CHAPTER 4

KINEMATICS OF FLOW

4.1 Introduction:

In hydrostatics one deals with liquids at rest in which there is no relative motion between fluid particles and therefore no shear stresses exist. Since no friction is involved the fluid may be assumed to be either ideal or real.

In fluid flow problems one often refers to the flow of an ideal fluid. An ideal fluid flow as the name implies, is an idealized situation in which the fluid is assumed to have no viscosity and therefore no shear stresses exist. Boundary effects are ignored and velocities assumed to be uniform. Such simplification is sometimes useful in solving some engineering problems. When dealing with real fluid flow, however, the effects of viscosity are introduced thus leading to considerations of the developed shear stresses between neighbouring fluid layers that are moving at different velocities. The flow picture thus becomes complex and can not be easily formulated mathematically as in the idealized situation. It requires the combination of mathematical theory with experiments. Study of fluids in motion thus requires consideration of fluid properties (such as specific weight, viscosity etc), kinematics and force and energy relationships. *Kinematics* deals only with the geometry of motion i.e. space time relationships of fluids only without regard to the forces causing the motion.

4.2 Velocity Field

There are two methods or frames of reference by which the motion of a fluid can be described:

(i) *The lagrangian Method* and
(ii) *The Eulerian Method*

The Lagrangian Method: In this method the observer focusses his attention on a single fluid particle during its motion through space to find out the path it traces and to describe its characteristics such as velocity, acceleration, density etc as it moves in the flow field with the passage of time. In the cartesian coordinate system, the position of a fluid particle in space (x,y,z) is expressed with respect to a coordinate system (a,b,c) at time to. Thus, a,b,c and t_0 are independent variables while x,y,z are dependent variables in this method. If u is the velocity of the fluid particle at time t, then its position x = a + ut where a is its x coordinate at time t_0. Thus the position of the particle will be:

$$\begin{aligned} x &= f_1(a,b,c,t) \\ y &= f_2(a,b,c,t) \\ z &= f_3(a,b,c,t) \end{aligned} \tag{4.1}$$

The corresponding velocities u, v and w and accelerations a_x, a_y and a_z in the x,y, and z directions will be:

$$\begin{aligned} u &= \frac{\partial x}{\partial t} & a_x &= \frac{\partial^2 x}{\partial t^2} \\ v &= \partial y/\partial t & a_y &= \partial^2 y/\partial t^2 \\ z &= \partial z/\partial t & a_z &= \partial^2 z/\partial t^2 \end{aligned} \tag{4.2}$$

The Lagrangian Method of analysis is difficult in fluid mechanics since it is not easy to identify a fluid particle and because every particle has a random motion.

The Eulerian Method: In this method the observer's concern is to know what happens at any given point in the space which is filled with a fluid. One is interested in what the velocities, accelerations, pressures etc are at various points in the flow field at any given time. This method is extensively used in fluid mechanics because of its simplicity and due to the fact that one is more interested in flow parameters at different points in a flow and not in what happens to individual fluid particles. The position of a particle in this method is expressed with respect to a fixed coordinate system x,y,z at a given time t. The Eulerian velcity field is thus given by:

$$
\begin{aligned}
u &= f_1(x,y,z,t) \\
v &= f_2(x,y,z,t) \\
w &= f_3(x,y,z,t)
\end{aligned}
\tag{4.3}
$$

Since equation (4.2) describes the motion of a single fluid particle, the relationship between the Lagrangian and the Eulerian equations will be:

$$
\begin{aligned}
\frac{dx}{dt} &= u(x,y,z,t) \\
\frac{dy}{dt} &= v(x,y,z,t) \\
\frac{dz}{dt} &= w(x,y,z,t)
\end{aligned}
\tag{4.4}
$$

The integration of Equations (4-4) leads to the Lagrangian equations (4-1) with the initial conditions $x = x_o = a$; $y = y_o = b$; $z = z_o = c$ and $t = t_o$. Hence, the Lagrangian Method can be derived from the Eulerian Method.

4.3 Velocity and Acceleration:

The motion of fluid particles in a particular flow phenomenon is expressed in terms of a vector quantity known as velocity. In figure (4-1), if Δs is the distance travelled by a fluid particle in time Δt, then $\lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t}$ is called the velocity V_s and it is tangential to the path s.

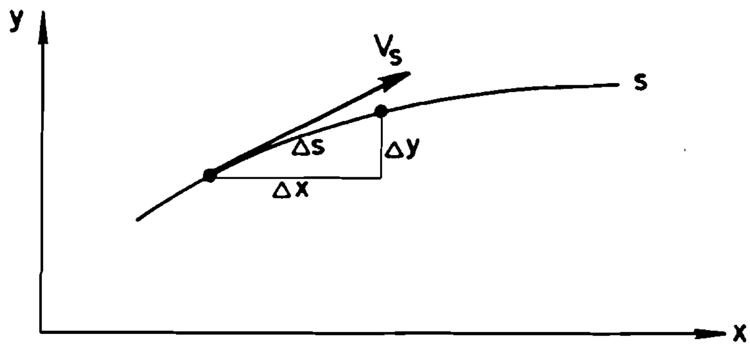


Figure 4.1 Tangential velocity

Since the distance Δs can be resolved in general into distances Δx, Δy and Δz in the x,y and z directions respectively, the velocity vector V_s inturn can be resolved into components u, v and w in the x,y and z directions respectively. The magnitudes of each of the component velocities will in general depend on the location of the point under consideration i.e on x,y and z and also on time t depending upon the type of flow. Thus the velocity V_s and components can be written in the following functional form:

$$
\begin{aligned}
V_s &= f_1(x,y,z,t) \\
u &= f_2(x,y,z,t) \\
v &= f_3(x,y,z,t) \\
w &= f_4(x,y,z,t)
\end{aligned}
\tag{4.5}
$$

Consider now a particle of fluid moving from A to B on a stream line as shown in figure 4.2

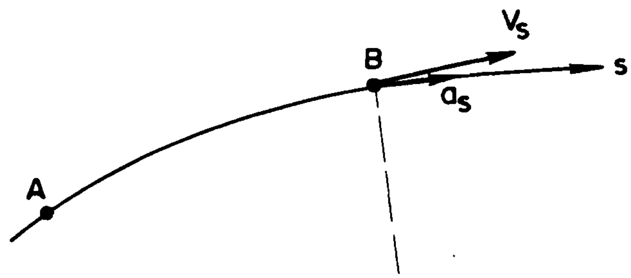


Figure 4.2 Tangential acceleration

The velocity of the particle may change for two reasons: At a particular instance, the velocity at A may be different from the velocity at B and also during the period the given particle moves from A to B, the velocity at B might change. Thus the total change in the velocity of the particle, dv_s, will be the sum of its change due to change in position and its change due to passage of time interval dt.

$$\text{i.e.} \qquad dv_s = \frac{\partial v_s}{\partial s} ds + \frac{\partial v_s}{\partial t} \,.\, dt.$$

The tangential acceleration a_s in the flow direction will be:

$$a_s = \frac{dv_s}{dt} = \frac{\partial v_s}{\partial s} \cdot \frac{ds}{dt} + \frac{\partial v_s}{\partial t} \cdot \frac{dt}{dt}$$

$$= \frac{\partial v_s}{\partial s} \cdot \frac{ds}{dt} + \frac{\partial v_s}{\partial t}$$

$$= V_s \frac{\partial v_s}{\partial s} + \frac{\partial v_s}{\partial t}$$

$$\textit{Thus} \quad a_s = v_s \frac{\partial v_s}{\partial s} + \frac{\partial v_s}{\partial t} \tag{4.6}$$

where: a_s = local tangential acceleration

$\frac{\partial v_s}{\partial t}$ = the local or temporal component

= the rate of change of velocity with respect to time at a particular point

$v_s \frac{\partial v_s}{\partial t}$ = the convective component

= the rate of change of velocity due to the particle's change of position

The normal acceleration, which is the result of change in direction of velocity may be obtained by considering the curved stream line and the velocity vector diagram shown in Figure (4-3).

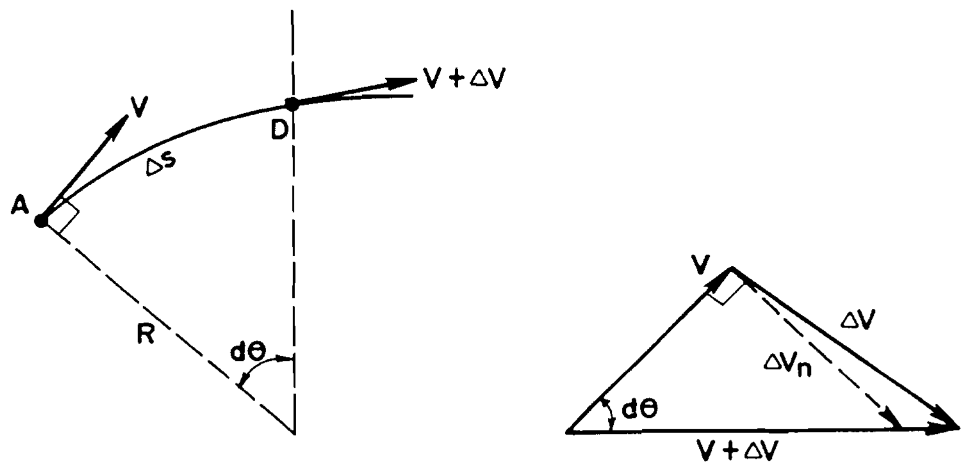


Figure 4.3

V is the tangential velocity at A

V + Δv is the tangential velocity at D at a distance Δs from A.

ΔV_n is the velocity in the normal direction at A.

R is the radius of curvature of the stream line.

From the velocity vector diagram,

$$\Delta V_n = V d\theta = V . \frac{\Delta s}{R}$$

The convective acceleration a_{n_1} in the normal direction is given by:

$$a_{n_1} = \frac{\Delta V_n}{\Delta t} = \frac{\Delta V_n}{\Delta s} . \frac{\Delta s}{\Delta t} = \frac{V}{R} . V = \frac{V^2}{R}$$

The normal velocity can also change with time. Hence, the temporal component of the normal acceleration will be $\frac{\partial V_n}{\partial t}$.

Therefore, the normal acceleration with its temporal and convective components will be:

$$a_n = \frac{\partial V_n}{\partial t} + \frac{V^2}{R} \tag{4.7}$$

4.4 Pathline, Streak line, Streamline and Steam tube

Pathline: If an individual particle of fluid is coloured, it will describe a pathline which is the trace showing the position at successive intervals of times of a particle which started from a given point.

Streakline or Filament line: If, instead of colouring an individual particle, the flow pattern is made visible by injecting a stream of dye into a liquid, or smoke into a gas, the result will be a streakline or filament line, which gives an instantaneous picture of the positions of all the particles which have passed through the particular point at which the dye is being injected. Since the flow pattern may vary from moment to moment, a streak line will not necessarily be the same as a pathline.

Streamline: A streamline is defined as an imaginary line drawn through a flow field such that the tangent to the line at any point on the line indicates the direction of the velocity vector at that instant. A streamline thus gives a picture of the average direction of flow in a flow field.

Since the velocity vector at any point on a streamline is tangential it will not have a component normal to the streamline. Hence there can not be any flow across a streamline. Thus the flow between any two streamlines remains constant. A smooth flow boundary can also be considered as a streamline. If conditions are steady and the flow pattern does not change from moment to moment, pathlines and streamlines are identical.

Consider a streamline shown in figure (4-4). The velocity vecotr V_s at point P(s,y) has components u and v in the x and y directions respectively.

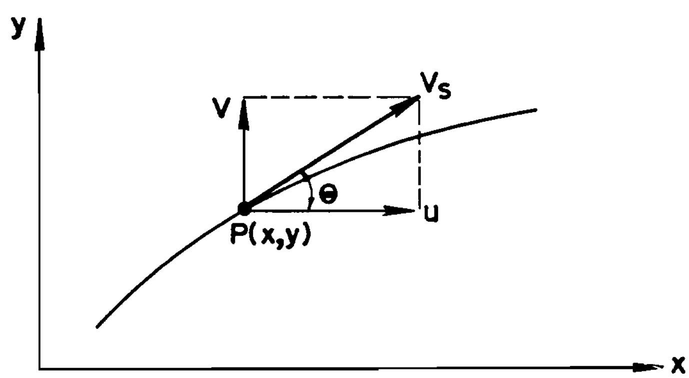


Figure 4.4 Steamline

Taking θ as the angle between V_s and the x axis,

$$\tan\theta = \frac{dy}{dx} = \frac{v}{u}$$

Thus, the equation of a streamline in two dimensional flow at any instant t_o is:

$$\frac{dx}{u} = \frac{dy}{v} \qquad \textit{Where} \qquad u = f_1(x, y, t_o)$$

$$v = f_2(x, y, t_o)$$

Example 4.1 If u = +x, and v = 2y, find the equation of the streamline through (1,1).

$$\frac{dx}{u} = \frac{dy}{v}$$

$$\text{or} \quad \frac{dx}{x} = \frac{dy}{2y}$$

$$\ell n x = \frac{1}{2} \ell n y + C$$

$$\text{at} \quad x = 1 \quad \text{and} \quad y = 1, \quad C = 0$$

$\therefore$ $x = \sqrt{y}$ *is the equation of the stream line*

Streamtube: If a series of streamlines are drawn through every point on the perimeter of a small area of a stream crossection, they will form a streamtube. Since there is no flow across a streamline, there will not be flow across a streamtube and the fluid inside a streamtube cannot excape through its walls. The flow thus behaves as if it were contained in an imaginary pipe. The concept of a streamtube is useful in dealing with the flow of fluids since it allows elements of the fluid to be isolated for analysis.

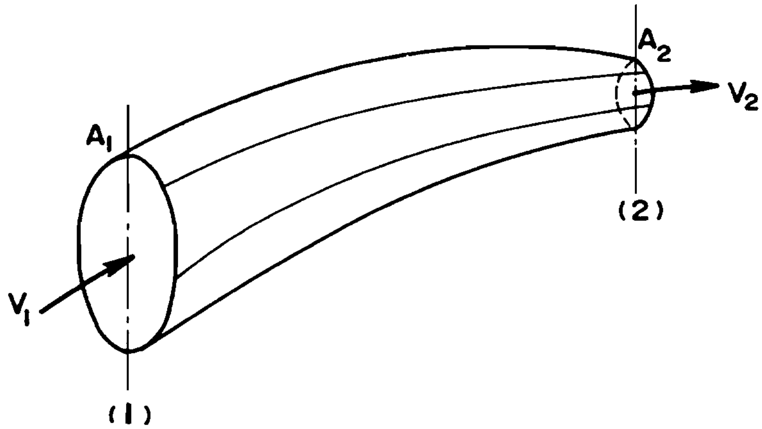


Figure 4.5 Stream tube

4.5 Classification of flows

In a general flow field, velocity, pressure, density etc. can vary from place to place or can change with respect to time or both variations can occur simultaneoussly. It is convenient to classify flows on the basis of change in velocity only. Accordingly, a flow may be classified as steady or unsteady depending upon whether the velocity at a point varies with time or not and as uniform or non-uniform depending upon whether the velocity at different points on a streamline in a flow field at an instant is the same or not.

When any of the flow parameters at a point do not change with time, the flow is said to be *steady*. Variations of any of the flow parameters with time at a point would cause the flow *unsteady*.
Steady and unsteady conditions refer only to average temporal velocity in a flow field and turbulent fluctuations are not considered.

Velocities in a flow field depend on the geometry of the boundary. Consider the three situations in Figure 4-6.

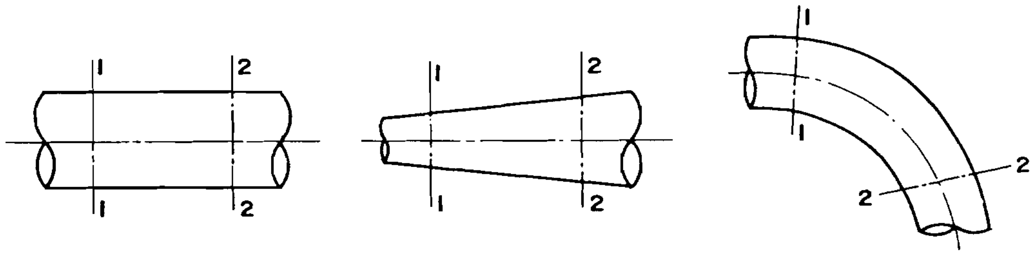


Figure 4.6 Uniform and non-uniform flows

If the rate of flow does not vary with time i.e the flow is steady, then the average velocities at any two sections 1-1 and 2-2 in Figure 4-6(a) would be the same. In the expanding pipe of Figure 4-6(b), the velocities at sections 1-1 and 2-2 are different since the cross-sectional areas vary. In the bend of constant diameter shown in Figure 4-6(c), eventhough the magnitudes of the velocities at sections 1-1 and 2-2 are the same, their directions are different and hence there is variation in velocity.

A flow is considred *uniform* if velocities at different points in a steamline (or average velocities at different sections in a conduit) in a flow field at an instant are the same both in magnitude and direction. If there is variation either in magnitude or direction or both, then the flow is said to be *non-uniform*. Thus flow in the straight pipe of uniform diameter is classified as uniform while those in the expanding pipe and the bend are non-uniform.

Considering both temporal and spatial (convective) variations in the flow parameters, the following four combinations of flow are possible:

i) Steady uniform flow - flow through a uniform diameter pipe with a constant rate of flow.

ii) Steady non-uniform flow - flow through a straight pipe with changing diameter (expanding or reducing) and a bend with uniform or non-uniform diameter at constant rate of flow.

iii) Unsteady uniform flow - flow through a uniform diameter pipe at changing rates of flow.

iv) Unsteady non-uniform flow - flow as in (ii) but with changing rate of flow.

4.6 One, Two and Three-Dimensional Flows

When the velocity components transverse to the main flow direction is neglected and only average conditions of flow are considered at a section then the flow is said to be one-dimensional. The assumption of one dimensional flow can be made where there is no wide variation of cross-section, where stream lines are not highly curvelinear and where the velocity variation across a section is not appreciable. Many engineering problems such as flows through a pipe and open channel flows are handled by one dimensional analysis by taking average values of the flow characteristics at sections.

In actual flows of real fluids, the presence of fluid viscosity and the no slip condition at the boundary require that the velocity vary from zero at the boundary to a maximum value somewhere in the flow field depending upon the boundary conditions. Such a flow where the velocity vector is a function of two co-ordinates is known as two-dimensional. Flow past a wide flat plate or over a long weir can be considered two- dimensional.

Figure 4.7 illustrates one and two-dimensional flows. In figure 4.7(a) there is a velocity variation only in the flowdirection and velocities are consatant at each of the sections 1-1 and 2-2. In figure 4.7(b) velocity variations occur in both x and y directions.

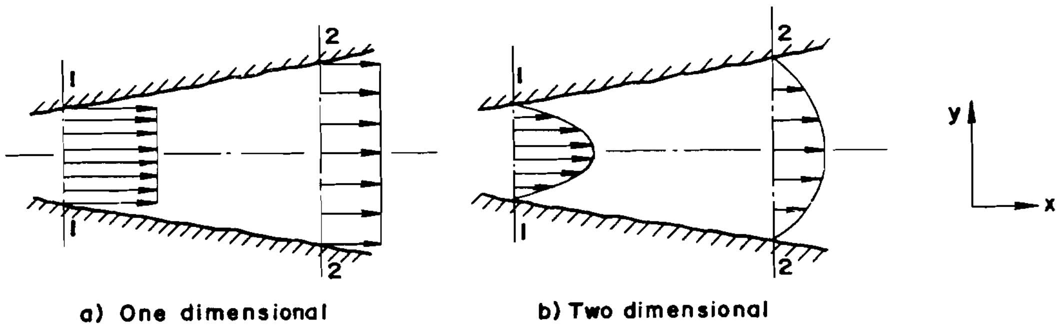


Figure 4.7 One and two dimensional flow

Three dimensional flow is the most general type of flow in which the velocity vector varies in the three coordinate directions x, y and z and is generally complex.

Thus in terms of the velocity vector V, the following apply:

	Unsteady	Steady
One-dimensional flow	v = f(x,t)	v = f(x)
Two-dimenstional flow	v = f(x,y,t)	v = f(x,y)
Three-dimesninal flow	v = f(x,y,z,t)	v = f(x,y,z)

4.7 Discharge and Mean Velocity

The total quantity of fluid flowing in unit time past any particular cross-section of a stream is called the discharge or flow at that section. It can be measured either in terms of mass, in which case it is referred to as the mass rate of flow $\dot{m}$ (eg. in kg/s), or it can be measured in terms of volume, when it is known as the volumetric rate of flow Q (eg. in m_3/s).

In many problems, the variation of velocity over the cross-section can be ignored and the velocity is assumed to be constant and equal to the mean velocity $\overline{V}$. If the cross-sectional area normal to the direction of flow is A, the volume passing the cross-section in unit time would be $A.\overline{V}$. Thus:

$$Q = A\overline{V}$$

$$\text{or The mean velocity } \overline{V} = \frac{Q}{A}$$

In a real fluid flow, the velocity adjacent to a solid boundary will be zero. The velocity profile across a section of a pipe for laminar and turbulent flows are as shown in Figure 4.8.

If u is the velocity at any radius r, the flow dQ through an annular element of radius r and thickness dr will be:

$$dQ = \text{Area of element} \times \text{velocity}$$

$$= 2\pi r dr.u$$

$$\text{Hence,} \quad Q = \int dQ = \int_o^R 2\pi r.u.dr$$

$$\text{or} \quad Q = 2\pi \int_o^R ur dr$$

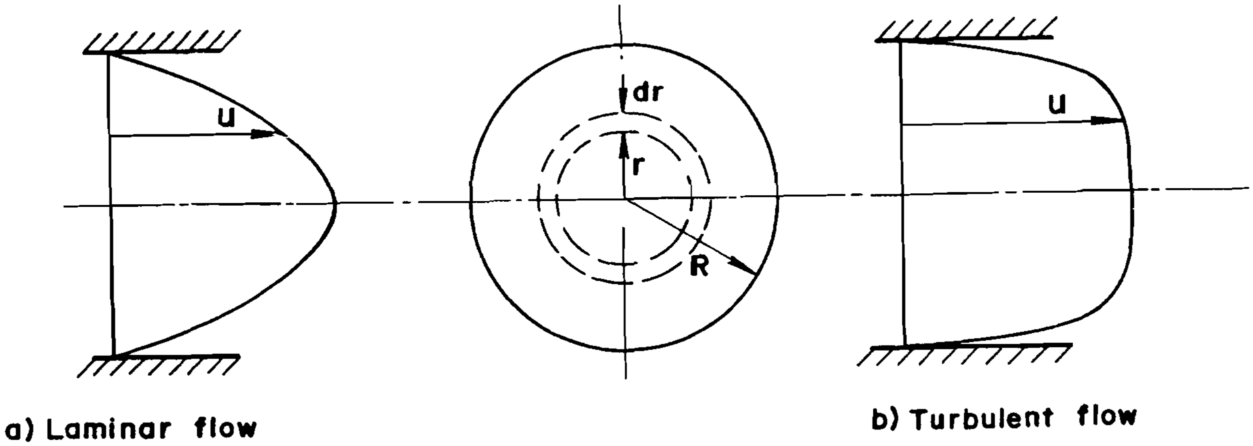


Figure 4.8 Velocity profiles

The above integral can be evaluated if the reltation between u and r can be extablished.

In general, if u is the velocity at any arbitrary location in the profile and A the total flow area, then the average velocity is given by:

$$\bar{V} = \frac{\int_A u dA}{A} \tag{4.8}$$

Example 4.1

The velocity distribution for laminar flow between parallel plates is given by: $u = K(Dy - y^2)$ where u is the velocity at distance y from the bottom plate, D is the distance between the plates and K is a constant. Determine the average velocity of flow.

Solution:

Taking a unit width of the plates, and letting the average velocity be $\bar{V}$,

$$\bar{V}.D.1 = \int_o^D u.dy.1$$

$$\text{or} \qquad \bar{V} = \frac{1}{D}\int_o^D k(Dy-y^2)\,dy$$

$$= \frac{K}{D}\left[\frac{Dy^2}{2} - \frac{y^3}{3}\right]_o^D = \frac{kD^2}{6}$$

4.8 Continuity Equation

The equation of continuity is the mathematical expression for the principle of conservation of mass flow.

4.8.1 *One Dimension, Steady Flow*:

Consider a steam tube through which passes a steady flow of fluid.

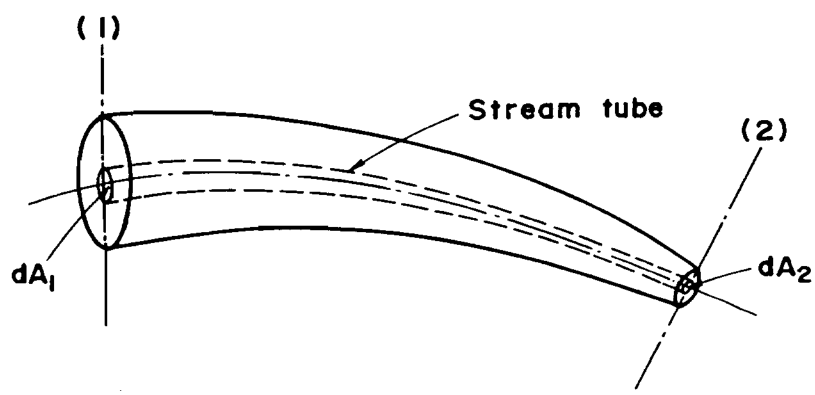


Figure 4.9

At Section (1):	At Section (2):
dA1 = cross-sectional area of stream tube	dA_2 = Cross-sectional area of stream tube
v_1 = avg velocity through stream tube	v_2 = avg. velocity through stream tube
ϱ_1 = fluid density	ϱ_2 = fluid density

Mass rate of flow through $dA_1 = \varrho_1\ dA_1\ v_1$

For the entire cross-section at (1), it will be $\int_{A_1} \rho_1 V_1 dA_1$

Similarly mass rate of flow through entire section (2) will be

$$\int_{A_1} \rho_2 V_2 dA_2$$

For steady flow, principle of conservation of mass gives:

$$\int_{A_1} \rho_1 V_1 dA_1 = \int_{A_2} \rho_2 V_2 dA_2 = Constant$$

For steady one dimensional flow where V_1 and A_1 represent the average velocity and cross-sectional area of section(1) and similarly v_2 and A_2 for section(2), then

$$\rho_1 V_1 A_1 = \rho_2 V_2 A_2$$

For incompressible flow $\rho_1 = \rho_2$

Therefore,

$$v_1 A_1 = v_2 A_2 = Q = Constant \tag{4.8}$$

Equation 4.8 is the continuity equation for steady, incompressible, one-demensional flow.

$$Q = A.V = [\frac{L^3}{T}] = \text{volume rate of flow = discharge.}$$

units of Q : m^3/s, ℓ/sec, ft^3/sec, etc.

Example 4.2

A conical pipe has a diameter of 10 cm and 15 cm at the two ends respectively. If the velocity at the 10 cm end is 2m/sec, what is the velocity at the other end and what is the discharge through the pipe?

Solution:

Continuity Equation:

$$Q = A_1 v_1 = A_2 v_2$$

$$given: \quad v_1 = 2m/\text{sec}, \quad A_1 = \frac{\pi}{4}(0.1)^2, \quad A_2 = \frac{\pi}{4}(0.15)^2$$

$$v_2 = v_1 \frac{A_1}{A_2} = 2 \times \frac{0.1^2}{0.15^2} = \underline{0.89\ m/s}$$

$$discharge, \quad Q = v_1 A_1 = 2 \times \frac{\pi}{4}(0.1)^2 = 0.0157\ m^3/\text{sec}$$

$$= \underline{15.7\ell/\text{sec}}$$

4.8.2 Two and Three Dimensional Flows

For the most general three dimensional case the continuity equation may be derived by considering an elemental volume of sides Δx, Δy and Δz in the cartesian co-ordinate system as shown in Fig. 4.10. Let the density at the centroid if the elemental space be ϱ and the components of velocity be u, v and w in the x, y, and z directions respectively.

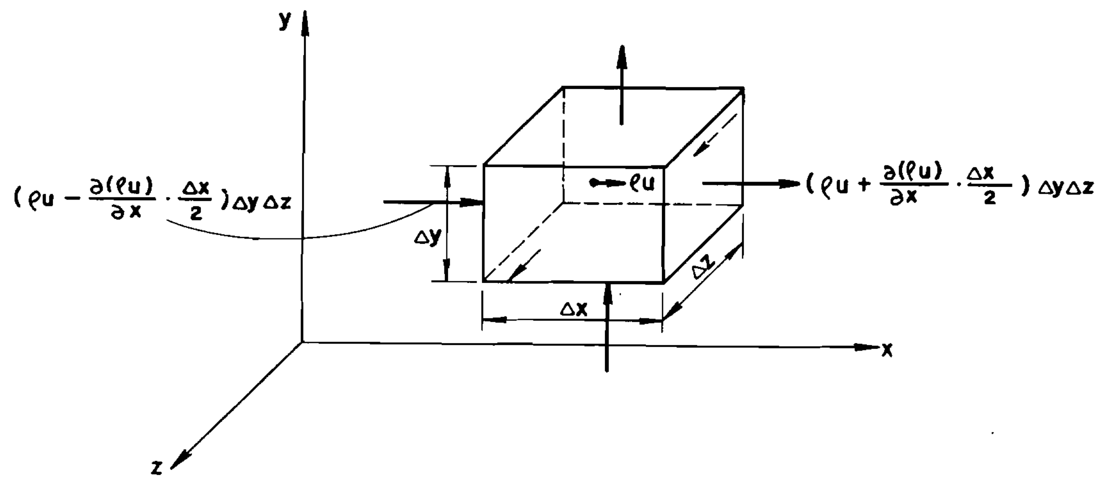


Figure 4.10 Flow through a three-demensional element

Consider first the mass inflow and out flow through the face normal to the x-axis:

Total mass inflow through the face on the left:

$$[\rho u - \frac{\partial(\rho u)}{\partial x}\frac{\Delta x}{2}]\,\Delta y \Delta z$$

Mass out flow through the opposite face:

$$[\rho u + \frac{\partial(\rho u)}{\partial x}\frac{\Delta x}{2}]\,\Delta y \Delta z$$

Hence the net rate of mass influx into the element through these faces is: $-\frac{\partial(\rho u)}{\partial x}\Delta x \Delta y \Delta z$

Similarly, net rate of mass influx through faces perpendicular to y axis is

$$-\frac{\partial(\rho v)}{\partial y}\Delta x \Delta y \Delta z$$

net rate of mass influx through faces perpendicular to z axis

$$-\frac{\partial(\rho w)}{\partial z}\Delta x \Delta y \Delta z$$

Total excess of mass passing into the element per unit time is:

$$-[\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z}]\Delta x \Delta y \Delta z$$

The rate of change of mass contained in the element is given by:

$$+\frac{\partial[\rho \Delta x \Delta y \Delta z]}{\partial t}$$

According to the prinicple of conservation of mass, the total excess rate of mass passing into the element should be equal to the rate of change of mass in the elemental volume:

i.e $$-[\frac{\partial(\rho y)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z}]\Delta x \Delta y \Delta z = \frac{\partial(\rho \Delta x \Delta y \Delta z)}{\partial t}$$

If the elemental volume is allowed to shrink, i.e $\Delta x\ \Delta y\ \Delta z \rightarrow 0$, then the general equation of continuity in Cartesian coordinates becomes:

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = \frac{-\partial\rho}{\partial t}$$

For steady flow, it becomes:

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

For steady, incompressible flow; it will be:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{4.10}$$

For two dimensional, steady , incompressible flow one gets:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{4.11}$$

Example 4.3

Determine the value of v in a two-dimensional flow field when $u = ax$.

Solution:

A possible flow should satisfy continuity equation,

$$\text{Since} \quad u = ax, \quad \frac{\partial u}{\partial x} = a$$

Two dimensional continuity equation is $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

$$\textit{Therefore,} \quad \frac{\partial v}{\partial y} = -\frac{\partial u}{\partial x} = -a$$

$$\partial v = -a\partial y$$

$$\textit{Integrating,} \quad v = -\int a\partial y = -ay + f_1(x)$$

$$\therefore \quad v = \underline{-ay + f_1(x)}$$

Example 4.4

Does the velocity field given by $\overline{U} = 5x^3{}_i - 15x^2{}_{ji} + tk$ represent a possible fluid motion?

Solution:

Here, $u = 5x^3$, $v = -15x^2y$, $w = t$

In order to check for a physically possible fluid notion, one needs to look for complaince with the Continuity equation.

For three-dimensional incompressible fluid, the continuity equation is:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

From the above,

$$\frac{\partial u}{\partial x} = 15x^2$$

$$\frac{\partial v}{\partial y} = -15x^2$$

$$\frac{\partial w}{\partial z} = 0$$

Substitution in the continuity equation gives:

$$15x^2 - 15x^2 + 0 = 0$$

Since continuity equation is satisfied, the given velocity field represents a possible fluid motion.

4.9 Rotational and Irrotational Flows

The angular velocity of the fluid elements about their mass centres should be considered while discussing the kinematics of fluid flow. Accordingly, fluid motion in which the fluid particles do not rotate about their own axes is known as irrotational flow while fluid motion in which the fluid particles rotate about their own axes is known as rotational flow. These two types of flow are illustrated in figure 4.11

Figure 4.11(a) refers to an ideal (non-viscous) fluid flow between two parallel plates. The velocity distribution is uniform. If a stick abc is laid normal to a stream line 0-0, it does not undergo rotation about an axis normal to the plane of the paper through b and hence it does not change its orientation as it moves in the flow direction. Such a flow is termed as *irrotational*.

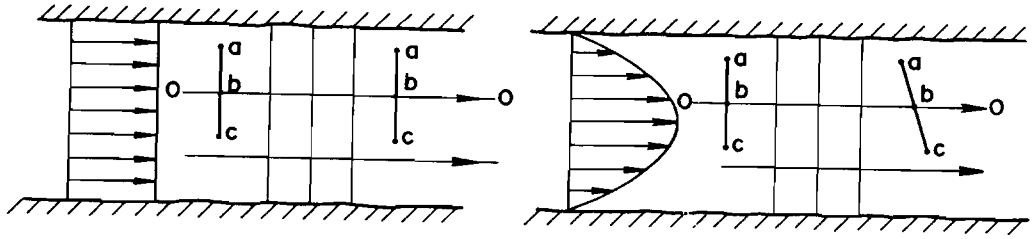


Figure 4.11 Irrotational and Rotational flows

Figure 4.11(b) shows a two dimensional real fluid flow with non-uniform velcity distribution with the velocities near the bounder being smaller than in the region close to the centre. A stick abc kept normal to 0-0 initially rotates about axis at b. Since the belocity at c is higher than at a, the stick will attain an inclined position after moving through a short distance. The stick has started rotating about an axis at b and its orientation has changed. Such a flow is termed as *rotational*.

Rotational and *irrotational* flows can be identified by determining the rotation of the fluid element at every point in a flow field.

Rotation: consider two elementary lengths such as OA and OB of length δx and δy respectively in a fluid as shown in Figure 4.12.

Let v = velocity at o in the y direction.
    u = velocity at o in the x direction.

Thus, velocity at A in the y direction will be $v + \frac{\partial v}{\partial x}\delta x$

velocity at B in the x direction will be $u + \frac{\partial u}{\partial y}\delta y$

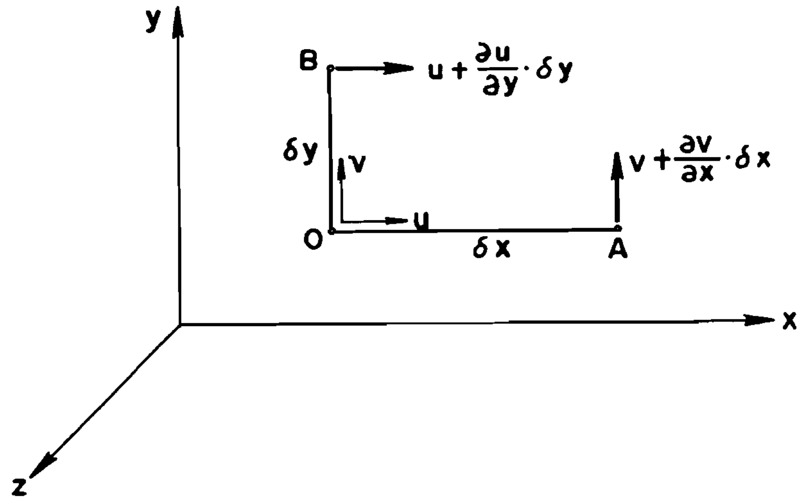


Figure 4.12 Rotation

Since velocities at O and A are different in the y direction, OA will rotate in the counter clockwise (+ive) direction.

Thus, the angular velocity of OA $= \dfrac{(v + \dfrac{\partial v}{\partial x}\delta x) - v}{\delta x} = \dfrac{\partial v}{\partial x}$

Similarly OB will rotate in the clock wise (-ive) direction..

The angular velocity of OB $\dfrac{u + \dfrac{\partial u}{\partial y}\delta y) - u}{\delta y} = -\dfrac{\partial u}{\partial y}$

If the rotation about the z-axis (i.e in the x-y plane), ϖ_z, is defined as the average rotation of the two elements OA and OB, then:

$$\varpi_z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right] \qquad (4.12)$$

Similarly:

$$\varpi_x = \frac{1}{2}\left[\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right] \qquad (4.13)$$

$$\varpi_y = \frac{1}{2}\left[\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right] \qquad (4.14)$$

If at every point in a flow field the rotations ω_x, ω_y and ω_z are zero, then the flow is known as irrotational; otherwise, the flow is rotational.

Thus for a two dimenstional flow in the x-y plan, $\omega_z = 0$

leading to $\frac{\partial v}{\partial x} = \frac{\partial u}{\partial y}$ as the condition of irrotationality.

Example 4.5

The velocity components of a two dimensional flow in the x-y plane are: $u = -3y^2$ *and* $v = -4x$. Does this represent a possible flow? If the flow is possible is it rotational or irrotational?

Solution:

Continuity equation in two-dimensional incompressible flow is:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Here, $\frac{\partial u}{\partial x} = 0$, $\frac{\partial v}{\partial y} = 0$

Therefore , continuity is satisfied and the flow is possible. To check whether the flow is rotational or irrotational, use the equation for rotation about the z-axis which is:

$$If \quad \omega_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0, \qquad \text{then the flow is irrotational}$$

otherwise, it is rotational.

$$Thus \quad \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = -4 - (-6y) = -4 + 6y \neq 0$$

Therefore, the flow is rotational.

4.10 Stream Function

Consider the streamline pattern of a two dimensional, steady, incompressible flow shown in Figure 4.13.

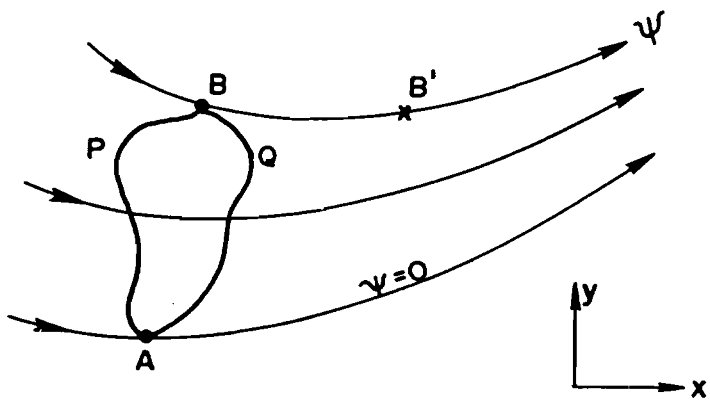


Figure 4.13 Stream function

Let A be a fixed position and B a variable position in the flow field. A and B may be joind by arbitrary curves such as APB and AOB. Let the thickness of the flow field in the Z direction be unity.

Then, the rate of flow through curve APB is equal to the rate of flow through AQB since the flow between two streamlines must remain unchanged. But the rate of flow depends upon the positions of A and B. If A is fixed, the rate of flow becomes a function of the position of B only. This function is known as stream function and denoted by Ψ. If the value of Ψ at A is zero, then the value of Ψ at B represents the flow rate between positions A and B. Consider any other point B' along the streamline through B. Since no flow occurs across BB', the flow through AB' should be the same as the flow through APB. Hence, the value of Ψ at B' should be the same as at B. Thus, the value of the stream function Ψ is constant along a streamline. Each streamline will have a different value of Ψ such that the difference in Ψ values of two streamlines gives the flow rate between the two streamlines.

The relationship between the velocity components in the x and y directions of a two dimensional flow and the stream function Ψ may be developed by considering the two streamlines shown in Figure 4.14.

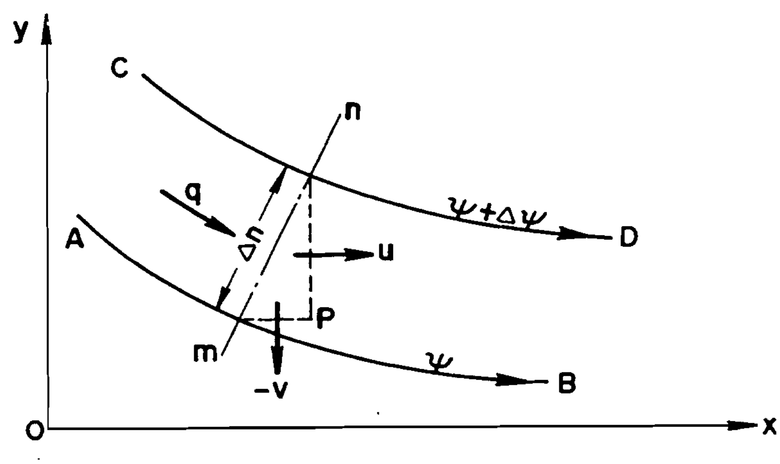


Figure 4.14

Let the value of the stream function for streamline AB be Ψ and the value of the stream function for streamline CD be $\Psi + \Delta\Psi$. The normal distance between the two stream lines is Δn. Then,

$\Delta\psi = q.\Delta n$, where q = average velocity of flow at section nm.

$$As \quad \Delta n \to 0, \qquad q = \frac{\partial\psi}{\partial n}$$

Note that what is required is the difference in the value of the stream functions between two streamlines, and not the absolute value of Ψ, in the determination of the velocity of the discharge. Hence the value $\Psi = 0$ can be assigned to any streamline. Flow rate across length mn equals the sum of the flow rates across mp and np.

$$Since \quad \psi = \psi(x,y),$$

$$d\psi = \frac{\partial\psi}{\partial x}\delta x + \frac{\partial\psi}{\partial y}\delta y$$

Taking the velocity of flow across np (= δy) to be u and that across mp (= δx) to be $-v$, it is clear that:

$$d\psi = q.\partial n = u.\delta y - v.\delta x$$

Comparing this equation with the above total differential of Ψ, it is clear that:

$$u = \frac{\partial\psi}{\partial y} \quad and \quad v = -\frac{\partial\psi}{\partial x} \tag{4.15}$$

Equation 4.15 is the relationship between the stream function in the x-y plane and the velocity components in the x and y directions. Examination of the continuity equation for two-demensional incompressible flow i.e

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

in light of the relations expressed in equation 4.15 and substitution gives:

$$\frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial^2 \psi}{\partial y \partial x} = \frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial^2 \psi}{\partial x \partial y} = 0$$

which shows that the continuity equation is identically satisfied. Therefore, the existence of a stream function Ψ for a flow implies a possible flow and conversely, for any possible flow, a stream function Ψ must exist. Considering the equation of a streamline in the x-y plane.

$$i.e. \quad \frac{dx}{u} = \frac{dy}{v}$$

$$or \quad udy - vdx = 0$$

Substituting the values of u and Ψ from equation 4.15,

$$\frac{\partial \psi}{\partial y} . dy + \frac{\partial \psi}{\partial x} . dx = 0$$

The left hand side of the above equation is the total differential $d\Psi$ of $\Psi = f(x,y)$.

Since $d\Psi = 0$, $\Psi = c =$ constant along a streamline.

Example 4.6

A stream function is given by $\psi = x + y^2$. Determine the magnitude of the velocity components in the x and y directions at (1,3).

Solution:

$$u = \frac{\partial \psi}{\partial y} = \frac{\partial}{\partial y}(x + y^2) = 2y$$

$$v = -\frac{\partial \psi}{\partial x} = -\frac{\partial}{\partial x}(x + y^2) = -1$$

The above stream function represents a possible flow since the continuity equation $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 + 0 = 0$.

At point (1,3)

$$u = 2 \times 3 = 6$$

$$v = -1$$

4.11 Velocity Potential

Analogous to the principle that electric current flows in the direction of decreasing voltage and that the rate of flow of current is proportional to the difference in voltage potential between two points, the velocity of flow of a fluid in a particular direction would depend on certain potential difference called *velocity potential*. The velocity potential, denoted by Φ *(phi)*, decreases in the direction of flow. It has no absolute value and is simply a scalar function of position and time. For steady flow, the velocity components u,v and w in the x, y and z directions respectively, in terms of velocity potential are:

$$u = \frac{-\partial\phi}{\partial x}, \quad v = \frac{\partial\phi}{\partial y} \quad and \quad w = \frac{-\partial\phi}{\partial z} \tag{4.16}$$

A potential line is a line along which the potential Φ is constant. Thus if a potential function exists for a certain flow, then it is possible to draw lines of constant potential.

Some of the properties of the potential function Φ may be derived by substituting it in the equations of rotation and continuity. Substituting the values of the velocity components u, v and w of equation 4.16 in the expression for rotation, one obtains:

$$\omega_x = \frac{1}{2}\left[\frac{\partial w}{\partial z} - \frac{\partial v}{\partial z}\right] = \frac{1}{2}\left[\frac{\partial^2\phi}{\partial z\partial y} - \frac{\partial^2\phi}{\partial y\partial z}\right]$$

$$\omega_y = \frac{1}{2}\left[\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right] = \frac{1}{2}\left[\frac{\partial^2\phi}{\partial x\partial z} - \frac{\partial^2\phi}{\partial z\partial x}\right]$$

$$\omega_z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right] = \frac{1}{2}\left[\frac{\partial^2\phi}{\partial y\partial x} - \frac{\partial^2\phi}{\partial x\partial y}\right]$$

If Φ is a continuous function, $\frac{\partial^2\phi}{\partial z\partial y} = \frac{\partial^2\phi}{\partial y\partial z}$ etc.

Hence $\omega_x = \omega_y = \omega_z = 0$, which is the condition for irrotationality. Therefore, if a velocity potential Φ exists then the flow should be irrotational and vice versa.

Substitution of the velocity components given in equation 4.16 in the three dimensional continuity equation 4.10 leads to the Laplace Equation:

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = 0 \tag{4.17}$$

Hence, any function Φ which satisties the Laplace Equation is a case of steady, incompressible, irrotational flow and such a flow is known as *potential flow*.

It should be noted that the stream function Ψ applies both for rotational and irrotational flows. However the potential function Φ is applicable only for irrotational flow. Equations 4.15 and 4.16 may be used to establish the relationship between stream function Ψ and potential function Φ for an irrotational, steady, incompressible flow leading to the following:

$$\begin{aligned} \frac{\partial\psi}{\partial y} &= -\frac{\partial\phi}{\partial x} \\ \frac{\partial\psi}{\partial x} &= +\frac{\partial\phi}{\partial y} \end{aligned} \tag{4.18}$$

Equations 4.18 are known as the Cauchy-Riemann Equations.

Example 4.7

Show that $\psi = x^2 - y^2$ represents a case of two dimensional flow and find its potential function.

Solution:

$$\text{From} \quad \psi = x^2 - y^2,$$
$$u = \frac{\partial\psi}{\partial y} = -2y$$
$$v = -\frac{\partial\psi}{\partial x} = -2x$$

The two dimensional continuity equation will be:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 + 0 = 0$$

Thus continuity is satisfied and the stream function represents a case of two dimensional flow.

Further:

$$\frac{\partial^2\psi}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial\psi}{\partial x}\right) = \frac{\partial}{\partial x}(2x) = 2$$

$$\frac{\partial^2\psi}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial\psi}{\partial y}\right) = \frac{\partial}{\partial y}(-2y) = -2$$

$$\text{Therefore,} \quad \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} = 2 - 2 = 0$$

Thus, since Ψ satisfies the Laplace equation the flow is alsc irrotational.

$$\text{From} \quad u = -\frac{\partial\phi}{\partial x} = \frac{\partial\psi}{\partial y} = -2y, \qquad \text{(a)}$$
$$\phi = 2xy + f_1(y)$$

$$\text{also from} \quad v = -\frac{\partial\phi}{\partial y} = -\frac{\partial\psi}{\partial x} = -2x, \qquad \text{(b)}$$
$$\phi = 2xy + f_2(x)$$

The velocity potential that satisfies both (a) and (b) is:

$\phi = 2xy + C$, where C is constant.

Example 4.8

In a two dimensional, incompressible flow velocity components are given by: $u = x - 4y$ *and* $v = -y - 4x$. Sow that the flow satisfies the continuity equation and obtain the expression for the stream function. If the flow is potential obtain also the expression for the velocity potential.

Solution:

For an incompressible, two dimensional flow, the continuity equation is:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\text{Here,} \quad u = x - 4y \quad \text{and} \quad v = -(y + 4x)$$

$$\text{Therefore,} \quad \frac{\partial u}{\partial x} = 1, \quad \text{and} \quad \frac{\partial v}{\partial y} = -1$$

$$\text{Thus,} \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 1 + (-1) = 0$$

i.e. the flow satisfies continuity equation.

To obtain the stream function,

$$u = \frac{\partial\psi}{\partial y} = x - 4y \qquad \text{(i)}$$

$$v = -\frac{\partial\psi}{\partial x} = +(y + 4x) \qquad \text{(ii)}$$

from (i): $\psi = \int (x - 4y)\,dy = xy - 2y^2 + f(x) + C$

But if $\psi_o = 0$ *at* $x = o$ *and* $y = 0$, then the reference streamline passes through the origin, then C = 0

$$\textit{Then} \quad \psi = xy - 2y^2 + f(x) \qquad \text{(iii)}$$

Differentiating (iii) with respect to x and equation to -v,

$$\frac{\partial\psi}{\partial x} = y + \frac{\partial}{\partial x}(f(x)) = y + 4x$$

$$\therefore \quad f(x) = \int 4x\,dx = 2x^2$$

$$\textit{Thus,} \quad \psi = 2x^2 + xy - 2y^2$$

To check whethere the flow is potential, the Laplace equation must be satisfied i.e.

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} = 0$$

$$\textit{from} \quad \psi = 2x^2 + xy - 2y^2,$$

$$\frac{\partial^2\psi}{\partial x^2} = 4, \quad \textit{and} \quad \frac{\partial^2\psi}{\partial y^2} = -4$$

Thus 4 - 4 = 0 showing that the flow is potential.

To obtain the velocity potential,

$$\frac{\partial\phi}{\partial x} = -u = -(x - 4y) = 4y - x$$

$$\textit{Therefore,} \quad \phi = \int (4y - x)\,dx = 4yx - \frac{x^2}{2} + f(y) + C$$

$$\textit{But} \quad \phi_o = 0 \ \textit{at} \ x = 0 \ \textit{and} \ y = 0, \ \textit{so that} \ C = 0$$

$$\textit{Thus} \quad \phi = 4yx - \frac{x^2}{2} + f(y)$$

$$\frac{\partial\phi}{\partial y} = 4x + \frac{d}{dy}(f(y)) = -v = 4x + y$$

$$\textit{Therefore,} \quad \frac{d}{dy}(f(y)) = y$$

$$\textit{or} \quad f(y) = \frac{y^2}{2}$$

$$\textit{Thus} \quad \phi = \frac{y^2}{2} + 4yx - \frac{x^2}{2}$$

4.12 Flow net

For any two-dimensional irrotational flow of an ideal fluid, two series of lines may be drawn as shown in Figure 4.15. These are: streamlines i.e. lines along which Ψ is constant and equipotential lines i.e. lines along which Φ is constant.

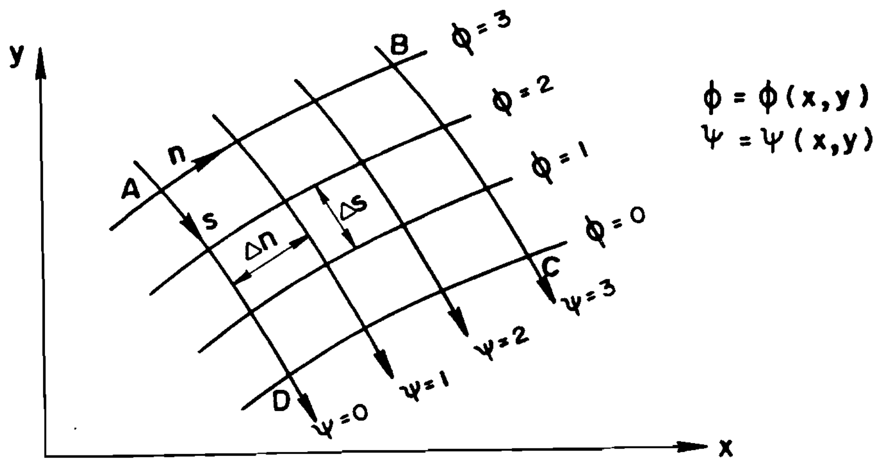


Figure 4.15 Flow net

Consider the equipotential line AB along which Φ is constant and equal to say 3. Since Φ is constant along such a line, the velocity tangential to such a line, $\frac{-\partial\phi}{\partial n} = v_n = 0$. However, since Φ is varying in the s direction, the velocity normal to the equipotential line, $\frac{\partial\phi}{\partial s} = v_s$ exists. Equipotential lines thus have the property that the flow is always at right angle to them.

Along streamlines such as CD, the stream function Ψ is constant. Therefore, $\frac{\partial\psi}{\partial s} = v_n = 0$ i.e. there is no flow at right angles to the Ψ = constant line. But $-\frac{\partial \phi}{\partial n} = v_s$ exists.

Hence, the flow is always tangential to the Ψ = constant line. Thus, at every point along a streamline, the velocity is always tangential to it. This shows that tangents to a streamline and an equipotential line intersect at 90°. A series of streamlines and equipotential lines form an orthogonal grid. Such a system represented graphically by finite number of equipotential and stream lines is called a *flow net*. The flow net is composed of a family of equipotential lines and a corresponding family of streamlines with the constants varying in arithmetic progression.

It is customary to let the change in constant between adjacent equipotential lines and adjacent streamlines be the same. If, at some small region of the flow net, the distance between adjacent streamlines = Δn and that between adjacent equipotential lines = Δs, then the approximate velocity v_s (in the s direction) is given by:

$$v_s \approx -\frac{\Delta \phi}{\Delta s} \text{, in terms of the spacing of equipotential lines}$$

and $$v_s \approx \frac{\Delta \psi}{\Delta n} \text{, in terms of the spacing of streamlines.}$$

Thus: $\Delta s = \Delta n$, since $\Delta \phi = -\Delta \psi$.

The flow net thus consists of an orthogonal grid that reduces to perfect squares in the limit as the grid size approaches zero. For a given set of boundary conditions there is only one possible pattern of flow of an ideal fluid.

Example 4.9

The potential function of a two dimensional irrotational flow is given by $\Phi = Ax$ where A is a constant. Determine the stream function Ψ and draw a set of streamlines and equipotential lines.

Solution:

First check if $\Phi = Ax$ satisfies the Laplace Equation.

$$\frac{\partial\phi}{\partial x} = A \quad and \quad \frac{\partial^2\phi}{\partial x^2} = 0 \ ; \quad \frac{\partial\phi}{\partial y} = 0 \quad and \quad \frac{\partial^2\phi}{\partial y^2} = C$$

$$Therefore, \quad \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{2y^2} = 0$$

Hence, $\Phi = Ax$ represents a fluid flow case.

Next, find the stream function Ψ

$$Since \quad -\frac{\partial\phi}{\partial x} = u = -A \quad and \quad u = -\frac{\partial\psi}{\partial y},$$

$$then \quad \frac{\partial\psi}{\partial y} = A$$

integrating, $\Psi = Ay + f(x)$

Differentiating the above expression for Ψ with respect to x,

$$\frac{\partial\psi}{\partial x} = f'(x)\,. \quad But \quad \frac{\partial\psi}{\partial x} = v \quad and \quad -\frac{\partial\phi}{\partial y} = v = 0$$

$$Therefore, \quad f'(x) = v = 0$$

Hence, $f(x) = \text{constant}$

Thus $\Psi = Ay + C$, where C is a constant

The corresponding streamlines and equipotential lines are shown in Figure 4.16 representing the case of parallel flow where the streamlines are parallel to the x-axis.

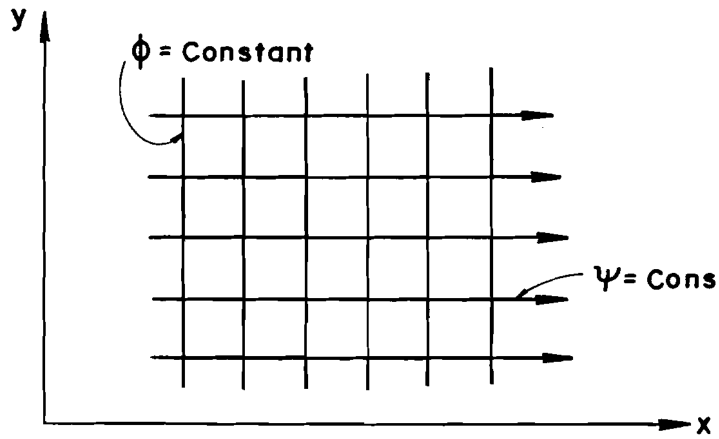


Figure 4.16

4.12.1 Construction of Flow Nets

It is clear from the foregoing discussions that obtaining the flow pattern or flow net for steady two dimensional irrotational flow involves the solution of the Laplace equation with given boundary conditions. The Laplace equation is a second order linear partial differential equation. Thus for flow patterns that can be interpreted as results of combinations of simple patterns, the superpostion of the solutions of the Laplace equation for the simple patterns can lead to the determination of the given flow pattern.

However, there are complex flow patterns of practical interest that are two involved for analytical solutions. In such cases fairly good approximations may be obtained by using mostly graphical method and electrical analogy method. The graphical method of construction of flow nets is presented and discussed below.

Graphical Method:

The graphical method is illustrated by considering the flow through the transition shown in Figure 4.17.

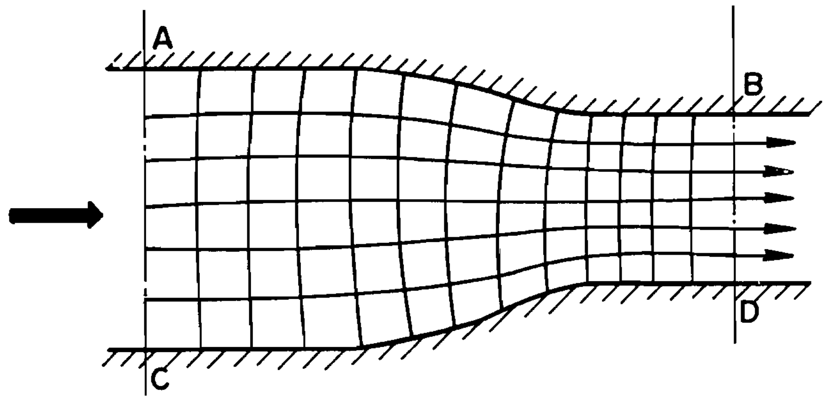


Figure 4.17 Graphical Construction of Flow Net

Since there is no flow at right angles to the boundary, the fixed boundaries AB and CD coincide wirh streamlines. Far to the left of section AC and to the right of section BD, the flow is uniform and therefore the streamlines will be equally spaced in these regions. Decide first on the number of streamlines. The more the number of streamlines, the more accurate will be the result, but more time will be spent in constructing the net. AC and BD represent equipotential lines. Mark equally spaced points on AC and BD representing the intersection of the streamlines with the equipotential lines through these sections. Between these corresponding points, streamlines can be joined by smooth curves. In the narrower section BD, the spacing between the stream lines is narrower than the wider section AC and from continuity, the velocities at section BD will be higher than at section AC. Equipotential lines can now be drawn such that:

- they intersect the boundary and other streamlines at right angles.
- the distance between consecutive streamlines and the

distance between consecutive equipotential lines are equal so that the two form squares.

- the diagonals of the squares form smooth curves which intersect each other normally.

At the curved boundary, the latter two conditions may not be completely satisfied unless the spacings are very close or the mesh is very fine. Successive trials may be required to arrive at a satisfactory flownet. The flownet so drawn will be the same for various discharges and for geometrically similar transitions of various sizes. Typical flow nets are shown in Figure 4.18.

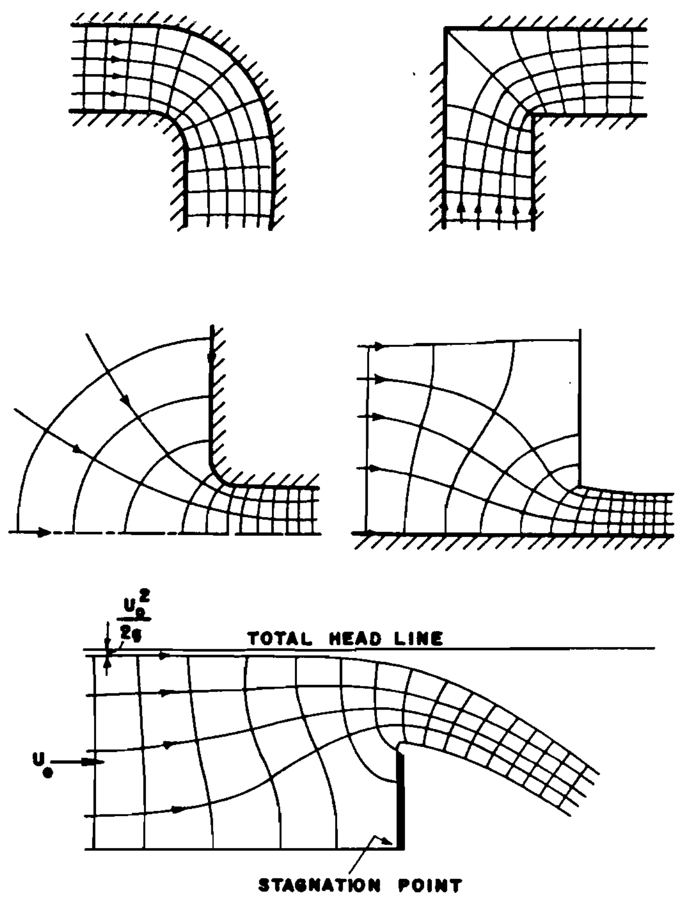


Figure 4.18

4.12.2 Uses of the Flow Net

A flow net of a two dimensional flow field under a given boundary condition, drawn to represent the flow pattern using a finite set of stream and equipotential lines will have the following uses:

i) The velocity at any point in the flow field can be determined if the velocity at a given point is known using continuity of flow between two streamlines.

ii) The flow net enables the determination of the velocity distribution and the pressure distribution the knowledge of which is necessary to calculate drag forces and uplift forces.

iii) It makes the visualization of flow pattern possible thus enabling the modification of boundaries to avoid undesirable effects such as separation and stagnation.

EXERCISE PROBLEMS

4.1 A 200 mm diameter pipe bifurcates into a 120 mm diameter pipe and a 100 mm diameter pipe. If the flow through the 200 mm diameter pipe is 100 ℓ/s assuming the velocity in the branch pipes are equal, find the rate of flow through each of the branch pipes. (Ans. 59.1 ℓ/s; 40.1 ℓ/s)

4.2 A diffuser at the end of a 100 mm diameter pipe is as show in Figure P 4.2. If the rate of flow throguh the pipe is 0.1 m^3/s, find the exit velocity at the diffuser. What is the ratio between the exit velocity at diffuser and the velocity in the 100 mm diameter pipe?

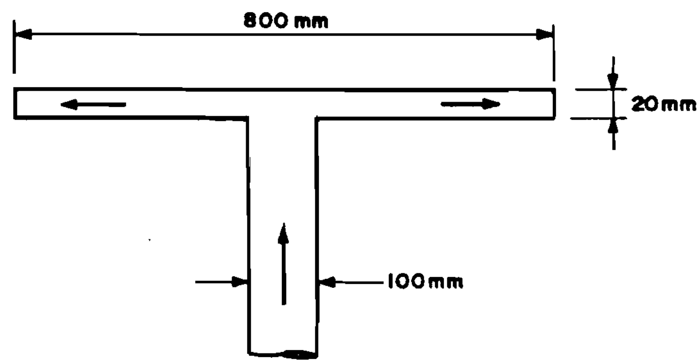


Fig. P 4.2

4.3 In a two dimensional incompressible flow the x-component of the velocity is given by $u = 3x-y$. Using the continuity equation, find the velocity component in the y-direction. (Ans. $v = -3y$)

4.4 The velocity components in a two-dimensional flow are expressed as:

$u = y^3/3 + 4x - x^2y$; and $v = xy^2 - 4y - x^3/3$

show that these functions represent a possible case of irrotational flow.

4.5 For a three dimensional flow, $u = x^2 + z^2 + 5$ and $v = y^2 + z^2$. Determine w. (Ans. $w = -2(x + y)z$)

4.6 Determine the stream function for a fluid flow if $u = 2x$ and $v = -2y$. Determine also the potential function.

4.7 If for a two dimensional potential flow, the velocity potential is given by $\phi = x(2y - 1)$

i) Determine the velocity at point p(4,5)

ii) What is the value of the stream function Ψ at point p?

(Ans. (i) $u = 9$ and $v = 8$ units
(ii) $\Psi = y^2 - x^2 - y$, 4 units)

4.8 State if the flow represented by: $u = 3x + 4y$ and $v = 2x - 3y$ is rotational or irrotational. Find the potential function if the flow is irrotational and the vorticity if it is rotational.

CHAPTER 5

DYNAMICS OF FLUID FLOW

5.1 Introduction

Dynamics of fluid flow deals with the forces respossible for fluid motion, the resulting accelerations and the energy change involved in the flow Phenomenon.

Just as in mechanics of solids, the mechanics of fluids is also governed by Newton's Second Law of motion i.e

Force = Mass x acceleration

The force and the acceleration are in the same direction. However, since liquids do not possess regidity of form, their mass center changes unlike that of solids. Therefore, for fluids mass per unit volume is more important than the total mass.

Thus in the *x* direction, Newton's Equation of motion will be:

$$\sum F_x = \rho a_x$$

where: ΣF_x = sum of x components of all forces per unit volume acting on the fluid mass.

a_x = total acceleration in the x direction.

ϱ = mass per unit volume of the fluid.

5.2 Forces Influencing Motion

In general the following different types of forces influence fluid motion: Force due to gravity, Pressure, viscosity, Surface tension, Compressibility, and Turbulence.

Gravity force F_g: is due to the weight of the fluid. Its component in the direction of motion causes acceleration in problems where gravity is important such as in open channel flow. F_g is proportional to the volume of the fluid mass under consideration. The gravity force per unit volume, $f_g = \varrho g$ and acts vertically downwards.

Fluid pressure force F_p: This is the force exerted by a fluid mass on any surface in a direction normal to the surface. The pressure intensity p is the force per unit area and indicates a local intensity of pressure force. Fluid pressure produces acceleration in a given direction only if the pressure decreases in that direction.

To determine the magnitude of the pressure force per unit volume, consider a small fluid element of cross-sectional area dA and length dx as shown in Fig 5.1.

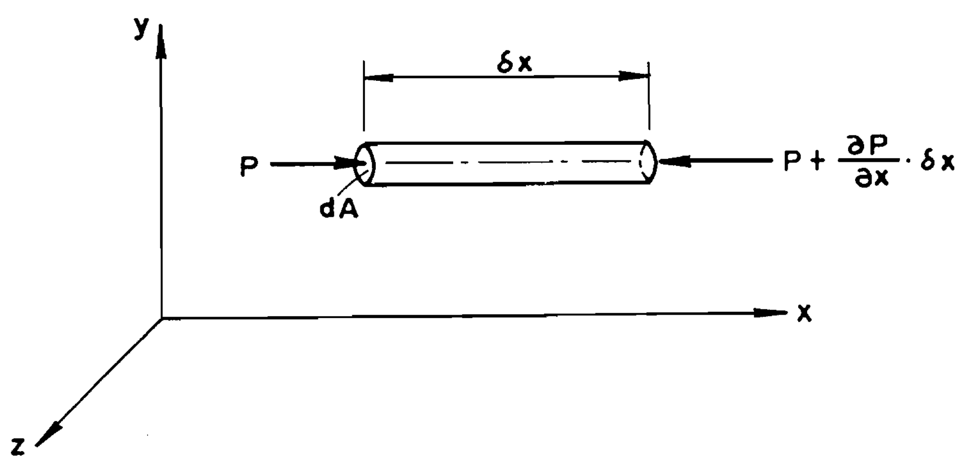


Figure 5.1 Pressure forces

P is the pressure on the left face

$\frac{\partial p}{\partial x}$ is rate of change of pressure in the x direction

$$P + \frac{\partial p}{\partial x}dx$$ is pressure on the right face

Since there is a difference in pressure between the two faces, there exists a pressure force F_{px} in the x direction which can cause the fluid to move in the x direction;

$$F_{px} = pdA - (p + \frac{\partial p}{\partial x}dx)\,dA = -\frac{\partial p}{\partial x}.dx.dA$$

Thus the pressure force per unit volume is $F_{px} = -\frac{\partial p}{\partial x}$

The negative sign indicates that F_{px} acts in the direction of decreasing pressure.

Viscouse force F_v: This force exists in all real fluids. When there is relative motion between two layers of a fluid, a tangential force is created due to the effects of viscosity. The shear resistance, F_v, acts in a direction opposite to that of motion thus retarding the flow.

Surface tension force F_σ: This force is important when the depths of flow or the related length dimensions are extremely small. F_σ = surface tension force/volume.

Force due to compressibility F_e: For incompressible fluids this becomes significant in problems of unsteady flow like water hammer where the elastic properties of fluids come into the picture.

In most problems, F_σ and F_c are neglected

Forces due to turbulence F_t: In highly turbulent flows, there is a continious momentum transfer between adjacent layers which causes normal and shear stresses due to turbulence. These are known as Reynolds stresses. These stresses, disignated by F_t must be taken into consideration in cases of turbulent flow.

5.3 Euler's Equation of Motion

Fluid motion is influenced by all the forces mentioned above. Motion of a fluid in any direction is thus caused by the components of all the forces in that direction. Thus for the x direction, Newton's Second Law will give the following:

$$F_{gx} + F_{px} + F_{vx} + F_{\sigma x} + F_{ex} + F_{tx} = \rho a_x \qquad 5.1$$

Similar equations can be written for the other two coordinate directions.

For ideal fluids which have no viscosity and neglecting $F_{\sigma x}$, F_{ex} and F_{tx}, the equation 5.1 reduces to:

$$F_{gx} + F_{px} = \rho a_x \qquad (5.2)$$

When proper expression for F_{gx}, F_{px} and a_x are substituted in Eqn. 5.2 it becomes what is known as Euler's Equation of Motion in the x direction.

When viscous forces are considered, Equation 5.2 will be:

$$F_{gx} + F_{px} + F_{vx} = \rho a_x \qquad (5.3)$$

This equation gives the Navier-Stoke's Equation in the x direction when the proper expressions for the forces and accelerations are substituted in it. For turbulent flow, the force due to turbulence must also be considered in addition to the forces in Equation 5.3. Thus, we have

$$F_{gx} + F_{px} + F_{vx} + F_{tx} = \rho a_x \tag{5.4}$$

This gives the Reynold's Equation of motion in the x direction.

In this discussion, we will consider Euler's Equation in some detail.

In Equation 5.2, introducing $-\frac{\partial p}{\partial x}$ to replace F_{px} and letting F_{gx} represent the gravity force per unit volume, we will have:

$$F_{gx} - \frac{\partial p}{\partial x} = \rho \left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\right] \tag{5.5}$$

Equation 5.5 is Euler's Equation for one dimensional flow.

The general three dimensional form of Euler'S Equation of Motion can be written as:

$$\begin{aligned}
\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} &= X - \frac{1}{\rho}\frac{\partial p}{\partial x} \\
\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} &= Y - \frac{1}{\rho}\frac{\partial p}{\partial y} \\
\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} &= Z - \frac{1}{\rho}\frac{\partial p}{\partial z}
\end{aligned} \tag{5.6}$$

u,v and w are velocity components in the x, y and z directions and X, Y and Z are components of fluid weight per unit mass in the x, y and z directions respectively.

5.4 Integration of Euler's Equation of Motion

The integration of Euler's Equation along a streamline results in an important equation in fluid mechanics known as Bernoulli's Equation.

Consider the forces acting on a fluid element of cross-sectional area dA and length ds along a stream line as shown in figure 5.2

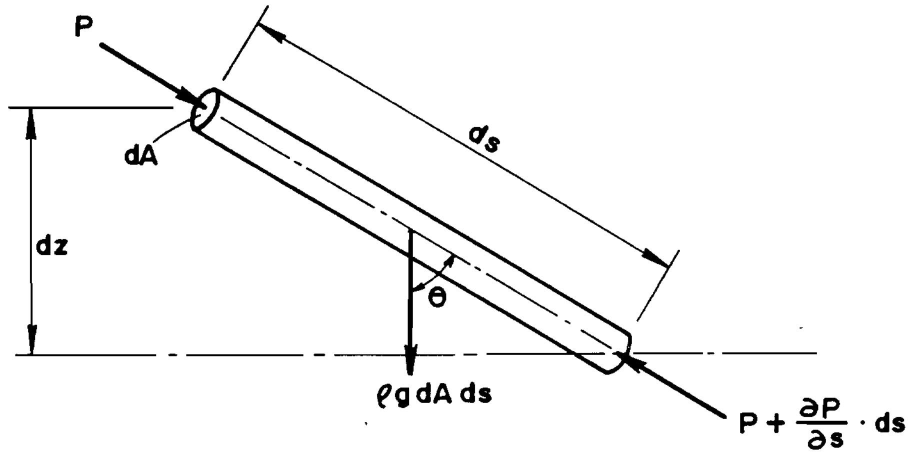


Figure 5.2 Forces acting on a fluid element

The forces acting on the fluid element are those due to gravity and due to pressure gradient. It is assumed that the fluid is frictionless and all minor forces are neglected. Thus:

Gravity force in the direction of motion = $\rho g dA ds . Cos\theta$

Pressure force in the direction of motion = $-\frac{\partial p}{\partial s} ds . dA$

If v = velocity in the direction of motion, then Euler's Equation 5.5 becomes:

$$\rho g dA . ds . Cos\theta - \frac{\partial p}{\partial s} . ds . dA = \rho dA ds \left(\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial s}\right) \qquad (5.7)$$

Dividing Equation 5.7 by $\rho dA.ds$, one gets:

$$g\cos\theta - \frac{1}{\rho}\frac{\partial p}{\partial s} = \left(\frac{\partial v}{\partial t} = v\frac{\partial v}{\partial s}\right)$$

Considering For steady flow, $\frac{\partial v}{\partial t} = 0$ and substituting

$Cos\theta = -\frac{\partial z}{\partial s}$ the above equation becomes:

$$-g\frac{\partial z}{\partial s} - \frac{1}{\rho}\frac{\partial p}{\partial s} = v\frac{\partial v}{\partial s} \qquad (5.8)$$

Equation 5.8 can also be written as:

$$\rho v\frac{\partial v}{\partial s} = -\frac{\partial}{\partial s}(P + \gamma z) \qquad (5.8a)$$

Equation 5.8 can be integrated along a streamline after multiplying each term by ds. Hence:

$$-\int gdz - \int\frac{dp}{\rho} = \int vdv$$

$$Thus: \quad \frac{v^2}{2} + gz + \int\frac{dp}{\ell} = Constant \qquad (5.9)$$

Equation 5.9 is Bernoulii's Equation for both compressible and incompressible fluids.

For incompressible fluid, ϱ is independent of pressure i.e. ϱ is constant.

$$\text{Hence,} \quad \frac{v^2}{2} + gz + \frac{p}{\ell} = \text{Constant} \tag{5.10}$$

Equation 5.10 is Bernoulli's Equation for incompressible fluids It can be written in the following two alternative forms:

$$\frac{v^2}{2g} + z + \frac{p}{\gamma} = \text{Constant} \tag{5.11}$$

$$\text{or} \quad \frac{\rho v^2}{2} + \gamma z + p = \text{Constant} \tag{5.12}$$

Each term in Equation 5.10, 5.11 and 5.12 represents energy of the fluid.

The terms in Equation 5.10 describe the energy per unit mass.
The terms in Equation 5.11 describe the energy per unit wt.
The terms in Equation 5.12 describe the energy per unit volume.

Mostly, however, Equation 5.11 is used in pipe and open channel flows. Each terms in Equation 5.11 has the dimension of length.

$\frac{v^2}{2g}$ is known as the velocity head (Kinetic Energy per unit weight),

$\frac{p}{\gamma}$ is known as the pressure head (pressure energy per unit weight),
and
z is known as the elevation or potential head (potential head per unit weight).

Bernoulli's Equation states that in a steady flow of an ideal fluid, the sum of velocity head, pressure head and potential head along a stream line is constant. Applying it between two sections,

$$\frac{V_1^2}{2g} + \left(\frac{p_1}{\gamma} + z_1\right) = \frac{V_2^2}{2g} + \left(\frac{p_2}{\gamma} + z_2\right) \tag{5.13}$$

or Bernoulli's Equation can also be stated as: The total energy per unit weight for a steady flow of an ideal fluid remains constant along a stream line.

5.5 The Energy Equation

For real fluids, some energy is converted into heat due to viscous shear and consequently there is a certain amount of energy loss. Thus, for real fluids, equation 5.13 becomes:

$$\frac{V_1^2}{2g} + \frac{p_1}{\gamma} + z_1 = \frac{V_2^2}{2g} + \frac{p_2}{\gamma} + z_2 + h_{L_{1-2}} \tag{5.14}$$

Total energy at (1) = Total energy at (2) + Loss of energy between (1) and (2)

where subscripts (1) and (2) refer to the two sections under consideration and h_{L1-2} is the energy loss per unit weight of fluid between the two sections. Section (1) is the upstream and section (2) is the downstream section and flow takes place from section (1) towards section (2). Equation 5.14 is the enrgy equation for real fluid flow.

If energy is added to the fluid between sections (1) and (2) such as by a pump, then the energy equation will be:

$$\frac{v_1^2}{2g} + \frac{p_1}{\gamma} + z_1 + H_p = \frac{v_2^2}{2g} + \frac{p_2}{\gamma} + z_2 + h_{L_{1-2}} \tag{5.15}$$

Where H_p is the energy head added by the pump.

If energy is taken out of the system between sections (1) and (2) by a turbine, the energy equation will be:

$$\frac{v_1^2}{2g} + \frac{p_1}{\gamma} + z_1 - H_t = \frac{v_2^2}{2g} + \frac{p_2}{\gamma} + z_2 + h_{L_{1-2}} \tag{5.16}$$

Where H_t is the head supplied to the turbine.

In all these cases, the velocity is assumed to be constant throughout the cross-section. This assumption may be valid in turbulent flows where the average velocity is not very different from the maximum. With varying velocities across a cross-section, a kinetic energy correction factor α should be applied to the kinetic energy head term. Referring to the velocity distribution at a cross-section shown in Figure 5.3:

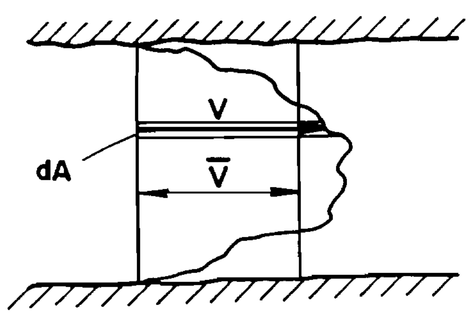


Figure 5.3

Let v = the velocity at a particular point in a cross-section where the elemental flow area is dA.

Then, the kinetic energy per unit weight = $\dfrac{v^2}{2g}$

Weight of fluid passing through dA per unit time = $\gamma . v . dA$

Kinetic energy passing through dA per unit time = $\gamma . v . dA . \dfrac{v^2}{2g}$

The integral of the above expression gives the total kinetic energy passing through the whole cross-sectional area A per unit time.

Using the mean velocity $\bar{v}$ and the kinetic energy correction factor α, the kinetic energy per unit time passing through the section will be $\gamma . \bar{v} . A . \alpha \dfrac{\bar{v}^2}{2g}$.

Thus:

$$\gamma . \bar{v} . A . \alpha \frac{\bar{v}^2}{2g} = \gamma . \int_A \frac{v^2}{2g} . v . dA$$

or $$\alpha = \frac{1}{A\bar{v}^3} \int_A v^3 . dA$$

Once the kinetic energy correction factor for a particular velocity distribution is known, then Bernoulli's equation between two sections (1) and (2) becomes:

$$z_1 + \frac{p_1}{\gamma} + \alpha_1 \frac{v_1^2}{2g} = z_2 + \frac{p_2}{\gamma} + \alpha_2 \frac{v_2^2}{2g}$$

For laminar flow in a pipe, $\alpha = 2$. For turbulent flow in a pipe, α varies from about 1.01 to 1.10 and is usually taken as unity except for precise work.

Example 5.2

Calculate the kinetic energy correction factor α for a parabolic velocity distribution in a pipe flow of radius r_o given by:

$$v = v_{max}\left(1 - \frac{r^2}{r_o^2}\right)$$

Solution:

Referring to the figure below:

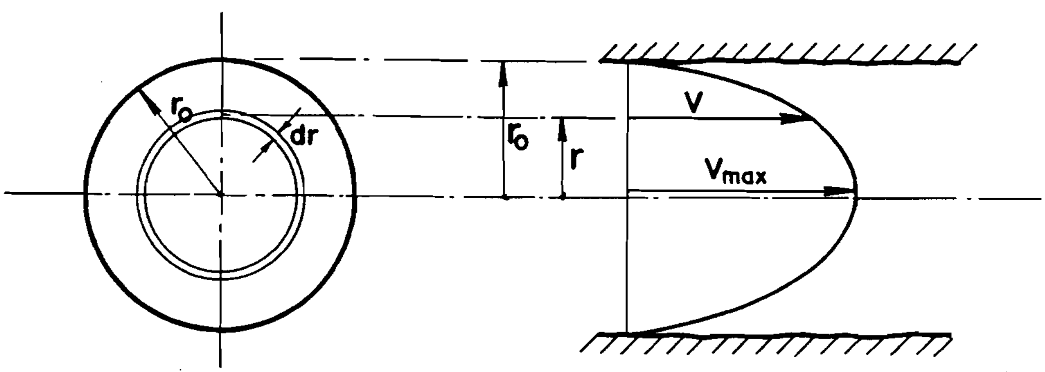


Figure E 5.2

$$\textit{Mean Velocity} \quad \bar{v} = \frac{Q}{A} = \frac{\int_A v\,dA}{A}$$

$$= \frac{\int_o^{r_o} v_{max}\left(1 - \frac{r^2}{r_o^2}\right).2\pi r.dr}{\pi r_o^2}$$

$$= \frac{2v_{max}}{r_o^2}.\int_o^{r_o}\left(r - \frac{r^3}{r_o^2}\right)dr$$

$$= \frac{2v_{max}}{r_o^2}\left(\frac{r_o^2}{2} - \frac{r_o^2}{4}\right) = \frac{v_{max}}{2}$$

$$\alpha = \frac{1}{A\bar{V}^3}\int_A v^3\,dA$$

$$\int_A v^3\,dA = \int_o^{r_o} v_{max}{}^3\left(1 - \frac{r^2}{r_o^2}\right)^3.2\pi r dr$$

$$= 2\pi\, v_{max}{}^3\int_o^{r_o}\left(r - \frac{3r^3}{r_o^2} + \frac{3r^5}{r_o^4} - \frac{r^7}{r_o^6}\right)dr$$

$$= 2\pi\, v_{max}{}^3\, r_o^2\left(\frac{1}{2} - \frac{3}{4} + \frac{3}{6} - \frac{1}{8}\right) = \frac{\pi}{4}\, v_{max}{}^3\, r_o^2$$

$$\therefore \qquad \alpha = \frac{\frac{\pi}{4}\, v_{max}^3\, r_o^2}{\pi r_o^2\left(\frac{V_{max}}{2}\right)^3} = 2$$

Example 5.3

A closed tank of a fire engine is partly filled with water, the air space above the water being under pressure. A 5 cm diameter hose connected to the tank discharges on the roof of a building 4.0 m above the level of water in the tank. The frictional head loss in the hose is equivalent to 40 cm head of water. What air pressure must be maintained in the air in the tank to deliver 12ℓ/s on the roof?

Solution:

Referring to Figure E 5.3

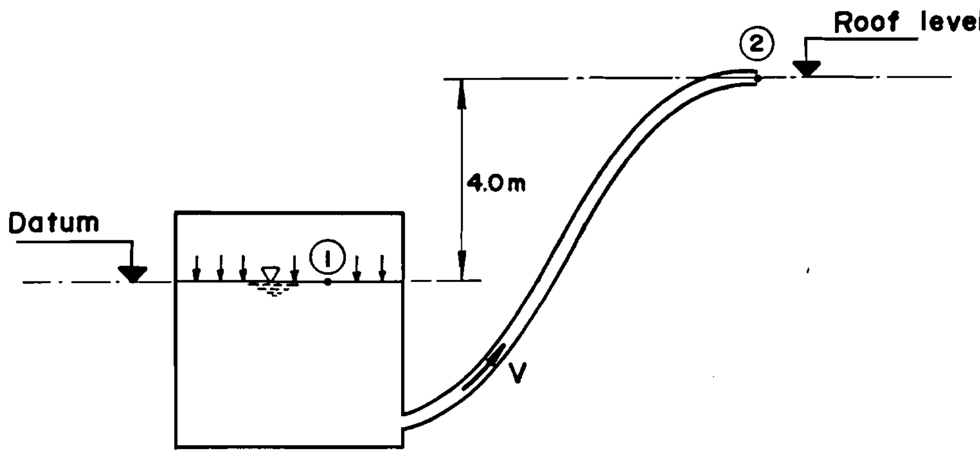


Figure E 5.3

The discharge Q in the hose = 12ℓ/s = 0.012 m^3/s

Velocity V in the 5 cm hose = $\dfrac{0.012}{\frac{\pi}{4}(0.05)^2} = 6.1\ m/s$

$\therefore$ The velocity head $\dfrac{V^2}{2g} = \dfrac{6.1^2}{2 \times 9.81} = 1.9\ m$

Applying the energy equation between (1) and (2) taking the water level in the tank as datum:

$$H_1 = H_2 + losses_{1-2}$$

$$z_1 + \frac{p_1}{\gamma_w} + \frac{v_1^2}{2g} = z_2 + \frac{p_2}{\gamma_w} + \frac{v_2^2}{2g} + h_{L_{1-2}}$$

$$0 + \frac{p_1}{\gamma_w} + 0 = 4.0 + 0 + 1.9 + 0.4$$

$$\therefore \quad \frac{p_1}{\gamma_w} = 6.3 \ m \ of \ water$$

$$p_1 = \gamma_w \ .6.3 = 9.81 \ kN/m^3 \times 6.3 \ m = 61.80 \ kN/m^2$$

Thus, a pressure of 61.80 kN/m^2 must be maintained in the air above the water in the tank to deliver 12 ℓ/s of water on the roof.

5.6 Power Considerations

When water under pressure is lead through a turbine, hydraulic energy is converted to mechanical energy, in the form of turbine rotation, which may then be converted to electrical energy by means of a generator coupled to the turbine. Thus energy is extracted from the water. Converely, a pump adds mechanical energy to the water which enables it to be lifted from a lower level to a higher level reservoir or makes the transportation of the water from one location to another possible by overcoming resistance to flow in the piping system. The power extracted from or added to the water may be calculated from the following:

$$\begin{aligned} Power\ \ P &= Work\ done\ per\ unit\ time \\ &= G.H \qquad (5.17) \\ &= \gamma_w Q.H \end{aligned}$$

Where G is the weight rate of flow in N/s

H is the energy head extracted or added in m

Q is the discharge in m^3/s

γ_w is specific weight of water in N/m^3

The power P is in Nm/s. Since 1 nm/s is equal to 1 watt, the power P in Equation 5.17 is in watts when γ_w is in N/m^3, Q is in m^3/s and H is in meters. But in the MKS system γ_w is in kgf/m^3 and γ_w = 1000 kgf/m^3.

Since 1 metric Horse power = 75 kg-m/s, then

$$P(\text{in Horse power}) = \frac{\gamma_w Q H}{75}, \quad \text{where} \quad \gamma_w = 1000\ kgf/m^3$$

H is in m

Q is in m^3/s

If the efficiency of a turbine to convert hydraulic energy to mechanical energy in η_t and the head of water extracted from the flowing water by the turbine is H_t metres of water, then the power in Horse power supplied to the generator is:

$$P = \frac{\gamma_w Q H_t}{75} . \eta_t$$

If the efficiency of a pump to convert mechanical energy to hydraulic energy of the water is η_p and the head of water supplied to the water by the pump is H_p metres of water, then the pump Horse power required is:

$$P = \frac{\gamma_w . QH_p}{75\,\eta_p}$$

Since $P(in\ KW) = 9.81\ Q\ H$

and $P(in\ Horse\ power) = \frac{1000\,QH}{75}$, *then*

$$1\ Horse\ power = 0.736\ kW$$

Example 5.4

Determine the Horepower supplied by the pump is 100 ℓ/s of water is flowing through the system shown in Figure 5.4. The gauge reading is 100 cm and the gauge liquid is mercury, s = 13.6. What is the pump Horsepower required if its efficiency is 91.5%?

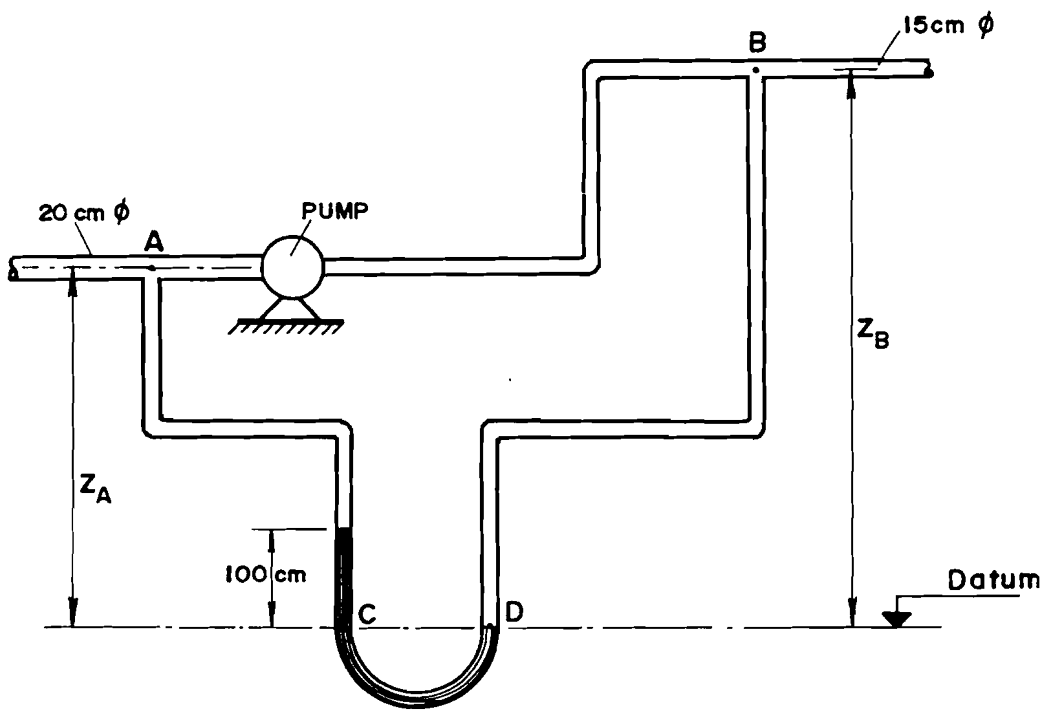


Figure E 5.4

Solution:

Since pressure at c = pressure at D,

$$p_A + (z_A - 1.0)\gamma_w + 1.0 \times 13.6 \times \gamma_w = p_B + z_B\gamma_w$$

Dividing throughout by γ_w,

$$\frac{p_A}{\gamma_w} + z_A - 1.0 + 13.6 = \frac{p_B}{\gamma_w} + z_B$$

Thus $$\frac{p_A - P_B}{\gamma_w} + z_A - z_B = -12.6$$

$$v_A = \frac{0.1 \times 4}{\pi(0.2)^2} = 3.18\ m/s \quad and \quad \frac{v_A^2}{2g} = \frac{3.18^2}{2 \times 9.81} = 0.52\ m$$

$$v_B = \frac{0.1 \times 4}{\pi(0.15)^2} = 5.65\ m/s \quad and \quad \frac{v_B^2}{2g} = \frac{5.65^2}{2 \times 9.81} = 1.63\ m$$

Let the head supplied by the pump to the system = h_p. Applying the energy equation between A and B (neglecting losses) and taking the plane through CD as datum:

$$z_A + \frac{p_A}{\gamma_w} + \frac{v_A^2}{2g} + h_p = z_B + \frac{p_B}{\gamma_w} + \frac{v_B^2}{2g}$$

Thus $$h_p = \frac{p_B - p_A}{\gamma_w} + z_A - z_B + \frac{v_B^2}{2g} - \frac{v_A^2}{2g}$$

$$= -\left[\left(\frac{p_A - p_B}{\gamma_w}\right) + z_B - z_A\right] + \frac{v_B^2}{2g} - \frac{v_A^2}{2g}$$

$$= 12.6 + 1.63 - 0.52 = 13.71\ m\ of\ water$$

$\therefore$ Horsepower supplied by the pump =

$$= \frac{\gamma_w Q h_p}{75}$$

$$= \frac{1000 \times 0.1 \times 13.71}{75} = 18.3$$

The pump Horsepower required $= \dfrac{\gamma_w Q h_p}{75 \times \eta_p} = \dfrac{18.3}{0.915} = 20$

Example 5.5

A flow of 450 ℓ/s of water enters a turbine through a 60 cm diameter pipe under a pressure of 147.1 kN/m^2. The water leaves the turbine through a 90 cm diameter pipe under a pressure of 34.32 kN/m^2. If a vertical distance of 2.0 m separates the centre lines of the two pipes, how much poser is supplied to the turbine? If the turbine is 90% efficient, how much power is made available to the generator?

Solution:

Referring to Figure E 5.5

$$A_1 = \frac{\pi}{4}(0.6)^2 = 0.283\ m^2, \qquad V_1 = \frac{Q}{A_1} = \frac{0.45}{0.283} = 1.59\ m/s$$

$$A_2 = \frac{\pi}{4}(0.9)^2 = 0.636\ m^2, \qquad V_2 = \frac{Q}{A_2} = \frac{0.45}{0.636} = 0.708\ m/s$$

Applying the energy equation (neglecting losses) between sections (1) and(2):

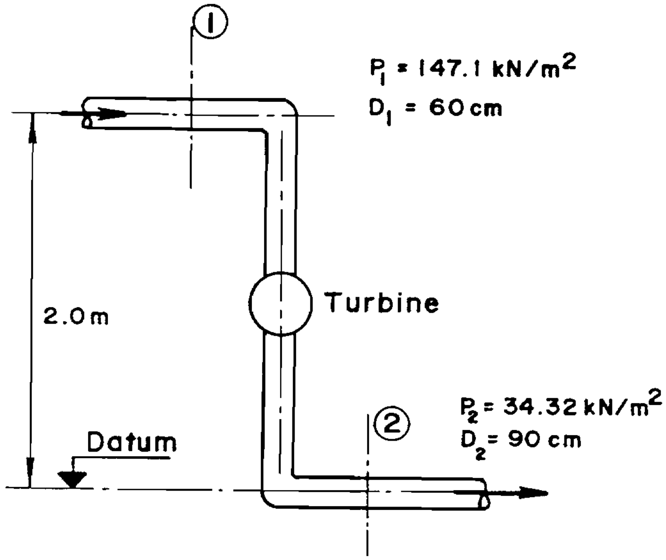


Figure E 5.5

$$z_1 + \frac{p_1}{\gamma_w} + \frac{v_1^2}{2g} - h_t = z_2 + \frac{p_2}{\gamma_w} + \frac{v_2^2}{2g}$$

$$2.0 + \frac{147.1}{9.81} + \frac{1.59^2}{2 \times 9.81} - h_t = 0 + \frac{34.32}{9.81} + \frac{0.708^2}{2 \times 9.81}$$

$$2 + 14.99 = 0.129 - h_t = 0 + 3.499 + 0.026$$

$$\therefore \qquad h_t = 13.594 \; m \; of \; water$$

Power supplied to the turbine = 9.81 x 0.45 x 13.594 = 60.01kW

$$= \frac{60.01}{0.736} = 81.54 \quad \textit{Horsepower}$$

Power made available to the generator = 60.01 x 0.90 = 54.1 kW

= 73.38 Horsepower

5.7 Piezometric Head and Total Head

Consider Bernoulli's equation i.e.

$$z + \frac{p}{\gamma} + \frac{v^2}{2g} = constant$$

Each term in the above equation represents energy per unit weight of fluid and has the dimension of length. The sum of the elevation head and pressure head i.e. $(z + p/\gamma)$ is called prezometric head and the sum of all the three is called total head, where the elevation head Z is measured with respect to an arbitrary datum. A piezometer is a simple device used to measure possitive pressures of liquids. It consists of a glass tube connected to the pipe wall in which the liquid can rise freely without overflowing. If piezometers were to be installed at different sections of the condiut shown in Figure 5.4, the liquid will rise to different levels above the centre line of the conduit.

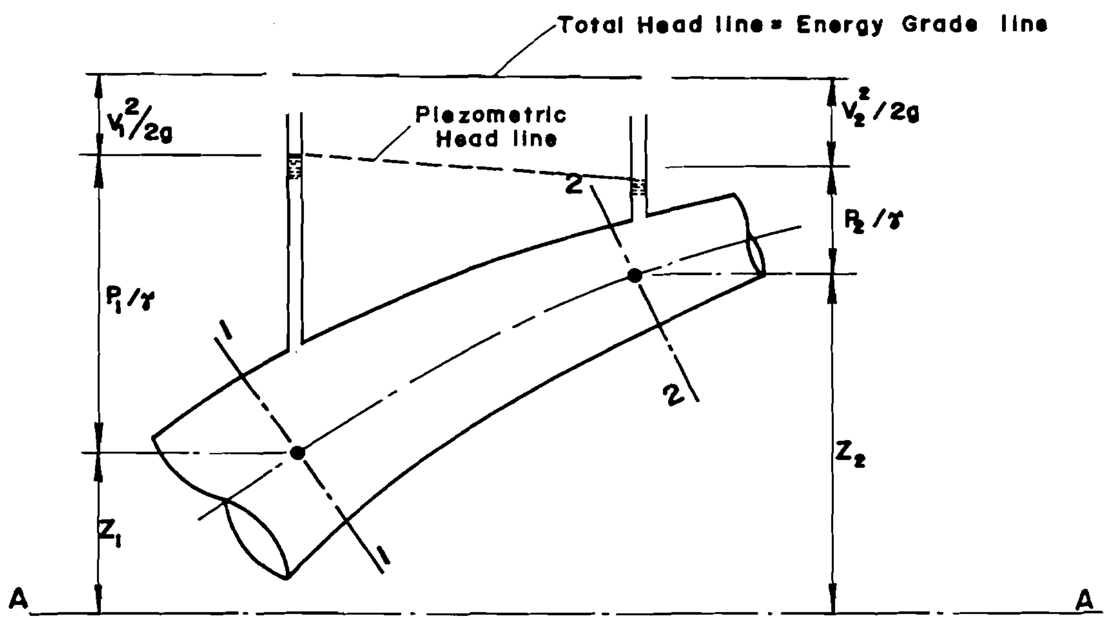


Figure 5.4 Schematic representation of Bernoulli's equation

Figure 5.4 shows a graphical representation of Bernoulli's equation for a frictionless flow through a streamtube. If AA is a horizontal datum, the elevation of the centerline of the tube with respect to the datum at any section is the elevation head. The vertical distance, at any section, from the tube centerline to the level up to which the liquid rises in a piezometer tube represents the pressure head at that section. The locus of all points at a distance $(z + p/\gamma)$ from the datum A-A is called the piezometric headline or the hydraulic gradeline. The locus of all points at a distance $(z + \frac{p}{\gamma} + \frac{V^2}{2g}) = H$ above the datum is called the total headline or the energy gradeline. Thus at any section the total headline is always above the piezometric headline by an amount equal to the velocity head $\frac{V^2}{2g}$. If the pressure at a section is sub atmospheric i.e. negative pressure or partial vacuum, the hydraulic grade line will be below the centreline of the cross-section by an amount equal to this negative pressure head.

In real fluid flow, there is reduction of energy along the flow due to friction, minor losses due to local changes in velocity etc.. If a pump is installed in a piping system, there will be an abrupt rise in the energy gradeline by an amount equal to the head supplied by the pump. Likewise, there will be an abrupt drop in the energy grade line at a turbine by an amount equal to the head extracted by the turbine. It may also be noted that both the hydraulic and energy, gradelines are straight sloping lines irrespective of the pipeline being straight or curved since the slopes of these lines are referred to per unit length of the pipe and not unit length in any specified direction.

In the following are shown typical Hydraulic gradeline (HGL) and energy gradeline [EGL] sketches for various flow conditions.

5.7.1 Gravity flow between two reservoirs through a straight pipeline.

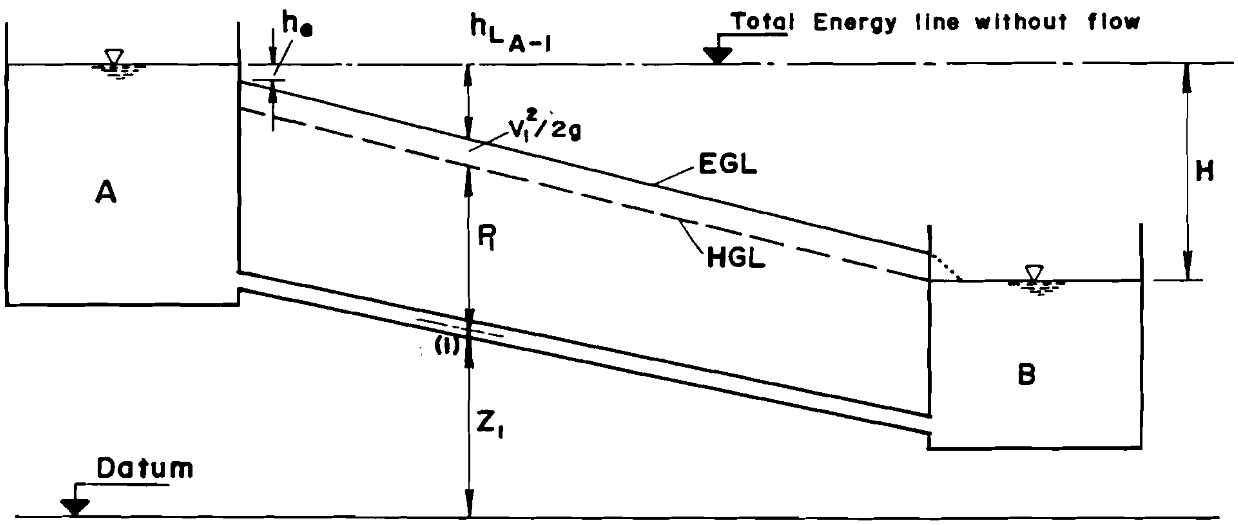


Figure 5.5 HGL and EGL for flow between two reservoirs connected by a uniform diameter pipe

For a straignt and uniform diameter pipe, the EGL and HGL will be straight, parallel lines and their slope will represent the rate of head loss. There are local (minor) losses at exit from reservoir A and at entrance to reservoir B. H is the total head loss between reservoir A and B or it is the head causing the flow.

5.7.2 Pipe discharging freely into the atmosphere from a reservoir

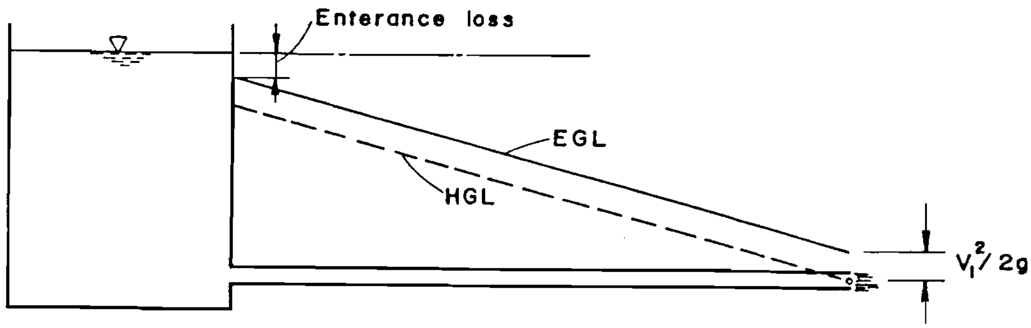


Figure 5.6 HGL and EGL for pipe discharging freely into the atmosphere

5.7.3 Two reservoirs connected by varying diameter pipes.

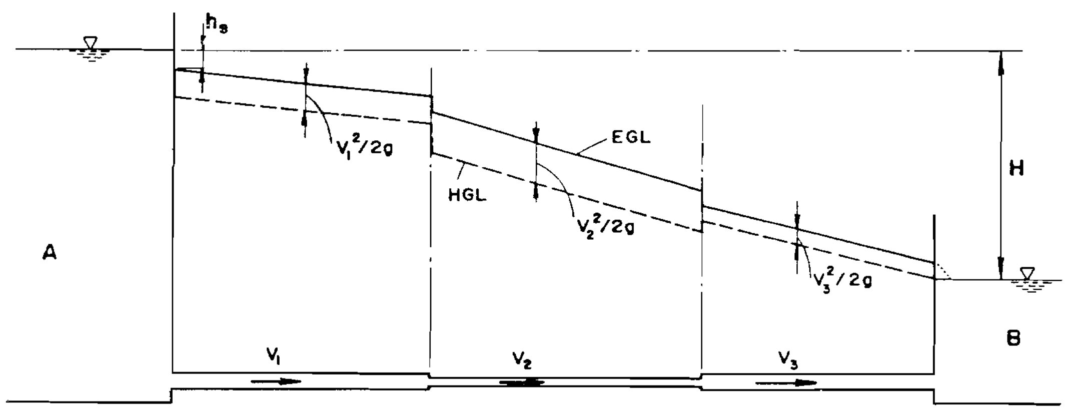


Figure 5.7 HGL and EGL for varying diameter pipes connecting two reservoirs

The smaller diameter pipe in the central portion introduces a contraction at its beginning and an enlargement at its end in the direction of flow. Thus in addition to frictional losses in the straight pipes, there will be local losses called entrance loss, contraction loss, enlargement loss, and exit loss and the HGL and the EGL will be as shown. It is seen that the HGL may rise in the direction of flow when the flow passes from a smaller to a larger diameter pipe since there will be an increase in pressure with a decrease in velocity and since the rate of energy loss and velocity head will be smaller in the larger pipe. But the EGL can never rise in the direction of flow (unless there is an external input of energy) as there is always a continuous loss of energy.

5.7.4 Free discharge through a nozzle in a pipeline containing a meter and a value.

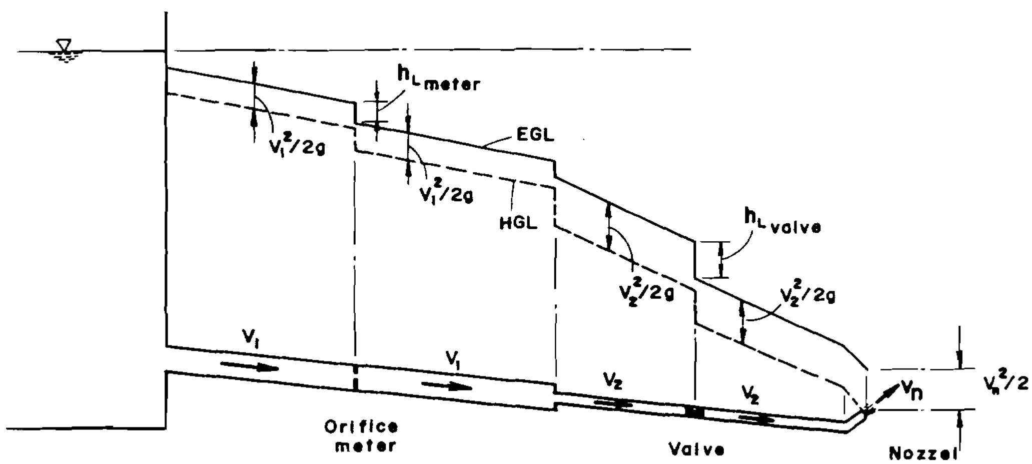


Figure 5.8

5.7.5 Pipeline with a pump

A pump in a pipeline system adds energy to the fluid flow and thus produces a sudden vertical rise in the hydraulic gradient.

h_e = entrance loss

h_{f_s} = head loss in suction pipe

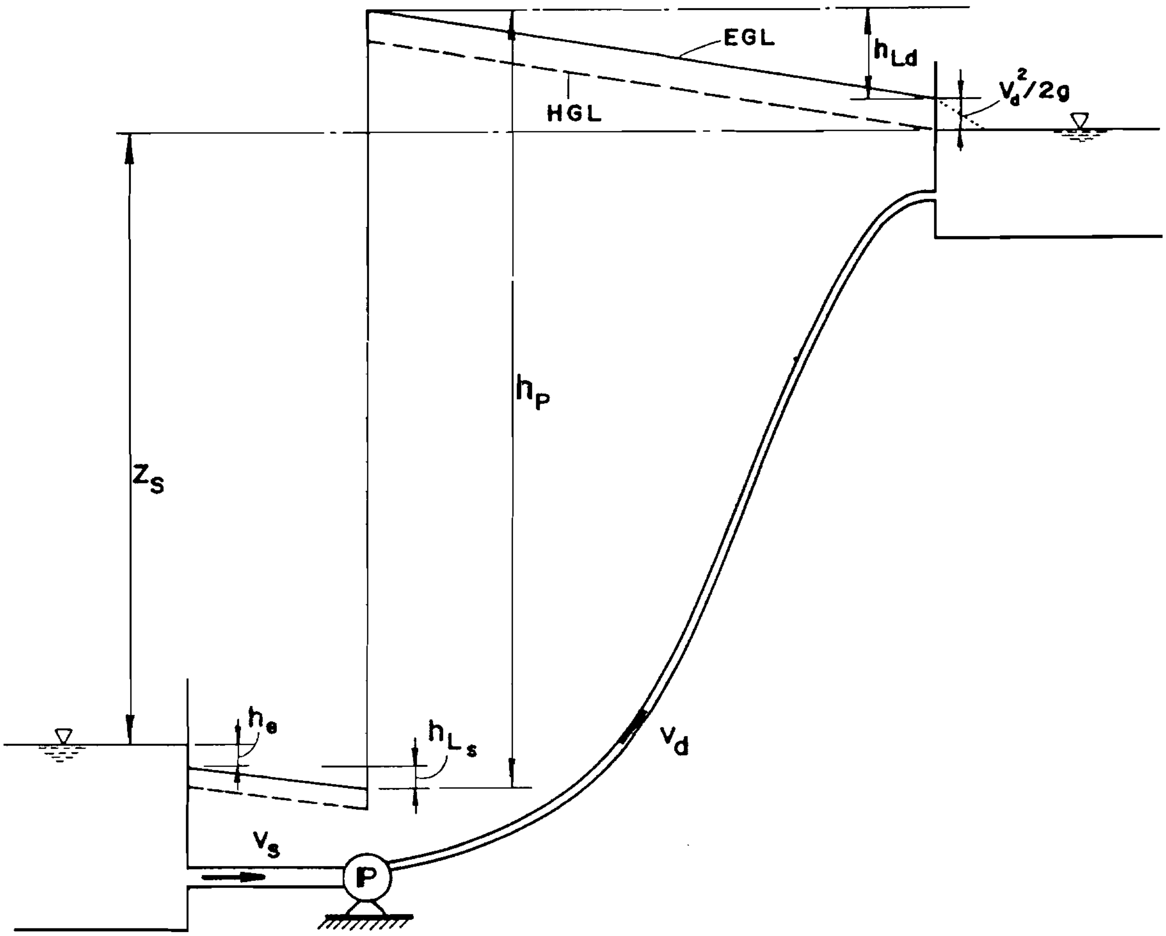


Figure 5.9 HGL and EGL pipe line with a pump

hl_d = head loss in delivery pipe

$\frac{V_d^2}{2g}$ = exit velocity head = exit head loss

h_p = head supplied by the pump

z_s = static lift = level difference between the reservoirs.

It is clear from Figure 5.9 that:

$$h_p = z_s + h_e + hl_s + hl_d + \frac{V_d^2}{2g}$$

i.e h_p = static lift + head loss in suction pipe + head loss in delivery pipe.

5.7.6 Discharge through a siphon

The hydraulic gradeline for a full flow condition in a siphon will be a straight line as if the pipe were taken straight from reservoir to reservoir.

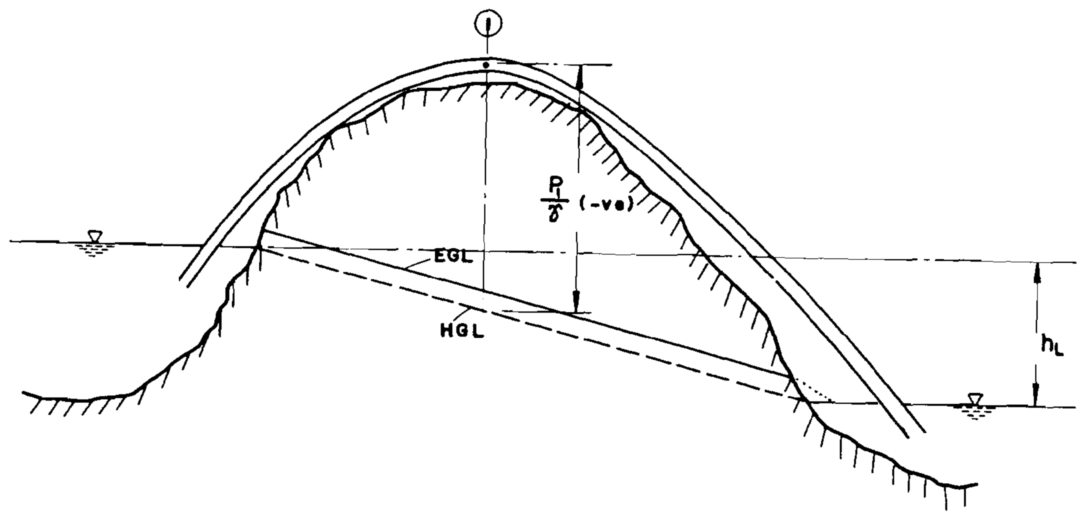


Figure 5.10 HGL and EGL for siphone flow

5.7.7 Discharge in an open channel

In open channel flow, atmospheric pressure acts on the water surface. Therefore the water surface and the hydraulic grade line should coincide.

5.7.8 Discharge over an ogee spillway

Figure 5.12 show the HGL and EGL for flow over an Ogee spillway with a hydraulic jump on the downstream apron.

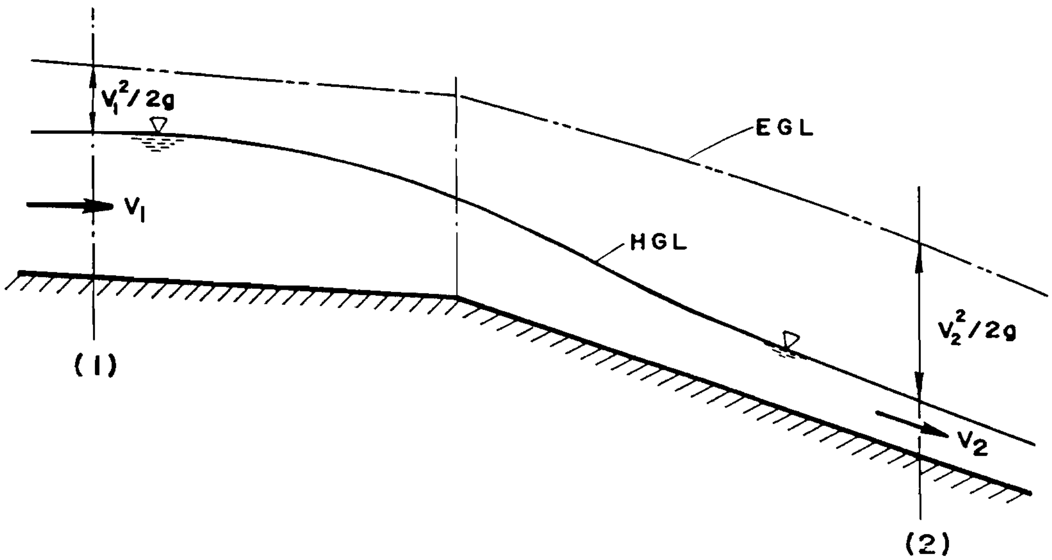


Figure 5.11 HGL and EGL in open channel flow

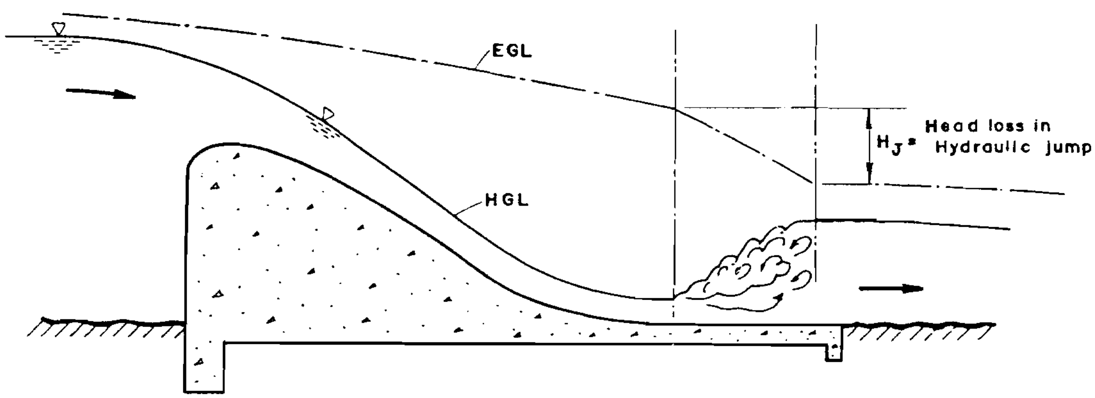


Figure 5.12 HGL and EGL for flow over a spillway

5.8 Impulse - Momentum Equation

The impulse momentum equation, along with the continuity equation and Bernoulli's Equation is the third basic tool for the solution of flow problems. Its application leads to the solution of problems in fluid mechanics involving forces and changes in veloctiy and which cannot be solved by the energy principle alone.

In the discussions that follow, steady one dimensional flow case will be used to develop the momentum equation since this approach has been found to be sufficient in the majority of cases.

The impluse momentum equation for fluids can be derived form Newton's 2nd Law of motion which states that the resultant external force acting on a fluid mass in any direction is equal to the time rate of change of linear momentum in that direction.

This may be obtained from Newton's Second Law of motion as follows:

$$F = ma = m.\frac{dv}{dt}$$

$$\text{or} \qquad F = \frac{d}{dt}(mv) = \text{rate of change of Momentum}$$

The above may written for steady flow as:

$$F = m\frac{dv}{dt} = \rho Q dt.\frac{dv}{dt} = \rho Q dv$$

dv is the change in velocity between the exit and entrance to a control volume considered.

$$\text{Thus:} \qquad F = \rho Q(v_2 - v_1) \qquad (5.18)$$

Where v_2 is the velocity at exit and v_1 is the velocity at entrance to the control volume and F, v_2 and v_1 are in the same direction.

Consider the flow in the stream tube shown in Figure 5.13

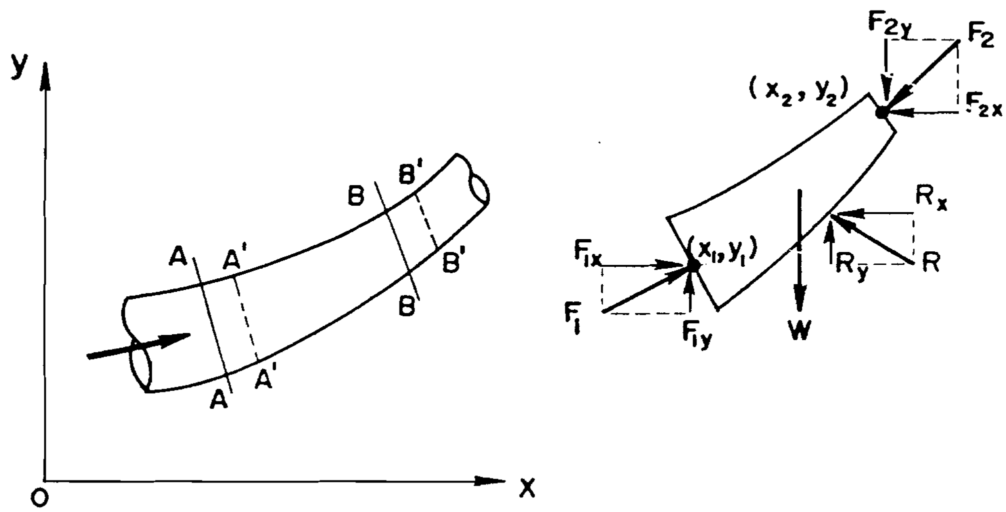


Figure 5.13 Development of the momentum principle

Let AA BB be the control volume.
Within the control volume the internal forces cancel out.

Summation of forces on the control volume will yield only the external forces at the control surfaces.

Forces acting on the control surfaces are:

1. Forces F_1 and F_2 at ends.
2. Weight W
3. Reaction force R on the Whole Control Volume

Equilibrium equations in the x and y directions will be

$$\sum F_x = F_{1x} - F_{2x} - R_x \tag{5.19}$$

$$\sum F_y = F_{1y} - F_{2y} + R_y - W \tag{5.20}$$

Equation (5.19) and (5.20) give the net force in the x and y directions.

To determine the change of Momentum of the fluid as it passes through the control volumes:

Let in a small internal of time the fluid move from postion AA BB to A'A' B'B'. The Mass A'A' BB does not experience any change in Momentum and may be takes as stationary.
Thus the differential change in Momentum is equal to the change in Momentum as the fluid moves form AA to A'A' and BB to B'B'. Mass of fluid entering the control volume in time dt is Qρdt. Assuming the fluid to be incompressible, the mass of fluid leaving the control volume in the same time interval dt is Qρ.dt.

Thus the change in momentum as the fluid moves into and out of the control volume will be:

Change in Momentum = Momentum at end BB - Momentum at end AA

$$i.e. \quad d(Mv) = (Q\rho\, dt)\, v_2 - (Q\rho\, dt)\, v_1 \qquad (5.21)$$

The rate of change of momentum in the x and y directions may be obtained from the general equation 5.21. Thus:

$$\frac{d}{dt}(Mv)_x = Q\rho\, v_{2x} - Q\rho\, v_{1x} \qquad (5.22)$$

$$\frac{d}{dt}(Mv)_y = Q\rho\, v_{2y} - Q\rho\, v_{1y} \qquad (5.23)$$

The final momentum equations in the two coordinate directions may be obtained from equations 5.18, 5.22 and 5.23 as:

In the x direction:

$$\sum F_x = \rho Q(v_{2x} - v_{1x}) \qquad (5.24)$$

In the y driection:

$$\sum F_y = \rho Q(v_{2y} - v_{1y}) \tag{5.25}$$

For a specific situation, the expressions for ΣF_x and ΣF_y are substituted in the momentum equation from equations 5.19 and 5.20 respectively.

The velocity components in the momentum equation is assumed to be constant and is the average velocity at the cross-section considered. In situations where the velocity is not constant across a cross-section, a correction factor β called the momentum correction factor, similar to the kinetic energy correction factor explained in section 5.5, needs to be applied to the average velocity components. Thus for non-uniform velocity of flow,

$$\sum F_x = \rho Q(\beta_2 V_{x2} - \beta_1 V_{x1})$$

$$\sum F_y = \rho Q(\beta_2 V_{y2} - \beta_1 V_{y1})$$

Where β_1 *and* β_2 are the momentum correction factors at section 1 and section 2 respectively. To obtain the value of β at a section, the momentum based on the average velocity is equated to the integral of the momentum of the elemental stream tubes over the entire cross section.

Thus:

$$\rho Q q \beta V_x = \int_A \rho \, dQ \, . \, v_x = \rho \int_A dQ \, . \, v_x$$

$$where: \quad dQ = v_x dA \quad and \quad Q = VA$$

Substituting,

$$\rho\, V_x . A . \beta . V_x = \rho \int_A v_x dA . v_x$$

$$\textit{Therefore,} \quad \beta = \frac{1}{A}\int_A \left(\frac{v_x}{V_x}\right)^2 dA$$

of more genergally,

$$\beta = \frac{1}{A}\int_A \left(\frac{v}{V}\right)^2 dA \tag{5.26}$$

EXERCISE PROBLESMS

5.1 Water flows through a horizontal 150 mm pipe under a pressure of 4.14 bar. Assuming no losses, what is the flow if the pressure at a 75 mm diameter reduction is 1.38 bar? (Ans. Q = 0.11 m^3/s)

5.2 Water flows upwards in a vertical 300 mm pipe at the rate of 0.22 m^3/s. At point A in the pipe the pressure is 2.1 bar. At B, 4.6 m above A, the diameter is 600 mm and the head loss from A to B equals 1.8 m of water. Determine the pressure at B.

5.3 The pressure inside the pipe at S (fig. P 5.3) must not fall below 0.24 bar absolute. Neglecting losses, how high above water level A may point S be located? (Ans. 6.6 m)

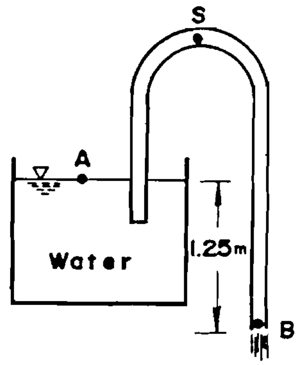


Fig. P 5.3

5.4 In Fig. P 5.4 the flow was found to be inadequate and it was decided to install a pump near the tank to increase the flow by 25%. Neglecting losses calculate the required horsepower of the pump.

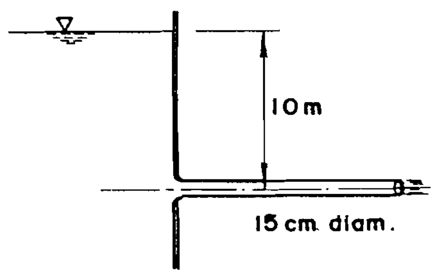


Fig. P 5.4

5.5 A turbine is supplied with water from a reservoir which is 200 m above the level of the discharge pipe. The discharge through the pipe is 0.20 m^3/s. If the power output from the shaft of the turbine is 310 kW and it has a mechanical efficiency of 90 per cent, calculate (a) the power drawn from the reservoir, (b) the hydraulic power delivered to the turbine. (Ans. 392.4 kW, 344.4 kW)

5.6 A closed tank contains water with air above it. The air is maintained at a pressure of 103 kPa and 5 m below the water surface a nozzle discharges into the atmosphere. At what velocity will water emerge from the nozzle?

5.7 How much power must be supplied for the pump in Fig. P 5.7 to maintain readings of 250 mm of Mercury vacuum and 275 kPa on gages (1) and (2) respectively, while delivering a flowrate of 0.15 m^3/s of water? (Ans. 54.36 kW)

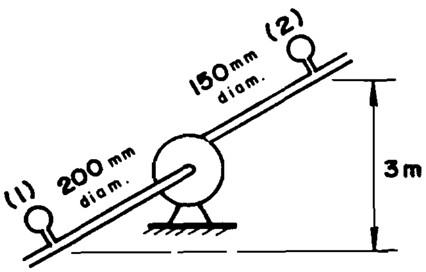


Fig. P 5.7

5.8 Calculate the discharge per unit width through the frictionless sluice gate, shown in Fig. P 5.8, when the depth h is 1.5 m. Also calculate the depth h for a flow rate of 3.25 $m^3/s/m$.

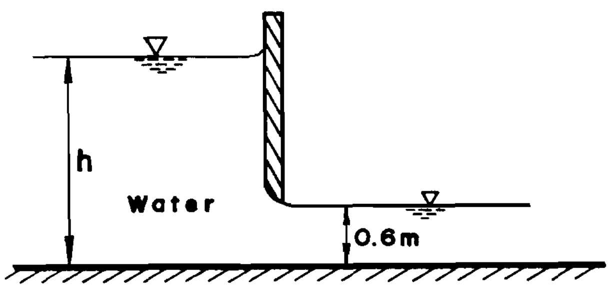


Fig. P 5.8

5.9 The turbine in Fig P 5.9 develops 75 kW when the flow rate is 0.6 m^3/s. What flow rate may be expected if the turbine is removed? (An. 1.118 m^3/s)

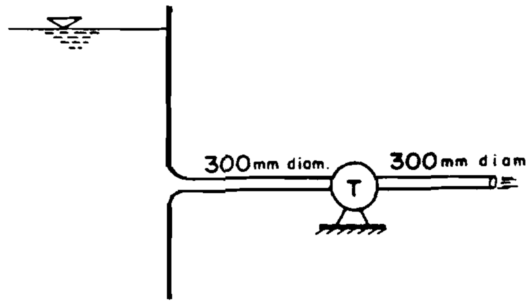


Fig. P 5.9

5.10 Determine the shaft horsepower for an 80 percent efficient pump to discharge 30 ℓ/s through the system of Fig P 5.10. The system losses, exclusive of pump losses, are 12 $V^2/2g$, and H = 16 m.

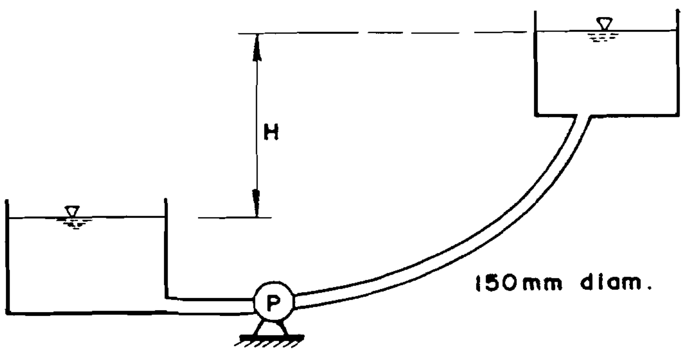


Fig. P 5.10

CHAPTER 6

APPLICATIONS OF BERNOULLI'S AND MOMENTUM EQUATION

6.1 Applications of Bernoulli's Equation

6.1.1 Introduction:

Bernoulli's equation is one of the important tools for solving many problems in fluid mechanics. It is applied either singly or in combination with the continuity and momentum equations depending upon the desired result. However, the following assumptions that were made in the derivation of Bernoulli's equation should be carefully borne in mind while applying the equation,

i) The flow is assumed to be steady i.e. there is no variation in the pressure, velocity and the density of the fluid at any point with respect to time. However, Bernoulli's equation can be applied without appreciable error in problems of unsteady flow with gradually changing conditions. Thus a problem of emptying a large reservoir, where the liquid level does not drop too rapidly, can be solved by applying Bernoulli's equation inspite of the fact that the flow is strictly unsteady.

ii) Bernoulli's equation holds true strictly only along a streamline since it is derived by integrating Euler's equation of motion along a streamline. However, in fully turbulent flows where variations in velocity across a section is not appreciable, the use of the mean velocity enables the application of Bernoulli's equation without appreciable error.

iii) The flow is assumed to be incompressible . Since liquids are generally considered incompressible, Bernoulli's equation is applicable for liquids. However, the equation can also be applied to gas flow problem when there is little variation in pressure and temperature.

iv) The equation is derived for ideal fluid where loss of energy due to friction does not exist. For real fluid flow in which frictional head loss occurs, this loss must be considered and included in Bernoulli's equation. But when the two sections considered are close to each other, frictional losses may be neglected.

Applications of Bernoulli's equation in some important devices in both closed conduit and open channel flows will be discussed in the sections that follow. In all cases loss of energy occurring is ignored in the derivation of the equations and then the theoretical results are corrected by experimentally determined coefficients to allow for the ignored loss of head.

6.1.2 The Pitot Tube

The pitot tube is used to measure the velocity of a stream. It consists of a simple L-shaped tube facing the oncoming flow (Fig. 6.1(a)). If u is the velocity of the stream at A, a particle moving from A to B will be brought to rest so that u_o at B is zero. B is called the stagnation point.

From Bernoulli's equation between A and B, datum through AB,

Total energy per unit weight at A = Total energy per unit weight at B

$$Z_A + \frac{p_A}{\gamma_w} + \frac{u^2}{2g} = Z_B + \frac{p_B}{\gamma_w} + \frac{u_o^2}{2g} \quad , \quad Z_A = Z_B = 0$$

$$\text{Thus} \quad \frac{p_B}{\gamma_w} = \frac{p_A}{\gamma_w} + \frac{u^2}{2g} \quad , \quad \text{since } u_o = o$$

$$\text{Since} \quad \frac{p_A}{\gamma_w} = Z \quad \text{and} \quad \frac{p_B}{\gamma_w} = h + Z$$

$$\frac{u^2}{2g} = \frac{p_B - p_A}{\gamma_w} = h$$

velocityatA$=u\sqrt{2gh}$

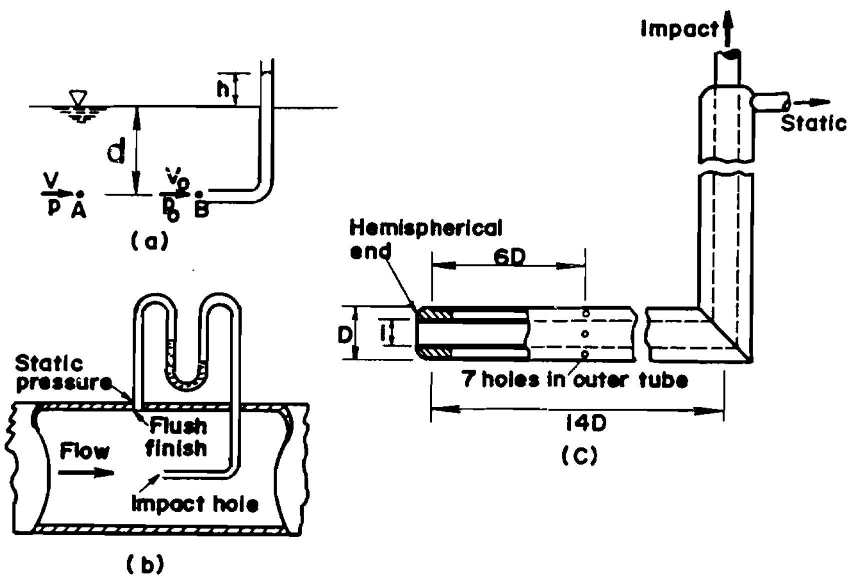


Figure 6.1 Pitot tube

When the Pitot tube is used in a channel, the value of h can be determined directly, as in Fig. 6.1(a). But if it is to be used in a pipe, the difference between the static pressure (p_A) and the pressure at the impact hole (i.e. the stagnation pressure p_B) must be measured with a differential pressure gauge, using a static pressure tapping in the pipe wall [Fig 6.1(b)] or a combined Pitot static tube [Fig 6.1(c)]. In the Pitot static tube, the inner tube is used to measure the impact (stagnation) pressure while the outer sheath has holes in its surface to measure the static pressure.

The theoretical velocity $u = \sqrt{2gh}$ requires calibration to obtain the real velocity.

The real velocity $u_r = C\sqrt{2gh}$, where C is the Pitot-tube Coefficient and h is the difference of head measured in terms of the flowing fluid. C usually varies between 0.95 and 1.00. For the Pitot-static tube [Fig. 6.1(c)] the value of C is unity for Reynolds number $\rho uD/\mu > 3000$, where D is the diameter of the tip of the tube.

Example 6.1

A pitot-static tube used to measure air velocity along a wind tunnel is coupled to a water manometer which shows a difference of head of 5 mm of water. The density of air is 1.2 kg/m^3. Determine the air velocity assuming the pitot-tube coefficient is unity.

Solution:

$$Velocity\ of\ air = C\sqrt{2gh}$$

where C = tube coefficient = 1.00

h = the difference in head expressed in terms of head of the flowing fluid i.e. air.

$$= 0.005\left(\frac{\rho_w}{\rho_a}\right) = 0.005\left(\frac{1000}{1.2}\right) = 4.167\ m\ of\ air$$

$$\therefore \quad Velocity\ of\ air = 1.0\sqrt{2 \times 9.81 \times 4.167} = 9.04\ m/s$$

6.1.3 The Venturi Meter

The Venturi meter is a device used to measure the rate of flow or the discharge Q in a pipe. It consists of a short converging conical tube leading to a cylindrical and straight portion called the "throat" which is followed by a diverging section, Fig. 6.2. The entrance and exit diameter is the same as that of the pipe line into which it is inserted. The size of a venturi meter is specified by the pipe and the throat diameter; for instance a 6 by 4 cm venturi meter fits a 6 cm diameter pipe and has a throat diameter of 4 cm. The angle of the convergent cone is between 20° and 40°, the length of the throat is equal to the throat diameter, and the angle of the divergent cone is 7° to 15°. Pressure tappings are taken at the entrance and at the throat and the pressure difference is measured by a suitable gauge. The pressure difference created as a result of the constriction is dependent on the rate of flow through the meter.

The expression for the discharge is obtained by considering sections 1 and 2 at the inlet and throat respectively.

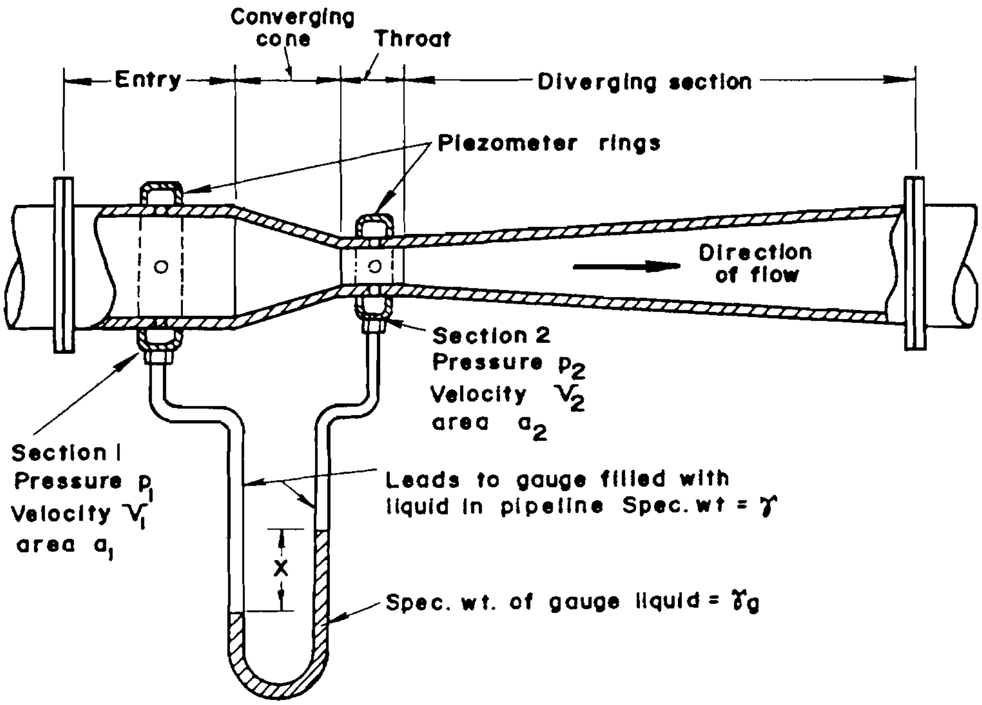


Figure 6.2 Venturi meter

Neglecting losses between inlet and throat and applying Bernoulli's equation between sections 1 and 2, datum through the centerline gives:

$$z_1 + \frac{p_1}{\gamma} + \frac{v_1^2}{2g} = z_2 + \frac{p_2}{\gamma} + \frac{v_2^2}{2g}$$

For a horizontal meter, $z_1 = z_2$.

$$\textit{Thus:} \quad \frac{v_2^2 - v_1^2}{2g} = \frac{p_1 - p_2}{\gamma} \tag{6.2}$$

From continuity equation: $a_1 v_1 = a_2 v_2$

$$\therefore \quad v_2 = \frac{a_1}{a_2} v_1$$

Substituting in equation 6.2,

$$v_1^2\left(\frac{a_1^2}{a_2^2} - 1\right) = 2g\left(\frac{p_1 - p_2}{\gamma}\right)$$

$$\therefore \quad v_1 = \frac{a_2}{\sqrt{a_1^2 - a_2^2}} \cdot \sqrt{2g\,\frac{p_1 - p_2}{\gamma}}$$

$$\text{Discharge } Q = a_1 v_1 = \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \cdot \sqrt{2gH} \qquad (6.3)$$

where $H = \dfrac{(P_1 - P_2)}{\gamma}$ = pressure difference expressed as head of the flowing fluid with specific weight γ.

In equation 6.3, Q is the theoretical discharge.

If $m = \dfrac{a_1}{a_2}$ is substituted, equation 6.3 becomes:

$$Q = a_1\sqrt{\frac{2gH}{(m^2 - 1)}}$$

The actual discharge is obtained by multiplying the theoretical discharge Q by the coefficient of discharge C_d obtained experimentally.

$$\text{Thus, Actual discharge} = C_d Q = c_d a_1 \sqrt{\frac{2gH}{m^2 - 1}} \qquad (6.4)$$

The value of C_d is a function of the ratio of throat to inlet diameter and the Reynolds number. For low diameter ratios and high Reynolds number, C_d is between 0.97 and 0.99. For the set up shown in Figure 6.2, the value of the pressure difference H is obtained by writing the manometric equation starting from the entry section 1 as:

$$p_1 + h_1 . \gamma - x . \gamma_g - (h_1 - x)\gamma = p_2$$

$$\frac{p_1 - p_2}{\gamma} = x\left(\frac{\gamma_g}{\gamma} - 1\right)$$

It can be shown that where the pressure difference is measured by a U-tube differential manometer, the value of x is independent of the inclination of the Venturi-meter.

Example 6.2

Water flows upwards in a 200 mm by 100 mm vertical Venturi-meter. The U-tube manometer with a gauge liquid of specific gravity 1.25 connected to the entrance and the throat registers a difference of 1 m of the gauge liquid. Taking the coefficient of the meter to be 0.99, determine the rate of flow.

Solution:

From the given data:

$d_1 = 0.2\ m,\quad d_2 = 0.1\ m,\quad x = 1\ m,\quad S_g = 1.25$
and $C_d = 0.99$

Thus:

$$a_1 = \frac{\pi \times 0.2^2}{4} = 0.0314\ m^2$$

$$a_2 = \frac{\pi \times 0.1^2}{4} = 0.00785\ m^2$$

$$H = x\left(\frac{\gamma_g}{\gamma_w} - 1\right) = 1\left(\frac{1.25.\gamma_w}{\gamma_w} - 1\right)$$

$$= 0.25\ m\ of\ water$$

$$m = \frac{a_1}{a_2} = \frac{d_1^2}{d_2^2} = 4.0$$

Using Equation 6.4,

$$Q = C_d . a_1\sqrt{\frac{2gH}{m^2 - 1}}$$

$$= 0.99 \times 0.0314\sqrt{\frac{2 \times 9.81 \times 0.25}{4 \times 4 - 1}}$$

$$= 0.0178\ m^3/s = 17.8\ \ell/s$$

6.1.4 The Orifice Meter

The orifice meter consists of a concentric, sharp-edged circular orifice made in a thin plate which is clamped between the flanges of a pipe, Figure 6.3. It is used for measuring the flow in a pipe, by relating the pressure difference between a section immediately upstream of the plate (section 1) and the vena Contracta of the issuing get downstream of the plate {section 2).

Figure 6.3 shows an orifice plate inserted in a pipeline. The fluid passing through the orifice contracts in area. The section of the stream where the cross-sectional area is minimum is called the Vena Contracta and forms at a distance of about $d_1/2$ down stream from the plane of the plate, where d_1 is the pipe

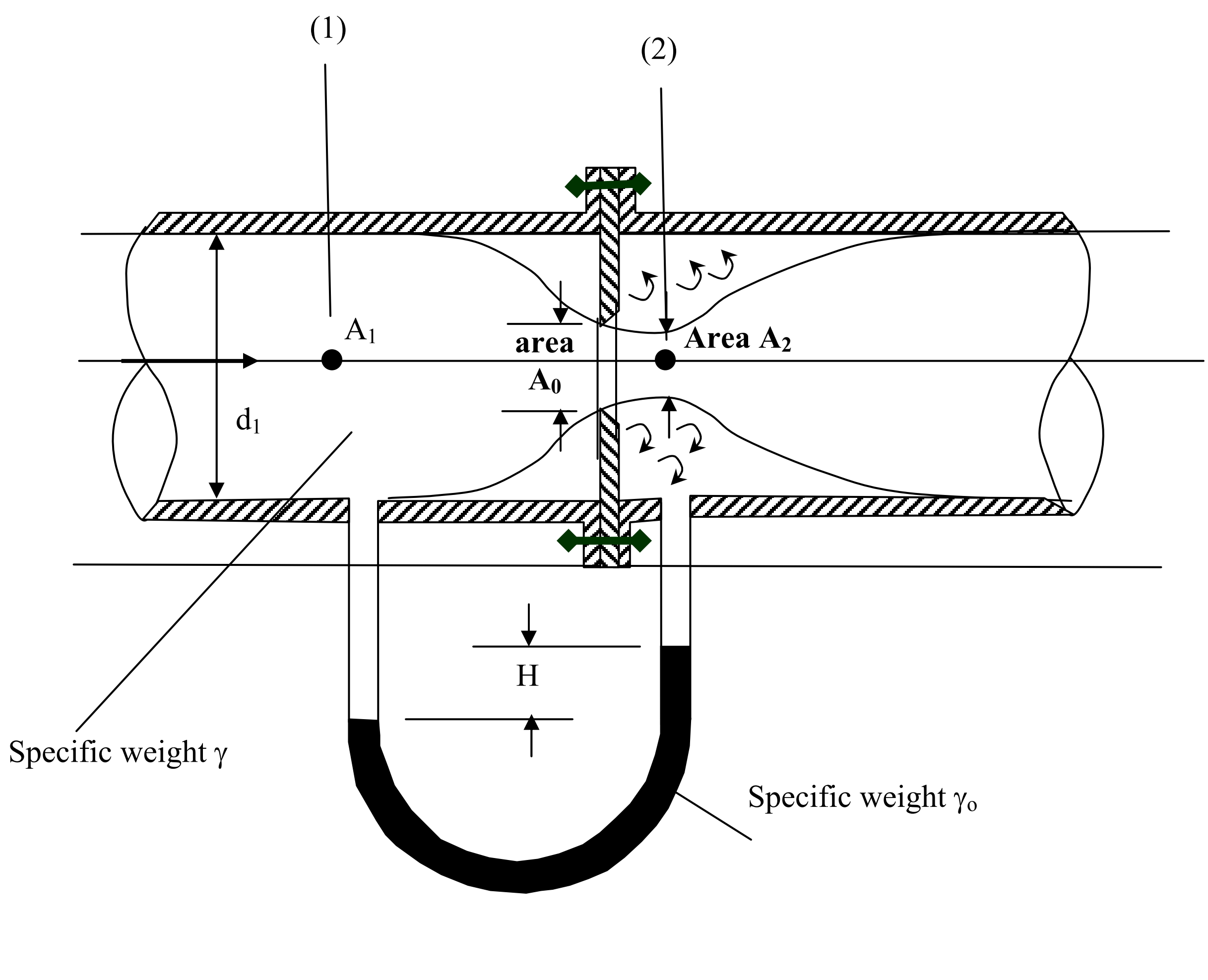


Figure 6.3 Orifice meter

The flow cross-sectional area at the vena contracta is minimum and the velocity is maximum and hence the pressure is minimum. Thus, as in the case of the venturi meter, the discharge may be calculated by measuring the pressure difference between the centers of the sections (1) and (2).

Applying Bernoulli's equation between the centers of the sections (1) and (2), datum through the centre line of the pipe:

$$\frac{p_1}{\gamma} + \frac{v_1^2}{2g} = \frac{p_2}{\gamma} + \frac{v_2^2}{2g}$$

$$or \qquad \frac{v_2^2 - v_1^2}{2g} = \frac{p_1 - p_2}{\gamma} = H$$

From continuity equation, $a_1 v_1 = a_2 v_2$

$$\therefore \qquad v_1 = \frac{a_2}{a_1} . v_2 = c_c \frac{a_o}{a_1} . v_2, \quad where \quad c_c = \frac{a_2}{a_o}$$

$$Thus: \quad \frac{v_2^2}{2g}\left[1 - c_c^2 \frac{a_o^2}{a_1^2}\right] = h$$

$$and \quad v_2 = \left[\frac{2gH}{1 - c_o^2 . a_o^2/a_1^2}\right]^{\frac{1}{2}}$$

Introducing the coefficient of velocity c_v, the actual velocity

$$v_{2a} = c_v\left[\frac{2gH}{1 - c_c^2 . a_o^2/a_1^2}\right]^{\frac{1}{2}}$$

The actual discharge Q will be

$$Q = a_2 \cdot v_{2a} = c_c \cdot a_o \cdot v_{2a}$$

$$or \quad Q = c_d \cdot a_o \left[\frac{2gH}{1 - c_c^2 \dfrac{a_o^2}{a_1}} \right]^{\frac{1}{2}}$$

$$where \quad c_c a_o = a_2 \quad and \quad c_c c_v = C_d$$

The above equation for Q may further be simplified by absorbing the two coefficients and the other constants into a single coefficient to give:

$$Q = c A_o \sqrt{2gH}$$

If the differential manometer reads a gauge difference x, it can be shown that:

$$H = \frac{p_1 - p_2}{\gamma} = x\left(\frac{\gamma_g}{\gamma} - 1\right)$$

$$\therefore \quad Q = CA_o \cdot \sqrt{2gx\left(\frac{\gamma_g}{\gamma} - 1\right)} \qquad (6.5)$$

The orifice coefficient C depends upon the ratios of the orifice and pipe areas and the Reynolds number of the flow. The coefficient C of the orifice meter is much lower than that of the venturi meter with values normally ranging from 0.6 to 0.65.

Example 6.3

An orifice meter, fixed to a 25 cm diameter pipe, has a diameter of 10 cm. The pipe conveys oil of specific gravity 0.9. Calculate the discharge if a mercury differential manometer reads a difference of 80 cm and C = 0.65.

Solution:

From equation 6.5

$$Q = CA_o \cdot \sqrt{2gx\left(\frac{\gamma_g}{\gamma} - 1\right)}$$

$$= 0.65 \times \frac{\pi}{4}(0.10)^2\sqrt{2 \times 9.81 \times 0.8\left(\frac{13.6}{1} - 1\right)}$$

Then $Q = 0.0718\ m^3/S = 71.8\ \ell/s$

6.1.4 Flow Through an Orifice

An orifice is an opening in the side or bottom of a tank or reservoir through which liquid is discharged in the form of a jet, normally into the atmosphere. Normally orifices are circular in cross-section. The flow velocity and thus the discharge through an orifice depend upon the head of the liquid above the level of the orifice. The flow is thus strictly speaking unsteady since the flow velocity varies with varying head as the outflow continues. However, for large tanks where the drop in level is small compared to the velocity of outflow through the orifice, steady flow may be assumed and Bernoulli's equation applied without appreciable error. Distinction will be made between small orifice and large orifice.

Small Orifice: The term 'small orifice' is used for an orifice which has diameter or vertical dimension small compared to the head producing the flow so that the head is assumed not to vary appreciably from point to point across the orifice. Figure 6.4 shows a small opening in the side of a large tank containing a liquid with specific weight γ and with a free surface open to the atmosphere. At point A, the pressure P_A is atmospheric and the velocity V_A i.e the rate of drop of the reservoir level will be negligibly small if the tank is large. Point B is a point in the vena contracta where P_B is atmospheric and the velocity v_B is equal to the velocity v of the jet.

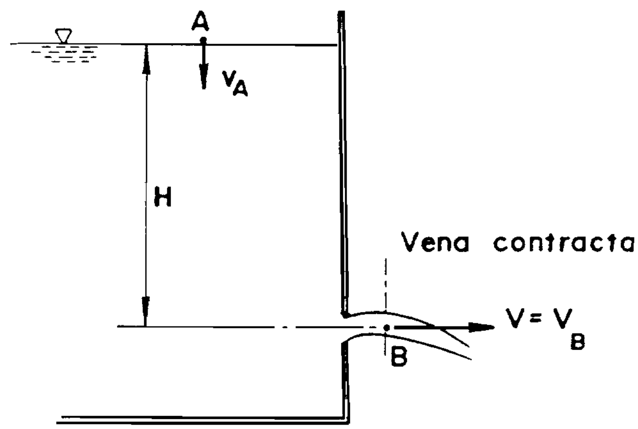


Figure 6.4 Flow through a small orifice

Applying Bernoulli's equation between A and B, datum through the center of the orifice and neglecting loss of energy between A and B:

$$Z_A + \frac{p_A}{\gamma} + \frac{v_A^2}{2g} = Z_B + \frac{p_B}{\gamma} + \frac{v_B^2}{2g}$$

$$H + 0 + 0 = 0 + 0 + \frac{v^2}{2g} \tag{6.6}$$

$$\therefore \quad velocity\ of\ jet \quad v = \sqrt{2gH}$$

Equation 6.6 is a statement of *Torricelli's theorem*, which is that the velocity of the issuing jet is proportional to the square root of the head producing the flow. If A is the cross-sectional area of the opening, then

$$Theoretical\ discharge\ \ Q = A.\sqrt{2gH}$$

The actual discharge through the orifice is much less than the theoretical discharge and must be obtained by introducing a discharge coefficient C_d, so that

$$\begin{aligned} Q_{actual} &= C_d.Q \\ &= C_d.A\sqrt{2gH} \end{aligned} \qquad (6.7)$$

The actual discharge is less than the theoretical discharge because:

i) The actual velocity of the jet is less than that given by equation 6.6 since there is a loss of energy between A and B. Thus

$$actual\ velocity\ at\ B = c_v.v = c_v.\sqrt{2gH} \qquad (6.8)$$

where c_v is coefficient of velocity, which has to be determined experimentally. c_v is of the order of 0.97.

ii) The area of the issuing jet at the vena contracta i.e. at B, is less than the area of the orifice opening

$$Actual\ area\ of\ jet\ at\ B = c_c A \qquad (6.9)$$

where c_c is coefficient of contraction and depends on the profile of the orifice. For sharp edged orifice c_c is of the order of 0.64.

Thus, from equation 6.8 and 6.9, the actual discharge will be:
Actual discharge = Actual velocity at B x Actual area at B

$$= c_v . \sqrt{2gH} . c_c . A = c_v c_c A \sqrt{2gH}$$

Comparison of the above with equation 6.7 shows that:

$$c_d = c_c c_v \qquad (6.10)$$

To determine the discharge coefficient c_d, the actual volume passing through the orifice in a given time is collected and compared with the theoretical discharge.

$$\text{Then,} \quad c_d = \frac{\text{Actual measured discharge}}{\text{Theoretical discharge}}$$

Similarly, by measuring the actual area of the jet and the velocity at the vena contracta, the coefficients of contraction C_c and coefficient of velocity C_v could be determined as:

$$C_c = \frac{\text{Area of jet at vena contracta}}{\text{Area of orifice}}$$

$$C_v = \frac{\text{Velocity at vena contract}}{\text{Theoretical velocity}}$$

For the case where the orifice is in the side of a tank (i.e. not at the bottom), measurement of the profile of the jet enables the determination of the actual velocity of the jet and thus that of C_v. Referring to Figure 6.5:

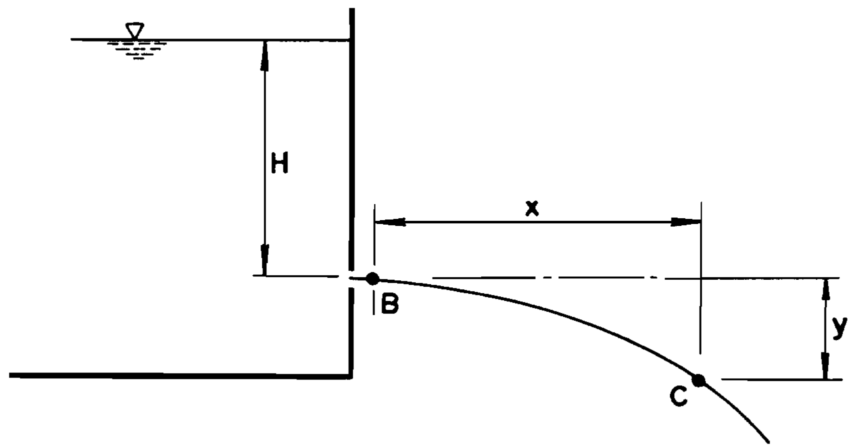


Figure 6.5 Profile of a jet

In Figure 6.5, point B is the vena contracta and at point C the jet falls a distance y vertically in a horizontal distance x from the vena contracta. Let t be the time taken for a fluid particle to travel from B to C. If air resistance is negligible and the horizontal component of the velocity v remains unchanged, then the distance travelled in time t will be:

$$x = v.t$$

and since the initial vertical velocity component at B is zero, the vertical distance y travelled in the same time t will be:

$$y = \frac{1}{2} g t^2$$

from which: $$v = \frac{x}{t} \quad and \quad t = \sqrt{\frac{2y}{g}}$$

so that $v = \sqrt{\dfrac{gx^2}{2y}}$ = actual velocity of jet at B.

Since theoretical velocity at B = $\sqrt{2gH}$, then

$$C_v = \frac{Actual\ velocity}{Theoretical\ velocity} = \frac{\sqrt{gx^2/2y}}{\sqrt{2gH}} = \sqrt{x^2/4yH}$$

Hydraulic coefficients of some typical orifices and mouth pieces are given in figure 6.6 below.

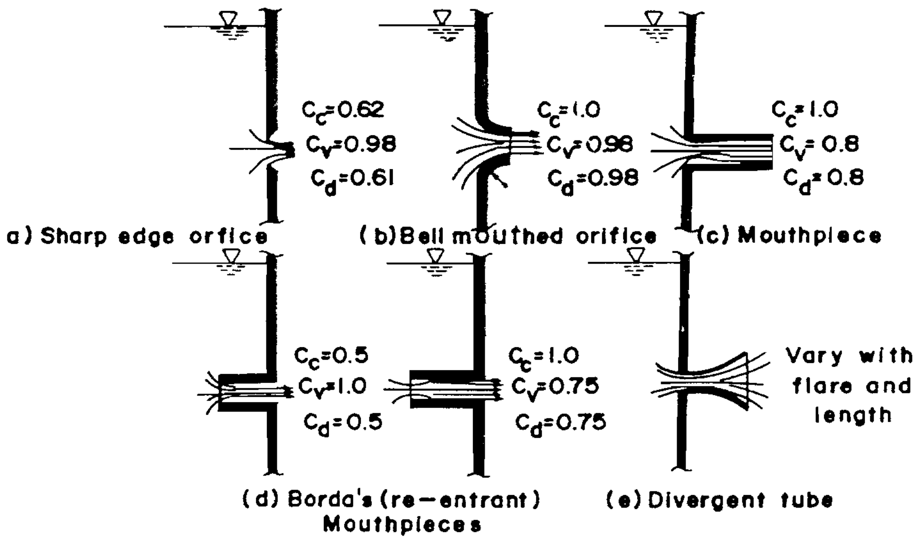


Figure 6.6 Hydraulic coefficients for some typical orifices and mouthpieces

Example 6.4

Find the diameter of a circular orifice to discharge 0.015 m^3/s under a head of 2.4 m using a coefficient of discharge of 0.6.

If the orifice is in a vertical plane and the jet falls 0.25 m in a horizontal distance of 1.3 m from the vena contracta, find the value of the coefficient of contraction.

Solution:

From the given data:

$Q = 0.015\ m^3/s, \quad H = 2.4\ m \quad C_d = 0.6$

and $y = 0.25, \quad x = 1.3\ m.$

$$Q = C_d A \sqrt{2gH} = C_d \cdot \pi \frac{d^2}{4} \sqrt{2gH}$$

$$\therefore \quad d = \left(\frac{4Q}{C_d \pi \sqrt{2gH}} \right)^{\frac{1}{2}} = \left(\frac{4 \times 0.015}{0.6 \pi \sqrt{19.62 \times 2.4}} \right)^{\frac{1}{2}}$$

$$= 0.0681\ m = 6.81\ cm$$

i.e diameter of the orifice = 6.81 cm

$$C_v = \sqrt{\frac{X^2}{4yH}} = \sqrt{\frac{1.3^2}{4 \times 0.25 \times 2.4}} = 0.8391$$

Since $\quad C_d = C_c C_v$,

$$C_c = \frac{C_d}{C_v} = \frac{0.6}{0.8391} = \underline{0.715}$$

Large Orifice: An orifice is classified as 'large' when the vertical height of the orifice is large so that the head producing the flow is substantially less at the top of the opening then at the bottom. The discharge calculated using the formula of small orifice, where the head H is measured to the centre of the orifice, will not be the true value since the velocity the will vary substantially from top to bottom of the opening. In this case theoretical discharge is calculated by integrating from top to bottom the flow through thin horizontal strips across the orifice.

Consider the large rectangular orifice of width B and depth D shown in Figure 6.7.

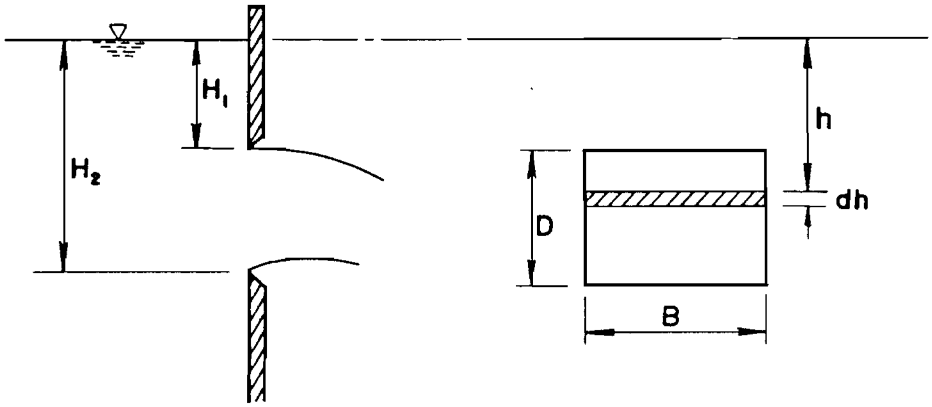


Figure 6.7 Flow through a large orifice

As shown in Figure 6.6 the top and bottom of the orifice opening are at depth H_1 and H_2 respectively below the free surface.

Consider a horizontal strip across the opening of height dh at a depth h below the free surface.

Area of the strip = Bdh

Velocity of flow through the strip = $\sqrt{2gh}$

Discharge through strip, $dQ = B\sqrt{2g}\,.\,h^{\frac{1}{2}}\,dh$

To obtain the discharge through the whole opening, integrate dQ from $h = H_1$ to $h = H_2$

$$\therefore \text{ Discharge } Q_t = B\sqrt{2g}\int_{H_1}^{H_2} h^{\frac{1}{2}}\, dh = \frac{2}{3}B\sqrt{2g}\left(H_2^{3/2} - H_1^{3/2}\right)$$

$$\text{The actual discharge } Q = \frac{2}{3}C_d \,.\, B\sqrt{2g}\left[H_2^{3/2} - H_1^{3/2}\right].$$

Example 6.5

Water flows from a reservoir through a rectangular opening 2 m high and 1.2 m wide in the vertical face of a dam. Calculate the discharge in m^3/s when the free surface in the reservoir is 0.5 m above the top of the opening assuming a coefficient of discharge of 0.64.

Solution

Referring go Figure 6.6:

$D = 2$ m, $B = 1.2$ m, $H_1 = 0.5$ m $C_d = 0.64$

$\therefore$ $H_2 = H_1 + D = 0.5 + 2 = 2.5$ m.

$$\therefore \quad Q = \frac{2}{3}C_d B\sqrt{2g}\left[H_2^{3/2} - H_1^{3/2}\right]$$

$$= \frac{2}{3} \times 0.64 \times 1.2\sqrt{19.62}\,[2.5^{3/2} - 0.5^{3/2}]$$

$$= 2.2679\,(3.9528 - 0.3536)$$

$$= 8.16\ m^3/s$$

Thus, the discharge is $8.16\ m^3/s$

Unsteady Flow Through Small Orifice: Problem of discharge through an orifice under varying head strictly fall under unsteady flow. But if the rate of fall of the head is very small compared to the velocity of efflux, Bernoulli's equation may be conveniently applied without appreciable error. The following two cases of unsteady flow through small orifice are of practical importance and will be considered:

i) Time required for a desired fall of liquid level in a tank due to efflux from an orifice.
ii) Flow from one tank to another.

Time required to empty a tank of uniform cross-section: Consider a tank of uniform cross-sectional area A discharging liquid through an orifice of cross-sectional area *a* installed at its bottom as shown in Fig. 6.8.

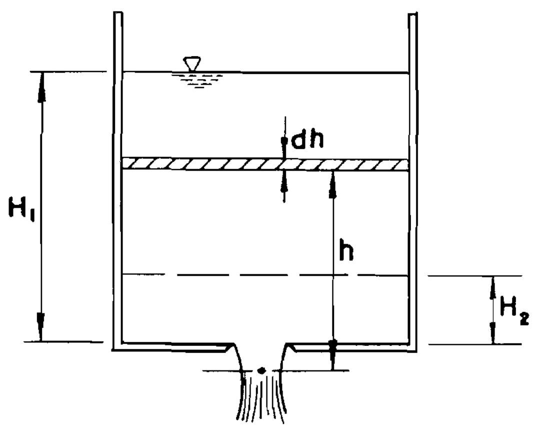


Figure 6.8 Flow through a small orifice at tank bottom

Let the height of the liquid be at h above the vena contracta at some instant. The theoretical outflow velocity at that instant will be:

$$v = \sqrt{2gh}$$

Let the liquid level fall by an amount *dh* during a time interval *dt*. The volume of liquid that has flown out in time *dt* will be:

$$dV = -A\,dh$$

Volume of liquid that has passed through the orifice in the same time interval *dt* will be:

$$dV = C_d \,.\, a \,.\, \sqrt{2gh} \,.\, dt$$

$$\text{Thus:} \quad C_d a \sqrt{2gh} \,.\, dt = -A\,dh$$

$$\therefore \quad dt = \frac{-A\,dh}{C_d \,.\, a \,.\, \sqrt{2gh}} = \frac{-A\left(h^{-\frac{1}{2}}\right)dh}{C_d a \,.\, \sqrt{2gh}}$$

The time T required for the liquid level to drop from H_1 to H_2 may be found by integrating the above equation between the limits H_1 and H_2.

$$T = \int dt = \frac{-A}{C_d \,.\, a\sqrt{2g}} \int_{H_1}^{H_2} h^{-\frac{1}{2}}\, dh$$

$$\text{i.e} \quad T = \frac{-2A}{C_d \,.\, a \,.\, \sqrt{2g}} \left(H_2^{1/2} - H_1^{1/2}\right)$$

Since $H_1 > H_2$, the term in brackets is negative, thus T will be positive. Taking the minus sign out of the bracket;

$$T = \frac{2A}{C_d \,.\, a \,.\, \sqrt{2g}} \left(H_1^{1/2} - H_2^{1/2}\right) \qquad (6.11)$$

The tank will be fully emptied when $H_2 = 0$.

Equation 6.11 gives the required time in seconds.

Flow from one tank to another through an orifice:

Consider two adjacent tanks of uniform cross-sectional area A_1 and A_2 connected by an orifice of cross-sectional area ***a*** as shown in Figure 6.9.

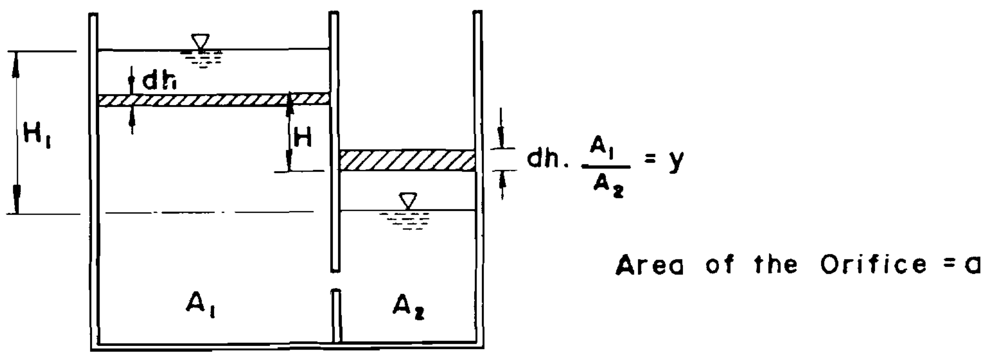


Figure 6.9 Flow between adjcent vessels through an orifice

Let H_1 = initial difference between the liquid levels in the two tanks

H_2 = final difference in level between the liquid levels in the two tanks

At any instant, let the difference in levels be H.
The theoretical velocity of the liquid through the orifice at this instant is

$$v = \sqrt{2gH}$$

After a small time interval dt, let the fall in head in tank A_1 be dh. The volume that has gone out of tank A_1 will be dhA_1. If y is the change in level of tank A_2, then the volume comming into tank A_2 in time dt will be $y.A_2$.

From continutiy, $y.A_2 = dhA,$

$$\therefore \quad y = dh.\frac{A_1}{A_2}$$

Then the total change in head difference between A_1 and A_2 will be $dH = dh + y = dh\left(1 + \frac{A_1}{A_2}\right)$

Equating the flow through the orifice for the time dt to the volume of displacement:

$$-A_1\,dh = C_d a\sqrt{2gH}\,.\,dt$$

$$or \qquad dt = \frac{-A_1\,dh}{C_d a\sqrt{2gH}} = \frac{-A_1\,dH}{C_d a\sqrt{2gH}\,.\left(1 + \frac{A_1}{A_2}\right)}$$

Integrating the above between H_1 and H_2, the time T required for the level difference in the two tanks to drop from H_1 to H_2 is

$$T = \frac{2A_1\left(H_1^{1/2} - H_2^{1/2}\right)}{C_d a\sqrt{2g}\left(1 + \frac{A_1}{A_2}\right)} \qquad (6.12)$$

The time required for the level between the two tanks to equalize is obtained when $H_2 = 0$.

Example 6.6

A rectangular tank 10 m x 6 m has an orifice with 10 cm diameter fitted at its bottom. It water stands initially at a height of 5 m above the orifice, what time is required for the level to drop to 1 m above the orifice. Take the orifice coefficient to be 0.64.

Solution:

The time required for the level to drop from H_1 to H_2 due to flow through an orifice fitted at the bottom of a tank is given by Equation 6.11 as:

$$T = \frac{2A\left(H_1^{1/2} - H_2^{1/2}\right)}{C_d\, a\sqrt{2g}}$$

Here; $A = 10 \times 6 = 60\ M^2$

$$H_1 = 5\ m, \quad H_2 = 1\ m$$

$$a = \frac{\pi d^2}{4} = \frac{\pi}{4}(0.1)^2 = 0.00785\ m^2$$

$$\therefore \quad T = \frac{2 \times 60\left(5^{\frac{1}{2}} - 1^{\frac{1}{2}}\right)}{0.64 \times 0.00785\sqrt{2 \times 9.81}} = 6665.37 \text{ sec}$$

i.e time required is : 1.851 *hrs.*

Example 6.7

Two tanks having plan areas of 6 m x 3 m and 1.5 m x 2 m are connected by a circular orifice 20 cm in diameter. Before the flow through the orifice began, the difference in water levels in the tanks was 4 m, with the higher level in the larger tank. Determine the time required to bring the difference down to 1.2 m. Take C_d = 0.61.

Solution:

From equation 6.12, the time required to bring the level difference between the two tanks from H_1 to H_2 is given by

$$T = \frac{2A_1\left(H_1^{1/2} - H_2^{1/2}\right)}{C_d\, a\sqrt{2g}\left(1 + \dfrac{A_1}{A_2}\right)}$$

where, A_1 = area of larger tank = 6 x 2 = 12 m^2

A_2 = area of smaller tank = 1.5 x 2 = 3 m^2

a = area of orifice = $\frac{\pi}{4}(0.2)^2 = 0.0314\ m^2$

H_1 = 4 m

H_2 = 1.2 m

C_d = 0.61

$$\therefore\quad T = \frac{2 \times 12\,(4^{1/2} - 1.2^{1/2})}{0.61 \times 0.0314\sqrt{2 \times 9.81}\left(1 + \dfrac{12}{3}\right)} = 51.18 \text{ sec}$$

6.1.6 Notches and Weirs

A notch is a geometrical opening in the side of a tank or reservoir extending above the free surface or it may be defined as any regular obstruction in open stream over which the flow takes place. It is in effect a large orifice which has no upper edge so that it has a variable flow area depending on the level of the free surface. A weir is a notch on a large scale, used, for example, to measure the discharge of a stream or a river.

A notch or a wier may be classified according to

a) Shape of the opening: as rectangular, triangular, trapezoidal etc.

b) Shape of the edge: as sharp-crested and broad-crested.

c) Discharge condition: as free or submerged.

d) End condition: as weir with end contractions and weir without end contraction i.e suppressed weir.

End Conditions: If the length of the crest of the notch or weir is equal to the width of the approach channel, then there will be no end contractions of the stream at the sides and the width of the nappe or jet flowing over the crest will be equal to the length of the crest. This type of weir in which end contractions are suppressed is called suppressed weir (Figure 6.10). Figures 6.10(b) and 6.10(c) illustrate weirs with one and two end contractions respectively.

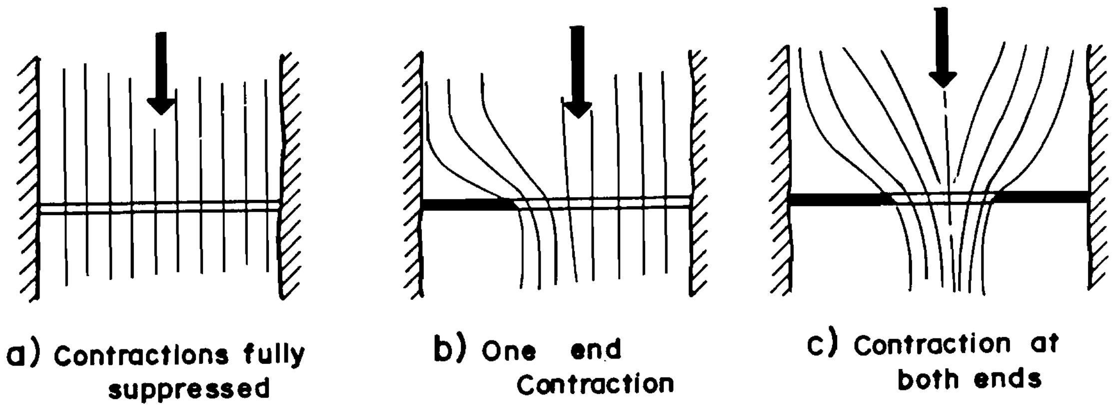


Figure 6.10 Suppressed and contracted weirs

Discharge Over Sharp Crested Rectangular Weir:

The sharp-crested rectangular weir, shown in Figure 6.11, has a sharp-edged horizontal crest which is normal to the flow. The nape falling over the crest is contracted at top and bottom as shown.

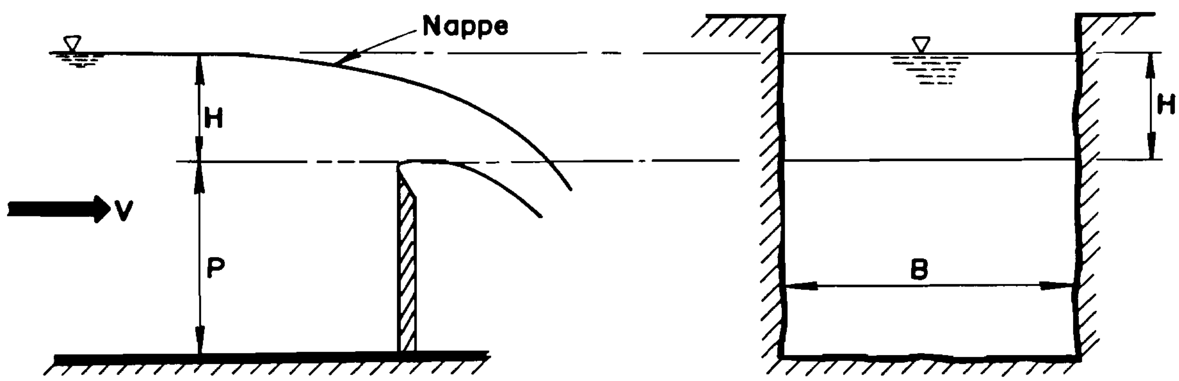


Figure 6.11 Sharp-crested rectangular weir

An equation for the discharge over the sharp-crested rectangular weir can be derived by neglecting the contractions of the nappe. Without contractions, the flow appears as shown in Figure 6.12 with the nappe having parallel streamlines with atmospheric pressure throughout.

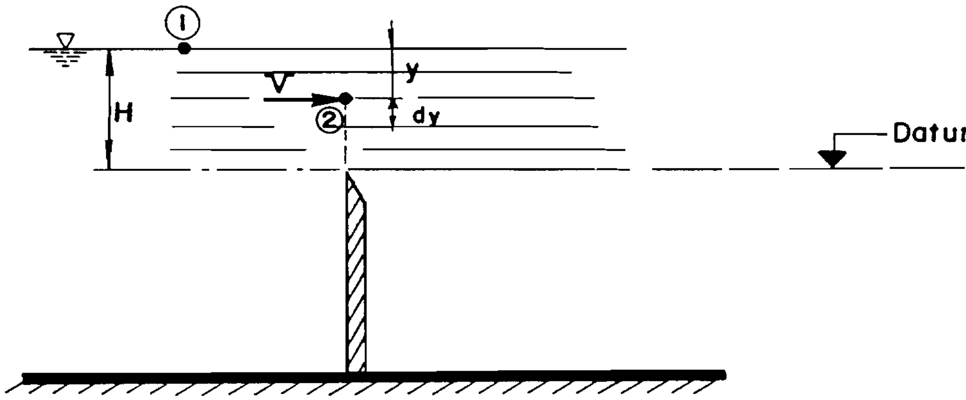


Figure 6.12 Weir nappe without contraction

Neglecting the approach velocity and losses and applying Bernoulli's equation between (1) and (2), datum through the crest,

$$H + 0 + 0 = \frac{v^2}{2g} + (H - y) + 0$$

Solving for v,

$$v = \sqrt{2gy}$$

The theoretical discharge Q_t is:

$$Q_t = \int v.dA = \int_o^H v.B.dy = \sqrt{2g}.B\int_o^H y^{1/2}\,dy$$

$$\text{or} \quad Q_t = \frac{2}{3}\sqrt{2g}\,B.H^{3/2}$$

where B = length of the weir crest.

The actual discharge Q_a is obtained by introducing a discharge coefficient C_d to the theoretical discharge.
Thus:

$$Q_a = C_d Q_t$$

$$Q_a = \frac{2}{3} C_d . \sqrt{2g} . B . H^{3/2}$$

The discharge coefficient C_d is a function of H and P and can be estimated from:

i) Bazin's formula:

$$C_d = \left(0.607 + \frac{0.00451}{H}\right)\left[1 + 0.55\left(\frac{H}{P + H}\right)^2\right]$$

where H = head over crest in metres
P = height of crest above channel floor in metres

ii) Rehbock formula:

$$C_d = 0.605 + \frac{1}{1048H - 3} + \frac{0.08H}{P}$$

where H and P are in metres.
This formula is valid for a notch with no end contractions.

Generally, however, $C_d \approx 0.62$ where by:

$$Q_a = 1.84 B H^{3/2} \tag{6.13}$$

Rectangular Weir with end Contraction:

When the weir crest does not extend completely across the full width of the channel it is said to have end contractions as shown in Figure 6.13. In this case the effective width, B_e, of the crest is less than B as a result of the end contraction. Francis found that the end contraction for each contraction is about 0.1 H, where H is the head over the weir crest.

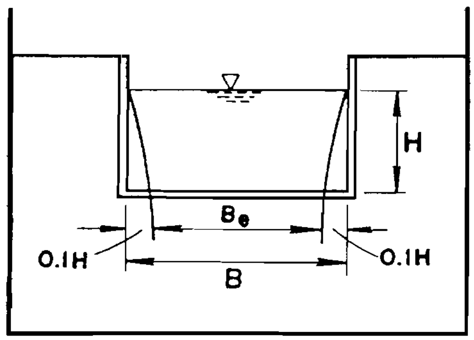


Figure 6.13 Rectangular weir with end contraction

Thus $B_e = (B - 0.1nH)$, where n = number of end contractions.

For a fully contracted rectangular weir, n = 2

$$\therefore \quad B_e = B - 0.2H$$

$$\text{Thus} \quad Q_a = 1.84(B - 0.2H)H^{3/2}$$

When the height P of a weir is small, the approach velocity head at point (1) cannot be neglected. In such a situation, a correction may be added to the head as:

$$Q = \frac{2}{3} C_d B.\sqrt{2g}\left(H + \alpha \frac{v^2}{2g}\right)$$

$$\text{where} \quad v = \frac{Q}{B(P + H)} \quad \text{and} \quad \alpha \approx 1.4$$

The above equation must be solved for Q by trial since both v and Q are unknown. As a first trial the term $\alpha \frac{v^2}{2g}$ may be neglected to approximate Q. With this trial discharge, a value of v is computed.

The V-notch or Triangular Weir:

The V-notch or triangular weir, shown in Figure 6.14, is particularly convenient for measuring small discharges. The contraction of the nappe is neglected and the discharge is computed as follows:

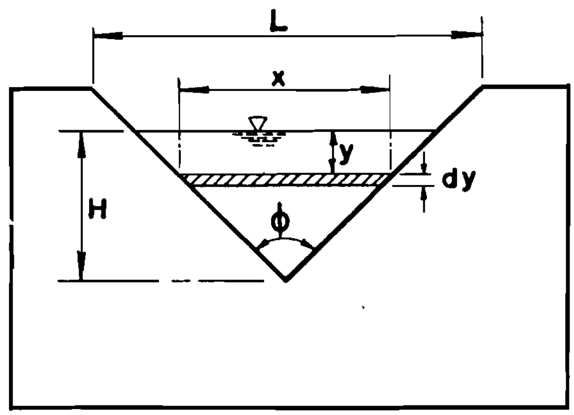


Figure 6.14 V-notch or Triangular weir

The velocity at depth y in Figure 6.14 is given by

$$v = \sqrt{2gy}$$

The theoretical discharge Q_t is

$$Q_t = \int v.dA = \int_o^H v.x\,dy$$

From similar triangles, x may be related to y:

$$\frac{x}{H - y} = \frac{L}{H}$$

$$\therefore \quad Q_t = \sqrt{2g} \, . \, \frac{L}{H} \int_o^H y^{1/2} \, (H - y) \, dy$$

$$= \frac{4}{15} \sqrt{2g} \, . \, \frac{L}{H} \, . \, H^{5/2}$$

$$But \quad \frac{L}{2H} = \tan \frac{\phi}{2}$$

$$\therefore \quad Q_t = \frac{8}{15} \sqrt{2g} \tan \frac{\phi}{2} \, . \, H^{5/2}$$

The actual discharge is obtained by introducing a discharge coefficient C_d. Thus:

$$Q_a = \frac{8}{15} C_d \sqrt{2g} \, . \tan \frac{\phi}{2} \, . \, H^{5/2} \tag{6.14}$$

$$where \quad C_d = f\left(\frac{H}{P}, \frac{P}{B}, \phi\right)$$

$$for \quad \phi = 90^\circ, \quad C_d \approx 0.58 \quad so\ that$$

$$Q_a = 1.38 \, H^{2.5} \tag{6.15}$$

Trapezoidal Weir: A Trapezoidal Weir, shown in Figure 6.15, can be considered to be made up of a rectangular weir of width B and a triangular weir of apex angle ϕ.

If C_d and C_d' represent the discharge coefficients of the rectangular weir and the triangular weir respectively, the

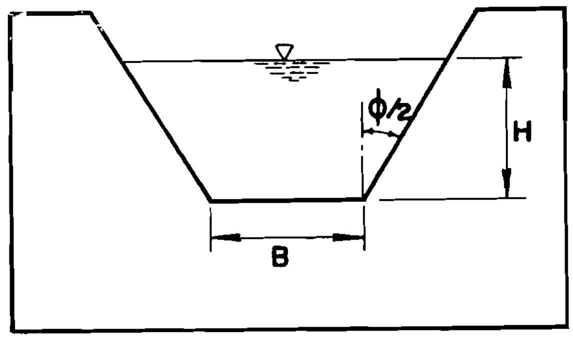


Figure 6.15 Trapezoidal weir

discharge Q flowing through the trapezoidal weir under head H will be

$$Q = \frac{2}{3} C_d \sqrt{2g} \, . \, B \, . \, H^{3/2} + \frac{8}{15} C_d' \sqrt{2g} \tan \frac{\phi}{2} \, . \, H^{5/2} \qquad (6.16)$$

Since the rectangular weir is contracted, the discharge will be reduced by $\frac{2}{3} C_d \, . \, \sqrt{2g} \, . \, (0.2H) \, H^{3/2}$. If this reduction in discharge due to contraction is thought of as being compensated by increase in discharge due to the triangular portion, then

$$\frac{2}{3} C_d \, . \, \sqrt{2g} \, (0.2H) \, H^{3/2} = \frac{8}{15} C_d \sqrt{2g} \tan \frac{\phi}{2} \, . \, H^{5/2}$$

from which one obtains $\tan \frac{\phi}{2} = \frac{1}{4}$

This particular trapezoidal weir in which the side slopes with 1 horizontal to 4 vertical is known as *Cippoletti Weir*. Thus, for a Cippoletti weir,

$$Q = \frac{2}{3} C_d . \sqrt{2g} . B . H^{3/2} \qquad (6.17)$$

The Broad-Crested Weir: A broad-crested weir is one where the crest of the weir is broad i.e. the crest width B > 0.4 H (Figure 6.16)

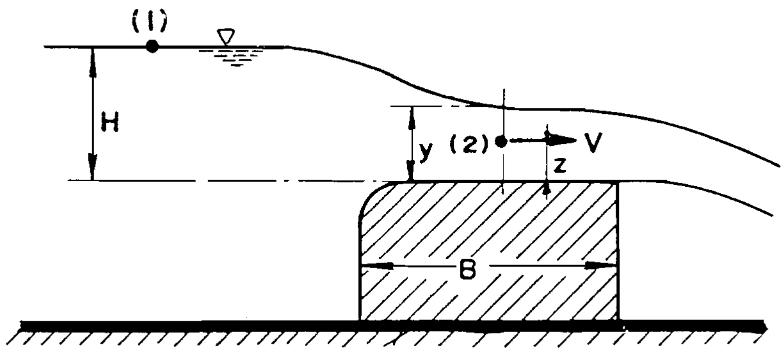


Figure 6.16 Broad-crested weir

The upstream edge of the weir is rounded to avoid separation. Neglecting losses and assuming a parallel stream of flow with hydrostatic pressure distribution over the crest, Bernoulli's equation applied between points (1) and (2), neglecting approach velocity, gives:

$$H + 0 + 0 = \frac{V_2^2}{2g} + z + (y - z)$$

$$\therefore \quad V_2 = \sqrt{2g(H - y)}$$

For a weir of width L normal to the plane of the figure, the theoretical discharge is

$$Q_t = V_2 . L . y = Ly\sqrt{2g(H - y)}$$

For y = H, Q = 0. Maximum Q occurs for a particular value of y. This value is obtained by differentiating Q_t with respect to y and equating the result to zero for maximum Q. Thus

$$\frac{dQ_t}{dy} = 0 = L\sqrt{2g(H-y)} + Ly \cdot \frac{1}{2} \cdot \frac{-2g}{\sqrt{2g(H-y)}}$$

$$\therefore \quad [2g(H-y)] = L \cdot y \cdot g$$

$$\text{from which} \quad y = \frac{2}{3}H = y_c$$

$$\text{Thus} \ : \ V_2 = \sqrt{2g\left(\frac{3}{2}y - y\right)} = \sqrt{gy}$$

$$\text{and} \quad Q_t = 1.705 L H^{3/2}$$

The actual discharge is obtained by introducing a discharge coefficient C_d. Thus

$$Q_a = 1.705\, C_d L H^{3/2}$$

For a well-rounded upstream edge, C_d = 0.98.

$$\therefore \quad Q_a = 1.67\, L H^{3/2} \qquad (6.18)$$

Example 6.8

The discharge over a suppressed rectangular weir is to be 0.20 m^3/s when the head over the crest is 30 cm. If the discharge coefficient is 0.6, calculate the length of weir crest required.

Solution:

$$Q = \frac{2}{3} C_d . B\sqrt{2g} . H^{3/2}$$

Substituting $Q = 0.20\ m^3/s$, $C_d = 0.6$ *and* $H = 0.30\ m$,

$$B = \frac{Q}{\frac{2}{3} C_d\sqrt{2g} . H^{3/2}} = \frac{0.20}{\frac{2}{3} \times 0.6 \times \sqrt{19.62}\,(0.3)^{3/2}} = 0.69\ m$$

Example 6.9

Determine the discharge over a sharp-crested rectangular weir with 8 m crest length and a head of 2.4 m. The width of the approach channel is 10 m. Take $C_d = 0.622$.

Solution:

This is a rectangular weir with end contractions. Thus, neglecting the approach velocity,

$$Q = \frac{2}{3} C_d (B - 0.2H)\ \sqrt{2g} . H^{3/2}$$

Substituting $C_d = 0.622$, $B = 8\ m$, $H = 2.4\ m$,

$$Q = \frac{2}{3} \times 0.622\,(8 - 0.2 \times 2.4)\ \sqrt{2g} . 2.4^{3/2}$$

$$= 51.36\ m^3/s$$

Example 6.10

A 90° V-notch has a discharge coefficient of 0.60. Calculate the discharge when the observed head is 0.65 m.

Solution:

For a V-notch, the discharge is given by:

$$Q = C_d \cdot \frac{8}{15}\sqrt{2g} \cdot \tan\frac{\phi}{2} \cdot H^{5/2}$$

Substituting $C_d = 0.60$, $H = 0.65\ m$ *and* $\phi = 90^o$,

$$Q = 0.60 \times \frac{8}{15}\sqrt{19.62}\,\tan 45^o \cdot (0.65)^{5/2}$$

$$= 0.483\ m^3/s$$

6.2 Applications of the Momentum Equation

6.2.1 Introduction

The momentum equation is used in the solution of the following two classes of problems:

i) To determine the resultant force acting on the boundary of a flow passage by a stream of fluid as the stream changes its direction or magnitude or both. Problems of this type are forces on pipe bends, reducers, stationary and moving vanes, jet propulsion etc.

ii) To determine the characteristics of flow when there is an abrupt change of flow section such as in sudden enlargement in a pipe and hydraulic jump in channels and also to determine the forces involved instructures in open channel flow.

Typical cases of the two types of problems will be discussed in the sections that follow.

6.2.2 Dynamic force due to a jet impinging on a stationary surface.

Force on a flat plate:

Consider a stationary, smooth plate on which a jet of cross-sectional area '*a*' impinges with a velocity V_o inclined at an angle θ with the plate. Let ϱ be the density of the fluid and assume the plate and the jet to be in a horizontal plane. Assume also no frictional and impact losses at the plate so that the velocity V_o remains unchanged.

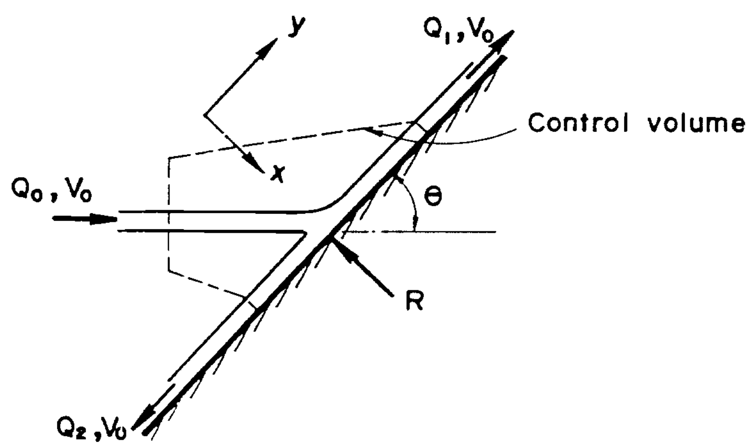


Figure 6.17 A jet impinging on a flat plate

When a jet strikes a solid surface a stream of fluid is formed which moves over the surface and it leaves the surface tangentially.

For the control volume shown,

Continuity equation gives: $Q_o = Q_1 + Q_2$ (a)

In the y direction no force is exerted by the plate on the fluid. Thus:

$$\sum F_y = 0 = \rho Q_1 V_o - \rho Q_2 V_o - \rho Q_o V_o Cos\theta = 0$$

$$\text{from which:} \quad Q_1 - Q_2 = Q_o Cos\theta \tag{b}$$

Solving (a) and (b)

$$Q_1 = \frac{Q_o}{2}(1 + Cos\theta)$$

$$Q_2 = \frac{Q_o}{2}(1 - Cos\theta)$$

In the x-direction, the plate exerts a force F on the fluid in the control volume in a direction normal to the plate as shown. Thus:

$$\sum F_x = -R = \rho Q_o \times 0 - \rho Q_o V_o Sin\theta$$

$$\therefore \quad R = \rho Q_o V_o Sin\theta \tag{6.19}$$

Hence the jet exerts an equal and opposite force to R on the plate in the positive x-direction.

If the inclined plate moves with a velocity v say in the same direction as the jet, then the mass of fluid impinging on the plate per unit time will be $\rho A(V_o - v)$ and it will be less than the mass impinging on a stationary plate.

Summing forces in the x-direction:

$$-R = \rho A(V_o - v)[0 - (V_o - v) Sin\theta]$$

$$= -\rho A(V_o - v)^2 Sin\theta$$

$$or \quad R = \rho A(V_o - v)^2 Sin\theta \tag{6.20}$$

Force on a curved vane:

Fixed vanes:

Since momentum is a vector quantity, change in momentum would occur across a control volume when there is change in direction only, with or without change in the magnitude of velocity. Consider the flow of jet of area A of fluid of density ϱ impinging on a fixed curved vane shown in Figure 6.18 with a constant velocity V_o.

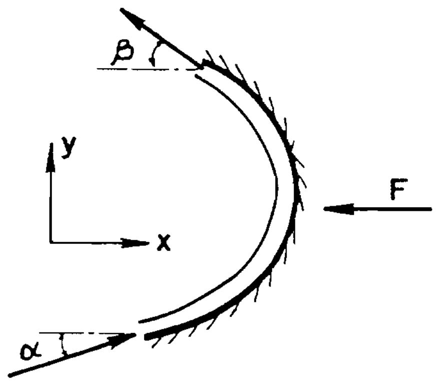


Figure 6.18 Force on a fixed curved vane

The entrance and exit angles of the curved vane are α and β with respect to the x-axis.

The Momentum of the fluid at inlet in the x direction is $\rho A V . v \cos\alpha$ and at exit it is $-\rho A v_o . V_o \cos\beta$. Hence,

$$\sum F_x = -F = -\rho A V_o^2 \cos\beta - \rho A V_o^2 \cos\beta$$

$$\therefore \ F = \rho A V_o^2 (\cos\alpha + \cos\beta) \qquad (6.21)$$

The force F shown in the figure is the force exerted by the vane on the fluid. An equal and opposite force is exerted by the fluid on the vane. The force will be maximum when $\alpha = \beta = 0$ i.e. when the vane is semi-circular.

Moving Vane:

Consider a curved vane moving at a velocity v in the x-direction as shown in Fig. 6.19.

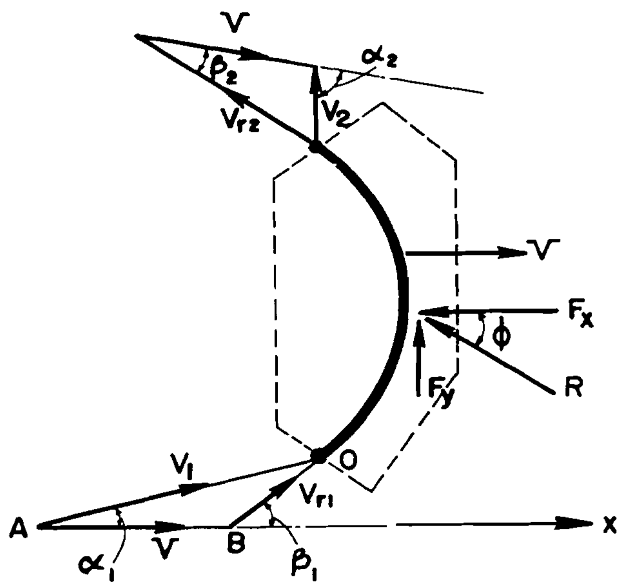


Figure 6.19 Force on a moving curved vane

A jet of absolute velocity V_1 (= AO) of a fluid density ϱ and area A is directed at the vane at an angle α_1 to the x direction i.e. the direction of motion of the vane. The jet will enter the vane with a velocity Bo, which is the relative velocity of the absolute velocity V_1 with respect to the vane velocity v(=AB). BO is called relative velocity Vr_1. This relative velocity v_{r1}r1 of the jet needs to be tangential to the inlet blade angle β_1 for the jet to enter the vane smoothly. For smooth vane surface, the jet moves along the vane without change in the magnitude of the relative velocity. Let it, however, be assumed that there is some change in velocity and the velocity of the jet becomes V_{r2} as it emerges from the vane with the vane outlet angle β_2. The absolute velocity of the jet leaving the vane will be V_2. V_{r2} is the relative velocity of V_2 with respect to the vane velocity v. V_2 is at an angle α_2 with the x-axis.

Considering the control volume (shown in dashed lines, Figure (6.19)) of the fluid enclosed by the inlet, the vane and the outlet, the mass of water entering the control volume per unit time is $\rho A V_{r1}$. Applying the momentum equation in the x direction:

$$\sum F_x = -F_x = \rho A V_{r1} (-V_2 \cos\alpha_2 - V_1 \cos\alpha_1)$$

which reduces to:

$$F_x = \rho A V_{r1} (V_1 \cos\alpha_1 + V_2 \cos\alpha_2) \qquad (6.22)$$

The force F_x shown in Figure 6.19 is the force exerted by the vane on the fluid.

Similarly,

$$\sum F_y = F_y = \rho A V_{r1} (V_2 \sin\alpha_2 - V_1 \sin\alpha_1$$

$$\therefore \qquad F_y = \rho A V_{r1} (V_2 \sin\alpha_2 - V_1 \sin\alpha_1 \qquad (6.23)$$

The resultant force R acting on the fluid by the vane is:

$$R = \sqrt{F_x^2 + F_y^2}$$

$$\text{and} \quad \tan\phi = \frac{F_y}{F_x}$$

An equal and opposite force to the force R shown in Figure 6.19 will be exerted by the water on the vane.

Example 6.11

A jet of water 4.0 cm in diameter and having a velocity of 20.0 m/s impinges on a flat plate normally. Find the force exerted on the plate if

a) The plate is stationary
b) The plate is moving with velocity of 3.0 m/s in the same direction and what is the work done?

Solution:

a) Mass of water striking the plate when the plate is stationary is:

$$\rho A V = 1000 \times \frac{\pi}{4}(0.04)^2 \times 20 = 25.13\ kg/s$$

Force exerted on the plate is:

$$F = M(V - 0) = 25.13 \times 20 = 502.6\ N$$

b) When the plate is moving in the same direction as the jet, the mass of water striking the plate will be

$$\rho A(V - v) = 1000 \times \frac{\pi}{4}(0.04)^2(20 - 3.0) = 21.36\ kg/s$$

Force exerted on the plate will be:

$$F = M(V - v) = 21.36 \times 17 = 363.13\ N$$

$$Work\ done = F.v = 363.13 \times 3 = 1.089.39\ W$$
$$= 1,089\ kW$$

Example 6.12

A 5.0 cm diameter jet of water having a velocity of 40 m/s strikes a vane (see Figure E) having a deflection angle of 135° and moving at a velocity of 15 m/s in the same direction. Assuming no friction, compute:

i) The x and y components of the force exerted by the fluid on the vane.

ii) Absolute velocity of the jet when it leaves the vane and

iii) The power developed.

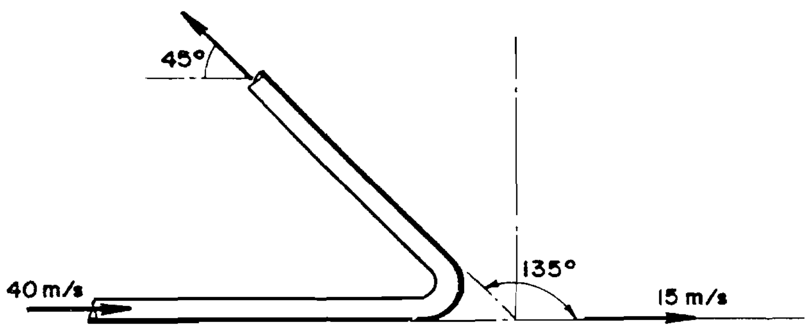


Figure E 6.12

Solution:

i) The relative velocity of the jet with respect to the vane is (40-15) = 25 m/s

Therefore, discharge striking the vane will be:

$$\frac{\pi}{4}(0.05)^2 \times 25 = 0.0491\ m^3/s$$

Since there is no friction the relative velocity as the jet leaves the vane will also be 25 m/s

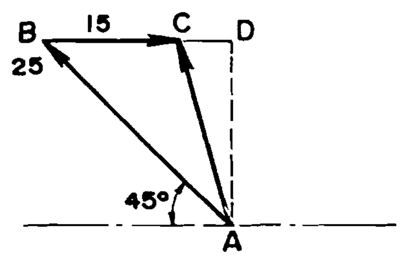
Thus:

$$\sum F_x = \rho Q[-25\cos 45 - 25]$$
$$= 1000 \times 0.0491[-17.68 - 25] = -2095.59\ N$$
$$\sum F_y = \rho Q[V_r \cos 45 - o]$$
$$= 1000 \times 0.0491[25\cos 45 - 0] = 868.09\ N$$

Therefore, the force components exerted by the fluid on the vane will be:

$$F_x = +2095.59\ N$$
$$F_y = -868.09\ N$$

ii) The absolute velocity of the jet when it leaves the vane at exit can be obtained by combining the relative velocity with vane velocity vectorially as shown by the velocity triangle.



AB = relative velocity at exit = 25 m/s

BC = vane velocity = 15 m/s

AC = Absolute velocity of jet at exit.

Since AD = AB sin 45° = 25 x sin 45° = 17.68 m/s

and CD = BD - BC = AB cos 45 - 15 = 17.68 - 15

= 2.68 m/s

Then $AC = \sqrt{17.68^2 + 2.68^2} = \sqrt{319.76} = 17.88\ m/s$

i.e. The absolute velocity as the jet leaves the vane is 17.88 m/s

iii) Power developed will be

$$F_x \cdot V = \frac{2095.59 \times 15}{1000}$$
$$= 31.434\ kW$$

Note that no work is done by the force F_y since the vane is not moving in that direction.

6.2.2 Dynamic Force due to Flow Around a Bend

Flow in a pipe bend, in a vertical or horizontal plane, and with or without change in diameter, experiences change in momentum. As a result of this change in momentum, a dynamic force is exerted by the fluid on the bend which has to be resisted by a thrust block or other suitable means. The force could be evaluated by a simple application of the momentum equation to the fluid mass in the control volume between the entrance and exit of the bend.

Consider a reducing bend in a vertical plane shown in Figure 6.20 (a)

Figure 6.20 (b) shows the control volume and the forces acting on the fluid mass within the control volume. Assuming steady flow Q of a fluid of density ϱ the momentum equation applied in the x and y directions gives the force components R_x and R_y exerted by the bed on the fluid as follows:

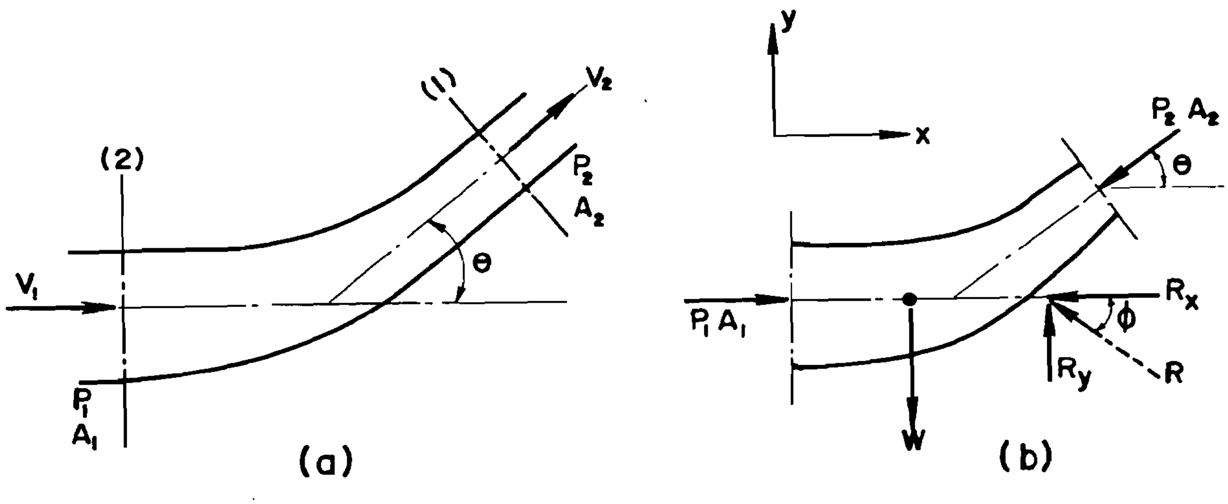


Figure 6.20 Force exerted on a bend

In the x-direction:

$$\sum F_x = P_1 A_1 - P_2 A_2 \cos\theta - R_x = \rho Q (v_2 \cos\theta - v_1)$$

$$\therefore \quad R_x = P_1 A_1 - P_2 A_2 \cos\theta - \rho Q (v_2 \cos\theta - v_1)$$

In the y-direction:

$$\sum F_y = R_y - W - P_2 A_2 \sin\theta = \rho Q (v_2 \sin\theta - 0)$$

$$\therefore \quad R_y = \rho Q v_2 \sin\theta + W + P_2 A_2 \sin\theta$$

Thus the resultant force R will be

$$R = \sqrt{R_x^2 + R_y^2}$$

$$\text{and} \quad \phi = \tan^{-1} \frac{R_y}{R_x}$$

However, for a horizontal bend, the weight W of the fluid mass between sections (1) and (2) of the bend will drop out of the momentum equation in the y-direction.

Example 6.13

The following data are given for a 60° reducer bend in a horizontal plane shown in Figure E 6.13. D_1 = 15 cm, D_2 = 10 cm, Q = 0.106 m^3/s and P_1 = 205.94 kN/m^2 . Assuming no loss as water flows from section (1) to section (2), determine the force required to hold the bend in place.

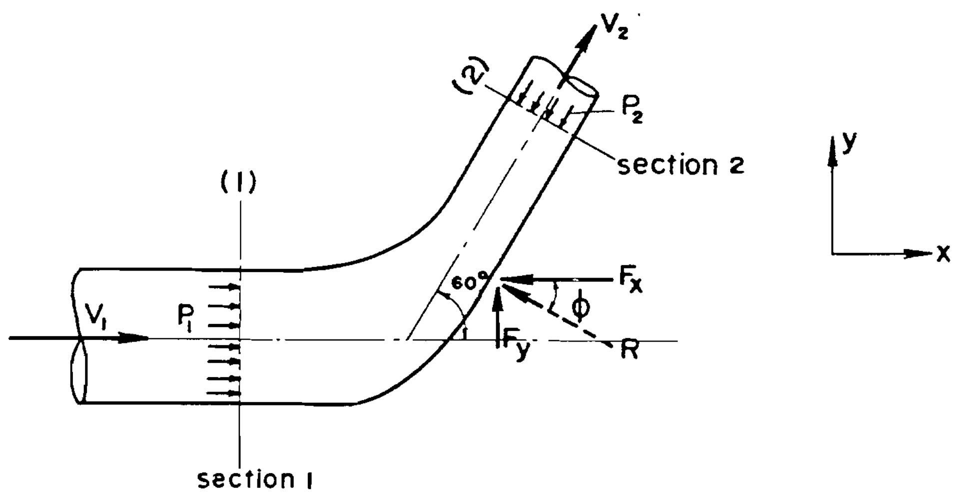


Figure E 6.13

Solution:

From the given data:

$$V_1 = \frac{Q}{A_1} = \frac{0.106 \times 4}{\pi \times (0.15)^2} = 5.998\ m/s\ ;\ \frac{V_1^2}{2g} = 1.834\ m$$

$$V_2 = \frac{Q}{A_2} = \frac{0.106 \times 4}{\pi (0.1)^2} = 13.496\ m/s\ ;\ \frac{V_2^2}{2g} = 9.284\ m$$

To determine the pressure at section (2) apply Bernoulli's equation between sections (1) and (2):

$$\frac{P_1}{\gamma_w} + \frac{v_1^2}{2g} + Z_1 = \frac{P_2}{\gamma_w} + \frac{v_2^2}{2g} + Z_2$$

$$\frac{205.94}{9.81} + 1.834 + 0 = \frac{P_2}{\gamma_w} + 9.284 + 0$$

$$\therefore \quad \frac{P_2}{\gamma_w} = 20.993 + 1.834 - 9.284 = 13.543 \ m \ of \ water$$

$$\therefore \quad P_2 = 13.543 \times \gamma_w = 132.857 \ kN/m^2$$

Applying the Momentum equation in the coordinate directions;

x-direction:

$$\sum F_x = P_1A_1 - P_2A_2\cos 60^o - F_x = \rho Q(v_2\cos 60^o - v_1)$$

$$i.e \quad F_x = P_1A_1 - P_2A_2\cos 60^o - \rho Q(v_2\cos 60^o - v_1)$$

$$P_1A_1 = 205.94 \times 10^3 \times \frac{\pi}{4}(0.15)^2 = 3639.26 \ N$$

$$P_2A_2 = 132.857 \times 10^3 \times \frac{\pi}{4}(0.1)^2 = 1043.46 \ N$$

$$\therefore \quad F_x = 3639.26 - 1043.46\cos 60^o$$

$$- 1000 \times 0.106(13.496\cos 60^o - 5.998)$$

$$= 3639.26 - 521.73 - 79.5 = 3038.03 \ N$$

y-direction:

$$\sum F_y = F_y - P_2 A_2 \sin 60^\circ = \rho Q(v_2 \sin 60^\circ - 0)$$

$$\therefore \quad F_y = P_2 A_2 \sin 60^\circ + \rho Q(v_2 \sin 60^\circ)$$

$$= 1043.46 \sin 60^\circ + 1000 \times 0.106(13.496 \sin 60^\circ)$$

$$= 903.66 + 1238.92 = 2142.58\ N$$

The force required to hold the bend in place is R

$$R = \sqrt{3038.03^2 + 2142.58^2} = 3717.56\ N$$

$$\phi = \tan^{-1} \frac{2142.58}{3038.03} = 35.19^\circ$$

6.2.4 Dynamic Force at a Nozzle

A nozzle, attached to a pipeline, and discharging to the atmosphere provides a good example of a rapid change in velocity. The fluid exerts a force on the nozzle and in accordance with Newton's third law there is a similar force, of opposite sign, exerted by the nozzle on the fluid. This is the force which the tension bolts holding the nozzle with the pipe must be designed to withstand.

Consider the nozzle shown in Figure 6.21 discharging a fluid of density ρ with a velocity v_2 to the atmosphere. The flow velocity at the entrance into the nozzle i.e. at section (1) is v_1.

Application of the momentum equation between upstream section (1) and downstream section (2) will yield the force R_x exerted by the nozzle on the fluid.

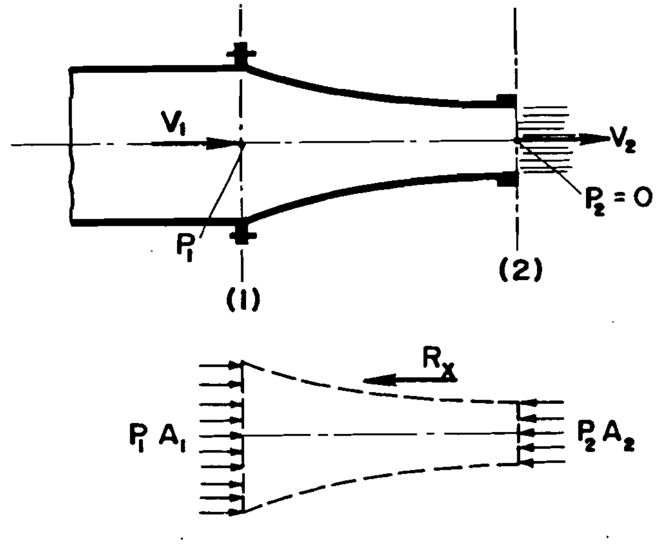


Figure 6.21 Nozzle discharging to atmosphere

In Figure 6.21 the component forces acting on the control volume are the pressure forces P_1 A_1 and P_2 A_2 and the force R_x exerted by the nozzle on the fluid. The rate of change of momentum is ρQ $(v_2 - v_1)$.

The momentum equation in the horizontal direction gives:

$$\sum F_x = P_1 A_1 - P_2 A_2 - R_x = \rho Q (v_2 - v_1)$$

Since the nozzle discharges to the atmosphere, $p_2 = 0$. Thus:

$$R_x = P_1 A_1 - \rho Q (v_2 - v_1) \tag{6.24}$$

Example 6.14

Calculate the tension force on the flanged connection between a 64 mm diameter pipe and a nozzle discharging a jet of water with velocity of 30 m/s and diameter of 19 mm.

Solution:

Let section (1) be the entrance and section (2) be the exit from the nozzle.

$$\text{The discharge } Q = \frac{\pi}{4} D_2^2 \, . \, v_2 = \frac{\pi}{4} (0.019)^2 \times 30 = 0.0085 \; m^3/s$$

From continuity, $D_1^2 v_1 = D_2^2 v_2$, so that:

$$v_1 = \left(\frac{D_2}{D_1}\right)^2 . \, v_2 = \left(\frac{19}{64}\right)^2 \times 30 = 2.65 \; m/s$$

Applying Bernoulli's equation between entry to and exit from the nozzle, and neglecting losses, one obtains:

$$\frac{p_1}{\gamma_w} + \frac{v_1^2}{2g} = \frac{v_2^2}{2g}$$

$$\text{Thus} \quad p_1 = \frac{\gamma_w}{2g}\left(v_2^2 - v_1^2\right) = \frac{9.81}{2 \times 9.81}(30^2 - 2.65^2) = 4446.5 \; kN/m^2$$

Substituting in Equation 6.24,

$$R_x = 446.5 \times 10^3 \times \frac{\pi}{4} (0.064)^2 - 10^3 \times 0.0085 (30 - 2.65)$$

$$= 1436.39 - 232.48 = 1203.9 \; N = 1.204 \; kN$$

Thus, the tension force on the flanged connection is 1.204 kN.

6.2.5 Force Exerted on a Sluice Gate.

Water issues at relatively high velocity from the opening caused by the raising of a sluice gate such as the one shown in Figure 6.22. The flow behavior resembles that of a jet issuing from an orifice. The difference is, however, in that the presence of the bed prevents the pressure inside the issuing jet downstream from a sluice gate from becoming atmospheric throughout. The pressure distribution a short distance from the opening, i.e at section (2), may be approximated to be hydrostatic.

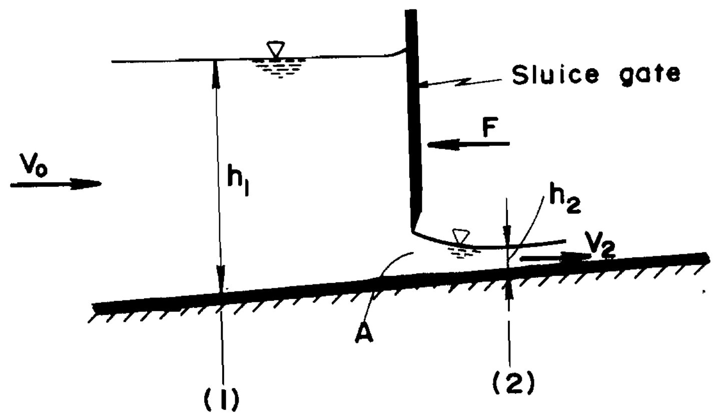


Figure 6.22 Flow under a vertical sluice gate

Application of Bernoulli's equation between section (1) and (2) gives the discharge as:

$$Q = CA\sqrt{h_1} \qquad (6.25)$$

Where, C is an overall coefficient of discharge incorporating the coefficient of contraction, the effects of downstream head, velocity of approach and energy loss, and A is the area of the opening of the gate.

Since there is a change in velocity between section (1) and (2), there is a change in momentum leading to a force F exerted on the gate. The momentum equation may be conveniently applied to the control volume of fluid between sections (1) and (2).

Assuming hydrostatic pressure distribution at sections (1) and (2) and that the gate is installed in a wide rectangular channel where the discharge per unit width is q, the forces acting on the control volume are:

- The hydrostatic force per unit width at section (1)

$$= \gamma_w \cdot \frac{h_1^2}{2}$$

- The hydrostatic force per unit width at section (2)

$$= \gamma_w \cdot \frac{h_2^2}{2}$$

- The force F per unit width of channel exerted by the gate on the fluid.

The change in momentum between section (1) and (2) per unit width of channel is $\rho q(v_2 - v_1)$.

Thus:

$$\sum F_x = \gamma_w \frac{h_1^2}{2} - \gamma_w \frac{h_2^2}{2} - F = \rho q(v_2 - v_1), \quad \textit{so that:}$$

$$F = \frac{\gamma_w}{2}(h_1^2 - h_2^2) - \rho q(v_2 - v_1) \tag{6.26}$$

The force exerted by the fluid on the gate is equal and opposite to the force F shown in Figure 6.22.

Example 6.15

The sluice gate in Figure 6.22 spans a wide rectangular channel and is raised 0.25 m above the channel floor. The upstream depth h_1 is 3 m. Estimate the horizontal force per metre width of channel exerted on the gate. Take C = 2.5 in Equation 6.25.

Solution:

The discharge per unit width is q.

$$q = 2.5 \times 0.25 \times 3^{1/2} = 1.08\ m^3/s$$

Since $q = v_1 . h_1, \quad v_1 = q/h_1 = 1.08/3 = 0.36\ m/s$

Applying Bernoulli's equation between (1) and (2):

$$3 + \frac{v_1^2}{2g} = h_2 + \left(\frac{1.08}{h_2}\right)^2 \frac{1}{2g}$$

from which, $h_2 = 0.14\ m$

Thus $v_2 = 1.08/0.14 = 7.71\ m/s$

Substituting in Equation 6.26

$$F = \frac{9810}{2}(3^2 - 0.14^2) - 1000 \times 1.08\,(7.71 - 0.36)$$

$$= 44048 - 7938 = 36110\ N = 36.11\ kN$$

EXERCISE PROBLEMS

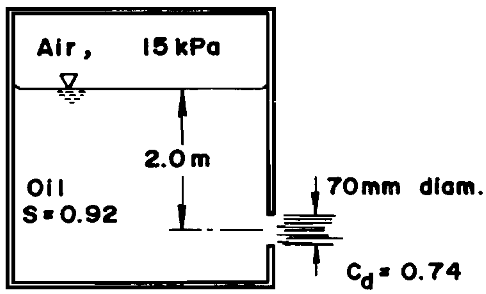
6.1 A venture meter installed in a horizontal water main has a throat diameter of 75 mm and a pipe diameter of 150 mm. The coefficient of discharge is 0.97. Calculate the flow rate if the difference of level in a mercury U-tube gauge connected to the throat and full bore tappings is 178 mm, the mercury being in contact with the water.
(Ans. 0.0292 m^3/s)

6.2 A 200 mm diameter water pipe has a venturi meter of throat diameter 12.5 cm, which is connected to a mercury manometer showing a gauge level difference of 87.8 cm. Find the velocity in the throat and the discharge. If the upstream pressure is 60 kN/m, what power would be given up by the water if it were allowed to discharge to atmospheric pressure?

6.3 A vertical cylinrical tank, 0.6 m in diameter and 1.5 m high, has an orifice of 25 mm diameter in the bottom. The discharge coefficient is 0.61. If the tank is originally full of water, what time is required to lower the level by 0.9 m? (Ans. 192 sec.)

6.4 Determine the equation of trajectory of a jet of water discharging horizontally from a small orifice with head of 5.0 m and a velocity coefficient of 0.96. Neglect air resistance.

6.5 Compute the discharge from the tank shown in Fig. P 6.5.
(Ans. 0.0241 m^3/s)



P 6.5

6.6 A 90° v-notch and a rectangular weir and placed in series. The length of the rectangular weir is 0.6 m and its coefficient is 1.81. If the discharge coefficient of the v-notch is 0.61, what will be its working head when the head on the weir is 0.15 m?

6.7 A closed tank partially filled with water discharges through an orifice of 12.5 mm diameter and has a coefficient of discharge of 0.70. If air is pumped into the upper part of the tank, dtermine the pressure required to produce a discharge of 0.6 ℓ/s when the water surface is 0.90 m above the outlet. (Ans. 15.7 kN/m^2)

6.8 How long does it take to raise the water surface of the tank in Fig P 6.8 by 2.0 m? The left had surface is that of a large reservoir of constant water surface elevation.

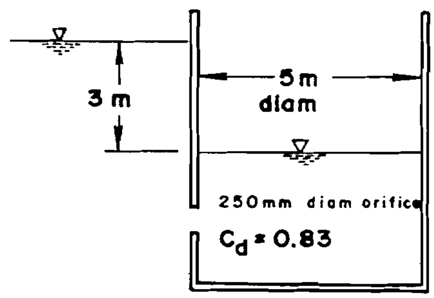


Fig. P 6.8

6.9 A jet of water, 6.5 cm^2 in cross-sectional area, moving at 12 m/s, it turned through an angle of 135° by a curved vane. The vane is moving at 4.5 m/s in the same direction as the jet. Neglecting any loss of velocity by shock or friction, find the amount of work done on the plate per sec. (Ans. 281 W)

6.10 Determine the head on a 60° v-notch for a discharge of 170 ℓ/s.

6.11 A 100 mm diameter orifice disarges 44.6 ℓ/s of water under a head of 2.75 m. A flat plate held normal to the jet just downstream from the vena contracta requires a force of 320 N to resist impact of the jet. Find C_d, C_v and C_c.
(Ans. C_d = 0.733, C_v = 0.977, C_c = 0.791)

6.12 Calculate the magnitude and direction of the vertical and horizontal componenets and the total force exerted on the stationary vane, Fig. P 6.12, by a 50 mm jet of water moving at 15 m/s.

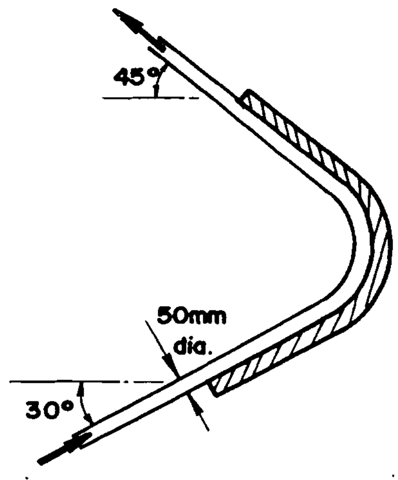


Fig. P 6.12

6.13 The blade shown in Fig. P 6.13 is one of a series. Calculate the force exerted by the jet on the blade system.
(Ans: 2651 N)

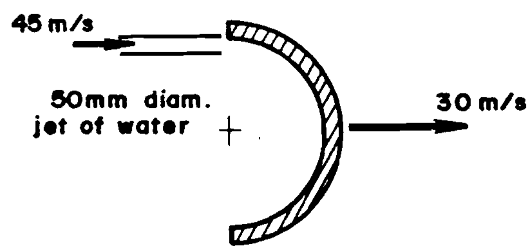


Fig. P 6.13

6.14 Calculate the magnitude and direction of the horizontal and vertical components of the force exerted by the flowing water on the 'flip bucket' AB. Assume that the water between sections A and B weighs 2.70 kN and that downstream from B, the moving fluid may be considered to be a free jet. (Fig. P 6.14).

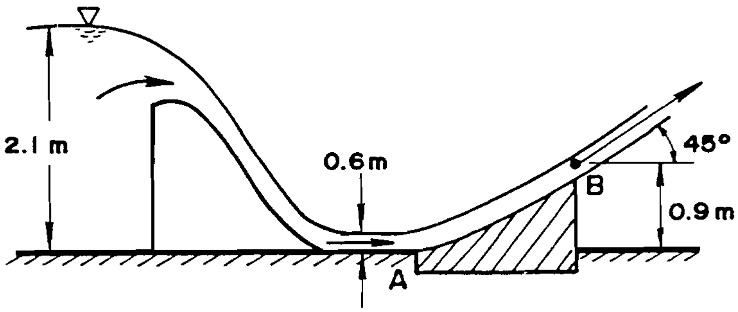


Fig. P 6.14

6.15 When 300 ℓ/s of water flows through the vertical 300 mm by 200 mm pipe reducer bend, the pressure at the entrance is 70 kPa. Calculate the force by the fluid on the bend if the volume of the bend is 0.85 m^3.

Ans: $F_x = 8172$ N

$F_y = 4218$ N

$\theta = 27.3°$

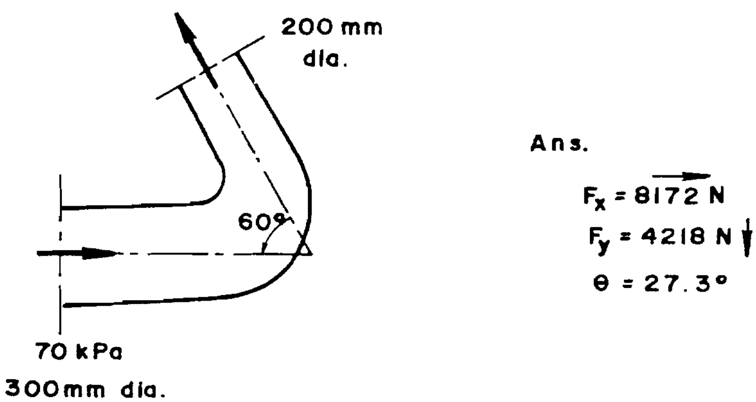


Fig. P 6.15

6.16 The plate is Fig. P 6.16 covers the 125 mm diameter hole. What is the maximum H that can be maintained without leaking?

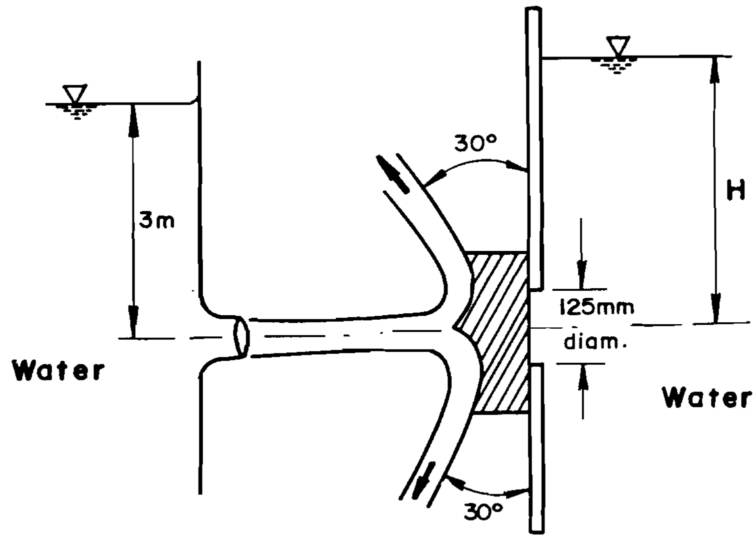


Fig. P 6.16

6.17 For the weir shown in Fig. P 6.17, determine the magnitude and direction of the horizontal component of the force on the structure.
(Ans: 18.937 kN/m) in the downstream direction)

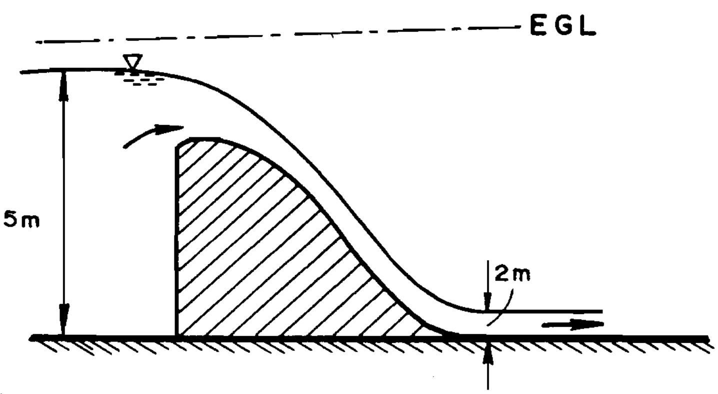


Fig. P 6.17

6.18 The jet of water of 50 mm diameter moving at 30 m/s is divided in half by a "splitter" on the stationary flat plate (Fig. P 6.18). Calculate the magnitude and direction of the force on the plate. Assume that the flow is in a horizontal plane.

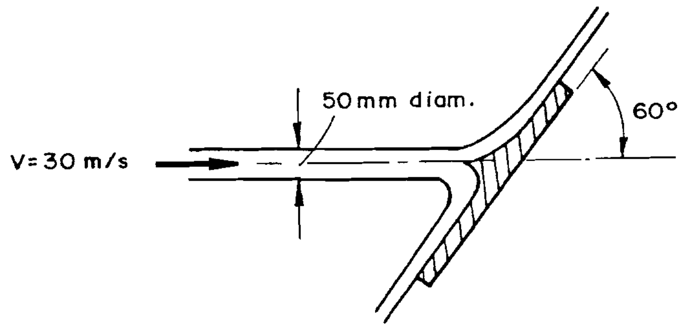


Fig. P 6.18

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