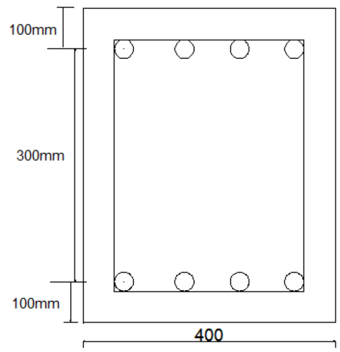
Example 4.1: [Column Interaction Chart]

Draw the interaction diagram for a given column if the column is made up of C25/30 and S460, Show at least a minimum of 6 points in the interaction diagram.



$$A_{s,total} = 6800\ mm^2$$

$$\omega = \frac{A_s f_{yd}}{f_{cd} bh} = \frac{6800*400}{14.1666*400*400} = 0.96$$

Solution

Step 1. Material property

Concrete

C25/30 $f_{cu} = 30\ mpa$

$f_{ck} = 25\ mpa$

$f_{cd} = 14.16667\ mpa$

Rebar

$S - 460$

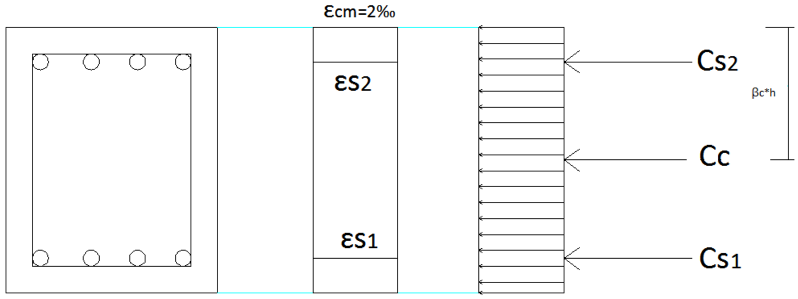
$f_{yk} = 460\ mpa$

$f_{yd} = 400\ mpa$

$\varepsilon_{yd} = 2‰$

Step 2. Interaction diagram points

a) <u>Pure axial compression</u>



$\varepsilon_{s1} = \varepsilon_{s2} = \varepsilon_{cm} = \mathbf{2‰}\ \boldsymbol{use}\ f_s = f_{yd}$

$C_c = \alpha_c f_{cd} bh$

$$\alpha_c = \frac{1}{189}(125 + 64\varepsilon_{cm} - 16{\varepsilon_{cm}}^2) = 1$$

$C_c = 1 * 14.1667 * 400 * 500 = 2833.333\ KN$

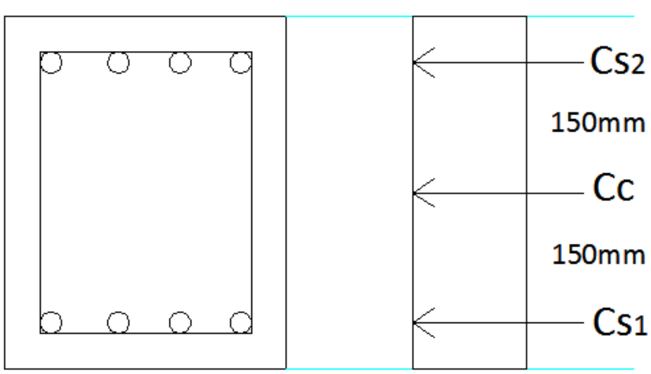
$$\beta_c = 0.5 - \frac{40}{7}\frac{(\varepsilon_{cm} - 2)^2}{125 + 64\varepsilon_{cm} - 16{\varepsilon_{cm}}^2} = 0.5 \qquad \beta_c h = 250\ mm\ from\ top$$

$C_{s1} = A_{s1} f_{yd} = 3400 * 400 = 1360\ KN$

$C_{s2} = A_{s2} f_{yd} = 3400 * 400 = 1360\ KN$

$p = C_c + C_{s1} + C_{s2} = 2833.33 + 1360 + 1360 = 5553.333\ KN$

Find moment at center of cross-section

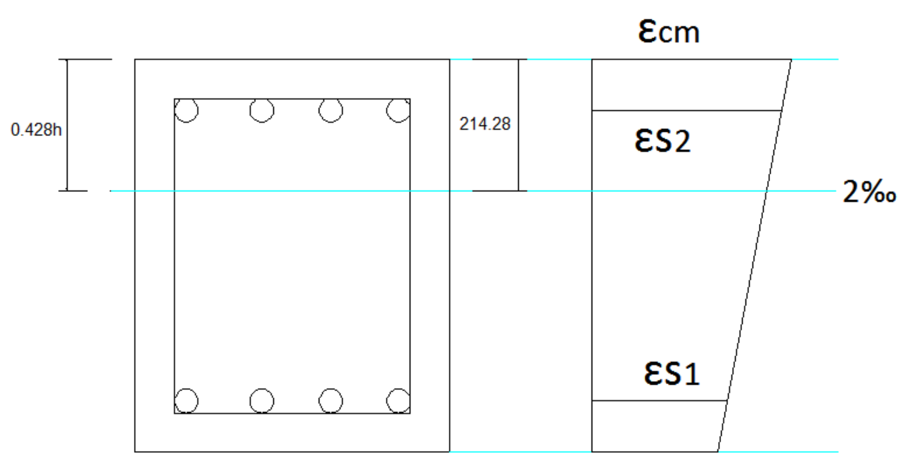


$$M = 0\ KNm$$

$$\mathcal{V}_{sd} = \frac{P}{f_{cd}bh} = \frac{5553.333 * 10^3}{14.16666 * 400 * 500} = 1.96$$

$$\mu_{sd} = \frac{M}{f_{cd}bh^2} = 0$$

b) Point between pure compression and on set of cracking



Assume ε_{cm} to be **3‰** by similarity of triangle $\boldsymbol{\varepsilon_{s1} = 1.1333}$‰ $\boldsymbol{\varepsilon_{s2} = 2.5333}$‰

$\varepsilon_{s1} < \varepsilon_{yd}$ $f_s = E\varepsilon_{s1} = 226.667\ mpa$ $\varepsilon_{s2} > \varepsilon_{yd}$ $f_s = f_{yd} = 400\ mpa$

$$C_c = \alpha_c f_{cd} bh \qquad \alpha_c = \frac{1}{189}(125 + 64 * 3 - 16 * 3^2) = 0.91534$$

$$C_c = \alpha_c f_{cd} bh = 0.91534 * 14.1666 * 400 * 500 = 2593.474\ KN$$

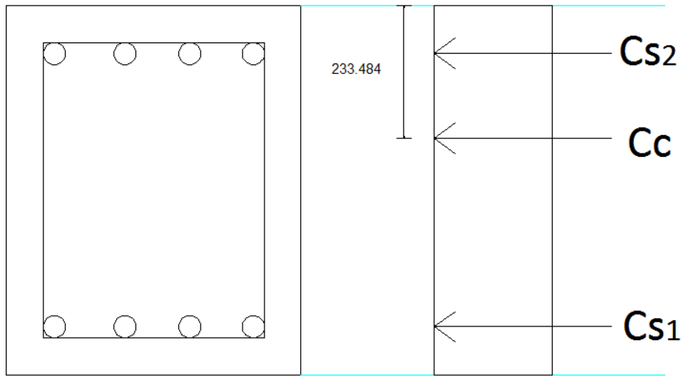
$$C_{s1} = A_{s1} f_s = 3400 * 226.667 = 770.666\ KN$$

$$C_{s2} = A_{s2} f_{yd} = 3400 * 400 = 1360\ KN$$

$$P = C_c + C_{s1} + C_{s2} = 4724.1406\ KN$$

$$\beta_c = 0.5 - \frac{40}{7} \frac{(\varepsilon_{cm} - 2)^2}{125 + 64\varepsilon_{cm} - 16{\varepsilon_{cm}}^2} = 0.467$$

$$\beta_c h = 233.484\ mm\ from\ top$$

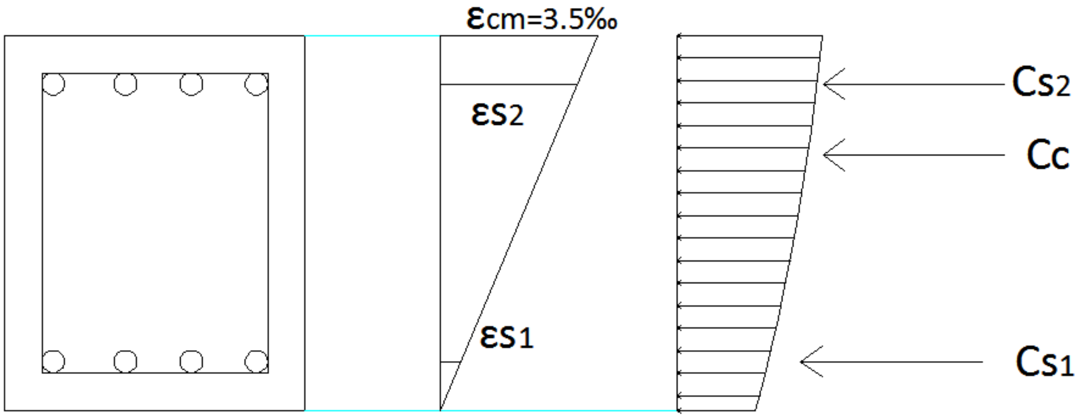


$$M = C_c * \left(\frac{250 - 233.484}{1000}\right) + C_{s2} * 0.15 - C_{s1} * 0.15 = 131.232\ KNM$$

$$\mathcal{V}_{sd} = \frac{P}{f_{cd}bh} = \frac{4724.1406 * 10^3}{14.16666 * 400 * 500} = 1.667$$

$$\mu_{sd} = \frac{M}{f_{cd}bh^2} = \frac{131.2339 * 10^6}{14.1666 * 400 * 500^2} = 0.0926$$

C) On set of cracking



From similarity of triangle $\boldsymbol{\varepsilon_{s1} = 0.7}$ ‰ $\boldsymbol{\varepsilon_{s2} = 2.8}$‰

$\varepsilon_{s1} < \varepsilon_{yd}$ $f_s = E\varepsilon_{s1} = 140\ mpa$ $\varepsilon_{s2} > \varepsilon_{yd}$ $f_s = f_{yd} = 400\ mpa$

$$C_c = \alpha_c f_{cd} bh \qquad \alpha_c = \frac{1}{189}(125 + 64 * 3.5 - 16 * 3.5^2) = 0.8095$$

$$C_c = \alpha_c f_{cd} bh = 0.8095 * 14.1666 * 400 * 500 = 2293.6508\ KN$$

$$C_{s1} = A_{s1} f_s = 3400 * 140 = 476\ KN$$

$$C_{s2} = A_{s2} f_{yd} = 3400 * 400 = 1360\ KN$$

$$P = C_c + C_{s1} + C_{s2} = 4129.6508\ KN$$

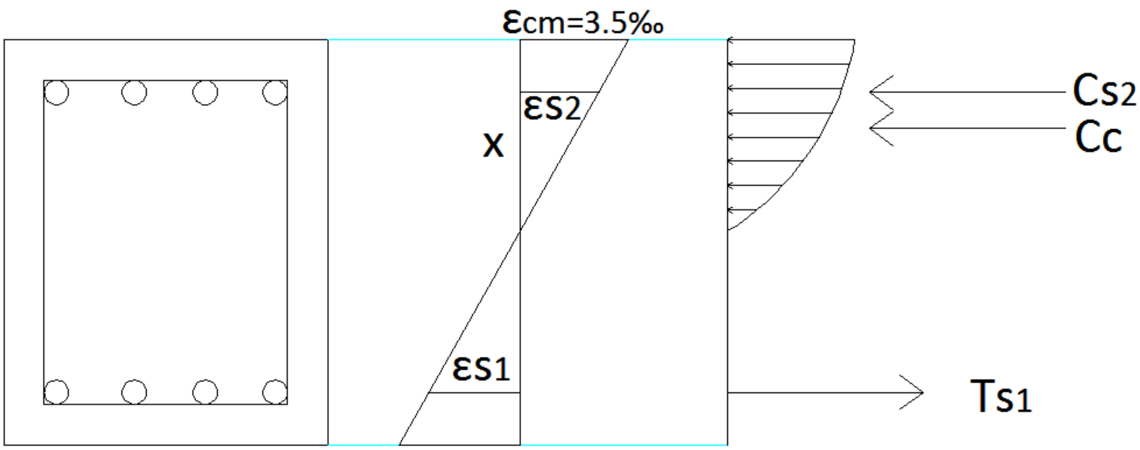
$$\beta_c = 0.5 - \frac{40}{7}\frac{(\varepsilon_{cm} - 2)^2}{125 + 64\varepsilon_{cm} - 16{\varepsilon_{cm}}^2} = 0.41596 \qquad \beta_c h = 207.983\ mm\ from\ top$$

$$M = C_c * \left(\frac{250 - 207.983}{1000}\right) + C_{s2} * 0.15 - C_{s1} * 0.15 = 228.972\ KNM$$

$$\mathcal{V}_{sd} = \frac{P}{f_{cd} bh} = \frac{4129.6508 * 10^3}{14.16666 * 400 * 500} = 1.4575$$

$$\mu_{sd} = \frac{M}{f_{cd} bh^2} = \frac{228.972 * 10^6}{14.1666 * 400 * 500^2} = 0.16163$$

D) Balanced failure



From similarity of triangle

$$\boldsymbol{\varepsilon_{s1}} = \mathbf{2}\ ‰ \quad \boldsymbol{\varepsilon_{s2}} = \mathbf{2.12}‰ \quad K_x = \frac{x}{d} = \frac{3.5}{3.5 + 2} = 0.6363 \quad X = 254.545mm$$

$$\varepsilon_{s1} > \varepsilon_{yd} \quad f_s = f_{yd} = 400\ mpa \qquad \varepsilon_{s2} > \varepsilon_{yd} \quad f_s = f_{yd} = 400\ mpa$$

$$C_c = \alpha_c f_{cd} bh \qquad \alpha_c = \frac{3 * 3.5 - 2}{3 * 3.5} * 0.6363 = 0.5151$$

$$C_c = \alpha_c f_{cd} bh = 0.5151 * 14.1666 * 400 * 500 = 1167.559\ KN$$

$$T_{s1} = A_{s1}f_{yd} = 3400 * 400 = 1360\ KN$$

$$C_{s2} = A_{s2}f_{yd} = 3400 * 400 = 1360\ KN$$

$$P = C_c + C_{s2} - T_{s1} = 1167.559\ KN$$

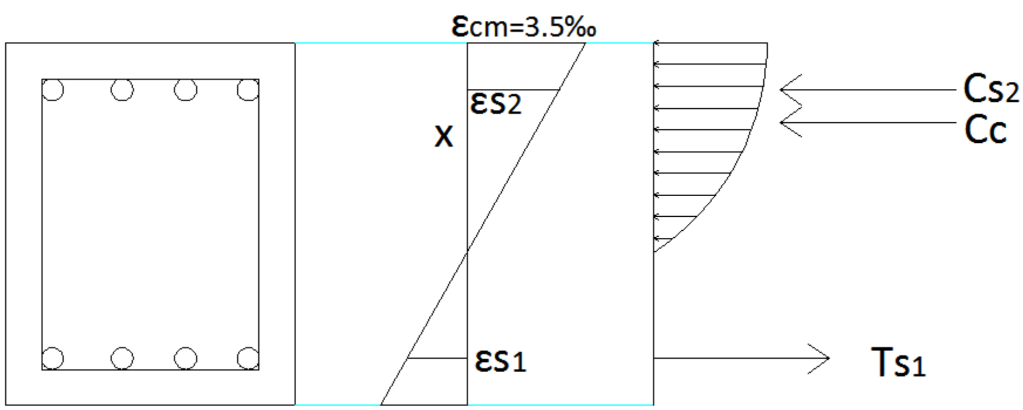
$$\beta_c = \frac{3.5(3*3.5-4)+2}{2*3.5(3*3.5-2)} * 0.6363 = 0.26467 \qquad \beta_c d = 105.871\ mm\ from\ top$$

$$M = C_c * \left(\frac{250-105.871}{1000}\right) + C_{s2} * 0.15 + T_{s1} * 0.15 = 576.279\ KNM$$

$$\mathcal{V}_{sd} = \frac{P}{f_{cd}bh} = \frac{1167.559 * 10^3}{14.16666 * 400 * 500} = 0.412$$

$$\mu_{sd} = \frac{M}{f_{cd}bh^2} = \frac{576.279 * 10^6}{14.1666 * 400 * 500^2} = 0.4067$$

E) <u>Between balanced and pure bending</u>



$assume\ \boldsymbol{\varepsilon_{s1} = 4}$‰

From similarity of triangle

$$\varepsilon_{s1} = 4\ ‰ \quad \varepsilon_{s2} = 1.625\ ‰ \quad K_x = \frac{x}{d} = \frac{3.5}{3.5+4} = 0.4666 \quad X = 186.667mm$$

$$\varepsilon_{s1} > \varepsilon_{yd} \quad f_s = f_{yd} = 400\ mpa \qquad \varepsilon_{s2} < \varepsilon_{yd} \quad f_s = E\varepsilon_{s2} = 325\ mpa$$

$$C_c = \alpha_c f_{cd} bh \qquad \alpha_c = \frac{3*3.5-2}{3*3.5} * 0.4666 = 0.3777$$

$$C_c = \alpha_c f_{cd} bh = 0.3777 * 14.1666 * 400 * 400 = 856.296\ KN$$

$$T_{s1} = A_{s1}f_{yd} = 3400 * 400 = 1360\ KN$$

$$C_{s2} = A_{s2}f_s = 3400 * 325 = 1105\ KN$$

$$P = C_c + C_{s2} - T_{s1} = 601.296\ KN$$

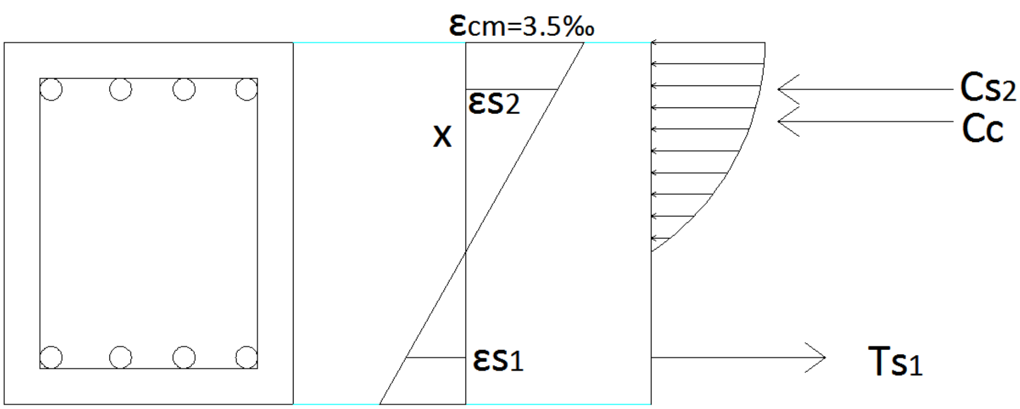
$$\beta_c = \frac{3.5(3*3.5-4)+2}{2*3.5(3*3.5-2)} * 0.4666 = 0.194117 \qquad \beta_c d = 77.6469\ mm\ from\ top$$

$$M = C_c * \left(\frac{250-77.6469}{1000}\right) + C_{s2} * 0.15 + T_{s1} * 0.15 = 517.3352\ KNM$$

$$\mathcal{V}_{sd} = \frac{P}{f_{cd}bh} = \frac{601.296 * 10^3}{14.16666 * 400 * 500} = 0.2122$$

$$\mu_{sd} = \frac{M}{f_{cd}bh^2} = \frac{517.3352 * 10^6}{14.1666 * 400 * 500^2} = 0.365$$

F) <u>Pure bending</u>



Assume $\varepsilon_{s1} > \varepsilon_{yd}$ $\varepsilon_{s2} < \varepsilon_{yd}$ So $f_{s1} = f_{yd}$ $f_{s2} = 200 * \varepsilon_{s2}$

$$K_x = \frac{3.5}{\varepsilon_{s1}+3.5} \quad \alpha_c = \frac{3\varepsilon_{cm}-2}{3\varepsilon_{cm}} * k_x = \frac{2.8333}{\varepsilon_{s1}+3.5} \quad C_c = \alpha_c f_{cd} bd$$

$$= \frac{2.8333}{\varepsilon_{s1}+3.5} * 14.166 * 400 * 400$$

$$C_c = \frac{6422.146}{\varepsilon_{s1}+3.5}\ KN$$

$$T_{s1} = A_{s1}f_{yd} = 3400 * 400 = 1360\ KN$$

$$C_{s2} = A_{s2}f_s = 3400 * 200 * \varepsilon_{s2} = 680\varepsilon_{s2}$$

Next relate ε_{s1} *and* ε_{s2} $x = \left(\frac{3.5}{\varepsilon_{s1}+3.5}\right) * 400 = \frac{1400}{\varepsilon_{s1}+3.5}$

From similarity of triangle $\varepsilon_{s2} = \frac{3675-350\varepsilon_{s1}}{1400}$ *so* $C_{s2} = 680\left(\frac{3675-350\varepsilon_{s1}}{1400}\right) = \frac{2499000-238000\varepsilon_{s1}}{1400}$

$$P = \frac{6421.146}{\varepsilon_{s1}+3.5} + \frac{2499000-238000\varepsilon_{s1}}{1400} - 1360 = 0$$

$\varepsilon_{s1} = 6.339‰$ $K_x = 0.3557$ $\alpha_c = 0.28796$ $C_c = 652.731\,KN$ $C_{s2} = 707.37\,KN$ $T_s = 1360\,KN$

$$M = 652.731 * \left(\frac{250-59.1836}{1000}\right) + 707.37 * 0.15 + 1360 * 0.15 = 434.657\,KNM$$

$$\mathcal{V}_{sd} = \frac{P}{f_{cd}bh} = \frac{0}{14.16666 * 400 * 500} = 0$$

$$\mu_{sd} = \frac{M}{f_{cd}bh^2} = \frac{434.657 * 10^6}{14.1666 * 400 * 500^2} = 0.3068$$