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Empirical Correlations

Laminar Flow over an Isothermal Plate

Laminar Flow over an Isothermal Plate:

$$Nu_x \equiv \frac{h_x x}{k} = 0.332\, Re_x^{1/2}\, Pr^{1/3} \qquad Pr \gtrsim 0.6$$

$$\overline{Nu}_x \equiv \frac{\overline{h}_x x}{k} = 0.664\, Re_x^{1/2}\, Pr^{1/3} \quad Pr \gtrsim 0.6$$

For fluids with Small Prandtl no (liquid metals)

$$Nu_x = 0.565\, Pe_x^{1/2} \quad Pr \lesssim 0.05, \quad Pe_x \gtrsim 100$$

Where:

$$Pe_x \equiv Re_x\, Pr \text{ is the } Peclet\ number$$

A single correlating equation, which applies for all Prandtl numbers, has been recommended by Churchill and Ozoe, For laminar flow over an isothermal plate, the local convection coefficient may be obtained from:

$$Nu_x = \frac{0.3387\, Re_x^{1/2}\, Pr^{1/3}}{[1 + (0.0468/Pr)^{2/3}]^{1/4}} \quad Pe_x \gtrsim 100$$

with $\overline{Nu}_x = 2Nu_x$.

Turbulent Flow over an Isothermal Plate

With the modified Reynolds, or Chilton–Colburn, analogy, the local Nusselt number for turbulent flow is:

$$Nu_x = St\, Re_x\, Pr = 0.0296\, Re_x^{4/5}\, Pr^{1/3} \quad 0.6 \lesssim Pr \lesssim 60$$

Mixed (Transition occurs from laminar to Turbulent) Boundary Layer

$$\overline{Nu}_L = (0.037\, Re_L^{4/5} - A)\, Pr^{1/3}$$

$$\begin{bmatrix} 0.6 \lesssim Pr \lesssim 60 \\ Re_{x,c} \lesssim Re_L \lesssim 10^8 \end{bmatrix}$$

where the bracketed relations indicate the range of applicability and the constant A is determined by the value of the critical Reynolds number, $Re_{x,c}$. That is,

$$A = 0.037\, Re_{x,c}^{4/5} - 0.664\, Re_{x,c}^{1/2}$$

A = 871 for $Re_{x,c} = 5(10)^5$

Flat Plate with Constant Heat Flux Conditions

For Laminar Flow:

$$Nu_x = 0.453 Re_x^{1/2}\, Pr^{1/3} \qquad Pr \gtrsim 0.6$$

For Turbulent Flow:

$$Nu_x = 0.0308 Re_x^{4/5}\, Pr^{1/3} \qquad 0.6 \lesssim Pr \lesssim 60$$

Circular Cylinder in Cross Flow

The empirical correlation due to Hilpert

$$\overline{Nu}_D \equiv \frac{\bar{h}D}{k} = C\, Re_D^m\, Pr^{1/3}$$

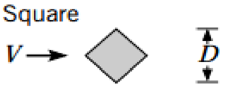
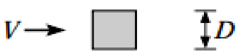
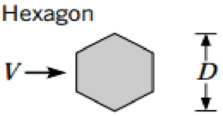
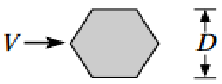
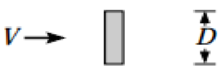
is widely used for Pr ≥ 0.7, where the constants C and mare listed in Table 7.2

TABLE 7.2 Constants of Equation 7.52 for the circular cylinder in cross flow [11, 12]

Re_D	C	m
0.4–4	0.989	0.330
4–40	0.911	0.385
40–4000	0.683	0.466
4000–40,000	0.193	0.618
40,000–400,000	0.027	0.805

The empirical correlation due to Hilpert may also be used for flow over cylinders of noncircular cross section,with the characteristic length D and the constants obtained from Table 7.3

TABLE 7.3 Constants of Equation 7.52 for noncircular cylinders in cross flow of a gas [13]

Geometry	Re_D	C	m
Square			
	5×10^3–10^5	0.246	0.588
	5×10^3–10^5	0.102	0.675
Hexagon			
	5×10^3–1.95×10^4	0.160	0.638
	1.95×10^4–10^5	0.0385	0.782
	5×10^3–10^5	0.153	0.638
Vertical plate			
	4×10^3–1.5×10^4	0.228	0.731

The correlation due to Zukauskas is of the form:

$$\overline{Nu}_D = C\,Re_D^m\,Pr^n\left(\frac{Pr}{Pr_s}\right)^{1/4}$$

$$\left[\begin{array}{l} 0.7 \lesssim Pr \lesssim 500 \\ 1 \lesssim Re_D \lesssim 10^6 \end{array}\right]$$

If Pr ≤ 10, n = 0.37; if Pr ≥ 10, n = 0.36
Where all properties are evaluated at T_∞, except Pr_s, which is evaluated at T_s. Values of C and m are listed in Table 7.4.

TABLE 7.4 Constants of Equation 7.53 for the circular cylinder in cross flow [16]

Re_D	C	m
1–40	0.75	0.4
40–1000	0.51	0.5
10^3–2×10^5	0.26	0.6
2×10^5–10^6	0.076	0.7

Churchill and Bernstein have proposed a single comprehensive equation that covers the entire range of Re_D for which data are available, as well as a wide range of Pr. The equation is recommended for all Re_D, Pr ≥ 0.2 and has the form:

$$\overline{Nu}_D = 0.3 + \frac{0.62\,Re_D^{1/2}\,Pr^{1/3}}{[1 + (0.4/Pr)^{2/3}]^{1/4}}\left[1 + \left(\frac{Re_D}{282{,}000}\right)^{5/8}\right]^{4/5}$$

Where all properties are evaluated at the film temperature

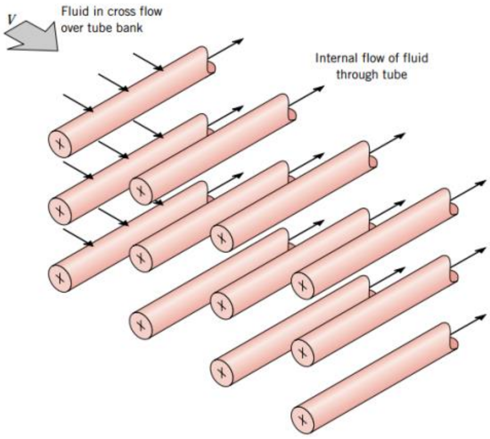
The Sphere

$$\overline{Nu}_D = 2 + (0.4\,Re_D^{1/2} + 0.06\,Re_D^{2/3})\,Pr^{0.4}\left(\frac{\mu}{\mu_s}\right)^{1/4}$$

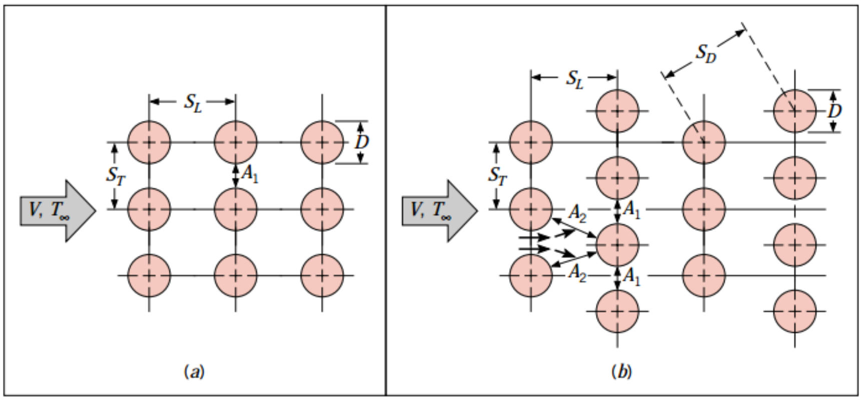
$$\left[\begin{array}{l} 0.71 \lesssim Pr \lesssim 380 \\ 3.5 \lesssim Re_D \lesssim 7.6 \times 10^4 \\ 1.0 \lesssim (\mu/\mu_s) \lesssim 3.2 \end{array}\right]$$

All properties except μ_s are evaluated at T_∞,

Flow across Bank of Tubes



Two Arrangements:



Tube arrangements in a bank. (*a*) Aligned. (*b*) Staggered.

Generally, we wish to know the average heat transfer coefficient for the entire tube bundle. For airflow across tube bundles composed of 10 or more rows ($N_L \geq 10$), Grimison has obtained a correlation of the form:

where C_1 and mare listed in Table 7.5 and $Re_{D,\max} \equiv \frac{\rho V_{\max} D}{\mu}$

TABLE 7.5 Constants of Equations 7.58 and 7.60 for airflow over a tube bank of 10 or more rows [19]

	S_T/D							
	1.25		1.5		2.0		3.0	
S_L/D	C_1	m	C_1	m	C_1	m	C_1	m
Aligned								
1.25	0.348	0.592	0.275	0.608	0.100	0.704	0.0633	0.752
1.50	0.367	0.586	0.250	0.620	0.101	0.702	0.0678	0.744
2.00	0.418	0.570	0.299	0.602	0.229	0.632	0.198	0.648
3.00	0.290	0.601	0.357	0.584	0.374	0.581	0.286	0.608
Staggered								
0.600	—	—	—	—	—	—	0.213	0.636
0.900	—	—	—	—	0.446	0.571	0.401	0.581
1.000	—	—	0.497	0.558	—	—	—	—
1.125	—	—	—	—	0.478	0.565	0.518	0.560
1.250	0.518	0.556	0.505	0.554	0.519	0.556	0.522	0.562
1.500	0.451	0.568	0.460	0.562	0.452	0.568	0.488	0.568
2.000	0.404	0.572	0.416	0.568	0.482	0.556	0.449	0.570
3.000	0.310	0.592	0.356	0.580	0.440	0.562	0.428	0.574

It has become common practice to extend this result to other fluids through insertion of the factor $1.13Pr^{1/3}$, in which case

$$\overline{Nu}_D = 1.13 C_1 Re_{D,\max}^m Pr^{1/3}$$

$$\begin{bmatrix} N_L \geq 10 \\ 2000 \lesssim Re_{D,\max} \lesssim 40{,}000 \\ Pr \gtrsim 0.7 \end{bmatrix}$$

All properties appearing in the above equations are evaluated at the film temperature. If $N_L<10$, a correction factor may be applied such that:

$$\overline{Nu}_D\Big|_{(N_L<10)} = C_2 \overline{Nu}_D\Big|_{(N_L \geq 10)}$$

Where C_2 is given in Table 7.6

TABLE 7.6 Correction factor C_2 of Equation 7.61 for $N_L < 10$ [20]

N_L	1	2	3	4	5	6	7	8	9
Aligned	0.64	0.80	0.87	0.90	0.92	0.94	0.96	0.98	0.99
Staggered	0.68	0.75	0.83	0.89	0.92	0.95	0.97	0.98	0.99

The Reynolds number $Re_{D,max}$ for the foregoing correlations is based on the maximum fluid velocity occurring within the tube bank. For the aligned arrangement, Vmax occurs at the transverse plane $A_1$of Figure 7.11a, and from the mass conservation requirement for an incompressible fluid.

$$V_{\max} = \frac{S_T}{S_T - D} V \qquad \text{.......7.62}$$

For the staggered configuration, the maximum velocity may occur at either the transverse plane A_1 or the diagonal plane A_2 of Figure 7.11*b*. It will occur at A_2 if the rows are spaced such that

$$2(S_D - D) < (S_T - D)$$

The factor of 2 results from the bifurcation experienced by the fluid moving from the A_1 to the A_2 planes. Hence $V_{\max}$ occurs at A_2 if

$$S_D = \left[S_L^2 + \left(\frac{S_T}{2} \right)^2 \right]^{1/2} < \frac{S_T + D}{2}$$

in which case it is given by

$$V_{\max} = \frac{S_T}{2(S_D - D)} V \qquad (7.63)$$

If $V_{\max}$ occurs at A_1 for the staggered configuration, it may again be computed from Equation 7.62.

$$V_{\max} = \frac{S_T}{S_T - D} V$$

More recent results have been obtained and Zukauskas [15] has proposed a correlation of the form:

$$\overline{Nu}_D = C\, Re_{D,\max}^{m}\, Pr^{0.36}\left(\frac{Pr}{Pr_s}\right)^{1/4}$$

$$\left[\begin{array}{l} N_L \geq 20 \\ 0.7 \lesssim Pr \lesssim 500 \\ 1000 \lesssim Re_{D,\max} \lesssim 2\times 10^6 \end{array}\right]$$

N_L = No of tubes in row (Longitudinal Direction).

where all properties except Pr_s are evaluated at the arithmetic mean of the fluid inlet and outlet temperatures and the constants C and mare listed in Table 7.7.

TABLE 7.7 Constants of Equation 7.64 for the tube bank in cross flow [15]

Configuration	$Re_{D,\max}$	C	m
Aligned	10–10^2	0.80	0.40
Staggered	10–10^2	0.90	0.40
Aligned	10^2–10^3	Approximate as a single (isolated) cylinder	
Staggered	10^2–10^3		
Aligned $(S_T/S_L > 0.7)$[a]	10^3–2×10^5	0.27	0.63
Staggered $(S_T/S_L < 2)$	10^3–2×10^5	$0.35(S_T/S_L)^{1/5}$	0.60
Staggered $(S_T/S_L > 2)$	10^3–2×10^5	0.40	0.60
Aligned	2×10^5–2×10^6	0.021	0.84
Staggered	2×10^5–2×10^6	0.022	0.84

[a]For $S_T/S_L < 0.7$, heat transfer is inefficient and aligned tubes should not be used.

If N_L<20, a correction factor may be applied such that:

$$\overline{Nu}_D\Big|_{(N_L<20)} = C_2\, \overline{Nu}_D\Big|_{(N_L\geq 20)}$$

where C_2 is given in Table 7.8.

TABLE 7.8 Correction factor C_2 of Equation 7.65 for $N_L < 20$ $(Re_{D,\max} \gtrsim 10^3)$ [15]

N_L	1	2	3	4	5	7	10	13	16
Aligned	0.70	0.80	0.86	0.90	0.92	0.95	0.97	0.98	0.99
Staggered	0.64	0.76	0.84	0.89	0.92	0.95	0.97	0.98	0.99

the fluid moves through the bank, its temperature approaches T_s and $|\Delta T|$ decreases. In Chapter 11 the appropriate form of ΔT is shown to be a *log-mean temperature difference,*

$$\Delta T_{\text{lm}} = \frac{(T_s - T_i) - (T_s - T_o)}{\ln\left(\frac{T_s - T_i}{T_s - T_o}\right)} \tag{7.66}$$

where T_i and T_o are temperatures of the fluid as it enters and leaves the bank, respectively. The outlet temperature, which is needed to determine ΔT_{lm}, may be estimated from

$$\frac{T_s - T_o}{T_s - T_i} = \exp\left(-\frac{\pi D N \bar{h}}{\rho V N_T S_T c_p}\right) \tag{7.67}$$

where N is the total number of tubes in the bank and N_T is the number of tubes in the transverse plane. Once ΔT_{lm} is known, the heat transfer rate per unit length of the tubes may be computed from

$$q' = N(\bar{h}\pi D \Delta T_{\text{lm}}) \tag{7.68}$$

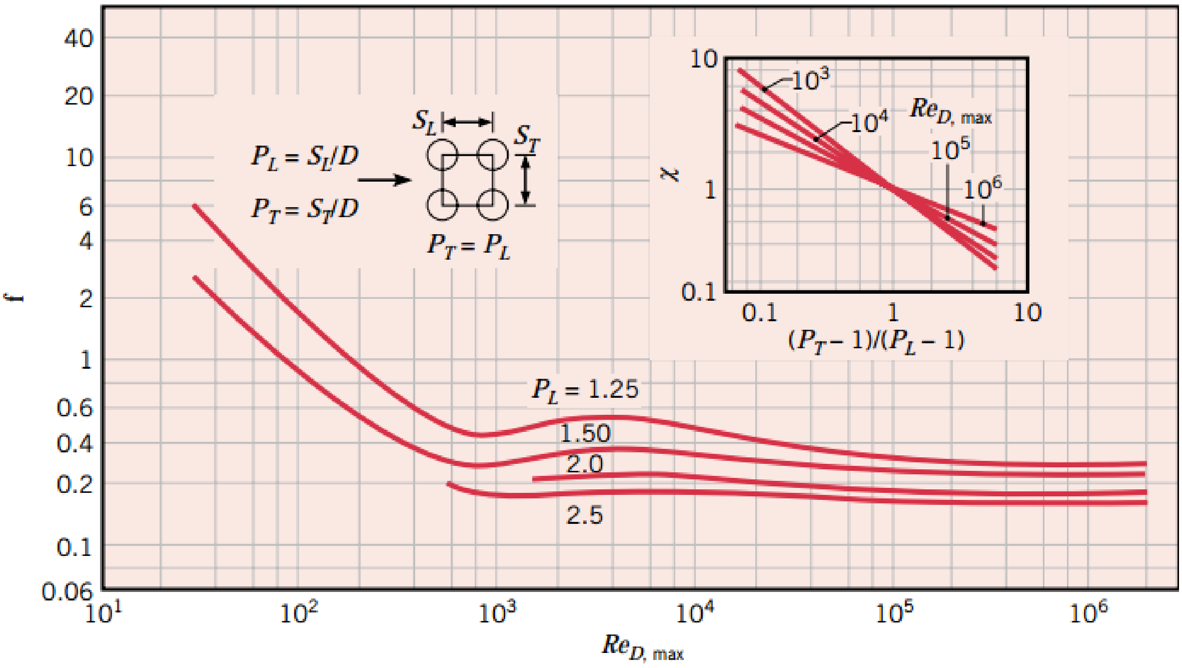


FIGURE 7.13 Friction factor f and correction factor χ for Equation 7.69. In-line tube bundle arrangement [15]. Used with permission.

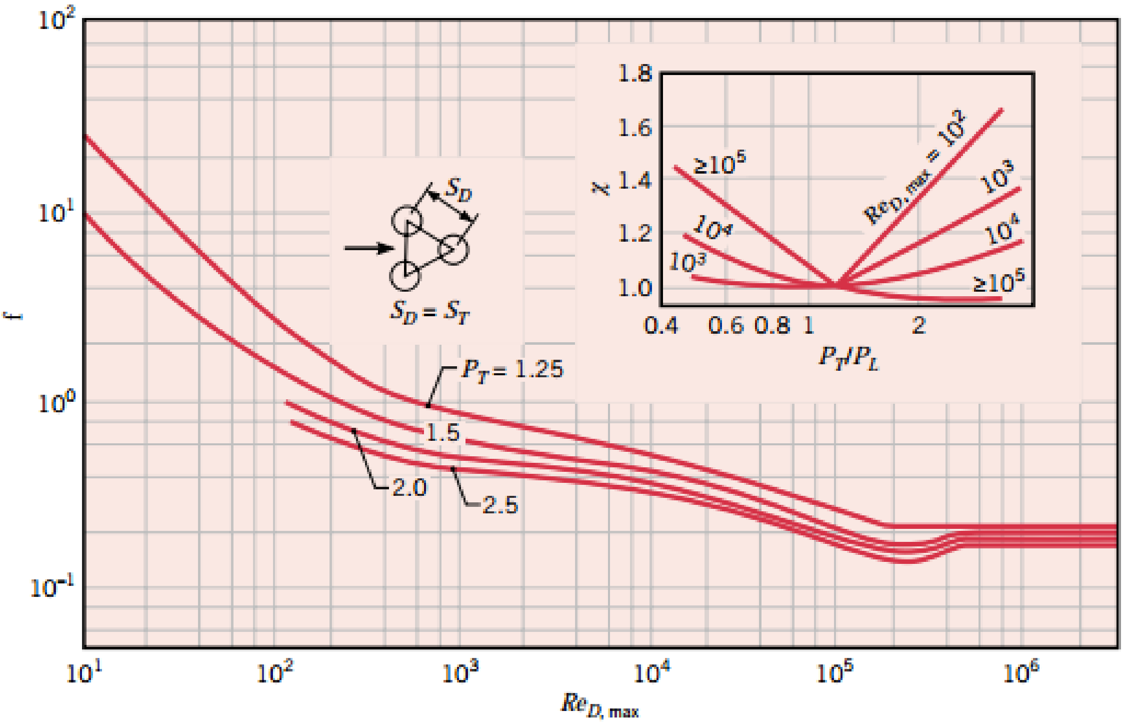
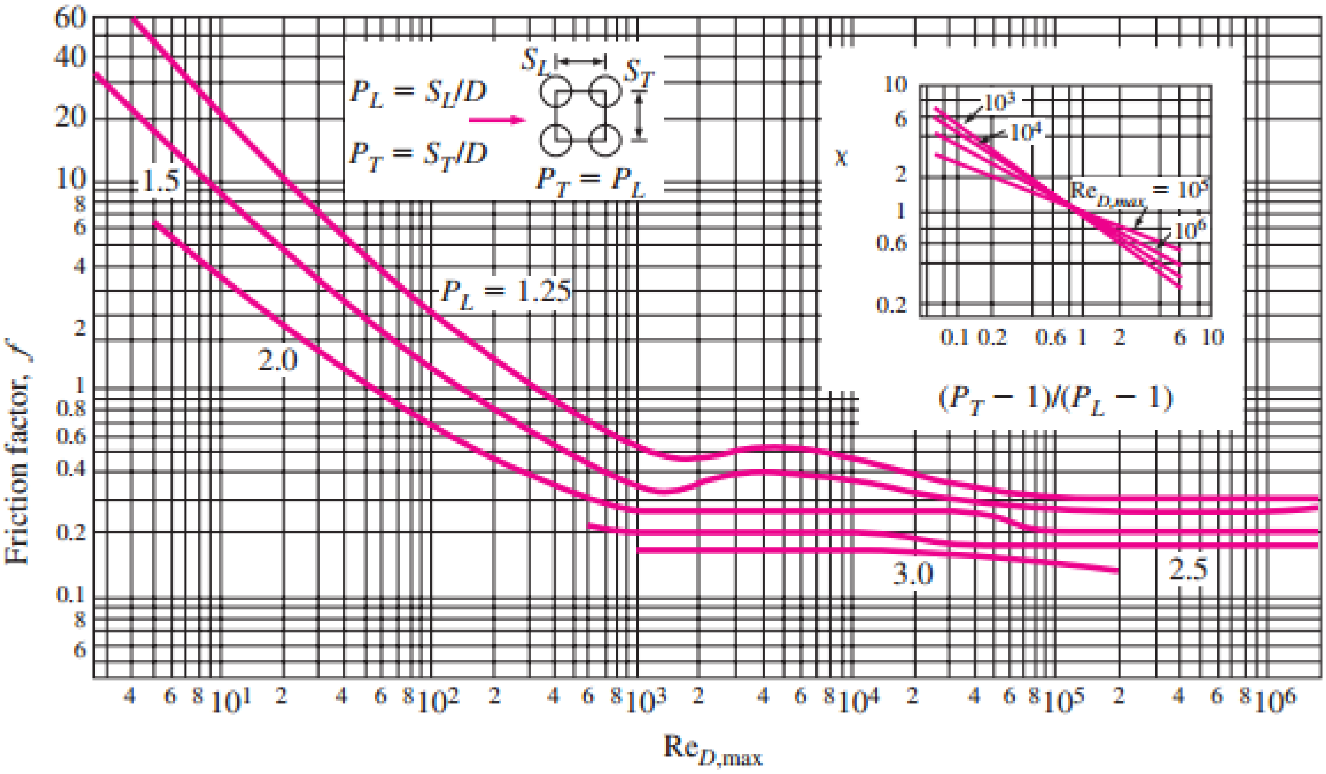


FIGURE 7.14 Friction factor f and correction factor χ for Equation 7.69. Staggered tube bundle arrangement [15]. Used with permission.

$$\Delta p = N_L \chi \left(\frac{\rho V_{max}^2}{2} \right) f \tag{7.69}$$

The friction factor f and the correction factor χ are plotted in Figures 7.13 and 7.14. Figure 7.13 pertains to a square, in-line tube arrangement for which the dimensionless longitudinal and transverse pitches, $P_L \equiv S_L/D$ and $P_T \equiv S_T/D$, respectively, are equal. The correction factor χ, plotted in the inset, is used to apply the results to other in-line arrangements. Similarly, Figure 7.14 applies to a staggered arrangement of tubes in the form of an equilateral triangle ($S_T = S_D$), and the correction factor enables extension of the results to other staggered arrangements. Note that the Reynolds number appearing in Figures 7.13 and 7.14 is based on the maximum fluid velocity V_{max}.



$P_L = S_L/D$
$P_T = S_T/D$
S_L
S_T
$P_T = P_L$
1.5
$P_L = 1.25$
2.0
3.0
2.5
Friction factor, f
$Re_{D,max}$
χ
10^3
10^4
$Re_{D,max} = 10^5$
10^6
$(P_T - 1)/(P_L - 1)$