Addis Ababa University
Addis Ababa Institute of Technology
School of Electrical and Computer Engineering
Electromagnetic Fields ECEG – 2122 Assignment – 3

Solve the following problems. *Show all relevant steps clearly and neatly. Justify your assumptions. Your results should include units. Unrelated and irrelevant text may result in deduction of marks.*

1. Region 1 ($z < 0$) contains a dielectric for which $\varepsilon_r = 2.5$, while region 2 ($z > 0$) is characterized by $\varepsilon_r = 4$. Let $\mathbf{E}_1 = -30\mathbf{a}_x + 50\mathbf{a}_y + 70\mathbf{a}_z$ V/m and find: (a) $\mathbf{D}_2$, (b) $\mathbf{P}_2$, (c) the angle between $\mathbf{E}_1$ and the normal to the surface.

2. Two homogeneous dielectric regions 1 ($\rho \leq 4$ cm) and 2 ($\rho \geq 4$ cm) have dielectric constants 3.5 and 1.5, respectively. If $\mathbf{D}_2 = 12\mathbf{a}_\rho - 6\mathbf{a}_\phi + 9\mathbf{a}_z$ nC/m^2, calculate: (a) $\mathbf{E}_1$ and $\mathbf{D}_1$, (b) $\mathbf{P}_2$ and ρ_{pv2}, (c) the energy density for each region.

3. (a) Given that $\mathbf{E} = 15\mathbf{a}_x - 8\mathbf{a}_z$ V/m at a point on a conductor surface, what is the surface charge density at that point? Assume $\varepsilon = \varepsilon_o$.
   (b) Region $y \geq 2$ is occupied by a conductor. If the surface charge on the conductor is -20 nC/m^2, find $\mathbf{D}$ just outside the conductor.

4. A parallel-plate capacitor has plate area 200 cm^2 and plate separation 3 mm. The charge density is 1 μC/m^2 with air as dielectric. Find
   (a) The capacitance of the capacitor
   (b) The voltage between the plates
   (c) The force with which the plates attract each other

5. A parallel-plate capacitor has its plates at $x = 0, d$ and the space between the plates is filled with an inhomogeneous material with permittivity $\varepsilon = \varepsilon_o\left(1 + \frac{x}{d}\right)$. If the plate at $x = d$ is maintained at V_o while the plate at $x = 0$ is grounded, find:
   (a) V and $\mathbf{E}$
   (b) $\mathbf{P}$
   (c) ρ_{ps} at $x = 0, d$

6. (a) State Ampere's circuit law.
   (b) A hollow conducting cylinder has inner radius a and outer radius b and carries current I along the positive z-direction. Find $\mathbf{H}$ everywhere.

7. If $\mathbf{H} = y\mathbf{a}_x - x\mathbf{a}_y$ A/m on plane $z = 0$, (a) determine the current density and (b) verify Ampere's law by taking the circulation of $\mathbf{H}$ around the edge of the rectangle $z = 0, 0 < x < 3, -1 < y < 4$.

8. For a current distribution in free space,

$$\mathbf{A} = (2x^2y + yz)\mathbf{a}_x + (xy^2 - xz^3)\mathbf{a}_y - (6xyz - 2x^2y^2)\mathbf{a}_z \text{ Wb/m}$$

(a) Calculate **B**.

(b) Find the magnetic flux through a loop described by $x = 1, 0 < y, z < 2$.

(c) Show that $\nabla \cdot \mathbf{A} = 0$ and $\nabla \cdot \mathbf{B} = 0$.

9. A block of iron ($\mu = 5000\mu_o$) is placed in a uniform magnetic field with 1.5 Wb/m^2. If iron consists of 8.5×10^{28} atoms/m^3, calculate: (a) the magnetization **M**, (b) the average magnetic current.

10. In a ferromagnetic material ($\mu = 4.5\mu_o$),

$$\mathbf{B} = 4y\mathbf{a}_z \text{ mWb/m}^2$$

calculate: (a) χ_m, (b) **H**, (c) **M**, (d) $\mathbf{J}_b$.

11. Region $0 \leq z \leq 2$ m is filled with an infinite slab of magnetic material ($\mu = 2.5\mu_o$). If the surfaces of the slab at $z = 0$ and $z = 2$, respectively, carry surface currents $30\mathbf{a}_x$ A/m and $-40\mathbf{a}_x$ A/m as in Figure 8.37, calculate **H** and **B** for

(a) $z < 0$

(b) $0 < z < 2$

(c) $z > 2$

12. Prove that the mutual inductance between the closed wound coaxial solenoids of length ℓ_1 and ℓ_2 ($\ell_1 \gg \ell_2$), turns N_1 and N_2, and radii r_1 and r_2 with $r_1 \simeq r_2$ is

$$M_{12} = \frac{\mu N_1 N_2}{\ell_1} \pi r_1^2$$