Chapter 3: Angle (Nonlinear) Modulation Techniques

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Angle Modulation

- Angle modulation
  - Frequency modulation (FM)
  - Phase modulation (PM)
- Basic idea
  - Vary frequency (FM) or phase (PM) of a carrier signal according to the message signal
- While AM is (almost) linear, FM or PM is highly nonlinear
  - Linear => Superposition applies

Angle Modulation

- In amplitude modulation
  - Spectrum of the modulated signal is the translated message spectrum
  - Transmission bandwidth never exceeds twice the message bandwidth
- In angle modulation
  - Spectrum of the modulated signal are **not related** in any simple fashion to message spectrum
  - Transmission bandwidth are much greater than twice the message bandwidth

Angle Modulation

- FM/PM provide many advantages
  - Main – noise immunity over AM
- At a cost of
  - Larger bandwidth and
  - Increased system complexity

- Demodulation may be complex, but modern ICs allow cost-effective implementation
- Example: FM radio (high quality, not expensive receivers)

Angle Modulation: Basic Definitions

- Angle-modulated signal (PM or FM) can be expressed as

$$x(t) = A_c \cos(\psi(t))$$

- Phase modulation $\psi(t) = \omega_c t + \varphi(t), \quad \varphi(t) = \Delta\varphi \cdot m(t)$
- $\Delta\varphi$ - phase deviation constant
  - Radians per unit of m(t)
- Frequency modulation $\psi(t) = \omega_c t + \int_0^t \Omega(\tau) d\tau, \quad \Omega(t) = \Delta\Omega \cdot m(t)$
- $\Delta\Omega$ - angular frequency deviation constant
  - In radians per second per unit of m(t)
  - $\Delta\Omega = 2\pi . \Delta f$; in Hz per second per unit of m(t)

Angle Modulation: Basic Definitions

- Max phase deviation: $\Delta\varphi = \mathrm{Max}\left\{\left|\varphi(t)\right|\right\} = \mathrm{Max}\left\{\left|\psi(t) - \omega_c t\right|\right\}$
- Max frequency deviation: $\Delta\Omega = \mathrm{Max}\left\{\left|\Omega(t)\right|\right\} = \mathrm{Max}\left\{\left|\omega(t) - \omega_c\right|\right\}$
- Normalized message signal: $\left|m(t)\right| \le 1$
- Note: deviation is w.r.t. unmodulated value

Angle Modulation: Analysis

- Instantaneous frequency

$$\omega(t) = \frac{d\psi(t)}{dt} = \begin{cases} \omega_c + \frac{d\varphi(t)}{dt} = \omega_c + \Delta\varphi \frac{dm(t)}{dt}, & PM \\ \omega_c + \Omega(t) = \omega_c + \Delta\Omega \cdot m(t), & FM \end{cases}$$

- Instantaneous phase

$$\psi(t) = \int_0^t \omega(\tau) d\tau = \begin{cases} \omega_c t + \varphi(t) = \omega_c t + \Delta\varphi \cdot m(t), & PM \\ \omega_c t + \int_0^t \Omega(\tau) d\tau = \omega_c t + \Delta\Omega \int_0^t m(\tau) d\tau, & FM \end{cases}$$

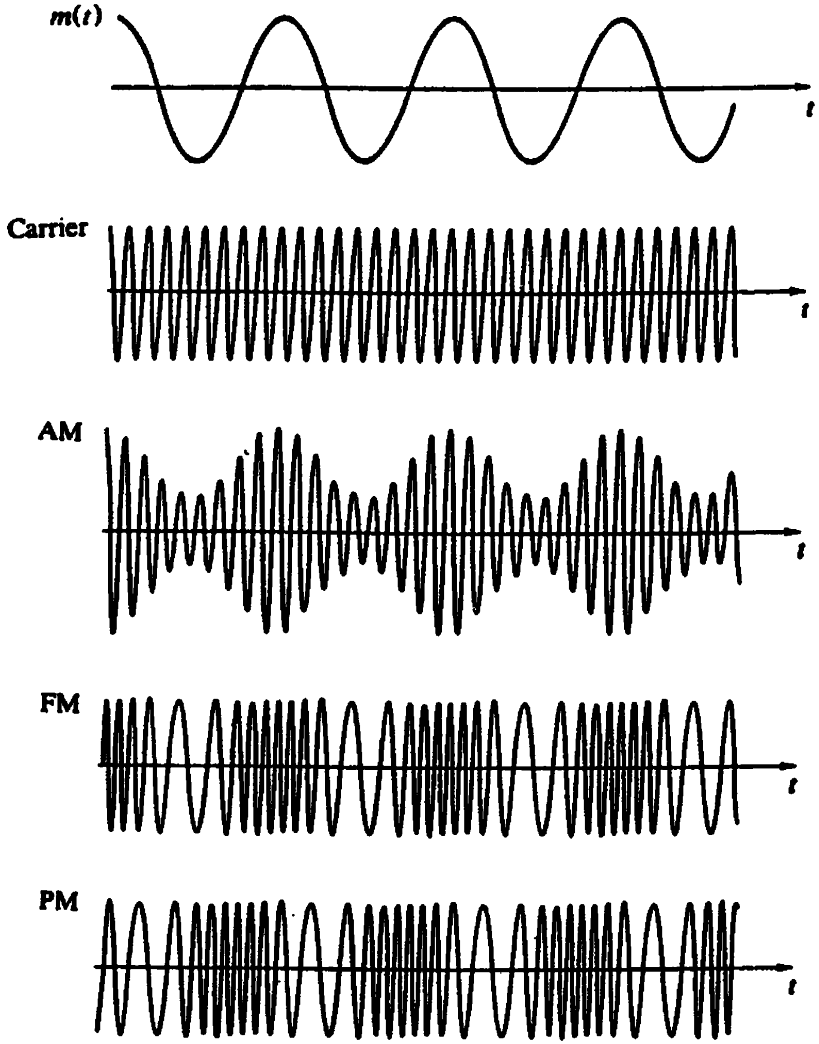
- Effect of mod. signal amplitude: $M(t) = A \cdot m(t), \ \max\left[|m(t)| \right] = 1$

$$\begin{cases} \Delta\varphi = k_p A, & PM \\ \Delta\Omega = 2\pi k_f A & FM \end{cases}$$

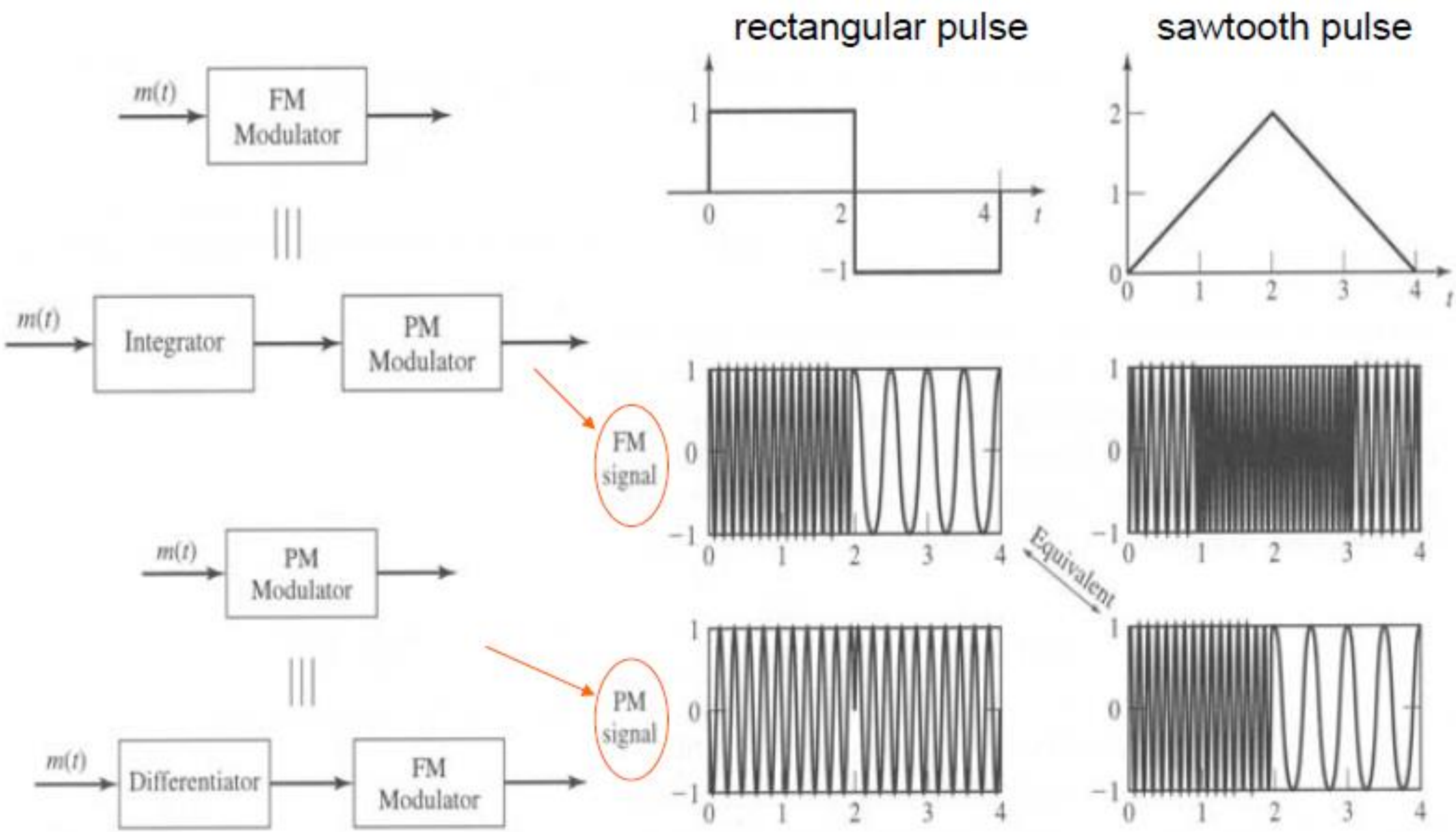
k_f, k_p - modulation constants,

Hz/V & rad./V

Angle Modulation: Analysis



Angle Modulation: Analysis



Example: Sinusoidal Modulating Signal

- Assume that $m(t) = \cos(\omega_m t)$
- Instantaneous phase: $\psi(t) = \begin{cases} \omega_c t + \Delta\varphi \cdot \cos(\omega_m t), & PM \\ \omega_c t + \dfrac{\Delta\Omega}{\omega_m} \sin(\omega_m t), & FM \end{cases}$
- Modulated signal: $x(t) = \begin{cases} A_c \cos\left[\omega_c t + \Delta\varphi \cdot \cos(\omega_m t)\right], & PM \\ A_c \cos\left[\omega_c t + \dfrac{\Delta\Omega}{\omega_m} \sin(\omega_m t)\right], & FM \end{cases}$
- Modulation indices: $\begin{cases} \beta_p = \Delta\varphi, & PM \\ \beta_f = \dfrac{\Delta\Omega}{\omega_m}, & FM \end{cases}$ ⇦ Valid in general case as well, with ω_m -> max. modulating frequency

Spectrum of Angle-Modulated Signal

- Consider sinusoidal modulating signal

$$x(t) = A_c \cos\left[\omega_c t + \beta \cdot \sin(\omega_m t)\right] = \mathrm{Re}\left[A_c e^{j\beta\cdot\sin(\omega_m t)} e^{j\omega_c t}\right]$$

- Complex envelope is expanded in Fourier series

$$C(t) = A_c e^{j\beta\cdot\sin(\omega_m t)} = A_c \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_m t}$$

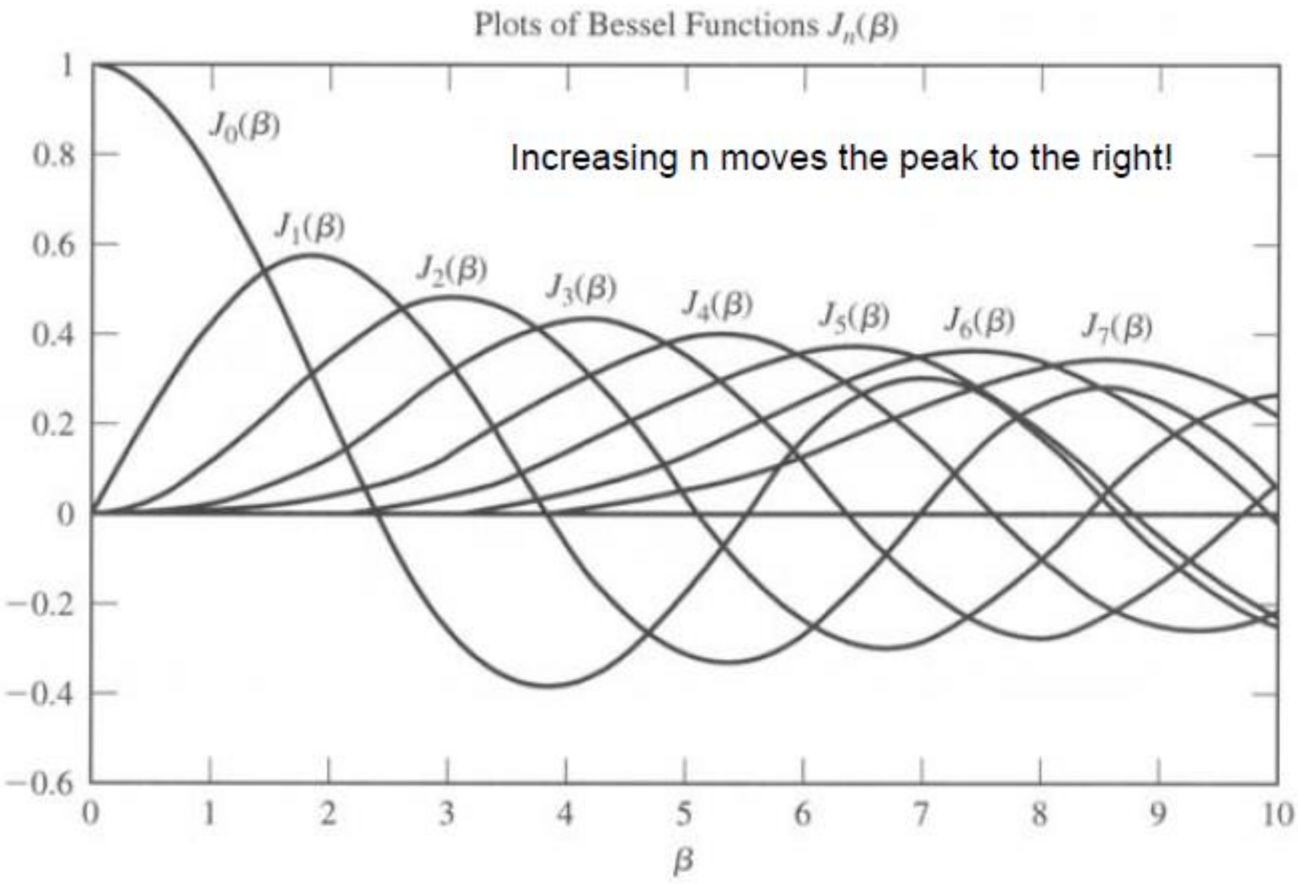
- Expansion coefficients are

$$c_n = \frac{1}{T_m}\int_0^{T_m} e^{j\beta\sin\omega_m t} e^{-jn\omega_m t}\,dt \overset{u=\omega_m t}{=} \frac{1}{2\pi}\int_0^{2\pi} e^{j(\beta\sin u - nu)}\,du = J_n(\beta)$$

- Finally, $$x(t) = \sum_{n=-\infty}^{\infty} A_c J_n(\beta)\cos\left[(\omega_c + n\omega_m)t\right]$$

$J_n(\beta)$ - Bessel function of 1st kind & n-th order, $J_{-n}(\beta) = (-1)^n J_n(\beta)$

Spectrum of Angle-Modulated Signal



Spectrum of Angle-Modulated Signal

n	$\beta=0.1$	$\beta=0.2$	$\beta=0.5$	$\beta=1$	$\beta=2$	$\beta=5$	$\beta=8$	$\beta=10$	n
0	0.998	0.990	0.938	0.765	0.224	-0.178	0.172	-0.246	0
1	0.050	0.100	0.242	0.440	0.577	-0.328	0.235	0.043	1
2	0.001	0.005	0.031	0.115	0.353	0.047	-0.113	0.255	2
3				0.020	0.129	0.365	-0.291	0.058	3
4				0.002	0.034	0.391	-0.105	-0.220	4
5					0.007	0.261	0.186	-0.234	5
6					0.001	0.131	0.338	-0.014	6
7						0.053	0.321	0.217	7
8						0.018	0.223	0.318	8
9						0.006	0.126	0.292	9
10						0.001	0.061	0.207	10
11							0.026	0.123	11
12							0.010	0.063	12
13							0.003	0.029	13
14							0.001	0.012	14
15								0.004	15
16								0.001	16

the last significant spectral component:

$$n = [\beta + 1]$$

Spectrum of Angle-Modulated Signal

- The spectrum consists of a carrier-frequency component plus an infinite number of sidebands components at frequencies $\omega_c \pm n\omega_m$ (n=1,2,3,…)
- The relative amplitude of the spectral lines depend on the value of $J_n(\beta)$
  - The value of $J_n(\beta)$ becomes very small for larger of n
- The number of significant spectral lines (i.e., is having appreciable relative amplitude) is a function of the modulation index β
  - With β <<1, only J_0 and J_1 are significant, so the spectrum will consists of carrier and two sideband lines
  - But if β >>1, there will be many sideband lines

Amplitude Spectrum Sinusoidally Modulated FM Signal

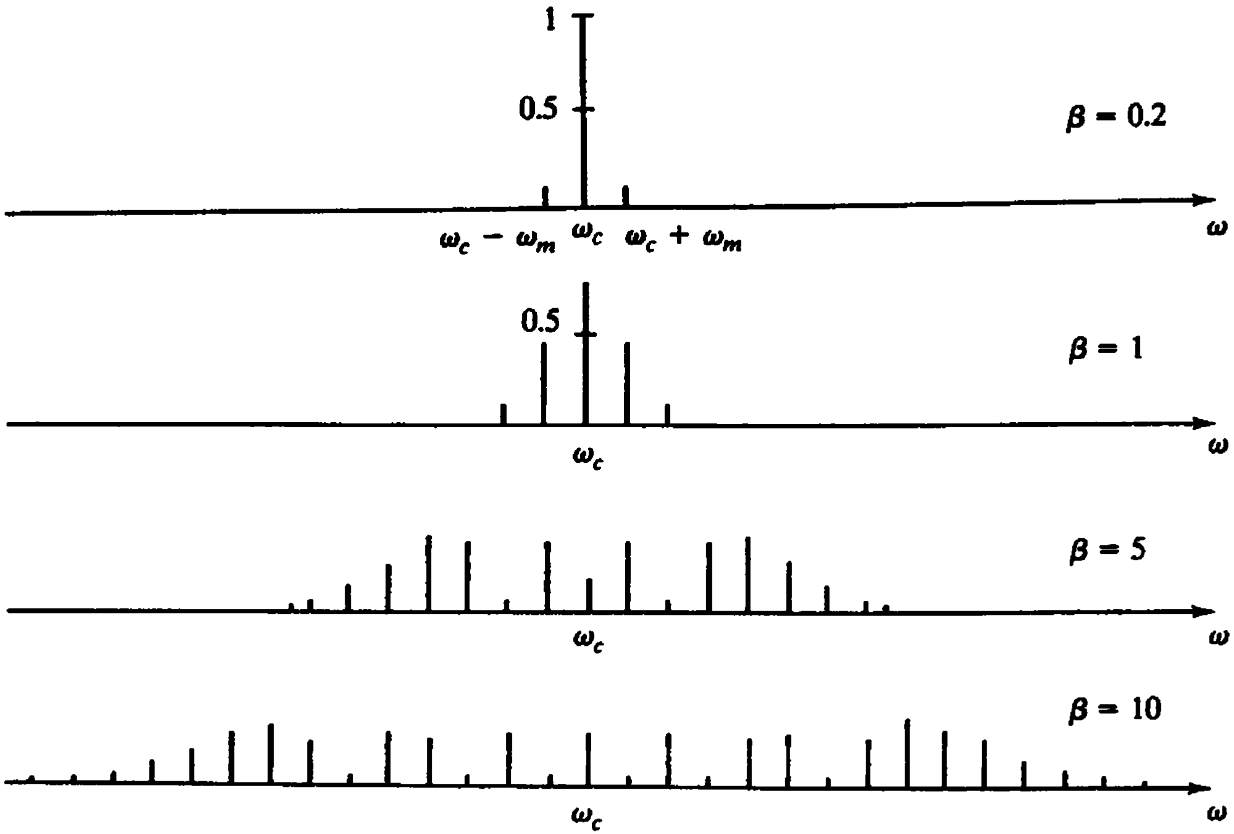
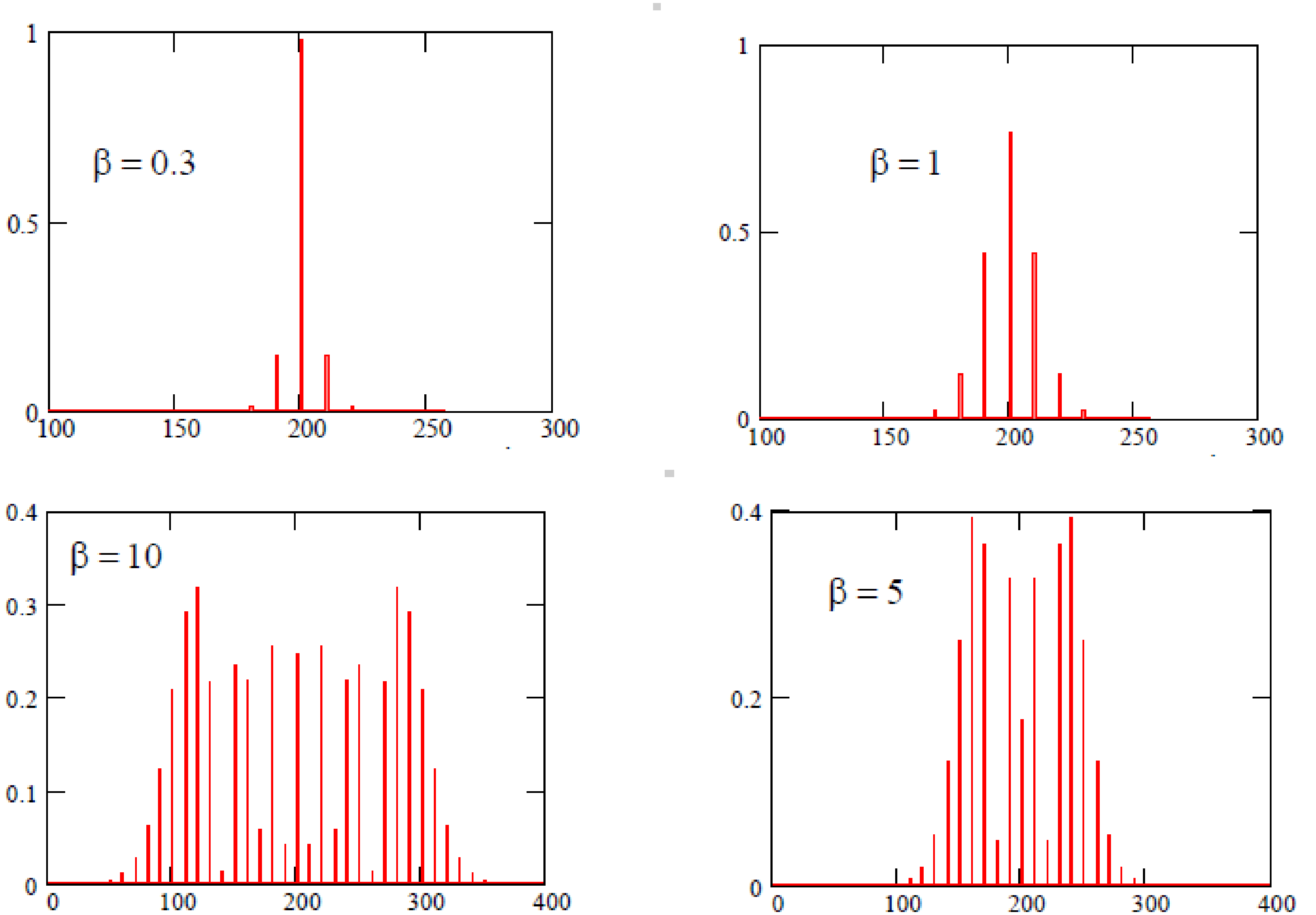


Fig. 4-2 Amplitude spectra of sinusoidally modulated FM signals (ω_m fixed)

Spectrum: Examples



Bandwidth of Angle-Modulated Signal

- Power bandwidth (98% of the power) of angle-modulated signal (Carson's rule)

$$\Delta\omega \approx 2(\beta+1)\omega_m$$

- Power bandwidth of PM and FM signals

$$\Delta\omega \approx 2(\beta+1)\omega_m = \begin{cases} 2(\Delta\varphi+1)\omega_m, & PM \\ 2(\Delta\Omega+\omega_m), & FM \end{cases}$$

- These expressions hold for a general modulating signal as well
  - ω_m - the max. modulating frequency
- Angle modulation with large index expands spectrum!

Arbitrary Modulation

- For arbitrary angle modulating signal m(t) bandwidth limited to ω_M rad/s, the deviation ratio is defined as

$$D = \frac{\text{maximum frequency deviation}}{\text{bandwidth of } m(t)} = \frac{\Delta\omega}{\omega_M}$$

- D plays the same role as the modulation index β plays for sinusoidal modulation
- Replacing β by D and ω_m by ω_M we have

$$W_B \approx 2(D+1)\omega_M$$

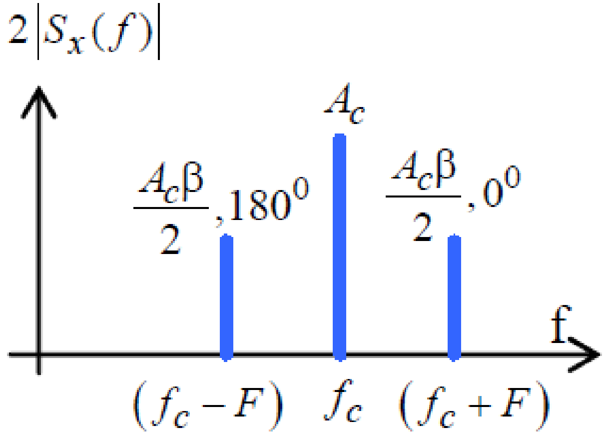
- This expression is referred to as Carson's rule
  - If D<<1, the bandwidth is approximately 2 ω_M : Narrowband signal
  - If D>>1, the bandwidth is approximately 2 $D\omega_M$: Wideband signal

Narrowband Angle Modulation

- Modulation index is low, β << 1
- Modulated signal can be expressed as:

$$x(t) = A_c \cos\left[\omega_c t + \beta \cdot \sin(\omega_m t)\right] =$$
$$= A_c \cos\omega_c t + \frac{A_c\beta}{2}\cos(\omega_c + \omega_m)t - \frac{A_c\beta}{2}\cos(\omega_c - \omega_m)t$$

- The bandwidth (both, PM & FM) is similar to AM signal



$$\Delta\omega \approx 2\omega_m$$

Wideband Angle Modulation

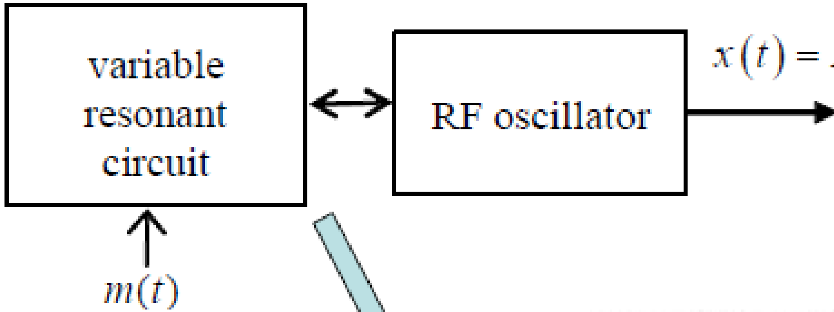
- Modulation index is high, $\beta >> 1$
- The signal bandwidth is:

$$\Delta\omega \approx 2\beta\omega_m = \begin{cases} 2\Delta\varphi \cdot \omega_m, & PM \\ 2\Delta\Omega, & FM \end{cases}$$

- Different for PM and FM!
- Wideband FM
  - The bandwidth is twice the frequency deviation
  - Does not depend on the modulating frequency
- Wideband PM
  - The bandwidth depends on modulating frequency

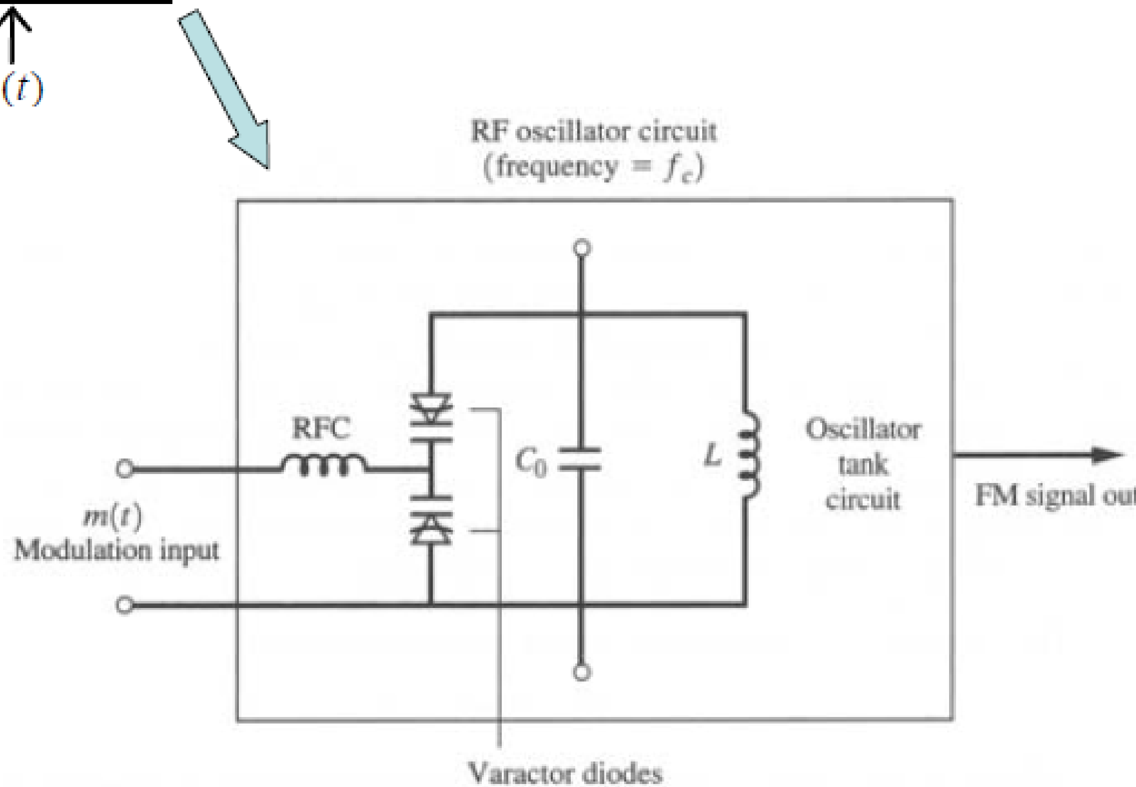
FM Modulator

General principle:



$$x(t) = A_c \cos\left[\omega_c t + \int_0^t \Omega(\tau)d\tau + \varphi_0\right]$$

Practical implementation:

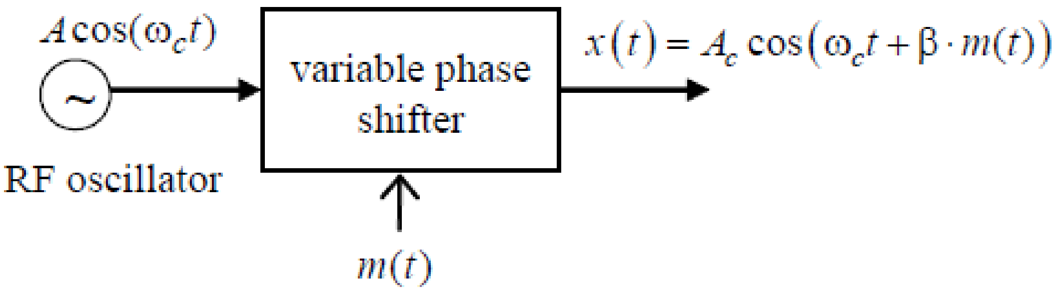


Difficulty: frequency stability.

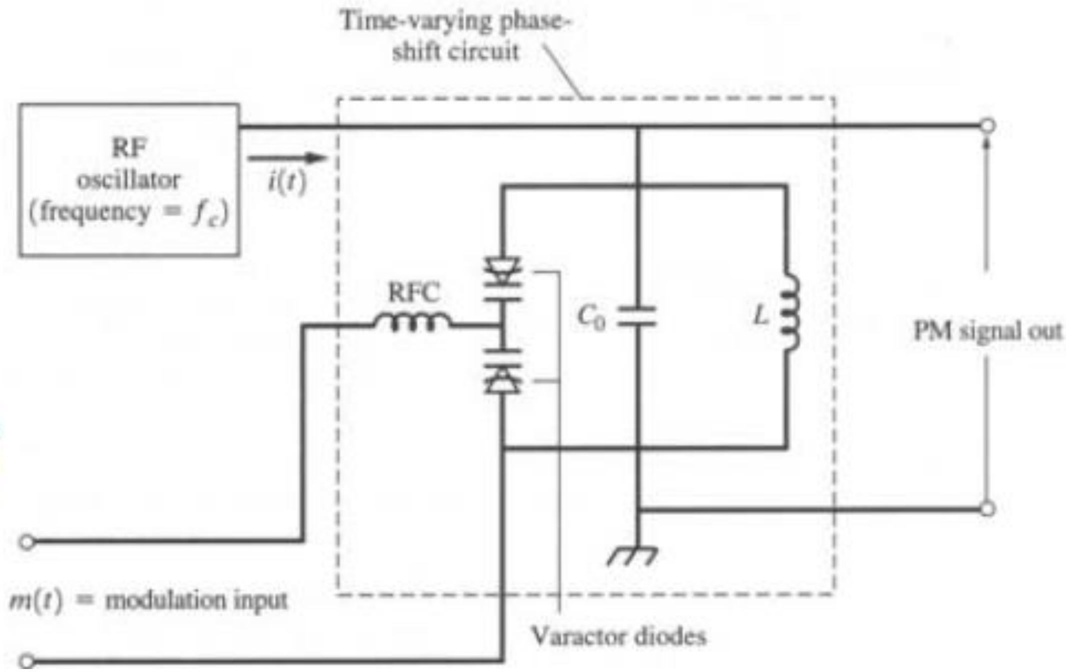
Suitable for narrowband FM only.

PM Modulator

General principle:



Practical implementation:



$$\Delta\varphi \approx k\Delta f, \ \Delta f = f_c - f_0$$

k = modulation constant

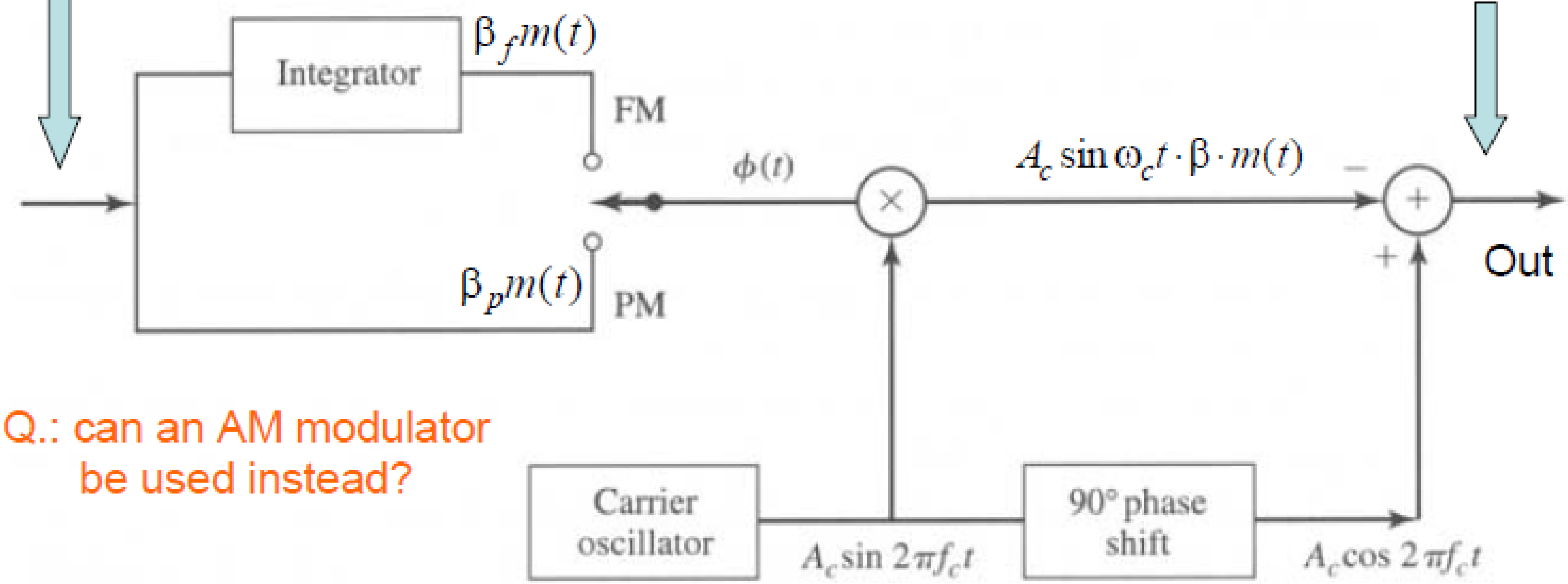
f_0 = resonant frequency

Narrowband Angle Modulator

Small modulation index: $\beta \ll 1$

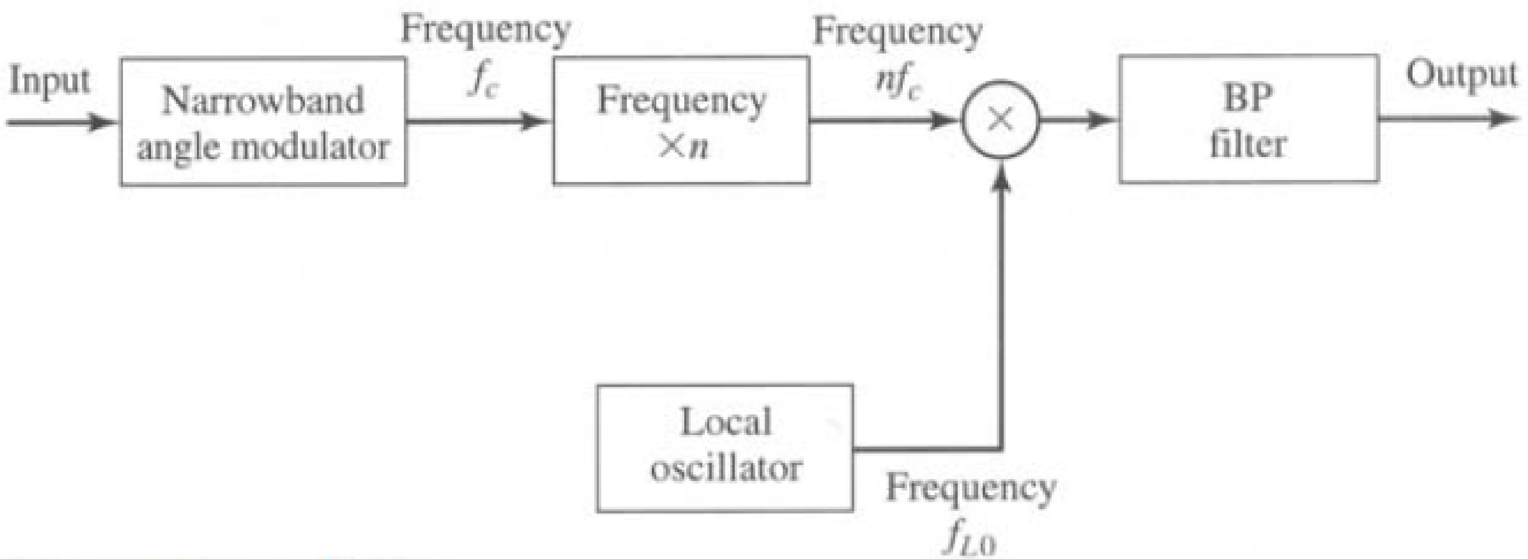
$$x(t) = A_c \cos\left[\omega_c t + \beta \cdot m(t)\right] \approx$$
$$\approx A_c \cos \omega_c t - A_c \sin \omega_c t \cdot \beta \cdot m(t)$$

Message signal

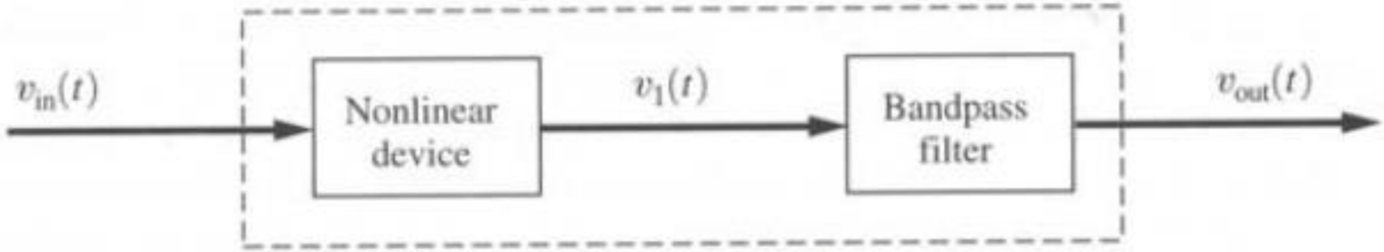


Q.: can an AM modulator be used instead?

Indirect Wideband Angle Modulator

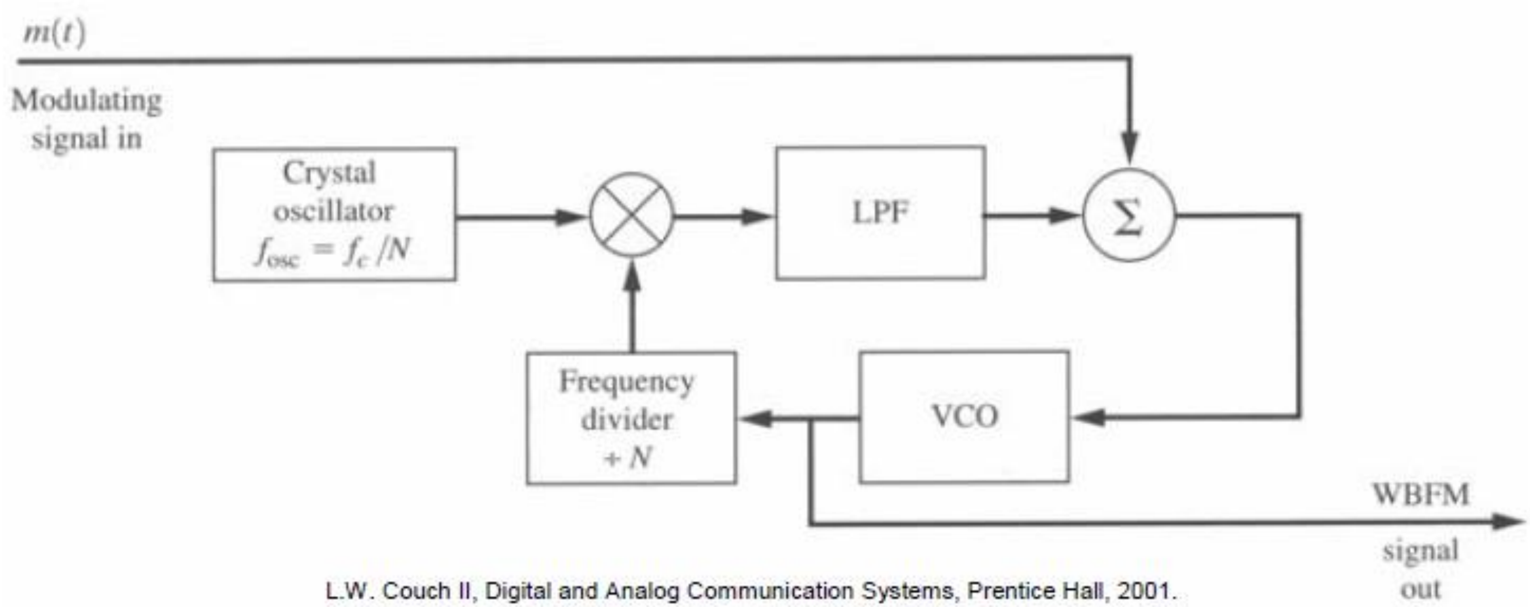


Frequency multiplier:



$$\cos^2\psi(t) = \frac{1}{2}\left[1+\cos\left(2\psi(t)\right)\right] \xrightarrow{\text{BPF}} \frac{1}{2}\cos\left(2\psi(t)\right)$$

Direct Wideband Angle Modulator



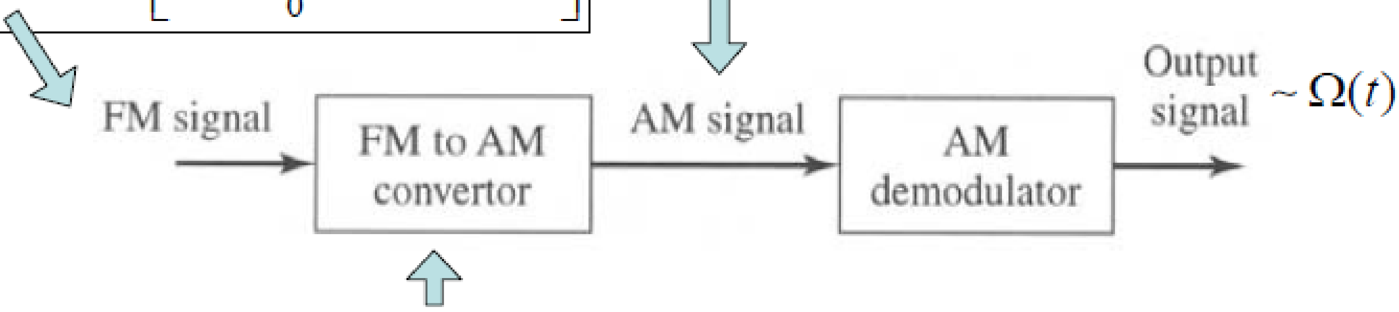
L.W. Couch II, Digital and Analog Communication Systems, Prentice Hall, 2001.

- Explain how it operates
  - Hint: consider it without feedback first
  - Explain why feedback is required
  - Explain why frequency divider is required

FM Demodulators

$$x(t) = A_c \cos\left[\omega_c t + \int_0^t \Omega(\tau)\, d\tau + \varphi_0\right]$$

$$y(t) = A_c\left(H_0 + \alpha\Omega(t)\right)\cos[*]$$



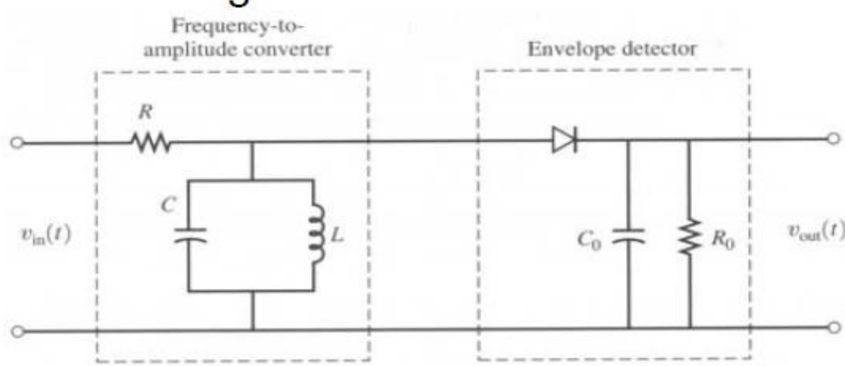
$$|H(f)| = H_0 + \alpha(f - f_c) \quad \text{for } |f - f_c| < \frac{B_c}{2}$$

- FM-to-AM conversion
  - Possible candidate: $|H(f)| = 2\pi f$ (differentiator)

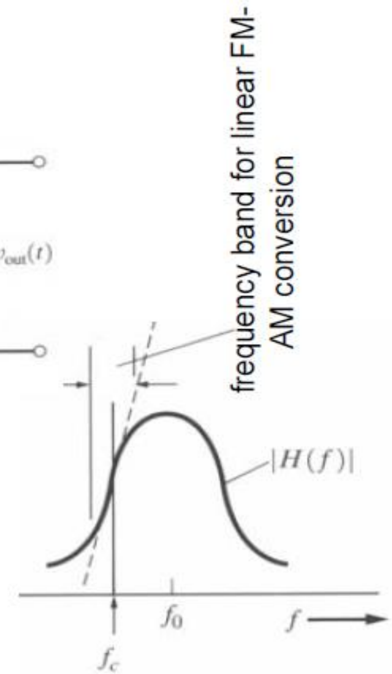
FM Demodulators

- Anther possible candidate: FM Slope Detector

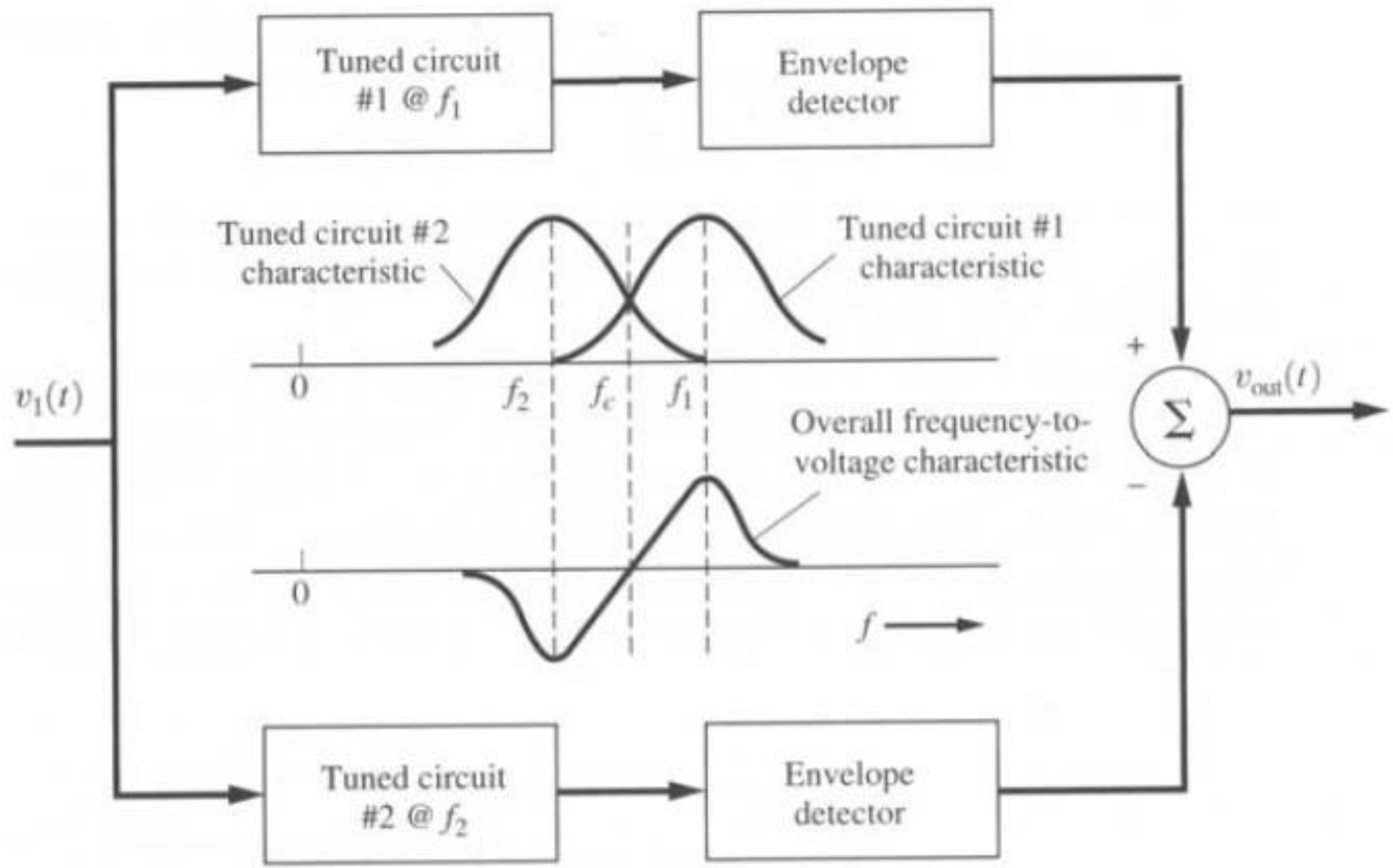
  - Circuit diagram:



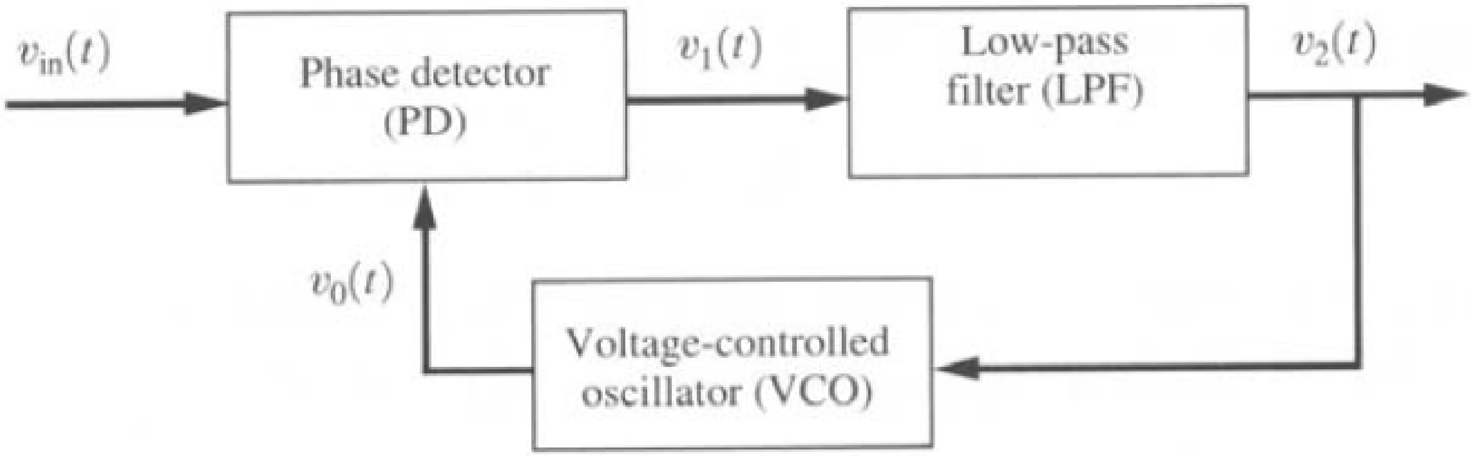
  - Magnitude frequency response:



Balanced Discriminator: Block Diagram



Phased Locked Loop (PLL) Detector



$$v_{in}(t) = A_{in}\sin[\omega_c t + \varphi_{in}(t)]$$

$$v_0(t) = A_0\cos[\omega_c t + \varphi_0(t)]$$

$$\omega_{VCO}(t) = \frac{d}{dt}(\omega_c t + \varphi_0(t)) = \omega_c + \alpha v_2(t)$$

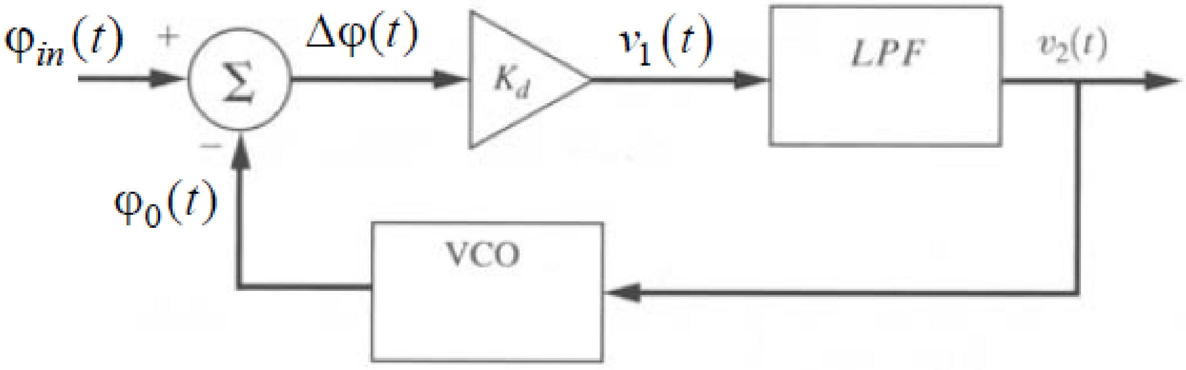
$$v_1(t) = \frac{A_1 A_2}{2}\sin[\varphi_{in}(t) - \varphi_0(t)] + (2\omega_c)\text{term}$$

Informally,

$$\varphi_{in}(t) \approx \varphi_0(t)$$

$$v_2(t) \approx \frac{1}{\alpha}\frac{d}{dt}\varphi_{in}(t)$$

PLL Detector: Linear Model



$$v_1(t) = \frac{A_1 A_2}{2}\sin\left[\varphi_{in}(t) - \varphi_0(t)\right] + +(2\omega_c)\text{term}$$

$\Rightarrow$

$$v_2(t) = \frac{A_1 A_2}{2}\sin\left[\varphi_{in}(t) - \varphi_0(t)\right] \approx \approx \frac{A_1 A_2}{2}\left(\varphi_{in}(t) - \varphi_0(t)\right) = K_d \Delta\varphi(t)$$

$$\frac{d}{dt}\varphi_0(t) = \alpha v_2(t)$$

$\Rightarrow$

$$v_2(t) = \frac{1}{\alpha}\frac{d}{dt}\left[\varphi_{in}(t) - \Delta\varphi(t)\right] \approx \frac{1}{\alpha}\frac{d}{dt}\varphi_{in}(t)$$

Comparison of AM and FM/PM

- Amplitude modulation
  - Is simple (envelope detector) but no noise/interference immunity (low quality)
  - Bandwidth is twice or the same as the modulating signal (no bandwidth expansion)
  - Power efficiency is low for conventional AM
  - DSB-SC & SSB – good power efficiency, but complex circuitry
- FM/PM
  - Spectrum expansion & noise immunity
  - Good quality
  - More complex circuitry
  - However, ICs allow for cost effective implementation

Important Properties of Angle-Modulated Signals: Summary

- FM/PM signal is a nonlinear function of the message
- The signal's bandwidth increases with the modulation index
- The carrier spectral level varies with the modulation index, being 0 in some cases
- Narrowband FM/PM
  - Signal's bandwidth is twice that of the message (same as for AM)
- Amplitude of the FM/PM signal is constant
  - Hence, the power does not depend on the message

Summary

- Angle modulation: PM & FM
- Spectra of angle-modulated signals. Modulation index.
- Narrowband (low-index) & wideband (large-index) modulation.
- Signal bandwidth.
- Relation between PM and FM.
- Generation of angle-modulated signals. Narrowband & wideband modulators.
- Demodulation of PM and FM signals. Slope detector & balanced discriminator. PLL detector.
- Comparison of AM and FM/PM.