

# Addis Ababa Institute of Technology

Department of civil and Environmental  
Engineering

Hydraulics-II (CENG-2162)

By Selam Belay

2020 A.C.

# Open Channel flow



# Main Topics

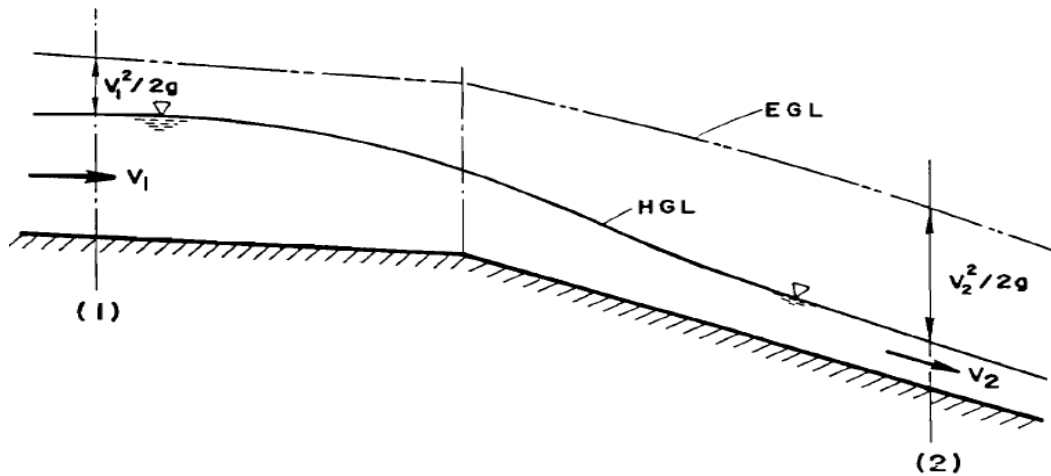
- ▶ General Characteristics of Open-Channel Flow
- ▶ Energy Considerations
- ▶ Uniform Depth Channel Flow
- ▶ Gradually Varies Flow
- ▶ Rapidly Varies Flow

# Introduction

- ▶ Open channel flow involves the flows of a liquid in a channel or conduit that is not completely filled.
- ▶ There exists a free surface between the flowing fluid (usually water) and fluid above it (usually the atmosphere).
- ▶ The main deriving force is the fluid weight-gravity forces the fluid to flow downhill.
- ▶ Under steady, fully developed flow conditions, the component of the weight force in the direction of flow is balanced by equal and opposite shear force between the fluid and the channel surface.

# Open Channel Flow vs. Pipe Flow

- ▶ There can be no pressure force driving the fluid through the channel or conduit.
- ▶ For steady, fully developed channel flow, the pressure distribution within the fluid is merely hydrostatic.
- ▶ The Hydraulic grade line of open channel flow is exactly coinciding with the water surface



HGL and EGL in open channel flow

# Examples of Open Channel Flow

- ▶ The natural drainage of water through the numerous creek and river systems.
- ▶ The flow of rainwater in the gutters of our houses.
- ▶ The flow in canals, drainage ditches, sewers, and gutters along roads.
- ▶ The flow of small rivulets, and sheets of water across fields or parking lots.
- ▶ The flow in the chutes of water rides.

# Variables in Open-Channel Flow

- ▶ Cross-sectional shape.
- ▶ Bends.
- ▶ Bottom slope variation.
- ▶ Character of its bounding surface.

➡ Most open-channel flow results are based on correlation obtained from model and full-scale experiments.

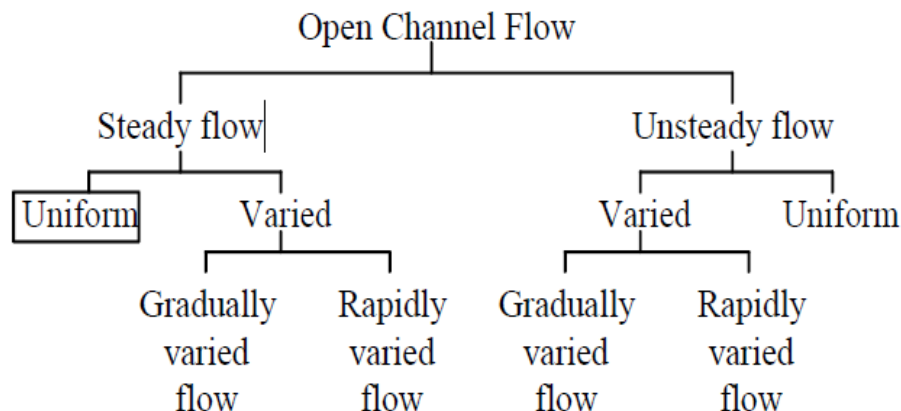
➡ Additional information can be gained from various analytical and numerical efforts.

# General Characteristics of Open-Channel Flow



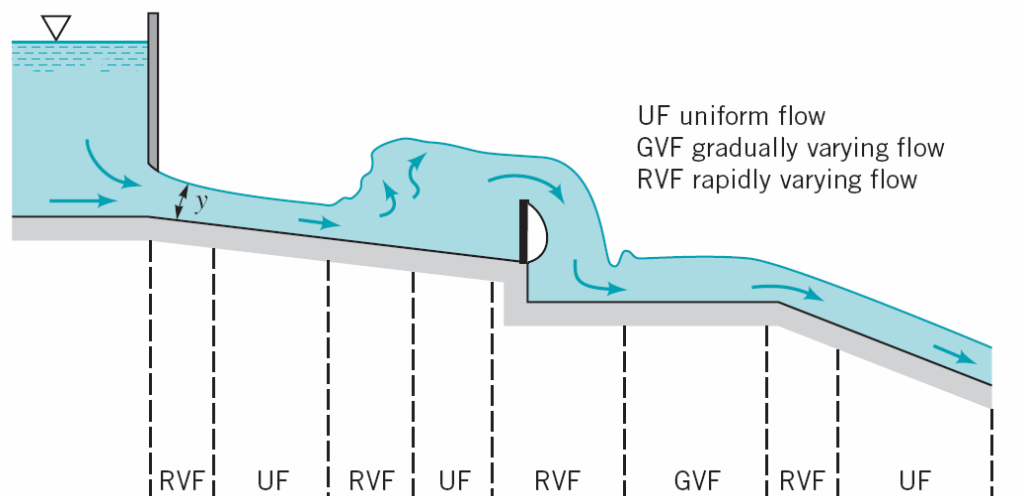
# Classification of Open-Channel Flow

- ▶ For open-channel flow, the existence of a free surface allows additional types of flow.
- ▶ The extra freedom that allows the fluid to select its free-surface location and configuration allows important phenomena in open channel flow that cannot occur in pipe flow.
- ▶ The fluid depth,  $y$ , varies with time,  $t$ , and distance along the channel,  $x$ , are used to classify open-channel flow:



# Classification - Type I

- ▶ Uniform flow (UF): The depth of flow does not vary along the channel ( $dy/dx=0$ ).
- ▶ Non uniform flows:
  - ▶ Rapidly varying flows (RVF):  
The flow depth changes considerably over a relatively short distance  $dy/dx \sim 1$ .
  - ▶ Gradually varying flows (GVF):  
The flow depth changes slowly with distance  $dy/dx \ll 1$ .



# Classification - Type II

$$R_e = \rho V R_h / \mu$$

$V$  is the average velocity of the fluid.  
 $R_h$  is the hydraulic radius of the channel.

- ❖ Laminar flow:  $Re < 500$ .
- ❖ Transitional flow:
- ❖ Turbulent flow:  $Re > 12,500$ .

Most open-channel flows involve water (which has a fairly small viscosity) and have relatively large characteristic lengths, it is uncommon to have laminar open-channel flows.

# Classification - Type III

$$Fr = V / \sqrt{gl}$$

- ▶ Critical Flow: Froude number  $Fr = 1$ .
- ▶ Subcritical Flow: Froude number  $Fr < 1$ .
- ▶ Supercritical Flow: Froude number  $Fr > 1$

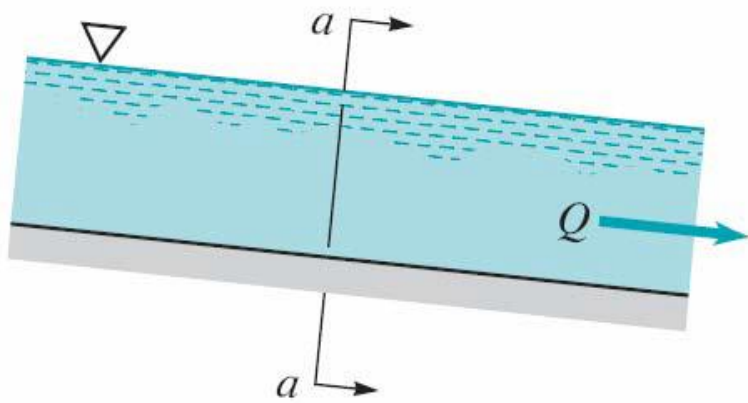
# Uniform Depth Channel Flow



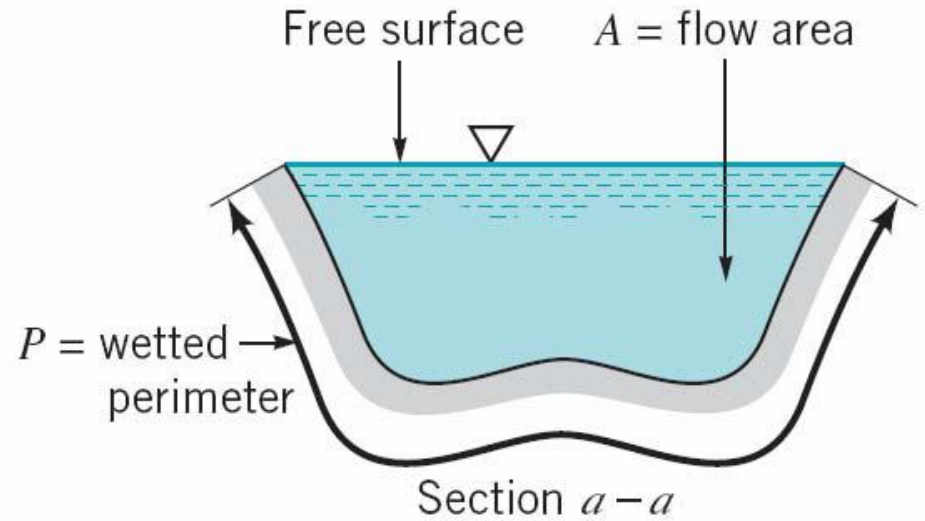
# Uniform Depth Channel Flow

- ▶ Many channels are designed to carry fluid at a uniform depth all along their length.
  - Irrigation canals.
  - Nature channels such as rivers and creeks.
- ▶ Uniform depth flow ( $dy/dx=0$ ) can be accomplished by adjusting the bottom slope,  $S_0$ , so that it precisely equal the slope of the energy line,  $S_f$ .
- ▶ A balance between the potential energy lost by the fluid as it coasts downhill and the energy that is dissipated by viscous effects (head loss) associated with shear stress throughout the fluid.

# Contd...



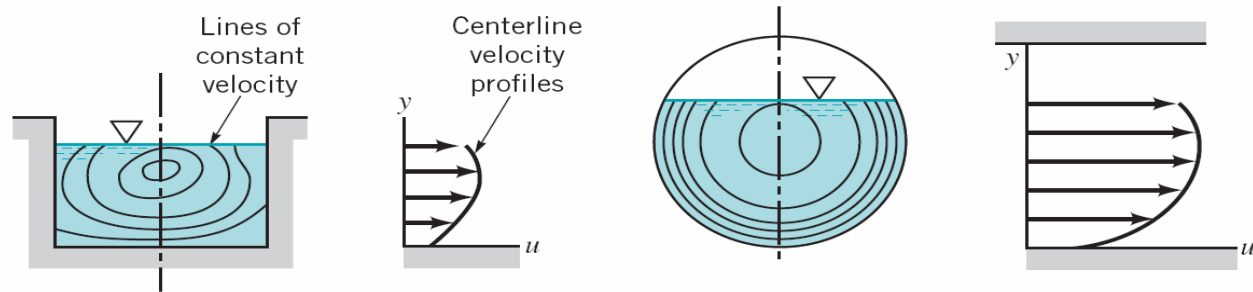
(a)



(b)

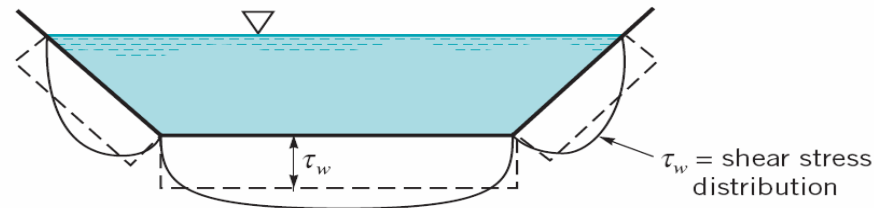
Uniform flow in an open channel.

# Contd...



(a)

— Actual  
- - - Uniform

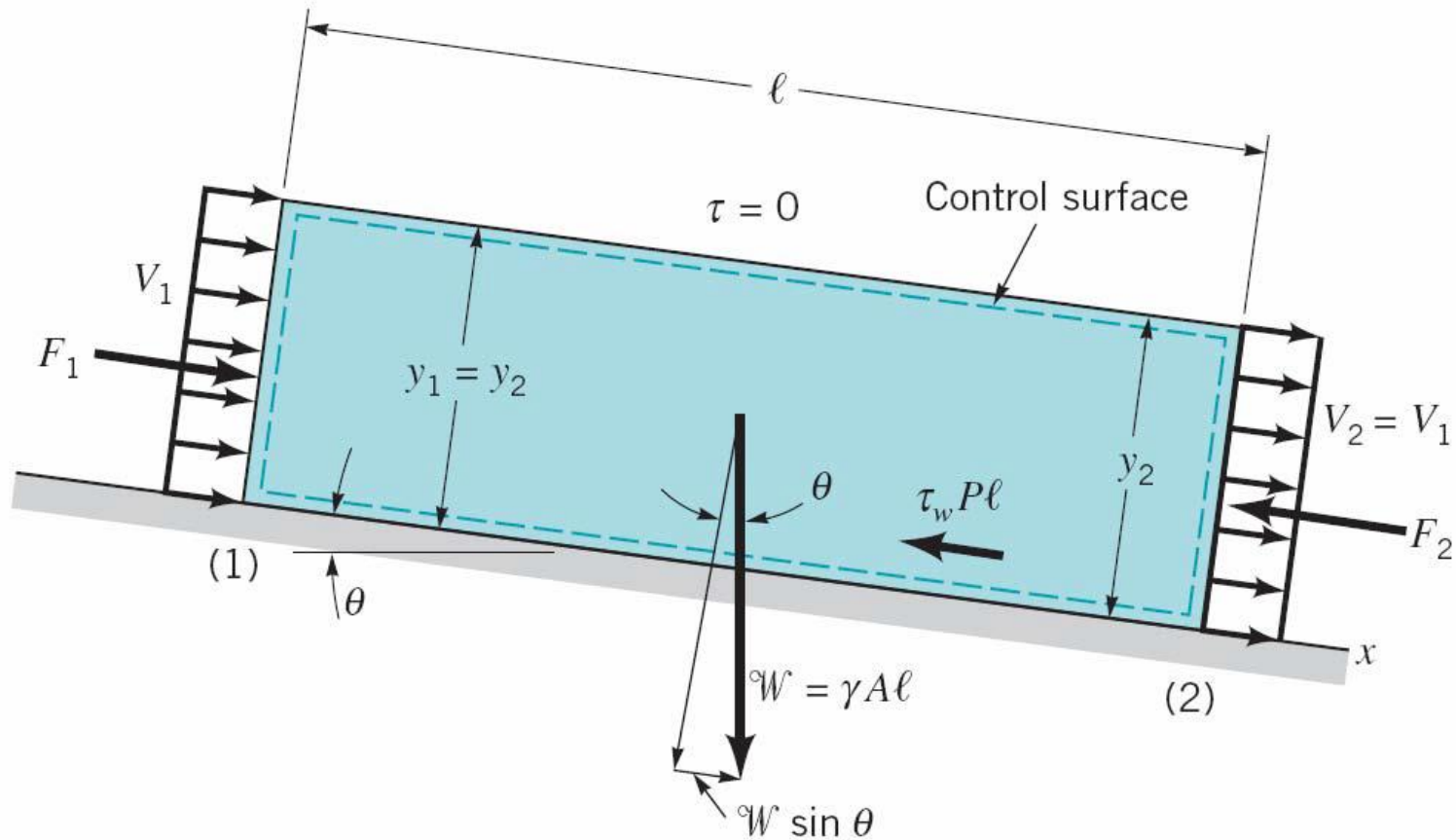


(b)

- ▶ Typical velocity and shear stress distributions in an open channel:
- ▶ (a) velocity distribution throughout the cross section. (b) shear stress distribution on the wetted perimeter.



# The Chezy & Manning Equation



Control volume for uniform flow in an open channel

## Contd...

- ▶ Under the assumption of steady uniform flow, the x component of the momentum equation

$$\sum F_x = \rho Q(V_2 - V_1) = 0$$

$$F_1 - F_2 - \tau_w P \ell + W \sin \theta = 0 \quad (15)$$

where  $F_1$  and  $F_2$  are the hydrostatic pressure forces across either end of the control volume.

$P$  is wetted perimeter.

## Contd...

$$y_1 = y_2 \rightarrow F_1 = F_2$$

$$(15) \quad \Rightarrow \quad -\tau_w Pl + W \sin \theta = 0$$

$$\tau_w = \frac{W \sin \theta}{Pl} \quad W = \gamma Al \quad R_h = \frac{A}{P}$$

$$\tau_w = \frac{\gamma Al S_o}{Pl} = R_h S_o \quad (16)$$

Wall shear stress is proportional to the dynamic pressure  $\tau_w \propto \rho \frac{V^2}{2}$

$$(Chapter 8) \quad \Rightarrow \quad \tau_w = K \rho \frac{V^2}{2}$$

**K** is a constant dependent upon the roughness of the pipe

# Contd...

$$(16) \implies K\rho \frac{V^2}{2} = rR_h S_o \implies V = C\sqrt{R_h S_o} \quad (17)$$

C is termed the Chezy coefficient

Chezy equation

Was developed in 1768 by A. Chezy (1718-1798), a French engineer who designed a canal for the Paris water supply.

$$(17) \implies V \propto S_o^{1/2} \quad \text{Reasonable}$$

$$V \propto \sqrt{R_h} \longrightarrow V \propto R_h^{2/3}$$

**Manning Equation**

## Contd...

- ▶ In 1889, R. Manning (1816-1897), an Irish engineer, developed the following somewhat modified equation for open-channel flow to more accurately describe the  $R_h$  dependence:

$$V = \frac{R_h^{2/3} S_o^{1/2}}{n} \quad (18) \quad \text{Manning equation}$$

$n$  is the Manning resistance coefficient. Its value is dependent on the surface material of the channel's wetted perimeter and is obtained from experiments.

**It has the units of  $\text{s/m}^{1/3}$  or  $\text{s./ft}^{1/3}$**

## Contd...

$$(18) \Rightarrow V = \frac{\kappa}{n} R_h^{2/3} S_o^{1/2} \quad (19) \quad \text{Where } \kappa = 1 \text{ if SI units are used, } \kappa = 1.49 \text{ if BG units are used.}$$

$$Q = \frac{\kappa}{n} A R_h^{2/3} S_o^{1/2} \quad (20)$$

The best hydraulic cross section is defined as the section of minimum area for a given flowrate  $Q$ , slope,  $S_o$ , and the roughness coefficient,  $n$ .

$$Q = \frac{\kappa}{n} A \left( \frac{A}{P} \right)^{2/3} S_o^{1/2} = \frac{\kappa}{n} \frac{A^{5/3} S_o^{1/2}}{P^{2/3}} \Rightarrow A = \left[ \frac{nQ}{\kappa S_o^{1/2}} \right]^{3/5} P^{2/5}$$

constant

A channel with minimum  $A$  is one with a minimum  $P$ .

# Values of the Manning Coefficient, $n$

■ **TABLE 10.1**

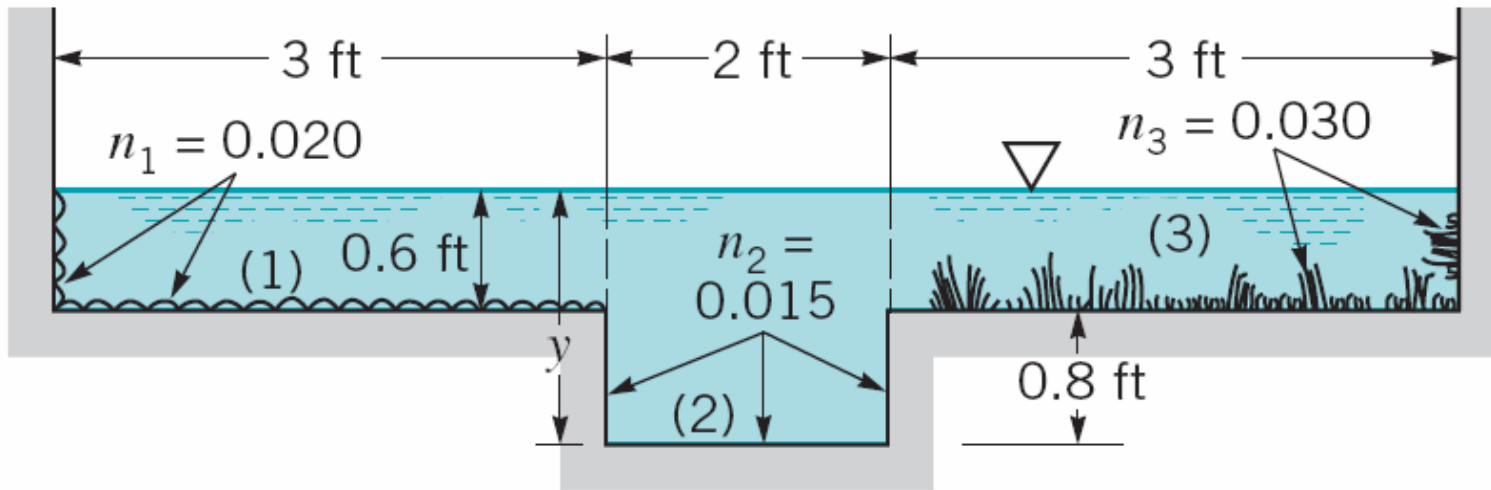
**Values of the Manning Coefficient,  $n$  (Ref. 6)**

Wetted Perimeter	$n$	Wetted Perimeter	$n$
<b>A. Natural channels</b>		<b>D. Artificially lined channels</b>	
Clean and straight	0.030	Glass	0.010
Sluggish with deep pools	0.040	Brass	0.011
Major rivers	0.035	Steel, smooth	0.012
		Steel, painted	0.014
<b>B. Floodplains</b>		Steel, riveted	0.015
Pasture, farmland	0.035	Cast iron	0.013
Light brush	0.050	Concrete, finished	0.012
Heavy brush	0.075	Concrete, unfinished	0.014
Trees	0.15	Planed wood	0.012
		Clay tile	0.014
<b>C. Excavated earth channels</b>		Brickwork	0.015
Clean	0.022	Asphalt	0.016
Gravelly	0.025	Corrugated metal	0.022
Weedy	0.030	Rubble masonry	0.025
Stony, cobbles	0.035		

# Uniform Flow, Variable Roughness

## Example 2

- ▶ Water flows along the drainage canal having the properties shown in the Fig below, If the bottom slope is  $S_0 = 1 \text{ ft}/500 \text{ ft} = 0.002$ , estimate the flow rate when the depth is  $y = 0.8 \text{ ft} + 0.6 \text{ ft} = 1.4 \text{ ft}$ .

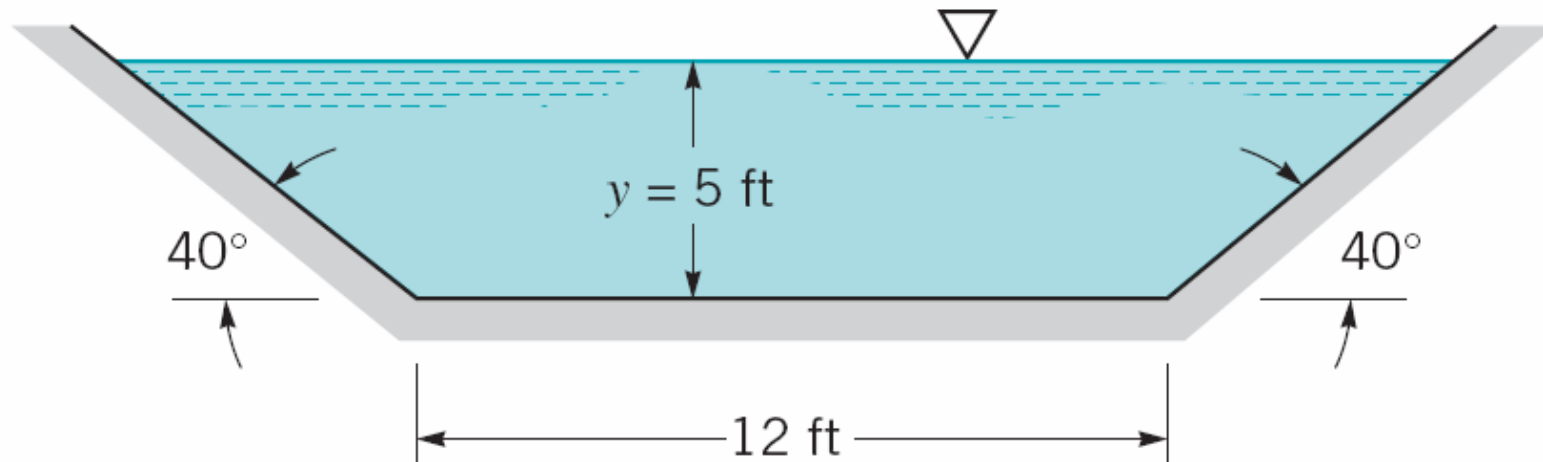




# *Uniform Flow, Determine Flow Rate*

## Example 1

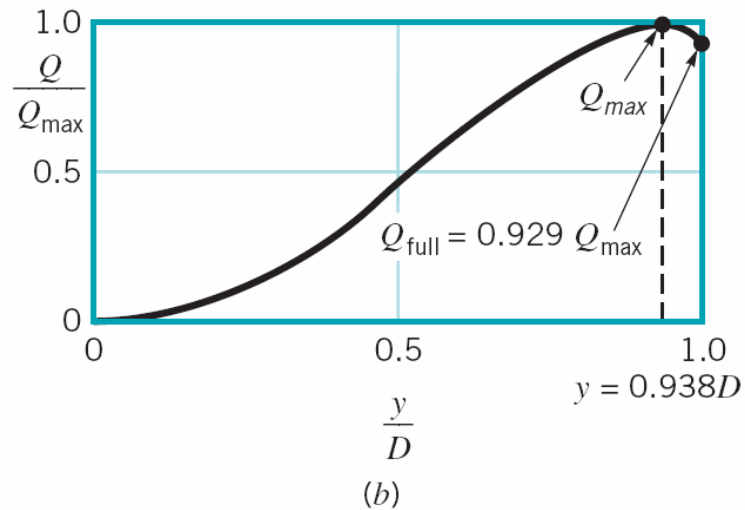
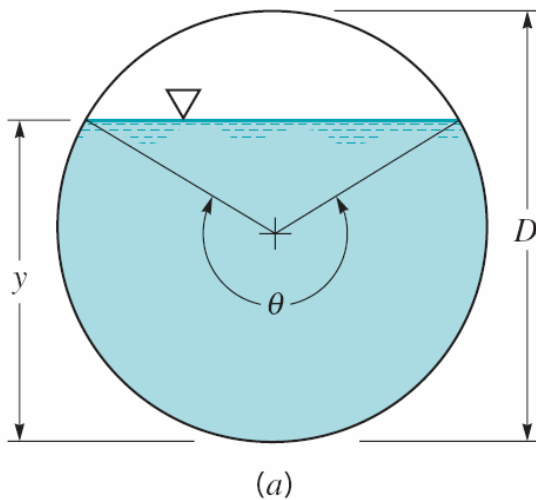
- ▶ Water flows in the canal of trapezoidal cross section shown in the Fig below. The bottom drops 1.4 ft per 1000 ft of length. Determine the flowrate if the canal is lined with new smooth concrete. Determine the Froude number



# Uniform Flow, Maximum Flow Rate

## Example 3

- ▶ Water flows in a round pipe of diameter  $D$  at a depth of  $0 \leq y \leq D$ , as shown in the Fig below. The pipe is laid on a constant slope of  $S_0$ , and the Manning coefficient is  $n$ . At what depth does the maximum flow rate occur? Show that for certain flow rate there are two depths possible with the same flow rate. Explain this behavior.



# Optimum Hydraulic Cross-sections

From Manning equation,

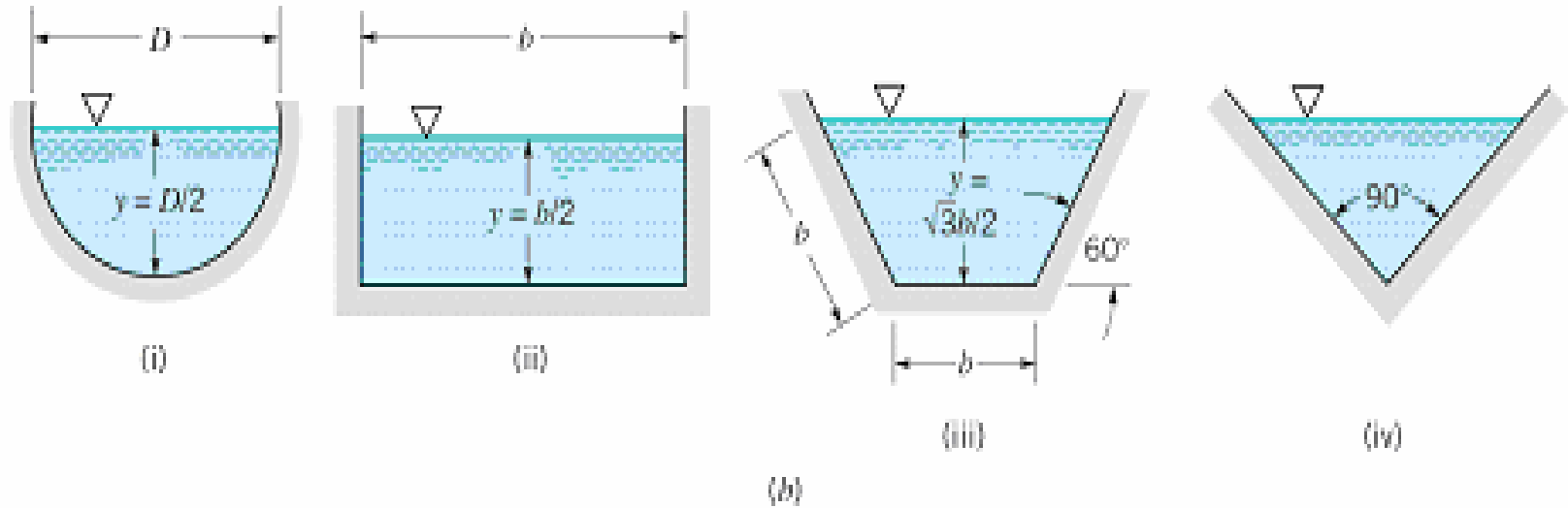
$$Q = \frac{1}{n} * \frac{A^{5/3} * \sqrt{S}}{P^{2/3}}$$

Hence, Q will be **maximum** when P is a minimum

- ▶ For a given cross-sectional area, A of an open channel, the discharge, Q is maximum when the wetted perimeter, P is minimum. Hence if the
- ▶ wetted perimeter, P for a given flow area is minimised, the area, A will give the least expensive channel to be construct.
- ▶ This corresponding cross-section is the **optimum hydraulic section** or the **best hydraulic section**.

# Contd...

*Find optimum cross section for Rectangular, Trapezoidal, Triangular and circular cross sectional channels*



The best hydraulic cross section for different shapes of cross sections

# Energy Considerations

The background features abstract, overlapping geometric shapes in various shades of green, ranging from light lime to dark forest green. These shapes are primarily located on the right side of the slide, creating a modern, layered effect. The text 'Energy Considerations' is centered in a blue, sans-serif font.



## Contd...

- ❖ With the assumption of a uniform velocity profile across any section of the channel, the one-dimensional energy equation become

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L \quad (5)$$

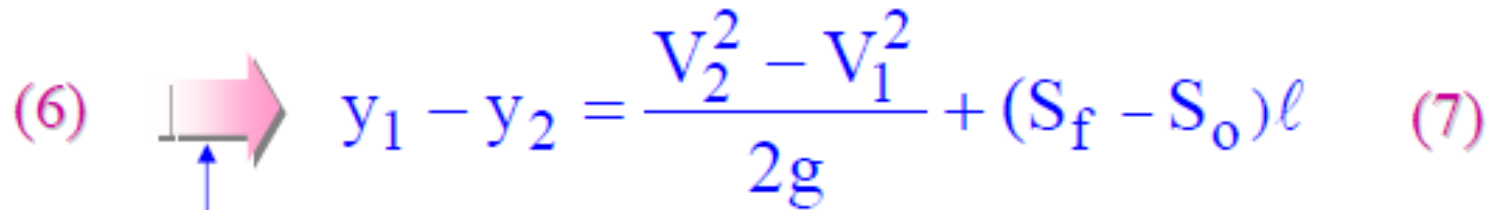
$h_L$  is the head loss due to viscous effects between sections (1) and (2).

$$(5) \quad \begin{array}{c} \text{---} \rightarrow \\ \uparrow \\ \text{---} \end{array} y_1 + \frac{V_1^2}{2g} + S_0 l = y_2 + \frac{V_2^2}{2g} + h_L \quad (6)$$

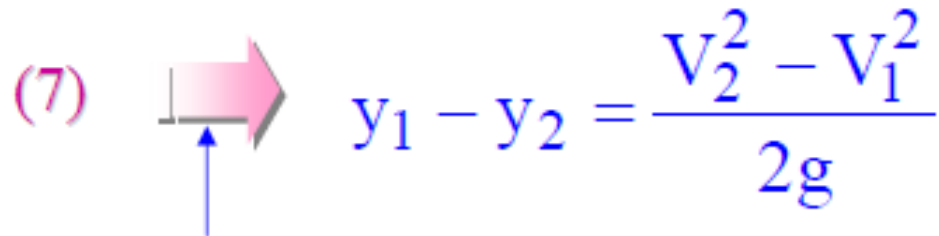
$$z_1 - z_2 = S_0 l$$

$$p_1 / \gamma = y_1 \quad p_2 / \gamma = y_2$$

## Contd...


$$(6) \quad y_1 - y_2 = \frac{V_2^2 - V_1^2}{2g} + (S_f - S_o)\ell \quad (7)$$

$$S_f = h_L / \ell$$


$$(7) \quad y_1 - y_2 = \frac{V_2^2 - V_1^2}{2g}$$

For a horizontal channel bottom ( $S_o=0$ )  
and negligible head loss ( $S_f=0$ )

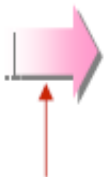


# Specific Energy

❖ Define specific energy,  $E$

$$E = y + \frac{V^2}{2g} \quad (8)$$

$$E_1 = E_2 + (S_f - S_o)\ell \quad (9)$$

(9)   $E_1 + z_1 = E_2 + z_2$

Head losses are negligible,  $S_f=0$

$$-S_o\ell = z_2 - z_1$$

The sum of the specific energy and the elevation of the channel bottom remains constant.

This a statement of the Bernoulli equation.

# Contd...

- ❖ If the cross-sectional shape is a rectangular of width  $b$

$$E = y + \frac{q^2}{2gy^2} \quad (10)$$

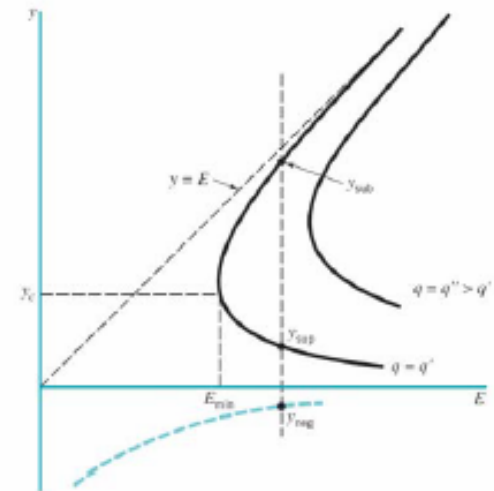
Where  $q$  is the flowrate per unit width,  $q=Q/b=Vy b/b=Vy$

- ❖ For a given channel

$b = \text{constant}$

$q = \text{constant}$

$E = E(y) \rightarrow$  Specific energy diagram



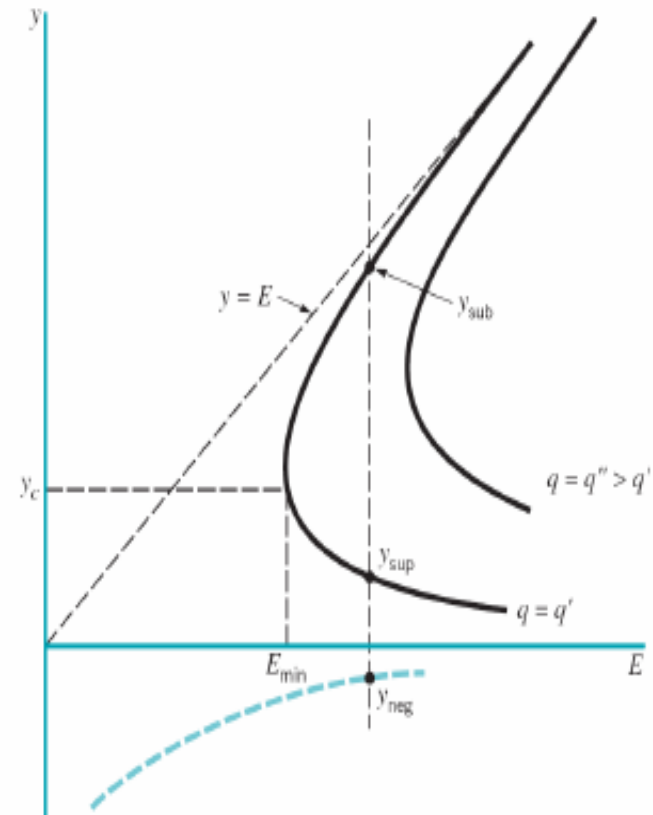
## Contd...

$$E = y + \frac{q^2}{2gy^2} \quad (10)$$

For a given  $q$  and  $E$ , equation (10) is a cubic equation with three solutions,  $y_{\text{sup}}$ ,  $y_{\text{sub}}$ , and  $y_{\text{neg}}$ .

If  $E > E_{\text{min}}$ , two solutions are positive and  $y_{\text{neg}}$  is negative (has no physical meaning and can be ignored).

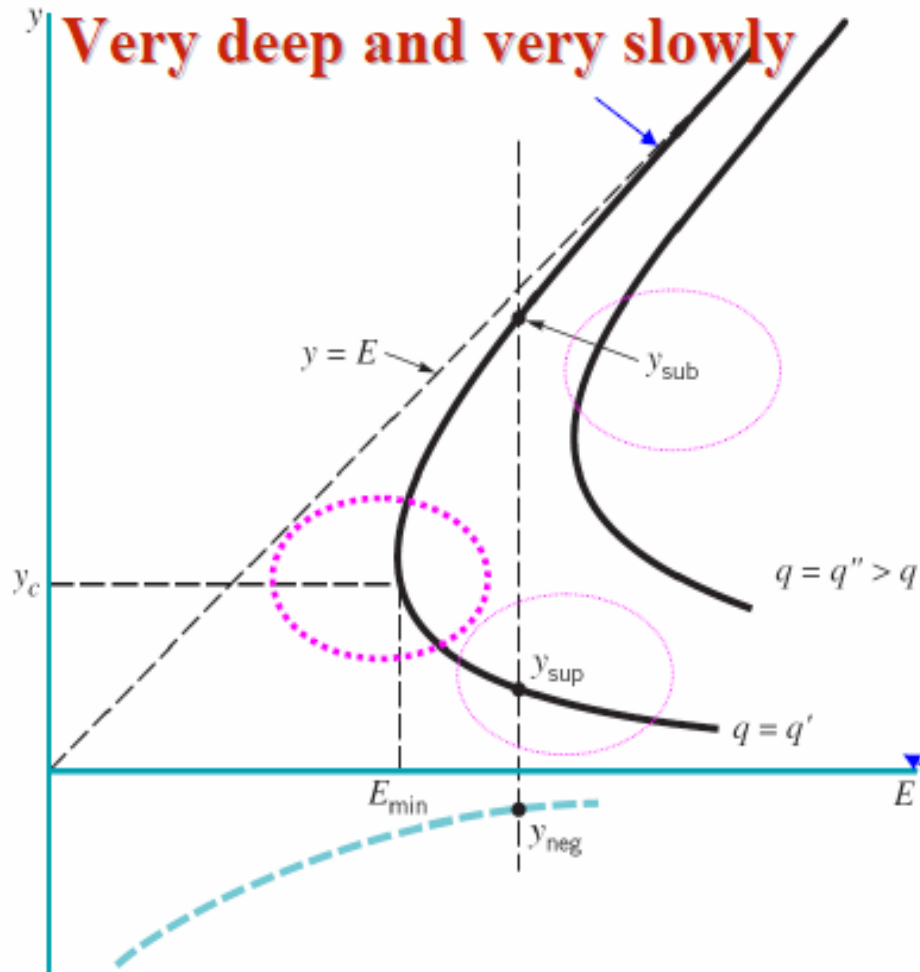
These two depths are term alternative depths.



# Contd...

Approach  $y=E$

**Very deep and very slowly**



$$Y_{sup} < Y_{sub}$$

$$V_{sup} > V_{sub}$$

$$E > E_{min}$$

Two possible depths of flow,  
one subcritical and the other  
supercritical

Approach  $y=0$

**Very shallow and  
very high speed**

# Determine Emin

❖ To determine the value of  $E_{\min}$

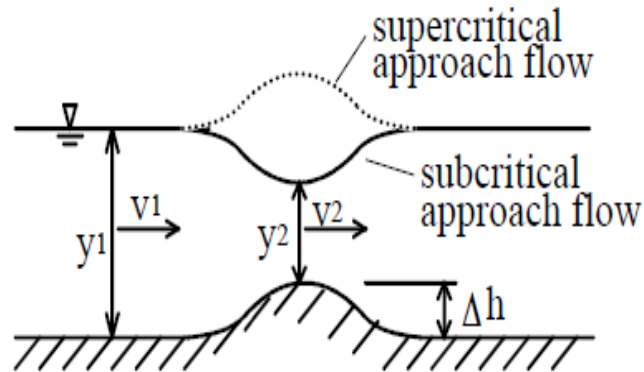
$$\frac{dE}{dy} = 0 \Rightarrow \frac{dE}{dy} = 1 - \frac{q^2}{gy^3} = 0 \Rightarrow y_c = \left( \frac{q^2}{g} \right)^{1/3} \quad (11)$$

Sub. (11) into (10)  $\Rightarrow$   $E_{\min} = \frac{3y_c}{2} \Rightarrow V_c = \frac{q}{y_c} = \sqrt{gy_c} \Rightarrow Fr_c = 1$

1. The critical conditions ( $Fr=1$ ) occur at the location of  $E_{\min}$ .
2. Flows for the upper part of the specific energy diagram are subcritical ( $Fr<1$ )
3. Flows for the lower part of the specific energy diagram are supercritical ( $Fr>1$ )

Apply critical flow computations for different cross sections of an open channel

# Frictionless Flow over a Bump



- ▶ When fluid is flowing over a bump, the behavior of the free surface is sharply different according to whether the approach flow is subcritical or supercritical.
- ▶ The height of the bump can change the character of the results.
- ▶ Applying Continuity and Bernoulli's equations to sections 1 and 2

$$v_1 * y_1 = v_2 * y_2$$
$$\frac{v_1^2}{2g} + y_1 = \frac{v_2^2}{2g} + y_2 + \Delta h$$

## Contd...

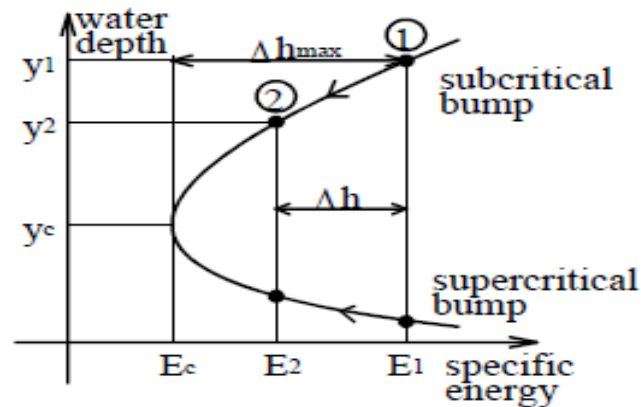
- ▶ Eliminating  $v_2$  between these two gives a cubic polynomial equation for the water depth  $y_2$  over the bump,

$$y_2^3 - E_2 y_2^2 + \frac{v_1^2 y_1^2}{2g} = 0 \quad (8.19)$$

$$\text{where } E_2 = \frac{v_1^2}{2g} + y_1 - \Delta h \quad (8.20)$$

- ▶ This equation has one negative and two positive solutions if  $\Delta h$  is not too large.
- ▶ The free surface's behavior depends upon whether condition 1 is in subcritical or supercritical flow.

## Contd...



The specific energy  $E_2$  is exactly  $\Delta h$  less than the approach energy,  $E_1$ , and point 2 will lie on the same leg of the curve as  $E_1$ .

- ◆ A subcritical approach,  $Fr_1 < 1$ , will cause the water level to decrease at the bump.
- ◆ Supercritical approach flow,  $Fr_1 > 1$ , causes a water level increase over the bump.
- ◆ If the bump height reaches  $\Delta h_{\max} = E_1 - E_c$ , the flow at the crest will be exactly critical ( $Fr = 1$ ).
- ◆ If the bump  $> \Delta h_{\max}$ , there are no physical correct solution. That is, a bump too large will choke the channel and cause frictional effects, typically a hydraulic jump.



# Channel Depth Variations

- ❖ Consider gradually varying flows.
- ❖ For such flows,  $dy/dx \ll 1$ , and it is reasonable to impose the one-dimensional velocity assumption.
- ❖ At an section the total head

$$H = \frac{V^2}{2g} + y + z$$

and the energy equation  $H_1 = H_2 + h_L$

The slop of the energy line

$$\frac{dH}{dx} = \frac{dh_L}{dx} = S_f \quad \frac{dz}{dx} = S_o \quad \Rightarrow \quad \frac{dH}{dx} = \frac{d}{dx} \left( \frac{V^2}{2g} + y + z \right) = \frac{V}{g} \frac{dV}{dx} + \frac{dy}{dx} + \frac{dz}{dx}$$

## Contd...


$$\Rightarrow \left\{ \begin{array}{l} \frac{dh_L}{dx} = \frac{V}{g} \frac{dV}{dx} + \frac{dy}{dx} + S_0 \\ \frac{V}{g} \frac{dV}{dx} + \frac{dy}{dx} = S_f - S_0 \end{array} \right. \quad (12)$$

For a given flowrate per unit width,  $q$ , in a rectangular channel of constant width  $b$ , we have

$$V = \frac{q}{y} \Rightarrow \frac{dV}{dx} = -\frac{q}{y^2} \frac{dy}{dx} = -\frac{V}{y} \frac{dy}{dx}$$

$$(12) \Rightarrow \frac{V}{g} \frac{dV}{dx} = -\frac{V^2}{gy} \frac{dy}{dx} = -F_r^2 \frac{dy}{dx} \quad (13)$$

## Contd...

Sub. (13) into (12)  
$$\frac{dy}{dx} = \frac{S_f - S_o}{(1 - F_r^2)} \quad (14)$$

**Depends on the local slope of the channel bottom, the slope of the energy line, and the Froude number.**

# Gradually Varied Flow

$$\frac{dy}{dx} \ll 1$$

# Gradually Varied Flow

- ▶ Open channel flows are classified as uniform depth, gradually varying or rapidly varying.
- ▶ If the channel bottom slope is equal to the slope of the energy line,  $S_o = S_f$ , the flow depth is constant,  $dy/dx = 0$ .
  - ➔ The loss in potential energy of the fluid as it flows downhill is exactly balanced by the dissipation of energy through viscous effects.
- ▶ If the bottom slope and the energy line slope are not equal, the flow depth will vary along the channel.

$$\frac{dy}{dx} = \frac{S_f - S_o}{(1 - F_r^2)} \quad (14)$$

The sign of  $dy/dx$ , that is, whether the flow depth increase or decrease with distance along the channel depend on  $S_f - S_o$  and  $1 - F_r^2$

# Classification of Surface Shapes

- ▶ The character of a gradually varying flow is often classified in terms of the actual channel slope,  $S_0$ , compared with the slope required to produce uniform critical flow,  $S_{oc}$ .
- ▶ The character of a gradually varying flow depends on whether the fluid depth is less than or greater than the uniform normal depth,  $y_n$ .

12 possible surface configurations 

# Classification of Surface Shapes

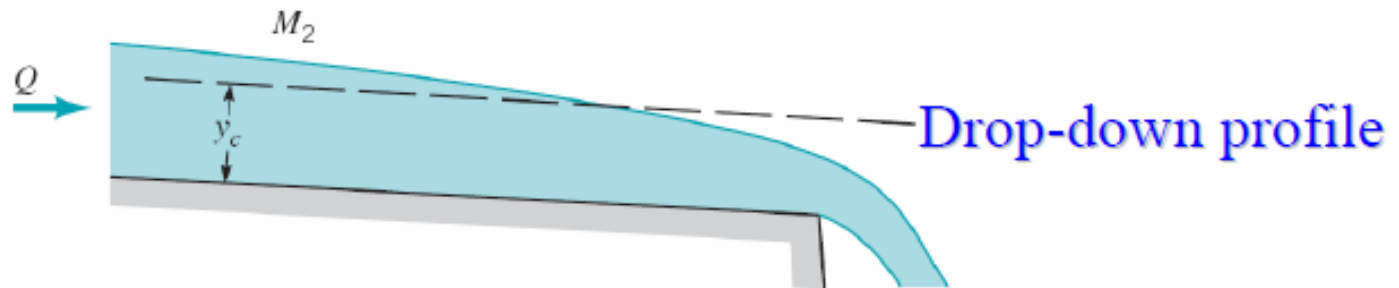
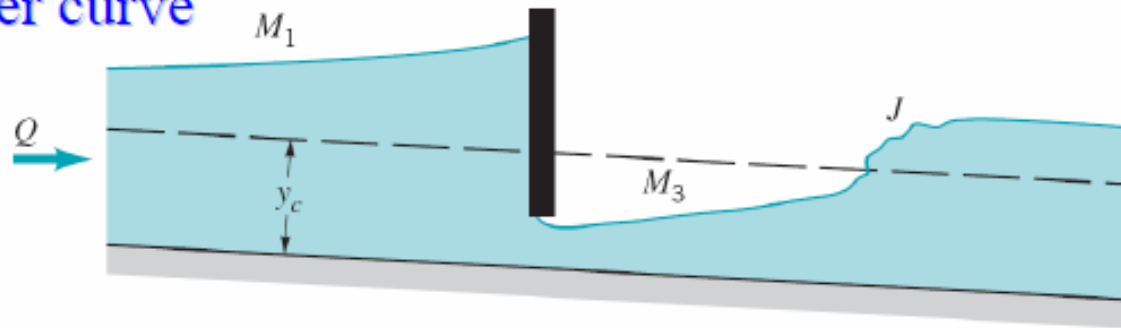
■ **TABLE 10.2**  
Possible Free-Surface Configurations

Slope Type	Slope Notation	Froude No.	Surface Shape Designation
$S_0 < S_{0c}$	Mild ( <i>M</i> )	Fr < 1	<i>M</i> -1
		Fr < 1	<i>M</i> -2
		Fr > 1	<i>M</i> -3
$S_0 = S_{0c}$	Critical ( <i>C</i> )	Fr < 1	<i>C</i> -1
		Fr > 1	<i>C</i> -3
$S_0 > S_{0c}$	Steep ( <i>S</i> )	Fr < 1	<i>S</i> -1
		Fr > 1	<i>S</i> -2
		Fr > 1	<i>S</i> -3
$S_0 = 0$	Horizontal ( <i>H</i> )	Fr < 1	<i>H</i> -2
		Fr > 1	<i>H</i> -3
$S_0 < 0$	Adverse ( <i>A</i> )	Fr < 1	<i>A</i> -2
		Fr > 1	<i>A</i> -3

Fr < 1 : y > y<sub>c</sub>  
Fr > 1 : y < y<sub>c</sub>

# Examples of Gradually Varied Flows

Backwater curve

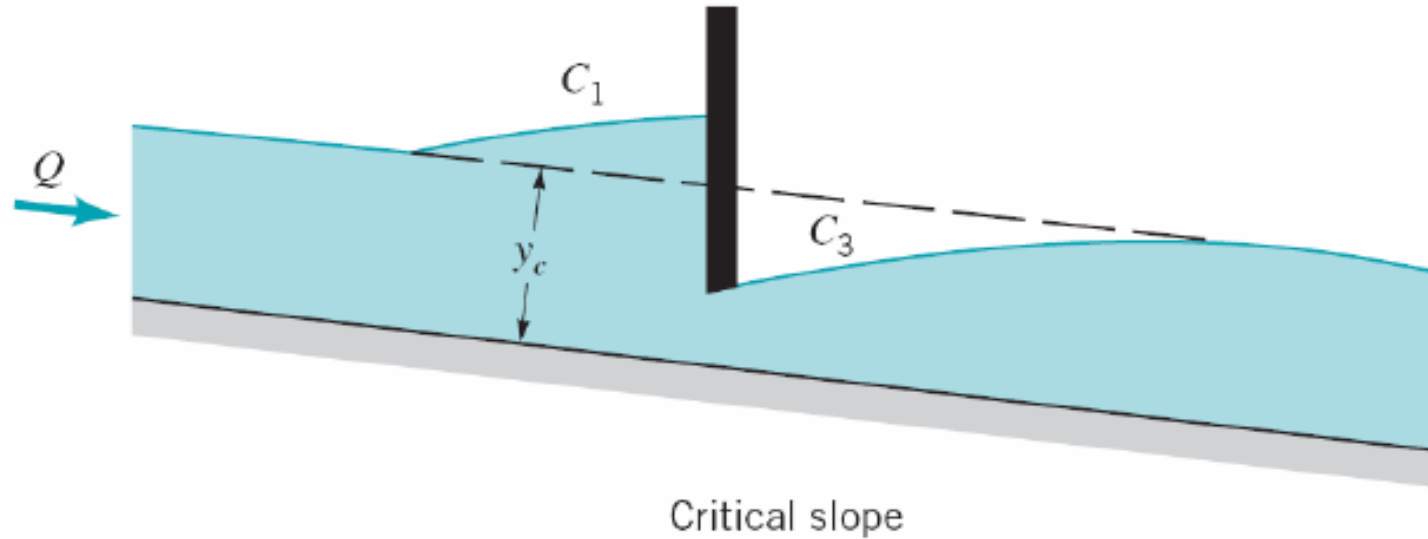


Mild slope

Typical surface configurations for nonuniform depth flow with a mild slope.  $S_0 < S_{0c}$ .

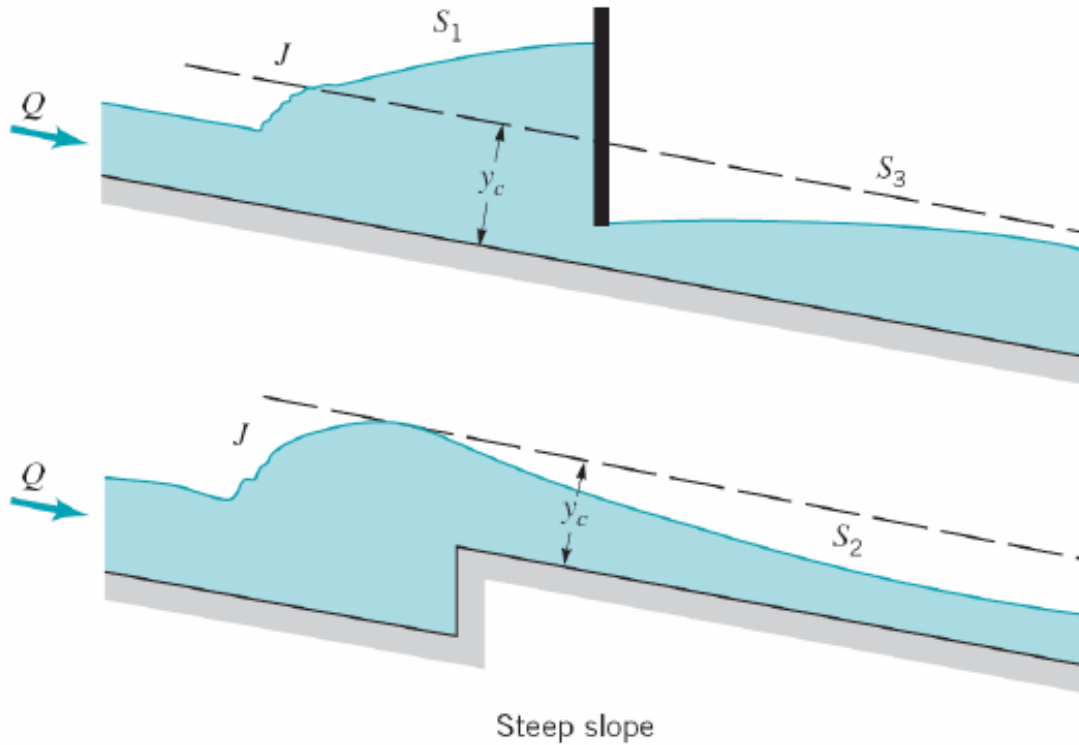


Contd...



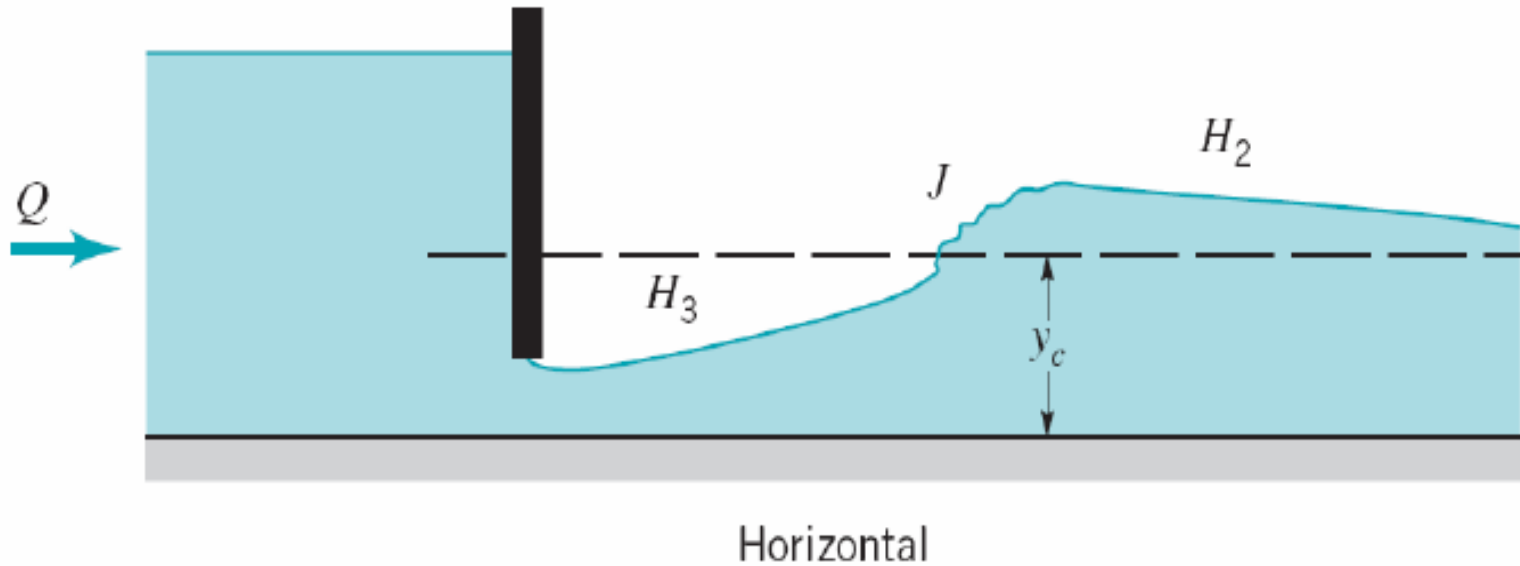
Typical surface configurations for nonuniform depth flow with a critical slope.  $S_0 = S_{0c}$ .

# Contd...



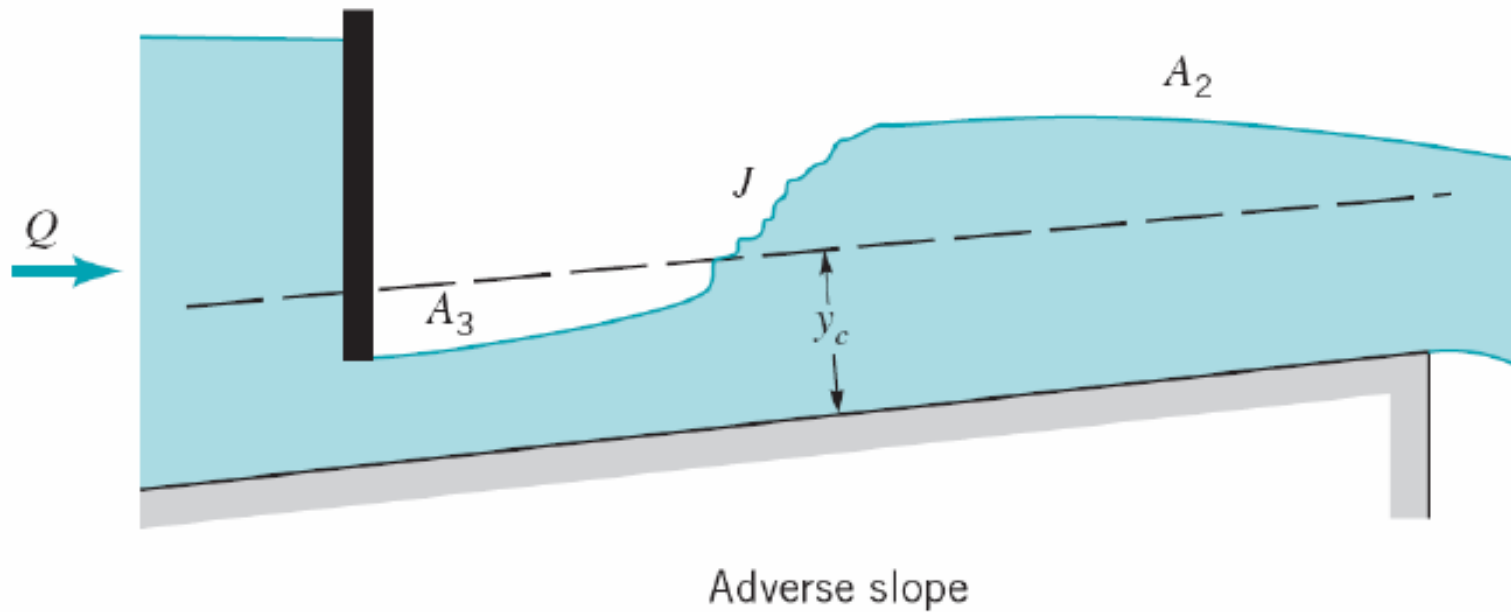
Typical surface configurations for nonuniform depth flow with a steep slope.  $S_0 > S_{0c}$ .

Contd...



Typical surface configurations for nonuniform depth flow with a horizontal slope.  $S_0 = 0$ .

Contd...



Typical surface configurations for nonuniform depth flow with a adverse slope.  $S_0 < 0$ .

# Classification of Surface Shapes

- ▶ The free surface is relatively free to conform to the shape that satisfies the governing mass, momentum, and energy equations.
- ▶ The actual shape of the surface is often very important in the design of open-channel devices or in the prediction of flood levels in natural channels.
- ▶ ☉ **The surface shape,  $y=y(x)$ , can be calculated by solving the governing differential equation obtained from a combination of the Manning equation (20) and the energy equation (14).**

**Numerical techniques have been developed and used to predict open-channel surface shapes.**

# Rapidly Varied Flow

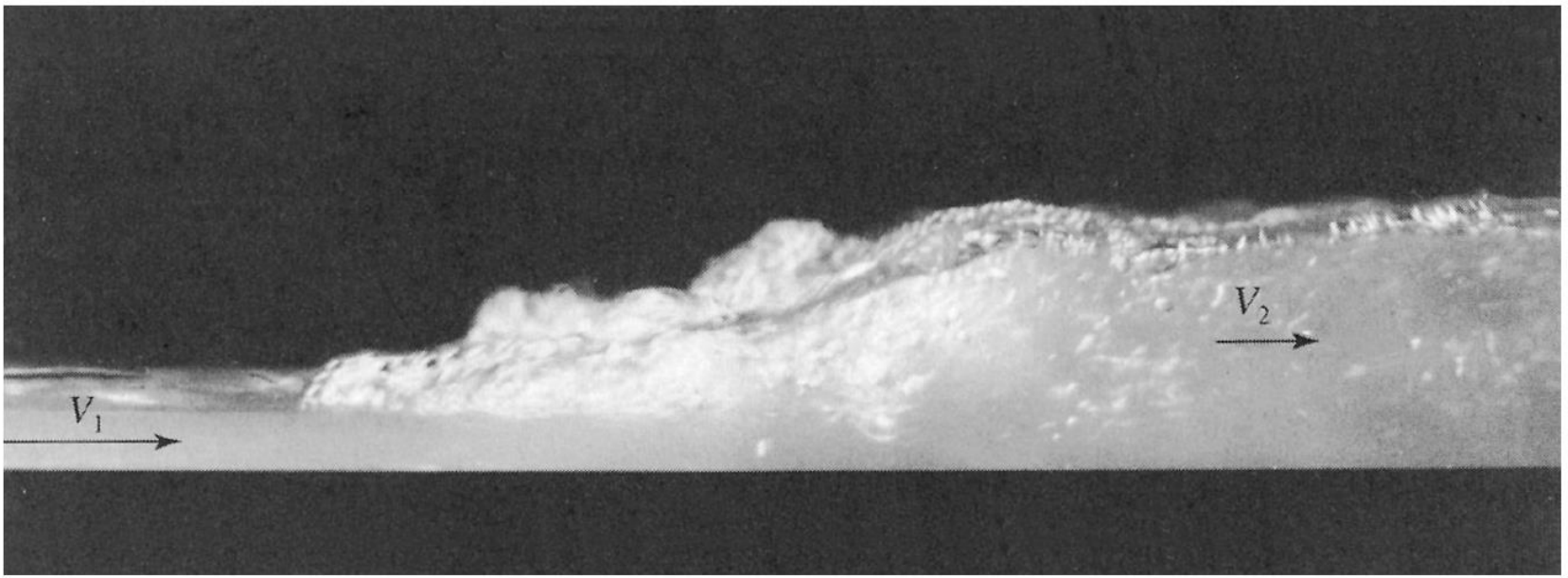
$$\frac{dy}{dx} \sim 1$$

# Rapidly Varied Flow

- ▶ Rapidly varied flow: flow depth changes occur over a relatively short distance.
  - Quite complex and difficult to analyze in a precise fashion.
  - Many approximate results can be obtained by using a simple one-dimensional model along with appropriate experimentally determined coefficients when necessary.

# Occurrence of Rapidly Varied Flow

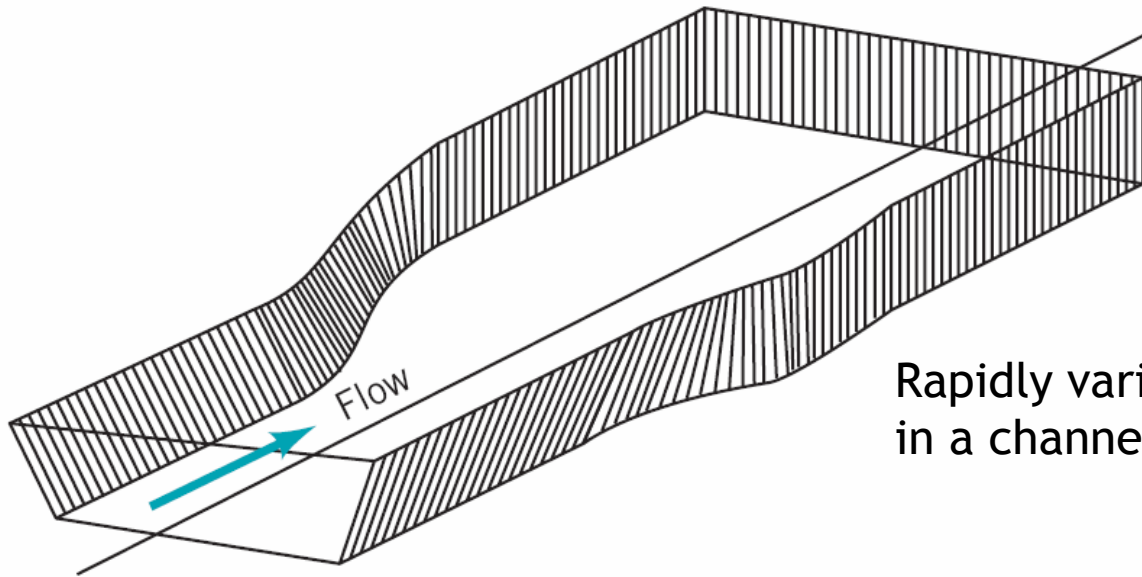
- ▶ Flow depth changes significantly un a short distance: The flow changes from a relatively shallow, high speed condition into a relatively deep, low speed condition within a horizontal distance of just a few channel depths.





## Contd...

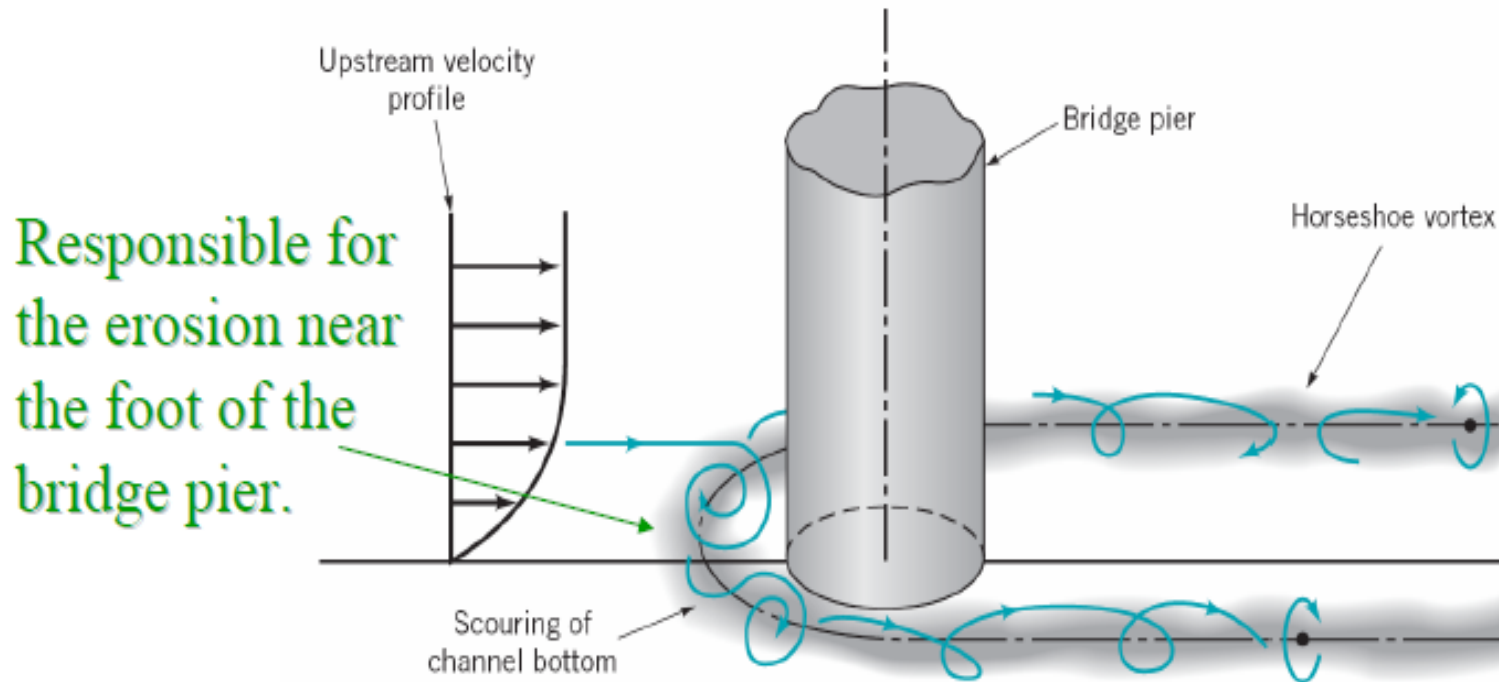
- ▶ Sudden change in the channel geometry such as the flow in an expansion or contraction section of a channel.



Rapidly varied flow may occur in a channel transition section.

# Example of Rapidly Varied Flow

- ❖ The scouring of a river bottom in the neighborhood of a bridge pier.



The complex three-dimensional flow structure around a bridge pier.

## Contd...

- ▶ Many open-channel **flow-measuring devices** are based on principles associated with rapidly varied flows.
  - ➔ Broad-crested weirs.
  - ➔ Sharp-crested weirs.
  - ➔ Critical flow flumes.
  - ➔ Sluice gates.

# Hydraulic Jump

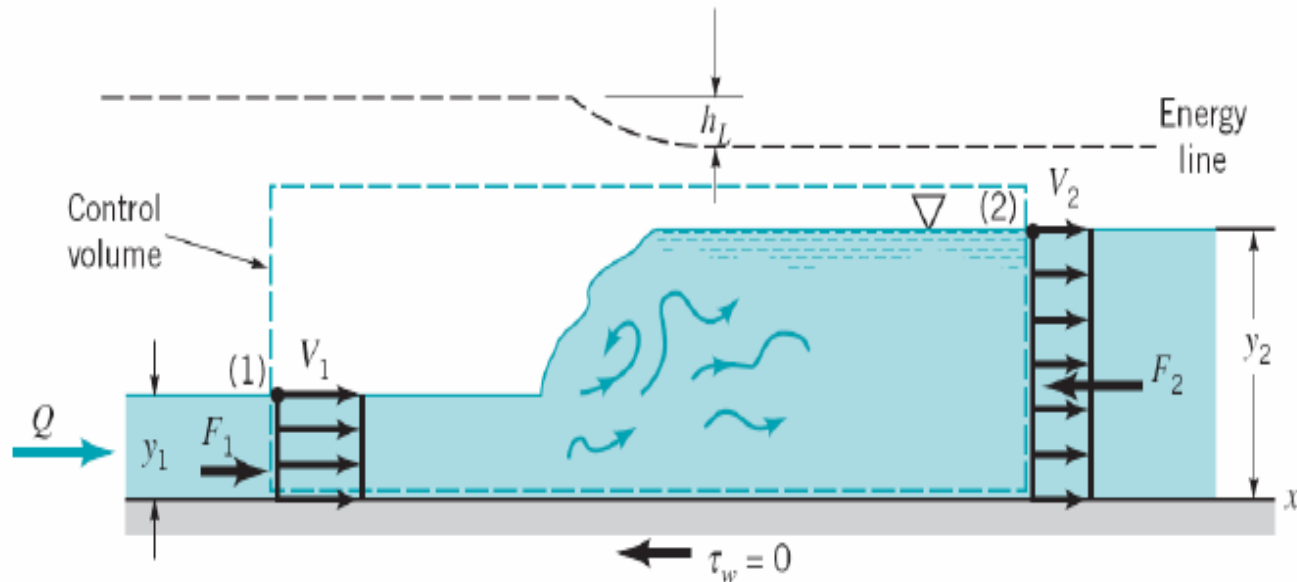


# Hydraulic Jump

- ▶ Under certain conditions it is possible that the fluid depth will change very rapidly over a short length of the channel without any change in the channel configuration.
  - ➔ Such changes in depth can be approximated as a discontinuity in the free surface elevation ( $dy/dx=\infty$ ).
  - ➔ This near discontinuity is called a *hydraulic jump*.

## Contd...

A simplest type of hydraulic jump in a horizontal, rectangular channel.



Assume that the flow at sections (1) and (2) is nearly uniform, steady, and one-dimensional.

## Contd...

The x component of the momentum equation

$$F_1 - F_2 = \rho Q(V_2 - V) = \rho V_1 y_1 b(V_2 - V)$$

$$F_1 = p_{c1} A_1 = \gamma y_1^2 b / 2$$

$$F_2 = p_{c2} A_2 = \gamma y_2^2 b / 2$$

$$\Rightarrow \frac{y_1^2}{2} - \frac{y_2^2}{2} = \frac{V_1 y_1}{g} (V_2 - V) \quad (21)$$

The conservation of mass equation  $y_1 b V_1 = y_2 b V_2 = Q \quad (22)$

The energy equation  $y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g} + h_L \quad (23)$

The head loss is due to the violent turbulent mixing and dissipation.

## Contd...

(21)+(22)+(23)  $\Rightarrow$  **Nonlinear equations**

One solution is  $y_1=y_2$ ,  $V_1=V_2$ ,  $h_L=0$

**Other solutions?**

$$(21)+(22) \Rightarrow \frac{y_1^2}{2} - \frac{y_2^2}{2} = \frac{V_1 y_1}{g} \left( \frac{V_1 y_1}{y_2} - V_1 \right) = \frac{V_1^2 y_1}{g y_2} (y_1 - y_2)$$

$$\Rightarrow \left( \frac{y_2}{y_1} \right)^2 + \left( \frac{y_2}{y_1} \right) - 2F_{r1}^2 = 0 \quad F_{r1} = \frac{V_1}{\sqrt{g y_1}}$$

$$\text{Solutions} \Rightarrow \frac{y_2}{y_1} = \frac{1}{2} \left( -1 \pm \sqrt{1 + 8F_{r1}^2} \right) \quad \frac{y_2}{y_1} = \frac{1}{2} \left( -1 + \sqrt{1 + 8F_{r1}^2} \right) \quad (24)$$



## Contd...

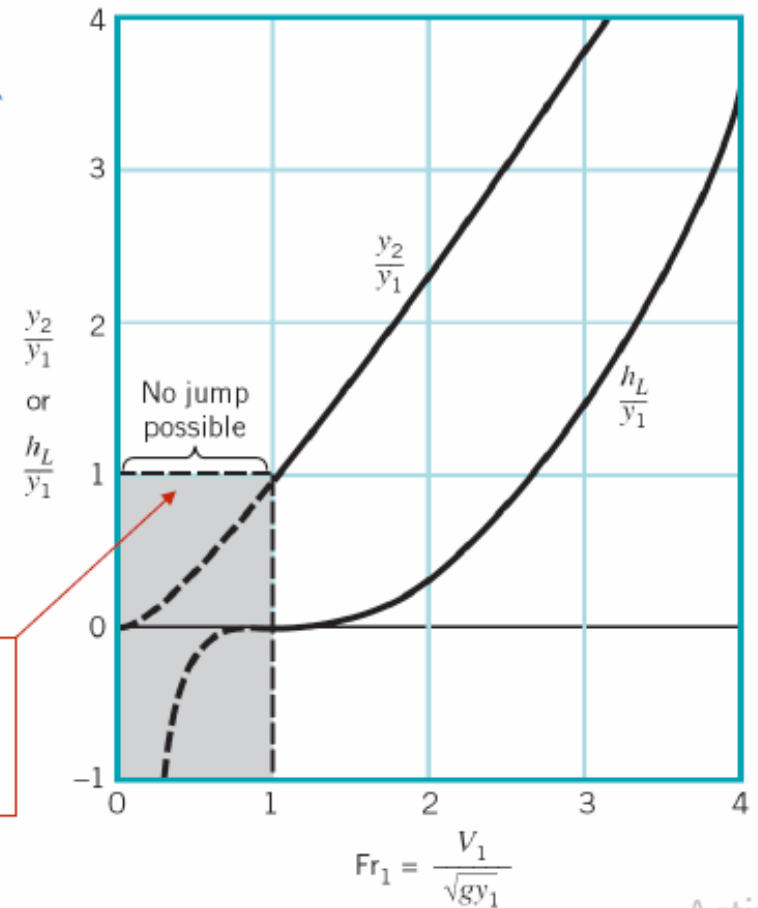
$$(23) \implies \frac{h_L}{y_1} = 1 - \frac{y_2}{y_1} + \frac{F_{r1}^2}{2} \left[ 1 - \left( \frac{y_1}{y_2} \right)^2 \right] \quad (25)$$

(24)+(25)  $\implies$  Depth ratio and dimensionless head loss across a hydraulic jump as a function of upstream Froude number.

## Contd...

The head loss is negative if  $Fr_1 < 1$ .  
→ Violate the second law of thermodynamics

**Not possible to produce a hydraulic jump with  $Fr_1 < 1$ .**



# Classification of Hydraulic Jump

- ▶ The actual structure of a hydraulic jump is a complex function of  $Fr_1$ , even though the depth ratio and head loss are given quite accurately by a simple one-dimensional flow analysis.
- ▶ A detailed investigation of the flow indicates that there are essentially five type of surface and jump conditions.

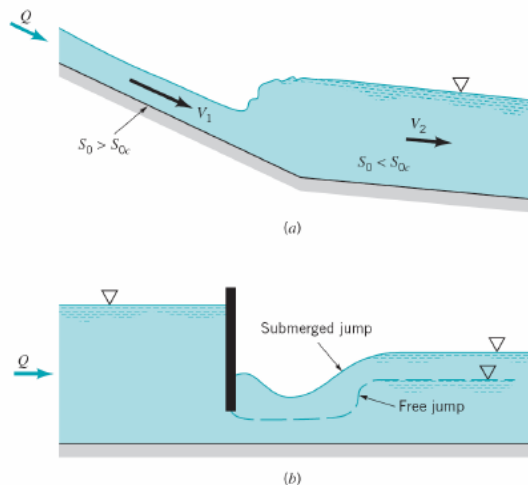
# Contd...

## Classification of Hydraulic Jumps (Ref. 12)

$Fr_1$	$y_2/y_1$	Classification	Sketch
$<1$	1	Jump impossible	
1 to 1.7	1 to 2.0	Standing wave or undulant jump	
1.7 to 2.5	2.0 to 3.1	Weak jump	
2.5 to 4.5	3.1 to 5.9	Oscillating jump	
4.5 to 9.0	5.9 to 12	Stable, well-balanced steady jump; insensitive to downstream conditions	
$>9.0$	$>12$	Rough, somewhat intermittent strong jump	

# Hydraulic Jump Variations

- ▶ Hydraulic jumps can occur in a variety of channel flow configurations, not just in horizontal, rectangular channels as discussed above.
- ▶ Other common types of hydraulic jumps include those that occur in sloping channels and the submerged hydraulic jumps that can occur just downstream of a sluice gate.



Hydraulic jump variations:  
(a) jump caused by a change  
in channel slope,  
(b) submerged jump

# Example

## Example

- ▶ Water on the horizontal apron of the 100-ft-wide spillway shown as in the Fig below has a depth of 0.60 ft and a velocity of 18 ft/s. Determine the depth,  $y_2$ , after the jump, the Froude numbers before and after the jump,  $Fr_1$  and  $Fr_2$ , and the power dissipated,  $P_d$ , with the jump.

