HEAT EXCHANGERS

Heat exchangers are devices that facilitate the *exchange of heat* between *two fluids* that are at different temperatures while keeping them from mixing with each other. Heat exchangers are commonly used in practice in a wide range of applications, from heating and air-conditioning systems in a household, to chemical processing and power production in large plants. Heat exchangers differ from mixing chambers in that they do not allow the two fluids involved to mix. In a car radiator, for example, heat is transferred from the hot water flowing through the radiator tubes to the air flowing through the closely spaced thin plates outside attached to the tubes.

Heat transfer in a heat exchanger usually involves *convection* in each fluid and *conduction* through the wall separating the two fluids. In the analysis of heat exchangers, it is convenient to work with an *overall heat transfer coefficient U* that accounts for the contribution of all these effects on heat transfer. The rate of heat transfer between the two fluids at a location in a heat exchanger depends on the magnitude of the temperature difference at that location, which varies along the heat exchanger. In the analysis of heat exchangers, it is usually convenient to work with the *logarithmic mean temperature difference LMTD,* which is an equivalent mean temperature difference between the two fluids for the entire heat exchanger.

Heat exchangers are manufactured in a variety of types, and thus we start this chapter with the *classification* of heat exchangers. We then discuss the determination of the overall heat transfer coefficient in heat exchangers, and the LMTD for some configurations. We then introduce the *correction factor F* to account for the deviation of the mean temperature difference from the LMTD in complex configurations. Next we discuss the effectiveness–NTU method, which enables us to analyze heat exchangers when the outlet temperatures of the fluids are not known. Finally, we discuss the selection of heat exchangers.

CONTENTS

13–1 ▪ TYPES OF HEAT EXCHANGERS

Different heat transfer applications require different types of hardware and different configurations of heat transfer equipment. The attempt to match the heat transfer hardware to the heat transfer requirements within the specified constraints has resulted in numerous types of innovative heat exchanger designs.

The simplest type of heat exchanger consists of two concentric pipes of different diameters, as shown in Figure 13–1, called the **double-pipe** heat exchanger. One fluid in a double-pipe heat exchanger flows through the smaller pipe while the other fluid flows through the annular space between the two pipes. Two types of flow arrangement are possible in a double-pipe heat exchanger: in **parallel flow,** both the hot and cold fluids enter the heat exchanger at the same end and move in the *same* direction. In **counter flow,** on the other hand, the hot and cold fluids enter the heat exchanger at opposite ends and flow in *opposite* directions.

Another type of heat exchanger, which is specifically designed to realize a large heat transfer surface area per unit volume, is the **compact** heat exchanger. The ratio of the heat transfer surface area of a heat exchanger to its volume is called the *area density* β. A heat exchanger with $\beta > 700$ m^2/m^3 (or 200 ft^2/ft^3) is classified as being compact. Examples of compact heat exchangers are car radiators ($\beta \approx 1000$ m^2/m^3), glass ceramic gas turbine heat exchangers ($\beta \approx 6000$ m^2/m^3), the regenerator of a Stirling engine ($\beta \approx 15,000$ m^2/m^3), and the human lung ($\beta \approx 20,000$ m^2/m^3). Compact heat exchangers enable us to achieve high heat transfer rates between two fluids in

FIGURE 13–1
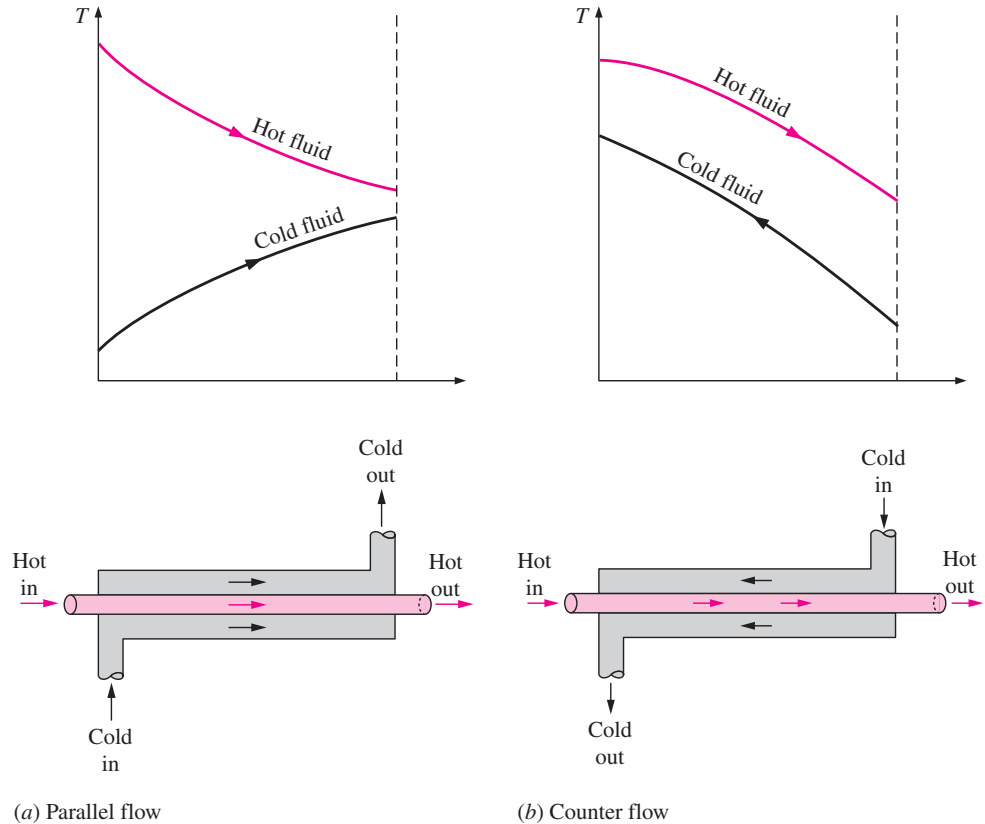Different flow regimes and associated temperature profiles in a double-pipe heat exchanger.

(*a*) Parallel flow

(*b*) Counter flow

a small volume, and they are commonly used in applications with strict limitations on the weight and volume of heat exchangers (Fig. 13–2).

The large surface area in compact heat exchangers is obtained by attaching closely spaced *thin plate* or *corrugated fins* to the walls separating the two fluids. Compact heat exchangers are commonly used in gas-to-gas and gas-to-liquid (or liquid-to-gas) heat exchangers to counteract the low heat transfer coefficient associated with gas flow with increased surface area. In a car radiator, which is a water-to-air compact heat exchanger, for example, it is no surprise that fins are attached to the air side of the tube surface.

In compact heat exchangers, the two fluids usually move *perpendicular* to each other, and such flow configuration is called **cross-flow.** The cross-flow is further classified as *unmixed* and *mixed flow,* depending on the flow configuration, as shown in Figure 13–3. In (*a*) the cross-flow is said to be *unmixed* since the plate fins force the fluid to flow through a particular interfin spacing and prevent it from moving in the transverse direction (i.e., parallel to the tubes). The cross-flow in (*b*) is said to be *mixed* since the fluid now is free to move in the transverse direction. Both fluids are unmixed in a car radiator. The presence of mixing in the fluid can have a significant effect on the heat transfer characteristics of the heat exchanger.
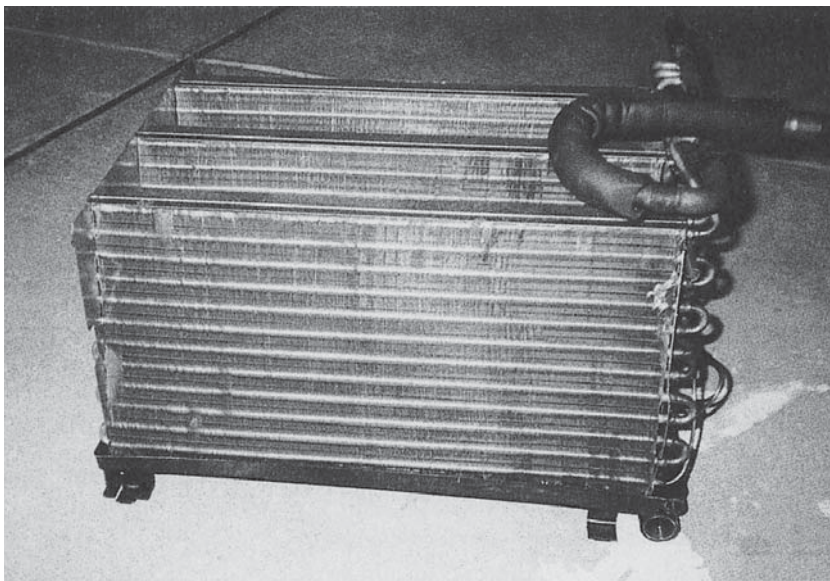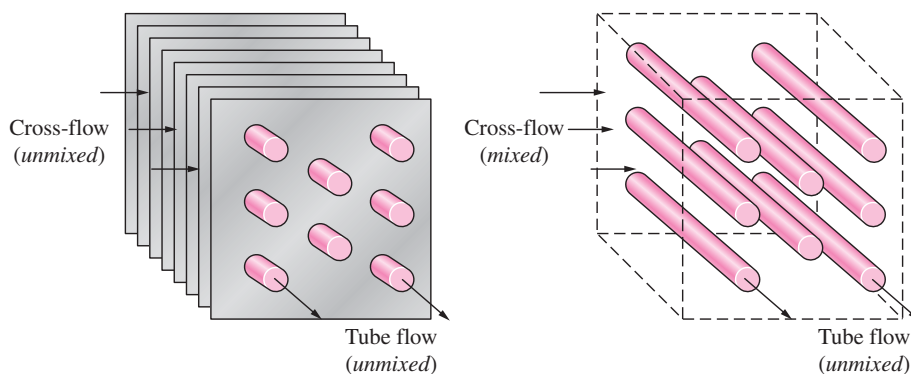

FIGURE 13–2
A gas-to-liquid compact heat exchanger for a residential air-conditioning system.
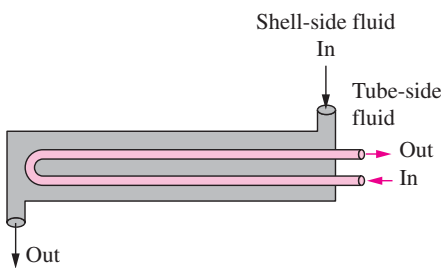

Cross-flow
(*unmixed*)

Tube flow
(*unmixed*)

(*a*) Both fluids unmixed

Cross-flow
(*mixed*)
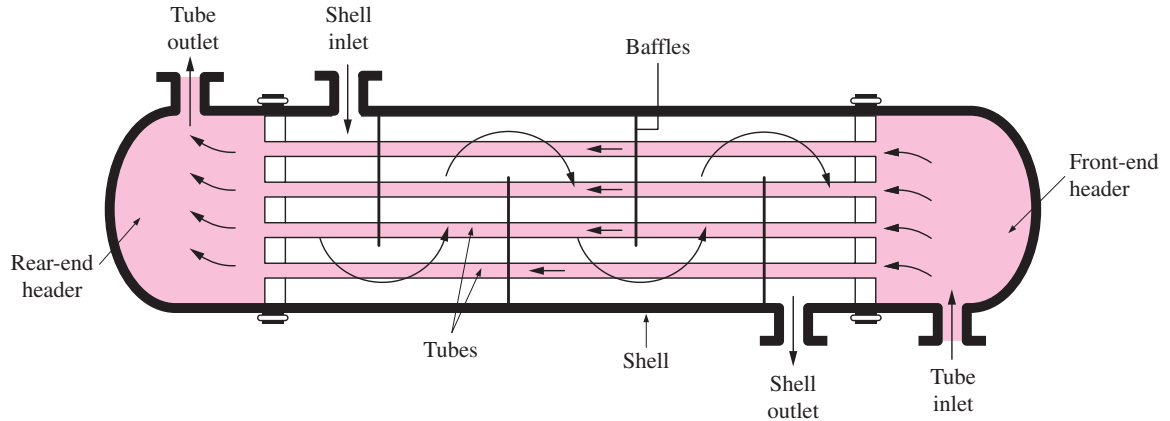
Tube flow
(*unmixed*)

(*b*) One fluid mixed, one fluid unmixed

FIGURE 13–3
Different flow configurations in cross-flow heat exchangers.

(*a*) One-shell pass and two-tube passes
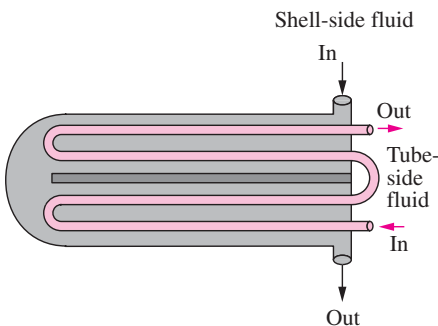
(*b*) Two-shell passes and four-tube passes

FIGURE 13–5
Multipass flow arrangements in shell-and-tube heat exchangers.

Perhaps the most common type of heat exchanger in industrial applications is the **shell-and-tube** heat exchanger, shown in Figure 13–4. Shell-and-tube heat exchangers contain a large number of tubes (sometimes several hundred) packed in a shell with their axes parallel to that of the shell. Heat transfer takes place as one fluid flows inside the tubes while the other fluid flows outside the tubes through the shell. *Baffles* are commonly placed in the shell to force the shell-side fluid to flow across the shell to enhance heat transfer and to maintain uniform spacing between the tubes. Despite their widespread use, shell-and-tube heat exchangers are not suitable for use in automotive and aircraft applications because of their relatively large size and weight. Note that the tubes in a shell-and-tube heat exchanger open to some large flow areas called *headers* at both ends of the shell, where the tube-side fluid accumulates before entering the tubes and after leaving them.

Shell-and-tube heat exchangers are further classified according to the number of shell and tube passes involved. Heat exchangers in which all the tubes make one U-turn in the shell, for example, are called *one-shell-pass and two-tube-passes* heat exchangers. Likewise, a heat exchanger that involves two passes in the shell and four passes in the tubes is called a *two-shell-passes and four-tube-passes* heat exchanger (Fig. 13–5).

An innovative type of heat exchanger that has found widespread use is the **plate and frame** (or just plate) heat exchanger, which consists of a series of plates with corrugated flat flow passages (Fig. 13–6). The hot and cold fluids flow in alternate passages, and thus each cold fluid stream is surrounded by two hot fluid streams, resulting in very effective heat transfer. Also, plate heat exchangers can grow with increasing demand for heat transfer by simply mounting more plates. They are well suited for liquid-to-liquid heat exchange applications, provided that the hot and cold fluid streams are at about the same pressure.

Another type of heat exchanger that involves the alternate passage of the hot and cold fluid streams through the same flow area is the **regenerative** heat exchanger. The *static*-type regenerative heat exchanger is basically a porous mass that has a large heat storage capacity, such as a ceramic wire mesh. Hot and cold fluids flow through this porous mass alternatively. Heat is transferred from the hot fluid to the matrix of the regenerator during the flow of the hot fluid, and from the matrix to the cold fluid during the flow of the cold fluid. Thus, the matrix serves as a temporary heat storage medium.
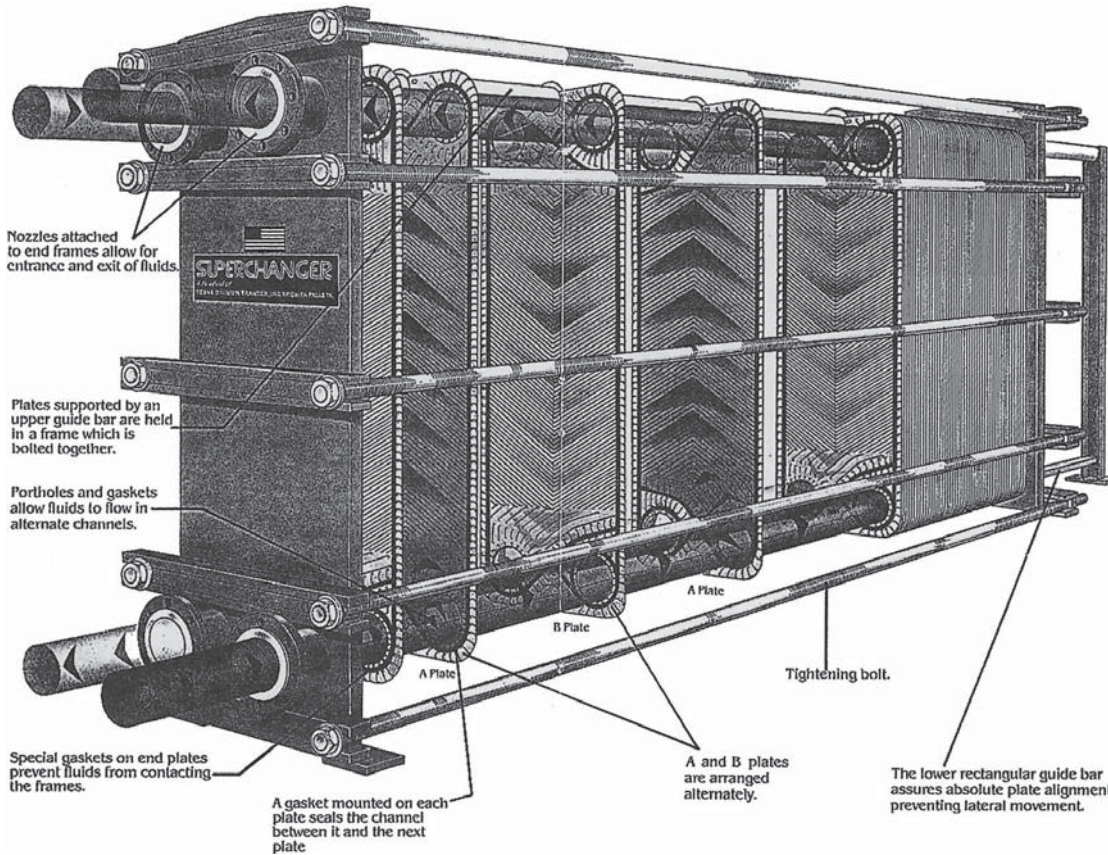
FIGURE 13–6
A plate-and-frame liquid-to-liquid heat exchanger (courtesy of Trante Corp.).

The *dynamic*-type regenerator involves a rotating drum and continuous flow of the hot and cold fluid through different portions of the drum so that any portion of the drum passes periodically through the hot stream, storing heat, and then through the cold stream, rejecting this stored heat. Again the drum serves as the medium to transport the heat from the hot to the cold fluid stream.

Heat exchangers are often given specific names to reflect the specific application for which they are used. For example, a *condenser* is a heat exchanger in which one of the fluids is cooled and condenses as it flows through the heat exchanger. A *boiler* is another heat exchanger in which one of the fluids absorbs heat and vaporizes. A *space radiator* is a heat exchanger that transfers heat from the hot fluid to the surrounding space by radiation.

13–2 ▪ THE OVERALL HEAT TRANSFER COEFFICIENT

A heat exchanger typically involves two flowing fluids separated by a solid wall. Heat is first transferred from the hot fluid to the wall by *convection,* through the wall by *conduction,* and from the wall to the cold fluid again by *convection.* Any radiation effects are usually included in the convection heat transfer coefficients.

The thermal resistance network associated with this heat transfer process involves two convection and one conduction resistances, as shown in Figure 13–7. Here the subscripts i and o represent the inner and outer surfaces of the
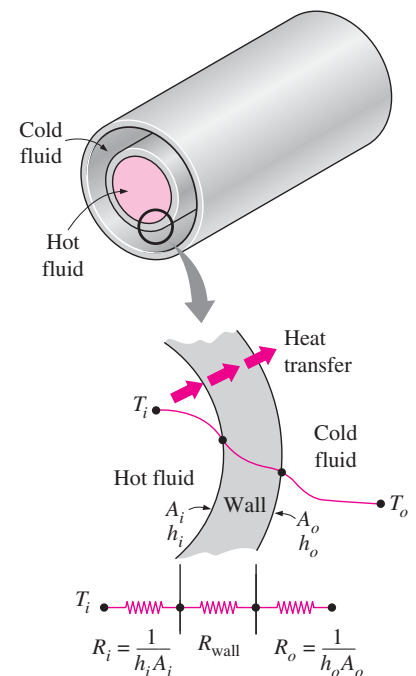

FIGURE 13–7
Thermal resistance network associated with heat transfer in a double-pipe heat exchanger.

inner tube. For a double-pipe heat exchanger, we have $A_i = \pi D_i L$ and $A_o = \pi D_o L$, and the *thermal resistance* of the tube wall in this case is

$$R_{wall} = \frac{\ln (D_o/D_i)}{2\pi kL} \tag{13-1}$$

where k is the thermal conductivity of the wall material and L is the length of the tube. Then the *total thermal resistance* becomes

$$R = R_{total} = R_i + R_{wall} + R_o = \frac{1}{h_i A_i} + \frac{\ln (D_o/D_i)}{2\pi kL} + \frac{1}{h_o A_o} \tag{13-2}$$

The A_i is the area of the *inner surface* of the wall that separates the two fluids, and A_o is the area of the outer surface of the wall. In other words, A_i and A_o are surface areas of the separating wall wetted by the inner and the outer fluids, respectively. When one fluid flows inside a circular tube and the other outside of it, we have $A_i = \pi D_i L$ and $A_o = \pi D_o L$ (Fig. 13–8).

In the analysis of heat exchangers, it is convenient to combine all the thermal resistances in the path of heat flow from the hot fluid to the cold one into a single resistance R, and to express the rate of heat transfer between the two fluids as

$$\dot{Q} = \frac{\Delta T}{R} = UA\,\Delta T = U_i A_i\,\Delta T = U_o A_o\,\Delta T \tag{13-3}$$

where U is the **overall heat transfer coefficient,** whose unit is W/m² · °C, which is identical to the unit of the ordinary convection coefficient h. Canceling ΔT, Eq. 13-3 reduces to

$$\frac{1}{UA_s} = \frac{1}{U_i A_i} = \frac{1}{U_o A_o} = R = \frac{1}{h_i A_i} + R_{wall} + \frac{1}{h_o A_o} \tag{13-4}$$

Perhaps you are wondering why we have two overall heat transfer coefficients U_i and U_o for a heat exchanger. The reason is that every heat exchanger has two heat transfer surface areas A_i and A_o, which, in general, are not equal to each other.

Note that $U_i A_i = U_o A_o$, but $U_i \neq U_o$ unless $A_i = A_o$. Therefore, the overall heat transfer coefficient U of a heat exchanger is meaningless unless the area on which it is based is specified. This is especially the case when one side of the tube wall is finned and the other side is not, since the surface area of the finned side is several times that of the unfinned side.

When the wall thickness of the tube is small and the thermal conductivity of the tube material is high, as is usually the case, the thermal resistance of the tube is negligible ($R_{wall} \approx 0$) and the inner and outer surfaces of the tube are almost identical ($A_i \approx A_o \approx A_s$). Then Eq. 13-4 for the overall heat transfer coefficient simplifies to

$$\frac{1}{U} \approx \frac{1}{h_i} + \frac{1}{h_o} \tag{13-5}$$

where $U \approx U_i \approx U_o$. The individual convection heat transfer coefficients inside and outside the tube, h_i and h_o, are determined using the convection relations discussed in earlier chapters.
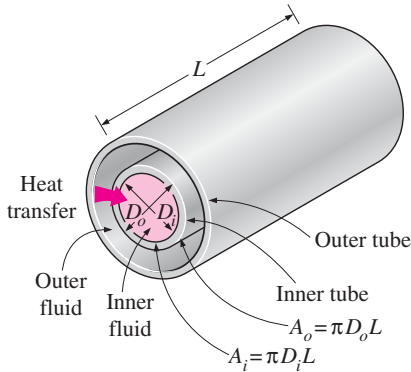

FIGURE 13–8
The two heat transfer surface areas associated with a double-pipe heat exchanger (for thin tubes, $D_i \approx D_o$ and thus $A_i \approx A_o$).

The overall heat transfer coefficient U in Eq. 13-5 is dominated by the *smaller* convection coefficient, since the inverse of a large number is small. When one of the convection coefficients is *much smaller* than the other (say, $h_i \ll h_o$), we have $1/h_i \gg 1/h_o$, and thus $U \approx h_i$. Therefore, the smaller heat transfer coefficient creates a *bottleneck* on the path of heat flow and seriously impedes heat transfer. This situation arises frequently when one of the fluids is a gas and the other is a liquid. In such cases, fins are commonly used on the gas side to enhance the product UA_s and thus the heat transfer on that side.

Representative values of the overall heat transfer coefficient U are given in Table 13–1. Note that the overall heat transfer coefficient ranges from about 10 W/m² · °C for gas-to-gas heat exchangers to about 10,000 W/m² · °C for heat exchangers that involve phase changes. This is not surprising, since gases have very low thermal conductivities, and phase-change processes involve very high heat transfer coefficients.

When the tube is *finned* on one side to enhance heat transfer, the total heat transfer surface area on the finned side becomes

$$A_s = A_{total} = A_{fin} + A_{unfinned} \tag{13-6}$$

where A_{fin} is the surface area of the fins and $A_{unfinned}$ is the area of the unfinned portion of the tube surface. For short fins of high thermal conductivity, we can use this total area in the convection resistance relation $R_{conv} = 1/hA_s$ since the fins in this case will be very nearly isothermal. Otherwise, we should determine the effective surface area A from

$$A_s = A_{unfinned} + \eta_{fin} A_{fin} \tag{13-7}$$

TABLE 13–1

Representative values of the overall heat transfer coefficients in heat exchangers

Type of heat exchanger	U, W/m² · °C*
Water-to-water	850–1700
Water-to-oil	100–350
Water-to-gasoline or kerosene	300–1000
Feedwater heaters	1000–8500
Steam-to-light fuel oil	200–400
Steam-to-heavy fuel oil	50–200
Steam condenser	1000–6000
Freon condenser (water cooled)	300–1000
Ammonia condenser (water cooled)	800–1400
Alcohol condensers (water cooled)	250–700
Gas-to-gas	10–40
Water-to-air in finned tubes (water in tubes)	30–60[†]
	400–850[†]
Steam-to-air in finned tubes (steam in tubes)	30–300[†]
	400–4000[‡]

*Multiply the listed values by 0.176 to convert them to Btu/h · ft² · °F.

[†]Based on air-side surface area.

[‡]Based on water- or steam-side surface area.

where η_{fin} is the fin efficiency. This way, the temperature drop along the fins is accounted for. Note that $\eta_{fin} = 1$ for isothermal fins, and thus Eq. 13-7 reduces to Eq. 13-6 in that case.

Fouling Factor

The performance of heat exchangers usually deteriorates with time as a result of accumulation of *deposits* on heat transfer surfaces. The layer of deposits represents *additional resistance* to heat transfer and causes the rate of heat transfer in a heat exchanger to decrease. The net effect of these accumulations on heat transfer is represented by a **fouling factor** R_f, which is a measure of the *thermal resistance* introduced by fouling.

The most common type of fouling is the *precipitation* of solid deposits in a fluid on the heat transfer surfaces. You can observe this type of fouling even in your house. If you check the inner surfaces of your teapot after prolonged use, you will probably notice a layer of calcium-based deposits on the surfaces at which boiling occurs. This is especially the case in areas where the water is hard. The scales of such deposits come off by scratching, and the surfaces can be cleaned of such deposits by chemical treatment. Now imagine those mineral deposits forming on the inner surfaces of fine tubes in a heat exchanger (Fig. 13–9) and the detrimental effect it may have on the flow passage area and the heat transfer. To avoid this potential problem, water in power and process plants is extensively treated and its solid contents are removed before it is allowed to circulate through the system. The solid ash particles in the flue gases accumulating on the surfaces of air preheaters create similar problems.

Another form of fouling, which is common in the chemical process industry, is *corrosion* and other *chemical fouling*. In this case, the surfaces are fouled by the accumulation of the products of chemical reactions on the surfaces. This form of fouling can be avoided by coating metal pipes with glass or using plastic pipes instead of metal ones. Heat exchangers may also be fouled by the growth of algae in warm fluids. This type of fouling is called *biological fouling* and can be prevented by chemical treatment.

In applications where it is likely to occur, fouling should be considered in the design and selection of heat exchangers. In such applications, it may be

FIGURE 13–9
Precipitation fouling of
ash particles on superheater tubes
(from *Steam, Its Generation, and Use,*
Babcock and Wilcox Co., 1978).

necessary to select a larger and thus more expensive heat exchanger to ensure that it meets the design heat transfer requirements even after fouling occurs. The periodic cleaning of heat exchangers and the resulting down time are additional penalties associated with fouling.

The fouling factor is obviously zero for a new heat exchanger and increases with time as the solid deposits build up on the heat exchanger surface. The fouling factor depends on the *operating temperature* and the *velocity* of the fluids, as well as the length of service. Fouling increases with *increasing temperature* and *decreasing velocity*.

The overall heat transfer coefficient relation given above is valid for clean surfaces and needs to be modified to account for the effects of fouling on both the inner and the outer surfaces of the tube. For an unfinned shell-and-tube heat exchanger, it can be expressed as

$$\frac{1}{UA_s} = \frac{1}{U_iA_i} = \frac{1}{U_oA_o} = R = \frac{1}{h_iA_i} + \frac{R_{f,i}}{A_i} + \frac{\ln(D_o/D_i)}{2\pi kL} + \frac{R_{f,o}}{A_o} + \frac{1}{h_oA_o} \quad \textbf{(13-8)}$$

where $A_i = \pi D_i L$ and $A_o = \pi D_o L$ are the areas of inner and outer surfaces, and $R_{f,i}$ and $R_{f,o}$ are the fouling factors at those surfaces.

Representative values of fouling factors are given in Table 13–2. More comprehensive tables of fouling factors are available in handbooks. As you would expect, considerable uncertainty exists in these values, and they should be used as a guide in the selection and evaluation of heat exchangers to account for the effects of anticipated fouling on heat transfer. Note that most fouling factors in the table are of the order of 10^{-4} m$^2 \cdot$ °C/W, which is equivalent to the thermal resistance of a 0.2-mm-thick limestone layer ($k = 2.9$ W/m $\cdot$ °C) per unit surface area. Therefore, in the absence of specific data, we can assume the surfaces to be coated with 0.2 mm of limestone as a starting point to account for the effects of fouling.

TABLE 13–2

Representative fouling factors (thermal resistance due to fouling for a unit surface area)

(*Source:* Tubular Exchange Manufacturers Association.)

Fluid	R_f, m$^2 \cdot$ °C/W
Distilled water, sea water, river water, boiler feedwater:	
Below 50°C	0.0001
Above 50°C	0.0002
Fuel oil	0.0009
Steam (oil-free)	0.0001
Refrigerants (liquid)	0.0002
Refrigerants (vapor)	0.0004
Alcohol vapors	0.0001
Air	0.0004

EXAMPLE 13–1 **Overall Heat Transfer Coefficient of a Heat Exchanger**

Hot oil is to be cooled in a double-tube counter-flow heat exchanger. The copper inner tubes have a diameter of 2 cm and negligible thickness. The inner diameter of the outer tube (the shell) is 3 cm. Water flows through the tube at a rate of 0.5 kg/s, and the oil through the shell at a rate of 0.8 kg/s. Taking the average temperatures of the water and the oil to be 45°C and 80°C, respectively, determine the overall heat transfer coefficient of this heat exchanger.

SOLUTION Hot oil is cooled by water in a double-tube counter-flow heat exchanger. The overall heat transfer coefficient is to be determined.

Assumptions **1** The thermal resistance of the inner tube is negligible since the tube material is highly conductive and its thickness is negligible. **2** Both the oil and water flow are fully developed. **3** Properties of the oil and water are constant.

Properties The properties of water at 45°C are (Table A–9)

$$\rho = 990 \text{ kg/m}^3 \qquad Pr = 3.91$$
$$k = 0.637 \text{ W/m} \cdot \text{°C} \qquad \nu = \mu/\rho = 0.602 \times 10^{-6} \text{ m}^2\text{/s}$$
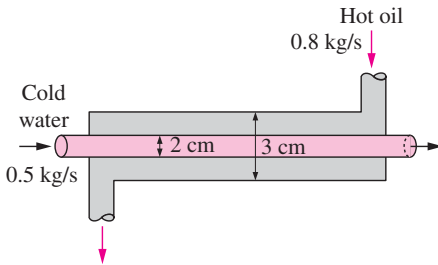
FIGURE 13–10
Schematic for Example 13–1.

TABLE 13–3

Nusselt number for fully developed laminar flow in a circular annulus with one surface insulated and the other isothermal (Kays and Perkins, Ref. 8.)

D_i/D_o	Nu_i	Nu_o
0.00	—	3.66
0.05	17.46	4.06
0.10	11.56	4.11
0.25	7.37	4.23
0.50	5.74	4.43
1.00	4.86	4.86

The properties of oil at 80°C are (Table A–16).

$$\rho = 852 \text{ kg/m}^3 \qquad Pr = 490$$
$$k = 0.138 \text{ W/m} \cdot \text{°C} \qquad \nu = 37.5 \times 10^{-6} \text{ m}^2/\text{s}$$

Analysis The schematic of the heat exchanger is given in Figure 13–10. The overall heat transfer coefficient U can be determined from Eq. 13-5:

$$\frac{1}{U} \approx \frac{1}{h_i} + \frac{1}{h_o}$$

where h_i and h_o are the convection heat transfer coefficients inside and outside the tube, respectively, which are to be determined using the forced convection relations.

The hydraulic diameter for a circular tube is the diameter of the tube itself, $D_h = D = 0.02$ m. The mean velocity of water in the tube and the Reynolds number are

$$\mathcal{V}_m = \frac{\dot{m}}{\rho A_c} = \frac{\dot{m}}{\rho(\frac{1}{4}\pi D^2)} = \frac{0.5 \text{ kg/s}}{(990 \text{ kg/m}^3)[\frac{1}{4}\pi(0.02 \text{ m})^2]} = 1.61 \text{ m/s}$$

and

$$Re = \frac{\mathcal{V}_m D_h}{\nu} = \frac{(1.61 \text{ m/s})(0.02 \text{ m})}{0.602 \times 10^{-6} \text{ m}^2/\text{s}} = 53{,}490$$

which is greater than 4000. Therefore, the flow of water is turbulent. Assuming the flow to be fully developed, the Nusselt number can be determined from

$$Nu = \frac{h D_h}{k} = 0.023 \, Re^{0.8} Pr^{0.4} = 0.023(53{,}490)^{0.8}(3.91)^{0.4} = 240.6$$

Then,

$$h = \frac{k}{D_h} Nu = \frac{0.637 \text{ W/m} \cdot \text{°C}}{0.02 \text{ m}}(240.6) = 7663 \text{ W/m}^2 \cdot \text{°C}$$

Now we repeat the analysis above for oil. The properties of oil at 80°C are

$$\rho = 852 \text{ kg/m}^3 \qquad \nu = 37.5 \times 10^{-6} \text{ m}^2/\text{s}$$
$$k = 0.138 \text{ W/m} \cdot \text{°C} \qquad Pr = 490$$

The hydraulic diameter for the annular space is

$$D_h = D_o - D_i = 0.03 - 0.02 = 0.01 \text{ m}$$

The mean velocity and the Reynolds number in this case are

$$\mathcal{V}_m = \frac{\dot{m}}{\rho A_c} = \frac{\dot{m}}{\rho[\frac{1}{4}\pi(D_o^2 - D_i^2)]} = \frac{0.8 \text{ kg/s}}{(852 \text{ kg/m}^3)[\frac{1}{4}\pi(0.03^2 - 0.02^2)] \text{ m}^2} = 2.39 \text{ m/s}$$

and

$$Re = \frac{\mathcal{V}_m D_h}{\nu} = \frac{(2.39 \text{ m/s})(0.01 \text{ m})}{37.5 \times 10^{-6} \text{ m}^2/\text{s}} = 637$$

which is less than 4000. Therefore, the flow of oil is laminar. Assuming fully developed flow, the Nusselt number on the tube side of the annular space Nu_i corresponding to $D_i/D_o = 0.02/0.03 = 0.667$ can be determined from Table 13–3 by interpolation to be

$$Nu = 5.45$$

and

$$h_o = \frac{k}{D_h} \text{Nu} = \frac{0.138 \text{ W/m} \cdot {}^\circ\text{C}}{0.01 \text{ m}} (5.45) = 75.2 \text{ W/m}^2 \cdot {}^\circ\text{C}$$

Then the overall heat transfer coefficient for this heat exchanger becomes

$$U = \frac{1}{\dfrac{1}{h_i} + \dfrac{1}{h_o}} = \frac{1}{\dfrac{1}{7663 \text{ W/m}^2 \cdot {}^\circ\text{C}} + \dfrac{1}{75.2 \text{ W/m}^2 \cdot {}^\circ\text{C}}} = \textbf{74.5 W/m}^2 \cdot {}^\circ\textbf{C}$$

Discussion Note that $U \approx h_o$ in this case, since $h_i \gg h_o$. This confirms our earlier statement that the overall heat transfer coefficient in a heat exchanger is dominated by the smaller heat transfer coefficient when the difference between the two values is large.

To improve the overall heat transfer coefficient and thus the heat transfer in this heat exchanger, we must use some enhancement techniques on the oil side, such as a finned surface.

EXAMPLE 13–2   **Effect of Fouling on the Overall Heat Transfer Coefficient**

A double-pipe (shell-and-tube) heat exchanger is constructed of a stainless steel ($k = 15.1$ W/m $\cdot$ °C) inner tube of inner diameter $D_i = 1.5$ cm and outer diameter $D_o = 1.9$ cm and an outer shell of inner diameter 3.2 cm. The convection heat transfer coefficient is given to be $h_i = 800$ W/m^2 $\cdot$ °C on the inner surface of the tube and $h_o = 1200$ W/m^2 $\cdot$ °C on the outer surface. For a fouling factor of $R_{f,\,i} = 0.0004$ m^2 $\cdot$ °C/W on the tube side and $R_{f,\,o} = 0.0001$ m^2 $\cdot$ °C/W on the shell side, determine (*a*) the thermal resistance of the heat exchanger per unit length and (*b*) the overall heat transfer coefficients, U_i and U_o based on the inner and outer surface areas of the tube, respectively.

SOLUTION The heat transfer coefficients and the fouling factors on the tube and shell sides of a heat exchanger are given. The thermal resistance and the overall heat transfer coefficients based on the inner and outer areas are to be determined.

Assumptions The heat transfer coefficients and the fouling factors are constant and uniform.

Analysis (*a*) The schematic of the heat exchanger is given in Figure 13–11. The thermal resistance for an unfinned shell-and-tube heat exchanger with fouling on both heat transfer surfaces is given by Eq. 13-8 as

$$R = \frac{1}{UA_s} = \frac{1}{U_i A_i} = \frac{1}{U_o A_o} = \frac{1}{h_i A_i} + \frac{R_{f,\,i}}{A_i} + \frac{\ln(D_o/D_i)}{2\pi k L} + \frac{R_{f,\,o}}{A_o} + \frac{1}{h_o A_o}$$

where

$$A_i = \pi D_i L = \pi (0.015 \text{ m})(1 \text{ m}) = 0.0471 \text{ m}^2$$
$$A_o = \pi D_o L = \pi (0.019 \text{ m})(1 \text{ m}) = 0.0597 \text{ m}^2$$

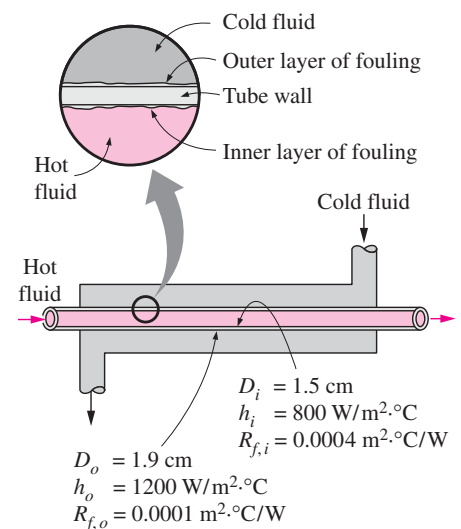Substituting, the total thermal resistance is determined to be

FIGURE 13–11
Schematic for Example 13–2.

Cold fluid
Outer layer of fouling
Tube wall
Inner layer of fouling
Hot fluid
Cold fluid
Hot fluid

$D_i = 1.5$ cm
$h_i = 800$ W/m$^2\cdot$°C
$R_{f,\,i} = 0.0004$ m$^2\cdot$°C/W

$D_o = 1.9$ cm
$h_o = 1200$ W/m$^2\cdot$°C
$R_{f,\,o} = 0.0001$ m$^2\cdot$°C/W

$$R = \frac{1}{(800 \text{ W/m}^2 \cdot °\text{C})(0.0471 \text{ m}^2)} + \frac{0.0004 \text{ m}^2 \cdot °\text{C/W}}{0.0471 \text{ m}^2}$$

$$+ \frac{\ln(0.019/0.015)}{2\pi(15.1 \text{ W/m} \cdot °\text{C})(1 \text{ m})}$$

$$+ \frac{0.0001 \text{ m}^2 \cdot °\text{C/W}}{0.0597 \text{ m}^2} + \frac{1}{(1200 \text{ W/m}^2 \cdot °\text{C})(0.0597 \text{ m}^2)}$$

$$= (0.02654 + 0.00849 + 0.0025 + 0.00168 + 0.01396)°\text{C/W}$$

$$= \mathbf{0.0532°C/W}$$

Note that about 19 percent of the total thermal resistance in this case is due to fouling and about 5 percent of it is due to the steel tube separating the two fluids. The rest (76 percent) is due to the convection resistances on the two sides of the inner tube.

(*b*) Knowing the total thermal resistance and the heat transfer surface areas, the overall heat transfer coefficient based on the inner and outer surfaces of the tube are determined again from Eq. 13-8 to be

$$U_i = \frac{1}{RA_i} = \frac{1}{(0.0532 \text{ °C/W})(0.0471 \text{ m}^2)} = \mathbf{399 \text{ W/m}^2 \cdot °C}$$

and

$$U_o = \frac{1}{RA_o} = \frac{1}{(0.0532 \text{ °C/W})(0.0597 \text{ m}^2)} = \mathbf{315 \text{ W/m}^2 \cdot °C}$$

Discussion   Note that the two overall heat transfer coefficients differ significantly (by 27 percent) in this case because of the considerable difference between the heat transfer surface areas on the inner and the outer sides of the tube. For tubes of negligible thickness, the difference between the two overall heat transfer coefficients would be negligible.

13–3 · ANALYSIS OF HEAT EXCHANGERS

Heat exchangers are commonly used in practice, and an engineer often finds himself or herself in a position to *select a heat exchanger* that will achieve a *specified temperature change* in a fluid stream of known mass flow rate, or to *predict the outlet temperatures* of the hot and cold fluid streams in a *specified heat exchanger.*

In upcoming sections, we will discuss the two methods used in the analysis of heat exchangers. Of these, the *log mean temperature difference* (or LMTD) method is best suited for the first task and the *effectiveness–NTU* method for the second task as just stated. But first we present some general considerations.

Heat exchangers usually operate for long periods of time with no change in their operating conditions. Therefore, they can be modeled as *steady-flow* devices. As such, the mass flow rate of each fluid remains constant, and the fluid properties such as temperature and velocity at any inlet or outlet remain the same. Also, the fluid streams experience little or no change in their velocities and elevations, and thus the *kinetic* and *potential energy changes* are negligible. The *specific heat* of a fluid, in general, changes with temperature. But, in

a specified temperature range, it can be treated as a constant at some average value with little loss in accuracy. *Axial heat conduction* along the tube is usually insignificant and can be considered negligible. Finally, the outer surface of the heat exchanger is assumed to be *perfectly insulated,* so that there is no heat loss to the surrounding medium, and any heat transfer occurs between the two fluids only.

The idealizations stated above are closely approximated in practice, and they greatly simplify the analysis of a heat exchanger with little sacrifice of accuracy. Therefore, they are commonly used. Under these assumptions, the *first law of thermodynamics* requires that the rate of heat transfer from the hot fluid be equal to the rate of heat transfer to the cold one. That is,

$$\dot{Q} = \dot{m}_c C_{pc}(T_{c,\,out} - T_{c,\,in}) \tag{13-9}$$

and

$$\dot{Q} = \dot{m}_h C_{ph}(T_{h,\,in} - T_{h,\,out}) \tag{13-10}$$

where the subscripts c and h stand for *cold* and *hot* fluids, respectively, and

$$\dot{m}_c,\ \dot{m}_h = \text{mass flow rates}$$
$$C_{pc},\ C_{ph} = \text{specific heats}$$
$$T_{c,\,out},\ T_{h,\,out} = \text{outlet temperatures}$$
$$T_{c,\,in},\ T_{h,\,in} = \text{inlet temperatures}$$

Note that the heat transfer rate $\dot{Q}$ is taken to be a positive quantity, and its direction is understood to be from the hot fluid to the cold one in accordance with the second law of thermodynamics.

In heat exchanger analysis, it is often convenient to combine the product of the *mass flow rate* and the *specific heat* of a fluid into a single quantity. This quantity is called the **heat capacity rate** and is defined for the hot and cold fluid streams as

$$C_h = \dot{m}_h C_{ph} \qquad \text{and} \qquad C_c = \dot{m}_c C_{pc} \tag{13-11}$$

The heat capacity rate of a fluid stream represents the rate of heat transfer needed to change the temperature of the fluid stream by 1°C as it flows through a heat exchanger. Note that in a heat exchanger, the fluid with a *large* heat capacity rate will experience a *small* temperature change, and the fluid with a *small* heat capacity rate will experience a *large* temperature change. Therefore, *doubling* the mass flow rate of a fluid while leaving everything else unchanged will *halve* the temperature change of that fluid.

With the definition of the heat capacity rate above, Eqs. 13-9 and 13-10 can also be expressed as

$$\dot{Q} = C_c(T_{c,\,out} - T_{c,\,in}) \tag{13-12}$$

and

$$\dot{Q} = C_h(T_{h,\,in} - T_{h,\,out}) \tag{13-13}$$

That is, the heat transfer rate in a heat exchanger is equal to the heat capacity rate of either fluid multiplied by the temperature change of that fluid. Note that *the only time the temperature rise of a cold fluid is equal to the temperature drop of the hot fluid is when the heat capacity rates of the two fluids are equal to each other* (Fig. 13–12).
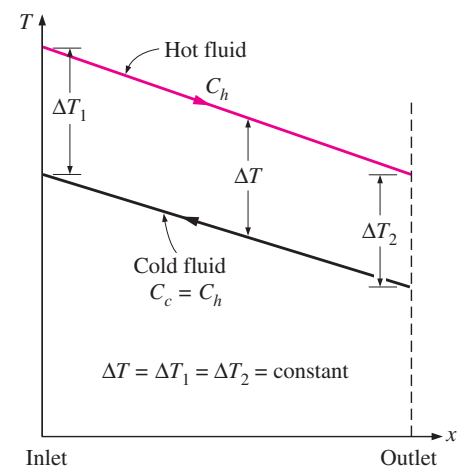

FIGURE 13–12
Two fluids that have the same mass flow rate and the same specific heat experience the same temperature change in a well-insulated heat exchanger.

Two special types of heat exchangers commonly used in practice are *condensers* and *boilers.* One of the fluids in a condenser or a boiler undergoes a phase-change process, and the rate of heat transfer is expressed as

$$\dot{Q} = \dot{m}h_{fg} \tag{13-14}$$

where $\dot{m}$ is the rate of evaporation or condensation of the fluid and h_{fg} is the enthalpy of vaporization of the fluid at the specified temperature or pressure.

An ordinary fluid absorbs or releases a large amount of heat essentially at constant temperature during a phase-change process, as shown in Figure 13–13. The heat capacity rate of a fluid during a phase-change process must approach infinity since the temperature change is practically zero. That is, $C = \dot{m}C_p \rightarrow \infty$ when $\Delta T \rightarrow 0$, so that the heat transfer rate $\dot{Q} = \dot{m}C_p \, \Delta T$ is a finite quantity. Therefore, in heat exchanger analysis, a condensing or boiling fluid is conveniently modeled as a fluid whose heat capacity rate is *infinity.*

The rate of heat transfer in a heat exchanger can also be expressed in an analogous manner to Newton's law of cooling as

$$\dot{Q} = UA_s \, \Delta T_m \tag{13-15}$$

where U is the overall heat transfer coefficient, A_s is the heat transfer area, and ΔT_m is an appropriate average temperature difference between the two fluids. Here the surface area A_s can be determined precisely using the dimensions of the heat exchanger. However, the overall heat transfer coefficient U and the temperature difference ΔT between the hot and cold fluids, in general, are not constant and vary along the heat exchanger.

The average value of the overall heat transfer coefficient can be determined as described in the preceding section by using the average convection coefficients for each fluid. It turns out that the appropriate form of the mean temperature difference between the two fluids is *logarithmic* in nature, and its determination is presented in Section 13–4.
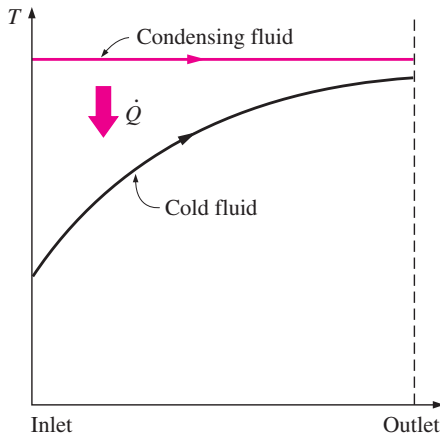
(a) Condenser ($C_h \rightarrow \infty$)
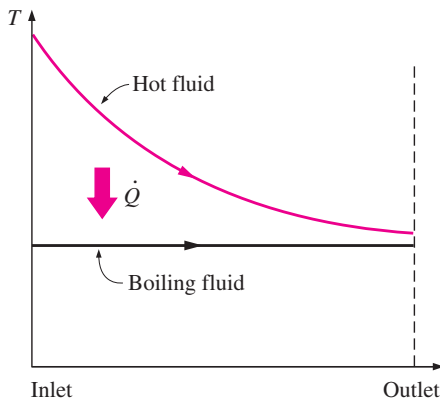
(b) Boiler ($C_c \rightarrow \infty$)

FIGURE 13–13
Variation of fluid temperatures in a heat exchanger when one of the fluids condenses or boils.

13–4 ▪ THE LOG MEAN TEMPERATURE DIFFERENCE METHOD

Earlier, we mentioned that the temperature difference between the hot and cold fluids varies along the heat exchanger, and it is convenient to have a *mean temperature difference* ΔT_m for use in the relation $\dot{Q} = UA_s \, \Delta T_m$.

In order to develop a relation for the equivalent average temperature difference between the two fluids, consider the *parallel-flow double-pipe* heat exchanger shown in Figure 13–14. Note that the temperature difference ΔT between the hot and cold fluids is large at the inlet of the heat exchanger but decreases exponentially toward the outlet. As you would expect, the temperature of the hot fluid decreases and the temperature of the cold fluid increases along the heat exchanger, but the temperature of the cold fluid can never exceed that of the hot fluid no matter how long the heat exchanger is.

Assuming the outer surface of the heat exchanger to be well insulated so that any heat transfer occurs between the two fluids, and disregarding any

changes in kinetic and potential energy, an energy balance on each fluid in a differential section of the heat exchanger can be expressed as

$$\delta\dot{Q} = -\dot{m}_h C_{ph}\, dT_h \tag{13-16}$$

and

$$\delta\dot{Q} = \dot{m}_c C_{pc}\, dT_c \tag{13-17}$$

That is, the rate of heat loss from the hot fluid at any section of a heat exchanger is equal to the rate of heat gain by the cold fluid in that section. The temperature change of the hot fluid is a *negative* quantity, and so a *negative sign* is added to Eq. 13-16 to make the heat transfer rate $\dot{Q}$ a positive quantity. Solving the equations above for dT_h and dT_c gives

$$dT_h = -\frac{\delta\dot{Q}}{\dot{m}_h C_{ph}} \tag{13-18}$$

and

$$dT_c = \frac{\delta\dot{Q}}{\dot{m}_c C_{pc}} \tag{13-19}$$

Taking their difference, we get

$$dT_h - dT_c = d(T_h - T_c) = -\delta\dot{Q}\left(\frac{1}{\dot{m}_h C_{ph}} + \frac{1}{\dot{m}_c C_{pc}}\right) \tag{13-20}$$

The rate of heat transfer in the differential section of the heat exchanger can also be expressed as

$$\delta\dot{Q} = U(T_h - T_c)\, dA_s \tag{13-21}$$

Substituting this equation into Eq. 13-20 and rearranging gives

$$\frac{d(T_h - T_c)}{T_h - T_c} = -U\, dA_s\left(\frac{1}{\dot{m}_h C_{ph}} + \frac{1}{\dot{m}_c C_{pc}}\right) \tag{13-22}$$

Integrating from the inlet of the heat exchanger to its outlet, we obtain

$$\ln\frac{T_{h,\,\text{out}} - T_{c,\,\text{out}}}{T_{h,\,\text{in}} - T_{c,\,\text{in}}} = -UA_s\left(\frac{1}{\dot{m}_h C_{ph}} + \frac{1}{\dot{m}_c C_{pc}}\right) \tag{13-23}$$

Finally, solving Eqs. 13-9 and 13-10 for $\dot{m}_c C_{pc}$ and $\dot{m}_h C_{ph}$ and substituting into Eq. 13-23 gives, after some rearrangement,

$$\dot{Q} = UA_s\, \Delta T_{\text{lm}} \tag{13-24}$$

where

$$\Delta T_{\text{lm}} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} \tag{13-25}$$

is the **log mean temperature difference,** which is the suitable form of the average temperature difference for use in the analysis of heat exchangers. Here ΔT_1 and ΔT_2 represent the temperature difference between the two fluids
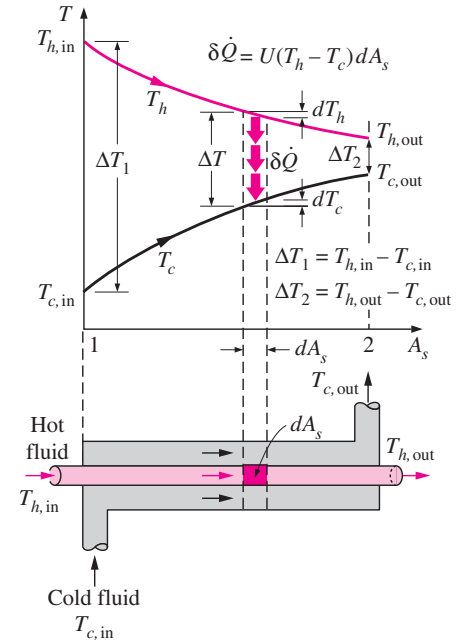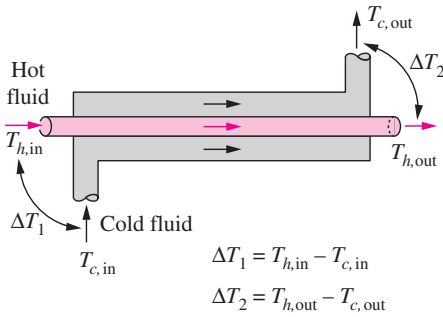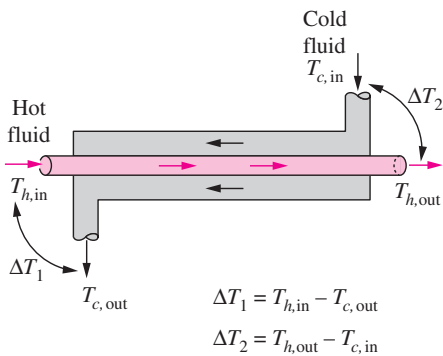
FIGURE 13–14

Variation of the fluid temperatures in a parallel-flow double-pipe heat exchanger.

$\Delta T_1 = T_{h,\text{in}} - T_{c,\text{in}}$

$\Delta T_2 = T_{h,\text{out}} - T_{c,\text{out}}$

(a) Parallel-flow heat exchangers

$\Delta T_1 = T_{h,\text{in}} - T_{c,\text{out}}$

$\Delta T_2 = T_{h,\text{out}} - T_{c,\text{in}}$

(b) Counter-flow heat exchangers
FIGURE 13–15
The ΔT_1 and ΔT_2 expressions in parallel-flow and counter-flow heat exchangers.

at the two ends (inlet and outlet) of the heat exchanger. It makes no difference which end of the heat exchanger is designated as the *inlet* or the *outlet* (Fig. 13–15).

The temperature difference between the two fluids decreases from ΔT_1 at the inlet to ΔT_2 at the outlet. Thus, it is tempting to use the arithmetic mean temperature $\Delta T_{\text{am}} = \frac{1}{2}(\Delta T_1 + \Delta T_2)$ as the average temperature difference. The logarithmic mean temperature difference ΔT_{lm} is obtained by tracing the actual temperature profile of the fluids along the heat exchanger and is an *exact* representation of the *average temperature difference* between the hot and cold fluids. It truly reflects the exponential decay of the local temperature difference.

Note that ΔT_{lm} is always less than ΔT_{am}. Therefore, using ΔT_{am} in calculations instead of ΔT_{lm} will overestimate the rate of heat transfer in a heat exchanger between the two fluids. When ΔT_1 differs from ΔT_2 by no more than 40 percent, the error in using the arithmetic mean temperature difference is less than 1 percent. But the error increases to undesirable levels when ΔT_1 differs from ΔT_2 by greater amounts. Therefore, we should always use the *logarithmic mean temperature difference* when determining the rate of heat transfer in a heat exchanger.

Counter-Flow Heat Exchangers

The variation of temperatures of hot and cold fluids in a counter-flow heat exchanger is given in Figure 13–16. Note that the hot and cold fluids enter the heat exchanger from opposite ends, and the outlet temperature of the *cold fluid* in this case may exceed the outlet temperature of the *hot fluid.* In the limiting case, the cold fluid will be heated to the inlet temperature of the hot fluid. However, the outlet temperature of the cold fluid can *never* exceed the inlet temperature of the hot fluid, since this would be a violation of the second law of thermodynamics.

The relation above for the log mean temperature difference is developed using a parallel-flow heat exchanger, but we can show by repeating the analysis above for a counter-flow heat exchanger that is also applicable to counter-flow heat exchangers. But this time, ΔT_1 and ΔT_2 are expressed as shown in Figure 13–15.

For specified inlet and outlet temperatures, the log mean temperature difference for a *counter-flow* heat exchanger is always *greater* than that for a parallel-flow heat exchanger. That is, $\Delta T_{\text{lm, }CF} > \Delta T_{\text{lm, }PF}$, and thus a smaller surface area (and thus a smaller heat exchanger) is needed to achieve a specified heat transfer rate in a counter-flow heat exchanger. Therefore, it is common practice to use counter-flow arrangements in heat exchangers.

In a counter-flow heat exchanger, the temperature difference between the hot and the cold fluids will remain constant along the heat exchanger when the *heat capacity rates* of the two fluids are *equal* (that is, $\Delta T =$ constant when $C_h = C_c$ or $\dot{m}_h C_{ph} = \dot{m}_c C_{pc}$). Then we have $\Delta T_1 = \Delta T_2$, and the last log mean temperature difference relation gives $\Delta T_{\text{lm}} = \frac{0}{0}$, which is indeterminate. It can be shown by the application of l'Hôpital's rule that in this case we have $\Delta T_{\text{lm}} = \Delta T_1 = \Delta T_2$, as expected.

A *condenser* or a *boiler* can be considered to be either a parallel- or counter-flow heat exchanger since both approaches give the same result.

Multipass and Cross-Flow Heat Exchangers: Use of a Correction Factor

The log mean temperature difference ΔT_{lm} relation developed earlier is limited to parallel-flow and counter-flow heat exchangers only. Similar relations are also developed for *cross-flow* and *multipass shell-and-tube* heat exchangers, but the resulting expressions are too complicated because of the complex flow conditions.

In such cases, it is convenient to relate the equivalent temperature difference to the log mean temperature difference relation for the counter-flow case as

$$\Delta T_{lm} = F \Delta T_{lm, CF} \tag{13-26}$$

where F is the **correction factor,** which depends on the *geometry* of the heat exchanger and the inlet and outlet temperatures of the hot and cold fluid streams. The $\Delta T_{lm, CF}$ is the log mean temperature difference for the case of a *counter-flow* heat exchanger with the same inlet and outlet temperatures and is determined from Eq. 13-25 by taking $\Delta T_1 = T_{h, in} - T_{c, out}$ and $\Delta T_2 = T_{h, out} - T_{c, in}$ (Fig. 13–17).

The correction factor is less than unity for a cross-flow and multipass shell-and-tube heat exchanger. That is, $F \leq 1$. The limiting value of $F = 1$ corresponds to the counter-flow heat exchanger. Thus, the correction factor F for a heat exchanger is *a measure of deviation of the* ΔT_{lm} *from the corresponding values for the counter-flow case.*

The correction factor F for common cross-flow and shell-and-tube heat exchanger configurations is given in Figure 13–18 versus two temperature ratios P and R defined as

$$P = \frac{t_2 - t_1}{T_1 - t_1} \tag{13-27}$$

and

$$R = \frac{T_1 - T_2}{t_2 - t_1} = \frac{(\dot{m}C_p)_{tube\ side}}{(\dot{m}C_p)_{shell\ side}} \tag{13-28}$$

where the subscripts 1 and 2 represent the *inlet* and *outlet,* respectively. Note that for a shell-and-tube heat exchanger, T and t represent the *shell-* and *tube-side* temperatures, respectively, as shown in the correction factor charts. It makes no difference whether the hot or the cold fluid flows through the shell or the tube. The determination of the correction factor F requires the availability of the *inlet* and the *outlet* temperatures for both the cold and hot fluids.

Note that the value of P ranges from 0 to 1. The value of R, on the other hand, ranges from 0 to infinity, with $R = 0$ corresponding to the phase-change (condensation or boiling) on the shell-side and $R \to \infty$ to phase-change on the tube side. The correction factor is $F = 1$ for both of these limiting cases. Therefore, the correction factor for a *condenser* or *boiler* is $F = 1$, regardless of the configuration of the heat exchanger.
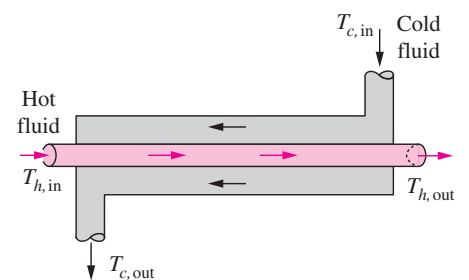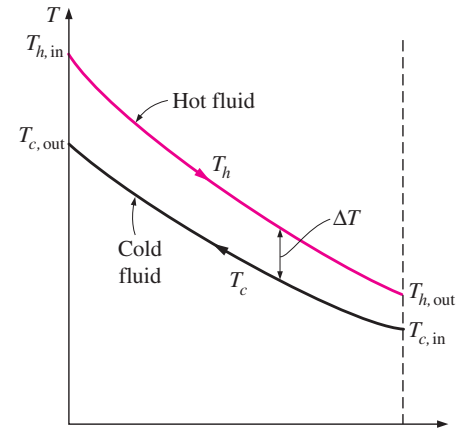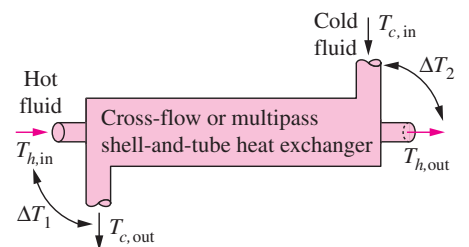

FIGURE 13–16

The variation of the fluid temperatures in a counter-flow double-pipe heat exchanger.

Heat transfer rate:

$$\dot{Q} = UA_s F \Delta T_{lm, CF}$$

where

$$\Delta T_{lm, CF} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)}$$
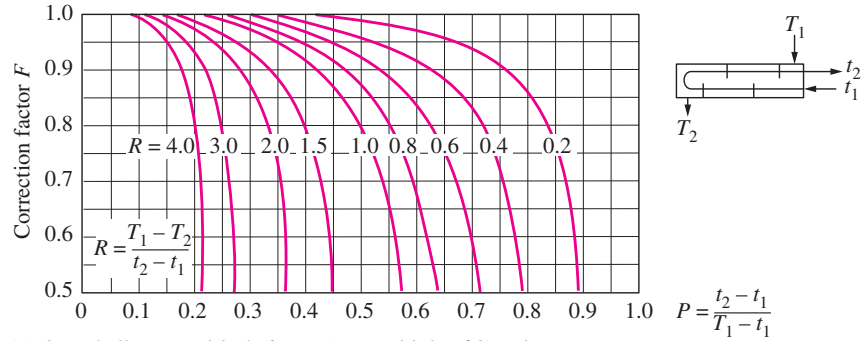
$$\Delta T_1 = T_{h, in} - T_{c, out}$$
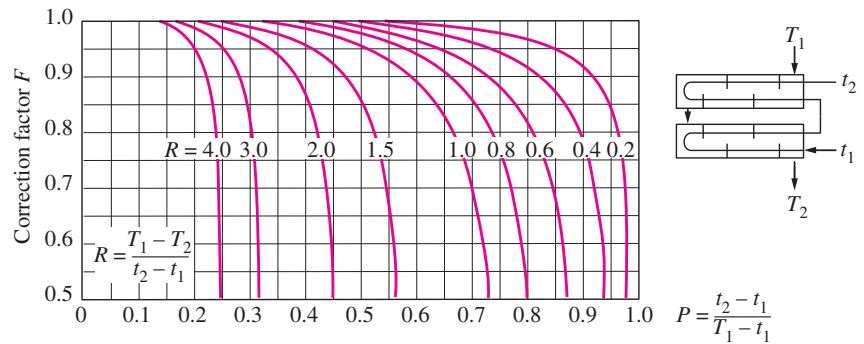
$$\Delta T_2 = T_{h, out} - T_{c, in}$$

and

$$F = \dots \text{(Fig. 13–18)}$$
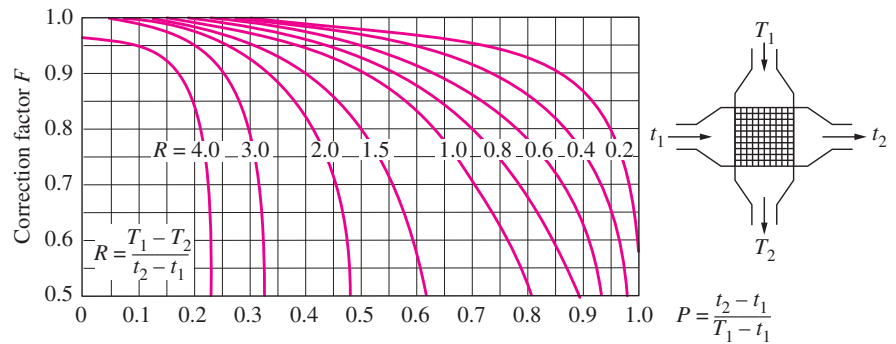
FIGURE 13–17

The determination of the heat transfer rate for cross-flow and multipass shell-and-tube heat exchangers using the correction factor.
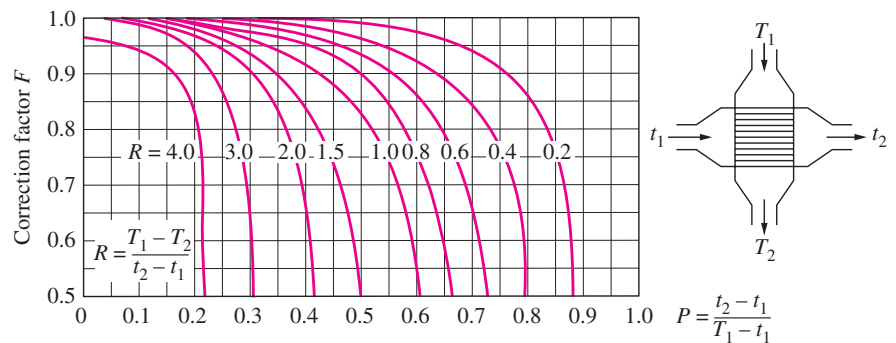
(a) One-shell pass and 2, 4, 6, etc. (any multiple of 2), tube passes

(b) Two-shell passes and 4, 8, 12, etc. (any multiple of 4), tube passes

(c) Single-pass cross-flow with both fluids *unmixed*

(d) Single-pass cross-flow with one fluid *mixed* and the other *unmixed*

FIGURE 13–18

Correction factor F charts for common shell-and-tube and cross-flow heat exchangers (from Bowman, Mueller, and Nagle, Ref. 2).

EXAMPLE 13–3   The Condensation of Steam in a Condenser

Steam in the condenser of a power plant is to be condensed at a temperature of 30°C with cooling water from a nearby lake, which enters the tubes of the condenser at 14°C and leaves at 22°C. The surface area of the tubes is 45 m², and the overall heat transfer coefficient is 2100 W/m² · °C. Determine the mass flow rate of the cooling water needed and the rate of condensation of the steam in the condenser.

SOLUTION   Steam is condensed by cooling water in the condenser of a power plant. The mass flow rate of the cooling water and the rate of condensation are to be determined.

Assumptions   **1** Steady operating conditions exist. **2** The heat exchanger is well insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** There is no fouling. **5** Fluid properties are constant.

Properties   The heat of vaporization of water at 30°C is $h_{fg} = 2431$ kJ/kg and the specific heat of cold water at the average temperature of 18°C is $C_p = 4184$ J/kg · °C (Table A–9).

Analysis   The schematic of the condenser is given in Figure 13–19. The condenser can be treated as a counter-flow heat exchanger since the temperature of one of the fluids (the steam) remains constant.

The temperature difference between the steam and the cooling water at the two ends of the condenser is

$$\Delta T_1 = T_{h,\,in} - T_{c,\,out} = (30 - 22)°C = 8°C$$
$$\Delta T_2 = T_{h,\,out} - T_{c,\,in} = (30 - 14)°C = 16°C$$

That is, the temperature difference between the two fluids varies from 8°C at one end to 16°C at the other. The proper average temperature difference between the two fluids is the *logarithmic mean temperature difference* (not the arithmetic), which is determined from

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \frac{8 - 16}{\ln(8/16)} = 11.5°C$$

This is a little less than the arithmetic mean temperature difference of $\frac{1}{2}(8 + 16) = 12°C$. Then the heat transfer rate in the condenser is determined from

$$\dot{Q} = UA_s \, \Delta T_{lm} = (2100 \text{ W/m}^2 \cdot °C)(45 \text{ m}^2)(11.5°C) = \mathbf{1.087 \times 10^6 \text{ W} = 1087 \text{ kW}}$$

Therefore, the steam will lose heat at a rate of 1,087 kW as it flows through the condenser, and the cooling water will gain practically all of it, since the condenser is well insulated.

The mass flow rate of the cooling water and the rate of the condensation of the steam are determined from $\dot{Q} = [\dot{m}C_p(T_{out} - T_{in})]_{cooling\ water} = (\dot{m}h_{fg})_{steam}$ to be

$$\dot{m}_{cooling\ water} = \frac{\dot{Q}}{C_p(T_{out} - T_{in})}$$

$$= \frac{1,087 \text{ kJ/s}}{(4.184 \text{ kJ/kg} \cdot °C)(22 - 14)°C} = \mathbf{32.5 \text{ kg/s}}$$
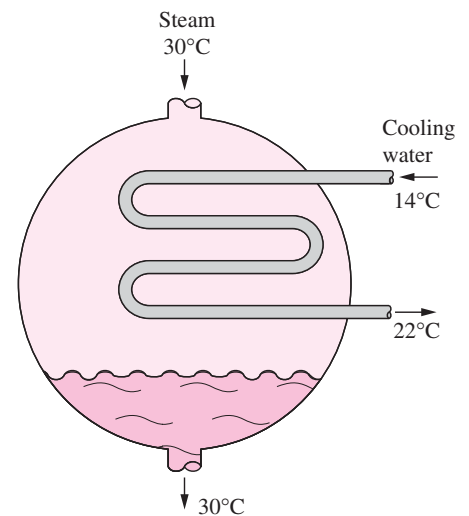

Steam
30°C

Cooling
water
14°C

22°C

30°C

FIGURE 13–19
Schematic for Example 13–3.

and

$$\dot{m}_{\text{steam}} = \frac{\dot{Q}}{h_{fg}} = \frac{1{,}087 \text{ kJ/s}}{2431 \text{ kJ/kg}} = \textbf{0.45 kg/s}$$

Therefore, we need to circulate about 72 kg of cooling water for each 1 kg of steam condensing to remove the heat released during the condensation process.

EXAMPLE 13–4   Heating Water in a Counter-Flow Heat Exchanger

A counter-flow double-pipe heat exchanger is to heat water from 20°C to 80°C at a rate of 1.2 kg/s. The heating is to be accomplished by geothermal water available at 160°C at a mass flow rate of 2 kg/s. The inner tube is thin-walled and has a diameter of 1.5 cm. If the overall heat transfer coefficient of the heat exchanger is 640 W/m² · °C, determine the length of the heat exchanger required to achieve the desired heating.

SOLUTION   Water is heated in a counter-flow double-pipe heat exchanger by geothermal water. The required length of the heat exchanger is to be determined.

Assumptions   **1** Steady operating conditions exist. **2** The heat exchanger is well insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** There is no fouling. **5** Fluid properties are constant.

Properties   We take the specific heats of water and geothermal fluid to be 4.18 and 4.31 kJ/kg · °C, respectively.

Analysis   The schematic of the heat exchanger is given in Figure 13–20. The rate of heat transfer in the heat exchanger can be determined from

$$\dot{Q} = [\dot{m} C_p (T_{\text{out}} - T_{\text{in}})]_{\text{water}} = (1.2 \text{ kg/s})(4.18 \text{ kJ/kg} \cdot °\text{C})(80 - 20)°\text{C} = 301 \text{ kW}$$

Noting that all of this heat is supplied by the geothermal water, the outlet temperature of the geothermal water is determined to be

$$\dot{Q} = [\dot{m} C_p (T_{\text{in}} - T_{\text{out}})]_{\text{geothermal}} \longrightarrow T_{\text{out}} = T_{\text{in}} - \frac{\dot{Q}}{\dot{m} C_p}$$

$$= 160°\text{C} - \frac{301 \text{ kW}}{(2 \text{ kg/s})(4.31 \text{ kJ/kg} \cdot °\text{C})}$$

$$= 125°\text{C}$$

Knowing the inlet and outlet temperatures of both fluids, the logarithmic mean temperature difference for this counter-flow heat exchanger becomes

$$\Delta T_1 = T_{h,\text{in}} - T_{c,\text{out}} = (160 - 80)°\text{C} = 80°\text{C}$$
$$\Delta T_2 = T_{h,\text{out}} - T_{c,\text{in}} = (125 - 20)°\text{C} = 105°\text{C}$$

and

$$\Delta T_{\text{lm}} = \frac{\Delta T_1 - \Delta T_2}{\ln (\Delta T_1/\Delta T_2)} = \frac{80 - 105}{\ln (80/105)} = 92.0°\text{C}$$

Then the surface area of the heat exchanger is determined to be

$$\dot{Q} = U A_s \Delta T_{\text{lm}} \longrightarrow A_s = \frac{\dot{Q}}{U \Delta T_{\text{lm}}} = \frac{301{,}000 \text{ W}}{(640 \text{ W/m}^2 \cdot °\text{C})(92.0°\text{C})} = 5.11 \text{ m}^2$$

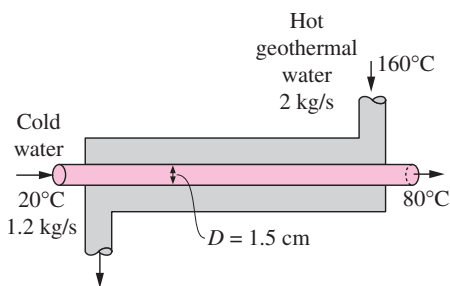Hot geothermal water 160°C 2 kg/s

Cold water 20°C 1.2 kg/s

80°C

$D = 1.5$ cm

FIGURE 13–20
Schematic for Example 13–4.

To provide this much heat transfer surface area, the length of the tube must be

$$A_s = \pi DL \quad \longrightarrow \quad L = \frac{A_s}{\pi D} = \frac{5.11 \text{ m}^2}{\pi(0.015 \text{ m})} = \mathbf{108 \text{ m}}$$

Discussion The inner tube of this counter-flow heat exchanger (and thus the heat exchanger itself) needs to be over 100 m long to achieve the desired heat transfer, which is impractical. In cases like this, we need to use a plate heat exchanger or a multipass shell-and-tube heat exchanger with multiple passes of tube bundles.

EXAMPLE 13–5 **Heating of Glycerin in a Multipass Heat Exchanger**

A 2-shell passes and 4-tube passes heat exchanger is used to heat glycerin from 20°C to 50°C by hot water, which enters the thin-walled 2-cm-diameter tubes at 80°C and leaves at 40°C (Fig. 13–21). The total length of the tubes in the heat exchanger is 60 m. The convection heat transfer coefficient is 25 W/m² · °C on the glycerin (shell) side and 160 W/m² · °C on the water (tube) side. Determine the rate of heat transfer in the heat exchanger (*a*) before any fouling occurs and (*b*) after fouling with a fouling factor of 0.0006 m² · °C/W occurs on the outer surfaces of the tubes.

SOLUTION Glycerin is heated in a 2-shell passes and 4-tube passes heat exchanger by hot water. The rate of heat transfer for the cases of fouling and no fouling are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** Heat transfer coefficients and fouling factors are constant and uniform. **5** The thermal resistance of the inner tube is negligible since the tube is thin-walled and highly conductive.

Analysis The tubes are said to be thin-walled, and thus it is reasonable to assume the inner and outer surface areas of the tubes to be equal. Then the heat transfer surface area becomes

$$A_s = \pi DL = \pi(0.02 \text{ m})(60 \text{ m}) = 3.77 \text{ m}^2$$

The rate of heat transfer in this heat exchanger can be determined from

$$\dot{Q} = UA_s F \, \Delta T_{\text{lm}, CF}$$

where F is the correction factor and $\Delta T_{\text{lm}, CF}$ is the log mean temperature difference for the counter-flow arrangement. These two quantities are determined from

$$\Delta T_1 = T_{h, \text{in}} - T_{c, \text{out}} = (80 - 50)°\text{C} = 30°\text{C}$$

$$\Delta T_2 = T_{h, \text{out}} - T_{c, \text{in}} = (40 - 20)°\text{C} = 20°\text{C}$$

$$\Delta T_{\text{lm}, CF} = \frac{\Delta T_1 - \Delta T_2}{\ln (\Delta T_1/\Delta T_2)} = \frac{30 - 20}{\ln (30/20)} = 24.7°\text{C}$$
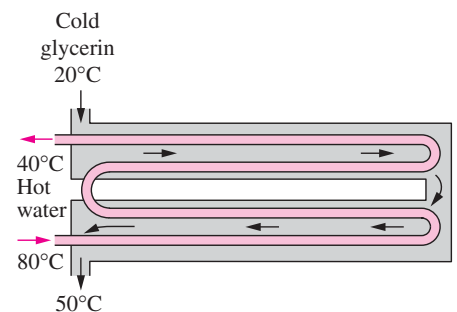
Cold
glycerin
20°C

40°C
Hot
water

80°C

50°C

FIGURE 13–21
Schematic for Example 13–5.

and

$$P = \frac{t_2 - t_1}{T_1 - t_1} = \frac{40 - 80}{20 - 80} = 0.67$$
$$R = \frac{T_1 - T_2}{t_2 - t_1} = \frac{20 - 50}{40 - 80} = 0.75$$
$$\left. \right\} F = 0.91 \qquad \text{(Fig. 13–18b)}$$

(a) In the case of no fouling, the overall heat transfer coefficient U is determined from

$$U = \frac{1}{\dfrac{1}{h_i} + \dfrac{1}{h_o}} = \frac{1}{\dfrac{1}{160 \text{ W/m}^2 \cdot {}^\circ\text{C}} + \dfrac{1}{25 \text{ W/m}^2 \cdot {}^\circ\text{C}}} = 21.6 \text{ W/m}^2 \cdot {}^\circ\text{C}$$

Then the rate of heat transfer becomes

$$\dot{Q} = UA_s F \, \Delta T_{\text{lm, } CF} = (21.6 \text{ W/m}^2 \cdot {}^\circ\text{C})(3.77 \text{m}^2)(0.91)(24.7{}^\circ\text{C}) = \textbf{1830 W}$$

(b) When there is fouling on one of the surfaces, the overall heat transfer coefficient U is

$$U = \frac{1}{\dfrac{1}{h_i} + \dfrac{1}{h_o} + R_f} = \frac{1}{\dfrac{1}{160 \text{ W/m}^2 \cdot {}^\circ\text{C}} + \dfrac{1}{25 \text{ W/m}^2 \cdot {}^\circ\text{C}} + 0.0006 \text{ m}^2 \cdot {}^\circ\text{C/W}}$$
$$= 21.3 \text{ W/m}^2 \cdot {}^\circ\text{C}$$

The rate of heat transfer in this case becomes

$$\dot{Q} = UA_s F \, \Delta T_{\text{lm, } CF} = (21.3 \text{ W/m}^2 \cdot {}^\circ\text{C})(3.77 \text{ m}^2)(0.91)(24.7{}^\circ\text{C}) = \textbf{1805 W}$$

Discussion Note that the rate of heat transfer decreases as a result of fouling, as expected. The decrease is not dramatic, however, because of the relatively low convection heat transfer coefficients involved.

EXAMPLE 13–6   Cooling of an Automotive Radiator

A test is conducted to determine the overall heat transfer coefficient in an automotive radiator that is a compact cross-flow water-to-air heat exchanger with both fluids (air and water) unmixed (Fig. 13–22). The radiator has 40 tubes of internal diameter 0.5 cm and length 65 cm in a closely spaced plate-finned matrix. Hot water enters the tubes at 90°C at a rate of 0.6 kg/s and leaves at 65°C. Air flows across the radiator through the interfin spaces and is heated from 20°C to 40°C. Determine the overall heat transfer coefficient U_i of this radiator based on the inner surface area of the tubes.

SOLUTION During an experiment involving an automotive radiator, the inlet and exit temperatures of water and air and the mass flow rate of water are measured. The overall heat transfer coefficient based on the inner surface area is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Changes in the kinetic and potential energies of fluid streams are negligible. **3** Fluid properties are constant.
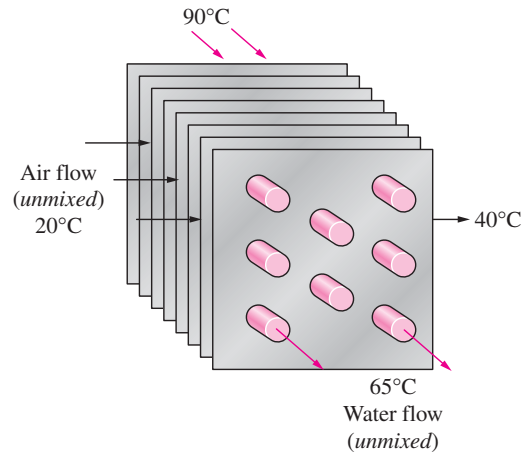
90°C

Air flow
(*unmixed*)
20°C

40°C

65°C
Water flow
(*unmixed*)

FIGURE 13–22
Schematic for Example 13–6.

Properties The specific heat of water at the average temperature of $(90 + 65)/2 = 77.5°C$ is 4.195 kJ/kg · °C.

Analysis The rate of heat transfer in this radiator from the hot water to the air is determined from an energy balance on water flow,

$$\dot{Q} = [\dot{m}C_p(T_{in} - T_{out})]_{water} = (0.6 \text{ kg/s})(4.195 \text{ kJ/kg} \cdot °C)(90 - 65)°C = 62.93 \text{ kW}$$

The tube-side heat transfer area is the total surface area of the tubes, and is determined from

$$A_i = n\pi D_i L = (40)\pi(0.005 \text{ m})(0.65 \text{ m}) = 0.408 \text{ m}^2$$

Knowing the rate of heat transfer and the surface area, the overall heat transfer coefficient can be determined from

$$\dot{Q} = U_i A_i F \Delta T_{lm, CF} \longrightarrow U_i = \frac{\dot{Q}}{A_i F \Delta T_{lm, CF}}$$

where F is the correction factor and $\Delta T_{lm, CF}$ is the log mean temperature difference for the counter-flow arrangement. These two quantities are found to be

$$\Delta T_1 = T_{h, in} - T_{c, out} = (90 - 40)°C = 50°C$$
$$\Delta T_2 = T_{h, out} - T_{c, in} = (65 - 20)°C = 45°C$$
$$\Delta T_{lm, CF} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \frac{50 - 45}{\ln(50/45)} = 47.6°C$$

and

$$\left.\begin{array}{l} P = \dfrac{t_2 - t_1}{T_1 - t_1} = \dfrac{65 - 90}{20 - 90} = 0.36 \\[2mm] R = \dfrac{T_1 - T_2}{t_2 - t_1} = \dfrac{20 - 40}{65 - 90} = 0.80 \end{array}\right\} F = 0.97 \qquad \text{(Fig. 13–18}c)$$

Substituting, the overall heat transfer coefficient U_i is determined to be

$$U_i = \frac{\dot{Q}}{A_i F \Delta T_{lm, CF}} = \frac{62,930 \text{ W}}{(0.408 \text{ m}^2)(0.97)(47.6°C)} = \textbf{3341 W/m}^2 \cdot \textbf{°C}$$

Note that the overall heat transfer coefficient on the air side will be much lower because of the large surface area involved on that side.

13–5 ▪ THE EFFECTIVENESS–NTU METHOD

The log mean temperature difference (LMTD) method discussed in Section 13–4 is easy to use in heat exchanger analysis when the inlet and the outlet temperatures of the hot and cold fluids are known or can be determined from an energy balance. Once ΔT_{lm}, the mass flow rates, and the overall heat transfer coefficient are available, the heat transfer surface area of the heat exchanger can be determined from

$$\dot{Q} = UA_s\, \Delta T_{lm}$$

Therefore, the LMTD method is very suitable for determining the *size* of a heat exchanger to realize prescribed outlet temperatures when the mass flow rates and the inlet and outlet temperatures of the hot and cold fluids are specified.

With the LMTD method, the task is to *select* a heat exchanger that will meet the prescribed heat transfer requirements. The procedure to be followed by the selection process is:

1. Select the type of heat exchanger suitable for the application.
2. Determine any unknown inlet or outlet temperature and the heat transfer rate using an energy balance.
3. Calculate the log mean temperature difference ΔT_{lm} and the correction factor F, if necessary.
4. Obtain (select or calculate) the value of the overall heat transfer coefficient U.
5. Calculate the heat transfer surface area A_s.

The task is completed by selecting a heat exchanger that has a heat transfer surface area equal to or larger than A_s.

A second kind of problem encountered in heat exchanger analysis is the determination of the *heat transfer rate* and the *outlet temperatures* of the hot and cold fluids for prescribed fluid mass flow rates and inlet temperatures when the *type* and *size* of the heat exchanger are specified. The heat transfer surface area A of the heat exchanger in this case is known, but the *outlet temperatures* are not. Here the task is to determine the heat transfer performance of a specified heat exchanger or to determine if a heat exchanger available in storage will do the job.

The LMTD method could still be used for this alternative problem, but the procedure would require tedious iterations, and thus it is not practical. In an attempt to eliminate the iterations from the solution of such problems, Kays and London came up with a method in 1955 called the **effectiveness–NTU method,** which greatly simplified heat exchanger analysis.

This method is based on a dimensionless parameter called the **heat transfer effectiveness** ε, defined as

$$\varepsilon = \frac{\dot{Q}}{Q_{max}} = \frac{\text{Actual heat transfer rate}}{\text{Maximum possible heat transfer rate}} \tag{13-29}$$

The *actual* heat transfer rate in a heat exchanger can be determined from an energy balance on the hot or cold fluids and can be expressed as

$$\dot{Q} = C_c(T_{c,\,out} - T_{c,\,in}) = C_h(T_{h,\,in} - T_{h,\,out}) \tag{13-30}$$

where $C_c = \dot{m}_c C_{pc}$ and $C_h = \dot{m}_c C_{ph}$ are the heat capacity rates of the cold and the hot fluids, respectively.

To determine the maximum possible heat transfer rate in a heat exchanger, we first recognize that the *maximum temperature difference* in a heat exchanger is the difference between the *inlet* temperatures of the hot and cold fluids. That is,

$$\Delta T_{max} = T_{h,\,in} - T_{c,\,in} \qquad \text{(13-31)}$$

The heat transfer in a heat exchanger will reach its maximum value when (1) the cold fluid is heated to the inlet temperature of the hot fluid or (2) the hot fluid is cooled to the inlet temperature of the cold fluid. These two limiting conditions will not be reached simultaneously unless the heat capacity rates of the hot and cold fluids are identical (i.e., $C_c = C_h$). When $C_c \neq C_h$, which is usually the case, the fluid with the *smaller* heat capacity rate will experience a larger temperature change, and thus it will be the first to experience the maximum temperature, at which point the heat transfer will come to a halt. Therefore, the maximum possible heat transfer rate in a heat exchanger is (Fig. 13–23)

$$\dot{Q}_{max} = C_{min}(T_{h,\,in} - T_{c,\,in}) \qquad \text{(13-32)}$$

where C_{min} is the smaller of $C_h = \dot{m}_h C_{ph}$ and $C_c = \dot{m}_c C_{pc}$. This is further clarified by the following example.

EXAMPLE 13–7   Upper Limit for Heat Transfer in a Heat Exchanger

Cold water enters a counter-flow heat exchanger at 10°C at a rate of 8 kg/s, where it is heated by a hot water stream that enters the heat exchanger at 70°C at a rate of 2 kg/s. Assuming the specific heat of water to remain constant at $C_p = 4.18$ kJ/kg · °C, determine the maximum heat transfer rate and the outlet temperatures of the cold and the hot water streams for this limiting case.

SOLUTION   Cold and hot water streams enter a heat exchanger at specified temperatures and flow rates. The maximum rate of heat transfer in the heat exchanger is to be determined.

Assumptions   **1** Steady operating conditions exist. **2** The heat exchanger is well insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** Heat transfer coefficients and fouling factors are constant and uniform. **5** The thermal resistance of the inner tube is negligible since the tube is thin-walled and highly conductive.
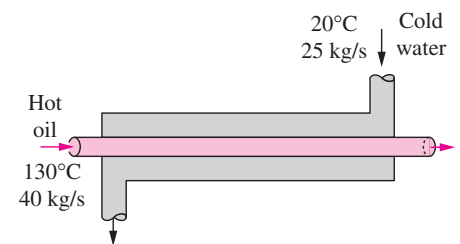
Properties   The specific heat of water is given to be $C_p = 4.18$ kJ/kg · °C.

Analysis   A schematic of the heat exchanger is given in Figure 13–24. The heat capacity rates of the hot and cold fluids are determined from

$$C_h = \dot{m}_h C_{ph} = (2 \text{ kg/s})(4.18 \text{ kJ/kg} \cdot °C) = 8.36 \text{ kW/°C}$$

and

$$C_c = \dot{m}_c C_{pc} = (8 \text{ kg/s})(4.18 \text{ kJ/kg} \cdot °C) = 33.4 \text{ kW/°C}$$

$$C_c = \dot{m}_c C_{pc} = 104.5 \text{ kW/°C}$$
$$C_h = \dot{m}_c C_{ph} = 92 \text{ kW/°C}$$
$$C_{min} = 92 \text{ kW/°C}$$
$$\Delta T_{max} = T_{h,in} - T_{c,in} = 110°C$$
$$\dot{Q}_{max} = C_{min}\Delta T_{max} = 10{,}120 \text{ kW}$$

FIGURE 13–23
The determination of the maximum rate of heat transfer in a heat exchanger.

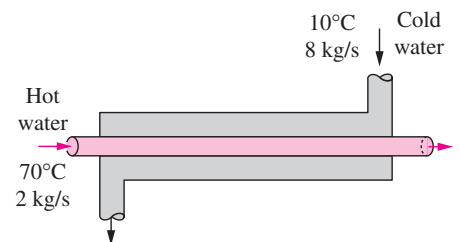**FIGURE 13–24**
Schematic for Example 13–7.

Therefore

$$C_{min} = C_h = 8.36 \text{ kW/°C}$$

which is the smaller of the two heat capacity rates. Then the maximum heat transfer rate is determined from Eq. 13-32 to be

$$\dot{Q}_{max} = C_{min}(T_{h,\,in} - T_{c,\,in})$$
$$= (8.36 \text{ kW/°C})(70 - 10)\text{°C}$$
$$= \mathbf{502\ kW}$$

That is, the maximum possible heat transfer rate in this heat exchanger is 502 kW. This value would be approached in a counter-flow heat exchanger with a *very large* heat transfer surface area.

The maximum temperature difference in this heat exchanger is $\Delta T_{max} = T_{h,\,in} - T_{c,\,in} = (70 - 10)\text{°C} = 60\text{°C}$. Therefore, the hot water cannot be cooled by more than 60°C (to 10°C) in this heat exchanger, and the cold water cannot be heated by more than 60°C (to 70°C), no matter what we do. The outlet temperatures of the cold and the hot streams in this limiting case are determined to be

$$\dot{Q} = C_c(T_{c,\,out} - T_{c,\,in}) \longrightarrow T_{c,\,out} = T_{c,\,in} + \frac{\dot{Q}}{C_c} = 10\text{°C} + \frac{502 \text{ kW}}{33.4 \text{ kW/°C}} = \mathbf{25\text{°C}}$$

$$\dot{Q} = C_h(T_{h,\,in} - T_{h,\,out}) \longrightarrow T_{h,\,out} = T_{h,\,in} - \frac{\dot{Q}}{C_h} = 70\text{°C} - \frac{502 \text{ kW}}{8.38 \text{ kW/°C}} = \mathbf{10\text{°C}}$$

Discussion Note that the hot water is cooled to the limit of 10°C (the inlet temperature of the cold water stream), but the cold water is heated to 25°C only when maximum heat transfer occurs in the heat exchanger. This is not surprising, since the mass flow rate of the hot water is only one-fourth that of the cold water, and, as a result, the temperature of the cold water increases by 0.25°C for each 1°C drop in the temperature of the hot water.

You may be tempted to think that the cold water should be heated to 70°C in the limiting case of maximum heat transfer. But this will require the temperature of the hot water to drop to −170°C (below 10°C), which is impossible. Therefore, heat transfer in a heat exchanger reaches its maximum value when the fluid with the smaller heat capacity rate (or the smaller mass flow rate when both fluids have the same specific heat value) experiences the maximum temperature change. This example explains why we use C_{min} in the evaluation of $\dot{Q}_{max}$ instead of C_{max}.

We can show that the hot water will leave at the inlet temperature of the cold water and vice versa in the limiting case of maximum heat transfer when the mass flow rates of the hot and cold water streams are identical (Fig. 13–25). We can also show that the outlet temperature of the cold water will reach the 70°C limit when the mass flow rate of the hot water is greater than that of the cold water.
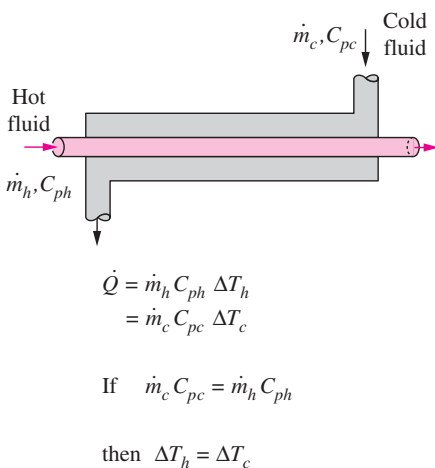

$$\dot{Q} = \dot{m}_h\,C_{ph}\,\Delta T_h$$
$$= \dot{m}_c\,C_{pc}\,\Delta T_c$$

If $\dot{m}_c\,C_{pc} = \dot{m}_h\,C_{ph}$

then $\Delta T_h = \Delta T_c$

FIGURE 13–25
The temperature rise of the cold fluid in a heat exchanger will be equal to the temperature drop of the hot fluid when the mass flow rates and the specific heats of the hot and cold fluids are identical.

The determination of $\dot{Q}_{max}$ requires the availability of the *inlet temperature* of the hot and cold fluids and their *mass flow rates,* which are usually specified. Then, once the effectiveness of the heat exchanger is known, the actual heat transfer rate $\dot{Q}$ can be determined from

$$\dot{Q} = \varepsilon\dot{Q}_{max} = \varepsilon C_{min}(T_{h,\,in} - T_{c,\,in}) \tag{13-33}$$

Therefore, the effectiveness of a heat exchanger enables us to determine the heat transfer rate without knowing the *outlet temperatures* of the fluids.

The effectiveness of a heat exchanger depends on the *geometry* of the heat exchanger as well as the *flow arrangement*. Therefore, different types of heat exchangers have different effectiveness relations. Below we illustrate the development of the effectiveness ε relation for the double-pipe *parallel-flow* heat exchanger.

Equation 13-23 developed in Section 13–4 for a parallel-flow heat exchanger can be rearranged as

$$\ln \frac{T_{h,\,out} - T_{c,\,out}}{T_{h,\,in} - T_{c,\,in}} = -\frac{UA_s}{C_c}\left(1 + \frac{C_c}{C_h}\right) \tag{13-34}$$

Also, solving Eq. 13-30 for $T_{h,\,out}$ gives

$$T_{h,\,out} = T_{h,\,in} - \frac{C_c}{C_h}(T_{c,\,out} - T_{c,\,in}) \tag{13-35}$$

Substituting this relation into Eq. 13-34 after adding and subtracting $T_{c,\,in}$ gives

$$\ln \frac{T_{h,\,in} - T_{c,\,in} + T_{c,\,in} - T_{c,\,out} - \dfrac{C_c}{C_h}(T_{c,\,out} - T_{c,\,in})}{T_{h,\,in} - T_{c,\,in}} = -\frac{UA_s}{C_c}\left(1 + \frac{C_c}{C_h}\right)$$

which simplifies to

$$\ln \left[1 - \left(1 + \frac{C_c}{C_h}\right)\frac{T_{c,\,out} - T_{c,\,in}}{T_{h,\,in} - T_{c,\,in}}\right] = -\frac{UA_s}{C_c}\left(1 + \frac{C_c}{C_h}\right) \tag{13-36}$$

We now manipulate the definition of effectiveness to obtain

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{max}} = \frac{C_c(T_{c,\,out} - T_{c,\,in})}{C_{min}(T_{h,\,in} - T_{c,\,in})} \quad \longrightarrow \quad \frac{T_{c,\,out} - T_{c,\,in}}{T_{h,\,in} - T_{c,\,in}} = \varepsilon\frac{C_{min}}{C_c}$$

Substituting this result into Eq. 13-36 and solving for ε gives the following relation for the effectiveness of a *parallel-flow* heat exchanger:

$$\varepsilon_{parallel\ flow} = \frac{1 - \exp\left[-\dfrac{UA_s}{C_c}\left(1 + \dfrac{C_c}{C_h}\right)\right]}{\left(1 + \dfrac{C_c}{C_h}\right)\dfrac{C_{min}}{C_c}} \tag{13-37}$$

Taking either C_c or C_h to be C_{min} (both approaches give the same result), the relation above can be expressed more conveniently as

$$\varepsilon_{parallel\ flow} = \frac{1 - \exp\left[-\dfrac{UA_s}{C_{min}}\left(1 + \dfrac{C_{min}}{C_{max}}\right)\right]}{1 + \dfrac{C_{min}}{C_{max}}} \tag{13-38}$$

Again C_{min} is the *smaller* heat capacity ratio and C_{max} is the larger one, and it makes no difference whether C_{min} belongs to the hot or cold fluid.

Effectiveness relations of the heat exchangers typically involve the *dimensionless* group UA_s/C_{min}. This quantity is called the **number of transfer units NTU** and is expressed as

$$\text{NTU} = \frac{UA_s}{C_{min}} = \frac{UA_s}{(\dot{m}C_p)_{min}} \tag{13-39}$$

where U is the overall heat transfer coefficient and A_s is the heat transfer surface area of the heat exchanger. Note that NTU is proportional to A_s. Therefore, for specified values of U and C_{min}, the value of NTU *is a measure of the heat transfer surface area A_s*. Thus, the larger the NTU, the larger the heat exchanger.

In heat exchanger analysis, it is also convenient to define another dimensionless quantity called the **capacity ratio c** as

$$c = \frac{C_{min}}{C_{max}} \tag{13-40}$$

It can be shown that the effectiveness of a heat exchanger is a function of the number of transfer units NTU and the capacity ratio c. That is,

$$\varepsilon = \text{function }(UA_s/C_{min}, C_{min}/C_{max}) = \text{function (NTU, } c)$$

Effectiveness relations have been developed for a large number of heat exchangers, and the results are given in Table 13–4. The effectivenesses of some common types of heat exchangers are also plotted in Figure 13–26. More

TABLE 13–4

Effectiveness relations for heat exchangers: NTU = UA_s/C_{min} and $c = C_{min}/C_{max} = (\dot{m}C_p)_{min}/(\dot{m}C_p)_{max}$ (Kays and London, Ref. 5.)
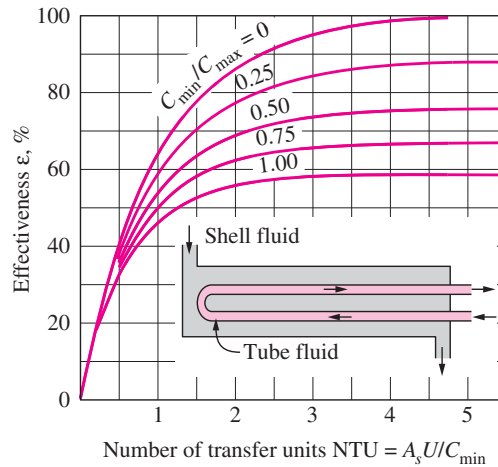
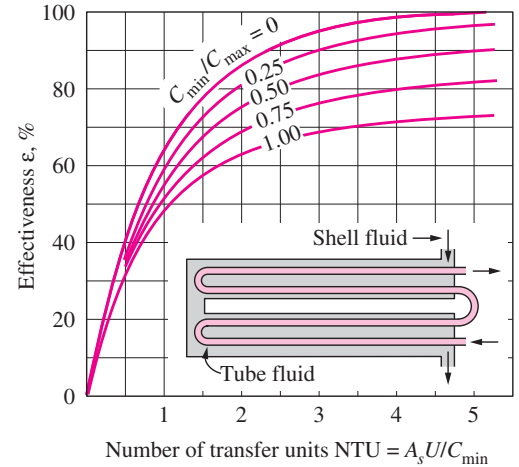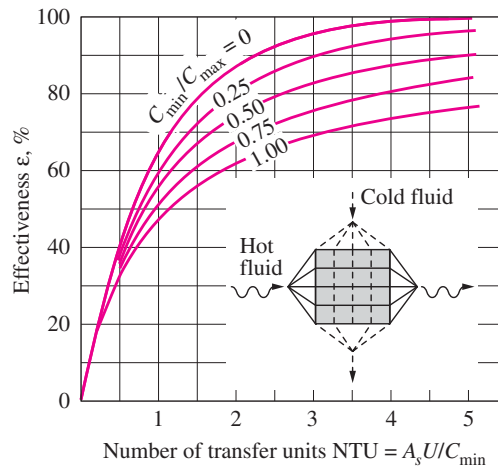| Heat exchanger type	Effectiveness relation
1 *Double pipe:*	
Parallel-flow	$\varepsilon = \dfrac{1 - \exp\left[-\text{NTU}(1 + c)\right]}{1 + c}$
Counter-flow	$\varepsilon = \dfrac{1 - \exp\left[-\text{NTU}(1 - c)\right]}{1 - c\exp\left[-\text{NTU}(1 - c)\right]}$
2 *Shell and tube:*	
One-shell pass 2, 4, . . . tube passes	$\varepsilon = 2\left\{1 + c + \sqrt{1 + c^2}\,\dfrac{1 + \exp\left[-\text{NTU}\sqrt{1 + c^2}\right]}{1 - \exp\left[-\text{NTU}\sqrt{1 + c^2}\right]}\right\}^{-1}$
3 *Cross-flow (single-pass)*	
Both fluids unmixed	$\varepsilon = 1 - \exp\left\{\dfrac{\text{NTU}^{0.22}}{c}\left[\exp\left(-c\ \text{NTU}^{0.78}\right) - 1\right]\right\}$
C_{max} mixed, C_{min} unmixed	$\varepsilon = \dfrac{1}{c}(1 - \exp\{1 - c[1 - \exp(-\text{NTU})]\})$
C_{min} mixed, C_{max} unmixed	$\varepsilon = 1 - \exp\left\{-\dfrac{1}{c}[1 - \exp(-c\ \text{NTU})]\right\}$
4 *All heat exchangers with $c = 0$*	$\varepsilon = 1 - \exp(-\text{NTU})$
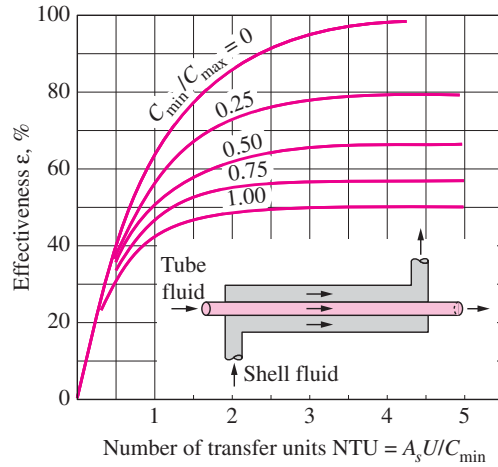
(a) Parallel-flow
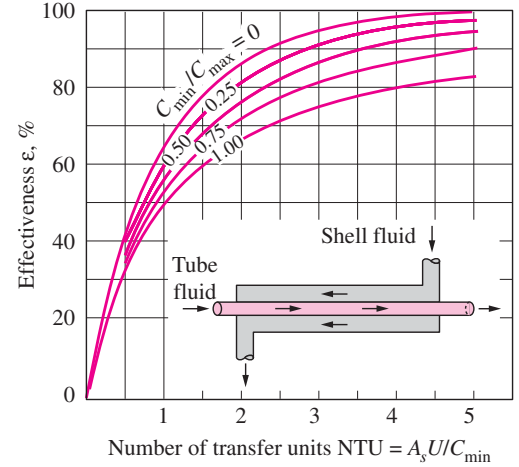
(b) Counter-flow
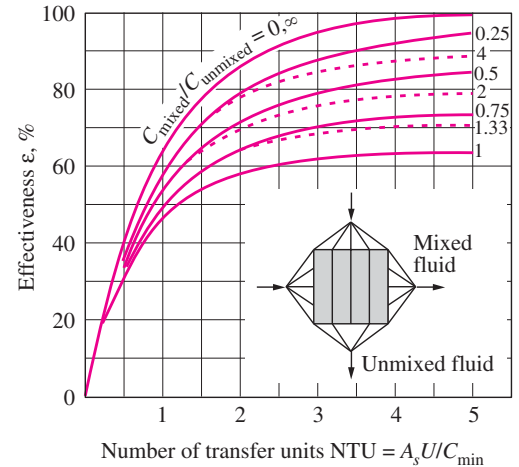
(c) One-shell pass and 2, 4, 6, … tube passes

(d) Two-shell passes and 4, 8, 12, … tube passes

(e) Cross-flow with both fluids unmixed

(f) Cross-flow with one fluid mixed and the other unmixed

FIGURE 13–26

Effectiveness for heat exchangers (from Kays and London, Ref. 5).

extensive effectiveness charts and relations are available in the literature. The dashed lines in Figure 13–26f are for the case of C_{min} unmixed and C_{max} mixed and the solid lines are for the opposite case. The analytic relations for the effectiveness give more accurate results than the charts, since reading errors in charts are unavoidable, and the relations are very suitable for computerized analysis of heat exchangers.

We make these following observations from the effectiveness relations and charts already given:

1. The value of the effectiveness ranges from 0 to 1. It increases rapidly with NTU for small values (up to about NTU = 1.5) but rather slowly for larger values. Therefore, the use of a heat exchanger with a large NTU (usually larger than 3) and thus a large size cannot be justified economically, since a large increase in NTU in this case corresponds to a small increase in effectiveness. Thus, a heat exchanger with a very high effectiveness may be highly desirable from a heat transfer point of view but rather undesirable from an economical point of view.

2. For a given NTU and capacity ratio $c = C_{min}/C_{max}$, the *counter-flow* heat exchanger has the *highest* effectiveness, followed closely by the cross-flow heat exchangers with both fluids unmixed. As you might expect, the lowest effectiveness values are encountered in parallel-flow heat exchangers (Fig. 13–27).

3. The effectiveness of a heat exchanger is independent of the capacity ratio c for NTU values of less than about 0.3.

4. The value of the capacity ratio c ranges between 0 and 1. For a given NTU, the effectiveness becomes a *maximum* for $c = 0$ and a *minimum* for $c = 1$. The case $c = C_{min}/C_{max} \rightarrow 0$ corresponds to $C_{max} \rightarrow \infty$, which is realized during a phase-change process in a *condenser* or *boiler*. All effectiveness relations in this case reduce to

$$\varepsilon = \varepsilon_{max} = 1 - \exp(-NTU) \qquad \text{(13-41)}$$

regardless of the type of heat exchanger (Fig. 13–28). Note that the temperature of the condensing or boiling fluid remains constant in this case. The effectiveness is the *lowest* in the other limiting case of $c = C_{min}/C_{max} = 1$, which is realized when the heat capacity rates of the two fluids are equal.

Once the quantities $c = C_{min}/C_{max}$ and $NTU = UA_s/C_{min}$ have been evaluated, the effectiveness ε can be determined from either the charts or (preferably) the effectiveness relation for the specified type of heat exchanger. Then the rate of heat transfer $\dot{Q}$ and the outlet temperatures $T_{h, out}$ and $T_{c, out}$ can be determined from Eqs. 13-33 and 13-30, respectively. Note that the analysis of heat exchangers with unknown outlet temperatures is a straightforward matter with the effectiveness–NTU method but requires rather tedious iterations with the LMTD method.

We mentioned earlier that when all the inlet and outlet temperatures are specified, the *size* of the heat exchanger can easily be determined using the LMTD method. Alternatively, it can also be determined from the effectiveness–NTU method by first evaluating the effectiveness ε from its definition (Eq. 13-29) and then the NTU from the appropriate NTU relation in Table 13–5.
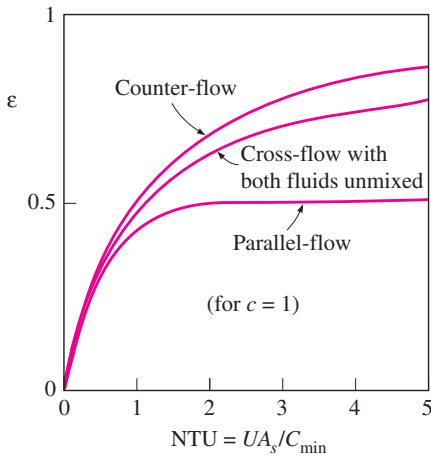

FIGURE 13–27
For a specified NTU and capacity ratio c, the counter-flow heat exchanger has the highest effectiveness and the parallel-flow the lowest.
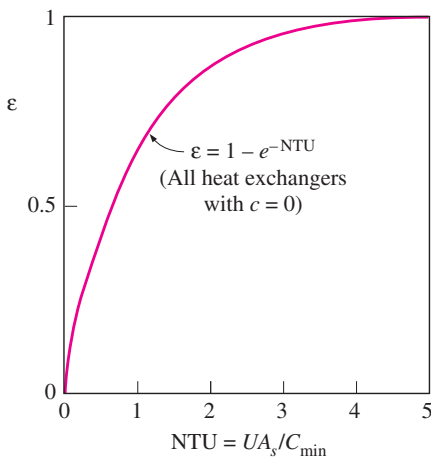

FIGURE 13–28
The effectiveness relation reduces to $\varepsilon = \varepsilon_{max} = 1 - \exp(-NTU)$ for all heat exchangers when the capacity ratio $c = 0$.

TABLE 13–5

NTU relations for heat exchangers $NTU = UA_s/C_{min}$ and $c = C_{min}/C_{max} = (\dot{m}C_p)_{min}/(\dot{m}C_p)_{max}$ (Kays and London, Ref. 5.)

Heat exchanger type	NTU relation
1 *Double-pipe:* Parallel-flow	$NTU = -\dfrac{\ln\,[1 - \varepsilon(1 + c)]}{1 + c}$
Counter-flow	$NTU = \dfrac{1}{c - 1}\,\ln\left(\dfrac{\varepsilon - 1}{\varepsilon c - 1}\right)$

2 *Shell and tube:*
   One-shell pass
   2, 4, . . . tube passes
	$NTU = -\dfrac{1}{\sqrt{1 + c^2}}\,\ln\left(\dfrac{2/\varepsilon - 1 - c - \sqrt{1 + c^2}}{2/\varepsilon - 1 - c + \sqrt{1 + c^2}}\right)$

3 *Cross-flow (single-pass)*
   C_{max} mixed,
   C_{min} unmixed
	$NTU = -\ln\left[1 + \dfrac{\ln\,(1 - \varepsilon c)}{c}\right]$

C_{min} mixed, C_{max} unmixed	$NTU = -\dfrac{\ln\,[c \ln\,(1 - \varepsilon) + 1]}{c}$
4 *All heat exchangers* with $c = 0$	$NTU = -\ln(1 - \varepsilon)$

Note that the relations in Table 13–5 are equivalent to those in Table 13–4. Both sets of relations are given for convenience. The relations in Table 13–4 give the effectiveness directly when NTU is known, and the relations in Table 13–5 give the NTU directly when the effectiveness ε is known.

EXAMPLE 13–8   Using the Effectiveness–NTU Method

Repeat Example 13–4, which was solved with the LMTD method, using the effectiveness–NTU method.

SOLUTION   The schematic of the heat exchanger is redrawn in Figure 13–29, and the same assumptions are utilized.
Analysis   In the effectiveness–NTU method, we first determine the heat capacity rates of the hot and cold fluids and identify the smaller one:

$$C_h = \dot{m}_h C_{ph} = (2\ kg/s)(4.31\ kJ/kg \cdot °C) = 8.62\ kW/°C$$
$$C_c = \dot{m}_c C_{pc} = (1.2\ kg/s)(4.18\ kJ/kg \cdot °C) = 5.02\ kW/°C$$

Therefore,

$$C_{min} = C_c = 5.02\ kW/°C$$

and

$$c = C_{min}/C_{max} = 5.02/8.62 = 0.583$$

Then the maximum heat transfer rate is determined from Eq. 13-32 to be

$$\dot{Q}_{max} = C_{min}(T_{h,\ in} - T_{c,\ in})$$
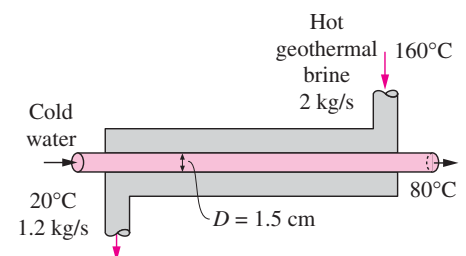$$= (5.02\ kW/°C)(160 - 20)°C$$
$$= 702.8\ kW$$

FIGURE 13–29
Schematic for Example 13–8.

That is, the maximum possible heat transfer rate in this heat exchanger is 702.8 kW. The actual rate of heat transfer in the heat exchanger is

$$\dot{Q} = [\dot{m}C_p(T_{out} - T_{in})]_{water} = (1.2 \text{ kg/s})(4.18 \text{ kJ/kg} \cdot °C)(80 - 20)°C = 301.0 \text{ kW}$$

Thus, the effectiveness of the heat exchanger is

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{max}} = \frac{301.0 \text{ kW}}{702.8 \text{ kW}} = 0.428$$

Knowing the effectiveness, the NTU of this counter-flow heat exchanger can be determined from Figure 13–26b or the appropriate relation from Table 13–5. We choose the latter approach for greater accuracy:

$$\text{NTU} = \frac{1}{c-1} \ln \left(\frac{\varepsilon - 1}{\varepsilon c - 1} \right) = \frac{1}{0.583 - 1} \ln \left(\frac{0.428 - 1}{0.428 \times 0.583 - 1} \right) = 0.651$$

Then the heat transfer surface area becomes

$$\text{NTU} = \frac{UA_s}{C_{min}} \longrightarrow A_s = \frac{\text{NTU } C_{min}}{U} = \frac{(0.651)(5020 \text{ W/°C})}{640 \text{ W/m}^2 \cdot °C} = 5.11 \text{ m}^2$$

To provide this much heat transfer surface area, the length of the tube must be

$$A_s = \pi DL \longrightarrow L = \frac{A_s}{\pi D} = \frac{5.11 \text{ m}^2}{\pi(0.015 \text{ m})} = \textbf{108 m}$$

Discussion Note that we obtained the same result with the effectiveness–NTU method in a systematic and straightforward manner.
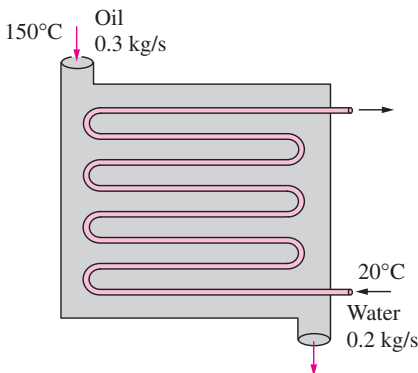

EXAMPLE 13–9   **Cooling Hot Oil by Water in a Multipass Heat Exchanger**

Hot oil is to be cooled by water in a 1-shell-pass and 8-tube-passes heat exchanger. The tubes are thin-walled and are made of copper with an internal diameter of 1.4 cm. The length of each tube pass in the heat exchanger is 5 m, and the overall heat transfer coefficient is 310 W/m² · °C. Water flows through the tubes at a rate of 0.2 kg/s, and the oil through the shell at a rate of 0.3 kg/s. The water and the oil enter at temperatures of 20°C and 150°C, respectively. Determine the rate of heat transfer in the heat exchanger and the outlet temperatures of the water and the oil.

SOLUTION   Hot oil is to be cooled by water in a heat exchanger. The mass flow rates and the inlet temperatures are given. The rate of heat transfer and the outlet temperatures are to be determined.

Assumptions   **1** Steady operating conditions exist. **2** The heat exchanger is well insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** The thickness of the tube is negligible since it is thin-walled. **4** Changes in the kinetic and potential energies of fluid streams are negligible. **5** The overall heat transfer coefficient is constant and uniform.

Analysis   The schematic of the heat exchanger is given in Figure 13–30. The outlet temperatures are not specified, and they cannot be determined from an

Oil
150°C   0.3 kg/s

20°C
Water
0.2 kg/s

FIGURE 13–30
Schematic for Example 13–9.

energy balance. The use of the LMTD method in this case will involve tedious iterations, and thus the ε–NTU method is indicated. The first step in the ε–NTU method is to determine the heat capacity rates of the hot and cold fluids and identify the smaller one:

$$C_h = \dot{m}_h C_{ph} = (0.3 \text{ kg/s})(2.13 \text{ kJ/kg} \cdot °\text{C}) = 0.639 \text{ kW/°C}$$
$$C_c = \dot{m}_c C_{pc} = (0.2 \text{ kg/s})(4.18 \text{ kJ/kg} \cdot °\text{C}) = 0.836 \text{ kW/°C}$$

Therefore,

$$C_{\min} = C_h = 0.639 \text{ kW/°C}$$

and

$$c = \frac{C_{\min}}{C_{\max}} = \frac{0.639}{0.836} = 0.764$$

Then the maximum heat transfer rate is determined from Eq. 13-32 to be

$$\dot{Q}_{\max} = C_{\min}(T_{h,\text{ in}} - T_{c,\text{ in}})$$
$$= (0.639 \text{ kW/°C})(150 - 20)°\text{C} = 83.1 \text{ kW}$$

That is, the maximum possible heat transfer rate in this heat exchanger is 83.1 kW. The heat transfer surface area is

$$A_s = n(\pi D L) = 8\pi(0.014 \text{ m})(5 \text{ m}) = 1.76 \text{ m}^2$$

Then the NTU of this heat exchanger becomes

$$\text{NTU} = \frac{UA_s}{C_{\min}} = \frac{(310 \text{ W/m}^2 \cdot °\text{C})(1.76 \text{ m}^2)}{639 \text{ W/°C}} = 0.853$$

The effectiveness of this heat exchanger corresponding to $c = 0.764$ and NTU $= 0.853$ is determined from Figure 13–26c to be

$$\varepsilon = 0.47$$

We could also determine the effectiveness from the third relation in Table 13–4 more accurately but with more labor. Then the actual rate of heat transfer becomes

$$\dot{Q} = \varepsilon \dot{Q}_{\max} = (0.47)(83.1 \text{ kW}) = \textbf{39.1 kW}$$

Finally, the outlet temperatures of the cold and the hot fluid streams are determined to be

$$\dot{Q} = C_c(T_{c,\text{ out}} - T_{c,\text{ in}}) \longrightarrow T_{c,\text{ out}} = T_{c,\text{ in}} + \frac{\dot{Q}}{C_c}$$
$$= 20°\text{C} + \frac{39.1 \text{ kW}}{0.836 \text{ kW/°C}} = \textbf{66.8°C}$$

$$\dot{Q} = C_h(T_{h,\text{ in}} - T_{h,\text{ out}}) \longrightarrow T_{h,\text{ out}} = T_{h,\text{ in}} - \frac{\dot{Q}}{C_h}$$
$$= 150°\text{C} - \frac{39.1 \text{ kW}}{0.639 \text{ kW/°C}} = \textbf{88.8°C}$$

Therefore, the temperature of the cooling water will rise from 20°C to 66.8°C as it cools the hot oil from 150°C to 88.8°C in this heat exchanger.

13–6 ▪ SELECTION OF HEAT EXCHANGERS

Heat exchangers are complicated devices, and the results obtained with the simplified approaches presented above should be used with care. For example, we assumed that the overall heat transfer coefficient U is constant throughout the heat exchanger and that the convection heat transfer coefficients can be predicted using the convection correlations. However, it should be kept in mind that the uncertainty in the predicted value of U can even exceed 30 percent. Thus, it is natural to tend to overdesign the heat exchangers in order to avoid unpleasant surprises.

Heat transfer enhancement in heat exchangers is usually accompanied by *increased pressure drop,* and thus *higher pumping power.* Therefore, any gain from the enhancement in heat transfer should be weighed against the cost of the accompanying pressure drop. Also, some thought should be given to which fluid should pass through the tube side and which through the shell side. Usually, the *more viscous fluid is more suitable for the shell side* (larger passage area and thus lower pressure drop) and *the fluid with the higher pressure for the tube* side.

Engineers in industry often find themselves in a position to select heat exchangers to accomplish certain heat transfer tasks. Usually, the goal is to heat or cool a certain fluid at a known mass flow rate and temperature to a desired temperature. Thus, the rate of heat transfer in the prospective heat exchanger is

$$\dot{Q}_{max} = \dot{m}C_p(T_{in} - T_{out})$$

which gives the heat transfer requirement of the heat exchanger before having any idea about the heat exchanger itself.

An engineer going through catalogs of heat exchanger manufacturers will be overwhelmed by the type and number of readily available off-the-shelf heat exchangers. The proper selection depends on several factors.

Heat Transfer Rate

This is the most important quantity in the selection of a heat exchanger. A heat exchanger should be capable of transferring heat at the specified rate in order to achieve the desired temperature change of the fluid at the specified mass flow rate.

Cost

Budgetary limitations usually play an important role in the selection of heat exchangers, except for some specialized cases where "money is no object." An off-the-shelf heat exchanger has a definite cost advantage over those made to order. However, in some cases, none of the existing heat exchangers will do, and it may be necessary to undertake the expensive and time-consuming task of designing and manufacturing a heat exchanger from scratch to suit the needs. This is often the case when the heat exchanger is an integral part of the overall device to be manufactured.

The operation and maintenance costs of the heat exchanger are also important considerations in assessing the overall cost.

Pumping Power

In a heat exchanger, both fluids are usually forced to flow by pumps or fans that consume electrical power. The annual cost of electricity associated with the operation of the pumps and fans can be determined from

$$\text{Operating cost} = (\text{Pumping power, kW}) \times (\text{Hours of operation, h}) \\ \times (\text{Price of electricity, \$/kWh})$$

where the pumping power is the total electrical power consumed by the motors of the pumps and fans. For example, a heat exchanger that involves a 1-hp pump and a $\frac{1}{3}$-hp fan (1 hp = 0.746 kW) operating 8 h a day and 5 days a week will consume 2017 kWh of electricity per year, which will cost \$161.4 at an electricity cost of 8 cents/kWh.

Minimizing the pressure drop and the mass flow rate of the fluids will *minimize* the operating cost of the heat exchanger, but it will *maximize* the size of the heat exchanger and thus the initial cost. As a rule of thumb, doubling the mass flow rate will reduce the initial cost by *half* but will increase the pumping power requirements by a factor of roughly *eight*.

Typically, fluid velocities encountered in heat exchangers range between 0.7 and 7 m/s for liquids and between 3 and 30 m/s for gases. Low velocities are helpful in avoiding erosion, tube vibrations, and noise as well as pressure drop.

Size and Weight

Normally, the *smaller* and the *lighter* the heat exchanger, the better it is. This is especially the case in the *automotive* and *aerospace* industries, where size and weight requirements are most stringent. Also, a larger heat exchanger normally carries a higher price tag. The space available for the heat exchanger in some cases limits the length of the tubes that can be used.

Type

The type of heat exchanger to be selected depends primarily on the type of *fluids* involved, the *size* and *weight* limitations, and the presence of any *phase-change* processes. For example, a heat exchanger is suitable to cool a liquid by a gas if the surface area on the gas side is many times that on the liquid side. On the other hand, a plate or shell-and-tube heat exchanger is very suitable for cooling a liquid by another liquid.

Materials

The materials used in the construction of the heat exchanger may be an important consideration in the selection of heat exchangers. For example, the thermal and structural *stress effects* need not be considered at pressures below 15 atm or temperatures below 150°C. But these effects are major considerations above 70 atm or 550°C and seriously limit the acceptable materials of the heat exchanger.

A temperature difference of 50°C or more between the tubes and the shell will probably pose *differential thermal expansion* problems and needs to be considered. In the case of corrosive fluids, we may have to select expensive

corrosion-resistant materials such as stainless steel or even titanium if we are not willing to replace low-cost heat exchangers frequently.

Other Considerations

There are other considerations in the selection of heat exchangers that may or may not be important, depending on the application. For example, being *leak-tight* is an important consideration when *toxic* or *expensive* fluids are involved. Ease of servicing, low maintenance cost, and safety and reliability are some other important considerations in the selection process. Quietness is one of the primary considerations in the selection of liquid-to-air heat exchangers used in heating and air-conditioning applications.

EXAMPLE 13–10   **Installing a Heat Exchanger to Save Energy and Money**

In a dairy plant, milk is pasteurized by hot water supplied by a natural gas furnace. The hot water is then discharged to an open floor drain at 80°C at a rate of 15 kg/min. The plant operates 24 h a day and 365 days a year. The furnace has an efficiency of 80 percent, and the cost of the natural gas is $0.40 per therm (1 therm = 105,500 kJ). The average temperature of the cold water entering the furnace throughout the year is 15°C. The drained hot water cannot be returned to the furnace and recirculated, because it is contaminated during the process.

In order to save energy, installation of a water-to-water heat exchanger to preheat the incoming cold water by the drained hot water is proposed. Assuming that the heat exchanger will recover 75 percent of the available heat in the hot water, determine the heat transfer rating of the heat exchanger that needs to be purchased and suggest a suitable type. Also, determine the amount of money this heat exchanger will save the company per year from natural gas savings.

SOLUTION   A water-to-water heat exchanger is to be installed to transfer energy from drained hot water to the incoming cold water to preheat it. The rate of heat transfer in the heat exchanger and the amount of energy and money saved per year are to be determined.

Assumptions   **1** Steady operating conditions exist. **2** The effectiveness of the heat exchanger remains constant.

Properties   We use the specific heat of water at room temperature, $C_p = 4.18$ kJ/kg · °C (Table A–9), and treat it as a constant.

Analysis   A schematic of the prospective heat exchanger is given in Figure 13–31. The heat recovery from the hot water will be a maximum when it leaves the heat exchanger at the inlet temperature of the cold water. Therefore,

$$\dot{Q}_{max} = \dot{m}_h C_p (T_{h, in} - T_{c, in})$$
$$= \left(\frac{15}{60} \text{ kg/s}\right)(4.18 \text{ kJ/kg} \cdot °C)(80 - 15)°C$$
$$= 67.9 \text{ kJ/s}$$

That is, the existing hot water stream has the potential to supply heat at a rate of 67.9 kJ/s to the incoming cold water. This value would be approached in a counter-flow heat exchanger with a *very large* heat transfer surface area. A heat exchanger of reasonable size and cost can capture 75 percent of this heat
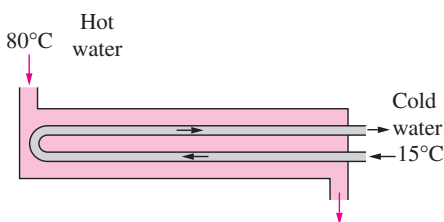

FIGURE 13–31
Schematic for Example 13–10.

transfer potential. Thus, the heat transfer rating of the prospective heat exchanger must be

$$\dot{Q} = \varepsilon \dot{Q}_{max} = (0.75)(67.9 \text{ kJ/s}) = \mathbf{50.9 \text{ kJ/s}}$$

That is, the heat exchanger should be able to deliver heat at a rate of 50.9 kJ/s from the hot to the cold water. An ordinary plate or *shell-and-tube* heat exchanger should be adequate for this purpose, since both sides of the heat exchanger involve the same fluid at comparable flow rates and thus comparable heat transfer coefficients. (Note that if we were heating air with hot water, we would have to specify a heat exchanger that has a large surface area on the air side.)

The heat exchanger will operate 24 h a day and 365 days a year. Therefore, the annual operating hours are

$$\text{Operating hours} = (24 \text{ h/day})(365 \text{ days/year}) = 8760 \text{ h/year}$$

Noting that this heat exchanger saves 50.9 kJ of energy per second, the energy saved during an entire year will be

$$\begin{aligned}\text{Energy saved} &= (\text{Heat transfer rate})(\text{Operation time})\\ &= (50.9 \text{ kJ/s})(8760 \text{ h/year})(3600 \text{ s/h})\\ &= 1.605 \times 10^9 \text{ kJ/year}\end{aligned}$$

The furnace is said to be 80 percent efficient. That is, for each 80 units of heat supplied by the furnace, natural gas with an energy content of 100 units must be supplied to the furnace. Therefore, the energy savings determined above result in fuel savings in the amount of

$$\begin{aligned}\text{Fuel saved} &= \frac{\text{Energy saved}}{\text{Furnace efficiency}} = \frac{1.605 \times 10^9 \text{ kJ/year}}{0.80}\left(\frac{1 \text{ therm}}{105{,}500 \text{ kJ}}\right)\\ &= 19{,}020 \text{ therms/year}\end{aligned}$$

Noting that the price of natural gas is $0.40 per therm, the amount of money saved becomes

$$\begin{aligned}\text{Money saved} &= (\text{Fuel saved}) \times (\text{Price of fuel})\\ &= (19{,}020 \text{ therms/year})(\$0.40/\text{therm})\\ &= \mathbf{\$7607/year}\end{aligned}$$

Therefore, the installation of the proposed heat exchanger will save the company $7607 a year, and the installation cost of the heat exchanger will probably be paid from the fuel savings in a short time.

SUMMARY

Heat exchangers are devices that allow the exchange of heat between two fluids without allowing them to mix with each other. Heat exchangers are manufactured in a variety of types, the simplest being the *double-pipe* heat exchanger. In a *parallel-flow* type, both the hot and cold fluids enter the heat exchanger at the same end and move in the same direction, whereas in a *counter-flow* type, the hot and cold fluids enter the heat exchanger at opposite ends and flow in opposite directions. In *compact* heat exchangers, the two fluids move perpendicular to each other, and such a flow configuration is called *cross-flow*. Other common types of heat exchangers in industrial applications are the *plate* and the *shell-and-tube* heat exchangers.

Heat transfer in a heat exchanger usually involves convection in each fluid and conduction through the wall separating the two fluids. In the analysis of heat exchangers, it is convenient to

work with an *overall heat transfer coefficient U* or a *total thermal resistance R*, expressed as

$$\frac{1}{UA_s} = \frac{1}{U_i A_i} = \frac{1}{U_o A_o} = R = \frac{1}{h_i A_i} + R_{\text{wall}} + \frac{1}{h_o A_o}$$

where the subscripts *i* and *o* stand for the inner and outer surfaces of the wall that separates the two fluids, respectively. When the wall thickness of the tube is small and the thermal conductivity of the tube material is high, the last relation simplifies to

$$\frac{1}{U} \approx \frac{1}{h_i} + \frac{1}{h_o}$$

where $U \approx U_i \approx U_o$. The effects of fouling on both the inner and the outer surfaces of the tubes of a heat exchanger can be accounted for by

$$\frac{1}{UA_s} = \frac{1}{U_i A_i} = \frac{1}{U_o A_o} = R$$
$$= \frac{1}{h_i A_i} + \frac{R_{f,\,i}}{A_i} + \frac{\ln (D_o/D_i)}{2\pi k L} + \frac{R_{f,\,o}}{A_o} + \frac{1}{h_o A_o}$$

where $A_i = \pi D_i L$ and $A_o = \pi D_o L$ are the areas of the inner and outer surfaces and $R_{f,\,i}$ and $R_{f,\,o}$ are the fouling factors at those surfaces.

In a well-insulated heat exchanger, the rate of heat transfer from the hot fluid is equal to the rate of heat transfer to the cold one. That is,

$$\dot{Q} = \dot{m}_c C_{pc}(T_{c,\,\text{out}} - T_{c,\,\text{in}}) = C_c(T_{c,\,\text{out}} - T_{c,\,\text{in}})$$

and

$$\dot{Q} = \dot{m}_h C_{ph}(T_{h,\,\text{in}} - T_{h,\,\text{out}}) = C_h(T_{h,\,\text{in}} - T_{h,\,\text{out}})$$

where the subscripts *c* and *h* stand for the cold and hot fluids, respectively, and the product of the mass flow rate and the specific heat of a fluid $\dot{m}C_p$ is called the *heat capacity rate*.

Of the two methods used in the analysis of heat exchangers, the *log mean temperature difference* (or LMTD) method is best suited for determining the size of a heat exchanger when all the inlet and the outlet temperatures are known. The *effectiveness–NTU* method is best suited to predict the outlet temperatures of the hot and cold fluid streams in a specified heat exchanger. In the LMTD method, the rate of heat transfer is determined from

$$\dot{Q} = UA_s \,\Delta T_{\text{lm}}$$

where

$$\Delta T_{\text{lm}} = \frac{\Delta T_1 - \Delta T_2}{\ln (\Delta T_1/\Delta T_2)}$$

is the *log mean temperature difference,* which is the suitable form of the average temperature difference for use in the analysis of heat exchangers. Here ΔT_1 and ΔT_2 represent the temperature differences between the two fluids at the two ends (inlet and outlet) of the heat exchanger. For cross-flow and multipass shell-and-tube heat exchangers, the logarithmic mean temperature difference is related to the counter-flow one $\Delta T_{\text{lm, }CF}$ as

$$\Delta T_{\text{lm}} = F \,\Delta T_{\text{lm, }CF}$$

where *F* is the *correction factor,* which depends on the geometry of the heat exchanger and the inlet and outlet temperatures of the hot and cold fluid streams.

The *effectiveness* of a heat exchanger is defined as

$$\varepsilon = \frac{\dot{Q}}{Q_{\text{max}}} = \frac{\text{Actual heat transfer rate}}{\text{Maximum possible heat transfer rate}}$$

where

$$\dot{Q}_{\text{max}} = C_{\text{min}}(T_{h,\,\text{in}} - T_{c,\,\text{in}})$$

and C_{min} is the smaller of $C_h = \dot{m}_h C_{ph}$ and $C_c = \dot{m}_c C_{pc}$. The effectiveness of heat exchangers can be determined from effectiveness relations or charts.

The selection or design of a heat exchanger depends on several factors such as the heat transfer rate, cost, pressure drop, size, weight, construction type, materials, and operating environment.

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