1 Open Channel Flow

1.1 Introduction

An open channel is a flow system in which the top surface of the fluid is exposed to the atmosphere. The term open channel refers to liquid flow that is not completely enclosed by solid boundaries (such as in a river). In open channel flow therefore, the flowing liquid has a free surface, and thus the liquid is not under gauge pressure at the surface.
open channel is takes place due to slope of bed of the channel only. The Hydraulic grade line of open channel flow is exactly conceding with the water surface but the total energy lines lie at a distance of ($V^2/2g$) above the hydraulic grade line at every section as in closed channel.
Since the pressure on the top surface of water in a channel is constant, no pressure difference can be built up between any two sections along the channel it will always be subjected to same resistance. As such in order to overcome the resistance and to cause the flow of water in a channel, it is constructed with its bottom sloping towards the direction of flow, so that the component of weight of the flowing water in the direction of flow is developed which causes the flow of water in channel.

Open channel flow occurs where ever the flow proceeds with the liquid surface exposed to constant pressure. In practice this pressure is the atmospheric pressure, and the flow proceeds with free surface (exposed to the atmosphere). Thus open channel flow may occur regardless of the type of conduit in which it is occurring i.e. an open channel flow may exist in a pipe, if it is flowing partially full. In practice flow in sewers, canals, streams and gutters is exposed to atmospheric pressure and hence is an example of open channel flow.

The longitudinal profile of the free surface in an open channel flow defines the hydraulic gradient and determines the cross-sectional area of flow, as is shown in Figure 1.1. It also necessitates the introduction of an extra variable, the stage, to define the position of the free surface at any point in the channel.

In consequence, problems in open channel flow are more complex, and the solutions are more varied, making the study of such problems both interesting and challenging.

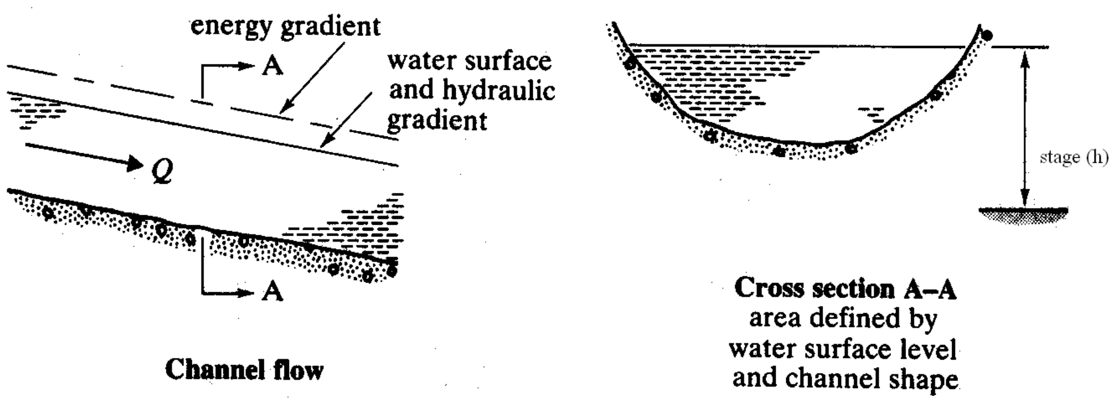


Fig. 1.1 Flow in open channel

Flow classification

Recalling that flow may be steady or unsteady and uniform or non-uniform, the major classifications applied to open channels are as follows:

Steady and Unsteady: Time is the criterion. Flow is said to be steady if the depth of flow at a particular point does not change or can be considered constant for the time interval under consideration. The flow is unsteady if depth changes with time.

Uniform and non-uniform Flow: Space as the criterion. Open Channel flow is said to be uniform if the depth and velocity of flow are the same at every section of the channel. Hence it follows that uniform flow can only occur in prismatic channels. Flow in channels is termed as non-uniform or varied if the depth of flow y, changes from section to section $\left(\frac{\partial y}{\partial s}\right) \neq 0$. Non-uniform flow is rapidly varied flow If the depth of flow changes a abruptly over a comparatively short distance; eg. Hydraulic Jump Non-Uniform flow is gradually varied flow. If the change in depth of flow takes place gradually in a long reach of the channel.

Steady uniform flow, in which the depth is constant, both with time and distance. This constitutes the fundamental type of flow in an open channel in which the gravity forces are in equilibrium with the resistance forces.

Steady non-uniform flow, in which the depth varies with distance, but not with time. The flow may be either (a) gradually varied or (b) rapidly varied.

Type (a) requires the joint application of energy and frictional resistance equations. Type (b) requires the application of energy and momentum principles.

Unsteady non uniform flow, in which the depth varies with both time and distance (unsteady uniform flow is very rare). This is the most complex flow type, requiring the solution of energy, momentum and friction equations through time. The various flow types are all shown in Figure 1.2.

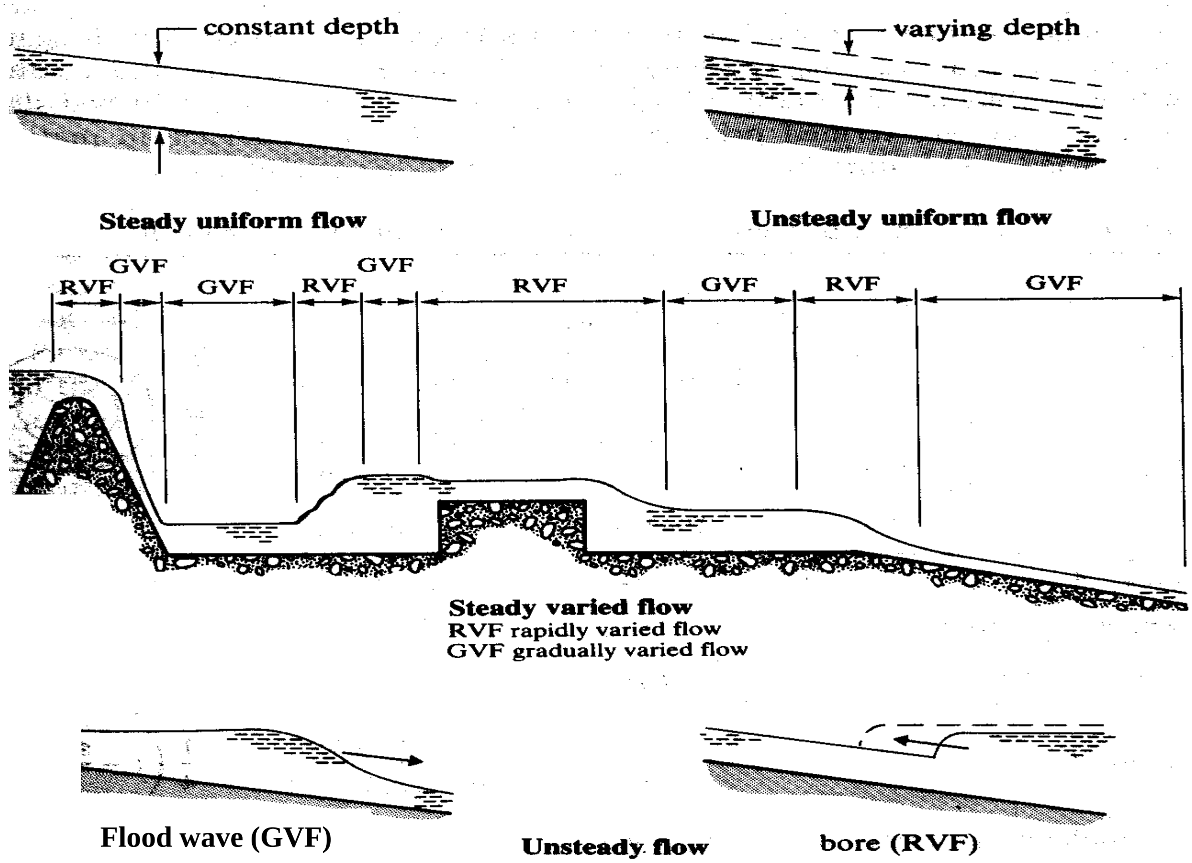


Fig. 1.2 Types of flow

$$R_e = \frac{\rho VR}{\mu}, \text{ for open channel}$$

$$R_e = \frac{\rho DV}{\mu}, \text{ for closed channel}$$

Where R_e = Reynolds number

ρ = mass density of the fluid

D = Pipe diameter

V = Velocity of the fluid

μ = dynamic viscosity of the fluid

R = hydraulic mean radius

R = A/p , where A – area of cross section of the channel

P – Wetted perimeter of the channel.

On this experimental data it has been found that

If RV 500 ≤ Re ≤ 600, the flow is considered laminar

If Re > 2000, the flow is considered turbulent

If 500 ≤ Re ≤ 2000, the flow is transition state.

Sub critical, critical and supercritical flow

Gravity is a predominant force in the case of channel flow. As such depending on the relative effect of gravity the inertia forces, the channel flow may be designed as sub critical, critical or supercritical.

Determination of such flow depend on the dimensionless parameter called fraud number (Fr) which is defined as the ratio of inertia the gravity forces.

$Fr = \frac{V}{\sqrt{gy}}$, Where V = the mean velocity of flow

g = acceleration due to gravity

y = hydraulic depth of the channel

y = A/T, T = top width, A = cross sectional area

This experiment indicate that

When Fr = 1, the channel flow is said to be critical state

If Fr < 1, or $V < \sqrt{gD}$, the flow is sub critical (tranquil or streaming)

If Fr > 1, or $V > \sqrt{gD}$, the flow is said to be supercritical or rapid or shooting or torrential flow.

1.2 *Types of Open Channel*

Channels where flow occurs under free surface can either be natural, such as rivers and streams, or artificial. Artificial channels comprise all man-made channels, including irrigation and navigation canals, spillway channels, sewers, culverts and drainage ditches. They are normally of regular cross-sectional shape and bed slope, and as such are termed *prismatic channels**. Their construction materials are varied, but commonly used

* A prismatic channel is characterized by unvarying cross section, constant bottom slope, and relatively straight alignment.

materials include concrete, steel and earth. The surface roughness characteristics of these materials are normally well defined within engineering tolerances. In consequence, the application of hydraulic theories to flow in artificial channels will normally yield reasonably accurate results. Various terms are used to refer to channels built under different conditions.

Canal: a channel built on ground, i.e excavated to the desired shape and slope with or without lining, usually having a mild slope. The lining could be made of concrete, stone masonry, cement, wood or bituminous material.

Flume: a channel built (or supported) above the ground to convey fluid from one point to another. In the field flumes are made of concrete, wood, sheet metal or masonry. Laboratory flumes are usually made of wood, metal, glass or a composite of these materials.

Chute: is a channel of steep slopes. If the change in elevation in the direction of flow occurs in a relatively short distance the channel is called a drop.

Culvert: is a relatively short and usually buried conduit that is commonly used for drainage purposes, as in highways and embankments. Open channel prevails whenever the culvert is flowing partially full.

In contrast, natural channels are normally very irregular in shape, and their materials are diverse. The surface roughness of natural channels changes with time, distance and water surface elevation. Therefore, it is more difficult to apply hydraulic theory to natural channels and obtain satisfactory results. Many applications involve man-made alterations to natural channels (e.g. river control structures and flood alleviation measures). Such applications require an understanding not only of hydraulic theory, but also of the associated disciplines of sediment transport, hydrology and river morphology.

Various geometric properties of natural and artificial channels need to be determined for hydraulic purposes. In the case of artificial channels, these may all be expressed algebraically in terms of the depth (y), as is shown in Table 1.1. This is not possible for natural channels, so graphs or tables relating them to stage (h) must be used.

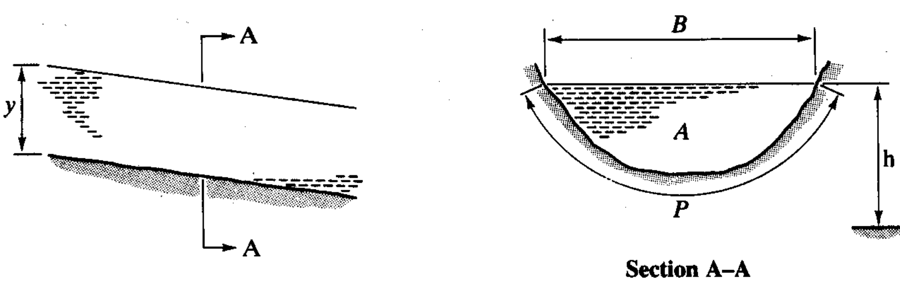


Figure 1.3 Definition sketch of geometric channel properties

1.3 *Important terms in open channel flow*

Depth (y) - the vertical distance of the lowest point of a channel section from the free surface;

Stage (h) - the vertical distance of the free surface from an arbitrary datum;

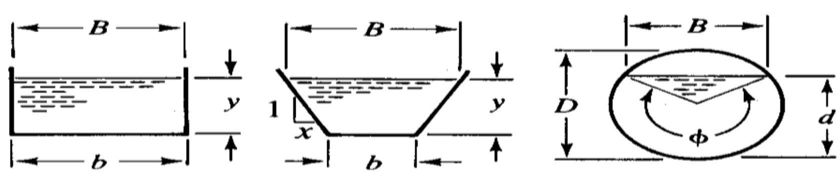
Area (A) - the cross-sectional area of flow normal to the direction of flow;

Wetted perimeter (P) - the length of the wetted surface measured normal to the direction of flow;

Surface width (B) - the width of the channel section at the free surface;

Hydraulic radius (R) - the ratio of area to wetted perimeter (A / P);

Hydraulic mean depth (Dm) - the ratio of area to surface width (A / B).



	Rectangle	Trapezoid	Circle
area, A	by	$(b+xy)y$	$\frac{1}{8}(\phi - \sin\phi)D^2$
wetted perimeter, P	$b+2y$	$b+2y\sqrt{1+x^2}$	$\frac{1}{2}\phi D$
top width, B	b	$b+2xy$	$\left(\sin\frac{\phi}{2}\right)D$
hydraulic radius, R	$\frac{by}{b+2y}$	$\frac{(b+xy)y}{b+2y\sqrt{1+x^2}}$	$\frac{1}{4}\left(1-\frac{\sin\phi}{\phi}\right)D$
hydraulic mean depth, D_m	y	$\frac{(b+xy)y}{b+2xy}$	$\frac{1}{8}\left(\frac{\phi-\sin\phi}{\sin(1/2\,\phi)}\right)D$

Table 1.1 Definition and sketches of some Geometric channel properties

1.4 *Velocity distribution in open channels*

The measured velocity in an open channel will always vary across the channel section because of friction along the boundary. Neither is this velocity distribution usually axisymmetric (as it is in pipe flow) due to the existence of the free surface. It might be expected to find the maximum velocity at the free surface where the shear force is zero but this is not the case. The maximum velocity is usually found just below the surface. The explanation for this is the presence of secondary currents which are circulating from the boundaries towards the section centre and resistance at the air/water interface. These have been found in both laboratory measurements and 3d numerical simulation of turbulence.

The figure below shows some typical velocity distributions across some channel cross sections. The number indicates percentage of maximum velocity.

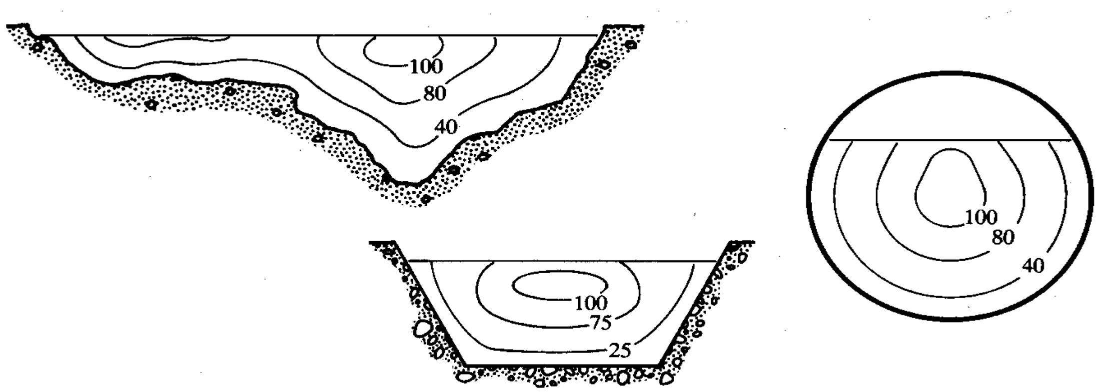


Fig. 1.5 velocity distribution in open channels

Determination of energy and momentum coefficients

To determine the values of α and β the velocity distribution must have been measured (or be known in some way). In irregular channels where the flow may be divided into distinct regions α may exceed 2 and should be included in the Bernoulli equation.

The figure below is a typical example of this situation. The channel may be of this shape when a river is in flood - this is known as a *compound channel.*

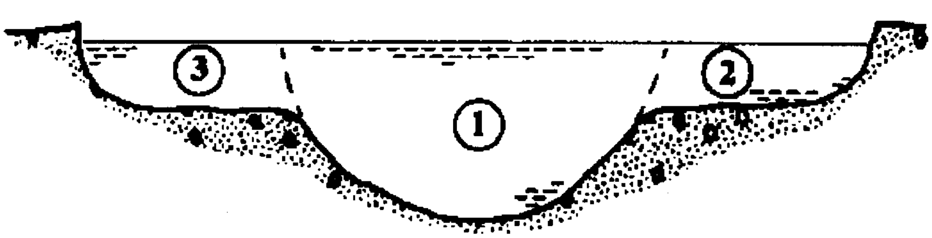


Fig 1.6 compound channel with three regions of flow

If the channel is divided as shown into three regions and making the assumption that α = 1 for each then

$$\alpha = \frac{\int u^3 dA}{\bar{V}^3 A} = \frac{V_1^3 A_1 + V_2^3 A_2 + V_3^3 A_3}{\bar{V}^3 (A_1 + A_2 + A_3)} = \frac{\sum V_i^3 \Delta A_i}{\bar{V}^3 \sum \Delta A_i}$$

Where

$$\bar{V} = \frac{Q}{A} = \frac{V_1 A_1 + V_2 A_2 + V_3 A_3}{A_1 + A_2 + A_3} = \frac{\sum V_i \Delta A_i}{\sum \Delta A_i}$$

Similarly,

$$\beta = \frac{\int u^2 dA}{\bar{V}^2 A} = \frac{V_1^2 A_1 + V_2^2 A_2 + V_3^2 A_3}{\bar{V}^2 (A_1 + A_2 + A_3)} = \frac{\sum V_i^2 \Delta A_i}{\bar{V}^2 \sum \Delta A_i}$$

1.5 Uniform flow and its governing equations in open channel

Uniform flow can occur in long straight runs of constant slope and constant channel cross section. Steady uniform flow is the simplest type of open channel flow to analyse, although in practice it is not of such frequent occurrence as might at first be supposed. Uniform conditions over a length of the channel are achieved only if there are no influences to cause a change of depth, there is no alteration of the cross-section of the stream, and there is no variation in the roughness of the solid boundaries.

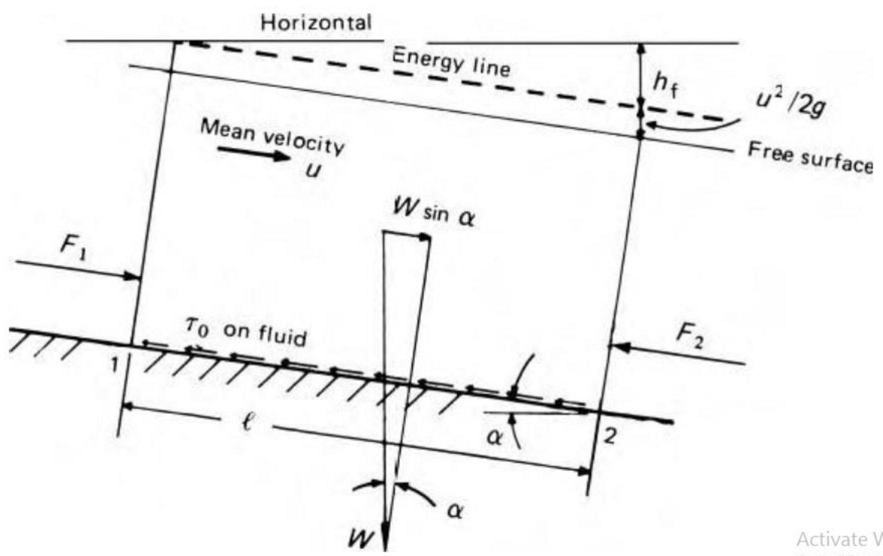
Indeed, strictly uniform flow is scarcely ever achieved in practice, and even approximately uniform conditions are more the exception than the rule. Nevertheless, when uniform flow is obtained the free surface is parallel to the bed of the channel (sometimes termed the *invert*) and the depth from the surface to the bed is then termed the *normal* depth.

Uniform flow equations

The basic formula describing uniform flow is due to the French engineer Antoine de Chezy (1718–98). He deduced the equation from the results of experiments conducted on canals and rivers. The formula can be derived analytically.

In steady uniform (or normal) flow there is no change of momentum, and thus the net force on the liquid is zero. Figure 1.7 represents a stretch of a channel in which these conditions are found. The slope of the channel is constant, the length of channel between the planes 1 and 2 is ***l*** and the (constant) cross-sectional area is A. It is assumed that the stretch of the channel considered is sufficiently far from the inlet (or from a change of slope or of other conditions) for the flow pattern to be fully developed. Now the control volume of liquid between sections 1 and 2 is acted on by hydrostatic forces F1 and F2 at the ends. However, since the cross-sections at 1 and 2 are identical, F1 and F2 are equal in magnitude and have the same line of action; they thus balance and have no effect on the motion of the liquid. Hydrostatic forces acting on the sides and bottom of the control volume are perpendicular to the motion, and so they too have no effect.

The only forces we need consider are those due to **gravity** and the **resistance** exerted by the bottom and sides of the channel. If the average stress at the boundaries is **τ0**, the total resistance force is given by the product of **τ0** and the area over which it acts, that is, by **τ0Pl** where **P** represents the wetted perimeter



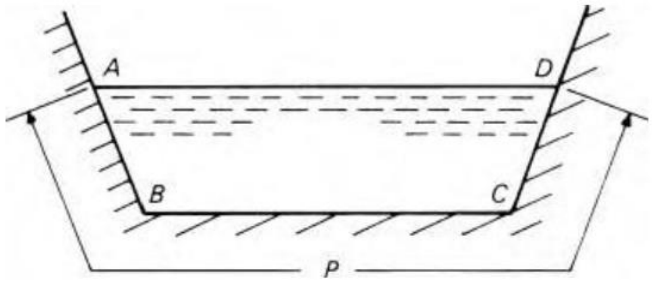


Fig. 1.7 Forces on a channel length in uniform flow

It is important to notice that *P* does *not* represent the total perimeter of the cross-section since the free surface is not included. Only that part of the perimeter where the liquid is in contact with the solid boundary is relevant here, for that is the only part where resistance to flow can be exerted. (The effect of the air at the free surface on the resistance is negligible compared with that of the sides and bottom of the channel.)

For zero net force in the direction of motion, the total resistance must exactly balance the component of the weight **W**. That is

$\tau_0 Pl = W \sin\alpha = A*l*\rho*g \sin\alpha$..........................Eqn 1.1

Hence,

$$\tau_0 = \frac{A}{P}\rho g \sin\alpha \qquad \text{......................................Eqn 1.2}$$

For uniform flow, however, **sin *α* = *hf/l***, the energy gradient. Denoting this by ***So*** we may therefore write:

$$\tau_0 = \left(\frac{A}{P}\right)\rho g S_o, \qquad \tau_0 = R\gamma S_o$$

We now require an expression to substitute for the average stress at the boundary, ***τ*0**. In almost all cases of practical interest, the Reynolds number of the flow in an open channel is sufficiently high for conditions to correspond to the turbulent rough flow regime in which the stress at the boundary is proportional to the square of the mean velocity.

Thus, we can therefore take

$$\tau_o = \frac{1}{2}\rho u^2 f$$

Where the value of the friction factor ***f*** depends on the roughness of the walls of the channel and is independent of ***u***. substituting for $\tau 0$ in eqn 1.2 gives:

$$\frac{1}{2} g u^2 f = \left(\frac{A}{P}\right)\rho g S_o \quad \text{Hence,}$$

$$u^2 = \frac{2g}{f}\frac{A}{P}S_o = \frac{2g}{f}R S_o \quad \text{.....................................Eqn 1.3}$$

Where R = A/P is termed the *hydraulic radius*.
Taking square roots in Eqn 1.3 and putting

$$C = \sqrt{\left(2g/f\right)} \quad \text{................................Eqn 1.4}$$

We arrive at Chezy's equation

$$u = C\sqrt{(R S_o)} \quad \text{..............................Eqn 1.5}$$

Since *u* is the average velocity of flow over the cross-section, the discharge through the channel is given by:

$$Q = Au = AC\sqrt{R S_o} \quad \text{........................Eqn 1.6}$$

The factor C is usually known as Chezy's coefficient. Its dimensional formula is
$[g^{1/2}] = [L^{1/2}T^{-1}]$
Since *f* is dimensionless magnitude. It varies with channel roughness and hydraulic radius.
Many attempts have been made to correlate the large amount of available experimental data and so enable the value of *C* for a particular channel to be predicted. Representative values of C are also estimated for different channel materials and presented in Table 1.1. many formulae have been also derived to predict the value of C. All are based on analyses of experimental results. The simplest expression, and one that is very widely used, is Manning's formula. This formula gives:

$$C = \frac{R^{1/6}}{n}$$

In other words, when combined with Chezy's equation (1.6), Manning's expression becomes:

$$u = \frac{1}{n} R^{2/3} S_o^{1/2} \qquad \text{.......................Eqn 1.7}$$

This formula is often known as Strickler's formula and *1/n* as Strickler's coefficient. It is also known as Gauckler, Kutter, Gauguillet and Hagen formula.

The *n* in equation 1.7 is often known as Manning's roughness coefficient. Representative values of *n* are as given in Table1.2, but it should be realized that they are subject to considerable variation.

Several other names have been associated with the derivation of this formula - or ones similar and consequently in some countries the same equation is named after one of these people. Some of these names are; Strickler, Gauckler, Kutter, Gauguillet and Hagen. The Manning's *n* is also numerically identical to the Kutter *n*

The Manning equation has the great benefits that it is simple, accurate and due to it long extensive practical use; there exists a wealth of publicly available values of *n* for a very wide range of channels.

The value of C can also be estimated using the Ganguillet and Kutter formula, which has been developed based on measurements in open channels of various types.

$$C = \frac{41.65 + \frac{0.00281}{S} + \frac{1.811}{n}}{1 + \left(41.65 + \frac{0.00281}{S}\right)\frac{n}{\sqrt{R}}}$$

Table 1.2. Selected values of C

Type of channel bed	Mean value of C
Smooth cement	90
Well-laid brickwork	70
Cement concrete	70
Natural channel (in good condition)	35
Natural channel (in bad condition)	25

Table 1.3. Selected values of Manning's *n*

Channel type	Surface material and form	Manning's *n* range
River	earth, straight	0.02-0.025
	earth, meandering	0.03-0.05
	gravel (75-150mm), straight	0.03-0.04
	gravel (75-150mm), winding	0.04-0.08
unlined canal	earth, straight	0.018-0.025
	rock, straight	0.025-0.045
lined canal	concrete	0.012-0.017
lab. models	mortar	0.011-0.013
	Perspex	0.009

1.6 Conveyance

Channel conveyance, *K*, is a measure of the carrying capacity of a channel. The *K* is really an agglomeration of several terms in the Chezy or Manning's equation:

$$Q = AC\sqrt{RS_o}$$
$$Q = KS_o^{1/2}$$

So $$K = ACR^{1/2} = \frac{A^{5/3}}{nP^{2/3}}$$

Use of conveyance may be made when calculating discharge and stage in compound channels and also calculating the energy and momentum coefficients in this situation.

Computations in uniform flow

We can use Manning's formula for discharge to calculate steady uniform flow. Two calculations are usually performed to solve uniform flow problems.

1. Discharge from a given depth

2. Depth for a given discharge

In steady uniform flow the flow depth is know as *normal depth.*

As we have already mentioned, and by definition, uniform flow can only occur in channels of constant cross-section (prismatic channels) so natural channel can be excluded. However we will need to use Manning's equation for gradually varied flow in natural channels - so application to natural/irregular channels will often be required.

Example

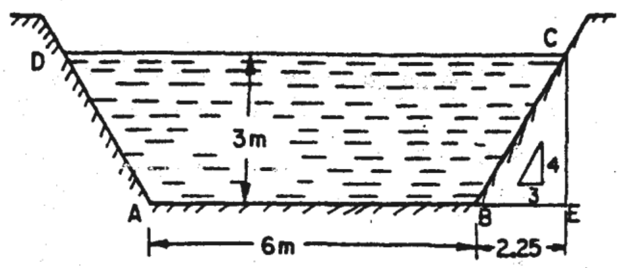
Find the bed slope of trapezoidal channel of bed width 6 m, depth of water 3 m and side slope of 3 horizontal to 4 vertical, when the discharge through the channel is 30 m^3/s.

Take Chezy's constant, C = 70.

Given: b = 6.0 m, d=3.Om, Side slope = 3 horizontal to 4 vertical

Discharge Q =30 m^3/s, Chezy's constant C =70

For depth of flow =3 m= CE: Distance BE = 3x3/4 = 9/4 =2.25m



Top width, CD=AB + 2BE=6.0+2x2.25= 10.50m

Wetted perimeter, P =AD+AB +BC = AB + 2BC

$$= AB + 2\sqrt{BE^2 + CE^2} = 6.0 + 2\sqrt{(2.25)^2 + (3)^2} = 13.5 \text{ m}$$

Area of flow, A = Area of trapezoid ABCD

A = [½ (AB +C D)X CE]

A = [½ (6 + 10.5)X 3]

A = 24.75 m^2

Hydraulic mean depth, R = A / P = 24.75 m^2 / 13.5m

. R =1.833m

$V = C\sqrt{RS}$

$S = V^2/ C^2R$

$S = V^2/ C^2R$, V = Q/A

$S = (30/24.75)^2 / 70^2 \times 1.833$

S = 1/ 6133

Exercises

1) Prove that for a channel of circular section: the depth of flow

   d = 0.81 D for maximum velocity.

   d = 0.95 D for maximum discharge

   Where D = Diameter of circular channel, d = depth of flow.

2) Consider a rectangular channel, built of rubble cement (Manning's *n* _ 0.020) which is 3 m wide, laid on a 1° slope. If the water depth is 2 m, what is the uniform-flow rate?

1.7 Design of most economical section

A Channel section is considered to be the most Economical when it can pass a maximum discharge for the given cross sectional area and when cost of construction of the channel is minimum. But the cost of construction of the channel depends up on Excavation and lining works.

To keep the cost of the excavation and lining minimum the wetted perimeter for a given discharge should be minimum. This condition is utilized for determining the dimension of economical section, of different forms of channels. Most Economical section is also called the best hydraulic section (Most efficient section)

$$Q = AV, \; but, \; V = C\sqrt{RS}, \; chezy's \quad formula$$

$$Q = AC\sqrt{RS}$$

$$= AC\sqrt{\frac{A}{P}S}$$

For a given A, S, C, Q is maximum when P is minimum. This condition will be used for determining the dimensions of most efficient section.

As such this condition determine the dimensions of economical sections of the following

1. Most economical Rectangular channel section
2. Most economical trapezoidal channel section
3. Most economical circular channel section.

Most efficient cross section

A cross-section having such a shape that the wetted perimeter is a minimum is thus, from a hydraulic point of view, the most efficient. Not only is it desirable to use such a section for the sake of obtaining the maximum discharge for a given cross-sectional area, but a minimum wetted perimeter requires a minimum of lining material, and so the most efficient section tends also to be the least expensive.

It may be shown that, of all sections whose sides do not slope inwards towards the top, the semicircle has the maximum hydraulic radius. This mathematical result, however, is not usually the only consideration. Although semicircular channels are in fact built from prefabricated sections, for other forms of construction the semicircular shape is impractical. Trapezoidal sections are very popular, but when the sides are made of a loose granular material its angle of repose may limit the angle of the sides.

Another point is, the most efficient section will give the maximum discharge for a given area and, conversely, the minimum area for a given discharge. This does not, however, necessarily imply that such a channel, if constructed below ground level, requires the minimum excavation. After all, the surface of the liquid will not normally be exactly level with the tops of the sides. Nevertheless the minimum excavation may, in certain instances, be an overriding requirement.

Factors other than the hydraulic efficiency may thus determine the best cross-section to be used for an open channel. However, when the hydraulic efficiency is the chief concern, determining the most efficient shape of section for a given area is simply a matter of obtaining an expression for the hydraulic radius, differentiating it and equating to zero to obtain the condition for the maximum. For example, for a channel section in the form of a symmetrical trapezium with horizontal base (Fig. below), the area A and wetted perimeter P are given by:

$A = bh + h^2 \cot \alpha$ and $P = b + 2h \operatorname{cosec} \alpha$

Since $b = (A/h) - h \cot \alpha$,

$$R = \frac{A}{P} = \frac{A}{(A/h) - h\cot\alpha + 2h\operatorname{cosec}\alpha}$$

For a given value of A, this expression is a maximum when its denominator is a minimum that is when:

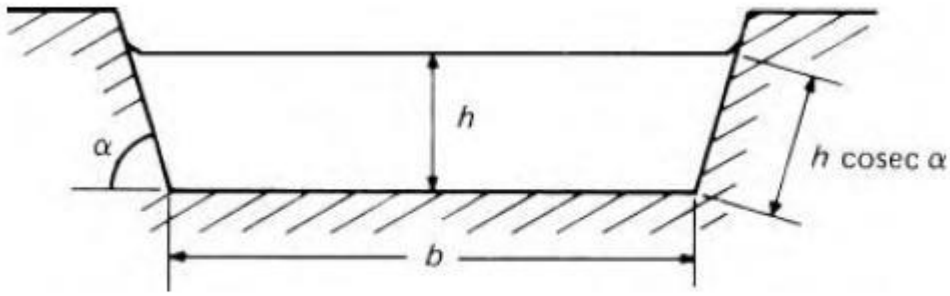
$$\frac{dA}{dh} = (-A/h^2) - \cot\alpha + 2\operatorname{cosec}\alpha = 0.$$

(The second derivative, $2A/h3$, is clearly positive and so the condition is indeed that for a minimum). Thus,

$$A = h^2(2\operatorname{cosec}\alpha - \cot\alpha) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{Eqn. 1.6}$$

Substituting this value in the expression for R gives $R_{max} = h/2$

In other words, for maximum efficiency a trapezoidal channel should be so proportioned that its hydraulic radius is half the central depth of flow. Since a rectangle is a special case of a trapezium (with $\alpha = 90^\circ$) the optimum proportions for a rectangular section are again given by $R = h/2$; taking $A = bh = 2h^2$ (from eqn 1.6) we get $b = 2h$. A further exercise in differential calculus shows that, if α may be varied, a minimum perimeter and therefore maximum R is obtained when $\alpha = 60^\circ$. This condition in conjunction with the first, shows that the most efficient of all trapezoidal section is half a regular hexagon section. The concept of the most efficient section as considered here applies only to channels with rigid boundaries. For channels with erodible boundaries, for example, of sand, the design must take account of the maximum shear stress, τ_o, on the boundary.



Example

1) A trapezoidal channel has side slopes 1 to 1. It is required to discharge 13.75 m^3/s of water with a bed gradient of 1 in 1000. If unlined the value of Chezy's C is 44. If lined with concretes, its value is 60. The Cost per m^3 of excavation is four times the cost per m^2 of lining. The channel is to be the most efficient one. Find whether the lined canal or the unlined canal will be cheaper. What will be the dimensions of that economical canal.

Solution

Cost per m^3 of excavation = 4 x cost per m^2 of lining

Let the cost per m^2 of lining = X

Then cost per m^3 of excavation = 4X

As the channel is most efficient,

Hydraulic mean depth, R = D / 2, D = depth of flow

Also for the most efficient trapezoidal channel we have:

Half of top width = length of sloping side

$$D\sqrt{n^2 + 1} = (B + 2nD) / 2$$

$$D\sqrt{1^2 + 1} = (B + 2 \times 1D) / 2$$

From this, B = 0.828D

$A = (B + nD)D = (0.828D + 1xD)D = 1.828D^2$

i) For the unlined channel: C = 44

$Q = A\,V = A\;C\sqrt{RS}$

$13.75 = 1.828D^2 \times 44\sqrt{(D/2)(1/1000)}$

D = 2.256m

B = 0.828D = 0.828(2.256m) = 1.868m

Now cost of excavation per meter length of unlined channel

= Volume of channel x cost per m^3 of excavation

=(Area of channel x 1) x 4X

=[(B + nD) D x 1] x 4X

=(1.868 + 1 x2.256)x 2.256 x 4X = 37.215X

ii) For the lined channel: C = 60

$Q = A\ V = A\ C\sqrt{RS}$

$13.75 = 1.828D^2 \times 60\sqrt{(D/2)(1/1000)}$

D = 1.992 m

B = 0.828D = 0.828(2.256m) = 1.649m

In case of lined channel, the cost of lining as well as cost of excavation is to be considered.

Now cost of excavation per meter length of lined channel

= Volume of channel x cost per m^3 of excavation

=(Area of channel x 1) x 4X

=[(B + nD) D x 1] x 4X

=(1.649 + 1 x 1.992)x 1.992 x 4X = 29.01X

Cost of lining = Area of lining x cost per rn^2 of lining

= (Perimeter of lining x 1) X

$= (2D\sqrt{n^2 + 1} + B)\ X$

$= 2\times1.992\sqrt{1^2 + 1} + 1.649)\ X$

= 7.283X

Total cost = 29.01X + 7.283X = 36.293X

The total cost of lined channel is 36.293X whereas the total cost of unlined channel is 37.215X.

Hence, lined channel will be cheaper. The dimensions are b = 1.649 m and d = 1.992

Exercise

1) A trapezoidal channel has side slopes of 1 horizontal to 2 vertical and the slope of the bed is 1 in 2000. The area of the section is 42 m2. Find the dimensions of the section if it is most economical. Determine the discharge of the most economical section if C = 60
2) Derive the design equation for the case of efficient triangular cross-section.
3) Why do we optimize wetted perimeter while deriving design equations for most economical and efficient section?

Specific Energy

Specific energy, *Es*, is defined as the energy of the flow with reference to the channel bed as the datum. The concept of specific energy was first introduced by Bakmenteff (1918). With reference to the figure below the total energy of flow with respect to the channel bottom is given by

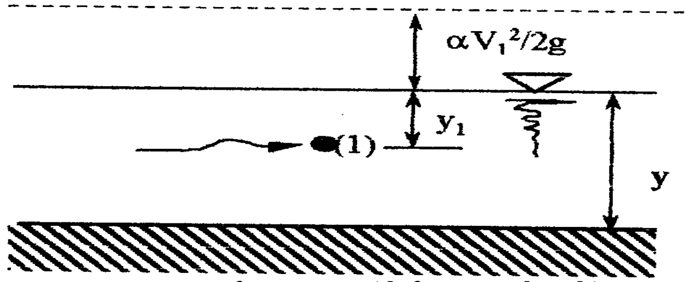


Fig. 1.10 Specific energy (definition sketch)

$$E_{s1} = (y - y_1) + \frac{p_1}{\rho g} + \frac{V_1^2}{2g}$$

$$= y + \frac{V_1^2}{2g}$$

Thus the specific energy at an open channel section is equal to the sum of the flow depth and the velocity head. In the above equation V_1 denotes the velocity of flow at the point of interest, in the figure above at point 1. In practice it is easier to use the average velocity of flow at the section and speak about the specific energy of the flow at a section. How ever the velocity of flow changes from point to point with in the flow and as a result the specific energy changes from stream line to stream line. It is common to use the average velocity of flow with a correction factor. The specific energy computed using the average velocity is taken to apply for all points in the section, i.e. is taken as the specific energy of the section. For steady flow this can be written in terms of discharge Q

$$E_s = y + \frac{\alpha (Q/A)^2}{2g}$$

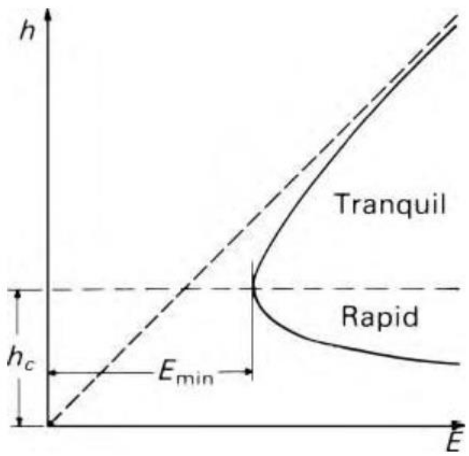
For a rectangular channel of width *b, Q/A* = *q/y*

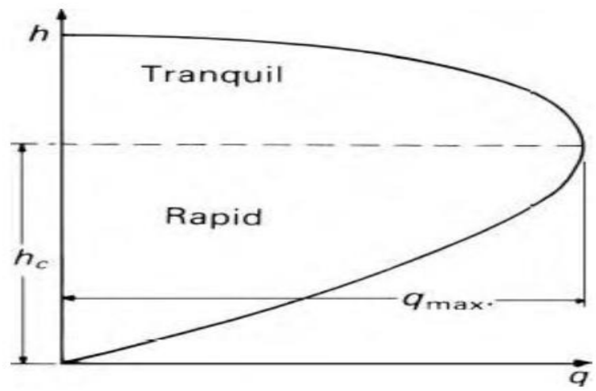
$$E_s = y + \frac{\alpha q^2}{2gy^2}$$

$$(E_s - y)y^2 = \frac{\alpha q^2}{2g} = cons\tan t$$

$$(E_s - y) = \frac{cons\tan t}{y^2}$$

It can be observed that the specific energy is a function of depth of flow, y, only. If one plots the depth of flow as ordinate against the specific energy for a constant Q, the energy diagram is obtained, which is a very useful curve in open channel hydraulics.





For each value of *E* other than the minimum it is seen that there are two possible values of *h*, one greater and one less than *h*c. These two values are known as *alternative depths*. The conditions for the critical depth, for a channel of rectangular section, are those for minimum E which, for a channel of rectangular section, may be found by differentiating the specific energy equations for specific cross section.

$$\frac{\partial E}{\partial h} = 1 + \frac{q^2}{2g}\left(-\frac{2}{h^3}\right)$$

This expression is zero when $q^2/gh^3 = 1$ that is when $h = (q2/g)^{1/3}$. This value of h is the critical depth hc and is written as:

$$h_c = \left(\frac{q^2}{g}\right)^{1/3}$$

$E_m = h_c + gh^3c/2gh^2c = 3/2hc$

Since u = Q/bh = q/h, the velocity (critical velocity) corresponding to the critical depth may be determined from eqn 1.9.

$uc = q/hc = (ghc^3)^{1/2}/hc$

Critical flow and its computation

It can be seen from the specific energy curve that, there is one point C on the curve which has a minimum specific energy, thereby indicating that below this values of the specific energy the given discharge cannot occur. The depth of flow at which the specific energy is minimum is called critical depth yc. Similarly the velocity of flow at the critical depth is known as critical velocity, Vc.

For a given discharge the condition for minimum specific energy can be obtained by differentiating equation 1 with respect to y and then considering $\frac{dE}{dy} = 0$

Therefore,

$$\frac{dE}{dy} = \frac{d}{dy}\left(y + \frac{Q^2}{2gA^2}\right) = \left[1 - \frac{2Q^2}{2gA^3}\left(\frac{dA}{dy}\right)\right] = 0$$

Since Q is a constant and A is a function of y. As shown in figure 4.1, in a channel section if T is the top width of flow then differential water area d_A near the free surface is equal to Td_y i.e d_A = Td_y or $\left(\frac{d_A}{d_y}\right) = T$

Therefore, $1 - \frac{Q^2 T}{gA^3} = 0$ ==➔ $\frac{Q^2}{g} = \frac{A^3}{T}$ 2

Since $V = \frac{Q}{A}$ and hydraulic mean depth $D = \frac{A}{T}$, equation 2 may be written as $\frac{V^2}{g} = 0$, or $V^2 = gD$ 3

Equation may also be written as $\left(\frac{V}{\sqrt{gD}} = 1\right)$. since $\frac{V}{\sqrt{gD}}$ represents for flow the Froude number, Fr = 1.

From equation 2, $Q_c = \left(\sqrt{\frac{A^3}{T}}\right) * \sqrt{g}$ 4

For the channel sections of different shapes the computation of critical flow may be carried out by adopting the procedure as indicated below.

a. Critical flow in Rectangular channel

Since for a channel of rectangular section the top width T is equal to the bottom width B, equation 2 can be further simplified. If it is assumed that q represents the discharge per unit width of the channel section then the total discharge Q passing through a channel of rectangular section of bottom width B may be expressed as Q=B*q or q= Q/B. Corresponding to a critical depth of flow yc the area of rectangular channel section A=Byc.

By substituting these values in equation 2 it becomes

$$\frac{qB}{\sqrt{g}} = By_c\sqrt{\frac{By_c}{B}}$$

$$\frac{q^2}{g} = \left(\frac{q^2}{g}\right)^{\frac{1}{3}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 5$$

b. critical flow in channel sections of other shapes

i. Triangular channel section

For a channel of triangular section with side slope z horizontal to 1 vertical the critical depth yc is given by the following expressions

$$y_c = \left(\frac{2Q^2}{gz^2}\right)^{\frac{1}{5}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 6$$

And $$y_c = \frac{4}{5}E \qquad \square\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 7$$

ii. **Parabolic channel section**

For a channel of parabolic section if the top width at the water surface is expressed as , the critical depth yc is given by the following expressions. 21 ky T □

$$y_c = \left(\frac{27Q^2}{8gk^2}\right)^{\frac{1}{4}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 8$$

$$y_c = \frac{3}{4}E \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 9$$

iii. Trapezoidal channel section

For a channel of trapezoidal section no explicit expressions for yc can be obtained, but the following expressions in terms of dimensionless parameters may be developed which can be used for the computation of the critical depth yc. Thus for a channel section of bottom width B and side slope z horizontal to 1 vertical, we have

$$\left(\frac{Q^2 z^3}{gB^5}\right)=\frac{\left[\frac{B}{zy_c}+1\right]^3}{\left(\frac{B}{zy_c}\right)^5\left(\frac{B}{zy_c}+2\right)} \quad \text{...10}$$

$$\frac{Ez}{B}=\frac{\left[3\frac{B}{zy_c}+5\right]}{2\frac{B}{zy_c}\left(\frac{B}{zy_c}+2\right)} \quad \text{...11}$$

Example1

For a constant specific energy of 1.8 N.m/N, calculate the maximum discharge that may occur in a rectangular channel 5m wide.

Solution

For a given specific energy the discharge is maximum when the flow is in critical state.

Thus

$$y = y_c = \frac{2}{3}E = \frac{2}{3}*1.8 = 1.2m$$

Further for a rectangular channel,

$$y_c = \left(\frac{q^2}{g}\right)^{1/3}$$

$$1.2 = \left(\frac{q^2}{g}\right)^{1/3}$$

$$q = \left[1.2^3 * 9.81\right]^{1/2}$$

$$q = 4.12m^3 / ses\ per\ m$$

Therefore,

$$Q = (4.12 * 5)$$

$$\mathbf{Q = 20.6\ m^3/sec}$$

Flow in channel transition

Rapid changes in stage and velocity occur whenever there is a sudden change in cross-section, a very steep bed-slope or some obstruction in the channel. This type of flow is termed **rapidly varied flow.** Typical examples are flow over sharp-crested weirs and flow through regions of greatly changing cross-section (Venturi flumes and broad-crested weirs).

Rapid change can also occur when there is a change from super-critical to sub-critical flow (see later) in a channel reach at a hydraulic jump.

In these regions the surface is highly curved and the assumptions of hydro static pressure distribution and parallel streamlines do not apply. However it is possibly to get good approximate solutions to these situations yet still use the energy and momentum concepts outlined earlier. The solutions will usually be sufficiently accurate for engineering purposes.

Flow over a raised hump – revisited: Application of the Specific energy equation.

The specific energy equation may be used to solve the raised hump problem. The figure below shows the hump and stage drawn alongside a graph of Specific energy *Es* against *y.*

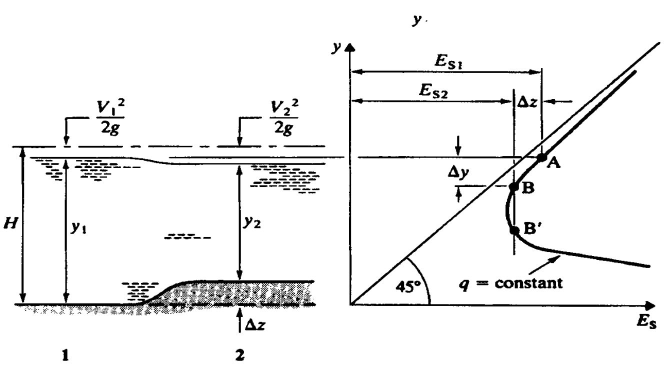


Figure of raised bed hump and graph of specific energy

The Bernoulli equation was applied earlier to this problem and the equation from that example may be written in terms of specify energy:

$$E_{s1} = E_{s2} + \Delta z$$

These points are marked on the figure. Point A on the curve corresponds to the specific energy at section 1 in the channel, but Point B or Point B' on the graph may correspond to the specific energy at point 2 in the channel.

All point in the channel between point 1 and 2 **must** lie on the specific energy curve between point A and B or B'. To reach point B' then this implies that $E_{s1} - E_{s2} > \Delta z$ which is not physically possible. So point B on the curve corresponds to the specific energy and the flow depth at section 2.

Critical, Sub-critical and super critical flow

The specific energy change with depth was plotted above for a constant discharge *Q*, it is also possible to plot a graph with the specific energy fixed and see how *Q* changes with depth. These two forms are plotted side by side below.

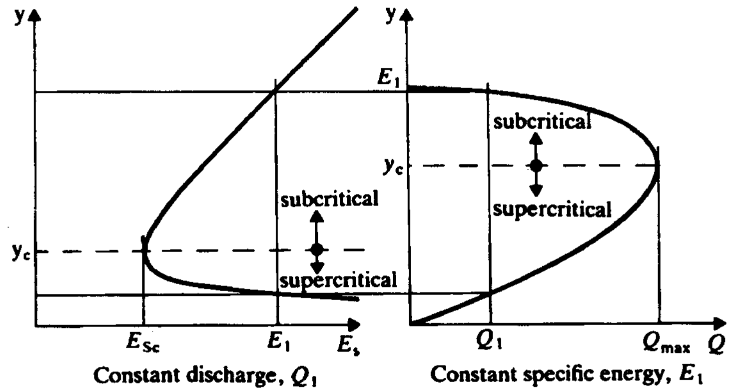


Figure of variation of Specific Energy and Discharge with depth.

From these graphs we can identify several important features of rapidly varied flow.

For a fixed discharge:

1. The specific energy is a minimum, Esc, at depth Yc, This depth is known as critical depth.
2. For all other values of Es there are two possible depths. These are called alternate depths. For

   subcritical flow $y > y_c$

   supercritical flow $y < y_c$

For a fixed Specific energy :

1. The discharge is a maximum at critical depth, *Yc.*
2. For all other discharges there are two possible depths of flow for a particular Es i.e. there is a sub-critical depth and a super-critical depth with the same Es.

An equation for critical depth can be obtained by setting the differential of *Es* to zero:

$$E_s = y + \frac{\alpha (Q/A)^2}{2g}$$

$$\frac{dE_s}{dy} = 0 = 1 + \frac{\alpha Q^2}{2g} \frac{d}{dA}\left(\frac{1}{A^2}\right)\frac{dA}{dy}$$

Since δA=Bδy, in the limit dA/dy = B and

$$0 = 1 - \frac{\alpha Q^2}{2g} B_c 2A_c^{-3}$$

$$\frac{\alpha Q^2 B_c}{g A_c^3} = 1$$

For a rectangular channel $Q = qb$, $B = b$ *and* $A = by$, and taking $\alpha = 1$ this equation becomes

$$y_c = \left(\frac{q^2}{g}\right)^{1/3} \quad \text{as } V_c\, y_c = q$$

$$V_c = \sqrt{g y_c}$$

Substituting this in to the specific energy equation

$$E_{sc} = y_c + \frac{V_c^2}{2g} = y_c + \frac{y_c}{2}$$

$$y_c = \frac{2}{3} E_{sc}$$

The Froude number

The Froude number is defined for channels as:

$$F_N = \frac{V}{\sqrt{g D_m}}$$

Its physical significance is the ratio of inertial forces to gravitational forces squared

$$F_N^2 = \frac{Inertial\ Force}{Gravitational\ Force}$$

It can also be interpreted as the ratio of water velocity to wave velocity

$$F_N = \frac{Water\ Velocity}{Wave\ Velocity}$$

This is an extremely useful non-dimensional number in open-channel hydraulics.

Its value determines the regime of flow - sub, super or critical, and the direction in which disturbances travel

Fr < 1 sub-critical

- water velocity > wave velocity
- upstream levels affected by downstream controls

Fr = 1 critical

Fr > 1 super-critical

- water velocity < wave velocity
- upstream levels not affected by downstream controls

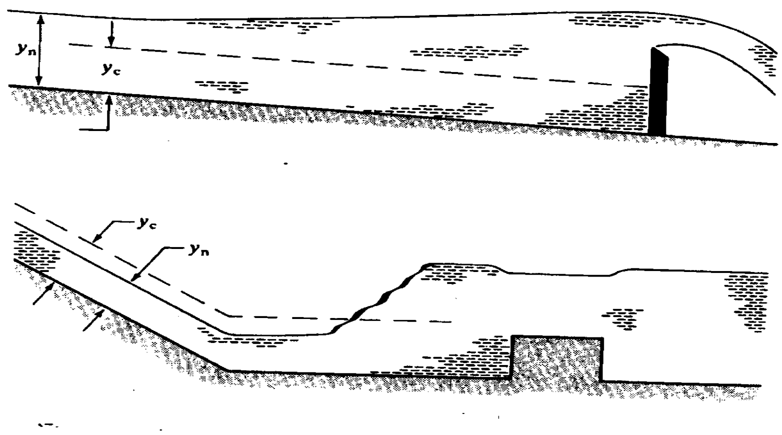


Figure of sub and super critical flow and transmission of disturbances

The Hydraulic jump

The hydraulic jump is an important feature in open channel flow and is an example of rapidly varied flow. A hydraulic jump occurs when a super-critical flow and a sub-critical flow meet. The jump is the mechanism for the two surfaces to join. They join in an extremely turbulent manner which causes large energy losses.

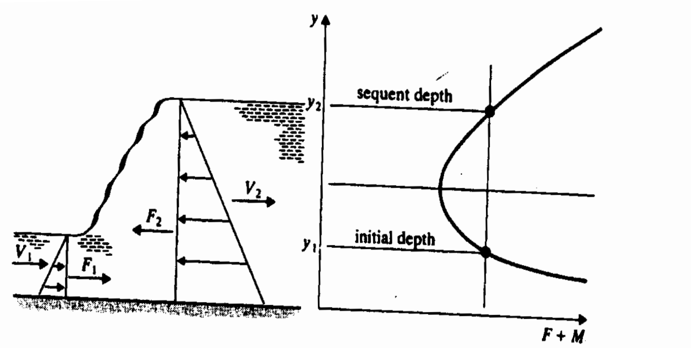


Figure of forces applied to the control volume containing the hydraulic jump

Because of the large energy losses the energy or specific energy equation cannot be use in analysis, the momentum equation is used instead.

Resultant force in x- direction = $F_1 - F_2$
Momentum change = $M_2 - M_1$

$$F_1 - F_2 = M_2 - M_1$$

Or for a constant discharge

$$F_1 + M_1 = F_2 + M_2 = \text{constant}$$

For a rectangular channel this may be evaluated using

$$F_1 = \rho g \frac{y_1}{2} y_1 b \qquad F_2 = \rho g \frac{y_2}{2} y_2 b$$

$$M_1 = \rho Q V_1 \qquad M_2 = \rho Q V_2$$

$$= \rho Q \frac{Q}{y_1 b} \qquad = \rho Q \frac{Q}{y_2 b}$$

Substituting for these and rearranging gives

$$y_2 = \frac{y_1}{2}\left(\sqrt{1+8F_{N1}^2} - 1\right)$$

or

$$y_1 = \frac{y_2}{2}\left(\sqrt{1+8F_{N2}^2} - 1\right)$$

So knowing the discharge and either one of the depths on the upstream or downstream side of the jump the other - or *conjugate depth* - may be easily computed.

More manipulation with the above equation and the specific energy give the energy loss in the jump as

$$\Delta E = \frac{(y_2 - y_1)^3}{4 y_1 y_2}$$

These are useful results and which can be used in gradually varied flow calculations to determine water surface profiles.

A hydraulic jump will only occur if the upstream flow is super-critical. The higher the upstream Froude number the higher the jump and the greater the loss of energy in the jump.

Purposes of hydraulic jump:-

- ✓ To increase the water level on the d/s of the hydraulic structures
- ✓ To reduce the net up lift force by increasing the downward force due to the increased depth of water,
- ✓ To increase the discharge from a sluice gate by increasing the effective head causing flow,
- ✓ For aeration of drinking water
- ✓ For removing air pockets in a pipe line

Analysis of hydraulic jump

Assumptions

1) The length of the hydraulic jump is small, consequently, the loss of head due to friction is negligible,

2) The channel is horizontal as it has a very small longitudinal slope. The weight component in the direction of flow is negligible.

3) The portion of channel in which the hydraulic jump occurs is taken as a control volume & it is assumed the just before & after the control volume, the flow is uniform & pressure distribution is hydrostatic.

Let us consider a small reach of a channel in which the hydraulic jump occurs.
The momentum of water passing through section (1) per unit time is given as:

$$\frac{p_1}{t} = \frac{rQV_1}{g} = \rho Q V_1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

Momentum at section (2) per unit time is:

$$\frac{p_2}{t} = \frac{rQV_2}{g} = \rho Q V_2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(ii)}$$

Rate of change of momentum b/n section 1 & 2

$$\frac{\Delta P}{t} = \rho Q (V_2 - V_1) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(iii)}$$

The net force in the direction of flow = F1-F2 ……………….(iv)

$F_1 = \gamma A_1 \overline{Y}_1$, $F_2 = \gamma A_2 \overline{Y}_2$

$\overline{Y}_1$ & $\overline{Y}_2$ are the center of pressure at section (1) & (2)

Therefore $F_1 - F_2 = \Delta M = \rho Q (V_2 - V_1)$

$$\gamma A_1 \overline{Y}_1 - \gamma A_2 \overline{Y}_2 = \frac{\gamma Q}{g}(V_2 - V_1) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(v)}$$

From continuity eqn. Q= A*V, ⟹ V= Q/A, so

$$\gamma A_1 \overline{Y}_1 - \gamma A_2 \overline{Y}_2 = \frac{\gamma Q}{g}\left(\frac{Q}{A_2} - \frac{Q}{A_1}\right)$$

$$A_1 \overline{Y}_1 - A_2 \overline{Y}_2 = \frac{Q^2}{g}\left(1/A_2 - 1/A_1\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (iv)$$

Rearranging this eqn.:

$$\underbrace{\left[\frac{Q^2}{gA_1} + A_1\overline{Y}_1\right]}_{M_1} = \underbrace{\left[\frac{Q^2}{gA_2} + A_2\overline{Y}_2\right]}_{M_2} = \text{Constant. ....(vii)}$$

M_1 and M_2 are the specific forces at section (1) & (2) indicates that these forces are equal before & after the jump.

Y_1= initial depth
Y_2 = sequent depth

Hydraulic jump in a rectangular channel

$A_1=By_1$
$A_2= By_2$ — the section has uniform width (B)

$\overline{Y}_1 = \frac{Y_1}{2}$, $\overline{Y}_2 = \frac{Y_2}{2}$

Now from eqn. (Vii) above:

$$\frac{Q^2}{gBy_1}+By_1\left(\frac{y_1}{2}\right)=\frac{Q}{gBy_2}+By_2*\left({}^{y_2}\!/_2\right)$$

$$\frac{Q^2}{gBy_1}+\frac{By_1^2}{2}=\frac{Q^2}{Bgy_2}+\frac{By_2^2}{2}\text{...}(viii)$$

Flow per unit width of q= Q/B ⟹ Q=qB, then eqn. (viii) becomes

$$\frac{q^2B^2}{Bgy_1}+\frac{By_1^2}{2}=\frac{q^2B^2}{Bgy_2}+\frac{By_2^2}{2}$$

$$\frac{q^2}{g}\left[\frac{1}{y_1}-\frac{1}{y_2}\right]=\frac{y_2^2-y_1^2}{2}\text{ ....}(.ix)$$

$$\frac{2q^2}{g}=y_1y_2\ \frac{\left(y_2^2-y_1^2\right)}{\left(y_2-y_1\right)}$$

$$\frac{2q^2}{g}=y_1y_2(y_1+y_2)\text{..}(x)$$

$$y_2y_1^2+y_1y_2^2-\frac{2q^2}{g}=0\text{..}(xi)$$

This is quadratic eqn. & the solution is given as

$$y_1=\frac{-y_2}{2}+\left(\sqrt{\left(\frac{y_2}{2}\right)^2+\frac{2q^2}{gy_2}}\right)\text{...}(xii)(a)$$

$$y_2=\frac{-y_1}{2}+\left(\sqrt{\left({}^{y_1}\!/_2\right)^2+\frac{2q^2}{gy_2}}\right)\text{...}(b)$$

$$y_1=\frac{y_2}{2}\left(-1+\sqrt{1+\frac{8q^2}{gy_2^3}}\right)\text{..}(c)$$

$$y_2=\frac{y_1}{2}\left(-1+\sqrt{1+\frac{8q^2}{gy_1^3}}\right)\text{..}(xii)(d)$$

The ratio of conjugate depths;

$$y_1/y_2 = 1/2\left(-1+\sqrt{1+\frac{8q^2}{gy_2^3}}\right)\text{...}(xii)(e)$$

$$y_2/y_1 = 1/2\left(-1+\sqrt{1+\frac{8q^2}{gy_1^3}}\right)\text{..}(f)$$

$$F_1 = \frac{V_1}{\sqrt{gy_1}} =, \qquad F_2 = \frac{V_2}{\sqrt{gy_2}} = \frac{q/y_2}{\sqrt{gy_2}} = \frac{q}{\sqrt{gy_2^3}}$$

$$Therefore \quad \frac{y_1}{y_2} = 1/2\left(-1+\sqrt{1+8F_2^2}\right)\text{.......................}(g)$$

$$\frac{y_2}{y_1} = 1/2\left(-1+\sqrt{1+8F_1^2}\right)\text{...}(h)$$

Energy dissipation in a Hydraulic Jump

The head loss $h_{l.f}$ caused by the jump is the drop in energy from section (1) to (2) or:

$h_{lf} = \Delta E = E_1 - E_2$

$$=\left(y_1+\frac{V_1^2}{2g}\right)-\left(y_2+\frac{V_2^2}{2g}\right)\text{.....................................}(1)a$$

$$=\left(y_1+\frac{q^2}{2gy_1^2}\right)-\left(y_2+\frac{q^2}{2gy_2^2}\right)\text{...................................}(b)$$

$$=\frac{q^2}{2g}\left(\frac{y_2^2-y_1^2}{y_1^2y_2^2}\right)-(y_2-y_1)\text{.....................................}(c)$$

From eqn. (x) substituting: $\frac{2q^2}{g} = y_1y_2(y_1+y_2)$ in to this eqn. & by rearranging:

$$h_{lf} = \Delta E = \frac{(y_2-y_1)^3}{4y_1y_2}\text{..}(2)$$

Therefore power lost = γ Q h_{lf} (kw)…………………(3)

Types of Hydraulic jump

Hydraulic jumps are classified according to the upstream Froude number and depth ratio.

F_1	Y_2/y_1	Classification
<1	1	Jump impossible

1-1.7	1-2	Undular jump (standing wave)
1.7-2.5	2-3.1	Weak jump
2.5-4.5	3.1-5.9	Oscillating jump
4.5-9.0	5.9-12	Steady jump (45-70% energy loss)
>9.0	>12	Strong or chopping jump (=85% energy loss)

Example

1) A sluice gate discharges water into a horizontal rectangular channel with a velocity of 6 m / s and depth of flow is 0.4 m. The width of the channel is 8 m. Determine whether a hydraulic jump will occur, and if so, find its height and loss of energy per kg of water. Also determine the power lost in the hydraulic jump.

Solution

Discharge per unit width, q = Q / b = (V1 x b x d1) / d1 = V1 x b

$$q = V1 \times b = 6 \times 0.4 = 2.4 m^2/s$$

Froude number on the upstream side, $Fr1 = \frac{V1}{\sqrt{gd1}}$,

$$Fr1 = \frac{6}{\sqrt{9.81x0.4}},$$

$$Fr1 = 3.09$$

As Froude number is more than one, the flow is shooting on the upstream side. Shooting flow is unstable flow and it will convert itself into streaming flow by raising its height and hence hydraulic jump will take place.

Let the depth of hydraulic jump = d2

$$\frac{d_2}{d_1} = \frac{1}{2}\left(\sqrt{1+8F^2{}_1} - 1\right)$$

$$\frac{d_2}{0.4} = \frac{1}{2}\left(\sqrt{1+8x3.09^2} - 1\right)$$

$d_2 = 1.525m$

Height of hydraulic jump = d2 — d1 = 1.525 —0.4 = 1.125 m.

Loss of energy per kg of water is, $\Delta E = \frac{(y_2 - y_1)^3}{4y_1y_2}$

$$\Delta E = \frac{(1.525 - 0.4)^3}{4(1.525)(0.4)}$$

$\Delta E = 0.5835$ m-kg/kg of water.

Power lost in kW, P = $\rho g Q h_L / 1000$,

Where Q = V x area=V1xd1xb= 6x0.4x 8=19.2m^3/s

Power lost in kW, P = (1000 x 9.81 x 19.2 x 0.5835) / 1000
Power lost, P = 109.9 kW.

Exercise

1) A sluice gate controls the flow in a channel of width 800 mm. When the flow rate is 1.65 $m^3\ s^{-1}$ supercritical flow occurs just downstream of the gate which undergoes transition almost immediately through a hydraulic jump to a depth of 1.16 m. Assuming no energy loss through the gate, calculate:
(a) The depths just upstream and downstream of the gate;
(b) The total head through the gate and the total head downstream of the hydraulic jump.

2) A rectangular channel carrying 10 $m^3\ s^{-1}$ undergoes an abrupt expansion from width 4 m to width 8 m, triggering a hydraulic jump. The upstream depth is 0.5 m. Stating assumptions clearly, find the downstream depth.