Chapter 2 Hydrostatics

Buoyancy, Floatation and Stability

Hydraulics I

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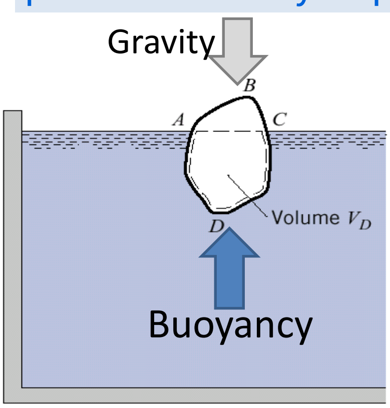
Rm. E119B

AAiT

Archimedes Principle

Force of buoyancy an upward force exerted by a fluid pressure on fully or partially floating body

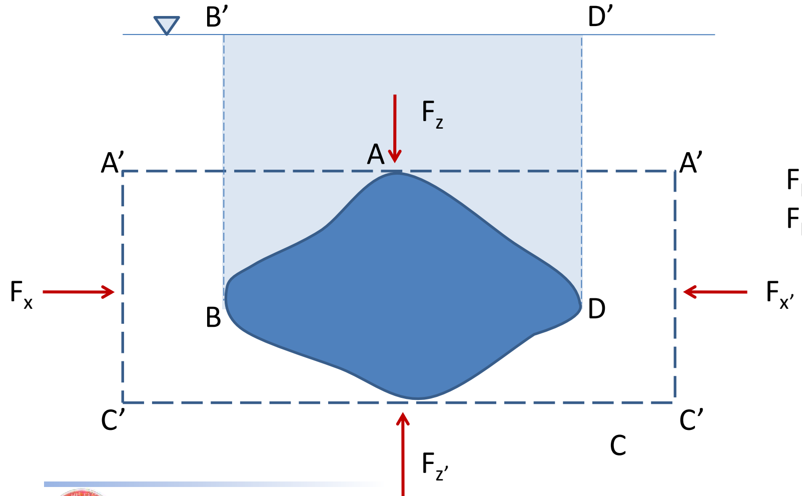


- Archimedes Principle

  F_B = weight displaced fluid

- A floating body displaces its own weight of the fluid in which it floats
- Line of action passes through the centroid of displaced volume

Buoyant Force



$F_B = F_{z'} - F_z$
$F_B = \rho g$ Vol (ABCD)

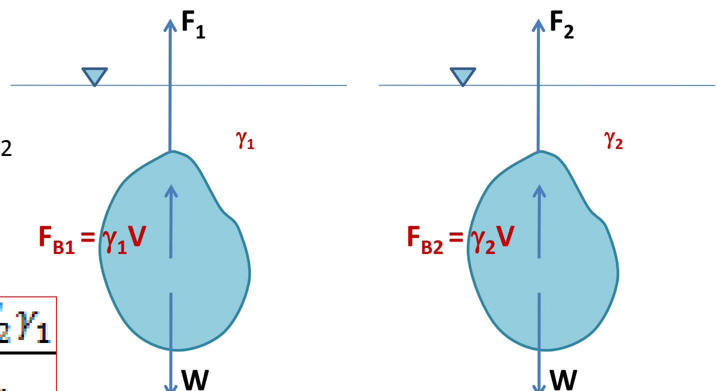
Application of Buoyancy

$F_1 + \gamma_1 V = W$
$F_2 + \gamma_2 V = W$
$\rightarrow V(\gamma_1 - \gamma_2) = F_1 - F_2$

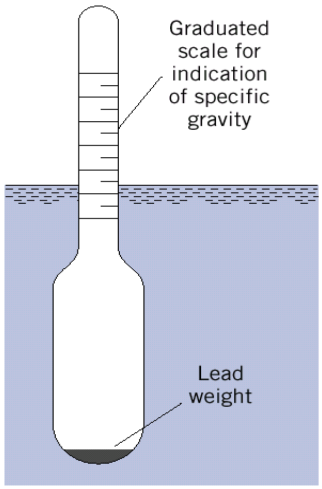
$$V = \frac{F_1 - F_2}{\gamma_2 - \gamma_1}$$

$$\boxed{W = \frac{F_1\gamma_2 - F_2\gamma_1}{\gamma_2 - \gamma_1}}$$

$\gamma = W/V$



Hydrometer



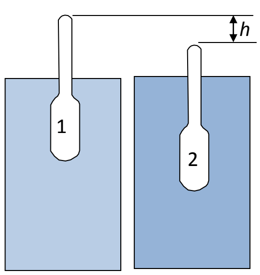
- Buoyant force

  F_B = weight of the hydrometer

  must remain constant

- Hydrometer floats deeper or shallower depending on the specific weight of the fluid

Example



A hydrometer weighs 0.0216 N and has a stem at the upper end that is cylindrical and 2.8 mm in diameter.

How much deeper will it float in oil of S=0.78 than in alcohol of S=0.821?

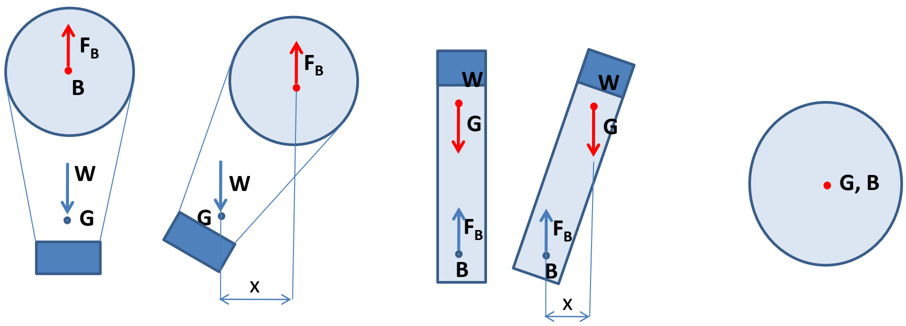
For position 1:

$$W_{hydrometer} = W_{displaced\ water}$$
$$0.0216 = 0.821 * 9810 * V_1$$
$$V_1 = 2.68x10^{-6}\ m^3$$

For position 2:

$$W_{hydrometer} = W_{displaced\ water}$$
$$0.0216 = 0.780 * 9810 * (V_1 + Ah)$$
$$= 0.780 * 9810 * [2.68x10^{-6} + \frac{\pi}{4}(0.0028)^2 h]$$
$$\boxed{h = 0.0232\, m = 23.2\, mm}$$

Stability of Submerged Bodies



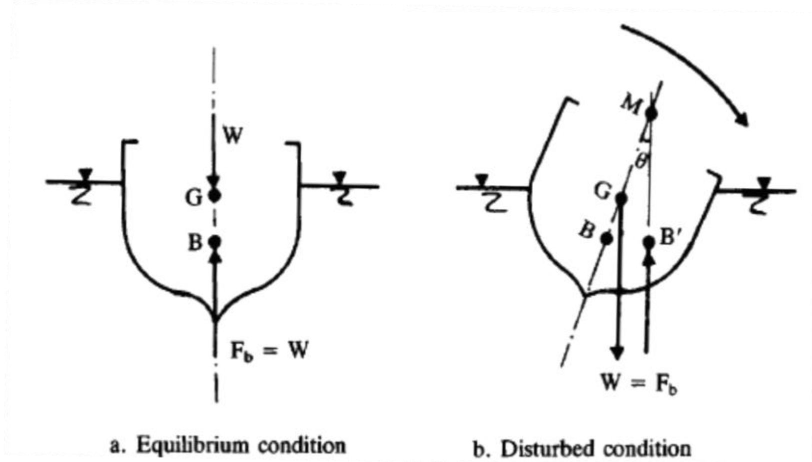
Stable Equilibrium: B always above G

Unstable Equilibrium: B always below G

Neutral Equilibrium: B and G

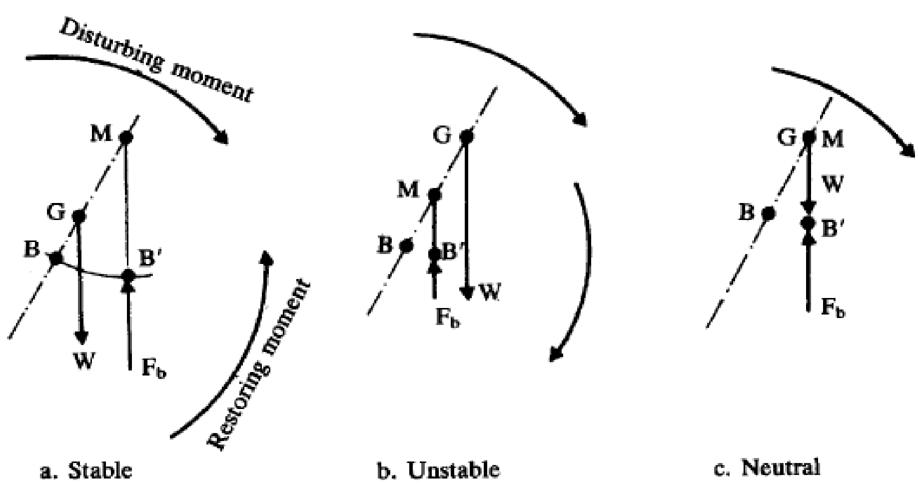
(i) G and B must lie on the same vertical line in the undisturbed position
(ii) B must always be above G for stable equilibrium

Stability of Floating Bodies

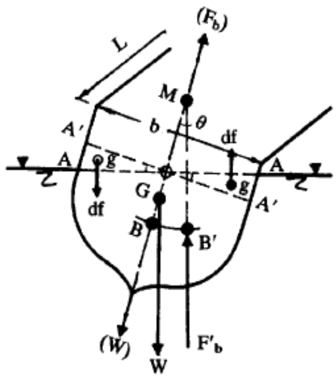


M = Metacenter
MG = Metacenteric height

Conditions of Stability



Determination of Metacenter



By moments about B, $F'_b \times BB' = df \times \overline{gg}$

$$\therefore BB' = df \times \overline{gg}/F'_b = df \times \overline{gg}/W = df \times \overline{gg}/\rho g V$$

where V is the volume of the displaced fluid.

$$\overline{MB} = \frac{I_{yy}}{V}$$

MG = MB - GB

RELATIVE EQUILIBRIUM OF LIQUIDS

Relative Equilibrium

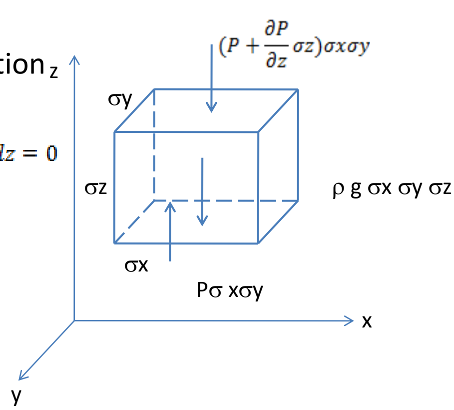
- Constant Velocity → Hydrostatics pressure
- Uniform acceleration/Rotation → Relative equilibrium

No relative motion between
the liquid particles and the container

- Uniform Linear Acceleration
- Uniform rotation about a vertical axis

Uniform Linear Acceleration

Considering equilibrium in the vertical direction



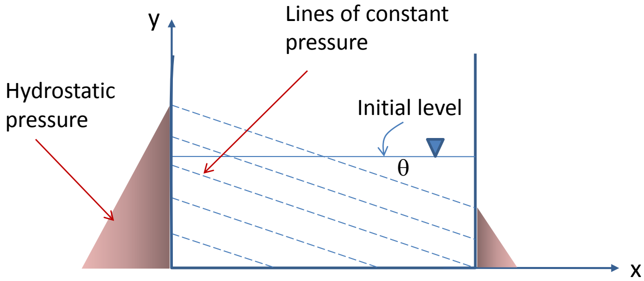
$$+p\,dx\,dy - \left(p + \frac{\partial p}{\partial z}dz\right)dxdy + \rho g dxdydz - a_z.\rho\,dxdydz = 0$$

Which reduces to $\frac{\partial p}{\partial z} = -\rho(a_z + g)$

Similarly in the other direction we get

$\frac{\partial p}{\partial y} = -\rho a_y$ and $\frac{\partial p}{\partial x} = -\rho a_x$

Uniform Linear Acceleration

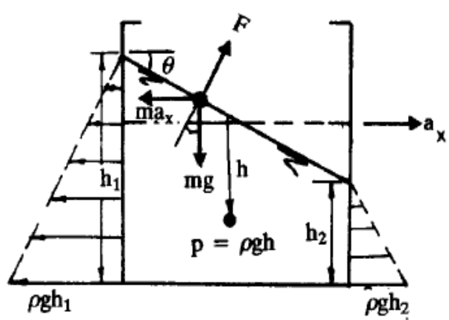


$$\frac{\partial p}{\partial x}dx + \frac{\partial p}{\partial x}dy = 0$$ ← On lines of constant pressure

$$\frac{dy}{dx} = -\frac{\partial p/\partial x}{\partial p/\partial y} = -\frac{\rho a_x}{\rho(g + a_y)} = -\frac{a_x}{(g + a_y)} = tan\theta$$

Horizontal Acceleration

$a_x \neq 0$ and $a_y = 0$ → $$\frac{dy}{dx} = -\frac{a_x}{g} = tan\theta$$

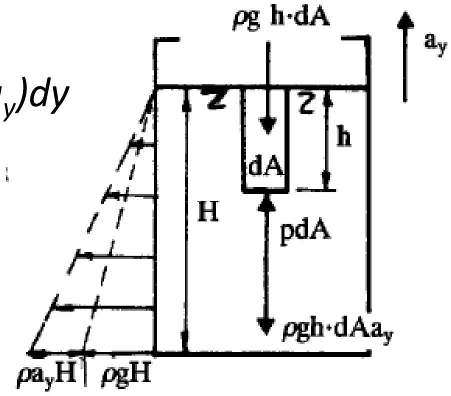


Vertical Acceleration

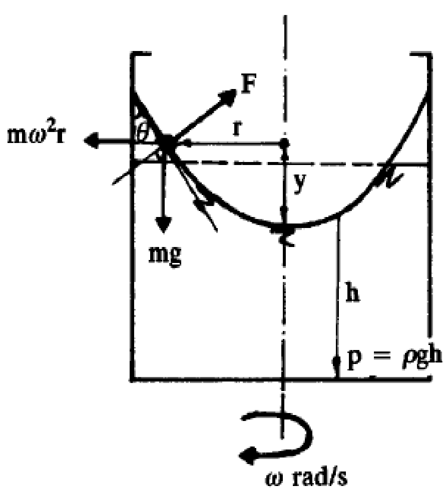
$a_x = 0$ and $a_y \neq 0$ ➔ $\partial p = -\rho(g + a_y)dy$

$p = -\rho(g + a_y)y$ *where* $h = -y$

$p = \rho(g + a_y)h$



Radial acceleration



The gradient of the free surface,

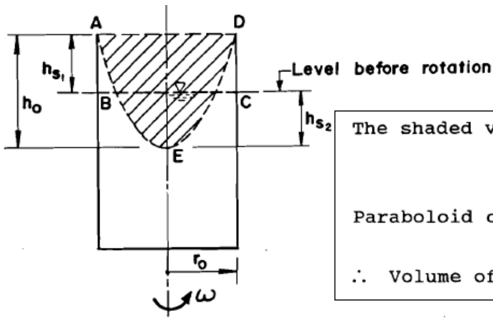
$\tan\theta = dy/dr = m\omega^2 r/mg = \omega^2 r/g$

$\therefore\ y = \omega^2 r^2/2g + \text{Constant, } C$

When $r = 0$, $y = 0$ and hence $C = 0$

$\therefore\ y = \omega^2 r^2/2g$

Radial acceleration



The shaded volume AED = paraboloid of revolution
= volume of the empty space ABCD

Paraboloid of revolution = 1/2(volume of circumscribing cylinder)

∴ Volume of empty space ABCD = 1/2 ($\pi r_o^2 . h_o$)

$$i.e. \quad \pi r_o^2 . hs_1 = \frac{1}{2} \pi r_o^2 . h_o$$

$$\therefore \quad hs_1 = \frac{1}{2} h_o = hs_2$$

Example 1

A rectangle block of wood floats in water with 50 mm projecting above the water surface. When placed in glycerine of relative density 1.35, the block projects 75 mm above the surface of glycerine. Determine the relative density of the wood.

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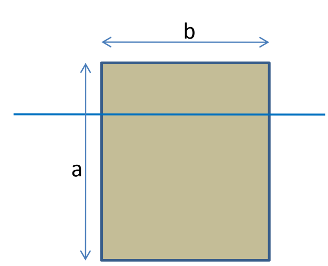
Solution

- Weight of wooden block, W = uptrust in water = uptrust in glycerine =weight of the fluid displaced
- W = $\rho gAh = \rho_w\, g\, A(h - 50 \times 10^{-3}) = \rho_G gA\,(h - 75 \times 10^{-3})$
- The relative density of glycerine= $\frac{\rho_G}{\rho} = \frac{h - 50 \times 10^{-3}}{h - 75 \times 10^{-3}} = 1.35$
- h = 146.43×10^{-3} m or 146.43 mm
- Hence the relative density of wood,
- $\boxed{\rho/\rho_w = (146.43-50)/146.43 = 0.658}$

Example 2

Determine the maximum ratio of a to b for the stability of a rectangular block of mass density ρ_b for a small angle of tilt when it floats in a liquid of mass density ρ_1. The dimension a is greater than b.



Solution

Consider 1 m length of block

Weight of the block = weight of liquid displaced

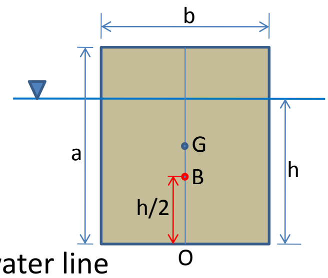
$\rho_b g \times ab \times 1 = \rho_1\, g \times bh \times 1$

$\rightarrow h = a \times (\rho_b/\rho_1)$

$OB = h/2 = a \times (\rho_b/\rho_1)/2$

$BG = OG - OB$

$= a/2 - (\rho_b a)/2\rho_1 = a(1-\rho_b/\rho_1)/2$



$$BM = \frac{\text{Moment of inertia of surface area at the water line}}{\text{Volume of body immersed in liquid}}$$

$= (1/12 \times 1 \times b^3)/(b \times h \times 1) = b^2/12h$

$BM = (\rho_1 b^2)/(12a\,\rho_b)$

For stability $BM \geq BG$ ➔ $\dfrac{b^2}{12a} \times \dfrac{\rho_1}{\rho_2} \geq \dfrac{a}{2} - (1 - \rho_b/\rho_1)$ ➔ $\boxed{\dfrac{a}{b} \geq \dfrac{\rho_1}{\sqrt{6\rho_b(\rho_1 - \rho_b)}}}$

Example 3

A barge 20 m long x 7 m wide has a draft of 2 m when floating in upright position. Its C.G. is 2.25 m above the bottom, (a) what is it's initial metacenteric height? (b) If a 5 tone weight is shifted 4 m across the barge, to what distance does the water line rise on the side? Find the rightening moment for this lift.

Solution

(a) BM = I/V

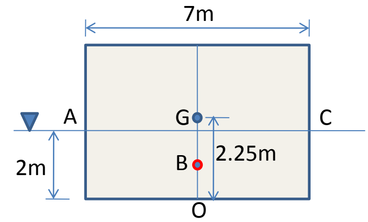
$= [1/12 \times 20 \times 7^3/(20 \times 7 \times 2)]$

$= 2.045$ m

OB = ½ x 2 = 1

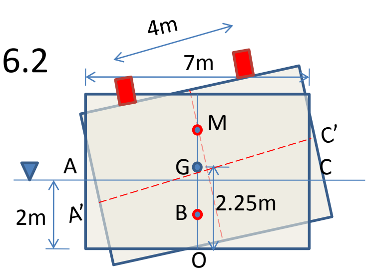
BG = OG – OB = 2.25 – 1.0 = 1.25 m

MG = BM – BG = 2.045 – 1.25 = $\boxed{0.795 \text{ m}}$



Solution

moment heeling the ship = 5 x 9.81x4 = 196.2
= moment due to the shifting of G to G'
= W x GG'
But GG' = GM sin θ
sin θ = 196.2/(9.81 x 20 x 7 x 2 x 0.795)
= 0.0904 = 5°12'
Rise of water line on one side = 3.5 tanθ
= 3.5 tan 5°12' = 0.318 m
Rightening moment = W x MG tan θ
= 9.81 x 20 x 7 x 2 x 0.795 x tan 5°12'
=197.1 kNm

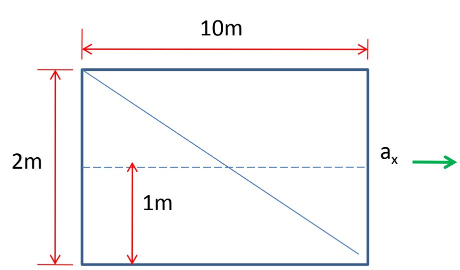


Example 4

An oil tanker 3 m wide, 2 m deep and 10 m long contains oil of density 900 kg/m^3 to a depth of 1 m. Determine the maximum horizontal acceleration that can be given to the tanker such that the oil just reaches its top end.

If this tanker is closed and completely filled with the oil and accelerated horizontally at 3 m/s^2 determine that total liquid thrust (i) on the front end, (ii) on the rear end, and (iii) on one of its longitudinal vertical sides.

Solution



maximum possible surface slope = 1/5 = a_x/g

a_x, the maximum horizontal acceleration = 9.81/5

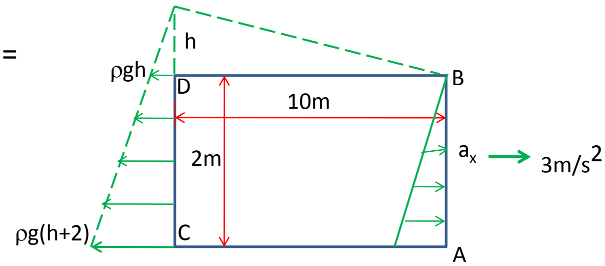
a_x = 1.962 m/s^2

Solution

(i) total thrust on front end AB =
½ ρg x 2^2 x 3 = 58.86 kN

(ii) total thrust on rear end CD:

h = 10 x tan θ= 10 x a_x/g
= 10 x 3/9.81 = 3.06 m



$$\therefore \text{ Total thrust on CD} = \frac{\rho g\,(3{\cdot}06) + \rho g\,(2 + 3{\cdot}06)}{2} \times 2 \times 3 = 239 \text{ kN}$$

(iii) total thrust on side ABCD = volume of the pressure prism

$$= \frac{1}{2}\rho g \times 2^2 \times 10 + \frac{1}{2}\rho g \times 3{\cdot}06 \times 10 \times 2$$

$$= \frac{1}{2}\rho g\,(2 + 3{\cdot}06) \times 2 \times 10$$

$$= 496 \text{ kN.}$$

Example 5

- A vertical hoist carries a square tank of 2m x 2m containing water to the top of a construction scaffold with a varying speed of 2 m/s per second. If the water depth is 2m, calculate the total hydrostatic trust on the bottom of the tank.
- If this tank of water is lowered with an acceleration equal to that of gravity, what are the thrusts on the floor and sides of the tank?

Solution

Vertical upward acceleration, $a_y = 2\ m/s^2$

Pressure intensity at a depth $h = \rho gh\,(1 + a_y/g)$

$= \rho gh\,(1 + 2/9{\cdot}81)$

$= 1{\cdot}204 \times gh\ kN/m^2$

$\therefore$ Total hydrostatic thrust on the floor

$=$ intensity $\times$ area $= 1{\cdot}204 \times 9{\cdot}81 \times 2 \times 2 \times 2 = 94{\cdot}5\ kN$

Downward acceleration $= -\ 9{\cdot}81\ m/s^2$

Pressure intensity at a depth $h = \rho gh\,(1 - 9{\cdot}81/9{\cdot}81)$

$= 0$

$\therefore$ There exists no hydrostatic thrust on the floor nor on the sides.

Example 6

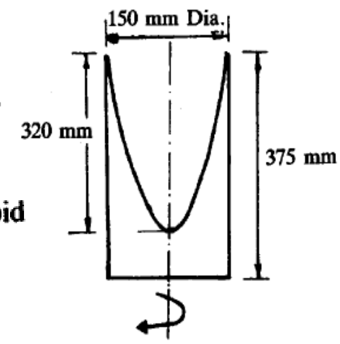
A 375 mm high open cylinder, 150 mm in diameter, is filled with water and rotated about its vertical axis at an angular speed of 33.5 rad/s. Determine (i) the depth of water in the cylinder when it is brought to rest, and (ii) the volume of water that remains in the cylinder if the speed is doubled.

Solution

Height of the paraboloid (fig. 2.32a), $y = \omega^2 r^2/2g$

$= (33{\cdot}5 \times 0{\cdot}075)^2/19{\cdot}62$

$= 0{\cdot}32$ m



a. $\omega = 33{\cdot}5$ rad/s

Amount of water spilled out = volume of the paraboloid

$= \frac{1}{2} \times$ volume of circumscribing cylinder

$= \frac{1}{2} \pi (0{\cdot}075)^2 \times 0{\cdot}32 = 2{\cdot}83 \times 10^{-3}$ m^3

Original volume of water $= \pi(0{\cdot}075)^2 \times 0{\cdot}375$

$= 6{\cdot}63 \times 10^{-3}$ m^3

$\therefore$ Remaining volume of water $= (6{\cdot}63 - 2{\cdot}83) \times 10^{-3}$

$= 3{\cdot}8 \times 10^{-3}$ m^3

Hence depth of water at rest

$= 3{\cdot}8 \times 10^{-3}/\pi(0{\cdot}075)^2$

$= 0{\cdot}215$ m

Solution

If the speed is doubled, $\omega = 67$ rad/s

$\therefore$ Height of paraboloid $= (67 \times 0{\cdot}075)^2/2g$

$= 1{\cdot}287$ m

The free surface assumes the shape shown in the figure

$1{\cdot}287 - 0{\cdot}375 = \omega^2 r^2/2g$

$\therefore r = \sqrt{2g \times 0{\cdot}912/67^2} = 0{\cdot}063$ m

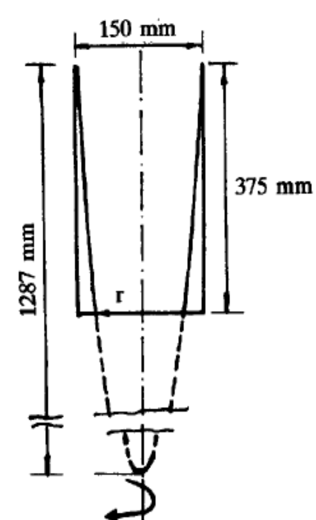
$\therefore$ Volume of water spilled out

$= \frac{1}{2} \pi (0{\cdot}075)^2 \times 1{\cdot}287 - \frac{1}{2} \pi (0{\cdot}063)^2 \times 0{\cdot}912$

$= 5{\cdot}684 \times 10^{-3}$ m^3

Hence volume of water left $= (6{\cdot}63 - 5{\cdot}684) \times 10^{-3}$

$= 0.946 \times 10^{-3}$ m^3.



b. $\omega = 67{\cdot}0$ rad/s