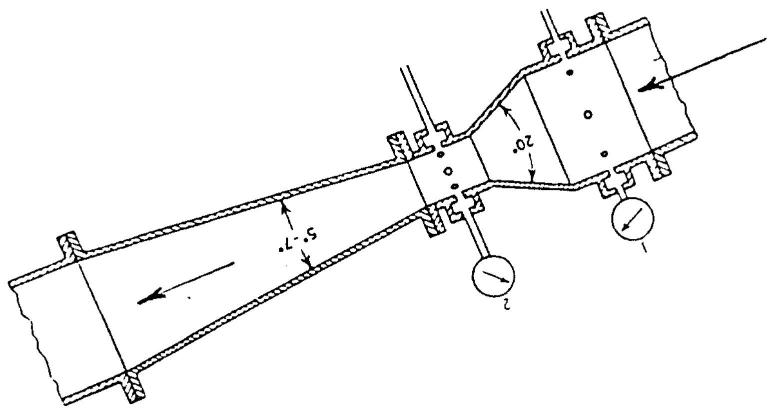
ESSENTIALS OF HYDRAULICS
(PART I)



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By

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CHAPTER 1

INTRODUCTION

1.1 Definition of a Fluid

Matter is recognized to exist in everyday life in three states: solid, liquid and gas. Liquids and gases are called fluids since they are characterized by their ability to flow. The existence of matter in these states is governed by the spacing between different molecules and the intermolecular attractive forces. The molecules in the solid state are spaced very closely and the intermolecular attractive forces are very strong thus imparting to solids the property of compactness and rigidity of form. On the other hand, as a result of weaker intermolecular attractive forces, liquid molecules can move more freely within the liquid mass and consequently liquids do not possess any rigidity of form but take the shape of the container in which they are kept. However, a definite mass of a liquid occupies a definite volume. The intermolecular forces are extremely weak in gases and the molecules are so farther apart spaced that gases do not have a definite volume like liquids and solids. Consequently a given mass of gas fills the container in which it is placed regardless of the size of the container. A liquid offers greater resistance to volumetric change (compression) and is not greatly affected by temperature changes. A gas, on the other hand, is easily compressible and responds markedly to temperature changes.

It is more appropriate to classify matter as fluids and solids on the basis of its response to the application of external forces. On this basis, a fluid may be defined as a substance which deforms continuously under the action of shear forces, however small these forces may be. This implies that if a fluid is at rest there can be no shearing forces acting. This property distinguishes fluids from solids. Solids acquire an equilibrium deformation corresponding to an internal stress that develops to just balance the applied external stress. Liquids do not acquire an equilibrium distortion but continue to deform as long as the stress acts.

1.2. The subject Matter of Hydraulics:

The branch of mechanics which treats the equilibrium and motion of liquids and gases and the force interactions between them and the bodies through or around which they flow is called hydromechanics or fluid mechanics. Hydraulics is an applied division of fluid mechanics governing a specific range of engineering problems and methods of their solution.

The principal concern of hydraulics is the study of fluids at rest and fluid flow constrained by surrounding surfaces, i.e., flow in open and closed channels and conduits, including rivers, canals and flumes, as well as pipes, nozzles and hydraulic machines with internal flow of fluids. It investigates what might be called "internal" problems, as distinct from "external" problems involving the flow of continuous medium about submerged bodies as in the case of a solid body moving in water or in the air. These "external" problems are treated in hydrodynamics and aerodynamics in connection with ship and aircraft design.

The science of hydraulics concerns itself mainly with the motion of liquids. However, under certain conditions, the laws of motion of liquids and gasses are practically identical as for example in the study of internal flows of gases with velocities much lower than that of sound in which case their compressibility can be disregarded. Hydraulics provides the methods of designing a wide range of hydraulic structures (dams, canals weirs pipelines etc), machinery (pumps, turbines, fluid couplings) and other devices in many branches of engineering.

Fluid flow problems in hydraulics are investigated by first simplifying and idealizing the phenomenon under investigation and applying the laws of theoretical mechanics. The results are then compared with experimental data, discrepancies are established and the theoretical formulae and solutions adjusted so as to make them suitable for practical application. Some phenomena are so involved as to defy theoretical analysis and are investigated in hydraulics on the sole basis of experimental measurement. Thus, hydraulics can be called a semi-empirical science.

1.3 Dimensions and Units of Measurement:

Physical quantities such as displacement, velocity, force etc are represented by dimensions. A unit is a particular way of describing the magnitude of a dimension. Thus length is a dimension associated with variables such as distance, displacement, width, deflection and height while centimeters and inches are both units used to describe the magnitude of the dimension length.

The dimensions length {L}, time {T}, mass {M} and force {F} are of fundamental interest in fluid mechanics. The dimensions of other, derived, physical quantities may be established by applying the above dimensions to the definition of the physical quantity under consideration as follows:

Physical quantity	Definition	Derived dimension
Velocity	Displacent/Time	$\{L\}/\{T\} = LT^{-1}$
Acceleration	Velocity/Time	$LT^{-1}/T = LT^{-2}$
Force	Mass x Acceleration	$M.LT^{-2}$
Mass	Force ÷ Acceleration	$F.L^{-1}T^{2}$

From the four fundamental dimensions given earlier only three need be selected as basic since force and mass are related through Newton's Second Law of motion. Thus if one is chosen as a fundamental dimension in any consistent dimensional system the other becomes a derived dimension. According to the choice made, two systems of measurement result. These are the force (or gravitational) system in which the basic dimensions are Force, Length and Time and the Mass (or Absolute) system in which the basic dimensions are Mass, Length and Time.

Thus the FPS (British) system is a force (gravitational) system while the MKS (metric) system is a mass (Absolute) system of measurement.

The S.I. System

The Absolute Metric System of units in which kilogram is the unit of mass, meter is the unit of length and second is the unit of time, forms the basis of an internationally agreed system of units - the Systeme Internationale d' unites - designated SI, which is now being adopted by almost all countries.

The basic SI units are the following		
Quantity	SI Units	Abbreviations in SI
Length	Meter	m
Mass	Kilogram	kg
Time	Second	s
Electric current	Ampere	A
Temperature	Kelvin	^{o}K
Luminous intensity	Candela	Cd.
Amount of substance	Mole	Mol.
Plane angle	radian	rad.

The following are the derived units of interest in fluid mechanics:

Quantity	SI Units	Abbreviation
Force	Newton	N {1N = 1kg m/s^2}
Pressure (stress)	Pascal	P_a {1P_a = 1 N/m^2}
Work, energy, quantity of heat	Joule	J {1J = 1 N.m}
Power	Watt	W {1W = 1 J/s}
Dynamic viscosity	Pascal-second	P_a.s
Kinematic viscosity	Squaremetre per second	m^2/s
Surface Tension	Newtons per meter	N/m
Momentum	Kilogram x meter/second	Kg.m/s

Regarding the unit of force, 1N is the force required to give a 1 kg mass an acceleration of 1 m/s^2. Hence 1N = 1 kg m/s^2. Since W = mg, the weight or the force of gravity of 1 kg mass is 1 kg 9.806 m/s^2 = 9.806 kg m/s^2 = 9.806 N. Standard acceleration due to gravity is 9.806 m/s^2.

Abbreviations of SI units are written in small letters eg. hours (h), meters (m). When a unit is named after a person, the abbreviation (but not the spelled form) is capitalized, examples are watt (W), pascal (P_a), newton (N). Common prefixes are shown below:

Multiple	SI Prefix	Abbre-viation	Multiple	SI Prefix	Abbre-viation
10^9	giga	G	10^{-3}	milli	m
10^6	mega	M	10^{-6}	micro	μ
10^3	kilo	K	10^{-9}	nano	n
10^{-2}	centi	C	10^{-12}	pico	p

CHAPTER 2
PROPERTIES OF FLUIDS

2.1. Introduction:

The properties of fluids vary from fluid to fluid and have a decisive influence on the motion of a fluid. Thus, it is not necessary to deal with each fluid separately while studying fluid motion. One needs to study only the variation of these properties and the manner in which they influence the fluid motion. This chapter discuss fluid properties and their significance.

2.2 Density, Specific Weight, Specific Volume and Specific Gravity

Density of a fluid, designated by the symbol ϱ (Rho), is probably the most important property. It is defined as the fluid mass per unit volume. In the S.I. system density is expressed in kg/m^3. Generally the density of a fluid is dependent on temperature and pressure. For water at 4°c and standard pressure (i.e. 760 mm of mercury), ϱ = 1000 kg/m^3.

Specific Weight (or Unit Weight) is defined as the weight of fluid per unit volume. It is designated by γ (Gama). It could also be seen as representing the force exerted by gravity on a unit volume of fluid. The unit of specific weight in the SI system is N/m^3. Density and specific weight may be related as follows:

$$Since\ \gamma = \frac{W}{V} = \frac{mg}{V}, \quad then\ \gamma = \rho g.$$

The specific weight of water at 4°c is 9810 N/m^3.

Specific Volume V is the volume of the fluid per unit weight. It is the reciprocal of specific weight so that $V = 1/\gamma$ with units of m^3/N. It is a property commonly used in gas flow problems.

Specific Gravity S (also known as relative density) is the ratio of the mass of a fluid to the mass of an equal volume of pure water at standard temperature and pressure. It may also be defined as the density of the fluid to the density of pure water at standard conditions. As a ratio, specific gravity is dimensionless. The specific gravity of pure water is unity while that of mercury is about 13.60.

2.3 Pressure, Compressibility, Viscosity:

Pressure: The normal force exerted against a plane area divided by the area is the average pressure on the area. Fluids exert pressure on the walls and the bottom of containers in which they are stored. If ΔF is the force exerted over an area ΔA, then the pressure P is given by:

$$P = \frac{\lim}{\Delta A \rightarrow 0} = \frac{\Delta F}{\Delta A}$$

Pressure P has the dimension force per unit area. In the SI system, the unit of P is N/m^2 which is called pascal (P_a). Pressure is also expressed in bars, where 1 bar = 100,000 N/m^2.

Compressibility: All fluids may be compressed by the application of pressure, elastic energy being stored in the process. As a result of compressibility, fluid density changes with pressure. Gases are highly compressible and hence are treated as such. In liquids, the change in density (and therefore in volume) is very small even under large pressure changes. Therefore, liquids are ordinarily considered as incompressible. But in special problems such as Water Hammer involving sudden or great changes in pressure, the compressibility of the liquid becomes important and should be taken into consideration.

The compressibility of a fluid is expressed by defining a modulus of elasticity as in done for solids. But since fluids do not possess rigidity of form, the modulus of elasticity must be defined on the basis of volume; such a modulus being termed Bulk Modulus of Elasticity K.

In order to define the Bulk Modulus of Elasticity, consider a compressible fluid in a cylinder of cross-sectional area A, which is being compressed by a piston as shown in Figure 2.1. The cylinder and the piston are considered rigid.

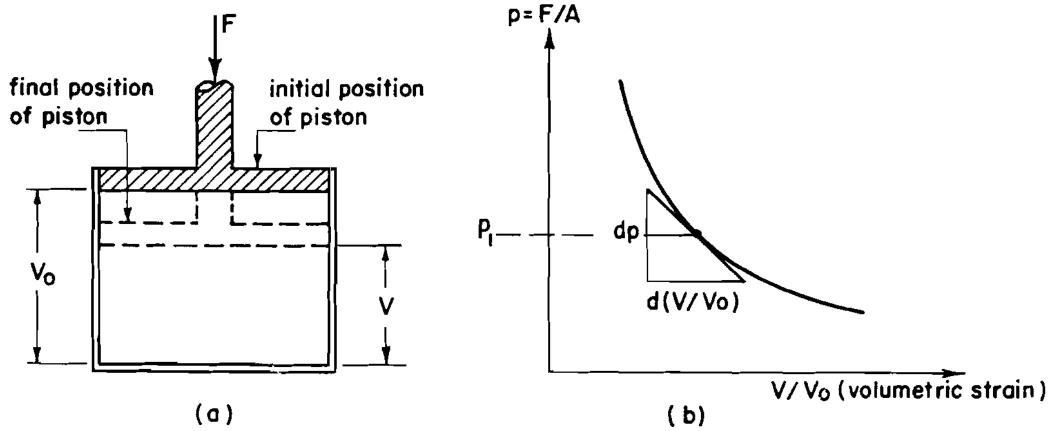


Figure 2.1

Let the original volume of the fluid be V_o. The application of a force F results in the pressure P = F/A exerted on the fluid. This pressure reduces the fluid volume to V. A plot of V/V_o (which is a measure of volumetric strain) against the pressure P results in a curve of negative slope as shown in Figure 1(b) . The Bulk Modulus of Elasticity K of the fluid corresponding to a pressure P_1 is defined as:

$$K = -\frac{dP}{d(v/V_o)} = -\frac{dP}{dv/V_o} \qquad (2.1)$$

The negative sign indicates the decrease in dv/V_o with increase in pressure. Since dv/V_o is dimensionless, the dimension of K is the same as that of the pressure P. Water has an average value of K = 2.1 GPa. This shows that water is about 100 times more compressible than steel, but it is ordinarily considered incompressible.

Table 2.1 Bulk Modulus of Elasticity of Water
K (GPa)

Pressure MPa	Temperature			
	0° C	10° C	20° C	50° C
0.1 - 2.5	1.93	2.03	2.07	
2.5 - 5.0	1.96	2.06	2.13	
5.0 - 7.5	1.99	2.14	2.23	
7.5 - 10.0	2.02	2.16	2.24	
10.0 - 50.0	2.13	2.27	2.34	2.43
50.0 - 100.0	2.43	2.57	2.67	2.77
100 - 150	2.84	2.91	3.00	3.11

Example 2.1. What pressure increase is required to reduce the volume of 100 c.c of water by 0.5%? K = 2.1 GPa

Solution:

$$K = \frac{-dP}{\frac{dV}{V_o}} = \frac{-dP}{(-0.5/100)} = \frac{100\,dP}{0.5} = 200\,dP$$

$$Thus \quad dP = \frac{K}{200} = \frac{2.1 \times 10^9}{200} P_a = 1.05 \times 10^7 P_a$$

Since 1 atmosphere ≈ 1 x 10^5 Pa, the required increase in pressure is 105 atmospheres. This is an extremely high pressure which is required to produce a volume change of only 0.5%. Hence the reasonableness of the assumption that liquids are practically incompressible under ordinary changes in pressure.

Viscosity: Viscosity is one of the most important physical properties of fluids. It is a measure of the resistance of a fluid to relative motion such as shear and angular deformation within the fluid. Viscosity is due to interchange of molecules between adjacent layers of fluids moving at different velocities and also to the cohesion between fluid particles. Viscosity plays a decisive role in laminar flow and fluid motion near solid boundaries.

The relationship between viscous shear stress and viscosity is expressed by Newton's law of viscosity. Consider a fluid confined between two plates separated by a small distance y as shown in Figure 2.2. The lower plate is stationary while the upper plate is moved with a velocity v.

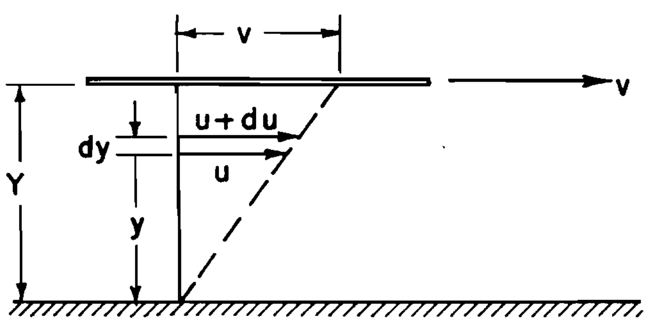


Figure 2.2

Since there will not be slippage between the plates and the fluid, particles of fluid in contact with each plate will adhere to it i.e. fluid particles in contact with the moving plate will have a velocity V while those in contact with the stationary plate will have zero velocity. The effect is as if the fluid were made up of a series of thin, parallel layers each moving a little faster relative to the adjacent, lower layer.

For a large number of fluids, the shear stress developed between adjacent layers of fluid is found to be directly proportional to the rate of change of velocity with respect to Y, which is the velocity gradient. For a layer of thickness dy at a distance y from the stationary plate this becomes;

$$\tau \propto \frac{du}{dy}$$

Introducing a constant of proportionality μ, one obtains:

$$\tau = \mu \frac{du}{dy} \qquad (2.2)$$

The proportionality constant μ expresses the property of the particular fluid and is called dynamic viscosity. Equation 2.2 is called Newton's Law of viscosity.

Fluids may be classified on the basis of the relationship between the shear stress τ and the rate of deformation (velocity gradient) as shown in Figure 2.3.

Fluids may be classified as Newtonian and non-Newtonian. In Newtonian fluids there is a linear relation between the magnitude of the applied shear stress τ and the resulting rate of deformation i.e. μ is constant. In non-Newtonian fluids there is a non-linear relation between the applied shear stress and the rate of angular deformation. An ideal plastic has a

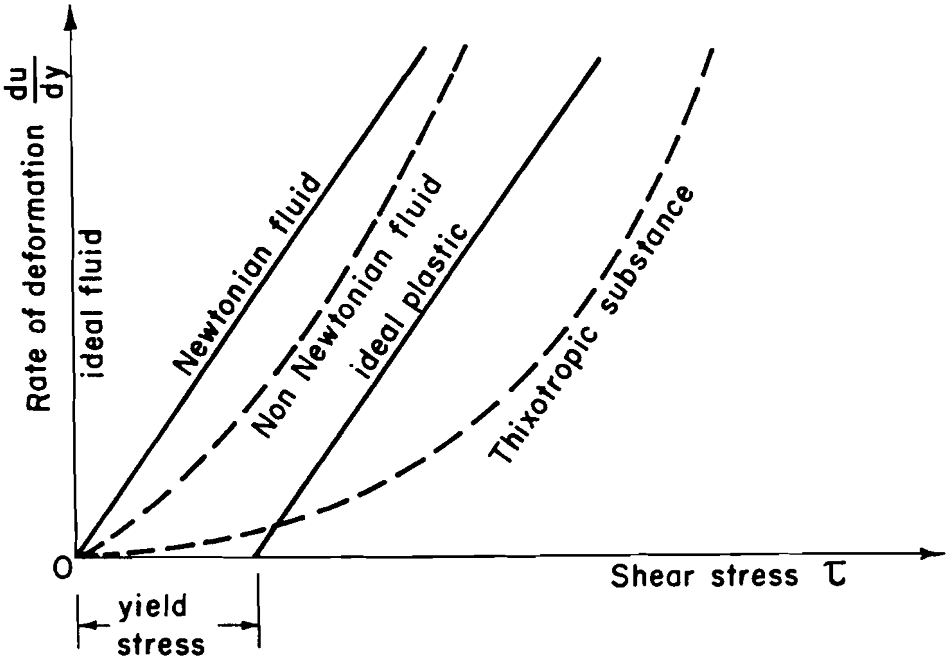


Figure 2.3 Rheological diagram

definite yield stress and a constant linear relation between τ and du/dy thereafter. A thixotropic substance, such as printer's ink, has a viscosity that is dependent upon the prior angular deformation of the substance and has a tendency of setting when at rest. Gases and thin liquids such as water, kerosene, glycerin etc are Newtonian fluids.

The viscosity of a fluid is a function of temperature. Since the viscosity of liquids is governed by cohesive forces between the molecules, it decreases with increase in temperature. In gases, however, molecular momentum transfer plays a dominant role in viscosity and as a result the viscosity of gases increases with increase in temperature.

The unit of dynamic viscosity μ is Ns/m^2 or kg/ms. A smaller unit of dynamic viscosity is called the poise. 1 poise = 1 gm/cm.s. Thus 1 poise = 0.1 kg/ms.

Kinematic viscosity (υ) is the ratio of dynamic viscosity to density. $\upsilon = \mu/\varrho$ and has units of m^2/s. A smaller unit of

υ is the stoke. 1 stoke = 1 cm^2/s. Thus 1 stoke = 1 x 10^{-4} m^2/s.

Table 2.2 Dynamic and Kinematic Viscosities of Water and Carbon Tetrachloride.

Temp ^oC	Water			Carbon Tetra Chloride		
	ρ (kg/m^3)	μ (10^{-4}P$_a$s)	υ (10^{6}m^2/s)	ρ (kg/m^3)	μ (10^{-4}P$_a$s)	υ (10^{-6}m^2/s)
0	999.8	17.53	1.75	1633	13.46	0.824
10	999.70	13.00	1.30	1613	11.34	0.703
20	998.21	10.02	1.004	1594	9.708	0.609
30	995.7	7.972	0.801	1575	8.418	0.534
40	992.2	6.514	0.657	1555	7.379	0.475
50	989.0	5.542	0.555	1535	6.529	0.425

2.4. Surface Tension, Capillarity and Vapour Pressure

These are strictly liquid properties.

Surface Tension and Capillarity are due to properties called cohesion and adhesion. Cohesion is the property as a result of which molecules of a liquid stick to each other whereas adhesion is the property that enables liquids to stick or adhere to another body. As a result of cohesion an imaginary film capable of resisting some tension is created at a free liquid surface. The liquid property that creates this capability is called surface tension. It is because of surface tension that a small pin placed gently on water surface will not sink but remain floating being supported by the tension at the water surface. The spherical shape of water drops is also due to surface tension. Surface tension force, designated by σ, is defined as force per unit length and has the unit N/m.

For water in contact with air δ varies from about 0.074 N/m at 0°C to 0.059 N/m at 100°C. Surface tension force is so small that it is neglected in ordinary hydraulic problems. It is, however, a factor to be taken into consideration in flows at small depths that occur in model studies. *Capillarity* is due to both adhesion and cohesion. If a glass tube of small diameter and open at both ends is dipped in a container of water, the water rises in the tube to some height above the level of water in the container (Figure 2.4 a). If the same tube is dipped in a container of mercury, the level inside the tube will be lower than that in the container (Figure 2.4 b). In the former, adhesion of water to glass is predominant in comparison to the cohesion whereas in the latter cohesion between mercury molecules is predominant.

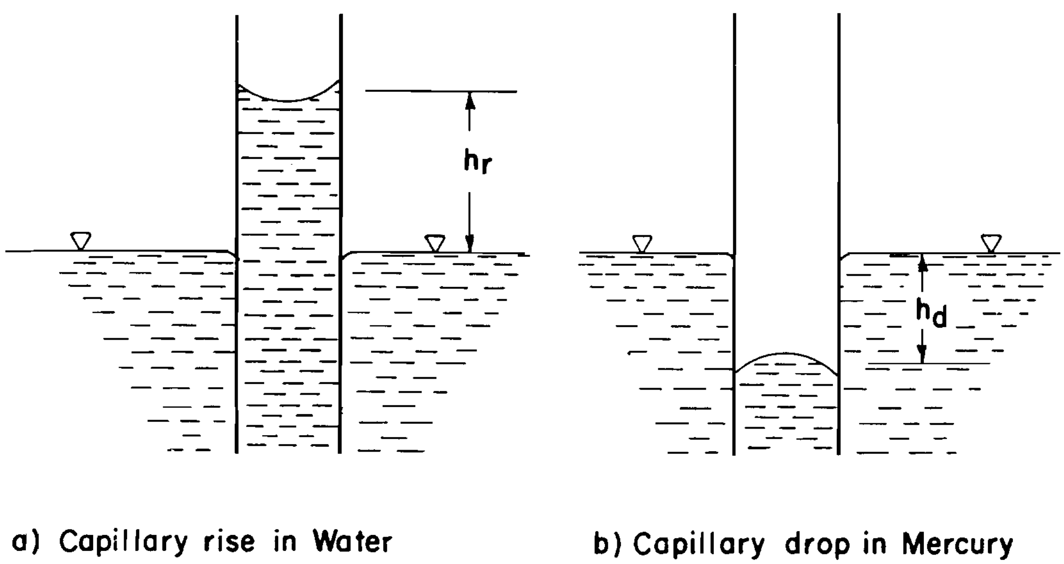


Figure 2.4 Capillarity Effects

Capillary rise or Capillary drop can be estimated by considering the equilibrium of the liquid column of height h as follows:

Consider a tube of small internal diameter D = 2r dipped in a liquid of specific weight γ and surface tension force σ.

Let the liquid rise to a height h in the tube as a result of capillarity (Figure 2.5).

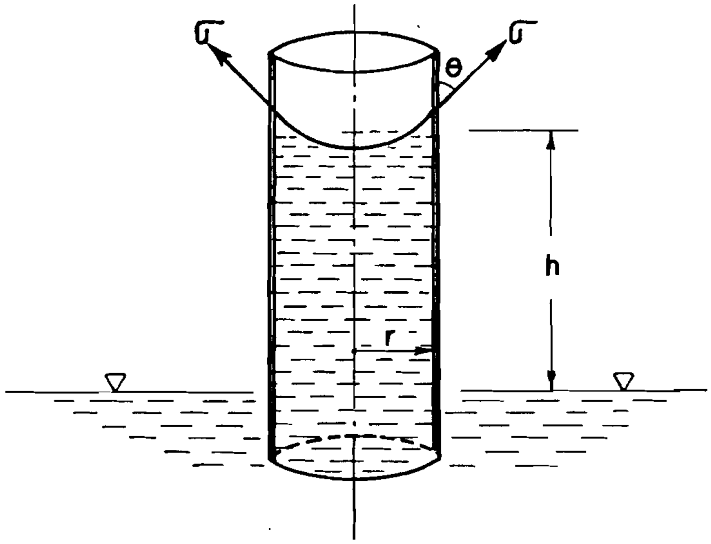


Figure 2.5

The liquid column of height h is supported by vertical component of the surface tension force at the liquid-air interface. For vertical equilibrium:

$$2\pi r.\sigma.\cos\theta = \pi r^2 h.\gamma$$

from which, $$h = \frac{2\sigma\cos\theta}{\gamma r} = \frac{4\sigma\cos\theta}{\gamma.D} \qquad (2.3)$$

This shows that the capillary rise is inversely proportional to the diameter of the tube. Hence for tubes of very small diameter, the capillary rise can be considerable. Therefore, to minimize the effect of capillarity, the tubes of manometers and piezometers should not be less that about 10 mm in diameter. If the surface is clean, the contact angle is zero for water and about 140° for mercury.

Table 2.3 Surface Tension of Water (N/m)

Temperature °C	0°	10°	20°	30°	40°	50°	60°
σ	0.0742	0.0728	0.0713	0.0698	0.0682	0.069	0.0661

Vapour Pressure: Liquids evaporate when their surface is exposed to the atmosphere. The nature of the liquid, its temperature and the prevailing atmospheric pressure determine the rate of evaporation. If a closed container is partially filled with a liquid maintained at a constant temperature, evaporation will take place and the vapor molecules accumulating in the space above the liquid exert a pressure called vapour pressure P_v on the liquid surface. In the process of evaporation, some of the vapour molecules get reabsorbed into the liquid. With the passage of time an equilibrium situation is established whereby the number of molecules released from the liquid become equal to the number of molecules reabsorbed and the vapour pressure becomes constant. This vapour pressure is called the saturation vapour pressure. Increase of temperature hastens evaporation because of increased molecular activity and consequently saturation vapour pressure is increased with temperature. A liquid having high vapour pressure evaporates more easily than a liquid with low vapour pressure. Thus Carbon Tetra Chloride, with a saturation vapour pressure of 1.275×10^4 N/m^2 at 20°C, evaporates easily compared to Mercury, which has a saturation vapour pressure of only 0.17 N/m^2 at 20°C. This is one of the reasons that make mercury an ideal liquid for barometers and manometers.

Boiling of a liquid will takes place, at any temperature, when the external absolute pressure impressed on a liquid surface is equal to or less than the saturation vapour pressure of the liquid. In liquid flow system, very low pressures may be produced at certain points in the system. If these pressures are less than or equal to the vapour pressure of the liquid, the liquid flashes into vapour creating vapour pockets. These vapour pockets collapse as they are swept into regions of higher pressure. This phenomenon is called *Caviation* and can result in damages of conduit walls and propeller runner blade tips where low pressures are likely to develop. Table 2.4 gives some values of saturation vapour pressure of water.

Table 2.4 Saturation Vapour Pressure of Water

Temperature ^{o}C	0	10	20	30	40	50
$P_v (x10^3\ N/m^2)$	0.6108	1.227	2.337	4.242	7.377	12.33

Example 2.2. An oil has a density of 850 kg/m^3 at 20°C. Find its specific gravity and Kinematic viscosity if the dynamic viscosity is 6 x 10^{-3} kg/ms.

Solution: Specific gravity of oil, $S_o = \varrho$ oil/ϱ water

$= 850/1000 = 0.85$

Kinematic viscosity, $\upsilon_o = \mu/\varrho = 6 \times 10^{-3} /850$

$= 7.06 \times 10^{-6}\ m^2/s.$

Example 2.3. The velocity distribution over a plate in a fluid (μ = 8.63 Poise) is given by $u = \frac{2}{3}y - y^2$ where

u is the velocity in m/s at a distance y metres above the plate. Determine the shear stress at the plate and at a distance of 0.15 metres from the plate.

Solution:

$$u = \frac{2}{3}y - y^2$$

$$\text{Therefore } du/dy = \frac{2}{3} - 2y$$

$$\text{Hence } du/dy = \frac{2}{3} \text{ at } y = 0$$

$$\text{and } du/dy = \frac{2}{3} - 0.30 = 0.367 \text{ at } y = 0.15\ m$$

$$\mu = 8.63 \text{ poise} = 0.863\ Ns/m^2$$

The shear stress $\tau = \mu\, du/dy$

Thus,

$$\text{at } y = 0,\ \tau = 0.863 \times \frac{2}{3} = 0.575\ N/m^2$$

$$\text{at } y = 0.15,\ \tau = 0.863 \times 0.367 = 0.317\ N/m^2$$

Example 2.4. What should be the diameter of a droplet of water in mm at 20°C if the pressure inside is to be 170 N/m^2 greater than the outside?

Solution:

Let the diameter of the droplet be d and the internal pressure be p. If the droplet is cut into two halves forces acting on one half will be those due to pressure intensity p on the area $\frac{\pi d^2}{4}$ and the force due to surface tension σ acting around the circumference πd. These two force will be equal and opposite under equilibrium condition.

$$p \times \frac{\pi d^2}{4} = \sigma . \pi d$$

$$\therefore \quad d = \frac{4\sigma}{p}$$

at $20^0 C$, $\sigma = 0.074\ N/m$

$$\text{Therefore } d = \frac{4 \times 0.074}{170} = 1.74 \times 10^{-3} m = 1.74\ mm$$

Example 2.5. The density of a certain oil at 20°C is 800 kg/m^3. Find its specific gravity and kinematic viscosity if the dynamic viscosity is 5×10^{-3} kg/m.s.

Solution:

Specific gravity, $s = \varrho$ of oil/ϱ of water
$= 800/10^3 = 0.80$

Kinematic viscosity, $\upsilon = \mu/\varrho$
$= 5 \times 10^{-3}/800 = 6.3 \times 10^{-6}\ m^2/s$

Example 2.6. The velocity destribution of a viscous fluid over a fixed boundary is given by $u = 0.72y-y^2$ in which u is the velocity in m/s at a distance y metres above the boundary surface. If the dynamic viscosity μ of the fluid is 0.92 Ns/m^2, determine the shear stress at the surface and at y = 0.36 m.

Solution:

$u = 0.72y-y^2$
$\therefore du/dy = 0.72-2y$
At the surface, $y = 0$ and $du/dy = 0.72\ s^{-1}$
At $y = 0.36$, $du/dy = 0.72-2(0.36) = 0$
Since $\tau = \mu\ du/dy$
At the surface, $\tau = 0.92 \times 0.72 = 0.662\ N/m^2$
At $y = 0.36$, $\tau = 0.92 \times 0 = 0$

Example 2.7. At a depth of 6.8 Km in the ocean the pressure is 72 MN/m^2. The specific weight of sea water at the surface is 10.2 KN/m^3 and its average bulk modulus is 2.4 x^{6}10 KN/m^2. Determine (a) The specific volume (b) The change in specific volume over the depth, and (c) the specific weight of sea water at 6.8 Km depth.

Solution:

change in pressure dp from surface to 6.8 Km depth

$= 72\ MN/m^2$

$= 7.2 \times 10^4\ KN/m^2$

$$\text{Bulk modulus,} \quad k = -\frac{dp}{dv/V}$$

$$\therefore \quad \frac{dv}{V} = -\frac{dp}{K} = 7.2 \times 10^4/2.4 \times 10^6 = 3 \times 10^{-}$$

a) Specific volume = volume per unit weight = 1/Y

$\therefore$ specific volume at the surface = 1/10.2 = $9.8 \times 10^{-2}\ m^3/KN$.

b) Change in specific volume between that at the surface and at 6.8 Km depth, $dv = 3 \times 10^{-2} \times 9.8 \times 10^{-2} = 29.4 \times 10^{-4}\ m^3/kN$

c) The specific voulme at 6.8 Km depth = $9.8 \times 10^{-2} - 29.4 \times 10^{-4}$

$= 9.51 \times 10^{-2}\ m^3/kN$

$\therefore$ The specific weight at 6.8 Km depth = 1/specific volume

$= 1/9.51 \times 10^{-2}$

$= 10.52\ kN/m^3$

Example 2.8. Calculate the capillary effect in mm in a glass tube of 6 mm diameter when immersed in (a) Water $\sigma = 73 \times 10^{-3}$ N/m and (b) Mercury, $\sigma = 0.5$ N/m. The contact angles for Water and Mercury are zero and 130° respectively. Take specific weights of water and mercury to be 9810 N/m^3 and $1.334 \times 10^5\ N/m^3$ respectively.

Solution:

Capillary rise (drop), $h = \dfrac{4\sigma\cos\alpha}{\gamma . d}$ **, where d is tube diameter.**

For Mercury: Capillary drop,

$$h = \frac{4 \times 0.5 \times \cos 130^o}{1.334 \times 10^5 \times 6 \times 10^{-3}} = -1.61 \times 10^{-3}\ m$$

$$= -1.61\ mm = 1.61\ mm\ depression$$

For Water: Capillary rise

$$h = \frac{4 \times 73 \times 10^{-3} \times \cos 0^o}{9810 \times 6 \times 10^{-3}} = 4.96 \times 10^{-3}\ m$$

$$= 4.96\ mm$$

Exercise Problems

1. A block of dimensions 300 mm x 300 mm x 300 mm and mass 30 kg slides down a plane inclined at 30^0 to the horizontal on which there is a thin film of oil of viscosity 2.3×10^{-3} Ns/m^2. Determine the speed of the block if the film thickness is 0.03 mm. (Ans. 21.3 m/s)

2. Calculate the capillary effect in mm in a glass tube of 6mm diameter when immersed in (i) water, and (ii) mercury, both liquids being at 20^0 C. Assume σ to be 73×10^{-3} N/m for water and 0.5 N/m for mercury. The contact angles for water and mercury are $0°$ and 130^0 respectively.

3. Calculate the internal pressure of a 25 mm diameter soap bubble if the tension in the soap film is 0.5 N/m. (Ans. 80 N/m^2)

4. A hydraulic ram 200 mm in diameter and 1.2 m long moves wholly within a concentric cylinder 100.2 mm in diameter, and the annular clearance is filled with oil of specific gravity 0.85 and kinematic viscosity 400 mm^2/s. What is the viscous force resisting the motion when the ram moves at 120 mm/s?

5. Eight kilometers below the surface of the ocean the pressure is 81.7 MN/m^2. Determine the specific weight of sea water at this depth if the specific weight at the surface is 10.06 kN/m^3 and the average bulk modulus of elasticity is 2.34 GN/m^2. Assume that g does not vary significantly. (Ans. 10.42 kN/m^3)

6. A one square metre then plate is dragged at a velocity of 3 m/s on the top of a 5mm deep liquid of dynamic viscosity 20 centipoises. Assuming linear velocity variation in the liquid, find the drag force.

7. If the velocity distribution over a plate is given by $u = \frac{3}{4}y - y^2$ where u is the velocity in m/s at distance y metres above the plate determine the shear stress at a distance of 0.15 m from the plate. Take the dynamic viscosity of the fluid as 0.834 Ns/m^2.
(Ans. 0.375 N/m^2)

8. The volume of a liquid is reduced by 1% by increasing the pressure from 5 atmospheres to 125 atmospheres. Estimate the modulus of elasticity of the liquid.

9. A sliding fit cylindrical body 14.9 cm in diameter and 15 cm long and having a 1 kg mass drops vertically at a constant velocity of 5 cm/s inside a cylinder with 15 cm inside diameter, the space between the body and the cylinder is filled with oil. Estimate the viscosity of the oil.
(Ans. u = 1.4 Ns/m^2)

CHAPTER 3

HYDROSTATICS

3.1 Introduction

Hydrostatics deals with the study of fluids that are at rest or are moving with uniform velocity as a solid body so that there is no relative motion between fluid elements. When there is no relative motion between fluid layers there is no shear stress in fluids at rest whatever the viscosity of the fluid. Hence only normal pressure forces are present in hydrostatics. Engineering applications of hydrostatic principles include the study of forces acting on submerged bodies such as gates, submarines, dams etc. and the analysis of stability of floating bodies such as ships, pontoons etc..

3.2 Pressure at a Point in a Static Fluid

In a fluid at rest, no tangential stresses can exist. The only forces between adjacent surfaces are pressure forces that are normal to the surfaces. Therefore the pressure at any point in a fluid at rest is the same in every direction. This is known as Pascal's Law. Pascal's Principle can be proved by considering a small wedge shaped fluid element at rest as shown in fig. 2.1. The thickness of the wedge perpendicular to the plane of the paper is dy.

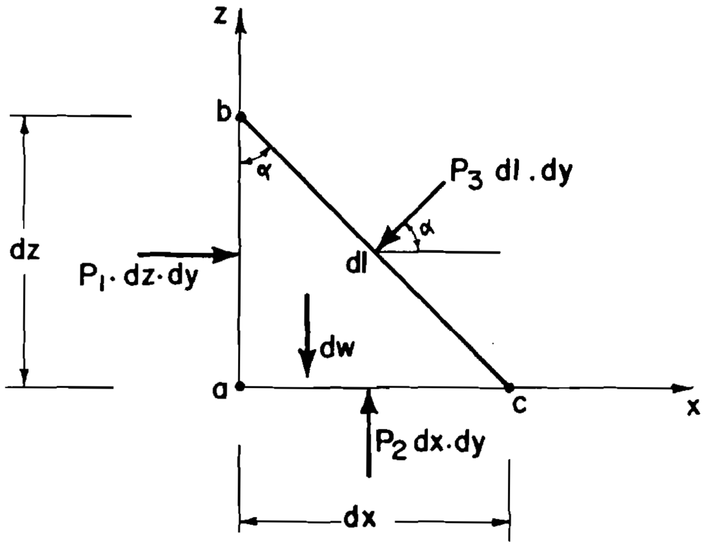


Figure 3.1 Free-body of a fluid wedge

Let P_1, P_2, and P_3 be the average pressures acting on the faces ab, ac and bc of the prism respectively. The weight of the fluid prism is $\frac{1}{2}\gamma$ dx dy dz where γ is the specific weight of the fluid.

Since the fluid prism is in equilbrium, the equations of equilibrium will be;

X direction: $P_1\ d_z\ d_y - P_3\ d\ell\ dy \cos\alpha = 0$

but $dz = d\ell \cos\alpha$

so that, $P_1\ d\ell \cos\alpha\ dy - P\ d\ell\ dy \cos\alpha = 0$

$\therefore\ P_1 = P_3$

Z direction: $P_2\ dx\ dy - P_3\ d\ell\ dy \sin\alpha - \gamma . \frac{dx.dy.dz}{2} = 0$

$dx = d\ell \sin\alpha$ and as dx, dy and dz all shrink to zero, the third term in the above equation becomes zero.

Thus $P_2 - P_3 = 0$

$\therefore\ P_2 = P_3$

Then $P_1 = P_2 = P_3$

This shown that the pressure at a point in a static fluid is the same in all directions.

3.3 Basic Equation of Hydrostatics

The basic equation of Hydrostatics may be derived by considering the infinitesimal fluid parallepiped in a static fluid shown in fig. 3.2. below.

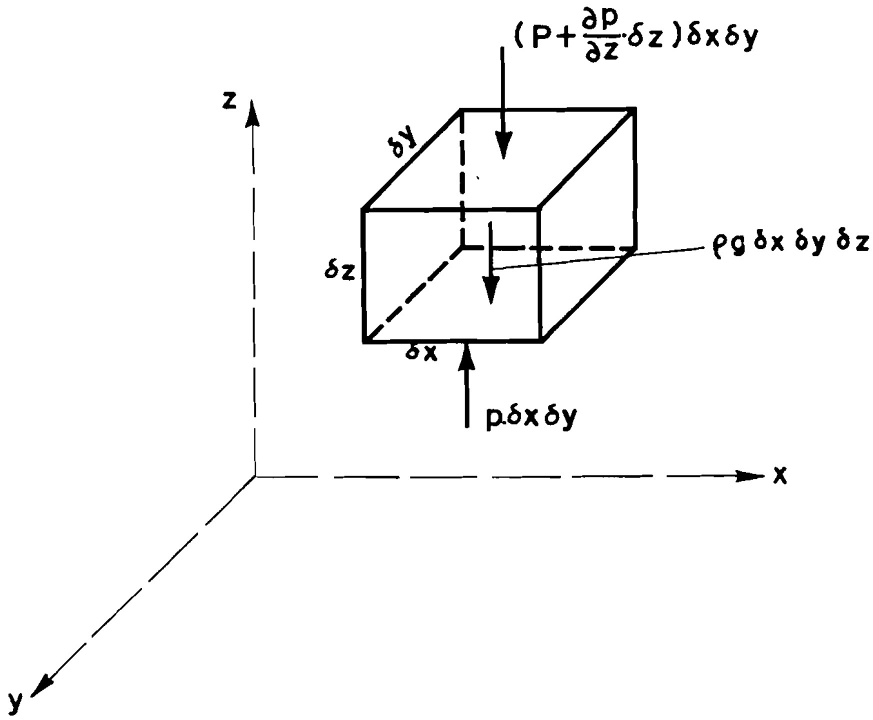


Figure 3.2 A rectangualr fluid parallelepiped

Assuming the density of the fluid ρ in the infinitesimal cube to be constant, the mass of the fluid is ρ.dx.dy.dz. Let the pressure variation in the x,y and z directions be $\frac{\partial p}{\partial x}$, $\frac{\partial p}{\partial y}$ and $\frac{\partial p}{\partial z}$ respectively and let the entire fluid mass be subjected to acceleration of a_x, a_y and a_z in the x, y and z directions respectively.

Considering the equilibrium in the vertical (Z) direction:

$$+p\ dx.dy - (P + \frac{\partial P}{\partial Z}\ dz)\ dx.dy - \rho g\ dx\ dy\ dz - a_z . \rho\, dxdydz = 0$$

which reduces to

$$\frac{\partial p}{\partial z}\ -\rho\,(a_z + g)$$

Similarly it can be shown that

$$\frac{\partial p}{\partial y} = -\rho a_y \text{ and } \frac{\partial p}{\partial x} = -\rho a_x$$

The total change in pressure is given by the total differential as follows:

$$dp = \frac{\partial p}{\partial x} . dx + \frac{\partial p}{\partial y} . dy + \frac{\partial p}{\partial z} . dz$$

$$\text{or} \qquad dp = -\rho a_x . dx - \rho a_y . d_y - \rho\,(a_z + g) . dz$$

$$\therefore \quad dp = -\rho[a_x dx + a_y dy + (a_z + g) dz] \qquad (3.1)$$

Equation 3.1 is the basic equation of fluid statics applicable for both compressible and incompressible fluids.

3.4 Variation of Pressure with Elevation in a Static Incompressible Fluid

For a fluid at rest and subjected only to graviational force, the accelerations ax, ay and az are zero. Eqn 3.1 thus reduces to:

$$dp = -\rho g\ dz \qquad (3.2)$$

Equation 3.2 holds true for both compressible and incompressibile fluids. However for homogeneous and incompressible fluids, ρ is constant and eqn 3.2 may be integrated to give;

$$p = -\rho g z + c$$

where c is a constant of integration and is equal to the pressure at z = 0. In hydrostatics the law of variation of pressure with depth is usually written as;

$$p = \rho g h + p_o \tag{3.3}$$

In Equation 3.3, h is measured vertically downward (i.e. h =-z) from a free surface, p is the pressure at a depth h below the free surface and p_o is the pressure at the free surface.

Equation 3.3 shows that for a fluid at rest, the pressures at the same depth from the free surface are equal. Hence in a homogeneous continuous fluid a surface of equal pressures is a horizontal plane.

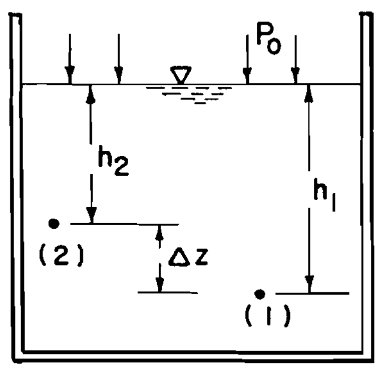


Figure 3.3

Consider two points (1) and (2) at a depth of h_1 and h_2 in a tank containing a liquid, with density ρ, at rest as shown in Figure 3.3. The pressure at (1) is $p_1 = p_o + \rho g h_1$. The pressure at (2) is $p_2 = p_o + \rho g h_2$. If $h_1 = h_2$, then $p_1 = p_2$.

For $h_1 > h_2$, the pressure difference between (1) and (2) is $p_1 - p_2 = \Delta p = \rho g h_1 - \rho g h_2 = \rho g(h_1 - h_2) = \rho g \Delta z$.

$\Delta z = \dfrac{\Delta p}{\rho g}$ is the pressure difference between (1) and (2)

expressed as a height of the liquid. This difference is also refered to as the pressure head difference. Thus, by dividing a pressure by the specific weight $\gamma = \rho g$ of a fluid, the pressure can be expressed as height of fluid column.

3.5 Variation of Pressure with Elevation in Static Compressible Fluids

Since density varies with pressure in compressible fluids, the relation between density and pressure must be known in order to integrate the basic equation of fluid statics and obtain expressions for the variation of pressure with elevation in compressible fluids. The relation between pressure and density is dependent on the prevailing conditions. These conditions are; Constant temperature (i.e. isothermal), adiabatic and constant temperature gradient conditions.

Isothermal Condition: The relation between pressure, density and temperature for constant temperature condition is given by the perfect gas law: $P/\rho = RT$. Substituting this in the basic equation of hydrostatics i.e. Equation 3.2:

$$\frac{dp}{dz} = -\rho g = \frac{-pg}{RT}$$

or $$\frac{dp}{p} = \frac{-g}{RT}.dz$$

Integrating from $p = p_1$ where $z = z_1$ to $p = p^2$ where $z = z_2$,

$$\log_e\left(\frac{p_2}{p_1}\right) = -\left(\frac{g}{RT}\right)(z_2 - z_1)$$

or

$$p_2/p_1 = \exp(-g/RT)(z_2 - z_1) \qquad (3.4)$$

Adiabatic Condition: Under adiabatic condition the relationship between pressure and density is given by P/ϱ^k = constant = p_1/ϱ_1^k,
so that

$$\rho = \rho_1 (p/p_1)^{1/k}$$

Substitution of the above in the basic equation 3.2 gives:

$$\frac{dp}{dz} = -(\rho_1 g/p_1^{1/k})$$

$$\text{or } dz = -(p_1^{1/k}/\rho_1 g) . p^{1/k} . dp$$

Integrating from $p = p_1$ when $z = z_1$, to $p = p_2$ when $z = z_2$,

$$z_2 - z_1 = -\left(p_1^{1/k}/\rho_1 g\right)\left[\frac{p^{(k-1)/k)}}{(k-1)/k}\right]_{p_1}^{p_2}$$

$$= -(k/(k-1) . \left(P_1^{1/K}/\rho_1 g\right)\left(P_2^{(K-1)/K} - P_1^{(K-1)/K)}\right)$$

The above may be written as:

$$z_2 - z_1 = -(k/(k-1))(p_1/\rho_1 g)\left\{(p_2/p_1)^{(k-1)/k} - 1\right\}$$

or, since $p_1/\rho_1 = RT$ for any gas,

$$z_2 - z_1 = -(k/_{(k-1)}\ RT_{1/g}\left\{(p_2/_{p_1})^{(k-1)/k} - 1\right\}$$

$$\text{or} \quad p_2/p_1 = \left\{1 - g(z_2 - z_1)/_{RT_1}(\frac{k-1}{k})\right\}^{k/(k-1)} \qquad (3.5)$$

In the above equation, T is absolute temperature in ^oK, R = 288 J kg^{-1} k^{-1} = 288 $m^2/s^2\,^oC$ and k = 1.4 for adiabatic condition. The temperature lapse rate - the rate of change of temperature with altitude - can be found for a diabatic conditions as follows:

Substituting the characteristic equation, $\varrho = p/RT$ in Equation 3.2 and rearranging,

$$dz = -(RT/gp)\,dp$$

For adiabatic condition, $p/\rho^k = p_1/\rho_1^k$, and since $p/\rho = RT$,

substitution and rearranging gives:

$$p = p_1(T_1/T)^{k/(1-k)}$$

Differentiating the above,

$$dp = -\{k/(1-k)\}P_1 . T_1^{k/(1-k)}\,T^{-1/(1-k)}\,dT$$

Substituting the values of p and dp in the equation for dz,

$$dz = \{k/(1-k)\}(R/g)\,dT$$

Therefore, the temperature gradient is given by:

$$dT/dZ = -\{(k-1)/k\}(g/R) \tag{3.6}$$

Constant Temperature Gradient Condition: Assuming that there is a constant temperature lapse rate (i.e. dT/dZ = constant) with elevation in a gas, so that its temperature drops by an amount δT for a unit change in elevation, then if T_1 = temperature at elevation Z_1, then T = temperature at elevation Z is given by:

$$T = T_1 - \delta T\ (Z - Z_1)$$

Putting this in Equation 3.2 and noting that $p/\varrho = RT$,

$$dp/dZ = -pg/RT$$

or

$$dp/p = (-g/RT)dZ$$

Substituting the above value of T,

$$dp/p = -\{g/(R(T_1 - \delta T(Z-Z_1)))\}dZ$$

Integrating the above between limits $P_1$and P_2 and Z_1 and Z_2,

$$p_2/p_1 = \{1 - (\delta T/T_1)(Z_2-Z_1)\}^{g/(R\delta T)} \tag{3.7}$$

On the average, there is a temperature gradient of about 6.5°C per 1000 m in the atmosphere i.e. δT = 6.5°C per 1000 m = $0.0065°C.m^{-1}$.

3.6 Absolute and Gage Pressur

A pressure may be expressed with reference to any arbitrary datum. It is usually expressed with respect to

Absolute zero (perfect vacuum) and local atmospheric pressure. When a pressure is expressed with respect to Absoulte zero, the pressure is called Absolute pressure, Pabs. If a pressure is expressed with respect to local atmospheric pressure, it is called gage pressure, Pgage.

Figure 3.4 ilustrates the concept of pressure datum.

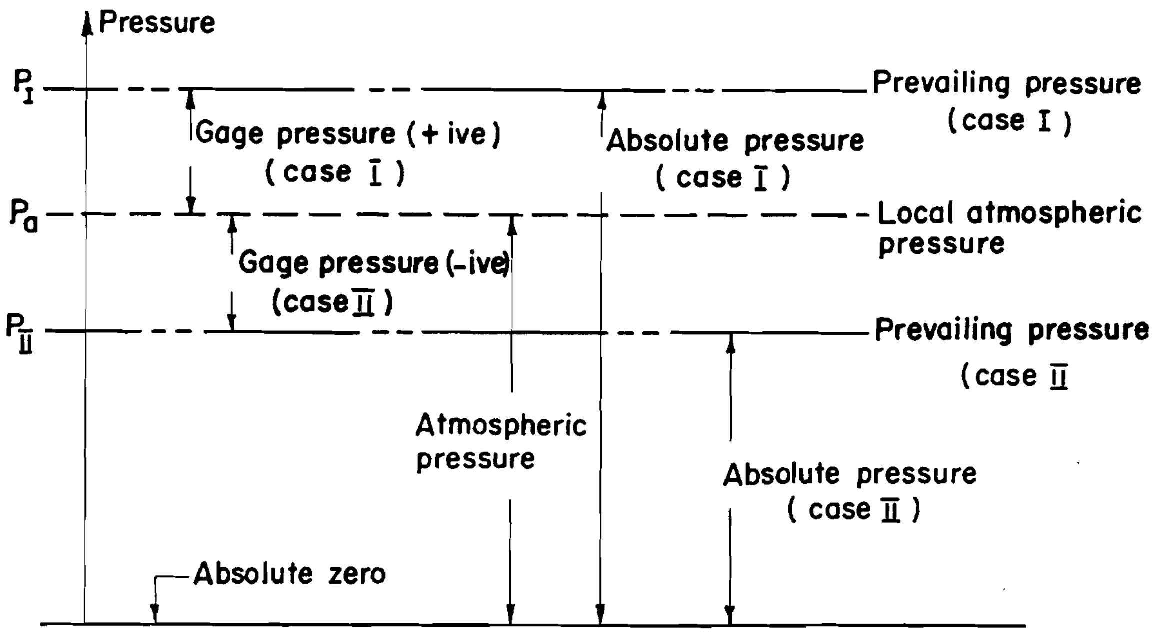


Figure 3.4 Pressure and Pressure Datum

It is evident from fig. 3.4 that absolute pressure is always positive since there cannot be any pressure below absolute zero.

Gage pressure is positive if the pressure is greater than atmospheric pressure and negative if the pressure is lower than the atmospheric pressure. The following equation expresses the relationship between absolute, gage and atmospheric pressure;

$$p_{abs} = p_{atm} + p_{gage} \tag{3.8}$$

In equation 3.8 P_{gage} may be positive or negative as the case may be. In hydrostatics, pressures are usually expressed as gage pressures unless otherwise specified.

Atmospheric pressure is also called Barometric pressure because the Barometer is the instrument used to measure the absolute pressure of the atmosphere. The simple barometer consists of an inverted tube closed at one end and immersed in a liquid with the open end down (Figure 3.5). If air is exhausted form the closed end of tube, the atmospheric pressure on the surface of the liquid in the container forces the liquid to rise in the tube.

If air is completely exhausted from the top portion of the tube, the liquid will rise in the tube to a height y and the only pressure on the liquid surface in the tube is the vapor pressure of the liquid, P_v.

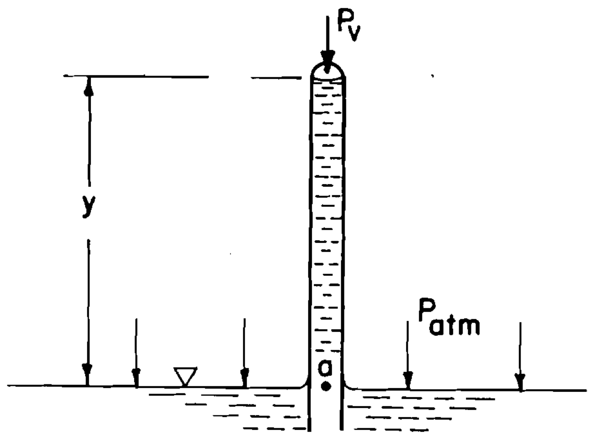


Figure 3.5 The simple Barometer

If ρ is the density of the liquid then the following equation is obtained from the variation of pressure in a static liquid.

$$p_a = P_v + \rho g y = P_{atm}$$

$$i.e. \quad P_{atm} = \rho g y + P_v \qquad (3.9)$$

The vapour pressure P_v is very small compared to the atmospheric pressure. Hence equation 3.9 may be approximated to $P_{atm} = \rho g y$. Thus the atmospheric pressure when expressed as the depth of the liquid becomes $y = \frac{p_{atm}}{\rho g}$ and y is called the pressure head. It follows from this that if a liquid with low density is used, y will be excessively large. Therefore, mercury is usually used in barometers mainly because its specific weight is very high thus enabling the use of short tube and also because its vapour pressure is negligibly small.

At sea level, y is 760 mm of mercury or 10.33 m of water. Atmospheric pressure at sea level is equal to 101.325 KN/m^2 and is also called standard atmospheric pressure.

3.7 Measurement of Pressure

Pressure is always measured by the determination of a pressure difference. As mentioned earlier, liquid pressures are normally expresed with respect to the prevailing atmospheric pressure and are called gage pressures. Several devices are employed for measuring pressures. Some of these are discussed here.

3.7.1 The Bourdon Gauge

The Bourdon gauge is a commercial instrument used to measure pressure differences (gage pressures) by the deformation of an elastic solid and may be employed when high precision is not required. It consists of a curved tube of elliptical cross-section closed at one end. The closed end is free to move while the other open end through which the fluid enters is rigidly fixed to the frame as shown in fig. 3.6.

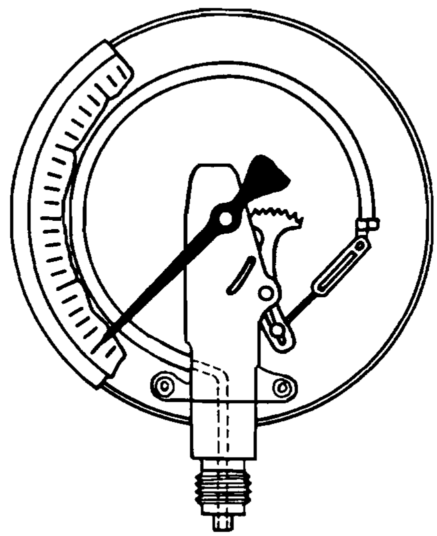


Figure 3.6 The Bourdon gauge

The internal pressure intensity of the fluid tends to straighten the curved tube by an amount proportional to the pressure intensity. The deflection of the tube cause a pointer moving over a scale to undergo a corresponding angular displacement by menas of a suitable gear and linkage arrangement. Zero reading is calibrated to correspond to local atmospheric pressure. All such gages required calibration.

3.7.2 Piezometer Column

A piezometer may be used to measure moderate positive pressures of liquids. It consists of a simple transparent tube open to the atmosphere in which the liquid can freely rise without overflowing as shown in Fig. 3.7. The height to which the liquid will rise in the tube indicates the pressure.

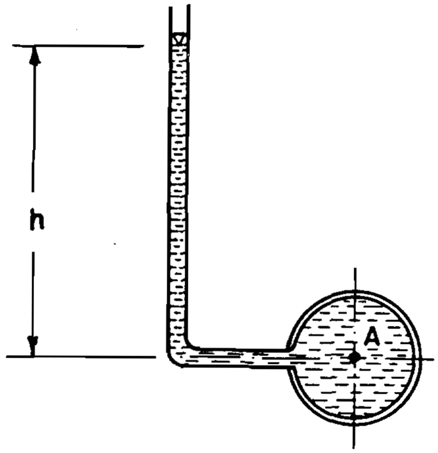


Figure 3.7 A simple piezometer

If the density of the liquid is ρ, then the pressure at A (gage pressure) is given by $P_A = \rho g h$. To reduce capillarity effects, the tube diameter should be at least 15 mm. Piezometers can not be used to measure negative pressure since air will be sucked into the container.

3.7.3 Manometers

Manometers are devices used to measure the defference in pressure between a certain point and the atmosphere, or between two points neither of which is necessarily at atmospheric pressure. They are suitable for measuring high pressure differences both positive and negative, in liquids and gases.

The Common (simple) Manometer:

The simple manometer consists of a transparent U - tube connected to a pipe or other container containing fluid N (figure 3.8). The lower part of the U - tube contains liquid M which should be immiscible with N and is of greater specific gravity. The most frequently used manometer liquids are mercury (specific gravity 13.6) and Alcohol (speific gravity 0.9).

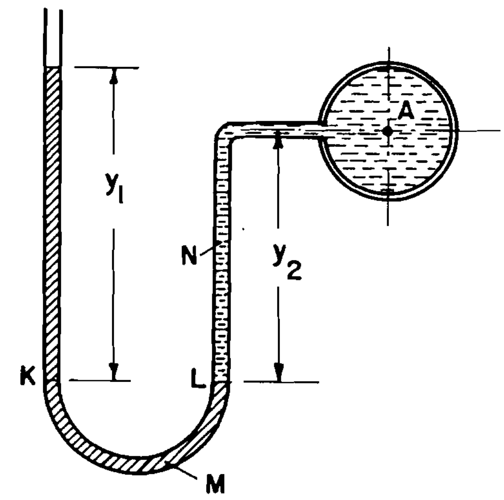


Figure 3.8 The Simple Manometer

Since the pressure in a continuous and homogenous fluid is the same at any two points in a horizontal plane, the pressures at K and L are equal for the equilibrium condition shown in Fig. 3.8. Thus if the specific weight of liquids N and M is γ_n and γ_m respectively one obtains the following;

$$P_L = P_K$$

$$or \quad P_A + \gamma_N y_2 = \gamma_M y_1$$

$$Thus \quad P = \gamma_M y_1 - \gamma_N y_2 \tag{3.10}$$

If liquid N is a gas, γ_N is negligible compared to γ_M and then $P = \gamma_M y_1$. In situations where the pressure to be measured is sub-atmospheric the arrangement may look like in Fig. 3.9.

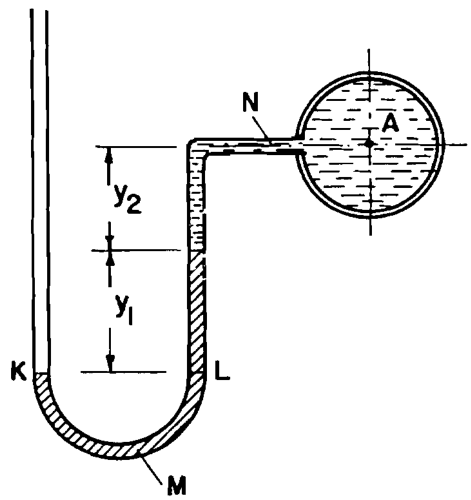


Figure 3.9

The manometric equation will now be $p_A + y_2 \cdot \gamma_N + y_1 \gamma_M = 0$.

Differential Manometer

A U - tube manometer is often used for measuring the difference in pressures between two containers as shown in Fig. 3.10. Such a manometer is sometimes refered to as differential manometer.

Considering points K and L an a horizontal plane in liquid M, $P_K = P_L$ and this may be written as

$$P_A + (y_3 + y_1)\gamma_N = P_B + (y_1 - y_2)\gamma_o + y_2\gamma_M$$

$$\therefore \quad P_A - P_B = (y_1 - y_2)\gamma_o + y_2\gamma_M - (y_3 + y_1)\gamma_N$$

Micromanometers

Micromanometers are used for measuring very small differences in pressure or precise determination of lare pressure differences. A typical arrangement, shown in Fig.

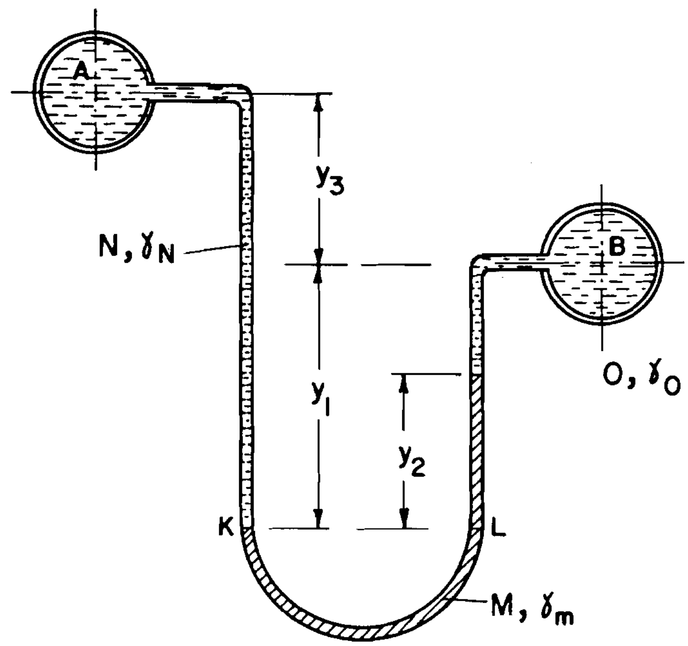


Figure 3.10 Differential Manometer

3.11, consists of two immiscible gage liquids A and B which are also immiscible with the fluid C to be measured. Prior to connection to the two containers m and n, the heavier gauge liquid A fills the lower protion of the U-tube to the level 1-1 and liquid B is at level 0-0. Fig. 3.11 shows the equilbrium situation when the pressure at m is higher than at n.

Writing the manometric equation starting from m:

$$P_m + (K_1 + \Delta y)\gamma_c + (K_2 - \Delta y + h/2)\gamma_B - h\gamma_A$$
$$- (K_2 - h/2 + \Delta y)\gamma_B - (K_1 - \Delta y)\gamma_c = P_n$$

The above simplifies to:

$$P_m + 2\Delta y\gamma_c - 2\Delta y\gamma_B + h\gamma_B - h\gamma_A = P_n$$

but $\Delta y.A = \frac{h}{2}a \quad or \quad 2\Delta y = \frac{a}{A}h$

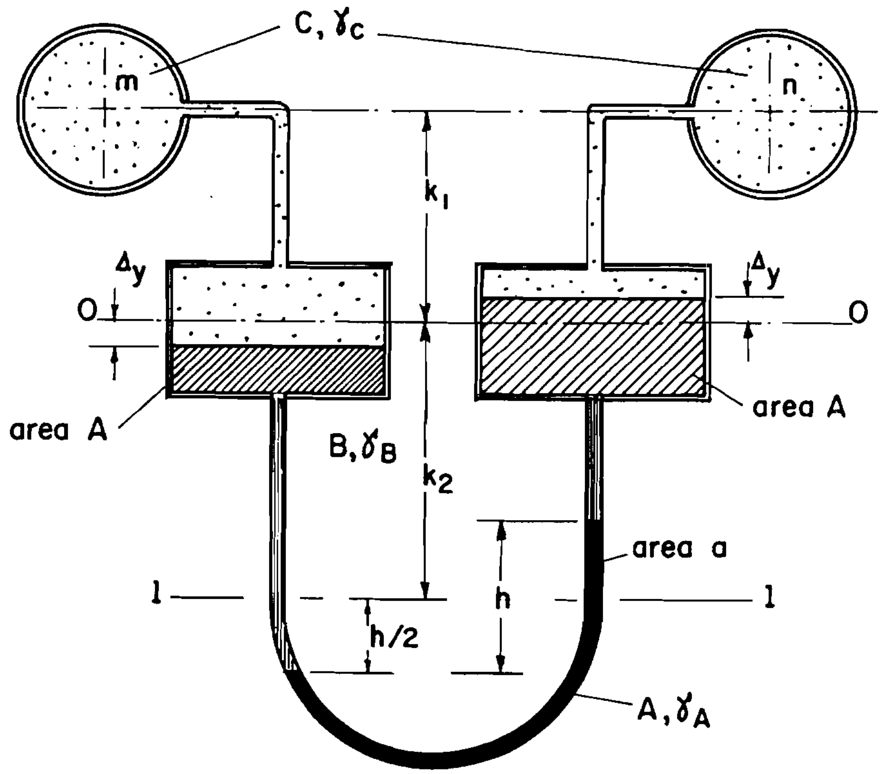


Figure 3.11 Micromanometer

Substituting and rearranging;

$$P_m - P_n = h\{\gamma_A - \gamma_B(1 - \frac{a}{A}) - \gamma_C\frac{a}{A}\}$$

The term in brackets is constant for a specific gauge and fluids and hence the pressure difference is directly proportional to h.

<u>Example 3.1</u>

A closed tank is partly filled with water and connected to the manometer containing mercury (S = 13.6) as shown in the figure below. A gauge is connected to the tank at a depth of 4 m below the water surface. If the manometer reading is 20 cm, determine the gauge reading in N/m^2. What will be the gauge reading when expressed as head of water in m?

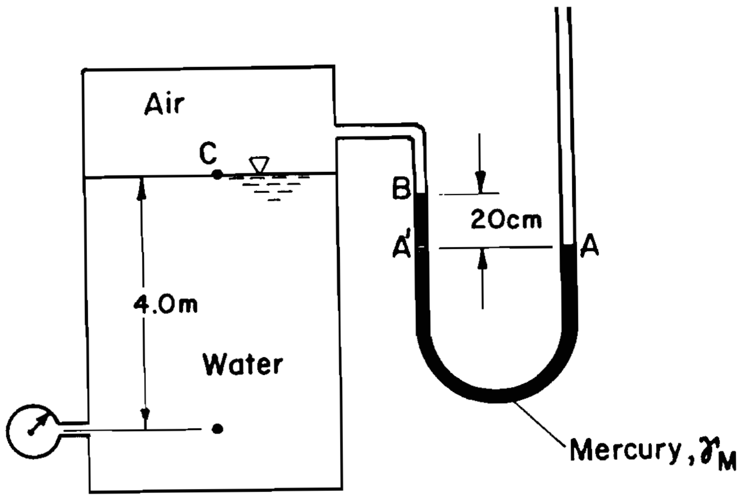


Figure E 3.1

Solution:

Using the letter designation in the Figure, $p_A = p_A'$

$$P_B = P_A' - 0.20\ \gamma_M$$

$$P_C = P_B \text{ and } P_D = P_{gauge} = P_C + 4\ \gamma_w$$

$$\therefore \quad P_D = P_A' - 0.20\gamma_m + 4\gamma_w$$

$$= 0 - 0.20 x \gamma_w . S_m + 4\gamma_w = \gamma_w(-0.2 S_m + 4)$$

$$= 9810 \frac{N}{m^3} (-0.2 \times 13.6 + 4) m = 9810(-2.72 + 4) N/m^2$$

$$P_D = 9810 \times 1.28 = 12556.8\ n/M^2$$

Therefore, the gauge reading is 12556.8 N/m_2
When expressed as head of water, the gauge reading will be

$$h = \frac{P}{\gamma_w} = \frac{12556.8\ N/m^2}{9810\ N/m^2} = 1.28\ m$$

Example 3.2

A manometer is mounted in a city water supply main pipe to monitor the water pressure in the pipe as shonw below. Determine the water pressure in the pipe.

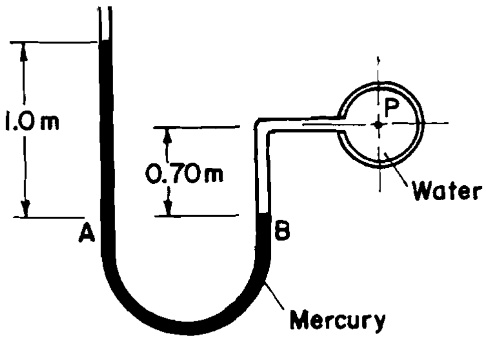


Figure E 3.2

Solution:

$$P_A = P_B$$

$$\gamma_{Hg} \cdot\ 1 = P_p + 0.70\ \gamma_w$$

$$P_p = \gamma_{Hg} \times 1 - 0.7\gamma_w + 13.6\gamma_w - 0.7\gamma_w$$

$$= 12.9\ \gamma_w = (12.9 \times 9810)\ N/m^2$$

$$= 1,2655 \times 10^5\ N/m = 1.249 atmospheres$$

(*Note*: 1 *standard atmosphere* = $1.01325 \times 10^5\ N/m^2$)

Example 3.3 Calculate the height of liquid columns from the bottom of the tank in the three piezometer tubes shown in Figure E 3.3.

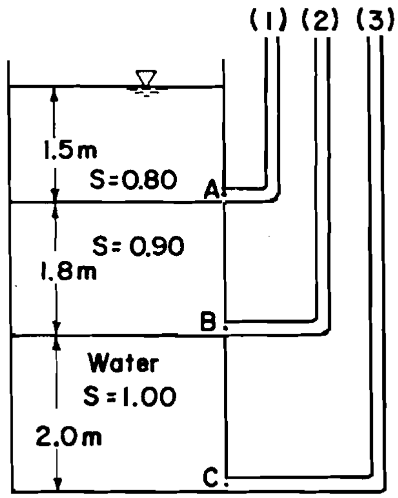


Figure E 3.3

Solution:

Pressure at C = P_c

$$P_c = 1.5 \times 9.81 \times 0.8 + 1.8 \times 9.81 \times 0.9 + 2 \times 9.81 \times 1.0$$

$$= 11.772 + 15.892 + 19.620 = 47.284\ kN/m^2$$

Pressure head in terms of water = h_3

$$h_3 = \frac{47.284}{9.81} = 4.82\ m$$

Pressure at B = P_B

$$P_B = 1.5 \times 9.81 \times 0.8 + 1.8 \times 9.81 \times 0.9$$
$$= 11.772 + 15.892 = 27.664\ kN/m^2$$

Pressure head in terms of liquid with s = 0.9 is h_2

$$h_2 = \frac{27.664}{0.9 \times 9.81} = 3.13\ m$$

Pressure at A = P_A

$$P_A = 1.5 \times 9.81 \times 0.8 = 11.772\ kN/m^2$$

Pressure head in terms of liquid with s = 0.8 is h_1

$$h_1 = \frac{11.772}{0.8 \times 9.81} = 1.5\ m$$

Therefore:

Height of liquid surface in tube 1 from tank bottom
= 1.5 + 1.8 + 2 = 5.3 m

Height of liquid surface in tube 2 from tank bottom
= 3.13 + 2.0 = 5.13 m

Height of liquid surface in tube 3 from tank bottom
= 4.82 m

Example 3.4 Calculate the pressure at point A in Fig. E 3.4 and express it in terms of head of water.

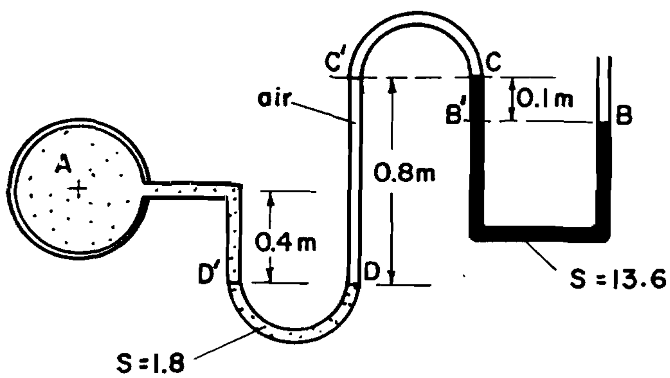


Figure E 3.4

Solution:

Starting from B,

$$0 - 0.1 \times 9810 \times 13.6 + 0.8\,\gamma_{air} - 0.4 \times 1.8 \times 9810 = P_A$$

$$-13{,}341.6 + 0.8\,\gamma_{air} - 7{,}063.2 = P_a$$

neglecting γ_{air},

$$P_A = -20{,}404.8\ N/m^2 \qquad \text{(vacuum)}$$

In terms of head of water, $h_A = \dfrac{-20404.8}{9810} = -2.08\ m$ of water

Example 3.5 Calculate the pressure difference between points A and B in the differential manometer shown in Figure E 3.5.

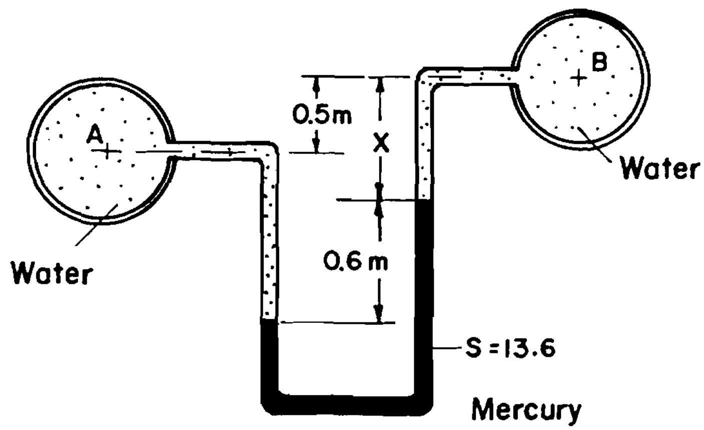


Figure E 3.5

Solution:

Starting from A,

$$P_A + (x - 0.5)\gamma_w + 0.6\gamma_w - 0.6 \times 13.6\gamma_w - x\gamma_w = P_B$$

$$PA + x\gamma_w - 0.5\gamma_w + 0.6\gamma_w - 8.16\gamma_w - x\gamma_w = P_B$$

$$\therefore \quad P_A - PB = (8.16 - 0.1)\gamma_w = 8.06\gamma_w$$

$$= 8.06 \times 9.81 = 79.07\ kN/m^2$$

Example 3.6 In the two compartment closed tank shown in Fig. E 3.6, the pressure in the air in the left compartment is -26.7

kN/m² while that in the air in the right compartment is 19.62 kN/m². Determine the difference h in the levels of the legs of the mercury manometer. Specific gravity of mercury is 13.6.

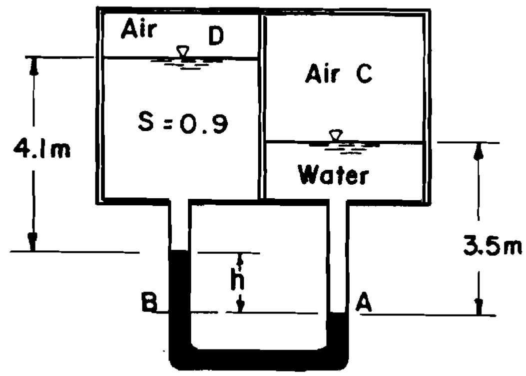


Figure E 3.6

Solution:

Since the pressure in a static fluid is the same in a horizontal plane, $P_A = P_B$

$$P_A = P_c + 3.5 \times \gamma_w = 19.62 + 3.5 \times 9.81 = 53.955 kN/m^2$$

$$P_B = P_D + 4.1 \times 0.9\gamma_w + h \times 13.6\gamma_w$$

$$= -26.7 + 4.1 \times 0.9 \times 9.81 + h \times 13.6 \times 9.81$$

$$= -26.7 + 36.20 + 133.42\ h$$

$$= 9.5 + 133.42\ h$$

$$\therefore \quad 9.5 + 133.42\ h = 53.955$$

$$Thus \quad h = (53.955 - 9.5)/133.42 = 0.333\ m$$

$$= 33.3\ cm$$

Example 3.7 At an altitude Z_1 of 11,000 m the atmospheric temperature T is -56.6°C and the pressure P is 22.4 kN/m^{-2}.

Assuming that the temperature remains the same at higher altitudes, calculate the density of the air at an altitude of Z_2 of 15000 m. Assume R = 287 J kg^{-1} K^{-1}.

Solution:

Let P_2 be the absolute pressure at Z_2
Since the temperature is constant:

$$P_2/P_1 = \rho^{-(g/RT)(Z_2 - Z_1)}$$

$$Here, \quad P_1 = 22.4kN\ m^{-2} = 22400N\ m^{-2}, \quad Z_1 = 11,000\ m, \quad Z_2 = 15000\ m$$

$$R = 287\ \delta\ kg^{-1}\ k^{-1}, \quad T = -56.6^o C = 216.5^o\ k:$$

$$P_2 = 22.4 \times 10^3. \quad \exp\left\{\frac{-9.81 \times (15000 - 11,000)}{287 \times 216.5}\right\}$$

$$= 22.4 \times 10^3. \quad \exp\ (-0.631) = 11.91 \times 10^3\ NM^{-2}$$

From the equation of state of a perfect gas, $P_2 = \varrho_2 RT$
Therefore, the density of air at 15000 m is $\varrho_2 = P_2/RT$

or $\varrho_2 = 11.92 \times 10^3/(287 \times 216.5) = 0.192$ kg m^{-3}.

Example 3.8 Assuming that the temperature of the atmosphere drops with increasing altitude at the rate of 6.5° C per 1000 m, find the pressure and density at a height of 5000 m if the corresponding values at sea level are 101 kN m^{-2} and 1.235 kg m^{-3} respectively when the temperature is 15° C.
Take R = 287 J kg^{-1} K^{-1}.

Solution:

From Equation:

$$P_2 = P_1 \left[1 - (\partial T/T_1)(Z_2 = Z_1)\right] g/R\delta T$$

$$\delta T = 6.5^o C \ per \ 1000 \ m = 0.0065 \ K \ m^{-1}$$

$$T_1 = 15^o \ c = 288 \ K$$

$$Z_2 - Z_1 = 5000 - 0 = 5000 \ m.$$

$$g/(R\delta T) = 9.81/(287 \times 0.0065) = 5.259$$

$$\therefore \quad P_2 = 101 \times 10^3 \ [1 - (0.0065/288) \times 5000]^{5.259} = 53.82 \times 10^3 \ N \ m^{-2}$$

From the equation of state

$$Density \quad \rho_2 = P_2/RT_2 = P_2/R(T_1 - \delta T(Z_2 - Z_1))$$

$$= 53.82 \times 10^3/287(288 - 0.0065 \times 5000)$$

$$= 0.734 \ kg \ m^{-3}$$

3.8 Hydrostatic Forces on Surfaces

Plane and curved surfaces, immersed fully or partly in liquids, are subjected to hydrostatic pressure forces. It is, therefore, essential to determine the magnitudes, directions and locations of the hydrostatic pressure forces on surfaces as a first step in the analysis of the stability of a body fully or partly immersed in a liquid and in the design of hydraulic structures such as dams and gates.

3.8.1 Hydrostatic Force on Plane Surfaces

a) Horizontal Plane Surfaces:

The pressure intensity in a static fluid is the same at any two points in a horizontal plane surface. Therefore, a plane surface in a horizontal position at a depth h below the free surface in a fluid at rest will be subjected to a constant pressure intensity equal to $\gamma.h$, where γ is the specific weight of the fluid. The total pressure force on a small differential area is given by:

$$dF_p = \gamma . h \ dA$$

The total pressure force on the entire horizontal plane surface with area A will be

$$F_p = \int^{A} \gamma h \, dA = \gamma \bar{h} A$$

The force F_p acts normal to the surface and towards the surface. Since the pressure intensity is distributed uniformly over the plane surface, the total resultant force F_p acts through the centroid of the area and h = $\bar{h}$, where $\bar{h}$ is the depth from the free surface to the centroid. Thus, for horizontal plane surfaces, the centre of pressure C coincides with the centroid G. The centre of pressure is the point on the immersed surface at which the resultant pressure force on the entire area is assumed to act.

b) Vertical Plane Surface

Consider a plane vertical surface of area A immersed vertically in a liquid (Fig. 3.12). Since the depth from the free surface to the various points on the surface varies, the pressure intensity on the surface is not constant and varies directly with depth.

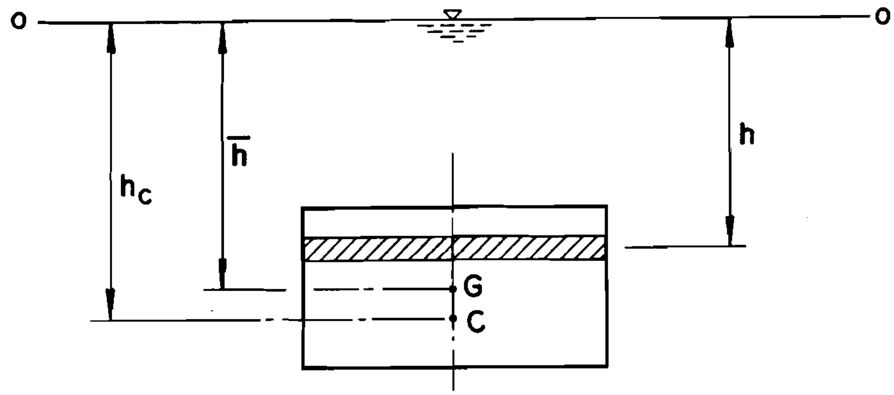


Figure 3.12

Consider also a narrow strip of horizontal area dA, shown shaded in Fig. 3.12, at a depth h below the free surface. The pressure intensity on this area dA is γ.h and is uniform. The total pressure force on one side of the strip is thus

$$dF_p = \gamma . h . dA$$

The total pressure force on one side of the entire area A is

$$F_p = \int \gamma h . dA = \gamma . \int h \; dA$$

or $$F_p = \gamma . \bar{h} A$$

where $\bar{h}$ is the depth from the free surface to the centroid G of the area. Thus, as for a horizontal plane area, the magnitude of the resultant hydrostatic pressure force on a vertical plane area is obtained by multiplying the pressure intensity at the centroid G, i.e $\gamma . \bar{h}$, by the total area A.

If the vertical area is not of a regular shape, the area may be divided into a finite number of small regular areas and the total hydrostatic pressure force determined as the sum of the pressure forces acting on these small areas.

The total pressure force F_p acts normal to the vertical plane area and towards the area through the centre of pressure C. Since the pressure distribution on the area is not uniform, the centre of pressure and the centroid will not coincide. The depth h_c to the centre of pressure may be obtained from the principle of moments. The moment of the elementary force dF_p acting on the area dA (Fig. 3.12) about axis 0-0 on the free surface is

$$dM = dF_p . h = (\gamma . h \; dA) h$$

The total moment of all elementary forces on the whole area is:

$$M = \int dM = \int \gamma . h^2 . dA$$

From the principle of moments, the sum of the moments of a number of forces about an axis is equal to the moment of their resultant about the same axis. Thus:

$$F_p . h_c = M = \gamma \int h^2 dA$$

The term $\int h^2 dA$ may be recognized as the second moment of area about the free surface i.e. I_{oo}.

i.e $$h_c = \frac{\gamma . I_{oo}}{F_p}$$

using the parallel axis theorem of second moment of area,

$$I_{oo} = I_G + A(\bar{h})^2$$

where I_G is the second moment of area about the axis parallel to 0-0 and passing through the centroid G. Therefore,

$$h_c = \frac{\gamma(I_G + A(\bar{h})^2)}{\gamma.\bar{h}.A}$$

or

$$h_c = \frac{I_G}{Ah-} + \bar{h}$$

Thus, the centre of pressure C for vertical plane area is below the centroid by an amount equal to:

$$I_G/A\bar{h}$$

. The moment of F_p about the centroid is:

$$F_p \times GC = \rho gh\text{-}A \times \frac{I_c}{h\text{-}A} = \rho g I_c$$, which is independent of depth of submergence.

c) Inclined Plane Surface

The analysis of the hydrostatic force on an inclined plane surface will be made by considering a plane surface of arbitrary shape and total area A inclined at an arbitrary angle θ to the free surface as shown in Fig. 3.13. AB is the trace of the inclined surface the extension of which intersects with the free surface at 0. h_c and h_p are the depths from the free surface to the centroid C and centre of pressure CP of the area respectively. Y_c and Y_p are the corresponding distances from 0 to C and CP respectively, measured along the inclined surface. It is required to determine the magnitude, direction and line of action of the resultant hydrostatic force F_p acting on one side of the area.

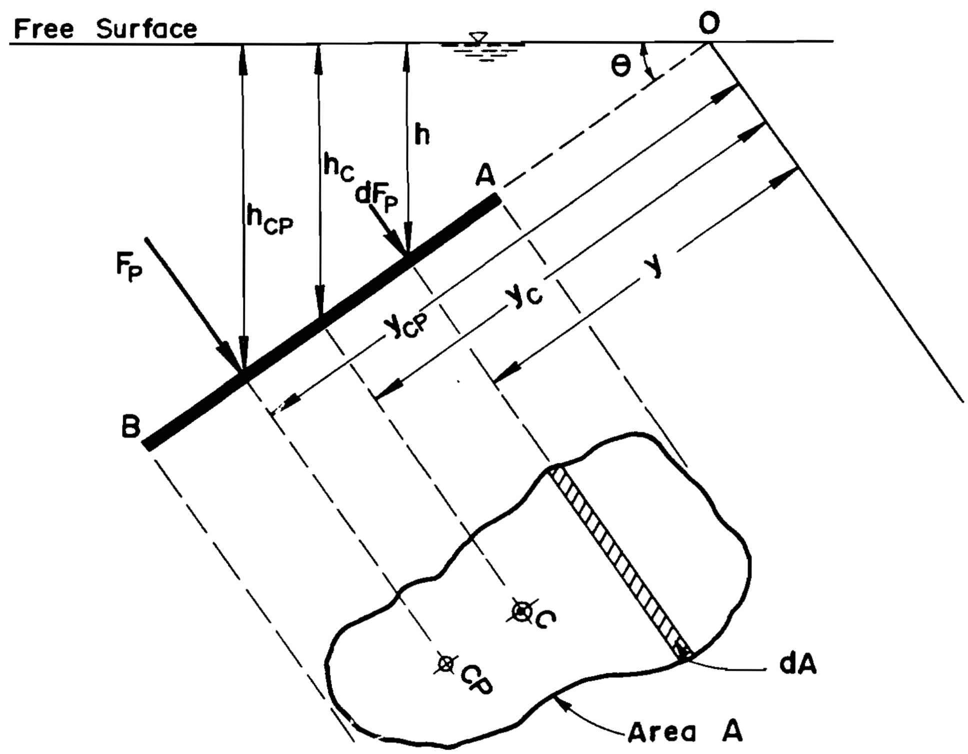


Figure 3.13 Hydrostatic force on an inclined plane surface

The magnitude of the force dF_p acting on an elementary area dA at a depth h below the free surface is given by

$$dF_p = p.dA = \rho gh.dA = \rho g.y\sin\theta \, dA$$

The force dF_p acts normal to the plane surface. The resultant hydrostatic force F_p is the sum of all elementary forces dF_p which are parallel to each other.

Thus $F_p = \int dF_p = \rho g \; Sin\theta \int y dA$

But $\int y.dA$ is the first moment of area A about axis through 0 and is equal to y_c.A and since y Sin θ $=_c$ h , the above equation for F_p becomes:

$$F_p = \gamma \; Sin\theta \; y_c \; A = \gamma h_c \; A \qquad 3.11$$

γh_c is the pressure intensity at the centroid of the inclined plane area. This shows that the magnitude of the resultant hydrostatic force on an inclined plane area is equal to the product of the area and the pressure intensity at the centroid of the area. The force F_p acts normal to the plane surface and towards the surface.

The resultant force F_p acts through the centre of pressure CP of the submerged plane area. The location of CP is determined using the principle of moments for a parallel force system. In Fig. 3.13 let the axis through 0 coinciding with the free surface be the axis of moments. The moment of force dF_p about this axis is equal to dM_o which is given by

$$dM_o = y.dF_p = y.\rho g.y \; Sin\theta \; dA = \rho g \; Sin\theta \; y^2 dA$$

The moment of the resultant force F_p about the axis of moments will be equal to the sum of all elemental moment dM_o. i.e.

$$F_p \; y_{cp} = \int dM_o = \rho g \; Sin\theta \int y^2 dA = \gamma \; Sin\theta \; I_{oo}$$

Where I_{oo} is the second moment of the plane area about axis 0-0.

Thus $$y_{cp} = \frac{\gamma\ Sin\theta\ I_{oo}}{F_p} = \frac{\gamma\ Sin\theta\ I_{oo}}{\gamma\ Sin\theta\ y_c\ A} = \frac{I_{oo}}{y_c\ A}$$

Using the parallel axis theorem,

$$I_{oo} = I_c + y_c^2\ A$$

Where I_c is the second moment of area about an axis parallel to 0-0 and passing through the centroid c.

Thus $$y_{cp} = \frac{I_c + y_c^2\ A}{y_c\ A} = y_c + \frac{I_c}{y_c\ A} \qquad (3.12)$$

This shows that the centre of pressure is always below the centroid of the area. The same has been shown for vertical plane surfaces.

The depth of the centre of pressure below the free surface is $h_{cp} = y_{cp}\ Sin\theta$. Substituting this and the value of $y_c = h_c/Sin\theta$ in Eqn. , the following equation is obtained for the depth to the centre of pressure.

$$h_{cp} = h_c + \frac{I_c\ Sin^2\ \theta}{h_c\ A} \qquad (3.13)$$

When the surface area is symmetrical about its vertical centroidal axis, the centre of pressure CP always lies on this symmetrical axis but below the centroid of the area. If the area is not symmetrical, an additional coordinate, x_{cp}, must be fixed to locate the centre of pressure completely.

Referring to Figure 3.14, and using moments,

$$X_{cp} \int_A dF_p = \int_A dF_p \, x$$

or $$X_{cp} \, \rho g \, y_c \, Sin\theta \, A = \int_A \rho g y \, Sin \, \theta \, dA \, x$$

$$\therefore \quad X_{cp} = \frac{1}{Ay_c} \int_A x \, y$$

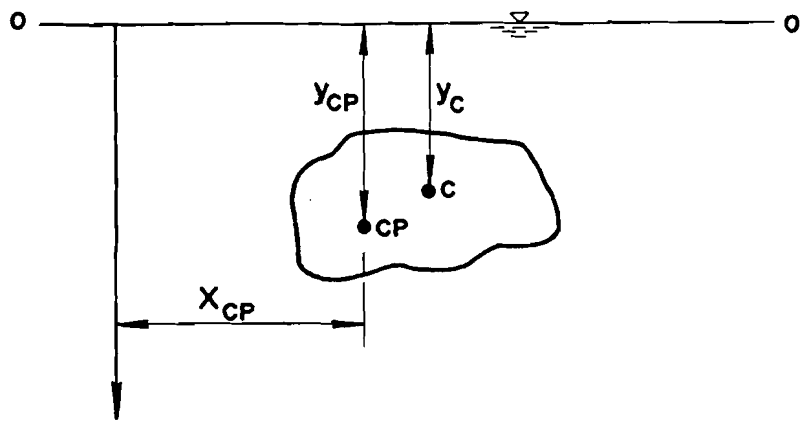


Figure 3.14 Centre of pressure of an asymmetrical plane surface

The location of the centroid C and the magnitude of the second moment of area about the centroidal axis of some common geometrical shapes is given in Table 3.1.

3.8.2 Hydrostatic Force on Curved Surfaces

The total hydrostatic force on a curved surface immersed in a liquid can not be directly determined by the methods developed for plane surfaces. For plane surfaces, the pressure forces on elementary areas act perpendicular to the surface and hence are parallel to each other. Consequently, it is easier to obtain the resultant force by a simple summation of the elementary forces. In the case of a curved surface each elementary force acts perpendicular to the tangent of the elementary area and because of the curvature of the surface the direction of each elementary force is different. As a result, the usual procedure is to determine the horizontal and vertical components of the resultant force and then add them vectorially to obtain the magnitude, direction and location of the line of action of the resultant hydrostatic force.

Consider the curved surface BC of unit width shown in Figure 3.15.

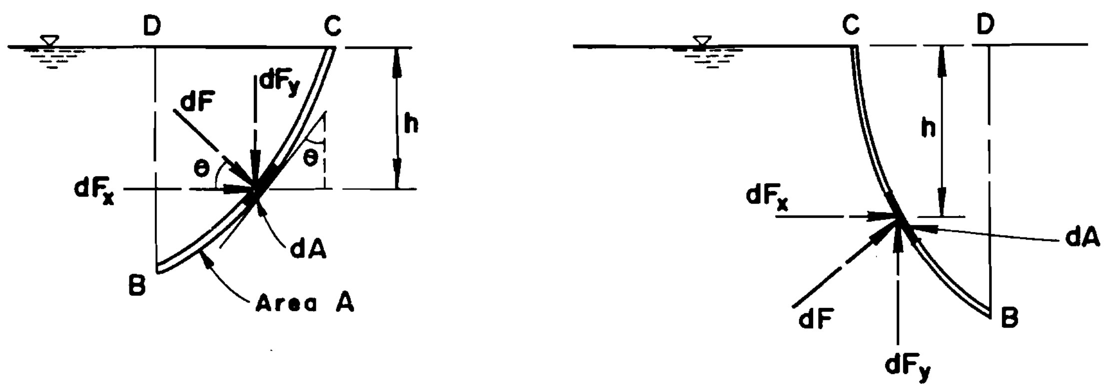


Figure 3.15 Hydrostatic force components on curved surfaces.

The elementary force dF acting on the elementary area dA has a horizontal component dF_x and a vertical component dF_y. The pressure intensity on dA is ϱgh.

The total hydrostatic force on dA = dF $= \varrho gh\ dA$

The horizontal component of dF $= dF_x = \varrho gh\ dA \cos\theta$

The vertical component of dF $= dF_y = \varrho gh\ dA \sin\theta$

But $dA \cos\theta = dA_v$ = The projection of dA on the vertical plane and $dA\ Sin\theta = dA_h$ = The projection of dA on the horizontal plane.

The components of the total hydrostatic force in the x and y directions are F_x and F_y respectively and are given by:

$$F_x = \int_A dF_x = \int_A \rho gh\ dA\ Cos\theta = \rho g h_c\ A_v$$

$$F_y = \int_A dF_y = \int_A \rho gh\ dA\ Sin\theta = \rho g \int_A dV$$

where: A_v is the projection of the whole curved surface BC on the vertical plane, i.e. BD

dV is the volume of the water prism (real or virtual) extending over the area dA to the free surface.

i.e $F_y = \varrho gV$

Thus:

The horizontal component, F_x, of the resultant hydrostatic force on a curved surface BC is equal to the product of the vertically projected area of BD and the pressure intensity at the centroid of the vertical area BD. The Force F_x passes through the centre of pressure of the vertically projected area BD.

The vertical component, F_y, of the resultant hydrostatic force on a curved surface BC is equal to the weight of the water (real or virtual) enclosed between the curved surface BC, the vertical BD and the free surface CD. The force component F_y acts through the centre of gravity of the volume.

The resultant force F is given by:

$$F = \sqrt{F_x^2 + F_y^2} \quad (3.14)$$

F acts normal to the tangent at the contact point on the surface at an angle α to the horizontal, where

$$\alpha = \tan^{-1} (f_y/F_x) \quad (3.15)$$

3.8.3 Pressure Diagrams:

The resultant hydrostatic force and centre of pressure for regular plane areas could be determined from pressure distribution diagrams such as those shown in Figure 3.16

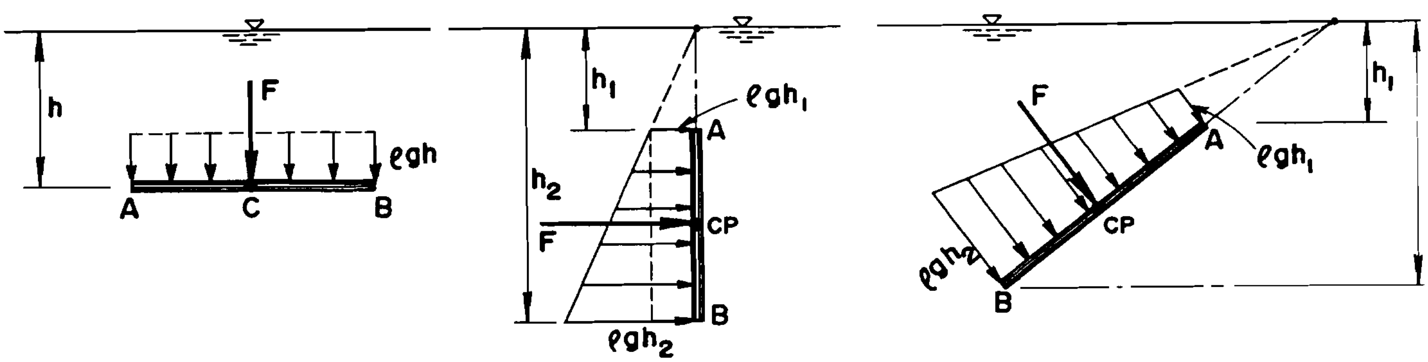


Figure 3.16 Pressure diagrams

In Fig. 3.16(a) the surface AB is horizontal and the pressure intensity is uniform over the area of the horizontal surface AB. The total hydrostatic thrust on AB is equal to the volume of the pressure prism, which is the product of the uniform pressure intensity ϱgh and the area A, and acts through the centroid of the area.

In Fig. 3.16(b), AB may be assumed to be rectangular with width b perpendicular to the plane of the paper. The pressure distribution is trapezoidal with intensity ρgh_1 at A and ρgh_2 at B. The total hydrostatic force on AB is equal to the volume of the pressure prism and is given by:

$$F = \frac{(\rho g h_1 + \rho g h_2)}{2} (h_2 - h_1) . b$$

The centre of pressure is the centroid of the pressure prism. It may be located by dividing the prism into a rectangular and triangular prism. For the rectangular prism, the centroid is at $(h_2 - h_1)/2$ above B and for the triangular prism it is at $(h_2-h_1)/3$ above b. The centroid of the trapezoidal prism can then be found from the principle of moments.

3.8.4 Tensile Stress in a Pipe

Pipes are conduits of circular cross-section that are used to transport fluids. During this transport process, a certain amount of internal pressure is necessary to make the flow possible. This pressure may be supplied by gravity flow or by an external input of energy by means of a pump. The internal pressure produces tensile stresses in the pipe walls. Both longitudinal and circumferencial (or hoop) stresses exist in pipes. However, the circumferential stresses are more important since they are twice the longitudinal stresses. In pipe flows, the problem is to determine either the required wall thickness of the pipe necessary to resist a certain pressure or the allowable pressure for a given wall thickness of a given pipe material. A circular pipe with internal radius r, wall thickness t and having a horizontal axis is in tension around its perihery as shown in Fig. 3.17. A 1 metre long section of pipe, i.e. the ring between two planes normal to the axis and 1 metre apart, is considered for the analysis of the problem. Considering one-half of this ring as a free body, the tensile forces per metre length at the top and bottom are T_1 and T_2 respectively.

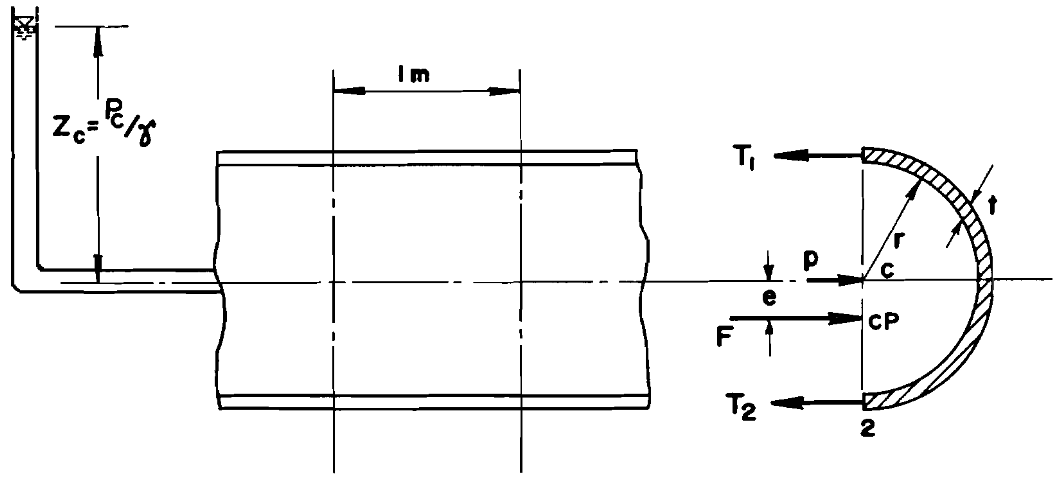


Figure 3.17 Tensile stress in pipes.

The horizontal component of the pressure force acts through the centre of pressure CP of the projected area and is given by :

$$F = p.2r.1$$

Where p is the pressure intensity at the pipe centre and r is the internal radius r.

Strictly speaking, T_1 is smaller than T_2. But for high internal pressure, the centre of pressure CP may be taken to coincide with the pipe centre c and T_1 may be approximated to be equal T_2 without serious error. Thus summing forces in the horizontal direction:

$$T_1 + T_2 = F = 2pr$$

since $T_1 = T_2 = T,\quad 2T = 2pr \quad and \quad T = pr$

where T is the tensile force per metre length of pipe. For wall thickness t, the circumferential stress σ in the pipe wall will be

$$\sigma = \frac{T}{t \times 1} = \frac{p.r}{t} \qquad (3.16)$$

For an allowable tensile stress σ_{all}, the required wall thickness t will be:

$$t = \frac{p.r}{\sigma_{all}}$$

Where p is in N/m^2, σ_{all} is in N/m^2, r is in cm and t is in cm.

For large variations in pressure between top and bottom of pipe, i.e. when $Z_c = p_c/\gamma \leq 10r$, the centre of pressure has to be computed for which the following two equations are necessary:

From $\quad \Sigma F_h = 0 \quad : \quad T_1 + T_2 = F = 2p.r$

$$\Sigma M_2 = 0 \quad : \quad 2rT_1 - 2pr(r - e) = 0$$

From which:

$$T_1 = p(r - e)$$

and $\quad T_2 = 2pr - T_1 = 2pr - p(r - e) = p(r + e)$

Obviously, $T_2 > T_1$ and T_2 must be used for further computations. The eccentricity e may be obtained as follows:

$$e = \frac{I_{xc}}{y_c A} = \frac{1.(2r)^3}{12.y_c.(2r.1)}$$

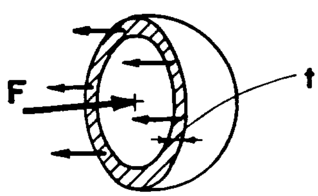
But, $y_c = Z_c = \frac{p_c}{\gamma}$, taking the area as a vertical area.

$$\therefore\ e = \frac{8r^3.\gamma}{12.p_c.2r} = \frac{r^2\gamma}{3.p_c}$$

from $\sigma = \frac{T_2}{t} = \frac{p(r + e)}{t}$,

$$t = \frac{p(r + e)}{\sigma_{all}} = \frac{p(r + r^2\gamma/3p_c)}{\sigma_{all}} \qquad (3.17)$$

In a thin spherical shell subjected to an internal pressure the stress in its wall may be found, neglecting the weight of the fluid in the sphere, by considering the forces on a free body consisting of a hemisphere, cut from the sphere by a vertical plane as shown below.



The component of the pressure force F is: $F = p.\Pi r^2$, where r is the internal radius of the sphere and p is the internal pressure.

If σ_t is the stress in the wall, then for equilibrium:

$$\sigma_t - 2\pi r.t - p\pi r^2 = 0$$

$$or \quad \sigma_\ell = \frac{pr}{2t} \qquad (3.18)$$

σ_ℓ is just half of the circumferential stress σ given by Eqn.3.16. For a pipe closed at one end, σ_ℓ will be the longitudinal stress in the pipe wall.

Example 3.7

A vertical rectangular gate AB shown in Figure E.37 has a width of 1.5 m. The gate is hinged on its upper edge at A. Determine the moment M at A required to just hold the gate from opening.

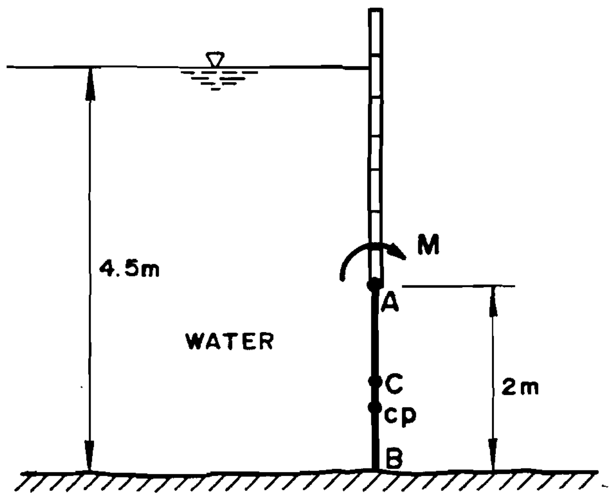


Figure E 3.7

Solution: Referring to Figure. E 3.7

The centroid C of AB is at (4.5 - 1) = 3.5 m below the water surface i.e. h_c = 3.5 m.

The hydrostatic force on gate AB is F_p and will act through CP normal to AB.

$$Fp = \gamma_w \, h_c \, A = 9.81 \; kN/m^3 . 3.5 \; m . (2 \times 1.5) \; m^2 = 103.005 \; kN$$

The location of CP is obtained from:

$$Y_{cp} = Y_c + \frac{I_c}{Y_c A} = 3.5 + \frac{1.5 \times 2^3}{12 \times 3.5 \times 2 \times 1.5} = 3.595\ m$$

Taking sum of moments about A,

$$\sum M_A = M - 103.005 \times 3.595 = 0$$

$$\therefore \quad M = 370.3\ kN.m \quad Clockwise$$

Example 3.8

The 2 m wide and inclined rectangular gate AB shown in Figure E 3.8 is hinged at B. The gate is unifrom and weighs 24 kN. Determine

a) The magnitude and location of the hydrostatic forces on each side of the gate.
b) The resultant of the hydrostatic forces.
c) The force F required to just open the gate.

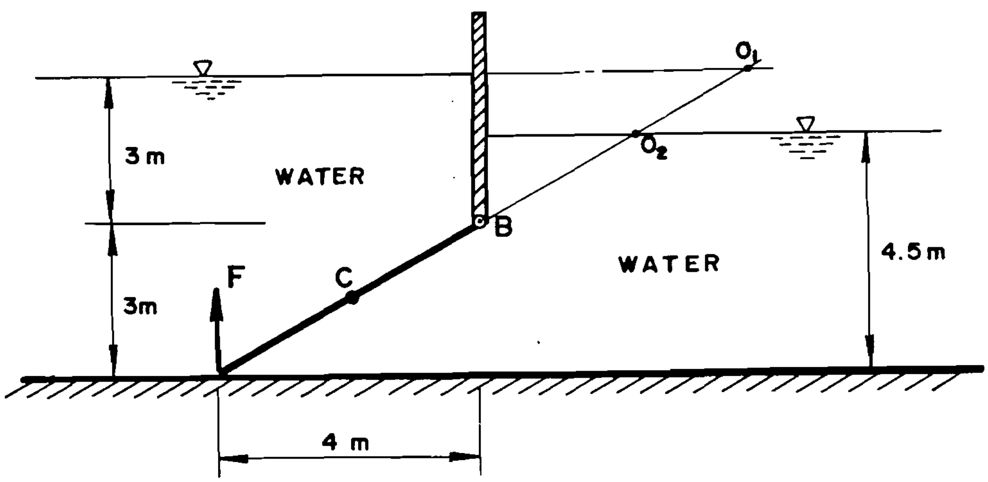


Figure E 3.8

Solution:

a) Let the hydrostatic force on the left side of gate AB be F_l and that on the rigth side of F_r.

The gate AB is 5 m long and its centroid C is at a depth of 1.5 from B.

Thus, $F_l = \gamma_w h_{cl} A = 9.81 \times (3 + 1.5) \times (5 \times 2) = 441.45\ kN$

F_l acts normal to AB through the center of pressure of the left side of AB which is located at Y_{cpl} from 01.

$$Y_{cpl} = Y_{cl} + \frac{I_c}{Y_{cl} A}$$
$$Y_{cl} = CO_1 = 7.5\ m$$

Therefore,

$$Y_{cpl} = 7.5 + \frac{2 \times 5^3}{12 \times 7.5 \times 5 \times 2} = 7.5 + 0.278 = 7.778\ m.$$

Similarly,

$$F_r = \gamma_w h_{cr} A$$
$$= 9.81 \times (1.5 + 1.5) \times (5 \times 2) = 294.3\ kN$$

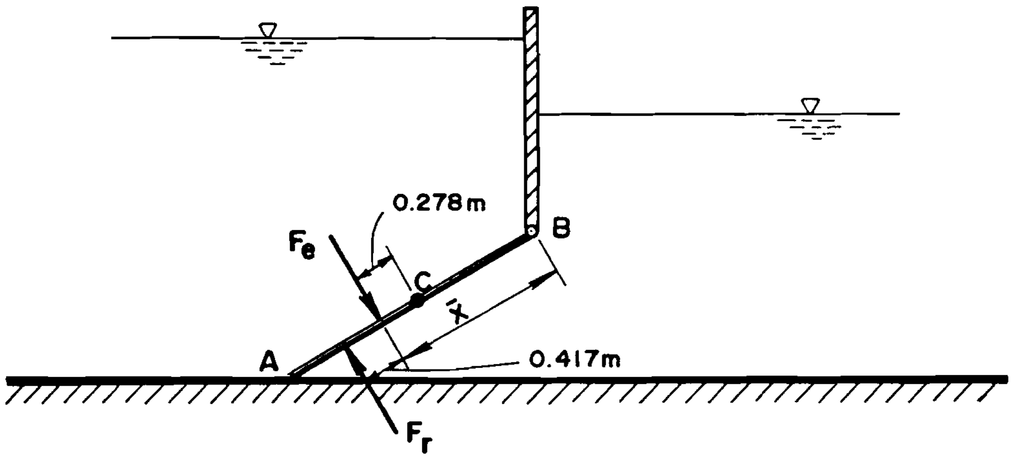
F_r acts normal to AB through the center of pressure of the left side of AB which is located at Y_{cpr} from O_2.

$$Y_{cpr} = Y_{cr} + \frac{I_c}{Y_{cr} A}$$

$$Y_{cr} = CO2 = 5\ m$$

Therefore $\quad Y_{cpr} = 5 + \dfrac{2 \times 5^3}{12 \times 7.5 \times 5 \times 2} = 5 + 0.417 = 5.417\ m$

The positions of the forces $F\ell$ and F_r shown below:



b) The resultant of F_ℓ and F_r is F_R and acts parallel to F_ℓ and F_r in the direction of the greater force F_ℓ normal to AB.

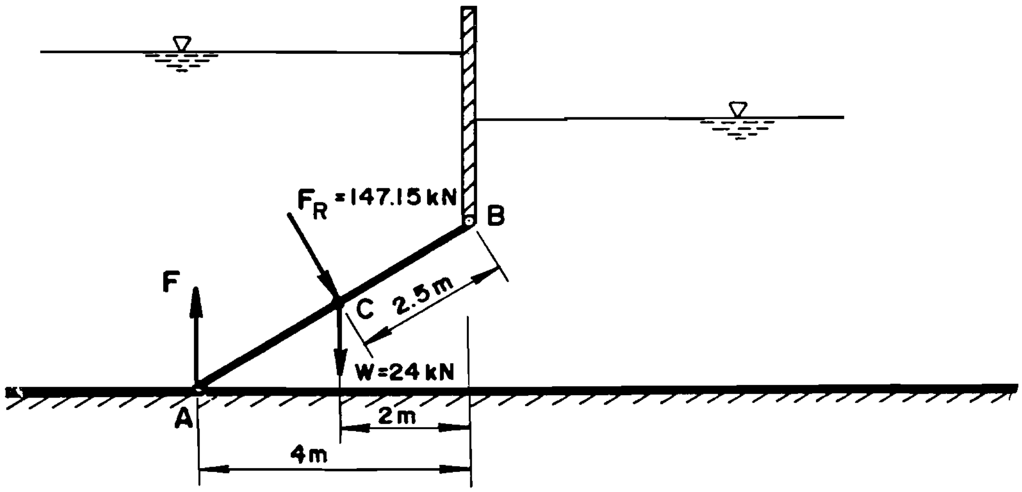
Thus, $\quad F_R = 441.45 - 294.3 = 147.15$ kN

Taking moments about the hinge B, the location of F_R from B is found as

$$\bar{x} = \frac{441.45 \times (2.5 + 0.278) - 294.3 \times (2.5 + 0.417)}{147.15}$$
$$= 2.5\ m$$

i.e F_R acts through C.

c) The force F required to just open the gate will be obtained by taking moments of the forces shown in the sketch below about the hinge B.



$$\sum M_B = F \times 4 - 147.15 \times 2.5 - 24 \times 2 = 0$$

Therefore, $F = \dfrac{147.15 \times 2.5 + 48}{4} = 103.97\ kN$

Example 3.9

A triangular opening in the form of an isosceles triangle, with dimensions shown in Figure E 3.9 and with its axis of symmetry horizontal, is closed by a plate. Water stands at 9 m from the axis of symmetry. Determine the resultant hydrostatic force on the plate and its centre of pressure.

Solution:

Plate area = 1/2 x 6 x 3 = 9 m^2.

Depth to centroid of plate = 9 m = h_c

Total hydrostatic force on plate = F = $\gamma_w h_c A$

= 9.81 x 9 x 9 = 794.61kN

The force F acts through the centre of pressure CP and normal to the plate.

To determine the vertical location of the centre of pressure,

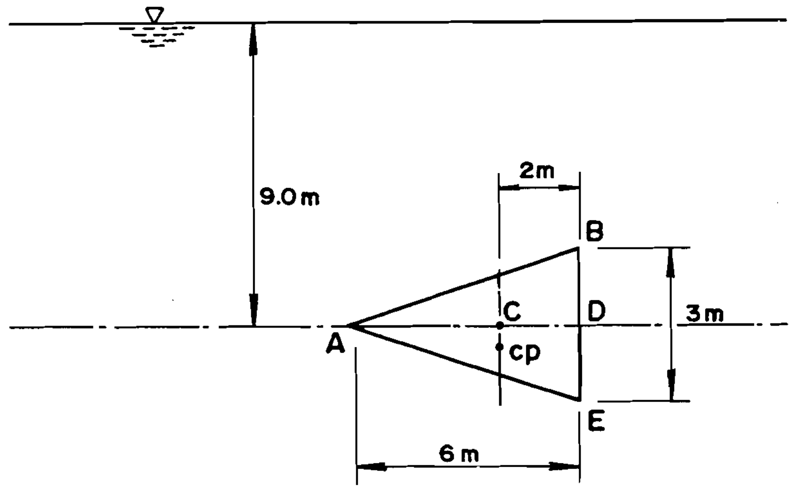


Figure E 3.9

it is necessary to determine the second moment of area of the triangle about axis AD since $y_{cp} = Y_c + \dfrac{I_c}{Y_c A}$.

The following steps will be used to determine I_c.

i) Split the plate into two triangles: ABD and ADE.

ii) Determine the second moment of area of the two triangles ABD and ADE separately about line AD.

iii) Add the results in (ii) to obtain the second moment of area I_c of triangle ABC about axis AD.

Thus, second moment of area of triangle ABD about AD is given by

$$\frac{bh^3}{12} = \frac{6 \times 1.5^3}{12} = 1.6875\ m^4$$

Second moment of area of triangle ADE about AD is

$$\frac{bh^3}{12} = \frac{6 \times 1.5^3}{12} = 1.6875\ m^4$$

Therefore, the second moment of area of triangle ABC about the centroidal axis AD is:

$$I_c = 1.6875 + 1.6875 = 3.375\ m^4$$

The depth h_{cp} to the centre of pressure CP will be:

$$h_{cp} = h_c + \frac{I_c}{h_c A} = 9 + \frac{3.375}{9 \times 9} = 9.042\ m.$$

In the horizontal direction the centre of pressure CP is located on the vertical passing through the centroid C i.e. the vertical at 6/3 = 2 m from BE.

Example 3.10

A vertical, symmetrical trapezoidal gate with its upper edge located 5 m below the free surface is shown in Figure E 3.10, Determine the total hydrostatic force and its centre of pressure.

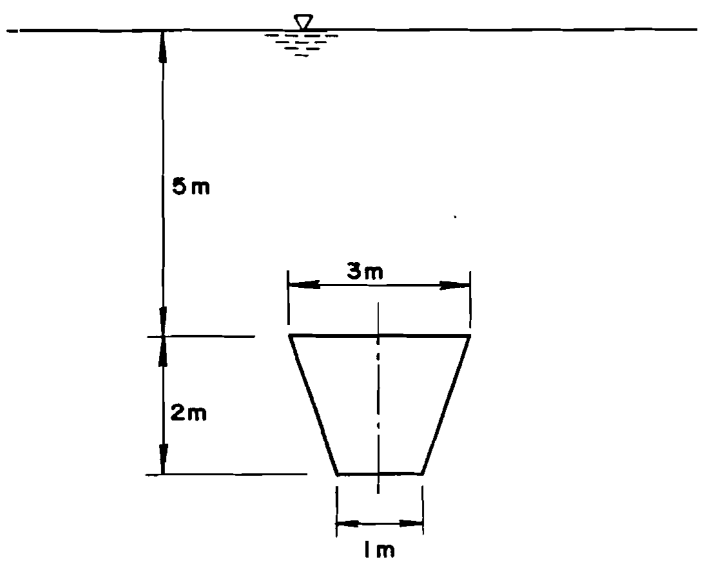


Figure E 3.10

Solution:

The total hydrostatic force = $F = \gamma_\omega h_c A$

The depth to centroid C = $h_c = 5 + \dfrac{2(2 \times 1 + 3)}{3(1 + 3)} = 5.833\ m$

The area = $A = 2 \times (3 + 1)/2 = 4\ m^2$

Therefore, F = 9.81 x 5.833 x 4 = 228.89 kN

The location of the centre of pressure is obtained from:

$$y_{cp} = y_c + I_c/(A.y_c)$$

$$I_c = \frac{h^3(a^2 + 4ab + b^2}{36\ (a + b)}$$

where a is the length of the shorter side, b the length of the longer side and h the distance between these parallel sides.

Thus,

$$I_c = \frac{2^3(1^2 + 4 \times 1 \times 3 + 3^2)}{36\ (1 + 3)} = \frac{8(22)}{36 \times 4} = 1.222\ m^4$$

$$y_c = h_c = 5.833\ m$$

Therefore, $y_{cp} = 5.833 + \dfrac{1.222}{4 \times 5.833} = 5.885\ m$ below the free surface.

Example 3.11

An inverted semicircular plane gate shown in Figure E 3.11 is installed at 45° inclination as shown. The top edge of the gate is at 3 m below the water surface. Determine the total hydrostatic force and the centre of pressure.

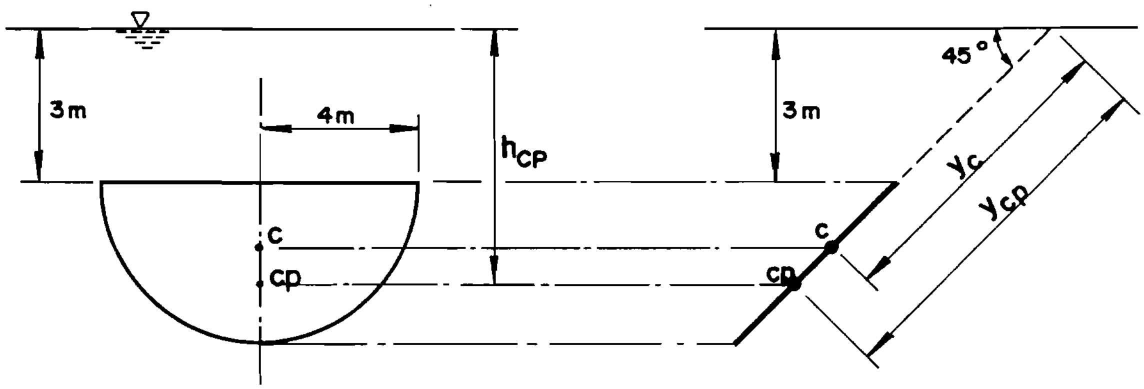


Figure E 3.11

Solution:

The total hydrostatic pressure = $F = \gamma_\omega h_c . A$

$$h_c = y_c \, Sin \, 45°$$

$$y_c = 3/Sin \, 45° + \frac{4r}{3\pi} = 4.24 + \frac{4 \times 4}{3\pi} = 5.94 \; m.$$

Therefore, $h_c = 5.94$ Sin 45° = 4.20 m.

$$Area\ of\ gate = A = \frac{1}{2}\pi r^2 = \frac{1}{2} \times \pi \times 4^2 = 25.13\ m^2$$

$$Thus, \quad F = 9.81 \times 4.20 \times 25.13 = 1.035\ MN$$

$$y_{cp} = y_c + \frac{I_c}{y_c A} = 5.94 + \frac{0.11 \times 4^4}{5.94 \times 25.13} = 6.13\ m.$$

$$and \quad h_{cp} = y_{cp}\ Sin\ 45^o = 4.33\ m$$

Example 3.12

A log hods water as shown in Figure E 3.12. Determine

a) The force pushing against the obstruction (dam) per metre length of log.

b) The weight of the log per metre length

c) The specific gravity of the log.

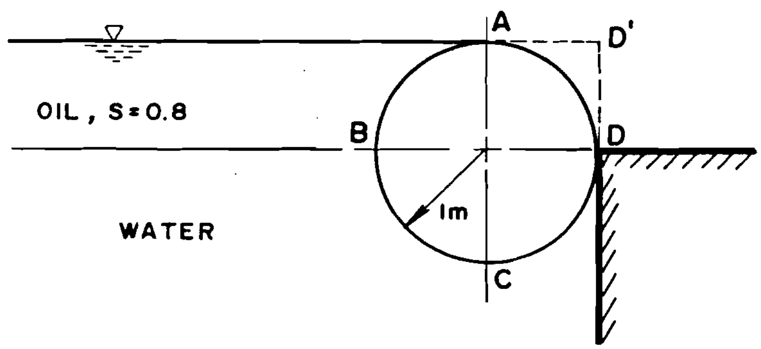
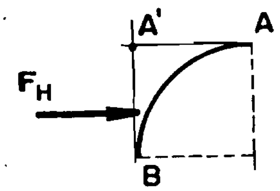


Figure E 3.12

Solution:

a) The force pushing against the dam is equal to the horizontal component of the hydrostatic force acting on the long.

Since the hydrostatic forces acting on surfaces BC and CD are equal and opposite to each other, they cancel out. Hence, the net hydrostatic force acting on the log is the horizontal component acting on curved surface AB.



$$F_H = \gamma_o h_c A$$

$$= (9.81 \times 0.8)0.5 \times 1 \times 1 = 3.924\ kN$$

b) The weight of the log is equal to the net vertical hydrostatic force on the log. The vertical component of the hydrostatic force is composed of the vertically upward force on surface BCD and the vertically downward force on surface AB.

On surface AB, the vertical force F_1 is equal to the weight of the oil supported by it. i.e.

$$F_1 = [(1 \times 1) - \frac{\pi}{4} \times 1^2] \times 1 \times 0.8 \times 9.81 = 1.684\ kN \text{ downwards}$$

On surface BCD the vertical force F_2 equals the weight of water and oil (real and virtual) supported by it. Thus

$$F_2 = (2 \times 1 \times 0.8 \times 9.81) + \frac{\pi}{2} \times 1^2 \times 1 \times 9.81$$

$$= 15.696 + 15.409 = 31.105\ kN\ ,\ upwards$$

Net vertical hydrostatic force = $F_2 - F_1$,

$F_2 - F_1 = 31.105 - 1.684 = 29.421$ kN upwards

Therefore, the weight of the log per metre is <u>29.421</u> kN

c) To determine the specific gravity of the log, determine its density first.

$$\rho_\ell \times 9.81 \times \pi \times 1^2 \times 1 = 29421\ N$$

from which , $\rho_\ell = \dfrac{29421}{\pi \times 9.81} = 954.6\ kg$

Specific gravity of log, $S_\ell = \rho_\ell/\rho_w = \dfrac{954.6}{1000} = \underline{0.955}$

Example 3.13

Referring to Figure E 3.13, calculate the force F required to hold the 1.4 m wide gate AB in a closed position if y = 0.8 m.

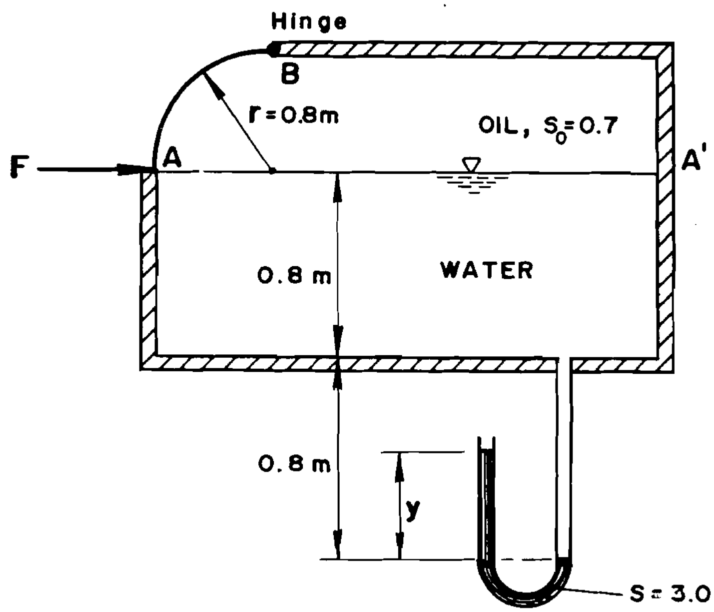


Figure E 3.13

Solution:

Let the pressure at the interface AA' between oil and water be p_A.
Starting from the open end of the manometer, the hydrostatic pressure variation gives:

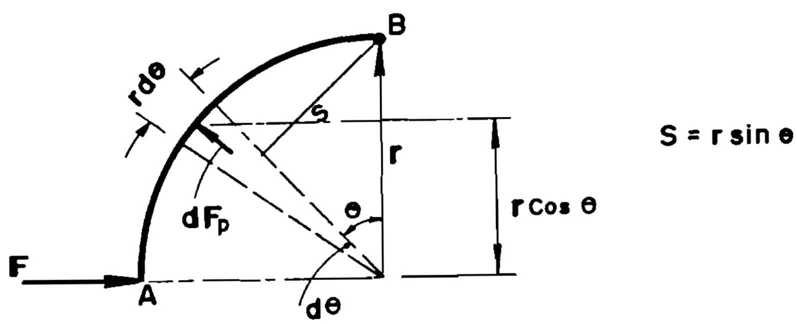
$$0 + y \times 3\gamma_w - 1.6\gamma_w = P_A$$

$$\text{or} \quad 0.8 \times 3\gamma_w - 1.6\gamma_w = P_A$$

$$\text{Thus:} \quad P_A = (2.4 - 1.6)\gamma_w = 0.8\ \gamma_w$$

i.e. head at interface = 0.8 m of water = 0.8/0.7 = 1.143 m of oil.

Referring to the following sketch:



The elementary hydrostatic force dF_p acting normal to the elementary area of length $rd\theta$ of the gate is given by:

$$dF_p = \gamma_o . h_c . A = 0.7\gamma_w(1.143 - r\cos\theta)(1.4 \times rd\theta)$$

$$= 0.7\gamma_w[1.143 - 0.8\cos\theta] \times 1.4 \times 0.8\,d\theta$$

For equilibrium, the sum of moments about the hinge B is zero.

$$\sum M_B = 0.8F - \int_o^{\pi/2} dF_p.s = 0$$

$$or \quad 0.8F = \int_o^{\pi/2} dF_p.s$$

$$= \int_o^{\pi 2} 0.7 \times 9810[1.143 - 0.8\cos\theta] \times 1.12\, d$$

$$= 6152.83 \int_o^{\pi/2} [1.143\, Sin\theta - 0.8\, Cos\theta\, Sin\theta]$$

$$= 6152.83[-1.143\, Cos\theta + 0.4\, Cos^2\theta]_o^{\pi/2}$$

$$= 6152.83[1.143 - 0.4]$$

$$= 4571.55$$

Thus: $F = \underline{5714\ N}$

Example 3.14

Neglecting the weight of the 4.2 m wide gate AB shown in figure E 3.14, determine

a) The vertical and horizontal components of the hydrostatic force on the gate including the location of the line of action.
b) The minimum moments M required to hold the gate.

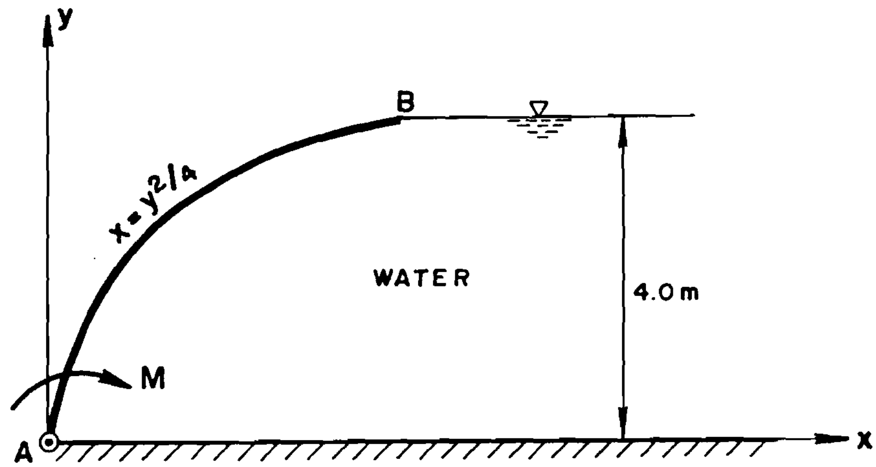
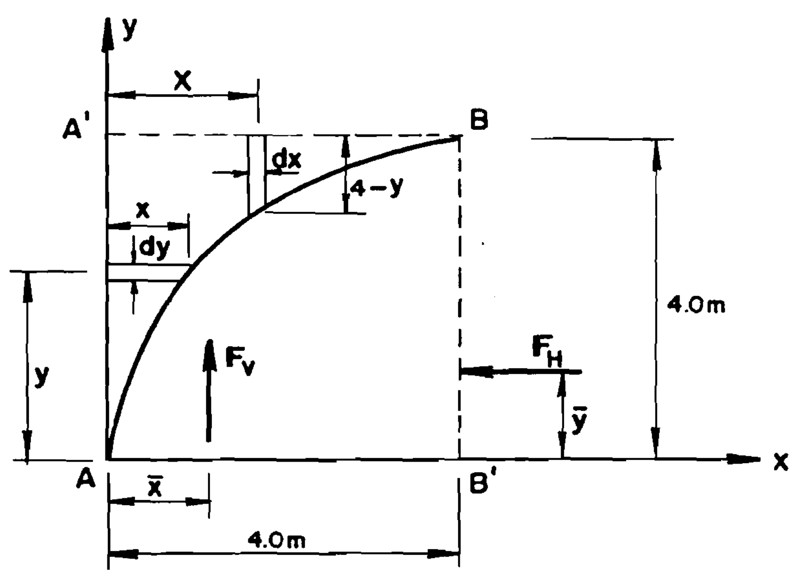


Figure E 3.14

Solution

Referring to the following sketch:



a) The horizontal component of the hydrostatic force, F_H, is the force acting on the projected area BB'

Therefore,

$$F_H = \gamma_w . h_c . A = 9.81 \times 2 \times (4 \times 4.2) = 329.616\ kN$$

$$y_{cp} = y_c + \frac{I_c}{y_c A} = 2 + \frac{4.2 \times 4^3}{12 \times 2 \times 4.2 \times 4} = 2.667\ m$$

Thus: $\bar{y} = 4 - 2.667 = 1.333\ m.$

The vertical component, F_v, of the hydrostatic force is equal to the weight of the volume of fluid bounded in AA'B and acts through the centroid of this volume. i.e.

$$F_v = \int_o^4 9.81 . x . d_y \times 4.2$$
$$= \int_o^4 41.202 . \frac{y^2}{4} d_y = 10.301 \frac{y^3}{3} |_o^4 = 219.744\ kN$$

To locate the line of action of F_v ,

$$F_v . \bar{x} = \int_o^4 (4 - y)\, d_x . 4.2 \times 9.81 . x = 41.202 \int_o^4 (4 - y)\, x d_x$$

$$= 41.202 \int_o^4 (4 - 2 . x^{1/2})\, x d_x\ , \quad since\ \ y = 2\sqrt{x}$$

$$= 41.202\, [2x^2 |_o^4 - \frac{2}{5} \times 2 . x^{5/2} |_o^4]$$

$$= 41.202\, [32 - 25.6] = 263.693$$

Therefore, $\bar{x} = \frac{263.693}{F_v} = \frac{263.693}{219.744} = \underline{1.2\ m}$

b) Summing moments about A to obtain the moment M required to hold the gate AB,

$$\sum M_A = M - F_v.\bar{x} - F_H.\bar{y} = o$$

$$\textit{from which,} \quad M = F_v\bar{x} + F_H\bar{y} = 263.693 + 329.616 \times 1.333$$

$$\therefore \quad M = 703.07 \ kN\text{-}m$$

3.9 Buoyancy and Stability of Submerged and Floating Bodies

Since the pressure in a fluid at rest increases with depth, the fluid exerts a resultant upward force on any body which is fully or partially immersed in it. This force is known as the Buoyant Force.

The principles of buoyancy and floatation, established by Archimedes (288-212 B.C), state that

i) a body immersed in a fluid in buoyed up by a force equal to the weight of the fluid displaced by the body and

ii) A floating body displaces its own weight of the fluid in which it floats.

These principles can be easily proven using the principle of hydrostatic force on surfaces.

3.9.1 Buoyant Force

Consider a body ABCD, shown in Fig. 3.18, submerged in a liquid of constant density ρ.

Referring to Fig. 3.18, A'C' is the projection of the body on a horizontal plane and B'D' is its projection on a vertical

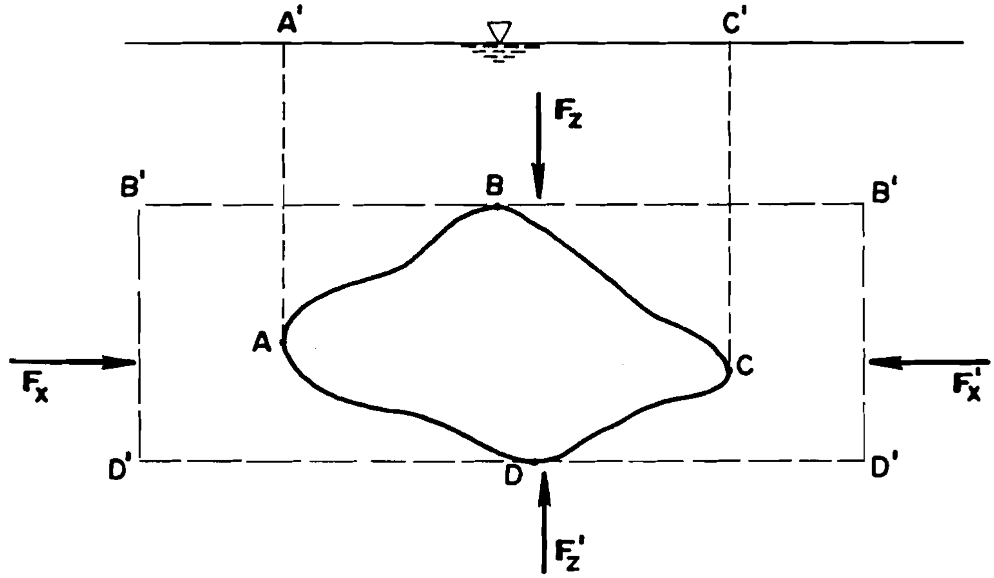


Figure 3.18 Buoyant Force

plane. Force F_x acting to the right is the horizontal component of the hydrostatic force on surface BAD and force F'_x acting to the left is the horizontal component of hydrostatic force on surface BCD. Both F_x and F'_x are equal to the force acting on vertical plane surface B'D' and since they are equal, opposite and collinear, they cancel each other. Hence, the resultant horizontal hydrostatic force on a submerged body is zero.

Force F_z is the downward, vertical component of the hydrostatic force acting on surface ABC. F_z is equal to the weight of the liquid volume A'A BC C'A' i.e. $F_z = \varrho g$. Vol. A'A BCC'A'.

Similarly, F'_z is the upward vertical component of the hydrostatic force on surface ADC and is equal to the weight of the liquid volume A'A DC C'A', i.e. $F'_z = \varrho g$. Vol. A'A DC C'A'.

The net upward force is the buoyant force F_B, which is

$F_B = F_Z' - F_Z$. i.e

$FB = F_Z' - F_Z = \varrho g$. Vol.A'A DC C'A' $- \varrho g$. Vol A'A BC C'A'

or $F_B = \varrho g$. Vol. ABCD (3.19)

Thus, the buoyant force F_B is the weight of the liquid displaced by the body and acts vertically upwards through the centre of buoyancy which is coincident with the centroid of the volume of the displaced liquid. Similar considerations show that for a body partially immersed in a liquid, the buoyant force is equal to the weight of the displaced liquid.

Considering the vertical equilibrium of a body submerged in a fluid, the condition of floatation of the body depends upon the relative magnitude of the weight of the body and the buoyant force. If the body is heavier than the weight of the fluid it can displace, it will sink to the bottom unless it is prevented from doing so by the application of an upward supporting force. If the weight of the body is lighter than the weight of the liquid it can displace when completely submerged in the fluid, it will rise above the surface to a position such that the weight of the displaced liquid is equal to the weight of the body.

The principle of buoyancy can be used to determine the weight, volume and consequently the specific weight and specific gravity of an object by weighing the object in two different fluids of known specific weights. Consider an object suspended and weighed in two fluids with specific weights γ_1 and γ_2 as shown in Fig. 3.19. Let the weight of the object be W and its volume V.

Vertical equilibrium of forces in Figure 3.19(a) gives:

$$F_1 + \gamma_1 V = W$$

Vertical equilibrium of forces in Figure 3.19(b) gives:

$$F_1 + \gamma_2 V = W$$

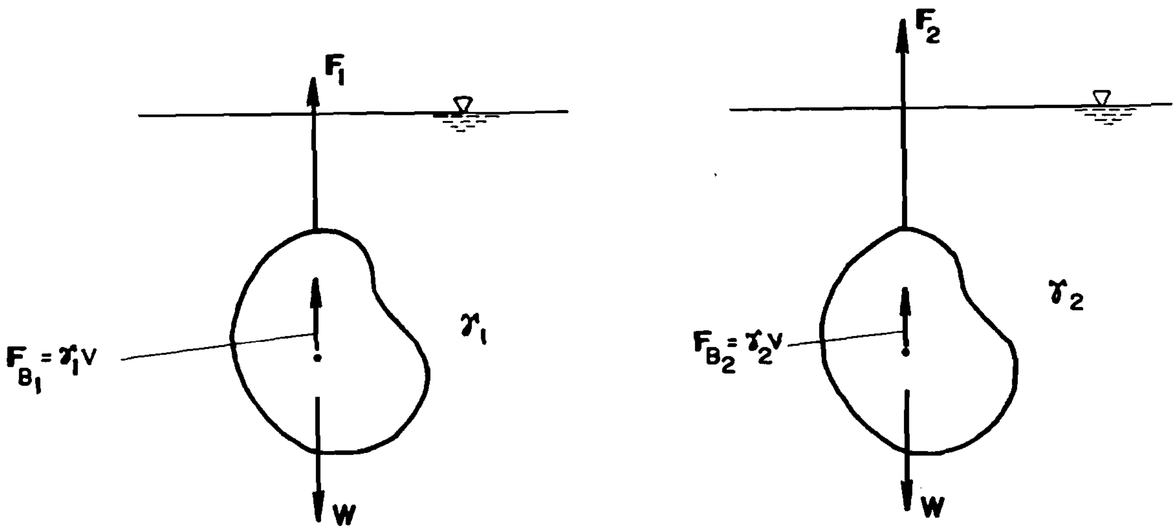


Figure 3.19

Equating the above two equations and rearranging:

$$V(\gamma_2 - \gamma_1) = F_1 - F_2$$

From which $$F = \frac{F_1 - F_2}{\gamma_2 - \gamma_1} \qquad 3.20$$

Substituting the value of V from the above equation in any of the two equilibrium equations, the following equation for the weight of the body may be obtained.

$$W = \frac{F_1\gamma_2 - F_2\gamma_1}{\gamma_2 - \gamma_1}$$

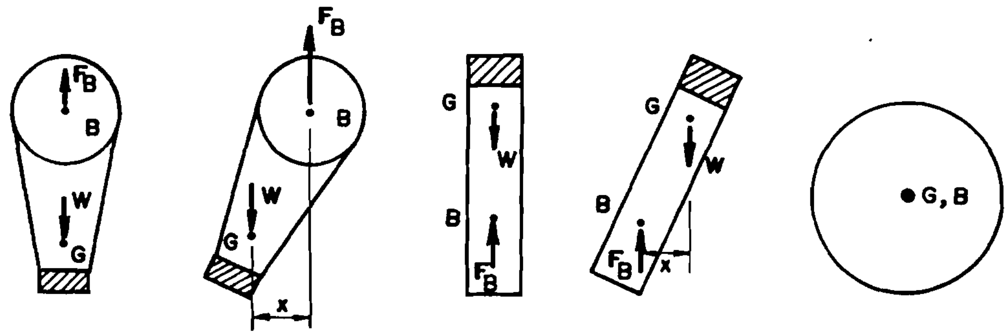
The specific weight of the body will be $\gamma = W/V$. It should be noted that the body should not be weighed in a liquid in which it dissolves.

The hydrometer, which is an instrument used to determine the specific gravity of liquids, is constructed on the basis of the principle of buoyancy. It consists of a closed glass tube with an enlarged bulb shape at the bottom in which lead shots are kept to allow it to float vertically when immersed in a liquid. The hydrometer sinks to different depth when immersed in liquids of different specific gravities, sinking deeper in lighter liquids than in heavier liquids. The graduations on the stem, from which the specific gravities are read directly at the meniscus, are obtained by calibration in liquids of known specific gravities. The reading 1.00 corresponds to that of distilled water.

3.9.2 Stability of Submerged Bodies

A submerged or a floating body is said to be stable if it comes back to its original position after a slight disturbance. The stability of a submerged body depends upon the relative position of its centre of gravity and its centre of buoyancy both of which have fixed positions.

Consider the three possible relative positions of centre of buoyancy B and centre of gravity G of submerged bodies shown in Figure 3.20.
Fig. 3.20 (a) shows a ballon where the centre of buoyancy is always above the centre of gravity. A small angular displacement generates a restoring couple, between the buoyant force F_B and the weight W, which brings the balloon back to its original position. This is an example of a stable equilibrium of a submerged body. In Fig. 3.20 (b) is shown a submerged body where the centre of buoyancy is below the centre of gravity. In this case, a small angular displacement generates a couple which further increases the displacement. This is a situation of unstable equilibrium. For a submerged, homogeneous spherical object shown in Fig...(c) the centre of gravity and the centre of buoyancy coincide and any angular



a) Stable Equilibrium: B always above G

b) Unstable Equilibrium: B always below G

c) Neutral Equilibrium B and G coinciding

Figure 20

displacement does not result in development of a couple. In this case neutral equilibrium is said to occur.

The above considerations show that for completely submerged bodies the requirements for stability are:

(i) The centre of buoyancy and centre of gravity must lie on the same vertical line in the undisturbed position and

(ii) The centre of buoyancy must be located above the centre of gravity for stable equilibrium

3.9.3 Stability of Floating Bodies

For a floating body, the centre of buoyancy need not be located above the centre of gravity for stability. When a floating body which is partially submerged in a liquid is given a small angular displacement about a horizontal axis, the shape of the displaced volume of liquid changes and consequently the centre of buoyancy moves relative to the body. As a result, restoring couple can be generated and stable equilibrium achieved even when the centre of gravity G of the body is above the centre of buoyancy B.

Figure 3.21 may be used to illustrate the situation. Position (a) is the undisturbed position where the centre of buoyancy and the centre of gravity are on the same vertical. The weight W of the boat and the buoyant force F_B are equal, opposite and collinear. Hence the boat is in equilibrium.

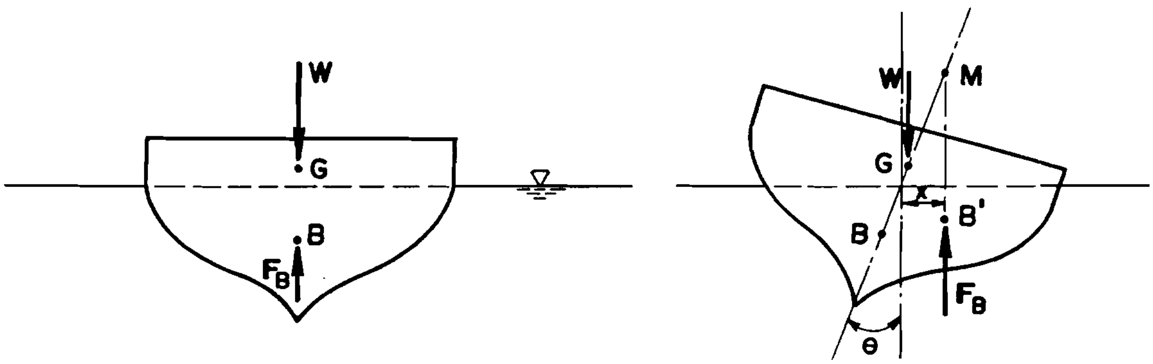


Figure 3.21 Stability of a floating body

Position (b) shows the boat just as it has undergone through a small angular displacement θ. It is here assumed that the location of the centre of gravity G remains unchanged (this is true only for situation of unshifting cargo). In this position, the displaced volume on the right hand side increases and that on the left hand side decreases as a result of the displacement and the centre of bouyancy shifts to the right to a new position B'. The buoyant force F_B (still equal to W) now acts vertically upwards through B' and the weight W acts downwards through G. F_B and W now constitute a restoring counter-clockwise couple which brings the boat back to its original position. The line of action of F_B now intersects the axis BG at M. This point M is known as the Metacentre. Thus, as long as M is above G, a restoring couple will be generated and the floating object is in stable equilibrium. If M falls below G, the generated couple will be an overturning couple and the equilibrium would be unstable.

Thus, for floating objects, stability would be achieved even when B is below G as long as the metacentre M is above G. The special case where G and B coincide constitutes a situation of neutral equilibrium.

The distance of the metacentre M above G i.e. MG, is known as the metacentric height. It must be positive (i.e. M must be above G) for stable equilibrium. For small values of heel angle θ, the metacentric height is practically constant. The concept of metacentre and metacentric height is very useful in the design of ship profiles, barrages and caissons and the estimation of the metacentric height under various conditions of loading is important to ensure stability of the floating body.

Metacentric Height: An expression for the metacentric height may be obtained by considering the cross-section of a ship through its centre of gravity as shown Figure 3.22. The plan view at the water line is also shown.

In Figure 3.22, AB is the original water line when the floating object was in the undisturbed, upright position with the centre of buoyancy B and the centre of gravity G in the same vertical axis of symmetry BG. CD is the new water line after the floating object has experienced a small rotation through an angle θ. As a result of the rotation, the triangular wedge BOD on the right side has come out of the liquid while an identical wedge AOC has gone inside. The total displaced volume does not change but its shape has changed and consequently the position of the centre of buoyancy shifts from B to B'. The triangular wedges AOC and BOD correspond to a gain and a loss respectively in buoyant forces ΔF_B.

The moment caused by these two forces is $\Delta F_B.S$ and has a clockwise sense. This must be equal to the opposite moment resulting from the shifting of the total buoyant force F_B to B'. This moment is counterclockwise and is equal to $\rho g.V.\delta$,

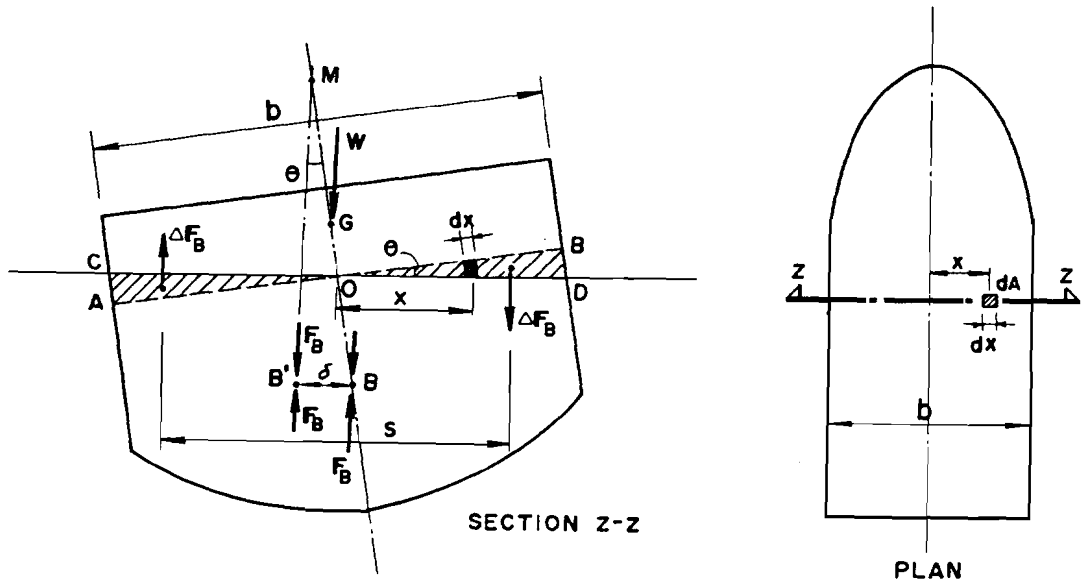


Figure 3.22 Centre of buoyancy and metacentre of a floating body.

where V is the total volume displaced by the floating object and ρ is the liquid density.

Thus $$\Delta F_B . S = \rho g . V . \delta$$

Therefore $$\delta = \frac{\Delta F_B . S}{\rho g . V}$$

Since $\delta = \overline{MB} . Sin\theta$,

$$\overline{MB} = (\Delta F_B . S) / (\rho g V . Sin\theta)$$

The buoyancy force produced by wedge AOC (see Figure 3.22) can be estimated by considering a small prism of the wedge. Assume that the prism has a horizontal area dA and is located at a distance x from the axis of rotation 0. The height of the prism is $x.(\tan\theta)$. For small angle θ it may be approximated by $x.\theta$. Thus the buoyancy force produced by the small prism is $\rho g.x\theta.dS$. The bouyancy force ΔF_B of the wedge AOC will be the sum of all these forces i.e.

$$\Delta F_B = \int \rho g x.\theta.dA$$

The moment produced by the couple is:

$$\Delta F_B.S = \int_{-b/2}^{b/2} \rho g x.\theta.dA.x = \rho g\theta \int_{b/2}^{b/2} x^2 dA$$

or $\quad \Delta F_B.S = \rho g.\theta.I_{yy}$

Where I_{yy} is the second moment of area about axis y-y.

Substituting, $\quad \overline{MB} = (\rho g\theta.I_{yy})/\rho g.V.Sin\theta = \dfrac{I_{yy}.\theta}{V.Sin\theta}$

But limit $\theta/_{sin\theta} = 1,$
$\theta \rightarrow 0$

Therefore, $\quad \overline{MB} = \dfrac{I_{YY}}{V}$ 3.21

The Metacentric height $\overline{MG} = \overline{MB} \pm \overline{GB}$

or $$\overline{MG} = \frac{I_{yy}}{V} \pm \overline{GB} \qquad 3.22$$

Since the position of G and B is known from the sectional geometry or design data of the vessel, the distance GB can be determined. In Eqn. 3.22, the (+) sign is used when G is below B and the (-) sign used when G falls above B. If the value of $\overline{MG}$ as determined above is positive, then the floating object is in stable equilibrium. If MG is negative, the floating object is unstable and if MG is zero, the object is in neutral equilibrium.

Example 3.15

A concrete block that has a total volume 1.5 m^3 and specific gravity of 1.80 is tied to one end of a long hollow cylinder. The cylinder is 3 m long and has a diameter of 80 cm. When the assembly is floated in deep water, 15 cm of the cylinder remain above the water surface. Determine the weight of the cylinder.

Solution:

Referring to the following sketch: (Fig. E 3.15)

let W_c = Weight of the cylinder
W_B = Weight of the concrete block
F_B = Buoyant force of the assembly.

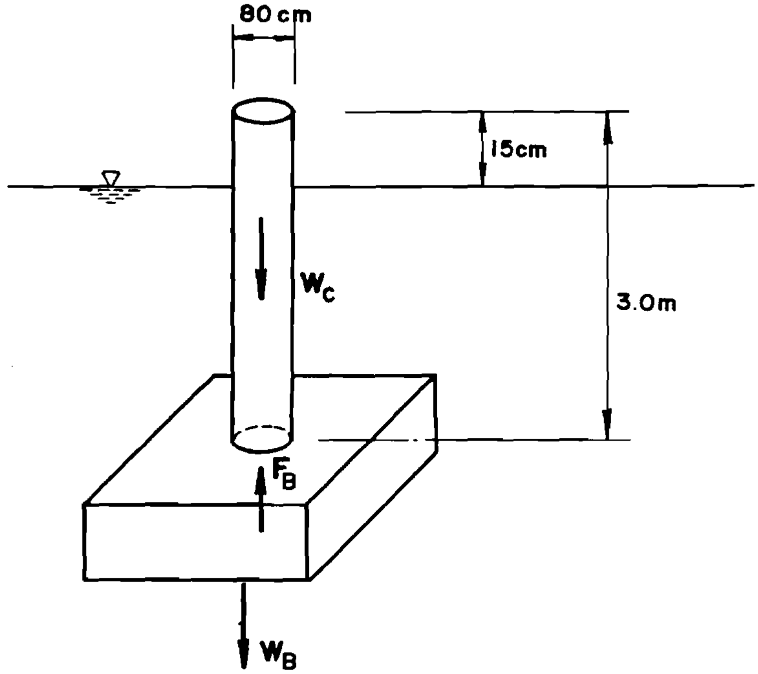


Figure E 3.15

Volume of Water displaced = V

$$V = \frac{\pi (0.8)^2}{4} (3 - 0.15) + 1.5 \; m^3$$

$$= 1.433 + 1.5 = 2.933 \; m^3$$

Therefore, $F_B = \gamma_w V = 9810 \times 2.933 = 28{,}772.7 \; N$

$$W_B = 1.80 \times 9810 \times 1.5 = 26{,}487.0 \; N$$

For equilibrium:

$$W_c + W_B - F_B = 0$$

Thus $W_c = F_B - W_B$

$$= 28772.7 - 26487.0 = \underline{2285.7 \; N}$$

Example 3.16

Two cubes of the same size, 1 m^3 each, one of specific gravity 0.80 and the other of specific gravity 1.1, are connected by a short wire and placed in water. What portion of the lighter cube is above the water surface? What is the tension in the wire?

Solution:

Referring to Figure E 3.16:

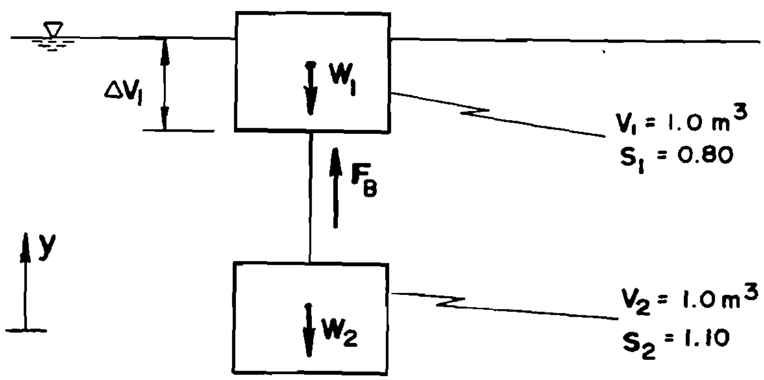


Figure E 3.16

Let ΔV_1 be the volume of submergence of the lighter cube.

W_1 = Weight of lighter cube

W_2 = Weight of heavier cube

F_B = Total Buoyant force of the assembly

Then $F_B = (V_2 + \Delta V_1)\gamma_w = (1 + \Delta V_1)9810 = 9810 + \Delta V_1 . 9810$

For equilibrium,

$$\sum F_y = 0$$

$$i.e. \quad F_B - W_1 - W_2 = 0$$

$$9810 + 9810\,\Delta V_1 - 0.8 \times 9810 \times 1 - 1.1 \times 9810 \times 1 = 0$$

$$9810(1 + \Delta v_1 - 0.8 - 1.1) = 0$$

$$1 + \Delta v_1 - 1.9 = 0$$

$$Therefore, \quad \Delta V_1 = 0.9$$

Thus 0.1 of the volume or 10% of the lighter cube is above the water surface.

To determine the tension in the wire:
The buoyant force due to the heavier cube is:

$$F_{B_2} = \gamma_w V_2 = 9810 \times 1 = 9810\ N$$

Weight of heavier cube = W_2 1.1 x 9810 x 1 = 10,791 N

Equilibrium of the heavier cube requires that:

$T + F_{b_2} - W_2 = 0$, where T = tension in the wire.

Thus $T = W_2 - F_{B_2} = 10{,}791 - 9810 = \underline{981\ N}$

Example 3.17

A rectangular barge 20 m long has a 5 m wide cross-section. The water line is 1.5 m above the bottom of the barge when it floats in the upright position. If the centre of gravity is 1.8 m above the bottom, determine the metacentric height.

Solution:

Referring to Fig. E 3.17

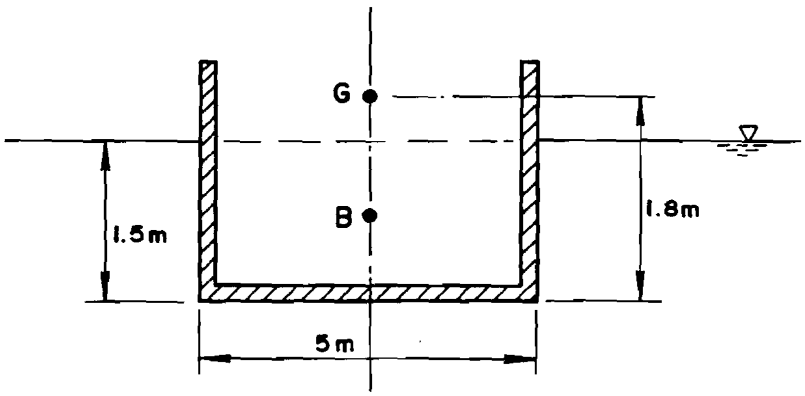


Figure E3.17

The position of B will be at $\frac{1.5}{2} = 0.75\ m$ above the bottom.

Since G is above B, the equation of the metacentric height will be:

$$\overline{MG} = \frac{I_{yy}}{V} - GB$$

$$V = Volume\ displaced = 20 \times 5 \times 1.5 = 150\ m^3$$

$$\frac{I_{yy}}{V} = \frac{bd^3}{12V} = \frac{20 \times 5^3}{12 \times 150} = 1.39\ m$$

$$GB = 1.8 - 0.75 = 1.05\ m$$

Therefore, the metacentric height MG will be

$$\overline{MG} = 1.39 - 1.05 = 0.34\ m$$

Example 3.18

A uniform wooden circular cylinder 400 mm in diameter and having a specific gravity of 0.6 is required to float in oil of specific gravity 0.8. Determine the maximum length of the cylinder in order that the cylinder may float vertically in the oil.

Solution:

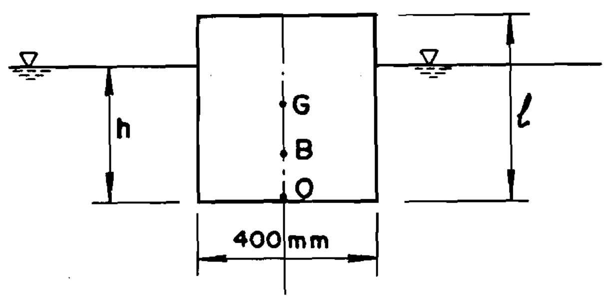


Figure E 3.18

let length of the cylinder be ℓ

Weight of cylinder $= 0.6\gamma_w . \frac{\pi d^2}{4} \times \ell$

Weight of displaced volume of oil $= h . 0.8\gamma_w \frac{\pi d^2}{4}$

Equating the two, depth of immersion h will be:

$$h = \frac{0.6}{0.8}\ell = \frac{3\ell}{4}$$

$$OB = \frac{h}{2} = \frac{3}{8}\ell$$

$$OG = \frac{\ell}{2}$$

Therefore, $BG = OG - OB = \frac{\ell}{2} - \frac{3}{8}\ell = \frac{\ell}{8}$

Second moment of area of the circular section is

$$I = \frac{\pi d^4}{64}$$

$$I = \frac{\pi(400)^4}{64} = 400 \times 10^6 \ \pi \ mm^4$$

Volume of oil displaced = V,

$$V = \frac{\pi}{4} d^2 . \frac{3\ell}{4} = \frac{3\pi d^2 \ell}{16} \ mm^3$$

$$= \frac{3\pi(400)^2 \ell}{16} = 30 \times 10^3 \pi \ell \ mm^3$$

The metacentric height $\overline{MG}$ is given by:

$$\overline{MG} = \frac{I}{V} - BG$$

For the cylinder to float vertically in oil,

$$\overline{MG} \geq 0$$

$$or \quad \frac{I}{V} - BG \geq 0$$

$$\frac{I}{V} = \frac{400 \times 10^6 \ \pi}{30 \times 10^3 \ \pi \ell} = \frac{40}{3\ell} \times 10^3 \ mm$$

$$\frac{40}{3\ell} \times 10^3 \geq \frac{\ell}{8}$$

$$or \quad \ell^2 \leq \frac{8 \times 40 \times 10^3}{3}$$

Therefore, $\ell \leq 326.6 \ mm$

Example 3.19

A ship of 50 MN displacement floating in water has a weight of 100 kN moved 10 m across the deck causing a heel angle of 5°. Find the metacentric height of the ship.

Solution:

Referring to Figure E 3.19

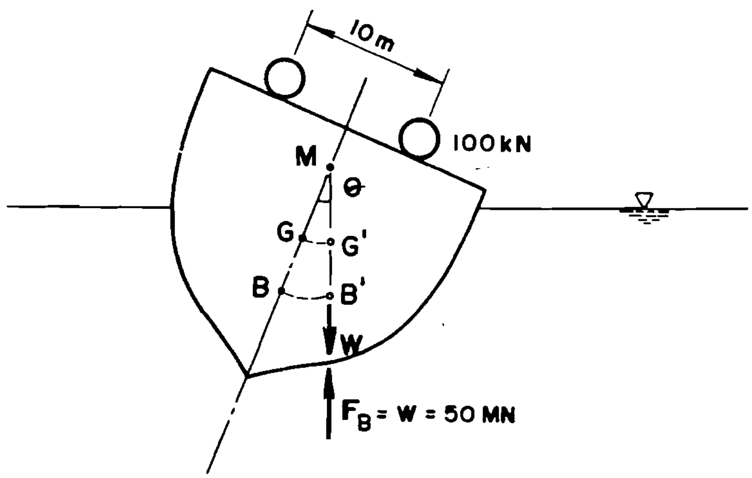


Figure E 3.19

Moment causing the ship to heel = 100 x 10 = 1000 kN m.
= Moment due to shift of W from G to G'
= W x GG'

But, $GG' = \overline{GM}\,Sin\,\theta = \frac{1000}{W} = \frac{1000}{50 \times 10^3} = \frac{1}{50}\ m$

Hence, the Metacentric height

$$\overline{GM} = \frac{1}{50\,Sin\,\theta} = \frac{1}{50\,Sin\,5^o} = 0.23\ m$$

3.10 Relative Equilibrium of Liquids

If a liquid is contained in a vessel which is at rest, or moving with constant linear velocity, it is not affected by the motion of the vessel and the pressure distribution is hydrostatic. But if the container is given a continuous and constant linear acceleration or is rotated about a vertical axis with uniform angular velocity (resulting in a constant, inward acceleration), the liquid will eventually reach an equilibrium situation and move as a solid body with no relative motion between the fluid particles and the container. Such equilibrium of liquids is referred to as relative equilibrium of liquids. The two cases of practical interest are:

i) Uniform linear acceleration
ii) Uniform rotation about a vertical axis.

In both cases, since there is no relative motion between fluid particles, shear stress does not exist and the laws of fluid statics still apply, but in a modified form to allow for the effect of acceleration.

3.10.1 Uniform Linear Acceleration:

Consider the fluid element, shown in Fig. 3.23, in a vessel containing a liquid with density ϱ. Let the vessel be given a uniform linear acceleration with components a_x, a_y and a_z along the x, y, and z directions respectively. Let the pressure at the centre of the element be P and pressure gradients $\partial p/\partial x$, $\partial p/\partial y$, $\partial p/\partial z$ are assumed to exist in the x,y and z directions respectively. The forces acting on the fluid element are shown.

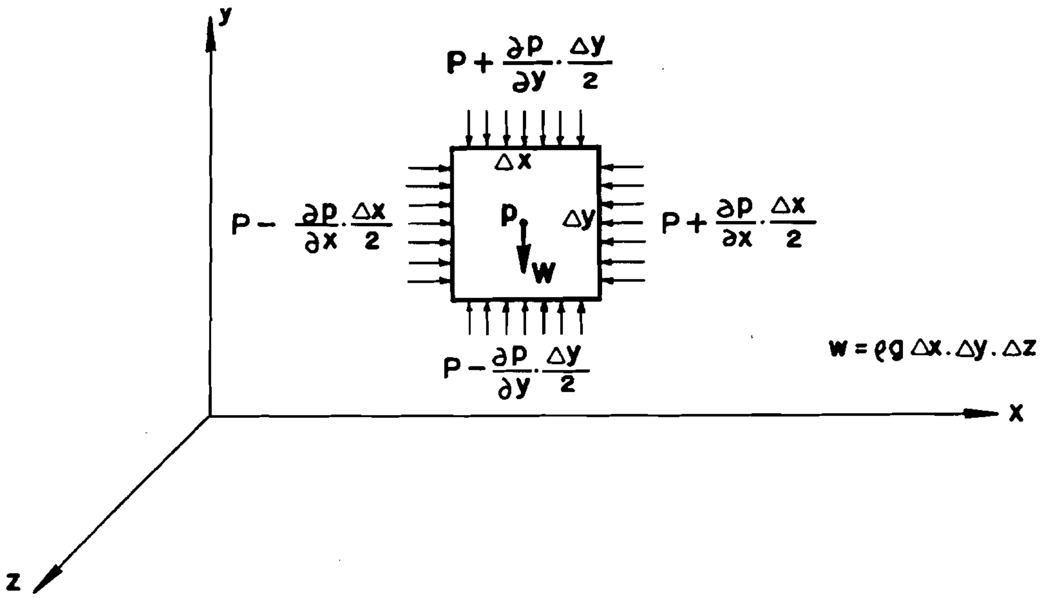


Figure 3.23 Forces on fluid element under linear acceleration

Applying Newton's Second Law, the net force in the x-direction is:

$$(p - \frac{\partial p}{\partial x}.\frac{\Delta x}{2})\Delta y.\Delta z - (p + \frac{\partial p}{\partial x}.\frac{\Delta x}{2})\Delta y.\Delta z = \rho\Delta x.\Delta y.\Delta z.a_x$$

which reduces to:

$$-\frac{\partial p}{\partial x} = \rho a_x \qquad (3.23)$$

In the y-direction, considering the weight of the fluid element, the net force will be

$$(p - \frac{\partial p}{\partial y}.\frac{\Delta y}{2})\Delta x.\Delta z - (p + \frac{\partial p}{\partial y}.\frac{\Delta y}{2})\Delta x.\Delta z - \rho g\Delta x\Delta y\Delta z = \rho a_y\Delta x\Delta y\Delta z$$

which reduced to:

$$-\frac{\partial p}{\partial y} = \rho(g + a_y) \qquad (3.24)$$

Similar considerations in the Z direction lead to:

$$-\frac{\partial p}{\partial z} = \rho a_z \qquad (3.25)$$

Consider a vessel shown in Fig. 3.24 having uniform linear acceleration in the x-y plane with components a_x and a_y in the x and y directions respectively.

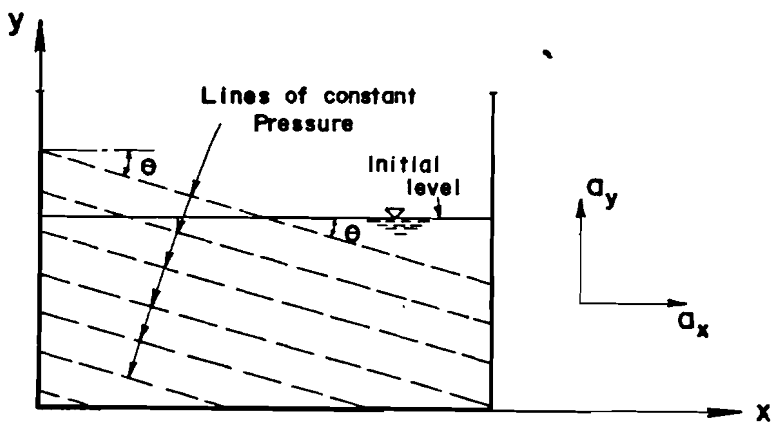


Figure 3.24 A vessel under uniform linear acceleration

The total pressure differential is given by:

$$dp = \frac{\partial p}{\partial x}.dx + \frac{\partial p}{\partial y}.dy$$

On lines of constant pressure, the total pressure differential will be zero.

Thus:

$$\frac{\partial p}{\partial x}dx + \frac{\partial p}{\partial y}.dy = 0$$

from which:

$$\frac{dy}{dx} = -\frac{\partial p/\partial x}{\partial p/\partial y} = -\frac{\rho a_x}{\rho (g + a_y)} = -\frac{a_x}{a_y + g} = \tan\theta \qquad (3.26)$$

This shows that the lines of constant pressure have a constant slope of $\tan\theta = -a_x/(a_y + g)$. Since the free surface is a line of constant pressure, the above conclusion also shows that lines of constant pressure are parallel to the free surface.

Once the position of the free surface is determined for a given acceleration, then the hydrostatic variation of pressure with depth applies as in fluid statics.

Horizontal Acceleration:

If a vessel containing a liquid moves with a constant linear horizontal acceleration a_x, say in the positive x direction, then $a_y = 0$. Then the slope of the line of constant pressure i.e. the slope of the free surface will be

$$\frac{dy}{dx} = -a_x/g = \tan\theta \qquad (3.27)$$

The variation of pressure with depth will be given by eqn.3.24 , with ay = 0, as:

$$\frac{dp}{dy} = -\rho g$$

Integrating,

$$p = -\rho g y + c$$

Measuring the depth h from the free surface vertically down, y will be replaced by (-h). Taking the free surface pressure as zero, the pressure p at a depth h from the free surface and at any section will be, as in hydrostatics,

$$p = \rho g h$$

Example 3.20

A rectangular tank 5m long, 2m wide and 3m deep contains water filled to 1.5m depth. It is accelerated horizontally at 4 m/sec^2 in the direction of its length. Compute a) the total hydrostatic force acting on each side, (b) the force needed to impart the acceleration.

If the tank is completely filled with water and accelerated in the direction of its length at the rate of 2.5 m/sec^2, how many liters of water will be spilled?

Solution:

Referring to Figure E 3.20

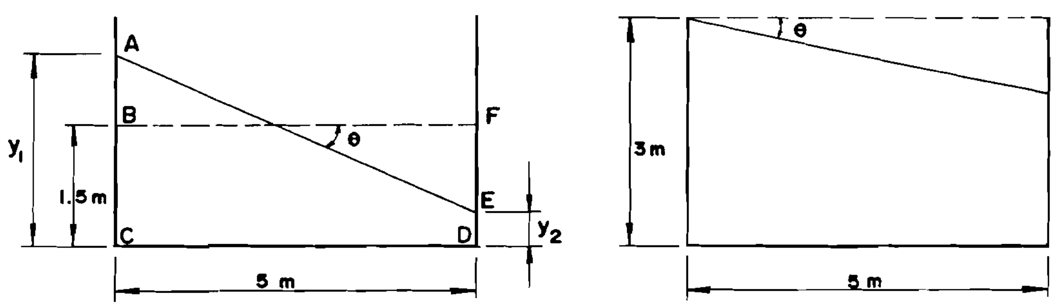


Figure E 3.20

$$\tan\theta = \frac{a_x}{g} = \frac{4}{9.81} = 0.408$$

$$Intercept\ AB = 2.5\tan\theta = 1.02m. = EF$$

$$Depth\ y_1 = 1.5 + 1.02 = 2.52m.$$

$$Depth\ y_2 = 1.5 - 1.02 = 0.48m.$$

a) Hydrostatic Force on end AB = F_{AB}

$$F_{AB} = \gamma \bar{h} A$$
$$= 9.81 \times \frac{2.52}{2}(2.52 \times 2) = \underline{62.297\ kN}$$

Hydrostatic Force on end ED = F_{ED}

$$F_{ED} = \gamma \bar{h} A$$

$$= 9.81 \times \frac{0.48}{2} \times (0.48 \times 2) = 2.260\ kN$$

b) Force needed to impart acceleration = F_{AB} - F_{ED}

$$F_{AB} - F_{ED} = 62.297 - 2.260$$
$$= 60.037\ kN$$

Inertial force of accelerated mass = mass x acceleration
= (1.5 x 5 x 2) x 1000 x4
= 60 kN

Hence, difference between force on each end is equal to the inertial force.

When the tank is full,

$$\tan\theta = \frac{a_x}{y} = \frac{2.5}{9.81} = 0.255$$

Drop in water surface on front side = 5 $\tan\theta$ = 1.275 m

$\therefore$ volume of water spilled

$$= \frac{1}{2} \times 1.275 \times 5 \times 2$$

$$= 6.375\ m^3 = 6375\ litres$$

Example 3.21 A rectangular oil tanker 3 m wide, 2.0 m deep and 10 m long contains oil, ϱ = 800 Kg/m^3, which stands at 1.0 m from the top of the tanker. Determine the maximum horizontal acceleration that can be given to the tanker without spilling the oil. If this tanker is closed and completely filled with oil and accelerated horizontally at 3 m/s^2 determine the total liquid thrust
i) on the front end (ii) on the rear end and (iii) on one of its longitudinal vertical sides.

Solution:

Referring to figure E 3.16

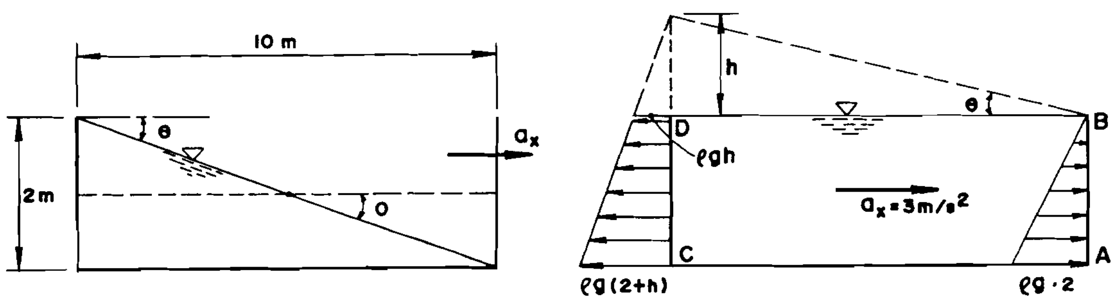


Figure E 3.21

For maximum acceleration without spilling, the level drops by 1 m from the original level.

$$Tan\theta = \frac{a_x}{g} = \frac{1}{5}$$

$$\therefore \quad a_x = g/5 = 9.81/5 = \underline{1.962 m/s^2}$$

When the tanker is completely filled and closed, there will be pressure built up at the rear end equivalent to the virtual oil column h that would assume a slope of a_x/g (Fig. E 3.21(b))

i) Total thrust on front AB $= \frac{1}{2}\rho g.2 \times 2 \times 3 = \underline{58.86\ kN}$

ii) Total thrust on rear end CD
Virtual rise of oil level at rear end is h

$$h = \tan\theta.10 = \frac{a_x}{g} \times 10 = \frac{3}{9.81} \times 10 = 3.06\ m$$

$\therefore$ Total thrust on CD

$$= \frac{\rho g(3.06) + \rho g(2 + 3.06)}{2}.(2\times 3)$$
$$= \underline{239\ kN}$$

iii) Total thrust on side ABCD = Volume of pressure prism which is equal to:

$$\frac{1}{2}\rho g(2 \times 2\times 10) + \frac{1}{2}\rho g(3.06)2\times 10$$

$$= \rho g(20 = 30.6) = \underline{496.4\ kN}$$

Example 3.22 Calculate the slope of the free surface when an open container of liquid accelerates at 4.2 m/s^2

i) In the horizontal direction
ii) Down a 30° inclined plane.

Solution:
i)

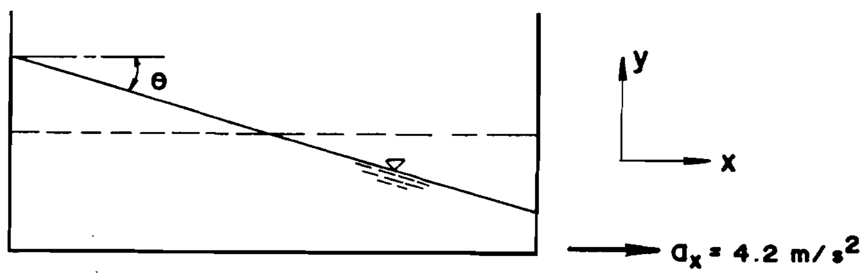


Figure E 3.22a

Slope of free surface is given by dy/dx

$$\frac{dy}{dx} = \tan\theta = -\frac{a_x}{a_y + g}$$

$$\text{Here,} \quad a_x = 4.2\ m/s^2, \quad a_y = 0$$

$$\therefore \quad \theta = \tan^{-1}\frac{4.2}{9.81} = \underline{23.18^o}$$

ii)

When the acceleration down the 30° inclined plane is 4.2 m/s^2,

a_x = -4.2 Cos 30° = -3.64 m/s^2

a_y = -4.2 x Sin 30° = 2.1 m/s^2

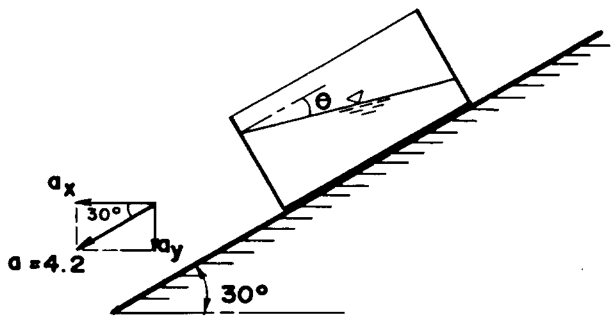


Figure E 3.22b

$$\therefore \quad \theta = \tan^{-1} \frac{a_x}{ay + g} = \tan^{-1} \frac{3.64}{-2.10 + 9.81} = 25.27^c$$

Example 3.23 The U-tube shown in the figure below is filled with a liquid having a specific gravity of 2.40 and accelerated horizontally at 2.45 m/s^2. The leg of the U-tube is closed at the right end and open at the left end. Draw the imaginary free surface and determine the pressure at A. If the cross-sectional area of the tube is 6.28 cm^2, what volume of the liquid will be spilled?

Solution:

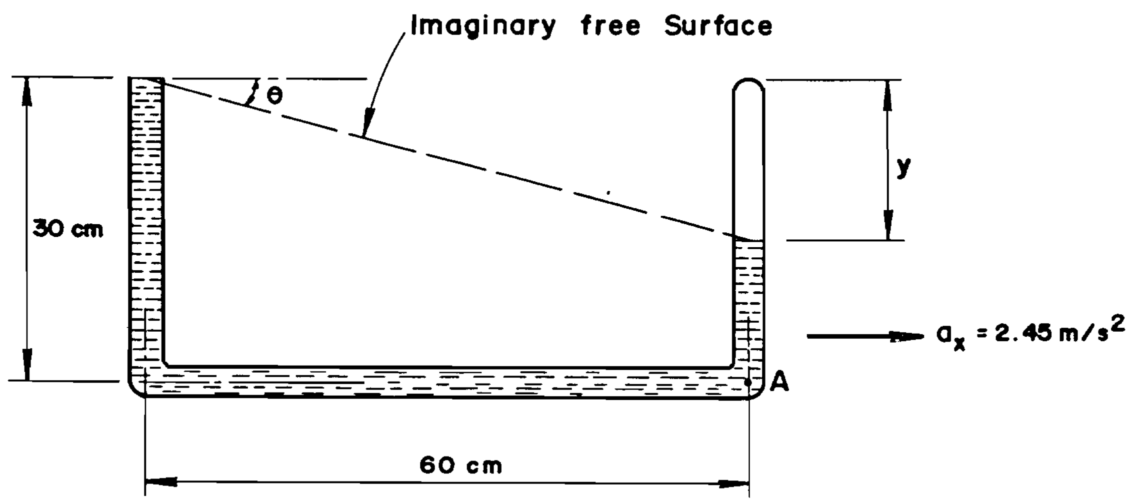


Figure E 3.23

Referring to the above figure:

$$\tan\theta = \frac{a_x}{g} = \frac{2.45}{9.81} = \frac{1}{4} = \frac{Y}{60}$$

$\therefore$ y = 60×1/4 = 15 cm.

$\therefore$ $P_A = \gamma.h = 2.4\gamma_W.0.15 =$ <u>3.53 KN/m^2</u>

Volume spilled = y × cross-sectional area of tube

= 15 × 6.28 = <u>94.20 cm^3</u>

Vertical Acceleration

If a liquid in a vessel is subjected to a constant vertical acceleration only, then a_x = 0. From Equation 3.23, $\frac{\partial p}{\partial x} = 0$ and from equation 3.26, $\frac{\partial y}{\partial x} = 0$. This means that the line of constant pressure is horizontal under vertical linear acceleration, i.e. the free surface remains horizontal. The pressure at any point in the liquid may be determined by integrating equation 3.24, i.e.,

$$\partial p = \rho(g + a_y)dy$$

Or

$$p = -\rho(g + a_y)y \text{ ,}$$

Where y is measured vertically upwards in the positive y direction. To determine the pressure at any depth h below the free surface, y will be replaced by (-h) and the pressure intensity is given by:

$$p = \rho(g + a_y)h \qquad 3.28$$

For a vessel that is accelerated vertically upwards, a_y will be positive and for vertically downward acceleration, a_y will be negative.

Example 3.24

A vertical hoist carries a square tank 2m x 2m containing water to the top of a construction scaffold with an acceleration of $2m/s^2$. If the water depth is 2m, calculate the total hydrostatic force on the bottom of the tank.

If this tank is lowered with an acceleration equal to that of gravity, what are the thrusts on the floor and sides of the tank?

Solution:

Since this is a case of vertical acceleration, the free surface and hence the lines of constant pressure remain horizontal.

Vertical upward acceleration = $a_y = 2m/s^2$

Pressure intensity at a depth h = $\rho(g + a_y)h$

$$= \rho g\left(1 + \frac{a_y}{g}\right)h$$

$$= \rho gh\left(1 + \frac{2}{9.81}\right)$$

$$= 1.204\ gh\ kN/m^2$$

∴ Total hydrostatic thrust on the floor

= intensity x area

= 1.204 x 9.81 x 2 x (2 x 2) = <u>94.49 kN</u>

Vertical downward acceleration = -9,81 m/s^2

Pressure intensity at depth h = ρgh(1-9.81/9.81) = 0

∴ There exists no hydrostatic force on the floor and on the side.

3.10.2 Rotation about a vertical axis

When a vessel containing a liquid is rotated about a vertical axis at constant angular velocity, the liquid will, after a small adjustment period, rotate as solid body. Since there is no relative motion between adjacent layers of the liquid and between the liquid and the container, there are no shear stresses. Such a motion is called forced-vortex motion. As a result of the constant angular velocity w, a constant, radially inward directed centripetal acceleration ($-w^2\gamma$) acts on the fluid mass towards the axis of rotation. Consequently the pressure will vary in the radial direction because of the centrifugal effects.

In order to determine the variation of pressure in the radial direction, consider a small element of fluid of length dr and cross sectional area dA at radial distance r in liquid mass which is contained in a cylinder of internal radius r_o shown in figure 3.25. The cylinder is rotated at constant angular velocity w rad/s about its vertical axis.

The mass of the element is ρ.dr.dA. This mass is subjected to a radially inward acceleration $-\omega^2\gamma$.

Newton's Second Law applied to the element will be:

$$p \cdot dA - (p - \frac{\partial p}{\partial r} \cdot dr)dA = \rho dr \cdot dA(-\omega^2 r)$$

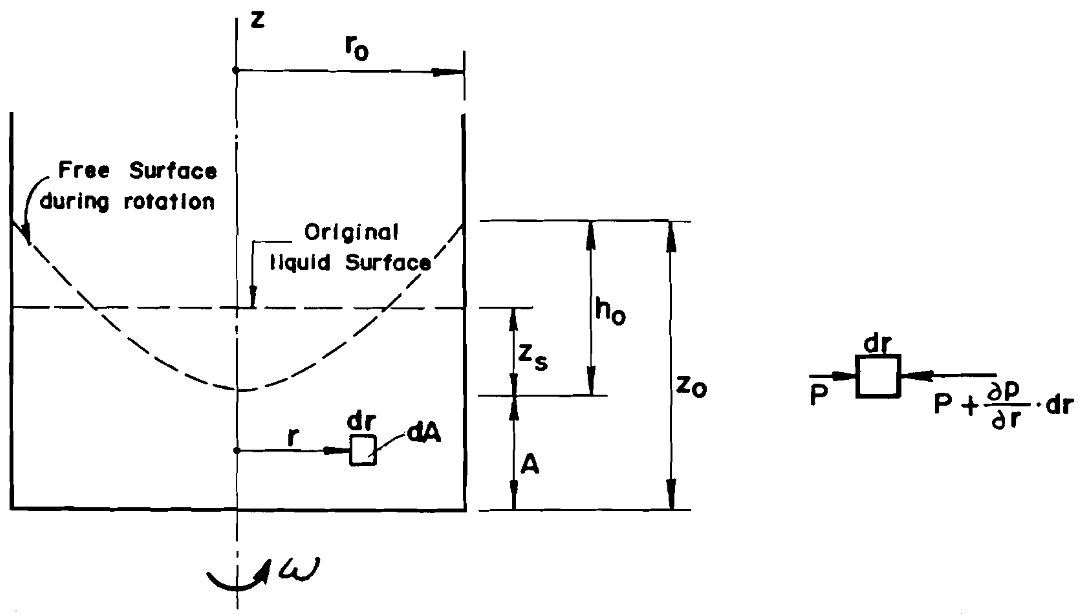


Figure 3.25 Rotation with constant angular velocity

Which when simplified reduced to:

$$\frac{\partial p}{\partial r} = \rho\,\omega^2 r \qquad (3.29)$$

The variation of pressure with depth will be obtained by considering the forces in the vertical direction with gravitational acceleration acting on the fluid element. This leads to the variation of pressure with depth to be the same as when the liquid is at rest i.e.

$$\frac{\partial p}{\partial z} = \rho g \qquad (3.30)$$

For the surface of constant pressure, the total pressure differential will be zero.

i.e $$dp = \frac{\partial p}{\partial r}.dr + \frac{\partial p}{\partial z}.dz = 0$$

or, $0 = \rho \omega^2 r . dr - \rho g dz$

Leading to:

$$\frac{dz}{dr} = \frac{\omega^2 r}{g} \qquad (3.31)$$

Integrating the above,

$$z = \frac{\omega^2 r^2}{2g} + c \qquad (3.32)$$

This shows that the constant pressure lines are parabolic. Considering the free surface which is constant pressure surface,
r = 0 at z = 0, which make c = A in equation 3.32

Thus: $Z - A = \frac{\omega^2 r^2}{2g}$

At the container's wall, $r = r_o$ and $z = z_o$. Therefore,

$$Z_o - A = h_o = \frac{\omega^2 r_o^2}{2g} \qquad (3.33)$$

Equation 3.33 shows that for a circular cylinder rotating about its axis, the rise of liquid along the wall from the vertex is $\omega^2 r_o^2/2g$.

Consider a cylindrical tank partially filled with a liquid and rotated about its vertical axis at constant angular velocity ω rad/sec so that no liquid is spilled as shown in Figure 3.26

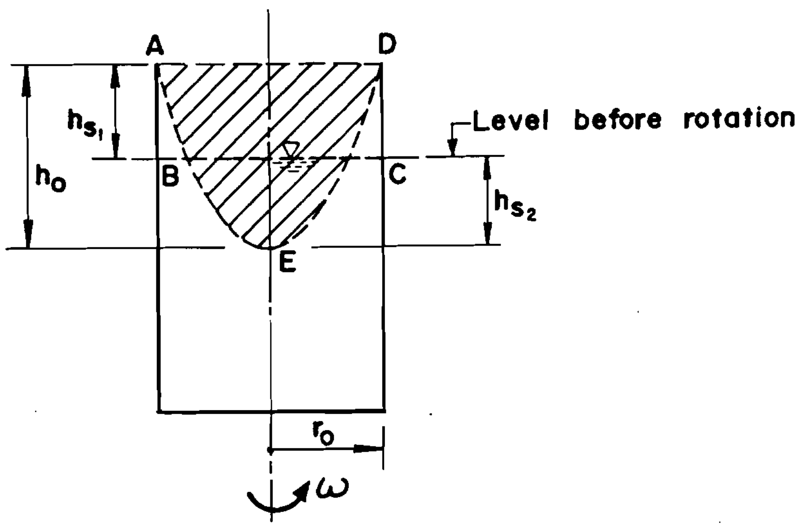


Figure 3.26

The shaded volume AED = paraboloid of revolution
= volume of the empty space ABCD

Paraboloid of revolution = 1/2(volume of circumscribing cylinder)

∴ Volume of empty space ABCD = 1/2 $(\pi r_o^2 . h_o)$

$$i.e. \quad \pi r_o^2 . hs_1 = \frac{1}{2} \pi r_o^2 . h_o$$

$$\therefore \quad hs_1 = \frac{1}{2} h_o = hs_2$$

This shows that during rotation about a vertical axis at constant angular velocity, the liquid rises along the walls the same amount above the rest level as the centre drops at the axis below the rest level.

Example 3.25

An open cylindrical tank, 2 m high and 1 m in diameter, contains 1.5 m depth of water. If the cylinder rotates about its vertical geometric axis,

a) What is the maximum constant angular velocity that can be attained without spilling any water?

b) What is the pressure intensity at the centre and corner of the bottom of the tank i.e. at C and D (fig. E 3.25) when the angular velocity is

Solution:

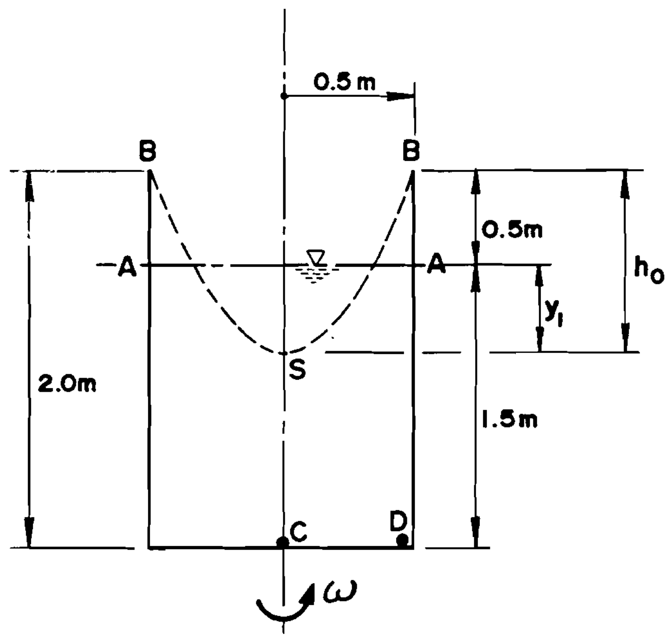


Figure E 3.25

a) If no liquid is to be spilled, the maximum angular velocity ω will have such a magnitude that will enable the liquid to rise to level B at the wall of the cylinder.

Under this condition,

Volume of paraboloid of revolution = Volume of original empty space

$$\frac{1}{2}(\pi \times 0.5^2)\,(y_1 + 0.5) = \pi \times 0.5^2 \times 0.5$$

From the above, $y_1 = 0.5$ m

Thus: $h_o = 1m = \dfrac{\omega^2 \times 0.5^2}{2 \times 9.81}$

i.e $\omega = [(2 \times 9.81)/0.5^2]^{1/2} =$ $\underline{8.86\ rad/s}$

b) For $\omega = 8$ rad/s,

$$h_o = \frac{8^2 \times 0.5^2}{2 \times 9.81} = 0.816\,m$$

S drops by 1/2 h_o = 0.405 from level A-A.

Thus: at C, depth from free surface = 1.5 - 0.408 = 1.092m

at D, depth from free surface = 1.5 + 0.408 = 1.908m

$\therefore \quad P_C = \rho g h_C = 9810 \times 1.092 = 10{,}713\ N/m^2 = 10.713\ kPa$

$P_D = \rho g h_D = 9810 \times 1.908 = 18{,}717\ N/m^2 = 18.72\ kPa$

Example 3.26

If the tank in the above example is closed at the top and the air subjected to a pressure of 1.07 bar (= 107 KN/m^2), determine the pressures at points C and D when the angular velocity is 115 rpm.

Solution:

Referring to Figure E 3.26:

Since there is no change in the volume of air within the tank,

volume above level A-A = volume of empty space

= volume of paraboloid of revolution

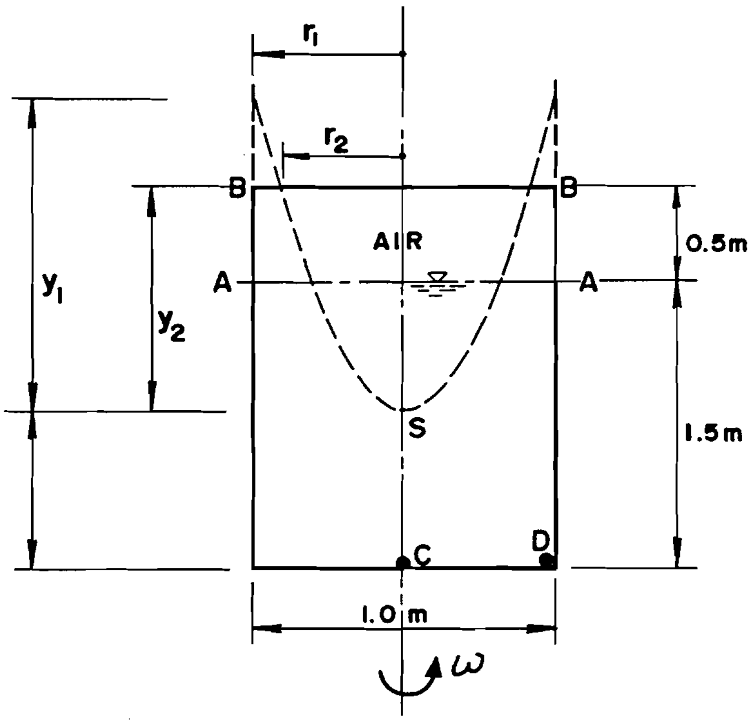


Figure E 3.26

i.e $$\frac{\pi}{4} \times 1^2 \times 0.5 = \frac{\pi \cdot r_2^2}{2} \times Y_2 \qquad (1)$$

also $$y_2 = \frac{\omega^2 r_2^2}{2g} = \frac{12.04^2 \cdot r_2^2}{2 \times 9.81} = 7.39\, r_2^2 \qquad (2)$$

Substituting the value of r_2^2 from (2) in (1):

$$\frac{7.39}{2} \cdot r_2^4 = 0.125$$

$$\therefore \quad r_2 = 0.43\ m$$

and $y_2 = 7.39 \times (0.43)^2 = 1.36$ m.

Thus, S is located (2-1.36) = 0.64 m above c

$$y_1 = \frac{\omega^2 \times r_1^2}{2g} = \frac{12.04^2 \times 0.5^2}{2 \times 9.81} = 1.847 \ m$$

∴ Pressure head at D = 0.64 + 1.847 = 2.487 m.

∴ Pressure at C = $P_c = P_{air} + \varrho g h_c$

$= 1.07 \times 10^5 + 9810 \times 0.64$

$= (1.07 + 0.063) \times 10^5 \ P_a$

$= 1.133 \times 10^5 \ P_a$

Pressure at D = $P_D = P_{air} + \varrho g h_D$

$= 1.07 \times 10^5 + 9810 \times 2.487$

$= (1.07 + 0.244) \times 10^5$

$= 1.314 \times 10^5 \ P_a$

Example 3.27

A closed cylindrical vessel 1 m in diameter and 1.8 m high contains water to a depth of 1.3 m. If the vessel is rotated at 18 rad/s, what is the radius of the circle that will be uncovered at the bottom of the vessel?

Solution:

Referring to Figure E 3.27

assume that the vertex s of the paraboloid is at a distance h_1 m below the bottom of the vessel.

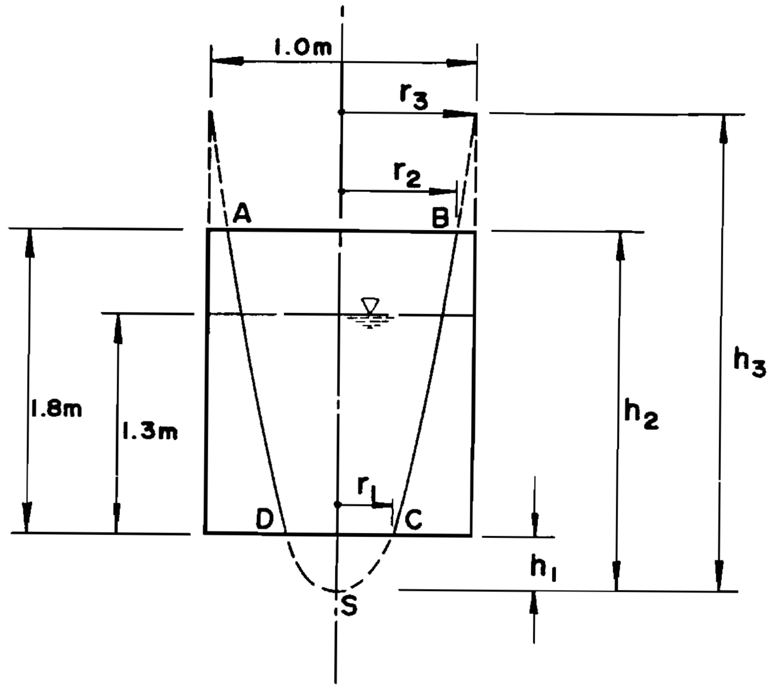


Figure E 3.27

Then: $$h_1 = \frac{\omega^2 r_1^2}{2 \times 9.81} = \frac{18^2 \times r_1^2}{19.62} = 16.51\ r_1^2$$

from which, $$r_1^2 = h_1/16.51 \qquad (1)$$

$$h_2 = (1.8 + h_1) = \frac{18^2 \times r_2^2}{19.62} = 16.51\ r_2^2$$

from which, $$r_2^2 = (1.8 + h_1)/16.51 \qquad (2)$$

$$h_3 = \frac{18^2 \times r_3^2}{19.62} = 16.51 \times 0.5^2 = 4.13\ m.$$

Volume of water in the vessel = Volume of cylinder - (volume of paraboloid ABS - volume of paraboloid DSC)

$$\frac{\pi}{4} \times 1^2 \times 1.3 = \frac{\pi}{4} \times 1^2 \times 1.8 - [\frac{1}{2}.\pi r_2^2.h_2 - \frac{1}{2}\pi r_1^2.h_1]$$

$$= \frac{\pi}{4}[1.8 - 2r_2^2 h_2 + 2r_1^2 h_1]$$

or $$1.3 = 1.8 - 2r_2^2 h_2 + 2r_1^2 h_1$$

Substituting the values of r_1^2 and r_2^2 from (1) and (2) above,

$$1.3 = 1.8 - 2(1.8 + h_1)(1.8 + h_1)/16.51 + 2h_1^2/16.51$$

$$-0.5 \times 16.51 = -2(3.24 + 3.6h_1 + h_1^2) + 2h_1^2$$

$$-8.255 = -6.48 - 7.2h_1$$

$$\therefore \quad h_1 = 0.247\ m.$$

$$and \quad r_1 = (0.247/16.5)^{1/2} = 0.122\ m$$

$$= \underline{12.2\ cm}$$

Example 3.28

The U-tube in Example 3.23 is rotated about a vertical axis 15 cm to the right of A at such a speed that the pressure at A is zero gauge. What is the rotational speed?

Solution:

Referring to Figure E 3.28

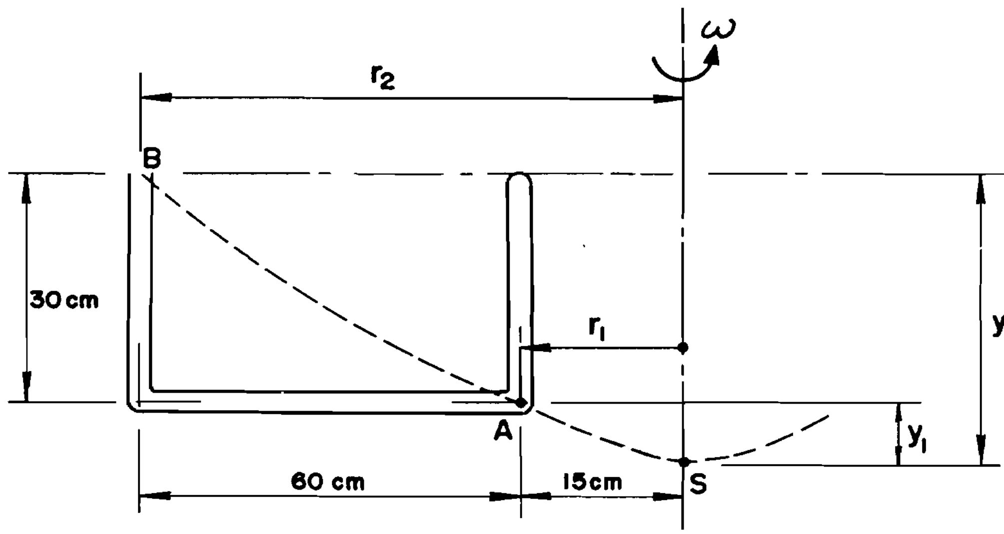


Figure 3.28

If the pressure at A is to be zero gauge i.e. atmospheric, then the paraboloid of revolution which passes through B must also pass through A. The vertex will be at S.

Thus: $$y_2 = y_1 + 0.30 = \frac{\omega^2 . r_2^2}{2g} = \frac{\omega^2 (0.75)^2}{2g} \qquad (1)$$

$$y_1 = \frac{\omega^2 r_1^2}{2g} = \frac{\omega^2 (0.15)^2}{2g} \qquad (2)$$

Substituting value of y_1 from (2) into (1):

$$\frac{\omega^2(0.5625)}{2g} = \frac{\omega^2 \times (0.0225)}{2g} + 0.3$$

$$\omega^2(0.5625 - 0.0225) = 0.3 \times 2g$$

$$\therefore \quad \omega = [(0.3 \times 19.62)/0.54]^{1/2} = 3.30 \; rad/s$$

$$= 31.5 \; rPm$$

Exercise Problems

3.1 What will be (a) the gauge pressure, (b) the absolute pressure of water at a depth of 20 m below the free surface. Assume the density of water to be 1000 kg/m^3 and the atmospheric pressure 101 kN/m^2. (Ans. 196.2 kN/m^2, 297.2 kN/m^2)

3.2 Calculate the pressure in the ocean at a depth of 2000 m assuming that salt water is (a) incompressible with a constant density of 1002 kg/m^3, (b) compressible with a bulk modulus of 2.05 GN/m^2 and a density at the surface of 1002 kg/m^3.

3.3 An inverted U-tube is used to measure pressure difference between A and B (Fig. P.33). If the top space in the tube is filled with air, what is the difference in pressure between A and B, when (a) water (b) oil of relative density 0.65 flows through the pipes.

3.4 Find the value of h in metres in Fig. P 3.4 when the air pressure above the surface is 3.5 m of water below atmospheric (The manometric liquid has s = 2.5)

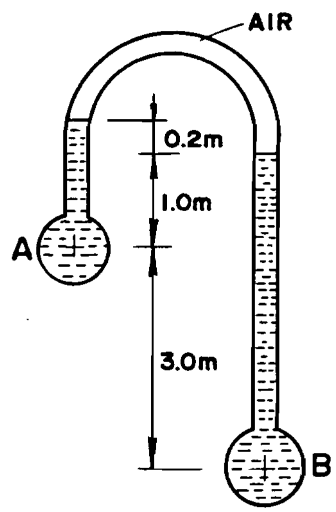


Figure P 3.3

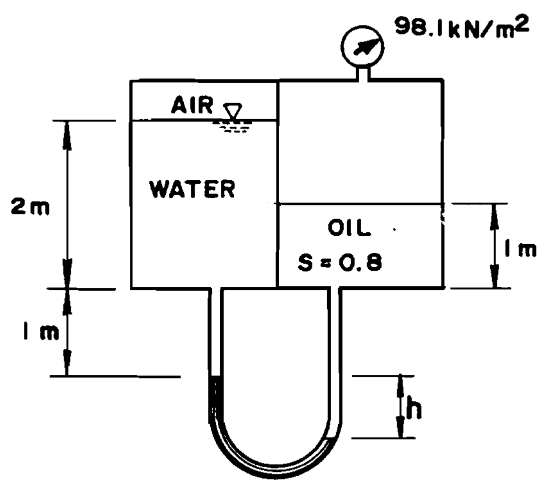


Figure P 3.4

3.5 At what height H of water will the conical valve in Fig. P 3.5 start to leak? The valve weighs 2.256 kN and assume the pulley to be frictionless. (Ans. H = 1.325 m)

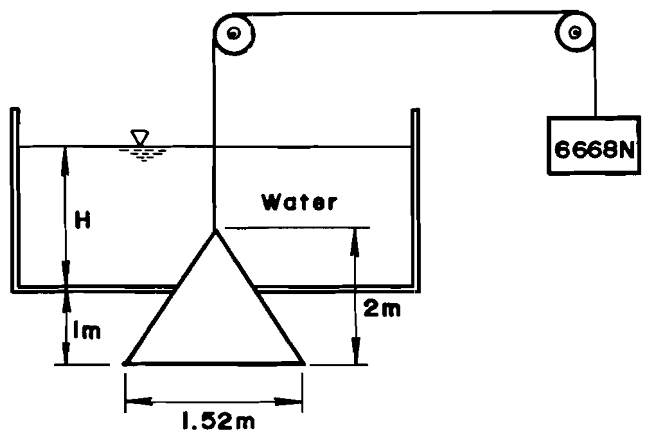


Fig P 3.5

3.6 A 6 m x 2 m rectangular gate is hinged at the base and is inclined at an angle of 60^0 (Fig P 3.6). If W = 39.2kNacting at angle of 90^0 to the gate find the depth of the water when the gate begins to fall. Neglect the weight of the gate and the friction of the pulley.

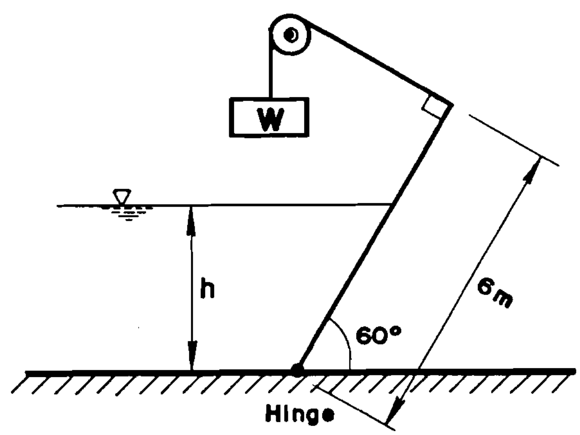


Fig. P 3.6

3.7 A gate consists of a quadrant of a circle of radius 1.5m pivoted at 0 (Fig. P 3.7). The centre of gravity of the

gate is at G. Calculate the magnitude and direction of the resultant force on the gate due to the water and the turning moment required to open the gate. The width of the gate is 3 m and it has a mass of 5000 kg.
(Ans. 61.6 kN, 57° 28', 29.417 kN m.)

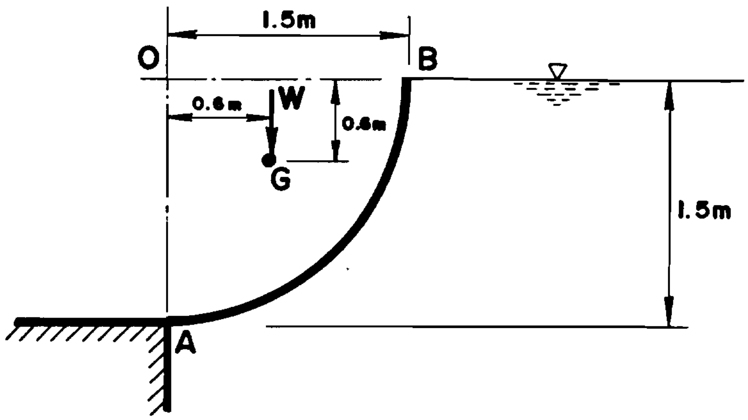


Fig. P 3.7

3.8 A submarine weighing 3.924 MN has an enclosed volume of 800 m^3. What volume of water should be taken in to submerge the vessel?

3.9 An open steel tank having a 3.3 m x 3.3 m plan section and a draft of 1.3 m has its centre of gravity at the water line. The tank has to be delivered by towing after fabrication to its final location. Determine whether it will float stably without adding ballast.
(Ans: Stable)

3.10 The open rectangular tank shown in Fig p 3.10 is 5 m wide, 6 m deep and 10 m long. It is filled to a depth of 4 m with water. If the tank is accelerated horizontally at 1/2g, calculate
a) The volume of water spilled (if any)
b) The force on the back and fron end

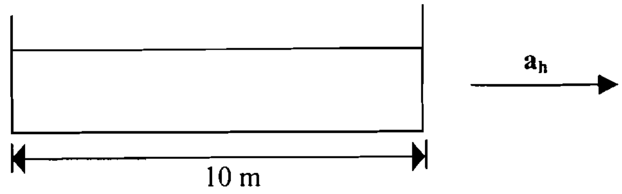


Fig. P 3.10

3.11 In figure P 3.11, calculate the minimum volume of the concrete block (γ_c = 22.5633 KN/m^3) which will hold the circular gate AB in place. The block is submerged in water. The pulley is frictionless. (Ans. 1.264 m^3)

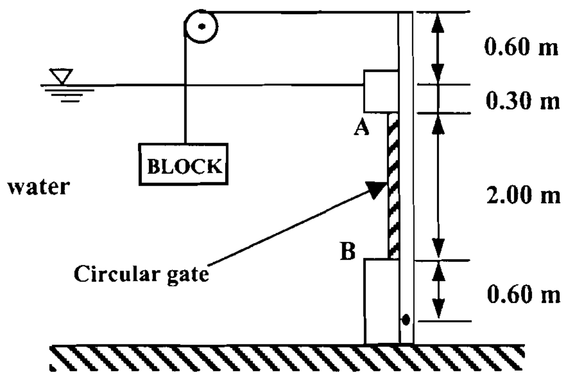


Fig: P 3.11

3.12 In figure P 3.12, a_x = 2.45 m/s^2, a_y = 4.90 m/s^2. Deetermine

a) The angle which the free surface makes with the horizontal
b) The pressure at B and C in N/m^2

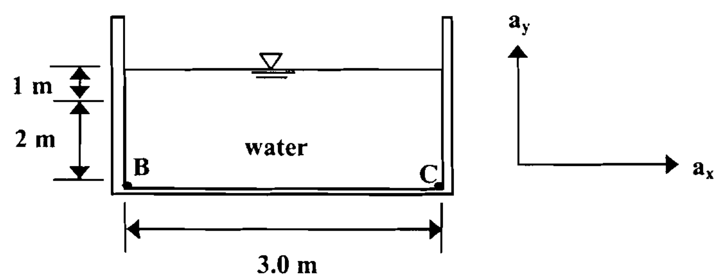


Fig. P 3.12

3.13 An open cylinderical tank 1.2 m in diameter and 1.8 m deep is filled with water and rotated about its axis at 60 rpm. How much liquid is spilled and how deep is the water at the axis ? (Ans. 0.43 m^3, 1.1 m)

3.14 At what speed should the tank in problem 3.13 be rotated in order that the center of the bottom of the tank have zero depth of water?

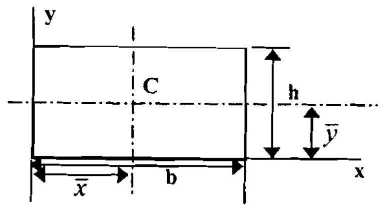
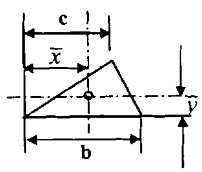
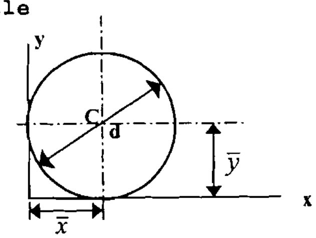
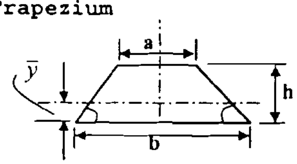
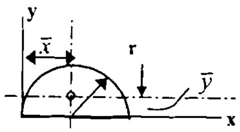
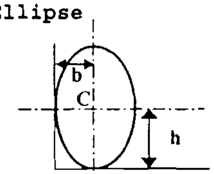
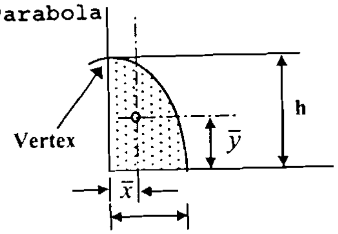
Table 3.1 : Surface Area, Centroid and Second Moment of Area of Some Simple Geometrical Shapes				
Shape	Area	Centroid	I_{cc}	I_{xx}
Rectangle 	b.h	$\bar{x} = \frac{1}{2}b$ $\bar{y} = \frac{1}{2}h$	$\frac{1}{2}bh^3$	$\frac{bh^3}{3}$
Triangle 	$\frac{1}{2}b.h$	$\bar{x} = \frac{b+c}{3}$ $\bar{y} = \frac{h}{3}$	$\frac{1}{36}bh^3$	$\frac{bh^3}{12}$
Circle 	$\frac{\pi d^2}{4}$	$\bar{x} = \frac{d}{2}$ $\bar{y} = \frac{d}{2}$	$\frac{\pi d^4}{64}$	

Table 3.1: (Cont'd)

Shape	Area	Centroid	I_{cc}	I_{xx}
Trapezium 	$\frac{(a+b)h}{2}$	$\bar{y} = \frac{h(2a+b)}{3(a+b)}$	$\frac{h^3(a^3+4ab+b^2)}{36(a+b)}$	$\frac{(3a+b)h^3}{12}$
Semicircle 	$\frac{1}{2}\pi r^2$	$\bar{y} = \frac{4r}{3\pi}$	0.115^4	
Ellipse 	πbh	$\bar{x} = b$ $\bar{y} = h$	$\frac{\pi}{4}bh^3$	
Parabola 	$\frac{2}{3}bh$	$\bar{y} = \frac{2}{5}h$ $\bar{x} = \frac{3}{8}b$	$\frac{8}{175}bh^3$	