Chapter 2 Hydrostatics

Hydraulics I

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Definitions

- **Hydrostatics:** the study of fluids that are at rest or moving with uniform velocity as a solid body (no relative motion)
- For fluids at rest or no relative motion
  - shear stress is zero between the fluid layers.
  - The only force acting will be hydrostatic: normal pressure force

o Application
  - Submerged bodies: gates, submarines, dams etc
  - Analysis of stability of floating bodies: ships, pontoons etc

Pascal's law

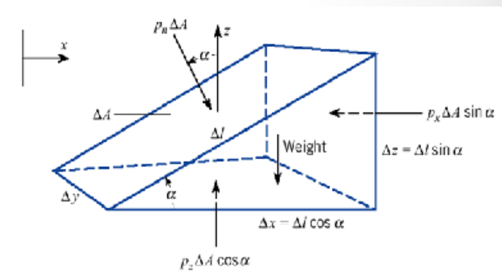
"the pressure at a point in a fluid at rest is the same in all directions."

$$\sum F_x = 0 = p_x \Delta y \Delta l \sin\alpha - p_n \Delta y \Delta l \sin\alpha \quad \Rightarrow \quad p_n = p_x$$

$$\sum F_z = 0 = p_z \Delta y \Delta l \cos\alpha - p_n \Delta y \Delta l \cos\alpha - \frac{1}{2}\gamma \Delta l \cos\alpha \Delta l \sin\alpha \Delta y$$

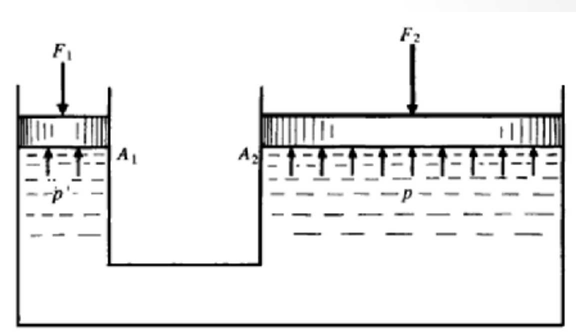
$$\Rightarrow \quad p_n = p_z$$

$$\therefore \quad p_n = p_x = p_y = p_z = p$$



Pascal's law

$P = F_1/A_1$
$F_2 = PA_2$
$F_2 = F_1\,(A_2/A_1)$



Hydraulic Piston

Basic Equation of Hydrostatics

The mass of the fluid = ρ.σx.σy.σz
Assume the fluid is subjected to
Acceleration of a_x, a_y and a_z

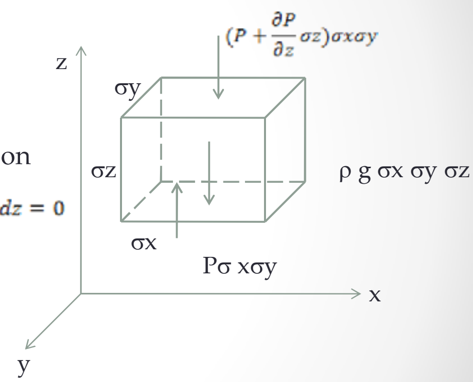
Considering equilibrium in the vertical direction

$$+p\,dx\,dy - \left(p + \frac{\partial p}{\partial z}dz\right)dxdy + \rho g dxdydz - a_z.\rho\, dxdydz = 0$$

Which reduces to $\frac{\partial p}{\partial z} - \rho(a_z + g)$

Similarly in the other direction we get

$\frac{\partial p}{\partial y} = -\rho a_y$ and $\frac{\partial p}{\partial x} = -\rho a_x$



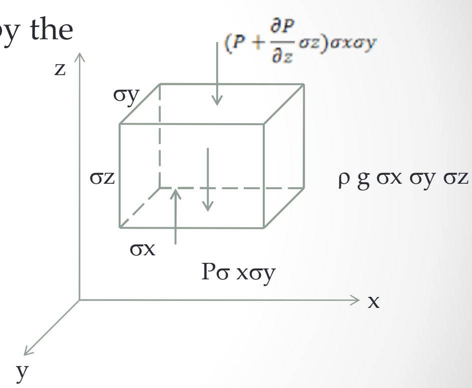
Basic Equation of Hydrostatics

The total change in pressure is given by the
Total differential as follows:

$$dp = \frac{\partial p}{\partial x}.dx + \frac{\partial p}{\partial y}.dy + \frac{\partial p}{\partial z}.dz$$

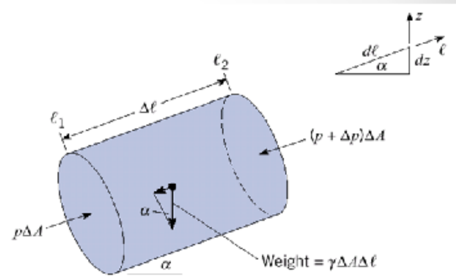
Or $dp = -\rho a_x.dx - \rho ay.dy - \rho(a_z + g).dz$

$$dp = -\rho[a_x.dx + a_y.dy + (a_z + g).dz]$$



Pressure Variation with Elevation

- **Static fluid** – pressure varies ***only*** with elevation in the fluid.



$$\sum F_l = 0$$
$$= F_{left} - F_{right} - F_{weight}$$
$$= p\Delta A - (p + \Delta p)\Delta A - \gamma \Delta A \Delta l \sin\alpha$$

$$\frac{\Delta p}{\Delta l} = -\gamma \sin\alpha = -\gamma \frac{\Delta z}{\Delta l} \quad \text{or} \quad \frac{dp}{dl} = -\gamma \frac{dz}{dl}$$

$$\boxed{\frac{dp}{dz} = -\gamma}$$

Pressure Variation with Elevation

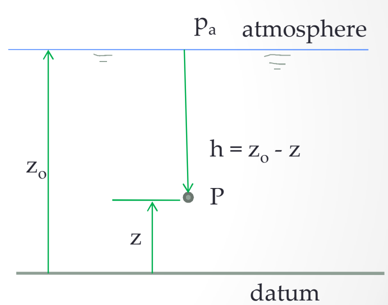
- If γ is a constant $\frac{dp}{dz} = -\gamma$

$$\int dp = -\rho g \int dz$$
$$p = -\rho g z + C$$

When $z = z_o$, $p = p_a$

Therefore, the pressure at a depth z is

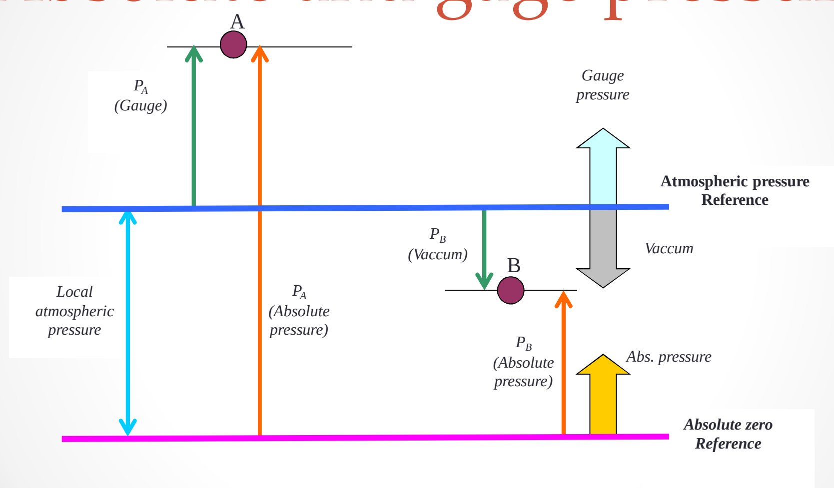
$p = p_a + \rho g h$



$$h = \frac{p}{\gamma} + z = \text{constant} \quad \text{(piezometric head)}$$

Elevation head
Pressure head
Piezometric head

Absolute and gage pressure



$$P_{abs} = P_{atm} + P_{gage}$$

Absolute and gage pressure

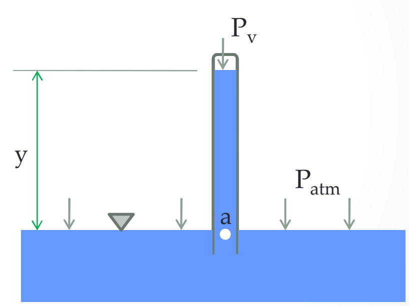
- Atmospheric pressure → Barometric pressure

$P_a = P_v + \rho g y = P_{atm}$

$\rightarrow P_{atm} = P_v + \rho g y$ ($P_v \cong 0$)

$\rightarrow P_{atm} = \rho g y$

$$y = \frac{P_{atm}}{\rho g}$$

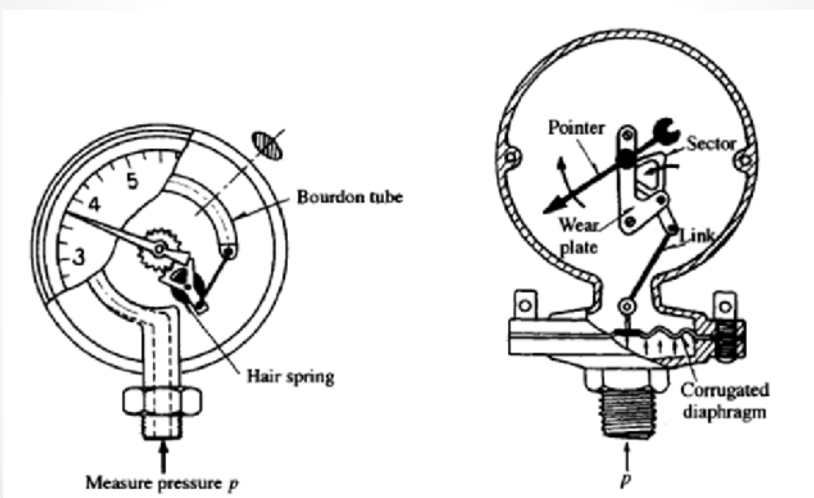


At sea level, y = 760 mm of Hg or 10.33 m of water

Measurement of Pressure

- Bourdon Gauge
- Piezometer column
- Manometers
- Micromanometer

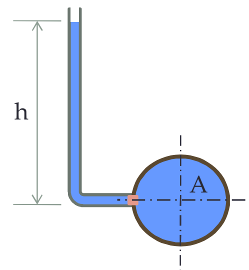
Bourdon Gauge



Piezometer column

Measures positive pressures

$$P_A = \rho g h$$

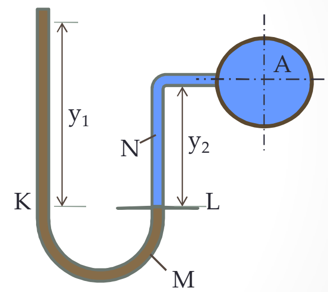


Manometers

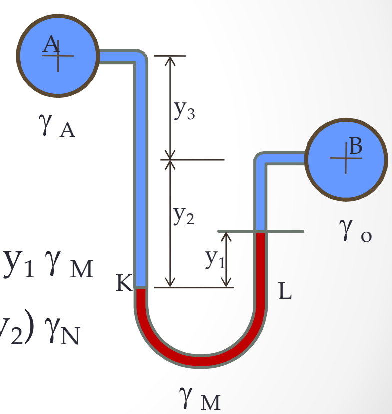
- Used to measure pressure difference between two points
- High positive and negative pressure differences

U-tube Manometer

$P_L = P_K$
$P_A + \gamma_N y_2 = \gamma_M y_1$
Thus, $P_A = \gamma_M y_1 - \gamma_N y_2$



Differential Manometer



$P_K = P_L$

$P_A + (y_3 + y_2)\gamma_N = P_B + (y_2 - y_1)\gamma_o + y_1\gamma_M$

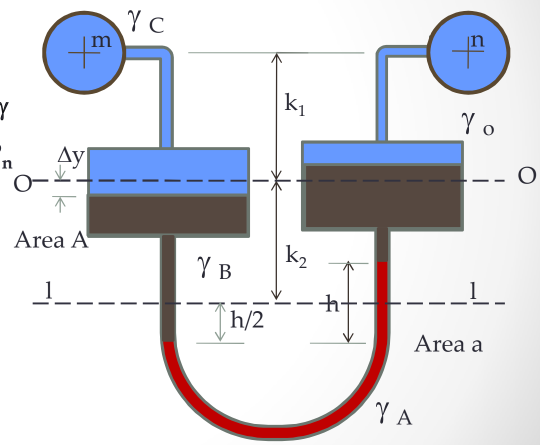
$P_A - P_B = (y_2 - y_1)\gamma_o + y_1\gamma_M - (y_3 + y_2)\gamma_N$

Micromanometers

$P_m + (k_1 + \Delta y)\,\gamma_c + (k_2 - \Delta y + h/2)\,\gamma_B - h\,\gamma_A - (k_2 - h/2 + \Delta y)\,\gamma_B - (k_1 - \Delta y)\,\gamma_C = P_n$

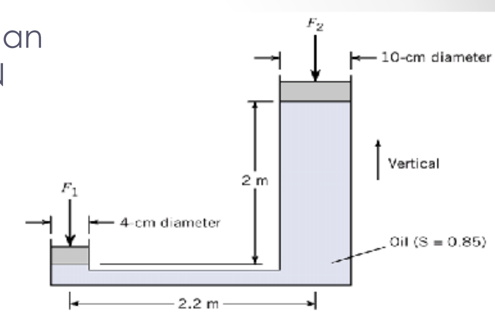
$\Delta y.\ A = h/2\ a$ or $2\,\Delta y = h\,(a/A)$

$P_m - P_n = h[\gamma_A - \gamma_B(1 - a/A) - \gamma_c\, a/A]$



Example 2.1

- What is maximum force F_2 that can be supported? Where F1 = 100 N



$$p_1 - p_2 = -\gamma_{oil}\,(z_1 - z_2)$$

$$p_2 = p_1 + \gamma_{oil}\,(z_1 - z_2)$$

$$= \frac{F_1}{A_1} + SG\;\gamma_w\,(z_1 - z_2)$$

$$= \frac{200}{\frac{\pi}{4}(0.04)^2} + (0.85)(9810)(0 - 2)$$

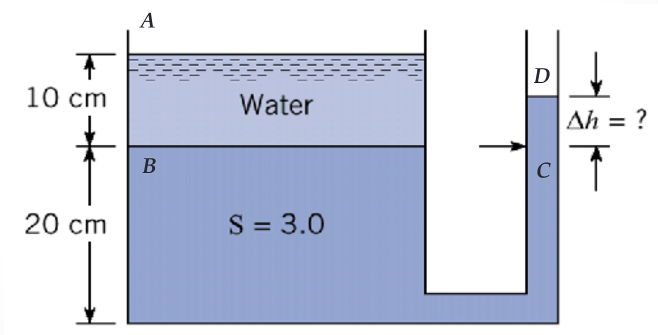
$$= 142{,}500\ N/m^2$$

$$F_2 = p_2 A_2 = 142{,}500\ (\frac{\pi}{4})(0.1)^2$$

$$F_2 = 1{,}119\ N$$

Example 2.2

- Find the location of the surface in the manometer



Solution

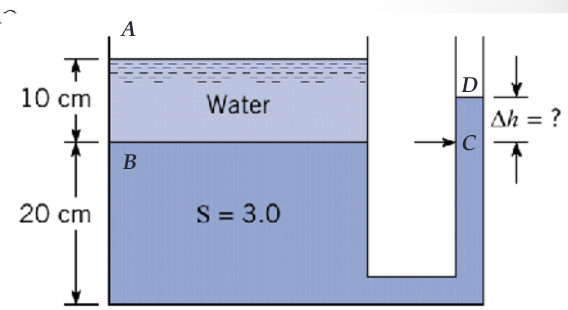
The distance Δh is the height of the liquid in the manomoter above the heavier liquid in the tank.

$$p_A + 0.1 * \gamma_w - \Delta h * \gamma_m = p_D$$

$$p_A = P_D = 0$$

$$\Delta h = 0.1 * \frac{\gamma_w}{\gamma_m}$$

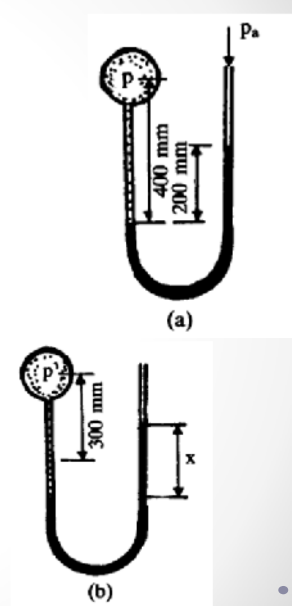
$$\Delta h = 0.1 * \frac{1}{3} = 3.33 \ cm$$



Example 2.3

A U-tube containing mercury (relative density 13.6) has its right-hand limb open to atmosphere and the left-hand limb connected to a pipe conveying water under pressure, the difference in levels of mercury in the two limps being 200 mm. If the mercury level in the left limb is 400 mm below the center line of the pipe, find the absolute pressure in the pipeline in kPa. Also find the new difference in levels of the mercury in the U-tube if the pressure in the pipe falls by 2 kN/m^2.

Solution



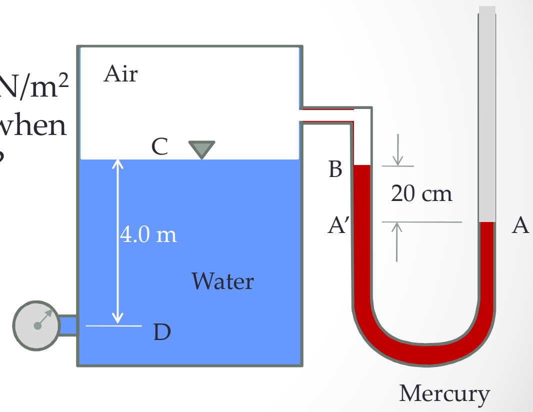
p/ρg + 0.40 - 13.6 x 0.20 = 0 (atmosphere)
➔ p/ ρg = 2.32 m of water
or p = 10^3 x 9.81 x 2.32 = 22.76 kN/m^2

The corresponding abs pressure = 101 + 22.76 = 123.76 kN/m^2

20.76 x 10^3/10^3 x 9.81 + 0.30 + x/2 - 13.6 x = 0
➔ x, the new difference is mercury levels = 0.184 m 0r 184 mm

Example 2.4

Determine the gauge reading in N/m^2
What will be the gauge reading when
Expressed as head of water in m?

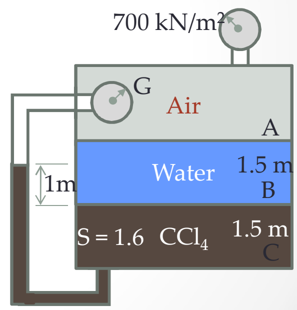


Solution

- $P_A = P_{A'}$
- $P_B = P_{A'} - 0.20\ \gamma_M$
- $P_C = P_B$ and $P_D = P_{gauge} = P_C + 4\ \gamma_W$
- ➔ $P_D = P_{A'} - 0.20\ \gamma_m + 4\ \gamma_W$
- $P_D = 0 - 0.20 \times \gamma_W . S_m + 4\ \gamma_W$
- $P_D = 12556.8\ N/m^2$
- Expressed as head of water
- $h = P/\ \gamma_W = 12556.8/9810 = 1.28$ m

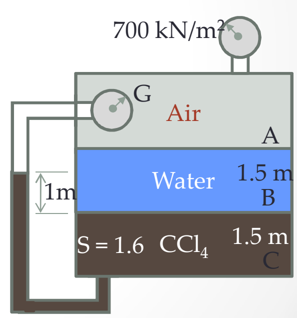
Example 2.5

The Bourden gauge connected to the manometer is inside The sealed tank. Atmospheric pressure is 100 kPa absolute. Calculate the gauge reading Px.



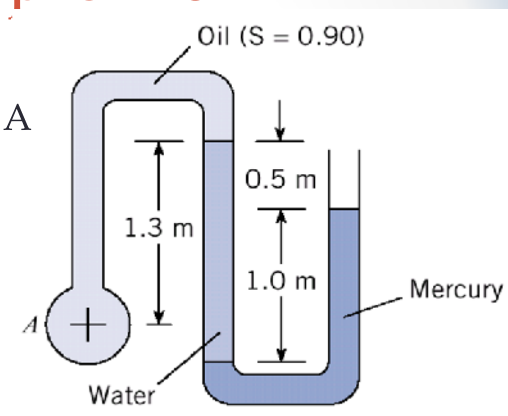
Solution

- P_A = atm. Pressure + 700 kPa gauge
- P_A = 800 kPa
- $P_B = P_A$ + pressure due to water column = 800 + 9.81 x 1.5 = 814.715 kPa abs.
- $P_C = P_B$ + pressure due to CCl4 = 814.715 + 9.81 x 1.6 x 1.5 = 838.259 kPa abs.
- Gauge Pressure at C with respect to A = 838.259 – 800 = 38.259 kPa
- The pressure in both limbs of U-tube should be equal
- P_x + 1.6 x 9.81 x (1.0 + 1.5) = 38.259
- P_x = 0.981 kPa vacuum



Example 2.6

Find the pressure at in pipe A



$P_A + 0.2 = 1.0 \times 13.6$

$P_A = 13.40$ m

$P_A = 13.4 \times 9.81 = 131.45$ kPa