,62 \text{, (load case 1)}$$

$$\eta = \frac{N_c}{N_{c,f}} = \frac{V_{L,Ed}}{N_{pl,a}} = \frac{2132}{3176} = 0,67 \text{, (load case 2)}$$

Both of these values are greater than $\eta_{\min} = 0,40$. Therefore the condition is satisfied.

4.4.4 Resistance moment of the composite cross-section

4.4.4.1 Resistance moment at mid-span

According to clause 6.2.1.3, EN 1994-1-1, the partial shear connection can be used in the region of sagging bending.

If ductile shear connectors are used, the resistance moment can be determined by means of rigid plastic theory in accordance with 6.2.1.2, EN 1994-1-1. However, the reduced value of the compressive force N_c must be taken into account instead of the force $N_{c,f}$.

It is very convenient to use the diagram of partial shear connection, shown in Figure 6.5, EN 1994-1-1, for determining the resistance moment. According to clause 6.2.1.3(5), EN 1994-1-1, the design resistance moment of the composite beam in sagging region can be conservatively calculated by the straight line AC in Figure 6.5, EN 1994-1-1:

$$M_{Rd} = M_{pl,a,Rd} + (M_{pl,Rd} - M_{pl,a,Rd}) \cdot \frac{N_c}{N_{c,f}}$$

where:

$M_{pl,a,Rd}$ is the design plastic resistance moment of the structural steel section alone in sagging region,
$M_{pl,Rd}$ is the design plastic resistance moment of the composite section with full shear connection to sagging bending,
$N_{c,f}$ is the design compressive force in the concrete flange with full shear connection,
N_c is the design compressive force in the concrete flange.

In this example, the simplified procedure is used. The procedure according to rigid plastic theory is used in examples B7 and B8. The number of shear studs between the supports and the point of maximum sagging bending moment varies between the two load cases. Accordingly, the degree of shear connection varies between load case 1 and load case 2. Theoretically, it is necessary to consider the separate values of sagging resistance moment corresponding to the two load cases.

However, in this example a conservative approach is adopted. The sagging resistance moment of the composite section is calculated using the minimum shear connection corresponding to load case 1. Then, this resistance moment is compared with the maximum sagging design moment corresponding to load case 2.

The resistance moment for the sagging regions of the composite beam is calculated in the same way as for the simply supported composite beam.

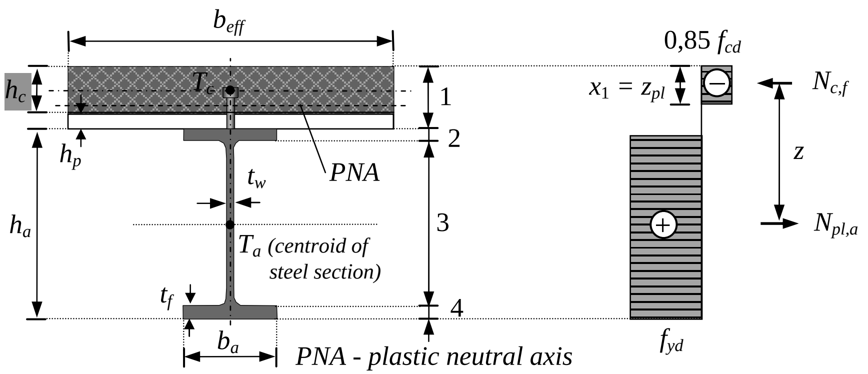


Figure B6.16 *Cross-section of composite beam at mid-span*

The plastic neutral axis lies within the thickness of the concrete flange if:

$$N_{c,f} > N_{pl,a}$$

The design plastic resistance of the structural steel section to normal force is:

$$N_{pl,a} = \frac{A_a \cdot f_{yd}}{\gamma_{M,0}} = \frac{115,5 \cdot 27,5}{1,0} = 3176 \text{ kN}$$

The design compressive resistance, $N_{c,f}$, neglecting the contribution of the reinforcement in compression according to clause 6.2.1.2(1), EN 1994-1-1, is:

$$N_{c,f} = h_c \cdot b_{eff} \cdot 0,85 f_{cd}$$

$$N_{c,f} = 99,0 \cdot 2125 \cdot 0,85 \cdot 26,7 \cdot 10^{-3} = 4774 \text{ kN}$$

$4774 > 3176$, the plastic neutral axis lies within the thickness of concrete flange.

Therefore, the design plastic resistance moment of the composite section with full shear connection in sagging region $M_{pl,Rd}$ is:

$$M_{pl,Rd} = N_{pl,a} \cdot z$$

$$M_{pl,Rd} = N_{pl,a} \cdot (0,5h_a + h_c + h_p - 0,5z_{pl})$$

$$z_{pl} = \frac{N_{pl,a}}{b_{eff} \cdot 0,85 f_{cd}} < h_c$$

$$z_{pl} = \frac{3176}{2125 \cdot 0,85 \cdot 2,67} = 0,66 \text{ cm} < h_c = 9,9 \text{ cm}$$

$$M_{pl,Rd} = 3176 \cdot (0,5 \cdot 50 + 9,9 + 5,1 - 0,5 \cdot 0,66) = 125992 \text{ kNcm} = 1260 \text{ kNm}$$

Thus, the design resistance moment of composite beam in the sagging region with the partial shear connection (using the minimum shear connection) is:

$$M_{Rd} = M_{pl,a,Rd} + (M_{pl,Rd} - M_{pl,a,Rd}) \cdot \frac{N_c}{N_{c,f}}$$

$$M_{Rd} = 603 + (1260 - 603) \cdot 0,62 = 1010 \text{ kNm}$$

The sagging bending moment is M_{Ed} = 392 kNm and the check is as follows:

$$\frac{M_{Ed}}{M_{pl,Rd}} \leq 1,0$$

$\frac{392}{1010} = 0,39 < 1,0$, the condition is satisfied

4.4.4.2 Resistance moment at the internal support

The design plastic resistance moment in hogging region is determined in accordance with clause 6.2.1.2, EN 1994-1-1, assuming that there is the full shear connection between structural steel, reinforcement and concrete. The appropriate shear connection needs to be provided to ensure yielding of the reinforcement in tension.

In other words, there need to be sufficient studs in the hogging region.

Therefore, the yielding of the reinforcement in tension occurs before failure of the studs.

The required number of studs to ensure that yielding of the reinforcement in tension is determined if the design tensile force in reinforcement N_s is divided by the governed design resistance of the studs:

$$\frac{N_s}{P_{Rd}^{(1)}} = \frac{502}{82} = 6$$

Therefore, in the hogging region of the beam there are sufficient studs to yield the reinforcement in tension.

The design plastic resistance moment in hogging region was calculated in Section 4.4.2.2 at the classification of the composite section and it is:

$$M_{pl,Rd} = 769 \text{ kNm}$$

The design bending moment at the internal support is M_{Ed} = 290 kNm and the check is as follows:

$$\frac{M_{Ed}}{M_{pl,Rd}} \leq 1{,}0$$

$\frac{290}{769} = 0{,}38 < 1{,}0$, the condition is satisfied

4.4.5 Lateral-torsional buckling of the composite beam

Clause 6.4.3, EN 1994-1-1, gives guidance for the verification of buckling resistance moment of continuous beam in buildings.

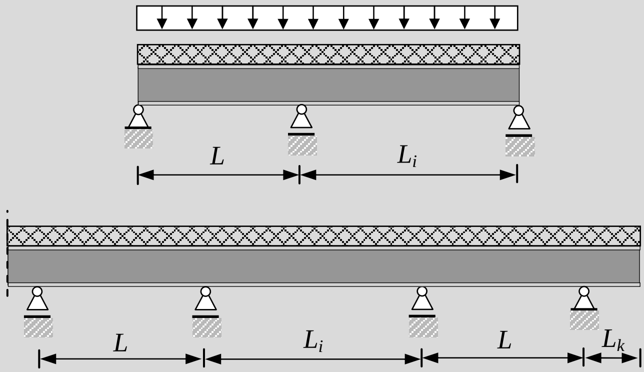
If the conditions, given in clause 6.4.3(1), EN 1994-1-1, are satisfied, the verification of lateral-torsional buckling is not necessary.

The conditions are as follows:

a) The ratio of adjacent spans

The difference in length of adjacent spans is less than 20% of the shorter span. The length of the cantilever is less than 15% of the adjacent span.

$0{,}8 \leq L/L_i \leq 1{,}20$ $\qquad$ $L_k/L \leq 0{,}15$



b) The ratio of permanent and total design loads

The loads are uniformly distributed on each span. The design permanent load is greater than the total design load by 40% or more.

$$\frac{\gamma_G \cdot G_k}{\gamma_G \cdot G_k + \gamma_Q \cdot Q_k} \geq 0{,}4$$

c) Shear connection

The shear connection between the upper flange of the steel beam and the concrete flange should be provided in accordance with clause 6.6 (EN 1994-1-1).

d) The inverted-U frame

The same concrete flange is also attached to one or more supporting steel beams so that they form an inverted-U frame.

e) The composite slab

The span of the composite slab between the two supporting beams of the inverted-U frame should be taken into consideration.

f) The lateral restraint of the bottom flange of the steel beam

The bottom flange of steel beam is laterally restrained at each support. Also, the web of the steel beam is stiffened.

g) Composite beams that are not partially encased

The limitations of depth for steel beams of IPE section or HE section that are not partially encased are given in Table B6.1.

h) Composite beams that are partially encased

The depth of partially encased composite beams does not exceed the limit given in Table B6.1 by more than 200 mm for steel grades up to S355 and by 150 mm for grades S420 and S460.

Table B6.1 *Maximum depth h (mm) of uncased steel beam, EN 1994-1-1*

Steel beam	*Nominal steel grade*			
	*S*235	*S*275	*S*355	*S*420 *and S*460
IPE	600	550	400	270
HE	800	700	650	500

According to clause 6.3.2.2(4), EN 1993-1-1, the verification of lateral-torsional buckling for the member in bending may be neglected if at least one of the following conditions is satisfied:

$\bar{\lambda}_{LT} \leq \bar{\lambda}_{LT,0}$ or $M_{Ed} / M_{cr} \leq \bar{\lambda}_{LT,0}$, with maximum value $\bar{\lambda}_{LT,0} = 0,4$

Verification of conditions:

a) The ratio of adjacent spans

The condition is satisfied because the spans L_1 and L_2 are the same.

b) The ratio of permanent and total design loads

$$\frac{\gamma_G \cdot g_{k,2} + \gamma_G \cdot g_{k,3}}{\gamma_G \cdot g_{k,2} + \gamma_G \cdot g_{k,3} + \gamma_Q \cdot q_k} = \frac{1,35 \cdot 4,11 + 1,35 \cdot 0,50}{1,35 \cdot 4,11 + 1,35 \cdot 0,50 + 1,5 \cdot 4,5} = 0,48 > 0,4$$

The condition is satisfied.

c) Shear connection

The condition is satisfied.

d) The inverted-U frame

The condition is satisfied.

e) The composite slab

The condition is satisfied.

f) Lateral restraint of the bottom flange of the steel beam

The condition is satisfied.

g) Composite beams that are not partially encased

The condition is satisfied.

h) Composite beams that are partially encased

In this case, the composite beam is not partially encased. The condition is not governed.

Since the conditions are satisfied, the additional verification to lateral-torsional buckling is not necessary.

4.4.6 Check of longitudinal shear resistance of the concrete flange

4.4.6.1 Check of transverse reinforcement

In practice it is usual to neglect the contribution of the steel sheeting, and the cross-sectional area of the transverse reinforcement A_{sf} at spacing s_f should satisfy:

$$\frac{A_{sf}}{s_f} \cdot f_{sd} \geq v_{L,Ed} \cdot \frac{h_f}{\cot\theta}$$

where:

A_{sf}/s_f is the transverse reinforcement expressed in mm^2/m,
h_f is the depth of concrete above the profiled sheeting, see Figures B6.17 and B6.18,
θ is the angle between the diagonal strut and the axis of the beam (strut-and-tie model),
$v_{L,Ed}$ is the design longitudinal shear flow in the concrete slab.

Remark:

In this example, the contribution of the profiled steel sheeting to the transverse reinforcement is neglected, although clause 6.6.6.4(4), EN 1994-1-1 allows that the contribution of the profiled steel sheeting with ribs transverse to the beam, continuous across the top flange of the steel beam and with mechanical interlocking to be taken into account.

The transverse reinforcement (A_{sf}/s_f) expressed in mm^2/m can be denoted as A_t, the cross-sectional area of the top transverse reinforcement, for failure due to shear in the failure plane shown in Figure B6.17 as section *a-a*.

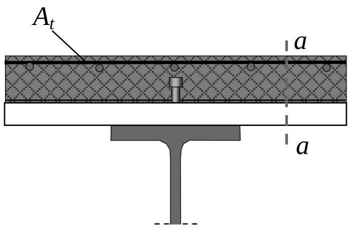


Figure B6.17 *Surface of potential failure in longitudinal shear*

When the concrete flange is in compression, longitudinal shear flow $v_{L,Ed}$ can be defined by the expression:

$$v_{L,Ed,1} = \frac{\Delta N_{c1}}{a_v} = \frac{V_{L,Ed}}{a_v}\frac{A_{c1,eff}}{A_{c,eff}}$$

where:

a_v is the critical length (the distance between two given sections, Figure B6.18),

ΔN_{c1} is the change in the longitudinal compressive forces in the slab over the critical length a_v, see Figure B6.18,

$V_{L,Ed}$ is the design longitudinal shear force in the steel-concrete interface or in the concrete flange,

$V_{L,Ed} = \min(N_{pl,a}, N_c, \Sigma P_{Rd})$

When the concrete flange is in tension, longitudinal shear flow $v_{L,Ed}$ can be defined by the expression:

$$v_{L,Ed,1} = \frac{\Delta N_{s1}}{a_v} = \frac{V_{L,Ed}}{a_v}\frac{A_{s1}}{A_{s1} + A_{s2}}$$

where:

a_v is the critical length (the distance between two given sections, Figure B6.18),

ΔN_{s1} is the change of the longitudinal tensile forces in the slab over the critical length a_v, see Figure B6.18,

$V_{L,Ed}$ is the design longitudinal shear force in the steel-concrete interface or in the concrete flange,

$$V_{L,Ed} = \min(N_s, \Sigma P_{Rd})$$

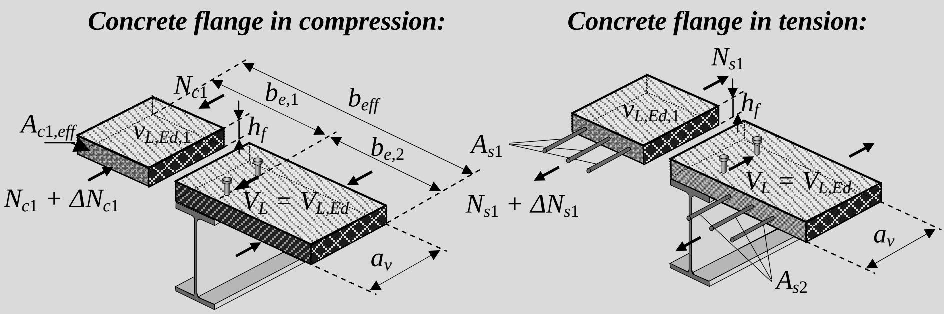


Figure B6.18 *Determination of longitudinal shear forces in the concrete flange*

In this example, only the region L_I = 3,75 m (load case 1) is taken into consideration. The design longitudinal shear force is determined from minimum resistance of the steel section, concrete and shear connectors:

$$V_{L,Ed} = \min(N_{pl,a}, N_c, \Sigma P_{Rd})$$

$$N_{pl,a} = 3176 \text{ kN}$$

$$\Sigma P_{Rd} = 1968 \text{ kN}$$

$$N_{c,f} = 4774 \text{ kN}$$

The maximum force that can be transferred is limited by the resistance of the shear connectors, and the design shear stress, $v_{L,Ed}$, is determined for one shear connector per trough of profiled sheeting:

$$a_v = 153 \text{ mm}$$

$$V_{L,Ed} = P_{Rd} = 82 \text{ kN}$$

$$f_{sd} = 400 \text{ N/mm}^2$$

$$h_f = h_c = 99 \text{ mm}$$

$$v_{L,Ed} = \frac{\Delta N_{c1}}{h_f \cdot a_v} = \frac{V_{L,Ed}}{2h_f a_v} = \frac{82 \cdot 10^3}{2 \cdot 99 \cdot 153} = 2{,}71 \text{ N/mm}^2$$

Remark:

In order to prevent splitting of the concrete flange, for the adopted "truss model", according to clause 6.2.4(4) EN 1992-1-1, the angle θ between the concrete diagonals and the longitudinal direction is limited to the value:

$26{,}5^\circ \leq \theta \leq 45^\circ$ concrete flange in compression

$38{,}6^\circ \leq \theta \leq 45^\circ$ concrete flange in tension

In order to minimize the cross-sectional area of the transverse reinforcement, the minimum angle θ is selected. For the concrete flange in tension (in the region of the internal support), the minimum angle θ is:

$$\theta = 38{,}6^\circ$$

$$\frac{A_{sf}}{s_f} \cdot f_{sd} \geq v_{L,Ed} \cdot \frac{h_f}{\cot\theta}$$

$$\frac{A_{sf}}{s_f} \geq \frac{v_{L,Ed}}{f_{sd}} \cdot \frac{h_f}{\cot\theta} = \frac{2{,}71}{400} \frac{99}{\cot 38{,}6} \cdot 10^3 = 535 \text{ mm}^2/\text{m}$$

The reinforcement provided is 12 mm bars at 200 mm, for which:

$$A_t = \frac{\pi \cdot 12^2}{4} \cdot \frac{1000}{200} = 565 \text{ mm}^2/\text{m} > 535 \text{ mm}^2/\text{m}$$

According to EN 1994-1-1, clause 6.6.6.3, the minimum area of transverse reinforcement is determined in accordance with EN 1992-1-1, clause 9.2.2(5), which gives the minimum area of reinforcement as a proportion of the concrete area. The ratio is:

$$\rho_{w,\min} = \frac{0{,}08\sqrt{f_{ck}}}{f_{yr,k}}$$

where:

f_{ck} is the characteristic compressive cylinder strength of the concrete at 28 days in N/mm^2,

$f_{yr,k} = f_{sk}$ is the characteristic yield strength of the reinforcement in N/mm^2.

The minimum area of transverse reinforcement is:

$$\rho_{w,\min} = \frac{0{,}08\sqrt{f_{ck}}}{f_{yr,k}} = \frac{0{,}08\sqrt{40}}{460} = 0{,}0011$$

$$A_c = h_c \cdot b = 99 \cdot 1000 = 99000 \ \text{mm}^2$$

$$A_{s,\min} = \rho_{w,\min} \cdot A_c = 0{,}0011 \cdot 99000 = 109 \ \text{mm}^2/\text{m}$$

Since $A_t = 565 \ \text{mm}^2/\text{m} > A_{s,\min} = 109 \ \text{mm}^2/\text{m}$, the requirement of minimum transverse reinforcement is satisfied.

4.4.6.2 Crushing of the concrete flange

To prevent crushing of the compression struts in the flange, the following condition should be satisfied according to EN 1992-1-1, expression 6.22:

$$v_{L,Ed} \le v_{Rd}$$

$$v_{L,Ed} \le \nu \cdot f_{cd} \cdot \sin\theta \cdot \cos\theta$$

where:

$$\nu = 0{,}6 \cdot (1 - \frac{f_{ck}}{250})$$

θ the angle between the concrete diagonals and the longitudinal direction.

In order to minimize the resistance of the concrete compression strut (in the mid-span region), the minimum angle θ is selected. For the concrete flange in compression, the minimum angle θ is:

$$\theta = 26{,}5°$$

$$v_{Rd} = \nu \cdot f_{cd} \cdot \sin\theta \cdot \cos\theta = 0{,}6 \cdot (1 - \frac{40}{250}) \cdot 26{,}7 \cdot \sin 26{,}5° \cdot \cos 26{,}5° = 5{,}37 \ \text{N/mm}^2$$

Check:

$$v_{L,Ed} = 2{,}71 \ \text{N/mm}^2 < v_{Rd} = 5{,}37 \ \text{N/mm}^2$$

Therefore the crushing resistance of the concrete compression strut is adequate.

5. Serviceability limit state

5.1 General

Chapter 7, EN 1994-1-1, is limited to provisions relating to serviceability that are specific to composite structures. Serviceability verifications in the case of composite structures generally include checks of stress, deflection and vibration as well as control of the crack width.

For buildings, stress limitation is not required for beams if, in the ultimate limit state, no verification of fatigue is required and no pre-stressing by tendons and/or by controlled imposed deformations is provided. However, if the stress limitation is required, clause 7.2, EN 1992-1-1 gives stress limits which may be applicable for buildings that have pre-stressing or fatigue loading.

Since the deflection is one of the most important verifications of the serviceability limit state, it is necessary to explain in detail the problems associated with the deflection calculation. Deflections due to loads applied to the composite member are calculated using elastic theory, taking into account the following effects:

a) Cracking of concrete

Cracking of concrete reduces the flexural stiffness in regions of hogging moments in the case of continuous composite beams. Several methods are proposed in EN 1994-1-1 for calculation of bending moments, internal forces and deformation, see Section 4.2.

b) Creep and shrinkage of concrete

The effect of creep and shrinkage may be taken into account by using the effective modulus of elasticity of concrete $E_{c,eff}$, or by using the modular ratio n_L which depends on the type of loadings. The expression for the effective modulus of elasticity of the concrete $E_{c,eff}$ is:

$$E_{c,eff} = \frac{E_{cm}}{1 + \psi_L \cdot \varphi(t,t_0)}$$

The expression for the modular ratio n_L is:

$$n_L = n_0\,(1 + \psi_L\,\varphi_t)$$

where:

n_0 is the modular ratio, E_a/E_{cm}, for short-term loading,

E_{cm} is the secant modulus of elasticity of the concrete for short-term loading,

φ_t is the creep coefficient $\varphi(t, t_0)$; this coefficient depends on both the age of the concrete on first loading, t_0, and at the time considered in the analysis t,

ψ_L is the creep multiplier depends on the type of loading.

Action	***Creep multiplier*** ψ_L
Short-term loading	$\psi = 0$
Permanent action constant in time	$\psi_P = 1{,}10$
Shrinkage	$\psi_S = 0{,}55$
Pre-stressing by controlled imposed deformations	$\psi_D = 1{,}50$
Time-dependent action effects	$\psi_{PT} = 0{,}55$

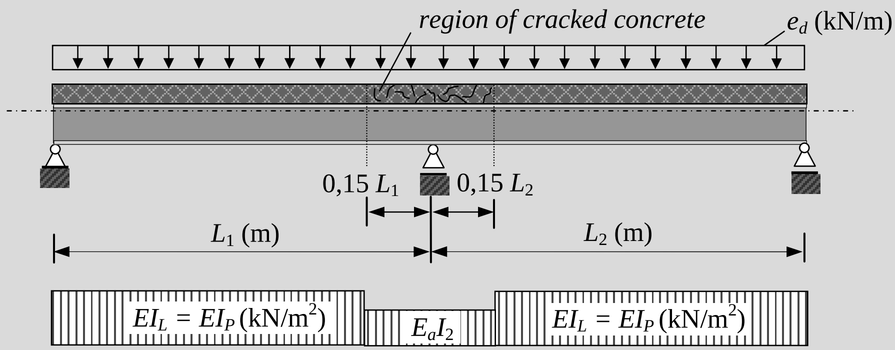


Figure B6.19 *Calculation of the deflection of the composite beam due to creep at time t = ∞*

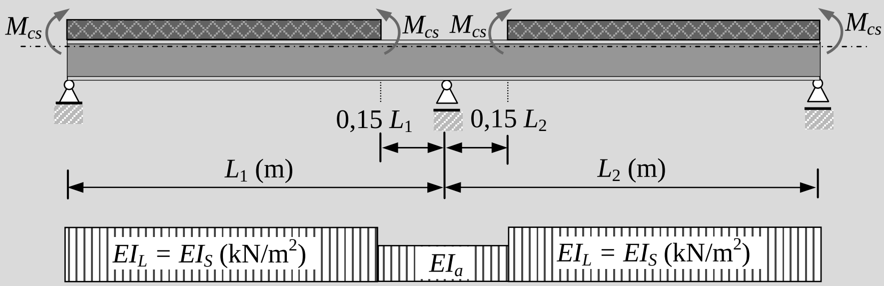


Figure B6.20 *Calculation of the deflection of the composite beam due to shrinkage*

c) Sequence of construction

The method of construction of composite beams can be propped or unpropped execution, Figure B6.21.

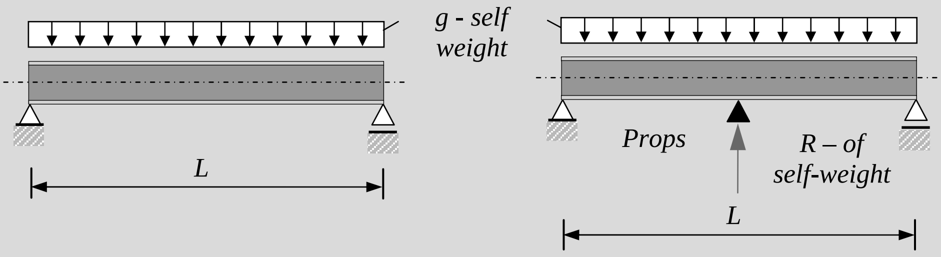


Figure B6.21 *Execution of the composite beam, unpropped and propped*

The method of construction affects the deflections of the composite beams. This influence may be governed especially for simply supported beams which are at the construction stage unpropped. In the case of propped execution, the props are retained until the concrete has achieved the compressive strength equivalent to grade C20/25, clause 6.6.5.2(4), EN 1994-1-1. In that case, elastic global analysis is sufficient, clause 7.3.1(2), EN 1994-1-1.

In the case of unpropped execution, the load acts only on the steel beam, and deflections are calculated according to EN 1993-1-1.

d) Influence of local yielding of the structural steel at the internal support

The serviceability load can cause yielding at the internal support when the continuous beam, with steel beam in classes 1 or 2, is unpropped in execution, Figure B6.22.

Local yielding is permitted for beams in buildings. However, yielding causes additional deflection, which should be taken into account. The method of calculation is given in clause 7.3.1(7), EN 1994-1-1. To calculate the bending moments at the internal support elastic analysis is used and effects of cracking of concrete are taken into account. In clause 7.3.1(7), EN 1994-1-1, the two values of reduction factors f_2 are recommended which correspond to different checks. The value of f_2 = 0,5 is recommended for permanent load: wet concrete on a steel beam. In this case the yield strength is reached before the concrete slab has hardened. The value of f_2 = 0,7 is recommended when the yield strength is reached due to extra loading applied after the concrete slab has hardened. Accordingly, the second check is carried out with the load additional to that for the first check acting on the composite beam.

The combination of actions for the second check are established depending on the function of the structure. According to EN 1990, there are three combinations: characteristic, frequent and quasi-permanent. For each analysis that is performed, appropriate assumptions are required for the adjacent spans with regard to their loading and execution.

The more accurate procedure for the determination of the effects of local yielding on deflections is shown in Figure B6.22.

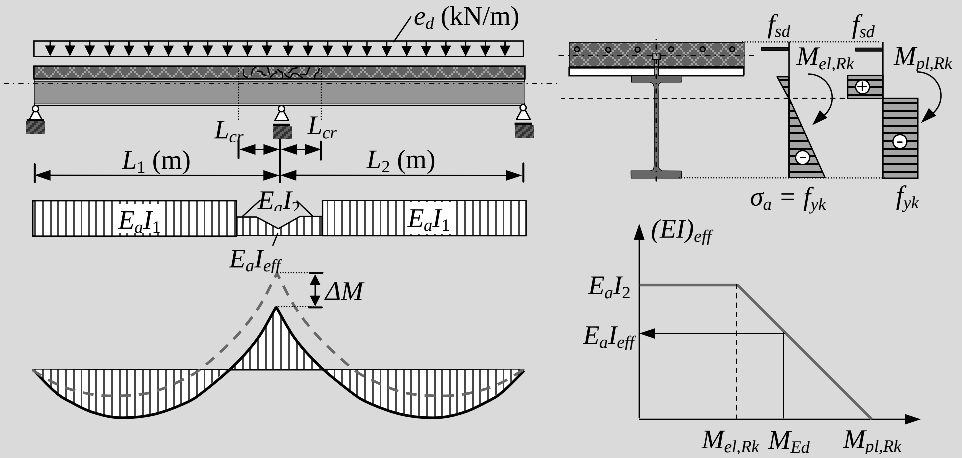


Figure B6.22 *Determination of the effects of local yielding on deflections by a more accurate procedure*

Influence of local yielding of structural steel at the internal support is considered in example B8.

e) Influence of incomplete interaction

Shear connectors are not rigid, and a small longitudinal slip occurs at the steel-concrete interface, as shown in Figure B6.23.

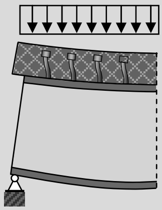


Figure B6.23 *Longitudinal slip between the steel beam and the concrete flange of a composite beam*

According to 7.3.1(4), EN 1994-1-1, the effects of incomplete interaction, i.e. the additional deflection caused by slip at the steel-concrete interface, can be neglected if:

a) the calculation of the shear connection is in accordance with clause 6.6, EN 1994-1-1,
b) either not less shear connectors than half the number for full shear connection are used, or the forces resulting from an elastic behaviour and which act on the shear connectors in the serviceability limit state do not exceed P_{Rd} (this condition relates to the minimum value of the degree of shear connection which gives a higher limit , η = 0,5, than the limit given in clause 6.6.1.2(1), η = 0,4).
c) in case of a ribbed slab with ribs transverse to the beam, the height of the ribs does not exceed 80 mm.

In the case where 0,4 ≤ η < 0,5, ENV 1994-1-1 gives the following expression for additional deflection due to incomplete interaction:

$$\delta = \delta_c + \alpha(\delta_a - \delta_c)(1-\eta)$$

where:

α = 0,5 or 0,3, for propped and unpropped construction, respectively,
δ_a is the deflection of the steel beam,
δ_c is the deflection of the composite beam with full shear connection,
η is the degree of the shear connection.

Both δ_a and δ_c are determined for the design load for the composite beam.

Since the serviceability criteria could be specified for each project and agreed with the client, several simplifications of the calculation of deflections may be applied. These simplifications are acceptable in practice, particularly for composite beams in buildings.

When the steel beam is unpropped at the construction stage, the calculation of deflection is performed separately for the construction stage and the composite stage.

The total deflections are obtained from the expressions:

$\delta = \delta_0$ (verification for the construction stage)

$\delta = \delta_{1,1} + \delta_{1,2} + \delta_{2,1} + \delta_{2,2} + \delta_{2,3}$ (verification for the composite stage)

where:

δ_0 is the deflection of the steel beam at the execution stage,

$\delta_{1,1}$ is the deflection of the steel beam due to permanent action immediately after casting concrete,
$\delta_{1,2}$ is the deflection due to the loads of floor finishes and partitions on the composite beam (the first loading),
$\delta_{2,1}$ is the deflection due to the frequent value of variable action at the time of first loading,
$\delta_{2,2}$ is the deflection due to creep under the quasi-permanent value of variable action at time $t = \infty$,
$\delta_{2,3}$ is the deflection due to shrinkage.

5.2 Calculation of deflections

5.2.1 Construction stage deflection

The deflection at the construction stage has been calculated by means of commercial software, using the flexural stiffness of the steel cross-section E_aI_a.

$$E_a = 21000 \text{ kN/cm}^2 \quad I_a = 48200 \text{ cm}^4$$

Recommended limiting values for deflection are:

$$\delta_{tot} \leq \frac{L}{250}$$

$$\delta_{\text{var}} \leq \frac{L}{360}$$

The total deflection due to permanent and variable actions, δ_{tot}, during execution is determined for the following total load e_d:

$$e_d = b \cdot (g_{k,1} + q_{k,1})$$

$$e_d = 3{,}00 \cdot (4{,}26 + 0{,}50) = 14{,}3 \text{ kN/m}$$

$$E_a I_a = 101220 \text{ kNm}^2$$

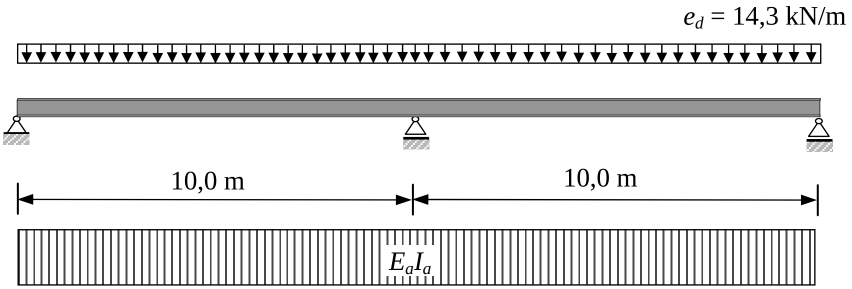


Figure B6.24 *Static system and load case 1 during execution*

The deflection (calculated using commercial software) is:

δ_{tot} = 7,6 mm < L/250 = 40,0 mm

The condition is satisfied.

The deflection due to variable actions, δ_{var}, during execution is determined for the following variable load e_d:

$$e_d = b \cdot q_{k,1}$$

$$e_d = 3{,}00 \cdot 0{,}50 = 1{,}50 \text{ kN/m}$$

$$E_a I_a = 101220 \text{ kNm}^2$$

The critical load case for deflection is where only one span is fully loaded:

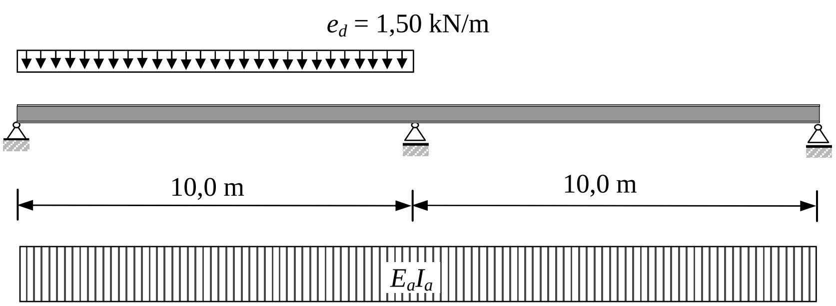


Figure B6.25 *Static system and load case 2 during execution*

The deflection (calculated using commercial software) is:

δ_{var} = 1,3 mm < L/360 = 27,8 mm

The condition is satisfied.

Remark:

The required conditions for deflections of the steel beam are satisfied. However, the buckling resistance moment of the steel beam is not adequate and the steel beam must be laterally restrained at the construction stage.

The limitation of deflection is adopted according to the recommendation given in EN 1990. This value can be changed in accordance with the recommendation given in the National Annex. Furthermore, an absolute limit of 25 mm could be recommended in order to limit the effects of ponding of wet concrete during execution.

5.2.2 Composite stage deflection

The deflection of the composite beam has been calculated by means of commercial software, using the flexural stiffness of the composite section which depends on the type of loading EI_L.

Determination of the creep coefficient and shrinkage

For the calculation of the creep coefficient $\varphi(t, t_0)$ the following is valid:

- the perimeter of that part which is exposed to drying, u

$$u = b$$

- the notional size of the cross-section, h_0

$$h_0 = \frac{2 \cdot A_c}{u} = \frac{b \cdot h_c}{b} = h_c = 99 \text{ mm}$$

- $t_0 = 1$ day, $t_0 = 28$ days,
- the ambient relative humidity, RH 50%,
- the concrete strength class C 40/50,
- the type of cement – cement class S, strength class 32,5 N.

The final value of creep coefficient $\varphi = \varphi(\infty, t_0)$ will be determined using the nomogram shown in Figure 3.1, EN 1992-1-1. Example B3, shows the detailed procedure for the determination of creep coefficients from nomograms.

The following creep coefficients are obtained:

$\varphi_t = \varphi(\infty,\, t_0 = 1 \text{ day}) = 5{,}2$
$\varphi_t = \varphi(\infty,\, t_0 = 28 \text{ days}) = 2{,}5$

The total shrinkage strain, according to clause 3.1.4, EN 1992-1-1, at the age of concrete at the beginning of drying shrinkage t_s = 3 days and at the age at the time considered in the analysis $t = \infty$, is:

$\varepsilon_{cs}(\infty) = 4{,}15 \cdot 10^{-4}$
$\varepsilon_{cs}(\infty) = 0{,}415$ ‰

Effective flexural stiffness of the composite section

The effective flexural stiffness of the composite section EI_L is:

$$EI = E_a \cdot I_a + E_L \cdot I_c + \frac{E_a \cdot A_a \cdot E_L \cdot A_c}{E_a \cdot A_a + E_L \cdot A_c} \cdot a^2$$

a) Short-term loading

$$E_a = 21000 \text{ kN/cm}^2 \quad I_a = 48200 \text{ cm}^4 \quad A_a = 115{,}5 \text{ cm}^2$$

$$I_c = \frac{b_{eff} \cdot h_c^3}{12} = \frac{212{,}5 \cdot 9{,}9^3}{12} = 17182 \text{ cm}^4$$

$$A_c = b_{eff} \cdot h_c = 212{,}5 \cdot 9{,}9 = 2104 \text{ cm}^2$$

The distance between the centroidal axes of the concrete flange and the steel section is:

$$a = \frac{h_a}{2} + h_p + \frac{h_c}{2} = \frac{50}{2} + 5{,}1 + \frac{9{,}9}{2} = 35{,}05 \text{ cm}$$

$$n_c = 1$$

$$E_0 = \frac{E_{cm}}{n_c} = \frac{3500}{1{,}0} = 3500 \text{ kN/cm}^2 \quad E_L = E_0$$

$$EI_0 = 21000 \cdot 48200 + 3500 \cdot 17182 + \frac{21000 \cdot 115{,}5 \cdot 3500 \cdot 2104}{21000 \cdot 115{,}5 + 3500 \cdot 2104} \cdot 35{,}5^2$$

$$EI_0 = 3371719672 \ \text{Ncm}^2 = 337172 \ \text{kNm}^2$$

b) Permanent loading, constant in time

$$n_c = 1 + 1{,}10 \cdot \varphi(\infty, t_0) = 1 + 1{,}10 \cdot 2{,}5 = 3{,}75$$

$$E_P = \frac{E_{cm}}{n_c} = \frac{3500}{3{,}75} = 933 \ \text{kN/cm}^2 \quad E_L = E_P$$

$$EI_P = 21000 \cdot 48200 + 933 \cdot 17182 + \frac{21000 \cdot 115{,}5 \cdot 933 \cdot 2104}{21000 \cdot 115{,}5 + 933 \cdot 2104} \cdot 35{,}5^2$$

$$EI_P = 2395537987 \ \text{Ncm}^2 = 239554 \ \text{kNm}^2$$

c) Secondary effects due to shrinkage (statically indeterminate structure)

$$n_c = 1 + 0{,}55 \cdot \varphi(\infty, t_0) = 1 + 0{,}55 \cdot 5{,}2 = 3{,}86$$

$$E_S = \frac{E_{cm}}{n_c} = \frac{3500}{3{,}86} = 907 \ \text{kN/cm}^2 \quad E_L = E_S$$

$$EI_S = 21000 \cdot 48200 + 907 \cdot 17182 + \frac{21000 \cdot 115{,}5 \cdot 907 \cdot 2104}{21000 \cdot 115{,}5 + 907 \cdot 2104} \cdot 35{,}5^2$$

$$EI_S = 2373766336 \ \text{Ncm}^2 = 237377 \ \text{kNm}^2$$

Remark:

As a simplification for composite beams in structures for buildings where first-order global analysis can be applied, clause 5.4.2.2(11), EN 1994-1-1, permits the modular ratio to be taken as $2 \cdot n_0$ for both short-term and long-term loading. The application of this simplification is shown in examples B3 and B8.

Effects of cracking of the concrete

Remark:

As explained in Section 4.2, effects of cracking of the concrete may be taken into account using the simplified method II for continuous composite beams as well as beams in braced frames. According to the simplified method the effect of cracking is taken into account by using the flexural stiffness E_aI_2 over 15% of the span on each side of each internal support, and as the un-cracked values E_aI_1 elsewhere. The reinforcement may be taken into consideration.

Therefore, in the region of the internal support the reduced flexural stiffness, E_aI_2, is used. The modulus of elasticity of steel is denoted as E_a. The second moment of area of the effective steel section calculated neglecting concrete in tension but including reinforcement is denoted as I_2. At the internal support the concrete is in tension and the second moment of area of the steel section and the reinforcement in the slab, I_2, is determined as follows.

The total area of steel section and reinforcement is:

$$A_s = 12{,}56 \ \text{cm}^2 \quad A_a = 115{,}5 \ \text{cm}^2$$

$$A_{st} = A_s + A_a = 12{,}56 + 115{,}5 = 128 \ \text{cm}^2$$

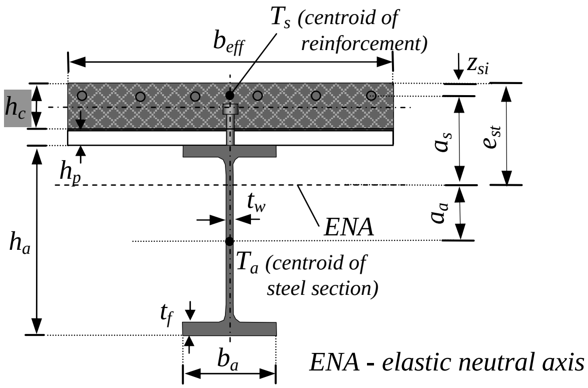


Figure B6.26 *Cross-section of composite beam at internal support*

The distance between the centroid of the cracked section and the top of concrete slab is:

$$e_{st} = \frac{A_s \cdot z_s + A_a(h_c + h_p + h_a / 2)}{A_{st}}$$

$$e_{st} = \frac{12{,}56 \cdot 2{,}5 + 115{,}5(9{,}9 + 5{,}1 + 50 / 2)}{128} = 36{,}3 \ \text{cm}$$

Therefore, the distance between the neutral axis and the centroid of the steel section, a_a, is:

$$a_a = \frac{h_a}{2} + h_p + h_c - e_{st} = \frac{50}{2} + 5{,}1 + 9{,}9 - 36{,}3 = 3{,}7 \text{ cm}$$

The distance between the neutral axis and the centroidal axis of the reinforcement, a_s, is:

$$a_s = h_c + h_p + \frac{h_a}{2} - z_{si} - a_a = 9{,}9 + 5{,}1 + \frac{50}{2} - 2{,}5 - 3{,}7 = 33{,}8 \text{ cm}$$

The second moment of area of the effective steel section neglecting concrete in tension but including reinforcement is:

$$I_{st} = I_2 = I_a + A_s \cdot z_s^2 + A_a (h_c + h_p + h_a / 2)^2 - A_{st} \cdot e_{st}^2$$

$$I_{st} = 48200 + 12{,}56 \cdot 2{,}5^2 + 115{,}5(9{,}9 + 5{,}1 + 50 / 2)^2 - 128 \cdot 36{,}3^2$$

$$I_{st} = 64414 \text{ cm}^4$$

The reduced flexural stiffness is:

$$EI_{st} = E_a I_2 = 1352694000 \text{ Ncm}^2 = 135269 \text{ kNm}^2$$

Calculation of deflections

The deflections have been calculated using commercial software. The concrete is cracked at a length of 0,15 L on each side of internal support. The reduced flexural stiffness $E_a I_2$ is allowed for in this region. At mid-span, the corresponding flexural stiffness is allowed for at the length of 0,85 L.

- Deflection due to permanent action at the time immediately after casting concrete:

$$e_d = b \cdot g_{k,1} = 3{,}00 \cdot 4{,}26 = 12{,}78 \text{ kN/m}$$

$$E_a I_a = 101220 \text{ kNm}^2$$

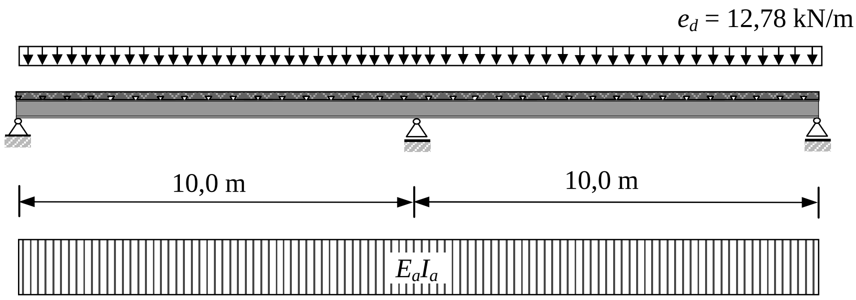


Figure B6.27 *Static system - permanent action at the time immediately after casting concrete*

$$\delta_{1,1} = 6,8 \text{ mm}$$

- Deflection due to loads of floor finishes, partitions on the composite beam at time of first loading:

$$e_d = b \cdot g_{k,3} = 3,00 \cdot 0,50 = 1,5 \text{ kN/m}$$

$$EI_L = EI_0 = 337172 \text{ kNm}^2$$

$$E_a I_2 = 135269 \text{ kNm}^2$$

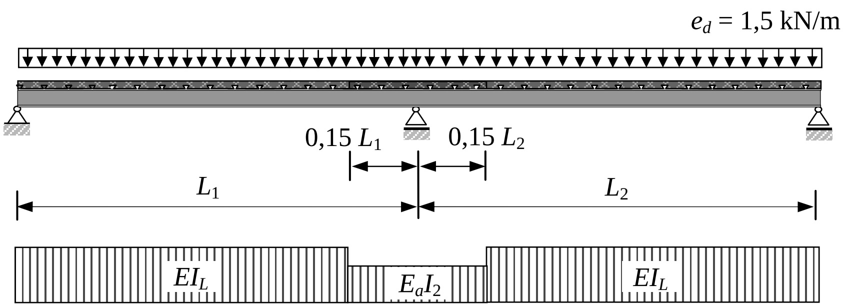


Figure B6.28 *Static system – loads of floor finishes and partitions at the time of first loading*

$$\delta_{1,2} = 0,3 \text{ mm}$$

- Deflection due to the frequent value of variable action at the time of first loading

For a building with floors in category B, office areas, the combination factor ψ is:

$\psi_1 = 0{,}5$

$$e_d = b \cdot \psi_1 \cdot q_{k,2} = 3{,}00 \cdot 0{,}5 \cdot 4{,}5 = 6{,}75 \text{ kN/m}$$

$$EI_L = EI_0 = 337172 \text{ kNm}^2$$

$$E_a I_2 = 135269 \text{ kNm}^2$$

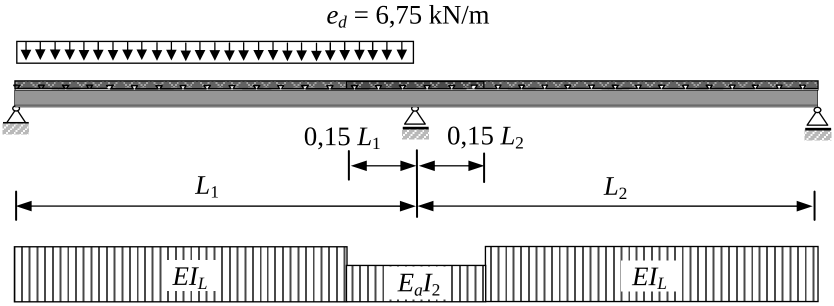


Figure B6.29 *Static system - frequent value of variable action at the time of first loading*

$$\delta_{2,1} = 2{,}0 \text{ mm}$$

- Deflection due to creep under the quasi-permanent value of the variable action at time $t = \infty$

This deflection is the difference of deflections at time $t = \infty$ and at the time of first loading t_0 = 28 days.

$$e_{d,1} = b \cdot (g_{k,3} + \psi_2 \cdot q_{k,2}) = 3{,}00 \cdot (0{,}5 + 0{,}3 \cdot 4{,}5) = 5{,}55 \text{ kN/m}$$

$$e_{d,2} = b \cdot g_{k,3} = 3{,}00 \cdot 0{,}50 = 1{,}5 \text{ kN/m}$$

$$EI_L = EI_0 = 337172 \text{ kNm}^2$$

$$EI_L = EI_P = 239554 \text{ kNm}^2$$

$$E_a I_2 = 135269 \text{ kNm}^2$$

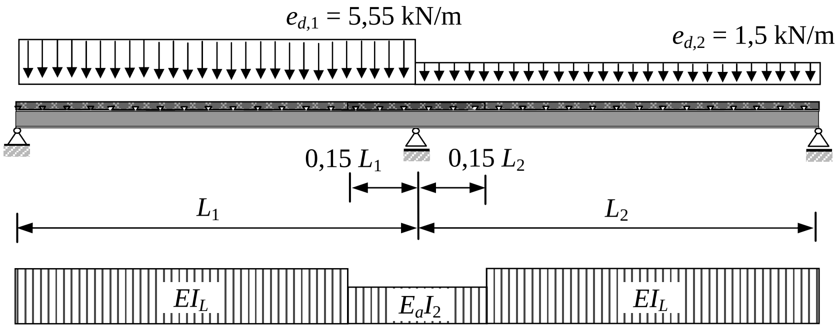


Figure B6.30 *Static system – deflection due to creep at time t = ∞ ($EI_L = EI_P$) and at the time of first loading ($EI_L = EI_0$)*

$\delta_{2,2} = 2,0 - 1,5 = 0,5$ mm

- Deflection due to shrinkage

$N_{cs} = \varepsilon_{cs}(\infty) \cdot E_S \cdot A_c = 4,15 \cdot 10^{-4} \cdot 907 \cdot 212,5 \cdot 9,9 = 792$ kN

$$a_c = \frac{E_a \cdot A_a}{E_a \cdot A_a + E_S \cdot A_c} \cdot a = \frac{21000 \cdot 115,5}{21000 \cdot 115,5 + 907 \cdot 2104} \cdot 19,05 = 10,66 \text{ cm}$$

$M_{cs} = N_{cs} \cdot a_c = 792 \cdot 10,66 = 8443$ kNcm $= 84,4$ kNm

$EI_L = EI_S = 237377$ kNm^2

$E_a I_2 = 135269$ kNm^2

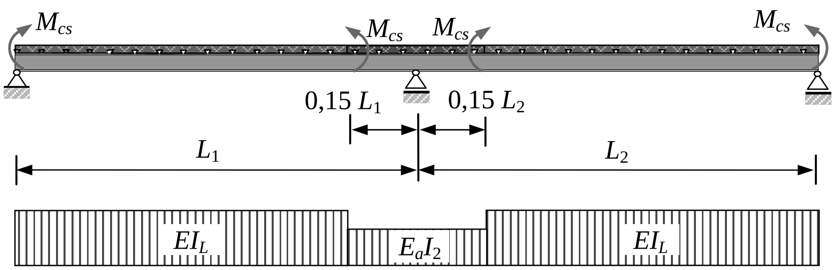


Figure B6.31 *Static system – deflection due to shrinkage*

$\delta_{2,3} = 2,3$ mm

The effects of shear connection on the deflection of the beam can be neglected because the condition $n/n_f \geq 0{,}5$ is satisfied.

Remark:

The limitations of deflections are adopted according to the recommendation given in EN 1990. These values can be changed in accordance with the recommendation given in the National Annex.

Deflection limits for composite beams are the same as for steel beams, and are determined by the National Annex.

Recommended limiting values for deflection of composite beams are:

$\delta_{tot} \leq \dfrac{L}{250}$, the deflection due to the total load

$\delta_{\mathrm{var}} \leq \dfrac{L}{360}$, the deflection due to the variable load

The deflection due to the permanent action is:

$$\delta_1 = \Sigma\delta_{1,i} = 6{,}8 + 0{,}3 = 7{,}1 \text{ mm}$$

The deflection due to the variable load, creep and shrinkage is:

$$\delta_2 = \Sigma\delta_{2,i} = 2{,}0 + 0{,}5 + 2{,}3 = 4{,}8 \text{ mm}$$

The total deflection due to the permanent and variable loads, creep and shrinkage is:

$$\delta_{tot} = \delta_1 + \delta_2 = 7{,}1 + 4{,}8 = 11{,}9 \text{ mm} \leq \frac{L}{250} = \frac{10000}{250} = 40{,}0 \text{ mm}$$

The total deflection meets the criterion $L/250$.

The deflection due to variable load, creep and shrinkage is:

$$\delta_{\mathrm{var}} = \delta_2 = 4{,}8 \text{ mm} \leq \frac{L}{360} = \frac{10000}{360} = 27{,}8 \text{ mm}$$

The deflection due to variable load, creep and shrinkage meets the criterion $L/360$.

5.3 Pre-cambering of the steel beam

In this example, the pre-cambering of the steel beam involves deflections due to permanent loads, creep and shrinkage:

$$\delta_p = \delta_1 + \delta_{2,2} + \delta_{2,3}$$

$$\delta_p = 7{,}1 + 0{,}5 + 2{,}3$$

The following value of pre-cambering is adopted:

$$\delta_p = 10{,}0 \text{ mm}$$

For more detailed explanation see example B3.

5.4 Check of vibration of the beam

The natural frequency may be calculated in terms of the well-known expression that is often used in design:

$$f = \frac{18}{\sqrt{\delta}}$$

where δ is the maximum deflection in millimetres due to self-weight and other permanent loads. This well-known natural frequency expression may be used as the expression for determining the natural frequency of individual members, even when they are not simply supported, providing that the appropriate value of δ is used.

For the calculation of the natural frequency, the characteristic values of the permanent loads for the composite stage are taken into account, and the effective flexural stiffness of the composite section for short-term loading EI_0.

$$e_d = b \cdot (g_{k,2} + g_{k,3}) = 3{,}00 \cdot (4{,}11 + 0{,}50) = 13{,}83 \text{ kN/m}$$

The deflection under this load is approximately:

$$\delta = \frac{5}{384} \cdot \frac{e_d \cdot L^4}{EI_0} = \frac{5}{384} \cdot \frac{13{,}83 \cdot 10^4}{337172} \cdot 100 = 0{,}53 \text{ cm}$$

The natural frequency of the beam is therefore:

$$f = \frac{18}{\sqrt{\delta}} = \frac{18}{\sqrt{5,3}} = 7,8 \text{ Hz} \geq 4 \text{ Hz} \quad \text{with } \delta \text{ in mm}$$

The criterion is satisfied for initial calculation purposes. However, the dynamic performance of the entire floor is carried out using a method such as the one in [51].

5.5 Control of crack width

5.5.1 Minimum reinforcement area

The exposure class is XC1. For reinforced concrete elements according to EN 1992-1-1 this means that the crack width should be limited to the maximum value w_{max} = 0,4 mm.

Remark:

For the exposure class XC1, the crack width has no influence on the durability of the structure, and this limit is set to guarantee acceptable appearance of structures.

The required minimum area of reinforcement A_s for the slab of composite beam, according to clause 7.4.2(1), EN 1994-1-1, is:

$$A_s = k_s \cdot k_c \cdot k \cdot f_{ct,eff} \cdot A_{ct} / \sigma_s$$

where:

$f_{ct,eff}$ is the mean value of the tensile strength of the concrete effective at the time when the first crack may be expected to occur. Values of $f_{ct,eff}$ can be taken as those for f_{ctm} (EN 1992-1-1, Table 3.1) or as f_{lctm} (EN 1992-1-1, Table 11.3.1) taking into account the concrete strength class at the time when the occurrence of the first crack in the concrete is expected. If the time of the occurrence of cracks cannot be established, it is possible to adopt the minimum tensile strength of 3 N/mm^2.

A_{ct} is the cross-sectional area of the tensile zone of the concrete (due to direct loading and the primary effects of shrinkage). For simplicity, the cross-sectional area of the concrete may be adopted as the area determined by its effective width.

σ_s is the maximum stress allowed in the reinforcement immediately after cracking of the concrete. This stress can be taken as the characteristic value of the yield strength f_{sk}. To satisfy the required width limits, lower values may be needed, depending on the diameter of the bar. These values are given in Table 7.1, EN 1994-1-1.

k, k_s, k_c are coefficients based on the calibration procedure. The magnitude of these coefficients, k, k_s and k_c, depend on the geometry of the cracked composite section. More detailed explanation is given below.

The meaning of the coefficients, k, k_s and k_c, is:

k allows for the effect of non-uniform self-equilibrating tensile stresses; it may be taken as 0,8.

k_s allows for the effect of the reduction of the normal force of the concrete slab due to initial cracking and local slip of the shear connection; it may be taken as 0,9.

k_c takes into account the stress distribution within the cross-section (the tensile zone of the concrete A_{ct}) immediately prior to cracking.

The coefficient k_c is calculated as:

$$k_c = \frac{1}{1 + h_c / (2z_0)} + 0,3 \le 1,0$$

where:

h_c is the thickness of the concrete flange, excluding any haunch or ribs,

z_0 is the vertical distance between the centroids of the uncracked concrete flange and the uncracked composite section, calculated using the modular ratio $n_0 = E_a/E_{cm}$ for short-term loading, i.e. at the time of first loading t_0.

Therefore, the values of coefficients k and k_s are:

$$k = 0,8$$

$$k_s = 0,9$$

Calculation of coefficient k_c

The modular ratio n_0 for short-term loading is:

$$n_L = n_0 = \frac{E_a}{E_{lcm}} = \frac{21000}{3500} = 6,00$$

The effective width of the concrete flange at internal support is:

$$b_{eff,2} = 1{,}25 \text{ m}$$

Transformed to the ideal steel section, the effective width is:

$$\frac{b_{eff}}{n_0} = \frac{125}{6{,}00} = 20{,}8 \text{ cm}$$

Geometrical properties of ideal cross-section:

The area of the ideal steel cross-section is:

$$A_{i,L} = A_{st} + A_{c,L} = A_a + A_{sa} + \frac{b_{eff,2}}{n_0} \cdot h_c = 115{,}5 + 12{,}56 + 20{,}8 \cdot 9{,}9 = 334 \text{ cm}^2$$

The distance between the centroid of the steel section and the top of concrete slab is:

$$a_{st} = \frac{h_a}{2} + h_p + \frac{h_c}{2} = \frac{50}{2} + 5{,}1 + \frac{9{,}9}{2} = 35{,}05 \text{ cm}$$

The vertical distance between the centroids of the uncracked concrete flange and the uncracked composite section is:

$$z_0 = z_{ic,L} = \frac{A_{st} \cdot a_{st}}{A_{i,L}} = \frac{(115{,}5 + 12{,}56) \cdot 35{,}05}{334} = 13{,}4 \text{ cm}$$

The coefficient k_c is:

$$k_c = \frac{1}{1 + h_c / (2z_0)} + 0{,}3 = \frac{1}{1 + 99 / (2 \cdot 134)} + 0{,}3 = 1{,}03 > 1{,}0$$

Adopted: k_c = 1,00.

If the time of the occurrence of the cracks cannot be established, the mean value of the tensile strength of the concrete effective at the time when the first crack may be expected to occur $f_{ct,eff}$, can be adopted as the minimum tensile strength of 3 N/mm^2.

The cross-sectional area of the tensile zone of the concrete is:

$$A_{ct} = b_{eff,2} \cdot h_c = 1250 \cdot 99 = 123750 \text{ mm}^2$$

The maximum stress allowed in the reinforcement immediately after cracking of concrete σ_s is chosen from Table 7.1, EN 1994-1-1:

$$\sigma_s = 280 \ \text{N/mm}^2$$

The maximum bar diameter ϕ^*, for design crack width w_k = 0,4 mm and for the chosen maximum stress allowed in reinforcement σ_s, according to Table 7.1, EN 1994-1-1, is:

ϕ^* = 16 mm

Minimum area of reinforcement

The required minimum area of reinforcement A_s is:

$$A_s = k_s \cdot k_c \cdot k \cdot f_{ct,eff} \cdot A_{ct} \,/\, \sigma_s = 0{,}9 \cdot 1{,}0 \cdot 0{,}8 \cdot 3{,}0 \cdot 123750 \,/\, 280 = 955 \ \text{mm}^2/b_{eff}$$

The required minimum area of reinforcement $A_s = 9{,}55 \ \text{cm}^2/b_{eff}$ is less than the cross-sectional area of the longitudinal reinforcement adopted in Section 4.2.2.2 as 12,56 cm^2/b_{eff}.

The initial selected reinforcement is adequate.

5.5.2 Control of cracking of the concrete due to direct loading

The bending moment at the internal support is calculated for the quasi-permanent combination of actions at time $t = \infty$. Since the steel beam is unpropped at the construction stage, the stresses in reinforcement are determined for the load of the floor finishes, $g_{k,3}$, the quasi-permanent load, $\psi_2 \, q_{k,2}$, and shrinkage.

The design load is:

$$e_d = b \cdot (g_{k,3} + \psi_2 \cdot q_{k,2}) = 3{,}00 \cdot (0{,}5 + 0{,}3 \cdot 4{,}5) = 5{,}55 \ \text{kN/m}$$

The corresponding flexural stiffnesses are:

$$EI_L = EI_P = 239554 \ \text{kNm}^2$$

$$E_a I_2 = 135269 \ \text{kNm}^2$$

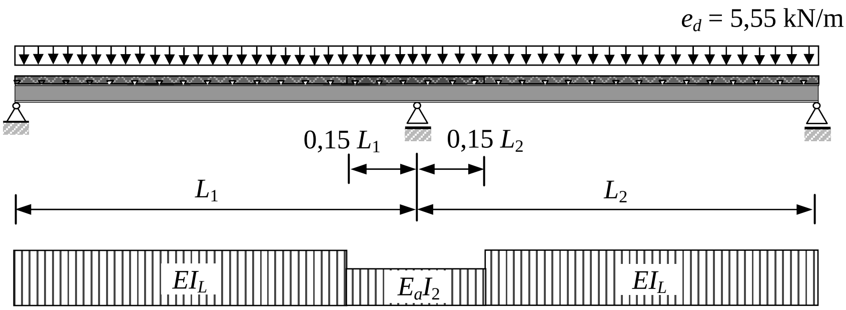


Figure B6.32 *Static system for calculation of the bending moment at the internal support of the composite beam for quasi-permanent actions at time $t = \infty$*

The maximum bending moment at the internal support, calculated using commercial software, is:

M_{max} = 58,0 kNm

The bending moment due to shrinkage is:

$$M_{cs} = N_{cs} \cdot a_c = 792 \cdot 10,66 = 8444 \text{ kNcm} = 84,4 \text{ kNm}$$

The corresponding flexural stiffnesses are:

$$EI_L = EI_S = 237377 \text{ kNm}^2$$

$$E_a I_2 = 135269 \text{ kNm}^2$$

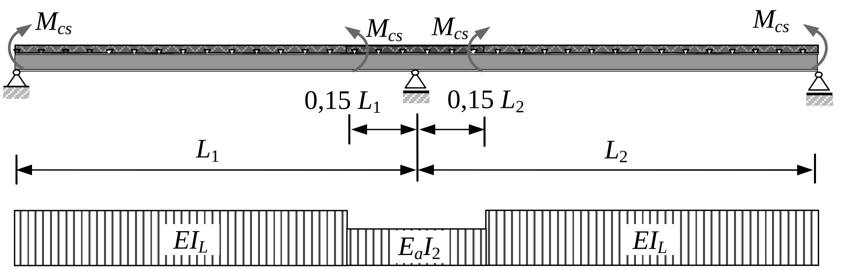


Figure B6.33 *Static system for calculation of the bending moment at the internal support of the composite beam due to shrinkage*

The maximum bending moment at the internal support, calculated using commercial software, is:

M_{max} = 70,8 kNm

The total bending moment is:

$$M_{st} = 58,0 + 70,8 = 129 \text{ kNm}$$

The distance between the neutral axis and the centroidal axis of reinforcement is:

$$z_{st} = e_{st} - z_{s1} = 36,3 - 2,5 = 33,8 \text{ cm}$$

The tensile stress in the reinforcement σ_s can be calculated for direct loading as:

$$\sigma_s = \sigma_{s,0} + \Delta\sigma_s$$

The stress in the reinforcement $\sigma_{s,0}$ caused by bending moment acting on the composite section is calculated on the assumption that the concrete in tension is neglected.

The geometrical properties of the cracked cross-section in accordance with Figure B6.26 are:

cross-sectional area

$$A_{st} = A_s + A_a = 12,56 + 115,5 = 128,0 \text{ cm}^2$$

second moment of area

$$I_{st} = 64414 \text{ cm}^4$$

The stress in the reinforcement $\sigma_{s,0}$ caused by the bending moment acting on the cracked section is:

$$\sigma_{s,0} = \frac{M_{st}}{I_{st}} z_{st} = \frac{129 \cdot 10^2}{64414} \cdot 33,8 = 6,77 \text{ kN/cm}^2 = 67,7 \text{ N/mm}^2$$

The correction of the stress in the reinforcement for tension stiffening is:

$$\Delta\sigma_s = \frac{0,4 \cdot f_{ctm}}{\alpha_{st} \cdot \rho_s}$$

with:

$$\alpha_{st} = \frac{AI}{A_a I_a} = \frac{128 \cdot 64414}{115{,}5 \cdot 48200} = 1{,}48$$

The reinforcement ratio ρ_s is:

$$\rho_s = \frac{A_s}{A_{ct}} = \frac{A_s}{h_c \cdot b_{eff,2}} = \frac{12{,}56}{9{,}9 \cdot 125} = 0{,}01015$$

The correction of the stress in the reinforcement for tension stiffening is:

$$\Delta\sigma_s = \frac{0{,}4 \cdot f_{ctm}}{\alpha_{st} \cdot \rho_s} = \frac{0{,}4 \cdot 0{,}35}{1{,}48 \cdot 0{,}01015} = 9{,}32 \ \text{kN/cm}^2 = 93{,}2 \ \text{N/mm}^2$$

Therefore, the tensile stress in the reinforcement σ_s is:

$$\sigma_s = \sigma_{s,0} + \Delta\sigma_s = 67{,}7 + 93{,}2 = 160{,}9 \ \text{N/mm}^2$$

$$\sigma_s = 160{,}9 \ \text{N/mm}^2$$

According to Table 7.1, EN 1994-1-1, the maximum bar diameter is $\phi_s \leq 32$ mm.

According to Table 7.2, EN 1994-1-1, the maximum bar spacing is ≤ 300 mm.

The use of 16 mm bars at 200 mm spacing at internal support, with the cross-sectional area 1005 mm^2/m, satisfies both conditions.

6. Commentary

Clause 7.3, EN 1994-1-1, the serviceability limit state related to the deformations of structural members in buildings, does not provide a detailed procedure for determining the deflections of structural members. In some cases of continuous composite beams, the effects of cracking of concrete can significantly affect the determination of the deflections. Therefore, this issue is worked out in more detail in this example, illustrated by two distinct procedures: for minimum reinforcement according to clause 7.4.2, EN 1994-1-1, and for reinforcement to control cracking due to direct loading according to clause 7.4.3, EN 1994-1-1. Generally, the required criteria for both ultimate limit state and serviceability limit state are met.

B7 Calculation of continuous beam over two spans by means of plastic–plastic procedure

1. Purpose of example

We need to design a two-span composite beam according to the plastic–plastic procedure. In this example, action effects are calculated by rigid plastic global analysis (plastic hinge analysis), and the resistance to bending is based on a plastic model. For this procedure, all cross-sections of the composite beam should be in class 1. For the given continuous beam under uniformly distributed load, the first plastic hinge is formed at the internal support. The continuous beam has changed the static system from statically indeterminate to statically determinate. According to the static theorem, it is necessary to prove that assuming the formation of the plastic hinge at internal support, $M_{pl,Rd}$ will not exceed the resistance of the section to bending at mid-span. The steel beam is fully propped at the construction stage, and the verification of lateral-torsional buckling is not necessary. However, the verification of lateral-torsional buckling of the continuous composite beam must be done in accordance with EN 1994-1-1. The total depth of the concrete slab, h, is 16 cm. The concrete slab was made with prefabricated elements of thickness, h_p = 5 cm where concrete has been laid on site with thickness, h_c, of 11 cm. In regions of sagging bending moments, partial shear connection is permitted, while in the region of hogging bending moments the full shear connection is required. The ductile headed stud connectors are applied with the diameter of the shank d = 22 mm and the overall height h_{sc} = 125 mm. The serviceability criteria are particularly strong, so that the deflection limit for variable action is δ_{req} = L/500. The continuous beam is the structural member of a multi-storey commercial building in which there are brittle floor finishes and fragile partitions. In this case, the characteristic combination is used because the limit state is not reversible. The quasi-permanent combination is used for long-term effects such as deformations from creep of the concrete. For a building with floors in category B, office areas, the combination factor for quasi-permanent combination is ψ_2 = 0,3. Since the steel beam is fully propped at the construction stage, it is necessary to perform the verification only for the composite stage.

Composite Structures according to Eurocode 4. Worked Examples.
First Edition. Darko Dujmović, Boris Androić, Ivan Lukačević.

2. Static system, cross-section and actions

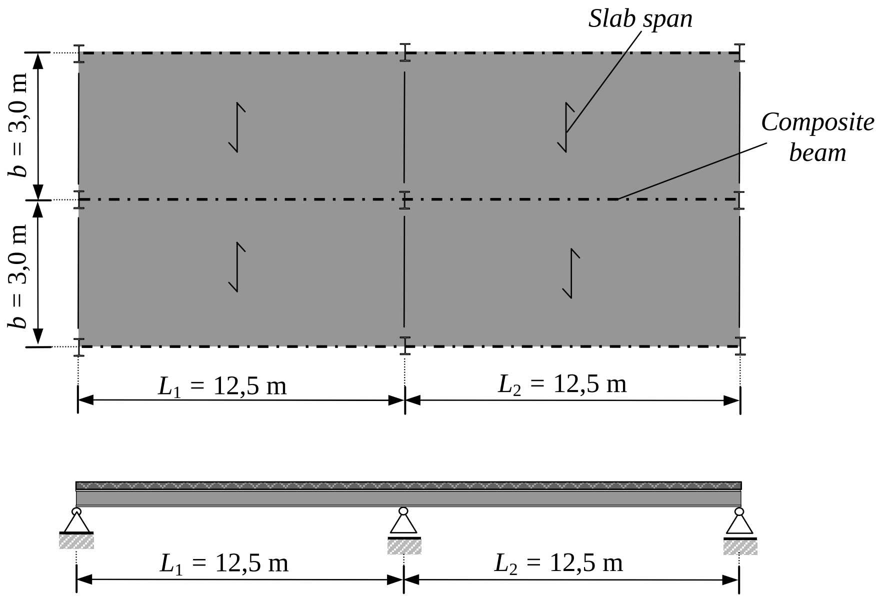
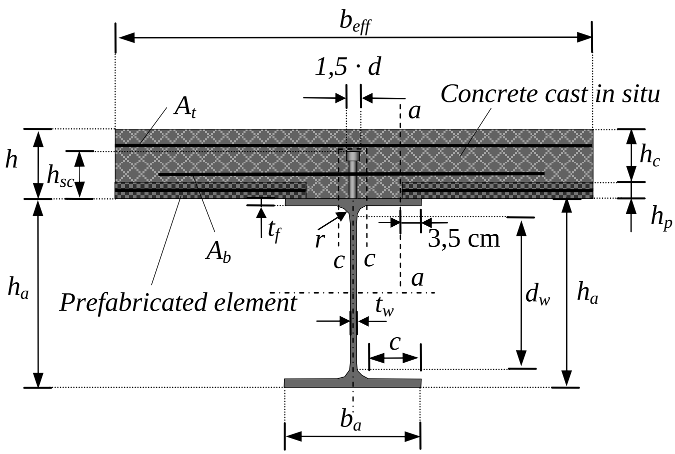


Figure B7.1 *Floor layout and static system*



h = 160 mm, h_c = 110 mm, h_p = 50 mm

Figure B7.2 *Cross-section of composite beam*

Actions

a) Permanent action

- *concrete slab and reinforcement (dry concrete)* $0,16 \cdot 25 = 4,0\ \text{kN/m}^2$

- *steel beam* $= 0,42\ \text{kN/m}^2$

Total $g_{k,1} = 4,42\ \text{kN/m}^2$

Floor finishes $g_{k,2} = 2,0\ \text{kN/m}^2$

b) Variable action

- *imposed floor load* $= 5,00\ \text{kN/m}^2$

Total $q_k = 5,00\ \text{kN/m}^2$

3. Properties of materials

Concrete strength class: C 25/30 $f_{ck} = 25\ \text{N/mm}^2$

$$f_{cd} = \frac{f_{ck}}{\gamma_c} = \frac{25}{1,5} = 16,7\ \text{N/mm}^2$$

$$0,85 f_{cd} = 0,85 \cdot 16,7 = 14,2\ \text{N/mm}^2$$

$$E_{cm} = 31000\ \text{N/mm}^2$$

$$f_{ctm} = 2,6\ \text{N/mm}^2$$

Reinforcement: *ductility class B or C (Table C.1, EN 1992-1-1))* $f_{sk} = 500\ \text{N/mm}^2$

$$f_{sd} = \frac{f_{sk}}{\gamma_s} = \frac{500}{1,15} = 435\ \text{N/mm}^2$$

$$E_s = 210000\ \text{N/mm}^2$$

Structural steel: S355 $f_{yk} = 355\ \text{N/mm}^2$

$$f_{yd} = \frac{f_{yk}}{\gamma_M} = \frac{355}{1,0} = 355\ \text{N/mm}^2$$

$$E_a = 210000\ \text{N/mm}^2$$

Shear connectors: *ductile headed studs* $f_u = 450\ \text{N/mm}^2$

$$d = 22\ \text{mm}$$

$$h_{sc} = 125\ \text{mm}$$

$$\frac{h_{sc}}{d} = \frac{125}{22} = 5,7 > 4,0 \rightarrow \alpha = 1,0$$

$$P_{Rd} = 98,9\ \text{kN}$$

4. Ultimate limit state

4.1 Design values of combined actions

The design load determined by a governed combination of actions is:

$$e_d = b \cdot (\gamma_G \cdot g_{k,1} + \gamma_G \cdot g_{k,2} + \gamma_Q \cdot q_k)$$

$b = 3{,}0$ m beam spacing

$$e_d = 3{,}00 \cdot (1{,}35 \cdot 4{,}42 + 1{,}35 \cdot 2{,}00 + 1{,}50 \cdot 5{,}0) = 48{,}50 \text{ kN/m}$$

4.2 Selection of steel cross-section

The approximate ratio of span to depth of the steel beam for a continuous composite secondary beam is:

$$\frac{L}{h_a} \approx 25$$

For span L = 12,5 m, the minimum depth of steel beam is:

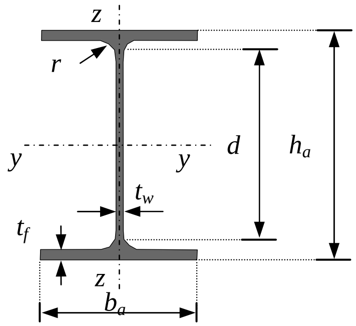
$$h_a = \frac{L}{25} = \frac{12{,}5 \cdot 10^3}{25} = 500 \text{ mm}$$

We can adopted section IPE 500 with the depth 500 mm. The cross-section of IPE 500 and the steel grade S355 in bending is class 1. In the section at the internal support, a region of hogging bending, the plastic neutral axis moves up and the depth of the web in compression is increased. In this case, the cross-section of IPE 500 is not in class 1. Therefore, the cross-section with lower depth of web is selected with approximately the same second moment of area as IPE 500.

Thus, section HEA 400 is selected.

Reinforcement

Top longitudinal	ϕ 10/150 mm, 5,24 cm^2/m, z_s = 2,5 cm
Bottom longitudinal	ϕ 10/150 mm, 5,24 cm^2/m, z_s = 10,5 cm
Top transverse	ϕ 10/150 mm, 5,24 cm^2/m, z_s = 3,5 cm
Bottom transverse	ϕ 10/150 mm, 5,24 cm^2/m, z_s = 13,5 cm



$W_{pl,y}$ = 2562 cm^3
$W_{el,y}$ = 2311 cm^3
A_a = 159,0 cm^2
h_a = 390 mm
b_a = 300 mm
t_w = 11,0 mm
t_f = 19 mm
r = 27 mm
$I_{y,a}$ = 45070 cm^4
d = 298 mm
i_z = 7,34 mm

Figure B7.3 *Cross-section of steel beam*

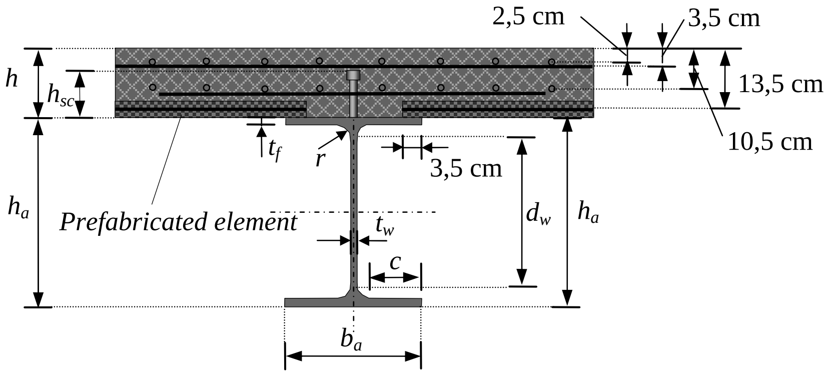


Figure B7.4 *Reinforcement in concrete slab*

4.3 Effective width of concrete flange

The effective width of the concrete flange is calculated according to the expression (5.3), EN 1994-1-1:

$$b_{eff} = b_0 + \Sigma b_{ei}$$

$b_0 = 0$ (there is only one row of shear connectors)

$$b_{ei} = \frac{L_e}{8} \leq b_i$$

The equivalent span of the beam in the mid-span region (L_1 = L_2 = 12,5 m) in accordance with Figure 5.1, EN 1994-1-1 is:

$$L_{e,1} = 0,85 \cdot L_1 = 0,85 \cdot 12,5 = 10,625 \text{ m}$$

$$b_1 = \frac{b}{2} = \frac{3,0}{2} = 1,5 \text{ m}$$

$$b_2 = \frac{b}{2} = \frac{3,0}{2} = 1,5 \text{ m}$$

$$b_{e,1} = \frac{L_{e,1}}{8} \leq b_1$$

$$b_{e,1} = \frac{10,625}{8} = 1,328 \leq b_1 = 1,5 \text{ m}$$

Adopted: $b_{e1} = 1,328 \text{ m}$

$$b_{e2} = \frac{L_{e,1}}{8} \leq b_2$$

$$b_{e2} = \frac{10,625}{8} = 1,328 \leq b_2 = 1,5 \text{ m}$$

Adopted: $b_{e2} = 1,328 \text{ m}$

$$b_{eff,1} = b_0 + b_{e1} + b_{e2} = 0 + 1,328 + 1,328 = 2,656 \text{ m}$$

The equivalent span of the beam for the region at internal support ($L_1 = L_2 = 12,5$ m) in accordance with Figure 5.1, EN 1994-1-1 is:

$$L_{e,2} = 0,25 \cdot (L_1 + L_2) = 0,25 \cdot (12,5 + 12,5) = 6,25 \text{ m}$$

$$b_1 = \frac{b}{2} = \frac{3,0}{2} = 1,5 \text{ m}$$

$$b_2 = \frac{b}{2} = \frac{3,0}{2} = 1,5 \text{ m}$$

$$b_{e1} = \frac{L_{e,2}}{8} \leq b_1$$

$$b_{e1} = \frac{6,25}{8} = 0,781 \leq b_1 = 1,5 \text{ m}$$

Adopted: $b_{e1} = 0{,}781$ m

$$b_{e2} = \frac{L_{e,2}}{8} \leq b_2$$

$$b_{e2} = \frac{6{,}25}{8} = 0{,}781 \leq b_2 = 1{,}5 \text{ m}$$

Adopted: $b_{e2} = 0{,}781$ m

$$b_{eff,2} = b_0 + b_{e1} + b_{e2} = 0 + 0{,}781 + 0{,}781 = 1{,}562 \text{ m}$$

4.4 Classification of the composite cross-section

The local buckling of cross-sections affects the resistance and rotation capacity of sections. Therefore, local buckling should be considered in the design. The classification of cross-sections of composite beams depends on the local slenderness of the flange (b/t) and of the web (c/t) of steel beams, the position of the plastic neutral axis and the area of longitudinal reinforcement in the slab at the internal support.

The section classification in EN 1993-1-1 is adopted for composite sections. Table 5.2 of EN 1993-1-1 gives limits for the width-to-thickness ratios for the compression parts of a section for each classification. In addition to the limitations on the local slenderness of the flange and the web of the steel beam, requirements for the ductility of reinforcement in tension are given for class 1 and class 2. The reinforcement should have ductility class B or C, Table C.1, EN 1992-1-1, and according to clause 5.5.1(5), EN 1994-1-1, the minimum area of reinforcement A_s should satisfy the following condition:

$$A_s \geq \rho_s \cdot A_c$$

with,

$$\rho_s = \delta \frac{f_y}{235} \frac{f_{ctm}}{f_{sk}} \sqrt{k_c}$$

where:

A_c is the effective area of the concrete flange,
f_y is the nominal (characteristic) value of the yield strength of the structural steel in N/mm^2,

f_{sk} is the characteristic yield strength of the reinforcement N/mm^2,
f_{ctm} is the mean tensile strength of the concrete,
δ is a factor which is:
$\delta = 1{,}1$ for the procedure plastic–plastic (cross-section class 1),
$\delta = 1{,}0$ for the procedure elastic–plastic (cross-section class 2),
k_c is a coefficient that takes into account the stress distribution within the section immediately prior to cracking and is $k_c = \dfrac{1}{1 + h_c / (2z_0)} + 0{,}3 \le 1{,}0$
h_c is the thickness of the concrete flange, excluding any haunch or ribs,
z_0 is the vertical distance between the centroids of the uncracked concrete flange and the uncracked composite section, calculated using the modular ratio $n_0 = E_a/E_{cm}$ for short-term loading, i.e. at the time of first loading t_0.

Since the action effects are calculated by rigid plastic global analysis (plastic hinge analysis), all cross-sections of the composite beam should be class 1.

4.4.1 Cross-section at mid-span

Classification of flange

The flange of the steel section is in compression, as shown in Figure B7.5.

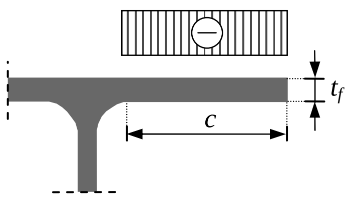


Figure B7.5 *Classification of flange (compressive stress is negative)*

$$c = \frac{b_a - t_w - 2 \cdot r}{2} = \frac{300 - 11{,}0 - 2 \cdot 27}{2} = 117{,}5 \text{ mm}$$

$$\frac{c}{t_f} = \frac{117{,}5}{19} = 6{,}18$$

The limiting value for class 1 is:

$$\frac{c}{t_f} \le 9\varepsilon = 9 \cdot 0{,}81 = 7{,}29$$

$6,18 < 7,29$ → Therefore, the flange in compression is class 1.

Classification of web

The class of a steel web depends on the proportion of its depth that is in compression. For the *I* or *HE* sections subjected to bending half of the depth is in compression. In regions of sagging bending moments, the depth of steel web in compression is decreased or could be in tension because the plastic neutral axis is usually within the steel top flange or slab. Therefore, if the cross-section is class 1 at internal support (region of hogging bending), then the cross-section is class 1 in the region of sagging bending. Thus, classification of the web is performed only for hogging bending.

4.4.2 Cross-section at the internal support

Classification of flange

At the internal support, a region of hogging bending, the bottom flange of the steel section is in compression. According to the classification of the flange from section 4.4.1, the flange satisfies the condition for class 1.

Classification of web

In composite beams subjected to hogging bending, the addition of longitudinal reinforcement in the concrete flange increases the depth of steel web in compression, $\alpha \cdot c$ in Figure B7.6.

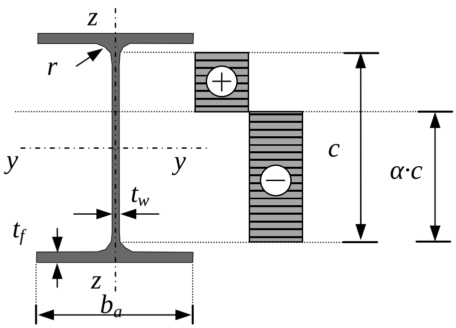


Figure B7.6 *Classification of web (compression is negative)*

For $\alpha > 0,5$, the limiting value for class 1 is:

$$\frac{c}{t} \leq \frac{396 \cdot \varepsilon}{13\alpha - 1}$$

$$c = h - 2t_f - 2r = 390 - 2 \cdot 19 - 2 \cdot 27 = 298 \text{ mm}$$

$$\frac{c}{t} = \frac{298}{11} = 27,09$$

To classify the web, the position of the plastic neutral axis should be determined.

The design resistance moment at internal support, M_{Rd}, can be calculated as:

$$M_{Rd} = M_{pl,Rd} = \Sigma N_{si} \cdot z_i + M_{a,V,Rd} - \frac{t_w \cdot d_0^2 (1-\rho) \cdot f_{yd}}{4}$$

The area of the longitudinal reinforcement within the effective width at internal support $b_{eff,2}$ = 1,562 m is:

$A_{s1} = 5,24 \cdot 1,562 = 8,185 \text{ cm}^2$ (top longitudinal reinforcement)

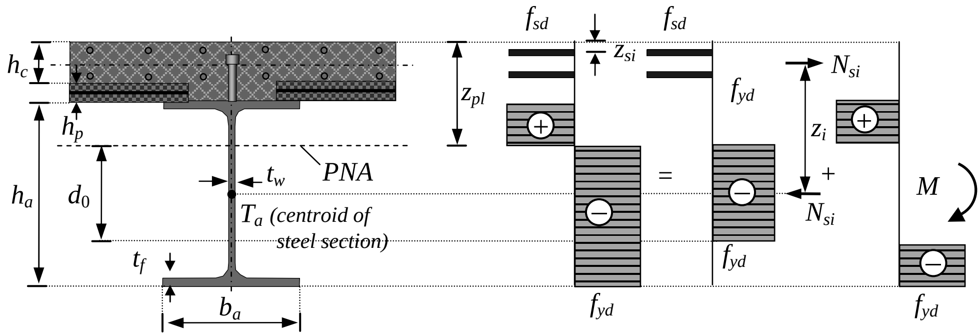
$A_{s2} = 5,24 \cdot 1,562 = 8,185 \text{ cm}^2$ (bottom longitudinal reinforcement)

The forces in these bars are:

$$N_{s1} = A_{s1} \cdot f_{sd} = 8,185 \cdot 43,5 = 356 \text{ kN}$$

$$N_{s2} = A_{s2} \cdot f_{sd} = 8,185 \cdot 43,5 = 356 \text{ kN}$$

In the case of hogging bending (moment at the internal support), the plastic neutral axis lies within the web of the steel beam, as shown on Figure B7.7:



PNA - plastic neutral axis

Figure B7.7 *Cross-section and stress distributions for composite beam in hogging bending*

According to 6.2.2.4, EN 1994-1-1, where the shear force exceeds half the shear resistance its effect on the resistance moment should be taken into account. The reduction factor for the design yield strength of the web is $(1 - \rho)$, where:

$$\rho = (\frac{2V_{Ed}}{V_{Rd}} - 1)^2$$

and V_{Rd} is the design shear resistance.

In this example there is no reduction in the resistance moment because the design shear force does not exceed half the design shear resistance, $V_{Ed} < 0{,}5\ V_{Rd}$.

From equilibrium, the design tensile force in reinforcement $N_{si} = N_{s1} + N_{s2}$ and the design compressive force in web $N_{si} = d_0 \cdot f_{yd} \cdot t_w$ which gives:

$$d_0 = \frac{N_{s1} + N_{s2}}{t_w \cdot f_{yd}} = \frac{356 + 356}{1{,}1 \cdot 35{,}5} = 18{,}23\ \text{cm}$$

The distance between plastic neutral axis and the top of the slab z_{pl} is:

$$z_{pl} = h_c + h_p + \frac{h_a}{2} - \frac{d_0}{2} = 11{,}0 + 5{,}0 + \frac{39}{2} - \frac{18{,}23}{2} = 26{,}39\ \text{cm}$$

The distances between the centroidal axes of top and bottom longitudinal reinforcement and the top of the slab are, respectively:

$$z_{s1} = 2{,}5\ \text{cm},\ z_{s2} = 10{,}5\ \text{cm}$$

The distances between the centroid of the steel section and the centroids of the top and bottom reinforcement are, respectively:

$$z_1 = h_c + h_p + \frac{h_a}{2} - z_{s1} = 11 + 5 + \frac{39}{2} - 2{,}5 = 33\ \text{cm}$$

$$z_2 = h_c + h_p + \frac{h_a}{2} - z_{s2} = 11 + 5 + \frac{39}{2} - 10{,}5 = 25\ \text{cm}$$

Where the shear force reduces the resistance moment of the steel section, the reduced design resistance moment is:

$$M_{a,V,Rd} = M_{pl,f,Rd} + (M_{pl,a,Rd} - M_{p,f,Rd})(1 - \rho)$$

In this case, there is no reduction in the resistance moment. Therefore, the design resistance moment of the steel section $M_{pl,a,Rd}$ is taken into account instead of the reduced design resistance moment $M_{a,V,Rd}$.

The design value of the plastic resistance moment of the steel section is:

$$M_{pl,a,Rd} = \frac{W_{pl,y} \cdot f_{yd}}{\gamma_{M0}} = \frac{2562 \cdot 35{,}5}{1{,}0} = 90951 \text{ kNcm} = 910 \text{ kNm}$$

Therefore, the design value of the plastic resistance moment of the composite section at internal support is:

$$M_{Rd} = M_{pl,Rd} = \Sigma N_{si} \cdot z_i + M_{pl,a,Rd} - \frac{t_w \cdot d_0^2 \cdot f_{yd}}{4}$$

$$M_{Rd} = M_{pl,Rd} = (356 \cdot 33 + 356 \cdot 25) + 91000 - \frac{1{,}1 \cdot 18{,}23^2 \cdot 35{,}5}{4}$$

$$M_{Rd} = M_{pl,Rd} = 108404 \text{ kNcm} = 1084 \text{ kNm}$$

For I-sections subject to major-axis bending and axial force with the neutral axis in the web, the parameter α_c can be calculated as:

$$\alpha_c = \frac{1}{c}\left(\frac{h_a}{2} + \frac{1}{2}\frac{N_{Ed}}{t_w \cdot f_{yd}} - (t_f + r)\right)$$

In this case, the design axial compressive force N_{Ed} is equal to the sum of the design tensile forces in reinforcement. The design axial compressive force N_{Ed} is:

$$N_{Ed} = N_{s1} + N_{s2} = 356 + 356 = 712 \text{ kN}$$

The parameter α_c is:

$$\alpha_c = \frac{1}{29{,}8}\left(\frac{39}{2} + \frac{1}{2}\frac{712}{1{,}1 \cdot 35{,}5} - (1{,}90 + 2{,}7)\right) = 0{,}81$$

For $\alpha > 0{,}5$, the limiting value for class 1 is:

$$\frac{c}{t} = \frac{29{,}8}{1{,}1} = 27{,}09 \leq \frac{396 \cdot \varepsilon}{13\alpha - 1} = \frac{396 \cdot 0{,}81}{13 \cdot 0{,}81 - 1} = 33{,}66$$

$27,09 < 33,66 \rightarrow$ the web is class 1

Therefore the cross-section is class 1.

Within the effective width of the composite section the ductile reinforcement is selected. According to clause 5.5.1(5), EN 1994-1-1, the minimum area of reinforcement A_s should satisfy the following condition:

$$A_s \geq \rho_s \cdot A_c$$

with:

$$\rho_s = \delta \frac{f_y}{235} \frac{f_{ctm}}{f_{sk}} \sqrt{k_c}$$

The area of reinforcement is:

$$A_s = A_{s1} + A_{s2} = 8,185 + 8,185 = 16,37 \text{ cm}^2$$

The effective area of concrete slab at internal support is:

$$A_c = b_{eff,2} \cdot h_c = 156,2 \cdot 11,0 = 1718,2 \text{ cm}^2$$

For the plastic–plastic procedure, the factor δ is 1,1.

The coefficient k_c takes into account the stress distribution within the section immediately prior to cracking is:

$$k_c = \frac{1}{1 + h_c / (2z_0)} \leq 1,0$$

The thickness of the concrete flange h_c is 11,0 cm. The vertical distance between the centroids of the uncracked concrete flange and the uncracked composite section is denoted by z_0. It is calculated using the modular ratio $n_0 = E_a/E_{cm}$ for short-term loading, i.e. at time of the first loading t_0.

The modular ratio n_0 is:

$$n_0 = \frac{E_a}{E_{cm}} = \frac{210}{31} = 6,77$$

The effective width of concrete flange at internal support is:

$$b_{eff,2} = 156,2 \ \text{cm}$$

Transformed to the ideal steel section, the effective width is:

$$\frac{b_{eff,2}}{n_0} = \frac{156,2}{6,77} = 23,1 \ \text{cm}$$

The area of the ideal steel cross-section is:

$$A = A_a + \frac{b_{eff,2}}{n_0} \cdot h_c = 159 + 23,1 \cdot 11,0 = 413,1 \ \text{cm}$$

The distance between the neutral axis and the centroid of the steel section is:

$$z_{n0} = \frac{\frac{b_{eff,2}}{n_0} \cdot h_c \cdot (\frac{h_a}{2} + \frac{h_c}{2} + h_p)}{A} = \frac{23,1 \cdot 11,0 \cdot (\frac{39}{2} + \frac{11}{2} + 5,0)}{413,1} = 18,5 \ \text{cm}$$

Thus, the vertical distance between the centroids of the uncracked concrete flange and the uncracked composite section is:

$$z_0 = (\frac{h_a}{2} + \frac{h_c}{2} + h_p) - z_{n0} = (\frac{39}{2} + \frac{11}{2} + 5,0) - 18,5 = 30,0 - 18,5 = 11,5 \ \text{cm}$$

Therefore:

$$k_c = \frac{1}{1 + h_c / (2z_0)} + 0,3 = \frac{1}{1 + 11 / (2 \cdot 11,5)} + 0,3 = 0,976 \leq 1,0$$

$$k_c = 0,976$$

$$\rho_s = \delta \frac{f_y}{235} \frac{f_{ctm}}{f_{sk}} \sqrt{k_c} = 1,1 \frac{355}{235} \frac{2,6}{500} \sqrt{0,976} = 0,854 \ \%$$

The final verification of the minimum reinforcement is:

$$A_s = 16,37 \ \text{cm}^2 \geq \rho_s \cdot A_c = 0,00854 \cdot 1718,2 = 14,67 \ \text{cm}^2.$$

The condition is satisfied.

All sections within the span of the composite beam, for both sagging and hogging regions, are in class 1.

4.5 Calculation of effects of actions

Rigid plastic analysis for the calculation bending moments and internal forces can be used if the conditions from clause 5.4.5, EN 1994-1-1, are satisfied. In the plastic design, structural failure (collapse) is presumed through the formation of a collapse mechanism. Collapse mechanisms are caused by the formation of one or more plastic hinges. At the plastic hinge the cross-section becomes fully plastic and may rotate at constant bending moment. Thus, the cross-sections where the plastic hinges occur, are capable of developing and sustaining their plastic resistance moment during the process of moment redistribution. The rotation capacity at each location of plastic hinge must exceed the required rotation. In clause 5.4.5, EN 1994-1-1, the provision on the sufficient rotation capacity of composite cross-section of continuous composite beams are given. The required capacity of rotation depends on the type of load, the ratio of adjacent spans and the ratio of the plastic resistance moment at internal support and the plastic resistance moment at mid-span. Since the plastic resistance moment in the hogging bending region is significantly lesser compared to that in the sagging bending region, the required rotation of section at internal support are obtained higher than the required rotation of only the steel beam. The steel compression flange at a plastic hinge location should be laterally restrained and the continuous beam should not be susceptible to lateral-torsional buckling. In continuous composite beams with uniformly distributed load the first plastic hinge is formed at internal support. It means that the bottom flange of steel section and the part of web are in compression, while the concrete flange is in tension. Therefore, the ductile reinforcement and the minimum reinforcement area must be provided in the concrete flange.

Because of these reasons, before determining bending moments and internal forces according to rigid plastic analysis, it is necessary to verify compliance with the conditions given in clause 5.4.5, EN 1994-1-1.

For the given continuous beam under uniformly distributed load, the first plastic hinge is formed at internal support. According to the static theorem, it is necessary to prove that assuming the formation of the plastic hinge at internal support, $M_{pl,Rd}$, will not be exceeded the resistance of section to bending in the mid-span.

Therefore, the statically determined system is considered which is subjected to uniformly distributed load e_d and the moment $M_{Ed} = M_{pl,Rd}$ at the end of beam, see Figure B7.8. In this case, it is necessary to calculate the moment at mid-span M_{Ed}, as the effect of action.

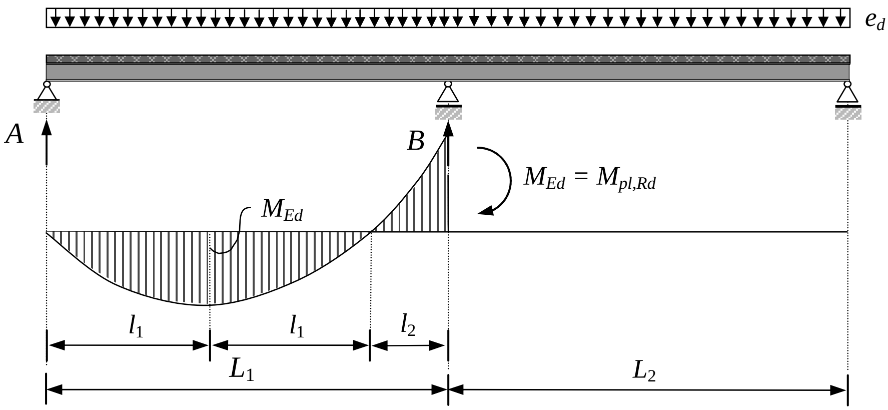


Figure B7.8 *Static system and loads needed to calculate plastic moment resistance at mid-span*

The design load determined by governed combination of actions is:

$$e_d = b \cdot (\gamma_{G,\sup} \cdot g_{k,1} + \gamma_{G,\sup} \cdot g_{k,2} + \gamma_Q \cdot q_k)$$

$b = 3{,}0$ m beam spacing

$$e_d = 3{,}00 \cdot (1{,}35 \cdot 4{,}42 + 1{,}35 \cdot 2{,}0 + 1{,}50 \cdot 5{,}0) = 48{,}5 \text{ kN/m}$$

The reaction at the support A is:

$$A = \frac{e_d \cdot L_1}{2} - \frac{M_{pl,Rd}}{L_1}$$

$$A = \frac{48{,}5 \cdot 12{,}5}{2} - \frac{1084}{12{,}5} = 303 - 86{,}7 = 216 \text{ kN}$$

The reaction at the support B is:

$$B = \frac{e_d \cdot L_1}{2} + \frac{M_{pl,Rd}}{L_1}$$

$$B = \frac{48{,}50 \cdot 12{,}5}{2} + \frac{1084}{12{,}5} = 303 + 86{,}7 = 390 \text{ kN}$$

The point of maximum moment is at the distance l_1 from the support A:

$$l_1 = \frac{A}{e_d} = \frac{216}{48{,}5} = 4{,}45 \text{ m}$$

$$l_2 = L_1 - 2 \cdot l_1 = 12{,}5 - 2 \cdot 4{,}45 = 3{,}6 \text{ m}$$

The maximum sagging moment for uniformly distributed load e_d and the moment $M_{Ed} = M_{pl,Rd}$ at the end of beam is:

$$M_{Ed} = A \cdot l_1 - \frac{e_d \cdot l_1^2}{2} = 216 \cdot 4{,}45 - \frac{48{,}50 \cdot 4{,}45^2}{2} = 481 \text{ kNm}$$

The shear forces are:

$V_{Ed} = 0$ (at the point of maximum sagging moment M_{Rd})

$V_{B,Ed} = B = 390$ kN (at the support B, at the hogging moment $M_{pl,Rd}$)

4.6 Check of shear connection

Remark:

The partial shear connection is permitted in sagging moment region.

The full shear connection is required in hogging moment region,

Region L_I

$$L_I = l_1 = 4{,}45 \text{ m}$$

For full shear connection in sagging moment region L_I, the minimum value of N_{cf} and $N_{pl,a}$ is governed:

$$V_{L,Ed} = \min(N_{pl,a}; N_{c,f}) = \min(A_a \cdot f_{yd}; b_{eff,1} \cdot h_c \cdot 0{,}85 \cdot f_{cd})$$

The design value of the plastic resistance of the structural steel section to normal force is:

$$N_{pl,a} = A_a \cdot f_{yd} = 159 \cdot 35{,}5 = 5645 \text{ kN}$$

The design value of the compressive normal force in the concrete flange with full shear connection is:

$$N_{c,f} = b_{eff,1} \cdot h_c \cdot 0,85 \cdot f_{cd} = 265,6 \cdot 11 \cdot 0,85 \cdot 1,67 = 4147 \text{ kN}$$

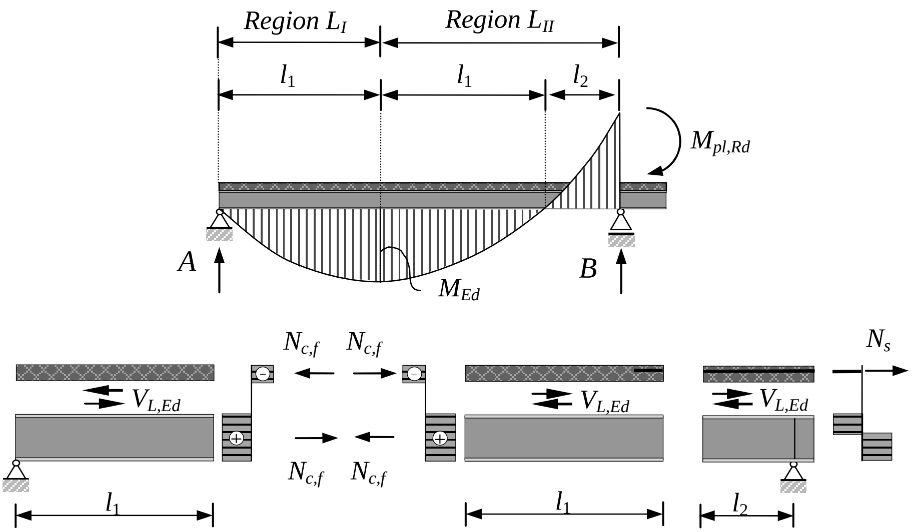


Figure B7.9 *Distribution of longitudinal shear for design of shear connection*

Thus, the design longitudinal force in the steel-concrete interface is equal to $N_{c,f}$:

$$V_{L,Ed} = N_{c,f} = 4147 \text{ kN}$$

The equivalent span of beam at mid-span ($L_1 = L_2 = 12,5$ m) according to Figure 5.1, EN 1994-1-1, is:

$$L_e = 0,85 \cdot L_1 \text{ for } b_{eff,1}$$

$$L_e = 0,85 \cdot 12,5 = 10,625 \text{ m}$$

For $L_e \leq 25$ m, according to clause 6.6.1.2 (3), EN 1994-1-1, the limit for the degree of shear connection is:

$$\eta_{\min} = 1 - (\frac{355}{f_y}) \cdot (0,75 - 0,03 \cdot L_e) \qquad \eta \geq 0,4$$

$$\eta_{\min} = 1 - (\frac{355}{355}) \cdot (0,75 - 0,03 \cdot 10,625) = 0,569 \qquad \eta \geq 0,4$$

Greater value is adopted:

$\eta_{\min} = 0{,}569$

The design longitudinal force for the minimum required degree of shear connection $\eta = 0{,}569$ is:

$$V_{L,Ed} = \eta \cdot 4147{,}2 = 0{,}569 \cdot 4147 = 2360 \text{ kN}$$

The required number of studs for region L_I = 4,45 m is:

$$n = \frac{V_{L,Ed}}{P_{Rd}} = \frac{2360}{98{,}9} = 23{,}9 \rightarrow \text{adopted 30 studs}$$

The actual longitudinal shear force in region L_I is:

$$V_{L,Ed} = n \cdot P_{Rd} = 30 \cdot 98{,}9 = 2967 \text{ kN} \geq 2360 \text{ kN}$$

The following spacing of studs in the longitudinal direction is selected:

$$e_L = \frac{L_I}{30} = \frac{4450}{30} = 148{,}3 \text{ mm} \rightarrow \text{adopted } e_L = 150 \text{ mm}$$

The verification of criteria for the spacing of studs:

e_L = 150 mm > 5d = 5 · 22 = 110 mm

e_L = 150 mm < 800 mm

e_L = 150 mm < 6 h_c = 6 · 160 = 960 mm

For uniform spacing of shear connectors the following condition must be satisfied:

$$\frac{M_{Rd}}{M_{pl,a,Rd}} \leq 2{,}5$$

Therefore, the design plastic resistance moment of the composite section does not exceed 2,5 times the design plastic resistance moment of the steel member alone.

The plastic neutral axis lies within the thickness of top steel flange if it is:

$$N_c < N_{pl,a}$$

The design value of the plastic resistance of the structural steel section to normal force is:

$$N_{pl,a} = A_a \cdot f_{yd} = 159 \cdot 35{,}5 = 5645 \text{ kN}$$

The design value of the compressive normal force in the concrete flange with full shear connection is:

$$N_c = N_{c,f} = b_{eff,1} \cdot h_c \cdot 0{,}85 \cdot f_{cd} = 265{,}6 \cdot 11 \cdot 0{,}85 \cdot 1{,}67 = 4147 \text{ kN}$$

Criterion:

$$4147 < 5645$$

The plastic neutral axis lies within the thickness of top steel flange.

When the plastic neutral axis lies within the thickness of top steel and the shear force is necessary to take into account, $N_{pl,a}$ is:

$$N_{pl,a} = (A_a - \rho \cdot A_V) f_{yd}$$

The design plastic resistance of the steel flange to normal force is:

$$N_{pl,f} = \frac{N_{pl,a} - N_c}{2}$$

The shear force does not exceed half the shear resistance and its effect on the resistance moment is neglected, i.e. $\rho = 0$. Therefore, the design plastic resistance of the structural steel section to normal force is:

$$N_{pl,a} = A_a \cdot f_{yd} = 159 \cdot 35{,}5 = 5645 \text{ kN}$$

The design plastic resistance of the steel flange to normal force is:

$$N_{pl,f} = \frac{N_{pl,a} - N_c}{2} = \frac{5645 - 4147}{2} = 749 \text{ kN}$$

In accordance with Figure B7.10, the following values are calculated:

$$x_1 = \frac{N_{c,f}}{b_{eff,1} \cdot 0{,}85 f_{cd}} = \frac{4147}{265{,}6 \cdot 0{,}85 \cdot 1{,}67} = 11{,}0 \text{ cm} \leq h_c = 11 \text{ cm}$$

$$x_2 = \frac{N_{pl,f}}{b_f \cdot f_{yd}} = \frac{749}{30 \cdot 35,5} = 0,7 \text{ cm} < t_f = 1,9 \text{ cm}$$

$$z = h_c + h_p + \frac{h_a}{2} - \frac{x_1}{2}$$

$$z = 11,0 + 5,0 + \frac{39,0}{2} - \frac{11,0}{2} = 11 + 5,0 + 19,5 - 5,5 = 30,0 \text{ cm} = 0,30 \text{ m}$$

The design resistance moment of the composite section in sagging region is:

$$M_{Rd} = N_{c,f} \cdot z + N_{pl,f}(h_a - x_2)$$

$$M_{Rd} = 4147 \cdot 0,30 + 749(0,39 - 0,007)$$

$$M_{Rd} = 1531 \text{ kNm}$$

The design plastic resistance moment of steel section $M_{pl,a,Rd}$ is:

$$M_{pl,a,Rd} = W_{pl,y} \cdot f_{yd} = 2562 \cdot 35,5 \cdot 10^{-2} = 910 \text{ kNm}$$

Check:

$$\frac{M_{pl,Rd}}{M_{pl,a,Rd}} = \frac{1531}{910} = 1,68 < 2,5$$

Region L_{II}

$$L_{II} = L_1 - l_1 = 12,5 - 4,45 = 8,05 \text{ m}$$

$$V_{L,Ed} = \Sigma P_{Rd} + N_s = n \cdot P_{Rd} + A_s \cdot f_{sd} = 30 \cdot 98,9 + 16,38 \cdot 43,5 = 3680 \text{ kN}$$

The required number of studs for region L_{II} = 8,05 m is:

$$n = \frac{V_{L,Ed}}{P_{Rd}} = \frac{3680}{98,9} = 37,2$$

In region L_{II} is adopted the same spacing between studs as in region L_I:

e_L = 150 mm

The number of studs for region L_{II} = 8,05 m is:

$$n = \frac{L_{II}}{e_L} = \frac{8050}{150} = 53,7 \ \rightarrow \text{ adopted 54 studs}$$

The actual longitudinal shear force in region L_{II} is:

$$V_{L,Ed} = n \cdot P_{Rd} = 54 \cdot 98,9 = 5341 \ \text{kN} > 3680 \ \text{kN}$$

The criteria for uniform spacing of shear connectors in region L_{II} are satisfied and there are the same as region L_I.

4.7 Resistance moment of composite section at mid-span

According to clause 6.2.1.3, EN 1994-1-1, the partial shear connection can be used in the region of sagging bending.

If the ductile shear connectors are used, the resistance moment can be determined by means of rigid plastic theory in accordance with 6.2.1.2, EN 1994-1-1. However, the reduced value of the compressive force N_c must be taken into account instead of the force $N_{c,f}$.

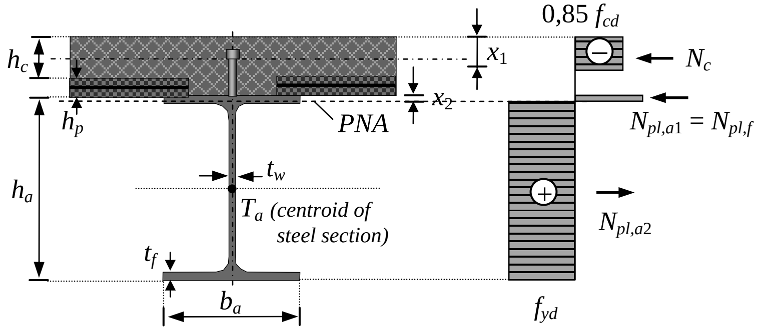
It is very convenient to use the diagram of partial shear connection, shown in Figure 6.5, EN 1994-1-1, for determining the resistance moment. According to clause 6.2.1.3(5), EN 1994-1-1, the design resistance moment of composite beam in sagging region can be conservatively calculated by the straight line AC in Figure 6.5, EN 1994-1-1:

$$M_{Rd} = M_{pl,a,Rd} + (M_{pl,Rd} - M_{pl,a,Rd}) \cdot \frac{N_c}{N_{c,f}}$$

where:

$M_{pl,a,Rd}$ is the design plastic resistance moment of the structural steel section alone in sagging region,
$M_{pl,Rd}$ is the design plastic resistance moment of the composite section with full shear connection in sagging region,
$N_{c,f}$ is the design compressive force in the concrete flange with full shear connection,
N_c is the design compressive force in concrete flange.

In this example, the resistance moment M_{Rd} is calculated by means of rigid plastic theory in accordance with clause 6.2.1.2, EN 1994-1-1, with taking into account the reduced value of the compressive force in the concrete flange N_c instead of $N_{c,f}$. The simplified procedure, with the conservative value of M_{Rd}, is illustrated in example B6.



PNA – plastic neutral axis

Figure B7.10 *Determination of the design resistance moment in the sagging region – PNA lies within flange of steel section*

The plastic neutral axis lies within the thickness of the top steel flange if it is:

$$N_c < N_{pl,a}$$

The design plastic resistance of the structural steel section to normal force is:

$$N_{pl,a} = \frac{A_a \cdot f_{yd}}{\gamma_{M,0}} = \frac{159 \cdot 35{,}5}{1{,}0} = 5645 \text{ kN}$$

The reduced design value of the compressive force in the concrete flange is:

$$N_c = n_I \cdot P_{Rd} = 30 \cdot 98{,}9 = 2967 \text{ kN}$$

$2967 < 5645$, the plastic neutral axis lies within the thickness of top steel flange.

When the plastic neutral axis lies within the thickness of the top steel flange and the shear force is necessary to take into account, $N_{pl,a}$ is:

$$N_{pl,a} = (A_a - \rho \cdot A_V) f_{yd}$$

The design plastic resistance of the steel flange to normal force is:

$$N_{pl,f} = \frac{N_{pl,a} - N_c}{2}$$

The shear force does not exceed half the shear resistance and its effect on the resistance moment is neglected, i.e. $\rho = 0$. Therefore, the design plastic resistance of the structural steel section to normal force is:

$$N_{pl,a} = A_a \cdot f_{yd} = 159 \cdot 35{,}5 = 5645 \text{ kN}$$

The design plastic resistance of the steel flange to normal force is:

$$N_{pl,f} = \frac{N_{pl,a} - N_c}{2} = \frac{5645 - 2967}{2} = 1339 \text{ kN}$$

In accordance with Figure B7.10, the following values are calculated:

$$x_1 = \frac{N_c}{b_{eff,1} \cdot 0{,}85 f_{cd}} = \frac{2967}{265{,}6 \cdot 0{,}85 \cdot 1{,}67} = 7{,}87 \text{ cm} < h_c = 11 \text{ cm}$$

$$x_2 = \frac{N_{pl,f}}{b_f \cdot f_{yd}} = \frac{1339}{30 \cdot 35{,}5} = 1{,}26 \text{ cm} < t_f = 1{,}9 \text{ cm}$$

$$z = h_c + h_p + \frac{h_a}{2} - \frac{x_1}{2}$$

$$z = 11{,}0 + 5{,}0 + \frac{39{,}0}{2} - \frac{7{,}87}{2} = 11 + 5{,}0 + 19{,}5 - 3{,}94 = 31{,}56 \text{ cm} = 0{,}316 \text{ m}$$

The design resistance moment of the composite section in sagging region is:

$$M_{Rd} = N_c \cdot z + N_{pl,f}(h_a - x_2)$$

$$M_{Rd} = 2967 \cdot 0{,}316 + 1339(0{,}39 - 0{,}0126)$$

$$M_{Rd} = 1443 \text{ kNm}$$

The design value of bending moment at mid-span, as the effect of action, M_{Ed} = 481 kNm and the check is as follows:

$$\frac{M_{Ed}}{M_{Rd}} \leq 1,0$$

$$\frac{481}{1443} = 0,33 < 1,0$$, the condition is satisfied

4.8 Vertical shear resistance of the cross-section

The shear buckling resistance of web should be verified, for unstiffened web when:

$$\frac{h_w}{t} > \frac{72}{\eta}\varepsilon$$

where:

$$\varepsilon = \sqrt{\frac{235}{f_y}} = \sqrt{\frac{235}{275}} = 0,92$$

$\eta = 1,2$, the factor defined in EN 1993-1-5

$$h_w = h_a - 2t_f = 390 - (2 \cdot 19) = 352 \text{ mm}$$

$$\frac{72}{\eta}\varepsilon = \frac{72}{1,2} \cdot 0,81 = 48,6$$

$$\frac{h_w}{t} = \frac{h_w}{t_w} = \frac{352}{11} = 32$$

Since 32 < 48,6 the condition is satisfied. The shear buckling resistance of the web need not be verified.

Remark:

The resistance of the composite beam to vertical shear is normally taken as the shear resistance of the steel section according to clause 6.2.6, EN 1993-1-1, which gives:

$$V_{pl,Rd} = V_{pl,a,Rd} = \frac{A_V (f_y / \sqrt{3})}{\gamma_{M0}}$$

For rolled I- and H-sections and the load applied parallel to the web, the shear area is calculated as:

$A_V = A_a - 2 \cdot b_a \cdot t_f + t_f \cdot (t_w + 2 \cdot r)$, but not less than $\eta \cdot h_w \cdot t_w$

The shear area A_V is:

$$A_V = 159 - 2 \cdot 30 \cdot 1{,}9 + 1{,}9(1{,}1 + 2 \cdot 2{,}7)$$

$$A_V = 57{,}35 \ \text{cm}^2$$

$$\eta = 1{,}2$$

$$\eta \cdot h_w \cdot t_w = 1{,}2 \cdot 35{,}2 \cdot 1{,}1 = 46{,}46 \ \text{cm}^2$$

57,35 cm^2 > 46,46 cm^2

Therefore A_V = 57,35 cm^2.

The design plastic shear resistance of the steel section is:

$$V_{pl,Rd} = V_{pl,a,Rd} = \frac{A_V (f_y / \sqrt{3})}{\gamma_{M0}}$$

$$V_{pl,Rd} = V_{pl,a,Rd} = 57{,}35 \frac{35{,}5}{\sqrt{3} \cdot 1{,}0} = 1175 \ \text{kN}$$

Verify that:

$$\frac{V_{Ed}}{V_{pl,Rd}} \leq 1{,}0$$

$$\frac{390}{1175} = 0{,}33 < 1{,}0$$

Therefore the shear resistance of the cross-section is adequate.

4.9 Interaction of M-V (bending and shear force)

Where the shear force is less than half the plastic shear resistance its effect on the resistance moment can be neglected.

$$0{,}5 \cdot V_{pl,Rd} = 0{,}5 \cdot 1175 = 588 \text{ kN}$$

$V_{Ed} = 390 \text{ kN} < 0{,}5\, V_{pl,Rd} = 588 \text{ kN}$ no reduction in the resistance moment

$$M_{y,V,Rd} = M_{pl,Rd} = 1443 \text{ kNm}$$

Therefore, the assumption for the classification of the cross-section that the reduction factor $\rho = 0$ is confirmed.

The condition at the internal support $M_{Ed} / M_{Rd} \leq 1{,}0$ is satisfied because in the plastic–plastic procedure the value $M_{Ed} = M_{pl,Rd}$ is adopted and confirmed in Section 4.5.

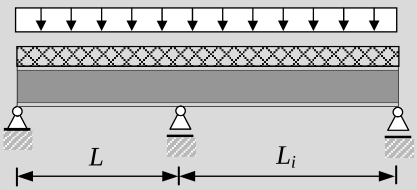
4.10 Lateral-torsional buckling of the composite beam

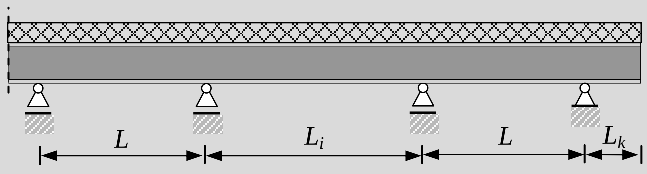
Clause 6.4.3, EN 1994-1-1, gives guidance for the verification of buckling resistance moment of continuous beams in buildings.

If the conditions, given in clause 6.4.3(1), EN 1994-1-1, are satisfied, the verification of lateral-torsional buckling is not necessary.

The conditions are as follows:

a) The ratio of adjacent spans





$0{,}8 \leq L/L_i \leq 1{,}20$ $L_k/L \leq 0{,}15$

The difference in length of adjacent spans is less than 20% of the shorter span. The length of the cantilever is less than 15% of the adjacent span.

b) The ratio of permanent and total design loads

The loads are uniformly distributed on each span. The design permanent load is greater than the total design load by 40% or more.

$$\frac{\gamma_G \cdot G_k}{\gamma_G \cdot G_k + \gamma_Q \cdot Q_k} \geq 0{,}4$$

c) Shear connection

The shear connection between the upper flange of the steel beam and the concrete flange should be provided in accordance with clause 6.6 (EN 1994-1-1).

d) The inverted-U frame

The same concrete flange is also attached to one or more supporting steel beams so that they form an inverted-U frame.

e) The composite slab

The span of the composite slab between the two supporting beams of the inverted-U frame should be taken into consideration.

f) The lateral restraint of the bottom flange of the steel beam

The bottom flange of the steel beam is laterally restrained at each support. Also, the web of the steel beam is stiffened.

g) Composite beams that are not partially encased

The limitations of depth for the steel beams of IPE section or HE section that are not partially encased are given in Table B7.1.

h) Composite beams that are partially encased

The depth of partially encased composite beams does not exceed the limit given in Table B7.1 by more than 200 mm for steel grades up to S355, and by 150 mm for grades S420 and S460.

Table B7.1 *Maximum depth h (mm) of uncased steel beam, EN 1994-1-1*

Steel beam	*Nominal steel grade*			
	*S*235	*S*275	*S*355	*S*420 *and S*460
IPE	600	550	400	270
HE	800	700	650	500

According to clause 6.3.2.2(4), EN 1993-1-1, the verification of lateral-torsional buckling for a member in bending may be neglected if at least one of the following conditions is satisfied:

$\bar{\lambda}_{LT} \leq \bar{\lambda}_{LT,0}$ or $M_{Ed}/M_{cr} \leq \bar{\lambda}_{LT,0}$ with maximum value $\bar{\lambda}_{LT,0} = 0{,}4$

Verification of conditions:

a) The ratio of adjacent spans

The condition is satisfied because the spans L_1 and L_2 are the same.

b) The ratio of permanent and total design loads

$$\frac{\gamma_G \cdot g_{k,2} + \gamma_G \cdot g_{k,3}}{\gamma_G \cdot g_{k,2} + \gamma_G \cdot g_{k,3} + \gamma_Q \cdot q_k} = \frac{1{,}35 \cdot 4{,}42 + 1{,}35 \cdot 2{,}00}{1{,}35 \cdot 4{,}42 + 1{,}35 \cdot 2{,}00 + 1{,}5 \cdot 5{,}0} = 0{,}54 > 0{,}4$$

The condition is satisfied.

c) Shear connection

The condition is satisfied.

d) The inverted-U frame

The condition is satisfied.

e) The composite slab

The condition is satisfied.

f) Lateral restraint of the bottom flange of the steel beam

The condition is satisfied.

g) Composite beams that are not partially encased

The condition is satisfied.

h) Composite beams that are partially encased

In this case, the composite beam is not partially encased. The condition is not governed.

Since the conditions are satisfied, the additional verification for lateral-torsional buckling is not necessary.

4.11 Check of longitudinal shear resistance of the concrete flange

4.11.1 Check of transverse reinforcement

The cross-sectional area of the transverse reinforcement is calculated according to the expression:

$$\frac{A_{sf}}{s_f} \cdot f_{sd} \geq v_{L,Ed} \cdot \frac{h_f}{\cot\theta}$$

where:

A_{sf}/s_f is the transverse reinforcement expressed in mm^2/m,
h_f is the thickness of the concrete flange, see Figures B7.11 and B7.12,
θ is the angle between the diagonal strut and the axis of the beam (strut-and-tie model),
$v_{L,Ed}$ is the design longitudinal shear flow in the concrete slab.

The transverse reinforcement (A_{sf}/s_f) expressed in mm^2/m can be denoted as A_t for the top transverse reinforcement and as A_b for the bottom transverse reinforcement. It is necessary to verify the failure due to shear in the failure plane shown in Figure B7.11 as sections *a*-*a* and *c*-*c*.

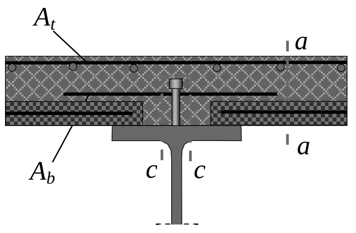


Figure B7.11 *Surfaces of potential failure in longitudinal shear*

The transverse reinforcement provided is 10 mm bars at 150 mm, for which:

$$A_t = A_b = \frac{\pi \cdot 10^2}{4} \cdot \frac{1000}{150} = 524 \ \text{mm}^2/\text{m}$$

It is necessary to ensure that the concrete flange can resist the longitudinal shear force transmitted to it by the shear connectors. At the steel-concrete interface, the distribution of longitudinal shear is influenced by yielding, by the spacing of the shear connectors, their load-slip properties, and shrinkage and creep of concrete. The design resistance to longitudinal shear for the relevant shear failure surfaces is given in clause 6.2.4, EN 1992-1-1. The model is based on considering the flange to act like a system of compressive struts combined with a system of ties in the form of the transverse reinforcement.

When the concrete flange is in compression, longitudinal shear flow $v_{L,Ed}$ can be defined as:

$$v_{L,Ed,1} = \frac{\Delta N_{c1}}{a_v} = \frac{V_{L,Ed}}{a_v} \frac{A_{c1,eff}}{A_{c,eff}}$$

where:

a_v is the critical length (the distance between two given sections) Figure B7.12,
ΔN_{c1} is the change of the longitudinal compressive forces in the slab over the critical length a_v, see Figure B7.12,
$V_{L,Ed}$ is the design longitudinal shear force in the steel-concrete interface or in the concrete flange,

$$V_{L,Ed} = \min(N_{pl,a}, N_c, \Sigma P_{Rd})$$

When the concrete flange is in tension, longitudinal shear flow $v_{L,Ed}$ can be defined as:

$$v_{L,Ed,1} = \frac{\Delta N_{s1}}{a_v} = \frac{V_{L,Ed}}{a_v} \frac{A_{s1}}{A_{s1} + A_{s2}}$$

where:

a_v is the critical length (the distance between two given sections) Figure B7.12,
ΔN_{s1} is the change of the longitudinal tensile forces in the slab over the critical length a_v, see Figure B7.12,
$V_{L,Ed}$ is the design longitudinal shear force in the steel-concrete interface or in the concrete flange,

$$V_{L,Ed} = \min(N_s, \Sigma P_{Rd})$$

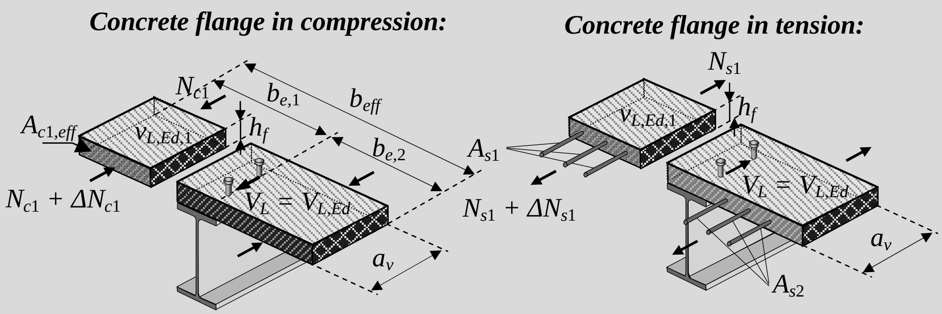


Figure B7.12 *Determination of the longitudinal shear forces in the concrete flange*

In the first case, the length a_v is L_I = 4,46 m, while in the second case the length is L_{II} = 8,04 m.

Region L_I

The design longitudinal shear force is determined from the minimum resistance of the steel section, concrete and shear connectors:

$$V_{L,Ed} = \min(N_{pl,a}, N_{c,f}, \Sigma P_{Rd})$$

$$N_{pl,a} = 5645 \text{ kN}$$

$$N_{c,f} = 4147 \text{ kN}$$

$$\Sigma P_{Rd} = 30 \cdot 98,9 = 2967 \text{ kN}$$

The maximum force between the point of maximum moment and the support A that can be transferred into the longitudinal shear flow is limited by the resistance of the shear connectors.

Section *a-a*

$$V_{L,Ed} = 2967 \text{ kN}$$

$$f_{sd} = 435 \text{ N/mm}^2$$

As there are two shear planes, see Figure B7.11, one on either side of the beam, running parallel to it, and with $h_f = h_c = 110$ mm (the prefabricated element is neglected), the design longitudinal shear stress is:

$$v_{L,Ed} = \frac{\Delta N_{c1}}{h_f \cdot a_v} = \frac{V_{L,Ed}}{2 \cdot h_f \cdot a_v} = \frac{2967 \cdot 10^3}{2 \cdot 110 \cdot 4450} = 3,03 \text{ N/mm}^2$$

Remark:

In order to prevent splitting of the concrete flange, for the adopted "truss model", according to clause 6.2.4(4) EN 1992-1-1, the angle θ between the concrete diagonals and the longitudinal direction is limited to the value:

$26,5^\circ \leq \theta \leq 45^\circ$ concrete flange in compression

$38,6^\circ \leq \theta \leq 45^\circ$ concrete flange in tension

In order to minimize the cross-sectional area of transverse reinforcement, the minimum angle θ is selected. For the concrete flange in compression (at mid-span), the minimum angle θ is:

$$\theta = 26,5^\circ$$

$$\frac{A_{sf}}{s_f} \cdot f_{sd} \geq v_{L,Ed} \cdot \frac{h_f}{\cot\theta}$$

$$\frac{A_{sf}}{s_f} \geq \frac{v_{L,Ed}}{f_{sd}} \cdot \frac{h_f}{\cot\theta} = \frac{3,03}{435} \frac{110}{\cot 26,5} \cdot 10^3 = 382 \text{ mm}^2\text{/m}$$

The reinforcement which is less than the selected reinforcement is obtained as $A_t + A_b = 524 + 524 = 1048$ mm^2.

Section *c-c*

$$V_{L,Ed} = 2967 \text{ kN}$$

$$f_{sd} = 435 \text{ N/mm}^2$$

The length h_f of the shear surface *c-c* passing round the studs as shown in Figure B7.11 is:

$$h_f = 2 \cdot h_{sc} + 1{,}5 \cdot d = 2 \cdot 125 + 1{,}5 \cdot 22 = 283 \text{ mm}$$

The design longitudinal shear stress is:

$$v_{L,Ed} = \frac{\Delta N_{c1}}{h_f \cdot a_v} = \frac{V_{L,Ed}}{h_f \cdot a_v} = \frac{2967 \cdot 10^3}{283 \cdot 4450} = 2{,}36 \text{ N/mm}^2$$

In order to minimize the cross-sectional area of the transverse reinforcement, the minimum angle θ is selected. For the concrete flange in compression (at mid-span), the minimum angle θ is:

$$\theta = 26{,}5^{\circ}$$

$$\frac{A_{sf}}{s_f} \cdot f_{sd} \geq v_{L,Ed} \cdot \frac{h_f}{\cot\theta}$$

$$\frac{A_{sf}}{s_f} \geq \frac{v_{L,Ed}}{f_{sd}} \cdot \frac{h_f}{\cot\theta} = \frac{2{,}36}{435} \frac{283}{\cot 26{,}5} \cdot 10^3 = 766 \text{ mm}^2\text{/m}$$

The reinforcement which is less than the selected reinforcement is obtained as $2 \cdot A_b = 2 \cdot 524 = 1048 \text{ mm}^2$.

Region L_{II}

Section *a-a*

In this region, there are both sagging and hogging bending. The design longitudinal shear force is the sum of the longitudinal shear forces from the two regions:

$$V_{L,Ed} = \Sigma P_{Rd} + N_s = 30 \cdot 98{,}9 + 16{,}38 \cdot 43{,}5 = 3680 \text{ kN}$$

$$f_{sd} = 435 \text{ N/mm}^2$$

$$h_f = h_c = 110 \text{ mm}$$

$$v_{L,Ed} = \frac{\Delta N_{c1}}{h_f \cdot a_v} = \frac{V_{L,Ed}}{2 \cdot h_f \cdot a_v} = \frac{3680 \cdot 10^3}{2 \cdot 110 \cdot 8050} = 2{,}08 \text{ N/mm}^2$$

In order to minimize the cross-sectional area of the transverse reinforcement, the minimum angle θ is selected. For the concrete flange, with the assumption that it is in tension along the whole length of region L_{II}, the minimum angle θ is:

$$\theta = 38{,}6^{o}$$

$$\frac{A_{sf}}{s_f} \cdot f_{sd} \geq v_{L,Ed} \cdot \frac{h_f}{\cot\theta}$$

$$\frac{A_{sf}}{s_f} \geq \frac{v_{L,Ed}}{f_{sd}} \cdot \frac{h_f}{\cot\theta} = \frac{2{,}08}{435} \frac{110}{\cot 38{,}6} \cdot 10^3 = 420 \text{ mm}^2\text{/m}$$

The reinforcement which is less than the selected reinforcement is obtained as $A_t + A_b = 524 + 524 = 1048 \text{ mm}^2$.

Section *c-c*

$$V_{L,Ed} = 3680 \text{ kN}$$

$$f_{sd} = 435 \text{ N/mm}^2$$

The length h_f of the shear surface *c-c* passing round the studs as shown in Figure B7.11 is:

$$h_f = 283 \text{ mm}$$

The design longitudinal shear stress is:

$$v_{L,Ed} = \frac{\Delta N_{c1}}{h_f \cdot a_v} = \frac{V_{L,Ed}}{h_f \cdot a_v} = \frac{3680 \cdot 10^3}{283 \cdot 8050} = 1{,}62 \text{ N/mm}^2$$

In order to minimize the cross-sectional area of the transverse reinforcement, the minimum angle θ is selected. For the concrete flange with the assumptions that it is in tension along the whole length of region L_{II}, the minimum angle θ is:

$$\theta = 38{,}6^{o}$$

$$\frac{A_{sf}}{s_f} \cdot f_{sd} \geq v_{L,Ed} \cdot \frac{h_f}{\cot\theta}$$

$$\frac{A_{sf}}{s_f} \geq \frac{v_{L,Ed}}{f_{sd}} \cdot \frac{h_f}{\cot\theta} = \frac{1{,}62}{435} \frac{283}{\cot 38{,}6} \cdot 10^3 = 841 \text{ mm}^2/\text{m}$$

The reinforcement which is less than the selected reinforcement is obtained as $2 \cdot A_b = 2 \cdot 524 = 1048 \text{ mm}^2$.

According to EN 1994-1-1, clause 6.6.6.3, the minimum area of transverse reinforcement is determined in accordance with EN 1992-1-1, clause 9.2.2(5), which gives the minimum area of reinforcement as a proportion of the concrete area. The ratio is:

$$\rho_{w,\min} = \frac{0{,}08\sqrt{f_{ck}}}{f_{yr,k}}$$

where:

f_{ck} is the characteristic compressive cylinder strength of the concrete at 28 days in N/mm^2,
$f_{yr,k} = f_{sk}$ is the characteristic yield strength of the reinforcement in N/mm^2.

The minimum area of transverse reinforcement is:

$$\rho_{w,\min} = \frac{0{,}08\sqrt{f_{ck}}}{f_{yr,k}} = \frac{0{,}08\sqrt{25}}{500} = 0{,}0008$$

$$A_c = h_c \cdot b = 110 \cdot 1000 = 110000 \text{ mm}^2$$

$$A_{s,\min} = \rho_{w,\min} \cdot A_c = 0{,}0008 \cdot 110000 = 88 \text{ mm}^2/\text{m}$$

Since $A_t + A_b = 1048 \text{ mm}^2/\text{m} > A_{s,\min} = 88 \text{ mm}^2/\text{m}$, the requirement for minimum transverse reinforcement is satisfied.

4.11.2 Crushing of the concrete flange

To prevent crushing of the compression struts in the flange, the following condition should be satisfied according to EN 1992-1-1, expression 6.22:

$$v_{L,Ed} \leq v_{Rd}$$

$$v_{L,Ed} \leq v \cdot f_{cd} \cdot \sin\theta \cdot \cos\theta$$

where:

$$\nu = 0{,}6 \cdot (1 - \frac{f_{ck}}{250})$$

θ is the angle between the concrete diagonals and the longitudinal direction.

In order to minimize the resistance of the concrete compression strut, the minimum angle θ is selected. For the concrete flange in compression (at mid-span), the minimum angle θ is:

$$\theta = 26{,}5^{\mathrm{o}}$$

$$v_{Rd} = \nu \cdot f_{cd} \cdot \sin\theta \cdot \cos\theta = 0{,}6 \cdot (1 - \frac{25}{250}) \cdot 16{,}7 \cdot \sin 26{,}5^{\mathrm{o}} \cdot \cos 26{,}5^{\mathrm{o}} = 3{,}60 \ \text{N/mm}^2$$

Check:

$$v_{L,Ed} = 3{,}03 \ \text{N/mm}^2 < v_{Rd} = 3{,}60 \ \text{N/mm}^2$$

Therefore the crushing resistance of the concrete compression strut is adequate.

5. Serviceability limit state

5.1 General

Chapter 7, EN 1994-1-1, is limited to provisions relating to serviceability that are specific to composite structures. Serviceability verifications in the case of the composite structures generally include checks of stress, deflection and vibration as well control of crack width.

For buildings, stress limitation is not required for beams if, in the ultimate limit state, no verification of fatigue is required and no pre-stressing by tendons and/or by controlled imposed deformations is provided. However, if the stress limitation is required, clause 7.2, EN 1992-1-1, gives the stress limits which may be applicable for buildings that have pre-stressing or fatigue loading.

Since the deflection is one of the most important verifications of the serviceability limit state, it is necessary to explain in detail the problems associated with the deflection calculation. Deflections due to loads applied to the composite member are calculated using the theory of elasticity taking into account the following effects:

a) cracking of concrete,
b) creep and shrinkage of concrete,
c) sequence of construction,
d) influence of local yielding of the structural steel at internal support (for continuous beams),
e) influence of incomplete interaction.

For a more detailed explanation of these effects, see example B6.

If the steel beam is fully propped, the total deflection of composite beams is obtained by summing the following deflections:

$$\delta = \delta_1 + \delta_{2,1} + \delta_{2,2} + \delta_{2,3}$$

where:

δ_1 is the deflection due to the permanent actions (the first loading is applied at the age of t_0 = 28 days),
$\delta_{2,1}$ is the deflection due to the frequent value of variable action at the time of first loading,
$\delta_{2,2}$ is the deflection due to creep under the quasi-permanent value of the variable action at time $t = \infty$,
$\delta_{2,3}$ is the deflection due to shrinkage.

5.2 Calculation of deflections

5.2.1 Construction stage deflection

In this example, the steel beam is fully propped at the construction stage and the deflection of the steel beam at the construction stage is:

$$\delta_0 = 0$$

5.2.2 Composite stage deflection

The deflection of the composite beam has been calculated with commercial software, using the flexural stiffness of the composite section which depends on the type of loadings EI_L.

Determination of the creep coefficient and shrinkage

For the calculation of the creep coefficient $\varphi(t, t_0)$ the following is valid:

- the perimeter of that part which is exposed to drying, u

$$u = b$$

- the notional size of the cross-section, h_0

$$h_0 = \frac{2 \cdot A_c}{u} = \frac{b \cdot h_c}{b} = h_c = 160 \text{ mm}$$

- $t_0 = 1$ day, $t_0 = 28$ days,
- the ambient relative humidity, RH 50%,
- the concrete strength class C 25/30,
- the type of cement – cement class S, strength class 32,5 N.

The final value of creep coefficient $\varphi(\infty, t_0)$ can be determined using the nomogram shown in Figure 3.1, EN 1992-1-1. Example B3, shows the detailed procedure for the determination of creep coefficients from nomograms.

The following creep coefficients are obtained:

$\varphi_t = \varphi(\infty, t_0 = 1 \text{ day}) = 5{,}8$
$\varphi_t = \varphi(\infty, t_0 = 28 \text{ days}) = 2{,}8$

The total shrinkage strain, according to clause 3.1.4, EN 1992-1-1, at the age of concrete at the beginning of drying shrinkage t_s = 3 days and at the time considered in the analysis $t = \infty$, is:

$$\varepsilon_{cs}(\infty) = 4{,}14 \cdot 10^{-4}$$

$$\varepsilon_{cs}(\infty) = 0{,}414 \text{ ‰}$$

Effective flexural stiffness of the composite section

The effective flexural stiffness of the composite section EI_L (in this case approximately is $h_c = h$) is:

$$EI = E_a \cdot I_a + E_L \cdot I_c + \frac{E_a \cdot A_a \cdot E_L \cdot A_c}{E_a \cdot A_a + E_L \cdot A_c} \cdot a^2$$

a) Short-term loading

$$E_a = 21000 \text{ kN/cm}^2, \quad I_a = 45070 \text{ cm}^4, \; A_a = 159{,}0 \text{ cm}^2$$

$$I_c = \frac{b_{eff} \cdot h^3}{12} = \frac{265{,}6 \cdot 16^3}{12} = 90658 \text{ cm}^4$$

$$A_c = b_{eff} \cdot h = 265{,}6 \cdot 16 = 4250 \text{ cm}^2$$

The distance between the centroidal axes of the concrete flange and the steel section is:

$$a = \frac{h_a}{2} + \frac{h_1}{2} = \frac{39}{2} + \frac{16}{2} = 27{,}5 \text{ cm}$$

$$n_c = 1$$

$$E_0 = \frac{E_{cm}}{n_c} = \frac{3100}{1{,}0} = 3100 \text{ kN/cm}^2, \; E_L = E_0$$

$$EI_0 = 21000 \cdot 45070 + 3100 \cdot 90658 + \frac{21000 \cdot 159 \cdot 3100 \cdot 4250}{21000 \cdot 159 + 3100 \cdot 4250} \cdot 27{,}5^2$$

$$EI_0 = 3242069539 \text{ kNcm}^2 = 324207 \text{ kNm}^2.$$

b) Permanent loading constant in time

$$n_c = 1 + 1{,}10 \cdot \varphi(\infty, t_0) = 1 + 1{,}10 \cdot 5{,}8 = 7{,}38$$

$$E_P = \frac{E_{cm}}{n_c} = \frac{3100}{7{,}38} = 420 \text{ kN/cm}^2 \quad E_L = E_P$$

$$EI_P = 21000 \cdot 45070 + 420 \cdot 90658 + \frac{21000 \cdot 159 \cdot 420 \cdot 4250}{21000 \cdot 159 + 420 \cdot 4250} \cdot 27{,}5^2$$

$$EI_P = 1864198384 \text{ kNcm}^2 = 186420 \text{ kNm}^2$$

c) Secondary effects due to shrinkage (statically indeterminate structure)

$$n_c = 1 + 0{,}55 \cdot \varphi(\infty, t_0) = 1 + 0{,}55 \cdot 2{,}8 = 2{,}54$$

$$E_S = \frac{E_{cm}}{n_c} = \frac{3100}{2{,}54} = 1220 \text{ kN/cm}^2 \quad E_L = E_S$$

$$EI_S = 21000 \cdot 45070 + 1220 \cdot 90658 + \frac{21000 \cdot 159 \cdot 1220 \cdot 4250}{21000 \cdot 159 + 1220 \cdot 4250} \cdot 27{,}5^2$$

$$EI_S = 2593058297 \ \text{kNcm}^2 = 259306 \ \text{kNm}^2$$

Remark:

As a simplification for composite beams in structures for buildings where first-order global analysis can be applied, clause 5.4.2.2(11), EN 1994-1-1, permits the modular ratio to be taken as $2 \cdot n_0$ for both short-term and long-term loading. The application of this simplification is shown in examples B3 and B8.

Effects of cracking of concrete

Remark:

As explained in Section 4.2, example B6, effects of cracking of the concrete may be taken into account using the simplified method II for continuous composite beams as well beams in braced frames. According to the simplified method the effect of cracking is taken into account by using the flexural stiffness E_aI_2 over 15% of the span on each side of each internal support, and as the uncracked values E_aI_1 elsewhere. The reinforcement may be taken into consideration.

Therefore, in the region of the internal support the reduced flexural stiffness, E_aI_2, is used. The modulus of elasticity of steel is denoted by E_a. The second moment of area of the effective steel section calculated neglecting concrete in tension but including reinforcement is denoted with I_2. At the internal support the concrete is in tension and the second moment of area of the steel section and the reinforcement in slab, I_2, is determined as follows.

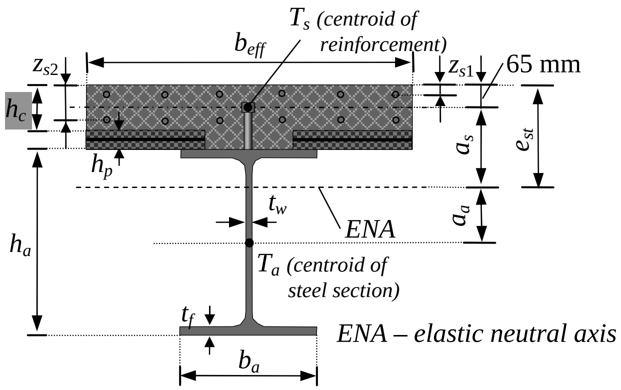


Figure B7.13 *Cross-section of the composite beam at the internal support*

The total area of steel section and reinforcement is:

$$A_{s1} = 8{,}185 \text{ cm}^2,\ A_{s2} = 8{,}185 \text{ cm}^2,\ A_a = 159 \text{ cm}^2$$

$$A_{st} = A_{s1} + A_{s2} + A_a = 8{,}185 + 8{,}185 + 159 = 175{,}4 \text{ cm}^2$$

The distance between the centroid of the cracked section and the top of concrete slab is:

$$e_{st} = \frac{A_{s1} \cdot z_{s1} + A_{s2} \cdot z_{s2} + A_a (h_c + h_p + h_a / 2)}{A_{st}}$$

$$e_{st} = \frac{8{,}185 \cdot 2{,}5 + 8{,}185 \cdot 10{,}5 + 159(11 + 5 + 39/2)}{175{,}4} = 32{,}8 \text{ cm}$$

Therefore, the distance between the neutral axis and the centroid of steel section, a_a, is:

$$a_a = \frac{h_a}{2} + h_p + h_c - e_{st} = \frac{39}{2} + 5{,}0 + 11{,}0 - 32{,}8 = 2{,}7$$

The distance between the neutral axis and the centroidal axis of reinforcement, a_s, is:

$$a_s = h_c + h_p + \frac{h_a}{2} - z_{si} - a_a = 11{,}0 + 5{,}0 + \frac{39}{2} - 6{,}5 - 2{,}7 = 26{,}3 \text{ cm}$$

The second moment of area of the effective steel section neglecting concrete in tension but including the reinforcement is:

$$I_{st} = I_2 = I_a + A_{s1} \cdot z_{s1}^2 + A_{s2} \cdot z_{s2}^2 + A_a (h_c + h_p + h_a / 2)^2 - A_{st} \cdot e_{st}^2$$

$$I_{st} = 45070 + 8{,}185 \cdot 2{,}5^2 + 8{,}185 \cdot 10{,}5^2 + 159(11 + 5 + 39/2)^2 - 175{,}4 \cdot 32{,}8^2$$

$$I_{st} = 57701 \text{ cm}^4$$

$$EI_{st} = E_a I_2 = 1211721000 \text{ kNcm}^2 = 121172 \text{ kNm}^2$$

Calculation of deflections

The deflections have been calculated using commercial software. The concrete is cracked at a length of 0,15 L on each side of the internal support. The reduced flexural stiffness E_aI_2 is allowed for in this region. At mid-span, the corresponding flexural stiffness E_aI_L is allowed for at a length of 0,85 L.

- Deflection due to permanent action, the first loading is applied at the age of t_0 = 28 days

$$e_d = b \cdot (g_{k,1} + g_{k,2}) = 3{,}00 \cdot (4{,}42 + 2{,}00) = 19{,}26 \text{ kN/m}$$

$$EI_L = EI_0 = 324207 \text{ kNm}^2$$

$$E_aI_2 = 121172 \text{ kNm}^2$$

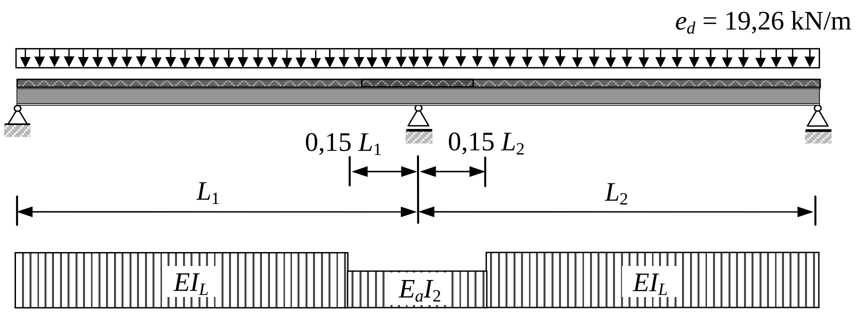


Figure B7.14 *Static system - permanent action at the time of first loading*

$$\delta_1 = 10{,}3 \text{ mm}$$

- Deflection due to the frequent value of the variable action at the time of first loading

For a building with floors in category B, office areas, the combination factor ψ is:

ψ_1 = 0,5

$$e_d = b \cdot \psi_1 \cdot q_k = 3{,}00 \cdot 0{,}5 \cdot 5{,}0 = 7{,}5 \text{ kN/m}$$

$$EI_L = EI_0 = 324207 \text{ kNm}^2$$

$$E_aI_2 = 121172 \text{ kNm}^2$$

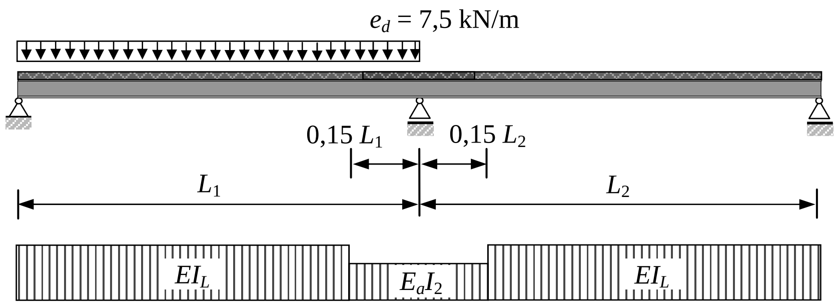


Figure B7.15 *Static system - frequent value of the variable action at the time of first loading*

$\delta_{2,1} = 5,8$ mm

- Deflection due to creep under the quasi-permanent value of the variable action at time $t = \infty$

This deflection is the difference of deflections at time $t = \infty$ and at time of first loading t_0 = 28 days.

$$e_{d,1} = b \cdot (g_{k,1} + g_{k,2} + \psi_2 \cdot q_k) = 3,00 \cdot (4,42 + 2,00 + 0,3 \cdot 5,0) = 23,76 \text{ kN/m}$$

$$e_{d,2} = b \cdot (g_{k,1} + g_{k,2}) = 3,00 \cdot (4,42 + 2,00) = 19,26 \text{ kN/m}$$

$$EI_L = EI_0 = 324207 \text{ kNm}^2$$

$$EI_L = EI_P = 186420 \text{ kNm}^2$$

$$E_a I_2 = 121172 \text{ kNm}^2$$

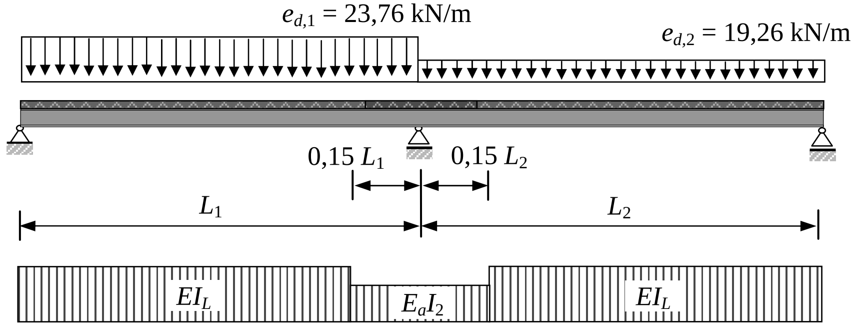


Figure B7.16 *Static system – deflection due to creep at time $t = \infty$ ($EI_L = EI_P$) and at the time of first loading ($EI_L = EI_0$)*

$\delta_{2,2} = 21{,}0 - 13{,}7 = 7{,}3 \text{ mm}$

- Deflection due to shrinkage

$N_{cs} = \varepsilon_{cs}(\infty) \cdot E_S \cdot A_c = 4{,}14 \cdot 10^{-4} \cdot 1220 \cdot 265{,}6 \cdot 16 = 2146 \text{ kN}$

$$a_c = \frac{E_a \cdot A_a}{E_a \cdot A_a + E_S \cdot A_c} \cdot a = \frac{21000 \cdot 159}{21000 \cdot 159 + 1220 \cdot 4250} \cdot 27{,}5 = 10{,}77 \text{ cm}$$

$M_{cs} = N_{cs} \cdot a_c = 2146 \cdot 10{,}77 = 23112 = \text{ kNcm} = 231 \text{ kNm}$

$EI_L = EI_S = 259306 \text{ kNm}^2$

$E_a I_2 = 121172 \text{ kNm}^2$

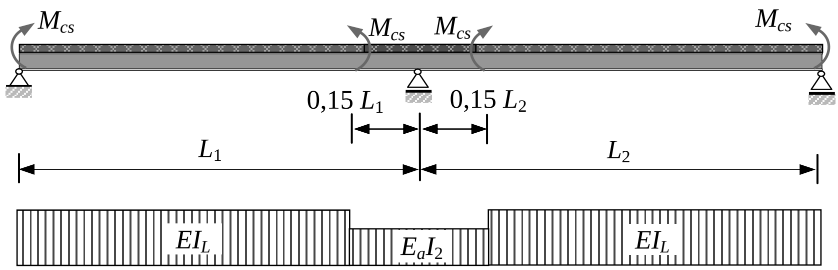


Figure B7.17 *Static system – deflection due to shrinkage*

$\delta_{2,3} = 9{,}6 \text{ mm}$

The effects of shear connection on the deflection of the beam can be neglected because the condition $n/n_f \geq 0{,}5$ is satisfied.

Remark:

The limitations of deflections are adopted according to the recommendation given in EN 1990. These values can be changed in accordance with the recommendation given in the National Annex.

Deflection limits for composite beams are the same as for steel beams, and are determined by the National Annex.

Recommended limiting values for deflection of composite beams are:

$\delta_{tot} \leq \dfrac{L}{250}$, the deflection due to total load

$\delta_{var} \leq \dfrac{L}{360}$, the deflection due to variable load

The deflection due to permanent action is:

$$\delta_1 = 10,3 \text{ mm}$$

The deflection due to variable load, creep and shrinkage is:

$$\delta_2 = \Sigma\delta_{2,i} = 5,8 + 7,3 + 9,6 = 22,7 \text{ mm}$$

The total deflection due to permanent and variable loads, creep and shrinkage is:

$$\delta_{tot} = \delta_1 + \delta_2 = 10,3 + 22,7 = 33,0 \text{ mm} \leq \frac{L}{250} = \frac{12500}{250} = 50,0 \text{ mm}$$

The total deflection meets the criterion *L*/250.

The deflection due to variable load, creep and shrinkage is:

$$\delta_{\text{var}} = \delta_2 = 22,7 \text{ mm} \leq \frac{L}{360} = \frac{12500}{360} = 35,0 \text{ mm}$$

The deflection due to variable load, creep and shrinkage meets the criterion *L*/360.

5.3 Pre-cambering of the steel beam

In this example, the pre-cambering of the steel beam involves deflections due to permanent loads, creep and shrinkage:

$$\delta_p = \delta_1 + \delta_{2,2} + \delta_{2,3}$$

$$\delta_p = 10,3 + 7,3 + 9,6$$

The following value of pre-cambering is adopted:

$$\delta_p = 27,2 \text{ mm}$$

For more detailed explanation see example B3.

5.4 Check of vibration of the beam

The natural frequency may be calculated in terms of the well-known expression that is often used in design:

$$f = \frac{18}{\sqrt{\delta}}$$

where δ is the maximum deflection in millimetres due to self-weight and other permanent loads. This well-known natural frequency expression may be used as the expression for determining the natural frequency of individual members, even when they are not simply supported, providing that the appropriate value of δ is used.

For the calculation of the natural frequency, the characteristic values of the permanent loads for the composite stage are taken into account and the effective flexural stiffness of the composite section for short-term loading EI_0.

$$e_d = b \cdot (g_{k,1} + g_{k,2}) = 3{,}00 \cdot (4{,}42 + 2{,}00) = 19{,}26 \ \text{kN/m}$$

The deflection under this load is approximately:

$$\delta = \frac{5}{384} \cdot \frac{e_d \cdot L^4}{EI_0} = \frac{5}{384} \cdot \frac{19{,}26 \cdot 12{,}5^4}{324207} \cdot 100 = 1{,}89 \ \text{cm}$$

The natural frequency of the beam is therefore:

$$f = \frac{18}{\sqrt{\delta}} = \frac{18}{\sqrt{18{,}9}} = 4{,}1 \ \text{Hz} \geq 4 \ \text{Hz} \quad \text{with} \quad \delta \ \text{in mm}$$

The criterion is satisfied for initial calculation purposes. However, the dynamic performance of the entire floor is carried out using a method such as the one in [51].

5.5 Control of crack width

5.5.1 Minimum reinforcement area

The exposure class is XC1. In the case of reinforced concrete elements according to EN 1992-1-1 this means that the crack width should be limited to the maximum value w_{max} = 0,4 mm.

Remark:

For exposure class XC1, crack width has no influence on durability of the structure, but this limit is set to guarantee acceptable appearance of structures.

The required minimum area of reinforcement A_s for the slab of composite beam, according to clause 7.4.2(1), EN 1994-1-1, is:

$$A_s = k_s \cdot k_c \cdot k \cdot f_{ct,eff} \cdot A_{ct} / \sigma_s$$

where:

$f_{ct,eff}$ is the mean value of the tensile strength of the concrete effective at the time when the first crack may be expected to occur. Values of $f_{ct,eff}$ can be taken as those for f_{ctm} (EN 1992-1-1, Table 3.1) or as f_{lctm} (EN 1992-1-1, Table 11.3.1) taking into account the concrete strength class at the time when the first crack of the concrete is expected to occur. If the time of occurrence of cracks cannot be established, it is possible to adopt the minimum tensile strength of 3 N/mm^2.

A_{ct} is the cross-sectional area of the tensile zone of the concrete (due to direct loading and primary effects of shrinkage). For simplicity, the cross-sectional area of the concrete may be adopted as the area determined by its effective width.

σ_s is the maximum stress allowed in the reinforcement immediately after cracking of the concrete. This stress can be taken as the characteristic value of the yield strength f_{sk}. To satisfy the required width limits, the lower values may be needed, depending on the diameter of the bar. This values are given in Table 7.1, EN 1994-1-1.

k, k_s, k_c are the coefficients based on the calibration procedure . The magnitude of these coefficients, k, k_s and k_c, depend on the geometry of the cracked composite section. More detailed explanation is given below.

The meaning of the coefficients, k, k_s, k_c, is the following:

k allows for the effect of non-uniform self-equilibrating tensile stresses; it may be taken as 0,8.

k_s allows for the effect of the reduction of the normal force of the concrete slab due to initial cracking and local slip of the shear connection; it may be taken as 0,9.

k_c takes into account the stress distribution within the cross-section (the tensile zone of the concrete A_{ct}) immediately prior to cracking.

The coefficient k_c is calculated as:

$$k_c = \frac{1}{1 + h_c / (2z_0)} + 0,3 \leq 1,0$$

where:

h_c is the thickness of the concrete flange, excluding any haunch or ribs,
z_0 is the vertical distance between the centroids of the uncracked concrete flange and the uncracked composite section, calculated using the modular ratio $n_0 = E_a/E_{cm}$ for short-term loading, i.e. at the time of first loading t_0.

Therefore, the values of coefficients k and k_s are:

$$k = 0,8$$
$$k_s = 0,9$$

Calculation of coefficient k_c

The modular ratio n_0 for short-term loading is:

$$n_L = n_0 = \frac{E_a}{E_{lcm}} = \frac{210000}{3100} = 6,77$$

The effective width of the concrete flange at the internal support is:

$$b_{eff,2} = 1,562 \text{ m}$$

Transformed to the ideal steel section, the effective width is:

$$\frac{b_{eff}}{n_0} = \frac{156,2}{6,77} = 23,1 \text{ cm}$$

Geometrical properties of ideal cross-section:

The area of the ideal steel cross-section is:

$$A_{i,L} = A_{st} + A_{c,L} = A_a + A_s + \frac{b_{eff,2}}{n_0} \cdot h_c = 159 + 16,37 + 23,1 \cdot 11 = 429,5 \text{ cm}^2$$

The distance between the centroid of steel section and the top of concrete slab is:

$$a_{st} = \frac{h_a}{2} + h_p + \frac{h_c}{2} = \frac{39}{2} + 5,0 + \frac{11}{2} = 30 \text{ cm}$$

The vertical distance between the centroids of the uncracked concrete flange and the uncracked composite section:

$$z_0 = z_{ic,L} = \frac{A_{st} \cdot a_{st}}{A_{i,L}} = \frac{(159 + 16,37) \cdot 30}{429,5} = 12,25 \text{ cm}$$

The coefficient k_c is:

$$k_c = \frac{1}{1 + h_c / (2z_0)} + 0,3 = \frac{1}{1 + 110 / (2 \cdot 122,5)} + 0,3 = 0,99 \leq 1,0$$

Adopted: $k_c = 0,99$.

If the time of occurrence of the cracks cannot be established, the mean value of the tensile strength of the concrete effective at the time when the first crack may be expected to occur $f_{ct,eff}$, can be adopted as the minimum tensile strength of 3 N/mm^2.

The cross-sectional area of the tensile zone of the concrete is:

$$A_{ct} = b_{eff,2} \cdot h_c = 1562 \cdot 110 = 171820 \text{ mm}^2$$

The maximum stress allowed in the reinforcement immediately after cracking of concrete σ_s is chosen from Table 7.1, EN 1994-1-1:

$$\sigma_s = 360 \text{ N/mm}^2$$

The maximum bar diameter ϕ^*, for design crack width w_k = 0,4 mm and for the chosen maximum stress allowed in reinforcement σ_s, according to Table 7.1, EN 1994-1-1, is:

ϕ^* = 10 mm

Minimum area of reinforcement

The required minimum area of reinforcement A_s is:

$$A_s = k_s \cdot k_c \cdot k \cdot f_{ct,eff} \cdot A_{ct} / \sigma_s = 0,9 \cdot 0,99 \cdot 0,8 \cdot 3 \cdot 171820 / 320 = 1148 \text{ mm}^2/b_{eff}$$

The required minimum area of reinforcement $A_s = 11{,}5\ \text{cm}^2/b_{eff}$ is less than the cross-sectional area of longitudinal reinforcement adopted in Section 4.4 as 16,37 cm^2/b_{eff}.

The initial selected reinforcement is adequate.

5.5.2 Control of cracking of the concrete due to direct loading

The bending moment at the internal support is calculated for the quasi-permanent combination of actions at time $t = \infty$. Since the steel beam is propped at the construction stage, the stresses in reinforcement are determined for permanent load $g_{k,1}$ and load of floor finishes, $g_{k,2}$, the quasi-permanent load, $\psi_2\ q_k$, and shrinkage.

$$e_d = b \cdot (g_{k,1} + g_{k,2} + \psi_2 \cdot q_k) = 3{,}00 \cdot (4{,}42 + 2{,}00 + 0{,}3 \cdot 5{,}0) = 23{,}76\ \text{kN/m}$$

$$EI_L = EI_P = 186420\ \text{kNm}^2$$

$$E_a I_2 = 121172\ \text{kNm}^2$$

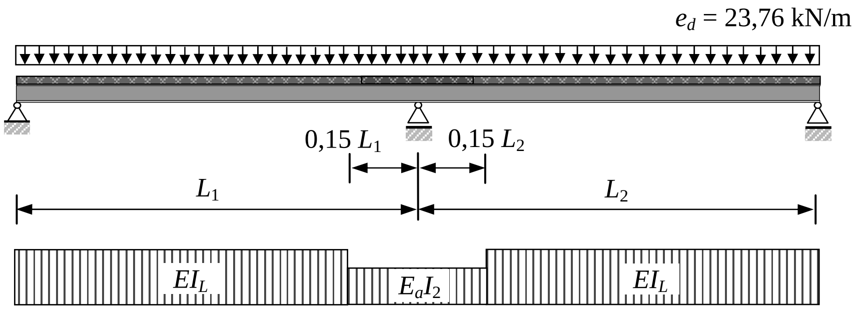


Figure B7.18 *Static system for calculation of the bending moment at the internal support of the composite beam for quasi-permanent actions at time $t = \infty$*

The maximum bending moment at the internal support, calculated using commercial software, is:

M_{max} = 407 kNm

The bending moment due to shrinkage is:

$$M_{cs} = N_{cs} \cdot a_c = 2146 \cdot 10{,}77 = 23112 =\ \text{kNcm} = 231\ \text{kNm}$$

$$EI_L = EI_S = 259306\ \text{kNm}^2$$

$$E_a I_2 = 121172 \ \text{kNm}^2$$

The maximum bending moment at the internal support due to shrinkage, calculated using commercial software, is:

M_{max} = 174 kNm

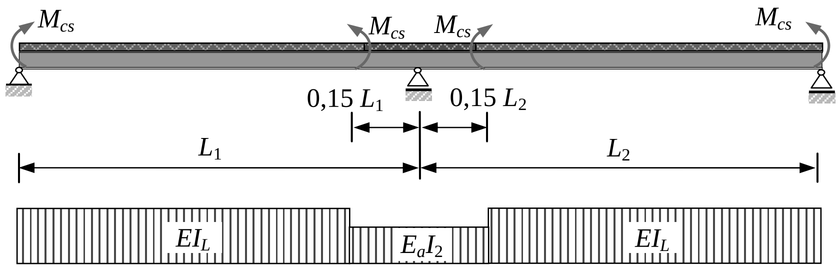


Figure B7.19 *Static system for calculation of the bending moment at the internal support of the composite beam due to shrinkage*

The total bending moment, including shrinkage, is:

$$M_{st} = 407 + 174 = 581 \ \text{kNm}$$

The distance between the neutral axis and the centroidal axis of reinforcement is:

$$z_{st} = e_{st} - z_{s1} = 32{,}8 - 2{,}5 = 30{,}3 \ \text{cm}$$

The tensile stress in reinforcement σ_s can be calculated for direct loading as:

$$\sigma_s = \sigma_{s,0} + \Delta\sigma_s$$

The stress in the reinforcement $\sigma_{s,0}$ caused by bending moment acting on the composite section is calculated on the assumption that the concrete in tension is neglected.

The geometrical properties of the cracked cross-section in accordance with Figure B7.13 are:

cross-sectional area

$$A_{st} = A_{s1} + A_{s2} + A_a = 8{,}185 + 8{,}185 + 159 = 175{,}4 \ \text{cm}^2$$

second moment of area

$$I_{st} = 57701 \text{ cm}^4$$

The stress in the reinforcement $\sigma_{s,0}$ caused by the bending moment acting on the cracked section is:

$$\sigma_{s,0} = \frac{M_{st}}{I_{st}} z_{st} = \frac{581 \cdot 10^2}{57701} \cdot 30{,}3 = 30{,}5 \text{ kN/cm}^2 = 305 \text{ N/mm}^2$$

The correction of the stress in the reinforcement for tension stiffening is:

$$\Delta\sigma_s = \frac{0{,}4 \cdot f_{ctm}}{\alpha_{st} \cdot \rho_s}$$

with:

$$\alpha_{st} = \frac{AI}{A_a I_a} = \frac{175{,}4 \cdot 57701}{159 \cdot 45070} = 1{,}41$$

The reinforcement ratio ρ_s is:

$$\rho_s = \frac{A_s}{A_{ct}} = \frac{A_{s,1} + A_{s,2}}{h_c \cdot b_{eff,2}} = \frac{8{,}185 + 8{,}185}{11 \cdot 156{,}2} = 0{,}00953$$

The correction of the stress in the reinforcement for tension stiffening is:

$$\Delta\sigma_s = \frac{0{,}4 \cdot f_{ctm}}{\alpha_{st} \cdot \rho_s} = \frac{0{,}4 \cdot 0{,}26}{1{,}41 \cdot 0{,}00953} = 7{,}7 \text{ kN/cm}^2 = 77 \text{ N/mm}^2$$

Therefore, the tensile stress in the reinforcement σ_s is:

$$\sigma_s = \sigma_{s,0} + \Delta\sigma_s = 305 + 77 = 382 \text{ N/mm}^2$$

$$\sigma_s = 382 \text{ N/mm}^2 > \sigma_s = 360 \text{ N/mm}^2$$

The tensile stress in the reinforcement is higher than the steel stress given in Table 7.1, EN 1994-1-1, for the maximum bar diameter $\phi^* = 10$ mm and the design crack width $w_k = 0{,}4$ mm.

For this reason, the maximum bar diameter $\phi^* = 8$ mm and the design crack width $w_k = 0{,}4$ mm are selected from Table 7.1, EN 1994-1-1, with the steel stress

$\sigma_s = 400$ N/mm^2. In order to maintain the same area of reinforcement in the considered section, it is necessary to reduce the spacing between bars.

The cross-sectional area of longitudinal reinforcement adopted in Section 4.4.2 is:

$$A_{s,1} = A_{s,2} = 8{,}185 \text{ cm}^2/b_{eff,2}$$

The cross-sectional area of the bar ϕ* = 8 mm is $A_{bar} = 0{,}5$ cm^2. The required number of bars is:

$$n = \frac{A_{s,1}}{A_{bar}} = \frac{8{,}185}{0{,}5} = 16{,}37$$

Therefore, spacing between bars is:

$$x = \frac{b_{eff,2}}{n} = \frac{156{,}2}{16{,}37} = 9{,}54 \text{ cm}$$

Thus, the final adopted reinforcement is:

Top longitudinal ϕ 8/90 mm, 5,59 cm^2/m, z_s = 2,5 cm,

Bottom longitudinal ϕ 8/90 mm, 5,59 cm^2/m, z_s = 10,5 cm,

Top transverse ϕ 10/150 mm, 5,24 cm^2/m, z_s = 3,5 cm,

Bottom transverse ϕ 10/150 mm, 5,24 cm^2/m, z_s = 13,5 cm.

6. Commentary

The design of a continuous beam according to the plastic–plastic procedure is illustrated in this example. In this case, action effects are calculated by rigid plastic global analysis (plastic hinge analysis), and the resistance to bending is based on a plastic model. Further, all cross-sections of the composite beam should be in class 1, i.e. the cross-sections of steel beams have sufficient rotation capacity. Rigid plastic global analysis can be used for ultimate limit state verifications other than fatigue. Also, this method is not applicable in cases where second-order effects have to be considered. For calculation internal forces, bending moments and deformations at serviceability limit state, the elastic global analysis should be used. The restrictions on the application of rigid plastic global analysis given in clause 5.4.5, EN 1994-1-1, are such that linear - elastic global analysis will often be used for composite beams.

B8 Two-span composite beam – more detailed explanations of provisions of EN 1994-1-1

1. Purpose of example

This example demonstrates the design of the continuous composite beam which is the structural member of floor of a warehouse, shown in Figure B8.1. The floor of warehouse consists of two bays each with a span of 10,0 m. The transverse composite beams at 2,5 m centres are assumed to be continuous over a central longitudinal wall and attached to columns in the outer walls. The composite beam consists of steel IPE 450 section acting structurally with a lightweight concrete slab, the total depth 130 mm, by means of shear connectors attached to the top flange of the steel section. The design of composite slab and beam-column joints are not considered in this example. The structural arrangements used in this notional building are not typical of building design. This is because the structural solutions have been chosen to illustrate the application of many of the provisions of EN 1994-1-1. The continuous beam is unpropped at the construction stage. The composite slab is cast in situ on profiled steel sheeting with profile height 50 mm, creating an overall slab thickness of 130 mm. This example deals with important aspects of the design of the considered beam for the persistent design situations for ultimate limit state. The serviceability verifications are performed taking into account secondary effects of shrinkage.

2. Static system, cross-section and actions

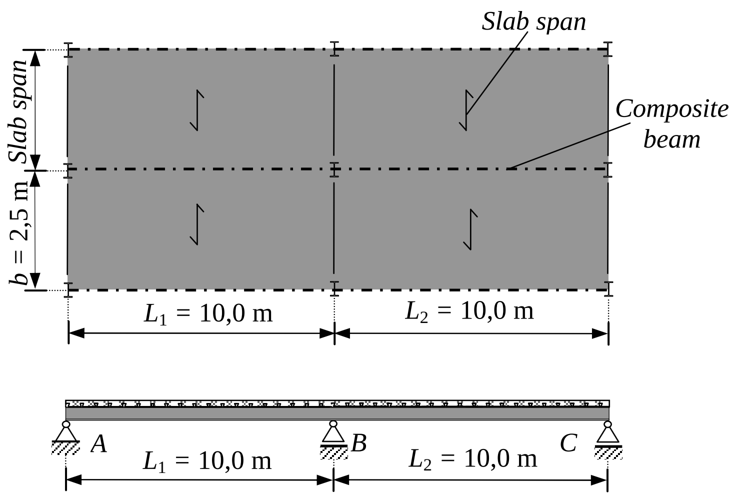
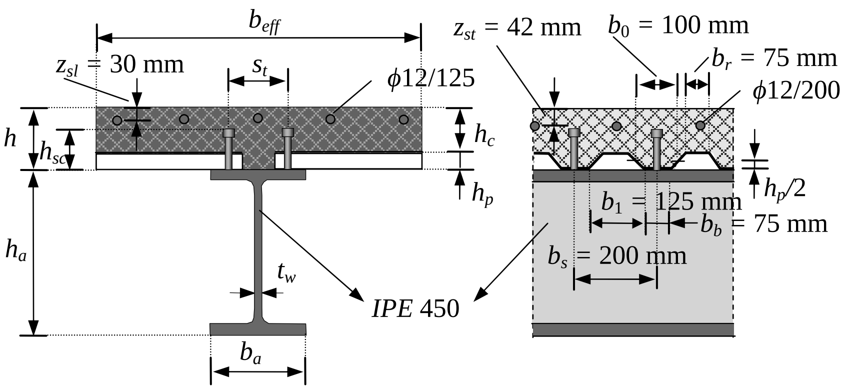


Figure B8.1 *Floor layout and static system*

Composite Structures according to Eurocode 4. Worked Examples.
First Edition. Darko Dujmović, Boris Androić, Ivan Lukačević.



$h = 130$ mm, $h_c = 80$ mm, $h_p = 50$ mm

Figure B8.2 *Cross-section of the composite beam*

Actions

a) Permanent action

Remark:

According to Table 11.1, clause 11.3, EN 1992-1-1 the density of lightweight aggregate concrete of density class 1.8 and strength class LC25/28 is 18,5 kN/m^3. According to EN 1991-1-1 the density is increased by 1 kN/m^3 for normal percentage reinforcement, and increased for the wet concrete by another 1 kN/m^3.

The concrete slab area per m width is:

$$A_c = 1000 \cdot h - (\frac{1000}{b_s} \cdot \frac{b_1 + b_r}{2} \cdot h_p)$$

$$A_c = 1000 \cdot 130 - (\frac{1000}{200} \cdot \frac{125 + 75}{2} \cdot 51) = 104500 \text{ mm}^2 = 1050 \text{ cm}^2$$

- *concrete slab and reinforcement (wet concrete)*

$$A_c \cdot 20{,}5 = 0{,}105 \cdot 20{,}5 = 2{,}15 \text{ kN/m}^2$$

- *concrete slab and reinforcement (dry concrete)*

$$A_c \cdot 19{,}5 = 0{,}105 \cdot 19{,}5 = 2{,}05 \text{ kN/m}^2$$

Construction stage

- *concrete slab* $= 2{,}15 \text{ kN/m}^2$

- *profiled steel sheeting* $= 0{,}17 \text{ kN/m}^2$

- *steel beam* $= 0{,}30 \text{ kN/m}^2$

Total $g_{k,1} = 2{,}62 \text{ kN/m}^2$

Composite stage

- *concrete slab* $= 2{,}05 \text{ kN/m}^2$

- *profiled steel sheeting* $= 0{,}17 \text{ kN/m}^2$

- *steel beam* $= 0{,}30 \text{ kN/m}^2$

Total $g_{k,2} = 2{,}52 \text{ kN/m}^2$

Floor finishes $g_{k,3} = 1{,}00 \text{ kN/m}^2$

b) Variable action

Construction stage

- *construction loads* $q_{k,1} = 0{,}50 \; kN/m^2$

Composite stage

- *imposed floor load (category of use C3) and movable partitions* $q_{k,2} = 7{,}00 \text{ kN/m}^2$

3. Properties of materials

Concrete strength class: LC 25/28 $f_{ck} = 25 \text{ N/mm}^2$

$$f_{cd} = \frac{f_{ck}}{\gamma_c} = \frac{25}{1{,}5} = 16{,}7 \text{ N/mm}^2$$

$$0{,}85 f_{cd} = 0{,}85 \cdot 16{,}7 = 14{,}2 \text{ N/mm}^2$$

$$E_{cm} = 31000 \text{ N/mm}^2$$
$$f_{ctm} = 2{,}6 \text{ N/mm}^2$$
$$E_{lcm} = 20752 \text{ N/mm}^2$$
$$f_{lctm} = 2{,}32 \text{ N/mm}^2$$

Reinforcement: *ductility class B or C (Table C.1, EN 1992-1-1)* $f_{sk} = 500 \text{ N/mm}^2$

$$f_{sd} = \frac{f_{sk}}{\gamma_s} = \frac{500}{1{,}15} = 435 \text{ N/mm}^2$$
$$E_s = 210\,000 \text{ N/mm}^2$$

Structural steel: S355 $f_{yk} = 355 \text{ N/mm}^2$

$$f_{yd} = \frac{f_{yk}}{\gamma_M} = \frac{355}{1{,}0} = 355 \text{ N/mm}^2$$
$$E_a = 210\,000 \text{ N/mm}^2$$

Shear connectors: *ductile headed studs* $f_u = 500 \text{ N/mm}^2$

$$d = 19 \text{ mm}$$
$$h_{sc} = 95 \text{ mm}$$
$$\frac{h_{sc}}{d} = \frac{95}{19} = 5{,}0 > 4{,}0 \;\rightarrow \alpha = 1{,}0$$

4. Properties of cracked and uncracked cross-sections

In this section, the calculation of elastic properties of cross-sections is illustrated taking into account changes of modular ratio, effective widths and the use cracked or uncracked sections.

- ***Calculation of the nominal modular ratio***

According to clause 11.3.1, EN 1992-1-1, the oven-dry density of the lightweight aggregate concrete of density class 1.8 is:

$$\rho = 1800 \text{ kg/m}^3$$

The tensile strength of lightweight aggregate concrete may be obtained by multiplying the f_{ctm} value given in Table 3.1, EN 1992-1-1, by a coefficient for determining the tensile strength:

$$\eta_1 = 0{,}4 + 0{,}6 \cdot \rho / 2200 = 0{,}4 + 0{,}6 \cdot 1800 / 2200 = 0{,}891$$

Thus, according to Table 11.3.1, EN 1992-1-1, the mean tensile strength is:

$$f_{lctm} = \eta_1 \cdot f_{ctm} = 0,891 \cdot 2,6 = 2,32 \text{ N/mm}^2$$

Clause 11.3.2, EN 1992-1-1, gives:

$$E_{lcm} = E_{cm} \cdot \eta_E = E_{cm} \cdot (\frac{\rho}{2200})^2$$

$$E_{lcm} = 31000 \cdot (\frac{18,0}{22})^2 = 20752 \text{ N/mm}^2$$

Therefore, the modular ratio for short-term loading is:

$$n_0 = \frac{E_a}{E_{lcm}} = \frac{210}{20,75} = 10,1$$

As a simplification for composite beams in structures for buildings where first-order global analysis can be applied, clause 5.4.2.2(11), EN 1994-1-1, permits the modular ratio to be taken as $2 \cdot n_0$ for both short-term and long-term loading.

Accordingly, the nominal modular ratio n is:

$$n = 2n_0 = 2 \cdot 10,1 = 20,2$$

The elastic properties of cross-sections will be calculated for the following cases:

- ***Effective width of the concrete flange***

- for the mid-span region $b_{eff} = 2,23$ m
- for the internal support region $b_{eff} = 1,35$ m
- at the end support $b_{eff} = 1,84$ m

Remark:

Calculation of the effective widths of the concrete flange is performed in Section 5.4.1.

- ***Properties of steel cross-section***

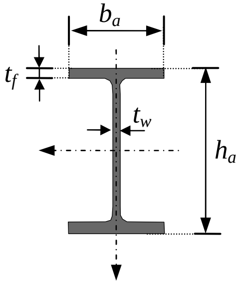
The approximate ratio of span-to-depth of the steel beam for a continuous composite beam is:

$$\frac{L}{h_a} \approx 25$$

For span L = 12 m, the minimum depth of the steel beam is:

$$h_a = \frac{L}{25} = \frac{10 \cdot 10^3}{25} = 400 \text{ mm}$$

The IPE 450 section is adopted.



$W_{pl,y}$ = 1702 cm^3
$W_{el,y}$ = 1500 cm^3
A_a = 98,82 cm^2
h_a = 450 mm
b_a = 190 mm
t_w = 9,4 mm
t_f = 14,6 mm
r = 21 mm
$I_{y,a}$ = 33740 cm^4
$I_{z,a}$ = 1676 cm^4
$I_{w,a}$ = 791000 cm^6
$I_{t,a}$ = 66,87 cm^4
g = 77,6 kg/m

Figure B8.3 *Cross-section of steel beam*

- ***Calculation of the properties of the cross-section at the internal support, cracked concrete neglected***

The section from Figure B8.4 is considered.

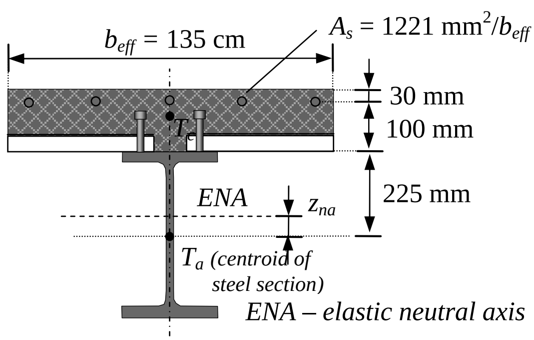


Figure B8.4 *Cross-section at the internal support, cracked concrete neglected*

Properties of the cross-section shown in Figure B8.4 are calculated as follows.
Area

$$A = A_a + A_s = 9880 + 1221 = 11101 \ \text{mm}^2$$

The distance between the neutral axis and the centroidal axis of steel section

$$z_{na} = 1221 \cdot \frac{100 + 225}{11101} = 36 \ \text{mm}$$

Second moment of area

$$I_y = \mathrm{I}_a + A_a \cdot z_{na}^2 + A_s \cdot (225 + 100 - z_{na})^2 \cdot 10^6$$

$$I_y = 337{,}4 + 9880 \cdot 0{,}036^2 + 1221 \cdot (0{,}225 + 0{,}100 - 0{,}036)^2 \cdot 10^6 = 452 \cdot 10^6 \ \text{mm}^4$$

Flexural stiffness

$$E_a I_y = 210 \cdot 452 \cdot 10^6 = 94920 \cdot 10^6 \ \text{kNmm}^2$$

- ***Calculation of the properties of the uncracked cross-section at the internal support with modular ratio $n_0 = 10{,}1$***

The effective width of the concrete flange in “steel” units is:

$$b_{eff} / n_0 = 1{,}35 / 10{,}1 = 0{,}134 \ \text{m}$$

The section from Figure B8.5 is considered.

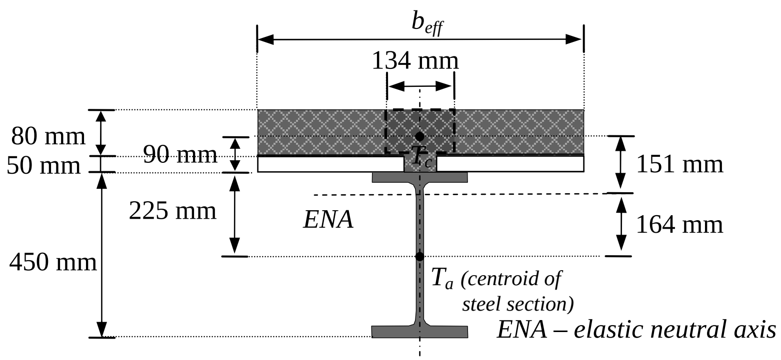


Figure B8.5 *Uncracked composite section at the internal support with $n_0 = 10{,}1$*

Properties of the cross-section shown in Figure B8.5 are calculated as follows.

Area

$$A = A_a + \frac{b_{eff}}{n_0} \cdot h_c = 9880 + 134 \cdot 80 = 9880 + 10720 = 20600 \ \text{mm}^2$$

The distance between the neutral axis and the centroidal axis of steel section

$$z_{na} = 10720 \cdot \frac{90 + 225}{20600} = 164 \ \text{mm}$$

Second moment of area

$$I_y = 337,4 + 9880 \cdot 0,164^2 + 10720 \cdot (0,225 + 0,090 - 0,164)^2 \cdot 10^6 = 848 \cdot 10^6 \ \text{mm}^4$$

Flexural stiffness

$$E_a I_y = 210 \cdot 848 \cdot 10^6 = 178080 \cdot 10^6 \ \text{kNmm}^2$$

Section modulus, top of slab, in concrete units

$$W_{c,top} = 848 \cdot 10^6 \cdot \frac{10,1}{(225 + 50 + 80 - 164)} = 44,8 \cdot 10^6 \ \text{mm}^3$$

- ***Calculation of the properties of the uncracked section at the internal support with modular ratio n = 20,2***

The effective width of concrete flange in “steel” units is:

$$b_{eff} / n = 1,35 / 20,2 = 0,067 \ \text{m}$$

The section from Figure B8.6 is considered.

Properties of the cross-section shown in Figure B8.6 are calculated as follows.

Area

$$A = A_a + \frac{b_{eff}}{n} \cdot h_c = 9880 + 67 \cdot 80 = 9880 + 5360 = 15240 \ \text{mm}^2$$

The distance between the neutral axis and the centroidal axis of steel section

$$z_{na} = 5360 \cdot \frac{90 + 225}{15240} = 111 \text{ mm}$$

Second moment of area

$$I_y = 337,4 + 9880 \cdot 0,111^2 + 5360 \cdot (0,225 + 0,090 - 0,111)^2 \cdot 10^6 = 682 \cdot 10^6 \text{ mm}^4$$

Flexural stiffness

$$E_a I_y = 210 \cdot 682 \cdot 10^6 = 143220 \cdot 10^6 \text{ kNmm}^2$$

Section modulus, top of slab, in concrete units

$$W_{c,top} = 682 \cdot 10^6 \cdot \frac{20,2}{(225 + 50 + 80 - 111)} = 56,5 \cdot 10^6 \text{ mm}^3$$

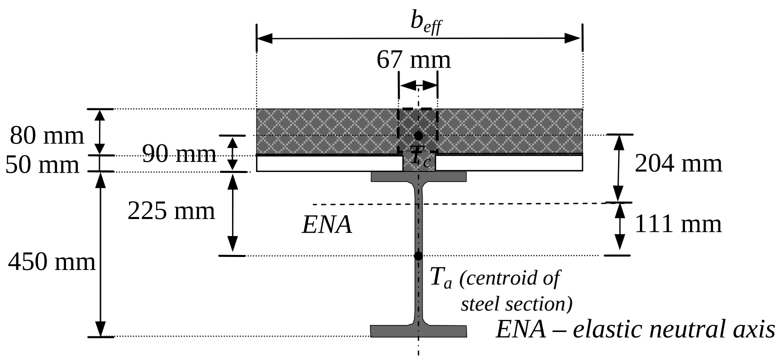


Figure B8.6 *Uncracked composite section at the internal support with n = 20,2*

- ***Calculation of the properties of the uncracked section at mid-span with modular ratio n_0 = 10,1***

The effective width of concrete flange in "steel" units is:

$$b_{eff} / n_0 = 2,23 / 10,1 = 0,221 \text{ m}$$

The section from Figure B8.7 is considered.

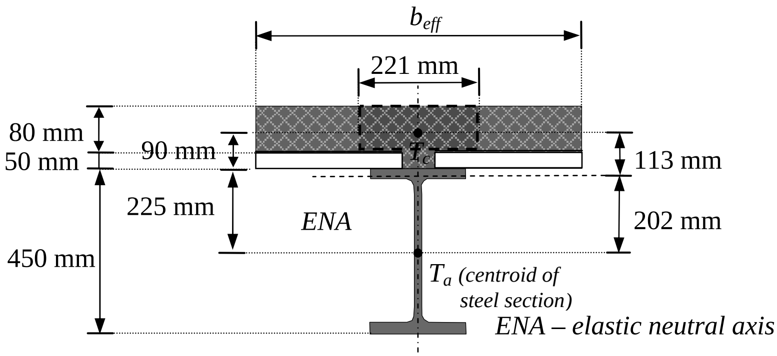


Figure B8.7 *Composite section at mid-span with n_0 = 10,1*

Properties of the cross-section shown in Figure B8.7 are calculated as follows.

Area

$$A = A_a + \frac{b_{eff}}{n} \cdot h_c = 9880 + 221 \cdot 80 = 9880 + 17680 = 27560 \text{ mm}^2$$

The distance between the neutral axis and the centroidal axis of steel section

$$z_{na} = 17680 \cdot \frac{90 + 225}{27560} = 202 \text{ mm}$$

Second moment of area

$$I_y = 337{,}4 + 9880 \cdot 0{,}202^2 + 17680 \cdot (0{,}225 + 0{,}090 - 0{,}202)^2 \cdot 10^6 = 966 \cdot 10^6 \text{ mm}^4$$

Flexural stiffness

$$E_a I_y = 210 \cdot 966 \cdot 10^6 = 202860 \cdot 10^6 \text{ kNmm}^2$$

Section modulus, top of slab, in concrete units

$$W_{c,top} = 966 \cdot 10^6 \cdot \frac{10{,}1}{(225 + 50 + 80 - 202)} = 63{,}8 \cdot 10^6 \text{ mm}^3$$

- ***Calculation of the properties of the uncracked section at mid-span with modular ratio n = 20,2***

The effective width of the concrete flange in “steel” units is:

$$b_{eff} / n = 2{,}23 / 20{,}2 = 0{,}110 \text{ m}$$

The section from Figure B8.8 is considered.

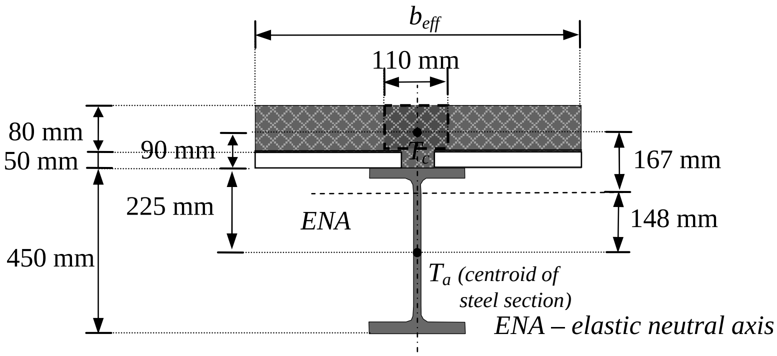


Figure B8.8 *Composite section at mid-span with n = 20,2*

Properties of the cross-section shown in Figure B8.8 are calculated as follows.

Area

$$A = A_a + \frac{b_{eff}}{n} \cdot h_c = 9880 + 110 \cdot 80 = 9880 + 8800 = 18680 \text{ mm}^2$$

The distance between the neutral axis and the centroidal axis of steel section

$$z_{na} = 8800 \cdot \frac{90 + 225}{18680} = 148 \text{ mm}$$

Second moment of area

$$I_y = 337{,}4 + 9880 \cdot 0{,}148^2 + 8800 \cdot (0{,}225 + 0{,}090 - 0{,}148)^2 \cdot 10^6 = 799 \cdot 10^6 \text{ mm}^4$$

Flexural stiffness

$$E_a I_y = 210 \cdot 799 \cdot 10^6 = 167790 \cdot 10^6 \text{ kNmm}^2$$

Section modulus, top of slab, in concrete units

$$W_{c,top} = 799 \cdot 10^6 \cdot \frac{20,2}{(225+50+80-148)} = 78,0 \cdot 10^6 \text{ mm}^3$$

- ***Modular ratio for effects of shrinkage***

In Figure 3.1, EN 1992-1-1, creep coefficients for normal-weight concrete are given in relation to the notional size of the cross-section. In this example, the concrete slab has only one surface exposed because the other is sealed by the profiled sheeting. Therefore, the notional size of the cross-section, h_0, is twice its thickness. The mean thickness of the slab is 104,5 mm (Figure B8.2) and the notional size of the cross-section is:

$$h_0 = 2 \cdot 104,5 = 209 \text{ mm}$$

According to clause 5.4.2.2(4), EN 1994-1-1, the age at loading can be presumed to be 1 day. The final value of the creep coefficient $\varphi(\infty, t_0)$ can be determined using the nomogram shown in Figure 3.1, EN 1992-1-1. Example B3, shows the detailed procedure for the determination of creep coefficients from nomograms.

The following creep coefficient is obtained for normally hardening cement and indoor conditions:

$$\varphi(\infty, t_0 = 1 \text{ day}) = 4,9$$

The correction factor for lightweight concrete is given in clause 11.3.3(1), EN 1992-1-1, which is $(\rho/2200)^2 = (1800/2200)^2$, giving the creep coefficient as:

$$\varphi = (\rho/2200) \cdot \varphi(\infty, t_0) = (18,0/22)^2 \cdot 4,9 = 3,28$$

The creep multiplier ψ_L from clause 5.4.2.2(2), EN 1994-1-1, which depends on the type of loading, is 0,55 for secondary effects of shrinkage. The modular ratio for the effects of shrinkage is:

$$n = n_0 \cdot (1 + \psi_L \cdot \varphi_t) = 10,1 \cdot (1 + 0,55 \cdot 3,28) = 28,3$$

- ***Calculation of the properties of the uncracked section at mid-span with modular ratio n = 28,7***

The effective width of the concrete flange in “steel” units is:

$b_{eff} / n = 2,23 / 28,3 = 0,079 \text{ m}$

The section from Figure B8.9 is considered.

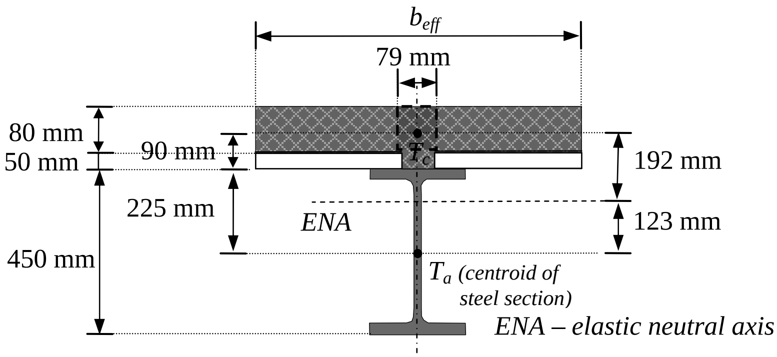


Figure B8.9 *Composite section at mid-span with n = 28,3*

Properties of the cross-section shown in Figure B8.9 are calculated as follows.

Area

$$A = A_a + \frac{b_{eff}}{n} \cdot h_c = 9880 + 79 \cdot 80 = 9880 + 6320 = 16200 \text{ mm}^2$$

The distance between the neutral axis and the centroidal axis of steel section

$$z_{na} = 6320 \cdot \frac{90 + 225}{16200} = 123 \text{ mm}$$

Second moment of area

$$I_y = 337,4 + 9880 \cdot 0,123^2 + 6320 \cdot (0,225 + 0,090 - 0,123)^2 \cdot 10^6 = 720 \cdot 10^6 \text{ mm}^4$$

Flexural stiffness

$$E_a I_y = 210 \cdot 720 \cdot 10^6 = 151200 \cdot 10^6 \text{ kNmm}^2$$

Section modulus, top of slab, in concrete units

$$W_{c,top} = 720 \cdot 10^6 \cdot \frac{28,3}{(225 + 50 + 80 - 123)} = 87,8 \cdot 10^6 \text{ mm}^3$$

The results are shown in Table B8.1.

Table B8.1 *Elastic section properties of the composite cross-section*

Cross-section	Modular ratio	b_{eff} (m)	Neutral axis (mm)	I_y (10^6 mm^4)	$W_{c,top}$ (10^6 mm^3)
1) Support, cracked, reinforced	—	1,35	36	452	—
2) Support, uncracked	10,1	1,35	164	848	44,8
3) Support, uncracked	20,2	1,35	111	682	56,5
4) Mid-span, uncracked	10,1	2,23	202	966	63,8
5) Mid-span, uncracked	20,2	2,23	148	799	78,0
6) Mid-span, uncracked	28,3	2,23	123	720	87,8

5. Ultimate limit state

5.1 Design values of the combined actions and of the effects of the actions for the construction stage

The design load determined by governed combination of actions is:

$$e_d = b \cdot (\gamma_{G,\sup} \cdot g_{k,1} + \gamma_Q \cdot q_{k,1})$$

$b = 2{,}5$ m beam spacing

$$e_d = 2{,}50 \cdot (1{,}35 \cdot 2{,}62 + 1{,}50 \cdot 0{,}50) = 10{,}7 \text{ kN/m}$$

a) Maximum design moment at the internal support

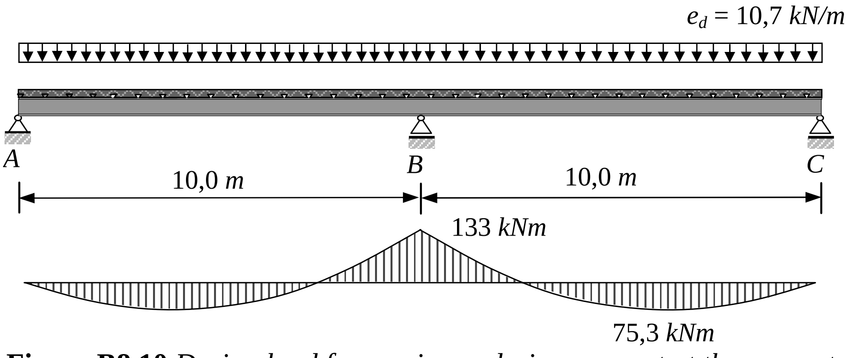


Figure B8.10 *Design load for maximum design moment at the support with the corresponding bending moment distribution*

$V_{Ed,A} = 40{,}2$ kN

$V_{Ed,B} = 66{,}8$ kN

b) Maximum design moment at mid-span

Remark:

For simplicity, in the determination of the maximum bending moment at mid-span, the self-weight of steel beam is neglected. This assumption produces a conservative result.

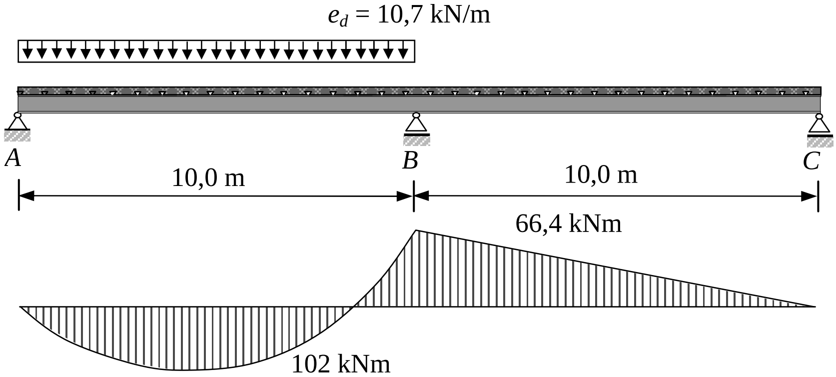


Figure B8.11 *Design load for the maximum design moment at mid-span with the corresponding bending moment distribution*

$V_{Ed,A} = 46{,}9\ \text{kN}$

$V_{Ed,B} = 60{,}1\ \text{kN}$

Considering both load cases, the maximum design moments on the steel section during the execution stage are:

- design negative moment $M_{Ed,B} = 133\ \text{kNm}$
- design positive moment $M_{Ed,1} = 102\ \text{kNm}$

The maximum design shear force on the steel section is:

$V_{Ed,B} = 66{,}8\ \text{kN}$

5.2 Design values of the combined actions and of the effects of the actions for the composite stage

For the design of continuous composite beams, the effects of cracking of the concrete should be taken into account. Cracking of concrete reduces the flexural stiffness at the internal support, a region of hogging bending moment, but not in the sagging regions. The reduction of the flexural stiffness should be taken into account in elastic global analysis. In EN 1994-1-1, the several different methods are proposed for allowing for cracking in beams. The two methods for calculation of action effects based on elastic theory are shown in Figure B8.12.

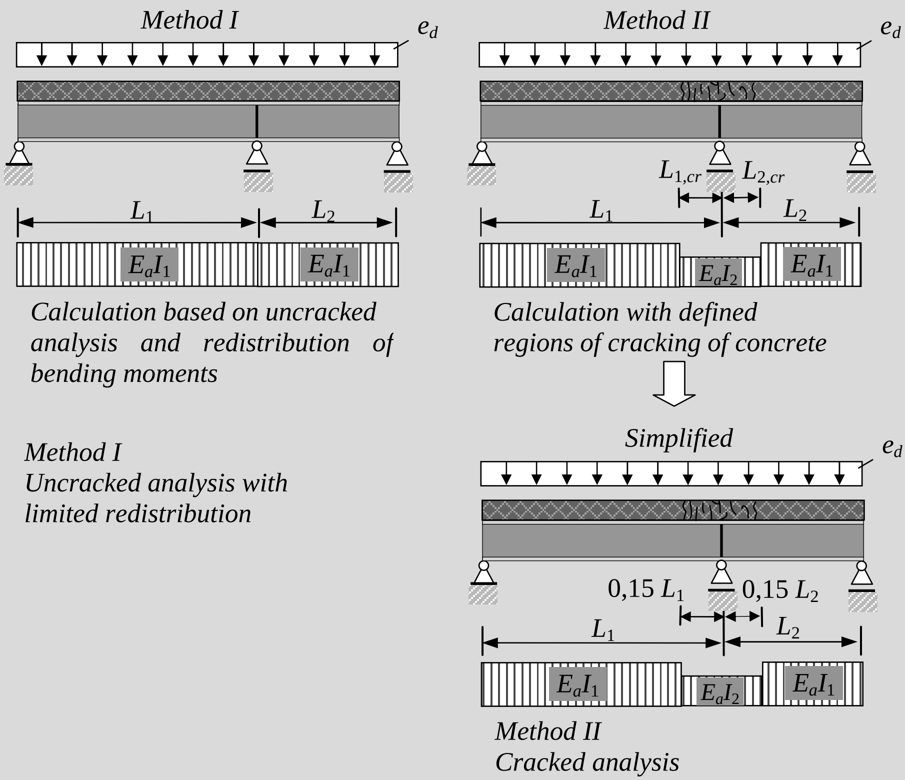


Figure B8.12 *Simplified methods for taking into account the effects of cracking based on elastic theory*

In Method I, Figure B8.12, the internal forces and bending moments are determined for the characteristic combination of actions with uncracked flexural stiffness E_aI_1. According to clause 5.4.4, EN 1994-1-1, the bending moments can be redistributed if the required conditions are satisfied. In Method II, effects of cracking in composite beam are taken into account as follows. The first step is to determine the regions of beam, L_{cr}, where the extreme fibre concrete tensile stress, σ_c, exceeds the limited value. For the ultimate limit state, the criterion is $\sigma_c > 2\ f_{ctm}$ (the extreme-fibre tensile stress in concrete exceeds twice the mean value of the axial tensile strength) and for the serviceability limit state $\sigma_c > 1{,}5\ f_{ctm}$, Figure B8.12. In the second step, the cracked stiffness is then adopted for such sections in cracked regions and the structure is re-analysed. In this analysis, the beams with cracked regions are treated as beams of non-uniform section. However, for continuous beams, we can use the simplification as follows. Where all the ratios of the length of adjacent continuous spans (shorter/longer) between supports are at least 0,6, the effects of cracking can be taken into account by using the flexural stiffness E_aI_2 over 15% of the span on each side of each internal support, and as the un-cracked values E_aI_1 elsewhere. The simplified Method II is shown in Figure B8.12.

In this example, the simplified Method II is used. Accordingly, it is assumed that cross-sections in the hogging region are cracked.

Analyses for the ultimate and serviceability limit states are performed by the same methods, in accordance with the rules for global analysis from Section 5, EN 1994-1-1, wherever possible. The effects of the behaviour of the joints, according to clause 5.1.2, EN 1994-1-1, are neglected in this case. According to clauses 5.2.2(1) and 5.4.1.1(1), EN 1994-1-1, first-order elastic global analysis is applied.

In this case, the lateral-torsional buckling is the only type of instability that need be considered. The member imperfections for lateral-torsional buckling are taken into account in the expressions for determining the resistance to lateral-torsional buckling. In this way, the requirement according to clause 5.2.2(4), EN 1994-1-1, is satisfied.

The analysis using Method I according to clause 5.4.2.3(2), EN 1994-1-1, shown in Figure 8.12, is carried out to illustrate the value of tensile stress in the beam at the internal support. The calculation is performed for uncracked sections and for the design imposed load, Figure B8.13.

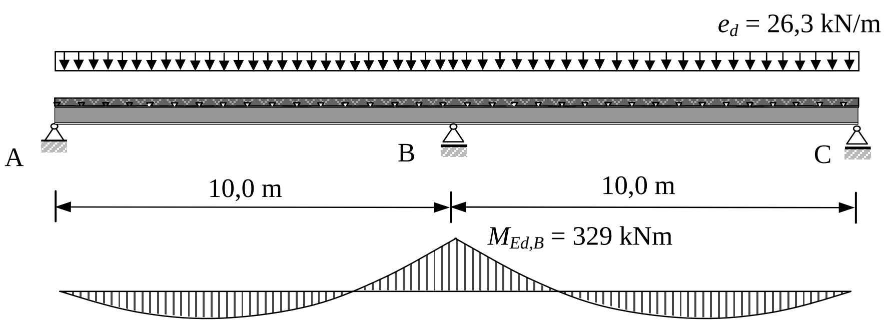


Figure B8.13 *Continuous beam subjected to design imposed load*

The design imposed load is:

$$q_d = b \cdot \gamma_Q \cdot q_{k,2}$$

$b = 2,5$ m beam spacing

$$q_d = 2,50 \cdot 1,50 \cdot 7,00 = 26,3 \text{ kN/m}$$

The design bending moment at internal support B, $M_{Ed,B}$, is:

$$M_{Ed,B} = \frac{q_d \cdot L^2}{8} = \frac{26{,}3 \cdot 10{,}00^2}{8} = 329 \text{ kNm}$$

The tensile stress in the concrete at the internal support for the uncracked section with $W_{c,top} = 44{,}8 \cdot 10^6$ mm^3, Table B8.1, is:

$$\sigma_c = \frac{M_{Ed,B}}{W_{c,top}} = \frac{329 \cdot 10^6}{44{,}8 \cdot 10^6} = 7{,}34 \text{ N/mm}^2 > f_{lctm} = 2{,}32 \text{ N/mm}^2$$

Thus, it is found that the tensile stress in the concrete at the internal support exceeds three times its tensile strength. It is important to note that shrinkage is neglected, which further increases the tensile stress. This method is impractical. Therefore, the analysis is carried out using the simplified Method II, according to clause 5.4.2.3(3), EN 1994-1-1, see Figure B8.12.

The effects of cracking are taken into account by using the flexural stiffness E_aI_2 over 15% of the span on each side of internal support, and as the uncracked values E_aI_1 elsewhere. According to clause 5.4.1.2(4), EN 1994-1-1, when elastic global analysis is used, the effective width at mid-span may be adopted over the whole of each span. However, in this example, the effective width b_{eff} for the cracked region is taken as 1,35 m. The adopted value of effective width is justified by the fact that the resistances are based on this value and the reinforcement outside this value can be quite light.

Method II is the non-iterative method. According to clauses 5.4.2.3(3) to 5.4.2.3(5), EN 1994-1-1, this method is applicable only for conventional continuous composite beam and beams in braced frames.

a) Maximum design moment at the internal support (load case 1)

The calculation is performed “by hand” according to the method given in [35]. Taking into account the symmetry of the structure and the load, the following statically indeterminate system is adopted for the calculation of the bending moment at point B, see Figure B8.14.

For the uniformly distributed load per unit length, e_d, and the ratio of the second moments of area λ, as shown in Figure B8.14, the expression for the elastic bending moment M_{Ed} at point B, according to [35], is:

$$M_{Ed,B} = (\frac{e_d \cdot L^2}{4}) \cdot \frac{(0{,}110 \cdot \lambda + 0{,}890)}{(0{,}772 \cdot \lambda + 1{,}228)}$$

$$\lambda = \frac{I_y(n)}{I_y(cracked)}$$

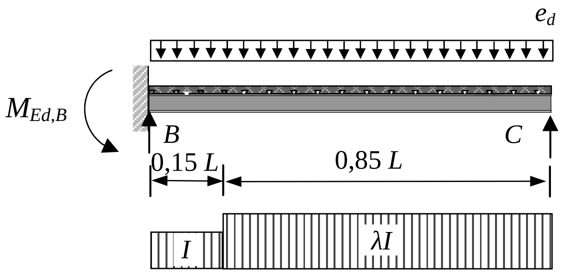


Figure B8.14 *Adopted static system – elastic propped cantilever with change of section at 0,15L*

The design values of permanent and variable actions for the governed combination of actions are:

$$g_d = b \cdot (\gamma_{G,\sup} \cdot g_{k,2} + \gamma_{G,\sup} \cdot g_{k,3})$$

$$g_d = 2{,}50 \cdot (1{,}35 \cdot 2{,}52 + 1{,}35 \cdot 1{,}0) = 11{,}9 \text{ kN/m}$$

$$q_d = b \cdot \gamma_Q \cdot q_{k,2}$$

$$q_d = 2{,}50 \cdot 1{,}50 \cdot 7{,}00 = 26{,}3 \text{ kN/m}$$

Taking into account the data of the elastic properties of composite cross sections from Table B8.1, the following values are obtained:

$$\lambda = \frac{966 \cdot 10^6}{452 \cdot 10^6} = 2{,}14 \quad M_{Ed,B} = (\frac{11{,}9 \cdot 10{,}00^2}{4}) \cdot \frac{(0{,}110 \cdot 2{,}14 + 0{,}890)}{(0{,}772 \cdot 2{,}14 + 1{,}228)} = 116 \text{ kN/m}$$

$$\lambda = \frac{799 \cdot 10^6}{452 \cdot 10^6} = 1{,}77 \quad M_{Ed,B} = (\frac{11{,}9 \cdot 10{,}00^2}{4}) \cdot \frac{(0{,}110 \cdot 1{,}77 + 0{,}890)}{(0{,}772 \cdot 1{,}77 + 1{,}228)} = 124 \text{ kN/m}$$

$$\lambda = \frac{966 \cdot 10^6}{452 \cdot 10^6} = 2{,}14 \quad M_{Ed,B} = (\frac{26{,}3 \cdot 10{,}00^2}{4}) \cdot \frac{(0{,}110 \cdot 2{,}14 + 0{,}890)}{(0{,}772 \cdot 2{,}14 + 1{,}228)} = 257 \text{ kN/m}$$

$$\lambda = \frac{799 \cdot 10^6}{452 \cdot 10^6} = 1,77 \qquad M_{Ed,B} = (\frac{26,3 \cdot 10,00^2}{4}) \cdot \frac{(0,110 \cdot 1,77 + 0,890)}{(0,772 \cdot 1,77 + 1,228)} = 275 \text{ kN/m}$$

The results are given in Table B8.2.

Table B8.2 *Design bending moments at the fixed end of the propped cantilever*

Action	e_d (kN/m)	n	I_y (10^6 mm^4)	λ	$M_{Ed,B}$ (kNm)
Permanent	11,9	10,1	966	2,14	116
Permanent	11,9	20,2	799	1,77	124
Variable	26,3	10,1	966	2,14	257
Variable	26,3	20,2	799	1,77	275

Remark:

The recommended value of ψ_2 factor is 0,6 for variable loading in a warehouse, clause A1.2.2(1), EN 1990. The value $\psi_2 \cdot b \cdot q_k$ = 0,6$\cdot b \cdot q_k$ (= 0,6·2,5·7,0 = 10,50 kN/m) is 40% of the value $\gamma_Q \cdot b \cdot q_k$ =1,5$\cdot b \cdot q_k$ (= 1,5·2,5·7,0 = 26,3 kN/m). Therefore, some creep of the composite beam is likely.

The design bending moment at internal support is governed for the dimensioning. Creep increases this bending moment, and the long-term effects of shrinkage are so significant for the case t, the time considered in the analysis, tends to ∞ and is critical.

Accordingly, the modular ratio, $n = 2 \cdot n_0 = 20,2$, is used in this example for all action effects except shrinkage.

Hence, the design bending moment at the internal support, excluding the effects of shrinkage, with n = 20,2, is:

$$M_{Ed,B} = 275 + 124 = 399 \text{ kNm}$$

The reaction at the end support with the full design load of both spans 38,2 kN/m is:

$$R_{Ed,A} = 38,2 \cdot 5,0 - 399 / 10,0 = 151 \text{ kN}$$

The point of maximum bending moment is at a distance of 151/38,2 = 3,95 m from the end support, and the maximum design sagging moment is:

$$M_{Ed} = 151 \cdot 3,95 - 38,2 \cdot 3,95 \cdot \frac{3,95}{2} = 298 \text{ kNm}$$

The vertical design shear force at internal support is:

$$V_{Ed,B} = (11,9 + 26,3) \cdot 5 + \frac{399}{10} = 231 \text{ kN}$$

Remark:

Redistribution of moments is not applied. According to clause 5.4.4(4), EN 1994-1-1, redistribution is not permitted if allowance for lateral-torsional buckling is required, see Section 5.4.6.

The calculation of bending moments and internal forces can be performed using commercial software for the static system and the design load shown in Figure B8.15. The concrete is cracked at a length of 0,15 L on each side of internal support. The flexural stiffness E_aI_2 in this region is 98910 kNm^2. At mid-span, the concrete is uncracked at a length of 0,85 L in both spans. The flexural stiffness, for the uncracked section and the modular ratio n = 20,2, is 174090 kNm^2.

The obtained results, shown in Figure B8.15, are the same as the results obtained by the method given in [35].

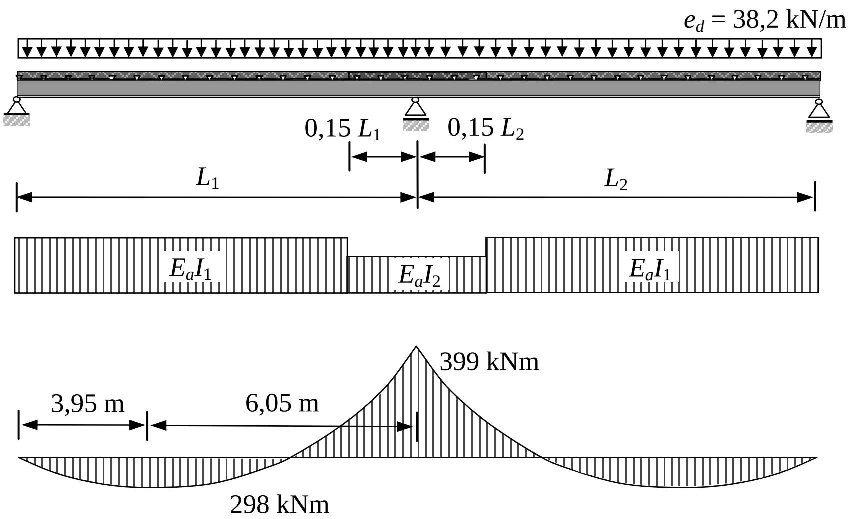


Figure B8.15 *Static system, design load and corresponding bending moment distribution (without redistribution)*

Secondary effects of shrinkage

Shrinkage of the concrete flange causes sagging curvature and shortening of the composite beam. These are the primary effects of shrinkage. However, in a continuous beam, the curvature causes bending moments and shear forces. These are then the secondary effects. Both the curvature and the stresses from the primary effects are neglected in regions assumed to be cracked, clauses 5.4.2.2(8) and 6.2.1.5(5), EN 1994-1-1.

The hogging bending moment at the internal support is the important secondary effect for the considered beam. It is calculated as follows. Since shrinkage is a permanent action, it is not reduced by a combination factor ψ_0.

It is assumed that the concrete flange is separated from the steel beam, Figure B8.16a). The concrete above the profiled sheeting is taken into account and denoted as A_c. The concrete flange is shrunk due to shrinkage. The force that would cause the opposite effect, N_{cs}, to extend the flange to its original length, is given as:

$$N_{cs} = \varepsilon_{cs}(\infty) \cdot E_S \cdot A_c$$

where:

$$E_S = \frac{E_a}{n_L} = \frac{E_{cm}}{n_c}$$

$$n_L = n_0 \cdot n_c = n_0 \cdot (1 + \psi_L \cdot \varphi_t)$$

$$n_c = 1 + 0{,}55 \cdot \varphi(\infty, t_0)$$

The force N_{cs} acts at the centre of the concrete flange, at a distance a_c above the centroid of the composite section. The parts of the beam are reconnected. To re-establish equilibrium, an opposite force N_{cs} and a bending moment $N_{cs} \cdot a_c$ are applied to the composite section, Figure B8.16a.

The radius of curvature of the uncracked part of the beam is given by:

$$R = \frac{E_a \cdot \lambda \cdot I_y}{N_{cs} \cdot a_c}$$

If the centre support is removed, from the geometry of the circle, the deflection δ at that point is (Figure B8.16b):

$$\delta = \frac{(0{,}85 \cdot L)^2}{2 \cdot R}$$

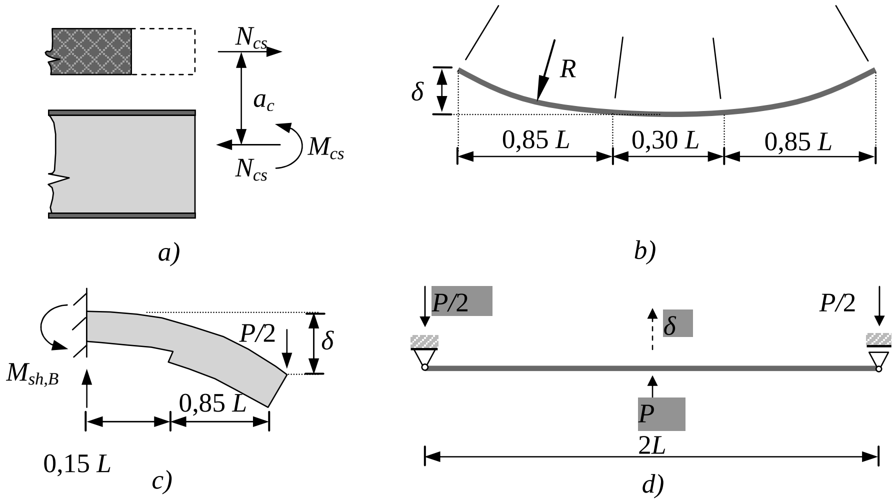


Figure B8.16 *Secondary effects due to shrinkage*

The actual deflection in the middle is equal to zero. It is necessary to calculate the force P, Figure B8.16c, applied at the point B, to reduce the deflection to zero, so that the internal support can be replaced, Figure B8.16d. The secondary hogging bending moment at B is:

$$M_{Ed,sh,B} = \frac{P \cdot L}{2}$$

Hence, the vertical shear force is $P/2$. For the cantilever, Figure B.8.16c, the force P can be found from:

$$\delta = \frac{P}{2} \cdot L^3 \cdot \frac{0{,}13 \cdot \lambda + 0{,}20}{E_a \cdot I_y \cdot \lambda}$$

Using this procedure, the secondary effect in this beam, the hogging bending moment at internal support $M_{Ed,sh,B}$, is calculated as follows:

$$A_c = 2{,}5 \cdot 0{,}08 = 0{,}20 \text{ m}^2$$

$$E_a = 210 \text{ kN/mm}^2$$

$$n = 28{,}3$$

According to Annex C, EN 1994-1-1, for a dry environment, the total shrinkage strain is:

$$\varepsilon_{cs} = -500 \cdot 10^{-6}$$

The force N_{cs} is:

$$N_{cs} = \varepsilon_{cs}(\infty) \cdot E_S \cdot A_c = 500 \cdot 10^{-6} \cdot \frac{210}{28,3} \cdot 0,2 \cdot 10^6 = 742 \text{ kN}$$

The force N_{cs} acts at the centre of the concrete flange, at a distance a_c above the centroid of the composite section:

$$a_c = a \cdot \frac{A_a}{A_{id}} = a \cdot \frac{E_a \cdot A_a}{E_a \cdot A_a + E_S \cdot A_c}$$

The distance between the centroidal axes of the concrete slab and the steel section, a, is:

$$a = \frac{450}{2} + \frac{80}{2} + 50 = 315 \text{ mm}$$

The ideal second moment of area of composite section is:

$$A_{id} = A_a + \frac{A_c}{n} = 9880 + \frac{2500 \cdot 80}{28,3} = 16947 \text{ mm}^2$$

The force N_{cs} acts at the centre of the concrete flange, at a distance a_c above the centroid of the composite section:

$$a_c = 315 \cdot \frac{9880}{16947} = 184 \text{ mm}$$

The second moment of area of the cracked cross-section at internal support, Table B8.1, is:

$$I_y(cracked) = 452 \cdot 10^6 \text{ mm}^4$$

The second moment of area of the uncracked cross-section at mid-span with the modular ratio n = 28,3, Table B8.1, is:

$$I_y(n = 28,3) = 720 \cdot 10^6 \text{ mm}^4$$

The ratio of the second moments of area λ is:

$$\lambda = \frac{I_y(n=28,3)}{I_y(cracked)} = \frac{720 \cdot 10^6}{452 \cdot 10^6} = 1,59$$

The radius of curvature of the uncracked part of the beam R is:

$$R = \frac{E_a \cdot \lambda \cdot I_y(cracked)}{N_{cs} \cdot a_c} = \frac{210 \cdot 1,59 \cdot 452 \cdot 10^6}{742 \cdot 184} = 1,105 \cdot 10^6 \text{ mm} = 1105 \text{ m}$$

With span L = 10 m, the deflection δ is:

$$\delta = \frac{(0,85 \cdot L)^2}{2 \cdot R} = \frac{(0,85 \cdot 10 \cdot 10^3)^2}{2 \cdot 1105 \cdot 10^3} = 32,7 \text{ mm}$$

From the expression for the deflection $\delta = \frac{P}{2} \cdot L^3 \cdot \frac{0,13 \cdot \lambda + 0,20}{E_a \cdot I_y(cracked) \cdot \lambda}$, the force P is:

$$\frac{P}{2} = \delta / (L^3 \frac{0,13 \cdot \lambda + 0,20}{E_a \cdot \mathrm{I}_y(cracked) \cdot \lambda}) = 32,7 / (10000^3 \cdot \frac{0,13 \cdot 1,59 + 0,20}{210 \cdot 452 \cdot 10^6 \cdot 1,59}) = 12,1 \text{ kN}$$

The hogging bending moment at the internal support $M_{Ed,sh,B}$ is:

$$M_{Ed,sh,B} = \frac{P \cdot L}{2} = 12,1 \cdot 10 = 121 \text{ kNm}$$

The design bending moment at the internal support, excluding the effects of shrinkage, with n = 20,2, is $M_{Ed,B}$ = 399 kNm. The value of the hogging bending moment at the internal support $M_{Ed,sh,B}$ is 23% of $M_{Ed,B}$. This high value is the result of the application of concrete with a large shrinkage and the continuous beam with two equal spans. According to clause 5.4.2.2(7), EN 1994-1-1, the hogging bending moment at the internal support $M_{Ed,sh,B}$ can be neglected if the resistance is not influenced by lateral-torsional buckling.

b) Maximum design moment at mid-span (load case 2)

The maximum design moment in the sagging region is obtained in a span when the other span is subjected to the minimum load, the design value of permanent load g_d = 11,9 kN/m, and is reduced by creep, so $n = n_0$ = 10,1 is presumed. In the considered continuous beam, the flexural stiffness at mid-span is reduced further due to creep than is the flexural stiffness at the internal support, where the concrete is cracked. Therefore, the sagging bending moment (for uniformly distributed load) and the longitudinal shear are reduced over time.

Since the variable load is removed from one span, the design bending moment at the internal support, using data from Table B8.2, is:

$$M_{Ed,B} = 116 + 257 / 2 = 245 \text{ kNm}$$

The reaction at the end support with full design load in one span 38,2 kN/m is:

$$R_{Ed,A} = 38,2 \cdot 5,0 - 245 / 10,0 = 167 \text{ kN}$$

The point of maximum bending moment is at a distance of 167/38,2 = 4,37 m from the end support, and the maximum design sagging moment is:

$$M_{Ed} = 167 \cdot 4,37 - 38,2 \cdot 4,37 \cdot \frac{4,37}{2} = 365 \text{ kNm}$$

The calculation of bending moments and internal forces can be performed using commercial software for the static system and design load shown in Figure B8.17. The concrete is cracked at a length of 0,15 L on each side of internal support. The flexural stiffness $E_a I_2$ in this region is 98910 kNm^2. At mid-span, the concrete is uncracked at a length of 0,85 L in both spans. The flexural stiffness, for uncracked section and the modular ratio n = 10,1, is 208320 kNm^2.

The obtained results, shown in Figure B8.17, are the same as the results obtained by the method given in [35].

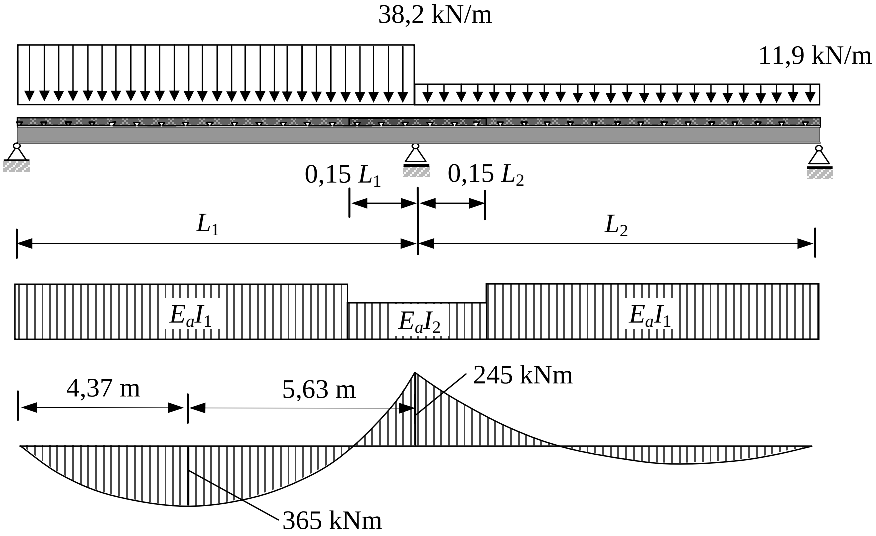


Figure B8.17 *Static system, design load and the corresponding bending moment distribution (without redistribution)*

5.3 Check for the construction stage

5.3.1 Classification of the steel cross-section

For t_f = 14,6 mm, the yield strength is f_y = 355 N/mm^2.

$$\varepsilon = \sqrt{\frac{235}{f_y}} = \sqrt{\frac{235}{355}} = 0,81$$

For the execution stage, the neutral axis is located in the half depth of the web of the steel section.

The classification of the steel cross-section is conducted according to Table 5.2, EN 1993-1-1.

Flange:

The outstand of compression flange, Figure B8.18 is:

$$c = \frac{b - t_w - 2 \cdot r}{2} = \frac{190 - 9,4 - 2 \cdot 21}{2} = 69,3 \text{ mm}$$

$$\frac{c}{t_f} = \frac{69,3}{14,6} = 4,75$$

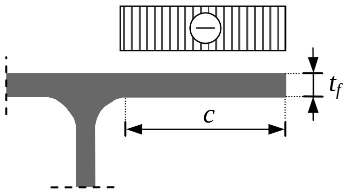


Figure B8.18 *Classification of the flange (compressive stress is negative)*

The limiting value for class 1 is:

$$\frac{c}{t_f} \leq 9 \cdot \varepsilon = 9 \cdot 0,81 = 7,29$$

$4,75 < 7,29$ $\rightarrow$ Therefore, the flange in compression is class 1.

Web:

The web subjected to bending, Figure B8.19 is:

$$c = d = h_a - 2 \cdot t_f - 2 \cdot r = 450 - 2 \cdot 14{,}6 - 2 \cdot 21 = 379 \text{ mm}$$

$$\frac{c}{t_w} = \frac{379}{9{,}4} = 40{,}3$$

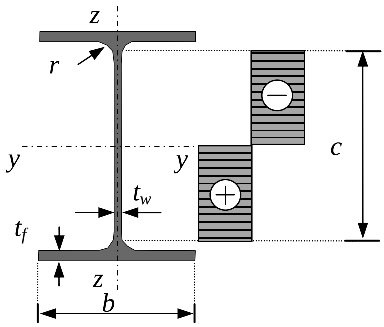


Figure B8.19 *Classification of the web (compressive stress is negative)*

The limiting value for class 1 is:

$$\frac{c}{t_w} \leq 72 \cdot \varepsilon = 72 \cdot 0{,}81 = 58{,}3$$

$40{,}3 < 58{,}3 \rightarrow$ Therefore the web in bending is class 1.

Therefore, the cross-section is class 1.

5.3.2 Plastic resistance moment of the steel cross-section

The design resistance moment for classes 1 and 2 cross-sections is:

$$M_{c,Rd} = M_{pl,a,Rd} = \frac{W_{pl,y} \cdot f_{yd}}{\gamma_{M0}}$$

$$M_{c,Rd} = M_{pl,a,Rd} = \frac{1702 \cdot 35{,}5 \cdot 10^{-2}}{1{,}0} = 604 \text{ kNm}$$

Verify that:

$$\frac{M_{y,Ed}}{M_{c,Rd}} \leq 1,0$$

$$\frac{133}{604} = 0,22 < 1,0$$

Therefore the resistance moment is adequate.

5.3.3 Shear resistance of the steel cross-section

According to 6.2.2.3, EN 1994-1-1, the shear buckling resistance of an un-encased web should be verified using Section 5, EN 1993-1-5, if:

$$\frac{h_w}{t} > \frac{72}{\eta}\varepsilon$$

where:

$$\varepsilon = \sqrt{\frac{235}{f_y}} = \sqrt{\frac{235}{355}} = 0,81$$

$\eta = 1,2$, the factor defined in EN 1993-1-5

$$h_w = h_a - 2 \cdot t_f = 450 - 2 \cdot 14,6 = 421 \text{ mm}$$

$$\frac{72}{\eta}\varepsilon = \frac{72}{1,2} \cdot 0,81 = 48,6$$

$$\frac{h_w}{t} = \frac{h_w}{t_w} = \frac{421}{9,4} = 44,8$$

Since 44,8 < 48,6 the condition is satisfied. The shear buckling resistance of the web need not be verified.

Remark:

The resistance of the composite beam to vertical shear is normally taken as the shear resistance of the steel section according to clause 6.2.6, EN 1993-1-1, which gives the following expression:

$$V_{pl,Rd} = V_{pl,a,Rd} = \frac{A_V (f_y / \sqrt{3})}{\gamma_{M0}}$$

For rolled I- and H-sections when the load is applied parallel to the web, the shear area is calculated as:

$A_V = A - 2 \cdot b_a \cdot t_f + t_f \cdot (t_w + 2 \cdot r)$, but not less than $\eta \cdot h_w \cdot t_w$

The shear area A_V is:

$$A_v = 9880 - 2 \cdot 190 \cdot 14{,}6 + (9{,}4 + 2 \cdot 21) \cdot 14{,}6$$

$$A_v = 5082 \text{ mm}^2$$

$$\eta = 1{,}2$$

$$\eta \cdot h_w \cdot t_w = 1{,}2 \cdot 421 \cdot 9{,}4 = 4749 \text{ mm}^2$$

5082 mm^2 > 4749 mm^2

Therefore, A_V = 5082 mm^2 = 50,8 cm^2

The design plastic shear resistance of the steel section is:

$$V_{pl,Rd} = V_{pl,a,Rd} = \frac{A_V (f_y / \sqrt{3})}{\gamma_{M0}}$$

$$V_{pl,Rd} = V_{pl,a,Rd} = \frac{50{,}8}{1{,}0} \cdot \frac{35{,}5}{\sqrt{3}} = 1041 \text{ kN}$$

Verify that:

$$\frac{V_{Ed}}{V_{pl,Rd}} \leq 1{,}0$$

$$\frac{66{,}8}{1041} = 0{,}06 < 1{,}0$$

Therefore, the shear resistance of the cross-section is adequate.

5.3.4 Interaction of M–V (bending and shear force)

Where the shear force is less than half the plastic shear resistance its effect on the resistance moment can be neglected.

$$0{,}5 \cdot V_{pl,Rd} = 0{,}5 \cdot 1041 = 521 \text{ kN}$$

$V_{Ed} = 66{,}8 \text{ kN} < 0{,}5\, V_{pl,Rd} = 521 \text{ kN}$ no reduction in the resistance moment

$$M_{y,V,Rd} = M_{c,Rd} = 604 \text{ kNm}$$

Verify that:

$$\frac{M_{y,Ed}}{M_{c,Rd}} \leq 1{,}0$$

$$\frac{133}{604} = 0{,}22 < 1{,}0$$

Since 0,22 < 1,00, the resistance to combined shear and bending is satisfactory.

5.3.5 Lateral-torsional buckling of the steel beam

The continuous beam is unpropped at the construction stage. It is necessary to verify the resistance to the lateral-torsional buckling of the steel beam according to EN 1993-1-1.

The elastic critical moment of lateral-torsional buckling is calculated using the following expression:

$$M_{cr} = C_1 \cdot \frac{\pi^2 \cdot E \cdot I_z}{(k \cdot L)^2} \cdot [\sqrt{(\frac{k}{k_w})^2 \cdot \frac{I_w}{I_z} + \frac{(k \cdot L)^2 \cdot G \cdot I_t}{\pi^2 \cdot E \cdot I_z} + (C_2 \cdot z_g)^2} - C_2 \cdot z_g]$$

L = 1000 cm is the length between points at which the compression flange is laterally restrained

$z_g = \frac{h}{2} = \frac{45{,}0}{2} = 22{,}5$ cm, is the distance of the shear centre from the point application of the load

$$G = \frac{E}{2\,(1+\nu)} = \frac{21\,000}{2\cdot(1+0{,}3)} = 8\,077 \ \text{kN/cm}^2$$

For the shape of the moment diagram and the load from Figure B8.10 we have:

- the effective length factors that depend on the support conditions at the end sections $k = 1{,}0$ and $k_w = 1{,}0$,

- the coefficient which takes into account the shape of the moment diagram C_1 and the coefficient which takes into account the destabilizing or stabilizing effect of the position of the load C_2 are found according to [3].

$$\mu = \frac{qL^2}{8M} = -\frac{10{,}7\cdot 10{,}0^2}{8\cdot 133} = -1{,}00$$

$$\psi = \frac{0}{133} = 0$$

$$C_1 = 2{,}22 \quad C_2 = 0{,}88$$

$$M_{cr} = 2{,}22\cdot\frac{\pi^2\ \cdot\ 21000\cdot 1676}{(1\cdot 1000)^2}\cdot$$
$$[\sqrt{(\frac{1}{1})^2\cdot\frac{791000}{1676} + \frac{(1\cdot 1000)^2\ \cdot\ 8077\ \cdot\ 66{,}87}{\pi^2\ \cdot\ 21000\ \cdot 1676} + (0{,}88\cdot 22{,}5)^2} - 0{,}88\cdot 22{,}5]$$

M_{cr} = 22658 kNcm = 227 kNm

Non-dimensional slenderness:

$$\bar{\lambda}_{LT} = \sqrt{\frac{W_y\ \cdot\ f_y}{M_{cr}}}$$

for classes 1 and 2 $W_y = W_{pl,y}$

$$\bar{\lambda}_{LT} = \sqrt{\frac{1702\ \cdot 35{,}5}{22658}} = 1{,}63 > \bar{\lambda}_{LT,0} = 0{,}4$$

The reduction factor for lateral-torsional buckling – General method:

$$\chi_{LT} = \frac{1}{\Phi_{LT} + \sqrt{\Phi_{LT}^2 - \bar{\lambda}_{LT}^2}} \text{ but } \chi_{LT} \leq 1{,}0$$

$$\Phi_{LT} = 0{,}5\,[1 + \alpha_{LT}\,(\bar{\lambda}_{LT} - 0{,}2) + \bar{\lambda}_{LT}^2\,]$$

$\dfrac{h}{b} = \dfrac{450}{190} = 2{,}37 > 2$, rolled I section → the buckling curve *b* is governed

For the buckling curve b → $\alpha_{LT} = 0{,}34$, (α_{LT} is the imperfection factor).

$$\Phi_{LT} = 0{,}5\,[1 + 0{,}34 \cdot (1{,}63 - 0{,}2) + 1{,}63^2\,] = 2{,}07$$

$$\chi_{LT} = \frac{1}{2{,}07 + \sqrt{2{,}07^2 - 1{,}63^2}} = 0{,}30$$

The design buckling resistance moment is:

$$M_{b,Rd} = \chi_{LT} \cdot \frac{W_y \cdot f_y}{\gamma_{M1}}$$

for classes 1 and 2 $W_y = W_{pl,y}$

$$M_{b,Rd} = 0{,}30 \cdot \frac{1702 \cdot 35{,}5}{1{,}0} = 18126 \text{ kNcm } = 181 \text{ kNm}$$

Verify that:

$$\frac{M_{Ed}}{M_{b,Rd}} \leq 1{,}0$$

$$\frac{133}{181} = 0{,}73 < 1{,}00$$

Therefore, the buckling resistance moment of the steel beam is adequate.

Remark:

Where the profiled steel sheeting spans perpendicularly to the beam and is attached to its top flange, the beam can be considered as restrained along its length. For this case, the verification is conducted according to EN 1993-1-1.

5.4 Check for the composite stage

5.4.1 Effective width of the concrete flange

In accordance with Figure 5.1, clause 5.4.1.2, EN 1994-1-1, for this beam we have:

$L_1 = L_2 = 10$ m

the equivalent span of the beam at mid-span region,

$L_e = 0{,}85 \cdot L_1 = 0{,}85 \cdot 10{,}0 = 8{,}5$ m for $b_{eff,1}$

If it is assumed that is $b_0 = 0{,}1$ m, then we get:

$$b_1 = b_2 = 2{,}5/2 - 0{,}1/2 = 1{,}20 \text{ m}$$

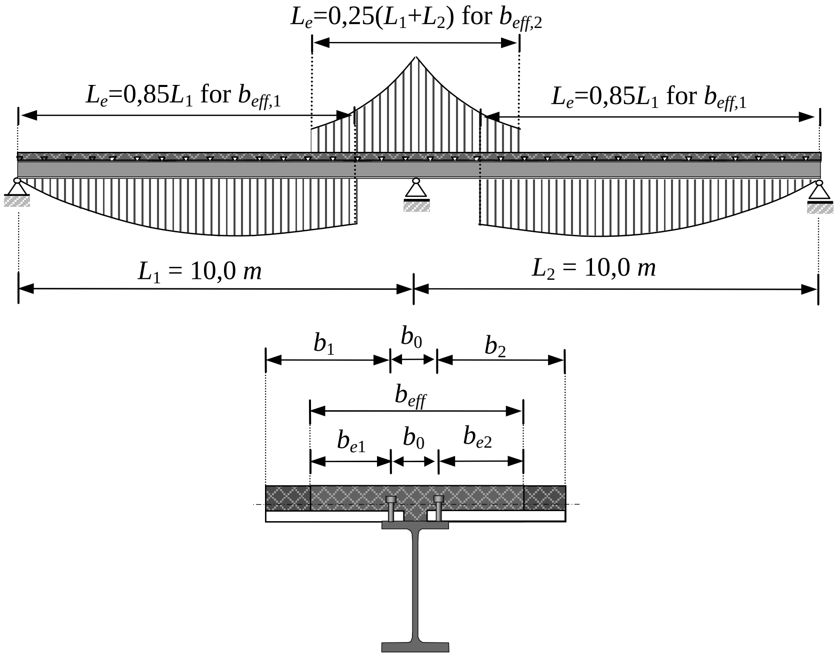


Figure B8.20 *Equivalent spans for the effective width of the concrete flange and the effective width dimensions*

At mid-span, the effective width of the concrete flange is:

$$b_{eff} = 0{,}1 + 2 \cdot 8{,}5/8 = 2{,}23 \text{ m (but} \leq 2{,}5 \text{ m)}$$

Adopted: $b_{eff} = 2{,}23\ \text{m}$

The equivalent span of the beam for the region at the internal support is:

$L_e = 0{,}25 \cdot 20{,}0 = 5{,}0$ m for $b_{eff,2}$

The effective width of the concrete flange for the region at the internal support is:

$$b_{eff} = 0{,}1 + 2 \cdot 5 / 8 = 1{,}35\ \text{m}$$

At the end support we have b_{ei} = 1,20 m, so that the effective width of the concrete flange is:

$$\beta_i = 0{,}55 + 0{,}025 \cdot 8{,}5 / 1{,}20 = 0{,}727$$

$$b_{eff} = 0{,}1 + 2 \cdot 0{,}727 \cdot 1{,}2 = 1{,}84\ \text{m}$$

The effective widths of the concrete flange b_{eff} of a continuous composite beam are:

- for the mid-span region $b_{eff} = 2{,}23\ \text{m}$
- for the internal support region $b_{eff} = 1{,}35\ \text{m}$
- at the end support $b_{eff} = 1{,}84\ \text{m}$

5.4.2 *Classification of the composite cross-section*

The local buckling of cross-sections affects the resistance and rotational capacity of sections. Therefore, the local buckling should be considered in the design. The classification of cross-sections of composite beams depends on the local slenderness of the flange (b/t) and of the web (c/t) of steel beams, the position of the plastic neutral axis and the area of longitudinal reinforcement in the slab at the internal support.

The section classification in EN 1993-1-1 is adopted for composite sections. Table 5.2 of EN 1993-1-1 gives limits for the width-to-thickness ratios of the compression parts of a section for each classification. In addition to the limitations of local slenderness of the flange and web of the steel beam, requirements for ductility of the reinforcement in tension are given for class 1 and class 2. The reinforcement should have ductility class B or C, Table C.1, EN 1992-1-1, and according to clause 5.5.1(5), EN 1994-1-1, the minimum area of reinforcement A_s should satisfy:

$A_s \geq \rho_s \cdot A_c$

with,

$$\rho_s = \delta \frac{f_y}{235} \frac{f_{ctm}}{f_{sk}} \sqrt{k_c}$$

where:

A_c is the effective area of the concrete flange,
f_y is the nominal (characteristic) value of the yield strength of the structural steel in N/mm^2,
f_{sk} is the characteristic yield strength of the reinforcement N/mm^2,
f_{ctm} is the mean tensile strength of the concrete,
δ is the factor which is:
δ = 1,1 for the procedure plastic–plastic (cross-section class 1),
δ = 1,0 for the procedure elastic–plastic (cross-section class 2),
k_c is a coefficient which takes into account the stress distribution within the section immediately prior to cracking and is $k_c = \dfrac{1}{1 + h_c / (2z_0)} + 0,3 \leq 1,0$
h_c is the thickness of the concrete flange, excluding any haunch or troughs,
z_0 is the vertical distance between the centroids of the uncracked concrete flange and the uncracked composite section, calculated using the modular ratio $n_0 = E_a/E_{cm}$ for short-term loading, i.e. at time of the first loading t_0.

5.4.2.1 Cross-section at mid-span

The cross-section in bending at the construction stage is class 1 and therefore, at the composite stage, the cross-section is also class 1.

5.4.2.2 Cross-section at the internal support

Classification of the flange

At the internal support, a region of hogging bending, the bottom flange of the steel section is in compression. According to the classification of the flange from Section 5.3.1, the flange satisfies the condition for class 1.

Classification of the web

For the classification of steel cross section, clause 5.5.1(1)P, EN 1994-1-1, refers to clause 5.5.2, EN 1993-1-1. In Table 5.2, EN 1993-1-1 the depth of the web, *c*, is defined as the straight portion of the web between the root radii.

In composite beams subjected to hogging bending, addition of longitudinal reinforcement in the concrete flange increases the depth of the steel web in compression, $\alpha \cdot c$, in Figure B8.21.

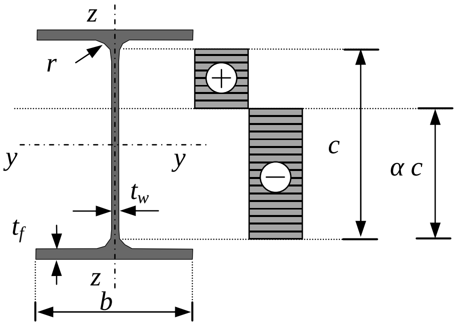


Figure B8.21 *Classification of the web (compression is negative)*

For $\alpha > 0{,}5$, the limiting value for class 1 is:

$$\frac{c}{t} \leq \frac{396 \cdot \varepsilon}{13\alpha - 1}$$

$$c = d = 450 - 2 \cdot 14{,}6 - 2 \cdot 21 = 379 \text{ mm}$$

To classify the web, the position of the plastic neutral axis should be determined.

The design resistance moment at the internal support, M_{Rd}, can be calculated as:

$$M_{Rd} = M_{pl,Rd} = \Sigma N_{si} \cdot z_i + M_{a,V,Rd} - \frac{t_w \cdot d_0^2 \cdot f_{yd}}{4}$$

The area of longitudinal reinforcement in the concrete flange at the internal support significantly affects the class of the web. It is necessary to choose a value for the area of longitudinal reinforcement. The reinforcement bars are assumed to be 12 mm at 125 mm, because larger-diameter bars may not give required control of crack widths. Therefore:

$$d_{bar} = 12 \text{ mm} \rightarrow A_{bar} = \pi \cdot d_{bar}^2 / 4 = 113{,}1 \text{ mm}^2 \rightarrow 113{,}1 \cdot 1000/125 = 904{,}8 \text{ mm}^2/\text{m}$$

$$A_s = b_{eff,2} \cdot 904{,}8 / 1000 = 1350 \cdot 904{,}8 / 1000 = 1221 \text{ mm}^2 = 12{,}21 \text{ cm}^2$$

The force in these bars is:

$$N_s = f_{sd} \cdot A_s$$

$$N_s = 435 \cdot 1221 \cdot 10^{-3} = 531 \text{ kN}$$

In the case of the hogging bending (moment at the internal support), the plastic neutral axis lies within the web of the steel beam, as shown in Figure B8.22:

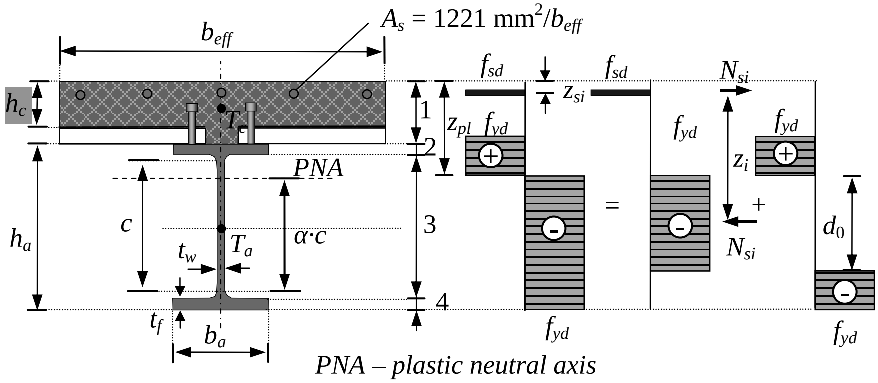


Figure B8.22 *Determination of the design resistance moment in the hogging region*

According to clause 6.2.2.4, EN 1994-1-1, where the shear force exceeds half the shear resistance its effect on the resistance moment should be taken into account. The reduction factor for the design yield strength of the web is $(1 - \rho)$, where:

$$\rho = \left(\frac{2V_{Ed}}{V_{Rd}} - 1\right)^2$$

and V_{Rd} is the design shear resistance.

For the design shear force $V_{Ed,B} = 231$ kN and the design shear resistance, determined in Section 5.3.3 for the steel section only, $V_{pl,Rd} = V_{pl,a,Rd} = 1041$ kN is:

$$\frac{V_{Ed}}{V_{Rd}} = \frac{231}{1041} = 0,22 < 0,50$$

Therefore, there is no reduction in the resistance moment.

From equilibrium the design tensile force in the reinforcement $N_{si} = N_s$ and the design compressive force in web $N_{si} = d_0 \cdot f_{yd} \cdot t_w$ is:

$$d_0 = \frac{N_s}{t_w \cdot f_{yd}} = \frac{531}{0{,}94 \cdot 35{,}5} = 15{,}9 \text{ cm}$$

The distance between plastic neutral axis and the top of the slab z_{pl} is:

$$z_{pl} = h_c + h_p + \frac{h_a}{2} - \frac{d_0}{2} = 8{,}0 + 5{,}0 + \frac{45}{2} - \frac{15{,}9}{2} = 27{,}6 \text{ cm}$$

The distance between the centroid of the steel section and the centroid of the reinforcement is:

$$z_i = h_c + h_p + \frac{h_a}{2} - z_{si} = 8{,}0 + 5{,}0 + \frac{45}{2} - 3{,}0 = 32{,}5 \text{ cm}$$

Where the shear force reduces the resistance moment of the steel section, the reduced design resistance moment is:

$$M_{a,V,Rd} = M_{pl,f,Rd} + (M_{pl,a,Rd} - M_{p,f,Rd})(1-\rho)$$

In this case, there is no reduction in the resistance moment. Therefore, the design resistance moment of the steel section $M_{pl,a,Rd}$ is taken into account instead of the reduced design resistance moment $M_{a,V,Rd}$.

The design value of the plastic resistance moment of the steel section is:

$$M_{pl,a,Rd} = 604 \text{ kNm}$$

Therefore, the design value of the plastic resistance moment of the composite section at the internal support is:

$$M_{pl,Rd} = N_s \cdot z_i + M_{pl,a,Rd} - \frac{t_w \cdot d_0^2 \cdot f_{yd}}{4}$$

$$M_{pl,Rd} = 531 \cdot 32{,}5 + 60400 - \frac{0{,}94 \cdot 15{,}9^2 \cdot 35{,}5}{4} = 75548 \text{ kNcm } = 755 \text{ kNm}$$

For I-sections subject to major-axis bending and axial force with the neutral axis in the web, the parameter α_c can be calculated as:

$$\alpha_c = \frac{1}{c}\left(\frac{h_a}{2} + \frac{1}{2}\frac{N_{Ed}}{t_w \cdot f_{yd}} - (t_f + r)\right)$$

In this case, the design axial compressive force N_{Ed} is equal to the design tensile force in reinforcement. The design axial compressive force N_{Ed} is:

$$N_{Ed} = N_s = 531 \text{ kN}$$

The parameter α_c is:

$$\alpha_c = \frac{1}{37,9}(\frac{45}{2} + \frac{1}{2}\frac{531}{0,94 \cdot 35,5} - (1,46 + 2,1)) = 0,71$$

For $\alpha > 0,5$, the limiting value for class 1, table 5.2, EN 1993-1-1, is:

$$\frac{c}{t} \leq \frac{396 \cdot \varepsilon}{13\alpha - 1}$$

$$\frac{c}{t} = \frac{379}{9,4} = 40,3 > \frac{396 \cdot \varepsilon}{13\alpha - 1} = \frac{396 \cdot 0,81}{13 \cdot 0,71 - 1} = 39,0$$

$40,3 > 39,0 \;\rightarrow$ the web is not class 1.

For $\alpha > 0,5$, the limiting value for class 2, table 5.2, EN 1993-1-1, is:

$$\frac{c}{t} \leq \frac{456 \cdot \varepsilon}{13\alpha - 1}$$

$$\frac{c}{t} = \frac{379}{9,4} = 40,3 < \frac{456 \cdot \varepsilon}{13\alpha - 1} = \frac{456 \cdot 0,81}{13 \cdot 0,71 - 1} = 44,9$$

$40,3 < 44,9 \;\rightarrow$ the web is class 2.

Therefore, the cross-section is class 2.

The considered beam is class 2 at the internal support region and in class 1 at the mid-span region.

Minimum reinforcement area A_s

Within the effective width of the composite section, the ductile reinforcement is selected. According to clause 5.5.1(5), EN 1994-1-1, the minimum area of reinforcement A_s should satisfy the following condition:

$$A_s \geq \rho_s \cdot A_c$$

with:

$$\rho_s = \delta \frac{f_y}{235} \frac{f_{ctm}}{f_{sk}} \sqrt{k_c}$$

The area of reinforcement is:

$A_s = 12{,}21$ cm^2, (bars 12 mm at 125 mm)

The effective area of concrete slab at the internal support is:

$$A_c = b_{eff,2} \cdot h_c = 135 \cdot 8{,}0 = 1080 \text{ cm}^2$$

For the elastic–plastic procedure, the factor δ is 1,0.

The coefficient k_c takes into account the stress distribution within the section immediately prior to cracking and is:

$$k_c = \frac{1}{1 + h_c / (2z_0)} \leq 1{,}0$$

The thickness of the concrete flange h_c is 8,0 cm. The vertical distance between the centroids of the uncracked concrete flange and the uncracked composite section is denoted by z_0. The vertical distance between the centroids of the uncracked concrete flange and the uncracked composite section is denoted by z_0.

The modular ratio n_0 is:

$$n_0 = \frac{E_a}{E_{lcm}} = \frac{210}{20{,}75} = 10{,}1$$

The effective width of the concrete flange at the internal support is:

$$b_{eff,2} = 135 \text{ cm}$$

Transformed to the ideal steel section, the effective width is:

$$\frac{b_{eff,2}}{n_0} = \frac{135}{10,1} = 13,4 \text{ cm}$$

The area of the ideal steel cross-section is:

$$A = A_a + \frac{b_{eff,2}}{n_0} \cdot h_c = 98,8 + 13,4 \cdot 8,0 = 206 \text{ cm}^2$$

The distance between the neutral axis and the centroid of the steel section is:

$$z_{n0} = \frac{\frac{b_{eff,2}}{n_0} \cdot h_c \cdot (\frac{h_a}{2} + \frac{h_c}{2} + h_p)}{A} = \frac{13,4 \cdot 8,0 \cdot (\frac{45}{2} + \frac{8,0}{2} + 5,0)}{206} = 16,4 \text{ cm}$$

Thus, the vertical distance between the centroids of the uncracked concrete flange and the uncracked composite section is:

$$z_0 = (\frac{h_a}{2} + \frac{h_c}{2} + h_p) - z_{n0} = (\frac{45}{2} + \frac{8,0}{2} + 5,0) - 16,4 = 31,5 - 16,4 = 15,1 \text{ cm}$$

Therefore:

$$k_c = \frac{1}{1 + h_c / (2 z_0)} + 0,3 = \frac{1}{1 + 8,0 / (2 \cdot 15,1)} + 0,3 = 1,09 \leq 1,0$$

$$k_c = 1,00$$

$$f_{lctm} = 2,32 \text{ N/mm}^2$$

$$\rho_s = \delta \frac{f_y}{235} \frac{f_{lctm}}{f_{sk}} \sqrt{k_c} = 1,0 \frac{355}{235} \frac{2,32}{500} \sqrt{1,0} = 0,70 \ \%$$

The final verification of the minimum reinforcement is:

$$A_s = 12,21 \text{ cm}^2 \geq \rho_s \cdot A_c = 0,0070 \cdot 1080 = 7,56 \text{ cm}^2$$

The condition is satisfied.

5.4.3 Resistance moment of composite cross-section

5.4.3.1 Resistance moment at mid-span

According to clause 6.2.1.3, EN 1994-1-1, the partial shear connection may be used in the regions of sagging bending.

If ductile shear connectors are used, the resistance moment can be determined by means of rigid plastic theory in accordance with 6.2.1.2, EN 1994-1-1. However, the reduced value of the compressive force N_c must be taken into account instead of the force $N_{c,f}$.

It is very convenient to use the diagram of partial shear connection, shown in Figure 6.5, EN 1994-1-1, for determining the resistance moment. According to clause 6.2.1.3(5), EN 1994-1-1, the design resistance moment of the composite beam in sagging region can be conservatively calculated by the straight line AC in Figure 6.5, EN 1994-1-1:

$$M_{Rd} = M_{pl,a,Rd} + (M_{pl,Rd} - M_{pl,a,Rd}) \cdot \frac{N_c}{N_{c,f}}$$

where:

$M_{pl,a,Rd}$ is the design plastic resistance moment of the structural steel section alone in sagging region,
$M_{pl,Rd}$ is the design plastic resistance moment of the composite section with full shear connection in sagging region,
$N_{c,f}$ is the design compressive force in the concrete flange with full shear connection,
N_c is the design compressive force in the concrete flange.

In this example, the calculation of plastic resistance moment of the composite section with full shear connection in sagging region was first implemented. To take into account the partial shear connection, the resistance moment M_{Rd} is calculated by means of rigid plastic theory in accordance with clause 6.2.1.2, EN 1994-1-1, taking into account the reduced value of the compressive force in the concrete flange N_c instead of $N_{c,f}$. The simplified procedure is illustrated in example B6.

The plastic neutral axis lies within the thickness of top steel flange if it is:

$$N_c < N_{pl,a}$$

The design plastic resistance of the structural steel section to normal force is:

$$N_{pl,a} = \frac{A_a \cdot f_{yd}}{\gamma_{M,0}} = \frac{9880 \cdot 0,355}{1,0} = 3507 \text{ kN}$$

In region of sagging bending, the reinforcement in compression is neglected and the available area of concrete is 2,23 m wide and 80 mm thick. The compressive force is:

$$N_c = N_{c,f} = 0,85 \cdot f_{cd} \cdot b_{eff} \cdot h_c = 14,2 \cdot 2,23 \cdot 80 = 2533 \text{ kN}$$

Since 2533 < 3507, the plastic neutral axis lies within the thickness of top steel flange.

Assuming full shear connection, with the plastic neutral axis in the steel top flange, the design plastic resistance moment of the composite section with full shear connection in sagging region is determined by the following expression:

$$M_{pl,Rd} = N_{c,f} \cdot z + N_{pl,f}(h_a - x_2)$$

The design plastic resistance of the steel flange to normal force is:

$$N_{pl,f} = \frac{N_{pl,a} - N_{c,f}}{2} = \frac{3507 - 2533}{2} = 487 \text{ kN}$$

In accordance with Figure B8.23, the following values are calculated:

$$x_1 = \frac{N_{c,f}}{b_{eff} \cdot 0,85 \cdot f_{cd}} = \frac{2533 \cdot 10^3}{2230 \cdot 14,2} = 80 \text{ mm} = h_c = 80 \text{ mm}$$

$$x_2 = \frac{N_{pl,f}}{b_f \cdot f_{yd}} = \frac{487}{190 \cdot 0,355} = 7,2 \text{ mm}$$

$$z = h_c + h_p + \frac{h_a}{2} - \frac{x_1}{2}$$

$$z = 80 + 50 + \frac{450}{2} - \frac{80}{2} = 80 + 50 + 225 - 40 = 315 \text{ mm}$$

The design plastic resistance moment of the composite section in sagging region is:

$$M_{pl,Rd} = N_{c,f} \cdot z + N_{pl,f}(h_a - x_2)$$

$$M_{pl,Rd} = 2533 \cdot 0{,}315 + 487 \cdot (0{,}450 - 0{,}0072)$$

$$M_{pl,Rd} = 1014 \text{ kNm}$$

The obtained value of design resistance will be reduced by the use of partial shear connection.

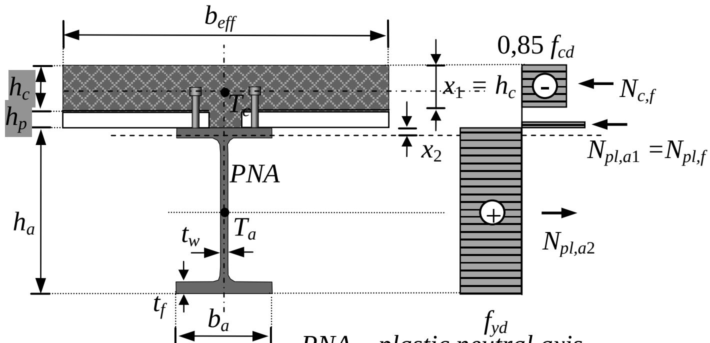


Figure B8.23 *Determination of design resistance moment in sagging region, PNA lies within flange of steel section (full shear connection)*

According to Section 5.2, the maximum design moment in sagging region is:

$$M_{Ed} = 365 \text{ kNm}$$

This value is significantly less than the design plastic resistance moment in sagging region $M_{pl,Rd}$ = 1014 kNm, so we will use the lowest permitted degree of shear connection. The maximum design moment in sagging region, M_{Ed}, is even less than $M_{pl,a,Rd}$, which is 604 kNm.

The equivalent span of beam at mid-span (L_1 = L_2 = 10,0 m) according to Figure 5.1, EN 1994-1-1, is:

$$L_{e,1} = 0{,}85 \cdot L_1 = 0{,}85 \cdot 10{,}0 = 8{,}5 \text{ m}$$

For $L_e \leq 25$ m, according to clause 6.6.1.2 (3), EN 1994-1-1, with f_y = 355 N/mm^2, the limit for the degree of shear connection is:

$$\eta = n / n_f = 1 - (\frac{355}{f_y}) \cdot (0{,}75 - 0{,}03 \cdot L_e) \quad \eta \geq 0{,}4$$

$$\eta = n / n_f = 1 - (\frac{355}{355}) \cdot (0,75 - 0,03 \cdot 8,5) = 0,51$$

Remark:

According to clause 6.6.1.2(3), EN 1994-1-1, a lower value is permitted if it satisfies the required conditions. However, the condition that there should be only one stud per trough of the profiled sheeting cannot be met. Therefore, the minimum number of studs in each half of the region of sagging bending is $0,51 \cdot n_f$, where n_f is the number of studs for full shear connection.

Calculation of the design plastic resistance moment $M_{pl,Rd}$ taking into account partial shear connection is performed by the procedure used above. A more detailed derivation of the method is given below.

In accordance with Figure B8.24, the depth of the compressive stress block in the concrete flange, x_1, is:

$$x_1 = \frac{N_c}{b_{eff} \cdot 0,85 \cdot f_{cd}}$$

The value of x_c is always less than thickness of slab h_c.

From the distribution of longitudinal strain in the cross-section shown in Figure B8.24, it can be seen that the neutral axis in the slab, denoted as x_n, is slightly greater than x_1.

It is generally assumed that the ratio x_1/x_n is between values 0,8 and 0,9 in the design of reinforced concrete slabs and beams. In the design of composite slabs and beams the less accurate assumption $x_1 = x_n$ is adopted. This assumption is on the unsafe side but the procedure is much simpler. The error introduced in the design of composite beams is negligible.

There is a second neutral axis, x_1. This axis lies within the steel section. If this axis lies within the steel top flange, the stress block is as shown in Figure B8.24.

In accordance with Figure B8.24, the depth x_2 is defined by the following expression:

$$x_2 = \frac{N_{pl,f}}{b_f \cdot f_{yd}}$$

where:

$$N_{pl,f} = \frac{N_{pl,a} - N_c}{2}$$

$$z = h_c + h_p + \frac{h_a}{2} - \frac{x_1}{2}$$

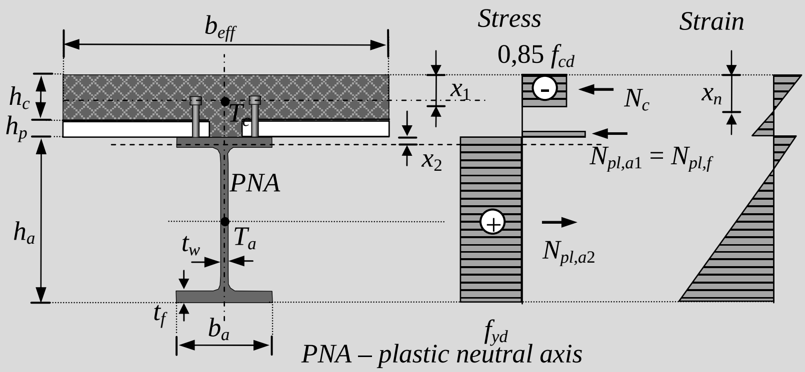


Figure B8.24 *Determination of the design resistance moment in the sagging region; the PNA lies within the flange of the steel section (partial shear connection)*

The design resistance moment of the composite section in sagging region is:

$$M_{Rd} = N_c \cdot z + N_{pl,f} (h_a - x_2)$$

The reduced design value of the compressive force in the concrete flange is:

$$N_c = \eta \cdot N_{c,f} = 0{,}51 \cdot 2533 = 1292 \text{ kN}$$

Since for $N_{c,f}$ is $x = h_c$ = 80 mm, then $x_1 = \eta \cdot h_c$ = 0,51·80 = 41 mm.

The design plastic resistance of the steel flange to normal force is:

$$N_{pl,f} = \frac{N_{pl,a} - N_c}{2} = \frac{3507 - 1292}{2} = 1108 \text{ kN}$$

In accordance with Figure B8.24, the following values are calculated:

$$x_2 = \frac{N_{pl,f}}{b_f \cdot f_{yd}} = \frac{1108}{0{,}355 \cdot 190} = 16{,}4 \text{ mm}$$

$$z = h_c + h_p + \frac{h_a}{2} - \frac{x_1}{2}$$

$$z = 80 + 50 + \frac{450}{2} - \frac{41}{2} = 80 + 50 + 225 - 20,5 = 334,5 \text{ mm}$$

The design resistance moment of the composite section in sagging region for partial shear connection is:

$$M_{Rd} = N_c \cdot z + N_{pl,f}(h_a - x_2)$$

$$M_{Rd} = 1292 \cdot 0,335 + 1108 \cdot (0,450 - 0,016)$$

$$M_{Rd} = 914 \text{ kNm}$$

The design value of the bending moment at mid-span, as the effect of action, M_{Ed} = 365 kNm and the check is:

$$\frac{M_{Ed}}{M_{pl,Rd}} \leq 1,0$$

$$\frac{365}{914} = 0,40 < 1,0 \text{, the condition is satisfied}$$

5.4.3.2 Resistance moment at the internal support

The design plastic resistance moment in hogging region is determined in accordance with clause 6.2.1.2, EN 1994-1-1, assuming that there is full shear connection between structural steel, reinforcement and concrete. The appropriate shear connection should be provided to ensure yielding of the reinforcement in tension.

In other words, we need to provide a sufficient number of studs in the hogging region. Therefore, the yielding of the reinforcement in tension occurs before the failure of the studs.

The design plastic resistance moment in hogging region was calculated in Section 5.4.2.2 at the classification of the composite section and is:

$$M_{pl,Rd} = 755 \text{ kNm}$$

The design bending moment at the internal support is M_{Ed} = 399 kNm and the check is:

$$\frac{M_{Ed}}{M_{pl,Rd}} \leq 1{,}0$$

$$\frac{399}{755} = 0{,}53 < 1{,}0$$, the condition is satisfied

Also, we need to calculate the characteristic plastic resistance moment, $M_{pl,Rk}$. The characteristic plastic resistance moment will be used in the check of lateral-torsional buckling. With the partial factor for reinforcement γ_s = 1 instead of γ_s = 1,15, the characteristic tensile force at yield increases to:

$$N_{s,k} = 531 \cdot 1{,}15 = 611 \text{ kN}$$

From equilibrium, the characteristic tensile force in reinforcement $N_{si} = N_{s,k}$ and the characteristic compressive force in web $N_{si} = d_0 \cdot f_{yk} \cdot t_w$ is:

$$d_0 = \frac{N_{s,k}}{t_w \cdot f_{yk}} = \frac{611}{0{,}94 \cdot 35{,}5} = 18{,}3 \text{ cm}$$

The distance between the plastic neutral axis and the top of the slab z_{pl} is:

$$z_{pl} = h_c + h_p + \frac{h_a}{2} - \frac{d_0}{2} = 8{,}0 + 5{,}0 + \frac{45}{2} - \frac{18{,}3}{2} = 26{,}4 \text{ cm}$$

The distance between the centroid of the steel section and the centroid of the reinforcement is:

$$z_i = h_c + h_p + \frac{h_a}{2} - z_{si} = 8{,}0 + 5{,}0 + \frac{45}{2} - 3{,}0 = 32{,}5 \text{ cm}$$

The characteristic value of the plastic resistance moment of the steel section is:

$$M_{pl,a,\mathrm{k}} = 604 \text{ kNm}$$

Therefore, the characteristic value of the plastic resistance moment of the composite section at the internal support is:

$$M_{pl,Rk} = N_{s,k} \cdot z_i + M_{pl,a,Rk} - \frac{t_w \cdot d_0^2 \cdot f_{yk}}{4}$$

$$M_{pl,Rk} = 611 \cdot 32,5 + 60400 - \frac{0,94 \cdot 18,3^2 \cdot 35,5}{4} = 77464 \text{ kNcm} = 775 \text{ kNm}$$

Remark:

All of these resistances may need to be reduced to take into account lateral-torsional buckling

5.4.4 Check of shear connection – ductile headed stud shear connectors

The degree of shear connection, η = 0,56 calculated in Section 5.4.3.1, enables the headed stud shear connectors to be treated as 'ductile'. An alternative design, using non-ductile shear connectors, is illustrated in Section 5.4.5.

According to clause 6.6.5.8(1), EN 1994-1-1, the height of the adopted stud of 19 mm diameter must be at least:

$h_p + 2 \cdot d = 50 + 2 \cdot 19 = 88$ mm

The adopted stud with the standard height of 95 mm, after welding, satisfies this rule.

5.4.4.1 Resistance of headed stud shear connectors

The design resistance of a single-headed shear connector in a solid concrete slab is determined by shank failure of the stud ($P_{Rd}^{(1)}$) or by concrete failure ($P_{Rd}^{(2)}$). The design resistance of a single-headed shear connector in a solid concrete slab, automatically welded in accordance with EN 14555, should be determined as the smaller of:

$$P_{Rd} = \min(P_{Rd}^{(1)}, P_{Rd}^{(2)})$$

$$P_{Rd}^{(1)} = \frac{0,8 \cdot f_u \cdot (\pi \cdot d^2)/4}{\gamma_V}$$

$$P_{Rd}^{(2)} = \frac{0,29 \cdot \alpha \cdot d^2 \cdot \sqrt{f_{ck} \cdot E_{cm}}}{\gamma_V}$$

where:

d is the diameter of the shank of the stud (16 mm ≤ d ≤ 25 mm),
h_{sc} is the overall nominal height of the stud,
f_u is the specified ultimate tensile strength of the material of the stud but not greater than 500 N/mm^2,
f_{ck} is the characteristic cylinder compressive strength of the concrete,
E_{cm} is the mean value of the secant modulus of elasticity of the concrete,
γ_V is the partial factor for stud (γ_V = 1,25),
α is the correction factor which takes into account the ratio of the height of the stud to the diameter of its shank.

The correction factor α is calculated as:

$$\alpha = 0,2[(\frac{h_{sc}}{d}) + 1] \quad \text{for} \quad 3 \leq \frac{h_{sc}}{d} \leq 4$$

$$\alpha = 1,0 \quad \text{for} \quad \frac{h_{sc}}{d} > 4$$

Since h_{sc}/d = 95/19 = 5,00, then α = 1.

Therefore:

$$P_{Rd}^{(1)} = \frac{0,8 \cdot f_u \cdot (\pi \cdot d^2)/4}{\gamma_V}$$

$$P_{Rd}^{(1)} = \frac{0,8 \cdot 500 \cdot (\pi \cdot 19^2)/4}{1,25} \cdot 10^{-3} = 90,7 \ \text{kN}$$

$$P_{Rd}^{(2)} = \frac{0,29 \cdot \alpha \cdot d^2 \cdot \sqrt{f_{ck} \cdot E_{cm}}}{\gamma_V}$$

$$P_{Rd}^{(2)} = \frac{0,29 \cdot 1 \cdot 19^2 \cdot \sqrt{25 \cdot 20,75 \cdot 10^3}}{1,25} \cdot 10^{-3} = 60,3 \ \text{kN}$$

The design resistance of a single-headed shear connector in a solid concrete slab is:

$$P_{Rd} = \min(P_{Rd}^{(1)}, P_{Rd}^{(2)})$$

$$P_{Rd} = P_{Rd}^{(2)} = 60,3 \, \text{kN}$$

Remark:

The resistance of headed studs used as shear connectors with profiled steel sheeting is less than the design resistance of headed studs used as shear connectors in a solid concrete slab.

The resistance of a headed stud within profiled sheeting is determined by multiplying the design resistance for a headed stud connector in a solid concrete slab (P_{Rd}) by a reduction factor k_l for profiled steel sheeting spanning parallel to the supporting beam and k_t for profiled steel sheeting spanning transverse to the supporting beam.

For profiled steel sheeting spanning parallel to the supporting beam, the design resistance a single-headed shear connector is:

$$P_{Rd} = k_l \cdot \min(P_{Rd}^{(1)}, P_{Rd}^{(2)})$$

The reduction factor k_l is calculated according to clause 6.6.4.1(2), EN 1994-1-1:

$$k_l = 0{,}6(\frac{b_0}{h_p})(\frac{h_{sc}}{h_p} - 1) \leq 1{,}0$$

In the above expression h_{sc} is the overall height of the stud, but not greater than h_p + 75 mm.

For profiled steel sheeting spanning transverse to the supporting beam, the design resistance a single-headed shear connector is:

$$P_{Rd} = k_l \cdot \min(P_{Rd}^{(1)}, P_{Rd}^{(2)})$$

The reduction factor k_t is calculated according to clause 6.6.4.2(1), EN 1994-1-1:

$$k_t = \frac{0{,}70}{\sqrt{n_r}} \frac{b_0}{h_p} (\frac{h_{sc}}{h_p} - 1) \leq k_{t,\max}$$

In the above expression n_r is the number of studs in one trough at a beam intersection, not to exceed two in the calculations, and $k_{t,max}$ is the maximum value of the reduction factor k_t which is given in Table 6.2, EN 1994-1-1.

Since in this example the profiled steel sheeting with ribs run transverse to the supporting beam, we need to check the effect of a reduction factor, k_t, on the shear connector resistance.

The reduction factor depends on the overall height of the stud, h_{sc}, the dimensions of the trough in the profiled sheeting (Figure B8.2), the thickness of the profiled sheeting (assumed to be 1,0 mm) and the number of studs per trough, n_r.

For n_r = 1, for one stud per trough, according to Table 6.2, EN 1994-1-1:

$$k_t = \frac{0,7}{\sqrt{n_r}} \cdot \frac{b_0}{h_p} \cdot (\frac{h_{sc}}{h_p} - 1) = \frac{0,7}{\sqrt{1}} \cdot \frac{100}{50} \cdot (\frac{95}{50} - 1) = 1,26 \text{, but} \leq 0,85$$

For n_r = 2, for two studs per trough, according to Table 6.2, EN 1994-1-1:

$$k_t = \frac{1,26}{\sqrt{n_r}} = \frac{1,26}{\sqrt{2}} = 0,89 \text{, but} \leq 0,70$$

Hence, the design resistance per stud in a trough where there is one stud per trough is:

$$P_{Rd,1} = 0,85 \cdot 60,3 = 51,3 \text{ kN}$$

The design resistance per stud in a trough where there are two studs per trough is:

$$P_{Rd,2} = 0,7 \cdot 60,3 = 42,2 \text{ kN}$$

Therefore, the trough with two studs provides the equivalent of $2 \cdot P_{Rd,2}/P_{Rd,1}$ = 2·42,2/51,3 = 1,65 single studs.

5.4.4.2 Arrangement of headed stud shear connectors and degree of shear connection

Remark:

Partial shear connection is permitted in sagging regions of composite beams.

Full shear connection is required in hogging regions of composite beams.

The assumed critical area is between the left-hand support and the point of maximum positive moment because between the point of maximum positive moment and the internal support there are significantly more studs, even if studs within the hogging region of the beam are neglected. This assumption should be verified when the number of studs needed to yield the slab reinforcement in the hogging region has been determined.

The point of maximum sagging moment is 4,37 m away from the far left-hand support, see Figure B8.17. The studs have to be provided within this length of 4,37 m. The troughs of profiled steel sheeting are spaced at 0,2 m, Figure B8.25, so 21 are available.

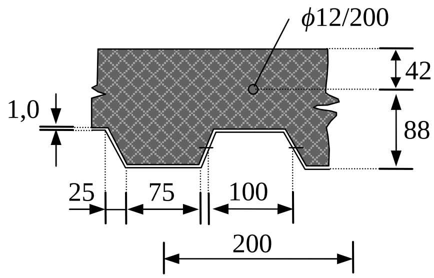


Figure B8.25 *Cross-section of the profiled steel sheeting*

The reduced design value of the compressive force in the concrete flange is N_c = 1292 kN, and the longitudinal shear force transfer, $V_{L,Ed}$, between the left-hand support A and the point of maximum positive moment is:

$$V_{L,Ed} = N_c = 1292\,\text{kN}$$

The longitudinal design shear force per unit length of the beam or the longitudinal design shear flow is:

$$v_{L,Ed} = 1292 / 4,37 = 296 \ \text{kN/m}$$

The required number of single studs is:

$$n_s = \frac{V_{L,Ed}}{P_{Rd,1}} = \frac{1292}{51,3} = 26 \ > \ \frac{4370}{200} = 21\text{, available troughs}$$

Therefore, two studs per trough have to be used over part of the span.

In EN 1994-1-1, there are no recommendations on how non-uniform shear connectors should be arranged. Generally, it could be recommended that more studs are provided in regions adjacent to supports, where shear is highest.

If the minimum number of troughs with two studs is n_{2s}, the number of single studs required 26, the number of available troughs 21 and trough with two studs provides the equivalent of 1,65 single studs, then the following equation is valid:

$$1,65 \cdot n_{2s} + 21 - n_{2s} = 26$$

Therefore, n_{2s} is:

$$n_{2s} \geq 7{,}7$$

For the cracked section in hogging bending, the design tensile force in reinforcement is:

$$N_s = 531 \text{ kN}$$

The number of studs needed to yield the slab reinforcement in the hogging region is:

$$\frac{N_s}{P_{Rd,1}} = \frac{531}{51{,}3} = 11$$

Therefore, the sufficient number of studs in hogging region is 11 so that the yielding of the reinforcement in tension occurs before the failure of the studs.

For the maximum hogging bending moment at the internal support, from the internal support to the point of maximum sagging bending moment is 6,05 m, see Figure B8.15. Therefore, the available number of troughs is 6,05/0,20 = 30 for a total of 26 + 11 = 37 single studs.

If the minimum number of troughs with two studs is n_{2h}, the number of single studs required is 37, the number of available troughs 30 and the trough with two studs provides the equivalent of 1,65 single studs, then the following equation is valid:

$$1{,}65 \cdot n_{2h} + 30 - n_{2h} = 37$$

Therefore, n_{2h} is:

$$n_{2h} \geq 10{,}8$$

The longitudinal design shear flow is:

$$v_{L,Ed} = \frac{1292 + 531}{6{,}02} = 303 \text{ kN/m}$$

The arrangement of studs is shown in Figure B8.26. This arrangement provides the equivalent of 27 (12 + 13 + 2) and 37 (18 + 17 + 2) studs within the lengths 4,4 m and 6,0 m, respectively.

From Figures B8.15 and B8.17 it can be seen that the maximum sagging and hogging bending moments are caused by different arrangements of variable load corresponding to load case 1 and load case 2. In this procedure, this arrangement of variable load is taken into account. Consequently, two studs near mid-span are effective for both sagging and hogging resistance.

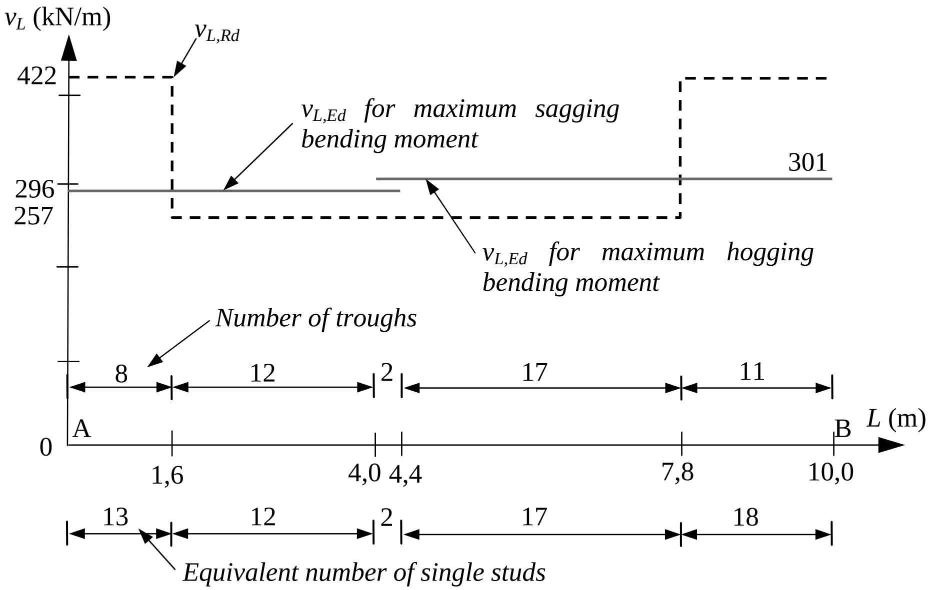


Figure B8.26 *Arrangement of studs*

5.4.5 Check of shear connection – non-ductile headed stud shear connectors

In the previous section the shear connection was calculated for shear connectors that satisfied the definition "ductile" given in clause 6.6.1.2, EN 1994-1-1. The number and arrangement of shear connectors is shown in Figure B8.26.

According to clause 6.6.1.3(5), EN 1994-1-1, it is possible to take into consideration the shear connectors that are not "ductile". In this case, the longitudinal design shear flow, $v_{L,Ed}$, is calculated by elastic theory.

The "inelastic redistribution of shear" is not required, so clause 6.6.1.3(5), EN 1994-1-1, on sufficient deformation capacity does not apply.

Since the calculation of the design resistance moment M_{Rd} according to clause 6.2.1.3(3), EN 1994-1-1, is not allowed, the stresses are calculated by elastic theory. According to clause 6.2.1.5(2), EN 1994-1-1, the limiting stresses are given as:

$$f_{cd} = \frac{f_{ck}}{\gamma_C} = \frac{25}{1,5} = 16,7 \ \text{N/mm}^2$$

$$f_{yd} = \frac{f_{yk}}{\gamma_M} = \frac{355}{1,0} = 355 \ \text{N/mm}^2$$

We need to calculate the maximum sagging bending moment which acts on the composite section.

Remark:

According to Table 5.1, EN 1994-1-1, the redistribution of the maximum hogging moment of 15% is permitted for cross-section class 2 and steel grade S355.

The maximum design moment in the sagging region was calculated in Section 5.2, Figure B8.17, without the redistribution of the hogging moment.

According to Figure B8.17, the bending moment at internal support is:

$$M_{Ed,B} = 245 \ \text{kNm}$$

With redistribution of 15%, the bending moment at the internal support is:

$$M_{Ed,B} = 0,85 \cdot 245 = 208 \ \text{kNm}$$

The reaction at the end support with full design load in one span 38,2 kN/m is:

$$R_{Ed,A} = 38,2 \cdot 5,0 - \frac{208}{10} = 170 \ \text{kN}$$

The point of maximum bending moment is at a distance of 170/38,2 = 4,45 m from the end support, and the maximum design sagging moment is:

$$M_{Ed} = 170 \cdot 4,45 - 38,2 \cdot 4,45 \cdot \frac{4,45}{2} = 378 \ \text{kNm}$$

The considered beam is unpropped at the construction stage. The steel beam at the construction stage is loaded by self-weight and the weight of the concrete slab. The static system and design loads are shown in Figure B8.27.

$e_d = b \cdot \gamma_{G,\sup} \cdot g_k$

$b = 2,5$ m beam spacing

$e_{d,1} = 2,50 \cdot 1,35 \cdot 2,32 = 7,83$ kN/m (concrete slab, profiled steel sheeting)

$e_{d,2} = 2,50 \cdot 1,35 \cdot 0,30 = 1,01$ kN/m (steel beam)

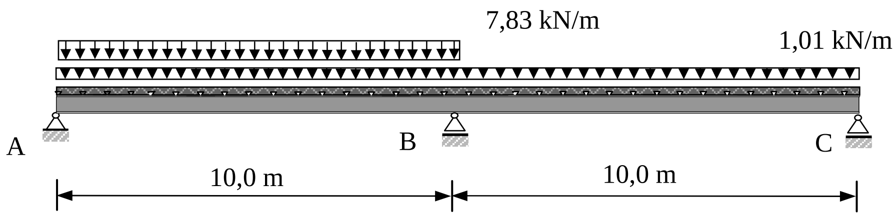


Figure B8.27 *Determination of the maximum sagging moment in the steel beam*

The design bending moment at internal support is:

$$M_{Ed,B} = 0,125 \cdot 1,01 \cdot 10,0^2 + 0,0625 \cdot 7,83 \cdot 10,0^2 = 61,6 \text{ kNm}$$

The reaction at the end support is:

$$R_{Ed,A} = (1,01 + 7,83) \cdot 5,0 - \frac{61,6}{10} = 38,0 \text{ kN}$$

The point of maximum bending moment is at a distance of 38,0/8,84 = 4,30 m from the end support, and the maximum design sagging moment is:

$$M_{a,Ed} = 38,0 \cdot 4,30 - (1,01 + 7,83) \cdot 4,30 \cdot \frac{4,30}{2} = 81,7 \text{ kNm}$$

Therefore, the maximum design sagging moment acting on the composite section $M_{c,Ed}$ is:

$$M_{c,Ed} = M_{Ed} - M_{a,Ed} = 378 - 81,7 = 296 \text{ kNm}$$

The mean stress in the 80 mm thick concrete slab, with b_{eff} = 2,23 m, is calculated in accordance with Figure B8.28.

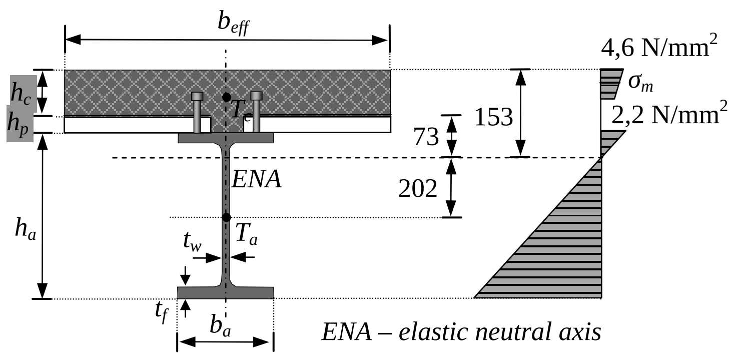


Figure B8.28 *Mean stress in the concrete slab*

In Section 4, the following properties of sections are calculated:

Second moment of area

$$I_y = 966 \cdot 10^6 \ \text{mm}^4$$

Section modulus, top of slab, in concrete units

$$W_{c,top} = 63,8 \cdot 10^6 \ \text{mm}^3$$

We need to calculate the section modulus at the bottom of the slab, 80 mm from the top of the slab. In accordance with Figure B8.28, the section modulus in concrete units is:

$$W_{c,bottom} = 966 \cdot \frac{10,1}{(225 + 50 - 202)} = 134 \cdot 10^6 \ \text{mm}^3$$

The design values of stresses at the top and bottom of the slab are:

$$\sigma_c = \frac{M_{c,Ed}}{W_{c,top}} = \frac{296 \cdot 10^6}{63,8 \cdot 10^6} = 4,6 \ \text{N/mm}^2 < f_{cd} = 16,7 \ \text{N/mm}^2$$

and

$$\sigma_c = \frac{M_{c,Ed}}{W_{c,bottom}} = \frac{296 \cdot 10^6}{134 \cdot 10^6} = 2,2 \ \text{N/mm}^2$$

The mean stress in the 80 mm thick concrete slab, with b_{eff} = 2,23 m, is:

$$\sigma_{c,sr} = \frac{4,6+2,2}{2} = 3,4 \text{ N/mm}^2$$

Remark:

Shrinkage reduces both the compressive stress in the concrete and the moments at mid-span. The effects of shrinkage are neglected due to simplification of the procedure, and the results are conservative.

Using the elastic section properties for the uncracked section at mid-span with n = 10,1 from Table B8.1, the calculated stresses are rather below the limiting stresses given in clause 6.2.1.5(2), EN 1994-1-1. The mean stress in the 80 mm thick concrete slab, with b_{eff} = 2,23 m, is 3,4 N/mm^2, and the longitudinal force in the slab is:

$$N_c = \sigma_{c,sr} \cdot b_{eff} \cdot h_c$$

$$N_c = 3,4 \cdot 2,23 \cdot 80 = 607 \text{ kN}$$

The force for full shear connection is:

$$N_{c,f} = 0,85 \cdot f_{cd} \cdot b_{eff} \cdot h_c = 14,2 \cdot 2,23 \cdot 80 = 2533 \text{ kN}$$

Therefore, the degree of shear connection needed is:

$$\eta = \frac{N_c}{N_{c,f}} = \frac{607}{2533} = 0,24$$

The diagram of the elastic shear flow is triangular. The design shear flow at the end support is:

$$v_{L,Ed} = 2 \cdot \frac{607}{4,45} = 273 \text{ kN/m}$$

The same headed stud shear connectors are used, as before. However, these studs are not “ductile” at this low degree of shear connection. The design resistances of studs, (Section 5.4.4.1) are:

$$P_{Rd,1} = 0,85 \cdot 60,3 = 51,3 \text{ kN (one stud per trough)}$$

$$P_{Rd,2} = 0,7 \cdot 60,3 = 42,2 \text{ kN (two studs per trough)}$$

For the used profiled sheeting, the width of trough is 0,2 m, Figure B8.25. There are five troughs per metre. One stud per trough ($v_{L,Rd}$ = 51,3/0,2 = 257 kN/m) is not sufficient near the end support. Two studs per trough provide 422 kN/m ($v_{L,Rd}$ = 2·42,2/0,2 = 422 kN/m).

One stud every other trough can be provided near mid-span ($v_{L,Rd}$ = 257/2 = 128 kN/m). Their spacing is 400 mm and it is less than the limiting values given in clause 6.6.5.3(3), EN 1994-1-1.

The arrangement of studs is shown in Figure B8.29 in comparison with the arrangement of studs obtained in Section 5.4.4. Relatively large differences are the result of different degrees of shear connection, η = 0,51 and η = 0,24, and the design sagging moment in this case is unusually low, in comparison to the plastic resistance moment in sagging region.

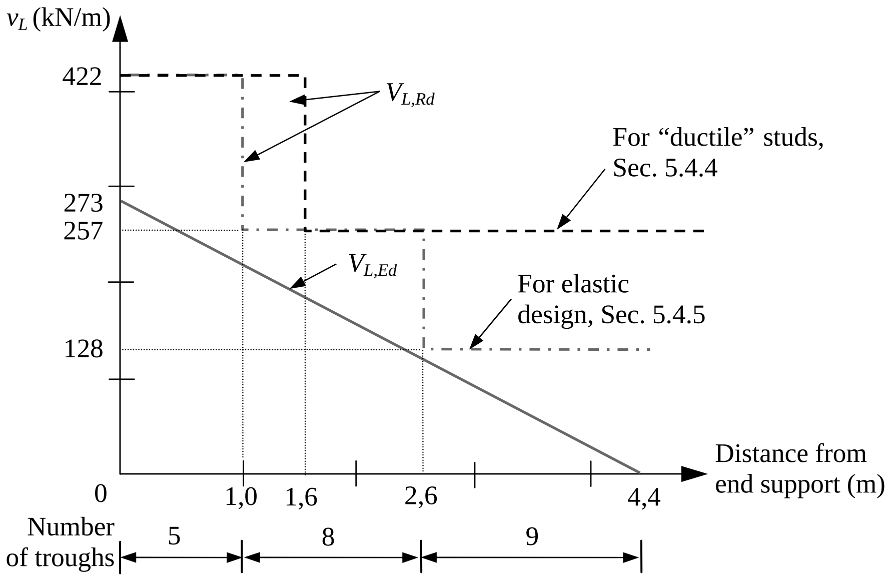


Figure B8.29 *Longitudinal shear flow and shear resistance for length 4,4 m (distance between the end support and the point of maximum sagging moment)*

5.4.6 Lateral-torsional buckling of the composite beam

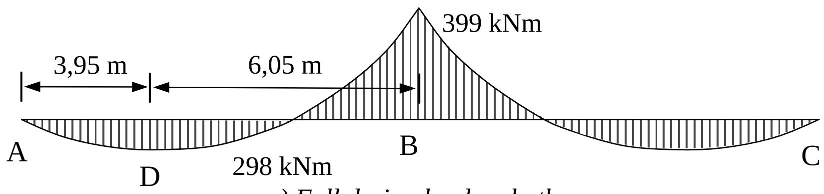
5.4.6.1 Introductory consideration

In a continuous composite beam, the top flange of the steel beam is laterally restrained by the concrete slab, while the bottom flange is restrained at the supports but not between the supports. In this case, beam buckling near the internal support may be expressed in terms of the distortion of the cross-section which results in the web bending. This failure is known as distortional buckling and the provisions in EN 1994-1-1 are related to distortional buckling.

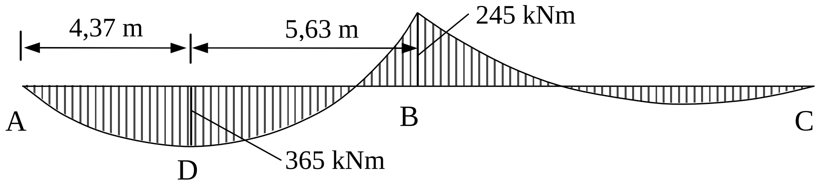
According to clause 6.4.1(3), EN 1994-1-1, it is permitted to use the provisions given in EN 1993-1-1 for steel beams. The simplified verification according to clause 6.4.3, EN 1994-1-1, is not applicable because the loading does not satisfy the condition given in clause 6.4.3(1), paragraph (b). In this example, the method according to clause 6.4.2, EN 1994-1-1, is used. The calculation of the elastic critical buckling moment at the internal support, M_{cr}, is required in this method. However, in EN 1994-1-1, there are neither recommendations nor information about the calculation of the elastic critical buckling moment. Expressions for M_{cr} can be found in [4], [34] or ENV 1994-1-1.

The most critical case is when both spans are loaded with full design load. This case is considered with n = 20,2.

The bending moments, given in Table B8.2, are shown in Figure B8.30. The effects of shrinkage are considered separately.



a) Full design load on both spans



b) Design permanent load on both spans and design variable load only on one span

Figure B8.30 *Distributions of bending moments for ultimate limit state, excluding shrinkage*

The bending moment for the simply supported case M_0 and for the full design load e_d is:

$$e_d = g_d + q_d = 11{,}9 + 26{,}3 = 38{,}2 \text{ kN/m}$$

$$M_0 = \frac{e_d \cdot L^2}{8} = \frac{38{,}2 \cdot 10^2}{8} = 478 \text{ kNm}$$

According to Figure 7.82, from [4], for C_4 is:

$$\psi = \frac{M_B}{M_0} = \frac{399}{478} = 0{,}83 \quad \text{and} \quad C_4 = 28{,}3$$

The elastic critical buckling moment at an internal support of the continuous beam is:

$$M_{cr} = \frac{k_c \cdot C_4}{L} \cdot [(G_a \cdot I_{at} + \frac{k_s \cdot L^2}{\pi^2}) \cdot E_a \cdot I_{afz}]^{1/2}$$

where:

L is the length of the beam between points at which the bottom flange of the steel member is laterally restrained (typically, the span length),

G_a is the shear modulus for steel,

$$G_a = \frac{E_a}{2 \cdot (1 + v_a)} = \frac{210}{2 \cdot (1 + 0{,}3)} = 80{,}8 \text{ kN/mm}^2$$

$I_{t,a}$ is the torsional moment of area of the steel section,

k_s is the rotational stiffness defined in clause 6.4.2(6), EN 1994-1-1,

I_{afz} is the minor-axis second moment of area of the steel bottom flange,

k_c is a property of the composite section.

For doubly symmetrical cross-section, the factor k_c is given by:

$$k_c = \frac{(h_s \cdot I_y / I_{ay})}{[(h_s^2 / 4 + i_x^2) / e + h_s]} \text{ with } e = \frac{A \cdot I_{ay}}{A_a \cdot z_c \cdot (A - A_a)}$$

where:

h_s is the distance between the centroids of the flanges of the steel section,

I_y is the second moment of area for major-axis bending of the cracked composite section of area A,

I_{ay} is the corresponding second moment of area of the steel section,

z_c is the distance between the centroid of the steel beam and the mid-depth of the slab,

$i_x^2 = (I_{ay} + I_{az}) / A_a$ where I_{az} and A_a are properties of the steel section.

For calculation of the rotational stiffness, k_s, the lesser of the ‘cracked’ flexural stiffnesses of the composite slab at a support and at mid-span, $(EI)_2$ is governed. The value of flexural stiffness at the internal support is less and is governed for calculation of the rotational stiffness. In accordance with Figure B8.31, the value of the ‘cracked’ flexural stiffnesses of the composite slab is calculated neglecting the profiled sheeting.

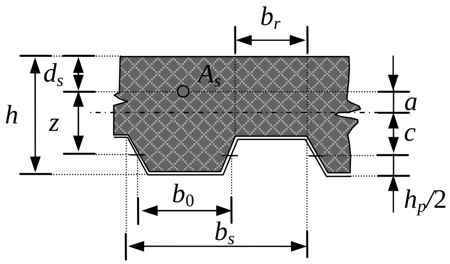


Figure B8.31 *Cross-section of the composite slab*

In Figure B8.31 the cross-section of composite slab is shown with denotations:

b_0 is the mean width of the troughs,
b_s is the spacing of the troughs,
h_p is the depth of the profiled sheeting,
A_s is the area of top reinforcement per unit width of slab,
z is the lever arm.

It is assumed that only the concrete in the trough is in compression. Its equivalent transformed area, A_e, in steel units, is:

$$A_e = \frac{b_0 \cdot h_p}{n \cdot b_s}$$

The position of the elastic neutral axis is defined by dimensions a and c, so:

$$A_e \cdot c = A_s \cdot a$$

$$a + c = z$$

The value of z is determined by the expression:

$$z = h - d_s - \frac{h_p}{2}$$

Assuming that each trough is rectangular, the second moment of area per unit width is:

$$I = A_s \cdot a^2 + A_e \cdot (c^2 + \frac{h_p^2}{12})$$

Using previously given expressions and after rearrangement, the following expression is obtained for the flexural stiffness:

$$(EI)_2 = E_a[\frac{A_s \cdot A_e \cdot z^2}{A_s + A_e} + \frac{A_e \cdot h_p^2}{12}]$$

5.4.6.2 Calculation of flexural stiffness $(EI)_2$ of composite slab and k_s

Assuming that the buckling is caused by short-term loading, the modular ratio is n = 10,1. In accordance with denotations from Figure A8.31 and for given dimensions of profiled sheeting we have:

A_s = 565 mm^2/m (the assumption: 12 mm bars at 200 mm, placed bellow the reinforcement)

z_{st} = 42 mm

$h_p = 50$ mm

Therefore A_e is:

$$A_e = \frac{b_0 \cdot h_p}{b_s \cdot n} = \frac{100}{200} \cdot \frac{50}{10{,}1} = 2{,}475 \text{ mm}^2 \rightarrow A_e = 2475 \text{ mm}^2\text{/m}$$

The value of z is:

$$z = h - z_{st} - \frac{h_p}{2}$$

$$z = 130 - 42 - \frac{50}{2} = 63 \text{ mm}$$

The flexural stiffness of the composite slab $(EI)_2$ is:

$$(EI)_2 = E_a[\frac{A_s \cdot A_e \cdot z^2}{A_s + A_e} + \frac{A_e \cdot h_p^2}{12}]$$

$$(EI)_2 = 210 \cdot 10^3 \cdot [\frac{565 \cdot 2475 \cdot 63^2}{565 + 2475} + \frac{2475 \cdot 50^2}{12}] = 4{,}92 \cdot 10^{11} \text{ Nmm}^2\text{/m}$$

$$(EI)_2 = 492 \text{ kNm}^2\text{/m}$$

According to clause 6.4.2(6), EN 1994-1-1, the rotational stiffness k_s is determined by:

$$k_s = \frac{k_1 \cdot k_2}{k_1 + k_2}$$

The flexural stiffness of the cracked concrete or the composite slab in the direction transverse to the steel beam k_1 is calculated as:

$$k_1 = \frac{\alpha (EI)_2}{a}$$

For unit width of slab continuous across the four or more steel beams at spacing a = 2,5 m and α = 4, the value of k_1 is:

$$k_1 = \frac{4(EI)_2}{a} = \frac{4 \cdot 492}{2{,}5} = 787 \text{ kNm/rad}$$

The flexural stiffness of the steel web k_2, with h_s = 435 mm for IPE 450, is:

$$k_2 = \frac{E_a \cdot t_w^3}{[4 \cdot h_s \cdot (1 - \nu_a^2)]} = \frac{210 \cdot 9{,}4^3}{4 \cdot 435 \cdot (1 - 0{,}3^2)} = 110 \text{ kN/rad}$$

The rotational stiffness k_s is:

$$k_s = \frac{k_1 \cdot k_2}{(k_1 + k_2)} = \frac{787 \cdot 110}{(787 + 110)} = 96{,}5 \text{ kN/rad}$$

5.4.6.3 Calculation of k_c

For doubly symmetrical cross-section, the factor k_c is:

$$k_c = \frac{(h_s \cdot I_y \;/\; I_{ay})}{[(h_s^2 \;/\; 4 + i_x^2)\,/\,e + h_s]}$$

with:

$$e = \frac{A \cdot I_{ay}}{A_a \cdot z_c \cdot (A - A_a)}$$

In the expressions for k_c and e, the symbols are as given earlier, except that A is the area of the cracked composite section and z_c is the distance between the centroid of the steel beam and mid-depth of the slab.

The area of the cracked composite section is:

$$A = A_a + A_s = 9880 + 1221 = 11101 \text{ mm}^2$$

The distance between the centroid of the steel beam and mid-depth of the slab is:

$$z_c = 225 + 130\,/\,2 = 290 \text{ mm}$$

In the definition for z_c, 'slab' means the 130 mm deep composite slab, not the 80 mm depth of concrete that contributes to the composite section because the stiffness of the composite slab prevents rotation of the steel top flange.

The value of e is:

$$e = \frac{A \cdot I_{ay}}{A_a \cdot z_c \cdot (A - A_a)} = \frac{11101 \cdot 337,4 \cdot 10^6}{9880 \cdot 290 \cdot 1221} = 1070 \text{ mm}$$

The value of i_x is:

$$i_x = \sqrt{\frac{I_{ay} + I_{az}}{A_a}} = \sqrt{\frac{33740 + 1676}{98,82}} = 18,9 \text{ cm} = 189 \text{ mm}$$

The value of k_c with $I_y = 452 \cdot 10^6$ mm^4, second moment of area of section, cracked and reinforced, at the internal support is:

$$k_c = \frac{(h_s \cdot I_y \;/\; I_{ay})}{[(h_s^2 \;/\; 4 + i_x^2)\,/\,e + h_s]} = \frac{(435 \cdot 452\,/\,337,4)}{[(435^2 \;/\; 4 + 189^2)\,/\,1070 + 435]} = 1,14$$

5.4.6.4 Calculation of M_{cr} and $M_{b,Rd}$

The torsional moment of area of the steel section is:

$$I_{t,a} = 0,669 \cdot 10^6 \text{ mm}^4$$

The minor-axis second moment of area of the steel bottom flange is:

$$I_{afz} = 190^3 \cdot \frac{14,6}{12} = 8,345 \cdot 10^6 \text{ mm}^4$$

The elastic critical buckling moment M_{cr} is:

$$M_{cr} = \frac{k_c \cdot C_4}{L} \cdot [(G_a \cdot I_{at} + \frac{k_s \cdot L^2}{\pi^2}) \cdot E_a \cdot I_{afz}]^{1/2}$$

$$M_{cr} = \frac{1,14 \cdot 28,3}{10,00} \cdot [(80,8 \cdot 0,669 + \frac{96,5 \cdot 10,00^2}{\pi^2}) \cdot 210 \cdot 8,345]^{1/2} = 4338 \text{ kNm}$$

According to clause 6.4.2(4), EN 1994-1-1, the relative slenderness is calculated as:

$$\bar{\lambda}_{LT} = \sqrt{\frac{M_{Rk}}{M_{cr}}}$$

where M_{Rk} is the characteristic resistance moment calculated in Section 5.4.3.

The relative slenderness is:

$$\bar{\lambda}_{LT} = \sqrt{\frac{M_{Rk}}{M_{cr}}} = \sqrt{\frac{775}{4338}} = 0,423$$

According to clause 6.4.2(1), EN 1994-1-1, the value of the reduction factor χ_{LT} may be obtained from EN 1993-1-1, clauses 6.3.2.2 and 6.3.2.3. If the alternative method for rolled section is used, clause 6.3.2.3, EN 1993-1-1, the reduction factor χ_{LT} is determined by the following expression:

$$\chi_{LT} = \frac{1}{[\Phi_{LT} + \sqrt{(\Phi_{LT}^2 - \beta \cdot \bar{\lambda}_{LT}^2)}]}$$

with:

$$\Phi_{LT} = 0{,}5 \cdot [1 + \alpha_{LT} \cdot (\bar{\lambda}_{LT} - \bar{\lambda}_{LT,0}) + \beta \cdot \bar{\lambda}_{LT}^2]$$

where:

α_{LT} is the imperfection factor that depends on the appropriate buckling curve,
$\bar{\lambda}_{LT}^2$ and β are parameters to be defined in the National Annexes.

The recommended values are:

$$\bar{\lambda}_{LT}^2 \leq 0{,}4 \text{ (minimum value)}$$

$$\beta \geq 0{,}75 \text{ (maximum value)}$$

The buckling curves to be adopted depend on the geometry of the cross section of the member and the buckling curve *c* is specified for IPE 450 with $\alpha_{LT} = 0{,}49$.

The calculation of the reduction factor χ_{LT} is:

$$\Phi_{LT} = 0{,}5 \cdot [1 + \alpha_{LT} \cdot (\bar{\lambda}_{LT} - \bar{\lambda}_{LT,0}) + \beta \cdot \bar{\lambda}_{LT}^2]$$

$$\Phi_{LT} = 0{,}5 \cdot [1 + 0{,}49 \cdot (0{,}423 - 0{,}4) + 0{,}75 \cdot 0{,}423^2] = 0{,}573$$

$$\chi_{LT} = \frac{1}{[\Phi_{LT} + \sqrt{(\Phi_{LT}^2 - \beta \cdot \bar{\lambda}_{LT}^2)}]}$$

$$\chi_{LT} = \frac{1}{0{,}573 + \sqrt{0{,}573^2 - 0{,}75 \cdot 0{,}423^2}} = 0{,}987$$

The design buckling resistance moment is:

$$M_{b,Rd} = \chi_{LT} \cdot M_{pl,Rd} = 0{,}987 \cdot 755 = 745 \text{ kNm}$$

The obtained value of $M_{b,Rd}$ is well above the value of the design bending moment at the internal support, including the effects of shrinkage design, M_{Ed} = (399 + 121) = 520 kNm. The result is quite sensitive to the recommended values for $\bar{\lambda}_{LT,0}$ and β.

According to clause 6.3.2.2, EN 1993-1-1, the lateral-torsional effects can be neglected if at least one of the following conditions is satisfied:

$$\bar{\lambda}_{LT} \leq \bar{\lambda}_{LT,0} \text{ or } M_{Ed} / M_{cr} \leq \bar{\lambda}^2_{LT,0}$$

In this case, M_{Ed} = 520 kNm and M_{cr} = 4338 kNm, gives:

$$\frac{M_{Ed}}{M_{cr}} = \frac{520}{4338} = 0{,}12 < \bar{\lambda}^2_{LT,0} = 0{,}4^2 = 0{,}16$$

The provision of bracing to the bottom flange is considered in the next section.

5.4.6.5 Calculation of M_{cr} and $M_{b,Rd}$ for laterally restrained bottom flange

The factor C_4 which takes into account the distribution of bending moment was used in the expression for M_{cr}. However, the values of this factor given in [4] are not suitable for design based on M_{cr} where the steel bottom flange is laterally restrained.

From Figure B8.30a, with full design load on both spans, the distance of a point of contraflexure from the internal support is:

$$10{,}00 - 2 \cdot 3{,}95 = 2{,}10 \text{ m}$$

At this point is provided the lateral bracing, and the length of the beam between the points at which the bottom flange of the steel member is laterally restrained is 2,10 m. Assuming that the distribution of bending moment over this 2,10 m length is linear, then from Figure 7.83, given in [4], the value of C_4 is 11,1. Substituting these values into the expression for M_{cr}, the following value of the elastic critical buckling moment is obtained:

$$M_{cr} = \frac{k_c \cdot C_4}{L} \cdot [(G_a \cdot I_{at} + \frac{k_s \cdot L^2}{\pi^2}) \cdot E_a \cdot I_{afz}]^{1/2}$$

$$M_{cr} = \frac{1{,}14 \cdot 11{,}1}{2{,}1}[(80{,}8 \cdot 0{,}669 + \frac{96{,}5 \cdot 2{,}1^2}{\pi^2}) \cdot 210 \cdot 8{,}345]^{1/2} = 2487 \text{ kNm}$$

The obtained value, M_{cr} = 2487 kNm, is much lower than the previous value M_{cr} = 4338 kNm, see Section 5.4.8.4., for the length of the beam between points at which the bottom flange of the steel member is laterally restrained L = 10,0 m. The value of M_{cr} is over-conservative for short lengths of beam between lateral bracing.

For this and similar cases, it is recommended to find M_{cr} from elastic critical analysis by means of computer.

5.4.7 Lateral-torsional buckling of the composite – simplified verification

Clause 6.4.3, EN 1994-1-1, gives the method of simplified verification without direct calculation. The method illustrated in Section 5.4.6 is very comprehensive. Therefore, the simplified verification can be used when the conditions given in clause 6.4.3, EN 1994-1-1, are satisfied. The simplified verification was used in examples B6 and B7. Limitations of the simplified method will be illustrated in this section with reference to the continuous beam considered in this example.

For further consideration the following results for hogging bending of the composite section at the internal support are relevant:

$$M_{pl,Rd} = 755 \text{ kNm}$$

$$\bar{\lambda}_{LT} = 0,423$$

$$\chi_{LT} = 0,987$$

$$M_{b,Rd} = \chi_{LT} \cdot M_{pl,Rd} = 0,987 \cdot 755 = 745 \text{ kNm}$$

The design loads for ultimate limit state per unit length of beam are:

permanent $g_d = 11,9$ kN/m

variable $q_d = 26,3$ kN/m

The steel section does not satisfy the condition in clause 6.4.3(1), g), EN 1994-1-1, because its depth, h = 450 mm, is higher than the limiting depth of 400 mm. The beam satisfies all other conditions except the one mentioned in clause 6.4.3(1), (b), EN 1994-1-1: the ratio of permanent to total loads is 11,9/38,2 = 0,31, far below the required minimum of 0,4.

For IPE 450 with concrete-encased web the limit of depth is 600 mm and this would be the simplest way to satisfy the condition in clause 6.4.3(1), g), EN 1994-1-1. In this case, the permanent load is increased to 15,6 kN/m due to additional concreting.

The condition in clause 6.4.3(1), (b), EN 1994-1-1, that the design permanent load exceeds 40% of the total design load, is quite severe. In this case, it is:

$15{,}6 \geq 0{,}4 \cdot (15{,}6 + q_d)$

Therefore, the value of q_d is:

$q_d \leq 23{,}4$ kN/m

This value corresponds to the characteristic imposed load on the floor of:

$23{,}4/(1{,}5 \cdot 2{,}5) = 6{,}2$ kN/m^2

The obtained value is significantly lower than the value of imposed floor load $q_k = 7$ kN/m^2.

The slenderness $\bar{\lambda}_{LT}$ is a function of the variation of bending moment along the span. The limitations in clause 6.4.3(1) (a) and (b) are based on the results obtained by varying loadings on continuous beams and taking into account different ratios of the length of adjacent spans. Comparing the calculated value $\bar{\lambda}_{LT} = 0{,}423$ with the limitation given in clause 6.4.3(1) (b), EN 1994-1-1, which is 0,4, the difference is less than 10%. According to this criterion, the lateral-torsional buckling is almost not governed. This result is inconsistent with the obtained value of characteristic imposed load according to the simplified method. The simplified methods are easy to apply but they have to cover such a wide a variety of situations that they are over-conservative for some of them.

5.4.8 Check of the longitudinal shear resistance of the concrete flange

5.4.8.1 Check of the transverse reinforcement

The profiled steel sheeting with ribs transverse to the beam, continuous across the top flange of the steel beam and with mechanical interlocking contributes to the transverse reinforcement. According to clause 6.6.6.4, EN 1994-1-1, its contribution to the transverse reinforcement for the shear surface can be calculated from:

$$\left(\frac{A_{sf}}{s_f} \cdot f_{sd}\right) + A_{pe} \cdot f_{yp,d} \geq v_{L,Ed} \cdot \frac{h_f}{\cot\theta}$$

where:

A_{sf}/s_f is the transverse reinforcement expressed in mm^2/m,
h_f is the depth of concrete above the profiled sheeting,
θ is the angle between the diagonal strut and the axis of the beam (strut-and-tie model),

$v_{L,Ed}$ is the design longitudinal shear flow in the concrete slab,
A_{pe} is the effective cross-sectional area of the profiled steel sheeting per unit length of the beam; for sheeting with holes, the net area should be used,
$f_{yp,d}$ is the design yield strength of the profiled steel sheeting.

If the profiled steel sheeting with ribs transverse to the beam is discontinuous across the top flange of the steel beam, and stud shear connectors are welded to the steel beam directly through the profiled steel sheets, the term $A_{pe} \cdot f_{yp,d}$ in above expression should be replaced by:

$$P_{pb,Rd} / s \quad \text{but} \quad \leq A_{pe} \cdot f_{yp,d}$$

where:

$P_{pb,Rd}$ is the design bearing resistance of a headed stud welded through the sheet according to clause 9.7.4, EN 1994-1-1,
s is the longitudinal spacing centre-to-centre of the studs effective in anchoring the sheeting.

According to clause 9.7.4, EN 1994-1-1, the design bearing resistance of a headed stud welded through the sheet is:

$$P_{pb,Rd} = k_{\varphi} \cdot d_{d0} \cdot t \cdot f_{yp,d}$$

with:

$$k_{\varphi} = 1 + a / d_{d0} \leq 6{,}0$$

where:

d_{d0} is the diameter of the weld collar which may be taken as 1,1 times the diameter of the shank of the stud,
a is the distance from the centre of the stud to the end of the sheeting, to be not less than 1,5 d_{d0},
t is the thickness of the profiled sheeting.

According to clause 6.6.6.4(2), EN 1994-1-1, shear surfaces that pass closely around a stud need not be considered. The detail of a stud welded through discontinuous profiled sheeting and the critical shear plane, labelled *a-a*, are shown in Figure B8.32.

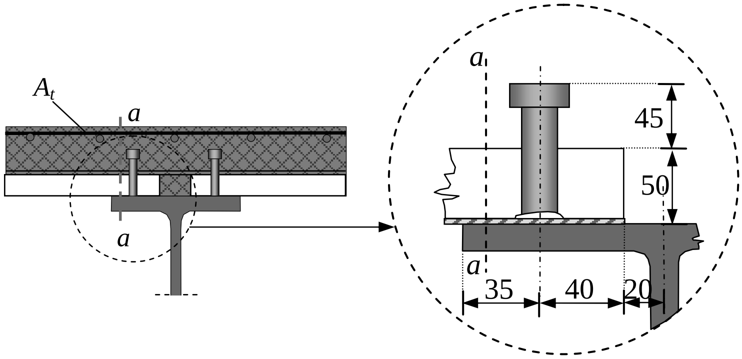


Figure B8.32 *Cross-section of composite beam and detail of a stud welded through discontinuous profiled sheeting (in mm)*

When the concrete flange is in compression, longitudinal shear flow $v_{L,Ed}$ can be defined by the expression:

$$v_{L,Ed,1} = \frac{\Delta N_{c1}}{a_v} = \frac{V_{L,Ed}}{a_v}\frac{A_{c1,eff}}{A_{c,eff}}$$

where:

a_v is the critical length (the distance between two given sections) Figure B7.12,

ΔN_{c1} is the change of the longitudinal compressive forces in the slab over the critical length a_v, Figure B8.33,

$V_{L,Ed}$ is the design longitudinal shear force in the steel-concrete interface or in the concrete flange,

$$V_{L,Ed} = \min(N_{pl,a}, N_c, \Sigma P_{Rd})$$

When the concrete flange is in tension, longitudinal shear flow $v_{L,Ed}$ can be defined by the expression:

$$v_{L,Ed,1} = \frac{\Delta N_{s1}}{a_v} = \frac{V_{L,Ed}}{a_v}\frac{A_{s1}}{A_{s1} + A_{s2}}$$

where:

a_v is the critical length (the distance between two given sections) Figure B7.12,

ΔN_{s1} is the change of the longitudinal tensile forces in the slab over the critical

	length a_v, Figure B8.33,
$V_{L,Ed}$	is the design longitudinal shear force in the steel-concrete interface or in the concrete flange,
	$V_{L,Ed} = \min(N_s, \Sigma P_{Rd})$

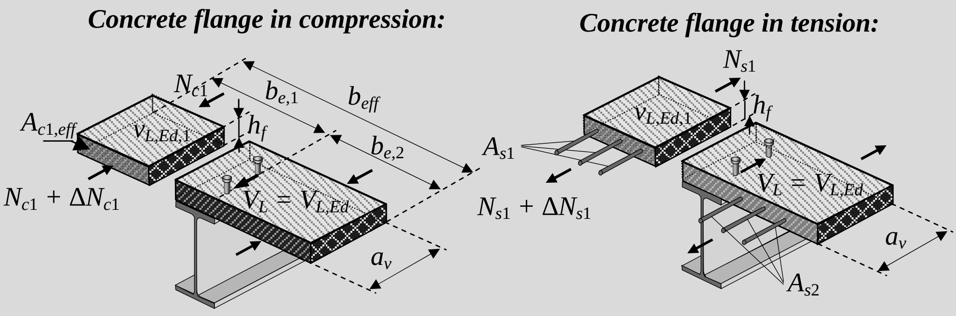


Figure B8.33 *Determination of the longitudinal shear forces in the conrete flange*

In clause 6.6.6.1(4), EN 1994-1-1, states that the design longitudinal shear for the concrete slab should be "*consistent with the design and spacing of the shear connectors*". This means that the resistance of the shear connection is determined by the longitudinal shear, rather than the design load.

The maximum of the design longitudinal shear is where there are two studs per trough, and is:

$$a_v = 200 \text{ mm}$$

$$V_{L,Ed} = 2 \cdot 42,2 = 84,4 \text{ kN}$$

$$h_f = h_c = 80 \text{ mm}$$

$$v_{L,Ed} = \frac{\Delta N_{c1}}{h_f \cdot a_v} = \frac{V_{L,Ed}}{2 h_f a_v} = \frac{84,4 \cdot 10^3}{2 \cdot 80 \cdot 200} = 2,64 \text{ N/mm}^2$$

For the design resistance of profiled sheeting (the bearing resistance of the sheeting) it refers to clause 9.7.4, EN 1994-1-1. The distance from the centre of the stud to the end of the sheeting a is 40 mm, Figure B8.32.

The diameter of the weld collar, d_{d0}, is taken as 1,1 times the diameter of the shank of the stud and is:

$d_{d0} = 1{,}1 \cdot 19 = 20{,}9$ mm

According to clause 9.7.4(3), EN 1994-1-1, the value of k_φ is:

$$k_\varphi = 1 + \frac{a}{d_{d0}} \le 6{,}0$$

$$k_\varphi = 1 + \frac{40}{20{,}9} = 2{,}91$$

Since the composite slab has not yet been designed, the assumed thickness of the profiled sheeting is at least 0,9 mm, although in Figure B8.2 the sheet is 1,0 mm thick. The yield strength of the profiled sheeting is 350 N/mm^2 and γ_M = 1,0. The cross-sectional area is A_p = 1178 mm^2/m. The design bearing resistance of a headed stud welded through the sheet according to clause 9.7.4, EN 1994-1-1, is:

$$P_{pb,Rd} = k_\varphi \cdot d_{d0} \cdot t \cdot f_{yp,d} = 2{,}91 \cdot 20{,}9 \cdot 0{,}9 \cdot 0{,}350 = 19{,}2 \text{ kN/shear stud}$$

According to clause 6.6.6.4(5), EN 1994-1-1, with a stud spacing of 200 mm, the shear resistance provided by the profiled sheeting is:

$$v_{L,pd,Rd} = \frac{P_{pb,Rd}}{s} = \frac{19{,}2}{0{,}2} = 96 \text{ kN/m}$$

The shear resistance must not exceed the value of $A_{pe} \cdot f_{yp,d}$, which is:

$A_p \cdot f_{yp,d}$, = 1178·0,350 = 412 kN/m

The condition given in clause 6.6.6.4(5), EN 1994-1-1, is satisfied.

Remark:

In order to prevent splitting of the concrete flange, for the adopted “truss model”, according to clause 6.2.4(4) EN 1992-1-1, the angle θ between the concrete diagonals and the longitudinal direction is limited to the values:

26,5º ≤ θ ≤ 45º concrete flange in compression

38,6º ≤ θ ≤ 45º concrete flange in tension

In order to minimize the cross-sectional area of the transverse reinforcement, the minimum angle θ is selected. For the concrete flange with the assumptions that it is in tension (regions at the internal support), the minimum angle θ is:

$$\theta = 38{,}6^{\circ}$$

$$\frac{A_{sf}}{s_f} \cdot f_{sd} + P_{pb,Rd} / s \geq v_{Ed} \cdot \frac{h_f}{\cot\theta}$$

$$\frac{A_{sf}}{s_f} \geq \frac{v_{Ed}}{f_{sd}} \cdot \frac{h_f}{\cot\theta} - \frac{P_{pb,Rd}}{s \cdot f_{sd}} = \frac{2{,}64}{435} \cdot \frac{80}{\cot 38{,}6} \cdot 10^3 - \frac{96}{435} \cdot 10^3 = 167 \ \text{mm}^2/\text{m}$$

The reinforcement is less than the selected reinforcement in the design for lateral-torsional buckling of 565 mm^2/m.

The assessment of the area of the transverse reinforcement may also be affected by the need for crack control above the beam, which arises in the design of the composite slab.

According to EN 1994-1-1, clause 6.6.6.3, the minimum area of transverse reinforcement is determined in accordance with EN 1992-1-1, clause 9.2.2(5), which gives the minimum area of reinforcement as a proportion of the concrete area. The ratio is:

$$\rho_{w,\min} = \frac{0{,}08\sqrt{f_{ck}}}{f_{yr,k}}$$

where:

f_{ck} is the characteristic compressive cylinder strength of the concrete at 28 days in N/mm^2,

$f_{yr,k} = f_{sk}$ is the characteristic yield strength of the reinforcement in N/mm^2.

The minimum area of transverse reinforcement is:

$$\rho_{w,\min} = \frac{0{,}08\sqrt{f_{ck}}}{f_{yr,k}} = \frac{0{,}08\sqrt{25}}{500} = 0{,}0008$$

$$A_c = h_c \cdot b = 80 \cdot 1000 = 80000 \ \text{mm}^2$$

$$A_{s,\min} = \rho_{w,\min} \cdot A_c = 0{,}0008 \cdot 80000 = 64 \ \text{mm}^2/\text{m}$$

Since $A_t = 565$ mm^2/m $> A_{s,\min} = 64$ mm^2/m, the requirement of minimum transverse reinforcement is satisfied.

5.4.8.2 Crushing of the concrete flange

To prevent crushing of the compression struts in the flange, the following condition should be satisfied according to EN 1992-1-1, expression 6.22:

$$v_{L,Ed} \leq v_{Rd}$$

$$v_{L,Ed} \leq v \cdot f_{cd} \cdot \sin\theta \cdot \cos\theta$$

where:

$$v = 0{,}6 \cdot (1 - \frac{f_{ck}}{250})$$

θ is the angle between the concrete diagonals and the longitudinal direction.

In order to minimize the resistance of the concrete compression strut, the minimum angle θ is selected. For the concrete flange in compression (at mid-span), the minimum angle θ is:

$$\theta = 26{,}5^{\circ}$$

$$v_{Rd} = v \cdot f_{cd} \cdot \sin\theta \cdot \cos\theta = 0{,}6 \cdot (1 - \frac{25}{250}) \cdot 16{,}7 \cdot \sin 26{,}5^{\circ} \cdot \cos 26{,}5^{\circ} = 3{,}60 \ \text{N/mm}^2$$

Check:

$$v_{L,Ed} < v_{Rd}$$

$$v_{L,Ed} = 2{,}64 \ \text{N/mm}^2 < v_{Rd} = 3{,}60 \ \text{N/mm}^2$$

Therefore the crushing resistance of the concrete compression strut is adequate.

6. Serviceability limit sate

6.1 General

If the steel beam at the construction stage is unpropped, the calculation of the deflection is performed separately for the construction stage and for the composite stage.

The total deflection for the construction stage is obtained from:

$\delta = \delta_0$ (verification for the construction stage)

where:

δ_0 is the deflection of the steel beam at the execution stage

The total deflection for the composite stage is obtained from:

$\delta = \delta_{1,1} + \delta_{1,2} + \delta_{2,1} + \delta_{2,2} + \delta_{2,3}$ (verification for the composite stage)

where:

$\delta_{1,1}$ is the deflection due to the permanent action immediately after casting the concrete (no shear connection),
$\delta_{1,2}$ is the deflection due to loads of the floor finishes and partitions on the composite beam (the first loading),
$\delta_{2,1}$ is the deflection due to the frequent value of the variable action at the time of first loading,
$\delta_{2,2}$ is the deflection due to creep under the quasi-permanent value of the variable action at time $t = \infty$,
$\delta_{2,3}$ is the deflection due to shrinkage.

Secondary effects due to shrinkage in this beam are significant. The important secondary effect in this beam, the hogging bending moment at the internal support, is $M_{Ed,sh,B}$ = 121 kNm. According to clause 7.3.1(8), EN 1994-1-1, the shrinkage effects should not be neglected at the serviceability verification of this beam because clause 7.3.1(8) does not refer to lightweight aggregate concrete, which is used in this case.

6.2 Stress limits

Serviceability stress verifications are not prescribed by EN 1994-1-1 and according to clause 7.2.2(1), EN 1994-1-1, there are no limitations on stresses. However, it is possible that serviceability loading may cause yielding at the internal supports. For the calculation of deflection of the unpropped continuous beam, the effect of local yielding of the structural steel over the internal support should be taken into account because it causes additional deflection, clause 7.3.1(7), EN 1994-1-1.

Yielding is an irreversible limit state. From a note to clause 6.5.3(2), EN 1990, the characteristic combination should be used for irreversible limit states. However, the actions for checking deflections depend on the requirements of serviceability.

For unpropped beam at the construction stage, the stresses are calculated first for the non-composite section subjected to loading at the construction stage, and then the stresses for the composite section should be added. Static systems and loadings for the construction stage and composite stage are shown in Figures B8.34, B8.35 and B8.36. For composite stage, we consider the characteristic combination with the variable load on both spans and 15% of each span cracked. The permanent load, except floor finishes, is assumed to act on the steel beam. The modular ratio *n* is taken as 20,2 for all of the loading except shrinkage, see clause 5.4.2.2(11), EN 1994-1-1. For shrinkage, the modular ratio *n* is 28,3. The obtained results are shown in Table B8.3.

Permanent action on steel beam – self-weight

$$e_d = b \cdot g_{k,1}$$

$$e_d = 2{,}5 \cdot 2{,}62 = 6{,}55 \text{ kN/m}$$

The bending moment at the internal support is:

$$M_{Ek,B} = 0{,}125 \cdot 6{,}55 \cdot 10{,}0^2 = 81{,}9 \text{ kNm}$$

The second moment of area, steel beam is:

$$I_{y,a} = 337{,}4 \cdot 10^6 \text{ mm}^4$$

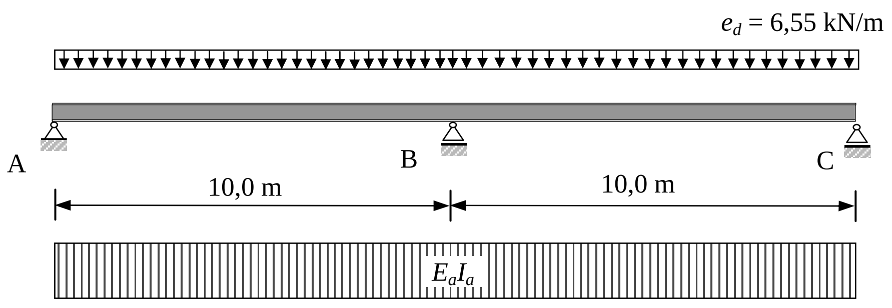


Figure B8.34 *Permanent load on the steel beam*

The section modulus of the bottom flange of the steel section is:

$$W_{a,bot} = \frac{I_{y,a}}{\frac{h_a}{2}} = \frac{337,4 \cdot 10^6}{\frac{450}{2}} = 1,50 \cdot 10^6 \text{ mm}^3$$

The stress in the bottom flange of the steel section is:

$$\sigma_{a,bot} = \frac{M_{Ek,B}}{W_{a,bot}} = \frac{81,9 \cdot 10^6}{1,50 \cdot 10^6} = 54,6 \text{ N/mm}^2$$

Permanent action on composite beam – floor finishes

$$e_d = b \cdot g_{k,3}$$

$$e_d = 2,5 \cdot 1,00 = 2,50 \text{ kN/m}$$

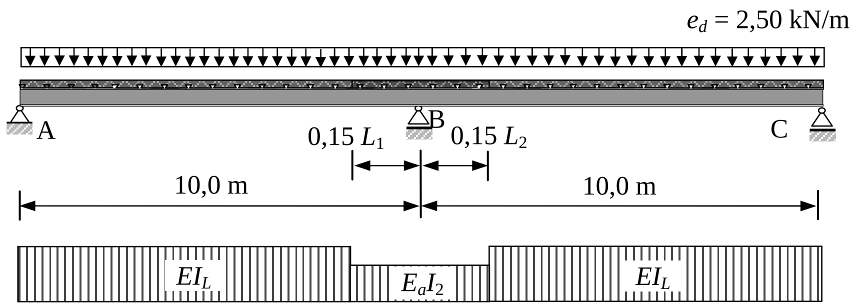


Figure B8.35 *Permanent load on the composite beam*

The second moment of area (Table B8.1):

$I_2 = I_{y,B} = 452 \cdot 10^6 \text{ mm}^4$, support, cracked, reinforced

$I_L = I_y = 799 \cdot 10^6 \text{ mm}^4$, mid-span, uncracked, n = 20,2

The bending moment at the internal support is:

$$M_{Ek,B} = \frac{e_d \cdot L^2}{4} \cdot \frac{0,110 \cdot \lambda + 0,890}{0,772 \cdot \lambda + 1,228}$$

$$\lambda = \frac{I_y}{I_{y,B}} = \frac{799 \cdot 10^6}{452 \cdot 10^6} = 1,76$$

$$M_{Ek,B} = \frac{2{,}50 \cdot 10{,}0^2}{4} \cdot \frac{0{,}110 \cdot 1{,}76 + 0{,}890}{0{,}772 \cdot 1{,}76 + 1{,}228} = 26{,}2 \text{ kNm}$$

The section at the internal support is considered, Figure B8.4, and the distance between the elastic neutral axis and the centroid of steel section is z_{na} = 36 mm.

The section modulus of the bottom flange of the steel section is:

$$W_{a,bot} = \frac{I_{y,B}}{\frac{h_a}{2} + z_{na}} = \frac{452 \cdot 10^6}{\frac{450}{2} + 36} = 1{,}73 \cdot 10^6 \text{ mm}^3$$

The stress in the bottom flange of the steel section is:

$$\sigma_{a,bot} = \frac{M_{Ek,B}}{W_{a,bot}} = \frac{26{,}2 \cdot 10^6}{1{,}73 \cdot 10^6} = 15{,}1 \text{ N/mm}^2$$

Variable action on the composite beam

$$e_d = b \cdot q_{k,2}$$

$$e_d = 2{,}50 \cdot 7{,}0 = 17{,}5 \text{ kN/m}$$

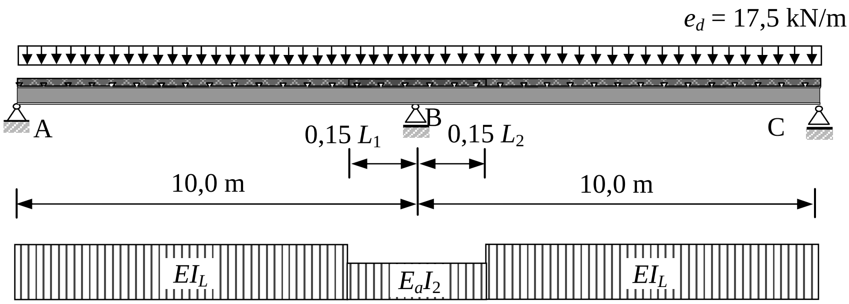


Figure B8.36 *Imposed load on composite beam*

The second moment of area (Table B8.1) is:

$I_2 = I_{y,B} = 452 \cdot 10^6$ mm^4, support, cracked, reinforced

$I_L = I_y = 799 \cdot 10^6$ mm^4, mid-span, uncracked, n = 20,2

The bending moment at internal support is:

$$M_{Ek,B} = \frac{e_d \cdot L^2}{4} \cdot \frac{0{,}110 \cdot \lambda + 0{,}890}{0{,}772 \cdot \lambda + 1{,}228}$$

$$\lambda = \frac{I_y}{I_{y,B}} = \frac{799 \cdot 10^6}{452 \cdot 10^6} = 1{,}76$$

$$M_{Ek,B} = \frac{17{,}5 \cdot 10{,}0^2}{4} \cdot \frac{0{,}110 \cdot 1{,}76 + 0{,}890}{0{,}772 \cdot 1{,}76 + 1{,}228} = 183 \text{ kNm}$$

The section at the internal support is considered, Figure B8.4, and the distance between the elastic neutral axis and the centroid of the steel section is z_{na} = 36 mm.

The section modulus of the bottom flange of the steel section is:

$$W_{a,bot} = \frac{I_{y,B}}{\frac{h_a}{2} + z_{na}} = \frac{452 \cdot 10^6}{\frac{450}{2} + 36} = 1{,}73 \cdot 10^6 \text{ mm}^3$$

The stress in the bottom flange of the steel section is:

$$\sigma_{a,bot} = \frac{M_{Ek,B}}{W_{a,bot}} = \frac{183 \cdot 10^6}{1{,}73 \cdot 10^6} = 106 \text{ N/mm}^2$$

Shrinkage

The second moment of area (Table B8.1) is:

$I_{y,B} = 452 \cdot 10^6 \text{ mm}^4$, support, cracked, reinforced

The bending moment at the internal support, according to Section 5.2, is:

$M_{Ed,sh,B} = 121 \text{ kNm}$, n = 28,3

The section at the internal support is considered, Figure B8.4, and the distance between the elastic neutral axis and the centroid of the steel section is z_{na} = 36 mm.

The section modulus of the bottom flange of the steel section is:

$$W_{a,bot} = \frac{I_{y,B}}{\frac{h_a}{2} + z_{na}} = \frac{452 \cdot 10^6}{\frac{450}{2} + 36} = 1{,}73 \cdot 10^6 \text{ mm}^3$$

The stress in the bottom flange of the steel section is:

$$\sigma_{a,bot} = \frac{M_{Ek,sh,B}}{W_{a,bot}} = \frac{121 \cdot 10^6}{1{,}73 \cdot 10^6} = 69{,}9 \text{ N/mm}^2$$

The total compressive stress in the bottom flange of the steel section is:

$$\sigma_{Ek,bot,a} = 54{,}6 + 15{,}1 + 106 + 69{,}9 = 246 \text{ N/mm}^2 \ (= 0{,}69 \cdot f_y)$$

Therefore, the effect of yielding in the steel section does not need to be taken into account for serviceability verification.

Table B8.3 *Hogging bending moment at the internal support and stresses in the bottom flange of the steel section, for the characteristic combination of actions*

Action	e_d (kN/m)	Modular ratio	$I_{y,B}$ (10^6 mm^4)	$M_{Ek,B}$ (kNm)	$W_{a,bot}$ (10^6 mm^3)	$\sigma_{a,bot}$ (N/mm^2)
1. Permanent (on steel beam)	6,55	—	337,4	81,9	1,50	54,6
2. Permanent (on composite beam)	2,2	20,2	452	26,2	1,73	15,1
3. Variable	17,5	20,2	452	183	1,73	106
4. Shrinkage	—	28,3	452	121	1,73	69,9

6.3 Calculation of deflections

6.3.1 Construction stage deflection

The deflection at the construction stage has been calculated by means of commercial software, using the flexural stiffness of the steel cross-section E_aI_a.

$$E_a = 21000 \text{ kN/cm}^2 \quad I_a = 33740 \text{ cm}^4$$

The recommended limiting values for deflection are:

$$\delta_{tot} \leq \frac{L}{250}$$

$$\delta_{\mathrm{var}} \leq \frac{L}{360}$$

The total deflection due to the permanent and variable actions, δ_{tot}, during execution is determined for the following total load e_d:

$$e_d = b \cdot (g_{k,1} + q_{k,1})$$

$$e_d = 2{,}5 \cdot (2{,}62 + 0{,}50) = 7{,}8 \text{ kN/m}$$

$$E_a I_a = 70854 \text{ kNm}^2$$

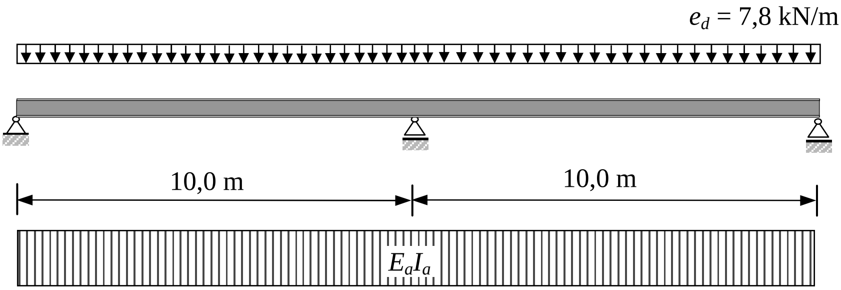


Figure B8.37 *Static system and load case 1 during execution*

The deflection (calculated using commercial software) is:

δ_{tot} = 5,9 mm < L/250 = 40,0 mm

The condition is satisfied.

The deflection due to the variable actions, δ_{var}, during execution is determined for the following variable load e_d:

$$e_d = b \cdot q_{k,1}$$

$$e_d = 2{,}50 \cdot 0{,}50 = 1{,}25 \text{ kN/m}$$

$$E_a I_a = 70854 \text{ kNm}^2$$

The critical load case for deflection is where only one span is fully loaded:

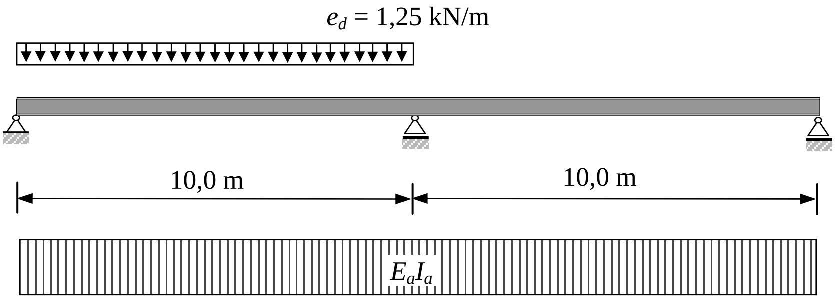


Figure B8.38 *Static system and load case 2 during execution*

The deflection (calculated using a commercial software) is:

δ_{var} = 1,6 mm < L/360 = 27,8 mm. The condition is satisfied.

Remark:

The limitation of the deflection is adopted according to the recommendation given in EN 1990. This value can be changed in accordance with the recommendation given in the National Annex. Furthermore, absolute limit of 25 mm could be recommended in order to limit the effects of ponding of wet concrete during execution.

6.3.2 Composite stage deflection

When variable load acts only on one span, the maximum deflection in the span of the beam will occur at about 4,0 m. The condition from clause 7.3.1(4), (a), EN 1994-1-1, is satisfied and the additional deflection caused by slip of the shear connection is neglected.

The deflection of the composite beam has been calculated by means of commercial software, with 15% of each span assumed to be cracked. The frequent combination of actions is used, for which the combination factor, a building with floors in category C, is ψ_1 = 0,7.

Frequent combination is used in the case of reversible limit states, such as the elastic deflection of composite beams due to variable actions. However, if the deformation causes cracking of brittle floor finish structures or damage to fragile partitions then the limit state is not reversible. In this case, the verification must be carried out for the higher, less probable, loading of the characteristic combination.

- Deflection due to the permanent action at time immediately after casting concrete

$$e_d = b \cdot g_{k,1} = 2{,}50 \cdot 2{,}62 = 6{,}55 \text{ kN/m}$$

$$E_a I_a = 70854 \text{ kNm}^2$$

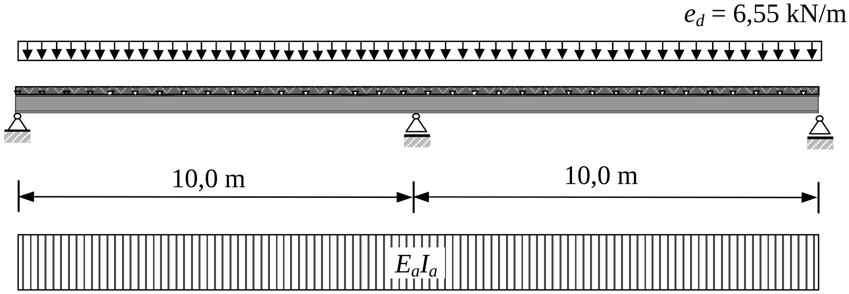


Figure B8.39 *Static system - permanent action at the time immediately after casting the concrete*

$$\delta_{1,1} = 5{,}0 \text{ mm}$$

- Deflection due to the loads of floor finishes and partitions on the composite beam at the time of first loading

$$e_d = b \cdot g_{k,3} = 2{,}5 \cdot 1{,}00 = 2{,}50 \text{ kN/m}$$

The value of the second moment of area for the cracked reinforced section at the internal support and for the uncracked section at mid-span with n = 10,1 are taken from Table B8.1:

$I_2 = 45200$ cm^4, at internal support, cracked, reinforced
$I_0 = 96600$ cm^4, at mid-span, uncracked, n = 10,1

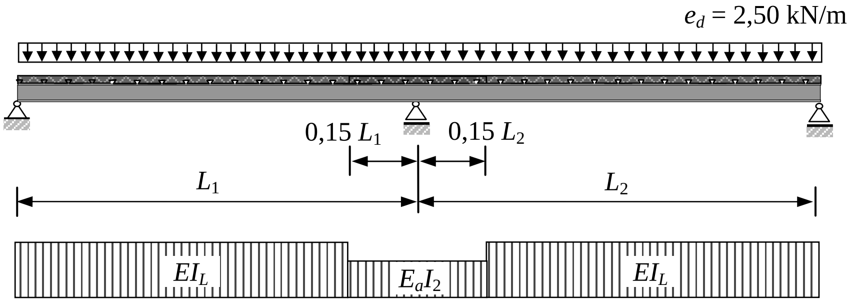


Figure B8.40 *Static system – loads of the floor finishes and partitions at the time of first loading*

Accordingly, the following flexural stiffnesses are used:

$E_a I_2 = 94920$ kNm^2, at internal support
$EI_L = EI_0 = 202860$ kNm^2, at mid-span

$\delta_{1,2} = 0{,}8$ mm

- Deflection due to the frequent value of variable action at the time of first loading

For a building with floors in category C, the combination factor ψ is.

$\psi_1 = 0{,}7$

$e_d = b \cdot \psi_1 \cdot q_{k,2} = 2{,}50 \cdot 0{,}7 \cdot 7{,}0 = 12{,}25$ kN/m

The value of second moment of area for the cracked reinforced section at the internal support and for the uncracked section at mid-span with n = 10,1 are taken from Table B8.1:

$I_2 = 45200$ cm^4, at internal support, cracked, reinforced
$I_0 = 96600$ cm^4, at mid-span, uncracked, n = 10,1

Accordingly, the following flexural stiffnesses are used:

$E_a I_2 = 94920$ kNm^2
$EI_L = EI_0 = 202860$ kNm^2

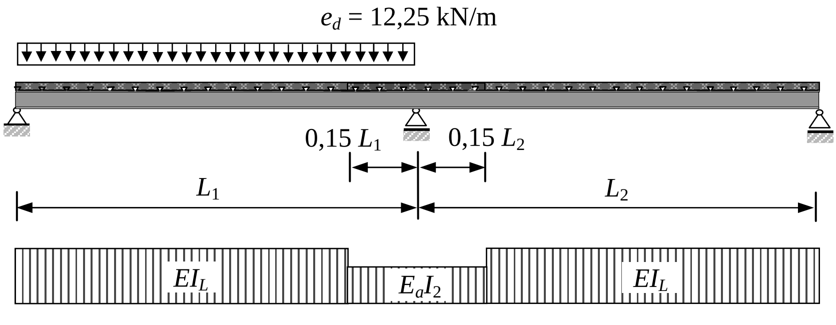


Figure B8.41 *Static system - frequent value of variable action at time on the first loading*

$\delta_{2,1} = 6{,}0$ mm

- Deflection due to creep under the quasi-permanent value of the variable action at time $t = \infty$

This deflection is the difference of the deflections at time $t = \infty$ and at the time of first loading t_0.

$$e_{d,1} = b \cdot (g_{k,3} + \psi_2 \cdot q_{k,2}) = 2{,}50 \cdot (1{,}0 + 0{,}6 \cdot 7{,}0) = 13{,}00 \ \text{kN/m}$$

$$e_{d,2} = b \cdot g_{k,3} = 2{,}50 \cdot 1{,}00 = 2{,}50 \ \text{kN/m}$$

The value of second moment of area for the cracked reinforced section at the internal support and for the uncracked section at mid-span with n = 10,1 and n = 20,2 are taken from Table B8.1:

$I_2 = 45200 \ \text{cm}^4$, at internal support, cracked, reinforced
$I_0 = 96600 \ \text{cm}^4$, at mid-span, uncracked, n = 10,1
$I_P = 79900 \, \text{cm}^4$, at mid-span, uncracked, n = 20,2

Accordingly, the following flexural stiffnesses are used:

$E_a I_2 = 94920 \ \text{kNm}^2$
$EI_L = EI_0 = 202860 \ \text{kNm}^2$
$EI_L = EI_P = 167790 \ \text{kNm}^2$

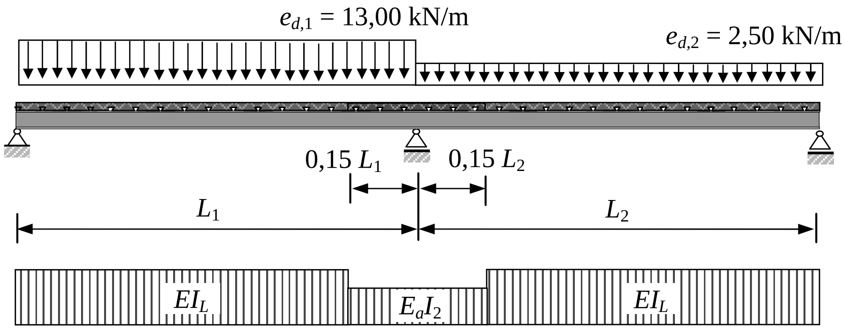


Figure B8.42 *Static system – deflection due to creep at time t = ∞ ($EI_L = EI_P$) and at the time of first loading t_0 ($EI_L = EI_0$)*

$$\delta_{2,2} = 7{,}0 - 5{,}9 = 1{,}1 \ \text{mm}$$

- Deflection due to shrinkage

The primary bending moment due to shrinkage is calculated in accordance with the model given in Figure B8.43.

According to Section 5.2, the axial force due to shrinkage is:

$$N_{cs} = 742 \text{ kN}$$

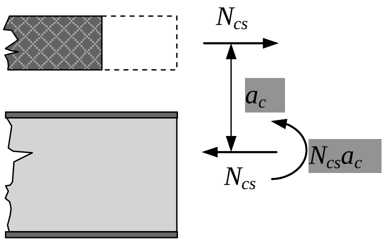


Figure B8.43 *Calculation of the primary bending moment due to shrinkage*

The force N_{cs} acts at the centre of the concrete flange, at a distance a_c above the centroid of the composite section:

$$a_c = a \cdot \frac{A_a}{A_{id}} = a \cdot \frac{E_a \cdot A_a}{E_a \cdot A_a + E_S \cdot A_c}$$

The distance between the centroidal axes of the concrete and the steel section, a, is:

$$a = \frac{450}{2} + \frac{80}{2} + 50 = 315 \text{ mm}$$

The ideal cross-sectional area of the composite section is:

$$A_{id} = A_a + \frac{A_c}{n} = 9880 + \frac{2500 \cdot 80}{28{,}3} = 16947 \text{ mm}^2$$

The force N_{cs} acts at the centre of the concrete flange, at a distance a_c above the centroid of the composite section, which is:

$$a_c = 315 \cdot \frac{9880}{16947} = 184 \text{ mm}$$

The primary bending moment due to shrinkage is:

$$M_{cs} = N_{cs} \cdot a_c = 742 \cdot 18{,}4 = 13653 \text{ kNcm} = 137 \text{ kNm}$$

The value of the second moment of area for the cracked reinforced section at the internal support and for the uncracked section at mid-span with n = 28,3 are taken from Table B8.1:

$I_2 = 45200\ \text{cm}^4$, at internal support, cracked, reinforced
$I_S = 72000\ \text{cm}^4$, at mid-span, uncracked, n = 28,3

Accordingly, the following flexural stiffnesses are used:

$$E_a I_2 = 94920\ \text{kNm}^2$$
$$EI_L = EI_S = 151200\ \text{kNm}^2$$

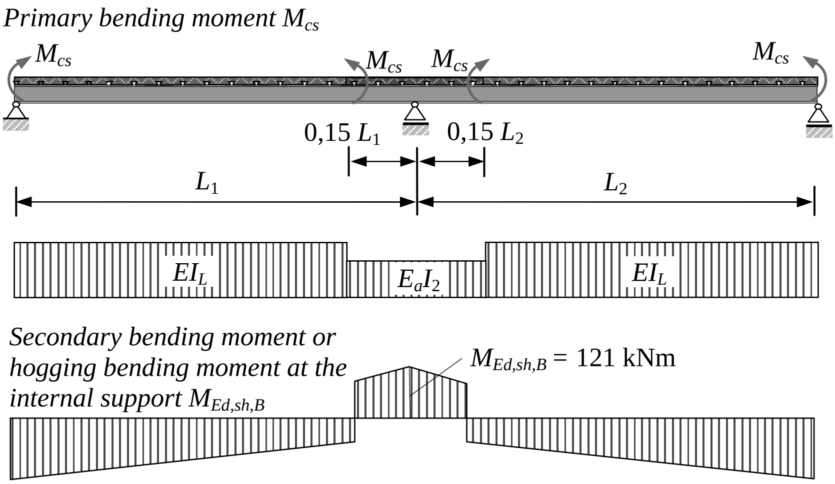


Figure B8.44 *Static system – deflection due to shrinkage*

$$\delta_{2,3} = 5{,}8\ \text{mm}$$

- Deflection due to shrinkage – alternative procedure

For calculation of deflection due to shrinkage, the alternative procedure is illustrated using the results from Section 5.2. From Section 5.2 and Figure B8.16, the primary effect is uniform sagging curvature at radius R = 1105 m, with deflection δ = 32,7 mm at the internal support. The secondary reaction at the internal support is 24,2 kN ($M_{Ed,sh,B} = P \cdot L/2$). The primary deflection at point E, Figure B8.45a, can be calculated from the geometry of the circle. The primary deflection at point E is:

$$\delta_{1,E} = 32{,}7 - (1105 - \sqrt{1105^2 - 4{,}5^2}) \cdot 1000 = 24\ \text{mm}$$

The reaction of 24,2 kN at the internal support causes the upwards displacement at E and moves point *B'* back to point B. This displacement of 17,8 mm is calculated by elastic analysis of the model shown in Figure B8.45b, with 15% of each span cracked.

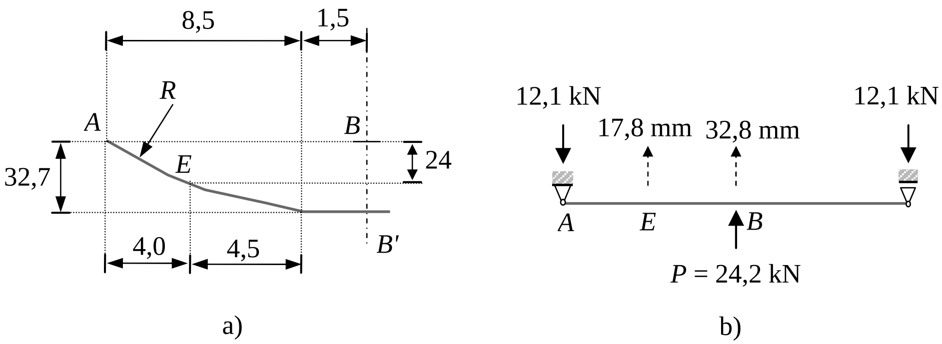


Figure B8.45 *Sagging deflection at point E due to shrinkage*

The obtained results are shown in Table B8.4.

Table B8.4 *Deflections at a distance of 4,0 m from the end support, due to shrinkage*

	Modular ratio	Deflection (mm)
Primary shrinkage	28,3	24
Secondary shrinkage	28,3	–17,8

The total shrinkage deflection is only 6,2 mm because the secondary shrinkage compensates most of primary shrinkage. The primary shrinkage (high free shrinkage strain) could not be neglected in a simply supported beam.

Remark:

The limitations of deflections are adopted according to the recommendation given in EN 1990. These values can be changed in accordance with the recommendation given in the National Annex.

Deflection limits for composite beams are the same as for steel beams, and are determined by the National Annex.

Recommended limiting values for deflection of composite beams are:

$\delta_{tot} \leq \dfrac{L}{250}$, the deflection due to the total load

$\delta_{\text{var}} \leq \dfrac{L}{360}$, the deflection due to the variable load

The deflection due to the permanent action is:

$$\delta_1 = \Sigma\delta_{1,i} = 5,0 + 0,8 = 5,8 \text{ mm}$$

The deflection due to the variable load, creep and shrinkage is:

$$\delta_2 = \Sigma\delta_{2,i} = 6,0 + 1,1 + 5,8 = 12,9 \text{ mm}$$

The total deflection due to permanent and variable loads, creep and shrinkage is:

$$\delta_{tot} = \delta_1 + \delta_2 = 5,8 + 12,9 = 18,7 \text{ mm} \leq \frac{L}{250} = \frac{10000}{250} = 40,0 \text{ mm}$$

The total deflection meets the criterion *L*/250.

The deflection due to the variable load, creep and shrinkage is:

$$\delta_{\text{var}} = \delta_2 = 12,9 \text{ mm} \leq \frac{L}{360} = \frac{10000}{360} = 27,8 \text{ mm}$$

The deflection due to the variable load, creep and shrinkage meets the criterion *L*/360.

6.4 Control of crack width

6.4.1 Minimum reinforcement area

Clause 7.4.1(1), EN 1994-1-1, states that the limitation of crack width depends on the exposure classes according to EN 1992-1-1. Concrete in tension in a composite beam or slab for a building will usually be in exposure class XC3, for which a note to clause 7.3.1(5), EN 1992-1-1, gives the design crack width as 0,3 mm. The procedure according to clause 7.4.1(3), EN 1994-1-1, is used.

The required minimum area of reinforcement A_s for the slab of a composite beam, according to clause 7.4.2(1), EN 1994-1-1, is:

$$A_s = k_s \cdot k_c \cdot k \cdot f_{ct,eff} \cdot A_{ct} / \sigma_s$$

where:

$f_{ct,eff}$ is the mean value of the tensile strength of the concrete, effective at the time when the first crack may be expected to occur. Values of $f_{ct,eff}$ can be taken as those for f_{ctm} (EN 1992-1-1, Table 3.1) or as f_{lctm} (EN 1992-1-1, Table 11.3.1) taking into account the concrete strength class at the time when the first crack of the concrete is expected the occurrence. If the time of occurrence of cracks cannot be established, it is possible to adopt the minimum tensile strength of 3 N/mm^2.

A_{ct} is the cross-sectional area of the tensile zone of the concrete (due to direct loading and primary effects of shrinkage). For the sake of simplicity, the cross-sectional area of the concrete may be adopted as the area determined by its effective width.

σ_s is the maximum stress allowed in the reinforcement immediately after cracking of the concrete. This stress can be taken as the characteristic value of the yield strength f_{sk}. To satisfy the required width limits, the lower values can be needed, depending on the diameter of the bar. These values are given in Table 7.1, EN 1994-1-1.

k, k_s, k_c are the coefficients based on the calibration procedure. The magnitude of these coefficients, k, k_s, and k_c, depend on the geometry of the cracked composite section. More detailed explanation is given below.

The meaning of the coefficients, k, k_s, k_c, is as follows:

k is the coefficient that allows for the effect of non-uniform self-equilibrating tensile stresses, which may be taken as 0,8.

k_s is the coefficient that allows for the effect of the reduction of the normal force of the concrete slab due to initial cracking and local slip of the shear connection, which may be taken as 0,9.

k_c is the coefficient that takes into account the stress distribution within the cross-section (the tensile zone of concrete A_{ct}) immediately prior to cracking.

The coefficient k_c is calculated as:

$$k_c = \frac{1}{1 + h_c / (2z_0)} + 0,3 \leq 1,0$$

where:

h_c is the thickness of the concrete flange, excluding any haunch or ribs,

z_0 is the vertical distance between the centroids of the uncracked concrete flange and the uncracked composite section, calculated using the modular ratio $n_0 = E_a/E_{cm}$ for short-term loading, i.e. at the time of first loading t_0.

The cracks near the internal support caused by hogging bending of the beam are considered in this case. The cracks along the beam caused by hogging bending of the composite slab supported on the beam should also be taken into consideration.

According to clause 5.4.1.2, EN 1994-1-1, the effective width at mid-span is 2,23 m and at the internal support the effective width is 1,35 m. At the internal support at a length of 2,5 m (0,25 L) in each span, the slab should be in tension. It is very difficult to prove that sections 2,5 m from the internal support are never subjected to tension. Therefore, calculations are done for both effective widths. Uncracked unreinforced sections are assumed. According to the definition of z_0, the modular ratio is n_0 = 10,1.

Therefore, the values of coefficients k and k_s are:

$$k = 0{,}8$$

$$k_s = 0{,}9$$

Calculation of coefficient k_c – composite section at the internal support

The modular ratio n_0 for short-term loading is:

$$n_L = n_0 = \frac{E_a}{E_{lcm}} = \frac{210000}{20752} = 10{,}1$$

The effective width of the concrete flange at the internal support is:

$$b_{eff} = 1{,}35 \text{ m}$$

Transformed to the ideal steel section, the effective width is:

$$\frac{b_{eff}}{n_0} = \frac{135}{10{,}1} = 13{,}4 \text{ cm}$$

The cross-section shown in Figure B8.5 is considered. In accordance with Figure B8.5 the vertical distance between the centroids of the uncracked concrete flange and the uncracked composite section is:

$$z_0 = 151 \text{ mm}$$

The coefficient k_c is:

$$k_c = \frac{1}{1 + h_c / (2z_0)} + 0,3 = \frac{1}{1 + 80 / (2 \cdot 151)} + 0,3 = 1,09 > 1,0$$

Adopted: k_c = 1,00.

Calculation of coefficient k_c – composite section at mid-span

The modular ratio n_0 for short-term loading is:

$$n_L = n_0 = \frac{E_a}{E_{lcm}} = \frac{210000}{20752} = 10,1$$

The effective width of the concrete flange at mid-span is:

$$b_{eff} = 2,23 \text{ m}$$

Transformed to the ideal steel section, the effective width is:

$$\frac{b_{eff}}{n_0} = \frac{223}{10,1} = 22,1 \text{ cm}$$

The cross-section shown in Figure B8.7 is considered. In accordance with Figure B8.7 the vertical distance between the centroids of the uncracked concrete flange and the uncracked composite section:

$$z_0 = 113 \text{ mm}$$

The coefficient k_c is:

$$k_c = \frac{1}{1 + h_c / (2z_0)} + 0,3 = \frac{1}{1 + 80 / (2 \cdot 113)} + 0,3 = 1,04 < 1,0$$

Adopted: k_c = 1,0.

If the time of occurrence of the cracks cannot be established, the mean value of the tensile strength of the concrete effective at the time when the first crack may be expected to occur, $f_{ct,eff}$, can be adopted as the minimum tensile strength of 3 N/mm^2.

The maximum stress allowed in the reinforcement immediately after cracking of concrete σ_s is chosen from Table 7.1, EN 1994-1-1:

$$\sigma_s = 280 \text{ N/mm}^2$$

The maximum bar diameter ϕ^*, for a design crack width w_k = 0,3 mm and for the chosen maximum stress allowed in reinforcement σ_s, according to Table 7.1, EN 1994-1-1, is:

ϕ^* = 12 mm

Minimum area of reinforcement

The required minimum area of reinforcement A_s for the slab of composite beam, according to clause 7.4.2(1), EN 1994-1-1, is:

$$A_s = k_s \cdot k_c \cdot k \cdot f_{ct,eff} \cdot A_{ct} / \sigma_s$$

From this expression, we obtain the following ratio:

$$100 \cdot \frac{A_s}{A_{ct}} = 100 \cdot \frac{k_s \cdot k_c \cdot k \cdot f_{ct,eff}}{\sigma_s} = 100 \cdot \frac{0,9 \cdot 1,0 \cdot 0,8 \cdot 3,0}{280} = 0,77 \text{ \%}$$

However, clause 5.5.1(5), EN 1994-1-1, gives the limit, as the condition for the use of plastic resistance moment, which was calculated in Section 5.4.2.2. This limit is:

$$\rho_s = 0,70 \text{ \%}$$

Therefore, for the slab thickness, which is 80 mm above the profiled sheeting, the minimum reinforcement is calculated for the higher percentage of reinforcement $\rho_s = 0,77$. The minimum area of reinforcement is:

$$A_{s,\min} = 0,0077 \cdot 80 \cdot 1000 = 616 \text{ mm}^2\text{/m}$$

One layer of 12 mm bars at 125 mm spacing provides 904 mm^2/m.

6.4.2 Control of cracking of concrete due to direct loading

According to clause 7.4.3(4), EN 1994-1-1, the bending moment at the internal support is calculated for the quasi-permanent combination of actions at time $t = \infty$.

$$e_d = b \cdot (g_{k,3} + \psi_2 \cdot q_{k,2}) = 2,50 \cdot (1,0 + 0,6 \cdot 7,0) = 13,00 \text{ kN/m}$$

$$EI_L = EI_P = 167790 \text{ kNm}^2$$

$E_a I_2 = 94920 \text{ kNm}^2$

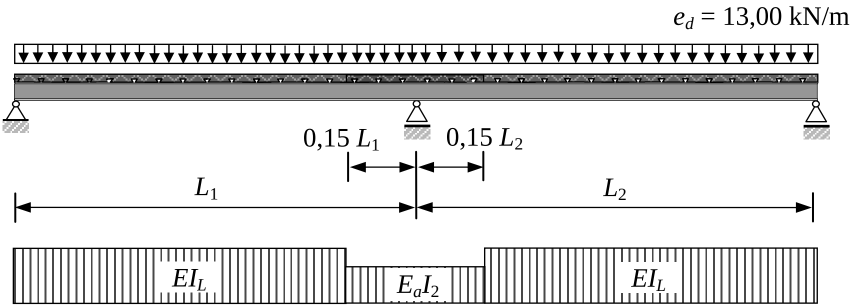


Figure B8.46 *Static system for calculating the bending moment at the internal support of the composite beam for the quasi-permanent actions at time t = ∞*

The maximum bending moment at the internal support, calculated using commercial software, is:

$M_{Ed,max}$ = 136 kNm

The bending moment due to shrinkage according to Section 5.2 is:

$$M_{Ed,sh,B} = 121 \text{ kNm}$$

The total bending moment, including shrinkage, is:

$$M_{Ed} = 136 + 121 = 257 \text{ kNm}$$

According to Figure B8.4, the neutral axis of the cracked section is 319 mm below the top of the slab.

The distance between the neutral axis and the centroidal axis of reinforcement is:

$$z_{st} = e_{st} - z_{sl} = 31{,}9 - 3{,}0 = 28{,}9 \text{ cm}$$

The tensile stress in reinforcement σ_s can be calculated for direct loading as:

$$\sigma_s = \sigma_{s,0} + \Delta\sigma_s$$

The stress in the reinforcement $\sigma_{s,0}$ caused by the bending moment acting on the composite section is calculated on the assumption that the concrete in tension is neglected.

The geometrical properties of the cracked cross-section in accordance with Figure B8.4 are:

Cross-sectional area

$$A_{st} = A_a + A_s = 9880 + 1221 = 11101 \text{ mm}^2$$

Second moment of area

$$I_{st} = 452 \cdot 10^6 \text{ mm}^4$$

The stress in the reinforcement $\sigma_{s,0}$ caused by the bending moment acting on the cracked section is:

$$\sigma_{s,0} = \frac{M_{Ed}}{I_{st}} z_{st} = \frac{257 \cdot 10^6}{452 \cdot 10^6} \cdot 289 = 164 \text{ N/mm}^2$$

By means of the calculated values of A_a, A_s, I_a and I (internal support, cracked, reinforced, Table B8.1), the following value for α_{st} is obtained:

$$\alpha_{st} = \frac{A \cdot I}{A_a \cdot I_a} = \frac{(A_a + A_s) \cdot I}{A_a \cdot I_a} = \frac{(9880 + 1221) \cdot 452 \cdot 10^6}{9880 \cdot 337,4 \cdot 10^6} = \frac{11101 \cdot 452 \cdot 10^6}{9880 \cdot 337,4 \cdot 10^6} = 1,51$$

For 12 mm bars at 125 mm spacing with A_s = 1221 mm^2 and the effective width of 1,35 m and the thickness of the concrete slab of 80 mm with $A_c = 108 \cdot 10^3$ mm^2, the following ratio is obtained:

$$\rho_s = \frac{A_s}{A_c} = \frac{1221}{108 \cdot 10^3} = 0,0113$$

According to expression (7.5), EN 1994-1-1, the correction of the stress in the reinforcement for tension stiffening is:

$$\Delta\sigma_s = \frac{0,4 \cdot f_{lctm}}{\alpha_{st} \cdot \rho_s} = \frac{0,4 \cdot 2,32}{1,51 \cdot 0,0113} = 54,4 \text{ N/mm}^2$$

According to expression (7.4), EN 1994-1-1, the tensile stress in the reinforcement σ_s due to direct loading is:

$$\sigma_s = \sigma_{s,0} + \Delta\sigma_s = 164 + 54,4 = 218 \text{ N/mm}^2$$

According to Table 7.1, EN 1994-1-1, the maximum bar diameter is $\phi_s \leq 16$ mm.

According to Table 7.2, EN 1994-1-1, the maximum bar spacing is ≤ 200 mm.

The use of 12 mm bars at 125 mm spacing at internal support, with the cross-sectional area 904 mm^2/m, satisfies both conditions.

7. Commentary

This example illustrates the application of many of the provisions of EN 1994-1-1. In accordance with the provisions of EN 1994-1-1, the following problems were studied in detail:

- buckling resistance moment of laterally unrestrained continuous composite beam and with laterally restrained bottom steel flange,
- partial shear connection with non-ductile connectors.

Serviceability stress verifications are especially explained. According to EN 1994-1-1, there are no limitations on stress. However, stress checks should be included as part of the serviceability criteria, because yielding under serviceability loading results with the additional deflection.

C Composite columns

C1 Composite column with concrete-filled circular hollow section subject to axial compression and verified using European buckling curves

1. Purpose of example

This example demonstrates the design of a composite column subject to axial compression. It is assumed to be pinned top and bottom. The concrete-filled column consists of a circular hollow section filled with concrete. In concrete-filled hollow sections no longitudinal reinforcement is normally necessary. In this example, the reinforcement is selected for educational reasons because the limitations of reinforcement are illustrated in this example. However, if the design for fire resistance is required, the longitudinal reinforcement can be used, clause 6.7.5.2 (1), EN 1994-1-1. Since the considered member is subjected only to end compression, clause 6.7.3.5(2) enables buckling curves to be used.

The columns without end moments are very rare in practice. However, the application of the simplified method of design for a column in axial compression is essential for understanding the behaviour of column in combined compression and bending.

2. Static system, cross-section and design action effects

Actions

Permanent action $N_{G_k} = 4019 \text{ kN}$

Variable action $N_{Q_k} = 1550 \text{ kN}$

Design action effect:

$$N_{Ed} = N_{G,Ed} + N_{Q,Ed},$$

$$N_{Ed} = 1,35 \cdot N_{G_k} + 1,50 \cdot N_{Q_k},$$

$$N_{Ed} = 1,35 \cdot 4019 + 1,50 \cdot 1550 = 5425 + 2325 = 7750 \text{ kN}.$$

Composite Structures according to Eurocode 4. Worked Examples.
First Edition. Darko Dujmović, Boris Androić, Ivan Lukačević.

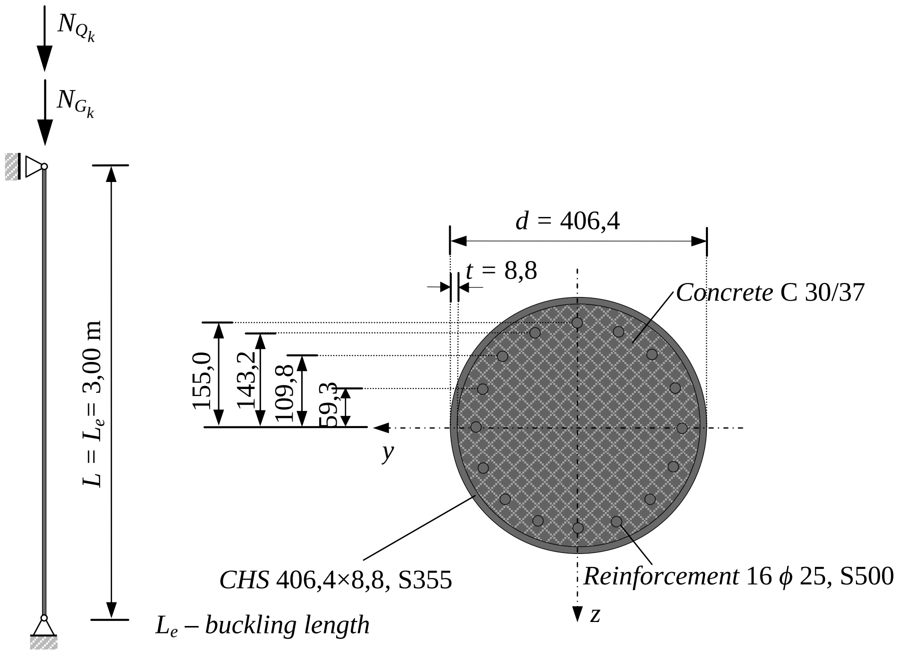


Figure C1.1 *Static system and cross-section*

3. Properties of materials

Concrete strength class: C 30/37

$$f_{ck} = 30 \text{ N/mm}^2$$

$$f_{cd} = \frac{f_{ck}}{\gamma_c} = \frac{30}{1,5} = 20 \text{ N/mm}^2$$

$$E_{cm} = 33000 \text{ N/mm}^2$$

Structural steel: S355

$$f_{yk} = 355 \text{ N/mm}^2$$

$$f_{yd} = \frac{f_{yk}}{\gamma_M} = \frac{355}{1,0} = 355 \text{ N/mm}^2$$

$$E_a = 210000 \text{ N/mm}^2$$

Reinforcement: *ductility class B or C*

$$f_{sk} = 500 \text{ N/mm}^2$$

$$f_{sd} = \frac{f_{sk}}{\gamma_s} = \frac{500}{1,15} = 435 \text{ N/mm}^2$$

$$E_S = 210000 \text{ N/mm}^2$$

4. Geometrical properties of the cross-section

4.1 Selection of the steel cross-section and reinforcement

It is assumed that only the axial compression load is applied, and the required fire resistance is R60. Since the composite columns are used in building, it is expected to set requirements for fire resistance.

According to the literature [40] the trial column diameter, d_{trial}, is determined approximately according to the following formula:

$$d_{trial} = [\frac{t_{fire} \cdot (L_e - 1000)}{0,08 \cdot (f_{ck} + 20)} \cdot \sqrt{N_{Ed}}]^{0,4}$$

With t_{fire} = 60 min, which corresponds to the required fire resistance R60 in minutes, the trial diameter is obtained as follows:

$$d_{trial} = [\frac{60 \cdot (3000 - 1000)}{0,08 \cdot (30 + 20)} \cdot \sqrt{7750}]^{0,4} = 370 \text{ mm}$$

The CHS with diameter d = 406,4 mm is selected.

For a concrete-filled circular hollow section, the minimum wall thickness is determined from the condition of the local buckling, Table 6.3, EN 1994-1-1:

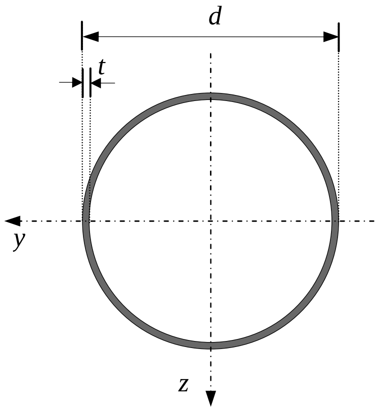
$$\max(\frac{d}{t}) = 90 \cdot \frac{235}{f_y}$$

$$\max(\frac{d}{t}) = 90 \cdot \frac{235}{355} = 59,6$$

The minimum wall thickness is obtained as:

$$t_{\min} = \frac{d}{59,6} = \frac{406,4}{59,6} = 6,82$$

Thus, the minimum required wall thickness of the circular tube is 7,0 mm. The CHS 406,4x8,8 is selected. The selected circular hollow section with dimensions is shown in Figure C1.2.



d = 406,4 mm
t = 8,8 mm
A_a = 110,0 cm^2
I_a = 21732 cm^4
$W_{pl,a}$ = 1391 cm^3

Figure C1.2 *Circular hollow section*

Remark:

In concrete-filled hollow sections no longitudinal reinforcement is normally necessary. However, if the design for fire resistance is required, which is the case in this example, longitudinal reinforcement can be used, clause 6.7.5.2 (1), EN 1994-1-1.

Initially, the assumed reinforcement is 16 bars with the diameter of 25 mm. The cross-sectional area of reinforcement is 78,6 cm^2. This initial selection of reinforcement is made for educational reasons with the aim of explaining the limitations of reinforcement according to clause 6.7.3.1(3), EN 1994-1-1.

The cross-sectional area of the structural steel section 406,4×8,8 is:

$$A_a = 110 \ \text{cm}^2$$

The cross-sectional area of the reinforcement with 16 bars of 25 mm is:

d_{bar} = 25 mm A_{bar} = 4,91 cm^2

$$A_s = 16 \cdot A_{bar} = 16 \cdot 4{,}91 = 78{,}6 \ \text{cm}^2$$

The cross-sectional area of the concrete is:

$$A_c = \pi \cdot (d - 2 \cdot t)^2 \cdot \frac{1}{4} - A_s$$

$$A_c = \pi \cdot (40{,}64 - 2 \cdot 0{,}88)^2 \cdot \frac{1}{4} - 78{,}6$$

$$A_c = 1187 - 78,6 = 1109 \text{ cm}^2$$

The ratio of reinforcement area to concrete area is:

$$\rho_s = \frac{A_s}{A_c} = \frac{78,6}{1109} = 0,071 \qquad \rho_s = 7,1\,\%$$

$$\rho_s = 7,1\% > 6\%$$

Remark:

According to clause 6.7.3.1(3), EN 1994-1-1, the ratio of reinforcement area to concrete area, ρ_s, should not exceed 6%.

Since the ratio of reinforcement area to concrete area, ρ_s, is higher than 6%, the reinforcement should be reduced. This can be achieved in two ways, as follows.

a) The same number of bars but a smaller diameter of bar

With the ratio of reinforcement area to concrete area of 6%, the diameter of bar is:

$$d_{bar} = \sqrt{\frac{A_c \cdot 0,06}{16} \cdot \frac{4}{\pi}}$$

$$d_{bar} = \sqrt{\frac{1109 \cdot 0,06}{16} \cdot \frac{4}{3,14}} = 2,30 \text{ cm} = 23,0 \text{ mm}$$

A diameter of 22 mm is adopted.

The cross-sectional area of the reinforcement with 16 bars of 22 mm diameter is:

d_{bar} = 22 mm A_{bar} = 3,80 cm^2

$$A_s = 16 \cdot A_{bar} = 16 \cdot 3,80 = 60,8 \text{ cm}^2$$

The ratio of reinforcement area to concrete area ρ_s is:

$$A_c = 1187 - 60,8 = 1126,2 \text{ cm}^2$$

$$A_s = 60,8 \text{ cm}^2$$

$$\rho_s = \frac{A_s}{A_c} = \frac{60,8}{1126,2} = 0,054 \qquad \rho_s = 5,4\ \%$$

$$\rho_s = 5,4\% < 6\%$$

b) Fewer bars but the same diameter of bar

The adopted number of bars is 12 and the diameter of the bar is 25 mm.

The cross-sectional area of the reinforcement with 12 bars of 25 mm diameter is:

d_{bar} = 25 mm A_{bar} = 4,91 cm^2

$$A_s = 12 \cdot A_{bar} = 12 \cdot 4,91 = 58,9 \ \text{cm}^2$$

The ratio of reinforcement area to concrete area ρ_s is:

$$A_c = 1187 - 58,9 = 1128 \ \text{cm}^2$$

$$A_s = 58,9 \ \text{cm}^2$$

$$\rho_s = \frac{A_s}{A_c} = \frac{58,9}{1128} = 0,052 \qquad \rho_s = 5,2\ \%$$

$$\rho_s = 5,2\% < 6\%$$

In this example, the adopted reinforcement is 12 bars with the diameter of 25 mm. The cross-section of the composite column is shown in Figure C1.3.

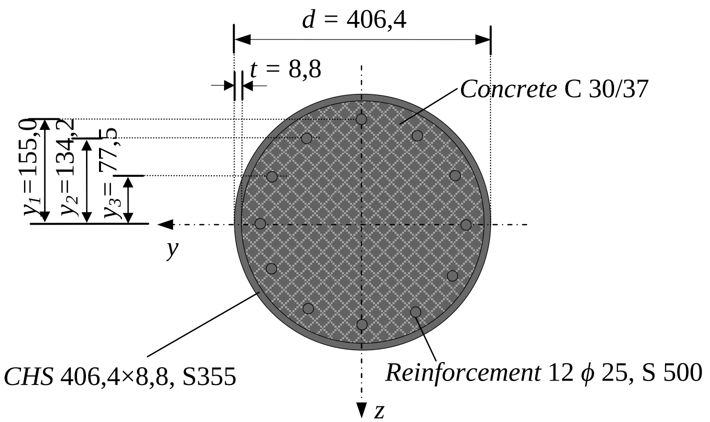


Figure C1.3 *Cross-section of the composite column*

4.2 Cross-sectional areas

Structural steel

$$A_a = 110\ \text{cm}^2$$

Reinforcement

$$A_s = 12 \cdot \frac{\pi \cdot d_{bar}^2}{4}$$

$$A_s = 12 \cdot \frac{\pi \cdot 2{,}5^2}{4} = 58{,}9\ \text{cm}^2$$

Concrete

$$A_c = \pi \cdot \frac{d^2}{4} - A_a - A_s$$

$$A_c = \pi \cdot \frac{40{,}64^2}{4} - 110 - 58{,}9 = 1128\ \text{cm}^2$$

4.3 Second moments of area

Structural steel

$$I_a = 21732\ \text{cm}^4$$

Reinforcement (in accordance with Figure C1.3)

$$I_s = 2 \cdot A_{bar} \cdot y_1^2 + 4 \cdot A_{bar} \cdot y_2^2 + 4 \cdot A_{bar} \cdot y_3^2$$

$$I_s = 2 \cdot 4{,}91 \cdot 15{,}5^2 + 4 \cdot 4{,}91 \cdot 13{,}42^2 + 4 \cdot 4{,}91 \cdot 7{,}75^2$$

$$I_s = 7076\ \text{cm}^4$$

Concrete

$$I_c = \pi \cdot \frac{(d - 2 \cdot t)^4}{64} - I_s$$

$$I_c = \pi \cdot \frac{(40,64 - 2 \cdot 0,88)^4}{64} - 7076 = 105094 \text{ cm}^4$$

5. Steel contribution ratio

According to clause 6.7.3.3(1), EN 1994-1-1, the steel contribution ratio, δ, is defined as:

$$\delta = \frac{A_a \cdot f_{yd}}{N_{pl,Rd}}$$

The term $A_a \cdot f_{yd}$ is the contribution of the structural steel section to the plastic resistance of the composite section to the axial force. The design plastic resistance of the composite section to the axial force $N_{pl,Rd}$ is calculated according to clause 6.7.3.2(1), EN 1994-1-1.

According to 6.7.1(4), EN 1994-1-1, the steel contribution ratio, δ, must satisfy the following conditions:

$0,2 \leq \delta \leq 0,9$

If δ is less than 0,2, the column should be designed as a reinforced concrete member according to EN 1992-1-1. If δ is larger than 0,9, the concrete is ignored in the calculations, and the column is designed as a structural steel member according to EN 1993-1-1.

The term $A_a \cdot f_{yd}$ is the contribution of the structural steel section to the plastic resistance of the composite section to the axial force:

$$A_a \cdot f_{yd} = 110 \cdot 35,5 = 3905 \text{ kN}$$

The plastic resistance of the composite section to the axial force is:

$$N_{pl,Rd} = A_a \cdot f_{yd} + A_c \cdot f_{cd} + A_s \cdot f_{sd}$$

$$N_{pl,Rd} = 110 \cdot 35,5 + 1128 \cdot 2,0 + 58,9 \cdot 43,5$$

$$N_{pl,Rd} = 3905 + 2256 + 2562 = 8723 \text{ kN}$$

The steel contribution ratio, δ, is:

$$\delta = \frac{A_a \cdot f_{yd}}{N_{pl,Rd}} = \frac{3905}{8723} = 0,45$$

The steel contribution ratio, δ, must satisfy the following conditions:

$0,2 \le \delta \le 0,9$

Since the limits $0,2 < \delta = 0,45 < 0,9$ are satisfied, the column can be classified as a composite column, and the provisions of EN 1994-1-1 can be used for the dimensioning.

Remark:

The confinement effects for concrete-filled circular tube can result in an increase of the cross-sectional plastic resistance, $N_{pl,Rd}$. However, the value of δ is not usually significantly changed. This means that for the calculation of steel contribution ratio, δ, we can use the following expression for the cross-sectional plastic resistance, $N_{pl,Rd}$:

$$N_{pl,Rd} = A_a \cdot f_{yd} + A_c \cdot f_{cd} + A_s \cdot f_{sd}$$

6. Local buckling

According to clause 6.7.1(9), EN 1994-1-1, for concrete-filled circular hollow cross-section, the effect of local buckling can be ignored if the following condition is satisfied:

$$\max\left(\frac{d}{t}\right) = 90 \cdot \frac{235}{f_y}$$

For the selected cross-section, the maximum slenderness is:

$$\max\left(\frac{d}{t}\right) = \frac{406,4}{8,8} = 46,2$$

The required condition is:

$$90 \cdot \frac{235}{f_y} = 90 \cdot \frac{235}{355} = 59,6$$

Since $46{,}2 < 59{,}6$, the condition is satisfied. The effect of local buckling can be neglected.

7. Effective modulus of elasticity for concrete

For long-term loading the creep and shrinkage are taken into account in the design by a reduced flexural stiffness of the composite cross-section. Due to the influence of long-term creep effects on the effective elastic stiffness, the modulus of elasticity of the concrete, E_{cm}, should be reduced to the value $E_{c,eff}$:

$$E_{c,eff} = \frac{E_{cm}}{1 + (\frac{N_{G,Ed}}{N_{Ed}}) \cdot \varphi_t}$$

where:

$\varphi_t = \varphi(t, t_0)$ is the creep coefficient, defining the creep between times t and t_0, related to the elastic deformation at 28 days,

$\varphi_t = \varphi(\infty, t_0)$ is the final creep coefficient,

t is the age of concrete at the time considered,

t_0 is the age of concrete at loading,

N_{Ed} is the axial design force,

$N_{G,Ed}$ is the permanent part of the axial design force N_{Ed}, $N_{G,Ed} = \gamma_G \cdot N_{Gk}$.

For the calculation of the creep coefficient $\varphi(t, t_0)$, the following is valid:

- the perimeter of that part which is exposed to drying, u

$$u = d \cdot \pi$$

$$u = 40{,}64 \cdot \pi = 127{,}7 \text{ cm}$$

- the notional size of the cross-section, h_0

$$h_0 = \frac{2 \cdot A_c}{u} = \frac{2 \cdot 1128}{\pi \cdot 40{,}64} = 17{,}7 \text{ cm} = 177 \text{ mm}$$

- $t_0 = 7$ days,
- inside conditions, the ambient relative humidity RH 50%,
- the concrete strength class C 30/37,
- the type of cement – cement class N, strength class 32,5 R.

The final value of the creep coefficient $\varphi(\infty, t_0)$ is determined using the nomogram shown in Figure 3.1, EN 1992-1-1. The process of determining the final value of the creep coefficient, taking into account these assumptions, is given in Figure C1.4.

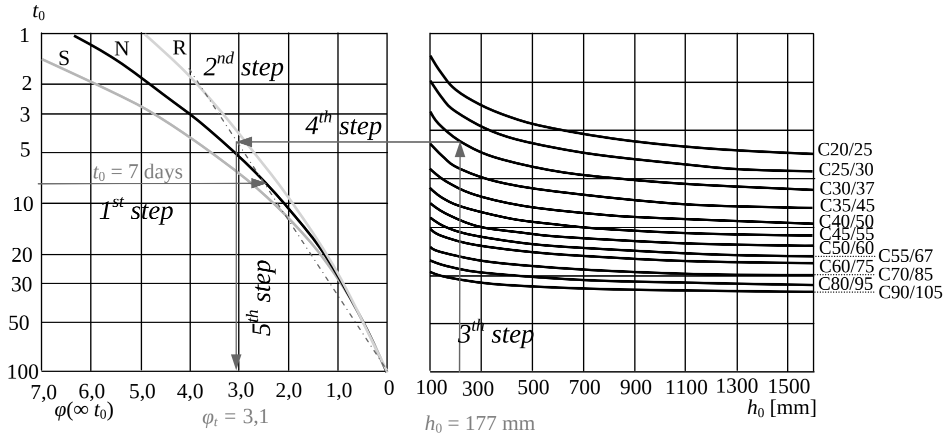


Figure C1.4 *Method for determining the creep coefficient*

The final value of creep coefficient $\varphi(\infty, t_0)$, found from Figure C1.4, is:

$\varphi_t = \varphi(\infty, t_0) = 3{,}1$

The design force for the permanent load, $N_{G,Ed}$, and the total design force, N_{Ed}, are:

$$N_{G,Ed} = \gamma_G \cdot N_{G_k}$$

$$N_{G,Ed} = 1{,}35 \cdot 4019 = 5425 \text{ kN}$$

$$N_{Ed} = 1{,}35 \cdot N_{G_k} + 1{,}50 \cdot N_{Q_k}$$

$$N_{Ed} = 1{,}35 \cdot 4019 + 1{,}50 \cdot 1550 = 5425 + 2325 = 7750 \text{ kN}$$

Accordingly, the value of $E_{c,eff}$ is:

$$E_{c,eff} = \frac{E_{cm}}{1 + (\frac{N_{G,Ed}}{N_{Ed}}) \cdot \varphi_t} = \frac{3300}{1 + (\frac{5425}{7750}) \cdot 3{,}1} = 1041 \text{ kN/cm}^2$$

Remark:

The obtained value of effective modulus of elasticity is conservative. For concrete-filled hollow section, the drying of the concrete is significantly reduced by the steel section. Taking into consideration this favourable effect, the sufficient good estimation of the creep coefficient can be achieved if 25% of that creep coefficient is used:

$\varphi_{t,eff} = 0{,}25 \cdot \varphi(\mathrm{t},\, t_0)$

The same recommendation was adopted in [12].

Therefore, the following value of creep coefficient, is adopted:

$$\varphi_{t,eff} = 0{,}25 \cdot \varphi(t, t_0) = 0{,}25 \cdot 3{,}1 = 0{,}775$$

In this case, the effective modulus of elasticity for concrete is:

$$E_{c,eff} = \frac{E_{cm}}{1 + (\frac{N_{G,Ed}}{N_{Ed}}) \cdot \varphi_{t,eff}} = \frac{3300}{1 + (\frac{5425}{7750}) \cdot 0{,}775} = 2139 \text{ kN/cm}^2$$

Further calculation will be performed with this effective modulus of elasticity of concrete, $E_{c,eff}$ = 2139 kN/m^2.

Remark:

It should be noted that a conservative estimation of the effective modulus of elasticity is obtained according to EN 1994-1-1 because in the case of concrete-filled hollow section the drying of the concrete is significantly reduced by the steel section. However, in the case of concrete-filled circular hollow section, the dimensioning of composite columns is rarely sensitive to the influence of creep coefficient φ_t on the effective modulus elasticity of concrete $E_{c,eff}$. With this statement the conservative estimation of $E_{c,eff}$, according to EN 1994-1-1, can be justified in the case of concrete-filled hollow sections.

8. Resistance of the cross-section to compressive axial force

8.1 Plastic resistance of the cross-section without confinement effect

The design plastic resistance of composite cross-section to compressive axial force, $N_{pl,Rd}$, is given by the sum of the design resistances of components as:

$$N_{pl,Rd} = A_a \cdot f_{yd} + 0,85 \cdot A_c \cdot f_{cd} + A_s \cdot f_{sd}$$

Remark:

The coefficient of 0,85 can be replaced with the value 1,0 due to better curing conditions in the case of concrete-filled hollow sections.

The design plastic resistance of the composite cross-section to compressive axial force, $N_{pl,Rd}$, is calculated according to the corrected expression:

$$N_{pl,Rd} = A_a \cdot f_{yd} + A_c \cdot f_{cd} + A_s \cdot f_{sd}$$

$$N_{pl,Rd} = 110 \cdot 35,5 + 1128 \cdot 2,0 + 58,9 \cdot 43,5$$

$$N_{pl,Rd} = 3905 + 2256 + 2562 = 8723 \text{ kN}$$

The characteristic value of the plastic resistance of the composite cross-section to compressive axial force, $N_{pl,Rk}$, is determined by:

$$N_{pl,Rk} = A_a \cdot f_{yk} + A_c \cdot f_{ck} + A_s \cdot f_{sk}$$

$$N_{pl,Rk} = 110 \cdot 35,5 + 1128 \cdot 3,0 + 58,9 \cdot 50$$

$$N_{pl,Rk} = 10234 \text{ kN}$$

8.2 Plastic resistance of the cross-section taking into account confinement effect

For composite columns with concrete-filled circular hollow sections, the increased strength of the concrete due to the confinement effect of the circular hollow section may be taken into account. Figure C1.5 illustrates the confinement effect of the circular hollow section.

When the concrete-filled circular hollow sections are under axial compression, the concrete is not able to expand laterally, and triaxial stresses are developed in the concrete. For stresses of concrete $\sigma_c > 0,8\, f_{ck}$ the Poisson's ratio of concrete is higher than the Poisson's ratio of structural steel. The confinement of the circular tube causes radial compressive stresses $\sigma_{c,r}$. This results in increased concrete strength and higher strains of the concrete. At the same time, the circumferential tensile stresses $\sigma_{a\varphi}$ in the steel tube also arise and reduce the axial resistance of the steel tube. The radial stresses also cause friction in the steel-concrete interface, which increase the longitudinal shear resistance. The increased resistance of concrete must be taken into account only in the case of concrete-filled circular hollow section.

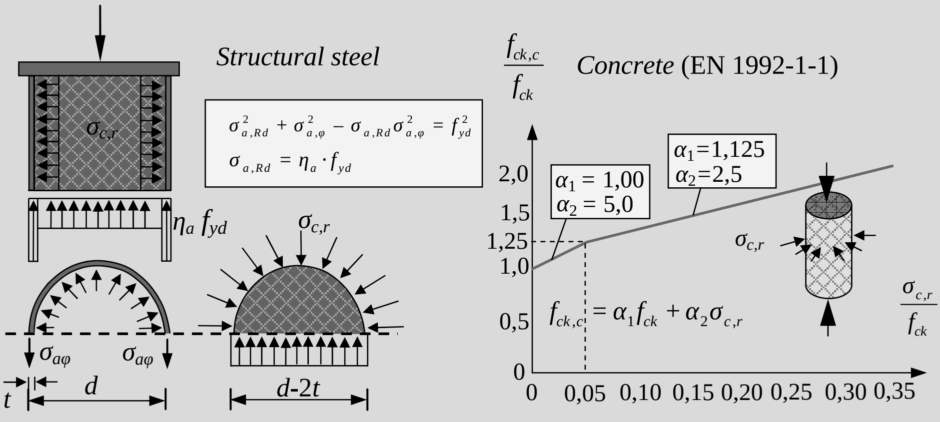


Figure C1.5 *Confinement effects for concrete-filled tubes of circular cross-section*

The increase in strength of concrete caused by confinement may be taken into account if the relative slenderness, $\bar{\lambda}$, does not exceed 0,5 and e/d < 0,1. The eccentricity of loading, e, is defined as:

$$e = \frac{M_{Ed}}{N_{Ed}}$$

where:

M_{Ed} is the maximum design moment (second-order effects are ignored),
N_{Ed} is the design axial force,
d is the outer diameter of the circular hollow section.

According to clause 6.7.3.2(6), EN 1994-1-1, when these conditions are satisfied, the design plastic resistance to compression taking into account the confinement effects may be obtained as:

$$N_{pl,Rd} = \eta_a \cdot A_a \cdot f_{yd} + A_c \cdot f_{cd}(1 + \eta_c \cdot \frac{t}{d} \cdot \frac{f_y}{f_{ck}}) + A_s \cdot f_{sd}$$

where:

t is the wall thickness of the steel hollow section.

For members with e = 0, the values $\eta_a = \eta_{a0}$ and $\eta_c = \eta_{c0}$ are given by:

$$\eta_a = \eta_{a0} = 0,25 \cdot (3 + 2 \cdot \bar{\lambda}) \text{, (but} \le 1,0)$$

$$\eta_c = \eta_{c0} = 4,9 - 18,5 \cdot \bar{\lambda} + 17 \cdot \bar{\lambda}^2 \text{, (but} \ge 0)$$

For members in combined compression and bending with $0 < e/d \le 0,1$, the values η_a and η_c should be determined from the following expressions:

$$\eta_\mathrm{a} = \eta_\mathrm{a0} + (1 - \eta_\mathrm{a0}) \cdot \frac{10 \cdot e}{d}$$

$$\eta_\mathrm{c} = \eta_\mathrm{c0} (1 - 10 \cdot \frac{e}{d})$$

For $e/d > 0,1$, $\eta_a = 1,0$ and $\eta_c = 0$.

The relative slenderness is $\bar{\lambda} = 0,355$, calculated in Section 9.1, and the relative slenderness does not exceed 0,5.

The eccentricity of loading is:

$$e = \frac{M_{Ed}}{N_{Ed}} = \frac{0}{7750} = 0 < 0,1$$

The conditions are satisfied and the values η_a and η_c are:

$$\eta_a = \eta_{a0} = 0,25 \cdot (3 + 2 \cdot 0,355) = 0,928$$

$$\eta_c = \eta_{c0} = 4,9 - 18,5 \cdot 0,355 + 17 \cdot 0,355^2 = 0,475$$

Therefore, the design plastic resistance to compression taking into account the confinement effects is:

$$N_{pl,Rd} = \eta_{a0} \cdot A_a \cdot f_{yd} + A_c \cdot f_{cd} (1 + \eta_{c0} \cdot \frac{t}{d} \cdot \frac{f_y}{f_{ck}}) + A_s \cdot f_{sd}$$

$$N_{pl,Rd} = 0,928 \cdot 110 \cdot 35,5 + 1128 \cdot 2,0 (1 + 0,475 \cdot \frac{0,88}{40,64} \cdot \frac{35,5}{3,0}) + 58,9 \cdot 43,5$$

$$N_{pl,Rd} = 0,928 \cdot 3905 + 2256 \cdot 1,122 + 2562$$

$$N_{pl,Rd} = 3624 + 2531 + 2562$$

$$N_{pl,Rd} = 8717 \text{ kN}$$

Remark:

Taking into account confinement effects according to the expression (6.33), EN 1994-1-1, the design plastic resistance of the cross-section to compression is $N_{pl,Rd}$ = 8717 kN. The design plastic resistance calculated in Section 8.1 is $N_{pl,Rd}$ = 8723 kN. The higher design plastic resistance, which is $N_{pl,Rd}$ = 8723 kN, is adopted for further calculation

Comparing the expressions for the plastic resistance with confinement effects and without confinement effects, it can be seen that the plastic resistance of structural steel is decreased for 7% while the plastic resistance of concrete is increased by 12%. It follows that the increase in plastic concrete resistance has not contributed to the overall increase in the plastic resistance of the cross-section $N_{pl,Rd}$.

9. Resistance of the member in axial compression

9.1 Verification of conditions for using simplified design method

The cross-section of the composite column should be doubly symmetrical and uniform along the entire length of the column.

This condition is satisfied.

Relative slenderness

To apply the simplified method it is necessary to satisfy the following condition:

$$\bar{\lambda} \leq 2,0$$

Relative slenderness, $\bar{\lambda}$, is determined by the following expression:

$$\bar{\lambda} = \sqrt{\frac{N_{pl,Rk}}{N_{cr}}}$$

For the determination of relative slenderness $\bar{\lambda}$ and elastic critical force N_{cr}, it is necessary to calculate the value of effective flexural stiffness of the cross-section of the composite column, $(EI)_{eff}$, according to:

$$(EI)_{eff} = E_a \cdot I_a + E_s \cdot I_s + K_e \cdot E_{c,eff} \cdot I_c$$

With the correction factor K_e = 0,6, the value $(EI)_{eff}$ is:

$$(EI)_{eff} = 21000 \cdot 21732 + 21000 \cdot 7076 + 0,6 \cdot 2139 \cdot 105094$$

$$(EI)_{eff} = 739,85 \cdot 10^6 \text{ kNcm}^2$$

The elastic critical force, N_{cr}, for the pin-ended column and the buckling length L_e, is determined by:

$$N_{cr} = \frac{(EI)_{eff} \cdot \pi^2}{L_e^2}, \; L_e = L$$

$$N_{cr} = \frac{739,85 \cdot 10^6 \cdot \pi^2}{300^2} = 81134 \text{ kN}$$

The relative slenderness, $\bar{\lambda}$, is:

$$\bar{\lambda} = \sqrt{\frac{10234}{81134}} = 0,355$$

Accordingly, $\bar{\lambda} = 0,355 < 2,0$, and the condition is satisfied.

Remark:

For the effective modulus of elasticity $E_{c,eff}$ = 1041 kN/cm^2, calculated according to EN 1994-1-1, the relative slenderness $\bar{\lambda}$ is 0,373, so the condition $\bar{\lambda} \leq 2,0$ is also satisfied.

The maximum permitted cross-sectional area of the longitudinal reinforcement

The maximum cross-sectional area of the longitudinal reinforcement $A_{s,max}$ that can be used in the calculation should not exceed 6% of the concrete area. This condition is satisfied, see Section 4.

Remark:

All conditions from clause 6.7.3.1, EN 1994-1-1, are satisfied, so this allows the use of the simplified design method for composite columns.

9.2 Check of resistance of the member in axial compression

The resistance of the member subjected only to axial compression can be checked by second-order analysis according to clause 6.7.3.5, EN 1994-1-1, so as to take into account member imperfections. As a simplification in the case of the member subjected only to axial compression, the design value of the axial force N_{Ed} should satisfy the check based on European buckling curves, which can be written in the following format:

$$\frac{N_{Ed}}{\chi \cdot N_{pl,Rd}} \leq 1{,}0$$

The reduction factor χ is given by:

$$\chi = \frac{1}{\Phi + \sqrt{\Phi^2 - \bar{\lambda}^2}}, \text{ but } \chi \leq 1{,}0$$

and

$$\Phi = 0{,}5 \cdot [1 + \alpha \cdot (\bar{\lambda} - \bar{\lambda}_0) + \bar{\lambda}^2] \text{ , s } \bar{\lambda}_0 = 0{,}2$$

Remark:

The relevant buckling curves for cross-sections of composite columns are given in Table 6.5, EN1994-1-1, according to which, circular or rectangular hollow section columns filled with concrete or containing up to 3% reinforcement can be designed using buckling curve *a* with an imperfection factor α = 0,21. However, concrete-filled hollow section columns containing 3% to 6% can be designed using buckling curve *b* with an imperfection factor α = 0,34.

The reinforcement ratio ρ_s is 5,2 %, within the limitations $3\% < \rho_s \leq 6\%$. Therefore from Table 6.5, EN1994-1-1, buckling curve *b* should be used.

From Table 6.3, EN1993-1-1, α = 0,34 for buckling curve *b* so that Φ is:

$$\Phi = 0{,}5 \cdot [1 + 0{,}34 \cdot (0{,}355 - 0{,}2) + 0{,}355^2] = 0{,}589$$

The reduction factor χ is:

$$\chi = \frac{1}{0{,}589 + \sqrt{0{,}589^2 - 0{,}355^2}} = 0{,}94 < 1{,}0$$

Check:

$$\frac{N_{Ed}}{\chi \cdot N_{pl,Rd}} \leq 1{,}0$$

$$\frac{7750}{0{,}94 \cdot 8723} = 0{,}95 < 1{,}0$$

Since $0{,}95 < 1{,}0$, the check of composite column subjected to axial compression, which is implemented by means of the simplified method with the application of European buckling curves, is satisfied.

10. Commentary

The buckling resistance of concrete-filled circular hollow section was calculated from the plastic resistance of the cross-section and the elastic critical load (Euler buckling load) using the EN 1993-1-1 buckling curve *b*. The buckling curves take into account the member imperfections, geometric and structural imperfections, implicitly. In this example, the utilization is 95%.

In this example, the increase in plastic concrete resistance due to confinement effects has not contributed to the overall increase in the plastic resistance of the cross-section $N_{pl,Rd}$. However, when the plastic resistance of the cross-section, taking into account the confinement effects, is governed, then the buckling verification is performed with this plastic resistance. In this case, the relative slenderness $\bar{\lambda}$ is calculated using the characteristic plastic resistance of cross-section as:

$$N_{pl,Rk} = \eta_a \cdot A_a \cdot f_{yk} + A_c \cdot f_{ck}(1 + \eta_c \cdot \frac{t}{d} \cdot \frac{f_y}{f_{ck}}) + A_s \cdot f_{sk}$$

and the iterative procedure is needed for its calculation.

To avoid the iterative procedure for calculation the relative slenderness $\bar{\lambda}$, it is permitted to use the following expression for the characteristic plastic resistance:

$$N_{pl,Rk} = A_a \cdot f_{yk} + A_c \cdot f_{ck} + A_s \cdot f_{sk}$$

The effects of confinement on the resistance enhancement of concrete depend on the slenderness of the composite columns and are significant only in stocky columns, $\bar{\lambda} < 0,2$. Also, the effects of confinement decrease as the bending moments are applied and are significant only for $e/d < 0,05$.

It is important to note that imperfections are unavoidable and that they result in deviations from the behaviour of member subject to “perfectly centred load”. Imperfections may be divided into geometrical imperfections and structural imperfections. The influence of geometrical and structural imperfections can be allowed for by equivalent geometrical imperfections which are given in Table 6.5, EN 1994-1-1, and depend on the buckling curve. Equivalent imperfections can be used directly as an eccentricity of the axial force for calculating the design moment. In that case the axially loaded column can be verified using second-order analysis according to clause 6.7.3.6, EN 1994-1-1. The use of buckling curves is limited to axially loaded members.

C2 Composite column with concrete-filled circular hollow section subject to axial compression, verified using European buckling curves and using second-order analysis taking into account member imperfections

1. Purpose of example

The composite column height 4,5 m is subjected to axial compression. The concrete-filled column consists of the circular hollow section (CHS) filled with concrete. It is assumed to be pinned top and bottom. The column is calculated using the simplified method of design in two ways. The first method is based on the application of European buckling curves, and the second is based on second-order analysis, taking into account the imperfections. The purpose of this example is to compare the two methods, which are based on a simplified procedure verification of the composite column. According to EN 1994-1-1 both methods are permitted, and it is considered to be equivalent. However, according to clause 6.7.3.5(1), EN 1994-1-1, the resistance of a member subjected to only axial compression can be verified primarily according to the second-order analysis taking into account the imperfections. Alternatively, the verification of the member is allowed using the European buckling curves.

The calculation is based on these assumptions:

a) When calculating the resistance of the member subjected to axial compression applying second-order analysis taking into account the imperfections, the check of cross-section resistance is calculated for combined compression and uniaxial bending. In that case, the interaction polygon ABCD is used.

b) The second-order effects in determining the action effects are taken into account by means of an approximate method according to clause 6.7.3.4, EN 1994-1-1. This means that design bending moments are calculated according to first-order analysis multiplied by the correction factor *k*.

2. Static system, cross-section and design action effects

Actions

Permanent action $N_G = 3000$ kN

Variable action $N_Q = 1300$ kN

Composite Structures according to Eurocode 4. Worked Examples.
First Edition. Darko Dujmović, Boris Androić, Ivan Lukačević.

Design action effect:

$$N_{Ed} = \gamma_G \cdot N_G + \gamma_G \cdot N_Q$$

$$N_{Ed} = 1{,}35 \cdot 3000 + 1{,}50 \cdot 1300 = 6000 \text{ kN}$$

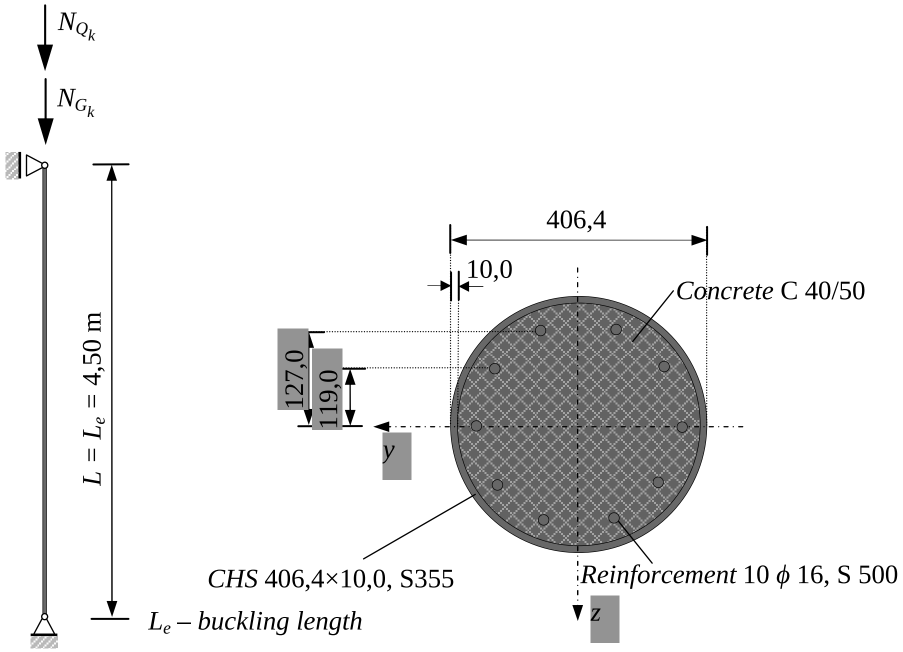


Figure C2.1 *Static system and cross-section*

3. Properties of materials

Concrete strength class: C 40/50 $f_{ck} = 40 \text{ N/mm}^2$

$$f_{cd} = \frac{f_{ck}}{\gamma_c} = \frac{40}{1{,}5} = 26{,}7 \text{ N/mm}^2$$

$$E_{cm} = 35000 \text{ N/mm}^2$$

Structural steel: S355 $f_{yk} = 355 \text{ N/mm}^2$

$$f_{yd} = \frac{f_{yk}}{\gamma_M} = \frac{355}{1{,}0} = 355 \text{ N/mm}^2$$

$$E_a = 210000 \text{ N/mm}^2$$

Reinforcement: *ductility class B or C* $f_{sk} = 460 \text{ N/mm}^2$

$$f_{sd} = \frac{f_{sk}}{\gamma_s} = \frac{460}{1,15} = 400 \text{ N/mm}^2$$

$$E_S = 210000 \text{ N/mm}^2$$

4. Geometrical properties of the cross-section

4.1 Selection of the steel cross-section and reinforcement

It is assumed that only axial compression load is applied, and the required fire resistance is R30.

Since the composite columns are used in building, it is expected settings for fire resistance will be required.

According to the literature [40] the trial column diameter, d_{trial}, is determined approximately according to the following formula:

$$d_{trial} = [\frac{t_{fire} \cdot (L_e - 1000)}{0,08 \cdot (f_{ck} + 20)} \cdot \sqrt{N_{Ed}}\,]^{0,4}$$

With t_{fire} = 30 min, which corresponds to the required fire resistance R30 in minutes, the trial diameter is obtained as follows:

$$d_{trial} = [\frac{30 \cdot (4500 - 1000)}{0,08 \cdot (40 + 20)} \cdot \sqrt{6000}\,]^{0,4} = 310 \text{ mm}$$

The CHS with the diameter d = 406,4 mm is selected. For a concrete-filled circular hollow section, the minimum wall thickness is determined from the condition of the local buckling, Table 6.3, EN 1994-1-1:

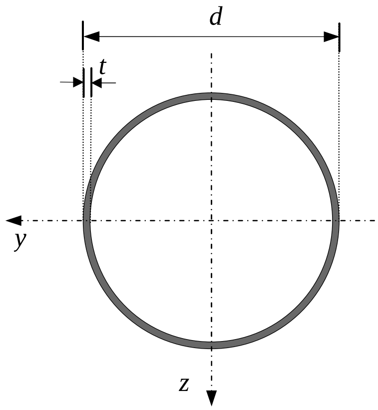
$$\max(\frac{d}{t}) = 90 \cdot \frac{235}{f_y}$$

$$\max(\frac{d}{t}) = 90 \cdot \frac{235}{355} = 59,6$$

The minimum wall thickness is obtained as:

$$t_{\min} = \frac{d}{59,6} = \frac{406,4}{59,6} = 6,82 \text{ mm}$$

Thus, the minimum required wall thickness of the circular tube is 7,0 mm. The CHS 406,4×10,0 is selected. The selected circular hollow section with the dimensions is shown in Figure C2.2.



d = 406,4 mm
t = 10,00 mm
A_a = 124,5 cm^2
I_a = 24476 cm^4
$W_{pl,a}$ = 1572 cm^3

Figure C2.2 *Circular hollow section*

Remark:

In concrete-filled hollow sections no longitudinal reinforcement is normally necessary. However, if the design for fire resistance is required, which is the case in this example, longitudinal reinforcement can be used, clause 6.7.5.2 (1), EN 1994-1-1.

In this example, the selected reinforcement is 10 bars with the diameter of 16 mm.

The cross-sectional area of the structural steel section 406,4×10,0 is:

$$A_a = 124{,}5 \text{ cm}^2$$

The cross-sectional area of the reinforcement with 10 bars of 16 mm diameter is:

d_{bar} = 16 mm A_{bar} = 2,01 cm^2

$$A_s = 10 \cdot A_{bar} = 10 \cdot 2{,}01 = 20{,}1 \text{ cm}^2$$

The cross-sectional area of the concrete is:

$$A_c = \pi (d - 2 \cdot t)^2 \cdot \frac{1}{4} - A_s$$

$$A_c = \pi(40,64 - 2 \cdot 1,00)^2 \cdot \frac{1}{4} - 20,10$$

$$A_c = 1172,6 - 20,10 = 1153 \text{ cm}^2$$

The ratio of reinforcement area to concrete area is:

$$\rho_s = \frac{A_s}{A_c} = \frac{20,1}{1153} = 0,017 \qquad \rho_s = 1,7\,\%$$

$$\rho_s = 1,7\,\% < 6\,\%$$

The limit of 6% in clause 6.7.3.1 (1), EN 1994-1-1, on the reinforcement is satisfied.

Remark:

According to clause 6.7.3.1(3), EN 1994-1-1, the ratio of reinforcement area to concrete area, ρ_s, should not exceed 6%.

4.2 Cross-sectional areas

Structural steel

$$A_a = 124,5 \text{ cm}^2$$

Reinforcement

$$A_s = 10 \cdot \frac{\pi \cdot d_{bar}^2}{4}$$

$$A_s = 10 \cdot \frac{\pi \cdot 1,6^2}{4} = 20,1 \text{ cm}^2$$

Concrete

$$A_c = \pi \cdot \frac{d^2}{4} - A_a - A_s$$

$$A_c = \pi \cdot \frac{40,64^2}{4} - 124,5 - 20,1 = 1153 \text{ cm}^2$$

4.3 Second moments of area

Structural steel

$$I_a = 24476 \ \text{cm}^4$$

Reinforcement

It is assumed that there are stirrups 8 mm in diameter around the longitudinal reinforcement and that the concrete cover is 50 mm, see Figure C2.3.

$$d_c = d - 2 \cdot t = 406,4 - 2 \cdot 10 = 386,4 \,\text{mm}$$

$$R = \frac{d_c}{2} - 50 - \frac{d_{bar}}{2} - 8$$

$$R = \frac{386,4}{2} - 50 - \frac{16}{2} - 8 = 127 \ \text{mm}$$

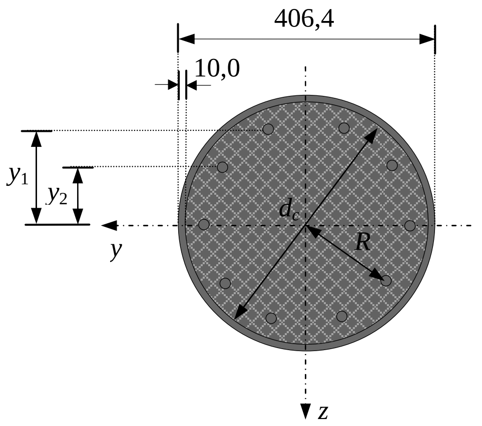


Figure C2.3 *Composite column cross-section*

$$y_1 = R - \frac{d_{bar}}{2} = 127 - \frac{16}{2} = 119 \ \text{mm}$$

$$y_2 = R \cdot \sin\frac{360°}{10} = 127 \cdot \sin 36° = 75 \ \text{mm}$$

$$I_s = 4 \cdot A_{bar} \cdot y_1^2 + 4 \cdot A_{bar} \cdot y_2^2$$

$$I_s = 4 \cdot 2{,}01 \cdot 11{,}9^2 + 4 \cdot 2{,}01 \cdot 7{,}5^2$$

$$I_s = 1591 \text{ cm}^4$$

Concrete

$$I_c = \frac{\pi \cdot (d - 2 \cdot t)^4}{64} - I_s$$

$$I_c = \frac{\pi \cdot (40{,}64 - 2 \cdot 1{,}0)^4}{64} - 1591 = 107834 \text{ cm}^4$$

4.4 Plastic section moduli

Structural steel

$$W_{pl,a} = 1572 \text{ cm}^3$$

Reinforcement

$$W_{pl,s} = 4 \cdot y_1 \cdot A_{bar} + 4 \cdot y_2 \cdot A_{bar}$$

$$W_{pl,s} = 4 \cdot 11{,}9 \cdot 2{,}01 + 4 \cdot 7{,}5 \cdot 2{,}01 = 156 \text{ cm}^3$$

Concrete

$$W_{pl,c} = \frac{(d - 2 \cdot t)^3}{6} - W_{pl,s}$$

$$W_{pl,c} = \frac{(40{,}64 - 2 \cdot 1{,}0)^3}{6} - 156 = 9459 \text{ cm}^3$$

5. Steel contribution ratio

According to clause 6.7.3.3(1), EN 1994-1-1, the steel contribution ratio, δ, is defined as:

$$\delta = \frac{A_a \cdot f_{yd}}{N_{pl,Rd}}$$

The term $A_a \cdot f_{yd}$ is the contribution of the structural steel section to the plastic resistance of the composite section to axial force. The design plastic resistance of the composite section to axial force $N_{pl,Rd}$ is calculated according to clause 6.7.3.2(1), EN 1994-1-1.

According to 6.7.1(4), EN 1994-1-1, the steel contribution ratio, δ, must satisfy the following conditions:

$0{,}2 \leq \delta \leq 0{,}9$

If δ is less than 0,2, the column should be designed as a reinforced concrete member according to EN 1992-1-1. If δ is larger than 0,9, the concrete is ignored in the calculations, and the column is designed as a structural steel member according to EN 1993-1-1.

The term $A_a \cdot f_{yd}$ is the contribution of the structural steel section to the plastic resistance of composite section to axial force:

$$A_a \cdot f_{yd} = 124{,}5 \cdot 35{,}5 = 4420 \text{ kN}$$

The plastic resistance of the composite section to axial force is:

$$N_{pl,Rd} = A_a \cdot f_{yd} + A_c \cdot f_{cd} + A_s \cdot f_{sd}$$

$$N_{pl,Rd} = 124{,}5 \cdot 35{,}5 + 1153 \cdot 2{,}67 + 20{,}1 \cdot 40{,}0$$

$$N_{pl,Rd} = 8302 \text{ kN}$$

According to 6.7.1(4), EN 1994-1-1, the steel contribution ratio, δ, must satisfy the following conditions:

$0{,}2 \leq \delta \leq 0{,}9$

The steel contribution ratio, δ, is:

$$\delta = \frac{A_a \cdot f_{yd}}{N_{pl,Rd}} = \frac{4420}{8302} = 0{,}532$$

Since the limits $0{,}2 < \delta = 0{,}532 < 0{,}9$ are satisfied, the column can be classified as a composite column, and the provisions of EN 1994-1-1 can be used for the dimensioning.

Remark:

The confinement effects of the concrete-filled circular tube can result in an increase of the cross-sectional plastic resistance, $N_{pl,Rd}$. However, the value of δ is usually not significantly changed. This means that for the calculation of steel contribution ratio, δ, we can use the following expression for the cross-sectional plastic resistance, $N_{pl,Rd}$:

$$N_{pl,Rd} = A_a \cdot f_{yd} + A_c \cdot f_{cd} + A_s \cdot f_{sd}$$

6. Local buckling

According to clause 6.7.1(9), EN 1994-1-1, for concrete-filled circular hollow cross-section, the effect of local buckling can be ignored if the following condition is satisfied:

$$\max(\frac{d}{t}) = 90 \cdot \frac{235}{f_y}$$

For the selected cross-section, the maximum slenderness is:

$$\max(\frac{d}{t}) = \frac{406,4}{10,0} = 40,64$$

The required condition is:

$$90 \cdot \frac{235}{f_y} = 90 \cdot \frac{235}{355} = 59,6$$

Since $40,64 < 59,6$, the condition is satisfied. The effect of local buckling can be neglected.

7. Effective modulus of elasticity for concrete

For long-term loading creep and shrinkage are taken into account in design by a reduced flexural stiffness of the composite cross-section. Due to the influence of long-term creep effects on the effective elastic stiffness, the modulus of elasticity of the concrete, E_{cm}, should be reduced to the value $E_{c,eff}$ in accordance with following equation:

$$E_{c,eff} = \frac{E_{cm}}{1+(\frac{N_{G,Ed}}{N_{Ed}})\cdot\varphi_t}$$

where:

$\varphi_t = \varphi(t, t_0)$ is the creep coefficient, defining the creep between times t and t_0, related to elastic deformation at 28 days,
$\varphi_t = \varphi(\infty, t_0)$ is the final creep coefficient,
t is the age of the concrete at the time considered,
t_0 is the age of the concrete at loading,
N_{Ed} is the axial design force,
$N_{G,Ed}$ is the permanent part of the axial design force N_{Ed}, $N_{G,Ed} = \gamma_G \cdot N_{Gk}$.

For the calculation of the creep coefficient φ (t, t_0), the following is valid:

- the perimeter of that part which is exposed to drying, u

$u = d \cdot \pi$

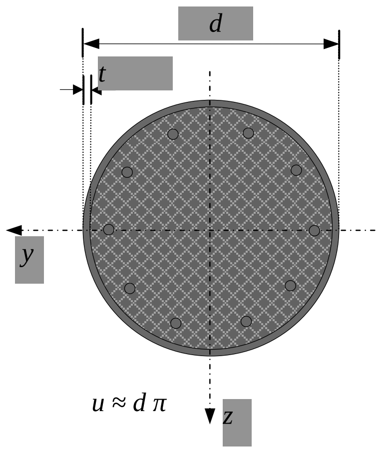


Figure C2.4 *Perimeter which is “exposed” to drying*

$u = 40{,}64 \cdot \pi = 127{,}7\,\text{cm}$

- the notional size of the cross-section, h_0

$$h_0 = \frac{2\cdot A_c}{u} = \frac{2\cdot 1153}{127{,}7} = 18{,}1\,cm = 181\,\text{mm}$$

- t_0 = 28 days,

- inside conditions, the ambient relative humidity RH 50 %,
- the concrete strength class C 40/50
- the type of cement – cement class N, strength class 32,5 R.

The final value of creep coefficient $\varphi(\infty, t_0)$ is determined using the nomogram shown in Figure 3.1, EN 1992-1-1. The process of determining the final value of the creep coefficient, taking into account these assumptions, is given in Figure C2.5.

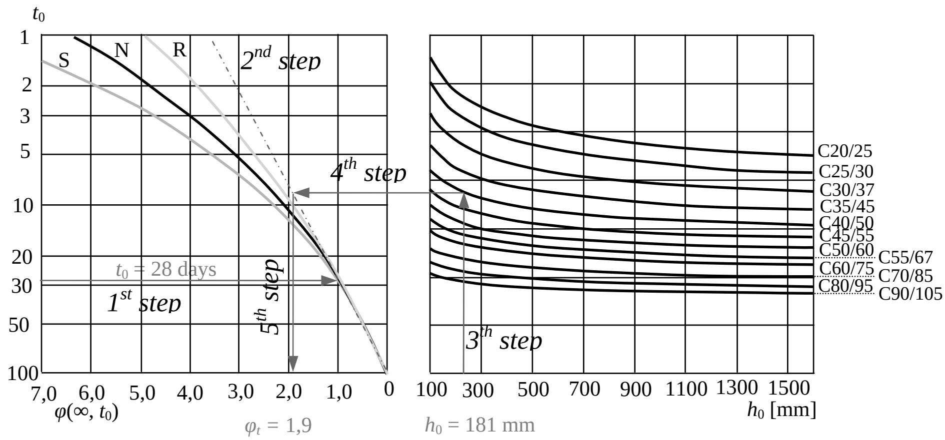


Figure C2.5 *Method for determining the creep coefficient*

The final value of creep coefficient $\varphi(\infty, t_0)$, found from Figure C2.5, is:

$$\varphi_t = \varphi(t, t_0) = 1,9$$

The design force of the permanent load, $N_{G,Ed}$, and the total design force, N_{Ed}, are:

$$N_{G,Ed} = \gamma_G \cdot N_{Gk}$$

$$N_{G,Ed} = 1,35 \cdot 3000 = 4050 \text{ kN}$$

$$N_{Ed} = 1,35 \cdot N_{Gk} + 1,50 \cdot N_{Qk}$$

$$N_{Ed} = 1,35 \cdot 3000 + 1,50 \cdot 1300 = 4050 + 1950 = 6000 \text{ kN}$$

Accordingly, the value of $E_{c,eff}$ is:

$$E_{c,eff} = \frac{E_{cm}}{1+(\frac{N_{G,Ed}}{N_{Ed}})\cdot\varphi_t} = \frac{3500}{1+(\frac{4050}{6000})\cdot 1,9} = 1533 \text{ kN/cm}^2$$

Further calculation is performed with the effective modulus of elasticity of concrete $E_{c,eff}$ = 1533 kN/m^2.

Remark:

It should be noted that a conservative estimation of effective modulus of elasticity is obtained according to EN 1994-1-1 because in the case of concrete-filled hollow section the drying of the concrete is significantly reduced by the steel section (see example C1). However, in the case of concrete-filled circular hollow section the dimensioning of composite columns is rarely sensitive to the influence of creep coefficient φ_t on the effective modulus elasticity of concrete $E_{c,eff}$. With this statement the conservative estimation of $E_{c,eff}$, according to EN 1994-1-1, can be justified in the case of concrete-filled hollow sections.

8. Resistance of the cross-section to compressive axial force

8.1 Plastic resistance of the cross-section without confinement effect

The design plastic resistance of the composite cross-section to axial compressive force, $N_{pl,Rd}$, is given by the sum of the design resistances of components as follows:

$$N_{pl,Rd} = A_a \cdot f_{yd} + 0,85 \cdot A_c \cdot f_{cd} + A_s \cdot f_{sd}$$

Remark:

The coefficient of 0,85 can be replaced with the value 1,0 due to better curing conditions in the case of concrete-filled hollow sections.

The design plastic resistance of composite cross-section to compressive axial force, $N_{pl,Rd}$, is calculated according to the corrected expression:

$$N_{pl,Rd} = A_a \cdot f_{yd} + A_c \cdot f_{cd} + A_s \cdot f_{sd}$$

$$N_{pl,Rd} = 124,5 \cdot 35,5 + 1153 \cdot 2,67 + 20,1 \cdot 40,0$$

$$N_{pl,Rd} = 4419 + 3079 + 804 = 8302 \text{ kN}$$

The characteristic value of the plastic resistance of the composite cross-section to compressive axial force, $N_{pl,Rk}$, is:

$$N_{pl,Rk} = A_a \cdot f_{yk} + A_c \cdot f_{ck} + A_s \cdot f_{sk}$$

$$N_{pl,Rk} = 124,5 \cdot 35,5 + 1153 \cdot 4,00 + 20,1 \cdot 46,0$$

$$N_{pl,Rk} = 4420 + 4612 + 925 = 9957 \text{ kN}$$

8.2 Plastic resistance of the cross-section taking into account the confinement effect

For concrete-filled tubes of circular cross-section, the concrete component develops a higher strength because of the confinement from the steel section, see clause 6.7.3.2, EN 1994-1-1 and example C1.

This increase in the strength of concrete can be taken into account in the calculation if the relative slenderness, $\bar{\lambda}$, does not exceed 0,5 and the ratio e/d < 0,1. The eccentricity of loading, e, is determined by the ratio M_{Ed}/N_{Ed}, and d is the outer diameter of the column. In this case is $e = 0$.

The relative slenderness, $\bar{\lambda}$, is calculated in Section 9.1 and is $\bar{\lambda} = 0,562$.

Since $\bar{\lambda} = 0,562 > 0,5$, the condition is not satisfied. This means that the increase in strength of the concrete due to the confinement from the steel section is not taken into account.

Further calculation is performed with the plastic resistant of the cross-section to compression $N_{pl,Rd}$ = 8302 kN.

9. Resistance of the member in axial compression – using European buckling curves

9.1 Verification of conditions for using the simplified design method

The cross-section of the composite column should be doubly symmetrical and uniform along the entire length of the column.

This condition is satisfied.

Relative slenderness

To apply the simplified method it is necessary to satisfy the following condition:

$$\bar{\lambda} \leq 2{,}0$$

Relative slenderness, $\bar{\lambda}$, is determined by the following expression:

$$\bar{\lambda} = \sqrt{\frac{N_{pl,Rk}}{N_{cr}}}$$

For the determination of the relative slenderness $\bar{\lambda}$ and the elastic critical force N_{cr}, it is necessary to calculate the value of the effective flexural stiffness of the cross-section of the composite column, $(EI)_{eff}$, according to the expression:

$$(EI)_{eff} = E_a \cdot I_a + E_s \cdot I_s + K_e \cdot E_{c,eff} \cdot I_c$$

With the correction factor $K_e = 0{,}6$, the value $(EI)_{eff}$ is:

$$(EI)_{eff} = 21000 \cdot 24476 + 21000 \cdot 1591 + 0{,}6 \cdot 1533 \cdot 107834$$

$$(EI)_{eff} = 646{,}59 \cdot 10^6 \text{ kNcm}^2$$

The elastic critical force, N_{cr}, for the pin-ended column and the buckling length L_e, is determined by:

$$N_{cr} = \frac{(EI)_{eff} \cdot \pi^2}{L^2}, \; L_e = L$$

$$N_{cr} = \frac{646{,}59 \cdot 10^6 \cdot \pi^2}{450^2} = 31514 \text{ kN}$$

$$N_{pl,Rk} = 9957 \text{ kN}$$

The relative slenderness, $\bar{\lambda}$, is:

$$\bar{\lambda} = \sqrt{\frac{9957}{31514}} = 0{,}562$$

Accordingly $\bar{\lambda} = 0{,}562 < 2{,}0$, and the condition is satisfied.

Maximum permitted cross-sectional area of the longitudinal reinforcement

The maximum cross-sectional area of the longitudinal reinforcement $A_{s,max}$ that can be taken in the calculation should not exceed 6% of the concrete area. This condition is satisfied, see Section 4.1.

Remark:

All conditions from clause 6.7.3.1, EN 1994-1-1, are satisfied, so this allows the use of the simplified design method for composite columns.

9.2 Check of resistance of the member in axial compression

The resistance of the member subjected only to axial compression can be checked by second-order analysis according to clause 6.7.3.5, EN 1994-1-1, so as to take into account member imperfections. As a simplification in the case of the member subjected only to axial compression, the design value of the axial force N_{Ed} should satisfy the check based on European buckling curves, which can be written in the following format:

$$\frac{N_{Ed}}{\chi \cdot N_{pl,Rd}} \leq 1,0$$

The reduction factor χ is given by:

$$\chi = \frac{1}{\Phi + \sqrt{\Phi^2 - \bar{\lambda}^2}}, \text{ but } \chi \leq 1,0$$

and

$$\Phi = 0,5 \cdot [1 + \alpha \cdot (\bar{\lambda} - \bar{\lambda}_0) + \bar{\lambda}^2], \text{ with } \bar{\lambda}_0 = 0,2$$

Remark:

The relevant buckling curves for cross-sections of composite columns are given in Table 6.5, EN1994-1-1. According to Table 6.5, EN 1994-1-1, circular or rectangular hollow section columns filled with concrete or containing up to 3% reinforcement can be designed using buckling curve *a* with an imperfection factor α = 0,21. However, concrete-filled hollow section columns containing from 3% to 6% can be designed using buckling curve *b* with an imperfection factor α = 0,34.

The reinforcement ratio ρ_s is 1,7 %. Therefore from Table 6.5, EN 1994-1-1, buckling curve *a* should be used.

From Table 6.3, EN1993-1-1, α = 0,21 for buckling curve *a* so that Φ is:

$$\Phi = 0,5 \cdot [1 + 0,21 \cdot (0,562 - 0,2) + 0,562^2] = 0,696$$

The reduction factor χ is:

$$\chi = \frac{1}{0,696 + \sqrt{0,696^2 - 0,562^2}} = 0,90 < 1,0$$

Check:

$$\frac{N_{Ed}}{\chi \cdot N_{pl,Rd}} \leq 1,0$$

$$\frac{6000}{0,90 \cdot 8302} = 0,80 < 1,0$$

Since 0,80 < 1,0, the check of the composite column subjected to axial compression, which is implemented by means of the simplified method with the application of European buckling curves, is satisfied.

Figure C2.6 shows the principle of verification of the member in axial compression using European buckling curves.

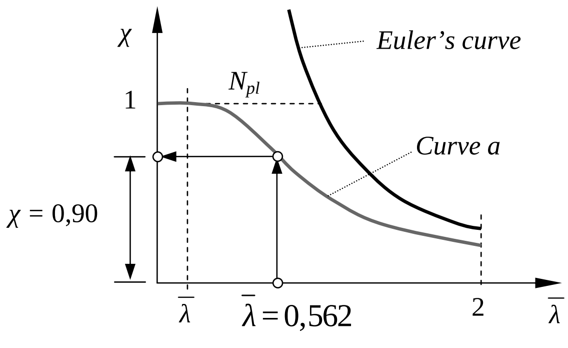


Figure C2.6 *Principle of verification of the member in axial compression using European buckling curves*

10. Resistance of the member in axial compression – using second-order analysis, taking into account member imperfections

10.1 General

In accordance with clause 6.7.3.5 (1), EN 1994-1-1, the verification of the axially loaded column can be carried out using a second-order analysis taking into account the member imperfections. Figure C2.7 illustrates the assessment of the column resistance in accordance with clause 6.7.3.6, EN 1994-1-1. It is necessary to satisfy the following condition:

$$\frac{M_{Ed}}{M_{pl,N,Rd}} = \frac{M_{Ed}}{\mu_d \cdot M_{pl,Rd}} \leq \alpha_M$$

In this expression, M_{Ed} is the greatest of the end moments and the maximum bending moment within the column length. This moment is calculated according to clause 6.7.3.4, EN 1994-1-1, including imperfections (Table 6.5, EN 1994-1-1) and second-order effects if necessary ($\alpha_{cr} > 10$).

The condition can be written in the following form, in accordance with Figure C2.7:

$$M_{Ed} \leq M_{Rd} = \alpha_M \cdot \mu_d \cdot M_{pl,Rd}$$

where:

$M_{pl,N,Rd}$ is the plastic resistance moment taking into account the axial force N_{Ed}, given by $\mu_d{\cdot}M_{pl,Rd}$, see Figure C2.7,

α_M is the coefficient related to bending of a composite column and is taken as 0,9 for steel grades between S235 and S355.

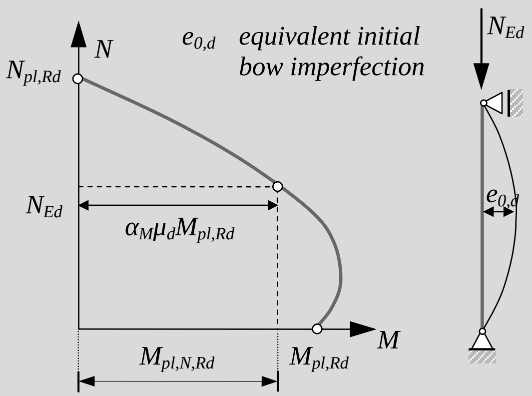


Figure C2.7 *Verification for combined compression and uniaxial bending*

Geometrical and structural imperfections are represented by an equivalent geometrical imperfections $e_{0,d}$ as the initial curvature of member.

10.2 Verification of conditions for using the simplified design method

See Section 9.1.

10.3 Resistance of the cross-section in combined compression and uniaxial bending

Remark:

In order to determine the resistance of the composite cross-section to combined compression and uniaxial bending, it is necessary to produce an axial load – bending moment (*N*–*M*) interaction curve. As a simplification, the interaction curve is replaced by an interaction polygon ACDB, clause 6.7.3.2 (5), EN 1994-1-1. The *N*–*M* interaction polygon ACDB is shown in Figure 6.19, EN 1994-1-1. A modified version of the interaction polygon, which refers to the composite column with concrete-filled circular hollow cross-section, is shown in Figure C2.8.

In order to produce the *N*–*M* interaction polygon, the cross-sectional capacities at points A to D should be determined, assuming the stress distributions indicated, see Figure C2.8.

It should be noted that EN 1994-1-1 does not provide expressions for circular cross-sections filled with concrete.

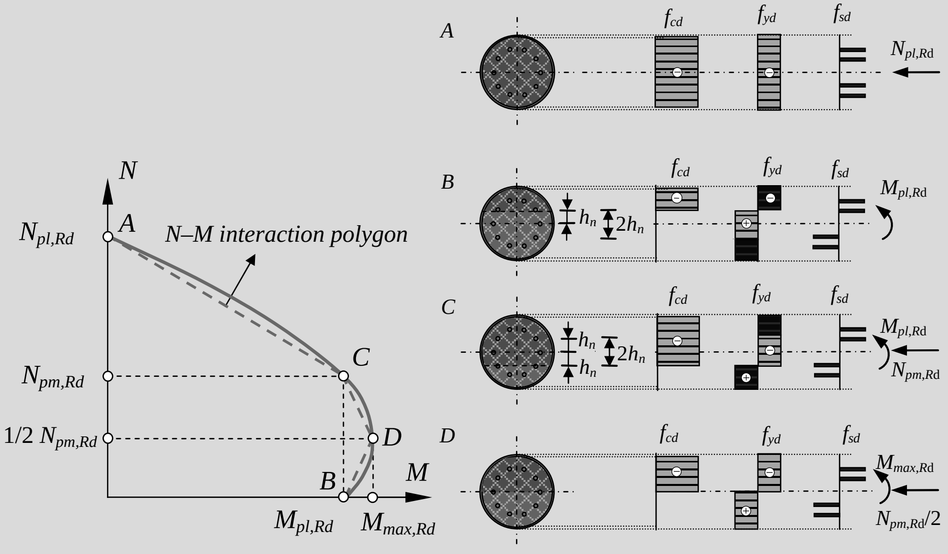


Figure C2.8 *N–M interaction polygon and corresponding stress distributions*

Point A

At point A, only the design plastic resistance of the cross-section is taken into account:

$$N_{pl,Rd} = A_a \cdot f_{yd} + 0,85 \cdot A_c \cdot f_{cd} + A_s \cdot f_{sd}$$

Remark

For concrete-filled hollow sections, the coefficient of 0,85 can be replaced with a value of 1,0 due to better curing conditions.

The design plastic resistance of the composite cross-section to compression, $N_{pl,Rd}$, is calculated according to the corrected expression:

$$N_{pl,Rd} = A_a \cdot f_{yd} + A_c \cdot f_{cd} + A_s \cdot f_{sd}$$

$$N_{pl,Rd} = 124,5 \cdot 35,5 + 1153 \cdot 2,67 + 20,1 \cdot 40,0 = 8302 \text{ kN}$$

Point D

The maximum design plastic resistance moment is determined as:

$$M_{\max Rd} = W_{pl,a} \cdot f_{yd} + W_{pl,c} \cdot f_{cd} + W_{pl,s} \cdot f_{sd}$$

The maximum design plastic resistance moment, M_{maxRd}, at point D is:

$$M_{\max Rd} = W_{pl,a} \cdot f_{yd} + 0,5 \cdot W_{pl,c} \cdot f_{cd} + W_{pl,s} \cdot f_{sd}$$

$$M_{\max Rd} = (1572 \cdot 35,5 + 0,5 \cdot 9459 \cdot 2,67 + 156 \cdot 40,0) \cdot 10^{-2} = 747 \text{ kNm}$$

The design value of the resistance of the concrete to compression, $N_{pm,Rd}$, is:

$$N_{pm,Rd} = \frac{\pi \cdot (d - 2t)^2}{4} \cdot f_{cd} = \frac{\pi \cdot (40,64 - 2 \cdot 1,0)^2}{4} \cdot 2,67 = 3130 \text{ kN}$$

The design axial force at the point of maximum design plastic resistance moment is $0,5 \cdot N_{pm,Rd}$, and therefore is:

$$0,5 \cdot N_{pm,Rd} = 0,5 \cdot 3130 = 1565 \text{ kN}$$

Point C

Calculation of the design plastic resistance moment of the composite section, $M_{pl,Rd}$, is carried out as shown below.

Determination of the position of the neutral axis depth, h_n, when the axial force is zero:

$$h_n = \frac{N_{pm,Rd} - A_{s,n} \cdot (2 \cdot f_{sd} - f_{cd})}{2 \cdot d \cdot f_{cd} + 4 \cdot t \cdot (2 \cdot f_{yd} - f_{cd})}$$

where $A_{s,n}$ is the reinforcement area within h_n. Because it is at this point unknown, assumed to be (initial guess):

$$A_{s,n} = 2 \cdot A_{bar} = 2 \cdot 2{,}01 = 4{,}02 \ \text{cm}^2$$

Thus, for the case when the axial force is equal to zero, h_n is:

$$h_n = \frac{3130 - 4{,}02 \cdot (2 \cdot 40{,}0 - 2{,}67)}{2 \cdot 40{,}64 \cdot 2{,}67 + 4 \cdot 1{,}0 \cdot (2 \cdot 35{,}5 - 2{,}67)} = 5{,}749 \ \text{cm} = 57{,}49 \ \text{mm}$$

Plastic section moduli in region 2·h_n

Generally, the value of $W_{pl,s,n}$ is equal to the value of $W_{pl,s}$. However, the worst case is where only two bars occur within h_n, and these are on the centre line. Accordingly, the value of the plastic section modulus of reinforcement is:

$W_{pl,s,n} = 0 \text{ cm}^3$

The effective plastic section modulus of the concrete is:

$$W_{pl,c,n} = (d - 2 \cdot t) \cdot h_n^2 - W_{pl,s,n}$$

$$W_{pl,c,n} = (40{,}64 - 2 \cdot 1{,}0) \cdot 5{,}749^2 - 0 = 1277 \ \text{cm}^3$$

The plastic section modulus of the steel section is:

$$W_{pl,a,n} = d \cdot h_n^2 - W_{pl,c,n} - W_{pl,s,n}$$

$$W_{pl,a,n} = 40{,}64 \cdot 5{,}749^2 - 1277 - 0 = 66 \ \text{cm}^3$$

The design plastic resistance moment of the composite section, $M_{pl,Rd}$, is calculated as follows:

$$M_{pl,Rd} = M_{\max,Rd} - M_{n,Rd}$$

where:

$$M_{n,Rd} = W_{pl,a,n} \cdot f_{yd} + W_{pl,s,n} \cdot f_{sd} + \frac{W_{pl,c,n} \cdot f_{cd}}{2}$$

$$M_{n,Rd} = (66 \cdot 35{,}5 + 0 \cdot 40{,}0 + \frac{1277 \cdot 2{,}67}{2}) \cdot 10^{-2} = 40{,}48 \text{ kNm}$$

The design plastic resistance moment of the composite section, $M_{pl,Rd}$, is:

$$M_{pl,Rd} = 747 - 40{,}48 = 707 \text{ kNm}$$

Point B

The design value of $M_{pl,Rd}$ has previously been calculated in order to define point C on the *N*–*M* interaction polygon:

$$M_{pl,Rd} = 707 \text{ kNm}$$

Previously calculated values at points A to D should be plotted to produce the *N*–*M* interaction polygon (Figure 6.19, EN 1994-1-1).

According to the interaction polygon ACDB, Figure C2.9, plotted in accordance with Figure 6.19, EN 1994-1-1, the following value $M_{pl,N,Rd}$ is obtained:

$$M_{pl,Rd} : \mu_d \cdot M_{pl,N,Rd} = (N_{pl,Rd} - N_{pm,Rd}) : (N_{pl,Rd} - N_{Ed})$$

$$M_{pl,N,Rd} = \frac{N_{pl,Rd} - N_{Ed}}{N_{pl,Rd} - N_{pm,Rd}} M_{pl,Rd}$$

$$M_{pl,N,Rd} = \frac{8302 - 6000}{8302 - 3130} 707 = 315 \text{ kNm}$$

The value of μ_d is:

$$\mu_d = \frac{M_{pl,N,Rd}}{M_{pl,Rd}} = \frac{315}{707} = 0,445 < 1,0$$

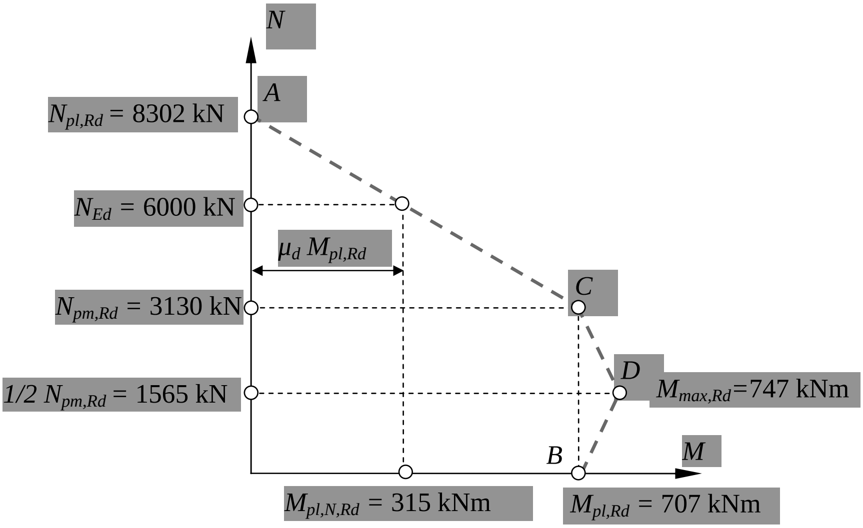


Figure C2.9 *N–M interaction polygon*

All values are calculated that are needed for drawing the N–M interaction polygon, shown in Figure C2.9.

10.4 Calculation of action effects according to second-order analysis

According to clause 6.7.3.4 (3), EN 1994-1-1, which refers to clause 5.2.1(3), EN 1994-1-1, second-order effects can therefore be neglected if the load factor α_{cr}, which is the ratio between the elastic critical load and the corresponding applied loading, for elastic instability of the member exceeds 10.

To calculate α_{cr}, the ends of the column are assumed to be pinned, and α_{cr} is found using the Euler formula for the elastic critical force $N_{cr,eff}$:

$$N_{cr,eff} = \frac{\pi^2 (EI)_{eff,II}}{L_e^2} \quad L_e = L$$

The design value of the effective flexural stiffness $(EI)_{eff,II}$, used to determine the internal forces and moments by second-order analysis, according to clause 6.7.3.4(2), EN 1994-1-1, is defined as:

$$(EI)_{eff,II} = K_0 \cdot (E_a \cdot I_a + E_s \cdot I_s + K_{e,II} \cdot E_{cm} \cdot I_c)$$

where:

$K_{e,II}$ is a correction factor which should be taken as 0,5,
K_0 is a calibration factor which should be taken as 0,9.

The value $E_{c,eff}$ has been used in place of E_{cm} in the expression for $(EI)_{eff,II}$ in order to take into account the long-term effects, in the same way as calculated in Section 7. Accordingly, the value of $E_{c,eff}$ is:

$$E_{c,eff} = 1533 \text{ kN/cm}^2$$

The design value of the effective flexural stiffness $(EI)_{eff,II}$ is:

$$(EI)_{eff,II} = 0,9 \cdot (21000 \cdot 24476 + 21000 \cdot 1591 + 0,5 \cdot 1533 \cdot 107834)$$

$$(EI)_{eff,II} = 567,06 \cdot 10^6 \text{ kNcm}^2$$

The elastic critical force, N_{cr}, for the pin-ended column, is:

$$N_{cr,eff} = \frac{567,06 \cdot 10^6 \cdot \pi^2}{450^2} = 27638 \text{ kN}$$

To check whether the effects of second-order analysis can be neglected, the value of α_{cr} must be higher than 10:

$$\alpha_{cr} = \frac{N_{cr,eff}}{N_{Ed}} = \frac{27638}{6000} = 4,61 < 10$$

The value of α_{cr} is less than 10, so second-order effects must be considered.

According to clause 6.7.3.4(5), EN 1994-1-1, the second-order effects can be calculated by multiplying the greatest first-order design bending moments by a factor k.

Thus, the second-order effects may be considered according to the expression:

$$M_{Ed,II} = M_{Ed,I} \cdot k$$

The factor k is given by:

$$k = \frac{\beta}{1 - N_{Ed} / N_{cr,eff}} \geq 1,0$$

where:

β is an equivalent moment factor given in Table 6.4, EN 1994-1-1,

$N_{cr,eff}$ is the critical axial force for the relevant axis and corresponding to the effective flexural stiffness $(EI)_{eff,II}$, with the effective length taken as the physical length of the column.

The design bending moment from the member imperfections is determined as:

$$M_{Ed,I} = N_{Ed} \cdot e_{0,d}$$

where:

N_{Ed} is the design value of the axial force,

$e_{0,d}$ is the equivalent member imperfection which is given in Table 6.5, EN 1994-1-1, depending on the buckling curve.

Thus, the equivalent member imperfection, for the buckling curve *a*, is:

$$e_{0,d} = \frac{L}{300}$$

$$e_{0,d} = \frac{450}{300} = 1,5 \text{ cm}$$

The mid-length design bending moment due to N_{Ed} is:

$$M_{Ed,I} = N_{Ed} \cdot e_{0,d} = 6000 \cdot 0,015 = 90 \text{ kNm}$$

For the design bending moment from the member imperfection, the equivalent moment factor is:

$$\beta = 1,0$$

The correction factor is therefore:

$$k = \frac{\beta}{1 - N_{Ed} / N_{cr,eff}} = \frac{1,0}{1 - 6000 / 27638} = 1,28 > 1,0$$

Thus, the maximum design bending moment according to the second-order analysis is:

$$M_{Ed,II} = M_{Ed,I} \cdot k = 90 \cdot 1,28 = 115,2 \;\text{ kNm}$$

The effects of the action for the column, calculated according to second-order analysis taking into account the equivalent member imperfection $e_{0,d}$, are shown in Figure C2.10.

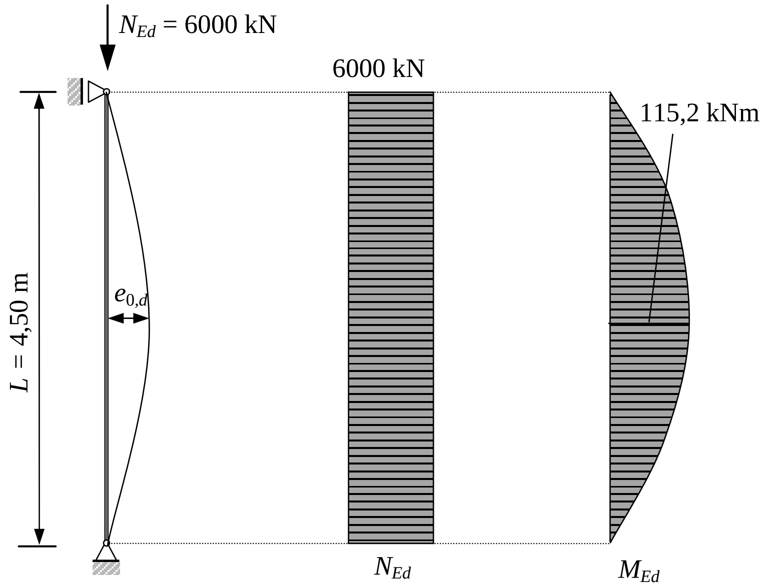


Figure C2.10 *Effects of action for the column, calculated according to second-order analysis*

10.5 Check of the resistance of the member in combined compression and uniaxial bending

It is necessary to satisfy the following condition:

$$\frac{M_{Ed}}{M_{pl,N,Rd}} = \frac{M_{Ed}}{\mu_d \cdot M_{pl,Rd}} \leq \alpha_M$$

The coefficient α_M is taken as 0,9 for steel grades between S235 and S355.

The condition can be written in the following form:

$$\frac{M_{Ed}}{M_{Rd}} \leq 1,0$$

where:

$$M_{Ed} = M_{Ed,II}$$

$$M_{Rd} = \alpha_M \cdot \mu_d \cdot M_{pl,Rd}$$

The maximum design moment according to second-order analysis is:

$$M_{Ed} = M_{Ed,II} = 115,2 \text{ kNm}$$

The design resistance moment M_{Rd} is, see Figure C2.9:

$$M_{Rd} = \alpha_M \cdot \mu_d \cdot M_{pl,Rd} = 0,9 \cdot 0,445 \cdot 707,0 = 283,2 \text{ kNm}$$

Condition:

$$\frac{M_{Ed}}{M_{Rd}} = \frac{115,2}{283,2} = 0,41$$

Since $0,41 < 1,0$, the condition is satisfied.

11. Commentary

Dimensioning of the composite column in axial compression was carried out using the simplified method in two ways recommended in EN 1994-1-1:

- design based on the application of European buckling curves in which are “embedded” imperfections,
- design based on second-order analysis taking into account the member imperfections.

In the first case the utilization is 80%, while in the second case the utilization is 41%.

For most composite columns, the simplified method requires the application of second-order analysis taking into account the member imperfections, clause 6.7.3.4, EN 1994-1-1. For a member subject only to axial compression, clause 6.7.3.5(2), EN 1994-1-1, enables buckling curves to be used. For columns in which this kind of check can be implemented, this is a useful simplification because these curves allow for member imperfections. The buckling curves are also useful as a preliminary check for columns with end moment. If the resistance to the axial force N_{Ed} is not sufficient, the considered column is clearly inadequate.

C3 Composite column with concrete filled circular hollow section subject to axial compression and uniaxial bending

1. Purpose of example

This example demonstrates the design of a composite column subject to axial compressive load and bending moment. The concrete-filled column consists of the circular hollow section filled with concrete. The considered composite column is isolated from the framework. It is assumed to be pinned top and bottom. The design load for the considered column on the ground floor of a building is made up of the variable and permanent load on the floor area immediately over the column and the load transmitted by the columns above. The total design axial load and the design bending moment are applied at the top of column, see Figure C3.1.

The column is calculated using the simplified method of design in accordance with clause 6.7.3.6, EN 1994-1-1. Second-order effects are included in two ways, by using a first-order analysis modified with appropriate amplification according to clause 6.7.3.4(5), EN 1994-1-1, and by using exact second-order analysis. According to clause 5.3.2.3(1), EN 1994-1-1, a design value of equivalent initial bow imperfection has been taken from Table 6.5, EN 1994-1-1. The effects of the applied moment at the top of the column and the moment due to initial member imperfections have been combined. The maximum combined moment at either the mid-span or the support has been used as the design bending moment. For checking individual composite column, according to clause 5.3.2.1(3), EN 1994-1-1, an explicit treatment of imperfections is always required because the resistance expressions are for cross-sections only and action effects due to these imperfections are not allowed for.

Actions

Permanent action $N_{G_k} = 3000 \text{ kN}$

$M_{G_k} = 22{,}20 \text{ kNm}$

Variable action $N_{Q_k} = 1300 \text{ kN}$

$M_{Q_k} = 20{,}00 \text{ kNm}$

Composite Structures according to Eurocode 4. Worked Examples.
First Edition. Darko Dujmović, Boris Androić, Ivan Lukačević.

2. Static system, cross-section and design action effects

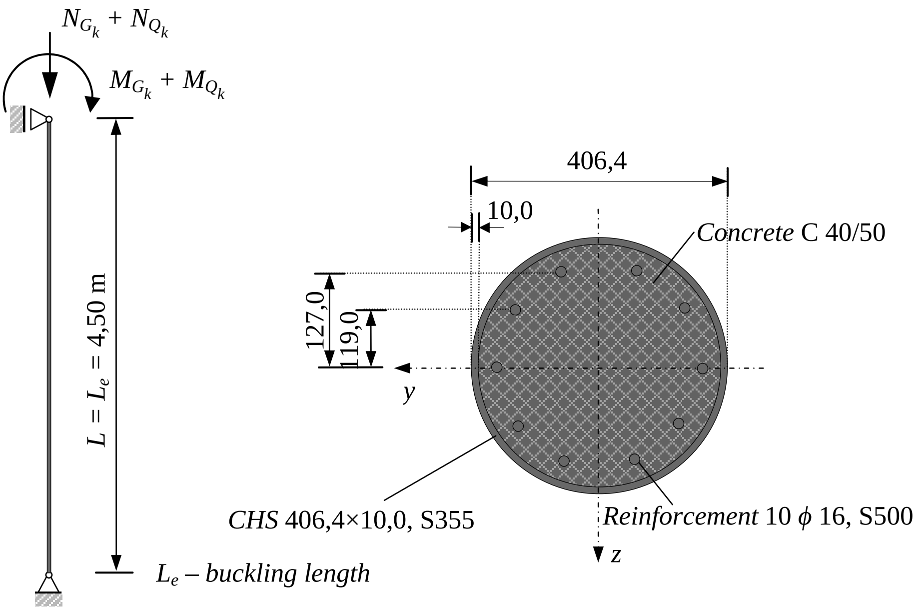


Figure C3.1 *Static system and cross-section*

Design action effect:

Axial force

$$N_{Ed} = \gamma_G \cdot N_{G_k} + \gamma_Q \cdot N_{Q_k}$$

$$N_{Ed} = 1{,}35 \cdot N_{G_k} + 1{,}50 \cdot N_{Q_k}$$

$$N_{Ed} = 1{,}35 \cdot 3000 + 1{,}50 \cdot 1300 = 4050 + 1950 = 6000 \text{ kN}$$

Bending moment at the top of the column:

$$M_{Ed} = \gamma_G \cdot M_{G_k} + \gamma_Q \cdot M_{Q_k}$$

$$M_{Ed} = 1{,}35 \cdot M_{G_k} + 1{,}50 \cdot M_{Q_k}$$

$$M_{Ed} = 1{,}35 \cdot 22{,}20 + 1{,}50 \cdot 20{,}00 = 60 \text{ kNm}$$

Bending moment at mid-height of the column:

$$M_{Ed} = 0,5 \cdot (\gamma_G \cdot M_{G_k} + \gamma_Q \cdot M_{Q_k})$$

$$M_{Ed} = 0,5 \cdot (1,35 \cdot M_{G_k} + 1,50 \cdot M_{Q_k})$$

$$M_{Ed} = 0,5 \cdot (1,35 \cdot 22,20 + 1,50 \cdot 20,00) = 30 \text{ kNm}$$

Shear force

$$V_{Ed} = (\gamma_G \cdot M_{G_k} + \gamma_Q \cdot M_{Q_k}) / L$$

$$V_{Ed} = (1,35 \cdot M_{G_k} + 1,50 \cdot M_{Q_k}) / L$$

$$V_{Ed} = (1,35 \cdot 22,20 + 1,50 \cdot 20,00) / 4,5 = 13,3 \text{ kN}$$

Denotation of imperfections

Imperfection about y-y axis is denoted by $e_{0,z}$.

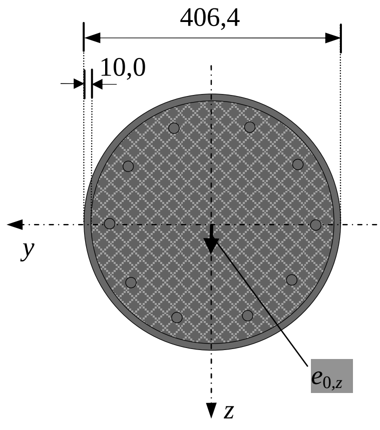


Figure C3.2 *Denotation of imperfection*

3. Properties of materials

Concrete strength class: C 40/50

$$f_{ck} = 40 \text{ N/mm}^2$$

$$f_{cd} = \frac{f_{ck}}{\gamma_c} = \frac{40}{1,5} = 26,7 \text{ N/mm}^2$$

$$E_{cm} = 35000 \text{ N/mm}^2$$

Structural steel: S355

$$f_{yk} = 355 \text{ N/mm}^2$$

$$f_{yd} = \frac{f_{yk}}{\gamma_M} = \frac{355}{1,0} = 355 \text{ N/mm}^2$$

$$E_a = 210000 \text{ N/mm}^2$$

Reinforcement: *ductility class B or C*

$$f_{sk} = 460 \text{ N/mm}^2$$

$$f_{sd} = \frac{f_{sk}}{\gamma_s} = \frac{460}{1,15} = 400 \text{ N/mm}^2$$

$$E_S = 210000 \text{ N/mm}^2$$

4. Geometrical properties of the cross-section

4.1 Selection of the steel cross-section and reinforcement

It is assumed that only the axial compression load is applied, and the required fire resistance is R30.

Since the composite columns are used in buildings, fire resistance is required.

According to the literature [40] the trial column diameter, d_{trial}, is determined approximately according to the following formula:

$$d_{trial} = [\frac{t_{fire} \cdot (L_e - 1000)}{0,08 \cdot (f_{ck} + 20)} \cdot \sqrt{N_{Ed}}]^{0,4}$$

With t_{fire} = 30 min, which corresponds to the required fire resistance R30 in minutes, the trial diameter is obtained as:

$$d_{trial} = [\frac{30 \cdot (4500 - 1000)}{0,08 \cdot (40 + 20)} \cdot \sqrt{6000}]^{0,4} = 310 \text{ mm}$$

The CHS with diameter d = 406,4 mm is selected.

For a concrete-filled circular hollow section, the minimum wall thickness is determined from the condition of the local buckling, Table 6.3, EN 1994-1-1:

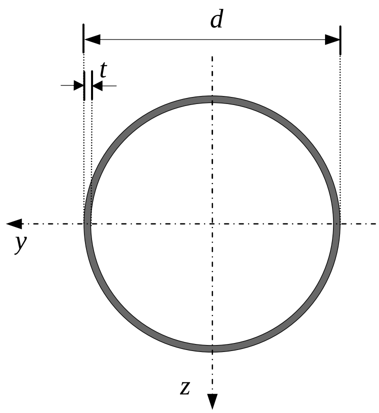
$$\max(\frac{d}{t}) = 90 \cdot \frac{235}{f_y}$$

$$\max(\frac{d}{t}) = 90 \cdot \frac{235}{355} = 59{,}6$$

The minimum wall thickness is obtained as:

$$t_{\min} = \frac{d}{59{,}6} = \frac{406{,}4}{59{,}6} = 6{,}82$$

Thus, the minimum required wall thickness of the circular tube is 7,0 mm. The CHS 406,4×10,0 is selected. The selected circular hollow section with the dimensions is shown in Figure C3.3.



d = 406,4 mm
t = 10,0 mm
A_a = 124,5 cm^2
I_a = 24476 cm^4
$W_{pl,a}$ = 1572 cm^3

Figure C3.3 *Circular hollow section*

Remark:

In concrete-filled hollow sections no longitudinal reinforcement is normally necessary. However, if the design for fire resistance is required, which is the case in this example, longitudinal reinforcement can be used, clause 6.7.5.2 (1), EN 1994-1-1.

In this example, the selected reinforcement is 10 bars with a diameter of 16 mm.

The cross-sectional area of the structural steel section 406,4×10,0 is:

$$A_a = 124{,}5 \text{ cm}^2$$

The cross-sectional area of reinforcement with 10 bars of 16 mm diameter is:

d_{bar} = 16 mm, A_{bar} = 2,01 cm^2

$$A_s = 10 \cdot A_{bar} = 10 \cdot 2{,}01 = 20{,}1 \ \text{cm}^2$$

The cross-sectional area of concrete is:

$$A_c = \pi (d - 2 \cdot t)^2 \cdot \frac{1}{4} - A_s$$

$$A_c = \pi (40{,}64 - 2 \cdot 1{,}00)^2 \cdot \frac{1}{4} - 20{,}10$$

$$A_c = 1172{,}6 - 20{,}10 = 1153 \ \text{cm}^2.$$

The ratio of reinforcement area to concrete area is:

$$\rho_s = \frac{A_s}{A_c} = \frac{20{,}1}{1153} = 0{,}017 \qquad \rho_s = 1{,}7\,\%$$

$$\rho_s = 1{,}7\,\% < 6\,\%$$

The limit of 6% in clause 6.7.3.1 (1), EN 1994-1-1, on the reinforcement is satisfied.

Remark:

According to clause 6.7.3.1(3), EN 1994-1-1, the ratio of reinforcement area to concrete area, ρ_s, should not exceed 6%.

4.2 Cross-sectional areas

Structural steel

$$A_a = 124{,}5 \ \text{cm}^2$$

Reinforcement

$$A_s = 10 \cdot \frac{\pi \cdot d_{bar}^2}{4}$$

$$A_s = 10 \cdot \frac{\pi \cdot 1{,}6^2}{4} = 20{,}1 \ \text{cm}^2$$

Concrete

$$A_c = \pi \cdot \frac{d^2}{4} - A_a - A_s$$

$$A_c = \pi \cdot \frac{40{,}64^2}{4} - 124{,}5 - 20{,}1 = 1153 \ \text{cm}^2$$

4.3 Second moments of area

Structural steel

$$I_a = 24476 \ \text{cm}^4$$

Reinforcement

It is assumed that there are stirrups 8 mm in diameter around the longitudinal reinforcement and a concrete cover of 50 mm, see Figure C3.4.

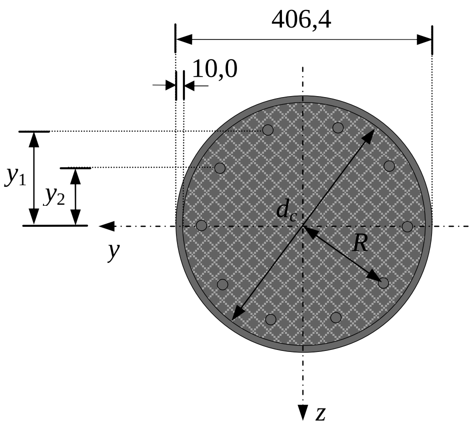


Figure C3.4 *Composite column cross-section*

$$d_c = d - 2 \cdot t = 406{,}4 - 2 \cdot 10 = 386{,}4 \ \text{mm}$$

$$R = \frac{d_c}{2} - 50 - \frac{d_{bar}}{2} - 8$$

$$R = \frac{386{,}4}{2} - 50 - \frac{16}{2} - 8 = 127 \ \text{mm}$$

$$y_1 = R - \frac{d_{bar}}{2} = 127 - \frac{16}{2} = 119 \text{ mm}$$

$$y_2 = R \cdot \sin\frac{360^\circ}{10} = 127 \cdot \sin 36^\circ = 75 \text{ mm}$$

$$I_s = 4 \cdot A_{bar} \cdot y_1^2 + 4 \cdot A_{bar} \cdot y_2^2$$

$$I_s = 4 \cdot 2{,}01 \cdot 11{,}9^2 + 4 \cdot 2{,}01 \cdot 7{,}5^2$$

$$I_s = 1591 \text{ cm}^4$$

Concrete

$$I_c = \frac{\pi \cdot (d - 2 \cdot t)^4}{64} - I_s$$

$$I_c = \frac{\pi \cdot (40{,}64 - 2 \cdot 1{,}0)^4}{64} - 1591 = 107834 \text{ cm}^4$$

4.4 Plastic section moduli

Structural steel

$$W_{pl,a} = 1572 \text{ cm}^3$$

Reinforcement

$$W_{pl,s} = 4 \cdot y_1 \cdot A_{bar} + 4 \cdot y_2 \cdot A_{bar}$$

$$W_{pl,s} = 4 \cdot 11{,}9 \cdot 2{,}01 + 4 \cdot 7{,}5 \cdot 2{,}01 = 156 \text{ cm}^3$$

Concrete

$$W_{pl,c} = \frac{(d - 2 \cdot t)^3}{6} - W_{pl,s}$$

$$W_{pl,c} = \frac{(40{,}64 - 2 \cdot 1{,}0)^3}{6} - 156 = 9459 \text{ cm}^3$$

5. Steel contribution ratio

According to clause 6.7.3.3(1), EN 1994-1-1, the steel contribution ratio, δ, is defined as:

$$\delta = \frac{A_a \cdot f_{yd}}{N_{pl,Rd}}$$

The term $A_a \cdot f_{yd}$ is the contribution of the structural steel section to the plastic resistance of the composite section to axial force. The design plastic resistance of composite section to axial force $N_{pl,Rd}$ is calculated according to clause 6.7.3.2(1), EN 1994-1-1.

According to 6.7.1(4), EN 1994-1-1, the steel contribution ratio, δ, must satisfy the following conditions:

$0{,}2 \le \delta \le 0{,}9$

If δ is less than 0,2, the column should be designed as a reinforced concrete member according to EN 1992-1-1. If δ is larger than 0,9, the concrete is ignored in the calculations, and the column is designed as a structural steel member according to EN 1993-1-1.

The term $A_a \cdot f_{yd}$ is the contribution of the structural steel section to the plastic resistance of composite section to axial force:

$$A_a \cdot f_{yd} = 124{,}5 \cdot 35{,}5 = 4420 \text{ kN}$$

The plastic resistance of the composite section to axial force is:

$$N_{pl,Rd} = A_a \cdot f_{yd} + A_c \cdot f_{cd} + A_s \cdot f_{sd}$$

$$N_{pl,Rd} = 124{,}5 \cdot 35{,}5 + 1153 \cdot 2{,}67 + 20{,}1 \cdot 40{,}0$$

$$N_{pl,Rd} = 8302 \text{ kN}$$

According to 6.7.1(4), EN 1994-1-1, the steel contribution ratio, δ, must satisfy the following conditions:

$0{,}2 \le \delta \le 0{,}9$

The steel contribution ratio, δ, is:

$$\delta = \frac{A_a \cdot f_{yd}}{N_{pl,Rd}} = \frac{4420}{8302} = 0{,}532$$

Since the limits $0{,}2 < \delta = 0{,}532 < 0{,}9$ are satisfied, the column can be classified as a composite column and the provisions of EN 1994-1-1 can be used for the dimensioning.

Remark:

The confinement effects for a concrete-filled circular tube can result in increasing the cross-sectional plastic resistance, $N_{pl,Rd}$. However, the value of δ usually is not significantly changed. This means that for the calculation of steel contribution ratio, δ, we can use the following expression for the cross-sectional plastic resistance, $N_{pl,Rd}$:

$$N_{pl,Rd} = A_a \cdot f_{yd} + A_c \cdot f_{cd} + A_s \cdot f_{sd}$$

6. Local buckling

According to Table 6.3, EN 1994-1-1, for concrete-filled circular hollow cross-section, the effect of local buckling can be ignored if the following condition is satisfied, see Figure C3.5:

$$\max\left(\frac{d}{t}\right) = 90 \cdot \frac{235}{f_y}$$

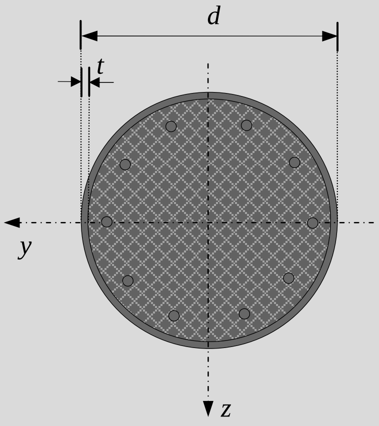


Figure C3.5 *Cross-section of the composite column and notations*

For the selected cross-section, the maximum slenderness is:

$$\max(\frac{d}{t}) = \frac{406,4}{10,0} = 40,64$$

The required condition is:

$$90 \cdot \frac{235}{f_y} = 90 \cdot \frac{235}{355} = 59,6$$

Since $40,64 < 59,6$, the condition is satisfied. The effect of local buckling can be neglected.

7. Effective modulus of elasticity for concrete

For long-term loading the creep and shrinkage are taken into account in design by a reduced flexural stiffness of the composite cross-section. Due to the influence of long-term creep effects on the effective elastic stiffness, the modulus of elasticity of the concrete, E_{cm}, should be reduced to the value $E_{c,eff}$ in accordance with following equation:

$$E_{c,eff} = \frac{E_{cm}}{1 + (\frac{N_{G,Ed}}{N_{Ed}}) \cdot \varphi_t}$$

where:

$\varphi_t = \varphi(t, t_0)$ is the creep coefficient, defining the creep between times t and t_0, related to the elastic deformation at 28 days,
$\varphi_t = \varphi(\infty, t_0)$ is the final creep coefficient,
t is the age of the concrete at the time considered,
t_0 is the age of the concrete at loading,
N_{Ed} is the axial design force,
$N_{G,Ed}$ is the permanent part of the axial design force N_{Ed}, $N_{G,Ed} = \gamma_G \cdot N_{Gk}$.

For the calculation of the creep coefficient $\varphi(t, t_0)$, the following is valid:

- the perimeter of that part which is exposed to drying, u

$$u = d \cdot \pi$$
$$u = 40,64 \cdot \pi = 127,7 \text{ cm}$$

- the notional size of the cross-section, h_0

$$h_0 = \frac{2 \cdot A_c}{u} = \frac{2 \cdot 1153}{127{,}7} = 18{,}1\,cm = 181 \text{ mm}$$

- t_0 = 28 days,
- inside conditions, the ambient relative humidity RH 50 %,
- the concrete strength class C 40/50
- the type of cement – cement class N, strength class 32,5 R.

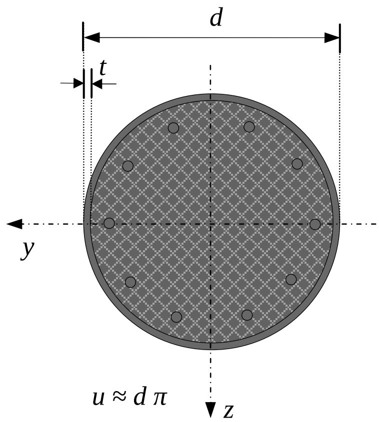


Figure C3.6 *Perimeter which is "exposed" to drying*

The final value of creep coefficient $\varphi(\infty,\ t_0)$ is determined using the nomogram shown in Figure 3.1, EN 1992-1-1. The process of determining the final value of the creep coefficient, taking into account these assumptions, is given in Figure C3.7.

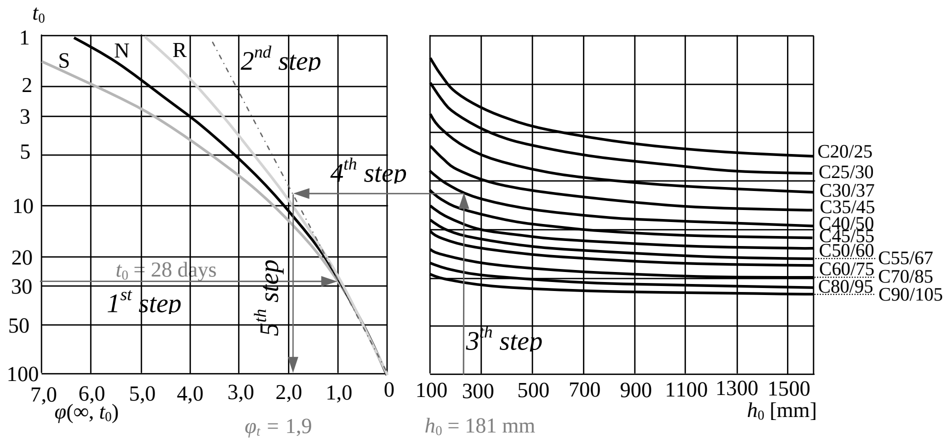


Figure C3.7 *Method for determining the creep coefficient*

The final value of creep coefficient $\varphi(\infty,\ t_0)$, found from Figure C3.7, is:

$\varphi_t = \varphi(\infty,\ t_0) = 1{,}9$

The design force of the permanent load, $N_{G,Ed}$, and the total design force, N_{Ed}, are:

$$N_{G,Ed} = \gamma_G \cdot N_{Gk}$$

$$N_{G,Ed} = 1{,}35 \cdot 3000 = 4050 \text{ kN}$$

$$N_{Ed} = 1{,}35 \cdot N_{Gk} + 1{,}50 \cdot N_{Qk}$$

$$N_{Ed} = 1{,}35 \cdot 3000 + 1{,}50 \cdot 1300 = 4050 + 1950 = 6000 \text{ kN}$$

Accordingly, the value of $E_{c,eff}$ is:

$$E_{c,eff} = \frac{E_{cm}}{1 + (\frac{N_{G,Ed}}{N_{Ed}}) \cdot \varphi_t} = \frac{3500}{1 + (\frac{4050}{6000}) \cdot 1{,}9} = 1533 \text{ kN/cm}^2$$

Further calculation is performed with the effective modulus of elasticity of concrete $E_{c,eff}$ = 1533 kN/m^2.

Remark:

Note that a conservative estimation of the effective modulus of elasticity is obtained according to EN 1994-1-1 because in the case of concrete-filled hollow section the drying of the concrete is significantly reduced by the steel section, (see example C1). However, in the case of concrete-filled circular hollow section the dimensioning of composite columns is rarely sensitive to the influence of creep coefficient φ_t on the effective modulus elasticity of concrete $E_{c,eff}$. With this statement the conservative estimation of $E_{c,eff}$, according to EN 1994-1-1, can be justified for concrete-filled hollow sections.

8. Resistance of the cross-section to compressive axial force

8.1 Plastic resistance of the cross-section without confinement effect

The design plastic resistance of the composite cross-section to axial compressive force, $N_{pl,Rd}$, is given by the sum of the design resistances of components as follows:

$$N_{pl,Rd} = A_a \cdot f_{yd} + 0{,}85 \cdot A_c \cdot f_{cd} + A_s \cdot f_{sd}$$

Remark:

The coefficient of 0,85 can be replaced by the value 1,0 due to better curing conditions in concrete-filled hollow sections.

The design plastic resistance of the composite cross-section to compressive axial force, $N_{pl,Rd}$, is calculated according to the corrected expression:

$$N_{pl,Rd} = N_{pl,a,Rd} + N_{c,Rd} + N_{s,Rd}$$

$$N_{pl,Rd} = A_a \cdot f_{yd} + A_c \cdot f_{cd} + A_s \cdot f_{sd}$$

$$N_{pl,Rd} = 124,5 \cdot 35,5 + 1153 \cdot 2,67 + 20,1 \cdot 40,0$$

$$N_{pl,Rd} = 4419 + 3079 + 804 = 8302 \text{ kN}$$

The characteristic value of the plastic resistance of the composite cross-section to compressive axial force, $N_{pl,Rk}$, is determined by the following expression:

$$N_{pl,Rk} = A_a \cdot f_{yk} + A_c \cdot f_{ck} + A_s \cdot f_{sk}$$

$$N_{pl,Rk} = 124,5 \cdot 35,5 + 1153 \cdot 4,00 + 20,1 \cdot 46,0$$

$$N_{pl,Rk} = 4420 + 4612 + 925 = 9957 \text{ kN}$$

8.2 Plastic resistance of the cross-section taking into account the confinement effect

For concrete-filled tubes of circular cross-section, the concrete component develops a higher strength because of the confinement from the steel section, see clause 6.7.3.2, EN 1994-1-1 and example C1.

This increase in the strength of concrete can be taken into account in the calculation if the relative slenderness, $\bar{\lambda}$, does not exceed 0,5 and the ratio e/d < 0,1. The eccentricity of loading, e, is determined by the ratio M_{Ed}/N_{Ed}, and d is the outer diameter of the column, see Figure C3.8.

The design axial force is:

$$N_{Ed} = 6000 \text{ kN}$$

The design bending moment at the top of column is:

$M_{Ed} = 60$ kNm

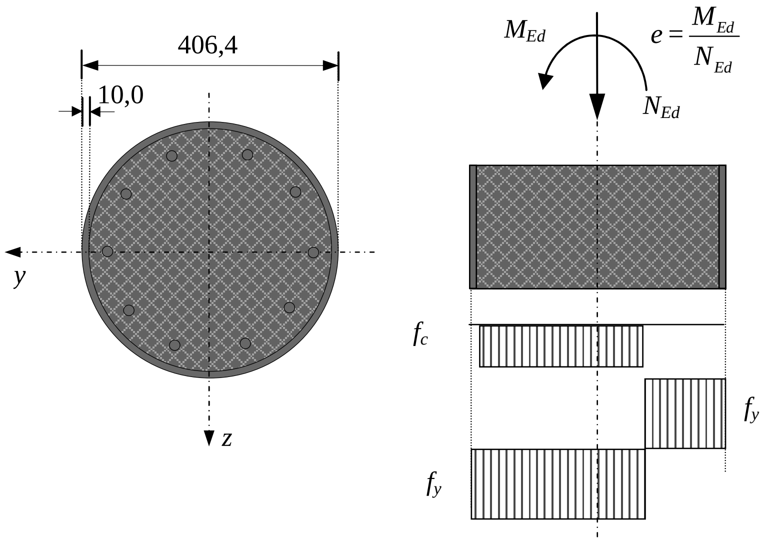


Figure C3.8 *Calculation of the eccentricity of loading, e*

The eccentricity of loading is:

$$e = \frac{M_{Ed}}{N_{Ed}} = \frac{60}{6000} = 0{,}01 \text{ m}$$

$$\frac{e}{d} = \frac{0{,}01}{0{,}406} = 0{,}025 < 0{,}1 \text{, this condition is satisfied}$$

The relative slenderness, $\bar{\lambda}$, is calculated in Section 9.1 and is $\bar{\lambda} = 0{,}562$.

Since $\bar{\lambda} = 0{,}562 > 0{,}5$, the condition is not satisfied. This means that the increase in strength of the concrete due to the confinement from the steel section is not taken into account.

Further calculation is performed with the plastic resistance of the cross-section to compression $N_{pl,Rd}$ = 8302 kN.

9. Verification of conditions for using the simplified design method

The cross-section of the composite column should be doubly symmetrical and uniform along the entire length of the column.

This condition is satisfied.

Relative slenderness

To apply the simplified method it is necessary to satisfy the following condition:

$$\bar{\lambda} \leq 2,0$$

Relative slenderness, $\bar{\lambda}$, is determined as:

$$\bar{\lambda} = \sqrt{\frac{N_{pl,Rk}}{N_{cr}}}$$

For the determination of the relative slenderness $\bar{\lambda}$ and the elastic critical force N_{cr}, it is necessary to calculate the value of the effective flexural stiffness of the cross-section of the composite column, $(EI)_{eff}$, according to the expression:

$$(EI)_{eff} = E_a \cdot I_a + E_s \cdot I_s + K_e \cdot E_{c,eff} \cdot I_c$$

With the correction factor K_e = 0,6, the value $(EI)_{eff}$ is:

$$(EI)_{eff} = 21000 \cdot 24476 + 21000 \cdot 1591 + 0,6 \cdot 1533 \cdot 107834$$

$$(EI)_{eff} = 646,59 \cdot 10^6 \text{ kNcm}^2$$

The elastic critical force, N_{cr}, for the pin-ended column and the buckling length L_e, is:

$$N_{cr} = \frac{(EI)_{eff} \cdot \pi^2}{L^2}, \; L_e = L$$

$$N_{cr} = \frac{646,59 \cdot 10^6 \cdot \pi^2}{450^2} = 31514 \text{ kN}$$

$$N_{pl,Rk} = 9957 \text{ kN}$$

The relative slenderness, $\bar{\lambda}$, is:

$$\bar{\lambda} = \sqrt{\frac{9957}{31514}} = 0{,}562$$

Accordingly $\bar{\lambda} = 0{,}562 < 2{,}0$, and the condition is satisfied.

The maximum permitted cross-sectional area of the longitudinal reinforcement

The maximum cross-sectional area of the longitudinal reinforcement $A_{s,max}$ that can be taken in the calculation should not exceed 6% of the concrete area. This condition is satisfied, see Section 4.1.

Remark:

All conditions from clause 6.7.3.1, EN 1994-1-1, are satisfied, so that allows the use of the simplified design method for composite columns.

10. Resistance of the member in axial compression

Remark:

Although the column is subjected to combined compression and bending, the check based on buckling curves is useful as the preliminary check for this column. If the resistance to the axial compressive force is not sufficient, the considered column is inadequate and it is necessary to select the stronger cross-section.

The resistance of a member subjected only to axial compression can be checked by second-order analysis according to clause 6.7.3.5, EN 1994-1-1, so as to take into account member imperfections. As a simplification in the case of the member subjected only to axial compression, the design value of the axial force N_{Ed} should satisfy the check based on the European buckling curves, which can be written as:

$$\frac{N_{Ed}}{\chi \cdot N_{pl,Rd}} \leq 1{,}0$$

The reduction factor χ is given by:

$$\chi = \frac{1}{\Phi + \sqrt{\Phi^2 - \bar{\lambda}^2}}$$, but $\chi \leq 1{,}0$

and

$\Phi = 0{,}5 \cdot [1 + \alpha \cdot (\bar{\lambda} - \bar{\lambda}_0) + \bar{\lambda}^2]$, with $\bar{\lambda}_0 = 0{,}2$

Remark:

The relevant buckling curves for cross-sections of composite columns are given in Table 6.5, EN1994-1-1. According to Table 6.5, EN 1994-1-1, circular or rectangular hollow section columns filled with concrete or containing up to 3% reinforcement can be designed using buckling curve *a* with an imperfection factor α = 0,21. However, concrete-filled hollow section columns containing from 3% to 6% can be designed using buckling curve *b* with an imperfection factor α = 0,34.

The reinforcement ratio ρ_s is 1,7 %. Therefore from Table 6.5, EN1994-1-1, buckling curve *a* should be used.

From Table 6.3, EN1993-1-1, α = 0,21 for buckling curve *a* so that Φ is:

$$\Phi = 0{,}5 \cdot [1 + \alpha \cdot (\bar{\lambda} - \bar{\lambda}_0) + \bar{\lambda}^2]$$

$$\Phi = 0{,}5 \cdot [1 + 0{,}21 \cdot (0{,}562 - 0{,}2) + 0{,}562^2] = 0{,}696$$

The reduction factor χ is:

$$\chi = \frac{1}{0{,}696 + \sqrt{0{,}696^2 - 0{,}562^2}} = 0{,}90 < 1{,}0$$

Check:

$$\frac{N_{Ed}}{\chi \cdot N_{pl,Rd}} \leq 1{,}0$$

$$\frac{6000}{0{,}90 \cdot 8302} = 0{,}80 < 1{,}0$$

Since 0,80 < 1,0, the check of composite column subjected to axial compression is satisfied. It is not necessary to select the stronger cross-section.

11. Resistance of the member in combined compression and uniaxial bending

11.1 General

According to clause 6.7.3.6, EN 1994-1-1, the member in combined compression and uniaxial bending has sufficient resistance if the following condition is satisfied:

$$\frac{M_{Ed}}{M_{pl,N,Rd}} = \frac{M_{Ed}}{\mu_d \cdot M_{pl,Rd}} \leq \alpha_M$$

where:

M_{Ed} is the greatest of the end moments and the maximum bending moment within the column length. This moment is calculated according to clause 6.7.3.4, EN 1994-1-1, including imperfections (Table 6.5, EN 1994-1-1) and second-order effects if necessary (α_{cr} > 10).

$M_{pl,N,Rd}$ is the plastic resistance moment taking into account the axial force N_{Ed}, given by $\mu_d{\cdot}M_{pl,Rd}$, see Figure 6.18, EN 1994-1-1.

$M_{pl,Rd}$ is the plastic resistance moment, given by point B in Figure 6.18, EN 1994-1-1.

μ_d is the factor related to design for compression and uniaxial bending.

α_M is the coefficient related to bending of a composite column and is taken as 0,9 for steel grades between S235 and S355.

The condition can be written in the following form:

$$\frac{M_{Ed}}{M_{Rd}} = \frac{M_{Ed}}{\alpha_M \cdot \mu_d \cdot M_{pl,Rd}} \leq 1{,}0$$

The calculation of the design bending moment M_{Ed} = $M_{Ed,II}$ taking the initial bending moment about *y*-*y* axis, the imperfection $e_{0,z}$, and second-order effects is shown in Figure C3.9.

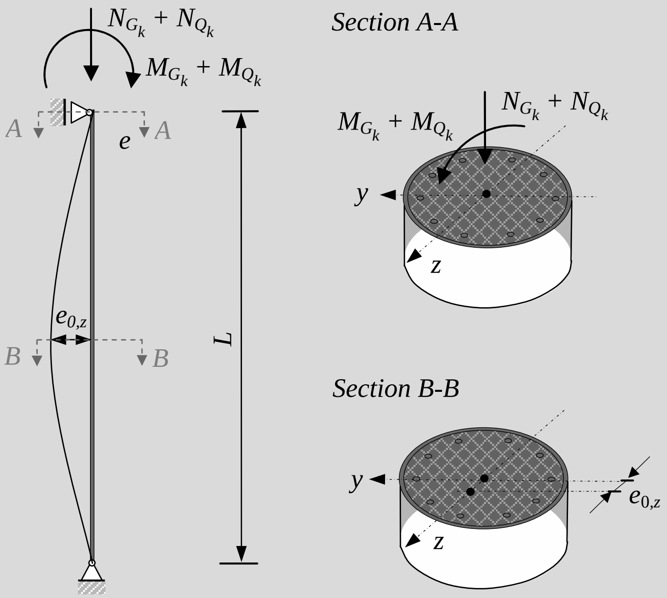


Figure C3.9 *Imperfection $e_{0,z}$ about the y-y axis*

11.2 Resistance of the cross-section in combined compression and uniaxial bending

Remark:

In order to determine the resistance of the composite cross-section to combined compression and uniaxial bending, it is necessary to produce an axial load – bending moment (*N*–*M*) interaction curve. As a simplification, the interaction curve is replaced by an interaction polygon *ACDB*, clause 6.7.3.2 (5), EN 1994-1-1.

The *N*–*M* interaction polygon *ACDB* is shown in Figure 6.19, EN 1994-1-1. The modified version of the interaction polygon, which refers to the composite column with concrete-filled circular hollow cross-section, is shown in Figure C3.10.

In order to produce the *N*–*M* interaction polygon, the cross-sectional capacities at points *A* to *D* should be determined assuming the stress distributions indicated, see Figure C3.10.

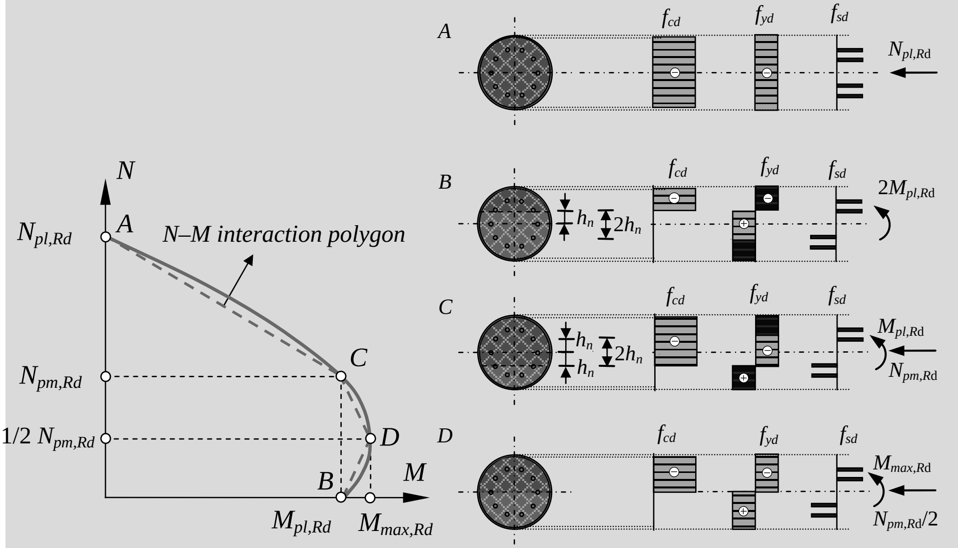


Figure C3.10 *N–M interaction polygon and corresponding stress distributions*

It should be noted that EN 1994-1-1 does not provide expressions for circular cross-sections filled with concrete.

Point A

At point A, only the design plastic resistance of the cross-section is taken into account:

$$N_{pl,Rd} = A_a \cdot f_{yd} + 0,85 \cdot A_c \cdot f_{cd} + A_s \cdot f_{sd}$$

Remark:

For concrete filled hollow sections, the coefficient of 0,85 can be replaced with a value of 1,0 due to better curing conditions.

The design plastic resistance of composite cross-section to compression, $N_{pl,Rd}$, is calculated according to the corrected expression:

$$N_{pl,Rd} = A_a \cdot f_{yd} + A_c \cdot f_{cd} + A_s \cdot f_{sd}$$

$$N_{pl,Rd} = 124,5 \cdot 35,5 + 1153 \cdot 2,67 + 20,1 \cdot 40,0 = 8302 \text{ kN}$$

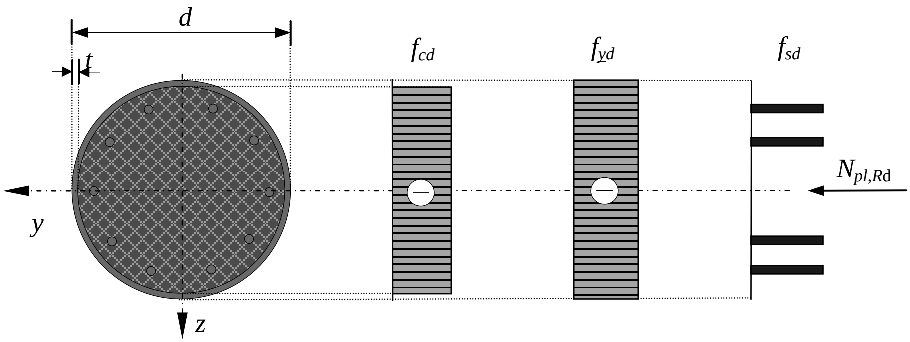


Figure C3.11 *Stress distributions for point A on the interaction polygon*

Point D

The position of the plastic neutral axis and the stress distributions are shown in Figure C3.12.

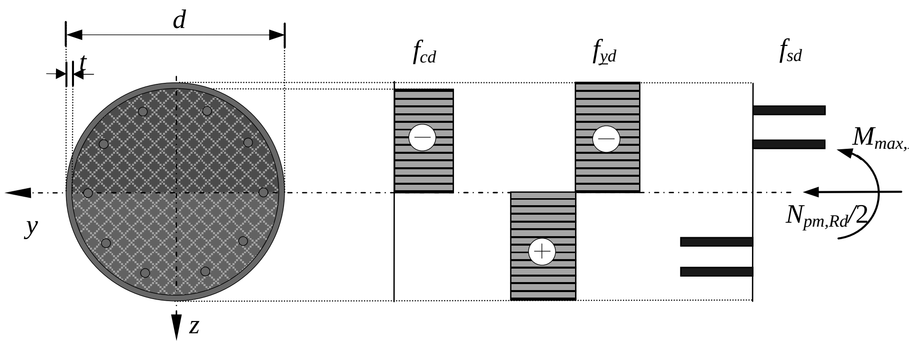


Figure C3.12 *Stress distributions for point D on the interaction polygon*

The maximum design plastic resistance moment is determined by the following expression:

$$M_{\max,y,Rd} = M_{pl,y,a,Rd} + M_{pl,y,c,Rd} + M_{pl,y,s,Rd}$$

The maximum design plastic resistance moment, M_{maxRd}, at point D is:

$$M_{\max Rd} = W_{pl,a} \cdot f_{yd} + 0{,}5 \cdot W_{pl,c} \cdot f_{cd} + W_{pl,s} \cdot f_{sd}$$

$$M_{\max Rd} = (1572 \cdot 35{,}5 + 0{,}5 \cdot 9459 \cdot 2{,}67 + 156 \cdot 40{,}0) \cdot 10^{-2}$$

$$M_{\max,Rd} = 747 \text{ kNm}$$

The design value of the resistance of the concrete to compression, $N_{pm,Rd}$, is:

$$N_{pm,Rd} = \frac{\pi \cdot (d-2t)^2}{4} \cdot f_{cd} = \frac{\pi \cdot (40,64 - 2 \cdot 1,0)^2}{4} \cdot 2,67 = 3130 \text{ kN}$$

The design axial force at the point of maximum design plastic resistance moment is $0,5 \cdot N_{pm,Rd}$, and therefore is:

$$0,5 \cdot N_{pm,Rd} = 0,5 \cdot 3130 = 1565 \text{ kN}$$

Point C

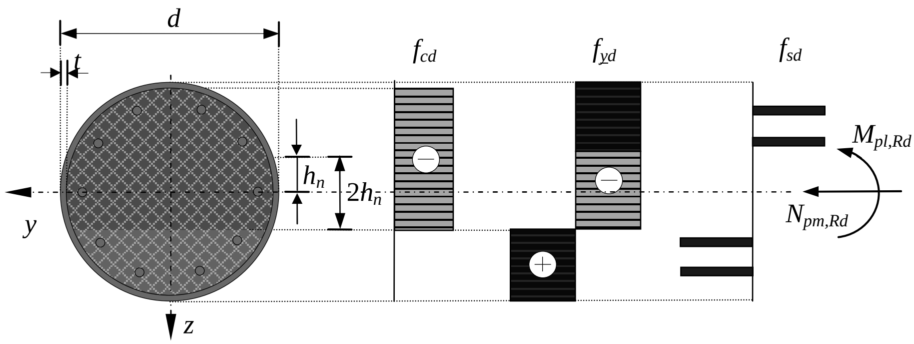


Figure C3.13 *Stress distributions for point C on the interaction polygon*

Calculation of the design plastic resistance moment of the composite section, $M_{pl,Rd}$, is carried out as shown below.

Determination of the position of the neutral axis depth, h_n, when axial force is zero:

$$h_n = \frac{N_{pm,Rd} - A_{s,n} \cdot (2 \cdot f_{sd} - f_{cd})}{2 \cdot d \cdot f_{cd} + 4 \cdot t \cdot (2 \cdot f_{yd} - f_{cd})}$$

where A_{sn} is the reinforcement area within h_n. Because it is at this point unknown, it is assumed to be (initial guess):

$$A_{s,n} = 2 \cdot A_{bar} = 2 \cdot 2,01 = 4,02 \text{ cm}^2$$

Thus, for the case when the axial force is equal to zero, h_n is:

$$h_n = \frac{3130 - 4,02 \cdot (2 \cdot 40,0 - 2,67)}{2 \cdot 40,64 \cdot 2,67 + 4 \cdot 1,0 \cdot (2 \cdot 35,5 - 2,67)} = 5,749 \text{ cm} = 57,49 \text{ mm}$$

Plastic section moduli in region $2 \cdot h_n$

Reinforcement

Generally, the value of $W_{pl,s,n}$ is equal to the value of $W_{pl,s}$. However, the worst case is where only two bars occur within h_n, and these are on the centre line. Accordingly, the value of the plastic section modulus of reinforcement is:

$W_{pl,s,n} = 0 \text{ cm}^3$

Concrete

Effective plastic section modulus of concrete:

$$W_{pl,c,n} = (d - 2 \cdot t) \cdot h_n^2 - W_{pl,s,n}$$

$$W_{pl,c,n} = (40,64 - 2 \cdot 1,0) \cdot 5,749^2 - 0 = 1277 \text{ cm}^3$$

Structural steel

Plastic section modulus of the steel section:

$$W_{pl,a,n} = d \cdot h_n^2 - W_{pl,c,n} - W_{pl,s,n}$$

$$W_{pl,a,n} = 40,64 \cdot 5,749^2 - 1277 - 0 = 66 \text{ cm}^3$$

The design plastic resistance moment of the composite section, $M_{pl,Rd}$, is calculated as:

$$M_{pl,Rd} = M_{\max,Rd} - M_{n,Rd}$$

where:

$$M_{n,Rd} = W_{pl,a,n} \cdot f_{yd} + W_{pl,s,n} \cdot f_{sd} + \frac{W_{pl,c,n} \cdot f_{cd}}{2}$$

$$M_{n,Rd} = (66 \cdot 35,5 + 0 \cdot 40,0 + \frac{1277 \cdot 2,67}{2}) \cdot 10^{-2} = 40,48 \text{ kNm}$$

The design plastic resistance moment of the composite section, $M_{pl,Rd}$, is:

$$M_{pl,Rd} = 747 - 40,48 = 707 \text{ kNm}$$

Point B

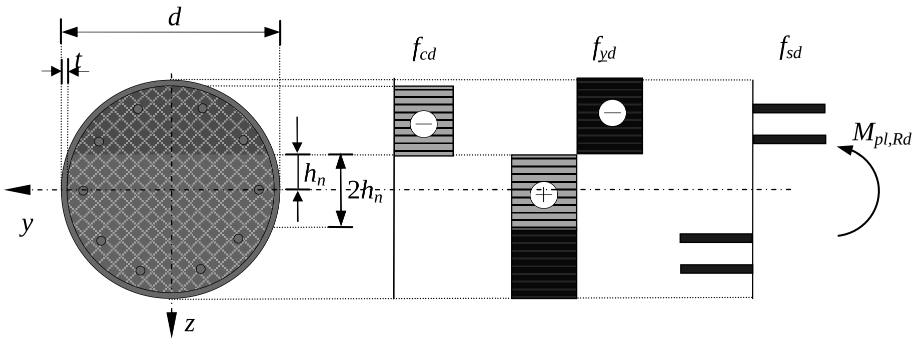


Figure C3.14 *Stress distributions for point B on the interaction polygon*

The design value of $M_{pl,Rd}$ has previously been calculated in order to define point C on the *N–M* interaction polygon:

$$M_{pl,Rd} = 707 \text{ kNm}$$

Previously calculated values at points A to D should be plotted to produce the *N–M* interaction polygon (Figure 6.19, EN 1994-1-1).

According to the interaction polygon ACDB, Figure C3.15, plotted in accordance with Figure 6.19, EN 1994-1-1, the following value $M_{pl,,N,Rd}$ is obtained:

$$M_{pl,Rd} : \mu_d \cdot M_{pl,N,Rd} = (N_{pl,Rd} - N_{pm,Rd}) : (N_{pl,Rd} - N_{Ed})$$

$$M_{pl,N,Rd} = \frac{N_{pl,Rd} - N_{Ed}}{N_{pl,Rd} - N_{pm,Rd}} M_{pl,Rd}$$

$$M_{pl,N,Rd} = \frac{8302 - 6000}{8302 - 3130} 707 = 315 \text{ kNm}$$

The value of μ_d is:

$$\mu_d = \frac{M_{pl,N,Rd}}{M_{pl,Rd}} = \frac{315}{707} = 0,445 < 1,0$$

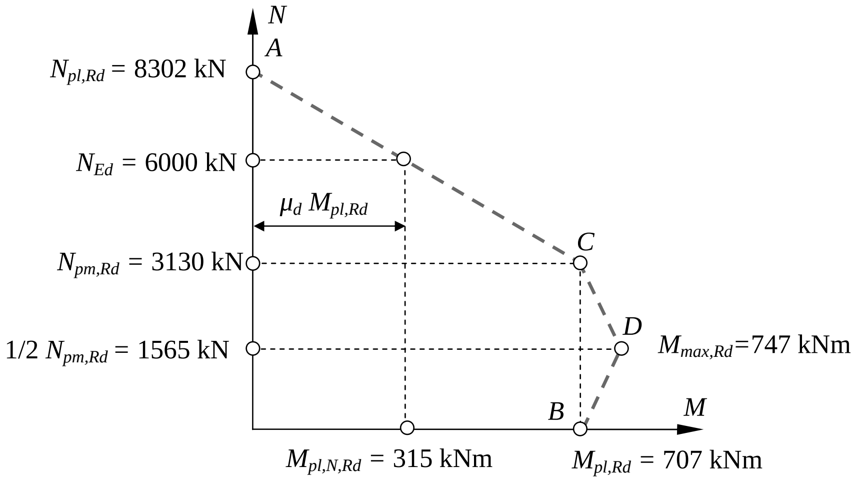


Figure C3.15 *N–M interaction polygon*

The check is carried out by the factor $\mu_d = 0,445$.

11.3 Calculation of action effects according to second-order analysis

11.3.1 General

According to clause 6.7.3.4 (3), EN 1994-1-1, which refers to clause 5.2.1(3), EN 1994-1-1, second-order effects can therefore be neglected if the load factor α_{cr}, which is the ratio between the elastic critical load and the corresponding applied loading, for elastic instability of the member exceeds 10.

To calculate α_{cr}, the ends of the column are assumed to be pinned, and α_{cr} is found using the Euler formula for the elastic critical force $N_{cr,eff}$:

$$N_{cr,eff} = \frac{\pi^2 (EI)_{eff,II}}{L_e^2} \quad L_e = L$$

The design value of the effective flexural stiffness $(EI)_{eff,II}$, used to determine the internal forces and moments by second-order analysis, according to clause 6.7.3.4(2), EN 1994-1-1, is defined by the following expression:

$$(EI)_{eff,II} = K_0 \cdot (E_a \cdot I_a + E_s \cdot I_s + K_{e,II} \cdot E_{cm} \cdot I_c)$$

where:

$K_{e,II}$ is a correction factor which should be taken as 0,5,
K_0 is a calibration factor which should be taken as 0,9.

The value $E_{c,eff}$ has been used in place of E_{cm} in expression for $(EI)_{eff,II}$ in order to take into account the long-term effects, in the same way as calculated in Section 7. Accordingly, the value of $E_{c,eff}$ is:

$$E_{c,eff} = 1533 \ \text{kN/cm}^2$$

The design value of the effective flexural stiffness $(EI)_{eff,II}$ is:

$$(EI)_{eff,II} = K_0 \cdot (E_a \cdot I_a + E_s \cdot I_s + K_{e,II} \cdot E_{c,eff} \cdot I_c)$$

$$(EI)_{eff,II} = 0,9 \cdot (21000 \cdot 24476 + 21000 \cdot 1591 + 0,5 \cdot 1533 \cdot 107834)$$

$$(EI)_{eff,II} = 567,06 \cdot 10^6 \ \text{kNcm}^2$$

The elastic critical force, N_{cr}, for the pin-ended column, is:

$$N_{cr,eff} = \frac{\pi^2 (EI)_{eff,II}}{L_e^2}$$

$$N_{cr,eff} = \frac{567,06 \cdot 10^6 \cdot \pi^2}{450^2} = 27638 \ \text{kN}$$

To check whether the effects of second-order analysis can be neglected, the value of α_{cr} must be higher than 10:

$$\alpha_{cr} = \frac{N_{cr,eff}}{N_{Ed}} = \frac{27638}{6000} = 4,61 < 10$$

The value of α_{cr} is less than 10, so the second-order effects must be considered.

Remark:

Second-order effects are included in two ways:

a) by using a first-order analysis modified with appropriate amplification according to clause 6.7.3.4(5), EN 1994-1-1,
b) by using exact second-order analysis, the procedure which is not included in EN 1994-1-1.

11.3.2 *Bending moments – approximate solution*

According to clause 6.7.3.4(5), EN 1994-1-1, the second-order effects can be calculated by multiplying the greatest first-order design bending moments by a factor *k*.

Thus, the second-order effects may be considered according to the expression:

$$M_{Ed,II} = M_{Ed,I} \cdot k$$

The factor *k* is given by:

$$k = \frac{\beta}{1 - N_{Ed} / N_{cr,eff}} \geq 1{,}0$$

where:

β is an equivalent moment factor given in Table 6.4, EN 1994-1-1,

$N_{cr,eff}$ is the critical axial force for the relevant axis and corresponding to the effective flexural stiffness $(EI)_{eff,II}$, with the effective length taken as the physical length of the column.

The design bending moment from the member imperfections is determined by the following expression:

$$M_{Ed,I} = N_{Ed} \cdot e_{0,d}$$

where:

N_{Ed} is the design value of the axial force,

$e_{0,d}$ is the equivalent member imperfection, which is given in Table 6.5, EN 1994-1-1, depending on the buckling curve.

Remark:

The reinforcement ratio ρ_s is 1,7 %. Therefore from Table 6.5, EN 1994-1-1, for $\rho_s \leq 3\%$ the buckling curve *a* should be used.

Therefore, for the buckling curve *a*, the equivalent member imperfection is:

$$e_{0,d} = \frac{L}{300}$$

$$e_{0,d} = \frac{450}{300} = 1{,}5 \text{ cm}$$

The design bending moments calculated according to first-order analysis are shown in Figure C3.16.

The design values of bending moments are:

The design bending moment at the top of the column is:

$$M_{Ed,I} = 60 \text{ kNm}$$

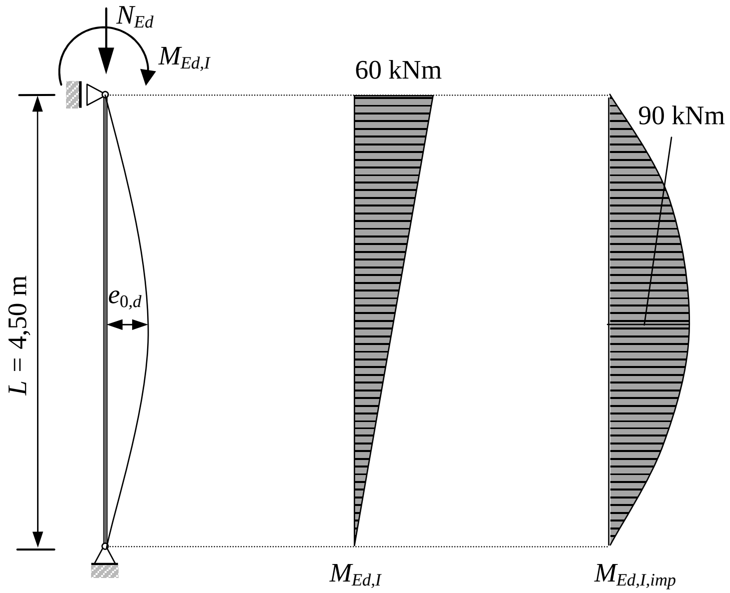


Figure C3.16 *First-order bending moments, design values*

The design bending moment at the bottom of the column is:

$$M_{Ed,I} = 0 \text{ kNm}$$

The design bending moment due to imperfection is:

$$M_{Ed,imp} = N_{Ed} \cdot e_{0,d} = 6000 \cdot 0{,}015 = 90 \text{ kNm}$$

Remark:

The factor β from Table 6.4, EN 1994-1-1, allows for the shape of the bending moment diagram. When bending is caused by lateral loading on the column, the value of factor β is 1,0. For a column subjected to end moments, the factor β is calculated as:

$$\beta_1 = 0,66 + 0,44 \cdot r \geq 0,44$$

where r is the ratio of the end-moments on the ends of the column ($-1 \leq r \leq +1$).

Therefore, the two values of factor k must be calculated:

- for the end moments, k_1,
- for the moment from the member imperfection, k_2.

Determination of factor k_1

The ratio of the end-moments on the ends of the column is:

$$r = \frac{0}{M_{Ed,I}} = 0,0$$

The equivalent moment factor β is:

$$\beta_1 = 0,66 + 0,44 \cdot r \geq 0,44$$

$$\beta_1 = 0,66 + 0,44 \cdot 0 = 0,66$$

Therefore, the factor k_1 is:

$$k_1 = \frac{\beta_1}{1 - N_{\mathrm{Ed}} / N_{\mathrm{cr,eff}}} = \frac{0,66}{1 - \frac{6000}{27638}} = 0,84 < 1,0$$

Remark:

According to clause 6.7.3.4(5), EN 1994-1-1, the value of factor k must be 1,0 or higher. It is over-conservative to use when combining two sets of second-order effects. Therefore, the calculated value of 0,84 is adopted.

Determination of factor k_2

For the bending moment from the member imperfection, according to Table 6.4, EN 1994-1-1, the equivalent moment factor β is:

$$\beta_2 = 1,0$$

Therefore, the factor k_2 is:

$$k_2 = \frac{\beta_2}{1 - N_{\mathrm{Ed}} / N_{\mathrm{cr,eff}}} = \frac{1,0}{1 - 6000/27638} = 1,28 > 1,0$$

The adopted value of the factor is:

k_2 = 1,28

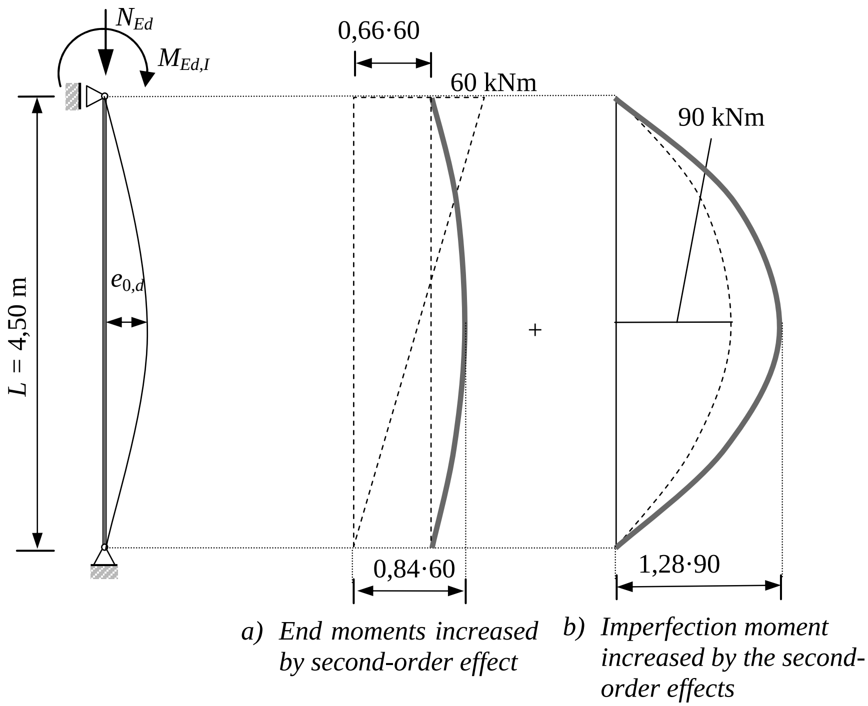


Figure C3.17 *Second-order bending moments, design values*

The design bending moment at mid-height, second-order effects are taken into account, is:

$$M_{Ed,II} = M_{Ed} \cdot k_1 + M_{Ed,imp} \cdot k_2 = 60 \cdot 0,84 + 90 \cdot 1,28 = 166 \text{ kNm}$$

The check is performed with this bending moment:

$$M_{Ed,II} = M_{\max} = 166 \text{ kNm}$$

11.3.3 Bending moments – exact solution

The calculation of the action effects will be done by exact second-order analysis.

The design bending moments calculated according to second-order analysis are shown in Figure C3.18.

For the buckling curve *a*, the equivalent member imperfection is:

$$e_{0,d} = \frac{L}{300}$$

$$e_{0,d} = \frac{450}{300} = 1{,}5 \text{ cm}$$

Second-order bending moments are calculated for the static system given in Figure C3.18 for the lateral load q_d, the end-moments M_{Ed} and the initial bow imperfection $e_{0,d}$. The coefficient r is the ratio of the end moments on the ends of the column.

Second-order bending moments at any section along the length of column, ξ, are calculated as:

$$M(\xi) = M_{Ed,I}\left(\frac{r\sin(\varepsilon\cdot(1-\xi)) + \sin(\varepsilon\cdot\xi)}{\sin\varepsilon}\right) + M_0\left(\frac{\cos(\varepsilon\cdot(0{,}5-\xi))}{\cos(\varepsilon/2)} - 1\right)$$

The maximum second-order bending moment at section ξ_M is calculated as:

$$M_{\max} = [0{,}5\cdot M_{Ed,I}\cdot(r+1) + M_0]\cdot\frac{\sqrt{1+c^2}}{\cos(\varepsilon/2)} - M_0$$

where:

$$\varepsilon = L_e\cdot\sqrt{\frac{|N_{Ed}|}{(EI)_{eff,II}}}$$

$$M_0 = (q_d \cdot L^2 + 8 \cdot N_{Ed} \cdot e_{0,d}) \cdot \frac{1}{\varepsilon^2}$$

The point of maximum bending moment is at a distance ξ_M from the bottom end of the column and it is calculated as:

$$c = \frac{M_{Ed,I} \cdot (1-r)}{M_{Ed,I} \cdot (r+1) + 2 \cdot M_0} \cdot \frac{1}{\tan(\varepsilon/2)}$$

$$\xi_M = 0{,}5 + \frac{\arctan(c)}{\varepsilon}$$

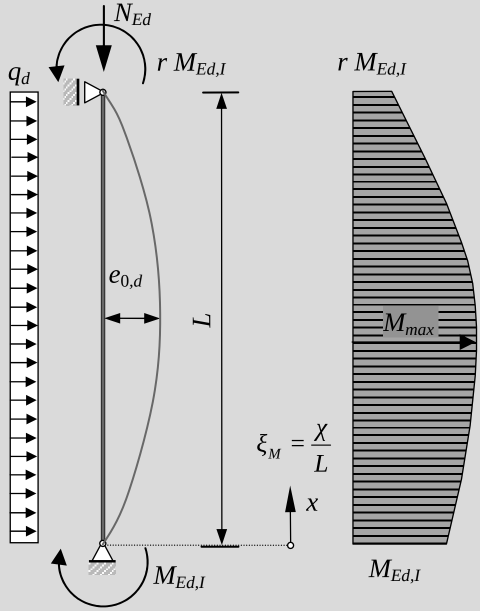


Figure C3.18 *Static system and actions*

The lateral load is $q_d = 0$, and the ratio of the end moments on the ends of the column is $r = 0$.

Accordingly:

$$\varepsilon = L_e \cdot \sqrt{\frac{|N_{Ed}|}{(EI)_{eff,II}}}$$

$$\varepsilon = 450 \cdot \sqrt{\frac{6000}{567{,}06 \cdot 10^6}} = 1{,}46$$

$$M_0 = (q_d \cdot L^2 + 8 \cdot N_{Ed} \cdot e_{0,d}) \cdot \frac{1}{\varepsilon^2}$$

$$M_0 = (0 \cdot 4{,}5^2 + 8 \cdot 6000 \cdot \frac{4{,}5}{300}) \cdot \frac{1}{1{,}46^2} = 338 \text{ kNm}$$

The coefficient c is:

$$c = \frac{M_{Ed,I} \cdot (1-r)}{M_{Ed,I} \cdot (r+1) + 2 \cdot M_0} \cdot \frac{1}{\tan(\varepsilon/2)}$$

$$c = \frac{60 \cdot (1-0)}{60 \cdot (0+1) + 2 \cdot 338} \cdot \frac{1}{\tan(1{,}46/2)} = 0{,}091$$

The point of maximum bending moment is:

$$\xi_M = 0{,}5 + \frac{\arctan(c)}{\varepsilon}$$

$$\xi_M = 0{,}5 + \frac{\arctan(c)}{\varepsilon} = 0{,}5 + \frac{\arctan(0{,}091)}{1{,}46} = 0{,}562$$

$$x = \xi_M \cdot L = 0{,}562 \cdot 4{,}50 = 2{,}53 \text{ m}$$

The maximum second-order bending moment is:

$$M_{\max} = [0{,}5 \cdot 60 \cdot (0+1) + 338] \cdot \frac{\sqrt{1+0{,}091^2}}{\cos(1{,}46/2)} - 338 = 158 \text{ kNm}$$

Remark:

The value of bending moment calculated by the exact method, $M_{\max} = 158$ kNm, is less than the value obtained by the approximate method $M_{Ed,II} = 166$ kNm. For the recommended value of k_1 = 1,0 according to clause 6.7.3.4(5), EN 1994-1-1, this difference is higher.

The check will be performed for the design value of second-order bending moment $M_{Ed,II} = 166$ kNm.

11.3.4 Shear forces – approximate solution

According to clause 5.2.2(5)B, EN 1993-1-1, second-order effects can be allowed for by multiplying the greatest first-order design bending moment by a factor k given by:

$$k = \frac{\beta}{1 - N_{Ed} / N_{cr,eff}} \geq 1{,}0$$

Accordingly, the approximate value of shear force can be obtained as:

$$V_{Ed,II} = V_{Ed} \cdot k$$

In accordance with Figure C3.19, the first-order design shear force at the bottom of the column is:

$$V_{Ed} = \frac{M_{Ed}}{L} + \frac{4 \cdot N_{Ed} \cdot e_{0,d}}{L} = \frac{60}{4{,}5} + \frac{4 \cdot 6000 \cdot 0{,}015}{4{,}5} = 13 + 80 = 93 \text{ kN}$$

In accordance with Figure C3.19, the first-order design shear force at the top of the column is:

$$V_{Ed} = -\frac{M_{Ed}}{L} + \frac{4 \cdot N_{Ed} \cdot e_{0,d}}{L} = -\frac{60}{4{,}5} + \frac{4 \cdot 6000 \cdot 0{,}015}{4{,}5} = -13 + 80 = 67 \text{ kN}$$

The diagram of shear forces, calculated by first-order analysis for bending moment $M_{Ed,I}$ and the equivalent lateral load due to imperfections, is shown in Figure C3.19.

Therefore, the factor k_1 is:

$$k_1 = \frac{\beta_1}{1 - N_{\mathrm{Ed}} / N_{\mathrm{cr,eff}}} = \frac{0{,}66}{1 - \dfrac{6000}{27638}} = 0{,}84 < 1{,}0$$

The factor k_2 is:

$$k_2 = \frac{\beta_2}{1 - N_{\mathrm{Ed}} / N_{\mathrm{cr,eff}}} = \frac{1{,}0}{1 - 6000/27638} = 1{,}28 > 1{,}0$$

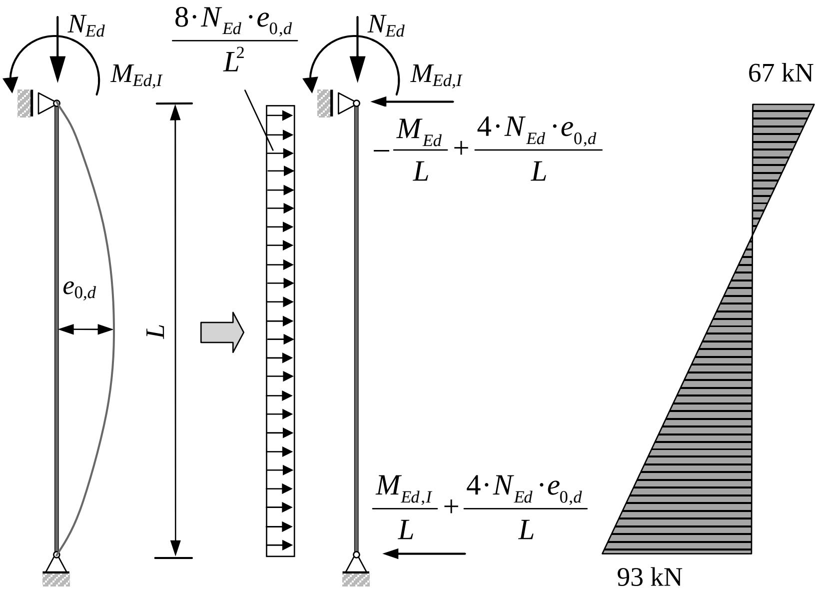


Figure C3.19 *First-order design shear forces*

Therefore, the maximum design shear force, calculated by approximative second-order analysis, is:

$$V_{Ed,II}=k_1\cdot\frac{M_{Ed}}{L}+k_2\cdot\frac{4\cdot N_{Ed}\cdot e_{0,d}}{L}$$

$$V_{Ed,II}=0,84\cdot\frac{60}{4,5}+1,28\cdot\frac{4\cdot 6000\cdot 0,015}{4,5}=11+102=113\ \text{kN}$$

11.3.5 Shear forces – exact solution

The design shear force calculated by exact second-order analysis is:

$$V_{Ed}(\xi)=\frac{M_{Ed,I}}{L}\cdot\varepsilon[\frac{-r\cdot\cos(\varepsilon\cdot(1-\xi))+\cos(\varepsilon\cdot\xi)}{\sin\varepsilon}]+\frac{M_0}{L}\cdot\varepsilon[\frac{\sin(\varepsilon\cdot(0,5-\xi))}{\cos(\varepsilon/2)}]$$

At the bottom of the column is ξ = 0, the ratio of the end moments is r = 0, then the design shear force is:

$$V_{Ed}(\xi=0)=\frac{60,0}{4,5}\cdot 1,46\cdot[\frac{\cos(1,46\cdot 0)}{\sin 1,46}]+\frac{338}{4,5}\cdot 1,46\cdot[\frac{\sin(1,46\cdot(0,5-0))}{\cos(1,46/2)}]$$

$$V_{Ed}(\xi = 0) = 19{,}6 + 98{,}1 = 117{,}7 \ \text{kN}$$

At the top of the column $\xi = 1{,}0$, and the ratio of the end moments is $r = 0$, then the design shear force is:

$$V_{Ed}(\xi = 1{,}0) = \frac{60{,}0}{4{,}5} \cdot 1{,}46 \cdot [\frac{\cos(1{,}46 \cdot 1{,}0)}{\sin 1{,}46}] + \frac{338}{4{,}5} \cdot 1{,}46 \cdot [\frac{\sin(1{,}46 \cdot (0{,}5 - 1{,}0))}{\cos(1{,}46 / 2)}]$$

$$V_{Ed}(\xi = 1{,}0) = 2{,}2 - 98{,}1 = -95{,}9 \ \text{kN}$$

The check will be performed for the design value of the second-order shear force $V_{Ed,II} = 113 \ \text{kN}$.

11.4 Check of the resistance of the member in combined compression and uniaxial bending

It is necessary to satisfy the following condition:

$$\frac{M_{Ed}}{M_{Rd}} = \frac{M_{Ed}}{\alpha_M \cdot \mu_d \cdot M_{pl,Rd}} \leq 1{,}0$$

The coefficient α_M is taken as 0,9 for steel grades between S235 and S355.

The design value of the maximum design bending moment by the approximative second-order analysis is:

$$M_{Ed} = M_{Ed,II} = 166 \ \text{kNm}$$

The design resistance moment M_{Rd} is, see Figure C3.15:

$$M_{Rd} = \alpha_M \cdot \mu_d \cdot M_{pl,Rd} = 0{,}9 \cdot 0{,}445 \cdot 707 = 283 \ \text{kNm}$$

Condition:

$$\frac{M_{Ed}}{M_{Rd}} = \frac{166}{283} = 0{,}59$$

Since $0{,}59 < 1{,}0$, the condition is satisfied.

The design value of the maximum design bending moment by the exact second-order analysis is:

$$M_{Ed} = M_{Ed,II} = 158 \text{ kNm}$$

The design resistance moment M_{Rd} is, see Figure C3.15:

$$M_{Rd} = \alpha_M \cdot \mu_d \cdot M_{pl,Rd} = 0{,}9 \cdot 0{,}445 \cdot 707 = 283 \text{ kNm}$$

Condition:

$$\frac{M_{Ed}}{M_{Rd}} = \frac{158}{283} = 0{,}56$$

Since $0{,}56 < 1{,}0$, the condition is satisfied.

11.5 Check of plastic resistance of composite section to transverse shear

In accordance with clause 6.7.3.2(4), EN 1994-1-1, for simplification V_{Ed} may be assumed to act on the structural steel section alone. According to clause 6.2.6(2), EN 1993-1-1, in the absence of torsion the design plastic shear resistance is given by:

$$V_{pl,a,Rd} = \frac{A_v \cdot (f_y / \sqrt{3})}{\gamma_{M0}}$$

The shear area, A_v, according to clause 6.2.6(3), EN1993-1-1, is calculated as:

$$A_v = \frac{2 \cdot A_a}{\pi}$$

where A_a is the cross-sectional area of the circular hollow section.

According to clause 6.2.2.4(1), EN 1994-1-1, where the shear force is less than half the plastic shear resistance its effect on the resistance moment can be neglected. Therefore, the condition is:

$$V_{Ed} < 0{,}5 \cdot V_{pl,a,Rd}$$

The design value of the second-order shear force is:

$$V_{Ed} = V_{Ed,II} = 113 \text{ kNm}$$

The shear area, A_v, is:

$$A_v = \frac{2 \cdot 124,5}{\pi} = 79,26 \text{ cm}^2$$

The design plastic shear resistance is:

$$V_{pl,a,Rd} = \frac{79,26 \cdot (35,5 / \sqrt{3})}{1,0} = 1625 \text{ kN}$$

Check:

$$V_{Ed} = 113 < 0,5 \cdot V_{pl,a,Rd} = 0,5 \cdot 1625 = 813 \text{ kN}$$

The condition is satisfied and there is no reduction in the resistance moment.

12. Check of the load introduction

It is assumed that this column is one of several similar columns in a multistorey braced frame. The beams of the composite floor are attached to the column. The assumed design shear forces from beams, which act on the column, are:

$V_{Ed,1} = 500 \text{ kN}$ and $V_{Ed,2} = 700 \text{ kN}$

The eccentricity at the pin joint is 0,3 m (Figure C3.20) and the bending moment applied to the column at the loaded floor level is:

$$M_{Ed} = V_{Ed,1} \cdot e_1 + V_{Ed,2} \cdot e_1 = 500 \cdot 0,3 - 700 \cdot 0,3 = 60 \text{ kNm}$$

The design axial load is:

$$N_{Ed} = V_{Ed,1} + V_{Ed,2} = 500 + 700 = 1200 \text{ kN}$$

The design axial load and the design bending moment are applied at the top of column, see Figure C3.20.

The structural detail, shown in Figure C3.20, is provided for load introduction. The steel plate is inserted through the steel section, which ensures the loading of the concrete. Since the concrete is confined by the steel circular hollow section, the stresses below the gusset plate can reach very high values.

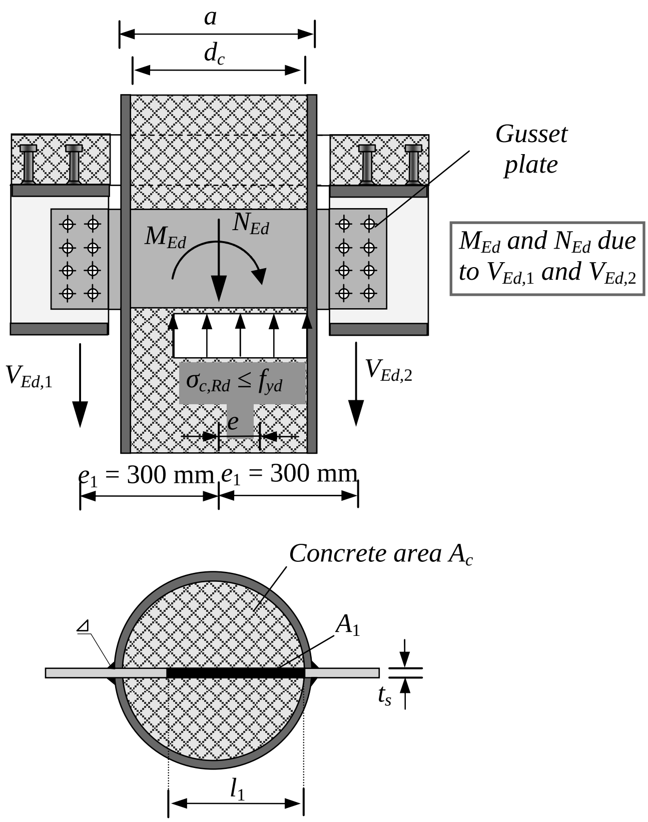


Figure C3.20 *Load introduction into the concrete through the gusset plate*

According to clause 6.7.4.2(6), EN 1994-1-1, when a concrete-filled circular hollow section is only partially loaded from a gusset plate through the profile, the local design strength of concrete, $\sigma_{c,Rd}$, under the gusset plate, resulting from the sectional forces of the concrete section, should be determined by:

$$\sigma_{\mathrm{c,Rd}} = f_{\mathrm{cd}} \cdot \left(1 + \eta_{\mathrm{cL}} \cdot \frac{t}{a} \cdot \frac{f_{\mathrm{y}}}{f_{\mathrm{ck}}}\right) \cdot \sqrt{\frac{A_{\mathrm{c}}}{A_1}}\ , \leq \frac{A_{\mathrm{c}} \cdot f_{\mathrm{cd}}}{A_1}\ , \leq f_{\mathrm{yd}}$$

where:

t is the wall thickness of the steel tube,
a is the diameter of the tube,
A_c is the cross-sectional area of the concrete,
A_1 is the loaded area under the gusset plate according to Figure C3.20,
η_{cL} is 4,9 for circular steel tubes.

The ratio A_c/A_1 should not exceed 20. Welds between the gusset plate and the steel hollow section should be designed according to Section 4 of EN 1993-1-8.

The eccentricity e is:

$$e = \frac{M_{Ed}}{N_{Ed}} = \frac{6000}{1200} = 5,0 \text{ cm}$$

The length l_1 is (Figure C3.20):

$$l_1 = 2\cdot(a/2 - t - e) = 2\cdot(40,06/2 - 1,0 - 5,0) = 28,06 \text{ cm}$$

In accordance with the ratio $A_c/A_1 < 20$**,** the wall thickness of the steel tube t_s is:

$$t_s = \frac{A_c}{20\cdot l_1} = \frac{1153}{20\cdot 28,06} = 2,05 \text{ cm}$$

The adopted wall thickness of the steel tube t_s is 2,2 cm.

The loaded area under the gusset plate, A_1, is:

$$A_1 = l_1 \cdot t_s = 28,06\cdot 2,2 = 61,7 \text{ cm}^2$$

Therefore, the design compressive stress below the gusset plate is:

$$\sigma_{\mathrm{c,Ed}} = \frac{N_{Ed}}{A_1} = \frac{1200}{61,7} = 19,4 \text{ kN/cm}^2$$

The local design strength of concrete, $\sigma_{c,Rd}$, under the gusset plate is:

$$\sigma_{\mathrm{c,Rd}} = f_{\mathrm{cd}} \cdot (1 + \eta_{\mathrm{cL}} \cdot \frac{t}{a} \cdot \frac{f_{\mathrm{y}}}{f_{\mathrm{ck}}}) \cdot \sqrt{\frac{A_{\mathrm{c}}}{A_1}} \,, \leq \frac{A_{\mathrm{c}} \cdot f_{\mathrm{cd}}}{A_1} \,, \leq f_{\mathrm{yd}}$$

$$\sigma_{\mathrm{c,Rd}} = 2,67 \cdot (1 + 4,9 \cdot \frac{1,0}{40,64} \cdot \frac{35,5}{4,0}) \cdot \sqrt{\frac{1153}{61,7}} = 23,9 \text{ kN/cm}^2$$

$$\sigma_{\mathrm{c,Rd}} = 23,9 < \frac{A_{\mathrm{c}} \cdot f_{\mathrm{cd}}}{A_1} = \frac{1153\cdot 2,67}{61,7} = 49,9 \text{ kN/cm}^2$$

$$\sigma_{\mathrm{c,Rd}} = 23,9 < f_{\mathrm{yd}} = 35,5 \text{ kN/cm}^2$$

The design compressive stress below the gusset plate $\sigma_{c,Ed} = 19,4$ kN/cm^2 is less than the local design strength of concrete under the gusset plate $\sigma_{c,Rd} = 23,9$ kN/cm^2. Accordingly, the condition is satisfied.

13. Commentary

The design of composite columns subject to compressive axial load and bending moment generally requires a second-order analysis. In this case, the analysis of the composite column was carried out in accordance with clause 6.7.3.4, EN 1994-1-1, taking into account the second-order effects and the member imperfections. The resistance of the column in combined compression and uniaxial bending was performed in accordance with clause 6.7.3.6, EN 1994-1-1.

This example illustrates the procedure of verification as follows:

a) The preliminary check was performed for the column, subject only to axial compression, by means of the method based on buckling curves. If the resistance to the axial force N_{Ed} is not sufficient, the considered column is clearly inadequate and the stronger cross-section should be adopted.

The utilisation is 80%.

b) In the next step, the column subject to compressive axial load and bending was treated. Since the value of α_{cr} was less than 10, the second-order effects were taken into account in two ways: approximate and exact. The resistance of the column was performed by the method based on the interaction polygon.

The utilisation is:

59% (for the design value of the maximum second-order bending moment $M_{Ed,II}$ calculated by the approximate method),

56% (for the design value of the maximum second-order bending moment $M_{Ed,II}$ calculated by the exact method).

C4 Composite column with concrete-filled rectangular hollow section subject to axial compression and uniaxial bending

1. Purpose of example

The concrete-filled column consists of the rectangular hollow section filled with concrete. It is assumed that this column is one of several similar columns in a multistorey braced frame. The beams of the composite floor are attached to the column. Global analysis provides for each column in a plane frame a design axial force, N_{Ed}, and applied end moments, M_{Ed}. Since the column consists of the rectangular hollow section filled with concrete, i.e. the double-symmetrical section with different geometrical properties about the principal axes, the check is carried out in several steps as described below.

For each axis of symmetry, the buckling resistance to compression should be checked with the relevant relative slenderness of the composite column. However, in this example the relative slenderness $\bar{\lambda}_z$ is higher than the relative slenderness $\bar{\lambda}_y$ and the check is carried out only about the *z-z* axis. It is the preliminary check.

In the presence of applied moment about the *y-y* axis, the resistance moment of the composite cross-section is checked with the relative slenderness $\bar{\lambda}_y$ of the composite column. Second-order effects are included by using a first-order analysis modified with appropriate amplification according to clause 6.7.3.4(5), EN 1994-1-1. The application of the method for buckling resistance based on second-order analysis taking into account the imperfections means that about the *z-z* axis there is bending due to the imperfections of the member. This means that the verification of the composite column should be performed as for the column in compression and in biaxial bending.

The first step is to check the column resistance under compression and uniaxial bending individually in each of the planes of bending. The second step is to check the column resistance in biaxial bending, taking into account imperfections in the plane in which failure is expected to occur. For the other plane of bending the effect of imperfections is neglected. If it is not obvious which plane is the more critical, checks should be made for both planes.

Composite Structures according to Eurocode 4. Worked Examples.
First Edition. Darko Dujmović, Boris Androić, Ivan Lukačević.

2. Static system, cross-section and design action effects

Actions

Permanent action $N_{G_{1,k}} = 410 \text{ kN}$

$N_{G_{2,k}} = 110 \text{ kN}$

Variable action $N_{Q_{1,k}} = 230 \text{ kN}$

$N_{Q_{2,k}} = 70 \text{ kN}$

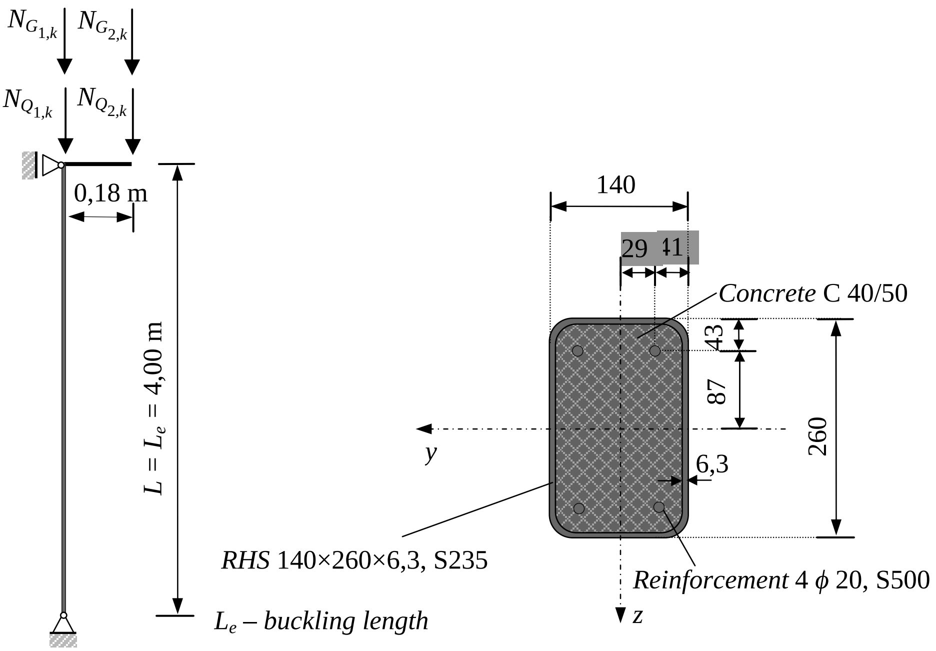


Figure C4.1 *Static system and cross-section (bending about the y-y axis)*

Design action effect

Axial force:

$$N_{Ed} = \gamma_G \cdot (N_{G_{1,k}} + N_{G_{2,k}}) + \gamma_Q \cdot (N_{Q_{1,k}} + N_{Q_{2,k}})$$

$$N_{Ed} = 1,35 \cdot (N_{G_{1,k}} + N_{G_{2,k}}) + 1,50 \cdot (N_{Q_{1,k}} + N_{Q_{2,k}})$$

$$N_{Ed} = 1,35 \cdot (410 + 110) + 1,50 \cdot (230 + 70) = 702 + 450 = 1152 \text{ kN}$$

Bending moment at the top of the column:

$$M_{y,Ed} = \gamma_G \cdot N_{G_{2,k}} \cdot 0{,}18 + \gamma_Q \cdot N_{Q_{2,k}} \cdot 0{,}18$$

$$M_{y,Ed} = 1{,}35 \cdot N_{G_{2k}} \cdot 0{,}18 + 1{,}50 \cdot N_{Q_{2,k}} \cdot 0{,}18$$

$$M_{y,Ed} = 1{,}35 \cdot 110 \cdot 0{,}18 + 1{,}50 \cdot 70 \cdot 0{,}18 = 27 + 19 = 46 \text{ kNm}$$

Denotation of imperfections

Imperfection about the *y-y* axis is denoted as $e_{0,z}$. Imperfection about the *z-z* axis is denoted by $e_{0,y}$.

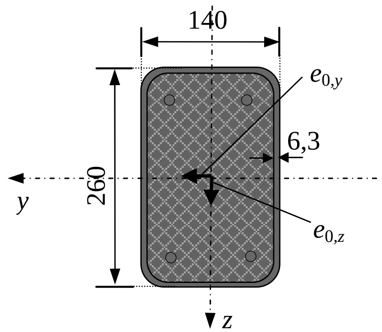


Figure C4.2 *Denotation of imperfections*

3. Properties of materials

Concrete strength class: C 40/50 $f_{ck} = 40 \text{ N/mm}^2$

$$f_{cd} = \frac{f_{ck}}{\gamma_c} = \frac{40}{1{,}5} = 26{,}7 \text{ N/mm}^2$$

$$E_{cm} = 35000 \text{ N/mm}^2$$

Structural steel: S235 $f_{yk} = 235 \text{ N/mm}^2$

$$f_{yd} = \frac{f_{yk}}{\gamma_M} = \frac{235}{1{,}0} = 235 \text{ N/mm}^2$$

$$E_a = 210000 \text{ N/mm}^2$$

Reinforcement: *ductility class B or C* $f_{sk} = 500 \text{ N/mm}^2$

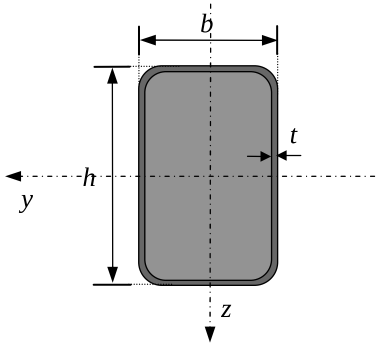
$$f_{sd} = \frac{f_{sk}}{\gamma_s} = \frac{500}{1,15} = 435 \ \text{N/mm}^2$$

$$E_S = 210000 \ \text{N/mm}^2$$

4. Geometrical properties of the cross-section

4.1 Selection of the steel cross-section and reinforcement

The rectangular hollow section RHS 260×140×6,3 is selected. The selected cross-section is shown in Figure C4.3.



h = 260,0 mm
b = 140,0 mm
t = 6,3 mm
A_a = 47,8 cm^2
$I_{y,a}$ = 4259 cm^4
$I_{z,a}$ = 1634 cm^4
$W_{pl,y,a}$ = 403 cm^3
$W_{pl,z,a}$ = 263 cm^3

Figure C4.3 *Rectangular hollow section*

Remark:

In concrete-filled hollow sections no longitudinal reinforcement is normally necessary. However, if the design for fire resistance is required, which is the case in this example, the longitudinal reinforcement can be used, clause 6.7.5.2 (1), EN 1994-1-1.

In this example, the selected reinforcement is four bars with a diameter of 20 mm. The cross-sectional area of the structural steel section RHS 260×140×6,3 is:

$$A_a = 47,8 \ \text{cm}^2$$

The cross-sectional area of the reinforcement with four bars of 20 mm diameter is:

d_{bar} = 20 mm, A_{bar} = 3,14 cm^2

$$A_s = 4 \cdot A_{bar} = 4 \cdot 3,14 = 12,6 \ \text{cm}^2$$

The cross-sectional area of the concrete, neglecting the rounded corners of the steel section is:

$$A_c = (h - 2 \cdot t) \cdot (b - 2 \cdot t) - A_s$$

$$A_c = (26,0 - 2 \cdot 0,63) \cdot (14,0 - 2 \cdot 0,63) - 12,6$$

$$A_c = 302,6 \ \text{cm}^2$$

The ratio of reinforcement area to concrete area is:

$$\rho_s = \frac{A_s}{A_c} = \frac{12,6}{302,6} = 0,042 \qquad \rho_s = 4,2\,\%$$

$$\rho_s = 4,2\% < 6\%$$

The limit of 6% in clause 6.7.3.1 (1), EN 1994-1-1, on the reinforcement is satisfied.

Remark:

According to clause 6.7.3.1(3), EN 1994-1-1, the ratio of reinforcement area to concrete area, ρ_s, should not exceed 6%.

4.2 Cross-sectional areas

Structural steel

$$A_a = 47,8 \ \text{cm}^2$$

Reinforcement

$$A_s = 12,6 \ \text{cm}^2$$

Concrete (rounded corners of the steel section neglected)

$$A_c = 302,6 \ \text{cm}^2$$

4.3 Second moments of area

Bending about the y-y axis

Structural steel

$$I_{y,a} = 4259 \ \text{cm}^4$$

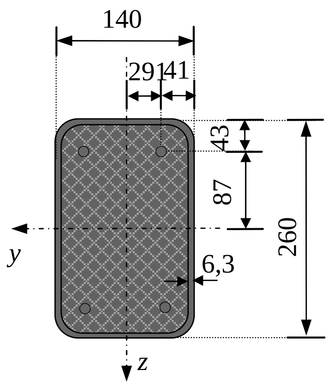


Figure C4.4 *Composite column cross-section*

Reinforcement

$$I_{y,s} = 4 \cdot A_{bar} \cdot 8{,}7^2$$

$$I_{y,s} = 4 \cdot 3{,}14 \cdot 8{,}7^2$$

$$I_{y,s} = 12{,}6 \cdot 8{,}7^2 = 954 \ \text{cm}^4$$

Concrete (rounded corners of the steel section neglected)

$$I_{y,c} = \frac{(b - 2 \cdot t) \cdot (h - 2 \cdot t)^3}{12} - I_{y,s}$$

$$I_{y,c} = \frac{(14{,}0 - 2 \cdot 0{,}63) \cdot (26{,}0 - 2 \cdot 0{,}63)^3}{12} - 954 = \frac{12{,}74 \cdot 24{,}74^3}{12} - 954 = 15122 \ \text{cm}^4$$

Bending about the z-z axis

Structural steel

$$I_{z,a} = 1634 \ \text{cm}^4$$

Reinforcement

$$I_{z,s} = 4 \cdot A_{bar} \cdot 2{,}9^2$$

$$I_{z,s} = 4 \cdot 3{,}14 \cdot 2{,}9^2$$

$$I_{z,s} = 12{,}6 \cdot 2{,}9^2 = 106 \ \text{cm}^4$$

Concrete (rounded corners of the steel section neglected)

$$I_{z,c} = \frac{(h - 2 \cdot t) \cdot (b - 2 \cdot t)^3}{12} - I_{z,s}$$

$$I_{z,c} = \frac{(26{,}0 - 2 \cdot 0{,}63) \cdot (14{,}0 - 2 \cdot 0{,}63)^3}{12} - 106 = \frac{24{,}74 \cdot 12{,}74^3}{12} - 106 = 4157 \ \text{cm}^4$$

4.4 Plastic section moduli

Bending about the y-y axis

Structural steel

$$W_{pl,y,a} = 403 \ \text{cm}^3$$

Reinforcement

$$W_{pl,y,s} = \sum_i A_{s,i} \cdot z_i = 4 \cdot 3{,}14 \cdot 8{,}7 = 109{,}3 \ \text{cm}^3$$

Concrete (rounded corners of the steel section neglected)

$$W_{pl,y,c} = \frac{(b - 2 \cdot t) \cdot (h - 2 \cdot t)^2}{4} - W_{pl,s,y}$$

$$W_{pl,y,c} = \frac{(14{,}0 - 2 \cdot 0{,}63) \cdot (26{,}0 - 2 \cdot 0{,}63)^2}{4} - 109{,}3 = 1840{,}1 \ \text{cm}^3$$

Bending about the z-z axis

Structural steel

$$W_{pl,z,a} = 263 \ \text{cm}^3$$

Reinforcement

$$W_{pl,z,s} = \sum_i A_{s,i} \cdot z_i = 12,6 \cdot 2,9 = 36,5 \text{ cm}^3$$

Concrete (rounded corners of the steel section neglected)

$$W_{pl,z,c} = \frac{(h-2\cdot t)\cdot(b-2\cdot t)^2}{4} - W_{pl,s,z}$$

$$W_{pl,z,c} = \frac{(26,0-2\cdot 0,63)\cdot(14,0-2\cdot 0,63)^2}{4} - 36,5 = 967,4 \text{ cm}^3$$

5. Steel contribution ratio

According to clause 6.7.3.3(1), EN 1994-1-1, the steel contribution ratio, δ, is defined as:

$$\delta = \frac{A_a \cdot f_{yd}}{N_{pl,Rd}}$$

The term $A_a \cdot f_{yd}$ is the contribution of the structural steel section to the plastic resistance of the composite section to axial force. The design plastic resistance of the composite section to axial force $N_{pl,Rd}$ is calculated according to clause 6.7.3.2(1), EN 1994-1-1.

According to 6.7.1(4), EN 1994-1-1, the steel contribution ratio, δ, must satisfy the following conditions:

$0,2 \leq \delta \leq 0,9$

If δ is less than 0,2, the column should be designed as a reinforced concrete member according to EN 1992-1-1. If δ is larger than 0,9, the concrete is ignored in the calculations, and the column is designed as a structural steel member according to EN 1993-1-1.

The term $A_a \cdot f_{yd}$ is the contribution of the structural steel section to the plastic resistance of the composite section to axial force:

$$A_a \cdot f_{yd} = 47,8 \cdot 23,5 = 1123,3 \text{ kN}$$

The plastic resistance of the composite section to axial force is:

$$N_{pl,Rd} = A_a \cdot f_{yd} + A_c \cdot f_{cd} + A_s \cdot f_{sd}$$

$$N_{pl,Rd} = 47,8 \cdot 23,5 + 302,6 \cdot 2,67 + 12,6 \cdot 43,5$$

$$N_{pl,Rd} = 1123,3 + 807,9 + 548,1 = 2479 \text{ kN}$$

According to 6.7.1(4), EN 1994-1-1, the steel contribution ratio, δ, must satisfy the following conditions:

$0,2 \leq \delta \leq 0,9$

The steel contribution ratio, δ, is:

$$\delta = \frac{A_a \cdot f_{yd}}{N_{pl,Rd}} = \frac{47,8 \cdot 23,5}{2479} = 0,45$$

Since the limits $0,2 < \delta = 0,45 < 0,9$ are satisfied, the column can be classified as a composite column and the provisions of EN 1994-1-1 can be used for the dimensioning.

Remark:

The containment effect is not present to the same extent in concrete-filled rectangular tubes because less circumferential tension can be developed. The increase of the compressive strength of the concrete is ignored.

6. Local buckling

According to Table 6.3, EN 1994-1-1, for concrete-filled rectangular hollow cross-section, the effect of local buckling can be ignored if the following condition is satisfied, see Figure C4.5:

$$\max\left(\frac{h}{t}\right) = 52 \cdot \sqrt{\frac{235}{f_y}}$$

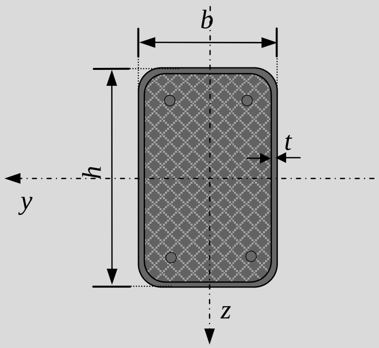


Figure C4.5 *Cross-section of composite column and denotations*

For the selected cross-section, the maximum slenderness is:

$$\max(\frac{h}{t}) = \frac{260}{6,3} = 41,3$$

The required condition is:

$$52 \cdot \sqrt{\frac{235}{f_y}} = 52 \cdot \sqrt{\frac{235}{235}} = 52$$

Since $41,3 < 52,0$, the condition is satisfied. The effect of local buckling can be neglected.

7. Effective modulus of elasticity for concrete

For long-term loading the creep and shrinkage are taken into account in the design by a reduced flexural stiffness of the composite cross-section. Due to the influence of long-term creep effects on the effective elastic stiffness, the modulus of elasticity of the concrete, E_{cm}, should be reduced to the value $E_{c,eff}$ in accordance with:

$$E_{c,eff} = \frac{E_{cm}}{1 + (\frac{N_{G,Ed}}{N_{Ed}}) \cdot \varphi_t}$$

where:

$\varphi_t = \varphi(t, t_0)$ is the creep coefficient, defining the creep between times t and t_0, related to elastic deformation at 28 days,

$\varphi_t = \varphi(\infty, t_0)$ is the final creep coefficient,

t is the age of the concrete at the time considered,

t_0 is the age of the concrete at loading,

N_{Ed} is the axial design force,

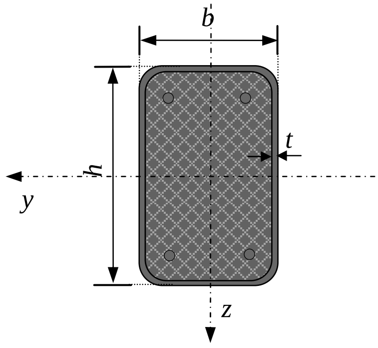
$N_{G,Ed}$ is the permanent part of the axial design force N_{Ed}, $N_{G,Ed} = \gamma_G \cdot N_{Gk}$.

For the calculation of the creep coefficient $\varphi(t, t_0)$, the following is valid:

- the perimeter of that part which is exposed to drying, u

$$u \approx 2 \cdot (h + b)$$

$$u \approx 2 \cdot (26 + 14) = 80 \text{ cm}$$



$u \approx 2(h + b)$

Figure C4.6 *Perimeter which is "exposed" to drying*

- the notional size of the cross-section, h_0

$$h_0 = \frac{2 \cdot A_c}{u} = \frac{2 \cdot 302,6}{80} = 7,6 \text{ cm} = 76 \text{ mm}$$

- $t_0 = 7$ days,
- inside conditions, the ambient relative humidity RH 50 %,
- the concrete strength class C 40/50,
- the type of cement – cement class N, strength class 32,5 R.

The final value of creep coefficient $\varphi(\infty, t_0)$ is determined using the nomogram shown in Figure 3.1, EN 1992-1-1. The process of determining the final value of the creep coefficient, taking into account these assumptions, is given in Figure C4.7.

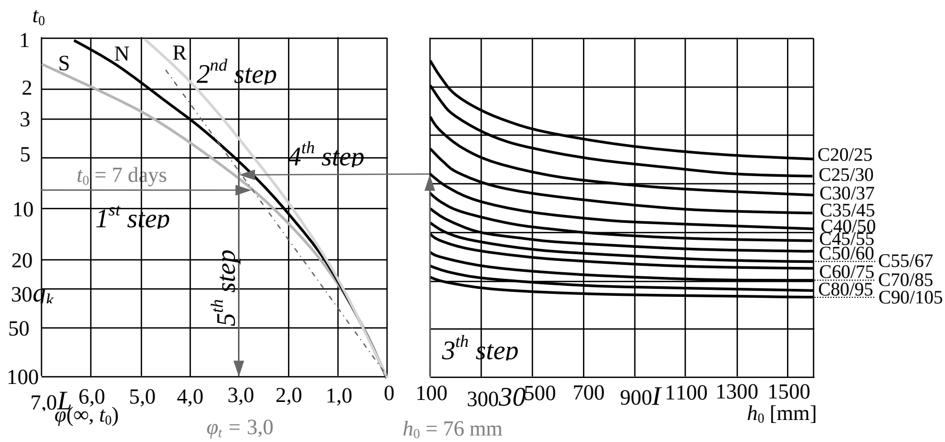


Figure C4.7 *Method for determining the creep coefficient*

The final value of creep coefficient $\varphi(\infty, t_0)$, found from Figure C4.7, is:

$\varphi_t = \varphi(\infty, t_0) = 3{,}0$

The design force of the permanent load, $N_{G,Ed}$, and the total design force, N_{Ed}, are:

$$N_{G,Ed} = \gamma_G \cdot (N_{G_{1,k}} + N_{G_{2,k}})$$

$$N_{G,Ed} = 1{,}35 \cdot (410 + 110) = 702 \text{ kN}$$

$$N_{Ed} = 1{,}35 \cdot (N_{G_{1,k}} + N_{G_{2,k}}) + 1{,}50 \cdot (N_{Q_{1,k}} + N_{Q_{2,k}})$$

$$N_{Ed} = 1{,}35 \cdot (410 + 110) + 1{,}50 \cdot (230 + 70) = 702 + 450 = 1152 \text{ kN}$$

Accordingly, the value of $E_{c,eff}$ is:

$$E_{c,eff} = \frac{E_{cm}}{1 + (\frac{N_{G,Ed}}{N_{Ed}}) \cdot \varphi_t} = \frac{3500}{1 + (\frac{702}{1152}) \cdot 3{,}0} = 1238 \text{ kN/cm}^2$$

Further calculation is performed with the effective modulus of elasticity of the concrete $E_{c,eff}$ = 1238 kN/m^2.

Remark:

Note that a conservative estimation of the effective modulus of elasticity is obtained according to EN 1994-1-1 because in the case of a concrete-filled hollow section the drying of the concrete is significantly reduced by the steel section (see example C1). However, in the case of concrete-filled rectangular hollow section the dimensioning of the composite columns is rarely sensitive to the influence of creep coefficient φ_t on the effective modulus of elasticity of the concrete $E_{c,eff}$. With this statement the conservative estimation of $E_{c,eff}$, according to EN 1994-1-1, can be justified in the case of concrete-filled hollow sections.

8. Resistance of the cross-section to compressive axial force

The design plastic resistance of the composite cross-section to axial compressive force, $N_{pl,Rd}$, is given by the sum of the design resistances of the components as follows:

$$N_{pl,Rd} = A_a \cdot f_{yd} + 0{,}85 \cdot A_c \cdot f_{cd} + A_s \cdot f_{sd}$$

Remark:

The coefficient of 0,85 can be replaced with value 1,0 due to better curing conditions of concrete-filled hollow sections.

The design plastic resistance of the composite cross-section to compressive axial force, $N_{pl,Rd}$, is calculated according to the corrected expression:

$$N_{pl,Rd} = A_a \cdot f_{yd} + A_c \cdot f_{cd} + A_s \cdot f_{sd}$$

$$N_{pl,Rd} = 47,8 \cdot 23,5 + 302,6 \cdot 2,67 + 12,6 \cdot 43,5$$

$$N_{pl,Rd} = 1123 + 808 + 548 = 2479 \text{ kN}$$

The characteristic value of the plastic resistance of the composite cross-section to compressive axial force, $N_{pl,Rk}$, is determined by the following expression:

$$N_{pl,Rk} = A_a \cdot f_{yk} + A_c \cdot f_{ck} + A_s \cdot f_{sk}$$

$$N_{pl,Rk} = 47,8 \cdot 23,5 + 302,6 \cdot 4,00 + 12,6 \cdot 50,0$$

$$N_{pl,Rk} = 1123,3 + 1210,4 + 630 = 2964 \text{ kN}$$

9. Verification of conditions for using the simplified design method

The cross-section of the composite column should be doubly symmetrical and uniform along the entire length of the column.

This condition is satisfied.

Relative slenderness

To apply the simplified method it is necessary to satisfy the following conditions:

$$\bar{\lambda}_y \leq 2,0$$

$$\bar{\lambda}_z \leq 2,0$$

In this example it is obvious that $\bar{\lambda}_z$ is higher than $\bar{\lambda}_y$, the verification of both relative slendernesses will be performed for educational reasons.

About the y-y axis

Relative slenderness, $\bar{\lambda}_y$, is determined as:

$$\overline{\lambda_y} = \sqrt{\frac{N_{pl,Rk}}{N_{cr,y}}}$$

For the determination of the relative slenderness $\bar{\lambda}_y$ and the elastic critical force $N_{cr,y}$, it is necessary to calculate the value of the effective flexural stiffness of the cross-section of the composite column, $(EI)_{eff,y}$, according to the expression:

$$(EI)_{eff,y} = E_a \cdot I_{y,a} + E_s \cdot I_{y,s} + K_e \cdot E_{c,eff} \cdot I_{y,c}$$

With the correction factor K_e = 0,6, the value $(EI)_{eff,y}$ is:

$$(EI)_{eff,y} = 21000 \cdot 4259 + 21000 \cdot 954 + 0,6 \cdot 1238 \cdot 15122$$

$$(EI)_{eff,y} = 120,71 \cdot 10^6 \text{ kNcm}^2$$

The elastic critical force, $N_{cr,y}$, for the pin-ended column and the buckling length $L_{e,y}$, is determined as:

$$N_{cr,y} = \frac{(EI)_{eff,y} \cdot \pi^2}{L_{e,y}^2} \qquad L_{e,y} = L$$

$$N_{cr,y} = \frac{120,71 \cdot 10^6 \cdot \pi^2}{400^2} = 7446 \text{ kN}$$

The relative slenderness, $\bar{\lambda}_y$, is:

$$\bar{\lambda}_y = \sqrt{\frac{2964}{7446}} = 0,63$$

Accordingly $\bar{\lambda}_y = 0,63 < 2,0$, and the condition is satisfied.

About the z-z axis

Relative slenderness, $\bar{\lambda}_z$, is determined as:

$$\bar{\lambda}_z = \sqrt{\frac{N_{pl,Rk}}{N_{cr,z}}}$$

For the determination of the relative slenderness $\bar{\lambda}_z$ and the elastic critical force $N_{cr,z}$, it is necessary to calculate the value of the effective flexural stiffness of the cross-section of the composite column $(EI)_{eff,z}$ according to the expression:

$$(EI)_{eff,z} = E_a \cdot I_{z,a} + E_s \cdot I_{z,s} + K_e \cdot E_{c,eff} \cdot I_{z,c}$$

With the correction factor K_e = 0,6, the value $(EI)_{eff,z}$ is:

$$(EI)_{eff,z} = 21000 \cdot 1634 + 21000 \cdot 106 + 0,6 \cdot 1238 \cdot 4157$$

$$(EI)_{eff,z} = 39,63 \cdot 10^6 \text{ kNcm}^2$$

The elastic critical force, $N_{cr,z}$, for the pin-ended column and the buckling length $L_{e,z}$, is determined as:

$$N_{cr,z} = \frac{(EI)_{eff,z} \cdot \pi^2}{L_{e,z}^2} \qquad L_{e,z} = L$$

$$N_{cr,z} = \frac{39,63 \cdot 10^6 \cdot \pi^2}{400^2} = 2445 \text{ kN}$$

The relative slenderness, $\bar{\lambda}_z$, is:

$$\bar{\lambda}_z = \sqrt{\frac{2964}{2445}} = 1,10$$

Accordingly $\bar{\lambda}_z = 1,10 < 2,0$, and the condition is satisfied.

The maximum permitted cross-sectional area of the longitudinal reinforcement

The maximum cross-sectional area of longitudinal reinforcement $A_{s,max}$ that can be taken in the calculation should not exceed 6% of the concrete area. This condition is satisfied, see Section 4.1.

The ratio of the depth to the width

The ratio of the depth to the width of the composite cross-section should be within the following limits:

$$0,2 \leq \frac{h}{b} \leq 5,0$$

$$\frac{h}{b} = \frac{26,0}{14,0} = 1,9$$

$0,2 < \frac{h}{b} = 1,9 < 5,0$, the condition is satisfied

Remark:

All conditions from clause 6.7.3.1, EN 1994-1-1, are satisfied, so this allows the use of the simplified design method for composite columns.

10. Resistance of the member in axial compression

Remark:

Although the column is subjected to combined compression and bending, the check based on buckling curves is useful as the preliminary check for this column. If the resistance to the axial compressive force is not sufficient, the considered column is inadequate and it is necessary to select the stronger cross-section.

The resistance of the member subjected only to axial compression can be checked by second-order analysis according to clause 6.7.3.5, EN 1994-1-1, so as to take into account member imperfections. As a simplification in the case of the member subjected only to axial compression, the design value of the axial force N_{Ed} should satisfy the check based on European buckling curves, which can be written as:

$$\frac{N_{Ed}}{\chi \cdot N_{pl,Rd}} \leq 1,0$$

The reduction factor χ is given by:

$$\chi = \frac{1}{\Phi + \sqrt{\Phi^2 - \bar{\lambda}^2}}, \text{ but } \chi \leq 1{,}0$$

and

$$\Phi = 0{,}5 \cdot [1 + \alpha \cdot (\bar{\lambda} - \bar{\lambda}_0) + \bar{\lambda}^2], \text{ with } \bar{\lambda}_0 = 0{,}2$$

Since $\bar{\lambda}_z = 1{,}10 > \bar{\lambda}_y = 0{,}63$, the buckling resistance about *z-z* axis is governed.

Remark:

The relevant buckling curves for cross-sections of composite columns are given in Table 6.5, EN1994-1-1. According to Table 6.5, EN 1994-1-1, circular or rectangular hollow section columns filled with concrete or containing up to 3% reinforcement can be designed using buckling curve *a* with an imperfection factor α = 0,21. However, concrete-filled hollow section columns containing from 3% to 6% can be designed using buckling curve *b* with an imperfection factor α = 0,34.

The reinforcement ratio ρ_s is 4,2 %. Therefore from Table 6.5, EN1994-1-1, buckling curve *b* should be used.

From Table 6.3, EN1993-1-1, α = 0,34 for buckling curve *b* so that Φ_z is:

$$\Phi_z = 0{,}5 \cdot [1 + \alpha \cdot (\bar{\lambda} - \bar{\lambda}_0) + \bar{\lambda}^2]$$

$$\Phi_z = 0{,}5 \cdot [1 + 0{,}34 \cdot (1{,}10 - 0{,}2) + 1{,}10^2] = 1{,}26$$

The reduction factor χ is:

$$\chi_z = \frac{1}{\Phi_z + \sqrt{\Phi_z^2 - \bar{\lambda}_z^2}}$$

$$\chi_z = \frac{1}{1{,}26 + \sqrt{1{,}26^2 - 1{,}10^2}} = 0{,}53 < 1{,}0$$

Check:

$$\frac{N_{Ed}}{\chi_z \cdot N_{pl,Rd}} \leq 1{,}0$$

$$\frac{1152}{0,53 \cdot 2479} = 0,88 < 1,0$$

Since $0,88 < 1,0$, the check of the composite column subjected to axial compression is satisfied. It is not necessary to select the stronger cross-section.

11. Resistance of the member in combined compression and uniaxial bending

11.1 Resistance of the member about the *y-y* axis taking into account the equivalent member imperfection $e_{0,z}$

11.1.1 General

According to clause 6.7.3.6, EN 1994-1-1, the member in combined compression and uniaxial bending has sufficient resistance if the following condition is satisified:

$$\frac{M_{Ed}}{M_{pl,N,Rd}} = \frac{M_{Ed}}{\mu_d \cdot M_{pl,Rd}} \leq \alpha_M$$

where:

M_{Ed} is the greatest of the end moments and the maximum bending moment within the column length. This moment is calculated according to clause 6.7.3.4, EN 1994-1-1, including imperfections (Table 6.5, EN 1994-1-1) and second-order effects if necessary ($\alpha_{cr} > 10$).

$M_{pl,N,Rd}$ is the plastic resistance moment taking into account the axial force N_{Ed}, given by $\mu_d \cdot M_{pl,Rd}$, see Figure 6.18, EN 1994-1-1.

$M_{pl,Rd}$ is the plastic resistance moment, given by point B in Figure 6.18, EN 1994-1-1.

μ_d is the factor related to the design for compression and uniaxial bending.

α_M is the coefficient related to the bending of a composite column and is taken as 0,9 for steel grades between S235 and S355.

The condition can be written in the following form:

$$\frac{M_{Ed}}{M_{Rd}} = \frac{M_{Ed}}{\alpha_M \cdot \mu_d \cdot M_{pl,Rd}} \leq 1,0$$

The calculation of the design bending moment $M_{Ed} = M_{Ed,II}$ taking the initial bending moment about y-y axis, the imperfection $e_{0,z}$, and second-order effects is shown in Figure C4.8.

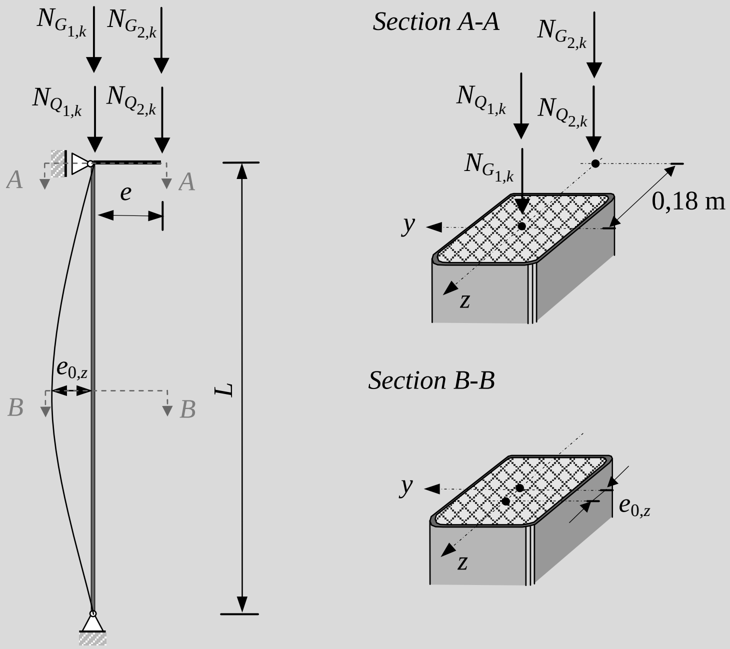


Figure C4.8 *Equivalent member imperfection $e_{0,z}$ about y-y axis*

11.1.2 *Resistance of cross-section in combined compression and bending about y-y axis*

Remark:

In order to determine the resistance of the composite cross-section to combined compression and uniaxial bending, it is necessary to produce an axial load – bending moment (*N*–*M*) interaction curve. As a simplification, the interaction curve is replaced by an interaction polygon *ACDB*, clause 6.7.3.2 (5), EN 1994-1-1.

The *N*–*M* interaction polygon *ACDB* is shown in Figure 6.19, EN 1994-1-1. For concrete-filled hollow sections, the interaction polygon of *AECDB*, (shown in Figure C4.9, may be preferred to the interaction polygon of *ACDB* shown in Figure 6.19, EN 1994-1-1. The introduction of the point *E* gives a more economical design, especially for columns with high axial force and low end moments. For better polygonal approximation to the interaction curve, the position of point *E* may be chosen to be closer to point *A* rather than being mid-way between points *A* and *C*. The introduction of the point *E* does not recommend in the case of steel section with low value of the shape factor α_{pl} (=

W_{pl}/W_{el}), *I*- section in bending about *y*-*y* axis.

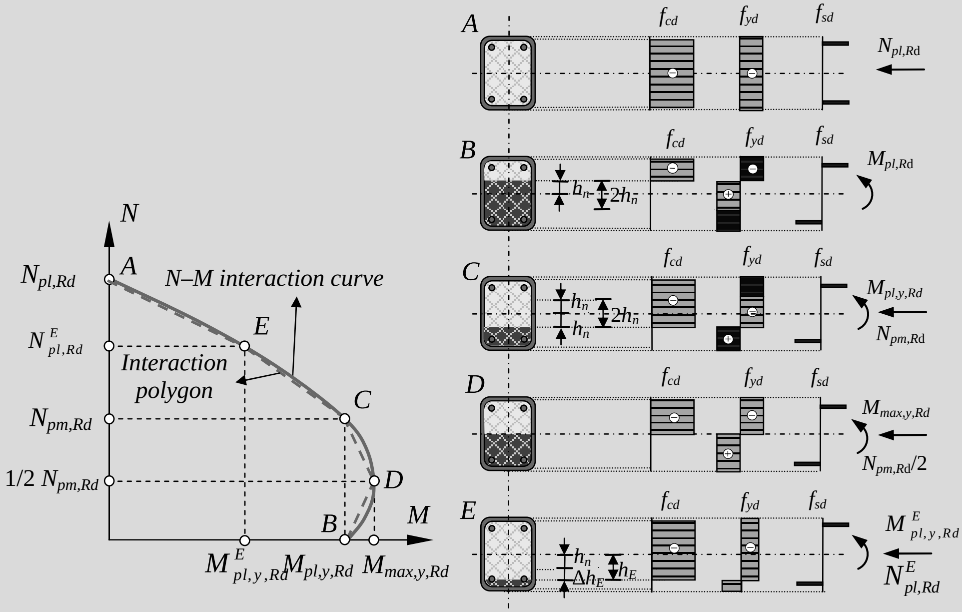


Figure C4.9 *N–M interaction polygon and corresponding stress distributions*

The modified version of the interaction polygon *AECDB*, which refers to the composite column with concrete-filled rectangular hollow section, is shown in Figure C4.9.

In order to produce the *N–M* interaction polygon, the cross-sectional capacities at points *A* to *D* should be determined assuming the stress distributions indicated, see Figure C4.9.

It should be noted that EN 1994-1-1 does not provide expressions for concrete-filled rectangular cross-sections.

Point A

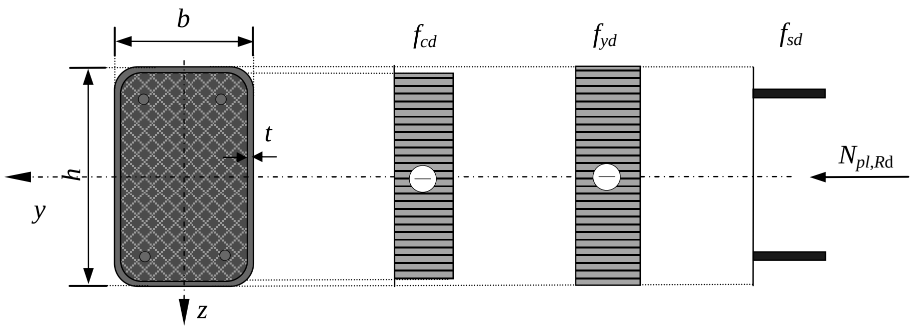


Figure C4.10 *Stress distributions for point A on the interaction polygon*

At point *A*, only the design plastic resistance of the cross-section is taken into account:

$$N_{pl,Rd} = A_a \cdot f_{yd} + 0,85 \cdot A_c \cdot f_{cd} + A_s \cdot f_{sd}$$

Remark:

For concrete filled hollow sections, the coefficient of 0,85 can be replaced with a value of 1,0 due to better curing conditions.

The design plastic resistance of composite cross-section to compression, $N_{pl,Rd}$, is calculated according to the corrected expression:

$$N_{pl,Rd} = A_a \cdot f_{yd} + A_c \cdot f_{cd} + A_s \cdot f_{sd}$$

The design plastic resistance of composite cross-section to compression is:

$$N_{pl,Rd} = 47,8 \cdot 23,5 + 302,6 \cdot 2,67 + 12,6 \cdot 43,5$$

$$N_{pl,Rd} = 1123 + 808 + 548 = 2479 \text{ kN}$$

Point D

The position of the plastic neutral axis and the stress distributions are shown in Figure C4.11.

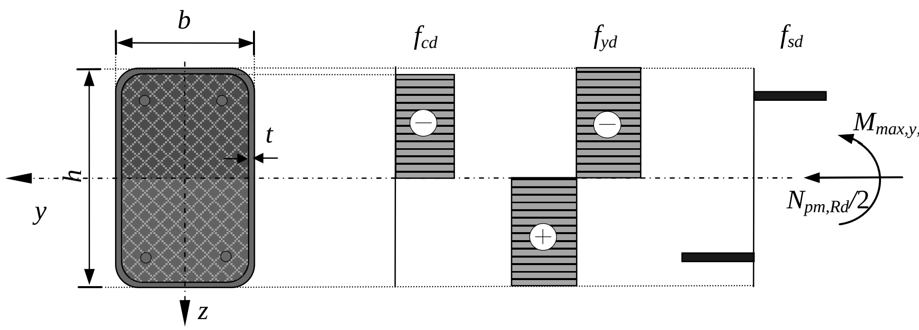


Figure C4.11 *Stress distributions for point D on the interaction polygon*

The maximum design plastic resistance moment is determined by the following expression:

$$M_{\max,y,Rd} = M_{pl,y,a,Rd} + M_{pl,y,c,Rd} + M_{pl,y,s,Rd}$$

The maximum design plastic resistance moment, $M_{max,y,Rd}$, at point D is:

$$M_{\max,y,Rd} = W_{pl,a,y} \cdot f_{yd} + 0{,}5 \cdot W_{pl,c,y} \cdot f_{cd} + W_{pl,s,y} \cdot f_{sd}$$

$$M_{\max,y,Rd} = (403 \cdot 23{,}5 + 0{,}5 \cdot 1840{,}1 \cdot 2{,}67 + 109{,}3 \cdot 43{,}5) \cdot 10^{-2}$$

$$M_{\max,y,Rd} = 167 \text{ kNm}$$

The design value of the resistance of the concrete to compression, $N_{pm,Rd}$, is:

$$N_{pm,Rd} = A_c \cdot f_{cd} = 302{,}6 \cdot 2{,}67 = 808 \text{ kN}$$

The design axial force at the point of maximum design plastic resistance moment is $0{,}5 \cdot N_{pm,Rd}$, and therefore is:

$$0{,}5 \cdot N_{pm,Rd} = 0{,}5 \cdot 808 = 404 \text{ kN}$$

Point C

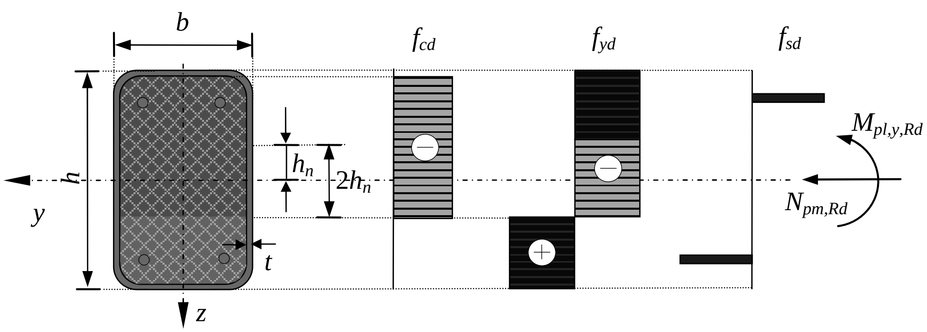


Figure C4.12 *Stress distributions for point C on the interaction polygon*

Calculation of the design plastic resistance moment of the composite section, $M_{pl,y,Rd}$, is carried out as shown below.

Determination of position of neutral axis depth, h_n, when axial force is zero:

$$h_n = \frac{N_{pm,Rd} - A_{s,n} \cdot (2 \cdot f_{sd} - f_{cd})}{2 \cdot d \cdot f_{cd} + 4 \cdot t \cdot (2 \cdot f_{yd} - f_{cd})}$$

where $A_{s,n}$ is the reinforcement area within h_n. Because it is at this point unknown, assumed to be (initial guess):

$$A_{s,n} = 0 \text{ cm}^2$$

Thus, for the case when the axial force is equal to zero, h_n is:

$$h_n = \frac{808,0 - 0 \cdot (2 \cdot 43,5 - 2,67)}{2 \cdot 14,0 \cdot 2,67 + 4 \cdot 0,63 \cdot (2 \cdot 23,5 - 2,67)} = 4,33 \text{ cm}$$

Since in this region there is no reinforcement, the assumption is correct.

Plastic section moduli in region 2·*h_n*

Reinforcement

$W_{pl,y,,s,n}$ = 0 cm^3

Concrete

$$W_{pl,y,c,n} = (d - 2 \cdot t) \cdot h_n^2 - W_{pl,y,s,n}$$

$$W_{pl,y,c,n} = (14,0 - 2 \cdot 0,63) \cdot 4,33^2 - 0 = 238,9 \text{ cm}^3$$

Structural steel

$$W_{pl,y,a,n} = d \cdot h_n^2 - W_{pl,y,c,n} - W_{pl,y,s,n}$$

$$W_{pl,y,a,n} = 14,0 \cdot 4,33^2 - 238,9 - 0 = 23,6 \text{ cm}^3$$

The design plastic resistance moment of the composite section, $M_{pl,y,Rd}$, is calculated as follows:

$$M_{pl,y,Rd} = M_{\max,y,Rd} - M_{n,y,Rd}$$

where:

$$M_{n,y,Rd} = W_{pl,y,a,n} \cdot f_{yd} + W_{pl,y,s,n} \cdot f_{sd} + \frac{W_{pl,y,c,n} \cdot f_{cd}}{2}$$

$$M_{n,y,Rd} = (23,6 \cdot 23,5 + 0 \cdot 40,0 + \frac{238,9 \cdot 2,67}{2}) \cdot 10^{-2} = 8,73 \text{ kNm}$$

The design plastic resistance moment of the composite section, $M_{pl,y,Rd}$, is:

$$M_{pl,y,Rd} = 167 - 8,73 = 158 \text{ kNm}$$

Point B

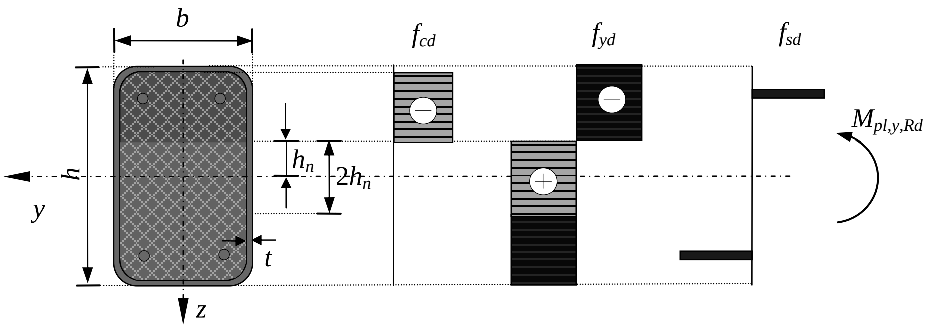


Figure C4.13 *Stress distributions for point B on the interaction polygon*

The design value of $M_{pl,y,Rd}$ has previously been calculated in order to define point C on the N–M interaction polygon:

$$M_{pl,y,Rd} = 158 \text{ kNm}$$

Point E

Remark:

If the enhancement of the resistance at point E is little more than that given by direct linear interpolation between A and C, the calculation can be omitted.

For the calculation of the design resistances at the point E, the neutral axis is located on the outer border of reinforcement bars, see Figure C4.14.

The distance between the centroidal axis of the composite section and the outer border of reinforcement bars, h_E, is obtained as follows:

$$h_E = z + \frac{d_{bar}}{2} = 8,7 + \frac{2,0}{2} = 9,7 \text{ cm}$$

In accordance with Figure C4.14, Δh_E is obtained as:

$$\Delta h_E = h_E - h_n = 9,7 - 4,33 = 5,37 \text{ cm}$$

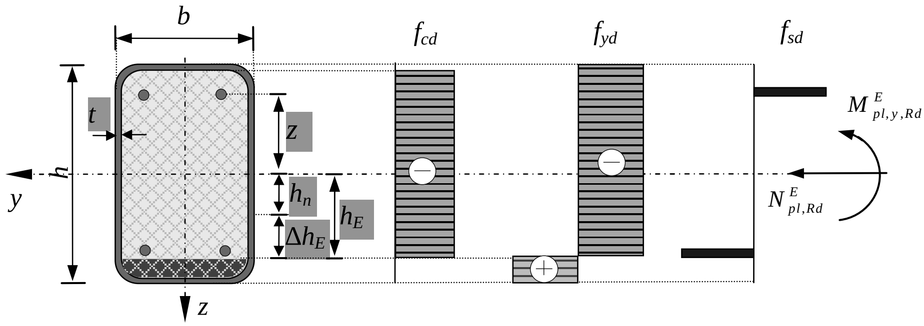


Figure C4.14 *Stress distributions for point E on the interaction polygon*

In region of Δh_E there is reinforcement (2 ϕ 20) of cross-sectional area $A_{s,n}$ = 6,28 cm^2. The stresses in the region $\Delta h_E = h_E - h_n$ provide the force $\Delta N^E_{pl,Rd}$ which is given by:

$$\Delta N^E_{pl,Rd} = b \cdot \Delta h_E \cdot f_{cd} + 2 \cdot t \cdot \Delta h_E \cdot (2 \cdot f_{yd} - f_{cd}) + A_{sn} \cdot (2 \cdot f_{sd} - f_{cd})$$

The force $\Delta N^E_{pl,Rd}$ is:

$$\Delta N^E_{pl,Rd} = 14,0 \cdot 5,37 \cdot 2,67 + 2 \cdot 0,63 \cdot 5,37 \cdot (2 \cdot 23,5 - 2,67) + 6,28 \cdot (2 \cdot 43,5 - 2,67)$$

$$\Delta N^E_{pl,Rd} = 1030 \text{ kN}$$

Therefore, the design resistance to axial force at point *E* is:

$$N^E_{pl,Rd} = \Delta N^E_{pl,Rd} + N_{pm,Rd} = 1030 + 808 = 1838 \text{ kN}$$

Plastic section moduli in region 2·h_E

Reinforcement

$$W_{pl,y,s,n} = \Sigma A_{s,i} \cdot z_i = 4 \cdot 3,14 \cdot 8,7 = 109,3 \text{ cm}^3$$

Concrete

$$W_{pl,y,c,n} = \frac{(b - 2 \cdot t) \cdot (2 \cdot h_E)^2}{4} - W_{pl,y,s,n} = \frac{(14,0 - 2 \cdot 0,63) \cdot (2 \cdot 9,7)^2}{4} - 109,3$$

$$W_{pl,y,c,n} = 1089,4 \ \text{cm}^3$$

Structural steel

$$W_{pl,y,a,n} = \frac{2 \cdot t \cdot (2 \cdot h_E)^2}{4} = \frac{2 \cdot 0,63 \cdot (2 \cdot 9,7)^2}{4}$$

$$W_{pl,y,a,n} = 118,6 \ \text{cm}^3$$

The design plastic resistance moment of the composite section, $M^E_{pl,y,Rd}$, is calculated as follows:

$$M^E_{pl,y,Rd} = M_{\max,y,Rd} - M^E_{n,y,Rd}$$

where:

$$M^E_{n,y,Rd} = W_{pl,y,a,n} \cdot f_{yd} + W_{pl,y,s,n} \cdot f_{sd} + \frac{W_{pl,y,c,n} \cdot f_{cd}}{2}$$

$$M^E_{n,y,Rd} = (118,6 \cdot 23,5 + 109,3 \cdot 43,5 + \frac{1089,4 \cdot 2,67}{2}) \cdot 10^{-2} = 90 \ \text{kNm}$$

The design plastic resistance moment of the composite section, $M^E_{pl,y,Rd}$, is:

$$M^E_{pl,y,Rd} = M_{\max,y,Rd} - M^E_{n,y,Rd} = 167 - 90 = 77 \ \text{kNm}$$

Previously calculated values at points *A* to *E* should be plotted to produce the *N–M* interaction polygon (Figure 6.19, EN 1994-1-1). With the obtained value at point *E* the better polygonal approximation to the interaction curve is achieved.

According to the interaction polygon *AECDB*, Figure C4.15, the following value $M_{pl,y,N,Rd}$ is obtained:

$$M_{pl,y,N,Rd} = M^E_{pl,y,Rd} + \Delta M_{y,Rd}$$

where:

$$(M_{pl,y,Rd} - M^E_{pl,y,Rd}) : \Delta M_{y,Rd} = (N^E_{Rd} - N_{pm,Rd}) : (N^E_{Rd} - N_{Ed})$$

$$\Delta M_{y,Rd} = \frac{N_{Rd}^{E} - N_{Ed}}{N_{Rd}^{E} - N_{pm,Rd}} \cdot (M_{pl,y,Rd} - M_{pl,y,Rd}^{E})$$

$$\Delta M_{y,Rd} = \frac{1838 - 1152}{1838 - 808} \cdot (158 - 77) = 53,9 \text{ kNm}$$

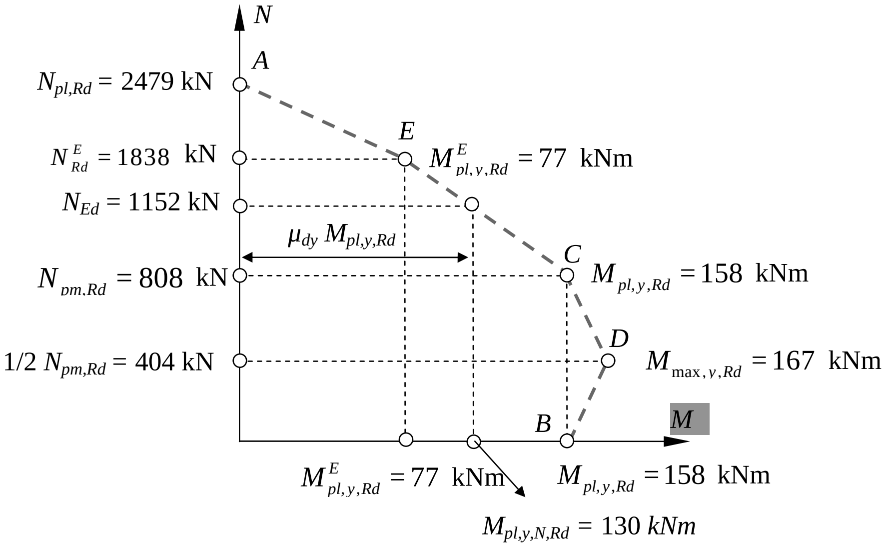


Figure C4.15 *N–M interaction polygon*

Therefore, the value $M_{pl,y,N,Rd}$ is:

$$M_{pl,y,N,Rd} = 77 + 53,9 = 131 \text{ kNm}$$

The value of μ_{dy} is:

$$\mu_{dy} = \frac{M_{pl,y,N,Rd}}{M_{pl,y,Rd}} = \frac{131}{158} = 0,83 < 1,0$$

The check is carried out by the factor $\mu_{dy} = 0,83$.

11.1.3 Calculation of the effects of actions about the y-y axis

11.1.3.1 General

According to clause 6.7.3.4 (3), EN 1994-1-1, which refers to clause 5.2.1(3), EN 1994-1-1, second-order effects can be therefore neglected if the load factor

α_{cr}, which is the ratio between the elastic critical load and the corresponding applied loading, for elastic instability of the member exceeds 10.

To calculate α_{cr}, the ends of the column are assumed to be pinned, and α_{cr} is found using the Euler formula for the elastic critical force $N_{cr,y,eff}$:

$$N_{cr,y,eff} = \frac{\pi^2 (EI)_{eff,y,II}}{L_{e,y}^2} \qquad L_{e,y} = L$$

The design value of the effective flexural stiffness $(EI)_{eff,y,II}$, used to determine the internal forces and moments by second-order analysis, pursuant to clause 6.7.3.4(2), EN 1994-1-1, is defined by the following expression:

$$(EI)_{eff,y,II} = K_0 \cdot (E_a \cdot I_{y,a} + E_s \cdot I_{y,s} + K_{e,II} \cdot E_{cm} \cdot I_{y,c})$$

where:

$K_{e,II}$ is a correction factor, which should be taken as 0,5,
K_0 is a calibration factor, which should be taken as 0,9.

The value $E_{c,eff}$ has been used in place of E_{cm} in expression for $(EI)_{eff,y,II}$ in order to take into account the long-term effects, in the same way as calculated in Section 7. Accordingly, the value of $E_{c,eff}$ is:

$$E_{c,eff} = 1238 \text{ kN/cm}^2$$

The design value of the effective flexural stiffness $(EI)_{eff,y,II}$, is:

$$(EI)_{eff,y,II} = K_0 \cdot (E_a \cdot I_{y,a} + E_s \cdot I_{y,s} + K_{e,II} \cdot E_{cm} \cdot I_{y,c})$$

$$(EI)_{eff,y,II} = 0,9 \cdot (21000 \cdot 4259 + 21000 \cdot 954 + 0,5 \cdot 1238 \cdot 15122)$$

$$(EI)_{eff,y,II} = 106,95 \cdot 10^6 \text{ kNcm}^2$$

The elastic critical force, $N_{cr,y,eff}$, for the pin-ended column, is:

$$N_{cr,y,eff} = \frac{\pi^2 (EI)_{eff,y,II}}{L_{e,y}^2}$$

$$N_{cr,eff,y} = \frac{106,95 \cdot 10^6 \cdot \pi^2}{400^2} = 6597 \text{ kN}$$

To check whether the effects of second-order analysis can be neglected, the value of α_{cr} must be higher than 10:

$$\alpha_{cr} = \frac{N_{cr,eff,y}}{N_{Ed}} = \frac{6597}{1152} = 5,7 < 10$$

The value of α_{cr} is less than 10, so the second-order effects must be considered.

Remark:

Second-order effects are included by using a first-order analysis modified with appropriate amplification according to clause 6.7.3.4(5), EN 1994-1-1.

11.1.3.2 Bending moments about the y-y axis

According to clause 6.7.3.4(5), EN 1994-1-1, the second-order effects can be calculated by multiplying the greatest first-order design bending moments by a factor *k*.

Thus, the second-order effects may be considered according to the expression:

$$M_{y,Ed,II} = M_{y,Ed,I} \cdot k$$

The factor *k* is given by:

$$k = \frac{\beta}{1 - N_{Ed} / N_{cr,y,eff}} \geq 1,0$$

where:

β is an equivalent moment factor given in Table 6.4, EN 1994-1-1,

$N_{cr,y,eff}$ is the critical axial force, about the *y*-*y* axis, obtained with the effective flexural stiffness $(EI)_{eff,y,II}$ and with the effective length taken as the physical length of the column.

The design bending moment from the member imperfections is determined by the following expression:

$$M_{y,Ed,I} = N_{Ed} \cdot e_{0,z}$$

where:

N_{Ed} is the design value of the axial force,

$e_{0,z}$ is the equivalent member imperfection, which is given in Table 6.5, EN 1994-1-1, depending on the buckling curve.

Remark:

The reinforcement ratio ρ_s is 4,4 %. Therefore from Table 6.5, EN 1994-1-1, for 3%< $\rho_s \leq$ 6%, the buckling curve *b* should be used.

Therefore, for the buckling curve *b*, the equivalent member imperfection is:

$$e_{0,z} = \frac{L}{200}$$

$$e_{0,z} = \frac{400}{200} = 2{,}0 \text{ cm}$$

The design bending moments calculated according to first-order analysis are shown in Figure C4.16.

The design values of bending moments are:

The design bending moment at the top of the column is:

$$M_{y,Ed} = 46 \text{ kNm}$$

The design bending moment at the bottom of the column is:

$$M_{y,Ed} = 0 \text{ kNm}$$

The design bending moment due to imperfection is:

$$M_{y,Ed,imp} = N_{Ed} \cdot e_{0,z} = 1152 \cdot 0{,}02 = 23 \text{ kNm}$$

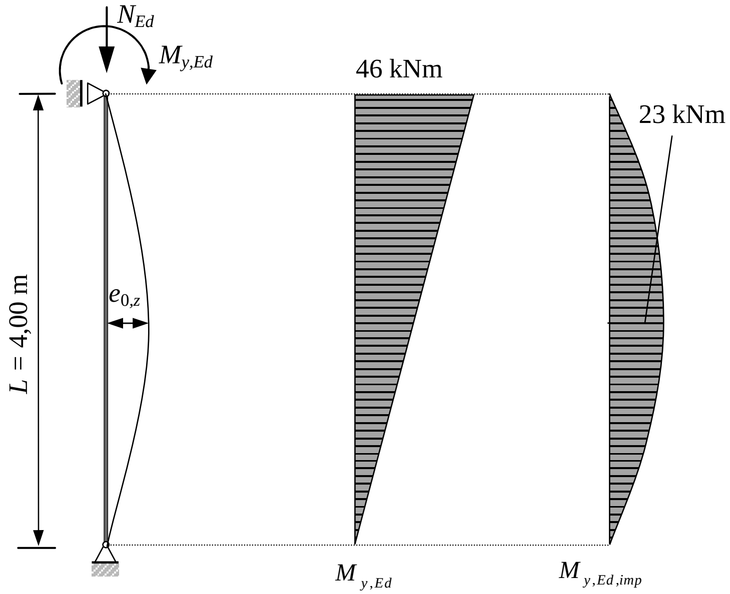


Figure C4.16 *First-order bending moments, design values*

Remark:

The factor β from Table 6.4, EN 1994-1-1, allows for the shape of the bending moment diagram. When bending is caused by lateral loading on the column, the value of factor β is 1,0. For a column subjected to end moments, the factor β is calculated as:

$$\beta_1 = 0{,}66 + 0{,}44 \cdot r \geq 0{,}44$$

where r is the ratio of the end-moments on the ends of the column ($-1 \leq r \leq +1$).

Therefore, the two values of factor k must be calculated:

- for the end moments, k_1,
- for the moment from the member imperfection, k_2.

Determination of factor k_1

The ratio of the end-moments on the ends of the column is:

$$r = \frac{0}{M_{y,Ed}} = \frac{0}{46} = 0{,}0$$

The equivalent moment factor β is:

$$\beta_1 = 0,66 + 0,44 \cdot r \geq 0,44$$

$$\beta_1 = 0,66 + 0,44 \cdot 0 = 0,66$$

Therefore, the factor k_1 is:

$$k_1 = \frac{\beta_1}{1 - N_{\mathrm{Ed}} / N_{\mathrm{cr,eff,y}}} = \frac{0,66}{1 - 1152 / 6597} = 0,80 < 1,0$$

Remark:

According to clause 6.7.3.4(5), EN 1994-1-1, the value of factor k must be 1,0 or higher. It is over-conservative to use when combining two sets of second-order effects. Therefore, the calculated value of 0,80 is adopted.

Determination of factor k_2

For the bending moment from the member imperfection, according to Table 6.4, EN 1994-1-1, the equivalent moment factor β is:

$$\beta_2 = 1,0$$

Therefore, the factor k_2 is:

$$k_2 = \frac{\beta_2}{1 - N_{\mathrm{Ed}} / N_{\mathrm{cr,eff,y}}} = \frac{1,0}{1 - 1152 / 6597} = 1,21 > 1,0$$

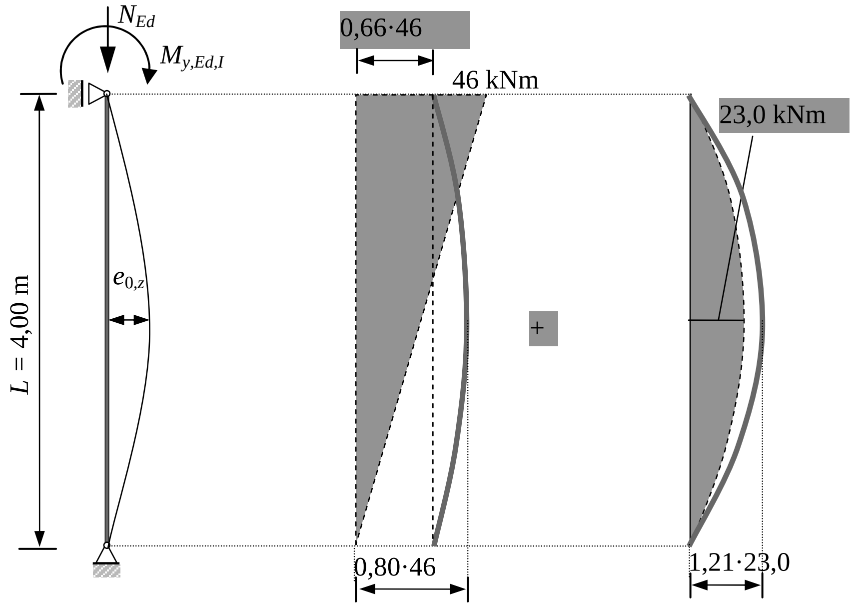
The adopted value of the factor is:

k_2 = 1,21

The design bending moment at mid-height, with second-order effects taken into account, is:

$$M_{y,Ed,II} = M_{y,Ed} \cdot k_1 + M_{y,Ed,imp} \cdot k_2 = 46 \cdot 0,80 + 23,0 \cdot 1,21 = 65 \text{ kNm}$$

The design bending moments calculated according to second-order analysis are shown in Figure C4.17.



a) *Second-order bending moments due to $N_{G2,Ed}$ and $N_{Q2,Ed}$*

b) *Imperfection moment increased by the second-order effects*

Figure C4.17 *Second-order bending moments about the y-y axis, design values*

The check is performed with the bending moment at mid-height:

$$M_{y,Ed,II} = M_{y,\max} = 65 \ \text{kNm}$$

11.1.3.3 Shear forces parallel to the z-z axis

According to clause 6.7.3.4(5), EN 1994-1-1, second-order effects can be allowed for by multiplying the greatest first-order design bending moment by a factor k given by:

$$k = \frac{\beta}{1 - N_{Ed} / N_{cr,eff}} \geq 1{,}0$$

Accordingly, the approximate value of shear force can be obtained as:

$$V_{Ed,II} = V_{Ed} \cdot k$$

In accordance with Figure C4.18, the first-order design shear force at the bottom of the column is:

$$V_{z,Ed} = \frac{M_{y,Ed}}{L} + \frac{4 \cdot N_{Ed} \cdot e_{0,z}}{L} = \frac{46}{4,0} + \frac{4 \cdot 1152 \cdot 0,02}{4,0} = 11,5 + 23,0 = 34,5 \text{ kN}$$

In accordance with Figure C4.18, the first-order design shear force at the top of the column is:

$$V_{z,Ed} = -\frac{M_{y,Ed}}{L} + \frac{4 \cdot N_{Ed} \cdot e_{0,z}}{L} = -\frac{46}{4,0} + \frac{4 \cdot 1152 \cdot 0,02}{4,0} = -11,5 + 23,0 = 11,5 \text{ kN}$$

The diagram of shear forces, calculated by first-order analysis for bending moment and the equivalent lateral load due to imperfections, is shown in Figure C4.18.

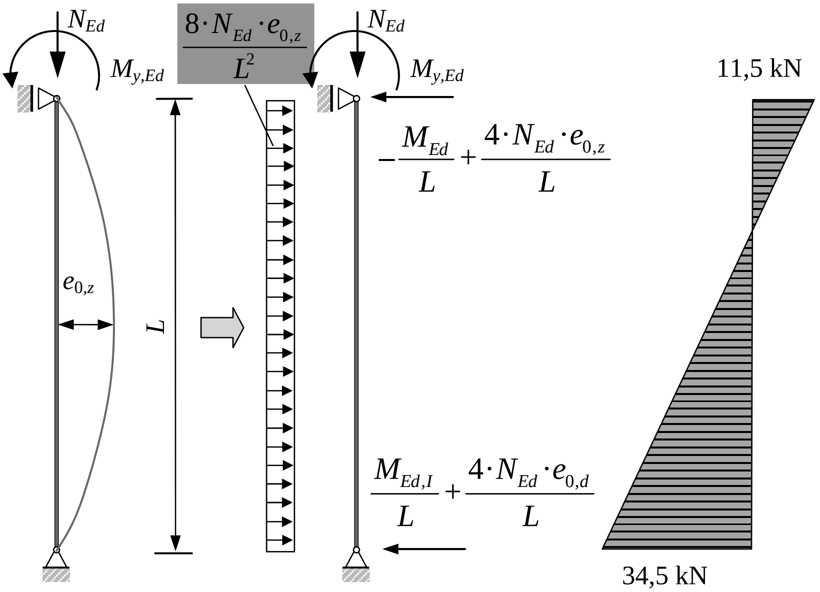


Figure C4.18 *First-order design shear forces parallel to the z-z axis*

The factor k_1 is:

$$k_1 = \frac{\beta_1}{1 - N_{Ed} / N_{cr,y,eff}} = \frac{0,66}{1 - 1152/6597} = 0,80 < 1,0$$

The factor k_2 is:

$$k_2 = \frac{\beta_2}{1 - N_{Ed} / N_{cr,y,eff}} = \frac{1}{1 - 1152/6597} = 1,21 > 1,0$$

Therefore, the maximum design shear force, calculated by approximative second-order analysis, is:

$$V_{z,Ed,II} = k_1 \cdot \frac{M_{y,Ed}}{L} + k_2 \cdot \frac{4 \cdot N_{Ed} \cdot e_{0,z}}{L}$$

$$V_{z,Ed,II} = 0,80 \cdot \frac{46}{4,0} + 1,21 \cdot \frac{4 \cdot 1152 \cdot 0,02}{4,0} = 9,2 + 27,9 = 37,1 \text{ kN}$$

11.1.4 Check of the resistance of the member in combined compression and bending about the y-y axis

It is necessary to satisfy the following condition:

$$\frac{M_{y,Ed}}{M_{y,Rd}} = \frac{M_{y,Ed}}{\alpha_{M,y} \cdot \mu_{dy} \cdot M_{pl,y,Rd}} \leq 1,0$$

The coefficient $\alpha_{M,y}$ is taken as 0,9 for steel grades between S235 and S355.

The design value of the maximum design bending moment by the approximative second-order analysis is:

$$M_{y,Ed} = M_{y,Ed,II} = 65 \text{ kNm}$$

The design resistance moment $M_{y,Rd}$ is (Figure C4.15):

$$M_{y,Rd} = \alpha_M \cdot \mu_{dy} \cdot M_{pl,y,Rd} = 0,9 \cdot 0,83 \cdot 158 = 118 \text{ kNm}$$

Condition:

$$\frac{M_{y,Ed}}{M_{y,Rd}} = \frac{65}{118} = 0,55 \leq 1,0$$

Since $0,55 < 1,0$, the condition is satisfied.

11.1.5 Check of the plastic resistance to transverse shear parallel to the z-z axis

In accordance with clause 6.7.3.2(4), EN 1994-1-1, for simplification V_{Ed} may be assumed to act on the structural steel section alone. According to clause 6.2.6(2), EN 1993-1-1, in the absence of torsion the design plastic shear resistance,

$V_{pl,z,a,Rd}$, is given by:

$$V_{pl,z,a,Rd} = \frac{A_{v,z} \cdot (f_y / \sqrt{3})}{\gamma_{M0}}$$

The shear area, $A_{v,z}$, according to clause 6.2.6(3), EN1993-1-1, is calculated as:

$$A_{v,z} = \frac{A_a \cdot h}{b + h}$$

where:

A_a is the cross-sectional area of the rectangular hollow section,
b is the width of the rectangular hollow section,
h is the depth of the rectangular hollow section.

According to clause 6.2.2.4(1), EN 1994-1-1, where the shear force is less than half the plastic shear resistance its effect on the resistance moment can be neglected. Therefore, the condition is:

$$V_{z,Ed} < 0{,}5 \cdot V_{pl,z,a,Rd}$$

The design value of second-order shear force is:

$$V_{z,Ed} = V_{z,Ed,II} = 37{,}1 \ \text{kNm}$$

The shear area, $A_{v,z}$, is:

$$A_{v,z} = \frac{47{,}8 \cdot 26{,}0}{14{,}0 + 26{,}0} = 31{,}07 \ \text{cm}^2$$

The design plastic shear resistance, $V_{pl,z,Rd}$, is:

$$V_{pl,a,z,Rd} = \frac{31{,}07 \cdot (23{,}5 / \sqrt{3})}{1{,}0} = 422 \ \text{kN}$$

Check:

$$V_{z,Ed} = 37{,}1 < 0{,}5 \cdot V_{pl,a,z,Rd} = 0{,}5 \cdot 422 = 211 \ \text{kN}$$

The condition is satisfied and no reduction in the resistance moment is needed.

11.2 Resistance of member about the *z-z* axis taking into account the equivalent member imperfection $e_{0,y}$

11.2.1 General

Remark:

Since that the column is subjected to bending about the *y-y* axis, initially given moment $M_{y,Ed}$, and the bending about the *z-z* axis, the bending moment due to imperfection $N_{Ed} \cdot e_0$, it is necessary to check the column resistance in combined compression and biaxial bending.

In accordance with clause 6.7.3.7, EN 1994-1-1, for combined compression and biaxial bending the following conditions should be satisfied:

Check for bending about the *y-y* axis:

$$\frac{M_{y,Ed}}{M_{pl,y,N,Rd}} = \frac{M_{y,Ed}}{\mu_{dy} \cdot M_{pl,y,Rd}} \leq \alpha_{My}$$

The condition can be written in the following form:

$$\frac{M_{y,Ed}}{\alpha_{M,y} \cdot \mu_{dy} \cdot M_{pl,y,Rd}} \leq 1{,}0$$

Check for bending about the *z-z* axis:

$$\frac{M_{z,Ed}}{M_{pl,z,N,Rd}} = \frac{M_{z,Ed}}{\mu_{dz} \cdot M_{pl,z,Rd}} \leq \alpha_{M,z}$$

The condition can be written in the following form:

$$\frac{M_{z,Ed}}{\alpha_{M,z} \cdot \mu_{dz} \cdot M_{pl,z,Rd}} \leq 1{,}0$$

Interaction of M_y–M_z–N:

$$\frac{M_{y,Ed}}{M_{pl,y,N,Rd}} + \frac{M_{z,Ed}}{M_{pl,z,N,Rd}} \leq 1,0$$

The condition can be written in the following form:

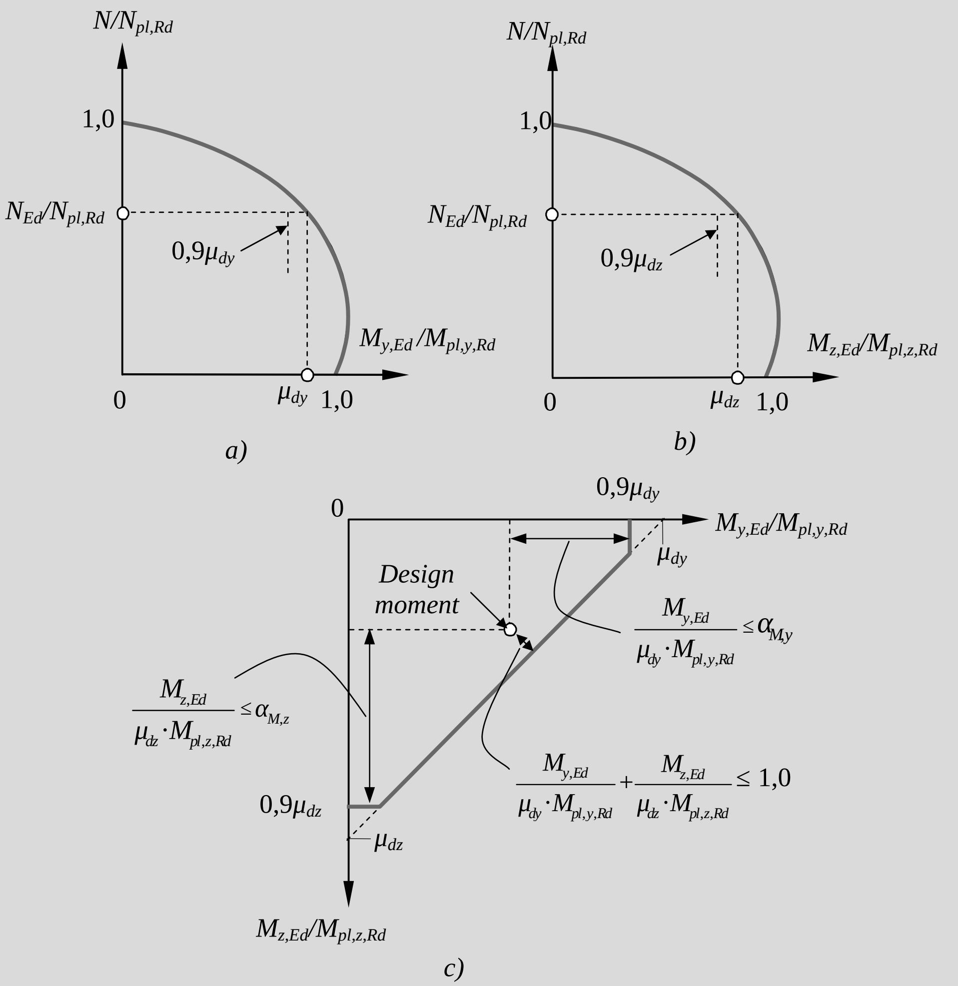
$$\frac{M_{y,Ed}}{\mu_{dy} \cdot M_{pl,y,Rd}} + \frac{M_{z,Ed}}{\mu_{dz} \cdot M_{pl,z,Rd}} \leq 1,0$$

These interaction expressions are shown in Figure C4.19 by means of interaction curves.

The check for bending about the *y-y* axis is carried out taking into account initially given bending moment $M_{y,Ed}$ without the equivalent member imperfection $e_{0,z}$, but including the second-order effects.

The check for bending about the *z-z* axis is carried out taking into account the bending moment $M_{z,Ed}$ (= $N_{Ed} \cdot e_{0,y}$) due to the equivalent member imperfection $e_{0,y}$, including the second-order effects.

Since the column is subjected to bending about the *y-y* axis and bending about the *z-z* axis, it is necessary to check the column resistance in combined compression and biaxial bending, the interaction of M_y–M_z–N.



a) *Section resistance interaction curve – non-failure axis (y-y axis). Neglect imperfections.*

b) *Section resistance interaction curve – axis of the anticipated failure (z-z axis). Consider imperfections.*

c) *Biaxial resistance moment of the column section under axial compression* N_{Ed}.

Figure C4.19 *Column resistance in combined compression and biaxial bending – assumed bending failure about the z-z axis*

Calculation of the bending moment about the *z-z* axis, $M_{z,Ed} = M_{z,Ed,II}$, taking into account the equivalent member imperfection $e_{0,y}$, and including the second-order effects, is shown in Figure C4.20.

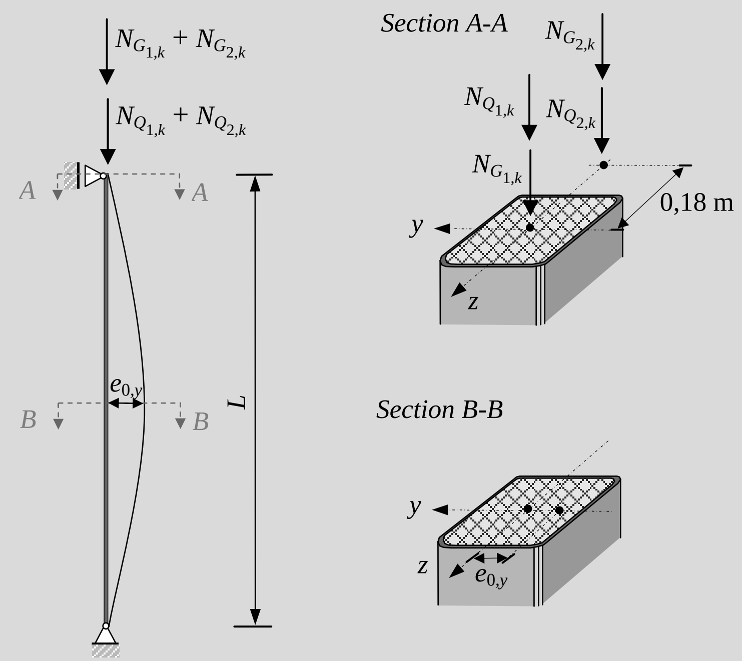


Figure C4.20 *Equivalent member imperfection $e_{0,y}$ about the z-z axis*

11.2.2 *Resistance of the cross-section in combined compression and bending about the z-z axis*

Remark:

In order to determine the resistance of the composite cross-section to combined compression and uniaxial bending, it is necessary to produce an axial load – bending moment (*N*–*M*) interaction curve. As a simplification, the interaction curve is replaced by an interaction polygon *ACDB*, clause 6.7.3.2 (5), EN 1994-1-1.

The *N*–*M* interaction polygon *ACDB* is shown in Figure 6.19, EN 1994-1-1. For concrete-filled hollow sections, the interaction polygon of *AECDB*, (shown in Figure C4.9, may be preferred to the interaction polygon of *ACDB* shown in Figure 6.19, EN 1994-1-1. The introduction of point *E* gives a more economical design, especially for columns with high axial force and low end moments. For better polygonal approximation to the interaction curve, the position of point *E* may be chosen to be closer to point *A* rather than being mid-way between points *A* and *C*. The introduction of the point *E* is not recommended in the case of steel section with low value of the shape factor α_{pl} (= W_{pl}/W_{el}), I-section in bending about the *y-y* axis.

The modified version of the interaction polygon *AECDB*, which refers to the composite column with concrete-filled rectangular hollow section, is shown in Figure C4.21.

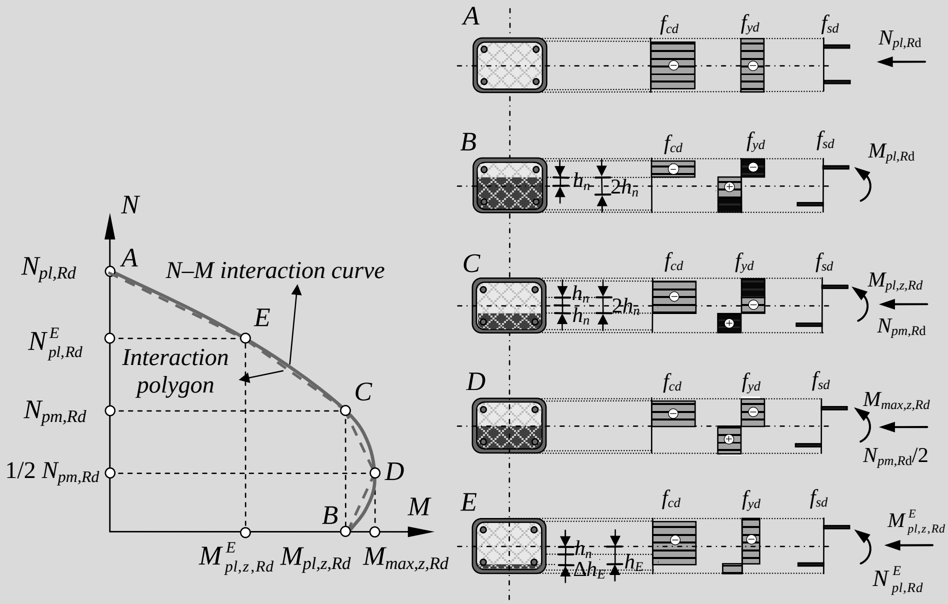


Figure C4.21 *N–M interaction polygon and corresponding stress distributions*

In order to produce the *N–M* interaction polygon, the cross-sectional capacities at points *A* to *D* should be determined assuming the stress distributions indicated, see Figure C4.21.

It should be noted that EN 1994-1-1 does not provide expressions for concrete-filled rectangular cross-sections.

Point A

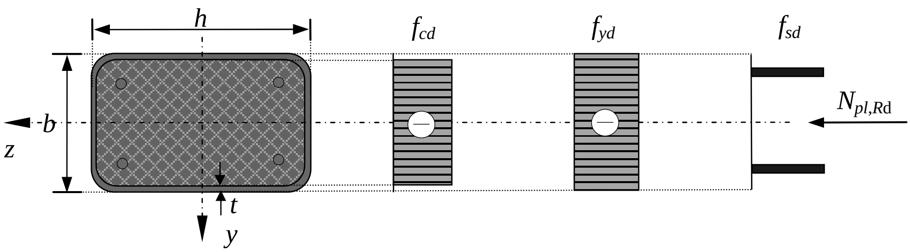


Figure C4.22 *Stress distributions for point A on the interaction polygon*

At point *A*, only the design plastic resistance of the cross-section is taken into account:

$$N_{pl,Rd} = A_a \cdot f_{yd} + 0{,}85 \cdot A_c \cdot f_{cd} + A_s \cdot f_{sd}$$

Remark:

For concrete filled hollow sections, the coefficient of 0,85 can be replaced with a value of 1,0 due to better curing conditions.

The design plastic resistance of the composite cross-section to compression, $N_{pl,Rd}$, is calculated according to the corrected expression:

$$N_{pl,Rd} = A_a \cdot f_{yd} + A_c \cdot f_{cd} + A_s \cdot f_{sd}$$

The design plastic resistance of the composite cross-section to compression is:

$$N_{pl,Rd} = 47{,}8 \cdot 23{,}5 + 302{,}6 \cdot 2{,}67 + 12{,}6 \cdot 43{,}5$$

$$N_{pl,Rd} = 1123{,}3 + 807{,}9 + 548{,}1 = 2479 \text{ kN}$$

Point D

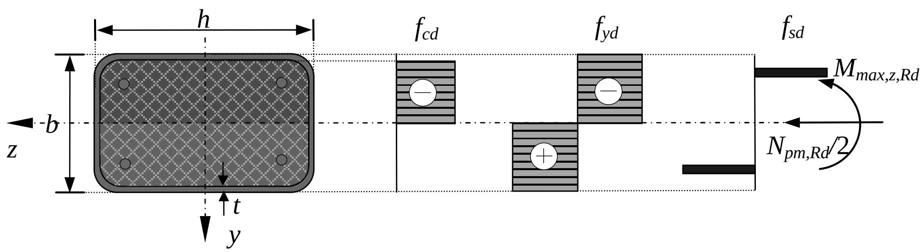


Figure C4.23 *Stress distributions for point D on the interaction polygon*

The maximum design plastic resistance moment is determined by the following expression:

$$M_{\max,z,Rd} = M_{pl,z,a,Rd} + M_{pl,z,c,Rd} + M_{pl,z,s,Rd}$$

The maximum design plastic resistance moment, $M_{max,z,Rd}$, at point D is:

$$M_{\max,z,Rd} = W_{pl,z,a} \cdot f_{yd} + 0{,}5 \cdot W_{pl,z,c} \cdot f_{cd} + W_{pl,z,s} \cdot f_{sd}$$

$$M_{\max,z,Rd} = (263 \cdot 23{,}5 + 0{,}5 \cdot 967{,}4 \cdot 2{,}67 + 36{,}5 \cdot 43{,}5) \cdot 10^{-2}$$

$$M_{\max,z,Rd} = 91,0 \text{ kNm}$$

The design value of the resistance of the concrete to compression, $N_{pm,Rd}$, is:

$$N_{pm,Rd} = A_c \cdot f_{cd} = 302,6 \cdot 2,67 = 808 \text{ kN}$$

The design axial force at the point of maximum design plastic resistance moment is $0,5 \cdot N_{pm,Rd}$, and therefore is:

$$0,5 \cdot N_{pm,Rd} = 0,5 \cdot 808 = 404 \text{ kN}$$

Point C

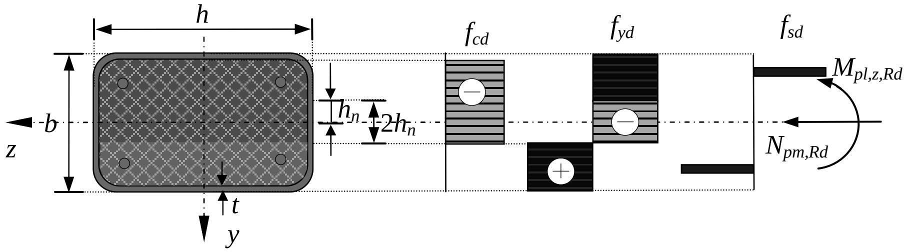


Figure C4.24 *Stress distributions for point C on the interaction polygon*

Calculation of the design plastic resistance moment of the composite section, $M_{pl,z,Rd}$, is carried out as shown below.

Determination of the position of the neutral axis depth, h_n, when axial force is zero:

$$h_n = \frac{N_{pm,Rd} - A_{s,n} \cdot (2 \cdot f_{sd} - f_{cd})}{2 \cdot h \cdot f_{cd} + 4 \cdot t \cdot (2 \cdot f_{yd} - f_{cd})}$$

where $A_{s,n}$ is the reinforcement area within h_n. Because it is at this point unknown, assumed to be (initial guess):

$$A_{s,n} = 0 \text{ cm}^2$$

Thus, for the case when the axial force is equal to zero, h_n is:

$$h_n = \frac{808 - 0 \cdot (2 \cdot 43,5 - 2,67)}{2 \cdot 26,0 \cdot 2,67 + 4 \cdot 0,63 \cdot (2 \cdot 23,5 - 2,67)} = 3,22 \text{ cm}$$

Since in this region there is reinforcement, the assumption is not correct. Accordingly, the neutral axis depth, h_n, is given by:

$$h_n = \frac{N_{pm,Rd} - 0{,}5 \cdot A_{s,n} \cdot (2 \cdot f_{sd} - f_{cd})}{2 \cdot h \cdot f_{cd} + 4 \cdot t \cdot (2 \cdot f_{yd} - f_{cd})}$$

$$h_n = \frac{808{,}0 - 0{,}5 \cdot 12{,}6 \cdot (2 \cdot 43{,}5 - 2{,}67)}{2 \cdot 26{,}0 \cdot 2{,}67 + 4 \cdot 0{,}63 \cdot (2 \cdot 23{,}5 - 2{,}67)} = 1{,}1 \text{ cm}$$

Plastic section moduli in region 2·h_n

Reinforcement

$W_{pl,z,s,n} = 0 \text{ cm}^3$

Concrete

$$W_{pl,z,c,n} = \frac{(h - 2 \cdot t) \cdot (2h_n)^2}{4} = \frac{(26{,}0 - 2 \cdot 0{,}63) \cdot 2{,}2^2}{4} = 29{,}9 \text{ cm}^3$$

Structural steel

$$W_{pl,z,a,n} = \frac{2 \cdot t \cdot (2h_n)^2}{4} = \frac{2 \cdot 0{,}63 \cdot 2{,}2^2}{4} = 1{,}5 \text{ cm}^3$$

The design plastic resistance moment of the composite section, $M_{pl,z,Rd}$, is calculated as follows:

$$M_{pl,z,Rd} = M_{\max,z,Rd} - M_{n,z,Rd}$$

where:

$$M_{n,z,Rd} = W_{pl,z,a,n} \cdot f_{yd} + W_{pl,z,s,n} \cdot f_{sd} + \frac{W_{pl,z,c,n} \cdot f_{cd}}{2}$$

$$M_{n,z,Rd} = (1{,}5 \cdot 23{,}5 + 0 \cdot 43{,}5 + \frac{29{,}9 \cdot 2{,}67}{2}) \cdot 10^{-2} = 0{,}8 \text{ kNm}$$

The design plastic resistance moment of the composite section, $M_{pl,z,Rd}$, is:

$$M_{pl,z,Rd} = 91,0 - 0,8 = 90 \text{ kNm}$$

Point B

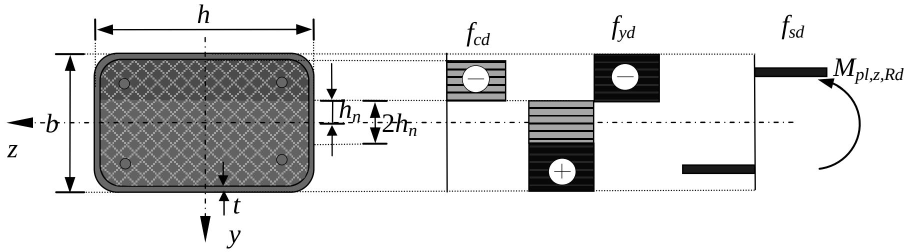


Figure C4.25 *Stress distributions for point B on the interaction polygon*

The design value of $M_{pl,z,Rd}$ has previously been calculated in order to define point C on the *N*–*M* interaction polygon:

$$M_{pl,z,Rd} = 90 \text{ kNm}$$

Point E

Remark:

If the enhancement of the resistance at point *E* is little more than that given by direct linear interpolation between *A* and *C*, the calculation can be omitted.

For the calculation of the design resistances at point *E*, the neutral axis is located on the outer border of reinforcement bars, see Figure C4.26.

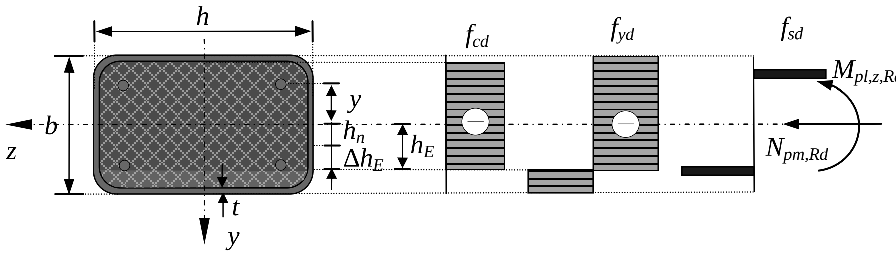


Figure C4.26 *Stress distributions for point E on the interaction polygon*

The distance between the centroidal axis of the composite section and the outer border of reinforcement bars, h_E, is obtained as:

$$h_E = y + \frac{d_{bar}}{2} = 2{,}9 + \frac{2{,}0}{2} = 3{,}9 \text{ cm}$$

In accordance with Figure C4.26, Δh_E is obtained as:

$$\Delta h_E = h_E - h_n = 3{,}9 - 1{,}1 = 2{,}8 \text{ cm}$$

In the region of Δh_E, there is reinforcement (2 ϕ 20) of cross-sectional area $A_{s,n}$ = 6,28 cm^2. The stresses in the region $\Delta h_E = h_E - h_n$ provide the force $\Delta N_{pl,Rd}^E$ which is given by:

$$\Delta N_{pl,Rd}^E = h \cdot \Delta h_E \cdot f_{cd} + 2 \cdot t \cdot \Delta h_E \cdot (2 \cdot f_{yd} - f_{cd}) + A_{s,n} \cdot (2 \cdot f_{sd} - f_{cd})$$

The force $\Delta N_{pl,Rd}^E$ is:

$$\Delta N_{pl,Rd}^E = 26{,}0 \cdot 2{,}8 \cdot 2{,}67 + 2 \cdot 0{,}63 \cdot 2{,}8 \cdot (2 \cdot 23{,}5 - 2{,}67) + 6{,}28 \cdot (2 \cdot 43{,}5 - 2{,}67)$$

$$\Delta N_{pl,Rd}^E = 880 \text{ kN}$$

Therefore, the design resistance to axial force at point E is:

$$N_{pl,Rd}^E = \Delta N_{pl,Rd}^E + N_{pm,Rd} = 880 + 808 = 1688 \text{ kN}$$

Plastic section moduli in region 2·h_E

Reinforcement

$$W_{pl,z,s,n} = A_s \cdot y_i = 12{,}6 \cdot 2{,}9 = 36{,}5 \text{ cm}^3$$

Concrete

$$W_{pl,z,c,n} = \frac{(h - 2 \cdot t) \cdot (2 \cdot h_E)^2}{4} - W_{pl,z,s,n}$$

$$W_{pl,z,c,n} = \frac{(26{,}0 - 2 \cdot 0{,}63) \cdot (2 \cdot 3{,}9)^2}{4} - 36{,}5$$

$$W_{pl,z,c,n} = 339{,}8 \text{ cm}^3.$$

Structural steel

$$W_{pl,z,a,n} = \frac{2 \cdot t \cdot (2 \cdot h_E)^2}{4} = \frac{2 \cdot 0,63 \cdot (2 \cdot 3,9)^2}{4}$$

$$W_{pl,z,a,n} = 19,2 \text{ cm}^3$$

The design plastic resistance moment of the composite section, $M^E_{pl,z,Rd}$, is calculated as follows:

$$M^E_{pl,z,Rd} = M_{\max,z,Rd} - M^E_{n,z,Rd}$$

where:

$$M^E_{n,z,Rd} = W_{pl,z,a,n} \cdot f_{yd} + W_{pl,z,s,n} \cdot f_{sd} + \frac{W_{pl,z,c,n} \cdot f_{cd}}{2}$$

$$M^E_{n,z,Rd} = (19,2 \cdot 23,5 + 36,5 \cdot 43,5 + \frac{339,8 \cdot 2,67}{2}) \cdot 10^{-2} = 24,9 \text{ kNm}$$

The design plastic resistance moment of the composite section, $M^E_{pl,z,Rd}$, is:

$$M^E_{pl,z,Rd} = M_{\max,z,Rd} - M^E_{n,z,Rd} = 91,0 - 24,9 = 66 \text{ kNm}$$

Previously calculated values at points *A* to *E* should be plotted to produce the *N–M* interaction polygon (Figure 6.19, EN 1994-1-1). With the obtained value at point *E* a better polygonal approximation to the interaction curve is achieved.
According to the interaction polygon *AECDB*, Figure C4.27, the following value $M_{pl,z,N,Rd}$ is obtained:

$$M_{pl,z,N,Rd} = M^E_{pl,z,Rd} + \Delta M_{z,Rd}$$

where:

$$(M_{pl,z,Rd} - M^E_{pl,z,Rd}) : \Delta M_{z,Rd} = (N^E_{Rd} - N_{pm,Rd}) : (N^E_{Rd} - N_{Ed})$$

$$\Delta M_{z,Rd} = \frac{N^E_{Rd} - N_{Ed}}{N^E_{Rd} - N_{pm,Rd}} \cdot (M_{pl,z,Rd} - M^E_{pl,z,Rd})$$

$$\Delta M_{z,Rd} = \frac{1688-1152}{1688-808} \cdot (90-66) = 14{,}6 \text{ kNm}$$

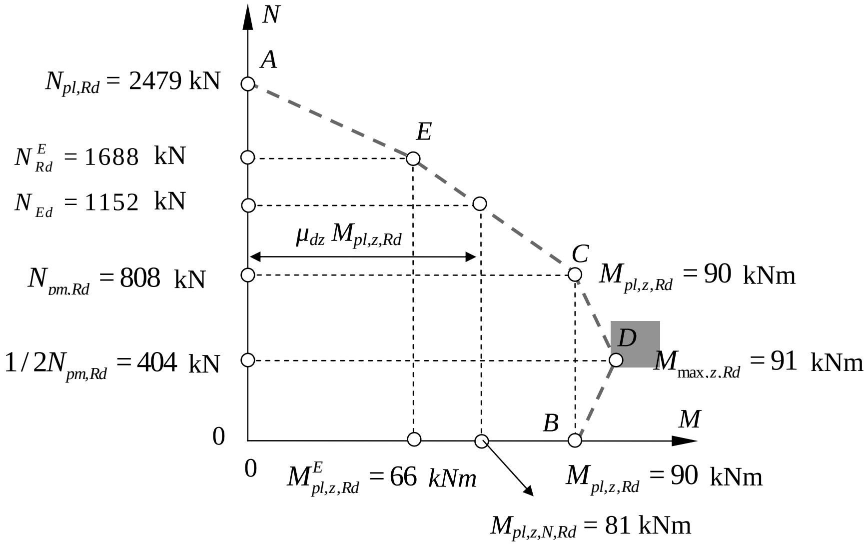


Figure C4.27 *N–M interaction polygon*

Therefore, the value $M_{pl,z,N,Rd}$ is:

$$M_{pl,z,N,Rd} = 66 + 14{,}6 = 81 \text{ kNm}$$

The value of μ_{dz} is:

$$\mu_{dz} = \frac{M_{pl,z,N,Rd}}{M_{pl,z,Rd}} = \frac{81}{90} = 0{,}90 < 1{,}0$$

The check is carried out by the factor $\mu_{dz} = 0{,}90$.

11.2.3 Calculation of action effects about the y-y axis

In accordance with the calculation given in Section 11.1.3.2, the value of the design bending moment at mid-height of the column, excluding the equivalent member imperfection, $e_{0,z}$, but including the second-order effects, is:

$$M_{y,Ed,II} = M_{y,Ed} \cdot k = 46{,}0 \cdot 0{,}80 = 37 \text{ kNm}$$

11.2.4 Calculation of action effects about the z-z axis

11.2.4.1 General

According to clause 6.7.3.4 (3), EN 1994-1-1, which refers to clause 5.2.1(3), EN 1994-1-1, second-order effects can therefore be neglected if the load factor α_{cr}, which is the ratio between the elastic critical load and the corresponding applied loading, for elastic instability of the member exceeds 10.

To calculate α_{cr}, the ends of the column are assumed to be pinned, and α_{cr} is found using the Euler formula for the elastic critical force $N_{cr,z,eff}$.

$$N_{cr,z,eff} = \frac{\pi^2 (EI)_{eff,z,II}}{L_{e,z}^2} \qquad L_{e,z} = L$$

The design value of the effective flexural stiffness $(EI)_{eff,z,II}$, used to determine the internal forces and moments by second-order analysis, according to clause 6.7.3.4(2), EN 1994-1-1, is defined by the following expression:

$$(EI)_{eff,z,II} = K_0 \cdot (E_a \cdot I_{z,a} + E_s \cdot I_{z,s} + K_{e,II} \cdot E_{cm} \cdot I_{z,c})$$

where:

$K_{e,II}$ is a correction factor, which should be taken as 0,5,
K_0 is a calibration factor, which should be taken as 0,9.

The value $E_{c,eff}$ has been used in place of E_{cm} in the expression for $(EI)_{eff,z,II}$ in order to take into account the long-term effects, in the same way as calculated in Section 7. Accordingly, the value of $E_{c,eff}$ is:

$$E_{c,eff} = 1238 \text{ kN/cm}^2$$

The design value of the effective flexural stiffness $(EI)_{eff,z,II}$, is:

$$(EI)_{eff,z,II} = K_0 \cdot (E_a \cdot I_{z,a} + E_s \cdot I_{z,s} + K_{e,II} \cdot E_{c,eff} \cdot I_{z,c})$$

$$(EI)_{eff,z,II} = 0{,}9 \cdot (21000 \cdot 1634 + 21000 \cdot 106 + 0{,}5 \cdot 1238 \cdot 4157)$$

$$(EI)_{eff,z,II} = 35{,}20 \cdot 10^6 \text{ kNcm}^2$$

The elastic critical force, $N_{cr,z,eff}$, for the pin-ended column, is:

$$N_{cr,z,eff} = \frac{\pi^2 (EI)_{eff,z,II}}{L_{e,z}^2}$$

$$N_{cr,z,eff} = \frac{35{,}20 \cdot 10^6 \cdot \pi^2}{400^2} = 2171 \text{ kN}$$

To check whether the effects of second-order analysis can be neglected, the value of α_{cr} must be higher than 10:

$$\alpha_{cr} = \frac{N_{cr,z,eff}}{N_{Ed}} = \frac{2171}{1152} = 1{,}9 < 10$$

The value of α_{cr} is less than 10, so second-order effects must be considered.

11.2.4.2 Bending moments about the z-z axis

According to clause 6.7.3.4(5), EN 1994-1-1, the second-order effects can be calculated by multiplying the greatest first-order design bending moments by a factor *k*.

Thus, the second-order effects may be considered according to the expression:

$$M_{z,Ed,II} = M_{z,Ed,I} \cdot k$$

The factor *k* is given by:

$$k = \frac{\beta}{1 - N_{Ed} / N_{cr,z,eff}} \geq 1{,}0$$

where:

β is an equivalent moment factor given in Table 6.4, EN 1994-1-1,

$N_{cr,z,eff}$ is the critical axial force, about the *z-z* axis, obtained with the effective flexural stiffness $(EI)_{eff,z,II}$ and with the effective length taken as the physical length of the column.

The design bending moment from the member imperfections is determined by the following expression:

$$M_{z,Ed,imp} = N_{Ed} \cdot e_{0,y}$$

where:

N_{Ed} is the design value of the axial force,

$e_{0,y}$ is the equivalent member imperfection which is given in Table 6.5, EN 1994-1-1, depending on the buckling curve.

Remark:

The reinforcement ratio ρ_s is 4,4 %. Therefore from Table 6.5, EN 1994-1-1, for 3%< $\rho_s \leq$ 6%,the buckling curve *b* should be used.

Therefore, for the buckling curve *b*, the equivalent member imperfection is:

$$e_{0,y} = \frac{L}{200}$$

$$e_{0,y} = \frac{400}{200} = 2,0 \text{ cm}$$

The design bending moments calculated according to first-order analysis are shown in Figure C4.28.

The design values of bending moments are as folows:

The design bending moment at the top of the column is:

$$M_{z,Ed} = 0 \text{ kNm}$$

The design bending moment at the bottom of the column is:

$$M_{z,Ed} = 0 \text{ kNm}$$

The design bending moment due to imperfection is:

$$M_{z,Ed,imp} = N_{Ed} \cdot e_{0,y} = 1152 \cdot 0,02 = 23,04 \text{ kNm}$$

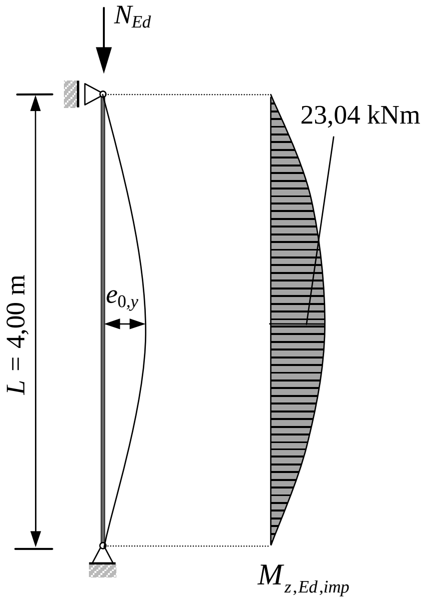


Figure C4.28 *First-order bending moments, design values*

Remark:

The factor β from Table 6.4, EN 1994-1-1, allows for the shape of the bending moment diagram. When bending is caused by lateral loading on the column, the value of factor β is 1,0. For a column subjected to end moments, the factor β is calculated as:

$$\beta_1 = 0{,}66 + 0{,}44 \cdot r \geq 0{,}44$$

where r is the ratio of the end-moments on the ends of the column ($-1 \leq r \leq +1$).

For the moment from the member imperfection, the factor k is given by:

$$k = \frac{\beta}{1 - N_{Ed} / N_{cr,z,eff}}$$

where β is the equivalent moment factor.

The equivalent moment factor β is:

$$\beta = 1{,}0$$

Therefore, the factor k is:

$$k = \frac{\beta}{1 - N_{Ed} / N_{cr,z,eff}} = \frac{1,0}{1 - 1152/2171} = 2,13 > 1,0$$

The design bending moment at mid-height, with second-order effects taken into account, is:

$$M_{z,Ed,II} = M_{z,Ed,imp} \cdot k = 23,04 \cdot 2,13 = 49 \text{ kNm}$$

The design bending moments calculated according to second-order analysis are shown in Figure C4.29.

The check is performed with the bending moment at mid-height:

$$M_{z,Ed,II} = M_{z,\max} = 49 \text{ kNm}$$

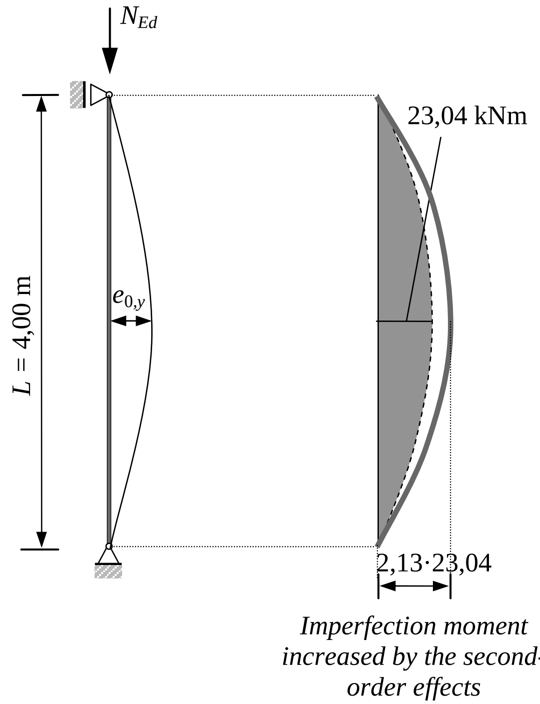


Figure C4.29 *Second-order bending moments about the z-z axis, design values*

11.2.4.3 Shear forces parallel to the y-y axis

According to clause 6.7.3.4(5), EN 1994-1-1, second-order effects can be allowed for by multiplying the greatest first-order design bending moment by a factor *k* given by:

$$k = \frac{\beta}{1 - N_{Ed} / N_{cr,eff}} \geq 1,0$$

Accordingly, the approximate value of the shear force can be obtained as:

$$V_{Ed,II} = V_{Ed} \cdot k$$

In accordance with Figure C4.30, the first-order design shear force at the bottom of the column is:

$$V_{y,Ed} = \frac{4 \cdot N_{Ed} \cdot e_{0,y}}{L} = \frac{4 \cdot 1152 \cdot 0,02}{4,0} = 23,04 \text{ kN}$$

In accordance with Figure C4.30, the first-order design shear force at the top of the column is:

$$V_{y,Ed} = \frac{4 \cdot N_{Ed} \cdot e_{0,y}}{L} = \frac{4 \cdot 1152 \cdot 0,02}{4,0} = 23,04 \text{ kN}$$

The diagram of the shear forces, calculated by first-order analysis for the equivalent lateral load due to imperfections, is shown in Figure C4.30.

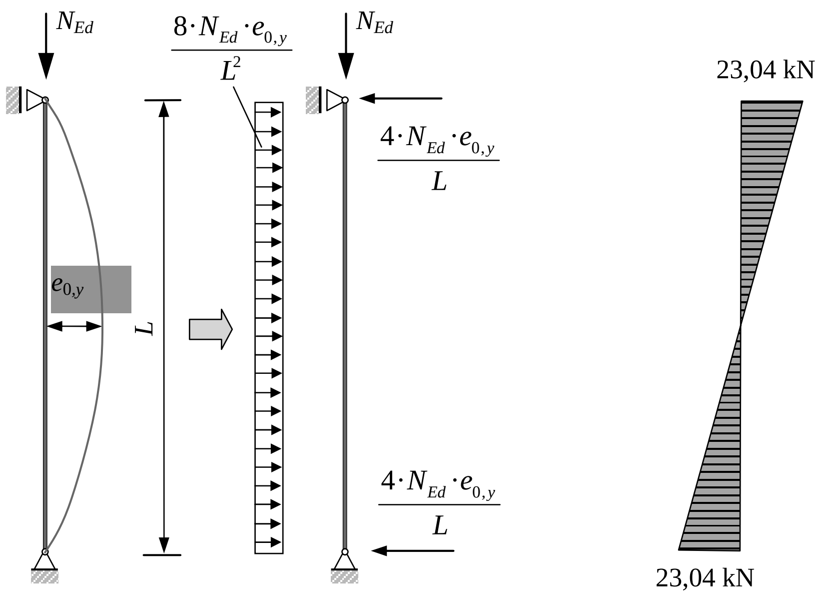


Figure C4.30 *First-order design shear forces parallel to the y-y axis*

The factor k is:

$$k = \frac{\beta}{1 - N_{Ed} / N_{cr,eff,z}} = \frac{1{,}0}{1 - 1152 / 2171} = 2{,}13 > 1{,}0$$

Therefore, the maximum design shear force, calculated by approximative second-order analysis, is:

$$V_{y,Ed,II} = V_{y,Ed} \cdot k = 23{,}04 \cdot 2{,}13 = 49{,}1 \text{ kN}$$

11.2.5 Check of the resistance of the member in combined compression and bending about the z-z axis

It is necessary to satisfy the following conditions:

$$\frac{M_{y,Ed}}{\alpha_{M,y} \cdot \mu_{dy} \cdot M_{pl,y,Rd}} \leq 1{,}0$$ → ($M_{y,Ed}$ – neglect imperfections but use second-order analysis)

$$\frac{M_{z,Ed}}{\alpha_{M,z} \cdot \mu_{dz} \cdot M_{pl,z,Rd}} \leq 1{,}0$$ → ($M_{z,Ed}$ – consider imperfections and second-order analysis)

$$\frac{M_{y,Ed}}{\mu_{dy} \cdot M_{pl,y,Rd}} + \frac{M_{z,Ed}}{\mu_{dz} \cdot M_{pl,z,Rd}} \leq 1{,}0$$ → (Interaction of $M_{y,\mathrm{Ed}}$ and $M_{z,Ed}$)

Substituting previously calculated values gives:

$$\frac{37}{0{,}9 \cdot 0{,}83 \cdot 158} = 0{,}31 < 1{,}0$$

$$\frac{49}{0{,}9 \cdot 0{,}90 \cdot 90} = 0{,}67 < 1{,}0$$

$$\frac{37}{0{,}83 \cdot 158} + \frac{49}{0{,}90 \cdot 90} = 0{,}28 + 0{,}60 = 0{,}88 < 1{,}0$$

Therefore, the resistance of the composite column to biaxial bending, taking into account the design axial force N_{Ed} = 1152 kN, is adequate.

11.2.6 Check of the plastic resistance to transverse shear parallel to the y-y axis

In accordance with clause 6.7.3.2(4), EN 1994-1-1, for simplification V_{Ed} may be assumed to act on the structural steel section alone. According to clause 6.2.6(2), EN 1993-1-1, in the absence of torsion the design plastic shear resistance, $V_{pl,y,a,Rd}$, is given by:

$$V_{pl,y,a,Rd} = \frac{A_{v,y} \cdot (f_y / \sqrt{3})}{\gamma_{M0}}$$

The shear area, $A_{v,y}$, according to clause 6.2.6(3), EN1993-1-1, is calculated as:

$$A_{v,y} = \frac{A_a \cdot b}{b + h}$$

where:

A_a is the cross-sectional area of the rectangular hollow section,
b is the width of the rectangular hollow section,
h is the depth of the rectangular hollow section.

According to clause 6.2.2.4(1), EN 1994-1-1, where the shear force is less than half the plastic shear resistance, its effect on the resistance moment can be neglected. Therefore, the condition is:

$$V_{y,Ed} < 0{,}5 \cdot V_{pl,y,a,Rd}$$

The design value of the second-order shear force is:

$$V_{y,Ed} = V_{y,Ed,II} = 49{,}1 \text{ kN}$$

The shear area, $A_{v,y}$, is:

$$A_{v,y} = \frac{47{,}8 \cdot 14{,}0}{14{,}0 + 26{,}0} = 16{,}7 \text{ cm}^2$$

The design plastic shear resistance, $V_{pl,a,y,Rd}$, is:

$$V_{pl,y,a,Rd} = \frac{A_{v,y} \cdot (f_y / \sqrt{3})}{\gamma_{M0}}$$

$$V_{pl,a,y,Rd} = \frac{16{,}7 \cdot (23{,}5 / \sqrt{3})}{1{,}0} = 226 \text{ kN}$$

Check:

$$V_{Ed} = 49{,}1 < 0{,}5 \cdot V_{pl,a,z,Rd} = 0{,}5 \cdot 226 = 113 \text{ kN}$$

The condition is satisfied and there is no reduction in the resistance moment.

12. Commentary

We have considered the concrete-filled tube of rectangular hollow section in combined compression and uniaxial bending. The member imperfections have been neglected in the global analysis. Therefore, it is necessary to include them in the analysis of the column. According to clause 6.7.3.7(1), EN 1994-1-1, imperfections should be considered only in the plane in which failure is expected to occur. If it is not obvious which plane is the more critical, checks should be made for both planes. This means that the verification of the composite column should be performed as for the column in compression and biaxial bending because the equivalent member imperfection $e_{0,y}$ causes the bending moment $M_{z,Ed}$. Accordingly, the following checks are needed:

a) The verification of the column resistance in axial compression only is carried out as the preliminary check. Since $\bar{\lambda}_z = 1{,}10 > \bar{\lambda}_y = 0{,}63$, the buckling resistance about the *z-z* axis is governed. The check of the composite column subjected to axial compression is satisfied. It is not necessary to select the stronger cross-section.

The utilization is 88%.

b) Check for bending about the *y-y* axis: The next step is to carry out the check of the column resistance in combined compression and uniaxial bending. The equivalent member imperfection $e_{0,z}$ is taken into account, which is in the same plane as the initial moment. In addition it was found that the second-order effects must be allowed for. The final step is to check that the cross-section can resist $M_{y,Ed}$ (consider imperfections and second-order analysis) with compression N_{Ed}.

The utilization is 55%.

c) Check for bending about the z-z axis: Finally, the check of the column resistance in combined compression and biaxial bending is carried out. The design bending moment about the y-y axis, $M_{y,Ed}$, is calculated neglecting the equivalent member imperfection. About the z-z axis, the design bending moment due to the equivalent member imperfection $M_{z,Ed}$,($=N_{Ed}\cdot e_{0,\mathrm{y}}$) is taken into account. In addition, it was found that the second-order effects must be allowed for. The final step is to check that the cross-section can resist $M_{y,Ed}$ (neglect imperfections but use second-order analysis) and $M_{z,Ed}$ (consider imperfections and second-order analysis) with compression N_{Ed}.

The utilization is:

31% ($M_{y,Ed}$ neglect imperfections but use second-order analysis),
67% ($M_{z,Ed}$ consider imperfections and second-order analysis),
88% (interaction of $M_{y,Ed}$ and $M_{z,Ed}$).

Remark:

According to clause 6.7.3.7(1), EN 1994-1-1, imperfections should be considered only in the plane in which failure is expected to occur, i.e. the bending moment $N_{Ed}\cdot e_0$ is included only for this plane. If it is not obvious, checks should be made for both planes.

C5 Composite column with partially concrete-encased H-section subject to axial compression and uniaxial bending

1. Purpose of example

This example demonstrates the design of a composite column with partially concrete-encased H-section subject to axial compressive load and bending moment. In order to ensure adequate force transfer between the steel and the concrete, stud connectors and reinforcement are provided. The design load for the considered column is made up of the variable and permanent load on the floor area immediately over the column, $N_{G2,k}$ and $N_{Q2,k}$, and the load transmitted by the columns above, $N_{G1,k}$ and $N_{Q1,k}$. The eccentricity moments due to the end reactions from the incoming beams are considered. The resistance of the cross-section to combined compression and bending is calculated in two ways: using the interaction curve of N–M (the exact approach) and using the interaction polygon of N–M (the approximate approach). Also, this example illustrates the design of load introduction for combined compression and bending. The forces at the interface are determined by elastic theory and by plastic theory.

2. Static system, cross-section and design action effects

Actions

Permanent action $$N_{G_{1,k}} = 1800 \text{ kN}$$

$$N_{G_{2,k}} = 300 \text{ kN}$$

Variable action $$N_{Q_{1,k}} = 750 \text{ kN}$$

$$N_{Q_{2,k}} = 150 \text{ kN}$$

Composite Structures according to Eurocode 4. Worked Examples.
First Edition. Darko Dujmović, Boris Androić, Ivan Lukačević.

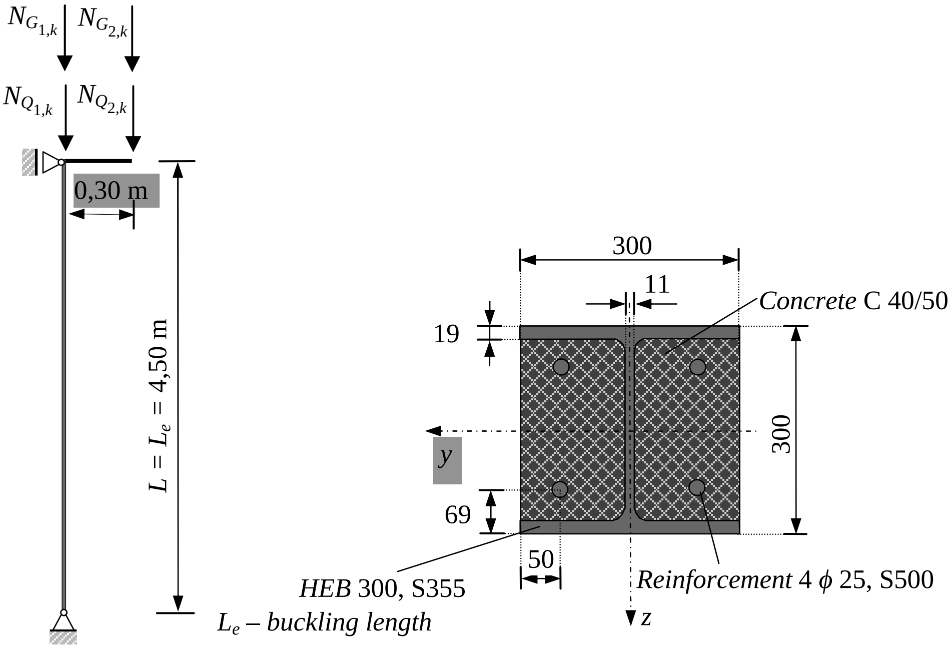


Figure C5.1 *Static system and cross-section (bending about the y-y axis)*

Design action effects

Axial force:

$$N_{Ed} = \gamma_G \cdot (N_{G_{1,k}} + N_{G_{2,k}}) + \gamma_Q \cdot (N_{Q_{1,k}} + N_{Q_{2,k}})$$

$$N_{Ed} = 1{,}35 \cdot (N_{G_{1,k}} + N_{G_{2,k}}) + 1{,}50 \cdot (N_{Q_{1,k}} + N_{Q_{2,k}})$$

$$N_{Ed} = 1{,}35 \cdot (1800 + 300) + 1{,}50 \cdot (750 + 150) = 2835 + 1350 = 4185 \text{ kN}$$

Bending moment at the top of the column:

$$M_{y,Ed} = \gamma_G \cdot N_{G_{2,k}} \cdot 0{,}30 + \gamma_Q \cdot N_{Q_{2,k}} \cdot 0{,}30$$

$$M_{y,Ed} = 1{,}35 \cdot N_{G_{2,k}} \cdot 0{,}30 + 1{,}50 \cdot N_{Q_{2,k}} \cdot 0{,}30$$

$$M_{y,Ed} = 1{,}35 \cdot 300 \cdot 0{,}30 + 1{,}50 \cdot 150 \cdot 0{,}30 = 121{,}5 + 67{,}5 = 189{,}0 \text{ kNm}$$

Denotation of imperfections

Imperfection about the *y*-*y* axis is denoted by $e_{0,z}$. Imperfection about the *z*-*z* axis is denoted by $e_{0,y}$.

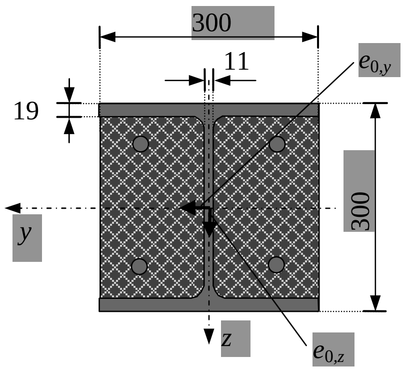


Figure C5.2 *Denotation of imperfections*

3. Properties of materials

Concrete strength class: C 40/50

$$f_{ck} = 40,0 \text{ N/mm}^2$$

$$f_{cd} = \frac{f_{ck}}{\gamma_c} = \frac{40,0}{1,5} = 26,7 \text{ N/mm}^2$$

$$E_{cm} = 35000 \text{ N/mm}^2$$

Structural steel: S355

$$f_{yk} = 355 \text{ N/mm}^2$$

$$f_{yd} = \frac{f_{yk}}{\gamma_M} = \frac{355}{1,0} = 355 \text{ N/mm}^2$$

$$E_a = 210000 \text{ N/mm}^2$$

Reinforcement: *ductility class B or C*

$$f_{sk} = 500 \text{ N/mm}^2$$

$$f_{sd} = \frac{f_{sk}}{\gamma_s} = \frac{500,0}{1,15} = 435 \text{ N/mm}^2$$

$$E_s = 210000 \text{ N/mm}^2$$

Shear connectors: *ductile headed studs*

$$f_u = 450 \text{ N/mm}^2$$

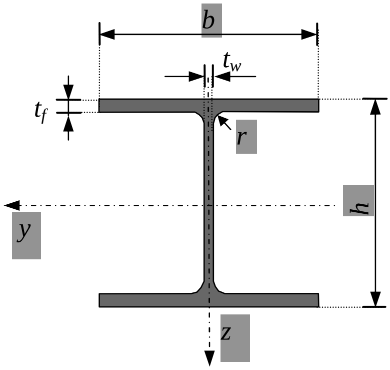
$$d = 19 \text{ mm} \qquad h_{sc} = 100 \text{ mm}$$

$$\frac{h_{sc}}{d} = \frac{100}{19} = 5,26 > 4,0$$

4. Geometrical properties of the cross-section

4.1 Selection of the steel cross-section and reinforcement

The cross-section HE 300 B is selected. The selected cross-section is shown in Figure C5.3.



$$h = 300 \text{ mm}$$
$$b = 300 \text{ mm}$$
$$t_f = 19 \text{ mm}$$
$$t_w = 11 \text{ mm}$$
$$r = 27 \text{ mm}$$
$$A_a = 149{,}1 \text{ cm}^2$$
$$I_{y,a} = 25170 \text{ cm}^4$$
$$I_{z,a} = 8563 \text{ cm}^4$$
$$W_{pl,y,a} = 1869 \text{ cm}^3$$
$$W_{pl,z,a} = 870{,}1 \text{ cm}^3$$

Figure C5.3 *Steel cross-section*

The cross-sectional area of the structural steel section HE 300 B is:

$$A_a = 149{,}1 \text{ cm}^2$$

The cross-sectional area of the reinforcement with four bars of 25 mm diameter is:

d_{bar} = 25 mm, A_{bar} = 4,91 cm^2

$$A_s = 4 \cdot A_{bar} = 4 \cdot 4{,}91 = 19{,}6 \text{ cm}^2$$

The cross-sectional area of the concrete is:

$$A_c = b \cdot h - A_a - A_s$$

$$A_c = 30 \cdot 30 - 149 - 19{,}6$$

$$A_c = 731{,}4 \, \text{cm}^2$$

The ratio of the reinforcement area to concrete area is:

$$\rho_s = \frac{A_s}{A_c} = \frac{19,6}{731,4} = 0,027 \qquad \rho_s = 2,7\ \%$$

$$\rho_s = 2,7\% < 6\%$$

The limit of 6% in clause 6.7.3.1 (1), EN 1994-1-1, on the reinforcement is satisfied.

Remark:

According to clause 6.7.3.1(3), EN 1994-1-1, the ratio of reinforcement area to concrete area, ρ_s, should not exceed 6%.

4.2 Cross-sectional areas

Structural steel

$$A_a = 149,1\ \text{cm}^2$$

Reinforcement

$$A_s = 19,6\ \text{cm}^2$$

Concrete

$$A_c = 731,4\,\text{cm}^2$$

4.3 Second moments of area

Bending about the y-y axis

Structural steel

$$I_{y,a} = 25170\ \text{cm}^4$$

Reinforcement

$$I_{y,s} = 4 \cdot A_{bar} \cdot 8,1^2$$

$$I_{y,s} = 4 \cdot 4,91 \cdot 8,1^2$$

$$I_{y,s} = 19{,}6 \cdot 8{,}1^2 = 1289 \text{ cm}^4$$

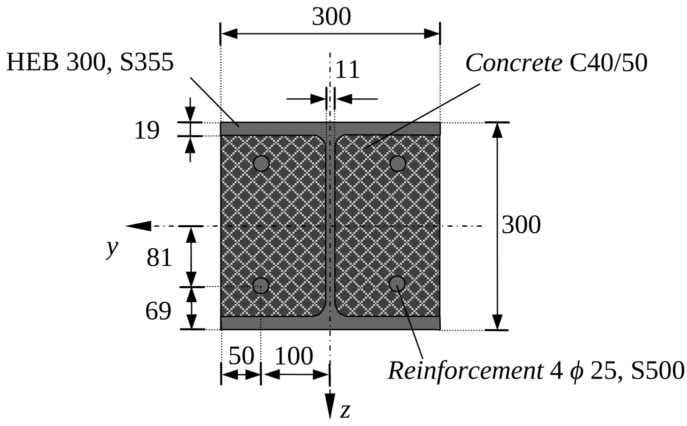


Figure C5.4 *Composite column cross-section*

Concrete

$$I_{y,c} = \frac{b \cdot h^3}{12} - I_{y,a} - I_{y,s}$$

$$I_{y,c} = \frac{30 \cdot 30^3}{12} - 25170 - 1289$$

$$I_{y,c} = 41041 \text{ cm}^4$$

Bending about the z-z axis

Structural steel

$$I_{z,a} = 8563 \text{ cm}^4$$

Reinforcement

$$I_{z,s} = 4 \cdot A_{bar} \cdot 10{,}0^2$$

$$I_{z,s} = 4 \cdot 4{,}91 \cdot 10{,}0^2$$

$$I_{z,s} = 12{,}6 \cdot 10{,}0^2 = 1964 \text{ cm}^4$$

Concrete

$$I_{z,c} = \frac{h \cdot b^3}{12} - I_{z,a} - I_{z,s}$$

$$I_{z,c} = \frac{30 \cdot 30^3}{12} - 8563 - 1964$$

$$I_{z,c} = 56973 \text{ cm}^4$$

4.4 Plastic section moduli

Bending about the y-y axis

Structural steel

$$W_{pl,y,a} = 1869 \text{ cm}^3$$

Reinforcement

$$W_{pl,y,s} = \sum_i A_{s,i} \cdot z_i = 4 \cdot 4{,}91 \cdot 8{,}1 = 159 \text{ cm}^3$$

Concrete

$$W_{pl,y,c} = \frac{b \cdot h^2}{4} - W_{pl,y,a} - W_{pl,y,s} = \frac{30{,}0 \cdot 30{,}0^2}{4} - 1869 - 159 = 4722 \text{ cm}^3$$

Bending about the z-z axis

Structural steel

$$W_{pl,z,a} = 870{,}1 \text{ cm}^3$$

Reinforcement

$$W_{pl,z,s} = \sum_i A_{si} \cdot z_i = 4 \cdot 4{,}91 \cdot 10{,}0 = 196 \text{ cm}^3$$

Concrete

$$W_{pl,z,c} = \frac{b \cdot h^2}{4} - W_{pl,z,a} - W_{pl,z,s} = \frac{30{,}0 \cdot 30{,}0^2}{4} - 870{,}1 - 196 = 5684 \text{ cm}^3$$

5. Steel contribution ratio

According to clause 6.7.3.3(1), EN 1994-1-1, the steel contribution ratio, δ, is defined as:

$$\delta = \frac{A_a \cdot f_{yd}}{N_{pl,Rd}}$$

The term $A_a \cdot f_{yd}$ is the contribution of the structural steel section to the plastic resistance of the composite section to axial force. The design plastic resistance of the composite section to axial force $N_{pl,Rd}$ is calculated according to clause 6.7.3.2(1), EN 1994-1-1.

According to 6.7.1(4), EN 1994-1-1, the steel contribution ratio, δ, must satisfy the following conditions:

$0{,}2 \leq \delta \leq 0{,}9$

If δ is less than 0,2, the column should be designed as a reinforced concrete member according to EN 1992-1-1. If δ is larger than 0,9, the concrete is ignored in the calculations, and the column is designed as a structural steel member according to EN 1993-1-1.

The term $A_a \cdot f_{yd}$ is the contribution of the structural steel section to the plastic resistance of composite section to axial force:

$$A_a \cdot f_{yd} = 149{,}1 \cdot 35{,}5 = 5293\,\text{kN}$$

The plastic resistance of the composite section to axial force is:

$$N_{pl,Rd} = A_a \cdot f_{yd} + 0{,}85 \cdot A_c \cdot f_{cd} + A_s \cdot f_{sd}$$

$$N_{pl,Rd} = 149{,}1 \cdot 35{,}5 + 0{,}85 \cdot 731{,}4 \cdot 2{,}67 + 19{,}6 \cdot 43{,}5$$

$$N_{pl,Rd} = 7806\ \text{kN}$$

According to 6.7.1(4), EN 1994-1-1, the steel contribution ratio, δ, must satisfy the following conditions:

$0{,}2 \leq \delta \leq 0{,}9$

The steel contribution ratio, δ, is:

$$\delta = \frac{A_a \cdot f_{yd}}{N_{pl,Rd}} = \frac{149,1 \cdot 35,5}{7806} = 0,68$$

Since the limits 0,2 < δ = 0,68 < 0,9, are satisfied, the column can be classified as a composite column and the provisions of EN 1994-1-1 can be used for the dimensioning.

6. Local buckling

The web of the steel section is encased in reinforced concrete. The encasement prevents local buckling of the web and rotation of the flange at the junction with the web. Due to this favourable effect, the higher ratio b/t_f can be used than for the web without the encasement.

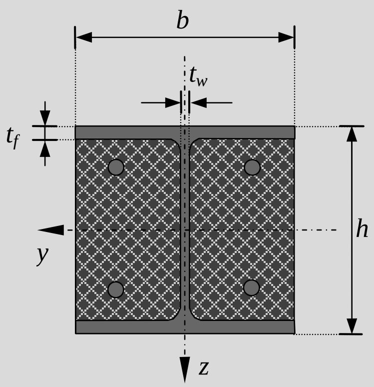


Figure C5.5 *Partially concrete encased H-section*

According to Table 6.3, EN 1994-1-1, for partially concrete encased I- or H-sections, the effects of local buckling can be ignored if the following condition is satisfied, see Figure C5.5:

$$\max(\frac{b}{t_f}) = 44 \cdot \sqrt{\frac{235}{f_y}}$$

For the selected cross-section, the maximum slenderness is:

$$\max(\frac{b}{t_f}) = \frac{300}{19,0} = 15,8$$

The required condition is:

$$44 \cdot \sqrt{\frac{235}{f_y}} = 44 \cdot \sqrt{\frac{235}{355}} = 35{,}8$$

Since $15{,}8 < 35{,}8$, the condition is satisfied. The effect of local buckling can be neglected.

7. Effective modulus of elasticity for concrete

For long-term loading, the creep and shrinkage are taken into account in design by a reduced flexural stiffness of the composite cross-section. Due to the influence of long-term creep effects on the effective elastic stiffness, the modulus of elasticity of the concrete, E_{cm}, should be reduced to the value $E_{c,eff}$ in accordance with following equation:

$$E_{c,eff} = \frac{E_{cm}}{1 + (\frac{N_{G,Ed}}{N_{Ed}}) \cdot \varphi_t}$$

where:

$\varphi_t = \varphi(t, t_0)$ is the creep coefficient, defining the creep between times t and t_0, related to the elastic deformation at 28 days,
$\varphi_t = \varphi(\infty, t_0)$ is the final creep coefficient,
t is the age of the concrete at the time considered,
t_0 is the age of the concrete at loading,
N_{Ed} is the axial design force,
$N_{G,Ed}$ is the permanent part of the axial design force N_{Ed}, $N_{G,Ed} = \gamma_G \cdot N_{Gk}$.

For the calculation of the creep coefficient $\varphi(t, t_0)$, the following is valid:

- the perimeter of that part which is exposed to drying, u, is determined in accordance with Figure C5.6.

$$u \approx 2 \cdot h + 0{,}5 \cdot b$$

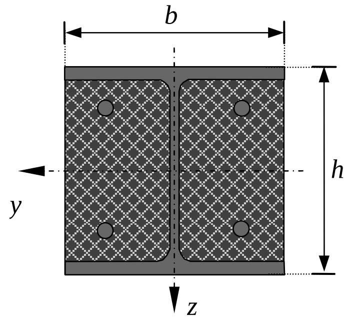
$$u \approx 2 \cdot 30 + 0{,}5 \cdot 30 = 75 \text{ cm}$$

- the notional size of the cross-section, h_0

$$h_0 = \frac{2 \cdot A_c}{u} = \frac{2 \cdot 731{,}4}{75} = 19{,}5 \text{ cm} = 195 \text{ mm}$$

- $t_0 = 28$ days,

- inside conditions, the ambient relative humidity RH 50 %,
- the concrete strength class C 40/50
- the type of cement – cement class N, strength class 32,5 R.



$u \approx 2h + 0,5\ b$

Figure C5.6 *Perimeter which is "exposed" to drying*

The final value of the creep coefficient $\varphi(\infty, t_0)$ is determined using the nomogram shown in Figure 3.1, EN 1992-1-1. The process of determining the final value of the creep coefficient, taking into account these assumptions, is given in Figure C5.7.

The final value of the creep coefficient $\varphi(\infty, t_0)$, found from Figure C5.7, is:

$\varphi_t = \varphi(\infty, t_0) = 1,9$

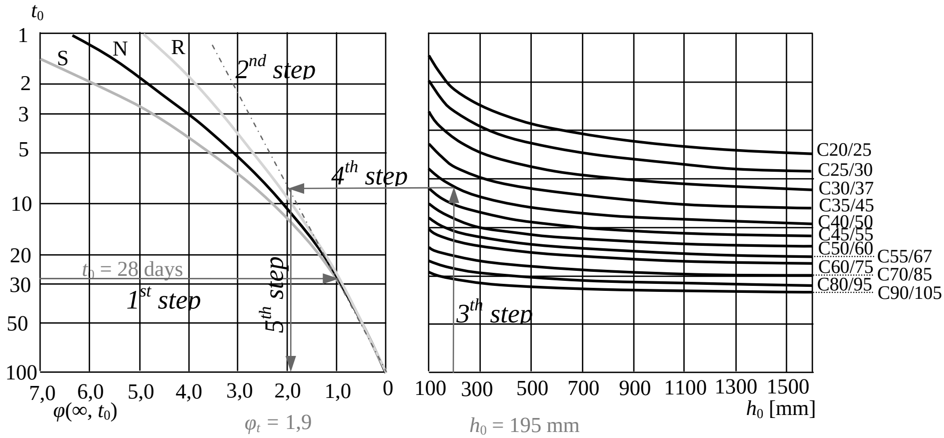


Figure C5.7 *Method for determining the creep coefficient*

The design force of the permanent load, $N_{G,Ed}$, and the total design force, N_{Ed}, are:

$$N_{G,Ed} = \gamma_G \cdot (N_{G_{k,1}} + N_{G_{k,2}})$$

$$N_{G,Ed} = 1{,}35 \cdot (1800 + 300) = 2835 \ \text{kN}$$

$$N_{Ed} = 1{,}35 \cdot (N_{G_{1,k}} + N_{G_{2,k}}) + 1{,}50 \cdot (N_{Q_{1,k}} + N_{Q_{2,k}})$$

$$N_{Ed} = 1{,}35 \cdot (1800 + 300) + 1{,}50 \cdot (750 + 150) = 2835 + 1350 = 4185 \ \text{kN}$$

Accordingly, the value of $E_{c,eff}$ is:

$$E_{c,eff} = \frac{E_{cm}}{1 + (\frac{N_{G,Ed}}{N_{Ed}}) \cdot \varphi_t} = \frac{3500}{1 + (\frac{2835}{4185}) \cdot 1{,}90} = 1530 \ \text{kN/cm}^2$$

Further calculation is performed with the effective modulus of elasticity of the concrete $E_{c,eff}$ = 1530 kN/m^2.

8. Resistance of the cross-section to compressive axial force

The design plastic resistance of composite cross-section to axial compressive force, $N_{pl,Rd}$, is given by the sum of the design resistances of components as follows:

$$N_{pl,Rd} = A_a \cdot f_{yd} + 0{,}85 \cdot A_c \cdot f_{cd} + A_s \cdot f_{sd}$$

$$N_{pl,Rd} = 149{,}1 \cdot 35{,}5 + 0{,}85 \cdot 731{,}4 \cdot 2{,}67 + 19{,}6 \cdot 43{,}5$$

$$N_{pl,Rd} = 5293 + 1660 + 853 = 7806 \ \text{kN}$$

The characteristic value of the plastic resistance of the composite cross-section to compressive axial force, $N_{pl,Rk}$, is determined by the following expression:

$$N_{pl,Rk} = A_a \cdot f_{yk} + 0{,}85 \cdot A_c \cdot f_{ck} + A_s \cdot f_{sk}$$

$$N_{pl,Rk} = 149{,}1 \cdot 35{,}5 + 0{,}85 \cdot 731{,}4 \cdot 4{,}0 + 19{,}6 \cdot 50$$

$$N_{pl,Rk} = 5293 + 2487 + 980 = 8760 \ \text{kN}$$

9. Verification of the conditions for using simplified design method

The cross-section of the composite column should be doubly symmetrical and uniform along the entire length of the column.

This condition is satisfied.

Relative slenderness

To apply the simplified method it is necessary to satisfy the following conditions:

$\bar{\lambda}_y \leq 2{,}0$

$\bar{\lambda}_z \leq 2{,}0$

About the y-y axis

Relative slenderness, $\bar{\lambda}_y$, is determined by the following expression:

$$\bar{\lambda}_y = \sqrt{\frac{N_{pl,Rk}}{N_{cr,y}}}$$

For the determination of the relative slenderness $\bar{\lambda}_y$ and the elastic critical force $N_{cr,y}$, it is necessary to calculate the value of the effective flexural stiffness of the cross-section of composite column, $(EI)_{eff,y}$, as:

$$(EI)_{eff,y} = E_a \cdot I_{y,a} + E_s \cdot I_{y,s} + K_e \cdot E_{c,eff} \cdot I_{y,c}$$

With the correction factor $K_e = 0{,}6$, the value $(EI)_{eff,y}$ is:

$$(EI)_{eff,y} = 21000 \cdot 25170 + 21000 \cdot 1289 + 0{,}6 \cdot 1530 \cdot 41041$$

$$(EI)_{eff,y} = 593{,}31 \cdot 10^6 \text{ kNcm}^2$$

The elastic critical force, $N_{cr,y}$, for the pin-ended column and the buckling length $L_{e,y}$, is determined as:

$$N_{cr,y} = \frac{(EI)_{eff,y} \cdot \pi^2}{L_{e,y}^2} \quad L_{e,y} = L$$

$$N_{cr,y} = \frac{593{,}31 \cdot 10^6 \cdot \pi^2}{450^2} = 28917 \text{ kN}$$

The relative slenderness, $\bar{\lambda}_y$, is:

$$\bar{\lambda}_y = \sqrt{\frac{8760}{28917}} = 0{,}55$$

Accordingly $\bar{\lambda}_y = 0{,}55 < 2{,}0$, and the condition is satisfied.

About the *z-z* axis

Relative slenderness, $\bar{\lambda}_z$, is determined by the following expression:

$$\bar{\lambda}_z = \sqrt{\frac{N_{pl,Rk}}{N_{cr,z}}}$$

For the determination of the relative slenderness $\bar{\lambda}_z$ and the elastic critical force $N_{cr,z}$, it is necessary to calculate the value of the effective flexural stiffness of the cross-section of the composite column $(EI)_{eff,z}$ according to the expression:

$$(EI)_{eff,z} = E_a \cdot I_{z,a} + E_s \cdot I_{z,s} + K_e \cdot E_{c,eff} \cdot I_{z,c}$$

With the correction factor K_e = 0,6, the value $(EI)_{eff,z}$ is:

$$(EI)_{eff,z} = 21000 \cdot 8563 + 21000 \cdot 1964 + 0{,}6 \cdot 1530 \cdot 56973$$

$$(EI)_{eff,z} = 273{,}37 \cdot 10^6 \text{ kNcm}^2$$

The elastic critical force, $N_{cr,z}$, for the pin-ended column and the buckling length $L_{e,z}$, is determined as:

$$N_{cr,z} = \frac{(EI)_{eff,z} \cdot \pi^2}{L_{e,z}^2} \quad L_{e,z} = L$$

$$N_{cr,z} = \frac{273,37 \cdot 10^6 \cdot \pi^2}{450^2} = 13324 \text{ kN}$$

The relative slenderness, $\bar{\lambda}_z$, is:

$$\bar{\lambda}_z = \sqrt{\frac{8760}{13324}} = 0,81$$

Accordingly $\bar{\lambda}_z = 0,81 < 2,0$, and the condition is satisfied.

The maximum permitted cross-sectional area of the longitudinal reinforcement

The maximum cross-sectional area of the longitudinal reinforcement $A_{s,max}$ that can be used in the calculation should not exceed 6% of the concrete area. This condition is satisfied, see Section 4.1.

The ratio of the depth to the width

The ratio of the depth to the width of the composite cross-section should be within the following limits:

$$0,2 \leq \frac{h}{b} \leq 5,0$$

$$\frac{h}{b} = \frac{30,0}{30,0} = 1,0$$

$0,2 < \frac{h}{b} = 1,0 < 5,0$, the condition is satisfied

Remark:

All the conditions from clause 6.7.3.1, EN 1994-1-1, are satisfied, so that allows the use of the simplified design method for composite columns.

10. Resistance of the member in axial compression

Remark:

Although the column is subjected to combined compression and bending, the check based on buckling curves is useful as the preliminary check for this column. If the resistance to axial compressive force is not sufficient, the considered column is inadequate and it is necessary to select the stronger cross-section.

The resistance of the member subjected only to axial compression can be checked by the second-order analysis according to clause 6.7.3.5, EN 1994-1-1, so as to take into account member imperfections. As a simplification in the case of the member subjected only to axial compression, the design value of the axial force N_{Ed} should satisfy the check based on European buckling curves, which can be written in the following format:

$$\frac{N_{Ed}}{\chi \cdot N_{pl,Rd}} \leq 1{,}0$$

The reduction factor χ is given by:

$$\chi = \frac{1}{\Phi + \sqrt{\Phi^2 - \bar{\lambda}^2}}, \text{ but } \chi \leq 1{,}0$$

and

$$\Phi = 0{,}5 \cdot [1 + \alpha \cdot (\bar{\lambda} - \bar{\lambda}_0) + \bar{\lambda}^2], \text{ with } \bar{\lambda}_0 = 0{,}2$$

Since $\bar{\lambda}_z = 0{,}81 > \bar{\lambda}_y = 0{,}55$, the buckling resistance about the *z-z* axis is governed.

Remark:

The relevant buckling curves for cross-sections of composite columns are given in Table 6.5, EN1994-1-1, according to which composite columns with partially concrete-encased section can be designed using buckling curve *b* for the *y-y* axis of buckling and using buckling curve *c* for the *z-z* axis of buckling.

From Table 6.5, EN1994-1-1, the buckling curve *c* with α = 0,49 is adopted for the *z-z* axis of buckling so that Φ_z is:

$$\Phi_z = 0{,}5 \cdot [1 + \alpha \cdot (\bar{\lambda}_z - \bar{\lambda}_0) + \bar{\lambda}_z^2]$$

$$\Phi_z = 0{,}5 \cdot [1 + 0{,}49 \cdot (0{,}81 - 0{,}2) + 0{,}81^2] = 0{,}98$$

The reduction factor χ_z is:

$$\chi_z = \frac{1}{\Phi_z + \sqrt{\Phi_z^2 - \bar{\lambda}_z^2}}$$

$$\chi_z = \frac{1}{0{,}98 + \sqrt{0{,}98^2 - 0{,}81^2}} = 0{,}65 < 1{,}0$$

Check:

$$\frac{N_{Ed}}{\chi_z \cdot N_{pl,Rd}} \leq 1{,}0$$

$$\frac{4185}{0{,}65 \cdot 7806} = 0{,}82 < 1{,}0$$

Since 0,82 < 1,0, the check of the composite column subjected to axial compression is satisfied. It is not necessary to select the stronger cross-section.

11. Resistance of the member in combined compression and uniaxial bending

11.1 Resistance of the member about the *y-y* axis taking into account the equivalent member imperfection $e_{0,z}$

11.1.1 General

According to clause 6.7.3.6, EN 1994-1-1, the member in combined compression and uniaxial bending has sufficient resistance if the following condition is satisfied:

$$\frac{M_{Ed}}{M_{pl,N,Rd}} = \frac{M_{Ed}}{\mu_d \cdot M_{pl,Rd}} \leq \alpha_M$$

where:

M_{Ed} is the greatest of the end moments and the maximum bending moment within the column length. This moment is calculated according to clause 6.7.3.4, EN 1994-1-1, including imperfections (Table 6.5, EN 1994-1-1) and second-order effects if necessary (α_{cr} > 10).

$M_{pl,N,Rd}$ is the plastic resistance moment taking into account the axial force N_{Ed}, given by $\mu_d \cdot M_{pl,Rd}$, see Figure 6.18, EN 1994-1-1.

$M_{pl,Rd}$ is the plastic resistance moment, given by point B in Figure 6.18, EN

1994-1-1.

μ_d is the factor related to design for compression and uniaxial bending.

α_M is the coefficient related to bending of a composite column and is taken as 0,9 for steel grades between S235 and S355.

The condition can be written as:

$$\frac{M_{Ed}}{M_{Rd}} = \frac{M_{Ed}}{\alpha_M \cdot \mu_d \cdot M_{pl,Rd}} \leq 1{,}0$$

The calculation of the design bending moment $M_{Ed} = M_{Ed,II}$ taking the initial bending moment about the y-y axis, the imperfection $e_{0,z}$, and second-order effects is shown in Figure C5.8.

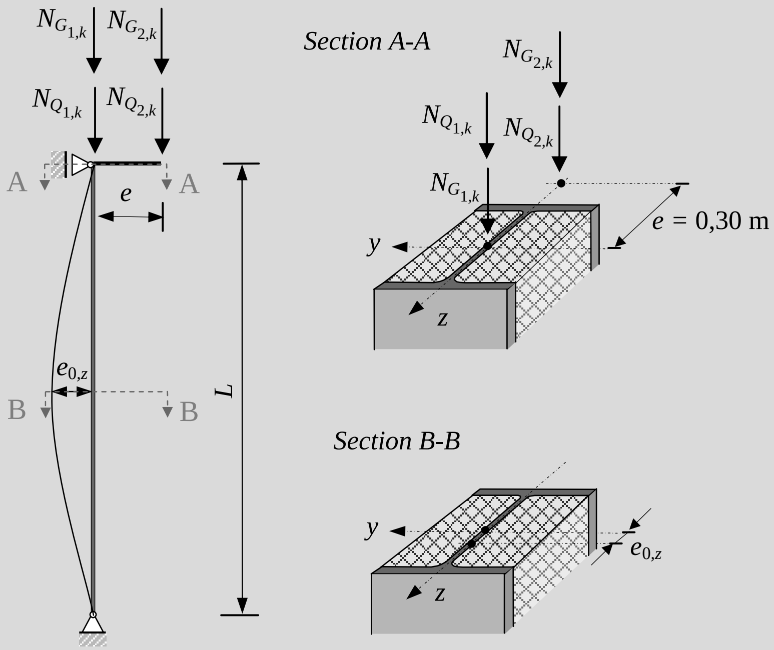


Figure C5.8 *Equivalent member imperfection $e_{0,z}$ about the y-y axis*

11.1.2 Resistance of the cross-section in combined compression and bending about the y-y axis

11.1.2.1 General

Remark:

In order to determine the resistance of the composite cross-section to combined compression and uniaxial bending, it is necessary to produce an axial load –

bending moment (N–M) interaction curve. As a simplification, the interaction curve is replaced by an interaction polygon *ACDB*, clause 6.7.3.2 (5), EN 1994-1-1.
The N–M interaction polygon *ACDB* is shown in Figure 6.19, EN 1994-1-1. Modified version of the interaction polygon, which refers to the composite column with partially concrete-encased H-section, is shown in Figure C5.9.

In order to produce the N–M interaction polygon, the cross-sectional capacities at points A to D should be determined assuming the stress distributions indicated, see Figure C5.9.

It should be noted that EN 1994-1-1 does not provide expressions for concrete-filled rectangular cross-sections.

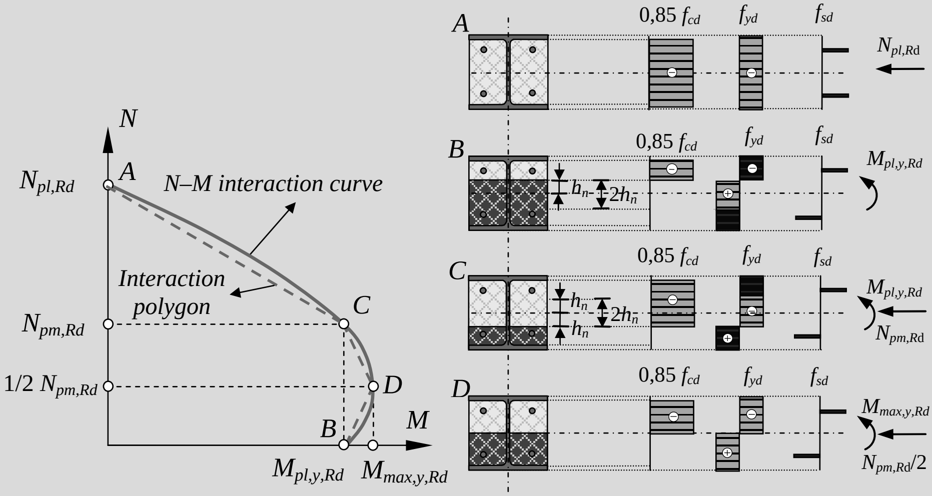


Figure C5.9 *N–M interaction polygon and corresponding stress distributions*

The resistance of the cross-section to combined compression and bending is calculated in two ways: using the interaction curve of N–M and using the interaction polygon of N–M.

11.1.2.2 Interaction curve

It is necessary to carry out the calculation of the reduced design resistance moment, $M_{pl,y,N,Rd}$, due to the design axial compressive force, N_{Ed}. The position of the plastic neutral axis is determined by the following condition:

$$N_{com} - N_{ten} = N_{Ed}$$

where:

N_{com} is the axial compressive force (from the compressive stress block),
N_{ten} is the axial tensile force (from the tensile stress block),
N_{Ed} is the design axial compressive force.

In the case of the high axial force, the neutral axis lies within the flange of the steel section. The position of the plastic neutral axis is in region 3, see Figure C5.10.
If the plastic neutral axis lies within region 3, the following expression is applicable:

$$N_{pl,Rd} = A_a \cdot f_{yd} + 0,85 \cdot A_c \cdot f_{cd} + A_s \cdot f_{sd}$$

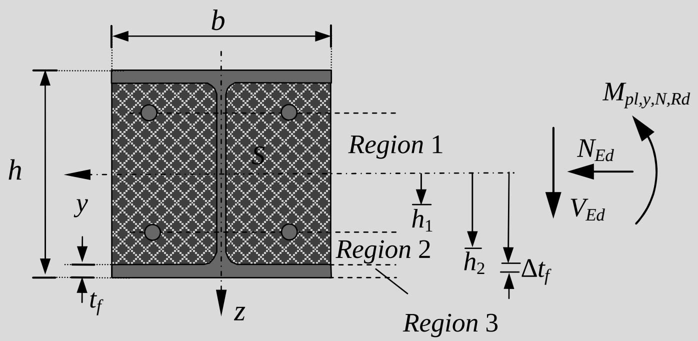


Figure C5.10 *Possible regions of positions of plastic neutral axis within the column section*

For the cross-section of the column, the axial compressive force and the axial tensile force are determined by:

$$N_{com} = N_{pl,Rd} - \Delta t_f \cdot b \cdot f_{yd}$$

$$N_{ten} = \Delta t_f \cdot b \cdot f_{yd}$$

From the condition $N_{com} - N_{ten} = N_{Ed}$, the expression for Δt_f is:

$$\Delta t_f = \frac{N_{pl,Rd} - N_{Ed}}{2 \cdot b \cdot f_{yd}}$$

The expression for $M_{pl,y,N,Rd}$ is:

$$M_{pl,y,N,Rd} = W_{pl,y,a,N} \cdot f_{yd} + W_{pl,y,c,N} \cdot 0,85 \cdot f_{cd} + W_{pl,y,s,N} \cdot f_{sd}$$

The symmetrical areas do not contribute to the bending so that the expression for $M_{pl,y,N,Rd}$ is:

$$M_{pl,y,N,Rd} = W_{pl,y,a,N} \cdot f_{yd} = \Delta t_f \cdot b \cdot (h - \Delta t_f) \cdot f_{yd}$$

Accordingly, the design plastic resistance of the composite cross-section to compression, $N_{pl,Rd}$, is:

$$N_{pl,Rd} = A_a \cdot f_{yd} + 0,85 \cdot A_c \cdot f_{cd} + A_s \cdot f_{sd}$$

$$N_{pl,Rd} = 149,1 \cdot 35,5 + 0,85 \cdot 731,4 \cdot 2,67 + 19,6 \cdot 43,5 = 7806 \text{ kN}$$

For calculation of $M_{pl,y,N,Rd}$, it is necessary to determine Δt_f:

$$\Delta t_f = \frac{N_{pl,Rd} - N_{Ed}}{2 \cdot b \cdot f_{yd}}$$

$$\Delta t_f = \frac{7806 - 4185}{2 \cdot 30 \cdot 35,5} = 1,70 \text{ cm}$$

Therefore, the reduced design resistance moment $M_{pl,y,N,Rd}$ is:

$$M_{pl,y,N,Rd} = \Delta t_f \cdot b \cdot (h - \Delta t_f) \cdot f_{yd}$$

$$M_{pl,y,N,Rd} = 1,70 \cdot 30 \cdot (30 - 1,70) \cdot 35,5 \cdot 10^{-2}$$

$$M_{pl,y,N,Rd} = 512 \text{ kNm}$$

If the design axial force is $N_{Ed} = 0$, the plastic neutral axis lies within region 1, and the exact design resistance moment is determined as:

$$N_{Ed} = 0 \quad \rightarrow \quad M_{pl,y,N,Rd} = M_{pl,y,Rd}$$

Thus, it is assumed that the plastic neutral axis lies within the web of the steel section ($h_n \leq h/2 - t_f$). The position of the plastic neutral axis, $h_1 = h_n$, is calculated as:

$$h_n = \frac{N_{pm,Rd} - A_{s,n} \cdot (2 \cdot f_{sd} - 0,85 \cdot f_{cd})}{2 \cdot b_c \cdot 0,85 \cdot f_{cd} + 2 \cdot t_w \cdot (2 \cdot f_{yd} - 0,85 \cdot f_{cd})}$$

The design value of the resistance of the concrete to compression, $N_{pm,Rd}$, is:

$$N_{pm,Rd} = A_c \cdot 0{,}85 \cdot f_{cd} = (b \cdot h - A_a - A_s) \cdot 0{,}85 \cdot f_{cd} \,.$$

The design resistance moment of the column section in combined bending about the y-y axis and the axial force $N_{Ed} = 0$ is obtained by:

$$M_{pl,y,N,Rd} = M_{pl,y,Rd} = W_{pl,y,a,N} \cdot f_{yd} + W_{pl,y,c,N} \cdot 0{,}85 \cdot f_{cd} + W_{pl,y,s,N} \cdot f_{sd}$$

Alternatively, the design resistance moment of the column section in bending about the y-y axis can be determined by:

$$M_{pl,y,Rd} = M_{\max,y,Rd} - M_{n,y,Rd}$$

where:

$$M_{n,y,Rd} = W_{pl,y,a,n} \cdot f_{yd} + W_{pl,y,s,n} \cdot f_{sd} + \frac{W_{pl,y,c,n} \cdot f_{cd}}{2}$$

The maximum design plastic resistance moment, $M_{\max,y,Rd}$, is determined by:

$$M_{\max,y,Rd} = W_{pl,y,a} \cdot f_{yd} + 0{,}5 \cdot W_{pl,y,c} \cdot 0{,}85 \cdot f_{cd} + W_{pl,y,s} \cdot f_{sd}$$

The design value of the resistance of the concrete to compression, $N_{pm,Rd}$, is:

$$N_{pm,Rd} = (b \cdot h - A_a - A_s) \cdot 0{,}85 \cdot f_{cd}$$

$$N_{pm,Rd} = (30{,}0 \cdot 30{,}0 - 149{,}1 - 19{,}6) \cdot 0{,}85 \cdot 2{,}67 = 1660 \text{ kN}$$

When the design axial force is $N_{Ed} = 0$, the plastic neutral axis lies within region 1, see Figure C5.10. The plastic neutral axis lies within the web of the steel section ($h_n \le h/2 - t_f$). and its position is determined by:

$$h_n = \frac{N_{pm,Rd} - A_{s,n} \cdot (2 \cdot f_{sd} - 0{,}85 \cdot f_{cd})}{2 \cdot b_c \cdot 0{,}85 \cdot f_{cd} + 2 \cdot t_w \cdot (2 \cdot f_{yd} - 0{,}85 \cdot f_{cd})}$$

where $A_{s,n}$ is the reinforcement area within h_n. Because it is at this point unknown, it is assumed to be (initial guess):

$$A_{s,n} = 0 \ \text{cm}^2$$

Thus, for the case when the axial force is equal to zero, h_n is:

$$h_n = \frac{1660 - 0}{2 \cdot 30,0 \cdot 0,85 \cdot 2,67 + 2 \cdot 1,1 \cdot (2 \cdot 35,5 - 0,85 \cdot 2,67)} = 5,78 \ \text{cm}$$

Since in this region there is no reinforcement, the assumption is correct.

Plastic section moduli in region 2·h_n

Structural steel

$$W_{pl,y,a,n} = \frac{t_w \cdot (2h_n)^2}{4} = \frac{1,1 \cdot 11,56^2}{4} = 36,7 \ \text{cm}^3$$

Reinforcement

$W_{pl,y,s,n}$ = 0 cm^3

Concrete

$$W_{pl,y,c,n} = \frac{b_c \cdot (2h_n)^2}{4} - W_{pl,y,a,n} = \frac{30 \cdot 11,56^2}{4} - 36,7 = 966 \ \text{cm}^3$$

The design plastic resistance moment of the composite section, $M_{pl,y,Rd}$, is calculated as:

$$M_{pl,y,Rd} = M_{\max,y,Rd} - M_{n,y,Rd}$$

where:

$$M_{n,y,Rd} = W_{pl,y,a,n} \cdot f_{yd} + W_{pl,y,s,n} \cdot f_{sd} + \frac{W_{pl,y,c,n} \cdot f_{cd}}{2}$$

$$M_{n,y,Rd} = (36,7 \cdot 35,5 + 0 \cdot 43,5 + \frac{966 \cdot 0,85 \cdot 2,67}{2}) \cdot 10^{-2} = 24,0 \ \text{kNm}$$

The maximum design plastic resistance moment is:

$$M_{\max,y,Rd} = W_{pl,y,a} \cdot f_{yd} + 0,5 \cdot W_{pl,y,c} \cdot 0,85 \cdot f_{cd} + W_{pl,y,s} \cdot f_{sd}$$

$$M_{\max,y,Rd} = (1869 \cdot 35{,}5 + 0{,}5 \cdot 4722 \cdot 0{,}85 \cdot 2{,}67 + 159 \cdot 43{,}5) \cdot 10^{-2}$$

$$M_{\max,y,Rd} = 786 \text{ kNm}$$

The design plastic resistance moment of the composite section, $M_{pl,y,Rd}$, is:

$$M_{pl,y,Rd} = 786 - 24{,}0 = 762 \text{ kNm}$$

The obtained values are the exact values on the interaction curve and they are shown in Figure C5.11.

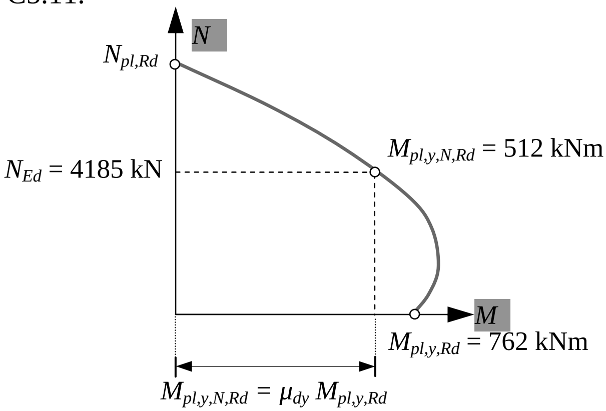


Figure C5.11 *Interaction curve*

The value of μ_{dy} is:

$$\mu_{dy} = \frac{M_{pl,y,N,Rd}}{M_{pl,y,Rd}} = \frac{512}{762} = 0{,}67 < 1{,}0$$

11.1.2.3 Interaction polygon

Point A

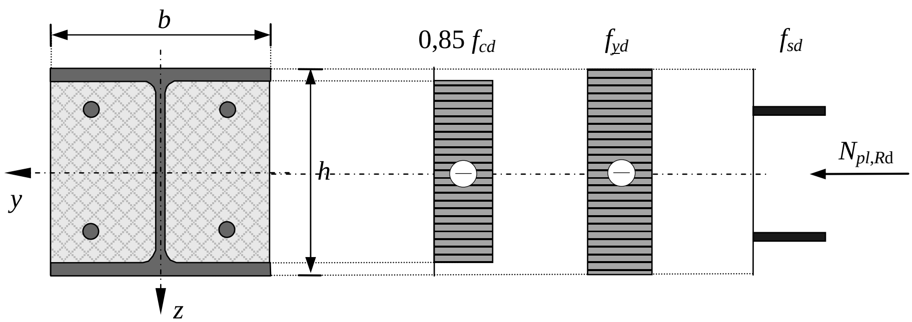


Figure C5.12 *Stress distributions for point A on the interaction polygon*

At point A, only the design plastic resistance of the cross-section is taken into account:

$$N_{pl,Rd} = A_a \cdot f_{yd} + 0,85 \cdot A_c \cdot f_{cd} + A_s \cdot f_{sd}$$

The design plastic resistance of composite cross-section to compression is:

$$N_{pl,Rd} = 149,1 \cdot 35,5 + 0,85 \cdot 731,4 \cdot 2,67 + 19,6 \cdot 43,5$$

$$N_{pl,Rd} = 7806 \text{ kN}$$

Point D

The positions of the plastic neutral axis and the stress distributions are shown in Figure C5.13.

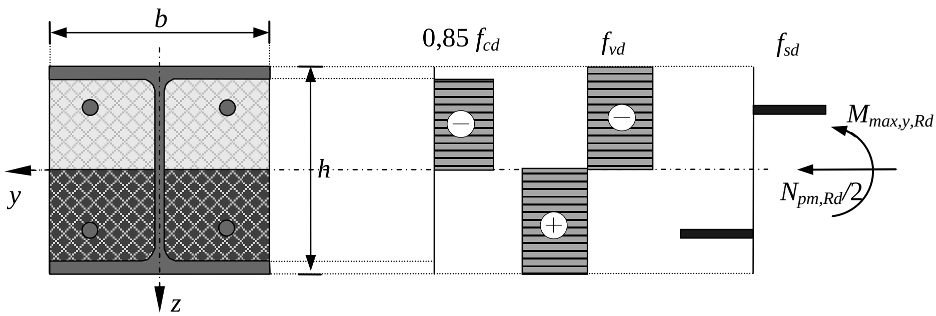


Figure C5.13 *Stress distributions for point D on the interaction polygon*

The maximum design plastic resistance moment is determined by:

$$M_{\max,y,Rd} = M_{pl,y,a,Rd} + M_{pl,y,c,Rd} + M_{pl,y,s,Rd}$$

The maximum design plastic resistance moment, $M_{max,y,Rd}$, at point D is:

$$M_{\max,y,Rd} = W_{pl,y,a} \cdot f_{yd} + 0,5 \cdot W_{pl,y,c} \cdot 0,85 \cdot f_{cd} + W_{pl,y,s} \cdot f_{sd}$$

$$M_{\max,y,Rd} = (1869 \cdot 35,5 + 0,5 \cdot 4722 \cdot 0,85 \cdot 2,67 + 159 \cdot 43,5) \cdot 10^{-2}$$

$$M_{\max,y,Rd} = 786 \text{ kNm}$$

The design value of the resistance of the concrete to compression, $N_{pm,Rd}$, is:

$$N_{pm,Rd} = A_c \cdot 0{,}85 \cdot f_{cd} = 731{,}4 \cdot 0{,}85 \cdot 2{,}67 = 1660 \text{ kN}$$

The design axial force at the point of maximum design plastic resistance moment is $0{,}5 \cdot N_{pm,Rd}$, and is therefore:

$$0{,}5 \cdot N_{pm,Rd} = 0{,}5 \cdot 1660 = 830 \text{ kN}$$

Point C

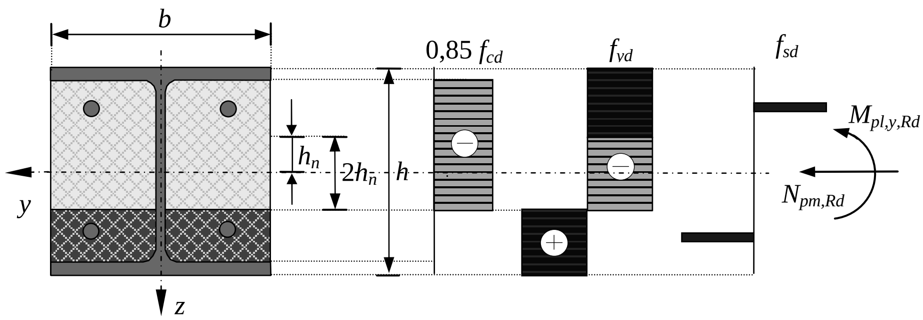


Figure C5.14 *Stress distributions for point C on the interaction polygon*

Calculation of the design plastic resistance moment of the composite section, $M_{pl,y,Rd}$, is carried out as shown below.

When the design axial force is equal to zero, the plastic neutral axis lies within the web of the steel section ($h_n \leq h/2 - t_f$) and its position is determined by:

$$h_n = \frac{N_{pm,Rd} - A_{s,n} \cdot (2 \cdot f_{sd} - 0{,}85 \cdot f_{cd})}{2 \cdot b_c \cdot 0{,}85 \cdot f_{cd} + 2 \cdot t_w \cdot (2 \cdot f_{yd} - 0{,}85 \cdot f_{cd})}$$

where $A_{s,n}$ is the reinforcement area within h_n. Because it is at this point unknown, it is assumed to be (initial guess):

$$A_{s,n} = 0 \text{ cm}^2$$

Thus, for the case when the axial force is equal to zero, h_n is:

$$h_n = \frac{1660 - 0}{2 \cdot 30{,}0 \cdot 0{,}85 \cdot 2{,}67 + 2 \cdot 1{,}1 \cdot (2 \cdot 35{,}5 - 0{,}85 \cdot 2{,}67)} = 5{,}78 \text{ cm}$$

Since in this region there is no reinforcement, the assumption is correct.

Plastic section moduli in region 2·h_n

Structural steel

$$W_{pl,y,a,n} = \frac{t_w \cdot (2h_n)^2}{4} = \frac{1,1 \cdot 11,56^2}{4} = 36,7 \ \text{cm}^3$$

Reinforcement

$W_{pl,y,s,n}$ = 0 cm^3

Concrete

$$W_{pl,y,c,n} = \frac{b_c \cdot (2h_n)^2}{4} - W_{pl,y,a,n} - W_{pl,y,s,n} = \frac{30 \cdot 11,56^2}{4} - 36,7 - 0 = 966 \ \text{cm}^3$$

The design plastic resistance moment of the composite section, $M_{pl,y,Rd}$, is calculated as:

$$M_{pl,y,Rd} = M_{\max,y,Rd} - M_{n,y,Rd}$$

where:

$$M_{n,y,Rd} = W_{pl,y,a,n} \cdot f_{yd} + W_{pl,y,s,n} \cdot f_{sd} + \frac{W_{pl,y,c,n} \cdot f_{cd}}{2}$$

$$M_{n,y,Rd} = (36,7 \cdot 35,5 + 0 \cdot 43,5 + \frac{966 \cdot 0,85 \cdot 2,67}{2}) \cdot 10^{-2} = 24,0 \ \text{kNm}$$

Point B

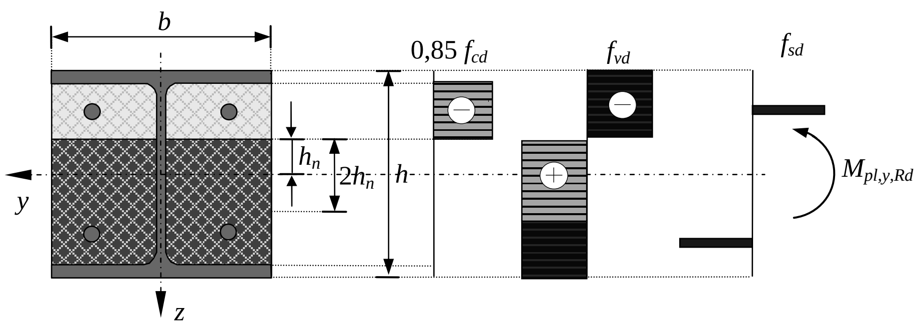


Figure C5.15 *Stress distributions for point B on the interaction polygon*

The design plastic resistance moment of the composite section, $M_{pl,y,Rd}$, is:

$$M_{pl,y,Rd} = 786 - 24{,}0 = 762 \text{ kNm}$$

The design value of $M_{pl,y,Rd}$ has previously been calculated in order to define point C on the N–M interaction polygon:

$$M_{pl,y,Rd} = 762 \text{ kNm}$$

Previously calculated values at points A to E should be plotted to produce the N–M interaction polygon (Figure 6.19, EN 1994-1-1). The interaction polygon $ACDB$ is shown in Figure C5.16.

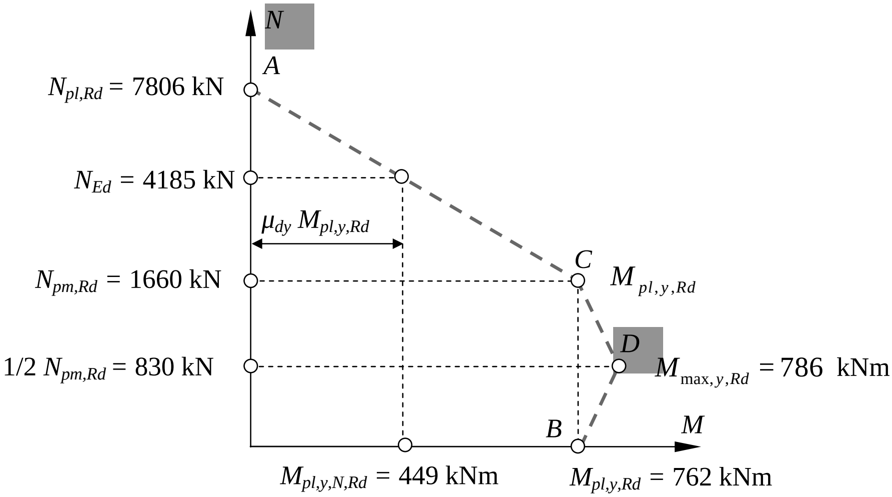


Figure C5.16 *N–M interaction polygon*

According to the interaction polygon $ACDB$, Figure C5.16, the following value $M_{pl,y,N,Rd}$ is obtained:

$$M_{pl,y,Rd} : M_{pl,y,N,Rd} = (N_{pl,Rd} - N_{pm,Rd}) : (N_{pl,Rd} - N_{Ed})$$

$$M_{pl,y,N,Rd} = \frac{N_{pl,Rd} - N_{Ed}}{N_{pl,Rd} - N_{pm,Rd}} \cdot M_{pl,y,Rd}$$

$$M_{pl,y,N,Rd} = \frac{7806 - 4185}{7806 - 1660} \cdot 762 = 449 \text{ kNm}$$

The value of μ_{dy} is:

$$\mu_{dy} = \frac{M_{pl,y,N,Rd}}{M_{pl,y,Rd}} = \frac{449}{762} = 0{,}59 < 1{,}0$$

Remark:

According to the approach with the interaction curve, Section 11.1.2.2., the value of μ_{dy} is 0,67. According to the approach with the interaction polygon, the value of μ_{dy} is 0,59. This value is too conservative. Accordingly, it is recommended to find an intermediate point, *E*, for better polygonal approximation to the interaction curve, see example C4. EN 1994-1-1 does not give this recommendation.

The check is carried out with the factor $\mu_{dy} = 0{,}59$, in accordance with EN 1994-1-1.

11.1.3 *Calculation of the effects of actions about the y-y axis*

11.1.3.1 General

According to clause 6.7.3.4 (3), EN 1994-1-1, which refers to clause 5.2.1(3), EN 1994-1-1, second-order effects can therefore be neglected if the load factor α_{cr}, which is the ratio between the elastic critical load and the corresponding applied loading, for elastic instability of the member exceeds 10.

To calculate α_{cr}, the ends of the column are assumed to be pinned, and α_{cr} is found using the Euler formula for the elastic critical force $N_{cr,y,eff}$:

$$N_{cr,y,eff} = \frac{\pi^2 (EI)_{eff,y,II}}{L_{e,y}^2} \quad L_{e,y} = L$$

The design value of the effective flexural stiffness $(EI)_{eff,y,II}$, used to determine the internal forces and moments by second-order analysis, according to clause 6.7.3.4(2), EN 1994-1-1, is defined as:

$$(EI)_{eff,y,II} = K_0 \cdot (E_a \cdot I_{y,a} + E_s \cdot I_{y,s} + K_{e,II} \cdot E_{cm} \cdot I_{y,c})$$

where:

$K_{e,II}$ is a correction factor, which should be taken as 0,5,
K_0 is a calibration factor, which should be taken as 0,9.

The value $E_{c,eff}$ has been used in place of E_{cm} in the expression for $(EI)_{eff,y,II}$ in order to take into account the long-term effects, in the same way as calculated in Section 7. Accordingly, the value of $E_{c,eff}$ is:

$$E_{c,eff} = 1530 \text{ kN/cm}^2$$

The design value of the effective flexural stiffness $(EI)_{eff,y,II}$, is:

$$(EI)_{eff,y,II} = K_0 \cdot (E_a \cdot I_{y,a} + E_s \cdot I_{y,s} + K_{e,II} \cdot E_{c,eff} \cdot I_{y,c})$$

$$(EI)_{eff,y,II} = 0,9 \cdot (21000 \cdot 25170 + 21000 \cdot 1289 + 0,5 \cdot 1530 \cdot 41041)$$

$$(EI)_{eff,y,II} = 528,33 \cdot 10^6 \text{ kNcm}^2$$

The elastic critical force, $N_{cr,y,eff}$, for the pin-ended column, is:

$$N_{cr,y,eff} = \frac{\pi^2 (EI)_{eff,y,II}}{L_{e,y}^2}$$

$$N_{cr,y,eff} = \frac{528,33 \cdot 10^6 \cdot \pi^2}{450^2} = 25750 \text{ kN}$$

To check whether the effects of second-order analysis can be neglected, the value of α_{cr} must be higher than 10:

$$\alpha_{cr} = \frac{N_{cr,y,eff}}{N_{Ed}} = \frac{25750}{4185} = 6,2 < 10$$

The value of α_{cr} is less than 10, so second-order effects must be considered.

11.1.3.2 Bending moments about the y-y axis

According to clause 6.7.3.4(5), EN 1994-1-1, the second-order effects can be calculated by multiplying the greatest first-order design bending moments by a factor *k*.

Thus, the second-order effects may be considered according to the expression:

$$M_{y,Ed,II} = M_{y,Ed,I} \cdot k$$

The factor *k* is given by:

$$k = \frac{\beta}{1 - N_{Ed} / N_{cr,y,eff}} \geq 1,0$$

where:

β is an equivalent moment factor given in Table 6.4, EN 1994-1-1,

$N_{cr,y,eff}$ is the critical axial force, about the *y*-*y* axis, obtained with the effective flexural stiffness $(EI)_{eff,y,II}$ and with the effective length taken as the physical length of the column.

The design bending moment from the member imperfections is determined as:

$$M_{y,Ed,imp} = N_{Ed} \cdot e_{0,z}$$

where:

N_{Ed} is the design value of the axial force,

$e_{0,z}$ is the equivalent member imperfection which is given in Table 6.5, EN 1994-1-1, depending on the buckling curve.

Remark:

According to Table 6.5, EN 1994-1-1, the composite columns with partially concrete-encased section can be designed using buckling curve *b* for the *y*-*y* axis of buckling.

Therefore, for buckling curve *b*, the equivalent member imperfection is:

$$e_{0,z} = \frac{L}{200}$$

$$e_{0,z} = \frac{450}{200} = 2,25 \text{ cm}$$

The design bending moments calculated according to first-order analysis are shown in Figure C5.17.

The design values of bending moments are as follows:

The design bending moment at the top of the column is:

$$M_{y,Ed} = 189 \text{ kNm}$$

The design bending moment at the bottom of the column is:

$$M_{y,Ed} = 0 \text{ kNm}$$

The design bending moment due to imperfection is:

$$M_{y,Ed,imp} = N_{Ed} \cdot e_{0,z} = 4185 \cdot 0{,}0225 = 94{,}2 \text{ kNm}$$

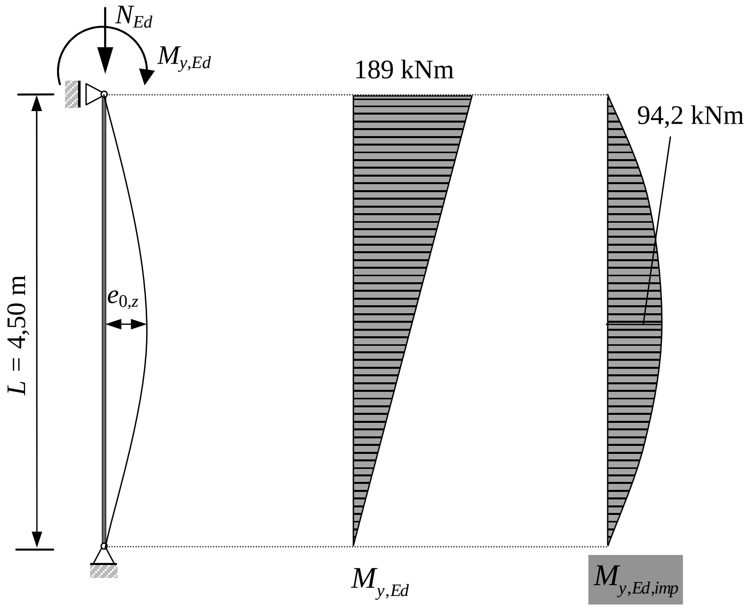


Figure C5.17 *First-order bending moments, design values*

Remark:

The factor β from Table 6.4, EN 1994-1-1, allows for the shape of the bending moment diagram. When bending is caused by lateral loading on the column, the value of factor β is 1,0. For a column subjected to end moments, the factor β is calculated as:

$$\beta_1 = 0{,}66 + 0{,}44 \cdot r \geq 0{,}44$$

where r is the ratio of the end-moments on the ends of the column (-1 ≤ r ≤ +1).

Therefore, the two values of factor k must be calculated:

- for the end moments, k_1,
- for the moment from the member imperfection, k_2.

Determination of factor k_1

The ratio of the end-moments on the ends of the column is:

$$r = \frac{0}{M_{y,Ed}} = \frac{0}{189} = 0,0$$

The equivalent moment factor β is:

$$\beta_1 = 0,66 + 0,44 \cdot r \geq 0,44$$

$$\beta_1 = 0,66 + 0,44 \cdot 0 = 0,66$$

Therefore, the factor k_1 is:

$$k_1 = \frac{\beta_1}{1 - N_{Ed} / N_{cr,y,eff}} = \frac{0,66}{1 - 4185 / 25750} = 0,79 < 1,0$$

Remark:

According to clause 6.7.3.4(5), EN 1994-1-1, the value of factor k must be 1,0 or higher. It is over-conservative to use when combining two sets of second-order effects. Therefore, the calculated value of 0,79 is adopted.

Determination of factor k_2

For the bending moment from the member imperfection, according to Table 6.4, EN 1994-1-1, the equivalent moment factor β is:

$$\beta_2 = 1,0$$

Therefore, the factor k_2 is:

$$k_2 = \frac{\beta_2}{1 - N_{\mathrm{Ed}} / N_{\mathrm{cr,y,eff}}} = \frac{1,0}{1 - 4185 / 25750} = 1,19 > 1,0$$

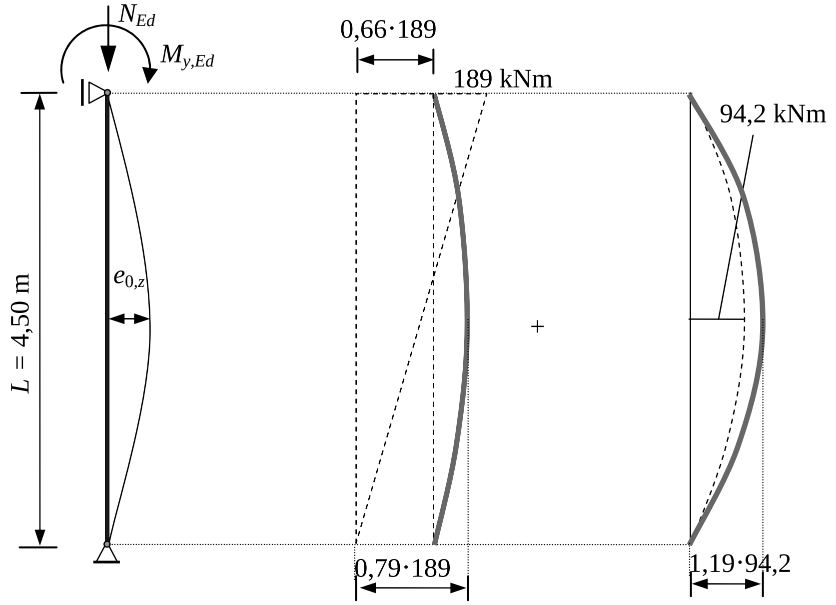
The adopted value of the factor is:

k_2 = 1,19

The design bending moment at mid-height, with second-order effects taken into account, is:

$$M_{y,Ed,II} = M_{y,Ed} \cdot k_1 + M_{y,Ed,imp} \cdot k_2 = 189 \cdot 0{,}79 + 94{,}2 \cdot 1{,}19 = 261 \text{ kNm}$$

The design bending moments calculated according to second-order analysis are shown in Figure C5.18.



a) Second-order bending moments due to $N_{G2,Ed}$ and $N_{Q2,Ed}$

b) Imperfection moment increased by the second-order effects

Figure C5.18 *Second-order bending moments about the y-y axis, design values*

The check is performed with the bending moment at mid-height:

$$M_{y,Ed,II} = M_{y,\max} = 261 \text{ kNm}$$

11.1.3.3 Shear forces parallel to the z-z axis

According to clause 6.7.3.4(5), EN 1994-1-1, second-order effects can be allowed for by multiplying the greatest first-order design bending moment by a factor *k* given by:

$$k = \frac{\beta}{1 - N_{Ed} / N_{cr,eff}} \geq 1{,}0$$

Accordingly, the approximate value of shear force can be obtained as:

$$V_{Ed,II} = V_{Ed} \cdot k$$

In accordance with Figure C5.19, the first-order design shear force at the bottom of column is:

$$V_{z,Ed} = \frac{M_{y,Ed}}{L} + \frac{4 \cdot N_{Ed} \cdot e_{0,z}}{L} = \frac{189}{4,5} + \frac{4 \cdot 4185 \cdot 0,0225}{4,5} = 42,0 + 83,7 = 126 \text{ kN}$$

In accordance with Figure C5.19, the first-order design shear force at the top of column is:

$$V_{z,Ed} = -\frac{M_{y,Ed}}{L} + \frac{4 \cdot N_{Ed} \cdot e_{0,z}}{L} = -\frac{189}{4,5} + \frac{4 \cdot 4185 \cdot 0,0225}{4,5} = -42,0 + 83,7 = 41,7 \text{ kN}$$

The diagram of shear forces, calculated by first-order analysis for bending moment and the equivalent lateral load due to imperfections, is shown in Figure C5.19.

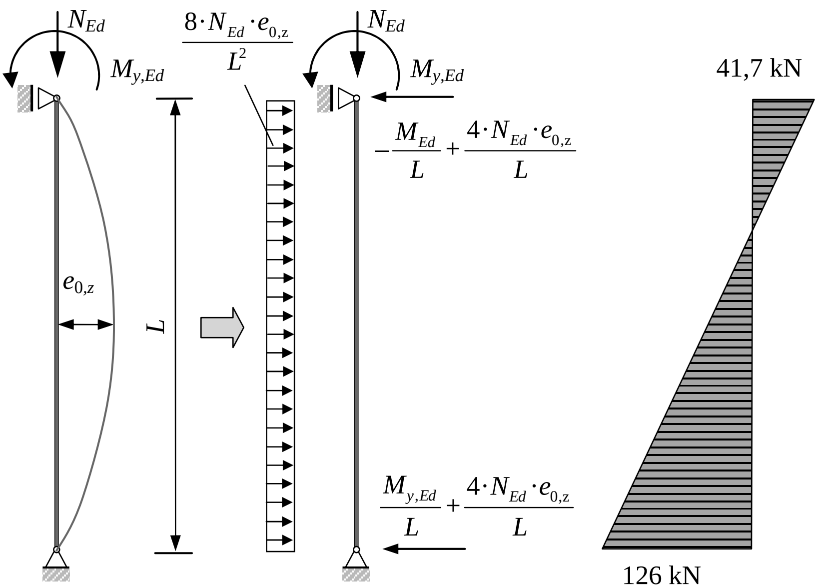


Figure C5.19 *First-order design shear forces parallel to the z-z axis*

The factor k_1 is:

$$k_1 = \frac{\beta_1}{1 - N_{Ed} / N_{cr,y,eff}} = \frac{0,66}{1 - 4185/25750} = 0,79 < 1,0$$

The factor k_2 is:

$$k_2 = \frac{\beta_2}{1 - N_{Ed} / N_{cr,y,eff}} = \frac{1}{1 - 4185/25750} = 1,19 > 1,0$$

Therefore, the maximum design shear force, calculated by approximate second-order analysis, is:

$$V_{z,Ed,II} = k_1 \cdot \frac{M_{y,Ed}}{L} + k_2 \cdot \frac{4 \cdot N_{Ed} \cdot e_{0,z}}{L}$$

$$V_{z,Ed,II} = 0,79 \cdot \frac{189}{4,5} + 1,19 \cdot \frac{4 \cdot 4185 \cdot 0,0225}{4,5} = 33,2 + 99,6 = 133 \text{ kN}$$

11.1.4 *Check of the resistance of the member in combined compression and bending about the y-y axis*

It is necessary to satisfy the following condition:

$$\frac{M_{y,Ed}}{M_{y,Rd}} = \frac{M_{y,Ed}}{\alpha_{M,y} \cdot \mu_{dy} \cdot M_{pl,y,Rd}} \leq 1,0$$

The coefficient $\alpha_{M,y}$ is taken as 0,9 for steel grades between S235 and S355.

The design value of the maximum design bending moment by the approximative second-order analysis is:

$$M_{y,Ed} = M_{y,Ed,II} = 261 \text{ kNm}$$

The design resistance moment $M_{y,Rd}$ is (Figure C5.16):

$$M_{y,Rd} = \alpha_{M,y} \cdot \mu_{dy} \cdot M_{pl,y,Rd} = 0,9 \cdot 0,59 \cdot 762 = 405 \text{ kNm}$$

Condition:

$$\frac{M_{y,Ed}}{M_{y,Rd}} = \frac{261}{405} = 0,64 \leq 1,0$$

Since $0{,}64 < 1{,}0$, the condition is satisfied.

11.1.5 Check of the plastic resistance to transverse shear parallel to the z-z axis

In accordance with clause 6.7.3.2(4), EN 1994-1-1, for simplification V_{Ed} may be assumed to act on the structural steel section alone. According to clause 6.2.6(2), EN 1993-1-1, in the absence of torsion the design plastic shear resistance, $V_{pl,z,a,Rd}$, is given by:

$$V_{pl,z,a,Rd} = \frac{A_{v,z} \cdot (f_y / \sqrt{3})}{\gamma_{M0}}$$

The shear area, $A_{v,z}$, according to clause 6.2.6(3), EN1993-1-1, is calculated as:

$$A_{v,z} = A_a - 2 \cdot b \cdot t_f + (t_w + 2r) \cdot t_f \geq \eta \cdot h_w \cdot t_w = \eta \cdot (h - 2 \cdot t_f - 2 \cdot r) \cdot t_w$$

According to clause 6.2.2.4(1), EN 1994-1-1, where the shear force is less than half the plastic shear resistance its effect on the resistance moment can be neglected. Therefore, the condition is:

$$V_{z,Ed} < 0{,}5 \cdot V_{pl,z,a,Rd}$$

The design value of the second-order shear force is:

$$V_{z,Ed} = V_{z,Ed,II} = 133 \text{ kNm}$$

The shear area, $A_{v,z}$, is:

$$A_{v,z} = A_a - 2 \cdot b \cdot t_f + (t_w + 2r) \cdot t_f \geq \eta \cdot h_w \cdot t_w = \eta \cdot (h - 2 \cdot t_f - 2 \cdot r) \cdot t_w$$

$$\eta \cdot h_w \cdot t_w = \eta \cdot (h - 2 \cdot t_f - 2 \cdot r) \cdot t_w = 1{,}0 \cdot (30{,}0 - 2 \cdot 1{,}9 - 2 \cdot 2{,}7) \cdot 1{,}1 = 22{,}9 \text{ cm}^2$$

$$A_{v,z} = 149{,}1 - 2 \cdot 30{,}0 \cdot 1{,}9 + (1{,}1 + 2 \cdot 2{,}7) \cdot 1{,}9 = 47{,}4 \text{ cm}^2 \geq 22{,}9 \text{ cm}^2$$

The design plastic shear resistance, $V_{pl,z,a,Rd}$, is:

$$V_{pl,z,a,Rd} = \frac{A_{v,z} \cdot (f_y / \sqrt{3})}{\gamma_{M0}}$$

$$V_{pl,z,a,Rd} = \frac{47,4 \cdot (35,5/\sqrt{3})}{1,0} = 972 \text{ kN}$$

Check:

$$V_{z,Ed,II} = 133 < 0,5 \cdot V_{pl,z,a,Rd} = 0,5 \cdot 972 = 486 \text{ kN}$$

The condition is satisfied and there is no reduction in the resistance moment.

11.2 Resistance of the member about the *z-z* axis taking into account the equivalent member imperfection $e_{0,y}$

11.2.1 General

Remark:

Since the column is subjected to bending about the *y-y* axis, initially given moment $M_{y,Ed}$, and the bending about the *z-z* axis, the bending moment due to imperfection $N_{Ed} \cdot e_{0,y}$, it is necessary to check the column resistance in combined compression and biaxial bending.

In accordance with clause 6.7.3.7, EN 1994-1-1, for combined compression and biaxial bending, the following conditions should be satisfied:

Check for bending about the *y-y* axis:

$$\frac{M_{y,Ed}}{M_{pl,y,N,Rd}} = \frac{M_{y,Ed}}{\mu_{dy} \cdot M_{pl,y,Rd}} \leq \alpha_{My}$$

The condition can be written in the following form:

$$\frac{M_{y,Ed}}{\alpha_{M,y} \cdot \mu_{dy} \cdot M_{pl,y,Rd}} \leq 1,0$$

Check for bending about the *z-z* axis:

$$\frac{M_{z,Ed}}{M_{pl,z,N,Rd}} = \frac{M_{z,Ed}}{\mu_{dz} \cdot M_{pl,z,Rd}} \leq \alpha_{M,z}$$

The condition can be written in the form:

$$\frac{M_{z,Ed}}{\alpha_{M,z} \cdot \mu_{dz} \cdot M_{pl,z,Rd}} \leq 1{,}0$$

Interaction of M_y–M_z–N:

$$\frac{M_{y,Ed}}{M_{pl,y,N,Rd}} + \frac{M_{z,Ed}}{M_{pl,z,N,Rd}} \leq 1{,}0$$

The condition can be written in the following form:

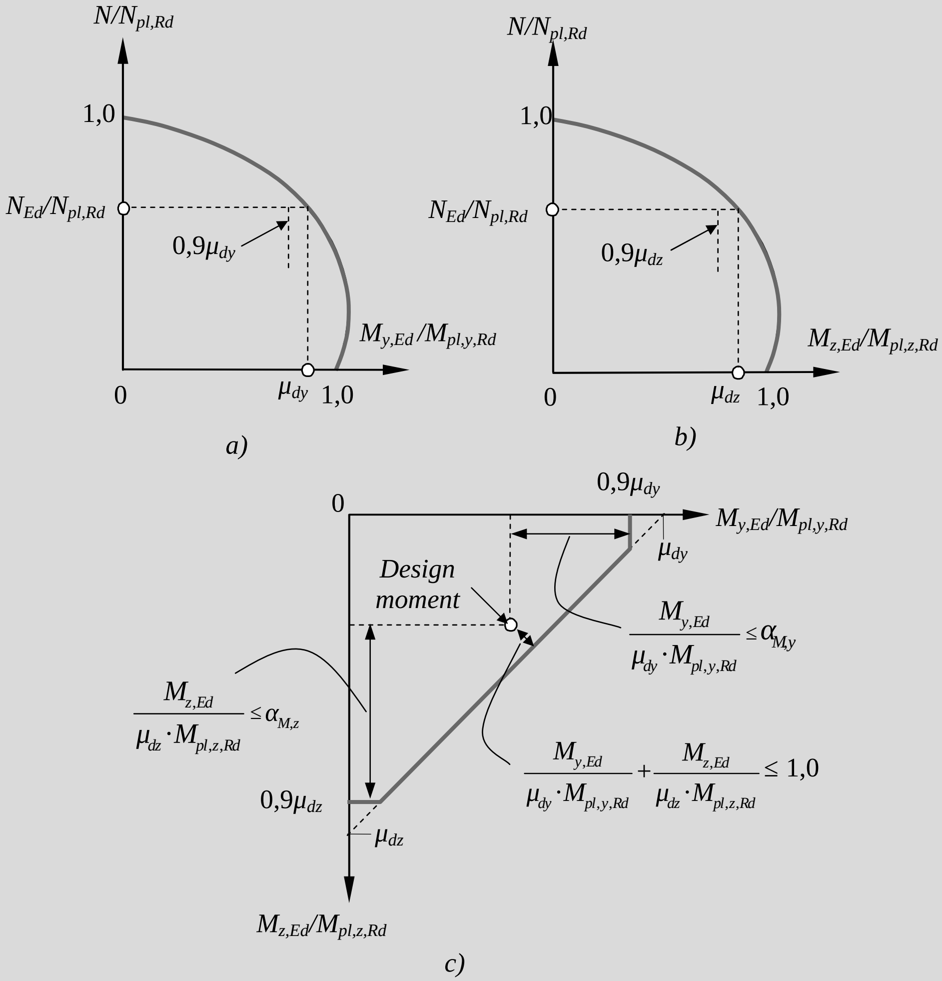
$$\frac{M_{y,Ed}}{\mu_{dy} \cdot M_{pl,y,Rd}} + \frac{M_{z,Ed}}{\mu_{dz} \cdot M_{pl,z,Rd}} \leq 1{,}0$$

These interaction expressions are shown in Figure C5.20 by means of interaction curves.

The check for bending about the *y*-*y* axis is carried out taking into account initially given bending moment $M_{y,Ed}$ without the equivalent member imperfection $e_{0,z}$, but including the second-order effects.

The check for bending about the *z*-*z* axis is carried out taking into account the bending moment $M_{z,Ed}$ (= $N_{Ed} \cdot e_{0,y}$) due to the equivalent member imperfection $e_{0,y}$, including the second-order effects.

Since the column is subjected to bending about the *y*-*y* axis and bending about the *z*-*z* axis, it is necessary to check the column resistance in combined compression and biaxial bending, the interaction of M_y–M_z–N.



a) Section resistance interaction curve – non-failure axis (y-y axis). Neglect imperfections.

b) Section resistance interaction curve – axis of the anticipated failure (z-z axis). Consider imperfections.

c) Biaxial resistance moment of the column section under axial compression N_{Ed}.

Figure C5.20 *Column resistance in combined compression and biaxial bending – assuming bending failure about the z-z axis*

Calculation of the bending moment about the *z-z* axis, $M_{z,Ed} = M_{z,Ed,II}$, taking into account the equivalent member imperfection $e_{0,y}$, and including the second-order effects, is shown in Figure C5.21.

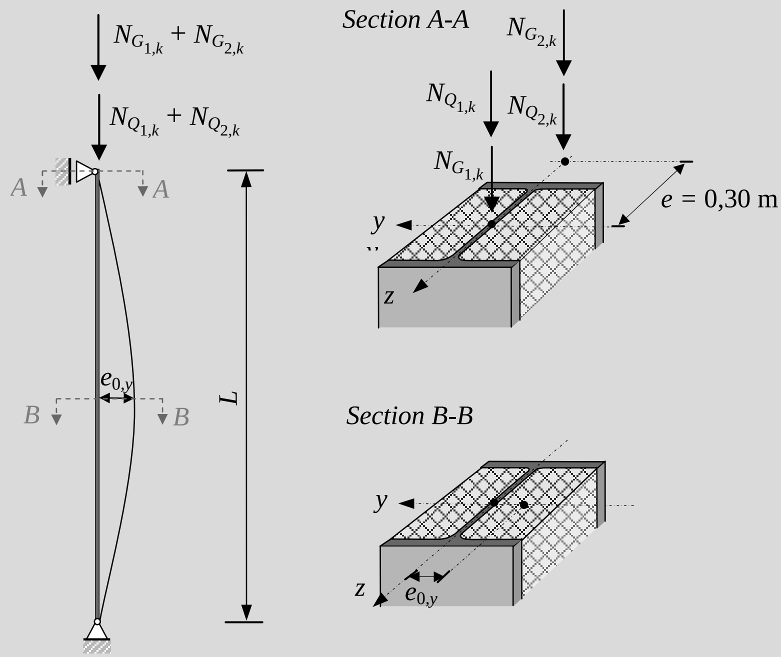


Figure C5.21 *Equivalent member imperfection $e_{0,y}$ about the z-z axis*

11.2.2 Resistance of the cross-section in combined compression and bending about the z-z axis

11.2.2.1 General

Remark:

In order to determine the resistance of the composite cross-section to combined compression and uniaxial bending, it is necessary to produce an axial load – bending moment (*N*–*M*) interaction curve. As a simplification, the interaction curve is replaced by an interaction polygon *ACDB*, clause 6.7.3.2 (5), EN 1994-1-1.

The *N*–*M* interaction polygon *ACDB* is shown in Figure 6.19, EN 1994-1-1. The modified version of the interaction polygon, which refers to the composite column with partially concrete-encased H-section, is shown in Figure C5.22.

In order to produce the *N*–*M* interaction polygon, the cross-sectional capacities at points *A* to *D* should be determined assuming the stress distributions indicated, see Figure C5.22.

It should be noted that EN 1994-1-1 does not provide expressions for concrete-filled rectangular cross-sections.

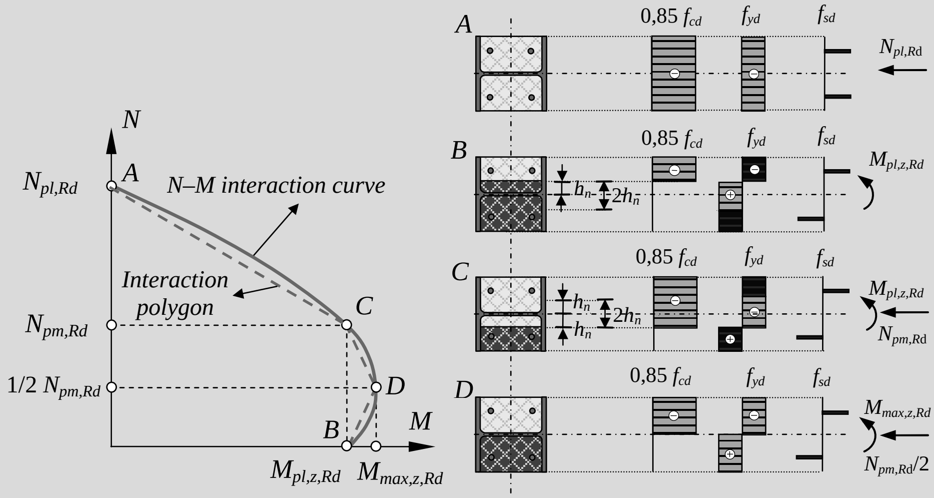


Figure C5.22 *N–M interaction polygon and corresponding stress distributions*

The resistance of the cross-section to combined compression and bending is calculated in two ways: using the interaction curve of N–M and using the interaction polygon of N–M.

11.2.2.2 Interaction curve

It is necessary to carry out the calculation of the reduced design resistance moment, $M_{pl,z,N,Rd}$, due to the design axial compressive force, N_{Ed}. The position of the plastic neutral axis is determined by the following condition:

$$N_{com} - N_{ten} = N_{Ed}$$

where:

N_{com} is the axial compressive force (from the compressive stress block),
N_{ten} is the axial tensile force (from the tensile stress block),
N_{Ed} is the design axial compressive force.

The position of the plastic neutral axis is in region 1 (Figure C5.23).

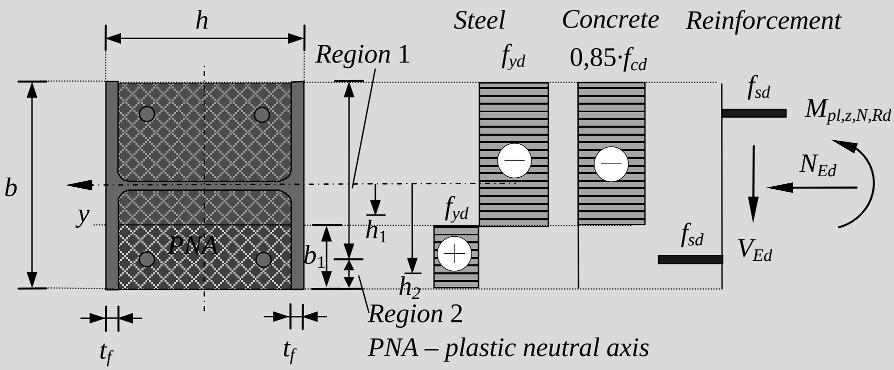


Figure C5.23 *Position of plastic neutral axis*

If the plastic neutral axis lies within region 1, the following expression is applicable:

$$N_{pl,Rd} = A_a \cdot f_{yd} + A_c \cdot 0{,}85 \cdot f_{cd} + A_s \cdot f_{sd}$$

For the cross-section of column, the axial compressive force and the axial tensile force are determined by the following expressions:

$$N_{com} = N_{pl,Rd} - N_{ten} = N_{pl,Rd} - b_1 \cdot 2 \cdot t_f \cdot f_{yd} - \sum_i A_{ten,si} \cdot f_{sd}$$

$$N_{ten} = b_1 \cdot 2 \cdot t_f \cdot f_{yd} + \sum_i A_{ten,si} \cdot f_{sd}$$

From the condition $N_{com} - N_{ten} = N_{Ed}$, the expression for b_1 is:

$$b_1 = \frac{N_{pl,Rd} - N_{Ed} - 2 \cdot \sum_i A_{ten,si} \cdot f_{sd}}{4 \cdot t_f \cdot f_{yd}}$$

The plastic resistance moment of the composite section in combined compression and bending $M_{pl,z,N,Rd}$ is determined by the following expression:

$$M_{pl,z,N,Rd} = W_{pl,z,a,N} \cdot f_{yd} + W_{pl,z,c,N} \cdot 0{,}85 \cdot f_{cd} + W_{pl,z,s,N} \cdot f_{sd}$$

Accordingly, the design plastic resistance of the composite cross-section to compression, $N_{pl,Rd}$, is:

$$N_{pl,Rd} = A_a \cdot f_{yd} + 0{,}85 \cdot A_c \cdot f_{cd} + A_s \cdot f_{sd}$$

$$N_{pl,Rd} = 149{,}1 \cdot 35{,}5 + 0{,}85 \cdot 731{,}4 \cdot 2{,}67 + 19{,}6 \cdot 43{,}5 = 7806 \text{ kN}$$

The position of the plastic neutral axis b_1 is:

$$b_1 = \frac{N_{pl,Rd} - N_{Ed} - 2 \cdot \sum_i A_{vl,si} \cdot f_{sd}}{4 \cdot t_f \cdot f_{yd}}$$

$$b_1 = \frac{7806 - 4185 - 2 \cdot 2 \cdot 4{,}91 \cdot 43{,}5}{4 \cdot 1{,}9 \cdot 35{,}5} = 10{,}3 \text{ cm}$$

Plastic section moduli

Structural steel

$$W_{pl,z,a,N} = 2 \cdot S_{z,N} = b_1 \cdot 2 \cdot t_f \cdot (b - b_1)$$

$$W_{pl,z,a,N} = 10{,}3 \cdot 2 \cdot 1{,}9 \cdot (30 - 10{,}3) = 771 \text{ cm}^3$$

Reinforcement

$$W_{pl,z,s,N} = \sum_i A_{si} \cdot | y_i | = 4 \cdot 4{,}91 \cdot 10 = 196 \text{ cm}^3$$

Concrete

$$W_{pl,z,c,N} = b_1 \cdot h \cdot \left(\frac{b}{2} - \frac{b_1}{2}\right) - \frac{W_{pl,z,a,N}}{2} - \sum_i A_{tl,si} \cdot | y_i |$$

$$W_{pl,c,N} = 10{,}3 \cdot 30 \cdot \left(\frac{30}{2} - \frac{10{,}3}{2}\right) - \frac{771}{2} - 2 \cdot 4{,}91 \cdot 10{,}0 = 2560 \text{ cm}^3$$

Therefore, the reduced design resistance moment $M_{pl,z,N,Rd}$ is:

$$M_{pl,z,N,Rd} = W_{pl,z,a,N} \cdot f_{yd} + W_{pl,z,c,N} \cdot 0{,}85 \cdot f_{cd} + W_{pl,z,s,N} \cdot f_{sd}$$

$$M_{pl,z,N,Rd} = (771 \cdot 35{,}5 + 2560 \cdot 0{,}85 \cdot 2{,}67 + 196 \cdot 43{,}5) \cdot 10^{-2} = 417 \text{ kNm}$$

If the design axial force is $N_{Ed} = 0$, the plastic neutral axis lies within region 1, and the exact design resistance moment is determined as:

$$N_{Ed} = 0 \quad \rightarrow \quad M_{pl,z,N,Rd} = M_{pl,z,Rd}$$

Thus, it is assumed that the plastic neutral axis lies within the web of the steel section ($h_n \leq t_w/2$). The position of the plastic neutral axis, $h_1 = h_n$, is calculated as:

$$h_n = \frac{N_{pm,Rd} - A_{s,n} \cdot (2 \cdot f_{sd} - 0,85 \cdot f_{cd})}{2 \cdot h_c \cdot 0,85 \cdot f_{cd} + 2 \cdot h \cdot (2 \cdot f_{yd} - 0,85 \cdot f_{cd})}$$

The design value of the resistance of the concrete to compression, $N_{pm,Rd}$, is:

$$N_{pm,Rd} = A_c \cdot 0,85 \cdot f_{cd} = (b \cdot h - A_a - A_s) \cdot 0,85 \cdot f_{cd}$$

The design resistance moment of the column section in combined bending about the z-z and the axial force $N_{Ed} = 0$ is obtained by:

$$M_{pl,z,N,Rd} = M_{pl,z,Rd} = W_{pl,z,a,N} \cdot f_{yd} + 0,85 \cdot W_{pl,z,c,N} \cdot f_{cd} + W_{pl,z,s,N} \cdot f_{sd}$$

Alternatively, the design resistance moment of the column section in bending about the z-z axis can be determined using:

$$M_{pl,z,Rd} = M_{\max,z,Rd} - M_{n,z,Rd}$$

where:

$$M_{n,z,Rd} = W_{pl,z,a,n} \cdot f_{yd} + W_{pl,z,s,n} \cdot f_{sd} + \frac{W_{pl,z,c,n} \cdot f_{cd}}{2}$$

The maximum design plastic resistance moment, $M_{\max,z,Rd}$, is determined as:

$$M_{\max,z,Rd} = W_{pl,z,a} \cdot f_{yd} + 0,5 \cdot W_{pl,z,c} \cdot 0,85 \cdot f_{cd} + W_{pl,z,s} \cdot f_{sd}$$

The design value of the resistance of the concrete to compression, $N_{pm,Rd}$, is:

$$N_{pm,Rd} = (b \cdot h - A_a - A_s) \cdot 0,85 \cdot f_{cd}$$

$$N_{pm,Rd} = (30,0 \cdot 30,0 - 149,1 - 19,6) \cdot 0,85 \cdot 2,67 = 1660 \text{ kN}$$

When the design axial force is $N_{Ed} = 0$, the plastic neutral axis lies within region 1, see Figure C5.23. The plastic neutral axis lies within the web of the steel section, $h_n \le t_w/2$, and its position is determined by:

$$h_n = \frac{N_{pm,Rd} - A_{s,n} \cdot (2 \cdot f_{sd} - 0,85 \cdot f_{cd})}{2 \cdot h_c \cdot 0,85 \cdot f_{cd} + 2 \cdot h \cdot (2 \cdot f_{yd} - 0,85 \cdot f_{cd})}$$

where $A_{s,n}$ is the reinforcement area within h_n. Because it is at this point unknown, it is assumed to be (initial guess):

$$A_{s,n} = 0 \text{ cm}^2$$

Thus, for the case when the axial force is equal to zero, h_n is:

$$h_n = \frac{1660 - 0}{2 \cdot 30 \cdot 0,85 \cdot 2,67 + 2 \cdot 30 \cdot (2 \cdot 35,5 - 0,85 \cdot 2,67)} = 0,39 \text{ cm}$$

Since in this region there is no reinforcement, the assumption is correct.

Plastic section moduli in region 2·h_n

Structural steel

$$W_{pl,z,a,n} = h \cdot h_n^2 = 30 \cdot 0,39^2 = 4,56 \text{ cm}^3$$

Reinforcement

$$W_{pl,z,s,n} = \sum_i A_{sn,i} \cdot |y_i| = 0$$

Concrete

$$W_{pl,z,c,n} = (h_c - h) \cdot h_n^2 - W_{s,0} = (30 - 30) \cdot 0,39^2 - 0 = 0$$

The design plastic resistance moment of the composite section, $M_{pl,z,Rd}$, is:

$$M_{pl,z,Rd} = M_{\max,z,Rd} - M_{n,z,Rd}$$

where:

$$M_{n,z,Rd} = W_{pl,z,a,n} \cdot f_{yd} + W_{pl,z,s,n} \cdot f_{sd} + \frac{W_{pl,z,c,n} \cdot f_{cd}}{2}$$

$$M_{n,z,Rd} = (4,56 \cdot 35,5 + 0 \cdot 40,0 + \frac{0 \cdot 0,85 \cdot 2,67}{2}) \cdot 10^{-2} = 1,62 \text{ kNm}$$

The maximum design plastic resistance moment is:

$$M_{\max,z,Rd} = W_{pl,z,a} \cdot f_{yd} + 0,5 \cdot W_{pl,z,c} \cdot 0,85 \cdot f_{cd} + W_{pl,z,s} \cdot f_{sd}$$

$$M_{\max,z,Rd} = (870,1 \cdot 35,5 + 0,5 \cdot 5684 \cdot 0,85 \cdot 2,67 + 196 \cdot 43,5) \cdot 10^{-2}$$

$$M_{\max,z,Rd} = 459 \text{ kNm}$$

The design plastic resistance moment of the composite section, $M_{pl,z,Rd}$, is:

$$M_{pl,z,Rd} = M_{\max,z,Rd} - M_{n,z,Rd}$$

$$M_{pl,z,Rd} = 459 - 1,62 = 457 \text{ kNm}$$

The obtained values are the exact values on the interaction curve and they are shown in Figure C5.24.

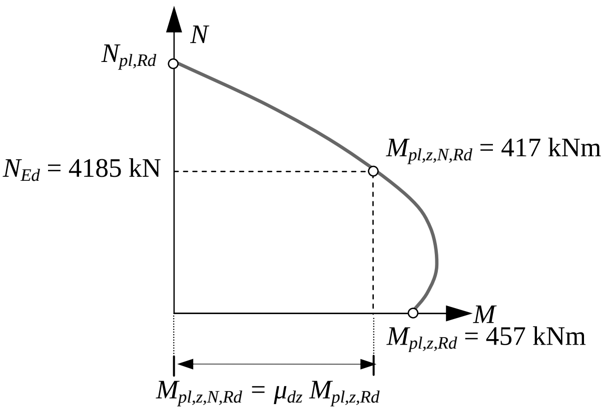


Figure C5.24 *Interaction curve*

The value of μ_{dz} is:

$$\mu_{dz} = \frac{M_{pl,z,N,Rd}}{M_{pl,z,Rd}} = \frac{417}{457} = 0,91 < 1,0$$

11.2.2.3 Interaction polygon

Point A

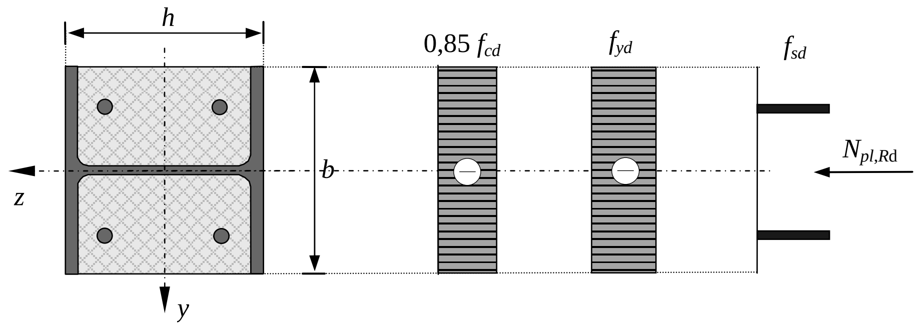


Figure C5.25 *Stress distributions for point A on interaction polygon*

At point *A*, only the design plastic resistance of the cross-section is taken into account:

$$N_{pl,Rd} = A_a \cdot f_{yd} + 0,85 \cdot A_c \cdot f_{cd} + A_s \cdot f_{sd}$$

The design plastic resistance of composite cross-section to compression is:

$$N_{pl,Rd} = 149,1 \cdot 35,5 + 0,85 \cdot 731,4 \cdot 2,67 + 19,6 \cdot 43,5$$

$$N_{pl,Rd} = 7806 \text{ kN}$$

Point D

The position of the plastic neutral axis and the stress distributions are shown in Figure C5.26.

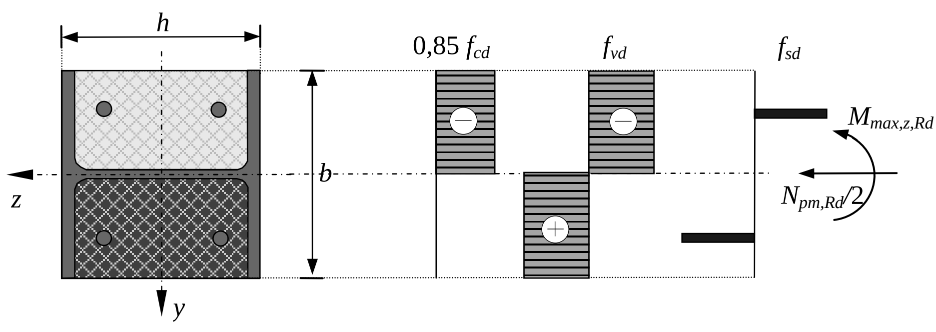


Figure C5.26 *Stress distributions for point D on the interaction polygon*

The maximum design plastic resistance moment is determined as:

$$M_{\max,z,Rd} = M_{pl,z,a,Rd} + M_{pl,z,c,Rd} + M_{pl,z,s,Rd}$$

The maximum design plastic resistance moment, $M_{max,z,Rd}$, at point *D* is:

$$M_{\max,z,Rd} = W_{pl,z,a} \cdot f_{yd} + 0,5 \cdot W_{pl,z,c} \cdot 0,85 \cdot f_{cd} + W_{pl,z,s} \cdot f_{sd}$$

$$M_{\max,z,Rd} = (870,1 \cdot 35,5 + 0,5 \cdot 5684 \cdot 0,85 \cdot 2,67 + 196 \cdot 43,5) \cdot 10^{-2}$$

$$M_{\max,z,Rd} = 459 \text{ kNm}$$

The design value of the resistance of the concrete to compression, $N_{pm,Rd}$, is:

$$N_{pm,Rd} = A_c \cdot 0,85 \cdot f_{cd} = 731,4 \cdot 0,85 \cdot 2,67 = 1660 \text{ kN}$$

The design axial force at the point of maximum design plastic resistance moment is $0,5 \cdot N_{pm,Rd}$, and is therefore:

$$0,5 \cdot N_{pm,Rd} = 0,5 \cdot 1660 = 830 \text{ kN}$$

Point C

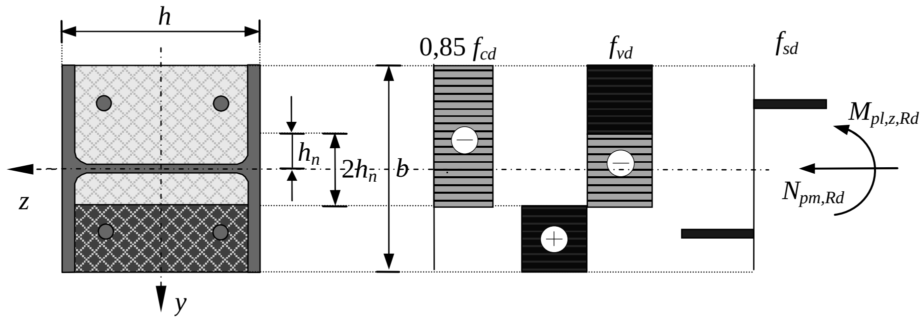


Figure C5.27 *Stress distributions for point C on the interaction polygon*

Calculation of the design plastic resistance moment of the composite section, $M_{pl,z,Rd}$, is carried out as shown below.

When the design axial force is equal to zero, the plastic neutral axis lies within the web of the steel section, $h_n \le t_w/2$, and its position is:

$$h_n = \frac{N_{pm,Rd} - A_{s,n} \cdot (2 \cdot f_{sd} - 0{,}85 \cdot f_{cd})}{2 \cdot h_c \cdot 0{,}85 \cdot f_{cd} + 2 \cdot h \cdot (2 \cdot f_{yd} - 0{,}85 \cdot f_{cd})}$$

where $A_{s,n}$ is the reinforcement area within h_n. Because it is at this point unknown, it is assumed to be (initial guess):

$$A_{s,n} = 0 \text{ cm}^2$$

Thus, for the case when the axial force is equal to zero, h_n is:

$$h_n = \frac{1660 - 0 \cdot (2 \cdot 43{,}5 - 0{,}85 \cdot 2{,}67)}{2 \cdot 30{,}0 \cdot 0{,}85 \cdot 2{,}67 + 2 \cdot 30{,}0 \cdot (2 \cdot 35{,}5 - 0{,}85 \cdot 2{,}67)} = 0{,}39 \text{ cm}$$

Since in this region there is no reinforcement, the assumption is correct.

Plastic section moduli in region 2·*h*$_n$

Structural steel

$$W_{pl,z,a,n} = h \cdot h_n^2 = 30{,}0 \cdot 0{,}39^2 = 4{,}56 \text{ cm}^3$$

Reinforcement

$$W_{pl,z,s,n} = 0 \text{ cm}^3$$

Concrete

$$W_{pl,z,c,n} = (h_c - h) \cdot h_n^2 - W_{pl,z,s,n} = (30{,}0 - 30{,}0) \cdot 0{,}39^2 - 0 = 0 \text{ cm}^3$$

The design plastic resistance moment of the composite section, $M_{pl,z,Rd}$, is calculated as follows:

$$M_{pl,z,Rd} = M_{\max,z,Rd} - M_{n,z,Rd}$$

where:

$$M_{n,z,Rd} = W_{pl,z,a,n} \cdot f_{yd} + W_{pl,z,s,n} \cdot f_{sd} + \frac{W_{pl,z,c,n} \cdot f_{cd}}{2}$$

$$M_{n,z,Rd} = (4{,}56 \cdot 35{,}5 + 0 \cdot 43{,}5 + \frac{0 \cdot 0{,}85 \cdot 2{,}67}{2}) \cdot 10^{-2} = 1{,}62 \text{ kNm}$$

The design plastic resistance moment of the composite section, $M_{pl,z,Rd}$, is:

$$M_{pl,z,Rd} = 459 - 1{,}62 = 457 \text{ kNm}$$

Point B

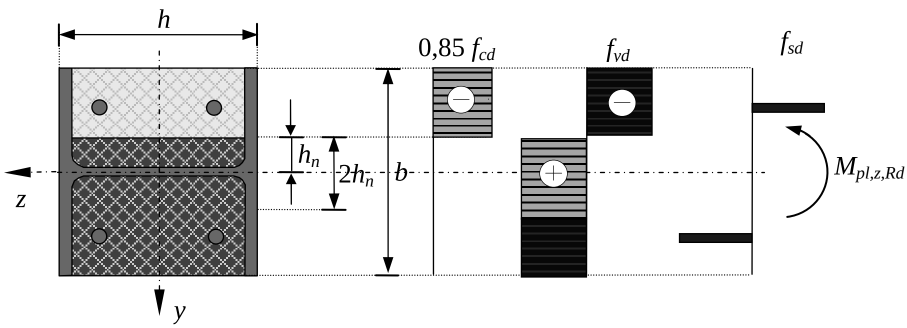


Figure C5.28 *Stress distributions for point B on the interaction polygon*

The design value of $M_{pl,z,Rd}$ has previously been calculated in order to define point *C* on the *N–M* interaction polygon:

$$M_{pl,z,Rd} = 457 \text{ kNm}$$

Previously calculated values at points *A* to *D* should be plotted to produce the *N–M* interaction polygon (Figure 6.19, EN 1994-1-1). The interaction polygon *ACDB* is shown in Figure C5.29.

According to the interaction polygon *AECDB*, Figure C5.29, the following value $M_{pl,z,N,Rd}$ is obtained:

$$M_{pl,z,Rd} : M_{pl,z,N,Rd} = (N_{pl,Rd} - N_{pm,Rd}) : (N_{pl,Rd} - N_{Ed})$$

$$M_{pl,z,N,Rd} = \frac{N_{pl,Rd} - N_{Ed}}{N_{pl,Rd} - N_{pm,Rd}} \cdot M_{pl,z,Rd}$$

$$M_{pl,z,N,Rd} = \frac{7806 - 4185}{7806 - 1660} \cdot 457 = 269 \text{ kNm}$$

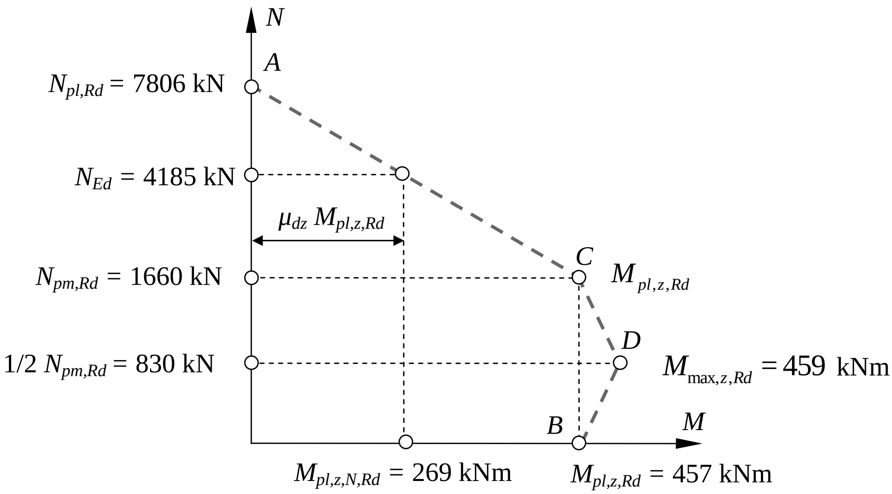


Figure C5.29 *N–M interaction polygon*

The value of μ_{dz} is:

$$\mu_{dz} = \frac{M_{pl,z,N,Rd}}{M_{pl,z,Rd}} = \frac{269}{457} = 0{,}59 < 1{,}0$$

Remark:

According to the approach with the interaction curve, Section 11.2.2.2, the value of μ_{dy} is 0,91. According to the approach with the interaction polygon, the value of μ_{dy} is 0,59. This value is too conservative. Accordingly, it is recommended to find an intermediate point, *E*, for better polygonal approximation to the interaction curve, see example C4. EN 1994-1-1 does not give this recommendation.

The check is carried out by the factor $\mu_{dz} = 0{,}59$, in accordance with EN 1994-1-1.

11.2.3 Calculation of the action effects about the y-y axis

In accordance with the calculation given in Section 11.1.3.2, the value of the design bending moment at mid-height of the column, excluding the equivalent member imperfection, $e_{0,z}$, but including the second-order effects, is:

$$M_{y,Ed,II} = M_{y,Ed} \cdot k_1 = 189 \cdot 0{,}79 = 149 \text{ kNm}$$

11.2.4 Calculation of the action effects about the z-z axis

11.2.4.1 General

According to clause 6.7.3.4 (3), EN 1994-1-1, which refers to clause 5.2.1(3), EN 1994-1-1, second-order effects can therefore be neglected if the load factor α_{cr}, which is the ratio between the elastic critical load and the corresponding applied loading, for elastic instability of the member exceeds 10.

To calculate α_{cr}, the ends of the column are assumed to be pinned, and α_{cr} is found using the Euler formula for the elastic critical force $N_{cr,z,eff}$.

$$N_{cr,z,eff} = \frac{\pi^2 (EI)_{eff,z,II}}{L_{e,z}^2} \quad L_{e,z} = L$$

The design value of the effective flexural stiffness $(EI)_{eff,z,II}$, used to determine the internal forces and moments by second-order analysis, according to clause 6.7.3.4(2), EN 1994-1-1, is defined by the following expression:

$$(EI)_{eff,z,II} = K_0 \cdot (E_a \cdot I_{z,a} + E_s \cdot I_{z,s} + K_{e,II} \cdot E_{cm} \cdot I_{z,c})$$

where:

$K_{e,II}$ is a correction factor, which should be taken as 0,5,
K_0 is a calibration factor, which should be taken as 0,9.

The value $E_{c,eff}$ has been used in place of E_{cm} in the expression for $(EI)_{eff,z,II}$ in order to take into account the long-term effects, in the same way as calculated in Section 7. Accordingly, the value of $E_{c,eff}$ is:

$$E_{c,eff} = 1530 \text{ kN/cm}^2$$

The design value of the effective flexural stiffness $(EI)_{eff,z,II}$, is:

$$(EI)_{eff,z,II} = K_0 \cdot (E_a \cdot I_{z,a} + E_s \cdot I_{z,s} + K_{e,II} \cdot E_{c,eff} \cdot I_{z,c})$$

$$(EI)_{eff,z,II} = 0,9 \cdot (21000 \cdot 8563 + 21000 \cdot 1964 + 0,5 \cdot 1530 \cdot 56973)$$

$$(EI)_{eff,z,II} = 238,19 \cdot 10^{-6} \text{ kNcm}^2$$

The elastic critical force, $N_{cr,z,eff}$, for the pin-ended column, is:

$$N_{cr,z,eff} = \frac{\pi^2 (EI)_{eff,z,II}}{L_{e,z}^2}$$

$$N_{cr,z,eff} = \frac{238{,}19 \cdot 10^6 \cdot \pi^2}{450^2} = 11609 \text{ kN}$$

To check whether the effects of second-order analysis can be neglected, the value of α_{cr} must be higher than 10:

$$\alpha_{cr} = \frac{N_{cr,z,eff}}{N_{Ed}} = \frac{11609}{4185} = 2{,}8 < 10$$

The value of α_{cr} is less than 10, so the second-order effects must be considered.

11.2.4.2 Bending moments about the z-z axis

According to clause 6.7.3.4(5), EN 1994-1-1, the second-order effects can be calculated by multiplying the greatest first-order design bending moments by a factor *k*.

Thus, the second-order effects may be considered according to the expression:

$$M_{z,Ed,II} = M_{z,Ed,I} \cdot k$$

The factor *k* is given by:

$$k = \frac{\beta}{1 - N_{Ed} / N_{cr,z,eff}} \geq 1{,}0$$

where:

β is an equivalent moment factor given in Table 6.4, EN 1994-1-1,

$N_{cr,z,eff}$ is the critical axial force, about the *z-z* axis, obtained with the effective flexural stiffness $(EI)_{eff,z,II}$ and with the effective length taken as the physical length of the column.

The design bending moment from the member imperfections is determined as:

$$M_{z,Ed,imp} = N_{Ed} \cdot e_{0,y}$$

where:

N_{Ed} is the design value of the axial force,

$e_{0,y}$ is the equivalent member imperfection which is given in Table 6.5, EN 1994-1-1, depending on the buckling curve.

Remark:

According to Table 6.5, EN 1994-1-1, composite columns with partially concrete-encased section can be designed using buckling curve *c* for *z-z* axis of buckling.

Therefore, for the buckling curve *c*, the equivalent member imperfection is:

$$e_{0,y} = \frac{L}{150}$$

$$e_{0,y} = \frac{450}{150} = 3{,}0 \ \text{cm}$$

The design bending moments calculated according to first-order analysis are shown in Figure C5.30.

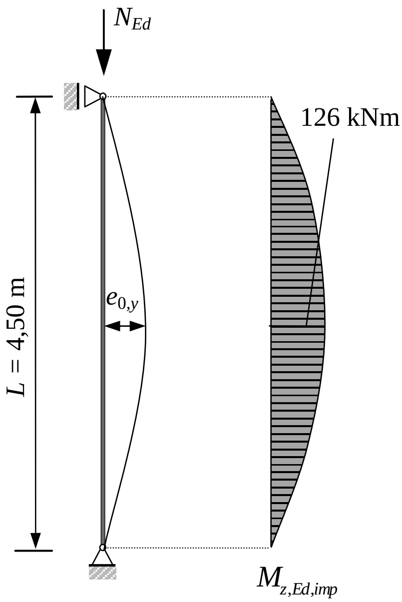


Figure C5.30 *First-order bending moments, design values*

The design values of bending moments are as follows:

The design bending moment at the top of the column is:

$$M_{z,Ed} = 0 \text{ kNm}$$

The design bending moment at the bottom of the column is:

$$M_{z,Ed} = 0 \text{ kNm}$$

The design bending moment due to imperfection is:

$$M_{z,Ed,imp} = N_{Ed} \cdot e_{0,y} = 4185 \cdot 0{,}03 = 126 \text{ kNm}$$

Remark:

The factor β from Table 6.4, EN 1994-1-1, allows for the shape of the bending moment diagram. When bending is caused by lateral loading on the column, the value of factor β is 1,0. For a column subjected to end moments, the factor β is calculated as:

$$\beta_1 = 0{,}66 + 0{,}44 \cdot r \geq 0{,}44$$

where r is the ratio of the end moments on the ends of the column ($-1 \leq r \leq +1$).

For the moment from the member imperfection, the factor k is given by:

$$k = \frac{\beta}{1 - N_{Ed} / N_{cr,z,eff}}$$

where β is the equivalent moment factor.

The equivalent moment factor β is:

$$\beta = 1{,}0$$

Therefore, the factor k is:

$$k = \frac{\beta}{1 - N_{Ed} / N_{cr,z,eff}} = \frac{1{,}0}{1 - 4185 / 11609} = 1{,}56 > 1{,}0$$

The design bending moment at mid-height, with second-order effects taken into account, is:

$$M_{z,Ed,II} = M_{z,Ed,imp} \cdot k = 126 \cdot 1{,}56 = 197 \text{ kNm}$$

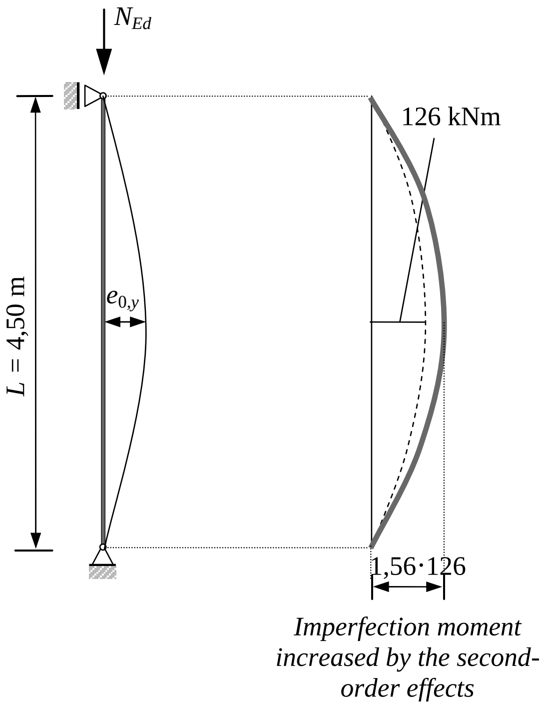


Figure C5.31 *Second-order bending moments about the z-z axis, design values*

The design bending moments calculated according to second-order analysis are shown in Figure C5.31.

The check is performed with the bending moment at mid-height:

$$M_{z,Ed,II} = M_{z,\max} = 197 \text{ kNm}$$

11.2.4.3 Shear forces parallel to the y-y axis

According to clause 6.7.3.4(5), EN 1994-1-1, second-order effects can be allowed for by multiplying the greatest first-order design bending moment by a factor k given by:

$$k = \frac{\beta}{1 - N_{Ed} / N_{cr,eff}} \geq 1{,}0$$

Accordingly, the approximate value of shear force can be obtained as:

$$V_{Ed,II} = V_{Ed} \cdot k$$

In accordance with Figure C5.32, the first-order design shear force at the bottom of the column is:

$$V_{y,Ed} = \frac{4 \cdot N_{Ed} \cdot e_{0,y}}{L} = \frac{4 \cdot 4185 \cdot 0,03}{4,5} = 112 \text{ kN}$$

In accordance with Figure C5.32, the first-order design shear force at the top of the column is:

$$V_{y,Ed} = \frac{4 \cdot N_{Ed} \cdot e_{0,y}}{L} = \frac{4 \cdot 4185 \cdot 0,03}{4,5} = 112 \text{ kN}$$

The diagram of shear forces, calculated by first-order analysis for the equivalent lateral load due to imperfections, is shown in Figure C5.32.

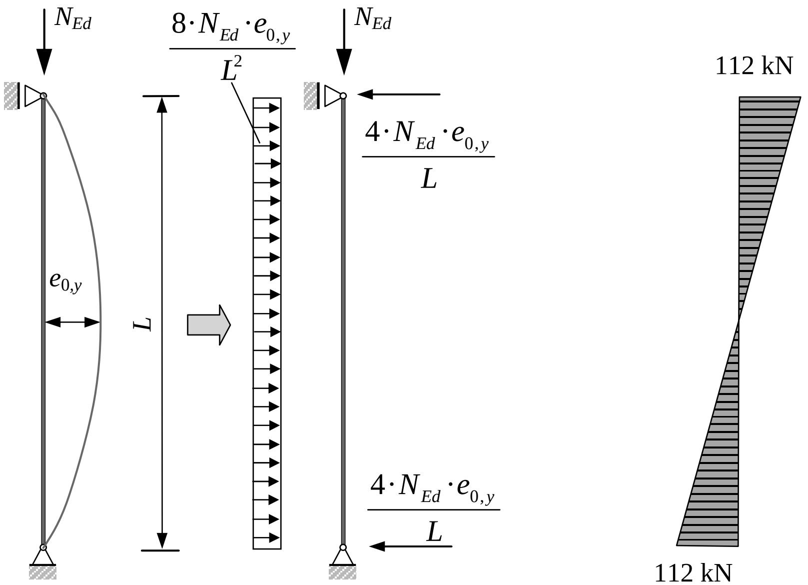


Figure C5.32 *First-order design shear forces parallel to the y-y axis*

The factor k is:

$$k = \frac{1}{1 - N_{Ed} / N_{cr,z,eff}} = \frac{1,0}{1 - 4185 / 11609} = 1,56 > 1,0$$

Therefore, the maximum design shear force, calculated by approximative second-order analysis, is:

$$V_{y,Ed,II} = V_{y,Ed} \cdot k = 112 \cdot 1{,}56 = 175 \text{ kN}$$

11.2.5 Check of the resistance of the member in combined compression and bending about the z-z axis

It is necessary to satisfy the following conditions:

$$\frac{M_{y,Ed}}{\alpha_{M,y} \cdot \mu_{dy} \cdot M_{pl,y,Rd}} \leq 1{,}0.$$ → ($M_{y,Ed}$ – neglect imperfections but use second-order analysis)

$$\frac{M_{z,Ed}}{\alpha_{M,z} \cdot \mu_{dz} \cdot M_{pl,z,Rd}} \leq 1{,}0.$$ → ($M_{z,Ed}$ – consider imperfections and second-order analysis)

$$\frac{M_{y,Ed}}{\mu_{dy} \cdot M_{pl,y,Rd}} + \frac{M_{z,Ed}}{\mu_{dz} \cdot M_{pl,z,Rd}} \leq 1{,}0.$$ → (interaction of $M_{y,Ed}$ and $M_{z,Ed}$)

Substituting previously calculated values gives:

$$\frac{149}{0{,}9 \cdot 0{,}59 \cdot 762} = 0{,}37 < 1{,}0$$

$$\frac{197}{0{,}9 \cdot 0{,}59 \cdot 457} = 0{,}81 < 1{,}0$$

$$\frac{149}{0{,}59 \cdot 762} + \frac{197}{0{,}59 \cdot 457} = 0{,}33 + 0{,}73 = 1{,}06 > 1{,}0$$

This exceeds 1,0 so the check is not satisfied.

Remark:

Therefore, the resistance of composite column to biaxial bending taking into account the design axial force N_{Ed} = 1152 kN does not satisfy the check. The check based on the interaction polygon was carried out in accordance with EN 1994-1-1. This approach is too conservative, see the comparison of the results obtained based on the interaction curve and on the interaction polygon in Section 11.2.2. Therefore, it is recommended to find an intermediate point, *E*, for better polygonal approximation to the interaction curve and a more accurate check of the resistance of the composite column, see example C4.

11.2.6 *Check of the plastic resistance to transverse shear parallel to the y-y axis*

In accordance with clause 6.7.3.2(4), EN 1994-1-1, for simplification V_{Ed} may be assumed to act on the structural steel section alone. According to clause 6.2.6(2), EN 1993-1-1, in the absence of torsion the design plastic shear resistance, $V_{pl,y,a,Rd}$, is given by:

$$V_{pl,y,a,Rd} = \frac{A_{v,y} \cdot (f_y / \sqrt{3})}{\gamma_{M0}}$$

The shear area, $A_{v,y}$, according to clause 6.2.6(3), EN1993-1-1, is calculated as:

$$A_{v,y} = 2 \cdot (b \cdot t_f)$$

where:

b is the width of the flange of the H-section,

t_f is the thickness of the flange of the H-section.

According to clause 6.2.2.4(1), EN 1994-1-1, where the shear force is less than half the plastic shear resistance its effect on the resistance moment can be neglected. Therefore, the condition is:

$$V_{y,Ed} < 0{,}5 \cdot V_{pl,y,a,Rd}$$

The design value of second-order shear force is:

$$V_{y,Ed} = V_{y,Ed,II} = 175 \text{ kN}$$

The shear area, $A_{v,y}$, is:

$$A_{v,y} = 2 \cdot (30{,}0 \cdot 1{,}9) = 114 \text{ cm}^2$$

The design plastic shear resistance, $V_{pl,a,y,Rd}$, is:

$$V_{pl,y,a,Rd} = \frac{A_{v,y} \cdot (f_y / \sqrt{3})}{\gamma_{M0}}$$

$$V_{pl,y,a,Rd} = \frac{114 \cdot (35,5/\sqrt{3})}{1,0} = 2337 \text{ kN}$$

Check:

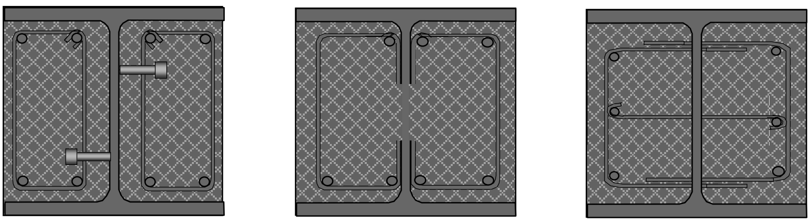
$$V_{y,Ed,II} = 175 < 0,5 \cdot V_{pl,a,y,Rd} = 0,5 \cdot 2337 = 1169 \text{ kN}$$

The condition is satisfied and there is no reduction in the resistance moment.

12. Check of the longitudinal shear outside the area of load introduction

The check of longitudinal shear outside the area of load introduction will not be carried out in this example. The shear resistance, developing at the interface between the concrete and the inner walls of the steel section, is provided by means of the appropriate structural details. The different structural details are shown in Figure C.5.33:

a) closed stirrups and studs welded to the web of the H-section,
b) stirrups welded to the web of the H-section,
c) bars and stirrups that pass through holes in the web of the H-section.



a) *b)* *c)*

Figure C5.33 *Resistance to longitudinal shear provided by different structural details*

13. Check of the load introduction

13.1 Load introduction for combined compression and bending

The beams of the composite floor are attached to the column. The characteristic values of shear forces from beams, which act on the column, see Figure C5.1, are:

$$N_{G_{2,k}} = 300 \text{ kN}$$

$$N_{Q_{2,k}} = 150 \text{ kN}$$

The design values of the effects of the actions are:

$$N_{Ed} = 1{,}35 \cdot N_{G_{2,k}} + 1{,}50 \cdot N_{Q_{2,k}}$$

$$N_{Ed} = 1{,}35 \cdot 300 + 1{,}50 \cdot 150 = 630 \text{ kN}$$

$$M_{y,Ed} = 1{,}35 \cdot N_{G_{2,k}} \cdot 0{,}3 + 1{,}50 \cdot N_{Q_{2,k}} \cdot 0{,}3$$

$$M_{y,Ed} = 1{,}35 \cdot 300 \cdot 0{,}3 + 1{,}50 \cdot 150 \cdot 0{,}3 = 189 \text{ kNm}$$

Load introduction for combined compression and bending is shown in Figure C5.34.

The load is applied through the joint attached to the steel section. The load is distributed between the individual components of the cross-section in proportion to their resistances. The distribution of the load into the components acting on the steel section, $N_{a,Rd}$ and $M_{a,Rd}$, and the reinforced concrete web encasement, $N_{c+s,Rd}$ and $M_{c+s,Rd}$, is determined as:

$$M_{a,Ed} = M_{Ed} \cdot \frac{M_{pl,a,Rd}}{M_{pl,Rd}}$$

$$M_{c+s,Ed} = M_{Ed} - M_{a,Ed}$$

$$N_{a,Ed} = N_{Ed} \cdot \frac{N_{pl,a,Rd}}{N_{pl,Rd}}$$

$$N_{c+s,Ed} = N_{Ed} - N_{a,Ed}$$

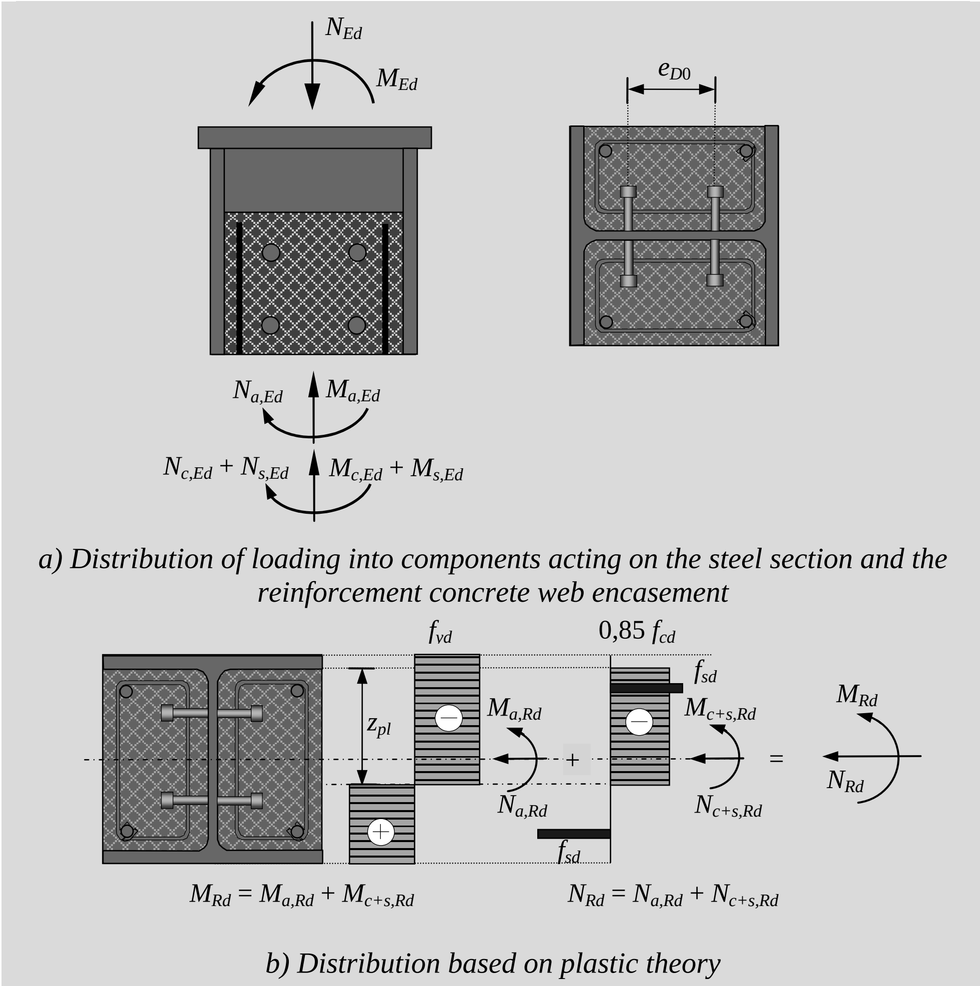


Figure C5.34 *Load introduction for combined compression and bending*

The design plastic resistance moment of the steel section, with the value $W_{pl,y,a,n}$ calculated in Section 11.1.2.3, Figure C5.14, is:

$$M_{pl,y,a,Rd} = (W_{pl,y,a} - W_{pl,y,a,n}) \cdot f_{yd} = (1869 - 36,7) \cdot 35,5 \cdot 10^{-2} = 650 \text{ kNm}$$

The design plastic resistance moment of the composite section, calculated in Section 11.1.2.3, Figure C5.16, is:

$$M_{pl,y,Rd} = 762 \text{ kNm}$$

Therefore:

$$\frac{M_{pl,y,a,Rd}}{M_{pl,y,Rd}} = \frac{650}{762} = 0,85$$

$$\frac{N_{pl,a,Rd}}{N_{pl,Rd}} = \frac{149,1 \cdot 35,5}{7806} = \delta = 0,68$$

Accordingly, the load acting on the steel section is:

$$M_{y,a,Ed} = M_{y,Ed} \cdot \frac{M_{pl,y,a,Rd}}{M_{pl,y,Rd}} = 189 \cdot 0,85 = 161 \text{ kNm}$$

$$N_{a,Ed} = N_{Ed} \cdot \frac{N_{pl,a,Rd}}{N_{pl,Rd}} = 630 \cdot 0,68 = 428 \text{ kN}$$

The load acting on the reinforced concrete web encasement is:

$$M_{c+s,Ed} = M_{y,Ed} - M_{y,a,Ed} = 189 - 161 = 28 \text{ kN}$$

$$N_{c+s,Ed} = N_{Ed} - N_{a,Ed} = 630 - 428 = 202 \text{ kN}$$

13.2 Calculation of the stud resistance

The headed stud shear connectors, with the diameter of the shank of the stud d = 19 mm and with the ultimate tensile strength f_u = 450 N/mm^2, are selected. The design resistance of a single-headed shear connector, automatically welded in accordance with EN 14555, should be determined as the smaller of:

$$P_{Rd} = \min\,(P_{Rd}^{(1)}, P_{Rd}^{(2)})$$

The design resistance $P_{Rd}^{(1)}$ is:

$$P_{Rd}^{(1)} = 0,8 \cdot f_u \cdot \frac{\pi \cdot d^2}{4} \cdot \frac{1}{\gamma_v} = 0,8 \cdot 45,0 \cdot \frac{\pi \cdot 1,9^2}{4} \cdot \frac{1}{1,25} = 81,7 \text{ kN}$$

The design resistance $P_{Rd}^{(2)}$ is:

$$P_{Rd}^{(2)} = 0,29 \cdot \alpha \cdot d^2 \cdot \sqrt{E_{cm} \cdot f_{ck}} \cdot \frac{1}{\gamma_v}$$

The overall nominal height of the stud h_{sc} = 100 mm so:

$$\frac{h_{sc}}{d} = \frac{100}{19} = 5,26$$

For $\frac{h_{sc}}{d} = 5,26 > 4$, α = 1,0, so the design resistance $P_{Rd}^{(2)}$ is:

$$P_{Rd}^{(2)} = 0,29 \cdot 1,0 \cdot 1,9^2 \cdot \sqrt{3500 \cdot 4,0} \cdot \frac{1}{1,25} = 99,1 \text{ kN}$$

Therefore, the governed design resistance is:

$$P_{Rd} = P_{Rd}^{(1)} = 81,7 \text{ kN}$$

The studs are welded to each side of the web into two rows. The number of studs within the load introduction length is n = 8 studs, with spacing e = 15 cm, see Figure C5.35.

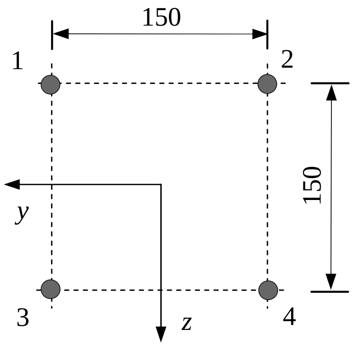


Figure C5.35 *Arrangement of the studs*

The adjacent steel flanges prevent the lateral expansion of the concrete. Due to the prevention of the lateral expansion of the concrete, the frictional forces are developed. This resistance can be added to the design resistance of the studs. This additional resistance is determined by the expression $\mu \cdot P_{Rd}/2$ on each flange and each horizontal row of studs as shown in Figure 6.21, EN 1994-1-1. For steel sections without painting, μ can be taken as 0,5.

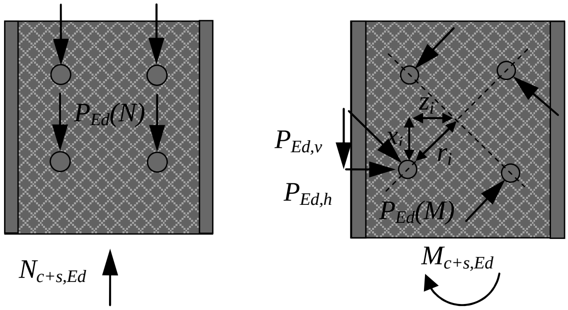
$$\frac{\mu \cdot P_{Rd}}{2} = \frac{0,5 \cdot 81,7}{2} = 20,4 \text{ kN on flange}$$

This additional resistance and the resistance of a single stud give the total design resistance, which is:

$$P_{Rd} = 81,7 + 20,4 = 102,1 \text{ kN}$$

13.3 Calculation of the shear forces on the studs based on elastic theory

The procedure based on elastic theory is shown in Figure C5.36.



$$\boxed{\max P_{Ed} = \sqrt{[\frac{N_{c+s,Ed}}{n} + \frac{M_{c+s,Ed}}{\Sigma r_i^2} \cdot x_i]^2 + [\frac{M_{c+s,Ed}}{\Sigma r_i^2} \cdot z_i]^2}}$$

Figure C5.36 *Shear forces on the studs based on elastic theory*

In accordance with Figure C5.36, the maximum design shear force for the single stud is:

$$\max P_{Ed} = \sqrt{[\frac{N_{c+s,Ed}}{n} + \frac{M_{c+s,Ed}}{\Sigma r_i^2} \cdot x_i]^2 + [\frac{M_{c+s,Ed}}{\Sigma r_i^2} \cdot z_i]^2}$$

According to Figures C5.35 and C5.36 we have:

$$x_i = z_i = 75 \text{ mm i } r_i = \sqrt{75^2 + 75^2} = 106 \text{ mm}$$

The maximum design shear force, $\max P_{Ed}$, is:

$$\max P_{Ed} = \sqrt{[\frac{202}{8} + \frac{28 \cdot 10^3}{8 \cdot 106^2} \cdot 75]^2 + [\frac{28 \cdot 10^3}{8 \cdot 106^2} \cdot 75]^2} = 53,9 \text{ kN}$$

The following condition must be satisfied:

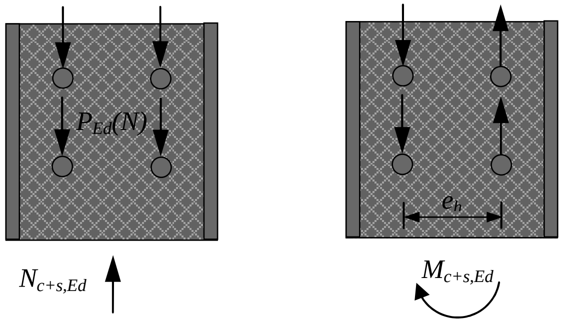
$$\frac{\max P_{Ed}}{P_{Rd}} \leq 1,0$$

$$\frac{\max P_{Ed}}{P_{Rd}} = \frac{53,9}{102,1} = 0,53$$

Since 0,53 < 1,0, the condition is satisfied.

13.4 Calculation of the shear forces on the studs based on plastic theory

The procedure based on plastic theory is shown in Figure C5.37.



$$\boxed{\max P_{Ed} = \frac{N_{c+s,Ed}}{n} + \frac{M_{c+s,Ed}}{e_h \cdot 0,5 \cdot n}}$$

Figure C5.37 *Shear forces on the studs based on plastic theory*

In accordance with Figure C5.37, the maximum design shear force for the single stud is:

$$\max P_{Ed} = \frac{N_{c+s,Ed}}{n} + \frac{M_{c+s,Ed}}{e_h \cdot 0,5 \cdot n} = \frac{202}{8} + \frac{28}{0,15 \cdot 0,5 \cdot 8} = 71,9 \text{ kN}$$

The following condition must be satisfied:

$$\frac{\max P_{Ed}}{P_{Rd}} \leq 1,0$$

$$\frac{\max P_{Ed}}{P_{Rd}} = \frac{71,9}{102,1} = 0,70$$

Since 0,70 < 1,0, the condition is satisfied.

14. Commentary

This example illustrates the design of the composite column with partially concrete-encased H-section subject to axial compressive load and bending moment. However, the bending moment about the *z-z* axis was caused by taking into account the equivalent member imperfection about the *z-z* axis. This means that the check of resistance should be carried out for combined compression and biaxial bending. Accordingly, the following checks are needed:

a) The verification of the column resistance in axial compression only is carried out as the preliminary check. Since $\bar{\lambda}_z = 0{,}81 > \bar{\lambda}_y = 0{,}55$, the buckling resistance about the *z-z* axis is governed. The check of the composite column subjected to axial compression is satisfied. It is not necessary to select the stronger cross-section.

The utilization is 82%.

b) Check for bending about the *y-y* axis: The next step is to carry out the check of the column resistance in combined compression and uniaxial bending. The equivalent member imperfection $e_{0,z}$ is taken into account, which is in the same plane as the initial moment. In addition it was found that the second-order effects must be allowed for. The final step is to check that the cross-section can resist $M_{y,Ed}$ (consider imperfections and second-order analysis) with compression N_{Ed}.

The utilization is 64%.

c) Check for bending about the *z-z* axis: Finally, the check of the column resistance in combined compression and biaxial bending is carried out. The design bending moment about the *y-y* axis, $M_{y,Ed}$, is calculated neglecting the equivalent member imperfection. About the *z-z* axis, the design bending moment due to the equivalent member imperfection $M_{z,Ed}$ ($=N_{Ed} \cdot e_{0,y}$) is taken into account. In addition it was found that the second-order effects must be allowed for. The final step is to check that the cross-section can resist $M_{y,Ed}$ (neglect imperfections but use second-order analysis) and $M_{z,Ed}$ (consider imperfections and second-order analysis) with compression N_{Ed}.

The utilization is:

37% ($M_{y,Ed}$ neglect imperfections but use second-order analysis),
81% ($M_{z,Ed}$ consider imperfections and second-order analysis),
1,06% (interaction of $M_{y,Ed}$ and $M_{z,Ed}$).

Since 1,06 > 1,0, the resistance of the composite column to biaxial bending taking into account the design axial force N_{Ed} = 1152 kN does not satisfy the check. The check based on the interaction polygon was carried out in accordance with EN 1994-1-1. This approach is too conservative, see the comparison of the results obtained based on the interaction curve (μ_{dy} = 0,67 and μ_{dz} = 0,91) and on the interaction polygon (μ_{dy} = 0,59 and μ_{dz} = 0,59), Section 11.2.2. Therefore, it is recommended to find an intermediate point, *E*, for better polygonal approximation to the interaction curve and the more accurate check of resistance of the composite column. EN 1994-1-1 does not state this recommendation.

C6 Composite column with fully concrete-encased H-section subject to axial compression and biaxial bending

1. Purpose of example

This example demonstrates the design of a composite column with fully concrete-encased H-section subject to axial compressive load and biaxial bending. The composite column consists of an H-column section with square concrete section encasement. Additional reinforcement is placed in the concrete cover around the steel section. The check of the composite column in combined compression and biaxial bending is carried out in several steps, as described below.

The resistance to axial compression is carried out separately for each axis. In general, based on the obtained result, it is possible to estimate which of the axes is more likely to fail.

In the next step, it is necessary to check the column resistance under compression and uniaxial bending individually in each of the planes of bending. Then, it is necessary to check the column resistance in biaxial bending taking into account imperfections in the plane in which failure is expected to occur. For the other plane of bending the effect of imperfections is neglected. If it is not obvious which plane is the more critical, checks should be made for both planes.

The verification of the considered composite column is carried out in accordance with clause 6.7.3, EN 1994-1-1.

2. Static system, cross-section and design action effects

Actions

Design action effects

Axial force:

$N_{Ed} = 1800$ kN (the total design axial force)

$N_{G,Ed} = 1200$ kN (the design axial force due to the permanent load)

Bending moments at the top of the column:

Composite Structures according to Eurocode 4. Worked Examples.
First Edition. Darko Dujmović, Boris Androić, Ivan Lukačević.

$$M_{y,Ed} = 380 \text{ kNm}$$

$$M_{z,Ed} = 50 \text{ kNm}$$

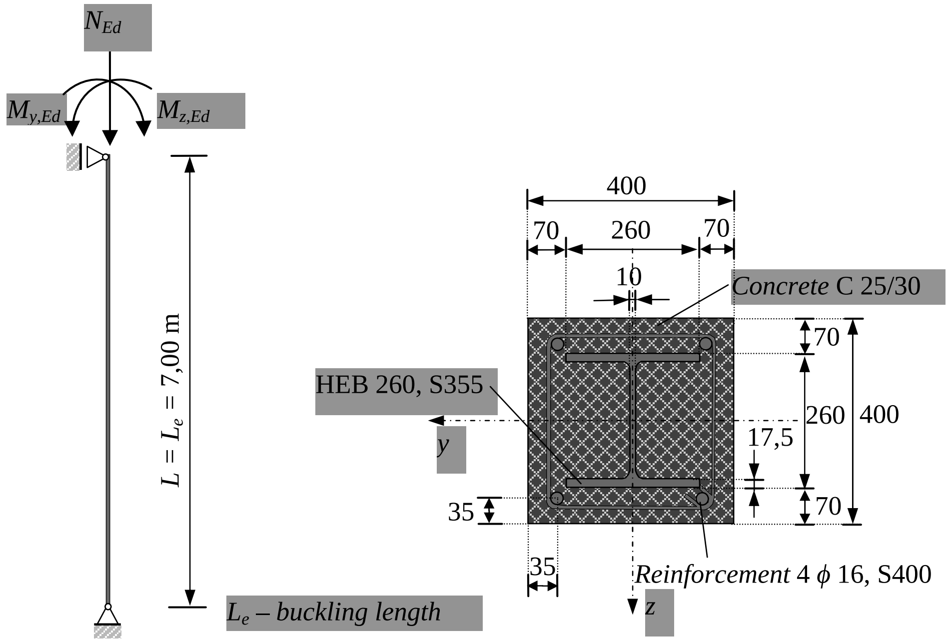


Figure C6.1 *Static system and cross-section (bending about the y-y axis and the z-z axis)*

Denotation of imperfections

Imperfection about the *y-y* axis is denoted with $e_{0,z}$. Imperfection about the *z-z* axis is denoted with $e_{0,y}$.

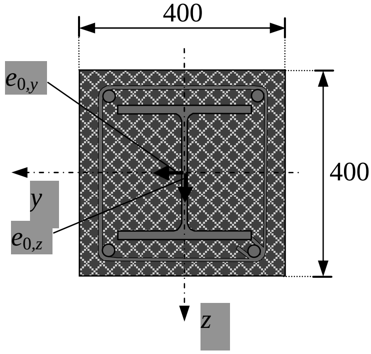


Figure C6.2 *Denotation of imperfections*

3. Properties of materials

Concrete strength class: C 25/30 $f_{ck} = 25 \ \text{N/mm}^2$

$$f_{cd} = \frac{f_{ck}}{\gamma_c} = \frac{25}{1{,}5} = 16{,}7 \ \text{N/mm}^2$$

$$E_{cm} = 31000 \ \text{N/mm}^2$$

Structural steel: S355 $f_{yk} = 355 \ \text{N/mm}^2$

$$f_{yd} = \frac{f_{yk}}{\gamma_M} = \frac{355}{1{,}0} = 355 \ \text{N/mm}^2$$

$$E_a = 210000 \ \text{N/mm}^2$$

Reinforcement: *ductility class B or C* $f_{sk} = 400 \ \text{N/mm}^2$

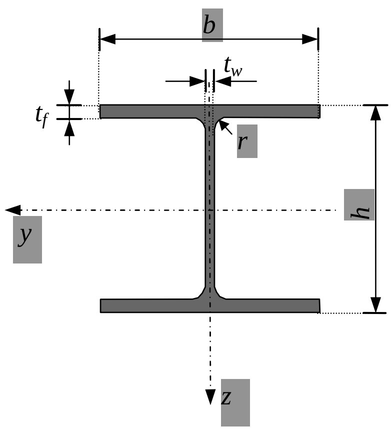
$$f_{sd} = \frac{f_{sk}}{\gamma_s} = \frac{400}{1{,}15} = 348 \ \text{N/mm}^2$$

$$E_s = 210000 \ \text{N/mm}^2$$

4. Geometrical properties of the cross-section

4.1 Selection of the steel cross-section and reinforcement

The cross-section HE 260 B is selected. The selected cross-section is shown in Figure C6.3.



$h = 260 \ \text{mm}$
$b = 260 \ \text{mm}$
$t_f = 17{,}5 \ \text{mm}$
$t_w = 10 \ \text{mm}$
$r = 24 \ \text{mm}$
$A_a = 118{,}4 \ \text{cm}^2$
$I_{y,a} = 14920 \ \text{cm}^4$
$I_{z,a} = 5135 \ \text{cm}^4$
$W_{pl,y,a} = 1283 \ \text{cm}^3$
$W_{pl,z,a} = 602{,}2 \ \text{cm}^3$

Figure C6.3 *Steel cross-section*

The cross-sectional area of the structural steel section HE 260 B is:

$$A_a = 118,4 \text{ cm}^2$$

The cross-sectional area of the reinforcement with four bars of 16 mm diameter is:

d_{bar} = 16 mm, A_{bar} = 2,01 cm^2

$$A_s = 4 \cdot A_{bar} = 4 \cdot 2,01 = 8,04 \text{ cm}^2$$

The cross-sectional area of the concrete is:

$$A_c = b_c \cdot h_c - A_a - A_s$$

$$A_c = 40 \cdot 40 - 118,4 - 8,04$$

$$A_c = 1473,6 \,\text{cm}^2$$

The ratio of reinforcement area to concrete area is:

$$\rho_s = \frac{A_s}{A_c} = \frac{8,04}{1473,6} = 0,005 \qquad \rho_s = 0,5\,\%$$

$$0,3\% < \rho_s = 0,5\% < 6\%$$

The limits of 0,3% in clause 6.7.5.2(1), EN 1994-1-1, and of 6% in clause 6.7.3.1 (1), EN 1994-1-1, on the reinforcement are satisfied.

Remark:

According to clause 6.7.3.1(3), EN 1994-1-1, the ratio of reinforcement area to concrete area, ρ_s, should not exceed 6%.

According to clause 6.7.5.2(1), EN 1994-1-1, the ratio of reinforcement area to concrete area, ρ_s, should be not less 0,3%.

4.2 Cross-sectional areas

Structural steel

$$A_a = 118,4 \text{ cm}^2$$

Reinforcement

$A_s = 8,04 \ \text{cm}^2$

Concrete

$A_c = 1473,6 \ \text{cm}^2$

4.3 Second moments of area

Bending about the y-y axis

Structural steel

$I_{y,a} = 14920 \ \text{cm}^4$

Reinforcement

$I_{y,s} = 4 \cdot A_{bar} \cdot 16,5^2$

$I_{y,s} = 4 \cdot 2,01 \cdot 16,5^2$

$I_{y,s} = 8,04 \cdot 16,5^2 = 2189 \ \text{cm}^4$

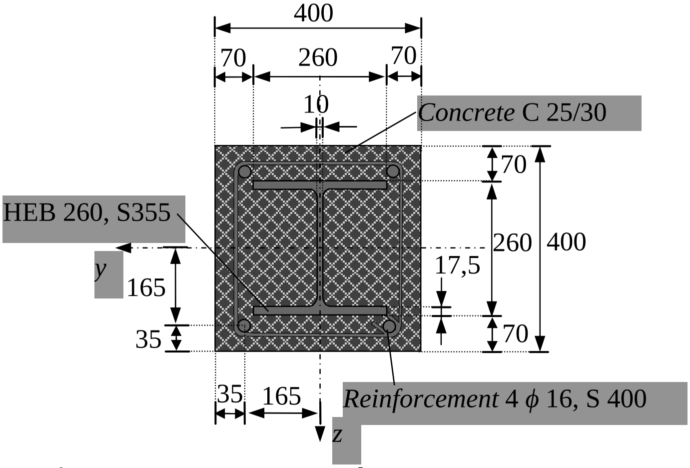


Figure C6.4 *Composite column cross-section*

Concrete

$$I_{y,c} = \frac{b_c \cdot h_c^3}{12} - I_{y,a} - I_{y,s}$$

$$I_{y,c} = \frac{40 \cdot 40^3}{12} - 14920 - 2189$$

$$I_{y,c} = 196224 \ \text{cm}^4$$

Bending about the z-z axis

Structural steel

$$I_{z,a} = 5135 \ \text{cm}^4$$

Reinforcement

$$I_{z,s} = 4 \cdot A_{bar} \cdot 16{,}5^2$$

$$I_{z,s} = 4 \cdot 2{,}01 \cdot 16{,}5^2$$

$$I_{z,s} = 8{,}04 \cdot 16{,}5^2 = 2189 \ \text{cm}^4$$

Concrete

$$I_{z,c} = \frac{h_c \cdot b_c^3}{12} - I_{z,a} - I_{z,s}$$

$$I_{z,c} = \frac{40 \cdot 40^3}{12} - 5135 - 2189$$

$$I_{z,c} = 206009 \ \text{cm}^4$$

4.4 Plastic section moduli

Bending about the y-y axis

Structural steel

$$W_{pl,y,a} = 1283 \ \text{cm}^3$$

Reinforcement

$$W_{pl,y,s} = \sum_i A_{si} \cdot z_i = 4 \cdot 2{,}01 \cdot 16{,}5 = 132{,}7 \ \text{cm}^3$$

Concrete

$$W_{pl,y,c} = \frac{b_c \cdot h_c^2}{4} - W_{pl,y,a} - W_{pl,y,s} = \frac{40,0 \cdot 40,0^2}{4} - 1283 - 132,7 = 14584,3 \text{ cm}^3$$

Bending about the z-z axis

Structural steel

$$W_{pl,z,a} = 602,2 \text{ cm}^3$$

Reinforcement

$$W_{pl,z,s} = \sum_i A_{si} \cdot z_i = 4 \cdot 2,01 \cdot 16,5 = 132,7 \text{ cm}^3$$

Concrete

$$W_{pl,z,c} = \frac{b_c \cdot h_c^2}{4} - W_{pl,y,a} - W_{pl,y,s} = \frac{40,0 \cdot 40,0^2}{4} - 602,2 - 132,7 = 15265,1 \text{ cm}^3$$

5. Steel contribution ratio

According to clause 6.7.3.3(1), EN 1994-1-1, the steel contribution ratio, δ, is defined as:

$$\delta = \frac{A_a \cdot f_{yd}}{N_{pl,Rd}}$$

The term $A_a \cdot f_{yd}$ is the contribution of the structural steel section to the plastic resistance of the composite section to axial force. The design plastic resistance of the composite section to axial force $N_{pl,Rd}$ is calculated according to clause 6.7.3.2(1), EN 1994-1-1.

According to 6.7.1(4), EN 1994-1-1, the steel contribution ratio, δ, must satisfy the following conditions:

$0,2 \le \delta \le 0,9$

If δ is less than 0,2, the column should be designed as a reinforced concrete member according to EN 1992-1-1. If δ is larger than 0,9, the concrete is ignored in the calculations, and the column is designed as a structural steel member according to EN 1993-1-1.

The term $A_a \cdot f_{yd}$ is the contribution of the structural steel section to the plastic resistance of the composite section to axial force:

$$A_a \cdot f_{yd} = 118,4 \cdot 35,5 = 4203 \,\text{kN}$$

The plastic resistance of the composite section to axial force is:

$$N_{pl,Rd} = A_a \cdot f_{yd} + 0,85 \cdot A_c \cdot f_{cd} + A_s \cdot f_{sd}$$

$$N_{pl,Rd} = 118,4 \cdot 35,5 + 0,85 \cdot 1473,6 \cdot 1,67 + 8,04 \cdot 34,8$$

$$N_{pl,Rd} = 6575 \;\text{ kN}$$

According to 6.7.1(4), EN 1994-1-1, the steel contribution ratio, δ, must satisfy the following conditions:

$0,2 \leq \delta \leq 0,9$

The steel contribution ratio, δ, is:

$$\delta = \frac{A_a \cdot f_{yd}}{N_{pl,Rd}} = \frac{118,4 \cdot 35,5}{6575} = 0,64$$

Since the limits $0,2 < \delta = 0,64 < 0,9$, are satisfied, the column can be classified as a composite column and the provisions of EN 1994-1-1 can be used for the dimensioning.

6. Local buckling

For fully encased steel section with the thickness of cover of reinforced concrete greater than 40 mm, or more than 1/6 of the width of the steel flange, the local buckling of steel elements can be neglected.

For the selected section, the thickness of cover of reinforced concrete is 70 mm. This thickness should not be less than the larger of the following two values:

40 mm

$$\frac{b}{6} = \frac{260}{6} = 43,3 \;\text{ mm}$$

The cover thickness, 70 mm, is larger than the required values, and local buckling can be neglected.

7. Effective modulus of elasticity for concrete

For long-term loading the creep and shrinkage are taken into account in design by a reduced flexural stiffness of the composite cross-section. Due to the influence of long-term creep effects on the effective elastic stiffness, the modulus of elasticity of the concrete, E_{cm}, should be reduced to the value $E_{c,eff}$ as:

$$E_{c,eff} = \frac{E_{cm}}{1 + (\frac{N_{G,Ed}}{N_{Ed}})\varphi_t}$$

where:

$\varphi_t = \varphi(t, t_0)$ is the creep coefficient, defining the creep between times t and t_0, related to elastic deformation at 28 days,
$\varphi_t = \varphi(\infty, t_0)$ is the final creep coefficient,
t is the age of the concrete at the time considered,
t_0 is the age of the concrete at loading,
N_{Ed} is the axial design force,
$N_{G,Ed}$ is the permanent part of the axial design force N_{Ed}, $N_{G,Ed} = \gamma_G \cdot N_{Gk}$.

For the calculation of the creep coefficient $\varphi(t, t_0)$, the following is valid:

- the perimeter of that part which is exposed to drying, u, is determined in accordance with Figure C6.5.

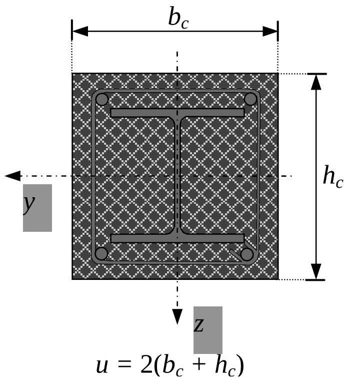


Figure C6.5 *Perimeter which is "exposed" to drying*

$$u = 2(b_c + h_c)$$
$$u = 2 \cdot (40 + 40) = 160 \text{ cm}$$

- the notional size of the cross-section, h_0

$$h_0 = \frac{2 \cdot A_c}{u} = \frac{2 \cdot 1473{,}6}{160} = 18{,}4 \text{ cm } = 184 \text{ mm}$$

- $t_0 = 30$ days,
- inside conditions, the ambient relative humidity RH 50 %,
- the concrete strength class C 25/30
- the type of cement – cement class N, strength class 32,5 R; 42,5 N.

The final value of creep coefficient $\varphi(\infty, t_0)$ is determined using the nomogram shown in Figure 3.1, EN 1992-1-1. The process of determining the final value of the creep coefficient, taking into account these assumptions, is given in Figure C6.6:

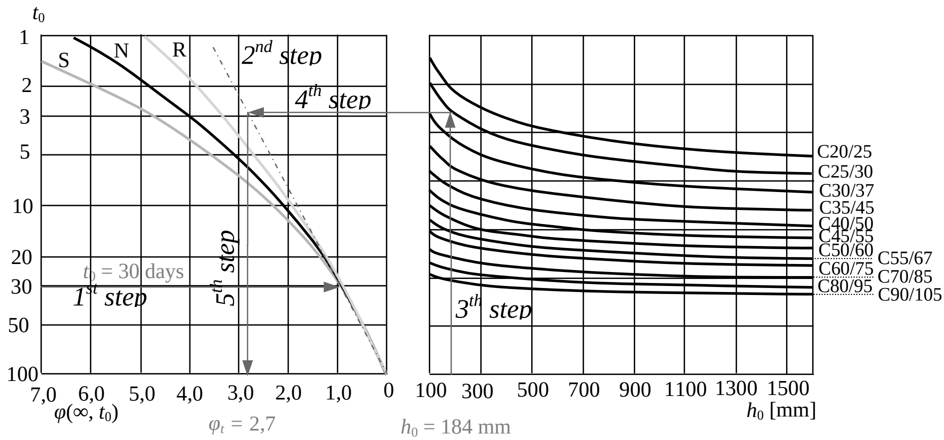


Figure C6.6 *Method for determining the creep coefficient*

The final value of creep coefficient $\varphi(\infty, t_0)$, found from Figure C6.6, is:

$\varphi_t = \varphi(\infty, t_0) = 2{,}7$

The design force of the permanent load, $N_{G,Ed}$, and the total design force, N_{Ed}, are:

$$N_{G,Ed} = 1200 \text{ kN}$$

$$N_{Ed} = 1800 \text{ kN}$$

Accordingly, the value of $E_{c,eff}$ is:

$$E_{c,eff} = \frac{E_{cm}}{1+(\frac{N_{G,Ed}}{N_{Ed}})\cdot\varphi_t} = \frac{3100}{1+\frac{1200}{1800}\cdot 2,7} = 1107 \ \text{kN/cm}^2$$

Further calculation is performed with the effective modulus of elasticity of concrete $E_{c,eff}$ = 1107 kN/m^2.

8. Resistance of the cross-section to compressive axial force

The design plastic resistance of the composite cross-section to axial compressive force, $N_{pl,Rd}$, is given by the sum of the design resistances of components as follows:

$$N_{pl,Rd} = A_a \cdot f_{yd} + 0,85 \cdot A_c \cdot f_{cd} + A_s \cdot f_{sd}$$

$$N_{pl,Rd} = 118,4 \cdot 35,5 + 0,85 \cdot 1473,6 \cdot 1,67 + 8,04 \cdot 34,8$$

$$N_{pl,Rd} = 4203 + 2092 + 280 = 6575 \ \text{kN}$$

The characteristic value of the plastic resistance of the composite cross-section to compressive axial force, $N_{pl,Rk}$, is determined as:

$$N_{pl,Rk} = A_a \cdot f_{yk} + A_c \cdot \alpha_c \cdot f_{ck} + A_s \cdot f_{sk}$$

$$N_{pl,Rk} = 118,4 \cdot 35,5 + 0,85 \cdot 1473,6 \cdot 2,5 + 8,04 \cdot 40$$

$$N_{pl,Rk} = 4203 + 3131 + 322 = 7656 \ \text{kN}$$

9. Verification of the conditions for using the simplified design method

The cross-section of the composite column should be doubly symmetrical and uniform along the entire length of the column.

This condition is satisfied.

Relative slenderness

To apply the simplified method it is necessary to satisfy the following conditions:

$\bar{\lambda}_y \leq 2{,}0$

$\bar{\lambda}_z \leq 2{,}0$

About the y-y axis

Relative slenderness, $\bar{\lambda}_y$, is determined as:

$$\bar{\lambda}_y = \sqrt{\frac{N_{pl,Rk}}{N_{cr,y}}}$$

For the determination of the relative slenderness $\bar{\lambda}_y$ and the elastic critical force $N_{cr,y}$, it is necessary to calculate the value of the effective flexural stiffness of the cross-section of composite column, $(EI)_{eff,y}$, as:

$$(EI)_{eff,y} = E_a \cdot I_{y,a} + E_s \cdot I_{y,s} + K_e \cdot E_{c,eff} \cdot I_{y,c}$$

With the correction factor K_e = 0,6, the value of $(EI)_{eff,y}$ is:

$$(EI)_{eff,y} = 21000 \cdot 14920 + 21000 \cdot 2189 + 0{,}6 \cdot 1107 \cdot 196224$$

$$(EI)_{eff,y} = 489{,}62 \cdot 10^6 \text{ kNcm}^2$$

Elastic critical force, $N_{cr,y}$,, for the pin-ended column and the buckling length $L_{e,y}$, is determined as:

$$N_{cr,y} = \frac{(EI)_{eff,y} \cdot \pi^2}{L_{e,y}^2} \qquad L_{e,y} = L$$

$$N_{cr,y} = \frac{489{,}62 \cdot 10^6 \cdot \pi^2}{700^2} = 9862 \text{ kN}$$

The relative slenderness, $\bar{\lambda}_y$, is:

$$\bar{\lambda}_y = \sqrt{\frac{7656}{9862}} = 0{,}88$$

Accordingly $\bar{\lambda}_y = 0{,}88 < 2{,}0$, and the condition is satisfied.

About the *z-z* axis

Relative slenderness, $\bar{\lambda}_z$, is determined as:

$$\bar{\lambda}_z = \sqrt{\frac{N_{pl,Rk}}{N_{cr,z}}}$$

For the determination of the relative slenderness $\bar{\lambda}_z$ and the elastic critical force $N_{cr,z}$, it is necessary to calculate the value of the effective flexural stiffness of the cross-section of composite column $(EI)_{eff,z}$ as:

$$(EI)_{eff,z} = E_a \cdot I_{z,a} + E_s \cdot I_{z,s} + K_e \cdot E_{c,eff} \cdot I_{z,c}$$

With the correction factor K_e = 0,6, the value of $(EI)_{eff,z}$ is:

$$(EI)_{eff,z} = 21000 \cdot 5135 + 21000 \cdot 2189 + 0,6 \cdot 1107 \cdot 206009$$

$$(EI)_{eff,z} = 290,64 \cdot 10^6 \text{ kNcm}^2$$

The elastic critical force, $N_{cr,z}$, for the pin-ended column and the buckling length $L_{e,z}$, is determined as:

$$N_{cr,z} = \frac{(EI)_{eff,z} \cdot \pi^2}{L_{e,z}^2} \qquad L_{e,z} = L$$

$$N_{cr,z} = \frac{290,64 \cdot 10^6 \cdot \pi^2}{700^2} = 5854 \text{ kN}$$

The relative slenderness, $\bar{\lambda}_z$, is:

$$\bar{\lambda}_z = \sqrt{\frac{7656}{5854}} = 1,14$$

Accordingly $\bar{\lambda}_z = 1,14 < 2,0$, and the condition is satisfied.

The maximum permitted cross-sectional area of the longitudinal reinforcement

The maximum cross-sectional area of longitudinal reinforcement $A_{s,max}$ that can be used in the calculation should not exceed 6% of the concrete area. This condition is satisfied, see in Section 4.1.

The minimum permitted cross-sectional area of the longitudinal reinforcement

The minimum cross-sectional area of longitudinal reinforcement $A_{s,min}$ that can be used in the calculation should be not less 0,3% of the concrete area. This condition is satisfied, see in Section 4.1.

The ratio of the depth to the width

The ratio of the depth to the width of the composite cross-section should be within the following limits, see Figure C6.4:

$$0,2 \leq \frac{h_c}{b_c} \leq 5,0$$

$$\frac{h}{b} = \frac{40,0}{40,0} = 1,0$$

$$0,2 < \frac{h_c}{b_c} = 1,0 < 5,0$$, the condition is satisfied

Maximum thickness of concrete cover

The maximum thicknesses of concrete cover that may be used in calculation are:

max $c_y = 0,4b_c$

max $c_z = 0,3h_c$

Thus, in accordance with Figure C6.4:

$\max c_y = 0,4 \cdot b_c = 0,4 \cdot 40,0 = 16\ cm > c_y = 7\ cm$, the condition is satisfied
$\max c_z = 0,3 \cdot h_c = 0,3 \cdot 40,0 = 12\ cm > c_z = 7\ cm$, the condition is satisfied

Remark:

All the conditions from clause 6.7.3.1, EN 1994-1-1, are satisfied, so this allows the use of the simplified design method for composite columns.

10. Resistance of the member in axial compression

Remark:

Although the column is subjected to combined compression and biaxial bending, the check based on buckling curves is useful as the preliminary check for this column. If the resistance to the axial compressive force is not sufficient, the considered column is inadequate and it is necessary to select a stronger cross-section.

The resistance of the member subjected only to axial compression can be checked by the second-order analysis according to clause 6.7.3.5, EN 1994-1-1, so as to take into account member imperfections. As a simplification in the case of the member subjected only to axial compression, the design value of the axial force N_{Ed} should satisfy the check based on European buckling curves, which can be written in the following format:

$$\frac{N_{Ed}}{\chi \cdot N_{pl,Rd}} \leq 1,0$$

The reduction factor χ is given by:

$$\chi = \frac{1}{\Phi + \sqrt{\Phi^2 - \bar{\lambda}^2}} \text{, but } \chi \leq 1,0$$

and

$$\Phi = 0,5 \cdot [1 + \alpha \cdot (\bar{\lambda} - \bar{\lambda}_0) + \bar{\lambda}^2] \text{ , with } \bar{\lambda}_0 = 0,2$$

Since $\bar{\lambda}_z = 1,14 > \bar{\lambda}_y = 0,88$, the buckling resistance about the *z-z* axis is governed.

Remark:

The relevant buckling curves for cross-sections of composite columns are given in Table 6.5, EN1994-1-1. according to which, composite columns with fully concrete encased section can be designed using buckling curve *b* for the *y-y* axis of buckling and using buckling curve *c* for the *z-z* axis of buckling.

From Table 6.5, the buckling curve *c* with α = 0,49 is adopted for the *z-z* axis of buckling so that Φ_z is:

$$\Phi_z = 0,5 \cdot [1 + \alpha \cdot (\bar{\lambda}_z - \bar{\lambda}_0) + \bar{\lambda}_z^2]$$

$$\Phi_z = 0{,}5 \cdot [1 + 0{,}49 \cdot (1{,}14 - 0{,}2) + 1{,}14^2] = 1{,}38$$

The reduction factor χ_z is:

$$\chi_z = \frac{1}{\Phi_{zz} + \sqrt{\Phi_z^2 - \bar{\lambda}_z^2}}$$

$$\chi_z = \frac{1}{1{,}38 + \sqrt{1{,}38^2 - 1{,}14^2}} = 0{,}46 < 1{,}0$$

Check:

$$\frac{N_{Ed}}{\chi_z \cdot N_{pl,Rd}} \leq 1{,}0$$

$$\frac{1800}{0{,}46 \cdot 6575} = 0{,}60 < 1{,}0$$

Since $0{,}60 < 1{,}0$, the check of the composite column subjected to axial compression is satisfied. It is not necessary to select the stronger cross-section.

11. Resistance of the member in combined compression and uniaxial bending

11.1 Resistance of the member about the *y-y* axis taking into account the equivalent member imperfection $e_{0,z}$

11.1.1 General

According to clause 6.7.3.6, EN 1994-1-1, the member in combined compression and uniaxial bending has sufficient resistance if the following condition is satisfied:

$$\frac{M_{y,Ed}}{M_{pl,y,N,Rd}} = \frac{M_{y,Ed}}{\mu_{dy} \cdot M_{pl,y,Rd}} \leq \alpha_{M,y}$$

where:

$M_{y,Ed}$ is the greatest of the end moments and the maximum bending moment within the column length. This moment is calculated according to clause 6.7.3.4, EN 1994-1-1, including imperfections (Table 6.5, EN 1994-1-1) and second-order effects if necessary ($\alpha_{cr} > 10$).

$M_{pl,y,N,Rd}$ is the plastic resistance moment taking into account the axial force N_{Ed}, given by $\mu_{dy} \cdot M_{pl,y,Rd}$, see Figure 6.18, EN 1994-1-1.

$M_{pl,Rd}$ is the plastic resistance moment, given by point *B* in Figure 6.18, EN 1994-1-1.

μ_{dy} is the factor related to the design for compression and uniaxial bending.

$\alpha_{M,y}$ is the coefficient related to the bending of a composite column and is taken as 0,9 for steel grades between S235 and S355.

The condition can be written as:

$$\frac{M_{y,Ed}}{M_{y,Rd}} = \frac{M_{y,Ed}}{\alpha_{M,y} \cdot \mu_{dy} \cdot M_{pl,y,Rd}} \leq 1{,}0$$

The calculation of the design bending moment $M_{y,Ed} = M_{Ed,II}$ taking the initial bending moment about the *y*-*y* axis, the imperfection $e_{0,z}$, and second-order effects is shown in Figure C6.7.

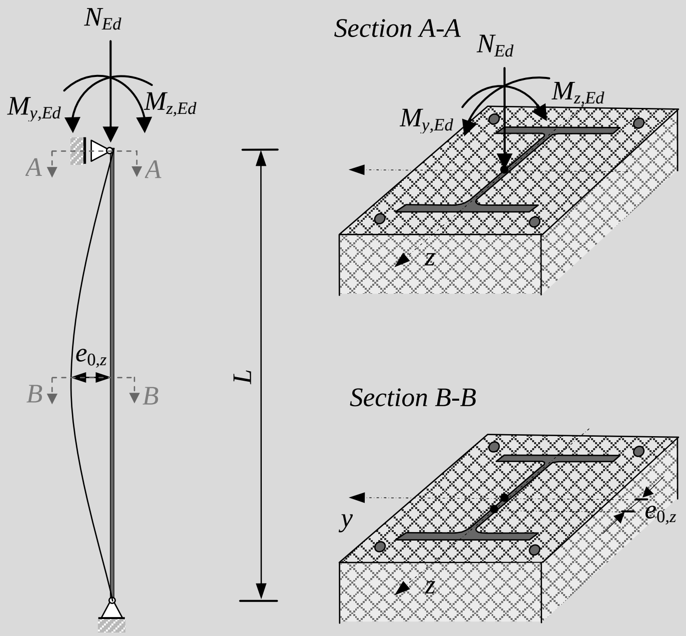


Figure C6.7 *Equivalent member imperfection $e_{0,z}$ about the y-y axis*

11.1.2 *Resistance of the cross-section in combined compression and bending about the y-y axis*

Remark:
In order to determine the resistance of the composite cross-section to combined compression and uniaxial bending, it is necessary to produce an axial load – bending moment (*N–M*) interaction curve. As a simplification, the interaction curve is replaced by an interaction polygon *ACDB*, clause 6.7.3.2 (5), EN 1994-1-1.

The *N–M* interaction polygon *ACDB* is shown in Figure 6.19, EN 1994-1-1. The modified version of the interaction polygon, which refers to the composite column with fully concrete-encased H-section, is shown in Figure C6.8.

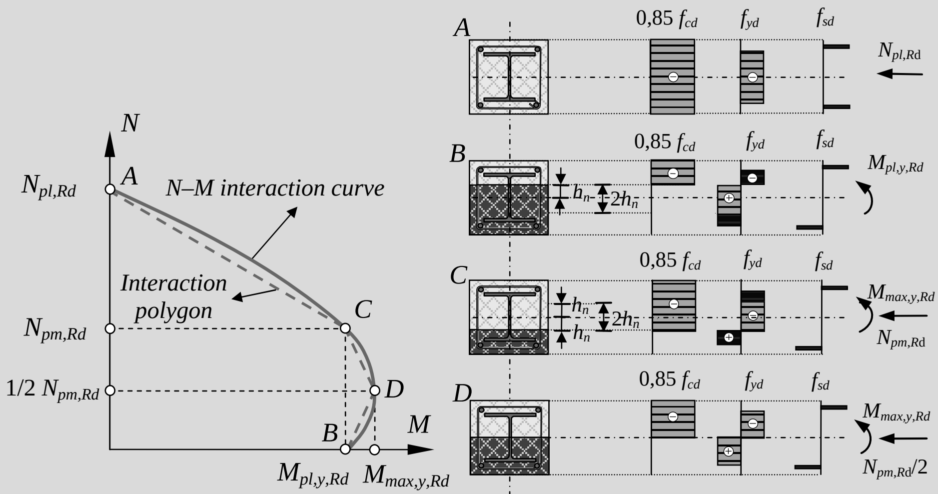


Figure C6.8 *N–M interaction polygon and corresponding stress distributions*

In order to produce the *N–M* interaction polygon, the cross-sectional capacities at points *A* to *D* should be determined assuming the stress distributions indicated, see Figure C6.8.

The resistance of the cross-section to combined compression and bending is calculated using the interaction polygon of *N–M*.

Point A

At point *A*, only the design plastic resistance of the cross-section is taken into account:

$$N_{pl,Rd} = A_a \cdot f_{yd} + 0{,}85 \cdot A_c \cdot f_{cd} + A_s \cdot f_{sd}$$

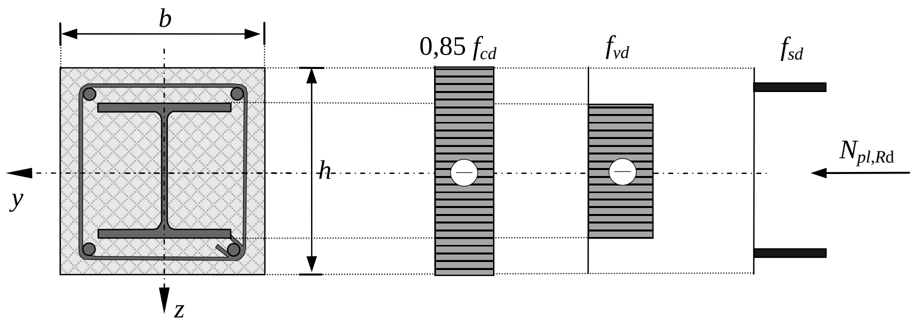


Figure C6.9 *Stress distributions for point A on the interaction polygon*

The design plastic resistance of the composite cross-section to compression is:

$$N_{pl,Rd} = 118,4 \cdot 35,5 + 0,85 \cdot 1473,6 \cdot 1,67 + 8,04 \cdot 34,8$$

$$N_{pl,Rd} = 6575 \text{ kN}$$

Point D

The position of the plastic neutral axis and the stress distributions are shown in Figure C6.10.

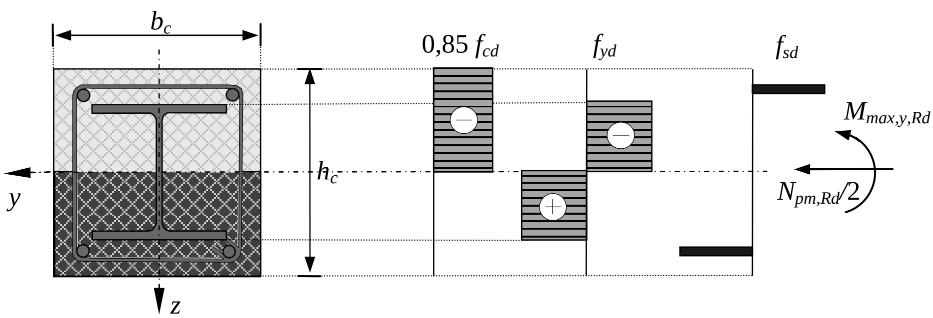


Figure C6.10 *Stress distributions for point D on the interaction polygon*

The maximum design plastic resistance moment is determined as:

$$M_{\max,y,Rd} = M_{pl,y,a,Rd} + M_{pl,y,c,Rd} + M_{pl,y,s,Rd}$$

The maximum design plastic resistance moment, $M_{max,y,Rd}$, at point *D* is:

$$M_{\max,y,Rd} = W_{pl,y,a} \cdot f_{yd} + 0,5 \cdot W_{pl,y,c} \cdot 0,85 \cdot f_{cd} + W_{pl,y,s} \cdot f_{sd}$$

$$M_{\max,y,Rd} = (1283 \cdot 35{,}5 + 0{,}5 \cdot 14584{,}3 \cdot 0{,}85 \cdot 1{,}67 + 132{,}7 \cdot 34{,}8) \cdot 10^{-2}$$

$$M_{\max,y,Rd} = 605 \text{ kNm}$$

The design value of the resistance of the concrete to compression, $N_{pm,Rd}$, is:

$$N_{pm,Rd} = A_c \cdot 0{,}85 \cdot f_{cd} = 1473{,}6 \cdot 0{,}85 \cdot 1{,}67 = 2092 \text{ kN}$$

The design axial force at the point of maximum design plastic resistance moment is $0{,}5 \cdot N_{pm,Rd}$, and therefore is:

$$0{,}5 \cdot N_{pm,Rd} = 0{,}5 \cdot 2092 = 1046 \text{ kN}$$

Point C

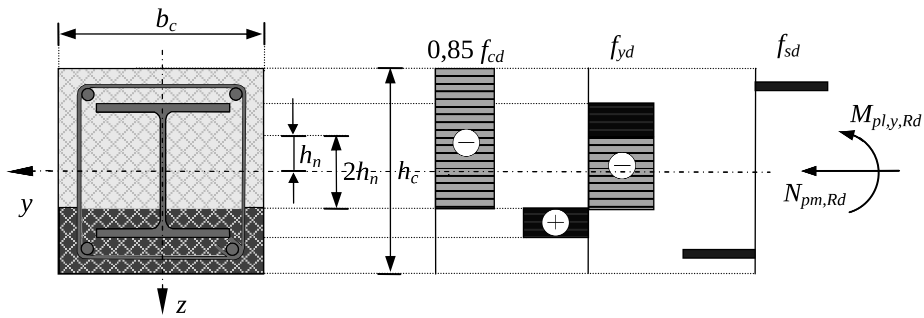


Figure C6.11 *Stress distributions for point C on the interaction polygon*

Calculation of the design plastic resistance moment of the composite section, $M_{pl,y,Rd}$, is carried out as shown below.

When the design axial force is equal to zero, the plastic neutral axis lies within the web of steel section ($h_n \leq h/2 - t_f$) and its position is determined as:

$$h_n = \frac{N_{pm,Rd} - A_{s,n} \cdot (2 \cdot f_{sd} - 0{,}85 \cdot f_{cd})}{2 \cdot b_c \cdot 0{,}85 \cdot f_{cd} + 2 \cdot t_w \cdot (2 \cdot f_{yd} - 0{,}85 \cdot f_{cd})}$$

where $A_{s,n}$ is the reinforcement area within h_n. Because it is at this point unknown, it is assumed to be (initial guess):

$$A_{s,n} = 0 \text{ cm}^2$$

Thus, for the case when the axial force is equal to zero, h_n is:

$$h_n = \frac{2092 - 0 \cdot (2 \cdot 34,8 - 0,85 \cdot 1,67)}{2 \cdot 40,0 \cdot 0,85 \cdot 1,67 + 2 \cdot 1,0 \cdot (2 \cdot 35,5 - 0,85 \cdot 1,67)} = 8,27 \text{ cm} = 82,7 \text{ mm}$$

Since in this region there is no reinforcement, the assumption is correct.

Plastic section moduli in region 2·h_n

Structural steel

$$W_{pl,y,a,n} = t_w \cdot h_n^2 = 1,0 \cdot 8,27^2 = 68,4 \text{ cm}^3$$

Reinforcement

$W_{pl,y,s,n} = 0 \text{ cm}^3$

Concrete

$$W_{pl,y,c,n} = b_c \cdot h_n^2 - W_{pl,y,a,n} - W_{pl,y,s,n} = 40,0 \cdot 8,27^2 - 68,4 - 0 = 2667 \text{ cm}^3$$

The design plastic resistance moment of the composite section, $M_{pl,y,Rd}$, is calculated as:

$$M_{pl,y,Rd} = M_{\max,y,Rd} - M_{n,y,Rd}$$

where:

$$M_{n,y,Rd} = W_{pl,y,a,n} \cdot f_{yd} + W_{pl,y,s,n} \cdot f_{sd} + \frac{W_{pl,y,c,n} \cdot f_{cd}}{2}$$

$$M_{n,y,Rd} = (68,4 \cdot 35,5 + 0 \cdot 34,8 + \frac{2667 \cdot 0,85 \cdot 1,67}{2}) \cdot 10^{-2} = 43,2 \text{ kNm}$$

The design plastic resistance moment of the composite section, $M_{pl,y,Rd}$, is:

$$M_{pl,y,Rd} = 605 - 43,2 = 562 \text{ kNm}$$

Point B

The design value of $M_{pl,y,Rd}$ has previously been calculated in order to define point *C* on the *N*–*M* interaction polygon:

$$M_{pl,y,Rd} = 562 \text{ kNm}$$

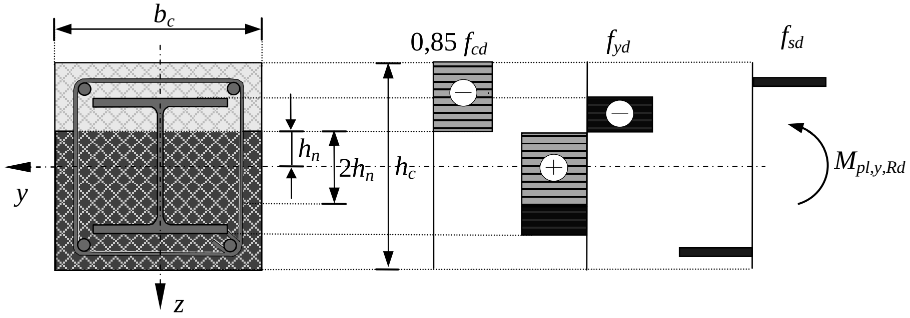


Figure C6.12 *Stress distributions for point B on the interaction polygon*

Previously calculated values at points *A* to *E* should be plotted to produce the *N–M* interaction polygon (Figure 6.19, EN 1994-1-1). The interaction polygon *ACDB* is shown in Figure C6.13.

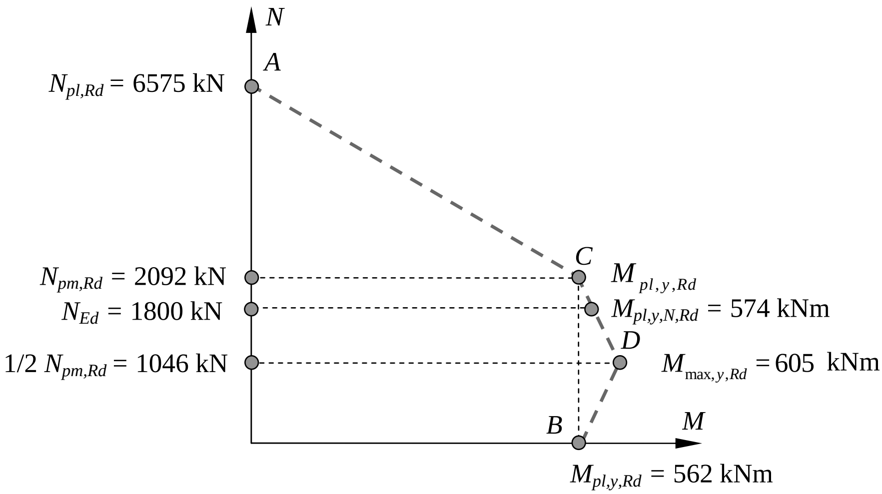


Figure C6.13 *N–M interaction polygon*

According to the interaction polygon *ACDB*, Figure C6.13, the following value $M_{pl,y,N,Rd}$ is obtained:

$$M_{pl,y,N,Rd} = M_{pl,y,Rd} + 2M_{n,y,Rd}\left(\frac{N_{pm,Rd} - N_{Ed}}{N_{pm,Rd}}\right)$$

$$M_{pl,y,N,Rd} = 562 + 2 \cdot 43{,}2\left(\frac{2092 - 1800}{2092}\right) = 574 \text{ kNm}$$

The value of μ_{dy} is:

$$\mu_{dy} = \frac{M_{pl,y,N,Rd}}{M_{pl,y,Rd}} = \frac{574}{562} = 1,02 > 1,0$$

Remark:

The value of μ_{dy} is greater than 1,0. According to clause 6.7.3.6(2), EN 1994-1-1, a value of μ_{dy} greater than 1,0 should be not used unless the design bending moment M_{Ed} is directly caused by the design axial force N_{Ed}, acting at the eccentricity on the considered column. If the design bending moment M_{Ed} does not depend on the action of the design axial force N_{Ed}, it is necessary to carry out the additional check in accordance with clause 6.7.1(7), EN 1994-1-1.

According to clause 6.7.1(7), EN 1994-1-1, for the composite column subjected to bending moment and axial force resulting from independent actions, the partial factor γ_F for this bending moment and axial force that lead to an increase of resistance should be reduced by 20%. It means that the design bending moment, $0,8 \cdot \gamma_F \cdot M_{Ek}$, coexists with the independent design axial force, $0,8 \cdot \gamma_F \cdot N_{Ek}$.

However, in this example, it makes no difference whether the bending moment and the axial force are from dependent actions or not, because the point $M_{pl,y,N,Rd}$ (= 574 kNm) lies on line *CD* in Figure C6.13, not on line *BD*, so the additional check according to clause 6.7.1(7), EN 1994-1-1, would not alter the result.

The check is carried out using the factor $\mu_{dy} = 1,02$, in accordance with clause 6.7.3.6(2), EN 1994-1-1, because the bending moment M_{Ed} results from the eccentricity of the design axial force N_{Ed}.

11.1.3 Calculation of the effects of actions about the y-y axis

11.1.3.1 General

According to clause 6.7.3.4 (3), EN 1994-1-1, which refers to clause 5.2.1(3), EN 1994-1-1, second-order effects can therefore be neglected if the load factor α_{cr}, which is the ratio between the elastic critical load and the corresponding applied loading, for elastic instability of the member exceeds 10.

To calculate α_{cr}, the ends of the column are assumed to be pinned, and α_{cr} is found using the Euler formula for the elastic critical force $N_{cr,y,eff}$:

$$N_{cr,y,eff} = \frac{\pi^2 (EI)_{eff,y,II}}{L_{e,y}^2} \qquad L_{e,y} = L$$

The design value of the effective flexural stiffness $(EI)_{eff,y,II}$, used to determine the internal forces and moments by second-order analysis, according to clause 6.7.3.4(2), EN 1994-1-1, is defined by:

$$(EI)_{eff,y,II} = K_0 \cdot (E_a \cdot I_{y,a} + E_s \cdot I_{y,s} + K_{e,II} \cdot E_{cm} \cdot I_{y,c})$$

where:

$K_{e,II}$ is a correction factor, which should be taken as 0,5,
K_0 is a calibration factor, which should be taken as 0,9.

The value $E_{c,eff}$ has been used in place of E_{cm} in the expression for $(EI)_{eff,y,II}$, in order to take into account the long-term effects, in the same way as calculated in Section 7. Accordingly, the value of $E_{c,eff}$ is:

$$E_{c,eff} = 1107 \text{ kN/cm}^2$$

The design value of the effective flexural stiffness $(EI)_{eff,y,II}$, is:

$$(EI)_{eff,y,II} = K_0 \cdot (E_a \cdot I_{y,a} + E_s \cdot I_{y,s} + K_{e,II} \cdot E_{c,eff} \cdot I_{y,c})$$

$$(EI)_{eff,y,II} = 0{,}9 \cdot (21000 \cdot 14920 + 21000 \cdot 2189 + 0{,}5 \cdot 1107 \cdot 196224) \cdot 10^{-2}$$

$$(EI)_{eff,y,II} = 421{,}11 \cdot 10^6 \text{ kNcm}^2$$

The elastic critical force, $N_{cr,y,eff}$, for the pin-ended column, is:

$$N_{cr,y,eff} = \frac{\pi^2 (EI)_{eff,y,II}}{L_{e,y}^2}$$

$$N_{cr,y,eff} = \frac{421{,}11 \cdot 10^6 \cdot \pi^2}{700^2} = 8482 \text{ kN}$$

To check whether the effects of second-order analysis can be neglected, the value of α_{cr} must be higher than 10:

$$\alpha_{cr} = \frac{N_{cr,y,eff}}{N_{Ed}} = \frac{8482}{1800} = 4{,}7 < 10$$

The value of α_{cr} is less than 10, so the second-order effects must be considered.

11.1.3.2 Bending moments about the y-y axis

According to clause 6.7.3.4(5), EN 1994-1-1, the second-order effects can be calculated by multiplying the greatest first-order design bending moments by a factor *k*.

Thus, the second-order effects may be considered according to the expression:

$$M_{y,Ed,II} = M_{y,Ed,I} \cdot k$$

The factor *k* is given by:

$$k = \frac{\beta}{1 - N_{Ed} / N_{cr,y,eff}} \geq 1{,}0$$

where:

β is an equivalent moment factor given in Table 6.4, EN 1994-1-1,

$N_{cr,y,eff}$ is the critical axial force, about the *y-y* axis, obtained with the effective flexural stiffness $(EI)_{eff,y,II}$ and with the effective length taken as the physical length of the column.

The design bending moment from the member imperfections is determined as:

$$M_{y,Ed,imp} = N_{Ed} \cdot e_{0,z}$$

where:

N_{Ed} is the design value of the axial force,

$e_{0,z}$ is the equivalent member imperfection, which is given in Table 6.5, EN 1994-1-1, depending on the buckling curve.

Remark:

According to Table 6.5, EN 1994-1-1, composite columns with fully concrete-encased section can be designed using buckling curve *b* for *y-y* axis of buckling.

Therefore, for the buckling curve *b*, the equivalent member imperfection is:

$$e_{0,z} = \frac{L}{200}$$

$$e_{0,z} = \frac{700}{200} = 3{,}5 \text{ cm}$$

The design values of the bending moments are as follows:

The design bending moment at the top of the column is:

$$M_{y,Ed} = 380 \text{ kNm}$$

The design bending moment at the bottom of the column is:

$$M_{y,Ed} = 0 \text{ kNm}$$

The design bending moment due to imperfection is:

$$M_{y,Ed,imp} = N_{Ed} \cdot e_{0,z} = 1800 \cdot 0{,}035 = 63{,}0 \text{ kNm}$$

The design bending moments calculated according to first-order analysis are shown in Figure C6.14.

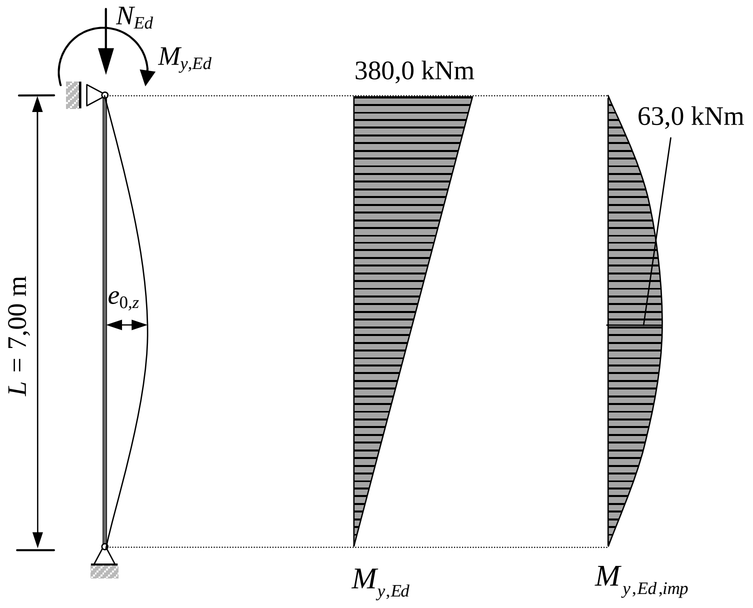


Figure C6.14 *First-order bending moments, design values*

Remark:

The factor β from Table 6.4, EN 1994-1-1, allows for the shape of the bending moment diagram. When bending is caused by lateral loading on the column, the value of factor β is 1,0. For a column subjected to end moments, the factor β is calculated as:

$$\beta_1 = 0,66 + 0,44 \cdot r \geq 0,44$$

where r is the ratio of the end moments on the ends of the column ($-1 \leq r \leq +1$).

Therefore, the two values of factor k must be calculated:

- for the end moments, k_1,
- for the moment from the member imperfection, k_2.

Determination of factor k_1

The ratio of the end moments at the ends of the column is:

$$r = \frac{0}{M_{y,Ed}} = \frac{0}{380} = 0,0$$

The equivalent moment factor β is:

$$\beta_1 = 0,66 + 0,44 \cdot r \geq 0,44$$

$$\beta_1 = 0,66 + 0,44 \cdot 0 = 0,66$$

Therefore, the factor k_1 is:

$$k_1 = \frac{\beta_1}{1 - N_{Ed} / N_{cr,y,eff}} = \frac{0,66}{1 - 1800 / 8482} = 0,84 < 1,0$$

Remark:

According to clause 6.7.3.4(5), EN 1994-1-1, the value of factor k must be 1,0 or higher. It is over-conservative to use when combining two sets of second-order effects. Therefore, the calculated value of 0,84 is adopted.

Determination of factor k_2

For the bending moment from the member imperfection, according to Table 6.4, EN 1994-1-1, the equivalent moment factor β is:

$\beta_2 = 1{,}00$

Therefore, the factor k_2 is:

$$k_2 = \frac{\beta_2}{1 - N_{\mathrm{Ed}}/N_{\mathrm{cr,eff,y}}} = \frac{1{,}0}{1 - 1800/8482} = 1{,}27 > 1{,}0$$

The adopted value of the factor is: $k_2 = 1{,}27$

The design bending moment at mid-height, second-order effects being taken into account, is:

$$M_{y,Ed,II} = M_{y,Ed} \cdot k_1 + M_{y,Ed,imp} \cdot k_2 = 380{,}0 \cdot 0{,}84 + 63{,}0 \cdot 1{,}27 = 399 \ \text{kNm}$$

The design bending moments calculated according to second-order analysis are shown in Figure C6.15.

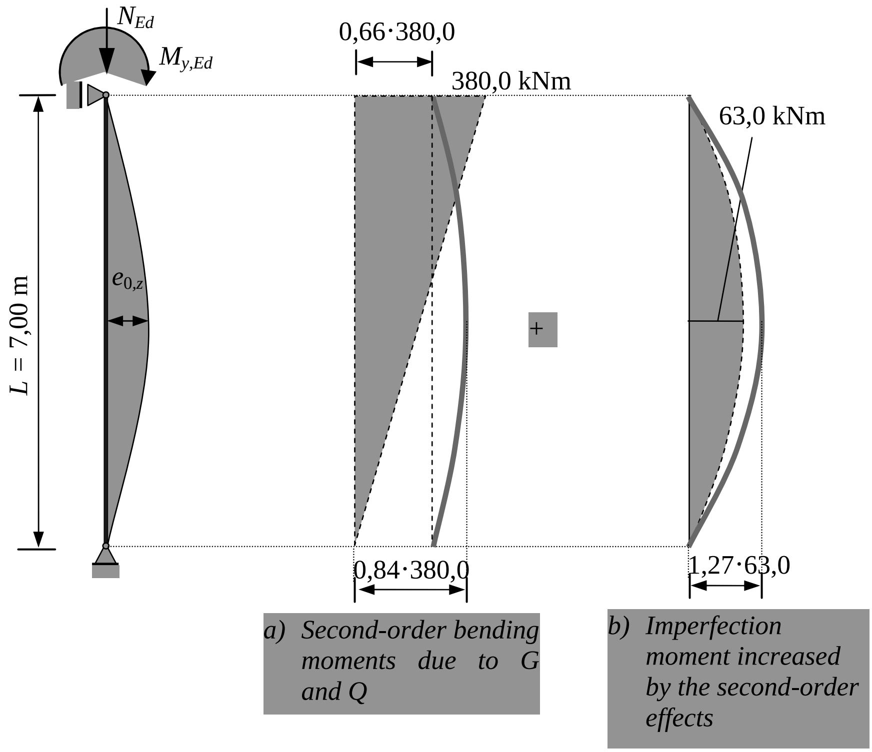


Figure C6.15 *Second-order bending moments about the y-y axis, design values*

The check is performed with the bending moment at mid-height:

$$M_{y,Ed,II} = M_{y,\max} = 399 \text{ kNm}$$

11.1.3.3 Shear forces parallel to the z-z axis

According to clause 6.7.3.4(5), EN 1994-1-1, second-order effects can be allowed for by multiplying the greatest first-order design bending moment by a factor k given by:

$$k = \frac{\beta}{1 - N_{Ed} / N_{cr,eff}} \geq 1{,}0$$

Accordingly, the approximate value of shear force can be obtained as:

$$V_{Ed,II} = V_{Ed} \cdot k$$

In accordance with Figure C6.16, the first-order design shear force at the bottom of column is:

$$V_{z,Ed} = \frac{M_{y,Ed}}{L} + \frac{4 \cdot N_{Ed} \cdot e_{0,z}}{L} = \frac{380{,}0}{7{,}0} + \frac{4 \cdot 1800 \cdot 0{,}035}{7{,}0} = 54{,}3 + 36 = 90{,}3 \text{ kN}$$

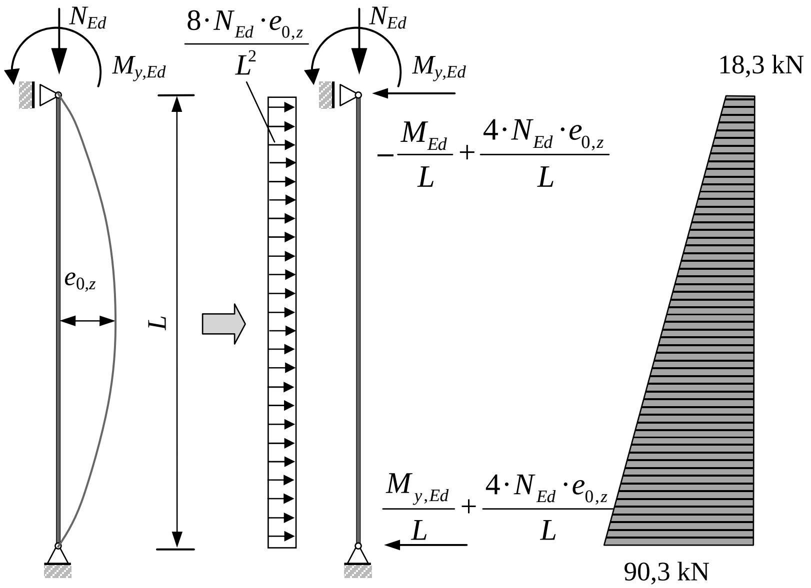


Figure C6.16 *First-order design shear forces parallel to the z-z axis*

In accordance with Figure C6.16, the first-order design shear force at the top of column is:

$$V_{z,Ed} = -\frac{M_{y,Ed}}{L} + \frac{4 \cdot N_{Ed} \cdot e_{0,z}}{L} = -\frac{380,0}{7,0} + \frac{4 \cdot 1800 \cdot 0,035}{7,0} = -54,3 + 36 = -18,3 \text{ kN}$$

The diagram of shear forces, calculated by first-order analysis for the bending moment and the equivalent lateral load due to imperfections, is shown in Figure C6.16.

The factor k_1 is:

$$k_1 = \frac{\beta_1}{1 - N_{Ed} / N_{cr,y,eff}} = \frac{0,66}{1 - 1800 / 8482} = 0,84 < 1,0$$

The factor k_2 is:

$$k_2 = \frac{1}{1 - N_{Ed} / N_{cr,y,eff}} = \frac{1,0}{1 - 1800/8482} = 1,27 > 1,0$$

Therefore, the maximum design shear force, calculated by approximate second-order analysis, is:

$$V_{z,Ed,II} = k_1 \cdot \frac{M_{y,Ed}}{L} + k_2 \cdot \frac{4 \cdot N_{Ed} \cdot e_{0,z}}{L}$$

$$V_{z,Ed,II} = 0,84 \cdot \frac{380,0}{7,0} + 1,27 \cdot \frac{4 \cdot 1800 \cdot 0,035}{7,0} = 45,6 + 45,7 = 91,3 \text{ kN}$$

11.1.4 Check of the resistance of the member in combined compression and bending about the y-y axis

It is necessary to satisfy the following condition:

$$\frac{M_{y,Ed}}{M_{y,Rd}} = \frac{M_{y,Ed}}{\alpha_{M,y} \cdot \mu_{dy} \cdot M_{pl,y,Rd}} \leq 1,0$$

The coefficient $\alpha_{M,y}$ is taken as 0,9 for steel grades between S235 and S355.

The design value of the maximum design bending moment by the approximate second-order analysis is:

$$M_{y,Ed} = M_{y,Ed,II} = 399 \text{ kNm}$$

The design resistance moment $M_{y,Rd}$ is (Figure C6.13):

$$M_{y,Rd} = \alpha_M \cdot \mu_{dy} \cdot M_{pl,y,Rd} = 0{,}9 \cdot 1{,}02 \cdot 562 = 516 \text{ kNm}$$

Condition:

$$\frac{M_{y,Ed}}{M_{y,Rd}} = \frac{399}{516} = 0{,}77$$

Since $0{,}77 < 1{,}0$, the condition is satisfied.

11.1.5 Check of the plastic resistance to transverse shear parallel to the z-z axis

In accordance with clause 6.7.3.2(4), EN 1994-1-1, for simplification V_{Ed} may be assumed to act on the structural steel section alone. According to clause 6.2.6(2), EN 1993-1-1, in the absence of torsion, the design plastic shear resistance, $V_{pl,z,a,Rd}$, is given by:

$$V_{pl,z,a,Rd} = \frac{A_{v,z} \cdot (f_y / \sqrt{3})}{\gamma_{M0}}$$

The shear area, $A_{v,z}$, according to clause 6.2.6(3), EN1993-1-1, is calculated as:

$$A_{v,z} = A_a - 2 \cdot b \cdot t_f + (t_w + 2r) \cdot t_f \geq \eta \cdot h_w \cdot t_w = \eta \cdot (h - 2 \cdot t_f - 2 \cdot r) \cdot t_w$$

According to clause 6.2.2.4(1), EN 1994-1-1, where the shear force is less than half the plastic shear resistance its effect on the resistance moment can be neglected. Therefore, the condition is:

$$V_{z,Ed} < 0{,}5 \cdot V_{pl,z,a,Rd}$$

The design value of the second-order shear force is:

$$V_{z,Ed} = V_{z,Ed,II} = 91{,}3 \text{ kN}$$

The shear area, $A_{v,z}$, is:

$$A_{v,z} = A_a - 2 \cdot b \cdot t_f + (t_w + 2r) t_f \geq \eta \cdot h_w \cdot t_w = \eta \cdot (h - 2 \cdot t_f - 2 \cdot r) \cdot t_w$$

$$\eta \cdot h_w \cdot t_w = \eta \cdot (h - 2 \cdot t_f - 2 \cdot r) \cdot t_w = 1,0 \cdot (26,0 - 2 \cdot 1,7 - 2 \cdot 2,4) \cdot 1,0 = 17,8 \text{ cm}^2$$

$$A_{v,z} = 118,4 - 2 \cdot 26,0 \cdot 1,7 + (1,0 + 2 \cdot 2,4) \cdot 1,7 = 39,9 \text{ cm}^2 \geq 17,8 \text{ cm}^2$$

The design plastic shear resistance, $V_{pl,z,a,Rd}$, is:

$$V_{pl,z,a,Rd} = \frac{A_{v,z} \cdot (f_y / \sqrt{3})}{\gamma_{M0}}$$

$$V_{pl,z,a,Rd} = \frac{39,9 \cdot (35,5 / \sqrt{3})}{1,0} = 818 \text{ kN}$$

Check:

$$V_{z,Ed,II} = 91,3 < 0,5 \cdot V_{pl,z,a,Rd} = 0,5 \cdot 818 = 409 \text{ kN}$$

The condition is satisfied and no reduction in the resistance moment is required.

11.2 Resistance of the member about the *z-z* axis taking into account the equivalent member imperfection $e_{0,y}$

11.2.1 General

According to clause 6.7.3.6, EN 1994-1-1, the member in combined compression and uniaxial bending has sufficient resistance if the following condition is satisfied:

$$\frac{M_{z,Ed}}{M_{pl,z,N,Rd}} = \frac{M_{z,Ed}}{\mu_{dz} \cdot M_{pl,z,Rd}} \leq \alpha_{M,z}$$

where:

$M_{z,Ed}$ is the greatest of the end moments and the maximum bending moment within the column length. This moment is calculated according to clause 6.7.3.4, EN 1994-1-1, including imperfections (Table 6.5, EN 1994-1-1) and second-order effects if necessary ($\alpha_{cr} > 10$).

$M_{pl,z,N,Rd}$ is the plastic resistance moment taking into account the axial force N_{Ed}, given by $\mu_{dz} \cdot M_{pl,z,Rd}$, see Figure C6.18.

$M_{pl,z,Rd}$ is the plastic resistance moment, given by point *B* in Figure 6.18, EN 1994-1-1.

μ_{dz} is the factor related to the design for compression and uniaxial bending.

$\alpha_{M,z}$ is the coefficient related to the bending of a composite column and is taken as 0,9 for steel grades between S235 and S355.

The condition can be written in the following form:

$$\frac{M_{z,Ed}}{M_{z,Rd}} = \frac{M_{z,Ed}}{\alpha_{M,z} \cdot \mu_{dz} \cdot M_{pl,z,Rd}} \leq 1,0$$

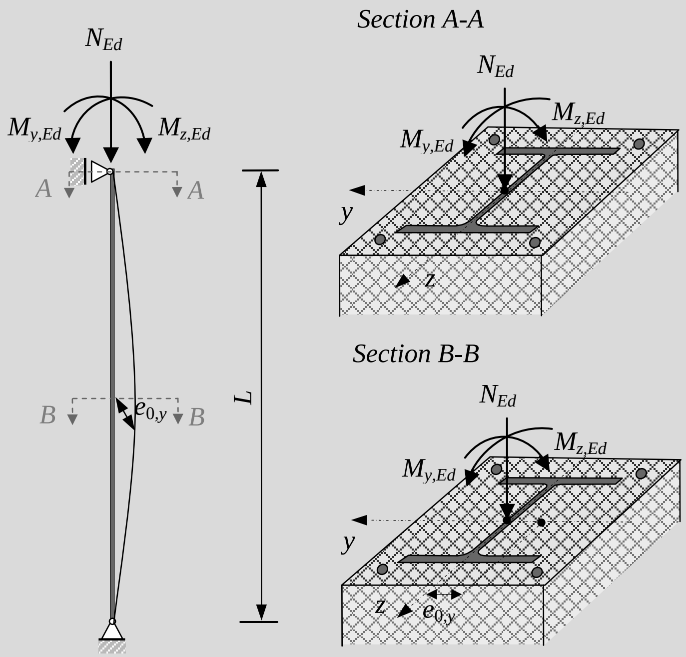


Figure C6.17 *Equivalent member imperfection $e_{0,y}$ about the z-z axis*

The calculation of the design bending moment $M_{z,Ed} = M_{z,Ed,II}$ taking the initial bending moment about the *z-z* axis, the imperfection $e_{0,y}$, and second-order effects is shown in Figure C6.17.

11.2.2 Resistance of the cross-section in combined compression and bending about the z-z axis

Remark:

In order to determine the resistance of the composite cross-section to combined compression and uniaxial bending, it is necessary to produce an axial load – bending moment (*N–M*) interaction curve. As a simplification, the interaction curve is replaced by an interaction polygon *ACDB*, clause 6.7.3.2 (5), EN 1994-

1-1.

The *N–M* interaction polygon *ACDB* is shown in Figure 6.19, EN 1994-1-1. A modified version of the interaction polygon, which refers to the composite column with fully concrete-encased H-section, is shown in Figure C6.18.

In order to produce the *N–M* interaction polygon, the cross-sectional capacities at points *A* to *D* should be determined assuming the stress distributions indicated, see Figure C6.18.

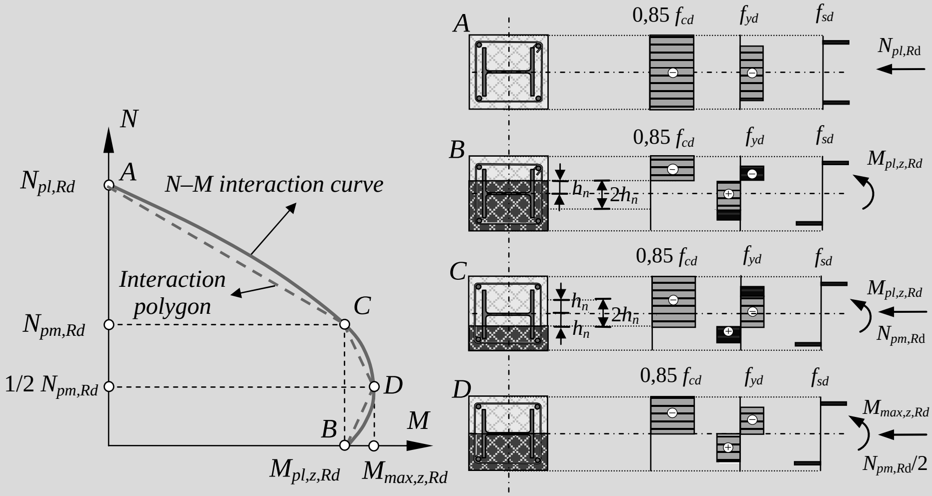


Figure C6.18 *N–M interaction polygon and corresponding stress distributions*

The resistance of the cross-section to combined compression and bending is calculated using the interaction polygon of *N–M*.

Point A

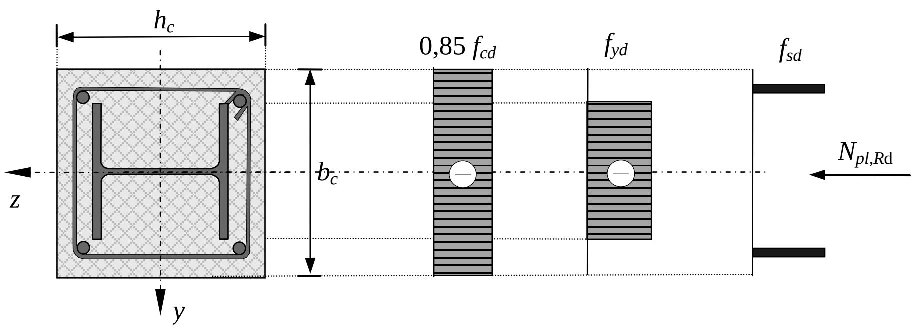


Figure C6.19 *Stress distributions for point A on the interaction polygon*

At point *A*, only the design plastic resistance of the cross-section is taken into account:

$$N_{pl,Rd} = A_a \cdot f_{yd} + 0{,}85 \cdot A_c \cdot f_{cd} + A_s \cdot f_{sd}$$

The design plastic resistance of the composite cross-section to compression is:

$$N_{pl,Rd} = 118{,}4 \cdot 35{,}5 + 0{,}85 \cdot 1473{,}6 \cdot 1{,}67 + 8{,}04 \cdot 34{,}8$$

$$N_{pl,Rd} = 6575 \text{ kN}$$

Point D

The position of the plastic neutral axis and the stress distributions are shown in Figure C6.20.

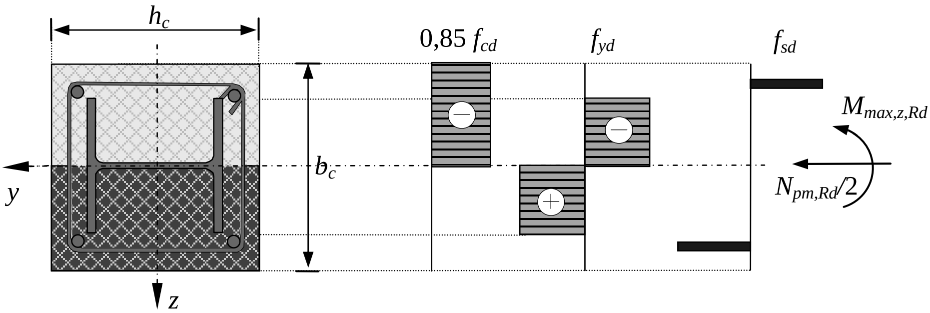


Figure C6.20 *Stress distributions for point D on the interaction polygon*

The maximum design plastic resistance moment is determined as:

$$M_{\max,z,Rd} = M_{pl,z,a,Rd} + M_{pl,z,c,Rd} + M_{pl,z,s,Rd}$$

The maximum design plastic resistance moment, $M_{max,z,Rd}$, at point D is:

$$M_{\max,z,Rd} = W_{pl,z,a} \cdot f_{yd} + 0{,}5 \cdot W_{pl,z,c} \cdot 0{,}85 \cdot f_{cd} + W_{pl,z,s} \cdot f_{sd}$$

$$M_{\max,z,Rd} = (602{,}2 \cdot 35{,}5 + 0{,}5 \cdot 15265{,}1 \cdot 0{,}85 \cdot 1{,}67 + 132{,}7 \cdot 34{,}8) \cdot 10^{-2}$$

$$M_{\max,z,Rd} = 368 \text{ kNm}$$

The design value of the resistance of the concrete to compression, $N_{pm,Rd}$, is:

$$N_{pm,Rd} = A_c \cdot 0{,}85 \cdot f_{cd} = 1473{,}6 \cdot 0{,}85 \cdot 1{,}67 = 2092 \text{ kN}$$

The design axial force at the point of maximum design plastic resistance moment is $0{,}5 \cdot N_{pm,Rd}$, and therefore is:

$$0{,}5 \cdot N_{pm,Rd} = 0{,}5 \cdot 2092 = 1046 \text{ kN}$$

Point C

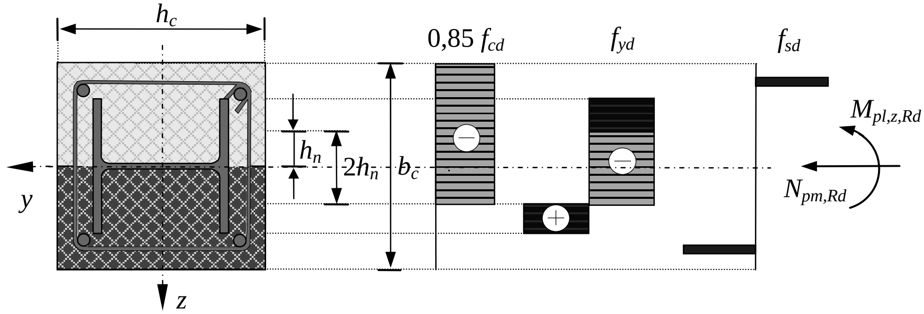


Figure C6.21 *Stress distributions for point C on the interaction polygon*

Calculation of the design plastic resistance moment of the composite section, $M_{pl,z,Rd}$, is carried out as shown below.

When the design axial force is equal to zero, the plastic neutral axis lies within the flanges of the steel section, $t_w/2 < h_n < b/2$, and its position is determined as:

$$h_n = \frac{N_{pm,Rd} - A_{s,n} \cdot (2 \cdot f_{sd} - 0{,}85 \cdot f_{cd}) + t_w \cdot (2 \cdot t_f - h) \cdot (2 \cdot f_{yd} - 0{,}85 \cdot f_{cd})}{2 \cdot h_c \cdot 0{,}85 \cdot f_{cd} + 4 \cdot t_f \cdot (2 \cdot f_{yd} - 0{,}85 \cdot f_{cd})}$$

where $A_{s,n}$ is the reinforcement area within h_n. Because it is at this point unknown, it is assumed to be (initial guess):

$$A_{s,n} = 0 \text{ cm}^2$$

Thus, for the case when the axial force is equal to zero, h_n is:

$$h_n = \frac{2092{,}0 - 0 \cdot (2 \cdot 34{,}8 - 0{,}85 \cdot 1{,}67) + 1{,}0 \cdot (2 \cdot 1{,}75 - 26{,}0) \cdot (2 \cdot 35{,}5 - 0{,}85 \cdot 1{,}67)}{2 \cdot 40{,}0 \cdot 0{,}85 \cdot 1{,}67 + 4 \cdot 1{,}75 \cdot (2 \cdot 35{,}5 - 0{,}85 \cdot 1{,}67)}$$

$$h_n = 0{,}88 \text{ cm}$$

Since in this region there is no reinforcement, the assumption is correct.

Plastic section moduli in region 2·h_n

Structural steel

$$W_{pl,z,a,n} = 2t_f \cdot h_n^2 + \frac{(h - 2 \cdot t_f) \cdot t_w^2}{4} = 2 \cdot 1{,}75 \cdot 0{,}88^2 + \frac{(26{,}0 - 2 \cdot 1{,}75) \cdot 1{,}0^2}{4} = 8{,}3 \ \text{cm}^3$$

Reinforcement

$W_{pl,z,s,n} = 0$ cm^3

Concrete

$$W_{pl,z,c,n} = h_c \cdot h_n^2 - W_{pl,z,a,n} - W_{pl,z,s,n} = 40{,}0 \cdot 0{,}88^2 - 8{,}3 - 0 = 22{,}7 \ \text{cm}^3$$

The design plastic resistance moment of the composite section, $M_{pl,z,Rd}$, is calculated as:

$$M_{pl,z,Rd} = M_{\max,z,Rd} - M_{n,z,Rd}$$

where:

$$M_{n,z,Rd} = W_{pl,z,a,n} \cdot f_{yd} + W_{pl,z,s,n} \cdot f_{sd} + \frac{W_{pl,z,c,n} \cdot f_{cd}}{2}$$

$$M_{n,z,Rd} = (8{,}3 \cdot 35{,}5 + 0 \cdot 34{,}8 + \frac{22{,}7 \cdot 0{,}85 \cdot 1{,}67}{2}) \cdot 10^{-2} = 3{,}1 \ \text{kNm}$$

Point B

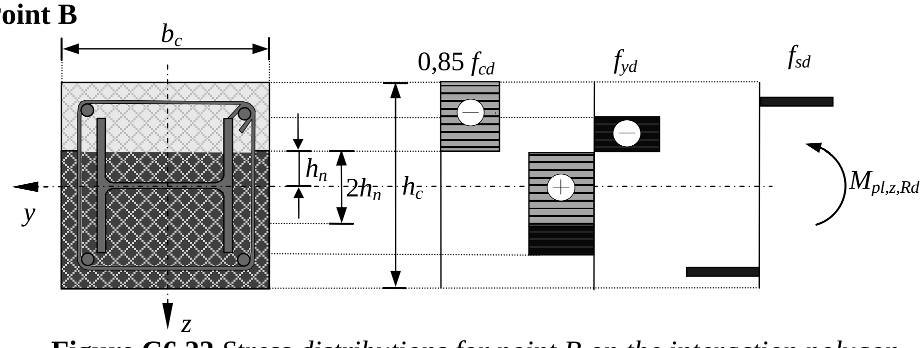


Figure C6.22 *Stress distributions for point B on the interaction polygon*

The design plastic resistance moment of the composite section, $M_{pl,z,Rd}$, is:

$$M_{pl,z,Rd} = 368 - 3{,}1 = 365 \text{ kNm}$$

The design value of $M_{pl,z,Rd}$ has previously been calculated in order to define point *C* on the *N–M* interaction polygon:

$$M_{pl,z,Rd} = 365 \text{ kNm}$$

Previously calculated values at points *A* to *D* should be plotted to produce the *N–M* interaction polygon (Figure 6.19, EN 1994-1-1). The interaction polygon *ACDB* is shown in Figure C6.23.

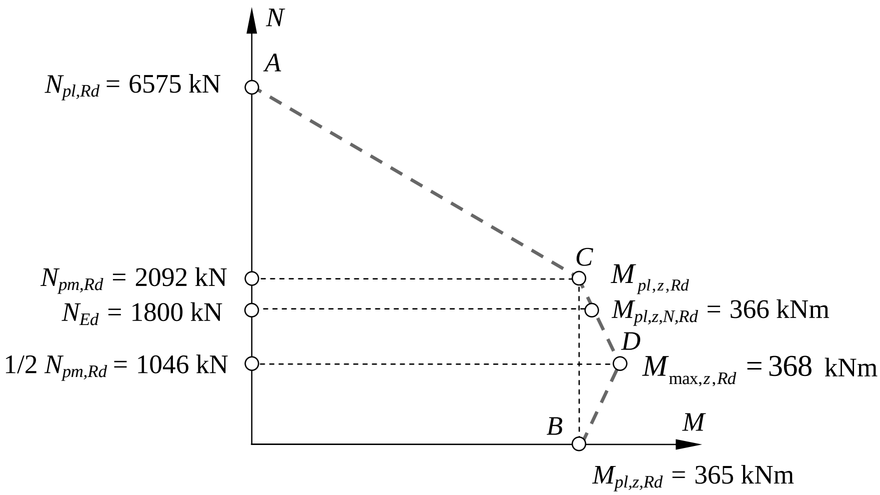


Figure C6.23 *N–M interaction polygon*

According to the interaction polygon *AECDB*, Figure C6.23, the following value $M_{pl,z,N,Rd}$ is obtained:

$$M_{pl,z,N,Rd} = M_{pl,z,Rd} + 2M_{n,z,Rd}\left(\frac{N_{pm,Rd} - N_{Ed}}{N_{pm,Rd}}\right)$$

$$M_{pl,z,N,Rd} = 365 + 2 \cdot 3{,}1\left(\frac{2092 - 1800}{2092}\right) = 366 \text{ kNm}$$

The value of μ_{dz} is:

$$\mu_{dz} = \frac{M_{pl,z,N,Rd}}{M_{pl,z,Rd}} = \frac{366}{365} = 1{,}003 \approx 1{,}0$$

The check is carried out by the factor $\mu_{dz} = 1,00$.

11.2.3 Calculation of the action effects about the z-z axis

11.2.3.1 General

According to clause 6.7.3.4 (3), EN 1994-1-1, which refers to clause 5.2.1(3), EN 1994-1-1, second-order effects can therefore be neglected if the load factor α_{cr}, which is the ratio between the elastic critical load and the corresponding applied loading, for elastic instability of the member exceeds 10.

To calculate α_{cr}, the ends of the column are assumed to be pinned, and α_{cr} is found using the Euler formula for the elastic critical force $N_{cr,z,eff}$.

$$N_{cr,z,eff} = \frac{\pi^2 (EI)_{eff,z,II}}{L_{e,z}^2} \qquad L_{e,z} = L$$

The design value of the effective flexural stiffness $(EI)_{eff,z,II}$, used to determine the internal forces and moments by second-order analysis, according to clause 6.7.3.4(2), EN 1994-1-1, is defined by:

$$(EI)_{eff,z,II} = K_0 \cdot (E_a \cdot I_{z,a} + E_s \cdot I_{z,s} + K_{e,II} \cdot E_{cm} \cdot I_{z,c})$$

where:

$K_{e,II}$ is a correction factor, which should be taken as 0,5,
K_0 is a calibration factor, which should be taken as 0,9.

The value $E_{c,eff}$ has been used in place of E_{cm} in the expression for $(EI\,)_{eff,z,II}$ in order to take into account the long-term effects, in the same way as calculated in Section 7. Accordingly, the value of $E_{c,eff}$ is:

$$E_{c,eff} = 1107 \text{ kN/cm}^2$$

The design value of the effective flexural stiffness $(EI)_{eff,z,II}$, is:

$$(EI)_{eff,z,II} = K_0 \cdot (E_a \cdot I_{z,a} + E_s \cdot I_{z,s} + K_{e,II} \cdot E_{c,eff} \cdot I_{z,c})$$

$$(EI)_{eff,z,II} = 0,9 \cdot (21000 \cdot 5135 + 21000 \cdot 2189 + 0,5 \cdot 1107 \cdot 206009)$$

$$(EI)_{eff,z,II} = 241,05 \cdot 10^6 \text{ kNcm}^2$$

The elastic critical force, $N_{cr,z,eff}$, for the pin-ended column, is:

$$N_{cr,z,eff} = \frac{\pi^2 (EI)_{eff,z,II}}{L_{e,z}^2}$$

$$N_{cr,z,eff} = \frac{241{,}05 \cdot 10^6 \cdot \pi^2}{700^2} = 4855 \text{ kN}$$

To check whether the effects of second-order analysis can be neglected, the value of α_{cr} must be higher than 10:

$$\alpha_{cr} = \frac{N_{cr,z,eff}}{N_{Ed}} = \frac{4855}{1800} = 2{,}7 < 10$$

The value of α_{cr} is less than 10, so the second-order effects must be considered.

11.2.3.2 Bending moments about the z-z axis

According to clause 6.7.3.4(5), EN 1994-1-1, the second-order effects can be calculated by multiplying the greatest first-order design bending moments by a factor *k*.

Thus, the second-order effects may be considered according to the expression:

$$M_{z,Ed,II} = M_{z,Ed,I} \cdot k$$

The factor *k* is given by:

$$k = \frac{\beta}{1 - N_{Ed} / N_{cr,z,eff}} \geq 1{,}0$$

where:

β is an equivalent moment factor given in Table 6.4, EN 1994-1-1,

$N_{cr,z,eff}$ is the critical axial force, about the *z-z* axis, obtained with the effective flexural stiffness $(EI)_{eff,z,II}$ and with the effective length taken as the physical length of the column.

The design bending moment from the member imperfections is determined as:

$$M_{z,Ed,imp} = N_{Ed} \cdot e_{0,y}$$

where:

N_{Ed} is the design value of the axial force,

$e_{0,y}$ is the equivalent member imperfection which is given in Table 6.5, EN 1994-1-1, depending on the buckling curve.

Remark:

According to Table 6.5, EN 1994-1-1, composite columns with fully concrete encased section can be designed using buckling curve *c* for the *z-z* axis of buckling.

Therefore, for the buckling curve *c*, the equivalent member imperfection is:

$$e_{0,y} = \frac{L}{150}$$

$$e_{0,y} = \frac{700}{150} = 4,7 \text{ cm}$$

The design bending moments calculated according to first-order analysis are shown in Figure C6.24.

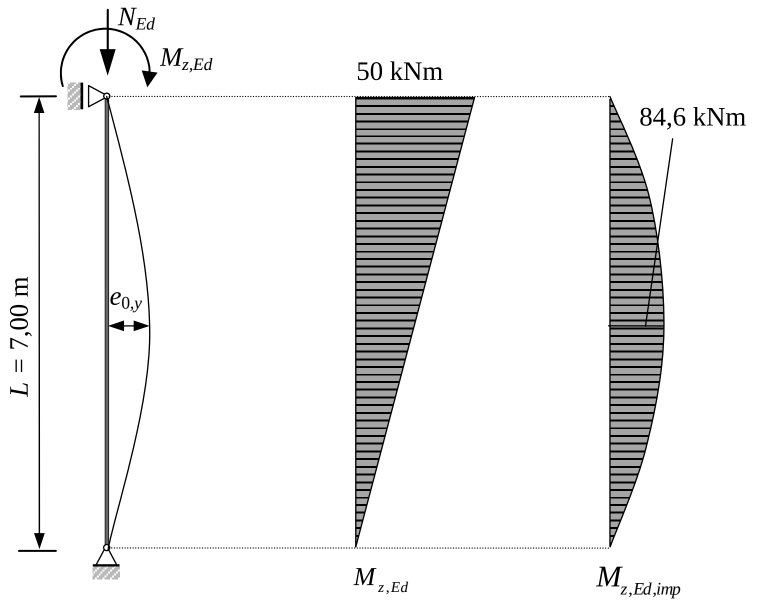


Figure C6.24 *First-order bending moments, design values*

The design values of bending moments are as follows:

The design bending moment at the top of the column is:

$$M_{z,Ed} = 50 \text{ kNm}$$

The design bending moment at the bottom of the column is:

$$M_{z,Ed} = 0 \text{ kNm}$$

The design bending moment due to imperfection is:

$$M_{z,Ed,imp} = N_{Ed} \cdot e_{0,y} = 1800 \cdot 0{,}047 = 84{,}6 \text{ kNm}$$

Remark:

The factor β from Table 6.4, EN 1994-1-1, allows for the shape of the bending moment diagram. When bending is caused by lateral loading on the column, the value of factor β is 1,0. For a column subjected to end moments, the factor β is calculated as:

$$\beta_1 = 0{,}66 + 0{,}44 \cdot r \geq 0{,}44$$

where r is the ratio of the end moments on the ends of the column ($-1 \leq r \leq +1$).

Therefore, the two values of factor k must be calculated:

- for the end moments, k_1,
- for the moment from the member imperfection, k_2.

Determination of factor k_1

The ratio of the end-moments on the ends of the column is:

$$r = \frac{0}{M_{z,Ed,I}} = 0{,}0$$

The equivalent moment factor β is:

$$\beta_1 = 0{,}66 + 0{,}44 \cdot r \geq 0{,}44$$

$$\beta_1 = 0{,}66 + 0{,}44 \cdot 0 = 0{,}66$$

Therefore, the factor k_1 is:

$$k_1 = \frac{\beta_1}{1 - N_{\mathrm{Ed}} / N_{\mathrm{cr,eff,z}}} = \frac{0,66}{1 - 1800 / 4855} = 1,05 > 1,0$$

The adopted value of the factor is:

$k_1 = 1,05$

Determination of factor k_2

For the bending moment from the member imperfection, according to Table 6.4, EN 1994-1-1, the equivalent moment factor β is:

$$\beta_2 = 1,0$$

Therefore, the factor k_2 is:

$$k_2 = \frac{\beta_2}{1 - N_{\mathrm{Ed}} / N_{\mathrm{cr,eff,z}}} = \frac{1,0}{1 - 1800/4855} = 1,59 > 1,0$$

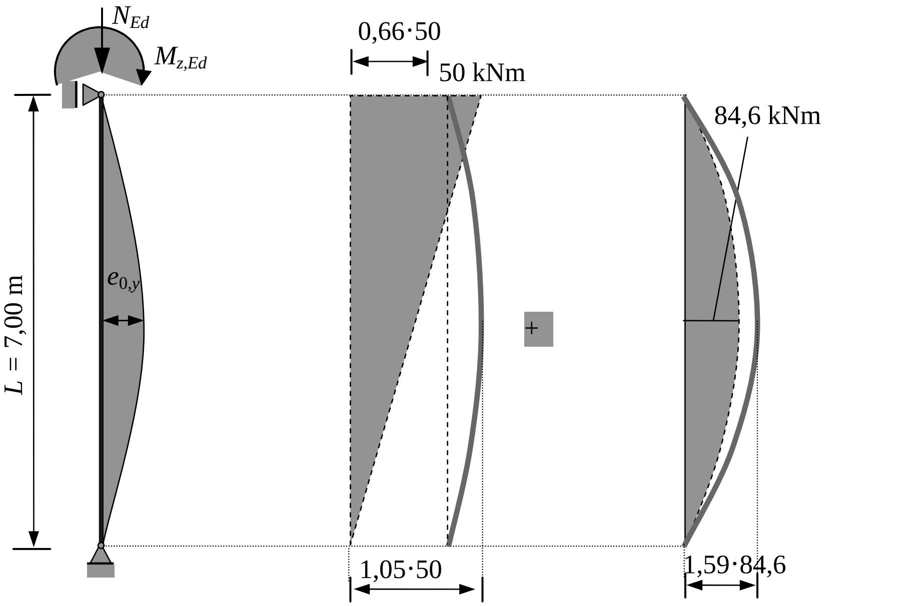
The adopted value of the factor is:

$k_2 = 1,59$

The design bending moment at mid-height, second-order effects being taken into account, is:

$$M_{z,Ed,II} = M_{z,Ed} \cdot k_1 + M_{z,Ed,imp} \cdot k_2 = 50 \cdot 1,05 + 84,6 \cdot 1,59 = 187 \text{ kNm}$$

The design bending moments calculated according to second-order analysis are shown in Figure C6.25.



a) *Second-order bending moments due to G and Q*

b) *Imperfection moment increased by the second-order effects*

Figure C6.25 *Second-order bending moments, design values*

The check is performed with the bending moment at mid-height:

$$M_{z,Ed,II} = M_{z,\max} = 187 \text{ kNm}$$

11.2.3.3 Shear forces parallel to the y-y axis

According to clause 6.7.3.4(5), EN 1994-1-1, the second-order effects can be allowed for by multiplying the greatest first-order design bending moment by a factor *k* given by:

$$k = \frac{\beta}{1 - N_{Ed} / N_{cr,eff}} \geq 1{,}0$$

Accordingly, the approximate value of shear force can be obtained as:

$$V_{Ed,II} = V_{Ed} \cdot k$$

In accordance with Figure C6.26, the first-order design shear force at the bottom of column is:

$$V_{y,Ed} = \frac{M_{z,Ed}}{L} + \frac{4 \cdot N_{Ed} \cdot e_{0,y}}{L} = \frac{50,0}{7,0} + \frac{4 \cdot 1800 \cdot 0,047}{7,0} = 7,1 + 48,3 = 55,4 \text{ kN}$$

In accordance with Figure C6.26, the first-order design shear force at the top of column is:

$$V_{y,Ed} = -\frac{M_{z,Ed}}{L} + \frac{4 \cdot N_{Ed} \cdot e_{0,y}}{L} = -\frac{50,0}{7,0} + \frac{4 \cdot 1800 \cdot 0,047}{7,0} = -7,1 + 48,3 = 41,2 \text{ kN}$$

The diagram of shear forces, calculated by first-order analysis for bending moment and the equivalent lateral load due to imperfections, is shown in Figure C6.26.

The factor k_1 is:

$$k_1 = \frac{\beta_1}{1 - N_{Ed} / N_{cr,z,eff}} = \frac{0,66}{1 - 1800/4855} = 1,05 > 1,0$$

The factor k_2 is:

$$k_2 = \frac{\beta_2}{1 - N_{Ed} / N_{cr,z,eff}} = \frac{1,0}{1 - 1800/4855} = 1,59 > 1,0$$

Therefore, the maximum design shear force, calculated by approximate second-order analysis, is:

$$V_{y,Ed,II} = k_1 \cdot \frac{M_{z,Ed}}{L} + k_2 \cdot \frac{4 \cdot N_{Ed} \cdot e_{0,y}}{L}$$

$$V_{y,Ed,II} = 1,05 \cdot \frac{50,0}{7,0} + 1,59 \cdot \frac{4 \cdot 1800 \cdot 0,047}{7,0} = 7,5 + 76,9 = 84,4 \text{ kN}$$

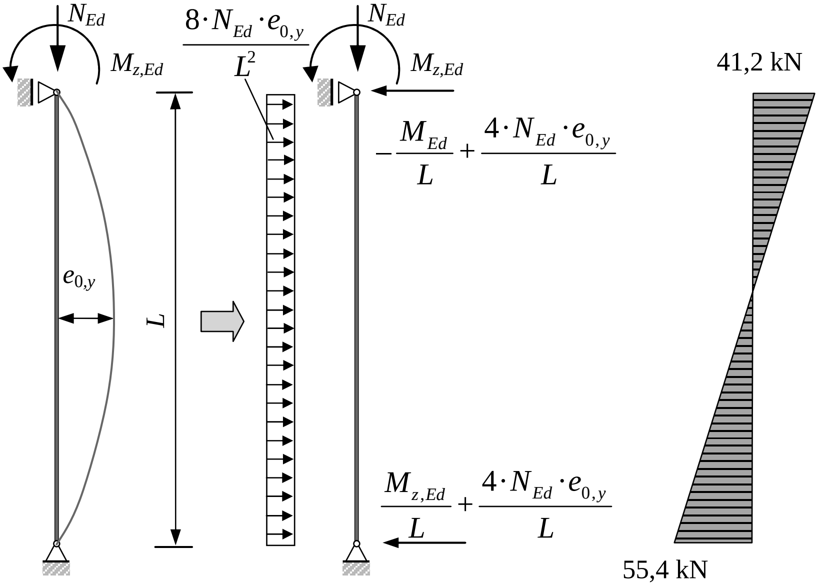


Figure C6.26 *First-order design shear forces parallel to the y-y axis*

11.2.4 Check of the resistance of the member in combined compression and bending about the z-z axis

It is necessary to satisfy the following condition:

$$\frac{M_{z,Ed}}{M_{z,Rd}} = \frac{M_{z,Ed}}{\alpha_{M,z} \cdot \mu_{dz} \cdot M_{pl,z,Rd}} \leq 1{,}0$$

The coefficient $\alpha_{M,z}$ is taken as 0,9 for steel grades between S235 and S355.

The design value of the maximum design bending moment by the approximative second-order analysis is:

$$M_{z,Ed} = M_{z,Ed;II} = 187 \text{ kNm}$$

The design resistance moment $M_{z,Rd}$ is (Figure C6.23):

$$M_{z,Rd} = \alpha_{M,z} \cdot \mu_{dz} \cdot M_{pl,z,Rd} = 0{,}9 \cdot 1{,}00 \cdot 365 = 329 \text{ kNm}$$

Condition:

$$\frac{M_{z,Ed}}{M_{z.Rd}} = \frac{187}{329} = 0,57$$

Since $0,57 < 1,0$, the condition is satisfied.

11.2.5 Check of the plastic resistance to transverse shear parallel to the y-y axis

In accordance with clause 6.7.3.2(4), EN 1994-1-1, for simplification V_{Ed} may be assumed to act on the structural steel section alone. According to clause 6.2.6(2), EN 1993-1-1, in the absence of torsion the design plastic shear resistance, $V_{pl,y,a,Rd}$, is given by:

$$V_{pl,y,a,Rd} = \frac{A_{v,y} \cdot (f_y / \sqrt{3})}{\gamma_{M0}}$$

The shear area, $A_{v,y}$, according to clause 6.2.6(3), EN1993-1-1, is calculated as:

$$A_{v,y} = 2(b \cdot t_f)$$

According to clause 6.2.2.4(1), EN 1994-1-1, where the shear force is less than half the plastic shear resistance its effect on the resistance moment can be neglected. Therefore, the condition is:

$$V_{y,Ed} < 0,5 \cdot V_{pl,y,a,Rd}$$

The design value of second-order shear force is:

$$V_{y,Ed} = V_{y,Ed,II} = 84,4 \text{ kNm}$$

The shear area, $A_{v,y}$, is:

$$A_{v,y} = 2(26,0 \cdot 1,75) = 91 \text{ cm}^2$$

The design plastic shear resistance, $V_{pl,a,y,Rd}$, is:

$$V_{pl,y,a,Rd} = \frac{A_{v,y} \cdot (f_y / \sqrt{3})}{\gamma_{M0}}$$

$$V_{pl,a,y,Rd} = \frac{91 \cdot (35{,}5/\sqrt{3})}{1{,}0} = 1865 \text{ kN}$$

Check:

$$V_{y,Ed,II} = 84{,}4 < 0{,}5 \cdot V_{pl,a,y,Rd} = 0{,}5 \cdot 1865 = 933 \text{ kN}$$

The condition is satisfied and no reduction in the resistance moment is required.

12. Resistance of the member in combined compression and biaxial bending

12.1 General

Remark:

For the design of a composite column subjected to axial compression and biaxial bending, it is necessary to carry out the following verifications:

- The resistance to axial compression separately for each axis is the first step.
- Then, it is necessary to check the column resistance under compression and uniaxial bending individually in each of the planes of bending.
- Also, it is necessary to check the column resistance in biaxial bending, taking into account imperfections in the plane in which failure is expected to occur. For the other plane of bending the effect of imperfections is neglected. If it is not obvious which plane is the more critical, checks should be made for both planes.

In accordance with clause 6.7.3.7, EN 1994-1-1, for combined compression and biaxial bending the following conditions should be satisfied:

Check for bending about the *y-y* axis:

$$\frac{M_{y,Ed}}{M_{pl,y,N,Rd}} = \frac{M_{y,Ed}}{\mu_{dy} \cdot M_{pl,y,Rd}} \le \alpha_{My}$$

The condition can be written in the following form:

$$\frac{M_{y,Ed}}{\alpha_{M,y} \cdot \mu_{dy} \cdot M_{pl,y,Rd}} \le 1{,}0$$

Check for bending about the *z-z* axis:

$$\frac{M_{z,Ed}}{M_{pl,z,N,Rd}} = \frac{M_{z,Ed}}{\mu_{dz} \cdot M_{pl,z,Rd}} \leq \alpha_{M,z}$$

The condition can be written in the following form:

$$\frac{M_{z,Ed}}{\alpha_{M,z} \cdot \mu_{dz} \cdot M_{pl,z,Rd}} \leq 1,0$$

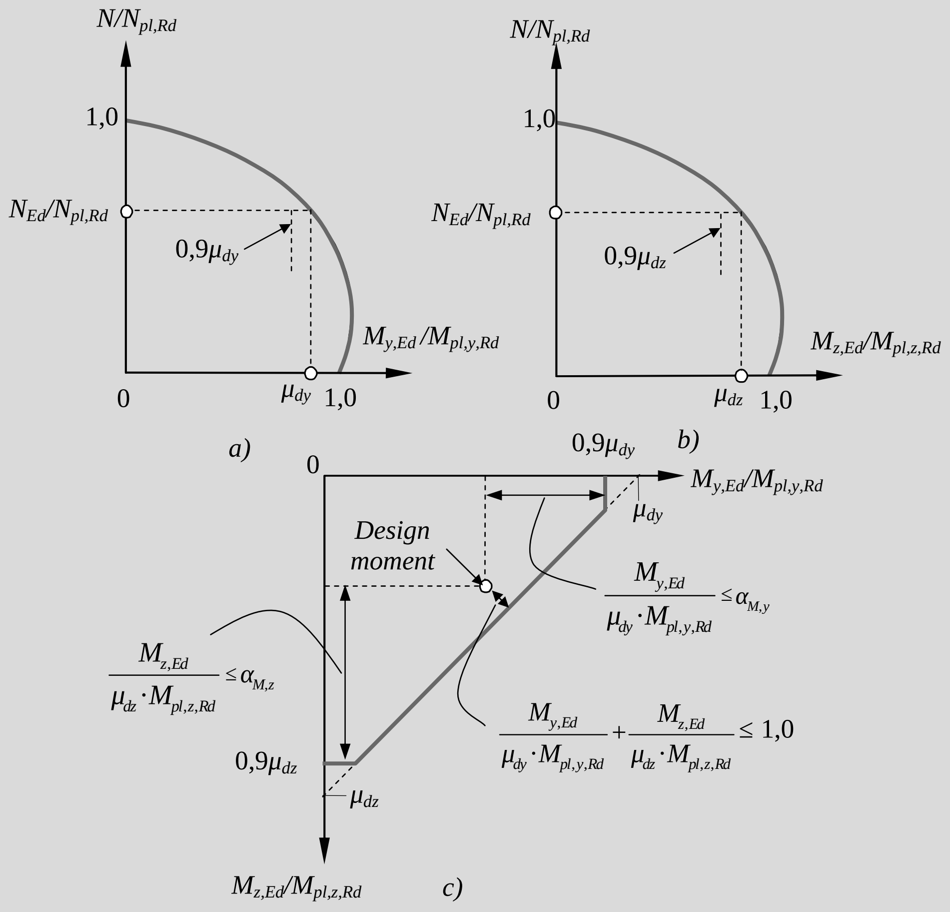
Interaction of M_y–M_z–N:

$$\frac{M_{y,Ed}}{M_{pl,y,N,Rd}} + \frac{M_{z,Ed}}{M_{pl,z,N,Rd}} \leq 1,0$$

The condition can be written in the following form:

$$\frac{M_{y,Ed}}{\mu_{dy} \cdot M_{pl,y,Rd}} + \frac{M_{z,Ed}}{\mu_{dz} \cdot M_{pl,z,Rd}} \leq 1,0$$

These interaction expressions are shown in Figure C6.27 by means of interaction curves.



a) *Section resistance interaction curve – non-failure axis (y-y axis). Neglect imperfections.*

b) *Section resistance interaction curve – axis of the anticipated failure (z-z axis). Consider imperfections.*

c) *Biaxial resistance moment of the column section under axial compression* N_{Ed}.

Figure C6.27 *Column resistance in combined compression and biaxial bending – assumed bending failure about the z-z axis*

12.2 Failure about the *y-y* axis is assumed

12.2.1 General

Assuming the failure about the *y-y* axis, the verification is carried out as follows:

- **Bending about the *y-y* axis**: It is necessary to check the resistance under compression, N_{Ed}, and bending, $M_{y,Ed}$, taking into account the equivalent member imperfection, $e_{0,z}$, and the second-order effects.

- **Bending about the *z-z* axis**: It is necessary to check the resistance under compression, N_{Ed}, and bending, $M_{z,Ed}$, neglecting the equivalent member imperfection, $e_{0,y}$, but taking into account the second-order effects.

- Finally, it is necessary to check the resistance under compression and biaxial bending in terms of the linear interaction curve M_y–M_z–N.

12.2.2 Calculation of the action effects about the y-y axis

In accordance with the detailed calculation given in Section 11.1.3.2, the design bending moment at mid-height $M_{y,Ed}$ taking into account the equivalent member imperfection, $e_{0,z}$, and the second-order effects is:

$$M_{y,Ed,II} = M_{y,Ed} \cdot k_1 + M_{y,Ed,imp} \cdot k_2 = 380,0 \cdot 0,84 + 63,0 \cdot 1,27 = 399 \text{ kNm}$$

12.2.3 Calculation of the action effects about the z-z axis

In accordance with the detailed calculation given in Section 11.2.3.2, the design bending moment at mid-height $M_{z,Ed}$ neglecting the equivalent member imperfection, $e_{0,y}$, but taking into account the second-order effects is:

$$M_{z,Ed,II} = M_{z,Ed} \cdot k_1 = 50 \cdot 1,05 = 53 \text{ kNm}$$

12.2.4 Check of the resistance of the member in combined compression and biaxial bending

It is necessary to satisfy the following conditions:

$$\frac{M_{y,Ed}}{\alpha_{M,y} \cdot \mu_{dy} \cdot M_{pl,y,Rd}} \leq 1,0$$

$$\frac{M_{z,Ed}}{\alpha_{M,z} \cdot \mu_{dz} \cdot M_{pl,z,Rd}} \leq 1,0$$

$$\frac{M_{y,Ed}}{\mu_{dy} \cdot M_{pl,y,Rd}} + \frac{M_{z,Ed}}{\mu_{dz} \cdot M_{pl,z,Rd}} \leq 1,0$$

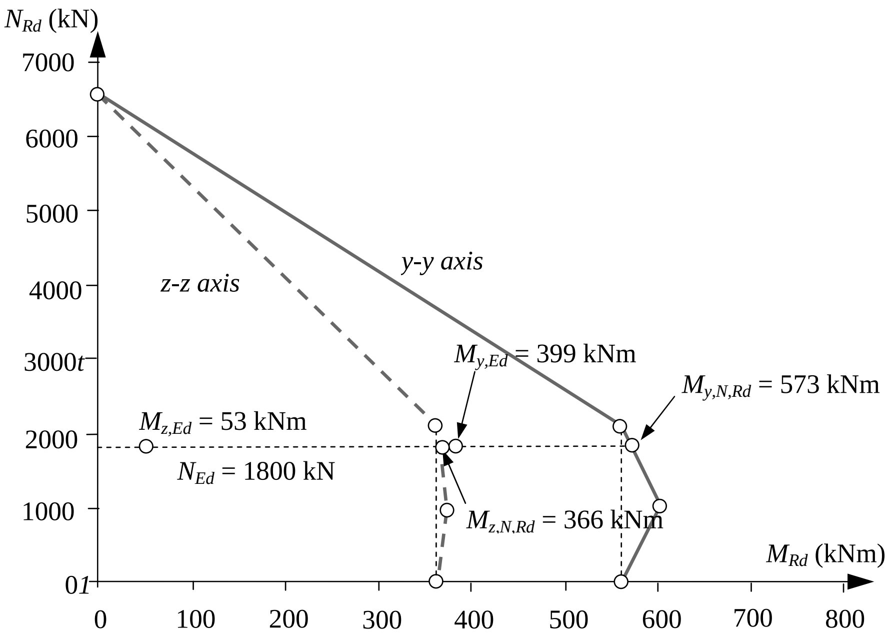


Figure C6.28 *Interaction polygon for bending about the y-y axis and the z-z axes - assumed bending failure about the y-y axis*

Substituting previously calculated values gives:

$$\frac{399}{0{,}9\cdot 1{,}02\cdot 562} = 0{,}77 < 1{,}0$$

$$\frac{53}{0{,}9\cdot 1{,}00\cdot 365} = 0{,}16 < 1{,}0$$

$$\frac{399}{1{,}02\cdot 562} + \frac{53}{1{,}00\cdot 365} = 0{,}70 + 0{,}15 = 0{,}85 < 1{,}0$$

Remark:

Therefore, the check of the resistance of the composite column to biaxial bending taking into account the design axial force N_{Ed} = 1800 kN and assuming bending failure about the *y-y* axis is satisfied.

12.3 Failure about the *z-z* axis is assumed

12.3.1 General

Failure about the *z-z* axis is assumed. The verification is carried out as follows:

- **Bending about the *y-y* axis**: It is necessary to check the resistance under compression, N_{Ed}, and bending, $M_{y,Ed}$, neglecting the equivalent member imperfection, $e_{0,y}$, but taking into account the second-order effects.

- **Bending about the *z-z* axis**: It is necessary to check the resistance under compression, N_{Ed}, and bending, $M_{z,Ed}$, taking into account the equivalent member imperfection, $e_{0,z}$, and the second-order effects.

- Finally, it is necessary to check the resistance under compression and biaxial bending in terms of the linear interaction curve M_y–M_z–N.

12.3.2 Calculation of the action effects about the y-y axis

In accordance with the detailed calculation given in Section 11.1.3.2, the design bending moment at mid-height $M_{y,Ed}$ neglecting the equivalent member imperfection, $e_{0,z}$, but taking into account the second-order effects is:

$$M_{y,Ed,II} = M_{y,Ed,I} \cdot k = 380,0 \cdot 0,84 = 319 \text{ kNm}$$

12.3.3 Calculation of the action effects about the z-z axis

In accordance with the detailed calculation given in Section 11.2.3.2, the design bending moment at mid-height $M_{z,Ed}$ taking into account the equivalent member imperfection, $e_{0,y}$, and the second-order effects is:

$$M_{z,Ed,II} = M_{z,Ed} \cdot k_1 + M_{z,Ed,imp} \cdot k_2 = 50 \cdot 1,05 + 84,6 \cdot 1,59 = 187 \text{ kNm}$$

12.3.4 Check of the resistance of the member in combined compression and biaxial bending

It is necessary to satisfy the following conditions:

$$\frac{M_{y,Ed}}{\alpha_{M,y} \cdot \mu_{dy} \cdot M_{pl,y,Rd}} \leq 1,0$$

$$\frac{M_{z,Ed}}{\alpha_{M,z} \cdot \mu_{dz} \cdot M_{pl,z,Rd}} \leq 1{,}0$$

$$\frac{M_{y,Ed}}{\mu_{dy} \cdot M_{pl,y,Rd}} + \frac{M_{z,Ed}}{\mu_{dz} \cdot M_{pl,z,Rd}} \leq 1{,}0$$

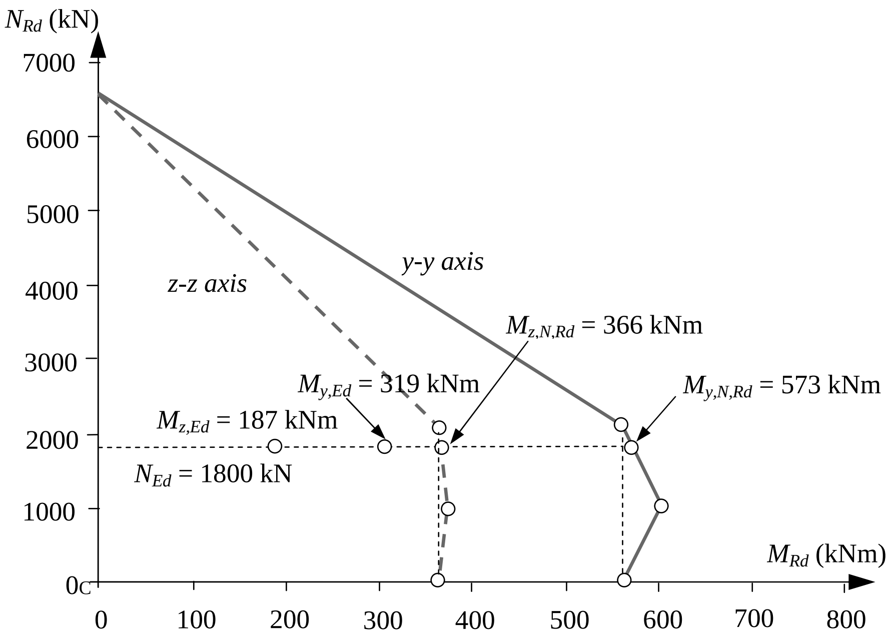


Figure C6.29 *Interaction polygon for bending about the y-y axis and the z-z axes - assumed bending failure about the z-z axis*

Substituting previously calculated values gives:

$$\frac{319}{0{,}9 \cdot 1{,}02 \cdot 562} = 0{,}62 < 1{,}0$$

$$\frac{187}{0{,}9 \cdot 1{,}00 \cdot 365} = 0{,}57 < 1{,}0$$

$$\frac{319}{1{,}02 \cdot 562} + \frac{187}{1{,}00 \cdot 365} = 0{,}56 + 0{,}51 = 1{,}07 > 1{,}0$$

Remark:

Therefore, the check of the resistance of the composite column to biaxial bending taking into account the design axial force N_{Ed} = 1800 kN and assuming bending failure about the *z-z* axis is not satisfied.

13. Commentary

This example illustrates the design of the composite column with fully concrete encased H-section subject to axial compressive load and biaxial bending. If it is not obvious which plane is the more critical, checks are made for both planes. Therefore, the following checks are needed:

a) The verification of the column resistance in axial compression only is carried out as the preliminary check. Since $\bar{\lambda}_z = 1{,}14 > \bar{\lambda}_y = 0{,}88$, the buckling resistance about the *z-z* axis is governed. The check of the composite column subjected to axial compression is satisfied. It is not necessary to select the stronger cross-section.

The utilization is 60%.

b) Check for bending about the *y-y* axis: The next step is to carry out the check of the column resistance in combined compression and uniaxial bending. The equivalent member imperfection $e_{0,z}$ is taken into account, which is in the same plane of the initial moment. In addition it was found that the second-order effects must be allowed for. The final step is to check that the cross-section can resist $M_{y,Ed}$ (consider imperfections and second-order analysis) with compression N_{Ed}.

The utilization is 77%.

c) Check for bending about the *z-z* axis: The next step is to carry out the check of the column resistance in combined compression and uniaxial bending. The equivalent member imperfection $e_{0,y}$ is taken into account, which is in the same plane of the initial moment. In addition it was found that the second-order effects must be allowed for. The final step is to check that the cross-section can resist $M_{z,Ed}$ (consider imperfections and second-order analysis) with compression N_{Ed}.

The utilization is 57%.

d) Check for biaxial bending: The failure about the *y-y* axis is assumed. For bending about the *y-y* axis, it is necessary to check the resistance under compression, N_{Ed}, and bending, $M_{y,Ed}$, taking into account the equivalent member imperfection, $e_{0,z}$, and the second-order effects. For bending about the *z-z* axis, it is necessary to check the resistance under compression, N_{Ed}, and bending, $M_{z,Ed}$, neglecting the equivalent member imperfection, $e_{0,y}$, but taking into account the second-order effects. Finally, it is necessary to check the resistance under compression and biaxial bending in terms of the linear interaction curve M_y–M_z–N.

The utilization is:

77% ($M_{y,Ed}$ with imperfection and second-order effects),
16% ($M_{z,Ed}$ without imperfection, but including second-order effects),
85% (interaction of $M_{y,Ed}$ and $M_{z,Ed}$).

e) Check for biaxial bending: The failure about the *z-z* axis is assumed. For bending about the *y-y* axis, it is necessary to check the resistance under compression, N_{Ed}, and bending, $M_{y,Ed}$, neglecting the equivalent member imperfection, $e_{0,y}$, but taking into account the second-order effects. For bending about the *z-z* axis, it is necessary to check the resistance under compression, N_{Ed}, and bending, $M_{z,Ed}$, taking into account the equivalent member imperfection, $e_{0,z}$, and the second-order effects. Finally, it is necessary to check the resistance under compression and biaxial bending in terms of the linear interaction curve M_y–M_z–N.

The utilization is:

62% ($M_{y,Ed}$ without imperfection, but including second-order effects),
57% ($M_{z,Ed}$ with imperfection and second-order effects),
107% (interaction of $M_{y,Ed}$ and $M_{z,Ed}$).

D Composite slabs

D1 Two-span composite slab unpropped at the construction stage

1. Purpose of example

This example demonstrates the design of a continuous composite slab over two spans. The composite slab consists of a cold-formed profiled steel sheeting covered with a concrete slab containing reinforcement. The composite slab is supported by steel beams, which act compositely with the concrete slab.

In composite slab design, we need to consider the construction stage and the composite stage. At the construction stage, the profiled steel sheeting acts as shuttering. The profiled sheeting has to carry its own weight, the wet concrete and the construction loads. In the composite stage, the slab is loaded with its own weight, the floor finishes and the variable load.

The composite slab is almost always continuous, because the profiled sheeting is provided in two-span lengths and the concrete is cast on the sheeting without joints. However, very often it is assumed that it is simply supported. According to clause 9.4.2(5), EN 1994-1-1, the continuous slab may be designed as a series of simply supported spans. In such cases, according to clause 9.8.1, EN 1994-1-1, the reinforcement for crack control is provided above internal supports.

The design resistance of the composite slab against longitudinal shear is carried out by the semi-empirical method called the *m*-*k* method. The method is based on two empirical factors, *m* and *k*. The design values of empirical factors *m* and *k* are based on slab tests and are provided by the manufacturer of the sheeting.

The partial connection method is an alternative to *m*-*k* method. This method also relies on tests on the composite slab to estimate the shear connection. Both of these methods can be applied in cases where the longitudinal shear behaviour is ductile. However, if the longitudinal behaviour is non-ductile, only the *m*-*k* method is permitted. According to clause B.3.5(1), EN 1994-1-1, in such cases the *m*-*k* method can be used but with an additional partial factor of 1,25, expressed by the reduction factor 0,8.

Composite Structures according to Eurocode 4. Worked Examples.
First Edition. Darko Dujmović, Boris Androić, Ivan Lukačević.

2. Static system, cross-section and actions

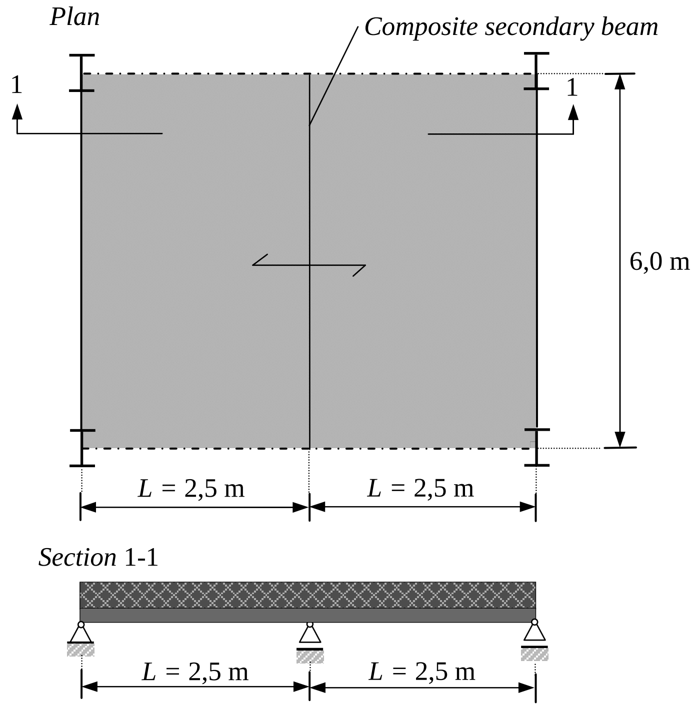


Figure D1.1 *Static system*

The profiled steel sheeting is continuous over two spans. For simplicity, it is assumed as simply supported span. This assumption is adopted only for the construction stage. The static system, adopted for verifications of profiled sheeting for ultimate limit state and serviceability limit state, is shown in Figure D1.2.

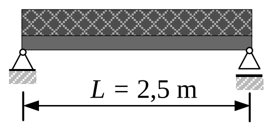


Figure D1.2 *Static system for the construction stage*

The cross-section of the composite slab and the cross-section of selected profiled sheeting with dimensions are shown in Figure D1.3.

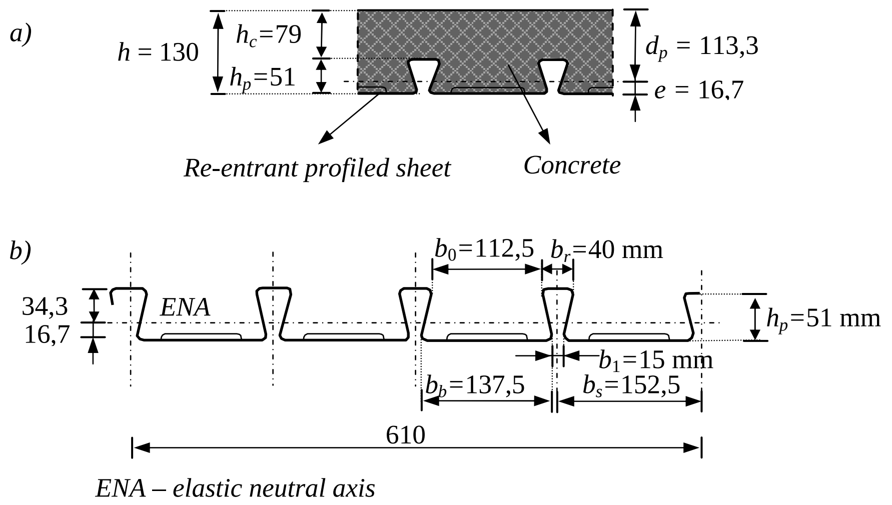


Figure D1.3 *Cross-sections: a) composite slab, b) profiled steel sheeting*

Actions

a) Permanent action

> **Remark:**
>
> According to EN 1991-1-1 the density of the normal weight concrete is 24 kN/m^3, increased by 1 kN/m^3 for normal percentage reinforcement, and increased for the wet concrete by another 1 kN/m^3.

Concrete slab area per m width:

$$A_c = 1000 \cdot h - (\frac{1000}{b_s} \cdot \frac{b_1 + b_r}{2} \cdot h_p)$$

$$A_c = 1000 \cdot 130 - (\frac{1000}{152{,}5} \cdot \frac{15 + 40}{2} \cdot 51) = 120800 \ \text{mm}^2 = 1208 \ \text{cm}^2$$

- *concrete slab and reinforcement (wet concrete):*

$$A_c \cdot 26 = 0{,}1208 \cdot 26 = 3{,}14 \ \text{kN/m}^2$$

- *concrete slab and reinforcement (dry concrete):*

$$A_c \cdot 25 = 0{,}1208 \cdot 25 = 3{,}02 \ \text{kN/m}^2$$

Construction stage

- *concrete slab* $= 3{,}14\ \text{kN/m}^2$

- *profiled steel sheeting* $= 0{,}16\ \text{kN/m}^2$

Total $g_{k,1} = 3{,}30\ \text{kN/m}^2$

Composite stage

- *concrete slab* $= 3{,}02\ \text{kN/m}^2$

- *profiled steel sheeting* $= 0{,}16\ \text{kN/m}^2$

Total $g_{k,2} = 3{,}18\ \text{kN/m}^2$

Floor finishes $g_{k,3} = 1{,}20\ \text{kN/m}^2$

b) Variable action

Construction stage

- *construction loads* $q_{k,1} = 1{,}50\ \text{kN/m}^2$

Composite stage

- *imposed floor load* $q_{k,2} = 7{,}0\ \text{kN/m}^2$

3. Properties of materials

Concrete strength class: C 25/30 $f_{ck} = 25\ \text{N/mm}^2$

$$f_{cd} = \frac{f_{ck}}{\gamma_c} = \frac{25}{1{,}5} = 16{,}7\ \text{N/mm}^2$$

$$0{,}85 f_{cd} = 0{,}85 \cdot 16{,}7 = 14{,}17\ \text{N/mm}^2$$

$$E_{cm} = 31000\ \text{N/mm}^2$$

Reinforcement: $f_{sk} = 500\ \text{N/mm}^2$

$$f_{sd} = \frac{f_{sk}}{\gamma_s} = \frac{500}{1{,}15} = 435\ \text{N/mm}^2$$

Profiled steel sheeting: $t = 1,1$ mm

$h_p = 51$ mm

$A_p = A_{pe} = 1938$ mm^2/m

$I_p = 68,5 \cdot 10^4$ mm^4/m

$E_p = E_a = 210\,000$ N/mm^2

$f_{yp,k} = 350$ N/mm^2

$$f_{yp,d} = \frac{f_{yp,k}}{\gamma_M} = \frac{350}{1,0} = 350 \text{ N/mm}^2$$

Resistance moment (provided by manufacturer): $M_{Rd} = 7,0$ kNm/m (sagging)

$M_{Rd} = 8,88$ kNm/m (hogging)

Empirical factors (provided by manufacturer): $m = 128,5$ N/mm^2

$k = 0$ N/mm^2

4. Structural details of composite slab

4.1 Slab thickness and reinforcement

The composite slab should satisfy the conditions given in clause 9.2, EN 1994-1-1.

a) The slab acts compositely with a beam, and the following conditions should be satisfied:

- the overall depth of slab $h \geq 90$ mm, $\rightarrow h = 130$ mm (satisfied),

- the thickness of concrete above the main flat surface of the top of the ribs of sheeting $h_c \geq 50$ mm, $\rightarrow h_c = 79$ mm (satisfied),

- the ratio of the width of the sheet rib to the rib spacing $\frac{b_r}{b_s} \leq$ 0,6, $\rightarrow$ $\frac{b_r}{b_s} = \frac{40}{152,5} = 0,26$ (satisfied).

b) The minimum amount of reinforcement in both directions should not be less than 80 mm^2/m. For the unpropped construction, the area of reinforcement, according to clause 9.8.1(2), EN 1994-1-1, is:

$$A_{s,\min} = 0,002 \cdot h_c \cdot b = 0,002 \cdot 79 \cdot 1000 = 158 \text{ mm}^2\text{/m} \rightarrow A_s = 80 \text{ mm}^2\text{/m}$$

The reinforcement bars are assumed to be 6 ϕ6/1000 mm. The cross-sectional area of reinforcement is:

$$A_s = 6 \cdot \frac{6^2 \cdot \pi}{4} = 170 \text{ mm}^2/\text{m}$$

c) Spacing of reinforcement bars

$$e < 2 \cdot h = 2 \cdot 130 = 260 \text{ or } < 350 \text{ mm}$$

4.2 Largest nominal aggregate size

$d_g \leq 0{,}4 \cdot h_c$ $\quad$ $0{,}4 \cdot 79 = 31{,}6$ mm
$d_g \leq b_0/3$ $\quad$ $112{,}5/3=37{,}5$ mm
$d_g \leq 31{,}5$ mm $\quad$ $= 31{,}5$ mm

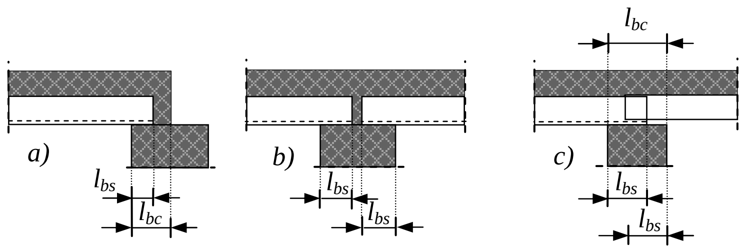
The minimum adopted value is $d_g = 31{,}5$ mm.

4.3 Minimum value for nominal thickness of steel sheet

In accordance with clause 3.5(2), EN 1994-1-1, the recommended value for the nominal thickness of steel sheet is 0,70 mm. The thickness of the selected profiled steel sheeting is 1,10 mm. The condition is satisfied.

4.4 Composite slab bearing requirements

According to clause 9.2.3(2), EN 1994-1-1, the recommended bearing lengths and support details differ depending upon the support material and they are different for internal supports and end supports, see Figure D1.4.



Bearing on	l_{bs} (mm)	l_{bc} (mm)
steel or concrete	50	75
other materials	70	100

Figure D1.4 *Minimum bearing lengths*

For composite slabs bearing on steel or concrete, the minimum bearing lengths are: l_{bc} = 75 mm and l_{bs} = 50 mm. The composite slab is supported by steel beams with the section IPE 500, and the top-flange width is 200 mm. Therefore, the condition for the bearing length is satisfied.

5. Ultimate limit state

5.1 Construction stage

At the construction stage, it is necessary to carry out verifications of profiled steel sheeting for the ultimate and serviceability limit states in accordance with EN 1993-1-3.

Profiled steel sheetings are normally continuous over two or more spans. However, in some cases, a single span is unavoidable due to the floor geometry.

Usually the manufacturer gives information on the properties of profiled steel sheeting. These properties are usually based on test results performed in accordance with EN 1993-1-3, Annex A. Characteristic and design values of resistance moment, crushing resistance, second moment of area etc. may be estimated using methods of reliability analysis in accordance with EN 1990. Properties of profiled steel sheeting estimated by calculation are more conservative than equivalent properties based on testing.

For simplicity, the profiled steel sheeting is considered as a simply supported span. The static system and the design load for the construction stage are shown in Figure D1.5.

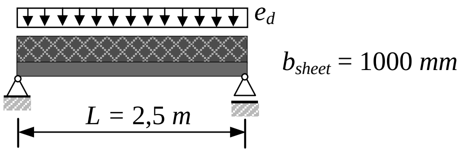


Figure D1.5 *Static system and design load for the construction stage*

Design load for ultimate limit state:

$$e_d = \gamma_G \cdot g_{k.1} + \gamma_Q \cdot q_{k,1}$$

$$e_d = 1,35 \cdot 3,30 + 1,5 \cdot 1,5 = 6,71 \text{ kN/m}^2$$

Therefore, the design values of bending moment and shear force are:

$$M_{Ed} = \frac{e_d \cdot L^2}{8} = \frac{6,71 \cdot 2,5^2}{8} = 5,24 \ \text{kNm/m}$$

$$V_{Ed} = \frac{e_d \cdot L}{2} = \frac{6,71 \cdot 2,5}{2} = 8,39 \ \text{kN/m}$$

Check for bending:

$$\frac{M_{Ed}}{M_{Rd}} \leq 1,0$$

$\frac{5,24}{7,0} = 0,75 < 1,0$, the condition is satisfied

In accordance with EN 1993-1-3, the following checks should be carried out:

- shear resistance of the cross-section according to clause 6.1.5, EN 1993-1-3,
- local resistance according to clause 6.1.7.3, EN 1993-1-3,
- combined bending and shear according to clause 6.1.10, EN 1993-1-3,
- combined web crushing and bending moment according to clause 6.1.11, EN 1993-1-3.

5.2 Composite stage

The continuous composite slab is designed as a series of simply supported spans, in accordance with clause 9.4.2(5), EN 1994-1-1, provided that the criterion for minimum reinforcement above internal supports of composite slab is satisfied, clause 9.8.1, EN 1994-1-1.

The static system and the design load for the composite stage are shown in Figure D1.6.

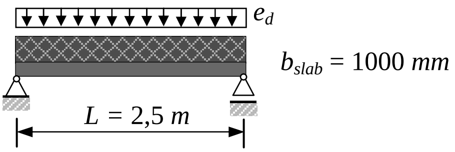


Figure D1.6 *Static system and design load for the composite stage*

Design load for ultimate limit state:

$$e_d = b \cdot (\gamma_G \cdot (g_{k,2} + g_{k,3}) + \gamma_Q \cdot q_{k,2})$$

$$e_d = 1{,}0 \cdot (1{,}35 \cdot (3{,}18 + 1{,}2) + 1{,}5 \cdot 7{,}0) = 16{,}4 \text{ kN/m}$$

Therefore, the design values of bending moment and shear force are:

$$M_{Ed} = \frac{e_d \cdot L^2}{8} = \frac{16{,}4 \cdot 2{,}5^2}{8} = 12{,}8 \text{ kNm/m}$$

$$V_{Ed} = \frac{e_d \cdot L}{2} = \frac{16{,}4 \cdot 2{,}5}{2} = 20{,}5 \text{ kN/m}$$

5.2.1 Plastic resistance moment in sagging region

It is assumed that the neutral axis lies above the sheeting. The assumed distribution of longitudinal bending stresses is shown in Figure D1.7. The design compressive force in concrete, $N_{c,f}$, is:

$$N_{c,f} = 0{,}85 \cdot f_{cd} \cdot h_c \cdot b \,, \quad b = 1000 \text{ mm}$$

$$N_{c,f} = 0{,}85 \cdot 16{,}7 \cdot 79 \cdot 1000 \cdot 10^{-3} = 1121 \text{ kN/m}$$

The design tensile force in the steel sheeting for a width of sheeting b is calculated with the characteristic of the effective steel section A_{pe}:

$$N_p = f_{yp,d} \cdot A_{pe}$$

$$N_p = 350 \cdot 1938 \cdot 10^{-3} = 678 \text{ kN/m}$$

Since $N_p < N_{c,f}$, the plastic neutral axis lies within the concrete. The design resistance moment in sagging region is calculated according to the distribution of stresses shown in Figure D1.7.

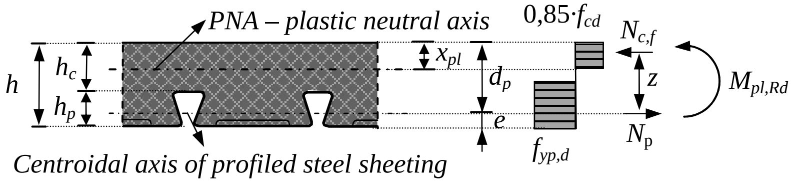


Figure D1.7 *Cross-section of composite slab and stress blocks for sagging bending*

The position of the plastic neutral axis of the composite section x_{pl} is:

$$x_{pl} = \frac{A_{pe} \cdot f_{yp,d}}{0,85 \cdot f_{cd} \cdot b}, \quad b = 1000 \text{ mm slab width}$$

$$x_{pl} = \frac{1938 \cdot 350}{0,85 \cdot 16,7 \cdot 1000} = 47,8 \text{ mm} < h_c = 79 \text{ mm}$$

For full shear connection, the design plastic resistance moment in sagging region $M_{pl,Rd}$ is calculated as:

$$M_{pl,Rd} = \min(N_{c,f}, N_p) \cdot z$$

$$M_{pl,Rd} = N_p \cdot (d_p - \frac{x_{pl}}{2})$$

$$M_{pl,Rd} = 678 \cdot (113,3 - \frac{47,8}{2}) \cdot 10^{-3} = 60,6 \text{ kNm/m}$$

Check:

$$\frac{M_{Ed}}{M_{pl,Rd}} \leq 1,0$$

$\frac{12,8}{60,6} = 0,21 < 1,0$, the condition is satisfied

The design plastic resistance moment in sagging region for full shear connection is adequate.

5.2.2 Longitudinal shear resistance

It is assumed that there is no end anchorage. Therefore, the longitudinal shear resistance is calculated according to clause 9.7.3, EN 1994-1-1. The design resistance of the composite slab against longitudinal shear is carried out by the semi-empirical method called the *m-k* method. According to clause 9.7.3(4), EN 1994-1-1, the maximum design vertical shear V_{Ed} for a width of slab b is limited due to the design longitudinal shear resistance $V_{l,Rd}$, given as:

$$V_{l,Rd} = \frac{b \cdot d_p}{\gamma_{vs}} \cdot (\frac{m \cdot A_p}{b \cdot L_s} + k)$$

where:

b, d_p are in mm,
A_p is the nominal cross-section of the sheeting in mm^2,
m, k are design values for the empirical factors in N/mm^2 obtained from slab tests meeting the basic requirements of the *m-k* method,
L_s is the shear span in mm, defined in clause 9.7.3(5), EN 1994-1-1,
γ_{vs} is the partial factor for ultimate limit state; the recommended value is 1,25.

If the *m-k* method is used, it should be verified that the maximum design vertical shear V_{Ed} does not exceed the design shear resistance $V_{l,Rd}$:

$$\frac{V_{Ed}}{V_{l,Rd}} \leq 1{,}0$$

Design values of empirical factors m and k are based on slab tests and are provided by the manufacturer of the sheeting:

$$m = 128{,}5 \text{ N/mm}^2$$

$$k = 0 \text{ N/mm}^2$$

According to clause 9.7.3(5), EN 1994-1-1, the shear span L_s for the uniform load applied to the entire span length is:

$$L_s = \frac{L}{4} = \frac{2500}{4} = 625 \text{ mm}$$

The design longitudinal shear resistance $V_{l,Rd}$ is:

$$V_{l,Rd} = \frac{b \cdot d_p}{\gamma_{vs}} \cdot \left(\frac{m \cdot A_p}{b \cdot L_s} + k\right)$$

$$V_{l,Rd} = [\frac{1000 \cdot 113{,}3}{1{,}25} \cdot (\frac{128{,}5 \cdot 1938}{1000 \cdot 625} + 0)] \cdot 10^{-3} = 36{,}1 \text{ kN/m}$$

This value, $V_{l,Rd}$ = 36,1 kN/m, must not be exceeded by the vertical shear in the slab.

Check:

$$\frac{V_{Ed}}{V_{l,Rd}} \leq 1{,}0$$

$$\frac{20{,}5}{36{,}1} = 0{,}57 < 1{,}0 \text{ , the condition is satisfied}$$

5.2.3 Check for vertical shear resistance

According to 9.7.5, EN 1994-1-1, the vertical shear resistance, $V_{v,Rd}$, should be determined according to the method given in EN 1992-1-1. According to clause 6.2.2., EN 1992-1-1, the design shear resistance $V_{v,Rd}$ is calculated as:

$$V_{v,Rd} = V_{Rd,c} = [C_{Rd,c} \cdot k \cdot (100 \cdot \rho_l \cdot f_{ck})^{1/3} + k_1 \cdot \sigma_{cp}]\ b_w \cdot d_p \geq V_{v,Rd,\min}$$

The minimum value of $V_{v,Rd,\min}$ is:

$$V_{v,Rd,\min} = (v_{\min} + k_1 \cdot \sigma_{cp}) \cdot b_w \cdot d_p$$

The minimum requirement for $V_{v,Rd}$ is related to the fact that the member without reinforcement still has some shear resistance.

Generally, the check is carried out as follows:

$$\frac{V_{Ed}}{V_{v,Rd}} \leq 1{,}0$$

According to clause 6.2.2(1), EN 1992-1-1, the values needed for calculation $V_{v,Rd}$ are:

$$C_{Rd,c} = \frac{0{,}18}{\gamma_c} = \frac{0{,}18}{1{,}5} = 0{,}12$$

$$k = 1 + \sqrt{\frac{200}{d_p}} \leq 2{,}0$$

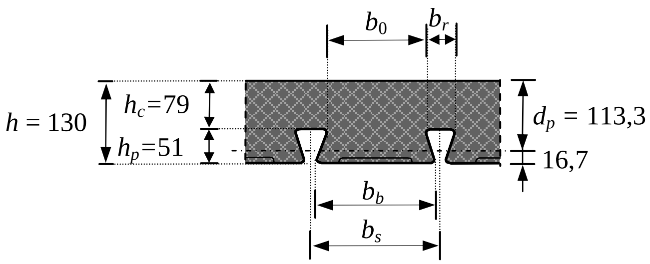
$$k = 1 + \sqrt{\frac{200}{113{,}3}} = 2{,}32 \ \rightarrow \text{ adopted } k = 2{,}0$$

$$\rho_l = \frac{A_{sl}}{b_w \cdot d_p} \leq 0{,}02$$

The resistance of the cross-section is dependent on the area of the tensile reinforcement, whose section has to be extended by an appropriate anchorage length, (l_{bd} + d) see – Figure 6.3, EN 1992-1-1 – where l_{bd} is the design anchorage length and d is the effective depth of the section, taken as the depth from the top surface to the centroid of the profile for a composite slab. The anchorage of the profiled sheeting was confirmed by the check on longitudinal shear, and the sheeting can be treated as reinforcement, i.e. $A_{sl} = A_{pe} = 1938$ mm^2.

In accordance with Figure D1.8, the smallest width of the cross-section in the tensile area b_w is calculated per metre width as follows:

$$b_w = \frac{b}{b_s} \cdot b_0 = \frac{1000}{152{,}5} \cdot 112{,}5 = 738 \text{ mm/m}$$



b_s = 152,5 mm
b_r = 40 mm
b_0 = 112,5 mm

Figure D1.8 *Determination of value b_w*

The percentage of longitudinal reinforcement is:

$$\rho_l = \frac{A_{sl}}{b_w \cdot d_p} \leq 0{,}02$$

$$\rho_l = \frac{1938}{738 \cdot 113{,}3} = 0{,}023 > 0{,}020$$

The value $\rho_l = 0{,}02$ is adopted.

The design axial force is $N_{Ed} = 0$ and therefore $\sigma_{cp} = \frac{N_{Ed}}{A_c} = 0$.

$k_1 = 0{,}15$, according to clause 6.2.2(1), EN 1992-1-1

The design shear resistance $V_{v,Rd}$ is:

$$V_{v,Rd} = V_{Rd,c} = [C_{Rd,c} \cdot k \cdot (100 \cdot \rho_l \cdot f_{ck})^{1/3} + k_1 \cdot \sigma_{cp}]\ b_w \cdot d_p$$

$$V_{v,Rd} = [0{,}12 \cdot 2{,}0 \cdot (100 \cdot 0{,}02 \cdot 25)^{1/3} + 0{,}15 \cdot 0]\ 738 \cdot 113{,}3 \cdot 10^{-3}$$

$$V_{v,Rd} = 73{,}9\ \text{kN/m}$$

The minimum value is:

$$V_{v,Rd,\min} = (v_{\min} + k_1 \cdot \sigma_{cp}) \cdot b_w \cdot d_p$$

$$v_{\min} = 0{,}035 \cdot k^{3/2} \cdot f_{ck}^{\ 1/2} = 0{,}035 \cdot 2{,}0^{3/2} \cdot 25^{1/2} = 0{,}49\ \text{N/mm}^2$$

$$V_{v,Rd,\min} = (0{,}49 + 0{,}15 \cdot 0) \cdot 738 \cdot 113{,}3 \cdot 10^{-3} = 41\ \text{kN/m} < V_{v,Rd} = 73{,}9\ \text{kN/m}$$

Check:

$$\frac{V_{Ed}}{V_{v,Rd}} \leq 1{,}0$$

$\frac{20{,}5}{73{,}9} = 0{,}28 < 1{,}0$, the condition is satisfied

Remark:

Since it is unlikely that the profiled steel sheet can satisfy the requirement “full anchorage”, the design shear resistance is equal to the minimum value:

$$V_{v,Rd} = V_{v,Rd,\min} = 41\ \text{kN/m}$$

Also, the required condition is satisfied since that is $V_{v,Rd} = V_{v,Rd,\min} = 41\ \text{kN/m} >$ $V_{Ed} = 20{,}5\ \text{kN/m}$.

6. Serviceability limit state

6.1 Control of cracking of concrete

Since the slab is designed as simply supported, it only requires reinforcement for crack width limitation.

According to clause 9.8.1(2), EN 1994-1-1, for unpropped construction the required cross-sectional area of reinforcement A_s is 0,2% of the area of concrete above the ribs:

$$A_s = \frac{0,2}{100} \cdot 1000 \cdot (h - h_p) \ \text{mm}^2/\text{m}$$

$$A_s = \frac{0,2}{100} \cdot 1000 \cdot (130 - 51) = 158 \ \text{mm}^2/\text{m}$$

The reinforcement bars are assumed to be 6 ϕ6/1000 mm. Therefore, the cross-sectional area of reinforcement is:

$$A_s = 6 \cdot \frac{6^2 \cdot \pi}{4} = 170 \ \text{mm}^2/\text{m} > 158 \ \text{mm}^2/\text{m}$$

The selected minimum amount of reinforcement may be insufficient to control cracking at the supports of continuous slabs for certain exposure classes. In such cases, the slab should be designed as continuous, and in hogging regions the crack widths should be estimated according to EN 1992-1-1.

6.2 Limit of span/depth ratio of slab

According to clause 9.8.2(4), EN 1994-1-1, calculation of the deflection of the composite slab can be omitted if the two conditions are satisfied. According to the first condition, the span/depth ratio of the slab should not exceed the limits given in EN 1992-1-1. These are:

- $\frac{L}{d} < 20$ for a simply supported span
- $\frac{L}{d} < 26$ for an external span of continuous slab
- $\frac{L}{d} < 30$ for an internal span of continuous slab

According to clause 9.8.2(6), EN 1994-1-1, the second condition is given as:

- the load causing an end slip of 0,5 mm in the tests on composite slab exceeds 1,2 times the design service load.

If the second condition is not satisfied, i.e. the end slip exceeds 0,5 mm at a load 1,2 times the design service load, two options exist:

- end anchors should be provided, or
- deflections should be calculated including the effect of end slip.

According to clause 9.8.2(8), EN 1994-1-1, in cases where the behaviour of the shear connection between the profiled sheeting and the concrete are not known from tests, the tied-arch model may be used, see [34].

For the considered slab, with L = 2500 mm and d_p = 113,3 mm, the following span/depth ratio is obtained:

- $\frac{L}{d} = \frac{2500}{113,3} = 22 < 26$ for the external span of continuous slab

Therefore it is not necessary to carry out the calculation of the deflection. However, the calculation is carried out for educational reasons.

6.3 Calculation of deflections

6.3.1 Construction stage deflection

According to clause 9.6(2), EN 1994-1-1, the deflection, δ_s, of the profiled sheeting due to its own weight and the weight of wet concrete should not exceed the following limit:

$$\delta_{s,\max} = \frac{L}{180} = \frac{2500}{180} = 14 \text{ mm}$$

For simplicity, the profiled steel sheeting is considered as a simply supported span. The static system and the design load for the construction stage are shown in Figure D1.9.

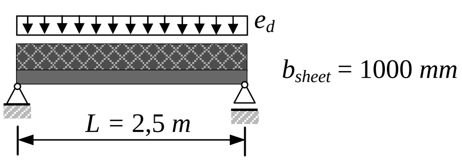


Figure D1.9 *Static system and design load for the construction stage*

The premature local buckling of the profiled sheeting under the weight of wet concrete and construction loading is checked to prevent irreversible deformation. This verification is important in regions of internal support.

Design load for the serviceability limit state is:

$$e_d = b \cdot g_{k.1}$$

$$e_d = 1,0 \cdot 3,30 = 3,30 \text{ kN/m}$$

Maximum sagging bending moment in the serviceability limit state is:

$$M_{Ed} = \frac{e_d \cdot L^2}{8} = \frac{3,30 \cdot 2,5^2}{8} = 2,58 \text{ kNm/m}$$

Maximum compressive stress in the top flange of the profiled sheeting is:

$$\sigma_{com} = \frac{M_{Ed}}{\mathrm{I}_p} \cdot z = \frac{2,58 \cdot 10^6}{68,5 \cdot 10^4} \cdot (51 - 16,7) = 129 \text{ N/mm}^2$$

In accordance with clause 4.4, EN 1993-1-5, the plate slenderness, $\bar{\lambda}_p$, is calculated as:

$$\bar{\lambda}_p = \sqrt{\frac{f_y}{\sigma_{cr}}} = \frac{\frac{b}{t}}{28,4 \cdot \varepsilon \cdot \sqrt{k_\sigma}}$$

$$\varepsilon = \sqrt{\frac{235}{\sigma_{com}}} = \sqrt{\frac{235}{129}} = 1,35$$

According to Table 4.1, EN 1994-1-1, for the stress ratio $\psi = 1$, the buckling factor is $k_\sigma = 4$.

Therefore, the plate slenderness, $\bar{\lambda}_p$, with the design thickness of the sheet $t = 1,06$ mm (not including coatings) and $b = b_r = 40$ mm, is:

$$\bar{\lambda}_p = \frac{\frac{40}{1,06}}{28,4 \cdot 1,35 \cdot \sqrt{4}} = 0,492$$

Since that is $\overline{\lambda}_p = 0,492 < 0,673$, the reduction factor is ρ = 1,0 and the cross-section is fully effective.

The deflection of the profiled steel sheeting for the simply supported span, Figure D1.9, is:

$$\delta_1 = \frac{5}{384} \cdot \frac{e_d \cdot L^4}{E_a \cdot I_p}$$

$$\delta_1 = \frac{5}{384} \cdot \frac{3,30 \cdot 2500^4}{210000 \cdot 68,5 \cdot 10^4} = 11,7 \text{ mm}$$

$$\delta_1 = 11,7 \text{ mm} < \delta_{s,\max} = \frac{L}{180} = \frac{2500}{180} = 14 \text{ mm}$$

The deflection due to the self-weight of profiled sheeting and the weight of the wet concrete meets the criterion L/180.

Since the deflection δ_1 is less than 10% of the slab depth, δ_1 = 11,7 mm < 0,10·h = 0,1·130 = 13 mm, according to clause 9.3.2(2), EN 1994-1-1, the ponding effects can be neglected at the construction stage.

The conditions for the serviceability limit state are satisfied, and the profiled steel sheeting can be used at the construction stage.

6.3.2 Composite stage deflection

For the calculation of the deflection at the composite stage, the slab is considered as continuous over two spans. According to clause 9.8.2(5), EN 1994-1-1, the following approximations can be applied:

- The second moment of area can be taken as the average of the values for the cracked and uncracked section.
- An average value of the modular ratio, n, for both short-term and long-term effects can be used:

$$n = \frac{E_a}{E'_{cm}} = \frac{E_a}{\frac{1}{2} \cdot (E_{cm} + \frac{E_{cm}}{3})} = \frac{210000}{\frac{2}{3} \cdot 31000} = 10,2$$

- Elastic analysis is used to calculate the deflection of the slab.

a) The second moment of area for the cracked section, I_{cc}, for slab width b is calculated in accordance with Figure D1.10.

The second moment of area for the cracked section and the slab width b is calculated as:

$$I_{cc} = \frac{b \cdot x_c^{\,3}}{3 \cdot n} + A_p \cdot (d_p - x_c)^2 + I_p$$

The position of the elastic neutral axis relative to the upper side of the slab is obtained as:

$$x_c = \frac{\Sigma A_i \cdot z_i}{\Sigma A_i} = \frac{n \cdot A_p}{b} \left(\sqrt{1 + \frac{2 \cdot b \cdot d_p}{n \cdot A_p}} - 1 \right)$$

$$x_c = \frac{10{,}2 \cdot 1938}{1000} \cdot \left(\sqrt{1 + \frac{2 \cdot 1000 \cdot 113{,}3}{10{,}2 \cdot 1938}} - 1 \right) = 50{,}0 \text{ mm}$$

The second moment of area for the cracked section is:

$$I_{cc} = \frac{1000 \cdot 50{,}0^3}{3 \cdot 10{,}2} + 1938 \cdot (113{,}3 - 50{,}0)^2 + 685000 = 12{,}54 \cdot 10^6 \text{ mm}^4\text{/m}$$

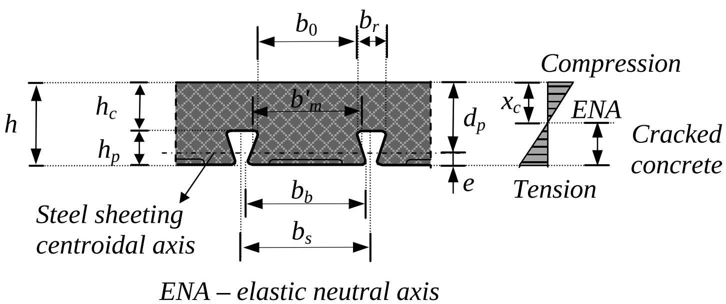


Figure D1.10 *Second moment of area calculation for cracked cross-section, I_{cc}*

b) The second moment of area for the uncracked section, I_{cu}, for slab width b is calculated in accordance with Figure D1.11.

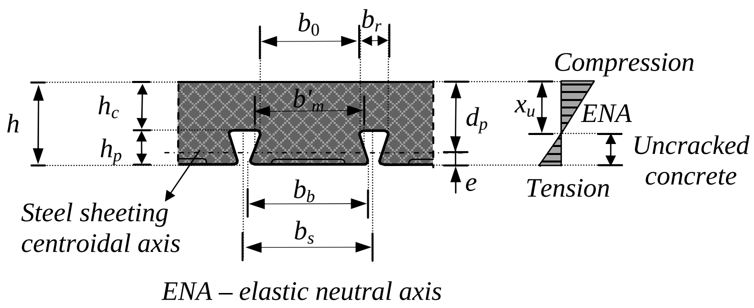


Figure D1.11 *Second moment of area calculation for uncracked cross-section, I_{cu}*

The second moment of area for the uncracked section and the slab width b is calculated as:

$$I_{cu} = \frac{b \cdot h_c^3}{12 \cdot n} + \frac{b \cdot h_c}{n} \cdot (x_u - \frac{h_c}{2})^2 + \frac{b_m \cdot h_p^3}{12 \cdot n} + \frac{b_m \cdot h_p}{n} \cdot (h_t - x_u - \frac{h_p}{2})^2 +$$
$$A_p \cdot (d_p - x_u)^2 + I_p$$

where:

$$x_u = \frac{b \cdot \frac{h_c^2}{2} + b_m \cdot h_p \cdot (h_t - \frac{h_p}{2}) + n \cdot A_p \cdot d_p}{b \cdot h_c + b_m \cdot h_p + n \cdot A_p}$$

In accordance with Figure D1.11, the value of b_m is:

$$b'_m = \frac{(b_b - 2 \cdot t) + b_0}{2} = \frac{(137{,}5 - 2 \cdot 1{,}10) + 112{,}5}{2} = 123{,}9 \ \text{mm}$$

$$b_m = \frac{b}{b_s} \cdot b'_m = \frac{1000}{152{,}5} \cdot 123{,}9 = 812 \ \text{mm/m}$$

The position of the elastic neutral axis relative to the upper side of the slab is:

$$x_u = \frac{1000 \cdot \frac{79^2}{2} + 812 \cdot 51 \cdot (130 - \frac{51}{2}) + 10{,}2 \cdot 1938 \cdot 113{,}3}{1000 \cdot 79 + 812 \cdot 51 + 10{,}2 \cdot 1938} = 69{,}1 \ \text{mm}$$

The second moment of area for the uncracked section is:

$$I_{cu} = \frac{1000 \cdot 79^3}{12 \cdot 10,2} + \frac{1000 \cdot 79}{10,2} \cdot (69,1 - \frac{79}{2})^2 + \frac{812 \cdot 51^3}{12 \cdot 10,2} + \frac{812 \cdot 51}{10,2} \cdot$$

$$\cdot (130 - 69,1 - \frac{51}{2})^2 + 1938 \cdot (113,3 - 69,1)^2 + 685000$$

$$I_{cu} = 21,25 \cdot 10^6 \text{ mm}^4/\text{m}$$

The mean value of I_{cc} and I_{cu} is:

$$I_c = \frac{I_{cc} + I_{cu}}{2}$$

$$I_c = \frac{12,54 \cdot 10^6 + 21,25 \cdot 10^6}{2} = 16,90 \cdot 10^6 \text{ mm}^4/\text{m}$$

Calculation of deflections

- Deflection due to permanent action

The design load of the weight of dry concrete, the weight of the profiled sheeting and the floor finishes is:

$$e_d = b \cdot (g_{k,2} + g_{k,3}) = 1,0 \cdot (3,18 + 1,20) = 4,38 \text{ kN/m}$$

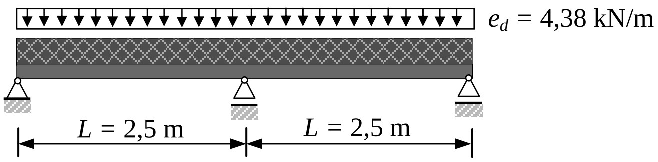


Figure D1.12 *Static system and load for calculation of deflection at the composite stage*

The deflection is:

$$\delta_1 = 0,0054 \cdot \frac{e_d \cdot L^4}{E \cdot I_c}$$

$$\delta_1 = 0,0054 \cdot \frac{4,38 \cdot 2500^4}{210000 \cdot 16,90 \cdot 10^6} = 0,26 \text{ mm} = L/9615$$

- Deflection due to frequent value of variable action and the selected combination factor is $\psi_1 = 0{,}7$

The design load is calculated for the frequent combination:

$$e_d = b \cdot \psi_1 \cdot q_{k,2} = 1{,}0 \cdot 0{,}7 \cdot 7{,}0 = 4{,}9 \text{ kN/m}$$

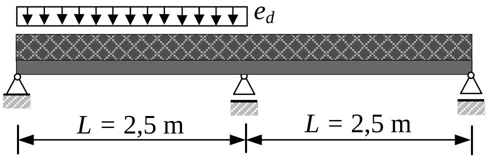


Figure D1.13 *Static system and load for calculation of deflection at the composite stage*

The deflection is:

$$\delta_2 = 0{,}0099 \cdot \frac{e_d \cdot L^4}{E_a \cdot I_c}$$

$$\delta_2 = 0{,}0099 \cdot \frac{4{,}9 \cdot 2500^4}{210000 \cdot 16{,}9 \cdot 10^6} = 0{,}53 \text{ mm} = \text{L/4716}$$

Remark:

The limit of the deflection is adopted according to clause 7.4.1(4), EN 1992-1-1. The recommended limitation is:

$$\delta_{total} \leq \frac{L}{250}$$

The total deflection is:

$$\delta_{total} = \delta_1 + \delta_2 = 0{,}26 + 0{,}53 = 0{,}79 \text{ mm} \leq \frac{L}{250} = \frac{2500}{250} = 10{,}0 \text{ mm}$$

The total deflection meets the criterion L/250.

7. Commentary

The design of composite slabs is mainly based on data provided by the supplier of the profiled sheeting. However, the reliability of the obtained data is very important for structural reliability.

At the construction stage, profiled steel sheetings act as both a working platform and also permanent formwork and they can stabilise beams during execution. The requirements for the construction stage are governed for the design of profiled steel sheetings.

The area of cross-section of profiled steel sheeting that satisfies the criteria for the construction stage usually provides enough bottom reinforcement for the composite slab. In such cases, the composite slabs are considered as simply supported. The top longitudinal reinforcement at internal supports should be provided to control the widths of cracks. It is recommended that reinforcement for crack control is provided in the form of a mesh over the full area of the slab. In this way, the mesh reinforcement has very favourable structural effects under fire conditions.

D2 Three-span composite slab propped at the construction stage

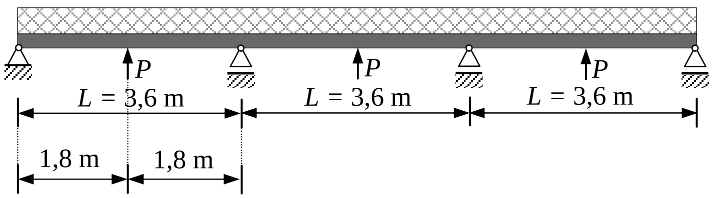
1. Purpose of example

This example demonstrates the design of continuous composite slab over three spans. We need to consider the construction stage, where the profiled steel sheeting acts as shuttering, and the composite stage, where the concrete and the steel sheeting form a composite unit. For practical reasons, the unpropped construction of composite slab is more acceptable solution; its speed and simplicity of execution is a great advantage. When unpropped construction is used, the profiled steel sheeting alone resists the selfweight of the wet concrete and construction loads. However, for educational reasons, in this example the design of a composite slab is illustrated where the profiled steel sheeting is temporarily propped during construction.

The design resistance of the composite slab against longitudinal shear is carried out by the partial interaction method. This method relies on tests on composite slabs to estimate the shear connection and it can be applied in cases where the longitudinal shear behaviour is ductile.

2. Static system, cross-section and actions

a) Construction stage – the profiled steel sheeting acts as shuttering



P – temporary props during casting of the concrete

Figure D2.1 *Static system of profiled sheeting for the construction stage*

b) Composite stage

The composite slab is continuous, see Figure D2.2. However, according to clause 9.4.2(5), EN 1994-1-1, the continuous slab may be designed as a series of simply-supported spans.

Composite Structures according to Eurocode 4. Worked Examples.
First Edition. Darko Dujmović, Boris Androić, Ivan Lukačević.

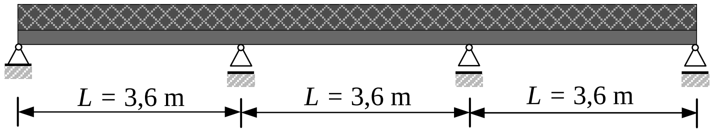


Figure D2.2 *Static system of the composite slab*

c) Cross-section of composite slab

The profiled steel sheeting, shown in Figure D2.3, used in unpropped construction, is optimised to suit beam centres in the region of 2,6 to 3,3 m. However, in this example, props are used for pedagogical reasons.

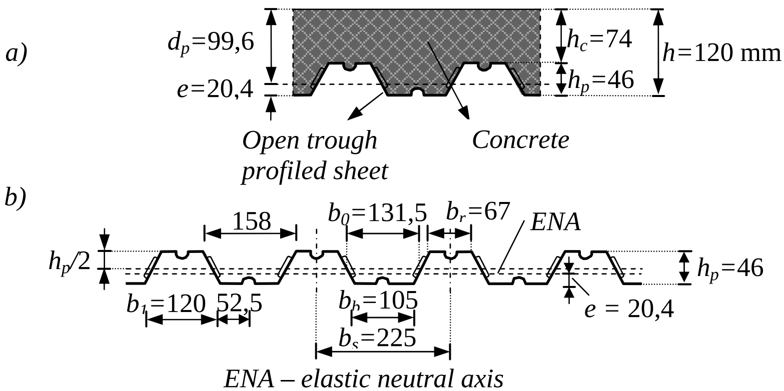


Figure D2.3 *Cross-sections: a) composite slab, b) profiled steel sheeting*

Actions

a) Permanent action

Remark:

According to EN 1991-1-1 the density of the normal weight concrete is 24 kN/m³, increased by 1 kN/m³ for normal percentage reinforcement, and increased for the wet concrete by another 1 kN/m³.

Concrete slab area per m width:

$$A_c = 1000 \cdot h - \left(\frac{1000}{b_s} \cdot \frac{b_1 + b_r}{2} \cdot h_p\right)$$

$$A_c = 1000 \cdot 120 - \left(\frac{1000}{225} \cdot \frac{120 + 67}{2} \cdot 46\right) = 100900 \ \text{mm}^2 = 1009 \ \text{cm}^2$$

- *concrete slab and reinforcement (wet concrete):*

$$A_c \cdot 26 = 0{,}1009 \cdot 26 = 2{,}62 \text{ kN/m}^2$$

- *concrete slab and reinforcement (dry concrete):*

$$A_c \cdot 25 = 0{,}1009 \cdot 25 = 2{,}52 \text{ kN/m}^2$$

Construction stage

- *concrete slab* $g_{c,1} = 2{,}62$ kN/m^2

- *profiled steel sheeting* $g_p = 0{,}09$ kN/m^2

Total $g_{k,1} = g_{c,1} + g_p = 2{,}71$ kN/m^2

Composite stage

- *concrete slab* $g_{c,2} = 2{,}52$ kN/m^2

- *profiled steel sheeting* $g_p = 0{,}09$ kN/m^2

Total $g_{k,2} = g_{c,2} + g_p = 2{,}61$ kN/m^2

Floor finishes $g_{k,3} = 1{,}20$ kN/m^2

b) Variable action

Construction stage

- *construction loads* $q_{k,1} = 1{,}50$ kN/m^2

Composite stage

- *imposed floor load* $q_{k,2} = 5{,}0$ kN/m^2

Remark:

Clause 9.3.2(1), EN 1994-1-1, refers to clause 4.11, EN 1991-1-6, for construction loads. According to clause 4.11.2(1), EN 1991-1-6, the actions from personnel and equipment, q_k, are referred to as "10% of the self-weight of the concrete, but not less than 0,75 and not more than 1,5 kN/m^2". Also, according to clause 4.11, EN 1991-1-6, the load of 1,50 kN/m^2 acts on the working area of 3,0 × 3,0 m, and outside the working area the load is 0,75 kN/m^2.

3. Properties of materials

Concrete strength class: C 25/30 $f_{ck} = 25 \text{ N/mm}^2$

$$f_{cd} = \frac{f_{ck}}{\gamma_c} = \frac{25}{1,5} = 16,7 \text{ N/mm}^2$$

$$0,85 \cdot f_{cd} = 0,85 \cdot 16,7 = 14,17 \text{ N/mm}^2$$

$$E_{cm} = 31\,000 \text{ N/mm}^2$$

Reinforcement: $f_{sk} = 500 \text{ N/mm}^2$

$$f_{sd} = \frac{f_{sk}}{\gamma_s} = \frac{500}{1,15} = 435 \text{ N/mm}^2$$

Profiled steel sheeting: $t = 0,9 \text{ mm}$

$$h_p = 46 \text{ mm}$$

$$A_p = A_{pe} = 1137 \text{ mm}^2/\text{m}$$

$$I_p = 41,5 \cdot 10^4 \text{ mm}^4/\text{m}$$

$$E_p = E_a = 210\,000 \text{ N/mm}^2$$

$$f_{yp,k} = 280 \text{ N/mm}^2$$

$$f_{yp,d} = \frac{f_{yp,k}}{\gamma_M} = \frac{280}{1,0} = 280 \text{ N/mm}^2$$

Plastic resistance moment (provided by manufacturer): $M_{pa,Rk} = 5,70 \text{ kNm/m}$

Resistance moment (provided by manufacturer): $M_{Rk} = 4,63 \text{ kNm/m (sagging)}$

$M_{Rk} = 4,67 \text{ kNm/m (hogging)}$

Resistance to support reaction (provided by manufacturer): $R_{w,k} = 34,0 \text{ kN/m}$

Resistance to horizontal shear (provided by manufacturer): $\tau_{u,Rk} = 0,306 \text{ N/mm}^2$

4. Structural details of composite slab

4.1 Slab thickness and reinforcement

The composite slab should satisfy the conditions given in clause 9.2, EN 1994-1-1.

a) The slab does not act compositely with a beam, nor is it used as a diaphragm, so the following conditions should be satisfied:

- the overall depth of slab $h \geq 80$ mm, $\rightarrow h = 120$ mm (satisfied),

- the thickness of concrete above the main flat surface of the top of the ribs of sheeting $h_c \geq 40$ mm, $\rightarrow h_c = 74$ mm (satisfied),
- the ratio of the width of the sheet rib to the rib spacing $\dfrac{b_r}{b_s} \leq$ 0,6, $\rightarrow$

  $\dfrac{67}{225} = 0,30 < 0,60$ (satisfied).

b) The minimum amount of reinforcement in both directions should not be less than 80 mm^2/m. For propped construction, the area of reinforcement, according to clause 9.8.1(2), EN 1994-1-1, is:

$$A_{s\min} = 0,004 \cdot h_c \cdot b = 0,004 \cdot 74 \cdot 1000 = 296 \text{ mm}^2\text{/m} \rightarrow A_s = 80 \text{ mm}^2\text{/m.}$$

The reinforcement bars are assumed to be ϕ8/160 mm. The cross-sectional area of reinforcement is:

$$A_s = \frac{8^2 \cdot \pi}{4} \cdot \frac{1000}{160} = 314 \text{ mm}^2\text{/m}$$

c) Spacing of reinforcement bars

$$e < 2 \cdot h = 2 \cdot 120 = 240 \text{ or} < 350 \text{ mm.}$$

4.2 Largest nominal aggregate size

$d_g \leq 0,4 \cdot h_c$	$0,4 \cdot 74 = 29,6$ mm
$d_g \leq b_0/3$	131,5/3=43,8 mm
$d_g \leq 31,5$ mm	= 31,5 mm

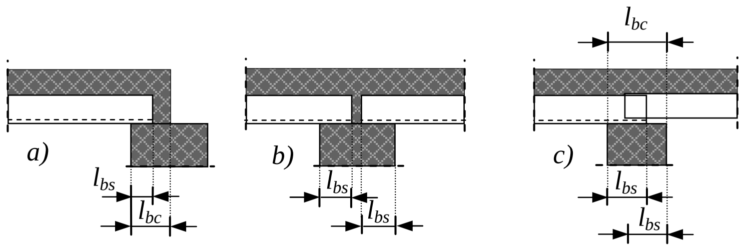
The minimum adopted value is $d_g = 29,6$ mm.

4.3 Minimum value for nominal thickness of steel sheet

In accordance with clause 3.5(2), EN 1994-1-1, the recommended value for the nominal thickness of steel sheet is 0,70 mm. The thickness of the selected profiled steel sheeting is 0,90 mm. The condition is satisfied.

4.4 Composite slab bearing requirements

According to clause 9.2.3(2), EN 1994-1-1, the recommended bearing lengths and support details differ depending upon the support material, and they are different for internal supports and end supports, see Figure D2.4.



Bearing on	l_{bs} *(mm)*	l_{bc} *(mm)*
steel or concrete	50	75
other materials	70	100

Figure D2.4 *Minimum bearing lengths*

For composite slabs bearing on steel or concrete, the minimum bearing lengths are: l_{bc} = 75 mm and l_{bs} = 50 mm. The composite slab is supported by steel beams with the top flange width larger than 75 mm. Therefore, the condition for the bearing length is satisfied.

5. Ultimate limit state

5.1 Construction stage

At the construction stage, it is necessary to carry out verifications of profiled steel sheeting for the ultimate and serviceability limit state in accordance with EN 1993-1-3.

Usually, the manufacturer gives information on the properties of profiled steel sheeting. These properties are usually based on test results performed in accordance with EN 1993-1-3, Annex A. Characteristic and design values of resistance moment, crushing resistance, second moment of area etc. may be estimated using methods of reliability analysis in accordance with EN 1990. Properties of profiled steel sheeting estimated by calculation are more conservative than equivalent properties based on testing.

Maximum sagging bending moment

The profiled steel sheeting acts as shuttering and carries its own weight, the wet concrete and the construction loads. The static system and loads are shown in Figure D2.5.

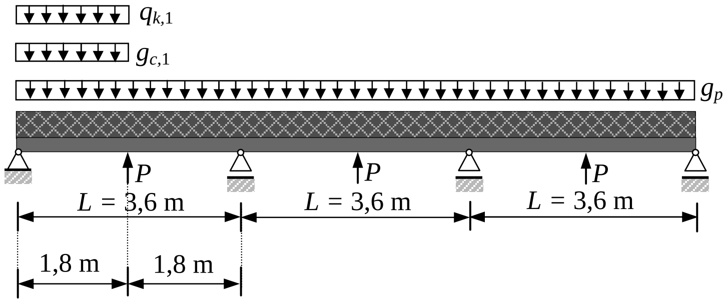


Figure D2.5 *Static system and loads for the construction stage*

The design value of sagging bending moment is:

$$M_{Ed} = \gamma_G \cdot M_g + \gamma_Q \cdot M_q$$

$$M_g = M_{g_p} + M_{g_{c,1}} = 0,078 \cdot 0,09 \cdot 1,8^2 + 0,094 \cdot 2,62 \cdot 1,8^2 = 0,82 \text{ kNm/m}$$

$$M_q = 0,094 \cdot 1,5 \cdot 1,8^2 = 0,46 \text{ kNm/m}$$

$$M_{Ed} = 1,35 \cdot 0,82 + 1,5 \cdot 0,46 = 1,80 \text{ kNm/m}$$

Verifications for the profiled steel sheeting are carried out in accordance with EN 1993-1-3. However, since the characteristic resistance moment is provided by the manufacturer, the check is carried out with this value:

$$M_{Rd} = \frac{M_{Rk}}{\gamma_{M_0}} = \frac{4,63}{1,0} = 4,63 \text{ kNm/m}$$

Check:

$$\frac{M_{Ed}}{M_{Rd}} \leq 1,0$$

$\frac{1,80}{4,63} = 0,39 < 1,0$, the condition is satisfied

Maximum hogging bending moment

The profiled steel sheeting acts as shuttering and carries its own weight, the wet concrete and the construction loads. According to clause 4.11, EN 1991-1-6, the construction load of 1,50 kN/m^2 acts on the working area of 3,0 × 3,0 m and outside the working area the construction load is 0,75 kN/m^2. The static system and loads are shown in Figure D2.6.

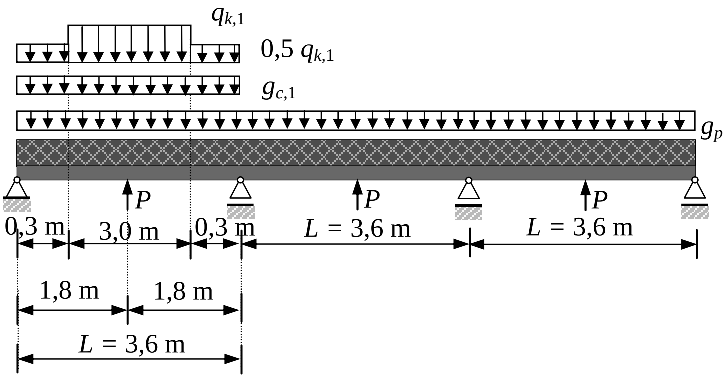


Figure D2.6 *Static system and loads for the construction stage*

The design value of hogging bending moment is:

$$M_{Ed} = \gamma_G \cdot M_g + \gamma_Q \cdot M_q = 1,35 \cdot 1,00 + 1,5 \cdot 0,54 = 2,16 \ \text{kNm/m}$$

The design value of support reaction is:

$$F_{Ed} = \gamma_G \cdot F_g + \gamma_Q \cdot F_q = 1,35 \cdot 5,80 + 1,50 \cdot 3,17 = 12,6 \ \text{kNm/m}$$

The values of M_g, M_q, F_g and F_q are calculated by computer.

The design resistance moment in hogging region is:

$$M_{Rd} = \frac{M_{Rk}}{\gamma_{M_0}} = \frac{4,67}{1,0} = 4,67 \ \text{kNm/m}$$

Check:

$$\frac{M_{Ed}}{M_{Rd}} \leq 1,0$$

$$\frac{2,16}{4,67} = 0,46 < 1,0 \text{, the condition is satisfied}$$

The design resistance to support reaction is:

$$R_{w,Rd} = \frac{R_{Rk}}{\gamma_{M0}} = \frac{34,0}{1,0} = 34,0 \text{ kN}$$

Check:

$$\frac{F_{Ed}}{R_{w,Rd}} \leq 1,0$$

$$\frac{12,6}{34,0} = 0,37 < 1,0 \text{, the condition is satisfied}$$

The check for combined bending moment and support reaction is carried out as (6.28), clause 6.1.11, EN 1993-1-3:

$$\frac{M_{Ed}}{M_{Rd}} + \frac{F_{Ed}}{R_{w,Rd}} \leq 1,25$$

$$\frac{2,16}{4,67} + \frac{12,6}{34,0} = 0,83 \leq 1,25$$

The condition is satisfied.

5.2 Composite stage

The continuous composite slab is designed as a series of simply supported spans, in accordance with clause 9.4.2(5), EN 1994-1-1, provided that the criterion for minimum reinforcement above internal supports of composite slab is satisfied, clause 9.8.1, EN 1994-1-1.

The static system and the design load for the composite stage are shown in Figure D2.7.

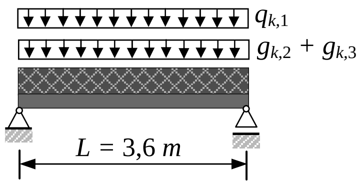


Figure D2.7 *Static system and loads for the composite stage*

The design values of bending moment and shear force are:

$$M_{Ed} = \frac{[\gamma_G \cdot (g_{k,2} + g_{k,3}) + \gamma_Q \cdot q_k] \cdot L^2}{8}$$

$$M_{Ed} = \frac{[1,35 \cdot (2,61 + 1,2) + 1,5 \cdot 5,0] \cdot 3,6^2}{8} = 20,5 \text{ kNm/m}$$

$$V_{Ed} = \frac{[\gamma_G \cdot (g_{k,2} + g_{k,3}) + \gamma_Q \cdot q_k] \cdot L}{2}$$

$$V_{Ed} = \frac{[1,35 \cdot (2,61 + 1,2) + 1,5 \cdot 5,0] \cdot 3,6}{2} = 22,7 \text{ kN/m}$$

5.2.1 Plastic resistance moment in sagging region

It is assumed that the neutral axis lies above the sheeting. The assumed distribution of longitudinal bending stresses is shown in Figure D2.8. The design compressive force in concrete, $N_{c,f}$, is:

$$N_{c,f} = 0,85 \cdot f_{cd} \cdot h_c \cdot b \,, \quad b = 1000 \text{ mm}$$

$$N_{c,f} = 0,85 \cdot 16,7 \cdot 74 \cdot 1000 \cdot 10^{-3} = 1050 \text{ kN/m}$$

The design tensile force in the steel sheeting for a width of sheeting b is calculated using the characteristic of the effective steel section A_{pe}:

$$N_p = f_{yp,d} \cdot A_{pe}$$

$$N_p = 280 \cdot 1137 \cdot 10^{-3} = 318 \text{ kN/m}$$

Since N_p < $N_{c,f}$, the plastic neutral axis lies within the concrete. The design resistance moment in sagging region is calculated according to the distribution of stresses shown in Figure D2.8.

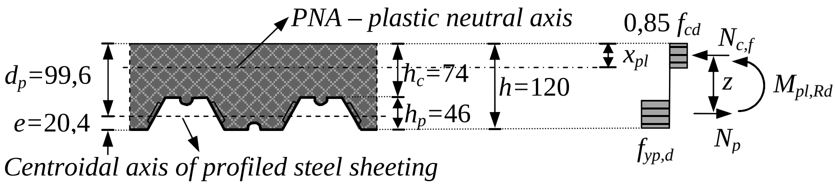


Figure D2.8 *Cross-section of composite slab and stress blocks for sagging bending*

The position of the plastic neutral axis of the composite section x_{pl} is:

$$x_{pl} = \frac{A_p \cdot f_{yp,d}}{0,85 \cdot b \cdot f_{cd}} \qquad b = 1000 \text{ mm slab width}$$

$$x_{pl} = \frac{1137 \cdot 280}{0,85 \cdot 1000 \cdot 16,7} = 22,4 \text{ mm} < h_c = 74 \text{ mm}$$

For full shear connection, the design plastic resistance moment in sagging region $M_{pl,Rd}$ is calculated as:

$$M_{pl,Rd} = \min(N_{c,f}, N_p) \cdot z$$

$$M_{pl,Rd} = N_p \cdot (d_p - \frac{x_{pl}}{2})$$

$$M_{pl,Rd} = 318 \cdot (99,6 - \frac{22,4}{2}) \cdot 10^{-3} = 28,1 \text{ kNm/m}$$

Check:

$$\frac{M_{Ed}}{M_{pl,Rd}} \le 1,0$$

$$\frac{20,5}{28,1} = 0,73 < 1,0 \text{ , the condition is satisfied}$$

The design plastic resistance moment in sagging region for full shear connection is adequate.

5.2.2 Longitudinal shear resistance

For composite slabs with ductile behaviour, the partial connection method can be used for the verification of the resistance to longitudinal shear, clause 9.7.3(8), EN 1994-1-1.

The shear span required for full shear connection is determined as:

$$N_c = \tau_{u,Rd} \cdot b \cdot L_x \leq N_{c,f}$$

The distance to the nearest support, L_x, required for full shear connection may be determined as:

$$L_x = \frac{N_{c,f}}{b \cdot \tau_{u,Rd}} = \frac{A_p \cdot f_{yd}}{b \cdot \tau_{u,Rd}}$$

The design shear strength, $\tau_{u,Rd}$, acting at the steel-concrete interface is:

$$\tau_{u,Rd} = \frac{\tau_{u,Rk}}{\gamma_{Vs}} = \frac{0{,}306}{1{,}25} = 0{,}245 \text{ N/mm}^2$$

The distance to the nearest support, L_x, required for full shear is:

$$L_x = \frac{1137 \cdot 280}{1000 \cdot 0{,}245} = 1299 \text{ mm} < L/2 = 3600/2 = 1800 \text{ mm}$$

Therefore, at a distance of 1299 mm from the support a full shear connection is fulfilled.

According to clause 9.7.3(7), EN 1994-1-1, the verification is carried out using the simplified partial interaction diagram and for any cross-section along the span it has to be shown that the corresponding design bending moment, M_{Ed}, does not exceed the design resistance moment M_{Rd}. This criterion can be written in the following format:

$$\frac{M_{Ed}(x)}{M_{Rd}(x)} \leq 1{,}0$$

The longitudinal shear resistance of the slab, expressed by its design resistance moment, M_{Rd}, found from the design shear strength, $\tau_{u,Rd}$, and the design bending moment from applied loads, are functions of x and they are plotted in Figure D2.9. Two characteristic points are found as follows.

The first characteristic point is where the degree of shear connection for sheeting η = 0. The sheeting has no longitudinal force, and the resistance moment is that of the sheeting only, $M_{pa,Rd} = M_{pa,Rk}/\ \gamma_M$ = 5,70 kNm/m.

The second characteristic point is determined as follows.

The design plastic resistance moment for full shear connection has been calculated in Section 5.2.1. Further calculation is carried out for pedagogical reasons.

The design tensile force in the steel sheeting for a width of sheeting b is (Figure D2.8):

$$N_p = A_p \cdot f_{yp,d} = 1137 \cdot 0,280 = 318 \text{ kN/m}$$

The position of the plastic neutral axis of the composite section x_{pl} is:

$$x_{pl} = \frac{A_p \cdot f_{yp,d}}{0,85 \cdot b \cdot f_{cd}}, \qquad b = 1000 \text{ mm slab width}$$

$$x_{pl} = \frac{1137 \cdot 280}{0,85 \cdot 1000 \cdot 16,7} = 22,4 \text{ mm} < h_c = 74 \text{ mm}$$

The lever arm z is calculated as (9.9), EN 1994-1-1:

$$z = h - 0,5 \cdot x_{\mathrm{pl}} - e_{\mathrm{p}} + (e_{\mathrm{p}} - e) \cdot \frac{N_{\mathrm{c}}}{A_{\mathrm{pe}} f_{\mathrm{yp,d}}}$$

The lever arm z, for $N_c = N_p$, see Figure D2.8, is:

$$z = h - e - \frac{x_{pl}}{2}$$

$$z = 120 - 20,4 - \frac{22,4}{2} = 88,4 \text{ mm}$$

Therefore, the design plastic resistance moment in sagging region, $M_{pl,Rd}$, is:

$$M_{pl,Rd} = N_p \cdot z$$

$$M_{pl,Rd} = 318 \cdot 0,0884 = 28,1 \text{ kNm/m}$$

The longitudinal shear resistance of the slab, expressed as its design resistance moment, M_{Rd}, found from the design shear strength, $\tau_{u,Rd}$, and the design bending moment from applied loads, are functions of x and they are plotted in Figure D2.9. From the simplified partial interaction diagram, shown in Figure D2.9, it can be seen that at any cross-section the design bending moment M_{Ed} does not exceed the design resistance moment M_{Rd}.

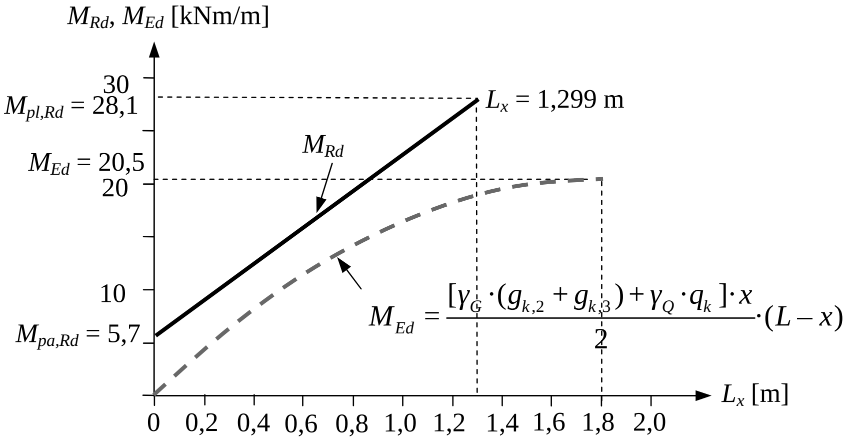


Figure D2.9 *Design partial-interaction diagram*

5.2.3 Check for vertical shear resistance

According to 9.7.5, EN 1994-1-1, the vertical shear resistance, $V_{v,Rd}$, should be determined according to the method given in EN 1992-1-1. According to clause 6.2.2, EN 1992-1-1, the design shear resistance $V_{v,Rd}$ is calculated as:

$$V_{v,Rd} = V_{Rd,c} = [C_{Rd,c} \cdot k \cdot (100 \cdot \rho_l \cdot f_{ck})^{1/3} + k_1 \cdot \sigma_{cp}] \; b_w \cdot d_p \geq V_{v,Rd,\min}$$

The minimum value of $V_{v,Rd,\min}$ is:

$$V_{v,Rd,\min} = (v_{\min} + k_1 \cdot \sigma_{cp}) \cdot b_w \cdot d_p$$

The minimum requirement for $V_{v,Rd}$ is related to the fact that the member without reinforcement still has some shear resistance.

Generally, the check is carried out as follows:

$$\frac{V_{Ed}}{V_{v,Rd}} \leq 1{,}0$$

According to clause 6.2.2(1), EN 1992-1-1, the values needed for calculation $V_{v,Rd}$ are:

$$C_{Rd,c} = \frac{0{,}18}{\gamma_c} = \frac{0{,}18}{1{,}5} = 0{,}12$$

$$k = 1 + \sqrt{\frac{200}{d_p}} \leq 2{,}0$$

$$k = 1 + \sqrt{\frac{200}{d_p}} = 1 + \sqrt{\frac{200}{99{,}6}} = 2{,}4 \quad \rightarrow \text{ adopted } k = 2{,}0$$

$$\rho_l = \frac{A_{sl}}{b_w \cdot d_p} \leq 0{,}02$$

The resistance of the cross-section is dependent on the area of the tensile reinforcement, whose section has to be extended by an appropriate anchorage length, (l_{bd} + d) see – Figure 6.3, EN 1992-1-1 – where l_{bd} is the design anchorage length and d is the effective depth of the section, taken as the depth from the top surface to the centroid of the profile for a composite slab. The anchorage of the profiled sheeting was confirmed by the check on longitudinal shear and the sheeting can be treated as reinforcement, i.e. $A_{sl} = A_{pe} = 1137$ mm^2.

In accordance with Figure D2.10, the smallest width of the cross-section in the tensile area b_w is calculated per metre width as follows:

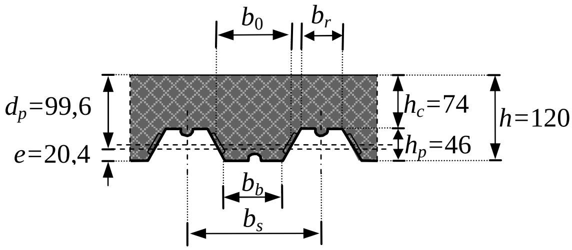
$$b_w = \frac{b}{b_s} \cdot b_0 = \frac{1000}{225} \cdot 128{,}5 = 571 \text{ mm/m}$$

The percentage of longitudinal reinforcement is:

$$\rho_l = \frac{A_{sl}}{b_w \cdot d_p} \leq 0{,}02$$

$$\rho_l = \frac{1137}{571 \cdot 99{,}6} = 0{,}02$$

The value $\rho_l = 0,02$ is adopted.



b_s = 225 mm
b_r = 67 mm
b_0 = 128,5 mm

Figure D2.10 *Determination of value* b_w

The design axial force is $N_{Ed} = 0$ and therefore $\sigma_{cp} = \dfrac{N_{Ed}}{A_c} = 0$.

$k_1 = 0,15$, according to clause 6.2.2(1), EN 1992-1-1

The design shear resistance $V_{v,Rd}$ is:

$$V_{v,Rd} = V_{Rd,c} = [C_{Rd,c} \cdot k \cdot (100 \cdot \rho_l \cdot f_{ck})^{1/3} + k_1 \cdot \sigma_{cp}]\ b_w \cdot d_p$$

$$V_{v,Rd} = [0,12 \cdot 2,0 \cdot (100 \cdot 0,02 \cdot 25)^{1/3} + 0,15 \cdot 0]\ \cdot 571 \cdot 99,6 \cdot 10^{-3}$$

$$V_{v,Rd} = 50,3 \text{ kN/m}$$

The minimum value is:

$$V_{v,Rd,\min} = (v_{\min} + k_1 \cdot \sigma_{cp}) \cdot b_w \cdot d_p$$

$$v_{\min} = 0,035 \cdot k^{3/2} \cdot f_{ck}{}^{1/2} = 0,035 \cdot 2,0^{3/2} \cdot 25^{1/2} = 0,49 \text{ N/mm}^2$$

$$V_{v,Rd,\min} = (0,49 + 0,15 \cdot 0) \cdot 571 \cdot 99,6 \cdot 10^{-3} = 27,9 \text{ kN/m} < V_{v,Rd} = 50,3 \text{ kN/m}$$

Check:

$$\frac{V_{Ed}}{V_{v,Rd}} \le 1,0$$

$$\frac{22,7}{50,3} = 0,45 < 1,0 \text{, the condition is satisfied}$$

Remark:

Since it is unlikely that the profiled steel sheet can satisfy the requirement of "full anchorage", the design shear resistance is equal to the minimum value:

$$V_{v,Rd} = V_{v,Rd,\min} = 27,9 \text{ kN/m}$$

Also, the required condition is satisfied since that is $V_{v,Rd} = V_{v,Rd,\min} = 27,9$ kN/m $>$ $V_{Ed} = 22,7$ kN/m.

6. Serivceability limit state

6.1 Control of cracking of concrete

Since the slab is designed as simply supported, it is only requires reinforcement for crack width limitation.

According to clause 9.8.1(2), EN 1994-1-1, for propped construction the required cross-sectional area of reinforcement A_s is 0,4% of the area of concrete above the ribs:

$$\min A_s = 0,004 \cdot b \cdot h_c = 0,004 \cdot 1000 \cdot 74 = 296 \text{ mm}^2\text{/m}$$

The reinforcement bar is assumed to be ϕ8/160 mm. Therefore, the cross-sectional area of reinforcement is:

$$A_s = \frac{8^2 \cdot \pi}{4} \cdot \frac{1000}{160} = 314 \text{ mm}^2\text{/m} > 296 \text{ mm}^2\text{/m}$$

The selected minimum amount of the reinforcement could be insufficient to control cracking at the supports of continuous slabs for certain exposure classes. In such cases, the slab should be designed as continuous, and in hogging regions the crack widths should be estimated according to EN 1992-1-1.

6.2 Limit of span/depth ratio of slab

According to clause 9.8.2(4), EN 1994-1-1, calculation of the deflection of the composite slab can be omitted if the two conditions are satisfied. According to the first condition, the span/depth ratio of the slab should not exceed the limits given in EN 1992-1-1. These are:

- $\frac{L}{d} < 20$ for a simply supported span
- $\frac{L}{d} < 26$ for an external span of continuous slab
- $\frac{L}{d} < 30$ for an internal span of continuous slab

According to clause 9.8.2(6), EN 1994-1-1, the second condition is given as follows:

- the load causing an end slip of 0,5 mm in the tests on composite slab exceeds 1,2 times the design service load.

If the second condition is not satisfied, i.e. the end slip exceeds 0,5 mm at a load 1,2 times the design service load, two options exist:

- end anchors should be provided, or
- deflections should be calculated including the effect of end slip.

According to clause 9.8.2(8), EN 1994-1-1, in cases when the behaviour of the shear connection between the profiled sheeting and the concrete are not known from tests, the tied-arch model may be used, see [34].

For the considered slab with L = 3600 mm and d_p = 99,6 mm, the following span/depth ratio is obtained:

- $\frac{L}{d} = \frac{3600}{99{,}6} = 36 > 26$ for the external span of continuous slab
- $\frac{L}{d} = \frac{3600}{99{,}6} = 36 > 30$ for the internal span of continuous slab

The span/depth ratio exceeds the limit, and the calculation of the deflection needs to be carried out.

6.3 Calculation of deflections

6.3.1 Construction stage deflection

According to clause 9.6(2), EN 1994-1-1, the deflection, δ_s, of the profiled sheeting due to its own weight and the weight of wet concrete should not exceed the following limit:

$$\delta_{s,\max} = \frac{L}{180} = \frac{1800}{180} = 10 \text{ mm}$$

The premature local buckling of the profiled sheeting under the weight of wet concrete and construction loading is checked to prevent irreversible deformation.

Maximum sagging bending moment in the serviceability limit state is:

$$M_{Ed} = 0{,}078 \cdot 0{,}09 \cdot 1{,}8^2 + 0{,}094 \cdot 2{,}62 \cdot 1{,}8^2 = 0{,}82 \text{ kNm/m}$$

Maximum compressive stress in the top flange of the profiled sheeting is:

$$\sigma_{com} = \frac{M_{Ed}}{\mathrm{I}_p} \cdot z = \frac{0{,}82 \cdot 10^6}{41{,}5 \cdot 10^4} \cdot (46 - 20{,}4) = 50{,}6 \text{ N/mm}^2$$

In accordance with clause 4.4, EN 1993-1-5, the plate slenderness, $\overline{\lambda}_p$, is calculated as:

$$\overline{\lambda}_p = \sqrt{\frac{f_y}{\sigma_{cr}}} = \frac{\frac{b}{t}}{28{,}4 \cdot \varepsilon \cdot \sqrt{k_\sigma}}$$

$$\varepsilon = \sqrt{\frac{235}{\sigma_{com}}} = \sqrt{\frac{235}{50{,}6}} = 2{,}2$$

According to Table 4.1, EN 1994-1-1, for the stress ratio $\psi = 1$, the buckling factor is $k_\sigma = 4$.

Therefore, the plate slenderness, $\overline{\lambda}_p$, with the design thickness of the sheet $t = 0{,}86$ mm (not including coatings) and $b = b_r = 67$ mm, is:

$$\overline{\lambda}_p = \frac{\frac{67}{0{,}86}}{28{,}4 \cdot 2{,}2 \cdot \sqrt{4}} = 0{,}623$$

Since that is $\overline{\lambda}_p = 0{,}623 < 0{,}673$, the reduction factor is $\rho = 1{,}0$ and the cross-section is fully effective.

The deflection, δ_s, is calculated as:

$$\delta_s = \frac{(2{,}65 \cdot g_p + 3{,}4 \cdot g_{c,1}) \cdot L^4}{384 \cdot E \cdot I_p}$$

The deflection, δ_s, is:

$$\delta_s = \frac{(2{,}65 \cdot 0{,}09 + 3{,}4 \cdot 2{,}62) \cdot 1800^4}{384 \cdot 210000 \cdot 41{,}5 \cdot 10^4} = 2{,}9 \text{ mm}$$

$\delta_s = 2{,}9$ mm < $\delta_{s,max} = 10$ mm

The deflection due to the self-weight of the profiled sheeting and the weight of the wet concrete meets the criterion $L/180$.

Since the deflection δ_s is less than 10% of the slab depth, $\delta_s = 2{,}9$ mm < $0{,}10 \cdot h = 0{,}1 \cdot 120 = 12$ mm, according to clause 9.3.2(2), EN 1994-1-1, the ponding effects can be neglected at the construction stage.

The conditions for the serviceability limit state are satisfied, and the profiled steel sheeting can be used at the construction stage.

6.3.2 *Composite stage deflection*

For the calculation of the deflection in composite stage, the slab is considered as continuous over two spans. According to clause 9.8.2(5), EN 1994-1-1, the following approximations can be applied:

- The second moment of area can be taken as the average of the values for the cracked and uncracked section.
- An average value of the modular ratio, n, for both short-term and long-term effects can be used:

$$n = \frac{E_a}{E'_{cm}} = \frac{E_a}{\frac{1}{2} \cdot (E_{cm} + \frac{E_{cm}}{3})} = \frac{210000}{\frac{2}{3} \cdot 31000} = 10{,}2$$

- Elastic analysis is used to calculate the deflection of the slab.

a) The second moment of area for the cracked section, I_{cc}, for slab width b is calculated in accordance with Figure D2.11.

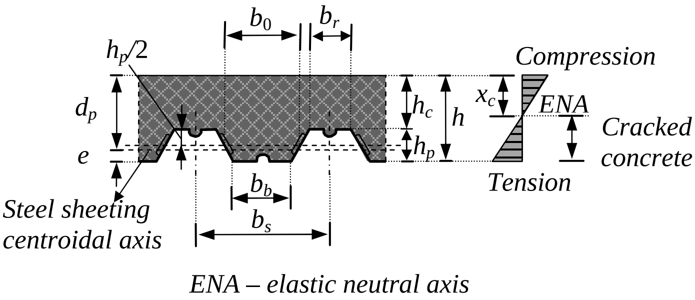


Figure D2.11 *Second moment of area calculation for cracked cross-section, I_{cc}*

The second moment of area for the cracked section and the slab width b is calculated as:

$$I_{cc} = \frac{b \cdot x_c^{\ 3}}{3 \cdot n} + A_p \cdot (d_p - x_c)^2 + I_p$$

The position of the elastic neutral axis relative to the upper side of the slab is:

$$x_c = \frac{\Sigma A_i \cdot z_i}{\Sigma A_i} = \frac{n \cdot A_p}{b} (\sqrt{1 + \frac{2 \cdot b \cdot d_p}{n \cdot A_p}} - 1)$$

$$x_c = \frac{10{,}2 \cdot 1137}{1000} (\sqrt{1 + \frac{2 \cdot 1000 \cdot 99{,}6}{10{,}2 \cdot 1137}} - 1) = 37{,}8 \text{ mm}$$

The second moment of area for the cracked section is:

$$I_{cc} = \frac{1000 \cdot 37{,}8^3}{3 \cdot 10{,}2} + 1137 \cdot (99{,}6 - 37{,}8)^2 + 41{,}5 \cdot 10^4 = 6{,}52 \cdot 10^6 \text{ mm}^4\text{/m}$$

b) The second moment of area for the uncracked section, I_{cu}, for slab width b is calculated in accordance with Figure D2.12.

The second moment of area for the uncracked section and the slab width b is calculated as:

$$I_{cu} = \frac{b \cdot h_c^{\ 3}}{12 \cdot n} + \frac{b \cdot h_c}{n} \cdot (x_u - \frac{h_c}{2})^2 + \frac{b_m \cdot h_p^3}{12 \cdot n} + \frac{b_m \cdot h_p}{n} \cdot (h - x_u - \frac{h_p}{2})^2 +$$

$$+ A_p \cdot (d_p - x_u)^2 + I_p$$

where:

$$x_u = \frac{b \cdot \frac{h_c^2}{2} + b_m \cdot h_p \cdot (h - \frac{h_p}{2}) + n \cdot A_p \cdot d_p}{b \cdot h_c + b_m \cdot h_p + n \cdot A_p}$$

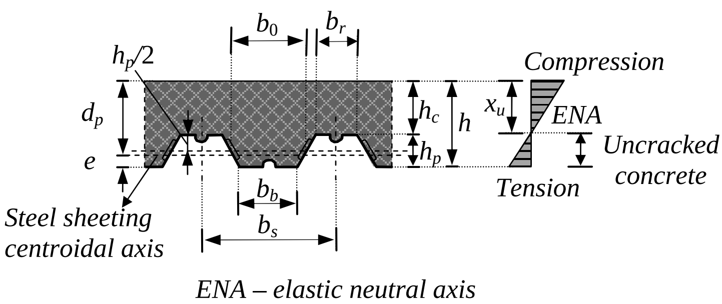


Figure D2.12 *Second moment of area calculation for uncracked cross-section,* I_{cu}

In accordance with Figure D2.12, the value of b_m is:

$$b_0 = \frac{158 + 105}{2} = 131{,}5 \ \text{mm}$$

$$b_m = \frac{b}{b_s} \cdot b_0 = \frac{1000}{225} \cdot 131{,}5 = 584 \ \text{mm/m}$$

The position of the elastic neutral axis relative to the upper side of the slab is:

$$x_u = \frac{1000 \cdot \frac{74^2}{2} + 584 \cdot 46 \cdot (120 - \frac{46}{2}) + 10{,}2 \cdot 1137 \cdot 99{,}6}{1000 \cdot 74 + 584 \cdot 46 + 10{,}2 \cdot 1137} = 57{,}8 \ \text{mm}$$

The second moment of area for the uncracked section is:

$$I_{cu} = \frac{1000 \cdot 74,0^3}{12 \cdot 10,2} + \frac{1000 \cdot 74,0}{10,2} \cdot (57,8 - \frac{74,0}{2})^2 + \frac{584 \cdot 46,0^3}{12 \cdot 10,2} +$$

$$\frac{584 \cdot 46,0}{10,2} \cdot (120 - 57,8 - \frac{46,0}{2})^2 + 1137 \cdot (99,6 - 57,8)^2 + 41,5 \cdot 10^4$$

$$I_{cu} = 13,36 \cdot 10^6 \text{ mm}^4/\text{m}$$

The mean value of I_{cc} and I_{cu} is:

$$I_c = \frac{I_{cc} + I_{cu}}{2} = \frac{6,52 \cdot 10^6 + 13,36 \cdot 10^6}{2} = 9,94 \cdot 10^6 \text{ mm}^4/\text{m}$$

Calculation of deflections

- Deflection due to permanent action – weight of floor finishes $g_{k,3}$ = 1,20 kN/m^2

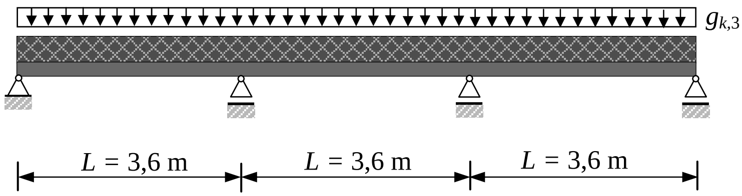


Figure D2.13 *Static system and load for calculation of deflection at the composite stage*

The deflection is:

$$\delta_1 = \frac{0,0068 \cdot g_{k,3} \cdot L^4}{E \cdot \mathrm{I}_c}$$

$$\delta_1 = \frac{0,0068 \cdot 1,2 \cdot 3600^4}{210000 \cdot 9,94 \cdot 10^6} = 0,66 \text{ mm}$$

- Deflection due to frequent value of variable action and the selected combination factor is ψ_1 = 0,7

The design load is calculated for the frequent combination:

$$q = \psi_1 \cdot q_{k,2}$$

The deflection is:

$$\delta_2 = \frac{0,0099 \cdot \psi_1 \cdot q_{k,2} \cdot L^4}{E \cdot I_c}$$

$$\delta_2 = \frac{0,0099 \cdot 0,7 \cdot 5,0 \cdot 3600^4}{210000 \cdot 9,94 \cdot 10^6} = 2,79 \text{ mm}$$

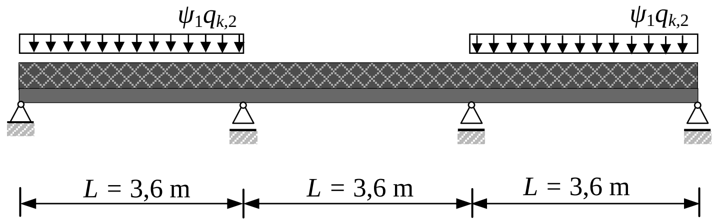


Figure D2.14 *Static system and load for calculation of deflection at the composite stage, the worst load case*

- Self-weight of the slab g_k, removal of the props

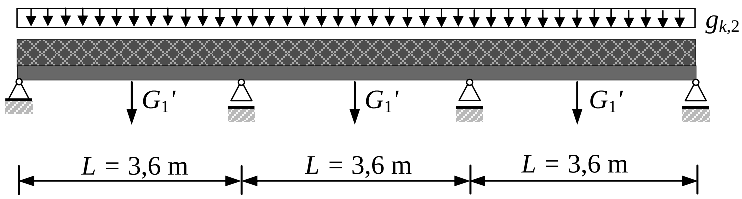


Figure D2.15 *Self-weight of slab, removal of the props*

The support reactions G_1' due to the self-weight of the slab are applied as point loads on the system:

$$G_1' = g_{k,2} \cdot \frac{L}{2} = 2,61 \cdot \frac{3,6}{2} = 4,70 \text{ kN/m}$$

The deflection is:

$$\delta_3 = \frac{0,01146 \cdot G_1' \cdot L^3}{E \cdot I_c}$$

$$\delta_3 = \frac{0,01146 \cdot 4700 \cdot 3600^3}{210000 \cdot 9,94 \cdot 10^6} = 1,20 \text{ mm}$$

Remark:

The limit of the deflection is adopted according to clause 7.4.1(4), EN 1992-1-1. The recommended limitation is:

$$\delta_{total} \leq \frac{L}{250}$$

The total deflection is:

$$\delta_{total} = \delta_1 + \delta_2 + \delta_3 = 0,66 + 2,79 + 1,20 = 4,65 \text{ mm} \leq \frac{L}{250} = \frac{3600}{250} = 14,4 \text{ mm}$$

The total deflection meets the criterion *L*/250.

7. Commentary

The usual spans for composite slabs are between 3 m and 4,5 m. For spans less than 3,5 m, it is not necessary to use temporary propping during concreting of the slab. In this case, the construction stage is governed for the design of the profiled sheeting. The stresses in the short span composite slab are very low. For such composite slabs, profiled steel sheeting with limited longitudinal shear resistance and ductility are usually used. For larger spans, props are necessary to support the profiled steel sheeting during execution. In this case, the stresses in the composite slab are very high, and usually the composite stage is governed for the design. Profiled steel sheeting requires good longitudinal shear resistance. Re-entrant steel profiles are often used in such cases.

In this example the slab is propped during construction. All of the loads have to be resisted by the composite section. The design resistance of the composite slab against longitudinal shear was carried out by the partial connection method. This method relies on tests on the composite slab to estimate the shear connection. The method is based on a diagram that relates the bending moment strength to the shear connection degree of the slab, Figure D2.9.

D3 Three-span composite slab propped at the construction stage – end anchorage and additional reinforcement

1. Purpose of example

This example shows the design of the composite floor slab that is supported by the composite beam at 3,6 m centres, Figure D3.1. The composite slab is 130 mm deep with the profiled steel sheeting of height 51 mm and 0,88 mm thick. The profiled steel sheeting is propped temporarily during construction. The composite slab is continuous because the profiled sheeting is provided in three-span lengths and the concrete is cast on the sheeting without joints. In accordance with EN 1994-1-1, two different static systems are considered, as simply supported and alternatively as continuous spans.

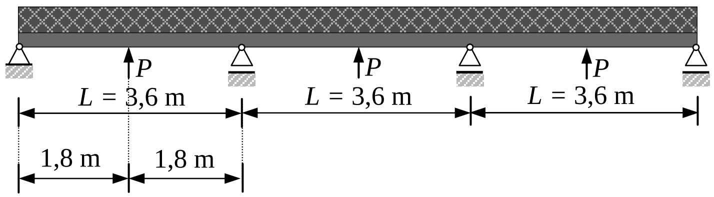
In this case, elastic global analysis is used based on the uncracked stiffness. The design bending moments at internal supports are very high. Due to very high resulting moments at internal supports, heavy reinforcement may be required. To avoid this problem, the composite slab may be designed as a series of simply supported spans, clause 9.4.2(5), EN 1994-1-1, or by using redistribution of moments, clause 9.4.2(3), EN 1994-1-1. In this case, the use of plastic analysis, clause 9.4.2(4), EN 1994-1-1, is not possible because the required conditions are not satisfied.

This example illustrates the design of the resistance of composite slab taking into account the end anchorage and the additional reinforcement provided in troughs of profiled sheeting. The first step is to carry out the verification of the longitudinal shear resistance without end anchorage. Since the check for longitudinal shear resistance is not satisfied, the end anchorage and the additional reinforcement are taking into account. The partial connection method is used, although the *m*-*k* method has been widely used in the design of composite slabs. The *m*-*k* method is not based on a mechanical model and is therefore less flexible than the partial connection method. In this example, the benefit of end anchorage and additional reinforcement cannot be quantified without additional tests that include these variables.

Composite Structures according to Eurocode 4. Worked Examples.
First Edition. Darko Dujmović, Boris Androić, Ivan Lukačević.

2. Static system, cross-section and actions

a) Construction stage – the profiled steel sheeting acts as shuttering



P – temporary props during casting of the concrete

Figure D3.1 *Static system of profiled sheeting for the construction stage*

b) Composite stage

The composite slab is continuous, Figure D3.2. However, according to clause 9.4.2(5), EN 1994-1-1, the continuous slab may be designed as a series of simply supported spans.

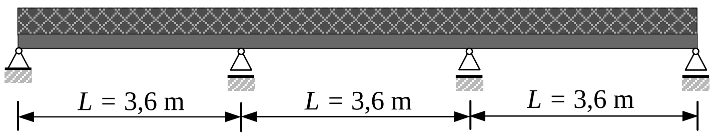
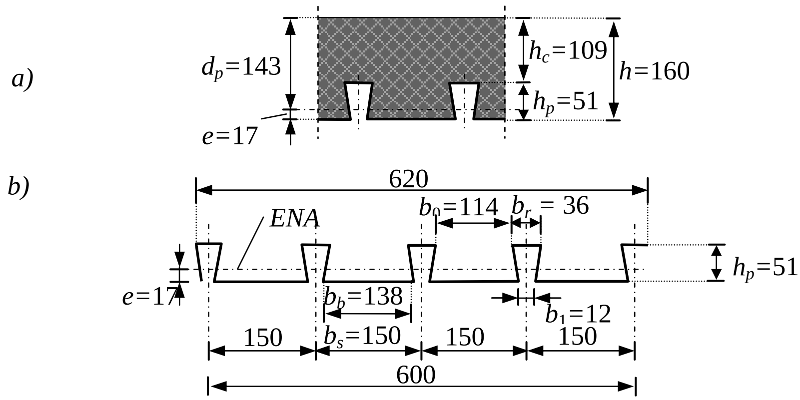


Figure D3.2 *Static system of the composite slab*

c) Cross-section of composite slab



ENA – elastic neutral axis

Figure D3.3 *Cross-sections: a) composite slab, b) profiled steel sheeting*

Actions

a) Permanent action

Remark:

According to EN 1991-1-1 the density of normal weight concrete is 24 kN/m^3, increased by 1 kN/m^3 for normal percentage reinforcement, and increased for the wet concrete by another 1 kN/m^3.

Concrete slab area per m width:

$$A_c = 1000 \cdot h - \left(\frac{1000}{b_s} \cdot \frac{b_1 + b_r}{2} \cdot h_p\right)$$

$$A_c = 1000 \cdot 160 - \left(\frac{1000}{150} \cdot \frac{12+36}{2} \cdot 51\right) = 151840 \text{ mm}^2 = 1518 \text{ cm}^2$$

- *concrete slab and reinforcement (wet concrete):*

$$A_c \cdot 26 = 0{,}152 \cdot 26 = 3{,}95 \text{ kN/m}^2$$

- *concrete slab and reinforcement (dry concrete):*

$$A_c \cdot 25 = 0{,}152 \cdot 25 = 3{,}80 \text{ kN/m}^2$$

Construction stage

- *concrete slab* $g_{c,1}$ = 3,93 kN/m^2

- *profiled steel sheeting* g_p = 0,13 kN/m^2

Total $g_{k,1}$ = 4,08 kN/m^2

Composite stage

- *concrete slab* $g_{c,2}$ = 3,80 kN/m^2

- *profiled steel sheeting* g_p = 0,13 kN/m^2

Total $g_{k,2}$ = 3,95 kN/m^2

Floor finishes $g_{k,3} = 1{,}20$ kN/m^2

b) Variable action

Construction stage

- *construction loads* $q_{k,1} = 1{,}50$ kN/m^2

Composite stage

- *imposed floor load* $q_{k,2} = 5{,}0$ kN/m^2

Remark:

Clause 9.3.2(1), EN 1994-1-1, refers to clause 4.11, EN 1991-1-6, for construction loads. According to clause 4.11.2(1), EN 1991-1-6, the actions from personnel and equipment, q_k, are referred to as "10% of the self-weight of the concrete, but not less than 0,75 and not more than 1,5 kN/m^2". Also, according to clause 4.11, EN 1991-1-6, the load of 1,50 kN/m^2 acts on the working area of 3,0 × 3,0 m, and outside the working area the load is 0,75 kN/m^2.

3. Properties of materials

Concrete strength class: C 25/30 $f_{ck} = 25 \text{ N/mm}^2$

$$f_{cd} = \frac{f_{ck}}{\gamma_c} = \frac{25}{1{,}5} = 16{,}7 \text{ N/mm}^2$$

$$0{,}85 f_{cd} = 0{,}85 \cdot 16{,}7 = 14{,}17 \text{ N/mm}^2$$

$$E_{cm} = 31\,000 \text{ N/mm}^2$$

Reinforcement: $f_{sk} = 500 \text{ N/mm}^2$

$$f_{sd} = \frac{f_{sk}}{\gamma_s} = \frac{500}{1{,}15} = 435 \text{ N/mm}^2$$

Profiled steel sheeting: $t = 0{,}88 \text{ mm}$

$$h_p = 51 \text{ mm}$$

$$A_p = A_{pe} = 1562 \text{ mm}^2\text{/m}$$

$$I_p = 62 \cdot 10^4 \text{ mm}^4\text{/m}$$

$$E_p = E_a = 210\,000 \text{ N/mm}^2$$

$$f_{yp,k} = 280 \text{ N/mm}^2$$

$$f_{yp,d} = \frac{f_{yp,k}}{\gamma_M} = \frac{280}{1,0} = 280 \ \text{N/mm}^2$$

Plastic resistance moment (provided by manufacturer): $M_{pa,Rk} = 6,30$ kNm/m

Resistance moment (provided by manufacturer): $M_{Rk} = 5,18$ kNm/m (sagging)

$M_{Rk} = 6,10$ kNm/m (hogging)

Resistance to support reaction (provided by manufacturer): $R_{w,k} = 22,8$ kN/m

Resistance to horizontal shear (provided by manufacturer): $\tau_{u,Rk} = 0,0425$ N/mm^2

4. Structural details of composite slab

4.1 Slab thickness and reinforcement

The composite slab should satisfy the conditions given in clause 9.2, EN 1994-1-1.

a) The slab does not act compositely with a beam, nor is it used as a diaphragm, so the following conditions should be satisfied:

- the overall depth of slab $h \geq 80$ mm, $\rightarrow h = 160$ mm (satisfied),

- the thickness of concrete above the main flat surface of the top of the ribs of sheeting $h_c \geq 40$ mm, $\rightarrow h_c = 109$ mm (satisfied),
- the ratio of the width of the sheet rib to the rib spacing $\frac{b_r}{b_s} \leq 0,6$, $\rightarrow$

$$\frac{36}{150} = 0,24 < 0,60 \ \text{(satisfied)}.$$

b) The minimum amount of reinforcement in both directions should not be less than 80 mm^2/m. For propped construction, the area of reinforcement, according to clause 9.8.1(2), EN 1994-1-1, is:

$$A_{s\min} = 0,004 \cdot h_c \cdot b = 0,004 \cdot 109 \cdot 1000 = 436 \ \text{mm}^2/\text{m} \rightarrow A_s = 80 \ \text{mm}^2/\text{m}$$

c) Spacing of reinforcement bars

$$e < 2 \cdot h = 2 \cdot 160 = 320 \ \text{or} < 350 \ \text{mm}$$

4.2 Largest nominal aggregate size

$d_g \leq 0,4 \cdot h_c$ $\quad$ $0,4 \cdot 109 = 43,6$ mm
$d_g \leq b_0/3$ $\quad$ $114/3 = 38,0$ mm

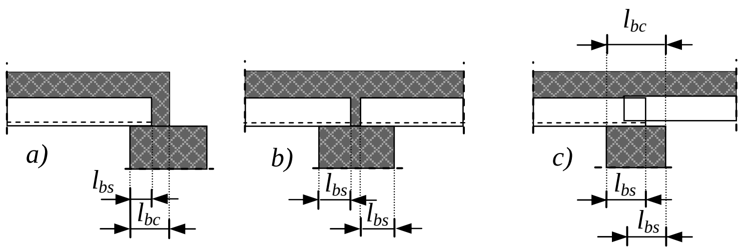
$d_g \leq 31{,}5$ mm $\qquad = 31{,}5$ mm

The minimum adopted value is $d_g = 31{,}5$ mm.

4.3 Minimum value for nominal thickness of steel sheet

In accordance with clause 3.5(2), EN 1994-1-1, the recommended value for the nominal thickness of steel sheet is 0,70 mm. The thickness of the selected profiled steel sheeting is 0,88 mm. The condition is satisfied.

4.4 Composite slab bearing requirements



Bearing on	l_{bs} *(mm)*	l_{bc} *(mm)*
steel or concrete	50	75
other materials	70	100

Figure D3.4 *Minimum bearing lengths*

According to clause 9.2.3(2), EN 1994-1-1, the recommended bearing lengths and support details differ depending upon the support material, and they are different for internal supports and end supports, see Figure D3.4.

For composite slabs bearing on steel or concrete, the minimum bearing lengths are l_{bc} = 75 mm and l_{bs} = 50 mm. The composite slab is supported by steel beams with the top flange width larger than 75 mm. Therefore, the condition for the bearing length is satisfied.

5. Ultimate limit state

5.1 Construction stage

At the construction stage, it is necessary to carry out verifications of profiled steel sheeting for the ultimate and serviceability limit state in accordance with EN 1993-1-3.

Usually, the manufacturer gives information on the properties of profiled steel sheeting. These properties are usually based on test results performed in accordance with EN 1993-1-3, Annex A. Characteristic and design values of resistance moment, crushing resistance, second moment of area etc. may be estimated using methods of reliability analysis in accordance with EN 1990. Properties of profiled steel sheeting estimated by calculation are more conservative than equivalent properties based on testing.

Maximum sagging bending moment

The profiled steel sheeting acts as shuttering and carries its own weight, the wet concrete and the construction loads. The static system and loads are shown in Figure D3.5.

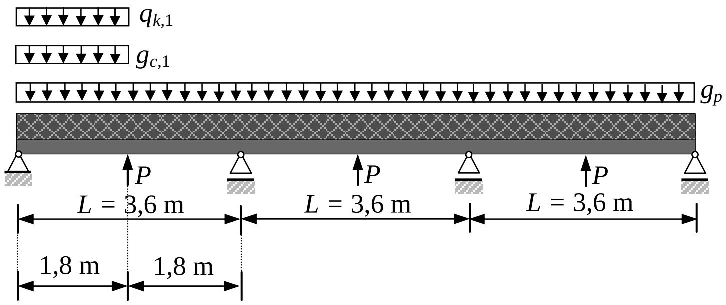


Figure D3.5 *Static system and loads for the construction stage*

The design value of sagging bending moment is:

$$M_{Ed} = \gamma_G \cdot M_g + \gamma_Q \cdot M_q$$

$$M_g = M_{g_p} + M_{g_{c,1}} = 0,078 \cdot 0,13 \cdot 1,8^2 + 0,094 \cdot 3,95 \cdot 1,8^2 = 1,24 \text{ kNm/m}$$

$$M_q = 0,094 \cdot 1,50 \cdot 1,8^2 = 0,46 \text{ kNm/m}$$

$$M_{Ed} = 1,35 \cdot 1,24 + 1,5 \cdot 0,46 = 2,36 \text{ kNm/m}$$

Verifications for the profiled steel sheeting are carried out in accordance with EN 1993-1-3. However, since the characteristic resistance moment is provided by the manufacturer, the check is carried out with this value:

$$M_{Rd} = \frac{M_{Rk}}{\gamma_{M_0}} = \frac{5,18}{1,0} = 5,18 \text{ kNm/m}$$

Check:

$$\frac{M_{Ed}}{M_{Rd}} \leq 1,0$$

$$\frac{2,36}{5,18} = 0,46 < 1,0 \text{ , the condition is satisfied}$$

Maximum hogging bending moment

The profiled steel sheeting acts as shuttering and carries its own weight, the wet concrete and the construction loads. According to clause 4.11, EN 1991-1-6, the construction load of 1,50 kN/m^2 acts on the working area of 3,0 × 3,0 m, and outside the working area the construction load is 0,75 kN/m^2. The static system and loads are shown in Figure D3.6.

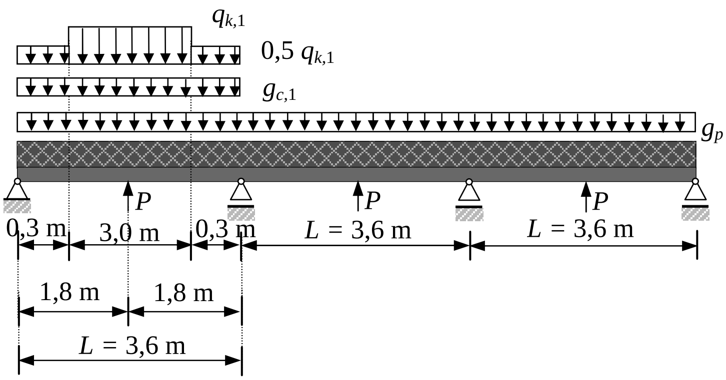


Figure D3.6 *Static system and loads for the construction stage*

The design value of hogging bending moment is:

$$M_{Ed} = \gamma_G \cdot M_g + \gamma_Q \cdot M_q = 1,35 \cdot 1,50 + 1,50 \cdot 0,54 = 2,84 \text{ kNm/m}$$

The design value of support reaction is:

$$F_{Ed} = \gamma_G \cdot F_G + \gamma_Q \cdot F_Q = 1,35 \cdot 8,73 + 1,50 \cdot 3,17 = 16,5 \text{ kNm/m}$$

The values of M_g, M_q, F_g and F_q are calculated by computer.

The design resistance moment in hogging region is:

$$M_{Rd} = \frac{M_{Rk}}{\gamma_{M_0}} = \frac{6,10}{1,0} = 6,10 \text{ kNm/m}$$

Check:

$$\frac{M_{Ed}}{M_{Rd}} \leq 1{,}0$$

$$\frac{2{,}84}{6{,}10} = 0{,}47 < 1{,}0$$, the condition is satisfied

The design resistance to support reaction is:

$$R_d = \frac{R_{Rk}}{\gamma_{M0}} = \frac{22{,}8}{1{,}0} = 22{,}8 \text{ kN}$$

Check:

$$\frac{F_{Ed}}{R_{w,Rd}} \leq 1{,}0$$

$$\frac{16{,}5}{22{,}8} = 0{,}72 < 1{,}0$$, the condition is satisfied

The check for combined bending moment and support reaction is carried out as (6.28), clause 6.1.11, EN 1993-1-3:

$$\frac{M_{Ed}}{M_{Rd}} + \frac{F_{Ed}}{R_{w,Rd}} \leq 1{,}25$$

$$\frac{2{,}84}{6{,}10} + \frac{16{,}5}{22{,}8} = 1{,}19 < 1{,}25$$, the condition is satisfied

5.2 Composite stage

The continuous composite slab is designed as a series of simply supported spans, in accordance with clause 9.4.2(5), EN 1994-1-1, provided that the criterion for minimum reinforcement above internal supports of composite slab is satisfied, clause 9.8.1, EN 1994-1-1.

The static system and the design load for the composite stage are shown in Figure D3.7.

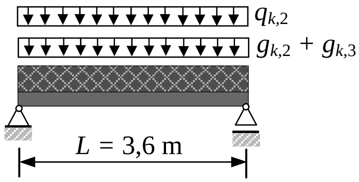


Figure D3.7 *Static system and loads for the composite stage*

The design values of bending moment and shear force are:

$$M_{Ed} = \frac{[\gamma_G \cdot (g_{k,2} + g_{k,3}) + \gamma_Q \cdot q_{k,2}] \cdot L^2}{8}$$

$$M_{Ed} = \frac{[1,35 \cdot (3,93 + 1,2) + 1,5 \cdot 5,0] \cdot 3,6^2}{8} = 23,4 \text{ kNm/m}$$

$$V_{Ed} = \frac{[\gamma_G \cdot (g_{k,2} + g_{k,3}) + \gamma_Q \cdot q_{k,2}] \cdot L}{2}$$

$$V_{Ed} = \frac{[1,35 \cdot (3,93 + 1,2) + 1,5 \cdot 5,0] \cdot 3,6}{2} = 26,0 \text{ kN/m}$$

5.2.1 Plastic resistance moment in sagging region

It is assumed that the neutral axis lies above the sheeting. The assumed distribution of longitudinal bending stresses is shown in Figure D3.8. The design compressive force in concrete, $N_{c,f}$, is:

$$N_{c,f} = 0,85 \cdot f_{cd} \cdot h_c \cdot b \qquad b = 1000 \text{ mm}$$

$$N_{c,f} = 0,85 \cdot 16,7 \cdot 109 \cdot 1000 \cdot 10^{-3} = 1547 \text{ kN/m}$$

The design tensile force in the steel sheeting for a width of sheeting b is calculated with the characteristic of the effective steel section A_{pe}:

$$N_p = f_{yp,d} \cdot A_{pe}$$

$$N_p = 280 \cdot 1562 \cdot 10^{-3} = 437 \text{ kN/m}$$

Since N_p < $N_{c,f}$, the plastic neutral axis lies within the concrete. The design resistance moment in sagging region is calculated according to the distribution of stresses shown in Figure D3.8.

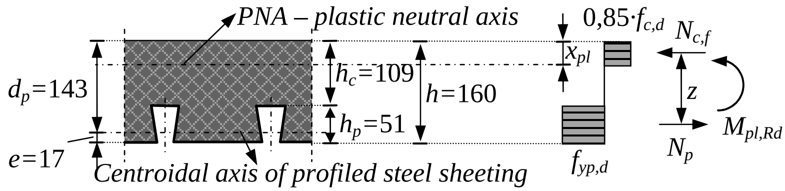


Figure D3.8 *Cross-section of composite slab and stress blocks for sagging bending*

The position of the plastic neutral axis of the composite section x_{pl} is:

$$x_{pl} = \frac{A_p \cdot f_{yp,d}}{b \cdot 0{,}85 \cdot f_{cd}} \qquad b = 1000 \text{ slab width}$$

$$x_{pl} = \frac{1562 \cdot 280}{1000 \cdot 0{,}85 \cdot 16{,}7} = 30{,}8 \text{ mm} < h_c = 109 \text{ mm}$$

For full shear connection, the design plastic resistance moment in sagging region $M_{pl,Rd}$ is calculated as:

$$M_{pl,Rd} = \min(N_{c,f}, N_p) \cdot z$$

$$M_{pl,Rd} = N_p \cdot (d_p - \frac{x_{pl}}{2})$$

$$M_{pl,Rd} = 437 \cdot (143 - \frac{30{,}8}{2}) \cdot 10^{-3} = 55{,}8 \text{ kNm/m}$$

Check:

$$\frac{M_{Ed}}{M_{pl,Rd}} \leq 1{,}0$$

$$\frac{23{,}4}{55{,}8} = 0{,}42 < 1{,}0 \text{, the condition is satisfied}$$

The design plastic resistance moment in sagging region for full shear connection is adequate.

5.2.2 Longitudinal shear resistance

5.2.2.1 Longitudinal shear resistance without end anchorage

For composite slabs with ductile behaviour, the partial connection method can be used for the verification of the resistance to longitudinal shear, clause 9.7.3(8), EN 1994-1-1.

The shear span required for full shear connection is determined as:

$$N_c = \tau_{u,Rd} \cdot b \cdot L_x \leq N_{c,f}$$

The distance to the nearest support, L_x, required for full shear connection may be determined by:

$$L_x = \frac{N_{c,f}}{b \cdot \tau_{u,Rd}} = \frac{A_p \cdot f_{yd}}{b \cdot \tau_{u,Rd}}$$

The design shear strength, $\tau_{u,Rd}$, acting at the steel-concrete interface, is:

$$\tau_{u,Rd} = \frac{\tau_{u,Rk}}{\gamma_{Vs}} = \frac{0{,}0425}{1{,}25} = 0{,}034 \text{ N/mm}^2$$

The distance to the nearest support, L_x, required for full shear, is:

$$L_x = \frac{1562 \cdot 280}{1000 \cdot 0{,}034} = 12864 \text{ mm} > L/2 = 3600/2 = 1800 \text{ mm}$$

Therefore, at a distance of 12864 mm from the support a full shear connection is fulfilled.

The length of shear span needed for full interaction L_{sf} = 12864 mm exceeds L/2 = 1800 mm, so full interaction is not achieved in a span of this length.

According to clause 9.7.3(7), EN 1994-1-1, the verification is carried out using the simplified partial interaction diagram and, for any cross-section along the span, it has to be shown that the corresponding design bending moment, M_{Ed}, does not exceed the design resistance moment M_{Rd}. This criterion can be written as:

$$\frac{M_{Ed}(x)}{M_{Rd}(x)} \leq 1{,}0$$

The longitudinal shear resistance of the slab, expressed by its design resistance moment, M_{Rd}, found from the design shear strength, $\tau_{u,Rd}$, and the design bending moment from applied loads, are functions of x and they are plotted in Figure D3.9. Two characteristic points of the simplified partial interaction diagram are found as follows.

The first characteristic point is where the degree of shear connection for sheeting η = 0. The sheeting has no longitudinal force, and the resistance moment is that of the sheeting only, $M_{pa,Rd} = M_{pa,Rk}/\gamma_M$ = 6,30 kNm/m.

The second characteristic point is determined as follows.

The design tensile force in the steel sheeting for a width of sheeting b is, Figure D3.8:

$$N_p = 280 \cdot 1562 \cdot 10^{-3} = 437 \text{ kN/m}$$

For full shear connection, the position of the plastic neutral axis of the composite section x_{pl} is:

$$x_{pl} = \frac{A_p \cdot f_{yp,d}}{0,85 \cdot b \cdot f_{cd}} \qquad b = 1000 \text{ mm slab width}$$

$$x_{pl} = \frac{1562 \cdot 280}{1000 \cdot 0,85 \cdot 16,7} = 30,8 \text{ mm}$$

This value of x_{pl} gives a slightly conservative result for the lever arm z. But, for simplicity this value is used in this example.

The lever arm z is calculated as (9.9), EN 1994-1-1:

$$z = h - 0,5 \cdot x_{\mathrm{pl}} - e_{\mathrm{p}} + (e_{\mathrm{p}} - e) \cdot \frac{N_{\mathrm{c}}}{A_{\mathrm{pe}} f_{\mathrm{yp,d}}}$$

The lever arm z, for $N_c = N_p$ (Figure D3.8), is:

$$z = h - e - \frac{x_{pl}}{2}$$

$$z = 160 - 17 - \frac{30,8}{2} = 127,6 \text{ mm}$$

Therefore, the design plastic resistance moment in sagging region, $M_{pl,Rd}$, is:

$$M_{pl,Rd} = N_p \cdot z$$

$$M_{pl,Rd} = 437 \cdot 0{,}1276 = 55{,}8 \text{ kNm/m}$$

The longitudinal shear resistance of the slab, expressed by its design resistance moment, $M_{Rd}(x)$, found from the design shear strength, $\tau_{u,Rd}$, and the design bending moment from applied loads, $M_{Ed}(x)$, are plotted in Figure D3.9.

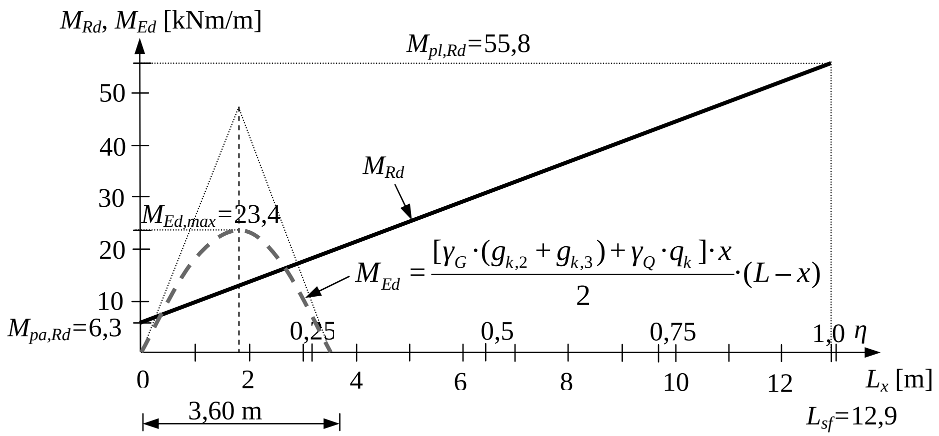


Figure D3.9 *Design partial interaction diagram*

According to the simplified partial interaction diagram, Figure D3.9, it is evident that the required condition, that for each cross-section along the span L:

$$\frac{M_{Ed}(x)}{M_{Rd}(x)} \leq 1{,}0$$

is not satisfied.

Remark:

The longitudinal shear resistance of the slab can be increased by the use of some form of end anchorage, such as studs or local deformations of the profiled sheeting, clause 9.7.4, EN 1994-1-1.

5.2.2.2 Longitudinal shear resistance with end anchorage

End anchorage is provided by welded studs:

- the stud diameter d = 19 mm, welded through the steel sheeting profile, the ultimate tensile strength $f_{u,k}$ = 500 N/mm^2 and the partial factor γ_V = 1,25,
- one stud connector in one rib – 6,67 studs/m.

According to clause 9.7.4, EN 1994-1-1, the design resistance of a headed stud welded through the steel sheeting profile used for end anchorage should be taken as the smallest of the design shear resistance of the stud in accordance with 6.6.4.2, EN 1994-1-1, or the bearing resistance of the sheet should be determined with the following expression:

$$P_{pb,Rd} = k_\varphi \cdot d_{d0} \cdot t \cdot f_{yp,d}$$

with:

$$k_\varphi = 1 + \frac{a}{d_{d0}} \leq 6{,}0$$

where:

d_{d0} is the diameter of the weld collar, which may be taken as 1,1 times the diameter of the shank of the stud,
a is the distance from the centre of the stud to the end of the sheeting, to be not less than $1{,}5 \cdot d_{d0}$,
t is the thickness of sheeting.

In this case, the bearing resistance of the sheet is governed. The following values needed for the calculation of the bearing resistance of the sheet are:

- the thickness of sheeting t = 0,88 mm,
- the diameter of the weld collar $d_{d0} = 1{,}1 \cdot d = 1{,}1 \cdot 19 = 20{,}9$ mm,
- the distance from the centre of the stud to the end of the sheeting $a = 2{,}0 \cdot d_{d0}$.

The value of k_φ is:

$$k_\varphi = 1 + \frac{a}{d_{d0}} \leq 6{,}0$$

$$k_\varphi = 1 + \frac{2 \cdot d_{d0}}{d_{d0}} = 3 < 6{,}0$$

The design bearing resistance of the sheet is:

$$P_{pb,Rd} = k_{\varphi} \cdot d_{d0} \cdot t \cdot f_{yp,d}$$

$$P_{pb,Rd} = 3 \cdot 20{,}9 \cdot 0{,}88 \cdot 280 \cdot 10^{-3} = 15{,}4 \text{ kN}$$

The design shear resistance of the end anchorage is:

$$V_{l,Rd} = 6{,}67 \cdot P_{pb,Rd} = 6{,}67 \cdot 15{,}4 = 102{,}7 \text{ kN/m}$$

The degree of shear connection is:

$$\eta_{l,d} = \frac{V_{l,Rd}}{N_{c,f}} = \frac{102{,}7}{437} = 0{,}24$$

The calculation is carried out using the linear interpolation method. The contribution of the end anchorage to the longitudinal shear resistance is taken into consideration by shifting the resistance curve to the left, Figure D3.10.

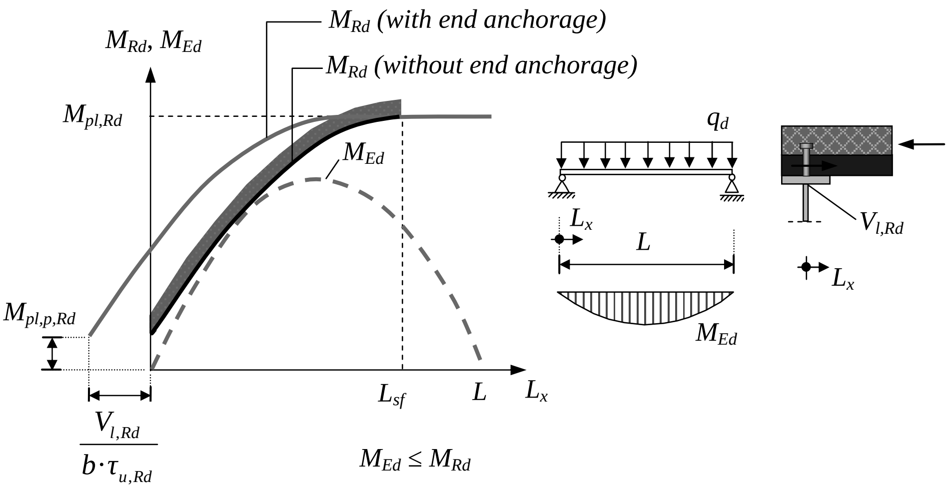


Figure D3.10 *End anchorage design*

Linear interpolation:

$$\frac{-V_{l,Rd}}{b \cdot \tau_{u,Rd}} = \frac{-102{,}7}{1{,}0 \cdot 34{,}0} = -3{,}02 \text{ m}$$

$$\frac{N_{c,f} - V_{l,Rd}}{b \cdot \tau_{u,Rd}} = \frac{437 - 102{,}7}{1{,}0 \cdot 34{,}0} = 9{,}83 \text{ m}$$

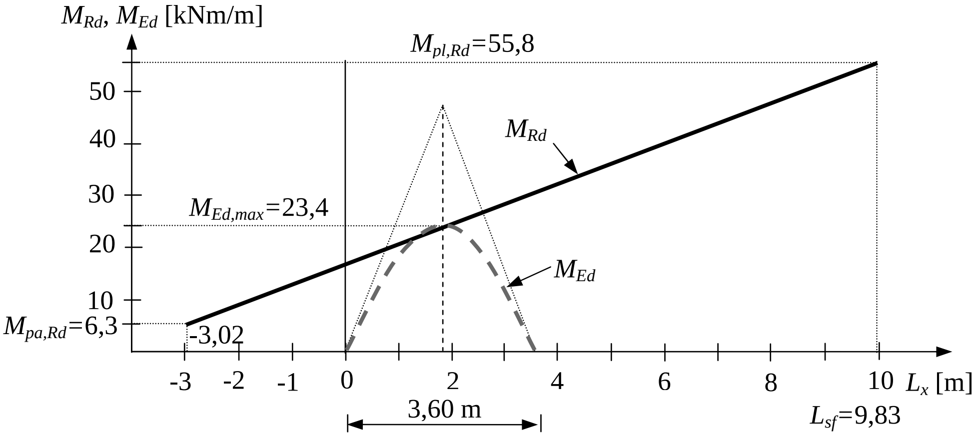


Figure D3.11 *Design partial interaction diagram, slab with end anchorage*

The simplified design partial interaction diagram for the slab with end anchorage is shown in Figure D3.11.

According to the simplified partial interaction diagram, Figure D3.11, it is evident that the required condition, that for each cross-section along the span *L*:

$$\frac{M_{Ed}(x)}{M_{Rd}(x)} \leq 1{,}0$$

is not satisfied.

Remark:

The longitudinal shear resistance of the slab can be increased by the use of additional reinforcement provided in troughs of the profiled sheeting, clause 9.7.3(10), EN 1994-1-1.

5.2.2.3 Longitudinal shear resistance with additional reinforcement

The following additional reinforcement is assumed:

- one ϕ 6 mm per rib $A_s = \frac{d^2 \cdot \pi}{4} \cdot \frac{b}{b_s} = \frac{6^2 \cdot \pi}{4} \cdot \frac{1000}{150} = 188\ \text{mm}^2\text{/m}$,
- yield strength of reinforcing steel $f_{yk,s} = 500\ \text{N/mm}^2$,
- distance between the reinforcement in tension and the extreme fibre of the composite slab in compression d_s = 130 mm.

In the analytical model it is assumed that the total resistance is that from composite action of the concrete with both the profiled sheeting and the bars, as for reinforced concrete. The resistance is calculated by plastic analysis of the cross-section.

At $L_x = 0$, there is no composite action and the load acts on the profiled sheeting. The distribution of stresses is shown in Figure D3.12.

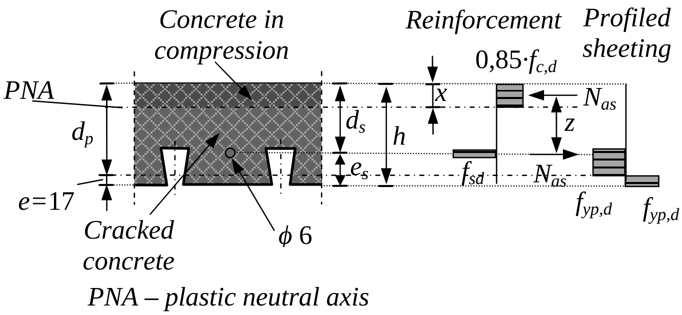


Figure D3.12 *Cross-section and stress distribution*

The design resistance of reinforcement to axial force is:

$$N_{as} = \frac{A_s \cdot f_{yk,s}}{\gamma_s} = \frac{188 \cdot 500}{1{,}15} \cdot 10^{-3} = 81{,}7 \ \text{kN/m}$$

At $L_x = 0$, there is no composite action, $\eta = 0$. It is assumed that the reinforcement is fully anchored at $x = 0$. The resistance moment is that of the profiled sheeting only, $M_{pa,Rd}$. The profiled sheeting has no longitudinal force, i.e.

$$N_p = 0 \ \text{kN}$$

Forces N_{as} and N_p give the longitudinal strains at two levels in the slab, for values of x. The depth of the neutral axis in the slab below the top surface is given by:

$$x = 1{,}25 \cdot \frac{b \cdot (N_{as} + N_p)}{b \cdot 0{,}85 \cdot f_{cd}}$$

assuming a rectangular stress block, see Figures D3.12 and D3.13, which extends to 80% of the depth to the neutral axis, [36].

The depth of the neutral axis in the slab below the top surface is:

$$x = 1,25 \cdot \frac{(81,7+0)}{0,85 \cdot 16,7} = 7,2 \text{ mm}$$

The lever arm z is determined in accordance with Figure D3.12:

$$z = h - e_s - \frac{x}{2}$$

$$z = 160 - 30 - \frac{7,2}{2} = 126 \text{ mm}$$

The design resistance moment, $M_{Rd}(\eta = 0)$, is calculated as:

$$M_{Rd}(\eta = 0) = M_{pr} + N_{as} \cdot z$$

With $M_{pr} = M_{pa,Rd} = 6,3$ kNm/m, the design resistance moment, $M_{Rd}(\eta = 0)$, is:

$$M_{Rd}(\eta = 0) = M_{pr} + N_{as} \cdot z$$

$$M_{Rd}(\eta = 0) = 6,3 + 81,7 \cdot 0,126 = 16,6 \text{ kNm/m}$$

At $L_x = L_{s,f}$, where $\eta = 1$ (full shear connection), the design plastic bending of the composite cross-section, $M_{pl,Rd}$, is reached. The design resistance moment is calculated in accordance with Figure D3.13.

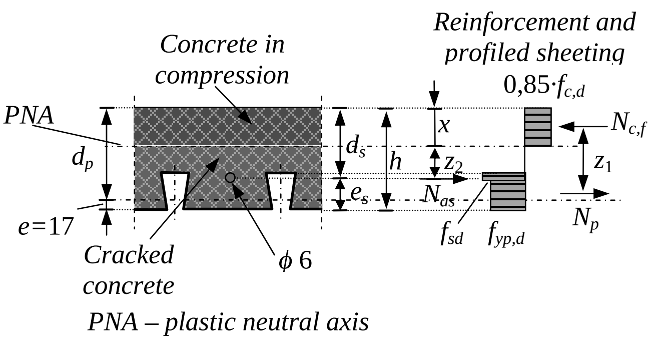


Figure D3.13 *Cross-section and stress distribution*

At this point, where $\eta = 1$, there is an axial tensile force N_p in the profiled sheeting but no bending moment, $M_{pr} = 0$. The axial tensile force N_p is:

$$N_p = A_p \cdot f_{yp,d} = 1562 \cdot 0,280 = 437 \text{ kN/m}$$

The depth of the neutral axis in the slab below the top surface is given by:

$$x = 1{,}25 \cdot \frac{b \cdot (N_p + N_{as})}{b \cdot 0{,}85 \cdot f_{cd}} = 1{,}25 \cdot \frac{(437 + 81{,}7)}{0{,}85 \cdot 16{,}7} = 45{,}7 \text{ mm}$$

The lever arms z_1 and z_2 are determined in accordance with Figure D3.13:

$$z_1 = h - e - \frac{x}{2}$$

$$z_1 = 160 - 17 - \frac{45{,}7}{2} = 120 \text{ mm}$$

$$z_2 = h - e_s - \frac{x}{2}$$

$$z_2 = 160 - 30 - \frac{45{,}7}{2} = 107 \text{ mm}$$

The design resistance moment, $M_{Rd}(\eta = 1)$, is calculated as:

$$M_{Rd}(\eta = 1) = N_p \cdot z_1 + M_{pr} + N_{as} \cdot z_2$$

With $M_{pr} = 0$, the design bending moment, $M_{Rd}(\eta = 1)$, is:

$$M_{Rd}(\eta = 1) = N_p \cdot z_1 + M_{pr} + N_{as} \cdot z_2$$

$$M_{Rd}(\eta = 1) = 437 \cdot 0{,}120 + 0 + 81{,}7 \cdot 0{,}107 = 61{,}2 \text{ kNm/m}$$

The simplified design partial interaction diagram is shown in Figure D3.14.

According to the simplified partial diagram, Figure D3.14, it is evident that the required condition is satisfied: for each cross-section along the span L:

$$\frac{M_{Ed}(x)}{M_{Rd}(x)} \leq 1{,}0$$

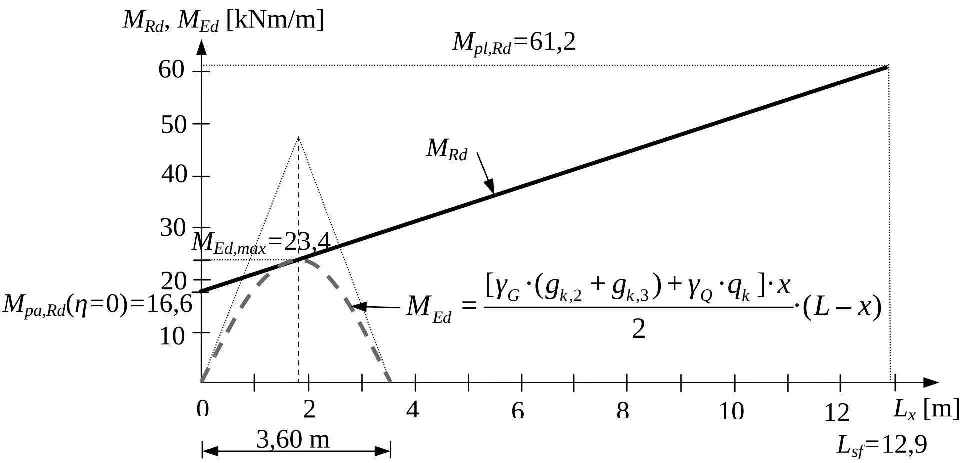


Figure D3.14 *Design partial interaction diagram, with additional reinforcement*

5.2.3 Check for vertical shear resistance

According to 9.7.5, EN 1994-1-1, the vertical shear resistance, $V_{v,Rd}$, should be determined according to the method given in EN 1992-1-1. According to clause 6.2.2, EN 1992-1-1, the design shear resistance $V_{v,Rd}$ is calculated as:

$$V_{v,Rd} = V_{Rd,c} = [C_{Rd,c} \cdot k \cdot (100 \cdot \rho_l \cdot f_{ck})^{1/3} + k_1 \cdot \sigma_{cp}]\ b_w \cdot d_p \geq V_{v,Rd,\min}$$

The minimum value of $V_{v,Rd,\min}$ is:

$$V_{v,Rd,\min} = (v_{\min} + k_1 \cdot \sigma_{cp}) \cdot b_w \cdot d_p$$

The minimum requirement for $V_{v,Rd}$ is related to the fact that the member without reinforcement still has some shear resistance.

Generally, the check is carried out as:

$$\frac{V_{Ed}}{V_{v,Rd}} \leq 1{,}0$$

According to clause 6.2.2(1), EN 1992-1-1, the values needed for calculation of $V_{v,Rd}$ are:

$$C_{Rd,c} = \frac{0{,}18}{\gamma_c} = \frac{0{,}18}{1{,}5} = 0{,}12$$

$$k = 1 + \sqrt{\frac{200}{d_p}} \leq 2{,}0$$

$$k = 1 + \sqrt{\frac{200}{143}} = 2{,}18 \ \rightarrow \text{ adopted } k = 2{,}0$$

$$\rho_l = \frac{A_{sl}}{b_w \cdot d_p}$$

The resistance of the cross-section is dependent on the area of the tensile reinforcement, whose section has to be extended by an appropriate anchorage length, (l_{bd} + d) see – Figure 6.3, EN 1992-1-1 – where l_{bd} is the design anchorage length and d is the effective depth of the section, taken as the depth from the top surface to the centroid of the profile for a composite slab. The anchorage of the profiled sheeting was confirmed by the check on longitudinal shear and the sheeting can be treated as reinforcement. Treating the profiled sheeting as the reinforcement and taking into account the additional reinforcement, the area of tensile reinforcement is:

$$A_{sl} = A_p + A_s = 1562 + 188 = 1750 \ \text{mm}^2$$

In accordance with Figure D3.15, the smallest width of the cross-section in the tensile area b_w is calculated per metre width as:

$$b_w = \frac{b}{b_s} \cdot b_0 = \frac{1000}{150} \cdot 114 = 760 \ \text{mm/m}$$

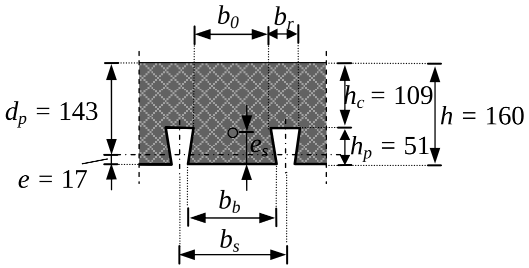
The percentage of longitudinal reinforcement is:

$$\rho_l = \frac{A_{sl}}{b_w \cdot d_p}$$

$$\rho_l = \frac{1562 + 188}{760 \cdot 143} = 0{,}016 < 0{,}02$$

The design axial force is $N_{Ed} = 0$ and therefore $\sigma_{cp} = \dfrac{N_{Ed}}{A_c} = 0$.

$k_1 = 0{,}15$, according to clause 6.2.2(1), EN 1992-1-1



b_s = 150 mm
b_r = 36 mm
b_0 = 114 mm
e_s = 30 mm

Figure D3.15 *Determination of value* b_w

The design shear resistance $V_{v,Rd}$ is:

$$V_{v,Rd} = V_{Rd,c} = [C_{Rd,c} \cdot k \cdot (100 \cdot \rho_l \cdot f_{ck})^{1/3} + k_1 \cdot \sigma_{cp}]\ b_w \cdot d_p$$

$$V_{v,Rd} = [0,12 \cdot 2,0 \cdot (100 \cdot 0,016 \cdot 25)^{1/3} + 0,15 \cdot 0]\ \cdot 760 \cdot 143 \cdot 10^{-3}$$

$$V_{v,Rd} = 89,2 \text{ kN/m}$$

The minimum value is:

$$V_{v,Rd,\min} = (v_{\min} + k_1 \cdot \sigma_{cp}) \cdot b_w \cdot d_p$$

$$v_{\min} = 0,035 \cdot k^{3/2} \cdot f_{ck}^{\ 1/2} = 0,035 \cdot 2,0^{3/2} \cdot 25^{1/2} = 0,49 \text{ N/mm}^2$$

$$V_{v,Rd,\min} = (0,49 + 0,15 \cdot 0) \cdot 760 \cdot 143 \cdot 10^{-3} = 53,3 \text{ kN/m} < V_{v,Rd} = 89,2 \text{ kN/m}$$

Check:

$$\frac{V_{Ed}}{V_{v,Rd}} \leq 1,0$$

$$\frac{26,0}{89,2} = 0,29 < 1,0 \text{ , the condition is satisfied}$$

5.3 Composite stage – alternatively, the composite slab is designed as continuous

The profiled sheeting is provided in three-span lengths. The concrete is cast on the sheeting without joints. Therefore, the composite slab is continuous over more spans.

The maximum design moment at the internal support is calculated for the design loads shown in Figure D3.16.

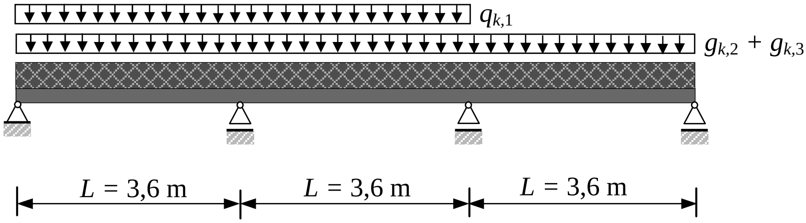


Figure D3.16 *Design load for maximum design moment at the internal support*

Linear elastic analysis with limited redistribution is applied, clause 5.4.4, EN 1994-1-1, for calculation of the action effects.

The maximum design moment at internal support, $M_{Ed,B}$, is:

$$M_{Ed,B} = \gamma_G \cdot M_g + \gamma_Q \cdot M_q$$

$$M_{Ed,B} = 1{,}35 \cdot [0{,}10 \cdot (3{,}93 + 1{,}20) \cdot 3{,}6^2] + 1{,}50 \cdot [0{,}117 \cdot 5{,}0] \cdot 3{,}6^2 = 20{,}3 \text{ kNm/m}$$

The maximum design shear force at internal support, $V_{Ed,B}$, is:

$$V_{Ed,B} = [1{,}35 \cdot (3{,}93 + 1{,}2) + 1{,}50 \cdot 5{,}0] \cdot \frac{3{,}6}{2} + \frac{20{,}3}{3{,}6} = 31{,}6 \text{ kN/m}$$

The resulting moments at the internal supports, calculated by elastic global analysis based on the uncracked stiffness, are high. To resist these moments the heavy reinforcement is required. According to clause 9.4.2(5), EN 1994-1-1, to avoid the heavy reinforcement the slab can be designed as a series of simply supported spans, provided that control of the cracking of concrete is not a problem. This approach has previously been carried out, see Section 5.2. The reduction in the quantity of hogging reinforcement is possible by the use of redistribution of moments, clause 9.4.2(3), EN 1994-1-1.

Clause 9.4.2(3), EN 1994-1-1, states that *"If the effects of cracking of concrete are neglected in the analysis for ultimate limit states, the bending moments at internal supports may optionally be reduced by up to 30%, and corresponding increases made to the sagging bending moments in the adjacent spans."*

The maximum design moment at the internal support, taking into account the redistribution, $M_{Ed,B,r}$, is:

$$M_{Ed,B,r} = (1-0{,}30)\cdot 20{,}3 = 14{,}2 \text{ kNm/m}$$

The maximum design shear force at the internal support, taking into account the redistribution, $V_{Ed,B,r}$, is:

$$V_{Ed,B,r} = [1{,}35\cdot(3{,}93+1{,}2)+1{,}50\cdot 5{,}0]\cdot\frac{3{,}6}{2}+\frac{14{,}2}{3{,}6} = 29{,}9 \text{ kN/m}$$

The reaction at the end support is:

$$R_{Ed,A,r} = [1{,}35\cdot(3{,}93+1{,}2)+1{,}50\cdot 5{,}0]\cdot\frac{3{,}6}{2}-\frac{14{,}2}{3{,}6} = 22{,}0 \text{ kN/m}$$

The point of maximum bending moment is at a distance of 22,0/[1,35·(3,93 + 1,2) + 1,50·5,0] = 1,53 m from the end support, and the maximum design sagging moment is:

$$M_{Ed,r} = 22{,}0\cdot 1{,}53-[1{,}35\cdot(3{,}93+1{,}2)+1{,}50\cdot 5{,}0]\cdot 1{,}53\cdot\frac{1{,}53}{2} = 16{,}8 \text{ kNm/m}$$

5.3.1 Plastic resistance moment in hogging region

The design resistance moment at the internal support is calculated taking into account the longitudinal reinforcement and according to the stress distribution shown in Figure D3.17.

The following additional reinforcement is assumed:

- ϕ 12/250 mm $\quad A_s = \frac{d^2\cdot\pi}{4}\cdot\frac{b}{b_s} = \frac{12^2\cdot\pi}{4}\cdot\frac{1000}{250} = 452 \text{ mm}^2\text{/m}$,
- yield strength of reinforcing steel $\quad f_{yk,s} = 500 \text{ N/mm}^2$,
- distance between the reinforcement and the top surface of the composite slab $\quad e_s$ = 20 mm,
- average width of concrete in compression $\quad b_c$ = 840 mm/m.

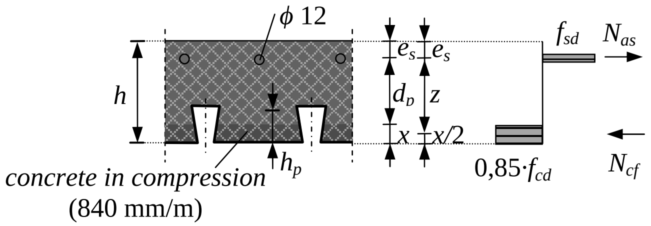


Figure D3.17 *Cross-section of composite slab and stress blocks for hogging bending*

$$b_c = (\frac{b_b + b_0}{2}) \cdot \frac{b}{b_s} = (\frac{138 + 114}{2}) \cdot \frac{1000}{150} = 840 \text{ mm/m}$$

The design resistance of the reinforcement bars is:

$$N_{as} = \frac{A_s \cdot f_{ys,k}}{\gamma_s} = \frac{452 \cdot 500}{1{,}15} \cdot 10^{-3} = 197 \text{ kN/m}$$

The design internal force in the concrete is:

$$N_{c,f} = 0{,}85 \cdot b_c \cdot x_{pl} \cdot f_{cd}$$

Equilibrium gives the depth of concrete in compression x_{pl} as:

$$x_{pl} = \frac{N_{cf}}{b_c \cdot 0{,}85 \cdot f_{cd}} = \frac{197 \cdot 10^3}{840 \cdot 0{,}85 \cdot 16{,}7} = 16{,}5 \text{ mm}$$

The design resistance moment is:

$$M_{Rd} = N_{c,f} \cdot z = N_{c,f} \cdot (h - e_s - \frac{x_{pl}}{2})$$

$$M_{Rd} = 197 \cdot (160 - 20 - \frac{16{,}5}{2}) \cdot 10^{-3} = 26{,}0 \text{ kNm/m}$$

Check:

$$\frac{M_{Ed,B,r}}{M_{Rd}} \leq 1{,}0$$

$$\frac{14,2}{26,0} = 0,55 < 1,0 \text{, the condition is satisfied}$$

To allow rotations in the yielded sections, the reinforcement should be sufficiently ductile. The selected high yield reinforcing steel satisfies this criterion since the depth of the concrete slab is not too great.

Since the design bending moment in the sagging region is less than the design bending moment for the simply supported slab, the verification of the sagging resistance moment is not necessary.

5.3.2 Longitudinal shear resistance

According to clause 9.7.3(6), EN 1994-1-1, where the composite slab is designed as continuous, it is permitted to use an equivalent isostatic span for the determination of the resistance. The span length should be taken as (Figure D3.18):

- 0,9·L for internal spans
- 0,8·L for external spans

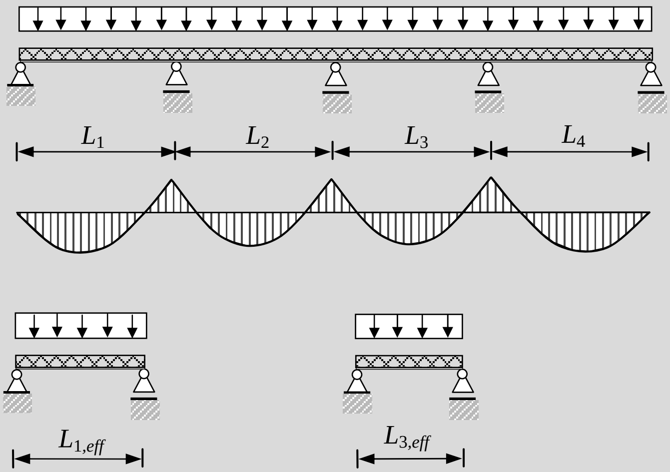


Figure D3.18 *Equivalent isostatic spans for determination of the resistance*

The design resistance against longitudinal shear is carried out as for the simply supported slab. From the simplified design partial interaction diagram shown in Figure D3.14, it is evident that the required condition is satisfied, that for each cross-section along the span L:

$$\frac{M_{Ed}(x)}{M_{Rd}(x)} \leq 1{,}0$$

5.3.3 Check for vertical shear resistance

End support

The vertical shear resistance, $V_{v,Rd}$, is the same as the vertical shear resistance for the simply supported slab. Therefore, the vertical shear resistance is:

$$V_{v,Rd} = 89{,}2 \text{ kN/m}$$

Check:

$$\frac{V_{Ed,A,r}}{V_{V,Rd}} \leq 1{,}0$$

$$\frac{22{,}0}{89{,}2} = 0{,}25 < 1{,}0 \text{, the condition is satisfied}$$

Internal support

According to 9.7.5, EN 1994-1-1, the vertical shear resistance, $V_{v,Rd}$, should be determined according to the method given in EN 1992-1-1. According to clause 6.2.2, EN 1992-1-1, the design shear resistance $V_{v,Rd}$ is calculated as:

$$V_{v,Rd} = V_{Rd,c} = [C_{Rd,c} \cdot k \cdot (100 \cdot \rho_l \cdot f_{ck})^{1/3} + k_1 \cdot \sigma_{cp}]\ b_w \cdot d_p \geq V_{v,Rd,\min}$$

The minimum value of $V_{v,Rd,\min}$ is:

$$V_{v,Rd,\min} = (v_{\min} + k_1 \cdot \sigma_{cp}) \cdot b_w \cdot d_p$$

The minimum requirement for $V_{v,Rd}$ is related to the fact that the member without reinforcement still has some shear resistance.

Generally, the check is carried out as:

$$\frac{V_{Ed}}{V_{v,Rd}} \leq 1{,}0$$

According to clause 6.2.2(1), EN 1992-1-1, the values needed for calculation $V_{v,Rd}$ are:

$$C_{Rd,c} = \frac{0,18}{\gamma_c} = \frac{0,18}{1,5} = 0,12$$

$$k = 1 + \sqrt{\frac{200}{d_p}} \leq 2,0$$

$$d_p = h - e_s - x_{pl}$$

$$d_p = 160 - 20 - 16,5 = 123,5 \text{ mm}$$

$$k = 1 + \sqrt{\frac{200}{123,5}} = 2,27 \rightarrow \text{adopted } k = 2,0$$

$$\rho_l = \frac{A_{sl}}{b_w \cdot d_p} \leq 0,02$$

The area of longitudinal reinforcement in the slab at the internal support is (Section 5.3.1):

$$A_{sl} = 452 \text{ mm}^2/\text{m}$$

In accordance with Figure D3.15, the smallest width of the cross-section in the tensile area b_w is calculated per metre width as:

$$b_w = \frac{b}{b_s} \cdot b_0 = \frac{1000}{150} \cdot 114 = 760 \text{ mm/m}$$

The percentage of longitudinal reinforcement is:

$$\rho_l = \frac{A_{sl}}{b_w \cdot d_p}$$

$$\rho_l = \frac{452}{760 \cdot 123,5} = 0,005 < 0,02$$

The design axial force is $N_{Ed} = 0$ and therefore $\sigma_{cp} = \dfrac{N_{Ed}}{A_c} = 0$.

$k_1 = 0{,}15$, according to 6.2.2, EN 1992-1-1

The design shear resistance $V_{v,Rd}$ is:

$$V_{v,Rd} = V_{Rd,c} = [C_{Rd,c} \cdot k \cdot (100 \cdot \rho_l \cdot f_{ck})^{1/3} + k_1 \cdot \sigma_{cp}]\ b_w \cdot d_p$$

$$V_{v,Rd} = [0{,}12 \cdot 2{,}0 \cdot (100 \cdot 0{,}005 \cdot 25)^{1/3} + 0{,}15 \cdot 0]\ \cdot 760 \cdot 123{,}5 \cdot 10^{-3}$$

$$V_{v,Rd} = 52{,}3\ \text{kN/m}$$

The minimum value is:

$$V_{v,Rd,\min} = (v_{\min} + k_1 \cdot \sigma_{cp}) \cdot b_w \cdot d_p$$

$$v_{\min} = 0{,}035 \cdot k^{3/2} \cdot f_{ck}{}^{1/2} = 0{,}035 \cdot 2{,}0^{3/2} \cdot 25^{1/2} = 0{,}49\ \text{N/mm}^2$$

$$V_{v,Rd,\min} = (0{,}49 + 0{,}15 \cdot 0) \cdot 760 \cdot 123{,}5 \cdot 10^{-3} = 46{,}0\ \text{kN/m} < V_{v,Rd} = 52{,}3\ \text{kN/m}$$

Check:

$$\frac{V_{Ed,B,r}}{V_{V,Rd}} \leq 1{,}0$$

$\dfrac{29{,}9}{52{,}3} = 0{,}57 < 1{,}0$, the condition is satisfied

6. Serviceability limit state

6.1 Control of cracking of concrete

When the composite slab is designed as a series of simply supported spans, it only requires reinforcement for crack width limitation.

According to clause 9.8.1(2), EN 1994-1-1, for propped construction the required cross-sectional area of reinforcement A_s is 0,4% of the area of concrete above the ribs:

$$A_s = \frac{0,4}{100} \cdot 1000 \cdot (h - h_p)$$

$$A_s = \frac{0,4}{100} \cdot 1000 \cdot (160 - 51) = 436 \ \text{mm}^2/\text{m}$$

The reinforcement bar is assumed to be ϕ 12/250 mm. Therefore, the cross-sectional area of reinforcement is:

$$A_s = \frac{12^2 \cdot \pi}{4} \cdot \frac{1000}{250} = 452 \ \text{mm}^2/\text{m} > 436 \ \text{mm}^2/\text{m}$$

The selected minimum amount of reinforcement could be insufficient to control cracking at the supports of continuous slabs for certain exposure classes. In such cases, the slab should be designed as continuous, and in hogging regions the crack widths should be estimated according to EN 1992-1-1.

6.2 Limit of span/depth ratio of slab

According to clause 9.8.2(4), EN 1994-1-1, calculation of the deflection of the composite slab can be omitted if the two conditions are satisfied. According to the first condition, the span/depth ratio of the slab should not exceed the limits given in EN 1992-1-1. These are:

- $\frac{L}{d} < 20$ for a simply supported span
- $\frac{L}{d} < 26$ for an external span of continuous slab
- $\frac{L}{d} < 30$ for an internal span of continuous slab

According to clause 9.8.2(6), EN 1994-1-1, the second condition is given as follows:

- the load causing an end slip of 0,5 mm in the tests on composite slab exceeds 1,2 times the design service load.

If the second condition is not satisfied, i.e. the end slip exceeds 0,5 mm at a load 1,2 times the design service load, two options exist:

- end anchors should be provided, or
- deflections should be calculated including the effect of end slip.

According to clause 9.8.2(8), EN 1994-1-1, in cases where the behaviour of the shear connection between the profiled sheeting and the concrete is not known from tests, the tied-arch model may be used, see [34].

For the considered slab with L = 3600 mm and d_p = 143 mm, the following span/depth ratio is obtained:

- $\frac{L}{d} = \frac{3600}{143} = 25 < 26$ for the external span of continuous slab
- $\frac{L}{d} = \frac{3600}{143} = 25 < 30$ for the internal span of continuous slab

Therefore, calculation of the deflection is not necessary. However, the calculation is carried out for educational reasons.

6.3 Calculation of deflections

6.3.1 Construction stage deflection

According to clause 9.6(2), EN 1994-1-1, the deflection, δ_s, of the profiled sheeting due to its own weight and the weight of the wet concrete should not exceed the following limit:

$$\delta_{s,\max} = \frac{L}{180} = \frac{1800}{180} = 10 \text{ mm}$$

The premature local buckling of the profiled sheeting under the weight of the wet concrete and the construction loading is checked to prevent irreversible deformation.

Maximum sagging bending moment in the serviceability limit state is:

$$M_{Ed} = 0{,}078 \cdot 0{,}13 \cdot 1{,}8^2 + 0{,}094 \cdot 3{,}95 \cdot 1{,}8^2 = 1{,}24 \text{ kNm/m}$$

Maximum compressive stress in the top flange of the profiled sheeting is:

$$\sigma_{com} = \frac{M_{Ed}}{\mathrm{I}_p} \cdot z = \frac{1{,}24 \cdot 10^6}{62 \cdot 10^4} \cdot (51 - 17) = 68 \text{ N/mm}^2$$

In accordance with clause 4.4, EN 1993-1-5, the plate slenderness, $\bar{\lambda}_p$, is calculated as:

$$\bar{\lambda}_p = \sqrt{\frac{f_y}{\sigma_{cr}}} = \frac{\frac{b}{t}}{28,4 \cdot \varepsilon \cdot \sqrt{k_\sigma}}$$

$$\varepsilon = \sqrt{\frac{235}{\sigma_{com}}} = \sqrt{\frac{235}{68}} = 1,9$$

According to Table 4.1, EN 1994-1-1, for the stress ratio ψ = 1, the buckling factor is k_σ = 4.

Therefore, the plate slenderness, $\bar{\lambda}_p$, with the design thickness of the sheet t = 0,84 mm (not including coatings) and $b = b_r$ = 36 mm, is:

$$\bar{\lambda}_p = \frac{\frac{36}{0,84}}{28,4 \cdot 1,9 \cdot \sqrt{4}} = 0,397$$

Since $\bar{\lambda}_p = 0,397 < 0,673$, the reduction factor is ρ = 1,0 and the cross-section is fully effective.

The deflection, δ_s, is:

$$\delta_s = \frac{(2,65 \cdot g_p + 3,4 \cdot g_{c,1}) \cdot L^4}{384 \cdot E \cdot \mathrm{I}_p}$$

The deflection, δ_s, is:

$$\delta_s = \frac{(2,65 \cdot 0,13 + 3,4 \cdot 3,95) \cdot 1800^4}{384 \cdot 210000 \cdot 62 \cdot 10^4} = 2,9 \text{ mm}$$

δ_s = 2,9 mm < $\delta_{s,max}$ = 15,5 mm

The deflection due to the self-weight of the profiled sheeting and the weight of the wet concrete meets the criterion L/180.

Since the deflection δ_s is less than 10% of the slab depth, δ_s = 2,9 mm < 0,10·h = 0,1·160 = 16 mm, according to clause 9.3.2(2), EN 1994-1-1, the ponding effects can be neglected at the construction stage.

The conditions for the serviceability limit state are satisfied, and the profiled steel sheeting can be used at the construction stage.

6.3.2 Composite stage deflection

To calculate the deflection at the composite stage, the slab is considered as continuous over three spans. According to clause 9.8.2(5), EN 1994-1-1, the following approximations can be applied:

- The second moment of area can be taken as the average of the values for the cracked and uncracked sections.
- An average value of the modular ratio, *n*, for both short-term and long-term effects, can be used:

$$n = \frac{E_a}{E'_{cm}} = \frac{E_a}{\frac{1}{2}\cdot\left(E_{cm} + \frac{E_{cm}}{3}\right)} = \frac{210000}{\frac{2}{3}\cdot 31000} = 10{,}2$$

- Elastic analysis is used to calculate the deflection of the slab.

a) The second moment of area for the cracked section, I_{cc}, for slab width *b* is calculated in accordance with Figure D3.19. The longitudinal reinforcement is not taken into account.

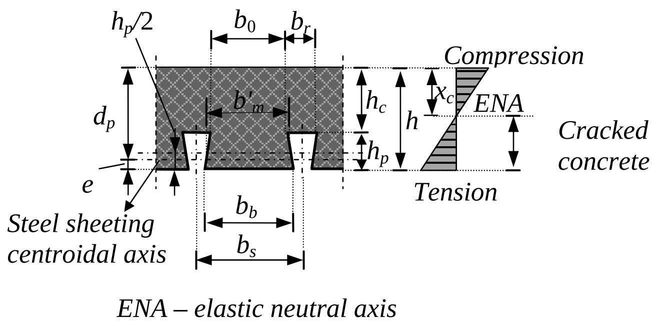


Figure D3.19 *Second moment of area calculation for cracked cross-section, I_{cc}*

The second moment of area for the cracked section and the slab width *b* is calculated as:

$$I_{cc} = \frac{b\cdot x_c^{\,3}}{3\cdot n} + A_p\cdot(d_p - x_c)^2 + I_p$$

The position of the elastic neutral axis relative to the upper side of the slab is obtained as:

$$x_c = \frac{\Sigma A_i \cdot z_i}{\Sigma A_i} = \frac{n \cdot A_p}{b}\left(\sqrt{1+\frac{2 \cdot b \cdot d_p}{n \cdot A_p}} - 1\right)$$

$$x_c = \frac{10,2 \cdot 1562}{1000}\left(\sqrt{1+\frac{2 \cdot 1000 \cdot 143}{10,2 \cdot 1562}} - 1\right) = 53,4 \text{ mm}$$

The second moment of area for the cracked section is:

$$I_{cc} = \frac{1000 \cdot 53,4^3}{3 \cdot 10,2} + 1562 \cdot (143 - 53,4)^2 + 62 \cdot 10^4 = 1814 \cdot 10^4 \text{ mm}^4/\text{m}$$

b) The second moment of area for the uncracked section, I_{cu}, for slab width b is calculated in accordance with Figure D3.20. The longitudinal reinforcement is not taken into account.

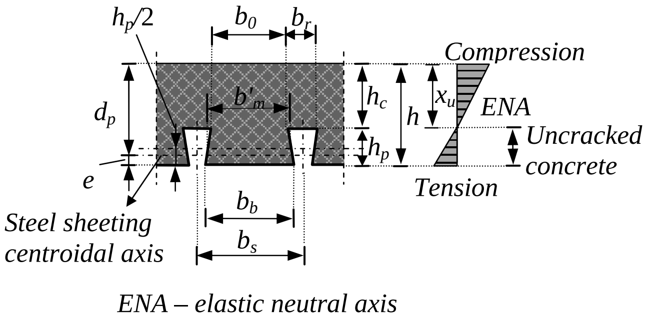


Figure D3.20 *Second moment of area calculation for uncracked cross-section, I_{cu}*

The second moment of area for the uncracked section and the slab width b is calculated as:

$$I_{cu} = \frac{b \cdot h_c^3}{12 \cdot n} + \frac{b \cdot h_c}{n} \cdot \left(x_u - \frac{h_c}{2}\right)^2 + \frac{b_m \cdot h_p^3}{12 \cdot n} + \frac{b_m \cdot h_p}{n} \cdot \left(h - x_u - \frac{h_p}{2}\right)^2 +$$
$$+ A_p \cdot (d_p - x_u)^2 + I_p$$

where:

$$x_u = \frac{b \cdot \frac{h_c^2}{2} + b_m \cdot h_p \cdot (h - \frac{h_p}{2}) + n \cdot A_p \cdot d_p}{b \cdot h_c + b_m \cdot h_p + n \cdot A_p}$$

In accordance with Figure D3.20, the value of b_m is:

$$b'_m = \frac{(b_b - 2 \cdot t) + b_0}{2} = \frac{(138 - 2 \cdot 0,88) + 114}{2} = 125,1 \text{ mm}$$

$$b_m = \frac{b}{b_s} \cdot b'_m = \frac{1000}{150} \cdot 125,1 = 834 \text{ mm/m}$$

The position of the elastic neutral axis relative to the upper side of the slab is:

$$x_u = \frac{1000 \cdot \frac{109^2}{2} + 834 \cdot 51 \cdot (160 - \frac{51}{2}) + 10,2 \cdot 1562 \cdot 143}{1000 \cdot 109 + 834 \cdot 51 + 10,2 \cdot 1562} = 83,2 \text{ mm}$$

The second moment of area for the uncracked section is:

$$I_{cu} = \frac{1000 \cdot 109^3}{12 \cdot 10,2} + \frac{1000 \cdot 109}{10,2} \cdot (83,2 - \frac{109}{2})^2 + \frac{834 \cdot 51^3}{12 \cdot 10,2} + \\ + \frac{834 \cdot 51}{10,2} \cdot (160 - 83,2 - \frac{51}{2})^2 + 1562 \cdot (143 - 83,2)^2 + 62 \cdot 10^4$$

$$I_{cu} = 3747 \cdot 10^4 \text{ mm}^4\text{/m}$$

The mean value of I_{cc} and I_{cu} is:

$$I_c = \frac{I_{cc} + I_{cu}}{2} = \frac{1814 \cdot 10^4 + 3747 \cdot 10^4}{2} = 2781 \cdot 10^4 \text{ mm}^4\text{/m}$$

Remark:

The longitudinal reinforcement was taken into account for the calculation of the design resistance moment at the internal support. Therefore, the second moment of area for both uncracked and cracked sections is calculated taking into account the longitudinal reinforcement.

c) The second moment of area for the cracked section, I_{cc}, for slab width b is calculated in accordance with Figure D3.21. The longitudinal reinforcement is taken into account.

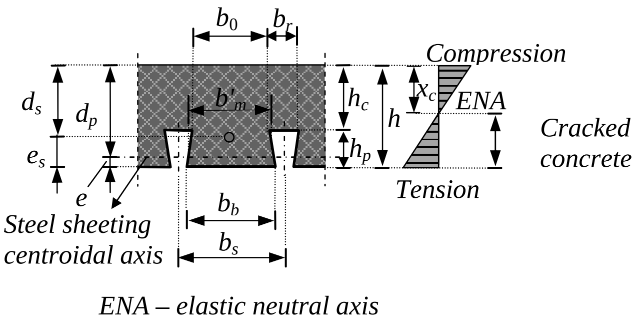


Figure D3.21 *Second moment of area calculation for cracked cross-section, I_{cc}*

The second moment of area for the cracked section and the slab width b is calculated as:

$$I_{bc} = \frac{b \cdot x_c^{\;3}}{3 \cdot n} + A_p \cdot (d_p - x_c)^2 + I_p + A_s \cdot (h - e_s - x_c)^2$$

The position of the elastic neutral axis relative to the upper side of the slab is obtained by the expression:

$$x_c = -\frac{n \cdot (A_p + A_s)}{b} + \sqrt{[\frac{n \cdot (A_p + A_s)}{b}]^2 + \frac{2 \cdot n \cdot [A_p \cdot d_p + A_s \cdot (h - e_s)]}{b}}$$

$$x_c = -\frac{10,2 \cdot (1562 + 188)}{1000} +$$
$$+ \sqrt{[\frac{10,2 \cdot (1562 + 188)}{1000}]^2 + \frac{2 \cdot 10,2 \cdot [1562 \cdot 143 + 188 \cdot (160 - 30)]}{1000}}$$

$$x_c = 55,6 \ \text{mm}$$

The second moment of area for the cracked section is:

$$I_{cc} = \frac{1000 \cdot 55,6^3}{3 \cdot 10,2} + 1562 \cdot (143 - 55,6)^2 + 62 \cdot 10^4 + 188 \cdot (160 - 30 - 55,6)^2$$

$$I_{cc} = 1921 \cdot 10^4 \ \text{mm}^4/\text{m}$$

d) The second moment of area for the uncracked section, I_{cu}, for the slab width b is calculated in accordance with Figure D3.22. The longitudinal reinforcement is taken into account.

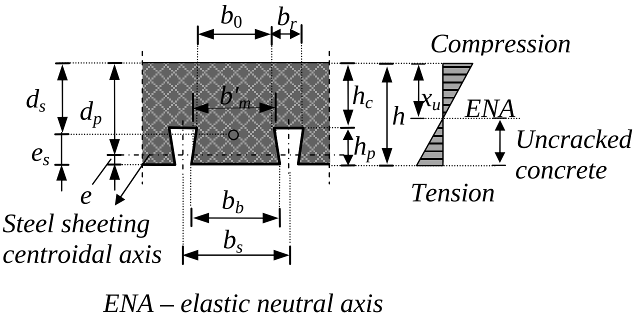


Figure D3.22 *Second moment of area calculation for uncracked cross-section, I_{cu}*

The second moment of area for the uncracked section and the slab width b is calculated as:

$$I_{cu} = \frac{b \cdot h_c^3}{12 \cdot n} + \frac{b \cdot h_c}{n} \cdot (x_u - \frac{h_c}{2})^2 + \frac{b_m \cdot h_p^3}{12 \cdot n} + \frac{b_m \cdot h_p}{n} \cdot (h - x_u - \frac{h_p}{2})^2 +$$
$$+ A_p \cdot (d_p - x_u)^2 + I_p + A_s \cdot (h - e_s - x_u)^2$$

where:

$$x_u = \frac{b \cdot \frac{h_c^2}{2} + b_m \cdot h_p \cdot (h - \frac{h_p}{2}) + n \cdot A_p \cdot d_p + n \cdot A_s \cdot (h - e_s)}{b \cdot h_c + b_m \cdot h_p + n \cdot A_p + n \cdot A_s}$$

In accordance with Figure D3.22, the value of b_m is:

$$b'_m = \frac{(b_b - 2 \cdot t) + b_0}{2} = \frac{(138 - 2 \cdot 0,88) + 114}{2} = 125,1 \text{ mm}$$

$$b_m = \frac{b}{b_s} \cdot b'_m = \frac{1000}{150} \cdot 125,1 = 834 \text{ mm/m}$$

The position of the elastic neutral axis relative to the upper side of the slab is:

$$x_u = \frac{1000 \cdot \frac{109^2}{2} + 834 \cdot 51 \cdot (160 - \frac{51}{2}) + 10,2 \cdot 1562 \cdot 143 + 10,2 \cdot 188 \cdot (160 - 30)}{1000 \cdot 109 + 834 \cdot 51 + 10,2 \cdot 1562 + 10,2 \cdot 188}$$

$x_u = 83,8$ mm

The second moment of area for the uncracked section is:

$$I_{cu} = \frac{1000 \cdot 109^3}{12 \cdot 10,2} + \frac{1000 \cdot 109}{10,2} \cdot (83,8 - \frac{109}{2})^2 + \frac{834 \cdot 51^3}{12 \cdot 10,2} +$$
$$+ \frac{834 \cdot 51}{10,2} \cdot (160 - 83,8 - \frac{51}{2})^2 + 1562 \cdot (143 - 83,8)^2 + 62 \cdot 10^4 +$$
$$+ 188 \cdot (160 - 30 - 83,8)^2$$

$I_{cu} = 3787 \cdot 10^4$ mm^4/m

The mean value of I_{cc} and I_{cu} is:

$$I_c = \frac{I_{cc} + I_{cu}}{2} = \frac{1921 \cdot 10^4 + 3787 \cdot 10^4}{2} = 2854 \cdot 10^4 \text{ mm}^4/\text{m}$$

Remark:

The values of the second moment of area are slightly higher taking into account the longitudinal reinforcement. The calculation of deflection is carried out with the mean value of the second moment of area taking into account the contribution of the longitudinal reinforcement.

Calculation of deflections

- Deflection due to permanent action – weight of floor finishes $g_{k,3}$ = 1,20 kN/m^2

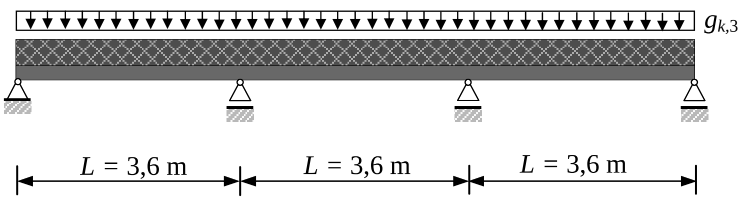


Figure D3.23 *Static system and load for calculation of deflection at the composite stage*

The deflection is:

$$\delta_1 = \frac{0{,}0068 \cdot g_{k,3} \cdot L^4}{E \cdot \mathrm{I}_c}$$

$$\delta_1 = \frac{0{,}0068 \cdot 1{,}2 \cdot 3600^4}{210000 \cdot 2854 \cdot 10^4} = 0{,}23 \text{ mm}$$

- Deflection due to the frequent value of variable action and the selected combination factor is $\psi_1 = 0{,}7$

The design load is calculated for the frequent combination:

$$q = \psi_1 \cdot q_{k,2}$$

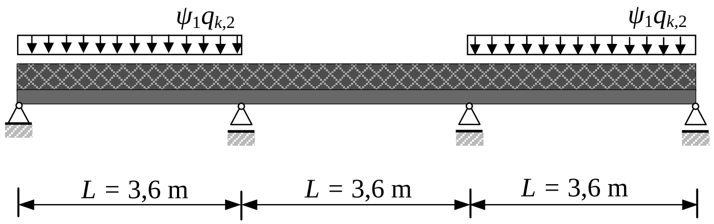


Figure D3.24 *Static system and load for calculation of deflection at the composite stage, for the worst load case*

The deflection is:

$$\delta_2 = \frac{0{,}0099 \cdot \psi_1 \cdot q_{k,2} \cdot L^4}{E \cdot I_c}$$

$$\delta_2 = \frac{0{,}0099 \cdot 0{,}7 \cdot 5{,}0 \cdot 3600^4}{210000 \cdot 2854 \cdot 10^4} = 0{,}97 \text{ mm}$$

- Self-weight of the slab g_k, removal of the props

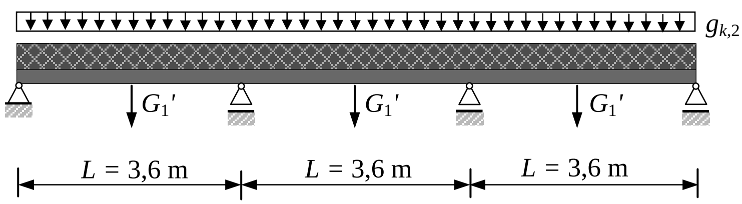


Figure D3.25 *Self-weight of slab, removal of the props*

The support reactions G_1' due to the self-weight of the slab are applied as point loads on the system:

$$G_1' = g_{k,2} \cdot \frac{L}{2} = 3,93 \cdot \frac{3,6}{2} = 7,07 \ \text{kN/m}$$

The deflection is:

$$\delta_3 = \frac{0,01146 \cdot G_1' \cdot L^3}{E \cdot I_c}$$

$$\delta_3 = \frac{0,01146 \cdot 7070 \cdot 3600^3}{210000 \cdot 2854 \cdot 10^4} = 0,63 \ \text{mm}$$

Remark:

The limit of the deflection is adopted according to clause 7.4.1(4), EN 1992-1-1. The recommended limitation is:

$$\delta_{total} \le \frac{L}{250}$$

The total deflection is:

$$\delta_{total} = \delta_1 + \delta_2 + \delta_3 = 0,23 + 0,97 + 0,63 = 1,83 \ \text{mm} \le \frac{L}{250} = \frac{3600}{250} = 14,4 \ \text{mm}$$

The total deflection meets the criterion *L*/250.

7. Commentary

Regarding the design of the continuous composite slab, the main disadvantage is the much more complex procedure compared with the design of the simply supported slabs. However, there are the corresponding advantages:

- The higher span/depth ratios can be applied for the given limit of deflection.
- It is possible to control the cracking of concrete at the internal support so that a brittle floor finish can be applied.
- Regarding the robustness of the structure, the continuity can have a positive effect.

According to clause 9.7.3(10), EN 1994-1-1, the design resistance against longitudinal shear can be carried out by the partial connection method taking into account the additional bottom reinforcement. But, it does not specify a method. Therefore, in this example, the following advantages of the partial connection method are illustrated:

- It may be applied to composite slabs with end anchorages.
- It may be modified to include additional reinforcing bars, which increase the resistance of the composite slabs.

Very often, the reinforcing bars are placed in the ribs of the slab due to improving its fire resistance. In the case of the calculation of the resistance of composite slab in fire conditions, the profiled steel sheeting offers little tensile resistance. Therefore, the reinforcement in the slab becomes the principal reinforcement. In accordance with EN 1994-1-2, the reinforcing bars are placed in the ribs of the slab. However, the corresponding tests have shown that adequate performance can also be achieved using mesh reinforcement without bottom bars, provided that the slab is continuous over at least one support.

D4 Two-span composite slab unpropped at the construction stage – commentaries on EN 1994-1-1

1. Purpose of example

The composite slab designed in this example is cast in situ on the profiled steel sheeting with a profile height of 75 mm and 0,9 mm thick, creating an overall slab thickness of 130 mm. Details of the geometry of this composite slab are shown in Figure D4.1. The spacing of the supporting composite beams is 2,75 m. The width of top flange of these beams is 190 mm.

The shear connection is provided by embossments in the profiled sheeting in accordance with clause 9.1.2.1(a), EN 1994-1-1. Some of the design data are taken from the manufacturer's publication.

Lightweight concrete is used in this composite slab. In the Eurocodes lightweight concrete is referred to as lightweight aggregate concrete. It is commonly used for the obvious advantage of 25% weight saving. Lightweight concrete also has better fire insulating qualities than normal weight concrete.

2. Static system, cross-section and actions

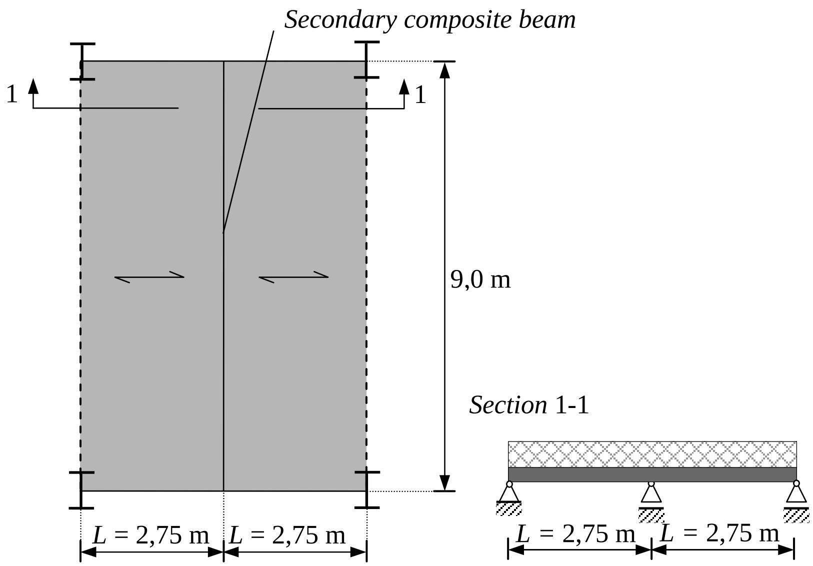


Figure D4.1 *Static system*

Composite Structures according to Eurocode 4. Worked Examples.
First Edition. Darko Dujmović, Boris Androić, Ivan Lukačević.

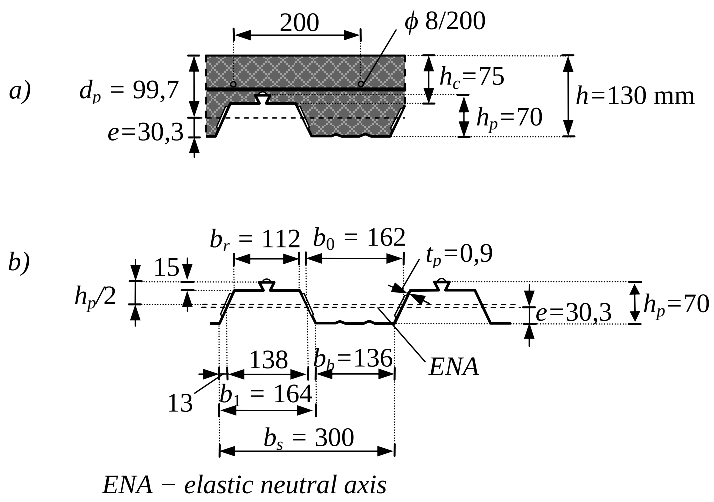


Figure D4.2 *Cross-sections: a) composite slab, b) profiled sheeting*

The shear connection is provided by embossments in the profiled sheeting in accordance with clause 9.1.2.1(a), EN 1994-1-1. According to clause 1.6, EN 1994-1-1, the overall depth of the profiled steel sheeting excluding embossments is denoted as h_p and the thickness of the concrete above the main flat surface of the top of the ribs of the sheeting is denoted as h_c. The profiled sheet and slab dimensions, in accordance with Figure 9.2, EN 1994-1-1, are shown in Figure D4.3.

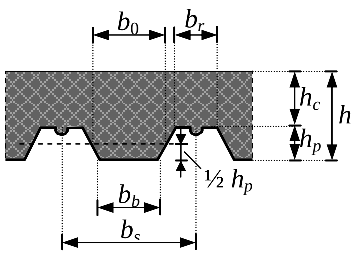


Figure D4.3 *Sheet and slab dimensions (open trough profile) shown in Figure 9.2, EN 1994-1-1*

In accordance with Figure D4.3 the dimensions of selected profiled steel sheeting have the following values:

h_c = 75 mm, concrete above “main flat surface” of the profiled sheeting,
h_p = 70 mm, “overall depth” of the profiled steel sheeting.

The thickness of the composite slab is 130 mm. This thickness is less than $h_c + h_p$. In such cases, the thickness of the slab, h_c = 75 mm, is appropriate for bending of the composite slab. For the calculation of the weight of the concrete slab, the depth of the profiled sheeting, h_p = 55 mm, is appropriate. However, for bending of the composite beam supporting the profiled sheeting and for in-plane shear in the slab, the relevant thickness is (h_c − 15) = (75 − 15) = 60 mm, see Figure D4.2.

Actions

a) Permanent action

Remark:

According to Table 11.1, clause 11.3, EN 1992-1-1, the density of the lightweight aggregate concrete of density class 1.8 and strength class LC25/28 is 18,5 kN/m^3. According to EN 1991-1-1 the density is increased by 1 kN/m^3 for normal percentage reinforcement, and increased for the wet concrete by another 1 kN/m^3.

Also, in Table 11.1, EN 1992-1-1, the oven-dry density is ρ = 1800 kg/m^3. This value is used for estimation of the mean value of the tensile strength, f_{lctm}, and of the secant modulus, E_{lcm}.

According to clause 11.3.1, EN 1992-1-1, the coefficient η_1 is:

$$\eta_1 = 0,4 + 0,6 \cdot \rho / 2200 = 0,4 + 0,6 \cdot 1800 / 2200 = 0,891$$

Thus, according to Table 11.3.1, EN 1992-1-1, the mean tensile strength is:

$$f_{lctm} = \eta_1 \cdot f_{ctm} = 0,891 \cdot 2,6 = 2,32 \text{ N/mm}^2$$

According to clause 11.3.2, EN 1992-1-1, the mean value of the secant modulus is:

$$E_{lcm} = E_{cm} \cdot \eta_E = E_{cm} \cdot (\frac{\rho}{2200})^2$$

$$E_{lcm} = 31000 \cdot (\frac{18}{22})^2 = 20752 \text{ N/mm}^2$$

Therefore, the modular ratio for short-term loading is:

$$n_0 = \frac{E_a}{E_{lcm}} = \frac{210}{20,7} = 10,1$$

Concrete slab area per m width, with the adopted depth of the profiled sheeting, h_p = 55 mm, is:

$$A_c = 1000 \cdot h - (\frac{1000}{b_s} \cdot \frac{b_1 + b_r}{2} \cdot h_p)$$

$$A_c = 1000 \cdot 130 - (\frac{1000}{300} \cdot \frac{164 + 112}{2} \cdot 55) = 104700 \text{ mm}^2 = 1050 \text{ cm}^2$$

- *concrete slab and reinforcement (wet concrete)*

$$A_c \cdot 20{,}5 = 0{,}105 \cdot 20{,}5 = 2{,}15 \text{ kN/m}^2$$

- *concrete slab and reinforcement (dry concrete)*

$$A_c \cdot 19{,}5 = 0{,}105 \cdot 19{,}5 = 2{,}05 \text{ kN/m}^2$$

Construction stage

- *concrete slab* $g_{c,1}$ = 2,15 kN/m^2

- *profiled steel sheeting* g_p = 0,10 kN/m^2

Total $g_{k,1} = g_{c,1} + g_p =$ 2,25 kN/m^2

Composite stage

- *concrete slab* $g_{c,2}$ = 2,05 kN/m^2

- *profiled steel sheeting* g_p = 0,10 kN/m^2

Total $g_{k,2} = g_{c,2} + g_p =$ 2,15 kN/m^2

Floor finishes $g_{k,3}$ = 1,00 kN/m^2

b) Variable action

Construction stage

- *construction loads* $q_{k,1}$ = 1,00 kN/m^2

Composite stage

- *imposed floor load and movable partitions* $q_{k,2} = 7{,}0$ kN/m^2

Remark:

Clause 9.3.2(1), EN 1994-1-1, refers to clause 4.11, EN 1991-1-6, for construction loads. According to clause 4.11.2(1), EN 1991-1-6, the actions from personnel and equipment, q_k, are given as "10% of the self-weight of the concrete, but not less than 0,75 and not more than 1,5 kN/m^2". Also, according to clause 4.11, EN 1991-1-6, the load of 1,50 kN/m^2 acts on the working area of 3,0 × 3,0 m and outside the working area the load is 0,75 kN/m^2.

In this example, the construction load is taken as 1,0 kN/m^2. This value allows for construction operatives, the heaping of concrete during placing, hand tools etc.

3. Properties of materials

Concrete strength class: LC 25/28 $f_{ck} = 25 \text{ N/mm}^2$

$$f_{cd} = \frac{f_{ck}}{\gamma_c} = \frac{25}{1{,}5} = 16{,}7 \text{ N/mm}^2$$

$$0{,}85 f_{cd} = 0{,}85 \cdot 16{,}7 = 14{,}2 \text{ N/mm}^2$$

$$E_{lcm} = 20752 \text{ N/mm}^2$$

$$f_{lcm} = 2{,}32 \text{ N/mm}^2$$

Reinforcement: $f_{sk} = 500 \text{ N/mm}^2$

$$f_{sd} = \frac{f_{sk}}{\gamma_s} = \frac{500}{1{,}15} = 435 \text{ N/mm}^2$$

Profiled steel sheeting: $t_p = 0{,}9 \text{ mm}$

$$h_p = 70 \text{ mm}$$

$$A_p = A_{pe} = 1178 \text{ mm}^2\text{/m}$$

$$I_p = 54{,}8 \cdot 10^4 \text{ mm}^4\text{/m}$$

$$E_p = E_a = 210\,000 \text{ N/mm}^2$$

$$f_{yp,k} = 350 \text{ N/mm}^2$$

$$f_{yp,d} = \frac{f_{yp,k}}{\gamma_M} = \frac{350}{1{,}0} = 350 \text{ N/mm}^2$$

Plastic resistance moment (provided by manufacturer): $M_{pa,Rk} = 6,18$ kNm/m

Resistance moment (provided by manufacturer): $M_{Rk} = 4,83$ kNm/m (sagging)

$M_{Rk} = 4,75$ kNm/m (hogging)

Empirical factors (provided by manufacturer): $m = 184$ N/mm^2

$k = 0,0530$ N/mm^2

Resistance to horizontal shear (provided by manufacturer): $\tau_{u,Rk} = 0,180$ N/mm^2

For the profiled steel sheeting, it is necessary to emphasize the following:

- The nominal thickness, including a zinc coating, is 0,9 mm and 0,86 without a zinc coating.
- The cross-sectional area of the profiled sheeting is A_p = 1178 mm^2/m.
- The self-weight of the profiled sheeting is 0,10 kN/m^2.
- The second moment of area is $I_{y,p}$ = 54,8 10^4 mm^4/m.
- The position of the plastic neutral axis above the bottom of the section is e_p = 33 mm, Figure 9.6, EN 1994-1-1.
- The position of the centroid above the bottom of the section is e = 30,3 mm, Figure 9.6, EN 1994-1-1.
- The modulus of elasticity E_a = 210 kN/mm^2, the yield strength $f_{yk,p}$ = 350 N/mm^2 and the partial factor $\gamma_{M,p}$ = 1,0.
- The characteristic value of the plastic resistance moment of the effective cross-section in hogging and sagging regions is M_{pa} = 6,18 kNm/m. It is assumed that in this value the effect of embossments is taken into account, clause 9.5(1), EN 1994-1-1.

4. Structural details of composite slab

4.1 Slab thickness and reinforcement

The composite slab should satisfy the conditions given in clause 9.2, EN 1994-1-1.

a) The slab acts compositely with a beam and the following conditions should be satisfied:

- the overall depth of slab $h \geq 90$ mm, $\rightarrow h = 130$ mm (satisfied),
- the thickness of concrete above the main flat surface of the top of the ribs of sheeting $h_c \geq 50$ mm, $\rightarrow h_c = 75$ mm (satisfied),
- the ratio of the width of the sheet rib to the rib spacing $\dfrac{b_r}{b_s} \leq 0,6$, $\rightarrow$

$$\frac{b_r}{b_s} = \frac{112}{300} = 0,37 \text{ (satisfied).}$$

b) The minimum amount of reinforcement in both directions should not be less than 80 mm^2/m. For unpropped construction, the area of reinforcement, according to clause 9.8.1(2), EN 1994-1-1, is:

$$A_{s,\min} = 0,002 \cdot 75 \cdot 1000 = 150 \ \text{mm}^2/\text{m} \ \rightarrow \ A_s = 80 \ \text{mm}^2/\text{m}$$

c) Spacing of reinforcement bars

$$e < 2 \cdot h = 2 \cdot 130 = 260 \ \text{or} < 350 \text{ mm}$$

4.2 Largest nominal aggregate size

$d_g \leq 0,4 \cdot h_c$	$0,4 \cdot 75 = 30,0$ mm
$d_g \leq b_0/3$	162/3=54,0 mm
$d_g \leq 31,5$ mm	= 31,5 mm

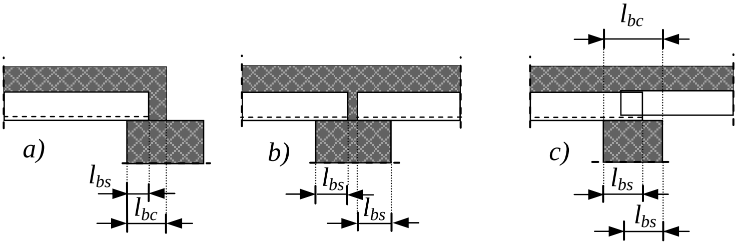
The minimum adopted value is $d_g = 30,0$ mm.

4.3 Minimum value for nominal thickness of steel sheet

In accordance with clause 3.5(2), EN 1994-1-1, the recommended value for the nominal thickness of steel sheet is 0,70 mm. The thickness of the selected profiled steel sheeting is 0,90 mm. The condition is satisfied.

4.4 Composite slab bearing requirements

According to clause 9.2.3(2), EN 1994-1-1, the recommended bearing lengths and support details differ depending upon the support material and they are different for internal supports and end supports, see Figure D4.4.



Bearing on	l_{bs} (mm)	l_{bc} (mm)
steel or concrete	50	75
other materials	70	100

Figure D4.4 *Minimum bearing lengths*

For composite slabs bearing on steel or concrete, the minimum bearing lengths are: l_{bc} = 75 mm and l_{bs} = 50 mm. The composite slab is supported by steel beams with the top flange width of 190 mm. Therefore, the condition for the bearing length is satisfied.

5. Ultimate limit state

5.1 Construction stage

At the construction stage, it is necessary to carry out verifications of profiled steel sheeting for the ultimate and serviceability limit states in accordance with EN 1993-1-3. To avoid brittle behaviour of the composite slabs, various mechanical means such as embossments or indentations are used, see clause 9.5(1), EN 1994-1-1. Clause 9.5(1), EN 1994-1-1, refers to the loss of effective cross-section caused by deep deformations of the sheeting. The loss of effective cross-section and the effects of local buckling are estimated by means of tests, and design recommendations are provided by manufacturers.

For simplicity, the profiled steel sheeting is considered as a simply supported span. The static system and the design load for the construction stage are shown in Figure D4.5.

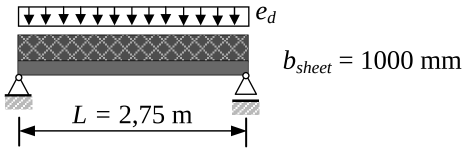


Figure D4.5 *Static system and design load for the construction stage*

The design load for ultimate limit state is:

$$e_d = \gamma_G \cdot g_{k,1} + \gamma_Q \cdot q_{k,1}$$

$$e_d = 1{,}35 \cdot 2{,}25 + 1{,}5 \cdot 1{,}0 = 4{,}54 \text{ kN/m}^2$$

Therefore, the design values of bending moment and shear force are:

$$M_{Ed} = \frac{e_d \cdot L^2}{8} = \frac{4{,}54 \cdot 2{,}75^2}{8} = 4{,}29 \text{ kNm/m}$$

$$V_{Ed} = \frac{e_d \cdot L}{2} = \frac{4{,}54 \cdot 2{,}75}{2} = 6{,}24 \text{ kN/m}$$

With the partial factor $\gamma_{M,0}$ = 1,0, the design resistance moment is:

$$M_{Rd} = \frac{M_{Rk}}{\gamma_{M,0}} = \frac{4,83}{1,0} \text{ kNm/m}$$

Check for bending:

$$\frac{M_{Ed}}{M_{Rd}} \leq 1,0$$

$$\frac{4,29}{4,83} = 0,89 < 1,0 \text{, the condition is satisfied}$$

The shear resistance of the profiled sheeting is calculated according to clause 6.1.5, EN 1993-1-3.

The design shear resistance of a single web, V_{Rd}, is determined as (6.8), EN 1993-1-3, with dimensions shown in Figure D4.6:

$$V_{Rd} = \frac{h_w}{\sin\theta} \cdot t \cdot \frac{1}{\sqrt{3}} \cdot \frac{f_{yk,p}}{\gamma_{M0}}$$

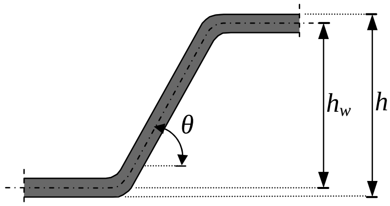


Figure D4.6 *Dimensions of steel sheeting profile*

In this case, there are 6,7 webs per metre with of sheeting. The depth of the profiled sheeting is 55 mm, Figure D4.7.

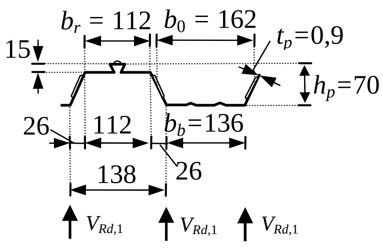


Figure D4.7 *Cross-section with more webs*

The design shear resistance of a single web is.

$$V_{Rd,1} = \frac{55}{\sin 64,7^{\mathrm{o}}} \cdot 0,9 \cdot \frac{1}{\sqrt{3}} \cdot \frac{0,350}{1,0} = 11,1 \text{ kN}$$

For 6,7 webs per metre width of sheeting, the design shear resistance is:

$$V_{Rd} = 6,7 \cdot V_{Rd,1} = 6,7 \cdot 11,1 = 74,4 \text{ kN/m}$$

Check:

$$\frac{V_{Ed}}{V_{Rd}} \leq 1,0$$

$$\frac{6,24}{74,4} = 0,08 < 1,0 \text{ (satisfied)}$$

Remark:

The slenderness of each web, λ_w, is:

$$\lambda_w = \frac{h_p / \sin\theta}{t} = \frac{55/\sin 64,7^{\mathrm{o}}}{0,9} = 68$$

This value is close to the limit value at which buckling must be taken into account. The design shear force, V_{Ed} = 6,24 kN/m, is far below the design shear resistance, V_{Rd} = 74,4 kN/m. This is usual for the construction stage and further calculation is not necessary.

In accordance with EN 1993-1-3, the following checks should be carried out:

- local resistance according to clause 6.1.7.3, EN 1993-1-3,
- combined bending and shear according to clause 6.1.10, EN 1993-1-3,
- combined web crushing and bending moment according to clause 6.1.11, EN 1993-1-3.

5.2 Composite stage

The continuous composite slab is designed as a series of simply supported spans, in accordance with clause 9.4.2(5), EN 1994-1-1, provided that the criterion for minimum reinforcement above internal supports of composite slab is satisfied, clause 9.8.1, EN 1994-1-1.

The static system and the design load for the composite stage are shown in Figure D4.8.

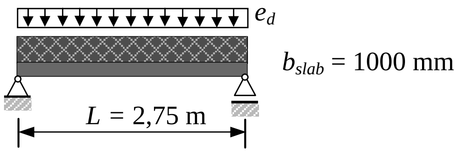


Figure D4.8 *Static system and design load for the composite stage*

Design load for ultimate limit state is:

$$e_d = \gamma_G \cdot (g_{k,2} + g_{k,3}) + \gamma_Q \cdot q_{k,1}$$

$$e_d = 1,35 \cdot (2,15 + 1,00) + 1,5 \cdot 7,0 = 14,8 \ \text{kN/m}^2$$

Therefore, the design values of bending moment and shear force are:

$$M_{Ed} = \frac{e_d \cdot L^2}{8} = \frac{14,8 \cdot 2,75^2}{8} = 14,0 \ \text{kNm}$$

$$V_{Ed} = \frac{e_d \cdot L}{2} = \frac{14,8 \cdot 2,75}{2} = 20,4 \ \text{kN/m}$$

5.2.1 Plastic resistance moment in sagging region

It is assumed that the neutral axis lies above the sheeting. The design compressive force in concrete, $N_{c,f}$, is:

$$N_{c,f} = 0,85 \cdot f_{cd} \cdot h_c \cdot b \qquad\qquad b = 1000 \ \text{mm}$$

$$N_{c,f} = 0,85 \cdot 16,7 \cdot 75 \cdot 1000 \cdot 10^{-3} = 1065 \ \text{kN/m}$$

The design tensile force in the steel sheeting per metre width of sheeting is:

$$N_p = f_{yp,d} \cdot A_{pe}$$

$$N_p = 350 \cdot 1178 \cdot 10^{-3} = 412 \ \text{kN/m}$$

Since N_p < $N_{c,f}$, the plastic neutral axis lies within the concrete. The design resistance moment in sagging region is calculated according to clause 9.7.2(5), EN 1994-1-1, see Figure D4.9.

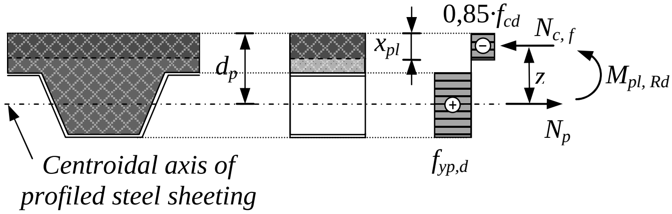


Figure D4.9 *Stress distribution for sagging bending when the neutral axis is above the steel sheeting*

The cross-section of the considered composite slab and the distribution of stresses are shown in Figure D4.10.

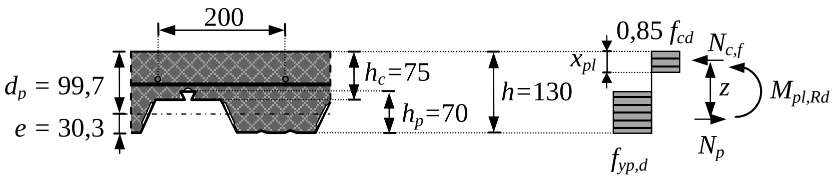


Figure D4.10 *Cross-section of the composite slab and the stress blocks for sagging bending*

The position of the plastic neutral axis of the composite section x_{pl} is:

$$x_{pl} = \frac{A_p \cdot f_{yp,d}}{b \cdot 0{,}85 \cdot f_{cd}} \qquad b = 1000 \text{ slab width}$$

$$x_{pl} = \frac{1178 \cdot 350}{0{,}85 \cdot 16{,}7 \cdot 1000} = 29 \text{ mm} < h_c = 75 \text{ mm}$$

For full shear connection, the design plastic resistance moment in sagging region $M_{pl,Rd}$ is:

$$M_{pl,Rd} = \min(N_{c,f}, N_p) \cdot z$$

$$M_{pl,Rd} = N_p \cdot (d_p - \frac{x_{pl}}{2})$$

$$M_{pl,Rd} = 412 \cdot (99{,}7 - \frac{29}{2}) \cdot 10^{-3} = 35{,}1 \text{ kNm/m}$$

Check:

$$\frac{M_{Ed}}{M_{pl,Rd}} \leq 1{,}0$$

$\dfrac{14{,}0}{35{,}1} = 0{,}40 < 1{,}0$, the condition is satisfied

The design plastic resistance moment in sagging region for full shear connection is adequate.

Remark:

The design bending moment, M_{Ed} = 14,0 kNm/m, is well below the design plastic resistance moment, $M_{pl,Rd}$ = 35,1 kNm/m. Accordingly, there is no need to consider the continuity of the composite slab or the resistance to hogging bending.

If it is necessary to consider the continuity of the composite slab, the provisions from clause 9.4.2, EN 1994-1-1, are applied for global analysis and from clause 9.7.2(4), EN 1994-1-1, for the resistance to hogging bending.

5.2.2 Longitudinal shear resistance

5.2.2.1 Longitudinal shear resistance – m-k method

It is assumed that there is no end anchorage. Therefore, the longitudinal shear resistance is calculated according to clause 9.7.3, EN 1994-1-1. The design resistance of the composite slab against longitudinal shear is carried out by the semi-empirical method called the *m*-*k* method. According to clause 9.7.3(4), EN 1994-1-1, the maximum design vertical shear V_{Ed} for a width of slab b is limited due to the design longitudinal shear resistance $V_{l,Rd}$ given as:

$$V_{l,Rd} = \frac{b \cdot d_p}{\gamma_{vs}} \cdot \left(\frac{m \cdot A_p}{b \cdot L_s} + k\right)$$

where:

b, d_p are in mm,
A_p is the nominal cross-section of the sheeting in mm^2,

m, k are design values for the empirical factors in N/mm² obtained from slab tests meeting the basic requirements of the *m-k* method,

L_s is the shear span in mm, defined in clause 9.7.3(5), EN 1994-1-1,

γ_{vs} is the partial factor for the ultimate limit state; the recommended value is 1,25.

If the *m-k* method is used it should be verified that the maximum design vertical shear V_{Ed} does not exceed the design shear resistance $V_{l,Rd}$:

$$\frac{V_{Ed}}{V_{l,Rd}} \le 1{,}0$$

It is assumed that tests have shown that the longitudinal shear behaviour may be considered as ductile in accordance with clause 9.7.3(3), EN 1994-1-1. Design values of empirical factors *m* and *k* are based on slab tests and are provided by the manufacturer of the sheeting:

$$m = 184 \ \text{N/mm}^2$$

$$k = 0{,}0530 \ \text{N/mm}^2$$

According to clause 9.7.3(5), EN 1994-1-1, the shear span L_s for the uniform load applied to the entire span length is:

$$L_s = \frac{L}{4} = \frac{2750}{4} = 688 \ \text{mm}$$

The design longitudinal shear resistance $V_{l,Rd}$ is:

$$V_{l,Rd} = \frac{b \cdot d_p}{\gamma_{vs}} \cdot \left(\frac{m \cdot A_p}{b \cdot L_s} + k\right)$$

$$V_{l,Rd} = \frac{1000 \cdot 99{,}7}{1{,}25} \cdot \left(\frac{184 \cdot 1178}{1000 \cdot 688} + 0{,}0530\right) \cdot 10^{-3} = 29{,}4 \ \text{kN/m}$$

This value, $V_{l,Rd}$ = 29,4 kN/m, must not be exceeded by the vertical shear in the slab.

Check:

$$\frac{V_{Ed}}{V_{l,Rd}} \leq 1{,}0$$

$$\frac{20{,}4}{29{,}4} = 0{,}69 < 1{,}0$$, the condition is satisfied

Remark:

For the two-span slab, the design shear force is a little higher at the internal support. However, it is evident that the design shear force does not exceed the design longitudinal shear resistance, $V_{l,Rd}$ = 29,4 kN/m.

5.2.2.2 Longitudinal shear resistance − partial connection method

For composite slabs with ductile behaviour, the partial connection method can be used for the verification of the resistance to longitudinal shear, clause 9.7.3(8), EN 1994-1-1.

The shear span required for full shear connection is determined as:

$$N_c = \tau_{u,Rd} \cdot b \cdot L_x \leq N_{c,f}$$

The distance to the nearest support, L_x, required for full shear connection may be determined as:

$$L_x = \frac{N_{c,f}}{b \cdot \tau_{u,Rd}} = \frac{A_p \cdot f_{yd}}{b \cdot \tau_{u,Rd}}$$

Based on tests of composite slabs with profiled sheeting of thickness t = 0,9 mm, the following design shear strength is obtained:

$$\tau_{u,Rd} = \frac{\tau_{u,Rk}}{\gamma_{vs}} = \frac{0{,}180}{1{,}25} = 0{,}144 \ \text{N/mm}^2$$

In clause 9.7.3(8), EN1994-1-1, the design compressive force in the slab is determined by:

$$N_c = \tau_{u,Rd} \cdot b \cdot L_x \leq N_{c,f}$$

Accordingly, the design compressive force in the slab at distance x(m) from an end support is:

$$N_c = \tau_{u,Rd} \cdot b \cdot x$$

$$N_c = 0{,}144 \cdot 1000 \cdot x = 144 \cdot x \text{ kN/m}$$

According to Figure D4.10, the design compressive force in the slab for full shear connection, $N_{c,f}$, is equal to the design tensile force in the steel sheeting. Therefore, the design compressive force in the slab for full shear connection is:

$N_{c,f} = A_p \cdot f_{yp,d} = 1178 \cdot 0{,}35 = 412$ kN/m

The distance to the nearest support, L_x, required for full shear, is:

$$L_x = \frac{N_{c,f}}{b \cdot \tau_{u,Rd}} = \frac{A_p \cdot f_{yd}}{b \cdot \tau_{u,Rd}}$$

$$L_x = \frac{1178 \cdot 350}{1000 \cdot 0{,}144} = 2863 \text{ mm} > L/2 = 2750/2 = 1375 \text{ mm}$$

The length of shear span needed for full interaction L_{sf} = 2863 mm exceeds $L/2$ = 1375 mm, so full interaction is not achieved in a span of this length.

According to clause 9.7.3(7), EN 1994-1-1, the verification is carried out using the simplified partial interaction diagram, and for any cross-section along the span it has to be shown that the corresponding design bending moment, M_{Ed}, does not exceed the design resistance moment M_{Rd}. This criterion can be written as:

$$\frac{M_{Ed}(x)}{M_{Rd}(x)} \leq 1{,}0$$

For full shear connection, the position of the plastic neutral axis of the composite section x_{pl} is 29,0 mm, see Section 5.2.1. This value of x_{pl} gives a slightly conservative result for the lever arm z. But, for simplicity, this value is used in this example.

The lever arm z is calculated as (9.9), EN 1994-1-1:

$$z = h - 0{,}5 \cdot x_{\mathrm{pl}} - e_{\mathrm{p}} + (e_{\mathrm{p}} - e) \cdot \frac{N_{\mathrm{c}}}{A_{\mathrm{pe}} f_{\mathrm{yp,d}}}$$

With h = 130 mm, e_p = 33 mm and e = 30,3 mm, the lever arm z is:

$$z = 130 - 0{,}5 \cdot 29 - 33 + (33 - 30{,}3) \cdot \frac{144 \cdot x}{412}$$

$$z = 82{,}5 + 0{,}94 \cdot x \text{ mm}$$

The reduced resistance moment of the composite slab is calculated as (9.6), EN 1994-1-1:

$$M_{pr} = 1{,}25 \cdot M_{pa} \cdot (1 - \frac{N_{c,f}}{A_{pe} \cdot f_{yp,d}}) \leq M_{pa}$$

In the above expression, $N_{c,f}$ is replaced by N_c, and M_{pa} is the design value of the plastic resistance moment of the effective cross-section in hogging and sagging regions is, $M_{pa,Rd}$ = 6,18 kNm/m.

The reduced resistance moment of the composite slab is:

$$M_{pr} = 1{,}25 \cdot 6{,}18 \cdot (1 - \frac{144 \cdot x}{412}) \leq 6{,}18$$

$$M_{pr} = 7{,}725 - 2{,}7 \cdot x \leq 6{,}18$$

so

$$x \geq \frac{7{,}725 - 6{,}18}{2{,}7} = 0{,}572 \text{ m}$$

According to Figure 9.6, EN 1994-1-1, the plastic resistance moment is given by:

$$M_{Rd} = N_{c,f} \cdot z + M_{pr}$$

$$M_{Rd} = 0{,}144 \cdot x \cdot (82{,}5 + 0{,}94 \cdot x) + (7{,}725 - 2{,}7 \cdot x)$$

$$M_{Rd} = 7{,}725 + 9{,}18 \cdot x + 0{,}135 \cdot x^2 \quad \text{for region } 0{,}572 \text{ m} \leq x \leq 1{,}375 \text{ m}$$

For x < 0,572 m, the plastic resistance moment is:

$$M_{Rd} = N_{c,f} \cdot z + M_{pr}$$

$$M_{Rd} = 0{,}144 \cdot x \cdot (82{,}5 + 0{,}94 \cdot x) + 6{,}18$$

$$M_{Rd} = 6{,}18 + 11{,}88 \cdot x + 0{,}135 \cdot x^2$$

The longitudinal shear resistance of the slab, expressed by its design resistance moment, $M_{Rd}(x)$, found from the design shear strength, $\tau_{u,Rd}$, and the design bending moment from applied loads, $M_{Ed}(x)$, are plotted in Figure D4.11. The curve $M_{Rd}(x)$, denoted as AB, lies above the curve $M_{Ed}(x)$, denoted as $0C$, for each cross-section along the span. Thus, there is sufficient resistance to longitudinal shear.

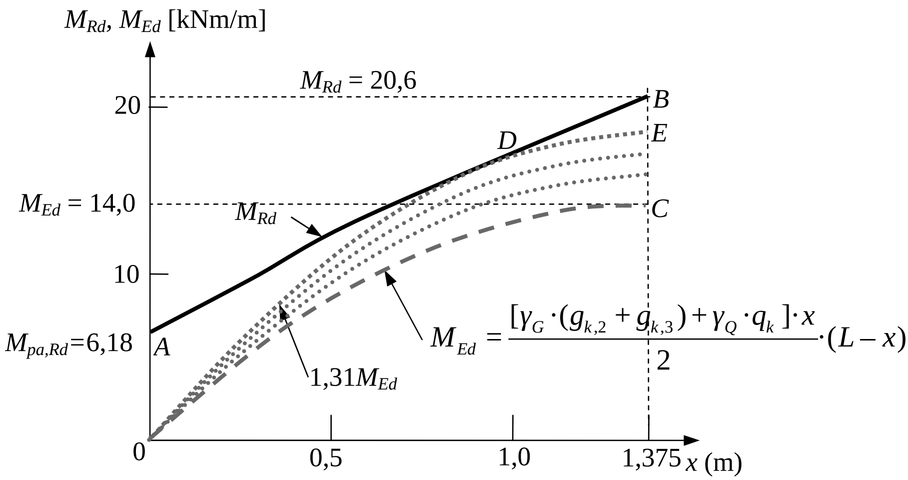


Figure D4.11 *Design partial-interaction diagram*

When the *m-k* method is used it should be verified that the maximum design vertical shear V_{Ed} does not exceed the design shear resistance $V_{l,Rd}$. Based on this criterion it is possible to compare the partial connection method and the *m-k* method. It is necessary to calculate V_{Ed} from the design partial interaction diagram shown in Figure D4.11. When the curve $M_{Ed}(x)$ touches the curve $M_{Rd}(x)$ at point x, the shear failure occurs along length x (m) at the end support. The reaction at this support can be compared with the design shear resistance $V_{l,Rd}$. After the calculation, it is found that the scale factor $\kappa = 1{,}31$, see the curve $0DE$ in Figure D4.11. The point of contact of these curves is 0,89 m away from the end support. The design reaction at the end support is:

$$V_{Ed} = \kappa \cdot \frac{e_d \cdot L}{2} = 1{,}31 \cdot \frac{14{,}8 \cdot 2{,}75}{2} = 26{,}7 \text{ kN/m}$$

The obtained value is 10% less than the design shear resistance, $V_{l,Rd}$ = 29,4 kN/m, obtained by the *m-k* method. In this example, these methods give similar results.

5.2.3 Check for vertical shear resistance

According to 9.7.5, EN 1994-1-1, the vertical shear resistance, $V_{v,Rd}$, should be determined according to the method given in EN 1992-1-1. According to clause 11.6.1, EN 1992-1-1, the design shear resistance $V_{lRd,c}$ is calculated as:

$$V_{lRd,c} = [C_{lRd,c} \cdot \eta_1 \cdot k \cdot (100 \cdot \rho_l \cdot f_{lck})^{1/3} + k_1 \cdot \sigma_{cp}] \cdot b_w \cdot d_p \geq V_{lv,Rd,\min}$$

The minimum value of $V_{lv,Rd,\min}$ is:

$$V_{lv,Rd,\min} = (v_{l,\min} + k_1 \cdot \sigma_{cp}) \cdot b_w \cdot d_p$$

The minimum value of $V_{lv,Rd}$ is related to the fact that the member without reinforcement still has some shear resistance.

Generally, the check is carried out as:

$$\frac{V_{Ed}}{V_{lRd,c}} \leq 1{,}0$$

According to clause 11.6.1(1), EN 1992-1-1, the values needed for calculation $V_{lRd,c}$ are:

$$C_{lRd,c} = \frac{0{,}15}{\gamma_c} = \frac{0{,}15}{1{,}5} = 0{,}10$$

According to clause 11.3.1, EN 1992-1-1:

$$\rho = 1800 \text{ kg/m}^3$$

$$\eta_1 = 0{,}4 + 0{,}6 \cdot \rho / 2200 = 0{,}4 + 0{,}6 \cdot 1800 / 2200 = 0{,}891$$

$$k = 1 + \sqrt{\frac{200}{d_p}} \leq 2{,}0$$

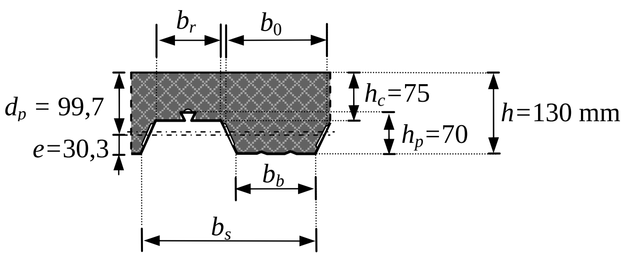
$$k = 1 + \sqrt{\frac{200}{99{,}7}} = 2{,}42 \rightarrow \text{adopted } k = 2{,}0$$

$$\rho_l = \frac{A_{sl}}{b_w \cdot d_p} \le 0{,}02$$

The resistance of the cross-section is dependent on the area of the tensile reinforcement, whose section has to be extended by an appropriate anchorage length, (l_{bd} + d) see – Figure 6.3, EN 1992-1-1 – where l_{bd} is the design anchorage length and d is the effective depth of the section, taken as the depth from the top surface to the centroid of the profile for a composite slab. The anchorage of the profiled sheeting was confirmed by the check on longitudinal shear, and the sheeting can be treated as reinforcement, i.e. $A_{sl} = A_{pe} = 1178\ \text{mm}^2$.

In accordance with Figure D4.12, the smallest width of the cross-section in the tensile area b_w is calculated per metre width as follows:

$$b_w = \frac{b}{b_s} \cdot b_0 = \frac{1000}{300} \cdot 162 = 540 \ \ \text{mm/m}$$



b_s = 300 mm
b_r = 112 mm
b_0 = 162 mm

Figure D4.12 *Determination of value* b_w

The percentage of longitudinal reinforcement is:

$$\rho_l = \frac{A_{sl}}{b_w \cdot d_p} \le 0{,}02$$

$$\rho_l = \frac{1178}{540 \cdot 99{,}7} = 0{,}022 > 0{,}020$$

The value $\rho_l = 0{,}02$ is adopted.

The design axial force is $N_{Ed} = 0$ and therefore $\sigma_{cp} = \dfrac{N_{Ed}}{A_c} = 0$.

$k_1 = 0{,}15$, according to clause 6.2.2(1), EN 1992-1-1.

The design shear resistance $V_{lRd,c}$ is:

$$V_{lRd,c} = [C_{lRd,c} \cdot \eta_1 \cdot k \cdot (100 \cdot \rho_l \cdot f_{lck})^{1/3} + k_1 \cdot \sigma_{cp}] \cdot b_w \cdot d_p$$

$$V_{lRd,c} = [0{,}10 \cdot 0{,}891 \cdot 2{,}0 \cdot (100 \cdot 0{,}02 \cdot 25)^{1/3} + 0{,}15 \cdot 0]\ 540 \cdot 99{,}7 \cdot 10^{-3}$$

$$V_{lRd,c} = 35{,}3\ \text{kN/m}$$

The minimum value is:

$$V_{lv,Rd,\min} = (v_{l\min} + k_1 \cdot \sigma_{cp}) \cdot b_w \cdot d_p$$

$$v_{l\min} = 0{,}03 \cdot k^{3/2} \cdot f_{lck}^{\ 1/2} = 0{,}03 \cdot 2{,}0^{3/2} \cdot 25^{1/2} = 0{,}42\ \text{N/mm}^2$$

$$V_{lv,Rd,\min} = (0{,}42 + 0{,}15 \cdot 0) \cdot 540 \cdot 99{,}7 \cdot 10^{-3} = 22{,}6\ \text{kN/m} < V_{lRd,c} = 35{,}3\ \text{kN/m}$$

Check:

$$\frac{V_{Ed}}{V_{v,Rd}} \leq 1{,}0$$

$\dfrac{20{,}4}{35{,}3} = 0{,}57 < 1{,}0$, the condition is satisfied

Remark:

Since it is unlikely that the profiled steel sheet can satisfy the requirement "full anchorage", the design shear resistance is equal to the minimum value:

$$V_{lRd,c} = V_{lv,Rd,\min} = 22{,}6\ \text{kN/m}$$

Also, the required condition is satisfied since that is $V_{lRd,c} = V_{lv,Rd,\min} = 22{,}6$ kN/m > $V_{Ed} = 20{,}4$ kN/m.

6. Serviceability limit state

6.1 Control of cracking of concrete

Clause 9.8.1(1), EN 1994-1-1, refers to continuous slab. For the considered slab, it can be assumed that the slab satisfies conditions according to clause 9.8.1(2), EN 1994-1-1, and that the slab is designed as simply supported in accordance with clause 9.4.2(5), EN 1994-1-1. Furthermore, clause 9.4.2(5), EN 1994-1-1, requires that the reinforcement in accordance with clause 9.8.1, EN 1994-1-1, should be provided over internal supports to control crack widths.

According to clause 9.8.1(2), EN 1994-1-1, for unpropped construction the required cross-sectional area of reinforcement A_s is 0,2% of the area of concrete above the ribs. In this case, the depth of concrete above the main flat surface of the profiled sheeting is 75 mm. Therefore, the required cross-sectional area of reinforcement is:

$$\min A_s = 0{,}002 \cdot b \cdot h_c = 0{,}002 \cdot 1000 \cdot 75 = 150 \ \text{mm}^2/\text{m}$$

Strictly, this reinforcement is not required at mid-span. It is recommended that reinforcement for crack control is provided as mesh over the full area of the slab.

The selected minimum amount of reinforcement could be insufficient to control cracking at the supports of continuous slabs for certain exposure classes. In such cases, the slab should be designed as continuous, and in hogging regions the crack widths should be estimated according to EN 1992-1-1.

6.2 Limit of span/depth ratio of slab

According to clause 9.8.2(4), EN 1994-1-1, calculation of the deflection of the composite slab can be omitted if the two conditions are satisfied. According to the first condition, the span/depth ratio of the slab should not exceed the limits given in EN 1992-1-1. These are:

- $\frac{L}{d} < 20$ for a simply supported span
- $\frac{L}{d} < 26$ for an external span of continuous slab
- $\frac{L}{d} < 30$ for an internal span of continuous slab

According to clause 9.8.2(6), EN 1994-1-1, the second condition is given as follows:

- the load causing an end slip of 0,5 mm in the tests on composite slab exceeds 1,2 times the design service load.

If the second condition is not satisfied, i.e. the end slip exceeds 0,5 mm at a load of 1,2 times the design service load, two options exist:

- end anchors should be provided, or
- deflections should be calculated including the effect of end slip.

According to clause 9.8.2(8), EN 1994-1-1, in cases when the behaviour of the shear connection between the profiled sheeting and the concrete is not known from tests, the tied-arch model may be used, see [34].

For the considered slab with L = 2750 m and d_p = 99,7 mm, the following span/depth ratio is obtained:

- $\frac{L}{d} = \frac{2750}{99{,}7} = 27{,}6 > 26$ for the external span of continuous slab

The span/depth ratio exceeds the limit, and the calculation of the deflection is necessary.

6.3 Calculation of deflections

6.3.1 Construction stage deflection

According to clause 9.6(2), EN 1994-1-1, the deflection, δ_s, of the profiled sheeting due to its own weight and the weight of the wet concrete should not exceed the following limit:

$$\delta_{s,\max} = \frac{L}{180} = \frac{2750}{180} = 15{,}3 \text{ mm}$$

For simplicity, the profiled steel sheeting is considered as simply supported span. The static system and the design load for the construction stage are shown in Figure D4.13.

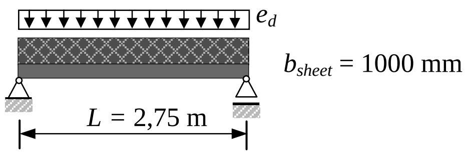


Figure D4.13 *Static system and design load for the construction stage*

The premature local buckling of the profiled sheeting under the weight of the wet concrete and the construction loading is checked to prevent irreversible deformation. This verification is important in the region of the internal support.

The design load for serviceability limit state is:

$$e_d = b \cdot g_{k.1}$$

$$e_d = 1{,}0 \cdot 2{,}25 = 2{,}25 \text{ kN/m}$$

The maximum sagging bending moment in serviceability limit state is:

$$M_{Ed} = \frac{e_d \cdot L^2}{8} = \frac{2{,}25 \cdot 2{,}75^2}{8} = 2{,}13 \text{ kNm/m}$$

The maximum compressive stress in top flange of profiled sheeting is:

$$\sigma_{com} = \frac{M_{Ed}}{\mathrm{I}_p} \cdot z = \frac{2{,}13 \cdot 10^6}{54{,}8 \cdot 10^4} \cdot (55 - 30{,}3) = 96 \text{ N/mm}^2$$

In accordance with clause 4.4, EN 1993-1-5, the plate slenderness, $\bar{\lambda}_p$, is calculated as:

$$\bar{\lambda}_p = \sqrt{\frac{f_y}{\sigma_{cr}}} = \frac{\frac{b}{t}}{28{,}4 \cdot \varepsilon \cdot \sqrt{k_\sigma}}$$

$$\varepsilon = \sqrt{\frac{235}{\sigma_{com}}} = \sqrt{\frac{235}{96}} = 1{,}56$$

According to Table 4.1, EN 1994-1-1, for the stress ratio $\psi = 1$, the buckling factor is $k_\sigma = 4$.

The width of the flange of profiled sheeting in compression, b, is determined by the intermediate flange stiffener of width 18 mm, see Figure D4.2b. Therefore, the width of the flange of the profiled sheeting in compression, the flat plate width between the stiffener and the web, is:

$$b = \frac{b_r - 18}{2} = \frac{112 - 18}{2} = 47 \text{ mm}$$

Therefore, the plate slenderness, $\bar{\lambda}_p$, with the design thickness of the sheet t = 0,86 mm (not including coatings) and $b = b_r$ = 47 mm, is:

$$\bar{\lambda}_p = \frac{\frac{47}{0,86}}{28,4 \cdot 1,56 \cdot \sqrt{4}} = 0,617$$

Since $\bar{\lambda}_p = 0,617 < 0,673$, the reduction factor is ρ = 1,0 and the cross-section is fully effective.

The deflection of the profiled steel sheeting for simply-supported span, Figure D4.13, is:

$$\delta_1 = \frac{5}{384} \cdot \frac{e_d \cdot L^4}{E_a \cdot I_p}$$

$$\delta_1 = \frac{5}{384} \cdot \frac{2,25 \cdot 2750^4}{210000 \cdot 54,8 \cdot 10^4} = 14,6 \text{ mm}$$

Since the deflection δ_1 is higher than 10% of the slab depth, δ_1 = 14,6 mm > 0,10·h = 0,1·130 = 13 mm, according to clause 9.3.2(2), EN 1994-1-1, the ponding effects should be taken into account at the construction stage.

In accordance with clause 9.3.2(2), EN 1994-1-1, the specified thickness of the additional concrete is 0,7·δ. The weight of the wet concrete is:

$$0,7 \cdot 0,0146 \cdot 20,5 = 0,21 \text{ kN/m}^2$$

This increases the deflection to:

$14,6 \cdot (2,25 + 0,21) / 2,25 = 16,0$ mm > $\delta_{s,\max} = 15,3$ mm, the condition is not satisfied.

Thus, the obtained value is L/172, which exceeds the limit value L/180. If the profiled sheeting is not continuous at either end of a 2,75 m span, propping must be used during construction.

To reduce the excessive deflection, the effects of continuity at one end span are taken into account. The worst load case is when the concrete in one span hardens, and there are no construction loads, before the other span is cast. Thus, on one span the load is 2,25 kN/m and on the adjacent span the load is 0,10 kN/m.

For a continuous beam of uniform section under uniformly distributed loading, the deflection δ_1 at the centre of a span can be calculated as:

$$\delta_1 = \delta_0[1 - 0{,}6(M_1 + M_2) / M_0]$$

where:

M_1 and M_2 are hogging end moments,
δ_0 is the deflection of the span when the end moments are zero,
M_0 is the mid-span moment of the span when the end moments are zero.

From elastic analysis, the following values are obtained:

$$M_0 = \frac{2{,}25 \cdot 2{,}75^2}{8} = 2{,}13 \ \text{kNm}$$

$$M_1 = \frac{1}{2} \cdot \frac{2{,}25 \cdot 2{,}75^2}{8} = 1{,}07 \ \text{kNm}$$

$$M_2 = 0$$

$$\delta_0 = \frac{5}{384} \cdot \frac{2{,}25 \cdot 2750^4}{210000 \cdot 54{,}8 \cdot 10^4} = 14{,}6 \ \text{mm}$$

Therefore, the deflection δ_1 is:

$\delta_1 = 14{,}6 \cdot [1 - 0{,}6 \cdot (1{,}07 + 0) / 2{,}13] = 10{,}2 \ \text{mm} < \delta_{s,\max} = 15{,}3$ mm, so the condition is satisfied.

6.3.2 Composite stage deflection

For the calculation of the deflection at the composite stage, the slab is considered as continuous over two spans. According to clause 9.8.2(5), EN 1994-1-1, the following approximations can be applied:

- The second moment of area can be taken as the average of the values for the cracked and uncracked sections.
- According to clause 11.3.2, EN 1992-1-1, the secant modulus is:

$$E_{lcm} = E_{cm} \cdot \eta_E = E_{cm} \cdot (\frac{\rho}{2200})^2$$

$$E_{lcm} = 31000 \cdot (\frac{18}{22})^2 = 20752 \ \text{N/mm}^2$$

- An average value of the modular ratio, n, for both short-term and long-term effects can be used:

$$n = \frac{E_a}{E'_{cm}} = \frac{E_a}{\frac{1}{2}\cdot(E_{lcm} + \frac{E_{lcm}}{3})} = \frac{210000}{\frac{2}{3}\cdot 20752} = 15,2$$

- Elastic analysis is used to calculate the deflection of the slab.

a) The second moment of area for the cracked section, I_{cc}, for the slab width b is calculated in accordance with Figure D4.14.

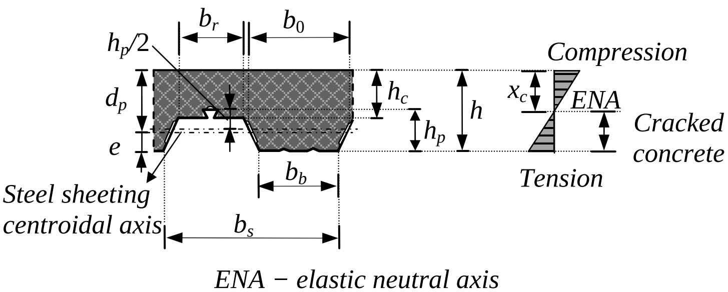


Figure D4.14 *Second moment of area calculation for the cracked cross-section, I_{cc}*

The second moment of area for the cracked section and the slab width b is calculated as:

$$I_{cc} = \frac{b\cdot x_c^{\,3}}{3\cdot n} + A_p\cdot(d_p - x_c)^2 + I_p$$

The position of the elastic neutral axis relative to the upper side of the slab is obtained from the expression:

$$x_c = \frac{\Sigma A_i\cdot z_i}{\Sigma A_i} = \frac{n\cdot A_p}{b}\left(\sqrt{1+\frac{2\cdot b\cdot d_p}{n\cdot A_p}} - 1\right)$$

$$x_c = \frac{15,2\cdot 1178}{1000}\left(\sqrt{1+\frac{2\cdot 1000\cdot 99,7}{15,2\cdot 1178}} - 1\right) = 44,5\ \text{mm}$$

The second moment of area for the cracked section is:

$$I_{cc} = \frac{1000 \cdot 44{,}5^3}{3 \cdot 15{,}2} + 1178 \cdot (99{,}7 - 44{,}5)^2 + 54{,}8 \cdot 10^4 = 6{,}07 \cdot 10^6 \text{ mm}^4\text{/m}$$

b) The second moment of area for the uncracked section, I_{cu}, for the slab width b is calculated in accordance with Figure D4.15.

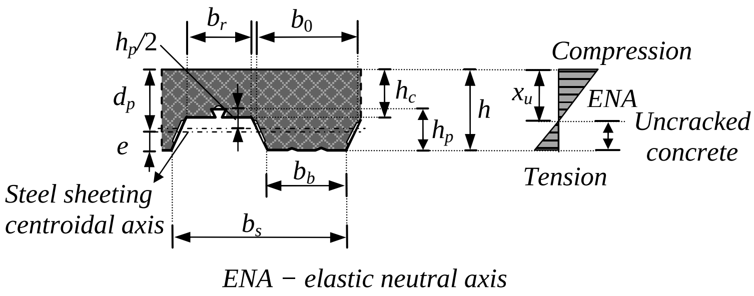


Figure D4.15 *Second moment of area calculation for the uncracked cross-section, I_{cu}*

The second moment of area for the uncracked section and the slab width b is calculated as:

$$I_{cu} = \frac{b \cdot h_c^3}{12 \cdot n} + \frac{b \cdot h_c}{n} \cdot (x_u - \frac{h_c}{2})^2 + \frac{b_m \cdot h_p^3}{12 \cdot n} + \frac{b_m \cdot h_p}{n} \cdot (h - x_u - \frac{h_p}{2})^2 +$$
$$+ A_p \cdot (d_p - x_u)^2 + I_p$$

where:

$$x_u = \frac{b \cdot \frac{h_c^2}{2} + b_m \cdot h_p \cdot (h - \frac{h_p}{2}) + n \cdot A_p \cdot d_p}{b \cdot h_c + b_m \cdot h_p + n \cdot A_p}$$

In accordance with Figure D4.15, the value of b_m is:

$$b_0 = 162 \text{ mm}$$

$$b_m = \frac{b}{b_s} \cdot b_0 = \frac{1000}{300} \cdot 162 = 540 \text{ mm/m}$$

For the depth of the profiled sheeting h_p = 55 mm, the position of the elastic neutral axis relative to the upper side of the slab is:

$$x_u = \frac{1000\cdot\frac{75^2}{2}+540\cdot55\cdot(130-\frac{55}{2})+15,2\cdot1178\cdot99,7}{1000\cdot75+540\cdot55+15,2\cdot1178} = 62 \text{ mm}$$

For the depth of the profiled sheeting h_p = 55 mm, the second moment of area for the uncracked section is:

$$I_{cu} = \frac{1000\cdot75,0^3}{12\cdot15,2}+\frac{1000\cdot75,0}{15,2}\cdot(62,0-\frac{75,0}{2})^2+\frac{540\cdot55,0^3}{12\cdot15,2}+$$
$$\frac{540\cdot55,0}{15,2}\cdot(130-62,0-\frac{55,0}{2})^2+1178\cdot(99,7-62,0)^2+54,8\cdot10^4$$

$$I_{cu} = 11,2\cdot10^6 \text{ mm}^4\text{/m}$$

The mean value of I_{cc} and I_{cu} is:

$$I_c = \frac{I_{cc}+I_{cu}}{2} = \frac{6,07\cdot10^6+11,2\cdot10^6}{2} = 8,64\cdot10^6 \text{ mm}^4\text{/m}$$

Calculation of deflections

Normally, deflection is a reversible limit state. According to clause 6.5.3(2), EN 1990, the frequent combination is recommended for the reversible limit state. The combination factor ψ_1 for this combination is dependent on the floor loading category. According to Table A1.1, EN 1990 the values of combination factor are in the range 0,5–0,9. In this example, the value of 0,7 is taken, which is recommended for congregation areas or shopping areas.

- Deflection due to permanent action

The design load of the weight of dry concrete, the weight of the profiled sheeting and the floor finishes is:

$$e_d = b\cdot(g_{k,2}+g_{k,3}) = 1,0\cdot(2,15+1,00) = 3,15 \text{ kN/m}$$

The deflection is:

$$\delta_1 = 0,0054\cdot\frac{e_d\cdot L^4}{E\cdot \mathrm{I}_c}$$

$$\delta_1 = 0{,}0054 \cdot \frac{3{,}15 \cdot 2750^4}{210000 \cdot 8{,}64 \cdot 10^6} = 0{,}54\,\text{mm} = L/5093$$

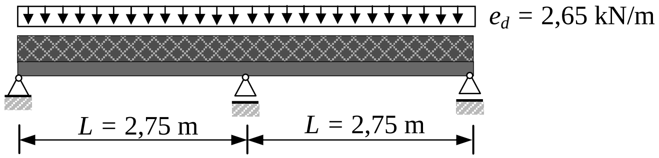


Figure D4.16 *Static system and load for the calculation of the deflection at the composite stage*

- The deflection due to the frequent value of variable action and the selected combination factor is $\psi_1 = 0{,}7$

The design load is calculated for the frequent combination:

$$e_d = b \cdot \psi_1 \cdot q_{k,2} = 1{,}0 \cdot 0{,}7 \cdot 7{,}0 = 4{,}9\ \text{kN/m}$$

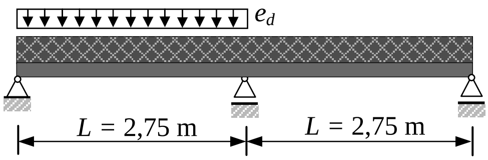


Figure D4.17 *Static system and load for calculation of the deflection at the composite stage*

The deflection is:

$$\delta_2 = 0{,}0099 \cdot \frac{e_d \cdot L^4}{E_a \cdot I_c}$$

$$\delta_2 = 0{,}0099 \cdot \frac{4{,}9 \cdot 2750^4}{210000 \cdot 8{,}64 \cdot 10^6} = 1{,}53\,\text{mm} = L/1797$$

Remark:

The limit of the deflection is adopted according to clause 7.4.1(4), EN 1992-1-1. The recommended limitation is:

$$\delta_{total} \le \frac{L}{250}$$

The total deflection is:

$$\delta_{total} = \delta_1 + \delta_2 = 0,54 + 1,53 = 2,07 \text{ mm} \leq \frac{L}{250} = \frac{2750}{250} = 11,0 \text{ mm}$$

The total deflection meets the criterion $L/250$.

7. Commentary

This example illustrates the use of lightweight aggregate concrete in a composite slab. The obvious advantage of lightweight aggregate concrete is weight saving. However, this advantage must be balanced against the effect of the reduced modulus of elasticity, which results in larger deflections of the slab.

The design resistance against longitudinal shear was carried out by means of both partial connection and the *m-k* method. Both methods are based on tests on composite slabs.

The partial connection method verifies the resistance moment of ductile slabs with ductile connections. Longitudinal shear resistance is related to the ultimate shear stress determined from full-scale tests.

The *m-k* method consists of determining two coefficients, *m* and *k*, per type of profiled steel sheeting by means of tests of composite slab specimens.

Both of these methods can be applied in cases where the longitudinal shear behaviour is ductile. However, if the longitudinal behaviour is non-ductile, only the *m-k* method is permitted.

D5 Hoesch Additive Floor

1. Purpose of example

The calculation and structural detailing of a Hoesch Additive Floor (HAF) in a car-park building is considered. The floor resistance is obtained by summing resistances of profiled steel sheet and ribbed reinforced concrete slab without composite action. Referring to EN 1994, the system is a combination of two materials, steel and concrete, with the degree of shear connection η = 0. The calculation is conducted using EN 1994-1-1 and Technical Approval. The bending moment is carried by the profiled sheet and the reinforced concrete ribbed slab. Their load-bearing capacities can be added. The bearing of shear force is provided at the supports via the profiled sheet and the patented cleat support. In this example a fire resistance is not considered.

2. Generally about the Hoesch Additive Floor system

The basic HAF system with its components is shown in Figure D5.1.

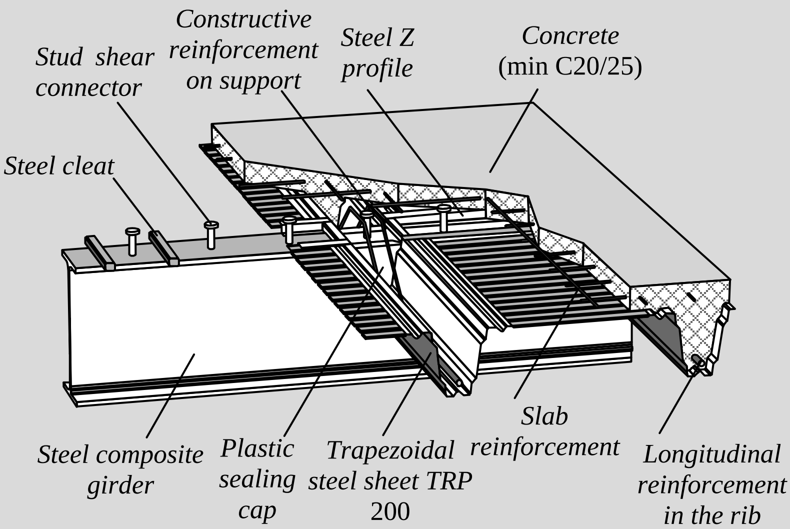


Figure D5.1 *The Hoesch Additive Floor*

The bending moment in the span is carried by a trapezoidal steel sheet and a reinforced concrete ribbed slab. The shear force at the support is transferred through trapezoidal steel sheet to the steel girder by means of patented cleats.

a) Verification of the ultimate limit states

Composite Structures according to Eurocode 4. Worked Examples.
First Edition. Darko Dujmović, Boris Androić, Ivan Lukačević.

The design value of resistance moment is the sum of resistances of the profiled steel sheet and the reinforced concrete ribbed slab given by the expression:

$$M_{Rd} = M_{PT,Rd} + M_{c,Rd}$$

M_{Rd}	design value of the resistance moment of HAF in the span of trapezoidal steel sheet,
$M_{PT,Rd} = \dfrac{M_{PT,Rk}}{\gamma_M}$	design value of the resistance moment of trapezoidal steel sheet in the span,
$M_{PT,Rk}$	characteristic value of the resistance moment of trapezoidal steel sheet according to table D5.1,
$\gamma_M = 1{,}1$	partial factor,
$M_{c,Rd} = \dfrac{M_{c,Rk}}{\gamma_C}$	design value of the resistance moment of ribbed reinforced concrete slab in accordance with EN 1992-1-1,
$M_{c,Rk}$	characteristic value of the resistance moment of ribbed reinforced concrete slab,
$\gamma_C = 1{,}5$	partial factor.

According to the design concept of load-bearing capacity, $M_{PT,Rd}$ is fully utilized, and the remaining bending moment is resisted by the concrete slab. Thus, the portion of resistance moment in the slab is given by expression:

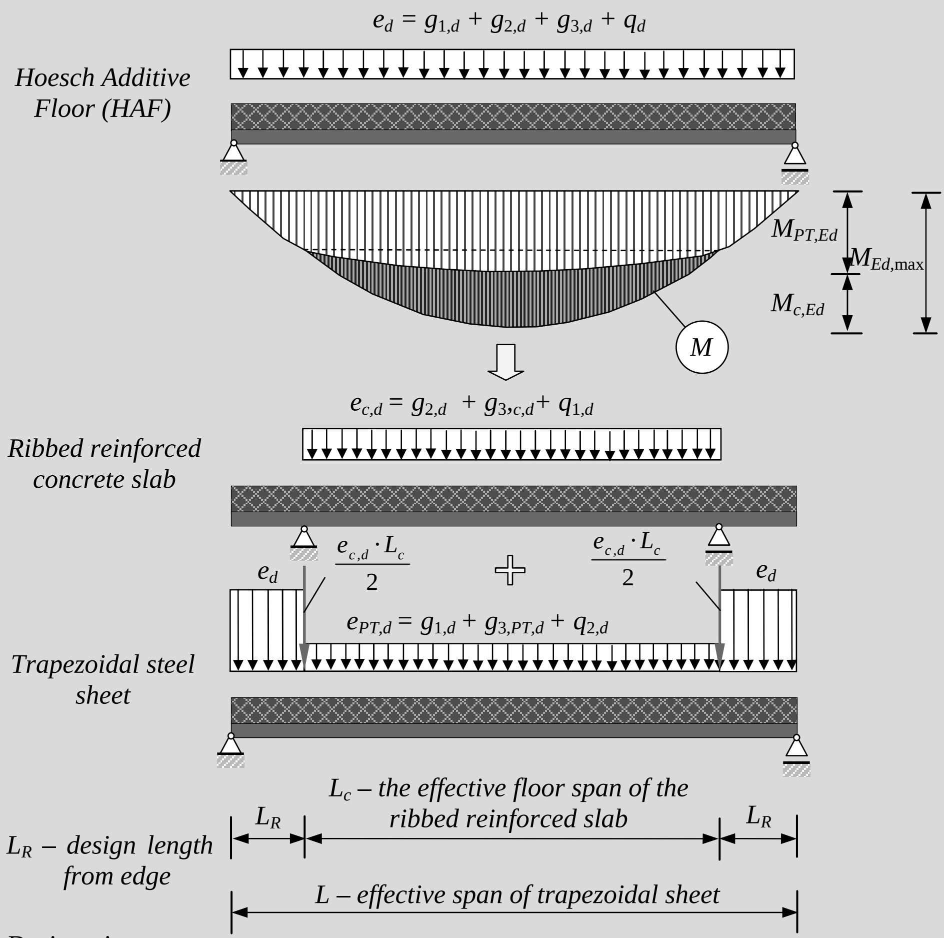
$$M_{c,Rd} = M_{Ed} - M_{PT,Rd}$$

Since the resistance moments of both “partners” are simply summed, it is assumed that resistance moments are reached at the same deflection, or that the stiffer “partner” has sufficient ductility to enable the flexible “partner” to reach its resistance.

This requirement can be fulfilled with experiments on a full-scale specimen of HAF according to following conditions:

- span $L \leq 6$ m,
- sheet thickness 1,0 mm $\leq t_N \leq$ 1,5 mm,
- concrete class C20/25 to C50/60,
- reinforcement of the rib $\leq$ 2,6 cm^2 per rib.

According to Figure D5.2, the trapezoidal sheet alone carries all the loads close to the supports.



Designation

PT	*trapezoidal steel sheet*
c	*reinforced concrete ribbed slab*
e_d	*overall loading on HAF*
$e_{c,d}$	*the portion of the concrete ribbed slab loading*
$e_{PT,d}$	*the portion of the trapezoidal steel sheet loading*
$g_{1,d}$	*design value of the dead load of the profiled sheet*
$g_{2,d}$	*design value of the dead load of concrete*
$g_{3,d}$	*design value of the dead load of finishes (flooring layers, asphalt, etc.)*
q_d	*design value of imposed load ($q_{1,d}$ construction stage, $q_{2,d}$ final stage)*

Figure D5.2 *Calculation model for verification of the resistance moment for the final stage*

For determination of the shear resistance it has to be verified that the profiled sheet support on the steel cleat can carry the support force on the slab alone. The relevant design value is $A_{K,Rd}$ which is the design resistance of the profiled sheet support per cleat. The design value of the shear force acting on the ribbed reinforced concrete slab $V_{c,Ed,\max}$ is determined as:

$$\frac{V_{c,Ed,\max}}{V_{c,Rd}} \leq 1{,}0$$

$$V_{c,Ed,\max} = e_{cd} \cdot \frac{L_c}{2}$$

where:

e_{cd} proportionate action of the ribbed reinforced concrete slab,

L_c the effective floor span of the ribbed reinforced concrete slab, which is smaller by $2 \cdot L_R$ than the span L of the profiled sheet is: $L_c = L - 2 \cdot L_R$.

According to Figure D5.2, the L_R represents the support length for the ribbed slab. Within this length the bending reinforcement in the span has to be fully anchored. The design length from the edge, $L_{R,}$ is calculated for load q_{PT} on the trapezoidal sheet and by using the calculation model from Figure D5.2 with fully utilized resistance moment $M_{PT,Rd}$ of trapezoidal sheet.

From the moment in span according to Figure D5.3, there are three parts of the resistance moment of the trapezoidal sheet:

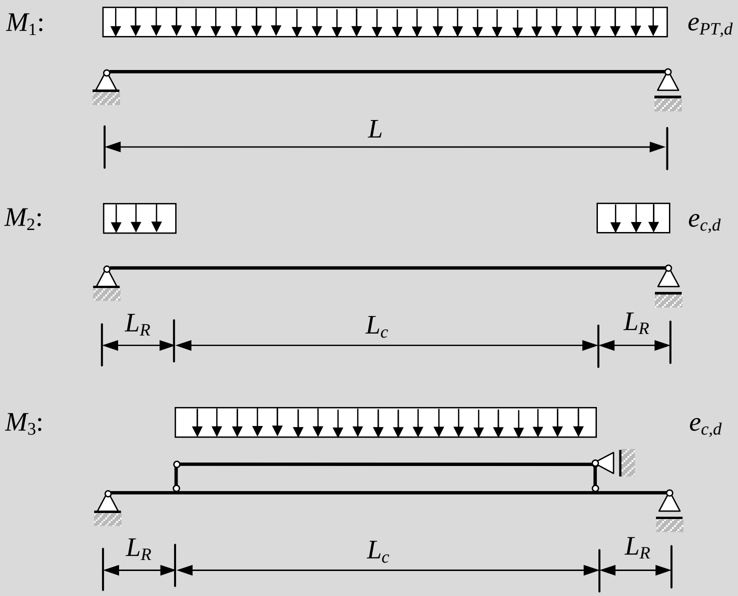


Figure D5.3 *Three parts of loading of one profiled steel sheet for calculation of the length from the edge, L_R*

$$M_{PT,Rd} = M_{1,Ed} + M_{2,Ed} + M_{3,Ed}$$

$$M_{1,Ed} = \frac{e_{PT,d} \cdot L^2}{8}$$

$$M_{2,Ed} = \frac{e_{c,d} \cdot L_R^2}{2}$$

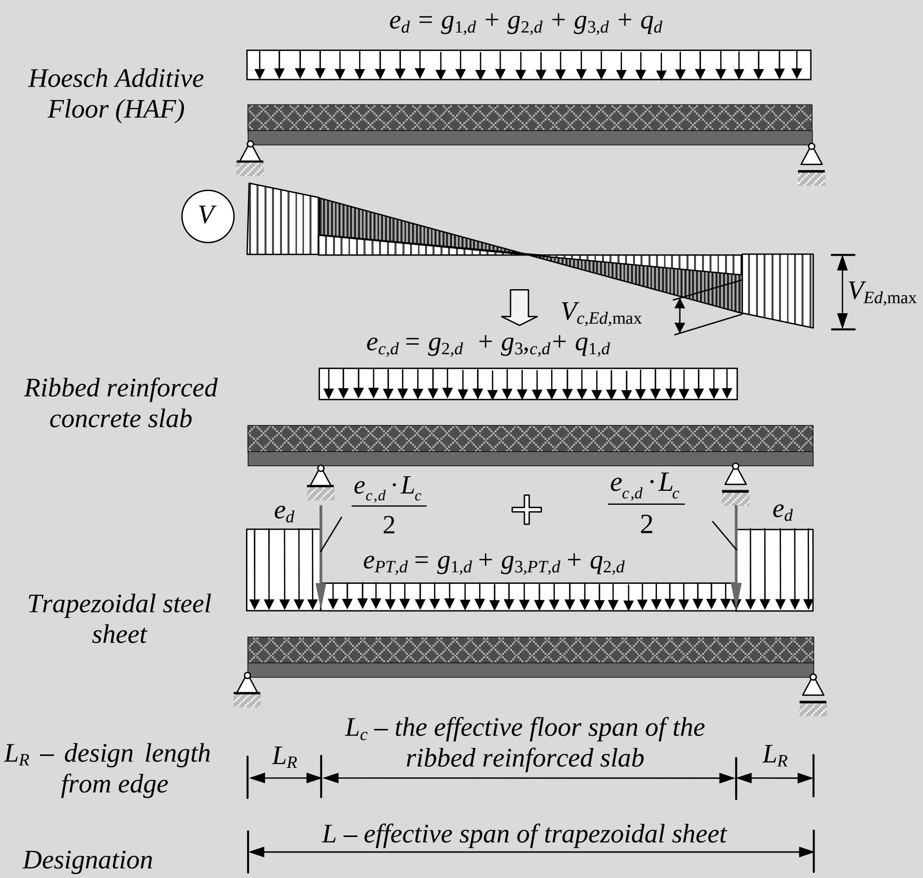
$$M_{3,Ed} = \frac{e_{c,d} \cdot L_c}{2} \cdot L_R$$

$$L_c = L - 2 \cdot L_R$$

After rearrangement, the design value of L_R is determined according to the following equation:

$$(\frac{L_R}{L})^2 - \frac{L_R}{L} + \frac{2 \cdot M_{PT,Rd} / L^2 - e_{PT,\mathrm{E}d} / 4}{e_{cd}} = 0$$

Calculation model for shear resistance is shown in Figure D5.4.



PT	*trapezoidal steel sheet*
c	*reinforced concrete ribbed slab*
e_d	*overall loading on HAF*
$e_{c,d}$	*the portion of the concrete ribbed slab loading*
$e_{PT,d}$	*the portion of the trapezoidal steel sheet loading*
$g_{1,d}$	*design value of the dead load of the profiled sheet*
$g_{2,d}$	*design value of the dead load of concrete*
$g_{3,d}$	*design value of the dead load of finishes (flooring layers, asphalt, etc.)*
q_d	*design value of imposed load ($q_{1,d}$ construction stage, $q_{2,d}$ final stage)*

Figure D5.4 *Calculation model for shear resistance*

b) Verification of the anchorage of the flexural tensile reinforcement in the rib.

Thereby, the required anchor length at the support L_R is verified from the required anchorage length summed with the effective depth of the reinforced concrete rib d:

$$L_R \geq l_{b,\mathrm{eq}} + d$$

where $l_{b,eq}$ is the anchorage length according to EN 1992-1-1.

c) Verification of slab as a flange of steel composite beam

The verification has to be done according to EN 1992-1-1 and/or EN 1994-1-1. The cleats may not be used as shear connectors for the composite beams.

d) Verification of the serviceability limit state

If no specific verification is carried out, a minimum reinforcement that is designed for the crack moment is installed above the inner girders (bearing moment) in case of predominantly bending forces. In case of predominant bending restraint the reinforcement will protrude over the edge of the flange for at least 25 cm in both directions. In case of predominantly tensile restraint a continuous minimum reinforcement is necessary if the tension exceeds the crack axial force. The verification of limitation of the crack widths as well as the acceptance of the crack moment and the crack axial force is made according to EN 1992-1-1. The deflections are limited in accordance with the provisions of the Member States.

e) Verification at the construction stage

Loads at the construction stage are shown in Figure D5.5.

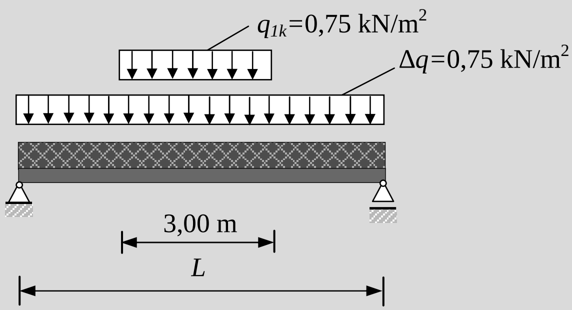


Figure D5.5 *Loads at the construction stage*

Designation:

g_{1k} dead load of trapezoidal steel sheet (in technical approval g_{PT}),
g_{2k} dead load of concrete (RC ribbed slab $g_{c,d}$ + floor layers $g_{AB,d}$),
$q_{1,k}$ live load value at the construction stage 1,50 kN/m^2 on area 3 × 3 m, see

Figure D5.5.
Δq live load value at the construction stage 0,75 kN/m^2 on the remaining area (outside 3 × 3 m), see Figure D5.5.

3. Structural system and cross-section

Composite beams IPE 550 S355 with a span of 16 m and a spacing of 5 m.

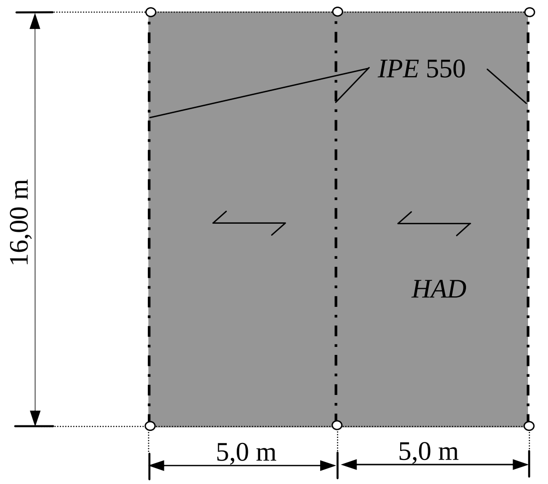


Figure D5.6. *Top view*

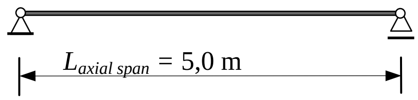


Figure D5.7 *Structural system – single span girder*

Trapezoidal sheet TRP 200, t = 1,25 mm with concrete slab thickness h_c = 9 cm is supported on girders.

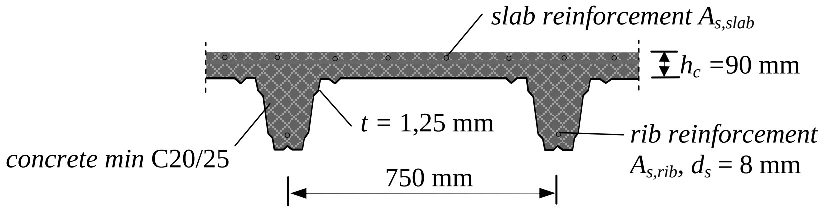


Figure D5.8 *Cross section*

Properties of Hoesch Additive trapezoidal sheet TRP 200 are given in Table D5.1.

Table D5.1 *Hoesch Additive trapezoidal sheet TRP 200*

Section properties							*Characteristic values of resistance moment*
Nominal sheet thickness	*Dead load*	*Moment of inertia*	*Axial force resistance*[1]				*Positive loads*
			Gross cross-section		*Effective cross-section*[2]		
t_N	g	I_{eff}	A_g	i_g	A_{eff}	i_{eff}	$M_{PT,Rk}$
[mm]	[kN/m^2]	[cm^4/m]	[cm^2/m]	[cm]	[cm^2/m]	[cm]	[kNm/m]
1,00	0,128	653	7,68	6,67	7,24	9,09	17,0
1,25	0,160	855	9,68	6,67	9,59	8,97	22,1
1,50	0,192	1030	11,70	6,67	11,70	8,79	26,5

[1] Calculation of the resistance value, see EN 1993-1-3: 2006, sections 6.1.2 and 6.1.3 considering the National Annex and/or the provisions of the Member State at the location where the product is incorporated in the works.

[2] Effective section for constant compressive stress: $f_{y,k}$ = 350 N/mm^2

Concrete cover

The longitudinal reinforcement one ϕ 8 is provided in each rib with the distance to the lower flange of trapezoidal sheet is given by:

$$u = c_{nom} + \frac{d_s}{2} = 50 + \frac{8}{2} = 54 \text{ mm}$$

The effective span of the ribbed reinforced concrete slab L, shown in Figure D5.9, is given by:

$$L = L_{axial\ span} - b_0 - 2 \cdot e - 2 \cdot L_K \cdot \frac{1}{2}$$

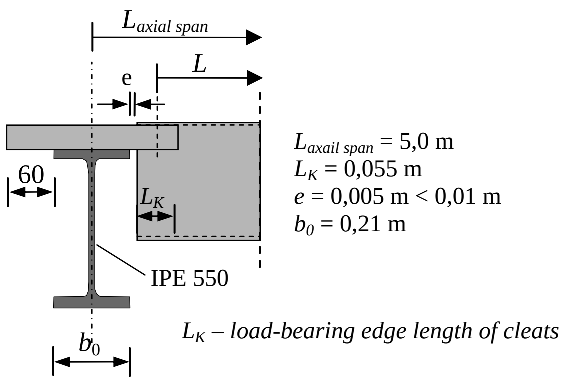


Figure D5.9 *Effective span of HAF*

$$L = 5{,}00 - 0{,}21 - 2 \cdot 0{,}005 - \frac{2 \cdot 0{,}055}{2} = 4{,}73 \text{ m}$$

4. Properties of materials

Concrete class: C 35/45 $f_{ck} = 3{,}5 \text{ kN/cm}^2$

f_{cd} → DIN 18800-5 $f_{cd} = \frac{0{,}85 \cdot f_{ck}}{\gamma_c} = \frac{0{,}85 \cdot 3{,}5}{1{,}5} = 1{,}98 \text{ kN/cm}^2$

Reinforcement: BSt 500S, BSt 500 M: $f_{sk} = 50 \text{ kN/cm}^2$

$$f_{sd} = \frac{f_{sk}}{\gamma_s} = \frac{50{,}0}{1{,}15} = 43{,}5 \text{ kN/cm}^2$$

Nails: Hilti X-ENP-19 L15: ETA-04/0101

Provided:

$$V_{Rk}(t_N = 1{,}25\, mm) = 8{,}0 \text{ kN and } \gamma_M = 1{,}25$$

$$V_{Rd} = \frac{V_{Rk}}{\gamma_M} = \frac{8{,}0}{1{,}25} = 6{,}4 \text{ kN}$$

5. Selection of effective span length without supporting at the construction stage

Input data:

Trapezoidal sheet TRP 200, $t_N = 1{,}25 \text{ mm}$

Reinforced concrete slab thickness: $h_c = 9 \text{ cm}$

Using Figure D5.10, maximal effective span without supporting is determined:

$$L_{\max} = 4{,}92 \text{ m}$$

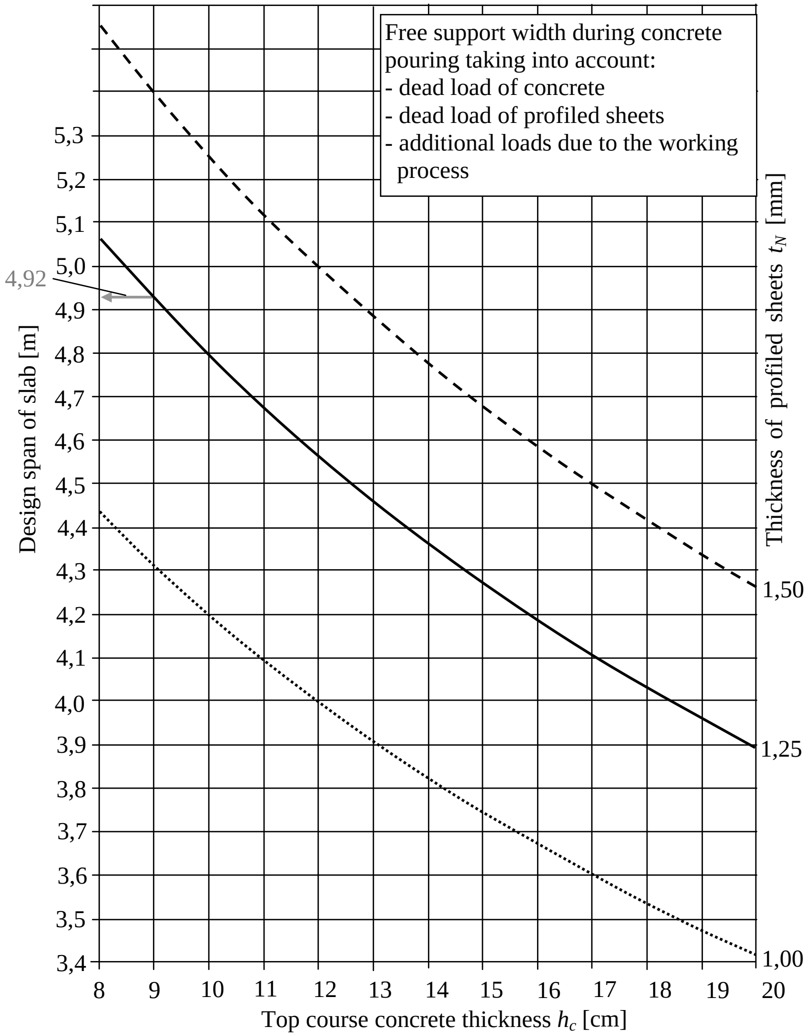


Figure D5.10 *Preliminary calculation of trapezoidal sheet as a function of effective slab span L*

Check:

$$\frac{L}{L_{\max}} = \frac{4,73}{4,92} = 0,961 < 1,0$$

6. Ultimate limit state

6.1 Calculation at the construction stage

6.1.1 Loads

Since the slab is not supported during concrete pouring, the trapezoidal sheet has to carry the self-weight of the sheet and concrete and the live load from the working process with the value g_{1k} = 0,75 kN/m^2 and additional load Δq_k = 0,75 kN/m^2 on the area 3 × 3 m.

Dead load of trapezoidal steel sheet (t_N = 1,25 mm):

$$g_{1k} = 0{,}16 \ \text{kN/m}^2$$

Dead load of wet concrete (ribbed slab):

$$g_{2k} = 0{,}87 + h_c \cdot 26 = 0{,}87 + 0{,}09 \cdot 26 = 3{,}21 \ \text{kN/m}^2$$

Design permanent load:

$$g_d = \gamma_F (g_{1,k} + g_{2,k}) = 1{,}35(0{,}16 + 3{,}21) = 4{,}55 \ \text{kN/m}^2$$

Live load:

$$q_{1,k} = 0{,}75 \ \text{kN/m}^2$$
$$\Delta q_k = 0{,}75 \ \text{kN/m}^2$$

Design live load

$$q_{1,d} = \Delta q_d = \gamma_Q \cdot q_{1,k} = 1{,}5 \cdot 0{,}75 = 1{,}13 \ \text{kN/m}^2$$

Overall loads

$$e_d = g_d + q_{1,d} = 4{,}55 + 1{,}13 = 5{,}68 \ \text{kN/m}^2$$

6.1.2 Action effects

For slab width 1 m:

$$M_{Ed} = e_d \cdot L^2 / 8 + \Delta q_d \cdot 3{,}00 \cdot (2 \cdot L - 3{,}00) / 8$$

$$M_{Ed} = 5{,}68 \cdot 4{,}73^2 / 8 + 1{,}13 \cdot 3{,}00 \cdot (2 \cdot 4{,}73 - 3{,}00) / 8 = 18{,}6 \text{ kNm/m}$$

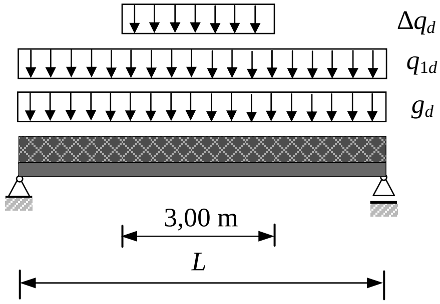


Figure D5.11 *Load arrangement during concreting resulting in maximum bending moment*

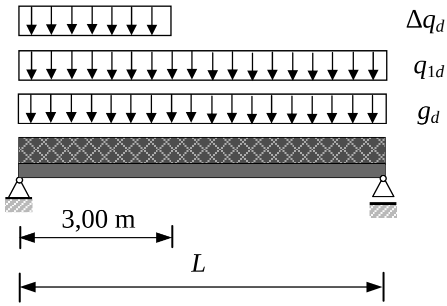


Figure D5.12 *Load arrangement during concreting resulting in maximum shear force*

$$V_{Ed} = e_d \cdot L / 2 + \Delta q_d \cdot 3{,}00 / L \cdot (L - 3{,}00 / 2)$$

$$V_{Ed} = 5{,}68 \cdot 4{,}73 / 2 + 1{,}13 \cdot 3{,}00 / 4{,}73 \cdot (4{,}73 - 3{,}00 / 2) = 15{,}7 \text{ kNm/m}$$

The force flow in the steel cleat is shown in Figure D5.13.

The shear force per one steel cleat is:

$$A_{K,Ed} = \frac{0{,}75 \cdot V_{Ed}}{2}$$

$$A_{K,Ed} = \frac{0{,}75 \cdot 15{,}7}{2} = 5{,}90 \text{ kN}$$

The shear force per one nail is:

$$V_{Ed} = 0{,}25 \cdot A_{K,Ed}$$

$$V_{Ed} = 0{,}25 \cdot 5{,}90 = 1{,}47 \text{ kN}$$

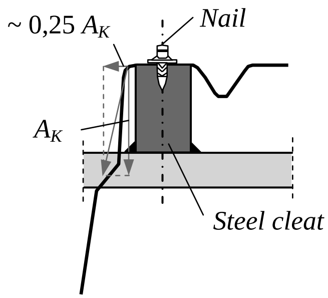


Figure D5.13 *Force flow in the steel cleat*

6.1.3 Design value of resistance moment

Verification of the profiled steel sheet is given by:

$$\frac{M_{Ed}}{M_{PT,Rd}} \leq 1{,}0$$

where:

M_{Ed} design value of bending moment,
$M_{PT,Rd}$ design value of resistance moment of the profiled steel sheet.

$$M_{PT,Rd} = \frac{M_{PT,Rk}}{\gamma_M} = \frac{22{,}1}{1{,}1} = 20{,}09$$

$M_{PT,Rk}$ characteristic resistance moment of profiled steel sheet according to Table D5.1.

Check:

$$\frac{M_{Ed}}{M_{PT,Rd}} = \frac{18{,}6}{20{,}09} = 0{,}926 < 1{,}0 \text{ (satisfactory)}$$

6.1.4 Shear resistance

Design shear resistance of one steel cleat $A_{k,Rd}$ according to table D5.2 is 10,7 kN.

$$A_{K,Rd} = \frac{11,80}{1,1} = 10,7 \text{ kN}$$

Table D5.2 *Characteristic values of resistance of the profiled sheet support on a steel cleat*

t_N [mm]	1,00	1,25	1,50
$A_{K,Rk}$ [kN]	8,60	11,80	15,50

Thus, verification is given by:

$$\frac{A_{K,Ed}}{A_{K,Rd}} = \frac{5,90}{10,7} = 0,551 < 1,0 \text{ (satisfactory)}$$

6.1.5 Design of nail

Provided: HILTI Setzbolzen X-ENP-19L 15

Characteristic shear resistance of nail:

$$V_{Rk}(t_N = 1,25 \text{ mm}) = 8,0 \text{ kN}$$

Design shear resistance of nail:

$$V_{Rd} = \frac{V_{Rk}}{\gamma_{M2}} = \frac{8}{1,25} = 6,40 \text{ kN}$$

Check of nail:

$$\frac{V_{Ed}}{V_{Rd}} = \frac{1,47}{6,40} = 0,23 < 1,0 \text{ (satisfactory)}$$

6.2 Calculation for final stage

6.2.1 Loads

Dead load of trapezoidal steel sheet:

$$g_{1k} = 0,16 \text{ kN/m}^2$$

Dead load of dry concrete:

$$g_{2k} = 0,83 + h_c \cdot 25 = 0,83 + 0,09 \cdot 25 = 3,08 \text{ kN/m}^2$$

Design permanent load:

$$g_d = \gamma_F (g_{1k} + g_{2k}) = 1{,}35(0{,}16 + 3{,}08) = 4{,}37 \text{ kN/m}^2$$

Live load:

$$q_{2,k} = 3{,}50 \text{ kN/m}^2$$

Design live load:

$$q_{2,d} = \gamma_Q \cdot q_{2,k} = 1{,}5 \cdot 3{,}50 = 5{,}25 \text{ kN/m}^2$$

Overall load:

$$e_d = g_d + q_{2,d} = 4{,}37 + 5{,}25 = 9{,}62 \text{ kN/m}^2$$

6.2.2 Action effects

$$M_{Ed} = \frac{e_d \cdot L^2}{8} = \frac{9{,}62 \cdot 4{,}73^2}{8} = 26{,}9 \text{ kNm/m}$$

$$V_{Ed} = \frac{e_d \cdot L}{2} = \frac{9{,}62 \cdot 4{,}73}{2} = 22{,}8 \text{ kN/m}$$

Shear force per one steel cleat:

$$A_{K,Ed} = \frac{0{,}75 \cdot V_{Ed}}{2}$$

$$A_{K,Ed} = \frac{0{,}75 \cdot 22{,}8}{2} = 8{,}55 \text{ kN}$$

Shear force per one nail:

$$V_{Ed} = 0{,}25 \cdot A_{K,Ed}$$

$$V_{Ed} = 0{,}25 \cdot 8{,}55 = 2{,}14 \text{ kN}$$

6.2.3 Resistance moment

Bending moment is carried by both the trapezoidal sheet and the RC ribbed slab.

Carrying capacity of the trapezoidal sheet is fully utilized, and the rest of the bending moment is distributed to the RC ribbed slab. Close to the supports the trapezoidal sheet carries all the loading.

The calculation model for verification of resistance moment is shown in Figure D5.2.

Verification is given by:

$$\frac{M_{Ed}}{M_{Rd}} \leq 1{,}0$$

where:

$$M_{Rd} = M_{PT,Rd} + M_{c,Rd}$$

$$M_{PT,Rd} = \frac{M_{PT,Rk}}{\gamma_M} = \frac{22{,}1}{1{,}1} = 20{,}09 \text{ kNm}$$

and $M_{c,Rd}$:

$$M_{c,Rd} = M_{Ed} - M_{PT,Rd} = 26{,}91 - 20{,}09 = 6{,}82 \text{ kNm/m}$$

The distance between the centre of reinforcement and the edge of lower rib:

$$u = c_{nom} + d_s / 2 = 54 \text{ mm}$$

Material properties:

Reinforcement

$$f_{sd} = 50 / 1{,}15 = 43{,}5 \text{ kN/cm}^2$$

Compressive strength of concrete

For calculating the composite slab cross-section resistance a simplified rectangular shape of stress block is considered instead of the parabolic–rectangular shape. Thus, the reduction factor for compressive strength of concrete with the value 0,95 may be introduced in the National Annex.

$$X \cdot f_{cd} = X \cdot \alpha \cdot f_{ck} / \gamma_c = 0{,}95 \cdot 0{,}85 \cdot 3{,}5 / 1{,}5 = 1{,}88 \text{ kN/cm}^2$$

$h_{TRP} = 205$ (the height of trapezoidal sheet)

Static height:

$d = h_{TRP} + h_c - u = 205 + 90 - 54 = 241$ mm

Slab width:

$b = 100$ cm

Distribution of strains (strain hypothesis):

$\varepsilon_c / \varepsilon_s = -0{,}197 / 25$ in ‰

With rectangular stress block

$\rightarrow \; x = d \cdot |\varepsilon_c| / (|\varepsilon_c| + \varepsilon_s) = 241 \cdot 0{,}197 / (0{,}197 + 25) = 1{,}88$ mm

Compressive force in concrete:

$F_{cd} = A_{cc,red} \cdot X \cdot f_{cd}$

where:

$A_{cc,red} = b \cdot 0{,}8 \cdot x = 100 \cdot 0{,}8 \cdot 0{,}188 = 15{,}04 \text{ cm}^2$

$F_{cd} = 15{,}04 \cdot 1{,}88 = 28{,}3$ kN

For the lever arm, see Figure D5.14:

$z = d - 0{,}5 \cdot x = 241 - 0{,}5 \cdot 1{,}88 = 240$ mm

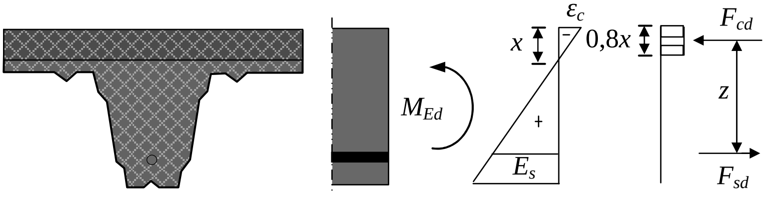


Figure D5.14 *Stresses in cross section*

The action effects per slab width 1 m are:

$M_{Ed} = F_{cd} \cdot z$

$M_{c,Rd} = 6,82$ kNm/m

$6,82 \approx 28,3 \cdot 0,24 = 6,79$ kNm/m

→ The distribution of strains correct!

The tensile force in steel:

$F_{sd} = F_{cd} = 28,3$ kN

Reinforcement:

required $A_s = F_{sd} / f_{yd} = 28,3 / 43,5 = 0,651$ cm^2/m

required $A_{s,rib} = 0,75 \cdot 0,651 = 0,488$ cm^2

provided: 1 ϕ 8 mm

existing reinforcement in rib $A_{s,rib} = 0,503$ cm^2

existing reinforcement for 1 m slab width $A_s = 0,671$ cm^2/m

Resistance moment provided by concrete:

$M_{c,Rd} = A_s \cdot f_{yd} \cdot (d - x / 2)$

$M_{c,Rd} = 0,671 \cdot 43,5 \cdot (24,1 - 0,188 / 2) = 701$ kNcm/m

$M_{c,Rd} = 7,01$ kNm/m

Check:

$$\frac{M_{Ed}}{M_{Rd}} = \frac{26,9}{M_{PT,Rd} + M_{c,Rd}} = \frac{26,9}{(20,09 + 7,01)} = 0,993 < 1,0$$

6.2.4 Shear resistance

The shear force is transferred to the supports using steel cleats. At the point where the concrete starts to carry load, the shear resistance of the concrete rib without shear reinforcement has to be verified (Figure D5.4)

Check of sheet: $$\frac{A_{K,Ed}}{A_{K,Rd}} = \frac{8,54}{10,7} = 0,798 \le 1$$

Check of nail: $$\frac{V_{Ed}}{V_{Rd}} = \frac{2,13}{6,40} = 0,333 \le 1$$

Shear resistance of RC ribbed slab:

$$\frac{V_{c,Ed,\max}}{V_{c,Rd}} \le 1,0$$

$$V_{c,Ed,\max} = q_{c,d} \cdot L_c / 2$$

where:

$$q_{c,d} = q_d - g_{PT,d} = 9,62 - 0,16 \cdot 1,35 = 9,41 \ \text{kN/m}^2$$

$$L_C = L - 2 \cdot L_R = L(1 - 2 \cdot L_R / L)$$

The length L_R is given as:

$$(L_R / L)^2 - L_R / L + (2 \cdot M_{PT,Rd} / L^2 - g_{PT,d} / 4) / q_{c,d} = 0$$

where $g_{PT,d} = g_{1d}$ (dead load of sheet)

$$(L_R / L)^2 - L_R / L + (2 \cdot 20,09 / 4,73^2 - 0,16 \cdot 1,35 / 4) / 9,41 = 0$$

$$(L_R / L)^2 - L_R / L + 0,185 = 0$$

$$L_R / L = 0,5 - \sqrt{(0,5^2 - 0,185)} = 0,245$$

$$L_c = 4,73(1 - 2 \cdot 0,245) = 2,41 \ \text{m}$$

$$V_{c,Ed,\max} = \frac{9,41 \cdot 2,41}{2} = 11,33 \text{ kN/m}$$

$$V_{c,Rd} = [0,1 \cdot \kappa \cdot \eta_1 \cdot (100 \cdot \rho_I \cdot f_{ck})^{1/3}] \cdot b_w \cdot d$$

where:

$$\kappa = 1 + \sqrt{\frac{200}{d}} = 1 + \sqrt{\frac{200}{241}} = 1,91$$

$\eta_1 = 1,0$ for normal concrete

$$\rho_I = \frac{A_{sI}}{b_w \cdot d}$$

$$A_{sI} = 0,503 \cdot 10^2 = 50,3 \text{ mm}^2$$

$$d = 241 \text{ mm}$$

$b_w(c_{nom} = 50 \text{ mm}) = 108$ mm → clause 10.3.3, DIN 1045-1:2001

$$\rho_I = \frac{50,3}{108 \cdot 241} = 1,93 \text{ ‰} < 20 \text{ ‰}$$

$$f_{ck} = 35 \text{ N/mm}^2$$

$$V_{c,Rd} = [0,1 \cdot 1,91 \cdot 1,0 \cdot (0,193 \cdot 35)^{1/3}] \cdot 108 \cdot 241 = 9398 \text{ N/Rib}$$

$$V_{c,Rd} = 9,40 \text{ kN/0,75m} = 12,54 \text{ kN/m}$$

Check:

$$\frac{V_{c,Ed}}{V_{c,Rd}} = \frac{11,33}{12,54} = 0,903 < 1,0$$

6.2.5 Verification of anchor of rib-reinforcement due to bending moment

$L_R \geq l_{b,eq} + d$ → clause 12.6.2, DIN 1045-1:2001

$$L_R = 0{,}245 \cdot L = 0{,}245 \cdot 4{,}73 = 1{,}16 \ \text{m}$$

The basic value of anchor length is:

$$l_b = \frac{d_s}{4} \cdot \frac{f_{sd}}{f_{bd}}$$

where:

$$f_{yd} = 500 / 1{,}15 = 435 \ \text{N/mm}^2$$

$f_{bd} = 3{,}4 \ \text{N/mm}^2 \rightarrow$ Table 25, DIN 1025-1:2007

$$d_s = 8 \ \text{mm}$$

$$l_b = \frac{d_s}{4} \cdot \frac{f_{sd}}{f_{bd}} = \frac{8}{4} \cdot \frac{435}{3{,}4} = 256 \ \text{mm}$$

The required anchor length is:

$$l_{b,eq} = \alpha_a \cdot l_b \cdot A_{s,required} \ / \ A_{s,existing} \geq l_{b,\min}$$

where:

$$\alpha_a = 1{,}0$$

$A_{s,required} = 0{,}490 \ \text{cm}^2$ ($A_{s,rib}$)

$A_{s,exisisting} = 0{,}503 \ \text{cm}^2$ ($A_{s,rib}$)

$$l_{b,\min} = \max(0{,}3 \cdot l_b ; 10 d_s) = \max(0{,}3 \cdot 256; 10 \cdot 8) = 80 \ \text{mm}$$

$$l_{b,eq} = 1{,}0 \cdot 256 \cdot 0{,}490 / 0{,}503 = 249 \ \text{mm} \geq 80 \ \text{mm}$$

$$l_{b,eq} + d = 249 + 241 = 490 \ \text{mm} = 0{,}490 \ \text{m}$$

Check:

$$L_R = 1{,}16 \ \text{m} > l_{b,eq} + d = 0{,}490 \ \text{m}$$

7. Serviceability limit state

7.1 Cracking of concrete

7.1.1 General

Generally, it is necessary to distinguish two types of structural members

- elements predominantly subjected to bending restraint
- elements predominantly subjected to tension restraint

In the first case restraint force occurs in practically every slab due to live load. In the second case restraint force occurs only in slabs with restrained thermal elongations with concrete cores or vertical bracings.

7.1.2 Design for bending restraint

$$M_R = k \cdot \eta \cdot f_{ct,eff} \cdot \frac{h_c^2}{6}$$

$$k = 0,8$$

$$\eta = 1 + 0,18 / \sqrt{h_c} = 1 + 0,18 / \sqrt{0,09} = 1,60 \,, \quad h_c \text{ [m]}$$

$$f_{ct,eff} = f_{ctm} = 3,2 \text{ N/mm}^2$$

$h_c = 0,09$ m slab thickness

$$M_R = 0,8 \cdot 1,6 \cdot 3,2 \cdot \frac{0,09^2}{6} = 5,53 \cdot 10^{-3} \text{ MN/m} = 5,53 \text{ kN/m}$$

Depending on the exposure class, specific requirements for minimum thickness of concrete cover apply. For car parks classified in XD3, minimal cover thickness 40+15 mm (DIN 1045-1, Table 4) is used. Crack control of HAF cannot be achieved with reinforcement installed that way. Therefore, for car parks chloride resistant reinforcement is used (for example made of stainless steel) with significant reduction of concrete cover.

$$A_{s,\min} = M_R / z / \sigma_s$$

where:

$$z = 0,9 \cdot d = 0,9 \cdot (h_c - c - d_s / 2) = 0,9 \cdot (90 - 18 - 6 / 2) = 62,1 \text{ mm} = 0,062 \text{ m}$$

$$\sigma_s = \sigma_{s,table} \cdot (c_0 + d_s / 2) / (c_{nom} + d_s / 2)$$

$$d_s = 6 \text{ mm}$$

$$\sigma_{s,table} = 370 \text{ N/mm}^2 \text{ (exposure class XD3)}$$

The reference value of concrete cover for calculation of reduced stress σ_s is:

$$c_0 = 15 \text{ mm}$$

Chosen: $c_{nom} = 18 \text{ mm}$

$$\sigma_s = 370 \cdot (15 + 6 / 2) / (18 + 6 / 2) = 317 \text{ N/mm}^2 = 31,7 \text{ kN/cm}^2$$

$$A_{s,\min} = 5,53 / 0,062 / 31,7 = 2,814 \text{ cm}^2\text{/m}$$

The reinforcement has to protrude above the edge of the flange for at least 25 cm in both directions. With limitations of stress in reinforcement, a characteristic cracked depth w_k = 0,13 mm is achieved.

7.1.3 Design for predominantly tensile restraint

Tensile restraint can result from deformation due prevented temperature elongation or concrete shrinkage. It can be constructively avoided, for example by placing the vertical stabilization system of the building centrally. This reinforcement is provided with a concrete cover requirement according to DIN 1045-1, analogous to shear introduction in composite beams. The reinforcement should be chloride resistance. Considering limit stresses in reinforcement σ_s [31], the minimum required reinforcement is calculated for concrete crack axial force N_R using:

$$N_R = k \cdot f_{ct,eff} \cdot h_c$$

where:

$$k = 0,8$$

$$f_{ct,eff} = f_{ctm} = 3,2 \text{ N/mm}^2$$

$h_c = 0{,}09\ \text{m}$

$A_{s,\text{rib,min}} = N_R / \sigma_s$

where:

$\sigma_s = \sigma_{s,table} = 240\ \text{N/mm}^2$, (for d_s = 8 mm and exposure class XD3)

$d_s = 8\ \text{mm}$

$A_{s,\text{rib,min}} = 230{,}4 / 24{,}0 = 9{,}60\ \text{cm}^2\text{/m}$

The constructive reinforcing of the slab is shown in Figures D5.16 and D5.17.

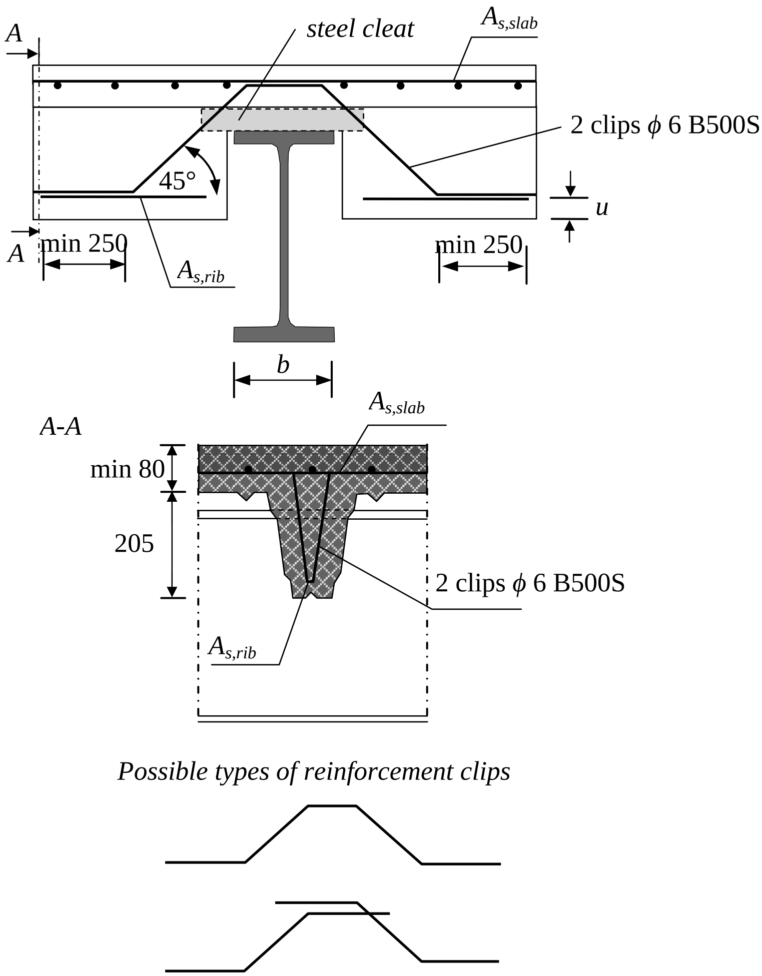


Figure D5.15 *Constructional reinforcement at intermediate support*

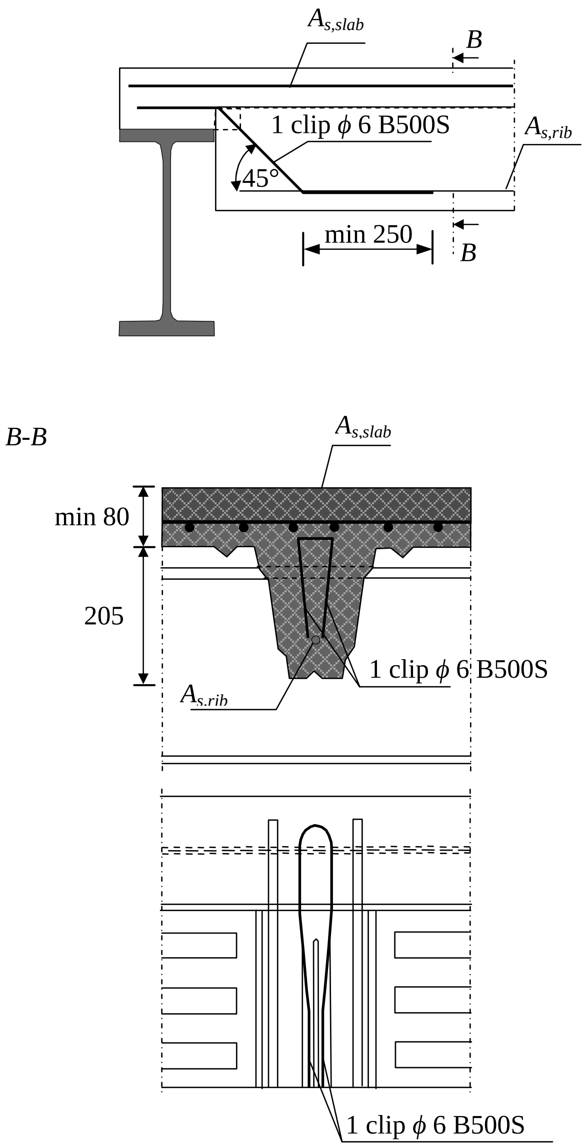


Figure D5.16 *Constructional reinforcement at end support*

7.2 Deflections

For deflection limits, clause 11.3 in DIN 1045-1: 2001 may be used. Deflection of an HAF slab consists of two parts. First part refers to the construction stage where all the loads are carried by the trapezoidal steel sheets. The second part refers to the final stage when profiled sheet and ribbed RC slab carry loads from floor finishes and live loads. The maximum deflection at the first stage (trapezoidal sheet + concrete) for sheet thickness 1,5 mm, slab thickness 80 mm and span of 5,55 m is

1,62 cm (*L*/342). For the final stage, deflections are not critical since the stiffness of the cross-section increases about 10 times.

8. Commentary

The HAF slab resistance at final stage is obtained by summing the resistances of profiled steel sheet and reinforced concrete ribbed slab. Shear connection between steel and concrete is not provided, so they act without composite action. This system is suitable for long spans as in the case of car park buildings.

E Fatigue

E1 Fatigue verification for composite highway bridge

1. Purpose of example

This example deals with the fatigue assessment of a composite highway bridge. The static system and cross-section of a composite bridge are shown in Figures E1.1 and E1.2. Analysis and dimensioning of the bridge were conducted in accordance with appropriate European standards. The bridge was verified for the following design situations: transient, permanent and accidental. The required criteria for static loading are met.

In this example, attention is focused on the fatigue assessment of the composite highway bridge subjected to simple vehicle load. However, this example does not aim to present the various actions on the bridge nor how they are modelled. Therefore, we only look at the design values of the internal forces and bending moments as well as the normal and shear stresses in selected cross-sections.

Fatigue verification is carried out for the structural steel part and its shear connectors as well as for the reinforcing steel bars in the concrete slab. Fatigue verification of the concrete and of the transverse reinforcement in the concrete slab is not carried out. The fatigue limit state is verified for the reference stress range due to the application of the simplified fatigue load model (single vehicle model – FLM 3). Fatigue verification is carried out only for certain key structural details.

2. Static system, cross-section and actions

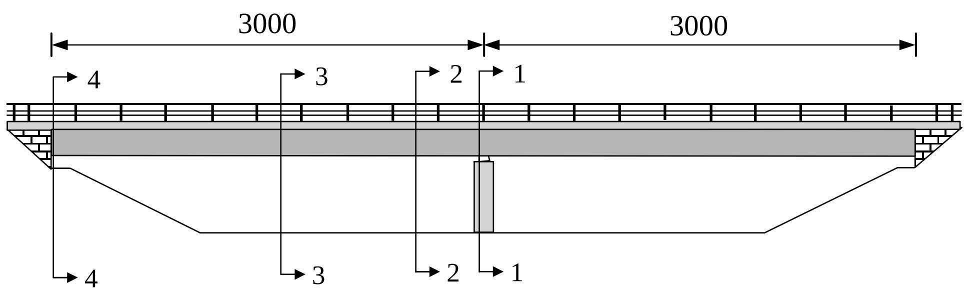


Figure E1.1 *Composite bridge – elevation with selected location of analysed cross-sections*

Composite Structures according to Eurocode 4. Worked Examples.
First Edition. Darko Dujmović, Boris Androić, Ivan Lukačević.

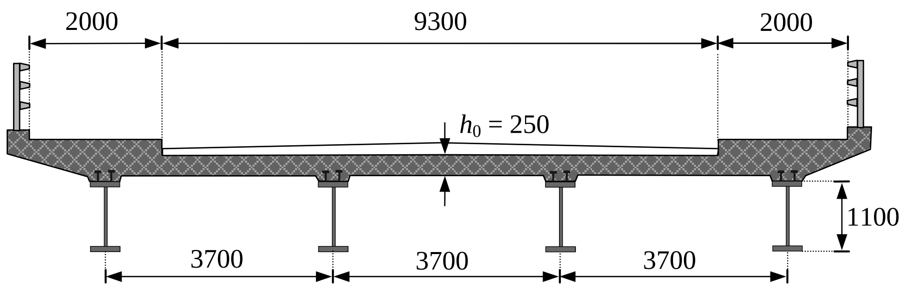


Figure E1.2 *Cross-section of composite bridge*

Fatigue load model

In clause 4.6, EN 1991-2, five fatigue load models are defined. Their use is given in corresponding Parts of EN 1992 to EN 1999. In this example, a single vehicle model, fatigue load model 3 (FLM 3), is used. This model consists of four axles, each having two identical wheels. The geometry of FLM 3 is shown in Figure E1.3. The weight of each axle is 120 kN. The contact surface of each wheel is a square with sides of 0,40 m.

This model is used for the calculation of the design value of the stress range at $2 \cdot 10^6$ cycles according to clauses 9.4.2(1)–(5), EN 1993-2, and according to clause 6, EN 1993-1-9. For the calculation of the design value of the stress range, elastic analysis is used and shear lag effect must be taken into account. In the case of cross-section class 4, stresses should be calculated taking into account the effective cross-section.

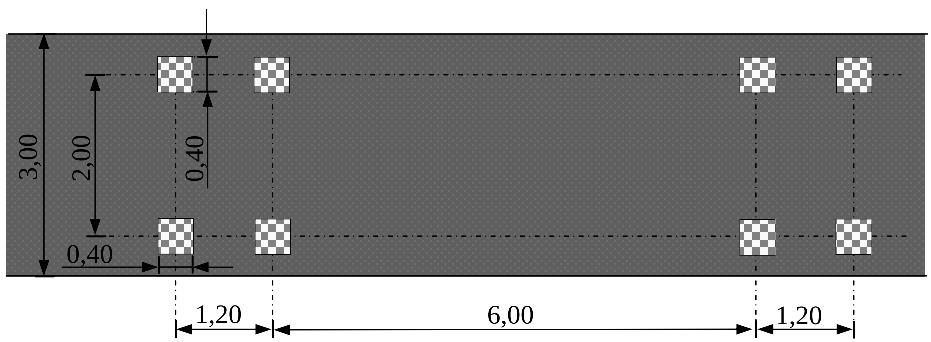


Figure E1.3 *Fatigue Load Model 3 (FLM 3)*

3. Properties of materials

Concrete strength class: C 40/50 $f_{ck} = 40{,}0$ N/mm²

$$f_{cd} = \frac{f_{ck}}{\gamma_c} = \frac{40,0}{1,5} = 26,7 \text{ N/mm}^2$$

$$f_{ctm} = 3,5 \text{ N/mm}^2$$

Structural steel: S355 $f_{yk} = 355 \text{ N/mm}^2$

$$f_{yd} = \frac{f_{yk}}{\gamma_M} = \frac{355}{1,0} = 355 \text{ N/mm}^2$$

Structural steel: *ductility class B or C (Table C.1., EN 1992-1-1)* $f_{sk} = 500 \text{ N/mm}^2$

$$f_{sd} = \frac{f_{sk}}{\gamma_s} = \frac{500}{1,15} = 435 \text{ N/mm}^2$$

4. Global analysis

The bending moments and internal forces are calculated by using an elastic analysis, see clauses 5.4.1 and 5.4.2, EN 1994-2. The analysis is carried out under the same conditions as the ones used to check the bridge design under basic traffic loads and taking into consideration the cracked zones in the region of the internal supports. The calculation of the bending moments and internal forces is carried out using the basic serviceability limit state (SLS) combination of the non-cyclic loads (permanent loads) to which the fatigue load is added, see clause 6.8.3, EN 1992-1-1. The stress range $\Delta\sigma$ is obtained by $\Delta\sigma = |\sigma_{\max,f} - \sigma_{\min,f}|$ where the stresses $\sigma_{\max,f}$ and $\sigma_{\min,f}$ are calculated from $M_{Ed,\max,f}$ and $M_{Ed,\min,f}$ with the short-term modular ratio. Three different situations are taken into consideration for the stress range calculation: a) $M_{Ed,\max,f}$ and $M_{Ed,\min,f}$ cause tensile stresses in the concrete slab, b) $M_{Ed,\max,f}$ and $M_{Ed,\min,f}$ cause compressive stresses in the concrete slab and c) $M_{Ed,\max,f}$ causes tensile stresses in the concrete slab and $M_{Ed,\min,f}$ causes compressive stresses in the concrete slab. The detailed explanation of determining stresses and stress ranges in steel and concrete composite structures is given in [46]. The similar procedure is used for calculation of shear stress ranges.

Selected cross-sections at locations from 1-1 to 4-4 are shown in Figures E1.4 and E1.5.

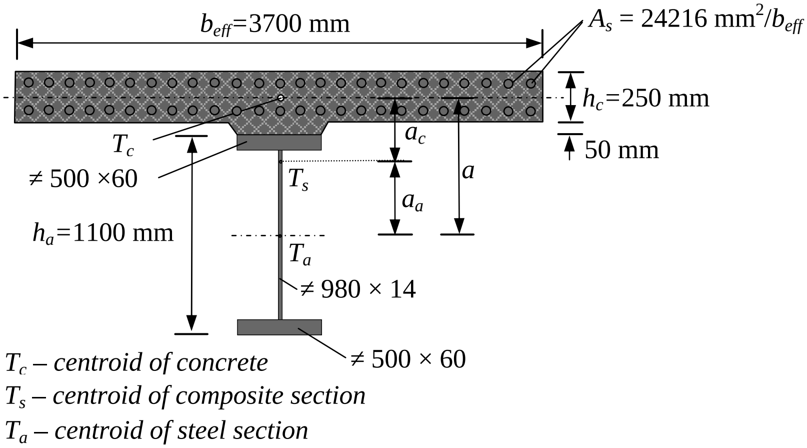


Figure E1.4 *Cross-sections 1-1 and 2-2*

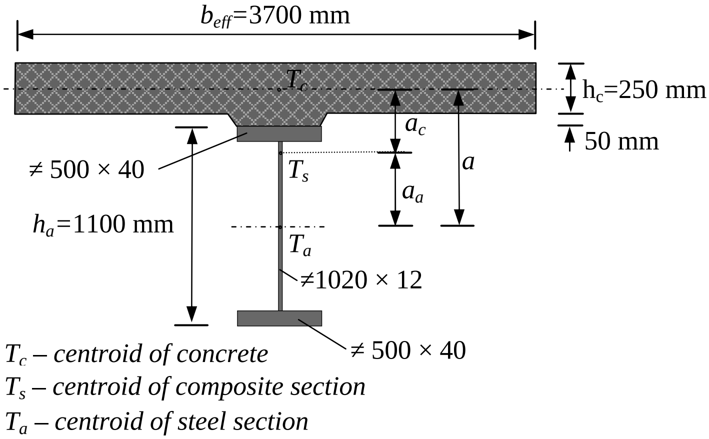


Figure E1.5 *Cross-sections 3-3 and 4-4*

The calculation of the bending moments and internal forces and of stresses in each cross-section due to fatigue load model FLM 3 is not presented here. Only their design values in selected cross-sections are shown in Table E1.1, Table E1.2 and Table E1.3.

Table E1.1 *Maximal bending moments*

	Section 1-1 (At pier)	Section 2-2 (At splice)	Section 3-3 (At midspan)
	M_y (kNm)	M_y (kNm)	M_y (kNm)
Lane 1, positive	0	410	660
Lane 1, negative	−430	−270	−140
Range ΔM	430	680	800
Lane 2, positive	0	390	630
Lane 2, negative	−403	−256	−132
Range ΔM	403	646	762

Table E1.2 *Maximal shear forces*

	Section 1-1	Section 2-2	Section 3-3	Section 4-4
	V_z (kN)	V_z (kN)	V_z (kN)	V_z (kN)
Lane 1, positive	265	95	53	15
Lane 1, negative	−10	−7	−27	−270
Range ΔV	275	102	80	285
Lane 2, positive	238	90	48	14
Lane 2, negative	0	−3	−24	−248
Range ΔV	238	93	72	262

The values of stress ranges, $\Delta\sigma$, given in Table E1.3 are calculated with bending moments, ΔM, from Table E1.1 and corresponding values of section moduli, W.

Table E1.3 *Stress ranges due to fatigue load model FLM 3*

		Bottom flange		*Top flange*		*Top bar*	
		W (10^6 mm^3)	$\Delta\sigma$ (N/mm^2)	W (10^6 mm^3)	$\Delta\sigma$ (N/mm^2)	W (10^6 mm^3)	$\Delta\sigma$ (N/mm^2)
Section 1-1	Range, lane 1	38,49	11,2	79,00	−5,4	42,02	−10,2
	Range, lane 2	38,49	10,5	79,00	−5,1	42,02	−9,6
Section 2-2	Range, lane 1	38,49	17,7	79,00	−8,6	—	—
	Range, lane 2	38,49	16,8	79,00	−8,2	—	—
Section 3-3	Range, lane 1	31,36	−25,5	2802	≈ 0	—	—
	Range, lane 2	31,36	−24,3	2802	≈ 0	—	—

Lanes for bridges are defined in EN 1991-2.

5. Fatigue assessment

5.1 Assessment of structural steel details

5.1.1 General

Fatigue assessment is carried out according to clause 9.5.1(1), EN 1993-2, as follows:

$$\gamma_{Ff} \cdot \Delta\sigma_{E,2} \leq \frac{\Delta\sigma_C}{\gamma_{Mf}}$$

where:

γ_{Ff} is the partial factor for equivalent constant amplitude stress ranges $\Delta\sigma_E$,
γ_{Mf} is the partial factor for fatigue strength $\Delta\sigma_C$.

The design value of the stress range in structural steel is given as follows:

$$\gamma_{Ff} \cdot \Delta\sigma_{E,2} = \gamma_{Ff} \cdot \lambda \cdot \Phi_2 \cdot \Delta\sigma_p$$

where $\Phi_2 = 1{,}0$ according to clause 9.4.1(5), EN 1993-2, γ_{Ff} = 1,0, $\Delta\sigma_p$ denotes stress range and λ is the damage equivalent factor.

Values of λ are determined according to clause 9.5.2(7), EN 1993-2 for road bridges up to 80 m:

$$\lambda = \lambda_1 \cdot \lambda_2 \cdot \lambda_3 \cdot \lambda_4$$

For intermediate supports in spans up to 30 m, the factor for the damage effect of traffic λ_1 should be obtained according to Figure 9.5, EN 1993-2, and is given by the expression:

$$\lambda_1 = (2 - 0{,}3 \cdot (L - 10)/20)$$

where L is the length of the critical influence line in metres and here L = 30 m, as the mean of the adjacent spans.

Thus, λ_1 for intermediate supports is:

$$\lambda_1 = (2 - 0{,}3 \cdot (L - 10)/20) = (2 - 0{,}3 \cdot (30 - 10)/20) = 1{,}70$$

For the span region, λ_1 should be determined according to Figure 9.5, EN 1993-2, and is given by:

$$\lambda_1 = (2{,}55 - 0{,}7 \cdot (L - 10) / 70)$$

For L = 30 m as before, λ_1 for the span region is:

$$\lambda_1 = (2{,}55 - 0{,}7 \cdot (30 - 10) / 70) = 2{,}35$$

The factor for the traffic volume λ_2 is calculated according to clause 9.5.2(3), EN 1993-2 as:

$$\lambda_2 = \left(\frac{Q_{m1}}{Q_0}\right) \cdot \left(\frac{N_{Obs}}{N_0}\right)^{0{,}2}$$

where:

$Q_0 = 480$ kN and $N_0 = 0{,}5 \cdot 10^6$

N_{Obs} is the total number of lorries per year in the slow lane, according to clause 4.6.1(3), Table 4.5, EN 1991-2; intermediate values of N_{Obs} are not excluded,

Q_{m1} is the average gross weight (kN) of the lorries in the slow lane obtained from the expression:

$$Q_{m1} = \left(\frac{\Sigma n_i Q_i^5}{\Sigma n_i}\right)^{0{,}2}$$

Q_i is the gross weight in kN of lorry i in the slow lane, as specified by the competent authority,

n_i is the number of lorries of gross weight Q_i in the slow lane, as specified by the competent authority.

The National Annex may give guidance on parameter λ_2.

In this example, the following values are adopted:

$$Q_{m1} = 260 \text{ kN}$$

$$N_{Obs} = 1 \cdot 10^6$$

Thus, λ_2 is:

$$\lambda_2 = (\frac{260}{480}) \cdot (\frac{1,0 \cdot 10^6}{0,5 \cdot 10^6})^{0,2} = 0,62$$

The factor for the design life of bridge λ_3 should be calculated as:

$$\lambda_3 = (\frac{t_{Ld}}{100})^{0,2}$$

where:

t_{Ld} is the design life of the bridge in years.

For the design life of 120 years, λ_3 is:

$$\lambda_3 = (\frac{120}{100})^{0,2} = 1,037$$

The value of λ_4 depends on the relative magnitude of stress range due to passage of FLM 3 in the second lane and is calculated as:

$$\lambda_4 = [1 + \frac{N_2}{N_1}(\frac{\eta_2 Q_{m2}}{\eta_1 Q_{m1}})^5 + \frac{N_3}{N_1}(\frac{\eta_3 Q_{m3}}{\eta_1 Q_{m1}})^5 + \cdots + \frac{N_k}{N_1}(\frac{\eta_k Q_{mk}}{\eta_1 Q_{m1}})^5]^{0,2}$$

where:

k is the number of lanes with heavy vehicles,
N_j is the number of lorries per year in lane j,
Q_{mj} is the average gross weight of the lorries in lane j,
η_j is the value of the influence line for the internal force that produces the stress range in the middle of lane j to be inserted in above equation with positive sign.

In this example for FLM 3 and two traffic lanes λ_4 is calculated as:

$$\lambda_4 = (1 + \frac{effect\ in\ line\ 2}{effect\ in\ line\ 1})^{0,2}$$

5.1.2 Design stress ranges – cross-section 1-1

According to Table E1.1, the ratio of ΔM in lane 2 to ΔM in lane 1 is 403/430 = 0,937. Thus, the value of λ_4 is:

$\lambda_4 = (1 + 0,937)^{0,2} = 1,14$

The damage equivalent factor λ is:

$\lambda = \lambda_1 \cdot \lambda_2 \cdot \lambda_3 \cdot \lambda_4 = 1,70 \cdot 0,62 \cdot 1,037 \cdot 1,14 = 1,25$

At cross-section 1-1, stress ranges $\Delta\sigma_p$ in the top and bottom flanges at their mid thickness according Table E1.3 are:

top flange 5,4 N/mm^2
bottom flange 11,2 N/mm^2

The design stress ranges are:

top flange $\Delta\sigma_{E,2} = \gamma_{Ff} \cdot \lambda \cdot \Phi_2 \cdot \Delta\sigma_p = 1,0 \cdot 1,25 \cdot 1,0 \cdot 5,4 = 6,8$ N/mm^2

bottom flange $\Delta\sigma_{E,2} = \gamma_{Ff} \cdot \lambda \cdot \Phi_2 \cdot \Delta\sigma_p = 1,0 \cdot 1,25 \cdot 1,0 \cdot 11,2 = 14,0$ N/mm^2

According to Table 3.1, EN 1993-1-9, the partial factor for fatigue strength for the safe life assessment method and high consequences of failure is recommended as:

$\gamma_{Mf} = 1,35$

The determination of the appropriate category for the actual structural detail, the bearing plate welded to the underside of the bottom flange, is as follows.

According to Table 8.5, EN 1993-1-9, constructional detail 6 can be applied to the bearing plate welded to the underside of the bottom flange, see Figure E1.6. For a flange plate over 50 mm thick, the worst detail category is category 36.

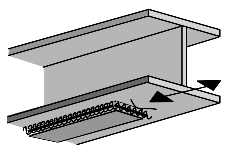


Figure E1.6 *Detail category 36*

- **Verification - plate welded to the underside of the bottom flange (detail category 36)**

The value of reference fatigue strength for detail category 36 is:

$\Delta\sigma_C = 36$ N/mm^2

Accordingly, the design fatigue strength is:

$\Delta\sigma_C / \gamma_{Mf} = 36 / 1,35 = 26$ N/mm^2

Check:

$$\gamma_{Ff} \cdot \Delta\sigma_{E,2} \leq \frac{\Delta\sigma_C}{\gamma_{Mf}}$$

Check for top flange:

$6,8 < 26$, the condition is satisfied

For bottom flange:

$14,0 < 26$, the condition is satisfied

5.1.3 Design stress ranges – cross-section 2-2

According to Table E1.1, the ratio of ΔM in lane 2 to ΔM in lane 1 is 646/680 = 0,950. The value of λ_4 is:

$\lambda_4 = (1 + 0,950)^{0,2} = 1,14$

The damage equivalent factor λ is:

$\lambda = \lambda_1 \cdot \lambda_2 \cdot \lambda_3 \cdot \lambda_4 = 2,35 \cdot 0,62 \cdot 1,037 \cdot 1,14 = 1,72$

At the cross-section 2-2, stress range $\Delta\sigma_p$ in top and bottom flanges at their mid thickness according Table E1.3 are:

top flange 8,6 N/mm^2
bottom flange 17,7 N/mm^2

The design stress ranges are:

top flange $\Delta\sigma_{E,2} = \gamma_{Ff} \cdot \lambda \cdot \Phi_2 \cdot \Delta\sigma_p = 1,0 \cdot 1,72 \cdot 1,0 \cdot 8,6 = 14,8$ N/mm^2

bottom flange $\Delta\sigma_{E,2} = \gamma_{Ff} \cdot \lambda \cdot \Phi_2 \cdot \Delta\sigma_p = 1,0 \cdot 1,72 \cdot 1,0 \cdot 17,7 = 30,4$ N/mm^2

The determination of the appropriate category for the actual structural detail, the bolted splice, is as follows.

According to Table 8.1, EN 1993-1-9, constructional detail 8 can be applied for a bolted splice, see Figure E1.7. For a bolted splice at the bolt holes, the worst detail category is category 112.

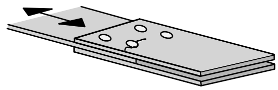


Figure E1.7 *Detail category 112*

- **Verification - bolted splice at the bolt holes (detail category 112)**

The value of reference fatigue strength for detail category 112 is:

$$\Delta\sigma_C = 112 \text{ N/mm}^2$$

Accordingly, the design fatigue strength is:

$$\Delta\sigma_c / \gamma_{Mf,s} = 112 / 1,35 = 83 \text{ N/mm}^2$$

Check:

$$\gamma_{Ff} \cdot \Delta\sigma_{E,2} \leq \frac{\Delta\sigma_C}{\gamma_{Mf}}$$

Check for top flange:

$14,8 < 83$, the condition is satisfied

For bottom flange:

$30,4 < 83$, the condition is satisfied

Further, it is necessary to determine the categories of other actual structural details. The determination of the appropriate categories for the actual structural details, the butt weld, welds with a cope hole, machining of the flange, full cross-section butt weld, is as follows.

According to Table 8.3, EN 1993-1-9, constructional detail 11 can be applied for a flange butt weld, see Figure E1.8. For a transverse splice in the flange, the worst detail category is category 80.

For a welded splice, a flange butt weld, category 80 is selected, and for t = 60 mm > 25 mm, with size effect factor $k_s = (25/t)^{0,2} = (25/60)^{0,2} = 0,84$.

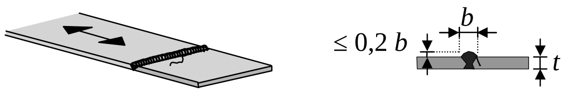
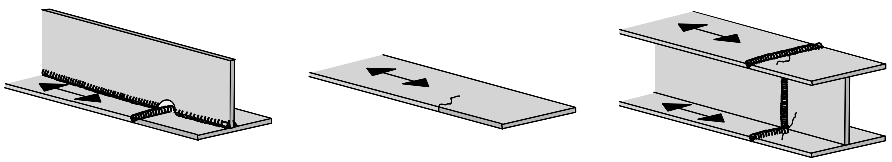


Figure E1.8 *Detail category 80*

According to Table 8.2, EN 1993-1-9, constructional detail 9 can be applied for a flange butt weld with a cope hole not greater than 60 mm in the flange, see Figure E1.9a. For a flange butt weld with a cope hole not greater than 60 mm, the corresponding detail category is category 71.

According to Table 8.1, EN 1993-1-9, constructional detail 5 can be applied for a web with machine gas cut edges, subsequently dressed to remove discontinuities, see Figure E1.9b. For such structural detail with appropriate stress concentration factor k_f = 2,4 according to [46], the corresponding detail category is category 125.

According to Table 8.3, EN 1993-1-9, constructional detail 3 can be applied for a full cross-section butt weld of rolled section, see Figure E1.9c. For this structural detail, the corresponding detail category is category 112.



a) 71, detail 9	b) 125, detail 5	c) 112, detail 3
EN 1993-1-9 Table 8.2	EN 1993-1-9 Table 8.1	EN 1993-1-9 Table 8.3

Figure E1.9 *Detail categories*

- **Verification - flange butt (detail category 80)**

The value of reference fatigue strength for detail category 80 is:

$$\Delta\sigma_C = 80 \ \text{N/mm}^2$$

Accordingly, the design fatigue strength, corrected with size effect factor $k_s = 0,84$ is:

$$\Delta\sigma_C / \gamma_{Mf} = 80 \cdot 0,84 / 1,35 = 50 \text{ N/mm}^2$$

Check:

$$\gamma_{Ff} \cdot \Delta\sigma_{E,2} \leq \frac{\Delta\sigma_C}{\gamma_{Mf}}$$

$30,4 < 50$, the condition is satisfied

- **Verification - flange at cope (detail category 71)**

The value of reference fatigue strength for detail category 71 is:

$$\Delta\sigma_C = 71 \text{ N/mm}^2$$

Accordingly, the design fatigue strength, corrected with size effect factor $k_s = 0,84$ is:

$$\Delta\sigma_C / \gamma_{Mf} = 71 \cdot 0,84 / 1,35 = 44 \text{ N/mm}^2$$

Check:

$$\gamma_{Ff} \cdot \Delta\sigma_{E,2} \leq \frac{\Delta\sigma_C}{\gamma_{Mf}}$$

$30,4 < 44$, the condition is satisfied

- **Verification - web at cope (detail category 125)**

The value of reference fatigue strength for detail category 125 is:

$$\Delta\sigma_C = 125 \text{ N/mm}^2$$

Accordingly, the design fatigue strength is:

$$\Delta\sigma_C / \gamma_{Mf} = 125 / 1,35 = 92,5 \text{ N/mm}^2$$

The design stress range corrected with the stress concentration factor k_f is:

$$k_f \cdot \Delta\sigma_{E,2} = 2,4 \cdot 30,4 = 73 \ \text{N/mm}^2$$

Check:

$$\gamma_{Ff} \cdot \Delta\sigma_{E,2} \leq \frac{\Delta\sigma_C}{\gamma_{Mf}}$$

$73 < 92,5$, the condition is satisfied

- **Verification – web butt at cope (detail category 112)**

The value of reference fatigue strength for detail category 112 is:

$$\Delta\sigma_C = 112 \ \text{N/mm}^2$$

Accordingly, the design fatigue strength is:

$$\Delta\sigma_C / \gamma_{Mf} = 112 / 1,35 = 83 \ \text{N/mm}^2$$

The design stress range corrected with the stress concentration factor k_f is:

$$k_f \cdot \Delta\sigma_{E,2} = 2,4 \cdot 30,4 = 73 \ \text{N/mm}^2$$

Check:

$$\gamma_{Ff} \cdot \Delta\sigma_{E,2} \leq \frac{\Delta\sigma_C}{\gamma_{Mf}}$$

$73 < 83$, the condition is satisfied

In cases where a transverse web stiffener is attached to the bottom flange, according to Table 8.4, EN 1993-1-9, the detail category would be 80, see Figure E1.10.

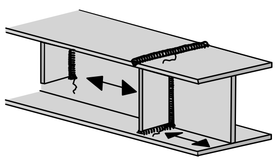


Figure E1.10 *Detail category 80*

- **Verification – transverse web stiffener (detail category 80)**

The value of reference fatigue strength for detail category 80 is:

$$\Delta\sigma_C = 80 \ \text{N/mm}^2$$

Accordingly, the design fatigue strength is:

$$\Delta\sigma_C \,/\, \gamma_{Mf} = 80 / 1{,}35 = 59 \ \text{N/mm}^2$$

Check:

$$\gamma_{Ff} \cdot \Delta\sigma_{E,2} \leq \frac{\Delta\sigma_C}{\gamma_{Mf}}$$

$30{,}4 < 59$, the condition is satisfied

5.1.4 Design stress ranges – cross-section 3-3

According to Table E1.1, the ratio of ΔM in lane 2 to ΔM in lane 1 is 762/800 = 0,953. The value of λ_4 is:

$$\lambda_4 = (1 + 0{,}953)^{0{,}2} = 1{,}14$$

The damage equivalent factor λ is:

$$\lambda = \lambda_1 \cdot \lambda_2 \cdot \lambda_3 \cdot \lambda_4 = 2{,}35 \cdot 0{,}62 \cdot 1{,}037 \cdot 1{,}14 = 1{,}72$$

At cross-section 3-3, the stress range in the top flange is negligible. The stress range in the bottom flange is found according to Table E1.3 is 25,6 N/mm^2.

The design stress range is:

bottom flange $\Delta\sigma_{E,2} = \gamma_{Ff} \cdot \lambda \cdot \Phi_2 \cdot \Delta\sigma_p = 1{,}0 \cdot 1{,}72 \cdot 1{,}0 \cdot 25{,}5 = 43{,}9 \ \text{N/mm}^2$

The determination of the appropriate category for the actual structural detail, the transverse web stiffener, is as follows.

According to Table 8.4, EN 1993-1-9, constructional detail 7 can be applied for a vertical stiffener welded to a plate girder. For the transverse web stiffener, the most onerous detail category is category 80.

- **Verification – transverse web stiffener (detail category 80)**

The value of reference fatigue strength for detail category 80 is:

$$\Delta\sigma_C = 80 \ \text{N/mm}^2$$

Accordingly, the design fatigue strength is:

$$\Delta\sigma_C / \gamma_{Mf} = 80 / 1{,}35 = 59 \ \text{N/mm}^2$$

Check:

$$\gamma_{Ff} \cdot \Delta\sigma_{E,2} \leq \frac{\Delta\sigma_C}{\gamma_{Mf}}$$

$43{,}9 < 59$, the condition is satisfied

5.2 Assessment of reinforcing steel

The fatigue assessment of the longitudinal reinforcement is carried out according to clause 6.8.5, EN 1992-1-1 as follows (the similar criterion as for the structural steel structure and therefore assuming the use of the fatigue load model FLM 3):

$$\gamma_{F,fat} \cdot \Delta\sigma_{S,equ}(N^*) \leq \frac{\Delta\sigma_{Rsk}(N^*)}{\gamma_{S,fat}}$$

where:

$N^* = 10^6$ cycles

$\Delta\sigma_{Rsk}$ = 162,5 N/mm^2	stress range for N^* cycles,
$\gamma_{F,fat}$ = 1,0	is the partial factor applied to the fatigue load model (FLM3),
$\gamma_{S,fat}$ = 1,15	is the partial factor for the material,
$\Delta\sigma_{S,equ} = \lambda_s \cdot \lvert \sigma_{s,\max,f} - \sigma_{s,\min,f} \rvert$	is the equivalent constant amplitude normal stress range in reinforcement,
λ_s	is the damage equivalent factor for reinforcement.

The equivalent stress range $\Delta\sigma_{S,equ}$ is referred to in EN 1994-2 as $\Delta\sigma_E$ and according to clause 6.8.6.1, EN 1994-2, it is given by:

$$\Delta\sigma_E = \lambda \cdot \Phi \cdot \lvert \sigma_{\max,f} - \sigma_{\min,f} \rvert$$

where:

$\sigma_{\max,f}$, $\sigma_{\min,f}$	are the maximum and minimum stresses determined in accordance with clauses 6.8.4 and 6.8.5, EN 1994-2,
λ	is the damage equivalent factor,
Φ	is the damage equivalent impact factor according to clause 6.8.6.1(7), EN 1994-2, for road bridges; this factor may be taken as equal to 1,0.

The value of $\lambda = \lambda_s$ and, according to clause NN2.1(102), EN 1992-2, λ_s is given by:

$$\lambda_s = \Phi_{fat} \cdot \lambda_{s,1} \cdot \lambda_{s,2} \cdot \lambda_{s,3} \cdot \lambda_{s,4}$$

where Φ_{fat} is the damage equivalent impact factor which can be taken for a road surface of good roughness as being equal to 1,2. It is important to note that the value Φ duplicated Φ_{fat}, but since is $\Phi = 1,0$ this is not significant.

Factor $\lambda_{s,1}$ takes into account the damage effects due to the traffic volume according to the length L of the influence line for the longitudinal bending moment. For intermediate support region and span of 30 m, according to Figure NN.1, clause NN 2.1, EN 1992-2, the value of factor $\lambda_{s,1}$ is:

$$\lambda_{s,1} = 0,97$$

The factor $\lambda_{s,2}$ takes into consideration the traffic volume and should be calculated as:

$$\lambda_{s,2} = \overline{Q}\sqrt[k_2]{\frac{N_{Obs}}{2,0}}$$

where:

N_{Obs}	is the total number of lorries per year in the slow lane, according to clause 4.6.1(3), Table 4.5, EN 1991-2. Intermediate values of N_{Obs} are not excluded.
k_2	is the slope of appropriate S-N curve according to Tables 6.3N and 6.4N, EN 1992-1-1.
$\overline{Q}$	is the factor for traffic type, according to Table NN.1, EN 1992-2.

In this example, the following values are adopted:

$N_{Obs} = 1 \cdot 10^6$ heavy vehicles per year and per slow lane, Table 4.5, EN 1991-2

$\overline{Q} = 0,94$, medium distance traffic and k_2 =9

Thus $\lambda_{s,2}$ is:

$$\lambda_{s,2} = \overline{Q} \cdot \sqrt[k_2]{\frac{N_{Obs}}{2,0 \cdot 10^6}} = 0,94 \cdot \sqrt[9]{\frac{1,0 \cdot 10^6}{2,0 \cdot 10^6}} = 0,87$$

The factor $\lambda_{s,3}$ takes into consideration the design life of bridge and should be calculated as:

$$\lambda_{s,3} = \sqrt[k_2]{\frac{N_{Years}}{100}}$$

where N_{Years} is the design life of the bridge in years.

Note:

According to EN 1993-2 the design life of the bridge is marked with t_{Ld}.

The design life of bridge in this example is chosen as 120 years so that $\lambda_{s,3}$ is:

$$\lambda_{s,3} = \sqrt[k_2]{\frac{N_{Years}}{100}} = \sqrt[9]{\frac{120}{100}} = 1,02$$

The factor $\lambda_{s,4}$ takes into account the effects of the heavy traffic on the other slow lanes defined in the design and it can be calculated as:

$$\lambda_{s,4} = \sqrt[k_2]{\frac{\Sigma N_{Obs,i}}{N_{Obs,1}}}$$

where:

$N_{obs,i}$ is the number of lorries expected on lane i per year,
$N_{obs,1}$ is the number of lorries on the slow lane per year.

In the case of two slow lanes with the same traffic, $\lambda_{s,4}$ is:

$$\lambda_{s,4} = \sqrt[k_2]{\frac{\Sigma N_{Obs,i}}{N_{Obs,1}}} = \sqrt[9]{\frac{2,0\cdot 10^6}{1,0\cdot 10^6}} = 1,08$$

The damage equivalent factor λ_s is:

$$\lambda_s = \Phi_{fat} \cdot \lambda_{s,1} \cdot \lambda_{s,2} \cdot \lambda_{s,3} \cdot \lambda_{s,4} = 1,2\cdot 0,97\cdot 0,87\cdot 1,02\cdot 1,08 = 1,12$$

The check of reinforcements under fatigue is dealt with for the cross-section at internal support – cross-section 1-1. The stress calculations in the reinforcing steel bars are therefore calculated based on the cracked section properties. The stress in the top bars due to permanent load is 110 N/mm^2. The stress range due to the fatigue load model, FLM 3, in lane 1 is from 0 to 10,2 N/mm^2, Table E1.3.

Remark:

Since the calculation of these values is very comprehensive, in this example we will only show the obtained values relevant for fatigue assessment.

According to clause NN.2.1(101), EN 1992-2, to determine the equivalent stress range for verification of reinforcing steel, the axle loads of FLM 3 should be multiplied by the following factors:

- for verification in regions of internal supports by 1,75,
- for verification in other regions by 1,40.

In this example the calculation is carried out for verification in the region of internal support, the stress range due to fatigue load model, FLM 3, in lane 1 should be multiplied with 1,75 giving a range of 18 N/mm^2, (1,75·10,2 ≈18 N/mm^2).

Therefore, neglecting tension stiffening, the maximal stress in reinforcement is:

$\sigma_{\max,f}$ = 110+18 = 128 N/mm^2

Since FLM 3 does not cause sagging bending minimal stress in reinforcing steel, the minimum stress in the top bars is only due to permanent actions. Therefore, the minimum stress in reinforcement is:

$\sigma_{\min,f}$ = 110 N/mm^2

To determine the effect of tension stiffening, the following parameters are needed:

- Cross-section of structural steel girder at internal support**,** Figure E1.4:

Cross-sectional area: $A_a = 73720 \text{ mm}^2$

Second moment of area $I_a = 1{,}734 \cdot 10^{10} \text{ mm}^4$

- Cross-section of reinforcement bar at internal support**,** Figure E1.4:

Cross sectional area: $A_s = 24216 \text{ mm}^2$

- Cracked composite cross-section – hogging region**,** Figure E1.4:

Cross-sectional area: $A = 97936 \text{ mm}^2$
Second moment of area: $I = 2{,}692 \cdot 10^{10} \text{ mm}^4$

The reinforcement ratio is:

$$\rho_s = A_s \,/\, A_{ct} = 24216 \,/\, (3700 \cdot 250) = 0{,}0262$$

and

$$\alpha_{st} = AI \,/\, A_a I_a = (97936 \cdot 2{,}692 \cdot 10^{10}) \,/\, (73720 \cdot 1{,}734 \cdot 10^{10}) = 2{,}06$$

For concrete C40/50, the mean value of tensile strength is:

$$f_{ctm} = 3{,}5 \text{ N/mm}^2$$

According to clause 6.8.5.4(1), EN 1994-2, with the factor 0,2, the effect of tension stiffening is:

$$\Delta\sigma_s = \frac{0{,}2 \cdot f_{ctm}}{\alpha_{st} \cdot \rho_s} = \frac{0{,}2 \cdot 3{,}5}{2{,}06 \cdot 0{,}0262} = 13 \text{ N/mm}^2$$

Thus, the maximum and minimum stresses with tension stiffening are:

$$\sigma_{s,\max,f} = \sigma_{\max,f} + \Delta\sigma_s = 128 + 13 = 141 \text{ N/mm}^2$$

$$\sigma_{s,\min,f} = \sigma_{s,\max,f} \cdot \frac{M_{Ed,\min,f}}{M_{Ed,\max,f}}$$

The minimum stress with tension stiffening can be calculated using the ratio of stresses:

$$\sigma_{s,\min,f} = \sigma_{s,\max,f} \cdot \frac{\sigma_{\min,f}}{\sigma_{\max,f}}$$

$$\sigma_{s,\min,f} = 141 \cdot \frac{110}{128} = 121 \text{ N/mm}^2$$

The equivalent stress range is:

$$\Delta\sigma_E = \lambda \cdot \Phi \cdot |\,\sigma_{\max,f} - \sigma_{\min,f}\,|$$

$$\Delta\sigma_E = 1{,}12 \cdot 1{,}0 \cdot |\,141 - 121\,| = 1{,}12 \cdot 20 = 22{,}4 \text{ N/mm}^2$$

The design value of stress range in the reinforcing steel is:

$$\gamma_{Ff} \cdot \Delta\sigma_{S,equ} = 1{,}0 \cdot 22{,}4 = 22{,}4 \text{ N/mm}^2$$

According to Table 6.3.N, EN 1992-1-1, the value of $\Delta\sigma_{Rsk}$ for straight bars is:

$$\Delta\sigma_{Rsk} = 162{,}5 \text{ N/mm}^2$$

Thus, the design value is:

$$\frac{\Delta\sigma_{Rsk}}{\gamma_{S,fat}} = \frac{162{,}5}{1{,}15} = 141 \text{ N/mm}^2$$

Check:

$$\gamma_{Ff} \cdot \Delta\sigma_{S,equ} \le \frac{\Delta\sigma_{Rsk}}{\gamma_{S,fat}}$$

$22{,}4 < 141$, the condition is satisfied

Note:

For the bent bars, such as at abutment, the value of $\Delta\sigma_{Rsk}$ is significantly reduced, see Table 6.3.N, EN 1992-1-1.

5.3 Assessment of shear connection

5.3.1 General

Fatigue verification of the stud connectors is carried out according to clause 6.8.7.2, EN 1994-2, as follows:

$$\gamma_{Ff} \cdot \Delta\tau_{E,2} \leq \frac{\Delta\tau_C}{\gamma_{Mf,s}}$$

The design value of the shear stress range in shear studs is calculated as:

$$\gamma_{Ff} \cdot \Delta\tau_{E,2}$$

The equivalent constant shear stress range $\Delta\tau_{E,2}$ at $2 \cdot 10^6$ cycles is given by:

$$\Delta\tau_{E,2} = \lambda_v \cdot \Delta\tau$$

where $\Delta\tau$ is the shear stress range in the cross-section of the stud with nominal value of stud diameter d.

The recommended value of the partial factor for action $\gamma_{Ff} = 1{,}0$.

The value of the damage equivalent factor should be determined in accordance with clause 6.8.6.2(3), EN 1994-2:

$$\lambda_v = \lambda_{v,1} \cdot \lambda_{v,2} \cdot \lambda_{v,3} \cdot \lambda_{v,4}$$

According to clause 6.8.6.2 (4), EN 1994-2, for road bridges of the span up to 100 m the factor $\lambda_{v,1} = 1{,}55$.

The values of factors $\lambda_{v,2}$, $\lambda_{v,3}$ and $\lambda_{v,4}$ are calculated in the same manner as for structural steel, but with an exponent of 0,125 rather than 0,2. This is because the slope of the S-N curve for shear studs is $m = 8$.

The factor $\lambda_{v,2}$ can be calculated as:

$$\lambda_{v,2} = \left(\frac{Q_{m1}}{Q_0}\right) \cdot \left(\frac{N_{Obs}}{N_0}\right)^{0,125}$$

where:

$Q_0 = 480$ kN and $N_0 = 0{,}5 \cdot 10^6$

N_{Obs} is the total number of lorries per year in the slow lane, according to clause 4.6.1(3), Table 4.5, EN 1991-2; intermediate values of N_{Obs} are not excluded,

Q_{m1} is the average gross weight (kN) of the lorries in the slow lane obtained from the expression:

$$Q_{m1} = \left(\frac{\Sigma n_i Q_i^8}{\Sigma n_i}\right)^{0,125}$$

Q_i is the gross weight in kN of the lorry *i* in the slow lane as specified by the competent authority,

n_i is the number of lorries of gross weight Q_i in the slow lane as specified by the competent authority.

The National Annex may give guidance on parameter λ_2.

In this example, the following values are adopted:

$$Q_{m1} = 260 \text{ kN}$$

$$N_{Obs} = 1 \cdot 10^6$$

Thus, $\lambda_{v,2}$ is:

$$\lambda_{v,2} = \left(\frac{260}{480}\right) \cdot \left(\frac{1{,}0 \cdot 10^6}{0{,}5 \cdot 10^6}\right)^{0,125} = 0{,}591$$

The factor $\lambda_{v,3}$ should be calculated as follows:

$$\lambda_{v,3} = \left(\frac{t_{Ld}}{100}\right)^{0,125}$$

t_{Ld} is the design life of the bridge in years.

For the design life of 120 years, $\lambda_{v,3}$ is:

$$\lambda_{v,3} = \left(\frac{120}{100}\right)^{0,125} = 1{,}02$$

The value of $\lambda_{v,4}$ depends on the relative magnitude of stress range due to the passage of FLM 3 in the second lane and is calculated as:

$$\lambda_{v,4} = [1 + \frac{N_2}{N_1}(\frac{\eta_2 Q_{m2}}{\eta_1 Q_{m1}})^8 + \frac{N_3}{N_1}(\frac{\eta_3 Q_{m3}}{\eta_1 Q_{m1}})^8 + \cdots + \frac{N_k}{N_1}(\frac{\eta_k Q_{mk}}{\eta_1 Q_{m1}})^8]^{0,125}$$

where:

k is the number of lanes with heavy vehicles,
N_j is the number of lorries per year in lane j,
Q_{mj} is the average gross weight of the lorries in lane j,
j is the value of the influence line for the internal force that produces the stress range in the middle of lane j to be inserted in the equation with a positive sign.

In this example for FLM 3 and two traffic lanes $\lambda_{v,4}$ is calculated as:

$$\lambda_4 = (1 + \frac{effect\ in\ line\ 2}{effect\ in\ line\ 1})^{0,125}$$

5.3.2 *Design shear stress – cross-section 1-1*

According to Table E1.2, the ratio of ΔV in lane 2 to ΔV in lane 1 is 238/275 = 0,865. Thus, the value of $\lambda_{v,4}$ is:

$$\lambda_{v,4} = (1 + 0,865)^{0,125} = 1,081$$

The damage equivalent factor λ_v is:

$$\lambda_v = \lambda_{v,1} \cdot \lambda_{v,2} \cdot \lambda_{v,3} \cdot \lambda_{v,4} = 1,55 \cdot 0,591 \cdot 1,023 \cdot 1,081 = 1,013$$

The longitudinal shear flow for uncracked composite cross-section without reinforcement can be calculated as:

$$\frac{V_{Ed} \cdot A_c \cdot \overline{z}}{n \cdot I_y}$$

where:

V_{Ed} is the shear force,
A_c is the area of concrete flange,
$\overline{z}$ is the distance between the neutral axis and the centroidal axis of concrete section (mm),

n is the modular ratio,
I_y is the second moment of area of the effective cross-section of the member.

The shear stress range can be calculated as:

$$\Delta\tau_E = \frac{shear\ flow\ for\ cyclic\ loading}{(number\ of\ studs\ per\ unit\ length)\cdot(d^2\cdot\pi/4)}$$

At cross-section 1-1, the studs are in rows of two at 150 mm spacing. The diameter of shear studs is 19 mm.

For the considered section, Figure E1.4, the obtained value of longitudinal shear flow is:

$$\frac{V_{Ed}\cdot A_c\cdot\overline{z}}{n\cdot I_y} = 275\cdot\frac{950000\cdot 229}{6\cdot 4,438\cdot 10^{10}}\cdot 10^3 = 275\cdot 0,817 = 225 \text{ kN/m}$$

The stress range in a shear stud is:

$$\Delta\tau_E = \frac{shear\ flow\ for\ cyclic\ loading}{(number\ of\ studs\ per\ unit\ length)\cdot(d^2\cdot\pi/4)}$$

$$\Delta\tau_E = \frac{225000}{1/0,15\cdot 2\cdot(19^2\cdot\pi/4)} = 59,5 \text{ N/mm}^2$$

The equivalent constant shear stress range $\Delta\tau_{E,2}$ at $2\cdot 10^6$ cycles is:

$$\Delta\tau_{E,2} = \lambda_v\cdot\Delta\tau$$

$$\Delta\tau_{E,2} = 1,013\cdot 59,5 = 60,3 \text{ N/mm}^2$$

According to clause 6.8.3(3), EN 1994-2, the value of reference fatigue strength for a shear stud is:

$$\Delta\tau_C = 90 \text{ N/mm}^2$$

According to clause 2.4.1.2(6), EN 1994-2, the recommended value of the partial factor for fatigue strength of shear studs is:

$\gamma_{Mf,s}$ = 1,0

Accordingly, the design fatigue strength for the shear studs is:

$$\Delta\tau_C \,/\, \gamma_{Mf,s} = 90 / 1{,}0 = 90 \text{ N/mm}^2$$

Check:

$$\gamma_{Ff} \cdot \Delta\tau_{E,2} \leq \frac{\Delta\tau_C}{\gamma_{Mf,s}}$$

$60{,}3 < 90$, the condition is satisfied

Note:

According to clause 6.8.7.2(2), EN 1994-2, where the maximum stress in the steel flange to which stud connectors are welded is tensile, the interaction between shear stress range in the weld of a stud and the normal stress range in the steel flange must be verified as:

$$\frac{\gamma_{Ff} \cdot \Delta\sigma_{E,2}}{\Delta\sigma_c \,/\, \gamma_{Mf}} + \frac{\gamma_{Ff} \cdot \Delta\tau_{E,2}}{\Delta\tau_c \,/\, \gamma_{Mf,s}} \leq 1{,}3$$

Stresses which occur at the same time must be used in verification. However, the most onerous values of $\Delta\sigma_C$ and $\Delta\tau_C$ can be conservatively used. The most onerous detail category in the steel flange is $\Delta\sigma_C = 80$ N/mm^2.

Check:

$$\frac{1{,}0 \cdot 6{,}8}{80 / 1{,}0} + \frac{1{,}0 \cdot 60{,}3}{90 / 1{,}0} = 0{,}76 < 1{,}3$$, the condition is satisfied

5.3.3 Design shear stress range – cross-section 2-2

According to Table E1.2, the ratio of ΔV in lane 2 to ΔV in lane 1 is 93/102 = 0,912. Thus, the value of $\lambda_{v,4}$ is:

$$\lambda_{v,4} = (1 + 0{,}912)^{0{,}125} = 1{,}084$$

The damage equivalent factor λ_v is:

$$\lambda_v = \lambda_{v,1} \cdot \lambda_{v,2} \cdot \lambda_{v,3} \cdot \lambda_{v,4} = 1{,}55 \cdot 0{,}591 \cdot 1{,}023 \cdot 1{,}084 = 1{,}016$$

At the cross-section 2-2, the studs are in rows of two at 150 mm spacing. The diameter of shear studs is 19 mm.

For the considered section, Figure E1.4, the obtained value of longitudinal shear flow is:

$$\frac{V_{Ed} \cdot A_c \cdot \overline{z}}{n \cdot I_y} = 102 \cdot \frac{950000 \cdot 229}{6 \cdot 4,438 \cdot 10^{10}} \cdot 10^3 = 102 \cdot 0,817 = 83,3 \text{ kN/m}$$

The stress range in a shear stud is:

$$\Delta\tau_E = \frac{\textit{shear flow for cyclic loading}}{(\textit{number of studs per unit length}) \cdot (d^2 \cdot \pi / 4)}$$

$$\Delta\tau_E = \frac{83300}{1 / 0,15 \cdot 2 \cdot (19^2 \cdot \pi / 4)} = 22,0 \text{ N/mm}^2$$

The equivalent constant shear stress range $\Delta\tau_{E,2}$ at $2 \cdot 10^6$ cycles is:

$$\Delta\tau_{E,2} = \lambda_v \cdot \Delta\tau$$

$$\Delta\tau_{E,2} = 1,016 \cdot 22,0 = 22,4 \text{ N/mm}^2$$

According to clause 6.8.3(3), EN 1994-2, the value of reference fatigue strength for a shear stud is:

$$\Delta\tau_C = 90 \text{ N/mm}^2$$

According to clause 2.4.1.2(6), EN 1994-2, the recommended value of the partial factor for fatigue strength of the shear studs is:

$$\gamma_{Mf,s} = 1,0$$

Accordingly, the design fatigue strength for the shear studs is:

$$\Delta\tau_C / \gamma_{Mf,s} = 90 / 1,0 = 90 \text{ N/mm}^2$$

Check:

$$\gamma_{Ff} \cdot \Delta\tau_{E,2} \le \frac{\Delta\tau_C}{\gamma_{Mf,s}}$$

$22,4 < 90$, the condition is satisfied

5.3.4 Design shear stress range – cross-section 3-3

According to Table E1.2, the ratio of ΔV in lane 2 to ΔV in lane 1 is 72/80 = 0,800. Thus, the value of $\lambda_{v,4}$ is:

$$\lambda_{v,4} = (1 + 0,800)^{0,125} = 1,076$$

The damage equivalent factor λ_v is:

$$\lambda_v = 1,55 \cdot 0,591 \cdot 1,023 \cdot 1,076 = 1,008$$

At the cross-section 3-3, the studs are in rows of two at 150 mm spacing. The diameter of shear studs is 19 mm.

For the considered section, Figure E1.5, the obtained value of longitudinal shear flow is:

$$\frac{V_{Ed} \cdot A_c \cdot \overline{z}}{n \cdot I_y} = 80 \cdot \frac{950000 \cdot 179}{6 \cdot 3,362 \cdot 10^{10}} \cdot 10^3 = 80 \cdot 0,843 = 67,4 \text{ kN/m}$$

The stress range in a shear stud is:

$$\Delta\tau_E = \frac{shear\ flow\ for\ cyclic\ loading}{(number\ of\ studs\ per\ unit\ length) \cdot (d^2 \cdot \pi / 4)}$$

$$\Delta\tau_E = \frac{67400}{1/0,15 \cdot 2 \cdot (19^2 \cdot \pi / 4)} = 17,8 \text{ N/mm}^2$$

The equivalent constant shear stress range $\Delta\tau_{E,2}$ at $2 \cdot 10^6$ cycles is:

$$\Delta\tau_{E,2} = \lambda_v \cdot \Delta\tau$$

$$\Delta\tau_{E,2} = 1,015 \cdot 17,8 = 18,1 \text{ N/mm}^2$$

According to clause 6.8.3(3), EN 1994-2, the value of reference fatigue strength for a shear stud is:

$$\Delta\tau_C = 90 \text{ N/mm}^2$$

According to clause 2.4.1.2(6), EN 1994-2, the recommended value of the partial factor for fatigue strength of shear studs is:

$\gamma_{Mf,s} = 1,0$

Accordingly, the design fatigue strength for the shear studs is:

$$\Delta\tau_C \,/\, \gamma_{Mf,s} = 90 / 1,0 = 90 \ \text{N/mm}^2$$

Check:

$$\gamma_{Ff} \cdot \Delta\tau_{E,2} \leq \frac{\Delta\tau_C}{\gamma_{Mf,s}}$$

$18,1 < 90$, the condition is satisfied

5.3.5 Design shear stress – cross-section 4-4

According to Table E1.2, the ratio of ΔV in lane 2 to ΔV in lane 1 is 262/285 = 0,919. Thus, the value of $\lambda_{v,4}$ is:

$$\lambda_{v,4} = (1 + 0,919)^{0,125} = 1,085$$

The damage equivalent factor λ_v is:

$$\lambda_v = 1,55 \cdot 0,591 \cdot 1,023 \cdot 1,085 = 1,017$$

At the cross-section 4-4, the studs are in rows of two at 150 mm spacing. The diameter of shear studs is 19 mm.

For the considered section, Figure E1.5, the obtained value of longitudinal shear flow is:

$$\frac{V_{Ed} \cdot A_c \cdot \overline{z}}{n \cdot I_y} = 285 \cdot \frac{950000 \cdot 179}{6 \cdot 3,362 \cdot 10^{10}} \cdot 10^3 = 285 \cdot 0,843 = 240 \ \text{kN/m}$$

The stress range in shear stud is:

$$\Delta\tau_E = \frac{\textit{shear flow for cyclic loading}}{(\textit{number of studs per unit length}) \cdot (d^2 \cdot \pi / 4)}$$

$$\Delta\tau_E = \frac{240000}{1/0{,}15 \cdot 2 \cdot (19^2 \cdot \pi / 4)} = 63{,}5 \ \text{N/mm}^2$$

The equivalent constant shear stress range $\Delta\tau_{E,2}$ at $2 \cdot 10^6$ cycles is:

$$\Delta\tau_{E,2} = \lambda_v \cdot \Delta\tau$$

$$\Delta\tau_{E,2} = 1{,}017 \cdot 63{,}5 = 64{,}6 \ \text{N/mm}^2$$

According to clause 6.8.3(3), EN 1994-2, the value of reference fatigue strength for a shear stud is:

$$\Delta\tau_C = 90 \ \text{N/mm}^2$$

According to clause 2.4.1.2(6), EN 1994-2, the recommended value of the partial factor for fatigue strength of shear studs is:

$$\gamma_{Mf,s} = 1{,}0$$

Accordingly, the design fatigue strength for the shear studs is:

$$\Delta\tau_C / \gamma_{Mf,s} = 90 / 1{,}0 = 90 \ \text{N/mm}^2$$

Check:

$$\gamma_{Ff} \cdot \Delta\tau_{E,2} \leq \frac{\Delta\tau_C}{\gamma_{Mf,s}}$$

$64{,}6 < 90$, the condition is satisfied

6. Commentary

In accordance with EN 1994-2, the equivalent stress ranges simplified method was used for the fatigue calculations. This example shows the determination of the parameters involved in a fatigue verification: applied stress ranges, fatigue strength of details and damage equivalent factors. In order to assist a structural designer in choosing the correct detail category, this example illustrates how the details can be classified.

E2 Fatigue assessment for a composite beam of a floor structure

1. Purpose of example

One purpose of this example is to check the fatigue resistance of reinforcement and shear studs of a composite beam. A floor structure for a storage building consists of composite beams over two spans of 10 m that act compositely with the slab. The composite slab consists of profiled steel sheeting with an in-situ reinforced lightweight concrete topping. The characteristic imposed floor load is $q_k = 7{,}0$ kN/m^2. Also it is assumed that one transport vehicle (a forklift) moves on the floor along a fixed path. The actions are determined from the pattern of the vehicle's wheel loads. The static value of wheel loads is determined from permanent weights and hoisting loads, and the spectra of loads is used to define the appropriate combination factors and fatigue loads.

The example demonstrates the calculation of stresses and stress ranges for a composite structure using elastic global analysis with cracked concrete.

2. Static system, cross-section and actions

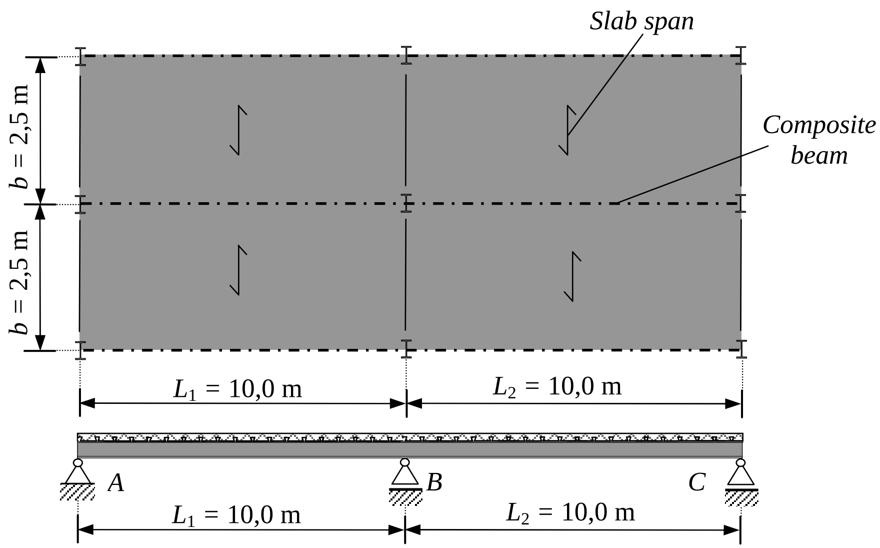
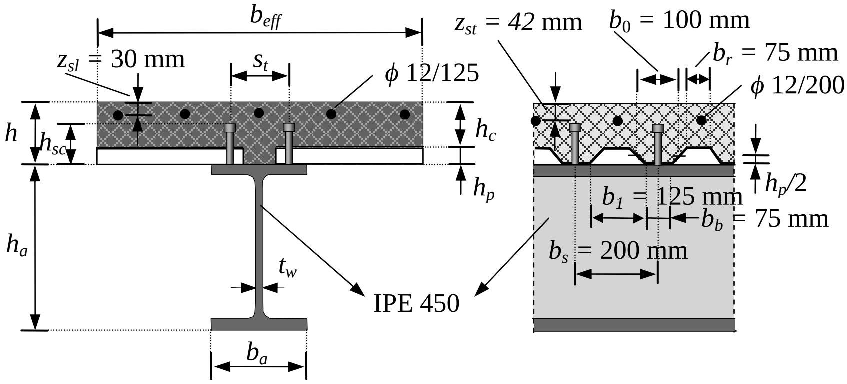


Figure E2.1 *Floor layout and static system*

Composite Structures according to Eurocode 4. Worked Examples.
First Edition. Darko Dujmović, Boris Androić, Ivan Lukačević.



$$h = 130 \text{ mm}, \ h_c = 80 \text{ mm}, \ h_p = 50 \text{ mm}$$

Figure E2.2 *Cross-section of composite beam*

Actions

a) Permanent action

Remark:

According to Table 11.1, clause 11.3, EN 1992-1-1 the density of the lightweight aggregate concrete of density class 1.8 and strength class LC25/28 is 18,5 kN/m^3. According to EN 1991-1-1 the density is increased by 1 kN/m^3 for normal percentage reinforcement, and increased for the wet concrete by an additional 1 kN/m^3.

Concrete slab area per m width:

$$A_c = 1000 \cdot h - \left(\frac{1000}{b_s} \cdot \frac{b_1 + b_r}{2} \cdot h_p\right)$$

$$A_c = 1000 \cdot 130 - \left(\frac{1000}{200} \cdot \frac{125 + 75}{2} \cdot 51\right) = 104500 \text{ mm}^2 = 1050 \text{ cm}^2$$

- *concrete slab and reinforcement (dry concrete)*

$$A_c \cdot 19{,}5 = 0{,}105 \cdot 19{,}5 = 2{,}05 \text{ kN/m}^2$$

Composite stage

- *concrete slab* $= 2{,}05$ kN/m^2

- *profiled steel sheeting* = 0,17 kN/m^2

- *steel beam* = 0,30 kN/m^2

Total $g_{k,1} = 2{,}52$ kN/m^2

Floor finishes $g_{k,2} = 1{,}00$ kN/m^2

b) Variable action

- *imposed floor load (category of use C3) and movable partitions* $q_k = 7{,}00$ kN/m^2

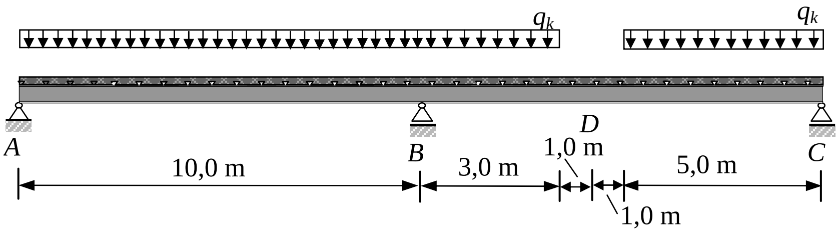


Figure E2.3 *Imposed load (non cyclic)*

c) Fatigue load

The four-wheeled forklift with two characteristic axle loads of 35 kN each is taken as the cyclic load (fatigue loading). The forklift moves along a fixed path 2,0 m wide. This path is free from other imposed loads and it is at right angles to the composite beam ABC, Figure E2.3.

The spacing of composite beams is 2,5 m and it is less than the axle spacing. Accordingly, each passage of the forklift can be represented by two cycles of point load, 0 – 35 – 0 kN. The load is applied at point D, Figure E2.4. The adopted design working life is 25 years. We assume 25 passages per hour and 4000 h/year. Thus, these data give $N_{Ed} = 2{,}5 \cdot 10^6$ cycles of each point load. The partial factor for fatigue load is $\gamma_{Ff} = 1{,}0$.

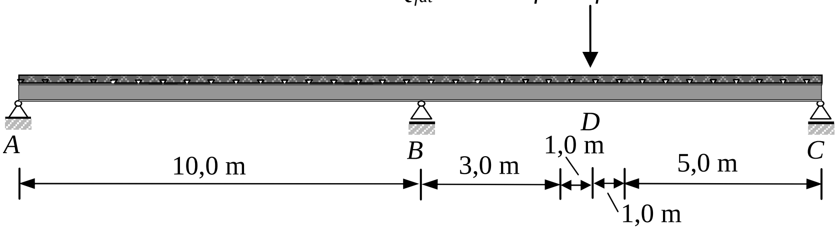


Figure E2.4 *Fatigue load*

3. Properties of materials

Concrete strength class: LC 25/28 $f_{ck} = 25 \text{ N/mm}^2$

$$f_{cd} = \frac{f_{ck}}{\gamma_c} = \frac{25}{1,5} = 16,7 \text{ N/mm}^2$$

$$0,85 f_{cd} = 0,85 \cdot 16,7 = 14,2 \text{ N/mm}^2$$

$$E_{cm} = 31000 \text{ N/mm}^2$$

$$f_{ctm} = 2,6 \text{ N/mm}^2$$

$$E_{lcm} = 20752 \text{ N/mm}^2$$

$$f_{lctm} = 2,32 \text{ N/mm}^2$$

Reinforcement: *ductility class B or C (Table C.1., EN 1992-1-1)* $f_{sk} = 500 \text{ N/mm}^2$

$$f_{sd} = \frac{f_{sk}}{\gamma_s} = \frac{500}{1,15} = 435 \text{ N/mm}^2$$

$$E_s = 210000 \text{ N/mm}^2$$

Structural steel: S355 $f_{yk} = 355 \text{ N/mm}^2$

$$f_{yd} = \frac{f_{yk}}{\gamma_M} = \frac{355}{1,0} = 355 \text{ N/mm}^2$$

$$E_a = 210000 \text{ N/mm}^2$$

Shear connectors: *ductile headed studs* $f_u = 500 \text{ N/mm}^2$

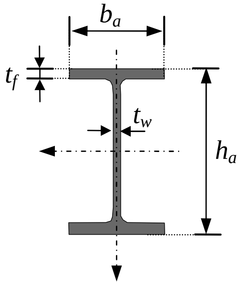
$$d = 19 \text{ mm}$$

$$h_{sc} = 95 \text{ mm}$$

$$\frac{h_{sc}}{d} = \frac{95}{19} = 5 > 4,0 \rightarrow \alpha = 1,0$$

4. Properties of the IPE 450 cross-section

The IPE 450 section is adopted with the cross-section and the dimensions shown in Figure E2.5.



$W_{pl,y} = 1702\ \text{cm}^3$
$W_{el,y} = 1500\ \text{cm}^3$
$A_a = 98{,}82\ \text{cm}^2$
$h_a = 450\ \text{mm}$
$b_a = 190\ \text{mm}$
$t_w = 9{,}4\ \text{mm}$
$t_f = 14{,}6\ \text{mm}$
$r = 21\ \text{mm}$
$I_{y,a} = 33740\ \text{cm}^4$
$I_{z,a} = 1676\ \text{cm}^4$
$I_{w,a} = 791000\ \text{cm}^6$
$I_{t,a} = 66{,}87\ \text{cm}^4$
$g = 77{,}6\ \text{kg/m}$

Figure E2.5 *Cross-section and dimensions of IPE 450*

5. Effective widths of concrete flange

Effective widths of concrete flange are taken from example B8:

- for the mid-span region $b_{eff} = 2{,}23\ \text{m}$
- for the internal support region $b_{eff} = 1{,}35\ \text{m}$
- at the end support $b_{eff} = 1{,}84\ \text{m}$

6. Classification of composite cross-section

The classification of the composite cross-section was done in example B8. The cross-section was classified as class 2.

7. Flexural properties of elastic cross-section

The flexural properties were calculated in example B8 and the obtained results are given in Table E2.1.

Table E2.1 *Elastic section properties of the composite cross-section*

Cross-section	Modular ratio	b_{eff} (m)	*Neutral axis*[1] (mm)	*Neutral axis*[2] (mm)	I_y (10^6 mm^4)
1) Support, cracked, reinforced	—	1,35	36	—	452
2) Mid-span, uncracked	10,1	2,23	202	113	966
3) Mid-span, uncracked	20,2	2,23	148	167	799
4) Mid-span, uncracked	28,3	2,23	123	192	720

[1] *The distance between the neutral axis and the centroidal axis of steel section*
[2] *The distance between the neutral axis and the centroidal axis of concrete section*

8. Global analysis

8.1 Introductory considerations

The calculation of internal forces and moments should be carried out by elastic global cracked analysis in accordance with clauses 5.4.1 and 5.4.2, EN 1994-1-1 and for combination of actions given in clause 6.8.3, EN 1992-1-1.

The effects of cracking are taken into account by using the flexural stiffness of cracked section over 15% of the span on each side of the internal support, and the flexural stiffness of uncracked section elsewhere.

The effects of creep and shrinkage of concrete are taken into account by using corresponding modular ratio:

n_0 = 10,1 for fatigue load

n = 20,2 for permanent and variable action

n = 28,3 for shrinkage

8.2 Calculation of bending moment at support B

Permanent load on composite beam

The continuous beam is unpropped at the construction stage. Only the load from floor finishes is relevant for the calculation of stress ranges. Thus, the characteristic load from floor finishes $g_{k,2}$ = 1,0 kN/m^2 is taken into account. The calculation of bending moments and internal forces is performed using commercial software for the static system and the design load shown in Figure E2.6. The concrete is cracked at a length of 0,15 L on each side of internal support.

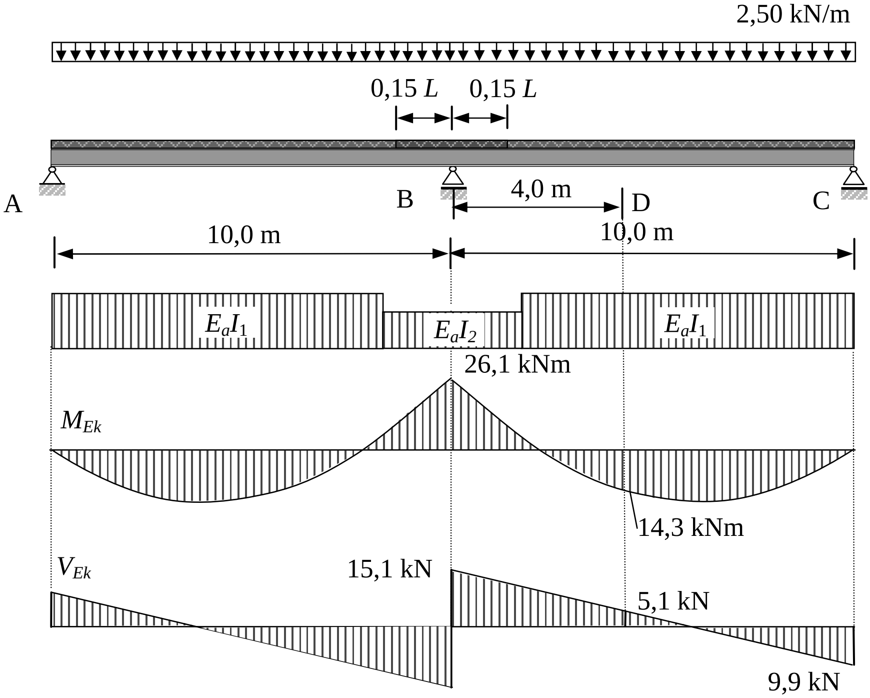


Figure E2.6 *Permanent action on the composite beam (weight of floor finishes only)*

Second moments of area are given in Table E2.1:

$I_1 = 799 \cdot 10^6$ mm^4, mid-span, uncracked, n = 20,2
$I_2 = 452 \cdot 10^6$ mm^4, support, cracked, reinforced

Therefore, the corresponding flexural stiffnesses are:

$$E_a I_1 = 167790 \text{ kNm}^2$$
$$E_a I_2 = 94920 \text{ kNm}^2$$

The characteristic load from the floor finishes is:

$$g_k = b \cdot g_{k,2} = 2{,}5 \cdot 1{,}0 = 2{,}50 \text{ kN/m}$$

Bending moments and shear forces are shown in Figure E2.6.

Imposed load on composite beam

The continuous beam with cracked sections in hogging region and the characteristic imposed load are shown in Figure E2.7. The calculation of bending moments and shear forces is carried out for the characteristic imposed load q_k = 7,0 kN/m^2 acting on span AB and on 10 m only of span BC.

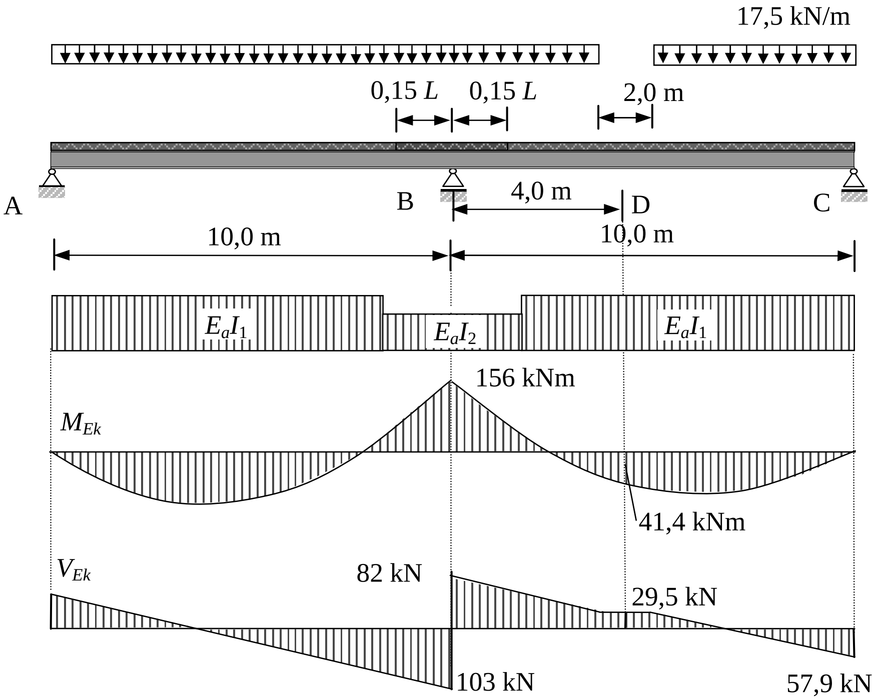


Figure E2.7 *Imposed load on composite beam (non cyclic)*

Second moments of area are given in Table E2.1:

$I_1 = 799 \cdot 10^6$ mm^4, mid-span, uncracked, n = 20,2
$I_2 = 452 \cdot 10^6$ mm^4, support, cracked, reinforced

Therefore, the corresponding flexural stiffnesses are:

$$E_a I_1 = 167790 \text{ kNm}^2$$
$$E_a I_2 = 94920 \text{ kNm}^2$$

The characteristic load from floor finishes is:

$$q_k = b \cdot q_k = 2{,}5 \cdot 7{,}0 = 17{,}5 \text{ kN/m}$$

Bending moments and shear forces are shown in Figure E2.7.

Shrinkage

The secondary effect of shrinkage in this beam, the hogging bending moment at internal support, is calculated as follows.

The primary bending moment due to shrinkage is calculated in accordance with the model given in Figure B8.43, Example B8. The primary bending moment due to shrinkage is:

$$M_{cs} = 137 \text{ kNm}$$

The second moment of area for the cracked section at internal support I_2 and the second moment of area for uncracked section I_1 at mid-span with the modular ratio n = 28,3 are given in Table E2.1, and they are:

$I_1 = 45200 \text{ cm}^4$ and $I_1 = 72000 \text{ cm}^4$

Therefore, the corresponding flexural stiffnesses are:

$$E_a I_1 = 151200 \text{ kNm}^2$$
$$E_a I_2 = 94920 \text{ kNm}^2$$

In the region of the cracked concrete, the shrinkage effects are neglected and the primary bending moments act on the beam at the points shown in Figure E2.8.

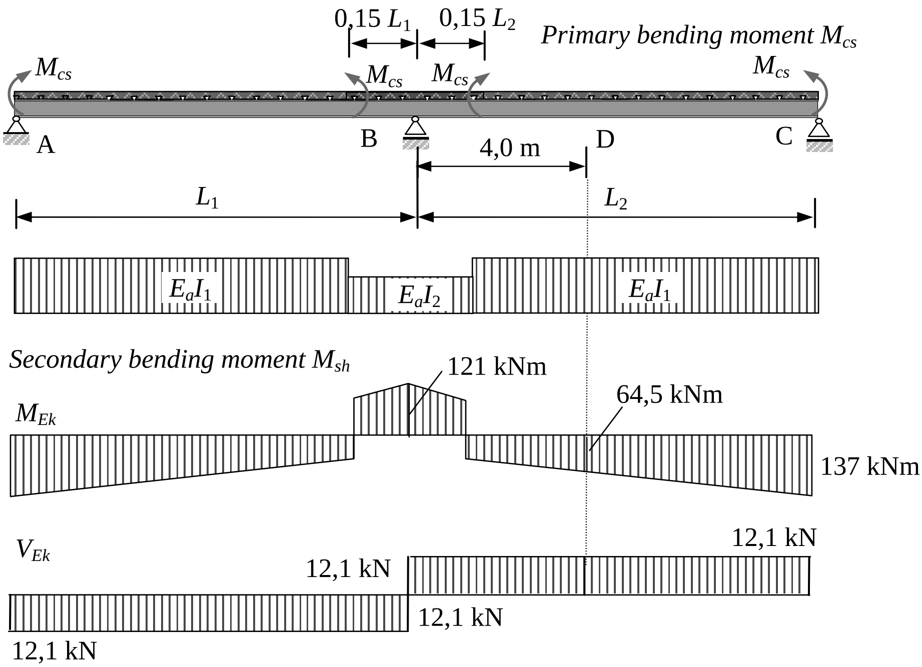


Figure E2.8 *Calculation of secondary bending moment due to shrinkage*

The secondary sagging bending moment due to shrinkage at point B is:

$$M_{Ek,sh,B} = 121 \text{ kNm}$$

Fatigue load

The analysis is carried out for the fatigue load Q_{fat} alone, with 15% of each span cracked, as shown in Figure E2.9. The second moments of area are given in Table E2.1:

$I_2 = 452 \cdot 10^6 \text{ mm}^4$, support, cracked, reinforced
$I_1 = 966 \cdot 10^6 \text{ mm}^4$, mid-span, uncracked, $n_0 = 10{,}1$

Therefore, the corresponding flexural stiffnesses are:

$$E_aI_1 = 202860 \text{ kNm}^2$$
$$E_aI_2 = 94920 \text{ kNm}^2$$

Bending moments and shear forces are shown in Figure E2.9.

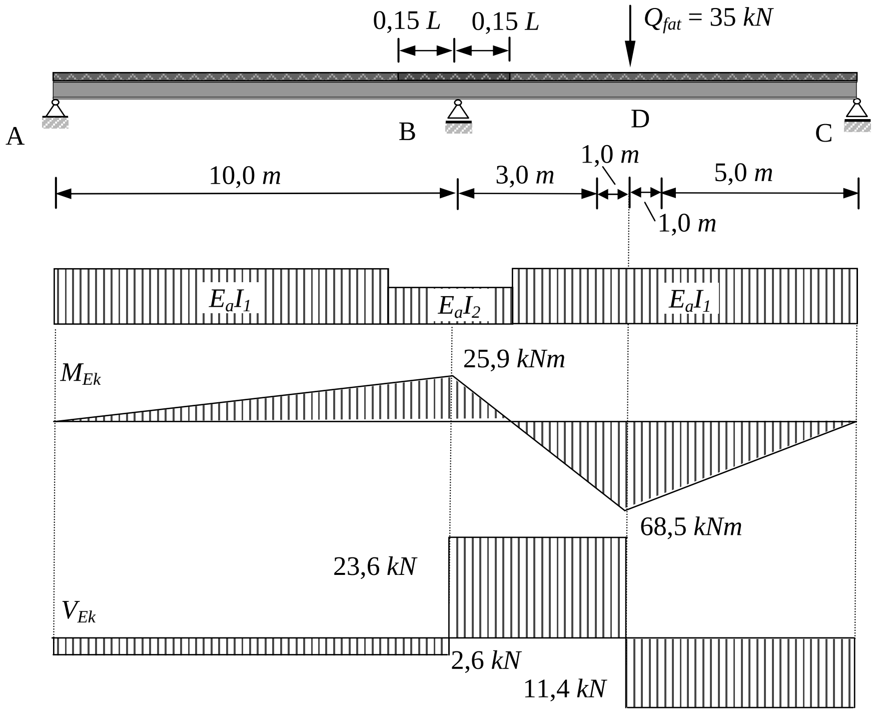


Figure E2.9 *Fatigue load and bending moments on the composite beam*

The characteristic values of bending moments at support B calculated for particular action are given in Table E2.2.

Table E2.2 *Characteristic values of hogging bending moments at support B*

Action	*Load* (kN/m)	Q_{fat} (kN)	Modular ratio	I_1 (10^6 mm^4)	I_2 (10^6 mm^4)	$M_{Ek,B}$ (kNm)
Permanent (on composite beam)	2,5	—	20,2	799	452	26,1
Variable, imposed load, static	17,5	—	20,2	799	452	156
Shrinkage	—	—	28,3	720	452	121
Cyclic load (fatigue)	—	35,0	10,1	966	452	25,9

9. Fatigue assessment

9.1 General

Clause 6.8.4(1), EN 1994-1-1, refers to clause 6.8.3, EN 1992-1-1 for the combination of actions. According to clause 6.8.3(3), EN 1992-1-1, the cyclic action should be combined with the unfavourable basic combination. This combination of actions, called the basic combination plus the cyclic action, can be expressed as:

$$\sum_{j>=1} G_{k,j} \text{"+"} P \text{"+"} \psi_{1,1} \cdot Q_{k,1} \text{"+"} \sum_{j>1} \psi_{2,i} \cdot Q_{k,i} \text{"+"} Q_{fat}$$

where $Q_{k,1}$ and $Q_{k,i}$ are non-cyclic variable actions and Q_{fat} is the cyclic load. In this case, only one G and one Q are relevant and there is no pre-stress, P = 0. Accordingly, the design combination is:

$$G_k + \psi_1 \cdot Q_k + Q_{fat}$$

In this combination, it is assumed that the non-cyclic loading co-exists with the design value of cyclic load. Then, it is the frequent combination.

Therefore, the frequent combination of variable actions (non-cyclic) with the combination factor of ψ_1 = 0,7 is considered. From Table E2.2, the characteristic value of imposed load is q_k = 17,5 kN/m.

Therefore, the load acting on span AB and on 10,0 m only of span BC is $\psi_1 \cdot q_k$ = 0,7·17,5 = 12,25 kN/m. In this case, the characteristic value of bending moment at support B is:

$$M_{Ek,B} = \psi_1 \cdot M_{Ek,B}$$

$$M_{Ek,B} = 0{,}7 \cdot 156 = 109 \text{ kNm}$$

The characteristic values of bending moments of the permanent action, due to shrinkage and of the cyclic load are given in Table E2.2.

The minimum design bending moment is calculated as:

$$M_{Ed,\min,f} = \gamma_{Ff} \cdot M_{Ek,B}^{permanent,composite} + \gamma_{Ff} \cdot M_{Ek,B}^{imposed} + \gamma_{Ff} \cdot M_{Ek,B}^{shrinkage}$$

The minimum design bending moment is:

$$M_{Ed,\min,f} = 1,0 \cdot 26,1 + 1,0 \cdot 109 + 1,0 \cdot 121 = 256 \text{ kNm}$$

The maximum design bending moment is calculated as:

$$M_{Ed,\max,f} = M_{Ed,\min,f} + \gamma_{Ff} \cdot M_{Ek,B}^{cyclic}$$

The maximum design bending moment is:

$$M_{Ed,\max,f} = 256 + 1,0 \cdot 25,9 = 282 \text{ kNm}$$

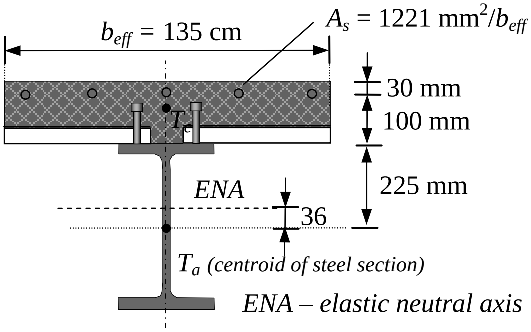


Figure E2.10 *Cross-section at internal support, cracked concrete is neglected*

The tensile stresses in the longitudinal reinforcement at support B are calculated for the cross-section shown in Figure E2.10, as:

$$\sigma_{s,0} = \frac{M_{Ek,B}}{W_{s,cr}}$$

The values of hogging bending moments at support B $M_{Ek,B}$, for the characteristic values of actions, are given in Table E2.3.

The section modulus, longitudinal reinforcement at support, $W_{s,cr}$, is calculated according Figure E2.10:

$$W_{s,cr} = \frac{452}{(225 + 100 - 36)} \cdot 10^6 = 1,56 \cdot 10^6 \text{ mm}^3$$

The obtained tensile stresses in the longitudinal reinforcement at support B are given in Table E2.3.

Table E2.3 *Stresses in longitudinal reinforcement at support B*

Action	n	load (kN/m)	$M_{Ek,B}$ (kNm)	$W_{s,cr}$ (10^6 mm^3)	$\sigma_{s,0}$ (N/mm^2)
Permanent, (on composite beam)	20,2	2,50	26,1	1,56	16,7
Imposed (ψ_1 = 0,7)	20,2	12,25	109	1,56	69,9
Shrinkage	28,3	—	121	1,56	77,6
Cyclic load	10,1	—	25,9	1,56	16,6
Total			282		180,8

9.2 Verification for reinforcement at cross-section B

According to clause 6.8.5.4 (1), EN 1994-1-1, the effects of tension stiffening can be taken into account by the simplified procedure that is used for other limit states. Thus, the maximum tensile stress in the fully cracked section $\sigma_{s,0}$ is increased by $\Delta\sigma_s$, that is independent of $\sigma_{s,0}$ and of the limit state.

The correction of the stress in the reinforcement for tension stiffening as (7.5), EN 1994-1-1, is:

$$\Delta\sigma_s = \frac{0,4 \cdot f_{ctm}}{\alpha_{st} \cdot \rho_s}$$

where:

f_{ctm} is the mean tensile strength of the concrete, for normal concrete taken as f_{ctm} from Table 3.1, EN 1992-1-1, or for lightweight concrete as f_{lctm} from Table 11.3.1, EN 1992-1-1.
ρ_s is the reinforcement ratio, given by ρ_s=(A_s/A_{ct}),
A_{ct} is the effective area of the concrete flange within the tensile zone; for simplicity the area of the concrete section within the effective width should be used,
A_s is the total area of all layers of longitudinal reinforcement within the effective area A_{ct},
$\alpha_{st} = \dfrac{A \cdot I}{A_a \cdot I_a}$, with appropriate cross-section properties.

In equation 7.5, EN 1994-1-1, the factor 0,2 should be used in place of the factor 0,4.

In accordance with Figure E2.10, the following values of properties of cross-sections are obtained:

$$A = A_a + A_s = 9880 + 1221 = 11101 \text{ mm}^2$$

$$I_y = (I_a + A_a \cdot z_{na}^2 + A_s \cdot (225 + 100 - z_{na})^2) \cdot 10^6$$

$$I_y = (337,4 + 9880 \cdot 0,036^2 + 1221 \cdot (0,225 + 0,100 - 0,036)^2) \cdot 10^6 = 452 \cdot 10^6 \text{ mm}^4$$

Thus, α_{st} is:

$$\alpha_{st} = \frac{A \cdot I}{A_a \cdot I_a} = \frac{11101 \cdot 452 \cdot 10^6}{9880 \cdot 337,4 \cdot 10^6} = 1,51$$

The reinforcement ratio ρ_s is:

$$\rho_s = \frac{A_s}{A_{ct}} = \frac{1221}{80 \cdot 1600} = 0,0095$$

The correction of the stress in the reinforcement for tension stiffening $\Delta\sigma_s$ is:

$$\Delta\sigma_s = \frac{0,4 \cdot f_{lctm}}{\alpha_{st} \cdot \rho_s}$$

$$\Delta\sigma_s = \frac{0,4 \cdot 2,32}{1,51 \cdot 0,0095} = 64,7 \text{ N/mm}^2$$

However, according to clause 6.8.5.4(1), EN 1994-1-1, in the expression for $\Delta\sigma_s$ a factor 0,2 should be used in place of the factor 0,4. Therefore, the correction of the stress in the reinforcement for tension stiffening $\Delta\sigma_s$ is:

$\Delta\sigma_s$ = 0,2/0,4·64,7 = 32,4 N/mm^2

With the value of the total stress $\sigma_{s,0}$ = 180,8 N/mm^2 from Table E2.3, the maximum stress in reinforcement $\sigma_{s,\max,f}$ is:

$$\sigma_{s,\max,f} = 180,8 + 32,4 = 213 \text{ N/mm}^2$$

The simplified rules for calculating stresses are given in clauses 6.8.5.4(2), EN 1994-1-1 and 6.8.5.4(3), EN 1994-1-1, with reference to Figure 6.26, EN 1994-1-1. The determination of the stresses $\sigma_{s,\max,f}$ and $\sigma_{s,\min,f}$ in the cracked region in accordance with Figure 6.26, EN 1994-1-1, is shown in Figure E2.11.

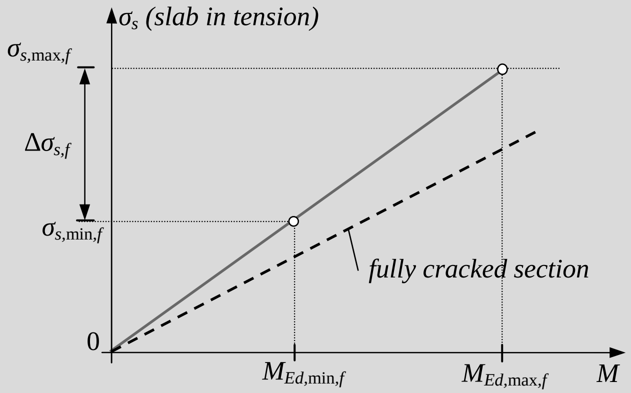


Figure E2.11 *Stress ranges in reinforcement in concrete cracked regions*

According to Figure E2.11, the minimum stress $\sigma_{s,\min,f}$ in the reinforcement due to $M_{Ed,\min,f}$ can be calculated as:

$$\sigma_{s,\min,f} = \sigma_{s,\max,f} \cdot \frac{M_{Ed,\min,f}}{M_{Ed,\max,f}}$$

In accordance with Figure E2.11, the minimum stress in the reinforcement is:

$$\sigma_{s,\min,f} = \sigma_{s,\max,f} \cdot \frac{M_{Ed,\min,f}}{M_{Ed,\max,f}}$$

$$\sigma_{s,\min,f} = 213 \cdot \frac{256}{282} = 193 \ \text{N/mm}^2$$

Thus, the stress range, $\Delta\sigma_{s,f}$, is:

$$\Delta\sigma_{s,f} = \sigma_{s,\max,f} - \sigma_{s,\min,f} = 213 - 193 = 20 \ \text{N/mm}^2$$

Clause 6.8.3(2), EN 1994-1-1, refers to clause 6.8.4 in EN 1992-1-1, which gives the verification procedure for reinforcing steel. In Table 6.3N, EN 1992-1-1, the recommended value N^* for straight bars is 10^6. The corresponding value for structural steel N_C is $2 \cdot 10^6$, according to EN 1993-1-1. This value is used also for shear connectors, clause 6.8.6.2(1), EN 1994-1-1.

For reinforcing steel, the fatigue assessment is carried out as (6.71), EN 1992-1-1:

$$\gamma_{F,fat} \cdot \Delta\sigma_{E,equ}(N^*) \leq \frac{\Delta\sigma_{Rsk}(N^*)}{\gamma_{s,fat}}$$

where:

$\Delta\sigma_{Rsk}(N^*)$	is the stress range at N^* cycles from the appropriate S-N curves given in Figure 6.30, EN 1992-1-1 or Table 6.3N, EN 1992-1-1,
$\Delta\sigma_{E,equ}(N^*)$	is the damage equivalent stress range for different types of reinforcement, and considering the number of loading cycles N^*. For building construction $\Delta\sigma_{s,equ}(N^*)$ may be approximated by $\Delta\sigma_{s,\max}$.
$\Delta\sigma_{s,\max}$	is the maximum steel stress range under the relevant load combinations,
$\gamma_{F,fat}$	is the partial factor for fatigue actions (the recommended value is 1,0),
$\gamma_{s,fat}$	is the partial factor for reinforcing or pre-stressing steel under fatigue loading (the recommended value is 1,15, Table 2.1N, EN 1992-1-1).

With the values of partial factors $\gamma_{F,fat}$ and $\gamma_{s,fat}$ recommended in EN 1992-1-1, the expression (6.71), EN 1992-1-1, can be shown as:

$$\Delta\sigma_{E,equ}(N^*) \leq \frac{\Delta\sigma_{Rsk}(N^*)}{1{,}15}$$

In cases where a stress range $\Delta\sigma_E(N_E)$ can be calculated, the resistance $\Delta\sigma_{Rsk}(N_E)$ can be determined from the S-N curve for reinforcing steel. The verification is carried out as:

$$\Delta\sigma_E(N_E) \leq \frac{\Delta\sigma_{Rsk}(N_E)}{1{,}15}$$

According to clause 6.8.4, EN 1992-1-1, the following values are adopted:

$m = 9$ for $N_E > 10^6$, $N^* = 10^6$, $\Delta\sigma_{Rsk}(N^*) = 162{,}5$ N/mm^2

The damage equivalent stress range can be determined according to Palmgren–Miner rule as follows:

$$(\Delta\sigma_{s,f})^m \cdot (N_{Ed}) = (\Delta\sigma_{E,equ})^m \cdot (N^*)$$

$$\Delta\sigma_{E,equ} = [\frac{(\Delta\sigma_{s,f})^m \cdot (N_{Ed})}{(N^*)}]^{1/m}$$

The damage equivalent stress range for $N_{Ed} = 2 \cdot 2{,}5 \cdot 10^6 = 5 \cdot 10^6$ cycles of stress range $\Delta\sigma_{s,f} = 20$ N/mm^2 is:

$$\Delta\sigma_{E,equ} = [\frac{(\Delta\sigma_{s,f})^m \cdot (N_{Ed})}{(N^*)}]^{1/m}$$

$$\Delta\sigma_{E,equ} = [\frac{(20)^9 \cdot (5 \cdot 10^6)}{(10^6)}]^{1/9} = 24 \text{ N/mm}^2$$

The verification is carried out as:

$$\Delta\sigma_E(N_E) \le \frac{\Delta\sigma_{Rsk}(N_E)}{1{,}15}$$

With $\Delta\sigma_E(N_E) = \Delta\sigma_{E,equ} = 24$ N/mm^2 and the resistance $\Delta\sigma_{Rsk}(N_E) = 162{,}5$ N/mm^2, we have:

$$24 \le \frac{162{,}5}{1{,}15}$$

24 N/mm^2 < 141,3 N/mm^2 the reinforcement is verified

9.3 Verification for shear connection near point D

The diagram of vertical shear forces due to the fatigue load is shown in Figure E2.9. Vertical shear is higher on the left of point D than on the right. From Figure E2.9 the value of shear force is $V_{Ed,f} = 23{,}6$ kN for each axle load.

The values of vertical shear for other actions are:

$V_{Ed} = 5{,}1$ kN, permanent load on composite beam, Figure E2.6

$V_{Ed} = \psi_1 \cdot 29{,}5 = 0{,}7 \cdot 29{,}5 = 20{,}7$ kN, imposed load on composite beam, Figure E2.7

$V_{Ed} = 12{,}1$ kN, shrinkage, Figure E2.8

The longitudinal shear flow for the uncracked composite cross-section without reinforcement can be calculated as:

$$\frac{V_{Ed} \cdot A_c \cdot \overline{z}}{n \cdot I_y}$$

where:

V_{Ed} is the shear force,
A_c is the area of concrete flange,
$\overline{z}$ is the distance between the neutral axis and the centroidal axis of concrete section (mm), Table E2.1,
n is the modular ratio,
I_y is the second moment of area of the effective cross-section of the member.

The obtained values of longitudinal shear flow are shown in Table E2.4.

Table E2.4 *The longitudinal shear per unit length (shear flow) near point D*

Action	Modular ratio	A_c (10^6 mm^2)	I_y (10^6 mm^4)	$\overline{z}$ (mm)	V_{Ed} (kN)	$\frac{V_{Ed} \cdot A_c \cdot \overline{z}}{n \cdot I_y}$ (kN/m)
Permanent on composite beam	20,2	0,2	799	167	5,1	10,6
Imposed $\psi_1 = 0{,}7$	20,2	0,2	799	167	20,7	42,8
Shrinkage	28,3	0,2	720	192	12,1	22,8
Cyclic load	10,1	0,2	966	113	23,6	54,7
Total					61,5	130,9

The maximum vertical shear at point D, including all actions from Table E2.4, is 61,5 kN.

The maximum longitudinal shear flow for the uncracked unreinforced composite cross-section is 130,9 kN/m. The longitudinal shear flow due to the cyclic load is 54,7 kN/m, Table E2.4.

According to Example B8, the shear connection is five shear studs per unit length, with P_{Rd} = 51,3 kN/per shear stud, Figure E2.12.

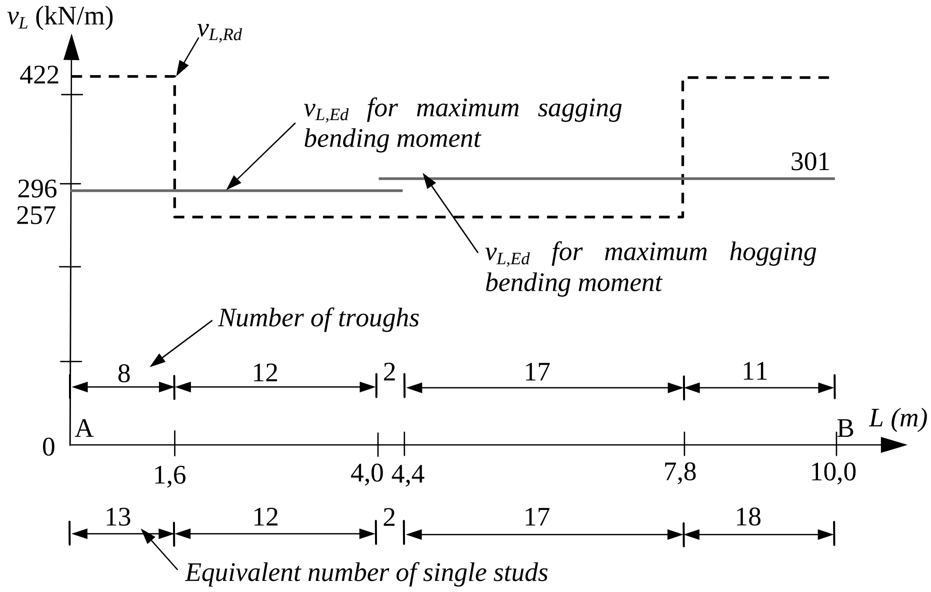


Figure E2.12 *Arrangement of stud connectors in one 10 m span*

According to clause 6.8.1(3), EN 1994-1-1, the maximum longitudinal shear force per connector should not exceed to $0{,}75 \cdot P_{Rd}$ for the characteristic combination of actions. For the characteristic combination, the shear flow from non-cyclic variable action is increased from 54,7 kN/m to $54{,}7/\psi_1 = 54{,}7/0{,}7 = 78{,}1$ kN/m. Thus, the increase is 23,4 kN/m.

The value of shear flow for the characteristic combination is P_{Ek} = 130,9 + 23,4 = 154,3 kN/m.

Accordingly, the ratio of P_{Ek} to P_{Rd} is:

$$P_{Ek} / P_{Rd} = 154{,}3 / (5 \cdot 51{,}3) = 0{,}60$$

The obtained value is below the limit, 0,75, given in clause 6.8.1(3), EN 1994-1-1.

The shear stress range can be calculated as:

$$\Delta\tau_E = \frac{shear\ flow\ for\ cyclic\ loading}{(number\ of\ studs\ per\ unit\ length) \cdot (d^2 \cdot \pi / 4)}$$

The shear stress range is:

$$\Delta\tau_E = \frac{54700}{5 \cdot 19^2 \cdot \pi / 4} = 38{,}6 \ \text{N/mm}^2$$

In the expression (6.50), EN 1994-1-1, $\Delta\tau_C$ is the reference value at $2 \cdot 10^6$ cycles with $\Delta\tau_C$ equal to 90 N/mm^2. According to clause 6.8.3(4), EN 1994-1-1, for studs in lightweight concrete with the density class 1.8, the reference value $\Delta\tau_C$ at $2 \cdot 10^6$ cycles is:

$$\Delta\tau_C = \eta_E \cdot \Delta\tau_C = (\rho / 2200)^2 \cdot \Delta\tau_C = (1{,}8 / 2{,}2)^2 \cdot 90 = 60 \ \text{N/mm}^2$$

The design cyclic loading consists of a single load cycle repeated N_E times. The damage equivalent factor λ_v used in clause 6.8.6.2, EN 1994-1-1 on shear connection is calculated using the Palmgren–Miner rule, as follows.

The load cycle causes a shear stress range $\Delta\tau$ in a stud connector and the slope of the fatigue strength curve $m = 8$. Then, the following expression can be given:

$$(\Delta\tau_E)^8 N_E = (\Delta\tau_{E,2})^8 \cdot N_C \ \text{ with } N_C = 2 \cdot 10^6 \text{ cycles}$$

Hence:

$$\Delta\tau_{E,2} / \Delta\tau_E = \lambda_v = (N_E / N_C)^{1/8}$$

From above expression, the equivalent constant range of shear stress $\Delta\tau_{E,2}$ for $2 \cdot 10^6$ cycles is given by:

$$\Delta\tau_{E,2} = \Delta\tau_E (N_E / N_C)^{1/8}$$

With $N_E = N_{Ed} = 2 \cdot 2{,}5 \cdot 10^6 = 5 \cdot 10^6$, the equivalent constant range of shear stress is:

$$\Delta\tau_{E,2} = 38{,}6 \cdot [5 \cdot 10^6 / (2 \cdot 10^6)]^{1/8} = 43{,}3 \ \text{N/mm}^2$$

For $\gamma_{Mf,s} = 1{,}0$, the design fatigue strength $\Delta\tau_C = 60$ N/mm^2. For the verification of shear connection, the following condition must be satisfied:

$$\Delta\tau_{E,2} \leq \Delta\tau_C$$

Check:

$$\Delta\tau_{E,2} \leq \Delta\tau_C$$

$43{,}3\ N/mm^2 < 60\ N/mm^2$ the shear connection is verified

10. Commentary

Fatigue verification is mainly needed for bridges, but this example demonstrates the application for buildings with composite floors on which a forklift is travelling.

Generally, fatigue problems are treated in EN 1993-1-9 in great detail. In EN 1993-1-9, the unified European rules for fatigue verifications are given. Fatigue in reinforcement and concrete is covered in EN 1992-1-1. For a building, fatigue of concrete is unlikely to influence design. Fatigue failure of a shear stud connector is covered by EN 1994-1-1.

In the analysis of the fatigue limit state the following effects should not be neglected:

- primary and secondary effects due to shrinkage and creep of concrete flange,
- effects of construction type,
- effects due to temperature.

For building structures, fatigue verification is not required except for specific cases. These specific cases are given in clauses of the particular EN as follows:

- for concrete, clauses 6.8.1 (1) and 6.8.1 (2), EN 1992-1-1. Fatigue verification should be carried out for crane rails, bridges subjected to high traffic loads, i.e. for structures and structural components that are subjected to cyclic loading. In these cases, verification is carried out separately for concrete and reinforcement.
- for structural steel, clause 4(4)B, EN 1993-1-1. There are several cases where fatigue should be considered: where members support cranes, vibrating machinery or rolling loads, and where members are subjected to wind-induced vibrations or crowd-induced oscillations.
- for composite structures, clause 6.8.1 (4), EN 1994-1-1. This clause gives guidance for types of building where fatigue assessment may be required. According to clause 4(4)B, EN 1993-1-1, this includes buildings where members support cranes, vibrating machinery or rolling loads, and where members are subjected to wind-induced vibrations or crowd-induced oscillations. In accordance with clause 6.8.1, EN 1992-1-1, this includes reinforcing steel and concrete components which are subjected to large numbers of repetitive loading cycles.

F Types of composite joints

F1 Beam to beam joints

Figure F1.1 illustrates four types of secondary beam to primary beam pinned joints.

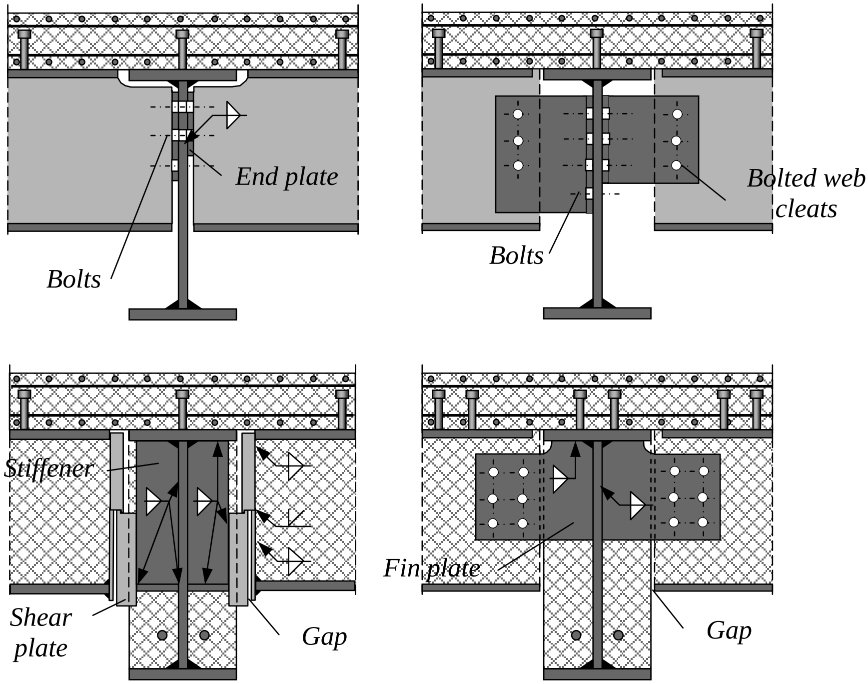


Figure F1.1 *Types of secondary to primary beam pinned joints*

In cases where speed of erection is an important consideration, the joint shown in Figure F1.2 can be used. In this case, the secondary beam is connected to the primary beam by means of the single extended end plate which is welded onto the cleat. The cleat is welded onto the top flange of the primary beam. The extended end plate is slotted and welded onto the cleat, so the transmission of tensile force from the secondary to the primary beam is ensured.

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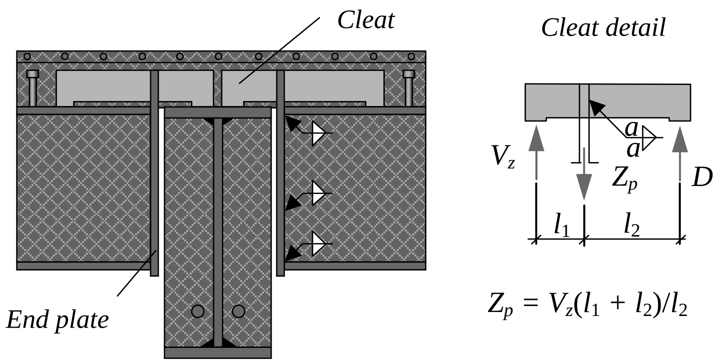

Figure F1.2 *Secondary to primary beam pinned joint*

Two variants of continuous beam to beam joints are shown in Figure F1.3. Continuity is achieved by means of cover plates welded onto the top flanges of the secondary beams and contact plates. Alternatively, the continuity can also be ensured by steel reinforcement in the concrete slab.

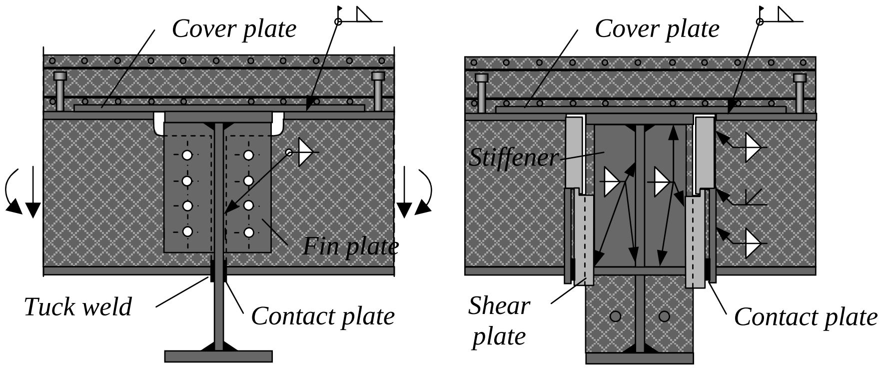

Figure F1.3 *Types of secondary to primary beam rigid joints (continuous)*

F2 Beam to column joints

Pinned joints of composite beams to steel or composite columns are often used in multi-storey buildings. Pinned joints are normally assumed to give vertical support and to be able to rotate without damage. Figure F2.1 illustrates a pinned joint of a composite beam to a steel column with angle cleats.

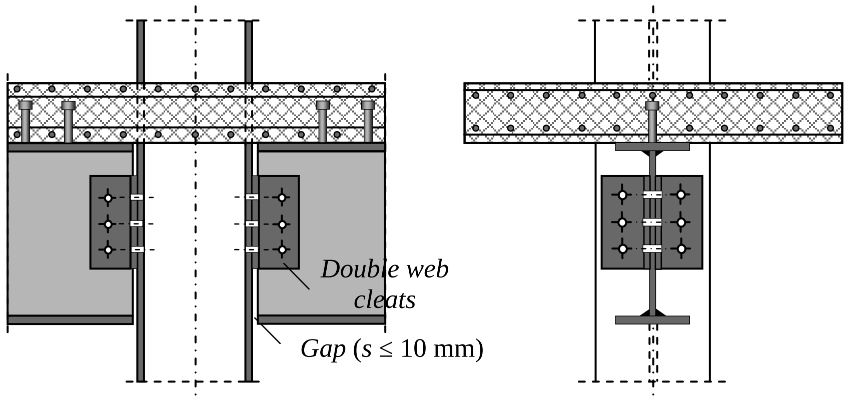

Figure F2.1 *Composite beam to steel column pinned joint with angle cleats*

The composite joint of partially encased composite beam to steel column is shown in Figure F2.2.

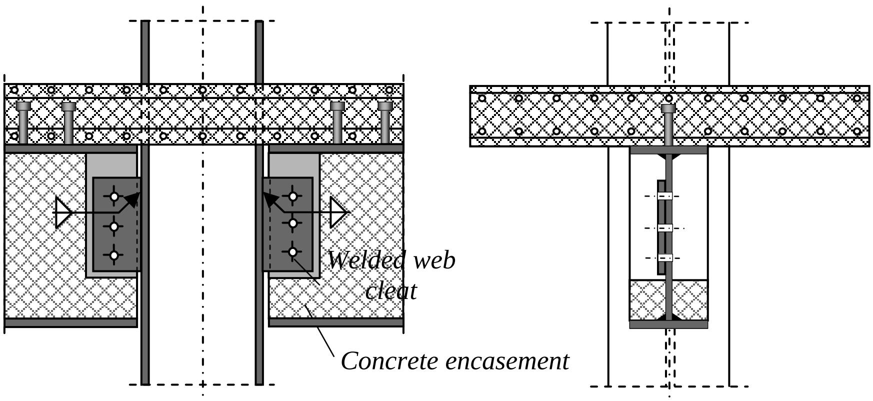

Figure F2.2 *Partially encased composite beam to steel column pinned joint with welded web cleat*

The joint shown in Figure F2.2 is a very economic joint. A seating cleat may be used to help erection. The web cleat is welded to the steel column in the workshop.

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Figure F2.3 illustrates the type of shear plate connection. This type of joint is particularly suitable where beam shear is high and/or speed of erection is an important consideration. The specific detail of the junction between the shear plate and the end plate ensures that the connection integrity is maintained.

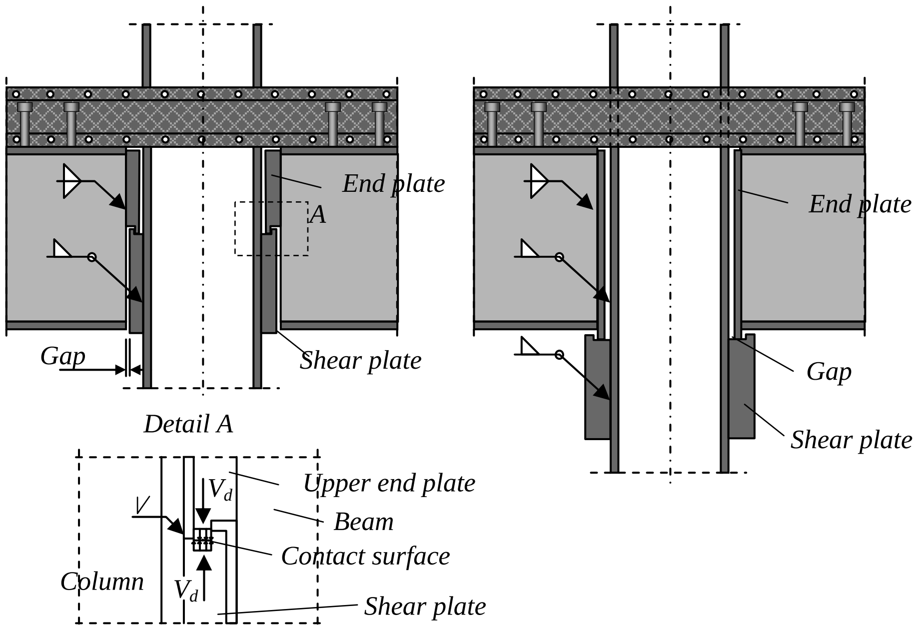


Figure F2.3 *Beam to column pinned joint*

Moment connections are those that are assumed to give vertical support, provide a degree of restraint against rotation and develop some moment capacity. Generally, joints are classified by stiffness (rigid, nominally pinned or semi-rigid) and by strength (full-strength, nominally pinned or partial-strength). Further, clause 5.1, EN 1993-1-8 defines the links between the types of global analysis and the types of models used for joints. In this way, it is possible to determine whether the resistance of the joint, its stiffness or both properties are relevant to the analysis.

Figure F2.4 illustrates the beam to column end plate joint. For frames where the depth of construction is limited and where stiffness rather than resistance governs design, the semi-rigid joints shown in Figure F2.4 are a very economical solution.

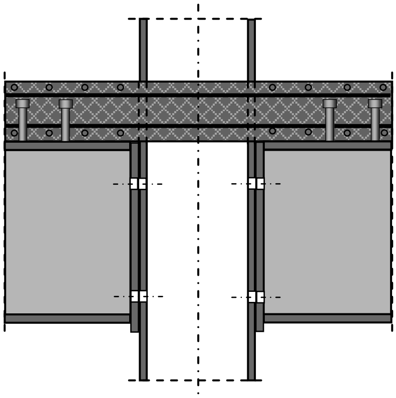
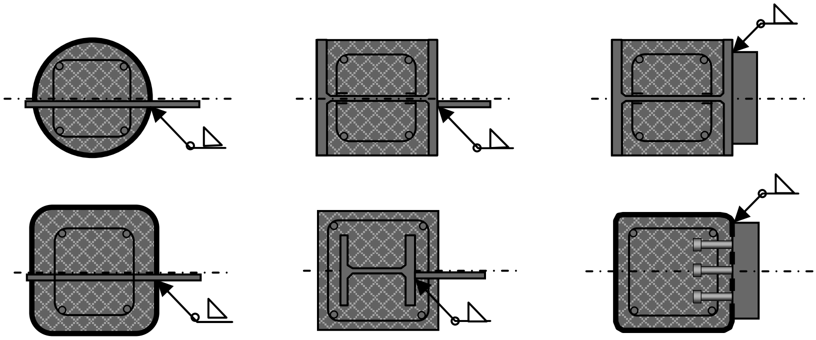


Figure F2.4 *Beam to column joint with bolted end plates*

Appropriate structural details ensure that the forces transmitted from a beam through the beam-column connection are distributed between the steel and concrete parts of the composite columns. Typical structural details are shown in Figure F2.5.



a) Through-plate passed through slots and welded to each face of CHS or RHS column

b) Welded fin plate

c) Support

Figure F2.5 *Typical structural details*

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