Chapter 4
Bearing Capacity of Soils

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General Outline

- Introduction
- Ultimate Bearing Capacity
- Additional Considerations for UBC
- Developments in UBC Equation
- Bearing Capacity from Field Test

1. Introduction

- Definition & Implication
- Importance
- Basic Concepts

Introduction

Foundation is a structure that transmits loads to underlying soils.

Footing is a foundation consisting of a small slab for transmitting the structural load to the underlying soil.

Embedment depth **(Df)** is the depth below the ground surface where the base of the foundation rests.

Shallow foundation is one in which the ratio of the embedment depth to the minimum plan dimension, which is usually the width (B), is $D_f/B \leq 2.5$.

Ultimate bearing capacity is the maximum pressure that the soil can support.

Ultimate net bearing capacity **(qu)** is the maximum pressure that the soil can support above its current overburden pressure.

Introduction cntd

***Ultimate gross bearing capacity* (qult)** is the sum of the ultimate net bearing capacity and the overburden pressure above the footing base.

Allowable bearing capacity or ***safe bearing capacity* (qa)** is the working pressure that would ensure a margin of safety against collapse of the structure from shear failure. The allowable bearing capacity is usually a fraction of the ultimate net bearing capacity.

Factor of safety or ***safety factor* (*FS*)** is the ratio of the ultimate net bearing capacity to the allowable net bearing capacity or to the applied maximum net vertical stress.

In geotechnical engineering, a factor of safety between 2 and 5 is used to calculate the allowable bearing capacity.

Introduction cntd

Loads from a structure are transferred to the soil through a foundation and a geotechnical engineers use the knowledge of the properties of soils and their response to loadings to design foundations.

A geotechnical engineer must ensure that a foundation satisfies the following two stability conditions:

1. The foundation must not collapse or become unstable under any conceivable loading. This is called ULS.

2. Settlement of the structure must be within tolerable limits so as not to impair the design function of the structure. This is called SLS.

2. Ultimate Bearing Capacity

- Earth Pressure Theory
- Slip Circle Methods
- Plastic Failure Theory
- Bearing Capacity Formula

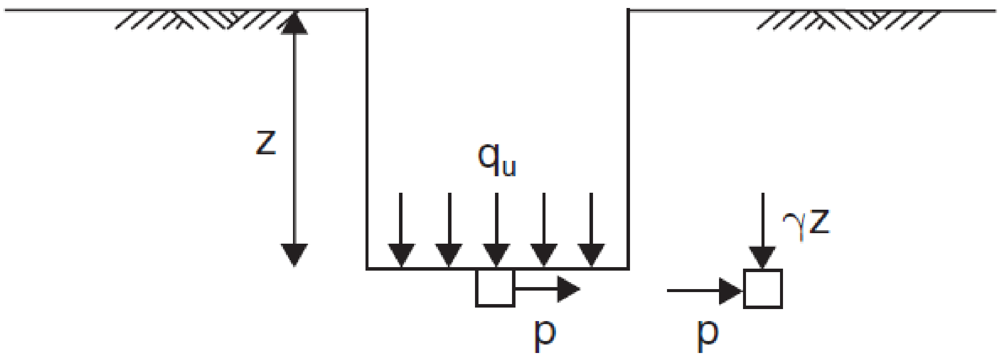
Ultimate Bearing Capacity

Earth Pressure Theory

Consider an element of soil under a foundation.

The vertical downward pressure of the footing, qu, is a major principal stress causing a corresponding Rankine active pressure, p.

For particles beyond the edge of the foundation this lateral stress can be considered as a major principal stress (i.e. passive resistance) with its corresponding vertical minor principal stress γz (the weight of the soil).



Ultimate Bearing Capacity cntd

$$p = q_u \frac{1 - \sin \emptyset'}{1 + \sin \emptyset'}$$

$$p = \gamma z \frac{1 + \sin \emptyset'}{1 - \sin \emptyset'}$$

$$q_u = \gamma z \left(\frac{1 + \sin \emptyset'}{1 - \sin \emptyset'} \right)^2$$

Obviously this is not satisfactory for shallow footings because when z = 0 then, according to the formula, qu also = 0.

Ultimate Bearing Capacity cntd

Bell's development of the Rankine solution for c–φ soils gives the following equation:

$$q_u = \gamma z\left(\frac{1+\sin\emptyset'}{1-\sin\emptyset'}\right)^2 + 2c'\sqrt{\left(\frac{1+\sin\emptyset'}{1-\sin\emptyset'}\right)^3} + 2c'\sqrt{\frac{1+\sin\emptyset'}{1-\sin\emptyset'}}$$

For, the undrained state, $\varphi_u = 0°$,

$$q_u = \gamma z + 4c_u$$

Or

$$q_u = 4c_u \; for\ surface\ footing.$$

Ultimate Bearing Capacity cntd

Slip Circle Methods

With slip circle methods the foundation is assumed to fail by rotation about some slip surface, usually taken as the arc of a circle.

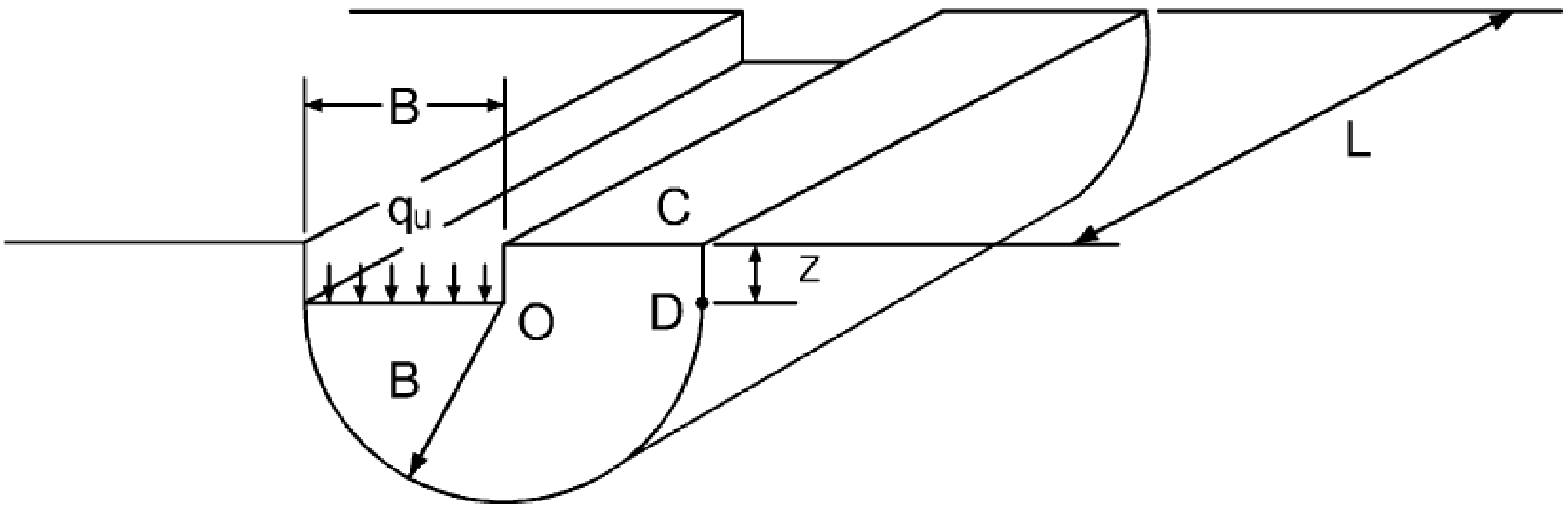
Almost all foundation failures exhibit rotational effects, and Fellenius (1927) showed that the centre of rotation is slightly above the base of the foundation and to one side of it.

He found that in a saturated cohesive soil the ultimate bearing capacity for a surface footing is

$$q_u = 5.52c_u$$

Ultimate Bearing Capacity cntd

Consider a foundation failing by rotation about one edge and founded at a depth z below the surface of a saturated clay of unit weight γ and undrained strength c_u



Ultimate Bearing Capacity cntd

Disturbing moment about O: $q_u \times LB \times \frac{B}{2} = \frac{q_u LB^2}{2}$

Resisting moments about O:

Cohesion along cylindrical sliding surface $= c_u \, \pi LB$

Moment $= \pi c_u \, LB^2$

Cohesion along CD $= c_u \, ZL$

Moment $= c_u \, ZLB$

Weight of the soil above the foundation $= \gamma ZLB$

Moment $= \frac{\gamma ZLB^2}{2}$

Ultimate Bearing Capacity cntd

For equilibrium

$$\frac{q_u L B^2}{2} = \pi c_u L B^2 + c_u Z L B + \frac{\gamma Z L B^2}{2}$$

$$q_u = 2\pi c_u + \frac{2 c_u z}{B} + \gamma Z$$

$$= 2\pi c_u \left(1 + \frac{1}{\pi}\frac{z}{B} + \frac{1}{2\pi}\frac{\gamma Z}{c_u}\right)$$

$$= 6.28 c_u \left(1 + 0.32\frac{z}{B} + 0.16\frac{\gamma Z}{c_u}\right)$$

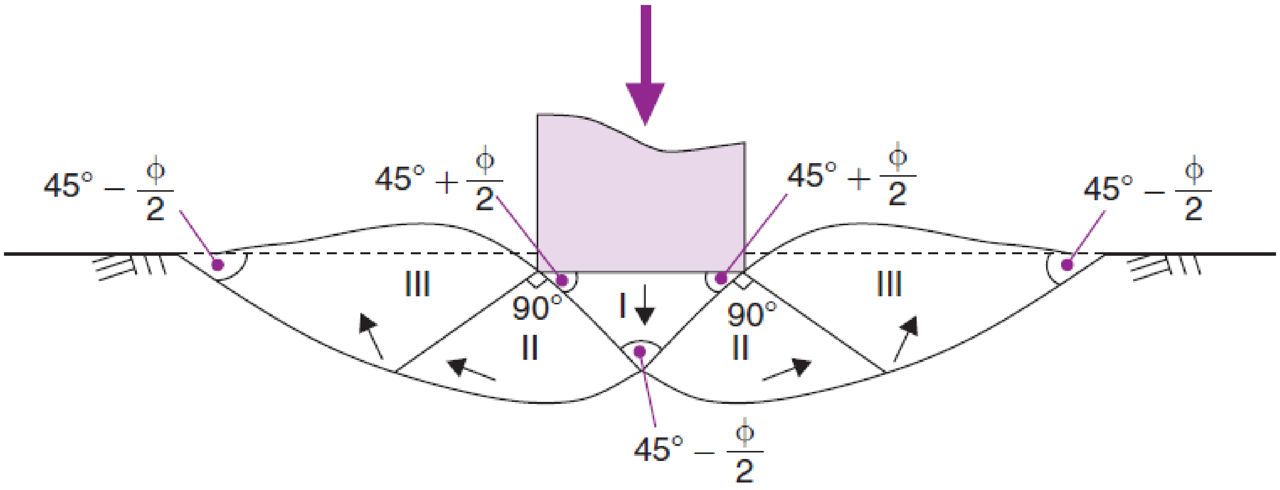
NB. This formula only applies to a strip footing, and if the foundation is of finite dimensions then the effect of the ends must be included.

Ultimate Bearing Capacity cntd

Plastic Failure Theory

Terzaghi (1943) stated that the bearing capacity failure of a foundation is caused by either a general soil shear failure or a local soil shear failure.

Vesic (1963) listed punching shear failure as a further form of bearing capacity failure.



Ultimate Bearing Capacity cntd

1) General shear failure

The failure pattern is clearly defined and it can be seen that definite failure surfaces develop within the soil.

A wedge of compressed soil (I) goes down with the footing, creating slip surfaces & areas of plastic flow (II). These areas are initially prevented from moving outwards by passive resistance of the soil wedges (III).

Once this passive resistance is overcome, movement takes place and bulging of the soil surface around the foundation occurs.

With general shear failure collapse is sudden and is accompanied by a tilting of the foundation.

Ultimate Bearing Capacity cntd

2) Local shear failure

The failure pattern developed is of the same form as for general shear failure but only the slip surfaces immediately below the foundation are well defined. Shear failure is local and does not create the large zones of plastic failure which develop with general shear failure.

Some heaving of the soil around the foundation may occur but the actual slip surfaces do not penetrate the surface of the soil and there is no tilting of the foundation.

Ultimate Bearing Capacity cntd

(3) Punching shear failure

This is a downward movement of the foundation caused by soil shear failure only occurring along the boundaries of the wedge of soil immediately below the foundation. There is little bulging of the surface of the soil and no slip surfaces can be seen.

For both punching and local shear failure, settlement considerations are invariably more critical than those of bearing capacity so that the evaluation of the ultimate bearing capacity of a foundation is usually obtained from an analysis of general shear failure.

Ultimate Bearing Capacity cntd

Prandtl's analysis

Prandtl (1921) was interested in the plastic failure of metals and one of his solutions (for the penetration of a punch into metal) can be applied to the case of a foundation penetrating downwards into a soil with no attendant rotation.

The analysis gives solutions for various values of φ, and for a surface footing with φ = 0, Prandtl obtained:

$$\boldsymbol{q_u = 5.14 c_u}$$

Ultimate Bearing Capacity cntd

Terzaghi's analysis

Terzaghi (1943) produced a formula for qu which allows for the effects of cohesion and friction between the base of the footing and the soil and is also applicable to shallow (z/B ≤ 1) and surface foundations.

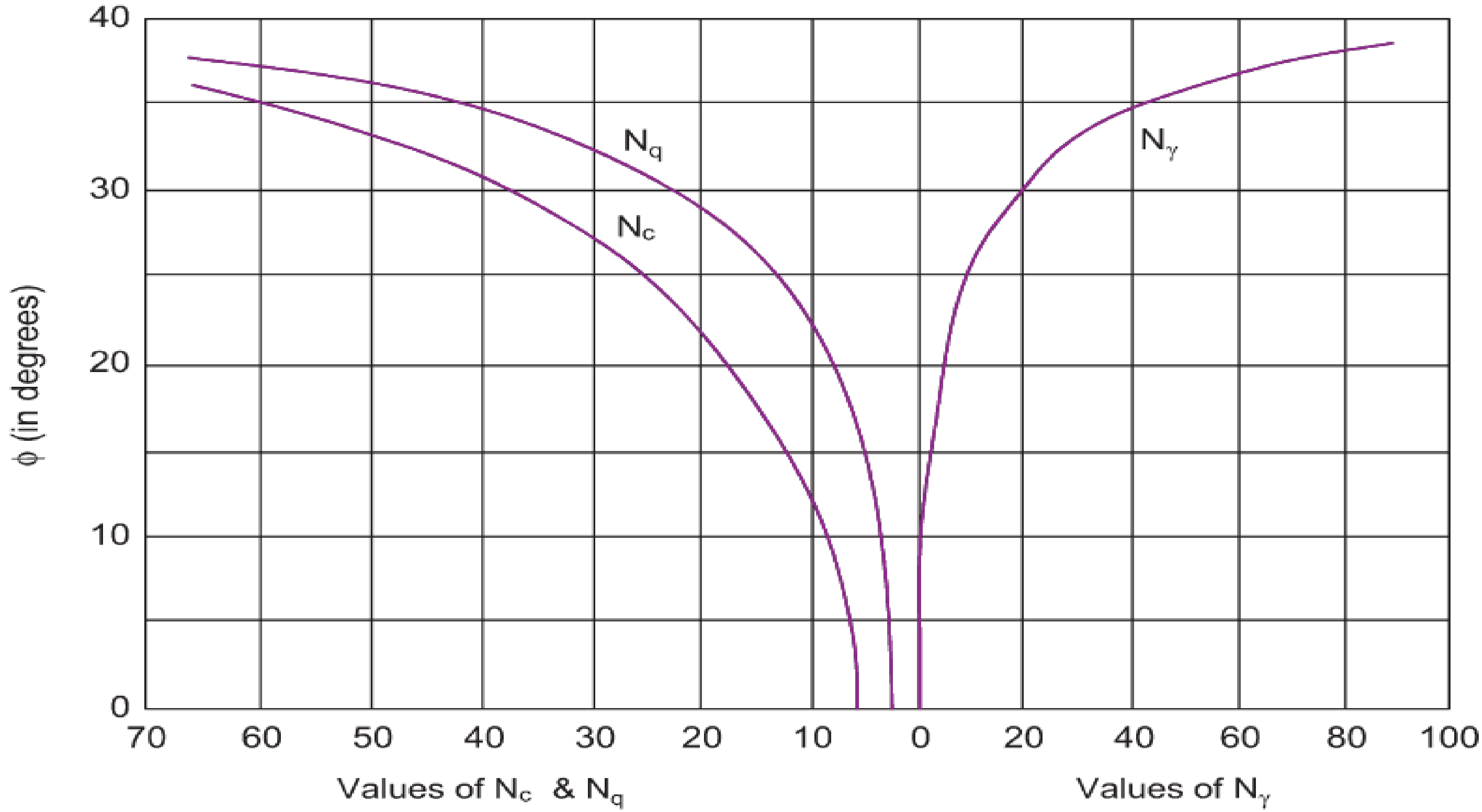
His solution for a strip footing is:

$$q_u = cN_c + \gamma z N_q + 0.5\gamma B N_\gamma$$

The coefficients Nc, Nq and N_γ depend upon the soil's angle of shearing resistance.

When $\varphi = 0^\circ$, Nc = 5.7; Nq = 1.0; Nγ = 0.

$q_u = 5.7c + \gamma z$ or qu = 5.7c for a surface footing.



φ	0°	5°	10°	15°	20°	25°	30°	35°	40°	45°
N_c	5.7	7.3	9.6	12.9	17.7	25.1	37.2	57.8	95.7	172
N_q	1.0	1.6	2.7	4.4	7.4	12.7	22.5	41.4	81.3	173
N_γ	0.0	0.5	1.2	2.5	5.0	9.7	19.7	42.4	100	298

Ultimate Bearing Capacity cntd

The increase in the value of Nc from 5.14 to 5.7 is due to the fact that Terzaghi allowed for frictional effects between the foundation and its supporting soil.

The coefficient Nq allows for the surcharge effects due to the soil above the foundation level, and $N\gamma$ allows for the size of the footing, B.

The effect of $N\gamma$ is of little consequence with clays, where the angle of shearing resistance is usually assumed to be the undrained value, φu, and assumed equal to 0° , but it can become significant with wide foundations supported on cohesionless soil.

Ultimate Bearing Capacity cntd

Terzaghi's solution for a circular footing is:

$$q_u = 1.3cN_c + \gamma z N_q + 0.3\gamma B N_\gamma \quad \text{(where } B = \text{diameter)}$$

For a square footing:

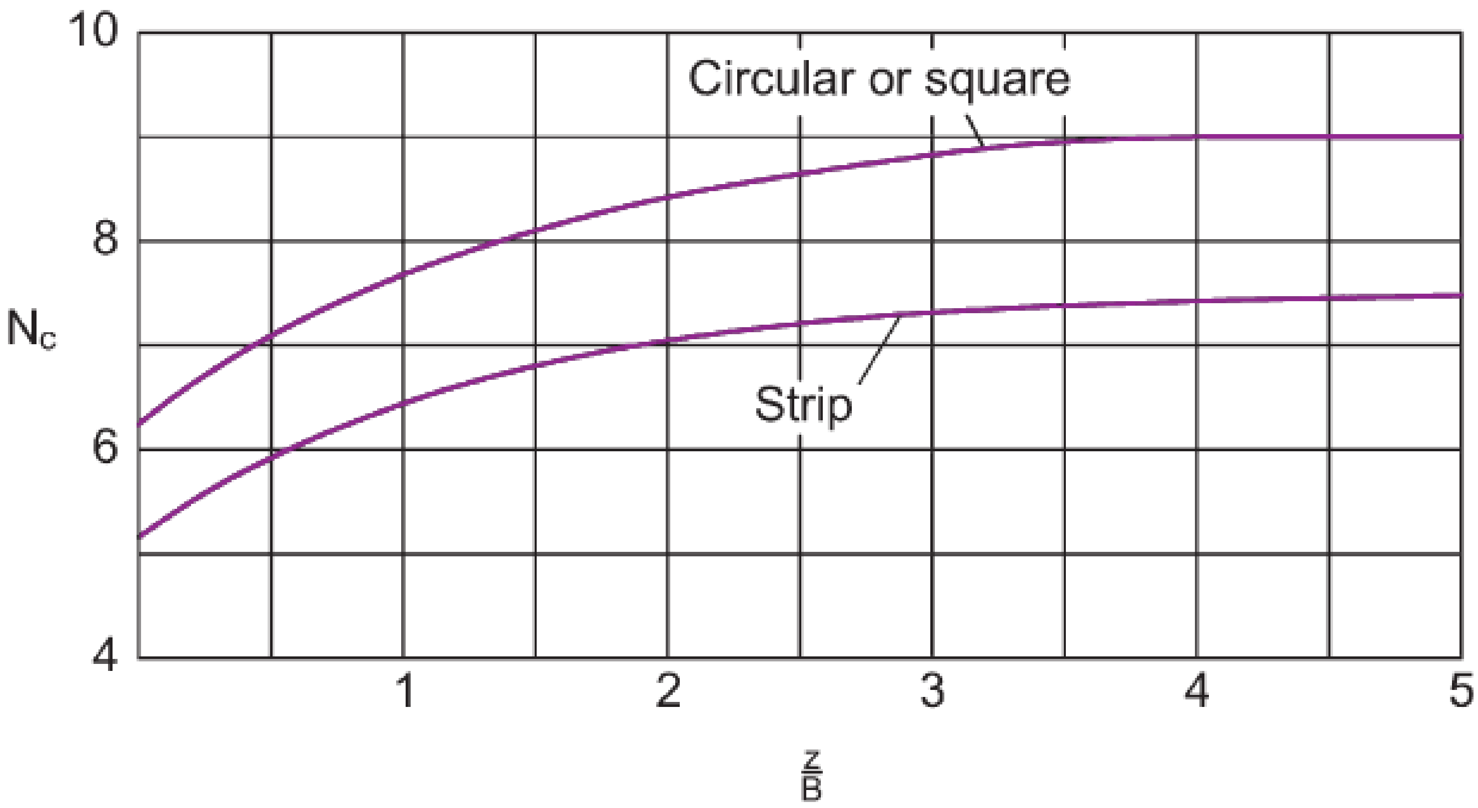
$$q_u = 1.3cN_c + \gamma z N_q + 0.4\gamma B N_\gamma$$

and for a rectangular footing:

$$q_u = cN_c\left(1 + 0.3\frac{B}{L}\right) + \gamma z N_q + 0.5\gamma B N_\gamma\left(1 - 0.2\frac{B}{L}\right)$$

Ultimate Bearing Capacity cntd

Skempton (1951) showed that for a cohesive soil ($\varphi = 0$) the value of the coefficient Nc increases with the value of the foundation depth, z.



Ultimate Bearing Capacity cntd

Choice of soil parameters

As with earth pressure equations, bearing capacity equations can be used with either the undrained or the drained soil parameters. As granular soils operate in the drained state at all stages during and after construction, the relevant soil strength parameter is φ'.

Saturated cohesive soils operate in the undrained state during and immediately after construction and the relevant parameter is cu. If required, the long-term stability can be checked with the assumption that the soil will be drained and the relevant parameters are c′ and φ' (with c′ generally taken as equal to zero).

Ultimate Bearing Capacity cntd

Example 4.1: Ultimate bearing capacity (Terzaghi) in shortand long-term

A rectangular foundation, 2 m × 4 m, is to be founded at a depth of 1 m below the surface of a deep stratum of soft saturated clay (unit weight = 20 kN/m3).

Undrained and consolidated undrained triaxial tests established the following soil parameters: cu = 24 kPa, $\varphi' = 25^\circ$, c′ = 0.

Determine the ultimate bearing capacity of the foundation, (i) immediately after construction

and, (ii) some years after construction.

Ultimate Bearing Capacity cntd

Example 4.2: Ultimate bearing capacity (Terzaghi); effect of $\boldsymbol{\varphi}'$

A continuous foundation is 1.5 m wide and is founded at a depth of 1.5 m in a deep layer of sand of unit weight 18.5 kN/m^3.

Determine the ultimate bearing capacity of the foundation if the soil strength parameters are c′ = 0 and φ' = (i) 35°, (ii) 30°.

3. **Additional Considerations**

- Effect of Ground Water Table
- Non-homogeneous soil conditions
- Effect of Eccentric Loading
- Effect of Inclined Loading

Additional Considerations

Water table below the foundation level

If the water table is at a depth of not less than B below the foundation, the expression for net ultimate bearing capacity is the one given above, but when the water table rises to a depth of less than B below the foundation the expression becomes:

$$q_{u\,net} = cN_c + \gamma z(N_q - 1) + 0.5\gamma B N_\gamma$$

where

γ = unit weight of soil above groundwater level ; γ' = effective unit weight.

For cohesive soils φ' is small and the term $0.5\gamma' B N_\gamma$ is of little account, and the value of the bearing capacity is virtually unaffected by groundwater. With sands, however, the term cN_c is zero and the term $0.5\gamma' B N_\gamma$ is about one half of $0.5\gamma B N_\gamma$, so that groundwater has a significant effect.

Additional Considerations

Water table above the foundation level

For this case Terzaghi's expressions are best written in the form:

qu net = cNc + σ'v (Nq −1)+0.5BN

where σv = effective overburden pressure removed.

From the expression it is seen that, in these circumstances, the bearing capacity of a cohesive soil can be affected by groundwater.

Unless an adequate drainage system and maintenance plan are ensured, the ground water table should be taken as the maximum possible level.

Additional Considerations

Non-homogeneous soil

Reading Assignment

Additional Considerations

Eccentric loads

Effective foundation width and length i.e. that part of the foundation that is symmetrical about the point of application of the load is considered to be useful, or effective, and is the area of the rectangle of effective length $L' = L - 2e_L$ and of effective width $B' = B - 2e_B$.

In the case of a strip footing of width B, subjected to a line load with an eccentricity e, then $B' = B - 2e$ and the ultimate bearing capacity of the foundation is found from either equation or the general equation with the term B replaced by B′.

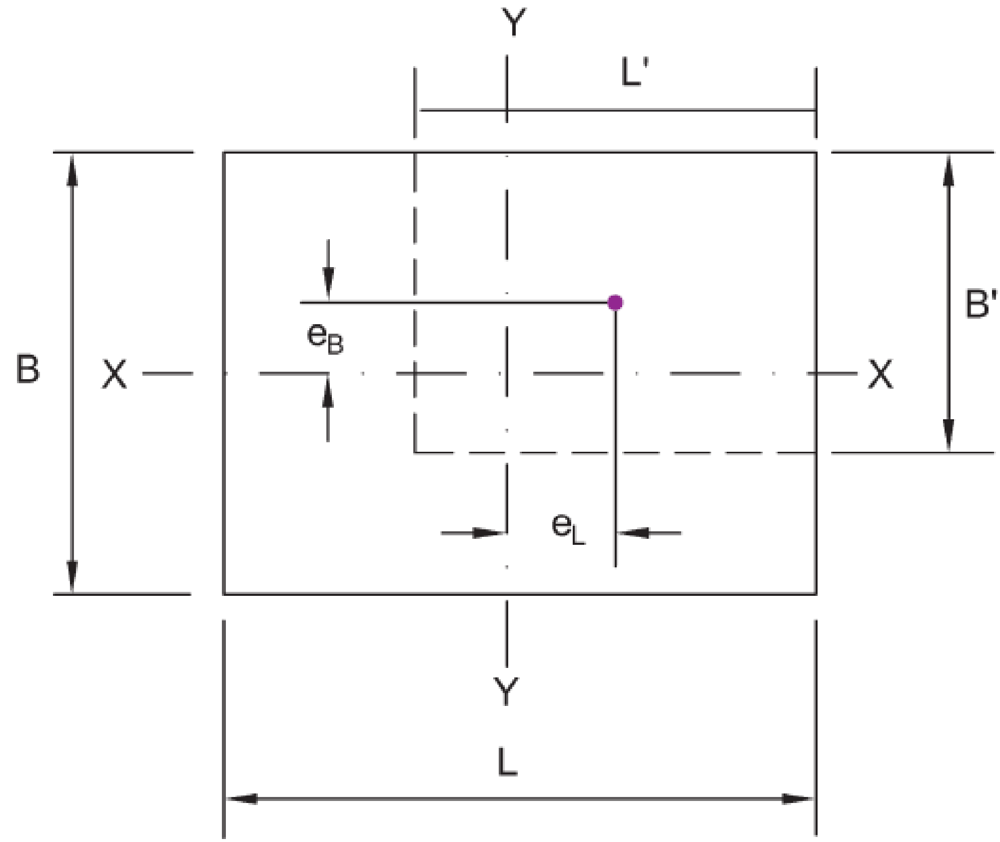
Additional Considerations

The overall eccentricity of the bearing pressure, e, must consider the self-weight of the foundation and is equal to:

$$e = \frac{P \times e_P}{P + W}$$

where

P = magnitude of the eccentric load
W = self-weight of the foundation
e_p = eccentricity of P.



Additional Considerations

Inclined loads

The usual method of dealing with an inclined line load, is to first determine its horizontal and vertical components PH and PV and then, by taking moments, determine its eccentricity, e, in order that the effective width of the foundation B′ can be determined from the formula B′ = B − 2e.

The ultimate bearing capacity of the strip foundation (of width B) is then taken to be equal to that of a strip foundation of width B′ subjected to a concentric load, P, inclined at α to the vertical.

Additional Considerations

Various methods of solution have been proposed for this problem, e.g. Janbu (1957), Hansen (1957), but possibly the simplest approach is that proposed by Meyerhof (1953) in which the bearing capacity coefficients Nc, Nq and Nγ are reduced by multiplying them by the factors ic, iq and iγ in his general equation.

Meyerhof's expressions for these factors are:

$$i_c = i_q = (1 - \alpha/90^\circ)^2$$

$$i_\gamma = (1 - \alpha/\phi)^2$$

4. Developments in BC Equations

- General form of the bearing capacity equation
- Shape factors
- Depth Factors

Developments in BC Equations

Terzaghi's bearing capacity equations have been successfully used in the design of numerous shallow foundations throughout the world and are still in use.

However, they are viewed by many to be conservative as they do not consider factors that affect bearing capacity such as

- ✓ inclined loading,
- ✓ foundation depth and
- ✓ the shear resistance of the soil above the foundation.

Developments in BC Equations

General form of the bearing capacity equation

Meyerhof (1963) proposed the following general equation for qu:

$$q_u = cN_c s_c i_c d_c + \gamma z N_q s_q i_q d_q + 0.5\gamma B N_\gamma s_\gamma i_\gamma d_\gamma$$

where

s_c, s_q and s_γ are shape factors
i_c, i_q and i_γ are inclination factors
d_c, d_q and d_γ are depth factors.

Developments in BC Equations

$$N_c = (N_q - 1)\cot\phi, \quad N_q = \tan^2\left(45^\circ + \frac{\phi}{2}\right)e^{\pi\tan\phi}$$

$N_\gamma = (N_q - 1)\tan 1.4\phi$ Meyerhof (1963)

$N_\gamma = 1.5(N_q - 1)\tan\phi$ Hansen (1970)

$N_\gamma = 2(N_q + 1)\tan\phi$ Vesic (1973)

$N_\gamma = 2(N_q - 1)\tan\phi$ where friction between foundation base and soil, $\delta \geq \phi/2$

ϕ (°)	N_c	N_q	N_γ
0	5.14	1.00	0.00
5	6.49	1.57	0.1
10	8.34	2.47	0.52
15	10.98	3.94	1.58
20	14.83	6.40	3.93
25	20.72	10.66	9.01
30	30.14	18.40	20.09
35	46.12	33.30	45.23
40	75.31	64.20	106.05
45	133.87	134.87	267.75
50	266.88	319.06	758.09

Developments in BC Equations

Shape factors

These factors are intended to allow for the effect of the shape of the foundation on its bearing capacity.

The factors have largely been evaluated from laboratory tests and the values in present use are those proposed by De Beer (1970):

$$s_c = 1 + \frac{B}{L} \cdot \frac{N_q}{N_c}$$

$$s_q = 1 + \frac{B}{L} \tan\phi$$

$$s_\gamma = 1 - 0.4\frac{B}{L}$$

Developments in BC Equations

Depth factors

These factors are intended to allow for the shear strength of the soil above the foundation.

Hansen (1970) proposed the following values:

	$z/B \leq 1.0$	$z/B > 1.0$
d_c	$1 + 0.4(z/B)$	$1 + 0.4 \arctan(z/B)$
d_q	$1 + 2 \tan\phi(1 - \sin\phi)^2(z/B)$	$1 + 2 \tan\phi(1 - \sin\phi)^2 \arctan(z/B)$
d_γ	1.0	1.0

Note: The arctan values must be expressed in radians, e.g. if z = 1.5 and B = 1.0 m then arctan (z/B) = arctan (1.5) = 56.3° = 0.983 radians.

Developments in BC Equations

Example 4.3: Ultimate bearing capacity (Meyerhof) in shortand long-term

Recalculate Example 4.1 using Meyerhof’s general bearing capacity formula.

5. BC from Field Tests

- Presumptive Values
- Plate Load Test
- Standard Penetration Test

	q_s (kPa)
Rocks	
(Values based on assumption that foundation is carried down to unweathered rock)	
Hard igneous and gneissic	10000
Hard sandstones and limestones	4000
Schists and slates	3000
Hard shale and mudstones, soft sandstone	2000
Soft shales and mudstones	1000–600
Hard chalk, soft limestone	600
Cohesionless soils	
(Values to be halved if soil submerged)	
Compact gravel, sand and gravel	>600
Medium dense gravel, or sand and gravel	600–200
Loose gravel, or sand and gravel	<200
Compact sand	>300
Medium dense sand	300–100
Loose sand	<100
Cohesive soils	
(Susceptible to long-term consolidation settlement)	
Very stiff boulder clays and hard clays	600–300
Stiff clays	300–150
Firm clays	150–75
Soft clays and silts	<75
Very soft clays and silts	Not applicable

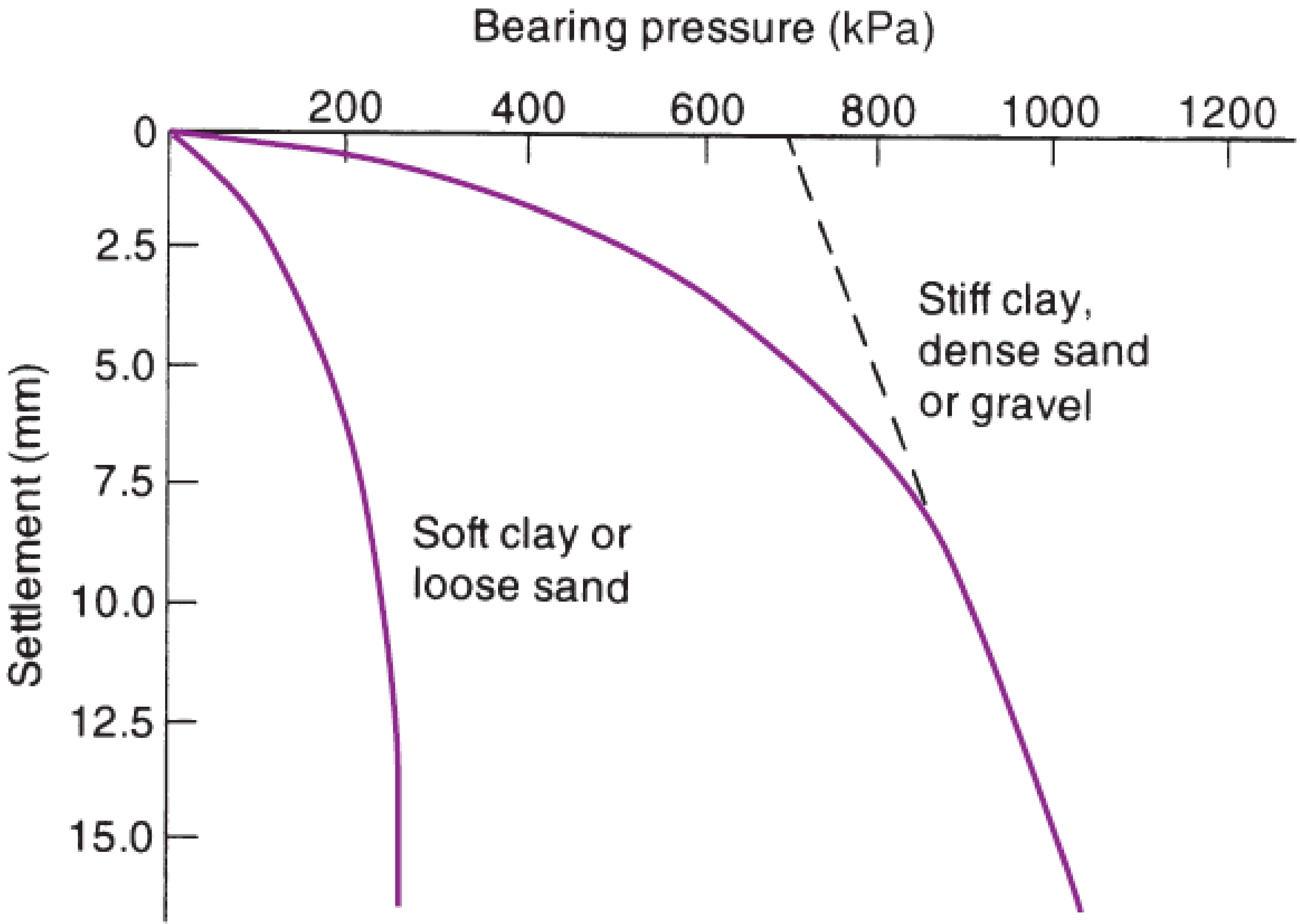
BC from Field Tests

Plate Loading Test

In the test an excavation is made to the expected foundation level of the proposed structure and a steel plate, usually from 300 to 750 mm square, is placed in position and loaded by means of a hydraulic loading system or kentledge.

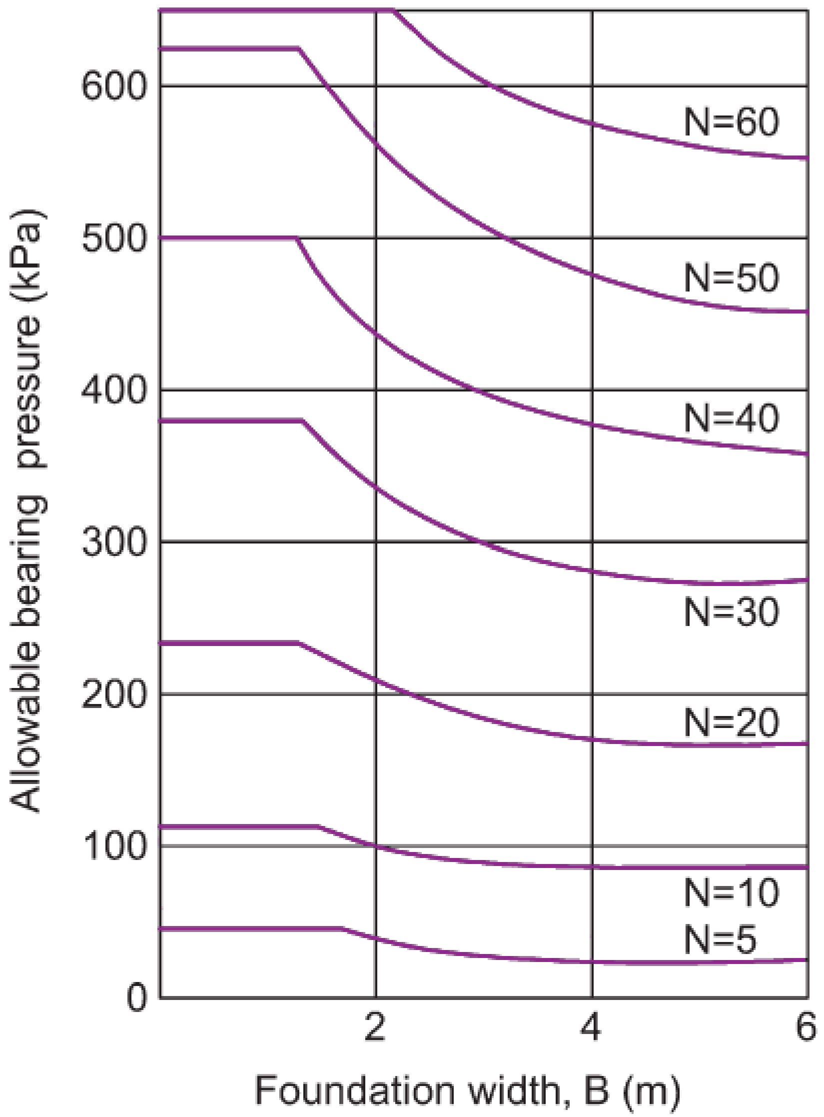
-can only assess a metre or two of the soil layer below the test level, but the method can be extremely helpful in stony soils where undisturbed sampling is not possible provided it is preceded by a boring programme, to prove that the soil does not exhibit significant variations.

BC from Field Tests



BC from Field Tests

Standard Penetration Test



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