Addis Ababa Institute of Technology

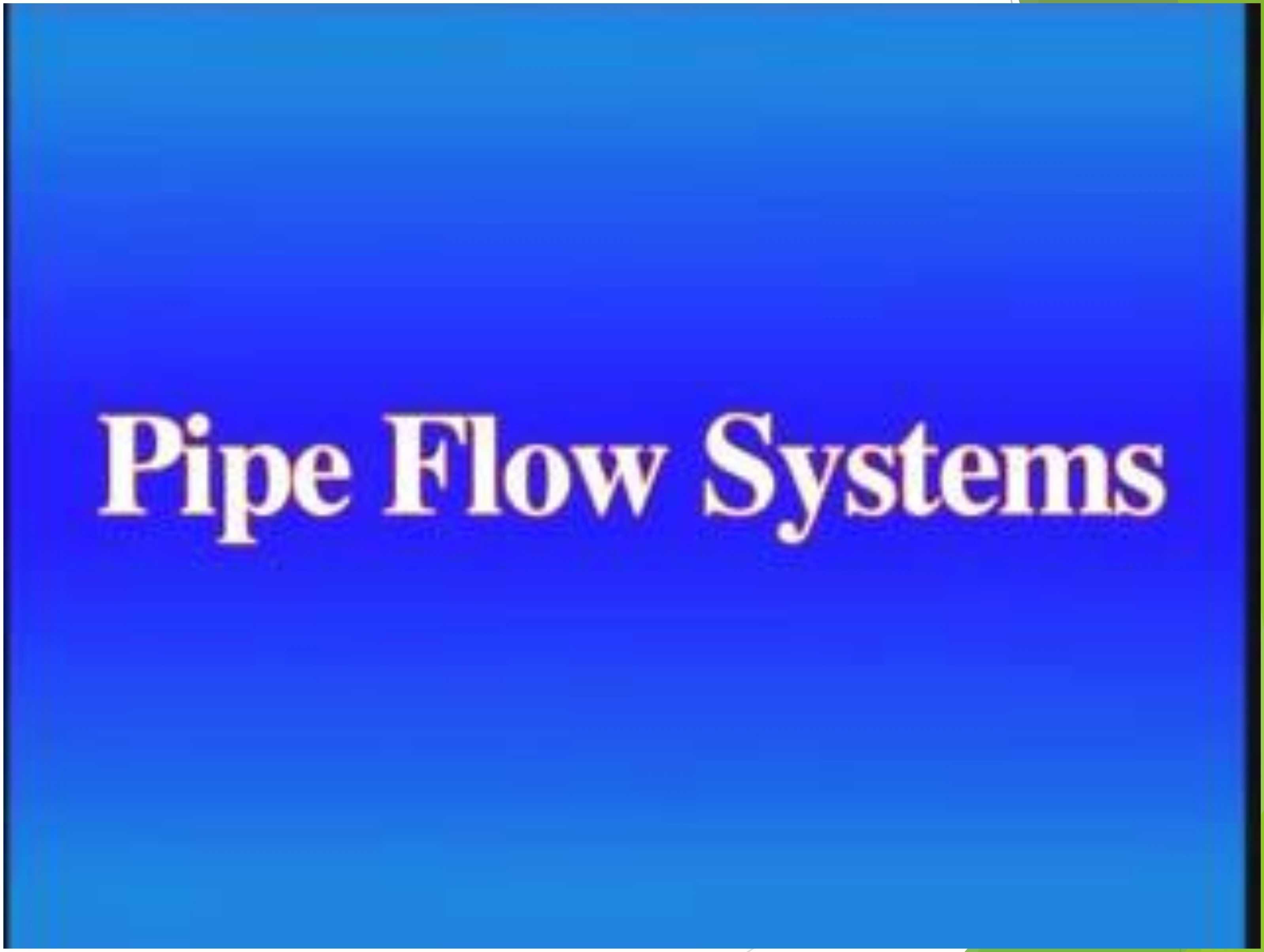
Department of civil and Environmental Engineering

Hydraulics-II (CENG-2162)

By Selam Belay

2020 A.C.

Closed conduit flow

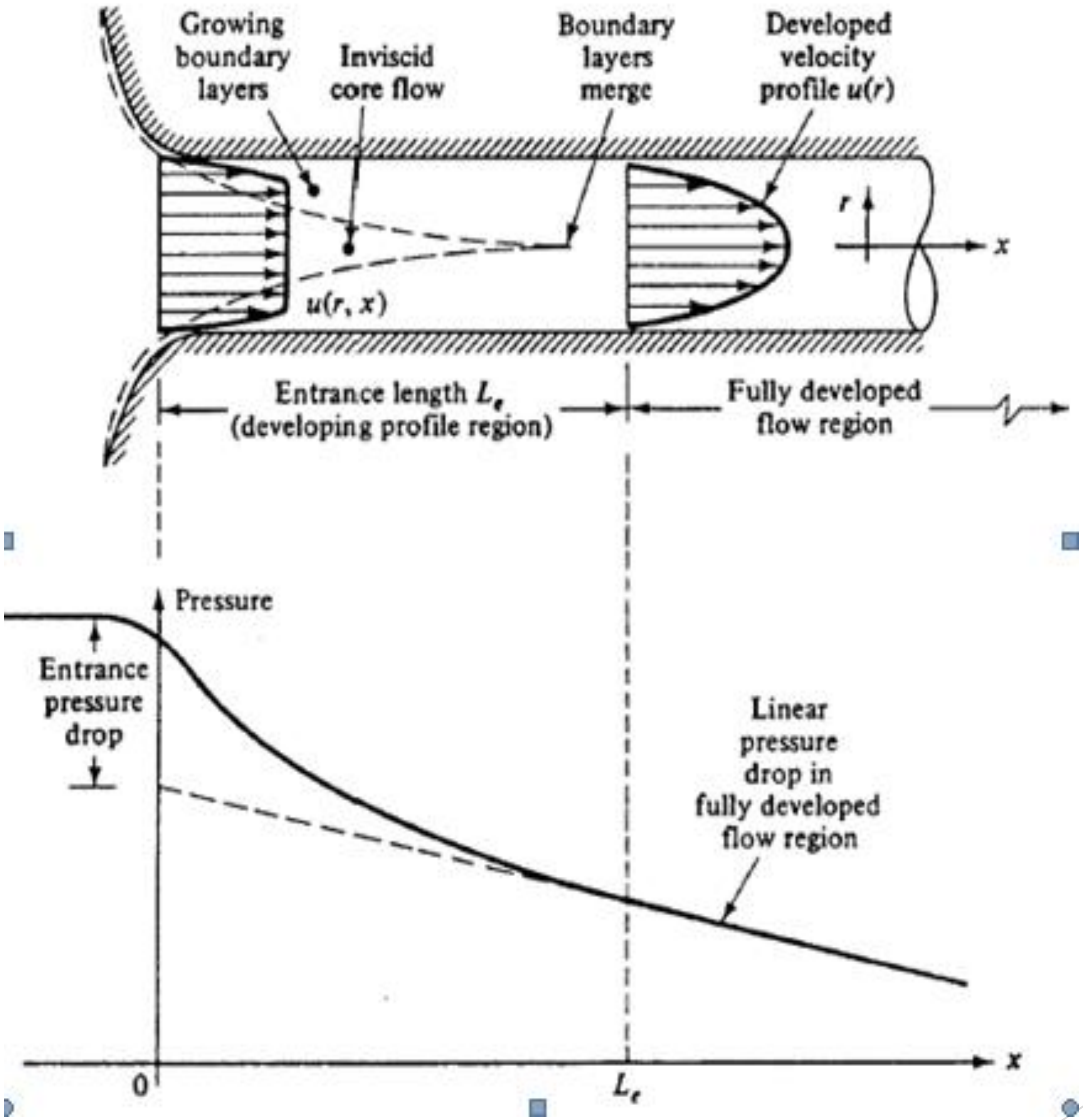


Closed conduit flow

Introduction

- Flow in closed conduits (pipe, if conduit is circular in section, and duct otherwise)
- A pipe is defined as a closed conduit of circular in cross-section through which the fluid is flowing full
- Flow in pipes is an example of internal flow, i.e., the flow is bounded by the walls.
  - For internal flows, the fluid enters the conduit at one point and leaves at the other
  - At the entrance to the conduit there appears what is known as **entrance region** with in which the viscous boundary layer grows and finally at the downstream end of this region covers the entire cross section

Contd…



The entrance length is a function of Reynolds number and is given by relations below:

$$\frac{Le}{d} \approx 0.06\,Re$$

for laminar flow, and

$$\frac{Le}{d} \approx 4.4\,Re^{1/6}$$

for turbulent flow.

Where

$$Re = \frac{\rho v d}{\mu}$$

Characteristics of laminar, transitional and turbulent flows:

Laminar flow

- Re < 2000
- 'low' velocity
- Dye does not mix with water
- Fluid particles move in straight lines
- Simple mathematical analysis possible
- Rare in practice in water systems.

Transitional flow

- 2000 > Re < 4000
- 'medium' velocity
- Dye stream wavers in water - mixes slightly.

Contd…

Turbulent flow

- Re > 4000
- 'high' velocity
- Dye mixes rapidly and completely
- Particle paths completely irregular
- Average motion is in the direction of the flow
- Cannot be seen by the naked eye
- Changes/fluctuations are very difficult to detect. Must use laser.
- Mathematical analysis very difficult - so experimental measures are used
- Most common type of flow.

Energy loss in pipe flows

- When water flows in a pipe, it experiences some resistance to its motion

→ its velocity and ultimately the head of water available is reduced.

This loss of energy (head) is classified as follows

1) Major loss of energy
2) minor loss of energy

Contd…

- The loss of head or energy due to friction in a pipe is known as major loss of energy
- The loss of energy due to change of velocity of the following fluid in magnitude or direction is called minor loss of energy
- Local losses are usually expressed in terms of the velocity head, i.e.

$$h_i = k_i \frac{V^2}{2g} \quad \text{where } k_i \text{ is the minor loss coefficient}$$

Major energy losses

These losses which are due to friction are calculated by

1) Darcy-weisbach formula

2) Chezy's formula

Darcy-weisbach formula

The loss of head or energy in pipes due to friction is calculated from Darcy-weisbach's formula which is given by

$$h_f = \frac{4fLV^2}{D \times 2g}$$

Where

h_f = loss of head due to friction,

f = co-efficient of friction, (a function of Reynolds number, Re)

$= \frac{16}{Re}$ for $Re < 2000$ (laminar/viscous flow)

Contd…

$$= \frac{0{\cdot}0791}{(Re)^{1/4}} \quad \text{for } Re \text{ varying from 4000 to } 10^6$$

$L =$ length of the pipe,

$V =$ mean velocity of flow, and

$D =$ diameter of the pipe.

Chezy's formula for loss of head due to friction

An equilibrium between the propelling force due to pressure difference and the frictional resistance gives:

$$(p_1 - p_2)\,A = f'\,PLV^2$$

or
$$\frac{(p_1 - p_2)}{w}\cdot A = \frac{f'}{w}\,PLV^2$$

or
$$h_f = \frac{f'}{w}\,\frac{P}{A}\,LV^2$$

$\therefore$ *Mean velocity,* $V = \sqrt{\dfrac{w}{f'}} \times \sqrt{\dfrac{A}{P} \times \dfrac{h_f}{L}}$

where, the factor $\sqrt{\dfrac{w}{f}}$, is called the Chezy's constant, C;

Contd...

- The ratio A/P (= area of flow/Wetted perimeter) is called the hydraulic radius denoted by R or m

The ratio hf/L prescribes the loss of head per unit length of pipe & denoted by I or s (Slope)

Mean Velocity, $V = C\sqrt{m\,i}$

known as the Chezy's formula.

This formula helps to find the head loss due to friction if the mean flow velocity through the pipe and also the values of Chezy's constant C is known

Contd...

Darcy-Weisbach formula (for loss of head)

↓

Flow through pipes

Chezy's formula (For loss of head)

↓

Flow through open channels

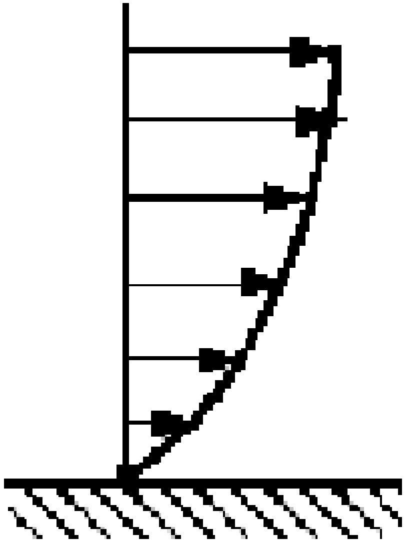
The value of Hydraulic mean depth for a circular pipe

$$m = \frac{D}{4}\left[\because \quad m = \frac{\text{area}}{\text{perimeter}} = \frac{\frac{\pi}{4} \times D^2}{\pi D} = \frac{D}{4}\right]$$

Contd…

- In reality, because fluids are viscous
  - energy is lost by flowing fluids due to friction which must be taken into account
  - The effect of the friction shows itself as a pressure (or head) loss

At the wall there is a shearing stress retarding the flow



Contd…

i) Head loss during laminar flow in a pipe

- The shear stress τ_w. is almost impossible to measure
- For laminar flow it is possible to calculate a theoretical value for a given velocity, fluid and pipe dimension
- consider a cylinder of fluid, length L, radius r, flowing steadily in the centre of a pipe
- We are in equilibrium, so the shearing forces on the cylinder equal the pressure forces

$$\tau\, 2\pi r\, L = \Delta p\, A = \Delta p \pi r^2$$

$$\tau = \frac{\Delta p}{L}\frac{r}{2}$$

Contd…

By Newtons law of viscosity we have $\tau = \mu \frac{du}{dy}$ where y is the distance from the wall.

As we are measuring from the pipe centre then we change the sign and replace y with r distance from the centre, giving $\tau = -\mu \frac{du}{dr}$

This can be combined with the equation above to give

$$\frac{\Delta p}{L}\frac{r}{2} = -\mu \frac{du}{dr}$$

$$\frac{du}{dr} = -\frac{\Delta p}{L}\frac{r}{2\mu}$$

In an integral form this gives an expression for velocity,

$$u = -\frac{\Delta p}{L}\frac{1}{2\mu}\int r\,dr$$

Integrating gives the value of velocity at a point distance r from the centre

$$u_r = -\frac{\Delta p}{L}\frac{r^2}{4\mu} + C$$

Contd…

At $r = 0$, (the centre of the pipe), $u = u_{max}$, at $r = R$ (the pipe wall) $u = 0$, giving

so, an express $C = \frac{\Delta p}{L}\frac{R^2}{4\mu}$ elocity at a point r from the pipe centre when the flow is laminar is

The discharge $u_r = \frac{\Delta p}{L}\frac{1}{4\mu}\left(R^2 - r^2\right)$

$$Q = u_m A$$

$$u_m = \int_0^R u_r \, dr$$

$$= \frac{\Delta p}{L}\frac{1}{4\mu}\int_0^R \left(R^2 - r^2\right) dr$$

$$= \frac{\Delta p}{L}\frac{R^2}{8\mu} = \frac{\Delta p\, d^2}{32\mu L}$$

Contd…

So the discharge can be written

$$Q = \frac{\Delta p\, d^2}{32\,\mu L}\frac{\pi d^2}{4}$$
$$= \frac{\Delta p}{L}\frac{\pi\, d^2}{128\mu}$$

This is the Hagen-Poiseuille equation for laminar flow in a pipe. It expresses the discharge Q in terms of the pressure gradient ($\frac{\partial p}{\partial x} = \frac{\Delta p}{L}$), diameter of the pipe and the viscosity of the fluid

We are interested in the pressure loss (head loss) and want to relate this to the velocity of the flow. Writing pressure loss in terms of head loss hf, i.e. p =ρghf

Contd…

$$u = \frac{\rho g h_f d^2}{32 \mu L}$$

$$h_f = \frac{32 \mu L u}{\rho g d^2}$$

This shows that pressure loss is directly proportional to the velocity when flow is laminar

ii) Head loss during Turbulent flow in a pipe

Darcy, Weisbach and others as a result of pipe experiments,

deduced a formula for pipe friction loss which may be expressed in the form

$$h_f = f \frac{L}{D} \frac{v^2}{2g}$$

The above expression is known as the Darcy-Weisbach formula

Contd…

Where, h_f = Head loss due to friction in the pipe length, L (m)

g = Acceleration due to gravity (m/s^2)

v = average velocity (m/sec.)

D = inside diameter of pipe (m)

f = A dimensionless coefficient called the friction factor

The friction factor, f is the function of the following parameter

$$f = F\left(v, D, \rho, \mu, \varepsilon, \varepsilon', m\right)$$

Where

v = velocity

D = diameter

ρ = density

μ = dynamic viscosity

ε = measure of size of the roughness projections

ε' = measure of the arrangement or spacing of the roughness elements

m = form factor i.e. dependent upon the shape of the individual roughness elements and it is dimensionless

$\varepsilon = \varepsilon' = m = 0$ for smooth pipes

Contd…

The friction factor, f can be determined for different flow condition as follows

a. For laminar flow, $f = \frac{64}{R_e}$ $\quad R_e$ = max. value 2000

b. For Turbulent flow

i) for smooth pipes

Empirical relation developed by Blasius

$$f = \frac{0.316}{R_e^{1/4}} \quad , R_e = \text{Ranges from 4000 to 100,000}$$

For values of R_e up to 300,000 *Von Karmas-Pranditl* modified as

$$\frac{1}{\sqrt{f}} = 2 \log \left(R_e \sqrt{f}\right) - 0.8$$

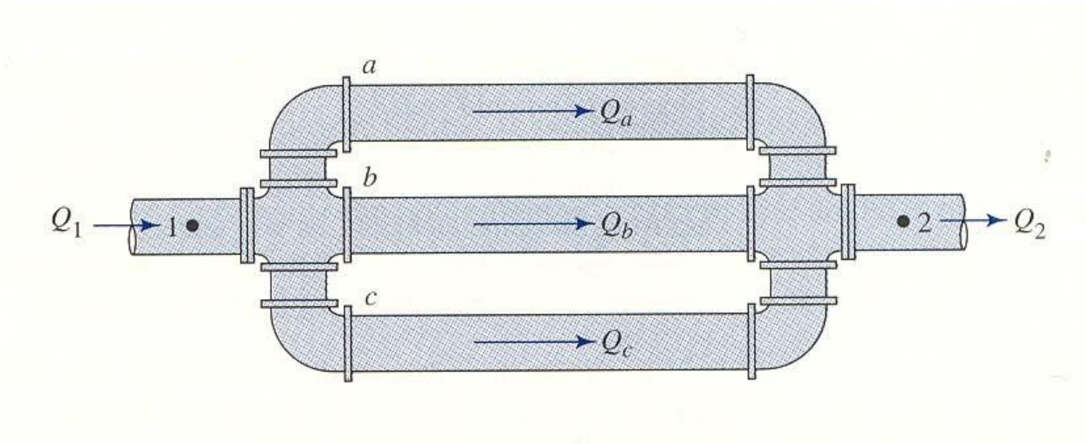
ii)for rough pipes in a complete turbulence zone,

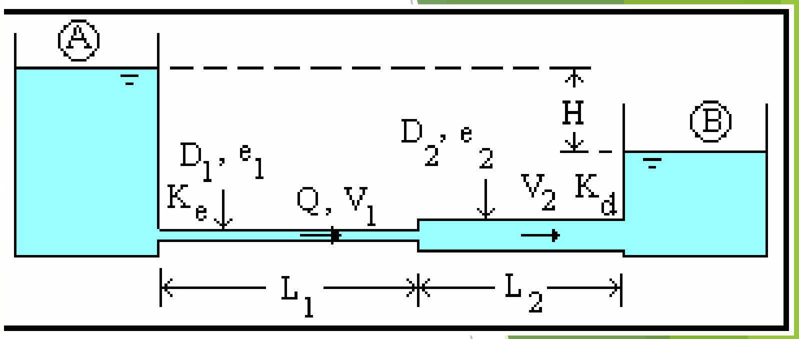
$$\frac{1}{\sqrt{f}} = 1.14 - 2 \log \left(\varepsilon/_D\right)$$

Contd...

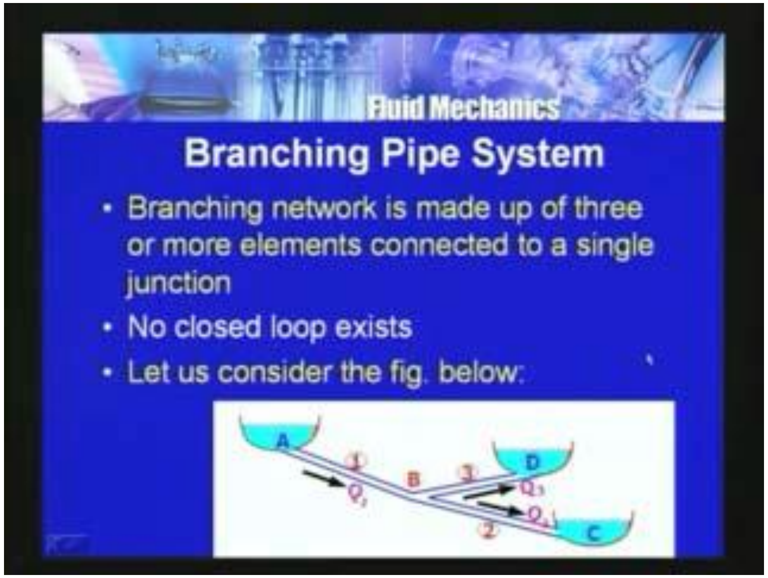
For all pipes

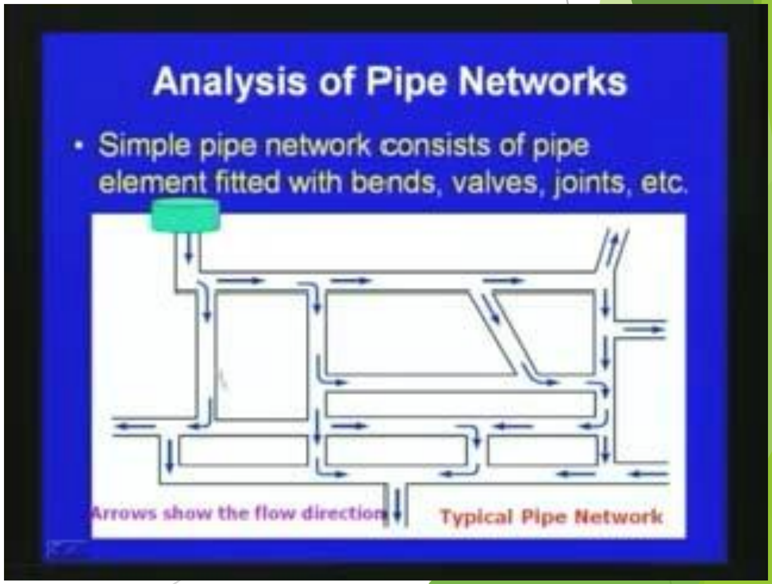
$$\frac{1}{\sqrt{f}} = -2\log\left[\frac{\varepsilon/D}{3.7} + \frac{2.523}{R_e\sqrt{f}}\right]$$





Pipe System (Network)

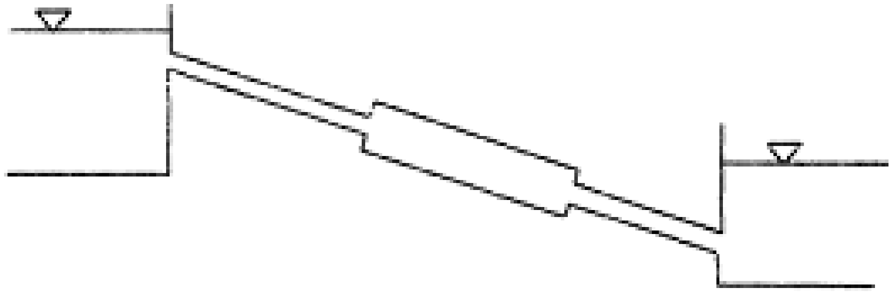




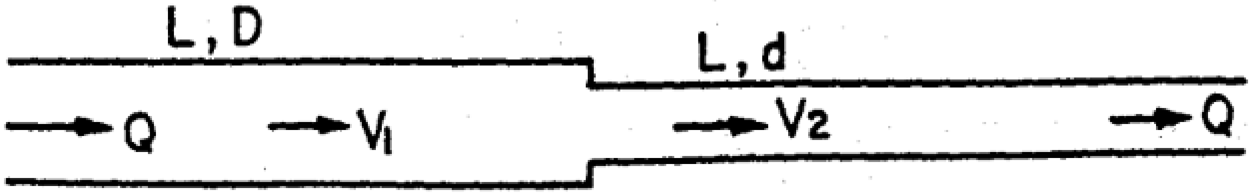
Pipe System (Network)

Pipe in series

- If two or more pipes of different sizes or roughness are connected that the fluid flows through one pipe and then through the other, they are said to be connected in series
- In the case of pipe in series the discharge is the same through each pipe and the total head loss is the sum of all losses in all the individual pipes and fittings



Contd…

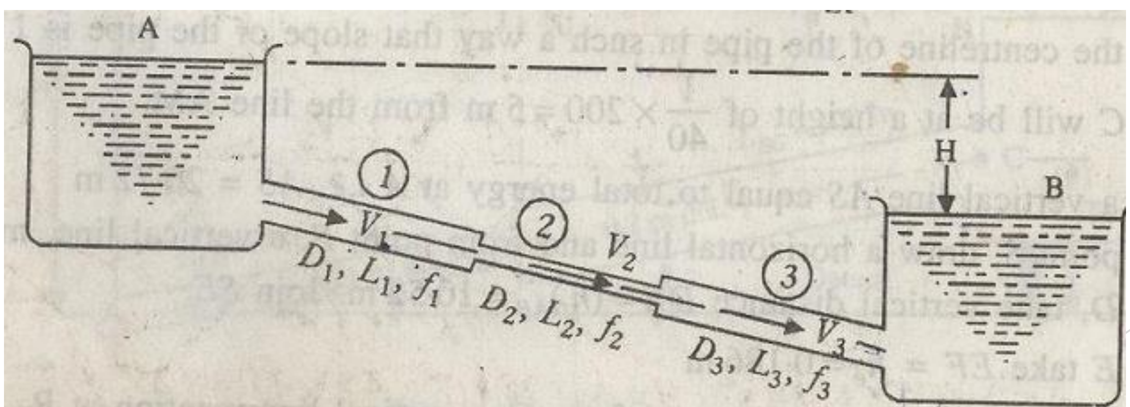


Two pipes of different diameters connected in series

$$Q_1 = Q_2$$

$$hf = h_{f1} + h_{f2}$$

A system of pipes in series



Contd…

As the rate of flow Q of water through each pipe is same therefore,

$$Q = A_1V_1 = A_2V_2 = A_3V_3$$

Also, the difference in liquid surface levels = sum of the various head losses in the pipes

i.e.
$$H = h_i + h_{f_1} + h_c + h_{f_2} + h_e + h_{f_3} + \frac{V_3^2}{2g} \qquad \text{(i)}$$

Where

$h_i =$ head loss at entrance $= \dfrac{0{\cdot}5V_1^2}{2g}$

$h_{f_1} =$ head loss due to friction in pipe $1 = \dfrac{4f_1L_1V_1^2}{D_1 \times 2g}$

$h_c =$ head loss at contraction $= \dfrac{0{\cdot}5V_2^2}{2g}$

Contd…

h_{f_2} = head loss due to friction in pipe 2 $= \dfrac{4f_2L_2V_2^2}{D_2 \times 2g}$

h_e = head loss due to enlargement $= \dfrac{(V_2 - V_3)^2}{2g}$

h_{f_3} = head loss due to friction in pipe 3 $= \dfrac{4f_3L_3V_3^2}{D_3 \times 2g}$

Substituting the values in (i), we have

$$H = h_i + h_{f_1} + h_c + h_{f_2} + h_e + h_{f_3} + \frac{V_3^2}{2g}$$

$$= \frac{0 \cdot 5V_1^2}{2g} + \frac{4f_1L_1V_1^2}{D_1 \times 2g} + \frac{0 \cdot 5V_2^2}{2g} + \frac{4f_2L_2V_2^2}{D_2 \times 2g} + \frac{(V_2 - V_3)^2}{2g} + \frac{4f_3L_3V_3^2}{D_3 \times 2g} + \frac{V_3^2}{2g}$$

If minor losses are neglected, then the above equation becomes,

Contd...

$$H = \frac{4f_1L_1V_1^2}{D_1 \times 2g} + \frac{4f_2L_2V_2^2}{D_2 \times 2g} + \frac{4f_3L_3V_3^2}{D_3 \times 2g}$$

If $f_1 = f_2 = f_3 = f$, then

$$H = \frac{4fL_1V_1^2}{D_1 \times 2g} + \frac{4fL_2V_2^2}{D_2 \times 2g} + \frac{4fL_3V_3^2}{D_3 \times 2g}$$

$$= \frac{4f}{2g}\left[\frac{L_1V_1^2}{D_1} + \frac{L_2V_2^2}{D_2} + \frac{L_3V_3^2}{D_3}\right]$$

Contd...

Equivalent pipe

It is a pipe of uniform diameter having loss of head and discharge equal to the loss of head

discharge of a compound pipe consisting of several pipes of different lengths and diameters

The uniform diameter of an equivalent pipe is known as the equivalent diameter of the series or compound pipe

Let, L_1, L_2, L_3 etc. = lengths of pipes 1, 2, 3, etc

D_1, D_2, D_3 etc. = diameters of pipes 1, 2, 3, etc.,

H = total head loss,

L = length of the equivalent pipe, and

D = diameter of the equivalent pipe.

Then, neglecting minor losses, total head loss

$$h_f = h_{f1} + h_{f2} + h_{f3} + \cdots$$

$$H = \frac{4f_1L_1V_1^2}{D_1 \times 2g} + \frac{4f_2L_2V_2^2}{D_2 \times 2g} + \frac{4f_3L_3V_3^2}{D_3 \times 2g} + \cdots\cdots$$

(where f_1, f_2 and f_3 etc. are co-efficients of friction)

Contd…

Also, from continuity considerations

$$Q = A_1 V_1 = A_2 V_2 = A_3 V_3$$

$$= \frac{\pi}{4} \times D_1^2 V_1 = \frac{\pi}{4} \times D_2^2 V_2 = \frac{\pi}{4} \times D_3^2 V_3$$

$$V_1 = \frac{4Q}{\pi D_1^2},\ V_2 = \frac{4Q}{\pi D_2^2},\ V_3 = \frac{4Q}{\pi D_3^2}$$

Substituting these values to the above equation, assuming f1 = f2 = f3 etc = f, we get

$$H = \frac{4fL_1 \times \left(\frac{4Q}{\pi D_1^2}\right)^2}{D_1 \times 2g} + \frac{4fL_2 \times \left(\frac{4Q}{\pi D_2^2}\right)^2}{D_2 \times 2g} + \frac{4fL_3 \times \left(\frac{4Q}{\pi D_3^2}\right)^2}{D_3 \times 2g} + \ldots\ldots$$

$$= \frac{4 \times 16 f Q^2}{\pi^2 \times 2g}\left(\frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \frac{L_3}{D_3^5} + \ldots\ldots\right)$$

Contd...

Head loss in the equivalent pipe,

$$H = \frac{4fLV^2}{D \times 2g}$$ (assuming the same value of f as in compound pipe)

Where

$$V = \frac{Q}{A} = \frac{Q}{\frac{\pi}{4} \times D^2} = \frac{4Q}{\pi D^2}$$

$$H = \frac{4fL\left(\frac{4Q}{\pi D^2}\right)^2}{D \times 2g} = \frac{4 \times 16fQ^2f}{\pi^2 \times 2g}\left[\frac{L}{D^5}\right]$$

From the above two equations we have

$$\frac{4 \times 16fQ^2}{\pi^2 \times 2g}\left(\frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \frac{L_3}{D_3^5} + \cdots\cdots\right) = \frac{4 \times 16fQ^2}{\pi^2 \times 2g}\left(\frac{L}{D^5}\right)$$

$$\frac{L}{D^5} = \frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \frac{L_3}{D_3^5} + \cdots$$

Contd…

- The Above equation is known as Dupit's equation

- If the length of the equivalent pipe is equal to the length of the compound pipe i.e. L = (L1+L2+L3…), the diameter D of the equivalent pipe may be determined by using this equation.

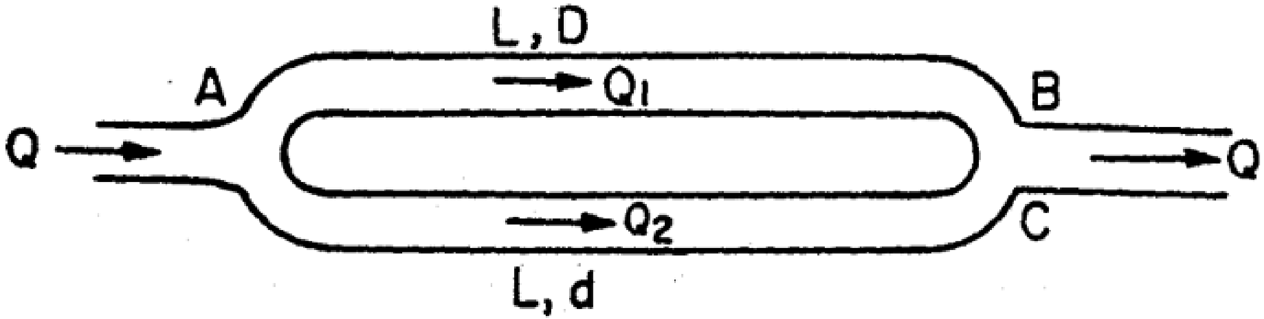
- Sometimes a pipe of a given diameter D which is available may be required to be used as equivalent pipe to replace a compound pipe;

  In this case the length of the equivalent pipe may be required to be determined and the same may also be determined by using the above equation

Contd…

iii. Pipes in parallel

When two or more pipes are connected as shown below. The regions u/s of A and d/s of B are connected in series with the parallel pipe system

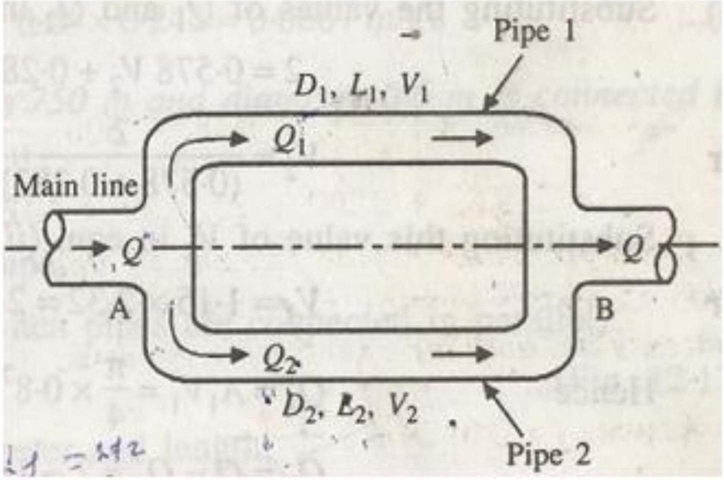


Two pipes between A and B connected in series

Contd…

- The pipes are said to be in parallel (Figure below) when a main pipe line divides into two or more parallel pipes which again join together downstream and continues as a main line
- The rate of discharge in the main line is equal to the sum of the discharges in each of the parallel pipes

Thus $$Q = Q_1 + Q_2$$



Contd…

- When the pipes are arranged parallel, the loss of head in each pipe (branch) is the same

Loss of head in pipe1 = Loss of head in pipe2

or
$$h_f = \frac{4f_1 L_1 V_1^2}{D_1 \times 2g} = \frac{4f_2 L_2 V_2^2}{D_2 \times 2g}$$

When $f_1 = f_2$, then

$$\frac{L_1 V_1^2}{D_1 \times 2g} = \frac{L_2 \times V_2^2}{D_2 \times 2g}$$

Contd...

- The discharge arriving at point A, says Q, must be distributed among the various branches. Thus, continuity requires that Q = Q1 + Q2
- All branches leading from A have the piezometric head, namely hA, and when they rejoin at B they must again have common piezometric head, hB. Consequently, the head loss must be the same in each branch $h_{l1} = h_{l2} = h_{l3}$
- If the piezometric head at point A and B is known, the discharge in each branch may be calculated as if the branch were a single line; and the total discharge obtained by summing. Trial and error will be required only to the extent that the resistance coefficient is unknown and trial values have to be assumed

Contd…

- If instead the discharge Q is given, both head loss and flow distribution are unknown and the problem is more difficult. For example in the case of three parallel pipes, we may write

$$h_{l1} = h_{l2}$$

$$f_1 \frac{L_1}{D_1^5} \frac{Q_1^2}{2g} \left(\frac{4}{\pi} \right)^2 = f_2 \frac{L_2}{D_2^5} \frac{Q_2^2}{2g} \left(\frac{4}{\pi} \right)^2$$

- Assume the length and diameter are known, this may be written as $c_1 f_1 Q_1^2 = c_2 f_2 Q_2^2$

Where $c_i = \frac{L_i}{D_i^5}$

- For computational purposes an equivalent pipe may be substituted for a number of pipes in parallel. The Darcy-Weisbach equation $h_l = f \frac{L}{D} \frac{V^2}{2g} = f \frac{L}{D^5} \frac{Q^2}{2g} \left(\frac{4}{\pi} \right)^2$

Contd…

Rearranges and gives as

$$Q = \left(\frac{h_l 2 g \pi^2}{16}\right)^{1/2} \left(\frac{D^5}{fL}\right)^{1/2}$$

Therefore, for three parallel pipes

$$Q = Q_1 + Q_2$$

$$Q = \left(\frac{h_l 2 g \pi^2}{16}\right)^{1/2} \left[\left(\frac{D_1^5}{f_1 L_1}\right)^{1/2} + \left(\frac{D_2^5}{f_2 L_2}\right)^{1/2}\right]$$

$$Q = \left(\frac{h_l 2 g \pi^2}{16}\right)^{1/2} \left(\frac{D_e^5}{f_e L_e}\right)^{1/2}$$

Where the subscript e indicates equivalent pipe component. Thus, the equivalent pipe may be chosen to satisfy

$$\left(\frac{D_e^5}{f_e L_e}\right)^{1/2} = \left[\left(\frac{D_1^5}{f_1 L_1}\right)^{1/2} + \left(\frac{D_2^5}{f_2 L_2}\right)^{1/2}\right]$$

Contd…

Head loss, $h_l = h_{l1} = h_{l2} = h_{l3} = \left(\frac{p_A}{\gamma} + z_A\right) - \left(\frac{p_B}{\gamma} + z_B\right)$

And, discharge, $Q = Q_1 + Q_2$

Two type of problem occur

I. With known elevation of the Hydraulic grade line A and B, the discharge Q need to be determined

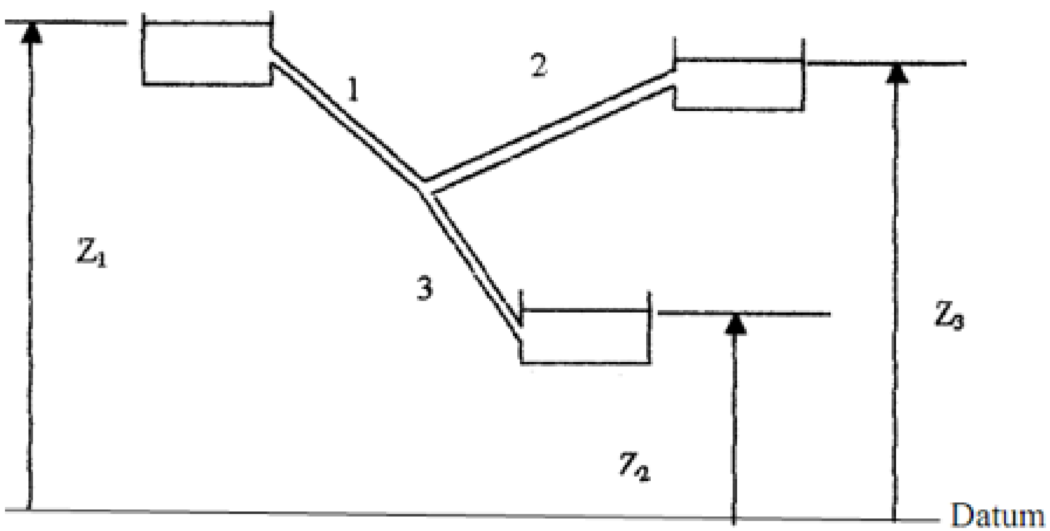
II. With Q known the distribution of flow and head loss need to be determined

The first type is simple, since the head loss is known. From which the discharges can be added to determine the total discharge. The second type is quite complex, as neither the head loss nor the discharge for any pipe is not known

Contd…

iii) Branching pipes (Three reservoir problems)

Branching pipes: such pipes branch off from the main and may return to it. Typical example is pipes that convey flow from multiple reservoirs

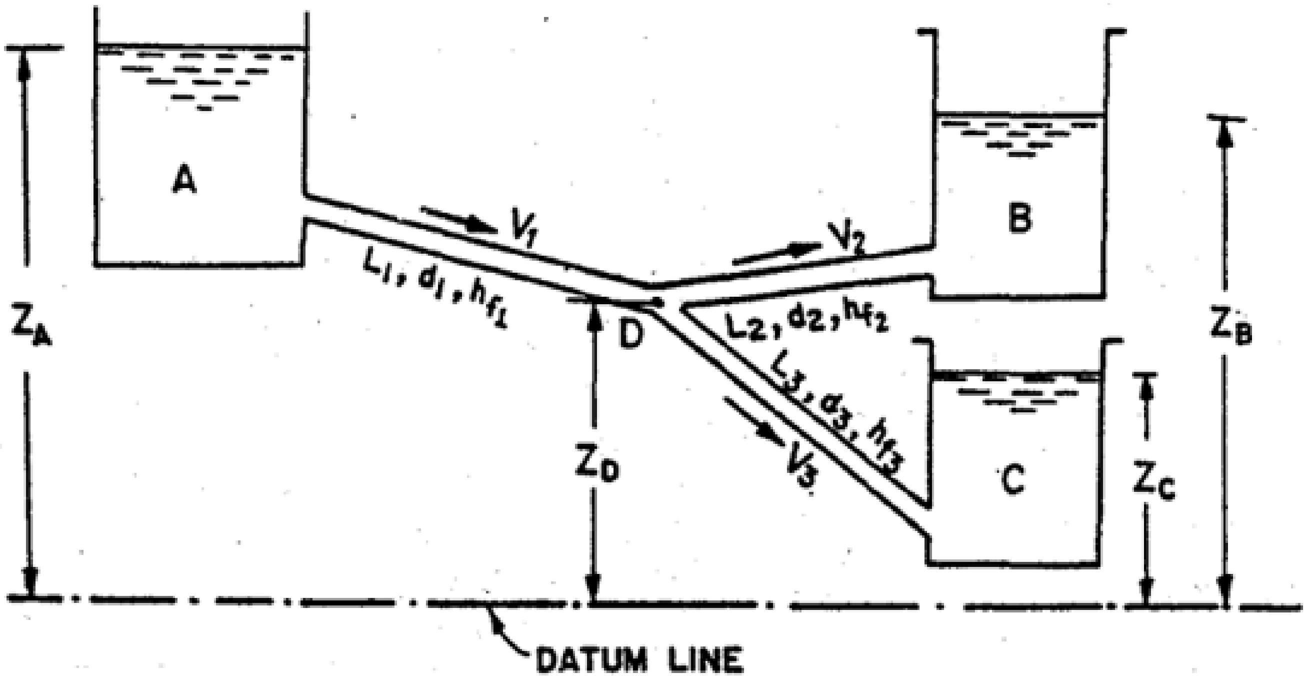


The flow through each pipe is to be determined when the reservoir elevations are given

Contd…

The size and types of pipes and fluid properties are assumed known

The Darcy-Weisbach equation must be satisfied to each pipe, and the continuity equation must also be satisfied



Contd…

Given the lengths, diameters, and coefficient of friction of each pipe , it is required to find the discharge and the direction of flow in each pipe

The basic equations used to solve such problems are:

1. Continuity equation
2. Bernoulli equation
3. Darcy-Weisbach equation

Also it is assumed that the reservoirs are very large and the water surface levels in the reservoirs are constant so that steady conditions exist in the pipes,

Also minor losses are assumed to be very small

Contd…

The flow from reservoir A takes the place to junction D. The flow from junction D is toward reservoir C

Now the flow from junction D toward B will take place only when pizometric head at B (PD /ρg + Elv. of point D) is more than pizometric head at B (Zb)

Example

HARDY CROSS METHOD for 3 branches or more (NETWORKS)

- You have to express the head loss h in terms of Q

  h = kQ^2

  We already express h in terms of v; and v = Q/A

Contd…

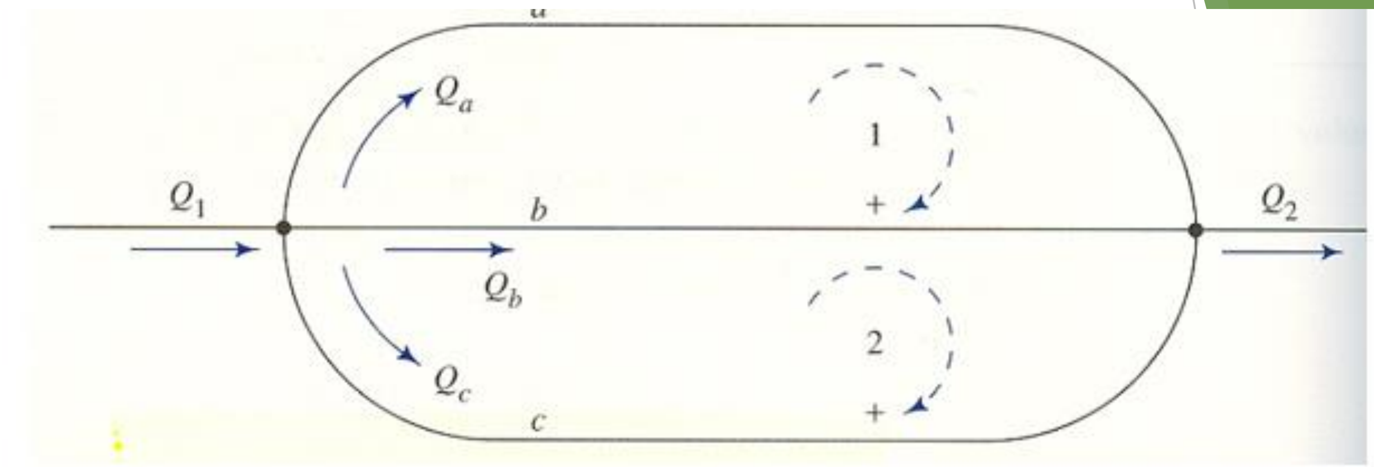
We have estimate the volume Q in the pipes. Estimation is based on the principle of conservation of volume and the likely losses that may occur in the pipe.

You have to divide the network into closed circuits and assign +ve and –ve values to head loss in the pipes.

Rule –

If flow in a given pipe of a circuit is clockwise, Q and h are positive. If flow is counterclockwise, Q and h are negative.

Contd…



Stepwise procedure for Hardy Cross Method –

1. Express energy loss in each pipe by h = kQ^2
2. Assume values of Q for each pipe.
3. Divide the network into closed circuits.
4. For each pipe calculate head loss using h = kQ^2
5. For each circuit, algebraically sum the values of h using the direction convention/rule.

Contd…

Find $\sum h$.

6. For each pipe calculate 2kQ

7. Sum all values of 2kQ for each circuit, assuming all are positive = $\sum 2kQ$

8. For each circuit calculate –

$$\Delta Q = \frac{\sum h}{\sum 2kQ}$$

9. for each pipe calculate –

$$Q' = Q - \Delta Q$$

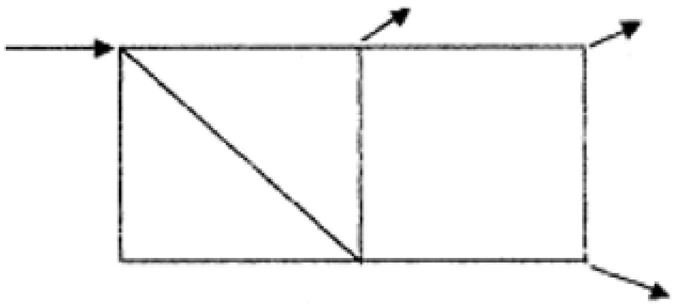
10. Repeat until ΔQ becomes very small

Contd...

Pipe Networks

- Interconnected pipes through which flow to a given outlet may come from different circuits are called networks of pipes.
- Problems on these require trial solutions
- Pipes that are interconnected in such a way that they make loops (or circuits) form a network
- In such systems the flow in any of the pipes may come from different circuits and as such it is not simple to know the direction of flow by observation

Contd…



- Pipe network problems involve the analysis of existing systems, i.e. the determination of flow rate in each pipe, pressure at junctions (or nodes), the head losses in the pipes and the selection of appropriate material and size
- The solution of network problems always uses iterative procedures that make use of the following two facts:

  1) the flow into a junction must equal the flow out of the junction, i.e. at each node (and for the entire system) continuity must be satisfied,

  2) the algebraic sum of the head losses around any circuit must add up to zero