

Chapter 4: Effect of Noise on Analog Communication Systems



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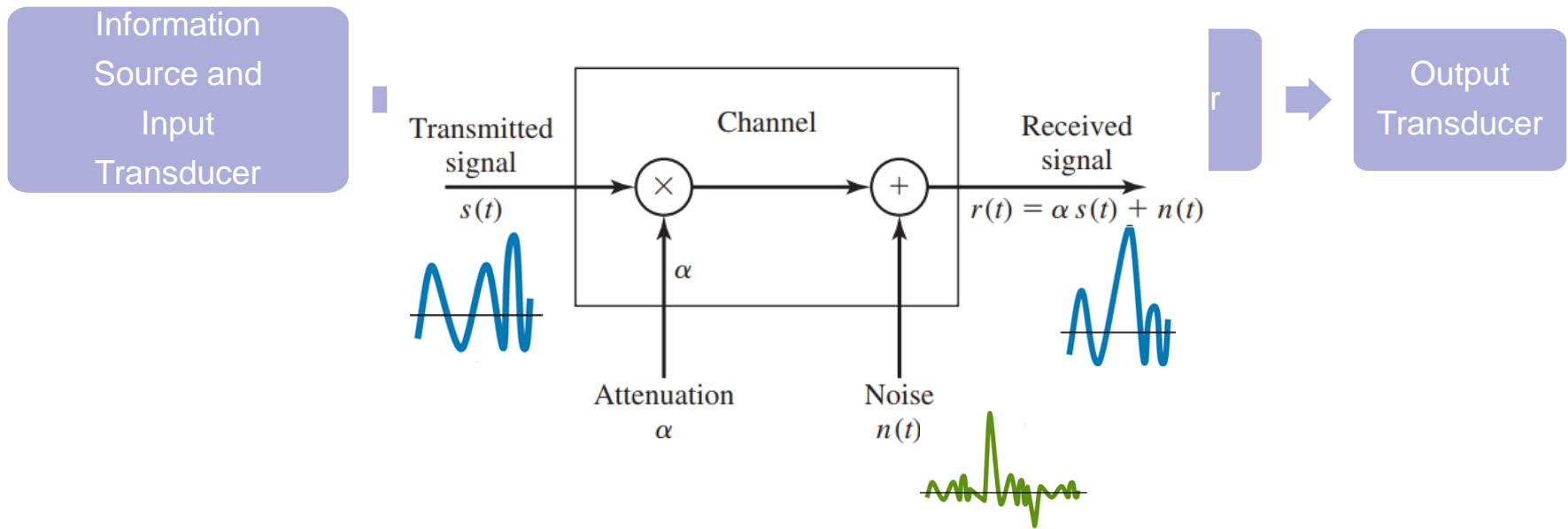
Undergraduate Program
School of Electrical and Computer Engineering

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Transmission Losses and Noise



- Two dominant factors that limit the performance of the system are *Attenuation* and *Additive Noise*.

- Lossy channels

- Electronic devices that are used to filter and amplify, wired media



Transmission Losses

- The amount of signal attenuation generally depends on
 - The physical medium,
 - The frequency of operation,
 - The distance between the transmitter and the receiver

$$\mathcal{L} = \frac{P_t}{P_r}$$

or, in decibels, as

$$\mathcal{L}_{dB} = 10 \log_{10} \mathcal{L} = 10 \log_{10} P_t - 10 \log_{10} P_r$$

- **Example 1:**

Determine the transmission loss for a 10-Km and a 20-Km coaxial cable if the loss/Km is 2 dB at the frequency operation.



Cont....

- Example 2:

In line of site wireless systems the transmission loss is given as

$$\mathcal{L} = \left(\frac{4\pi d}{\lambda} \right)^2$$

: also known as free-space path loss

Determine the free-space path loss for a signal transmitted at $f = 900\text{MHz}$ over distances of 1 Km and 2 Km.

- *How will the increases in distance affect the received signal strength?*
- *If the frequency is changed to 1800MHz what will be the new transmission loss at 1 Km and 2 Km? how does it change?*

Challenge:

If a mobile tower antenna is transmitting at 16dBw, at what maximum distance will your phone still can be able to receive a signal? (take the transmission loss = free-space path loss, GSM transmission at 900MHz)



Noise

- Noise is undesired or unwanted signal
 - **Thermal noise**, which is due to the random motion of electrons in a wire which creates an extra signal not originally sent by the transmitter.
 - **Induced noise** comes from sources such as motors and appliances. These devices act as a sending antenna, and the transmission medium acts as the receiving antenna.
 - **Crosstalk** is the effect of one wire on the other. One wire acts as a sending antenna and the other as the receiving antenna.
 - **Impulse noise** is a spike (a signal with high energy in a very short time) that comes from power lines, lightning,...



Review : Random Processes

- Important concept in modeling the randomness of noise.
- Random process (signal) $X(t)$ can be viewed as collection on random variables $\{X(t_1), X(t_2), X(t_3)\dots\}$ at $t_1, t_2, t_3 \dots$. All $t \in \mathbb{R}$
- Can we quantity it?
 - Yes, with Statistical descriptions
- Can it be filtered?
 - Yes , LTI filters



Cont....

- Autocorrelation Function

$R_{xx}(t, \tau)$ is defined by $R_{xx}(t, \tau) = E[X(t)X(t + \tau)]$

Where the expectation $E[X(t)] = \int_{-\infty}^{\infty} x f_{X(t)}(x) dx$

- If $R_{xx}(t, \tau) = R_{xx}(\tau)$ and $E[X(t)] = m_x$.. (the autocorrelation is dependent on τ & mean is constant) then the process is **Wide sense stationary**.

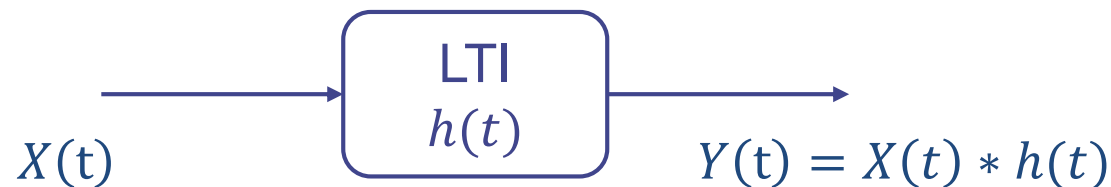
Power Spectral density (w\Hz): $S_{xx}(f) = \int_{-\infty}^{\infty} R_{xx}(\tau) e^{-j2\pi f\tau} d\tau$

Signal Power: $\int_{-\infty}^{\infty} S_{xx}(f) d(f)$



Cont...

- For WSS Process $X(t)$,



- Proof: Ex!!

$$R_{yy}(\tau) = h(\tau)R_{xx}(\tau)h(-\tau)$$

$$S_{yy}(f) = \int_{-\infty}^{\infty} R_{yy}(\tau) e^{-j2\pi f\tau} d\tau = S_{xx}(f) * |H(f)|^2$$



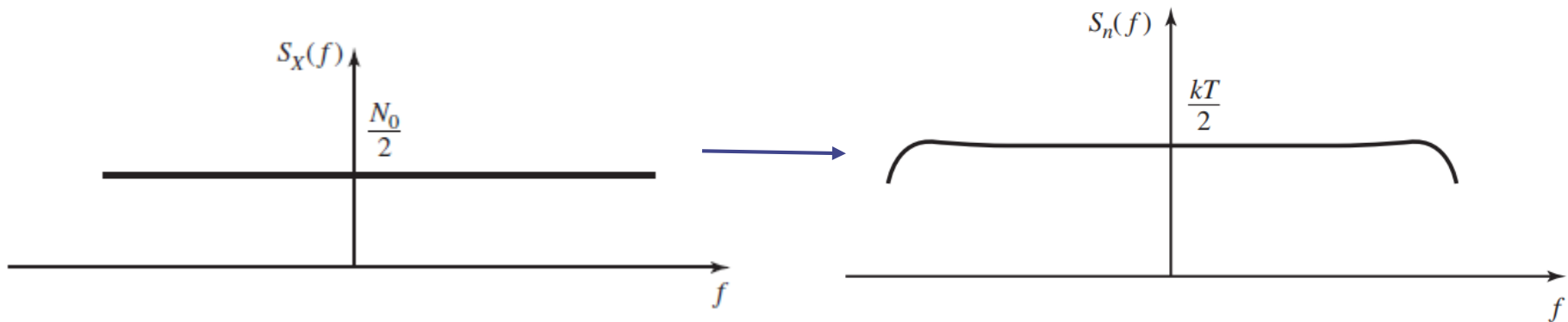
Ex!

- For a given random signal $X(t) = A\cos(2\pi f_c t +$



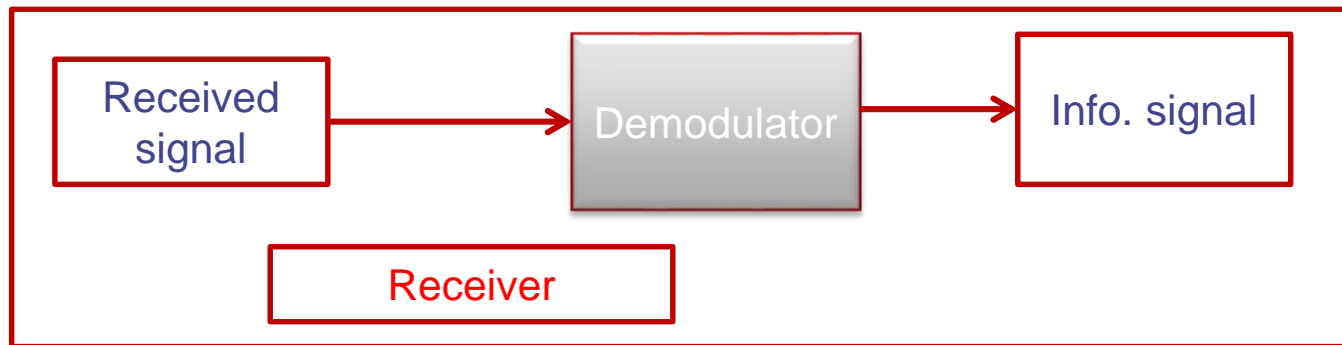
Noise characterization

- Thermal noise can be closely modeled as Gaussian Process.
- Noise process exist in all frequency components
 - appear with equal power; i.e., the power-spectral density is a constant for all frequencies → white noise
- Thus, we can refer it as **Additive white Gaussian Noise**
- The spectral density of AWGN where $N_o = \kappa \cdot T$



Effect of Noise at the Receiver

- Main function: to recover the message from the received signal
 - *Somewhat inverse of the transmitter function*
- Demodulate, decode and extract the information content of the received signal.
- Operates in the presence of noise, interference, attenuation
 - Hence, some distortions are unavoidable
- Some other functions: filtering, suppression of noise and interference
- Error detection and correction.



Signal to Noise Ratio (SNR)

- To measure the quality of a system the SNR is often used.
- It indicates the strength of the signal w.r.t. the noise power in the system.

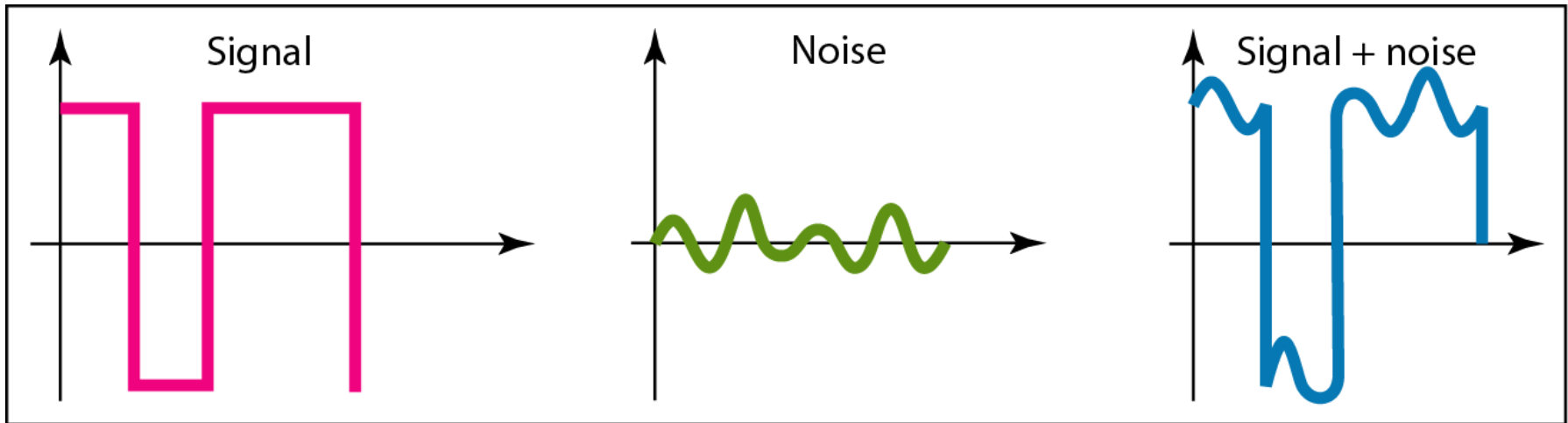
$$SNR = \frac{P_s}{N}$$

- It is usually given in dB and referred to as SNR_{dB} .

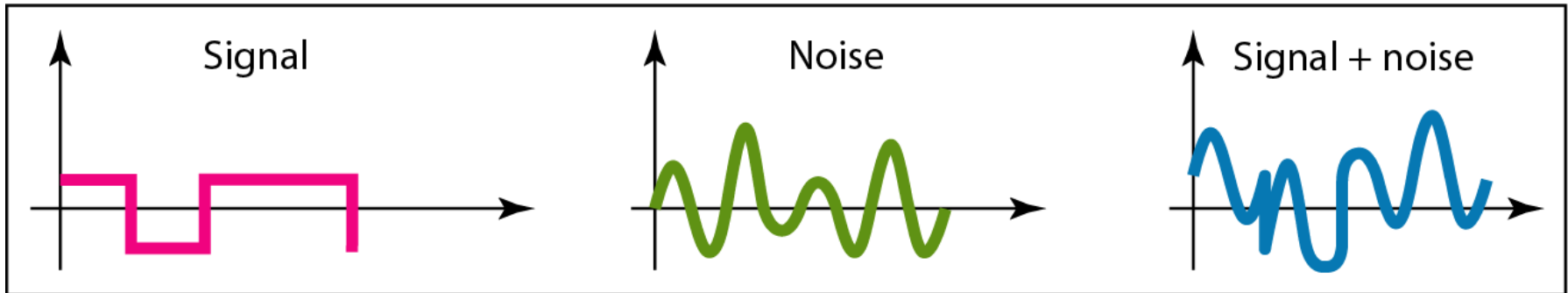
$$SNR_{dB} = 10 \log_{10} SNR = 10 \log_{10} P_s - 10 \log_{10} N_o$$

Where $N = \kappa \cdot Bw \cdot T = \text{Thermal Noise power}$





a. Large SNR



b. Small SNR



Effect of Noise on a Baseband System

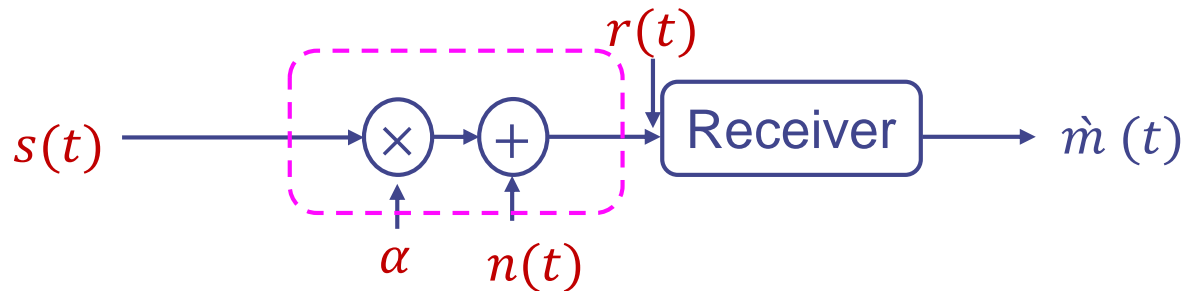
- Since baseband systems serve as a basis for comparison of various modulation systems, we begin with a noise analysis of a baseband system.
- In this case, there is no carrier demodulation to be performed.
- The receiver consists only of an ideal lowpass filter with the bandwidth W .
- The noise power at the output of the receiver, for a white noise input, is

$$P_{n_0} = \int_{-W}^W \frac{N_0}{2} df = N_0 W$$

- If we denote the received power by P_R , the baseband SNR is given by
- $$\left(\frac{S}{N} \right)_b = \frac{P_R}{N_0 W}$$



Effect of Noise on Linear-Modulation Systems



- The transmitted signal, $s(t) =$
 - $A_c m(t) \cos 2\pi f_c t \dots$ **DSB-SC**
 - $A_c (1 + m(t)) \cos 2\pi f_c t \dots$ **C. AM**
 - $A_c m(t) \cos 2\pi f_c t \mp A_c \hat{m}(t) \sin 2\pi f_c t \dots$ **SB-SC**
- The received signal at the output of the receiver noise-limiting filter : Sum of this signal and filtered noise

$$r(t) = \alpha S(t) + n(t)$$



Cont....

- The filtered noise process can be expressed in terms of its in-phase and quadrature components as

$$\begin{aligned}n(t) &= A(t) \cos[2\pi f_c t + \theta(t)] = A(t) \cos \theta(t) \cos(2\pi f_c t) - A(t) \sin \theta(t) \sin(2\pi f_c t) \\ &= n_c(t) \cos(2\pi f_c t) - n_s(t) \sin(2\pi f_c t)\end{aligned}$$

(where $n_c(t)$ is in-phase component and $n_s(t)$ is quadrature component)



Effect of Noise on DSB-SC AM

- Received signal (Adding the filtered noise to the modulated signal)

$$\begin{aligned}r(t) &= \alpha S(t) + n(t) = u(t) + n(t) \\ &= Am(t) \cos(2\pi f_c t) + n_c(t) \cos(2\pi f_c t) - n_s(t) \sin(2\pi f_c t)\end{aligned}$$

- Demodulate the received signal by first multiplying $r(t)$ by a locally generated sinusoid $\cos(2\pi f_c t + \phi)$, where ϕ is the phase of the sinusoid.
- Then passing the product signal through an ideal lowpass filter having a bandwidth W .



Cont....

- The multiplication of $r(t)$ with $\cos(2\pi f_c t + \phi)$ yields

$$\begin{aligned} r(t) \cos(2\pi f_c t + \phi) &= u(t) \cos(2\pi f_c t + \phi) + n(t) \cos(2\pi f_c t + \phi) \\ &= Am(t) \cos(2\pi f_c t) \cos(2\pi f_c t + \phi) \\ &\quad + n_c(t) \cos(2\pi f_c t) \cos(2\pi f_c t + \phi) - n_s(t) \sin(2\pi f_c t) \cos(2\pi f_c t + \phi) \\ &= \frac{1}{2} Am(t) \cos(\phi) + \frac{1}{2} A_c m(t) \cos(4\pi f_c t + \phi) \\ &\quad + \frac{1}{2} [n_c(t) \cos(\phi) + n_s(t) \sin(\phi)] + \frac{1}{2} [n_c(t) \cos(4\pi f_c t + \phi) - n_s(t) \sin(4\pi f_c t + \phi)] \end{aligned}$$

- The lowpass filter rejects the double frequency components and passes only the lowpass components.

$$y(t) = \frac{1}{2} Am(t) \cos(\phi) + \frac{1}{2} [n_c(t) \cos(\phi) + n_s(t) \sin(\phi)]$$



Cont....

- The effect of a phase difference between the received carrier and a locally generated carrier at the receiver is a drop equal to $\cos^2(\phi)$ in the received signal power.
- If we assume that $\phi = 0$

$$m'(t) = \frac{1}{2} \left[A m(t) + n_c(t) \right]$$



Cont....

- Therefore, at the receiver output, the message signal and the noise components are additive and we are able to define a meaningful SNR. The message signal power is given by

$$P_o = \frac{A^2}{4} P_M$$

- power P_M is the content of the message signal
- The noise power is given by

$$P_{n_0} = \frac{1}{4} P_{n_c} = \frac{1}{4} P_n$$

- The power content of $n(t)$ can be found by noting that it is the result of passing $n_w(t)$ through a filter with bandwidth W .



Cont....

- Therefore, the power spectral density of $n(t)$ is given by

$$S_n(f) = \begin{cases} \frac{N_0}{2} & |f - f_c| < W \\ 0 & \text{otherwise} \end{cases}$$

- The noise power is

$$P_n = \int_{-\infty}^{\infty} S_n(f) df = \frac{N_0}{2} \times 4W = 2WN_0$$

- Now we can find the output SNR as

$$\left(\frac{S}{N} \right)_0 = \frac{P_0}{P_{n_0}} = \frac{\frac{A^2}{4} P_M}{\frac{1}{4} 2WN_0} = \frac{A^2 P_M}{2WN_0}$$

- The received signal power, given by

$$P_R = A^2 P_M / 2.$$



Cont....

- The output SNR for DSB-SC AM may be expressed as

$$\left(\frac{S}{N}\right)_{0_{DSB}} = \frac{P_R}{N_0 W}$$

- which is identical to baseband SNR
- In DSB-SC AM, the output SNR is the same as the SNR for a baseband system
 \Rightarrow DSB-SC AM does not provide any SNR improvement over a simple baseband communication system



Example 3:

In a broadcasting communication system the transmitter power is 40 KW, the channel attenuation is 80 dB, and the noise power-spectral density is 10^{-10} W/Hz. The message signal has a bandwidth of 10^4 Hz.

- Find the output SNR if the modulation is DSB.
- *Find the pre-detection SNR (SNR in $r(t) = ku(t) + n(t)$)*



Effect of Noise on SSB AM

- Input to the demodulator

$$\begin{aligned}r(t) &= A m(t) \cos(2\pi f_c t) \mp A \hat{m}(t) \sin(2\pi f_c t) + n(t) \\ &= A m(t) \cos(2\pi f_c t) \mp A \hat{m}(t) \sin(2\pi f_c t) + n_c(t) \cos(2\pi f_c t) - n_s(t) \sin(2\pi f_c t) \\ &= [A m(t) + n_c(t)] \cos(2\pi f_c t) + [\mp A \hat{m}(t) - n_s(t)] \sin(2\pi f_c t)\end{aligned}$$

- Assumption :

- Demodulation with an ideal phase reference ($\phi = 0$).

- Hence, the output of the lowpass filter is the in-phase component (with a coefficient of $\frac{1}{2}$) of the preceding signal.

$$m'(t) = \frac{1}{2} [A m(t) + n_c(t)]$$



Cont...

- Parallel to our discussion of DSB, we have

$$\begin{aligned} P_o &= \frac{A^2}{4} P_M \\ P_{n_0} &= \frac{1}{4} P_{n_c} = \frac{1}{4} P_n \\ P_n &= \int_{-\infty}^{\infty} S_n(f) df = \frac{N_0}{2} \times 2W = WN_0 \end{aligned}$$

$\left(\frac{S}{N}\right)_0 = \frac{P_o}{P_{n_0}} = \frac{A^2 P_M}{WN_0}$

$P_R = P_U = A^2 P_M$

$\left(\frac{S}{N}\right)_{0_{SSB}} = \frac{P_R}{N_0 W} = \left(\frac{S}{N}\right)_b$

- The signal-to-noise ratio in an SSB system is equivalent to that of a DSB system.



Effect of Noise on Conventional AM

- Received signal at the input to the demodulator

$$\begin{aligned}r(t) &= A[1 + am_n(t)]\cos(2\pi f_c t) + n(t) \\ &= A [1 + am_n(t)]\cos(2\pi f_c t) + n_c(t)\cos(2\pi f_c t) - n_s(t)\sin(2\pi f_c t) \\ &= \left[A [1 + am_n(t)] + n_c(t) \right]\cos(2\pi f_c t) - n_s(t)\sin(2\pi f_c t)\end{aligned}$$

- a is the modulation index
- $m_n(t)$ is normalized
- If a synchronous demodulator is employed, the situation is basically similar to the DSB case, except that we have $1 + am_n(t)$ instead of $m(t)$.
- After mixing and lowpass filtering
$$\hat{m}(t) = \frac{1}{2} \left[A am_n(t) + n_c(t) \right]$$



Cont...

- Received signal power

$$P_R = \frac{A^2}{2} [1 + a^2 P_{M_n}]$$

- Now we can derive the output SNR as

$$\begin{aligned} \left(\frac{S}{N} \right)_{0_{AM}} &= \frac{\frac{1}{4} A^2 a^2 P_{M_n}}{\frac{1}{4} P_{n_c}} = \frac{A^2 a^2 P_{M_n}}{2N_0W} = \frac{a^2 P_{M_n}}{1 + a^2 P_{M_n}} \frac{\frac{A^2}{2} [1 + a^2 P_{M_n}]}{N_0W} \\ &= \frac{a^2 P_{M_n}}{1 + a^2 P_{M_n}} \frac{P_R}{N_0W} = \frac{a^2 P_{M_n}}{1 + a^2 P_{M_n}} \left(\frac{S}{N} \right)_b = \eta \left(\frac{S}{N} \right)_b \end{aligned}$$

- η denotes the modulation efficiency
- Since $a^2 P_{M_n} < 1 + a^2 P_{M_n}$ the SNR in conventional AM is always smaller than the SNR in a baseband system.



EX:

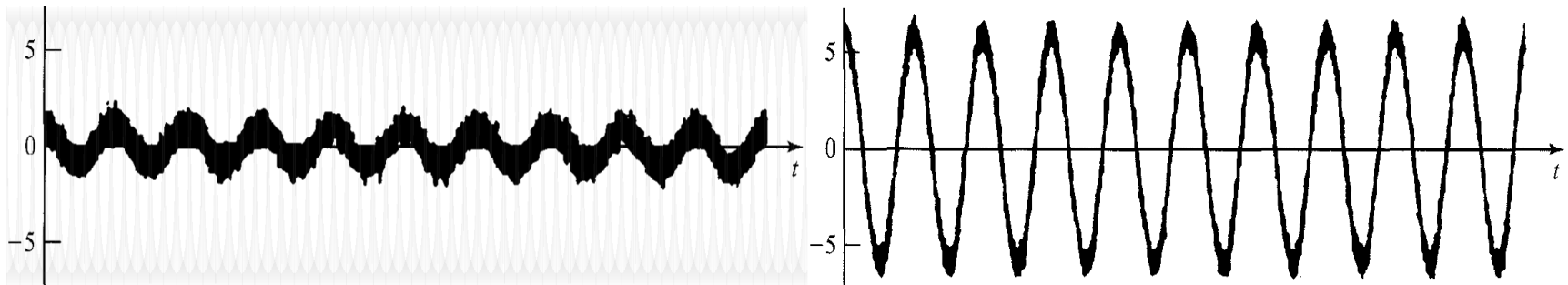
From Example 3;

- Find the output SNR if the modulation is conventional AM with a modulation index of 0.85 and normalized message power of 0.2.



Effect of Noise on Angle Modulation

- A figure shown in below is the effect of additive noise on zero crossings of two FM signals, one with high power and the other with low power.
- From the previous discussion and also from the figure it should be clear that the effect of noise in an FM system is different from that for an AM system.
- **We also observe that the effect of noise in a low-power FM system is more severe than in a high-power FM system.**
 - In a low power signal, noise causes more changes in the zero crossings.
- The analysis that we present next verifies our intuition based on these observations.

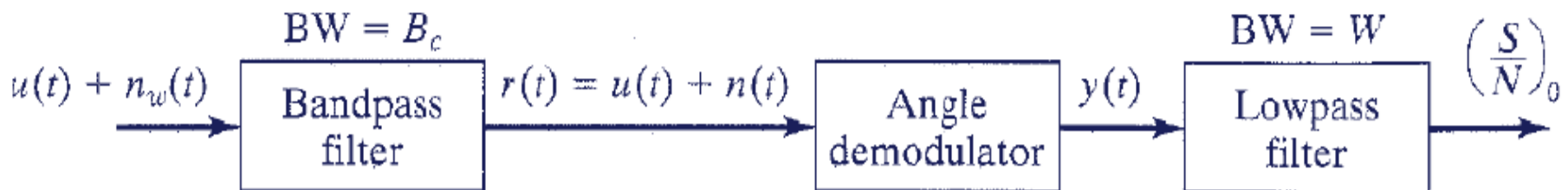


Cont....

- The receiver for a general angle-modulated signal is shown in below
- The angle-modulated signal is represented as

$$u(t) = A_c \cos(2\pi f_c t + \phi(t)) = \begin{cases} A_c \cos\left(2\pi f_c t + 2\pi k_f \int_{-\infty}^t m(\tau) d\tau\right) & FM \\ A_c \cos(2\pi f_c t + k_p m(t)) & PM \end{cases}$$

- The AWGN $n_w(t)$ is added to $u(t)$, and the result is passed through a noise-limiting filter whose role is to remove the out-of-band noise.
- The bandwidth of this filter is equal to that of the modulated signal
- Therefore, it passes the modulated signal without distortion.
- However, it eliminates the out-of-band noise.
- Hence, the noise output of the filter is a filtered noise denoted by $n(t)$.



Cont....

- The output of this filter is

$$r(t) = u(t) + n(t) = u(t) + n_c(t) \cos(2\pi f_c t) - n_s(t) \sin(2\pi f_c t)$$

- A precise analysis is complicated due to the nonlinearity of demodulation .
- Let us assume that the signal power is much higher than the noise power.
- Then, the bandpass noise is represented as

$$n(t) = \sqrt{n_c^2(t) + n_s^2(t)} \cos\left(2\pi f_c t + \arctan \frac{n_s(t)}{n_c(t)}\right) = V_n(t) \cos(2\pi f_c t + \Phi_n(t))$$

- where $V_n(t)$ and $\Phi_n(t)$ represent the envelope and the phase of the bandpass noise process, respectively.



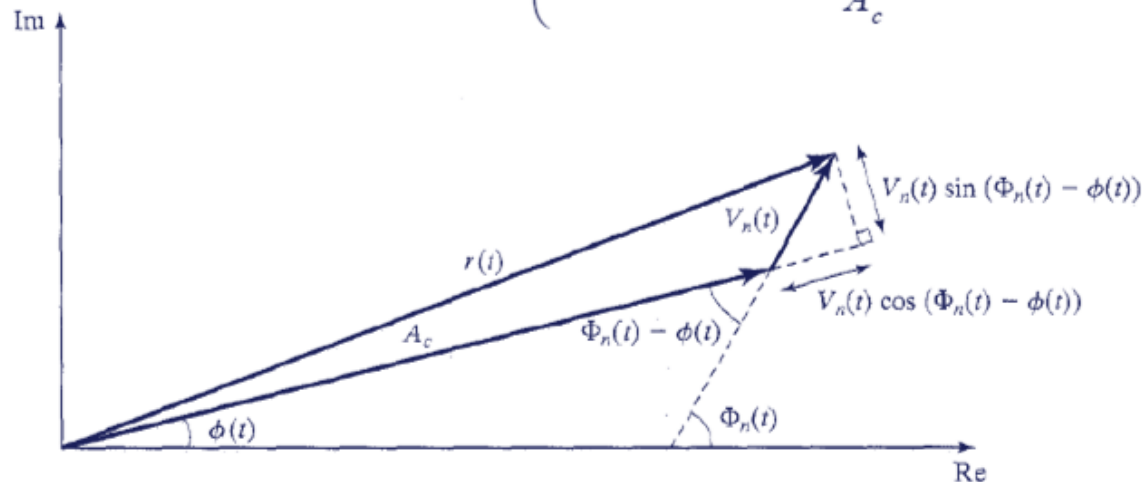
- Assume that the signal is much larger than the noise, that is,

$$P(V_n(t) \ll A_c) \approx 1$$

- The phasor diagram of signal and noise are shown in below.
- From this figure, it is obvious that we can write

$$r(t) \approx [A_c + V_n(t) \cos(\Phi_n(t) - \phi(t))] \cos \left(2\pi f_c t + \phi(t) + \arctan \frac{V_n(t) \sin(\Phi_n(t) - \phi(t))}{A_c + V_n(t) \cos(\Phi_n(t) - \phi(t))} \right)$$

$$\approx [A_c + V_n(t) \cos(\Phi_n(t) - \phi(t))] \cos \left(2\pi f_c t + \phi(t) + \frac{V_n(t)}{A_c} \sin(\Phi_n(t) - \phi(t)) \right)$$



- Noting that $\phi(t) = \begin{cases} k_p m(t), & PM \\ 2\pi k_f \int_{-\infty}^t m(\tau) d\tau, & FM \end{cases}$

- We see that the output of the demodulator is given by

$$y(t) = \begin{cases} \phi(t) + \frac{V_n(t)}{A_c} \sin(\Phi_n(t) - \phi(t)) & PM \\ \frac{1}{2\pi} \frac{d}{dt} \left[\phi(t) + \frac{V_n(t)}{A_c} \sin(\Phi_n(t) - \phi(t)) \right] & FM \end{cases}$$

$$= \begin{cases} k_p m(t) + \frac{V_n(t)}{A_c} \sin(\Phi_n(t) - \phi(t)) & PM \\ k_f m(t) + \frac{1}{2\pi} \frac{d}{dt} \frac{V_n(t)}{A_c} \sin(\Phi_n(t) - \phi(t)) & FM \end{cases} = \begin{cases} k_p m(t) + Y_n(t) & PM \\ k_f m(t) + \frac{1}{2\pi} \frac{d}{dt} Y_n(t) & FM \end{cases}$$

□ where we define

$$Y_n(t) \stackrel{\text{def}}{=} \frac{V_n(t)}{A_c} \sin(\Phi_n(t) - \phi(t))$$



Cont....

$$y(t) = \begin{cases} k_p m(t) + \frac{V_n(t)}{A_c} \sin(\Phi_n(t) - \phi(t)) & PM \\ k_f m(t) + \frac{1}{2\pi} \frac{d}{dt} \frac{V_n(t)}{A_c} \sin(\Phi_n(t) - \phi(t)) & FM \end{cases} = \begin{cases} k_p m(t) + Y_n(t) & PM \\ k_f m(t) + \frac{1}{2\pi} \frac{d}{dt} Y_n(t) & FM \end{cases}$$

- The first term in above equation is the desired signal component.
- The second term is the noise component.
- The noise component is inversely proportional to the signal amplitude A_c .
- Hence, the higher the signal level, the lower the noise level.



$$S_{Y_n}(f) = (a^2 + b^2)S_{n_c}(f) = \frac{S_{n_c}(f)}{A_c^2}$$

- $S_{n_c}(f)$ is the power spectral density (psd) of the in-phase component of the filtered noise given :
- Therefore

$$S_{Y_n}(f) = \begin{cases} \frac{N_0}{A_c^2} & |f| < \frac{B_c}{2} \\ 0 & \textit{otherwise} \end{cases}$$



Cont...

- This equation provides an expression for the power spectral density of the filtered noise at *the front end of the receiver*.
- After demodulation, another filtering is applied; this reduces the noise bandwidth to W , which is the bandwidth of the message signal.
- Note that in the case of FM modulation, the process $Y_n(t)$ is differentiated and scaled by $1/2\pi$.
- The PSD of the process $(1/2\pi) (dY_n(t)/dt)$ is given by

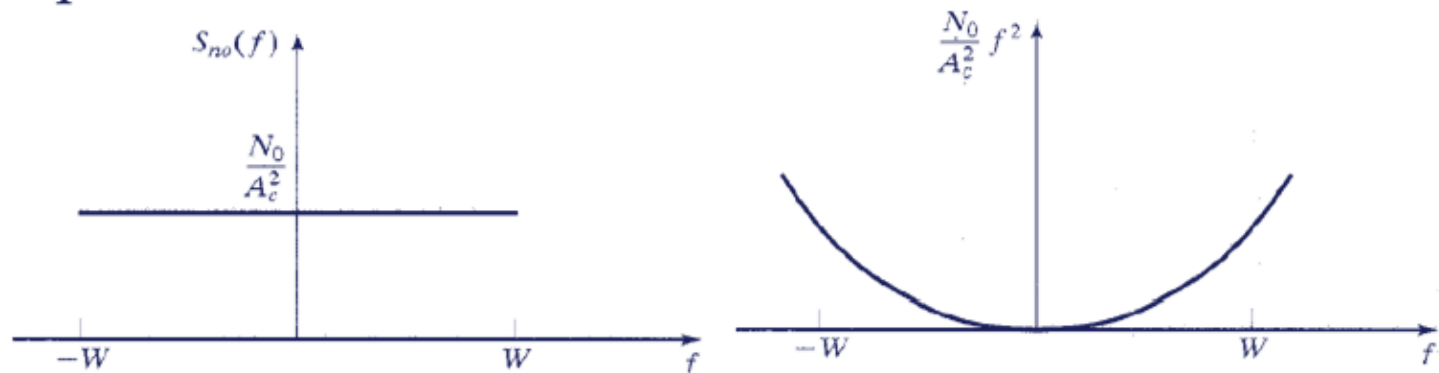
$$\frac{4\pi^2 f^2}{4\pi^2} S_{Y_n}(f) = f^2 S_{Y_n}(f) = \begin{cases} \frac{N_0}{A_c^2} f^2 & |f| < \frac{B_c}{2} \\ 0 & \text{otherwise} \end{cases}$$



- Hence, for $|f| < W$

$$S_{n_0}(f) = \begin{cases} \frac{N_0}{A_c^2} & PM \\ \frac{N_0}{A_c^2} f^2 & FM \end{cases}$$

- Fig. 6.4 shows the power spectrum of the noise component at the output of the demodulator for PM and FM.



Noise power spectrum at demodulator output for $|f| < W$ in (a) PM (b) and (b) FM.

- It is interesting to note that **PM has a flat noise spectrum and FM has a parabolic noise spectrum.**
- Therefore, the effect of noise in FM for higher frequency components is much higher than the effect of noise on lower frequency components.
- The noise power at the output of the lowpass filter is the noise power in the frequency range $[W, +W]$.
- Therefore, it is given by

$$P_{n_0} = \int_{-W}^W S_{n_0}(f) df = \begin{cases} \int_{-W}^W \frac{N_0}{A_c^2} df & PM \\ \int_{-W}^W \frac{N_0}{A_c^2} f^2 df & FM \end{cases} = \begin{cases} \frac{2WN_0}{A_c^2} & PM \\ \frac{2N_0W^3}{3A_c^2} & FM \end{cases}$$



SNR

- the output SNR in angle modulation.
- First, we have the output signal power

$$P_{S_o} = \begin{cases} k_p^2 P_M & PM \\ k_f^2 P_M & FM \end{cases}$$

- Then the SNR, which is defined as

$$\left(\frac{S}{N}\right)_o \stackrel{\text{def}}{=} \frac{P_{S_o}}{P_{n_o}} \longrightarrow \left(\frac{S}{N}\right)_o = \begin{cases} \frac{k_p^2 A_c^2}{2} \frac{P_M}{N_0 W} & PM \\ \frac{3k_f^2 A_c^2}{2W^2} \frac{P_M}{N_0 W} & FM \end{cases}$$

- Noting that $A_c^2/2$ is the received signal power, denoted by P_R , and

$$\begin{cases} \beta_p = k_p \max [|m(t)|] & PM \\ \beta_f = \frac{k_f \max [|m(t)|]}{W} & FM \end{cases} \longrightarrow \left(\frac{S}{N}\right)_o = \begin{cases} P_R \left(\frac{\beta_p}{\max [|m(t)|]} \right)^2 \frac{P_M}{N_0 W} & PM \\ 3P_R \left(\frac{\beta_f}{\max [|m(t)|]} \right)^2 \frac{P_M}{N_0 W} & FM \end{cases}$$



NOTE:

■ Observations

- In both PM and FM, the output SNR is proportional to β^2 . Therefore, increasing β increases the output SNR.
- Increasing β increase the bandwidth (from Carson's rule).
So angle modulation provides a way to trade off bandwidth for transmitted power.



Quiz

1. Briefly explain advantage of angle modulation over AM modulation /list only two.
2. Why do we need to modulate /list only two.
3. An FM modulating signal has 500Hz frequency, 3.2volt amplitude and 6.4 KHz frequency deviation.
 - a. If the baseband signal voltage is now increased to 8.4volt, determine the new frequency deviation, modulation index and Carson's bandwidth.
 - b. If the message signal voltage is raised to 20volts while the audio frequency is dropped to 200Hz, determine the frequency deviation, modulation index and Carson's bandwidth.



Quiz

1. For a given random signal $X(t) = A\cos(2\pi f_c t + \theta)$

