CHAPTER 4. ANALYSIS AND DESIGN OF COLUMNS

4.1. INTRODUCTION

A column is a vertical structural member transmitting axial compression loads with or without moments. The cross sectional dimensions of a column are generally considerably less than its height. Column support mainly vertical loads from the floors and roof and transmit these loads to the foundation.

In a typical construction cycle, the reinforcement and concrete for the beam and slabs in a floor system are placed first. Once this concrete has hardened, the reinforcement and concrete for the columns over that floor are placed. The longitudinal (vertical) bars protruding from the column will extend through the floor into the next-higher column and will be lap spliced with the bars in that column. The longitudinal bars are bent inward to fit inside the cage of bars for the next-higher column.

The more general terms compression members subjected to combined axial and bending are sometimes used to refer to columns, walls, and members in concrete trusses or frames. These may be vertical, inclined, or horizontal. A column is a special case of a compression member that is vertical.

Columns may be classified based on the following criteria:

a. Classification on the basis of geometry; rectangular, square, circular, L-shaped, T-shaped, etc. depending on the structural or architectural requirements.
b. Classification on the basis of composition; composite columns, in-filled columns, etc.
c. Classification on the basis of lateral reinforcement; tied columns, spiral columns.
d. Classification on the basis of manner by which lateral stability is provided to the structure as a whole; braced columns, un-braced columns.
e. Classification on the basis of sensitivity to second order effect due to lateral displacements; sway columns, non-sway columns.
f. Classification on the basis of degree of slenderness; short column, slender column.
g. Classification on the basis of loading: axially loaded column, columns under uni-axial moment and columns under biaxial moment

4.2. TIED/SPIRAL COLUMNS

a) Tied Columns: Columns where main (longitudinal) reinforcements are held in position by separate ties spaced at equal intervals along the length. Tied columns may be, square, rectangular, L-shaped, circular or any other required shape. And over 95% of all columns in buildings in non-seismic regions are tied columns.

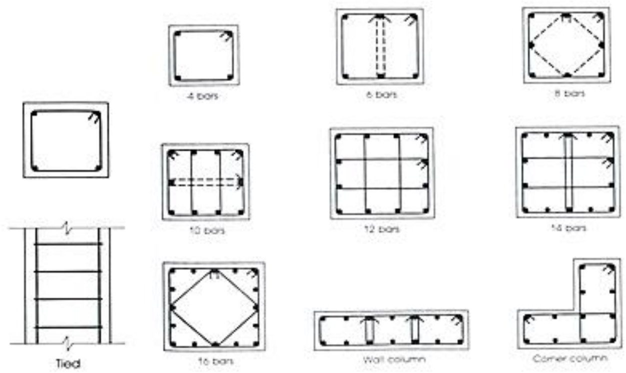

Figure 4-1 Tied Columns

b) Spiral Columns: Columns which are usually circular in cross section and longitudinal bars are wrapped by a closely spaced spiral.

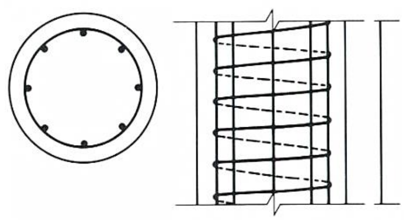

Figure 4-2 Spiral Columns

Behavior of Tied and Spiral columns

The load deflection diagrams (see Figure 4-3) show the behavior of tied and spiral columns subjected to axial load.

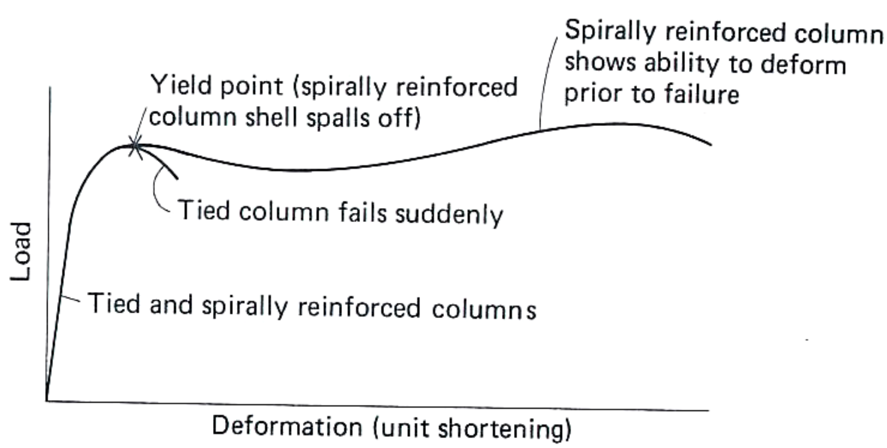

Figure 4-3 Load deflection behavior of tied and spiral columns

The initial parts of these diagrams are similar. As the maximum load is reached vertical cracks and crushing develops in the concrete shell outside the ties or spirals, and this concrete spalls off.

When this happens in a tied column, the capacity of the core that remains is less than the load and the concrete core crushes and the reinforcement buckles outward between the ties. This occurs suddenly, without warning, in a brittle manner.

When the shell spalls off in spiral columns, the column doesn't fail immediately because the strength of the core has been enhanced by the tri axial stress resulting from the confinement of the core by the spiral reinforcement. As a result the column can undergo large deformations before collapses (yielding of spirals). Such failure is more ductile and gives warning to the impending failure.

Accordingly, ductility in columns can be ensured by providing spirals or closely spaced ties.

4.3. CLASSIFICATION OF COMPRESSION MEMBERS

4.3.1. BRACED/UN-BRACED COLUMNS

a) Un-braced columns

An un-braced structure is one in which frames action is used to resist horizontal loads. In such a structure, the horizontal loads are transmitted to the foundations through bending action in the beams and columns. The moments in the columns due to this bending can substantially reduce their axial (vertical) load carrying capacity. Un-braced structures are generally quit flexible and allow horizontal displacement (see Figure 4-4). When this displacement is sufficiently large to influence significantly the column moments, the structure is termed a sway frame.

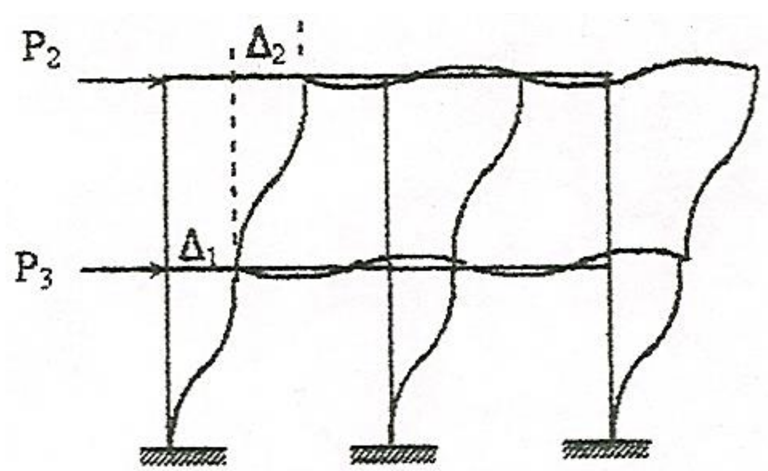

Figure 4-4 Sway Frame/ Un-braced columns

b) Braced columns:

Although, fully non sway structures are difficult to achieve in practice, building codes allow a structure to be classified as non-sway if it is braced against lateral loads using substantial bracing members such as shear walls, elevators, stairwell shafts, diagonal bracings or a combination of these (See Figure 4-5). A column with in such a non-sway structure is considered to be braced and the second order moment on such column, P-Δ, is negligible.

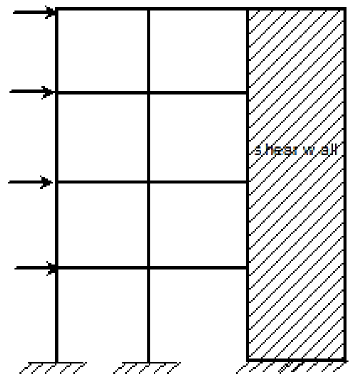

Figure 4-5 Non-sway Frame / Braced columns

4.3.2. SHORT/SLENDER COLUMNS

a) Short columns

They are columns with low slenderness ratio and their strengths are governed by the strength of the materials and the geometry of the cross section.

b) Slender columns

They are columns with high slenderness ratio and their strength may be significantly reduced by lateral deflection.

When an unbalanced moment or as moment due to eccentric loading is applied to a column, the member responds by bending as shown in Figure 4-6. If the deflection at the center of the member is, δ, then at the center there is a force P and a total moment of M + Pδ. The second order bending component, Pδ, is due to the extra eccentricity of the axial load which results from the deflection. If the column is short δ is small and this second order moment is negligible. If on the other hand, the column is long and slender, δ is large and Pδ must be calculated and added to the applied moment M.

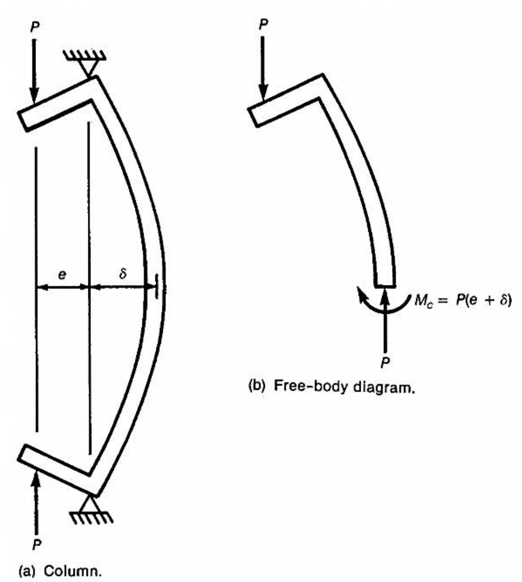

Figure 4-6 Forces in slender column

4.4. CLASSIFICATION OF COLUMNS ON THE BASIS OF LOADING

4.4.1. AXIALY LOADED COLUMNS

They are columns subjected to axial or concentric load without moments. They occur rarely. When concentric axial load acts on a short column, its ultimate capacity may be obtained, recognizing the nonlinear response of both materials, from:

$$P_{do} = f_{cd}\left(A_g - A_{st}\right) + A_{st} f_{yd} \tag{1}$$

Where

A_g is gross concrete area

A_{st} is total reinforcement area

4.4.2. COLUMN UNDER UNI-AXIAL BENDING

Almost all compression members in concrete structures are subjected to moments in addition to axial loads. These may be due to the load not being centered on the column or may result from the column resisting a portion of the unbalanced moments at the end of the beams supported by columns.

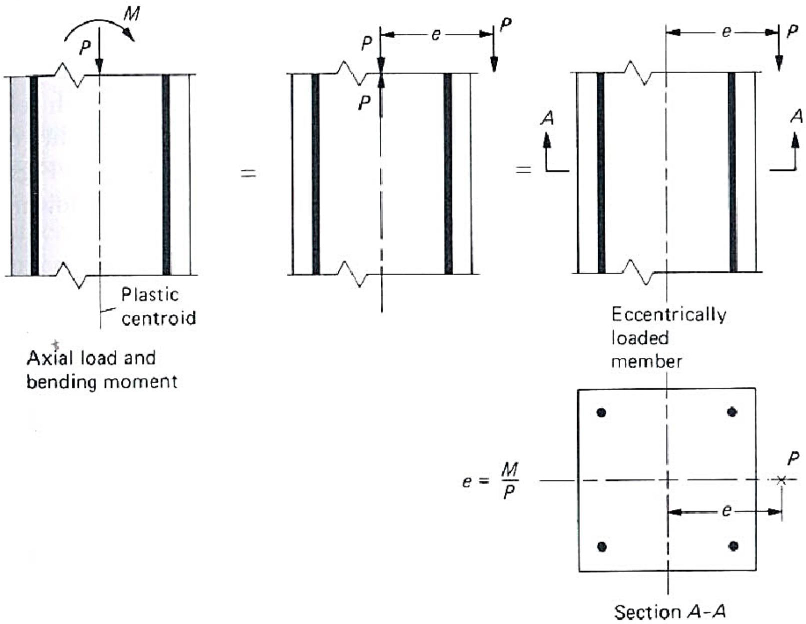

Figure 4-7 Equivalent eccentricity of column load

When a member is subjected to combined axial compression P_d and moment M_d, it is more convenient to replace the axial load and the moment with an equivalent P_d applied at eccentricity e_d as shown in Figure 4-7.

4.5. INTERACTION DIAGRAM

The presence of bending in axially loaded members can reduce the axial load capacity of the member.

To illustrate conceptually the interaction between moment and axial load in a column, an idealized homogenous and elastic column with a compressive strength, f_{cu}, equal to its tensile strength, f_{tu}, will be considered. For such a column failure would occur in a compression when the maximum stresses reached f_{cu} as given by:

$$f_{cu} = \frac{P}{A} + \frac{My}{I} \qquad (2)$$

Where

A, I area and moment of inertia of the section
y distance from the centroidal axis to the most highly compressed surface
P Axial load, positive in compression
M Moment, positive as shown in Figure 4-8c

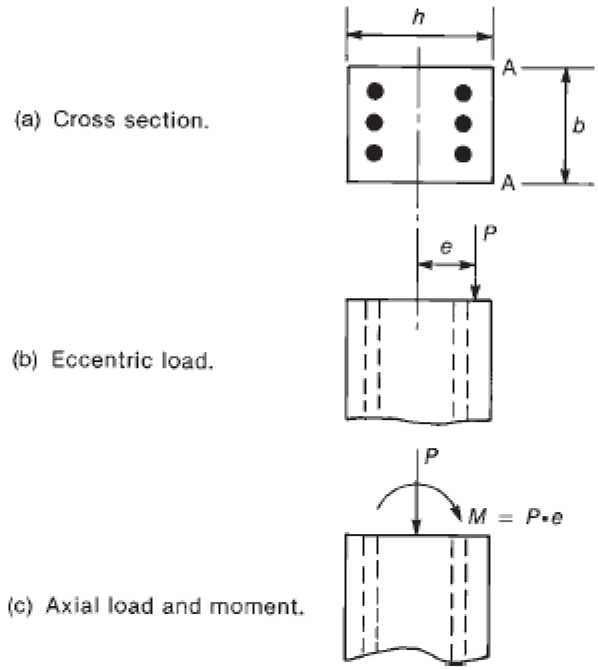

Figure 4-8 –Load and Moment on a column

Dividing both sides by f_{cu} gives:

$$1 = \frac{P}{f_{cu}A} + \frac{My}{f_{cu}I} \quad (3)$$

The maximum axial load the column can support is obtained when M = 0, and is $P_{max} = f_{cu}A$. Similarly the maximum moment that can be supported occurs when P=0 and is $M_{max} = f_{cu}I/y$. Substituting P_{max} and M_{max} gives:

$$1 = \frac{P}{P_{max}} + \frac{M}{M_{max}}$$

This is known as an interaction equation, because it shows the interaction of, or relationship between, P and M at failure. It is plotted as line AB (see Figure 4-9). A similar equation for a tensile load, P, governed by f_{tu}, gives line BC in the figure, and the lines AD and DC result if the moments have the opposite sign.

Figure 4-9 is referred to as an interaction diagram. Points on the lines plotted in this figure represent combination of P and M corresponding to the resistance of the section. A point inside the diagram such as E, represents a combination of P and M that will not cause failure. Load combinations falling on the line or outside the line, such as point F, will equal or exceed the resistance of the section and hence will cause failure.

Figure 4-9 is plotted for an elastic material with $f_{tu} = -f_{cu}$. Figure 4-10a shows an interaction diagram for an elastic material with a compressive strength f_{cu}, but with the tensile strength, f_{tu}, equal to zero, and Figure 4-10b shows a diagram for a material with $|-f_{tu}| = 0.5|f_{cu}|$. Lines AB and AD indicate load combinations corresponding to failure initiated by compression (governed by f_{cu}), while lines BC and DC indicate failures initiated by tension. In each case, the points B and D in Figure 4-9 and Figure 4-10 represent balanced failures, in which the tensile and compressive resistances of the material are reached simultaneously on opposite edges of the column.

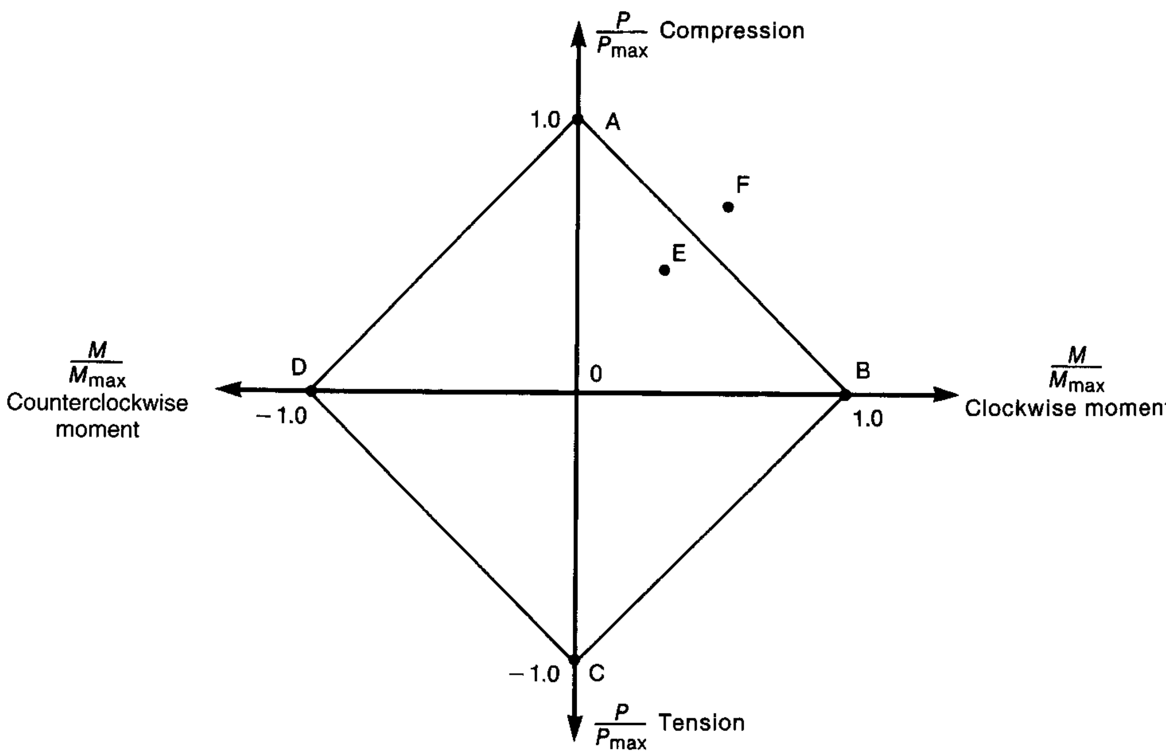

Figure 4-9 Interaction Chart for an elastic column

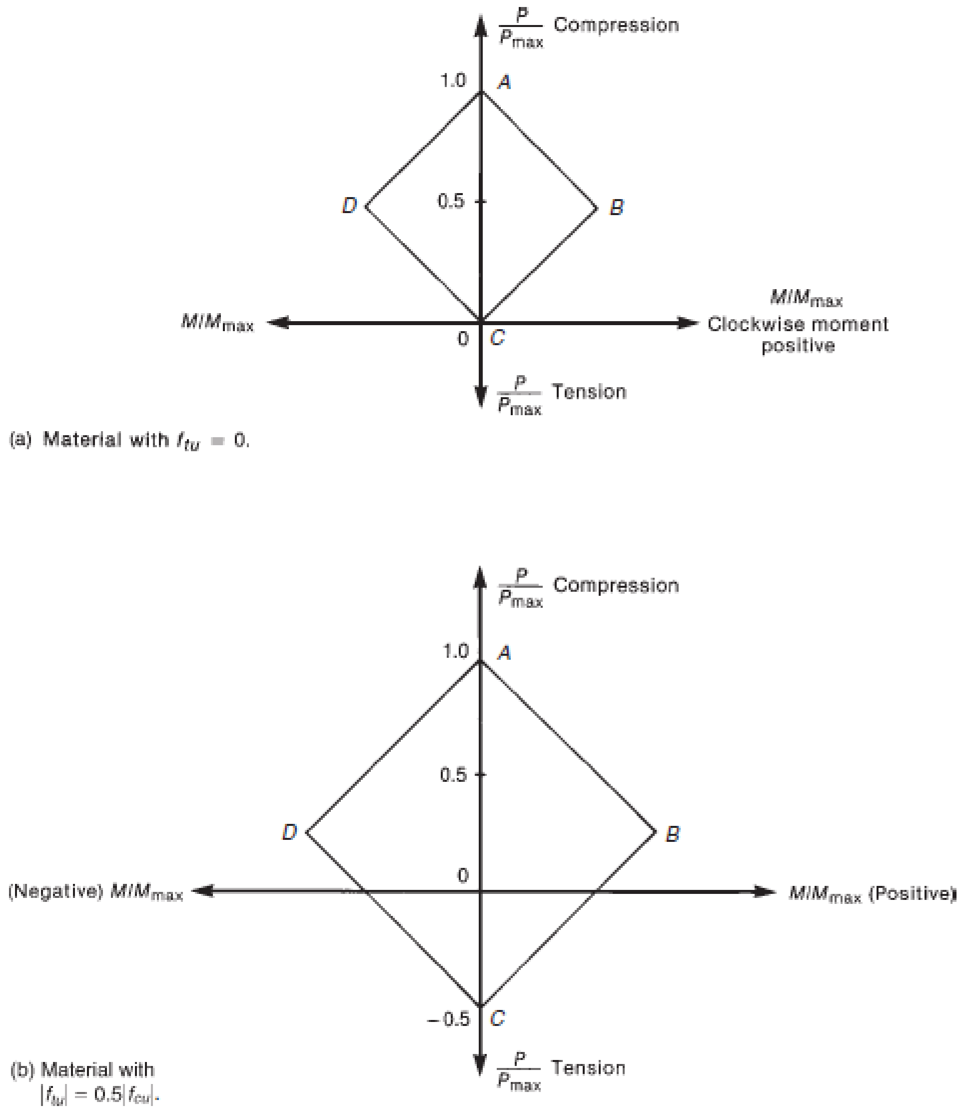

Figure 4-10 – Interaction diagrams for elastic columns, $|f_{cu}|$ not equal to $|f_{tu}|$

Reinforced concrete is not elastic and has a tensile strength that is much lower than its compressive strength. An effective tensile strength is developed, however, by reinforcing bars on the tension face of the member. For these reasons, the calculation of an interaction diagram for reinforced concrete is more complex than that for an elastic material. However, the general shape of the diagram resembles Figure 4-10b.

4.5.1. INTERACTION DIAGRAMS FOR REINFORCED CONCRETE COLUMNS

Since reinforced concrete is not elastic and has a tensile strength that is lower than its compressive strength, the general shape of the diagram resembles Figure 4-11.

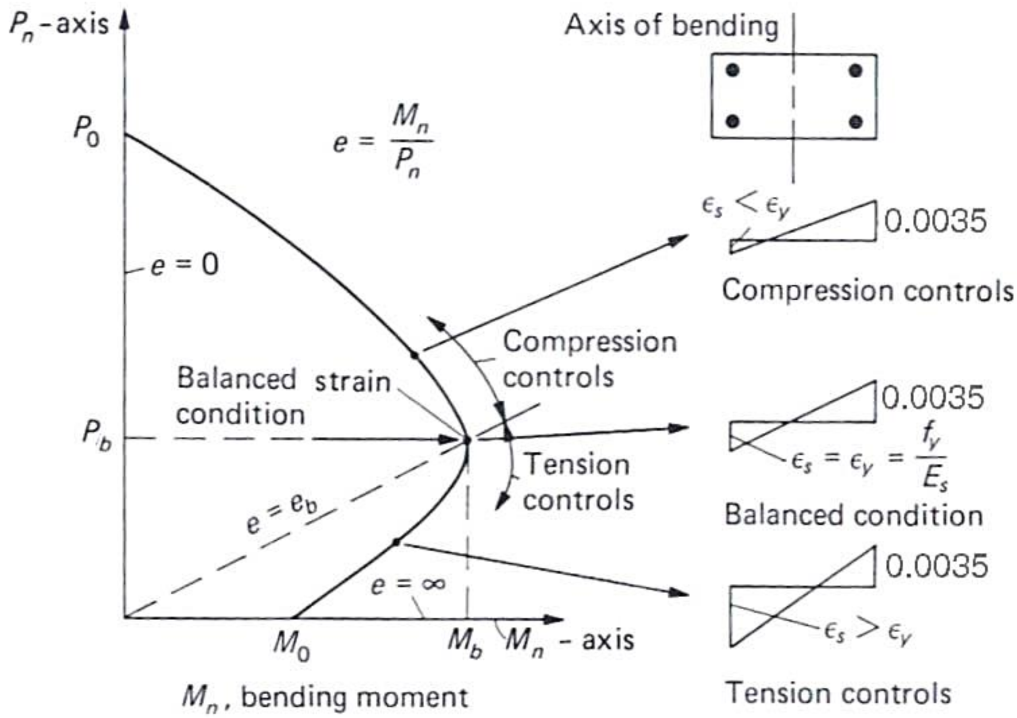

Figure 4-11 Interaction diagram for column in combined bending and axial load

Although it is possible to derive a family of equations to evaluate the strength of columns subjected to combined bending and axial loads, these equations are tedious to use. For this reason, interaction diagrams for columns are generally computed by assuming a series of strain distributions, each corresponding to a particular point on the interaction diagram, and computing the corresponding values of P and M. Once enough such points have been computed, the results are plotted as an interaction diagram.

4.5.2. SIGNIFICANT POINTS ON THE COLUMN INTERACTION DIAGRAM

Figure 4-12 illustrate a series of strain distributions and the corresponding points on an interaction diagram for a typical tied column. As usual for interaction diagrams, axial load is plotted vertically and moment horizontally.

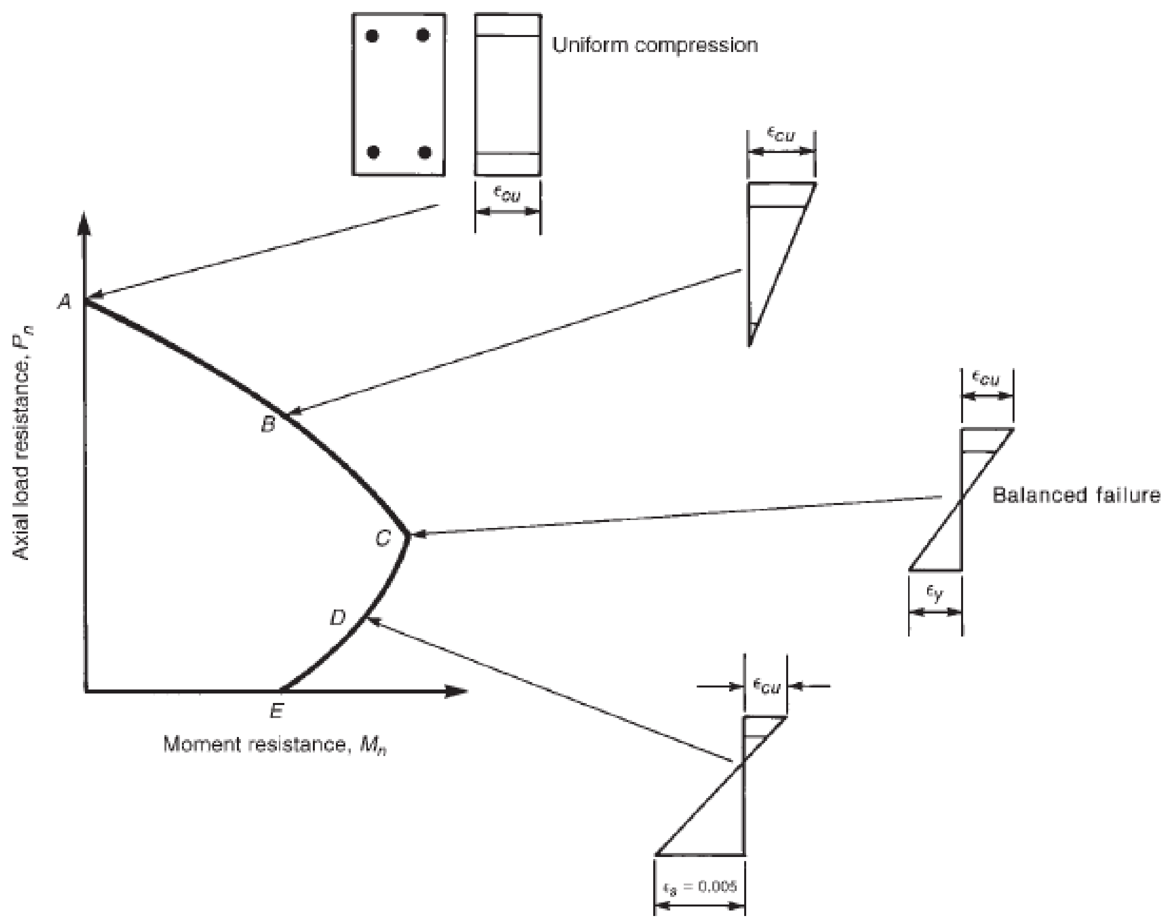

Figure 4-12 – Strain distribution corresponding to points on the interaction diagram

1. Point A – Pure Axial Load. Point A in Figure 4-12 and the corresponding strain distribution represent uniform axial compression without moment, sometimes referred to as pure axial load. This is the largest axial load the column can support.

2. Pont B- Zero Tension, Onset of Cracking. The strain distribution at B in Figure 4-12 corresponds to the axial load and moment at the onset of crushing of the concrete just as the strains in the concrete on the opposite face of the column reach zero. Case B represents the onset of cracking of the least compressed side of the column. Because tensile stresses in the concrete are ignored in the strength calculations, failure load below point B in the interaction diagram represent cases where the section is partially cracked.

3. Region A-C – Compression – Controlled Failures. Columns with axial loads and moments that fall on the upper branch of the interaction diagram between points A and C initially fail due to crushing of the compression face before the extreme tensile layer of reinforcement yields. Hence, they are called compression-controlled columns.

4. Point C- Balanced Failure, Compression-Controlled Limit Strain. Point C in Figure 4-12 corresponds to a strain distribution with a maximum compressive strain on one face of the section, and a tensile strain equal to the yield strain in the layer of reinforcement farthest from the compression face of the column.

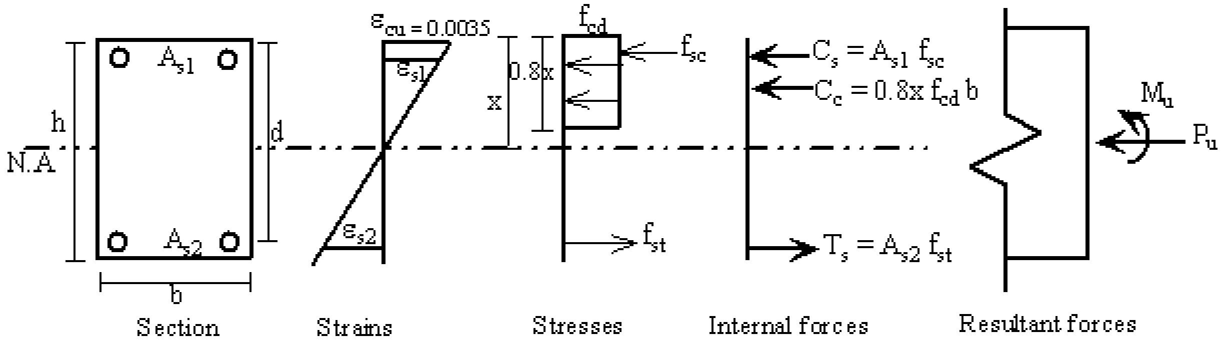

Figure 4-13 Stress-Strain relationship for column

In the actual design, interaction charts prepared for uniaxial bending can be used. The procedure involves:

- Assume a cross section, d' and evaluate d'/h to choose appropriate chart
- Compute:
  - Normal force ratio: $\nu = N_u/f_{cd}bh$
  - Moment ratios: $\mu = M_u/f_{cd}bh^2$
- Enter the chart and pick ω (the mechanical steel ratio), if the coordinate (ν, μ) lies within the families of curves. If the coordinate (ν, μ) lies outside the chart, the cross section is small and a new trail need to be made.
- Compute $A_{s,tot} = \omega A_c f_{cd}/f_{yd}$
- Check A_{tot} satisfies the maximum and minimum provisions
- Determine the distribution of bars in accordance with the charts requirement

4.6. COLUMN UNDER BI-AXIAL BENDING

Up to this point in the chapter we have dealt with columns subjected to axial loads accompanied by bending about one axis. It is not unusual for columns to support axial forces and bending about two perpendicular axes. One common example is a corner column in a frame. For a given cross section and reinforcing pattern, one can draw an interaction diagram for axial load and bending about either principal axis. As shown in Figure 4-14, these interaction diagrams form two edges of a three-dimensional interaction surface for axial load and bending about two axes. The calculation of each point on such a surface involves a double iteration: (1) the strain gradient across the section is varied, and (2) the angle of the neutral axis is varied. The neutral axis will generally not be parallel to the resultant moment vector.

Consider the RC column section shown under axial force P acting with eccentricities e_x and e_y, such that $e_x = M_y/p$, $e_y = M_x/P$ from centroidal axes (Figure 4-14c).

In Figure 4-14a the section is subjected to bending about the y axis only with eccentricity e_x. The corresponding strength interaction curve is shown as Case (a) (see Figure 4-14d). Such a curve can be established by the usual methods for uni-axial bending. Similarly, in Figure 4-14b the section is subjected to bending about the x axis only with eccentricity e_y. The corresponding strength interaction curve is shown as Case (b) (see Figure 4-14d). For case (c), which combines x and y axis bending, the orientation of the resultant eccentricity is defined by the angle λ

$$\lambda = \arctan\frac{e_x}{e_y} = \arctan\frac{M_{ny}}{M_{nx}}$$

Bending for this case is about an axis defined by the angle θ with respect to the x-axis. For other values of λ, similar curves are obtained to define the failure surface for axial load plus bi-axial bending.

Any combination of P_u, M_{ux}, and M_{uy} falling outside the surface would represent failure. Note that the failure surface can be described either by a set of curves defined by radial planes passing through the P_n axis or by a set of curves defined by horizontal plane intersections, each for a constant P_n, defining the load contours (see Figure 4-14).

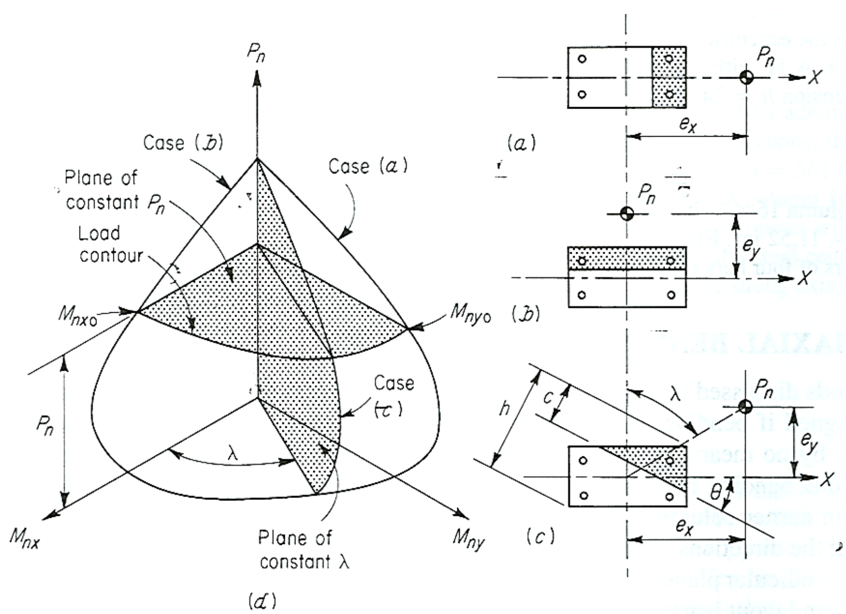

Figure 4-14 Interaction diagram for compression plus bi-axial bending

Computation commences with the successive choice of neutral axis distance c for each value of q. Then using the strain compatibility and stress-strain relationship, bar forces and the concrete compressive resultant can be determined. Then P_n, M_{nx}, and M_{ny} (a point on the interaction surface) can be determined using the equation of equilibrium

Since the determination of the neutral axis requires several trials, the procedure using the above expressions is tedious. Thus, the following simple approximate methods are widely used.

1. **Load contour method:** It is an approximation on load versus moment interaction surface. Accordingly, the general non-dimensional interaction equation of family of load contours is given by:

$$\left(\frac{M_{dx}}{M_{dxo}}\right)^{\alpha_n} + \left(\frac{M_{dy}}{M_{dyo}}\right)^{\alpha_n} = 1$$

$$\alpha_n = 0.667 + 1.667\left(\frac{P_{da}}{P_{do}}\right) and\ 1.15 \le \alpha_n \le 2$$

where: $M_{dx} = P_d e_y$
$M_{dy} = P_d e_x$
$M_{dxo} = M_{dx}$ when $M_{dy} = 0$ (design capacity under uni-axial bending about x)
$M_{dyo} = M_{dy}$ when $M_{dx} = 0$ (design capacity under uni-axial bending about y)

2. **Reciprocal method/Bresler's equation**: It is an approximation of bowl shaped failure surface by the following reciprocal load interaction equation.

$$\frac{1}{P_{dx}} = \frac{1}{P_{dxo}} + \frac{1}{P_{dyo}} - \frac{1}{P_{do}}$$

where: P_d = design (ultimate) load capacity of the section with eccentricities e_{dy} and e_{dx}
P_{dxo} = ultimate load capacity of the section for uni axial bending with e_{dx} only ($e_{dy} = 0$)
P_{dyo} = ultimate load capacity of the section for uni axial bending with e_{dy} only ($e_{dx} = 0$)
P_{do} = concentric axial load capacity ($e_{dx} = e_{dy} = 0$)

However interaction charts prepared for biaxial bending can be used for actual design. The procedure involves:

- Select cross section dimensions h and b and also h' and b'
- Calculate h'/h and b'/b and select suitable chart
- Compute:
    - Normal force ratio: $\nu = N_u/f_{cd}bh$
    - Moment ratios: $\mu_h = M_h/f_{cd}A_C h$ and $\mu_b = M_b/f_{cd}A_C b$
- Select suitable chart which satisfy and ratio:
- Enter the chart to obtain ω
- Compute $A_{s,tot} = \omega A_c f_{cd}/f_{yd}$
- Check A_{tot} satisfies the maximum and minimum provisions
- Determine the distribution of bars in accordance with the charts requirement

4.7. SLENDER COLUMNS

4.7.1. DEFINITION OF A SLENDER COLUMN

An eccentrically loaded, pin-ended column is shown in Figure 4-15a. The moments at the ends of the column are

$$M_e = Pe \qquad (4)$$

When the loads P are applied, the column deflects laterally by an amount δ, as shown. For equilibrium, the internal moment at midheight is (Figure 4-15b)

$$M_c = P\left(e + \delta\right) \qquad (5)$$

The deflection increases the moments for which the column must be designed. In the symmetrical column shown here, the maximum moment occurs at midheight, where the maximum deflection occurs.

Figure 4-16 shows an interaction diagram for a reinforced concrete column. This diagram gives the combinations of axial load and moment required to cause failure of a column cross section or a very short length of column. The dashed radial line O-A is a plot of the end moment on the column in Figure 4-15. Because this load is applied at a constant eccentricity, e, the end moment, M_e , is a linear function of P, given by Eq (4). The curved, solid line O-B is the moment M_c at midheight of the column, given by Eq(5). At any given load P, the moment at midheight is the sum of the end moment, Pe, and the moment due to the deflections, $P\delta$. The line O-A is referred to as a load-moment curve for the end moment while the line O-B is the load – moment curve for the maximum column moment.

Failure occurs when the load-moment curve O-B for the point of maximum moment intersects the interaction diagram for the cross section. Thus the load and moment at failure are denoted by point B in Figure 4-16. Because the increase in maximum moment due to deflections, the axial – load capacity is reduced from A to B. This reduction in axial-load capacity results from what is referred to as slenderness effects.

A slender column is defined as a column that has a significant reduction in its axial-load capacity due to moments resulting from lateral deflections of the column. In the derivation of the ACI Code, “a significant reduction” was arbitrarily taken as anything greater than about 5 percent.

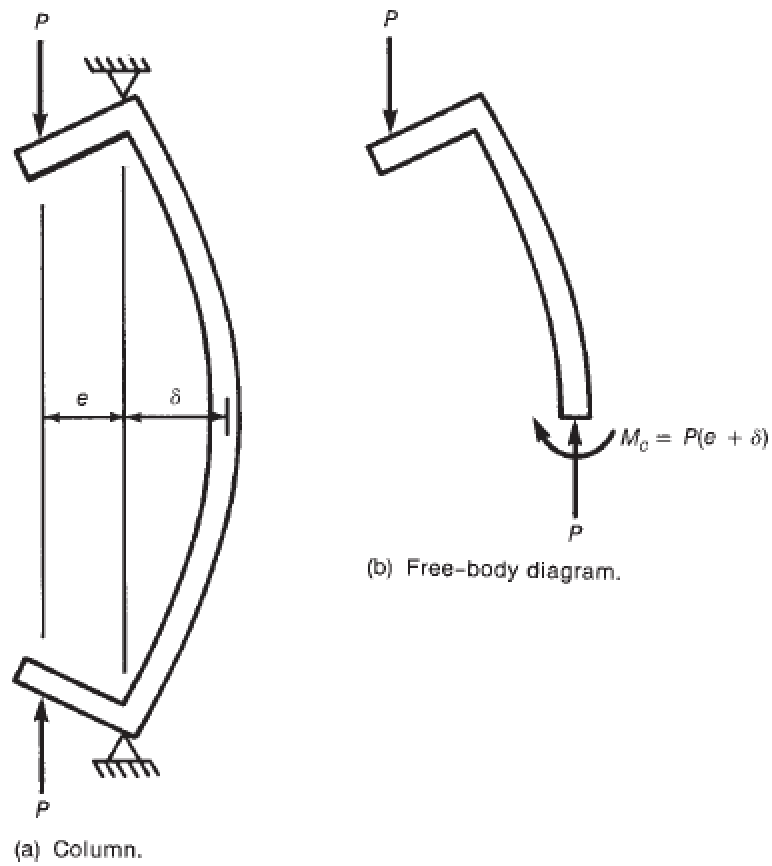

Figure 4-15 – Forces in a deflected column

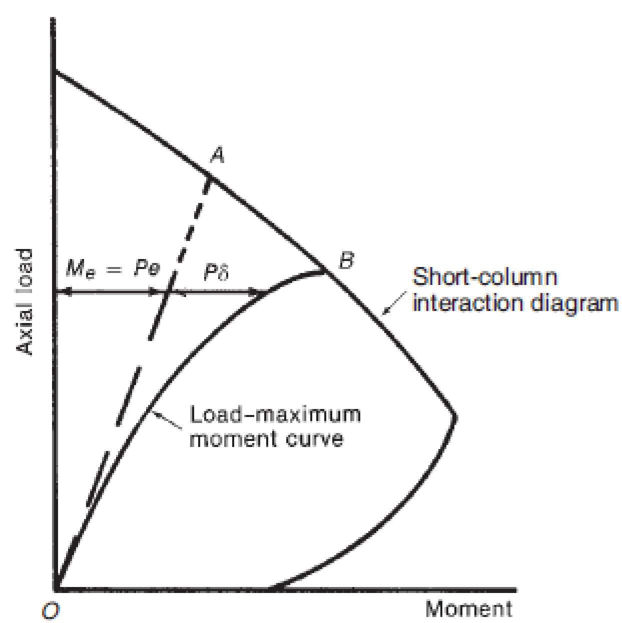

Figure 4-16 – Load and moment in a column

Buckling of Axially Loaded Elastic Columns

Figure 4-17 illustrates three states of equilibrium. If the ball in Figure 4-17a is displaced laterally and released, it will return to its original position. This is stable equilibrium. If the ball in Figure 4-17c is displaced laterally and released, it will roll off the hill. This is unstable equilibrium. The transition between stable and unstable equilibrium is neutral equilibrium, illustrated in Figure 4-17b. Here, the ball will remain in the displaced position. Similar states of equilibrium exist for the axially loaded column in Figure 4-18a. If the column returns to its original position when it is pushed laterally at midheight and released, it is in stable equilibrium; and so on.

Figure 4-18b shows a portion of a column that is in a state of neutral equilibrium. The differential equation for this column is

$$EI\frac{d^2y}{dx^2} = -Py \tag{6}$$

In 1744, Leonhard Euler derived Eq(6) and its solution,

$$P_c = \frac{n^2\pi^2 EI}{l^2} \tag{7}$$

where: EI = flexural rigidity of column cross section
l = length of the column
n = number of half – sine waves in the deformed shape of the column

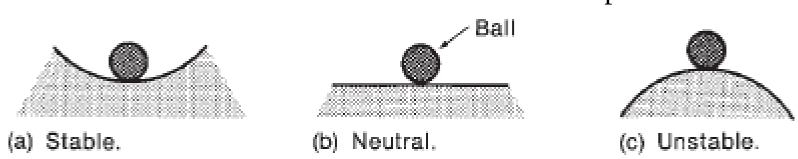

Figure 4-17 – States of equilibrium

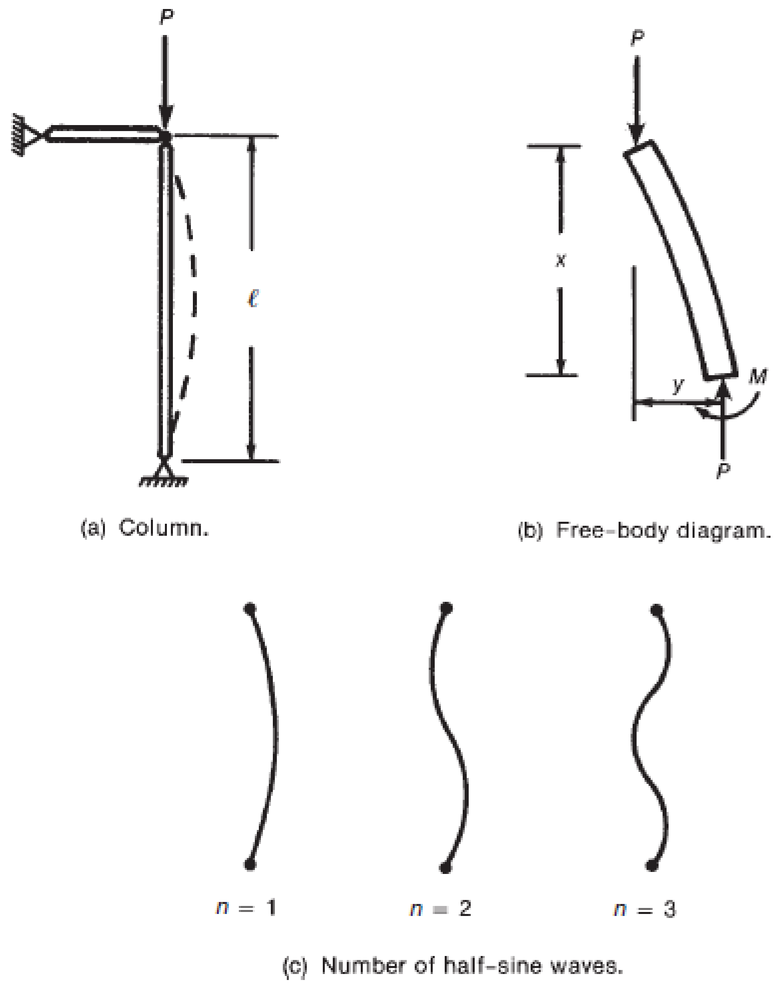

Figure 4-18 – Buckling of a Pin ended column

Cases with n = 1,2, and 3 are illustrated in Figure 4-18c. The lowest value of P_c will occur with n = 1.0. This gives what is referred to as the Euler buckling load:

$$P_E = \frac{\pi^2 EI}{l^2} \tag{8}$$

Such a column is shown in Figure 4-19a. If this column were unable to move sideways at midheight, as shown in Figure 4-19b, it would buckle with n = 2, and the buckling load would be

$$P_c = \frac{2^2 \pi^2 EI}{l^2} \tag{9}$$

which is four times the critical load of the same column without the midheight brace.

Another way of looking at this involves the concept of the effective length of the column. The effective length is the length of a pin-ended column having the same buckling load. Thus the column in Figure 4-19c has the same buckling load as that in Figure 4-19b. The effective length of the column is l/2 in this case, where l/2 is the length of each of the half-sine waves in the deflected shape of the column in Figure 4-19b. The effective length, kl, is equal to l/n. The effective length factor is k = 1/n. Equation(8) is generally written as

$$P_c = \frac{\pi^2 EI}{(kl)^2} \tag{10}$$

Four idealized cases are shown in Figure 4-20, together with the corresponding values of the effective length, kl. Frames a and b are prevented against deflecting laterally. They are said to be braced against sidesway. Frames c and d are free to sway laterally when they buckle. They are called unbraced or sway frames. The critical loads of the columns shown in Figure 4-20 are in the ratio 1:4:1:1/4.

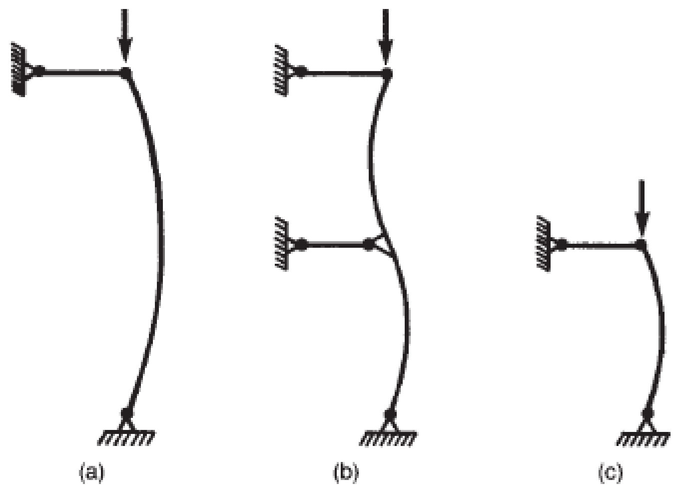

Figure 4-19 – Effective lengths of columns

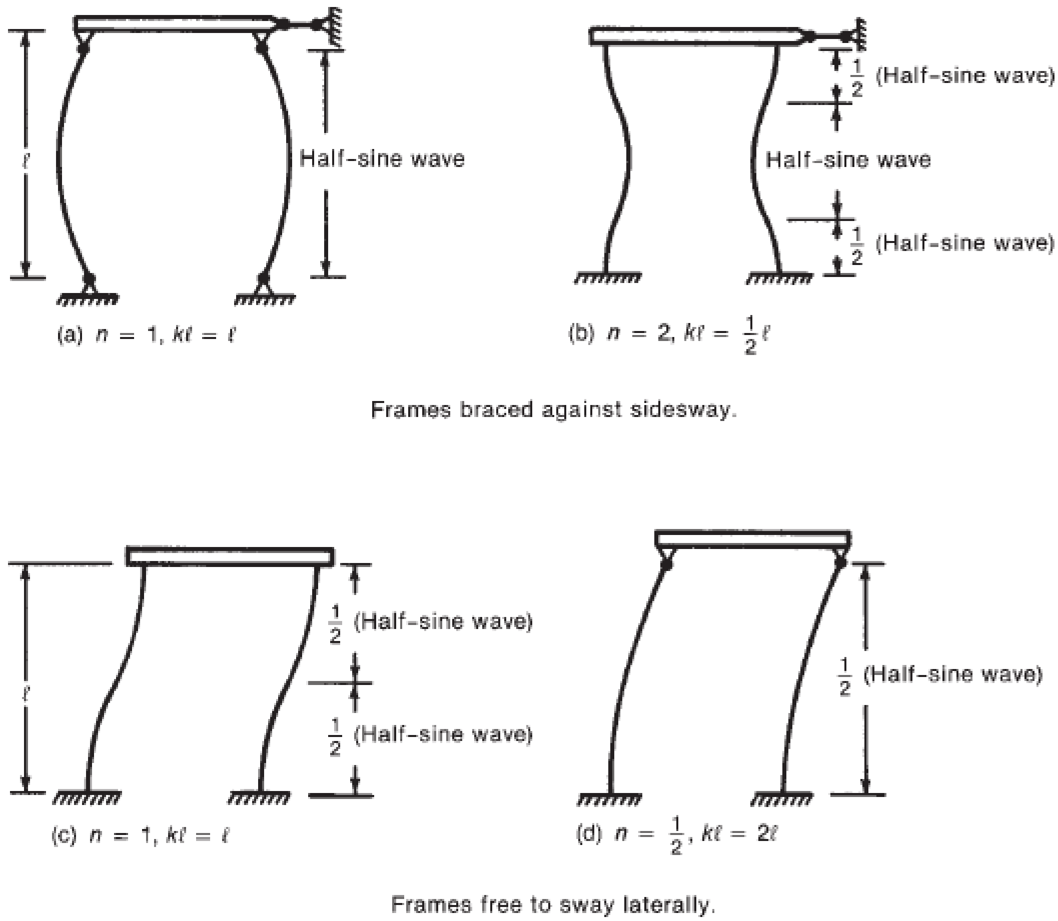

Figure 4-20 – Effective lengths of idealized columns

Thus it is seen that the restraints against end rotation and lateral translation have a major effect on the bucking load of axially loaded elastic columns. In actual structures fully fixed ends, such as those shown in Figure 4-20 b to d, rarely, if ever, occur.

4.7.2. BEHAVIOR AND ANALYSIS OF PIN-ENDED COLUMNS

Lateral deflections of a slender column cause an increase in the column moments, as illustrated in Figure 4-15 and Figure 4-16. These increased moments cause an increase in the deflections, which in turn lead to an increase in the moments. As a result, the load-moment line O-B in Figure 4-16 is nonlinear. If the axial load is below the critical load, the process will converge to a stable position. If the axial load is greater than the critical load, it will not. This is referred to as a second-order process, because it is described by a second-order differential equation.

In a first-order analysis, the equations of equilibrium are derived by assuming that the deflections have a negligible effect on the internal forces in the members. In a second-order analysis, the equations of equilibrium consider the deformed shape of the structure. Instability can be investigated only via a second-order analysis, because it is the loss of equilibrium of the deformed structure that causes instability. However, because many engineering calculations and computer programs are based on first-order analyses, methods have been derived to modify the results of a first-order analysis to approximate the second-order effects.

P – δ Moments and P – Δ Moments

Two different types of second-order moments act on the columns in a frame:

1. **P – δ moments.** These result from deflections, δ, of the axis of the bent column away from the chord joining the ends of the column. The slenderness effects in pin-ended columns and in nonsway frames result from P- δ effects.
2. **P – Δ moments.** These results from lateral deflections, Δ, of the beam-column joints from their original undeflected locations. The slenderness effects in sway frames result from P – Δ moments.

Material Failures and Stability Failures

Load-moment curves are plotted in Figure 4-21 for columns of three different lengths, all loaded (as shown in Figure 4-15) with the same end eccentricity, e. The load-moment curve O-A for a relatively short column is practically the same as the line M = Pe. For a column of moderate length, line O-B, the deflections become significant, reducing the failure load. This column fails when the load-moment curve intersects the interaction diagram at point B. This is called a material failure and is the type of failure expected in most practical columns in braced frames. If a very slender column is loaded with increasing axial load, P, applied at a constant end eccentricity, e, it may reach a deflection δ at which the value of the $\partial M/\partial P$ approaches infinity or becomes negative. When this occurs, the column becomes unstable, because, with further deflections, the axial load capacity will drop. This type of failure is known as a stability failure and occurs only with very slender braced columns or with slender columns in sway frames.

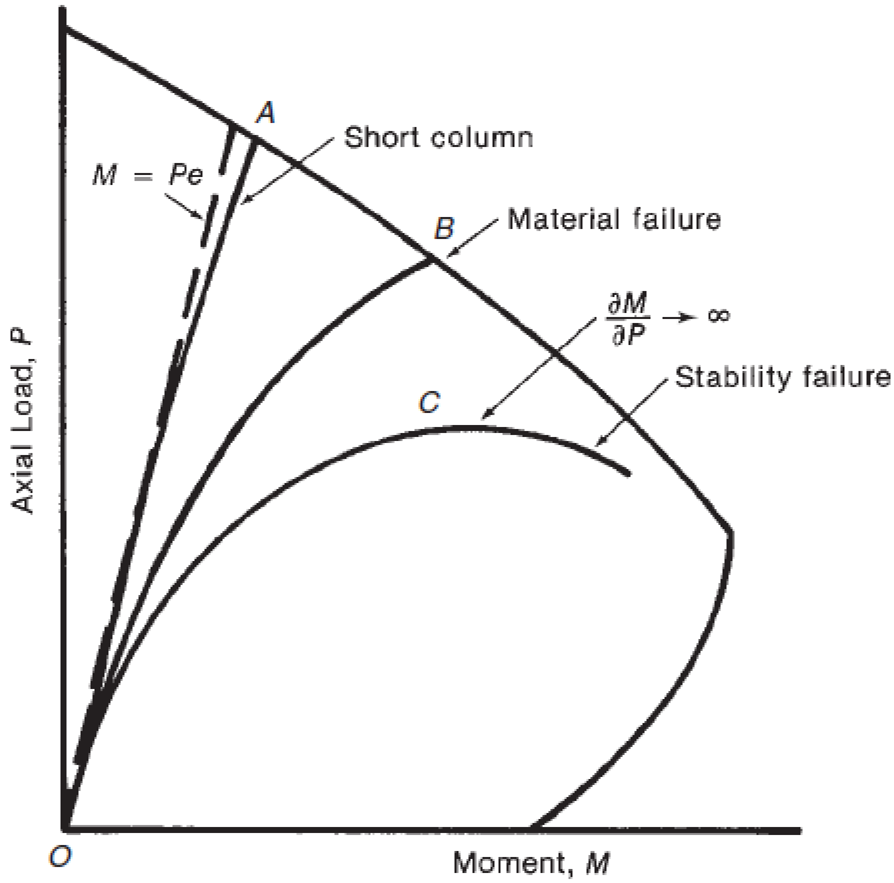

Figure 4-21 – Material and stability failures

Effect of Unequal end moments on the strength of a slender column

Up to now, we have considered only pin-ended columns subjected to equal moments at the two ends. This is a very special case, for which the maximum deflection moment, $P\delta$, occurs at a section where the applied load moment, Pe, is also a maximum. As a result, these quantities can be added directly, as done in Figure 4-15 and Figure 4-16.

In the usual case, the end eccentricities, $e_1 = M_1/P$ and $e_2 = M_2/P$, are not equal and so give applied moment diagrams as shown shaded in Figure 4-22b and c for the column shown in Figure 4-22a. The maximum value of δ occurs between the ends of the column while the maximum e occurs at one end of the column. As a result, e_{max} and δ_{max} cannot be added directly. Two different cases exist. For a slender column with small end eccentricities, the maximum sum of $e+\delta$ may occur between the ends of the column, as shown in Figure 4-22b. For a shorter column, or a column with large end eccentricities, the maximum sum of $e+\delta$ will occur at one end of the column, as shown in Figure 4-22c.

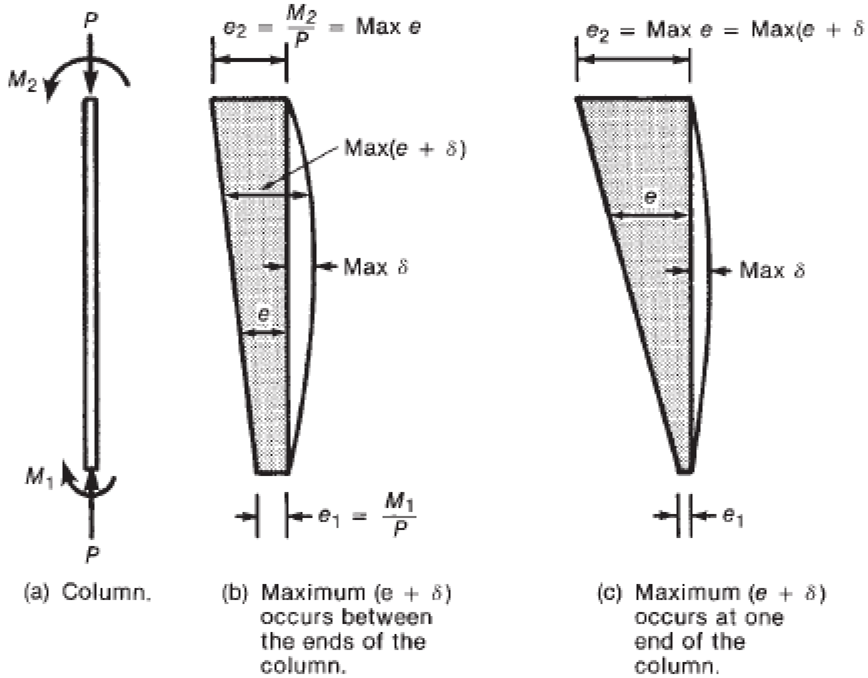

Figure 4-22 – Moments in columns with unequal end moments

In the moment-magnifier design procedure, the column subjected to unequal end moments shown in Figure 4-23a is replaced with a similar column subjected to equal moments of $C_m M_2$ at both ends, as shown in Figure 4-23b. The moments $C_m M_2$ are chosen so that the maximum magnified moment is the same in both columns. The expression for the equivalent moment factor C_m was originally derived for use in the design of steel beam – column and was adopted without change for concrete design.

$$C_m = 0.6 + 0.4\frac{M_1}{M_2} \qquad (11)$$

Thus in the above equation, M_1 and M_2 are the smaller and larger end moments, respectively, calculated from a conventional first-order elastic analysis. The sign convention for the ratio M_1/M_2 is illustrated in Figure 4-23c and Figure 4-23d. If the moments M_1 and M_2 cause single curvature bending without a point of contraflexure between the ends, as shown in Figure 4-23c, M_1/M_2 is positive. If the moments M_1 and M_2 bend the column in double curvature with a point of zero moment between the two ends, as shown in Figure 4-23d, then M_1/M_2 is negative.

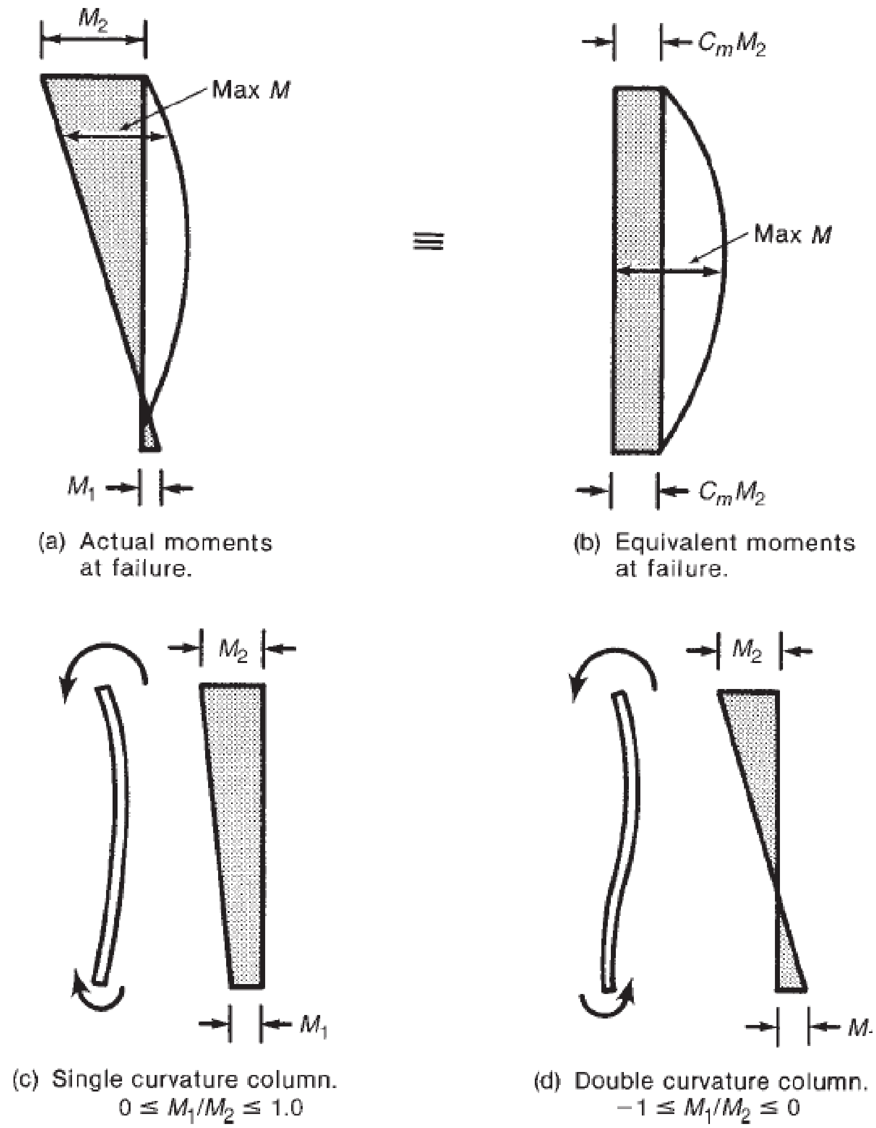

Figure 4-23 – Equivalent moment factor, C_m

4.8. DESIGN OF COLUMNS ACCORDING TO ES EN 1992:2015

4.8.1. SECOND ORDER EFFECTS WITH AXIAL LOAD

Second order effects may be ignored if they are less than 10% of the corresponding first order effects. Simplified criteria are given for isolated members below.

4.8.1.1. Simplified criteria for second order effects

Slenderness criterion for isolated members

Second order effects may be ignored if the slenderness λ is below a certain value λ_{lim} . The following may be used:

$$\lambda_{lim} = 20A \bullet B \bullet C\sqrt{n}$$

Where:

λ	is the slenderness ratio
A	$= 1/(1+0.2\varphi_{ef})$ (if φ_{ef} is not known, A = 0.7 may be used
B	$= \sqrt{1+2\omega}$ (if ω is not known, B = 1.1 may be used)
C	$= 1.7 - r_m$ (if r_m is not known, C = 0.7 may be used)
φ_{ef}	Effective creep ratio
ω	$= A_s f_{yd} / A_c f_{cd}$;mechanical reinforcement ratio;
A_s	is the total area of longitudinal reinforcement
n	$= N_{Ed} / (A_c f_{cd})$; relative normal force
r_m	$= M_{01}/M_{02}$; moment ratio
M_{01} and M_{02}	are the first order end moments, $\lvert M_{02}\rvert \geq \lvert M_{01}\rvert$

If the end moments M_{01} and M_{02} give tension on the same side, r_m should be taken positive (i.e. $C \leq 1.7$), otherwise negative (i.e. $C > 1.7$).

In the following cases, r_m should be taken as 1.0 (i.e. $C = 0.7$).

– For braced members with first order moments only or predominantly due to imperfections or transverse loading
– For unbraced members in general

In cases with biaxial bending, the slenderness criterion may be checked separately for each direction. Depending on the outcome of this check, second order effects:

a) May be ignored in both directions
b) Should be taken into account in one direction
c) Should be taken into account in both directions

4.8.1.2. Slenderness and effective length of isolated members

The slenderness ratio is defined as follows:

$$\lambda = l_0 / i \qquad (12)$$

where:

l_0	is the effective length
i	is the radius of gyration of the uncracked concrete section

Effective length is a length used to account for the shape of the deflection curve; it can also be defined as buckling length, i.e. the length of a pin-ended column with constant normal force, having the same cross section and buckling load as the actual member.

Examples of effective length for isolated members with constant cross section are given in Figure 4-24.

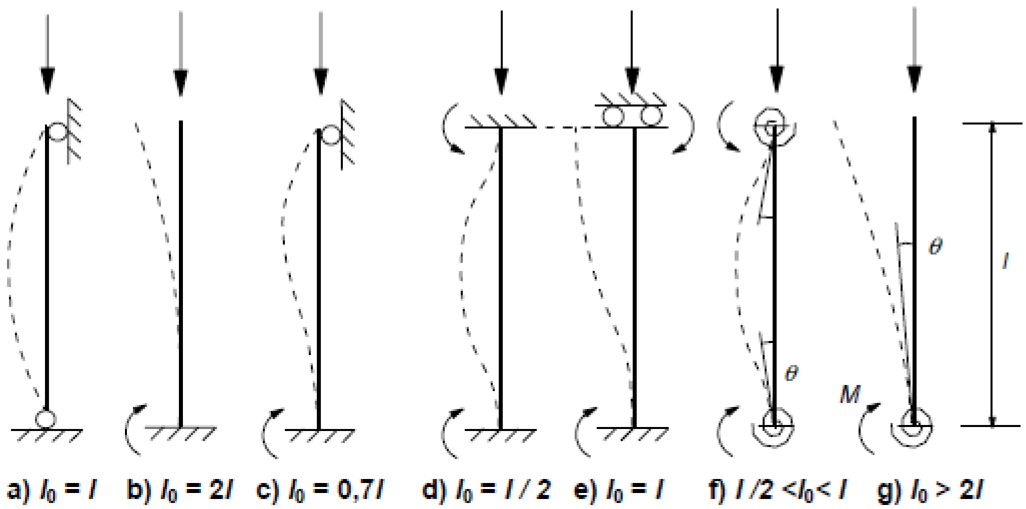

Figure 4-24 – Examples of different buckling modes and corresponding effective lengths for isolated members

For compression members in regular frames, the slenderness criterion should be checked with an effective length l_0 determined in the following way.

Braced members

$$l_0 = 0.5l \bullet \sqrt{\left(1 + \frac{k_1}{0.45 + k_1}\right) \bullet \left(1 + \frac{k_2}{0.45 + k_2}\right)} \qquad (13)$$

Unbraced members

$$l_0 = l \bullet \max\left\{\sqrt{1 + 10 \cdot \frac{k_1 \cdot k_2}{k_1 \cdot k_2}}; \left(1 + \frac{k_1}{1 + k_1}\right) \cdot \left(1 + \frac{k_2}{1 + k_2}\right)\right\} \qquad (14)$$

where;

k_1 , k_2	are the relative flexibilities of rotational restraints at ends 1 and 2 respectively:
k	$= (\theta / M) \cdot (EI / l)$
θ	is the rotation of restraining members for bending moment M;
EI	is the bending stiffness of compression member
l	is the clear height of compression member between end restraints

$k = 0$ is the theoretical limit for rigid restraint, and $k = \infty$ represents the limit for no restraint at all. Since fully rigid restraint is rare in practice, a minimum value of 0.1 is recommended for k_1 and k_2.

If an adjacent compression member (column in a node is likely to contribute to the rotation at buckling, then (EI/l) in the definition of k should be replaced by $\left[(EI/l)_a + (EI/l)_b\right]$, a and b representing the compression member (column) above and below the node.

In the definition of effective lengths, the stiffness of restraining members should include the effect of cracking, unless they can be shown to be uncracked in ULS.

4.8.2. CREEP

The effect of creep shall be taken into account in second order analysis, with due consideration of both the general conditions for creep and the duration of different loads in the load combination considered.

The duration of loads may be taken into account in a simplified way by means of an effective creep ratio, φ_{ef}, which, used together with the design load, gives a creep deformation (curvature) corresponding to the quasi-permanent load:

$$\varphi_{ef} = \varphi_{(\infty,t0)} \cdot M_{oEqp} / M_{0Ed} \quad (15)$$

Where:

$\varphi_{(\infty,t0)}$	is the final creep coefficient according to 3.1.4 of ES EN 1992:2015
M_{oEqp}	is the first order bending moment in quasi-permanent load combination (SLS)
M_{0Ed}	is the first order bending moment in design load combination (ULS)

If M_{oEqp}/M_{0Ed} varies in a member of structure, the ratio may be calculated for the section with maximum moment, or a representative mean value may be used.

The effect of creep may be ignored, i.e. $\varphi_{ef} = 0$ may be assumed, if the following three conditions are met:

- $\varphi_{(\infty,t0)} \leq 2$
- $\lambda \leq 75$
- $M_{oEd}/N_{Ed} \geq h$

Here M_{oEd} is the first order moment and h is the cross section depth in the corresponding direction.

4.8.3. METHODS OF 2ND ORDER MOMENT ANALYSIS FOR SLENDER COLUMNS

The methods of analysis include a general method, based on non-linear second order analysis, and the following two simplified methods

a) Second order analysis based on nominal stiffness
b) Method based on estimation of curvature

Method (a) may be used for both isolated members and whole structures, if nominal stiffness values are estimated appropriately but Method (b) is mainly suitable for isolated members.

a) Second order analysis based on nominal stiffness

In a second order analysis based on stiffness, nominal values of the flexural stiffness should be used, taking into account the effects of cracking, material non-linearity and creep on the overall behavior.

The nominal stiffness should be defined in such a way that total bending moments resulting from the analysis can be used for design of cross sections to their resistance for bending moment and axial force.

Nominal Stiffness

The following model may be used to estimate the nominal stiffness of slender compression member with arbitrary cross section:

$$EI = K_c E_{cd} I_c + K_s E_s I_s \quad (16)$$

where:

E_{cd} is the design value of the modulus of elasticity of concrete
I_c is the moment of inertia of concrete cross section
E_s is the design value of the modulus of elasticity of reinforcement
I_s is the second value of the modus of elasticity of reinforcement, about the center of area of the concrete
K_c is a factor for effects cracking, creep etc,
K_s is a factor for contribution of reinforcement

The following factors may be used in the above expression, provided $\rho \geq 0.002$:

$K_s = 1$

$K_c = k_1 k_2 / (1 + \varphi_{ef})$

Where:

ρ is the geometric reinforcement ratio, A_s / A_c

A_s is the total area of reinforcement

A_c is the area of concrete section

φ_{ef} is the effective creep ratio

k_1 is a factor which depends on concrete strength class

k_2 is a factor which depends on axial force and slenderness

$k_1 = \sqrt{f_{ck}/20}$ (MPa)

$$k_2 = n \bullet \frac{\lambda}{170} \leq 0.20$$

where:

n is the relative axial force, $N_{Ed} / (A_c f_{cd})$

λ is the slenderness ratio

If the slenderness ratio λ is not defined, k_2 may be taken as

$$k_2 = n \bullet 0.3 \leq 0.20$$

As a simplified alternative, provided $\rho \geq 0.01$, the following factors may be used in equation (17):

$$K_s = 0$$

$$K_c = 0.3 / (1 + 0.5\varphi_{ef})$$

In statically indeterminate structures, unfavourable effects of cracking in adjacent members should be taken into account. The above expressions are not generally applicable to such members. As a simplification, fully cracked sections may be assumed. The stiffness should be based on an effective concrete modulus:

$$E_{cd,eff} = E_{cd} / (1 + \varphi_{ef}) \quad (17)$$

where:

E_{cd} Is the design value according to 5.8.6 (3)

φ_{ef} Is the effective creep ratio; same value as for columns may be used

$$E_{cd} = E_{cm} / \gamma_{cE} \quad (18)$$

Method based on moment magnification factor

The total design moment, including second order moment, may be expressed as a magnification of the bending moments resulting from a linear analysis, namely:

$$M_{Ed} = M_{0Ed}\left[1+\frac{\beta}{(N_B/N_{Ed})-1}\right] \quad (19)$$

where:

M_{0Ed} is the first order moment

β is a factor which depends on distribution of 1st and 2nd order moments

N_{Ed} is the design value of axial load

N_B is the buckling load based on nominal stiffness

For isolated members with constant cross section and axial load, the second order moment may normally be assumed to have a sine-shaped distribution. Then

$$\beta = \pi^2/c_0 \quad (20)$$

Where:

c_0 is a coefficient which depends on the distribution of first order moment (for instance, $c_0 = 8$ for a constant first order moment, $c_0 = 9.6$, for a parabolic and 12 for a symmetric triangular distribution etc.).

For members without transverse load, differing first order end moments M_{01} and M_{02} may be replaced by an equivalent constant first order moment M_{0e}. Consistent with the assumption of a constant first order moment, $c_0 = 8$ should be used.

$$M_{0e} = 0.6M_{02} + 0.4M_{01} \geq 0.4M_{02} \quad (21)$$

M_{01} and M_{02} should have the same sign if they give tension on the same side, otherwise opposite signs. Furthermore, $|M_{02}| \geq |M_{01}|$.

The value of $c_0 = 8$ also applies to members bent in double curvature. It should be noted that in some cases, depending on slenderness and axial force, the end moment(s) can be greater than the magnified equivalent moment.

If the above expressions for calculation of β is not applicable, $\beta = 1$ is normally a reasonable simplification. Expression (18) can then be reduced to:

$$M_{Ed} = \frac{M_{0Ed}}{1-(N_{Ed}/N_B)} \quad (22)$$

b) Method based on nominal curvature

This method is primarily suitable for isolated members with constant normal force and a defined effective length l_0 . The method gives a nominal second order moment based on a deflection, which in turn is based on the effective length and an estimated maximum curvature. The resulting design moment and axial force.

Bending moments

The design moment is :

$$M_{Ed} = M_{0Ed} + M_2 \tag{23}$$

where:

M_{0Ed} is the 1st order moment, including the effect of imperfections

M_2 Is the nominal 2nd order moment

The nominal second order moment M_2 is

$$M_2 = N_{Ed} e_2 \tag{24}$$

where:

N_{Ed} is the design value of axial force

e_2 is the deflection $= (1/r) l_0^2 / c$

$1/r$ is the curvature

l_0 is the effective length

c is a factor depending on the curvature distribution

For constant section, $c = 10(\approx \pi^2)$ is normally used. If the first order moment is constant, a lower value should be considered (8 is a lower limit, corresponding to constant total moment).

Curvature

For members with constant symmetrical cross sections, the following may be used:

$$1/r = K_r K_\varphi 1/r_0 \tag{25}$$

where:

K_r is a correction factor depending on axial load

K_φ is a factor for taking account of creep

$1/r_0$ $= \varepsilon_{yd} / (0.45d)$

ε_{yd} $= f_{yd} / E_s$

d is the effective depth

If all reinforcement is not concentrated on opposite sides, but part of it is distributed parallel to the plane of bending, d is defined as

$$d = (h/2) + i_s \tag{26}$$

where i_s is the radius of gyration of the total reinforcement area

K_r in Expression (18) should be taken as:

$$K_r = (n_u - n)/(n_u - n_{bal}) \leq 1 \tag{27}$$

where:

n	$= N_{Ed}/(A_c f_{cd})$, relative axial force
N_{Ed}	is the design value of axial force
n_u	$= 1 + \omega$
n_{bal}	is the value of n at maximum moment resistance; the value 0.4 may be used
ω	$= A_s f_{yd}/(A_c f_{cd})$
A_s	is the total area of reinforcement
A_c	is the area of concrete cross section

The effect of creep should be taken into account by the following factor

$$K_\varphi = 1 + \beta\varphi_{ef} \geq 1 \tag{28}$$

where:

φ_{ef}	is the effective creep ratio
β	$= 0.35 + f_{ck}/200 - \lambda/150$
λ	is the slenderness ratio

4.8.4. DETAILING RULES FOR COLUMNS ACCORDING TO ES EN 1992:2015

Longitudinal Reinforcement

- Bars should have a diameter of not less than $\phi_{\min}$ and the recommended value of $\phi_{\min}$ is 8 mm.
- The total amount of longitudinal reinforcement should not be less than $A_{s,\min}$ and the recommended value is

$$A_{s,\min} = \frac{0.1N_{Ed}}{f_{yd}} \text{ or } 0.002A_c \text{ whichever is the greater} \tag{29}$$

Where:

f_{yd}	Is the design yield strength of the reinforcement
N_{Ed}	Is the design axial compression force

- The area of reinforcement should not exceed $A_{s,\max}$. The recommended value of $A_{s,\max}$ is $0.04A_c$ outside lap locations unless it can be shown that the integrity of concrete is not affected, and that the full strength is achieved at ULS. This limit should be increased to $0.08A_c$ at laps.
- For columns having a polygonal cross-section, at least one bar should be placed at each corner. The number of longitudinal bas in a circular column should not be less than four.

Transverse reinforcement

- The diameter of the transverse reinforcements should not be less than 6 mm or one quarter of the maximum diameter of the longitudinal bars, whichever is the greater. The diameter of the wires or welded mesh fabric for transverse reinforcement should not be less than 5 mm.
- The transverse reinforcement should be anchored adequately.
- The spacing of the transverse reinforcement along the column should not exceed $S_{cl,t\max}$. The recommended value of $S_{cl,t\max}$ is the least of the following three distances:
  - 20 times the minimum diameter of the longitudinal bars
  - The lesser dimension of the column
  - 400 mm
- Every longitudinal bar or bundle of bars placed in a corner should be held by transverse reinforcement. No bar within a compression zone should be further than 150 mm from a restrained bar.