Chapter 2: Linear Modulation Techniques

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Undergraduate Program
School of Electrical and Computer Engineering

Chapter Objective

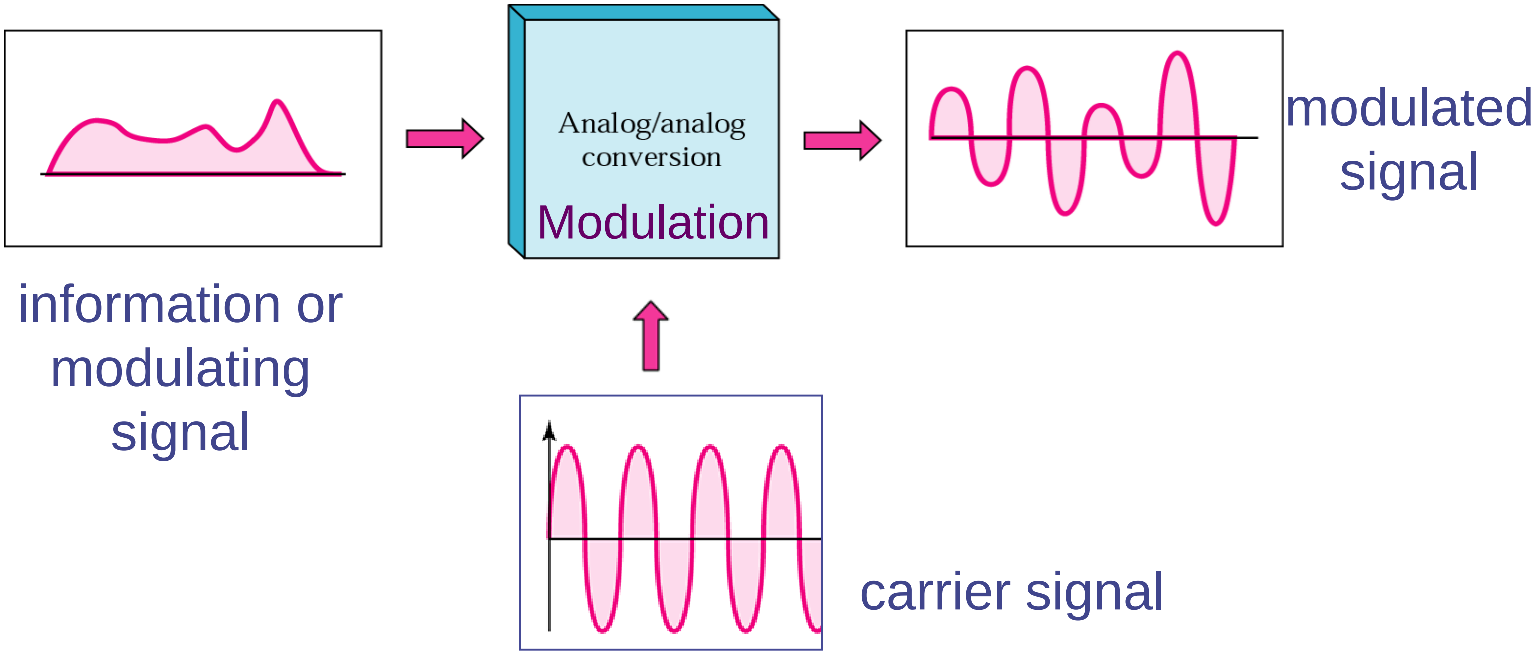
- Study four linear modulation strategies that constitute the amplitude modulation family.
- Identified system complexity and the two primary communication resources—namely, *transmitted power and channel bandwidth*—as the central issues involved in the design of a communication system.

Overview

- Introduction to Modulation
- Conventional AM
- Double-Sideband Suppressed Carrier
- Single Sideband
- Vestigial-sideband

Modulation Process

- Modulation: transforming an information-bearing signal m(t) (lowpass) into a narrowband bandpass signal x(t)
  - The process by which some characteristic of a carrier wave is varied in accordance with an information-bearing signal.
  - m(t) is also called the modulating signal



Analog/analog conversion

Modulation

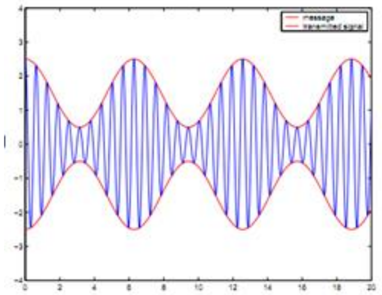
information or modulating signal

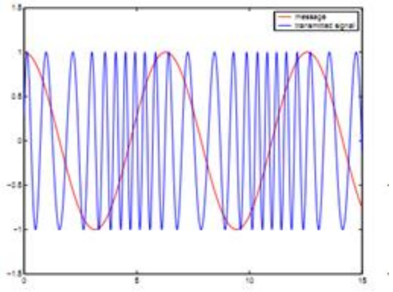
modulated signal

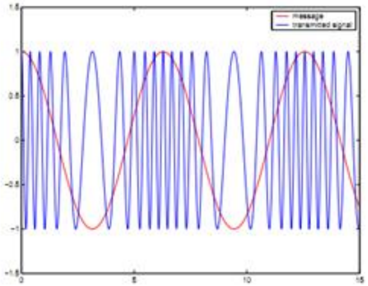
carrier signal

Types of Analog Modulation

- Starting with a sinusoidal signal (carrier) $\boxed{A\cos\left(2\pi ft+\varphi_0\right)}$
  - ***Amplitude Modulation (AM):*** varying the amplitude of the carrier A=A(t) based on the information signal m(t) as done for radio channels that are transmitted in the AM radio band.
  - ***Phase Modulation (PM):*** varying the phase of the carrier $\varphi = \varphi(t)$ based on the information signal.
  - ***Frequency Modulation (FM):*** varying the frequency of the carrier f=f(t) based on the information signal as done for channels transmitted in the FM radio band.







- FM and PM can be viewed as angle modulation

Why do we need Modulation/Demodulation?

- Frequency Translation
  - Narrowband communication
  - Adjustments in the bandwidth is allowed.
- Reception quality improves.
  - Reduction of noise/interference
  - Atmospheric/cable properties
- Frequency characteristics of antennas
  - Antenna size gets reduced.
- Multiplexing
  - Frequency division multiplexing (FDM)
  - Time division multiplexing (TDM)
  - Code division multiplexing (CDM)
  - *Space division Multiplexing (SDM)*

Frequency Translation

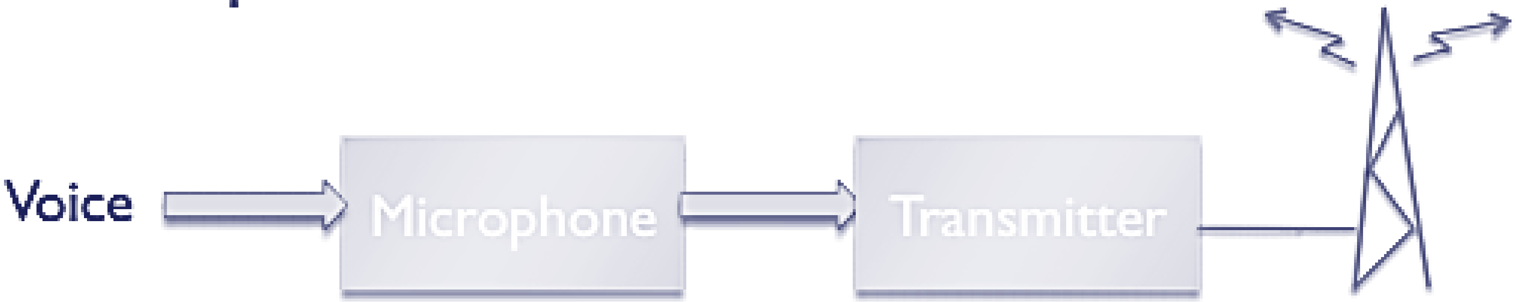
- The modulation process shifts the modulating frequency to a higher frequency, which in turn depends on the carrier frequency, thus producing upper and lower sidebands.

- Hence, signals are *up-converted* from low frequencies to high frequencies and *down-converted* from high frequencies to low frequencies in the receiver.

- The process of converting a frequency or a band of frequencies to another location in the frequency spectrum is called *frequency translation*.

Frequency characteristics of antennas

- Example: Radio transmission



Electric signal,
20 Hz – 20
KHz

Antenna:
Size requirement
> 1/10 wavelength

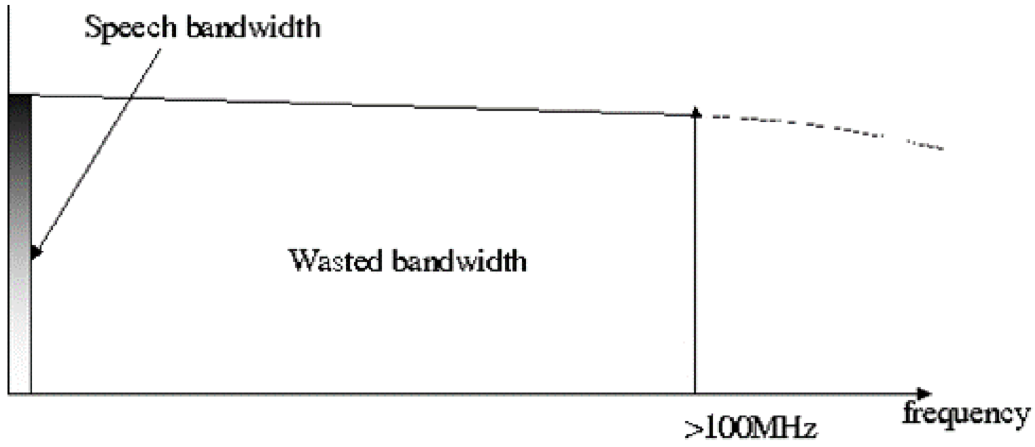
At 3 KHz: $\lambda = \frac{c}{f} = \frac{3\times10^8}{3\times10^3} = 10^5 = 100km$

$\Rightarrow .1\lambda = 10km$

Antenna too large!
Use modulation to
transfer
information to a
higher frequency

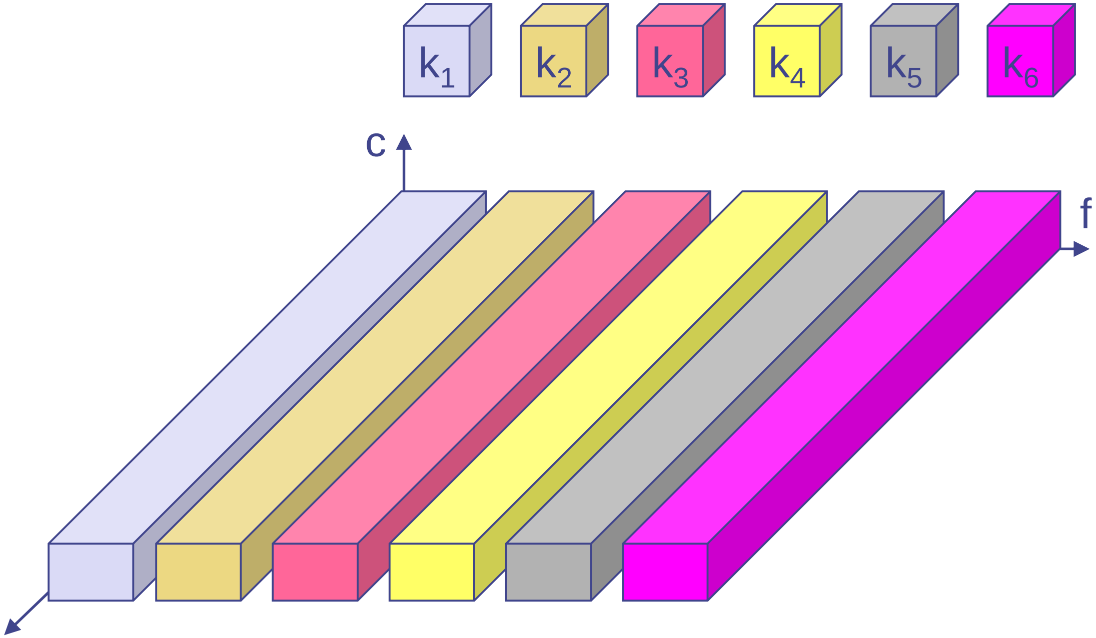
Multiplexing : Sharing a Medium

- When more than one application or connection share the capacity of one channel, it is called multiplexing.
- This results in better utilization of resources.
  - A typical example is, many conversations over telephone line, trunk line, wireless channel, etc
  - For example, a copper cable has a bandwidth of 100's of Mhz. Baseband speech is a only a few kHz



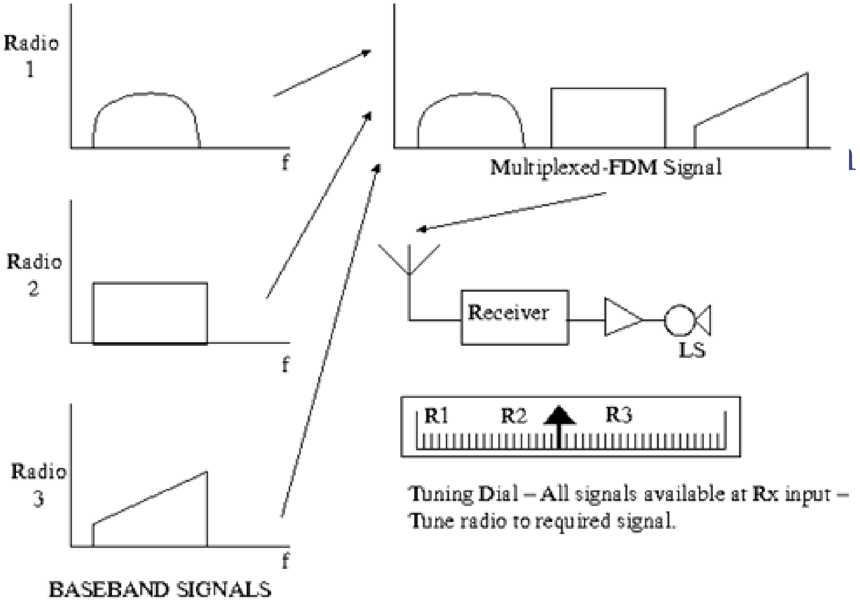
Frequency Division Multiplexing (FDM)

- Assignment of non-overlapping frequency ranges to each "user" or signal on a medium. Thus, all signals are transmitted at the same time, each using different frequencies.
  - Broadcast radio and television, cable television, and cellular phone systems use frequency division multiplexing.



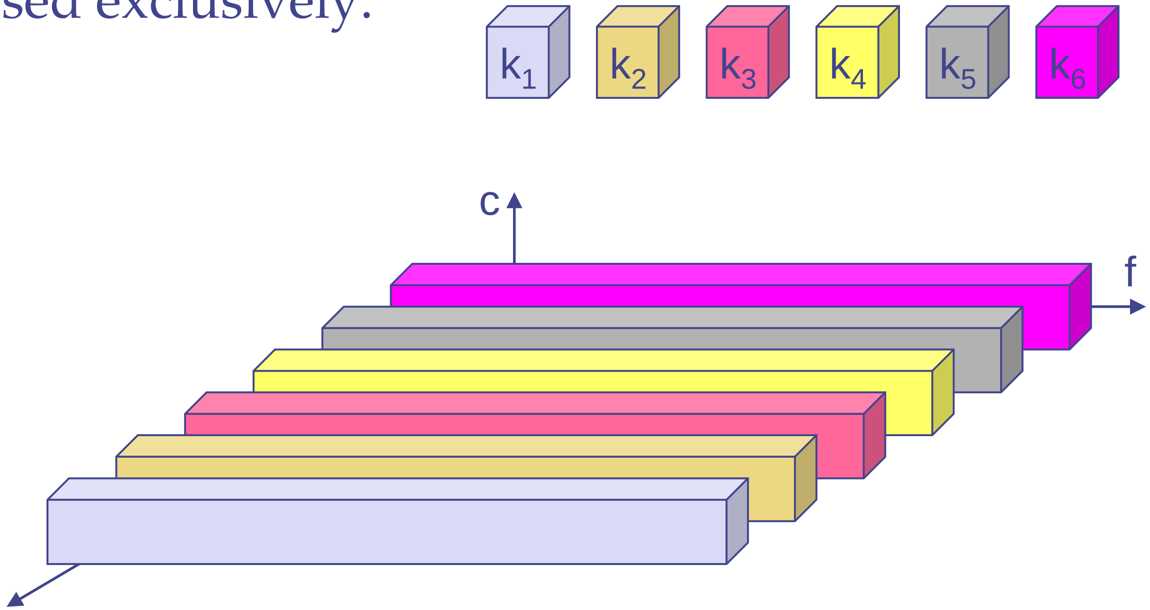
FDM (Cont….)

- A user gets a certain band of the spectrum for the whole time
- Advantages:
  - No dyna necessary
  - Works also for analog signals
- Disadvantages:
  - Inflexible
  - Guard spaces
  - more susceptible to noise



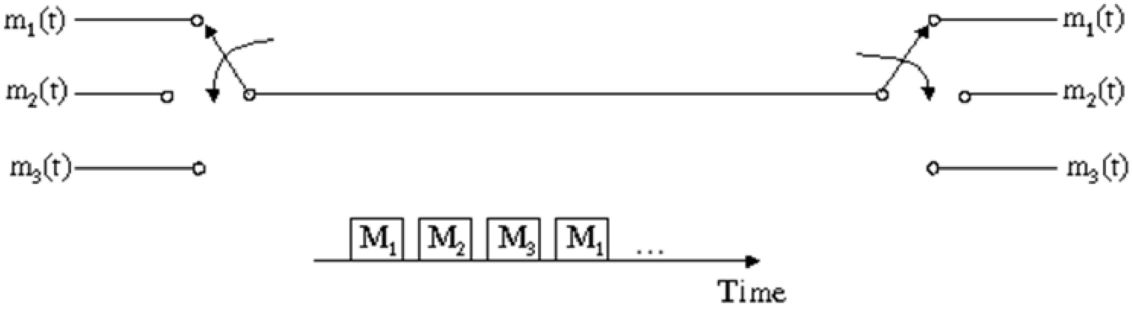
Time Division Multiplexing (TDM)

- Sharing of the signal is accomplished by dividing available transmission time on a medium among users.
- A channel gets the whole spectrum for a certain amount of time
  - Digital signaling is used exclusively.



TDM (Cont….)

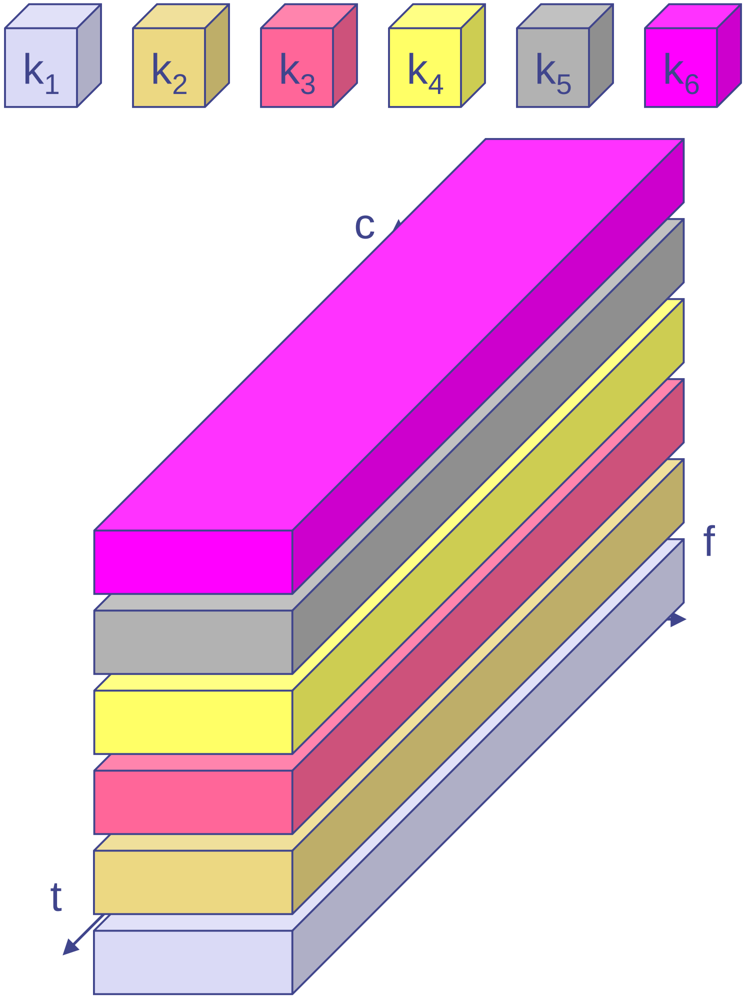
- T-1 telephone lines are common examples of synchronous TDM
- Advantages:
  - Only one carrier in the medium at any time
  - Throughput at high utilization



- Disadvantages:
  - synchronization necessary
  - Long delays at low utilization
  - Line will require as much bandwidth as all the bandwidths of the sources

Code Division Multiplexing (CDM)

- An advanced technique that allows multiple devices to transmit on the *same* frequencies at the *same* time using different codes
  - all users share same frequency, but each user has own "chipping" sequence (i.e., code) to encode data
  - allows multiple users to " coexist " and transmit simultaneously with minimal



- Used in Mobile Communication

Amplitude Modulation (AM)

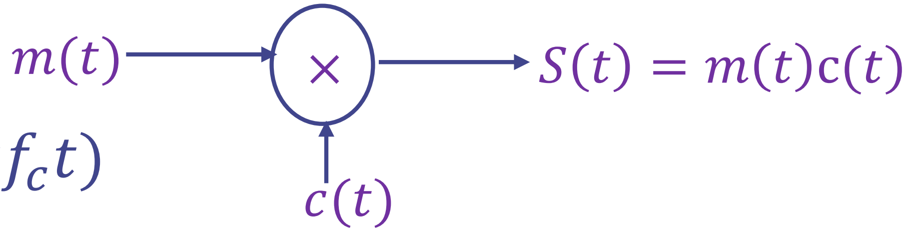
- Information-bearing signal m(t) is impressed onto the carrier amplitude.
- Four types of AM
  1. Conventional
  2. Double sideband suppressed carrier (DSB-SC)
  3. Single sideband (SSB)
     - Can be lower or upper (LSB/USB)
  4. Vestigial sideband (VSB)
- Relevant parameters
  - Spectral characteristics and bandwidth
  - Modulation index
  - Power efficiency

Overview

- Introduction
- Double-Sideband Suppressed Carrier AM
- Conventional AM
- Single Sideband AM
- Vestigial-sideband AM

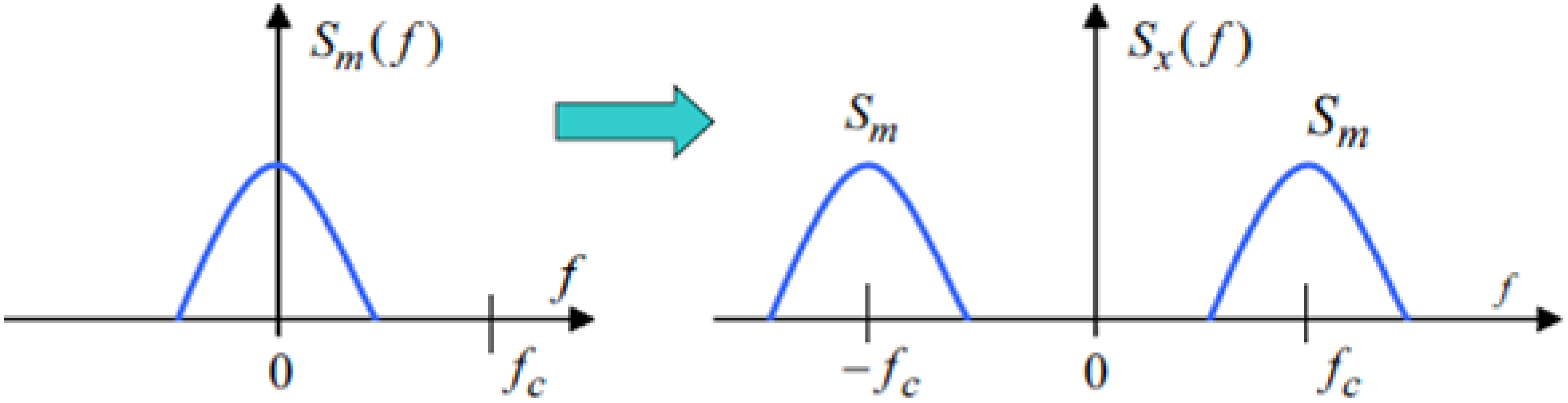
Double-Sideband Suppressed Carrier (DSB-SC)

- A DSB-SC AM signal is obtained by multiplying the message signal $m(t)$ with the carrier $c(t) = A_c Cos(2\pi f_c t + \varphi_c)$



$$S(t) = A_c m(t) Cos(2\pi f_c t)$$

- Spectrum: $S_x(f) = \frac{A_c}{2}\left[S_m(f - f_c) + S_m(f + f_c)\right]$



Double-Sideband Suppressed Carrier (DSB-SC)

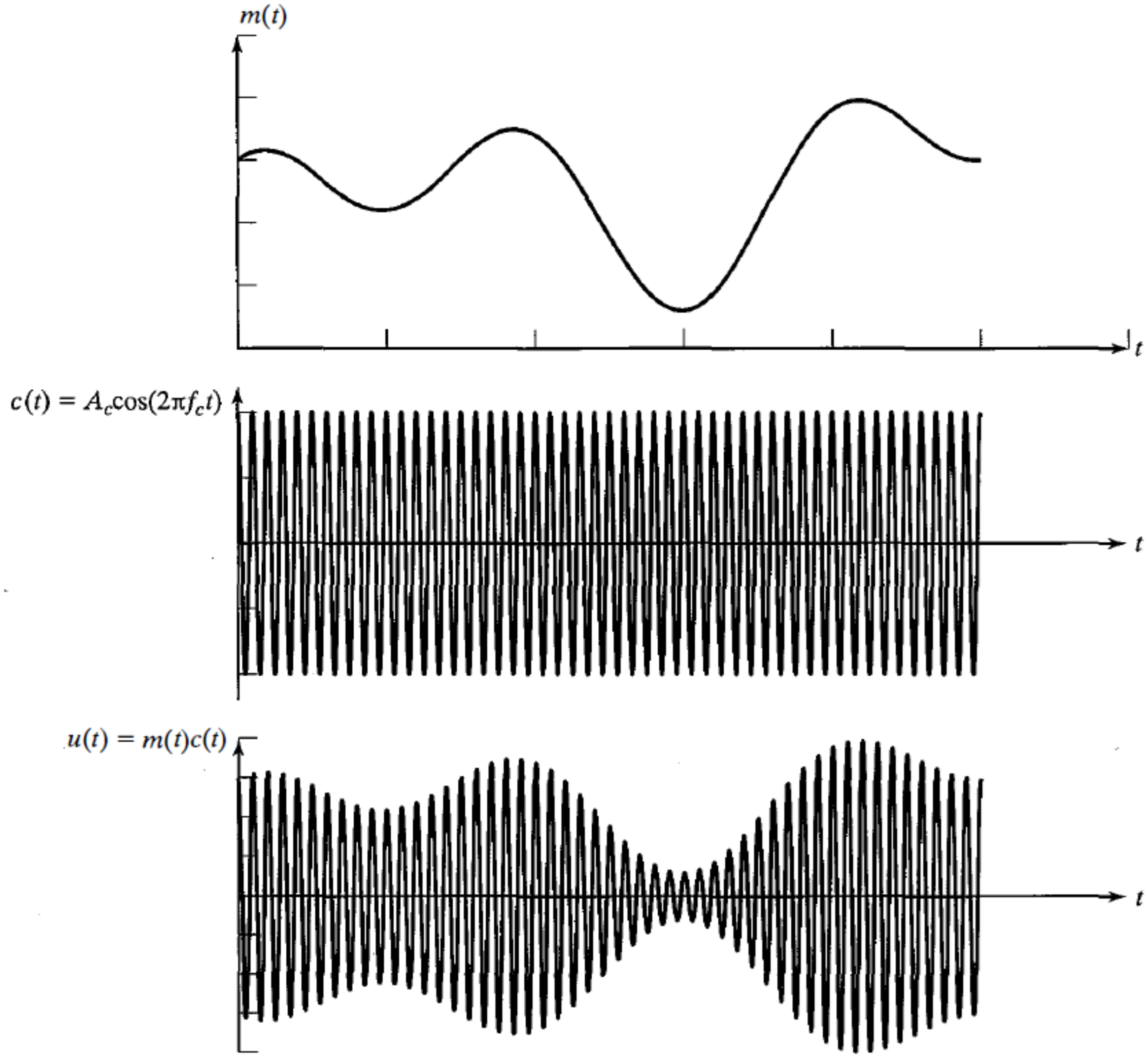
- Magnitude of the spectrum of the message signal m(t) has been translated or shifted in frequency by an amount f_c.

- What do you see on spectrum analyzer?
  - Bandwidth? Power efficiency? PSD?

  - Bandwidth occupancy: $\boldsymbol{B_W = f_{max} - f_{min} = 2Bw_m}$

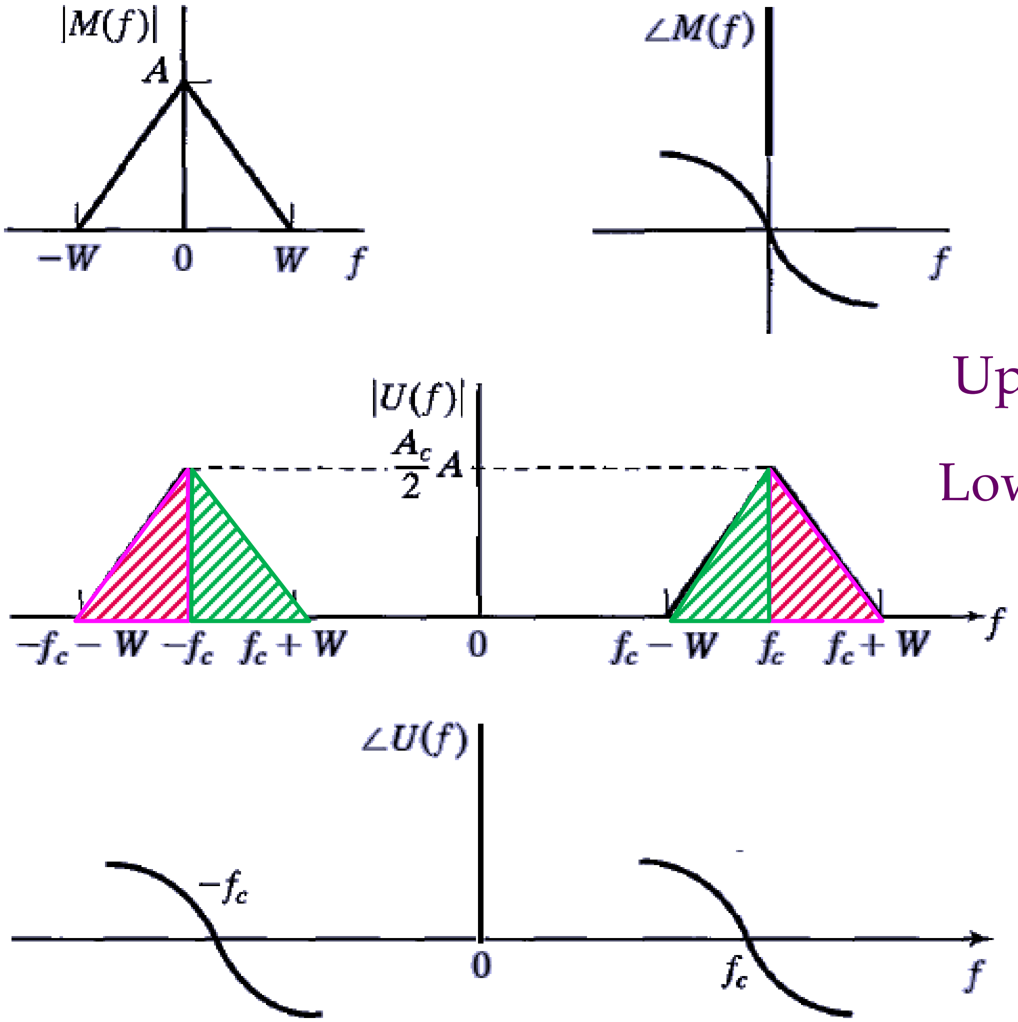
Where the bandwidth of the message signal $m\,(t)$ is Bw_m and $f_c \gg Bw_m$.

DSB-SC AM…



- Examples of message, carrier, and DSB-SC-modulated signals.

DSB-SC AM…



Upper sideband (USB) of S (*f*)

Lower sideband (LSB) of S (*f*)

- Magnitude and phase spectra of the message signal $m(t)$ and the DSB AM-modulated signal $u(t)$.

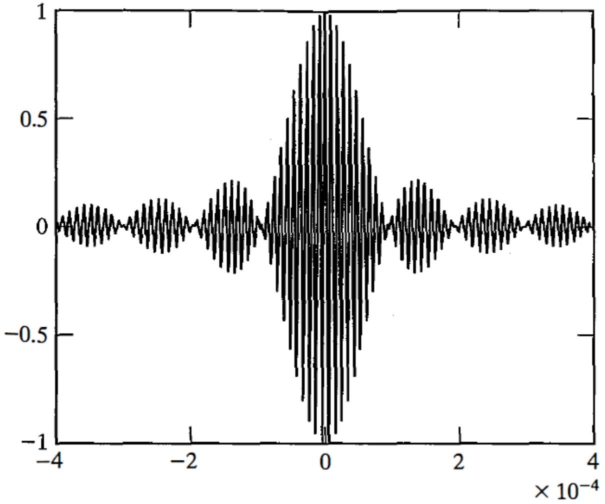
Example:

Suppose that the modulating signal $m\ (t)$ is a sinusoid of the form $m(t) = mcos(2\pi f_m t)$ where $f_m \ll f_c$.

- Determine the DSB-SC AM signal and its upper and lower sidebands.

$$\boxed{S(t) = A_c mcos(2\pi f_m t)Cos(2\pi f_c t)}$$

$$\boxed{S(t) = \frac{A_c m}{2}[cos(2\pi(f_c - f_m)t) + Cos(2\pi(f_c + f_m)t)\ ]}$$

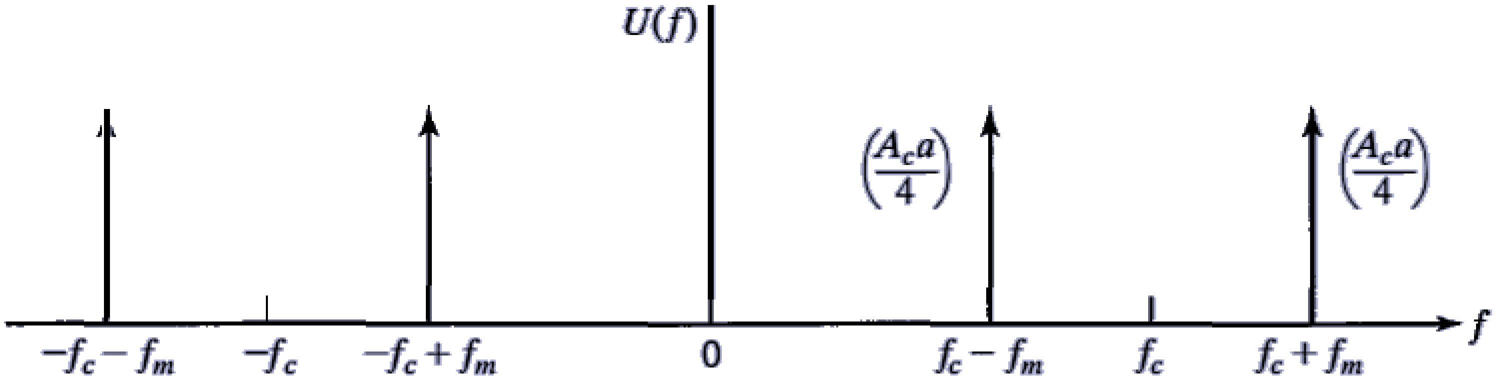


$$f_m = 10^4 H_z,\ f_c = 10^6 H_z$$

Example…(cont.)

- The modulated signal in the frequency domain will have the following form:

$$S(f) = \frac{A_c m}{4}\left[\delta(f-(f_c-f_m)) + \delta(f+(f_c-f_m)) + \delta(f-(f_c-f_m))\right)$$



$$\boldsymbol{B_W = f_{max} - f_{min} = 20KHz}$$

Power Content of DSB-SC Signals

- $P_s = \lim_{T\to\infty} \frac{1}{T} \int_{\frac{-T}{2}}^{\frac{T}{2}} S^2(t)dt$

$$= \lim_{T\to\infty} \frac{1}{T} \int_{\frac{-T}{2}}^{\frac{T}{2}} A_c{}^2 m^2(t) Cos^2\,(2\pi f_c t)dt$$

$$= \frac{Ac^2}{2} p_m$$

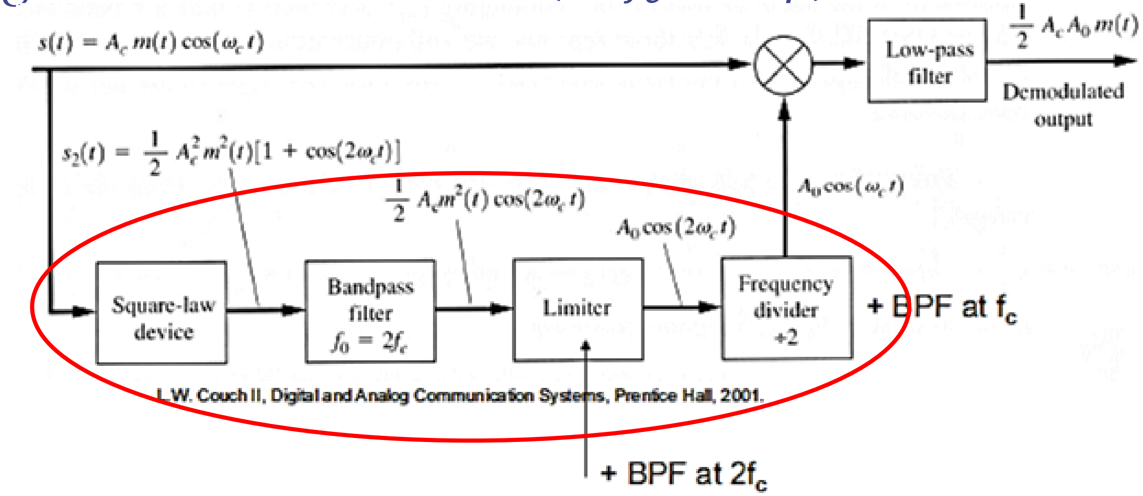
where P_m indicates the power in the message signal $m(t)$.

- *In Example 1,* determine the power in the modulated signal and the power in each of the sidebands.

Demodulation of DSB-SC

- Suppose that the DSB-SC AM signal $S(t)$ is transmitted through an **ideal channel** *(with no channel distortion and no noise)*,
  - Then the received signal is equal to the modulated signal, i.e., $\mathrm{r}(t) = S(t)$

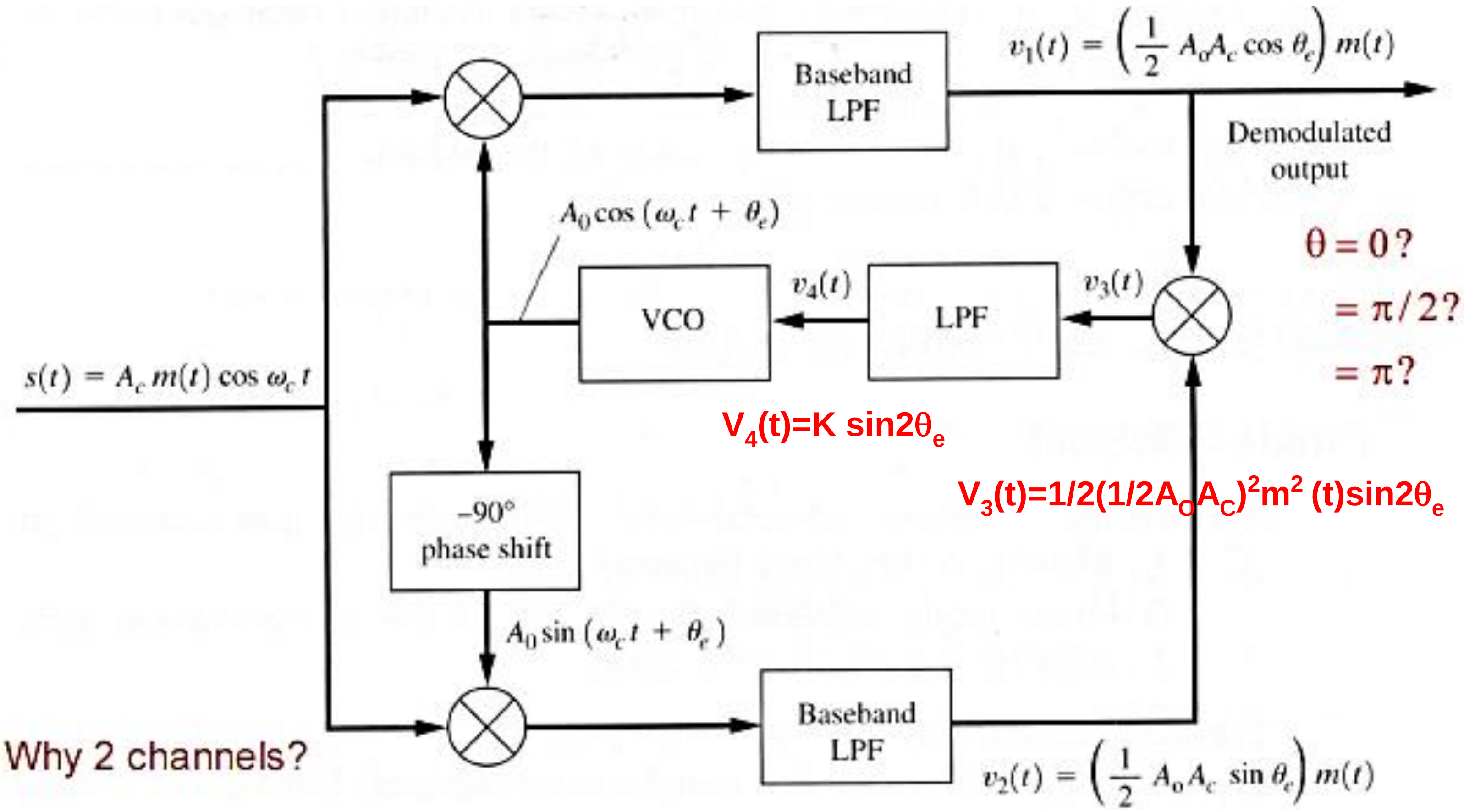
  - To demodulate the received signal by first multiplying r (t) by a locally generated sinusoid $\cos(2\pi f_c t\ +\ \varphi)$- ***Product detector***



L.W. Couch II, Digital and Analog Communication Systems, Prentice Hall, 2001.

Demodulation of DSB-SC

- Demodulation – Costas loop



$\theta = 0$?

$= \pi/2$?

$= \pi$?

$V_4(t)=K \sin 2\theta_e$

$V_3(t)=1/2(1/2A_O A_C)^2 m^2 (t) \sin 2\theta_e$

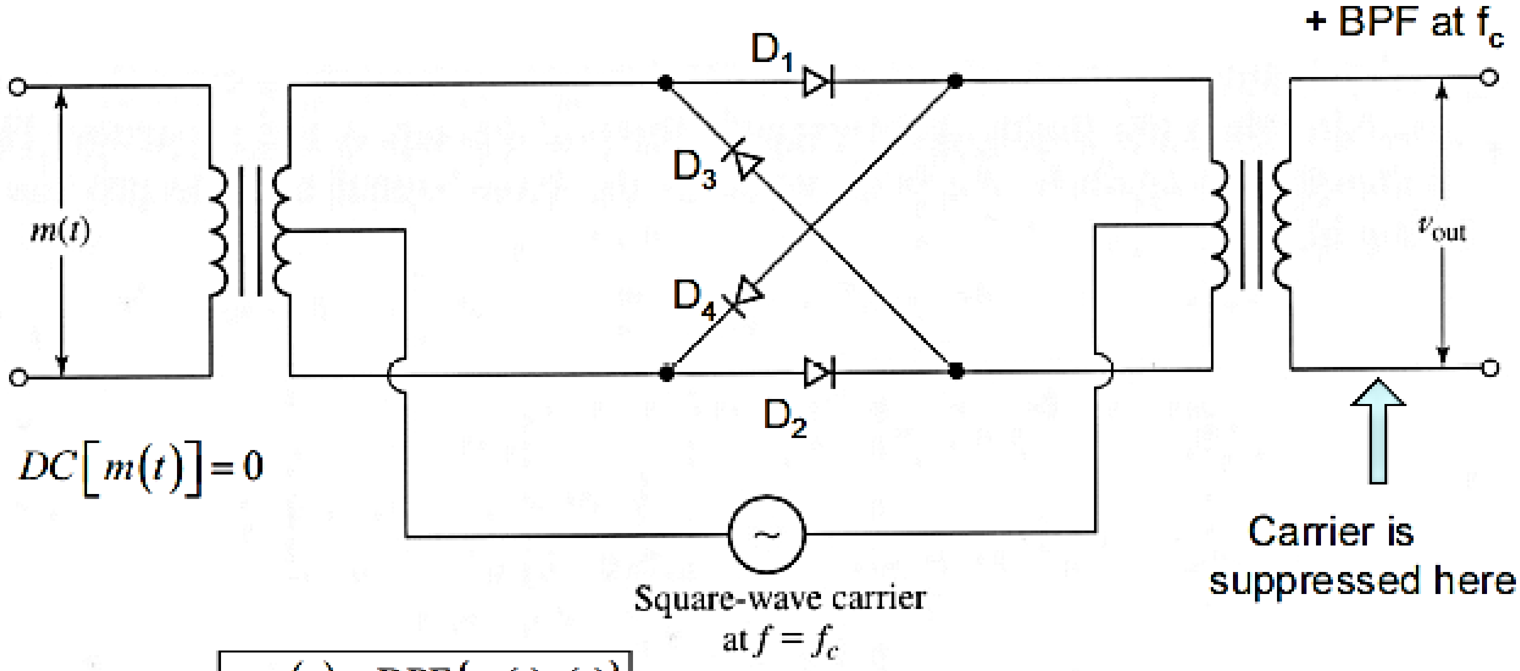
Why 2 channels?

Advantages/Disadvantages of DSB-SC

- Advantages
  - High power efficiency
  - If message m(t) > 0, envelope detection is possible
- Disadvantages
  - Double the baseband bandwidth
  - Complex modulation/demodulation (some form of carrier recovery is required)
  - Pilot tone may be required to simplify demodulation

Generation of DSB-SC

- Ring modulator



+ BPF at f_c

$DC[m(t)] = 0$

Carrier is suppressed here

$$\boxed{\begin{aligned} v_{out}(t) &= \text{BPF}\{m(t)s(t)\} \\ &= \frac{2A_c}{\pi} m(t)\cos 2\pi f_c t \end{aligned}}$$

$$s(t) = A_c \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos\left[2\pi f_c t(2n-1)\right]$$

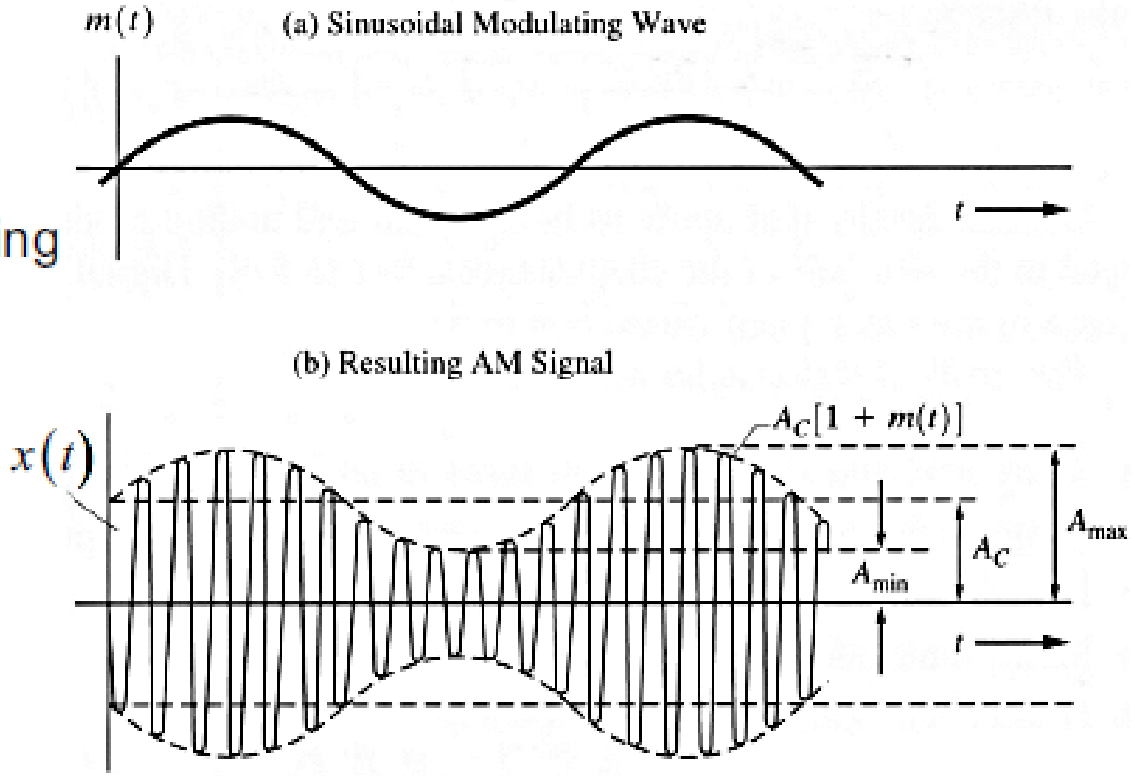
- Large-amplitude sinusoidal signal may be used instead of the square wave

Overview

- Introduction
- Double-Sideband Suppressed Carrier AM
- Conventional AM
- Single Sideband AM
- Vestigial-sideband AM

Conventional AM

- General form: $\boxed{x(t) = A_c\left[1+m(t)\right]\cos\left(2\pi f_c t+\phi_c\right)}$
- m(t) must be constrained: -1 ≤ m(t)



  - A_c – carrier amplitude without modulation;
  - $A_c\left[1+m(t)\right]$- time-varying (modulated) carrier amplitude.
  - Example: sinusoidal modulating signal:

$$m(t) = a\cos(\Omega t)$$

Conventional AM - General Case

- General form: $x(t) = A_c\left[1 + m(t)\right]\cos\left(2\pi f_c t + \phi_c\right)$
- Modulated signal spectrum

$$S_x(f) = \frac{A_c}{2}\left[\delta(f - f_c)e^{j\phi_c} + \delta(f + f_c)e^{-j\phi_c} + S_m(f - f_c)e^{j\phi_c} + S_m(f + f_c)e^{-j\phi_c}\right]$$

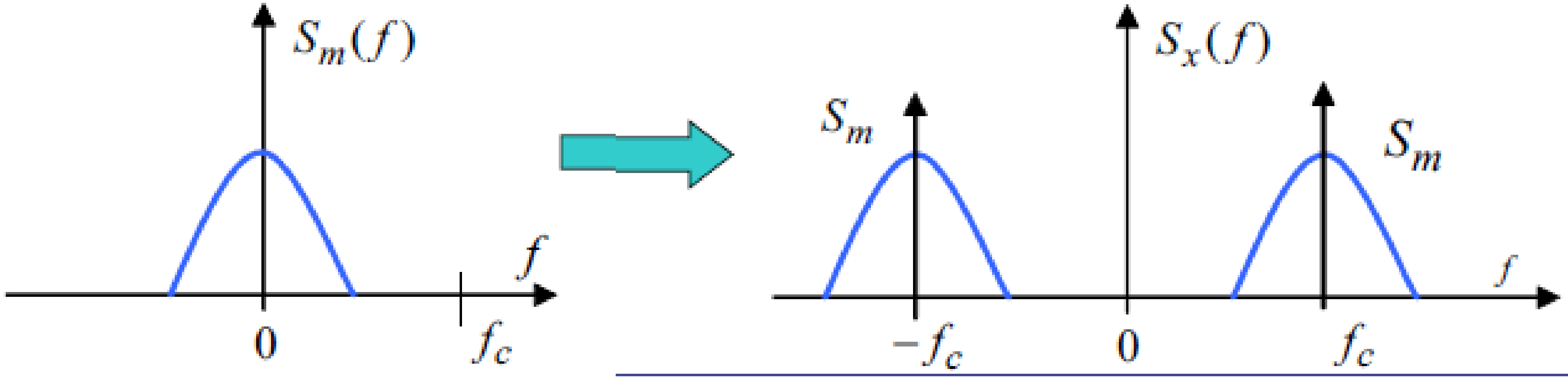
$$2\left|S_x^+(f)\right| = A_c\left[\delta(f - f_c) + \left|S_m(f - f_c)\right|\right]$$

- Power ?
- Power efficiency?

- Measured by spectrum analyzer: no infinite height for delta function in practice

$\delta(f - f_c) \leftrightarrow \Delta(f - f_c)$

Bandwidth???

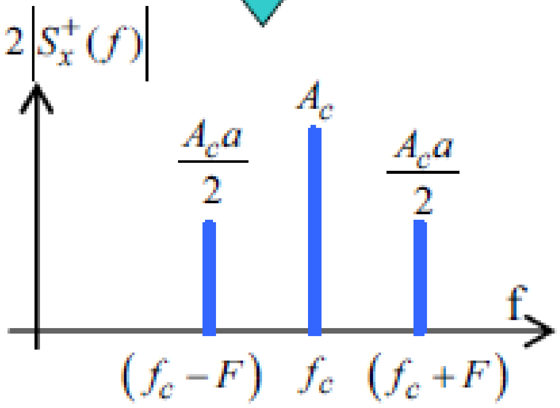


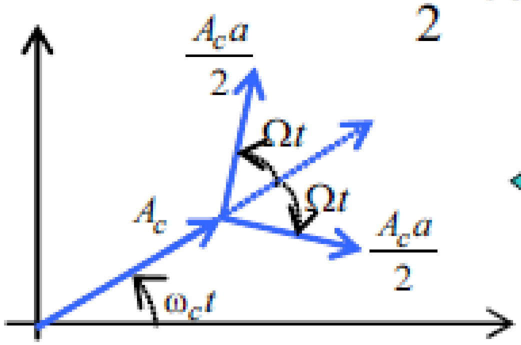
Conventional AM - Sinusoidal Modulation

- Modulated signal $x(t) = A_c\left[1 + a\cos(2\pi F t)\right]\cos\left(2\pi f_c t + \phi_c\right)$
- Minimum and maximum carrier amplitudes $\begin{aligned} A_{\min} &= A_c[1-a] \\ A_{\max} &= A_c[1+a] \end{aligned}$
- Modulation index: $M = \dfrac{A_{\max} - A_{\min}}{2A_c} \le 1$

- x(t) spectrum: $x(t) = A_c\cos[2\pi f_c t + \phi_c] + \dfrac{A_c a}{2}\cos[2\pi(f_c - F)t + \phi_c] + \dfrac{A_c a}{2}\cos[2\pi(f_c + F)t + \phi_c]$

Bandwidth?

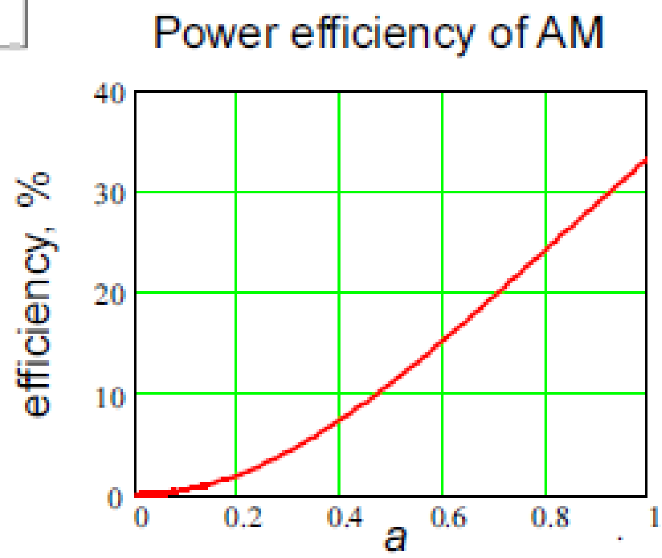




Geometrical representation

Conventional AM - Sinusoidal Modulation

- Signal power (average): $$\boxed{P_x = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} x^2(t)\,dt = \overline{x^2(t)} = \underbrace{\frac{A_c^2}{2}}_{\text{carrier}} + \underbrace{\frac{a^2A_c^2}{4}}_{\text{sidebands}}}$$
- Power efficiency: $$\boxed{\eta = \frac{P_{SB}}{P_{tot}} = \frac{a^2}{2+a^2}}$$
- Bandwidth: $\boxed{\Delta f = 2F}$



- What is the best power efficiency?

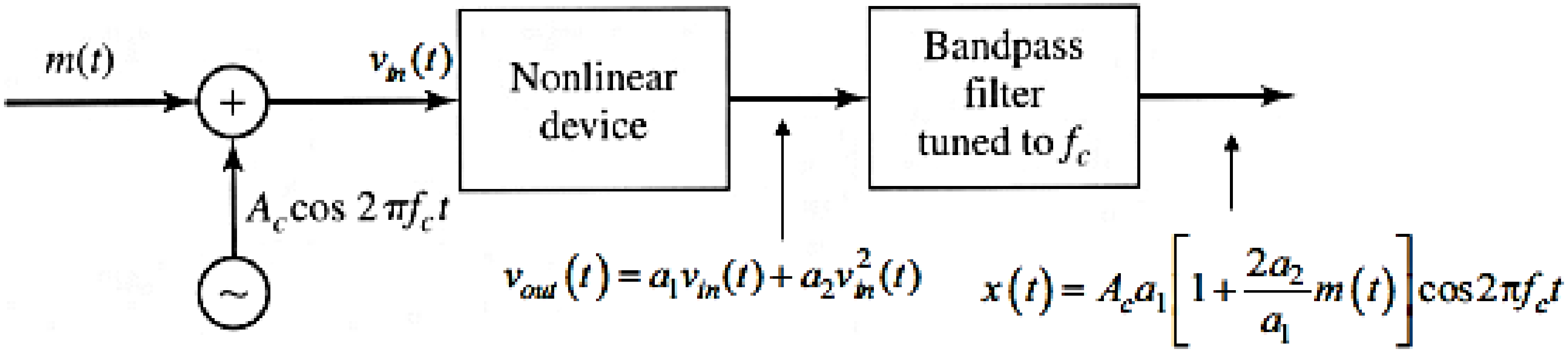
Example 2

The double tone message signal $m(t) = 3\cos(200\pi t) + \sin(600\pi t)$ is used to modulate the carrier $c(t) = \cos(2\,x10^5 t)$.

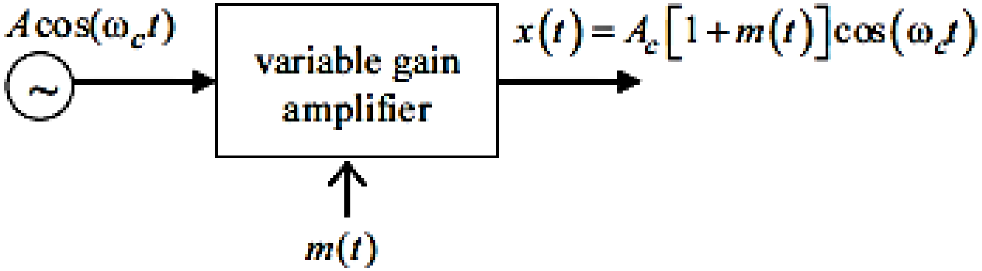
- If the modulation index is a = 0.85, determine the power in the carrier component and in the sideband components of the modulated signal.

Generation of Conventional AM

- Power-law(square-law) modulator

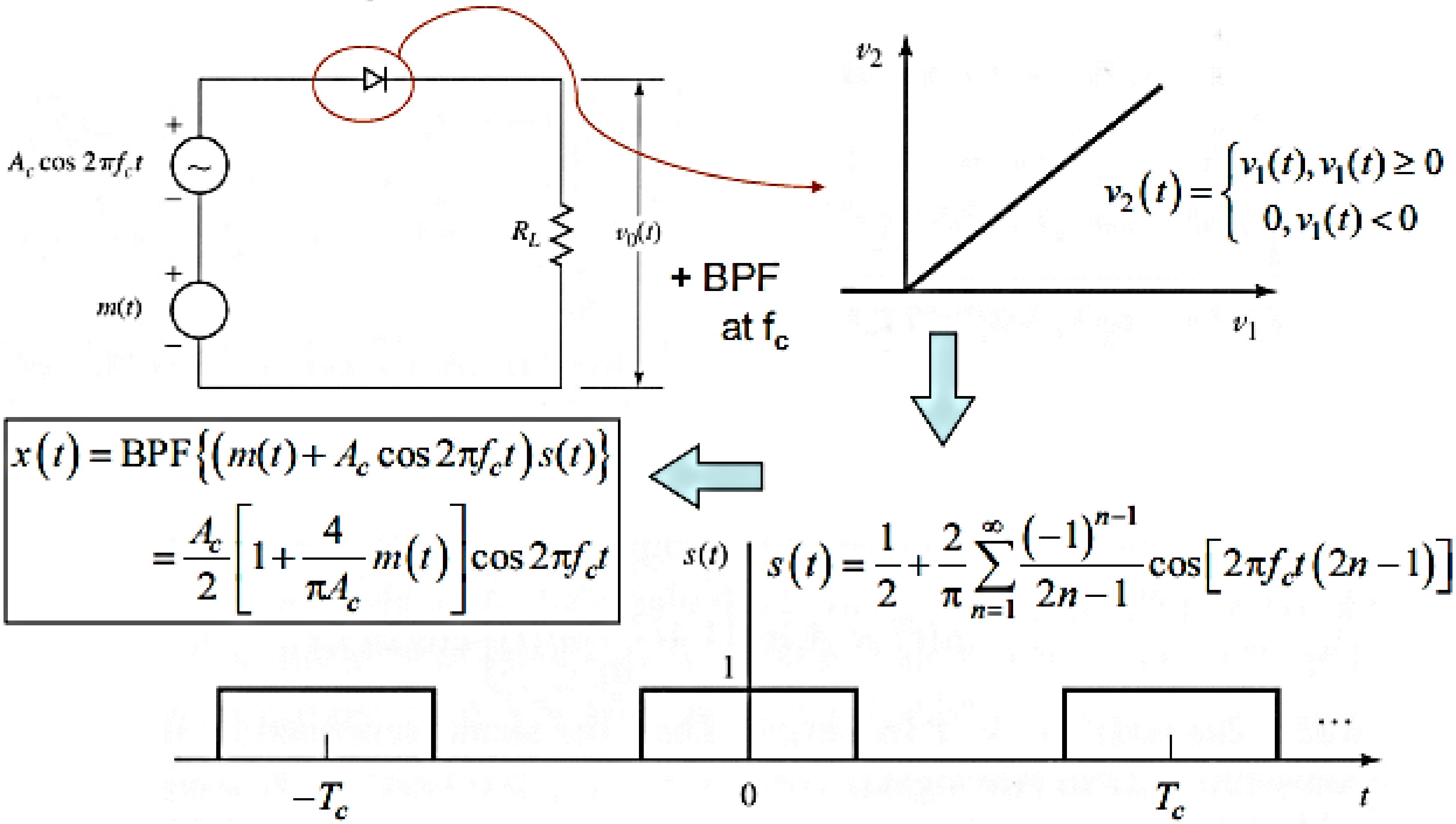


- Using variable-gain amplifier (modulator)



Generation of Conventional AM

- Switching modulator



+ BPF at f_c

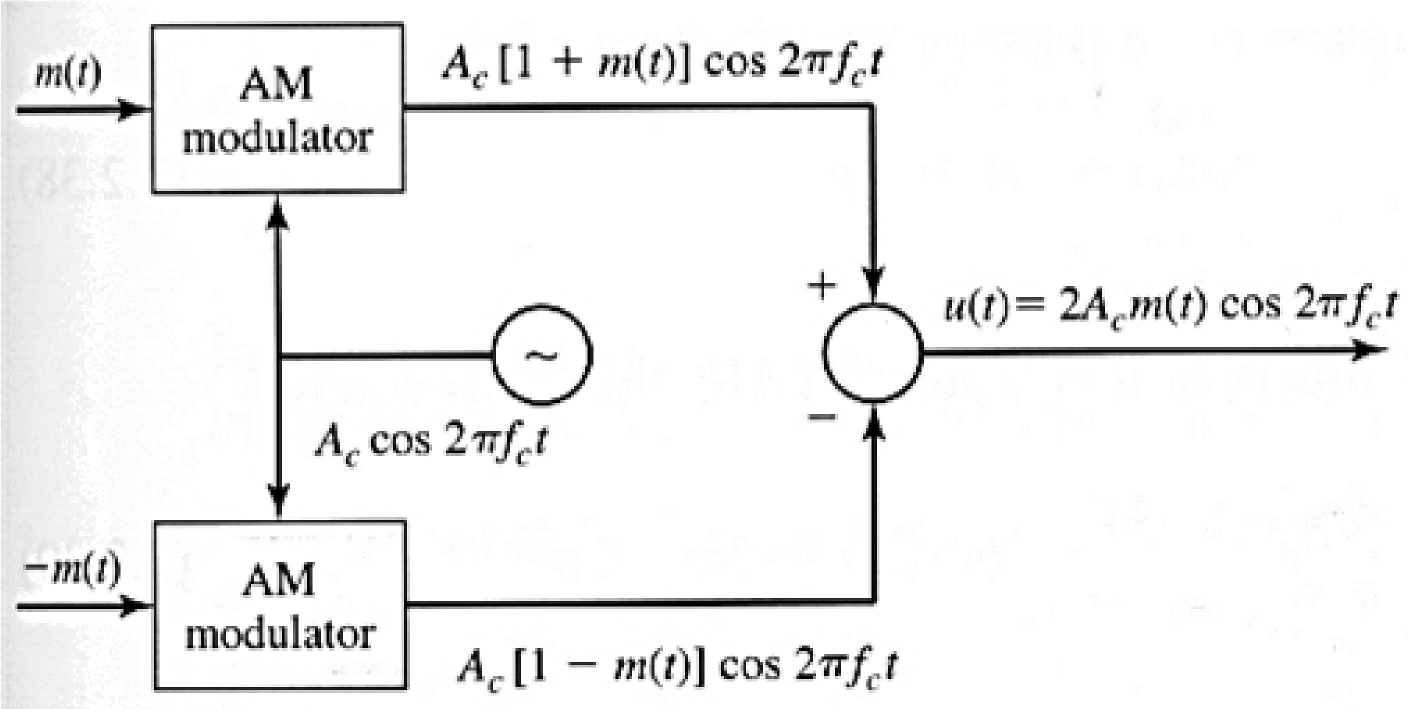
$$v_2(t)=\begin{cases} v_1(t), & v_1(t)\geq 0 \\ 0, & v_1(t)<0 \end{cases}$$

$$s(t)=\frac{1}{2}+\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2n-1}\cos\left[2\pi f_c t(2n-1)\right]$$

$$\begin{aligned} x(t) &= \text{BPF}\left\{\left(m(t)+A_c\cos 2\pi f_c t\right)s(t)\right\} \\ &= \frac{A_c}{2}\left[1+\frac{4}{\pi A_c}m(t)\right]\cos 2\pi f_c t \end{aligned}$$

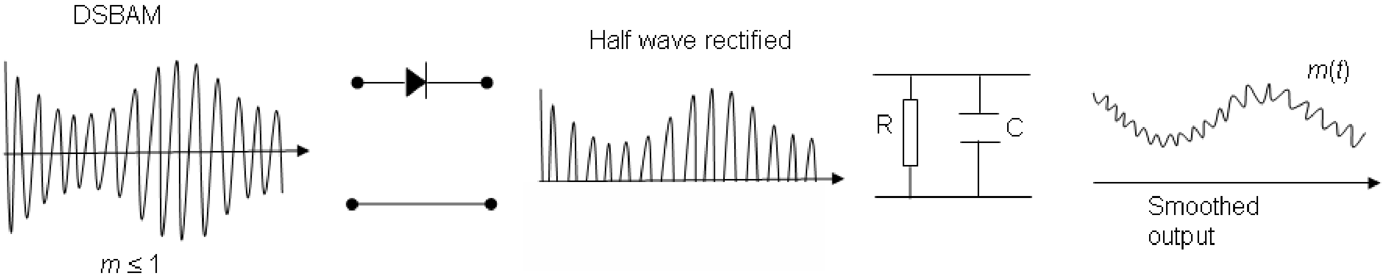
Generation of DSB-SC with AM modulators

- Balanced modulator: use of two square-law AM modulators

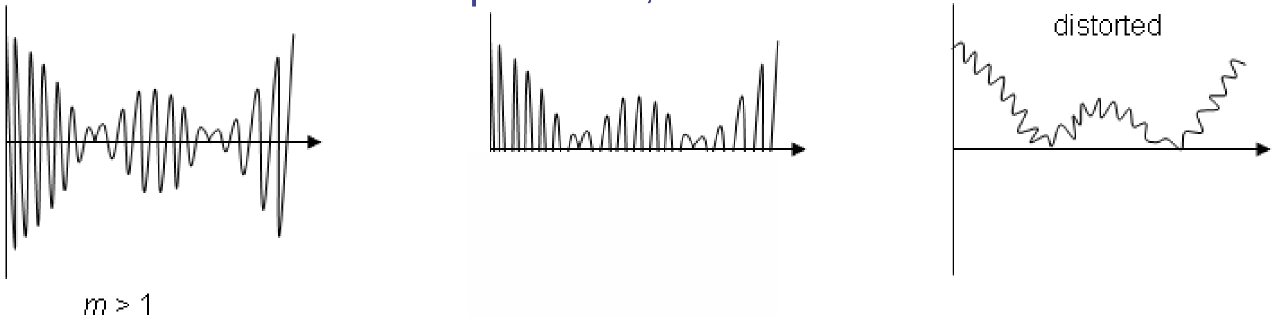


Demodulation of Conventional AM -

- Envelop Detector
- For large inputs signal
  - The diode is switched i.e. forward biased ≡ ON,
  - Reverse biased ≡ OFF, and acts as a half wave rectifier.
- The 'RC' combination acts as a 'smoothing circuit' and the output is m(t)plus 'distortion'.

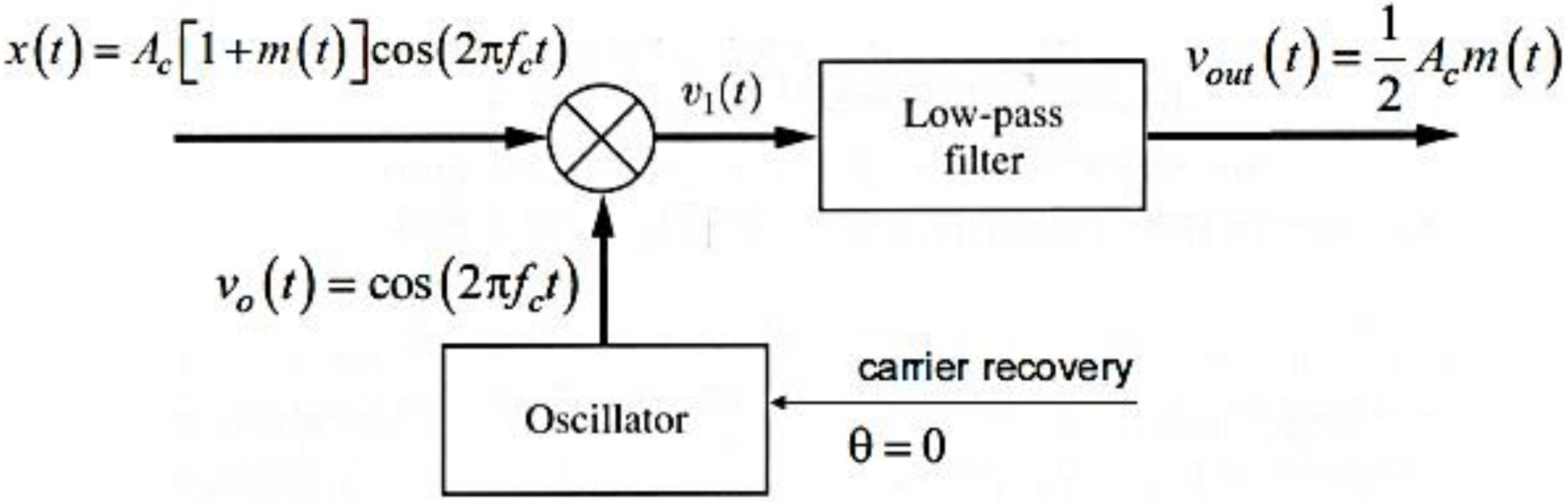


If the modulation depth is > 1, the distortion below occurs



Demodulation of Conventional AM

- Product detector (Coherent detector)



- What happens if $\theta \neq 0$?

Advantages/Disadvantages of Conventional AM

- Advantages
  - Very simple demodulation (envelope detector)
  - “Linear” modulation
- Disadvantages
  - Low power efficiency
  - Double the baseband bandwidth

Summary

- Conventional AM
    - Time-domain and frequency-domain representations
    - Power efficiency and bandwidth
- Generation (modulation) and demodulation of conventional AM
- Double sideband suppressed carrier (DSB-SC)
    - Spectrum
    - Bandwidth
    - Generation and demodulation of DSB-SC
- Advantages/disadvantages of conventional & DSB-SC AM

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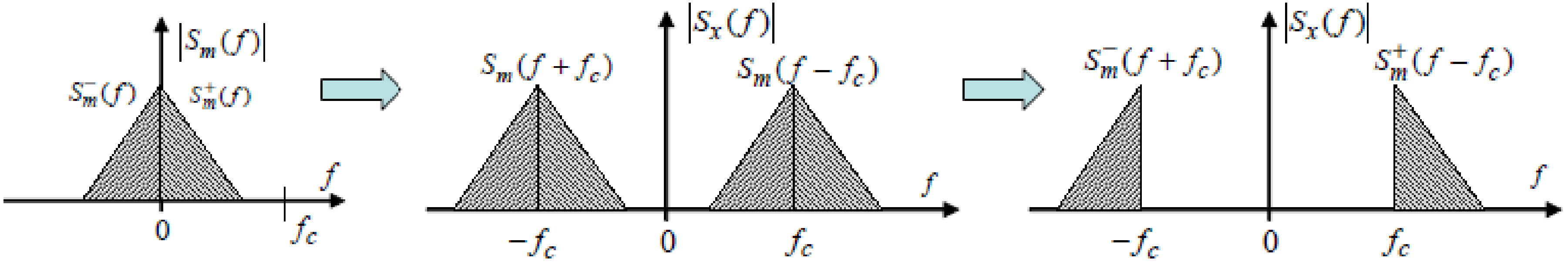
Single Sideband (SSB) AM

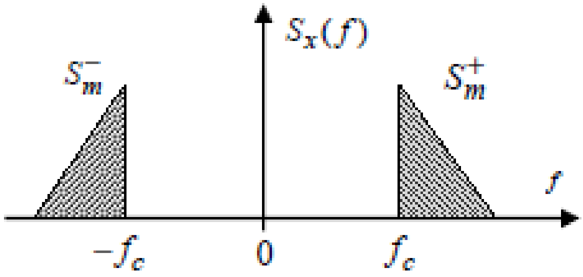
- Why SSB-AM?
  - Spectral efficiency is of great importance
- Conventional and DSB-SC occupy **twice** the message bandwidth
- All the information is contained in either half
  - The other is redundant
- Spectral efficiency can be greatly (twice) increased by transmitting one half



Generation of SSB: Analysis

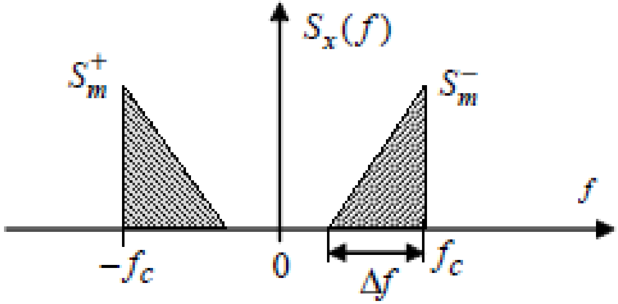
- Hilbert transform can be effectively used
- Start with the message m(t) and show that USB (Upper SSB) is given by

$$\boxed{x(t) = A_c m(t)\cos 2\pi f_c t - A_c \hat{m}(t)\sin 2\pi f_c t}$$



- Similarly, LSB can be expressed as

$$\boxed{x(t) = A_c m(t)\cos 2\pi f_c t + A_c \hat{m}(t)\sin 2\pi f_c t}$$



- In-phase and quadrature channels are required to generate SSB

USB: Frequency-Domain Viewpoint

- Time-domain signal $\boxed{x(t) = A_c m(t)\cos 2\pi f_c t - A_c \hat{m}(t)\sin 2\pi f_c t}$

- Spectra of individual components

$$\boxed{\begin{array}{l} m(t) \leftrightarrow S_m^+(f) + S_m^-(f), \quad \hat{m}(t) \leftrightarrow -jS_m^+(f) + jS_m^-(f) \\ \cos(\omega_c t) \leftrightarrow \frac{1}{2}(\delta(f - f_c) + \delta(f + f_c)), \quad \sin(\omega_c t) = \frac{1}{2j}(\delta(f - f_c) - \delta(f + f_c)) \end{array}}$$

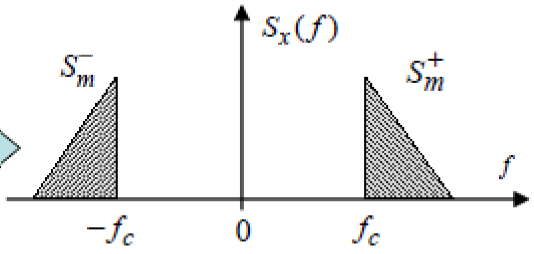
- Use multiplication property of FT

$$\boxed{\begin{array}{l} m(t)\cos\omega_c t \leftrightarrow \frac{1}{2}\left(S_m^+(f - f_c) + S_m^-(f - f_c) + S_m^+(f + f_c) + S_m^-(f + f_c)\right) \\ \hat{m}(t)\sin\omega_c t \leftrightarrow \frac{1}{2}\left(-S_m^+(f - f_c) + S_m^-(f - f_c) + S_m^+(f + f_c) - S_m^-(f + f_c)\right) \end{array}}$$

USB: Frequency-Domain Viewpoint

- Combine the two expressions above

$$\boxed{x(t) \leftrightarrow S_m^+(f - f_c) + S_m^-(f + f_c)}$$



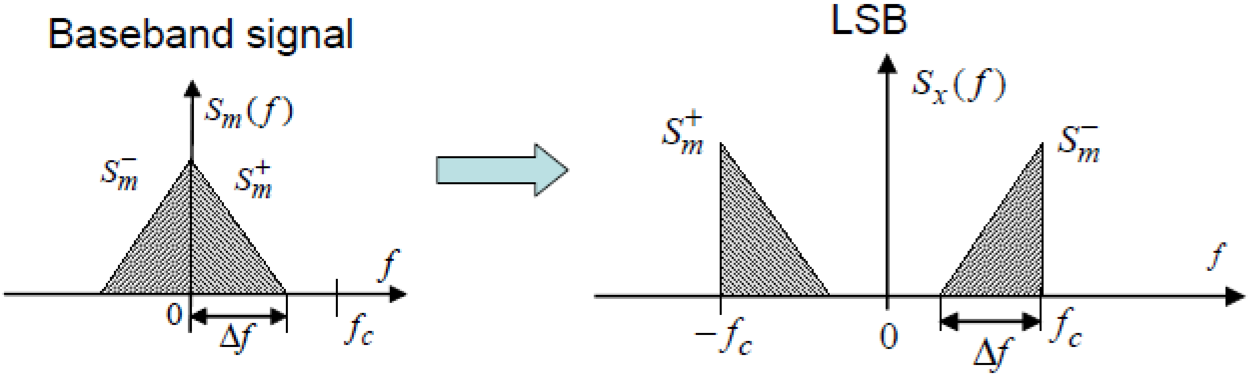
Lower SSB (LSB)

- Analysis method is the same as for USB
- Time-domain signal is

$$\boxed{x(t) = A_c m(t)\cos 2\pi f_c t + A_c \hat{m}(t)\sin 2\pi f_c t}$$

- Its spectrum is

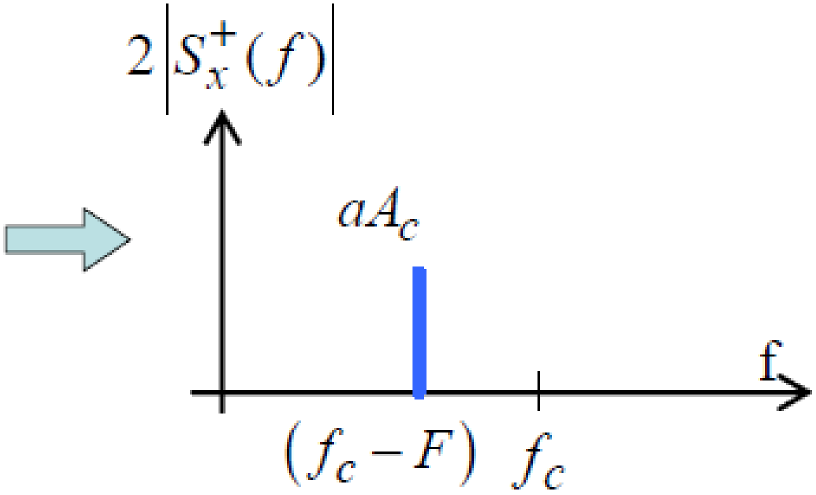
$$\boxed{x(t) \leftrightarrow S_m^-(f - f_c) + S_m^+(f + f_c)}$$



Example: Sinusoidal Modulating Signal

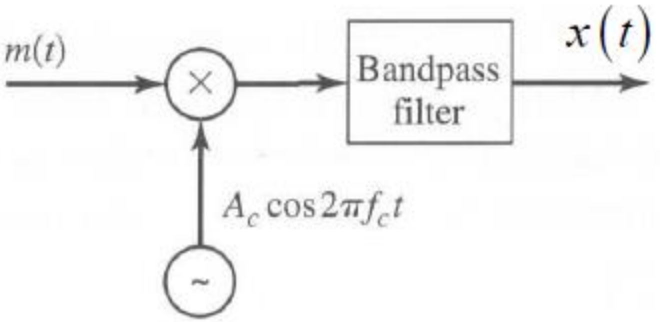
- Assume that $\boxed{m(t) = a\cos\Omega t}$
- Then
$$\boxed{\begin{aligned} x(t) &= aA_c\cos\Omega t\cos\omega_c t + aA_c\sin\Omega t\sin\omega_c t = \\ &= aA_c\cos(\omega_c - \Omega)t \end{aligned}}$$
- Obviously, this is LSB signal with one spectral component only at (ω_C-Ω)
- Modulated signal

  is just a sinusoidal

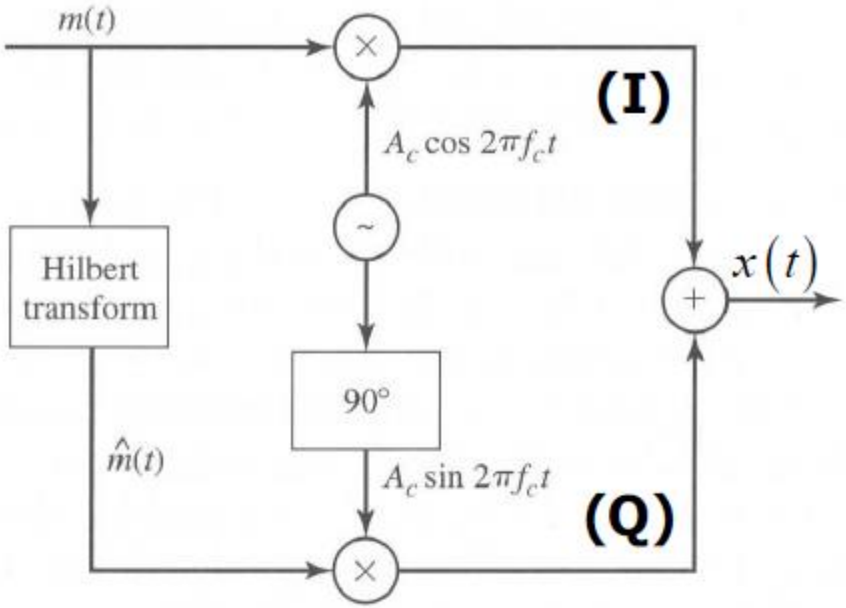


Generation of SSB

- Filtering method



- Using balanced modulators



Hilbert transform is a linear filter (phase shifter):

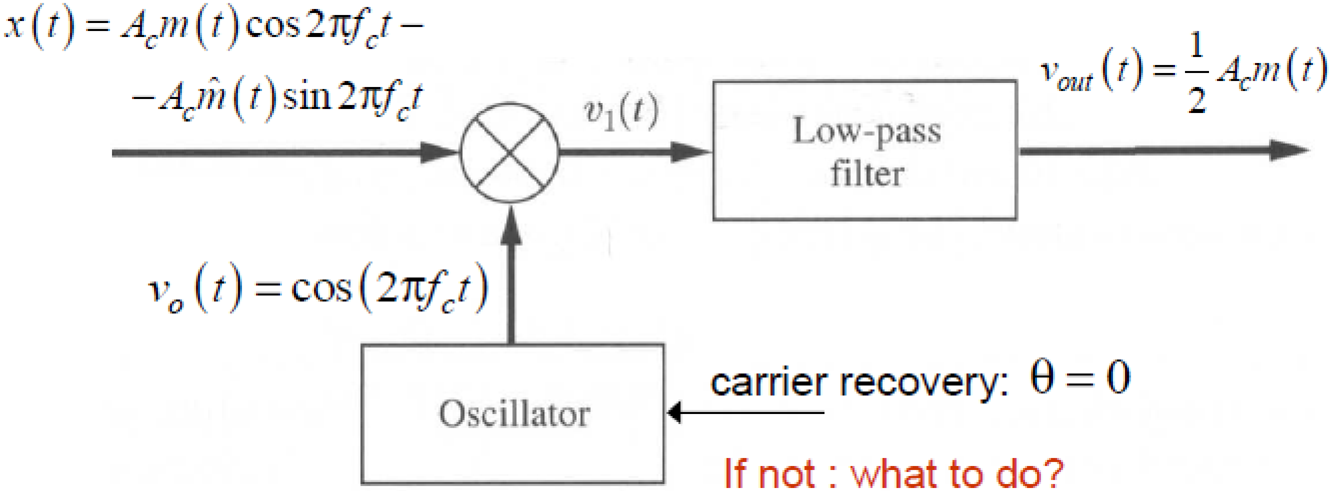
$$H(f) = \begin{cases} -j, f > 0 \\ j, f < 0 \end{cases}$$

Demodulation of SSB

- Product detector

$$\boxed{x(t)\cos(2\pi f_c t+\theta)=\frac{1}{2}A_c m(t)\cos\theta+\frac{1}{2}A_c \hat{m}(t)\sin\theta \text{ + 2}f_c \text{ terms}}$$

- After low-pass filter, only 1st two terms remain
- Coherent demodulation: θ=0



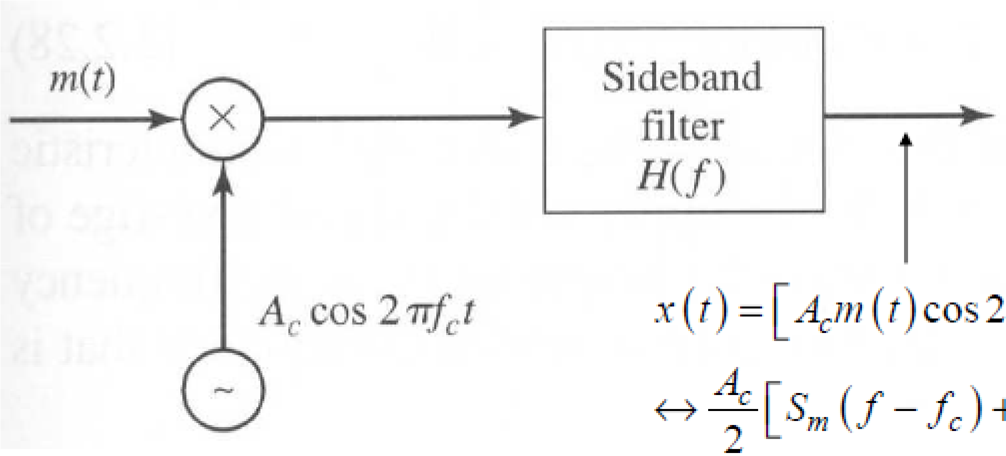
If not : what to do?

Overview

- Introduction
- Double-Sideband Suppressed Carrier AM
- Conventional AM
- Single Sideband AM
- Vestigial-sideband AM

Vestigial-sideband (VSB) AM

- SSB can be simplified by allowing a part of the other sideband to appear
- A filter implementation is feasible



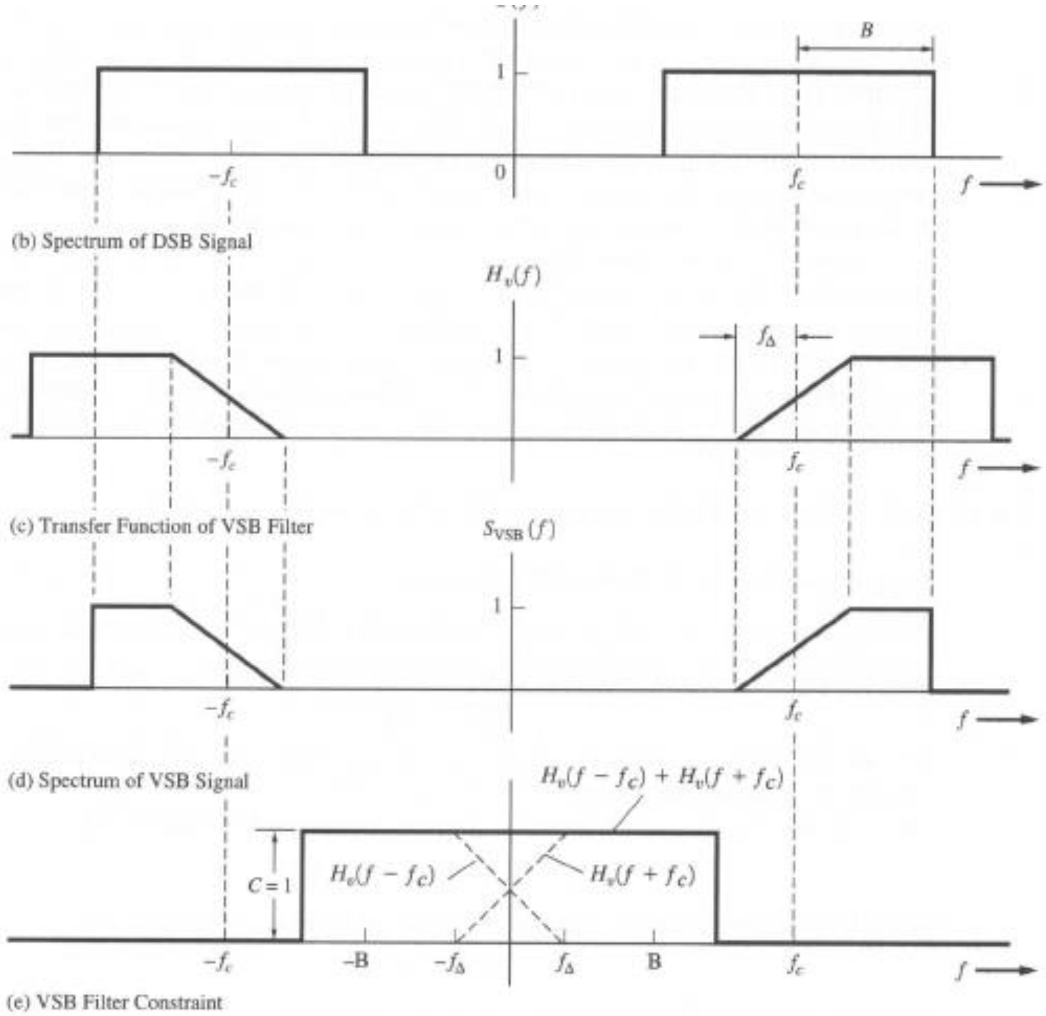
$$x(t) = \left[A_c m(t)\cos 2\pi f_c t\right] * h(t) \leftrightarrow$$

$$\leftrightarrow \frac{A_c}{2}\left[S_m(f-f_c) + S_m(f+f_c)\right]H(f)$$

Filter requirement:

$$H(f-f_c) + H(f+f_c) = \text{constant}, \quad |f| \le \mathrm{W} \quad \text{+ linear phase}$$

VSB spectrum and Filter Response



DSB signal

Filter response

VSB signal spectrum

Filter constrain

(b) Spectrum of DSB Signal

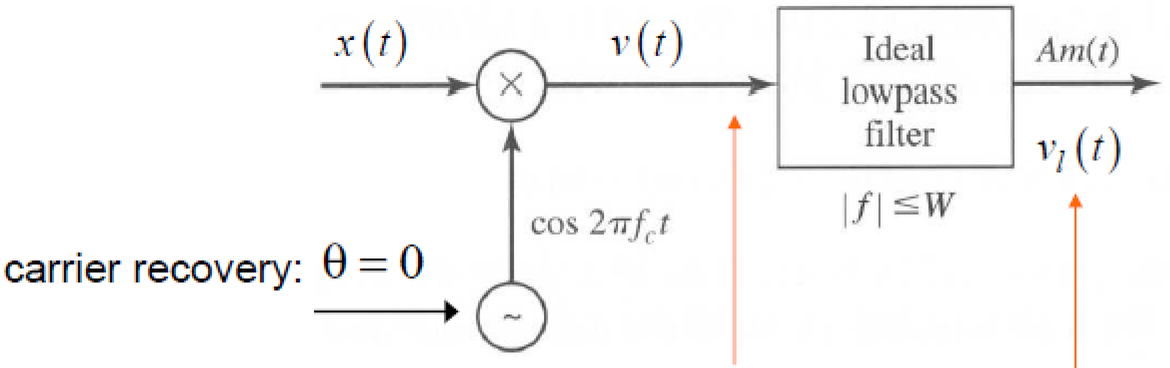
(c) Transfer Function of VSB Filter

(d) Spectrum of VSB Signal

(e) VSB Filter Constraint

Demodulation of VSB

- Multiplier (coherent) demodulator



$$\boxed{\begin{aligned} S_v(f) = {} & \frac{A_c}{4}\left[S_m(f-2f_c)+S_m(f)\right]H(f-f_c)+ \\ & +\frac{A_c}{4}\left[S_m(f)+S_m(f+2f_c)\right]H(f+f_c) \end{aligned}}$$

$$\boxed{S_l(f)=\frac{A_c}{4}S_m(f)\left[H(f-f_c)+H(f+f_c)\right]}$$

Comparison of Conventional AM

- Conventional AM
  - Simple to modulate and to demodulate, but low power efficiency (33-50% max) and double the bandwidth
- DSB-SC
  - High power efficiency, but more complex to modulate and demodulate, doubles the bandwidth
- SSB
  - High power efficiency, the same (message) bandwidth, but more difficult to modulate and demodulate
- VSB
  - Lower power efficiency and larger bandwidth but easier to implement