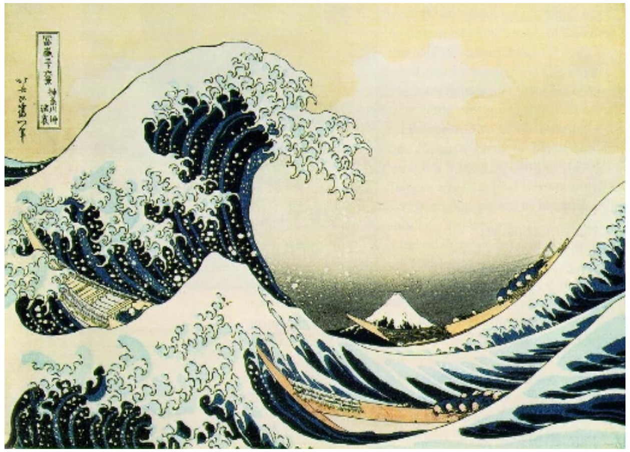

Chapter 3 Fluid Kinematics

Hydraulics I

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AAiT

Kinematics of Fluids

- **Ideal fluid flow**
  - No viscosity → no shear stresses
  - Boundary effects ignored and uniform velocity
- **Real fluid flow**
  - Fluid properties
  - Kinematics and dynamics

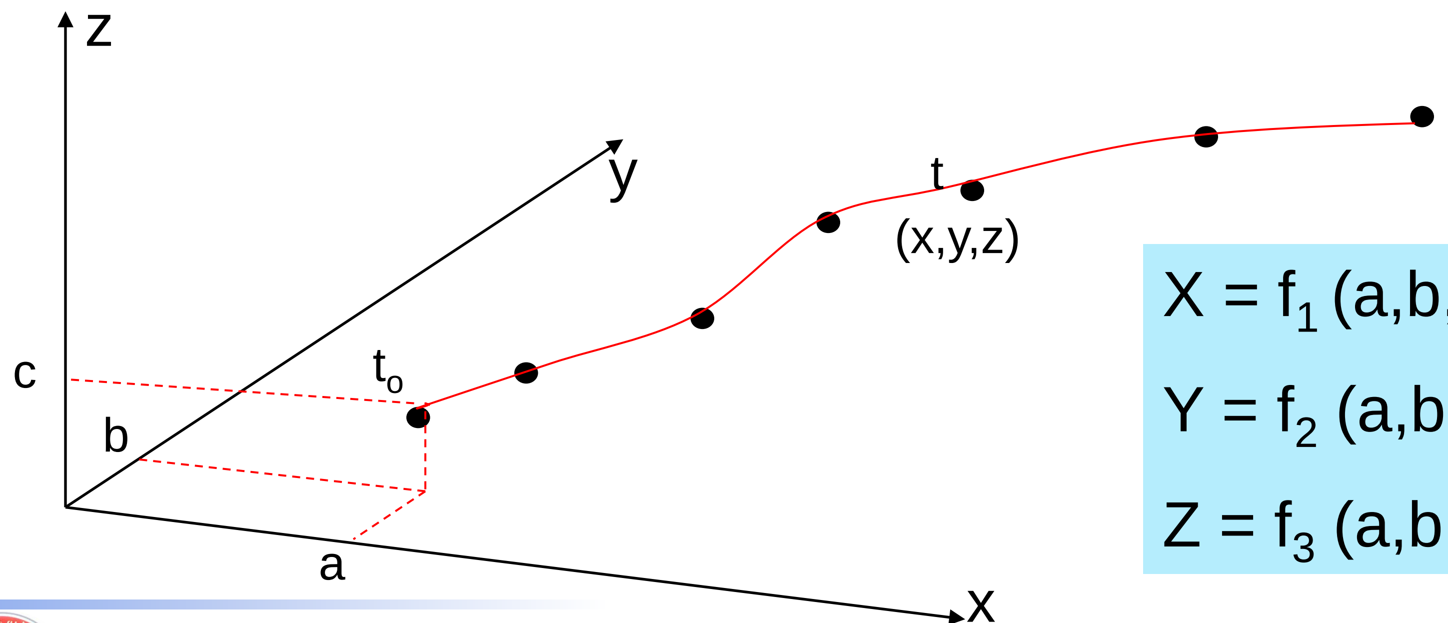
Kinematics of fluids deal with geometry of motion (space-time relationships) for fluids in motion.

Velocity field

- Two ways to describe fluid motion
  - Lagrangian method
  - Eulerian method

Lagrangian Method

- Follows a single particle and describe its characteristics (velocity and acceleration)



$$X = f_1\,(a,b,c,t)$$

$$Y = f_2\,(a,b,c,t)$$

$$Z = f_3\,(a,b,c,t)$$

Lagrangian method (contd.)

- Velocity and acceleration

$$u = \frac{\partial x}{\partial t} \qquad a_x = \frac{\partial^2 x}{\partial t^2}$$

$$v = \frac{\partial y}{\partial t} \qquad a_y = \frac{\partial^2 y}{\partial t^2}$$

$$w = \frac{\partial z}{\partial t} \qquad a_z = \frac{\partial^2 z}{\partial t^2}$$

Eulerian Method

- Focus on a point in space filled with fluid and describe the characteristics of flow at various points in the flow field at any time.

Velocity

$$u = f_1(x,y,z,t)$$
$$v = f_2(x,y,z,t)$$
$$w = f_3(x,y,z,t)$$

The relationship between Lagrangian and Eulerian method

$$\frac{dx}{dt} = u(x,y,z,t)$$
$$\frac{dy}{dt} = v(x,y,z,t)$$
$$\frac{dz}{dt} = w(x,y,z,t)$$

Lagrangian Vs Eulerian

Lagrangian Method

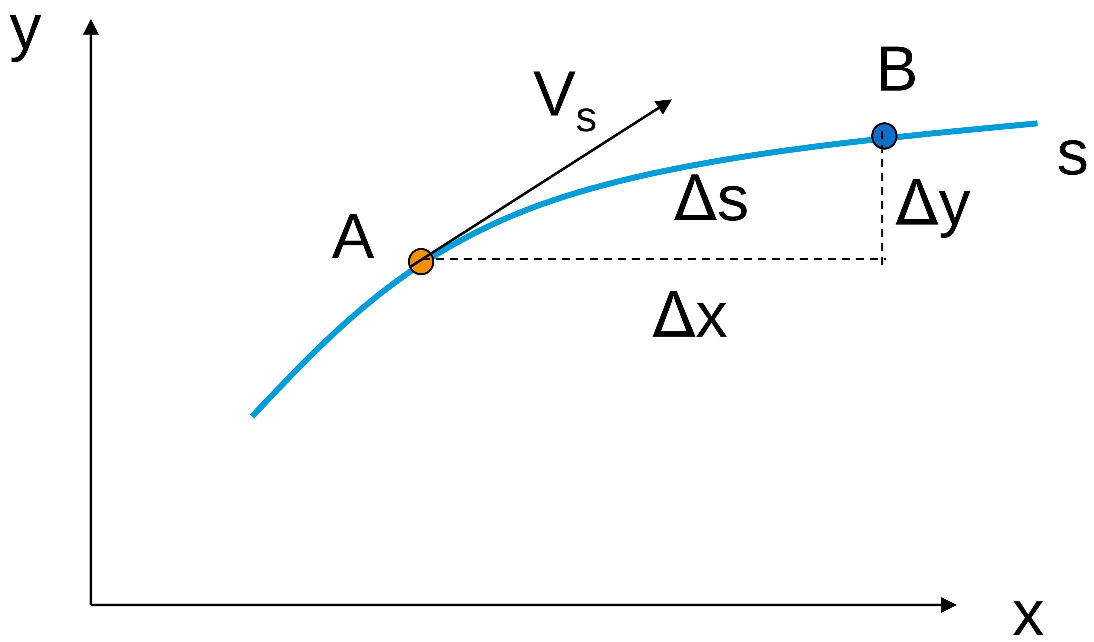
- difficult because it is not easy to identify a fluid particle and trace its path
- Each particle has a random path

Eulerian Method

- Easier and more practical because usually one is interested in flow parameters at certain points than what happens to the individual particles

Velocity and Acceleration

- Velocity is the rate of change of displacement



$$V_s = Lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t}$$

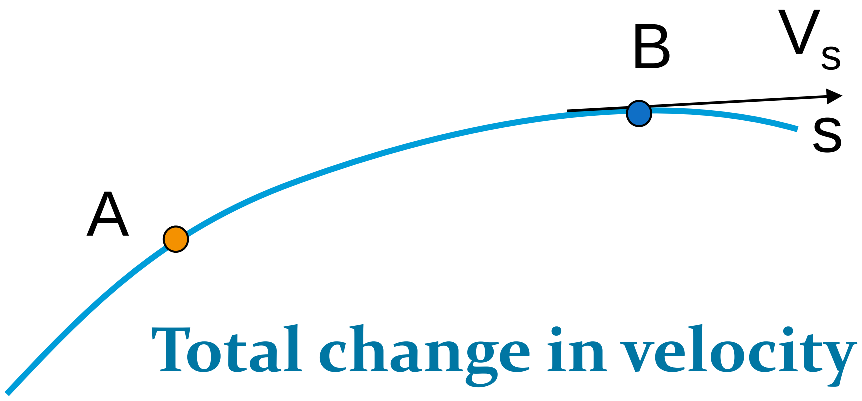
$$u = Lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

$$v = Lim_{\Delta t \to 0} \frac{\Delta y}{\Delta t}$$

$$w = Lim_{\Delta t \to 0} \frac{\Delta z}{\Delta t}$$

Acceleration

Acceleration is rate of change of velocity. It has two components: Tangential acceleration and normal acceleration



Total change in velocity

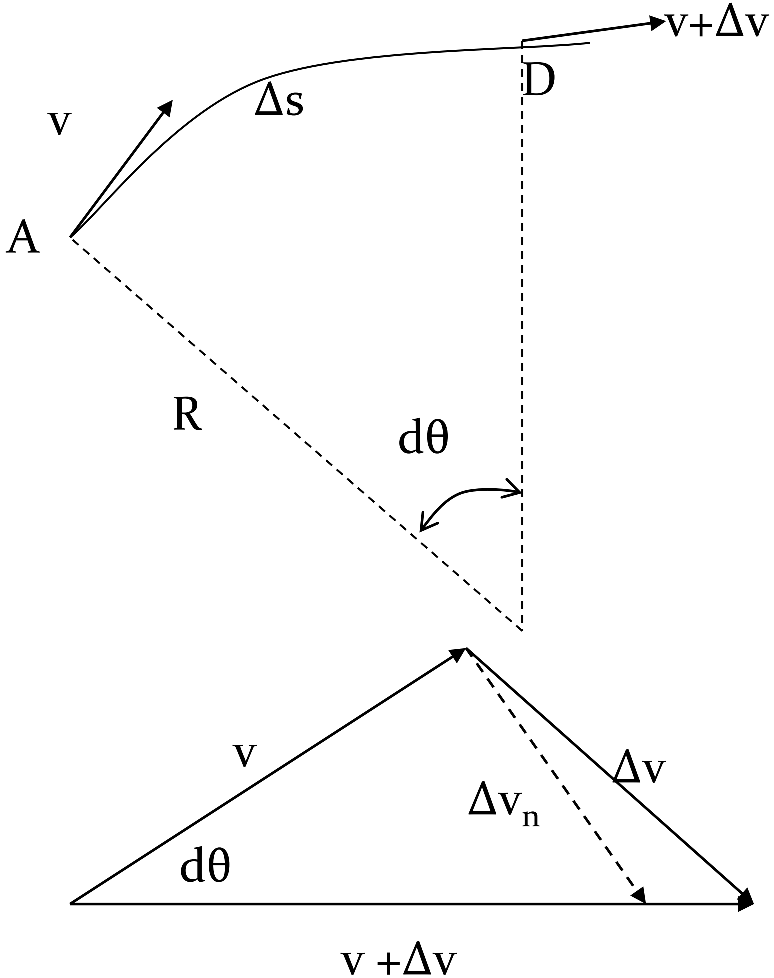
$$dV_s = \frac{\partial V_s}{\partial s} ds + \frac{\partial V_s}{\partial t} dt$$

Tangential acceleration

$$a_s = \frac{dV_s}{dt} = \frac{\partial V_s}{\partial s}\frac{ds}{dt} + \frac{\partial V_s}{\partial t}\frac{dt}{dt}$$

$$a_s = \frac{\partial V_s}{\partial s} V_s + \frac{\partial V_s}{\partial t}$$

The Normal Acceleration



$$dV_n = Vd\theta = V\frac{ds}{R}$$

$$a_{n1} = \frac{dV_n}{dt} = \frac{dV_n}{ds}\frac{ds}{dt} = \frac{V}{R}V = \frac{V^2}{R}$$

$$a_n = \frac{V^2}{R} + \frac{\partial V_n}{\partial t}$$

$$a_s = \frac{\partial V_s}{\partial t} + V_s\frac{\partial V_s}{\partial s}$$

$$a_n = \frac{\partial V_n}{\partial t} + \frac{V^2}{R}$$

Temporal component   Convective component

Stream Patterns and Type of Accelerations

a. No accelerations exist

b. Tangential convective accelerations

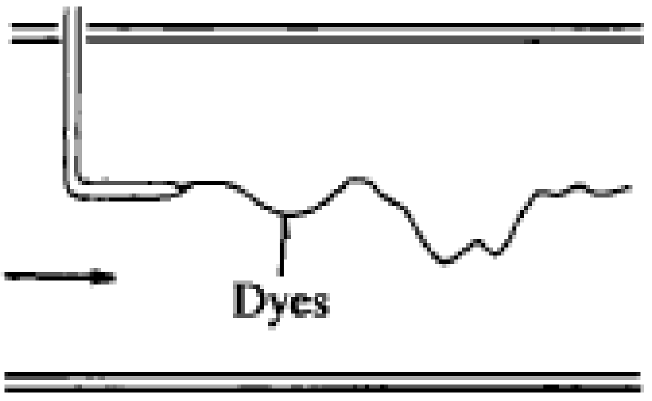
c. Normal convective accelerations

d. Tangential and normal convective accelerations

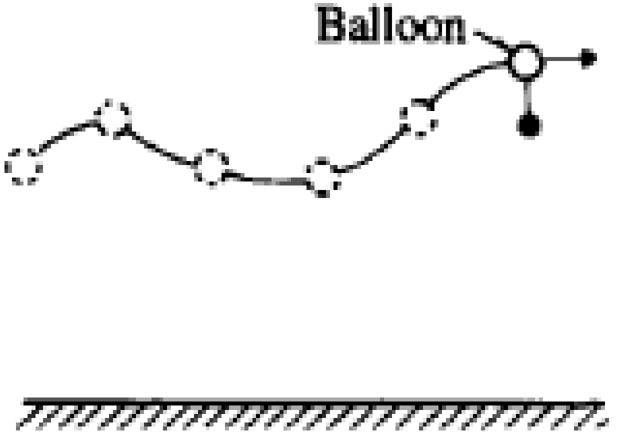
Pathline and streakline

Pathline: the path traced by a fluid particle during a given time interval

Streakline: path showing the positions of all particles that passed through a given point at a given time interval



(b) Streak line

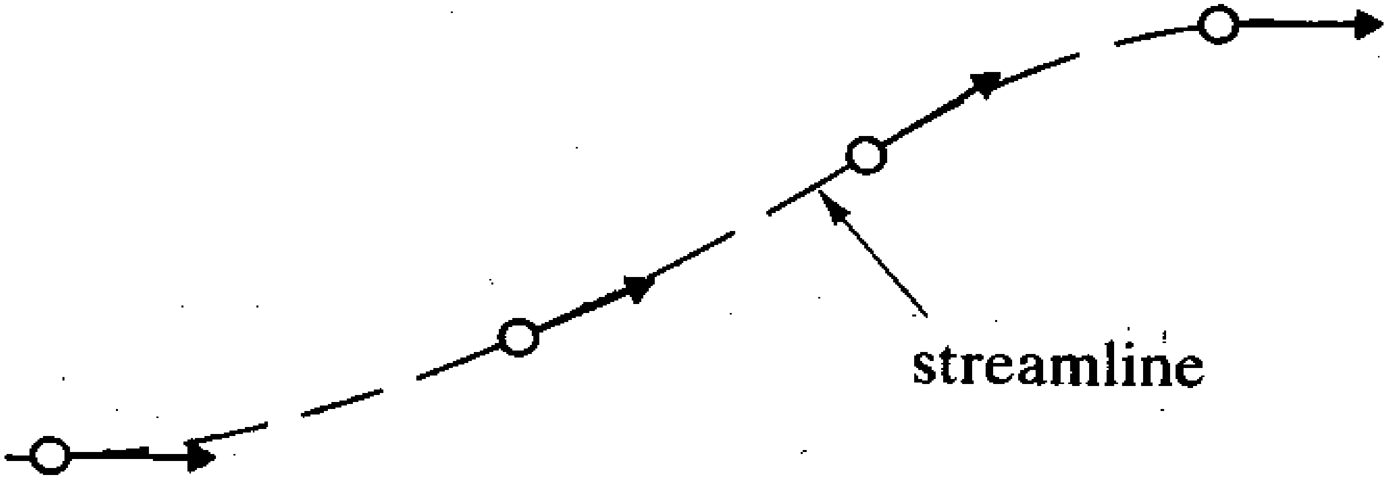


(c) Path line

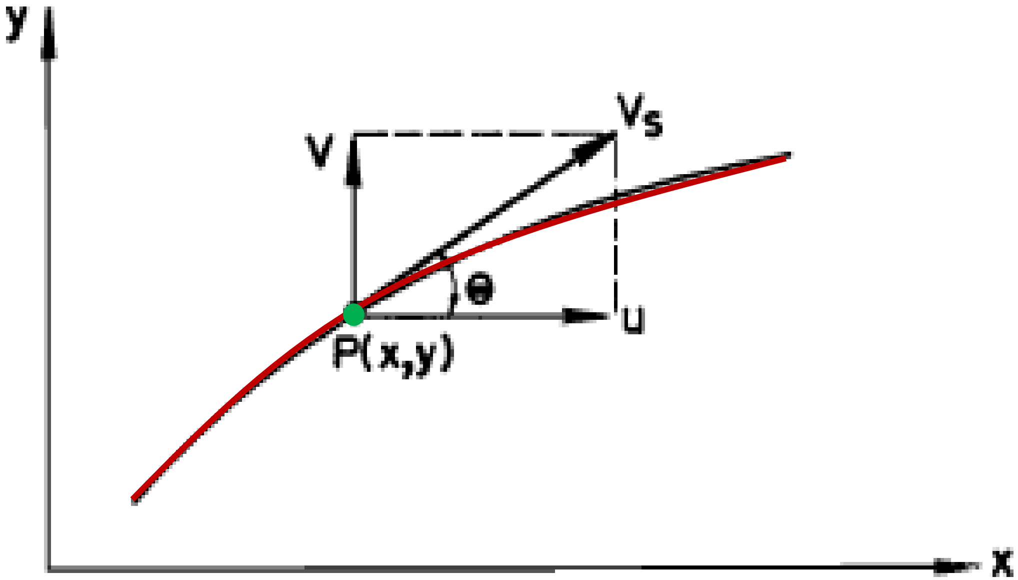
Streamline

Streamline: An imaginary line drawn through a flow field such that the tangent to the line at any point on the line indicates the direction of the velocity vector at that instant.

Note: since the flow is tangent to the stream line there will be no flow normal to a stream line, i.e., flow between any two stream lines remains constant



Streamline Equation



$$\tan\theta = \frac{dy}{dx} = \frac{v}{u}$$

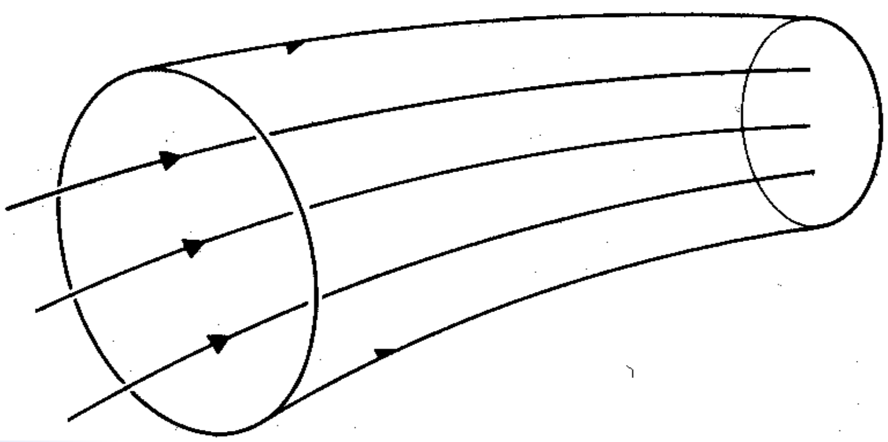
$$\frac{dx}{u} = \frac{dy}{v}$$

Where $u = f_1(x, y, t_o)$

$v = f_2(x, y, t_o)$

Stream tube

Stream tube: is a collection of streamlines drawn around the perimeter of a small area of stream cross section. Since the perimeter is composed of streamlines, there will not be any flow across a streamtube as well.



Classification of flow

- Flow is classified based on change in velocity
- Depending on temporal variations of velocity
  - Steady or unsteady
- Depending on spatial variations of velocity
  - uniform or non-uniform

Classification of flow

Steady flow: average flow velocity at a particular point does not change with time

$$\frac{\partial \mathbf{V}}{\partial t} = 0$$

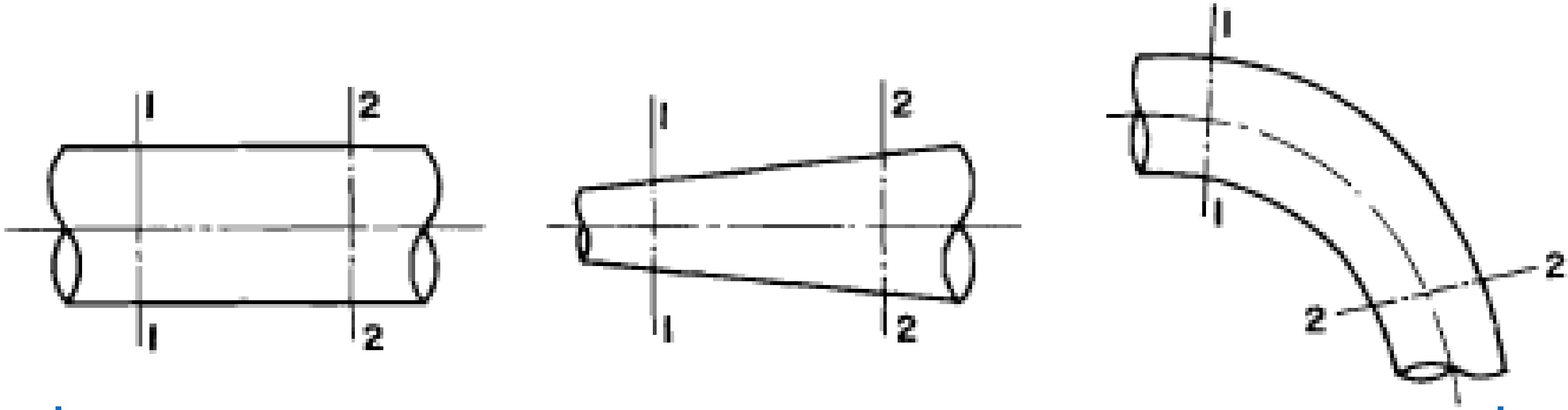
Unsteady flow $\frac{\partial \mathbf{V}}{\partial t} \neq 0$

Uniform flow: velocity at any point on a stream line remains the same at any given instant

$$\frac{\partial \mathbf{V}}{\partial s} = 0$$

Non- uniform flow $\frac{\partial \mathbf{V}}{\partial s} \neq 0$

Classification of flow



Steady if flow rate is constant

Uniform

Non-uniform

Unsteady non-uniform

One Dimensional flows

One dimensional:

When velocity components normal to the main flow are neglected and only average conditions at a section are considered

Applicable when:

- No wide variation of cross- section
- Stream lines are not highly curvilinear
- Velocity variation across a section is negligible

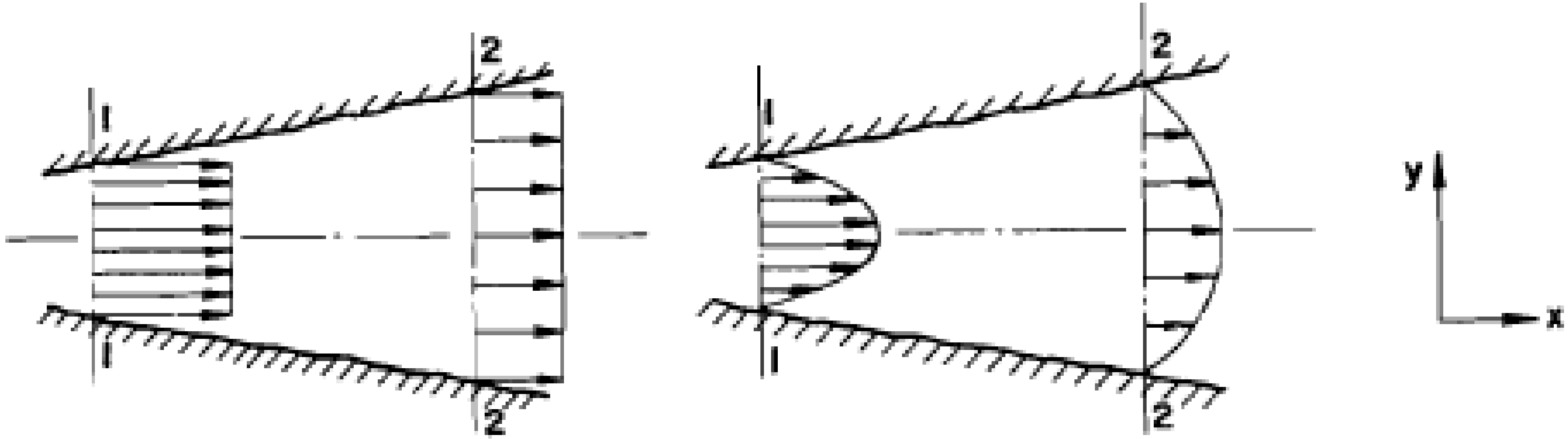
Two and Three Dimensional flows

Two Dimensional:

When real fluids are considered, the effects of boundaries and viscosity brings about variations in velocity at different sections

Three Dimensional:

Velocity vector varies in the three principal directions x, y and z.

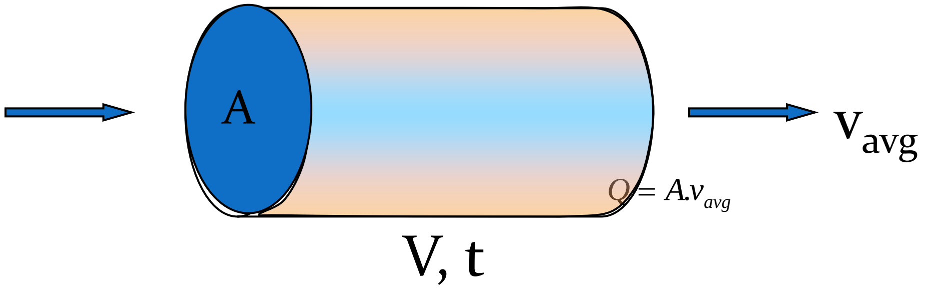


a) One dimensional   b) Two dimensional

	Unsteady	Steady
One-dimensional flow	v = f(x,t)	v = f(x)
Two-dimenstional flow	v = f(x,y,t)	v = f(x,y)
Three-dimesninal flow	v = f(x,y,z,t)	v = f(x,y,z)

Discharge and Mean velocity

Discharge: The volume of fluid passing a certain cross section in unit time. Measured in m^3/s



If velocity distribution is assumed to be uniform,

$$Q = AV_{avg}$$

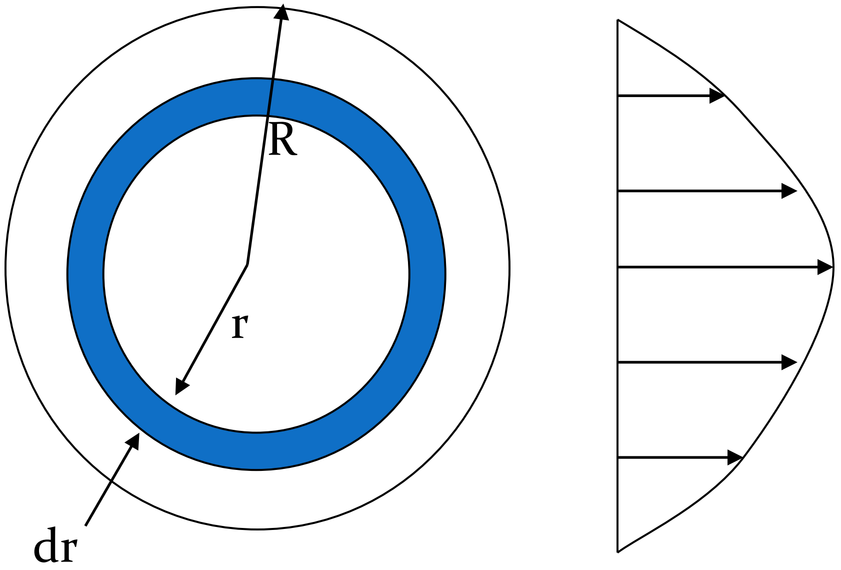
Note: A should be perpendicular to the direction of flow

Discharge and Mean velocity

When the actual velocity distribution is considered,

$$dQ = u.dA$$

$$Q = \int dQ = \int_A u.dA$$



For a circular section, the annular element

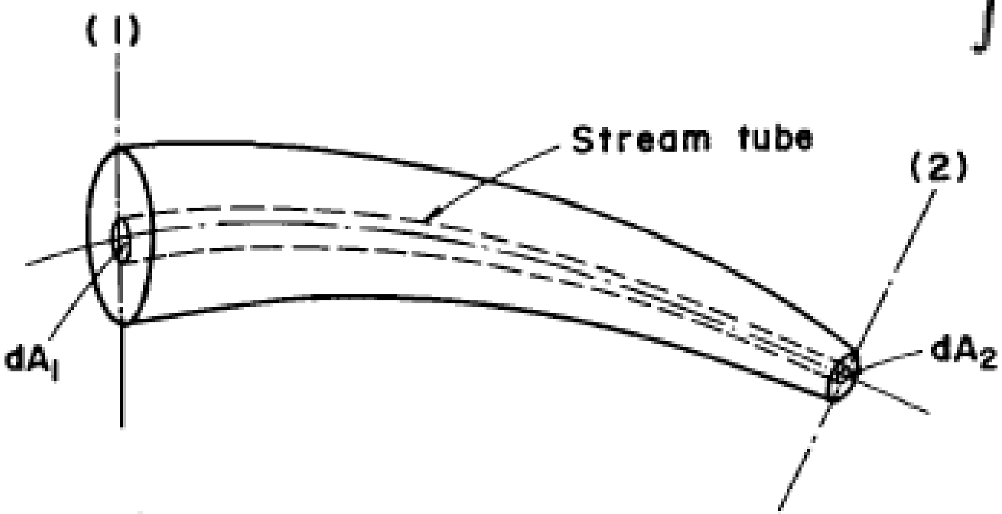
$$dA = 2\pi .r.dr$$

and

$$Q = 2\pi \int_0^R u.r.dr$$

Continuity Equation

- law of conservation of mass :In steady flow, the mass flow per unit time passing through each section does not change, even if the pipe diameter changes.



$$\int_{A_1} \rho_1 V_1 dA_1 = \int_{A_2} \rho_2 V_2 dA_2 = Constant$$

$$\bar{\rho}_1 \bar{V}_1 A_1 = \bar{\rho}_2 \bar{V}_2 A_2$$

For incompressible flow $\rho_1 = \rho_2$

$$A_1 \bar{V}_1 = A_2 \bar{V}_2 = Q$$

Q is the volumetric rate of flow called discharge, expressed in m^3/s

Example 3.1

If $u = 1.1 + 2.8\,x + 0.65\,y$ and $v = 0.95 - 2.1\,x - 2.8\,y$ calculate the acceleration at $(x, y) = (-2, 3)$

Example 3.2

$$\text{Given}: \vec{V} = 3t\vec{i} + xz\vec{j} + ty^2\vec{k}$$

$$\text{Find}: \text{Acceleration}, \vec{a}$$

$$u = 3t; \quad v = xz; \quad w = ty^2$$

$$a_x = \frac{\partial u}{\partial x}u + \frac{\partial u}{\partial y}v + \frac{\partial u}{\partial z}w + \frac{\partial u}{\partial t} = 0(3t) + 0(xz) + 0(ty^2) + 3 = 3$$

$$a_y = \frac{\partial v}{\partial x}u + \frac{\partial v}{\partial y}v + \frac{\partial v}{\partial z}w + \frac{\partial v}{\partial t} = z(3t) + 0(xz) + x(ty^2) + 0 = 3zt + xy^2t$$

$$a_z = \frac{\partial w}{\partial x}u + \frac{\partial w}{\partial y}v + \frac{\partial w}{\partial z}w + \frac{\partial w}{\partial t} = 0(3t) + 2ty(xz) + 0(ty^2) + y^2 = 2xyzt + y^2$$

$$\vec{a} = a_x\vec{i} + a_y\vec{j} + a_z\vec{k} = 3\vec{i} + (3tz + txy^2)\vec{j} + (2xyzt + y^2)\vec{k}$$

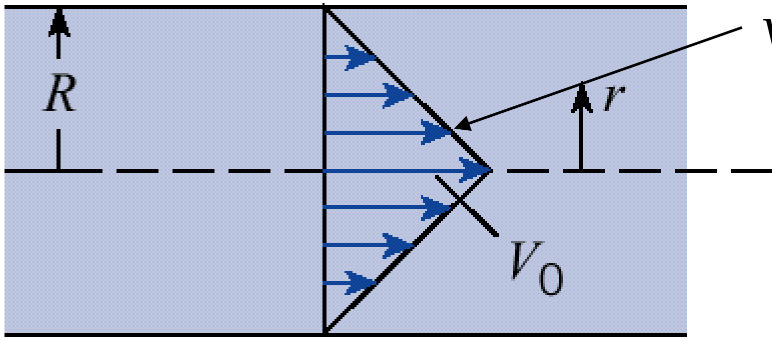
Examples 3.3

Discharge in a 2-cm pipe is 0.03 m³/s. What is the average velocity?

$$Q = VA$$

$$V = \frac{Q}{A} = \frac{Q}{\frac{\pi}{4}d^2} = \frac{0.03}{\frac{\pi}{4}(0.25)^2} = 0.611 m/s$$

Example 3.4



$$v(r) = V_o(1 - \frac{r}{R})$$

Find: $\frac{\bar{V}}{V_o}$

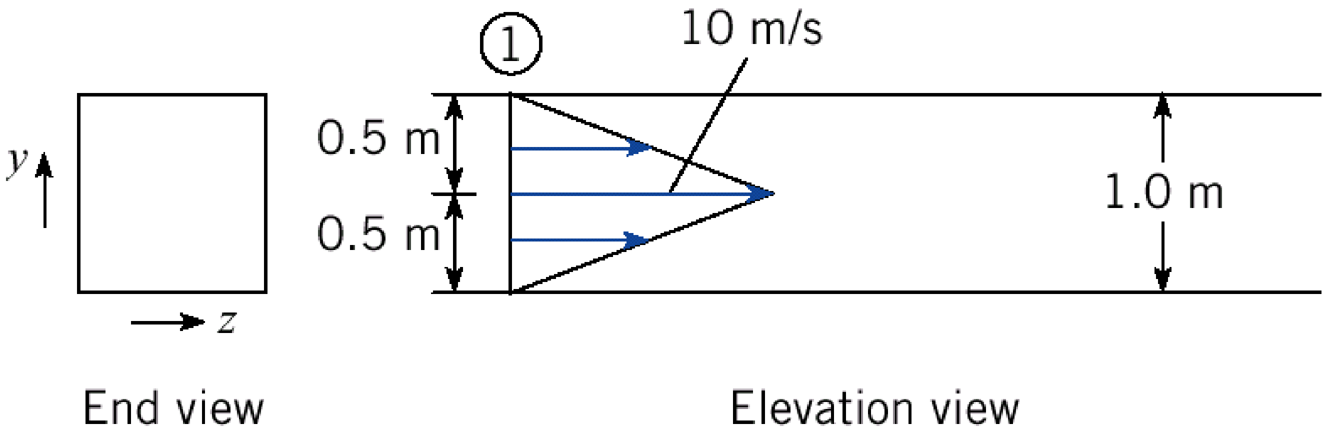
$$Q = \int_A V dA = \int_0^R V_o(1 - r/R) 2\pi r dr$$

$$= 2\pi V_o (\frac{r^2}{2} - \frac{r^3}{3R})\Big|_0^R = 2\pi V_o (\frac{R^2}{2} - \frac{R^2}{3})$$

$$= \frac{1}{3}\pi V_o R^2$$

$$\frac{\bar{V}}{V_o} = \frac{Q}{A V_o} = \frac{\frac{1}{3}\pi V_o R^2}{\pi R^2 V_o} = \frac{1}{3}$$

Example 3.5



Find: $Q, \overline{V}$

$$Q = 2\int_0^{0.5} V dA = 2\int_0^{0.5} 20 y dy$$

$$= 40 \frac{y^2}{2}\Bigg|_0^{0.5} = 5 m^3/s$$

$$\overline{V} = \frac{Q}{A} = \frac{5}{1} = 5 m/s$$