Michel Bitbol
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Editors

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Constituting Objectivity

Transcendental Perspectives on Modern Physics

Springer

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Introduction

Michel Bitbol, Pierre Kerszberg, and Jean Petitot

An appropriate starting point for this introduction consists in providing the reader with a short definition of the adjectives "transcendent" and "transcendental". All too often, these adjectives are mixed up (especially in the English-speaking philosophical tradition), and this leads to many misunderstandings. In a book entirely devoted to transcendental epistemology and its applications to physics, such misunderstandings could easily blur how each idea is perceived. This is why we must try to avoid them from the outset.

"Transcendent" and "transcendental" somehow point towards opposite directions. True, both words share a common component of meaning, which is "exceeding experience". But "exceeding" can be achieved in two antithetical ways. A transcendent *object* exceeds experience insofar as it allegedly exists *beyond* experience, as a remote (and intellectually reconstructed) external cause of experienced phenomena. By contrast, a transcendental structure exceeds experience because it is a *background precondition* of experience. Since transcendental structures concern the methods of access to experience, they have been thought of as pertaining to the *subject* of this experience by the classical tradition. But the latter notion of subject has nothing to do with psychology; it can rather be construed as a precursor of the cognitive notion of "access consciousness" in the sense of Ned Block. So, a transcendent object is supposed to wait for us "out there", and is indifferent to our intervention. By contrast, transcendental preconditions prescribe rules of active definition and selection of phenomena in such a way that one may consider them *as if* they were appearances of an object. This is the difference between merely believing in the existence of objects, and being aware of the procedure through which we *constitute* them. This also accounts for the difference between an ordinary and a critical definition of objectivity: objectivity in the first sense refers to that which possesses transcendent being; whereas objectivity in the second sense refers to what can be *made* valid for any one of us, independently of our situation, but *not independently of the fact of being situated*.

Kant was the primary source of the distinction we have just stressed between "transcendent" and "transcendental". The contrast develops thus:

M. Bitbol et al. (eds.), *Constituting Objectivity,*

> (…) As soon as we posit the unconditioned (…) in what is entirely outside the world of sense and hence outside all possible experience, the ideas become *transcendent*".[1]
> "I call *transcendental* all cognition that deals not so much with objects as rather with our way of cognizing objects in general insofar as that way of cognizing is to be possible *a priori*.[2]

Despite this clear distinction, Kant's own use of the word "transcendental" is sometimes misleading. This is the case when he writes e.g. the expression "transcendental realism", which could roughly be interpreted as "transcendent realism". The reason why he still uses the word "transcendental" instead of "transcendent" in this context is that he wishes to make a distinction between two misuses of our intelligence. The fist misuse consists in extrapolating the application of the principles or categories of pure understanding (a major component of the transcendental preconditions of knowledge) beyond the limits of possible experience; it gives rise to what Kant calls "transcendental illusion". The second misuse consists in manipulating entirely new speculative principles "(…) requiring us to tear down all these boundary posts"[3]; it gives rise to the representation of fake transcendent realms.

To recapitulate, "transcendent" connotes an attempt at breaking up the limits of experience, whereas "transcendental" refers to a reflective move in which one examines the (subjective) conditions of possibility of this experience. "Transcendent" points towards the farthest, whereas "transcendental" brings us back to the closest (which is usually inapparent due to its being too close). Accordingly, elaborating a transcendental epistemology of physics does not mean looking for hidden entities beyond empirical knowledge, but rather undertaking a reflective research about the indispensible preconditions of our knowledge and their relevance to the structure of physical theories.

1 Bringing Transcendental Epistemology Back to Life

As indicated in the title, this book concerns transcendental approaches of *modern* physics. This may seem surprising as it has become commonplace to assume that transcendentalism has been invalidated by the successive developments of physics after Newton. Most philosophers of science think that "transcendental" and "modern physics" are two terms which have long since become incompatible. Their idea is

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[1] I. Kant, *Critique of Pure Reason*, B 593, in: *The Cambridge Edition of the Works of Immanuel Kant*, Cambridge: Cambridge University Press, 1999.

[2] I. Kant, *Critique of Pure Reason*, B 25, in: *The Cambridge Edition of the Works of Immanuel Kant*, op. cit.

[3] I. Kant, *Critique of Pure Reason*, B 352–353, in: *The Cambridge Edition of the Works of Immanuel Kant*, op. cit.

that the limits of Kant's philosophy of science indicate the limits of critical philosophy. One may counter this strong prejudice by mentioning three points.

(i) As indicated by the title, the central problem of this book is the *constitution of objectivity*.
(ii) Transcendental approaches therefore intervene as a general philosophy of constitution, not as a special inventory of fixed mental «faculties».
(iii) There is no reason which prevents us from thinking that, in this respect, transcendentalism can be generalized far beyond its kantian version, even if updating it means distancing oneself from a literal reading of Kant. Kant initiated an approach which has many more resources than those he himself developed.

In the same way as the original version of empiricism which came about during the Scottish enlightenment has been generalized and deepened to a considerable extent, well beyond what its founders had envisaged, by modern epistemologies such as the logical empiricism of the Vienna circle or Bas van Fraassen's constructive empiricism, the original version of transcendentalism formulated by critical rationalism can also be generalized and deepened to a considerable extent, well beyond what its founder was able to imagine. True, according to some researchers, this distancing strategy distorts the Kantian perspective so much that it no long deserves the name "transcendental". This pushed them to espouse the advances of physics against a philosophy which nevertheless offers the best epistemology of classical mechanics. But, as this book aims to show, adapting transcendentalism is much more fruitful than rejecting it; and such an adaptation turns out to be very faithful to its Kantian sources, in its spirit and even sometimes in its letter.

As it is well known, the transcendental question arises as soon as one realizes that the central and specific epistemological problem of physics is that of *mathematical* physics. Indeed, fundamental equations are able to generate myriads of precise mathematical models of the variety of observable phenomena, out of universal principles and general concepts. One can express this by saying that these models realize a "computational synthesis" of phenomena. This is a modern form of what Kant called "mathematical construction", when he pointed out in his time (see *Prolegomena*, AK, IV, 272) that Hume empiricism tended to underrate the problem of mathematics.

Actually, there exists a radical contrast between conceptual abstraction (which is a subject for an Analytic) and computational synthesis; a contrast that can be seen as regarding the difference between a direct problem and a *reverse problem*. The direct problem consists in abstracting from the manifold of intuition; it consists in "subordinating" this manifold to what Kant called "the unity of a concept" and what we would call today a categorizing concept. By contrast, the reverse problem consists in *constructing* the referents of concepts by transforming conceptual contents into algorithms for computing these referents. The reverse problem starts from concepts and points towards the manifold of intuition, not the other way around. Mathematics, helped today by methods of numerical simulation, are the essential tool of computational synthesis.

The mere fact that physics involves a computational synthesis of observable phenomena means that physical objectivity cannot be tantamount to an ontology of some independent substantial reality. Indeed, the possibility of a mathematical reconstruction of such an *ontological reality* would ascribe the human mind excessive intellectual capacities which transcend its finiteness. This leaves only two options:

(a) Physics is purely descriptive. It conceptually organizes the empirical manifold by means of an Analytic, and it can thereby pretend it describes an ontological independent reality, but without reconstructing this reality mathematically and without doing any job other than picturing it passively (empiricism + nominalism).
(b) Physics can reconstruct the empirical manifold mathematically, and it must then accept to partake of a "weak" form of objectivity which *de jure* can only concern relations between observable phenomena, namely a reality filtered by ineliminable conditions of experimental or sensory *accessibility*, and by intellectual criteria of selection. The condition of possibility of computational synthesis is the principle of restriction of physical knowledge to laws of observables, and the decoupling between a "strong" ontology and a "weak" objectivity.

The general assumption of this book is that modern physics is dominated by the second attitude, and that it raises an increasing number of questions on the processes of constitution of objects connected with the mathematization of observable data. In our opinion, the term "transcendental" essentially refers to that concern. The use of this term is still justified insofar as it can be shown (see Section 3 of this introduction) that appropriate extensions of Kant's transcendentalism push most of the apparently definitive criticisms which had been formulated against it in the name of the "revolutions" represented by General Relativity and Quantum Mechanics to obsolescence.

This introduction is not the right place to develop the basis of the physical transcendentalism which generalizes Kant's analysis of newtonian mechanics in the *Metaphysische Anfangsgründe der Naturwissenschaft (Metaphysical Foundations of Natural Science, abbrevieted by MFNS*, Kant, 1786). Yet, it is useful to think of it as a generic model for other transcendental readings of mathematical physics. We will therefore outline it by enumerating the following points:

1. Mathematical physics is an objective theory of *observable phenomena*. The conditions of observability are therefore constitutive of the very concept of a physical object. Since the concept of a phenomenon is relational, namely relative to structures of accessibility, to conditions of observations and to measurement results, physical objectivity cannot *de jure* bear on an independent reality. Due to its principle of reduction to observable phenomena, physical objectivity cannot, here again, be an ontology but only a "weak" objectivity.
2. Although it is non-ontological, physical objectivity is not naively subjective-relative either. This is due to the fact that it consists in an act of universal legalization of

phenomena. It expresses a prescriptive law-like order which imposes a norm onto any description of phenomena.

3. Prescribing a law-like order imposes using an apparently paradoxical procedure. This procedure must indeed take the conditions of accessibility to observables into account, but without including the theory of instruments of observation into the theory of physical objects.
4. The categories and principles of physical objectivity - "system", "state", "property", "causality", "interaction", etc. – must then be interpreted mathematically according to the former points. They are not ontological categories, they are prescriptive rather than descriptive, and they incorporate their conditions of accessibility.

In classical mechanics as interpreted by Kant in *MFNS*, point (1) is expressed by the reduction of the scope of physics to sensory phenomena, point (2) is expressed by the Analytic of concepts, point (3) is expressed by the transcendental Aesthetic which explains why mechanics consists of a differential geometry of motions in space–time, and point (4) is expressed by the procedure of schematism, or the construction of categories. But there is no reason to restrict this transcendental analysis to classical mechanics. In quantum mechanics, for instance, one can consider that: point (1) is expressed by Heisenberg's reduction to observables, point (2) remains a transcendental Analytic, but with some alterations, point (3) corresponds to probability amplitudes and operator algebras in Hilbert spaces of states, and point (4) is a reinterpretation of the categorial Analytic in this new framework.

In his *MFNS*, Kant then exposes the following features of classical mechanics, by using a one–one correspondence with his table of categories as described in the *Critique of Pure Reason*:

(i) *Phoronomy (Kinematics)*. The measurement of the phenomena of motion is derived from the metric of space–time. In other terms, space as a form of presentation and manifestation of phenomena (conditions of observability = forms of intuition) becomes geometry (what Kant calls "formal intuition") in the context of physics. Kant discovered that Euclidean space is a background structure for mechanics and that, due to Galilean relativity, this Euclidean structure cannot be dissociated from the principle of inertia (more about this later). The symmetry group of Galilean relativity is therefore expressed philosophically by the transcendental ideality of space. Thus, in his book about Kant's conception of physics, Jules Vuillemin insists on the phoronomic meaning of the transcendental ideality of space: "It is the principle of phoronomy which offers the true demonstration of transcendental aesthetic (…). It is the relativity of motion which makes the subjectivity of space [its transcendental ideality] transcendentally necessary".[4] Kant was the first philosopher who identified – as soon as 1758 with his *New Theory of Motion and Rest … (Neuer Lehrbegriff der Bewegung und Ruhe…)*, and in 1768 with his *Ultimate Foundation of the*

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[4] J. Vuillemin, *Physique et Métaphysique kantiennes*, Presses Universitaires de France, 1955, pp. 59–60.

Distinction of the Directions in Space (Von dem ersten Grunde des Unterschiedes der Gegenden im Raume) – the philosophical consequences of the fact that symmetries of space (e.g. chirality) which are irreducibly "non conceptual" exist.

(ii) *Dynamics*. Motion is described by means of intensive magnitudes, such as velocities and accelerations (i.e. "moments"). Therefore, mechanics is *a priori* a differential geometry, and the differential descriptions must be compatible with phoronomic relativity: this is an outline of the concept of covariance. J. Vuillemin also insists on this, and draws a major philosophical conclusion: "that dynamics presupposes phoronomy means the possibility of a Copernican revolution about the concept of substance, a revolution which is likely to be at the heart of Kant's idealism".[5]

(iii) *Mechanics*. By way of temporal schematism which defines it as a principle of permanence, the category of substance is the source of any principle of conservation of physical magnitudes, namely of physical principles of invariance (conservation of energy, momentum, etc.). Besides, causality is expressed by forces.

(iv) *Phenomenology*. Galileo's principle of relativity stems from the fact that absolute motion cannot be an object of experience. In kinematics, this means that the state of motion cannot be a *real* predicate, but only a *possible* predicate. It cannot be interpreted as a real transformation of the real internal state of the system, and of some of its properties taken as intrinsic mechanical properties. Hence, one can both assert and negate motion without any contradiction. In other terms, the relativity of motion invalidates the spontaneous ontological interpretation of statements such as "the body S *has* such and such position and velocity" in terms of a verb "to have" which would mean "to possess (a property)". Neither a spatial or temporal absolute position, nor the absolute velocity (of a uniform motion in straight line) are observable. Dynamics however affords criteria of reality of motion, since forces are real predicates. This reality is ruled by laws of mechanics which are *necessary*. Here, necessity is not to be understood from the standpoint of logic, but from a transcendental standpoint: it is a conditional necessity, relative to the radical contingency of experience.

Another important feature of Kant's approach is the "construction" of categories, when they are applied to a regional object such as motion. It is well-known that, in the *Critique of Pure Reason*, there is a difference between the so-called "mathematical" and "dynamical" categories. Unlike "mathematical" categories (which, by schematization, give rise to the "axioms of intuition" and to the "anticipations of perception" in the *Analytic of Principles*), "dynamical" categories (such as the categories of relation which, by schematization, give rise to the "analogies of experience") *posit* existence and *condition* it, while leaving it *undetermined*. This means that they are *not* constructible. Since they only apply to the object in the most general sense, they are "mere forms of thought", and are therefore only schematizable. But they become "constructible" – and thereby acquire "objective reality", "meaning", and "truth" – when they are applied to an "an additional determination", such as motion, which

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[5] J. Vuillemin, *Physique et Métaphysique kantiennes*, op. cit., p. 87.

"contains a pure intuition". This is a crucial point to understand the relation between the *Critique of Pure Reason* and the *MFNS*, between a transcendental theory of knowledge and a transcendental approach of physics.

To sum it up, Kant was the first thinker who developed the heart of modern physics constituted by the correlation between: *relativity, symmetry, covariance, invariance, and conservation* as a philosophical theme. It is precisely this correlation that has been generalized, diversified and deepened in fundamental modern physics (see Section 4 of this Introduction). It is therefore astonishing to see that a philosophy such as transcendental philosophy, which is so relevant to the essence of mathematical physics, has been rejected instead of being steadily improved along with the advances of science.

2 Various Interpretations of Kant's Project for Constituting Objectivity: A Short Historical Outline

We will now briefly focus our attention on the history of Transcendentalism after Kant. This will help us to realize that appropriate generalizations of Transcendentalism were hindered by a combination of over-speculative interpretations and rigidly Kantian interpretations. This unfortunately led to the adoption of other epistemological traditions which were not as well adapted to the essence of mathematical physics as transcendentalism. But, at the same time, this history shows that another path could have been followed. The carefully scientific and flexible version of Transcendentalism advocated by the various neo-Kantian schools of the turn of the nineteenth and twentieth century was a good starting point for this alternative way.

But let's first come back to our basic question. We have just seen that, as many authors from Hermann Cohen to Michael Friedman pointed out, it was his remarkable vision of the scientific theories of his time that enabled Kant to form the project of transcendental philosophy. If contemporary science rejects these theories, is transcendental philosophy bound to collapse as well? One common idea is that the historical limits of Kant's philosophy of science indicate the limits of critical philosophy altogether. The consensus until now has basically been that Kant might have been right in claiming that rules exist ahead of experience, but he was faulty inasmuch as he seems to have believed that some rules are *definitive* as they reflect immutable structures of human reason. A short (and therefore incomplete) outline of the historical development that led to the sciences being disentangled from a Kantian *foundation* will now help us understand why and how some kind of rapprochement between the sciences and Kant's general project can be obtained.

Let us first highlight some of the limitations of Kant's system. Natural science and the theory of knowledge are closely interrelated in Kant. Whereas modern science has progressively disconnected the perceptual object from the scientific object, the whole of Kant's original version of critical philosophy seems to be bound to some fixed balance between perception and cognition. Kant then brought together: (i) a statically conceived metaphysics of nature and (ii) an advance in empirical knowledge of nature, which is in principle endless. As a result, Kant could not give us the means to fully apprehend knowledge in its historical development. He perfectly accepted the idea of a historical evolution of the empirical content of science, but not an alteration of principles. Accordingly, many features of science are missing in his system. He did not make room for leibnizian principles of least action from which Lagrangian formalisms are derived. In his mechanics, Kant also lacks the concept of Work, which is why his epistemology cannot be applied to thermodynamics. Besides, Kant's laws of nature are related to dynamics, and it would appear that they have no bearing on statistical laws. As a result, the allegedly immutable system of categories turns out to be both narrow and false.

This is precisely the challenge Kant faces today: How can we preserve the ideal of unity of knowledge, without ignoring the widening gap between common and scientific experience? Is there a way of vindicating Kant's theories despite the fact that in the present state of physics the *a priori* (normative component of knowledge) is virtually impossible to separate from the empirical?

But this task took time to even be defined as such. The initial phase in Kant's reception was operated by the idealistic movement. Fichte was the first author to emphasize the need for the primacy of practical reason over theoretical reason in his philosophy, and to assert that this reversal made the completion of Kant's system possible. This strategy culminated in German Idealism, particularly through Hegel who argued for a totalizing view of knowledge which includes comprehensive concepts of natural and historical processes. But Kant's views were also supported and reinterpreted by the pioneers of *Naturphilosophie* in Germany. Since mechanistic materialism was commonly taken as a necessary consequence of classical mechanics and mathematical physics in general, there was a search for alternative sorts of natural science which would in turn offer a vindication of the anti-materialist concepts of natural philosophy. Kant's *Critique of Pure Reason* was thus interpreted as opening up the possibility of divorcing classical mechanics from materialistic dogmatism for the first time. As for the *Critique of Judgment*, with its reflection about aesthetics and about teleology in biology, it provided resources for an anti-mechanistic conception of nature influenced not only by physics but also by biology. From Kant's description of the formal *a priori* background of knowledge there arose, as a result of the objective turn which Schelling gives to the Fichtean notion of intellectual intuition, a new metaphysics of nature. The subjective formal *a priori* was converted into a formative power at work in nature. The power of understanding was replaced with a creative force shaping organic development.

After the demise of this metaphysical natural philosophy (which took place around 1830), when this speculation could no longer be taken seriously from the scientific point of view, the fundamental tendency in science can be described as one of partial unification of theories and methods combined with a simultaneous explosion of experimental knowledge. To be sure, the mere idea of a completely unified natural science was unimaginable at the time. But the adventure of metaphysical natural philosophy left its traces: mechanism, as a total explanation of nature, became either a mere program or a philosophical dogma. The theories of heat, optics, magnetism, and electricity were largely independent divisions of physics, with a remote perspective of a unified mechanical interpretation and a more immediate urge for partial unification under appropriate principles. Most of the important innovations then arose as the result of a project of integration of these separate branches of physics; a project in which one can still feel the influence of Kant's philosophical impulse. For instance, the integration of magnetic and electrical phenomena by Oersted was motivated by the application of Kant's metaphysical claim concerning the duality and interaction of two fundamental forces (attraction and repulsion) to physics. This led to the theory of electromagnetism, which Faraday connected to mechanics, and Maxwell and Hertz to optics. The project of innovative integration transcended the limits of physics itself, also affecting chemistry and other disciplines; something that Kant had anticipated in his later *Opus Postumum*.

After Hegel, Schopenhauer rediscovered Kant's need for distinction between phenomena and things in themselves. Accordingly, the vindication of Kant in the second half of the nineteenth century concerned his epistemological contribution as expressed in the *Critique of Pure Reason*, rather than his *Metaphysical Principles of Natural Science*. However, even Kant's epistemology was subjected to intense scrutiny. After all, the key notion for post-idealist, anti-metaphysical philosophy in the nineteenth century was inductivism. From an inductivist standpoint, the Kantian *a priori*, along with all concepts, laws and theories, was conceived as nothing more than the result of empirical generalization. Thus, according to Helmholtz, the point at which natural science and metaphysics come into contact with each other is the theory of human sense-perception. Helmholtz therefore presented the results of enquiry into the physiology of perception in such a way that they fitted perfectly with transcendental philosophy. Science could now be seen as an open system of knowledge: a totality which is constantly growing and changing as a result of experience, so that science as a system of true judgments about the world is projected in the future; instead of delivering truth *via* fixed categories and intuitions, science is understood as a gradual approximation of truth. This is perfectly expressed in the view that came to be called a descriptivist or phenomenological view of natural science. The exclusion of metaphysics compelled physics to confine itself strictly to what is given, and what is given are phenomena. Concepts of substance or force were accordingly eliminated from science (Wundt, Hertz).

According to this view (in good agreement with the spirit of Kant's epistemology), the only concepts which should be used are those which make it possible to express functional connections between phenomena, so that the search for an underlying ontology is abandoned in favor of increasingly abstract mathematical representations of observables. Boltzmann, who supported this view to a certain extent, was convinced that the laws of thought arose by internal ideas' being applied to actually existing objects, so that the existing laws of thought are inherited habits in a Darwinian sense. Current evolutionary epistemology considerably developed this approach. In it, the transcendental basis of knowledge is entirely re-interpreted in terms of the biological preconditions of experience. And the *a priori* is construed as the byproduct of an experience of the human species that became innate in the individual. This is a short step to abandoning the Kantian *a priori* as precondition of experience, since considering the *a priori* as an "organ" (something that resulted from phylogenetic adaptation to the experienced external world) destroys the very concept of the *a priori* in Kant's original sense, namely as a *precondition* of experience. This is also not very easy to reconcile with several of Kant's explicit statements (especially in his *Response to Eberhard)*, according to which *a priori* does not mean "innate".[6] However, those who defend a Darwinian and naturalized conception of transcendental philosophy can still rely on the fact that, even though Kant insists that *a priori* forms themselves are "originally acquired", and therefore not innate, he also considers that the *foundation* of this cognitive process of original acquisition is itself innate.

But it should now be borne in mind that there is more to Kant than his strictly critical system. For instance, the pre-critical *Universal Natural History and Theory of the Heavens* was the first coherent cosmogonical model compatible with suitably revised basic tenets of Newtonian mechanics. This theory can be seen today as pioneering the kind of evolutionary models in natural science, which became fashionable long before Darwin.

The physiological and Darwinian interpretation of Kant's intuitions and categories was countered by neo-Kantianism, even though historicizing the *a priori* seemed from now on to be an inescapable route for any plausible revival of transcendental philosophy. At the turn of the twentieth century, Neo-Kantianism was the most important philosophical movement which developed in the intellectual climate of positivism. Its aim was to forge a new philosophy as an exact science, on the basis of the principles of Kant's theory of knowledge. The central argument was that the essential aim of transcendental philosophy is to identify the fundamental methods and concepts of natural science. Hermann Cohen, who founded the Marburg School (later developed by Natorp and Cassirer), substituted a strictly logical conception of the Kantian program for the physiological interpretation inherited from Helmholtz. Here, intuition must be understood as a source of knowledge rather than as a psychological faculty implemented on a physiological substrate. Insofar as critical philosophy restricts philosophical reflection to the conditions of possibility of *science*, the fall into the psychological or physiological interpretation of the categories is completely avoided. After all, the function of the transcendental subject is to provide the necessary conditions without which "nature", *including the part of nature referred to by the physiological reading of Kant*, means nothing at all. However, the Marburg School replaced Kant's original "static" or timeless version of the synthetic *a priori* with what they perceived as an essentially developmental or "genetic" conception of scientific knowledge. The crucial point is that, in this case, development is represented by the ongoing history of science rather than the past history of our species.

The most famous representative of the Marburg school was Cassirer, who developed his early thesis about the relational-functional character of scientific laws in the

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[6] H. Allison, *Kant's Theory of Taste*, Cambridge: Cambridge University Press, 2001, p. 17, AK VIII 221

context of classical physics, and his conception of the functional and historicized *a priori* in light of the then recent developments of the theory of Relativity (see Section 3 of this introduction). Cassirer argued that the genetic process of science is such that general laws at an earlier stage, are exhibited as approximate special cases of the still more general laws at a later stage, one obvious example being the road from Newton to Einstein. This being granted, many features of scientific theories that claim to be representations of things "out there" are reinterpreted as mere tools for this open task of generalization. For instance, non-Euclidean geometry as it intervenes in General Relativity does not express the nature of things themselves, but rather the laws and relations appropriate to a given stage of the systematic organization of science. One should not, says Cassirer, speculate about the *being* of space, but rather inquire into how scientists *use* geometrical structures.

Cassirer also retained from Kant that the meaning of a concept is not tantamount to a mere abstraction out of the variety of its applications; the meaning of a concept must rather be identified ahead of application. Hence the idea, developed by some successors of Cassirer (e.g. G. Buchdahl), that Kant's theories can be salvaged if the locus of the transcendental is not the constitutive dimension of the categories of understanding, but the regulative ideas of reason. In this case, the value of transcendental philosophy has shifted from the laws to the organization of these laws. This was a good way to *go beyond Kant while grounding the move on Kantian premises*, according to Cassirer's famous slogan.

The theme of the flexibility of *a priori* forms, on which the neo-Kantian Marburg school insisted so strongly, was developed in many other ways outside this school. Perhaps the most extreme (yet a-historical) way of advocating flexibility while preserving the basics of Kant's philosophy in light of contemporary mathematics and natural science, was advocated by Poincaré. Poincaré considered that: (i) the idea of a system of fixed categories as a foundation of natural science contradicts the history of natural science; (ii) a conventional (free) choice in the determinations of space and time have to supersede space and time as *a priori* forms of sensibility. In spite of this radical criticism of Kant's foundationalism, Poincaré still perceived his own epistemology as Kantian. Indeed, he merely shifted Kant's issue concerning the synthesis of the objects of knowledge to the problem of whether objective *relations* between objects can be described in terms of subjective capacities (including the visual, tactile and motor faculties that, according to him, underly our notion of space). He also thought that a generalization of Kant's theory of space to spaces of constant curvature is possible provided one replaces Euclid's axioms with a more general principle: the principle of free mobility allowing for the arbitrary continuous motion of rigid bodies.

In another investigation of the structure and function of natural science, Kant's transcendentalism was confronted with history even more brutally than in neo-Kantianism. According to E. Meyerson, stronger than the rational demand for lawfulness, is the demand for *identity*. The development of modern natural science, he says, reflects a perpetual dialectical opposition between: (i) the mind's *a priori* demand for substantiality, and thus absolute identity through time, and (ii) nature's irrational *a posteriori* resistance to such a demand. Interesting developments can be derived from this remark. Indeed, identity is more precisely instantiated by the concept of *invariance*, which is highly relevant for the symmetry groups that have played an increasingly prominent role in contemporary physics (e.g. the Lorentz group in special relativity). In agreement with his neo-Kantian conception of science, Cassirer argued that group theory does not represent “reality”, but is an instrument endowed with transcendental function, insofar as it provides the active link between the demands of the knowing subject and the definition of its object. By and large, invariance posits a new concept of objectivity disconnected from any ontological claim. Here, an object (or a class of objects) of a theory is specified as nothing else and nothing more than a bundle of invariant features.

3 Constituting Objectivity in Relativity and Quantum Physics

The accusation according to which Kant’s epistemology had become irrelevant to modern physics, was developed in intricate details as a reaction to the relativistic and quantum revolutions. Hence the need for a more detailed study of the role of these two theories in the debate about the possibility of a renewed transcendental approach.

To begin with, Relativity seemed to discard Kant’s *Transcendental Aesthetic* with its doctrine of space as an intuitive *a priori* form. Einstein stressed that, in view of the newly established status of non-euclidean geometry in the theory of gravitation, Kant’s thesis that a three-dimensional euclidean space is an *a priori* form of the human faculty of knowledge must be wrong. In Einstein’s own words, «Unlike one is ready to declare that relativity theory is averse to reason, one cannot stick any longer to Kant’s system of *a priori* concepts and norms».[7]

Similarly, quantum mechanics seemed to discard Kant’s *Transcendental Analytic*, with its doctrine of substance and causality as categories, namely as conceptual *a priori*. Heisenberg was especially instrumental in denouncing both concepts as inapplicable to the quantum domain. He first claimed, in his *uncertainty relations* paper of 1927, that «quantum mechanics establishes the final failure of causality». Later, in 1929, Heisenberg became both more nuanced and more accurate. He no longer claimed that there was no room for causality in quantum physics. He rather pointed out that applying the law of causality and locating phenomena in space–time were *complementary* approaches, namely approaches that mutually *exclude* each other. But if causal laws cannot apply to spatio-temporal phenomena, Kant’s theory of knowledge is no longer valid, since his crucial category of causality has no other legitimate domain than appearances in space–time. In his book *Physics and philosophy*, of 1958, Heisenberg then explicitly stated that «Kant’s arguments for the *a priori* character of the law of causality no longer apply».[8]

Yet, at the same time as Kant’s conception of knowledge was thus challenged, several neo-kantian philosophers found many reasons in modern physics to not

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[7] A. Einstein, *Oeuvres choisies, 5*, Seuil, 1991, p. 221.

[8] W. Heisenberg, *Physics and philosophy*, Penguin, 1990, p. 78.

only stick to the basic ideas of transcendental epistemology as formulated by Kant, but to generalize and even to amplify them. The central motivation of this return to Kant was that both relativistic and quantum theories reactualized the basic move of the so-called «Copernican revolution». In both theories, one could no longer focus exclusively on a description of objects, but had to seriously consider the cognitive, or at least instrumental, pre-conditions of this description. In other terms, a reflective attitude, that is typical of transcendental epistemology, was required.

In the theory of relativity, Ernst Cassirer thus noticed that one must: (i) investigate how measurements of length and duration are obtained and coordinated, and (ii) formulate a systematic method of extracting invariants from them.

In quantum mechanics as understood by Bohr, the conclusion to be drawn was even more general. Here, considerations about *contextuality*, about how any micro-phenomenon whatsoever is both relative to and indissociable from an experimental context, are central. Grete Hermann, a German philosopher who had extensive discussions with Heisenberg in 1934, concluded that not only had Kant's philosophy not been refuted by quantum mechanics, it had also been made more *indispensible* and pushed to its most radical consequence in the new physics.

So, at this point, we must list and discuss some strategies for promoting the essential ideas of Kant's theory of knowledge, without sticking to the historical features of the doctrine that were clearly made obsolete by relativistic and quantum theories. We believe that there are essentially three such strategies.

1. The first strategy consists in restricting the validity of Kant's original synthetic *a priori* to the direct environment of mankind, in which classical physics remain a good approximation.
2. The second strategy amounts to formulating new pre-conditions of knowledge that are general enough to encompass the extended domains of phenomena which are accounted for by modern physical theories and, hopefully, any future physical theory as well.
3. Finally, the third strategy consists in «relativizing» the *a priori*, namely making it relative to a certain situation of science that can change from one step to another of its history.

Restricting the domain of validity of Kant's forms of intuition and categories is what Einstein, Bohr and Heisenberg did almost spontaneously after they had formulated their revolutionary theories. All these authors expressed the idea that Kant's *a priori* forms remain unshakable anthropocentric foundations of physical knowledge.

Thus Einstein pointed out in 1921 that Riemannian geometry is grounded on the presupposition that there are rigid bodies which behave as if Euclidean geometry were *locally* valid. But he also warned against any reification of this local validity. As he wrote, "The concepts which proved useful in order to establish a certain order easily acquire for us such an authoritative status that we forget their earthly origin and that we come to construe them as immutable data".[9] So, according to Einstein,

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[9] A. Einstein, *Oeuvres choisies, 5*, Seuil, 1991, pp. 75, 226.

Kant's forms of intuition are nothing else and nothing more than local principles of order which act as minimal presuppositions for any further attempt at extending physics beyond the limited environment of mankind.

As for Bohr and Heisenberg, they promoted the same idea, but applied it to Kant's categories, especially substance and causality, rather than to the forms of intuition. According to both of them, the classical organization of macroscopic experience is a precondition for any further theoretical development, including Quantum Mechanics. But Kant's categories are clearly preconditions for this classical organization. These categories therefore work *de facto* as second-order anthropocentric presuppositions of quantum mechanics, even though they cannot work as general first-order presuppositions that are directly applied to microscopic phenomena. Heisenberg thus remarked that: "What Kant had not foreseen was that these *a priori* concepts can be the conditions for science and at the same time have a limited range of applicability".[10]

Accepting that the constitutive role of the categories of the *Critique of Pure Reason* only applies to the meso-macroscopic domain looks like a partial renunciation of the Kantian project. Yet, one must not forget that Kant's philosophy has enough resources to *also* formulate constructive propositions on what does not directly fall under the joint rule of forms of intuition and categories of pure understanding. Let us take an example. Kant claimed that certain figures of non-euclidean geometry are impossible insofar as the possibility of *constructing* them in *intuitive* space is concerned. But he also accepted that "(…) there is no contradiction in the *concept* of a figure enclosed by two straight lines".[11] It can be inferred from this that Kant did not exclude using such concepts in order to fulfill the need of a systematic unity of the laws of physics according to what is prescribed by the power of judgment.[12]

This resource has recently been used to make sense of Quantum Mechanics in a strictly Kantian framework.[13] The approach here consists in understanding quantum theoretical structures, not as direct expressions of the constitutive function of categories, typical of the *Critique of Pure Reason*, but as a formal transcription of a project of unity of the system of nature, typical of the *Critique of Judgment*. Let us see how this can be done. We know that any prospect of conceptual unity appeared to be blocked in the period of edification of quantum theories, between 1900 and 1924, when one had to accept that using mutually exclusive representations such as the corpuscule and wave pictures, cannot be avoided. Some sort of unity was restored only when Bohr formulated his concept of "complementarity", according to which these two exclusive representations (i) are relative to different types of

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[10] W. Heisenberg, *Physics and philosophy*, op. cit., p. 78.

[11] I. Kant, *Critique of Pure Reason*, A 220, Hackett, 1996, p. 284.

[12] S. Palmquist «Kant on Euclid: geometry in perspective», *Philosophia Mathematica II* 5:1/2, 88–113, 1990.

[13] H. Pringe, *Critique of the Quantum Power of Judgment: A Transcendental Foundation of Quantum Objectivity*, De Gruyter, 2007. See also in this volume: H. Pringe, «A transcendental view on correspondence and complementarity».

experimental devices and different types of correlative classical concepts, and (ii) jointly characterize "one and the same object". However, it must be realized that the hypothetical object towards which the two complementary representations are supposed to converge cannot be said to simultaneously *possess* the two corres-ponding properties. No *constituted* object can therefore be said to be "behind" the contextual phenomena. Bohr's "objects" are only *regulative* devices used as unifying *symbols*, with a merely "as if" causal role. In the same way as in Kant's *Critique of Judgment*, one must here use a purely «symbolic analogy», instead of the normal constitutive «analogy of experience» which would only be available for proper objects of intuition.[14]

But this attempt at finding resources in strict accordance with Kant's texts, including the *Critique of Judgment*, is by no means the only way of maintaining and developing the relevance of transcendental epistemology in modern physics. Let us then turn to the second available strategy, which consists in generalizing the synthetic *a priori*. Here, the hope is to succeed where Kant failed, namely finding some *really necessary* preconditions for *any* empirical knowledge at *any* time of history. Along with this perspective, the project aims to show that the basic structures of physical laws essentially express the structures of these very broad presuppositions. Demonstrating that, is what one may call 'giving a transcendental justification' of a physical theory. But these two aims are likely to be conflicting. Indeed, a set of preconditions general enough to be universal and perennial is likely to be so poor in content that very little of the law-like structures can be justified by it. While a reasonably large part of the law-like structure of Classical Mechanics could be transcendentally justified by Kant's preconditions, a much smaller part of any physical theory would be justified by truly general preconditions.

This strategy was especially advocated by C. F. von Weizsäcker,[15] in his most recent work. He formulated two central preconditions for any scientific knowledge, far more general than Kant's. The first precondition is that it must be possible to *discriminate* between at least two phenomena. The second precondition is that one must be able to distinguish between *potential* and *actual* phenomena, namely between future and past, between prediction and possession of information. Undoubtedly one can hardly conceive any item of scientific knowledge that does not rely on a possibility of discrimination and on the prospect of gaining information in the course of time. But, in view of these presuppositions' being so elementary, it is not surprising that Von Weizsäcker's project of deriving modern physics from them failed, except for some very broad features.

Let us then turn to the third strategy for giving transcendental epistemology new relevance in modern physics and recognizing its constitutive features: the strategy of relativized and historicized *a priori*.

Here are first some arguments in favor of the relativized *a priori*.

To begin with, it is clear that the relativized *a priori* is fully compatible with the two previous options. Bringing out specialized relative *a priori* structures does not

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[14] I. Kant, *Critique of Judgment*, section 59, Hackett, 1987, p. 227.

[15] C.F. Von Weizsäcker, *The structure of physics*, Heidelberg: Springer, 2006.

prevent one from extracting more universal preconditions. Indeed, the most general and poorest preconditions of empirical knowledge (such as Von Weizsäcker's) might easily be construed as invariants of the many special and richer preconditions for each region of experience. Furthermore, saying as Einstein, Bohr, and Heisenberg did, that the validity of Kant's forms of intuition and thought is restricted to our mesoscopic environment, can also be taken to mean that Kant's forms are relative to the range of rules and procedures taking place within this environment. They are preconditions for the most familiar region of experience. This being granted, Einstein's, Bohr's and Heisenberg's restriction can be taken by contrast as an incentive to identify new anticipative forms which are relevant to the new range of phenomena and procedures explored by microphysics.

Another point in favor of the relativized *a priori* is that, as we will now see, it is not too difficult to confute the accusation according to which it is empty, arbitrary, and amounts to little more than a restatement of basic scientific methodologies.

This accusation was first formulated by Einstein against Cassirer's reading of the theory of Relativity. According to Einstein, "One can always set up a system of *a priori* elements in such a way that it is not contradictory with a given physical system".[16] If this trend towards relativization is pushed to its ultimate consequences, Einstein concludes, one lands into little more than the hypothetico-deductive method. The only component of Kantianism which still seems to be retained at this point is a recognition of the spontaneity of reason, namely the fact that our reason always tends to anticipate phenomena with a set of constructive hypotheses. Transcendental methodology would then be reduced to Peirce's Abduction, or to Popper's conjecture of regularities.

So, if a transcendental epistemology is to retain any specificity at all, one must not push relativization to a point where it can no longer be distinguished from an ongoing dialectic of conjectures and tests. But is this possible? We think the answer is «yes»: this is indeed possible. There is a key difference between an *a priori* background and a mere conjecture. The difference bears on *necessity*. An *a priori* form is somehow necessary; not a conjecture. But of course, the concept of necessity must here be seriously qualified if we do not want to fall back into the absolute and eternal *a priori* forms of Kant.

Let us illustrate this idea of a qualified necessity with an example.

Hans Reichenbach was probably the first author who formulated, in 1920, the idea of a relativized *a priori* in direct response to modern physics (in his *Theory of relativity and a priori knowledge*). But according to him one must carefully separate: (1) the constitutive components, and (2) the apodictic (or necessary) components, of the *a priori* in physics. Along with point (1), Reichenbach insists: (i) that one can isolate «coordinating principles» which are crucial to any physical theory[17]; and (ii) that these coordinating principles *are constitutive of* the objects

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[16] A. Einstein, *Oeuvres choisies, 5*, op. cit. p. 222.

[17] For instance, the Lorentz transformation, which was still an empirical law in Lorentz' physics, became a true background coordinating principle in Einstein's physics.

of this theory, because they prescribe the framework against which some phenomena can be interpreted as fleeting appearances of permanent objects. But, says Reichenbach, they are by no means *necessary*, unlike Kant's categories of pure understanding. The component of the *a priori* referred to in point (2) must then, according to him, be relinquished.

In this version of the relativized *a priori*, there is clearly more than in the hypothetico-deductive method, since the coordinating principles which are genuinely *constitutive* are carefully separated from the connecting principles which only state the relations between the properties of the constituted objects. By contrast, the usual conjecture-refutation method would merge both principles into a single category: that of corroborated conjectures or hypotheses. In spite of this difference, however, the idea that the coordinating principles lack any necessity was taken by Reichenbach, a few years after having written his *Theory of relativity and* a priori *knowledge*, as a good reason to abandon any reference to transcendental philosophy and to revert to empiricism.

Let us now have a closer look at why Reichenbach decided to drop any claim of necessity in his view of the relativized *a priori*. His basic reason was of course that, if the *a priori* is to be historicized, one cannot retain any principle which would be "valid for all times". Since Reichenbach identified "necessary" and "valid for all times", it was obvious to him that the historically drifting constitutive principles are not and cannot be necessary. But we do not have to accept this identification of "necessary" and "valid for all times". Less stringent definitions of necessity are available, and they can be used in the context of transcendental epistemology. One of them is *conditional necessity*: certain constitutive principles are necessary under the *condition* that a certain practice of research is implemented. But practices may evolve and a new network of presuppositions may then become conditionally necessary. Then, surprising as it may seem, a set of constitutive principles can be *necessary* and *provisional* at the same time!

If we accept this, the procedure of transcendental justification can be activated again, though of course not in the same sense as Kant's. Here, a transcendental justification would no longer be a regression from the fact of objective knowledge to certain concepts and principles which are taken to be "*a priori* conditions of the possibility of *all* experience". It would only be a regression from a given historical project of intersubjective knowledge, to a set of preconditions which are necessary *if* this particular project is to be successfully carried out. A transcendental justification of certain general structures of a physical theory is thus possible in such a restrictive acceptation.

In this sense, it now proves quite easy to justify transcendentally a large part of the structure of Quantum Mechanics. One can for instance derive a crucial part of the quantum formalism from assumptions about the limits of accessible experimental information[18]; or from assumptions about contextuality of phenomena, combined with a demand of unity of the mathematical tools used for

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[18] A. Grinbaum, The Significance of information in quantum theory, Ph.D. thesis, Ecole Polytechnique, 2004, http://www.imprimerie.polytechnique.fr/Theses/Files/ Grinbaum.pdf

predicting these phenomena.[19] This means a lot for the interpretation of quantum theories. This means that one is no longer compelled to understand quantum theories as a representation of the «external», «independent» world, with all the strangeness and paradoxes that are associated with such a representation. Rather, quantum theories can very naturally be understood as expressing the constraints and bounds of (experimental) knowledge. This is very much in the spirit of Kant, if not in the letter of his original texts.

Now, let us inquire further into how the procedure of constitution of objectivity can be applied to quantum physics.

In everyday life and in using classical physics, considering that objects have been «constituted» may sound superfluous. After all, if such a constitution has taken place, it was in the ontogenic (or, possibly, phylogenic) past of human beings. The basic conditions of the constitution of objects have been permanently available since then, and they do not have to be questioned. Therefore, at present, everything looks as if the material bodies of everyday life and classical physics were given out there.

But in microphysics, things are very different. The basic conditions of the constitution of objects in space–time are no longer available, and this forces us to think afresh about constituting new types of objects. To begin with, what exactly are these conditions of the constitution of objects in space–time? They essentially consist in clauses of active imposition of continuity and reversibility of the temporal sequences of phenomena. These clauses, when they are successfully implemented, give ground to the idea that there is *something* permanent or substantial retaining its own *identity* across space–time[20]: a "something" which is endowed with *properties*, and which can *cause* events. But none of these clauses can be enforced on the micro-scale[21]:

1. The scheme of identity requires the possibility of restoring the continuity of spatio-temporal trajectories in order to follow them; but, in view of Heisenberg's uncertainty relations, no such trajectory is accessible to experience. At most, we can have access to a fuzzy trajectory. The continuity criterion, which defines identity, can then only be used with reasonable efficiency in situations of very low density.
2. The scheme of definition of properties requires reproducibility of phenomena across a large range of variation of perceptive or experimental history. But in quantum physics, when some pairs of measurements (those which bear on conjugate variables) are performed sequentially, the result of each type of measurement crucially depends on the order of the sequence.

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[19] J.L. Destouches, *Principes fondamentaux de physique théorique*, Hermann, 1942; M. Bitbol, *Mécanique quantique, une introduction philosophique*, Flammarion, 1996; M. Bitbol, «Some steps towards a transcendental deduction of quantum mechanics», *Philosophia naturalis*, 35, 253–280, 1998.

[20] J. Piaget, *La construction du réel chez l'enfant*, Delachaux et Niestlé, 1977.

[21] M. Bitbol, *L'aveuglante proximité du réel*, Flammarion, 1998; M. Bitbol, *Schrödinger's Philosophy of Quantum Mechanics*, Kluwer, 1996.

3. The scheme of definition of ordinary causality requires free substitution of well-defined antecedent conditions in order to check that a certain effect is determined (or at least probabilistically promoted) by a certain antecedent. But, in quantum physics, this definition cannot be applied to its usual mechanical domain, to wit *motion*. For, here again due to Heisenberg's uncertainty relations, it is impossible to completely specify the spatial and kinematic antecedent conditions of a certain process of motion.

This means that *all the schemes of reversibility which justify our belief in the existence of spatio-temporal objects called material bodies at our scale*, are *missing* at the microscopic scale. What can we do at this point? Return to Kant's method of constituting objectivity, but applying it differently and to a different pattern of phenomena.

Let us first remember what motivated Kant's conception according to which objects of perception as well as objects of science are *constituted*. Kant's primary aim was to take a middle course between dogmatism and empiricism, between the view that objects are real entities independent of us and the opposite view that objects are merely imaginations of the human mind. A constituted object is neither isomorphic to a real object existing *in itself*, nor reducible to a figment of the imagination. So, what is it exactly? Let us read one of Kant's clearest statements about this point. He wrote: " (…) insofar as (…) presentations are connected and determinable (in space and time) according to the laws of the unity of experience, they are called *objects* ".[22] Here, nothing other than presentations, namely appearances, is required. But these appearances are embedded within a structural framework provided in advance by our understanding: the laws of the unity of experience. This structural framework is what *must* be presupposed in order to organize the presentations into manifold complexes made independent with respect to any particular situation or to any particular subjective state. In other terms, the structural framework of our understanding provides us with cognitive invariants. This definition of (constituted) objects needs no reference to exteriority, except in the weaker sense of spatial exteriority; no reference to reality either, except in the weaker sense of empirical reality (a sense that has been revived in a modern version by Putnam under the name "internal realism"). Objects are by no means construed as part of external reality in the strongest sense; yet objects are as independent of particular subjects as one may wish.

A crucial point is that, here, objectivity no longer means complete *detachment* of entities and properties with respect to the cognitive apparatus, but coordination of phenomena into several *strata* of invariants across a variety of subjective and instrumental circumstances. The fact that the usual types of spatio-temporal invariants, namely material corpuscles, are no longer available in quantum physics should not prevent one from attempting some sort of coordination.

This quest of a radically renewed constitution of objectivity can be carried out in two steps. Firstly coming back to classical mechanics in this spirit and analyzing

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[22] Kant, *Critique of Pure Reason*, B 522, op. cit. p. 508.

how objects were in fact defined in this theory, beyond the superficial claim that they are merely given to us. Secondly extending this mode of definition to microphysics, with some suitable alterations.

When we perform an analysis of the status of objects in classical mechanics, we find that they are nothing else than the boolean lattice of those experimental propositions that are embedded in a covariance diagram corresponding to Galileo's group. They ultimately play no other role in the theory than an invariant of Galileo's group.[23] Any further statement according to which a classical object is a carrier of properties, beyond the level of these properties, is just a metaphysical addition, without any bearing on the way classical mechanics operates.

How can we transpose this procedure to quantum physics? Peter Mittelstaedt made a very interesting suggestion[24] after Schrödinger. He first noticed that if something holds the role Kant ascribes to a «substance», this something can only be the *state* Ψ itself, because (i) *state* Ψ gathers in its preparation a complete set of commuting observables, (ii) *state* Ψ is *permanent*, in good agreement with Kant's first analogy of experience. He then added that, by contrast, those putative entities that are conceived as carriers of the same spatial and kinematic properties as classical bodies, namely particles, "can only be considered as *fictional* objects".[25] One reason for this is that, given a certain state Ψ, only *commuting* observables can be taken as jointly "objective", in the sense of their being mutually accessible without alteration. In contrast, since the spatial and kinematic observables that are united by the concept of classical bodies do not commute, they are not jointly objective. This is a clear incentive to dispense with the old type of objects called material bodies altogether, and adopt a new type of object instead. We have no need for fictional objects which are only able to generate paradoxes.

However, in spite of this, few people cross the line, and replace the traditional body-like domain of objectivity with a new domain of objectivity such as the Hilbert space. Why is this so? We think this is due to the dominant realist attitude in epistemology. Realist philosophers of science are not content with invariant structures: they want "elements of external reality". Now, the problem with states Ψ is that, although they are indeed abstract invariants (by Dirac transformation) across the whole range of observables, they are quite poor candidates, taken in isolation, to the title of "elements of reality". Indeed, they are little more than mathematical generators of probabilities. And since they are generators of probabilities, they connect only indirectly, by means of Born's algorithm, with genuine experimental invariants such as values of spatial or kinematic observables, whereas good old material bodies are supposed to carry them directly.

But unlike realists, transcendental epistemologists do not care at all whether an invariant represents reality or not. What they require is only that these invariants be

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[23] See E. Castellani, «Galilean particles, an example of constitution of objects», in: E. Castellani (ed.), *Interpreting bodies*, Princeton, NJ: Princeton University Press, 1998.

[24] P. Mittelstaedt, *Philosophical problems of modern physics*, Boston, MA: Reidel, 1976.

[25] P. Mittelstaedt, *Philosophical problems of modern physics*, op. cit. p. 129–130.

completely free of any paradoxical feature, as general as possible, and able to unify the largest conceivable domain of knowledge. If those conditions are fulfilled, they feel free to say that they have reached an optimal state of objectivity in the effective sense of maximal independence with respect to any subjective, spatial and instrumental situation. This is more than enough for them.

4 Constituting Objectivity in Contemporary Physics

We will now see how these ideas about constitution of objectivity can be extended to the most recent advances of theoretical physics, from Quantum Field Theories to Quantum Gravity.

As already mentioned in section 1 of this introduction, in the *MFNS*, the "mathematical" categories are specialized into "phoronomy" and "dynamics", whereas the "dynamical" categories are specialized into "mechanics" and "phenomenology". But the constructibility of the latter does not result in a true geometrization of physical contents. In other terms, in this reading of the original version of classical mechanics as formulated by Newton, constructibility does not result in an ascension of the "dynamical" into the "mathematical", nor in what Hermann Weyl called the transformation of kinematical principles into dynamical principles.

Now, it is clear that a large fraction of the subsequent advances of mathematical physics consisted in a "stronger" progressive construction of the dynamical categories by means of stepwise extensions of the field of applicability of mathematical categories, namely by means of stepwise extensions of relativity groups and other symmetries, and therefore of covariance constraints as well as of conservation principles. One can thus give a natural transcendental interpretation of the generalizations of classical mechanics that were developed throughout the nineteenth century.

Lagrangian and Hamiltonian formalisms (underpinned by symplectic geometry) allow two types of advances.

Firstly, they make it possible to reformulate Kant's spatio-temporal synthetic *a priori* (constitutively correlative of Galileo's relativity group and of the principle of inertia which states that geodesics are Euclidean straight lines and that inertial motions are uniform motions along a straight line) by considering that the Euclidean metric of space is a "background structure" of Newtonian mechanics. In a variational Lagrangian formalism, one calls "background structure" a structure that appears in the Lagrangian, but which do not have to be varied in order to obtain the Euler-Lagrange equations.

Secondly, another advance allowed by symplectic formalisms is represented by Noether's theorem which connects relativity principles (i.e. principles of *inobservability* of absolute kinematical magnitudes) and symmetries (invariance of the Lagrangian), with the laws of conservation of corresponding physical magnitudes (principles of *observability* of the latter magnitudes). This theorem is somehow *the* transcendental theorem, which vindicates Kant beyond what he could have hoped, and beyond what he could figure out. Indeed, it develops to an unsuspected extent

Kant's project in the Phenomenology and Mechanics of his *MFNS* (see Section 1): deriving a definition of observable magnitudes from principles of inobservability implied by Galilean relativity.

Thirdly, in General Relativity, the *content* of transcendental principles is changed, but far from being weakened, the architectonic of transcendental philosophy is actually reinforced by this change. The "axioms of intuition" (with the corresponding kinematics) and the "anticipations of perception" (with the corresponding dynamics) are transferred from the global and metric level, which was typical of newtonian mechanics, to the underlying local and differentiable level. The relativity group of the theory then becomes the group of space–time diffeomorphisms. Accordingly, the constraints of covariance become more important. This makes it possible to reduce forces, along with the category of causality, to a generalized principle of inertia. Here, the geometrical synthetic *a priori* is no longer located on a metric level, but rather on the differentiable level; it concerns, e.g. the cohomology of differential forms. This change can be expressed by saying that the metric is no longer a "background structure" (be it Euclidian or Minkowskian), but becomes a *dynamical* element of the theory. This new stage of the geometrization of physics can be interpreted from the standpoint of transcendental philosophy as a chiasm between a generalized "phoronomy" (relativity) which becomes dynamical, and a "mechanics" which becomes kinematical (inertial). An important consequence of this is that diffeomorphism invariance deprives location from any physical meaning.

Fourthly, in quantum field theory (gauge theories) one introduces "internal" degrees of freedom, and this yields broadened symmetry principles which considerably enrich the geometrization of physics by geometrizing interactions. As Yuri Manin (1988) claimed: "From a philosophical point of view, one can speak of a new wave of geometrization of physical thought which for the first time is sweeping far beyond the boundaries of general relativity".

Since the pioneering research of Chen Ning Yang and Robert Mills in the 1950s, two classes of fields were then distinguished in gauge theories:

(i) Matter fermionic fields, which are interpreted as fiber bundles over space–time (the coordinates of fibers are internal degrees of freedom, and the symmetry group of fibers express internal symmetries of particles).
(ii) Bosonic gauge fields, which represent interaction fields mediated by exchanged virtual particles (bosons). These are interpreted as connections over these fiber bundles.

The particles which mediate interactions are therefore the quanta of connection fields over matter fiber bundles. The Yang-Mills Lagrangian is the norm of the curvature of connections. It is an invariant of the gauge group, and space–time contribute to it as a gauge field by means of the scalar curvature of its connection. Covariant derivatives then offer the possibility of expressing interactions geometrically. In this situation, gauge theories were able to "construct" interactions by introducing a dependence of internal symmetries of systems (which are apparently non spatio-temporal global symmetries associated with particles quantum numbers) on space–time. If these internal symmetries are thus localized, and if the invariance of the theories is required, corrective terms must be introduced. It then appears that the latter precisely generate the interaction terms. This means that forces and interactions can generally be derived from local conservation principles.

After a very long evolution, physical formalisms are thus able to take into account a very difficult metaphysical debate in which Kant challenged the Leibnizians. This debate is developed in Kant's *Physical Monadology* (1756), whose central thesis combines physical atomism and geometrical continuism. In this early book, Kant claims that a monad "fills" a determined, non-punctual, portion of space, and that this is not incompatible with its infinite divisibility, as the monad has "internal determinations". Indeed, Kant writes in proposition VII that internal determinations are not in space precisely because they are internal; they are therefore not divided when spatial extension is divided. The minimal space occupied by a monad is a sphere of activity, and not an extension. One had to wait for the modern concept of a fiber bundle to see a rigorous geometrical expression of this metaphysical intuition.

Fifthly, in General Relativity, the absence of any "background metric" is connected with the research of global invariants. In contrast, in Quantum Fields Theory, the metric remains a "background structure". Imposing the unification of the two theories is the problem of Quantum Gravitation. It is striking to see that the deepest proposition in this domain, namely *Noncommutative Geometry*, can be given a natural transcendental interpretation (see J. Petitot in this volume).

In summary, it has been possible to gradually construct a true *formal ontogenesis* of observable physical reality, by means of variational formalisms, Noether's theorems, Riemann's geometry, connections on fiber bundles, Feynman's path integrals, and Noncommutative Geometry. This mathematical construction converted Kant's synthetic *a priori* into algorithms of computational synthesis allowing to generate ever-expanding explicit models of measurable or observable phenomena. Transcendental approaches of physics allow us to elucidate the philosophical significance of this remarkable historical achievement.

5 Presentation of the Book

This collective book covers the whole span of issues which have just been indicated. It relies on an improved understanding of Kant's historical views of mathematics and physics in order to see how a transcendental reading of relativistic and quantum physics can be carried out. It focuses on what is allegedly the core of the transcendental method in epistemology, i.e. the procedure of constitution of objectivity. Furthermore, it investigates the renewed forms this method must take in order to make sense of the latest developments in theoretical physics, from quantum field theory to quantum gravitation.

Let us now give a short outline of the chapters of this book.

The first part is concerned by the history of transcendental epistemology, from Kant to the successive waves of neo-Kantianism.

As an indispensible initial step, M. Friedman and P. Kerszberg both examine Kant's approach of Newtonian mechanics. M. Friedman studies the impact of the metaphysical and theological background of Newton's thought on Kant's conception of space and gravitational action at a distance. But he also shows in exquisite details how this pre-critical background was thoroughly transformed by transcendantal philosophy. From then on, the Newtonian God was construed as a regulative idea of reason, and the conception of God's (space-like) omnipresence in the universe as a "sublime analogy" which only has a "practical meaning". P. Kerszberg then describes Kant's transcendental reinterpretation of Newtonian kinematics and dynamics, from the *Metaphysical Foundations of Natural Science* to the *Opus Postumum*. He points out that, between Newton and Kant, there is a decisive shift: Newton loosely assumes that a physicist is free to choose the most convenient set of assumptions, whereas Kant shows that there are some general principles from which physics *necessarily* starts.

In the second subsection of the historical inquiry, the central strategy for moving beyond Kant while remaining faithful to his core epistemological program is addressed. As we mentioned in Section 3 of the present introduction, this central strategy is relativization and historicization of the *a priori*. Relativization is studied in Cassirer and Carnap, while Schlick's criticism is also documented.

C. Schmitz-Rigal emphasizes Cassirer's remarkable intellectual freedom in his adaptation of Kant's most basic insight. Kant's static of constitution of objectivity by way of *a priori* forms, is reinterpreted as a dynamic of constitution of *meaning* by way of a multiplicity of symbolic forms. Here, objects are relative to an *objectivation-project* that can develop and change in history. Interestingly, Cassirer's neo-transcendental conception was stimulated rather than hindered by the onrise of relativity and quantum mechanics. Indeed, according to Cassirer, relativity and quantum mechanics represent a remarkable advance towards self-awareness of the procedure of objectivation by physics. At the same time, the very procedures disclosed by modern physics represent a welcome occasion for the philosopher to understand the detailed workings of objectivation, and therefore to "reveal the constitutive conditions of the *means of constitution themselves*". A. Cei and S. French then focus on Cassirer's philosophical analysis of quantum mechanics. Cassirer's crucial move consists in a redefinition of the causality principle comprehensive enough to encompass statistical physics and quantum "indeterminacy". But another dimension which is at least as important as the former one, is Cassirer's structuralist redefinition of the concept of object as an 'intersection of relations'. C. Bonnet and R. de Calan turn our attention from neo-Kantian thinkers to logical empiricists who were still thinking under the spell of Kant. They concentrate on Moritz Schlick's philosophy of knowledge. Of course, as every logical empiricist, Schlick denies the existence of anything like Kant's synthetic *a priori* judgments (judgments that do not derive from experience, yet are indispensible preconditions of experience). But he also radicalizes Kant's philosophy by declaring that the true *a priori* reduces to pure logical form. And he holds that the *role* of the denied synthetic *a priori* is played by conventions and hypotheses. Finally, P. Parrini goes further in the examination of Kant's influence on logical empiricism, by focusing on Carnap. Carnap accepted and amplified Schlick's rejection of the synthetic *a priori*, and his adoption of logical form instead. But he simultaneously developed a personal conception of the relativized *a priori*, which was able to vindicate Kant's basic intuition in the context of modern science. His distinction between 'external questions' and 'internal questions' (namely questions that are external or internal to a certain framework of presuppositions) was especially important in this respect. Indeed, Carnap assumed that the boundary between 'internal' and 'external' (and therefore the whole framework of presuppositions) can be shifted whenever the evolution of knowledge makes it expedient to do so.

In the second part of the book, we shift our attention from history to current philosophy of physics. But before each particular theory is dealt with for its own sake, one must ask a series of precise questions about the methods to be used and the general issues to be addressed if transcendental epistemology is to be adapted to modern physics.

R. Harré examines the possibility that the standard corpuscularian ontology of the initial phase of classical physics is underpinned by a dispositional stratum. Dispositions come either in a classical form (especially fields), or in a quantum form which is even more radical. An analogy is developed between this two (or many-) *strata* structure of the domain of physics and Kant's duality of phenomena and things in themselves. Can one consider that emergent instrumental "affordances" manifest in terms of categories, just as Kant's phenomena do, whereas the "world as it is" is made of pure acategorical *dispositions*? G. Brittan then tackles the issue of causality and indeterminacy, which is crucial for any Transcendental account of quantum physics. According to him, the required outcome of any procedure of constitution of objectivity is being able to consider various appearances or descriptions as aspects of the *same* item. Therefore, the problem of objectivation is a problem of *identification*. This being granted, (quantum) indeterminacy can be taken into account provided one constitutes unsharp objects which can be identified across space–time. But another conflict, between the urge to identify and non-separability, must be addressed. Are the requirements of objectivation really missing in the quantum domain? Another central question is then addressed by P. Mittelstaedt: which conditions have to be fulfilled by observable data in order to count as appearances *of an object*? He states these conditions in technical detail, and evaluates the ability of macroscopic and microscopic observable data to be ordered according to the norms of the knowledge of objects. His conclusion is that whereas macroscopic data (corresponding to the domain of classical physics) can be ordered according to the norms of the knowledge of *individual* objects, microscopic data (corresponding to the domain of quantum physics) can only be ordered according to the norms of the knowledge of *classes* of objects. Therefore, Mittelstaedt claims, in quantum physics, the constitution of objectivity is restricted to *kinds*. Now, lawlikeness is the crucial feature of objectivation, in the Kantian tradition. G. Boniolo thus examines critically the debate about the "laws of nature" in the empirical tradition and points out that Kantian ideas have strangely been neglected for decades. He first explains Kant's own stratified view of lawlikeness, with *a priori* principles of understanding as a universal ground, particular empirical laws embedded in these principles, and regulative ideals of systematic unity of laws on top. He finally offers a Kantian solution of the problem of discrimination between nomological universals and accidental universals. P. Kauark-Leite then insists on a class of principles of understanding that is usually forgotten in the discussions about Transcendental epistemology: the anticipations of perceptions. This class of principles is crucial, however, because it aims at expressing the role infinitesimal calculus, and especially differential equations, play in physics. But there is a major difference between classical and quantum mechanics in this respect: in classical mechanics, infinitesimal calculus is used for both descriptive and predictive purposes (it is meant to describe the present states of systems and to predict their future states as a direct consequence of this description); but in quantum mechanics, infinitesimal calculus can only be used for the sake of probabilistic prediction. However, P. Kauark-Leite points out that if the *Critique of Pure Reason* is correctly understood, objectivity is not really defined by a set of invariant properties, but rather by an invariant anticipative structure. This being granted, Kant's strategy of constitution of objectivity can easily be extended to quantum mechanics. Then, M. Stöltzner turns our attention to another neglected aspect of Transcendental epistemology, initiated by Kant in the *Critique of Judgment*. In philosophy, this aspect concerns teleology, and in physics it manifests itself by the use of a *Principle of Least Action (PLA)*. Does the *PLA* reintroduces finality in physics in the ordinary metaphysical sense? In the spirit of Kant, M. Stöltzner argues against such a simple reading of the *PLA*, and rather points out that teleological constraints act as regulative principles imposed onto the formalism of physical theories. They are thereby indirectly constitutive of particular laws of nature, and should be studied together with other varieties of the historically relativized constitutive *a priori*. B. Falkenburg then provides us with a final classification of the major principles of Kant's theory of nature. With this chart of principles in mind, she enquires into whether the ordinary belief in subatomic processes underpinning microscopic phenomena can still be maintained in modern physics. The applicability of each principle to quantum physics is examined in detail, and it turns out that the status of the belief in a subatomic reality is more nuanced than usually accepted by either realist or empiricist philosophers. An important point is that, here again, the subatomic world cannot be construed as some piece of an independent reality, but rather as a fraction of the empirical domain which is relative to a set of macroscopic experimental contexts.

In the second subsection of this second part of the book, we shift from general issues to more focused studies of modern theories of physics, starting with special and general Relativity.

M. Friedman first exemplifies the idea of a relativized and historicized constitutive *a priori* by applying it to the theories of Relativity. To achieve this aim, he undertakes a historical analysis of the connection between geometry and kinematics by way of the principle of relativity of motion, from Kant to Einstein. He advocates the idea that, far from hindering the onrise of Relativity, Kant's conception served as a matrix for further development of physics after Newton, *including Einstein's theory*. Y. Balashov then turns to the problems of relativistic *cosmology*. Applying a Kantian frame of thought to cosmology may look paradoxical, in view of Kant's sharp criticism of the concepts of totality, such as the concept of "the world as a whole". However, it appears that cosmology (including Einstein's relativistic cosmology of 1917) relies extensively on transcendental arguments, namely on an examination of the conditions of possibility for describing global properties of the universe. An inventory of these transcendental cosmological arguments is undertaken by Y. Balashov. Finally, T. Ryckman examines the program consisting of extending the ideas of General Relativity to a unitary field theory including electromagnetism. Here, Hermann Weyl is a major reference. It is therefore striking that, throughout his theoretical research, Weyl was guided by principles derived from transcendental philosophies from Kant to Husserl. These principles were instrumental in his formulation of gauge theories. One then realizes that transcendental epistemology is not only a good strategy for making retrospective sense of theories, but also an excellent prospective path towards major theoretical breakthroughs.

The following sequence of chapters deals with the role played by transcendental epistemology in the genesis and history of quantum mechanics.

S. Brock first describes Kant's and Helmholtz' influence on Bohr's thought and theoretical program. This influence can be seen when Bohr considered that complementarity, as an organizing principle of objective knowledge, is a generalization of the 'ideal of causality'. Indeed, according to Bohr, causality can be analyzed into two complementary aspects: 'space–time coordination' and 'dynamic conservation laws'. In the second half of his paper, S. Brock develops another (partial) parallel between Cassirer's and Bohr's views about physics. The connection between Kant's and Bohr's philosophies of physics is also the central theme of H. Pringe's chapter. A central idea here is that in order to understand this connection one should rely more on the *Critique of Judgment* than on the *Critique of Pure Reason*. For instance, Bohr's principle of correspondence can easily be seen as a maxim for *reflective judgment* when empirical analogies (between the classical and quantum domains) are looked for. As for complementarity, it plays the role of a 'symbolic analogy', namely a way to afford a twofold symbolization of objects (as waves and as corpuscles) in situations where complete unification of phenomena under a single picture of objects is not possible. L. Soler then studies one of the most accomplished and heroic attempt at vindicating the whole of Kant's epistemology in the face of quantum mechanics. This attempt was made by Grete Hermann, a German neo-Kantian philosopher who spent several weeks with Heisenberg and Von Weiszäcker in 1935, trying to persuade them that the principle of causality could by no means be relinquished. Her claim was that it is always possible to reconstruct a causal chain *a posteriori*, even when (as it is the case in quantum mechanics) no predictive use of causality can be made. She then undertook a complete transcendental reinterpretation of quantum mechanics along the lines of the neo-Kantian psychology of J.F. Fries and L. Nelson.

At this point we focus on some ideas that can be used to make sense of recent developments of quantum physics in the framework of transcendental epistemology.

M. Bitbol first develops a transcendental reading of *decoherence theories*. He presents it as a middle way between the usual realist reading and a strictly empiricist reading. According to realist philosophers, the macro-world is a mere appearance arising from quantum reality by way of decoherence. And according to empiricist philosophers, only macroscopic experimental outcomes are real, whereas the quantum formalism is only a predictive symbolism. But for a transcendental philosopher, both readings are biased. The first one is biased in favor of formal theoretical constructs taken as descriptive of a putative reality more real than phenomena; whereas the second one is biased in favor of phenomena, thus forgetting that phenomena only acquire their meaning from the formalism in which they are embedded. Along with the transcendental middle way, decoherence is situated in a two-step scheme of constitution of objectivity appropriate for quantum mechanics. S. Osnaghi then attempts a radical move indispensible to a transcendental construal of quantum mechanics. He shows that in the framework of this theory, one must conceive objectivity without (individual) objects. He also addresses the issue of decoherence, undertaking to dismantle its realist reading by way of a careful analysis of quantum entanglement, as illustrated by QED-cavity experiments. With G. Catren's chapter, we are presented with the deepest formal operations of constitution of objects, applied to quantum theories. Here, objective properties are defined as transformations, and constrained to be automorphism. However, the most crucial condition is imposed not on each property taken in isolation but on a set of properties: if they are to be taken as properties of *one and the same object*, each one of them must be invariant with respect to the transformations associated to the other properties of the same set. This condition is found to be extremely discriminative. It allows one to interpret Heisenberg's uncertainty principle as an expression of the much more general fact that not all the spatio-kinematic properties can be simultaneously objective. Retrospectively, it then appears that classical mechanics went astray by wrongly considering certain properties as simultaneously objective. In view of this rigorous analysis of the constitution of objective properties, quantum mechanics is complete, whereas classical mechanics is misleadingly overabundant.

But beyond Relativity and Quantum Mechanics, there are many modern theories which aim at broadening their scope by unifying them as far as possible. These theories are also liable to a transcendental interpretation.

I.O. Stamatescu adopts a thoroughly Cassirerian pattern of analysis: here, one is concerned by the evolution of "symbolic forms", as needed by the development of modern physics. Objectivation is thus conceived as a dynamical process expanding across the history of physics. This idea is especially applied to the issue of the successive redefinitions of a standard object of microphysics such as the electron. Then, H.G. Dosch applies a similar Cassirerian outlook to our understanding of particle physics and quantum field theories. Here, the creative aspect of the construction of the standard model of particle physics is highlighted, and it is pointed out that the large amount of freedom of this creative process can hardly be accounted for by a realist construal of physics. Finally, J. Petitot provides a detailed transcendental analysis of one of the most important proposals for unifying General

Relativity and Quantum Theories: Noncommutative Geometry (NCG). He starts from a conception of constitution of objectivity as mathematical construction of categories. Then, he points out that, in general Relativity and quantum field theories, symmetry groups enable one to construct mathematically both the physical content of the categories of substance and the physical content of the categories of force and interaction. This procedure is then extended to NCG, which introduces quantum concepts in the very definition of geometry. Equivalents in NCG of every single step of Kant's transcendental analysis of classical mechanics in the *Metaphysical Foundations of Natural Science*, from phoronomy to phenomenology, are provided.

But this book would have been incomplete without some dissonant voices, and without the opportunity to confront transcendental epistemology with alternative epistemologies, from constructive empiricism to realism.

B. Van Fraassen first carefully specifies the terms of the debate between constructive empiricism and transcendentalism. Both doctrines reject metaphysics, but at the same time each one of them appear to the other as deceitfully engaging into the rejected metaphysics. As seen from an empiricist standpoint, transcendentalism appears to grant too much to a metaphysics of the knowing subject, whereas as seen from a transcendentalist standpoint, empiricism appears to make too many concessions to transcendent realism (though with a strong dose of agnosticism about "reality out there"). Van Fraassen then undertakes to meet the standard transcendentalist objection that he formulates thus: "It does not make sense to say that *there could be* things that are not describable (in our language in use) and hence not knowable, and thus also certainly not known". His residual realism is aknowledged by him, but carefully distinguished from any metaphysical version of it that could be criticized by a transcendentalist. This empiricist variety of realism only amounts to accepting that describing scientific activity (and scientific product) *de facto* involves common sense realism. B. d'Espagnat then undertakes a nuanced defense of realism from the standpoint of a practicing physicist. He accepts that modern physics (especially quantum physics) represents a fatal blow to any *direct* realist epistemology claiming that physics is meant to describe reality as it is. But at the same time, he rejects the extreme neo-Kantian view according to which any discourse about the "thing in itself" is meaningless. He rather sticks to a position similar to Kant's original conception, for reasons he draws from a detailed discussion of the meaning of quantum theories. H. Lyre then offers a systematic defense of structural realism against epistemic structuralism and ontic realism, yet basing this defense on transcendental arguments. He illustrates his point by discussing the status of Gauge theories. P. Teller finally ponders about what we add to (or what we tend to withdraw from) empirical knowledge when we frame theories and explanatory strategies. If we want neither to stick to empiricist agnosticism, nor to accept a transcendental interpretation of this added structure as a condition of possibility of experience, we must wonder about its status. Is the added material merely a distortion, or can it sometimes count as (provisional) knowledge (of some external reality)?

Acknowledgments This book relies on a long-term research program developed by the informal collaboration of several teams, in France and in many other countries. A series of conferences and seminars served as milestones for this collective work: the Cerisy conference *Rationality and Objectivity* (1988), the UNESCO conference *Philosophical and Scientific Rationalism* (1989), the Cerisy conference *The Fate of Transcendental Philosophy* (1990), the conference and book *Physique et Réalité, un débat avec Bernard d'Espagnat, éditions Frontières-Diderot*, (1997), the special issue *Philosophie et Sciences: Objectivité scientifique et logique transcendantale*, of *Archives de Philosophie* (2000), the workshop *Physics and Constitution of Objectivity* (2002), the CREA seminars *Transcendental Arguments in the Philosophy of Physics* and *Constitutions of Objectivity*, the workshop *Kant, Cosmology, and Quantum Physics* (2003), the workshop *Structuralism in Philosophy of Physics III* (2005), and finally the conference *Philosophical and formal foundations of modern physics* at Fondation des Treilles (2007). We thank all the participants to these events who contributed to our better understanding of the permanent value of transcendental epistemology in physics, and the many institutions that supported them, from the CNRS to the Fondation des Treilles.

Last but not least, we wish to thank Ms. Fredline Laryea, who carefully edited our introduction as well as several papers in the volume, by correcting the English whenever it was suitable.

Part I
Historical Survey of Transcendental Readings of Physics

A

Kant and Newtonian Mechanics

Newton and Kant on Absolute Space: From Theology to Transcendental Philosophy

Michael Friedman

Abstract I argue that Kant's methodological differences with Newton over absolute space and gravitational action at a distance are importantly related to metaphysical and theological issues about God and the creation of the material world in space. These differences constitute an essential part of Kant's radical transformation of the very meaning of metaphysics as practiced by the predecessors – from ontological and theological issues to transcendental philosophy.

In my previous work on Newton and Kant I have primarily emphasized methodological issues: why Kant takes the Newtonian Laws of Motion (as well as certain related propositions of what he calls "pure natural science") as synthetic a priori constitutive principles rather than mere empirical laws, and how this point is intimately connected, in turn, with Kant's conception of absolute space as a regulative idea of reason – as the limit point of an empirical constructive procedure rather than a self-subsistent "container" existing prior to and independently of all perceptible matter. I have also argued that these methodological differences explain the circumstance that Kant, unlike Newton, asserts that gravitational attraction *must* be conceived as an "action at a distance through empty space," and even formulates a (rare) criticism of Newton for attempting to leave the question of the "true cause" of gravitational attraction entirely open. In this paper I emphasize the importance of metaphysical and theological issues – about God, his creation of the material world in space, and the consequences different views of such creation have for the metaphysical foundations of physics. I argue, in particular, that Kant's differences with Newton over these issues constitute an essential part of his radical transformation of the very meaning of metaphysics as practiced by his predecessors. I also suggest that the metaphysical and theological issues in question form an essential part of the intellectual context for the methodological issues I have emphasized previously – especially the issue of action at a distance.

It is now well known that the main target of Newton's rejection of "relationalism" in favor of an "absolutist" metaphysics of space was Descartes, and the *locus classicus* for Newton's own metaphysics of space is his unpublished

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M. Bitbol et al. (eds.), *Constituting Objectivity*,

De Gravitatione.[1] What was most important for Newton was decisively to reject Descartes's identification of matter with extension and to defend, accordingly, the concept of absolute (empty) space existing prior to and independently of matter. Yet Newton, like Descartes before him, also appropriated philosophical ideas from the neo-Platonic tradition,[2] which he incorporated into his own metaphysics. For Newton, the most salient source of such ideas was the Cambridge Platonism represented especially by Henry More, and Newton employs them in his doctrine that absolute space is neither a substance nor an accident, but what he calls "an emanative effect of God and an affection of every kind of being" (*De Grav*., p. 21).[3] In particular, absolute space or pure extension is even an affection of God himself, since God is omnipresent or everywhere. God can thereby create matter or body (as something quite distinct from pure extension) by endowing certain determined regions of space with the conditions of mobility, impenetrability, and obedience to the laws of motion. God can do this anywhere in space, in virtue of his omnipresence, by his immediate thought and will, just as our souls can move our bodies by our immediate thought and will. It is essentially this doctrine which surfaces in Newton's well-known published statements, in the General Scholium to the *Principia* and the Queries the *Optics*, that space is the "sensorium" of God.[4]

The sharp differences between Descartes's and Newton's metaphysics of space – their different conceptions of the relationships among space, God, and matter – are of fundamental importance. For Descartes, since space is simply identical with matter, God creates matter by creating space itself, and it is precisely this act of creation of space at successive moments of time that is responsible for the laws of motion. In particular, the conservation of what Descartes called the total "quantity of motion" results from the unity and simplicity of God, whereby God continually recreates the entire universe

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[1] This point was first made in Stein (1967), and we can now also cite Stein (2002) for an authoritative account of Newton's metaphysics. *De Gravitatione* first appeared, together with an English translation, in Hall and Hall (1962). An improved translation by Christian Johnson, made with the assistance of Andrew Janiak, and consulting an earlier unpublished translation by Stein, appears in Janiak (2004): my parenthetical page references to *De Grav*. – and to Newton's writings more generally – are to this edition.

[2] For Descartes's appropriation of neo-Platonic metaphysics, as mediated by Augustine, see Menn (1998).

[3] Some of the most important writings of the Cambridge Platonists are collected in Patrides (1980). For discussion of the idea that space is an emanative effect of God see the exchange between J. E. McGuire and John Carriero in Bricker and Hughes (1990). See also the very careful discussion in Stein (2002, pp. 266–272). In the course of his discussion Stein is led to claim (p. 269) that "the grounds for thinking that Newton's theory of emanation is neo-Platonic, or 'Cambridge Platonic,' are very weak." Whatever one may think of Stein's particular reasons for this claim, it seems to me very hard to deny, in any case, that Newton is *appropriating* neo-Platonic (and, indeed, 'Cambridge Platonic') ideas for his own purposes here.

[4] In Query 31, for example, Newton describes God as (p. 138) "a powerful ever-living agent, who being in all places, is more able by his will to move the bodies within his boundless uniform sensorium, and thereby to form and reform the parts of the universe, than we are by our will to move the parts of our own bodies."

(the whole of pure extension, whose various parts may have different instantaneous tendencies to motion at any given time) at each instant while constantly expressing the very same divine essence. For Newton, by contrast, matter and space have radically different statuses vis-à-vis God's creation. Space is "an emanative effect of God and an affection of every kind of being," including God, while matter is the result of God's creative activity in space, wherein certain determined regions are then endowed with the conditions of mobility, impenetrability, and obedience to the laws of motion. By instituting the laws of motion, in particular, God thereby endows certain regions of space with Newtonian *mass* or quantity of matter (*vis inertiae*), and the presence of this quantity, specifically, clearly distinguishes matter from empty space. This not only leads, following earlier work of Wren, Wallis, and Huygens, to a much more adequate formulation of the laws of impact (whereas Descartes's inadequate formulation had no room for the quantity of mass, and thus no room for momentum or quantity of motion in the Newtonian sense), it eventually leads to the theory of universal gravitation of Book III of the *Principia*. And this theory, in turn, puts the notions of absolute space, time, and motion to real physical work in determining the center of mass of the solar system as the true "center of the world."

Nevertheless, despite these fundamental differences, both Descartes and Newton are using neo-Platonic ideas to support an essentially mathematical approach to physics over the older qualitative approach of Aristotelian physics. For Descartes, the world described by physics is, in its essence, the object of pure geometry. God, in creating this world, not only brings about (what Descartes takes to be) the (mathematical) laws of motion of the new physics, he also, in creating us as mind-body composites located within this world, guarantees that we can use our purely intellectual mathematical knowledge in successively correcting and refining our knowledge of the material world – as we apply pure mathematics, that is, to the initially misleading deliverances of our senses.[5] For Newton, although the world described by physics is not, in its essence, the object of pure geometry, space (which *is* the object of pure geometry) nonetheless constitutes the "frame of the world" – an emanative effect of the divine existence wherein God then creates matter by an immediate act of his will. The bare existence of space suffices for the existence of all the shapes and figures studied in pure geometry (*De Grav*., p. 22): "there are everywhere all kinds of figures, everywhere spheres, cubes, triangles, straight lines, everywhere circular, elliptical, parabolical, and all other kinds of figures, and those of all shapes and sizes, even though they are not disclosed to sight." And thus pure geometry is *ipso facto* applicable to all material bodies as well (pp. 22–23): "the delineation of any material figure is not a new production of that figure with respect to space, but only a corporeal representation of it, so that what was formerly insensible in space now appears before the senses." Therefore, in virtue of their (differently) neo-Platonic conceptions of a metaphysics of space, neither Descartes nor Newton has any room for a necessary gap (as there was in Plato's original "Platonism") between pure mathematics, on the one side, and the sensible and material world, on the other.

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[5] This, in a nutshell, is how I read the argument for the existence of matter of the Sixth Meditation: see Friedman (1997, 2008).

The significance of this point becomes clearer if we contrast the conceptions of both Descartes and Newton with the quite distinct approach of Leibniz, who was explicitly opposed to both Descartes and Newton in correspondingly different ways. Leibniz began, in fact, by reacting to Descartes's failure adequately to formulate the basic laws of impact, which were supposed to govern, according to the then dominant paradigm of the mechanical natural philosophy, all phenomena in the material or corporeal world. Leibniz responded to this problem, in his "Brief Demonstration of a Notable Error of Descartes and Others Concerning a Natural Law" (1686), by emphasizing the importance of a new, essentially dynamical quantity, which he called *vis viva* or living force (in modern terms, mass multiplied by the square of the velocity), where the basic law of motion is now formulated as the conservation of the total quantity of *vis viva*. Beginning with his *Discourse on Metaphysics* (written in the same year), Leibniz also strongly emphasized that living force is not purely geometrical or mechanical, so that, in particular, this quantity (unlike Descartes's purely mechanical "quantity of motion") reintroduces an element of Aristotelian teleology into the mechanical philosophy. For *vis viva*, on Leibniz's view, is the counterpart of the Aristotelian notion of *entelechy*: namely, that internal (non-spatial) principle by which an ultimate simple substance or monad determines (by a kind of "appetition") the entire future development of its own internal state. Moreover, in accordance with this same renewed emphasis on Aristotelian teleology, Leibniz then articulated a doctrine of divine creation in terms of God's choice of the best among all merely logically possible worlds. The distinction between what is logically possible and what is actual – between all merely thinkable worlds available to the divine intellect and the best and most perfect of these worlds as determined by the divine will – thereby corresponds to the distinction between principles of pure mathematics (including geometry) and principles of natural science or physics (the laws of motion). In particular, the laws of motion, unlike the principles of pure mathematics, precisely express the divine wisdom in actualizing or creating the best and most perfect of all possible worlds.

Leibniz thereby breaks decisively with Descartes's metaphysics of space, for the actual world of material substances results from a special act of the divine will which introduces additional non-spatial, and essentially teleological elements into the mechanical laws of motion. Indeed, Leibniz's break with Descartes on this issue is deeper still, for, on Leibniz's view, the entire mechanical physical world (including the space in which bodies move) is a secondary appearance or phenomenon (a "well-founded phenomenon" like the rainbow) of an underlying metaphysical reality of mind-like simple substances or monads – substances which, at this level, are not spatial at all, but rather have only purely internal properties and no external relations. This point, in turn, is closely connected with a fundamental disagreement with Descartes about the nature of the intellect: whereas Descartes entirely rejects traditional Aristotelian logic and takes purely intellectual knowledge to be exemplified by the procedure of his new analytic geometry instead, Leibniz self-consciously returns to the idea that purely intellectual knowledge is essentially logical. And, although Leibniz appears to have envisioned some sort of extension of Aristotelian logic capable of embracing the new algebraic methods of his calculus, there is no doubt that the traditional subject-predicate structure of this logic pervades his monadic metaphysics: it is precisely because ultimate metaphysical reality is essentially intellectual in the logical sense that the entire mechanical world, including space, is a merely secondary reality or phenomenon. Thus, although Leibniz, like everyone else in the period, holds that there are exact mathematical laws governing the sensible and material world, he reintroduces a new kind of necessary gap between reality as known by the intellect and this sensible world.

For Newton, by contrast, space – the very space in which bodies exist and move – is metaphysically fundamental, for, as we have seen, it is "an affection of every kind of being," including God himself. Indeed, Newton puts the point even more strongly several pages later (*De Grav*., p. 25): "Space is an affection of a being just as a being. No being exists or can exist which is not related to space in some way." In particular, God, through his omnipresence, creates matter in space by endowing certain determined regions with mass (*vis inertiae*), and God thereby institutes the (Newtonian) Laws of Motion by singling out momentum (mass multiplied by velocity) as the fundamental dynamical quantity governing all changes of motion of matter. For Newton, moreover, impressed force (*vis impressa*) is a further dynamical quantity involved in such changes – where this refers to any action on the body in question by which a change of momentum is produced. Impressed force, in the Newtonian sense, is an external action on a body by something else, not an internal principle of change like Leibnizean *vis viva*, and, what is more, the changes it effects are not intrinsically limited to the condition of contact. On the contrary, the principal instantiation of this concept, in the *Principia*, is precisely the force of universal gravitation, whereby one body exchanges momentum with another body immediately and at a distance; and it is the theory of universal gravitation, as we have said, which then puts the notions of absolute space, time, and motion to real physical work in determining the true "center of the world."

It is by no means surprising, therefore, that Newton also rejects the traditional Aristotelian notion of substance, and replaces it, in effect, with space itself – or, more precisely, with space plus God (*De Grav*., p. 29): "For the existence of these beings [bodies] it is not necessary that we suppose some unintelligible substance to exist in which as subject there may be an inherent substantial form; extension and an act of the divine will are enough. Extension takes the place of the substantial subject in which the form of the body is conserved by the divine will; and that product of the divine will is the form or formal reason of the body denoting every dimension of space in which the body is to be produced." For Leibniz, by contrast, space, as we have seen, is a mere "well-founded phenomenon," and pure intellectual knowledge is explicitly modelled on Aristotelian subject-predicate logic: (a modified version of) the Aristotelian concept of substance *must* be metaphysically fundamental.

Newton's struggles with the problem of action at a distance result in significant complications here. Although later Newtonians (including Kant) were happy to conceive gravitation as an immediate action of one body on another body across empty space, Newton himself was seriously troubled. He appeared deliberately to leave it open in the first (1787) edition of the *Principia* that gravity may ultimately be explained by mechanical impact; and he also speculated in the *Optics* about an interplanetary aetherial medium as the cause of gravity.[6] Moreover, Newton famously declared that the idea of action at a distance is an "absurdity" in his well-known letter to Bentley of February 5, 1693 (pp. 102–103):

> It is inconceivable that inanimate brute matter should, without the mediation of something else, which is not material, operate upon and affect other matter without mutual contact, as it must be, if gravitation in the sense of Epicurus, be essential and inherent in it. And this is one reason why I desired you would not ascribe innate gravity to me. That gravity should be innate, inherent, and essential to matter, so that one body may act upon another at a distance through a vacuum without the mediation of anything else, by and through which their action and force may be conveyed from one to another, is to me so great an absurdity, that I believe that no man who has in philosophical matters a competent faculty of thinking can ever fall into it. Gravity must be caused by an agent acting constantly according to certain laws; but whether this agent be material or immaterial, I have left to the consideration of my readers.

And what is most striking, from our present point of view, is the suggestion that the true cause of gravity may be an *immaterial* agent – perhaps even God himself.

It is natural, in the first place, that the mediating agent between distantly gravitating bodies be immaterial, for it is essential to Newton's argument for universal gravitation in Book III of the *Principia* that such mutually attracting bodies – Jupiter and Saturn, for example – directly and immediately exchange momentum with one another, entirely independently of any other matter that may be located in between. Whatever is playing this mediating role must therefore experience negligible exchanges of momentum with the two attracting bodies themselves, and the most natural way to achieve this, in general, is to conceive the mediating agent as massless or immaterial. Moreover, in the second place, since God exists or is omnipresent everywhere in space, and he thereby creates matter and its fundamental laws by an immediate act of the divine will, it is natural to suppose that the ubiquitous immaterial agent ultimately responsible for gravitational attraction is either God himself or an ubiquitous immaterial spirit directly resulting from God's own ubiquity.[7] Finally, in the third place, God is described in the

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[6] Thus, for example, in the Scholium to section 11 of Book I of the *Principia*, after discussing the three-body problem at some length, Newton says (p. 86, my emphasis): "I use the word 'attraction' here in a general sense for any endeavor whatever of bodies to approach one another, whether that endeavor occurs as a result of the action of the bodies either drawn toward one another or acting on one another by means of spirits emitted or whether it arises from the action of the aether or air or of any medium whatsoever – whether corporeal or incorporeal – *in any way* impelling toward one another the bodies floating therein." However, as explained in note 17 below (which also discusses the aetherial medium proposed in the *Optics*), Newton definitely appears to exclude mechanical impact from the possible candidates in the second (1713) edition of the *Principia*.

[7] In Query 31 to the *Optics* (unlike *De Grav.*), Newton suggests that God's act of creating matter in space is responsible not only for impenetrability and mass (in accordance with the three "passive" laws of motion), but also for specific forces or "active principles," including gravity (pp. 136–137): "[I]t seems probable to me, that God in the beginning formed matter in solid, massy, hard, impenetrable, moveable particles, of such sizes and figures, and with such other properties, and in such proportion to space, as most conduced to the end for which he formed them;… It seems to me farther, that these particles have not only a *vis inertiae*, accompanied with such passive laws of motion as naturally result from that force, but also that they are moved by certain active principles, such as that of gravity, and that which causes fermentation, and the cohesion of bodies. These principles I consider, not as occult qualities, supposed to result from the specific forms of things, but as general laws of nature, by which the things themselves are formed; their truth appearing to us by phenomena, though their causes be not yet discovered." Compare also the passage quoted in note 4 above, which can easily be taken to suggest that God himself "moves" the bodies interacting in accordance with universal gravitation.

General Scholium added to the second edition of the *Principia* in 1713 as an omnipresent acting substance (p. 91): "God is one and the same God always and everywhere. He is omnipresent not only *virtually* but also *substantially*; for action requires substance."[8] Therefore, Newton does not so much entirely reject the traditional notions of substance and active agency, but reinterprets them in light of his metaphysics of space. He continues to conceive of efficient causality as the (local) action of one substance on another, and God, in particular, is the ultimate substantial agent underlying all causal action in the material world. His true opposition to Descartes concerns the notion of specifically *material* substance, and he uses his neo-Platonic (Cambridge Platonic) metaphysics of space to craft a further argument against Descartes's metaphysics from the (apparent) phenomena of gravitational attraction at a distance.

The importance of Newton's metaphysics of space in underwriting his principled rejection of the mechanical philosophy has not, I believe, been sufficiently appreciated. For, from a post-Newtonian perspective, the requirement that all causal interaction in the material world be limited to the communication of motion by impact may appear as an entirely arbitrary restriction on the basic principles governing the exchange of momentum, and there is then no reason, from this point of view, that a direct (equal and opposite) exchange of momentum at a distance via universal gravitation may not be viewed as a perfectly legitimate example of causal interaction.[9] At the time when Newton was first formulating this theory, however, everyone took it for granted that one substance could act on another by efficient causality only if the one is locally present to the other: this principle was shared by contemporary Aristotelians, by mechanical philosophers, and (as we have just seen) by Newton himself. Everyone also took it for granted that the clearest and most fundamental example of causal agency is the creative activity of God. Newton's metaphysics of space then made it possible for him to maintain that universal gravitation involves an immediate exchange of momentum across empty space (as his physics requires) while, at the same time, preserving the more traditional ideas

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[8] Immediately following this passage Newton adds (ibid.): "In him all things are contained and moved, but he does not act on them nor they on him." And, at the very end of the General Scholium, after pointing out that he has "not yet assigned a cause to gravity," and that, nonetheless, it is not to be reckoned among the "occult qualities," but is rather derived by induction from the phenomena, Newton continues (p. 93): "A few things could now be added concerning a certain very subtle spirit pervading gross bodies and lying hidden in them; by its force and actions, the particles of bodies attract one another at very small distances and cohere when they become contiguous; and electrical bodies act at greater distances, repelling as well as attracting neighboring corpuscles;…" If, in accordance with the above passage from Query 31 of the *Optics*, we suppose that this "very subtle spirit" is also the cause of gravitational attraction, it would follow that it is this (presumably immaterial or massless) "spirit" which mediates gravitational action in line with the letter to Bentley. God's own active agency would then be confined to creating both matter and the spirit in question, which then *interact* with one another to produce the phenomena of gravitational attraction.

[9] This, for example, is how Robert DiSalle views the matter in his excellent recent philosophical history of space-time physics. In particular, according to DiSalle (2006, p. 42): "[I]n the Newtonian view, any interaction is physically intelligible as long as, and just to the extent that, it conforms to the laws of motion." This, however, was not the view of Newton himself; rather, it is a (certainly very natural) conception arising in a post-Newtonian context where Newton's physics itself is then taken as a reliable guide to metaphysics – for example, as we shall soon see, in Kant.

of causality and agency he shared with his contemporaries. Indeed, from Newton's own point of view, his conception of the creation of matter by God makes maximal room for divine creative activity, and thereby avoids the threat of atheism opened up by the Cartesian conception of material substance.[10]

Kant, in the pre-critical period, attempts to fashion a direct unification of Leibnizean and Newtonian ideas, by starting with a Leibnizean metaphysics of monads and then building a Newtonian metaphysics of space, as it were, on top of this monadic metaphysics. The primary reality remains a non-spatial realm of ultimate simple substances, but these substances, for Kant, now have *both* purely internal, intrinsic properties *and* external or extrinsic relations. Such external relations among the monads are not necessary for them to be the simple substances which they are, but they are necessary for them to exist – or, more precisely, to co-exist – together in a common world. In this way, God's creative activity has two distinguishable aspects: one act by which the simple substances themselves are created in the first place, and a second by which a number of such simple substances are joined together into a single world. This second act occurs in conformity with what Kant calls a "schema of the divine intellect," and it is in virtue of just such a schema, in the end, that what we know as the laws of nature then arise. More precisely, what we know as the fundamental forces of matter (attraction and repulsion) – together with the laws that govern them – are a direct expression of the divinely instituted external relations (of *co*-existence) between monads; and what we know as space is then the phenomenal expression of this same system of divinely instituted relations. Space is thus a secondary reality, derivative from the monads and their external relations, but, since the external relations between monads, for Kant, are just as real as their internal properties, it is a reality nonetheless – and not, as in Leibniz, a merely ideal "well-founded phenomenon." Indeed, since the fundamental force of attraction, for Kant, is explicitly modelled on Newtonian universal gravitation (as an immediate action at a distance through empty space), Kant explicitly links his pre-critical conception of space with the Newtonian conception of divine omnipresence.[11]

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[10] Of course Newton's conception of divine agency is highly unorthodox, and, from a more traditional point of view, one would certainly not constrain God's creative activity by the requirement of local presence governing the interactions of material substances. From this point of view, it is Newton who opens up the threat of atheism (or rather pantheism) by seeming to materialize God. However, although Leibniz, for example, thus stands on firmer theological ground than Newton, he does not have a competing metaphysics adequate for natural philosophy and physics. Kant's problem was precisely to construct such a competing metaphysics along broadly Leibnizean lines, while simultaneously doing full justice to Newtonian physics.

[11] Kant makes this connection in the *New Exposition of the First Principles of Metaphysical Cognition* and the *Universal Natural History and Theory of the Heavens*, both appearing in 1755. For discussion, and references, see Friedman (1992, pp. 5–14). As I point out there, an echo of the Newtonian doctrine of divine omnipresence occurs as late as the Scholium to section 22 of the *Inaugural Dissertation* (1770). (Kant of course had no knowledge of Newton's unpublished *De Gravitatione*, but, as observed above, essentially the same metaphysics of space surfaces in such well-known published writings as the General Scholium to the *Principia* and the Queries to the *Optics*.) For further recent discussions of Kant's pre-critical metaphysics see Laywine (1993), Schönfeld (2000), Watkins (2005). A recent volume of translations is Walford and Meerbote (1992).

It is in the *Inaugural Dissertation* of 1770 that Kant makes a fundamental break with the Leibnizean philosophy – and, in a somewhat different fashion, with the Newtonian philosophy as well. Kant here first articulates his characteristic distinction between two independent rational faculties of the human mind – the pure understanding or pure intellect, on the one side, and pure sensibility or pure intuition, on the other. The former embodies the traditional categories and concepts of rational (Leibnizean) metaphysics, but it is the latter, for Kant, which now embodies the concepts and principles of pure mathematics. In particular, Kant now holds that mathematical knowledge is in no way purely intellectual, but is rather essentially intuitive or sensible, requiring the forms of pure sensibility, space and time. The world as we know it therefore bifurcates into two: the intellectual world described by traditional metaphysics (the Leibnizean metaphysics of ultimate simple substances as modified by the earlier Kant), and the sensible world as described by mathematics and mathematical physics in space and time. Although something like Newtonian space therefore remains as the foundation of this sensible world, space can no longer be conceived, as in Newton, as the sensorium of God – it is rather, as it were, the form of *our* sensorium, the form of our pure sensibility. Yet it is an unresolved problem, in the *Inaugural Dissertation*, how these two worlds are now supposed to be related, and, in particular, how the world described by mathematics and mathematical physics (the world as it appears to us) is related to the ultimate metaphysical reality of the intellectual world.

It is precisely this problem which finally gives birth to the critical philosophy in 1781. Kant now declares that purely intellectual, metaphysical knowledge – whether of immaterial things like God and the soul or of the ultimate simple substances which (according to both Leibniz and the pre-critical Kant) underlie the material world – is completely impossible, at least from a theoretical point of view. The pure intellect, considered entirely on its own and independently of any possible relation to sensibility, can issue only in the empty logical forms of Aristotelian syllogistic: in what Kant calls the "logical forms of judgement." And, while it is true that these forms then yield, in what Kant calls the "metaphysical deduction," the pure concepts or categories of the understanding (substance, causality, community, possibility, actuality, necessity, and so on), such pure concepts of the understanding are themselves entirely empty and without any "relation to an object" (again from a purely theoretical point of view) considered independently of our particular (human) forms of sensibility – space and time.[12]

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[12] Kant takes particular pains, in the second (1787) edition of the *Critique*, to emphasize that his conception of space and time as pure forms of sensibility is the only real alternative to the – theologically disastrous (compare note 10 above) – Newtonian view (B 71–72): "In natural theology, where one thinks an object that is not only no object of sensible intuition for us, but cannot even be an object of sensible intuition for itself, one takes care to remove the conditions of space and time from all of its intuition (for all of it cognition must be intuition and not *thought*, which is always a manifestation of limitations). But with what right can one do this, if one has previously made both into forms of things in themselves – and, indeed, into forms which, as a priori conditions of the existence of things, even remain when one has annihilated the things themselves? (For, as conditions of all existence in general, they must also be conditions for the existence of God.)

In short, it is only in virtue of spatio-temporal "schemata" produced by our pure intellect that rational knowledge of the phenomenal world is possible, and the task of showing how the pure intellect thereby injects itself into pure sensibility (space and time) so as to apply the pure categories of the understanding to sensible experience then becomes the problem of the transcendental deduction.[13] Such an injection of *our* pure intellect into *our* pure forms of sensibility now takes the place, as it were, of Kant's pre-critical doctrine that a schema of the divine intellect, by an analogue of Newtonian divine omnipresence, is ultimately responsible for the order we perceive in the physical world.[14]

Pure metaphysical concepts – pure concepts of the understanding – can now be used for genuine (theoretical) knowledge only when applied to spatio-temporal

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There is therefore no alternative, if one does not pretend to make them into objective forms of all things, except to make them into subjective forms of our outer and inner mode of intuition. [This kind of intuition] is called sensible, because it is *not original* – i.e., it is not such that the existence of objects of intuition is itself given through it (which, as far as we can comprehend, can only pertain to the primordial being), but it depends on the existence of the objects, and is thus only possible in so far as the representative faculty of the subject is affected by them." (All translations from Kant's writings are my own, and I cite all writings – except for the first *Critique* – by volume and page numbers of the standard Akademie edition of *Kant's gesammelte Schriften*.).

[13] Since, for Kant, the pure mathematician inscribes figures in space – in the process of Euclidean construction – by this same activity of the understanding, we thereby obtain, at the same time, an explanation of why all empirical objects in the phenomenal world (appearances) are necessarily subject to pure mathematics. This explanation essentially involves the categories of quantity and, in particular, the Axioms of Intuition (A 165–166/B 206): "The synthesis of spaces and time, as the essential form of all intuition, is that which also makes possible the apprehension of the appearance, and thus all outer experience, and therefore all cognition of the objects of experience; and what mathematics in its pure use demonstrates of the former [the essential form of all intuition], it is also necessarily valid for the latter [all outer experience, etc.]." And it is in precisely this way, too, that Kant demonstrates the necessary applicability of mathematics to sensible experience (and forestalls any possible Platonic gap between the two) which Newton secured by his metaphysics of space: compare the paragraph to which note 5 above is appended.

[14] See note 11 above, together with the paragraph to which it is appended. As I observed, there is an echo of the pre-critical theory of divine omnipresence even in the *Inaugural Dissertation*, where Kant has already drawn a fundamental distinction between understanding and sensibility. The question Kant raises there (in the Scholium to section 22) concerns precisely the *causes* of our sensible intuitions, and, in particular, the relationship between our sensible intuitions and the assumed ultimate substances constituting the intelligible world. The answer Kant (tentatively) suggests is that, since both our mind and these "external things" are sustained by a single infinite being, space, as the "sensibly cognized universal and necessary condition for the co-presence of all things" can thus be characterized as (God's) *phenomenal omnipresence*. In light of section 22 itself, it appears that Kant is thereby invoking a pre-established harmony (instituted by God) between the purely intellectual reality of ultimate substances and our spatio-temporal sensibility to explain the necessary connection between this reality as it is in itself and as it appears to us. In section 27 of the second edition transcendental deduction, Kant explicitly rejects such an explanation of the agreement between experience and its objects (which he calls a "**preformation-system** of pure reason") in favor of his new, critical explanation (which he calls an "**epigenesis** of pure reason") – where, as I understand it, the understanding rather creates the a priori order of sensible experience by injecting *itself* into the pure forms of sensibility.

"appearances," and thus only when "schematized" in terms of space and time: substance in terms of permanence, causality in terms of succession, and so on. When we do this, moreover, we find that specifically outer or spatial intuitions are also necessarily required, so that, in particular, "in order to give something *permanent* in intuition corresponding to the concept of substance (and thereby to verify the objective reality of this concept), we require an intuition *in space* (of matter), because space alone is determined as permanent, but time, and thus everything in inner sense, is continually flowing" (B 291). There is no longer any room (among the objects of theoretical knowledge) for mind-like or spiritual substances in the traditional sense, and there is no such room, therefore, for Leibnizean simple substances having only purely internal properties:

> Only that is internal in an object of pure understanding which has no relation at all (with respect to its existence) to anything different from itself. By contrast, the internal determinations of a substantia phaenomenon in space are nothing but relations, and it itself is nothing but a totality of mere relations. We are only acquainted with substance in space through forces that are active in space, either driving others into [this space] (attraction) or stopping their penetration into it (repulsion and impenetrability). We are acquainted with no other properties constituting the concept of a substance which appears in space and which we call matter. As object of the pure understanding, on the other hand, every substance must have internal determinations and powers, which pertain to [its] internal reality. However, what can I entertain as internal accidents except those which my inner sense presents to me—namely, that which is either itself a thought or is analogous to it? Therefore, Leibniz, after he had taken away everything that may signify an external relation, and therefore also composition, made of all substances, because he represented them as noumena, even the constituents of matter, simple substances with powers of representation—in a word, **monads**. (A 265–266/B 321–322)

The entire conception of the Leibnizean monadology – along with the more traditional conception of purely mental or spiritual substances – is now seen to rest on a fundamental mistake: neglecting the necessary spatio-temporal schematization of the pure concepts of the understanding.

But it now follows, similarly, that our basic concepts of action and efficient causality – by which one substance effects a change in another – must also be limited to the necessary conditions of specifically outer or spatial intuition (B 66–67): "[E]verything belonging to intuition in our cognition (and thus excluding the feeling of pleasure and displeasure, and the will, which are certainly not cognitions) contains nothing but mere relations—[relations] of position in an intuition (extension), change of position (motion), and laws in accordance with which this change is determined (moving forces). But what may be present in the position, or what may be active in the thing itself aside from the change of position, is not thereby given." Aside from the intuitively presented laws governing the spatio-temporal changes of phenomenal substances, in other words, we have absolutely no conception of inter-substantial efficient causality at all – at least, once again, from a purely theoretical point of view.

It is in the *Metaphysical Foundations of Natural Science* of 1786 (appearing between the first and second editions of the first *Critique*) that Kant develops the "special metaphysics of corporeal nature" governing matter or material substance.[15] In particular, in the second or Mechanics chapter, the three Analogies of Experience governing the pure categories of substance, causality, and community are here specifically instantiated or realized by what Kant calls the three "laws of mechanics" – the conservation of the total quantity of matter, the law of inertia, and the equality of action and reaction – which Kant takes to be very close to (although not completely identical with) the three Newtonian Laws of Motion. In the case of matter or material substance, therefore, its possible changes and interactions are entirely delimited by these laws, in the sense that what it now *means* for one (material) substance to exert a causal action on another (so as, in this case, to effect a change of motion in it) is simply for a well-defined exchange of momentum to take place between the two. Thus, if two bodies exchange momentum at a distance across empty space (as, in Newton's theory of universal gravitation, they must), then they do in fact causally interact with one another at a distance, and there are absolutely no remaining grounds for raising metaphysical or theological objections.[16]

The second or Dynamics chapter introduces the two fundamental forces of repulsion and attraction – the one responsible for impenetrability, the other for gravitation. Proposition 7 states (Ak. 4, 512): "The *attraction essential to all matter* is an immediate action of matter on other matter through empty space." And, in the first remark to this proposition, Kant argues that to confine the activity of matter by the condition of contact would be an entirely arbitrary restriction (Ak. 4, 513):

> [T]o say that matters cannot act immediately on one another at a distance, would amount to saying that they cannot act immediately on one another except through the forces of impenetrability. But this would be as much as to say that repulsive forces are the only ones whereby matter can be active, or that they are at least the necessary conditions under which alone matters can act on one another, which would declare attractive force to be either completely impossible or always dependent on the action of repulsive forces. But these are both groundless assertions.

Once we conceive both impenetrability and gravitation as impressed forces in the Newtonian sense, governed solely by the Newtonian laws of motion, then there is no longer any reason to take one to be more intrinsically intelligible than the other.

In the second remark to the same proposition, however, Kant goes on to make a much stronger claim—that, in explicit opposition to Newton, gravitational attraction *must* be conceived as an essential active power of matter, operating immediately at a distance through empty space (Ak. 4, 515):

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[15] All translations from this work are taken from Friedman (2004).

[16] Leibniz's main theological objection to the Newtonian force of gravity, it will be recalled, was that it would be a "perpetual miracle" if a body could persist in orbital motion (without flying off along the tangent in accordance with the law of inertia) unless the material in a celestial vortex acted upon it by impact or pressure to maintain this orbital motion. Since Newton himself shared the wide-spread rejection of action at a distance at the time, he could not give the straightforward rejoinder later available to Kant: the sun itself causes the planets to persist in their orbits, by precisely its immediate attraction across empty space. Compare notes 9 and 10 above, together with the paragraph to which they are appended.

> [O]ne cannot adduce this great founder of the theory of attraction as one's predecessor, if one takes the liberty of substituting an apparent attraction for the true attraction he did assert, and assumes the *necessity* of an impulsion through *impact* to explain the phenomenon of [gravitational] approach. He rightly abstracted from all hypotheses purporting to answer the question of the cause of the universal attraction of matter, for this question is physical or metaphysical, but not mathematical. And, even though he says in the advertisement to the second edition of his *Optics*, "to show that I do not take *gravity* for an *essential* property of bodies, I have added one question concerning its cause," it is clear that the offense taken by his contemporaries, and perhaps even by Newton himself, at the concept of an original attraction set him at variance with himself. For he could not say that the attractive forces of two planets, those of Jupiter and Saturn, for example, manifested at equal distances of their satellites (whose mass is unknown), are proportional to the quantity of matter of these heavenly bodies, if he did not assume that they attracted other matter merely as matter, and thus according to a universal property of matter.[17]

Kant's point here, specifically, is that Newton cannot leave the question of the "true cause" of universal gravitation entirely open, without fatally compromising the fundamental property of this interaction that the mutual accelerations in question are directly proportional to the masses or quantities of matter of the two interacting bodies.

I have considered Kant's argument in detail elsewhere,[18] so let me simply state it briefly here. Consider the system consisting of Jupiter, Saturn, and two of their respective moons. Newton's argument in Book III of the *Principia* crucially involves the idea that one can determine the masses of the primary bodies in question by the gravitational accelerations produced in their satellites. Newton assumes, in order to make this determination, that there are also gravitational accelerations of Saturn produced by Jupiter and *vice versa*. Then, in the most important step, Newton applies the equality of action and reaction directly to these two accelerations, so that the acceleration of Jupiter towards Saturn, multiplied by the mass of Jupiter, is equal and opposite to the acceleration of Saturn towards Jupiter, multiplied by the mass of Saturn. Newton assumes, in other words, that we can apply the conservation of momentum directly to this particular exchange, entirely independently of what other matter may or may not be found in between.[19] For Kant, this amounts, from

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[17] The "one question concerning its cause" added to the second edition of the *Optics* is of course Query 21, where Newton famously speculates that a universal "Aetherial Medium" growing denser at greater distances from the heavenly bodies might explain the gravitational interactions between these bodies. However, this aether does not act by impact (as in the vortex theory favored by the mechanical philosophers), but is rather governed by short-range repulsive forces (between the particles of aether) responsible for its pressure (and thus density). So far, therefore, this particular speculation about a possible cause for gravity is consistent with Newton's remarks in the General Scholium to the second edition of the *Principia* (compare note 6 above), where he denies that such a cause can be mechanical (p. 92): [T]his force arises from some cause that penetrates as far as the centers of the sun and the planets without any diminution of its power to act, and that acts not in proportion to the quantity of the *surfaces* of the particles on which it acts (as mechanical causes are wont to do) but in proportion to the quantity of *solid* matter, and whose action is extended everywhere to immense distances, always decreasing as the squares of the distances."

[18] See Friedman (1992, pp. 153–159), and compare Friedman (1990).

[19] As is well-known, Roger Cotes objected to Newton on this score in their correspondence, and argued that Newton himself must therefore assume that gravitational attraction – as an immediate action at a distance – is in fact essential to matter. See Koyré (1965, chapter 7), and also Stein (1967).

a methodological point of view, to assuming, in effect, that no other matter is in fact involved, and that conservation of momentum within such an exchange is both necessary and sufficient for true causal action. So it is at precisely this point, therefore, that any metaphysical conception of cause pretending to compete with the conservation of momentum must now most definitely fall away.[20]

The importance of this argument is underscored, for Kant, by the circumstance that Newton's own inductive inference to the law of universal gravitation crucially involves such direct applications of conservation of momentum to gravitational interactions at a distance (in showing, for example, that Saturn's gravitational acceleration towards Jupiter is proportional to the mass of Jupiter and *vice versa*).[21] And it is further underscored, in particular, by the fact that the resulting determinations of the masses of the primary bodies in the solar system play a central role in Kant's parallel constructive procedure, articulated in the fourth chapter or Phenomenology, for arriving at the true motions of bodies from their apparent motions. We begin with our parochial perspective here on earth, from which we can record both the observable phenomena governed by Galileo's law of fall and the observable relative motions of a variety of satellites in the solar system with respect to their primary bodies (the moon relative to the earth, the planets relative to the sun, the moons of Jupiter and Saturn relative to their planets). The latter are just the phenomena expressed in Kepler's laws, and what we now find is that we can first determine the true state of rotation of the earth (using small deviations from the law of fall manifesting what we now call Coriolis forces), and we can then determine the masses of all the primary bodies in the solar system (at least those actually having satellites) – with the result (as Newton shows) that the center of mass of the solar system is always very close to the center of the sun.

Hence we can empirically determine, from the observable phenomena themselves, the true center of motion of the solar system, and this thereby counts as an approximation, for Kant, of Newtonian absolute space. However, since it is also true, for Kant, that the solar system itself rotates around the center of the Milky Way galaxy, this galaxy rotates around the center of a larger system of such galaxies, and so on *ad infinitum*, absolute space (the true center of motion of the entire universe) is in the end what he calls an "idea of reason" – a forever unreachable regulative ideal we can only successively approximate in experience but never fully attain.[22]

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[20] By contrast, for Newton himself, as we have seen, this particular problem is solved by taking the ultimate causal agent here to be immaterial and, indeed, divine (compare again notes 9 and 10 above, together with the paragraph to which they are appended).

[21] Thus, Newton's adherence to a neo-Platonic (Cambridge Platonic) metaphysics of space is not simply an additional (and arbitrary) assumption on his part, one which could easily be dropped. On the contrary, his own inductive argument for universal gravitation, in the context of the prevailing ideas about efficient causation and ultimate (divine) agency, more or less uniquely single out this metaphysics among the available alternatives.

[22] This constructive procedure for approximating absolute space in experience is analogous, in important respects, to the constructive method of Euclidean geometry (compare note 13 above). But the circumstance that the former can never be completed marks an essential difference between the two, closely related to Kant's view that the mathematical principles of pure understanding are *constitutive* with respect to intuition while the dynamical principles are merely *regulative* with respect to intuition (but constitutive with respect to experience): for further discussion see Friedman (1992, pp. 159–164).

Finally, since Newtonian absolute space is thus viewed as a regulative idea of reason, there is also an associated reconfiguration, for the critical Kant, of the relationships among space, the interactions of matter, and the idea of God. For the idea of God, too, is a regulative idea of reason. Indeed, there is an important sense in which it is the ultimate such regulative idea, since all human activity, together with the whole of nature, is ultimately subject to the idea of the Highest Good – the idea of a perfect community of all rational beings in a moral realm of ends, for which our only ground even to hope this could actually be achieved in nature (or, more precisely, successively approximated) is the idea of God (or, more precisely, divine providence). Moreover, Kant saw a deep analogy between the community of all rational beings in a moral realm of ends and the thoroughgoing community effected among all material bodies in the universe by universal gravitation, and this is the basis, in fact, for his late (and very striking) re-interpretation of the Newtonian doctrine of divine omnipresence in a footnote appended to the General Remark to the Third Part of *Religion Within the Limits of Reason Alone* (1793):

> When Newton represents [the universal gravitation of all matter in the world] as, so to speak, divine universal presence in the appearance (*omnipaesentia phenomenon*), this is not an attempt to explain it (for the existence of God in space contains a contradiction), but rather a sublime analogy, in which it is viewed merely as the unification of corporeal beings into a world-whole, in so far as we base this upon an incorporeal cause. The same would happen in the attempt to comprehend the self-sufficient principle of the unification of the rational beings in the world into an ethical state and to explain the latter from the former. We know only the duty that draws us towards this; the possibility of the intended effect, even when we obey this [duty], lies entirely beyond the limits of all our insight. (Ak. 6, 138–139)

For the critical Kant, in other words, the only possible meaning the idea of divine omnipresence (and divine providence) can now have is a purely *practical* meaning, in so far as we unconditionally obey the command of morality to strive to realize the realm of ends here on earth, and, accordingly, we take the whole of that material nature of which we are a part to be in principle *capable* of such a realization (or, more precisely, its successive approximation). Kant thereby brings the characteristic mode of metaphysical investigation into the relationships among space, God, and matter practiced by his predecessors to a close, and transforms it – without remainder – into transcendental philosophy.

References

Bricker, P. and R. I. G. Hughes, eds. (1990). *Philosophical Perspectives on Newtonian Science*. Cambridge, MA: MIT Press.

DiSalle, R. (2006). *Understanding Space–Time: The Philosophical Development of Physics from Newton to Einstein*. Cambridge: Cambridge University Press.

Friedman, M. (1990). “Kant and Newton: Why Gravity is Essential to Matter.” In Bricker and Hughes, pp. 185–202.

Friedman, M. (1992). *Kant and the Exact Sciences*. Cambridge, MA: Harvard University Press.

Friedman, M. (1997). “Descartes on the Real Existence of Matter.” *Topoi* 16: 1–10.

Friedman, M., ed. (2004). *Immanuel Kant: Metaphysical Foundations of Natural Science*. Cambridge: Cambridge University Press.

Friedman, M. (2008). “Descartes and Galileo: Copernicanism and the Metaphysical Foundations of Physics.” In J. Broughton and J. Carriero, eds. *A Companion to Descartes*. Oxford: Blackwell.

Hall, A. R. and M. B. Hall, eds. (1962). *Unpublished Scientific Papers of Isaac Newton*. Cambridge: Cambridge University Press.

Janiak, A., ed. (2004). *Isaac Newton: Philosophical Writings*. Cambridge: Cambridge University Press.

Koyré, A. (1965). *Newtonian Studies*. Cambridge, MA: Harvard University Press.

Laywine, A. (1993). *Kant’s Early Metaphysics and the Origins of the Critical Philosophy*. Atascadero, CA: Ridgeview Press.

Menn, S. (1998). *Descartes and Augustine*. Cambridge: Cambridge University Press.

Patrides, C. A., ed. (1980). *The Cambridge Platonists*. Cambridge: Cambridge University Press.

Schönfeld, M. (2000). *The Philosophy of the Young Kant*. Oxford: Oxford University Press.

Stein, H. (1967). “Newtonian Space–Time.” *Texas Quarterly* 10: 174–200.

Stein, H. (2002). “Newton’s Metaphysics.” In I. B. Cohen and G. E. Smith, eds. *The Cambridge Companion to Newton*. Cambridge: Cambridge University Press, pp. 256–307.

Walford, D. and R. Meerbote, eds. (1992). *Immanuel Kant: Theoretical Philosophy*, 1755–1770. Cambridge: Cambridge University Press.

Watkins, E. (2005). *Kant and the Metaphysics of Causality*. Cambridge: Cambridge University Press.

On Kant's Transcendental Account of Newtonian Mechanics

Pierre Kerszberg

Abstract Kant's account of Newtonian science in terms of a priori structures of the mind has been generally interpreted as too restrictive. If Newtonian science is an instantiation of the system of categories, then, in order to retain any value, they need to be dynamized in accordance with the development of science beyond Newton. This paper suggests that the restriction in best understood as Kant attempt to provide a primary matrix of sense for any possible natural science, inasmuch as it reflects the "first idea" contained in the Copernican Revolution.

1 Introduction

In the *Critique of Pure Reason*, Kant explained that a transcendental proof is not meant to "explain" natural phenomena (A174/B215–B216).[1] Its merit is to free the understanding so as to allow it to think them in ways other than those acknowledged by natural science in its own practice. Consider the possibility for space to be filled with matter, which is the cause of our sensation of the real. According to the transcendental principle of the Anticipations of Perception, space is always filled with matter, even if the filling does not reach the degree at which sensation begins. Color, heat, moment of gravity are examples of phenomena which show an infinite gradation of ever smaller degrees, without ever reaching the smallest possible degree. All levels of experience are represented in this gradation, from ordinary empirical judgment to a fundamental instance of modern physics. However, as Kant goes on to

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[1] Citations to Kant's texts are given in parentheses. Citations from the *Critique of Pure Reason* are located by reference to Kant's first (A) and/or second (B) editions. All other passages from Kant's works are cited by the volume and page number in the standard edition of Kant's works.

M. Bitbol et al. (eds.), *Constituting Objectivity*,

show, in the properly physical explanation of the filling of space, a presupposition of metaphysical type cannot be done away with. How is the physicist to explain the variety in the quantity of matter in bodies that have the same volume? In order to lend itself to mathematical language, the explanation rests on the assumption that the real in space is everywhere homogeneous, but the volume is empty in varying proportions. Against his own will or unconsciously, the physicist is also metaphysician, since the hypothesis according to which the real is everywhere of the same kind cannot be verified empirically, and the proof of empty space has no foundation in experience. Such an explanation is thus compelled to contradict the famous Newtonian imperative against such hypothesis ("hypotheses non fingo"). The transcendental proof is not metaphysically dogmatic in the precise sense that it allows for an alternative assumption, according to which matter fills space continuously by infinitesimal degrees: there is no place where matter is not present, even at the smallest possible degree. Kant leads metaphysics in a new direction, referred to as "general metaphysics" or transcendental philosophy, inasmuch as he is well aware that the latter assumption cannot give us insight into what actually occurs in material bodies as they differ in specific gravity. That is, the transcendental proof is not meant to step over the limits of natural science, but to free the understanding by dispensing physics with the burden of proving its own metaphysical presuppositions.

In fact this burden turns out to be too heavy. In the *Metaphysical Foundations of Natural Science*, Newton is accused by Kant to misinterpret the metaphysical implications of his own theoretical model (4:514). He was not willing to, or simply could not, equate the methodology of physical science with metaphysics. Newton is at variance with himself when he gives physicists too much freedom to explain the mechanism of the attractive force. Indeed, in a famous letter to Bentley, Newton said that this mechanism could be explained by the action of corpuscles, for example the pressure exerted by the ether. But the agent causing the propagation of force could still be viewed as immaterial, or it could still be ascribed to some immediate divine action. Metaphysics in Kant's new sense requires resolution. Against Newton's reluctance to assume instantaneous action at a distance for its own sake, Kant argues that gravity is an essential property of matter, which acts universally and immediately without the support of an intervening medium; moreover, the full universality of Newtonian physics is achieved by means of yet another universal force, repulsion, whose action Newton confined to the level of minute interactions between masses. As a matter of fact, action at a distance plays a fundamental role in the operations needed to measure the masses of celestial bodies, hence in the construction of a privileged reference frame capable of distinguishing true from apparent motions.[2] Newton asserts that the mass values of the planets can be derived from the gravitational attraction of their satellites; Kant then argues for the essentiality of the force of gravitation, that is, bodies attract each other merely as matter, because this is needed in order to say that the attractive forces of two planets are proportional to the quantity of matter of these heavenly bodies. Consequently,

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[2] See M. Friedman, *Kant and the Exact Sciences* (Cambridge, MA: Harvard University Press, 1992).

when Kant says that gravity is essential, he states the condition of a metaphysics of universal gravitation which would be naturalistic from beginning to end, in accordance with the actual aims of physics. Newton’s disagreement with himself is thus not freely assumed. He is in fact compelled to fall into such a disagreement, because he mistakenly believes that the extension from operational means to the being of phenomena (such as the cause of attractive force) requires metaphysical decision. The alternative task that transcendental philosophy sets for itself is to determine the metaphysical principles with which natural science is compelled to *begin*. This beginning, rather than the far-reaching implications of natural science beyond itself, is the proper place of metaphysical decision concerning natural science. Do the operational means of physics result from this beginning, and if so, how?

Showing that they do is the main core of the *Metaphysical Foundations*. The work divides into four sections. Each section is to consider the concept of matter in one of its aspects and each is said to add a new determination to it. In defining matter in its basic sense, Kant states that motion is fundamental, and all other predicates of matter are said to find their ground in it. ‘Das Bewegliche’ means something which is capable of moving or of being moved; it includes both the fact of motion and a ‘that which’ moves. The subject-matter of the four sections is determined accordingly. The definition of matter according to Phoronomy (what we today would call kinematics) states that matter is the movable in space (pure quantum of motion); this is the purely geometrical aspect of motion. The definition according to the Dynamics states that matter is the movable inasmuch as it fills a space: it has the quality of an original power of motion, since this filling contains the fundamental forces which reside in matter. In the Mechanics, matter is regarded as having a moving force as a consequence of the motion of material bodies and their mutual actions. In the Phenomenology, matter is seen as having motion or rest relative to a mode of representation, that is, as appearance of outer sense. These four sections are thus the application of the four groups of categories that Kant had outlined in the *Critique*.

2 Phoronomy

Kant’s modification of Newton’s strategy reflects his concern throughout his intellectual journey about the limitations of natural philosophy based on Newtonian mechanics. A grounding in metaphysics is necessary for any such natural philosophy. Early in his precritical period Kant identified this grounding with the Leibnizean notions of substance and active force. Metaphysically speaking reality would thus consist in unextended, simple and non-interacting substances, determined by purely internal principles. But this worldview clashes with the teachings of Newtonian mechanics, which Kant always accepted as paradigmatic: material reality is spatio-temporal and essentially interactive. From the outset Kant’s examination of the natural sciences aimed at reconciling Leibniz (reality is deduced from some first general principles of natural bodies) and Newton’s “deduction from phenomena.” Kant’s formulation of those metaphysical principles with which science begins is his final breakthrough toward such a synthesis. However problematical it may be in its own terms, this strategy creates a number of problems within transcendental philosophy itself. Kant conscientiously speaks about *Anfangsgründe* rather than a priori principles. The metaphysical principles of natural science will not be a priori in the sense suggested in the *Critique*. Whereas Kant argued in the *Critique* that the categories of pure understanding are empty when deprived of content, whereas intuitions without concepts are blind (that is, meaningless) (A51/B75), in *Metaphysical Foundations* he said the "separated metaphysics of corporeal nature [enable us] … to realize the concepts and propositions of [transcendental philosophy], that is, to give a mere form of thought sense and meaning" (4:478). Thus the concrete instances provided by the special metaphysics of nature realize the pure concepts and principles of the understanding, i.e., they give them meaning in addition to filling them with content. This new relation between thought and content, form and matter, imposes a number of constraints on both. The first instance of a filling of a pure thought that is also meaningful for thought is empirical space.

The consideration of empirical space is forced on us by the movability of an object in space, which must be examined in accordance with a priori principles even though this concept is *per se* empirical and cannot be given a priori at all. Empirical space is not opposed to the pure form of outer sensible intuition, but rather is *added* to it (4:481). Empirical space, Kant writes, is "a space as a property of the things we are considering, namely, corporeal entities" (4:484). Since the metaphysical explication of matter is not concerned with a predicate that belongs to it as object, but rather proceeds in relation to the cognitive faculty in which its representation is given, it must be asked: How does general metaphysics accommodate such an addition to space as a subjective form?

The Phoronomy contains the principles of the application of the category of quantity to matter in motion. Under what conditions does matter in motion fall within the requirements of an extensive magnitude, according to which several parts of a given whole can be juxtaposed? Any matter in motion is composed of other motions which must be homogeneous to one another. The problem underlying the Phoronomy is thus the following: what are the minimum conditions of possibility for the composition of the smallest number of parts, that is, any two motions? Kant's answer is that the sought-for composition requires *two spaces*. This is stated in the Proposition (*Lehrsatz*): "The composition of two motions of one and the same point can only be thought of by one of them being represented in absolute space, but instead of the second motion being so represented, a motion of the relative space in the opposite direction and with the same velocity is represented as being identical with the first motion" (4:490). Motion as quantity includes the concept of direction, which is spatial, and velocity, which is both spatial and temporal. But the pure doctrine of motion as quantum is to be arrived at away from time, "according to all three moments furnished by space" (4:495), namely: unity, plurality, and totality.

Whereas empty space is the formal condition of intuition for nature in general, material or empirical space is the condition for matter in motion to become an object of experience. Where does empirical space itself come from, what does it specify? In other words: what is the thought condition that allows us to posit a reference frame for the description of a body's motion? Since this space is movable, it must be embedded in an even larger frame at rest thanks to which it can be perceived, and so on indefinitely as the frames become more encompassing. Newton's mathematical theory of motion applied to the celestial bodies in the solar system, but Kant's examination of this theory inserts it in the much larger field of the entire cosmos. One conclusion that first comes to mind is that all these larger frames at rest tend to approximate absolute space in the ontological sense of Newton. But since any frame in particular is still movable, absolute space as limit is a nothing for possible experience, that is, an idea of reason. As Kant puts it: "To assume an absolute space … as itself given means to assume something that cannot be perceived *either in itself or in its consequences* (motion in absolute space) for the sake of the possibility of experience, which must in actuality always be constituted without such a space" (4:481 emphasis added). The immediately sensible effects of absolute space (such as the centrifugal force) do not compel us to accept Newton's switch away from them. The leap from the material to the purely spatial frame rests on a confusion between merely possible frames of reference and actually given frames. The enlarged frame beyond each given space, "though still material space," is apprehended only in thought. New enlarged frames cannot be posited as better and better approximations of the largest possible frame, because this possibility of thought can never coincide with an actual object of experience (which is too large), and therefore must remain a pure possibility (a logical universality). When Newton absolutizes space, he mistakes "the logical universality of any space, with which I can compare each empirical space as being included in it, for the physical universality of a real container." Newton argues for a transition from the relative (the possibility of measurement in relative spaces) to the absolute (the true magnitudes) via the recognition that apparent motions are differences of true motions. But inasmuch as a possibility of thought is meaningful only when filled with an actual object of experience, Kant assumes that the absolute and the relative are with us right from the start; their conjoined presence, all the way through every possible experience, forbids any leap to the absolute from within the relative.

Now, the confusion of an object of thought with a real object is a process that Kant has described most generally as transcendental subreption. This is a process according to which one holds his own subjective representation as something objective, i.e., a cognition of what is really in the object. Through subreption the subjective representation is hypostasized into a thing thought as the ground of all things. The occurrence of such a mechanism is of course most typical in the ideas of reason. These are necessary (not arbitrary) concepts of reason inasmuch as they consider every cognition as determined by the absolute totality of conditions (soul, world, God), but their objective employment is "transcendent" rather than "transcendental" since no object corresponding to them can be given in experience. Since Kant's discussion of space in Newton's physics seems to refer to reason rather than the understanding, one interpretation of Kant's project has been to argue that particular mathematical–physical theories such as Newton's belong to the regulative use of reason, whereas the understanding constitutes experience as either ordinary experience or experience in general. This has become a rather common interpretation of Kant's project in twentieth-century scholarship.[3] Obviously enough, this kind of interpretation was meant to remove Kant from too close an adherence to Euclidean geometry and Newtonian mechanics, thereby making transcendental philosophy immune to the later transformations of science brought about by Riemann or Einstein. More recently, in response to this view, Friedman has argued quite the opposite, namely, that Kant's most general account of the possibility of experience in terms of a priori synthetic judgments is itself dependent on the particular examples provided by the exact sciences of nature. Therefore Kant's metaphysical grounding of absolute space in his doctrine of pure natural science would be the primary example showing that his entire system is exposed to further progress of empirical natural science just as the principles of pure understanding are subject to refutation. The regulative use of reason is compatible with various theoretical models, whereas the outright refutation of Kant's actual system would still leave it as an example of what needs to be done today, since a priori constitutive principles are still needed in order to ensure mathematical first principles in physics.

Is the core of transcendental philosophy of science to be assessed in relation to the (otherwise legitimate) worry that it might dissolve in light of such theories as general relativity or quantum mechanics? Both interpretations agree at least on the following: If first principles are mathematical, then whether they retain their allegedly constitutive function or whether they are purely regulative and hover above empirical natural knowledge does not change anything to the fact that they can be historicized, generalized and relativized at will in accordance with the development of the mathematical and physical sciences themselves. This view echoes a rather standard interpretation inherited from neo-Kantianism, according to which Kant seeks to provide a legitimate (transcendental) grounding of Newtonian science, as if the latter was an instantiation of the system of categories of pure understanding.[4]

3 The Role of Mathematics

To be sure, Kant claims that "although a pure philosophy of nature in general … may indeed be possible even without mathematics, a pure doctrine of nature concerning determinate natural things … is only possible by means of mathematics. And, since in any doctrine of nature there is only as much proper science as there is a priori knowledge therein, a doctrine of nature will contain only as much proper science as there is mathematics capable of application there" (4:470). What is at stake here is not the trivial observation that the only possible science of nature must be expressed

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[3] See in particular G. Buchdahl, "Gravity and Intelligibility: Newton to Kant", in *The Methodological Heritage of Newton*. Eds. R. Butts and J. Davis (Oxford: Blackwell, 1970), pp. 74–102.

[4] For an alternative account emphasizing Kant's rationalistic background, see E. Watkins, "Kant's Justification of the Laws of Mechanics", in E. Watkins (ed.), *Kant and the Sciences* (Oxford: Oxford University Press, 2000), pp. 136–159.

in mathematical language. Kant has shown anyway that there will always be completely naturalistic interpretations of physical theories, as long as they are formulated mathematically. In the Preface to *Metaphysical Foundations*, Kant argues that the concept of matter is given a "complete analysis" when presented in accordance with the four groups of categories of pure understanding, fully justifying the application of mathematics to physics. However, if this application is to succeed, the science of nature must be founded on something *pure*, for otherwise it would lose its character as necessity. How can science grow empirically without limit, and yet preserve its non-empirical and necessary foundation? In order to demonstrate this possibility, Kant shows that the something pure lying at the foundation of natural science (what he calls *physica generalis*) is made up of *both* mathematical and metaphysical propositions. Does this mean that the a priori knowledge of the objects of physica generalis *combines* mathematics and metaphysics so as to melt them together in some way?

In the *Critique* mathematics as knowledge by construction of concepts in intuition is clearly contrasted with philosophy as knowledge by concepts (A713/B741). Thus there is nothing in the examples provided by Kant to support the possibility of a transcendental synthesis of imagination (B154) to indicate that thinking a line by drawing it in thought, or a circle by describing it, requires a mere mathematical point, though it could be compatible with it; and even if the motion of a mathematical point were to be involved in the drawing of a straight line, this motion is still not necessarily the rectilinear inertial motion which grounds the principle of inertia in modern physics.[5] All true metaphysics, Kant says, that is, any metaphysics whether general or special, "is taken from the essential nature of the thinking faculty itself and therefore is by no means invented" (4:472). But a concept such as the mathematical concept, which is constructed in an intuition which is itself a priori, is an arbitrary concept which has been invented in order to be defined; such a concept "does not assure me of the existence or of the possibility of its object" (A729/B757). The contrast between the two kinds of knowledge is now to be re-assessed, however, since we are interested in the knowledge of *determinate* natural things, not nature in general. Inasmuch as they derive from the logical functions of unity in a judgment, the categories are themselves too general to yield the knowledge of determinate things. What are we to do with this generality? Kant goes so far to say that, when transposed from nature in general to determinate natural things, the transcendental possibility of thought loses its synthetic power and converts back into something analytical: "that it does not contradict itself" (4:470). How, then, can we find the a priori synthesis which will be appropriate to the new context? In the case of the knowledge of nature in general, mathematics did not have a specific function in transcendental synthesis. But as a construction of concepts in pure intuition, it can now be taken precisely as that which allows thought to be projected outside of its sphere. Pure mathematics proceeds synthetically in just the way needed by the a priori concept of matter: for "if pure intuition be wanting, there is nothing in which the matter for synthetic judgments a priori can be given" (4:471). That is, in the mathematical synthesis, the form may substitute for the matter of construction. Therefore, there must

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[5] Compare M. Friedman, "Matter and Motion in the Metaphysical Foundations and the First Critique", in E. Watkins (ed.), *Kant and the Sciences*, op. cit., pp. 62–64.

at the very least be a correspondence between mathematical construction (which takes us outside of the concept) and "the necessity of what belongs to the existence of a thing" (4:469), even though existence itself is of course not constructible. The latter necessity is the properly metaphysical one, which has as its purpose to present first the "principles of the construction of concepts that belong to the possibility of matter in general" (4:472). Kant's *Metaphysical Foundations* presents the systematic exposition of those metaphysical propositions containing the general conditions of representability for determinate things in intuition. The principles of any construction (by which concepts are presented a priori) must themselves be presented, i.e., constructed. To that extent, since the transcendental conditions of possibility are now themselves equated with pure exteriority (constructibility), there is no other place in Kant's theoretical philosophy where these conditions come closer to that which in outer intuition is actually made possible; they lose any reference to time, and become as anonymous – and therefore universal – as mathematics. In mathematics the pure concept of understanding is tied up with pure intuition, so that this concept must only be presented (externalized) in intuition in order to be thought (A719/B747), whereas the synthetic principles of pure understanding "cannot exhibit a priori any one of their concepts in a specific instance; they can only do this a posteriori, by means of experience" (A721/B749). To be sure, Kant also requires in the context of transcendental knowledge that for the objective reality of the categories to be verified (*darzutun*), always only outer intuitions are needed (B291). Thus "in order to supply something permanent in intuition corresponding to the concept of substance … we require an intuition in space (of matter)." But requirement for verification is not the same thing as exhibition of a built-in possibility, which belongs to mathematical concepts only.

That the form may substitute for the matter, in the mathematical synthesis, parallels the way the idea of reason schematizes a manifold of intuition even when an object cannot be given in experience at all.[6] The metaphysical synthesis of determinate nature is thus the exclusive property of the faculty of reason, in consequence of which the ideal of absolute constructibility cannot come full circle. As we progress into the *Metaphysical Foundations*, we attend to the *limits* of such absolute constructibility. Thus in the Phoronomy, we are told that construction can be carried out at best as "mediate composition" (4:494), removing any sense of immediate (non-conceptual) evidence proper to pure mathematical synthesis. Worse, in the case of Dynamics, the initial datum which is the moving force reveals how much the arbitrariness of mathematical synthesis (i.e., the fact that it may pick *anything* to serve as initial datum) remains an obstacle to the metaphysics of nature (4:498):

> Here the mathematician has assumed something as an initial datum of the construction of the concept of matter, but this something does not admit of being further constructed. Now, he can indeed begin his construction from any datum he pleases, without involving himself also in explicating this datum in turn; but he is nevertheless not thereby authorized to explicate this datum as something wholly incapable of any mathematical construction in order thereby to prevent a return to the first [metaphysical] principles of natural science.

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[6] *Prolegomena to Any Future Metaphysics* (4:328).

Mathematics is the construction of the concept in pure intuition, but mathematical physics falls into metaphysical dogmatism when such or such initial datum is declared to be no more susceptible to further construction. This dogmatism rests on the conflation of the initial datum with the unconditioned in the sense of the highest aims of reason. The unconditioned of the mathematician rears its head just when the power of mathematical synthesis gives out. That is why the initial datum should be understood as proceeding from reason, not understanding. Because it originates from the faculty of reason, the mathematical construction of the concept of matter accomplishes *too much* with respect to the dynamical properties of matter. Indeed, the mathematical treatment postulates these properties as "unconditioned original positions," which explains why limits must be set to this mathematical construction. To do just that is the main business of the metaphysical investigation of natural science. The initial datum must be carefully distinguished from the unconditioned according to the proper metaphysics which we are to spell out now, because mathematics conflates the two as it is blinded by its own power. When does synthesis, first mathematical, become properly metaphysical?

4 Back to Phoronomy

Physical space is the ever renewed possibility of extending the class of reference frames beyond any given limit and below any absolute limit (plurality-within-totality). On the other hand we know that geometrical space contains the ground for the possibility of constructing figures in pure intuition. If we now go a step further and ask what is the possibility of *physico*-geometrical space, we are left with the following problem: How can we *construct* – rather than simply postulate – the possibility of further frames of reference beyond the limits imposed by the confines of a physical system? Kant's answer is that we need precisely *two* spaces in order to achieve such a synthesis – the absolute *and* the relative, which always interact in some way.

Let a given velocity of a body in one direction be represented by means of two separate, smaller but equal velocities AB and CD; they can be regarded as the component-velocities of the given velocity AD. The question is: does the line AB + CD = AD represent the sum of the two velocities? If that were so, then the body should go twice as fast from A to D than it does from A to B or C to D. But then, the portion AB or CD of the line AD will of course no longer represent the velocity AB or CD taken separately one from another as components. In order for the composition to be possible in spatial intuition, we need to satisfy the following requirement: 2AB should be traversed in the same time as AB. Kant's solution is to say that one of the two motions will be attributed to the body itself, whereas the other will be attributed to the space (frame of reference) in the other direction; the latter has the effect of doubling, as it were, the velocity of the separate component. The construction of the two velocities is thus intuitively possible only by means of two spaces moving in opposite directions, one absolute and one relative. It is this mixing of progression and regression within the same synthesis that bestows sense upon the overlap of the absolute and the relative. In Newton's conception, the innumerability of relative motions in intuition was irresolvable in terms of kinematics alone.

It was not reducible to something itself either intuitive or numerable. Kant has thus just achieved such a reduction while preserving the arbitrariness of mathematical synthesis (which frame counts as absolute, and which is relative, depends on the constructive process itself).

The doubling of space, and the full interchangeability of the two spaces, defines the metaphysical principle of Phoronomy. The interchangeability of the motion of a body in space and the motion of the reference frame in the opposite direction is nothing other than the principle of relativity of modern physics, viewed as condition of possibility for the experience of matter in motion. Thus, as soon as the object has been specified as material body in motion, the empirical or material space relative to it allows it to become an object of experience. The addition of this space to the body generates equivocity in intuition: whether the body moves in the space at rest, or whether the body is at rest in the space moving in the opposite direction, intuition cannot decide. This equivocity is the price to pay for granting the relativity principle a metaphysical status appropriate to transcendental reason, as opposed to the dogmatic metaphysics of the mathematician.

5 Various Senses of Nature

As far as the examination of matter-in-motion is concerned, it can be argued that here too, beyond his own interpretation of the cause of gravity, Newton is again at variance with himself. Indeed his “deduction from phenomena” tolerates two different epistemological strategies. In order to understand motion, Newton says, we have to “abstract from our senses, and consider things themselves, distinct from what are only sensible measures of them.”[7] This view concerning the intelligibility of motion clashes with any theory of the universal qualities of matter. For in the case of the structure of matter, Newton argues that “we no other way know the extension of bodies than by our senses.”[8] What is true of extension is also true of the other qualities such as hardness, impenetrability, mobility, and inertia; in contradistinction to the facts of motion, all these qualities are established “not from reason, but from sensation.” Where the power of sensation gives out, mathematics and experimentation take over, but nothing is changed in the continuous chain of sensibly experienced objects. A whole conception of nature is at stake in Newton’s conception of sensible continuity in the case of atoms. Things could not have a “nature” of their own unless they were made up of ultimately indivisible atoms: “should they wear or break in pieces, the nature of things depending on them would be changed.”[9] Gravity, on the other hand, as a force related to motion, is not an

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[7] I. Newton, *Mathematical Principles of Natural Philosophy*, trans. A. Motte, rev. F. Cajori (Berkeley, CA: University of California Press, 1934), p. 8.

[8] *Ibid.*, p. 399.

[9] I. Newton, *Opticks* (New York: Dover, 1952), p. 376.

essential property of bodies because their nature is not changed whether or not they are subject to it. Since Kant denies that the intelligibility of motion requires that we abstract at some point from the senses, the following problem arises: what does this inseparable connection imply for our understanding of the forces and the inner structure of matter?

Kant's definition of nature in accordance with modern mathematical physics states that it is the totality of rules under which all appearances must come in order to be thought, provided that appearances are cognized by means of a necessary connection of them in experience (*Prolegomena* §36). This is nature in the *formal* sense, which is only thinkable in terms of the necessary unity of the connection between phenomena: conformity of things to the a priori (transcendental) conditions of the understanding in its empirical employment. But who is to say that this conformity exhausts the sense of nature? The objects of empirical cognition will always be "determinable in all sorts of additional ways,"[10] and there is no absolute guarantee that the understanding will always identify some necessity in them which reflects its own a priori structure. It cannot be a priori true that all experience leads to a recognizable connection in it. That is why Kant allows for another sense of nature which, while still formal, is not transcendental but metaphysical: "the primal, internal principle of everything that belongs to the existence of a thing" (4:467), i.e. that which constitutes the essence of a thing, the quality or set of qualities which characterize a thing and belong to it necessarily and specifically. The problem with this view of nature is that it echoes the now banished pre-Newtonian view dealing with the inner, nonmathematizable qualities of things. There will be as many sciences of nature as there are specifically different objects in nature. Since he is infatuated with the Newtonian method, does Kant simply leave behind this conception of scattered natures? His definition of nature as material throws out hints about such a rejection: "the sum total of all things insofar as they can be objects of our senses and hence also objects of experience." Material nature includes the two senses of formal nature, transcendental and metaphysical, so that finally conformity to laws (formal transcendental nature) and the sum total of objects of experience (material nature) are present in the definition of nature as "conformity to law of all the objects of experience" (4:296). What does Kant actually do in *Metaphysical Foundations*? Obviously Newton's methodological cunundrum is at the heart of the two foregoing senses of nature. In the Preface Kant further observes that the internal principle of a thing does not refer exclusively to its nature (or its existence); it can also refer to its possibility, in which case the principle belongs to the essence, not the nature, of the thing. Geometrical figures, or mathematical properties in general, fall within the range of the essence (see also the note in §63 of the *Critique of Judgment*).

Gravitation is the overarching concept between the two senses of nature, because it acts both within matter (ensuring the filling of space) and outside matter (between parts of matter).

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[10] I. Kant, *Critique of Judgment*, Introduction (5:183).

6 Dynamics

In the Dynamics these two senses of nature are dealt with in turn. First the distinction is made between two types of impenetrability, which refer to two different concepts of matter. According to the mathematical concept, matter resists all penetration with absolute necessity (4:502), which gives rise to the mathematical–mechanical mode of explanation of matter; if one follows this explanation, only atoms and the void are needed to account by composition for the constitution of matter. Mechanics is concerned with the actions of nature as arising from the shape of atoms as machines. However useful it may be for the practical purposes of exact science, this mode of intelligibility still reflects the mind's old desire to transcend the limits of possible experience, because it has an empty concept at its foundation (absolute solidity) which can never be given in sensibility. In the dynamical concept of impenetrability, by contrast, the metaphysical–dynamical mode of explanation allows for an expanding force which "first makes matter itself possible." The main point is that the dynamical concept cannot be comprehended or demonstrated a priori, that is, constructed in pure intuition. In fact, Kant opposes the dynamical and the mechanical mode of explanation as metaphysical versus mathematical. But the metaphysics of dynamics is critical and not dogmatic since, as we shall see, matter is conceived as relatively, rather than absolutely, impenetrable. Thus Kant explores here what critical reason is able to represent by its own means about the unattainable interiority of matter.

Whereas in Newton the geometrical and the dynamical properties intertwine in some way in order to produce a coherent analytical mechanics, Kant sharply opposes kinematics and dynamics. In the dynamical explication of the concept of matter, a property is added to the phoronomic one, namely, the capacity of resisting a motion within a certain space. The transcendental relationship between thought and sensibility is changed accordingly. According to the Anticipations of Perception in the *Critique* all reality has a degree in sensation which defines its reality. The object of sensation is given in an instant, as a shock, as it were, that is more or less intense. This intensity may be diminished in thought so as to gradually vanish from X to O, which is what it means to anticipate the continuity of perception. Opposed to the concept of reality, the concept of negation refers to the complete absence of reality from a sensible intuition. By complete absence, Kant does not mean the non-perceptible degrees from O to X, but rather a completely empty space or time. Kant opposes reality and negation in terms of being and not-being, which transcendentally means the distinction of one and the same time as filled or empty. By means of the fullness of time, the concept of reality can be schematized, i.e., applied to appearances. Kant states that the schema of a reality is precisely the continuous production of that which fills time, but he does not explicitly states what the schema of negation is, as if the concept of negation could have no schema: it cannot be applied to appearances essentially because it refers to the complete absence of appearances. Now, mathematical physics achieves what is transcendentally problematical, namely, it provides a schema which can hardly be specified transcendentally.

The schema in question is the property of filling a space, not a time. Filling a space means that matter is capable of resisting everything movable that strives to press into the space filled by a body. Kant sets out to show that this impenetrability is constituted by moving force as the cause of matter's motion. In turn, he proves that there are only two possible moving forces, repulsion and attraction. In so far as matter consists in or maintains itself through repelling other forces, in the absence of attraction it would have no cohesion; it would be infinitely scattered so that the consistency of matter occupying a given space could not be explained. Taken together as the limit of one another, these forces constitute the quality of matter in motion. Prior to Kant, impenetrability (the property of filling a space) would be explicitly distinguished from a body's having a force. Thus Euler thought of impenetrability as fundamental in the sense that all other properties could be derived from it; any force was an effect of impenetrability. For Boscovitch the essence of matter was force, and impenetrability was its effect. But for Kant matter fills a space in virtue of being endowed with a force of repulsion. Kant favors the dynamical explication even in the case of impenetrability, which is the very property which was thought to be central to the mechanistic interpretation. According to the latter, matter is absolutely impenetrable, that is, impenetrability is greater than any finite degree; variations in the density of bodies are explained in terms of relative amounts of filled and empty space that they contain.

Now, Kant calls attraction a fundamental force because it cannot be derived from reversed repulsion. Starting from a given expansive force in a given direction, an expansive force originating from another body could always be found, which is both greater and in the opposite direction. That is, the original expansive force turns, via a change of direction and intensity, into what Kant calls a compressive force. The greater compressive force is needed in order for expanding matter to avoid complete dispersion or dissolution in empty space. Why, then, do we need attraction, which is a qualitatively different force? Why could we not keep identifying attractive force with the possibility of compressive force exceeding any given expansive force? Kant resists the suggestion that one force could be deduced from the other. They are both fundamental in the sense that each is unconditioned, but unconditionality as understood here differs in essential respects from the mathematician's sense of arbitrary initial datum. Indeed, a difference prevails between repulsion and attraction at the level of the possibility of experience of matter. While the property of filling a space is apprehended immediately as a surface phenomenon, so that repulsion is action by contact, attractive force is not immediately sensible, and is rather given mediately, or "inferred" (4:513) because the center of attraction is always concealed within the body. The concept of negation is thus rendered determinate as attractive force; this concept ceases to be empty, since the action of universal attraction in all parts of the universe is nothing other than the force of gravitation as identified and made calculable by Newton. But whereas attraction acts immediately at a distance according to mathematical physics, transcendentally it is not immediately sensible. Facing the attractive force as an unconditioned, critical reason observes how the possibility or essence of attraction is at variance with its own nature.

Kant distinguishes true from apparent attraction in the following way. When one body X collides with another Y, and body Y is impelled toward a third body

Z, it appears that Y is attracted to Z. This appearance, however, is merely an epiphenomenon of impact (4:514). True attractive force, on the other hand, belongs to the essence or possibility of matter. The reasoning using a minimum of three matters confuses accidental impact and essential force. Attraction at a distance is made evident by any instantaneous distribution of matter, while impulsion cannot be represented without an account of the genesis of that distribution. Now, since attraction is immediately infinite whereas repulsion is always confined within the limits of a solid, in what sense can we speak of a relation between the two forces? A determinate degree of filling of space results from the limitation of repulsion by attraction, not a reciprocal limitation between two equal terms. The bridge between the two forces is secured by what Kant calls derivative forces (4:526–527). One such derivative force is cohesion, which is attraction thought as active only for particles in contact. As a derivative force, cohesion is not linked to the concept of matter by a priori synthesis; it can be known by experience only. Taken together, these forces in the dynamical synthesis provide a view of material nature quite different from the mechanical one, in which absolute solids are separated from one another by empty spaces.

A central point in the Dynamics is that attraction, which acts at a distance, is more fundamental than repulsion. Even contact action between bodies must be thought of as a mode of action at distance, since the inner determination of gravity is concealed *in*, not beyond, sensible experience itself; two bodies in contact still occupy different spaces. But showing how action at a distance is possible remains quite problematical for the critical philosopher, since the inner determination of gravity is concealed in sensible experience itself. In some deep sense attraction is a fundamental force in that, if we follow the dynamical mode of explanation, we shall never really understand how it is possible in accordance with its own nature. This does not prevent the mechanical philosophy to still ask for a knowledge of its inner determinations, which would explain the mechanisms of gravity. Do the two modes of explanation, then, fall into some kind of antinomical reasoning?

There is yet another reason why the attractive force is even more fundamental than the repulsive. This has to do with the fact that it is not confined to intensive quality. Attraction, Kant says, rests on quantity, i.e., the quantity of matter contained in a given space, whereas the expansive force rests essentially on the intensity with which it fills the given space. Therefore attraction supports the possibility of a synthesis of quantity and quality. The successful schematization of negation is thus a bit tricky, because it borrows what it needs for its purposes from the categories of quantity. The implicit presence of quantity in the Dynamics draws attention to the significance of kinematics for Kant's whole project. Indeed it is important to bear in mind that the Phoronomy contains the *one single metaphysical principle* of the whole of natural science: this principle is the *Grundsatz* according to which the motion of a body can always be interchanged with the motion of the corresponding material space (reference frame) in the opposite direction with equal velocity. All the other chapters of the *Metaphysical Foundations* consists in definitions, theorems, or laws.

Furthermore, quality in natural science cannot be conceived apart from the category of relation, since dynamically the body is characterized in terms of relations to its possible effects in other bodies, where these effects are described in terms of a rule that governs them. This is the distinctive character of the dynamical explanation, since mechanists conceive of space-filling (solidity) as a property a body has apart from its relations to others; absolute impenetrability is not ascribed by them to a thing on account of that thing's possible causal relations.

7 Mechanics

Kant considers three laws of mechanics, none of them being identical to any of Newton's three laws of motion. The first law states that the quantity of matter taken as a whole remains the same, and is neither increased nor diminished. This law refers to motion only secondarily, inasmuch as motion comes into play only as far as the measurement of the quantity of motion is concerned. The quantity of matter for a given body cannot be determined in isolation from the other bodies, for example by measuring its density and volume (Newton's procedure). Outside measurement, this first law seems to embody in some way the ancient atomistic insight according to which nothing arises out of nothing. The second law is reminiscent of Newton's law of inertia, except that it refers to change of matter, not change of motion: it states that every change of matter has an external cause. Most notable is the absence of any explicit reference to Newton's own second law of motion, which is the pillar of Newtonian mechanics. Only Kant's third law looks similar to Newton's own third law: this is the law of action and reaction, according to which in all communication of motion action and reaction are always equal to one another. There is no clear indication that Kant thought that Newton's second law can be derived from his own, though in fact he may have thought so. In any event it is still best to think of his relation to Newtonian mechanics in terms of the critical concern for setting limits to reason, assuming that reason in mathematical physics operates without making the experience of time explicitly intelligible. Time in physics is a purely operational tool that transcendental consciousness does not or cannot interrogate. From this viewpoint, since Newton's second law makes use of the concept of acceleration and thus includes a reference to time, there is no way in which it can or should be made intelligible transcendentally.

The experience of time is accessible within the realm of transcendental consciousness only. Each schema of imagination is a particular determination in time (*Zeitbestimming*). In the Second Analogy of Experience the law of causality is proved to be ground of all changes, so that "progression in time determines everything" (A210/B255). Now, Kant also argues in the Second Analogy that causality brings us back to substance, the permanence of which is stated as a principle in the First Analogy of Experience: the permanent in all appearances (as identical substratum) is nothing other than the object referred to in all possible experiences. This is achieved via a series of intermediary concepts: from causality down to action, from action down to force, from force down to substance. How do action and force secure the connection between causality and substance? In order to have objective cognition of a change, we need to know the real forces which make it possible, something which can only be given empirically. These real forces will be the moving forces in the metaphysics of nature, where Kant writes (Second Law of Mechanics in *Metaphysical Foundations*) that any change of matter has an *external* cause. Beginning with the conservation of substance (First Law of Mechanics), natural science goes on to deal with force, but since force is essentially external, it fails to provide us with the expected link between substance and action. Hence an important restriction to objective causality in the Second Analogy: it does not apply to those cases where causality does not lead to an action, for example the perception of a house, as opposed to the apprehension of a happening such as a ship moving downstream. The Third Analogy widens the scope of causal action, and so it provides the missing link. This principle states the simultaneity of substances in the following manner. Two objects exist simultaneously if the perception of A followed by B is equivalent to the perception of B followed by A. In the latter half of this proposition the analogy of succession is taken backwards, implying what was forbidden by the Second Analogy, namely, that progression in time does *not* determine everything. For example the static perception of the parts of a house is now sufficient to determine an action: as soon as floors have been added to a basement, this basement is not quite what it was before, when it was perceived in isolation as a mere moment in succession. Why does this twist imposed on causality lead to action? Because the one substance from which we began as ultimate identical substratum can now be fragmented into many substances without losing its identity, provided that a physical influence travels from one fragmented substance to the other. In this way, the dynamical relation in the universal reciprocal action allows us to conceive phenomena external to one another as forming a compound. But physical influence cannot occur in the void. The condition of possibility for the simultaneity of substances is that space be necessary full, by contrast with the empty form of sensibility which space was as a purely formal condition of intuition. This full space is needed as a character of transcendental experience, if the latter is still to belong to a coherent world. How does it relate to the empirical space of physical science?

If Kant's third law of Mechanics in *Metaphysical Foundations* looks identical to Newton's own third law, its proof is not. Kant proves the law of action and reaction by asserting that action and reaction are equal in the communication of motion. Something rather weird occurs here, because the proof is based on impact, which makes it difficult to extend it to action at a distance. Kant argues that the communication of motion cannot be understood in such a way that one body is in motion and a second body is completely at rest until the moment of impact, after which the situation is reversed. If both bodies are in motion, then each one must be the cause of the other's motion, that is, action and reaction are required in the communication of motion. The Third Law of Mechanics thus provides a rule for constructing in intuition the communication of motion. Kant is attempting to show how the fullness of space (Third Analogy) is used in physics as a condition of possibility for the intelligibility of the laws of motion, which is why communication of motion is understood in terms of impact, and also why Kant adds that the proof could be extended to action at a distance – if only the fullness of space could be made intuitable at all degrees of perception.

8 Phenomenology

In the Phoronomy, absolute space was an idea of reason, not a physical reality. Always the final leap to the largest possible space is an act of pure thought, not a mathematical construction. In the last chapter of *Metaphysical Foundations*, matter is now defined as the movable inasmuch as it is an object of possible experience.

The connection between motion of a body and the space in which it takes place has now to be understood in terms of possible experience. Experience itself is the connection between appearances. Kant begins by saying that the principle of relativity in the Phoronomy does not raise above appearances, since the free choice between motion of the body in absolute space or the opposite motion of the relative space excludes their connection.

He goes on to explain what it means to view absolute space as an operational rule for the determination of true magnitudes, or as a cause of the acceleration of moving bodies in the context of dynamics. Absolute space, as he puts it, is a "strange concept," and we must understand the reason why it is nevertheless the foundation of all possible motion. As an idea, it can be used as a rule thanks to which all motion can be regarded as relative. This can be seen in circular motion, in which the connection between appearances is now fixed: since the effects of motion with respect to space become significant in dynamics, the opposite motion of a relative space cannot be freely exchanged with the motion of the body. The continuous change of rectilinear motion (circular motion) is thus a connection between appearances inasmuch as it contradicts the mere appearance of rectilinear motion.

Consider the two hypotheses: the earth at rest and the stars in motion (Ptolemy) versus the rotating earth and the stars at rest (Copernicus). Again from the phoronomic standpoint the two hypotheses are equivalent, because the two appearances contradict each other, and they cannot be interpreted in terms of some connection between them. The absence of connection is a consequence of the fact that appearances are given passively: the parts of the earth remain attached, things do not spontaneously fly off in outer space. But we can also relate actively to our environment; experimentation on or near the surface of the earth allows us to establish the rotating motion of the earth about its own axis. Since all that can be established is a continuous change in the relation between matters, this motion is true motion, i.e., motion that is not contradictory of another motion; this is still different from absolute motion. Now, Kant says, true motion could still be *represented* in absolute space – something we can do, for example, by using Newton's two globe experiment, or any experiment in which centrifugal effects clearly compensate for the effect of gravitation. True motion in the dynamic sense is relative, that is, it is the relation to one another of the parts of the movable body. The only thinkable absolute motion in a consistent Newtonian theory would be the motion of the whole universe in empty space (which can never become an object of experience), because then true motion would seem to be irrespective of other matter (4:562–3).

Though not meaningful by itself, absolute space is meaningful when it is used as rule for the connection between appearances. Indeed, between the appearances and their connection in terms of experience, there is no continuity but contradiction. In this way, Kant expresses his opinion that in natural science an unbridgeable gap exists between the passive accumulation of sensory data and their rational organization in a coherent system of nature. In the final analysis, contradiction awaits Kant's own metaphysics of nature, since *Metaphysical Foundations* concludes with the observation that the metaphysical theory of natural bodies is unable to decide between the possibility or the impossibility of empty space. The ultimate mystery of nature is how matter itself sets limits to its own extension; this ultimate mystery is no longer in the hands of reason and its self-imposed limits. The metaphysics of nature cannot probe into this question. This is the point at which physics begins.

9 Opus Postumum

If Newton was found to be at variance with himself as regards the interpretation of absolute space and the status of universal gravity, in his last, postcritical and unfinished work Kant objected to him in an even more radical way. Here Newton's work is presented as a rival to Kant's own *Metaphysical Foundations*. The very title of Newton's book is declared to be contradictory, since there can be no philosophical principles of mathematics any more than there can be mathematical principles of philosophy (21:208 and 22:512). There could be no hybrid science of both, yet Kant points out that they can still be associated to one another in some way, which is what happens when knowledge is scientific knowledge. In short, in the *Opus Postumum* Kant says that Newton's achievement consisted in realizing something impossible by means of a bold stroke, namely, a philosophical use of mathematics (22:522).

If the *Metaphysical Foundations* were to be regarded as a specification of nature in general in terms of the empirical properties of matter, then it would of course be tempting to interpret Kant's post-critical investigations of empirical aspects of nature other than gravitation (such as heat and electricity) as further specifications of the same kind. On this assumption Kant would be exposed to the objection that Hegel levelled against all idealistic philosophies of nature, namely, that the application of the same mould or formula (categorial structure of the understanding/reason) to the most diverse materials would condemn the original idea from which we started to remain always in its primitive condition.[11] Even more dramatically, if Kant's view was indeed that natural phenomena can only be subsumed through the ever more specific descriptions supplied by the never ending progress of scientific knowledge, then the whole project could certainly not succeed, since through this further development of the exact sciences themselves, a point is finally reached where the basic principles of Newtonian physics can no longer be consistently

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[11] G.W.F. Hegel, *Phenomenology of Spirit*, trans. A.V. Miller (Oxford: Oxford University Press, 1977), p. 8.

maintained. Assuming that the transcendental principles of pure understanding include a strict conservation principle for the total quantity of substance and an equally strict determinism, assuming also that the a priori geometry of outer intuition is necessarily Euclidean, then the failure of Kant's system is obvious. In fact, turning an apparent drawback to his own advantage, Kant is precisely eager to show that the transcendental account of the scientific investigation of nature cannot but remain in its primitive condition. This primitive condition is its true state. The fertility of the original idea is then all the more visible and effective when it is confined to its primitiveness.

The original idea is the metaphysical principle of the Phoronomy, the only *Grundsatz* of the entire *Metaphysical Foundations* according to which the motion of a body can always be interchanged with the motion of the corresponding space. Interchangeability ensures mathematical construction. In Newton's system all motions precede the moving forces, so that the latter are effects of the former. The moving forces are thus entirely subdued to mathematics and the formal conditions of appearances (XXII:513). Now, if the whole of natural science could be reduced to Phoronomy, then, Kant points out, Newton's mathematical principles of natural science would be possible despite being self-contradictory, that is, they would be possible *as philosophy* (XXII:523). On Kant's interpretation, Newton acted as a philosopher precisely in that the mathematical principles at work in the case of such forces as the centripetal or the centrifugal force compelled him to deal with an original force such as universal gravitation; thereby mathematics became the proper instrument of the moving forces. In so doing Newton turned the whole of his own mechanics into a phoronomy; here it should be recalled that Newton thought of the force of gravitation he had discovered as "mathematical" in essence. Newton thus accomplished a crucial step when, from apparent motion, he deduced force as the cause of motion. But the step involves more than whatever the mathematical tool is able to reveal about it. Moving forces are also discovered empirically in experience. Kant thinks about the whole body of knowledge referred to as the experimental physics of the eighteenth century: forces involved in chemical phenomena, the cohesion of fluids and solids, magnetism, etc. In the case of the metaphysics of nature, mathematics combined with the categories of understanding provided the fundamental criterion of scientificity, namely, systematic knowledge. But if the other moving forces are given in experience, they cannot lend themselves to a systematic plan. Kant's project in the *Opus Postumum* is referred to as "transition" or *Übergang*: that is, there exists a smooth transition from mathematics and metaphysics to empirical physics, despite the fact that the latter looks unsystematic because of the astonishing diversity of physical and chemical phenomena.

Physics is now defined as "the system of the moving forces of matter inasmuch as it can be exhibited in experience" (XXII:511). That is, in apparent opposition to *Metaphysical Foundations*, moving force is taken as the basis of motion so that motion is derived from it. Opposition does not mean rejection, however, nor does it mean that Kant would at last do full justice to the Newtonian conception of force as the cause of motion. Kant's purpose is to reveal the philosophical part in Newton's thought that Newton could not recognize, or that he identified mistakenly as metaphysics of nature. In the final page of *Metaphysical Foundations*, which announces the *Opus Postumum*, Kant had already returned to the possibility that attraction might be merely apparent (cohesion). In line with the previous demonstration according to which space cannot be empty, he calls *aether* any matter below the threshold of perception. It would be scattered continuously over all cosmic spaces, exerting a compression such that matter is always full by virtue of its sole expansive force.

Kant now proposes to add physical principles to the metaphysical/mathematical principles of natural science. The principles are no longer the moving forces, because they remain dependent on the existence of a given motion. Rather, they are "forces which would never be present in matter without an external moving cause" (XXI:356). This external cause is the aether, which as matter is compressed so as to generate the phenomenon of attraction. On this basis, an attempt to deduce the two moving forces from one fundamental force (the aether) is now permitted.

It is not the association of space with moving objects in it, but the deduction itself which has the effect of making space a sensible object. The concepts of physics so understood can no longer be simply given by reason or experience (given in the sense of *conceptus dati*). Rather, they are *fabricated (conceptus factitii)* (21:358) quite on purpose, as it were, in order to make possible the search for the physical principles of nature. Fabricated concepts are, Kant says, "regulative principles which are at the same time constitutive" (22:241). As a result of the fabrication, any partitioning of the appearances of the world in terms of specific categories is to be dropped; Kant speaks of a filling of space which can be either extensive or intensive. A new concept, or "third thing," emerges from their conflation: this is the aether, which is a "continual … agitation, by attraction and repulsion" (22:211–213). As a third term, the aether as a putatively real object in the world corresponds to the transcendental schematism in the cognitive faculties. The transition (or *Übergang*) from mathematics and metaphysics to physics is thus something like the working out of the schematism of nature itself, disclosing what Kant called the possibility of the possibility of experience. The Critique had merely demonstrated that schematizable categories are needed to account for the possibility of experience. The question as to how they actually do that was left outside of it. By probing deeply into empirical nature, the mind probes in turn into its most original powers, and reaches a point where it has nothing to learn about new developments in science. Let us see how Kant reaches the puzzling conclusion that the aether is an object produced by the mind, which affects the mind while bypassing the mediation of empirical experience.

Despite their non-systematic character, the moving forces given in experience can be known systematically *as a whole*. The concepts appropriate for this kind of knowledge will not be a part of physical science itself, since they will merely provide the formal framework for this science to be a true science. What is it that can be said about this form which is not included in Kant's previous works? The metaphysics of nature is an intermediary step between the *Critique* and the *Opus Postumum* in the following precise sense. While in the former work Kant addressed the question of how to follow Copernicus's "first idea" (Bxvi) in order to bring out the conditions whereby the mind has access to the outer natural world, *Metaphysical Foundations* presented systematically the "first principles" of the metaphysics of nature without which natural science could not be regarded as a science. Hence the various formal modes of access to the natural world are complemented with all that is real in the objects of our external senses, namely, all that is determinable in space plus the moving forces (4:523). Now, the need to give a coherent description of the moving forces which are discovered empirically captures what is at least a pretention (*Tendenz* at 22:166, *Aufforderung* at 21:635) constitutive of physical science. This pretention is even more fundamental that the principle with which science begins, since it sends us back to a form which is more primitive than any other form: by means of new a priori principles, physics is given "the sketch (*Umris*) of the form" (21:360) whereby the manifold of physical perceptions is organized in accordance with their predefined place in the architectonic of reason. The task of the Transition project is to provide such a formal sketch. Kant argues that this task can be accomplished if a *material* principle of the origin of motion can now be found *within* possible experience itself. This principle is the aether, which is "primary matter" (*erste Materie*) or "elementary matter" (*Elementarstoff*), or again "an element which refers to the mere existence of matter without its particular forces" (21:312, 217; 22:610). This matter is cause of its own motion, so that one must assume "uniform and constant persistence of this motion" (21:217). On account of the rational need for systematicity in the whole of experience, Kant thus ends up by postulating something like a generalized principle of inertia at the basis of the mind's most intimate relation to the natural world. In this way a quasi-organic connection is finally brought out between the outer and the inner dimension of nature, a connection which appears as a duality in natural science via the principle of relativity and the principle of inertia.

The problem with this final step is that Kant thought that the same formal structure of the mind in the *Critique* operates at the level of the more primitive state of the form as well. Indeed Kant goes on to argue that what makes possible the systematic unity of the moving forces is also what makes possible the material unity of experience. The aether is supposed to do this because the totality of matter is unified by means of constant activity into a whole which is both spatial *and* dynamic.

10 Conclusion

Kant's merit is to be gauged against the fact that the "first idea" (Copernicus) leading to science, the "first principle" of natural science, and the "primary matter" of the mind-dependent world are all constrained by the same set of categories and ideas. Whether or not this merit is still great enough, it should be borne in mind that Kant's interest in mathematics and physics is motivated by his concern for metaphysics, not primarily by these sciences as such. Thus in *Prolegomena* §40 he states that, as far as their own safety and certainty are concerned, both pure mathematics and pure natural science stood in need of transcendental investigation, "not for themselves, but for the sake of another science, namely, metaphysics." The *Metaphysical Foundations* describe the lesson that transcendental philosophy is to retain from their unique combination successfully achieved by Newton in mathematical physics: i.e., an access to the a priori conditions of possibility of natural knowledge via their outer representation. Hence the principles of pure understanding are not to be seen as specialized in accordance with intuition and empirical experience. Rather, through contact with the propositions of natural science, they open themselves up to an outer representation of what they are as pure ideas.

Bibliography

Brittan, G. *Kant's Theory of Science*. Princeton, NJ: Princeton University Press, 1978.

Butts, R., ed. *Kant's Philosophy of Physical Science*. Dordrecht, The Netherlands: Reidel, 1986.

Edwards, J. *Substance, Force, and the Possibility of Knowledge: On Kant's Philosophy of Material Nature*. Berkeley, CA: University of California Press, 2000.

Friedman, M. *Kant and the Exact Sciences*. Cambridge, MA: Harvard University Press, 1992.

Gloy, K. *Die Kantische Theorie der Naturwissenschaft*. Berlin: de Gruyter, 1976.

Hoppe, H. *Kants Theorie der Physik*. Frankfurt a.M.: Klostermann, 1969.

Kerszberg, P. *Kant et la Nature*. Paris: Belles-Lettres, 1999.

Mudroch, V. *Kants Theorie der physikalischen Gesetze*. Berlin: de Gruyter, 1987.

Plaass, P. *Kant's Theory of Natural Science*. Trans. A.E. Miller and M.G. Miller. Dordrecht, The Netherlands: Reidel, 1994.

Pollok, K. *Kants Metaphysische Anfangsgründe der Naturwissenschaft: Ein kritischer Kommentar*. Hamburg, Germany: F.Meiner, 2001.

Schäfer, L. *Kants Metaphysik der Natur*. Berlin: de Gruyter, 1966.

Tuschling, B. *Metaphysische and transzendentale Dynamik in Kants opus postumum*. Berlin: de Gruyter, 1971.

Vuillemin J. *Physique et métaphysique kantiennes*. Paris: Presses Universitaires de France, 1955.

Warren, D. *Reality and Impenetrability in Kant's Philosophy of Nature*. New York: Routledge, 2001.

Watkins, E., ed. *Kant and the Exact Sciences*. Oxford: Oxford University Press, 2001.

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The Relativized *A Priori*: Cassirer and the Founders of Logical Positivism

Ernst Cassirer: Open Constitution by Functional A Priori and Symbolical Structuring

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Abstract This article aims at presenting

1. The innovations of Ernst Cassirer's open epistemological model, transforming the concept of 'constituting objectivity' by founding it on the dynamical 'constitution of meaning'
2. The entailed conception of physics' objectivation as a process of 'symbolic formation', studying its transcendental functions and their roles for the major tasks of creative constitution and pragmatic justification of knowledge
3. The clarifying and convincing solutions this view offers for the interpretation of relativity theory and quantum mechanics[1]

It leads to the conclusion that the transcendental method might be the best way to fulfil the realist's claim.

1 A Second Copernican Turn: Constitution Beyond Representationalism

Ernst Cassirer's oeuvre can be seen as a unique, radical and most fertile continuation of the original ideas of transcendentalism. It overcomes the historical forms of the Kantian as well as the Neokantian schemes, even if it is greatly indebted to them and remains entirely truthful to their founding philosophical orientation. The comprehension of physics, especially of relativity theory and quantum mechanics – the challenging contemporary theories of his time – has played a decisive role in the development of his conception. Cassirer has discussed his epistemological views directly with Einstein and Bohr, and it is astounding to see that in the midst of the intellectual uproar caused by their innovations, he was one of the rare thinkers able

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[1] For a detailed discussion of Cassirer's philosophy and the interpretation of modern physics see C. Schmitz-Rigal, Die Kunst offenen Wissens.

M. Bitbol et al. (eds.), *Constituting Objectivity*,

to receive and to appreciate their revolutionary changes with open arms as a clear confirmation of his own analysis of science and its evolution. He even claimed "that his fundamental outlook could be formulated more precisely and corroborated more successfully than before thanks to the development in modern physics"[2]: a tendency and claim which can be still further substantiated by the latest developments.[3] This remarkable fact deserves attention and confers great credibility to Cassirer's philosophical position and its explanatory power.

In Cassirer's view relativity theory and quantum mechanics had touched upon the defining limits of physics as a specific form of objectivation and it was precisely this discovery of their own epistemic preconditions that caused much incomprehension if not refusal – which to some extent lasts until today. It could be held that modern physics had thus reached the stage of 'transcendental reflection' by itself, and – as history shows – this crucial change in its auto-appreciation provoked a profound crisis, for it shattered the convictions inherent in the still dominant, representational view of knowledge. Some of its protagonists – like Bohr, Schrödinger or Pauli – fully grasped this epistemological dimension and it is revealing to see how Bohr succeeded in countering all of Einstein's obstinate efforts to refute quantum mechanics only by elucidating the hidden, highly questionable, often ontological assumptions behind his arguments. "I therefore tried […] to explain [to Einstein] that the only question was an endeavour to *clarify the conditions*, in each field of knowledge, for the analysis and synthesis of experience".[4] In Cassirer's analysis it was to be expected that physics would have to face the task of acknowledging the role played by the conditions of its own possibility as a science and that it had to integrate them explicitly into its reasoning and research.

Already his early work on the foundations of science – "Concept of substance and concept of function"[5] – studies science's inherent tendency to rediscover the open transcendental *function* – as stability, definiteness or unity – lying behind each of the *fixed historical forms* – i.e. particular notions of basic 'entities', categories or principles – that temporarily succeed in fulfilling them. As long as their use remains unproblematic these forms naturally assume a 'substantial', realistic appearance and obscure the open tasks to which they are the momentaneous answer. This is not prejudicial in itself and can help to motivate the research efforts, but it becomes a powerful obstacle, if their transitory status as a 'working hypothesis' is entirely forgotten. For the blind belief in them then creates strong resistance to the very change, in which progress inevitably consist and which is necessary to adapt and to assure the fulfilment of these fundamental functions in the constantly evolving context of new evidence. For Cassirer this was exactly what happened regarding relativity theory and quantum mechanics: both theories caused considerable confusion, because they put into question central concepts of classical physics,

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[2] E. Cassirer, ZMP, 131: "meine Grundanschauung […] auf Grund der Entwicklung der modernen Physik schärfer formulieren und besser begründen zu können als es früher der Fall war" Cp. a. ZMP, 277.

[3] See H.G. Dosch: Renormalized Quantum Field Theory and Cassirer's Epistemological System.

[4] N. Bohr: Discussion with Einstein, p. 63, italics C.S.

[5] E. Cassirer: Substanzbegriff und Funktionsbegriff.

which seemed to be beyond doubt, as ‘time and space’, and worse even, ‘object’, ‘causality’ and ‘predetermination’. Thus they dared to abandon the classical invariants, which had long served as firm reference points for the system of physics. The resulting destabilisation and disorientation was aggravated by the fact that the new foundations they introduced did not fulfil the needs of immediate perception and diverged from the intuitive world view, safely inscribed into our languages, both of which – according to Cassirer – habitually play the role of the reassuring “mother soil”[6] from which our common understanding grows. In Cassirer’s position these difficulties are a direct consequence of questionable representational and foundationalist expectations, and they can be solved by differentiating and disentangling the conflicting claims between perception, language and physics through a deeper epistemological analysis of their origin.

Cassirer’s main epistemological motivation is to grasp, how such a historic process and progress of knowledge is *possible*, that is, how our knowledge can be *inherently* changing and contingent *on all levels*, without loosing its claim of veracity. It seems as insufficient to him to simply separate ‘temporal’ and ‘atemporal’ elements – as the idealist–rationalist version attempts – as it is to merge the two poles of ‘transitoriness and truth’ – so the empiricist–realist tendency – thus dissolving their fruitful tension. He tries to construe a third, intermediate position in which he can fully meet this major epistemological challenge by maintaining their opposition yet explaining, why ‘temporality’ and ‘truth’ are not only compatible, but why *they even mutually demand and depend on each other*.

He finds the key to a solution in a consequent continuation of Kant’s critical effort, reapplying the transcendental search for preconditions onto those elements in the work of their *own* proponents which still keep a ‘given’, isolated character and carry the traces of the very epistemological dualism and metaphysics they tried to overcome. Fully acknowledging the inexhaustible dimension of temporally open, contingent experience, Cassirer does no longer try to find ‘final’ answers to the perpetually open “*problem* of knowledge”,[7] but centers on the concrete question how the ‘order’ arises in which our knowledge actually consists. Therein pursuing reflexions from Kant’s ‘Critique of Judgement’ and integrating numerous other philosophical influences – especially Leibnizean, Pragmatist[8] and ‘Gestaltist’ – Cassirer arrives at a thoroughly *dynamised* view of knowledge which eliminates pretendedly atemporal and absolute components by understanding all formerly fixed conceptual elements – as the form-matter scheme, the categories, judgements and principles of the Kantian a priori – as results of an open process of structuring and organization. Cassirer strives to reveal its functions and presuppositions, its *open tasks*, thus presenting knowledge almost like an ‘operator’, able to yield different solutions depending on the initial conditions. Thanks to this performative, purely *functional* and *operational* perspective he is not only capable to trace back every claim of ‘objectivity’ to its concrete process of objectivation, but he completes the Kantian analysis by revealing the constitutive conditions of the *means of constitution themselves*. He thus performs a kind

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[6] E. Cassirer, PSF III, 398: “Mutterboden der Anschauung”, “Mutterboden der Sprache”.

[7] E. Cassirer: Das Erkenntnisproblem in der Philosophie und Wissenschaft der neueren Zeit.

[8] Cassirer refers to J. Dewey, but particularly to W. James (see f. ex. SFB 319, 382, 388, 424, 441).

of ‘second Copernican turn’ and brings into sight a fundamental aspect of constituting objectivity which has not been discussed by his predecessors: the *constitution of meaning*. Cassirer claims that “we must conceive the problem of knowledge and the problem of truth as particular cases of the more general problem of meaning”.[9] This crucial turn leads to an innovative, open, inherently plural and holistic model of knowledge which offers maximal adaptability of *all* of its components – the fundament included – as well as sound safeguards against arbitrariness. And it proposes nothing less than a reconciliation of the traditional dichotomy between idealistic–rationalistic and realist or empirical paradigms.

It is essential to grasp that it is *not* the recognition of the semiotic dimension *in itself* that leads to a solution for the epistemological problems of dualism. For as long as symbols are still looked upon as ‘representations of pre-existing predetermined things’ the problematic dualist pattern is simply *reproduced* on this level, offering no intellectual progress whatsoever. What Cassirer proposes is not a semiotic, but a Copernican turn, because it implies an *inversion* of the seeming epistemological priorities. His rigorous analysis reveals that it is impossible to justify a ‘symbol’ by referring to the designated, allegedly independent pre-existing ‘entity’ and a bijective relation of ‘cause’ and ‘effect’. But *inversely* it is in fact the elementary *possibility of ‘symbolic reference’* without which we would not be able to refer to ‘some-thing’ as ‘this definite object’ at all. In the same way as Kant solved the inconsistencies of the dogmatic approaches by renouncing direct ontological hopes and founding knowledge humbly on what is truly ours, Cassirer reiterates and completes this critical movement for the means of constitution themselves. We cannot explain ‘symbolic reference’ with reference to other phenomena, we cannot go beyond or behind it, because it is the necessary condition which enables us in the first place to *address something as a phenomenon at all*. It is what we have to start with. That is why for Cassirer this transcendental condition is a “primary phenomenon” or “primary function”,[10] equivalent to our primary experience of consciousness and phenomenality itself. All ‘consciousness’ is as such ‘symbolic’, in the fundamental sense that it is ‘consciousness *of*’, ‘of someone’ and ‘of something’; indeed it only *is consciousness, insofar as* it is this *relation*, a unity of inseparable, yet clearly distinct poles, the bipolar *relation of reference* which the symbol *is*. It is precisely because of this rare and unique structural quality to be *as such* a unity of unity and difference, an *inherently plural*, yet *inseparable unity*, that Cassirer uses his key-term ‘symbol’ as an incorruptible alternative to the omnipresent, Aristotelian ‘form-matter’-scheme and its intrinsic dualism. Even if Bohr, Schrödinger and other ingenious interpreters of quantum mechanics already discovered the level of constitution and meaning and thereby discussed the limits of the predominant ontology and language, they are still standing

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[9] E. Cassirer: Erkenntnistheorie nebst den Grenzfragen der Logik und Denkpsychologie, p. 34: “müssen wir […] das Erkenntnisproblem und das Wahrheitsproblem als Sonderfälle des allgemeinen Bedeutungsproblems begreifen”. See also E. Cassirer, PSF III, 229. J.M. Krois also stresses this underestimated fact. Cp: Symbolic Forms and History, p. 44ff.

[10] See E. Cassirer, PSF III, 102 (“Urphänomen”), PSF I, 34 (“Urfunktion”), PSF III, 61 (“Urfaktum”), cp. also PSF III, 117, 229, 462, 458f., 524.

on its ground and remain its prisoners, because they were lacking a working epistemological alternative to representationalism, able to reveal the liberating dimension of the constitution *of* meaning. ‘Meaning’ is not ‘given’ by a ‘pre-existing thing’; ‘meaning’ is what enables us to focus and fix reference and to relate to ‘something’ as a ‘thing’. When a full comprehension of this *constitutive* role of our symbols is reached, the object-intention appears as an *integral part of the bipolar structure of reference itself*, enabling us to construe productive anticipations of order, which can then be tested and confirmed – or rejected – pragmatically.

2 Physics as a Symbolic Formation: Objectivity Beyond Foundationalism

What then – according to Cassirer who proposes such an alternative to representationalism – are the open tasks and constitutive functions that need to be fulfilled in order to make the objectivation of physics possible?

1. Differing from classical epistemologies, Cassirer asserts that an objectivation-process cannot even begin before a particular aim has been delimited, which defines its direction, provides its driving force and serves as a criterion for its success. So the first open task consists in some sort of symmetry-breaking, in which one aspect, one determinate respect has to be singled out, which is already a product of a first particular constitution of meaning. For Cassirer it is self-contradictory to strive for ‘unconditioned’, ‘absolute’ knowledge, for partiality and interrelatedness describe the very nature of determinateness. In his analysis knowledge is on the contrary on all of its levels a matter of concrete interest, precise determination and engaged choice. It therefore appears as a lack of depth and understanding to imagine physics as ‘a true image of the world in itself’ as the traditional model of correspondence suggests. Instead Cassirer refers to a physicist himself, Henri Poincaré, to specify physics’ particular aim of objectivation: “‘Jean sans terre’ has passed through here: there you have something which is admirable, something for which I would give all theories of the world. That is the way the historian speaks. The physicist would rather say: ‘Jean sans terre’ has passed through here, well, what do I care, for he will not pass again.”[11] Poincaré thus clearly expresses that the physicist is not interested in the unique, individual and unrepeatable aspect of events – even if it is perfectly, even enviably, objectifiable too. On the contrary it restricts its investigation to the repeatability, the regularity and lawfulness of observable physical events – a view of the epistemic range of physics equally shared by Pauli.[12]

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[11] H. Poincaré: La science et l’hypothèse, p. 158: “Jean sans terre a passé par ici: voilà ce qui est admirable, voilà une réalité pour laquelle je donnerais toutes les théories du monde. C’est là le langage de l’historien. Le physicien dirait plutôt: Jean sans terre a passé par ici; cela m’est bien égal, puisqu’il n’y repassera plus.”

[12] W. Pauli: Phänomen und physikalische Realität, p. 94.

Since this initial choice of an aim allows for various diverging directions, it follows that objectivation can take place in a *diversity* of ways and has actually done so historically. This introduces another significant difference to the representational conception of knowledge. With every different orientation is born what Cassirer calls a 'Symbolic form' which is in fact an open ongoing objectifying organization. Science – along with language, history, art, religion, myth, technique, custom etc. – is one of them. In this productive as well as pragmatic view, each of these "different modalities of 'sense-giving'",[13] of 'making sense', contribute in an irreplaceable manner to our understanding and none of them can pretend to exhaust the notion of 'reality'. Taking into account this fundamental "multidimensionality of knowledge"[14] Cassirer concretises and widens the one-sided 'critique of pure reason' to a plural 'critique of culture' which he attempts to accomplish in his main work: 'The Philosophy of Symbolic forms'.[15]

2. The second open, constitutive task of objectivation concerns its means. Cassirer holds that a Symbolic form cannot simply rely on any 'given, predetermined' framework of logical, psychological or ontological orders – this impression only occurs, because our main orientation system, the Symbolic form 'language', occupies a predominant place in our understanding and seems to furnish unquestionable orders which we can draw from. But even if a Symbolic form can borrow and integrate notions from other forms – as physics has done with language – in the end the physicists have to actively choose, conceive and create all conceptual tools adequate to achieve their specific goal themselves – from the basic notions, the appropriate images and laws to the principles and experimental testing methods. That is in fact what a 'Symbolic formation' *is* and does and it presents another aspect of the permanent 'constitution of meaning'. Since this applies to any of the different directions of objectivation, it follows that – strictly speaking – each Symbolic form has its own symbolic system and creates its own, particular, irreducible 'universe of sense'.

Investigating concretely how the symbolic articulation of experience takes place that constitutes physics and its objectivity, Cassirer draws our attention to an initial paradox: the very physicality of the phenomena it wants to study actually has to *disappear* before it can begin its work. For the lawlike, regular relations it is interested in only become visible, once the multitude of dissimilar sensitive observations have been made comparable by finding a common denominator: that is why physics *measures* and this demonstrates, how the driving aim shapes the symbolic form to be adopted. Cassirer has coined the telling term of "trans-substantiation"[16] to characterize this first necessary step of physics' objectivation: the seeming sensory

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[13] E. Cassirer, PSF III, 234: "verschiedener Modalitäten der Sinngebung".

[14] Cp. E. Cassirer: Zur Logik der Kulturwissenschaften, p. 101: "Mehrdimensionalität der Erkenntnis".

[15] E. Cassirer, PSF I, 11: "Die Kritik der Vernunft wird damit zur Kritik der Kultur."

[16] E. Cassirer, PSF III, 510. Cp. a. 503ff., especially 513.

‘substance’ of its observations is completely ‘transfused’ into abstract mathematical concepts, the plural phenomenality of perception is abandoned in favour of the sameness of mere numbers, keeping as only trace of their origin the accompanying dimension-sign. This constitutive step of quantification also implies that *from the very beginning* – far from our common sense beliefs – the actual ‘objects’ of physics are *no longer* those of our everyday perception and spatio-temporal intuition which we tend to generalize spontaneously as a model for all objectivity. But analysed thoroughly each of them is nothing more and nothing less than a complex and abstract “notion integrating determinations of number and measure”,[17] a unique focus of attention and intention, inseparable from an entire system of interrelated meanings, known as ‘physics’. The illusion of ‘the same objects’ persists mainly because our habitual structuring of perception and language – unjustly – claims uniqueness and priority, but also because the originally borrowed terms from language – the great majority of physical terms in fact – have preserved the same name, although in their new context and use they no longer have the same *meaning* at all. Bohr already wondered about this disturbing discrepancy when noticing that “Our basic tool is, of course, plain language, which serves the needs of practical life and social intercourse.”[18] As physics progresses the gap to ordinary experience and its convenient order patterns inevitably widens, for ‘language’ and ‘physics’ do not pursue the same aim. It is hence perfectly natural that finally the physicists have to create their own original notions and orders, being more apt to obey the requirements of their specific quest – like ‘isospin’, ‘CP-violation’, ‘strangeness’ or ‘boson’ – and it is natural too, that they no longer resemble our cherished ‘everyday objects’, for in fact they have never been the same. Nonetheless they are perfectly objective – in a certain sense even more so than ‘sense-objects’ – since they successfully occupy their place within physics’ extremely rigorous, well-experimented and working knowledge system.

This ‘trans-’ or ‘de-substantiation’ distinguishes physics from other Symbolic formations and confers a particular potential to it, since it entails that physics expresses itself almost exclusively in mathematical terms. Its major symbolical tools are basically two: (1) ‘number’ – resp. vector, matrix, tensor – to unambiguously fix the measuring results and (2) ‘function’ to clearly express their lawful relations. These mathematical terms, free of any material connotations, belong to a specific type of symbol Cassirer calls “signs of pure relation and order”.[19] They introduce a particularly strict, unequivocal type of reference, which does not permit shifting polyvalence and ambiguous suggestiveness and thus differs considerably from ‘image-signs’ or ‘word-signs’, typical of perception and language. Therefore physics is exposed to precise and powerful inner constraints that act as a motor for its perpetual transformation, because they are able to contradict and counter the pressure

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[17] E. Cassirer, PSF III, 510: “Inbegriff von *Zahl-* und *Maßbestimmungen*”, Cassirer’s italics.

[18] N. Bohr: Unity of Knowledge, p. 67.

[19] E. Cassirer, PSF III, 389, cp. also 396, 400, 408.

of our habitual, linguistically laden beliefs – as modern physics has demonstrated so impressively.

But he who wants to quantify, first needs a measure. Besides these mathematical tools, physics employs ‘word-signs’ – as ‘mass’, ‘charge’ or ‘temperature’ – which specify the needed respects of measuring. These notions define the actual, presumedly recurrent and stable ‘entities’, the ‘objects’ and ‘qualities’ of physics. They establish the first of three main levels of inner articulation of the knowledge system of physics which Cassirer characterizes as “notions of measure”, “notions of law” and “propositions of principles”.[20] These notions correspond to different epistemological functions that indicate conditions for structuring determination and the constitution of physics’ specific objectivity:

1. The ‘notions of measure’ determine the fundamental reference units, providing the needed stability and basic articulation of the symbolic system.
2. The ‘notions of law’ serve to integrate the measurement results into notions of higher order and thus determine the actually sought relations between the elementary notions.
3. The ‘principles’ anticipate regulatory ideals of coherent unity for a sub-group of these notions or – as the highest heuristic principle – for all of them. Should there be different, competing options how to reach such a unity for the entire symbolic system, then the principles of univocity and simplicity – i.e. explaining a maximum of phenomena with a minimum of principles, “plurima ex paucissimis” as Cassirer formulates with Kepler – specify the notion of unity itself by indicating an extremum. Principles thus serve as criteria for overall orientation and evaluation.

In order to arrive at any of these organizing notions we have to make a “leap into the void”,[21] a constructive guess and effort, which anticipates an order that might or might not be confirmed by later experience. Neither can the laws be simply inferred from the measuring results, nor can the principles be deduced from the postulated laws. It requires active ‘Einbildungs*kraft*’ and ‘Urteils*kraft*’ – which already in Kant’s fine analysis name the same ‘force’, the same capacity of anticipative structuring, of creating ‘sense-units’ – to fill the conceptual gap between these different degrees of order and to generate these orders themselves. In Cassirer’s framework all three types of notions are principally equal: they are ‘symbols’, ‘foci imaginarii’[22] – to use the famous expression that Kant invented for the ‘ideas’ – i.e. imagined centers for our orientation, fixed intentions, organizing anticipations of order, tentatively fulfilling a specific function, yet remaining entirely questionable and transformable. And it makes no difference, what particular function they happen to assume: be it providing the elementary stability of basic units or the integration into

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[20] E. Cassirer, ZMP, 161–194: “Maßbegriffe”, “Gesetzesbegriffe” und “Prinzipienaussagen”.

[21] E. Cassirer, ZMP, 194: “Sprung ins Leere”.

[22] I. Kant, KrV, A 644.

highest unity. In either case our use of reason cannot claim to be more than hypothetical. Even if we are ready to admit, that this hypothetical status is applicable for the overall principles – since they 'only' extrapolate concrete experience to a horizon of its possible completeness – we are not used to consider 'laws' and especially not the basic 'notions of measure' in this way. The commonsense belief is that there have to be 'anchoring', 'corresponding' elements, which connect our symbolic system directly to 'reality' and thus guarantee its 'truthfulness'. It is indubitable that we need operational criteria which guarantee the objectivity of our knowledge. The decisive question is however, how we can *really* attain them. The tempting idea of 'direct correspondence', seeing knowledge as a 'one-to-one mapping of pregiven 'elements of reality' to 'elements of our symbolic representation' ' turns out to be untenable, because it yields no viable criteria. It is natural to charge the notion of 'reality' with the most important epistemic function to warrant 'truth' and serve as ultimate security against arbitrariness and empty speculation. But it does not help to invoke it with much emphasis and engagement, because it only designates the aim of our investigation, the *unknown*, and not a given means we could readily employ, and it is not able to assume the crucial role we would like it to play. For Cassirer the classical 'foundational effort' – be it through 'deduction' or 'direct correspondence to reality' – is an integral part of the dualist model of knowledge and its validity stands and falls with it. For as long as we conceive of knowledge as a hierarchical structure, reposing on an untouchable fundament, the linear dependence of its epistemic layers inevitably leads back all claims of 'truth' to the foundational basis, which then needs to be justified as 'true' in order to assure the legitimacy of the whole structure built on it. Already the habitual metaphors employed – 'structure', 'fundament', 'underlying', 'build up' etc. – betray the allpervasiveness of this static epistemological pattern. In this view the entire system and its epistemic value 'breaks down' when its fundament changes – which could very well describe the impression of crisis called forth by modern physics.

In Cassirer's analysis these 'foundational' problems are artificially created by an inadequate model of knowledge, which confuses the separate tasks of 'constitution' and 'justification', because it violates the symmetry between all elements of any symbolic system, forcing it into a linear order. If we study the concrete symbolic system of physics and e.g. its 'notions of measure', we find that they define their 'basic entities' by providing a precise definition of how to measure them. So a 'second', as a unit of 'time', is defined to be 9,192,631,770 times $T(^{133}Cs)$, i.e. the period of a Caesium-maser oscillation. The significance of this basic definition already presupposes nothing less than an understanding of the complex theory of atoms, which itself remains incomprehensible outside the entire system of physics. In fact the three 'levels' of articulation – notions of measure, law and principle – cannot be understood as a linear structure, resting on a 'solid basis'. Instead they turn out to be inseparable, for they *mutually* define each other and form *one*, completely interconnected whole, structured internally by different functions of order, none of which can claim to be more important than the other. This reveals the internal symmetry, the principally relational and holistic character of any symbolic system and of 'meaning' itself. The *determinateness* in which individual sense *consists* can only be constituted as a relational ‘limit’, a ‘relative internal difference’ arising within a network of mutual de-limit-ations and differentiations *from which it cannot be abstracted.* It is of prime importance to grasp that ‘meaning’ therefore has no independent ‘substantial’ character, based on a mysterious transcendent ‘one-to-one correspondence’ to ‘given entities’, but an interdependent or co-dependent character, based on concrete, rational mutual constitution.[23] To acknowledge this irreducibly holistic character of a symbolic system implies that it does not make sense to formulate the question of ‘truth’ for a single element, *but only for the system as a whole.*

3. Thanks to this entire internal structuring the physicists can anticipate and calculate theoretically a particular measurement result and invent practical procedures on how to test it as a third open task of objectivation. Now, if the actual result does not coincide with the anticipated one, a tension is introduced into the symbolic system of physics. Since it aims at coherent unity – which is not the case for all Symbolic forms – this disagreement is evaluated as a ‘contradiction’ that needs to be removed. However, the result in itself does not indicate, how to undo the tension, and the system’s holism allows for a multitude of possible changes to dissolve it: so again the physicist’s creativity is challenged to make a clever ‘leap into the void’. Yet the structuring transcendental conditions of his Symbolic form serve him as indispensable guide-lines: so the driving principles of coherence and unity are to be satisfied in such a way, that they allow for a maximally simple integration of the respective basic notions and laws best satisfying the evidence and all disturbing contradictions. But *vice versa* – diametrically opposed to the Kantian categorical conception – in Cassirer’s holistic view the basic notions and laws can *equally* be re-conceived and changed in such a way, that they allow for an optimal fulfilment of the principles: it is in fact the productive *interaction* between the different actual *equally important transcendental demands* – determinateness, univocity, stability, simplicity, unity – which generates a fertile *dynamics* and leads to the inner transformations that create new insights. Surprisingly close to recent theories of self-organization Cassirer’s model thus envisages a nonlinear feed-back-loop between all interacting elements of the symbolic system. It gives rise to a movement of self-correction by constantly searching a dynamic *equilibrium* between these constitutive functional demands in the permanently changing context of new evidence. Relativity theory and quantum mechanics give a striking example for the great catalytic power of these abstract exigencies: intrepidly following the principles of coherence and unification the physicists were driven – even against their personal convictions – to opt for a thoroughly different configuration to find a new equilibrium, which asked to abandon the ancient fix-points of the system, the classical ’entities’ that had so long been able to play the stabilising role, but, given the new situation, could do so no longer.

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[23] Even more profoundly it can be shown that it is in fact this holistic structure of *mutual constitution*, which makes the phenomenon of *mutual reference*, of symbolic reference *possible*. For if every element is a function of all the others, it can also *stand for another one or for the whole* and thus act as the ‘sign of’. Cp. C. Schmitz-Rigal, Die Kunst offenen Wissens, p. 112 – 115.

In Cassirer's model it appears as just another inevitable open constitutive *task* of each specific objectivation-project to decide and define which should be its most adequate 'fix-points', serving as 'putatively invariant entities'. And the answers depend (1) on the particular objectivation-goal in question, but also (2) on the entire evolving structure of the respective symbolic system – as the history of physics corroborates. The physicist has to advance skilful anticipations, which local choice of 'stable reference units' will lead to an optimal transformation of the entire system in direction of the global aim of unification. But he needs to have the humility to leave the judgement about their epistemic value to time, contingency and experience rather than to his personal ontological predilections. For if these notions will be able or not *to fulfil this function of 'invariants'* does not depend on him, but only and entirely on their pragmatic success. It is not the rigid, atemporal fixity of 'substantial elements' that will grant us the stability knowledge necessarily requires, but the adaptability and strength of mutually supporting interrelatedness we attain in a concrete symbolic system.

Cassirer finds this functional view of stability and invariance confirmed in the work of the German mathematician Felix Klein. Working on comparative geometry at the end of the nineteenth century, he outlined that the so-called 'in-variants' clearly depend on the type of 'variation' one studies: so the invariants of rotation are not the same as those of lateral translation or of a mirroring bijection etc. It seems evident, but is often neglected or ignored that the notion of invariance itself is *relational* – as all notions are – and that it only makes sense with reference to a specific framework. In the same way, the invariants of our knowledge systems cannot be considered to be absolutes, but they *depend on the framework of the specific Symbolic form they are an integral part of*. Hence for Cassirer it is not shocking, but even to be expected, that the invariants of our worldly kinaesthetic perceptual orientation and the invariants of the search for the fundamental laws between quantifiable observables are most probably not going to be the same.

4. Thus we reach the fourth open task of knowledge: its justification. Even if the choices and infinite options of organizing constitution are our task, their actual epistemic value and usefulness, given the chosen aim, then no longer depend on us. It is up to us to formulate the questions, but then we do not decide upon the answers. "The first step: we are free to choose – but on the second and all following steps we are servants."[24] Thus Cassirer dares to surrender *all* elements of our symbolic systems without exception – from the anticipations of 'basic entities' to the highest formal principles – to the judgement of their pragmatic success, for knowledge cannot pretend to 'justify itself' *concerning its actual epistemic efficiency*. Cassirer's dynamic view of knowledge liberates the multiple tasks of 'constitution' from the unfulfilable constraint of 'direct correspondence', but with the same theoretical

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[24] E. Cassirer, PSF III, 492 f.: "Das Erste: die Wahl […] steht uns frei – aber beim Zweiten und bei allem Folgenden sind wir Knechte."

movement ‘justification’ becomes entirely a matter of pragmatism. An element is justified in its epistemic function, *because* it is able to fulfil it and *as long* as it is able to fulfil it: and that is all we have – and all we need. Those who dream about ‘final justification’ by correspondence or deduction adhere to an ‘absolutist’, static vision of knowledge that takes the risk of paralysing itself by clinging to limiting prejudices. For Cassirer the only form of legitimation able to meet future, uncontrollable contingency without restriction is the temporal justification ‘by function and efficiency’. This type of ‘open’ justification is not Cassirer’s own invention, but other than in the pragmatist tradition it can already be found in the ‘Critique of pure reason’, following Kant’s arguments to justify the ‘ideas’. For the critical analysis of the Transcendental Dialectics explores the insoluble incoherencies that arise if one wants to affirm – as the metaphysical tradition before Kant did – that ideas are ‘true’, because they ‘correspond’ to pre-existing entities. Nonetheless it in no way undermines their epistemic value and utility, if one admits that ideas *do not possess a categorical, but only a hypothetical and heuristic status*. They are still perfectly justified, because they are “an indispensable condition for the practical use of reason”,[25] that is, because they fulfil a transcendental function. One could draw the parallel that the antimetaphysical critique Kant undertook for the ‘ideas’, Cassirer has completed for the categorical ‘basic entities’ and ‘invariants’, showing that both are fundamentally the same: symbols, ‘foci imaginarii’. They are fully objective and justifiable, not because they ‘correspond’ to a given, enigmatic entity, but because they likewise fulfil an indispensable epistemic function which this time is the function of providing adequate stabilising fix-points.

The great achievement of this functional approach is that it gives us *perfectly concrete criteria* to judge. So, if the physicist wants to decide about the epistemic value, about the ‘truth’ of a symbolic element, he can test, whether *within his constituted symbolic system* (1) it fulfils its own, local function and (2) whether this allows the whole of the system to fulfil its global goal and (3) whether this development shows an overall direction towards greater unification: together these three criteria clearly inform him about the pragmatic success of the knowledge system. “We call a proposition ‘true’ not because it corresponds to a fixed reality beyond all thought and all thinkability, but because it proves a success during the process of thinking itself and because it leads to new fertile consequences. Its actual justification is the effectiveness it unfolds in direction of the progressive unification.”[26] It is this functional and pragmatic conception of ‘truth’ that succeeds in including transitoriness and temporality as its own essential elements.

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[25] I. Kant, KrV, A 328; see also KrV, A 671.

[26] E. Cassirer, SFB, 423: “ ‘Wahr’ heißt uns ein Satz, nicht weil er mit einer festen Realität jenseits alles Denkens und aller Denkbarkeit übereinstimmt, sondern weil er sich im Prozeß des Denkens selbst bewährt und zu neuen fruchtbaren Folgerungen hinleitet. Seine eigentliche Rechtfertigung ist die Wirksamkeit, die er in Richtung auf die fortschreitende Vereinheitlichung entfaltet.”

3 Interpreting Modern Physics: Transcendental Functions Beyond Transitory Forms

One of the finest examples of the transformative power of transcendental reasoning is special relativity. One can hold that Einstein's ingeniosity resides precisely in the fact that he dared to give absolute priority to the *conditions of the possibility of physics*, even if this implied apparently 'unacceptable' consequences. He did not hesitate to sacrifice the conventional concepts of time and space, although they seemed to be untouchable, in favour of these abstract principles, because he deeply understood that these requirements are the true core value of physics, for without them there would be no physics at all. Hence consciously integrating them and using the transcendental presuppositions as his guide-lines Einstein *postulated*, or as he himself formulated "elevated the supposition to a presupposition",[27] that the velocity of light *should be* a constant, independently of the observational status, and that the 'principle of relativity', the claim that all inertial systems are equivalent, *should* hold. For without this demand of equality and symmetry of all observational situations, the laws of physics would no longer be the same in all domains, and thus the conditions of *unity* and *coherence* of physics could no longer be fulfilled. And Einstein reapplied the same method to arrive at general relativity. The consequence of this apparently simple demand was a revolutionary change in our world-view, able to overthrow venerable convictions about the 'nature' of time and space. Using this approach he reached a viewpoint in which formerly unconnected, independent 'substantial' theoretical items reappear as interdependent observables, as quantifiable aspects of one and the same process of determination of movement: be it time, space, energy or mass. But the fact that they thus lost the 'last rest of their status of independent objects', as Einstein himself affirmed,[28] does not in the least threaten their status of 'objectivity'. The very opposite is the case: for the newly discovered dynamic relations between them concretise, enrich and strengthen the meaning, importance and position of each of them for the whole of physical knowledge and they constitute the specific *knowledge-gain*, the new objectivation that relativity theory actually consists in. In fact, the claim of 'time', 'space', 'energy' and 'mass' to be 'rational' or 'real' can only be formulated, affirmed and justified *thanks to this 'relational', 'relative' status*.

Provokingly one can even assert that all progress in physics is nothing other than that: relativizing seemingly given 'absolutes' by integrating them into a larger context and thus '*understanding*' them; for that is precisely what 'com-prehension' *is*. The 'invariants' of the old theory then appear as dependant and variable with respect to other, more fundamental conditions, now taking on their role of 'invariants'. Progress thus appears as a "deepening of the foundations"[29] as Hilbert put it, which takes

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[27] A. Einstein: Zur Elektrodynamik bewegter Körper, p. 276f.: "Wir wollen diese Vermutung (deren Inhalt im folgenden 'Prinzip der Relativität' genannt werden wird) zur Voraussetzung erheben."

[28] Cp. E. Cassirer, ZMP, 71: "letzten Rest physikalischer Gegenständlichkeit".

[29] Cp. E. Cassirer, ZMP, 179.

place when we succeed in replacing former fix-points in favour of new, more performing, more *encompassing* ones, thus expanding the scope of explicable phenomena while concentrating at the same time the theoretical core. In reality the much dreaded or denigrated 'relativization' is only the visible sign of a deeper level of understanding, equivalent to the discovery of further preconditions, leading to a fruitful 'relationalization', *which expresses the very essence of 'rationalization'*, of our fundamental endeavour to 'com-prehend'. This comprehensive progression describes precisely what the transcendental method *actually amounts to* and thus we can understand, why it is so apt to analyse the parallel abstract quest of physics.

In Cassirer's analysis relativity theory as well as quantum mechanics did not cause a crisis of physics, but rather a crisis of our habitual worldview, inscribed in most of the world's languages, and a crisis of intuition, for their results could no longer be apprehended within the unity of a unique spatio-temporal picture. But why should it? "It may be remarked that the main object of physical science is not the provision of pictures, but is the formulation of laws governing phenomena and the application of these laws to the discovery of new phenomena."[30] In accordance with Cassirer Dirac here insists, that above all we have to be aware of the *particular* objectivation-aim of physics and clearly distinguish it from external, heterogeneous demands. The impression of a crisis disappears when we clarify the epistemological situation thanks to Cassirer's antimetaphysical 'Occam's razor' par excellence: by searching the actual transcendental functions hidden behind the apparently failing forms. Thus we are able to realize that it is not intuition and imagery, which are put into question, since relativity theory as well as quantum mechanics still successfully use a multitude of images. But it is the *function of 'unity' and 'continuity'* which can no longer be fulfilled in a spatio-temporal form. Yet nothing in these abstract demands themselves obliges us to understand them in this way: that is only the form in which they appear *under the conditions of our perception*. Cassirer teaches us to observe, that 'continuity' has not been *lost* in these theories, but that it only had to be *transferred* to another, more *appropriate* theoretical location *which allows for its satisfying fulfilment in the new context*. Concerning relativity theory the 'unity' and 'uniqueness' lost on the level of the classical, individual space–time-parameters is restored through the equations of the Lorentz-Transformations, which permit a complete re-integration and transformability, taking into account the new dynamic parameter, their relative velocity. In terms of physics' symbolical tools the 'unity' has thus been *elevated* from the level of 'number' to that of 'function', of 'relation'. In terms of our understanding it has thus been elevated from the level of sense-perception to the level of thought – which is not surprising if we remember its initial goal. In the same way one can analyse the 'failure' of strictly determined individual laws, the 'failure' of continuous causal relations, the 'failure' of continuous space–time paths for particles in quantum mechanics as the expression of a new,

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[30] P.A.M. Dirac: Principles of Quantum Mechanics, p. 10. W. Pauli defends the same point of view, see 'Raum, Zeit und Kausalität in der modernen Physik', p. 68f., 74.

successful adaptation and progress, now satisfying the necessary demand for 'continuity' by elevating it from the level of mere numbers describing the individual outcomes of single experiments to a higher, more abstract, more *encompassing* theoretical level: that of the Schrödinger-equation and the evolution of the probabilities for the *entire space of possible outcomes. Therein consists* in fact the undeniable *progress in understanding* these theories offer us, demonstrating a degree of coherence, unity, explanatory power and pragmatic success never equalled before.

A parallel analysis holds for the so-called 'Uncertainty-relations'. In spite of their name – betraying above all our 'classical' expectations and prejudices – they do not leave us in a deplorable state of uncertainty or indeterminateness, announcing the end of our search for unambiguous scientific determination. On the contrary they constitute a remarkable progress, likewise reached by revealing further conditions of objectivation. For these equations quantify with precision the very limits of possible quantification and thus of *objectifiability itself*. In this way they succeed in determining the physical core-activity of determination itself: hence they furnish us *more*, and not *less* information than classical mechanics.

In Cassirer's view even the frightening 'failure' of the key-notion of 'localised object' – which appears though as the very 'incarnation' of the scientific project and subject itself, as the guarantor of 'objectivity' as such – can be accepted and understood as a liberating achievement. Using again his transcendental 'Occam's razor' we have to determine, what is the constitutive function this notion actually fulfils? Being the prototype of all 'notions of measure' it provides the elementary reference units that fix and structure our knowledge. It thus fulfils the basic functions of stability and differentiation and serves the needs of 'identification' and 're-identification'. Moreover the 'foci' it offers can be used as support for further determinations: we ascribe 'variable qualities' to 'invariant objects'. It thus presents a specific pattern of order, conceived to grasp change conceptually, that corresponds to – and most probably stems from – the linguistic schema of 'subject and predicate'.

Now the question is, can this specific pattern of order, can this notion of 'localised object endowed with intrinsic qualities' still assure its stabilising role as an 'invariant' in the light of the quantum mechanical evidence? If we analyse whereupon exactly its use is founded, we find the basic fact, that consecutive measurements can repeat certain localised results. This has led to the assumption, that an invariant, individual 'carrier' supports these observable 'qualities'. But do we really need that particular assumption to account for the evidence? Let us investigate inhowfar the fundamental physical objects, i.e. 'elementary particles', still fulfil these epistemic requirements to be stable, differentiable, identifiable and re-identifiable. Most of them show an extremely short life-time, so the claim of 'stability' has become a matter of our own arbitrary definition of an 'appropriate time-span'. We can still differentiate one type of elementary particle from another one, e.g. a K-meson from an electron. But within a certain 'type' all particles have *identical* qualities and can no longer be differentiated from one another and a fortiori cannot be identified or re-identified by internal attributes. We can try to avoid this problem and save our 'object-idea' by applying an external differentiation-process using the space–time-coordinates of our particles and following their continuous path and history. But even if this is still working in classical mechanics, it is no longer possible in quantum mechanics, since for consecutive measurements of the conjugate variables the previous results can *not* be reproduced. That is where and why the – already weakened – concept of the 'localised, invariant object' looses its usefulness and plausibility as a stabilising invariant within this domain of experience – which does not mean that it cannot continue to fulfil other helpful functions. Would not the state-vector in Hilbert-Space be a more adequate candidate for its vacant vital post in this realm, assuring the needed stability, differentiatedness and unambiguous identifiability? As in the cases we discussed before, for Cassirer it is to be *expected* that mathematical, relational concepts will take over the role we have assigned to directly perceivable items as a 'first guess and approximation', guided by our daily habits and the elementary need to 'see' in order to 'grasp'. But just consider that – following either Bohr or Bohm, Everett or Schrödinger etc. – one can adopt very different ontological 'colourings' to understand quantum mechanics. Yet these choices do not alter the equations, nor the physical constants, nor the symmetry-principles and conservation laws. This clearly shows that the 'basic notions' – suggesting certain ontological options – are not at all as important as the dualist-representational model tries to make us believe. Instead the true core of physical knowledge consists in these *systemic, relational* values. This fact can also be confirmed by the way in which we construe physical identity. Take visible light, tangible infrared-heat, imperceptible radio-waves and x-rays: these phenomena incompatible for the witnessing of our senses are all identified by physics as being 'the same': electromagnetical waves at different wavelengths. The postulated 'identity' is based on a complex mathematical judgement, affirming that these phenomena obey the same laws – Maxwell's equations – agree in central numerical determinations and fall under the same constant: the velocity of light. Fixing individual 'identity' is thus a complex logical achievement which can only take place thanks to the same systemic values and the holistically interrelated network of physics' symbolic system – and does not in the least give us a simple 'basis' of 'independent substances' we could start with. It is the whole that constitutes the 'elements' and not vice versa.[31]

Cassirer's functional model of the constitution of meaning can thus help us to solve the interpretational problems of modern physics:

1. By differentiating Symbolic forms according to their aim of orientation – as physics, language and the linguistically shaped perception – we can disentangle their diverging claims and admit that their respective symbolic system might

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[31] Cassirer has learned from Leibniz's struggle to conceptualize dynamical processes mathematically that epistemically speaking the determinateness of the 'single element' is the result of a process of discretisation. Therefore Cassirer no longer starts with 'basic elements', the seemingly 'simple', in order to arrive at a higher order by 'syn-thesis' as Kant does. But on the contrary he considers that it is already part of the open 'problem of knowledge' to arrive at the determination of something as discrete by differentiation. For we can only rightly claim and reason logical determinateness if it is itself the product of a precise process of determination.

differ in any of their components including their fix-points, without any contradiction to other Symbolic forms, each of them being perfectly valid for their respective domain.

2. By becoming aware of the actual constitutive functions of the objectivation process we can distance ourselves from its historically given forms and perceive changes on all levels as a necessary and welcome ingredient of successful and truthful objectivation.

It would thus be possible to view Cassirer's pluralistic position as a kind of 'relativity theory of knowledge' – it is not by accident that he first mentions his idea of a 'philosophy of symbolic forms' in his essay on relativity theory[32] – for his holistic model re-installs (1) the equivalence between all elements – fundament and principles – within a specific knowledge system itself and (2) the equivalence of different types of objectifying knowledge, affirming that their claim of veracity is equally justified, just as Einstein has claimed the equivalence of all possible inertial systems. There as here it is the discovery of deeper preconditions and the underlying dynamics which allows for the unifying, more encompassing view of the ancient parameters.

As a summary Cassirer shows that the objectivation of physics presupposes:

1. Symbolic reference as the primary condition which enables the physicist to constitute the differentiatedness, stability, identifiability and re-identifiability without which he would not be able to refer to 'something' as 'this phenomenon' at all
2. The choice of a particular aim which provides the direction, the driving force and the criterion for progress
3. The constitution of the symbolic means that structure and organize its specific symbolic system, fulfilling the transcendental functions of finding adequate fix-points (constitute 'basic entities'), of determining their relations and of anticipating a coherent and maximally simple unity of all components
4. The choice of quantifying experiments that will operate physics' crucial 'trans-substantiation' and can put to test the concrete questions its entire symbolic system allows to anticipate
5. The search for an equilibrium, driven and guided by the different transcendental demands of its symbolic system – determinateness, univocity, stability, coherence, simplicity, unity – satisfying the measurement results which have been objectified thanks to this entire process of constitution
6. Concrete criteria for its pragmatic success and justification, given by the functional, transcendental demands of its symbolic system

Having thus made the vital, concrete role of the transcendental conditions explicit, Cassirer can liberate us from limiting beliefs, surreptitiously conveyed by language and cultural tradition, and thus set free our full potential of comprehension and

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[32] E. Cassirer, ZMP, 108–110.

objectivation. Physics shows a similar critical potential - sometimes even in spite of itself – because it uses the neutral, prejudiceless language of mathematics following its own unambiguous rules: both can thus be precious allies, leading to an ever more concrete and demystified vision of objectivation. Refining our understanding by tracing back objectivity to objectivation, form to formation and ‘Gestalt’ to ‘Gestaltung’ Cassirer completes the critical ‘desubstantialisation’ of our conceptual means that Kant had begun. He succeeds in fluidifying our cognitive reifications by leading us back to the living source of all knowledge: the non-rationalizable capacity to choose, to give rules, to create orders, and to fearlessly abandon the judgement about their epistemic value to time and the circumstances. Thus Cassirer’s position does not only relate back each structured objectified element to its initial process of open symbolical structuring, but it also relates back abstract epistemology to the concrete situation of the ‘conditio humana’ and its worldliness. In this view knowledge appears as the result of a complex creative ordering process, brought forward by the anthropological need for orientation, driven by different freely chosen aims of objectivation, devising sensitive symbolical tools and order-patterns to achieve these goal and finally exposing the results of all these efforts to their relentless pragmatic testing. The human is thus characterized as an ‘animal symbolicum’,[33] free yet forced to forge his own understanding of himself and the world he experiences, relying on free creativity and honest pragmatism – as well as cultural tradition – to face this fact.

Interestingly enough it is precisely when we give up the hope of a ‘direct grasp’ of ‘reality as such’, of a ‘final justification’, and when we accept the irremediable openness of the ‘conditio humana’, our ‘being on the way’, that we gain access to a mode of understanding that offers us all we could ask for: perfect determinateness of our objectified knowledge, clarity, unlimited adaptability, testability and concrete criteria for success. Ironically it is when we humbly concentrate on our own indirect symbolic mediation and adopt a standpoint of sober immanence, that we can really enter a process of true discovery of the unknown.

It looks as if the transcendental method is the best way to fulfil the realist’s dream.

References

Bohr, N.: The philosophical writings of Niels Bohr, vol II, Essays 1932–1957, Woodbridge, Ox Bow Press 1987

Bohr, N.: Discussion with Einstein on epistemological problems of atomic physics. In: The philosophical writings of Niels Bohr, Vol II, Essays 1932–1957, Woodbridge 1987

Bohr, N.: Unity of Knowledge. In: The philosophical writings of Niels Bohr, Vol II, Essays 1932–1957, Woodbridge 1987

Cassirer, E.: SFB: Substanzbegriff und Funktionsbegriff. Berlin 1910

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[33] E. Cassirer: Versuch über den Menschen, p. 51.

Cassirer, E.: Erkenntnistheorie nebst den Grenzfragen der Logik und Denkpsychologie, in: Jahrbücher der Philosophie, ed. by Willy Moog, 1927

Cassirer, E.: ZMP: Zur modernen Physik, Darmstadt 1964; (a) Zur Einstein'schen Relativitätstheorie. Erkenntnistheoretische Betrachtungen, Berlin 1921; (b) Determinismus und Indeterminismus in der modernen Physik. Historische und systematische Studien zum Kausalproblem, Göteborg 1936

Cassirer, E.: PSF: Philosophie der symbolischen Formen (1923–1929), Darmstadt 1953/1954

Cassirer, E.: Versuch über den Menschen. Einführung in eine Philosophie der Kultur, Frankfurt/M 1990

Cassirer, E.: Das Erkenntnisproblem in der Philosophie und Wissenschaft der neueren Zeit, Darmstadt 1971/1973, 1991

Dirac, P.A.M.: Principles of Quantum Mechanics, Oxford 1958

Dosch, H.G.: Cassirers Erkenntnistheorie, Kommentar eines Physikers. In: Determinismus – Indeterminismus, ed. by W. Marx, Frankfurt/Main 1990

Dosch, H.G.: Renormalized Quantum Field Theory and Cassirer's Epistemological System, in: Philosophia Naturalis, vol. 28, 1991, part 1

Einstein, A.: Zur Elektrodynamik bewegter Körper, in: The collected papers of Albert Einstein, ed. by J. Stachel, Bd. 2, The Swiss Years: writings 1900–1909, Princeton, NJ 1989

Kant, I.: KrV, Kritik der reinen Vernunft (1781/1787), ed. by W. Weischedel, Frankfurt/Main 1968

Krois, J. M.: Symbolic Forms and History, New Haven/London 1987

Pauli, W.: Aufsätze und Vorträge über Physik und Erkenntnistheorie, Braunschweig 1961

Pauli, W.: Phänomen und physikalische Realität. In: Aufsätze und Vorträge über Physik und Erkenntnisthedie, Braunschweig 1961

Pauli, W.: Raum, Zeit und Kausalität in der modernen Physik. In: Aufsätze und Vorträge über Physik und Erkenntnisthedie, Braunschweig 1961

Poincaré, H.: La science et l'hypothèse, Paris 1968

Schmitz-Rigal, C.: Die Kunst offenen Wissens, Hamburg 2002

On the Transposition of the Substantial into the Functional: Bringing Cassirer's Philosophy of Quantum Mechanics into the Twenty-First Century

Angelo Cei and Steven French

Abstract Cassirer's neo-Kantian philosophy of scientific knowledge has been the subject of renewed interest recently, in particular with regard to the interpretation of General Relativity. However, Cassirer's analysis of Quantum Mechanics, found in *Determinism and Indeterminism in Modern Physics*, has not received the attention it deserves. Our aim in this paper is to sketch out the central themes of this work and illustrate its relevance for contemporary discussions of structuralism in the quantum context. Cassirer's general philosophy of physics is outlined before presenting the analysis of the nature and role of the causality principle. We place particular emphasis on the hierarchical view of scientific laws and principles which set causality at the apex and expressed it in abstract functional terms. Through such notions, transcendental philosophy can accommodate statistical laws and hence it can render harmless the apprent threat of quantum indeterminism. It is also shown that the uncertainty principle, quantum holism and the implications of quantum statistics are the grounds for Cassirer's conclusion that the true import of quantum mechanics was the reconceptualisation of our notion of object. Such reconceptualisation is structural, with point particles understood as 'intersections of relations'. A brief comparison of Cassirer's neo-Kantian structuralism with some modern forms concludes our analysis.

1 Introduction

Although Cassirer's neo-Kantian philosophy in general and its application to General Relativity in particular have been the subject of renewed interest recently, his analysis of quantum theory in *Determinism and Indeterminism in Modern Physics* has not received the attention it deserves. Our aim in this paper is to sketch out the central themes of this work and illustrate its relevance for contemporary discussions of structuralism in the quantum context.

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We begin with an outline of Cassirer's neo-Kantian approach to the philosophy of physics in general, before presenting his analysis of the nature and role of the causality principle. In particular we shall emphasise Cassirer's hierarchical view of scientific laws and principles which set causality at the apex and expressed it in abstract functional terms. This made room within his transcendental philosophy for statistical laws and hence the apparent threat of quantum indeterminism was rendered harmless. Indeed, the true import of quantum mechanics, as far as Cassirer was concerned, was the reconceptualisation of our notion of object. Drawing on the uncertainty principle, quantum holism and the implications of quantum statistics, Cassirer argued that the appropriate reconceptualisation was structural, with point particles understood as 'intersections of relations'.

We conclude with a brief comparison of Cassirer's neo-Kantian structuralism with some modern forms and consider how much of his position can be projected onto today's discussions.

2 From Kant to Cassirer

Cassirer's philosophy of science is a form of *transcendental idealism*, in which the fundamental principles of theoretical natural science express the universal patterns by means of which thought orders the manifold of phenomena. Cassirer's version of kantianism evolved from the Marburg School's interpretation of Kant.[1] We shall not pursue the issue of the extent to which Cassirer can be considered an epigone of that view. We will rather focus on three features of such a legacy relevant to the present discussion. These elements also allow us to appreciate the distance that Cassirer places between himself and Kant. In the reading of the Marburg School:

(a) The main lesson of Kantianism is that science and its objectivity are *facts*. Such facts are the explananda of a philosophical theory of knowledge whose questions are how we have knowledge of nature and why such knowledge is objective. In this view, foundational issues in science have primarily an epistemological dimension.
(b) In such a picture thought plays a "constructive" role and broadly speaking objectivity is to be understood as emerging from this constructive activity.
(c) The kantian notion of pure intuition as distinct from understanding, as well as the relative doctrine of mathematics as resulting from the injection of the logical forms of the categories in the pure intuition of space and time has to be rejected since it is denied by the development of modern mathematics in which it is shown that intuition does not play any role.

In the light of (a) and (b) it is very natural to adopt a relativized view of the *a priori* principles of science. After all if the history of science presents us with profoundly different theoretical frameworks it is sound to expect different *a priori* principles to be instantiated to grant in some form the universal unity and objectivity that those frameworks enjoy (Ryckman, 1999). In this sense, we will see that Cassirer's analysis

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[1] For Cassirer's relationship with Cohen and the Marburgh School see Friedman (2000).

of Quantum Mechanics will aim to highlight precisely the kind of assumptions allowing the construction of objectivity in the quantum domain.

The epistemological framework that Cassirer projects onto Quantum Mechanics is grounded in his peculiar appreciation of the significance of (c). Historically the rejection of the idea of pure intuition dates back to the crisis of the understanding of mathematics as based on intuition springing from the rise of the non-Euclidean geometries (Ryckman, 1991; Friedman, 2000). The role played by developments in the foundation of mathematics of the late nineteenth century in shaping Cassirer's approach is twofold. On the one hand, from (c) we draw the idea that mathematical concepts are logical in nature and in this sense they play with respect to natural knowledge the same role that in the Kantian picture was played by categories.[2] They structure the manifold of experience allowing for our knowledge of it. On the other hand, the developments in the foundations of mathematics are the source of the concept around which revolves the revised notion of *synthetic a priori* employed to analyse quantum theory.

The rejection of pure intuition involves the dismissal of the model of Kantian Schematism. Roughly speaking the idea to be rejected is that pure intuition of space and time represents the "place" in which the pure logic of understanding encounters the manifold of perception.[3] Neo-Kantianism now has to explain how this encounter, this synthesis, takes place if there is no pure intuition to act as the general "theatre". Cassirer's answer (Cassirer, 1907) relies on the notion of *Zuordnung* or *functional coordination*.[4] Such a notion is taken as a primitive, fundamental one and "has no other meaning than that of relation and mutual coordination of one thing to another"(Ryckman, 1991, p. 63).

An interesting element to emphasize here is that Cassirer is using this notion to mimic precisely the Kantian move in the Schematism without making use of the idea of pure intuition:

> "[these] same basic syntheses upon which mathematics and logic rest, also govern the scientific structure of empirical knowledge and first enable us, by a fixed lawful ordering of phenomena to speak of its objective significance" (Cassirer, 1907, p. 45; quoted in English in Ryckman, 1991, p. 65). We could think of it as a generalized form of schematization.

This notion is modeled on that of function in analysis and according to Cassirer its key role in allowing us to form the fundamental concepts of science has to do with the fact that a function instantiates a general rule or law that relate all the members of the series and that law, rather than being inducible by enumeration of each of the members, can be seen as the fundamental form of each of them (Cassirer, 1953).

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[2] This is particularly evident in Cassirer (1907), where he responds to logicist criticisms of the Kantian idea of synthetic *a priori*. Cassirer claims that the logical and thus analytical nature of mathematics does not compromise the fundamental Kantian idea of the *a priori* synthesis of the understanding. Mathematics is in fact synthetic when considered in its role in ordering the manifold of perceptual experience in theoretical natural science.(see Friedman, 2000).

[3] This is, also, the core of Kant's explanation of the mathematical nature of Physics since the schematization of categories in the pure intuition of time determines the conditions of possibility of arithmetic and the schematization of categories in the pure intuition of space to geometry.

[4] Ryckman (Ryckman, 1991) explores the extent to which the notion of *coordination* was in the early twentieth century the focus of a wide variety of analyses of science and identifies in it a further element of commonality between Neo-Kantianism and Logical Empiricism.

The extent and importance of the notion of functional coordination and the role it plays in generalizing the Kantian idea of schematism, setting it free from pure intuition, can be seen in the role that the mathematical notion of group plays for Cassirer with respect to perceptual experience.

3 Group Theory and Perception

Proceeding ahistorically, the central importance of group theory for his work is nicely displayed in Cassirer's discussion of the psychology of perception (Cassirer, 1944). Here the concept of a group is referred to at the very beginning as the 'universal instrument of mathematical thought' (p. 1), insofar as it acts as an 'organizing and clarifying principle', across the range of mathematical fields.

Adopting a historical stance, Cassirer begins with Helmholtz's 'empiristic' approach to perception and in particular the latter's – fundamentally group-theoretic – analysis of the notion of space. However, this then takes us a further epistemological step back to the question, 'what is the foundation for the notion of group?' Poincaré's answer, famously (but surprisingly ignored by recent philosophers of science, it seems), was to take this notion as *a priori* and as arising from an 'original intuition' underpinning all experience. Thus the general concept of a group 'pre-exists' in the mind (at least potentially) and this understanding underpins Poincaré's conventionalism: from the possible groups which are latent in the mind one must be chosen to form 'a kind of standard' in terms of which natural phenomena can be compared. The role of experience is then limited to helping to indicate which choice 'adapts itself best to the properties of our body' (p. 4). Interestingly, Poincaré himself applied a group-theoretic understanding to the psychological problem of how, in the great flux of sense impressions, we manage to differentiate the spatial movements of an object from qualitative alteration. The answer, of course, is that displacements can be compensated for, something represented group-theoretically by the inverse operation. Although it is experience that reveals such compensation, it acts not as the source of the relevant geometrical concepts but only as the 'occasional source of their formation' (p. 5).

Now, Cassirer takes Poincaré's attempt to construct a bridge between mathematics and psychology as not merely an original and stimulating conjunction of ideas by a brilliant thinker but as indicative of a fundamental and epistemological 'inner connection' between the concept of group and fundamental problems in the psychology of perception. As he insists, the nature of the game here is logical, rather than ontological, in that he is not suggesting that psychology can be *reduced* to group theory and hence mathematics, but rather that insofar as perception cannot be understood in the absence of some organising and coordinative scheme, and insofar as such a scheme is provided by the intellect, group theoretical type concepts will be found to be applicable.

As Cassirer notes, the group theoretical analysis of geometry begins with a shift in focus from the '*hic et nunc*' or individuality of an object, to those properties describable in terms of (crucially) invariant formulae. On this basis the equivalence of spatial concepts can be established, so that the 'essence' of a triangle, for example, remains unchanged, even if its 'individuality' is altered by displacement or expansion. With the realisation regarding the choice of the group of transformations in terms of which this 'essence' can be delineated, we arrive at Klein's view that 'the characteristic properties of a multiplicity must not be defined in terms of the *elements* of which the multiplicity is composed, but solely in terms of the *group* to which the multiplicity is related' (p. 7). A different choice of group will obviously lead to a different determination of what is taken to be (geometrically) identical or different. In particular, shifting to a different group structure – from Euclidean to projective geometry, say – results in a shift of what are taken to be the 'independent geometrical individualities' of the objects concerned. Thus the distinct conics of Euclidean geometry are reduced to just the one in projective geometry. The geometrical figures themselves are only a kind of 'plastic material' (p. 8) and the true foundation of mathematical certainty lies in the rule by which the (group theoretical) elements are related. In this manner, the elements – geometrical figures in this case – lose their individuality but a sense of definiteness is still retained, in terms of the relevant group-theoretic context.

It is here that there appears to be a clear difference from perception, concerned as it apparently is with the '*hit et nunc*' and individual content. Granted this, granted that perception cannot attain that sense of 'universality' towards which geometrical thought progresses, nevertheless the 'sensualistic thesis' of modern psychology, which insist that perception is nothing but a bundle of sense impressions, must be abandoned in the face of the phenomenological facts.

Consider, for example, the fact of perceptual constancy, whereby quite dramatic changes in the intensity of illumination and – to a degree – the colour of illumination, do not apparently affect our perception of colours. Similarly, our perception of spatial shape and size remains broadly constant – within limits – despite changes undergone by the object itself. What can be extracted from such facts is the fundamental significance of concepts of invariance and transformation, and even granted that, as noted above, psychology cannot be reduced to mathematics, it is remarkable that such group theoretic concepts should appear in the exposition of such psychological facts. What this suggests is some kind of 'mediate connection' (p. 12) whereby the 'form of universality' represented by group theory may be seen to be present with respect to both mathematical and perceptual concepts.

A nice distinction between Cassirer and Russell can be drawn at this point, a distinction which has some significance for our understanding of both the history and foundations of structuralism. We recall the central role played in Russell's epistemology by his causal theory of perception, which assumes a straightforward correspondence between 'percepts' and stimuli, such that differences in our percepts imply differences in the stimuli. Indeed, Russell rejected the views of the Gestalt psychologists and insisted that claims that the continuity of the percept is neither that of the mathematical continuum nor that of 'deceptive' vagueness 'go beyond what the evidence warrants' (Russell, 1927, p. 280). It is on this basis that Russell's structuralism – which has proven so influential – is constructed. The above assumption, together with that of spatio-temporal continuity, suffice '… to give a great deal of knowledge as to the *structure* of stimuli (ibid., p. 227; his emphasis), leaving the 'intrinsic characters' of the stimuli unknown. Hence, all that we know about the external world is its *structure*:

> When we are dealing with inferred entities, as to which … we know nothing beyond structure, we may be said to know the equations, but not what they mean: so long as they lead to the same results as regards percepts, all interpretations are equally legitimate. (ibid., p. 287)

As Cassirer notes, this whole approach depends on what he calls the 'constancy hypothesis' of an immediate correspondence between stimulus and sensation. However, experiments such as those demonstrating 'perceptual constancy' as well as others which support the claims of gestalt psychology undermine such a hypothesis. There can be no mere reflection of the 'external' by the internal' (p. 19). Indeed, Cassirer insists, the real problem is that 'we do not stick to the given', the '*hit et nunc*' or particularity of a stimulus, but rather go beyond this so that perception is integrated into the total experience. It is in virtue of such integration that perception becomes 'objective'. And the means by which it becomes such is through analogy with the concept of transformation:

> Psychology dismisses the dogma of the strict one-to-one correspondence between physical stimuli and perceptions. It is, on the contrary, the "transformed" impression, i.e., the impression as modified with respect to the various phenomena of constancy, which is regarded as the "true" impression, since we can on these grounds construct knowledge of reality. (p. 35)

We recall that in the domain of mathematics and geometry in particular, this concept of transformation, as represented group-theoretically, allows us to transcend the particularity of geometrical intuition and move towards universality via the choice of an appropriate group, each group yielding a different set of invariants under the relevant transformations and hence represent different geometrical properties. Likewise, in perception we go beyond the particular and integrate the perception in a given context. And in that context, the 'apprehension' of the particular involves an apprehension of the possibilities of transformation which it contains (p. 15). These possibilities can be ordered, as it were, along certain dimensions, such as the conditions of illumination for example. Thus, '[t]he perceptual image … [also] … involves … reference to certain possible groups of transformation. It changes when we refer it to a different group and determine the "invariants" of perception accordingly' (p. 16). Consider the observations made by Gestalt psychologists, for example, which show how changes in the distribution of light and shadow can lead to a shift in perception from one mode of 'apprehension' to another (ibid.). According to Cassirer, this shift – made by a 'free choice' – between perceptual structures is analogous to the shift between different geometries when we move from one group to another. Perception thus involves a process of 'objectification', to be understood via the formation of invariants (p. 20).

Of course, this analogy cannot be taken too far, principally because the kinds of determination we find in perception are always only approximate, rather than absolute. Increase the intensity of the colour of illumination too far, for example, and the perceptual constancy of colour tone breaks down. Perception is always vague and imprecise and at best only tends towards the ideal attained by mathematics.

Hence the concepts of group and invariant function as 'mediating principles of a higher order' (p. 19) which help bring mathematical and psychological problems under a 'common denominator'. And the nature of this common denominator can be understood if we shift our epistemological focus from objects per se to the process of objectification. Consider not just mathematics, but science as a whole: adding the physical and chemical constants to geometrical invariants, it is in these terms that '… we formulate the "existence" of physical objects' (p. 20), just as we effectively construct the "true" colour in perception through a similar process of objectification. In general, '[t]he *positing* of something endowed with objective existence and nature depends on the formation of constants …' of this sort, whether that 'something' be a physical or perceptual object.

The rejection of 'sensationistic' psychology with its reliance on imagistic mechanics, has consequences which extends beyond a rejection of the kind of structuralism we find in Russell's *Analysis of Matter*, into Cassirer's neo-kantian form. The group-theoretic interpretation of the foundations of geometry of course constitutes an integral component of Cassirer's neo-kantian position in general. General concepts, such as that of a triangle, say, are represented not by an image but only via a rule, since the latter incorporates the generality and universality which the image cannot capture and which so bothered Berkeley, for example. The same is the case for perceptual concepts as well, so that the concept 'dog', for example, is not to be understood as represented by some bundle of properties, but by an appropriate rule (p. 22). And group theory, of course, gives us a more precise 'handle' on the nature of such rules: 'The rule may, in simple and exact terms, be defined as that *group of transformations* with regard to which the variation of the particular image is considered' (ibid.). The history of geometry as Cassirer sees it, from Euclid to Poncelet, is a history of emancipation, from elements given in intuition, to the *relations* between such elements. From a group-theoretic perspective, '[t]he "nature" or "essence" of a figure is defined in terms of the *operations* which may be said to *generate* the figure' (p. 24). And these operations are characterised in terms of the relevant group, of course.

As in the history of geometry, so in the history of the psychology of perception, as the sensationistic views of Berkeley and Hume, inherited by Russell, are replaced by those of Ehrenfels and Koehler and the Gestalt psychologists in general, with their emphasis on 'form-qualities' and physical Gestalten (see French forthcoming). In both cases, a crucial role is played by the relevant invariances and just as certain heterogeneous geometrical figures come to be seen as identical, by virtue of being inter-transformable via certain group operations, so in the case of perception, a similar 'identity' allows one to grasp the relevant structures. Corresponding to the mathematical notion of transformability, Gestalt psychologists in particular have emphasised the notion of 'transposability', and corresponding to the shift away from geometrical elements to relations, within the domain of perception we see a shift away from bundles of simple sense impressions to forms, taken as primitive. Of course, as has been noted, the analogy cannot be taken too far. In mathematics the invariants and transformations are subject to a logical systematization which is not available in the psychology of perception, although the Gestalt psychologists' notion of 'laws of understanding' goes some way towards formalising this understanding.

Furthermore, both geometry and perception, '… share the function of objective knowledge' (p. 31) and in these terms a form of 'mediation' can obtain between them, as suggested above. This allows for what Cassirer calls an 'upward and downward reference':

> If we proceed in the upward direction we come to the all-comprehensive geometrical systematization achieved by group theory; if we proceed in the downward direction, we encounter those "schemata" that are present already in perception and immediate intuition. (p. 31)

> On such a basis, he concludes, '… psychology and epistemology may meet and cooperatively attack the numerous problems still to be solved'. (p. 35)

There is much more to be said about this paper, of course, in particular with regard to its place within Cassirer's corpus but having outlined the group-theoretic context, we shall move on to consider his approach to the foundations of physics, and quantum theory in particular.

4 Space–Time, Structures and Group Theory

The central theme which runs through Cassirer's writings in this area is the analysis of the concept of object (Ihmig, 1999). And the fundamental perspective from which this analysis should proceed is, of course, epistemological:

> … epistemological reflection leads us everywhere to the insight that what the various sciences call the "object" is nothing in itself, fixed once for all, but that it is first determined by some standpoint of knowledge. (Cassirer, 1953, p. 356)

We recall that Cassirer's interest in this issue can be traced back to his reflections on the nature of space and the influence of Klein's Erlanger programme, with its emphasis on group-theoretic notions. What this yields, of course, is a structural conception of geometrical objects which shifts the focus from individual geometrical figures, grasped intuitively, to the relevant geometrical transformations and the associated laws. This shift is then manifested in Cassirer's neo-Kantian assertion of 'the priority of the concept of law over the concept of object.'

This assertion in turn forms an integral component of Cassirer's interpretation of the Kantian understanding of objectivity:

> For objectivity itself - following the critical analysis and interpretation of this concept - is only another label for the validity of certain connective relations that have to be ascertained separately and examined in terms of their structure. The tasks of the criticism of knowledge ("Erkenntniskritik") is to work backwards from the unity of the general object concept to the manifold of the necessary and sufficient conditions that constitute it. In this sense, that which knowledge calls its "object" breaks down into a web of relations that are held together in themselves through the highest rules and principles. (Cassirer 1913, trans. in Ihmig, op. cit., p. 522)

These 'highest rules and principles' are the symmetry principles which represent that which is invariant in the web of relations itself. And these principles, in turn, are represented group-theoretically; thus the relevant group effectively lays down the general conditions in terms of which something can be viewed as an object. We shall return to the analysis of such principles below.

Cassirer's 'application' of this framework to the foundations of relativity theory is well known (Ihmig, op. cit., pp. 524–528). According to Ihmig, what it does is restore the *unity* of the concept of object which is apparently undermined by the relativistic transformations. From the structuralist perspective, this unity is 'reinstated on a higher level.' (ibid., p. 525) via the 'lawful unity' of inertial systems offered by the Lorentz transformations. The process of abstraction from a substantivalist conception of objects to a structuralist one is furthered by the General Theory of Relativity and what we are left with is an understanding of the objects of a theory as defined by those transformations which leave the relevant physical magnitudes invariant. Thus Cassirer saw General Relativity as the natural conclusion of the structuralist tendency.

Cassirer's understanding of the foundations of General Relativity has been further pursued by Ryckman (1999), who points to the central importance of the principle of general covariance in this understanding. According to Ryckman, Cassirer viewed general covariance as a principle of objectivity which offers a 'deanthropomorphized' conception of a physical object. Furthermore, he (Ryckman) claims, this view of Cassirer's meshed with Einstein's own and underpinned the latter's objections to quantum mechanics through its implementation in the separability principle.

As the requirement that the laws of nature be formulated so that they remain valid in any frame of reference, general covariance '… is a further manifestation of the guiding methodological principle of "synthetic unity" necessary to the concept of the object of physical knowledge.' (ibid., p. 604). Regarded as a synthetic requirement, general covariance comes to be seen as both a formal restriction and a heuristic guide for the discovery of general laws of nature (ibid.). Physical objectivity – apparently lost by space and time themselves – re-emerges in deanthropomorphised form in terms of the functional forms of connection and coexistence:

> With the demand that laws of nature be generally covariant, physics has completed the transposition of the substantial into the functional - it is no longer the existence of particular entities, definite permanencies propagating in space and time, that form "the ultimate stratum of objectivity" but rather "the invariance of relations between magnitudes". (ibid., p. 606, citing Cassirer, 1957, p. 467)

5 Quantum Mechanics, Causality and Objects

There has been comparatively little discussion of Cassirer's analysis of the other major revolution of the twentieth century, namely quantum mechanics, as presented most famously in his classic work *Determinism and Indeterminism in Modern Physics* (Cassirer, 1936).[5]

The focus of the work is the notion of causality and Cassirer can be characterised as attempting to protect Kant from the impact of quantum theory by demonstrating how a neo-Kantian understanding of causality can be preserved in this new context. In a nutshell, this understanding takes causality to be a general, 'transcendental' principle which refers not to objects, of course, but to our cognition of them

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[5] A sketch is given in Itzkoff (1997, pp. 83–98).

(1936/56, p. 58). As such, it is a '… guide-line which leads us from cognition to cognition and thus only indirectly from event to event, a proposition which allows us to reduce individual statements to general and universal ones and to represent the former by the latter' (ibid., p. 65). And from this standpoint, the concepts of chance and causality do not stand in opposition, but rather 'side by side' (ibid., p. 104), in a 'complementary relationship' (ibid. p. 103) which is as it must be if we are to determine an event as completely as possible. In classical physics the relationship is represented by that between 'the course of an event' and knowledge of its initial conditions, or more generally, by that between 'nomological' laws and 'ontological' laws which 'nowhere contradict each other' but, rather, 'interweave', giving rise to the universal form of 'order according to law' (ibid., p. 105).

Cassirer's effort to rescue causality can thus be represented as follows: If one wishes to express it in the language of Kant "the law of causality belongs, according to Cassirer, to the modal principles, it is a postulate of empirical thought" (quoted in Rudolph, p. 241). Thus, that which was taken to be constructive is now elevated to the status of a regulative principle.

Quantum physics, of course, poses a more serious challenge to such a view, standing as it does in 'far greater contrast' to classical physics than General Relativity does (op. cit., p. 105). However, this challenge can be met as long as we cleave to the essential idea that causality expresses 'something about the structure of empirical knowledge' (p. 114). In particular, quantum mechanics does not dispense with conformity to law, even if 'law' must now be understood as 'statistical' rather than 'dynamical', as in the classical case. The challenge is to our characterisation of 'the physical concept of reality' (ibid., p. 128) and in particular, it is the classical concept of object which is undermined, a shift which Cassirer portrays as jumping from the frying pan into the fire! In other words, the true import of quantum physics lies not in the apparent implication of some kind of indeterminism but in its further support for a shift away from the notion of object, to that of fundamental laws and principles, understood, ultimately, in a structural sense.

What does such a shift amount to? To address this question we need to engage in a deeper analysis of the case for causality.

The notion of causality that Cassirer is defending can be "expressed in the language of Kant" but it is certainly not Kantian. He is here characterizing causality as a postulate of empirical thought. Postulates are meant to specify the kind of attitude that thought entertains with regard to representation. For example if you have the representation of Italian people having cappuccino in a coffee bar in Leeds and find it nice you might think that such representation is not possible. Kant would say that the representation conforms to the law of the understanding, and thus it is indeed possible (Kant definitely is not a big name in coffee expertise!). But for Kant the Causal Law is one of the Analogies of Experience; in other terms that law is meant to structure the representation not to tell us what is the modality in which the understanding conceives it.

So why is that? Why does Cassirer want to show that causality still plays somehow a Kantian role and at the same time wants to twist the very notion so much?

The answer to this question involve an analysis of Cassirer's conception of the body of knowledge of mathematical physics. He draws a distinction between three 'basic' types of statements in physics: statements of the results of measurements (Ch. 3); statements of laws (Ch. 4) and statements of principles (Ch. 5).

The first represent 'that decisive transformation' (p. 31) from immediate perceptual data to experimental observation, where the latter must be understood as a determination into which concepts of measure and number enter. This transformation is highly complex and in Cassirer's discussion we may perhaps see a 'foreshadowing' of Suppes's characterisation of the 'conceptual grinder' which takes us from sense data to data models.

Statements of laws effectively join the particular to the whole and they are able to do this through the mathematical concept of function. This notion plays a key role in Cassirer's architecture of science. There are a few things to note about his discussion here. The first is that with regard to the relationship between these two kinds of statements, Cassirer rejects induction as an appropriate way of characterising it. Indeed, he sees the problem of induction as the 'chief stumbling block' for the philosophy of science in general (ibid., p. 39). Instead, what we have in the move from statements of measurement to statements of laws is a 'characteristic transformation' from a 'here-thus' to an 'if-then' (ibid., p. 41) and the hypothetical judgments embodied in the latter cannot be regarded as mere summaries of individual facts since they pertain to classes of magnitudes which typically consist of infinitely many elements. Not only are classes of a different nature from that of their elements, according to Russell's theory of types (ibid.), but we cannot say anything about such a class of elements if each one has not been previously tested and examined (ibid.). What a statement of law represents is an 'abrogation' of the space–time realm in which individual facts are situated and this 'change of dimension' cannot be captured as mere induction. What grants the passage here from the particular to the general? And further from these laws to the principle? In answering this question one answers the first part of the preceding one. Why is causality still around?

From the traditional Kantian standpoint the model of causality is formulated in the second analogy and it is the core of Kant's response to Hume. Hume says that we at most represent events as successive in time and constantly conjoined and this is all causality is about and in the second analogy Kant points out that the very possibility of such representation implies the working of a rule of ordering that makes possible that succession. Now, temporal ordering is certainly not something we grasp by perception so it must necessarily come from somewhere else. Thus it is provided by the understanding; hence it is possible to attach to it some form of *a priori* necessity. Cassirer has certainly abandoned that standpoint especially because of his rejection of a role for intuition. In other terms he need not have any principle to grant permanence, succession or coexistence in time because in his view the intuition of space and time are not playing any role in modern science. The analogies are nonetheless playing another role in Kantian philosophy: they ground in the activity of the understanding the fundamental Laws of Newtonian mechanics (Friedman 1992). It is our contention that Causality interpreted as a general principle is granting for Cassirer the possibility to apply universally the idea of functional coordination according to a law.

Let's put the issue in slightly different terms: Cassirer views the causal principle as a guarantee of the indefinite growth of the body of natural knowledge since he needs a criterion that make sense of the fact that "the process of conversion of the observational data into exact statements of measure, gathering together of the measurements in equations between functions, and the systematic unification of these equations through general principles" can never be completed. The reason for this new role for causality has to do, in the first instance, with the entangled notions of law and function.

What is noteworthy here is the similarity of Cassirer's consideration of the mathematical aspect of laws with more recent structuralist discussions, Thus, he notes that once placed in this form, phenomena are effectively established as 'enduring thoughts' (ibid., p. 38), in the sense that their duration extends far beyond their original representation in this form. As an example, he gives that of Fourier's theory of heat which was developed in the context of a view of heat as a fluid but whose mathematical description – in terms of which the phenomena were represented as the results of 'purely geometrical relations' – came to be seen as independent from these particular hypothetical presuppositions. It is this separation of the fundamental structure, as represented by the mathematical equations, from the underlying metaphysical commitments – which may of course play a crucial heuristic role – which was also noted by Poincaré, for example, and which lies at the heart of modern forms of 'structural realism' (Ladyman, 1998). Even more interestingly, perhaps, Cassirer goes on to point out how Fourier's formulae were subsequently resurrected by Heisenberg in the development of quantum mechanics, Saunders uses this example to illustrate the 'heuristic plasticity' of such formulae (1993), and we take this to be broadly the same as what Cassirer calls their 'indwelling sagacity' (Spürkraft). The central idea is that it is by means of this plastic mathematics that fundamental structural aspects of classical dynamics are isolated, become entrenched and are thereby preserved in subsequent developments. In particular, as Saunders notes, certain of these features (those which are group-theoretic in particular), provide '… over-arching abstract frameworks … within which one dynamical structure may be embedded in another' (op. cit., p. 308). Both Cassirer and Saunders see this feature as indicative of the significant independence of the relationships represented by the equations and formulae, from the hypothetical/metaphysical presuppositions which led to their elaboration in the first place. The characteristic mathematical formulation is thus a crucial component of the notion of law and in this sense Cassirer is very close to Kant who expressed clearly is scepticism towards the scientific standards of a discipline which did not achieve a mathematical formulation at least of its basic laws and principles.

In the case of Kant, mathematics and the objectivity of the laws are constructed out of the same source. Mathematics depends upon the injection of the categorical order into the pure intuition of time. In particular this has to do with the order of succession, permanence and simultaneity, a process which is crucial for the construction of objectivity itself and starts precisely with the determination of something persisting in the flowing of experience: "…the substratum of everything real, i.e. everything that belong to the existence of things, is *substance*, of which everything that belongs to existence can be thought only as a determination" (*Critique of Pure Reason*, CE B225, p. 300).

The Kantian notion of objectivity has to do with the necessity of meshing the logical order of categories with the perceptual content of sensibility within the framework provided by space and time. Mathematics comes to being in this process when the intuition of time is considered in its purity independently from any perception. In this sense, physical objects are substrata logically required as holders of the various determinations and mark one of the crucial elements of our comprehension of the world: permanence in time. Cassirer's deeper departure from Kant starts precisely here and from this standpoint it possible to grasp the gist of his radical form of structuralism about physics.

For Cassirer mathematics plays a crucial architectonic role as well, but it does it alongside logic in a context in which the landscape of logic has been enriched by the results of Frege and his followers. Cassirer rejects completely the idea that mathematics is constructed out of pure intuition and logic. He maintains that such structural features embody a synthetic element as far as mathematical natural knowledge is concerned (see Friedman, op. cit., p. 92). Now the rejection of the role of pure intuition here is combined with the idea that the Kantian distinction between Sensibility and Understanding as faculties is untenable. Thought in generating our knowledge of reality acts as a whole.

How is objectivity to be constructed in this picture? Again the logical element provided by mathematics and logic (regarded as on a par) has to provide order in the appearance. But the first step in the introduction of an element of permanence does not take place *via* the adoption of the notion of a substance as a substratum. For Cassirer physics characteristically and progressively assigns this role to the notion of function. He presents this point for the first time in an early writing concerned with the interpretation of Leibniz's concept of force that Cassirer takes to be a model for the notion of substance: "the continuous persistence of force is not to be understood as "material constancy of a thing" but a persisting identity of a superordinate law" (Rudolf, 1994, p. 236). The picture we end up with then, is that laws are represented by equations expressing relations between functions.

Moving on and upwards, as it were, statements of principle, seen as 'statements of third order', arise when one begins to consider how the laws are inter-related and the process here is what underlies the great moments of unification in science, for example. As a typical example, Cassirer gives the principle of least action and notes again, that as it was developed and made more precise through history, the metaphysical basis for it was increasingly lost from view (ibid., p. 48). However, the price for universality is the apparent loss of the subject of the principle (Cassirer nicely refers to its 'iridescent indeterminateness' (ibid., p. 51), but rather than seeing this as a defect, Cassirer insists that it points to the real import and methodological character of such principles: they function as heuristic rules for seeking and finding laws (ibid., p. 52). And they do this by presupposing 'certain common determinations' which hold for all natural phenomena, and then effectively consider what, in a particular domain, corresponds to these determinations. Thus their power and value lie in this 'capacity for "synopsis"' (ibid.), which affords an overview of more than one physical domain. Unlike the laws themselves, the principles do not refer directly to phenomena, but to '… the form of laws according to which we order these phenomena' (ibid.). The relationship between principles and laws is akin to that between laws and measurement results in that a 'new dimension' is entered and the purposes of a 'pure immanence' are served, in the sense of 'the inner construction and securing of experience' (ibid., p. 54).

Putting it a little crudely perhaps, 'statements of measurements are individual, statements of laws general, and statements of principle universal' (ibid.). However, Cassirer emphasises that the relationships between them should not be characterised in terms of any kind of spatial metaphor since these statements all mutually condition and support one another (ibid., p. 35) in a kind of 'reciprocal interweaving and bonding' (ibid.). Consider the relationship between statements of measurement and statements of laws, for example: the former, as already indicated, do not constitute some bedrock of 'facts' since, as Cassirer claims, in an early reference to theory-ladenness, that 'everything significantly factual is already theory' (ibid.). Thus we should not see these statements as forming the structure of a pyramid; this would suggest that the top 'layers'; could somehow be removed without affecting the bottom, but such a suggestion is simply untenable since the truth of all such statements at whatever 'level' is due to their mutual interconnection (one might draw the obvious analogy with the coherence theory of truth here). Rather than a pyramid, Cassirer likens this structure to a Parmenidean 'well rounded' sphere, wherein the various elements can be logically distinguished, even though they cannot be ascribed any kind of independent existence. Significantly, Cassirer insists that within such a structure there is '… no proper substantial carrier, nothing that *per se est et per se concipitur*' (ibid.); rather there is '… only a functional coordination in which all the elements, all the determining factors of physical truth, uniformly participate' (ibid.).

Likewise, from Cassirer's structuralist perspective, there are no substantial carriers of physical properties, but only functional coordinations to which our metaphysical notion of a physical object is ultimately reduced.

Indeed, it is only through the mediation of the results of measurements that the 'concepts and judgments' of physics can refer to an object and acquire objectivity. It is at this level of statements that we find the 'feature of individuality' that objects are typically taken to have, in the sense that such statements pertain to a definite here and now. In other words, what we have here is a form of what has been called 'space–time individuality', in the sense that the individuality (and distinguishability) of objects is ultimately grounded via their location in space–time (see French and Krause, 2006). It is precisely this that quantum mechanics will undermine. To use Eddington's phrase, this level of statement yields only a 'legend' of individuality', which results when our 'ordinary' frames of thought are transformed by the mathematics relevant to quantum theory (French, 2003). In this sense, in which the statements of the results of measurements are the beginning and end of physics, '[**w**]hat physics calls an "object" is nothing ultimately but an aggregate of characteristic numbers' (ibid., p. 36). Of course, as far as Cassirer is concerned, such an aggregate is determined and informed by the other elements of the structure, namely the laws and principles. Physical knowledge must not be thought of as a mere aggregate of data, since the data are mutually conditioned and interrelated. What is important is that '… we do not need to posit objects as sundered beings-in-themselves behind these determinations' (ibid.).

The overall framework, then, is the same as in the space–time case, at least insofar as it involves a shift from things-as-substances to relations as the ground of objectivity in science; or as Cassirer put it, '[w]e are concerned not so much with the existence of things as with the objective validity of relations; and all our knowledge of atoms can be led back to, and depends on, this validity' (Cassirer, 1936, p. 143). In classical mechanics objectivity rests on the spatio-temporal persistence of individual objects and here, ' "[o]bjective" denotes a being which can be recognized as the same in spite of all changes in its individual determinations, and this recognition is possible only if we posit a spatial substratum.' (ibid., p. 177). As Cassirer points out, 'The entire axiomatic system of classical mechanics is based on this presupposition.' (ibid.). As is well known, this presupposition features explicitly in Boltzmann's axioms for example and it forms the basis of the 'world-view' of classical (particle) physics in which we have individual objects possessing at all times well-defined properties and traversing well-defined spatio-temporal trajectories. It is this world-view that is apparently overturned by quantum mechanics (at least under the orthodox interpretation) and in the new situation in which we find ourselves, we cannot say that the particles unambiguously possess definite properties at all times, even beyond measurement interactions, or that they travel along well-defined trajectories. It is at this juncture that Cassirer asks a pair of crucial questions: '… what *are* these electrons whose path we can no longer follow? Is there any sense in ascribing to them a definite, strictly determined existence, which, however, is only incompletely accessible to us?' (ibid., p. 178). In answering these questions, Cassirer makes the fundamental demand of the ontic form of structural realism (Ladyman, 1998), namely that we take the 'conditions of accessibility' as 'conditions of the objects of experience'. If we do that, then '… there will no longer exist an empirical object that in principle can be designated as utterly inaccessible; and there may be classes of presumed objects which we will have to exclude from the domain of empirical existence because it is shown that with the empirical and theoretical means of knowledge at our disposal, they are not accessible or determinable' (ibid., p. 179). There are no epistemically inaccessible objects laying behind the structures which we can know.

What is an electron then? Not, Cassirer insists, an individual object (ibid., p. 180) and he cites Born's comment (from 1926) that from the perspective of quantum statistics, the particles cannot be identified as individuals at all (ibid., p. 184). Cassirer writes,

> The impossibility of delimiting different electrons from one another, and of ascribing to each of them an independent individuality, has been brought into clear light through the evolution of the modern quantum theory, and particularly through the considerations connected with the Pauli exclusion principle. (ibid., p. 184, fn. 17)[6]

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[6] And here Cassirer follows Weyl in associating the Exclusion Principle with Leibniz's Principle of Identity of Indiscernibles (see French and Krause, 2006).

Of course, this is to follow the ‘received view’ regarding the indistinguishability of quantum particles which draws the conclusion that they are non-individuals in some sense. Nevertheless, Cassirer takes it to further support the shift away from particles as substantival ‘things’. If we want to continue to talk, in everyday language, about electrons as objects – because we lack the logico-linguistic resources to do otherwise – then we can do so ‘only indirectly’, ‘… not insofar as they themselves, as individuals, are given, but so far as they are describable as “points of intersection” of certain relations’ (ibid.). And this relational conception of an object is taken straight from Kant himself: ‘All we know in matter is merely relations… but among these relations some are self-subsistent and permanent, and through these we are given a determinate object’ (Kant, *Critique of Pure Reason* B 341, CE, p. 379; in Cassirer, 1956, p. 182). Charge, understood as an intrinsic or state-independent property of particles, is just such a ‘self-subsistent and permanent relation’ but as Cassirer points out, in an acute rebuttal of the assumption made by the ‘standard’ realist, ‘… the constancy of a certain relation is not at all sufficient for the inference of a constant carrier’ (ibid.). The permanence of charge justifies our regarding the electron, say, as a ‘determinate object’, where the scare quotes indicate that the sense is that of an entity prior to reconceptualisation in structural terms, but it is does not justify what Cassirer calls the ‘substantialization and hypostasis’ of the electron in the sense of an entity which is not so reconceptualised.

Charge, like the other intrinsic properties, features in the relevant laws of physics and according to Cassirer, what we have here is a reversal of the classical relationship between the concepts of object and law (ibid., pp. 131–132): instead of beginning with a ‘definitely determined entity’ which possess certain properties and which then enters into definite relations with other entities, where these relations are expressed as laws of nature, what we now begin with are the laws which express the relations in terms of which the ‘entities’ are constituted. From the structuralist perspective, the entity ‘… constitutes no longer the self-evident starting point but the final goal and end of the considerations: the *terminus a quo* has become a *terminus ad quem*’ (ibid., p. 131). Objectivity, therefore, is determinable through law, which is prior to it (ibid., p. 176) and the boundaries of law mark the boundaries of objective knowledge (ibid., p. 132).

The significance of quantum physics for epistemology lies precisely with these considerations regarding the nature of objects. As already indicated, the ‘principle’ of causality can be retained, since it should be regarded not as a proposition pertaining to events themselves, but, rather, ‘… a stipulation concerning the means through which things and events are constituted in experience.’ (Werkmeister, 1949, p. 789). As such, the principle is not undermined by quantum mechanics; indeed, Cassirer insists, understood as a demand for strict functional dependence, the essence of causality remains untouched (op. cit., p. 188). At best the formulation of the principle must be corrected, following the articulation of the uncertainty relations. Cassirer examines the impact of the latter in the following manner: the logical form of the causality principle is that of ‘If x, then y’. Now, what can we say about this form if uncertainty has ‘crept’ into x? Logically, of course, we are not entitled to infer any uncertainty in the y and hence the statement ‘If x, then y’ is not invalid.

All that we can say is that in order for it to be useful in the quantum domain, the values of x must be 'permissible', in the sense that they can be determined by an appropriate mode of measurement. The causal relation as such is not affected, only its domain of legitimate application, and this is now further delineated by the uncertainty relations.

Moving back to our sketch of Cassirer's view of General Relativity, the retention of causality provides of course a further connection between Cassirer and Einstein. Ryckman notes that general covariance underpins Einstein's criterion of observer independent objectivity in terms of his famous and much discussed principle of separability (Ryckman, op. cit.). Put simply, this embodies the idea that spatially separated systems possess distinct states. The origins of the principle lie in field theory, where field quantities are defined at space–time points and the (mathematical) separability of the points is inherited (ontologically) by these field quantities which in turn represent properties of systems. The connection to General Relativity is provided by Schlick who claimed that only general covariance can adequately satisfy the Maxwellian requirement that causal differences between two events should not depend upon the particular spatio-temporal locations of the events (Ryckman, ibid., p. 609). This further requires a way of distinguishing causal occurrences so that they may be regarded as similar but not identical and *this* is what the principle of separability allows.

As is now well known, separability was central to the EPR objection and in one of his famous letters to Born, Einstein insisted that,

> [**u**]nless one makes this kind of assumption about the independence of the existence (the 'being-thus') of objects which are far apart from one another in space – which stems in the first place from everyday thinking – physical thinking in the familiar sense would not be possible. It is also hard to see any way of formulating and testing the laws of physics unless one makes a clear distinction of this kind. This principle has been carried to extremes in the field theory by localising the elementary objects on which it is based and which exist independently of each other, as well as the elementary laws which have been postulated for it, in the infinitely small (four-dimensional) elements of space. (Born-Einstein Letters, pp. 170–171)

As Ryckman puts it, by '… distinguishing physical systems by virtue of causal independence of measurement interactions, [separability] serves as a principle of individuation in lieu of the usual identification of physical systems by reference to a fixed background of space and time…' (ibid., p. 615). The lesson to be drawn is that Einstein's criterion of 'observer objectivity' is not the expression of a 'simple minded realism', '… but rather a presupposition for the application of causal laws in the physical description of the world.' (ibid., p. 616).

Howard has famously understood separability both in spatio-temporal terms and as a sufficient condition for the individuality of physical systems (1984). The failure of separability in quantum mechanics was then taken to imply a kind of non-individuality for quantum systems. Elsewhere (French, 1989), this move has been resisted, on the grounds that it presupposes that spatio-temporal location is the 'Principle of Individuality'. On an alternative understanding of the latter, one could accommodate the failure of separability through the introduction of Teller's 'non-supervenient' relations holding between the particles. Ryckman, however, takes Howard to have simply missed the point, since '… it is *not* possible to use the bare points of the manifold… as a means of individuating separate physical systems' (ibid., p. 617, fn. 51), because – and this is the 'central message' of general covariance – the bare manifold is not space–time. Thus the principle of separability is *not* to be understood as a form of spatio-temporal principle of individuality.

How is it to be understood then? And if, as Ryckman suggests, it does function as some kind of principle of individuation, how does this mesh with Cassirer's apparent realisation that quantum particles should not be regarded as individuals? Our suggestion is that it acts as a principle of 'pseudo-individuality' which allows us to distinguish systems – in a limited and localised way – in terms of their independent causal effects but does not give us licence to effectively import this principle beyond the observable effects and regard the systems as full-blown individual objects.[7] Citing Heisenberg, Cassirer writes, 'The process of observation cannot be simply objectified; its results cannot be turned immediately into real objects.' (1937, p. 142). The apparent failure of separability in EPR situations should then be read, not as a failure of the principle as a 'Principle of Pseudo-Individuality' but as a failure of the attempt to regard it as a Principle of (Full-Blown) Individuality and import it beyond the immediate measurement situation. In line with Ladyman's ontic structural realism (Ladyman, 1998; French and Ladyman, 2003), how this failure in turn should be understood is not in terms of the systems being non-individual objects, but in terms of their not being objects at all. In this way, structuralism as informed by Cassirer's approach, may offer a different ontological perspective on the implications of the Bell/EPR results.

6 Conclusion: Cassirer in the Twenty-First Century

In order to highlight the relevance of Cassirer's analysis for the contemporary debate on structuralism it is useful to isolate the main tenets of his view:

1. *Holism.* As we have seen Cassirer distinguishes three kinds of statements in the architecture of physics characterized by a different level of generality. Not only are the fundamental concepts of a theory implicitly defined by the mutual interplay of all the features of the theoretical framework in which they appear, but each single statement makes complete sense as such only in relation to the others.
2. *Functional Coordination.* This is the core concept in Cassirer's account. Modelled on the notion of series as employed in calculus, the notion of functional coordination grounds the abovementioned shift from objects to relations

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[7] This notion of a kind of 'pseudo-individuality' has been introduced by Toraldo di Francia (see French and Krause, op. cit.).

since it replaces the idea of an object as the bearer of the properties and the element that is constant through change, with the constancy of a rule that relates various elements always in the same way. The concept captures the sense in which the data, laws and principles relate to each other.
3. *The centrality of the notion of Law*. The laws are the features that in the theoretical set-up bring about the coordinative component. They express the pattern that we then find instantiated in the various singular cases. In this sense the principles just replicate this coordinative "move" at the more general level of the laws themselves.
4. *Neo-kantian conception of laws*. Cassirer relies on a concept of law that evolves from the kantian tradition (the Marburg school in particular). Accordingly a law consists of a logico-mathematical element expressing the synthetic constructive role of thought in determining our knowledge and of empirical perceptual elements resulting from our experience.

From this general framework two conclusions about the nature of theoretical physics follow straightforwardly. In the framework of theoretical physics:

(a) *Relations are conceptually prior to objects.*
(b) *Objectivity has nothing to do with the existence of objects independent from our thought.*

The objects of the theory emerge from the interplay of the laws and the principles of the theory itself because they express the kind of constant pattern that ties together the empirical features that in different ways we consider properties of the object or consequences of the dynamics that the theory ascribes to its objects. In this sense, a working theory "generates" its own objects, and their objectivity is grounded in the very ground of the universality of laws and principles: the universal logical validity of mathematics.

As we have shown above, Cassirer' diagnosis about the nature of the conceptual challenge represented by quantum mechanics conforms to these principles. Quantum mechanics does not question the ideal of a nature ordered according to accessible laws and principles; rather, it presents us with a profoundly different picture of these items. In particular quantum objects appear to lack individuality as a consequence of the laws and the principles of the framework. Nonetheless this framework provides us with perfectly objective knowledge of quantum phenomena. Put in different terms, any understanding of phenomena that involves the adoption of the principles of quantum mechanics will lead us to ascribe individuality to the items of the theory only in a problematic sense.

Notice that Cassirer tends to stick with the view of individuality that is traditionally endorsed by Born and the standard interpretation. Recent accounts have questioned the force of arguments against the individuality of quantum objects based on the structure of the theory (see French and Krause, 2006) which indicate that the status of individuality in quantum physics is far more problematic than it appears in the standard interpretation. Nevertheless, as these accounts are based on a thorough conceptual analysis of the fundamental principles of the theory, their implication for any interpretation can be seen as independently refining the results of

Cassirer's reading rather than questioning his basic framework. Most importantly, these more recent accounts can be seen as agreeing with conclusion a) at least when it comes to the quantum domain.

This might sound like an attempt to suggest that that the debate on structuralism should move in a Kantian direction. It is our contention that the peculiarities of Cassirer's picture and the similarities between his account and the terms of the contemporary debate are such that Neo-Kantianism could be viewed as a relevant option. However, Cassirer's account in our view presents elements of interest in a more general sense for the structuralist agenda.

First of all, Cassirer's conclusions about quantum mechanics are not a consequence of his Neo-Kantianism. All that is needed to conclude that quantum objects are not individuals are assumptions 1–3. The only remaining commitment is to a notion of law compatible with idea of functional coordination, but in Cassirer's work there are no elements that lead to the conclusion that the Neo-Kantian notion of law is the only possible candidate although it is certainly a very natural one, conceptually speaking.

This allows us to look at Cassirer's other tenets from a different perspective. In our view, if the relational notions of laws and principles can be detached from the neo-kantian background, there are interesting consequences for the idea of objectivity. From the transcendental idealist standpoint this notion is profoundly linked with the universality of laws and results in particular from the logical nature of mathematics. The objects that a naïve realist reading of the theory would like to postulate as independent from the theory itself are simply independent from the mind setup of a particular human as such. They are not beyond the representation provided by the theory. Rather the representation itself as a piece of objective scientific work is *invariant for representation users* as it would be if they were "out there".

The adoption of a different understanding of the laws may lead to a view of objectivity closer to the classical realist ideal of a mind independent reality to which knowledge gives us access but linked to the fundamentally relational nature of our knowledge.

More generally we take all of the above points to suggest that a crucial aspect of Cassirer's lesson has to do with the role that an account of the laws of nature should play in the debate on structuralism.

References

Cassirer, E (1907) Kant und die moderne Mathematik *Kantstudien*, 12, 1–49.

Cassirer, E. (1944) The Concept of Group and the Theory of Perception, *Philosophy and Phenomenological Research* 5, 1–36.

Cassirer, E. (1913) "Erkenntnistheorie nebst der Grenzfragen der Logik" *Jahrbücher der Philosophoie.*

Cassirer, E. (1953) *Substance and Function & Einstein's Theory of Relativity* Trans. by W.C. Swabey and M.C. Swabey.

Cassirer, E. (1956) *Determinism and Indeterminism in Modern Physics* Trans. by O.T. Benfey, Yale University Press, New Haven.

Cassirer, E. (1957) *Das Erkenntnisprobleme in der Philosophie und Wissenschaft der neueren* Zeit. Stuttgart: Kohlhammer.

French (1989) 'Identity and Individuality in Classical and Quantum Physcis', Australasian Journal of Philosophy 67, 432–446.

French, S. (2003) Scribbling on the Blank Sheet: Eddington's Structuralist Conception of Objects, *Studies in History and Philosophy of Modern Physics* 34, 227–259.

French, S., & Krause, D., (2006) *Identity in Physics: A Historical, Philosophical, Formal Analysis*, Oxford University Press, New York.

French, S., & Ladyman, J. (2003), Remodelling Structural Realism: Quantum Physics and the Metaphysics of Structure, and The Dissolution of Objects: A Reply to Cao, *Synthese* 136, 31–56 and 73–77.

Friedman, M., (1992) Causal Laws and the Foundations of Natural Science. In P. Guyer (ed.) *The Cambridge to Kant*, Cambridge University Press, Cambridge.

Friedman, M., (2000) *A Parting of the Ways. Carnap, Cassirer, and Heidegger* Open Court, Chicago, IL.

Ihmig, K-N. (1999) Ernst Cassirer and the Structural Conception of Objects in Modern Science: The Importance of the "Erlanger Program" *Science in Context* 12, 4, 513–529.

Itzkoff, S.W. (1997) *Ernst Cassirer: Scientific Knowledge and the Concept of Man*, University of Notre Dame Press, Notre Dame, IN.

Kant, I. (1998) *Critique of Pure Reason*. Trans. by P. Guyer and A.W. Wood, Cambridge University Press, Cambridge.

Ladyman, J. (1998), What is Structural Realism?, *Studies in History and Philosophy of Science* 29, 409–424.

Rudolph, E. (1994) Substance as Function: Ernst Cassirer's Interpretation of Leibniz as criticism of Kant. In Rudolph, E., Stamatescu I.O., (Eds) *Philosophy, Mathematics and Modern Physics. A Dialogue*, Springer, Heidelberg.

Russell, B. (1927) *The Analysis of Matter*, Kegan Paul, Trench, Trubner & Co, London.

Ryckman, T.A. (1991) *Conditio sine qua non? Zuordnung* in the early epistemologies of Cassirer and Schlick *Synthese* 88, 57–95.

Ryckman, T.A. (1999) Einstein, Cassirer, and General Covariance – Then and Now *Science in Context* 12, 4, 585–619.

Saunders, S.W. (1993), To What Physics Corresponds. In French, S., and Kaminga, H. (eds.), *Correspondence, Invariance, and Heuristics; Essays in Honour of Heinz Post*, Kluwer, Dordrecht, pp. 295–326.

Werkmeister, W.H., 1949. [1973]. Cassirer's advance beyond neo-Kantianism. In: Schilpp, P.A., The Philosophy of Ernst Cassirer. Open Court, La Salle, IL, pp.757–798.

Moritz Schlick: Between Synthetic *A Priori* Judgment and Conventionalism[1]

Christian Bonnet and Ronan de Calan

Abstract The present article aims at tracing Moritz Schlick's theoretical route from 1915 to 1936 – the year he was assassinated. The authors describe this route as Schlick's attempt at successively evading what one could define as two flaws in modern philosophy – the Charybdis of Kantian epistemology and the Scylla of radical conventionalism.

Such an original and daring guideline also deviates from all great epistemological philosophies dating from the beginning of the century with which the Vienna Circle's founder engaged in fruitful dialogue – among which Neokantianism and phenomenology, as well as Hilbertian Axiomatic or Poincaré and Duhem's doctrines.

In Otto Neurath's opinion, what made Austrian philosophy, and more particularly the Vienna Circle, so singular, was that Austria "had been spared any Kantian interlude.[2]" However relevant this analysis may be in certain respects, it can scarcely pertain to Moritz Schlick who insisted on confronting himself with Kant, underlining the convergent issues between his own doctrine and transcendental philosophy not only in his early texts, which commentators have regularly labelled as Kantian or neo-Kantian,[3] but also in his texts dating from the 1930s.

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[1] Translated into English by Estelle Folest (Université Paris III).

[2] Cf. *Le développement du Cercle de Vienne et l'avenir de l'empirisme logique*, Actualités scientifiques et industrielles, Paris, Hermann, 1936, p. 8.

[3] Alberto Coffa underlines that in Schlick's early works, before he arrived in Vienna, "we are back in the world of Kantian questions and semi-Kantian answers" (*The Semantic Tradition from Kant to Carnap. To the Vienna Station*, Cambridge, Cambridge University Press, 1991, p. 171). Michael Friedman also argues that "Schlick was not a positivist or strict empiricist in 1918, but a neo-Kantian or 'critical' realist" (*Reconsidering Logical Positivism*, Cambridge, Cambridge University Press, 1999, p. 20).

M. Bitbol et al. (eds.), *Constituting Objectivity*,

Indeed, in 1932 in *Form and Content*, if Schlick objected – as all the logical empiricists[4] did – to the Kantian idea according to which synthetic *a priori* judgments existed and allowed us to grasp reality without relying on experience at all, he nevertheless gave credit to the concept of *a priori*. More precisely, Schlick agreed with Kant that only the *a priori* was endowed with an absolute validity, anywhere at any time, for he "saw clearly, of course, that all analytic propositions must be *a priori*" and that "the validity of a tautology is quite independent of experience, as it rests on nothing but the definitions of the concepts occurring in it.[5]" To Schlick's mind, Kant's relevance was therefore twofold: on the one hand, the latter was right "to insist that the term *a priori* must not be understood psychologically, but logically: that is to say, a judgment *a priori* is not one that is generated in the mind without any previous experience […], but it is one whose *truth* is not based on experience; it would not have come into existence without experience, but does not derive its validity from it.[6]" As a matter of fact, though Schlick had chosen to keep aloof from his early Kantian background, after having been tempted, quite like Helmoltz,[7] to regard the *a priori* as a psycho-physiological concept, in 1932, rather paradoxically, he seemed to have adopted Kant's ideas as regards the purely transcendental conception of the *a priori*. On the other hand, Schlick gave Kant credit for having stated "that if a proposition is valid *a priori* it must owe its validity to the *form* of knowledge, not to its material, because our understanding cannot possibly know beforehand what material will present itself to the mind in experience, while it might very well impress its own form on any material.[8]" In other words, the *a priori* is strictly formal: "Tautologies (or analytic judgements) are the only propositions *a priori*, they have absolute validity, but they owe it to their own form, not to a correspondence to facts, they tell us nothing about the world, they represent empty structures.[9]"

Despite Kant's acknowledgment of the purely formal nature of the *a priori* and his positing of the truly formal character of synthetic *a priori* propositions – which, he believed, were evinced in our cognitive activity – as well as of their being devoid of any ambition, in principle, to provide information about the content or about the matter of reality, from Schlick's point of view, Kantian synthetic *a priori* propositions *do* have an intrinsic content: "Space, time and the categories are spoken of as 'pure forms' in Kant's philosophy, but they are used as if they were a strange mixture of form and content. There is no such mixture, of course, and as soon as one realizes that only the Logical deserves to be called pure Form, one will easily get rid of the confusion which seems to give some plausibility to Kant's explanation of the supposed possibility of synthetic judgements *a priori*.[10]"

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[4] This objection, shared by all logician empiricists, is even presented in the 1929 Manifest as "*the* fundamental thesis of modern empiricism '*die Grundthese des modernen Empirismus*' ".

[5] *Form and Content*: Schlick, *Philosophical Papers*, edited by Henk L. Mulder and Barbara F.B. van de Velde-Schlick, vol. 2, Dordrecht, The Netherlands, Reidel, 1979, p. 343.

[6] *Form and Content: op. cit.*, p. 342.

[7] See Coffa, *The Semantic Tradition*, p. 171–172.

[8] *Form and Content: op. cit.*, p. 351.

[9] *Ibid.*

[10] *Ibid.*

If Schlick was at odds with Kant over the existence of a synthetic *a priori* knowledge, he was nevertheless more Kantian than Kant himself as he literally proposed to hollow the Kantian notion of *a priori* out of all its material content. In fact, as Schlick saw it, not only did the very idea of an *a priori* intuition unveil the hybridism which was at the heart of the Kantian notion of form, but it was also meaningless insofar as the *a priori* could be nothing but either intellectual or conceptual. So, if Kant was right in stating that knowledge was purely conceptual, Schlick was convinced that some consequences of this idea eluded Kant since the latter considered intuition as a reliable element leading to knowledge, as can be construed from his conception of geometry.

Yet, at the time, the latest scientific discoveries such as the axiomatisation of geometry or the theory of relativity tended to suggest against Kantism that the 'transcendental' tenet on which both geometric and physical knowledge relied might very well be a strictly conceptual *a priori*. If Kant believed in such baffling things as synthetic *a priori* judgments, it is undoubtedly because "among both the definitions and the empirical propositions of the exact sciences we find statements that are deceptively similar to synthetic judgments *a priori*. In the class of definitions, which by their very nature possess a validity independent of experience and thus are *a priori*, there are a great many conventions that, viewed superficially, seem not to be derivable from definitions and hence to be synthetic.[11]"

1 Schlick and Kantism: Finding One's Philosophical Bearing

Schlick altered the various philosophical courses he chartered *vis-à-vis* Kantism. If we follow Alberto Coffa,[12] Schlick was one of the first philosophers who drew a lesson from the theory of relativity. His first reaction, shared by many others, was to examine the theory of relativity in a light which still owed much to Kantism. Broadly speaking, as Michael Friedman puts it, if Schlick and most of the logical positivists kept discarding the Kantian conception of science, and more particularly Kant's theory of synthetic *a priori* judgments, they were however not won over to the radically opposed empirical conception of geometry and they even thought, at first, that "although Kant was wrong to think that Euclidean Geometry is synthetic *a priori* and we can in fact use non-Euclidean geometry instead in physical theory, the question whether space is Euclidean or non-Euclidean is nonetheless not a straightforwardly empirical question. Indeed, the logical positivists here agree with Kant in rather maintaining a sharp distinction between the underlying spatiotemporal framework of physical theory, on the one hand, and the empirical laws then formulated within this framework, on the other.[13]"

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[11] *General Theory of Knowledge*, translated by Albert E. Blumberg, La Salle, Ill., Open Court, 1974, p. 74.

[12] Cf. *The Semantic Tradition*, p. 196.

[13] Michael Friedman, *Reconsidering Logical Positivism*, p. 60.

In 1915, in *The Philosophical Significance of the Principle of Relativity*, Schlick noticed that though restricted theory of relativity was not contradictory in itself, it "might yet – in Kant's terms – be in conflict with our *a priori* intuition, and would then have no validity for the objective world, because the latter is subject to the laws of our intuition.[14]" And once the transcendental guarantee of a continuity between the intuitive space and the physical one has been forlorn – that is, once pure intuition has been dismissed – then the compatibility between the two spaces can no longer be taken for granted: "the theory is perfectly compatible with our immediate awareness of time, for the simple reason that the latter tells us nothing whatever about those properties of time that are dealt with in relativity theory. The time of our intuition is psychological time – a qualitative, unmeasurable thing – whereas Einstein's theory deals with the *measurement* of time.[15]"

In 1918, in the *General Theory of Knowledge*, Schlick went on asserting that "there is only one form in which apodictic knowledge of reality is still discussable, namely, the one discovered by Kant.[16]" Still, "it is clear that the Kantian solution, even if correct, would not signify a resounding triumph for rationalism. For the *a priori* knowledge that his theory allows us has not concrete, material meaning in any individual case either in scientific research or in our daily life. Propositions that express merely the forms in which (according to Kant) all of our experience must appear are quite general.[17]" In other words, these *a priori* principles are so broad that they do not intervene directly in the physicist's work. Their apodicticity does not allow them to have any scientific bearing as it is unable to modify the nature of any scientific proposition. "For example, we might assert with apodictic certainty that each single real event has a cause. But in no case would we be in a position to decide *a priori* which cause belongs to which event.[18]"

This concept of *a priori* can have different meanings. In a letter to Reichenbach written back in 1920,[19] Schlick pointed that Kant had identified the most universal laws of nature with the object-constituting principles. As regards Kantian criticism, Schlick considered this identification so fundamental that "one cannot undermine it without distancing oneself from Kantian philosophy as a whole". Of course, Schlick granted Kant that knowledge had to be based on object-constituting principles and he acknowledged that the Kantian distinction between constituting principles and empirical laws was reliable. Yet, he definitely dismissed the solution which likened those principles to synthetic *a priori* judgments.

In 1921, Schlick's "Critical or Empiricist Interpretation of Modern Physics" delved into Cassirer's *Einstein's Theory of Relativity*.[20] Schlick's disquisition was a major

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[14] *Philosophical Papers*, vol. 1, p. 162.

[15] *Ibid*.

[16] *General Theory of Knowledge*, p. 344.

[17] *Op. cit*., p. 346.

[18] *Ibid*.

[19] Schlick to Reichenbach, November 26, 1920, Archives of Scientific Philosophy, University of Pittsburgh, HR-015-63-22.

[20] *Philosophical Papers*, Dordrecht, The Netherlands, Reidel, 1978, vol. 1, p. 322–334.

step in his assessment of the Kantian conception of science and of its ability to account for the theory of relativity. Though Kant could not be blamed for being unaware not only of non-Euclidian geometry but also of the theory of relativity, it remained that from then on, the theoretical framework of his epistemology could only be upheld provided that the bases of the theory of relativity were synthetic propositions universally valid for all kinds of experience. And it seems that such synthetic propositions can be found nowhere, not even in the neo-Kantian Cassirer's works.

The theory of coincidences on which both Schlick and Cassirer relied to think out relativity – or, more precisely, the idea according to which physics as a whole has to be seen as a set of laws which accounts for spatiotemporal coincidences between two objects or more – can play the role of a methodological abstraction, but it can absolutely not play that of a synthetic *a priori* principle in the Kantian meaning. Indeed, the theory of coincidences needs not propound any precise determination of the form of space and time, even as far as the core conditions of the possibilities of experience are concerned. As he equated "pure intuition, which is a methodological presupposition", with the concept of coincidence, Cassirer emptied the notion of intuition from its content (that is, from the *a priori* forms on which geometrical axioms are based), making it pointless. Yet, according to Schlick, this new method of objectification wanted new procedures and Cassirer could hardly implement any: "Cassirer declares Kant's pure intuition to be a 'specific method of objectification', which indeed it is *as well*, but its nature is not exhausted by this. Kant certainly wanted to purge it of everything psychological – but I shall never be able to persuade myself that he succeeded. For no such success is possible, without employing the sole method which permits us to separate the purely conceptual elements of geometry from the psychologically intuitive, namely, the method of implicit definition, first framed in modern mathematics.[21]"

If the principles constitutive of the *Naturwissenschaft* cannot be defined as synthetic *a priori* judgments, then what are they? "There are two other possibilities", Schlick said,[22] "they can be considered either as hypotheses or as conventions". Does this allow us to view Schlick as a conventionalist?

2 Conventionalism?

The term *conventionalism* first needs to be defined, as it was used to qualify so many different doctrines that it ended up giving rise to much confusion. Indeed, resorting to conventions does not necessarily lead to conventionalism and therefore the status of these conventions as well as their field of application also have to be further delineated: are these conventions thoroughly arbitrary stipulations? To what constraints are they subjected? Can they be applied to all sciences or to some very restricted fields only?

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[21] *Op. cit.*, p. 331.

[22] Schlick to Reichenbach, November 26, 1920.

The reason why the doctrine of conventionalism remains hard to circumscribe seems to spring mostly from a failure to understand its paternity. This confusion is quite obvious if one peruses the section devoted to the foundation of physics in the Vienna Circle manifesto: not only are the distinctions between the very singular doctrines of Mach, Poincaré and Duhem blurred, but their doctrines are also fused into a single one. "Originally, the Vienna Circle's strongest interest was in the method of empirical science. Inspired by the ideas of Mach, Poincaré, and Duhem, the problems of mastering reality through scientific systems, especially through systems of hypotheses and axioms, were discussed. A system of axioms, cut loose from all empirical application, can at first be regarded as a system of implicit definitions; that is to say, the concepts that appear in the axioms are fixed, or as it were defined, not from their content but only from their mutual relations through the axioms. Such a system of axioms attains a meaning for reality only by the addition of further definitions, namely the 'coordinating definitions', which state what objects of reality are to be regarded as members of the system of axioms. The development of empirical science, which is to represent reality by means of as uniform and simple a net of concepts and judgments as possible, can now proceed in one of two ways, as history shows. The changes imposed by new experience can be made either in the axioms or in the coordinating definitions. Here we touch the problem of conventions, particularly treated by Poincaré.[23]"

The interest of this exemplary text is twofold: on the one hand, it introduces the main theses of Schlick's *General Theory of Knowledge*; on the other hand, it unveils the three highly questionable influences under which Schlick seemed to be. Two conclusions should be drawn at this point: first, Schlick obviously contributed to lay the theoretical foundations that would lead to what he would himself call an extreme form of conventionalism – with which he would however not side; secondly, both Schlick's extreme conventionalism and his realism were fostered by the latter's paradoxical and even ill-construed reading of the French philosophers' works. As regards this legacy, the link Schlick operated between Duhem and Mach can be all the more taken for granted as Duhem himself commented on Mach's *Mechanics* to praise and approve of it.[24] On the contrary, even by any stretch of the imagination, comparing Duhem and Poincaré seems rather far-fetched given their radical disagreement as far as the tenets on which the foundation of physical theory is based are concerned, that is to say the nature of the mathematical reasoning, the role of definitions, the existence of a continuity or not between the mathematical and physical methods, and the status of hypotheses in physics.

When he showed that the reasoning by recurrence might be used to prove the rules of algebraic calculus, Poincaré also held that it equated the exact type of the synthetic

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[23] «The Scientific Conception of the World: The Vienna Circle», *in* Otto Neurath, *Empiricism and Sociology*, edited by Marie Neurath and Robert S. Cohen, Dordrecht, The Netherlands, Reidel, 1973, p. 311–312.

[24] «Analyse de l'ouvrage de Ernst Mach, *La Mécanique», in* P. Duhem, *L'évolution de la mécanique* (1903), Vrin, Mathesis, 1992.

a priori judgment. He thus refused to contemplate the mathematical reasoning as a deductive one. Yet, he acknowledged that geometrical axioms could be granted the status of conventions, or "disguised definitions", given the fact that these axioms were the result of the free adoption of a group of transformation through which properties that are approximately induced from our experience were redefined in the field of mathematical idealities. He thus argued for a kind of mathematisation of nature which allowed a continuity as well as a hierarchy linking arithmetic to geometry, geometry to mechanics, and mechanics to physics. Eventually, in the field of physics, he discovered a set of hypotheses which partook neither of the conventional nature of geometrical postulates nor of the principles of mechanics.

As for Duhem, he judged that "the reasoning by recurrence is nothing but a form of deductive reasoning among others[25]" – in other words, that thinkers demonstrations are relying on syllogisms. In fact, as he was restrained by his very academic view of mathematics, Duhem never investigated the conventional status of either arithmetic or geometrical formulations. Moreover, he never acknowledged any continuity between the nature of the mathematical reasoning and that of the physical one. Added to that, he insisted that, just like Poincaré's principles of mechanics, many of the hypotheses on which physical theory is founded have no experimental basis at all, that "there can be no question of either confirming or contradicting them by experiment,[26]" that these hypotheses are nothing but conventions.

The conclusion that should be drawn at this point is that far from reading Poincaré through Duhem's eyes – which was what most logician empiricists did, considering that this was the canonical reading – Schlick was unfaithful to both thinkers. So, we should now enquire into the status of the implicit definitions as well as into the way they may be linked to Poincaré's "disguised definitions" or conventions, and further into the regulating role of the "coordinating definitions" the realism of which seems incompatible with Duhem's theory. Eventually, we will see what may set Schlick apart from extreme conventionalists.

3 Implicit Definitions as Conventions

The concept of implicit definition implies a form of reasoning which is utterly dissimilar to the process of mathematisation as Poincaré saw it – and against which the latter often argued – namely the Hilbertian axiomatic. In the *Foundations of Geometry*, Hilbert considered that the meaning of expressions such as 'point', 'straight 'line', 'to be located on …' and 'to be located between … and …' is neither presupposed by the axioms of geometry nor explicated by separated definitions. Therefore, the only reliable definition for these terms is precisely the one that the axioms in which they appear give them. The function of these axioms is to establish relations between

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[25] Pierre Duhem, «la nature du raisonnement mathématique», *Revue de Philosophie*, tome 21, p. 533.

[26] Pierre Duhem, *The Aim and Structure of Physical Theory* (1906/1914), translated by Philip P. Wiener, Princeton University Press, 1991, p. 215.

concepts. They define a structure which is utterly independent from any experience or intuition and which can be studied in itself so that the geometrical reasoning needs no longer be founded on, or justified by, any specific intuition. In the *General Theory of Knowledge*, Schlick acknowledged that the Hilbertian notion of 'implicit definition' had a broad epistemological bearing and he applied it to *Naturwissenschaft*. In this field, implicit definitions allowed him to define the system of concepts without relying on intuition at all: the concepts are not determined by any intuitive or empirical contents but only by the relations they share in the system of signs in which they appear. As a result, the realms of concept and intuition are not linked any more as they are in the Kantian system and "the bridges between them are down.[27]"

What is basically characteristic of an axiomatic system is the fact that its axioms implicitly define their primitive terms – which is not the case at all as regards Poincaré's 'disguised definitions' since they are exact mathematical conventional reformulations of 'approximate' intuitive links, as has been gathered by now. What needs to be grasped at this point is that the way the different propositions are linked to one another in an axiomatic is strictly deductive, that is, analytic. Once any series of implicit definitions has been circumscribed within a system by axioms, all propositions can be deduced logically. Schlick applied this idea to the astronomical field: "For example, astronomy can report purely descriptively the positions of the planets at various times and thus describe events in the solar system by means of an immense number of historical judgments. But it can also designate the planets by means of the concepts of bodies that move in accordance with certain equations, which amounts to an implicit definition. From these basic equations of astronomy we can then obtain purely deductively all the desired assertions about the past and future locations of the bodies that make up the solar system.[28]"

Schlick and Poincaré thus diverged on one precise point, namely the kind of reasoning at work as far as astronomy and, more generally, physics are concerned. If one take sides with Poincaré's perspective of mathematisation, then astronomical propositions do not fall within the province of discursive thought but within that of a certain type of application of reasoning by recurrence to phenomena (so, within that of a theory of measurement) to which physical hypotheses must be added, none of them having anything to do with the strictly analytic model that may be found in syllogisms. In this case, astronomical and geometrical formulations derive neither from postulates nor from logical deductions but they are a matter for *calculus*.

As for Schlick, he saw mathematical concepts (combinatorial operations) as discursive concepts (subsumption operations), which Poincaré had explicitly opposed. The former denied neither that mathematical concepts worked according to the same logic as generic concepts, nor that geometrical and astronomical propositions, as they are mathematised or arithmetised, were analytic propositions. Thus, on these points, Schlick's conclusions were not far from Duhem's except that they relied on a modern conception of mathematics to which the French philosopher owed nothing.

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[27] *General Theory of Knowledge*, p. 38.

[28] *General Theory of Knowledge*, p. 70.

4 Realism-Compatible Hypotheses

This axiomatic construction of knowledge, which Schlick seemed then to be promoting, might appear as a form of extreme conventionalism, be it not counterbalanced by an utterly realistic theory of meaning. The latter is characterised by the affixing of "coordinating definitions" to implicit definitions, which gives an axiomatic its meaning. The link between the concept and reality is seen as a link between a sign and what it refers to and knowledge, just like truth, is conceived of as the 'univocal coordination' (*eindeutige Zuordnung*) of concepts with reality, that is, as a fixed correspondence between the signs and the objects or the state of affairs to which they refer. Starting from these premises, Schlick discriminated between two kinds of judgments: on the one hand, those which issue from arbitrary decisions and introduce new signs through basic stipulations; on the other hand, those which are formed by concepts that have already been defined and which create new relations with one another: "We do not need to learn separately which fact is designated by a particular judgment; we can tell this from the judgment itself. A cognitive judgment is a *new* combination made up exclusively of *old* concepts. […] Only the primitive concepts and judgments – those to which knowledge reduces all the others – depend on conventions and have to be learned as arbitrary signs.[29]" This theory of univocal coordination led Schlick to question the alleged isomorphism of both the conceptual system and the world of experience: "Obviously, to suppose that the world is intelligible is to assume the existence of a system of implicit definitions that corresponds exactly to the system of empirical judgments. And our knowledge of reality would be better off if we knew that concepts always exist which are generated by implicit definitions and which guarantee a strictly unambiguous designation of the world of fact. But on this point we have already had to adopt a skeptical attitude, and we shall not go beyond it in the course of our study. Thus the claim that a particular conceptual system provides perfect knowledge in the sense described – or even the claim that such system exists – cannot itself be proved to be a true judgment. Rather, it is an *hypothesis*, and for precisely this reason every judgment about real facts that is neither a definition nor a purely descriptive judgments bears the character of an hypothesis.[30]"

When he decided that physical theories could be equated with 'hypotheses' and underlined their inability to be validated by experience, Schlick seemed to follow Duhem's holistic doctrine. Yet, his realism of principle – which is based on the introduction of coordinating definitions – is actually utterly impervious to the processes at work in *The Aim and Structure of Physical Theory*. In effect, this theory of definitions supposes a monadic or atomic correspondence as regards primary concepts and world properties, which Duhem's doctrine of 'theoretical fact', as a symbolic construction, wholly proscribes.

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[29] *General Theory of Knowledge*, p. 67.

[30] *General Theory of Knowledge*, § 11, p. 70–71.

This same realism of principle gave birth to a later argument with Eddington's and Carnap's views of extreme conventionalism. During the First International Congress for the Unity of Science in Paris in 1935, Schlick made a presentation – which was symptomatically called "Are the laws of nature conventions?" – in which he made himself clear on a point that had so far remained much ambiguous in his works. Indeed, he definitely opposed the role both theoreticians ascribed to the laws of nature, that is to say the role of Eddington's 'implicit definitions' and that of Carnap's 'syntactic rules'. According to Schlick, both interpretations were wrong because they had been induced by major mistakes as regards logic: "It is due to the formulation of the law – usually in the form of an equation – as it is written on paper, but without enough consideration given to the definitional explications through which the expression attains its meaning and which are usually not sufficiently explicitly or completely formulated.[31]" In other words, both Eddington and Carnap misjudged the determining role of the coordinating definitions which give the laws of nature an empirical content and allow the law of conservation of energy as well as the law of inertia, among others, to express facts and not conventions: "The energy principle, for example, is generally considered to express that 'objective' order of facts which makes it impossible to produce work out of nothing – an impossibility which is continually impressed upon us in our daily experience and which is certainly quite independent of the manner in which we care to formulate it.[32]"

The specificity of natural sciences, which Eddington as well as Carnap failed to see, is that they cannot bypass the meaning of their own terms, which is usually allowed inside an axiomatic that thinks of signs only as far as their relations to one another are concerned: "The proper content of a natural law may be seen in the fact that to certain grammatical rules (for instance, of a geometry) some quite definite propositions correspond as true descriptions of reality. And this fact is completely invariant with respect to any arbitrary changes in notation.[33]" The invariance of natural law thus has to be taken into account not as regards experience but as regards conceptual systems as well as variable, interchangeable axiomatics through which science is built.

Although Schlick's axiomatic doctrine tends to converge on conventionalism, his realism consisted in maintaining a world of facts or intuitions as bases for the world of knowledge or for concepts, the latter ones being contemplated as an ideal superstructure that has an intrinsic formal autonomy. Because of his hypothesis according to which coordinating definitions may be a link between these two worlds, Schlick closed the door on conventionalism without returning to a metaphysical realism.

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[31] «Are Natural Law Conventions?», *in* Schlick, *Philosophical Papers*, vol. 2, p. 440.

[32] Schlick, *op. cit.*, p. 443.

[33] Schlick, *ibid.*

Carnap’s Relativised A Priori and Ontology

Paolo Parrini

Abstract The paper reconstructs Carnap’s epistemological and ontological ideas stressing the link between these ideas and the most general tenets of Logical Empiricism (negation of the Kantian theory of synthetic a priori judgments, linguistic theory of the a priori, influence of Poincaré’s conventionalism, principle of verification, refusal of metaphysical absolutism). From this point of view it also discusses both the Carnap/Quine debate on analyticity and ontology and the difference between Carnap and the ‘young’ Reichenbach on the nature of the relativised a priori.

1 General Remarks

Carnap can be seen as the philosopher who made the most significant contribution to the development of the Neo-empiricistic theory of the *relativised a priori*, by systematising the standard *linguistic* version of this conception. Among the essential components of this theory, we find the thesis of the linguistic foundation of logical and mathematical truths and the conviction that it is possible to trace a distinction between analytic statements (coinciding with the a priori ones) and synthetic statements (coinciding with the a posteriori ones); in other words, between sentences whose truth value will depend only on the rules of the language they belong to and sentences whose truth value will depend on language and experience.

Carnap’s work is particularly relevant for two reasons. Firstly, he had more time than the other Logical Empiricists (ranging from Hahn to Frank, from Schlick to Reichenbach) to develop his views. He could therefore take epistemological holism (already *explicitly* considered in the *Logische Syntax der Sprache* [see Parrini, 2001/04]), as well as Quine’s criticism of the analytic/synthetic distinction into consideration. Secondly, Carnap tried to clearly define the relativised a priori within a general theory of formal languages, which was to become the realisation

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M. Bitbol et al. (eds.), *Constituting Objectivity*,

of the *Wissenschaftslogik* and 'scientific philosophy'. In the *Logische Syntax der Sprache* (1934), Carnap tried to formulate this general theory in conventionalistic-syntactic terms. Later on, he introduced the semantic-extensional formulations of *Introduction to Semantics* (1942), then the semantic-intensional characterizations of *Meaning and Necessity* (1947), and finally the semantic-pragmatic explications he first evoked in his answers to Church, Quine and Chisholm in the first half of the 1950s (Carnap, 1956, Supplement B, C, D, E, 1963, pp. 919–921).

Today, we tend to see the strong attention given to the logical-formal aspect of the problem as a weakness of Carnap's approach. The trend towards logical formalism leads him to suppose that the analytic/synthetic distinction is purely a formal or logical distinction, and Quine showed that logically speaking all sentences of a given formal system are of equal value. Thus «Carnap's attempt further to characterize» some components of the system as semantical rules, meaning postulates or analytic sentences «ultimately amounts to nothing more than an otherwise arbitrary label» (Friedman, 2001, p. 49; Parrini, 2001/04, § 5).

Moreover, in the last few decades it has emerged that the Carnap/Quine debate cannot be exhausted in the question regarding the possibility of providing a purely formal clarification of the theses and distinctions evoked above. Behind such debate, there are also epistemological problems linked to the ideas that, according to the new historical-critical perspectives, should be considered as the main inspirational motives of Logical Empiricism. In order to better highlight this, I will illustrate Carnap's position whilst putting its logical-formal aspects to one side and instead link it to the general epistemological and ontological issues which those aspects are meant to rigorously express.

2 Carnap's Relationship with Neo-empiricistic Epistemology

Carnap essentially adheres to the three main epistemological theses that characterise Neo-empiricists' conception of scientific knowledge and the theory/experience relationship.

2.1 The Negation of Kant's Theory of Synthetic a Priori Judgments

Carnap accepts the essential nucleus of the Neo-empiricistic position on the a priori, the nucleus already individuated by Schlick in the *Allgemeine Erkenntnislehre*: «There are no synthetic judgments *a priori*»: «Apodictic truths about reality go beyond the power of the human faculty of cognition and are not accessible to it» (Schlick, 1918/25, p. 384). This negation of the synthetic a priori – also adopted by Einstein in the famous 1921 essay "Geometrie und Erfahrung" – is taken up by Carnap who adds: «If the whole of Empiricism is to be compressed into a nutshell, this is one way of doing it» (Carnap, 1966/74, p. 180). From the negation of the synthetic a priori he then derives the identification of the a priori with the analytic, and the a posteriori with the synthetic.

For Logical Empiricists, saying that synthetic a priori judgments do not exist is tantamount to saying – in line with Hume – that there is no way we can guarantee a priori the possibility of the 'cognitive synthesis'. As it emerges from the controversy on the philosophical meaning of the theory of relativity between Schlick and Reichenbach, on one side, and Cassirer on the other, the possibility of coordinating concepts to experience – a possibility that is the very condition of knowledge – depends on contingent reasons that cannot be founded a priori on arguments similar to Kant's transcendental deduction of categories (see Parrini, 1999/2003, pp. 352–356).

2.2 The Refusal of Radical Empiricism

For Carnap and the other Logical Empiricists, the rejection of apodictically certain synthetic judgments must not be pushed as far as to negate the weaker idea that the human mind «can describe natural phenomena only by using a certain pattern, certain forms of thinking that are produced by the observing mind and are not provided by the observed physical objects» (Frank, 1941/49, p. 7).

Similarly to the members of the original Vienna group (see Frank, 1941/49, p. 1 ff.), Schlick and Reichenbach started off by distancing themselves from radical forms of empiricism. Schlick criticised Mach's sensualism (Schlick, 1921, p. 324), and Reichenbach refused the «naïve» empiricist conception which, neglecting «the problems of conceptualization» (Reichenbach, 1922, p. 37), characterized «all scientific statements indifferently by the notion "derived from experience"» (Reichenbach, 1920, p. 93). In his early works, Carnap had the same approach. In 1921, he wrote an essay on space influenced by Kant's work and Husserl's ideas and in the 1923 essay "Über die Aufgabe der Physik", he stated that «pure empiricism has lost its dominion» since Poincaré and Dingler showed that «the construction of physics» cannot «be sustained exclusively on experiments results»: it must apply «also principles that are not experimental (*nichterfahrungsmässige Grundsätze; Festsetzungen*)» (Carnap, 1923, p. 90).

Admitting that there are peculiar *Festsetzungen* that cannot be traced back to experience is even present in Carnap's work which is closest to reductionism: *Der logische Aufbau der Welt* (1928). Putting aside some unresolved questions linked to the purely structural notion of objectivity proposed in this work, Carnap undoubtedly undertook his project by presupposing some constitutive rules for attributing qualities to the cosmic point-instants and constructing physical objects. As Quine summarises in "Two Dogmas of Empiricism" (1951), these rules are aimed at maximising some characteristics pertaining to simplicity, compactness, elegance, etc. «in such a way as to achieve the laziest world compatible with our experience». Their application is of a holistic nature in the sense that the judgments on the reality or unreality of something (for instance, of brick houses in Elm Street) depend on «considerations of equilibrium affecting the field as a whole». It is no coincidence that Quine himself recognised that his «Empiricism without the Dogmas» has its roots not only in Duhem's epistemological holism, but also in the «doctrine of the physical world in the *Aufbau*» (Quine, 1953/61, pp. 40–43; see Carnap's example, in Carnap, 1928, p. 274).

Later on, after the discussion on the protocols and the publication of "Testability and Meaning" (1936), which heralded the beginning of the liberalisation process of empiricism, Carnap abandoned the definitional method for the introduction of concepts that he had *explicitly* used in the *Aufbau* and instead adopted the partial interpretation method based on postulates, correspondence rules and (bilateral) reduction sentences. In the Preface to the second edition of the *Aufbau* (1961), though, he underlines that, without being aware of it, he had already *implicitly* abandoned the definitional method in treating the construction of the physical world in the first edition (Carnap, 1928/61, p. viii f.). The systematic application of the new method led Carnap and Hempel, around the middle of the century, to the full development of the so-called Received view (or Standard conception) of scientific theories, Hempel's abandonment of the empiricist criterion of cognitive significance and Carnap's proposal of a new fully relativised formulation of both this criterion and the analytic/synthetic distinction (see Parrini, 2001/04, § 2).

2.3 Historical Changes and Conventionality

Carnap accepts the Neo-empiristic view according to which the «forms of thinking» presupposed by scientific knowledge: (i) may «change with the advance of science» (Frank, 1941/49, p. 8); (ii) are of a conventional nature. This thesis in its dual aspect was put forward initially by Frank (Frank, 1941/49, pp. 10–12) and Schlick in particular. Schlick defines such forms of thinking as «conventions in Poincaré's sense» (Schlick, 1921, p. 333) and in the *Allgemeine Erkenntnislehre* calls the assumptions that establish a connection between the implicitly defined scientific concepts and some aspects of experience «coordinative conventions». At a later stage, this thesis was also fully accepted by Reichenbach, who had only accepted part (i) at the beginning of the 1920s. In the 1920 book *Relativitätstheorie und Erkenntnis apriori* and in other works published between 1921–1922, he did in fact defend the thesis that Schlick's «coordinative conventions» had to be considered as «constitutive principles» similar to Kant's synthetic a priori judgments in every aspect except for eternal and apodictic validity (Reichenbach, 1920, ch. V, 1922, p. 30, pp. 36–41, n. 21). Later on, though, he refers to these principles as coordinative *definitions* (Reichenbach, 1924, § 2, 1928, § 4).

Carnap also asserted the conventional as well as the historically variable nature of several components of knowledge. In the § 103 of the *Aufbau*, he states that «the general rules of construction could be called a priori rules, since the construction and cognition of the object is logically dependent upon them». At the same time, he specifies that such rules «are not to be designated "a priori knowledge", for they do not represent knowledge, but postulations (*Festsetzungen*)» (Carnap, 1928, p. 163).

The utilisation of constitutive rules does not therefore contrast with the general thesis of the § 106 according to which the «statements or theorems of a constructional system are divided» in analytic a priori and in synthetic a posteriori (or empirical) with the exclusion of those «"synthetic judgments a priori" which are essential for Kant's approach to epistemological problems» (Carnap, 1928, p. 176).

2.4 Empiricism, Reductionism, Conventionalism

In the years following the publication of the *Aufbau*, Carnap discussed the distinctions between analytic statements and synthetic statements, and between the *Realwissenschaften* and the *Formalwissenschaften* (logic and mathematics) using the formal tools of the *Wissenschaftslogik*, including the Ramsey sentence. In fact, in the 1950s, he was forced to resort to the Ramsey sentence to solve the problem, posited by Hempel, regarding the possibility of keeping «the specification of meanings and the description of facts» separate not only in observational, but also in theoretical language (Hempel, 1963, p. 691; Carnap, 1963, pp. 963–966; see Parrini, 2001/04, § 2). If we set these logical aspects aside, there are two main characteristics in Carnap's conception which should be highlighted here.

(i) From the years of the *Logische Syntax* to the last answers given to Quine and Hempel, Carnap has always supported the relative nature of the notion of analyticity (and thus of the a priori). According to him, we can only speak of analytical sentences in relation «to a particular system of assumptions and methods of reasoning (primitive sentences and rules of inference), that is to say, in our terminology, to a particular language» (Carnap, 1934, p. 44, 1963, p. 921). This thesis is linked to the conventionalism theorised in the *Logische Syntax* and the criticism advanced in this same book against «Wittgenstein's absolutist conception of language, which leaves out the conventional factor in language-construction» (Carnap, 1934, p. 186). It is important to note that Carnap is not simply saying that what he is interested in elaborating is a relativised conception of analyticity, or more precisely, of logical validity (L-validity). He is making a stronger statement: we should only recognise a relative validity to the notion of analyticity.

(ii) Reducing some of the components of knowledge to conventions, coordinative definitions, semantic rules, or meaning postulates is perfectly in line both with the way in which Logical Empiricists had distanced themselves from Kant, and the two dogmas of empiricism as they will be intended and criticised by Quine.

According to Quine, the dogma of reductionism, or better still the dogma of the reductionist interpretation of the verification principle, asserts, in its *radical* version, that «every meaningful statement is [...] translatable into a statement (true or false) about immediate experience», or, in an *attenuated* form, «that each statement, taken in isolation from its fellows, can admit of confirmation or infirmation at all». The dogma of reductionism and the dogma of analyticity support each other, they are even «at root identical», because if we accept reductionism «it seems significant to speak also of a limiting kind of statement which is vacuously confirmed, *ipso facto*, come what may; and such a statement is analytic» (Quine, 1953/61, p. 38f., 41). This formulation of the reductionist thesis suits Carnap's thought well and finds an almost literal correspondence in the way in which Reichenbach states his position in the paper presented at the Paris Conference on scientific philosophy (1935). In this paper Reichenbach asserts the possibility of allowing for the pragmatic justification (or vindication) of induction and gives the following formulation of his criticism of Kant's theory of synthetic a priori judgments: «Today's science does not believe any longer in the legislative capacity of a pure reason. All that we know of the world is taken from experience, and the transformations of empirical data are purely tautological, analytical» (Reichenbach, 1935/36, p. 32).

Formulations such as these though, must not make us forget that for Logical Empiricists, anti-Kantianism and the adherence to empiricism does not mean going as far as to deny that the transformations to which empirical data are submitted are ruled not only by logical-mathematical principles, but also by non-empirical assumptions: rules of the *Konstitutionssystem*, conventions in Poincaré's sense, coordinative definitions, semantic rules, meaning postulates, and so on. Also in the most reductionist versions of Logical Empiricism, reductionism does not mean denial of the existence, within the cognitive process, of «forms of thought» that cannot be traced back to experience. It only means that these forms of thought are interpreted as assumptions devoid of empirical-factual content; thus their acceptance does not undermine the classification of statements into only two categories, the analytic a priori and synthetic a posteriori.

For this reason, Logical Positivism in general, and Carnap's epistemology in particular, cannot be defined, not even briefly, by H. P. Price's famous words «Hume plus mathematical logic». In saying this, I do not intend to endorse an interpretation that favours the connection between Logical Empiricism and the Kantian and Neo-Kantian line of thought to such an extent as to leave the empiricist components that connect it with Hume's philosophy, Mach's empiriocriticism and some aspects of Russell's and Wittgenstein's thought in the shadows. Logical Positivism must be considered as a philosophical movement that tried to synthetize elements coming from all these different traditions in a unitary vision, using conventionalism and in particular Poincaré's conventionalism as a unifying agent. If Neo-empiricistic philosophy of knowledge, and Carnap's in particular, cannot be considered as an anachronistic lapse back into seventeenth and eighteenth century Empiricism, it is not because they do not have a genuinely empiricistic inspiration, but because they wanted to construct an empiricism capable of taking into consideration the teachings provided by Kantian, conventionalist and Neo-Kantian epistemology (Parrini, 1983, 2001/03).

3 Objectivity and Ontology

The link which exists between Carnap's position and that of Logical Empiricism on the epistemological level, can also be found when problems of the objectivity of knowledge and ontology are considered. For both Carnap and other Neo-empiricists, the presence in science of presupposed forms of thinking, not derived from experience, should not endanger the objective value of knowledge. On the contrary: this presence should allow us to defend an anti-metaphysical conception of objectivity that – as it was in Mach's *desiderata* regarding mechanics and mechanicism – frees our discourses on the objects from any form of absolutisation or ontological hypostatization (see Frank, 1941/49, pp. 1–6, 17–19). Rightly, under Carnap's influence, Schlick comes to fully develop a similar conception by following a path that starts from the *Allgemeine Erkenntnislehre* (1918/25) and leads to the 1932 essay "Positivismus und Realismus". Reichenbach expresses the same idea in a way that is more similar to Neo-Kantianism, and then goes in the direction of a peculiar form of realism linked to his views on probability and induction. As he writes in the *Philosophie der Raum-Zeit-Lehre* for example «The description of nature is not stripped of arbitrariness by naive absolutism, but only by recognition and formulation of the points of arbitrariness. The only path to objective knowledge leads through conscious awareness of the role that subjectivity plays in our methods of research» (Reichenbach, 1928, p. 37).

Carnap, for his part, tries to develop anti-metaphysical objectivism using the theory of the ontological-metaphysical *Neutralität* of the *Konstitutionssystem*. This thesis appeared during the years that saw the preparation of *Der logische Aufbau der Welt* and *Scheinprobleme in der Philosophie*, published in 1928 as one volume. In some letters written during the same period, Reichenbach describes Carnap's aspiration to this ontological-metaphysical *Neutralität* «ein schöner Traum» and asserts the impossibility of renouncing the «*Realitätsaxiom*» or «Axiom des Realismus». Carnap's reply to this is that the «*Realitätsaxiom*» could not be expressed within the *Konstitutionssystem* (see Parrini, 2002, p. 290 f.). He remained loyal to this interpretation and in the following years devoted a great deal of effort in showing that, if metaphysically intended, the main ontological and epistemological options are devoid of any cognitive meaning. He did so both in the case of the classical contrapositions such as realism/idealism, materialism/spiritualism, etc., and in the case of contrapositions that have become particularly relevant in the course of the process of liberalisation of empiricism, such as the contrast between scientific realists and instrumentalists regarding the existence of unobservable entities, or the contrast between nominalists and platonic realists regarding the existence of semantic entities (it is well known that this very controversy inspired the 1950 essay "Empiricism, Semantics, and Ontology" [Carnap, 1956, Supplement A, pp. 205–222]).

Throughout the whole development of his thought, Carnap tried to defend his anti-metaphysical objectivism with 'clusters' of variously interrelated arguments that are subjected to changes over the course of time and that also, in more or less contemporary texts, do not always present themselves in the same form. He uses different tools: the verification principle in its more or less strong formulations; the maxim according to which «to be real in the scientific sense means to be an element of the system» and that «hence this concept cannot be meaningfully applied to the system itself» (Carnap, 1956, p. 207); the analytic/synthetic dichotomy; the distinctions between metaphysical reality and empirical reality (*Aufbau* and *Scheinprobleme*), between the formal and material way of speaking (*Logische Syntax*), and between external and internal questions of existence ("Empiricism, Semantics, and Ontology”). As Schlick immediately detected, the fundamental inspiration of this vast network of ideas is clearly permeated by Kant. Schlick draws a parallel between Carnap’s notion of empirical realism and Kant’s teaching that empirical reality and unreality are not things that can immediately be given, but have to do with the possibility of an object becoming a part of a comprehensive set of beliefs governed by rules. In “Positivismus und Realismus” (1932) Schlick uses this idea to criticise both the realist components of the conception he had advanced himself a few years earlier in the *Allgemeine Erkenntnislehre* (1918/25), and the older positivistic idea of the absoluteness of the experience datum (see Parrini, 1991/94, p. 263 f.).

The fact that Carnap, despite the substantial unity of inspiration of his position, formulated it in many ways using different arguments, led to numerous interpretations that have made the description of his philosophy considerably more complex. Without denying the specific merits of these interpretative analysis – from Quine’s (1951) to the more recent ones by Stroud (1987, ch. V) and Bird (2003) – it seems to me, all things considered, that the truly relevant distinction for Carnap is that between *absolute* and *relative*. He essentially intended to support the thesis that the cognitive problems – both the *general* and the *specific* or *particular* ones – regarding the existence and properties of objects are only genuinely theoretical and meaningful when posited as *internal* questions, i.e. as questions *relative* to a previously accepted conceptual reference framework. In accordance with the linguistic turn taken by philosophy and the linguistic conception of the a priori, Carnap describes this structure as a language form or linguistic framework (see Parrini, 1991/94, pp. 267–274).

It is only by considering the cognitive problems as questions within a linguistic framework, that it is possible to give them answers, whether they be logical-analytical or empirical-factual depending on the question and the issue discussed. In the case of physical entities, for example, it is only within, or relative to, the linguistic reference system through which we can talk about ordinary things or microphysics entities that we can pronounce ourselves regarding the existence and the properties of King Arthur, or the existence and the properties of electrons. In questions such as these, we can see the intervention of «an empirical, scientific, non-metaphysical concept» of reality (Carnap, 1956, p. 207) that must be kept separate from the metaphysical, non-scientific concept that intervenes in external or absolute questions. For Carnap – as Schlick had already understood in the 1930s – «To recognize something as a real thing or event means to succeed in incorporate it into the system of things at a particular space–time position so that it fits together with the other things recognized as real, according to the rules of the framework» (Carnap, 1956, p. 207).

Undoubtedly, external questions also seem to posit problems relating to existence, both of a *particular* or *specific* nature (for example, the problem of the existence of number five or King Arthur), and a *general* nature (for example, the problem of the existence of classes or numbers or physical objects); but, differently from internal questions, external ones pose these problems in an absolute form, non-relative to a given linguistic framework. For Carnap, these external questions can be intended in two different ways: *metaphysical* or *pragmatic*. In the first case, they are meant as questions regarding the absolute correctness of a linguistic framework or a statement on entities. In this form, external questions apparently raise problems of truth/falsity to be solved either by empirical-factual or by logical-analytical means, but they are really metaphysical questions devoid of cognitive meaning. As I said before, for Carnap «To be real in the scientific sense means to be an element of the system; hence this concept cannot be meaningfully applied to the system itself» (Carnap, 1956, p. 207). He had already maintained in the 1932 essay "Überwindung der Metaphysik durch logische Analyse der Sprache" that metaphysical discourse does not refer to the sphere of theoretical elaboration, but serves «for the *expression of the general attitude of a person towards life (Lebenseinstellung, Lebensgefühl)*» (Carnap, 1932, p. 78).

But if external questions are intended in a pragmatic sense, they, despite still being empirically and thus cognitively vacuous, can be reformulated as genuine practical problems regarding the suitability of choosing a certain form of language rather than another in relation to our goals. They can be transformed, for example, in the question regarding the suitability of accepting the theoretical language as well as the observational one, or the physicalist language as opposed to the phenomenalist one. In this case, we can also not give external questions an answer of an absolute type, «because various [language] forms have different advantages in different respect» and so «one cannot speak of "the correct language form"» (Carnap, 1963, p. 68). Nevertheless, we can give them a practical-conventional answer. In this respect, Carnap himself is willing to acknowledge that these practical-conventional choices are influenced at various levels by the empirical-factual knowledge we possess, exactly as the answers to the internal questions of an empirical kind are influenced by pragmatic considerations that indicate, for example, which hypothesis it is best to adopt among various alternatives that are all logically admissible and empirically adequate.

The external/internal distinction and this type of anti-absolutism are integral parts of Carnap's conception of philosophy. For him, the philosopher's task consists in proposing clear and precise explications of the problematic concepts by means of the invention and articulation of appropriate linguistic frameworks: «Our task is one of planning forms of language. Planning means to envisage the general structure of a system and to make, at different points in the system, a choice among various possibilities, theoretically an infinity of possibilities, in such a way that the various features fit together and the resulting total language system fulfills certain given desiderata» (Carnap, 1963, p. 68).

4 The Carnap/Quine Debate and the Refusal of Metaphysical Absolutism

From their first formulations, Carnap's epistemological and ontological conceptions attracted much criticism about their main theoretical tenets: the verification principle, the axiomatic-formal conception of scientific theories, the dichotomy between theoretical and observational language, the analytic/synthetic distinction and the whole linguistic theory of the relativised a priori. In the 1952 essay "On Carnap's Views on Ontology" in particular, Quine established a strong connection between Carnap's ontological and epistemological views, and stated that the abandonment of the dichotomy between analytic and synthetic statements brings with it the abandonment of the distinction between internal and external questions of existence.

Quine's «Empiricism without the Dogmas» was favourably received. Many accepted his refusal of analyticity and the type of realism and naturalism that Quine, from a certain point onwards, linked to such refusal. In truth, many also followed paths he had also criticised. I think, for example, of the retrieval of strong forms of logical and mathematical Platonism, and mainly of that Aristotelic essentialism that, with Kripke, has accompanied the development of quantified modal logic. Moreover, for some scholars Quine's criticism had the merit to help analytical philosophy in freeing itself of the 'prohibitions' of Neo-empiricistic origin «and to sever a main route to rationalism and metaphysics» (Burge, 2003, p. 199).

From a strictly historical point of view, this way of positing the question seems correct. Nevertheless, I think that, whilst still remaining in the field of empiricism and anti-metaphysics (as I prefer to do), Quine's objections to Carnap cannot be considered so conclusive as to mark the 'end of the story'. I do not want to deny that these objections contributed to corroding a large part of the neo-empiricistic and Carnap's theses, but rather intend to suggest that they, as well as often depending on a very controversial behaviouristic conception of language, present two weaknesses that considerably undermine their effect.

The first weakness regards the connection between reductionism and analyticity. Contrarily to Quine's statement, the two dogmas are *not* «at root identical». Grice and Strawson showed in 1956 that it is possible to abandon reductionism without renouncing the synonymy and the analytic/synthetic distinction (Grice and Strawson, 1956).

The second weakness is much more complex and regards the discussion of the ontological problem. In his criticism, Quine interprets the external/internal distinction not only in terms of the analytic/synthetic one, but also in terms of the general/specific one. From this second point of view, it is enough to reject reductionism to affirm that the external (general) questions are continuous with the internal (particular) ones. If we abandon reductionism, both the answers to the general questions (for example, the choice of an ontology or a canonical notation) and the answers to the specific questions (for example, the existence of brick houses in Elm Street) end up depending on «considerations of equilibrium» of a logical, pragmatic and empirical nature that affect «the field as a whole». As a consequence, Quine is right in saying that there are only differences of degree and not of type: in the case of some questions/answers logical considerations play the most important role, in others pragmatic considerations prevail, and in some others data from experience are most important.

However, it also appears true that abandoning reductionism only allows the refusal of Carnap's position when the external/internal distinction is intended in the general/particular sense, but not when it is intended in the absolute/relative sense. From the point of view of the absolute/relative dichotomy, Quine himself agreed, in the 1950 essay "Identity, Ostension, and Hypostasis" (virtually contemporary to "Two Dogmas of Empiricism"), with one of the main theses at the base of Carnap's ontological neutrality. For Quine, as for Carnap, we cannot establish *in an absolute sense* «How much of our science is merely contributed by language and how much is a genuine reflection of reality», because in order to answer that question «we must talk about the world as well as about language, and to talk about the world we must already impose upon the world some conceptual scheme peculiar to our own special language». So «we cannot detach ourselves» from our conceptual scheme «and compare it objectively with an unconceptualized reality. Hence it is meaningless […] to inquire into the absolute correctness of a conceptual scheme as a mirror of reality. Our standard for appraising basic changes of conceptual scheme must be, not a realistic standard of correspondence to reality, but a pragmatic standard» [Quine, 1953/61, p. 78f.).

After the crisis of the verification principle, it seems difficult to find a meaning for the word "sense" which is consistent with the purpose of considering as senseless or meaningless the question of the absolute correctness of a conceptual scheme or, in Carnap's terms, of a linguistic form. I also deem that Quine's philosophical development was not completely coherent with what was stated in the concluding pages of "Identity, Ostension, and Hypostasis", in that Quine moved more towards realism than pragmatism. The argument, though, still carries a certain weight even when freed from the verification principle, because it presents the advantage of highlighting the problematic nature of talking about reality in ways that are not epistemically conditioned, and therefore the difficulty in comparing our pretended descriptions of reality with reality *per se*. In this form, the argument is independent of the verification principle, and is very similar to both Kant's observations on the object of representations and Einstein's observations on the relationship between physical theories and real world, none of which I can deal with here.

Naturally, in its non-verificational form, the argument cannot be considered decisive against a metaphysical realist position. The realism/antirealism controversy is extremely multifaceted, and it would be wrong to pass from the assertion that it is not possible to have an epistemic access to reality which is conceptually non-conditioned, to the conclusion of the meaningless nature of the notion of absolute reality and metaphysical realism (see Parrini, 1999/2002). Despite this, such an argument certainly goes in favour of an anti-metaphysical conception, according to which our scientific and cognitive efforts are unable to justify statements regarding a more or less close correspondence between our pretended descriptions of reality and reality as it is. As I said at the beginning of § 3, this aspect was the most characteristic feature of the conception of objectivity that Logical Empiricists and Carnap intended to defend. The objectivity they aimed at was, precisely, an objectivity completely free from the metaphysical idea of the absoluteness, an objectivity that – to quote Reichenbach again – had to pass through the recognition of the role played in the cognitive process by subjective assumptions, historically changeable and dependent on the changing of theories (see Reichenbach, 1928, p. 37).

This brings us back to the question of analyticity and the relativised a priori. In fact, the second serious limitation of Quine's objections precisely relates to the *relative* nature of the epistemic justification of our attributions of a truth value, or confirmation or empirical adequacy to the statements regarding the existence of entities and their properties. Despite what Hempel also feared at one point (see Hempel, 1963, p. 705), Quine's objections did not succeed in showing the full epistemological irrelevance of the analytic/synthetic and a priori/a posteriori distinctions. As time passed, it became increasingly clear that the contrast between Quine and Carnap is linked to two mainly alternative epistemological projects (see Creath, 1991). Both projects seek to eliminate intuition from the basis of epistemic justification, but otherwise, they are profoundly different. Quine's project is centred mainly on a holistic evaluation of the way in which our set of beliefs changes and calls into question holistically applied considerations regarding empirical adequacy, simplicity and conservation. Carnap's model, on the contrary, aims at understanding not so much the dynamics of knowledge, but the logical structure of epistemic justification and is centred mainly on the constitution of objectivity. Carnap realises that justification proceeds through 'outpourings' of many reasons one after the other, and that there is never the possibility of individuating a point of correspondence between our statements and reality in itself that is not epistemically conditioned. Consequently, he asks himself which other source of justification is available to us in addition to empirical observation and gives an answer that fits in the «semantic tradition» (Coffa, 1991, p. 22): he tries to avoid the defects of the traditional answers based on intuition by calling into question conventions, definitions and semantic rules or meaning postulates.

If we consider the question from this point of view, though, at least the main idea at the root of Carnap's theory of the relativised a priori seems to survive Quine's epistemological criticism. From the point of view of epistemic justification, we can even suggest that such an idea has taken a new plausibility rightly because of the developments of the philosophy of science that led to the crisis of the Standard conception of scientific theories. Not only does a relativistic theory of the a priori and analyticity have nothing to fear from the idea that all our statements are susceptible to revision in the light of the developments of experience (Quine's 'revision argument' [see Carnap, 1963, p. 921]), but we can also assert – *contra* Quine (see, e.g., Quine, 1986, p. 207) – that epistemological holism does not exclude, but demands, the relevance of the distinction between relatively a priori and a posteriori statements.

The analysis of the logical structure of experimental testing and the so called "coordinative definitions" (see Parrini, 1976, §§ II/5–6), the analysis of theories such as Newton's physics and relativistic physics (see, e.g., Friedman, 2000, pp. 374–6), the study of the dynamics of science, as it was developed mainly under the influence of Kuhn, all converge towards the conclusion that the understanding of scientific knowledge requires a functional distinction between the different components of scientific theories. For example, according to Kuhn, the disciplinary matrixes (originally called «paradigms») that characterise the so called «normal science» do not only include manuals and exemplars (or «paradigms» in the strict sense of the term), but also what have been called «paradigmatic propositions». Such propositions «are neither analytic, [...] nor eternal truths», «nor empirical in the usual sense» exactly because «they are protected from straightforward empirical refutation. [...] They constitute an epistemically distinct class in that they do not fit the traditional division of all propositions into a priori and empirical. Rather, they are propositions which are accepted as a result of scientific experience but which come to have a constitutive role in the structure of scientific thought» (Brown, 1977, p. 105; see Kuhn, 1983, pp. 566–567).

There are also obviously some important differences between Carnap's and Kuhn's conception of scientific theories, but despite this, we cannot, in both cases, understand the structure and changes of science without taking the presuppositions that provide the framework for the scientific activity into account. Whilst still considering the holistic conception of the theory/experience relationship and the idea of the revisionability of any scientific statement as valid, the two conceptions do not accept as adequate a vision of science that puts all scientific statements on the same level, without setting any functional distinction between those *directly* dependent on experience and those that depend on it only *indirectly* and play a presuppositional role.

5 Carnap, Reichanbach and the Nature of the Relativised a priori. Ontological Implications

If we adopt the epistemic justification point of view, it is thus still possible today to see a validity to Carnap's (and Neo-empiricistic) theory of the relativised a priori, or more precisely, to the fundamental idea on which this theory is based. I am talking about the fundamental idea, and not the whole theory, because the crisis of reductionism and epistemological holism impose the abandonment of a distinctive component of that theory, i.e. the thesis of the exclusively linguistic (analytic) nature of the relativized a priori statements. This, in its turn, makes it necessary to make more extensive concessions to Kantianism than Carnap and the other Logical Empiricists thought it was necessary, although not extensive enough to trespass that boundary of empiricism that is represented by thesis 2.1 mentioned above: the negation of Kant's theory of synthetic a priori judgments.

The concessions to Kantianism that have become necessary today were clearly anticipated in the conception put forward by Reichenbach in his 1920 book, *Relativitätstheorie und Erkenntnis apriori* and in other essays published during the same period. In these works, Reichenbach states that scientific knowledge needs constitutive principles that, as well as being very similar to Kuhn's paradigmatic propositions, are distinguished by almost all the main characteristics of Carnap's relativised a priori: (i) unlike the usual principles of physics, the constitutive or coordinative principles do not act as connectors of specific physical quantities to one another, but as general rules to establish such a connection: «they do not say *what* is known in the individual case, but *how* knowledge is obtained; they […] indicate the conditions the logical satisfaction of which leads to knowledge» (Reichenbach, 1920, p. 104); (ii) coordinative principles form a particular group of statements in the sense that, albeit being «determined by experience», «their validity does not depend only upon the judgment of particular experiences, but also upon the possibility of the whole system of knowledge: this is the sense of the a priori» (Reichenbach, 1920, p. 104); (iii) constitutive principles are relative to the state that scientific knowledge assumes at a certain historical moment, and as such, change with the changing of such knowledge under the stimulus of experience: «"A priori" means "before knowledge", but not "for all time" and not "independent of experience"» (Reichenbach, 1920, p. 105).

Despite these similarities between Reichenbach's 1920 conception and that defended by Carnap, there is also an important difference. By expressing the prevailing ideas in Logical Empiricism, Carnap maintains the linguistic or logical-analytical nature of all the relativised a priori: this a priori is defined by referring to «semantical rules» or «meaning postulates» that serve the very purpose of constituting the class of meanings and logically valid assertions, i.e. the linguistic tools by means of which we describe the world or reality. Reichenbach's coordinative principles, on the other hand, are principles that serve to constitute – in Kantian terms – the object of knowledge, i.e. the world or reality as we know them. For this very reason in the early 1920s Reichenbach did not agree with Schlick, who had declared that in science there is no place but for «*hypotheses* or *conventions*» and that Reichenbach's constitutive principles had to be considered «conventions, in Poincaré's sense» (Schlick, 1921, p. 324, 333). Only in the following years did Reichenbach also adopt the standard Neo-empiristic conception of «coordinative *definitions*» instead of «coordinative or constitutive principles» (see, e.g., Parrini, 2001/03, 2001/04, §§ 5–6; for a different interpretation see Friedman, 2000, 2001, pp. 79–90). We can, therefore, notice how relevant failing to see the differences between Poincaré's and Duhem's positions was in the history of Logical Empiricism. In fact, one of the main criticisms made by Duhem to Poincaré aimed precisely at establishing that the aspects of subjectivity present in scientific discourse cannot be reduced – as Poincaré stated – to the linguistic component of that discourse (Parrini, 1983, §§ 2–3). The awareness of the differences between Poincaré and Duhem might ring an important alarm bell for Logical Empiricists!

The different interpretations for the coordinative assumptions given in the 1920s by Schlick and Reichenbach have very important implications for ontology. In Schlick's *Allgemeine Erkenntnislehre*, the assimilation of coordinative assumptions to conventions was perfectly in line with his designative, or «semiotic» (Howard, 1991/94, p. 63), conception of knowledge. In fact, Schlick saw in cognitive coordination a form of *designation* of objects and facts that are already there, independently of the cognitive process, whereas in the Reichenbach of the early 1920s, the interpretation of coordinative assumptions as constitutive principles brought with it the idea that these assumptions are necessary for the constitution of physical reality, of the object of knowledge. According to him, in the cognitive process, one of the two sides of the coordination – «the "real"» or the object of knowledge – is constituted in an 'immanent' way with respect to the coordination principles, and its transcendence is simply due to the fact that it is susceptible to a potentially endless number of empirical determinations (Reichenbach, 1920, p. 38, 42; see Parrini, 2001/03, §§ 2–3). Nevertheless, there is an important difference between Kant and Reichenbach. Kant believed in the uniqueness and immutability of constitutive principles, and so thought that «only the determination of a *particular concept* is an infinite task». Reichenbach, instead states the historical variability of coordinative assumptions, and so maintains that: «*even our concepts of the very object of knowledge, that is, of reality and the possibility of its description, can only gradually become more precise*» (Reichenbach, 1920, p. 88).

The recovery of Duhem's vision of the theory/experience relationship leads us to reconsider the theory Reichenbach developed in the 1920s. As I said before, if we consider the point of view of epistemic justification, epistemological holism does not exclude, but demands an opportunely relativised distinction between the a priori and the a posteriori; but from the same epistemic justification viewpoint, epistemological holism is incompatible with the idea that all the relatively a priori components can be reduced to assumptions only regarding the linguistic structure of science. The holistic conception asserts that when presented with certain experimental results, we cannot speak of the empirical adequacy or non-adequacy of an individually considered hypothesis, if not relative to a network of collateral assumptions taken for granted, assumptions which do not only apply to the linguistic rules which fix the meaning of the sentence by means of which the considered hypothesis is formulated (see Parrini, 1976, § II/2, 1998, §§ II/2, 5). So, if we continue stating that the ontological questions are subdivided in external and internal questions with respect to a conceptual frame of reference, we must also acknowledge – differently from Carnap – that such a frame cannot be solely constituted by the language in which the questions are formulated. The framework will have to include a more or less wide set of theoretical principles that, in a given context, will work as relatively a priori assumptions of a constitutive nature like Reichenbach's old coordinative principles.

This raises the epistemological problem of relativism. If we state the possibility of having alternative linguistic frameworks and the purely linguistic nature of the relativised a priori, it is possible to state – to paraphrase Reichenbach after the 1920s – that relativity does not regard the objectivity and truth of our descriptions of the world, but only our different *formulations* of that truth and objectivity (see Reichenbach, 1949, pp. 294–296). If, on the other hand, we talk about constitutive principles of objectivity, and we assert that knowledge also requires subjective assumptions of a theoretical-synthetic nature, the situation changes considerably. The very notions of objectivity and truth run the risk of becoming relative, and the possibility of avoiding relativism requires another set of conceptual tools. These new tools too are of a *latu sensu* Kantian derivation, but they are different from the ones set up by Carnap and the Logical Empiricists (see Parrini, 1995/98).

References

Bird, G., 2003, "Carnap's Internal and External Questions", in *Language, Truth and Knowledge. Contributions to the Philosophy of R. Carnap*, ed. by T. Bonk, Kluwer, Dordrecht, The Netherlands, 97–131

Brown, H. I., 1979, *Perception, Theory and Commitment. The New Philosophy of Science*, First edn. 1977, The University of Chicago Press, Chicago, IL

Burge, T., 2003, "Logic and Analyticity", in *Fifty Years of Quine's "Two Dogmas"*, ed. by H.-J. Glock, K. Glüer and G. Keil, *Grazer Philosophische Studien*, 66, 199–249

Carnap, R., 1923, "Über die Aufgabe der Physik", *Kant-Studien*, 28, 90–107

Carnap, R., 1928/61, *Der logische Aufbau der Welt* * *Scheinprobleme in der Philosophie* (Weltkreis-Verlag, Berlin Schlachtensee, 1918; Second edn., Felix Meiner, Hamburg, 1961), English translation, *The Logical Structure of the World* * *Pseudoproblems in Philosophy*, University of California Press, Berkeley/Los Angeles, CA, 1967

Carnap, R., 1932, "Überwindung der Metaphysik durch logische Analyse der Sprache" (*Erkenntnis*, 2), "The Elimination of Metaphysics through Logical Analysis of Language", in *Logical Positivism*, ed. by A. J. Ayer, Free Press, New York, 1959, 60–81

Carnap, R., 1934, *Logische Syntax der Sprache* (Springer, Wien), English edn., *The Logical Syntax of Language*, Routledge & Kegan Paul, London, 1937, Third impression, 1951

Carnap, R., 1956, *Meaning and Necessity. A Study in Semantics and Modal Logic*, First edn., 1947, University of Chicago Press, Chicago, IL

Carnap, R., 1963, "Carnap's Intellectual Autobiography" * "The Philosopher Replies", in *The Philosophy of Rudolf Carnap*, ed. by P. A. Schilpp, Open Court, La Salle (Ill.), 1–84, 857–1013

Carnap, R., 1966/74, *Philosophical Foundations of Physics*, Second edn. rev., *An Introduction to the Philosophy of Science*, ed. by M. Gardner, Basic Books, New York

Coffa, A. J., 1991, *The Semantic Tradition from Kant to Carnap. To the Vienna Station*, ed. by L. Wessels, Cambridge University Press, Cambridge

Creath, R., 1991, "Every Dogma Has Its Day", *Erkenntnis*, 35, 347–89

Frank, P., 1941/49, *Modern Science and Its Philosophy*, Harvard University Press, Cambridge, MA

Friedman, M., 2000, "Transcendental Philosophy and A Priori Knowledge: A Neo-Kantian Perspective", in *New Essays on A Priori*, ed. by P. Boghossian and C. Peacocke, Clarendon, Oxford, 367–383

Friedman, M., 2001, *Dynamics of Reason. The 1999 Kant Lectures at Stanford University*, CSLI Publications, Stanford, CA

Grice, H.P., P.F. Strawson, "In Defense of a Dogma" *Philosophical Review*, 65, 141–158

Hempel, C. G., 1963, "Implications of Carnap's Work for the Philosophy of Science", in *The Philosophy of R. Carnap* [see Carnap 1963], 685–709

Howard, D., 1991/94, "Einstein, Kant, and the Origins of Logical Empiricism", in *Logic, Language, and the Structure of Scientific Theories. Proceedings of the Carnap-Reichenbach Centennial, University of Konstanz, 21–24 May, 1991*, ed. by W. Salmon and G. Wolters, University of Pittsburgh Press/Universitätsverlag Konstanz, Pittsburgh, PA/Konstanz, Germany 1994, 45–105

Kuhn, T. S., 1983, "Rationality and Theory Choice", *The Journal of Philosophy*, 80, 563–570

Parrini, P., 1976, *Linguaggio e teoria. Due saggi di analisi filosofica*, La Nuova Italia, Firenze

Parrini, P., 1983, *Empirismo logico e convenzionalismo*, Angeli, Milano, Italy

Parrini, P., 1991/94, "With Carnap Beyond Carnap. Metaphysics, Science, and the Realist/Instrumentalism Controversy", in *Logic, Language, and the Structure of Scientific Theories* [see Howard 1994], 255–277 [Italian version in Parrini 2002: V]

Parrini, P., 1995/98, *Knowledge and Reality. An Essay in Positive Philosophy* (English edn. of *Conoscenza e realtà. Saggio di filosofia positiva*, Laterza, Roma-Bari, 1995), Kluwer, Dordrecht, The Netherlands

Parrini, P., 1999/2002, "Realism and Anti-Realism from an Epistemological Point of View", in *The Problem of Realism*, ed. by M. Marsonet, Ashgate, Bodmin, UK (Cornwall), 2002, 135–159

[Italian version in P. Parrini, *Sapere e interpretare. Per una filosofia e un'oggettività senza fondamenti*, Guerini, Milano, Italy, 2002, 67–85]

Parrini, P., 1999/2003, "Reason and Perception. In Defense of a Non-linguistic Version of Empiricism", in *Logical Empiricism. Historical and Contemporary Perspectives*, ed. by P. Parrini, M. Salmon and W. Salmon, University of Pittsburgh Press, Pittsburgh, PA, 2003, 349–374 [Italian version in Parrini 2002: IX]

Parrini, P., 2001/03, "On the Formation of Logical Empiricism", in *The Vienna Circle and Logical Empiricism. Re-Evaluation and Future Perspectives*, ed. by F. Stadler, Kluwer, Dordrecht, The Netherlands, 2003, 9–20 [Italian, enlarged version in Parrini 2002: VI]

Parrini, P., 2001/04, "The "Dogma" of Analyticity Fifty Years After", in *Logic and Metaphysics*, ed. by M. Marsonet and M. Benzi, Name edizioni, Genova, 103–131 [Italian version in Parrini 2002: X]

Parrini, P., 2002, *L'empirismo logico. Aspetti storici e prospettive teoriche*, Carocci, Roma

Quine, W. V., 1951, "On Carnap's Views on Ontology" (*Philosophical Studies*, 2), reprinted in W. V. Quine, *The Ways of Paradox and Other Essays* (1966), Revised and enlarged edition, Harvard University Press, Cambridge, MA, 1976, 203–211

Quine, W. V., 1953/61, *From a Logical Point of View. Logico-Philosophical Essays*, Harvard University Press, Cambridge, MA, 1953, Second edn., rev., Harper & Row, New York

Quine, W. V., 1986, "Reply to Geoffrey Hellman", in *The Philosophy of W. V. Quine*, ed. by L. E. Hahn and P. A. Schilpp, La Salle (Ill.), Open Court, 206–208

Reichenbach, H., 1920, *Relativitätstheorie und Erkenntnis apriori* (Springer, Berlin), English edn., *The Theory of Relativity and A Priori Knowledge*, ed. by M. Reichenbach, University of California Press, Berkeley/Los Angeles, CA, 1965

Reichenbach, H., 1922, "Der gegenwärtige Stand der Relativitätsdiskussion" (*Logos*, 10), English translation, "The Present State of the Discussion on Relativity", in H. Reichenbach, *Selected Writings (1909–1953)*, Vol. II, ed. by M. Reichenbach and R. S. Cohen, Reidel, Dordrecht, The Netherlands, 1978, 3–47

Reichenbach, H., 1924, *Axiomatik der relativistischen Raum-Zeit-Lehre* (Vieweg, Braunschweig), English translation, *Axiomatization of the Theory of Relativity*, University of California Press, Berkeley/Los Angeles, CA, 1969

Reichenbach, H., 1928, *Philosophie der Raum-Zeit-Lehre* (Walter de Gruyter, Berlin/Leipzig, Germany), English translation, *The Philosophy of Space and Time*, Dover, New York, 1958

Reichenbach, H., 1935/36, "L'empirisme logistique et la désagrégation de l'*a priori*", in *Actes du Congrès International de Philosophie Scientifique*, Paris 1935, I, Hermann, Paris, 1936, 29–35

Reichenbach, H., 1949, "The Philosophical Significance of the Theory of Relativity", in *Albert Einstein: Philosopher-Scientist*, ed. by P.A. Schilpp, Evanston, III, Second edition, Tudor, New York, 1951, 287–311

Schlick, M., 1918/25, *Allgemeine Erkenntnislehre* (Julius Springer, Berlin, 1918, Second edn. rev., 1925); English translation, *General Theory of Knowledge*, Springer, Wien/New York, 1974

Schlick, M., 1921, "Kritizistische oder empiristiche Deutung der neuen Physik?" (*Kant-Studien*, 26), English translation, "Critical or Empiricist Interpretation of Modern Physics?", in M. Schlick, *Philosophical Papers*, Vol. I (1909–1922), ed. by H. L. Mulder and B. F. B. Van De Velde-Schlick, Reidel, Dordrecht, The Netherlands, 1979, 322–334

Schlick, M., 1932, "Positivismus und Realismus" (*Erkenntnis*, 3), English translation, "Positivism and Realism", in M. Schlick, *Philosophical Papers* [see Schlick 1921], Vol. II, 259–284

Stroud, B., 1987, *The Significance of Philosophical Scepticism*, Clarendon, Oxford

Part II
Transcendental Epistemologies and Modern Physics

A

General Issues: Concepts and Principles

The Transcendental Domain of Physics

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Abstract The physical sciences display the world as a hierarchy of regresses, in which epistemological levels, observables and unobservables, are integrated with ontological levels, such as part-whole. Homogeneous regresses preserve generic ontologies, while heterogeneous regresses involve radical ontological transitions. Causal explanations map onto these regresses, transcending reference to causal mechanisms by hypothesizing causal powers. Faraday's physics can be adapted as the basis of a transcendental argument to support the necessity of supposing that the world consists of causal powers. The subject of causal powers attributions can not be the world but the world indissolubly linked to apparatus.

1 Regresses in the Sciences

The sciences have developed by sustaining one level or layer of reality by or on another, and that on yet another and so on. At least that is how the development of the sciences looks to a Realist. Diseases, as displays of symptoms are sustained in being by the activities of bacteria and viruses. The aurora borealis as a visible phenomenon is sustained in being by the solar wind passing through the rarefied gases of the upper atmosphere. Chemical phenomena, like the evolution of hydrogen and oxygen during the electrolysis of water are sustained in being by a flow of ions under a potential gradient. These are familiar regresses. They display relations such as 'Whole/Part'; 'Effect/Cause' and so on. In the preliminary scene-setting stage of this discussion scientific regresses will be examined ontologically, that is in terms of the categories of beings arranged according to the above relations and exemplified in cases like those sketched above.

It usually happens that the first stage of a regress transcends the boundary of unaided perception, though preserving the kinds and at least some of the determinables

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characteristic of the observable phenomena. This constraint makes the construction of sense extending instruments appropriate as a research strategy. Later stages of a regress usually transcend the boundary of possible experience. When this level is reached instrumentation is designed to manipulate imperceptible beings indirectly. It may still happen that the hypothetical beings to be pushed around are ontologically conservative, falling under some generic ontological prescription, such as that of the simple material body. For example radically unobservable entities with magnetic fields are manipulated in the Stern–Gerlach experiment. However, larger entities with magnetic fields such as common bar magnets are easily observable, and the shape of their fields made visible by the use of iron filings.

However, in the development of physics the lines of regression sometimes reach as far as beings of kinds not represented, or at least not represented overtly in the domain of common experience. In particular regresses sometimes lead to causal powers, tendencies and capacities. The effects of the activity of a causal power can be observed, but not the power itself. The ontology of powers is complex and will be tackled in detail below.

1.1 Homogeneous and Heterogeneous Regresses

In a homogeneous regress, the ontological type of the beings in the supporting level remains unchanged however far the regress is pursued. Wittgenstein's examination of the regress of rules supported by yet more rules, convinced him that this progress could not go on ad infinitum, since otherwise action according to a rule would be paralyzed by the need to consult yet more rules. His solution was to declare that forms of life are defined by the normative practices that the local folk carry out without further ado. If asked to justify a certain practice that is foundational for a culture a member can say only 'This is what I do'. This line of argument was worked out in detail in his last work, *On Certainty* (Wittgenstein, 1979).

In heterogeneous regresses, at some level in the regress, the ontological type of the supporting level changes from that of the supported levels to something different. For example, the regress of explanations of an agent's skilled performance in cognitive psychology proceeds a few steps by invoking unobservable cognitive processes, for example Jerome Bruner's schemata (Bruner, 1983), such as those we use when judging the value of coins. It is not long before neural processes in the brain are introduced as the mechanism or tool with which an agent acts skillfully.

The physical sciences seem to involve initially a homogeneous regress or regresses that are bounded by a descent (to follow the prevailing metaphor) into another homogeneous regress that is heterogeneous with respect to that which depends upon it.

Chemical reactions among material substances are explained by reference to the causal powers and mutual relations of their constituent material parts, molecules. These causal powers are explained by reference to the causal powers and mutual relations of their constituent material parts, atoms. And so on through further layers of the 'structure of constituents' ontology. However, research in physical chemistry soon comes to a boundary between the realm of material beings (Kant's empirical world) and the realm of charges, poles, fields, potentials and so on. Whatever these are, they are not just more material entities of the everyday sort with versions of everyday properties. Compare this with the fundamental role of Robert Boyle's 'corpuscles' which are defined by four determinables, 'bulk, figure, texture and motion'. Determinates of these determinables are everywhere to be seen in the world as people perceive it. One strand of physics has always been based on a heterogeneous regress, the physics of Gilbert, Boscovich, Kant and Faraday. I shall refer to the metaphysical basis of this strand of the ontology of physics as 'dynamism'.

Dynamism has been developed along two distinct lines. The metaphysics of Boscovich and Kant preserved the idea of a corpuscle, though drastically stripped down, and reduced to a continuously existing being with no extension in space, a point-particle individuatable by its location in space and time. Each point-particle was a centre of power, manifested in forces of attraction and repulsion, diagrammed as emanating from these centres. The manifested forces were used to account for the perceptible properties of many kinds of material things, such as size and shape, impenetrability, magnetic and electrical interactions and so on. The metaphysics of Gilbert and Faraday dispensed with even the shadow of corpuscularity retained by Boscovich and Kant. It was based on a world that consisted only of fields, continuously distributed causal powers.

2 Causality and Causal Powers

The line of argument to be followed in this discussion terminates in a sketch of a transcendental argument in favour of ontology of causal powers. However, to reach that point the concept of 'causal power' needs to be spelled out.

Causal powers are possessed by material particulars continuously, but whether or not they are manifested in occurrent properties of some observable particular, say the orientation of magnetic needle, depends on contexts, as, e.g. in Bohr's philosophy of experimentation, to be discussed later in detail (Brock, 2003). A material particular that possesses a causal power 'brings it about that …' some phenomenon occurs. Events do not possess causal powers, only things and stuffs. The root idea is that of *efficacy*.

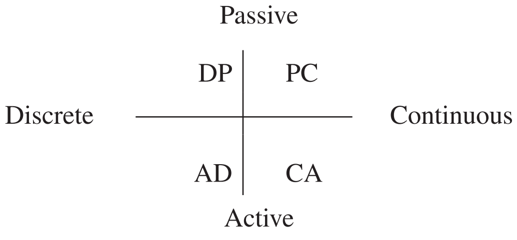
Efficacy cannot be analyzed in terms of the occurrent properties of material beings. As Hume long ago pointed out, the results of the activity of a causal power are observable, but not the power itself. Because he thought it must be an unobservable of which we do not have an impression, Hume casts it out. Psychologists have demonstrated that activity, the manifestation of a power, is perceptible, though not, of course, as a determinable property like colour or shape (Michotte, 1963). This explains how we can routinely tell whether some material being is an agent or a patient, acting or acted upon. It is a distinction we can all make in the observable world. We can certainly tell whether something has a causal power, as we watch it in action, though the ‘power itself’ is manifested only in the phenomena. Things start to happen which seem to emanate from some material being. The mist dissipates when the sun’s rays fall upon it. Even though we now know a good deal about the nuclear reactions inside the sun, nevertheless the sun’s power to warm things up gives us an exemplar of an active agent.

2.1 Ontological Considerations in Physics

The double regress, in which one homogeneous regress is supported in being by another homogenous regress, heterogeneous with the first, poses ontological problems: what are the kinds for beings which ground the second regress? To what sort of beings should fundamental ‘efficacies’ be ascribed?

1. The second regress terminates in beings (hereafter called ‘terminals’) the nature of which is given by reason, that is by theoretical developments.
2. In physics ‘terminals’ have a characteristic logical form:
    (a) Powerful particulars, e.g. charges (discrete and active) are integrated with
    (b) Distributed dispositions, e.g. fields (continuous and active)
    (c) The ‘energy of the field’ has yet to be found a secure ontological status
3. This suggests a Cartesian space of terminal kinds.

There are four quadrants, PC, CA, AD, and DP. Since the abandonment of mechanics as the deepest level of Physics, in favour of dynamism, whether it be Boscovichean or Faradayan, the implicit ontology of Physics has favoured AD and CA. In following the development of these ideas in the nineteenth century, we will find that a conflict emerges between them, as the format for the groundwork of physics. The result, in hindsight, was a victory for CA, as argued by Michael Faraday. The Boscovichean discrete point sources of causal efficacy lost out to energetics and field theories. Nevertheless, physics and chemistry, even in the twenty-first century persist with a kind of hybrid ontology in practice, in which powerful particulars and continuous fields are both permitted.



Reading contemporary physical chemistry reveals a cheerful eclecticism, in which Newtonian corpuscular talk, e.g. HCl molecules as vibrating, rotating and translating; fields, e.g. as potential surfaces transcending reactants and products; and Schödinger equations describing the whole set up, crop up on an ad hoc basis. This sort of thing can be seen throughout accounts of the Nobel Prize winning work of John Polanyi (1986), on the use of chemoluminescence to track the intimate details of chemical reactions, and in hundreds of other places.

In terms of the above schema we have a mix of concepts from quadrants DP, AD and CA.

2.2 To What Logical Subject Must Dispositional Concepts Be Ascribed?

This question can be answered only if we take account of the pervasive phenomenon of ‘complementarity’. This concept is required in order to maintain the rationality of heterogeneous regresses. ‘Complementarity’ is the basis of a third rule for forbidding incompatible predications in the discourse of the sciences, in addition to two kinds of rulings against contradiction. The traditional rules of incompatible predication delineate what complementarity is not.

Behind these rules stands the Aristotelian principle of non-contradiction that forbids the joint assertion of P and not-P. However, it is too general to fix the limits of predication for the sciences.

The first rule of incompatible predication forbids the joint ascription of more than one determinate under the same determinable. For example, ‘Nothing can be red and green all over’ looks like an empirical claim, but we have learned from Wittgenstein to treat it as a grammatical rule, laying down one of the conventions for using the words ‘red’ and ‘green’. However, this rule is a special case of the general rule that forbids joint predication of determinates under the same determinable, for example joint predication of ‘hues’ like ‘red’, ‘blue’ etc. under ‘colour’ are forbidden.

The second rule is more complex, forbidding the use of certain rules of reasoning beyond the limits of experience. This is the rule of antinomies, a formidable weapon in Kant’s criticism of illegitimate uses of pure reason. It seems we can find grounds for holding that the universe as a whole ‘has a beginning in time and is limited as regards space’, and also that it ‘has no beginning, and no limits in space’; neither a beginning nor an end (Kant, 1787, B 454 –455). The source of the antinomy is the use of a predicate to assign a property appropriate to an empirically given entity to attempt to describe a totality of such entities. The contradiction evident in juxtaposing the two characterizations of the universe remarks throws into doubt the assumption that properties of parts can be attributed to the wholes of which they are parts. As Kant says (1787, B 451–452) ‘This method of provoking a conflict of assertions [is] not for the purposes of deciding for one or the other side, but of investigating whether the object of the controversy is not perhaps a deceptive appearance.’

‘Complementarity’ is neither the rule of determinates nor the principle of antinomy. It can be defined as follows:

A and B are complementary if

1. They are determinates, say A and B, each under a different determinable. For example exact position and exact momentum are determinates under different determinables, spatial location and the product of mass and velocity respectively.
2. The conditions under which A can be realized preclude those under which B can be realized. For example, if an apparatus is set up to determine the exact location of a subatomic particle, it precludes setting up an apparatus at the same time and in the same place to determine the exact momentum of the particle.

Complementarity of predicates that cannot be ascribed together however, permits a common unified discourse, provided that predications are segregated with respect to conditions of display of the complementary attributes as they are ascribed to some common system. In this way a complete description of a being to which A and B type predicates are ascribable is possible.

This account will need to be refined. It is not the attributes that are jointly ascribable, but rather the tendencies to manifest the attributes. The *manifestations* are actualizations of possibilities, to which probabilities can be assigned. The *attributes* are tendencies to display such manifestations in the relevant circumstances.

Two deep questions remain to be resolved. To what kind of beings can complementary predicates be ascribed? In the conditions under which fundamental predicates fall under the Principle of Complementarity what sort of explanatory concepts are permissible?

A living brain affords electrical rhythms to an EEG machine and conscious experience to the person whose brain this is. The states of the brain are accessed through different media, so there is no logical conflict between predications according to each point of view. The concepts ‘beta wave’ and ‘conscious’ are complementary in the sense of Niels Bohr. The means of access under which a brain displays periodic electrical activity are incompatible with the means of access under which a brain displays thoughts, images, feelings and so on.

On analysis concepts such as these appear to be relational. They are ascribable only to complex entities, ‘brain/EEG machine’ and ‘brain/person’. Since the phenomena to which the above concepts refer are segregated, ‘conscious’ cannot be ascribed to the complex being ‘brain/EEG machine’. There are no criteria for deciding whether such a predication is appropriate or not. Each of these complex entities is logically independent of the other, though they share the same spatio-temporal location. People could report what they thinking, feeling, hearing and seeing, long before EEG recording was invented.

Each complex entity above can be characterized by a unique set of dispositions. However, the distinguishing dispositions, noting that the entities in question occupy the same spatio-temporal location, have a special character. The conditional clause refers to something available only to a human being. I follow James Gibson (1979) in calling this subclass of dispositions ‘affordances’. This suggests idioms such ‘A brain/EEG machine complex *affords* alpha, beta, delta, theta etc rhythms’.

2.3 Why Should We Adopt the Dispositions/Powers Ontology?

Argument 1: These concepts are already in use in physics and chemistry in well-accepted forms for the transcendent parts of the sciences.
Argument 2: By adopting this ontology the paradoxes that arise because of the weakness of empirical adequacy as a guide to theory choice can be made up.
Argument 3: The standard pattern of scientific explanation by citation of unobservable causal processes is retained throughout the domain of physics and chemistry.

All three arguments are based on local matters of fact about how science has and does proceed.

Could we devise a transcendental argument to justify this choice of terminal kinds?

However, we must first resolve the tension between DA and CA above.

3 Michael Faraday's Arguments for CA or Fields as Terminals

According to Roger Joseph Boscovich (1763) the world does not consist of extended material substances acting by contact, but of a sea of discrete point-centered powers of attraction and repulsion, in universal mutual interaction. Boscovichean metaphysics still requires atoms as a substantival ground, though 'matter' has given way to mutual forces as manifestations of the powers of point atoms. His account of solidity as a secondary quality in which the balance between attractive and repulsive forces is experientially manifested was echoed by Kant (1786). The surface of a solid body is the locus of all points at which the positive and negative forces are in equilibrium.

Faraday's final version of a general metaphysics for natural science needs to be carefully distinguished from that of Boscovich. Faraday had gradually come to believe that only powers were real. Though he was none too careful in keeping the necessary distinction between 'forces' as the activity of powers, from the powers themselves, and sometimes uses the word 'force' when he clearly means 'power', his formulation of the field metaphysics is complete, except for the lack of a mathematical representation (Berkson, 1974).

Faraday's introduction of the 'field' concept into the metaphysics of electromagnetism, revived ideas popular in the late sixteenth century. William Gilbert (1600) introduced a concept of the magnetic field that is essentially modern. Every magnetic body is surrounded by an *orbis virtutis*, a sphere of power differently disposed at each point. A compass needle orients to the intrinsic directionality of the *orbis* at each point. Indeed, Gilbert believed he had shown experimentally that the behaviour of a compass needle could not be explained in terms of the attraction and repulsion between poles.

The culmination of Faraday's theoretical-cum-metaphysical reflections was the paper 'On the Physical Character of the Lines of Force' *Experimental Researches*, section 3244 (1839–1855). Faraday had two main arguments to show that his 'lines of force' as continuous curvilinear distributions of powers over a three-dimensional space must be the fundamental constituents of the universe.

Lines of force are inherently curved. If their curvature can be displayed by two independent methods of research, this argues for their reality. The alternative, that the curvature is a mere artifact of the method employed, seems to make the agreement of two independent methods too much of a coincidence. The curved lines appear in the effects of a magnet on iron filings spread on a sheet of paper. The second experiment depends on the law that a wire has a current induced in it only when it cuts a line of force. If the wire is moving along the direction of the line of force, no current will be induced in it. Lines can be traced out by following the path of null current. The lines arrived at by this method are also curved. Here is another and independent method of displaying the real curvatures of lines of force.

The physical reality of the lines of force is also demonstrated by the fact that when a current is flowing through a wire the magnetic field does not appear instantaneously, nor does it collapse immediately the current is turned off. Faraday drew an explicit parallel with the time dependent propagation of light.

3.1 The Dynamicist Metaphysics

In the dynamicist metaphysics three concepts are in play: dispositions (tendencies), forces and powers. The patterning of these concepts is something like this:

Powers are manifested in forces, and their possibilities of action in specific conditions are expressed as dispositions.

The simplest mathematical expression of a disposition is a vector. For example in kinematics a vector represents the counterfactual conditional: 'this is the speed and direction a particle would have taken had it been released from constraint at that point'.

A field is a spatial distribution of powers. Powers are manifested as forces, which would cause the phenomena described in the associated dispositions. Vector representation is a natural choice for representing powers. This the force and the direction in which it would act if the relevant releasing conditions were met. Powers are represented as force vectors and displayed in what ever observable effects these forces would have. So Faraday's 'lines of force' should be interpreted as Newtonian fluxions, the locus of instantaneous force vectors modulating according to Maxwell's Laws.

3.2 Finding a Transcendental Argument

Does this choice of metaphysics reveal a necessary condition for the possibility of physics? We note that both Discrete-Passive and Continuous-Passive are terminal formats for Descartes regress of motions to the creation. The efficacy of moving bodies is not intrinsic. It was 'injected' by God, who alone has originating powers.

Causal efficacy is displayed by something when removing an inhibition allows a pre-existing activity (potential) to be realized, somehow. It seems that so far either Corpuscularian or Dynamicist metaphysics could serve as the grounding of physics. Neither could claim to express the totality of necessary conditions for the possibility of a rational science of the material world.

However, there is an opening for a stronger argument. Events cannot be terminals, that is regresses cannot reach a sea of events as the fundamental constituents of the world. Since events are logically distinct no event could be the effective cause of another. Events are always actual. They do not pre-exist the conditions which release the forces which brings about the changes which constitute events. Since no event can pre-exist its coming to be, so no event could have potentiality. At best, the releasing of a tendency or power could be an event. The released power cannot be an event, since it is presumed to exist before the event that led to its release. The only remaining possibility left from the general scheme set out above is Discrete Active or Continuous Active, that is either powerful particulars or Faradayan fields.

So much for the transcendental domain of classical physics. Can anything similar be developed for the world as it is described by Schrödinger's Equation? Is there a 'causal powers' metaphysics implicit in quantum physics too?

The first point to be noticed is that Schrödinger's Equation distributes possibilities in time as well as space. Electrons, dispatched one at a time through a small hole, slowly fill out the expected distribution for a diffracted wave. The possibilities represented in solutions to the equation are realized over time.

However, Bitbol (1996) and Humphreys (1997) have drawn attention to Schrödinger's interpretation of 'quantum entanglements'. According to this interpretation at the level of subatomic behaviour any form of corpuscularianism is untenable (Schrödinger, 1995, pp. 9–36). The reason is simply that there no place among the 'terminals', that is among the fundamental constituents of the deepest level of the common regress of physics, for entities individuated at specific locations in space and time, or by a definite trajectory as if of a corpuscle. Schrödinger argued for a continuous manifold, particularly for the case where a whole system is in a pure state, that is fully specified by a Schrödinger Equation, but that the 'states' of what we take to be its component parts depend on one another, that is are 'entangled'. Humphrey is right to treat such systems as emergent wholes. This in turn must rest on a Bohrian account of the practice of experimentation.

Bohr's analysis of experiments involves the principle that apparatus and the world cannot be separated with respect to the results of a program of experimentation (Bohr, 1958). The phenomena are attributes of an artificial entity, the complex unity formed by the fusion of apparatus and world in the practice of experimentation. It follows that none of the attributes of such phenomena can be projected back on to the world. There are no electrons in the world in the absence of suitable apparatus to bring them into *distinctive being*. Properties can be ascribed only to apparatus/world complexes. So while electron trajectories are attributes of cloud chambers, the world/cloud chamber complex can be ascribed no more than a propensity to afford trajectories in cloud chambers. In the absence of all apparatus, the world has no properties that we could recognize. The complex affords that which is realized in a piece of apparatus, as a definite state or perhaps even a corpuscle.

It is a short step to a new post neo-Kantian transcendental deduction. What are the necessary conditions that an experiment should be possible? That the apparatus/world complex currently in use should afford observable phenomena to those who use it and perhaps to others too. The world of phenomena is largely corpuscularian, while the world of the noumena, the unobservable ground of being, must be a continuous field of potencies, powers coupled with potentialities. The world consists of causal powers. Our apparatus and we ourselves can realize only affordances (Gibson, 1979).

4 Conclusion

Bitbol and Humphreys treatment of Schrödinger's philosophy of physics goes a long way to resolving some of the ontological puzzles of quantum mechanics, but it does not go far enough. Schrödinger's Equation represents the affordances of various apparatus/world complexes, including, of course those formed by the interaction of the nervous systems of human beings with the world. Just as the world as it manifests in the behaviour of scientific instruments manifests itself more or less in terms of the Kantian categories, so too does the world as manifests itself to people. The world as we know it is emergent from the world as it is. The concept of an 'affordance' allows us to carry the insights of Kant's account of the natural sciences to experience in general. Cloud chambers afford tracks, which are themselves affordances for human beings constituted in certain way.

References

Berkson, W. (1974) *Fields of Force: The Development of a World View from Faraday to Einstein*. London: Routledge & Kegan Paul.

Bitbol, M. (1996) *Schrödinger's Philosophy of Quantum Mechanics*. London: Kluwer.

Bohr, N. (1958) *Atomic Physics and Human Knowledge*. New York: Wiley.

Boscovich, R. J. (1763) [1966] *A Theory of Natural Philosophy*. Cambridge, MA: MIT Press.

Brock, S. (2003) *Niels Bohr's Philosophy of Quantum Physics*. Berlin: Logos.

Bruner, J. S. (1983) *In Search of Mind*. New York: Harper & Row.

Gibson, J. J. (1979) *The Ecological Approach to Visual Perception*. New York: Houghton Miflin.

Humphreys, P. (1997) 'How properties emerge' *Philosophy of Science* **64**, 1–17.

Kant, I. (1786) [2002] *Metaphysical Foundations of Natural Science*. Trans. M. Friedman, in: *Theoretical Philosophy after 1781*, H. Allison, R. Brandt, P. Guyer, R. Meerbote, C. D. Parsons, H. Robinson, J. B. Schneewind, A. W. Wood (eds.), *The Cambridge Edition of the Works of Immanuel Kant in Translation*. Cambridge: Cambridge University Press.

Kant, I. (1787) [1999] *Critique of Pure Reason*. Edited by Paul Guyer & Allen W. Wood, in: H. Allison, R. Brandt, P. Guyer, R. Meerbote, C. D. Parsons, H. Robinson, J. B. Schneewind, A. W. Wood (eds.), *The Cambridge Edition of the Works of Immanuel Kant in Translation*. Cambridge: Cambridge University Press.

Michotte, A. (1963) *The Perception of Causality*. London: Methuen.

Polanyi, J. (1986) 'Reaction dynamics' *Science* **234**, 673.

Schrödinger, E. (1995) *The Interpretation of Quantum Mechanics*. M. Bitbol (Ed.) Woodbridge, CT: Oxbow Press.

Wittgenstein, L. (1979) *On Certainty*. Trans D. Paul and G. E. M. Anscombe, Oxford: Blackwell.

Determinism, Determination, and Objectivity in Modern Physics

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Abstract Kant's case for the objectivity of at least some of our experience is more threatened by the indeterminate than the indeterministic character of modern physics. Indeterminancy is a complex notion. It can be understood, ultimately, in terms of the failure of a "separability" principle, that objects can be individuated only with respect to non-vanishing spatial-temporal intervals. Its failure seems to follow from the fact that it is indispensable to the derivation of Bell's Theorem and that the conclusion of the Theorem is incompatible with well-established empirical results. But Kant's case for objectivity depends on it. The result is unsettling.

Much of the controversy about Kant in the perspective of modern physics has to do with the advent of indeterministic theories, and turns on the claim that they involve a "breakdown of causality" which is incompatible with Kant's argument in the Second Analogy. Since the argument in the Second Analogy sets out an important presupposition of the objectivity of at least some of our experience, it has often been held that indeterministic theories demonstrate that his account of objectivity must be rejected as well.

In an earlier paper (1994) I argued that there is an interpretation of the quantum theory, and more particularly of the probabilistic operators embedded in its equations, which is compatible with Kant's account of causality, and, more generally, that indeterministic theories as such do not undermine his account of the objectivity of experience. At the time, I indicated in passing that there were deeper and more difficult problems for Kant in connection with Bell's Theorem, and as a result of the experiments carried out to test its implications. I now want, once again in a rather schematic and non-technical way, to take up these problems and to indicate, less optimistically than earlier, how they might be resolved. It turns out that indeterminism does not challenge Kant's position (and its plausible account of objectivity) nearly so much as indeterminacy does, and that indeterminacy is a very complex notion.

Let me begin with some general and uncontroversial remarks, then pass to a very general feature of Kant's position, and finally isolate the elements of modern physics which are problematic from the perspective of that position.

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Judgments are objective in that they are made true or false by the objects to which they refer. Objectivity requires objects, and objects require objective reference. There are some obvious requirements on objective reference. Among them is the fact that the truth or falsity of an objective judgment must be independent of the *manner* in which the reference is made. A judgment is objective in *this* sense when the replacement of co-extensive parts (singular terms in particular) does not alter truth-value.

Another way of putting this is to say that objects must be independent of my *conception* or *perception* of them, which is just to say that in order to be "independent" (in the appropriate sense) the *same* objects must be conceivable or perceptible in different ways. But to talk about the *same* objects is to presuppose a set of well-defined identity conditions for them. The problem of objectivity is, at least in large part, the problem of identity.

Now how is identity to be understood? The classic account, of course, is Leibniz's. Two objects are identical just in case they have all and only the same properties. Since Leibniz thought that space and time could be construed in terms of relations on objects and events, he understood his Law to read: two objects are identical just in case they have all and only the same non-spatial, non-temporal properties. The application of this Law entails that individual objects properly so-called, "substances" in Leibniz's preferred vocabulary, are completely determined.

> It is in the nature of an individual substance or of a complete being to furnish us a conception so complete that the concept alone suffices to understand and to deduce all of the predicates of which the substance is or will become the subject. (*Discours de metaphysique*, VII)

We could put this in a slightly more contemporary way by saying that "a complete individual concept is a concept which contains, for every simple attribute, either this attribute or its negation, but never the two together" (Mates, 1986, p. 63). Kant himself puts the principle thus, in terms of the "proposition", *Everything existing is thoroughly determined… (CPR*, A 573/B 605; Kant, 1998, p. 554), although, characteristically, he at once reformulates this semantic principle as a cognitive condition: "to know anything completely, we must know every possible [predicate], and must determine it thereby, either affirmatively or negatively." We can call this the "completeness principle."

The "completeness principle" is at the core of Kant's distinction between "transcendental idealism" and "transcendental idealism." The somewhat complicated line of thought is as follows. "Transcendental idealism," Kant tells us (*CPR*, A 369; Kant, 1998, p. 426), is "the doctrine that [appearances] are all together to be regarded as mere representations and not as things in themselves…" If we take "appearances" as the objects of experience, bracket out their (otherwise misleading) identification with "representations," and assume that we can have knowledge only of those objects we can experience, then it follows that we cannot have knowledge of things in themselves.

But what are "things in themselves" and why can they not be objects of experience and hence of knowledge? The most plausible answer is given in the sections added to the Transcendental Aesthetic in the second edition of the *Critique of Pure Reason*. Kant there indicates that "everything in our [knowledge] that belongs to intuition… contains nothing but mere relations" (*CPR*, B 66; Kant, 1998, pp. 188–189). Since all of our knowledge involves the having of intuitions, it follows that all of our knowledge is relational in character. "Now through mere relations," Kant continues, "no thing in itself is [known]" (*CPR*, B 67; Kant, 1998, p. 189). Therefore, things in themselves cannot be known.

The point is entirely general. To the extent that our knowledge is of the relational properties of objects, it is not of these objects as they are in themselves, but only as they can be compared, and eventually measured. But it applies in particular to our knowledge of the temporal and spatial properties of the objects of experience. On the one hand, the determination of such properties, "longer than" for example, involves a comparison of objects one with another, on the other hand, their determination involves a comparison of objects with our measuring devices. That is to say, the spatial and temporal properties of objects could only be determined if there were at least two objects in the world, and a ruler and a clock (which themselves presuppose our presence as well).

Now the concept of a thing in itself is the concept of an object all of whose properties are non-relational in character. A thing in itself has whatever properties it has in independence from anything else; whether the world has one object in it or many or a human presence is indifferent to the properties of anything in it considered as it is in itself.

Kant's general point that we can have knowledge only of the relational properties of objects, and therefore not of the objects as they are in themselves, has an important corollary. It is that "appearances," in so far as their properties are relational, are never completely determined. For the determination of their properties involves, as we have just seen, the comparison of objects and eventually their measurement, and, among other things, these activities of ours cannot be carried out completely in a finite amount of time. It is with this sort of point in mind that Kant asserts that "in the explanation of the appearances of nature,…, much must remain uncertain and many questions insoluble, because what we know about nature is in many cases far from sufficient for what we would explain" (*CPR*, A 476–477/B 504–B 505; Kant, 1998, pp. 503–504).

"Empirical realism" is the view that we can have knowledge of objects that exist independently of our perception and conception of them. Kant shows that such objects "are *never* given in themselves, but only in experience, and that they do not exist at all outside it" (*CPR*, A 492/B 521; Kant, 1998, p. 512). That is to say that such objects are "appearances." This is the first premise of the argument. But, for the reasons already set out, "appearances" are not completely determined, the second premise. If at this point we invoke the "completeness principle," the proposition that everything existing is empirically determined, then "appearances" do not exist. But if they do not exist, then empirical realism is impossible.

This argument is evidently valid. Since its second premise, that "appearances" are not completely determined, follows from the way in which the "appearance"/"thing in itself" distinction is spelled out, we must deny its third premise, the "completeness principle," if we are to avoid its conclusion. This is precisely what Kant does, noting two points in the process.

The first point is that "thoroughgoing determination is… a concept that we can never exhibit *in concreto* in its totality…" (*CPR*, A 573/B 601; Kant, 1998, p. 554). This is so because the exhibition of a concept *in concreto* is by way of an intuition which necessarily has spatial-temporal properties, a "sensible" intuition. But as we have seen, spatial-temporal properties are relational in character. From which it follows that they can never be completely determined. Yet we have knowledge only of objects whose concepts can be exhibited *in concreto*. In other words, if we have knowledge, then the objects of that knowledge cannot satisfy the "completeness principle." We do have knowledge. Therefore, the "completeness principle" must be rejected.

The second point Kant notes about the "completeness principle" is that it can, and should, be reconstrued. Although it is not true, it has an important use. Or, as he would put it, although the "completeness principle" is not "constitutive" with respect to human experience, it is nonetheless "regulative." In short, it is properly to be taken as an *ideal*, indeed "the one single genuine [*eigentliche*]" ideal of which human reason is capable (*CPR*, 576/B 604; Kant, 1998, p. 556). It enjoins us to pursue the further determination of experience, on the understanding that there are no *a priori* limits to such further determination. The complete determination of these objects is the goal of all human inquiry. But it is no more than a goal. "The aim of reason with its ideal is… a thoroughgoing determination in accordance with *a priori* rules; hence it thinks for itself an object that is to be thoroughly determinable in accordance with principles, even though the sufficient conditions for this are absent from experience, and thus the concept itself is transcendent" (*CPR*, A 571/B 599; Kant, 1998, p. 553).

The upshot is this: if the "completeness principle" were true (as Leibniz and transcendental realists generally hold), then "empirical realism" would be impossible. But the "completeness principle" is false, a corollary of "transcendental idealism." Therefore, Kant concludes, transcendental idealism makes empirical realism possible (*CPR*, A 370–371; Kant, 1998, pp. 426–427).

We are at last in position to take up certain questions concerning determinacy and objectivity in modern [read: contemporary] physics. As we proceed, the character of Kant's "empirical realism" will have to be made more precise. But it needs to be made clear at the outset that "empirical realism" embraces what is generally referred to as "scientific realism." Kant is a scientific realist in just this sense, that we have good reasons for thinking that many of the statements made about theoretical objects are true. So much is clear from his remarks about our knowledge of the "magnetic matter pervading all bodies" (*CPR*, A 226/B 273; Kant, 1998, pp. 325–326), knowledge which rests on an inference from what we do perceive (iron filings being attracted) to what we do not perceive, the "magnetic matter" postulated to explain this attraction. Of particular significance is the fact that these remarks immediately precede his Refutation of [Empirical] Idealism in the second edition of the *Critique*. The case made for empirical realism is of a piece with his scientific realism. This said, contemporary physics [read: the quantum theory] raises questions about indeterminacy and objectivity on three progressively more difficult levels.

On the first level, indeterminacy is very general. Indeed, it is endemic to the concept of what it is, at least in certain paradigm cases, to provide a theoretical explanation. In these cases, to provide a theoretical explanation is to provide a reductive account of the phenomena. Two principles might be thought to characterize such reductive explanations:

> P.1 (the principle of micro-reduction): the properties of wholes are to be explained in terms of the properties of their parts.

> P.2 (the principle of property-reduction): the properties of these parts must differ from those of the wholes they are invoked to explain (for otherwise no genuine "reduction" has been carried out, and the resulting explanations are vacuous).

It was in this way that Galileo, for example, explained the temperature of a given volume of gas in terms of the motion of the particles, none of which were hot or cold, comprising it. The intuition involved has quite plausibly been taken as an important source of the persistent search for atomistic theories in science, and of the accompanying belief that the phenomena have not really been explained until such theories have been found. It dates, perhaps, from the time of Democritus. Heisenberg, evidently attempting to rationalize the otherwise puzzling character of the indeterminate character of quantum mechanical entities, expresses it as follows:

> It is impossible to explain…quantities of matter except by tracing these back to the behavior of entities which themselves no longer posses these qualities. If atoms are really to explain the origin of color and smell of visible bodies, then they cannot possess properties like color and smell…Atomic theory consistently denies the atom any such perceptible properties. (1937 , p. 119)

If we think, as the completeness principle has it, that any existent object must be thoroughly determined, and we grant that any scientific object cannot be so determined if it is to play an explanatory role, then we must conclude that scientific objects do not, because they cannot, exist. To disarm the argument we have only to deny its first premise, as Kant does. To do so is, once again, to maintain the correctness of transcendental idealism, which in this way makes empirical, [now read: scientific] realism possible.

This is admittedly a very simple and compressed account. But it goes some way toward resolving the difficulties which the indeterminate character of theoretical entities in contemporary physics might otherwise seem, in a general sort of way, to pose for Kant's account of objective experience. But it leaves the identity conditions for these entities open. What is to be said about them? Kant makes two points in this connection.

The first point is that Leibniz's Law sets out neither necessary nor sufficient conditions for the identity of objects. Thus the fact that objects indeterminate with respect to some of their properties fail to satisfy it does not preclude the possibility of their identity nor does the fact that they satisfy it entail that they are in fact identical. In the Amphiboly of Concepts, Kant insists on the fact that "even if there is no difference as regards the concepts," that is, even if their (non-spatial, non-temporal) descriptions are identical, it is still possible that at least two distinct objects satisfy them, witness the existence of incongruent counterparts.

The second point is that, as once again the example of incongruent counterparts makes clear, the possibility of assigning objects to spatial-temporal locations is enough to distinguish them. In the case of *physical* objects, spatial-temporal location is necessary as well. Thus two physical objects are distinct just in case they occupy different regions of space at the same time. But to this point in the discussion, it is possible to individuate and identify theoretical entities in this way. Hydrogen atoms are all pretty much alike; if you have seen one, you have seen them all. But some are here and others are there.

On the second level, indeterminacy is tied more directly to the quantum theory. It follows from the character of the matrix algebra in which the theory is usually expressed that the values of certain pairs of conjugate properties of objects cannot be determined to a degree of accuracy equal to or greater than a function of Planck's constant. In the most frequently cited example, a precise and simultaneous determination of the position and momentum of an elementary particle is ruled out in principle. Again, there are two points to make in connection with the sort of indeterminacy at stake here.

The first point is that this consequence of the so-called Heisenberg Uncertainty Relations does not preclude the possibility of a determination of an object's position, both spatial and temporal, to an arbitrary degree of precision. I have already admitted in Kant's behalf that not all of the properties of an empirical object can be determined, and *a fortiori* that not all of the properties can be determined precisely and simultaneously.

The second point is that the possibility of individuating objects does not depend on the fact that any of their properties in particular is definite, including their spatial and temporal properties. It is enough that the properties are mathematically well-defined and, as I argued in my 1994 paper, *objective*. The latter condition requires that whatever indefiniteness is expressed by the use of probability operators to describe states of objects (or systems of such objects) is not to be construed as a measure of our ignorance, but rather reflects certain intrinsic but indefinite properties as propensities. In this respect, I agree with Mittelstaedt (1994, p. 128): on the contemporary interpretation of quantum theory, we have to deal with "unsharp objects," which are "constituted in an approximate way. However, this restriction does not invalidate the objectivity of physical knowledge."

On the third and deepest level, indeterminacy is tied to the apparent impossibility of making out any sort of spatial-temporal separability, probabilistic or not, on which the possibility of individuating and identifying objects depends. This impossibility, which stems from reflection on Bell's Theorem and the experiments designed to test some of its implications, is implicit in the formalism of the quantum theory, but raises more general questions about the very possibility of objectivity.

Put in its very simplest terms, Bell's Theorem (Bell, 1964) is to the effect that, on certain basic assumptions about what Bell called "locality" and causality, one can derive an inequality concerning the correlations between the spins of two electrons. This inequality is incompatible with the spin correlations predicted by quantum mechanics. Moreover, the spin correlations predicted by quantum mechanics have been verified by a series of very careful experiments. Taken together, the basic assumptions from which Bell's inequality can be derived characterize what is often called "local realism" [Jarrett, 1989, pp. 61–62]. Given the incompatibility of the predictions to which it leads, first with the well-entrenched quantum theory and, more importantly, with the experimental results, "local realism" must be given up.

There are a variety of philosophically perspicuous derivations of the Bell inequality (see Jarrett, 1984; Cushing, 1989; Wessels, 1989). For my purposes, it suffices to list three of its usual premises. They more than the others are constitutive of "local realism." The first premise is determinism: physical systems (e.g., the two-particle system considered by Bell) are deterministic in the sense that, for any one instant, its state is physically compatible with only one state at each other instant. The second premise is locality (properly so-called): the state of a system is unaffected by events so removed that no light signal could connect them. Locality thus understood rules out the possibility of action-at-a-distance. The third premise is what Howard calls "separability" and Jarrett calls "completeness" (the formal expressions of locality and completeness can be found in Jarrett [1989]). In Howard's words, "the separability principle asserts that the presence of a nonvanishing spatio-temporal interval is a *sufficient* [and also necessary] *condition* for the individuation of physical states, and that the states thus individuated exhaust the reality that physics aims to describe, that physical systems are no more than the 'sums' of their parts" (Howard, 1989, p. 226), i.e., that the joint state of two previously interacting systems is simply the product of their separate states. It remains to say something about each of these premises in turn from the perspective of the transcendental philosophy. There is not space to be anything more than schematic.

Determinism. There are three things to be noted in connection with the determinist premise. First, Bell's Theorem can be derived without it (indeed, Bell himself did so). The experimental failure of the inequalities does not show that determinism must be given up. Second, the standard quantum-mechanical laws invoked to predict what actually happens in Bell-type situations are indeterministic. But, third, and as already indicated, my 1994 paper makes the case that Kant can accommodate this sort of indeterminism without much difficulty. Just so long as the indeterminacy is placed in the objects of our experience, and not in the uncertainty of our belief with respect to them, the experience itself is objective. Despite the many (to date completely unsuccessful) efforts to save some version of it, determinism is not an issue from the perspective of contemporary physics, nor from Kant's.

Locality. Until Jarrett's decisive clarification of the issue, it was commonly thought that there was a deep incompatibility between the Special Theory of Relativity and the Bell results. This is no longer the case (see Howard, 1989, p. 233, note 16). In the appropriate sense, the standard version of quantum mechanics is a "local" theory. Moreover, Einstein, whose thought-experiment was the motive for Bell's work, did not really put the emphasis on locality. This much is made clear in a later, more careful, statement of the argument.

> …it is characteristic of these physical things that they are conceived of as being in a space–time continuum. Further, it appears to be essential for this arrangement of the things introduced in physics that, at a specific time, these things claim an existence independent of one another, insofar as these things 'lie in different parts of space.' (Einstein, 1948)

So far as Einstein is concerned, *this* is what is incompatible with the set-up depicted in his thought-experiment, and it is *this* (and not "locality") which captures the essence of "completeness." Finally, in his unwavering commitment to action-at-a distance (see Kant, 1970, pp. 67ff.), Kant insists that "non-local" phenomena are possible. There is thus, at least initially, no problem for him in reconciling the apparent "non-locality" of the Bell results with his account of the objectivity of experience. At the same time, the ostensibly "local" character of the standard version of quantum mechanics *is* incompatible with his account.

Separability. If the Bell inequality derives from the assumption of determinism, locality, and separability, then, since Kant can accommodate indeterminism and non-locality, it follows that for him separability is the issue. But there is more to it. I want to make two points in particular about it. First, the passage already quoted from Einstein continues as follows:

> Without such an assumption [what we are calling separability] of the mutually independent existence… of spatially distant things, an assumption which originates in everyday thought, physical thought in the sense familiar to us would not be possible.

For Einstein, separability in the intended sense is the presupposition of the formulation and testing, hence of the objectivity, of physical laws. Second, Kant simply makes precise one way in which this claim is to be understood. There is no objectivity without objects. But, given the failure of the completeness principle, there are no objects except on condition that we assign them particular places at particular times. Such location becomes the criterion of their identification. But if such assignment is no longer possible, as seems to be the case in the wake of the Bell results, then there are no objects. To put it briefly, Kant parts company with the standard version of quantum mechanics with respect to locality and separability. But it is the latter which is crucial to his account of objectivity (in fact, the case made for the necessity of action-at-a-distance limps very badly). To the extent that objects are "entangled," they are no longer objects.

I want, finally, to suggest two ways out of the conflict between the transcendental philosophy and the apparent need to give up the separability condition. Neither of them is very attractive.

The first is suggested by Linda Wessels. She writes (Wessels, 1989, pp. 95–96):

> While the results of experimental and philosophical analyses of the Bell inequalities do require a significant departure from the way we standardly model physical objects, they have only minimal consequences for our conception of everyday objects and of most objects studied by science. For the Bell theorems give no reason to doubt that *these* objects can be studied as bodies… with objective properties… The Bell theorems show only that our traditional models are not satisfied by *all* objects in nature – in particular, they fail for objects and processes at the *micro*level.

But Kant rejects this sort of bifurcation of nature in the interests of a thoroughgoing realism. Such is the point made at A 226/B 273 (Kant, 1998, p. 326) of the first *Critique*, in a passage immediately preceding the "refutation of idealism," where he declares that "The grossness of our senses does not in any way decide the form of possible experience in general." There can be, in principle, no distinction to be made between any alleged "micro" and "macro" levels. An empirically realistic position must treat both in exactly the same way, as part of a unified picture of nature. As Kant insists, universality is an indispensable condition of objectivity, in moral as much as in theoretical philosophy.

A second way out of the impasse is suggested by Bohr. "There is no quantum world," he says, only an abstract quantum description. It is wrong to think that the task of physics is to find out how nature is. Physics concerns only what we can say about nature.

Insofar as Kant's idealism is left vague and undefined, he could take this tack. But in fact it is quite precise. At one level, of course, that of things-in-themselves, we cannot find out how nature is, for reasons advanced earlier. At another level, that of nature as usually understood, the task *is* to find out how nature is, and not merely to describe it. This is the core of Kant's empirical realism. Indeed, the presupposition of the Critical enterprise is that we do have knowledge of nature (as usually understood, the object of our experience); the question is, how is such knowledge possible?

It is possible to be more precise. It is part of Kant's position concerning the ideality of space and time that spatial and temporal quantities do not exist unless and until we have defined procedures for measuring them; the Categories, which embody certain presuppositions of the measuring procedures, endow space and time with a metric. This position can be generalized; for Kant, I think it is fair to say, there are no quantities in general apart from the procedures (including the instruments) by which we determine them, no quantities without quantifiers. In this respect, not all of the characteristics of various physical entities are "independent of our knowledge of them," and in this sense "real." But this is not to say that the properties of objects to which these quantities are ascribed do not exist unless and until they are measured, still less to say that physics is no more than a "description" of these quantities. For Kant it would make no sense that the properties themselves are not properties of objects or that they arise as a result of our interaction with the world. In particular, on his account of "objectivity" the properties which a scientific theory attributes to objects must be properties of those objects (although the way in which these properties are quantified depends on the measuring procedures we employ), and not simply a feature of the stories we tell about them. On Kant's view, the objectivity of the scientific view of the world is in part a function of the fact that we can distinguish sharply between the objects that we experience and ourselves as subjects of that experience, between reality and our description of it.

Presumably there are ways out of the difficulties that the Bell results pose for Kant's account of objectivity other than to bifurcate or idealize nature. One which occurs to me (but to none of my more knowledgeable friends) is to re-metricize the space in which the elementary particles at stake in the Bell experiments are located. As just indicated, Kant rejects the view that the metric of space is intrinsic; rather, it is imposed by us, as a condition of the possibility of experience. It might be possible to find a metric on which separability is retained in the face of the Bell results. But I have not yet been able to work out questions concerning the restraints imposed on the choice of possible metrics in the physical situation described by Bell (i.e., in the EPR set-up), and thus have no idea whether it is possible to retain separability in this way.

Otherwise, there seems little option to rejecting one of the premises in Kant's initial argument. We might now re-phrase that argument as follows:

1. There is no objectivity without objects
2. There are no objects without identity conditions for them
3. There are no identity conditions for objects if the principle of separability is rejected

Since the Bell results seem to require the rejection of the principle of separability (at least insofar as we want to retain locality, as on the standard version of quantum mechanics), it follows that there is no objectivity. The problem with giving up any of the premises of Kant's argument is that they are so deeply entrenched. If nothing else, reflection on the transcendental philosophy shows how unsettling the Bell results really are. Taken seriously, they would seem to require a new metaphysics of experience. So far as I know, no serious and systematic contenders for this title have yet appeared.

References

Bell, J.S. 1964. "On the Einstein-Podolsky-Rosen Paradox," *Physics* 1:195–200.

Brittan , G.G. 1994. "Kant and the Quantum Theory," in *Kant and Contemporary Epistemology*, ed. P. Parrini. Dordrecht: Kluwer, pp. 131–155.

Cushing, J.T. 1989. "A Background Essay," in *Philosophical Consequences of Quantum Theory: Reflections on Bell's Theorem*, eds. J.T. Cushing and E. McMullin. Notre Dame, IN: University of Notre Dame, pp. 1–24.

Einstein, A. 1948. "Quantemmechanik und Wirklichkeit," *Dialectica* 2:320–324. Quoted by Howard, 1989, 233.

Heisenberg, W. 1939. "Gedanken der antiken Naturphilosophie in der modernen Physik," *Die Antike* 13:118–124.

Howard, D. 1989. "Holism, Separability, and the Metaphysical Implications of the Bell Experiments," in *Philosophical Consequences of Quantum Theory: Reflections on Bell's Theorem*, eds. J.T. Cushing and E. McmMullin. Notre Dame, IN: University of Notre Dame, pp. 224–253.

Jarrett, J.P. 1984. "On the Philosophical Significance of the Locality Conditions in the Bell Arguments," Nous 18, 569–589.

Jarrett, J.P. 1989. "Bell's Theorem: A Guide to the Implications," in *Philosophical Consequences of Quantum Theory: Reflections on Bell's Theorem*, eds. J.T. Cushing and E. McMullin. Notre Dame, IN: University of Notre Dame, pp. 60–79.

Kant, I. 1970. *Metaphysical Foundations of Natural Science*, trans. J. Ellington. Indianapolis, IN: The Bobbs-Merrill.

Kant, I. 1998. *Critique of Pure Reason*, trans. P. Guyer and A. Wood. Cambridge: Cambridge University Press.

Mates, B. 1986. *The Philosophy of Leibniz*. Oxford: Oxford University Press.

Mittelstaedt, P. 1994. "The Constitution of Objects in Kant's Philosophy and in Modern Physics," in *Kant and Contemporary Epistemology*, ed. P. Parrini. Dordrecht: Kluwer, pp. 115–130.

Wessels, L. 1989. "The Way the World Isn't: What the Bell Theorems Force Us to Give Up," in *Philosophical Consequences of Quantum Theory: Reflections on Bell's Theorem*, eds. J.T. Cushing and E. McMullin. Notre Dame, IN: University of Notre Dame, pp. 80–96.

The Constitution of Objects in Classical Physics and in Quantum Physics

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Abstract In quantum physics as well as in classical physics we are usually concerned with observable quantities and their time dependence, but not with objects as carriers of observable properties. However, for establishing objectivity of our cognition in addition to the observable properties objects must be constituted in classical mechanics as well as in quantum mechanics. This problem can be traced back to the critical philosophy of Kant. Surprisingly, it became obvious only in recent years that the way to introduce objects systematically into the physical theories mentioned is essentially an adoption and realization of Kant's transcendental way of reasoning.

1 Introduction

The present article is concerned with constitution of objects in physics. It leads from Kant's transcendental arguments in the *Critique of Pure Reason* to the concept of objects in classical physics and in quantum physics. The investigations are based on the surprising observation, that the method of constituting objects in Kant's critical philosophy can be applied almost literally to classical physics, and with some small but essential restrictions also to quantum physics. Hence, one could get the impression that there is a strong continuity in the history of the foundations of physics. This is, however, not the case. Neither in the highly developed formulation of classical mechanics in the nineteenth century, nor in the original version of quantum mechanics, which was formulated in 1925–1932, the method of constituting objects was applied. Instead, objects were considered in both cases as elementary entities, as mass points, massive bodies, or particles, and these objects were inserted *ad hoc* into the already completely formulated theories. Only during the last decades in became obvious how objects can be introduced systematically into the two fields of physics mentioned and that this incorporation is in fact an adoption and realisation of Kant's original transcendental way of reasoning.

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2 The Cognition of Objects in Kant's Philosophy

2.1 Historical Preliminaries

In the *Critique of pure reason*[1] Kant formulated his transcendental philosophy in contrast to two alternative positions, the metaphysics of Leibniz and Wolff, and the empiricism of David Hume and Locke. In particular, for the problem of constituting objects the sceptical philosophy of Hume is most interesting as an opposite project. As to the question whether in addition to our direct perceptions there are objects in the external reality and what we know about these entities, Hume presented his opinion at different places. The importance of this problem becomes obvious, if we realise that the directly observable and usually time dependent qualities or predicates are per se not a criterion of an object, since the various properties are in general different from each other. Accordingly, Hume writes that this difference

> obliges the imagination to feign an unknown something, or original substance and matter, as a principle of union an cohesion among these qualities, and as what may give the compound object a title to be call'd one thing […].[2]

Indeed, the qualities that we observe are first of all completely independent of another and independent of a carrier whose properties they probably are:

> Every quality being a distinct thing from another, may be conceiv'd to exist apart, and may exist apart, not only from every other quality, but from that unintelligible chimera of substance.[3]

Hence, we must ask, whether the search for an object is nothing but hunting a chimera. Kant has taken up this question and answered it within the framework of his transcendental philosophy. However, this question must also be answered within the framework of any field of science that claims to be a realistic description of nature.

2.2 The Constitution of Objects in Kant's Philosophy

Also Kant's way of reasoning begins with the argument that our perceptions lead per se merely to the cognition of qualities but not to objects whose properties correspond to the observed qualities, and which are the time-independent carriers of time-dependent properties. In this point, Kant agrees with Hume. However, Kant doubts that objects are merely the products of our imagination and he gives two reasons. First, it is not always possible to relate several observed qualities to an object as their referent. This is only the case, if the observed qualities fulfil some

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[1] Kant (1998), CpR.

[2] Hume (1978) (1748), p. 221

[3] Hume (1978) (1748), p. 222

necessary conditions. With respect to this argument, Kant is more cautious than Hume. However, if the necessary preconditions mentioned are fulfilled, then, according to Kant, the object that persists in time in contrast to the time-dependent properties is an element of the objective and external reality – and not a "chimera". Obviously, in this point Kant exceeds Hume's empiricist position.

The constitution of "objects of experience" from our perceptions and observations starts with the requirement of *objectivity*. The observed qualities should not refer to the perceiving subject but to the objective, external reality, which is clearly distinguished from the subject. In order to apply this realistic interpretation to our observations, some necessary conditions must be fulfilled. It should be possible to order and to interpret the observed qualities according to some conceptual prescriptions, the categories of substance and causality. It should be possible to consistently relate the time-dependent predicates to a substance as the time-independent carrier of the predicates in question. And in addition, it should be possible to interpreting the temporal changes of the predicates as causal alterations of the properties of the object.

Kant does not state that an interpretation of this kind can always be applied to our perceptions and sensations. But he claims that if this is impossible, then there is no cognition at all.[4] However, if our observations refer to an element of the external reality and not to the observing subject, then the observations in space and time must have been ordered and interpreted according to the categories of substance and causality. In this way, "objects of experience" are constituted and the categories mentioned are necessary preconditions of these objects that – for this reason – fulfil the *a priori* laws of substance and causality. In other words, there is a time-independent carrier of time-dependent properties, whose temporal alterations obey some causal regularity. Hence, an object of experience is an element of the external reality that is clearly distinguished from the observer. The observable and changeable qualities can be related to this object, which itself is determined by a few unchangeable, permanent features.[5]

The categories of substance and causality belong to the necessary preconditions of objects of experience. However, the constitution of objects by means of these categories determines in general merely the kind of objects that are characterised by some permanent properties, but not individuals. For the determination of individual objects, we must extend the *formal* preconditions of experience, in particular the categories mentioned, by *material* preconditions of experience. The material preconditions of experience correspond to the material possibilities to perform observations of predicates and they extend the possibilities for constituting objects. In this context, Kant's "principle of complete determination", which applies to "things", becomes relevant:

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[4] Kant (1998), CpR, pp. 227–228.

[5] Here we could think of the mass, the form, etc. Cf. also Kant (1998), CpR, p. 379.

> Every **thing**, however, as to its possibility, further stands under the principle of **thoroughgoing determination**; according to which, among **all possible** predicates of **things**, insofar as they are compared with their opposites, one must apply to it.[6]

This principle does not follow from the preconditions of experience. However, if it can be fulfilled, it allows for further determination of objects. In particular, the position property pertains to a "thing" at any time. Objects that possess the position-property at any time will be called here "continuously localizable".

Even if we presuppose "continuous localizability", the determination of individual objects by their positions is not possible in general, since two objects that are equal with respect to all other predicates, could still be at the same place. Hence, for the determination of individual objects we must assume in addition, that objects possess the contingent property of *impenetrability*. Kant mentioned the possibility of individuation by means of the position property only casually and without taking account of the impenetrability, when he put forward his critique of Leibniz' "principium identitatis indiscernibilium":

> […] then the issue is not the comparison of concepts, but rather, however identical everything may be in regard to that, the difference of places of these appearances at the same time is still an adequate ground for the **numerical difference** of the object (of the senses) itself.[7]

Kant's considerations show, which necessary and which contingent preconditions must be fulfilled in order to consistently relate the observed qualities to an object as their carrier. This method of constituting objects at all, and in particular individuals, must be concretised in the various fields of natural sciences. We should not expect, that in these fields, objects can be determined in a less complicated way.

3 Objects in Classical Physics

3.1 Historical Preliminaries

Kant's reaction to the empiricism of David Hume had shown, in which way cognition of "objects of experience" can be achieved and that in spite of the sceptical arguments mentioned. Hence, one could guess that in classical physics, and in particular in classical mechanics, the Kantian way of reasoning would have been adopted in order to guarantee the objectivity of cognition in physics and to characterise the concept of a mechanical object in the transcendental way. This was, however, not the case.

In 1787, when the *Critique of Pure Reason* appeared, the most important field of physics was Newton's mechanics, first published in 1687, and further elaborated by d'Alambert (1758), Lagrange (1788), etc. From a philosophical point of view, this theory was, however, still exposed to the objections against a theory in the

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[6] Kant (1998), CpR, p. 553.

[7] Kant (1998), CpR, p. 368. Cf. also pp. 372–373.

sense of empiricism. In particular, this means that objects are not constituted within the theory on the basis of observable qualities, but inserted into the theory of predicates as primitive entities.

Since Hume's scepticism was one of the starting points of Kant's critical philosophy, a reformulation of classical mechanics on the basis of Kant's transcendental way of reasoning would have suggested itself. However, this idea is confronted with serious difficulties. Within the framework of Lagrange's formulation of classical mechanics, the constitution of objects would have been extremely difficult, if not impossible. Only in the new and more advanced formulation of classical mechanics by Hamilton (c 1835), there was some chance for applying the idea of constituting objects in the new "canonical formalism" of mechanics.[8] But even in this "phase–space" formulation of classical mechanics, not all the tools necessary for constituting objects were already available. We mention here, in particular, the theory of "Continuous Groups of Transformations", which was developed by Sophus Lie not before the end of the nineteenth century.

Except from these technical questions, we should mention that the philosophical situation had changed very much in the last decades of the nineteenth century. Neither Hume's arguments against objects nor Kant's reaction to this position were seriously discussed in the philosophy of physics. Instead, empiricism and positivism were considered as an adequate philosophical basis of physics. Kant's critical arguments, and in particular the idea of constituting objects, were almost ignored at that time. For these reasons, it is no surprise that classical mechanics was not reformulated in the sense of Kant's critical philosophy but considered as a field of science that is based philosophically on empiricism or on positivism.

The open systematic problem, how we can get objective knowledge of things or objects in physics, was treated not before 1963 and that first as a mathematical problem of quantum mechanics, in spite of the fact that in quantum mechanics the formal problems are even more difficult than in classical mechanics.[9] Here, we will not follow the historical development of physics but investigate first (in Section 3.3) how objects can be constituted in classical mechanics. Kant's arguments can be applied here almost literally. In a second step (in Section 4.3), we treat the same problem in quantum mechanics and find that we are confronted here with new difficulties unknown in Kant's philosophy and in classical mechanics.

3.2 *Objectivity and Invariance*

Classical mechanics describes the properties of classical objects and in particular the time dependence of properties. The mathematical framework of mechanics – in

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[8] It should be added that W.R. Hamilton (1805–1865) was quite familiar with Kant's Critique of Pure Reason.

[9] Cf.: Makey, G. (1963); Sudarshan, E.C.G. and Mukunda, N. (1974); Piron, C. (1976).

the Hamiltonian formulation – is the space of possible states of an object system, the "phase–space". Observables are then given by convenient functions on this phase–space. In this Hamiltonian formulation, classical mechanics is still exposed to the critique of the empiricism. Since the theory is concerned merely with observables and their time dependence the concept of an object as the carrier of the observable properties is almost void. Indeed, Hume's critique applies to this theory almost literally since an object is an "unknown something", a product of our "imagination", but not an element of the theory.

According to Kant, we start with the requirement of objectivity of our cognition. Similarly, in physics, and in particular in classical mechanics, our goal is the cognition of the external reality and not of the observing subject. Accordingly, observations or measuring results should refer to the external reality, and not to the observer and his impressions. Hence, the cognition of the external reality must be independent in some sense of the preconditions of the observer. The subjective, observer dependent component of an observation or a measurement result is given by the space–time coordinates of the observer. Hence, the requirement of objectivity means that the laws of the external reality must fulfil some *invariance* properties.[10] If an observer changes his space–time coordinates, then the observations should be changed such that they refer to the same but equivalently changed object. The same changes can also be obtained, if the object is subject of an *active* transformation that corresponds to a *passive* transformation of the coordinate system. Weyl illustrates this symmetry of active and passive transformations by a simple geometrical example.[11] For geometrical objects like triangles in the Euclidean plane, we have always symmetry between active and passive transformations. However, for physical objects this symmetry is a necessary precondition of their objectivity.

Within the context of classical mechanics, these relations can be made more explicit. The fundamental laws of classical mechanics are invariant with respect to the transformations of the ten-parameter Galileo group G. For a given inertial frame of reference, these transformations consist of three translations in space, three rotations in space, three changes of the constant velocity of the inertial system and one translation in time. If the observer is "moved" in accordance with a Galileo transformation, then the observations, which refer to the external object, will transform "covariant" with respect to this transformation. Since also the observers, represented by measurement instruments are physical objects, they will be subject to the same invariance laws. This implies a symmetry between active and passive transformations: The transformation of the measurement results does not depend on whether the observer is moved according to a Galileo transformation or whether the object is moved according to the inverse transformation.

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[10] This point was emphasised first by Weyl (1927).

[11] Weyl (1927), pp. 88–89.

3.3 Covariance and Observables

The symmetry between active and passive transformations allows for clarification of the concept of an "observable". Intuitively, an observable is a measurable quantity or a property of an object system S, that belongs to the external reality and that is clearly distinguished from the observer and the apparatus, respectively. "Properties" (or predicates) may pertain to the object or not, and hence they correspond to value definite yes–no propositions P_i or to the most simple observables with only two values 0 and 1, say. The set $\{P_i\}$ of elementary propositions can be extended by introducing the logical operations $\wedge$, $\vee$, $\neg$, and the relation $\leq$. In this way, we arrive at the full propositional system of classical mechanics, which is given by the complete, atomic and Boolean lattice L_C of classical logic.

One can then define an "observable" as a relation between numbers on the reading scale of the apparatus and properties of the object system. Hence, an observable may be considered as a mapping Φ from the Borel sets $B(\mathfrak{R})$ of the real numbers $\mathfrak{R}$ (of the reading scale) to the Boolean lattice L_C of propositions. An observable is connected with the group G of Galileo transformations in a twofold way. On the one hand, the properties of the system *S* are changed by an *active* transformation, when the transformation group acts on the system and its propositional lattice. On the other hand, the coordinate system of the observer is changed by a *passive* transformation, when the transformation group acts on the measurement device *M*, i.e. on the Borel sets of the reading scale.

Within this conceptual framework, the symmetry between active and passive transformations leads to the following important *covariance postulate* (C), which must be fulfilled by an observable Φ that can be interpreted as a property of a really existing object: The properties $\Phi[B(\mathfrak{R})]$ of the object *S* that are *actively* transformed by a representation S(G) of the *Galileo* group must coincide with the properties $\Phi[B(\mathfrak{R})]$ that one obtains from Borel sets $B(\mathfrak{R})$ (of the reading scale of the apparatus *M*) that are *passively* transformed by a representation M(G) of the *Galileo* group. This means that the diagram in Fig. 1 must "commute". The covariance postulate (C) determines those functions Φ, which may be considered as "observables" and it shows, how these observables are transformed under a special transformation.[12]

On the basis of the covariance postulate (C) and the Galileo group, one can now define the fundamental observables p (momentum), q (position) and the observable t (time). In this way, the basic quantities (p, q, t) of the state space can be shown to be "observables" in the sense of the covariance postulate. If an object of classical mechanics is understood as a carrier of properties, then it is obviously sufficient, to require that it is a carrier of the fundamental observables p, q, and t.

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[12] Considerations of this kind can be found in Makey (1963) and in Piron (1976). The connection with Kant's philosophy is established in Mittelstaedt (1994) and (1995).

$$\begin{tikzcd} \underline{M} & \underline{S} \\ \mathrm{B} \arrow[r, "\Phi"] \arrow[d, "\mathrm{M(G)}"'] & L_{\mathrm{C}} \arrow[d, "\mathrm{S(G)}"] \\ \mathrm{B} \arrow[r, "\Phi"] & L_{\mathrm{C}} \end{tikzcd}$$

Fig. 1 Covariance diagram of classical mechanics

3.4 Classical Objects

A *classical* object is a carrier of the properties $P \in L_C$, not only in one contingent situation K given by the observers system of coordinates, but also in all other situations K' that evolve from K by Galileo transformations – where the properties are transformed under these transformations according to the covariance postulate. Mathematically, these objects are representations of the Galileo group. One can further specify this concept by considering different classes. Elementary systems, say, are given by irreducible representations of the Galileo group. For elementary systems that correspond to mass points without geometrical structure, there are no *true* but only *projective* representations of the group G. These representations are characterised by one continuous parameter *m*, which can be interpreted as the "mass" of the object. The next, slightly more general system is a rotating system, with three additional degrees of freedom which correspond to the components of the internal angular momentum.[13]

3.5 Individual Systems

The representations of the Galileo group characterise classes of objects with the same permanent properties. In order to denote an *individual* system one has to find additional properties that distinguish the system S in question from all the other systems of the same class. Firstly, one has to make clear, whether the triple (p, q, t) is a unique denotation of S, i.e. whether there is only one system with these properties. Secondly, if *uniqueness* is given, one has to find out in which way the system S defined at time t can be reidentified at some later time t'. In order to guarantee *uniqueness* of S one needs an additional dynamical principle that excludes that two systems are at the same time t at the same phase point (p, q). Clearly, this postulate

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[13] For more details cf. Sudarshan et al. (1974), pp. 389ff.

is fulfilled if *impenetrability* in position space is given. In order to guarantee also the *reidentifiability* of the system S uniquely defined at time t, at a later time value t′, one needs a convenient law which connects the point $(p, q)_t$ in phase space (at time t) with the phase point $(p, q)_t'$ (at any other time t′). In classical mechanics, a dynamical law of this kind is given by a Hamiltonian H(p, q) and the canonical equations. This means that an individual system S can be reidentified at any other time value t′ by the (p, q)-values on its dynamical trajectory in phase space. Both requirements for individual objects, the *uniqueness* and the *reidentifyability* are usually guaranteed in classical mechanics. Hence, we can name an individual system S permanently by an arbitrary point (p_t, q_t) on its trajectory.

4 Objects in Quantum Mechanics

4.1 General Remarks

In the "Copenhagen interpretation" of quantum mechanics Niels Bohr made use of an empiricist view and considered only measurement results but without assuming that the observed predicates can be attributed to an object as its properties. Bohr used this "Copenhagen interpretation" not for philosophical reasons, but since the assumption of objects, as carriers of properties is – in general – incompatible with quantum mechanics. The reason why the incorporation of objects is impossible is, that quantum systems are not subject to the "principle of complete determination". Quantum theory of measurement does not allow for determining jointly all possible properties of a given system. In any contingent situation, described by a state Ψ, only a subset P_Ψ of properties P^i can be measured jointly on the system S. The properties $P^i \in P_\Psi$ are mutually *commensurable*, i.e. they can be measured in arbitrary sequence without thereby changing the results of the measurements. The measured properties can be related to the object system just as in classical mechanics. Hence, we refer to these properties as the "objective" properties of the system in the state Ψ. However, for any state Ψ there are also non-objective properties $P^i \notin P_\Psi$ whose measurement changes the state Ψ of S.

In quantum physics as well as in classical physics for the constitution of objects we to begin with the requirement of objectivity. The observed predicates should refer to an object as its properties. Again, this requirement leads to the necessary preconditions of any objective experience, the categories of *substance* and *causality*. However, in the present case the *material* preconditions of classical experience are not fulfilled, since the systems are not "completely determined". From these arguments it follows that the causality law in quantum mechanics holds only for the set P_Ψ of objective properties of the state Ψ(t) at a time value t. The time development of this state is governed by the Schrödinger equation and the state Ψ(t) determines the state Ψ(t′) at any later time t′. However, since the state Ψ corresponds only to the restricted set P_Ψ of objective properties, at different time values we have different sets of objective properties. Hence, it will in general not be possible to establish a causal connection between a property P^a (t) at time t and the same property P^a (t′) at a later time t′. Consequently, there is only a very limited *quantum causality* law between the objective properties P_Ψ and P_Ψ' at different time values.[14] Also, Kant's law of the conservation of substance cannot be valid for "all appearances" and must be restricted to the objective properties $P_{\Psi(t)}$.

4.2 Objectivity and Invariance

In principle, the same way of reasoning which allows for the constitution of objects in classical mechanics can be applied to quantum mechanics. As in classical mechanics, also in quantum mechanics we are interested in the cognition of the external reality and not in the observing subject. This leads again to the requirement of *objectivity* which means that the fundamental laws of physics are subject to a group of symmetry transformations. Different observers, which are connected by transformations of the invariance group, will then describe the same object of the external reality. The invariance group is again the Galileo group G. The observer corresponds to a classical apparatus, which is associated with a space–time coordinate system. For this reason, the meaning of a passive Galileo transformation is quite similar to the classical case. Different observers are connected by transformations of the Galileo group and the measuring results will then transform "covariant" with respect to these transformations.

As in classical mechanics, observables will be characterised by their covariance with respect to the Galileo group. A Galileo covariant observable can be defined as self-adjoint operator or a projection valued measure Φ. Observables of this kind allow for measurements of properties, they are, however, subject to the well-known complementarity restrictions. The properties of a quantum system S at a some time that correspond to yes–no propositions P_i are given by subspaces of the Hilbert space of the system, or by projection operators. If the set $\{P_i\}$ of propositions is extended by the quantum logical operations $\wedge$, $\vee$, $\neg$, and the implication relation $\leq$, then one arrives at the complete, atomic and orthomodular lattice L_Q of *Quantum Logic*.[15]

A quantum mechanical observable Φ is a relation between pointer values Z on the reading scale of the apparatus M and properties of the object system S. Accordingly, an observable is a mapping $\Phi: B(\Re) \rightarrow L_Q$ from the Borel sets $B(\Re)$ on the real line $\Re$ to the propositional lattice L_Q, i.e. a projection valued measure. An observable is again connected with the invariance group G in a twofold way. The transformation group acts *actively* on the system changing its properties and it acts *passively* on the measuring outcomes corresponding to Borel sets $B(\Re)$. The principle of covariance implies again the equivalence of *active* and *passive* transformations.[16,17]

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[14] Cf. Mittelstaedt (1994).

[15] Cf. Mittelstaedt (1995).

[16] C. Piron (1976) p. 93 ff.

[17] Mittelstaedt (1995).

$$\begin{tikzcd}
\underline{M} & \underline{S} \\
B \arrow[r, "\Phi"] \arrow[d, "M(G)"'] & L_Q \arrow[d, "S(G)"] \\
B \arrow[r, "\Phi"] & L_Q
\end{tikzcd}$$

Fig. 2 Covariance diagram of quantum mechanics

Hence, the image $\Phi(Z')$ of a transformed pointer value Z' agrees with the transformed image $\Phi(Z')$ of the pointer value Z, i.e. the diagram in Fig. 2 “commutes”.

The difference between the covariance postulates of classical and quantum physics consists in the different propositional systems L_C and L_Q. The general concept of an observable can again be specified by the fundamental observables q (position), p (momentum) and t (time).

4.3 Quantum Objects

A quantum objects is a carrier of the properties $P \in L_Q$, not only in one contingent situation K, given by the observers space time coordinates, but also in all other situations K’ that can be obtained from K by Galileo-transformations – where the properties $P \in L_Q$, are transformed covariant under these transformations. In spite of the similarities in the method of constitution, between classical objects and quantum objects there are striking differences that come from the different lattices L_C and L_Q. The propositional system L_C is a complete atomic Boolean lattice. Hence, the object S possesses any property $P \in L_C$ either in the affirmative or in the negative sense, i.e. the object S is *completely determined*. In contrast to this well known situation, a quantum object S possesses at a certain time t simultaneously only a limited class of commensurable properties given by elements of a Boolean sublattice of L_Q. Hence, a quantum system is only carrier of a class of mutually commensurable properties. One can again specify this concept by considering different classes. Elementary quantum systems are given by irreducible unitary representations of the Galileo-group. For elementary objects, there are only projective representations that are characterised by one continuous parameter *m* which can be interpreted as the mass of the quantum object and which characterises a certain class of objects.

4.4 Individual Quantum Systems

The characterisation of individual objects in quantum mechanics provides problems, that are different from those discussed by Leibniz, Locke, and Kant. The reasons are that – in contrast to Leibniz – the essential properties are not sufficient for the characterisation of an object and that – in contrast to Locke and Kant – the totality of all accidental properties that were needed for the individualisation is not simultaneously available. Since only *some* classical properties pertain simultaneously to a quantum system, the determination of quantum systems by their accidental properties is never complete. Hence, the characterisation of individual quantum systems by their *permanent* properties fails since the permanent properties define classes of objects, and the characterisation of individual systems by their *accidental* properties cannot be applied, since the accidental properties are not simultaneously available.

In classical physics, the determination of individuals requires *uniqueness* and *reidentifiability. Uniqueness* can be achieved only by a property that is subject to some "generalised impenetrability" which means, that two numerically different objects cannot possess the same value of that property. *Reidentifiability* means that a measurement of the "individuation property" must be repeatable, since otherwise an object, which was determined by this property at a time t could not be re-identified at a later time t′. Since *impenetrability* is known to hold for the position observable, in classical physics the position property is used for the determination of individuals. Since *repeatability* does not provide serious problems, for the permanent characterisation of objects trajectories can be used.

In quantum mechanics, the position observable fulfils the impenetrability requirement too and it fulfils the covariance condition with respect to the Euclidean group. However, in the quantum theory of measurement it is well known that repeatability implies discreteness of the measured observable.[18] Since the position observable is continuous, it cannot be measured repeatable and hence it is not possible to re-identify an object by measurement of its position. There are, of course, procedures to discretize a continuous observable. However, a discretization of the position observable would destroy the covariance with respect to the Euclidean group. It is obvious, that the Euclidean covariance must be fulfilled if the position observable shall pertain to the system as an objective property. Hence, individual objects cannot be determined in quantum physics.

5 Concluding Remarks

The transcendental way of reasoning, which was applied by Kant, shows in which way constitution of objects can be achieved. Kant formulated the necessary preconditions that must be fulfilled by the received data if they represent the cognition of an object. We applied these arguments to two different domains.

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[18] Busch et al. (1996).

1. In the domain of classical physics the transcendental strategy can exactly be applied and it leads to the constitution of classical individual objects, provided these objects are impenetrable or characterised by another uniquely determined property.
2. In the domain of quantum physics, the same strategy leads to the constitution of quantum objects, but only to classes of the same kind and not to individual objects. In quantum physics, the constitution of individual objects in the strict sense is not possible.

References

Busch, P., Lahti, P. and P. Mittelstaedt (1996), *The Quantum Theory of Measurement*, 2nd ed., Heidelberg, Germany: Springer.

Hume, D. (1978), *A Treatise of Human Nature*, L. A. Selby-Bigge and P. H. Nidditch, eds., Oxford: Clarendon.

Kant, I. (1998), *Critique of Pure Reason*, Translated by P. Guyer and A. W. Wood. Cambridge: Cambridge University Press (Cited: CpR).

Makey, G. (1963), *The Mathematical Foundations of Quantum Mechanics*, New York: Benjamin.

Mittelstaedt, P. (1994), *The Constitution of Objects in Kant's Philosophy and in Modern Physics*, in: P. Parrini, ed., *Kant and Contemporary Epistemology*, Dordrecht, The Netherlands: Kluwer, pp. 115–129.

Mittelstaedt, P. (1995), *Constitution of Objects in Classical Mechanics and in Quantum Mechanics*, International Journal of Theoretical Physics, **34**, pp. 1615–1626.

Piron, C. (1976), *Foundations of Quantum Physics*, Reading, MA: W. A. Benjamin.

Sudarshan, E. C. G. and Mukunda, N. (1974), *Classical Dynamics: A Modern Perspective*, New York: Wiley.

Weyl, H. (1927), *Philosophie der Mathematik und Naturwissenschaft*, English transl.: *Philosophy of Mathematics and Natural Sciences* (1949), Princeton, NJ: Princeton University Press.

Laws of Nature: The Kantian Approach

Giovanni Boniolo

Abstract The problem of the laws of nature inside a Kantian perspective is analyzed. In this way I try also to cover a lack in the contemporary debate on this issue: a debate that, almost totally and incredibly, has neglected, and is neglecting, the Kantian position. In particular, I show how the three Kantian levels (the transcendental, the metaphysical, and the empirical one) are connected and how the problem of nomologicity versus accidentality could be solved at the empirical level.

1 Lawlikeness, Lawness, Lawfulness

In *The Structure of* Science, E. Nagel is extremely pessimistic about the possibility of explicating in a rigorous way the concept of "law of nature":

> There is […] more than an appearance of futility in the recurring attempts to define with great logical precision what is a law of nature – virtue of its possessing an inherent "essence" which the definition must articulate. For not only is the term 'law' vague in its current usage, but its historical meaning has undergone many changes. (Nagel, 1961, pp. 49–50)

If we seriously considered this suggestion, there would be little point in working on this topic. Actually Nagel's graphic and discouraging judgment can be considered as the honest conclusion that a good observer of the discussions on laws among the German and Austrian neo-positivists and the American post-positivists must arrive at.

After 1961 and Nagel's dismal epigraph on the failure of the neo-positivist and post-positivist attempts to grasp nomologicity, something apparently strange happened. While both neo-positivist and post-positivist attempts were strongly characterised by an emphasis on the logical structure of the statements supposed to

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be laws, and on a radical refusal of both modal approaches and metaphysical solutions, after the 1960s both the modal and the metaphysical way have been brought up again, as if all the previous caution was forgotten. This passage from a strong anti-metaphysical position to a strong metaphysical one was made possible by a sort of inter-kingdom during which supporters of the counterfactual approach played the role of joining ring.

Needless to say, the new metaphysical attempts have been severely criticised, not least because the memory of the anti-metaphysical heritage had not completely vanished. Nevertheless all this debate has been characterised by a surprising omission of the Kantian approach.[1] This is not the right place to inquire into the sociological and historical reasons for this oblivion. Our aim is rather to recall what Kant said in order to show the relevance of his suggestions.

To begin with, a terminological remark is in order. There is a difference between: (1) lawlikeness; (2) lawness; (3) lawfulness. *Lawlikeness* regards the fact that certain statements, in particular the universal conditionals, have the form of a law, even if it is not said that they are laws. *Lawness*[2] has to do with the necessary and sufficient conditions characterising a law. Therefore lawness is what characterises a lawlike statement to be a law. What I will call "Schlick's problem" is exactly the problem concerning lawness, that is, using different words, nomologicity. Finally *lawfulness* will be used to indicate, with Pearson (1892, p. 72), "what is not prohibited by the law". Thus I will speak of the lawfulness of nature to indicate what is allowed and not allowed in *nature*.[3]

2 The Failure of the Humean and Pre-Humean Attempts

One of the purposes of the philosophers belonging to the composite neo-positivist movement concerned the search for a rigorous criterion demarcating the not cognitively significant statements from the cognitively significant statements. As we know, it was thought that such a criterion could be identifiable by means of empirical verification. Immediately a new problem arose: the laws of nature are universal statements and therefore never verifiable in a conclusive way. What, then, should we do?

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[1] For Kant's writings, I use the English translations indicated in the references. However I have compared them with I. Kant, *Werkausgabe*, edited by W. von Weischedel, 1956–1964, Suhrkamp, Frankfurt a.M. 1968, voll. I–XII. In particular, I quote (1) *Kritik der reinen Vernunft* by (KdrV, p. xx, Byy); (2) *Prolegomena zu einer jeden künftigen Metaphysik, die als Wissenschaft wird auftreten können* by (P, p. xx, yy); (3) *Metaphysische Anfangsgründe der Naturwissenschaft* by (M, p. xx, yy); (4) *Kritik der Urteilskraft* by (KdU, p. xx, yy); (5) *Logik* by (L, pp. xx). Where xx indicates the pages of the English translation used, and yy the pages, if quoted, of the original text to which the translation refers. Note that I do not use the first edition of the *Kritik der reinen Vernunft*. Laws of nature and the relevance of a Kantian approach in the philosophy of science are discussed in more details in Boniolo (2007).

[2] This is my neologism. Some authors prefer to speak about *lawhood* (cf. Vallentyne, 1988).

[3] Note that 'nature' in this context must be interpreted in a Kantian way.

This was the problem tackled by M. Schlick in his 1931 ‘Die kausalität in der gegenwartigen Physik’. During his discussion he investigated the possibility of introducing what, since then, has been called *Maxwell’s requisite*, according to which in the laws of nature values of the space–time coordinates must not appear (cf. Maxwell, 1873, p. II). He went on to identify the following problem:

> So far as I can see, it would be imaginable, for example, that regular measurements of the elementary quantum of electricity (electron charge) would yield values for this quantity that fluctuate up and down quite uniformly by 5%, in say 7 hours, and then another 7 hours, and then 10 hours, without our being able to find even the slightest ‘cause’ for this; and perhaps there would be another fluctuation on the top of this, for which an absolute change of the earth’s position in space would be held responsible. The Maxwellian condition would then no longer be satisfied. (Schlick, 1931, pp. 181–182)

Then Schlick went on to suggest that the prediction of new data could be the right mark for a law to count as such. Though necessary, the criterion of the empirical confirmation reveals some problems, since the confirmation of a single prediction implies neither the definitive verification of the law nor the certainty that a causal relation really exists in the world (idem, p. 51).

Schlick’s way out of this conundrum was extremely interesting. On the one hand, laws cannot be considered real scientific statements since they cannot be conclusively verified. But on the other hand they cannot even be simply expunged as not cognitively significant. Instead they must be seen as schemes to build up singular statements: the only ones that can be conclusively verified.[4]

However, Schlick considers the empirical confirmation of the predictions as the core requisite to demarcate a statement expressing a nomological regularity from a statement expressing an accidental regularity (I call it *Schlick’s criterion*), that is, using a late terminology, to solve the problem of the difference between a nomological universal statement and an accidental universal statement (I call it *Schlick’s problem*).[5]

There are two main reasons why this 1931 essay remains extremely important for the contemporary setting of the question concerning laws of nature. *First*. Schlick completely changes the physiognomy of the problem related to the nature of laws. To be sure, in the pre-neo-positivist period, chiefly within the European epistemological debate at the end of the nineteenth century, that problem had been tackled, but it had been dealt with from an essentially gnoseological point of view, that is, mainly the cognitive role of laws was discussed. With Schlick the problem concerns the determination of the necessary and sufficient conditions to characterise lawness (*Schlick’s problem*).

Second. Schlick’s work brilliantly exemplifies the research tradition of the neo-positivists, the post-positivists of the “standard view”, and the new regularists. It is a strong Humean tradition, which refuses a causal structure of the world, and at the same time considers as necessary, in a purely epistemological sense, the nomological

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[4] Of course in 1931 Schlick cannot be aware of the critics by Popper and Carnap to the impossibility of conclusively verifying a singular statement.

[5] Schlick writes explicitly about the possibility that there are regularities due to chance (idem, p. 184).

statements describing the empirical regularities. The problem, for such a tradition, is not to understand why laws of nature are metaphysically necessary, but to differentiate the nomological regularities from the accidental regularities. Those who accept this approach, that is, the *classical regularists* (for example, Schlick, Reichenbach, Nagel, Pap) and the *new regularists* (for example, van Fraassen), will engage in a programme aimed at finding either logico-classical conditions (for example, Reichenbach, Nagel, Pap), or modal conditions (for example, A. Burks), or pragmatical conditions (for example, N. Goodman), or structural conditions (for example, van Fraassen). Those who do not accept this kind of program, that is, the anti-Humeans (the intensional realists such as D. Lewis; the necessitarists, such as W. Sellars, R. Pargetter, S. McCall, and P. Vallentyne; and the realists on the universals such as W. Kneale, D. Armstrong, F. Dretske, and M. Tooley) on the contrary take up a research programme addressed to show that laws are strictly connected to metaphysical nomologicity. It is this duality between Humean regularists and anti-Humean realists that has monopolised the debate on the laws of nature. Now, what about Kant? He was more or less neglected.

3 Back to Kant

3.1 The Status Quaestionis

Dealing with the topic of laws of nature in Kant's critical works is not an easy task, especially if we consider the difficulties regarding the connection of the various levels within which the discussion about laws is carried out. There is (1) the transcendental level, almost exclusively dealt with in the *Kritik der reinen Vernunft* (1781 and 1787); (2) the metaphysical level in the *Metaphysische Anfangsgründe der Naturwissenschaft* (1786); (3) the empirical level, particularly discussed in the *Kritik der Urteilskraft* (1790). In what follows, I will offer an overview of Kant's position concerning these three levels.

Let me make one last preliminary note. We must specify some lexical aspects, especially related to the terms *Gesetzlichkeit* and *Gesetzmässigkeit*. Both stem from *Gesetz*, a term which in the 1700 German and, thus, also in Kant's language, means 'law' in the sense of something imposed. It differs from *Recht*, i.e., from 'law' seen as a command resulting from an agreement among independent individuals. In other words, the difference between *Gesetz* and *Recht* is precisely the Latin difference between *lex* and *jus*. So Kant uses *Gesetz* when talking about law of nature and moral law, while he uses *Recht* when dealing with political issues (cf. Krieger, 1965).

Both *Gesetzlichkeit* and *Gesetzmässigkeit* are used by Kant in the sense of "to be in accordance with a law," "to be allowed by a law." Nevertheless, since the first is present only a few times in all his writings, this specification could be considered useless. Actually there are two different questions which Kant, because of obvious historical contextual limitations, does not distinguish. The first concerns the conformity to a law of something extralinguistical, i.e., its *lawfulness*. In this sense, nature is lawful, that is, it lies in accordance with a law. The second regards the universal statements. As we can see, while each law is a universal conditional statement, not each conditional universal statement is a law. While the term *Gesetzmässigkeit* is appropriate for the idea of lawfulness of something extralinguistical, the term *Gesetzlichkeit* is appropriate for the idea of nomological validity of an universal conditional statement, i.e., for its being law, or for its *lawness*. Both problems are present in Kant, with the difference that the first, concerning lawfulness of nature, is explicitly discussed, while the other, regarding lawness, i.e., the nomological validity, of a lawlike statement, is discussed only implicitly.

Considering these differences, it should be noted that the issue of lawfulness is present at the transcendental, at the metaphysical, and at the empirical level. The issue of the nomological validity is present only at the empirical level. In fact this is not surprising. Suffice it to recall that it is related to the distinction between accidental universal statements and nomological universal statements. This is not a problem at the transcendental level, because at this level, which universal statements are to be regarded as laws is well known: these are those which make the application of the categories possible, that is, the pure principles of understanding. Nor is it a problem at the metaphysical level: here the universal statements considered to be laws are precisely the metaphysical principles. The question takes on a different dimension at the empirical level, where it seems to be extremely problematical to sharply differentiate the empirical laws, i.e., the nomological universal statements, from the accidental universal statements.

3.2 The Transcendental Level

3.2.1 The Lawgiving Understanding

As is well known, Kant considers the faculty of understanding (*Verstand*) from many different angles: as spontaneity of knowledge, as faculty of thinking, as faculty of concepts, as faculty of judging (*Vermögen zu Urteilen*), as faculty of knowledge, and as faculty of rules. For understanding has the faculty of producing by itself the relevant representations. Moreover, if it is considered as faculty of thinking, since thinking, in this sense, means knowing by concepts, therefore it is, on the one hand, faculty of knowing, and, on the other hand, faculty of judging, because concepts are predicates of possible judgements (this is the *Leitfaden* that makes it possible to infer the table of the categories from the table of the judgements). Finally, it is a faculty of rules, since concepts are rules.

At this point it is worth recalling that in the mid-1700s, philosophers addressing the epistemological problem of knowledge-formation, proceeded along two possible ways: (1) the object is given to us inductively, in order to be represented in its theoretical bareness; (2) the knowing subject knows the object producing it completely through its own representations (cf. KdrV, pp. 264–265, B 167–168). Kant, as we read in his letter of February 21, 1772, to Marcus Herz, rejects both the first possibility, which would imply an *intellectus ectypus*, and the second, which would imply an *intellectus archetypus*.[6] He proposes a third way between the radical kind of empiricism (the first possibility) and extreme rationalism (the second possibility). This solution means that there are "two stems of the human cognition (*zwei Stämme der menschlichen Erkenntnis*)" (idem, p. 152, B 29)[7]: sensibility and understanding. In this way, "experience itself is a kind of cognition requiring the understanding, whose rules I have to presuppose in myself before any object is given to me, hence *a priori*, which rule is expressed in concepts *a priori*, to which all objects of experience necessarily conform, and with which they must agree" (idem, p. 111, B XVII).

Now we are at the very heart of Kant's *Copernican revolution*,[8] according to which:

> reason has insight only into what it itself produces according to its own design; that it must take the lead with principles for its judgments according to constant laws and compel nature to answer to its questions, rather than letting guide its movement by keeping reason, as it were, in leading-strings. (idem, pp. 108–109, B XIII)

All the above considerations lead us to the idea that human understanding imposes its laws a priori – its pure principles (*die Grundsätze der reinen Verstand*) – on the world, constituting it as nature in its lawful character:

> that the highest legislation for nature must lie in ourselves, i.e., in our understanding, and that we must not seek the universal laws of nature from nature by means of experience, but conversely, that we must seek nature, as regards its universal conformity to law, solely in the conditions of the possibility of experience that lie in our sensibility an understanding. (P, pp. 73, 319)
>
> *the understanding does not draw its (a priori) laws from nature, but prescribes them to it.* (idem, pp. 73–74, 320)

Briefly put, it is the human understanding that produces the lawfulness of experience.

3.2.2 *Nature* überhaupt *and Experience* überhaupt

The Transcendental Analytic (including the Analytic of Principles) may be seen as a long argument for the claim that *the understanding anticipates a priori the form of possible experience in general (überhaupt)*. This 'form' of possible *experience überhaupt* is nothing but the *regularity (Gesetzmässigkeit) of the phenomena in space and time* (cf. Scaravelli, 1968, p. 292). Therefore the lawfulness, the *Gesetzmässigkeit*, of *possible nature in general (möglichen Natur überhaupt)* is the result of the legislating understanding.

At this point, it should be easy to grasp the true meaning of the principle of the synthetic *a priori* judgements, according to which "Every object stands under the necessary conditions of synthetic unity of the manifold of intuition in a

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[6] Cf. Kant (1986, pp. 99–106). Note that there is also another possibility: Leibniz's preestablished harmony, that Kant rejects as well (cf. KdrV, pp. 264–265, B 167).

[7] Note that in the Cambridge edition of the KdrV that I use, the term '*Erkenntnis*' is translated by 'cognition', while I prefer the more classical term 'knowledge'.

[8] "This would be just like the first thoughts of Copernicus (*mit den ersten Gedanken des Kopernicus*)" (KdrV, p. 110, B XVI).

possible experience" (KdrV, p. 283, B 197), that is, "The conditions of the *possibility of experience* in general [*überhaupt*] are at the same time conditions of the *possibility of the objects of experience*" (ibidem). For the conditions of the possibility of experience are exactly the pure principles of the understanding, that is, what makes both the constitution of objects and the constitution of the relations among them possible.

It follows immediately that nature is not something bare given to human being, but something cognitively constituted both (1) regarding its elements, that is, phenomena, and (2) regarding the nomological relations connecting those phenomena. In brief, it is to be thought of both (1) as *natura materialiter spectata*, that is, as the class of the phenomena *constituted* as such by the forms of space and time and then by the categories, and (2) as *natura formaliter spectata*, that is, as the class of the nomological relations among phenomena (cf. idem, pp. 262–264, B 163–165) *constituted* by the pure principles of the understanding, especially by the analogies of experience.

Let us focus on the meaning of '*überhaupt*'. We know that "transcendental" means all knowledge "that is occupied not so much with objects but rather with our mode of cognition of objects insofar as this is to be possible *a priori*" (idem, p. 149, B 25). Therefore dealing with the possibility of nature *überhaupt* means exactly dealing with the way according to which it is made possible *a priori*.

To grasp the relation between possible experience *überhaupt* and possible nature *überhaupt*, it is worth mentioning the main two meanings of the term 'experience' (*Erfahrung*): (1) one regarding "with"[9] what knowledge begins; (2) one concerning the realisation of knowledge through the matching between the empirical datum and the categorial apparatus. It is exactly this second meaning which is now interesting for us:

> The possibility of *experience in general* is therefore at the same time the universal law of nature, and the principles of the former are themselves the laws of the latter. (P, pp. 72, 319)

Taking into account what has been said on nature *überhaupt* and on the law-giving understanding, since *natura formaliter spectata* is the product of the imposition of the pure laws of the understanding, it is nothing but a different way of considering the *Gesetzmässigkeit der Natur überhaupt*, that is, the lawfulness of nature at the transcendental level.

3.3 The Metaphysical Level

3.3.1 *The Problem of the* Anfangsgründe

As soon as we move from the transcendental level of the pure principles of understanding (*die Grundsätze der reinen Verstand*) to the metaphysical level of the first metaphysical principles (*die metaphysische Anfangsgründe*) we bump into one of the main problems raised by contemporary Kantian scholars: what is the logical connection (if there is one) between the two kinds of principles? Though this is not the right place to discuss this question in detail, something must be said about it (cf. for example, Buchdahl, 1971; Allison, 1994; Friedman, 1994; O'Shea, 1997).

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[9] It is important to be aware of the fact that "though all our knowledge begins *with [mit]* experience, it does not follow that it all arise *out of* [*aus*] experience" (KdrV, p. 136, B 1, my italics).

Both in chapter III of the Doctrine of Method and in the Introduction to the *Anfangsgründe*, Kant gives a taxonomy of the different philosophical fields. In particular he defines metaphysics as “the system of pure reason (science), the whole (true as well as apparent) philosophical cognition from pure reason in systematic interconnection” (KdrV, p. 696, B 869).

Metaphysics itself is divided into two parts: (1) *metaphysics of nature* that concerns the speculative employment of pure reason[10]; (2) *metaphysics of morals* that concerns the practical employment of pure reason. One of the two fields of the metaphysics of nature is the “doctrine of bodies” (*Körperlehre*) – as Kant calls it in the *Anfangsgründe* – which deals with the objects of external senses. This is “*physics* [*die Metaphysik der körperlichen Natur heisst Physik*]” (idem, p. 699, B 874), or “*physica rationalis*”, or “*physicam puram*” (idem, p. 147, fn., B 21).

Now since both the *Grundsätze der reinen Verstand* and the *metaphysische Anfangsgründe* are synthetic *a priori* judgements, what is the difference between them? To answer this question, it is necessary (though not sufficient) to grasp the difference between *general metaphysics* dealing with nature *überhaupt*, and the *particular metaphysics* concerning nature in particular, that is, that particularisation of nature limited to the particular (*besondere*) class of objects considered by that given science. This does not mean at all that general metaphysics and the particular metaphysics are independent from one another. Actually, the principles of the former, related to the possibility of an object *überhaupt*, make it possible that this object is also an object investigable by physics, in particular by pure physics.

The transition from the transcendental level to the metaphysical one is made possible by the introduction of a particular concept allowing us to delimit the search in question to a specific field. In the case of physics, since we are dealing with objects affecting the external senses, the empirical concept to be introduced is that of *matter* (cf. also KdU, pp. 20–21, 181–182 – II Introduction). Of course it must not be intended as this or that particular matter, but as “matter in general (*Materie überhaupt*)” (M, pp. 11, 475).

Between the *Grundsätze* and the *Anfangsgründe* there is no change in epistemological status, since both include synthetic *a priori* judgements. What occurs is a change in epistemological rank: although both classes are formed by universal statements, they have a different “rank in regard to generality” (KdrV, p. 697, B 871): the object of the *Grundsätze* is nature *überhaupt*, while the object of the *Anfangsgründe* is particular physical nature.

3.3.2 The Mathematical Lawfulness of Physical Nature

On several occasions, especially in the part of the Transcendental Doctrine of Method, in the section entitled ‘The Discipline of the Pure Reason in Its Dogmatic

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[10] It should be pointed out that we are dealing with the ‘metaphysics *of* nature’ and not with ‘metaphysics *in* nature’.

Use', Kant claims that while philosophical knowledge is discursive, that is, based on concepts, mathematical knowledge is based on the construction of concepts.

Let me give the reason why mathematics is so successful in physics, particularly in pure physics[11]. Pure physics deals with objects deprived of empirical intuition, and we can positively deal with them only by constructing their concepts, that is, by using mathematics. If one reflects on this aspect of mathematical physics, one becomes aware that Kant states something which will be easily agreed on: doing theoretical physics means representing objects conceptually, away from the empirical component.

Granted that mathematics allows us to construct the concepts of physical objects without having an intuition of them, the task is to understand how this happens: "But in order to make possible the application of mathematics to the doctrine of body, which only through this can become natural science, principles for the construction of the concepts [*Prinzipien der* Konstruktion *der Begriffe*] that belong to the possibility of matter in general [*Materie überhaupt*] must be introduced first" (M, pp. 8, 472).

These "principles of the *construction* of the concepts (*Prinzipien der* Konstruktion *der Begriffe*)" are nothing other than the first metaphysical principles of natural science. By analyzing the concept of matter *überhaupt, a priori* principles can be found, which allow for the application of mathematics and, thus, the construction of the concepts of rational physics. This is why "all natural philosophers who have wished to proceed mathematically in their occupation have always, and must have always, made use of metaphysical principles (albeit unconsciously), even if they themselves solemnly guarded against all claims of metaphysics upon their science […]. Thus these mathematical physicists could no way avoid metaphysical principles" (ibidem).

Therefore, the "possibility of a mathematical doctrine of nature (*der Möglichkeit einer mathematischen Naturlehre*)" is based on the "principles of the construction (*Prinzipien der Konstruktion*) of these concepts" (idem, pp. 9, 473) of objects of the external senses, independently of whether or not they are intuited.

To sum up, if at the transcendental level the law-giving understanding produces the lawfulness of nature *überhaupt* (the *Gesetzmässigkeit der Natur überhaupt*), that is, produces *natura formaliter spectata*, something different happens at the metaphysical level. We have seen that the first metaphysical principles make the mathematization of physics possible. That is, they allow us a particular way of representing both objects of the external senses and abstract objects (not to be confused with objects *überhaupt*). This means that they allow us a mathematical representation of the lawfulness of the physical nature which, as particularisation of nature *überhaupt*, is made cognitively significant by means of the pure principles. In other words, at the metaphysical level, we have the lawfulness of a mathematised particular nature.

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[11] As well-know this is the widely discussed problem of the relation between physics and mathematics; cf. Boniolo, Budinich, 2005.

3.4 The Empirical Level

3.4.1 The Empirical Laws and the Second Analogy of Experience

We know that all the phenomena, *in order to be given*, have to fall under the pure forms of intuition (space and time), and, *in order to be known*, they have to fall under the categories of the understanding, that is, under the pure principles of the understanding or, in other words, under the laws of nature *überhaupt*. However, apart from nature *überhaupt*, which is constituted and regulated by the pure principles of understanding (at the transcendental level), there is also nature in particular, which is constituted and regulated by the empirical laws (at the empirical level):

> Particular laws [*besondere Gesetze*, also called *empirische Gesetze* or *empirische Grundsätzse* (cf. KdrV, pp. 283–284, B 197–200)], because they concern empirically determined appearances, *cannot* be *completely derived* from the categories, although they all stand under them. Experience must be added in order to come to know particular laws *at all*; but about experience in general [*überhaupt*], and about what can be cognized as an object of experience, only those *a priori* laws offer instruction. (idem, p. 264, B 165)

The empirical laws are obviously different from the pure principles of understanding which apply to nature *überhaupt* and have the transcendental function both of constituting *natura materialiter spectata*, and of regulating *natura formaliter spectata* (cf. also P, pp. 71–74, 318–320). They cannot be derived from the pure principles of understanding, but they can be found by resorting to experience (cf. KdrV, p. 320, B 263; KdU, pp. 23–24, 184–85 – II Introduction). Nevertheless, although they cannot be established only on the basis of the pure principles of understanding, they depend on them, since such principles are the transcendental laws of nature *überhaupt*, that is, of nature for which they – the empirical laws – rule the particular relations. This means that beyond *Natur überhaupt*, there is a *Natur (im empirischen Verstande)* (KdrV, p. 320, B 263) particularising the former, but this is possible only thanks to the former.

At this point I must clarify the relation between the empirical laws and the pure principles which is articulated in the second Analogy of Experience. Let us put aside the huge secondary literature on the relation between the second Analogy of Experience and the empirical laws, and let us try to get a grip on Kant's texts directly. We are told that according to the principle of the Analogies of Experience:

> Experience is possible only through the representation of a necessary connection of perceptions. (idem, p. 295, B 218)

As the *Beweis* of this principle begins, experience is empirical knowledge, that is, objective knowledge which, in the case of the connections of perceptions, is made possible precisely thanks to the analogies of experience. These regulate necessarily the connections between perceptions, in function of the three modalities of time: permanence, succession, and coexistence. It should be noted that the analogies of experience concern neither the phenomena nor the synthesis of their intuition. Phenomena refer to the Axioms of Intuition and the Anticipations of Perception, which make possible the "*existence* [of such appearances] and their *relation* to one another with regard to this their existence" (idem, p. 297, B 220).

First the phenomena as such are constituted. Then, in order to objectively rule the relations among them, the analogies of experience come into play, i.e., the regulative principles of the understanding, which are prior to all experience and so make it possible (idem, pp. 295–296, B 218–2199). Thus, there is a *Regelmässigkeit* of nature *überhaupt* made possible exactly by the analogies of experience. In other words, the law-giving understanding, especially inasmuch as it imposes the analogies of experience (which are regulative, i.e., they provide rules), constitutes the *Gesetzmässigkeit* of nature.[12]

Among the three analogies, the second is particularly important, since it concerns the succession of perceptions in time, and thus their causal connection. With reference to causality, it should be recalled that the *category of causality* is one thing, while the *schema of causality is another*. Moreover *the principle of causality* is yet another thing.

But what is the category of causality?

> […] the concept of cause, which signifies a particular kind of synthesis, in which given something A something entirely different B is posited according to a rule [… in a way such that the latter] follows from it *necessarily* and *in accordance with an absolutely universal rule*. (KdrV, pp. 222–223, B 122–124)

The category of causality being specified, let us recall the schema of causality:

> The schema of cause [*Ursache*], and of causality [*Kausalität*] of a thing in general [*eines Dinges überhaupt*] is the real upon which, whenever it is posited, something else always follows. It therefore consists in the succession of the manifold insofar as it is subject to a rule. (idem, p. 275, B 183)

and the principle of the second Analogy of Experience, i.e., the synthetic *a priori* judgement expressing the principle of causality:

> All alterations occur in accordance with the law of the connection of cause and effect. (idem, p. 304, B 232)

Each time that a change of a certain thing in general (*überhaupt*) occurs, it is regulated by a necessary and universal law, that is, by the "law of connection of cause and effect": the *principle of causality*. Put differently, each time that an alteration occurs from a phenomenal situation A *überhaupt* to a phenomenal situation B *überhaupt*, there occurs a universal and necessary judgement which rules it:

> For each alteration leading to an event B *überhaupt*,
> there is an event A *überhaupt* such as
> A *überhaupt* is the cause of B *überhaupt*

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[12] Note that the Analogies of Experience, which are regulative, constitute the lawfulness of nature. As we will see later on, and as Kant himself specifies (KdrV, pp. 601–602, B 691), the analogies *regulate* in the sense that they impose laws, and thus *constitute* nature as *natura formaliter spectata*. Another remark should be in order. There are two meanings of "regulative" to be distinguished: (1) "regulative" in the sense of the pure laws (rules) which are constitutively imposed to rule nature; (2) "regulative" in the sense of the heuristic rules helping the scientific research.

If we indicate "A *überhaupt*" by **A** and "B *überhaupt*" by **B**, then the *principle of transcendental causality*, contained in the principle of the second Analogy of Experience, can be formulated as

$$\forall x \mathbf{A}x \overset{C^T}{\rightarrow} \mathbf{B}x$$

where C^T indicates that we are dealing with causality in the transcendental sense. It is important to point out that **A** and **B** are not particular events, neither tokens of event-type nor event-type, but events *überhaupt*, i.e., events which can be identified only within Kant's transcendental architectonic.

Here we are dealing with A and B *überhaupt*; we are at the transcendental level, where nature, even if *formaliter spectata*, has to be intended *überhaupt*. Obviously enough, at this level, the problem of lawness is trivial: the universal statement $\forall x \mathbf{A}x \overset{C^T}{\rightarrow} \mathbf{B}x$ is certainly a law, since it is produced as such by the legislating understanding.

Now we can move to the issue concerning the relation between the particular causal laws and transcendental causality. We know that in order to perceive a change not merely as a subjective succession, but as a temporal objective succession, a rule establishing what comes first and what comes after must come into play. We also know that this rule is *a priori. However, we do not know anything else, or, rather, we do not know anything else at this a priori level.*

We should not be deceived by the examples discussed by Kant: the sun warming up a stone, the sun melting wax, the river pushing a ship, the weight of the ball producing a concavity, etc. In all these cases, we know which causal force is involved, and so we could interpret the second Analogy as stating that not only is the succession of perceptions objectivised thanks to the causal rule, but also that all the similar cases fall under the same rule. However, this is not true. As was said before, Kant is absolutely clear in that respect: the particular causal empirical laws cannot be known *a priori*, nor deduced from the principles *a priori*, especially from the principle of the second Analogy. For

> how in general anything can be altered, how it is possible that upon a state in one point of time an opposite one could follow in the next – of these we have *a priori* not the least concept. For this acquaintance with actual forces is required, which can only be given empirically, e.g., aquaintance with moving forces, or, what comes to the same thing, with certain successive appearances (as motions) which indicate such forces. But the form of such an alteration, the condition under which alone it, as the arising of another state, can occur (whatever the content, i.e., the state, that is altered might be), consequently the succession of states itself (that which has happened), can still be considered *a priori* according to the law of causality and the conditions of time. (idem, p. 314, B 252)

Only the formal condition of possibility of knowledge of any alteration is given *a priori*, that is, we know *a priori* only that each time an alteration occurs, $\forall x \mathbf{A}x \overset{C^T}{\rightarrow} \mathbf{B}x$ holds, but certainly we do not know the particular instantiation of the causal law applying to that particular alteration. Only by *reflecting* (this verb is not fortuitous) on the particular phenomenal situation, the knowing subject can arrive at the determination of the universal statement causally connecting A-type event to B-type event, that is, to a statement such as

$$\forall x A x \xrightarrow{C^E} B x$$

Now, the causal relation between being A and being B is no more a transcendental relation (C^T), but an empirical relation (C^E).

3.4.2 The Unity of System

Although most Kant's commentators agree in claiming that the Analytic of Principles does not solve the problem of the empirical laws, some of them argue that it is solved in the Appendix to the Transcendental Dialectic of the *Kritik der reinen Vernunft*, and others affirm that it is solved in the *Kritik der Urteilskraft* (cf. Scaravelli, 1968, pp. 369–371; Guyer, 1990; Kitcher, 1994, p. 257; O'Shea, 1997, pp. 242–248).

The commentators arguing the first possibility base their interpretations on the consideration that the Appendix anticipates almost all of what is going to be at issue in the *Kritik der Urteilskraft*. As far as this interpretative proposal is concerned, I am inclined to consider it as forcing Kant's words well beyond their intention, especially in view of the architectonic of his critical philosophy, theorised in the Doctrine of Method of the *Kritik der reinen Vernunft*, and practically displayed firstly with the three *Critiques*, and particularised in the *Metaphysische Anfangsgründe der Naturwissenschaft*, as far as the topic on nature is concerned ("the starry heaven above me"), and in *Die Metaphysik der Sitten*, as far as the problem of freedom is concerned ("the moral law within me"). It should also be observed that Kant himself writes explicitly that the critique of pure reason without the critique of the capacity of judgment is incomplete (KdU, pp. 4–5, 167–168 – Preface).

The thesis granting the Appendix an anticipation of topics analysed in the third critique turns out to be problematical, especially if we compare the two writings. As Allison (1994, p. 305, note 5) pointed out, a mistake is made by some commentators when they fail to distinguish sharply between reason in its regulative use and the reflecting capacity of judgment. Other important differences are also neglected. Let us focus on the problems of unity of nature and the systematicy of knowledge.

Kant writes that it is a *requirement of reason (Forderung der Vernunft)* – which in the Appendix is called *interest of reason (Interesse der Vernunft)* – in its logical use, to find an unconditioned condition "with which its [of the understanding] unity will be completed" (KdrV, p. 392, B364). We know that the unconditioned can be found by reasoning prosyllogistically on the basis of the three forms of the syllogism: the categorical, the hypothetical, and the disjunctive syllogism. In that way, the three transcendental ideas can be identified. If they are considered objectively, they lead to dialectical conclusions, while, if considered as *focus imaginarius*, that is, as a regulative ideal, they are what Kant considers "an excellent and indispensably necessary regulative use" (idem, p. 591, B 672).

The Appendix ought to be read on this basis. What is it that is implied in a *focus imaginarius*? Nothing but an idea (in the subjective, regulative sense) concentrating, as it were, what we know by means of the categories. Just like the optical focus is the point toward which the rays of light converge, the transcendental idea allows for the convergence of what is already constituted by the intervention of the categorial apparatus, that is, what is already known. This means that the transcendental idea allows for the *unity (Einheit)* of knowledge; it allows us to have a set of synthetic judgements structured as a *system (System)* (idem, pp. 591–592, B 673).[13]

Therefore, "the systematic unity or the unity of reason of the manifold of the understanding's cognition is a logical principle [… which assists the understanding in those cases in which] the understanding alone does not attain to rules" (idem, p. 593, B 676). This occurs precisely because "human reason has a natural propensity to overstep" the limits of possible experience (idem, p. 590, B 670). Naturally enough, one must be careful to use it rigorously in a regulative way.

In short, understanding unifies (from the transcendental point of view) the empirical multiplicity through categories; reason unifies (from the regulative, that is, methodological, point of view) the "manifold of concepts through ideas" (idem, p. 591, B 672). Moreover since concepts are predicates of possible judgements, reason also unifies in a system the empirical judgments, or the empirical laws.

It follows that at the cognitive top, i.e., at the transcendental level, we have the eight pure principles of understanding (plus the pure principles derived from them), which make nature *überhaupt* possible. At the metaphysical level, we have the first 12 metaphysical principles allowing us to have a mathematised and almost-axiomatised discipline of the physical objects (those falling under the external senses). At the empirical level, we have both the infinite universal empirical judgments, i.e., the infinite empirical laws, and the infinite singular and particular empirical judgments, most of which are unified into a system.

This infinite multiplicity of empirical laws, which are contingent in comparison with the products of the pure understanding *but necessary precisely because they are laws*,[14] is unified into a system. Such a system is made possible only thanks to reason used regulatively: it is reason that searches for the unconditioned in the conditioned series.

At this point we should throw some light upon the ideas allowing for such unity, that is, the principles guiding the formation of the system of laws. Following this, we should analyze the epistemological status of these principles in order to justify them. As far as the first problem is concerned:

> Reason thus prepares the field for the understanding: 1. by a principle of *sameness of kind* in the manifold under higher genera, 2. by a principle of *variety* of what is same in kind under lower species; and in order to complete the systematic unity it adds 3. still another

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[13] It should be noted here that "unity" has not to be intended in the transcendental sense, since it was the unity allowed by the *I think*. Instead now it must be intended as a methodological unity.

[14] With reference to this point Kant writes that "the word nature already carries with it the concept of laws, and the latter carries with it the concept of necessity of all determinations of a thing belonging to its existence" (M, p. 4, 469).

> law of the *affinity* of all concepts, which offers a continuous transition from every species to every other through a gradual increase of varieties. (KdrV, p. 598, B 685–686)

Following Kant's suggestion, we call these principles

1. The principle of homogeneity (*entia praeter necessitatem non esse multiplicanda*); which is related to *the idea of unity*
2. The principle of specification (*entium varietates non temere esse minuendas*); which is related to *the idea of multiplicity*
3. The principle of continuity (*continui specierum – formarum logicarum –*); which is related to *the idea of affinity*

Rigorously speaking, only the first principle about the subsumption of species to genus, and genus to higher genus on the basis of homogeneity, which implies the use of Ockham's razor, is strictly related to reason's tendency to unity. Nevertheless, the second principle is necessary to limit the "rashness of the first principle" (idem, p. 596, B 682), that is, the "inclination to unanimity" (idem, p. 599, B 688). Our attention is thus focused on the fact that Porphyri's tree, relative to concepts, can be run bottom-up, that is, from what is more particular to what is more general, but it can be also run top-down, that is, toward the less general. In the former case we find the system, in the latter case we find the chaotic infinite multiplicity of the particular empirical laws and the singular empirical judgments. Lastly, the third principle fulfils Kant's intention to have three moments in any one partition, and allows us to find the connection between the first two principles. Indeed this principles invites us to search for the affinity which has to be found thanks to the fact that species "are all collectively descendent, through every degree of extended determination, from a single highest genus" (idem, p. 598, B 686).[15]

Next we need a better understanding of the content and status of these principles. First of all, Kant gives them a name:

> I call all subjective principles that are taken not from the constitution of the object but from the interest of reason in regards to a certain possible perfection of the cognition of this object, *maxims* of reason. (idem, p. 603, B 694)

Both in the Appendix and in the *Kritik der Urteilskraft*, there are passages where the status of the maxims (principles) of reason and the reflecting capacity of judgment is discussed. Nevertheless Kant seems to suggest many contradictory theses, and therefore some commentators (for instance, Kemp Smith, 1923, p. 547) have strongly questioned the consistency of the Prussian philosopher, in particular in the Analytic. This problem can be skipped for our own purposes (cf. O'Shea, 1997, pp. 229–237). It is best to move to another extremely relevant point of the Appendix, in particular where Kant makes an interesting remark related to the third idea "which contains a merely relative supposition of a being as the sole and all-sufficient cause of all cosmological series, [this] is the rational concept of *God*" (KdrV, p. 613,

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[15] From these three principles of reason, Kant derives also that one according to which *non datur vacuum formarum*, and its corollary: *datur continuum formarum*.

B 713). This idea, due to the interest of reason, according to which we have to think *as if* there were a sovereign reason which is the cause of any phenomenon and any series of phenomena, "opens out for our reason, as applied to the field of experience, entirely new prospects for connecting up things in the world in accordance with teleological laws [*teleologischen Gesetzen*], and thereby attaining to the greatest systematic unity among them" (idem, p. 614, B 714–715).

It should be noted that in this way "the *purposive* unity of things [zweckmässige *Einheit der Dinge*]" (ibidem) is introduced as a consequence of the fact that we must think the world *as if* it were caused by a supreme Being. This is nothing other than the external purposiveness, analyzed specifically in the *Kritik der Urteilskraft*. What is remarkable is that in this third *Critique*, Kant suggests something seemingly contrary to the above. For teleology and theology must be differentiated both in the sense of the non-derivability of purposiveness from the concept of God, and in the sense of non-derivability of the opposite possibility. Of course, we should not fall in a *circulus vitiosus in probando*, that is, in a diallelus, arguing firstly the purposiveness on the basis of the concept of God and then the concept of God on the basis of purposiveness (KdU, pp. 261–262, 381–382). However, since by starting from the fictionalistic idea of a supreme Being, cause of everything, the idea of purposiveness can be introduced even if in a "presumptuous (*vermessen*)" way (idem, p. 264, 383), similarly the same could be done the other way around, that is, it can be claimed that:

> *The peculiar character of my cognitive powers* is such that the only way I can judge [how] those things are possible and produced is by conceiving, [to account] for this production, a cause that acts according to intentions, and hence a being that produces [things] in a way analogous to the causality of an understanding. (idem, pp. 280, 397–398)

Note that this not a *Beweis* of the existence of a supreme Being, but only an indication of the fact that, due to the way according to which our cognitive capacities are structured, we need to resort to a supreme Being in order to have the concept of world encompassed in a system and, thus, to have the unity of nature (cf. idem, pp. 281, 399).

However, both in the Appendix and more systematically in the *Kritik der Urteilskraft*, Kant shows that referring to a final cause, or to a teleological relation, or to some *nexus finalis*, must absolutely not be intended as a denial of efficient causality, that is, as a denial of the possibility of having a mechanical or physical relation, i.e., a *nexus effectivus* (KdrV, p. 615, B 715–716). This must be considered with great care, failing which both the Appendix and the *Kritik der Urteilskraft* will be misread as Kant's attempt to replace efficient causality by teleology. These are two ways of approaching empirical knowledge that, for Kant, must be equally taken into consideration. Put differently, we can pose causal why-questions, but we can pose purposive why-questions as well, even if it is not said that we are obliged to look for both a causal and a purposive answer.

3.4.3 Lawfulness

The second Analogy of Experience shows that a causal lawfulness (*Gesetzmässigkeit*) is imposed on nature *überhaupt*, though this lawfulness does not come into play in the particular natures. The latter are constituted as such by the empirical causal laws and by the empirical purposive laws, which have been found thanks to the maxims of the reflecting capacity of judgment. Indeed these maxims, precisely by reflecting on the particular phenomenon, allow us to produce hypotheses which make that given particular cognitively significant *sub specie* instantiation of an empirical law. That is, they allow for the lawfulness of the particular natures by prompting us to produce the appropriate cognitively constituting empirical laws.

We have just one *Natur überhaupt*, but we have infinite *spezifisch-verschiedene Naturen*, which, according to the interest of reason in its regulative use, are to be unified into a systematic unity. Thus, nature at the empirical level is not made lawful by means of the pure laws of understanding, but by means of the particular empirical laws found by reflecting thanks to the maxims of the capacity of judgment. The *Gesetzmässigkeit* at the empirical level is made possible by the reflecting capacity of judgment which, analogously to the legislating understanding of nature *überhaupt*, could perfectly well be considered to be the legislator of the natures *besondere*.

I want to repeat again that the second Analogy does not state, or give us, the uniformity of nature, not even in the sense of nature *überhaupt*. The uniformity of nature (the *Ordnung der Natur*) does not result from finding the empirical laws, but it is an implicit *a priori* presupposition of the two maxims guiding us to discover particular laws:

> Hence, though the understanding cannot determine anything *a priori* with regard to these [objects], still it must, in order to investigate these empirical so-called laws, lay on an *a priori* principle at the basis of all the reflection on nature: the principle that a cognizable order of nature [*erkennbare Ordnung der Natur*] in terms of these laws is possible […] since without presupposing this harmony we would have no order of nature [*Ordnung der Natur*] in terms of empirical laws, and hence nothing to guide us in using empirical laws so as to experience and investigate nature in its diversity. (idem, pp. 24–25, 185 – II Introduction)

Therefore, the reflecting capacity of judgment presupposes the uniformity of nature at the empirical level, since it is exactly here that it plays its role. It follows that, still at the empirical level, nature is made lawful through the causal and purposive laws which are found hypothetically by applying the two maxims.

3.4.4 Schlick's Problem

Now the problem of the nomological validity of the empirical universal judgements, that is, Schlick's problem (the problem of lawness) is easily tractable. Given a particular 'a' of the sort A and a particular 'b' of the sort B, why can I state that the universal statement of the type $\forall x Ax \rightarrow Bx$ is really a causal law of the type $\forall x Ax \xrightarrow{C^E} Bx$, or a purposive law of the type $\forall x Ax \xrightarrow{pE} Bx$? In other words, what is it that assures me that I have a nomological universal and not an accidental universal? The answer is clear: this assurance is provided by the reflecting capacity of judgment! Its maxims spur me both to search for a (causal or purposive) universal subsuming under itself that particular, and to interpret it as an hypothetical law. We can formulate the wrong hypothetical law, both in the sense of having found the wrong nomological universal and in the sense of having confused an accidental universal with a nomological one. However, this is the price that the knowing subject must pay, since he has neither an *intellectus ectypus*, nor an *intellectus archetypus*.

Note, however, that by taking into account the meaning of the maxims of the reflecting capacity of judgment, to take as nomological what is in fact accidental is almost impossible. If I take a coin out of my pocket and see that it is a nickel, I would hardly claim that the statement "All coins in my pocket are nickels" is an empirical law. This is a universal statement which, however, is neither causal nor purposive, whereas the maxims of the reflecting capacity of judgment spur me to search for only causal and purposive laws.

The case in which we have statements such as "All ravens are black" is different. Is it accidental or nomological? Kant would have said without hesitation that it is a nomological statement, not because it proves simply that being raven causes being black, but because being raven in a given environmental circumstance causes being black, in that evironment.

Briefly put, nature *überhaupt* is lawful because the understanding is *a priori* law-giver; nature *besondere* is lawful because of the legislating reflecting capacity of judgment, even if only the possibility of generating hypothetical laws is granted thereby. Moreover, the pure principles of understanding and the first metaphysical principles are nomological, because they are produced as such by the understanding. To put it differently, the empirical laws are hypothetically nomological because only in this way does the reflecting capacity of judgment succeed in using them to subsume the particular, that is, to give it cognitive significance. This amounts to seeing it as a *sub specie* instantiation of an empirical law.[16]

References

Allison, H.E.: 1994, 'Causality and Causal Laws in Kant. A Critique of Michael Friedman', in P. Parrini (ed.), *Kant and Contemporary Epistemology*, Kluwer, Dordrecht, pp. 291–307.

Boniolo, G., Budinich, P.: 2005, 'The Role of Mathematics in Physical Sciences and Dirac's Methodological Revolution'; in G. Boniolo, P. Budinich, M. Trobok (eds.), *Mathematics and Physics: an Interdisciplinary Approach*, Springer, Dordrecht, pp. 75–96.

Boniolo, G.: 2007, *On Scientific Representations. From Kant to a New Philosophy of Science*, Palgrave Macmillan, Houndmills.

Buchdahl, G.: 1971, 'The Conception of Lawlikeness in Kant's Philosophy of Science', *Synthese* 23, pp. 24–46; now L.W. Beck (ed.), *Kant's Theory of Knowledge*, Reidel, Dordrecht, 1974, pp. 128–150.

Friedman, M.: 1994, 'Kant and the Twentieth Century', in P. Parrini (ed.), *Kant and Contemporary Epistemology*, Kluwer, Dordrecht, pp. 27–46.

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[16] Kitcher (1998) arrives at the same result by emphasising the role of the system. He speaks in term of 'lawfulness', but, as seen, this should not be the right term.

Guyer, P.: 1990, 'Reason and Reflective Judgment: Kant on the Significance of Systematicity', *Nous* 24, pp. 17–43.

Kant, I.: 1986, *Briefwechsel*, Felix Meiner Verlag, Hamburg.

Kant, I.: 1987, *Erste Einleitung* to the *Kritik der Urteilskraft*, Engl. transl. *First Introduction*, in *Critique of Judgement*, ed. by W.S. Pluhar, Hackett Publishing, Indianapolis, IN, pp. 385–441.

Kant, I.: 1781–1787^2, *Kritik der reinen Vernunft*, Engl. transl. *Immanuel Kant's Critique of Pure Reason*, ed. by P. Guyer and A.W. Wood, Cambridge University Press, Cambridge, 1998.

Kant, I.: 1783, *Prolegomena zur einer jeden künftigen Metaphysik die als Wissenschaft wird auftreten Können*, Engl. transl. *Prolegomena to Any Future Metaphysics*, ed. by G. Hatfield, Cambridge University Press, Cambridge, 1997.

Kant, I.: 1786, *Metaphysischen Anfangsgründe der Naturwissenschaft*, Engl. transl. *Metaphysical Foundations of Natural Science*, ed. by M. Friedman, Cambridge University Press, Cambridge, 2004.

Kant, I.: 1790, *Kritik der Urteilskraft*, Engl. transl. *Critique of Judgement*, ed. by W.S. Pluhar, Hackett Publishing, Indianapolis, IN, 1987.

Kant, I.: 1800, *Logik*, Engl. transl. *Logic*, ed. by R.S. Hartman and W. Schwarz, Dover, New York, 1974.

Kant, I.: 1956–1964, *Werkausgabe*, vols. I–XII, ed. by W. von Weishedel, Suhrkamp, Frankfurt a.M., 1968.

Kemp Smith, N.: 1923, *A Commentary to Kant's 'Critique of Pure Reason'*, Humanities Press, London, 1950.

Kitcher, P.: 1994, 'The Unity of Science and the Unity of Nature', in P. Parrini (ed.), *Kant and Contemporary Epistemology*, Kluwer, Dordrecht, pp. 253–272.

Kitcher, P.: 1998, 'Projecting the Order of Nature', in Patricia Kitcher (ed.), *Kant's Critique of Pure Reason. Critical Essays*, Rowan & Littlefield, Lanham, MD, pp. 219–238.

Krieger, L.: 1965, 'Kant and the Crisis of Natural Law', *Journal of the History of Ideas* 26, pp. 191–210.

Maxwell, J.C.: 1873, *Treatise on Electricity and Magnetism*, Dover, New York, 1952.

Nagel, E.: 1961, *The Structure of Science. Problems in the Logic of Scientific Explanation*, Harcour, Brace & World, New York.

O'Shea, J.R.: 1997, 'The Needs of Understanding: Kant on Empirical Laws and Regulative Ideals', *International Journal of Philosophical Studies* 5, pp. 216–254.

Pearson, K.: 1892, *The Grammar of Science*, J.M. Dent & Sons, London, 1943.

Scaravelli, L.: 1968, *Scritti kantiani*, La Nuova Italia, Firenze.

Schlick, M.: 1931, 'Die kausalität in der gegenwartigen Physik', *Die Naturwissenschaften* 19, pp. 145–162; Engl. transl. 'Causality in Contemporary Physics', in Schlick, 1979, vol. II, pp. 176–209.

Vallentyne, P.: 1988, 'Explicating Lawhood', *Philosophy of Science* 55, pp. 598–613.

The Transcendental Role of the Principle of Anticipations of Perception in Quantum Mechanics

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Abstract The aim of this work is to analyse the diffrerences between the formal structure of anticipation of perception in classical and in quantum context. I argue that a transcendental point of view can be supported in quantum context if objectivity is defined by an invariant anticipative structure, which has only a predictive character. The classical objectivity, which defined a set of properties having a descriptive meaning must be abandoned in quantum context. I will focus my analysis on Kant's Principle of the Anticipations of Perception.

1 Introduction

In the Neokantian tradition of the Marburg School, the Kantian principle of Anticipations of Perception is considered to be the most important tool for understanding the essence of the transcendental methodology and the difficult transition proper to the schematism of the pure concepts of understanding. Despite its importance, this principle has always been left out of the epistemological debate about the universal validity of the Kantian principles in relation to quantum mechanics. This debate is specifically centered in the universality of the principle of the permanence of substance and that of causality, as well as in the *a priori* forms of intuition. For the purpose of determining whether or not the *a priori* conditions are fulfilled in the quantum mechanical context, we propose to shift the focus of the discussion from the principles of permanence of substance and causality to the principle of Anticipations of Perception. This will hopefully lead to new ways of analyzing the correlation between the *a priori* and the quantum mechanical conditions as required to support a transcendental position in quantum mechanics.

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2 Anticipations of Perception in the Classical Physics Context

Classical systems can be formalized by means of Hamilton's equations, which are a set of differential equations equivalent to other formulations, such as Newton's law and Lagrange's equations. The difference between them is that in Hamilton's treatment of the classical systems, the formulation in terms of a field of forces is replaced by a formulation in terms of the energy of the system. The total energy of the system is described by a function called Hamiltonian, which is expressed in terms of the position and the momentum coordinates of the particles comprising the system. It is the Hamiltonian function that determines how the state of a system evolves through time.

As is well known, the state of a classical system, comprised of a single particle or of a set of particles, is specified by the values of position and momentum of each particle at time t. Since the physical space is three-dimensional, six real numbers – three to specify the position and another three to specify the momentum – are needed to determine the state of each particle. Thus, for a system of n particles, $6n$ numbers specify the state of the system. The abstract $6n$-dimensional space is called *phase space*, and a point in this real space represents the state of the system. Once we know the state of an isolated classical system, we can deduce all its properties at any point in time by solving Hamilton's equation. In this sense, we can anticipate the instantaneous values of physical quantities – which vary continuously in time – and express them through real numbers. To anticipate a perception with absolute certainty is to predict, with the highest degree of probability, the event we perceive. This is because the event is conceived as governed by a mathematical law expressed through a differential equation. Since the law and the initial conditions of natural events are known, we can predict the future and reconstitute the past with *apodictical* certainty. This is the main reason why classical systems are considered deterministic.

The deterministic prediction by means of classical equations has absolutely nothing to do with the experimental measurement of any of the physical quantities; whether the measurement has been made or not is irrelevant. Moreover, the differential equations of classical mechanics assume that we can determine the position and the momentum of a given particle, or of a given set of particles, simultaneously at any point in time. A deterministic theory, such as in classical physics, not only assumes that differential equations describing the evolution of the system through time have unique solutions at any given time, but also assigns sharp values to all the physical quantities of the system. This is possible because the Hamiltonian function is taken as continuously differentiable with regard to the position and momentum coordinates for any given time.

Another important characteristic of the classical predictive structure is that it is also descriptive, i.e., besides allowing the anticipation of any state of measurable objects, it describes the evolution of all the properties of the object/system in time. Given that knowing the state of the system is knowing the values of all its properties, classical states are called both descriptive and predictive.

3 The Anticipations of Perception in Kant's Philosophy

The mathematical formulation of classical mechanics was developed by Hamilton in the nineteenth century, much later than Kant's *Critique of Pure Reason*. Even so, Kant knew that it was the mechanical laws governing the interactions between particles in a field of forces that determine how the system evolves through time and, thus, how a future state can be predicted. It is the Kantian principle of Anticipations of Perceptions that aims at expressing in philosophical language the role played by differential calculus in the continuous constitution of the states of classical objects. This process will be described in detail below.

In the Kantian system, the principle of Anticipations of Perception, also called principle of intensive magnitude, is one of the four synthetic *a priori* judgments that constitute the system of all principles of pure understanding. Together with the Principle of Axioms of Intuition, that of Anticipations of Perception concerns mathematical principles of pure understanding distinct from dynamical principles, such as the principle of permanence of substance and the principle of causality.

In the second edition of *Critique of Pure Reason*, the principle of Anticipations of Perception is formulated as follows: "In all appearances the real, which is an object of the sensation, has intensive magnitude, i.e., a degree" (Kant, 1998, p. 290; B207). This principle results from the application of the categories of quality (reality, negation and limitation) to the manifold of intuition, through the mediation of the schema of each of these categories.

Hermann Cohen saw in the process of deduction of the principle of intensive magnitude, through the schematization of the category of reality within the continuity of time, the triumph of understanding over intuition. He also identified in this principle the heart of the Copernican revolution carried out by Kant. As he points out, the crucial moment of the *Critique of Pure Reason* occurs when Kant justifies how a mathematical object changes into a physical object. It is also the guideline of the transcendental method, a method which aims at providing the solution to the problem of how the mathematical knowledge of nature is possible. In Cohen's point of view, intensive continuity is presupposed by all the other transcendental principles. In this sense, the principle of Anticipations of Perception is not only more fundamental than the other principles that form the system of pure understanding, but, particularly, the mathematical principle of Axioms of Intuition and the dynamical principle of Causality are based on their relationships with intensive magnitudes. Thus, the chief question pertaining to transcendental logic, that is, how the unity of the synthesis of a possible experience can be constituted as a physical reality, is consequently answered when Kant introduces the second principle, which conveys the transcendental meaning of the highest principle of all synthetic judgments.

Contrary to current interpretations of the *Critique of Pure Reason*, Cohen claims (2001, p. 434–435) that the epistemic conditions of the mathematical objects are not given in their possible forms in the Transcendental Aesthetic, since the elements required to think them are not yet available. To produce the synthetic unity in the manifold of intuition, the elements of the understanding are required together with those of sensibility. Without the synthesis of the capacity of imagination – in conformity with the categories of the understanding – no consciousness of the unity of intuition is possible. Consequently, the Transcendental Aesthetic is fully clarified only in the Analytic of Principles, which elucidates the possibility of constructing geometrical and arithmetical knowledge. In this sense, the answer to the question "how are synthetic judgments in mathematics possible?" is not given until the introduction to the Axioms of Intuition. However, Cohen's "triumph of thought" should not lead us to forget that the proper employment of thought remains conditioned to sensibility.

Whilst it is in the principle of Axioms of Intuition – the principle of extensive magnitudes – that the genesis of mathematical objects is worked out, it is in the principle of intensive magnitude that genesis of physical objects is accounted for. The latter principle concerns the effective possibility of the mathematical science of nature, not of the constitution of science in general. The principle of intensive magnitude provides, therefore, the rule which allows the constitution of a necessary and universal science of experience.

The principle of Anticipations of Perception refers not only to the form of intuition of the appearance, but also its matter, which is given in sensation. However, as a synthetic *a priori* judgment, it cannot anticipate what will necessarily be given *a posteriori*. It does anticipate the matter of appearances in its formal aspect. In this sense, the principle of the Anticipations of Perception is related to the possibility of anticipating the state of an object that can be present in a possible experience. Anticipation is thus defined by Kant as "all cognition through which I can cognize and determine *a priori* what belongs to empirical cognition" (Kant, 1998, p. 290; A166/B208). This means that the formal quality of perception can be known *a priori*.

In the Kantian strategy, the principle of Anticipations of perception corresponds to the category of reality. The real has a quantity which is intensive, not extensive. Kant defines the intensive magnitude as a "magnitude which can only be apprehended as a unity, and in which multiplicity can only be represented through approximation to negation = 0" (Kant, 1998, p. 291; A168/B210). According to this definition, the degree of reality can be decreased continuously between zero and any other degree. The principle of anticipations clarifies precisely this continuous and uniform production of reality in time. This reality is perceived in different degrees of intensity, that is, the representation of the object in time is indicated by a more or less intense filling of perception. By decreasing perception from any particular degree to zero, only the pure intuitions of space and time remain. "Between reality and negation there is a continuous nexus of possible realities, and of possible smaller perceptions" (Kant, 1998, p. 292; A169/B211). Sensation, in general, can be taken into account only if it is the sensation of an object. In the transition from pure consciousness to empirical consciousness in the continuous progress from zero to any degree of sensation, reality is produced in sensation and, therefore, the double genesis of object and subject takes place. Without this degree there would be neither sensation nor reality.

Among all the qualities of perception, Kant claims that only continuity can be known *a priori*. It is thus a nonempirical characteristic pertaining to the *a priori* nature of perception. Indeed, continuity, as the single *a priori* characteristic of quality, and intensive magnitude, as an *a priori* determination of the quality of reality, are the same. Continuity is not perceived through sensation. For instance, the transmission of the quantity of movement, i.e., the passage from the state of rest to the state of movement, cannot be grasped by our perception. We can see these two states, but the transition from one to the other eludes our sensibility. Consequently empirical perception is not sufficient to account for the contents of experience. The a priori characteristic of intensive magnitude is required to play this role of constitution of intuition.

Just as number is the schema which allows the application of the category of quantity to appearances, infinitesimal calculus is the schema which allows the application of the category of quality to the empirical world. Infinitesimal calculus constitutes a rule which makes possible the gradual transition from empirical consciousness to pure consciousness. Thus, between reality and its negation, an infinite approximation is possible through an infinite and gradual sequence of ever smaller degrees. This is realized by infinitesimal calculus, where the approximation from 0 to 1 is infinite. This is why, according to Kant, “there is also possible a synthesis of the generation of the magnitude of a sensation from its beginning, the pure intuition = 0, to any arbitrary magnitude” (Kant, 1998, p. 290; B208).

The Kantian treatment of almost all fundamental concepts of understanding, such as the concepts of substance, causality, reality, space and time, refers to the philosophical tradition that began in ancient Greece. The infinitesimal concept, however, is a modern concept introduced in the seventeenth century to take into account the process of mathematicization of nature. Newton’s mechanical principles were possible through the advent of infinitesimal calculus, which was developed independently by Leibniz and Newton. However, the meaning and legitimacy of the infinitesimal concept is much more explicit in Leibniz’s differential calculus than in Newton’s fluxional calculus. The term originally used in Newton’s calculus is ‘fluxions’ instead of ‘derivative’. It is even possible, as Michael Friedman (1992, p. 76) showed, to explain the fundamental notions of fluxional calculus without resorting to differentials or to the infinitely small. Nevertheless, Hermann Cohen draws our attention to the fact that the essence of the principle of intensive magnitude lies in the differential concept. The importance of the principle of anticipations actually derives from the fact that it contains the transcendental foundation of differential calculus.

Although Kant does not explicitly mentioned the predictive role of differential equations in the process of Anticipations of Perception, the notion of differentiability introduced by infinitesimally calculus permeates his text. In contrast with the concept of causality, which Kant often used in his work, the infinitesimal concept was sparingly exploited. This, perhaps, is due to the fact that the employment of differential calculus in Kant’s time was so widely spread and uncontroversial that it was not necessary for him to either prove and/or explain its significance. But as it is the principle of anticipations that actually founds the relationship between mathematics and physics, the above concept cannot in any way be neglected. In an important passage of the Anticipations of Perception, Kant refers unequivocally to the Newtonian term ‘fluxions’ in order to characterize the continuity of intensive magnitudes:

> Magnitudes of this sort can also be called **flowing**, since the synthesis (of the productive imagination) in their generation is a progress in time, the continuity of which is customarily designated by the expression “flowing” (“elapsing”). (Kant, 1998, p. 292; A170/B211–B212)

Kant wanted to express that the rates of change of physical coordinates – like position and momentum coordinates – are intensive magnitudes. The derivative of a function represents an infinitesimal change in the function with respect to the parameters considered. In this sense, unlike extensive magnitudes, intensive magnitudes are infinitesimal magnitudes. Kant argued that the concept of continuity is closely associated with the concept of infinitesimal. According to him, intensive magnitude results from the principle of continuity.

A novelty in Kant's approach is that he shows that reality, as well as substance and causality, do not belong to the rough character of external experience, but ought to be regarded as a presupposition of thought that allows for the constitution of experience. Reality for Kant is, above all, a category of thought and not something independent from us. It belongs to the understanding and not to intuition. Reality is not what is immediately given to us in the perceptual experience, but, rather, an *a priori* category that allows the transformation of the subjective content of sensation into something objective. Thus, reality is a process inherent to knowledge, and it can be anticipated by virtue of a formal predictive structure which evolves according to a mental law. The main objective in the Anticipations of Perception is to show that the process of knowledge is a process of production of reality. It is by producing reality through a continuous derivative process that physics determines movement and, more generally, nature. This is why, according to Cohen, the true meaning of Kant's Copernican revolution is realized in the Anticipations of Perception: it is not our knowledge that stems from objects, but, rather, it is objects that stem from our knowledge.

Kant uses the term 'phenomenon' to talk about the character of this reality as it is represented to us. He did not want to postulate the existence of the phenomenon as a metaphysical entity distinct from the noumenon. We are thus incapable of understanding what the noumenon, this independent reality, actually is. For this reason, the noumenon is an empty concept. There is no biunivocal correspondence between the *things-for-us* (the phenomena) and the *things-in-themselves* (the noumena). As Henry Allison (2004) shows us, in opposition to Strawson (1966), it is misleading to interpret Kant's phenomena/noumena distinction in an ontological sense as if it established two different worlds.

Another important point to consider is the strong relationship between the principle of Anticipations of Perception and determinism. In the Kantian analytical system, it is this principle, rather than the principle of causality, that is associated with the determinism of the laws of nature. Even though, as Ernst Cassirer (1956, p. 19) pointed out, the Kantian determinism is not metaphysical mathematicism, as found in Leibniz's and in Laplace's conceptions, but, rather, "critical determinism", its significance cannot be overestimated.

In the epistemological debate about the problem of indeterminism in quantum theory, determinism and causality are often identified as the same principle. Grete Hermann (1996) and Ernst Cassirer (1956) were able to point out the difference between causality and determinism. In fact, in Kantian terminology, this difference can be expressed in terms of two distinct principles: they are, respectively, the Second Analogy of Experience and the intensive principle of

Anticipations of Perception. The problem is that in the classical mechanics context, and also in the Kantian system of principles, these two principles were conceived as conjoined.

In the Second Analogy of Experience, Kant (A209 /B254) shows the internal connection between the principle of causality and the principle of intensive magnitude. The process of continuous transformation is, according to him, inherent to the production of intensive parts of reality, which, in turn, can be anticipated. Constrained by the epistemological conditions of classical mechanics, Kant asserted that, in order to follow the change from one state to another, we must associate the principle of causality to the principle of intensive magnitude. This allows us to follow all the steps of the process of determination of an effect starting from its cause. Thus, the principle of intensive magnitude is the basis for the reconstruction of the process of physical change, i.e., the causal transformation of empirical phenomena. Thus we must consider that causal connection and deterministic prediction are two absolutely distinct principles, even though Kant considers them as intrinsically associated.

This intrinsic association is perfectly accepted in the context of classical physics. Nevertheless, as Grete Hermann argues, the epistemological rupture between classical physics and quantum mechanics affects the very entanglement between these two principles.

For Hermann, the major cause of misconception in the philosophical interpretation of the results of quantum mechanics lies in the erroneous acceptance of an equivalence between causality and determinism. As the metaphor of Laplace's Demon reveals symbolically, determinism has as its basis the principle of prediction through calculus. This means, in the context of classical mechanics, that once the physical laws and the initial conditions (position and momentum) of a given event are known, we can determine their entire past and future with absolute certainty. Determinism is thus an inherent characteristic of the principle of prediction through calculus. It remains to be seen whether this characteristic is also inherent to the principle of causality. For Hermann, not only the principle of causality has a wider scope than that the principle of deterministic prediction, but it is the latter that was refuted by the quantum theory.

Hermann tries to isolate the dynamic meaning of the principle of causality so that it can be applied to the context of quantum mechanics. This meaning remains strictly limited to the scope of relations between perceptions. In Kantian terms, the synthesis of the manifold of appearance is not itself included in the empirical matter of perceptions. The association of perceptions, as David Hume has already shown, is always accidental, and this is the reason why perceptions cannot themselves establish any necessary connection. In order to acquire a properly scientific knowledge of experience, that is, knowledge that entails an objective rather than a purely subjective synthesis, it is necessary to introduce a dynamic rule of relations between perceptions so as to objectively allow for the constitution of appearances in time. This dynamic rule is the principle of causality itself.

As a transcendental condition of knowledge, causality is based on an epistemic rather than an ontological condition. As G. Hermann emphasized, the principle of causality cannot, therefore, be applied to an extrasensory, microphysical reality of the physical quantities which are not measurable. In this sense, the principle in question cannot be taken as equivalent to the continuous wave function of the strictly formal structure of the theory. This is contrary to the Copenhagen interpretation of causality.

In light of this argument, we will now focus on the transcendental role played by the principle of Anticipations of Perception in the context of quantum mechanics.

4 Anticipations of Perception in the Quantum Mechanics Context

As is widely acknowledged in quantum mechanics, the law that governs the evolution of a system, when no measurement is performed, is given by Schrödinger's equation. Much like in classical mechanics, it is a differential equation which describes, by analogy, how the state of a system evolves through time. Nevertheless, the key role in this equation is played not by the Hamiltonian function, but by an operator, the so-called Hamiltonian operator. This operator represents the total energy of the system, which does not make it different from a classical system. According to the formalism introduced by Dirac, the pure state of the quantum system is not represented by a point in real space, but by a vector – a state vector – in a vector space called Hilbert space.

The state of the system in quantum theory evolves deterministically, and this does not differ from the classical theory. However, in relation to the nature of prediction given by differential equations, there is a great difference between the quantum and the classical approaches. Unlike the classical theory, we cannot assign to a particular quantum state values for all the physical quantities associated with the system. In contrast, the function of description, which is quite characteristic of the classical systems, can no longer be present in quantum systems. Therefore we can say that the law governing the latter is only predictive and not descriptive, the prediction being inherently probabilistic. The state of a quantum system simply points to the probabilities of a particular outcome being obtained in an experiment performed at a particular time. It does not characterize the properties of the system. It is not even clear whether properties (in the sense of classical physics) can be attributed to the system at all. In this sense, as Michel Bitbol (1996) points out, the kind of predictive structure obtained in quantum theory is always contextual. It depends on the conditions of preparatory procedures and on the outcomes gathered by measurements. The quantum theory predicts only conditionally and probabilistically the outcomes of future experiments. The dynamical evolution of the state vector, from the experimental preparation of the system to the measurement of an observable, is deterministic. Nevertheless, the very act of performing a measurement modifies the state of the system. For this reason, the content of the predictive structure itself is not deterministic.

I claim that a philosophically consistent interpretation of quantum mechanics may be associated with a transcendental program of objectivation, even at the cost of making some modifications to the framework of the Kantian system. Considering Kant's pioneering role in the constitution of an anti-realistic semantic, as suggested by Carl Posy (1981) and Hilary Putnam (1983), it is possible to harmonize the non-classical requirements of objectivation with the transcendental approach. This procedure involves, in fact, the need to envisage the quantum system as a contextual system which cannot be independent from our means of objectivation.

According to Kant, the distinction between our theoretical reconstructions and the events of the independent external world is purely methodological. We cannot compare the external reality with our theoretical idea of it because both, reality and theory, overlap. Reality in itself is a theoretical construction. As we have seen, in Kantian terms "reality" is a structure evolving temporally according to a law expressed in terms of partial differential equations which allow us to anticipate the content of perception. On the order hand, in quantum mechanics, the quantum state represented by the state vector fulfills this very same transcendental requirement. It is also a predictive structure expressed in terms of partial differential equations which, in turn, allow us to anticipate outcomes of measurement processes. Even for Kant reality is not defined by a set of properties that remains invariant, but by an anticipative structure whose predictive unity remains invariant. In this regard, there is a perfect agreement between Kantian epistemology and quantum mechanics.

Nevertheless, it is exactly within this relation between the conceptual thought in quantum mechanics and the sensible intuition that a problem arises concerning the transcendental method proposed by Kant. Part of the problem is to articulate the principle of anticipations with the other constitutive principles of pure understanding, such as the dynamical principles of substance and causality and the mathematical principle of the axioms of intuition.

It is important to consider the fact that numbers do not express physical quantities in quantum mechanics. They are expressions of a more complex mathematical entity: the diagonal terms of a matrix. As Heisenberg (1983, p. 82) argues:

> If, for example, the X-coordinate of the electron is no longer a « number », as can be concluded experimentally, according to equation (1) [$p_1 q_1 \sim h$], the simplest assumption conceivable [(that does not contradict (1)] is that this X-coordinate is a diagonal term of a matrix whose nondiagonal terms express themselves in an uncertainty term of a matrix whose nondiagonal terms express themselves in an uncertainty or - by transformations – in other ways. (…) As soon as one accepts that all quantum-theoretical quantities are "in reality" matrices, the quantitative laws follow without difficulty.

Heisenberg's explanation of quantum physical quantities imposes important restrictions on the Axioms of Intuition. This principle, which governs the application of mathematics to experience, preserves its indisputable validity in the classical context. However, in quantum mechanics, since physical quantities are not expressed simply by numbers, limitations must be set to the Axioms of Intuition.

In the proof of the Axioms of Intuition Kant (B206) clearly states his conviction that the transcendental validity of the Euclidean geometry implies the existence of a corresponding empirical intuition. Even though Kant says that the geometry of pure intuition is undeniably valid for empirical intuition, he is not denying that the physicist can conceive other spaces (e.g. vector spaces) which are also applicable to the physical systems that human beings can conceive but not perceive. However, the objects of possible experience – by which Kant means "objects which are within the range of human perception" – must conform to the ordinary space of the Euclidean geometry. We associate each physical quantity which corresponds to a property of the classical systems with a number and then represent them in the ordinary space of the Euclidean geometry. Nevertheless, quantum systems resist being represented by numbers expressing physical properties in any phase space. The principle of superposition in quantum theory prevents us from representing any quantum system as a set of well-defined physical properties. This, consequently, forces us to consider that the Axioms of Intuition is not a general transcendental condition of quantum systems in spite of its applicability to the classical context. The main reason for this is that we do not have any intuition of quantum systems. We should, perhaps, follow the recommendation of Jean Petitot (1992), for whom a radical change in the mathematical principles of pure understanding must be introduced not through the imposition of limitations, but, rather, by substituting more general principles for them.

Nevertheless, it is still possible to claim that, because vector states are no more than a predictive structure, they perfectly satisfy the condition of the Principle of Anticipations of Perception. As Michel Bitbol (2005) points out in a Bohrian spirit, the predictive structure of quantum mechanics – which is essentially probabilistic and must correspond to a given experimental preparation – concerns only contextual phenomena. This structure cannot be made to correspond to a state of the system endowed with its own particular properties, such as in classical physics. The state vector always relates to an experimental procedure and no longer describes the evolution of objects in space–time. However, this structure is unified for all of the measurement operations that follow such experimental preparation and for all experimental contexts. In this sense, we now face the *a priori* and universal predictive structure that enables us to anticipate the perception of phenomena which manifest themselves in space–time. The kind of synthesis allowed by such an anticipating structure is absolutely original. It has nothing to do with the traditional unity of anticipation of the state of objects defined from their inherent properties. If we take into consideration the fact that in quantum mechanics the anticipative structure is only predictive and no longer describes the nature of the system, we can conclude that quantum theory verifies the Kantian claim that the notion of reality belongs to anticipation. In this sense, Bohr, in the spirit of Kant's thought, claims that natural science describes the interplay between nature and ourselves, not nature itself.

Bibliography

Allison, H. (2004). *Kant's transcendental idealism. Revised and enlarged edition*. New Haven, CT/London: Yale University Press.

Bitbol, M. (1996). *Mécanique quantique: une introduction philosophique*. Paris: Flammarion.

Bitbol, M. (1998). 'Some steps towards a transcendental deduction of quantum mechanics', *Philosophia naturalis*, **35**, 253–280.

Bitbol, M. (2000a). ‘Relations, Synthèses, Arrière-Plans; sur la philosophie transcendantale et la physique moderne’, *Archives de Philosophie*, **63**, 595–620.

Bitbol, M. (2000b). ‘Arguments transcendantaux en physique moderne’, in S. Chauvier, F. Capeillères (ed.), *La querelle des arguments transcendantaux, Revue philosophique de l’Université de Caen*, **35**, 81–101.

Bitbol, M. (2005). ‘Anticiper l’unité: une méthode de connaissance’, in M. Cazenave (ed.), *Unité du monde, unité de l’être*. Paris: Dervy.

Bohr, N. (1928). ‘The quantum postulate and the recent development of atomic theory’, *Nature (Suppl.)*, **121**, 580–590 (*BCW6*, 147–158).

Cassirer, E. (1956). *Determinism and indeterminism in modern physics: historical and systematic studies of the problem of causality*. Trad. O. T. Benfey. New Haven, CT: Yale University Press.

Cohen, H. (1999). *Le principe de la méthode infinitésimale et son histoire*. Trad. M. Launay. Paris: J. Vrin. (publié originellement à Berlin: F. Dümmler, 1883).

Cohen, H. (2000). *Commentaire de la «Critique de la raison pure» de Kant*. Trad. E. Dufour. Paris: Les Editions du Cerf.

Cohen, H. (2001). *La théorie kantienne de l‘expérience*. Trad. E. Dufour et J. Servois. Paris: Les Editions du Cerf.

Friedman, M. (1992). *Kant and the exact sciences*. Cambridge/London: Harvard University Press.

Heisenberg, W. (1930). *The physical principles of the quantum theory*. Chicago, IL: The University of Chicago Press

Hermann, G. (1996). *Les fondements philosophiques de la mécanique quantique*. Trad. A. Schnell, introduction et postface par Lena Soler. Paris: Vrin.

Hermann, G. (1999). ‘The foundations of quantum mechanics in the philosophy of nature’, *Harvard review of philosophy*, **7**, 35–44.

Hughes, R. I. G. (1992). *The structure and interpretation of quantum mechanics*. Cambridge, MA: Harvard University Press.

Kant (1998). *Critique of pure reason*. Trans. P. Guyer, A. Wood. New York: Cambridge University Press.

Petitot, J. (1991). *La philosophie transcendantale et le problème de l’objectivité*. Paris: Osiris.

Petitot, J. (1992). ‘Actuality of transcendental aesthetics for modern physics’, in L. Boi, D. Flament, J.-M. Salanskis (eds.), *1830–1930: a century of geometry*. Berlin/Heidelberg, Germany/New York: Springer, 273–304.

Petitot, J. (1994). ‘Esthétique transcendantale et physique mathématique’, in E. W. Orth et H. Holzhey (eds.), *Der Neukantianismus. Perspektiven und probleme*. Würzburg, Germany: Königshausen & Neumann, 185–213.

Posy, C. (1992). ‘Kant’s mathematical realism’, in Carl J. Posy (ed.), *Kant’s philosophy of mathematics*. Dordrecht, The Netherlands: Kluwer, 293–313.

Putnam, H. (1983). ‘Why there isn’t a ready-made world’, *Realism and reason, Philosophical papers*, vol. 3. Cambridge: Cambridge University Press, 205–228.

Strawson, P. F. (1966). *The bounds of sense: an essay on Kant’s critique of pure reason*. London: Methuen.

Can the Principle of Least Action Be Considered a Relativized A Priori?

Michael Stöltzner

Abstract Hardly another principle of classical physics has to a larger extent nourished hopes into a universal theory and has simultaneously been plagued by mathematical counterexamples than the Principle of Least Action (PLA). I investigate whether the PLA can be interpreted as a historically relativized constitutive a priori principle of mathematical physics along the lines Michael Friedman has drawn in *Dynamics of Reason*, using the example of relativity theory. Such an interpretation suggests itself, historically, because two main advocates of the PLA, Max Planck and David Hilbert, considered relativity theory as a case in point for the PLA. But they were also aware of the mathematical pitfalls and that without physical specification the PLA only represented an empty form. I argue that the different levels required for a consistent application of the PLA in mathematical physics induce a stratification that bears close parallels to the one by which Friedman intends to overcome the joint challenges of epistemological holism and a relativist reading of Kuhnian incommensurability. Yet, two differences remain. First, the mathematical and physical levels of the PLA are more intertwined than in Friedman's case. Second, although the PLA has survived quite a few scientific revolutions, so has the formulation of physical theories in terms of differential equations.

If one wants to embark on a Kantian analysis of the Principle of Least Action (henceforth PLA), there are basically two routes. First, one may focus on the formal teleology the PLA has often been associated with and interpret the PLA in the perspective of the *Critique of Judgment* as a regulative principle of reflective judgment. Along this line, as I have argued elsewhere (Stöltzner, 2000, 2005), the distinctive features of the PLA are: (i) its globality as compared to the local differential equations; (ii) the systemic modal structure of the actual dynamics and the possible dynamics that are set up in order to obtain a mathematically well-defined extremal principle. The strength of a formal-teleological surplus of the PLA over the standard

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formulation in terms of differential equations crucially hinges on mathematical subtleties and the concept of causal explanation assumed, rather than express a metaphysical harmony of the world.

Second, one may focus on the PLA's role as a mathematical principle that, after appropriate specification, permits one to succinctly formulate physical theories as different as classical mechanics, electrodynamics, relativity theory, and quantum physics. Taken in its abstract generality, the PLA has survived many vicissitudes of the scientific world-view and the minor or major revolutions in physical theory. This suggests viewing the PLA as a historically relativized constitutive a priori principle in the sense recently advocated by Michael Friedman (2001). Such is the line taken by the present paper. Friedman's conception of the dynamics of reason reaches back to the interpretation of the Kantian categories of space and time outlined in Hans Reichenbach's (1920) early analysis of the theory of relativity and, thus, to neo-Kantian debates about a modernization of the *Critique of Pure Reason* in the face of twentieth century physics.

Both assessments of the PLA do not necessarily contradict one another. On the one hand, the Marburg neo-Kantians, most prominently Ernst Cassirer, considered the Kantian categories not as constitutive but as regulative principles precisely to allow for a historical evolution of the basic principles of science in which the a priori appeared as an absolute – not historically relativized – invariant attained only in the ideal limit of the scientific enterprise as a whole. On the other hand, in the case of a mathematical principle applied to physics, the regulative principle of formal teleology represents a norm imposed on the mathematical architecture of a physical theory. If the PLA incorporating this formal teleology acts as the core mathematical axiom of a physical theory so conceived, it may thus be considered as constitutive for the particular natural laws – the mathematical models – derived from this axiom system. In a historical perspective, one might perhaps say that the first approach departs from Kant's critical analysis of the debates on physical teleology and natural theology that surrounded the birth of the PLA in the eighteenth century, while the second is associated with two important debates in the second half of the nineteenth century. Hermann von Helmholtz not only successfully applied the PLA beyond the narrower context of mechanics, thus bringing about its renaissance among his fellow physicists, but he also gave the 'return to Kant' prevailing within the German philosophical community an influential scientific twist.

Helmholtz's scientific achievements prompted the question whether the PLA represented "a valuable heuristic principle and leitmotif in striving for a formulation of the laws of new classes of phenomena" (1886, p. 210) or whether it was it just – as Ernst Mach held – a more economical reformulation of the same facts and thus "new only in *form* and not in *matter*" (1989, p. 452) Through the mathematical investigations of Karl Weierstraß and David Hilbert, it became clear that any surplus of the PLA over the differential equations required mathematical precision rather than strong metaphysical commitments. Nonetheless, Logical Empiricists largely sided with Mach, while Max Planck and Hilbert firmly believed in the significance of the PLA for a unified conception of physics. Even more so, Planck and Hilbert's emphatic pronouncement played an important role in making the PLA a shibboleth for

Logical Empiricists (Stöltzner, 2003). This basic disagreement makes the PLA an interesting test case as to whether the Logical Empiricist conception of constitutive principles in natural science, which basically takes them as part of pragmatics, needs to be revised in the direction outlined by Friedman, in particular if one wants to understand the progress of modern mathematical physics.

After a brief characterization of the PLA and a rehearsal of Friedman's approach, I study Planck and Hilbert's interpretations of the PLA. Although both are in many respects water on Friedman's mills – and stand in historical vicinity to his primary example general relativity –, two major problems remain. First, the mathematical and physical levels of the PLA are more intertwined than Friedman assumes, and what counts as 'natural' or 'deep' according to the respective standards does not always coincide. Second, although the PLA has survived quite a few scientific revolutions, so has the formulation of physical theories in terms of differential equations. Hence, there have always been two different lines of constitutive principles that show little sign of convergence despite the fact that in many cases both formulations yield physically equivalent results.

1 The PLA in a Nutshell

For the purpose of the present paper, I understand the PLA as an umbrella term for all integral variational principles in theoretical physics, among them Hamilton's and Maupertuis' principles. Mathematicians treat all those principles within the discipline of variational calculus and speak of a variational problem rather than the PLA. In an abstract sense the PLA states that the actual dynamics u yields an extremal value of the action functional $W[u]$ in comparison to all possible dynamics $(u + \delta u) \in M_u$, where δ denotes the variation of a quantity, $u + \delta u$ the varied dynamics, and M_u a function space that includes both the actual and all possible dynamics. $W[u]$ is the integral of the Lagrangian L that incorporates the physics contained in the PLA. Since the PLA is an integral principle, the boundary conditions must be specified in order to arrive at a mathematically well-defined variational problem.

In classical mechanics, L is the difference of kinetic and potential energy, the PLA reads $W = \int_a^b L(t,u(t),\dot{u}(t))dt = \text{Extr.!}$, with δu, $\delta \dot{u} = 0$ at the boundary, and u belongs to a class of admitted solutions M (=C^2, PC^1,…) of the variational problem. Variation leads to the Euler-Lagrange equations,

$$\frac{d}{dt}\frac{\partial}{\partial \dot{u}}L(t,u(t),\dot{u}(t)) - \frac{\partial}{\partial u}L(t,u(t),\dot{u}(t)) = 0$$

which typically – but not always – correspond to the standard equations of motion. But they are only a necessary condition for the variational problem. Other necessary conditions include the continuity properties of u, the form of the constraints imposed on the system, and that there are no (conjugate) points between a and b through which all $(u + \delta u) \in M_u$ have to pass. To find sufficient conditions has been a most difficult task. The first one was obtained only by Weierstraß, and Hilbert's main achievements in variational calculus lay precisely in this domain. Roughly speaking, sufficient conditions correspond to embedding the extremals into a suitable field of extremals (Mayer fields). This embedding expresses the above-mentioned global features of the PLA; and if a sufficient condition is fulfilled, the PLA represents a stronger claim than the corresponding differential equations.

2 The Dynamics of Reason

Friedman's overall intention is to develop a modified Kantian epistemology that successfully answers the combined challenges of W.V. Quine's epistemological holism and the prevailing relativist readings of Thomas S. Kuhn's theory of scientific revolutions. Friedman's analysis departs from diagnosing a far-reaching resemblance between Rudolf Carnap's (1950) linguistic frameworks and Kuhn's (1962) scientific paradigms. For, both frameworks and paradigms provide the basic concepts and rules under which science is 'normally' performed and define a standard of scientific rationality. The transitions between different frameworks or paradigms that characterize scientific revolutions cannot be assessed in this way, even though the later framework typically permits one to reconstruct the pre-revolutionary paradigm as a limit case and justify the revolution as rational. But this retrospective reconstruction fails to reflect the historical situation before the revolution, when the new concepts to come were still unavailable to science. Instrumentalistically-minded practitioners may well use bits and pieces of a new paradigm as a black-box device for making predictions, however without ascribing to it explanatory value. Predictive success alone, however, cannot motivate shifting badly understood or even ill-defined concepts into the core of a new paradigm or positing them as the axioms of a new linguistic framework, let alone overcome the problem of Kuhnian incommensurability.

The only remedy, Friedman argues, is to understand the scientific enterprise not as a succession of radically distinct speech communities but as "a common tradition of cultural change" that contains "different evolutionary stages of a single language" (2001, p. 60). This single language does not correspond to a single all-encompassing Carnapian framework, but contains stratifications that allow for communication across and transition between different frameworks or paradigms and render the new paradigm a real possibility already before the revolution. This "is already more than half the battle" (ibid. p. 103).

Friedman's dynamics of reason involves three different strata: (i) empirical laws properly so-called; (ii) a set of constitutive a priori principles that (a) render these laws meaningful as mathematical entities and (b) relate these mathematical entities to possible empirical circumstances by coordinative definitions; (iii) philosophical meta-principles or meta-paradigms that serve as a guidance "in motivating and sustaining the transition from one paradigm or conceptual framework to another" (ibid., p. 46). How the first two strata relate in detail can best be seen at the examples of Newtonian mechanics, relativistic electrodynamics, and general relativity, while other disciplines are still lacking the degree of mathematization presupposed in Friedman's conception.

> [A]dvanced theories in mathematical physics … should be viewed as consisting of two asymmetrically functioning parts: a properly empirical part containing laws such as universal gravitation, Maxwell's equations of electromagnetism, or Einstein's equations for the gravitational field; and a constitutively a priori part containing both the relevant mathematical principles used in formulating the theory (Euclidean geometry, the geometry of Minkowski space—time, the Riemannian theory of manifolds) and certain particularly fundamental physical principles (the Newtonian laws of motion, the light principle, the equivalence principle) (ibid., p. 71).

Mathematical concepts are, for one, a condition of the possibility of physical theories, e.g., by allowing one to represent space-time in terms of Riemannian manifolds. But additionally pure mathematics has the remarkable property that across revolutions it usually tends to preserve the earlier concepts as a special case, such that in retrospect the new concepts appear as extensions or generalizations of the earlier ones. Euclidean geometry is, for instance, simply a Riemannian geometry with zero curvature. "Revolutionary transitions within pure mathematics, then, have the striking property of continuously (and, as it were, monotonically) preserving … *retrospective* communicative rationality" (ibid., p. 96). But mathematics alone cannot mediate across revolutions in physical science because the very same mathematical structures may be used to conceptualize incommensurable theories. Riemannian manifolds, for instance, are the mathematical basis of general relativity, but supplemented with a different set of coordinating principles they allow a mathematically elegant, albeit non-standard, reformulation of Newtonian mechanics.

It is the coordinative principles through which the formulas of mathematical physics acquire an empirical meaning. These principles cannot be tested in the usual sense because they are constitutive for those specific empirical laws that face the tribunal of experience. But they do have empirical content and are revisable in the course of history, although a scientific revolution is needed to unseat a well-established coordinative principle because of the failure of the particular laws constituted by it. The examples of the Michelson-Morley and the Eötvös experiment show that coordinative principles, such as the light principle and the principle of equivalence, often emerge out of well-corroborated empirical facts. But in elevating them to a constitutive a priori principle, "an essentially non-empirical element of 'convention' or 'decision' must necessarily intervene" (ibid., p. 91). Once the new paradigm is established and the old laws are reformulated in terms of it, the decision between the old and the new may appear as a plainly empirical fact.

The purely mathematical and the coordinative constitutive a priori principles, accordingly, not only define a space of logical and empirical possibility for physical science, but in virtue of their internal dynamics and mutual relationship they also suggest what counts as reason or justification for any such possibility. But even if a transformation can be justified retrospectively, the question remains how the new paradigm can at all develop from within the pre-revolutionary framework. Here the philosophical principles play a decisive role because they smoothen out the revolutionary transitions prospectively. To allow for a rational transition despite a Kuhnian incommensurability at the manifest level of the language spoken by the practitioners (cf. 101), Friedman requires

> first, that the new conceptual framework or paradigm should contain the previous constitutive framework as an approximate limiting case …; second that the new constitutive principles should also evolve continuously out of the old constitutive principles, by a series of natural transformations; and third, that this process of continuous conceptual transformation should be motivated and sustained by an appropriate new philosophical framework, which, in particular, interacts productively with both older philosophical meta-frameworks and new developments taking place in the sciences themselves. This new philosophical meta-framework thereby helps to define what we mean, at this point, by a natural, reasonable, or responsible conceptual transformation (ibid., p. 66).

Moreover, philosophy provides "a new source of ideas, alternative programs, and expanded possibilities.(ibid., p. 25)". Philosophy can fulfill both roles only if it does not limit itself to a mere logic of science that is always bound to a rigid conceptual framework. But then one may wonder how "a subject inevitably and permanently fraught with unresolved intellectual disagreements [can] possibly help us to achieve a new rational consensus." But Friedman only requires: first, "that the new constitutive framework become a reasonable and responsible live option"; second, that there exists "a relatively stable consensus on what are the important contributions to the debate"; third, that "characteristically philosophical reflection interacts with properly scientific reflection in such a way that controversial and conceptually problematic philosophical themes become productively intertwined with relatively uncontroversial and unproblematic scientific accomplishments" (All ibid., p. 107).

In the case of special relativity theory, for instance, there was common agreement that Mach's criticism of the Newtonian concept of absolute motion was pivotal and combined with other investigations about relative and inertial motions. Einstein succeeded in putting these insights together with "recently established empirical facts concerning the velocity of light in a striking and hitherto unexpected manner" (ibid., p. 108) because he had been familiar with late nineteenth century debates on the foundations of geometry and Poincaré's conventionalist resolution of the problem how to determine the proper physical geometry. The same philosophical meta-principles also played an important, though somewhat different role, in general relativity that showed that even philosophical principles as deeply entrenched as space and time can undergo transitions.

Friedman's dynamics of reason also contains a global perspective. Putting together that, in virtue of the mathematical constitutive principles, transitions between different paradigms correspond to well-defined conceptual extensions and that the constitutive philosophical meta-principles provide the inter-paradigm transitions with a measure of naturalness, he argues "that we can thus view the evolution of succeeding paradigms or frameworks as a convergent series, as it were, in which we successively refine our constitutive principles in the direction of ever greater generality and adequacy" (ibid., p. 63). But "this is explicitly not convergence to an entirely independent 'reality' (however conceived) but rather convergence *within* the evolving sequence of constitutive frameworks itself" (ibid, p. 118), that is, 'internal' or Kant's 'empirical' realism. For, any strong version of scientific realism presupposes a unique sequence of successor theories, which contradicts the obvious plurality of historical pathways.

3 Max Planck on Principles and Constants

In 1915 Max Planck wrote an encyclopedia entry on the PLA. It opened emphatically.

> As long as there exists physical science, its highest desirable goal had been the solution of the problem to integrate all natural phenomena observed and still to be observed into a single simple principle …. It is natural that this goal has not been reached to date, nor ever will it be reached entirely. [However,] … the history of theoretical physics demonstrates that … this ideal problem is not merely utopical, but eminently fertile … Among the more or less general laws which manifest the achievements of physical science in the course of the last centuries, the Principle of Least Action is probably the one which, as regards form and content, may claim to come nearest to that final ideal goal of theoretical research (1944, p. 68).

Planck was well aware of the mathematical pitfalls. Only after a precise mathematical specification of the Lagrangian and of the conditions for the virtual displacements the PLA ceased to be "an empty form" (ibid., p. 70). Moreover, when emphasizing that the PLA did not reintroduce any material teleology into physics but was consistent with a causal explanation of all natural phenomena, Planck took a surprisingly instrumentalist tack and compared the reference to events at a later time in the PLA to calculations in which one keeps redundant variables in order to maintain the symmetry of the equations. In both cases, "[t]he question of their legitimacy has nothing to do with teleology, but it is merely a practical one" (ibid., p. 72). And Planck even provided examples how the PLA led science astray if interpreted as instance of a universal teleology.

"The fundamental importance of the Principle of Least Action became generally recognized only when it proved its applicability to such systems whose mechanism is either completely unknown or too complex to think of a reduction to ordinary coordinates" (ibid., p. 76). For, the PLA as an integral principle was independent of any choice of coordinates. Around 1910, Planck became increasingly convinced that his law of black-body radiation required a fundamental break with classical electrodynamics because the latter unavoidably yielded Jeans's law, in blatant contradiction to everyday experience.

> [O]ne will not for this purpose have to give up the Principle of Least Action, which has so strongly attested its universal significance, but the universal validity of the Hamiltonian differential equations; for those are derived from the Principle of Least Action under the assumption that all physical processes can be reduced to changes occurring continuously in time. Once radiation processes do no longer obey the Hamiltonian differential equations, the ground is cut from Jeans's theory (1910, p. 239).

The PLA was not simply applicable to discontinuous functions as well, such functions had even been an important source of mathematical progress in variational calculus. The PLA was thus perfectly consistent with a different coordinative principle according to which atomic processes were quantized.

That the PLA was deeper than a merely heuristic principle can also be seen at the fact that it matched what Planck considered as the most basic distinction in the physical world. There were, on the one hand, reversible processes governed by strictly causal dynamical laws. "All of them can be subsumed without difficulty under a single dynamical law, the Principle of Least Action" (ibid., p. 59). "In the realm of irreversible processes, however, the Principle of Least Action is no longer sufficient because the principle of entropy increase introduces an entirely novel element into the physical world view that is in itself extraneous to the action principle" (ibid., p. 11).

Its pivotal status in the architecture of the physical world-view and the emphasis that the PLA represented a form to be completed by physical specifications suggest investigating whether Planck in effect treated the PLA as a relativized a priori in Friedman's sense. Since he did not attribute much importance to the mathematical architecture involved, we have to look whether setting up a suitable PLA represented a coordinative a priori principle on a par with Newton's laws or the principle of equivalence, while the respective Lagrangian corresponded to the empirical laws specific for each domain. Although the PLA had not developed out of well-known empirical facts, Planck had cited quantum discontinuity in its favor.

The main test for relativized constitutive principles is how they behave 'normally' and in times of turmoil. Here Planck was pretty explicit. "[I]n all recent conflicts [between facts and theories] the great general physical principles held the field, namely, the principle of conservation of energy, the principle of conservation of momentum, the Principle of Least Action, the main laws of thermodynamics" (ibid., p. 44), while well-accustomed intuitive foundations had to give way, among them the immutability of chemical atoms, the independence of space and time, and the continuity of all dynamical effects in nature. This insight was part of a general tendency of simultaneous de-anthropomorphization and unification that Planck diagnosed within modern physics. Moreover, our present picture already contained certain traits that most likely would remain constant forever. "This constancy … is that which we now call the real [*das Reale*]"(ibid., p. 22). In 1925, Planck even conveived relativity theory within an overall convergence to absolute reality. "Yet when space and time have been denied the character of being absolute, the absolute has not been blotted out, it has just been moved more backward, to wit, into the metric of the four-dimensional manifold" (ibid., p. 154). Outdated absolute concepts are relativized just in order to find deeper absolute concepts.

Planck's convergent realism seems to openly contradict any relativized a priori. But Planck himself intended to remain consistent with Kant's critical philosophy. Since there was no way to distinguish between 'world view' and 'world', he argued, we could interpret 'world' itself as the ideal aim of all scientific research. Moreover, the constant elements in our world-view were abstract principles, such as the PLA, that remained empty without physical specification while entities such as the indivisible chemical atoms were superseded by new ones.

And there was also another side to Planck's realism. His own quantum of action and Boltzmann's constant characterizing thermal radiation, and the gravitational constant provide a universal system of units that does not depend on a metric convention. While present-day physicists aspire at reducing the fundamental constants of nature to laws, Planck considered them as "the invariable building blocks from which the edifice of the physical world is composed" (ibid., p. 39). Each major step towards the ideal aim of absolute knowledge uncovered a hitherto unknown constant of nature. First, "[t]he modification brought into mechanics by the principle of relativity contains as its essential part the introduction of a new universal constant alien to classical mechanics, the velocity of light in vacuum" (ibid., p. 82). Second, "the laws of thermal radiation, specific heat, electron emission, radioactivity unanimously indicate that not only matter itself but also the effects originating from matter … possess discontinuous properties, which once again is characterized by a new constant of nature: the elementary quantum of action (ibid., p. 83f.).

Since the abstract principles held the field in each scientific revolution, it is the universal constants that represent a Kuhnian incommensurability, e.g., between the paradigms of classical mechanics and relativity theory or between Jeans's and Planck's laws of radiation. For, the fundamental constant of the latter theory cannot be expressed in terms of the former. If one formulates the latter theory by way of a PLA, the new constant, of course, has to enter. Thus the full-fledged action principle contains both constitutive and empirical elements.

To conclude, Planck treated the PLA in its abstract form as constitutive for the domain of reversible physics, while the coordination was established through a suitable Lagrangian. No doubt, Planck was well-informed about and derived major motivations from the philosophical debates surrounding the PLA, giving them a rather Kantian twist both as regards causality and formal teleology. Needless to say, Planck's talk about the ideal aim of theoretical physics did not exclude that the PLA one day could be integrated into a more comprehensive formal principle.

4 David Hilbert and the PLA as Core of the Axiomatic Method

Again in 1915, David Hilbert gave an independent derivation of the field equations of general relativity by means of a single action principle. The paper that was rushed out in two parts (1916, 1917) bore the ambitious title "The Foundations of Physics." In it, he combined what he had learned about the physical characteristics of Einstein's theory in the making with his top expertise in variational calculus. The correct form of Einstein's equations was entered only into the galley proofs, such that Hilbert cannot claim full priority. But they were not his primary objective. What is more, both "Hilbert and Einstein saw their achievements of November 1915 as the culmination of year-long efforts of scientific research along their respective research programs" (Sauer, 1999, p. 566). These were by no means identical, followed different heuristics, and attributed different weight to the PLA (Rowe, 2001).

In the "Foundations", Hilbert poses four axioms and two physicality conditions. (I) Mie's axiom of the world function H demands that the variation of $\int H\sqrt{g}\, d\omega$ vanishes for each gravitational potential $g_{\mu\nu}$ and each electromagnetic potential q_s, where g is the determinant of $g_{\mu\nu}$ and $d\omega = d\omega_1 d\omega_2 d\omega_3 d\omega_4$ is the differential of the world parameters ω_k uniquely fixing the world points. H contains gravitational arguments, the $g_{\mu\nu}$ and their first an second partial derivatives with respect to the ω_k, and electromagnetic arguments, the q_s and their first partial derivatives with respect to the ω_k. Axiom (II) states that H be invariant with respect to an arbitrary transformation of the world parameters ω_k. Hilbert considers this axiom as "the simplest mathematical expression of the requirement that the coordinates in themselves do not possess any physical significance" (1924, p. 50). And in a footnote, he connects it to Einstein's idea of general invariance (today: 'covariance'). Hilbert next formulates a theorem that he calls the "leitmotif for the construction of his theory" (1916, p. 396), but does not provide a proof. Its main objective had been to show that "*the electrodynamic phenomena are an effect of gravitation*" (1916, p. 397). In the 1924 reprint, this claim was tacitly dropped and a weakened version of Noether's second theorem was proven as theorem II.

Although sufficient for a derivation of geometrical properties, such as Noether's second theorem and the Bianchi identities, axioms (I) and (II) do not fix H uniquely, such that Hilbert introduced two further axioms of a more physical kind. Axiom (III) demands the additivity of pure gravity and electromagnetism $H = R + L$, with R being the Riemann scalar curvature and L not containing second derivatives of the $g_{\mu\nu}$. This guarantees that no higher than second order derivatives of the $g_{\mu\nu}$ appear in the field equations, such that one obtains a reasonable dynamics. Axiom (IV) specifies the signature of the metric in order to obtain the required 3 + 1 pseudo-geometry for space–time. In addition, there are two supplementary conditions requiring that the physical solutions respect causal order and are free of singularities. Gödel's universe, in which one can return into one's own past, and the boom of research into singularities since the 1960s have shown that both conditions of Hilbert's were too restrictive.

Hilbert's axiomatization of general relativity exhibits a three-layered structure that not only goes naturally with the different steps in specifying the PLA, but that is also typical for his axiomatic method as a whole. Hilbert did not treat an axiom system as a homogeneous conceptual framework in which only logical deductions operate. This can already be seen in the Sixth Problem of 1900, where he gave a programmatic outline of the axiomatization of physics.

> [W]e shall try first by a small number of axioms to include as large a class as possible of physical phenomena, and then by adjoining new axioms to arrive gradually at the more special theories .... The mathematician will have also to take account not only of those theories coming near to reality, but also, as in geometry, of all logically possible theories.
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> Further, the mathematician has the duty to test exactly in each instance whether the new axioms are compatible with the previous ones. The physicist, *as his theories develop*, often finds himself forced by the results of his experiments to make new hypotheses, while he depends, with respect to the compatibility of the new hypotheses with the old axioms, solely upon these experiments or upon a certain physical intuition (1900, p. 272f./454f.).

The task of the mathematician begins with establishing the completeness of the axioms, i.e., that they permit one to derive all laws of the respective domain. Next is the internal consistency of the axioms. For instance, in the theory based on Fourier's heat equation "it is necessary to prove that the familiar boundary-value problem of potential theory is always solvable; for only the solution of this boundary-value problem shows that a temperature distribution satisfying the equation of heat conduction is at all possible" (1918, p. 410/1111). Cast in Friedman's terms, internal consistency of a suitable axiom system is an a priori condition that a certain physical phenomenon is logically possible.

By interpreting an axiom system in terms of appropriate number fields, Hilbert played internal consistency back to the consistency of arithmetic, which was to be proven by the finite means of meta-mathematics. But in 1930, Gödel's incompleteness theorem demonstrated that it was impossible to reach the desired absolute foundation of mathematics along these lines. Interestingly, in the same year, Hilbert viewed precisely this attempt as the legitimate heir of the Kantian a priori that "still contains anthropological dross from which it must be liberated [such as the preference of Euclidean geometry]; afterwards only the *a priori* attitude is left over which also underlies pure mathematical knowledge: essentially it is the finite attitude which I have characterized in several works" (1930, p. 962/1163). Hilbert's third requirement was external consistency. Kinetic theory is consistent with thermodynamics, and Einsteinian gravity possesses a well-defined Newtonian limit, while quantum theory contradicts Maxwell's equations, such that a new foundation of electrodynamics is called for. Recall that Friedman saw the main role of mathematics in establishing such a precise relationship between conceptual frameworks.

The fourth requirement, which Hilbert called 'deepening the foundations', started from the analysis of the independence of the axioms and was an heir of the failed attempts to prove the fundamental presuppositions of science themselves. But these reductions "only make it possible to trace things back to certain deeper propositions, which in turn now to be regarded as new axioms …. The actual *axioms* of geometry, arithmetic, statics, mechanics, radiation theory, or thermodynamics arose in this way" (1918, p. 407/1109). There was no unique way of 'deepening the foundations' of a given theory. Hilbert lauded both Boltzmann and Hertz for having deepened the foundations of Lagrange's mechanics containing arbitrary forces and constraints to either forces without constraints or constraints without forces. Moreover, by 'deepening the foundations', one may arrive at a physically unintuitive formulation, given that one intends to keep a very deep mathematical concept, such as continuity.

> The axioms of classical mechanics can be deepened if, using the axiom of continuity, one imagines continuous motions to be decomposed into small uniform rectilinear motions caused by discrete impulses and following one another in rapid succession. One then applies Bertrand's maximum principle as the essential axiom of mechanics, according to which the motion that actually occurs after each impulse is that which always maximizes the kinetic energy of the system with respect to all motions that are compatible with the law of the conservation of energy (1918, p. 409/1111).

All four requirements together with the fact that a unique and realist interpretation was not aspired at, suggest considering Hilbert's axiomatic method as a mathematical reorganization and stratification of a physical theory aimed at casting as much as possible in terms of constitutive a priori principles. For, mathematics has the advantage that the relationship across frameworks is rigorous and that one can precisely spot coordinating principles, such as general covariance and – less convincingly – Bertrand's principle. Moreover, Hilbert's axiomatic treatment of phenomenological theories, among them Kirchhoff's law of radiation and continuum mechanics, shows that he tried to establish empirical facts as constitutive principles under the joint guidance of a general philosophical outlook on the axiomatization of science and of the well-known heuristic powers of the PLA. This did not involve realist or reductionist aspirations, especially after the road to an immutable foundation of mathematics was barred. In the case continuum mechanics, he simply argued as had Planck in the above mentioned case of radiation, that the PLA was applicable even though, as of 1907, knowledge about the molecular constitution of matter was insufficient.

But there are problems to such an interpretation, especially if one looks at the "Foundations". For one, Hilbert's first axiom already contained a strong physical claim insofar as all energy-matter was subsumed under Mie's theory, a claim that Einstein considered as highly premature. Even more, the whole rationale of the failed theorem I was to blur the boundary between mathematics and physics. For, Hilbert believed "that a reduction of all physical constants to mathematical constants should be possible" (1916, p. 407). Hence, in some cases, the 'deepening' was to unearth what Hilbert typically – and pretty vaguely – described as the non-Leibnizian pre-established harmony between mathematics and physics. Not least this repeated allusion made Hilbert's axiomatic method suspicious among Logical Empiricists.

But the problem of non-uniqueness is more generic. The mathematician's 'deepening' and the physicist's search for the 'deepest' principles may yield diverging results. This is problematic even if one does not heed realist or structural realist aspirations. What is more, throughout its history the PLA has always been accompanied by largely equivalent formulations in terms of differential equations. Even if one emphasizes the philosophical and mathematical differences between both approaches – as I have done in the present paper – there is little sign of convergence to constitutive principles "of ever greater generality and adequacy" (Friedman, 2001, p. 63).

References

Carnap, R. (1950), "Empiricism, Semantics, and Ontology", *Revue internationale de philosophie*, 4, 20–40.

Friedman, M. (2001), *Dynamics of Reason*, Stanford: CSLI Publications.

Hilbert, D. (1900), "Mathematische Probleme", *Göttinger Nachrichten. Mathematisch-Physikalische Klasse*, 253–297. English translation in *Bulletin of the American Mathematical Society*, 8 (1902), 437–479.

Hilbert, D. (1916), "Die Grundlagen der Physik (Erste Mitteilung)", *Göttinger Nachrichten. Mathematisch-Physikalische Klasse*, 395–407.

Hilbert, D. (1917), "Die Grundlagen der Physik (Zweite Mitteilung)", *Göttinger Nachrichten. Mathematisch-Physikalische Klasse*, 53–76.

Hilbert, D. (1918), "Axiomatisches Denken", *Mathematische Annalen*, 78, 405–415. English translation in W. Ewald (ed.), *From Kant to Hilbert: A Source Book in the Foundations of Mathematics,* vol. II. Oxford: Clarendon Press, 1996, 1105–1115.

Hilbert, D. (1924), "Die Grundlagen der Physik", reprinted in *Hilbertiana – Fünf Aufsätze von David Hilbert*, Darmstadt: Wissenschaftliche Buchgesellschaft, 1964, 47–78.

Hilbert, D. (1930), "Naturerkennen und Logik", *Die Naturwissenschaften*, 18, 959–963. English translation in W. Ewald (ed.), op.cit., vol. II., 1157–1165.

Kuhn, Thomas S. (1962), *The Structure of Scientific Revolutions*, Chicago, IL: University of Chicago Press.

Mach, E. (1989), *The Science of Mechanics. Account of Its Development*, La Salle, IL: Open Court.

Planck, M. (1910), “Zur Theorie der Wärmestrahlung”, in: *Physikalische Abhandlungen und Vorträge*, Braunschweig, Vieweg, 1958, vol. II, 237–247.

Planck, M. (1944), *Wege zur physikalischen Erkenntnis. Reden und Vorträge*, Leipzig: S. Hirzel.

Reichenbach, H. (1920), *Relativitätstheorie und Erkenntnis A Priori*, Berlin: Springer; English translation, Los Angeles: University of California Press, 1965

Rowe, D.E. (2001), “Einstein Meets Hilbert: At the Crossroads of Physics and Mathematics”, *Physics in Perspective*, 3, 379–424.

Sauer, T. (1999), “The Relativity of Discovery: Hilbert’s First Note on the Foundations of Physics”, *Archive for the History of Exact Sciences*, 53, 529–575.

Stöltzner, M. (2000), “Le principe de moindre action et les trois ordres de la téléologie formelle dans la Physique”, *Archives de Philosophie*, 63, 621–655.

Stöltzner, M. (2003), “The Least Action Principle as the Logical Empiricist’s Shibboleth”, *Studies in History and Philosophy of Modern Physics*, 34, 285–318.

Stöltzner, M. (2005), “Drei Ordnungen formaler Teleologie. Ansichten des Prinzips der kleinsten Wirkung”, in M. Stöltzner and P. Weingartner (eds.), *Formale Teleologie und Kausalität*, Paderborn: Mentis, 199–241.

Von Helmholtz, H. (1886), “Über die physikalische Bedeutung des Princips der kleinsten Wirkung”, in: *Wissenschaftliche Abhandlungen*, Leipzig, 1895, vol. III, 203–248.

A Critical Account of Physical Reality

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Abstract Several methodological *a priori* assumptions that underlie modern physics are investigated. They are transcendental in Kant's sense, i.e. necessary conditions of objective physical knowledge, in particular, of theory formation and experimental practice. In the transition from classical physics to quantum physics, their meaning weakened substantially. (1) The general methodological and metaphysical principles behind modern physics were closely related to rationalist metaphysics, above all the belief in knowledge-independent causal agents and fundamental laws of nature. This belief is an essential ingredient of Planck's and Einstein's *metaphysical realism* and of current positions of *scientific realism*, or what is called *classical realism* in the paper. (2) Empiricism criticises classical realism. However, this criticism misses the methodological indispensability of non-empirical principles such as the principles of substance, mereological and causal analysis, unity, and simplicity. (3) Kant criticised classical realism. His a *priori* is compatible with a methodological view of these principles. (4) In twentieth century physics, however, Kant's a *priori* turned out to be too strong. The touchstone of his *critical account of physical reality* is quantum theory. It is shown that a critical account of *subatomic reality*, which is slightly more liberal, comes close to central ideas of Niels Bohr's *complementarity vi*ew of quantum mechanics.

In this paper I will investigate several kinds of *a priori* assumptions that underlie modern physics from its very beginnings. I will discuss their methodological role and show that some of them are indispensable for theory formation and experimental practice. They are necessary conditions of objective physical knowledge, that is, they are transcendental in Kant's sense. However, their meaning weakens substantially in the transition from classical physics to quantum physics. (1) The general methodological and metaphysical principles that gave rise to modern physics were closely related to rationalist metaphysics. To the metaphysics behind modern physics belongs the belief in knowledge-independent causal agents and fundamental laws of nature. It survived

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up to the present day, in Planck's and Einstein's *metaphysical realism* as well as in current positions of *scientific realism*. Since this kind of realism is associated with the concepts of classical physics, I call it *classical realism* in the following. (2) Empiricism has always been criticising the metaphysical principles of classical realism. However, it misses the methodological indispensability of several non-empirical principles of physics, amongst them principles of substance, mereological and causal analysis, unity, and simplicity. (3) Kant criticised classical realism. His *a priori* does not miss the methodological character of these principles. In the *Critique of Pure Reason* and the *Metaphysical Foundations of Natural Science*, he elaborated a *relational view* of physics that is strikingly modern. His empirical realism opens a *third way* between classical realism and empiricism that does not collapse into constructivism. (4) However, in view of twentieth century physics his *a priori* turned out to be too strong. The touchstone of his *critical account of physical reality* is quantum theory. It turns out that a critical account of *subatomic* reality, which is slightly more liberal, comes close to central ideas of Niels Bohr's *complementarity vi*ew of quantum mechanics.

1 Classical Realism

Classical realism stems from classical point mechanics and was further elaborated in classical electrodynamics. Both theories were developed in the spirit of rationalist metaphysics. According to them, physical reality is completely determined. On this basis, Einstein was convinced that the quantum mechanical description of physical reality is not complete.[1] Classical realism is based on two presuppositions that are deeply rooted in rational metaphysics. The first is the principle that physical objects are *substances*. A substance in Descartes' or Leibniz' sense is a thing-in-itself, which is explained as follows. It is independent of the rest of the world, it is a carrier of primary qualities, and it can be defined in terms of monadic predicates. The second metaphysical presupposition is the assumption that all physical events and processes are completely determined by the laws of Nature, in accordance with the principle of *causality*. The traditional principles of substance and causality enter classical physics in terms of interaction-free physical objects and deterministic laws of their interactions.

In classical point mechanics, substance and causality come in terms of mass points and their trajectories. Mass points are carriers of physical properties such as mass, momentum, energy and charge. For these properties, conservation laws are valid.[2] A mass point has the following features in common with a substance in the

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[1] Einstein et al. (1935); Einstein (1949).

[2] In non-relativistic mechanics, the concept of mass splits into inertial and gravitational mass. Gravitational mass is the charge of gravitation. Mass, momentum and energy are conserved separately. In special relativity, inertial mass splits into rest mass and velocity-dependent relativistic mass. The non-relativistic conservation laws are replaced by combined conservation laws for mass–energy and energy–momentum.

traditional sense. It is regarded as a thing-in-itself that exists on its own and carries its physical properties independently of the rest of the world. This independence is expressed in Newton's law of inertia. Even though the law of inertia is a specific case of Newton's law of force (i.e., the case of a vanishing force), the force zero or interaction-free case is methodologically prior. All non-inertial motions are defined as deviations from the inertial motions. (Only Einstein's general relativity skips this distinction.) Deviations of the inertial trajectory are supposed to be caused by external forces. In a first approach, these forces are described as if they were substance-like entities, too, that act at mechanical bodies. The forces act on a *relational dynamic* property of a mass point, namely its *charge*. The charge of gravitation is the gravitational mass (as distinguished from the inertial mass).

For Newton, the gravitational mass did *not* belong to the primary qualities of a body. He did not yet have this concept but only the concepts of weight and mass (i.e., the inertial mass). The weight of a body is obviously a relational property. Newton thought that it is proportional to mass, where mass is an absolute quantity defined in terms of the number of atoms per volume.[3] The atoms were Newton's substances. Today, the charges of other interactions such as the electric charge add to the gravitational mass. For example, they are combined in the law of the classical Lorentz force $\mathbf{F} = q/c\ (\mathbf{E} + m\ \mathbf{v} \times \mathbf{B})$, which describes the deflection of massive charged particles in an electric and magnetic field.

In classical electrodynamics, substance comes in terms of space–time points as carriers of the electric and magnetic field strengths and in terms of the solutions of the Maxwell equations, i.e., fields and waves as certain functions of the electromagnetic field strengths. Causality comes in terms of the propagation of these fields and waves. Regarding the relation between substance-like physical objects that are conceived as things-in-themselves and their interactions, Maxwell's field theory is similar to Newton's mechanics. The simplest case are the homogeneous Maxwell equations. They describe free fields.

Classical mechanics and electrodynamics have in common that their basis is the description of non-interacting entities. Interactions between mass points or between matter and fields are taken into account later. Whenever the interactions become too complicated, perturbation theory is used in classical physics (and beyond). It starts from the unperturbed case, i.e., free mass points and uncoupled fields, and approximates their interactions stepwise. In this regard, the metaphysical concept of a substance that is independent of the rest of the world underlies the most familiar idealization of physics, namely the concept of a non-interacting, approximately isolated physical object or system. Every physicist knows that this idealisation is *not really* justified. The law of inertia neglects the universality of gravitation, i.e., the interaction of any mass point with celestial bodies that make up a *non*-inertial frame. Classical mechanics is based on the assumption that this idealization is in most cases a prettily good approximation. And perturbation theory is based on the assumption that the

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[3] Newton (1687).

deviations from this idealization can be taken into account in a *linear* approach by a *converging* series of corrections of *decreasing* size. In addition, classical mechanics and electrodynamics assume that different kinds of forces or interactions, namely gravitation and electric or magnetic fields, sum up linearly.

Hence, classical physics is built on the metaphysical presupposition that physical reality can be analysed in terms of independent objects and forces that sum up linearly to the complex phenomenological properties and interactions of the complex physical phenomena observed in nature. The experimental methods of modern physics, too, are based on this presupposition. Any experiment aims at isolating a physical system from its environment and studying its spatio-temporal and dynamic properties. And any experiment aims at causal analysis, that is, at isolating the relevant causal factors that determine its behaviour.

Indeed, the traditional metaphysical concepts of substance and causality are closely related to the methodological principles of Galileo's *resolutive-compositive method*, i.e., the assumption that physical phenomena can be decomposed and recomposed by means of experiments and mathematical methods. Newton called this the method of *analysis and synthesis*. The concept of substance underlies the attempts at analysing the phenomena into components (*mereological analysis*). For example, the spectral analysis of light by means of a prism results in the spectral decomposition of light, in rainbow colours (*analysis*). Newton explained it in terms of different light atoms. In addition, he showed how white light may be recomposed from two overlapping spectra (*synthesis*). Newton *would* have liked to decompose also matter in its constituent parts, the atoms, but he could not. However, the principles of mereological analysis remained fruitful in modern subatomic physics. Quantum theory explains the results of spectral analysis in terms of light quanta. Analogously, the experiments of atomic, nuclear physics result in analysing matter into its subatomic parts, i.e., in atom and subatomic particles. The concept of causality, in turn, identifies the 'substances' of physics with causal agents. Experiments aim at the isolation of causally relevant factors and the causal agents behind them (causal analysis), the mathematical descriptions of which in turn are assumed to combine to the contents of physical explanations.

Let me summarise these crucial features of classical physics. The classical concept of a non-interacting, isolated physical object or system corresponds to the traditional metaphysical concept of substance. Classical forces and fields are also modelled after this concept. And the interactions between physical objects are described in terms of forces or fields, which are assumed to sum up linearly as independent causal factors. The experimental methods of physics are based on related concepts of an isolated object and the causal analysis of its spatio-temporal and dynamic properties. Hence, the traditional metaphysical concepts of substance and causality are closely related to the mathematical and experimental methods of modern physics. They are indispensable *methodological principles* of physics. Without them, neither a physical dynamics nor any experimental test of it were possible.

To them add other metaphysical principles of modern physics, such as the principles of the unity of nature and the simplicity of its mathematical structure. Newton expressed them in his *Opticks* as follows: "Nature will be very conformable to her self and very simple".[4] These metaphysical assumptions about a rational order of the phenomena lie at the very heart of modern physics. They give rise to the methodological principles of the *unity* and *simplicity* of the laws of physics. The latter are closely related. Unification is a central goal of physics. Simplicity is an important principle for the construction of a unified theory. Newton, Maxwell, and Einstein constructed their theories based on them. In addition, these principles give rise to the symmetry principles of physics. Leibniz used them as powerful tools against Newton's concepts of absolute space and time, in the famous Leibniz–Clarke debate[5]; indeed, they underlie theory construction in physics up to the present day. All the principles mentioned here stem from rationalism. They express the belief that the world is rational. They claim that the physicists are able to decipher the book of Nature because it is written in mathematical letters which may be put in axiomatic terms.

Classical realism is just the assumption that Nature is *really like this*. It is the assumption that Nature is constituted by independent physical objects with causal properties that are completely determined by the laws of mathematical physics. Newton, Planck, and Einstein combined this assumption with the belief in the existence of an all-embracing mathematical structure behind the variety of physical phenomena, which may be grasped by universal physical principles and a unified theory of physics. Planck emphasised that the goal of physics is to discover a constant mathematical unity, which is actually the real, behind the phenomena.[6] And Einstein did not accept quantum theory because it violates the ideas of independent causal agents behind the phenomena and of a unified axiomatic description of these agents and their observable effects.

2 Empiricism and Metaphysics

Empiricist philosophers criticise classical realism and the underlying metaphysical principles. This criticism accompanied modern physics almost from its very beginnings. Locke criticised the non-empirical concept of a substance in the sense of an unobservable carrier of observable properties of things. He claimed that our experience of empirical substances only gives rise to the complex idea of empirical properties that constantly go together.[7] Nevertheless, he believed in the existence of unobservable causal agents in nature, and in particular, of atoms. In this point, his views still came close to the metaphysics behind Newton's physics. Hume was more radical (or consequent). He criticised any principle of causality that attributes the necessity of a law of nature to mere empirical regularities. In taking up their criticism, Kant understood both principles as conditions *a priori* for the possibility

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[4] Newton (1730, p. 397).

[5] Leibniz and Clarke (1715–1716), in particular Leibniz' third letter.

[6] Planck (1908, p. 49).

[7] Locke (1689, Book II, beginning of chapter XXIII).

of our experience. He attributed them to the cognitive capacities of our understanding rather than to the structure of a reality that exists independent of our empirical knowledge. Modern empiricism followed Hume and rejected Kant's *a priori*. Mach criticised Newton's concept of force and his atomistic foundation of the concept of mass as metaphysical and expressed Newton's laws in operational concepts. In view of the twentieth century scientific revolutions, Carnap and Reichenbach criticised Kant's *a priori* as bound to a Newtonian metaphysics.

From an empiricist point of view, any science needs *empirical* foundations and the quest for any *metaphysical* foundations of science is at odds with this requirement. Carnap initiated a research program of reconstructing physical theories from concepts with empirical significance. More recently, van Fraassen claimed that only the empirical substructure of a physical theory is a safe description of physical reality.[8] However, twentieth century empiricism failed in its attempts at finding a demarcation between empirical science and metaphysics. The distinction of the empirical and the metaphysical had to be revised, when the search for empiricist demarcation criteria turned out to be in vain. According to the Quine-Duhem thesis, isolated theoretical statements do not unambiguously correspond to isolated empirical data. This gave rise to a liberalised version of empiricism. According to it, an empirical theory may be subject to revision in front of the tribunal of *all* available empirical data; nevertheless, the principles of physical theories are partially and indirectly testable. Empirical underdeterminacy exists, but it is a *philosophical* problem rather than a problem of physical practice.

Indeed, closer examination of specific physical theories showed that the architectonics of physics is a complicated many-dimensional network of hierarchical structures. Its empirical and non-empirical elements are in many ways interwoven. One may identify the following three kinds of non-empirical or metaphysical elements in physical theories:

(i) *Conventional elements*: Many physical theories contain quantities which are empirically underdetermined and which can be fixed in favour of a mathematically simple and elegant structure of the theory. A famous example is the convention about the two-way speed of light, upon which Einstein's famous operational definition of simultaneity is based. In his seminal paper on special relativity, Einstein suggested that, when we synchronise clocks by means of light signals, light travels in both directions with the same speed.[9]

(ii) *The choice of a metric*: Measurement presupposes a metric. The choice of a metric, however, presupposes a theory of the measurement devices. Length and time may be measured by means of rigid rods and Newtonian clocks or by means of light signals, that is, with a Galilean or a Lorentzian metric. The same is true of velocity, which may be measured in a Galilean frame or a Lorentz frame.[10] Mass may be measured in classical terms, i.e. with a mass

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[8] van Fraassen (1980).

[9] Einstein (1905, p. 894).

[10] Krantz et al. (1971).

spectrograph which obeys the Lorentz force law, or in non-classical terms, e.g. in terms of the mean energy of the resonance of the decay of an unstable subatomic particle and Einstein's equivalence of mass and energy.

(iii) *Meta-theoretical principles* that stem from non-empirical assumptions about the structure of empirical reality and the laws of Nature. They fix classes of theories.[11] To them belong the metaphysical principles discussed in the preceding section, above all, the *symmetrie*s of physics. Classical mechanics is invariant under spatial reflection and time reversal. Non-relativistic mechanics is Galilean invariant. Electrodynamics is Lorentz invariant. The assumption that fundamental laws are CPT invariant[12] belongs to physics up to the present day. A basic principle of classical field theory hitherto not mentioned is the *locality* of interactions, i.e., the assumption that there are no actions-at-a-distance.

All these non-empirical elements of physical theories are closely related. The spatio-temporal symmetry of a theory defines the choice of a Galilean or Lorentzian metric. The choice of the conventional elements of a space–time theory affects the metric, too. To reject the Einstein convention of special relativity means to opt for a very complicated metric.[13] All three kinds of assumptions affect what is called the ontology of a theory, that is, the kind of entities to which a theory commits.[14] To believe in a preferred inertial frame means to re-establish an absolute, neo-Newtonian space–time and to reject Lorentz invariance, and vice versa. To believe in a quantum theory with hidden variables means to re-establish action-at-a-distance, to reject Lorentz invariance, and so on.

However, *for all practical purposes* the metaphysical principles (i)–(iii) are empirically testable. The choice of a metric is empirically justified by the operational foundations of a measurement method, the choice of adequate axioms of measurement, and the validity of a representation theorem, as empiricist measurement theory shows.[15] Once the metric is fixed, the conventional elements of physical theories no longer remain arbitrary.[16] And several symmetries as well as the locality

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[11] Here, 'meta-theoretical' is not used as a linguistic term but following Wigner. Wigner (1964, p. 16), observed "a great similarity between the relation of the laws of nature to the events on one hand, and the relation of symmetry principles to the laws of nature on the other." See Falkenburg (1988). A meta-theoretical principle fixes a certain structural feature of a theory. It can be expressed in the language of the theory as well as in an informal meta-language.

[12] See Streater and Wightman (1964, pp. 142–146). P is the parity transformation, the subatomic analogue of mirroring; C is charge conjugation, the transformation of particles into antiparticles; T is time reversal.

[13] See the discussion of non-standard simultaneity relations in Friedman (1983, pp. 165–176), and the related discussion of empirically equivalent theories, loc cit. pp. 266–339.

[14] Quine (1951, p. 15).

[15] Krantz et al. (1971).

[16] Friedman (1983, p. 317), points out "how firmly the standard simultaneity relation is embedded in relativity theory. One can not question the objectivity of this relation without also questioning significant parts of the rest of the theory. In particular, one cannot maintain that distant simultaneity is conventional without also maintaining that such basic quantities as the proper time metric are conventional as well."

assumption of classical field theory were rejected in subatomic physics. The non-locality of quantum phenomena, parity violation, and CP violations are supported by many experimental results. The experimental tests of symmetries or quantum non-locality have the nice feature that they directly test the respective meta-theoretical principle, by testing a *class* of theoretical structures rather than a specific theory.[17] In particular, the experiments of Bell's inequality exclude all hidden variable theories with local coupling.

Nevertheless, this nice liberalised empiricist view of physics is not the whole story. Some crucial *completely untestable* principles of modern physics remain. And they do not only concern the speculative foundations of rival theories that remain *empirically underdetermined for principal reasons*, like the interpretations of quantum mechanics beyond the probabilistic Born-von Neumann interpretation. Above all, there are the *general methodological principles* of physics discussed in the preceding section. To them belong the principles of:

1. S*ubstance*: Physical objects are (approximately) *non-interacting* carriers of physical properties (macroscopic matter, particles, forces, fields).
2. C*ausality*: These carriers of physical properties are *causal agents* which determine the phenomena completely.
3. A*nalysis*: The experimental methods of physics aim at *isolating* these causal agents and their properties. In particular, they aim at mereological and causal analysis.
4. S*ynthesis*: The properties of the causal agents of physics are assumed to *sum up linearly* to the properties of complex phenomena.
5. U*nity*: The observable phenomena, their constituents, and their composites finally obey the same laws of physics.
6. S*implicity*: Of two given theoretical explanations, the simpler one is preferable as presumably closer to truth.

All these principles are *methodologically* indispensable for physics, from Galileo's and Newton's day up to twenty-first century physics. To abandon them means to dispense with physics as an experimental and mathematical science that aims at objective knowledge. They stem from traditional metaphysics, they are non-empirical or *a priori*, and they are constitutive of physics as *the* paradigmatic exact empirical science. They constitute physical objects and their interactions in the sense that they make it possible to investigate them. They are transcendental principles of physics in Kant's sense, that is, conditions of the very possibility of physical experience.

In twentieth century physics, the causal agents and dynamic processes that underlie physical phenomena turned out to be much less substantial, causal, linear, symmetric, unified, and simple than the founders of modern physics thought. Nevertheless, the belief in the above principles remains indispensable for theory formation and experimental practice in physics. In particular, the quest for unification remains an indispensable methodological principle. Modern philosophy of science emphasises that

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[17] Franklin (1986, pp. 35–38), investigates this feature for the experimental tests of parity violation.

scientific explanation is unification and simplicity is a crucial criterion for the choice between empirically equivalent theories.[18] A further important point is completely neglected in the philosophy of science. In view of the current axiomatic disunity of classical and quantum physics, the construction of the *scales* of the fundamental physical quantities is *without empirical foundation*. In current physics, the length, time, and mass scale are constructed from the Planck scale (10^{-35} m) over current subatomic physics (10^{-18} m) and the mesocosmic scale (10^{-2}–10^{6} m) up to the horizon of the observable universe and beyond. From the quarks to the horizon of the universe, there is a *chain of overlapping measurement methods*. Each of these measurement methods has well-defined empirical foundations in the sense of the empiricist theory of measurement. However, below the quarks and beyond the horizon is *nothing* that has been measured. And the *unity* of the different measurements involved in between is *a priori* assumed. The chain of measurement methods mentioned above involves measurements with *incommensurable theoretical foundations*. Its supposed unity rests on two pillars: on the fact that measurement methods with overlapping domains do not give rise to contradictory measurement results *and* on the metaphysical assumption that the properties of the causal agents in Nature exhibit *more* unity than the axiomatic theories of current physics.[19]

3 Kant's *A Priori*

Classical realism as well as empiricism misses the methodological indispensability of the above non-empirical principles of physics. Classical realism assumes that Nature is *really like this*, i.e., constituted of isolated causal agents with linear properties that completely determine the phenomena. This view claims *too much* about physical objects, their properties, and their interactions, as quantum physics shows. Empiricism, on the other hand, assumes that Nature is defined in *empirically testable terms* only. This view does *not claim enough* about physical objects, their properties, and their interactions, as the above non-empirical principles show.

Kant's theory of nature is different. It criticises the rationalist metaphysics behind classical physics as follows. Rationalist metaphysics aims at giving a true description of the real world. It is a doctrine of independent entities such as Cartesian substances or Leibnizean monads and it embraces objects such as God or the whole world. According to the *Critique of Pure Reason*, all these entities are epistemically unaccessable. In particular, it is contradictory to postulate entities that are conceived as *independent* and as *objects of our knowledge* at the same time.[20] The first assumption gives to an entity the status of a thing-in-itself that does not

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[18] See Friedman, (1983, pp. 266–271).

[19] See Falkenburg (1997, 2007, chapter 5).

[20] This is the rationale of Kant's doctrine of the antinomy of pure reason. For a detailed analysis, see Falkenburg (2000, p. 177ff.)

belong to empirical reality, whereas according to the second an entity is given by means of experience and measurement. In this sense, rationalist metaphysics claims *too much* about the objects of our knowledge. In Kant's view, objective science only investigates the structure of a world of appearances, that is, empirical reality, and not a world of things in themselves. As a consequence, he attributes objective reality only to the objects of possible experience. However, he also emphasises that the claims of empiricism are *not sufficient* for the justification of objective knowledge. He claims that the structure of scientific experience depends on the principles of pure reason, in particular, on the principles of substance and causality. Indeed, his theory of nature explains the methodological role that these principles play in modern physics. Up to the present day, in particular the following general ideas and principles *a priori* of Kant's theory of nature deserve attention.[21]

A. Physical reality is *structured by our cognitive tools*. To them belong the concepts and principles of

   A.1 Extensive and intensive magnitudes
   A.2 Substance
   A.3 Causality

B. Physical reality is *thoroughly relational*.[22] It is given in terms of relations which hold between

   B.1 Appearances
   B.2 Unobservable entities that cause the appearances
   B.3 Experimental data

C. Physical reality *can not be known completely*. The idea of complete knowledge of the world only gives rise to regulative principles, in particular of the

   C.1 Mereological analysis of matter into infinitely many parts
   C.2 Causal analysis in search of a fundamental force of physics
   C.3 Systematicity of nature in its empirical laws

A. According to the *Critique of Pure Reason* and the *Metaphysical Foundations of Natural Science*, the *a priori* concepts of extensive and intensive magnitudes, substance, and causality give rise to indispensable *a priori* principles of modern physics. These concepts are constitutive for any knowledge of physical objects, their properties, and their interactions. A.1: The extensive and intensive quantities of modern physics are physical quantities such as length, time, mass, momentum, energy, charge, and temperature. The construction of their *scales* is indeed an indispensable *a priori* of physics, which is neglected by empiricist philosophy

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[21] I neglect Kant's theory of space and time as pure intuitions. The role that remains for intuition in quantum physics is discussed in Falkenburg (2006).

[22] See Kant (1781/1787, A 265/B 321).

of science.[23] A.2: Kant's category of substance is *not* identical with the traditional concept of substances as things-in-themselves. It is more modest. For Kant, substance is only *our* concept *a priori* of an entity with some stable, re-identifiable property. For him, the substance of classical mechanics is a physical *property* (and not a *carrier* of properties), namely the quantity of momentum. A substance of physics is subject to a conservation law.[24] A.3: Similarly, for him causality is only *our* concept *a priori* of a necessary link between subsequent events that helps to establish an objective time order.[25]

B. As a consequence of his criticism of the traditional concept of substance, for Kant empirical reality is *a relational structure*. It is given in terms of relations that hold between appearances. All appearances are relational entities, too. No appearance has primary qualities like Newton's atoms or internal properties like Leibniz' monads: "The inner determinations of a *substantia phaenomenon* in space[...] are nothing but relations, and it is itself a sum total of mere relations."[26] Once things-in-themselves are rejected, there remain only objects of physical knowledge related to each other. Things-in-themselves are relationless. They can neither be experienced nor measured and do not belong to empirical reality.

Kant admits three kinds of parts of empirical reality or objects of possible experience. The first is straightforward, the second invokes the postulates of empirical thinking, the third admits experimental data. B.1: The basic ingredients of empirical reality are *appearances*, that is, spatio-temporal objects or events which are immediately perceived and related according to the principles of pure understanding. B.2: In addition, Kant accepts *causes of appearances*, that is, things connected with appearances according to the three analogies of experience. According to the analogies of experience, the relations between all parts of empirical reality are based on *a priori* principles of conservation of substance, causality, and universality of interaction. These principles function like *a priori* guides to inferences to the best explanation. Kant's own example of the existence of an unobservable part of empirical reality is "the existence of magnetic matter penetrating all bodies" which we infer "from the perception of attracted iron filings, although an immediate perception of this matter is impossible for us given the constitution of our organs".[27] B.3: Most physical objects are neither immediately perceived nor simply related to perceptions. The appearances of empirical science are obtained by means of experimental investigation and measurement. From Kant's point of view, experiments are theory-laden in quite another way than any kind of non-scientific experience. For him, an experiment is a specific question put to a specific part of nature (or empirical reality) – "like an appointed judge who compels witnesses to answer the questions he puts to them."[28]

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[23] See my above remarks in Section 2, Falkenburg (1997, 2007, chapters 2 and 7).

[24] See Kant (1786, pp. 537–542), and von Weizsäcker (1971, pp. 383–404).

[25] See Kant (1781/1787, A 189–211/B 232–256).

[26] Kant (1781/1787, A 265/B 321).

[27] Kant (1781/1787, A 226/B 273, chapter on the "postulates of empirical thinking").

[28] Kant (1787, p. XIII).

Such a question to nature is put under certain further *a priori* presuppositions. They add to the general *a priori* principles of the pure understanding, concerning specific measuring devices and the processes investigated in experiments. Here, perhaps the term "relative a priori" may be employed.[29]

C. According to the doctrine of the antinomy of pure reason, it is impossible to complete the objective knowledge of the empirical world. The speculative idea of the world as a sum total of appearances gives only rise to *regulative principles* that serve the extension of our empirical knowledge. C.1–2: The concepts of *ultimate* substances and causes are the ideas that give rise to the methodological principles of analysing matter into smaller and smaller constituent parts and searching for a fundamental force of physics. Due to Kant's resolution of the *second* antinomy of reason, empirical reality is a mereological sum of relational parts without least parts. His ontology of appearances is a mereology without atoms.[30] Unrestricted separability of empirical reality into parts is a further *a priori* assumption of Kant's theory of nature. Needless to say that it is closely related to the Galilean resolutive–compositive method of empirical science. (However, it is only meaningful to talk about *parts* of empirical reality if we may assume that we may *separate* such parts somehow by experimental methods, at least on the basis of well-confirmed theoretical principles.) C.3 To them add the principles of the homogeneity, continuity, and specification of natural kinds and the related principle of the systematic unity of the empirical laws of nature explained in the *Critique of Judgement*.

Under (A.2–3) and (C.1–3), we indeed find most of the methodologically indispensable principles of the preceding section. They are put in a different order that employs Kant's distinction of constitutive and regulative principles. Only the principles of synthesis and simplicity are not explicitly given. However, they would belong to a methodology of experimental and mathematical physics that Kant never worked out in his systematic philosophy. Nevertheless, their methodological status is absolutely clear.

It remains to identify Kant's position between classical realism and empiricism. Due to (A.1–3 and B.1), Kant's view of empirical reality is obviously weaker than classical realism. His substances are methodological relational concepts rather than metaphysical relationless entities. Matter, as the empirical *substantia phaenomenon* in space, is for him a relational structure of appearances.[31] In this point, he comes close to empiricism. However, regarding the *a priori* status of the principles of the pure understanding, his position is stronger than empiricism. In particular, according to (B.2) the principle of causality admits the inference to unobservable causes

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[29] See Reichenbach's distinction between Kant's own *a priori* of space and time and the *a priori* of specific physical assumptions about space–time.

[30] See Falkenburg (2000). For the concepts of mereology, see Simons (1987).

[31] See Kant (1781, p. 265, 1787, p. 321) (quoted above).

of the phenomena. In this point, his position comes close to causal realism.[32] Concerning (B.3), Kant explains the relation between physical theory and observation in terms of a hypothetical-deductive approach that is strikingly modern. Even though he wanted to explain all physical objects in terms of Euclidean space–time, Newtonian forces, non-atomic relational substances, and an ether-like matter penetrating all bodies, his relational view of empirical reality is quite liberal. It can cope with the modern concepts of field and interaction as well as with complicated ways of tracing back from experimental data to theoretical explanations. In addition it has to be noted that due to (B.3) his view of empirical reality does not collapse into constructivism. The results of experiments are not determined by the principles of pure reason. They are contingent.[33]

4 Quantum Theory: The Touchstone

In face of relativity and quantum theory, classical realism turned out to be too strong.[34] In the relativistic domain the traditional concepts of space and time fail, whereas in the quantum domain the traditional concepts of substance and causality no longer apply. In general, the results of quantum measurements are irreducibly contingent. Non-commuting quantum observables such as position and momentum

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[32] Causal realism was first expressed in Newton's first rule of reasoning (Newton, 1687, p. 794), in a framework of classical metaphysical realism. Today, weaker versions such as Cartwright's or Hacking's are prominent. In search of a sufficient criterion for the existence of an entity, Hacking couples causal realism to the requirement of successfully using an entity as a technological device. According to his reality criterion, electrons exist because they can be successfully used as experimental devices with observable effects in a scattering experiment. Or, as Hacking (1983, p. 23), put it, "if you can spray them then they are real". Cartwright (1983, 1989) defends a version of causal realism according to which nature has causal powers or capacities that are subject to causal analysis. Her account of Nature's capacities is coupled to belief in the (approximate) truth of phenomenological laws.

[33] This point may be made precise in terms of Kant's distinction of the *real* and the *actual*; see Falkenburg (2007, section 1.3). The real consists in the *qualia*, in properties. The actual, as distinct from the possible and the necessary, is a *modal* category. To the *real*, the pure concept of an intensive quantity belongs, where any intensive quantity has a degree. Kant calls this degree the "real of the sensation". In empirical reality, the *actual* degrees of intensive quantities are *a posteriori* given by the sense data (Kant, 1787, B 207). The mark of the actual is that it is *not* determined by the categories and principles of pure reason. It is due to something that acts independently of our cognitive capacities. The English term "actual" as well as the German term "wirklich" preserves the idea that something is acting independently of our concepts and theories.

[34] In the following, "quantum theory" means any quantum theory currently used in physical practice, i.e., quantum mechanics in the usual probabilistic interpretation as well as the current quantum field theories. Mittelstaedt (2006) shows how special relativity and quantum theory criticise classical realism as an over-determined construal of empirical reality in two respects. Special relativity teaches that the classical concepts of space, time, and simultaneity are not general enough. Quantum theory teaches that the classical concept of substance is too strong.

obey Heisenberg's uncertainty relation and exhibit a non-Boolean algebraic structure. This non-Boolean structure of quantum phenomena replaces the classical construal of physical reality. It is minimal. It dispenses with classical particle trajectories and classical waves. What remains is the probabilistic interpretation of quantum theory developed by Born and von Neumann.[35] It reduces physical reality to the measured values of physical quantities and their probabilities.

But Kant's *a priori* also turns out to be too strong. In the quantum domain, the causal aspects of *whatever* realism of physical objects become highly problematic, be it classical metaphysical realism, Kant's empirical realism, or recent versions of causal realism such as defended by Cartwright or Hacking.[36] Except in the case of repeatable measurements, quantum theory precludes any explanation of individual events or measurement outcomes. Quantum theory commits us to dispense with individual causes.[37] In addition, the conservation laws of a quantum theory give rise to the prediction of non-causal law-like relations between individual events, the so-called EPR (Einstein–Podolsky–Rosen) correlations. EPR correlations are non-local. They are non-causal according to the causality condition of Einstein's special relativity, which tells that no signal can be transmitted over space-like distances. And they are incompatible with Bell's inequality that derives from the locality assumptions of classical realism.[38]

Due to these features, since the early days of quantum mechanics the question arose: Does this theory deal with an objective subatomic reality at all or is it grist for the mills of empiricism? Quantum theory is needed in order to explain the contingent events and measurement results of subatomic physics. But it is incompatible with classical realism, above all, with the claim that physical reality is made up of independent causal agents that are individuated and completely determined by their spatio-temporal and dynamic properties.[39] However, the algebraic structure and the probabilistic interpretation of quantum theory are *not* incompatible with *empiricism*. Empiricists claim that there are only relative frequencies of measurement outcomes and predictions of conditional probabilities but no subatomic processes underlying them.

Niels Bohr, however, objected that *both* options fall short of a satisfactory interpretation of quantum theory. According to his complementarity view of quantum

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[35] Born (1926a, b), von Neumann (1932, pp. 101–110).

[36] Cartwright (1983), Hacking (1983).

[37] Cassirer (1937) suggested that in quantum theory the principle of causality can be maintained only at the probabilistic level of the predictions from the Schrodinger equation, i.e. for the statistical ensemble.

[38] Einstein et al. (1935), Bell (1964), Aspect et al. (1982). In recent experiments, EPR correlations were observed over a distance of 12 km.

[39] Here, I neglect non-standard interpretations of quantum theory such as a Bohm-type hidden variable approach or the many-worlds interpretation. The former introduces a non-local potential, i.e., an action-at-a-distance that is at odds with special relativity. The latter is highly speculative.

mechanics, any classical construal of subatomic reality in terms of physical objects is impossible, whereas the classical concepts of position, momentum, mass, energy, etc. remain to be indispensable for the description of subatomic phenomena.[40] The diagnosis reminds of Kant's critical account of empirical reality: Classical realism is *too strong* to cope with the physical properties of quantum phenomena, whereas empiricism is *too weak* to admit any subatomic processes that cause the quantum phenomena. Bohr did not dispense with the concept of causality. He only emphasised that the spatio-temporal and causal processes of quantum physics *do no longer come together*. They are *complementary*, i.e., they exclude each other depending on the physical quantities that may be measured in a specific experimental situation, but they complement each other insofar as they only together give a complete account of the quantum domain. Some experimental arrangements generate quantum phenomena that only admit of a spatio-temporal description, i.e., in terms of position and time. Most typical is the particle detection by means of a scintillator or a photon counter. Other experimental arrangements generate quantum phenomena that only admit of a dynamic description in terms of momentum and energy. Bohr calls this description "causal".[41] His example is the energy loss of a photon and the kick-off of an observable electron in the Compton effect, which underlies momentum-energy conservation. However, here one should also think of the interference fringes obtained behind a double slit. Interference is a typical wave phenomenon and it is described in terms of a wavelength that corresponds to a certain momentum and energy of an electron or photon beam.

Before comparing Bohr's and Kant's views of physical reality, let me sketch the regards in which the metaphysical assumptions behind classical realism must be weakened in face of quantum theory. At this point, it should be recalled that they give rise to indispensable *methodological* principles of physics (a fact that empiricism does not take into account, whereas Kant's theory of nature does). To them belong the crucial principles of (1) substance, (2) causality, (3) analysis, (4) synthesis, (5) unity, and (6) simplicity. What remains of them in the quantum domain?[42]

1. S*ubstance*: Subatomic particles are still conceived as approximately *non-interacting* carriers of *permanent dynamic* properties. Quantum mechanics and quantum field theory start with non-interacting quantum waves or fields. Electrons, protons, neutrons, photons, etc. are considered to be collections of dynamic properties such as mass–energy, spin, and charge that obey conservation laws and correspond to the representations of symmetry groups. This results in an *operational* particle concept, but *not* in a unified *theoretical* particle concept.

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[40] Bohr (1928, 1948, 1949). See also Falkenburg (1998).

[41] Bohr (1928).

[42] For details, see Falkenburg (2007, in particular chapters 5–8).

2. C*ausality*: These collections of dynamic properties are *causal agents* which determine the phenomena *not completely*, but at least in several *very* distinct regards, amongst them:
    - (i) *Conservation laws* for momentum–energy, charge, and spin hold for any *individual* subatomic process (this was Bohr's use of the term "causal"). In addition, they give rise to sum rules for the way in which matter is built up from quarks and electrons.
    - (ii) *Einstein causality* tells that no signalling is possible over space-like distances by means of non-local quantum correlations.
    - (iii) The *unitary evolution of the wave function* according to the Schrödinger equation (or another wave or field equation) only determines the *probability* of measurement results.

3. A*nalysis*: The experimental methods of quantum physics still aim at mereological and causal analysis. But according to Bohr, Planck's constant is a *minimum action* that indicates the *limitations of experimental analysis*.[43] Nevertheless, atomic, nuclear, and particle physics successfully *isolate* subatomic particles and their dynamic properties. The structure of matter is explained in particle terms down to the quark model of the proton and neutron. However, the smaller parts of matter are investigated the harder is it to isolate them. Quarks can not be isolated. Their existence is inferred from scattering experiments and conservation laws.
4. S*ynthesis*: The dynamic properties of subatomic particles are assumed to sum up according to *simple sum rules* to the respective properties of quantum mechanical many-particle systems and macroscopic matter. However, quantum theory does *not completely* explain the spatio-temporal properties of matter. Due to the quantum measurement problem, the top-down approach (mereological analysis) is more successful than the bottom-up approach (mereological synthesis). In addition, in quantum field theory the usual perturbative approach of a physical dynamics does no longer work straightforward. The prize of this approach is the renormalization of the mass and charge of subatomic particles needed in order to remove infinities from higher-order terms.
5. *Unity*: The observable phenomena, their constituents, and their composites no longer obey the same laws of physics. The unity of physics is still an indispensable regulative principle. But no convincing unified theory of physics is in sight. In the decoherence approach,[44] the quantum measurement problem is only resolved at the probabilistic level. And no quantum gravity is available. In addition, the traditional concept of causality is replaced by several *very* distinct causal

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[43] Bohr emphasised that "the so-called quantum postulate attributes to any atomic process an essential discontinuity, or rather individuality, completely foreign to the classical theories and symbolized by Planck's quantum of action" (Bohr 1928, p. 580). Since 'individuality' for Bohr means 'indivisibility', this is closely related to the limitations of the analysis and synthesis of experience he repeatedly stated. See Chevalley (1991, pp. 373–378).

[44] Giulini et al. (1996).

concepts. What remains is the unity of the *language* of physics, i.e., of the *scales* of physical quantities such as length, time, and mass. They range from the Planck scale over the size of the quarks up to the event horizon and the age of the universe and beyond.

6. *Simplicity*: Quantum explanations are *not really* simple. Nevertheless, the principle remains that of two given theoretical explanations, the simpler one is preferable as presumably closer to truth. And this is a reason to believe in the usual probabilistic standard version of quantum theory rather than in one of the highly speculative interpretations beyond.

Hence quantum physics goes on with these *methodologically* indispensable principles, and it does so most successfully. Nevertheless, the structure of quantum phenomena substantially restricts their applicability. What does this mean for a Kantian account of subatomic reality?

As far as I can see, only Kant's principle of causality and his distinction of constitutive and regulative principles are affected. Kant's view of extensive and intensive magnitudes fits in well with the constructive features of the length, time, and mass scale[45] – and, above all, with Bohr's (and Heisenberg's) claim that the language of classical physics is indispensable for the physical interpretation of quantum phenomena. The way in which Kant's principle of substance applies to the conservation laws of physics does absolutely agree with the methodological use of this concept in quantum physics. In addition, Kant's claim that empirical reality is thoroughly relational agrees with several of Bohr's views. Subatomic reality is *context dependent*. The kind of quantum phenomena that is observed (particle detections, interference patterns, or in the recent *which way* experiments of quantum optics even both) depends on the experimental arrangement. In addition, all quantum phenomena occur in a classical world. And the physical meaning of quantum concepts is only defined *relative to classical concepts*. All quantum phenomena are finally described in terms of length, time, and mass. According to Bohr, the language of classical physics is indispensable. Bohr's complementarity view is based on his correspondence principle. For him, complementary quantum phenomena correspond to mutually exclusive classical phenomena, and in order to interpret them the language of classical physics must be employed. Even though this view can not really cope with genuine quantum phenomena without classical correspondence, Bohr's underlying ideas are valid up to the present day: All measurement results must be expressed in terms of the classical length, time, and mass scales. In addition, the scattering experiments of high energy physics are interpreted relative to classical models of scattering centres, charge distributions, etc.[46]

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[45] However, the construction is not arbitrary. As far as overlapping measurement methods and independent semi-empirical tests are available, the scales turn out to be empirically coherent. See Falkenburg (2007, chapter 5).

[46] Falkenburg (2007, chapters 4–7).

The regulative principles of mereological and causal analysis and the systematic unity of the empirical laws of nature still hold as indispensable methodological principles of physics. But they do no longer succeed in establishing unambiguous physical objects that are described in terms of unified theoretical principles. And Kant's principle of causality dissolves into various causal concepts that can not be brought together in any obvious way.

Nevertheless, Kant's principle of causality requires to *ask* for the causal agents behind individual quantum measurement results. The description of these causal agents in terms of a quantum mechanical wave function is only capable of explaining conservation laws and quantum probabilities. Individual measurement results occur in an irreversible non-deterministic process, the so-called reduction of the wave function, which is postulated *ad hoc* rather than explained. In accordance with Kant's concept of substance, it is assumed that particles are collections of conserved quantities or stable, re-identifiable bundles of properties. The assumption that *there is* a unique causal agent behind repeated particle detections is not objective, but *subjective* in Kant's sense. The physicist's belief that *there is* an electron that causes a curved particle track in a bubble chamber is the only "metaphysical glue" that makes the respective collection of properties stick together. It is the belief in a *non-local* causal agent. In the quantum domain, the transcendental status of such metaphysical belief is substantially weaker than in the classical. In the latter, the principles of substance and causality are constitutive for the knowledge of physical objects with completely determined properties. In the former, they only are regulative ideas that aim at understanding the systematic unity of quantum phenomena.[47] The principles of substance and causality can not be constitutive for the knowledge of quantum objects. They are only constitutive for unifying complementary quantum phenomena generated from the same kind of source, in different experimental arrangements. The unity of the quantum domain is limited. There is neither a unified particle concept nor a unified account of causality. In physical practice, the regulative use of the principles of substance and causality comes together with a plurality of theoretical particle concepts and meanings of causality. Therefore, the ways in which Kant's principle of causality and his regulative principles apply seem to be *empirically* restricted, even though their *methodological* use remains unrestricted.

The subjective belief in subatomic processes behind the quantum phenomena, in which certain dynamic quantities are conserved, is stronger than empiricism but weaker than classical realism. The scales of physical quantities, the conservation laws of physics, and the causal assumptions underlying the preparation procedures and the data analysis of quantum experiments belong to physical practice, not to an independent physical reality on its own. The genuine Kantian point is that such an *independent* reality does not exist *for us*. Physical reality is conceived to depend on our cognitive tools, and in particular, on the experimental and mathematical methods of physics. According to the critical account suggested here, subatomic reality is not a micro-world on its own but a part of empirical reality that only exists relative

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[47] See Pringe (2007) and Pringe's contribution to this volume.

to the macroscopic world, in given experimental arrangements and in well-defined physical contexts outside the laboratory. From a realist point of view, there are sufficient reasons to believe that there are entities such as electrons, quarks, and photons in the physics laboratories and in the world beyond. But any scientific realism about subatomic particles should take into account that they do *not really* behave like independent localized particles but rather like the local effects of non-local quantum processes within a classical world.

References

Aspect, A., Grangier, P. and Roger, G., 1982: Experimental Realization of Einstein-Podolsky-Rosen-Bohm *Gedankenexperiment*. A New Violation of Bell's Inequalities. Phys. Rev. Lett. 49, 91–94

Bell, J.S., 1964: On the Einstein-Podolsky-Rosen Paradox. Physics 1, 195–200

Bohr, N., 1928: The Quantum Postulate and the Recent Development of Atomic Theory. Nature 121, 580–590. Reprinted in: Wheeler and Zurek 1983, 87–126. Modified version of the Como Lecture (1927). Both versions in: Bohr's Collected Works 6, 109–158.

Bohr, N., 1948: On the Notions of Causality and Complementarity. Dialectica 2, 312–318. Reprinted in: Bohr's Collected Works 7, 330–337.

Bohr, N., 1949: Discussion with Einstein on Epistemological Problems of Atomic Physics. In: Schilpp 1949, 115–150. Reprinted in: Wheeler and Zurek 1983, 9–49.

Born, M., 1926a: Zur Quantenmechanik der Stoßvorgänge. Z. f. Physik 37, 863–867. English translation in: Wheeler and Zurek 1983, 52–55.

Born, M., 1926b: Quantenmechanik der Stoßvorgänge. Z. f. Physik 38, 803–827.

Carson, E. and Huber, R. (eds.), 2006: Intuition and the Axiomatic Method. Dordrecht: Springer

Cartwright, N., 1983: How the Laws of Physics Lie. Oxford: Clarendon Press.

Cartwright, N., 1989: Nature's Capacities and their Measurement. Oxford: Clarendon Press

Cassirer, E., 1937: Determinismus und Indeterminismus in der modernen Physik. Göteborg. Reprinted in: H.-W. Wendt (ed.), Zur modernen Physik. Darmstadt: Wissenschaftliche Buchgesellschaft (1957).

Chevalley, C., 1991: Introduction and Glossaire. Analyse et Synthèse. In: Niels Bohr: Physique atomique et connaissance humaine. Paris: Gallimard, 17–144 and 345–567.

Einstein, A., 1905: Über einen die Erzeugung und Verwandlung des Lichts betreffenden heuristischen Gesichtspunkt. Annalen der Physik 17, 132–148.

Einstein, A., 1949: Autobiographical Notes. In: Schilpp 1949, 1–94.

Einstein, A., Podolsky, B. and Rosen, N., 1935: Can Quantum Mechanical Description of Reality Be Considered Complete? Phys. Rev. 47, 777–780.

Falkenburg, B., 1988: The unifying role of symmetry principles in particle physics. Ratio (New Series) 1, 113–134.

Falkenburg, B., 1996: The Analysis of Particle Tracks. A Case for Trust in the Unity of Physics. Stud. Hist. Phil. Mod. Phys. 27, 337–371

Falkenburg, B., 1997: Incommensurability and Measurement. In: Theoria, 12 (30), 467–491.

Falkenburg, B., 1998: Bohr's Principles of Unifying Quantum Disunities. Phil. Nat. 35, 195–120.

Falkenburg, B., 2000: Kants Kosmologie. Die wissenschaftliche Revolution der Naturphilosophie im 18. Jahrhundert. Frankfurt: Klostermann.

Falkenburg, B., 2006: Functions of Intuition in Quantum Physics. In: Carson and Huber 2006, 267–292.

Falkenburg, B., 2007: Particle Metaphysics. A Critical Account of Subatomic Reality. New York: Springer.

Franklin, A., 1986: The Neglect of Experiment. Cambridge: Cambridge University Press.

Friedman, M., 1983: Foundations of Space–Time Theories. Princeton, NJ: Princeton University Press.

Hacking, I., 1983: Representing and Intervening. Cambridge: Cambridge University Press.

Giulini, D., Joos, E., Kiefer, C., Kupsch, J., Stamatescu, I.-O. and Zeh, H.D., 1996: Decoherence and the Appearance of a Classical World in Quantum Theory. Berlin: Springer. 2nd edition 2003.

Kant, I., 1781/1787: Kritik der reinen Vernunft. Riga: Hartknoch (ed. A/B) English Translation: Guyer, P., and Wood, A.W. (eds.): The Cambridge Edition of the Works of Immanuel Kant. Critique of Pure Reason. Cambridge: Cambridge University Press (1998).

Kant, I., 1786: Metaphysische Anfangsgründe der Naturwissenschaft. Riga English Translation: Friedman, M. (ed.): Metaphysical Foundations of Natural Science. Cambridge: Cambridge University Press (2004).

Krantz, D.H., Duncan, R., Suppes, L.P., and Tversky, A., 1971: Foundations of Measurement. Vol. 1. San Diego, CA: Academic.

Leibniz, G.W. and Clarke, S., 1715/1716: The Leibniz-Clarke Correspondence. Edited with introduction and notes by H.G. Alexander. Manchester: Manchester University Press (1956, Reprinted in 1998).

Locke, J., 1689: An Essay Concerning Human Understanding. London NewNidditch, P.H. (ed.). Oxford: Clarendon Press (1975).

Mittelstaedt, P., 2006: The Intuitiveness and Truth of Modern Physics. In: Carson and Huber 2006, 251–266.

Newton, I., 1687: Philosophiae naturalis principia mathematica. Londinum. Quoted after the new English translation: The Principia. Mathematical Principles of Natural Philosophy. Translated by I. Bernard Cohen and Anne Whitman with the assistance of Julia Budenz. Berkeley, London: University of California Press (1999).

Newton, I., 1730: Opticks. London (4th edn.) Reprinted with a Preface by I. B. Cohen. New York: Dover (1979).

Planck, M., 1908: Die Einheit des physikalischen Weltbildes. In: Planck 1965, 28–51. In: Vorträge und Erinnerungen. Darmstadt: Wissenschaftliche Buchgesellschaft (9th edn.).

Pringe, H., 2007: Critic of the Quantum Power of Judgement. Berlin: de Gruyter.

Quine, W.v.O., 1951: Two Dogmas of Empiricism. In: From a Logical Point of View. Harvard: Harvard University Press 1953.

Schilpp, P.A. (ed.), 1949: Albert Einstein: Philosopher – Scientist. Library of Living Philosophers. Evanston, IL.

Simons, P., 1987: Parts. A Study in Ontology. Oxford: Clarendon Press

Streater, R.F. and Wightman, A.S., 1964: PCT, Spin and Statistics, and all that. New York: Benjamin

van Fraassen, B.C.1980: The Scientific Image. Oxford: Clarendon Press.

von Neumann, J., 1932: Mathematische Grundlagen der Quantenmechanik. Berlin: Springer. Engl. Transl. by R. Beyer: Mathematical Foundation of Quantum Mechanics. Princeton, NJ: Princeton University Press (1955).

von Weizsäcker, C.F. 1971: Die Einheit der Natur. München, Wien: Hanser

Wheeler, J.A. and Zurek, W.H. 1983: Quantum Theory and Measurement. Princeton, NJ/ Guildford: Princeton University Press.

Wigner, E.P. 1964: Symmetry and Conservation Laws. Proceedings of the National Academy of Sciences USA 51, 956–965. Reprinted in: Philosophical Reflections and Syntheses. Berlin, Heidelberg: Springer (1997), 297–333.

B

The Scientific Revolutions of the Twentieth Century and Beyond

B.1
Relativity and Cosmology

Einstein, Kant, and the Relativized *A Priori*

Michael Friedman

Abstract I argue that Einstein's creation of both special and general relativity instantiates Reichenbach's conception of the relativized a priori. I do this by showing how the original Kantian conception actually contributes to the development of Einstein's theories through the intervening philosophical and scientific work of Helmholtz, Mach, and Poincaré.

Kant's original version of transcendental philosophy took both Euclidean geometry and the Newtonian laws of motion to be synthetic a priori constitutive principles – which, from Kant's point of view, function as necessary presuppositions for applying our fundamental concepts of space, time, matter, and motion to our sensible experience of the natural world.[1] Although Kant had very good reasons to view the principles in question as having such a constitutively a priori role, we now know, in the wake of Einstein's work, that they are not in fact a priori in the stronger sense of being fixed necessary conditions for all human experience in general, eternally valid once and for all. And it is for precisely this reason that Kant's original version of transcendental philosophy must now be radically reconceived.

It was Hans Reichenbach, in *Relativitätstheorie und Erkenntnis Apriori* (1920), who first proposed the idea of *relativizing* Kantian constitutively a priori principles of geometry and mechanics. Such principles still function, throughout the development from Newton to Einstein, as necessary presuppositions for applying our (changing) conceptions of space, time, and motion to our sensible experience, but they are no longer eternally valid once and for all. Instead of global necessary conditions for all human experience in general, we have merely local necessary conditions for the empirical application of a particular mathematical–physical theory at a given time and in a given historical context. For example, while Euclidean geometry and the

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[1] For details on Kant's understanding of Euclidean geometry and the fundamental principles of Newtonian mechanics see Friedman (1992).

M. Bitbol et al. (eds.), *Constituting Objectivity*,

Newtonian laws of motion are indeed necessary conditions for giving empirical meaning to the Newtonian theory of universal gravitation, the situation in Einstein's general theory relativity is quite different. The crucial mediating role between abstract mathematical theory and concrete sensible experience is now played by the light principle and the principle of equivalence, which together insure that Einstein's revolutionary new description of gravitation by a four-dimensional geometry of variable curvature in fact says something about concrete empirical phenomena: namely, the behavior of light and gravitationally interacting bodies.

In my recent book, *Dynamics of Reason* (2001), I have taken up, and further developed, Reichenbach's idea. But my implementation of this idea of relativized constitutively a priori principles (of geometry and mechanics) essentially depends on an historical argument describing the developmental process by which the transition from Newton to Einstein actually took place, as mediated, in my view, by the parallel developments in scientific philosophy involving, especially, Hermann von Helmholtz, Ernst Mach, and Henri Poincaré. However, since this argument depends on the concrete details of the actual historical process in question, it would therefore appear to be entirely contingent. How, then, can it possibly be comprehended within a properly *transcendental* philosophy? Indeed, once we have given up on Kant's original ambition to delineate in advance the a priori structure of all possible scientific theories, it might easily seem that a properly transcendental argument is impossible. We have no way of anticipating a priori the specific constitutive principles of future theories, and so all we can do, it appears, is wait for the historical process to show us what emerges a posteriori as a matter of fact. So how, more generally, can we develop a philosophical understanding of the evolution of modern science that is at once genuinely historical and properly transcendental?[2]

Let us begin by asking how Kant's original transcendental method is supposed to explain the sense in which certain fundamental principles of geometry and mechanics are, in fact, both a priori and necessary. This method, of course, appeals to Kant's conception of the two rational faculties of sensibility and understanding. The answer to the question "how is pure mathematics possible?" appeals to the necessary structure of our pure sensibility, as articulated in the Transcendental Aesthetic of the *Critique of Pure Reason*; the answer to the question "how is pure natural science possible?" appeals to the necessary structure of our pure understanding, as articulated in the Transcendental Analytic. Yet there is an obvious objection to this procedure: how can such proposed transcendental explanations inherit the (assumed) a priori necessity of the sciences whose possibility they purport to explain unless we can also somehow establish that they are the *unique* such explanations?[3] From our present point of view, for example, it does not appear that Kant's explanation of the possibility

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[2] I am especially indebted to Charles Parsons for raising this problem of historical contingency and stimulating me to take it very seriously.

[3] Kant often makes such claims to explanatory uniqueness, for example, in the Transcendental Exposition of the Concept of Space added to the second ["B"] edition (B 41): "Therefore, only our explanation makes the *possibility of geometry* as an a priori synthetic cognition comprehensible. Any mode of explanation that does not achieve this, even if it appeared to be similar to ours, can be most securely distinguished from ours by this criterion." I am indebted to Dagfinn Føllesdal for emphasizing to me the importance of the problem of uniqueness in this connection.

of pure mathematics is uniquely singled out in any way; on the contrary, our greatly expanded conception of purely logical or analytic truth suggests that an appeal to the faculty of pure sensibility may, after all, be entirely superfluous. Indeed, from the point of view of the anti-psychological approach to such questions that dominated much of twentieth-century analytic philosophy, it appears that all consideration of our subjective cognitive faculties is similarly explanatorily superfluous.

In Kant's own intellectual context, however, explanations of scientific knowledge in terms of our cognitive faculties were the norm – for empiricists, rationalists, and (of course) Aristotelians. Everyone agreed, in addition, that the relevant faculties to consider were the senses and the intellect; what was then controversial was the precise nature and relative importance of the two. Empiricist views, which denied the existence of the pure intellect or its importance for scientific knowledge, were, for Kant, simply out of the question, since they make a priori rational knowledge incomprehensible.[4] Moreover, the conception of the pure intellect that was most salient for Kant was that of Leibniz, where the fundamental structure of this faculty is delineated, in effect, by the logical forms of traditional Aristotelian syllogistic. But this conception of the pure intellect, Kant rightly saw, is entirely inadequate for representing, say, the assumed infinite extendibility and divisibility of geometrical space, which had recently proven itself to be both indispensable and extremely fruitful in Newtonian mathematical physics.[5] Nevertheless, Newton's own conception of space as the divine sensorium was also entirely unacceptable on theological and metaphysical grounds, and so the only live alternative left to Kant was the one he actually came up with: space is a pure form of our sensibility (as opposed to the divine sensibility), wherein *both* (infinitely iterable) geometrical construction *and* the perception of spatial objects in nature (like the heavenly bodies) then become first possible.[6]

Kant's answers to the questions "how is pure mathematics possible?" and "how is pure natural science possible" therefore operate against the background of an

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[4] Thus, in considering the questions "how is pure mathematics possible?" and "how is pure natural science possible?" in section VI of the Introduction to the second edition of the *Critique*, Kant simply takes it for granted that the actual existence of these sciences puts the existence of synthetic a priori knowledge entirely beyond all doubt. In particular, in considering Hume's skepticism concerning the necessity of the causal relation – which then leads to skepticism about the possibility of any a priori metaphysics – Kant blames this result on Hume's insufficiently general understanding of the problem (B 20): "[H]ume would never have arrived at this assertion, which destroys all pure philosophy, if he had kept our problem before his eyes in its [full] generality; for he would then have seen that, according to his argument, there could also be no pure mathematics (for it certainly contains synthetic a priori propositions), and his good sense would then surely have saved him from this assertion." Similarly, while considering (in section 14 of the second edition) the circumstance that neither Locke nor Hume posed the problem of the transcendental deduction, and instead attempted a psychological or empirical derivation of the pure concepts of the understanding, Kant concludes (B 127–128): "But the *empirical* derivation which both fell upon cannot be reconciled with the actuality of the a priori scientific cognition that we have—namely of *pure mathematics* and *universal natural science*—and is thus refuted by this fact [*Faktum*]."

[5] Again, see Friedman (1992, chapters 1 and 2) for details.

[6] See my "Newton and Kant on Absolute Space: From Theology to Transcendental Philosophy" (this volume) for details.

existing set of intellectual resources in a particular historical context. Geometry, for Kant, is limited to the classical system of Euclid; the pure understanding or pure intellect is delimited by the logical forms of Aristotle; the available conceptions of space and time are exhausted by the Leibnizean and Newtonian alternatives; and so on. Kant's theory of our faculties of sensibility and understanding can only be understood against the background of precisely these resources – mathematical, logical, metaphysical, and theological – as Kant delicately navigates within them and eventually radically transforms them. The revolutionary and completely unexpected result, that space and time are pure forms of our (human) faculty of sensibility and that, considered independently of sensibility, our faculty of understanding yields no (theoretical) cognition at all, then emerges as the practically unique solution to the problem set by the existing intellectual resources: it is the only available conception of our rational faculties that does simultaneous justice to both Newtonian mathematical physics and Leibnizean (as opposed to Newtonian) natural theology and metaphysics.

It is of course entirely contingent that Kant operated against the background of precisely these intellectual resources, just as it is entirely contingent that Kant was born in 1724 and died in 1804. Given these resources, however, and given the problems with which Kant was faced, the solution he came up with is not contingent. On the contrary, the intellectual situation in which he found himself had a definite "inner logic" –mathematical, logical, metaphysical, and theological – which allowed him to triangulate, as it were, on a practically unique (and in this sense necessary) solution.

Beginning with this understanding of Kant's transcendental method and its associated rational necessity, we can then see a way forward for extending this method to post-Kantian developments in both the mathematical exact sciences and transcendental philosophy. We can trace out how the "inner logic" of the relevant intellectual situation evolves and changes after Kant in response to both new developments in the mathematical exact sciences themselves and the manifold and intricate ways in which post-Kantian scientific philosophers attempted to reconfigure Kant's original version of transcendental philosophy in light of these developments. That each of these successive new intellectual situations has its own "inner logic" implies that the enterprise does not collapse into total contingency; that, in addition, they successively evolve out of, and in light of, Kant's original system suggests that it may still count as transcendental philosophy. In my reconceived version of transcendental philosophy, therefore, integrated intellectual history of both the exact sciences and scientific philosophy takes over the role of Kant's original transcendental faculty psychology.

Hermann von Helmholtz's neo-Kantian scientific epistemology, for example, had deep roots in Kant's original conception. In particular, Helmholtz developed a distinctive conception of space as a "subjective" and "*necessary* form of our external intuition" in the sense of Kant; and, while this conception was certainly developed within Helmholtz's *empirical* program in sensory psychology and psycho-physics, it nevertheless retained important "transcendental" elements.[7] More specifically,

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[7] For Helmholtz's characteristic combination of empirical (or "naturalistic") and transcendental (or "normative") elements see Hatfield (1990). For my reading of Helmholtz's conception of space and geometry see Friedman (1997, 2000).

space is "transcendental," for Helmholtz, in so far as the principle of free mobility (which allows arbitrary continuous motions of rigid bodies) is a necessary condition for the possibility of spatial measurement – and, indeed, for the very existence of space and spatial objects. Moreover, the condition of free mobility represents a natural generalization of Kant's original (Euclidean) conception of geometrical construction, in the sense that Euclidean constructions with straight-edge and compass, carried out within Kant's form of spatial intuition, are generated by the group of specifically Euclidean rigid motions (translations and rotations). The essential point, however, is that free mobility also holds for the classical non-Euclidean geometries of constant curvature (hyperbolic and elliptic), and so it is no longer a "transcendental" and "necessary" condition of our spatial intuition, for Helmholtz, that the space constructed from our perception of bodily motion obeys the specific laws of Euclidean geometry. Nevertheless, Helmholtz's generalization of the Kantian conception of spatial intuition is, in an important sense, the *minimal* (and in this sense unique) such generalization consistent with the nineteenth-century discovery of non-Euclidean geometries.[8]

The great French mathematician Henri Poincaré then transformed Helmholtz's conception in turn. In particular, Poincaré's use of the principle of free mobility (which plays a central role in his philosophy of geometry) is explicitly framed by a hierarchical conception of the mathematical sciences, beginning with arithmetic and proceeding through analysis, geometry, mechanics, and empirical physics; and, for Poincaré, it follows that one should thereby explain the application of pure mathematics to our perceptual experience precisely in terms of the hierarchy in question.[9]

Poincaré, to begin with, views pure arithmetic as a synthetic a priori science involving the ineliminable use of an essentially non-logical principle of reasoning by recurrence or mathematical induction. This principle, for Poincaré, rests on the fundamental intuition of indefinitely repeatable succession or iteration – a conception which is very close indeed to Kant's original philosophy of arithmetic.[10] At the next lower level of the hierarchy is analysis or the theory of mathematical magnitude (also explained with an eye towards its intuitive meaning and perceptual application); and, at a crucial intermediate level, below the sciences of arithmetic and analysis but above the sciences of mechanics and empirical physics, is the science of geometry. In particular, whereas the mathematical structure and empirical

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[8] Bernhard Riemann's general theory of manifolds includes spaces of *variable* curvature not satisfying the condition of free mobility, and it is for precisely this reason that Hermann Weyl later attempted to generalize Helmholtz's approach to comprehend the (four-dimensional) (semi-) Riemannian geometries of variable curvature used in Einstein's general theory of relativity. Moreover, as I explain in Friedman (2000, pp. 209–211), Weyl, too, conceived his work as a generalization of Kant's original theory of space as an (a priori) "*form of experience*." The important point here, however, is that Helmholtz is "closer" to Kant's original theory (in so far as his generalization preserves the possibility of geometrical constructions analogous to Euclid's), whereas Weyl's work arises only as a further generalization, in turn, of Helmholtz's.

[9] This hierarchical conception is developed especially clearly in *La Science et l'Hypothèse* (1902). For details see Friedman (1999, chapter 4, 2000).

[10] For Poincaré's philosophy of arithmetic, in particular, see Folina (1992).

meaning of the science of geometry presupposes the existence of the two preceding sciences, it is presupposed, in turn, by the two succeeding ones.

This hierarchical conception of the mathematical sciences underlies Poincaré's fundamental disagreement with Helmholtz. For Helmholtz, as we have seen, the principle of free mobility expresses the necessary structure of our form of external intuition, and, following Kant, Helmholtz views all empirical investigation as necessarily taking place within this already given form. Helmholtz's conception is Kantian, that is, in so far as space has a "necessary form" expressed in the condition of free mobility, but it is also empiricist in so far as which of the three possible geometries of constant curvature obtains is then determined by experience. For Poincaré, by contrast, although the principle of free mobility is still fundamental, our actual perceptual experience of bodily "displacements" arising in accordance with this principle is far too imprecise and indefinite to yield the empirical determination of a specific mathematical geometry: our only option, at this point, is to *stipulate* Euclidean geometry by convention, as the simplest and most convenient idealization of our actual perceptual experience. In particular, experiments with putatively rigid bodies, for Poincaré, involve essentially physical processes at the level of mechanics and experimental physics, and these sciences, in turn, *presuppose* that the science of geometry is already firmly in place. In the context of Poincaré's hierarchy, therefore, the principle of free mobility expresses our necessary freedom to choose – by a "convention or definition in disguise" – which of the three classical geometries of constant curvature is the most suitable idealization of physical space.

One of the most important applications of Poincaré's hierarchical conception involves his characteristic perspective on the problem of absolute space and the relativity of motion explained in his discussion of the next lower level in the hierarchy: (classical) mechanics. Poincaré's key idea is that what he calls the (physical) "law of relativity" rests squarely on the "relativity and passivity of space" and therefore reflects the circumstance, essential to free mobility, that the space constructed from our experience of bodily displacements is both homogeneous and isotropic: all points in space, and all directions through any given point, are, necessarily, geometrically equivalent.[11] Thus, Poincaré's conception of the relativity of motion depends

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[11] The "law of relativity" is first introduced in chapter V, "Experience and Geometry," of *La Science et l'Hypothèse* (1902, p. 96, 1913b, p. 83): "The laws of the phenomena which will happen [in a material system of bodies] will depend on the state of these bodies and their mutual distances; but, because of the relativity and passivity of space, they will not depend on the absolute position and orientation of this system. In other words, the state of the bodies and their mutual distances will depend only on the state of the same bodies and their mutual distances at the initial instant, but they will not depend at all on the absolute initial position of the system and its absolute initial orientation. This is what I shall call, for the sake of brevity, *the law of relativity*." Moreover, "in order fully to satisfy the mind," Poincaré continues, the phenomena in question should also be entirely independent of "the velocities of translation and rotation of the system, that is to say, the velocities with which its absolute position and orientation vary" (1902, p. 98, 1913b, p. 85). Thus, because of "the relativity and passivity of space," the absolute position or orientation of a system of bodies in space can have no physical effect whatsoever, and neither can any *change* (velocity) of such absolute position or orientation. In emphasizing that Poincaré's treatment of the relativity of motion rests squarely on his philosophy of space and geometry, I am in very substantial agreement with the excellent discussion in DiSalle (2006, section 3.7).

entirely on his philosophy of geometry, and this is especially significant, from our present point of view, because Poincaré's ideas on the relativity of motion were also inextricably entangled with the deep problems then afflicting the electrodynamics of moving bodies that were eventually solved (according to our current understanding) by Einstein's special theory of relativity.

I shall return to Einstein below, but I first want to emphasize that the connection Poincaré makes between his philosophy of geometry and the relativity of motion represents a continuation of a problematic originally prominent in Kant. Helmholtz, as we have seen, transformed Kant's philosophy of space and geometry, and Ernst Mach, among others, participated in a parallel transformation of Kant's approach to the relativity of motion – which finally eventuated in the modern concept of an inertial frame of reference.[12] Neither Helmholtz nor Mach, however, established any kind of conceptual connection between the foundations of geometry and the relativity of motion – which, at the time, appeared to be entirely independent of one another. Yet it was an especially central feature of Kant's original approach to transcendental philosophy that the two were in fact closely connected. While Kant's answer to the question "how is pure mathematics possible?" essentially involved his distinctive perspective on Euclidean constructive operations, his answer to the question "how is pure natural science possible" involved an analogous constructive procedure by which Newton, from Kant's point of view, arrived at successive approximations to "absolute space" via a definite sequence of rule-governed operations starting with our parochial perspective here on earth and then proceeding to the center of mass of the solar system, the center of mass of the Milky Way galaxy, the center of mass of a system of such galaxies, and so on *ad infinitum*.[13] Indeed, the way in which Kant thereby established a connection between the problem of space and geometry and the problem of the relativity of motion was intimately connected, in turn, with both the overarching conception of the relationship between sensibility and understanding that frames his transcendental method and his characteristic perspective, more generally, on the relationship between constitutive and regulative transcendental principles.[14]

Now it was Mach, as I have suggested, who first forged a connection between Kant's original solution to the problem of "absolute space" and the late nineteenth-century solution based on the concept of an inertial frame of reference.[15] And it is clear,

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[12] For the nineteenth-century development of the concept of an inertial frame see DiSalle (1988, 1991); for Mach's place in this development see DiSalle (2002).

[13] Kant develops this interpretation of "absolute space" in his *Metaphysische Anfangsgründe der Naturwissenschaft* (1786), published between the first (1781) and second (1787) editions of the *Critique of Pure Reason*. For details see Friedman (1992, chapters 3 and 4), and also the Introduction to my (2004) translation of Kant's work.

[14] In particular, "absolute space," for Kant, is a regulative idea of reason, defined by the forever unreachable "center of gravity of all matter" which we can only successively approximate but never actually attain.

[15] Kant's construction of "absolute space," from a modern point of view, yields better and better approximations to a cosmic inertial frame of reference defined by the "center of gravity of all matter." Such a cosmic frame, in which the fixed starts are necessarily at rest, also counts as a surrogate for Newtonian "absolute space" in Mach's treatment: for details see again DiSalle (2002).

moreover, that Poincaré was familiar with this late nineteenth-century solution as well. However, it is also clear that Poincaré's attempt to base his discussion of the relativity of motion on his philosophy of geometry runs into serious difficulties at precisely this point; for Poincaré is here forced to distinguish his "law of relativity" from what he calls the "principle of relative motion." The latter applies only to inertial frames of reference, moving uniformly and rectilinearly with respect to one another, while the latter applies, as well, to non-inertial frames of reference in a state of uniform rotation: it follows from the "relativity and passivity" of space, for Poincaré, that uniform rotations of our coordinate axes should be just as irrelevant to the motions of a physical system as uniform translations. Therefore, the full "law of relativity," as Poincaré says, "ought to impose itself upon us with the same force" as does the more restricted "principle of relative motion." Poincaré must also admit, however, that the more extended "law of relativity" does not appear to be in accordance with our experiments (e.g., Newton's famous rotating bucket experiment).[16]

It is for this reason that Einstein's appeal to what he calls the "principle of relativity" in his 1905 paper on special relativity is entirely independent of Poincaré's "law of relativity," and it is also independent, accordingly, of Poincaré's "conventionalist" philosophy of geometry. Einstein's principle is limited, from the beginning, to inertial frames of reference (moving relative to one another with constant velocity and no rotation), and his concern is rather to apply this (limited) principle of relativity to both electro-magnetic and mechanical phenomena. Thus, in particular, whereas Poincaré's "law of relativity" involves very strong a priori motivations deriving from his philosophy of geometry (based on the "relativity and passivity of space"), Einstein's "principle of relativity" rests on the emerging experimental evidence suggesting that electro-magnetic and optical phenomena do not in fact distinguish one inertial frame from another. Einstein "conjectures" that this experimentally suggested law holds rigorously (and for all orders), and he proposes to "elevate" it to the status of a presupposition or postulate upon which a consistent electrodynamics of moving bodies may then be erected:

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[16] Poincaré formulates "the principle of relative motion" in chapter VII, "Relative Motion and Absolute Motion," of *La Science et l'Hypothèse* (1902, p. 135, 1913b, p. 107): "The motion of any system whatsoever must obey the same laws, whether it be referred to fixed axes, or to movable axes transported by a rectilinear and uniform motion. This is the principle of relative motion, which imposes itself upon us for two reasons: first, the most common experience confirms it, and second, the contrary hypothesis is singularly repugnant to the mind." This, of course, is the principle of what we now call Galilean relativity, which was originally formulated by Newton as Corollary V to the Laws of Motion, and then played a central role in the recent literature on inertial frames of reference (see the references cited in note 12 above). However, as Poincaré is well aware, such Galilean relativity holds only for (uniform) rectilinear motions and does not extend, therefore, to the case of (uniform) rotational motion Poincaré also wishes to subsume under his "law of relativity." Nevertheless, Poincaré says, "it seems that [the principle of relative motion] ought to impose itself upon us with the same force, if the motion is varied, or at least if it reduces to a uniform rotation" (1902, pp. 136–137, 1913b, p. 108). Thus, Poincaré's a priori commitment to the law of relativity, derived from the homogeneity and isotropy of space, stands in *prima facie* contradiction with the well-known experimental limitations of the principle of relative motion. (Poincaré presents a sophisticated analysis of this apparent contradiction in the following discussion, which I shall have to pass over here.)

> Examples of this sort [the relatively moving conductor and magnet—MF], together with the unsuccessful attempts to discover any motion of the earth relative to the "light medium," suggest that the phenomena of electrodynamics as well as mechanics possess no properties corresponding to the idea of absolute rest. They suggest rather that, as has already been shown to the first order of small quantities, the same laws of electrodynamics and optics will be valid for all frames of reference for which the equations of mechanics are valid. We will elevate [*erheben*] this conjecture (whose content will be called the "principle of relativity" in what follows) to the status of a postulate [*Voraussetzung*], and also introduce another postulate, which is only apparently irreconcilable with it, namely, that light is always propagated in empty space with a definite velocity *c* which is independent of the state of motion of the emitting body. These two postulates suffice for attaining a simple and consistent theory of the electrodynamics of moving bodies based on Maxwell's theory for stationary bodies. (1905, pp. 891–892, 1923, pp. 37–38)

Hence, Einstein's understanding of the principle of relativity is also entirely independent of Poincaré's carefully constructed hierarchy of the mathematical sciences, and it is for precisely this reason, I suggest, that Poincaré himself could never accept Einstein's theory.[17]

Nevertheless, it appears overwhelmingly likely that, although Einstein did not embrace Poincaré's "conventionalist" philosophy of geometry, Einstein's use of the principle of relativity was explicitly inspired by Poincaré's more general methodology described in *La Science et l'Hypothèse* – according to which the fundamental principles of mechanics, in particular, are "conventions or definitions in disguise" arising from "experimental laws" that "have been elevated into principles to which our mind attributes an absolute value."[18] In Einstein's case, the experimental law in question comprises the

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[17] In his 1912 lecture on "Space and Geometry," appearing in Poincaré (1913a), Poincaré explicitly considers what we now call the four-dimensional geometry of Minkowski space–time, and he clearly states his preference for an alternative formulation of the Lorentzian type – where, in particular, both the Newtonian laws of mechanics and "the relativity and passivity of space" retain a foundational role. Thus, from a modern point of view, while Poincaré's most fundamental "law of relativity" is a purely geometrical principle, expressing the necessary symmetries of three-dimensional (homogeneous) space, Einstein's "principle of relativity" expresses the symmetry or invariance properties of the laws of Maxwell–Lorentz electrodynamics – which we now take to be the symmetries of Minkowski space–time. The central problem with Poincaré's hierarchy, from this point of view, is that it makes the three-dimensional geometry of space prior to the four-dimensional geometry of space–time: compare again DiSalle (2006, section 3.7) for a similar diagnosis.

[18] This idea is stated as a key part of Poincaré's "General Conclusions" to his discussion of (classical) mechanics (1902, p. 165, 1913b, p. 125): "[The fundamental principles of mechanics] are conventions or definitions in disguise. Yet they are drawn from experimental laws; these laws, so to speak, have been elevated [*érigées*] into principles to which our mind attributes an absolute value." Later, in *Geometrie und Erfahrung* (1921), Einstein explicitly uses the language of "elevation" [*erheben*] in connection with precisely Poincaré's "conventionalism" (1921, p. 8, 1923, p. 35): "Geometry (*G*) [according to Poincaré's standpoint] asserts nothing about the behavior of actual things, but only geometry together with the totality (*P*) of physical laws. We can say, symbolically, that only the sum (*G*) + (*P*) is subject to the control of experience. So (*G*) can be chosen arbitrarily, and also parts of (*P*); all of these laws are conventions. In order to avoid contradictions it is only necessary to choose the remainder of (*P*) in such a way that (*G*) and the total (*P*) together do justice to experience. On this conception axiomatic geometry and the part of the laws of nature that have been elevated [*erhobene*] to conventions appear as epistemologically of equal status." (I shall return to *Geometrie und Erfahrung* below.) To the best of my knowledge, this striking language in Einstein's 1905 paper (together with its reappearance in 1921) has not been previously noted in the literature.

recent results in electrodynamics and optics, and Einstein now proposes to “elevate” both the principle of relativity and the light principle (which together imply that the velocity of light is invariant in all inertial frames) to the status of “presuppositions” or “postulates.” These two postulates together then allow us to “stipulate” a new “definition of simultaneity” (based on the assumed invariance of the velocity of light) implying a radical revision of the classical kinematics of space, time, and motion. In particular, whereas the fundamental kinematical structure of an inertial frame of reference, in classical mechanics, is defined by the Newtonian laws of motion, (a revised version of) this same structure, in Einstein’s theory, is rather defined by his two postulates.[19]

A central contention of Kant’s original version of transcendental philosophy, as we know, is that the Newtonian laws of motion are not mere empirical laws but a priori constitutive principles on the basis of which alone the Newtonian concepts of space, time, and motion can then have empirical application and meaning. What we have just seen is that Einstein’s two fundamental “presuppositions” or “postulates” play a precisely parallel role in the context of special relativity. But we have also seen significantly more. For Poincaré’s conception of how a mere empirical law can be “elevated” to the status of a “convention or definition in disguise” is a continuation, in turn, of Kant’s original conception of the constitutive a priori. Whereas Helmholtz’s principle of free mobility generalized and extended Kant’s original theory of geometrical construction within our “subjective” and “*necessary* form of external intuition,” Poincaré’s idea that specifically Euclidean geometry is then imposed on this form by a “convention or definition in disguise” represents an extension or continuation of Helmholtz’s conception. In particular, specifically Euclidean geometry is applied to our experience by precisely such a process of “elevation,” in which the merely empirical fact that this geometry governs, very roughly and approximately, our actual perceptual experience of bodily displacements gives rise to a precise mathematical framework within which alone our properly physical theories can subsequently be formulated.[20]

This same process of “elevation,” in Einstein’s hands, then makes it clear how an extension or continuation of Kant’s original conception can also accommodate new and surprising empirical facts – in this case, the very surprising empirical discovery (to one

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[19] The crucial point, in this connection, is that Newton’s third law – the equality of action and reaction – implicitly defines the relation of absolute simultaneity in a classical inertial frame, in so far as it allows us to coordinate action-reaction pairs related by the Newtonian law of (instantaneously propagated) gravitation. Einstein’s two postulates take over precisely this role in the case of his new, relativized relation of simultaneity defined by (continuously propagated) electro-magnetic processes.

[20] Euclidean geometry is singled out, for Poincaré, in that it is both mathematically simplest and very naturally corresponds – roughly and approximately – to our pre-scientific experience of bodily displacements. Just as Helmholtz’s conception, as I have suggested, is the minimal extension of Kant’s original conception consistent with the discovery of non-Euclidean geometries (compare note 8 above), Poincaré’s conception is the minimal extension of Helmholtz’s consistent with the more sophisticated group-theoretic version of the principle of free mobility due to Sophus Lie, the new perspective on the relativity of motion due to the modern concept of an inertial frame, and, most importantly, the apparently paradoxical new situation in electrodynamics arising in connection with precisely this relativity of motion – where, in particular, Poincaré’s hierarchical conception of the mathematical sciences allows him to retain the foundational role of both Euclidean spatial geometry and the laws of Newtonian mechanics in the face of what we now call Lorentzian (as opposed to Galilean) relativity (compare note 17 above).

or another degree of approximation) that light has the same constant velocity in every inertial frame. It now turns out, in particular, that we can not only impose already familiar and accepted mathematical frameworks (Euclidean geometry) on our rough and approximate perceptual experience, but, in appropriate circumstances, we can also impose entirely unfamiliar ones (the kinematical framework of special relativity). Einstein's creation of special relativity, from this point of view, thus represents the very first instantiation of a relativized and dynamical conception of the a priori – which, in virtue of precisely its historical origins, has a legitimate claim to be considered as genuinely constitutive in the transcendental sense. And what vindicates this claim, accordingly, is a reconceived version of transcendental philosophy where precisely the kind of integrated intellectual history I have been trying to exemplify takes the place of Kant's original transcendental faculty psychology: in particular, that the "inner logic" of the successive intellectual situations in question proceeds against the background of, and explicitly in light of, Kant's original theory is what makes this enterprise properly "transcendental."

Yet Einstein's creation of the general theory of relativity in 1915 involved an even more striking engagement with Poincaré's "conventionalist" methodology, which, I contend, makes the transcendentally constitutive role of this theory's fundamental postulates (the light principle and the principle of equivalence) even more evident.

The first point to make, in this connection, is that the principle of equivalence (together with the light principle) plays the same role in the context of the general theory that Einstein's two fundamental "presuppositions" or "postulates" played in the context of the special theory: namely, they define a new inertial-kinematical structure for describing space, time, and motion. Because Newtonian gravitation theory involves an instantaneous action at a distance (and therefore absolute simultaneity: compare note 18 above), it was necessary after special relativity to develop a new theory of gravitation where the interactions in question propagate with the velocity of light. And Einstein solved this problem, via the principle of equivalence, by defining a new inertial-kinematical structure wherein the freely falling trajectories in a gravitational field replace the inertial trajectories described by free particles affected by no forces at all. The principle of equivalence, in this sense, replaces the classical law of inertia holding in both Newtonian mechanics and special relativity. But the principle of equivalence itself rests on a well-known empirical fact: that gravitational and inertial mass are equal, so that all bodies, regardless of their mass, fall with exactly the same acceleration in a gravitational field. In using the principle of equivalence to define a new inertial-kinematical structure, therefore, Einstein has "elevated" this merely empirical fact (recently verified to a quite high degree of approximation by Lorand von Eötvös) to the status of a "convention or definition in disguise" – just as he had earlier undertaken a parallel "elevation" in the case of the new concept of simultaneity introduced by the special theory.[21]

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[21] Friedman (2001, pp. 86–91) develops more fully the parallel between these two cases of "elevating" a mere empirical fact to the status of a (relativized) a priori principle by first examining the relationship between the invariance of the velocity of light (as recently verified in the Michelson–Morley experiment) and Einstein's new definition of simultaneity, and then the relationship between the equality of gravitational and inertial mass (as recently verified in the Eötvös experiments) and the principle of equivalence.

Nevertheless, Einstein did not reach this understanding of the principle of equivalence all at once. He first operated, instead, within an essentially three-dimensional understanding of special relativity, and he proceeded to develop relativistically acceptable models of the gravitational field by considering the inertial forces (like centrifugal and Coriolis forces) arising in non-inertial frames of reference within this framework.[22] It was in precisely this context, in particular, that Einstein came upon the example of the uniformly rotating frame (the rotating disk), and it is at this point (and only at this point) that he then arrived at the conclusion that the gravitational field may be represented by a non-Euclidean geometry. This use of non-Euclidean geometry, however, was essentially three-dimensional, limited to purely *spatial* geometry, and Einstein did not arrive at the idea of a four-dimensional non-Euclidean geometry – where *space-time* geodesics represent freely falling trajectories affected only by gravitation – until he had generalized his conception to what we now call the four-dimensional (semi-)Riemannian geometries of variable curvature.[23]

It was in precisely the context of this line of thought, finally, that Einstein found that he now had explicitly to oppose Poincaré's "conventionalist" philosophy of geometry. Yet Einstein's argument – as described in *Geometrie und Erfahrung* (1921) – was far from a simple rejection of Poincaré's methodology in favor of a straightforward "empiricism."[24] For Einstein also famously says, in the same work, that "*sub specie aeterni*" Poincaré is actually correct – so that, in particular, Einstein's reliance on a Helmholtzian conception of "practically rigid bodies" is here merely provisional. I have suggested, therefore, that we can best understand Einstein's procedure as one of delicately situating himself *between* Helmholtz and Poincaré. Whereas Einstein had earlier followed Poincaré's general "conventionalist" methodology in "elevating" the principle of relativity (together with the light principle) to the status of a "presupposition" or "postulate," he here follows Helmholtz's "empiricism" in rejecting Poincaré's more specific philosophy of geometry in favor of "practically rigid bodies."[25]

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[22] See Norton (1985) (1989) for the details of Einstein's early applications of the principle of equivalence.

[23] I discuss at length the crucial importance of the rotating disk example in the development of Einstein's thought – following Stachel (1980) (1989) – in Friedman (2001, 2002).

[24] Compare note 18 above; and again, for a detailed analysis of *Geometrie und Erfahrung*, against the background of both Helmholtz and Poincaré, see Friedman (2001, 2002).

[25] Einstein does not explicitly mention Helmholtz in *Geometrie und Erfahrung*. However, in a closely related article on "Non-Euclidean Geometry and Physics," Einstein makes it perfectly clear that the opposition he has in mind is precisely that between Helmholtz and Poincaré (1925, pp. 18–19): "Either one accepts that the 'body' of geometry is realized in principle by the solid bodies of nature, if only certain prescriptions are maintained regarding temperature, mechanical stress, and so on; this is the standpoint of the practicing physicist. Then a natural object corresponds to the 'interval' of geometry, and all propositions of geometry thereby attain the character of assertions about real bodies. This standpoint was represented especially clearly by Helmholtz, and one can add that without it establishing the [general—MF] theory of relativity would have been practically impossible. Or, one denies in principle the existence of objects that correspond to the fundamental concepts of geometry. Then geometry alone contains no assertions about objects of reality, but only geometry together with physics. This standpoint, which may be more perfect for the systematic

It does not follow, however, that Einstein is also rejecting his earlier embrace of Poincaré's general "conventionalist" methodology. Indeed, Einstein had already side-stepped Poincaré's philosophy of geometry in the case of special relativity, and for essentially the same reason he explicitly opposes it here: Poincaré's rigid hierarchy of the sciences, in both cases, stands in the way of the radical new innovations Einstein himself proposes to introduce.[26]

But why was it necessary, after all, for Einstein to engage in this delicate dance between Helmholtz and Poincaré? The crucial point is that Einstein thereby arrived at a radically new conception of the relationship between the foundations of (physical) geometry and the relativity of space and motion. These two problems, as we have seen, were closely connected in Kant, but they then split apart and were pursued independently in Helmholtz and Mach (compare the paragraph to which note 11 above is appended). In Poincaré, as we have also seen, the two were perceptively reconnected once again, in so far as Poincaré's hierarchical conception of the mathematical sciences incorporated both a modification of Helmholtz's philosophy of geometry and a serious engagement with the late nineteenth-century concept of inertial frame (compare note 15 above, together with the paragraph to which it is appended, and also note 19 above). Indeed, it is for precisely this reason, as we now see, that Poincaré's scientific epistemology was so important to Einstein. Einstein could not simply rest content with Helmholtz's "empiricist" conception of geometry, because the most important problem with which he was now faced was to connect the foundations of geometry with the relativity of motion. But Einstein could not rest content with Poincaré's conception either, because his new models of gravitation had suggested that geometry has genuine physical content.

Einstein's radically new way of reconfiguring the relationship between the foundations of geometry and the relativity of motion therefore represents a natural (but also entirely unexpected) extension or continuation of the same conception of dynamical and relativized constitutive a priori principles he had first instantiated in the creation of special relativity (compare note 20 above, together with the paragraph to which it is appended and the following paragraph). Just as he had

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presentation of a completed physics, was represented especially clearly by Poincaré. On this standpoint the total content of geometry is conventional; which geometry is to be preferred depends on how 'simple' a physics can, by its use, be established in agreement with experience." Ryckman (2005, section 3.3) emphasizes the importance of this passage in relation to the earlier argument of *Geometrie und Erfahrung*.

[26] See note 17 above, together with the paragraph to which it is appended. As I suggested, Einstein could not embrace Poincaré's philosophy of geometry even in 1905, since it privileges a priori the three-dimensional geometry of space over the *de facto* symmetries of the laws of motion (which, on our current understanding, express the four-dimensional geometrical symmetries of space–time). Einstein's divergence from Poincaré on this point is even stronger in general relativity; for, not only do we now use non-Euclidean geometries to describe both space and space–time, but we have also definitively given up (in both cases) the homogeneity and isotropy (constant curvature) of the underlying geometry. Einstein thereby ultimately arrived at a radically new conception of physical geometry envisioned by neither Helmholtz nor Poincaré (compare again note 8 above): for details see again Friedman (2002).

earlier shown how an extension or continuation of Kant's original conception could accommodate new and surprising empirical facts (the discovery of the invariance of the velocity of light), Einstein here shows how a further extension of this same tradition can do something very similar in facilitating, for the first time, the application of a non-Euclidean geometry to nature. In this case, however, it is not the relevant empirical fact (the well-known equality of gravitational and inertial mass) that is surprising, but the entirely unforeseen connection between this fact and the new geometry. And what makes this connection itself possible, for Einstein, is precisely the principle of equivalence – which thereby constitutively frames the resulting physical space-time geometry of general relativity in just the same sense that Einstein's two fundamental "presuppositions" or "postulates" had earlier constitutively framed his mathematical description of the electrodynamics of moving bodies in special relativity (within what we now call the geometry of Minkowski space–time). Whereas the particular geometry in a given general relativistic space–time is now determined entirely empirically (by the distribution of mass and energy in accordance with Einstein's field equation), the principle of equivalence itself is not empirical in this sense. This principle is instead *presupposed* – as a transcendentally constitutive condition – for any such geometrical description of space–time to have genuine empirical meaning in the first place.

The historicized version of transcendental philosophy I am attempting to exemplify therefore sheds striking new light, I believe, on the truly remarkable depth and fruitfulness of Kant's original version. Kant's particular way of establishing a connection between the foundations of geometry and the relativity of motion – which, as we have seen, lies at the heart of his transcendental method (compare the paragraph to which note 6 above is appended, together with the following paragraph) – has not only lead, through the intervening philosophical and scientific work of Helmholtz, Mach, and Poincaré, to a new conception of the relativized a priori first instantiated in Einstein's theories, it has also led, through this same tradition, to a radically new reconfiguration of the connection between geometry and physics in the general theory of relativity itself. There can be no question, of course, of Kant having "anticipated" this theory in any way. The point, rather, is that Kant's own conception of the relationship between geometry and physics (which was limited, of necessity, to Euclidean geometry and Newtonian physics) then set in motion a remarkable series of successive reconceptualizations of this relationship (in light of profound discoveries in both pure mathematics and the empirical basis of mathematical physics) that finally eventuated in Einstein's theory.

References

DiSalle, R. (1988). *Space, Time, and Inertia in the Foundations of Newtonian Physics*. Doctoral Dissertation. Chicago, IL: University of Chicago.

DiSalle, R. (1991). "Conventionalism and the Origins of the Inertial Frame Concept." *PSA 1990* 2: 139–147.

DiSalle, R. (2002). "Reconsidering Ernst Mach on Space, Time, and Motion." In D. Malament, ed. *Reading Natural Philosophy: Essays in the History and Philosophy of Science and Mathematics*. Chicago, IL: Open Court, pp. 167–191.

DiSalle, R. (2006). *Understanding Space-Time: The Philosophical Development of Physics from Newton to Einstein*. Cambridge: Cambridge University Press.

Einstein, A. (1905). "Zur Elektrodynamik bewegter Körper." *Annalen der Physik* 17: 891–921. Translated by G. Jeffrey and W. Perrett (1923) as *On the Electrodynamics of Moving Bodies* in Lorentz et al. *The Principle of Relativity: A Collection of Original Memoirs on the Special and General Theories of Relativity*. London: Methuen.

Einstein, A. (1921). *Geometrie und Erfahrung. Erweiterte Fassung des Festvortrages gehalten an der Preussischen Akademie der Wissenschaft zu Berlin am 27. Januar 1921*. Berlin: Springer. Translated by G. Jeffrey and W. Perrett (1923) as *Geometry and Experience* in *Sidelights on Relativity*. London: Methuen.

Einstein, A. (1925). "Nichteuklidische Geometrie und Physik." *Die neue Rundschau* 36: 16–20.

Folina, J. (1992). *Poincaré and the Philosophy of Mathematics*. New York: Macmillan.

Friedman, M. (1992). *Kant and the Exact Sciences*. Cambridge, MA: Harvard University Press.

Friedman, M. (1997). "Helmholtz's *Zeichentheorie* and Schlick's *Allgemeine Erkenntnislehre*: Early Logical Empiricism and Its Nineteenth-Century Background." *Philosophical Topics* 25: 19–50.

Friedman, M. (1999). *Reconsidering Logical Positivism*. Cambridge: Cambridge University Press.

Friedman, M. (2000). "Geometry, Construction, and Intuition in Kant and His Successors." In G. Scher and R. Tieszen, eds. *Between Logic and Intuition: Essays in Honor of Charles Parsons*. Cambridge: Cambridge University Press.

Friedman, M. (2001). *Dynamics of Reason. The 1999 Kant Lectures at Stanford University*. Stanford: CSLI Publications.

Friedman, M. (2002). "Geometry as a Branch of Physics: Background and Context for Einstein's 'Geometry and Experience.' " In D. Malament, ed. *Reading Natural Philosophy: Essays in the History and Philosophy of Science and Mathematics*. Chicago, IL: Open Court, pp. 193–229.

Friedman, M., trans. & ed. (2004). *Immanuel Kant: Metaphysical Foundations of Natural Science*. Cambridge: Cambridge University Press.

Hatfield, G. (1990). *The Natural and the Normative: Theories of Spatial Perception from Kant to Helmholtz*. Cambridge, MA: MIT Press.

Norton, J. (1985). "What Was Einstein's Principle of Equivalence?" *Studies in History and Philosophy of Science* 16: 203–246. Reprinted in D. Howard and J. Stachel, eds. (1989). *Einstein and the History of General Relativity*. Boston, MA: Birkhäuser.

Poincaré, H. (1902). *La Science et l'Hypothèse*. Paris: Flammarion. Translated as *Science and Hypothesis* in Poincaré (1913b).

Poincaré, H. (1913a). *Dernières Pensées*. Paris: Flammarion. Translated by J. Buldoc (1963) as *Mathematics and Science: Last Essays*. New York: Dover.

Poincaré, H. (1913b). *The Foundations of Science*. Translated and edited by G. Halstead. Lancaster: Science Press.

Reichenbach, H. (1920). *Relativitätstheorie und Erkenntnis Apriori*. Berlin: Springer. Translated by M. Reichenbach (1965) as *The Theory of Relativity and A Priori Knowledge*. Berkeley/Los Angeles, CA: University of California Press.

Ryckman, T. A. (2005). *The Reign of Relativity: Philosophy in Physics 1915–1925*. Oxford: Oxford University Press.

Stachel, J. (1980). "Einstein and the Rigidly Rotating Disk." In A. Held, ed. *General Relativity and Gravitation*. New York: Plenum. Reprinted as "The Rigidly Rotating Disk as the 'Missing Link' in the History of General Relativity" in D. Howard and J. Stachel, eds. (1989). *Einstein and the History of General Relativity*. Boston, MA: Birkhäuser.

A Cognizable Universe: Transcendental Arguments in Physical Cosmology

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Abstract Cosmology deals with a unique object which comprises everything and yet is self-contained and singular. To describe this object in the language of physics, certain conditions must be in place. The increased role of such conditions finds its manifestation in distinct argument patterns. One popular case in point has to do with the anthropic arguments, some of which can be looked upon as varieties of transcendental reasoning, broadly construed. After reviewing this aspect of anthropic arguments, I show that the scope of transcendental inference at work in twentieth-century cosmology has been more extensive. Indeed, one important thread of such inference – the claim that, in order to be mathematically tractable, the Universe as a whole has to be a certain way – can be traced back to the first relativistic cosmological model proposed by Einstein in 1917. A somewhat different strategy of the same broad sort played a major role in shaping the steady-state theory, the main rival of big-bang cosmology in 1948–1965. Finally, the famous "no-boundary" condition for quantum cosmology would (if it could bear the weight of far-reaching interpretations put on it) be another example of grounding the mere possibility of the physical description of the Universe in its global properties.

1 Introduction

It has been a recurrent topic in philosophical discussions of physical cosmology, both by philosophers and physicists, to emphasize the special nature of its object, the Universe as a whole, which comprises all that exists and yet is manifestly singular. The fact that the object of cosmology combines these features naturally gives rise to a number of intriguing questions about the relationship between the general and the particular in the physical description of the Universe. Such questions

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have been framed in somewhat different ways,[1] emphasizing the unusual and often surprising connection, revealed or at least suggested by the cosmological perspective, between the members of traditional dichotomies: (1) laws of physics and boundary conditions, (2) necessity and contingency, (3) physics and geometry of the Universe as a whole. This, in turn, has been argued to have interesting methodological implications, by influencing explanatory standards and expectations, introducing novel inference patters or even imposing new Big Principles.[2]

I add my voice to those who think that modern physical cosmology embodies some of these distinctive features. But I believe one aspect of its methodology – one that it shares with other areas of contemporary physics, as evidenced by this volume – has so far been neglected. I argue below that *transcendental reasoning*, broadly construed, has been a recurrent topic in the development of relativistic cosmology since its beginning. The reasoning of this sort seeks to infer features of the entire Universe from conditions that make its physical description possible or coherent.

Two disclaimers are in order. First, while I borrow the term 'transcendental' from a venerable historical tradition, I make no attempt to relate my use it to any major representatives of that tradition. My objective is more modest: to show that a certain peculiar *pattern* of reasoning, roughly characterized as above, has been employed in modern cosmology on several occasions. Second, my consideration will, of necessity, be brief and abstract from many complexities of the historical cases at hand.

I begin by reviewing a recent instance of transcendental reasoning in cosmology, the anthropic reasoning, but only to set it aside. My real interest is to argue that *non*-anthropic transcendental arguments have been at play at some crucial junctures in the history of twentieth-century cosmology. In Sections 3–5 I focus on three important episodes: (1) Einstein's first relativistic cosmological model, (2) the steady-state theory, and (3) Hartle-Hawking's "no-boundary" condition for quantum cosmology. Different as these developments are, they have something in common; they attempt, in their distinct ways and with varying degree of success, to incorporate the idea that the mere possibility of a coherent physical description of the Universe as a whole poses constraints on what kind of entity it could be.

2 The Anthropic Reasoning

If certain physical properties of the Universe were even slightly different, it would not contain complex material structures. In particular, it would not contain observers capable of posing questions about the physical properties of the Universe. Importantly, the properties at hand comprise both *nomic* and *non-nomic* properties of the Universe as a whole (or at least of a large physically isolated and self-contained part of it): those

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[1] See, in particular, Bondi (1960), Munitz (1962), North (1965), Merleau-Ponty (1965). For recent discussions, see Gale (1992, 1999), Gale and Urani (1993), McMullin (1993, 1994), Gale and Shanks (1996), Balashov (1994, 2002) and references therein.

[2] As was done by Edward Milne in his cosmological project (Milne, 1948). On Milne's "rationalist program," see, e.g., Gale (1992, 1999).

having to do with the fundamental physical *laws* operating across it as well as those having to do with its physical *state*. The first category includes the values of various physical constants (such as e, m_p, G, and α_e), while the second certain cosmological parameters (such as ρ, H, and Λ).[3] Why do we observe these particular values of such parameters and, hence, a particular physical makeup of the Universe, rather than some other? One answer is that we do it simply because a Universe with a different makeup would remain unobservable: there would be no one to observe it.

While certain interpretations of this situation may be questionable, one of them – involving the so-called *weak* anthropic reasoning – is valid. In order to perform its task, however, the reasoning needs to be supplemented with an additional assumption to the effect that an observable portion of the Universe is a small fragment of a larger portion, which, in turn, is a member of a huge ensemble of (lower-case) universes, each having a relevantly different physical makeup (where the latter may include both nomic and global non-nomic properties). If all relevantly different makeups are realized in such an ensemble the fact that we observe a very special one – that compatible with our existence – is not surprising. In order to be cognizable, the (lower-case) universe around us *must* be a certain way: it must allow cognizers. The real significance of this reasoning lies in modifying the antecedent likelihood of competing hypotheses about the universe (and the Universe). Suppose, on one such hypothesis, the actual value of a certain physical constant is antecedently unlikely, but the hypothesis is otherwise very successful.[4] One can support the hypothesis in the face of its initial implausibility by invoking a weak anthropic argument that the unlikely value of the constant is required for the observability of the universe (i.e., for the presence of observers in it) and, hence, the Universe must be structured accordingly, to allow for that value to be realized in one of its relatively isolated parts.

One can put this inference pattern in more formal terms and make its Bayesian pedigree more explicit (see Bostrom, 2002). However, the broadly transcendental nature of the inference is obvious: knowledge of the universe (and of the Universe) is constrained by the global physical conditions necessary for such knowledge to take place.[5] It would, however, be wrong to think that transcendental reasoning in physical cosmology is confined to occasional (and often problematic) applications of anthropic arguments. Even a cursory look at the history of twentieth-century cosmology suggests otherwise.

3 Einstein's "No-Boundary Proposal"

Soon after completion of his work on general relativity Einstein applied the new theory to the geometry of the Universe as a whole, thereby starting an entirely new chapter in the history of cosmology (Einstein, 1917). Einstein assumed that the large-scale

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[3] See Barrow and Tipler (1986) for a useful survey of various "anthropic constraints."

[4] One good example is the vacuum-energy driven cosmological constant Λ. See Vilenkin (2004) for a recent discussion.

[5] On the transcendental nature of weak anthropic reasoning, see Balashov (1992) and Roush (2003).

structure of the Universe must be unchanging and was led to his first relativistic cosmological model by considerations having to do with the difficulty of formulating boundary conditions at infinity similar to those that would obtain in a static Newtonian Universe with no matter at infinity. The problem with boundary conditions was twofold. Part of it had to do with Mach's principle. At that time, Einstein thought that any viable theory of gravitation had to incorporate this principle, but a model with a flat spatial metric at matter-devoid infinity would violate it (for on Mach's principle, all metrical properties of space must be due to the influence of matter). Secondly, Einstein was worried that boundary conditions of this sort would bring with them a "definite choice of the system of reference, which is contrary to the spirit of the relativity principle" (Einstein [1917], 1923, p. 183).

Einstein's solution was, as we know, ingenious: to get around the problem of boundary conditions, he proposed that the Universe had *no* boundary. Rather, it must be a spatially closed spherical world. "[I]f it were possible to regard the universe as a continuum which is finite (closed) with respect to its spatial dimensions, we should have no need at all of any … boundary conditions [at spatial infinity]" (ibid., p. 183). The idealized geometry of such a world would be described by the following metric and stress-energy tensor:

$$g_{14} = g_{24} = g_{34} = 0;\ g_{44} = 1$$

$$g_{mn} = -\left(\delta_{mn} + \frac{x_m x_n}{R^2 - (x_1^2 + x_2^2 + x_3^2)}\right) \tag{1}$$

$$T^{44} = \rho;\ T^{\mu\nu} = 0 \text{ unless } \mu = \nu = 4$$

Here ρ is the average density of matter in the Universe, R its radius of curvature, and c is set to 1. That was the model Einstein *wanted* to have. But it turned out to be inconsistent with his field equations of general relativity:

$$R_{\mu\nu} - \tfrac{1}{2}\mathrm{R}\mathrm{g}_{\mu\nu} = -\kappa T_{\mu\nu} \tag{2}$$

This prompted Einstein to modify his original equations by introducing the famous Λ-term:

$$R_{\mu\nu} - \tfrac{1}{2}\mathrm{R}\mathrm{g}_{\mu\nu} - \Lambda g_{\mu\nu} = -\kappa T_{\mu\nu} \tag{3}$$

Inserting the desired model (1) into the modified equations (3) yields the following relationships:

$$\Lambda = \frac{\kappa\rho}{2} = \frac{1}{R^2} \tag{4}$$

$$M = 2\pi^2 R^3 \rho \tag{5}$$

where M is the mass of the closed Universe.

Certain aspects of this derivation are notable. First, Einstein's train of thought in deriving the model seems to have been the following:

1. In order to be mathematically tractable (i.e., to absolve one of the necessity to deal with boundary conditions at infinity) and physically consistent (to satisfy the demands of Mach's principle: no geometry – not even a flat geometry – without matter), the Universe must possess a certain global property: be spatially closed and thus have no boundary in space (Einstein's "no-boundary proposal").
2. In order for the Universe to possess this property and be static, Einstein's field equations (i.e. a law of nature) must be modified.

I submit that these two steps embody, in a very clear sense, transcendental reasoning: to be describable in the language of physics, the Universe has to be a certain way. Remarkably, the way the Universe has to be includes both its geometry and physics. Despite the fact that Einstein himself took the connection between the geometry and physics of the Universe, manifested in Eqs. (4) and (5), for granted, it is very striking. Equations (4) and (5) relate quantities of two rather different sorts: Λ and κ, figuring in the fundamental laws of physics, which describe all possible ways the Universe could have been, and the quantities R, M, and ρ, representing a unique way the Universe actually is. Both kinds of quantities, however, pertain to the Universe as a whole and this gives some reason to treat them on a par. And yet the correlation between them is unusual. Although Einstein did not find it particularly remarkable, Hermann Weyl, for example, wrote that the correlation between Λ and M, as expressed in Eqs. (4) and (5), "obviously makes great demands on our credulity" (quoted in North, 1965, p. 83). And Eddington noted that the correlation had a strange consequence that "the creation of a new stellar system in a distant part of the world would have to propagate to us, not merely a gravitational field, but a modification of the law of gravitation itself" (ibid., p. 85). Moreover, the propagation would have to be instantaneous.

These implications of Eqs. (4) and (5) should not detract from the significance of the main thread which led Einstein to his cosmological model. Upon reflection, the demand that reality has to be structured in a certain way in order to be describable in the language of physics should not strike one as outrageous: we see it at work in different quarters of physics. Cosmology, however, adds the grandeur of scale to it, and, in some cases, an interesting connection between the material structure of the Universe and its nomic structure. Einstein set a notable precedent for thinking along these lines.

4 Steady-State Cosmology

And the precedent was not without its followers. In 1948–1965, the big-bang cosmology had to fight a major rival, the steady-state theory (SST). According to SST, the expanding Universe, instead of evolving from the hot big bang, is stationary on the large scale. The dilution of matter due to the cosmic expansion is compensated for by the creation of new matter, and any other global process operative in the Universe is regarded as being self-perpetuating.

The 1964–1965 discovery of the microwave background radiation, soon afterwards identified by the majority of cosmologists as a relic of the hot big bang, dealt a crushing blow to SST and vindicated the big-bang scenario. But the rivalry between the two competing theories of the Universe greatly stimulated theoretical and observational developments in the 1950s.

Both versions of SST (Bondi and Gold, 1948; Hoyle, 1948) were driven by methodological reflections on the nature of cosmology as a science,[6] in which transcendental motives played a major role. In the Bondi–Gold version, the guiding idea had to do with a possible influence the Universe at large may have on the local laws of physics. If the Universe changed radically in space or time, one could not, according to Bondi and Gold, apply physical principles discovered locally to other parts of the Universe. To guarantee the universal validity of physical laws, the Universe must be uniform in both space and time. The standard relativistic models fulfill this requirement only partially, in the form of the cosmological principle, which proclaims the large-scale homogeneity and isotropy of the Universe in space. But Bondi and Gold were convinced that one could not stop halfway here. The Universe must be constant on a large scale, not only in space, but also in time. Otherwise there would be no guarantee that the laws of physics discovered here and now could apply to the distant past of the Universe. In order to be describable by physical principles, discovered here and now, the past of the Universe must, in its gross features, be like its present. Bondi and Gold put these considerations in the form of the "perfect cosmological principle" (PCP). Their entire theory was then derived from this single principle, without relying on any particular field theory of gravitation.

The derivation proceeds as follows (see, e.g., Bondi, 1960, pp. 145–146). Bondi and Gold start with the generic Robertson-Walker metric:

$$ds^2 = c^2dt^2 - R^2(t)(dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\varphi^2)(1+\frac{kr^2}{4})^{-2} \tag{6}$$

The spatial curvature k/R^2 is responsible for certain observable effects (for example, the number of galaxies observable in the unit volume of space) and, therefore, according to PCP, must be constant. Since $R(t)$ is not constant (otherwise there would be no red shifts in the spectra of distant galaxies), this gives $k = 0$. The Hubble parameter H is also an observable quantity. From $H = \dot{R}/R = const.$, it follows that $R(t) = \exp(Ht)$. Thus the metric of the stationary Universe is

$$ds^2 = c^2dt^2 - (dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\varphi^2)\exp(2Ht) \tag{7}$$

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[6] See Balashov (1994) for a discussion of the methodological foundations of SST. For a detailed history of the big bang-steady state controversy, see Kragh (1996).

This model, which is formally similar to one of the early de Sitter solutions, represents the way the whole Universe has to be in order to secure the consistency of physics throughout its space–time volume. And just like in Einstein's model, the geometrical way the Universe has to be entails modifications in its basic physics. The Universe is expanding (here SST differs from Einstein's static model), but its density is constant and non-zero. Therefore there must be continuous creation of new matter, which should be incorporated into the basic laws of nature.

One way (due to Hoyle [1948]), in which this was done, takes a cue from Einstein's modification of the field equations of general relativity briefly discussed above (see Eq. 3). Like Einstein, Hoyle introduced into them an additional symmetrical tensor term $C_{\mu\nu}$:

$$R_{\mu\nu} - \tfrac{1}{2}\mathrm{R}g_{\mu\nu} - C_{\mu\nu} = -\frac{8\pi G}{c^4} T_{\mu\nu}, \tag{8}$$

where

$$C_{\mu\nu} = C_{\mu;\nu} = \frac{\partial C_\mu}{\partial x^\nu} - \Gamma^\alpha_{\mu\nu} C_\alpha \tag{9}$$

and

$$C_\mu = \frac{3c}{a}(1,0,0,0),\ a = const \tag{10}$$

Under the normal assumption that the only non-vanishing component of $T_{\mu\nu}$ is $T_{00} = \rho c^2$, a solution of Eq. (8):

$$ds^2 = c^2 dt^2 - (dx_1^2 + dx_2^2 + dx_3^2)\exp(\frac{2ct}{a}) \tag{11}$$

gives a de Sitter-type metric of the stationary Universe (Hoyle, 1948, pp. 375–377). Of course, the proper density of matter in SST, unlike that in the de Sitter model, is a constant non-zero quantity given by

$$\rho = \frac{3c^2}{8\pi G a^2} \tag{12}$$

It can be shown that the vector field C_μ is responsible for the "creation-of-matter" process. From Eq. (8) we have:

$$(C^{\mu\nu})_{;\nu} = -\frac{8\pi G}{c^4}(T^{\mu\nu})_{;\nu} \tag{13}$$

Since $(C^{0\nu})_{;\nu} \neq 0$, a continuous creation of matter and energy uniformly occurs.

The details of Hoyle's model are not of primary interest to us. But it is worth reflecting on the result. The modified field equations of gravitation (8) represent a *general* relation between physical quantities $g_{\mu\nu}$ and $T_{\mu\nu}$. Incorporated in this general relation, however, is another quantity, $C_{\mu\nu}$, having, it would seem, a purely *factual* significance, as it is constructed from the vector field C_μ, which has its origin in the features of a particular *model* of the Universe. Thus, in order to derive this model from the modified field theory of gravitation, one has first to ground the theory itself in the model at hand. What legitimizes creating such a "centaur," in which the nomic and apparently non-nomic features are blended together in a single relation, is, again, the idea that for the Universe as a whole, the distinction between the general and the particular fades away. Yet the particular "mixture" of them, which is represented by Eq. (8), must be in place to insure the consistency between the laws physics and the cosmological behavior of this very special Universe.

The two cases, Einstein's static model and the steady-state model, thus have much in common in that both centrally involve transcendental arguments, in the sense noted in Section 1. Both cases present considerable historical interest. Both, however, represent dead ends in cosmological theory. It would be interesting to see what else, besides the anthropic arguments (briefly considered in Section 2), may illustrate the contemporary value of transcendental reasoning in cosmology. I would like to look at one rather controversial case and end on a cautious note.

5 Euclidean Quantum Cosmology

The case in question is the Euclidean, or Riemannian, quantum cosmology (Hartle and Hawking, 1983).[7] Its central idea is to use the path-integral approach to quantum gravity to calculate the wave function of the Universe. The propagator of quantum gravity, $K(\Sigma_i, \gamma_i, \phi_i; \Sigma_f, \gamma_f, \phi_f)$, is supposed to integrate over the set of all 4D Lorentzian manifolds interpolating between the initial and final 3D configurations $(\Sigma_i, \gamma_i, \phi_i)$ and $(\Sigma_f, \gamma_f, \phi_f)$, which include Riemannian metric fields γ_i and γ_f and matter field configurations ϕ_i and ϕ_f:

$$K(\Sigma_i,\gamma_i,\phi_i;\Sigma_f,\gamma_f,\phi_f) = \int_{\wp_{\mathrm{L}}} e^{iA/\hbar} d\mu \tag{14}$$

But Lorentzian integration is not well defined (one reason being the oscillatory nature of the complex exponent). Accordingly, one follows Hawking's earlier proposal and replaces it with integration over the set of compact Riemannian 4-manifolds. The action A then becomes a "Euclidean" action A_E and is assigned a real-valued weight:

$$K(\Sigma_i,\gamma_i,\phi_i;\Sigma_f,\gamma_f,\phi_f) = \int_{\wp_{\mathrm{L}}} e^{-A_{\mathrm{E}}/\hbar} d\mu \tag{15}$$

Compact Riemannian geometries are geodesically complete, hence there are no singularities. This suggests the idea that one can avoid the initial cosmological singularity as well by eliminating the initial configuration (or replacing it, so to

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[7] My account of the Hartle–Hawking proposal is based on an excellent critical review by Gordon McCabe (2005, pp. 74–81). His notation is used throughout.

speak, with an empty set) and integrating over the compact 4-manifolds with a single boundary[8]:

$$K(\varnothing;\Sigma_f,\gamma_f,\phi_f)=\int_{\wp_{\mathrm{L}}} e^{-A_{\mathrm{E}}/\hbar}d\mu \tag{16}$$

The Hartle–Hawking program builds on this framework by identifying the propagator of this sort with the “wave function of the Universe”:

$$\Psi_0(\Sigma_f,\gamma_f,\phi_f)\equiv K(\varnothing;\Sigma_f,\gamma_f,\phi_f) \tag{17}$$

and claiming that it gives the probability amplitude of “creation of the Universe *ex nihilo*.” To quote from Hartle and Hawking’s seminal paper, “[t]his means that the Universe does not have any boundaries in space or time (at least in the Euclidean regime). There is thus no problem of boundary conditions” (Hartle and Hawking, 1983, p. 2961).

If this interpretation of Eq. (17) were plausible it would present another remarkable example of transcendental inference in cosmology. At the very least, one could say that, in order to be singularity-free and insensitive to boundary conditions, the Universe has to be a certain way: without a boundary, Euclidean in the past, and with a determinate probability of emerging from nothing.

Unfortunately, as noted by many critics (see, e.g., Butterfield and Isham, 1999, section 5.5; McCabe, 2005, pp. 79–81), Hartle and Hawking’s result does not warrant the interpretation they put on it. First, it is doubtful that the features noted above – not having a past boundary and being Euclidean in the past – have anything to do with the Universe in which we live, for Eq. (17) describes a *wave function*, not a single classical manifold. Second, there is no clear sense in which “emergence from nothing” or even “from a Euclidean regime” could be viewed as itself a process in time. Finally, it is unclear whether the proposal actually gets rid of boundary conditions or, rather, provides a recipe *for* a boundary condition (of the wave function of the Universe); in other words, it is unclear in what sense it is a “no-boundary” proposal.

This prompts one to end on a cautious note. Transcendental arguments have been crucially involved in the history of modern physical cosmology. One also finds a different variety of them at work in weak anthropic arguments. Given the importance of transcendental reasoning, one should not be surprised to see other examples of its application – or at least *attempted* application – in contemporary cosmological theory.[9]

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[8] Even so, the “Euclidean” action is in general not positive definite, so the exponent diverges. To tame it, one needs to integrate over a select subset of four-geometries.

[9] A version of this paper was presented at the conference “Cosmology: Physics and Philosophical Perspectives” held at University of Notre Dame, Indiana, USA in April 2005. My thanks to the audience for a stimulating discussion.

References

Balashov, Yuri (1992), “Transcendental Background to the Anthropic Reasoning in Cosmology,” *Man & World* 25: 115–132.

Balashov, Yuri (1994), “Uniformitarianism in Cosmology: Background and Philosophical Implications of the Steady-State Theory,” *Studies in History and Philosophy of Science* 25B: 933–958.

Balashov, Yuri (2002), “Laws of Physics and the Universe,” in Y. Balashov and V. Vizgin, eds., *Einstein Studies in Russia*. Boston, MA/Basel/Berlin: Birkhäuser, pp. 107–148.

Barrow, John and Tipler, Frank (1986), *The Anthropic Cosmological Principle*. Oxford: Clarendon.

Bondi, Hermann (1960), *Cosmology*. Cambridge: Cambridge University Press.

Bondi, Hermann and Gold, Thomas (1948), “The Steady-State Theory of the Expanding Universe,” *Monthly Notices of the Royal Astronomical Society* 108: 252–270.

Bostrom, Nick (2002), *Anthropic Bias: Observation Selection Effects in Science and Philosophy*. New York: Routledge.

Butterfield, Jeremy and Isham, Chris (1999), “On the Emergence of Time in Quantum Gravity,” in J. Butterfield, ed., *The Arguments of Time*. Oxford: Oxford University Press, pp. 111–168.

Einstein, Albert (1917), “Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie,” *Königlich Preussiche Akademie der Wissenschaften* (Berlin). *Sitzungsberichte*: 142–152. Page references in text are to the English translation in A. Einstein et al., *The Principle of Relativity*. London: Methuen, 1923, pp. 175–188.

Gale, George (1992), “Rationalist Programmes in Early Modern Cosmology,” *The Astronomy Quarterly* 8: 4–21.

Gale, George (1999), “Milne, Bondi and the ‘Second Way’ to Cosmology,” in H. Goenner, J. Renn, J. Ritter, and T. Sauer, eds., *The Expanding Worlds of General Relativity*. Boston, MA/Basel/Berlin: Birkhäuser, pp. 343–375.

Gale, George and Shanks, Niall (1996), “Methodology and the Birth of Modern Cosmological Inquiry,” *Studies in History and Philosophy of Modern Physics* 27B: 279–296.

Gale, George and Urani, John (1993), “Philosophical Midwifery and the Birth-Pangs of Modern Cosmology,” *American Journal of Physics* 61: 66–73.

Hartle, James and Hawking, Stephen (1983), “Wave Function of the Universe,” *Physical Review* D28: 2960–2975.

Hoyle, Fred (1948), “A New Model for the Expanding Universe,” *Monthly Notices of the Royal Astronomical Society* 108: 372–382.

Kragh, Helge (1996), *Cosmology and Controversy: The Historical Development of Two Theories of the Universe*. Princeton, NJ: Princeton University Press.

McCabe, Gordon (2005), “The Structure and Interpretation of Cosmology: Part II. The Concept of Creation in Inflation and Quantum Cosmology,” *Studies in History and Philosophy of Modern Physics* 36: 67–102.

McMullin, Ernan (1993), “Indifference Principle and Anthropic Principle in Cosmology,” *Studies in the History and Philosophy of Science* 24: 359–389.

McMullin, Ernan (1994), “Long Ago and Far Away: Cosmology as Extrapolation,” in R. Fuller, ed., *Bang: The Evolving Cosmos*. Lanham, MD: University Press of America, pp. 105–145.

Merleau Ponty, Jacques (1965), *Cosmologie du XX Siècle*. Paris: Gallimard.

Milne, Edward Arthur (1948), *Kinematic Relativity*. Oxford: Clarendon.

Munitz, Milton (1962), “The Logic of Cosmology,” *The British Journal for the Philosophy of Science* 13: 34–50.

North, John (1965), *The Measure of the Universe. A History of Modern Cosmology*. Oxford: Clarendon.

Roush, Sherrylin (2003), “Copernicus, Kant, and the Anthropic Cosmological Principles,” *Studies in History and Philosophy of Modern Physics* 34: 5–35.

Vilenkin, Alexander (2004), “Anthropic Predictions: The Case of the Cosmological Constant,” http://www.arxiv.org/abs/astro-ph/0407586.

Hermann Weyl and "First Philosophy": Constituting Gauge Invariance

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Abstract The current vogue of naturalism – whether of a pragmatist, instrumentalist or realist variety – in philosophy of physics is largely attributable to a fiction promulgated by logical empiricism, but surviving the latter's demise. It states that relatively theory (especially general relativity) comprised a decisive refutation of Kant, and transcendental idealism more broadly. A closer look at the early years of general relativity reveals a considerably different picture. Here we trace how transcendental idealism informed Weyl's construction of a "purely infinitesimal geometry" whose additional (gauge) degrees of freedom enabled incorporation of electromagnetism into the spacetime metric.

1 Naturalism's Hegemony

First Philosophy? Most of us will recall Quine's pronouncements on the subject: First Philosophy is the philosophical disease for which naturalism is the cure. Naturalism, the fifth and final milestone of empiricism, the cap crowning empiricism's triumphal arch, mandates the "abandonment of the goal of first philosophy", which is just to say that (Naturalism) sees natural science as an inquiry into reality, fallible and corrigible, but not answerable to any supra-scientific method, and not in need of any justification beyond observation and hypothetico–deductive method.[1]

By implication, First Philosophy subordinates natural science to "supra-scientific method" and to non-empirical constraints of justification. Quine notes that the sources of naturalism, thus understood, are two, and both negative: despair at the failure of attempts to define theoretical terms of science in terms of phenomena, with the resulting surrender of empirical meaning to semantic holism, and secondly,

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[1] Quine (1981), p. 72.

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an unregenerate realism, the robust state of mind of the natural scientist who has never felt any qualms beyond the negotiable uncertainties internal to science.

According to Quinean naturalism, "the scientific epistemologist" has become a deckhand on Neurath's leaky skiff, adrift in a turbulent sea of incoming sensory stimulation, with only the rough planks hewn by "naturalized epistemology" – empirical psychology, the neurology of perception and of language learning, evolutionary theory, and so on – to keep afloat the ever-shifting cargo of contemporary scientific theory.

Although few "scientific epistemologists" – if that term accurately describes philosophers of physics – have followed the letter of Quine's commandment to seek illumination of their perceived problems in the details of cognitive psychology and evolutionary biology, the wider Quinean conception of naturalism has global currency within philosophy of science. To be sure, some have puzzled that Quine should trouble to belabor what had been obvious since the time of Galileo and Newton, that science must not be yoked to an Aristotelian or Cartesian *Prima Philosophia*. Such objections miss the intended target of Quine's remarks. Penelope Maddy, in a recent paper enthusiastically endorsing Quinean naturalism, explicitly brings this out, by quoting the master himself.

> For the naturalist, there is no higher perspective, where transcendental or other extra-scientific considerations hold sway. The naturalist operates "from the point of view of our own science, which is the only point of view I can offer".[2]

Lest we forget, the non-existent "higher perspective where transcendental or other extra-scientific considerations hold sway" includes the remnants of a "relativized *a priori*" retained in the logical empiricists' recognition of non-empirical ("conventional") elements in scientific theories, in the analytic/synthetic distinction, and in particular in Carnap's conception of linguistic frameworks as a presupposition of inquiry. In any case, while naturalism would remain a subject of live debate within epistemological discussions *per se*, principally regarding the nature and sources of normativity, naturalism in Quine's broader sense, constraining philosophical reflections on science to lie within "the point of view of our own science", has for a generation been the implicit ideology in much of contemporary philosophy of science and in philosophy of physics.

There are, of course, exceptions, of which the program formulated in Michael Friedman's *Dynamics of Reason* (2001) is among the most articulated and influential. There, as against Quinean holism and Kuhnian relativism, it is argued that the cumulative and rational character of revolutionary change in mechanics and gravitational theory is grounded in a continuous succession of meta-empirical *a priori* principles, "relativized" to particular theories. On the other hand, except by implication, within the frame of the *Dynamics of Reason* an explicit alternative is not posed to scientific realism, itself the inevitable outgrowth of the "unregenerate realism" of natural science that is, for Quine, one of two primary sources of naturalism.

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[2] Maddy (2000), p. 107; citing Quine (1981), p. 181.

Merely the briefest perusal of the current literature (at least in English) on scientific realism, both pro- and con-, returns the verdict of naturalism's domination. This is readily apparent in the scientific realisms of Richard Boyd and Richard Miller, of Wesley Salmon and of Philip Kitcher, in the so-called "structural realism" of Tian Yu Cao, as well as in the mathematical realism of Penelope Maddy. On the other side of the coin, the principal alternatives to scientific realism similarly presuppose naturalism: van Fraassen's constructive empiricism, Laudan's "normative naturalism", Hilary Putnam's pragmatism, and Arthur Fine's "natural ontological attitude" (1984). Even the radical fringe of challenges to scientific realism posed by sociology of science, and by "science studies" are, in the main, motivated from within the naturalistic world view of the primacy of human practice and institutions, of social constraints and political biases. In contrast, non-naturalistic alternatives to scientific realism, such as Duhem's instrumentalism, have all but disappeared from the scene. Within philosophy of physics, the dominance of naturalism appears perhaps even more monolithic. Following Abner Shimony's lead in regarding the work of Bell, coupled with earlier work of Gleason, of Kochen and Specker and others, as comprising "near decisive results in experimental metaphysics", that is, as bringing out the explicit metaphysical content of quantum mechanics, mainstream philosophy of physics (again, at least in English) now comfortably speaks of the interrelatedness of physics and metaphysics in unveiling the ultimate nature of reality.[3]

In my view, the reasons for the hegemony of naturalism in philosophy of science and philosophy of physics have but little to do with Quine (who at any rate is little read by philosophers of physics) nor with any *triumph* of naturalism in the arena of debate (which has not occurred) but with historical, and contingent factors that have obscured the existence of any illuminating non-naturalistic alternative. The principal factor responsible for this occlusion is the pervasive and perduring myth that the general theory of relativity comprised an *experimentum crusis* disconfirming not merely Kant's Transcendental Aesthetic but all varieties of transcendental philosophy. Moritz Schlick, the *éminence gris* of logical empiricism, first fashioned this fiction in the early 1920s. Here is Schlick addressing the German Society of Natural Scientists in 1922, the same year he took over Ernst Mach's old chair in the Philosophy of the Inductive Sciences in Vienna.

> Now along comes the general theory of relativity, which finds itself obliged to use non-Euclidean geometry in order to describe the (physical) world. Through Einstein, therefore, what Riemann and Helmholtz claimed as a possibility has now become a reality, the Kantian position is untenable, and empiricist philosophy has gained one of its most brilliant triumphs.[4]

Today, long after logical empiricism's demise, it is widely assumed that relativity theory had shown the untenability of any "philosophy of the synthetic a priori", a belief that still finds its way into the curriculum of philosophical instruction. But

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[3] Shimony (1984); Redhead (1995).

[4] Schlick (1922), p. 63.

however rhetorically useful to logical empiricism's promoted public image, Schlick's claim that general relativity sounded the death knell of "the Kantian position" can be maintained only if one ignores, as Schlick knowingly and willfully did, not only neo-Kantian refinements of Kant's thought, but as well several of the most significant developments *within* general relativity in its first 10 years, i.e., from 1915–1925. Regarding the former, one need only mention the names of Ernst Cassirer and Edmund Husserl, regarding the latter, those of David Hilbert, Hermann Weyl, and Arthur Eddington.[5] While much more about Husserl and Weyl will be said in what follows, I simply cite here but one opinion countering Schlick's assessment, that of Nobel prize winner and fellow Planck student Max von Laue, from his textbook on general relativity, first published in 1921.

> It is, frankly, an identifying characteristic (*Kennzeichen*) for a correct epistemology, that it remains invariant against all transformations that the physical world picture experiences in the course of time. We would not conceal our conviction that Kant's critical idealism (although not every sentence of the "Critique of Pure Reason") satisfies this requirement even against the general theory of relativity.[6]

Von Laue was both a close personal friend and Berlin colleague of Einstein; his book was the first actual textbook on general relativity, reprinted as recently as 1982. (Neither Weyl's *Raum-Zeit-Materie* of 1918, nor Pauli's *Relativitätstheorie* of 1921, are books from which general relativity could be readily learnt or taught.)

Like most myths, Schlick's broad assertion is false, about as completely false as any philosophical claim can be. But why was it so successful? Here I can only point to the fickleness of philosophical fashion, and to the fact that the fundamental contributions to general relativity of Hilbert, Weyl, Eddington, and von Laue were mainly buried in technical works that had little resonance in philosophical circles. In the period in question, only one of them, Hermann Weyl, wrote anything like a comprehensive philosophical work, but Weyl's book, *The Philosophy of Mathematics and Natural Science* (1926), though difficult, is without parallel in its genre. As it did not appear in English until 1949, during the rise and consolidation of logical empiricism it was known only to those able to read German. In short, the logical empiricist myth has endured because it was not publicly challenged in the forum of philosophical debate, primarily because those with the expertise to speak with authority on general relativity were, by and large, not philosophers, and their philosophical views are intricately tied up with mathematics that in the 1920s was not widely known or even taught. On the other hand, Cassirer, still widely regarded within analytic philosophy as merely a historian of philosophy, did not attract a noticeable following in philosophy of science. As for Husserl, well, wasn't he followed by Heidegger?

On account of these historical contingencies, the non-existence of a credible *non-naturalistic alternative* to scientific realism or to any of the permitted alternatives within the hegemony of naturalism of contemporary philosophy of science

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[5] For details, see Ryckman (2005); Brading and Ryckman (2008).

[6] Von Laue (1921), p. 42.

and philosophy of physics, is simply an article of faith. As I have suggested, however, such a transcendental idealist alternative exists, at least programmatically, though it is has long remained nearly completely invisible. One key component lay in Cassirer's (1921) genetic account of general relativity as the latest stage within a historically progressing sequence of mathematical structures constituting "physical objectivity", a development that has led to the imposition of the requirement of general covariance governing the "constitution" of objects of fundamental physical theory. But here I want to outline a second, and equally crucial, line of approach of a revitalized transcendental idealism, already followed by *Husserl* in 1910 in opposition to the philosophical naturalism of his day, in the paper "*Philosophy as a Rigorous Science*" and subsequently in many other writings. If we are to envisage transcendental idealism as a promising alternative to the stalemate over scientific realism, that controversy has to be viewed from a quite unaccustomed standpoint where the salient issue is posed by the transcendental question: *How is mathematical natural science possible*? In Husserl's re-phrasing, this is to ask: *How are we to understand "the posit of reality" made in mathematical natural science? What* ***can*** *mathematical natural science mean*? This is to inquire as to the *possible* sense and meaning of scientific assertions regarding *a nature abstractly conceived as without any relation to consciousness*, i.e., as mind-independent. The crucial step, of course, lies in the infamous "phenomenological reduction" that "*suspends" or "places out of action"* the existential force of the assertions of our best scientific theories. Thus, it is necessary to leave the *natural attitude* of everyday life and the practice of science where realist assertions do have their ordinary, unquestioned and *perfectly acceptable* meanings.[7] Having done this, the sense and justification of the "posits of reality" made within the special sciences is then to be disclosed by a decidedly non-naturalistic intentional-analytic investigation, whose aim is to show that the posited ideal objects of theories of mathematical natural science are constituted in acts of *sense bestowal*. In short, for Husserl, the only legitimate source of meaning of any object of cognition, i.e., any object of an asserted proposition, also in mathematics and physics, is *experience*, broadly construed as *what is actually or potentially present to consciousness*, and in particular, within the *immediate experience that is directly given in intentional acts of consciousness*. As pertains to cognition of nature, this postulate means that mathematically characterized entities of

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[7] Fine (1984) has urged that science be "left alone", that we should take its theoretical pronouncements as they are, without the metaphysical lard of realism or the epistemological embellishment of antirealism. As with Fine's "Natural Ontological Attitude" (the term, of course, derives from Husserl) we recognize and respect the "natural attitude" of working science and its pronouncements regarding the furniture of the unobserved world, and indeed, as laymen, we take them on faith. Husserl stated this position very clearly, while also pointing out its *limitations*:
"No reasonable person will doubt the objective truth or the objectively grounded probability of the wonderful theories of mathematics and the natural sciences. Here there is, by and large, no room for private 'opinions', 'notions', or 'points of view'". Husserl (1910), 290, Engl. trans. 74
"When it is actually natural science that speaks, we listen gladly and as disciples. But it is not always natural science that speaks when natural scientists are speaking…." Husserl (1913), § 20

physical theory are, *according to their sense*, never objects with which "consciousness has nothing to do"; to consider them such is not only misleading but countersense (*Widersinn*) – an absurdity. Accordingly, the aim of transcendental phenomenological inquiry, as pertains to the claims of theoretical natural science, is to exhibit the sense of the objective world of physics as that of "being for a consciousness", i.e., a world constituted from what is "given to consciousness". Here is the program in compressed and schematic outline.

> The actual world (*wirkliche Welt*), (that is) each of its components, and all their determinations, is, and can only be, given as intentional objects of conscious acts. Conscious experiences that I have are absolutely given – just as I have them …. The immanent is absolute, that is, it is exactly what it is as I have it and can eventually bring its essence (*Wesen*) to givenness (*Gegebenheit*) before me in acts of reflection. … The given to consciousness (*Bewußtseins-Gegebene*) is the starting point in which we must place ourselves in order to comprehend the sense and the justification of the posit of reality (*Wirklichkeitsetzung*). … 'Pure consciousness' is the seat of the philosophical *a priori*.

It usually comes as a surprise to learn that this avowal of transcendental phenomenological method is not by Husserl, but by the mathematician Hermann Weyl, from the "Introduction" to his classic book on general relativity, *Raum-Zeit-Materie*, first published in Weyl (1918b), and then again in quickly successive editions, culminating in the fifth edition of 1923. Unbeknownst to nearly all contemporary philosophers, relativity physicists, and even historians of science, Weyl actually carried out a rich and detailed development of Husserl's schematic answer to the problem of naturalism and scientific realism in his writings on general relativity in the period 1918–1923.

2 First Philosophy (Husserl)

To set the stage for Weyl's achievement, let us first recall the fundamental *thesis of transcendental phenomenological idealism*, as stated at the end of § 49 of Husserl's *Ideen I* (1913).

> The whole spatiotemporal world, which includes man himself and the human Ego as subordinate single realities is, according to its sense, a merely intentional being, thus one having the merely secondary, relative sense of a being for a consciousness. It is a being consciousness in its experience posits that, in principle, is only determined and intuited as something identical by motivated manifolds of phenomena: beyond that it is nothing.

According to Husserl, the sense of the "objectifying" formalisms of theoretical physics, as indeed of logic and pure mathematics, are only to be understood as constituted within transcendental subjectivity, the domain of pure consciousness, with its intentional analytic structure. To show this is to come to a *rigorous* or *presuppositionless* knowledge of the world, the kind of knowledge that is attained only in philosophical, and in particular, in phenomenological reflection. A representative statement of that task is given in Husserl's last work, *The Crisis of the European Sciences and Transcendental Phenomenology*.

> Knowing the world in a seriously scientific way, "philosophically," can have meaning and be possible only if a method can be devised of *constructing*, systematically and in a sense in advance, the world, the infinitude of causalities, starting from the meager supply of what can be established only relatively in direct experience, and of compellingly *verifying* this construction in spite of the infinitude [of experience]. How is this thinkable?[8]

Husserl's answer in the *Crisis* to the latter question is that an "archaeological method" is required, an inquiry directed to uncovering in the accreted layers of formalism in the exact sciences of mathematics and physics, the successive "sedimentations" internally and autonomously generated by algebraic and other formal techniques, primitive symbols whose meanings are ultimately found in what has been directly "given to consciousness".

Is this not to ask the impossible? Obviously, no archaeological excavation can reach to the *experience* of any particular cognizing subject, let alone to the level of "transcendental solipsism", the domain of truths that hold exclusively "for me" the residual ego remaining on carrying out the "transcendental-phenomenological reduction" with its universal suspension regarding all worldly affairs that are given beforehand.

But it need not do so. For

> *any straightforwardly (geradehin) constituted objectivity*… in its ***essential manner*** *(Wesensart) points back … to a correlative* **essential form** of the manifold actual and possible … intentionality *…, which is constitutive for that objectivity.* … In the same fashion, the modes of consciousness that can make one aware of some ideal objectivity or other, and can become united as a synthetic consciousness of it, have a definite style, essential to *this* sort of objectivity.[9]

In so many words, archeological method can reach the intentional accomplishments underlying the *ideal objectivities of logic and mathematics* precisely because these objectivities are founded in the ideal or absolute being of eidetic necessity and independence from all factual contingencies that only "pure consciousness" can confer. The lawfully structured conditions of possible human cognition arise within this ideal being through the directed *essential analysis* of intending acts and their ideal objects. Among the most primitive of these would be the guiding idea of finitism, of the intuitability of finite sequences of symbols and the iterability of basic operations on them. Another would be, as we shall see, the directly evidenced relation of *congruence* (involving the superposition of figures) and of immediate comparison, involving direct discrimination of two immediately adjacent simple figures, including, in particular, directed magnitudes, or vectors. The evidence for each of these relations is *immediately given to me, but is nothing particular to me*. For this reason, these relations can be regarded as immediately but essentially perceived by a transcendental subject.

As is well known, in the *Crisis*, Husserl himself sought to disclose the "origin of geometry" in just this way. What is largely unknown is that Weyl himself,

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[8] Husserl (1962), pp. 29–30; Engl. trans., p. 32.

[9] Husserl (1929), p. 253; Engl. trans., p. 246.

already in the period 1918–1923, constructed such an archaeological understanding of the "sense and the justification of the posit of reality" made within Einstein's theory of general relativity – a theory dating only from late 1915 – by recasting that theory within the new framework of a "purely infinitesimal geometry". This is a geometry whose objects are built up from a basis of primitive operations directly and immediately evident within a phenomenological intuition localized to the here-and-now. Expressly carrying out this construction, Weyl's *epistemological* aim was to showcase *how* the physical world of general relativity, the framework for the tensor fields of gravitation and of electromagnetism, could be built up from a basis that is "given to consciousness". In this way, these formalisms acquire their objectifying sense as pertaining to physical objects, from the "given to consciousness", from what is ultimately immediately given in phenomenological intuition. Weyl would demonstrate that these objects, conceived, and rightly conceived with the natural attitude of theoretical practice as descriptive of a mind-independent nature, nonetheless have, *within phenomenological reflection*, the "sense of a merely intentional being", of a "being for a consciousness". As we have seen, this is precisely what Weyl himself stated in the "Introduction" to *Raum-Zeit-Materie* (1918). Moreover, as Weyl will show, phenomenological reflection upon the primitive operations underlying the fundamental geometric object of Einstein's theory, the metric tensor, reveals that it retains an unnecessary residue contrary to field theory's prohibition of "action-at-a-distance", removal of which adds new degrees of freedom to its initial characterization (it becomes only conformally invariant) and leads to the requirement that the objects of the theory must not only be generally covariant but also "gauge invariant", i.e., invariant with respect to freedom to locally chose different units of scale (magnitude).

3 First Philosophy (Weyl)

For his geometrical construction, Weyl restricted the homogeneous space of phenomenological intuition, the locus of phenomenological *Evidenz*, to what is given at, or neighboring, the experiencing ego.

> Only the spatio-temporally coinciding and the immediate spatial-temporal neighborhood have a directly clear meaning exhibited in intuition. […] The philosophers may have been correct that our space of intuition bears a Euclidean structure, regardless of what physical experience says. I only insist … that the ego-center (*Ich Zentrum*) belongs to this space of intuition, and that the coincidences, the relations of the space of intuition to that of physics, becomes vaguer the further one distances oneself from the ego-center.[10]

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[10] Weyl (1931), p. 49, 52. In this paper, his W. Rouse Ball lecture at Oxford, Weyl reminisced about his work in relativity theory in the period 1917–1923. In the cited passage, Weyl alludes to the fact that the tangent space at a point P of a Riemannian manifold M is a vector space associated with P, not a part of M itself. However, there is always a neighborhood of any vector in the tangent space to P that can be mapped diffeomorphically onto an open neighborhood of P in M, the so-called exponential mapping, traceable back to Riemann.

By delimiting what Husserl termed "the sharply illuminated circle of perfect givenness", the domain of "eidetic vision", to the infinitely small homogeneous space of intuition surrounding the "ego-centre", Weyl could limit his attention to linear relations, since only these need be considered in passing to the tangent space of a point in a manifold. Linearity, in turn, gave the expectation of "uniform elementary laws".[11] Thus Weyl initially restricted the concept of a coordinate system to the tangent space centering on each manifold point *P*, essentially assuming a four-dimensional manifold that is Hausdorff, simply connected, and differentiable. Imposition of this local coordinate system is regarded as the original constitutive act of "a pure, sense-giving ego". A necessary presupposition of any differential structure, a coordinate system always bears an indelible mark of transcendental subjectivity, it is "the unavoidable residue of the ego's annihilation in that geometrico-physical world which reason sifts from the given under the norm of 'objectivity'".[12] In this, Weyl recognized an intimation of the phenomenological postulate that "existence is only given and *can* only be given as the intentional content of the conscious experience of a pure, sense-giving ego".[13]

The next steps concern the immediately evident "purely infinitesimal" relations of comparison of direction and magnitude that depend on a specific choice of coordinates and unit of scale. The construction of purely infinitesimal geometry is laid out as taking place in three distinct stages of "connection": topological manifold or "continuous connection" (*stetiger Zusammenhang*), affine connection, and "metric (or, length) connection". The construction itself, "in which each step is executed in full naturalness, visualizability and necessity" (*in voller Natürlichkeit, Anschaulichkeit, und Notwendigkeit*), is "in all essential parts the final result" of the renewed investigation of the mathematical foundations of Riemannian geometry opened up by Levi–Civita's discovery of the concept of infinitesimal parallel displacement.[14] The physical world is then to be distinguished within this "world geometry" through the univocal choice of a gauge invariant action function $S(g_{\mu\nu},\varphi_{\mu})$, where $g_{\mu\nu}$ is the (only conformally invariant) metric tensor and φ_{μ} is the electromagnetic four potential.[15] However, to Weyl's dismay, it soon became apparent that, despite the restrictive condition of gauge invariance, a number of such functions could be constructed, choice among them being essentially arbitrary.[16]

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[11] Weyl (1926), p. 61; Engl. trans., p. 86.

[12] Weyl (1918a), p. 72; Engl. trans., p. 93: "The coordinate system is the unavoidable residuum of the ego's annihilation (*das unvermeidliche Residuum der Ich-Vernichtung*) in that geometrico-physical world which reason sifts from the given under the norm of 'objectivity' – a final scanty token in this objective sphere that existence (*Dasein*) is only given and *can* only be given as the intentional content of the conscious experience of a pure, sense-giving ego."

[13] Weyl (1918a), p. 72; Engl. trans., p. 94.

[14] "*Vorwort zur dritten Auflage*", in Weyl (1919), p. vi.

[15] Weyl (1918d), p. 385; reprinted in (1968), **2**, p. 2.

[16] See Weitzenböck (1920).

3.1 First Stage: Continuous Connection (Topology)

Weyl's several discussions of topology in the context of his geometry add little to topology *per se* but take over the modern topological concepts of "point" and "neighbourhood", first clarified in his own 1913 book on Riemann surfaces. There is a clearly identified reason for his reticence in extending phenomenological constitution to the manifold, and so to the concept of "continuous connection" itself. With reference to his discussion in *Das Kontinuum* of the "deep chasm" separating the intuitive and the mathematical continuum, Weyl observed that a "fully satisfactory analysis of the concept of the *n-dimensional manifold* is not possible today" in view of the "difficulty of grasping the intuitive essence (*anschauliche Wesen*) of continuous connection through a purely logical construction".[17] Setting that task aside, Weyl simply assumed that in the tangent space covering each manifold point P, there is an affine linear space of vectors centered on P in that line elements dx radiating from P are infinitely small vectors. In this way, functions at P and in its neighborhood (in particular, the displacement functions – see below) transform linearly and homogeneously. Weyl's attention then concentrated on the manifold's *Strukturfeld*, its metric, affine (and conformal, and projective) structures, originating the now familiar machinery of connections in a specifically philosophical context.

3.2 Second Stage: Affine Connected Manifold

The concept of parallel transport of a tangent vector in a Riemannian manifold M was first developed in 1917 by Levi–Civita (and independently by Schouten in 1918). It provided a geometric interpretation – as the parallel displacement of a vector along a path connecting a point P to another point P' in the infinitesimal neighbourhood (tangent space T_P) of P ($T_{P'}M = T_PM$) – to the hitherto purely analytical Christoffel symbols (of the second kind) of covariant differentiation. This enabled covariant differentiation to be understood as a means of comparing infinitesimal changes in vector or tensor fields with respect to a parallel transported vector or tensor at the point in question. Parallel transport is purely infinitesimal in the sense that directional comparison of vectors at finitely distant points P and Q can be made only by specifying a path of displacement from P to Q and "transporting" to Q a comparison vector defined as "parallel to" the original vector at P. In general, parallel displacement is not integrable, i.e. the new vector arising at Q will depend upon the path taken between P and Q.

In Weyl's assessment, Levi–Civita's concept marked a significant advance of "simplicity and visualizability (*Anschaulichkeit*) in the construction of Riemannian infinitesimal geometry."[18] But whereas Levi–Civita had employed an auxiliary con-

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[17] Weyl (1918d), p. 386; reprinted in (1968), **2**, p. 3.

[18] Weyl (1923b), p. 11.

struction, embedding *M* in a Euclidean space where parallel transport was defined, and then projecting it into the tangent space of *M*, Weyl gave the first intrinsic characterization in terms of bilinear functions $\Gamma(A^\mu, dx)$ since known as the components of a (symmetrical) affine connection.[19] In general, the change δA^μ in a given vector A^μ displaced from *P* to $P'_{(x^\nu + dx^\nu)}$ is defined

$$\delta A^\mu = -\Gamma^\mu_{\alpha\beta} A^\alpha dx^\beta, \tag{1}$$

while the *covariant derivative* of $A^\propto$ (a tensor, and so of objective significance) is defined

$$A^\mu{}_{;\alpha} = \frac{\partial A^\mu}{\partial x^\alpha} + \Gamma^\mu_{\beta\alpha} A^\beta. \tag{2}$$

Parallel transport occurs when the components of the affine connection vanish. Next followed the concept of a manifold with an affine connection. A point *P* is affinely connected with its immediate neighborhood just in case it is determined, for every vector at *P*, the vector at *P′* to which it gives rise under parallel transport from *P* to *P′*. If it is possible to single out a unique affine connection, among all the possible ones at each point *P*, then *M* is called a *manifold with an affine connection*. This is essentially a conception of space as stitched together in linear fashion from infinitely small homogeneous patches. To Weyl, parallel transport was the paradigm comparison relation of infinitesimal geometry for it satisfied the epistemological demand that all integral (and so, not immediately surveyable) relations between finitely separated points cannot be posited but must be constructed from a specified infinitesimal displacement along a given curve connecting them. He also introduced the idea of the curvature of a connection *R*(Γ), a (1, 3) tensor analogous to the Riemann–Christoffel tensor of Riemannian geometry, and showed that the calculus of tensors could be developed on the basis of the concept of infinitesimal parallel transport, without any reliance on a metric.[20] However, it was Eddington, not Weyl, who first fully exploited this idea in physics.[21]

The "essence of parallel displacement" (*das Wesen der Parallelverschiebung*) is expressed in that, in a given coordinate system covering *P* and its neighborhood, the components of an arbitrary vector A^μ do not change when A^μ is parallel-displaced from *P* to a neighboring point *P′*.[22] Unaltered displacement accordingly depends on a particular "geodetic" (at *P*) coordinate system, proleptically referring to the fact that at *P* the $g_{\mu\nu}$ have stationary values, $\frac{\partial g_{\mu\nu}}{\partial x^\sigma} = 0$, and so the components of the affine connection vanish. According to the principle of equivalence, such geodetic coordinates at a point always exist. In this dependence on a particular coordinate system, parallel displacement of a vector or tensor without "absolute change" is not an invariant or "objective" relation. But a specifically epistemological

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[19] Following Cartan, such a connection is now denominated "without torsion".

[20] Weyl (1923b), p. 17. A metric tensor is needed only to raise or lower indices.

[21] Eddington (1923); for discussion, see Ryckman (2005), Chapter 8.

[22] Weyl (1923a), p. 113, (1921c), p. 542, reprinted in (1968), **2**, p. 238. See also Scholz (1994).

and non-conventional meaning is intended for the statement that some vector at P' is "the same" as a given vector at P. Namely from the original vector at P, a new vector arises at P' that, in the purely local comparison made, as it were, by a particularly situated consciousness, is affirmed to be "without change". Despite the subjectivity of the "experienced" condition $\frac{\partial g_{\mu\nu}}{\partial x^{\sigma}} = 0$, required by this construction, such comparison is nonetheless the basis for the invariant relation of covariant differentiation. Obviously, the idea is an analogy formed from Einstein's theory in which the non-tensorial gravitational field strengths $\Gamma^{\sigma}_{\mu\nu}$ (in Weyl's suggestive terminology, the "guiding field" (*Führungsfeld*)) can be locally, but not generally, "transformed away", an observer-dependent "disappearance" of a gravitational field. At the same time, invariant space–time curvatures are derived from the $\Gamma^{\sigma}_{\mu\nu}$ that have an objective significance for all observers.

3.3 Third Stage: Metrically Connected Manifold

In Weyl's estimation:

> [A] truly infinitesimal geometry (wahrhafte Nahegeometrie) should know only a principle of displacement (Übertragung) of a length from one point to another infinitely close by.[23]

As the "essence of space" is metric, the fundamental metrical concept, congruence, also must be conceived "purely infinitesimally".[24] Enshrined as "the epistemological principle of relativity of magnitude", a postulate is laid down that direct comparison of vector magnitudes can be immediately made only at a given point P or at infinitesimally nearby points $P'(P' - P = \overrightarrow{P'P} \in T(M_p))$. Just as an affine connection governs direct infinitesimal comparisons of orientation, or parallelism, so a *length* or *metric connection* is required to determine infinitesimal comparisons of congruence. This also requires a vector to be displaced from $P$to P' and, in general, the (square of the) "length" l of the vector is altered. Thus if l is the (squared) length of a vector A^{μ} at $P_{(x)}$, $l_{P(x)}(A^{\mu}) = ds^2 = g_{\mu\nu}A^{\mu}A^{\nu}$, then on being displaced to P', the change of length is defined to be a definite fraction of l,

$$\frac{dl}{dx^{\mu}} := -l\frac{d\varphi}{dx^{\mu}}, \tag{3}$$

where $d\varphi = \sum_{\mu} \varphi_{\mu} dx^{\mu}$ is a homogeneous function of the coordinate differentials. The new vector at P', corresponding to A^{μ} at P, accordingly has the length

$$l_{P'_{(x+dx)}} = (1 - d\varphi)(g_{\mu\nu} + d\,g_{\mu\nu})A^{\mu}A^{\nu}, \tag{4}$$

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[23] Weyl (1918c), p. 466; reprinted in (1968), **2**, p. 30. Emphasis in original.

[24] Weyl (1923b), p. 47.

where (1 – $d\varphi$) is a proportionality factor, arbitrarily close to 1. In analogy to Eq. (1), the change in length of A^{μ} is defined as

$$\delta l := \frac{\partial l_P}{\partial x^{\mu}} dx^{\mu} + l\, d\varphi. \tag{5}$$

Then, just as the vanishing of its covariant derivative means that a vector has been parallel-displaced from P to P' without “absolute change”, so here the vanishing of δl indicates that A^{μ} has been *congruently displaced* from P to P':

$$\delta l = 0 \Leftrightarrow dl = \frac{\partial l}{\partial x^{\mu}} dx^{\mu} = -l\, d\varphi\ . \tag{6}$$

Up to this point, an arbitrary “gauge” (unit of scale) has been assumed. Re-calibrating the unit of length at P through multiplication by λ, an always positive function of the coordinates, multiplies the length $l_{P(x)}$ by λ, $l' = \lambda\, l$. Then the change in length at P', dl', corresponds to a transformation of the “length connection” $d\varphi$,

$$\begin{aligned} dl' &= d(\lambda l) = l\, d\lambda + \lambda dl = l\, d\lambda - \lambda l\, d\varphi \\ &= -\lambda l (d\varphi - \frac{d\lambda}{\lambda}) = -l' d\varphi', \end{aligned} \tag{7}$$

($d\lambda / \lambda = d \log \lambda$). A *metrically connected manifold* is then one in which each point P is metrically connected to every point P' is its immediate neighborhood through a *metric connection*. In general, length is not integrable for Eq. (8) follows from Eq. (5) by integration,

$$\log l|_P^Q = -\int_P^Q \varphi_{\mu}\, dx^{\mu}, \text{and so } l_Q = l_P^{\,-\int_P^Q \varphi_{\mu} dx^{\mu}}\ . \tag{8}$$

As Pauli demonstrated, displacement of a vector along different paths between finitely separated points P and Q will lead to arbitrarily different results at Q.[25] But when the linear form φ_{μ} vanishes, the magnitude of a vector is independent of the path along which it is displaced, which is just the case of Riemannian geometry. The necessary and sufficient condition for this is the disappearance of the “length curvature” (*Streckenkrümmung*) of Weyl’s geometry

$$F_{\mu\nu} = \frac{\partial \varphi_{\nu}}{\partial x^{\mu}} - \frac{\partial \varphi_{\mu}}{\partial x^{\nu}}, \tag{9}$$

just as the vanishing of the Riemann tensor is the necessary and sufficient condition for flat space.

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[25] Pauli (1921); Eng. trans., pp. 195–156.

Implementation of the local comparison condition means that the fundamental tensor $g_{\mu\nu}$ of Riemannian geometry induces only a local conformal structure on the manifold. There is then an immediate meaning given to the angle between two vectors at a point, or to the ratio of their lengths there, but not to their absolute lengths. These transform at a point$x$ as $g'_{\mu\nu}(x) = \lambda\, g_{\mu\nu}(x)$. This weakening of the metrical structure has two important ramifications. Such a metric no longer determines a unique linear (affine) connection, but only an equivalence class of connections. Yet Weyl required, as the "fundamental fact" of infinitesimal geometry, that there be unique affine compatibility in the sense that the transport of tangent vectors along curves associated with the connection, i.e. affine geodesics, leave the vectors congruent with themselves with respect to the metric. Weyl showed that a unique connection, coupled to given choice of a metric tensor, is found by incorporating into its definition the linear differential form φ of his length connection. Then, when the components of φ vanish identically at a point, the connection becomes identical to the "Levi–Civita" connection, as can be seen from comparison of the definitions of the two connections in components:

$$\text{Levi-Civita: } \Gamma^{\sigma}_{\mu\nu} = \frac{1}{2} g^{\sigma\tau}\left[\frac{\partial g_{\tau\mu}}{\partial x^{\nu}} + \frac{\partial g_{\nu\tau}}{\partial x^{\mu}} - \frac{\partial g_{\mu\nu}}{\partial x^{\tau}}\right]; \tag{10}$$

$$\text{Weyl: } \Gamma^{\sigma}_{\mu\nu} = \frac{1}{2} g^{\sigma\tau}\left[\frac{\partial g_{\tau\mu}}{\partial x^{\nu}} + \frac{\partial g_{\nu\tau}}{\partial x^{\mu}} - \frac{\partial g_{\mu\nu}}{\partial x^{\tau}}\right] + \frac{1}{2}\left(g_{\mu\sigma}\varphi^{\nu} + g_{\nu\sigma}\varphi^{\mu} + g_{\mu\nu}\varphi^{\sigma}\right). \tag{11}$$

Given this "Weyl connection", it is possible to speak of a manifold with an affine connection where, as in the Riemannian case, there is a unique determination of parallel displacement of a vector at every point. Only in the case of "congruent displacement" (*kongruente Verpflanzung*), or displacement without alteration of length, is parallel displacement possible, and so it is that infinitesimal length or "tract displacement" (*Streckenübertragung*), the "foundational principle of metric geometry", brings along also directional displacement (*Richtungsübertragung*). This is to say, according to Weyl, that *"according to its nature, a metric space bears an affine connection*."[26]

Justification for his "essential analysis" of infinitesimal geometry culminated in Weyl's purely mathematical group-theoretical proof of Riemann's posit of an "infinitesimal Pythagorean (Euclidean) metric", the capstone of his efforts to show that the supposition of the purely infinitesimal character of the geometry underlying field physics was not arbitrary.[27] Writing to Husserl on 26 March 1921, Weyl could report that he had finally captured the "*a priori* essence of space (*apriorische Wesen*

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[26] Weyl (1923a), p. 124: "*ein metrischer Raum tragt von Natur einen affinen Zusammenhang*". Laugwitz (1958) proved this conjecture, showing that this condition singles out infinitesimal Euclidean metrics from the wider class of Finsler metrics.

[27] Weyl (1921b), p. 497; reprinted in (1968), **2**, p. 235; the full group-theoretic proof appears in Weyl (1923b).

des Raumes), through a notable deepening of its mathematical foundations (*eine merkliche Tieferlegung der Fundamente*).”[28] This philosophical linkage was publicly announced in a newly appended fourth (1921) edition of *RZM*. There Weyl declared that his “investigations concerning space … appear to me to be a good example of the essential analysis (*Wesenanalyse*) striven for by phenomenological philosophy (Husserl)”.[29] Referring to this passage some 30 years later, Weyl observed that he still essentially held to its implicit characterization of the relation between cognition and reflection underlying his method of investigation, one that combined experimentally supported experience, analysis of essence (*Wesensanalyse*) and mathematical construction.[30]

3.4 Transition to Physics

Just as Einstein required the invariance of physical laws under arbitrary continuous transformation of the coordinates (general covariance), Weyl additionally demanded their invariance under the “gauge transformations”

$$g \Rightarrow g'_{\mu\nu} = \lambda\, g_{\mu\nu}, \text{and}\, \varphi \Rightarrow \varphi'_{\mu} = \varphi_{\mu} - \frac{1}{\lambda}\frac{\partial \lambda}{\partial x^{\mu}}. \tag{12}$$

And since the first system of Maxwell’s equations

$$\frac{\partial F_{\mu\nu}}{\partial x^{\sigma}} + \frac{\partial F_{\nu\sigma}}{\partial x^{\mu}} + \frac{\partial F_{\sigma\mu}}{\partial x^{\nu}} = 0 \tag{13}$$

follows immediately from Eq. (9) on purely formal grounds, Weyl made the obvious identifications of his length curvature $F_{\mu\nu}$ with the already gauge-invariant electromagnetic field tensor (of “gauge weight 0”), and his metric connection φ_{μ}

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[28] Schuhmann and Schuhmann (1994), p. 291.

[29] Weyl (1921a), p. 133; cf. Engl. trans. (1953), p. 148: “The investigations made concerning space in chapter two appear to me to be a good example of the essential analysis (*Wesenanalyse*) striven for by phenomenological philosophy (Husserl), an example that is typical for such cases where a non-immanent essence is dealt with. We see in the historical development of the problem of space, how difficult it is for us reality-prejudiced humans to hit upon what is decisive. A long mathematical development, the great unfolding of geometrical studies from Euclid to Riemann, the physical exploration of nature and its laws since Galileo, together with all its incessant boosts from empirical data, finally, the genius of singularly great minds – Newton, Gauss, Riemann, Einstein – all were required to tear us loose from the accidental, non-essential characteristics to which we at first remain captive. Certainly, once the true standpoint has been attained, Reason (*Vernunft*) is flooded with light, recognizing and accepting what is understandable out-of-itself (*das ihr aus-sich-selbst Verständliche*)”. “The example of space”, Weyl continued, “is most instructive for that question of phenomenology that seems to me particularly decisive: to what extent the delimitation of the essentialities (*Wesenheiten*) rising up to consciousness express a characteristic structure of the domain of the given itself and to what extent mere convention participates in it.”

[30] Weyl (1955), p. 161.

with the space–time four potential. As a mathematical consequence of his geometry, Eq. (13) are held to express "the essence of electricity"; they are an "essential law" (*Wesensgesetze*) whose validity is completely independent of the actual laws of nature.[31] Furthermore, Weyl could show that a vector density and contravariant second rank tensor density follow from the *general* form of a hypothetical action function invariant under local changes of gauge $\lambda = 1 + \pi$, where π is an arbitrarily specified infinitesimal scalar field. These are respectively identified with the four current density $\mathbf{j}^{\mu}$ and the electromagnetic field density $\mathbf{h}^{\mu\nu}$, through the relation

$$\frac{\partial \mathbf{h}^{\mu\nu}}{\partial x^{\nu}} = \mathbf{j}^{\mu}, \tag{14}$$

i.e. the second system of Maxwell equations. Thus Weyl claimed that, without having to specify a particular action function, "the entire structure of the Maxwell theory could be read off of gauge invariance".[32] Again, using only the general form of such a function, he demonstrated that conservation of energy-momentum and of charge follow from the field laws in two *distinct* ways.[33] Accordingly he asserted that, just as the Einstein theory had shown that the agreement of inertial and gravitational mass was "essentially necessary" (*wesensnotwendig*), his theory did so in regard to the facts finding expression in the structure of the Maxwell equations, and in the conservation laws. This appeared to him to be "an extraordinarily strong support" for the "hypothesis of the essence of electricity" (*Wesen der Elektrizität*).[34] The domain of validity of Einstein's theory of gravitation, with its assumption of a global unit of scale, was originally held to correspond to $F_{\mu} = 0$, the vanishing of the electromagnetic field tensor. By 1919, Weyl substituted his own "dynamical" account of the origin of "the natural gauge of the world" noted above. These details of Weyl's theory will suffice for present purposes; further discussion is available elsewhere.[35]

As just seen, guided by phenomenological reflection within a now localized space of phenomenological intuition, Weyl's "purely infinitesimal" geometrical framework for field physics rests upon two evidentially privileged geometrical relations of comparison: *parallel displacement of a vector* from one point to an immediately adjacent point, and modeled on this, *congruent displacement of a vector magnitude* from point to neighboring point. It is the latter relation that enables choice of a unit of scale at each point, and so prohibits "comparisons at a distance" as in Einstein's' pseudo-Riemannian general relativity. Weyl's geometry was therefore a non-Riemannian geometry constructed through a phenomenological "essential analysis" of the two basic comparison relations of vectors and tensors. But it yielded a new geometrical concept, that of a *connection*, that would come to serve as the new axiomatic basis for differential geometry, and indeed, for general relativity

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[31] Weyl (1919), p. 244.

[32] Weyl (1919), p. 251.

[33] Weyl (1919), p. 251–252; (1923a), p. 314–315; for discussion see Brading (2002).

[34] Weyl (1919), p. 253.

[35] See Ryckman (2005), Chapters 4–6.

itself. By mandating that the consistency and coherence of all relations field physics permits between magnitudes at *P* and *Q*, points at *finite separation*, arise through integrating *a next-to-next relation of comparison along a specific path connecting all points between P* and *Q*, Weyl’s geometry satisfied *by construction* both field theory’s prohibition against “action at a distance” as well as the requirement of phenomenological *Evidenz*, that these relations of comparison be *immediately inspectable*. As a reconstructive epistemological project, it coincided, as Weyl himself noted, with the explicitly metaphysical aspirations of Leibniz and Riemann to “understand the world from its behavior in the infinitesimally small”.[36]

It is widely, but wrongly, believed that Einstein showed that Weyl’s theory of “gravitation and electromagnetism” was incompatible with the data of observation, in particular, with the sharp spectral lines of the chemical elements. In fact, Weyl made a sustained and detailed response that turned the tables on Einstein, centering on the inconsistency of the independent postulate of rigid rods and perfect clocks that Einstein supposed necessary to link his gravitational theory to experience. Of course, such a posit physically manifests the comparisons of lengths and times “at a distance” that is inconsistent with the spirit of field theory. At the same time, Weyl showed how his theory offered an in-principle dynamical account of the observed behavior of rods and clocks, remarking that his theory, but not Einstein’s, provided the possibility of accounting for this “natural gauge of the world”. In the fifth (1923) edition of *Raum-Zeit-Materie*, the last to appear in his lifetime, the theory was defended not so much as a physical hypothesis, but as “a theoretically very satisfying amalgamation and interpretation of our whole knowledge of field physics.”[37]

Weyl would abandon his world geometry only in 1928, when he convinced himself that the “epistemological principle of relativity of length” was not valid in the realm of atomic physics. Even so, he retained his principle of “gauge invariance” by reinterpreting gauge as pertaining not to a space–time factor of scale, but where the local gauge transformations are imaginary phase transformations of the complex-valued quantum matter field in the setting of Dirac’s relativistic theory of the electron in 1929. Weyl derived the Maxwell equations from the requirement of local phase invariance, thus coupling charged matter to the electromagnetic field, and so originating the modern understanding of the principle of local gauge invariance (“*local symmetries dictate the form of the interaction*”) that lies at the basis of contemporary geometrical unification programs in fundamental physics.[38] In sum, the contemporary preference in field theory for theories that are “gauge invariant”, i.e., the viewpoint that local symmetries are the hallmark of physical objectivity, is the final fruit of Weyl’s quest in 1918 to uncover the transcendental subjectivity underlying the sedimented tensor formalism of general relativity.

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[36] Weyl (1926), p. 61; Engl. trans, p. 86.

[37] Weyl (1923a), p. 308.

[38] Weyl (1929). Weyl’s argument for his correct conclusion is, in fact, flawed, resting on an unnecessary assumption about the representation of spinor matter fields within tetrad formulations of arbitrary curved space–times.

4 Conclusion

Despite the unearned hegemony of naturalism in philosophy of science, the promise of a non-naturalistic and transcendental alternative to scientific realism and all its ilk is not at all an atavistic illusion, but can be extracted from a close examination of how Hermann Weyl, seeking such an understanding of general relativity, arrived at the postulate that our fundamental theories, our field theories, satisfy an *a priori* constraint of reasonableness, the requirement of local gauge freedom, or local symmetry. Although Weyl arrived at this conclusion by coupling mathematical construction with the difficult nexus of transcendental phenomenological idealism – or First Philosophy – his example is and remains a canonical demonstration of how and why *a priori* constraints of reasonableness can govern our best theories of nature, without the proud presumption of scientific realism that these constraints are inherent in nature itself. In a word, this is the program of a transcendental philosophy of science.[39]

References

K. Brading (2002). "Which Symmetry? Noether, Weyl, and the Conservation of Charge", *Studies in History and Philosophy of Modern Physics*, **33**, 3–22.

K. Brading and T. Ryckman (2008). "Hilbert's 'Foundations of Physics': Gravitation and Electromagnetism within the Axiomatic Method", Studies in History and Philosophy of Modern Physics, 39, 102–153.

E. Cassirer (1921). *Zur Einsteinschen Relativitätstheorie: Erkenntnistheoretische Betrachtungen*. Berlin: Bruno Cassirer. Pagination as reprinted in Cassirer, 1957, 1–125. Translated by W. Swabey and M. Swabey as *Substance and Function and Einstein's Theory of Relativity*, 1923. Reprint edition, New York: Dover, 1953, 347–460.

A. Eddington (1923). *The Mathematical Theory of Relativity*. Cambridge: Cambridge University Press.

A. Fine (1984). "The Natural Ontological Attitude", in J. Leplin (ed.), *Scientific Realism*. Berkeley/Los Angeles, CA: University of California Press, 83–107.

M. Friedman (2001). *Dynamics of Reason: The 1999 Kant Lectures at Stanford University*. Stanford, CA: Center for the Study of Language and Information Publications.

E. Husserl (1910). *"Philosophie als strenge Wissenschaft," Logos*, I, 1910–1911, 289–341. Translated by Q. Lauer as "Philosophy as Rigorous Science," in E. Husserl (ed.), *Phenomenology and the Crisis of Philosophy*. New York: Harper & Row, 1965.

E. Husserl (1913). "*Ideen zu einer reinen Phänomenologie und phänomenologischen Philosophie. Erstes Buch*". *Jahrbuch für Philosophie und phänomenologische Forschung*, Bd. I. Augmented text with latter insertions edited by W. Biemel, published in *Husserliana* v. 3. The Hague: M. Nijhoff, 1950. Translated by F. Kersten as *Ideas Pertaining to a Pure Phenomenology and to a Phenomenological Philosophy*. *First Book*. The Hague: M. Nijhoff, 1983.

E. Husserl (1929). *"Formale und transzendentale Logik: Versuch einer Kritik der logischen Vernunft", Jahrbuch für Philosophie und phänomenologische Forschung* **10**, 1–298. Translated by Dorion Cairns as *Formal and Transcendental Logic*. The Hague, The Netherlands: Martinus Nijhoff, 1969.

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[39] For further details, see Ryckman (2005).

E. Husserl (1962). *Die Krisis der europäischen Wissenschaften und die tranzendentale Phänomenologie. Eine Einleitung in die phänomenologische Philosophie*. W. Biemel (hrsg). 2. Auflage. Haag, Martins Nijhoff, 29–30. Parts originally published in Philosophia, **1**(1936). Translated by D. Carr as *The Crisis of European Sciences and Transcendental Phenomenology*. Evanston, IL: Northwestern University Press, 1970.

D. Laugwitz (1958). *"Über eine Vermutung von Hermann Weyl zum Raumproblem", Archiv der Mathematik*, **9**, 128–33.

P. Maddy (2000). "Naturalism and the *A Priori*", in P. Bogohassian and C. Peacocke (eds.), *New Essays on the A Priori*. New York: Oxford University Press, 92–116.

W. Pauli, Jr. (1921). *Relativitätstheorie*, in *Encyklopädie der mathematischen Wissenschaften*, **19**. Leipzig, Germany: B. G. Teubner. Translated as *The Theory of Relativity*. New York: Pergamon, 1958.

W. Quine (1981), *Theories and Things*. Cambridge, MA: Harvard University Press.

M. Redhead (1995). *From Physics to Metaphysics*. Cambridge: Cambridge University Press.

T. Ryckman (2005). *The Reign of Relativity: Philosophy in Physics 1915–1925*. New York: Oxford University Press.

M. Schlick (1922). *"Die Relativitätstheorie in der Philosophie", Verhandlungen der Gesellschaft Deutscher Naturforscher und Ärzte*. Leipzig, Germany, 58–69.

E. Scholz (1994). "Hermann Weyl's Contribution to Geometry, 1917–1923", in S. Chikara, S. Mitsuo, and J.W. Dauben (eds.), *The Intersection of History and Mathematics*, Basel, Switzerland/Boston, MA/Berlin: Birkhäuser Verlag, 203–30.

K. Schuhmann and E. Schuhmann (eds.) (1994). *Edmund Husserl Briefwechsel*, Vol. 7. Dordrecht, The Netherlands/Boston, MA/London: Kluwer.

A. Shimony (1984). "Contextual Hidden Variables Theories and Bell's Inequalities", *British Journal for the Philosophy of Science*, **35**, 25–45.

M. von Laue (1921). *Die Relativitätstheorie. Bd.II. Die Allgemeine Relativitätstheorie and Einsteins Lehre der Schwerkraft*. Braunschweig, Germany: Vieweg.

R. Weitzenböck (1920). *"Über die Wirkungsfunktion in der Weyl'schen Physik". Akademie der Wissenschaften in Wien. Sitzungsberichte, Abteilung IIa. Mathematisch-naturwissenschaftliche Klasse*, **129**, 683–708.

H. Weyl (1918a). *Das Kontinnum: Kritische Untersuchungen über die Grundlagen der Analysis*. Leipzig, Germany: Veit. Translated by S. Pollard and T. Bowl as *The Continuum: A Critical Examination of the Foundations of Analysis* (1994). New York: Dover.

H. Weyl (1918b). *Raum-Zeit-Materie*. Berlin: J. Springer.

H. Weyl (1918c). *"Gravitation und Elektrizität", Preußische Akademie der Wissenschaften (Berlin) Sitzungsberichte. Physikalisch-Mathematische Klasse*, 465–80. Reprinted in Weyl (1968), **2**, 29–42.

H. Weyl (1918d). *"Reine Infinitesimalgeometrie", Mathematische Zeitschrift*, **2**, 384–411. Reprinted in Weyl (1968), **2**, 1–28.

H. Weyl (1919). *Raum-Zeit-Materie*, 3rd edn. Berlin: J. Springer.

H. Weyl (1921a). *Raum-Zeit-Materie*, 4th edn. Berlin: J. Springer. Translated by H. L. Brose as *Space-Time-Matter*. London: Methuen, 1922; reprinted, Dover: New York, 1953.

H. Weyl (1921b). *"Über die physikalischen Grundlagen der erweiterten Relativitätstheorie". Physikalische Zeitschrift*, **22**, 473–80. Reprinted in Weyl (1968), **2**, 99–112.

H. Weyl (1921c). *"Feld und Materie," Annalen der Physik*, **65**, 541–63. Reprinted in Weyl (1968), **2**, 237–59.

H. Weyl (1923a). *Raum-Zeit-Materie*, 5th edn. Berlin: J. Springer.

H. Weyl (1923b). *Mathematische Analyse des Raumproblems: Vorlesungen gehalten in Barcelona und Madrid*. Berlin: J. Springer.

H. Weyl (1926). *"Philosophie der Mathematik und Naturwissenschaft,"* in A. Baeumler and M. Schröter (eds.), *Handbuch der Philosophie, Abteilung II*. Munich, Germany/Berlin: R. Oldenbourg Verlag. Revised and augmented English edition, Translated by O. Helmer as *Philosophy of Mathematics and Natural Science*. Princeton, NJ: Princeton University Press, 1949.

H. Weyl (1929). “Elektron und Gravitation”. *Zeitschrift für Physik*, **56**, 330–52. Reprinted in Weyl (1968), **3**, 245–67.

H. Weyl (1931). *“Geometrie und Physik”, Die Naturwissenschaften*, **18**, 49–58. Reprinted in Weyl (1968), **3**, 336–45.

H. Weyl (1955). *“Erkenntnis und Besinnung” (Vortrag, gehalten an der Universität Lausanne im Mai 1954). Studia Philosophia* (Jahrbuch der Schwiezerischen Philosophischen Gesellschaft), **15**, 153–71. Reprinted in Weyl (1968), **4**, 631–49.

H. Weyl (1968). *Gesammelte Abhandlungen*. Bd. **1–4**. S. Chandrasekharan (hrsg.). Berlin/Heidelberg, Germany/New York: Springer.

B.2

Quantum Mechanics: History

Old Wine Enriched in New Bottles: Kantian Flavors in Bohr's Viewpoint of Complementarity

Steen Brock

Abstract In this chapter, I discuss in what sense and to which extent Niels Bohr's philosophical interpretation of early Quantum Mechanics can be assessed as answering to some of the core features of Kant's Natural Philosophy, especially the Kantian idea of Schemata and the Kantian account of the relation and unity between kinematics and dynamics. In the later half of the chapter, my discussion focus on Bohr's two notions of "idealization" and "abstraction" in an attempt to explain his understanding of the essential link between theoretical concepts and the design and conduct of experiments. Finally, after giving a brief sketch of Ernst Cassirer's interpretation of early Quantum Mechanics, I explain in what sense the combination of Bohr's and Cassirer's respective interpretations lead to a more elaborate Natural Philosophy than the interpretations of Bohr and Cassirer taken in isolation.

In my opinion, Niels Bohr's understanding of quantum physics is clear and straightforward. The problems he wanted to focus on are clear as are the concepts or ideas he pointed to as central to the themes at stake. So in contrast to most other interpreters of Bohr, I think that *what he says* is clear. But the implications and points he makes may not be as clear. It may not be clear how quantum physics is seen either as a specific branch of physics or a paradigm for other sciences, neither is it clear in what sense the understanding of quantum physics along Bohrian lines relates to philosophical issues. In this chapter I will try to answer these questions. It turns out that the assessment of Bohr's understanding of quantum physics does not make much sense unless one refers to many themes and concepts in Kant's philosophy. This is not to say that Bohr's understanding of physics is "Kantian" nor to say that there is a systematic link between Bohr's thought and Kant's. However, both historical and philosophical links can easily

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be found between Bohr's and Kant's views. As I have argued elsewhere, the (neo)-Kantian features of the philosophy of physics mainly put forward by Helmholtz and Planck left their mark on the general understanding of physics and its possible developments for all leading physicists of Bohr's generation.[1] It would then come as a big surprise if Bohr's understanding of physics did not share many parallels with Kant's.

So what are Bohr's exact thoughts on quantum physics? First, Bohr held the traditional belief that physics was some sort of continuously developing epistemic undertaking, despite the historical fact that the conditions for pursuing this goal change radically as time goes on. Consequently Bohr thought that certain empirical findings, related to a new set of bold ideas, implied that physics – around 1925 – found itself in quite an unusual situation where the very idea of what it means to have a theoretical understanding of certain parts of Nature had become unclear. There was something about *what it means* to observe, to conduct experiments, to measure and to formulate theoretical thoughts that had been blurred. So the serious concern, in 1925, was that physics did not just need a new theory to account for a new surprising set of experimental findings; it rather needed a new *kind* of theory, where the links between observations, experiments, measurements, concepts and judgments was seen in a different way than it had previously been.

Accordingly, this was the perspective from which Bohr tried to assess both Heisenberg's and Schroedinger's "quantum mechanics". Here we must tread carefully. Bohr used the term "quantum physics" to refer to the whole world of physics after the discovery of the photon and the formulation of the Bohr model of the atom, and, last but not least, after the introduction of Planck's "quantum postulate" concerning the limited divisibility of energy. However, Heisenberg's and Schroedinger's formulas were the first that could be seen as examples of a new kind of *theory*, called "quantum theory". What, according to Bohr, constitutes such a theory? There are three crucial factors in his conception. Firstly, the new kind of theory was, by contrast to the vague definition of "quantum physics", formulated as *applicable* to a definite realm of physical items, namely *atoms*. The two examples of a new kind of theory were both "*atomic* theories". It should therefore be noted that the title that Bohr chose for his first compilation of philosophical essays was *Atomic Theory and the Description of Nature*. The title of the Como Lecture (Bohr, 1928) should also be noted, since it contains the same expression *Atomic theory*.

Secondly, Bohr considers theories to be normative constructs. They answer to general *principles for the description of Nature*. These principles have physical, methodological, metaphysical, and epistemological features: Physics is understood as an empirically informed, experimentally based, construction of both (a) concepts for the description of phenomena, and (b) theoretical formula (containing those concepts) that model the essential physical characteristics of certain kinds of physical systems. Many things could be said about this notion of the theory and the relation between theories and models; but the fact is that Bohr, more or less as a matter of course, followed the tradition and methods of Hamiltonian physics. The Canon here

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[1] Brock (2003).

was the Hamiltonian phase–space method of representing physical systems (associated with the Hamilton Principle, i.e. the mathematical integration formulation of the Principle of Least Action, and the mathematical use of Poisson Brackets in determining the possible constants of motion for given systems of objects). Bohr would add two distinctive features to this Hamiltonian approach. The first concerns the relation between two Hemholtzian points – one about the necessity of a unifying concept of physical change, i.e. the concept of energy, and another about the idea that physical systems (like an atom) possess a necessary "stability". Accordingly, Bohr's addition led to the following claims to the effect that the new quantum theory should both (a) express a generalization of the very concept of *energy*, and (b) introduce a new general idea about what it means to say that physical systems are "stable". In a sense, Bohr was here advocating his own original ideas about the "binding energy" and "energy states" of elementary particles *bound* within an atomic system, which precisely provided the two former ideas.

The third basic feature of the scientific theories that mattered to Bohr is in a sense the most important: Bohr reacted vividly to Heisenberg's early formulation of the Indeterminacy Relations (IR) in the paper *Über den Anschaulichen Inhalt der quantentheoretische Kinematik und Mekanik*.[2] As the title indicates, the issue was that the new kind of theory, atomic-quantum theory, had to be understood as applicable within an *intuitive* form of experience. Heisenberg had in effect formulated a bold new conception of such an intuition; namely the idea that one should relate the intuitive understanding of the application of physical formula to a *description of experimental design*. I believe it is extremely important to note that this was a basic issue for Bohr when he composed his Como Lecture in September 1927. Indeed, the main point of the Como Lecture was to show how the new set of quantum theoretical formula could be "adopted on to a new form of intuition."[3]

Here we face two basic problems: what Bohr meant by this, and how it relates to the original view of Kant concerning the unity of thought and intuition. Firstly, although Bohr maintained Heisenberg's ambition to provide a new form of intuition by developing an account of the description of experimental design, he radically changed the idea of such a design, as we shall see below. This change reveals Bohr as a forerunner to ideas that were later developed by some of Wittgenstein's pupils, W.H. Watson and S. Toulmin, and which later led to the formulation of the so called "semantic conception of theories" (known through the paradigms of B.C. van Fraassen and Ronald Giere).[4] These aspects of Bohr's ideas deserve attention – and I will say more about it below – but in the present context it is more important to note another more "classical" part of Bohr's theory. As Rom Harré has convincingly suggested, Bohr's understanding of the relation between the description of experiments and the theoretical representation of Nature can be seen as a bold new

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[2] Heisenberg (1927).

[3] See the concluding section of Bohr (1928).

[4] Watson (1938), Toulmin (1953), van Fraassen (1980), Giere (1988).

formulation of the *Kantian idea of schematism*.[5] Bohr formulated a new conception of what it means to *realize concepts within the framework of an experimental set-up*. In short, the required applicability of physical formula was conceived in a Kantian way by analyzing how the *means* of empirical physical experience correspond to a basic set of concepts. For Kant, the role of classical geometry was to construct recognizable patterns that could be postulated in advance to characterize the *manifest forms* of physical experience. These geometrical schemes formed the backbone of the idea that (Newtonian) mechanics is a way of *anticipating* physical experience and is not a conjecture about the true, ontological structure of the Universe. We shall see that Bohr in effect substituted a set of procedures concerning the adequate "space–time configuration" of physical apparatus for Kant's geometrical schemes.

We should be careful at this point: Kant's schemata were defined as "time-determinations a priori".[6] They made the basic categories of human Understanding applicable by "constructing", in an intuitive way, the way in which a temporal series of manifest spatial positions could be interpreted as an objective representation of the *possibility for objects of experience to entertain some sort of characteristic physical change*. The schemata, as it were, translate a temporal series of spatial manifestations into a description of the characteristic *kinematics and dynamics* of the physical objects posited in theoretical thought. So the Kantian idea of schematism forms the heart of a certain understanding of *what it means to establish a "mechanics"* in relation to a field of physical experience. There is a defining unity between "kinematics" and "dynamics" that constitutes a "mechanics": *This was also Heisenberg's view* when he successfully developed a new form of "mechanics": quantum mechanics.[7]

We now have three elements to consider (a) the conjecture of the importance of experimental design, (b) the link between temporal determinations within the intuition and the representation of objective forms of physical change, and (c) the idea of uniting kinematics and dynamics in a new non-classical way. In a technical, methodological sense, *this was what Bohr originally had in mind* when he formulated the idea of complementarity. Let us translate this into Bohr's own terms. Bohr first of all considered "complementarity" to be a generalization of the Kantian ideal of causality, where causality is a principle of knowledge and not an ontological claim. Bohr then claims that:

> [T]he trend of the whole argumentation presented in the Como lecture was to show that the viewpoint of complementarity might be regarded as a rational generalization of the very ideal of causality. (APHK, p. 41)

And he explains this as follows:

> The use of energy conservation in connection with the idea of stationary states (…) means an upholding of causality particularly striking when we realize that the very idea of motion,

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[5] Harré (1986, 1990).

[6] Kant Kr.d.r.V, Teil I, *Analytik der Grundsätze*, A145/B184.

[7] The logic of the structure of Heisenberg (1930) is the same as in Kant(*MAN*).

> on which the classical definition of kinetic energy rests, has become ambiguous in the field of atomic constitution. As I have stressed by the argumentation mentioned, space time co-ordination and dynamical conservation laws may be considered as *two complementary aspects of ordinary causality*. (Works VI, p. 37 – Faraday Lecture)

"Space–time coordination" of experimental phenomena can therefore no longer be seen as a representation of the kinematics of the atomic systems whose characteristics we try to measure. So the joint content of "dynamic conservation laws" and "space–time coordination" no longer expresses the unity characterizing a proper theoretical representation of the energy states of physical systems. The two do not form a unit but are "complementary". It should be noted that this is a *normative* statement, not a logical statement about a well-defined conceptual relation. The normative statement can be expressed thus:

> [I]n the description of atomic phenomena, the quantum postulate presents us with the task of developing a "complementarity" theory, the consistency of which can be judged only by weighing the possibilities of definition and observation. (ATDN, p. 55)

"The possibilities of definition", i.e. the possibility of defining the energy states of elementary particles bound within an atomic system must, as stated in the Como Lecture, be dependent upon the possibility of making a certain set of observations. Not just anything counts as a *description of an atomic phenomenon*. The mere report of either spatio-temporal change or a dynamic effect does not in itself qualify as such a description. To describe a phenomenon one has to know in advance *in what sense* this phenomenon is part of the conditions for making various claims about a certain kind of atomic system. The lines in a line spectrum, or the click in a Geiger counter, or the tracks in a cloud chamber, can be called manifestations of something. The aim is to be able to see such manifestations as *phenomena*, and this, Bohr says, requires some sort of *comprehension* of what is manifest.[8] So it is not the very manifest spatio-temporal changes that are significant in an experiment, it is the *form* of these manifest changes.

Here, and in the following couple of sections, I will try to characterize the aim and the methodological strategy of Bohr's viewpoint of Complementarity. However, I will first give a more detailed account of how the viewpoint "works", and what it implies, below. Let me begin the methodological part by noting that, for Bohr, all physical observations build upon some *idealizations* of a certain spatio-temporal form, such as the prolongation of a harmonic wave, or the continuous track of a particle. The observation of such forms is necessary. According to Bohr's understanding of Heseinberg's IR, the new thing in quantum physics is that such forms somehow mutually exclude one another. In respect to any significant "individual part" of an experimental set up, if the notion of, say, tracks, is essential for your description of the function of the experiment, then it does not make sense to also describe this function in terms of, say, the prolongation of a harmonic wave. If we put this in a very formal manner, we can see that the situation is quite the same as

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[8] More on this below.

that depicted by Kant: if you have a well-conceived algorithm concerning intuitive temporal determinations such that you have established, *a priori*, what it means to associate such determinations with *dynamic* features that characterize a system of causally related objects, then you have already established the conditions for the possibility of formulating a lawful, objective representation of these systems. Clearly, this is also Bohr's goal in introducing the idea of complementarity as a generalized form of causality. Heisenberg's IR constitute a *logic of discovery*, not the limits of a formerly established epistemic practice[9]:

> It is only the mutual exclusion of any two experimental procedures, permitting the unambiguous definition of complementary physical quantities, which provides room for new physical laws, the coexistence of which might at first appear irreconcilable with the basic principles of science. It is just this entirely new situation as regards the description of physical phenomena, that the notion of *complementarity* aims at characterizing. (Bohr, 1935 [the EPR-paper], p. 148)

We can see that the new kind of "definition" of physical quantities, i.e. the new kind of conceptually mediated account of quantum phenomena, provides room for a variety of new physical laws. The new Heisenbergian rules for the account of experimental data (the new "possibilities of observation") have thus been weighted, or balanced, with a new set of "definitions" concerning the understanding of the character of atomic objects. A new kind of intuition has been projected onto a set of new kinds of physical laws. This is strictly un-Kantian but definitely has a Kantian ring to it.

After we have thus pointed out how Bohr's view of complementarity is far from revolutionary in these respects, we can still note something radically new in his understanding of physics (all physics and not only quantum physics). I shall now try to formulate as briefly as possible how Bohr's philosophy of physics counts as a "modern" view of physics, with many parallels to the philosophy of the later Wittgenstein.

To begin with, it is clear from the early drafts of the Como paper, that Bohr introduced a totally new conception of the instruments of physical measurement.[10] Such instruments, like rulers, meters, scales, and clocks, are in no way independent, given, well-defined or, as Rom Harré has put it, "transparent" items.[11] In a very deep sense, there are no objectively given rulers, meters, scales, and clocks. There is only a set of apparatus that is what it is because of the *use we* give it.

This shall of course not be understood in the following two-steps way: (1) we have a kind of proto-idea about, say, the kind of determinations a ruler is meant to measure ("*length* of *distance*"), and (2) we then correct this by saying "but such determinations are in fact relative to 'uses' or intentions". We shall rather say that *what it means* for an object to be ascribed a certain physical measure is related *from*

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[9] This is an important point in (Cassirer, DIMP, p. 122ff). See also what I call Bohr's manifesto, below.

[10] Works VI, p. 75.

[11] Harré (1998), p. 353 ff.

the outset to the possibility of there being a certain procedure for utilizing some instrument. This is why Bohr calls the observable spatio-temporal determinations associated with a kind of measurement an “idealization”. The very idea of particles, waves, emissions, impacts, and all other kinds of movements in space–time, is our mathematical construct. However, these constructs adequately model the structure and function of a given experimental set up.

The methodological and physical core of this is that one can *design* measurement apparatus and whole experimental set-ups. Rulers, meters, scales, and clocks can now be seen as integrated into an experimental set-up; and their being so integrated is a main part of our designing the kinematics and the dynamics of the experimental design. It is *only in such a complete experimental context that you have measurement apparatus* at all. We should realize that there are no “isolated” rulers, meters, scales or clocks. If you use a rod as a ruler, there is a dynamic aspect of how you are allowed to perform the measurement of spatial positions (of length). There is no *a priori* sense of what it means to “place” the ruler against another object. The spatial determination thus has both a dynamic and a temporal aspect. It is the joint determination of these spatio-temporal and dynamic features that is called an “idealization”.

The “schematization” – as we might rightly call it – of such an integrated set of determinations means that three things have to be taken into account. Firstly, as we have already indicated, one and the same parts of an experimental design, say a plate, may in one case be an integrated part of a device that indicates a temporal (by contrast to a dynamic or a spatial) feature of the design, whereas in another case it is the latter feature that is the focal point. The “performance” of the experiment thus implies that one looks exclusively at one feature within the integrated set of determinations for the given item, say the plate. It is when one and the same such part of a device has two mutually exclusive focal points within the performance of an experiment that one can say the two performances are “complementary”. So complementarity is not *just* about particles versus waves, kinematics versus dynamics and so on. The things that are “complementary” are the *forms* of uniting a kinematic and a dynamic description of an experimental design in order to make sense of the phenomena which are produced accordingly.

This is our second implication: the intelligibility and significance of an experimental design aims at selecting *a priori* the kinds of *manifestations* that have explanatory value; be it clicks in a Geiger counter, tracks in a bubble chamber, spots on a screen, spectral lines in a spectrometer, or vibrations in a spring. The *physical* results of an experiment consist in such manifestations. I here use Bohr’s terminology; the first aim of an experiment is to “comprehend” such manifestations. Such comprehension is what constitutes what Bohr calls a *phenomenon*. We should namely:

> [R]eserve the word “phenomenon” for the comprehension of the effects observed under given experimental conditions. (Bohr, 1938, p. 24)

For instance, the track in the bubble chamber is considered to be the trajectory of a charged elementary particle, or the click in the Geiger counter is considered to be the emission of an elementary particle from an atomic system. These are phenomena because if one talks about the "trajectories" or "emissions" of elementary particles, one does not merely account for the workings of a physical experiment in ordinary or classical macro physical fashion; rather, one is using the best of quantum physical theories in order to make explanatory sense of the given experimental manifestations. In fact, Bohr's understanding of "phenomena" is even more theoretical and is far from reducing to empirical events:

> The essential lesson of the analysis of measurements in quantum theory is thus the emphasis on the necessity, in the account of phenomena, of taking the whole experimental arrangement into consideration, in complete conformity with the fact that all unambiguous interpretation of the quantum mechanical formalism involves the fixation of the external conditions, defining the initial state of the atomic system concerned and the character of the possible predictions as regards subsequent observable properties of that system. Any measurement in quantum theory can in fact only refer either to a fixation of the initial state or to the test of such predictions, and it is first the combination of measurements of both kinds which constitutes a well-defined phenomenon. (Bohr, 1938, p. 20)

Our third implication is that the comprehension of physical manifestations is expressed *in terms of abstractions* as Bohr calls it.[12] In our example, the "abstraction" in question was the idea of an elementary particle. This is not merely an idea about a particle, but an idea about how one and the same physical entity can both be *bound* within an atomic system and be able to be *freely* moving in space and time, which means that the item can be both bound and free within the *same* experimental set-up. This illustrates how an abstraction is the idea of an object *within* the phenomena that a given experimental set up "affords".[13] For instance we have made it clear that an "electron" is something that can be "emitted", either spontaneously or by some kind of induced influence. So the elementary particles are "abstractions" in two ways. They are literally speaking "abstracts" from atomic systems since they can be emitted from them, and they are abstract ideas *in terms of which* we can comprehend manifestations and thus recognize phenomena.

We saw that the performance of an experiment, involving two succeeding measurements, leads to the comprehension of a single phenomenon. We have now "analyzed" the possibility and significance of such single phenomena, and this brings us to Bohr's main thesis, that the task of a theoretical science is to combine such analysis with a proper kind of *synthesis*[14]: First, there is the direct methodological instruction about how to "synthesize" phenomena in terms of theoretical constructs, or "abstractions", in the sense that was explained above:

> The Viewpoint of Complementarity allows us indeed to avoid any futile discussion about determinism or indeterminism of physical events, by offering a straightforward generalization of the very ideal of causality, which can aim only at the synthesis of phenomena describable in terms of a behavior of objects independent of the means of observation. (Bohr, 1938, p. 25)

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[12] Bohr, *Works* VI, p. 98, ATDN, p. 69.

[13] In the sense of Harré (1990). This notion of "affordances" is in effect a variety of the notion of secondary properties interpreted in accordance with Gibson's account of perception.

[14] On the general relation between analysis of observations and synthesis of phenomena, see Bohr (1936).

The idea is that we should refrain from trying to see a causal link between experimental phenomena. Such an effort simply makes no sense if we understand a "phenomenon" as Bohr did. The aim is rather to *classify* experiences. For instance, the phenomenon in which an electron ray splits in two, tells us that such electrons can be both in a "spin up" and a "spin down" state within the bound system of an atom. Accordingly, my conjecture is that Bohr simply points to the ambition of mapping the electron configuration diagrams of all well known atomic systems.

> I need recall only briefly, how often the development of physics has taught us that a consistent application even of the most elementary concepts indispensable for the description of our daily experience, is based on assumptions initially unnoticed, the explicit considerations of which is, however, essential if we wish to obtain a classification of more extended *domains* of experience as clear and free from arbitrariness as possible. (Bohr, 1937, p. 290, italics mine!)

So the synthesis of phenomena is a means to an end. This end is the classification of physical domains, like that of atomic systems, where we map this domain by setting up a table of possible energy states of elementary particles bound within such systems, i.e. the electron configuration schemes of the periodic table including stable isotopes.

The second part of Bohr's idea about the "synthesis of phenomena" concerns the epistemic progress that characterizes the development of a given branch of science. Bohr claims that:

> The extension of our knowledge may lead to the recognition of relations between formerly unconnected groups of phenomena, the harmonious synthesis of which demands a renewed revision of the presuppositions for the unambiguous application of even our most elementary concepts. (Bohr, 1936, p. 28)

This is the goal of a scientific discipline like "atomic theory". However whenever such a goal is beginning to take form, an "epistemological lesson" appears. Here Bohr is definitely not Kantian. The lesson is that our previous understanding of how to use concepts in the description of phenomena has proved to be conditioned in a way that we should no longer maintain. The point is not so much that there are conditions for the application of concepts, at all. The point is different and twofold. Firstly, what we previously saw as forming a necessary background for the applicability of a concept has proven not to be so – at least in some new and special context. Secondly, realizing all this opens up the possibility that phenomena that one might have thought could and should never be related to one another, can nevertheless be "synthesized" in a certain way.

A clear example of this is the study of biological organisms, where two different approaches to that study, physicalism and vitalism respectively, seemed to be irreconcilable. In fact it was in this context that the father of Niels Bohr, the physiologist Christian Bohr, formulated the idea of complementarity.[15] Niels adopted his father's views and reformulated them in strict analogy with his account of quantum physical

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[15] See Brock (2003), p. 261 ff.

experimentation. So the argument here is not only the dual aspect view that biological organisms essentially have, namely both physiological properties and the ability to sustain and project a biological process of *life*. The point is more sophisticated, for the argument is that the study of a biological organism as a physiological system only makes sense because one is able to *treat the organism as a self-sustaining life process* when one is conducting the physiological study. First of all, the organism must not die or be in a special psychological condition when one wants to investigate its characteristic physiological processes. Similarly, the study of the organism as a life process presupposes that one can control some of the physiological features of it, e.g. provide the organism with food and air, or shield it from various influences like heat, bullets, or flooding waters.

Again, the crucial thing, in biology as in quantum physics, is that one learns that the *conditions* for the study of one aspect have connections with the conditions for the study of other aspects. As Bohr puts it, the viewpoint of complementarity is based on the possibility of "scrutinizing presuppositions" or conditions.[16] This possibility reveals, according to Bohr, that one has caught a glimpse of a "harmony" that lies deeper than the apparent opposition between the various aspects. As I have argued elsewhere, Bohr – on a very special private occasion – formulated as early as 1928 what I call his "manifesto" which, in the fullest way, expresses Bohr's vision of the far reaching philosophical implications of quantum physics for *all* fields of experience[17]:

> Everywhere, new forms of outlook are brought up and new fields and connections (*Zusammenhang*) dawn upon us. But as every road we choose is branching and curving, we soon lose our sense of direction and will sooner or later return to our point of departure. Despite of this we are always able to return home enriched (*mit einem Ertrag*). The richness of that which we can collect and piece together is unlimited. As we investigate into deeper and deeper presuppositions we realize broader and broader connections. In this way our lives are informed by still richer impressions of an eternal and unlimited harmony; only, the harmony itself can never be caught, it can only be glimpsed. And with any effort to catch hold on it, it slips by the nature of its essence through our fingers. Nothing is fixed. Every thought, even every word, is only suited to underline a connection, which can never be fully described, but always reflected deeper. *Such is, undeniably, the conditions for human thought*… we can only complete our picture of the conditions of life by recognizing the play of oppositions. It is precisely in this vertigo of facing the unlimited that we recollect those vague forebodings which form the background of the spontaneous excitement of the youth. (Bohr, 1985, p. 263, my translation from Danish, italics mine)

This view is not Kantian. Bohr is known to have associated it with a variety of Romantic philosophical ideas, especially those of Schiller and Goethe.[18] But there is still a Kantian ring to this Romantic Manifesto. At the heart of the view is the idea that human thought is bound to speculate about those transcendent things and relations that we can never have an explicit justifiable knowledge about. There is

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[16] Bohr (1985), pp. 261–262.

[17] Brock (2003), p. 269.

[18] Brock (2003), p. 257; Honner (1987), p. 6, 197; Chevalley (1994, 1995).

also the idea that "deep down" Nature possesses a harmony or unity, a Logos that does not correspond to a given World Order. It is rather a Logos that exhibits itself in the multitude of changes and forms of interaction that characterizes all domains of knowledge. So even if the basic features of Nature will forever escape our understanding, and Nature perhaps is so complex and disordered that we can never tell its "plan", we should still never say that Nature is unintelligible. Nature will never be like a closed door. Nature will forever be a ground, a place, or condition *of possibility*. This, one might argue, is the common metaphysical stance of Bohr and the German idealists, including Kant.

Bohr certainly did not see himself as a Kantian philosopher. He thought that any "conceptual frame" in any science would prove too narrow and would have to be replaced by new schemes.[19] Neither did Bohr agree with a dialectic view like that of Hegel; there was to Bohr no necessary kind of development of physical thought, no necessary direction. Now these two views do not challenge the possibility of describing the trend or pattern in the actual historical development of physics, so far. We have accordingly seen that Bohr did not believe that quantum physics was something entirely revolutionary. Even if, in his mature understanding, he no longer thought of his own early "principle of correspondence" as important, there is no doubt in my opinion that Bohr in some sense saw quantum physics as a rational continuation of older forms of physics. We have even seen, in respect to basic notions such as causality and unity, that Bohr saw quantum physics as a *generalization* of older kinds of thought.

Bohr is far from clear on these issues. Obviously, quantum physics, and any of the other kinds of "new possible sciences", in biology and elsewhere, that Bohr suggested, can all be said to "contain" their predecessors. The range of phenomena of the new theory simply exceeds the range of the predecessors. But is that all? Two things seem to be pressing. Firstly, Bohr's general idea about the possibility of an expansion of the sciences needs the support of some concept of *morphology*. Because, the different forms of phenomena with which the sciences are concerned are not only interrelated, according to the complementarity viewpoint, they are also *integrated* and united in a way we have to explain further. Secondly, the idea that the sciences become more "general" can be understood in many ways. I have for example tried to account elsewhere for the difference in the way in which Helmholtz and Planck conceived such a generalization.[20] There are three issues here. Firstly physical theories may be more and more *universal* (and that will involve an idea about how the various fields of applicability of older theories become "united" by means of the new universal theory). Secondly, theories can also become more *allgemein* (and let us call this "general"). This means that the *way in which* the theory covers a field of individual examples is more general than the way in which older theories did. Thus, whereas Newtonian mechanics dealt only with such and such

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[19] Conclusion of Bohr (1954).

[20] Brock (2003), p. 82 ff.

material particulars, and then later with field theories, the idea of point masses has become part of a more general idea about the spatial distribution of forces within the field. Finally, there is a third important aspect of the generalization of physical theory, namely the sense in which the *mathematical expressions* of physical theory become more and more general. So the whole issue about what generalization means in mathematics should also be accounted for.

Not many philosophers of science have managed to take these three kinds of generalization into account. Among the few who have, I will now discuss Ernst Cassirer's view about the development of physics. I will try to argue that his account makes Bohr's position stronger. I will not argue that Cassirer's account is *just* the support Bohr needs, nor will I claim that Cassirer's view is in any sense closer to Bohr's than to, say, Planck's. Cassirer is in fact very close to Planck.[21] I simply claim that the *joint* position of Cassirer and Bohr is stronger than Bohr's, or anybody else's, alone.

Cassirer describes the development of any branch of physics by mentioning three phases characterized by a certain kind of crucial *statement*. The first phase is characterized by statements of the results of measurement (*Massaussagen*). Cassirer describes this phase as a transformation from an everyday mode of understanding to a systematic-theoretical outlook. An example would be the successive introduction of new concepts for the description of experimental results by Coulomb, Ampere, Faraday, and others *before* the formulation of a theoretical system of laws; before, say, Maxwell. To use Planck's expressions, this transition from an ordinary "World of Sense" to a theoretically posited "Real World" has one important implication: the concepts introduced in the description of phenomena have a determinate function in relation to the kinds of judgment associated with theoretical thought. So this is an entirely Kantian view about the unity of Thought and Judgment and about the transition from "judgments of perception" to "judgments of experience".[22] In our context, the important thing is that Cassirer presents Heisenberg's IR as a late crucial example of how to introduce concepts in physical thought with the aim of stating the results of measurement in a new fashion. The special thing about IR, according to Cassirer, is that the mathematical relation between continuity and discrete elements is altered by comparison to older physical thought.[23] In the kind of experiments where we utilize IR we do not aim at tracing dynamically interacting material particulars in space and time. We try to grasp a complete, individual, phenomenon. Such a phenomenon is "continuous" with other phenomena we might observe. The "jumps" from one phenomenon to the other are movements not in space and time but within the *wholeness and unity* of the kind of atomic physical system in question.

Before his account of Heisenberg's IR, Cassirer has in his text carefully reminded his readers of some important parts of Kant's epistemology: The notion

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[21] Incidentally, Einstein was also in line with Planck. See Brock (2005, 2003, p. 142ff).

[22] Kant, Prol., § 1, p. 18.

[23] Cassirer, DIMP, p. 181.

of continuity in our observations is, Kantian wise, intimately connected with the issue of *causality*.[24] For that in a nutshell was the point of the Kantian Schemata. The idea was to make *some sort of* continuity a paradigm for the adequate causal representation of a range of physical items. In classical physics, the relevant kind of continuity is the intuitively represented movements of phenomena in space and time. In quantum physics, the kind of intuitiveness is different; but formally the two approaches have the same rational core: there is a specific formal account of the relation between continuity and causality in respect to individual features of given physical systems.[25] The difference is a difference in the mathematical formulation of the issue. We have gone from a Euclidean, intuitive method of representation, to a more abstract topology.[26]

According to Cassirer, this first phase of the development of a branch of physics is followed by a phase where the formulation of statements of *Laws* is crucial. Here the important thing is completeness, and not ontology. The formulation of natural laws is part of the task of circumscribing a specific *domain* of physical experience (DIMP 44). Again, what matters is the Kantian idea of a field of experience corresponding to a certain clarity concerning the *applicability* of a set of concepts; namely those that were associated with the already introduced statements of the results of measurement. What matters is the very exercise of being able to survey the range of relevant phenomena, and this requires a special kind of *comprehension*. We see that Cassirer here formulates what he takes to be a well taken Kantian insight, in a manner very close to how Bohr explained the viewpoint of complementarity in quantum physics.

The third phase in the development of a branch of science is the expression of *principles*, which for Cassirer means regulative principles for finding out, and expressing, new kinds of physical laws. Principles like the Principle of Least Action and the Principle of the Conservation of Energy are each to be recognized as "birthplaces from which new laws spring, again and again".[27] Similarly the General Principle of Causality is a regulative principle for establishing new methods of measurement in the natural sciences.[28] I have elsewhere traced the various formulations of the Principle of Least Action in detail, from Maupertuis to Planck, and the connection of this basic principle of physics to all other principles and fundamental concepts.[29] So I will here turn straight to Cassirer's main point about principles. For they are not only principles behind the formulation of new laws and methods, they are *themselves* developing, i.e. what it takes to formulate the principles adequately is changing. The Principle of Least Action, the Principle of the

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[24] Cassirer, DIMP, p. 155 ff.

[25] Cassirer, DIMP, p. 162 ff.

[26] Cassirer, DIMP, p. 187.

[27] Cassirer, DIMP, p. 53.

[28] Cassirer, DIMP, p. 60.

[29] Brock (2003), p. 74 ff.

Conservation of Energy, the Principle of Causality, the Principle of Continuity, all have to be reformulated as time goes on. A telling example would be that according to Bohr, energy is not just energy, since we have to distinguish between bound and free energy. We already saw that it is an important part of the Como Lecture in which Bohr characterizes Heisenberg's and Schroedinger's different forms of quantum mechanics as invoking different generalizations of the concept of energy. It is at this point that we see how Cassirer's account can lend support to Bohr's.

It is difficult to judge whether Bohr simply means that physics is better off after the advent of quantum physics because a lot of confusion that previously colored the enterprise has vanished, or whether he is prepared to say *in what sense* physics is better off. For whatever the merits of the physics of our time, it will always, according to Bohr, be in need of a correction, yet to come; a correction that will in principle be of the same fundamental kind as the correction of classical physics.[30] The critique will be as severe and all embracing. Could that be what the theory of super strings is already saying?

By contrast, Cassirer is able to point to a distinguished feature of the development of new laws and of new (expressions of) principles: it is as if the focal point of physics shifts, gradually, from material individuals to something more general. Here Cassirer is very precise: Every piece of physics aims at formulating a certain set of *constant values*. The development concerns *what has* such constant values. It was a first step forward when one formulated material constants like mass density and conductivity. A second step was when one formulated constants of *Nature*, as such, like the elementary charge and mass of the electron and the velocity of electromagnetic waves in vacuum. Today, we have taken the final step which deals with universal constants in an even more general sense. Cassirer here believes the *group* of natural constants has definite mathematical properties.[31]

On the face of it, this view seems to differ radically from Bohr's. Cassirer explicitly argues that modern physics is turning physical thought into a Neo-Platonic exercise, where our conceptual grasp of Nature converges with an absolutely given world of ideas or LOGOS. However as soon as we remind ourselves of another part of Cassirer's philosophy, a close parallel to Bohr emerges: Cassirer is also a neo-Fichtean philosopher. The disclosure of a real and intelligible world is *as such* an unfolding of the *subject* of thought and action, the unfolding of a human "I" within the posited Real World. Cassirer's neo-platonic view is not a view from nowhere about a converging insight into the absolute structure of being. His idea is not that the *content* of physical thought is converging towards absolute truth. Cassirer rather considers that the *situation and the conditions* in which modern physicists find themselves have become closely related to basic objective structures of Nature. Cassirer's view is a view about *freedom and possibility*. The modern physicist is free to unfold her theoretical effort in a way that no scientist before her has been.

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[30] Bohr (1954).

[31] Cassirer (1936), III, 2–3, pp. 106–134.

The possible lines of further research are not only more but "point in all directions", i.e. the manifold of relevant phenomena have no explicit boundaries.

These are very big metaphysical issues characterizing post-Fichtean, especially Goethean, philosophical speculation; and neither Bohr nor Cassirer can be said to have dealt with them properly. However, I believe that the combination of Cassirer's imposing version and Bohr's more modest version of neo-Fichtean thoughts provide us with some of the cornerstones of a new adequate kind of Natural Philosophy where the rational epistemic progress and the unfolding of subjective features of Human Understanding are seen as two sides of the same coin.

Bibliography

(Bohr, 1928): "The Quantum Postulate and the Recent Development of Atomic Theory", *Atomic Theory and the Description of Nature*, Cambridge University Press, Cambridge 1934

(Bohr, 1935): "Can Quantum-Mechanical description of Reality Be Considered Complete?", *Physical Review* **48**, 696–702

(Bohr, 1936): "Analysis and Synthesis in Science", *International Encyclopedia of Unified Science* **I**, no. 1, Chicago University Press, Chicago 1938, 28

(Bohr, 1937): "Causality and Complementarity", *Philosophy of Science* **4**, 289–298

(Bohr, 1938): "The Causality problem in Atomic Physics", *New Theories in Physics*, International Institute of Intellectual Co-operation, Paris 1939, 11–45

(Bohr, 1954): "The Unity of Knowledge", *Atomic Physics and Human Knowledge*, Wiley, New York 1958

(Bohr, 1986): *Niels Bohr's Collected Works*, Vol. VI, North-Holland, Amsterdam 1986

(Bohr, 1985): *Naturbeskrivelse og menneskelig erkendelse*, K¢benhavn, Rhodos 1985

(Brock, 2003): *Niels Bohr's Philosophy of Quantum Physics in the Light of the Helmholtzian Tradition of Theoretical Physics*, Logos Verlag, Berlin 2003

(Brock, 2005): "Einstein's and Planck's different adoption of the Helmholtzian Tradition of Theoretical Physics", translated into Russian language, in the proceedings from the Annual Meeting of the Russian Academy of Science, December 2005, on *Einstein, Relativity, and Philosophy*.

(Cassirer, 1936): *Ziele und Wege der Wirklichkeitserkenntnis*, Felix Meiner, Hamburg, Germany 1998

(DIMP) *Determinism and Indeterminism in Modern Physics*, Yale University Press, New Haven, CT 1956

(Chevalley, 1994): "Niels Bohr's Words and the Atlantis of Kantianism", in *Niels Bohr and Contemporary Philosophy* (eds. Faye and Folse), Boston Studies in the Philosophy of Science **153**, Dordrecht, The Netherlands 1994

(Chevalley, 1995): "Philosophy and the Birth of Quantum Theory", *Physics, Philosophy and the Scientific Community* (ed. K. Gavrogly et al.), Klüwer, Dordrecht, The Netherlands 1995

(Giere, 1988): *Explaining Science. A Cognitive Approach*, Chicago University Press, Chicago, IL 1988

(Harré, 1986): *Varieties of Realism. A Rationale for the Natural Sciences*, Blackwell, Oxford 1986

(Harré, 1990): "Exploring the Human Umwelt", in *Harré and His Critics* (ed. R. Bhaskar), Blackwell, Oxford 1990

(Harré, 1998): "Recovering the Experiment", *Philosophy* **73**, 1998, 353–777

(Heisenberg, 1927): "Über den Anschaulichen Inhalt der quantentheoretischen Kinematik und Mekanik", *Zeitschrift für Physik* **43**, 197

(Heisenberg, 1930): *The Physical Principles of the Quantum Theory*, Dover, Chicago, IL 1930.

(Honner, 1987): *The Description of Nature. Niels Bohr and the Philosophy of Quantum Physics*, Clarendon, Oxford 1987

(Kant, Prol.): *Prolegomenon zu einer jeden künftigen Metaphysik*, Philip Reclam Junior, Berlin 1979

(Kant, Kr. d. r. V): *Kritik der reinen Vernunft*, Felix Meiner, Hamburg, Germany 1990

(Kant, MAN): *Metaphysische Anfangsgründe der Naturwissenschaften*, Werkausgabe IX, Suhrkamp, Frankfurt am Main 1991

(Toulmin, 1953): *The Philosophy of Science: An Introduction*. Hutchinson, London 1953

(van Fraassen, 1980): *The Scientific Image*, Oxford University Press, Oxford 1980

(Watson, 1938): *On Understanding Physics*, Cambridge University Press, Cambridge 1938

A Transcendental Account of Correspondence and Complementarity

Hernán Pringe

Abstract The aim of this work is to analyse the epistemological role of the Bohrian notions of correspondence and complementarity from a transcendental perspective. We argue that the principle of correspondence is *regulative* in the strict Kantian sense. In particular, we maintain that until the introduction of complementarity this principle functions as a maxim for the reflection upon nature in the attempt to exhibit concepts of physical objects *directly* in intuition. On the contrary, from the point of view of complementarity, the principle of correspondence guides the reflection when *symbolic analogies* are established. This transcendental reading of Bohr's thought enables us to account for the conceptual development of his interpretation of quantum theory from 1913 to the Como Lecture in 1927.

In the first part of this paper we discuss the minimal Kantian framework necessary for our investigation. Secondly, we study the history of the notion of correspondence, from its origins in 1913 to the Bohr–Kramers–Slater's theory. We turn further to the notion of complementarity and its connection with the question of symbolic knowledge in quantum mechanics. Finally, we analyse the role of correspondence in the framework of complementarity.[1]

1 A Minimal Kantian Framework

In order to introduce the Kantian elements that we need for our analysis of Bohr's thought the study of the following example will suffice.[2] Consider the analogy:[3] "C is the cause of E, as A is the cause of B", where A, B and E are given events, but

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[1] The importance of these results for a transcendental foundation of quantum objectivity is discussed in Pringe (2007).

[2] See Pringe (2007), pp. 19ff.

[3] According to Kant, an analogy is "a perfect similarity between two relations in wholly dissimilar things." AA IV, pp. 357–358. Kant (1783), p. 147.

M. Bitbol et al. (eds.), *Constituting Objectivity*,

C is unknown. E being given, its cause C is searched for in experience under the assumption that the causal relation between C and E is identical in certain sense to the already known between A and B. This procedure is empirical, and thus the analogy so established is *a posteriori*. However, transcendental philosophy determines its *a priori* conditions of possibility. These are of two different kinds.

Firstly, for the search of the unknown cause C to have sense, we must presuppose that any event E has *in general* a cause, even though this may remain unknown for the meantime. Kant argues that each event *must* have a cause, because only by having it can it be represented in general as *objective*, i.e., as belonging to the necessary sequence of experience and not merely to the contingent sequence of our perceptions. This is the *a priori* principle of temporal sequence according to the law of causality. In Kant's own words:

> All alterations occur in accordance with the law of the connection of cause and effect.[4]

The *empirical* analogy C:E = A:B is hence based on the *a priori* analogy C:E = Cause:Effect. The a priori analogy guides our search for the unknown term C by pointing out the essential feature it must possess: C must contain the condition for a rule in accordance to which E always, necessarily follows.[5]

Secondly, the analogy C:E = A:B presupposes that the particular causal law between A and B is not just valid in respect to them, but may obtain in other cases as well. This is not a trivial assumption, since it could be the case that laws relating different pairs of causes and effects bore no resemblance at all. In this regard, Kant states:

> [T]he multiplicity and diversity of empirical laws could be so great that it might be possible for us to connect perceptions to some extent in accordance with particular laws discovered on various occasions into one experience, but never to bring these empirical laws themselves to the unity of kinship under a common principle.[6]

Were the multiplicity and diversity of empirical laws so great that they made up "a raw chaotic aggregate,"[7] no connection between them could be found and the analogy C:E = A:B could not be established. Thus, for this analogy to be possible, we must assume that particular causal laws may be brought under more general ones. In our case, the law connecting A and B, and the one relating C and E must be conceivable as falling under a common principle, i.e., as being able to receive *systematic unity*. This *logical* or *methodological* assumption concerning the multiplicity of *empirical laws* depends in turn on a *transcendental* presupposition regarding *nature* itself.[8] The presupposition that nature qualifies as an empirical system

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[4] B232. Kant (1781), p. 304.

[5] A193/B238–B239. Kant (1781), p. 307.

[6] AA XX, p. 209. Kant (1790), p. 13. See also AA V, p. 183. Kant (1790), p. 70.

[7] AA XX, p. 209. Kant (1790), p. 13.

[8] AA XX, p. 215. Kant (1790), pp. 18–19. Otherwise, by aiming at a system of empirical laws, we would set as a goal an idea that, albeit convenient for the economy of reason, contradicted the arrangement of nature. Thereby, our knowledge would make no progress.

through the affinity of particular laws under more general ones is what Kant calls "the transcendental principle of the power of judgment."[9]

Therefore, two different kinds of principles ground the analogy C:E = A:B. On the one hand, the principle of causality, which depends on the pure concept of *understanding* and, on the other hand, the principle of systematicity of nature, that is assumed by the (reflecting) *power of judgment*. By means of the first one, events are *constituted* as *objective* alterations.[10] On the contrary, the second principle *regulates* the subsumption of these objective cognitions under empirical laws of increasing generality. In the first case, the power of judgment *determines* the *a priori* spatio-temporal manifold of C and E according to the principle of causality, so that C is represented as being *objectively* before E and not just as a previous element in the subjective sequence of perceptions. In the second case, the power of judgment *reflects* upon the *empirical* manifold of events in order to find the empirical law under which they stand and, in turn, to subsume this law under more general ones.

In the next section we shall study how these constitutive and regulative principles performs their epistemological task in the "old quantum theory" by following Bohr's thought from 1913 to 1924. In particular we shall see how the principle of correspondence plays the role of a maxim for the *reflection* when empirical analogies of the form just discussed are searched for. Specifically, in this period Bohr aims at establishing analogies connecting the classical relation between electronic motion and radiation with the quantum-theoretical one. According to classical electrodynamics, the accelerated motion of an orbiting electron causes a radiation field. But, precisely for this reason, the theory seems to preclude a stable atomic model.[11] Thus, a quantum-theoretical law analogue to the classical is searched for, so that the electronic motion inside stable atoms may also be represented as causing the radiation spectra in a way to be determined. The core of the problem is that the analogies obtained remain merely formal because in quantum theory a *causal* connection between electronic motion and radiation cannot be exhibited in *space and time*. This will provoke a turning point in Bohr's thought, leading to the framework of complementarity.

2 Correspondence

The principle of correspondence states the *methodological* requirement of searching for analogies between quantum theory and classical physics:[12]

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[9] AA XX, p. 209. Kant (1790), p. 13.

[10] An exhaustive analysis of the constitution problem would demand the consideration of *all* principles of pure understanding and not just the principle of causality. But this is not necessary for our purposes.

[11] This was the problem of Rutherford's atomic model.

[12] See Pringe (2007), pp. 49ff.

> An attempt is made to elucidate the problems by means of a general principle which postulates a formal correspondence between the fundamentally different conceptions of the classical electrodynamics and those of the quantum theory.[13]

This means that one ought to trace the analogy between the quantum theory and the ordinary theory of radiation as closely as possible.[14]

More precisely, in the "old quantum theory" three assumptions are made by the correspondence principle. The first of them is the affinity between classical and quantum laws. In view of it, knowledge of the classical laws may provide clues to discovering the quantum ones. The second assumption is the claim that quantum laws are rational generalizations of classical laws. Finally, according to the correspondence principle classical laws may be regained in a certain limit. There should be a gradual transition between classical and quantum theories that makes a generalization of classical physics in the atomic realm possible, guaranteeing at the same time an asymptotic agreement of quantum and classical laws when the quantum of action vanishes. In this way, the correspondence principle expresses the demands of the Kantian principles of the hypothetical use of reason[15] or maxims of the power of judgment[16]: the principles of continuity, homogeneity and specification. In sum, Bohr's main presupposition is that quantum and classical theories may be brought into a system in spite of their intrinsic differences.[17]

Under the guidance of the correspondence principle, from 1913 to 1923 Bohr assumes that the quantum relation between radiation spectra and electronic motion inside the atom is in certain aspects and within certain limits identical to the corresponding relation between radiation and electronic motion in classical electrodynamics. In particular, the following relations are determined as identical in the limit of low frequencies: (i) the quantum and the classical relations between optical and mechanical frequencies.[18] (ii) The quantum relation between the probabilities of transitions among stationary states and the amplitudes of the Fourier expansion of the displacements of the particles in the corresponding stationary states, and the classical relation between the intensities of radiation and the amplitudes of the Fourier expansion of the displacements of the particles in multiperiodic systems.[19] (iii) The quantum relation between the polarisation of the radiation emitted in a transition among stationary states and the amplitudes of the Fourier expansion of

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[13] Bohr (1922), p. 2. Bohr used for the first time the expression *principle of correspondence* in 1920. Now he points out that "the first germ" of this principle may already be found in his trilogy of 1913.

[14] Bohr (1918–1922), p. 70.

[15] A657/B685. Kant (1781), p. 598. Each of these *logical* principles presupposes in turn a corresponding *transcendental* one.

[16] AA V, p. 182. Kant (1790), p. 69. These maxims are based on the transcendental principle of the reflecting power of judgment.

[17] Along Kantian lines, one may argue that this *methodological* assumption about the systematic unity of *theories* depends on a *transcendental* one about the systematic unity of *nature*.

[18] Bohr (1913), pp. 172ff.

[19] Bohr (1918–1922), pp. 97–98.

the displacements of the particles in the corresponding stationary states, and the classical relation between the polarisation of radiation and the amplitudes of the Fourier expansion of the displacements of the particles in multiperiodic systems.[20]

However, these analogies remain merely *formal*, because thereby no causal *and* spatio-temporal representation of the quantum process connecting electronic motion and radiation is given. For example, the quantum relation between optical and mechanical frequencies corresponds to the classical relation in the limit of low frequencies. But the classical relation is based on a causal *and* spatio-temporal mechanism of radiation. On the contrary, there is nothing of that sort in the quantum case. Moreover:

> [A]t the present state we do not possess any means of describing in detail the process of direct transition between two stationary states accompanied by an emission or absorption of radiation and cannot be sure beforehand that such a description will be possible at all by means of laws consistent with the application of the principle of conservation of energy.[21]

The quantum-theoretical analogue of the classical orbits may be exhibited in intuition by the stationary state picture. But its relation to radiation constantly moves away from the classical model. While classical electromagnetism establishes that the frequency of emission of an orbiting electron is equal to its frequency of motion, this simple relation is already abandoned in 1913. In the case of multiperiodic systems, in turn, the electronic motion grounds only the *probability* of radiation, as Bohr argues in 1918.

The main difficulty for determining the mechanism of radiation is the tension between the *continuous* and *discontinuous* aspects of the issue. On the one hand, the picture of stationary states provides a continuous representation of electronic motion. On the other hand, the transitions among these states are nevertheless conceived as discontinuous. A new side of this problem is illuminated in 1924 by the Bohr–Kramer–Slater (BKS) theory,[22] which entails that *if* a continuos spatio-temporal connection between electronic motion in the stationary states and radiation is assumed, so that this accounts for the statistical laws of transitions, *then* a causal relation between electronic motion and radiation in individual processes cannot be maintained.

An atom in a stationary state is in this case conceived in analogy to a corresponding system of classical charges that oscillate at the transition frequencies allowed by the state: the so-called "virtual" oscillators. The analogy has a very limited scope, since in fact these oscillators do not obey Maxwell's laws.[23] However, it suffices to provide a spatio-temporal description of the interaction between radiation and matter. The restriction to the use of the category of causality in regard to electronic motion and radiation corresponds in the "virtual" model to the violation of conservation principles in individual transitions occurring in distant atoms. By means of its virtual field an atom may induce a certain transition in another atom, but then undergo a different one, so that neither energy nor momentum remains

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[20] Idem.

[21] Bohr (1921), p. 372.

[22] Bohr, Kramers and Slater (1924).

[23] Darrigol (1992), p. 257.

conserved. Only if one considers a statistical average of such events are the conservation principles satisfied.[24] Thus, by assuming the continuous spatio-temporal character of the relation between the electronic motion in stationary states and radiation, and even in the absence of any further knowledge of this relation, the model of virtual oscillators precludes a causal account of the radiation involved in individual transitions.

But a causal connection between electronic motion and radiation is, in Kantian terms, a necessary condition of their being unified in experience. For this reason, the representation of electronic motion cannot be referred to the empirical content provided by radiation, remaining thereby a *formal* one. It merely possesses the mathematical sense of the classical equations of motion:

> At the present state of science it does not seem possible to avoid the formal character of the quantum theory which is shown by the fact that the interpretation of atomic phenomena does not involve a description of the mechanism of the discontinuous processes, which in the quantum theory of spectra are designated as transitions between stationary states of the atom.[25]

Bohr's main point is however that the analogies established through the correspondence principle not only *are* but *can only be* formal, for the BKS-theory implies that the spatio-temporal representation of the radiation mechanism impedes any causal description of individual processes. Radiation cannot be subsumed under the category of causality as the effect of electronic motion, and *therefore* electrons in stationary states cannot be constituted as objects of *possible experience* in the Kantian sense. In other words, the representation of the electronic motion in stationary states lacks the empirical content associated to radiation spectra and remains merely formal. *Experience* of electrons in stationary states, i.e., their spatio-temporal *and* causal representation, is *not* possible.

Even though some predictions of the BKS-theory were contradicted by experiments conducted by Bothe and Geiger[26] and the theory was hence abandoned, Bohr kept on claiming the incompatibility between a spatio-temporal continuous description of optical phenomena and a causal connection in individual transitions processes. If, in view of experimental results, this connection is to be accepted, then, Bohr argues, a continuos spatio-temporal description of optical phenomena cannot be achieved.[27]

The further development of Bohr's thought is addressed to reconsider how the formalism of quantum theory may acquire objective reference, given the incompatibility between a spatio-temporal and a causal account of phenomena. In this situation and in accordance with a strict Kantian way of thinking, Bohr is led to assert the necessity of *symbolic* analogies for a proper interpretation of the formalism of quantum theory:[28]

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[24] On the BKS theory see also Petruccioli (1993), pp. 111ff.

[25] Bohr, Kramers and Slater (1924), p. 101.

[26] Bothe and Geiger (1925).

[27] BCW 5, pp. 204–205.

[28] Pringe (2007). Although Chevalley acknowledges the Kantian origin of Bohr's concept of symbol, she argues that Bohr turns out to be a Kantian "heretic". See Chevalley (1995), p. 344. On the Bohrian notion of symbol see also Honner (1987), pp. 153–160.

> I feel … that we must take recourse to symbolic analogies to a still higher degree than before. Just lately I have been racking my brains trying to imagine such analogies.[29]

3 Complementarity

Bohr's solution to the problem of the objective reference of quantum theory bases on the consideration of *symbolic* analogies under the notion of *complementarity*.[30] The Bohrian position may be reconstructed as follows.

The objective reference of the theory is achieved when the mathematical formalism acquires empirical content. But empirical data are received in space and time. In a physical experiment these data must be constituted as objective cognitions for them to count as empirical *results* and thus to be more than illusions of mere subjective validity:

> The description of atomic phenomena has … a perfectly objective character, in the sense that no explicit reference is made to any individual observer.[31]

In order to obtain this objective character, Bohr underlines, the empirical data associated with atomic phenomena must be subsumed under the category of *causality*:

> [I]t should not be forgotten that the concept of causality underlies the very interpretation of each result of experiment, and that even in the co-ordination of experience one can never, in the nature of things, have to do with well-defined breaks in the causal chain.[32]

Specifically, *classical* concepts must be applied so that a spatio-temporal *and* causal description of the result of the observations is given,[33] for only classical concepts can simultaneously fulfil the demands of spatio-temporality and causality,[34] guaranteeing thereby the objectivity of the description:

> *However far the phenomena transcend the scope of classical physical explanation, the account of all evidence must be expressed in classical terms*. The argument is simply that by the word 'experiment' we refer to a situation where we can tell others what we have done and what we have learned and that, therefore, the account of experimental arrangement and of the results of the observations must be expressed in unambiguous language with suitable application of the terminology of classical physics.[35]

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[29] BCW 5, p. 85. Already in 1913 Bohr speaks of "the most beautiful analogi [sic] between the old electrodynamics and the considerations used in my paper" (BCW 2, pp. 584). In 1922 he discusses the issue of analogies in atomic physics with Høffding (BCW 10, pp. 513–514) and turns to this in his Nobel Lecture at the end of the year once again (BCW 4, p. 482). However, it is only after the failure of the BKS-theory that Bohr comes to consider these analogies as *symbolic* ones. See Chevalley (1994), pp. 37ff.

[30] See Pringe (2007), pp. 75ff.

[31] Bohr (1958b), p. 3.

[32] Bohr (1937), p. 87.

[33] "Strictly speaking, the idea of observation belongs to the causal spatio-temporal way of description." Bohr (1927), p. 67.

[34] "[T]he union of [the space-time co-ordination and the claim of causality] characterizes the classical theories." Bohr (1927), p. 54.

[35] Bohr (1949), p. 39.

Now the main issue arises: there is no single spatio-temporal and causal picture which can embrace the totality of the different images associated to an atomic system. Rather, a proper interpretation of all empirical data demands incompatible pictures, the paradigmatic example of which are the wave- and particle-pictures:

> Very striking illustrations are afforded by the well-known dilemmas regarding the properties of electromagnetic radiation as well as of material corpuscles, evidenced by the circumstances that in both cases contrasting pictures as waves and particles appear equally indispensable for the full account of experimental evidence.[36]

As long as all the different pictures are necessary for an exhaustive interpretation of the empirical data, while they exclude each other, the pictures are called *complementary*:

> [E]vidence obtained under different experimental conditions cannot be comprehended within a single picture, but must be regarded as *complementary* in the sense that only the totality of the phenomena exhausts the possible information about the objects.[37]

The classical pictures which we introduce to describe experimental results gained in different experimental contexts should be used as *symbols* of the quantum object. Accordingly, the quantum object is represented as behaving in certain experimental situations *as if* it *were* a particle or *as if* it *were* a wave:

> [W]e symbolize [the quantum object] by the abstractions of isolated particles and radiation.[38]

This symbolic character of the objective reference prevents the unjustified statements we would make if we affirmed that the object was *in fact* a particle in certain situations and *in fact* a wave in others, for in neither case does the object possess all the properties associated to the corresponding picture. In this respect, Heisenberg states:

> Both pictures (the particle- and the wave-picture) can only claim a right as analogies that sometimes apply and sometimes fail. In fact, it is, e.g., only experimentally proved that electrons behave like particles in certain experiments, but it is by no means shown that electrons possess all the attributes of the corpuscular picture. The same is valid mutatis mutandi for the wave-picture.[39]

These symbolic analogies, based on the *classical* pictures through which the experiments are described, provide the concepts of quantum objects with empirical content and, thus, the mathematical formalism of the theory with physical reference:

> [I]t continues to be the application of these [classical] concepts alone that makes it possible to relate the symbolism of the quantum theory to the data of experience.[40]

Moreover, precisely by taking part of these analogies, those incompatible classical descriptions of experiments acquire *systematic unity*. When we affirm that a quantum object behaves in certain situations as if it were a wave and in certain others as if it were a particle, we conceive the corresponding wave- and particle-like phenomena

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[36] Bohr (1956), p. 167.

[37] Bohr (1949), p. 40.

[38] Bohr (1927), p. 69.

[39] Heisenberg (1930), p. 7. Our translation.

[40] Bohr (1929), p. 16.

as phenomena *of the quantum object*. In this way, they become connected and subsumed under a common general representation. But thereby no spatio-temporal process by means of which the object causes the phenomena is assumed, for the demands of spatio-temporal co-ordination and causality are also *complementary*:

> The very nature of the quantum theory thus forces us to regard the space-time co-ordination and the claim of causality … as complementary but exclusive features of the description.[41]

In Kantian terms, the object is conceived as a cause, but the category of causality does not receive a spatio-temporal schematization.[42] This representation of the *quantum* object just brings about *systematic unity* among its phenomena and does not take part of their *constitution* as objective cognitions, which is rather the result of the application of *classical* concepts.

4 Conclusions: Correspondence and Complementarity

Let us consider the analogy with which we have begun our discussion once again: "C is the cause of E, as A is the cause of B." From 1913 to 1923 Bohr tries to establish analogies of this form in order to determine the quantum-theoretical relation between electronic motion and radiation on the basis of the knowledge of the classical laws. The principle of correspondence plays at this stage the role of a regulative maxim for our *reflection* upon nature. However, the analogies so obtained remain merely formal, because no causal and spatio-temporal exhibition of the mechanism of radiation is given. In Kantian terms, electronic motion is not *constituted* as the cause of radiation.

Moreover, the BKS-theory of 1924 implies that it is *impossible* to obtain a continuos *spatio-temporal* description of the way in which electronic motion grounds the probabilities of transitions, compatible with a *causal* relation between electronic motion and radiation in individual processes. In spite of the failure of the theory, Bohr does not reject this incompatibility between causal connections and spatio-temporal descriptions. Thus, Bohr's position entails now that electronic motion *cannot* be constituted as the cause of radiation in quantum theory, for this would demand a causal *and* spatio-temporal representation of the radiation mechanism. Electronic motion and radiation cannot be *directly* represented in a *single* intuitive picture.

The impossibility of a direct exhibition of quantum objects and processes in intuition leads Bohr to the introduction of the notion of complementarity in 1927. Within this new framework, the analogies of the type "C is the cause of E, as A is the cause of B" do not loose their central role. Rather, they are now used in order to exhibit the concepts of quantum objects *indirectly* in intuition. The analogies

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[41] Bohr (1927), pp. 54–55.

[42] Precisely for this reason, objective reality is accorded to the category by means of *symbols*.

possess in this case a mere *symbolic* character. Specifically, the relation between a quantum object and the empirical data is conceived in terms of one of the complementary pictures by means of which the experimental results are interpreted. Thereby, a quantum object is conceived as cause of the complementary phenomena, but no spatio-temporal representation of the causal relationship between object and phenomena is given.

In this situation, the correspondence principle retains its epistemological significance as a rule guiding an extensive application of classical concepts, but now in order to obtain *symbols* of quantum objects. These symbols are used as *complementary* pictures that, by exhausting the possible empirical information about the quantum objects, provide the mathematical formalism of the theory with complete physical reference. As Heisenberg indicates:

> The *Bohrian* correspondence principle states in its most general formulation that there is a qualitative analogy, which can be carried out in detail, between quantum theory and the classical theory belonging to the respective picture employed. This analogy does not just serve as a guide to the discovery of formal laws; its particular value is that it furnishes at the same time the physical interpretation of the discovered laws.[43]

By means of this use of the correspondence principle Bohr aims at guaranteeing the systematic unity of physics. Classical and quantum theories do not make up a "patchwork,"[44] but may be brought into a system. Their systematic unity is nevertheless not to be found at the end of a reduction program. But we must leave the discussion of this problem for another occasion.

References

Bohr's Collected Works (BCW), Rosenfeld, L., Rud Nielsen, J., Rüdinger, E., Aaserud, F. (eds.), North-Holland, Amsterdam; American Elsevier, New York, 1972 ff.
Bohr, N. (1913), "On the Constitution of Atoms and Molecules," in: BCW 2, pp. 161–233.
Bohr, N. (1918–1922), "On the Quantum Theory of Line Spectra," in: BCW 3, pp. 67–184.
Bohr, N. (1921), "On the Application of Quantum Theory to Atomic Problems," in BCW 3, pp. 364–380.
Bohr, N. (1922), *The Theory of Spectra and Atomic Constitution*, Cambridge University Press, Cambridge.
Bohr, N. (1927), "The Quantum Postulate and the Recent Development of Atomic Theory," in: Bohr (1934), pp. 52–91.
Bohr, N. (1929), "Introductory Survey," in: Bohr (1934), pp. 1–24.
Bohr, N. (1934), *Atomic Theory and the Description of Nature*, Cambridge University Press, Cambridge.
Bohr, N. (1937), "Causality and Complementarity," in: Bohr (1998), pp. 83–91.
Bohr, N. (1949), "Discussion with Einstein on Epistemological Problems in Atomic Physics," in: Bohr (1958a), pp. 32–66.

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[43] Heisenberg (1930), p. 78. Our translation.

[44] Cartwright (1999).

Bohr, N. (1956), “Mathematics and Natural Philosophy,” in: Bohr (1998), pp. 164–169.

Bohr, N. (1958a), *Atomic Physics and Human Knowledge*, Wiley, New York.

Bohr, N. (1958b), “Quantum Physics and Philosophy. Causality and Complementarity,” in: Bohr (1963), pp. 1–7.

Bohr, N. (1963), *Essays 1958–1962 on Atomic Physics and Human Knowledge*, Wiley, London.

Bohr, N. (1998), *Causality and Complementarity. The Philosophical Writings of Niels Bohr*, vol. IV, Faye, J. and Folse, H. (eds.), Ox Bow, Woodbridge, CT.

Bohr, N., Kramers, H. and Slater, J. (1924), “The Quantum Theory of Radiation,” in: BCW 5, pp. 99–118.

Bothe, W. and Geiger, H. (1925), “Über das Wesen des Comptoneffekts: ein experimenteller Beitrag zur Theorie der Strahlung”, *Zeitschrift für Physik*, 32, pp. 639–663.

Cartwright, N. (1999), *The Dappled World*, Cambridge University Press, Cambridge.

Chevalley, C. (1994), “Niels Bohr’s Words and the Atlantis of Kantianism,” in Faye, J. and Folse, H. (eds.), *Niels Bohr and Contemporary Philosophy*, Kluwer, Dordrecht, The Netherlands, pp. 33–55.

Chevalley, C. (1995), “On Objectivity as Intersubjective Agreement,” in Krüger and Falkenburg (eds.), *Physik, Philosophie und die Einheit der Wissenschaften*, Spektrum, Heidelberg, Germany, pp. 332–346.

Darrigol, O. (1992), *From C-numbers to Q-numbers. The Classical Analogy in the History of Quantum Theory*, University of California Press, Berkeley, CA.

Heisenberg, W. (1930), *Physikalische Prinzipien der Quantentheorie*, Hirzel, Leipzig, Germany.

Honner, J. (1987), *The Description of Nature. Niels Bohr and the Philosophy of Quantum Physics*, Clarendon, Oxford.

Kants gesammelte Schriften (AA), Königlichen Preußischen (Deutschen) Akademie der Wissenschaften, Berlin, 1902 ff.

Kant, I. (1781), *Critique of Pure Reason* (1781 = A, 1787 = B), Guyer, P. and Wood, A. (trans. and eds.), Cambridge University Press, Cambridge, 1998.

Kant, I. (1783), *Prolegomena to Any Future Metaphysics That Will Be Able to Come Forward as Science*, Hatfield, G. (trans.) in *Theoretical Philosophy after 1781*, Allison, H. and Heath, P. (trans. and eds.), Hatfield, G. and Friedman, M. (trans.), Cambridge University Press, Cambridge, 2002 (AA IV, pp. 253–384).

Kant, I. (1790), *Critique of the Power of Judgment*, Guyer, P. and Matthews, E. (trans.), Guyer, P. (ed.), Cambridge University Press, Cambridge, 2000 (AA V, 165–486 and AA XX, pp. 193–251).

Petruccioli, S. (1993), *Atoms, Metaphors and Paradoxes. Niels Bohr and the Construction of a New Physics*, Cambridge University Press, Cambridge.

Pringe, H. (2007), *Critique of the Quantum Power of Judgment. A Transcendental Foundation of Quantum Objectivity*, de Gruyter, Berlin.

The Convergence of Transcendental Philosophy and Quantum Physics: Grete Henry-Hermann's 1935 Pioneering Proposal[1]

Léna Soler

Abstract In the course of 1934, Grete Henry-Hermann, a young German pupil of Leonard Nelson trained in philosophy as well as in mathematics and physics, visited Heisenberg in Leipzig, with the intention to call Kant before the tribunal of history, hoping to be able to conciliate transcendental philosophy with the new quantum physics. Do Kant's propositions still hold good for a physics which seemed to break so radically and scandalously with the principles of the physics retrospectively called 'classical'? Grete Henry-Hermann's attempt to confront quantum physics with Kantian philosophy is, chronologically, one of the first: she

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[1] I came to hear about Grete Henry-Hermann's writings during my DEA study (the university year just before a Ph.D.) devoted to the relationship between Kantian philosophy and quantum mechanics. It was Michel Bitbol, then my DEA supervisor, who drew my attention upon this writings and encouraged me to work on it. I am happy to have the occasion to thank him warmly here, for this indeed but more fundamentally for the inspiration I always found in his way of doing philosophy and for his friendly support.

I initiated the translation of Grete Henry-Hermann's most important text into French and edited it with an introduction and a long critical postface. It was published in 1996, by Vrin (*mathesis* collection), as *Les fondements philosophiques de la mécanique quantique* (translation by Alexandre Schnell, presentation by Bernard d'Espagnat). The present paper is based on my analyses in this 1996 book. The English version is largely inspired from a conference speech, delivered on second March 2001 in Bremen (in German thanks to the collaboration of Alexander Schell), at the invitation of the Philosophisch-Politische Akademie on the occasion of the Grete Henry-Hermann Centenary Celebration. A French version of this lecture has been translated in English by Dr Edmund Jephcott (A & G Translations) at the request of the Society for the Furtherance of Critical Philosophy (SFCP). This English translation has been completed by professor Fernando Leal's translation of some of Grete Henry-Hermann's original work and has been edited by the SFCP ("The Contributions of Grete Henry-Hermann to the Philosophy of Physics", Occasional Working Papers in Ethics and the Critical Philosophy, ed. by P. Shipley and H. Mason for the SFCP, vol. 3, December 2004, 23–31. Interested readers can acquire a copy of this volume 3 from Keith Martin, SFCP Administrator, 148 Friern Park, London, N12 9LU).

M. Bitbol et al. (eds.), *Constituting Objectivity*,

was truly a pioneer in engaging in a philosophical interpretation of quantum physics. The paper presents and criticize the positions of Grete Henry-Hermann, and especially:

(1) Her original attempt to save the kantian category of causality, by on the one hand retaining the universal validity of the pure concept of causality, while on the other hand accepting, with Bohr and Heisenberg, the definitive character of statistical predictions. Her attempt consists in showing that once a result has been actually obtained, it is possible, by working backwards, to reconstitute, *retrospectively and completely*, the causal chain which has necessarily produced such a result.
(2) Her refutation of the so-called von Neumann's proof. Grete Henry-Hermann's essay of 1935 contains the first critique of von Neumann's argument which aimed to demonstrate the impossibility of completing quantum physics by means of hidden parameters, and which has been regarded as firmly established until 1964, that is, until the year Bell published his famous refutation. Yet, reading Grete Henry-Hermann's essay, one discovers that in 1935, *thirty years before Bell*, Grete Henry-Hermann had produced a refutation of von Neumann, based on arguments very similar to those of Bell in 1964. Had Grete Henry-Hermann's refutation not remained a dead letter, the history of the interpretations of quantum physics would certainly have been very different.

In the 1930s, Grete Henry-Hermann (1901–1984) was a young woman trained in physics, mathematics and philosophy. She was deeply convinced that transcendental philosophy is the best framework to articulate both ethical and epistemological issues. At the time, it was more and more said, here and there in philosophical and scientific circles, that the (then just born) quantum mechanics refuted Kantian philosophy, especially the Kantian table of categories and its concept of causality. Having heard about this, Grete Henry-Hermann decided to tackle the problem. She worked on it more than a year and submitted her proposals to Werner Heisenberg and Carl Von Weizsäcker. At the end she published arguments which aim to show, first that the Kantian category of causality still constitutes a necessary condition of possibility of quantum physics, and second that, more fundamentally, quantum physics and critical philosophy manifest an essential structural convergence that reinforce the strength of transcendental philosophy.

This paper will present, and sketch the significance, of Grete Henry-Hermann's interpretation of the relationships between transcendental philosophy and quantum physics.[2] Grete Henry-Hermann's contributions to the philosophy of physics, although almost unknown, especially outside Germany, will appear very important

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[2] For a more detailed account, see my 1996 introduction and postface (from the Vrin reference given in note 1).

in an historical perspective, as well as of great philosophical interest from a contemporary point of view, especially for a transcendentally-oriented philosopher.

1 The Philosophy of Science in Grete Henry-Hermann's Work: Motivation and Situation

Grete Henry-Hermann was a pupil and great admirer of Leonard Nelson, and she followed her teacher in concerning herself primarily with ethics and political philosophy. Comparatively, her works on the philosophy of science occupy a relatively marginal position. What caused Grete Henry-Hermann to take an interest in physics?

Grete Henry-Hermann, following Nelson, considered Kant's philosophy, or more precisely its reinterpretation by Fries, to be the basis on which twentieth-century philosophy should unfold. Yet, around the 1930s, a number of participants in the debates concerning quantum mechanics, still a very young science at that time, took the view that the new physics called into question, indeed definitively refuted, some fundamental aspects of the critical philosophy inaugurated by Kant. Given the centrality of transcendental philosophy for Grete Henry-Hermann, it was of crucial importance to her to investigate whether twentieth-century physics *does* or does *not e*ffectively refute the fundamental principles of such philosophy.

The principal published texts in which Grete Henry-Hermann sets out her reflections on this subject are relatively few in number and are confined to a limited period: they correspond to three relatively short essays produced during the years 1934–1937:

1. 'Die naturphilosophischen Grundlagen der Quantenmechanik' ('The Philosophic Foundations of Quantum Mechanics'), 1935[3]
2. *Die Bedeutung der modernen Physik für die Theorie der Erkenntnis* ('*The Significance of Modern Physics for the Theory of Knowledge*), 1937[4] and

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[3] Published in *Abhandlungen der Fries'schen Schule*, Neue Folge, 1935. Leonard Nelson edited this from 1904 (reviving the enterprise begun by two followers of Fries, Apelt and Schleiden, and which was continued from 1847 to 1849, until being interrupted during the 1848 Revolution because of political disagreements between the editors). A summary of this essay by Grete Henry-Hermann was also published in *Die Naturwissenschaften*, 23, no. 42, 1935, 718–721.

[4] Pages 1–44 from G. Hermann, E. May, Th. Vogel, *Die Bedeutung der modernen Physik für die Theorie der Erkenntnis: Drei mit dem Richard-Avenarius-Preis ausgezeichnete Arbeiten (The Significance of Modern Physics for the Theory of Knowledge: Three Essays awarded the Richard Avenarius Prize)*, Verlag von S. Hirzel, Leipzig, 1937.

3. 'Über die Grundlagen physikalischer Aussagen in den älteren und der modernen Theorien' ('On the Foundations of Physical Statements in Earlier and Modern Theories'), 1937[5]

The truly major text – the one which contains Grete Henry-Hermann's fundamental theses and arguments – is that of 1935, which deals with quantum mechanics. The later texts either revisit the developments of 1935 or apply analytical principles similar to those of 1935 to the theory of relativity, and arrive at convergent conclusions.[6] For this reason, the essay of 1935 will form the primary focus of this paper.[7]

2 Physics and Causality in the 1930s

As we saw, Grete Henry-Hermann's first concern is the compatibility/incompatibility between transcendental philosophy and twentieth century physics. In the 1935 essay, the reflection is more especially focused on the relationship between causality and quantum physics.

Grete Henry-Hermann stated the problem as follows:

- Kant listed the conditions of possibility of knowledge, and therefore, in particular, the conditions of possibility of *any future physics*.
- These conditions include the category of causality, which seems to indicate that the predictions of any science worthy of the name (for Kant, of any physics) must be strictly deterministic in the following sense: there must be a *univocal (one-to-one) connection between cause and effect* (between the initial conditions

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[5] Published by Öffentliches Leben, Leipzig 1937; also in: *Abhandlungen der Fries'schen Schule*, Neue Folge, 6, no. 3/4, 1937, 309–398.

[6] Grete Henry-Hermann's contributions to the philosophy of science comprise four other articles:

The first, from 1935, *Physikalishe Zeitschrift*, 36, 481–482, is a review of the work by Karl Popper *The Logic of Scientific Discovery*, which was written in 1934 and later became famous.

The second is the text of a short talk given in Copenhagen in June 1936, at the Second International Congress for the Unity of Science: 'Zum Vortrag Schlicks', *Erkenntnis*, vol 6, book 5/6, 342. This was a reply to a paper by Moritz Schlick regarding the causal problem, in which Grete Henry-Hermann brought to bear the theses of 1935.

The third, 'Die Naturphilosophische Bedeutung des Übergangs von der klassischen zur modernen Physik' *(*'The Philosophic Significance of the Transition from Classical to Modern Physics'), represents Grete Henry-Hermann's contribution to the 9th International Congress of Philosophy, 'Congrès Descartes', held in Paris in 1937 (in: *Travaux du IXe Congrès International de Philosophie 'Congrès Descartes'*, published by Raymond Bayer, Paris, VII, 99–101); in it the conclusions from the essay of 1935 are again summarised.

Finally, an article of about ten pages from 1948, entitled 'Die Kausalität in der Physik' ('Causality in Physics') (*Studium Generale*, vol. 1, book 6, pp. 375–383), provides an extremely clear synthesis of Grete Henry-Hermann's previous works.

[7] It is this essay that was translated into French in 1996, see note 1.

and the final conditions); one and the same cause can produce only *one single*, well-defined effect.
- Now, the predictions of quantum mechanics are *statistical*: one and the same initial state can be followed by *several different* final states, and one knows *in advance* (before any effective experiment) *only* the *probability* of each possible result.
- Should we therefore conclude that the Kantian category of causality is refuted by the new physics? That strict causality is actually not a *necessary* condition of any physics?

Grete Henry-Hermann was not the first to frame the problem in these terms.

The general strategy underlying such a framing was to call Kant before the tribunal of history, according to the following structure of reasoning: in listing the conditions determining the possibility of any future physics, Kant had drawn his inspiration from the physics of his time (that of Newton, retrospectively called 'classical'); now, do his propositions still hold good for post-kantian physics? Can the kantian conditions of sensibility and understanding be maintained, confronted in particular with twentieth century physics, which seemed to break so radically with the fundamental principles of classical physics?

A number of philosophers and physicists had already applied such a line of reasoning to the case of the non-Euclidian geometries and the theory of relativity.[8] They had notably inquired whether relativity theory refuted the conditions of sensibility set out by Kant, equated with Euclidian space and absolute time. With the emergence of quantum physics, it was the turn of the pure concepts of understanding to come under threat. And, at first sight at least, it was primarily the concept of causality which seemed to be refuted.

These were the questions on the side of Kantian philosophers. Now, philosophers were not the only ones to be concerned by causality in the 1930s. Causality was also preoccupying physicists themselves. On their side the debate essentially concerned the so-called 'hidden variables', and the corresponding problem was framed through the following alternative:

- Should it be admitted that quantum physics and its statistical predictions express no more than a deficiency of human knowledge? Or stated differently, should it be admitted that certain variables, as yet unknown to physicists, do actually exist and univocally determine all measurement results? Are there 'hidden variables' – or parameters – which, if known, would enable any given cause to be in a one-to-one correspondence with a single definite effect?
- Or should the statistical character of predictions, that is, the association of a plurality of possible effects with a given cause, be recognized as definitive because it expressed a fundamental aspect of phenomena, or a fundamental aspect of the relationship between human beings and the physical world?

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[8] See for example M. Schlick, *General Theory of Knowledge*, Springer, New York, 1974, especially sections 37–40.

Grete Henry-Hermann addressed both the above questions, those of the philosophers as well as those of the physicists. Here are the conclusions she finally reached, briefly summarized:

- Quantum physics does not refute the category of causality.
- There are no hidden variables; quantum physics is complete and its predictions will remain statistical.

Considered in detail, Grete Henry-Hermann's conclusions, and the arguments supporting them, are complex and subtle. A full understanding of them would require an extensive discussion drawing simultaneously on philosophy, physics and history.[9] In this paper, attention will be confined to the following:

- Some noteworthy points which give Grete Henry-Hermann's 1935 essay its major interest (part III)
- Indications of what makes up the central originality of Hermann's thesis concerning quantum physics (part IV)
- Some remarks concerning the strengths and weaknesses of Hermann's interpretation (part V) and
- Some general conclusions of Hermann concerning the relationship between critical philosophy and quantum physics (part VI)

3 Noteworthy Aspects of Grete Henry-Hermann's Work in the Philosophy of Science

1. Grete Henry-Hermann's attempt to confront quantum physics with Kantian philosophy is, chronologically, one of the first. Philosophers of science are often accused of lagging behind the advancement of science. For once, the accusation does not apply. Grete Henry-Hermann was truly a pioneer in engaging in a philosophical interpretation of quantum physics.[10]

Indeed, in 1934 quantum physics was a physical theory worthy of the name for only a few years. Since the Solvay Congress of 1927, the term 'quantum mechanics' had referred to something fairly definite and stable: a formalism which was a synthesis of Heisenberg's

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[9] See Léna Soler, postface to G. Hermann, *Les fondements philosophiques de la mécanique quantique*, Vrin, 1996.

[10] With a few others, for example: Alexandre Kojève, *L'idée du déterminisme dans la physique classique et dans la physique moderne* (*The Idea of Determinism in Classical and Modern Physics*), Paris, Le Livre de Poche (written in 1932, but only published poshtumously in 1990); Gaston Bachelard, *Le nouvel esprit scientifique*, Paris, Presses Universitaires de France, 1934 (published in English as *The New Scientific Spirit*; Beacon Press, 1986); Ernst Cassirer, 'Determinismus und Indeterminismus in der modernen Physik', *Göteborgs Högskolas Arssskrift*, XLII, 1937, (published in English as 'Determinism and Indeterminism in Modern Physics': *Historical and Systematic Studies in the Problems of Causality*, New Haven, Yale University Press, 1956), etc.

matrix mechanics, Schrödinger's wave mechanics and Dirac's theory of transformations – together with a global interpretation of this formalism, proposed by Bohr and Heisenberg and later called the 'Copenhagen interpretation' or the 'orthodox interpretation'. Grete Henry-Hermann was to adopt numerous ingredients of this interpretation in her own account: the complementarity of wave and particle representations, and of the conjugate variables of position and velocity; the idea that the results obtained have validity only in relation to an experimental situation, etc.

2. Grete Henry-Hermann had a twofold training, scientific (physics and mathematics) and philosophical. This is uncommon enough to deserve mention, and clearly is not without relevance to her contributions to the philosophy of science. Indeed, Grete Henry-Hermann had sufficient mastery of physics to be able to study in depth the theory and its formalism, and to engage in high-level dialogue with scientists.
3. Grete Henry-Hermann's text of 1935 is the outcome of discussions she held, during 1 year in Leipzig, with Heisenberg and a group of major physicists who were the originators of quantum physics.

Leipzig was one of the centres which contributed most, with Göttingen and Copenhagen, not only to the development of quantum theory but to the clarification of its philosophical foundations. Werner Heisenberg, the famous pioneer in this venture, organised a seminar there, bringing together a considerable number of eminent scientists, such as the Swiss Félix Bloch, the Soviet Landau, and from Germany, Peierls, Karl Friedrich Hund and Edward Teller, as well as Carl Friedrich von Weizsäcker who was still very young at that time. The latter, despite his training as a physicist, took a passionate interest in the philosophical questions raised by the new physics, and for this reason, he was to play a leading role in the dialogue with Grete Henry-Hermann.

In the course of 1934, Grete Henry-Hermann went to Leipzig in order to participate in Heisenberg's seminar. It seems that her decision resulted from the following events.[11] G. Hermann wrote a manuscript dealing with causality in quantum physics which she sent to Bohr and Heisenberg. Bohr asked von Weizsäcker to read the manuscript and, possibly, to respond to it. Von Weizsäcker did so, indicating in his letter in what way the theses of the article seemed erroneous to him. Having received a letter with very similar content from Heisenberg, Grete Henry-Hermann decided to travel to Leipzig, to discuss the matter with the two physicists in person.

It was at the end of one year of debate with these prestigious figures that she wrote the essay of 1935. By the end of these discussions it seems that she had succeeded in convincing Heisenberg of her point of view.[12] In his scientific autobiography, *Physics and Beyond*, Heisenberg devoted an entire chapter to the

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[11] According to an unpublished interview of von Weizsäcker by T.S. Kuhn, dated 9 July 1963 (for references to this interview, cf. Archives for the History of Quantum Physics, J.L Heilbron and T.S. Kuhn, *Sources for the History of Quantum Physics: an Inventory and Report*, The American Philosophical Society, Philadelphia, PA, 1967).

[12] To begin with, at least; he seems later to have changed his mind under the influence of Bohr.

discussions between Grete Henry-Hermann, von Weizsäcker and himself. This chapter, entitled "Quantum Mechanics and Kantian Philosophy", presents the content of the arguments and their progress, together with the compromise agreement which emerged from them. His tone at the end is quite positive: "*we had the feeling that we had all learned a good deal about the relationship between Kant's philosophy and modern science*".[13]

Von Weizsäcker, for his part, reviewed Grete Henry-Hermann's essay in highly eulogistic terms in an article published in 1936 in *Physikalische Zeitschrift*,[14] presenting Grete Henry-Hermann's essay as the first "*positive and indisputable contribution to elucidating the implications of quantum mechanics for the theory of knowledge*", and adding that "*a fruitful debate on this subject could hardly be opened in a clearer or more objective manner*". Moreover, in his book *The World View of Physics*,[15] von Weizsäcker develops a conception of the relationship between critical philosophy and quantum physics which in many respects is akin to that of Grete Henry-Hermann.[16]

4. In addition to the original ideas she developed on the relationship between the Kantian category of causality and quantum physics, to which I shall return below, Grete Henry-Hermann's essay of 1935 contains the first critique of von Neumann's argument which aimed to demonstrate the impossibility of completing quantum physics by means of hidden parameters.[17]

In 1931, von Neumann claimed to prove that the statistical character of quantum physics was not due to a deficiency in human knowledge, and that it was therefore pointless to hope to discover hidden variables. Quantum mechanics is complete, he claimed, in the sense that the physicist already knows everything there is to be known. According to him, the predictions of quantum physics are statistical, because quantum phenomena themselves are not deterministic (in the sense that given one and the same initial conditions, different final conditions can indeed follow).

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[13] Werner Heisenberg, *Physics and Beyond*, London: Allen & Unwin, 1971, 124. Original German published in 1969.

[14] C.F von Weizsäcker, review of Grete Henry-Hermann, 'Die naturphilosophischen Grundlagen der Quantenmechanik', *Physikalische Zeitschrift*, 37, 14, 1936, 527–528.

[15] C.F. von Weizsäcker, *The World View of Physics*, London: Routledge and Kegan Paul, 1952.

[16] In the 1963 unpublished interview of von Weizsäcker by T.S. Kuhn mentioned above in note 11, von Weizsäcker, addressing the question of the relationship between physics and philosophy, emphasised that around 1933–1934 the Leipzig group formed a unified bloc defending the new ideas associated with physics against attacks by philosophers. He then went on spontaneously to speak of Grete Henry-Hermann, emphasising above all, in the brief account he gave of her, her twofold training in mathematics and philosophy. Von Weizsäcker referred to her as an extremely intelligent person and remarked that her great clarity of mind made discussion with her easy. He added that Grete Henry-Hermann was probably right in maintaining that Kantian philosophy, correctly interpreted, was in no way placed in difficulty by modern physics, itself correctly interpreted. Then he alluded to Hermann's manuscript dealing with causality in quantum physics.

[17] Max Jammer seems to be the first (and as far as I know the only one) to notice this point. Cf. *The Philosophy of Quantum Mechanics*, A Wiley-Interscience Publication, 1974, p. 272.

Historically, the so-called ‘von Neumann’s proof’ played a very important role. Indeed, up to 1964 it was regarded as firmly established, and between 1931 and 1964, its existence and supposed validity deterred physicists from trying to develop theories of hidden variables. The situation changed in 1964, because in that year, John Bell attacked Neumann’s ‘proof’,[18] which has since then appeared to specialists to have been definitively refuted. Yet in 1935, *30 years before Bell*, Grete Henry-Hermann had produced a refutation of von Neumann, based on arguments very similar to those of Bell in 1964.

Indeed, Grete Henry-Hermann identified as problematic the premise of von Neumann’s reasoning on which, 30 years later, John Bell was to base his famous refutation. This premise, the condition of additivity, stipulates that the expectation value of the sum of two physical quantities is equal to the sum of each of their expectation values. Such a property, trivial for variables capable of being measured simultaneously (variables of classical physics or non-conjugate variables of quantum physics) needs to be proved in the case of the conjugate quantities of quantum physics.

Grete Henry-Hermann and Bell both insist on this point. This is really striking when their statements are put side by side.

In Grete Henry-Hermann’s terms:

> The sum of two of these quantities is by no means immediately definite. Because precise measurement of one of them excludes that of the other, so that these two quantities cannot simultaneously admit precise values, the conventional definition of the sum of two quantities is no longer valid. It is only by means of a detour through certain mathematical operators associated with these quantities that formalism introduces the concept of the sum of such quantities’.[19]

In John Bell’s terms:

> A measurement of a sum of noncommuting observables cannot be made by combining trivially the results of separate observations on the two terms – it requires a quite distinct experiment. […] The additivity of expectation values […] is a quite peculiar property of quantum mechanical states, not to be expected a priori.[20]

Or again, in the words of Grete Henry-Hermann, von Neumann’s argumentation

> Although indisputable from a mathematical point of view, introduces into its formal premises, without justifying it, a statement equivalent to the thesis which it is supposed to demonstrate. […] Expressed verbally: The expectation value of a sum of physical quantities is equal to the sum of the expectation values of the two quantities. Von Neumann’s entire demonstration rests on this presupposition and collapses with it.[21]

And in the words of Bell

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[18] J.S. Bell, ‘On the problem of hidden variables in quantum mechanics’, *Review of Modern Physics*, 38, 447–452, 1966.

[19] ‘Die naturphilosophischen Grundlagen der Quantenmechanik’ (‘The Philosophic Foundations of Quantum Mechanics’), 1935, section 7.

[20] J.S. Bell, *Speakable and Unspeakable in Quantum Mechanics*, Cambridge University Press, Cambridge, 1987, p. 4.

[21] Hermann, ibid., section 10.

V*on Neumann's essential assumption is*: Any real, linear combination of any two Hermitian operators represents an observable, and the same linear combination of expectation values is the expectation value of the combination'.[22]

Now, despite this similarity between Hermann and Bell's arguments, and despite the fact that Bell's paper rapidly convinced all physicists after its publication in the 1960s, Hermann's refutation had no impact: in fact it remained entirely unknown – and this is highly surprising if one bears in mind that physicists such as Heisenberg and von Weizsäcker must have known of it.

Whatever the reasons may be, the fact has important historical implications. Indeed, if Grete Henry-Hermann's refutation had not remained a dead letter, the history of the interpretations of quantum physics would certainly have been very different. Theories involving hidden variables, which have proliferated since the 1960s, would probably have flourished much earlier, and the Copenhagen interpretation, so long regarded as the only acceptable available interpretation, would perhaps have enjoyed a less exclusive monopoly.[23]

4 The Core of Grete Henry-Hermann's Original Interpretation

Let us now turn to Grete Henry-Hermann's way of conceiving the links between quantum physics and Kantian philosophy. With her refutation of von Neumann's proof Grete Henry-Hermann had re-opened the door to the possibility of discovering hidden variables. One might therefore believe that she was about to engage in an attempt to save the Kantian category of causality, by invoking, as others had done, the existence of hidden variables determining a unique effect for each cause. But that is not the case. Grete Henry-Hermann set out on another original path: that of retaining the universal validity of the pure concept of causality, while accepting, with Bohr and Heisenberg, the definitive character of statistical predictions.

The original core of her interpretation is, in essence, the following. The results of measurements actually obtained for quantum objects cannot be univocally predicted with certainty. However, *after* having effectively obtained a quantum measurement,

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[22] Bell, ibid., 4.

[23] Even the habits underlying the theoretical practices of physicists, including judgments of simplicity, could perhaps have been substantially transformed (and this is important, since the main argument, today, against the main available theory of hidden variables in quantum physics, namely the Bohmian interpretation of quantum physics, is its alleged lack of simplicity: it is said to be less simple than the standard interpretation of quantum mechanics. I develop this point, in relation to the question of the contingency of our history of science, in *Philosophie de la physique: dialogue à plusieurs voix autour de controverses contemporaines et classiques*, entre Michel Bitbol, Bernard d'Espagnat, Pascal Engel, Paul Gochet, Léna Soler et Hervé Zwirn, Léna Soler (ed.), L'Harmattan, collection 'Epistémologie et philosophie des sciences', 2006. See also James T. Cushing, *Quantum Mechanics. Historical Contingency and the Copenhagen Interpretation*, The University of Chicago Press, Chicago and London, 1994.

and *after* having gained knowledge of its result (previously not predictable with certainty), it is possible, by working backwards, to reconstitute, *retrospectively and completely*, the causal chain which has necessarily produced such a result.

To properly grasp what is meant by these causal chains reconstructed *a posteriori*, it is necessary to emphasise that the causal chains under consideration connect:

- On the one hand, a phenomenon (the phenomenon resulting from measurement – for example, a spot on a screen)
- And on the other hand, the value of a theoretical variable (for example, the value of the quantity of movement)

The effect is the phenomenon resulting from the measurement. The cause is the value of the variable.

This type of causal chain already plays a part in classical physics. For example, when you measure a weight, the observed phenomenon equated with the effect is the needle movement and its stopping on the scale at a certain graduation mark; and the value of the theoretical variable equated with the cause of such a phenomenon is the determinate weight of the object weighed. The link between the two is a causal scenario of the following kind: the weight causes the vertical displacement of the scale pan, which in turn causes, through a series of specifiable mechanical actions, the deflection of the needle.

Grete Henry-Hermann transposed this classical theory of measurement to the case of quantum measurements. For example, take the case of measuring the quantity of movement of an electron by illuminating this electron under a microscope. Because the electron is illuminated, there is an interaction between the electron and the incident light; this light is then captured on a screen; from it, information about the electron is derived. The effect, the phenomenon resulting from the experiment, is a discrete impact on the screen. The cause is the quantity of movement of the electron at the instant of interaction with the incident light.

There are two differences between the quantum and the classical cases:

1. Contrary to the situation in classical physics, in quantum physics the causal scenario cannot be anticipated; it is known only once the measurement has actually been performed and only once the phenomenon obtained has actually been observed.
2. In quantum physics, although the causal scenario connecting the cause to the effect continues to make use of classical concepts such as wave or particle, it involves, at the same time and with regard to the same object, representations which, according to the classical account, are antagonistic. Here Grete Henry-Hermann reverts to Bohr's idea of complementarity. The same physical system, depending on the moments of the same scenario, is treated now as a wave, now as a particle. For instance, in our example, the light which interacts with the electron is treated firstly as corpuscular (the situation is represented as an electron–photon collision), secondly as undulatory (after the interaction, the light is seen as a flat wave which passes through the lenses of the microscope and converges at a point on the screen).

For every measurement carried out on a quantum object it is possible, according to Grete Henry-Hermann, to reconstitute, *a posteriori*, a causal chain of this type. In addition, Grete Henry-Hermann proposes a verification procedure, which she calls a “mediate procedure”, for her *a posteriori* causal reconstitutions. By this procedure, which cannot be discussed at length here,[24] she believed she had proved that her causal scenarios were not only possible, but also necessary.

From the above elements taken altogether, Grete Henry-Hermann draws the following conclusions:

1. Because the causes of any phenomenon resulting from a quantum measurement can always be univocally determined (albeit only *a posteriori*); because a single causal scenario continues in quantum physics to connect the phenomena resulting from the measurement to the theoretical variables; then, the Kantian category of causality remains a necessary condition of quantum physics.
2. Because one is already in possession of all the causes which determine any result of measurement, the hypothesis of possible hidden variables loses all credibility; to seek additional parameters which are supposed to put an end to the statistical character of the quantum description becomes, in principle, pointless.

5 Strengths and Weaknesses of Grete Henry-Hermann’s interpretation

The main strength of Grete Henry-Hermann’s interpretation, the essential point which she establishes, is that, in order to exist, both classical physics and quantum physics require that the physicist be able to establish a *one-to-one* connection between:

- The great diversity of phenomena which constitute the results of measurement (impacts on screens, deflections of needles, etc.) and
- The values of a limited number of variables involved in theory (position, velocity, quantity of movement, etc.).

Judiciously, Grete Henry-Hermann places the accent on the *only one-to-one connection which remains necessary to the very existence of quantum physics*. If the physicist were unable to interpret univocally a given phenomenon resulting from measurement *as* the definite value of *this theoretical variable*, the phenomena constituting the results of measurement *would lose all meaning*, all connection with our theories. With this, Grete Henry-Hermann emphasises something crucial.

The weak point of Grete Henry-Hermann’s thesis, is that she is not content with asserting that the one-to-one character of the connection under consideration is a

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[24] See L. Soler, introduction and postface to G. Hermann, *Les fondements philosophiques de la mécanique quantique*, Vrin, 1996.

condition of the possibility of physics. She goes much further in interpreting this one-to-one connection, since she asserts:

- That this connection is *causal in type*
- That the concept of causality involved *is essentially similar* to the Kantian concept of causality and
- That the *a posteriori* causal scenario is *necessary*

Each of these three points is open to question.

Take the latter one. One can readily imagine, in linking the cause to the effect, other scenarios than the ones proposed by Grete Henry-Hermann. And since there is no means of deciding between the alternative propositions, it undermines the assumed necessity of Henry-Hermann's scenarios.

Turning to the two other points, one might stress that Grete Henry-Hermann's causal scenarios connect the phenomena resulting from measurement to *only one* of the two *conjugate variables*, and that it remains impossible *to bring together, at the same time, two of these causal chains*, one of which would culminate in one of the variables (for instance the position) and the other in the conjugate variable (for instance the velocity). Hence Grete Henry-Hermann's interpretation, in no way allows the conjugate variables to be combined, and therefore, in no way allows the reconstitution of the continuous trajectory of an object. Now, it is precisely on the basis of the possibility of gaining access to such continuous trajectories, that classical physicists conceived causality. For them, causal behaviour meant that the values of two conjugate variables of an object at a given time (position *and* velocity equated with *the* cause) univocally determined the subsequent trajectory (position *and* velocity at a later time equated with *the* effect). Here one can readily attack Grete Henry-Hermann's conclusions by claiming that the concept of causality involved is very different from the classical, Kantian concept of causality (or at least cannot be identified with it).

6 A General Comparison Between Transcendental Philosophy and Quantum Physics

Having shown that the category of causality – the Kantian category which seemed the most threatened by the advent of quantum physics – continued to constitute a condition of possibility of quantum physics, Grete Henry-Hermann went on to consider, on the most general level, the question of the relationship between quantum physics and transcendental (or critical) philosophy.

Her conclusion is that quantum physics and transcendental philosophy converge with regard to essentials, at least if 'transcendantal philosophy' is understood to mean, as Grete Henry-Hermann understood it, not the Kantian system taken in its precisely literal form, but Kantian philosophy as re-read, clarified and reinterpreted by Fries.

Grete Henry-Hermann compares the principal assertions of critical philosophy and quantum mechanics on three points:

(a) For critical philosophy, the Kantian categories "provide the theoretical schema needed to interpret perception".[25] Now, in order to interpret the results of measurement, quantum physics must necessarily make use of classical concepts. Thus, in classical physics, as in quantum mechanics, the *same* fundamental classical concepts mediate the transition from the diverse material of sense data to knowledge of nature, although in the second case their applicability is limited.[26] Quantum physics and classical physics therefore rest, once and for all, on the same conditions of possibility. The *a priori* forms listed by Kant are not specifically threatened by the advent of quantum physics.

(b) What has just been said implies that quantum physics also does not call specifically into question the assertion of critical philosophy that the table of Kantian categories is complete, i.e. that Kant's twelve pure concepts are *sufficient* to order the flux of sensations for knowledge. The advent of quantum theory obliges us neither to add a pure concept to the table, nor to remove or modify one.

(c) If one subscribes to the clarification, carried out by Fries, then by Nelson, of the true implications of the Kantian theses, critical philosophy also shows that the application of the categories to the diversity of phenomena remains limited, in the sense that the pure concepts are only ideal models which, as "simple analogies", provide the "guiding thread to the interpretation of perception".[27] This means that description extends only to the structures of connections, but *never*, properly speaking, isolates *absolute* substances, causes or effects. Description therefore remains relative. Nevertheless, the structures of connections represent spatio-temporal relationships which are objective and unequivocally determined.

Quantum mechanics confirms the limits of the application of the fundamental concepts which make knowledge possible: the classical concepts, like the categories,

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[25] Hermann, op. cit., section 17.

[26] Of course, to conclude from this that the Kantian categories continue to constitute the conditions of possibility of quantum physics, it would also be necessary to have demonstrated that the entire edifice of classical physics does actually rest on such categories. According to Grete Henry-Hermann, such a demonstration remains to be produced and goes outside the framework of her own essay. Kant apparently believed he had provided such a proof, at least with regard to the physics of his time. In *The Metaphysical Foundations of the Science of Nature*, 1786 (published as *Metaphysical Foundations of Natural Science*, London, Bobbs Merill, 1970) he sets out to demonstrate that the twelve categories listed in the *Critique of Pure Reason* do indeed constitute the necessary conditions of the possibility of physics. The demonstration, when examined, appears to posit, as the foundation of physics, laws which are essentially similar to the fundamental principles of Newtonian physics (conservation of matter, principle of inertia and law of action and reaction). Now, a modern epistemology could hardly have recourse to such a demonstration to prove that the Kantian categories continue to constitute the conditions of the possibility of post-Kantian physics, and this already holds for post-Kantian *classical* physics (for example electromagnetism).

[27] Hermann, op. cit., section 17.

are no more than analogies, which should not be understood literally. Classical physics is concerned only with differential equations within which nothing refers properly speaking to substances, causes or effects, although such concepts remain indispensable in guiding research and organising the diversity of perception into a knowledge of macroscopic objects. In the same way, quantum physics does not enable the systems it describes to be identified with waves or particles properly speaking, although it cannot do without such concepts in organising the diverse material of perception into a knowledge of atomic phenomena.

In fact, quantum physics goes still further than transcendental philosophy. It confirms that physics has access only to structures of connections, and shows *in addition* that these structures of connections *are in each case relative to the experimental situation* by means of which the experimenter gains knowledge of them. This, according to Grete Henry-Hermann, is the major teaching of the philosophy of the new physics: quantum mechanics, far from contradicting the fundamental principles of transcendental philosophy, radicalises them still further.

Finally, quantum mechanics, like transcendental philosophy but to a still greater degree, forces us to abandon the dream of a universal science capable of embracing all aspects of reality within a single description. Indeed, not only is knowledge, as Kant's analysis had already shown, divided into different types of description (psychology, physics, ethics, etc.), which constitute several perspectives on the world; but, in addition, as is shown by quantum theory, the disintegration of truth into a multitude of perspectives is now infiltrating the very heart of physics itself:

> The novelty introduced by quantum mechanics with regard to the philosophy of nature can be described as follows: the splitting of truth goes further than philosophy and natural science had thus far assumed. It penetrates into the very knowledge which physics has of nature. Instead of just setting a boundary between the latter and other possible ways of apprehending reality [e.g. axiological, ethical, aesthetic, etc.], it also separates different but equally justified representations within the mode of description in physics [e.g. waves and particles inter alia], which cannot be a synthesized single image of nature.[28]

Grete Henry-Hermann insists, however, that these convergences between quantum physics and critical philosophy should not mask the independence of the paths followed by each of these two kinds of discourse: quantum description "rests manifestly on the teachings of experiment and is entirely independent of philosophical speculation", while the critical system "rests integrally on mathematical and philosophical reflections".[29]

But, according to Grete Henry-Hermann, this observation confers still greater value on transcendental philosophy. For her, the fact that these convergences had been achieved at the end of wholly independent approaches based on distinct principles, underlines the credibility of the fundamental principles of trenscendantal philosophy. Such convergence, she wrote, "signifies, if not a justification, then at any rate a

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[28] Ibid., section 18. Quotation translated from the original German by Professor Fernando Leal.

[29] Ibid, section 18.

very important empirical corroboration"[30] of this philosophy. In short, the prestige attaching to the exact sciences is to an extent reflected back on critical philosophy.

Grete Henry-Hermann concludes her fundamental essay of 1935, which was to provide the inspiration for all her later writings on the philosophy of science, with these words:

> If the undeniable merit of physical research is to have decisively furthered the understanding of the philosophic foundations of our knowledge of nature, nevertheless such a progress implies as little a break with past philosophy as quantum mechanics does with respect to classical physics. On the contrary, careful examination of the issue shows that, in spite of the obvious discrepancies of quantum mechanics with the apparent conclusions of critical philosophy, the decisive discoveries of quantum mechanics are consistent with the principles of that philosophy, so that the latter illuminate the former in their significance for physical knowledge.[31]

7 Conclusion

This paper is confined to presenting, as faithfully as possible, the central elements of Grete Henry-Hermann's pioneering contributions to the philosophy of quantum physics. Of course, her conceptions, like any philosophical analyses, are open to criticism from various directions. Now these critiques, which have been barely sketched here, show the fecundity of Grete Henry-Hermann's position, and they provide an excellent springboard for subtle analyses of the relationships between causality and quantum physics.

All in all, Grete Henry-Hermann provides one of the first contributions to the philosophy of quantum physics – and an original one –, as well as interesting analysis of the relationships between modern physics and Kantian-inspired philosophy. One can only regret that her writings have not been, and are not, better known.

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[30] Ibid., section 17.

[31] Ibid., section 18. Quotation translated from the original German by Professor Fernando Leal.

B.3
Quantum Mechanics: Ideas

Decoherence and the Constitution of Objectivity

Michel Bitbol

Abstract A transcendental interpretation of decoherence theories is presented, as a middle way between the realist and empiricist interpretations. From a transcendental standpoint, the latter interpretations are both biased. The realist one is biased in favor of formal constructs taken as descriptive of a reality more real than phenomena; and the empiricist one is biased in favor of phenomena, thus forgetting that they acquire their meaning from the formalism in which they are embedded. By contrast with these two positions, transcendental epistemology sees decoherence as one step in a stratified process of constitution of objectivity adapted to microphysical phenomena.

According to Zurek (1982, 2003), the ideal of decoherence theories is to use quantum mechanics as a fundamental tool to clarify its own interpretation. But in spite of their remarkable achievements, decoherence theories have not completely fulfilled this program. Firstly, the measurement problem still retains an irreducibly philosophical component which is stubbornly resistant (Joos et al., 2003). This component is basically the issue of the transition from potentiality to actuality, from an extended potentiality to a particular, unique and local actuality. Secondly, it is wrong to think that a satisfactory interpretation of quantum mechanics may arise entirely from decoherence theories. Indeed, one should not forget that the results of decoherence theories, and of the experiments which tend to corroborate them (Brune et al., 1996; Dreyer, 1997; Hagley et al., 1997) depend, for their evaluation and their formulation, on a preliminary interpretation of quantum mechanics. In other terms, their putative interpretational contents are not independent of the interpretation they presuppose.

This means that the interpretational contents of decoherence theories are basically circular. But this circle does not necessarily have to be vicious. I shall argue that decoherence theories provide us with genuine proof of self-consistence: a proof of the mutual consistence between the interpretation they start from and their own interpretational contribution. Such proof is less than what their creators hoped for, but is not negligible.

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This importance of the preliminary interpretation on the status of decoherence is now beginning to attract attention. A good example is Ulrich Mohrhoff (2001). According to him « Decoherence is not a part of any interpretational strategy. It is a physical phenomenon predicted by quantum mechanics and, like quantum mechanics itself, in need of an interpretation ». In the same way as quantum mechanics itself, decoherence is already influenced by some underlying interpretations; usually a strange mixture of at least *two* interpretations. The most common mixture is made up of a large amount of realism of formal essences, and a touch of empiricism. My first task will therefore be to disentangle these two components, realist and empiricist, in the standard mixture of interpretations. I shall then undertake to reconcile the conclusions drawn from the two interpretational components by adopting the standpoint of transcendental epistemology. Decoherence will eventually be seen as an element in a multi-layered strategy of constituting objectivities in physics.

1 Formalist Realism of State Vectors: A Standard Approach to Decoherence

The realist component of the underlying interpretation of quantum mechanics almost entirely determines the vocabulary and the program of decoherence theories. But what is the specific motivation of this realist view? Its motivation is a twofold characterization of reality that a neo-Kantian philosopher would construe as a twofold characterization of objectivity. Reality, in this approach, is a structure which remains *invariant* across a given set of substitutions of experimental procedures. It is also, more specifically, a structure which evolves according to a *law* which takes on the form of a partial differential equation. Now, in quantum mechanics, state vectors obey these two conditions. Firstly, a single state vector is enough to derive probabilistic predictions for *any* type of measurement which may be performed after a given preparation: in this sense, it is a predictive invariant. Secondly, state vectors are ruled by a partial differential equation including time among its variables: the Schrödinger equation or the Dirac equation. These two reality-like characteristics make it tempting to endow the state vector with ontological status. The state vector is construed as a reflection of some intrinsic feature of physical systems called their « state », in virtue of a dubious but widespread analogy with the classical state.

If one takes this standard interpretation seriously, the aim of decoherence theories is to show that the structure of quantum micro-states entails the macro-state structure that can be witnessed in the laboratory and in ordinary life. The quantum states are usually *superpositions, linear combinations* of accessible states represented in formalism by eigenstates of the corresponding observables. By contrast, on the macroscopic scale, only *one* of these accessible states may be obtained at a time. In superpositions, the various accessible states interfere, whereas the states witnessed on a macroscopic scale are mutually exclusive and do not interfere. The solution provided by decoherence to this problem of existence of a gap between the quantum domain and the semi-classical macroscopic domain is called « environment-induced superselection », or « einselection ». This solution consists in showing that the phase coherences of the state of an apparatus correlated to a micro-system are rapidly diluted in its environment. The almost complete disappearance of the interference terms is then equivalent to a superselection rule which only retains the eigenstates of a given observable, itself selected by the decoherence process.

This well-known result is clearly important, but the realist interpretation of state vectors tends to overrate it. According to the perspective of a realism of formal essences, linear superpositions of vectors in a Hilbert space faithfully represent physical *reality*. Then, everything else, including the classical behavior of macroscopic objects is nothing but superficial *appearance*. According to this interpretation, decoherence is thus able to do nothing less than *derive* classical appearances from a quantum reality.

Accordingly, the authors who support this realist interpretation claim that decoherence is able to *explain* why a universe which is essentially ruled by quantum laws *appears* classical on a large scale of size and complexity. Decoherence theories are specifically ascribed the task of explaining the strange subdivision of the world established by Bohr between the classical and quantum domains retrospectively, in the framework of quantum formalism. Along with the realist interpretation of decoherence, the bohrian divide is only a practical distinction between what *appears* to us macro-observers, and what supposedly *exists* according to a quantum description. Appearance is explained by *Being*, in the most common one-directional and causal-like meaning of the term 'explanation'. *Appearance* as an *explanandum* is displayed by our measuring instruments, whereas *Being* as an *explanans* is revealed by our theoretical thought (in the same way as in Kant's inaugural dissertation of 1770, which belongs to his *pre-critical* philosophy, our senses show us things as they *appear* whereas reason represents things as they *are*). Experimental appearance can rationally be shown to be a consequence of what *is* in reality, provided *Being* is construed in the appropriate way.

However, some difficulties make us suspicious that although decoherence theories are a success insofar as they have defined and accurately predicted a whole new domain of phenomena which were detected experimentally a few years ago, they have much less foundational abilities than their realist interpreters assume. Let me examine three of these difficulties in turn.

The first is the persistence of residual interference effects, namely non-diagonal terms in the density matrix of the system object + apparatus. Since the process which leads to the decrease of these diagonal terms is in principle reversible, one might witness the resurgence of interference effects between macroscopic states. This possibility is not purely fanciful. Long-term experiments have been performed by the same team of the Ecole Normale Supérieure, in Paris, who detected decoherence effects for the first time on small mesoscopic systems (dubbed « Schrödinger's kittens »). These experiments (Raimond et al., 1997) resulted in beautiful curves of periodical recurrence, that show a virtually complete recovery of phase correlations before these correlations are lost again.

Of course, one may reply by pointing out that decoherence theorists do not claim to have proven that a genuinely classical world emerges out of the quantum world, but only, once again, that what emerges is a set of *quasi*-classical, and possibly temporary, appearances. This is credible enough, since residual terms are small, and recurrence periods are very long when large systems are concerned. Here, Boltzmann's argument against Loschmidt (Reichenbach, 1956) may be used with reasonable success: one does not prove that the macroscopic phenomena are the *only* consequence of a certain microscopic behaviour, but that they are the most *probable* consequence in the long run. However, it must be noticed that Boltzmann-like arguments point towards a variety of physical explanations that are weaker than strict derivability.

The second difficulty is more serious. Decoherence yields a density matrix with negligible non-diagonal terms. But, as Bernard d'Espagnat (1994) pointed out, it does not provide one with proper statistical mixtures; only with so-called improper mixtures bearing the trace of *entanglement*. Thus, Decoherence does not amount to a true *state reduction*. The gulf between potentiality and actuality has not been crossed by decoherence. To cope with this problem, many authors, including Zurek and Gell-Mann, suggested some combination of decoherence and Many-World interpretation: Many-World interpretations ascribe each term of a decoherence-induced improper mixture, the status of one actuality existing in parallel with many others; Conversely, decoherence solves the preferred basis problem that plagues the Many-World interpretations. But the very need of some version of the Many-World interpretation is enough to show that decoherence, in its realist version, is not able to solve the measurement problem as it is construed under realist premises, nor explain the univocity of the events of the macro-world by itself.

The third difficulty is more subtle but also deeper than the two previous ones. As Zurek (1982) pointed out, decoherence processes only occur as an effect of the interaction between appropriately defined sub-systems; usually *three* sub-systems called the object, the apparatus, and the environment. This procedure is supported by the existence of theorems of « tridecompositionality ». The theorems of tridecompositionality state the *uniqueness* of the decomposition of the state vector of a system made of three interacting sub-systems into a diagonal (« Schmidt ») superposition of tensor products of three state vectors, *if* such a decomposition exists at all (Bub, 1997). Now, on what grounds can one assume that the universe *is* somehow divided into sub-systems, that the decomposition *does* exist, and that the interest of the experimenter can be focused on *one* of these subsystems (the object)? If one thinks hard about this question, one soon realizes that the only ground and origin of this assumption is our macroscopic experience of spatially separated bodies. But is it legitimate to take our macroscopic experience as a basis for a theory which is supposed to *explain* how this experience is generated out of something truly deeper and truly different? Is it acceptable to take *appearances* as a starting point of a theory which is supposed to deal primarily with *Being*? This strategy is clearly circular. And such a circle is a threat for a realist reading of decoherence theories. For, in such a reading, one would like to get a one-directional derivation (and hence an *explanation* in the most common acceptation) of classical appearances from a microscopic reality allegedly represented in a faithful manner by quantum formalism. Circularity is disastrous for any realist interpretation of explanation. But if another, weaker interpretation is adopted, this feature of decoherence theories can become more acceptable. Even the circle between the presuppositions of decoherence and their outcome may turn out to make sense. One must only be willing to explore other, less common, varieties of explanation.

2 A Thoroughly Empiricist Reading of Decoherence

I shall now consider the second type of interpretation of decoherence theories: the *empiricist* interpretation. In an empiricist framework, the standard of reality is diametrically opposed to that of a realism of formal essences. What is real, according to an empiricist thinker, is the *fact*; the experimental fact as it is directly witnessed in a laboratory, as it is expressed in short descriptive propositions, and as it is sketchily interpreted by means of models borrowed from empirically valid theories. Instead of granting ontological value to abstract invariants and universal law-like formalisms, the empiricist puts ontological weight on concrete variations and particular events. In that respect, the empiricist philosophers of science are heirs to nominalism. According to empiricists, physical laws are only there to anticipate, by means of a structural scaffolding devoid of any picture-like significance, constant correlations between possible facts. The laws of classical mechanics thus anticipate *strict* correlations between possible phenomena. Then, as soon as one has selected the initial conditions by identifying them as a set of actual facts, these formal correlations are turned into definite predictions for later actual facts. As for the laws of quantum mechanics, they anticipate *probabilistic* correlations between possible phenomena. These formal correlations become testable as soon as an initial state vector has been associated to an actual preparation, since probabilities can then be calculated from this state vector and compared to actual experimental outcomes. A. Peres (1995) and J. Schwinger (2003) are among the physicists who recently and forcefully advocated this view. A. Peres stated what he believes is the true but highly restrictive status of quantum mechanics thus: « Quantum theory is a set of rules allowing the computation of *probabilities* for the outcome of tests which follow specified preparations ». All the terms borrowed from the ontological framework of classical physics, such as « system » and « state », are redefined accordingly:

- « A quantum system is defined by an equivalence class of preparations ».
- « We can define a state as follows: a state is characterized by the probabilities of the various outcomes of every conceivable test ».

Along with this class of views, which was also developed in a more nuanced way by B. Van Fraassen (1991), state vectors are by no means reflections of a reality deeper than the facts they enable us to predict. They are essentially sophisticated tools of probability valuation, adapted to situations where phenomena cannot be properly separated from their experimental context, which is precisely the case in microphysics.

What then is the meaning of decoherence according to such a thoroughly empiricist interpretation of quantum mechanics? Obviously, decoherence cannot be taken to have *explained* the emergence of classical appearances from a deeper reality allegedly represented by the symbols of quantum mechanics. For, in an empiricist framework of thought, reality is nothing else and nothing beyond the « classical appearances » themselves. According to an empiricist reading, if anything can be said to emerge as a consequence of a decoherence process, this is a purely formal, symbolic, abstract sort of emergence. The standard, kolmogorovian form of probability valuation, emerges from the quantum form of probability valuation. A probability theory adapted to predicting distributions of intrinsically occurring events, emerges from a more general probability theory adapted to predicting contextual events (Bitbol, 1996, 2000). Nothing more emerges by way of the decoherence process, but nothing less either.

The three difficulties of realist interpretations of decoherence are then seen in a very different light.

The first difficulty, namely the persistance of small interference terms is no longer harmful for an empiricist. It only means that complete detachment of experimental phenomena with respect to their context is an idealisation, and that the algorithm of probability valuation must take this into account. No weakening of the concept of explanation is involved here; rather, a strengthening of our grasp of the bounds of empirical science.

The second difficulty is even less of a problem for an empiricist thinker. S/he is even entitled to wonder how anybody ever entertained the strange belief that actual facts can somehow be derived from a purely formal manipulation of possibilities and their correlations. This is tantamount to thinking that the concrete reality of everyday life can arise from a pure thought process; or that existence can be derived from essence, in the same way as the ontological proof of God's existence provided by St Anselm and Descartes (Van Fraassen, 1980). In a strictly empiricist framework, the only thing which can be demonstrated by decoherence is that a classical system of probability valuations which lean(s) towards the 'ignorance' interpretation can emerge (approximately) from a quantum system of probability valuations which is definitely averse to any 'ignorance' interpretation. One has no good reason to expect anything more spectacular, let alone more metaphysical, from decoherence theories.

True, the ignorance interpretation of probability valuations enables one to figure out that a given property is realised in nature independently of its experimental manifestation. Therefore, it is tempting to believe that, as soon as decoherence has yielded a quasi-classical probabilistic structure, some true property is around, irrespective of and independently from any experimental test. But it would be absurd to suppose that the probabilistic formalism is able to single out this putative property *by itself*. And since, according to radical empiricist philosophers of science, the quantum formalism is but a special type of probabilistic formalism, it is equally absurd to suppose that it could select a given value by itself. No form of probability theory may have an explanation in store for the *unicity* and identity of each experimental fact. Providing probabilistic valuations of how frequently one should expect a certain type of experimental fact to occur, presupposes the unicity and identity of each fact. But *presupposing* it is quite another thing to *explaining* it.

Finally, the third difficulty can be provided with an interesting germ of solution in a thoroughly empiricist frame of thought. It is perfectly true that there is a kind of circle between the assumption of decoherence theories according to which the universe is divided into at least three sub-systems, and their conclusion according to which macro-objects behave like mutually separated entities endowed with intrinsic and stable properties. But this element of circularity is by no means shocking or even surprising for an empiricist. According to an empiricist, one cannot avoid starting from tangible, real and concrete mesoscopic facts when a physical theory is to be elaborated. The only condition which must be imposed onto the theory is to be *compatible*, at the end of the day, with this concrete reality which was naturally taken as its starting point. Now this is exactly what decoherence does by imposing a sort of feed-back process on quantum theory. Decoherence shows that the formal apparatus of quantum mechanics does not deny the structure of its real, factual contents, *provided certain assumptions are introduced in order to restrict the range of possible evolutions of its probabilistic predictive symbols*. This is much less than someone who believes in the reality of essences dreams of, yet neither is this negligible.

But can one contend that decoherence « explains » anything if this view is accepted? Of course not if the usual, linear, causal, acceptation of « explanation » is retained. But there may be some prospect for an explanatory reading of decoherence in a non-realist view of quantum mechanics as well, provided an alternative meaning is ascribed to the concept of « explanation ». This can be seen in a more convincing way by considering a third interpretation of quantum mechanics which is neither realist nor empiricist but *transcendental*.

3 Decoherence in a Transcendental Sense

The transcendental interpretation that I wish to develop at this point, is similar in some respects and different in some other respects to the former interpretations. It borrows something from both the formal realist and the empiricist interpretations, in the true spirit of Kant's transcendentalism that was elaborated as a middle way between Wolff's metaphysical dogmatism and Hume's empiricism. But the transcendental line of thought also rejects the distinctive ontological claims of the two former interpretations. In a transcendental approach, neither formal universals nor factual particulars taken in isolation are taken to be real. Instead, what is taken as fundamental is the very process of definition of formal universals and factual particulars by one another, of which scientific investigations offer the purest instantiation.

According to a transcendentalist philosopher, empiricists are right when they suggest that the classical organization of the macro-mesoscopic world is a precondition for quantum formalism. They are right to point out that our ordinary belief in the intrinsic occurrence of mutually exclusive elementary events, and in the intrinsic possession of properties, is bound to be the starting point of the elaboration of quantum formalisms (and of any other theoretical formalism as well). Indeed, if it were not the case, no one could tell what the quantum probabilistic valuations are *about*. Probabilities are probabilities *of* well-defined events, and therefore belief in these well-defined events *is* and *must* be a basic supposition of quantum theories. This is the reason for which Bohr insisted so much on the necessity of describing part of the experimental devices in a classical framework: this type of description, according to him, was a pragmatic and epistemological necessity. However, unlike empiricist philosophers committed to their « two dogmas », transcendentalists are definitely averse to endowing the classical mode of organization of directly observable events with a foundational status. Being a pragmatic precondition for is not tantamount to *Being* full stop. Having been used as an indispensible starting point of an epistemic process is not equivalent to having more ontological weight than the end product of this very epistemic process. One should realize that choosing a starting point has no ontological implication at all.

Transcendentalism also differs from empiricism by emphasizing that isolated actual facts are purely anecdotal. Actual facts only acquire their meaning from a certain formalism which connects them to other facts by means of law-like statements. A mere sequence of facts is no *experience* at all, in kantian terms: it is at most a rhapsody of phenomena. Facts are organized into an experience *stricto sensu* by their being embedded within a universally shared formal framework; a formal framework in which every single fact is construed as a special aspect or facet of some unifying invariant. Within our close mesoscopic environment, the natural organizing framework is what Kant called the system of the categories of understanding. This system underpins classical physics. But for domains that are increasingly distant from our direct neighborhood, the organizing framework may differ considerably from its kantian paradigm. As Cassirer pointed out, this difference is not a sign of failure of transcendental epistemology; it rather corresponds to an appropriate generalization of the objectifying function of kantian categories.

Now, as soon as one understands the crucial role of the organizing structures beyond and above the isolated empirical data, facts tend to be relegated to the background. Conversely, the universal meaning of facts, as provided by a formal framework, comes to the fore, and the speculative interest of reason is stimulated by this inversion in the order of priorities. One then seems to fall closer and closer to the doctrinal antithesis of empiricism, namely realism of formal essences.

However, transcendental philosophy is just as little attracted to realism of formal essences as to empiricism. According to it, form cannot be made completely independent from of content. It is even less likely for content to be derived from pure form. In particular, any attempt at grounding the facts that are presupposed by the quantum formalism on a theorem of this very formalism, looks awkward from a transcendentalist standpoint. For a modern transcendentalist, form and facts are interdependent, and none of them should be given any priority, let alone any autonomy with respect to the other. Just as much as facts need a formal framework in order to acquire their objective meaning, formalism needs facts in order to be filled in by them; formalism needs facts in order to be *about* something. To paraphrase Kant, facts without formalisms are blind (not to say meaningless) and formalisms without facts are empty.

Now what about decoherence in a transcendentalist context? The two first difficulties of the realist reading of decoherence are solved in the empiricist's way. But the solution of the third difficulty is more demanding, and also more significant, than in the empiricist frame of thought. The empiricists were content to say that decoherence proves that quantum formalism does not *contradict* its real basis made of actual facts. In contrast, according to a transcendentalist, facts are no more real than forms, and forms are no more real than factual appearances. Decoherence then has a much more important role to play in transcendentalism than in empiricism. Here, Decoherence proves the validity of one of the two directions of the dialectic by means of which facts and forms are mutually defined in the quantum paradigm. The first direction of the dialectic, which goes from facts, especially spectral data, to the quantum formalism, was progressively clarified during the first quarter of the twentieth century by Bohr and Heisenberg. It involves the *correspondence principle*, which extrapolates from the classical-like interpretation of the instruments' working and readings, towards a new theoretical formalism able to predict the instrument's readings in a coherent way. But one had to wait until the end of the twentieth century in order to clarify the second direction of the dialectic, the direction which starts from the formalism to disclose the structure of the facts about which it provides probabilistic predictions. As soon as this was done, by way of decoherence theories, the system made of the quantum formalism and its empirical presuppositions was shown to be completely self-consistent.

Clearly, the transcendentalist reading of quantum mechanics has gone beyond the empiricist agnosticism. In transcendentalism, formalism is no longer seen as a mere instrument for predicting facts that really exist; it is no longer a mere shadow of the system of facts. It is *constitutive*, in the sense that it endows facts with objective meaning by construing them as aspects of a structural invariant. In this respect, decoherence itself can rightly be said to have a constitutive role: it operates as a second step in a two-step process of constitution of objectivity. To understand this, we must state this stratified process as precisely as possible.

- In the first step, quantum formalism as a whole prescribes a statistical order to micro-events. Through this, clusters of contextual phenomena are anticipated by means of decontextualized probabilistic invariants called « state vectors », ruled by a universal law of evolution. This clearly corresponds to an act of constitution of objectivity, since decontextualizing anticipation means disconnecting it from particular circumstances, or making it invariant with respect to any change of these circumstances.
- In the second step, one shows (through decoherence) that, in certain well-specified circumstances, *traditional* types of invariants isomorphic to classical properties can be made compatible with the new types of invariants introduced by quantum formalism. This step also corresponds to an act of constitution of objectivity, though a higher-order type of objectivity, since the traditional universe of individual objects and detached features, which is necessarily presupposed by experimentalists, emerges as a possible domain for the objectified (but unusual, interference-like) probabilistic valuations of quantum mechanics. That a certain (interference-free) structure of probabilistic valuations is necessary if the latter are to bear on the behaviour of *classical* objects was pointed out long ago by G. Boole (1952), and then cogently commented by I. Pitowsky (1994) in quantum context. Boole's structure is the condition that must be fulfilled *if a certain set of numbers included in the interval [0, 1] is to be considered as a*

> *probabilistic valuation for the occurrence of intrinsic properties of classical objects* (Mittelstaedt, 1998). And decoherence is precisely the theoretical device which shows that Boole's classical probabilistic structure is (approximately) compatible, under certain assumptions, with the quantum theoretical probabilistic structure. It is no wonder that G. Boole called his classical probabilistic structures: « conditions of possible experience », with overtly Kantian undertones. They are indeed preconditions for objectifying the *domain* of probabilistic valuations; and they are therefore constitutive of objectivity in this sense.

To recapitulate, the first step in the constitution of objectivity in quantum mechanics consists in extracting a universal, invariant, tool of probabilistic valuation; and the second step (represented by decoherence) consists in showing how the structure of this probabilistic valuation can be made approximately compatible, at the macroscopic level, with the assumption that it bears on a domain of randomly distributed intrinsic properties of objects.

Now, what about *explanation* in this transcendentalist context? We have just seen that, unlike the empiricist's, the transcendentalist's reading of decoherence does not boils down to a proof of non-contradiction. It rather displays how facts and formalism can *shape* each other in a fully consistent way; and it shows that this mutual shaping is a precondition for a stratified, two-step, objective knowledge. On the other hand, unlike formal realists, transcendentalist philosophers do not claim that decoherence provides us with a one-directional derivation of the factual domain out of formalism. In the usual sense of causation, whose theoretical equivalent is one-way derivation, the formalism of quantum mechanics as construed by transcendentalists therefore does not *explain* the classical-like organization of facts. But in an other sense, in the structural sense of mutually defining relation, one can say that decoherence, together with the converse process from observables to formalism typical of Bohr's correspondence principle, is explanatory. Here, it is not decoherence alone, but the whole process which includes (i) theory formulation and (ii) the proof of the self-consistence of the system made of the theory and its empirical presuppositions, which can be called an explanation in the sense of a *structural* explanation. It should be remembered that what is taken as a proper « explanation » crucially depends on what is implied by a why-question (Van Fraassen, 1980), and that in many circumstances a why-question asked in a scientific context only implies embedding the *explanandum* within a coherent structural network.

Bibliography

Bitbol, M. (1996), *Mécanique quantique, une introduction philosophique*, Flammarion, Paris

Bitbol, M. (1998), «Some steps towards a transcendental deduction of quantum mechanics», *Philosophia Naturalis*, 35, 253–280

Bitbol, M. (2000), *Physique et philosophie de l'esprit*, Flammarion, Paris

Boole, G. (ed.) (1952), « On the theory of probabilities », *Studies in Logic and Probability*, Open Court Publishing, London

Brune, M., Hagley, E., Dreyer, J., Maître, X., Maali, A., Wunderlich, C., Raimond, J.M., and Haroche, S. (1996), « Observing the Progressive decoherence of the meter in a quantum measurement », *Physical Reviews Letters*, 77, 4887

Bub, J. (1997), *Interpreting the Quantum World*, Cambridge University Press, Cambridge

d'Espagnat, B. (1994), *Le réel voilé*, Fayard, Paris

Dreyer, J. (1997), *Atomes de Rydberg et cavités: observation et décohérence dans une mesure quantique*, Thèse de doctorat de l'Université Paris VI

Hagley, E., Maître, X., Wunderlich, C., Brune, M., Raimond, J.M., and Haroche, S. (1997), « Generation of Einstein-Podolsky-Rosen pairs of atoms », *Physical Reviews Letters*, 79, 1–5

Joos, E., Zeh, H.D., Kiefer, C., Giulini, D., Kupsch, K., and Stamatescu, I.O. (2003), *Decoherence and the Appearance of a Classical World in Quantum Theory*, Springer, Heidelberg

Kant, I. (2004), *Inaugural Dissertation of 1770*, Kessinger Publishing, Whitefish MT

Mittelstaedt, P. (1998), *The Interpretation of Quantum Mechanics and the Measurement Process*, Cambridge University Press, Cambridge

Mohrhoff, U. (2001), « Two theories of decoherence », arXiv: quant-ph/0108002

Omnès, R. (1994), *The Interpretation of Quantum Mechanics*, Princeton University Press, Princeton, NJ

Peres, A. (1995), *Quantum Theory, Concepts and Methods*, Kluwer, Dordrecht

Pitowsky, I. (1994), « George Boole's 'conditions of possible experience' and the quantum puzzle », *British Journal for the Philosophy of Science*, 45, 95–125

Raimond, J.M., Brune, M., and Haroche, S. (1997), « Reversible decoherence of a mesoscopic superposition of field states », *Physics Review Letters*, 79, 1964–1967

Reichenbach, H. (1956), *The Direction of Time*, University of California Press, Berkeley, CA

Schrödinger, E. (ed.) (1935), « La situation présente en mécanique quantique », *Physique quantique et représentation du monde*, Points-Seuil, Paris

Schwinger, J. (2003), *Quantum Mechanics, A Symbolism of Measurements*, Springer, Heidelberg

Van Fraassen, B. (1980), *The Scientific Image*, Oxford University Press, Oxford

Van Fraassen, B. (1991), *Quantum Mechanics, An Empiricist View*, Oxford University Press, Oxford

Zurek, W.H. (1982), « Environment-induced superselection rules », *Physical Review*, D26, 1862–1880

Zurek, W.H. (2003), « Decoherence, einselection, and the quantum origins of the classical », *Review of Modern Physics*, 75, 715–775

The Entangled Roots of Objective Knowledge

Stefano Osnaghi

Abstract If no model based on locally interacting *objects* fits quantum phenomena, how can knowledge grounded in the quantum theory be *objective*? According to a common view, the conditions which ensure the reproducibility of experiments and the predictability of results are fulfilled in the quantum world owing to the "appearance" of macroscopic objects through decoherence. Based on the analysis of some recent experiments on quantum entanglement, I will point out the circularity of this argument. More generally, I will suggest that the objective features of scientific knowledge do not need to reflect the structure of an "external world", and that they can be understood as the outgrowth of a systematic endeavour to organize experience in a way which makes prediction possible.

1 Quantum Entanglement and the Limits of Objectification

Let us consider a typical microscopic system, a so-called Rydberg atom. In several respects, Rydberg atoms are aptly described by a semi-classical picture (a compact core and an external electron which rotates on a circular orbit corresponding to either the *g*round or the *e*xcited energy level). However, when we make *two* Rydberg atoms interact in suitable conditions,[1] they become *entangled* (Schrödinger, 1935a). Two entangled atoms display a special kind of correlation, called "EPR"

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[1] The dipole–dipole interaction between Rydberg atoms is exceptionally strong and can be further enhanced by a superconducting cavity quasi-resonant with the relevant electronic transition. Under these conditions, it is sufficient for one atom to come within a *millimetric* distance of the other in order to observe non-classical correlations between their respective external electrons (Osnaghi et al., 2001).

M. Bitbol et al. (eds.), *Constituting Objectivity*,

(Einstein et al., 1935), which cannot be explained by any *local realistic* model, i.e. any model in which the results of measurement reveal the individual properties of two objects and in which these properties change only through local interactions (Bell, 1964; Shimony, 2005).[2]

If this fact appears puzzling, it is not only because it deceives the naïve expectation that the phenomena can be explained based on a natural ontology of objects and properties, but also because it prevents the literal construal of the very linguistic expressions that we use to denote microscopic phenomena. More generally, the possibility of acquiring knowledge based on *ob*servation seems to require an intentional correlate (*ob*jects) for our cognitive acts and experimental manipulations. But given the empirical consequences of entanglement, it is not clear how a coherent account of microscopic phenomena in terms of the contextual behaviour (if not the "real properties") of some *objects* can be provided.[3]

In addition to these difficulties, one must also consider those brought about by situations such as that described by Schrödinger in his famous cat example (Schrödinger, 1935b). According to quantum mechanics, in such situations, it should be possible to observe EPR correlations in measurements involving *macroscopic* systems. The virtual existence of such correlations gives rise to a latent contradiction between quantum theory and the rules that we use to anticipate ordinary experience, since such rules are modelled upon the dispositional properties of objects. A related problem arises when we consider the results of a measurement. Indeed, in order for experimental activity to be possible at all, the results of measurements must be "objective", which means for example that different observers must agree on the occurrence of a given result. These features are habitually explained by assuming that the results reflect the properties of some object, which exists independently of any observation. But such an explanation is at odds with the quantum mechanical account of the observation process, which predicts that the observed system S and the measurement apparatus M end up in an entangled state. Von Neumann's postulate of projection (1955, chapter 5), stating that the entangled state S+M is "reduced" abruptly by the act of observation, is a way of getting rid of this paradox. However, because of the *ad hoc* nature of the postulate, this solution is widely considered to be unsatisfactory. To explain the "reduction", some scholars have therefore conjectured that the state of a system under observation can be affected by the mind of the observer (Wigner, 1961).[4] Others have argued that it is possible to do away with the postulate of projection by taking into account the

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[2] Like the wave-like behaviour of particles and, more generally, the interference patterns exhibited by quantum systems, the entanglement is a consequence of the structural relations existing between quantum observables (or, more precisely, between the statistical results obtained when these observables are measured). As is well known, these relations are suitably represented in a vector space, in which each vector corresponds to a particular "preparation" of the system under study (see for instance Hughes, 1989, pp. 107–113).

[3] See Bitbol (1996, cap. 4). The attempts to provide such an account, by discarding one or the other feature of the "natural" ontology, include Bohr's complementarity (see Rosenfeld, 1961), "hidden variables" theories (Bohm, 1957), and non-standard logic (see Mittelstaedt, 1994).

[4] See Bitbol (2000a, cap. 1) for a critical discussion.

effects of *decoherence* within a theoretical framework which gives information a central role (see e.g. Zurek, 2004).

I believe that the notion of information can indeed be useful in understanding the implications of entanglement. But it is essential to state clearly *what* information is about (Bell, 1990). Niels Bohr's "instrumentalist" answer to this question was that the information provided by quantum states is about the *results* "obtainable under experimental conditions described in classical terms" (Bohr, 1948, p. 314). It is generally accepted that, owing to its lack of ontological commitment, this view avoids the paradoxes related to quantum entanglement. Yet it is not clear how such a view can also account for the objective features of physical experience. The instrumentalist view seems incapable of shedding any light on the fact that there is an intersubjective agreement on measurement outcomes, and that these are reproducible and predictable. Likewise, the effectiveness of the notion of object in ordinary experience and in the construction of physical models remains unexplained. Instrumentalists have sometimes replied to these objections by simply rejecting the request for explanation. Here, however, I will argue that instrumentalism can in fact provide a rational and consistent framework in which the objective features of physical knowledge can be understood. But in order to do this, the hypothesis that physics addresses (and theories describe) a pre-structured reality must be abandoned, and a constructivist point of view should be endorsed.

I will discuss these ideas in the light of some recent experiments of cavity-QED. This is not because I believe that the *results* of these experiments[5] provide grounds for my claim. I find these experiments instructive for other reasons. By their very nature, the *gedankenexperiments* that usually illustrate the exotic implications of quantum entanglement are not framed within laboratory practice: measurement protocols are not given a precise operational meaning and the possible results are not expressed in terms of experimentally distinguishable alternatives. Within the representationalist view of theories, in which observables reflect properties, such an idealization is perfectly harmless: granted that certain classes of properties are instantiated in the real world (for example that a cat *is* either living or dead), there is no need to state how the corresponding observable can be *measured* in order to know that it is *well defined*. But if this view is precisely what is being questioned, the analysis of real experiments presents the advantage of compelling us to take the operational meaning of the symbolism into account *while formulating* any question.

2 Objects Disentangled by the Observer

Let us consider a field C stored in a superconducting cavity (Raimond et al., 2001) and let N be the observable which corresponds to the photon number. Suppose that C is prepared in a *superposition* of two eigenstates of N:

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[5] For a review of recent experiments on quantum entanglement see Bouwmeester et al. (2001).

$$\frac{1}{\sqrt{2}}\left(|0\rangle+|1\rangle\right) \tag{1}$$

It is well known that no statistical ensemble of zero-photon fields and one-photon fields can reproduce the predictions of Eq. (1) for a set of measurements which involve observables incompatible with *N*. Hence we are not allowed to assert that the cavity contains either one or zero photons. However, the propositions by which we express the result of a measurement of *N* are exactly of this sort. Indeed, measuring *N means* to answer the question as to how many photons the cavity contains. More precisely, it means to provide an *objective* answer to such a question, i.e. an answer on which different observers would agree and which can, for example, be confirmed by the immediate repetition of the measurement. This remark lies at the bottom of the common idea that, after the measurement of a given observable has been carried out, the state vector associated with a system must be an eigenstate of that observable. For example, in the case discussed above, after the measurement and depending on the result, the field is attributed either state $|1\rangle$ or $|\,0\,\rangle$. This seems to imply that a process of "state reduction", in which the superposition (Eq. 1) "collapses" into the eigenstate of *N* corresponding to the observed result, must take place during the observation.[6] This section is devoted to showing that, if one advocates an instrumentalist interpretation of formalism, the very notion of state reduction is superfluous. Postulating such a process only makes sense if one assumes a descriptive interpretation of the state vector. Then, however, one has to deal with the difficulties arising from the fact that state reduction is at odds with the dynamical equations of standard quantum mechanics.

In order to establish if state reduction is really demanded by the phenomena, one has to build a model of the measurement process, draw experimental predictions from that model, and finally compare such predictions with the results obtained in the corresponding experimental context. As an example we can analyse an experiment in which an atom "measures" *non-destructively* the number of photons of a field prepared in state (Eq. 1) (Raimond et al., 2001). The first step of the experiment involves the preparation of the field state. An "excited" atom A_0 is sent through the empty cavity. The frequency of the electronic transition $e \rightarrow g$ is adjusted (using Stark effect) so as to be resonant with the cavity. This enables the atom decay and the corresponding emission of one photon. If the parameters are fixed in such a way that the decay rate is 1/2, the state of the compound system right after the A_0–C interaction reads:

$$\begin{aligned}&\frac{1}{\sqrt{2}}\left(|0\rangle\otimes|e\rangle-i|1\rangle\otimes|g\rangle\right)\\&=\frac{1}{\sqrt{2}}\left\{\frac{1}{\sqrt{2}}\left(|0\rangle-|1\rangle\right)\otimes\frac{1}{\sqrt{2}}\left(|e\rangle+i|g\rangle\right)+\frac{1}{\sqrt{2}}\left(|0\rangle+|1\rangle\right)\otimes\frac{1}{\sqrt{2}}\left(|e\rangle-i|g\rangle\right)\right\}\end{aligned} \tag{2}$$

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[6] More generally, it is the entangled state of the compound system field + measuring apparatus that is supposed to be "reduced" to a product state. See van Fraassen (1991) for a thorough discussion.

The atom states $\frac{1}{\sqrt{2}}(|e\rangle + i|g\rangle)$ and $\frac{1}{\sqrt{2}}(|e\rangle - i|g\rangle)$ are the eigenstates of an observable Q linked to the phase of the atomic dipole. Let us call the corresponding eigenvalues "+" and "–". "Preparing state (Eq. 1)" amounts to measuring Q and to disregarding the experimental sequences in which such a measurement has not yielded "–".

Once state (Eq. 1) has been prepared, the photon number is measured by a Rydberg atom A_1. The atom is first prepared in a superposition of $|e\rangle$ and $|g\rangle$ and then sent through the cavity. This time, the frequency of the atomic transition $g \rightarrow e$ is tuned off-resonance, hence the atom can neither emit nor absorb a photon. Nevertheless, due to the strong coupling between the confined field and Rydberg atoms, the oscillation of the atomic dipole is affected by the intensity of the cavity field. Consequently, the phase of the atomic dipole, which can be measured by atom interferometry, carries the information about the number of photons "seen" by the atom.[7] We can now return to the question formulated above. Firstly, we note that the "measuring interaction" between the atom and the field results in an *entangled* state, as can be seen by testing the correlations existing between C and A_1 after the interaction (which can be done by means of other atoms). Hence, no state reduction occurs within this model. Still no special problem arises when it comes to the objectivity of the results. This can be seen by sending a second atom A_2 through the cavity in order to check the reproducibility of the result "found" by A_1. In most situations this would not be possible, since the field's energy is generally measured by absorption. In this experiment, however, the field state is probed without disturbance: even if atom A_1 finds a photon, it does not absorb it. Thus the measurement can be repeated by a second atom A_2, and it turns out that there is a perfect correlation between the number of photons found by atom A_1 and that found by A_2. Obviously, such correlations can be predicted by a model which does not involve state reduction. Moreover, for measurements involving observables incompatible with N, the entangled state $A_1 + C + A_2$ predicts correlations which *cannot* be derived from a "reduced" state and which are in fact observed (Nogues et al., 1999).

Of course a real measuring chain is immensely more complicated than a couple of atoms. Indeed, in the above experiment, the final reading is obtained by ionising the atom, amplifying the signal electronically, etc. Furthermore, when we say that the atom recorded a given result, we actually mean that *someone* looked at the output of a complex apparatus, and inferred that the atom was in the atomic configuration corresponding to "having recorded" that result. From an instrumentalist point of view, however, these remarks are quite irrelevant. On the one hand, if we proceed "bit by bit", *any* step of the observation chain (including a human observer) can in principle be included in a model of the measurement process, and will (by construction) obey the linear dynamical equations of quantum mechanics, resulting in an entangled state. On the other hand, no such model will provide anything else

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[7] So far, the experiment has been carried out using a resonant version of this technique (Nogues et al., 1999).

than *predictions* about the results which can be found by an observer placed in a well-defined experimental context. In other words, one cannot expect that "at some point" the model will exhibit a link with the "objective world" which would "explain" the occurrence of a particular result.

Although (as is apparent from the preceding account of the cavity-QED experiment) the use of the term *state* in laboratory jargon resembles that of a predicate in ordinary language, all that the state *vector* provides in the instrumentalist interpretation is the formal link between an experimental preparation and its possible outcomes. Since the state vector is *defined* in terms of both a *given* preparation and a *given* measurement, asking what the state vector of the system is *before* the preparation, or *after* the measurement, simply does not make sense. Therefore, it is pointless to speculate on a transition occurring *during* the preparation or the measurement. It is important to stress that such remarks in no way imply that the instrumentalist account of the phenomena is incomplete: one is free to consider an extended experimental context, but then the state vector assigned to the (compound) system under study will reflect the new experimental conditions, and will refer to a new preparation and a new measurement.

The foregoing argument shows that, if one is prepared to acknowledge that a system does not have a state vector independently of the manipulations that are being considered, then postulating pre-constituted objects (in order to ground the objectivity of the results) and a process of state reduction (in order to enable the correspondence between such putative objects and the symbolism) is altogether unnecessary. This simplification, however, comes at a cost, since the instrumentalist interpretation of the state vector seems to lead to an epistemological impasse. We can wonder for example on what grounds (if we accept that interpretation) we assign a state vector to a system prepared in a specific way, or why we trust the predictions of that state vector concerning future measurements. For many scientists, the only reasonable answer to such questions is to postulate that the measurement outcomes reflect a "real" state of affairs "which exists objectively and independently of any observation or measurement" (Einstein, 1953), and to regard the state vector of a system as a description of such a reality.[8] Under these hypotheses, it is perfectly reasonable to suppose that, during a measurement, the system's state undergoes a transition from the linear superposition which describes it immediately before the observation to the eigenstate "found" in the measurement. Since such a transition cannot be described by the standard linear equations of motion, it may seem inevitable to conjecture that the reduction is brought about by some extra-physical agent, which introduces a "non-linearity" into the evolution of the state vector.[9] Along these lines, some scholars have suggested that the collapse of the observed system into a well-determined "objective" state should be related to the consciousness of the observer (Wigner, 1961).

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[8] See Park (1973, pp. 216–217), Bitbol (2000a, pp. 72–83).

[9] More generally, it is the entangled state of the compound system field + measuring apparatus that is supposed to be "reduced" to a product state. See van Fraassen (1991) for a thorough discussion.

It is important to emphasize that, even though in the instrumentalist approach there is an implicit reference to a "virtual" observer (better: to a specific context of observation), and this reference is regarded as a precondition for attributing a meaning to the theoretical symbols, there is still no room for any influence of the observer's *mind* on the physical world. Conversely, in the descriptivist approach, for which the state of the system exists independently of any pragmatic framework, the meaning of the state vector does not depend on the operations implemented by a virtual observer (it is fixed by the "reality" which the state vector is supposed to mirror). What the descriptivist approach *does* require, however, is a conscious observer capable of "breaking" the entanglement between the observed system and all the systems that have previously interacted with it (like the atom A_0 in the example above). The role attributed to mind within such an approach is not that of *constituting* (in a Kantian sense) the objects putatively underlying the measurement results. Rather, the observer's mind is supposed to act *upon* the physical world so as to restore the straightforward correspondence between such objects and the symbols which are supposed to objectively describe them.

3 Objects Emerging from Entanglement

Except for the proposals involving *ad hoc* modifications of the Schrödinger equation,[10] state reduction entails the splitting of quantum dynamics into two distinct laws of evolution. Usually, as we have seen, this splitting is in turn associated with a dualist ontology. In order to avoid dualism, one should exhibit a linear process capable of explaining how a predictive structure compatible with objectification can emerge when quantum theory is applied to the domain of ordinary "macroscopic" experience. It is generally believed that such an explanation can be provided based on decoherence (see e.g. Zurek, 2004), that is an effect which can be understood by taking into account the propagation of entanglement to all the systems surrounding the measuring agent, rather than postulating that the agent is responsible for the "reduction" of the overall entangled state.

To illustrate this idea, we can consider another cavity-QED experiment in which the electromagnetic field generated by a microwave source and injected into a superconducting cavity plays the role of the "macroscopic system". The high reflectivity of the cavity walls, together with a negligible thermal radiation (the cavity is cooled down to temperatures of less than 1 K), ensures a very good isolation of the field C from the environment E. The method of preparing a "macroscopic" superposition of field states exploits the dispersive interaction between the field and a Rydberg atom A_0 prepared in superposition of $|e\rangle$ and $|g\rangle$ (Raimond et al., 2001). The electronic configurations corresponding to $|e\rangle$ and $|g\rangle$ behave like two distinct optical media. Therefore, when interacting dispersively with

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[10] See e.g. Ghirardi et al. (1986).

the atom, the quasi-monochromatic cavity field splits into two components which accumulate different phase delays with respect to the same wave propagating in vacuum. The corresponding state evolution is:

$$|\alpha\rangle\otimes\frac{1}{\sqrt{2}}\left(|g\rangle+|e\rangle\right)=\frac{1}{\sqrt{2}}\left(|\alpha\rangle\otimes|g\rangle+|\alpha\rangle\otimes|e\rangle\right)\rightarrow$$
$$\rightarrow\ \frac{1}{\sqrt{2}}\left(|e^{-i\varphi}\alpha\rangle\otimes|g\rangle+e^{i\varphi}|e^{i\varphi}\alpha\rangle\otimes|e\rangle\right) \qquad (3)$$

where $|\alpha\rangle$ (called a "coherent state") represents a classical field whose phase and intensity are determined by the phase and modulus of the complex number α. The effect of the dispersive interaction upon the field phase is expressed by the factor $e^{\pm i\varphi}$. We can see from Eq. (3) that, if the parameters are set so as to have $\varphi = \pi/2$, the interaction produces an entangled state whose components represent classical fields which oscillate *with opposite phases*. Then, by measuring the observable Q of the atom (see Eq. 2), one can for example prepare the state:

$$\frac{1}{\sqrt{2}}\left(|i\alpha\rangle+|-i\alpha\rangle\right) \qquad (4)$$

Similarly to state (Eq. 1) in the preceding section, state (Eq. 4) predicts results which are at odds with what can be expected for any statistical ensemble of classical fields. These results can be observed with the help of an atom A_1. After state (Eq. 4) has been prepared at time t_0, A_1 enters the cavity at time t_1, and interacts dispersively with the field so as to produce an entangled state. As in the experiment discussed in the previous section, the information carried by A_1 can be manipulated in order to yield a complete reconstruction of the state of C at t_1. Finally, by repeating the experiment and varying the delay between t_1 and t_0, one can observe how the interference pattern predicted by Eq. (4) is progressively washed out by decoherence.

The effect of decoherence can be understood as follows. Notwithstanding the good isolation, dissipation is so effective that shortly after state (Eq. 4) has been prepared, C displays parasitic correlations with the environment E. Hence, when A_1 measures an observable pertaining to C, the corresponding statistical distributions are no longer those which can be deduced from the state (Eq. 4), but rather those predicted by an entangled state C+E. Remarkably, the *marginal* distributions deduced from such an entangled state *for measurements involving C alone* do *not* display the interference pattern predicted by Eq. (4). Instead, these distributions are quasi-identical to those predicted by a statistical mixture of coherent states of C, i.e. to those expected for a statistical ensemble of classical fields in which each field has a well-determined phase and intensity. More generally, it can be shown that, regardless of how C is prepared, its state (i.e. the state that must be used to predict the results of measurements involving C alone) collapses into a statistical mixture of coherent states well before relaxing towards thermal equilibrium (which, in the case outlined here, essentially corresponds to the vacuum state).

A suggestive interpretation of this experiment is the following. Imagine that A_1 plays a role analogous to the memory of an observer (an extraordinarily simplified memory, made up of a single *qubit*). Such a memory is obviously incapable of gathering and processing all the information disseminated in the environment. Rather, it has to rely on the information encoded in the marginal distributions derived from the entangled state C+E, which show no sign of the interference predicted by Eq. (4). Read in this way, the experiment provides a paradigmatic example of the idea according to which, should a superposition between state vectors representing different classical properties occur, its non-classical implications would cease to be observable almost instantaneously (for an ordinary observer). This simple idea suggests a possible explanation for the "emergence" of classical objects within a quantum universe. What appear to us as the properties of objects (starting with their localization in space) would really be clusters of stable correlations between our memories (rather: us, *qua* memories) and the complex systems surrounding us, once the selective action of the environment has been taken into account. Within such a framework, the very existence of microscopic observables which take "objective" values when measured would be explained by the fact that, in order to be observed, microscopic systems have to be coupled with macroscopic apparatus for which decoherence continuously selects a set of "preferred states" (corresponding to their putative properties) (Zurek, 2004).[11]

The constitution of objectivity in ordinary experience is thus pictured as a natural process entirely described by quantum mechanics. Moreover, a naturalistic explanation of how "the sentient beings we know" have emerged, and why they have "the particular concepts they do for describing their world" (Vaidman, 2002), can be put forward: because they allow deterministic predictions and an effective organization of experience, the objective structures selected by decoherence can be regarded as representing an opportunity of survival for biological organisms which are able to exploit the information thus made available.[12]

This argument is not *prima facie* at odds with the Bohrian conception of formalism. However, a closer analysis shows that the search for a "naturalistic" solution of the measurement problem is in fact incompatible with the instrumentalist approach. In his well-known reflection on the role and status of "classical" concepts, Bohr pointed out that experimentation *presupposes* a conceptual framework with certain characteristics. These characteristics must ensure the unambiguous definition of the context in which measurements are carried out on the one hand, and the communicability of the results on the other. For Bohr, ordinary language and classical models met these characteristics, hence their irreducible role in microscopic physics.[13] It should be stressed that, in so far as quantum mechanics is understood as a

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[11] For an experimental illustration of the latter point see Bertet et al. (2001).

[12] See Saunders (1993).

[13] The transcendental nature of this argument is analysed in Bitbol (1996, pp. 263–269). For a discussion of the Kantian influence and pragmatist strain in Bohr's ideas, see Murdoch (1987, pp. 225–235).

contextual predictive algorithm (Bitbol, 1996), Bohr's reflection on the role of classical concepts in no way questions the applicability of the theory to the macroscopic systems which represent the experimental context. Indeed, nothing prevents one from conceiving a physical model which would include, besides the degrees of freedom of a system S, also those of a system E representing some experimental context, and those of a system O representing an observer. However, the correlations between S and O that can be deduced from such a model will typically be understood as referring to the results of a series of possible observations made *upon* the compound system S+O+E.[14] The fact that such correlations are to some extent "isomorphic" to the results *experienced* by an observer when undertaking certain experiments is what allows us to call this "a model of observation". However, the isomorphism "holds by construction": it cannot serve to *prove* the consistency between ordinary experience and the quantum theory, nor does it provide a bridge between the "subjective" experiences of the observer and some "objective" reality. Rather, it connects two sorts of *situations*: those experienced by an observer who observes S and those experienced by an observer who observes S+O+E. The very definition of these experimental observations and of their possible outcomes rests on a network of acknowledged facts and operations whose structure and characteristics are *presupposed* by any *model* of observation (in the same way as Escher's *Drawing hands* presupposes a drawing hand).[15]

Nonetheless, the programmes aiming at a complete naturalisation of epistemology regard the possibility of providing a theoretical model of measurement, including the context and the observer, as a crucial issue of completeness. It is quite typical of such approaches to overlook the transcendental nature of Bohr's argument and to understand it as a *physical* assumption that the "classical world is physically distinct from the microsystems described by quantum mechanics" (Rovelli, 1996). Along the same lines, the *virtual* observer presupposed by the instrumentalist conception of formalism is improperly thematised as an *external* observer, i.e. a physical system which interacts with the observed system and yet is arbitrarily placed out of the range of the theory (Wheeler, 1957).[16] Based on these premises, providing a quantum mechanical description of the "external observer" and of the "macroscopic world" is presented as a way of getting rid of the arbitrary splitting of the physical world allegedly implied by Bohr's argument. However, what is really at stake in the attempts to "close the epistemological circle" via decoherence (Bacciagaluppi, 2005) is the possibility of providing an "objective description" of the universe in which any intrinsic dependence on the operations carried out by a virtual observer has been eliminated. The naturalised account of observation outlined above, in which the atom A_1 played the role of the observer, is meant to provide an elementary example of how such a programme could work. What is

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[14] See Bitbol (2000a, p. 275).

[15] See Bitbol (2000b, p. 99).

[16] Similar statements can be found in Zurek (2004). The ambiguities of Bohr's own formulation are highlighted in Bitbol (1996, pp. 263–269).

essential in that example is the assumption that the state vector of the observer describes the properties of a memory capable of storing information. Also, the values recorded by the memory are supposed to reflect the “subjective experiences” of the observer.

If one assumes such a descriptive interpretation of the state vector (which is sharply at odds with Bohr’s), the stable correlations between observed systems and naturalised observers can possibly serve as an objective correlate of macroscopic experience, i.e. the ensemble of facts on which experimental practice relies. It should however be stressed that, in deducing the effects of decoherence at the beginning of this section, we relied on a Bohrian conception of formalism. In order to see those results as implying the “emergence of objects” within a framework in which the cognitive processes have been naturalised, one should be able to show that the memory records of naturalised observers are in agreement with the contextual predictions of the instrumentalist interpretation. This is the programme initiated by Hugh Everett (1957). Unlike the approaches involving state reduction, this programme aims at providing a strictly *physical and unitary* account of what exists. The descriptivist conception of theories that underlies the two proposals is nonetheless the same.

4 Objectivity Disentangled from Objects

If the formulations of quantum mechanics involving state reduction are widely regarded as unsatisfactory, those based on Everett’s ideas encounter serious difficulties as well.[17] As for the instrumentalist approach, its capacity to dissolve the paradoxes associated with quantum entanglement depends on the fact that it is logically independent from the metaphysical assumptions that the observables of quantum theory reflect a pre-structured reality, and the results obtained when such observables are measured reflect some objective property. However, unless a theory of scientific knowledge containing no such assumptions is put forward, the instrumentalist approach cannot but appear as a provisional, partial and somewhat *ad hoc* solution (as it is generally considered to be indeed). In this section, I will sketch a few ideas that can be used to construct such an alternative epistemological framework.

To begin with, we note that denying that measurement protocols are determined by the purpose of detecting existing properties does not imply arguing that they spring from a free creative act of the researcher. New measurement protocols have to fit the existing conceptual and pragmatic frameworks, and to generalize them in a coherent way. To be sure, in order to structure experimental facts and operations,

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[17] See Kent (1990) and Barrett (1999).

the physicist relies on the notions of "system" and "observable", which are clearly a legacy of the ontological way of thinking. Yet, the existence of an isomorphism between physical systems and the objects of ordinary discourse, as well as between observables and properties, cannot be taken for granted.

What then, according to this view, is the status of objects? Although, from an operational standpoint, it would be quite misleading to assert that the objects of ordinary discourse (including microscopic ones) are what scientific knowledge is about, their central role in scientific practice is fully acknowledged.[18] Indeed, objects effectively sum up a series of instructions, operations and expectations which are essential in both defining experiments (Bohr, 1948) and working out new models and measurement protocols (Pickering, 1984). Incidentally, we note that, since macroscopic and microscopic objects are on the same footing inasmuch as those pragmatic functions are concerned, the traditional (and problematic) empiricist's distinction between "observable" and "unobservable" objects becomes irrelevant.

In the light of these considerations, the paradoxes mentioned in the introduction lose any *physical* import. Let us take the "collision between two atoms" for example. How can it be that, although we prepare *two atoms*, make *them* interact and finally measure *their* properties, we cannot anticipate the results thus obtained by means of any model involving the causal interaction between two individual objects? According to the instrumentalist view, the answer is that physics is committed to anticipating the correlations between acknowledged facts, and this can certainly be achieved regardless of whether the predictive algorithm fits the categories used to denote and coordinate those facts.

Similarly, the fact that the behaviour of ordinary *objects* seem not to conform to quantum mechanics does not prevent one from treating macroscopic *systems* quantum-mechanically, nor does it threaten the "objectivity" of the facts occurring in the laboratory (typically, measurement outcomes). Paradoxes such as that of Schrödinger's cat stem from the uncritical mixing of two "language games", that of ordinary experience and that of physics. If both are expected to provide representations of the world, the latter (which is supposed to be more accurate) must be able to include the former (which is indisputably corroborated by experience). However, according to a pragmatic view, there is no reason to expect that the syntactic and semantic structures which have proved effective in ordinary life admit of a translation into physical terms. For one thing, ordinary language serves a number of purposes which go beyond those of physics and, for another thing, it does not have to face the same requirements for precision and coherence when predictive implications are at stake. If we are concerned with the *physical* implications of a sentence, it must necessarily be framed in a well-specified experimental context and enunciated in a way that can be operationally understood.

Let us consider for instance the typical claim that "we don't observe cats in a superposition between life and death", which may seem to threaten the universal

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[18] See van Fraassen (1980, pp. 80–83) for a discussion.

validity of quantum theory (Leggett, 1987, p. 98; Zurek, 1991, p. 37).[19] If such a statement is formulated in operational terms, its paradoxical aspects disappear – at the cost, of course, of recognizing that it must be framed in well-defined physical contexts. This means that, firstly, one has to operationally define a quantum observable (i.e. a special kind of operator in a Hilbert space) whose eigenvalues l and d provide a satisfactory model of the infinite dispositional implications of the predicates “living” and “dead”. Then one must be able to control all the degrees of freedom involved in this definition, and to isolate those conventionally associated with the system C (representing the cat) from those associated with the system E (representing the environment), so as to avoid the effects of decoherence. Finally, one must explicitly state the procedure for preparing and testing C in a superposition of the eigenstates $|l\rangle$ and $|d\rangle$. If these preliminary operations are successfully completed, actually detecting a quantum interference pattern (itself defined in operational terms) will come as no surprise at all![20]

The experiment discussed in Section 3 provides a good example of how a superposition of “classical” states can be given an operational meaning. Of course, this is not achieved within the framework of a single experiment. Rather, it involves a complex constructive process by which the sequences of operations traditionally used to measure the putative properties of classical radiation are connected to the cavity QED protocols used to measure the quantum field observables.[21] This process *presupposes* a framework of facts and procedures whose definition and structure rely on the notion of “classical electromagnetic field”. And since (in the instrumentalist interpretation of formalism) the effects of decoherence measured by the experiment only acquire a meaning within such a framework, it is difficult to see in which sense they could provide an *explanation* of the emergence of the concept of classical electromagnetic field.

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[19] In the more concrete framework of the experiment discussed in Section 3, this claim would amount to asserting that we never observe one field which oscillates with two different phases at the same time. This statement is a tautology if literally construed. It is false if it refers to the empirical implications of a superposition of coherent states like Eq. (4). Indeed, such implications *are* observed, at least inasmuch as “to observe” means to collect the (statistical) results of a series of well-defined measurements.

[20] It might be objected that the phrasing commonly associated with experiments involving, say, atoms, shows no trace of such an operational reduction. But this happens because, in that case, there is a well-established practice in which the operational implications of expressions like “the atom is in a superposition state” are tacitly acknowledged by the actors involved.

[21] An important step in this constructive process was, for example, the study of the properties of the so-called “coherent states”. Among the state vectors which serve to anticipate the behaviour of a quantum field, the coherent states are those whose predictions conform (to some extent) with the laws of classical electromagnetism, provided that one identifies the expectation values of certain quantum observables with the “corresponding” classical quantities. Coherent states provide a theoretical bridge between the practice associated with classical electromagnetism, on the one hand, and the measurement protocols of cavity QED, on the other. Based on this correspondence, one can characterise a field prepared by a *classical* source through *quantum* measurements, typically carried out by means of individual atoms.

These considerations can straightforwardly be extended to the attempts to *derive* the objective features of measurement results from a formal account of the observation process. Since the state vector of a system S refers to the results one can expect from measurements performed on S, the assignment of a state vector to S *presupposes* that there are sets of facts which (within a well-established practice) can be consistently interpreted as the indication that such or such result occurred. “Consistently” means that, for example, one can expect there will be intersubjective agreement about the occurrence of one or the other result. It follows that the existence of facts which “objectively” indicate the occurrence of some result is presupposed by any physical model of the measurement process itself. Let us suppose for instance that M is the system that represents the apparatus (or the observer) which measures S. The observables of M will in this case be operationally defined so as to reproduce the “good” correlations between M and S, i.e. the correlations which enable the identification of M with a “measurement apparatus of S”. But once such a model has been constructed and given an operational meaning, all it can provide are predictions concerning the results of measurements carried out *upon* the compound system S+M. As we saw in the previous section, these predictions are unlikely to provide a better insight into the “foundations” of the theory than any other quantum model (actually, the opposite is true, since this particular model does not explore any border situation, and is entirely conceived so as to fit the existing factual framework).

The foregoing analysis suggests that the instrumentalist interpretation of the state vector can be fruitfully associated with an operationalist–constructivist account of scientific activity, according to which experimental facts and theoretical models (including the definition of observables) co-emerge through a dynamic process. Rather than being constrained by the purpose of providing a faithful representation of a pre-structured reality, such a process is driven by *our* need to establish stable correlations between facts, in order to efficiently anticipate the outcomes of our actions.[22] In this framework, quantum entanglement (like the other structural features of quantum mechanics) does not appear to reflect the structure of a putative reality underlying the phenomena, but rather the conditions of invariance which make the reproducibility and predictability of experimental operations possible. If these ideas proved correct, *adequatio ad rem* would no longer be required in order to have an effective coordination of experimental *situations*. The representationalist conception of knowledge that underlies both the formulation of the quantum paradoxes and their dualist or naturalistic solutions could be relinquished. And empirical adequacy would be understood as a consequence of the close link which exists between the structure of theoretical models and the conditions of possibility of an effective know-how.

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[22] See Bitbol (1996, 1998a, b). See also von Weizsäcker (1980).

References

Bacciagaluppi, G. “The role of decoherence in quantum mechanics”. In: E. N. Zalta (ed.), *The Stanford Encyclopedia of Philosophy*, 2005. Available online: http://plato.stanford.edu/archives/sum2005/entries/qm-decoherence/.

Barrett, J. *The Quantum Mechanics of Minds and Worlds*. Oxford, Oxford University Press, 1999.

Bell, J. S. “On Einstein–Podolsky–Rosen paradox”. *Physics*, 1, 1964, 195–200. (Reprinted in: Wheeler, J. A. & Zurek, W. H. (eds.), *Quantum Theory and Measurement*. Princeton, NJ, Princeton University Press, 1983, pp. 403–408.

Bell, J. S. “Against ‘measurement’ ”. *Physics World*, 8, 1990, 33–40.

Bertet, P., Osnaghi, S., Rauschenbeutel, A., Nogues, G., Auffeves, A., Brune, M., Raimond, J. M. & Haroche, S. “A complementarity experiment with an interferometer at the quantum-classical border”. *Nature*, 411, 2001, 166–170.

Bitbol, M. *Mécanique quantique, une introduction philosophique*. Paris, Champs Flammarion, 1996.

Bitbol, M. *L’aveuglante proximité du réel, anti-réalisme & quasi-réalisme en physique*. Paris, Flammarion, 1998a.

Bitbol, M. “Some steps towards a transcendental deduction of quantum mechanics”. *Philosophia naturalis*, 35, 1998b, 253–280.

Bitbol, M. *Physique et philosophie de l’esprit*. Paris, Flammarion, 2000a.

Bitbol, M. “Arguments transcendantaux en physique moderne”. *La querelle des arguments transcendantaux, Cahiers de philosophie de l’Université de Caen*, 35, 2000b, 81–101.

Bohm, D. *Causality and Chance in Modern Physics*. Londres, Routledge & Kegan Paul, 1957.

Bohr, N. “On the notions of causality and complementarity”. *Dialectica*, 2, 1948, 312–319.

Bouwmeester, D., Ekert, A. K. & Zeilinger, A. (eds.). *The Physics of Quantum Information*. Berlin, Springer, 2001.

Einstein, A. “Remarques préliminaires sur les concepts fondamentaux”. In: André, G. (ed.), *Louis de Broglie: Physicien et Penseur*. Paris, Albin Michel, 1953, pp. 4–15.

Einstein, A., Podolsky, B. & Rosen, N. “Can quantum mechanical description of physical reality be considered complete?”. *Physical Review*, 47, 1935, 777–780.

Everett III, H. “Relative state formulation of quantum mechanics”. *Reviews of Modern Physics*, 29, 1957, 454–462. (Reprinted in: Wheeler, J. A. & Zurek, W. H. (eds.), *Quantum Theory and Measurement*. Princeton, NJ, Princeton University Press, 1983, pp. 315–323.)

Ghirardi, G. C., Rimini, A. & Weber, T. “Unified dynamics for microscopic and macroscopic systems”. *Physical Review*, D34, 1986, 470–491.

Hughes, R. I. G. *Quantum Mechanics, Its Structure and Interpretation*. Cambridge, MA, Harvard University Press, 1989.

Kent, A. “Against many-world interpretation”. *International Journal of Modern Physics*, A5, 1990, 1745–1762.

Leggett, A. J. “Reflections on the quantum measurement paradox”. In: Hiley, B. J. & Peat, F. D. (eds.), *Quantum Implications: Essays in Honour of David Bohm*. London, Routledge & Kegan Paul, 1987.

Mittelstaedt, P. “The constitution of objects in Kant’s philosophy and in modern physics”. In: Parrini, P. (ed.), *Kant and Contemporary Epistemology*. Dordrecht, Kluwer, 1994, pp. 115–129.

Murdoch, D. *Niels Bohr’s Philosophy of Physics*. Cambridge, Cambridge University Press, 1987.

Nogues, G., Rauschenbeutel, A., Osnaghi, S., Brune, M., Raimond, J. M. & Haroche, S. “Seeing a single photon without destroying it”. *Nature*, 400, 1999, 239–242.

Osnaghi, S., Bertet, P., Auffeves, A., Maioli, P., Brune, M., Raimond, J. M. & Haroche, S. “Coherent control of an atom collision in a cavity”. *Physical Review Letters*, 87, 2001, 037902–037905.

Park, J. L. "The self-contradictory foundations of formalistic quantum measurement theories". *International Journal of Theoretical Physics*, 8(3), 1973, 211–218.

Pickering, A. *Constructing Quarks. A Sociological History of Particle Physics*. Chicago, IL, Chicago University Press, 1984.

Raimond, J. M., Brune, M. & Haroche, S. "Manipulating quantum entanglement with atoms and photons in a cavity". *Review of Modern Physics*, 73, 2001, 565–582.

Rosenfeld, L. "Foundations of quantum theory and complementarity". *Nature*, 190, 1961, 384–388.

Rovelli, C. "Relational quantum mechanics". *International Journal of Theoretical Physics*, 35, 1996, 1637–1678.

Saunders, S. "Decoherence, relative states and evolutionary adaptation". *Foundations of Physics*, 23, 1993, 1553–1585.

Schrödinger, E. "Discussion of probability relations between separated systems". *Proceedings of the Cambridge Philosophical Society*, 31, 1935a, 555–563.

Schrödinger, E. "Die gegenwärtige Situation in der Quantenmechanik". *Die Naturwissenschaften*, 23, 1935b, 807–812, 823–828, 844–849. (Translated into English in: Wheeler, J. A. & Zurek, W. H. (eds.), *Quantum Theory and Measurement*. Princeton, NJ, Princeton University Press, 1983, pp. 152–167.)

Shimony, A. "Bell's Theorem". In: Zalta, E. N. (ed.), *The Stanford Encyclopedia of Philosophy*, 2005. Available on-line: http://plato.stanford.edu/archives/sum2005/entries/bell-theorem.

Vaidman, L. "The many-worlds interpretation of quantum mechanics". In: Zalta, E. N. (ed.), *The Stanford Encyclopedia of Philosophy*, 2002. Available online: http://plato.stanford.edu/archives/sum2002/entries/qm-manyworlds.

van Fraassen, B. C. *The Scientific Image*. Oxford, Oxford University Press, 1980.

van Fraassen, B. C. *Quantum Mechanics, an Empiricist View*. Oxford, Oxford University Press, 1991.

von Neumann, J. *Mathematical Foundations of Quantum Mechanics (1932)*. Trans. by R. T. Beyer. Princeton, NJ, Princeton University Press, 1955.

von Weizsäcker, C. F. *The Unity of Nature*. Trans. by F. J. Zucker. New York, Farrar-Straus-Giroux, 1980.

Wheeler, J. A. "Assessment of Everett's 'Relative State' formulation of quantum theory". *Reviews of Modern Physics*, 29, 1957, 463–465. (Reprinted in: Wheeler, J. A. & Zurek, W. H. (eds.), *Quantum Theory and Measurement*. Princeton, NJ, Princeton University Press, 1983, pp. 324–326.)

Wigner, E. "Remarks on the mind-body question". In: Good, I. J. (ed.), *The Scientist Speculates*. London, Heinemann, 1961, pp. 284–302. (Reprinted in: Wheeler, J. A. & Zurek, W. H. (eds.), *Quantum Theory and Measurement*. Princeton, NJ, Princeton University Press, 1983, pp. 168–181.)

Zurek, W. H. "Decoherence and the transition from quantum to classical". *Physics Today*, 44, 1991, 36–44.

Zurek, W. H. "Decoherence, einselection and the quantum origins of the classical". *Review of Modern Physics*, 75, 2004, 715–775.

Can Classical Description of Physical Reality Be Considered Complete?

Gabriel Catren

Abstract We propose a definition of physical objects that aims to clarify some interpretational problems in quantum mechanics. We claim that the transformations induced by an objective property of a physical system must leave invariant all the other objective properties of the same system. The uncertainty principle is understood as a natural consequence of the imbrication between objective properties and non-objective properties. It follows from the proposed definition that in classical mechanics non-objective properties are wrongly considered objective. We conclude that, unlike classical mechanics, quantum mechanics provides a complete objective description of physical systems.

1 Introduction

According to Einstein, quantum mechanical description of physical reality cannot be considered complete. In his words, there are ‘elements of physical reality’ that do not ‘have a counterpart in the physical theory’.[1] In classical mechanics, the exact position and the exact momentum of a particle can be simultaneously predicted for all times from a given set of initial conditions. In quantum mechanics, on the other hand, the momentum of a system characterized by a well-defined position cannot be predicted by the theory (and vice versa). More generally, Heisenberg’s uncertainty principle states that canonically conjugated variables can be simultaneously

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[1] This is the conclusion of the seminal Einstein-Podolsky-Rosen article (Einstein et al., 1935). An historical account can be found in (Mittelsteadt, 2006).

M. Bitbol et al. (eds.), *Constituting objectivity*,

predicted up to some inversely correlated uncertainties. The conceptual content of this principle has been the object of a heated debate that remains unresolved to this day.[2]

In this paper, we argue that quantum mechanics can be understood as the formalization of a rigorous definition of physical objects. According to the standard characterization, the objective properties that define a physical object are invariants under a certain set of transformations (Auyang, 1995; Born, 1998; Nozick, 1998; Weyl, 1952). However, there is no general prescription for determining which transformations are needed in order to define the objective properties of a given physical system. Our definition of physical objects claims that these transformations are induced by the objective properties themselves. In other words, we argue that the transformations induced by the objective properties of a physical system must be *automorphisms* of the system. This definition imposes a compatibility condition on the set of objective properties of a given object. This condition requires that an objective property be invariant under the transformations induced by the other objective properties of the same object. The significant result is that this compatibility condition is not consistent with classical mechanics, but rather with quantum mechanics.

According to our definition of physical objects, the uncertainty principle is the formal translation of the imbrication between objective properties and non-objective properties. As we shall see, asking which position is objective in a quantum system with a well-defined momentum is as nonsensical as asking which side of a die is the objective (or privileged) one. This means that in classical mechanics non-objective elements of physical reality are wrongly considered objective. On the other hand, we claim that quantum mechanics provides a complete description of all the objective properties of a physical object. It follows that the quantum description of a physical object is not incomplete, but rather that classical states are specified by means of too many variables. Since quantum states only describe all the objective properties of the object, they depend on half of the classical variables.

This article develops, in more conceptual terms, the interpretation of quantum mechanics begun in Catren, (2008). This interpretation is founded on an analysis of the symplectic formulation of mechanics (Abraham and Marsden, 1978; Libermann and Marle, 1987; Marsden and Ratiu, 1999; Souriau, 1997) and the geometric quantization formalism (Brylinski, 1993; Kostant, 1970; Souriau, 1997; Woodhouse, 1992). In Section 2, we propose a definition of physical objects. In Section 3, we consider the dynamics of physical objects. In the last section we summarize the obtained results. Finally, in the appendix we give a brief description of the relevant mathematical structures from symplectic geometry.

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[2] Many interpretations were proposed for the uncertainty principle. It was alternatively interpreted as a consequence of the unpredictable perturbations in experimental measures of physical quantities, as a result of the mutual incompatibility of certain experimental contexts, in terms of a subjective lack of knowledge of well-defined objective states, as a description of the statistical spread in an ensemble of similarly prepared systems, as the manifestation of an ontological indeterminateness in the definition of physical quantities, etc. (see for example Hilgevoord and Uffink, 2006).

2 Phases of an Elephant

In this section, we propose a definition of physical objects by means of two postulates. It is possible to show that these postulates cannot be implemented in the framework of classical mechanics (see Appendix and Catren, 2008). On the contrary, quantum mechanics can be considered a satisfactory implementation of the proposed definition of physical objects.

Physical experience is not a chaotic swarm of disconnected empirical data. As Whitehead put it: ‘Sometimes we see an elephant, and sometimes we do not’ (Whitehead, 1978, p. 4). In other words, physical reality is organized in different objective configurations that can be identified and recognized. According to a standard characterization, an object is a physical configuration that can be completely characterized by specifying the set of the object’s *objective properties*. Such a set will be called the *eidos* ε of the physical object.[3] In order to unpack this characterization, it is necessary to specify what we understand by objective properties. As we shall see, the characterization of objective properties as invariants under a certain set of transformations does not suffice for defining the notion of objective properties. In order to achieve a satisfactory definition of physical objects, it is necessary to take into account that a physical object does not only have objective properties that allow us to identify and recognize it: it also has different *phases, aspects* or *profiles*. In general, various kinds of transformations can be performed in order to observe the different phases of an object. For instance, there are objects that exhibit different phases when rotated around a given axis. The transformations that interchange the phases of an object will be called *phase transformations* of the object. A set of phases connected by means of a given one-parameter family of phase transformations will be called *orbit of phases*. For instance, the different phases observed when the object is rotated around a given axis belong to the same orbit of phases. Since a phase transformation only modifies the observed phase, the objective properties that define the object are necessarily invariant under phase transformations. In order to stress this fact, phase transformations will also be called *automorphisms* of the object. In this way, we recover the idea that an object can be defined by means of the invariants under a certain group of transformations (see for example Auyang, 1995; Born, 1998; Nozick, 1998). Following H. Weyl, we can thus state that ‘[...] objectivity means invariance with respect to the group of automorphisms’ (Weyl, 1952). Nevertheless this standard characterization is insufficient for defining objectivity. This problem was clearly stated by R. Nozick (1998): ‘The notion of invariance under transformations cannot (without further supplementation) be a *complete* criterion of the objectivity of facts, for its application depends upon a selection of *which* transformations something is to be invariant under.’[4] The definition of physical objects that we will propose provides this ‘further supplementation’ by stating that the

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[3] This Husserlian terminology is borrowed from Heelan, (2004).

[4] Analogously, H. Weyl continues the preceding quotation as follows: ‘Reality may not always give a clear answer to the question what the actual group of automorphisms is [...]’ (Weyl, 1952).

object's automorphisms are induced by the objective properties of the object. Hence, not only is an objective property invariant under all the object's automorphisms, but it also induces a one-parameter family of automorphisms.

In order to formalize this idea, we will propose a definition of physical objects by means of two fundamental postulates. To do so, we will introduce some terminology. We will say that a physical object *realizes* a certain number of *universal operators* in a way that depends on the objet. For example, an object that can be rotated around the z-axis realizes (in a particular way that depends on the objet) the universal operator that generates universal rotations around the z-axis. We will sometimes say that a universal operator makes *ingression* into the object in a way that depends on the object. The important point is that two different objects can realize different universal operators and/or realize differently the same universal operator. Hence, an object can be characterized by the way in which it realizes a particular set of universal operators. Therefore, there are two ways of characterizing an object, namely by means of its objective properties or by specifying how it realizes certain universal operators. Our first postulate unifies these two ways of characterizing an object:

Postulate ♠: The value of an objective property of a given object specifies the particular way in which the object realizes an universal operator.

The ingression of an universal operator into an object defines what we will call an *eigenoperator* of the object. For example, there are objects defined (at least partially) by the objective property that specifies how the universal operator that generates universal rotations around the z-axis makes ingression into the object. This ingression defines an eigenoperator that generates the object's rotations around the z-axis. In this way, postulate ♠ states that the particular value p_0 that an objective property p takes on a given object $\mathcal{O}$ defines an ingression application $\imath_{p0}$ of the form:

$$\imath_{p_0} : \textbf{\textit{universal operator}}\ \xi_p \rightarrow \textbf{\textit{eigenoperator}}\ \hat{v}_p \tag{1}$$

Each possible value of the objective property p defines a different ingression of the same universal operator ξ_p, that is to say a different eigenoperator $\hat{v}_p$ of $\mathcal{O}$.

The second postulate of our definition of physical objects specifies the nature of the transformations generated by the object's eigenoperators:

Postulate ♣: The transformations generated by an object's eigenoperator are phase transformations.

In other words, a transformation generated by one of the object's eigenoperators is not an objective transformation of the object into another object. In the previous example, this means that the object's rotation around the z-axis is not an objective transformation of the object, but rather an automorphism that leaves the objective properties invariant. These two postulates can be assembled together by stating that an objective property specifies how a universal operator is realized by the object in the form of an eigenoperator that generates automorphisms of the object. We will sometimes summarize this characterization by saying that an objective property *induces* a one-parameter family of automorphisms. In this way, the object's *eidos* (i.e. the set of its objective properties) defines the identity of the object by inducing the phase transformations between its different phases. We can therefore propose the following definition:

Definition: An object is a physical configuration that can be completely characterized by specifying the values of the objective properties that induce all the object's automorphisms.

We could say that this definition provides a rigorous formalization of Weyl's prescription: 'Whenever you have to do with a structure-endowed entity Σ try to determine its group of automorphisms, the group of those element-wise transformations which leave all structural relations undisturbed. You can expect to gain a deep insight into the constitution of Σ in this way' (Weyl, 1952). In the case of the proposed definition of physical objects, the objective properties that define the object do induce the object's automorphisms.

One significant consequence of this definition is that the phase transformations induced by an objective property in the object's *eidos* cannot modify the other objective properties of the object. Objective properties must therefore be invariant under phase transformations induced by the other objective properties of the same object. Let's consider for example an object defined by the *eidos* $\varepsilon = \{p_1, p_2, \ldots, p_n\}$, where each p_i is an objective property of the object. This means that it is possible to completely characterize the object by the set of values that the properties $p_1, p_2, \ldots, p_n$ take on the object. The standard definition of objectivity requires that the value of each objective property p_i be invariant under a certain group of transformations. Nevertheless – as we have said before – it is not clearly stated which transformations have to be considered. Our definition bypasses this flaw by stating that each objective property p_i induces a one-parameter family of automorphisms of the object. Hence, we arrive at the conclusion that each objective property has to be invariant under the automorphisms induced by all the other properties in the same *eidos*. This fact imposes a restrictive condition on the *eidos* of an object. The *eidos* is not merely an arbitrary collection of objective properties. Each property has to satisfy the condition of being invariant under the phase transformations induced by all the others. If a property p_1 is invariant under the phase transformations induced by p_2, we will say that these properties are commensurable or compatible. If a property is modified by the phase transformations induced by an objective property in the object's *eidos*, we will say that the former is *phased out* by these phase transformations. Therefore, the *eidos* is characterized by an internal structure that guarantees the compatibility between the objective properties that define the object.[5] The object will be completely determined if the *eidos* contains the maximum number of mutually compatible independent properties. In particular, if a property q is modified by the phase transformations induced by an objective property p in the object's *eidos*, then q cannot also be an objective property of the object. This statement can

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[5] In technical terms, the action induced by a property g on a property f is given by the Poisson bracket $\delta_g f = \{f, g\}$. The requirement of internal consistency of the *eidos* ε imposes the condition $\{f, g\} = 0, \forall f, g \in \varepsilon$. In other words, the *eidos* is a *commutative Poisson algebra*.

be considered the conceptual translation of the uncertainty principle. In particular, the momentum p is a property that induces transformations in the position q (and vice versa).[6] Hence, if the momentum p is an objective property in the object's *eidos*, then the position q cannot also be an objective property. The position q is rather a phase that changes when the object is acted upon by the phase transformations induced by p. In other words, since q and p are incompatible, they cannot both be objective properties of the same object. Asking which position is objective in an object with a well-defined momentum p is as nonsensical as looking for the objective (or privileged) side of a die. Nevertheless, even if a die has no objective side, it will show a particular side when thrown. This does not mean that the resulting side was the objective but unknown side of the die, nor that it becomes the objective side of a new die produced by the toss. Analogously, even if a physical system with a well-defined momentum p has no objective position q, it will appear in a particular position q_1 if a measurement of the position is performed. This does not mean that q_1 was the objective but unknown position of the system, nor that q_1 becomes the objective position of a new object produced by the measurement.

Figure 1 resumes the proposed definition of physical objects. The property p is an objective property of the object represented. The value p_0 of this property specifies how the universal operator ξ_p makes ingression into the object. The ingression of the universal operator ξ_p defines the eigenoperator $\hat{v}_p$. This eigenoperator generates phase transformations that act upon the property q. Hence, the different values of this property are just different phases in the orbit of phases generated by the eigenoperator $\hat{v}_p$.

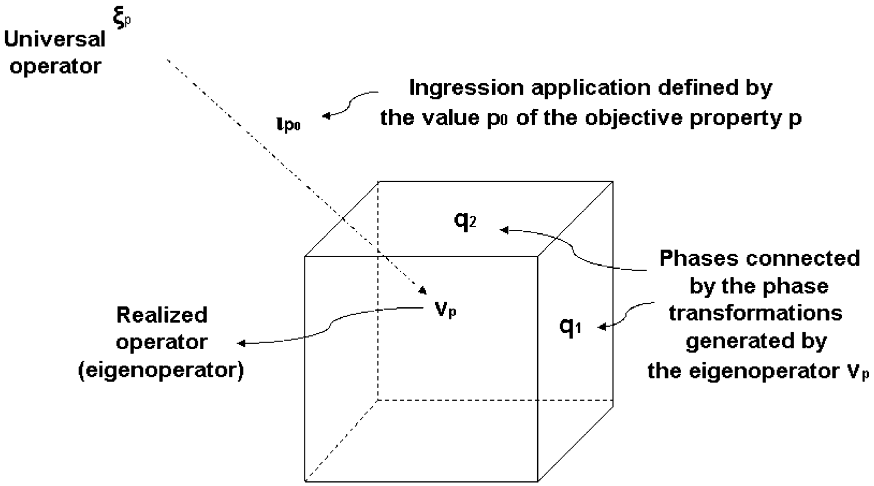

Fig. 1 Physical object defined by the objective property p.

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[6] The Poisson bracket $\{q, p\} = 1$ means that the momentum p is the generator of the infinitesimal canonical transformations of the position q (and vice versa).

According to postulate ♣, the transformations induced by the objective properties of an object are phase transformations. As we have shown, the uncertainty principle is a formal consequence of this postulate. If the momentum p is an objective property of an object, then the position q is completely phased out by the phase transformations induced by p. Since the classical definition of objective physical states comprises both the exact position and the exact momentum of the system, this postulate cannot be consistently implemented in the framework of classical mechanics. We can also argue differently. If both the position and the momentum were included in the object's *eidos*, then both the position and the momentum would be phased out by the phase transformations induced by the momentum and the position respectively. Therefore, both the position and the momentum would only be non-objective phases, and the physical system would have no objective properties at all. We can thus conclude that the classical definition of states by means of both q and p is incompatible with our definition of physical objects. The classical definition of a physical state is consistent only if we deny that the transformations induced by an objective property of the system are phase transformations. In fact, in classical mechanics the transformations induced by an objective property are not interpreted as phase transformations, but rather as transformations between states that are objectively different. For example, the transformations induced by the Hamiltonian are interpreted as temporal evolutions between different objective states. In this way, the definition of classical states becomes consistent. Nevertheless, objective properties can no longer be defined as quantities that induce the object's automorphisms. Hence, the classical definition of both objective properties and physical objects remains problematic. The situation has thus been conveniently reversed: the problem is no longer how to recover objectivity in quantum mechanics, but rather how to define classical objects in a consistent manner.

For the sake of simplicity we have only considered the case of an object with a well-defined momentum and a completely undetermined position. The reciprocal case – a well-defined position with an undetermined momentum – is completely analogous. In the general case, both the position and the momentum are subject to certain indeterminacies. In fact, the flexibility of quantum mechanics' formalism makes it possible to define physical objects characterized by properties which are neither objective properties nor phases, but rather a mixture of both. In these cases, neither q nor p are *sharp* objective properties of the object. For example, if q is an *unsharp* objective property of an object, then the induced phase transformations are *unsharp phase transformations*. Therefore, the conjugated momentum p is phased out only partially. Hence, p is in turn an unsharp objective property that partially phases out the coordinate q. Therefore, q cannot be a sharp objective property (as it was assumed at the beginning) and the circle closes consistently. This means that a certain property can be partially considered an unsharp objective property of the object and partially a phase. It follows that the mere distinction between objective properties and phases does not suffice for treating generic cases. The resulting subtle equilibrium between unsharp objective properties and unsharp non-objective phases is formally governed by the uncertainty principle.

3 The Revenge of Zeno

> [. . .] *comment l'objet qui se meut serait-il en un point de son trajet* ?
>
> H. Bergson, 1938, *La pensée et le mouvant*, (p. 158)

The analysis presented in the previous section makes no reference to temporal processes. Since physics, as it is usually understood, studies the temporal evolution of physical systems, we will now introduce a temporal parameter t. The consideration of temporal processes allows to shift the discussion from momenta p_i (observables that generate infinitesimal canonical transformations of positions q^i) to velocities $\dot{q}^i$ (infinitesimal temporal variations of q^i).

We will begin by noting that an object $\mathcal{O}$ characterized by a well-defined velocity lacks, by definition, a well-defined position. Analogously, a nomad is a person characterized by the property of not having a well-defined position. We claim that this trivial fact contains the conceptual kernel of the uncertainty principle for positions and velocities. One might argue that this lack of a well-defined position can be bypassed by decomposing the movement in instantaneous objects $\mathcal{O}_t$ that evolve in time, that is to say that change *objectively* as time passes. Even though the state of motion of the object $\mathcal{O}$ makes it impossible to assign it a constant position, it might still be possible to define the objective positions of the different instantaneous objects $\mathcal{O}_t$. According to the standard terminology, the objects $\mathcal{O}_t$ might be called the *instantaneous states* of the object $\mathcal{O}$. We will now analyze whether this strategy can be consistently pursued in the framework of our definition of physical objects.

In what follows, we will restrict the analysis to the simplest case. Let's consider an object $\mathcal{O}$ consisting of a free particle moving with a constant momentum p (or a constant velocity $\dot{q}$). In principle, we could decompose $\mathcal{O}$ in a sequence of instantaneous objects $\mathcal{O}_t$. Each of these instantaneous objects $\mathcal{O}_t$ would be characterized by the objective property p (which induces the displacements in q) and its position $q(t)$ at t. In other words, as in classical mechanics, both p and $q(t)$ would be objective properties of the instantaneous object $\mathcal{O}_t$. Even if $q(t)$ is not an objective property of $\mathcal{O}$, it might still be considered an objective property of the instantaneous object $\mathcal{O}_t$. Nevertheless, this decomposition of $\mathcal{O}$ in instantaneous objects $\mathcal{O}_t$ characterized by both p and $q(t)$ is inconsistent with the proposed definition of physical objects. Since the position $q(t)$ changes in time, the different instantaneous objects $\mathcal{O}_t$ are *objectively* different. This results from the fact that the position $q(t)$ is considered an objective property of $\mathcal{O}_t$. Hence, the different instantaneous objects $\mathcal{O}_t$ differ in the objective property $q(t)$. This means that temporal evolution is a non-trivial *objective* modification of the instantaneous objects $\mathcal{O}_t$. Therefore, the Hamiltonian h, which induces the transformations of t, cannot be an objective property of the instantaneous object $\mathcal{O}_t$. According to our definition of physical objects, if h were an objective property of $\mathcal{O}_t$, then the transformations induced by h would be phase transformations that could not *objectively* modify the object. Since temporal evolution objectively modifies the instantaneous objects $\mathcal{O}_t$, the Hamiltonian h cannot be an objective property of $\mathcal{O}_t$. Nevertheless, this conclusion contradicts the fact that if p is an objective property of $\mathcal{O}_t$, then $h = \frac{p^2}{2m}$ should also be an objective property of $\mathcal{O}_t$. We can also argue in the opposite sense. Since p is an objective property of $\mathcal{O}_t$, the Hamiltonian $h = \frac{p^2}{2m}$ is also an objective property of $\mathcal{O}_t$. Hence, according to our definition of physical objects, the transformations induced by h are phase transformations.[7] Therefore, the different $\mathcal{O}_t$ are not different instantaneous objects $\mathcal{O}_t$, but rather different phases of the same object $\mathcal{O}$. We can thus conclude that the object $\mathcal{O}$ cannot be consistently decomposed in different instantaneous objects $\mathcal{O}_t$. Hence, the object $\mathcal{O}$ is an indecomposable object with different non-objective temporal phases. In other words, what we observe at different times are not different instantaneous objects $\mathcal{O}_t$, but rather different non-objective temporal phases of the same object $\mathcal{O}$.

These considerations do not mean that it is impossible to define physical objects that change objectively in time. We simply claim that a system moving with a constant velocity cannot be analyzed in terms of instantaneous objects (or objective states) that change objectively in time. However, in principle, it is possible to define an instantaneous object $\mathcal{O}_t$ such that its *eidos* contains the property t.[8] Since the time t is an objective property of the instantaneous object $\mathcal{O}_t$, the property h is phased out by the phase transformations induced by t. Hence, the transformations induced by h are no longer phase transformations, but rather objective transformations of the object. Therefore, at different times t and t', there are instantaneous objects $\mathcal{O}_t$ and $\mathcal{O}_{t'}$ that are objectively different.[9]

4 Conclusion

We have defined a physical object as a set of mutually compatible objective properties such that each objective property induces a one-parameter family of automorphisms. The compatibility condition guarantees that the objective properties are invariant under the automorphisms induced by all the other objective properties of the same object. The uncertainty principle is a direct consequence of the mutual imbrication between objective properties and non-objective phases: if p is a sharp objective property of an object, then the property q (phased out by the phase transformations induced by p) cannot also be an objective property.

We could restate Einstein's requirement by saying that a satisfactory physical theory has to provide a complete *objective* description of physical reality

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[7] This statement is a rigorous interpretation of the fact that '[...] the motion of a mechanical system corresponds to the continuous evolution or unfolding of a canonical transformation' (Goldstein, 1981).

[8] We are supposing that it is possible to treat time and the Hamiltonian as another pair of conjugated canonical variables. In fact, this is possible in the framework of the so-called parameterized systems (see for example Lanczos, 1986 and Castagnino et al., 2002).

[9] The arguments presented in this section suggest that a satisfactory comprehension of the uncertainty principle for time and energy might be an essential component of a consistent interpretation of quantum mechanics.

(Einstein et al., 1935). Firstly, this means that every objective property of physical reality should have a counterpart in the theory. Secondly, non-objective properties should not be mistaken for objective properties by the theory. The classical description of a physical system includes both its objective properties and its non-objective phases. Unlike classical mechanics, quantum mechanics provides a complete objective description of physical systems.

5 Appendix

We will now give a brief account of the formal structures that underlie the proposed definition of physical objects (for more details see Abraham and Marsden, 1978; Catren, 2008; Libermann and Marle, 1987; Marsden and Ratiu, 1999). A *symplectic action* of a Lie group G (of Lie algebra g) on a symplectic manifold (M,ω) is a group action $\Phi : G \times M \to M$ that preserves the symplectic form ω, i.e. that satisfies $\Phi_g^*\omega = \omega$, where Φ_g^* is the pullback defined by the map $\Phi_g(\cdot) \doteq \Phi(g,\cdot)$. Such an action defines a map $\iota : g \to TM$ (that we have called *ingression*) between Lie algebra elements $\xi \in \mathrm{g}$ (that we have called *universal operators*) and *fundamental vector fields* v_ξ on M (that we have called *realized operators*). The fundamental vector field v_ξ is given by the expression

$$v_\xi(x) = \frac{d}{d\lambda}(exp(-\lambda\xi)\cdot x)|_{\lambda=0},$$

where $x \in M$ and $\xi \in g$. The symplectic action is said to be *Hamiltonian* if the ingression map $\iota : g \to TM$ can be "factorized" as follows

$$\begin{tikzcd} \mathrm{g} \arrow[r, "\tilde{\mu}"] \arrow[rr, bend left, "\iota"] & \mathcal{C}^\infty(M) \arrow[r, "\pi"] & TM, \end{tikzcd} \tag{2}$$

where $\tilde{\mu} : g \to \mathcal{C}^\infty(M)$ is the so-called *co-momentum map* and $\pi : \mathcal{C}^\infty(M) \to TM$ is the map between classical observables on M and the so-called *Hamiltonian vector fields*. A classical observable $f \in \mathcal{C}^\infty(M)$ defines a Hamiltonian vector field v_f by means of the expression $i_{v_f}\omega = df$, where $i_{v_f}\omega$ denotes the contraction of v_f with the symplectic two-form ω. The Hamiltonian vector field v_f is the generator of the symplectic diffeomorphisms $\phi_\lambda^f : M \to M$, that is to say of the canonical transformations induced by the observable f. In $\mathbb{R}^2$, the Hamiltonian vector field associated to an observable $f \in \mathcal{C}^\infty(M)$ is given by the expression

$$v_f = \frac{\partial f}{\partial p}\frac{\partial}{\partial q} - \frac{\partial f}{\partial q}\frac{\partial}{\partial p}.$$

In other words, a symplectic action is Hamiltonian if the fundamental vector field v_ξ that realizes on M the universal operator $\xi \in \mathrm{g}$ can also be obtained as the Hamiltonian vector field v_f associated to a physical observable f. It might seem that we have all the elements for implementing the proposed definition of physical objects. In fact, the sequence of maps (2) seems to be the formal implementation of the application (1) between universal operators and eigenoperators. According to (2), the objective properties $f \in \mathcal{C}^\infty(M)$ "factorize" the ingression on M of universal operators $\xi \in \mathrm{g}$. Nevertheless, the two postulates that we used for defining physical objects are not satisfied in the classical framework. The main problem is that the homomorphism π between the Poisson algebra of classical observables $f \in \mathcal{C}^\infty(M)$ and the Lie algebra of classical operators v_f (under the Lie bracket of vector fields) is not an isomorphism of Lie algebras. This is a consequence of the fact that the map $f \mapsto v_f$ is not injective (since $v_k = 0$ for any $k \in \mathbb{R}$). The fundamental consequence of this non-injectivity is that $\mathcal{L}ie_{v_p} v_q = [v_q, v_p] = v_{\{q,p\}} = v_1 = 0$, even if $\mathcal{L}ie_{v_p} q = \{q, p\} = 1$. This means that in classical mechanics, a non-trivial transformation (generated by the classical operator v_p) of the value of an objective property q does not necessarily modify the realized operator v_q. This means that in classical mechanics, different states (characterized by different values of q) do not realize differently the same universal operator in g. Hence, an objective property cannot be defined – as we did in postulate ♠ – as a quantity that specifies the particular way in which the object realizes a universal operator. As for postulate ♠ (according to which a realized operator generates automorphisms of the object), we have already shown why it cannot be consistently implemented in classical mechanics. In order to satisfy these postulates, it is necessary to extend the classical operators v_f to quantum operators $\hat{v}_f$ such that the latter satisfy Dirac's quantization conditions. In the framework of geometric quantization, this can be done by means of the so-called prequantization formalism (Brylinski, 1993; Kostant, 1970; Souriau, 1997; Woodhouse, 1992).

References

Abraham, R. and Marsden, J.E. [1978]: *Foundations of Mechanics*, second ed., Cambridge, MA: Addison-Wesley.

Auyang, S.Y. [1995]: *How Is Quantum Field Theory Possible?*, Oxford/New York: Oxford University Press.

Bergson, H. [1938]: *La pensée et le mouvant*, Paris: Presses Universitaires de France.

Born, M. [1998]: 'Physical Reality', in E. Castellani (ed.), *Interpreting Bodies. Classical and Quantum Objects in Modern Physics*, Princeton, NJ: Princeton University Press.

Brylinski, J.-L. [1993]: *Loop Spaces, Characteristic Classes, and Geometric Quantization*, Progr. Math. 107, Boston, MA: Birkhäuser Boston.

Castagnino, M., Catren, G. and Ferraro, R. [2002]: 'Time Asymmetries in Quantum Cosmology and Boundary Conditions to the Wheeler-DeWitt Equation', *Classical and Quantum Gravity*, Vol. 19, No. 18, pp. 4729–4746.

Catren, G. [2008]: 'On Classical and Quantum Objectivity', *Foundations of Physics*, Vol. 38, No. 5, pp. 470–487.

Einstein, A., Podolsky, B. and Rosen, N. [1935]: ‘Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?’, *Physical Review*, Vol. 47, pp. 777–780.

Goldstein, H. [1981]: *Classical Mechanics*, Cambridge, MA: Addison-Wesley.

Heelan, P. [2004]: ‘The Phenomenological Role of Consciousness in Measurement’, *Mind and Matter*, Vol. 2, No. 1, pp. 61–84.

Hilgevoord, J. and Uffink, J. [2008]: ‘The Uncertainty Principle’, The stanford Encyclopedia of Philosophy (Fall 2008 Edition), Edward N. Zalta (ed.), http://palto.stanford.edu/archives/fall2008/entries/qt-uncertainty.

Kostant, B. [1970]: ‘Quantization and Unitary Representations’, *Lecture Notes in Mathematics*, Vol. 170, Berlin/Heidelberg/New York: Springer.

Lanczos, C. [1986]: *The Variational Principles of Mechanics*, New York: Dover.

Libermann, P. and Marle, C.-M. [1987]: *Symplectic Geometry and Analytical Mechanics*, Dordrecht, Holland: D. Reidel Publishing.

Marsden, J.E. and Ratiu, T.S. [1999]: *Introduction to Mechanics and Symmetry*, second ed., New York: Springer.

Mittelsteadt, P. [2006]: ‘Einstein’s Objections Against Quantum Mechanics’, *Physics and Philosophy*, http://hdl.handle.net/2003/22995.

Nozick, R. [1998]: ‘Invariance and Objectivity’, *Proceedings and Addresses of the American Philosophical Association*, Vol. 72, No. 2, pp. 21–48.

Souriau, J.-M. [1997]: *Structure of Dynamical Systems. A Symplectic View of Physics*, Cambridge, MA: Birkhäuser Boston.

Weyl, H. [1952]: *Symmetry*, Princeton, NJ: Princeton University Press.

Whitehead, A.N. [1978]: *Process and Reality*, New York: Free Press.

Woodhouse, N. [1992]: *Geometric Quantization*, second ed., Oxford: Oxford University Press.

B.4

Beyond Relativity and Quantum Mechanics

A View of the Symbolic Structure of Modern Physics

Ion-Olimpiu Stamatescu

Abstract This essay discusses the character of the concepts used in modern physical theories and the symbolic structure they define. Are these symbols interchangeable free constructs and the theories artificial devices with in and out slots to accomodate observations? Are the symbols "read" up - that is, interpreted - from nature and are the physical theories directly referring to a final reality? Did our reason by itself fix immovable rules to build up these concepts? None of the "fundamentalist" stances hereto appears supported by the scientific process of developing physical knowledge. A more differentiate approach is suggested, which could offer for these questions a sustainable, less strong but may be more fruitful point of view.

1 On Physical Understanding, Symbols, and Reality

In discussing modern physics in the perspective of critical philosophy it is necessary to first ask what physical understanding involves. Can the structural scheme provided by transcendental philosophy (which deals with the development and interpretation of physical theories) be used in this discussion, and if so, how?

The symbolic, conceptual structures of physics are developed in accordance with a process endowed its own dynamics directed to physical knowledge. Periodically during its development physics leaves behind those very worldviews and philosophical schemes that it has contributed to build up. Examples are provided by Newtonian space–time as an a priori objectification frame, or classical mechanics as a guarantee for determinism and continuity. While the dialogue between physics and philosophy is necessary and rewarding, physics would not do its job if its primary concern were to agree with the latter. Hence the following

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discussion cannot be understood in terms of the simple dichotomy: "does the Kantian system apply; or does it not?" Irreducible incompatibilities must be taken into account, which do not allow a clear-cut answer to that kind of question.

In this essay some concepts related to physical understanding will be examined. The background of this discussion is modern and contemporary physics. Problems will be raised in relation to objectification, intuition, continuity and realism, justification and truth, where critical philosophy defines questions and provides schemes for analysing the answers. This study is not essentially philosophical since it proceeds from the point of view of physics.

In this kind of discussion physicists are sometimes confused by the use of the word reality. Some ideas should be fixed up, even if they sound philosophically trivial. A physicist typically assumes that "reality" (or "nature") is something external to mind, more or less independent of our cognitive activity, toward which this activity is directed. This is a very simplistic metaphysical assumption; the alternative assumption, according to which everything happens in my own mind, is not very convincing either, and it is even more inconsistent to assert that the real takes place in many minds: what are they to me if there is no external reality? What about the alleged independence? If I am at a party and someone leaves, I observe that the party usually goes on. Then it is natural to assume that if I am to leave the party will equally go on. Some of the future events may depend to a certain degree on my being there and even on what I am thinking about it. And there will be also events for which it seems safe to assume that they do not depend at all on my knowledge about them, such as what will happen to me if I were to fall from a high building. More generally learning the laws of nature is vital for us, and yet it seems indifferent to them.

At this point we raise the question of "realism" in the following way: do the concepts that we develop in order to speak about reality (or nature) and our encounter with it have some "things" to which they refer "there"? The safest attitude in this regard seems to be still that of Hertz:

"We construct internal appearances or symbols of external objects, and we make them such that what results by thought-necessity from such symbols will always be a symbol of that, what follows by nature-necessity from the symbolized objects […] – The symbols we speak of are our representations for things; they have with the things the one essential concordance which consists of satisfying the above requirement, but it is not necessary for their scope to have any other concordance with the things."[1]

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[1] "Wir machen uns innere Scheinbilder oder Symbole der äusseren Gegenstände, und zwar machen wir sie von solcher Art, dass die denknotwendigen Folgen der Bilder stets wieder Bilder seien von den naturnotwendigen Folgen der abgebildeten Gegenstände […] – Die Bilder, von welchen wir reden, sind unsere Vorstellungen von den Dingen; sie haben mit den Dingen die eine wesentliche Übereinstimmung, welche in der Erfüllung der genannten Forderung liegt, aber es ist für ihren Zweck nicht nötig, dass sie irgend eine weitere Übereinstimmung mit den Dingen äe." Hertz, 1894, p. 1. All translations are ad hoc.

The perspective associated with this attitude will be called "symbolic stance". It essentially assumes that there are external things to which concepts refer, that the concepts are not just "images" of these things and that we are free to make them, provided that the resulting conceptual structure parallels the relations in which these things show up. In this context we would speak of "physical understanding" as the development and establishment of theoretical conceptual structures in which symbols assumed to point out onto "things" are bound to phenomena in an interpretational network and to each other in an analytic network. A more extended discussion is provided elsewhere.[2] Here we shall concentrate on the character of the reference relation and the associated questions of necessity and definiteness.

In the background of what follows the above "symbolic stance" is assumed. This express a bias toward realism since interaction with "reality" is central for shaping our concepts through the "requirement" described by Hertz. Nevertheless the questions raised by the critical philosophy retain their full relevance.

2 On Forming Physical Concepts, Certainty and Objectification, and the Development of Physical Knowledge

Following Aristotle, "we consider that we know something if we think to know the cause based on which a thing is (and to know that this is its cause) and also to know that it cannot be otherwise". In order for this to be possible, knowledge must be based on "something which is true and primary and immediate and… which must precede the conclusion and be its cause."[3]

Philosophy of knowledge has always been concerned with the relevance of these questions: necessity, certainty, reduction to principles, and with their grounding. Physics, on the other hand, as a "knowledge building" enterprise is bound to operate within the frame of such questions which define its praxis and standards. But this also means that philosophical consideration and physical experience imply different perspectives.

In "Metaphysische Anfangsgründe der Naturwissenschaft" Kant specifies a central epistemological problem. He defines[4] science as apodictictally certain, since only in this way the necessary character of the laws of nature can be secured. The "pure part" of science which grounds this certainty consists of metaphysics, which is "rational knowledge from concepts for themselves", and mathematics, which is

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[2] I.-O. Stamatescu, 2002.

[3] Analytica posteriora I 2 71b. Aristotle develops these problems in both Physics and Metaphysics, where he discusses the question of "being", the foundation of knowledge, the role of logic, principles and categories and such basic concepts as "surplus and default" or "attraction and repulsion". We mention this to remember how old and variate this discussion is.

[4] I. Kant, 1957, pp. A V,VI,VII.

concerned with the construction of concepts from the representation of objects in an "Anschauung a priori" (a priori forms of intuition). The metaphysical system itself is based on the table of categories and the resulting construction operates by means of a tight mathematical basis. Emphasizing the a priori character of mathematics, Kant goes on to argue: "I contend that in every special natural science only so much science is to be found as mathematics can be found in it."[5]

Kant's strategy provides the adequate framework for discussing the relevance and character of physical knowledge. I will define physical concepts in terms of basic categories, and resort to mathematics to construct them and their relations on the basis of the fundamental forms of intuition which control and order the observations. One arrives in this way to the objectification by which empirical information is taken over into physical knowledge. This process can involve a hierarchy of statements, such as Cassirer's (data, laws, principles), assuming that their claim to certainty is a feature of this procedure. This does not mean that empirical proof is not *required* – it only means that empirical information cannot by itself support a positive claim to certainty. As Cassirer observes, the essence of this claim is the general principle of causality (der allgemeine Kausalsatz), which is "ein Sprung in Nichts" (a jump into void)[6] – or "a desperate hope"?[7]–, and its completion resides on clean metaphysics, reliable empirical information and correct mathematics.

However, this beautiful construction has its problems. Both its strength and its weakness rest in its rigidity. Is there a solid basis for it? Thus, for instance, according to Kant Euclidean geometry of space and linear, absolute time determine the a priori structure of the fundamental forms of intuition of space and time. However, already 1879 Helmholtz noticed that "Kant's teaching of the a priori forms of intuition is a very clear expression of the matters: but these forms must be empty of content and free enough to take in any content which could present itself to the corresponding form of perception."[8] This is not only a question of empirical adequacy. Indeed, if we declare, e.g., Euclidean geometry and Galilean relativity to be relevant in some way we run into serious contradictions: since the theory provides the instruments of objectification, and the correct theory is Einstein's relativity, how can contradicting criteria of objectification be acting simultaneously in the same object (some body in space and time)? But even if we reduce the fundamental forms of intuitions to ordering

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[5] "Ich behaupte aber, daß in jeder besonderen Naturlehre nur so viel eigentliche Wissenschaft angetroffen werden kann, als darin Mathematik anzutreffen ist." I. Kant, 1957, A VIII, IX. The role of mathematics in natural science is undoubtedly one of the most outstanding questions in the epistemology of science. For some remarks concerning this subject in the context of transcendental philosophy see H. Wismann and I.-O. Stamatescu, 1994.

[6] E. Cassirer, 1987, p. 200.

[7] See Peirce, 1991. For Peirce the understandability of the natural process is a postulate, or, just as well, a "desperate hope", since only in as far as this holds is knowledge possible.

[8] "Kants Lehre von den a priori gegebenen Formen der Anschauung ist ein sehr glücklicher und klarer Ausdruck des Sachverhältnisses; aber diese Formen müssen inhaltsleer und frei genug sein, um jeden Inhalt, der überhaupt in die betreffende Form der Wahrnehmung eintreten kann, aufzunehmen." H. von Helmholtz, 1971, p. 299.

tendencies, with which Helmholtz might agree, we run into problems, since these orderings cannot be trivially achieved throughout: they become intermingled because of relativity effects, and they are limited by quantum effects. Finally, we may try to put them into a hierarchy (in as much as, say, Newtonian mechanics is in some sense a good approximation in the frame of special relativity). But this means a very different understanding of the problem of how to achieve certainty, which in essence is different from that of Kant. The same holds for categories: something like quantum mechanical entanglement, for instance, cannot be set up in a classical scheme. The problem is not that our concepts must be changed, but that even the rules for constructing them evolve as a result of growing physical knowledge. Let us now take a closer look at the perspective brought about by physics itself.

Physics builds up symbolic structures under the constraints of empirical adequacy and mathematical consistency. By this we mean systems of concepts and rules, bound in relational and interpretational networks, in reference to both observations and what lies "behind the phenomena". We do not "read up" theories from the observations, nor do we construct them arbitrarily by simply summarising observations. Rather, we develop them on the basis of the latter, using mathematics as both a reservoir of concepts and analytic rules. This is a very tight process according to which already acquired knowledge sets theoretical lines allowing for the evolution of established schemes *and* the emergence of new ones – mostly in contradiction with these lines themselves. Most theoretical developments are evolutions and revolutions at the same time. Thus special relativity uses the relativity principles including homogeneity and isotropy of space but contradicts the simultaneity principle, yet both belong to pre-Einsteinean physics. In the praxis of physics there is no immovable a priori, not tied to the growth of this praxis itself. In as much as intuition is not just "canonization of common sense"[9] it must itself evolve (see also B. Falkenburg, 2002, I.-O. Stamatescu, 1995).

Thus we understand space by starting from our three-dimensional experience, but then also by extending this experience beyond its immediate limits, both by the use of instruments and by reasoning. There is no reason to feel secure of never needing to deal with higher dimensional spaces. We have already had to deal with "particles" having no well defined trajectories; in a not too distant future we will perhaps have to deal with "paradoxal" relativistic; none of these are daily intuitions. We steadily need to develop new intuitions and this involves observation and reasoning ("welche Reihe von Anschauung und Nachdenken verfolgte ich nicht…" – "what chain of intuitions and reasoning was I not following…"[10]).

Hence the physical concepts are built up under rules which themselves evolve as a result of the need to incorporate empirical accuracy and mathematical consistency in them. Quantum physics, for instance, does not only bring forth new concepts but also new rules for objectification and for categorical analysis. The validation rules are unchanged in their essence – empirical and mathematical correctness – but their

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[9] H. Reichenbach, 1965, p. 73.

[10] J.W. von Goethe. S.39.

realization can become more complex in order to cope with abstract theoretical schemes and the phenomenology.

This raises the question of continuity. Our knowledge grows not just by accumulation and refinement: we build up increasingly powerful theories, increasingly richer in their structures. We cannot just add a new feature to an old concept or theoretical scheme: this would require arbitrarily many ad hoc procedures, since it would have to be repeated for each phenomenon. It can therefore only be an intermediary step – such was, for instance Bohr's model of the hydrogen atom. Only when we have achieved to bind the new features in a new theoretical scheme, which then dictates how they have to be assigned to every phenomenon, do we reach a new understanding. Each step extends the horizon of explanations and bases the latter on fewer hypotheses. But this means that the new theories must involve genuinely new concepts and rules. How is the question of continuity in our knowledge process resolved? It turns out that, with few exceptions, the old conceptual structures are recognizable in the new ones, in the sense that the latter provide ways to make a relation to the old ones. The notion of controlled approximation is essential in this connection, by which we mean that we can unequivocally identify the physical situations in which the old theories provide a description which is nearly as good as that provided by the new theory, with "nearly" having a well-defined quantitative meaning. Of course, the reversed question is also raised: how strong is the bound to the old structure; does it not overinfluence the new ones? Is continuity not an artefact of our procedure? Note that the old conceptual scheme is not fully recovered. Only a part of it is recognizable, which will become a feature of the new theory. In fact the new structures are typically not approachable in the frame of the old ones. On the one hand we have to do with real incompatibilities, on the other hand the old concepts appear to be describable in the frame of the new theory in the relative sense now sketched. This seems to be a reasonable basis for identifying true progress in a scientific process. This also means that the conceptual structures are part of the same dynamics as physical knowledge in general. The structural questions of critical philosophy must therefore be treated in the frame of this dynamics.

3 What is Modern Physics?

Let us recall the general frame of our discussion. This is modern physics, that is, the theories of the standard model of fundamental interactions (SM) and the theory of general relativity (GRT), together with the theories of classical physics which represent general theoretical schemes, some of them being approximations to the more fundamental theories: thermodynamics, classical mechanics and statistical mechanics, electrodynamics.

These theories can also be viewed from a more global perspective: the structure of matter, space and time, complex systems, fundamental interactions, etc. They cover more or less without gaps the whole known physical universe, from the largest to the smallest observable scales. This does not mean that they explain everything.

Thus it is not meaningful to require them to explain life, etc., but also many physical phenomena remain as yet unexplained, and we do not know whether they can be dealt with in the proper in any foreseeable future. What I mean here is that within their domains of validity they do not seem to be contradicted, and moreover these domains are not disjunct from one another. They may refer to a certain class of phenomena (e.g., electromagnetic, gravitational) or they may be defined as approximations (to other theories) for phenomena fulfilling some conditions (non-relativistic theories for velocities much lower than c, non-quantum theories for "decohering" situations). Nevertheless, within these limits, these theories are understood as generally valid in the following sense: If we find phenomena which do not obey the theory but should belong to its validity domain, this cannot be cured by arbitrarily redefining the validity domain but it has to be taken as a statement for the theory as a whole. So, for instance, before special relativity was established, classical mechanics was claimed valid for all mechanical phenomena. The subsequent restriction of its validity domain implied that the theory itself can only be approximative.

Viewed as theoretical schemes all the above-mentioned theories are more or less self-consistent; viewed as knowledge about nature they lead to an open hierarchy. The standard model is a collection of quantum field theories, like Quantum Electrodynamics (QED), which themselves implement consistently the theoretical schemes of quantum mechanics and of the special relativistic, classical field theory such as Electrodynamics (ED). They are theories *in* space and time concerning all known interactions besides gravity. General relativity, on the other hand, is a theory *of* space and time, and it incorporates gravity. This hierarchy is open since these theories represent separate, unrelated schemes and therefore they can only apply to phenomena where the effects they describe are independent of each other. However, there are physical situations where this can no longer be expected to hold, and current research is directed toward a unified theoretical scheme. The names GUT (Grand Unified Theory) or FT (Final Theory) are now the current road signs in this enterprise.

Since the usual reduction goes from the macroscopic to the microscopic, a theory is typically valid for all length scales larger (or energy scales smaller) than those where it was established, while from the point of view of a theory set up for smaller scales the former may appear as "effective" or approximate. The standard model of elementary particles, for instance, appears valid at all scales between the subnuclear (hadronic) and the cosmological (up to uncertainties such as those concerning the so-called "dark matter", etc., which cannot be assessed at this time). On the other hand, only an "effective" theory is expected from the point of view of the more fundamental, "grand unified" theories, which we hope to develop in order to describe the phenomena at scales much smaller than the hadronic.

Is it not the case, however, that the reversed view also holds? Thermodynamic quantities and laws, for instance, emerge when we deal with very many degrees of freedom; some peculiar regularities, such as flow patterns, only appear at large scales. But this does not mean that mechanics is not valid here. It is merely the coherent application of microscopic laws which leads to such peculiar behaviour ("deterministic chaos" for instance). To be sure, this example discloses some limitations to our explanatory model which uses the reduction of large-scale phenomena to microscopic laws; moreover developing concepts dealing directly with complexity cannot be avoided.

4 How can we Understand the Evolution of Physical Symbols

We have been talking about conceptual dynamics, so let us try to gain some insight into it by considering one example.

What do we mean by the concept of *electron*? We can call *Electron*^{ED} the thing that electrodynamics speaks about, *Electron*^{QM} the thing quantum mechanics speaks about, and *Electron*^{QED} the thing quantum electrodynamics speaks about. They are all (more or less) fixed in their respective symbolic networks; but then they can only be said to point approximately to one thing in the real world, since the "objects" they define do not overlap. Thus *Electron*^{ED} is classical, relativistic particle without spin and with undefined internal structure (if it were point-like its self-energy would be divergent); *Electron*^{QM} is a non-relativistic, point-like quantum particle with spin, and *Electron*^{QED} is a relativistic, point-like quantum object with spin, which, in contradiction with the other two, can be created and annihilated.

The theoretical schemes in which we identify these objects differ quite a lot: not only do we observe different qualities (spin, self-energy) but even the conditions of objectification are incompatible.

In quantum mechanics we use Galilean relativity and Euclidean geometry, while in the other two cases we have special relativity and Minkowski space-time. In electrodynamics *Electron*^{ED} is a classical, distinguishable particle, with well-defined velocity and position at each point along a continuous trajectory – while of course this no longer holds for the quantum object. And both in electrodynamics and in quantum mechanics we deal with stable particles, while *Electron*^{QED} can appear and disappear, in conjunction with a new object, its antiparticle. It is only in the sense of "controlled approximation" as discussed in Section 2 that we can put all these objects into some relation to one another.

Now, up to a certain non-definiteness, each of these concepts is effective and justified in well defined physical situations. Moreover we can tune the physical conditions so as to produce gradual changes between the manifestations as *Electron*^{QED}, *Electron*^{QM} and *Electron*^{ED}. A Wilson or buble chamber can be used for this purpose. Of course, *Electron*^{QED} is the overriding concept since it has a richer structure, and it also ensures the continuity condition referred to in section 2. Hence one may be tempted to designate *Electron*^{QED} as the "true" concept, and consider the other two as approximations to two different, limited physical situations. But there is no guarantee that *Electron*^{QED} is not itself merely an approximation, an "effective concept". In fact there is good reason to expect that in the frame of a higher, "grand unified theory" this will be replaced by an *Electron*^{GUT} or some other, still richer concept.

Do these facts suggest that all we do is construct objects in one or the other theoretical frame? We should not forget that we do not speak of abstract exercises but of very precise physical situations, and that these theories are not merely happy thoughts, but that they are themselves outcomes of our endeavour: They are coherent conceptual systems with empirical basis, constructed such as to include the objects we are speaking of in a consistent symbolic network, and they are constructed in accordance with a growing process. The interesting observation, then, is that these $\mathit{Electron}^{TM}$ appear as marks on a path which seems "to be there" at least for a while, and which we shall simply call *Electron*. The "track" *Electron* represents a directed path, in the sense in which one theory overrides the other. The symbol *Electron* is of a different and much more ambiguous character than $\mathit{Electron}^{ED}$, $\mathit{Electron}^{QM}$ or $\mathit{Electron}^{QED}$, since it is related to all of them in an un-precise way. There may even appear further concepts in the future to which it will be correlated. Nevertheless it shows a "necessity" which in some sense transcends that of $\mathit{Electron}^{ED}$, $\mathit{Electron}^{QM}$ or $\mathit{Electron}^{QED}$ because it seems to hold beyond the confines of a particular symbolic network. A conceptual path such as *Electron* seems more directly related to the "alien element" which we need to deal with when we operate in accordance with our symbolic construction. On the one hand this alien element forces us to improve our theories and concepts, because it stays behind the track followed by the theories; on the other hand we put marks of our own in this construction, and it is precisely this which makes us recognize that there is a track to be followed in the first place.

Hence the reference relation appears well defined but conditional for each of these $\mathit{Electron}^{TM}$, while it is unconditional but undefined for the *Electron*. Of course we could see different "frames" of reality depending on the "scale" we look at, and we may consistently define the physical conditions which support these frames. For example a stone is a most "real" object, typically used in illustrations for the intuition of reality. But if we look at it through an electronic microscope we shall see shadows of atoms, and indeed we shall find those things we describe as atoms in quantum mechanics by "looking" through refined "instruments", such as electron diffraction or α-ray experiments. At still smaller scales we shall find (in the sense outlined above) the things and interactions taken in by the conceptual structure of the standard model. We can accept the "conditional reality" of each such frame – its reality as constrained at a given scale. The point is, however, that these frames, beyond their "surface" reality, seem to define a directed conceptual flow pointing to a "deeper" reality, one which must be existing, since otherwise there would be no necessary relation between these frames. This "deeper reality" may be accessible to a "final theory", something like $\mathit{Electron} = \mathit{Electron}^{FT}$, or even perhaps $FT = GUT$ so that physics would reach its end. But it may also be that a final theory does not exist, or that it cannot consistently associate a metaphysics but only provide a way to relate different, complementary theoretical frames. Thus, beyond accepting the conditional reality of each of these frames it is also interesting to ask about the status of that "deeper reality", which is only indicated by the fact that it drives the flow recognized by the marks set in each frame of the hierarchy: $\mathit{Electron}^{ED}$, $\mathit{Electron}^{QM}$, $\mathit{Electron}^{QED}$, $\mathit{Electron}^{GUT}$.

These frames are not merely arbitrary choices. Physical situations bring them forth, and the closure of the corresponding theoretical schemes define them in accordance with the criterion of self-consistency. By defining them consistently we mean that we can try to understand how they emerge – mostly a posteriori, using more steps of the theoretical development. So, for instance, in quantum field theory we know that particles can only be produced if the available energies are larger than their masses. This means that we shall not expect quantum field theoretical effects below some energy scale, and thus we can stay in the frame of quantum mechanics if all energies are low. On the other hand, classical, non-quantum physics can be approached from the frame of quantum mechanics within the so-called decoherence program,[11] again in well circumscribed physical conditions. In this way we can define the "conditionality" of the quantum mechanics frame, between classical mechanics and quantum field theory.

The reality status itself evolves as we go from frame to frame. For instance, in order to get to the quantum mechanical concept of system, we had to combine in a nontrivial way the concepts of particle and wave. However, quantum mechanics is not merely such a combination, but is a much richer structure, so that the quantum object lays claim to reality. This object is not only very different from the classical one, but even the status of "reality" described by quantum mechanics is different from that of the classical physics. We cannot distinguish, for instance, identical particles by following their paths, we have no continuous causal space-time chains, etc. Now quantum mechanics itself is not the final theory. We do not even know whether there is such a thing as a "final theory", and therefore the question of "reality" status for the *FT*-frame might turn out to be more dramatic than for quantum mechanics.

This example shows that not only concepts evolve, but so do the corresponding objectification rules and the "reality" status. Objectification cannot be based on an immovable scheme but can only be understood dynamically.

5 More on Necessity, Truth, and the Dynamics of Knowledge

As we have seen the analysis of physical concepts cannot lay claim to an absolute necessity for the association of symbols to things; we should have reached the asymptotic limit of a final theory for this necessity to be enforced! But this does not mean that the requirement for necessity should be left behind; on the contrary, this requirement seems to be an important stabilizing factor in a process based on "proposing and testing hypotheses", since it forces us to be more restrictive than mere "adequacy" would otherwise require. Also the observed development of science seems to support some concept of necessity as far the choice of theories and the production of hierarchies are concerned. The "conditional reality" of the physical concepts discussed in the last section corresponds to a "conditional necessity" of their association to things. We should be aware that the relations implied by these "necessities" cannot be simple and immovable. They have to be understood in an evolutionary process according to which objects are defined in

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[11] See, e.g., E. Joos et al., 2003.

a theoretical frame and found in the corresponding physical situation. What remains behind this conditional necessity is the agreement of theoretical definition and empirical embedding, the unambiguous and reproducible physical situation and its identification in the theory.

Similar observations apply to the notion of truth. A philosophical pragmatist may claim to be "suspicious about the distinction between justification and truth, for that distinction makes no difference to my decision about what to do."[12] A physicist would be inclined to think that surely this distinction makes no difference as far as the decision of an action is concerned, but it may make a great deal of difference to what happens after the action has been performed. If physical theories are justification systems, we should emphasize the following:

- The anticipation of possible unexpected reactions from the environment is the motivation for research programs.
- Ongoing research improves our knowledge, in that previously unexpected reactions are accommodated in terms of new justifications.
- Research programs lead in the long run to ordered justification structures (the previous level is either replaced or incorporated – e.g., recognized as an approximation with a well-defined domain of validity).
- Competition situations are solved in the long run in an "objective" way, in that the alternative with the best development capacities tends to take over.

In particular this is the reason why one continues to test the predictions of even well-established theories: the present "justification network", as solid as it may appear, can still be defective or it can miss further connections (see, for instance, the important, present day field of quantum mechanics tests).

The expectation of a possible discrepancy between our predictions (based on justifications) and the actual events is therefore pragmatically relevant (it makes us eager to learn). This expectation itself hides in it an "additional norm" besides justification, since we do not expect that whatever was behind the prior discrepancy depends on our improved justification but the other way around. Under the "postulate" of the intelligibility of the world the motor of scientific progress lies in the expectation of something *not yet justifiabl*e but *in principle intelligible*, and this implies in fact a notion of truth. Again, this cannot be understood in an absolute sense, since otherwise we would not be able to set up any theories. Each time an element of truth is to be tracked, as it were, and we can construct on this basis a consistent theory. But, as long as we have not reached the final theory, this procedure is restricted to a certain level, and therefore there must be a "truth" which, though conditional, is effective at this level. Truth, as well as reference, are therefore dynamical concepts. They are, however, robust, that is, well defined, related to reproducible physical situations and pragmatically relevant.

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[12] R. Rorty, 1998, p. 19.

6 Critical Philosophy and Physical Knowledge

Physical understanding, while always conditioned by the criteria of necessity and universality, as well as by the aim of reducing observations to basic principles, must be ready to accept, precisely for these reasons, developmental changes in its own rules, and even in the definition of how knowledge about the physical world is to be achieved. On the one hand, we can identify and describe the kind of continuity at work in scientific process, and rely on it to define scientific progress. On the other hand the accompanying epistemological process may seem more discontinuous. This may be due to the fact that usually philosophical systems are seen as alternative perspectives, and one is not inclined to accept a notion of progress such as the one that operates in the process of physical knowledge. None of these systems, however, can provide a unique scheme for accompanying the process of physics in all its forms. Nevertheless, via agreements and contradictions, they can help characterize physical understanding.

Ordinarily physicists do not feel overly burdened by philosophical accuracy. This pragmatic attitude is already visible in the realist/idealist alternative:

"In natural science the opposing world views of Realism and Idealism designate non-contradictory methodological principles…. We construct [in natural science] an objective world in which simultaneously two principles must hold: A 'realistic' principle [which, following Helmholtz, could be described as follows] – 'a difference in the perceptions reaching us is always due to a difference of the real conditions.'… [further], an 'idealistic' principle – 'the objective picture of the world should allow no differences which could not show up in perception; an existence which by principle is closed to perception is not accepted.'"[13]

These methodological principles, however, do not determine the character of the theoretical construction. What kind of light is thrown upon the knowledge process of physics in one or the other perspectives? What naive idealism cannot show is why our knowledge works – why we can describe reality and not merely ourselves. What naive realism cannot show is why it may fail – why explanations may be

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[13] "Innerhalb der Naturwissenschaft bezeichnen die weltanschaulichen Gegensätze von Realismus und Idealismus einander nicht widersprechende methodische Prinzipien…. Wir konstruieren in ihr eine objektive Welt, in der zugleich zwei Prinzipien gelten müssen: Ein 'realistisches' Prinzip, [das man mit Helmholtz so darstellen kann]: 'Eine Verschiedenheit der sich uns aufdrängenden Wahrnehmungen ist stets in einer Verschiedenheit der reellen Bedingungen fundiert.'… [Ferner] ein 'idealistisches' Prinzip: 'das objektive Weltbild darf keine Verschiedenheiten zulassen, die nicht in Verschiedenheiten der Wahrnehmung sich kundgeben können; ein prinzipiell der Wahrnehmung unzugängliches Sein wird nicht zugestanden'" (Weyl, 1976, p. 84.). Of course perception here is meant in a generalized sense: computer registered events in CERN detectors are perceptions, just as well as WMAP data. Even so we do not hesitate some times to introduce concepts which appear necessary to make the theoretical construction consistent, but – at least at a certain level – have no directly observable effects, such as the potentials in QED and gauge theories. One can argue that they have an indirect reality, as components of a successful theory, but this is surely in need of further discussion.

wrong and why we need to allow our concepts to grow. And what empiricism and constructivism cannot show is why there is a progressive scientific process rather than a collection of ad hoc explanations.

The "symbolic stance" mentioned in section 1 has been at the basis of our discussion. We have tried to describe a number of features characteristic of the process of developing physical knowledge: a certain kind of directed continuity, what we have called conditional necessity and reality connected to the reference of our concepts to things, a "dynamical" notion of truth. What is the significance of these features from the point of view of critical philosophy?

The Kantian program allows us to raise the questions of objectification and of the construction of concepts. We can in this way at any given moment try to disentangle settings from findings, and this is essential for understanding the evolution of physical concepts and theories and uncover the associated dynamics. We have tried to follow these questions in our discussion. But the Kantian program itself is not compatible with the view of this dynamics which we have suggested here, since it is based on a static scheme separating knowledge and conditions of knowledge. Therefore Kant's alleged solution to Hume's problem cannot be accepted either.

Certainty cannot be secured in the way proposed by Kant. A priori forms of intuition as conditions of objectification are themselves included in an evolutionary process, and they are subject to the same rules that are found to be effective for physical knowledge generally. Of course certainty cannot be secured empirically, simply because at no given moment do we have complete information. But do we really need certainty? Popper,[14] for example, renounces certainty while retaining necessity as an operational aspect in the construction of hypotheses: we claim necessity in order to be able to test and judge hypotheses.

Arguably certainty must have the same status as the causality principle on which it eventually depends. Then, under an intelligibility postulate, the conditional reality and necessity we have described, and the progress of the scientific process, would suffice to justify the use of an effective notion of certainty, going therefore beyond Popper's operational proposition.

We began this discussion in the frame of thinking opened up by critical philosophy, and we found that this allowed for a differentiate analysis. But we also found out that we must renounce the rigid scheme of the Kantian program and its unidirectional implication. It is undeniable that some parts of every possible knowledge come from ourselves. But in detail this does not always work. In fact, we are a result of evolution and there should be at least some compatibility between the way nature works and the way our mind works. But this does not mean we are fully aware of either one. This also concerns the analytic part: not only must we choose between various mathematical structures, but every chosen structure seems to close up too early, that is, it cannot reach the definitive truth: when they are mathematically closed, our theories are not yet physically completed. Hence we must admit

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[14] K. Popper, 1972.

for the a priori part of knowledge the same rules and the same evolutive character as for the a posteriori part. When they work in tandem something of the structure of the world, lying behind but showing up in the phenomena, can rightly be said to have been disclosed. Part of this is cast in the foundation of our concepts – and this is what we may call a priori – and part of it enters the content of these concepts. The former is the direct reflection of that interaction between the world and living things which we call thinking. It determines the latter (the content of the concepts) but its evolution itself is dependent on its realization as the latter: we would not have changed the rules for organizing events, for instance, were it not for the observations we have made and the theories in space and time that we have devised and tested. This intertwined evolution appears to be specific to the process of physics.

Bibliography

E. Cassirer (1987), "Determinismus und Indeterminismus in der modernen Physik", in *Zur Modernen Physik*, Wissenschaftliche Buchgesellschaft, Darmstadt.

E. Joos, H.-D. Zeh, C. Kiefer, D. Giulini, J. Kupsch and I.-O. Stamatescu (2003), *Decoherence and the Appearance of a Classical World in Quantum Theory*, II-nd ed., Springer, New York/ Heidelberg.

B. Falkenburg (2002), "Symbol and intuition in modern physics", in *Symbol and Physical Knowledge*, M. Ferrari and I.-O. Stamatescu (Eds.), Springer, Heidelberg/New York.

H. Hertz (1894), *Die Prinzipien der Mechanik*, Leipzig.

I. Kant (1957), "Metaphysische Anfangsgründe der Naturwissenschaft. Vorrede", in *Werke*, Band V, Wissenschaftliche Buchgesellschaft, Darmstadt.

C.S.S. Peirce (1991), Notes concerning the "Architecture of Theories", MS 956, in *Charles S. Peirce. Naturordnung und Zeichenprozess*, H. Pape (Ed.), Suhrkamp, Frankfurt am Main.

K. Popper (1972), *Objective Knowledge*, Clarendon Press, Oxford.

H. Reichenbach (1965), *The Theory of Relativity and A Priori Knowledge*, University of California Press, Berkeley.

R. Rorty (1998), "Truth and Progress" in *Philosophical papers*, vol. 3, Cambridge University Press, Cambridge, p. 19.

I.-O. Stamatescu (1995), "Anschauung und wissenschaftliche Erkenntnis" in *Kulturkritik nach Ernst Cassirer*, E. Rudolph and B.-O. Küppers (Eds.), Felix Meiner Verlag.

I.-O. Stamatescu (2002), "On the role and character of symbols in modern physical theories", in *Symbol and Physical Knowledge*, M. Ferrari and I.-O. Stamatescu (Eds.), Springer, Heidelberg/ New York.

J. W. von Goethe (1949), "Zur Naturwissenschaft im Allgemeinen", HA, Bd.13, Chr. Wegner Verlag, Hamburg.

H. von Helmholtz (1971), "Die Tatsachen in der Wahrnehmung", Rektoratsrede, in *Philosophische Vorträge und Aufsätze*, Akademie-Verlag, Berlin.

H. Weyl (1976), *Die Philosophie der Mathematik und der Naturwissenschaften*, Oldenbourg, München.

H. Wismann and I.-O. Stamatescu (1994), "Questions Concerning theory and Experience and the Role of Mathematics in Physical Science", in *Philosophy, Mathematics and Modern Physics – A Dialogue*, E. Rudolph and I.-O. Stamatescu (Eds.), Springer, Heidelberg/New York.

Symbolic Constructions in Quantum Field Theory

Hans Günter Dosch, Volkhard F. Müller, and Norman Sieroka

Abstract The aim of this short essay is to show that the view of physics as a symbolic construction of nature is specially suitable for the examination of epistemic problems of modern particle physics. We point out that the awareness of these problems arose already with the foundation of classical electrodynamics. Then we give a short description of salient features of quantum field theory, and finally we show the adequacy of the symbolic approach in some relevant cases.[1]

1 Symbolic Interpretation of Science

The ongoing success of Newtonian and Euler–Lagrangian mechanics led in the eighteenth and early nineteenth century to a widely accepted mechanistic *Weltbild*. This *Weltbild*, however, came to a crisis through the progress of electrodynamics which later culminated in the establishment of Maxwell's equations. The essentially new ingredient of these equations is the displacement current. Its derivation took place within a mechanistic model of the ether, which was the assumed carrier of electromagnetic phenomena. Thereby electrodynamics was formally embedded in the

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[1] For more details we refer to Dosch et al. (2005).

M. Bitbol et al. (eds.), *Constituting objectivity*,

mechanistic Euler–Lagrange field theory. It turned out, however, that a mechanistic interpretation of the ether became less and less tenable, so that the concept of a field was emerging as an assignment of space–time points to measurable quantities without assuming a material carrier of the field. This concept existed besides the old concepts of mechanics and allowed an adequate description of electrodynamic phenomena. Many of the epistemic problems related to quantum field theory already appeared with the rise of the electrodynamical field theory in the middle of the nineteenth century.

A short historical sketch of concepts will be given, beginning with the theory of signs (symbols) of Hermann von Helmholtz and Heinrich Hertz. Further development appears in the works of Henri Poincaré and Hermann Weyl.

Helmholtz's theory of signs[2] was influenced by the physiology of sensation. Starting from the fact that our sensation gives us a message only of the peculiarity of the evoking external influence Helmholtz concluded that sensation can be accepted as a *sign* of this external influence, but not as a copy. Though he insists that our sensual impressions are only signs they are not to be disposed of as empty phantoms (*leerer Schein*) but 'they are signs of *something*, be it existing or happening, and what is most important, they map the *law* of what is happening'.[3] For Helmholtz the relevant feature of science is not the particular set of signs, but that what he calls 'law' and which he defines as the unchanging relation between changing variables (signs).

A further reason why talking about symbols or functional relations seems fundamental for physics results from the fact that experiments reveal how things under investigation *act*, i.e. the way they impinge on measuring devices. Hence, one has to acknowledge the possibility that we might be unable to distinguish between two quite different intrinsic properties, namely if they happen to be exactly the same with respect to the way they impinge on our instruments. By the same token we must then accept the possibility that we know almost nothing about the intrinsic nature of physical objects.

Hertz considers as the principal aim of conscious natural science (*bewusster Naturerkenntnis*) to foresee future experiences. He proposes a sign theory which is more explicit than Helmholtz's. Hertz gives a set of rules, both formative and descriptive, the core of which is:

> We form for us phantom pictures (*Scheinbilder*) or symbols of the external objects and in such a way that the logical (*denknotwendig*) consequences of the pictures are always pictures of the physically necessary (*naturnotwendig*) consequences of the depicted objects.[4]

He does not take it for granted that such a procedure is possible, rather he observes that experience tells us that it is.

There is a crucial difference between the sign theory of Hertz and that of Helmholtz. Helmholtz's signs are related to the sensual impressions whereas those

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[2] Helmholtz (1892, 1921).

[3] Helmholtz (1921, p. 116).

[4] Hertz (1894, pp. 1–2).

of Hertz can be free creations of the mind. Here a semiotic distinction drawn by Charles Sanders Peirce is helpful to contrast between these two types of 'signs'; namely the distinction between indexical signs and symbols.[5] The former show some existential relation to their objects of reference; i.e. their meaning is based on a cause and effect relationship. A symbol on the other hand carries *pure* meaning and lacks such a relationship.

Thus, one can take Helmholtz's approach, to be a theory of indexical signs, whereas Hertz's theory should count as one of symbols.

The historical development went further away from a theory of indexical signs and moved gradually towards a theory of symbols. Both Helmholtz and Hertz were certainly motivated by the unreconciled differences of classical mechanics and of Maxwell's theory but they had no reason to doubt the unconditioned physical validity of both these theories. This was no longer the case for the next main contributor in this tradition, the great mathematician Henri Poincaré who lived long enough to see the beginning erosion of classical physics at the turn to the twentieth century. He expounded his views on the epistemological foundation of science in three works: *La Science et l'Hypothèse* (1903), *La Valeur de la Science* (1905), and *Science et Méthode* (1908).[6] Much like Helmholtz and Hertz he takes the success of science as a starting point and concludes – like today's miracle argument – that this success would not be possible if science would not give us knowledge of something real (*quelque chose de la réalité*). However, in line with what is today called the pessimistic meta-induction, he goes on to argue that what science can attain is not the things themselves, but only the relations between the things (*les rapports entre les choses*). Outside these relations there is no recognizable (*connaisable*) reality.

Poincaré stresses the importance of conventions in science – indeed he is notoriously credited as the very founding father of conventionalism. He lived until 1912 and saw the big changes occurring at the turn of the century. He seems not to have been impressed strongly enough by the birth of quantum physics, but the effects of special relativity on mechanics shook him considerably, as can be read from the last chapter of *La Science et l'Hypothèse*, entitled 'La fin de la matière'. He also foresaw the consequences of these theories for the then best established physical theory, Newton's theory of gravitation.

The fourth eminent figure whose interpretation of science we shall discuss briefly is the mathematician Hermann Weyl. He did not only live during the period of great changes in the first decades of the twentieth century but also contributed essentially to them and was fully aware of their epistemological impact.

In contrast to metaphysical inclinations of his earlier writings, Weyl refers in the philosophical publications of his later years to symbolic construction as the adequate approach to mathematics and physics. This becomes clear already from the titles of his articles; 'Science as a Symbolic Construction of Man'[7] and 'On the Symbolism

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[5] See Peirce (1998, pp. 273–274, 291–292).

[6] Poincaré (1903, 1905, 1908).

[7] 'Wissenschaft als symbolische Konstruktion des Menschen' (Weyl, 1949).

of Mathematics and Mathematical Physics'.[8] In the former paper, much like the philosopher Ernst Cassirer, he uses case studies to show the development from substantial to symbolic forms. He also quotes approvingly the symbolic approach of Hertz and comes to such definite statements as:

> It is the free spirit working in symbols that constructs for itself an objective frame in physics which it then uses to order the manifold of phenomena. For this it does not need to import such means as space, time and particles of substance: it takes everything from out of itself.[9]

This general development in physics fits well with the *Philosophy of Symbolic Forms* of Ernst Cassirer.[10] Cassirer even remarks that it were just the exact natural sciences which first became aware of the symbolic character of their means.[11] He refers explicitly to Maxwell's theory, which he calls an 'important and methodologically essential cut' in physics.[12] There already the circle of description of physical processes by reduction to intuitive relations was broken in favour of a purely mathematical determination and thus a transition was made from intuition into the realm of pure meaning (*Bedeutung*).[13]

Our aim is to corroborate this point of view analysing especially the developments of quantum field theory in the course of the twentieth century.

2 The Field as Basic Theoretical Concept

Phenomena ascribed to subnuclear particles and their interaction form the physical domain of relativistic quantum field theory. The salient feature that characterises the interaction of subnuclear particles is the possible transmutation of matter into energy and vice versa: particles can be created or annihilated, provided certain conservation laws are respected. Many of the objects, however, which are conveniently called subnuclear particles, are not stable but decay spontaneously into lighter particles. Thus, strictly speaking, they cannot appear in an asymptotic state. Nevertheless, it is theoretically appealing and proves to be empirically justified to treat an unstable particle also as forming asymptotic states, provided its lifetime is large compared with the reaction time in a scattering process. This indicates that the notion of a subnuclear particle is firmly based on the related theoretical perspective. The huge disparity in lifetimes of the various subnuclear particles demands particular attention to this dependence on the theoretical frame.

The basic concept of the theory are relativistic quantum fields, not particles. The quantum fields, in terms of which the theory is constructed, are operators that

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[8] 'Über den Symbolismus der Mathematik und mathematischen Physik' (Weyl, 1953).

[9] Weyl (1949, vol. 4, p. 327).

[10] See Schmitz-Rigal (2002).

[11] Cassirer (1923, vol. 1, p. 5).

[12] Cassirer (1995, p. 16).

[13] Cassirer (1923, vol. 1, p. 16–17).

depend on space–time and act on state vectors. Their dependence on the relativistic space–time variables shows the behaviour of a generalised function; therefore in general pointwise products of fields are not defined. Converting indiscriminately the products of classical fields into the corresponding products of quantum fields produces mathematically ill-defined objects, from which infinities emerge when used in the context of a quantum field theory.

Quantum field theory has grown out of classical field theory by the application of 'quantisation rules', which extend the rules that converted classical mechanics into quantum mechanics. Later on, a systematic reformulation of the originally ill-defined approach to relativistic quantum field theory was achieved – the so-called perturbative renormalisation theory – which provides a mathematically well-defined formal power series. This form, however, is only useful for interactions which are weak.

Classical electrodynamics can be directly formulated in terms of the electromagnetic field. An electromagnetic potential acting as an ancillary mathematical object can be substituted for this field. However, the relation between electromagnetic field and potential is not a one-to-one relation: a whole equivalence class of the latter corresponds to a given field. Nevertheless, quantum electrodynamics (QED), i.e. the quantum field theory describing the electromagnetic interaction, involves the operator version of the potential as a basic quantum field. Its interactions show the so-called *local gauge symmetry*. This symmetry implies that the interaction encoded in the theory is *local*, i.e. there is no action-at-a-distance, and all physical effects propagate with finite velocity (*Nahwirkungsprinzip*). All sectors of the Standard Model of particle physics[14] obey such a local gauge symmetry, which is why it is called a *quantum gauge field theory*. The basic matter fields of this model are the quark fields in its strong-interaction sector, as wells as additional lepton fields (electrons, e.g.) in the sector of weak and electromagnetic interactions. The gauge fields (photons and gluons, e.g.) mediate the interaction between the matter fields. The fields of protons and neutrons, the constituents of the atomic nucleus, are constructed from quark fields.

In the course of its evolution quantum field theory has expanded in several directions. Each direction represents a particular aspect of the theory. Taken together, these facets can be characterized very briefly as follows.

General Theory of Quantised Fields: Paying attention to the mathematical properties of a local quantum field, a general framework – but only a framework – of a relativistic quantum field theory has been formed by way of few well-defined postulates, usually called 'Wightman axioms'.[15] A number of physically important structural consequences, all well established, follow from this. A far-reaching mathematical consequence is that the theory allows to be transformed into an Euclidean formulation.

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[14] See Veltman (2003). Here and in the following we generally do not quote the original literature but monographs on the subject.

[15] Streater and Wightman (1980); Jost (1965).

Perturbative Renormalisation Theory: The well-defined theory of quantum fields without interaction forms the basis of this approach, the desired interaction being treated as a 'small' perturbation via a formal perturbation expansion.[16] This expansion involves an inductive procedure – called 'renormalisation' – which generates finite modified local products of field operators representing the interaction. Of all facets, perturbative renormalisation theory provides the most distinctive predictions, since it accounts for the electromagnetic and weak sectors of the Standard Model. Here, there is a direct correspondence between 'particles' and 'fields'.

Lattice Gauge Theory: The first step of this constructive approach is to replace (Euclidean) space–time by a lattice, which is a discrete structure.[17] It does not resort to a perturbation expansion and preserves the local gauge symmetry, aiming at the domain of strong interaction of the Standard Model. The emerging mathematical system is treated by numerical simulations, combined with a crucial transition procedure back to continuous space–time. Most remarkably, the basic quantum fields involved do not necessarily correspond to asymptotic particles.

Constructive Quantum Field Theory: In view of the unsatisfactory status of the formal renormalised perturbation theory, serious efforts have been made to construct rigorously a quantum field theory allowing for interaction.[18] Up till now this has been achieved for (fictitious) two- and three-dimensional space–times, but not yet for the physically relevant four-dimensional case.

Local Quantum Physics: This approach, often referred to as 'algebraic quantum field theory', aims at formulating a conceptual frame serving as a foundational basis of quantum field theory.[19] Its basic concept is local observables, whereas quantum fields only act as particular building blocks of observables.

3 The Different Facets of Quantum Field Theory

Our presentation of quantum field theory has been very brief, and we now present a philosophical framework in a similarly sketchy way. A much more detailed discussion of quantum field theory and of the sign-theoretic terminology in which it can be phrased has been dealt with elsewhere.[20] Suffice it to say here that the difference between 'intuition' and 'meaning' stressed by Cassirer finds its semiotic counterpart (which, of course, is much less philosophically laden) in the difference between connotation and denotation. The current paper might thus be viewed as an attempt to give a broader philosophical setting for our aforementioned work.

We suggest that the various facets of quantum field theory which have just been mentioned are related as different symbolic forms or constructions in the sense of

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[16] Itzykson and Zuber (1980); Faddeev and Slavnov (1991).

[17] Wilson (1974); Montvay and Münster (1994).

[18] Glimm and Jaffe (1987); Gawedzki and Kupiainen (1985).

[19] Buchholz and Haag (2000).

[20] Dosch et al. (2005).

Cassirer and Weyl. Some of the respective entities and concepts involved in the general theory of quantised fields, in perturbative renormalisation theory, and in lattice gauge theory differ markedly from each other.

Even though the various facets of quantum field theory also have common elements, their marked differences prevent a hierarchical order between them. Their status ought to be made intelligible as different symbolic forms. Three examples will help understand this.

First, recall that up till now no quantum field theory, which describes interaction between particles in four space–time dimensions, has been constructed on a sound mathematical footing. The Wightman axioms, forming the basis of the general theory of quantised fields, are considered to form the core structure of a theory based on field operators; but they do not formulate the dynamical evolution in terms of field operators. It is this dynamical evolution, however, that entails the specific physical outcome of the theory. Moreover, in contrast to perturbative gauge theory and lattice gauge theory the physically distinguished non-Abelian gauge symmetry is not present in the Wightman axioms. Nevertheless, the Wightman axioms allow to derive some stringent structural consequences as e.g. the symmetry between matter and antimatter or the connection between spin and statistics. These predictions are in fact experimentally very well satisfied and any violation of them would seriously call into question the concept of a local quantum field.

Second, concrete quantum field theories have been generated by 'quantising' a heuristic classical precursor theory. In the case of the Standard Model there are two complementary symbolic forms to perform a 'quantisation' of the classical precursor: perturbative renormalisation theory and lattice gauge theory. Perturbative renormalisation theory is only formulated as a formal power series based on free quantum fields and cannot create hadrons like protons and neutrons which are basic constituents of matter. Moreover, in order to implement (covariantly) the local gauge symmetry, it employs unphysical degrees of freedom, so called ghost fields. Lattice gauge theory avoids these unphysical entities and starts with a well-defined nonperturbative set-up albeit not on continuous space–time but on a discrete lattice. The respective predictive power of these two symbolic forms points to complementary physical domains. The perturbative theory accounts for electroweak processes and for the short-distance behaviour of the strong interactions. In contrast, the lattice gauge theory aims at the long-distance properties, i.e. the spectrum of hadrons dynamically generated from fundamental degrees of freedom of the theory (the quark and gluon fields). Perturbative renormalisation theory merely leads to a formal power series and lattice gauge theory encompasses only certain elements of the aforementioned Wightman axioms.

Third, in perturbation theory the concept of spontaneously breaking a local gauge symmetry has proven to be of great efficiency and led to the discovery of the weak gauge bosons. In contradistinction, within the realm of lattice gauge theory it was proven that local gauge symmetry cannot be spontaneously broken.[21]

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[21] Unless done explicitly by way of fixing a gauge (Elitzur, 1975).

If one looks at particle physics more generally, even more symbolic forms can be found at work. Consider, for instance, electrons, heavy and light quarks. The symbolic forms in which electrons occur include classical mechanics, non-relativistic quantum mechanics and quantum field theory. Heavy quarks only occur in the context of non-relativistic quantum mechanics and quantum field theory, whereas light quarks occur only in that of quantum field theory. For a philosophy of symbolic forms this is not a problem, but it would be a problem if one were to stick to intuition as the one firm ground from which questions like 'Does particle *xyz* exist?' must find a definite answer.

Now, think about the relation between 'higher' (more fundamental) and 'lower' theories. In general the former does not make the latter obsolete. This is quite obvious in the case of 'macroscopic' problems. The unresolved difficulties of quantum field theory, for instance, do not prevent us from solving most complicated problems concerning the propagation of radio waves. It might even be necessary to use the lower theory in the higher one in an essential way, as e.g. in the analytic description of bound states in quantum field theory. In order to calculate the Lamb-shift, which is one of the triumphs of relativistic quantum field theory, one starts with a quantum field theory in a given 'external' classical field, though this field should in principle also be described in quantum field theory. Thus, in order to calculate the Lamb-shift one has to make use of different symbolic forms at the same time; namely the 'higher' quantum and the 'lower' classical field theory.

Viewing different theories in physics and even the various facets of quantum field theory as different symbolic forms is a decisively non-realist position. Such a position puts special emphasis on the creative part played by the human mind in the generation of scientific theories. Already Weyl emphasised that only transcendental approaches in philosophy can account for this creativity, and this is why he uses the term 'symbolic *construction*'.[22] According to Cassirer, all knowledge inheres a 'primordially producing and not only re-producing power'.[23] Rather than merely teaching us something about facts, tools are manufactured to make the world understandable:

> The history of physics [...] is not a history of discovering a simple line of 'facts', but of discovering ever new *tools of thought*.[24]

The history of the gauge principle gives further evidence in favor of such a view. Brought forward initially by Weyl in 1918, the gauge principle was meant to unify gravitation and electromagnetism, the only two types of interaction known at that time. Although it turned out to be empirically invalid in that context, it reappeared about 10 years later in quantum mechanics and it is by now of greatest significance in quantum field theory. If one thinks of physics as simply discovering facts about nature this is hard to understand. One would have to talk about some historical

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[22] See, e.g., Weyl (1949), passim.

[23] Cassirer (1929, vol. 1, p. 9).

[24] Cassirer (1921, p. 88).

'ironies' or something alike.[25] However, from the point of view of a philosophy of symbolic forms this makes perfect sense and there is nothing ironic about it. If the creativity of the human mind plays a central role, then the reappearance of the gauge principle simply shows that this principle is a resourceful 'tool of thought'.[26]

Furthermore, a significant feature of a symbolic approach is that it acknowledges the usefulness and empirical success of the different facets of quantum field theory. If one looks at the publications within what may be called philosophy of quantum field theory,[27] most of it being rather realist in spirit, those coexisting different facets have largely been ignored. Most of the literature only takes perturbative renormalisation theory into account. This leads to what Whitehead would call a 'fallacy of misplaced concreteness'.[28] Discussing only one facet and then taking it for being 'the real one' is dangerous, since several important concepts and predictions stemming from the other facets cannot be taken into account; and this has led to several rather misguided discussions.[29]

The miracle argument is the cornerstone of realism. According to it, only realism can account for the empirical success of science in a satisfying way. But this argument fails in the case of quantum field theory. A realist position in some way or other has to single out one facet, so that the empirical success of the other facets would be turned into a miracle, unless finally an all-embracing theory which unifies all facets is found. Basing one's realism on the general theory of quantised fields, for instance, amounts to restricting the theoretical claim to the very core of local quantum field theory. The price to be paid for eschewing thereby the formulation of an explicit dynamics is that only few (albeit fundamental and experimentally very well satisfied) physical relations result. In order to arrive at a wealth of physical relations aiming to cope with the immense body of detailed measurements performed in the subnuclear domain, an adequate concrete quantum field theory has to be created, i.e. in terms of specific fields and their interaction, thus fixing the dynamical content of the theory. But here again different facets offer different forms of description and realms of prediction. Thus, to a large extent, the choice between facets is dictated by the questions asked. The facets are symbolic forms and there is no single facet 'depicting reality point by point'. Even if some day a 'completed version' of quantum field theory could be obtained this would not per se alter the symbolic construction of the theory and its concentration on *meaning*. This remains true even if beyond quantum field theory a new theory valid on smaller scales turned out to be developed in the future.

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[25] Cf. J.D. Dyson (1983): 'Unfashionable Pursuits'; quoted in Coleman and Korté (2001, pp. 312–313).

[26] Indeed the principle of a *Nahgeometrie* (geometry of vicinity), which lies at the base of Weyl's introduction of a gauge principle, can be traced back at least to Descartes, who grounded the concept of space and movement on it. See Descartes (1644) (*Principia Philosophiae*), II, 10ff., especially 25.

[27] See Cao (1996).

[28] Cf. Whitehead (1925, Chapter 3).

[29] One being the debate about 'quasi-autonomous domains' in Robinson (1992); Cao and Schweber (1993); Hartmann (2001).

4 Conclusions

We have presented arguments to the effect that a symbolic approach is appropriate for a full account of contemporary physics. Such an approach allows for the different and empirically successful facets of quantum field theory, as indeed any serious interpretation of contemporary physics should.

Like Cassirer we claimed that along with the establishment of Maxwell's theory a transition occurred away from intuition towards the realm of symbols and pure meaning. Hence, it is not astonishing that the aforementioned protagonists of a symbolic approach in science were themselves eminent physicists and mathematicians. Helmholtz, Hertz, Poincaré and Weyl creatively contributed to the further development of physics and were all in a position to experience a certain and particular permanence in the mathematical descriptions of their discipline. While certain everyday connotations arguably changed quite radically at the transition from classical mechanics to field physics or to quantum mechanics, the change in the symbolic constructions given in terms of mathematical relations was rather restrained.[30]

Cassirer must be credited for widening this symbolic approach to other human cultural enterprises like art, myth and religion. Of course, differences in degrees must be taken into account here. Moving from myth to science is certainly more radical than moving from one physical theory to another. However, as we have pointed out, there are crucial differences even between such related symbolic forms as the facets of quantum field theory regarding the concepts involved and the entities described.

References

Buchholz, D. and Haag, R. (2000). The quest for understanding in relativistic quantum physics. *J. Math. Phys.*, 41:3674–3697.

Cao, T. Y. (1996). *Conceptual Developments of 20th Century Field Theories*. Cambridge University Press, Cambridge.

Cao, T. Y. and Schweber, S. S. (1993). The conceptual foundations and the philosophical aspects of renormalization theory. *Synthese*, 97(1):33–108.

Cassirer, E. (1921). *Zur Einsteinschen Relativitätstheorie*. Bruno Cassirer, Berlin. Reprint Darmstadt, 1989.

Cassirer, E. (1923–1929). *Philosophie der Symbolischen Formen*. Bruno Cassirer, Berlin, 3 volumes.

Cassirer, E. (1995). Das Symbolproblem und seine Stellung im System der Philosophie. In Cassirer, E., editor, *Symbol, Technik, Sprache. Aufsätze aus den Jahren 1927–1933*, pages 1–21. Meiner, Hamburg.

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[30] Adhering mainly to the intuitive concepts of everyday life also seems to be the reason why people from outside physics tend to interpret transitions between theories much more dramatically than people judging from inside physics.

Coleman, R. and Korté, H. (2001). Hermann Weyl: mathematician, physicist, philosopher. In Scholz, E., editor, *Hermann Weyl's 'Raum-Zeit-Materie' and a General Introduction to His Scientific Work*, pages 157–386. Birkhäuser, Basel.

Descartes, R. (1644). *Principia Philosophiæ*. Elzevir, Amsterdam, Tannery edition.

Dosch, H. G., Müller, V. F., and Sieroka, N. (2005). *Quantum Field Theory in a Semiotic Perspective*. Springer, Berlin/Heidelberg.

Elitzur, S. (1975). Impossibility of spontaneously breaking local gauge theories. *Phys. Rev.*, D12:3978–3982.

Faddeev, L. D. and Slavnov, A. A. (1991). *Gauge Fields: An Introduction to Quantum Theory*. Addison-Wesley, Redwood, CA, 2nd edition.

Gawedzki, K. and Kupiainen, A. (1985). Gross-Neveu model through convergent perturbation expansion. *Commun. Math. Phys.*, 102:1–30.

Glimm, J. and Jaffe, A. (1987). *Quantum Physics*. Springer, New York, 2nd edition.

Hartmann, S. (2001). Effective field theories, reductionism and scientific explanation. *Stud. Hist. Philos. Mod. Phys.*, 32(2):267–304.

Helmholtz, H. v. (1892). *Handbuch der physiologischen Optik*. Voss, Hamburg, 2nd edition.

Helmholtz, H. v. (1921). Die Tatsachen in der Wahrnehmung. In Hertz, P. and Schlick, M., editors, *Hermann von Helmholtz, Schriften zur Erkenntnistheorie*, pages 109–152. Springer, Berlin.

Hertz, H. (1894). *Die Prinzipien der Mechanik*. Barth, Leipzig. Preface by H. v. Helmholtz.

Itzykson, C. and Zuber, J.-B. (1980). *Quantum Field Theory*. McGraw-Hill, New York.

Jost, R. (1965). *The General Theory of Quantized Fields*. American Mathematical Society, Providence, RI.

Montvay, I. and Münster, G. (1994). *Quantum Fields on a Lattice*. Cambridge University Press, Cambridge.

Peirce, C. S. (1998). *The Essential Peirce—Volume II*. Indiana University Press, Bloomington, IN.

Poincaré, H. (1903). *La Science et l'Hypothèse*. Flammarion, Paris.

Poincaré, H. (1905). *La Valeur de la Science*. Flammarion, Paris.

Poincaré, H. (1908). *Science et Méthode*. Flammarion, Paris.

Robinson, D. (1992). Renormalization and the effective field theory programme. In Hull, D., Forbes, M., and Okruhlik, K., editors, *Proceedings of the 1992 Biennial Meeting of the Philosophy of Science Association*, volume 1, pages 393–403. East Lansing, MI Philosophy of Science Association.

Schmitz-Rigal, C. (2002). *Die Kunst des offenen Wissens*. Cassirer-Forschungen, volume 7 Meiner, Hamburg.

Streater, R. F. and Wightman, A. S. (1980). *PCT, Spin and Statistics, and All That*. Benjamin, Reading, MA, 3rd edition.

Veltman, M. (2003). *Facts and Mysteries in Elementary Particle Physics*. World Scientific, Singapore.

Weyl, H. (1949). Wissenschaft als symbolische Konstruktion des Menschen. *Eranos-Jahrbuch*, pages 375–431. Reprinted in *Gesammelte Abhandlungen*, edited by K. Chandrasekharan, Berlin, 1968.

Weyl, H. (1953). Über den Symbolismus der Mathematik und mathematischen Physik. *Stud. Gen.*, 6:219–228. Reprinted in *Gesammelte Abhandlungen*, edited by K. Chandrasekharan, Berlin, 1968.

Whitehead, A. N. (1925). *Science and the Modern World*. Macmillan, New York.

Wilson, K. G. (1974). Confinement of quarks. *Phys. Rev.*, D10:2445–2459.

Noncommutative Geometry and Transcendental Physics

Jean Petitot

Abstract In our neo-transcendental approach, physical theories are built up from a categorial structure that is mathematically interpreted (what Kant called the "mathematical construction of categories"). The interpretation of physical categories provided by noncommutative geometry is presented in this perspective.

1 Introduction

In the early 1980s I began a research program which developed a new transcendental epistemology for modern theoretical physics. A synthetic summary of this approach can be found (in French) in my book *La Philosophie transcendantale et le problème de l'Objectivité* (1991) and (in English) in my paper "Actuality of Transcendental Aesthetics for Modern Physics" (1992) for the international Conference *1830–1930: Un siècle de géométrie, de C.F. Gauss et B. Riemann à H. Poincaré et E. Cartan : épistémologie, histoire, et mathématiques* held at the Institut Henri Poincaré in Paris the 18–23 September 1989. Further developements can be found in other papers cited in the bibliography.

The key idea is that, if physical theories are conceptually construed on the basis of categorial concepts such as "system", "state", "observable", etc. and geometrodynamical intuitions such as those of space, time or motion, these representations have to be *mathematically* interpreted in a specific way (what Kant called the "mathematical construction of categories") in order to constitute a well-behaved physical objectivity. In this way, physical objectivity cannot be an ontology, and the departure of objectivity from ontology is, I think, the basic justification for transcendentalism.

Even if objective categories remain fairly invariant in the history of physics, their mathematical interpretation has changed tremendously as physical theories have

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evolved, but this is by no means an argument against transcendentalism. On the contrary, the by now classical criticisms of Reichenbach, Carnap and many others are perhaps valid against a rigid, narrow minded, dogmatic reading of Kant but certainly not against a more general and open conception of transcendentalism. Incidentally, logical positivism is in great part a "grammatical" reinterpretation of classical transcendentalism.

In fact, Kant was the first philosopher to discover the *constitutive* nature of objectivity – "discovery of the constitutive element" that Hans Reichenbach himself called an "eminent philosophical result": objective principles are *prescriptive* rather than descriptive, and they are constitutive of physical reality. But in Kant, the constitutive components of objectivity were rooted in a cognitive representational theory. As Schlick pointed out, they were characteristic of our representational consciousness. That is why a form of transcendental subjectivism became the foundational basis for objectivity. Developments in physics (General Relativity and Quantum Mechanics) created a conflict between the objective components and their cognitive basis. However the appropriate response to this situation is not provided by logical positivism, but rather by a renewed transcendentalism where the objective and the cognitive components are methodologically separated. This transcendentalism is no longer founded on cognitive universals but on procedures of mathematical "construction".

In the previously cited papers I have shown how Hamiltonian (symplectic) mechanics, general relativity, non abelian gauge theories, and even superstring theories can be transcendentally interpreted in a very natural way. I aim at presenting in this perspective the deep and technical mathematical interpretation of physical categories provided by *noncommutative geometry*.

2 Generalizing and "Historicizing" Transcendentalism

As we have seen in the *Introduction* of this volume, a generalized and "historicized" transcendental perspective on modern physics can be based on very general principles:

1. Physics deals only with phenomena. Phenomena are relational entities that are inseparable from their conditions of observation: access conditions (observation, measurement, gathering of information, etc.) are constitutive of the very concept of physical object. In that sense, physical objectivity cannot be the ontology of a mind-independent substantial reality and any ontological realism has to be rejected.
2. But even if they lose ontological content, "categorial" concepts still have a theoretical function. In order to be transformed into scientific objects, phenomena must be conceptually lawful, "legalized" according to a categorial structure. The first philosophical thematization of this principle was Kant's *Metaphysische Anfangsgründe der Naturwissenschaft* (MAN). Kant explained how the four groups of categories and principles specialize in physics into Phoronomy (Kinematics),

Dynamics, Mechanics, and Phenomenology, and how they are mathematically interpreted in Newtonian Mechanics.

3. The essential feature of physics is the mathematical interpretation which transforms the categorial concepts into *algorithms* for the mathematical reconstruction of phenomena. This is a critical point. Physics has to solve an *inverse problem*, namely the inverse problem of the abstraction problem. Conceptual analysis must be supplemented by a *computational synthesis* of phenomena. In Kant, computational synthesis is first based on schematization and then on the "construction" of categories.

The main difficulty with a generalized transcendentalism is to understand the general meaning of *Transcendental Aesthetics*. The latter presents two aspects corresponding to what Kant called two "expositions" (*Erörterung* = "clear representation of what belongs to a concept") in the *Kritik der reinen Vernunft* (KRV): the metaphysical and the transcendental. First, phenomena are observable and therefore must appear to an observer. They appear in a specific medium of manifestation (space and time for sensible phenomena) which provides "forms of intuition". Second, these "forms" can be mathematically determined and converted into what Kant called "formal intuitions" (see the celebrated footnote to section 26 of KRV). To determine phenomena objectively, we need therefore a link between mathematically determined forms of observability (what is "gegeben") and categorial forms of lawfulness (what is "gedacht"). In Kant this link is worked out at two levels. At the level of KRV it is provided by transcendental *schematism* which converts the categories into principles ("*Grundsätze*"). At the level of MAN, it is provided by what Kant called the *construction* ("*Konstruktion*") of categories. The construction is a mode of presentation ("*Darstellung*"). It means that it is possible to interpret mathematically the schematized categorial contents by using mathematics stemming from the transcendental exposition of Transcendental Aesthetics. I think that it is in this very special sort of "mathematical hermeneutics" – not only for the intuitive forms of manifestation but also for the categorial forms of lawfulness themselves – that the synthetic a priori finds its true and deep transcendental meaning.

In the *Introduction* of the volume, we also reminded (in modern terms) the categorial moments of classical Mechanics according to the *Metaphysische Anfangsgründe der Naturwissenschaft*.

1. **Phoronomy (Kinematics).** "Mathematical" categories of quantity and "Axioms of Intuition" ("*Axiomen der Anschauung*") governing "extensive" magnitudes: the Euclidean metric of space is a background (a priori) geometrical structure and physical motion complies with Galilean relativity.
2. **Dynamics.** "Mathematical" categories of quality and "Anticipations of Perception" ("*Anticipationen der Wahrnehmung*") governing "intensive" magnitudes: physical dynamics has to be described in terms of differential entities (velocities, accelerations, etc.) varying covariantly (link with *Phoronomy*). Physics must therefore be a kind of differential geometry (not a "logic" in the traditional Aristotelian sense).

3. **Mechanics.** "Dynamical", i.e. physical, categories of relation (substance = *Inhärenz und Subsistenz*, causality = *Causalität und Dependenz*, community, reciprocity and interaction = *Gemeinschaft*) and "Analogies of Experience" ("*Analogien der Erfahrung"*): the category of substance is reinterpreted as the transcendental principle of conservation laws, the category of causality as that of forces, and the category of community as that of interactions.
4. **Phenomenology.** Categories of modality and "Postulates of empirical thought" ("*Postulate des empirischen Denkens überhaupt*"): because of relativity, motion cannot be a real but only a "possible" predicate of matter (it is a purely relational phenomenon). Position and velocity are not observable properties whose values could individuate dynamical states. The sentence "The body S "has" such position and such velocity" (in the sense of "having a property") is not a physical judgment. We find here the root of the transcendental ideality of space and time, which has nothing to do with a subjective idealism à la Berkeley. But forces (causality) are real and are governed by necessary laws. Necessity is not a logical but a transcendental modality. It is conditional, relative to the radical contingency of possible experience.

A striking modern example of such a transcendental structure is provided by the constitutive role of *symmetries*. In general relativity and non abelian gauge theories, the radical enlargment of the symmetry groups enables us to construct mathematically on the basis of relativity principles not only the physical content of the categories of substance, but also the physical content of the categories of force and interaction. As far as I am concerned (a view shared by Daniel Bennequin, a specialist of symplectic geometry and string theory) this is a far-reaching manifestation of the "Galoisian" essence of modern physics: *symmetries that express entities which cannot be physical observables act as principles of determination for the physical obervables themselves*.

The evolution of modern physics displays fairly stable categorial structures, together with many changes in their successive mathematical interpretations. I think that such a variability is by no means an argument against a transcendental approach. For instance, according to Kant, the a priori nature of space and time means essentially that the Euclidean metric of space–time and the Galilean group act as a background structure for Mechanics. This remains perfectly true. In GR, the metric is no longer a background structure and becomes a dynamical feature of the theory. The $\mathit{Diff}(M)$-invariance implies that localization becomes relational so that points lack any physical content. But this background independence is no refutation of transcendentalism. I have developped the thesis that the *differentiable* structure of space–time and the associated cohomology of differential forms remain a background structure in GR.

In Petitot (1992a) I gave a transcendental approach to:

1. Hamiltonian (symplectic) mechanics, in particular Noether's theorem and the formalism of the momentum map worked out by B. Kostant, J.M. Souriau, V. Arnold, A. Weinstein, R. Abraham, and J. Marsden (deep broadening of the construction of the category of substance).

2. General relativity and the a priori determination of Einstein equations proposed by Wheeler, Misner, and Thorne in their *Geometrodynamics* (construction of the category of force).
3. Non abelian gauge theories (construction of the category of interaction).

As it turns out, this perspective shares many theses with Friedman's works (*Dynamics of Reason*, 1999):

1. The developement of modern physics does not destroy the transcendental constitutive perspective:

   > We still need superordinate and highly mathematical first principles in physics – principles that must be injected into our experience of nature before such experience can teach us anything at all. (p. 14)

2. The conditions of possibility of physical theories (a priori synthetic principles of coordination) are not logico-analytic judgements.
3. Kant's a priori principles can be generalized, relativized and historicisized:

   > What we end up with (...) is thus a relativized and dynamical conception of a priori mathematical-physical principles, which change and develop along with the development of the mathematical and physical sciences themselves, but which nevertheless retain the characteristically Kantian constitutive function of making the empirical natural knowledge thereby structured and framed by such principles first possible. (p. 31)

4. The central role of constitutive principles:

   > What characterizes the distinguished elements of our theories is rather their special constitutive function: the function of making the precise mathematical formulation and empirical application of the theories in question first possible. (p. 40)

3 Noncommutative Geometry as a New Framework

Let me now comment on a new technical example of mathematical reinterpretation of the categorial structures of physics. This reinterpretation is achieved by using John Baez' requisite of *background independence* (less radical than Lee Smolin's). The problem is rather difficult, especially in Quantum Gravity. In GR general covariance implies that the metric is no longer a background structure and points of space–time M lose any physical meaning: GR observables must be $Diff(M)$ invariant and are therefore *non-local*. On the contrary (Carlip, 2001), in Quantum Field Theory there exists a fixed background space–time M and points have a physical meaning: the value $\varphi(x)$ of a field φ at a point $x \in M$ is in principle obervable. How are we to eliminate the background geometry in QFT while maintaining at the same time the computational efficiency of geometry? How are we to reconcile mathematically theories such as GR and QFT which are so heterogeneous to one another? Remarkable suggestions exist – in particular *loop quantum gravity* developped by Abhay Ashtekar, Lee Smolin, Carlo Rovelli, John Baez, etc. – for enlarging the formal framework of Riemann and Cartan geometry and quantize some of their components, but it seems that the problem is not a technical problem to be reckoned with only at the boundary of physical theories but a basic foundational difficulty. This means that we need a change of paradigm, much like GR in the case of Riemannian geometry.

It seems that the most interesting answer to this problem comes from *Noncommutative Geometry* (NCG) which introduces from the outset quantum concepts in the definition of the most fondamental geometrical concepts. I will present here how Connes and Lott achieved the deduction in NCG of the coupling of gravity (Einstein-Hilbert action[1]) with the Standard Model of Quantum Field Theory (QFT), how metric can be reinterpreted in purely spectral terms using the formalism of Clifford algebras and Dirac operators, and how a purely noncommutative generalization yields a natural interpretation of the Higgs phenomenon.

Philosophically speaking, NCG is a new paradigm – or framework – in as much as it includes *both* GR and the standard model of QFT as commutative approximations and provides the first deep theoretical meaning to the Higgs phenomenon. The breakthrough of NCG consists in starting from QM and "quantizing" all classical geometrical concepts. The conflict between geometry and QM disappears from the outset since quantum concepts are no longer subordinated to any prior background geometrical structure.

4 Gelfand Theory

To understand Alain Connes' NC Geometry we must first come back to Gelfand theory for commutative C^*-algebras.

4.1 C^*-algebras

Recall that a C^*-algebra $\mathcal{A}$ is a (unital) Banach algebra on $\mathbb{C}$ (i.e. a $\mathbb{C}$-algebra which is normed and complete for its norm) endowed with an involution $x \longrightarrow x^*$ s.t. $\|x\|^2 = \|x^*x\|$. The norm (the metric structure) is then deducible from the algebraic structure. Indeed, $\|x\|^2$ is the spectral radius of the positive element x^*x, that is, the *Sup* of the modulus of the spectral values of x^*x:[2]

$$\|x\|^2 = Sup\{|\lambda| : x^*x - \lambda I \text{ is not invertible}\}$$

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[1] It would be better to call this action the Hilbert-Einstein action since there is a priority of Hilbert (1915). See e.g. Majer-Sauer (2004).

[2] In the infinite dimensional case, the spectral values ($x - \lambda I$ is not invertible) are not identical with the eigenvalues ($x - \lambda I$ has a non trivial kernel). Indeed non invertibility no longer implies non injectivity (a linear operator can be injective and non surjective). For instance, if e_n, $n \in \mathbb{N}$, is a countable basis, the shift $\sum_n \lambda_n e_n \longrightarrow \sum_n \lambda_n e_{n+1}$ is injective but not surjective and is not invertible.

(where I is the unit of $\mathcal{A}$). In a C^*-algebra the norm becomes therefore a purely *spectral* concept.

An element $x \in \mathcal{A}$ is called self-adjoint if $x = x^*$, normal if $xx^* = x^*x$, and unitary if $x^{-1} = x^*$ ($\|x\| = 1$).

In this classical setting, the mathematical interpretation of the fundamental (categorial) concepts of

1. Space of states
2. Observable
3. Measure

is the following:

1. The space of states is a smooth manifold: the phase space M (in Hamiltonian mechanics, $M = T^*N$ is the cotangent bundle of the space of configurations N endowed with its canonical symplectic structure).
2. The observables are functions $f : M \to \mathbb{R}$ (interpreted as $f : M \to \mathbb{C}$ with $f = \bar{f}$) which measure some property of states and output a real number.
3. The measure of f in the state $x \in M$ is the evaluation $f(x)$ of f at x; but as $f(x) = \delta_x(f)$ (where δ_x is the Dirac distribution at x) a state can be dually interpreted as a continuous linear operator on observables.

The observables constitute a commutative C^*-algebra $\mathcal{A}$ and Gelfand theory explains that the *geometry* of the manifold M can be completely recovered from the *algebraic* structure of $\mathcal{A}$.

4.2 Gelfand's Theorem

Let M be a topological space and let $\mathcal{A} := \mathcal{C}(M)$ be the $\mathbb{C}$-algebra of continuous functions $f : M \to \mathbb{C}$ (the $\mathbb{C}$-algebra structure being inherited from that of $\mathbb{C}$ itself via pointwise addition and multiplication). Under very general conditions (e.g. if M is compact[3]), it is a C^*-algebra for complex conjugation $f^* = \overline{f}$.

The possible values of f – that is the possible results of a measure of f – can be defined in a purely algebraic way as the *spectrum* of f that is

$$sp_{\mathcal{A}}(f) := \{c : f - cI \text{ is not invertible in } \mathcal{A}\}\,.$$

Indeed, if $f(x) = c$ then $f - cI$ is not invertible in $\mathcal{A}$. $sp_{\mathcal{A}}(f)$ is the complementary set of what is called the *resolvent* of f,

$$r(f) := \{c : f - cI \text{ is invertible in } \mathcal{A}\}\,.$$

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[3] If M is non compact but only locally compact, then one take $\mathcal{A} = \mathcal{C}_0(M)$ the algebra of continuous functions vanishing at infinity but $\mathcal{A}$ is no longer unital since the constant function 1 doesn't vanish at infinity.

The main point is that the evaluation process $f(x)$ – that is measure – can be interpreted as a *duality* $\langle f, x\rangle$ between the space M and the algebra $\mathcal{A}$. Indeed, to a point x of M we can associate the *maximal ideal* of the $f \in \mathcal{A}$ vanishing at x:

$$\mathfrak{M}_x := \{f \in \mathcal{A} : f(x) = 0\}.$$

But the maximal ideals $\mathfrak{M}$ of $\mathcal{A}$ constitute themselves a space – called the *spectrum* of the algebra $\mathcal{A}$. They can be considered as the kernels of the *characters* of $\mathcal{A}$, that is of the morphisms (multiplicative linear forms) $\chi : \mathcal{A} \to \mathbb{C}$,

$$\mathfrak{M} = \chi^{-1}(0).$$

A character is by definition a coherent procedure for evaluating the elements $f \in \mathcal{A}$. The evaluation $\chi(f)$ is also a *duality* $\langle \chi, f\rangle$ and its results $\chi(f)$ belong to $sp_{\mathcal{A}}(f)$. Indeed, as *distributions* (continuous linear forms), the characters correspond to the Dirac distributions δ_x and if $\chi = \delta_x$, then $\chi(f) = f(x) = c$ and $c \in sp_{\mathcal{A}}(f)$.

The *spectrum* of the C^*-algebra $\mathcal{A}$ (not to be confused with the spectra $sp_{\mathcal{A}}(f)$ of the single elements f of $\mathcal{A}$) is by definition the space of characters $Sp(\mathcal{A}) := \{\chi\}$ endowed with the topology of simple convergence: $\chi_n \to \chi$ iff $\chi_n(f) \to \chi(f)\ \forall f \in \mathcal{A}$. It is defined uniquely from $\mathcal{A}$ without any reference to the fact that $\mathcal{A}$ is of the form $\mathcal{A} := \mathcal{C}(M)$. It is also the space of irreducible representations of $\mathcal{A}$ (since $\mathcal{A}$ is commutative, they are 1-dimensional).

Now, if $f \in \mathcal{A}$ is an element of $\mathcal{A}$, using duality, we can associate to it canonically a *function* $\tilde{f}$ on the space $Sp(\mathcal{A})$

$$\begin{aligned} \tilde{f} : Sp(\mathcal{A}) &\to \mathbb{C} \\ \chi &\mapsto \tilde{f}(\chi) = \chi(f) = \langle \chi, f\rangle. \end{aligned}$$

We get that way a map

$$\begin{aligned} \tilde{} : \mathcal{A} &\to \mathcal{C}(Sp(\mathcal{A})) \\ f &\mapsto \tilde{f} \end{aligned}$$

which is called the *Gelfand transform*. For every f we have

$$\tilde{f}(Sp(\mathcal{A})) = sp_{\mathcal{A}}(f).$$

The key result is then:

Gelfand-Neimark theorem. If $\mathcal{A}$ is a *commutative* C^*-algebra, the Gelfand transform $\tilde{}$ is an *isometry* between $\mathcal{A}$ and $\mathcal{C}(Sp(\mathcal{A}))$.

Indeed, the norm of $\tilde{f}$ is the spectral radius of f, $\rho(f) := \lim_{n\to\infty}\left(\|f^n\|^{\frac{1}{n}}\right)$ and we have $\left\|\tilde{f}\right\| = \rho(f) = \|f\|$. To see this, suppose first that f is self-adjoint ($f = f^* = \bar{f}$). We have $\|f\|^2 = \|f.f^*\| = \left\|f^2\right\|$. So, $\|f\| = \left\|f^{2^n}\right\|^{2^{-n}}$ and as

$\left\| f^{2^n} \right\|^{2^{-n}} \to \rho\left(f\right)$ by definition we have $\|f\| = \rho\left(f\right)$. Suppose now that f is any element of $\mathcal{A}$. Since $f.f^*$ is self-adjoint, we have $\|f\|^2 = \|f.f^*\| = \rho\left(f.f^*\right) = \left\|\widetilde{f.f^*}\right\|$. But $\left\|\widetilde{f.f^*}\right\| = \left\|\widetilde{f}.\widetilde{f^*}\right\| = \left\|\widetilde{f}\right\|^2$ and therefore $\|f\|^2 = \left\|\widetilde{f}\right\|^2$ and $\|f\| = \left\|\widetilde{f}\right\|$.

Gelfand theory shows that, in the classical case of commutative C^*-algebras $\mathcal{A} := \mathcal{C}(M)$ (M compact), there exists a complete *equivalence* between the geometric and the algebraic perspectives.

4.3 Towards a New (Functional) "Phoronomy"

We think that Gelfand theorem has a deep philosophical meaning. In classical mechanics "phoronomy" (kinematics) concerns the structure of the configuration space N and the phase space $M := T^*N$. Observables and measurements are defined in terms of functions on these basic spaces directly construed from the geometry of space–time (transcendental aesthetics). Gelfand theorem shows than we can *exchange* the primary geometrical background and the secondary algebraic moment of measure, take measure as a primitive fact and reconstruct the geometric background from it as a secondary moment. In one word, *we can substitute a "functional" transcendental aesthetics to a purely geometrical one*.

4.4 Towards Noncommutative Geometry

In Quantum Mechanics, the basic structure is that of the *noncommutative C^*-algebras* $\mathcal{A}$ of observables. In Petitot (1992a) I suggested that "phoronomy" operates at this level. It is challenging and natural to wonder if there could exist a *geometric correlate* of this noncommutative algebraic setting. The deepest answer is Connes' *Noncommutative Geometry* (NCG) also called *Spectral Geometry* or *Quantum Geometry*. In NCG the basic structure is the NC C^*-algebra $\mathcal{A}$ of obervables: any phenomenon is primarily something which is observable in the quantum sense, and not an event in space–time. But observables must be defined for states and are therefore represented in the space of states of the system, which is an Hilbert space and not the classical space. The associated NC space is then the space of *irreducible representations*.

NCG is a fundamentally new step toward a *geometrization* of physics. Instead of beginning with classical differential geometry and try to develop Quantum Mechanics on this backgrond, it begins with Quantum Mechanics and construct a new *quantum geometrical framework*. In that sense, Connes is the Einstein of Quantum Mechanics. The most fascinating aspect of his research program is how he succeeded in reinterpreting all the basic structures of classical geometry inside the framework of NC C^*-algebras operating on Hilbert spaces. *The basic concepts (with their categorial content) remain almost the same but their mathematical interpretation is significantly complexified*, since their classical meaning becomes a *commutative limit*. We meet here a new very deep example of the conceptual transformation of physical theories through mathematical enlargements, as it is the case in GR or QM. As explained by Daniel Kastler [NCG]:

> Alain Connes' noncommutative geometry (...) is a systematic quantization of mathematics parallel to the quantization of physics effected in the twenties. (...) This theory widens the scope of mathematics in a manner congenial to physics.

5 NCG and Differential Forms

Connes reinterpreted (in an extremely deep and technical way) the six classical levels:

1. Measure theory
2. Algebraic topology and topology (K-theory)
3. Differentiable structure
4. Differential forms and De Rham cohomology
5. Fiber bundles, connections, covariant derivations, Yang-Mills theories
6. Riemannian manifolds and metric structures.

Let us take as a first example the reinterpretation of the differential calculus.

5.1 A Universal and Formal Differential Calculus

How can we interpret differential calculus in the new NC paradigm? Connes wanted first to define *derivations* $D : \mathcal{A} \to \mathcal{E}$, that is $\mathbb{C}$-linear maps satisfying the *Leibniz rule* (which is the universal formal rule for derivations):

$$D(ab) = (Da)b + a(Db)$$

For that, $\mathcal{E}$ must be endowed with a structure of $\mathcal{A}$-bimodule (right and left products of elements of $\mathcal{E}$ by elements of $\mathcal{A}$). It is evident that $D(c) = 0$ for every scalar $c \in \mathbb{C}$ since $D(1.a) = D(1)a + 1D(a) = D(a)$ and therefore $D(1) = 0$.

Let $Der(\mathcal{A}, \mathcal{E})$ be the $\mathbb{C}$-vector space of such derivations. In $Der(\mathcal{A}, \mathcal{E})$ there exist very particular elements, the inner derivatives, associated with the elements m of $\mathcal{E}$, which express the difference between the right and left $\mathcal{A}$-module structures of $\mathcal{E}$:

$$D(a) := ad(m)(a) = ma - am.$$

Indeed,

$$\begin{aligned} ad(m)(a).b + a.ad(m)(b) &= (ma - am)b + a(mb - bm) \\ &= mab - abm \\ &= ad(m)(ab). \end{aligned}$$

In the case where $\mathcal{E} = \mathcal{A}$, $ad(b)(a) = [b,a]$ expresses the non commutativity of $\mathcal{A}$. By the way, $Der(\mathcal{A},\mathcal{A})$ is a Lie algebra since $[D_1, D_2]$ is a derivation if D_1, D_2 are derivations.

Now, the fact must be stressed that there exists *a universal derivation* depending only upon the algebraic structure of $\mathcal{A}$, and having therefore *absolutely nothing to do* with the classical "infinitesimal" intuitions underlying the classical concepts of differential and derivation. It is given by

$$\begin{aligned} d : \mathcal{A} &\to \mathcal{A} \otimes_{\mathbb{C}} \mathcal{A} \\ a &\mapsto da := 1 \otimes a - a \otimes 1. \end{aligned}$$

Let $\Omega^1\mathcal{A}$ be the sub-bimodule of $\mathcal{A} \otimes_{\mathbb{C}} \mathcal{A}$ generated by the elements $adb := a \otimes b - ab \otimes 1$, i.e. the kernel of the multiplication $a \otimes b \mapsto ab$.[4] $\Omega^1\mathcal{A}$ is isomorphic to the tensorial product $\mathcal{A} \otimes_{\mathcal{A}} \overline{\mathcal{A}}$, where $\overline{\mathcal{A}}$ is the quotient $\mathcal{A}/\mathbb{C}$, with $adb = a \otimes \overline{b}$. It is called the bimodule of *universal* 1*-forms* on $\mathcal{A}$ where "universality" means that

$$Der(\mathcal{A},\mathcal{E}) \simeq Hom_{\mathcal{A}} \left(\Omega^1\mathcal{A}, \mathcal{E} \right)$$

i.e. that a derivation $D : \mathcal{A} \to \mathcal{E}$ is the same thing as a morphism *of algebras* between $\Omega^1\mathcal{A}$ and $\mathcal{E}$. If $D : \mathcal{A} \to \mathcal{E}$ is an element of $Der(\mathcal{A},\mathcal{E})$, the associated morphism $\tilde{D} : \Omega^1\mathcal{A} \to \mathcal{E}$ is defined by

$$a \otimes b \mapsto aD(b).$$

So $da = 1 \otimes a - a \otimes 1 \mapsto 1.D(a) - a.D(1) = D(a)$ (since $D(1) = 0$).

We can generalize this construction to universal n-forms, which have the symbolic form[5]

$$a_0 da_1 ... da_n.$$

If $\Omega^n\mathcal{A} := \left(\Omega^1\mathcal{A}\right)^{\otimes n} = \mathcal{A} \otimes_{\mathcal{A}} \left(\overline{\mathcal{A}}\right)^{\otimes n}$ with $a_0 da_1 ... da_n = a_0 \otimes \overline{a_1} \otimes ... \otimes \overline{a_n}$, the differential is then

$$\begin{gathered} d : \Omega^n\mathcal{A} \to \Omega^{n+1}\mathcal{A} \\ a_0 da_1 ... da_n \mapsto da_0 da_1 ... da_n \\ a_0 \otimes \overline{a_1} \otimes ... \otimes \overline{a_n} \mapsto 1 \otimes \overline{a_0} \otimes \overline{a_1} \otimes ... \otimes \overline{a_n} \, . \end{gathered}$$

Since $d1 = 0$, it is easy to verify the fundamental cohomological property $d^2 = 0$ of the graduate differential algebra $\Omega\mathcal{A} := \bigoplus_{n \in \mathbb{N}} \Omega^n\mathcal{A}$. Some technical difficulties must be overcome (existence of "junk" forms) to transform this framework into a "good" formal differential calculus.

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[4] For $a \otimes b - ab \otimes 1$ the multiplication gives $ab - ab = 0$. Reciprocally if $ab = 0$ then $a \otimes b = a \otimes b - ab \otimes 1$ and $a \otimes b$ belongs to $\Omega^1\mathcal{A}$.

[5] $da_1 ... da_n$ is the exterior product of 1-forms, classically denoted $da_1 \wedge ... \wedge da_n$.

5.2 Noncommutative Differential Calculus or "Quantized" Calculus

Connes wanted to *represent* this universal differential algebra in spaces of physical states. Let us suppose therefore that the C^*-algebra $\mathcal{A}$ acts upon an Hilbert space $\mathcal{H}$ and we want to interpret in this representation the universal, formal, and purely symbolic differential calculus of the previous section. For that, we must interpret the differential df of the elements $f \in \mathcal{A}$, these f being now represented as *operators* on $\mathcal{H}$. Connes' main idea was to use the well-known formula of QM

$$\frac{df}{dt} = \frac{2i\pi}{h}[F,f]$$

where F is the Hamiltonian of the system and f any obervable.

Consequently, he interpreted the symbol df as

$$df := [F,f]$$

for an appropriate self-adjoint operator F. We want of course $d^2 f = 0$. But $d^2 f = \left[F^2, f\right]$ and therefore F^2 must commute with all observables.

The main constraint is that, once interpreted in $\mathcal{H}$, the symbol df must correspond to an *infinitesimal*. *The classical concept of infinitesimal ought to be reinterpreted in the NC framework*. Connes' definition is that an operator T is infinitesimal if it is *compact*, that is if the eigenvalues $\mu_n(T)$ of its absolute value $|T| = (T^*T)^{1/2}$ – called the *characteristic values* of T – converge to 0, that is if for every $\varepsilon > 0$ the norm $\|T\|$ of T is $< \varepsilon$ outside a subspace of *finite* dimension. If $\mu_n(T) \underset{n\to\infty}{\longrightarrow} 0$ as $\frac{1}{n^\alpha}$ then T is an infinitesimal of order α (α not necessarily an integer).

If T is compact, let ξ_n be a complete orthonormal basis of $\mathcal{H}$ associated to $|T|$, $T = U|T|$ the polar decomposition of T[6] and $\eta_n = U\xi_n$. Then T is the sum

$$T = \sum_{n\geq 0} \mu_n(T) \left|\eta_n\right\rangle \left\langle\xi_n\right| \,.$$

If T is a positive infinitesimal of order 1, its trace $\mathit{Trace}(T) = \sum_n \mu_n(T)$ has a logarithmic divergence. If T is of order >1, its trace is finite > 0. It is the basis for *NC integration* which uses the *Dixmier trace*, a technical tool for constructing a new trace extracting the logarithmic divergence of the classical trace. Dixmier trace is a technical way for giving a sense to the formula $\lim\limits_{N\to\infty} \frac{1}{\ln N} \sum_{n=0}^{n=N-1} \mu_n(T)$. It vanishes for infinitesimals of order >1.

Therefore, we interpret the differential calculus in the NC framework through triples $(\mathcal{A}, \mathcal{H}, F)$ where $[F,f]$ is compact for every $f \in \mathcal{A}$. Such a structure is called *a Fredholm module*.

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[6] The polar decomposition $T = U|T|$ is the equivalent for operators of the decomposition $z = |z|e^{i\theta}$ for a complex number. In general U cannot be unitary but only a partial isometry.

The differential forms $a_0 da_1 ... da_n$ can now be interpreted as operators on $\mathcal{H}$

$$a_0 da_1 ... da_n := a_0 [F, a_1] ... [F, a_n]$$

and we see how the second transcendental moment of physical objectivity, namely that of "dynamics", becomes interpreted in the NC framework.

It must be emphasized that the NC generalization of differential calculus is a wide and wild generalization since it enables us to extend differential calculus to fractals!

6 NC Riemannian Geometry, Clifford Algebras, and Dirac Operator

Another great achievement of Alain Connes was the complete and deep reinterpretation of the ds^2 in Riemannian geometry. Classically, $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$. In the NC framework, dx must be interpreted as $dx = [F, x]$ (where $(\mathcal{A}, \mathcal{H}, F)$ is a Fredholm module), and the matrix $(g_{\mu\nu})$ as an element of the $n \times n$ matrix algebra $M_n(\mathcal{A})$. The ds^2 must therefore become a *compact* and *positive* operator of the form

$$G = [F, x^\mu]^* g_{\mu\nu} [F, x^\nu].$$

6.1 A Redefinition of Distance

Connes' idea is to reinterpret the classical definition of distance $d(p,q)$ between two points p, q of a Riemannian manifold M as the *Inf* of the length $L(\gamma)$ of the paths $\gamma : p \to q$

$$d(p,q) = \underset{\gamma : p \to q}{Inf} \; L(\gamma)$$

$$L(\gamma) = \int_p^q ds = \int_p^q \left(g_{\mu\nu} dx^\mu dx^\nu \right)^{1/2}.$$

Using the equivalence between a point x of M and the pure state δ_x on the commutative C^*-algebra $\mathcal{A} := C^\infty(M)$, an elementary computation shows that this definition of the distance is equivalent to the dual algebraic definition using only concepts concerning the C^*-algebra $\mathcal{A}$

$$d(p,q) = Sup \left\{ |f(q) - f(p)| : \|grad(f)\|_\infty \leq 1 \right\}$$

where $\|...\|_\infty$ is the L^∞ norm, that is the *Sup* on $x \in M$ of the norms on the tangent spaces $T_x M$.[7]

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[7] Let $\gamma : I = [0,1] \to M$ be a C^∞ curve in M from p to q. $L(\gamma) = \int_p^q |\dot{\gamma}(t)| dt = \int_0^1 g\left(\dot{\gamma}(t), \dot{\gamma}(t)\right)^{1/2} dt$. If $f \in C^\infty(M)$, using the duality between df and $gradf$ induced by the metric, we find

6.2 Clifford Algebras

Now the core of the NC definition of distance uses the *Dirac operator*. In order to explain this key point, *which makes distance a quantum concept*, the so called *Clifford algebra* of a Riemannian manifold must be introduced.

Recall that the formalism of Clifford algebras relates *the differential forms and the metric* on Riemannian manifolds. In the classical case of the Euclidean space $\mathbb{R}^n$, the main idea is to encode the isometries $O(n)$ in an algebra structure. Since every isometry is a product of reflections (Cartan), we can associate to any vector $v \in \mathbb{R}^n$ the reflection $\overline{v}$ relative to the orthogonal hyperplane $v^\perp$ and introduce a multiplication $v.w$ which is nothing else than the composition $\overline{v} \circ \overline{w}$. We are then naturally led to the anti-commutation relations

$$\{v,w\} := v.w + w.v = -2(v,w)$$

where (v,w) is the Euclidean scalar product.

More generally, let V be a $\mathbb{R}$-vector space endowed with a quadratic form g. Its Clifford algebra $Cl(V,g)$ is its tensor algebra $\mathcal{T}(V) = \oplus_{k=0}^{k=\infty} V^{\otimes k}$ quotiented by the relations

$$v \otimes v = -g(v)1,\ \forall v \in V$$

(where $g(v) = g(v,v) = \|v\|^2$). In $Cl(V,g)$ the tensorial product $v \otimes v$ becomes a product $v.v = v^2$. It must be stressed that there exists always in $Cl(V,g)$ the constants $\mathbb{R}$ which correspond to the 0th tensorial power of V.

Using the scalar product

$$2g(v,w) = g(v+w) - g(v) - g(w)$$

one gets the *anti-commutation* relations

$$\{v,w\} = -2g(v,w)$$

Elementary examples are given by the $Cl_n = Cl(\mathbb{R}^n, g_{Euclid})$.

- $Cl_0 = \mathbb{R}$
- $Cl_1 = \mathbb{C}$ ($V = i\mathbb{R}$, $i^2 = -1$, $Cl_1 = \mathbb{R} \oplus i\mathbb{R}$)
- $Cl_2 = \mathbb{H}$ ($V = i\mathbb{R} + j\mathbb{R}$, $ij = k$, $Cl_2 = \mathbb{R} \oplus i\mathbb{R} \oplus j\mathbb{R} \oplus k\mathbb{R}$)
- $Cl_3 = \mathbb{H} \oplus \mathbb{H}$
- $Cl_4 = \mathbb{H}[2]$ (2×2 matrices with entries in $\mathbb{H}$)
- $Cl_5 = \mathbb{C}[4]$

$f(q) - f(p) = \int_0^1 df_{\gamma(t)}(\dot\gamma(t))\,dt = \int_0^1 g_{\gamma(t)}\left(grad_{\gamma(t)} f, \dot\gamma(t)\right) dt$. This shows that $|f(q) - f(p)| \leq \int_0^1 \left|grad_{\gamma(t)} f\right| |\dot\gamma(t)|\,dt \leq \|gradf\|_\infty L(\gamma)$. Therefore, if $\|grad(f)\|_\infty \leq 1$ we have $|f(q) - f(p)| \leq d(p,q)$. When we take the *Sup* we retreive $d(p,q)$ using the special function $f_p(x) = d(p,x)$ since $|f_p(q) - f_p(p)| = d(p,q)$.

- $Cl_6 = \mathbb{R}[8]$
- $Cl_7 = \mathbb{R}[8] \oplus \mathbb{R}[8]$
- $Cl_{n+8} = Cl_n \otimes \mathbb{R}[16]$ (Bott periodicity theorem)

If $g(v) \neq 0$ (which would always be the case for $v \neq 0$ if g is non degenerate) v is invertible in this algebra structure and

$$v^{-1} = -\frac{v}{g(v)}.$$

The multiplicative Lie group $Cl^{\times}(V,g)$ of the invertible elements of $Cl(V,g)$ act through *inner automorphisms* on $Cl(V,g)$. This yields the *adjoint representation*

$$\begin{aligned} Ad : Cl^{\times}(V,g) &\to Aut(Cl(V,g)) \\ v &\mapsto Ad_v : w \mapsto v.w.v^{-1}. \end{aligned}$$

But[8]

$$v.w.v^{-1} = -w + \frac{2g(v,w)v}{g(v)} = Ad_v(w).$$

As $-Ad_v$ is the reflection relative to $v^{\perp}$, this means that reflections act through the adjoint representation of the Clifford algebra. The derivative *ad* of the adjoint representation enables to recover the Lie bracket of the Lie algebra $cl^{\times}(V,g) = Cl(V,g)$ of the Lie group $Cl^{\times}(V,g)$

$$\begin{aligned} ad : cl^{\times}(V,g) = Cl(V,g) &\to Der(Cl(V,g)) \\ v &\mapsto ad_v : w \mapsto [v,w] \end{aligned}$$

Now there exists a fundamental relation between the Clifford algebra $Cl(V,g)$ of V and its exterior algebra $\Lambda^* V$. If $g = 0$ and if we interpret $v.w$ as $v \wedge w$, the anti-commutation relations become simply $\{v,w\} = 0$, that is the classical antisymmetry $w \wedge v = -v \wedge w$ of differential 1-forms. Therefore

$$\Lambda^* V = Cl(V,0).$$

In fact, $Cl(V,g)$ can be considered as a way of *quantizing* $\Lambda^* V$ using the metric g in order to get *non trivial* anti-commutation relations.

Due to the relations $v^2 = -g(v)1$ which decrease the degree of a product by 2, $Cl(V,g)$ is no longer a $\mathbb{Z}$-graded algebra but only a $\mathbb{Z}/2$-graded algebra, the $\mathbb{Z}/2$-gradation corresponding to the even/odd elements. But we can reconstruct a $\mathbb{Z}$-graded algebra $\mathcal{C} = \bigoplus_{k=0}^{k=\infty} C^k$ associated to $Cl(V,g)$, the C^k being the homogeneous terms of degree k: $v_1.\cdots.v_k$.

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[8] $v.w.v^{-1} = -v.w.\frac{v}{g(v)} = -(-w.v - 2g(v,w))\frac{v}{g(v)} = w.\frac{v^2}{g(v)} + \frac{2g(v,w)v}{g(v)} = -w + \frac{2g(v,w)v}{g(v)}$.

Theorem. The map of graded algebras $\mathcal{C} = \bigoplus_{k=0}^{k=\infty} C^k \longrightarrow \Lambda^* V = \bigoplus_{k=0}^{k=\infty} \Lambda^k$ given by $v_1 . \cdots . v_k \longrightarrow v_1 \wedge \cdots \wedge v_k$ is a *linear isomorphism* (but not an *algebra* isomorphism).

We consider now 2 operations on the *exterior* algebra $\Lambda^* V$:

1. The outer multiplication $\varepsilon(v)$ by $v \in V$:

$$\varepsilon(v)\left(\bigwedge_i u_i\right) = v \wedge \left(\bigwedge_i u_i\right).$$

We have $\varepsilon(v)^2 = 0$ since $v \wedge v = 0$.

2. The contraction (inner multiplication) $\imath(v)$ induced by the metric g:[9]

$$\imath(v)\left(\bigwedge_i u_i\right) = \sum_{j=1}^{j=k} (-1)^j g(v, u_j)\, u_1 \wedge \cdots \wedge \widehat{u_j} \wedge \cdots u_k.$$

We have also $\imath(v)^2 = 0$. The inner multiplication $\imath(v)$ is a supplementary structure involving the metric structure.

One shows that the following anti-commutations relations obtain:

$$\{\varepsilon(v), \imath(w)\} = -g(v,w)1\ .$$

Let now $c(v) = \varepsilon(v) + \imath(v)$. We get the anti-commutation relations of the Clifford algebra

$$\{c(v), c(w)\} = -2g(v,w)1$$

and $Cl(V,g)$ is therefore generated in $End_{\mathbb{R}}(\Lambda^* V)$ by the $c(v)$ (identified with v).

6.3 Spin Groups

The isometry group $O(n)$ is canonically embedded in $Cl(V,g)$ since every isometry is a product of reflections. In fact $Cl(V,g)$ contains also the *pin group* $Pin(n)$ which is a twofold covering of $O(n)$. If we take into account the orientation and restrict to $SO(n)$, the twofold covering becomes the *spin group* $Spin(n)$. $Spin(n)$ is generated by the even products of v s.t. $g(v) = \pm 1$, $SO(n)$ is generated by even products of $-Ad_v$ and the covering $Spin(n) \rightarrow SO(n)$ is given by $v \mapsto -Ad_v$. By restriction of the Clifford multiplication and of the adjoint representation $w \mapsto v.w.v^{-1}$ to $Spin(n)$, we get therefore a representation γ of $Spin(n)$ into the spinor space $\mathbb{S} = Cl(V,g)$.

---

[9] In the following formula $\widehat{u_j}$ means that the term u_j is deleted.

6.4 Dirac Equation

We can use the Clifford algebra, and therefore the metric, to change the classical exterior derivative of differential forms given by

$$d := \varepsilon\,(dx^{\mu})\,\frac{\partial}{\partial x^{\mu}}.$$

We then define the Dirac operator on spinor fields $\mathbb{R}^n \to \mathbb{S}$ as

$$\begin{aligned} D \ :\ &= c\,(dx^{\mu})\,\frac{\partial}{\partial x^{\mu}} \\ &= \gamma^{\mu}\,\frac{\partial}{\partial x^{\mu}} \end{aligned}$$

where c is the Clifford multiplication, and D acts on the spinor space $\mathbb{S} = Cl(V,g)$. As $\{c(v),c(w)\} = -2g(v,w)1$, the γ^{μ} satisfy standard Dirac relations of anticommutation $\{\gamma^{\mu},\gamma^{\nu}\} = -2\delta^{\mu\nu}$ in the Euclidean case.[10] On can check that $D^2 = \Delta$ is the Laplacian.

6.5 Dirac Operator

More generally, if M is a Riemannian manifold, the previous construction can be done for every tangent space T_xM endowed with the quadratic form g_x. In this way we get a *bundle* of Clifford algebras $Cl(TM,g)$. If S is a spinor bundle, that is a bundle of $Cl(TM)$ -modules s.t. $Cl(TM) \simeq End(S)$, endowed with a covariant derivative ∇, we associate to it the Dirac operator

$$D : \mathcal{S} = \Gamma(S) = C^{\infty}(M,S) \to \Gamma(S)$$

which is a first order elliptic operator interpretable as the "square root" of the Laplacian Δ, Δ interpreting itself the metric in operatorial terms. The Dirac operator D establishes a coupling between the covariant derivation on S and the Clifford multiplication of 1-forms. It can be extended from the $C^{\infty}(M)$-module $\mathcal{S} = \Gamma(S)$ to the Hilbert space $\mathcal{H} = L^2(M,S)$.

In general, because of chirality, S will be the direct sum of an even and an odd part, $S = S^+ \oplus S^-$ and D will have the characteristic form

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[10] The classical Dirac matrices are the $-i\gamma^{\mu}$ for $\mu = 0,1,2,3$.

$$D = \begin{bmatrix} 0 & D^- \\ D^+ & 0 \end{bmatrix}$$
$$D^+ : \Gamma(S^+) \to \Gamma(S^+)$$
$$D^- : \Gamma(S^-) \to \Gamma(S^-)$$

D^+ and D^- being adjoint operators.

6.6 NC Distance and Dirac Operator

In this classical framework, it easy to compute the bracket $[D,f]$ for $f \in C^\infty(M)$. First, there exists on M the *Levi-Civita connection*:

$$\nabla^g : \Omega^1(M) \to \Omega^1(M) \underset{C^\infty(M)}{\otimes} \Omega^1(M)$$

satisfying the Leibniz rule for $\alpha \in \Omega^1(M)$ and $f \in C^\infty(M)$:

$$\nabla^g(\alpha f) = \nabla^g(\alpha) f + \alpha \otimes df$$

(as $\nabla^g(\alpha) \in \Omega^1(M) \underset{C^\infty(M)}{\otimes} \Omega^1(M)$, $\nabla^g(\alpha)f \in \Omega^1(M) \underset{C^\infty(M)}{\otimes} \Omega^1(M)$ and as α and $df \in \Omega^1(M)$, $\alpha \otimes df \in \Omega^1(M) \underset{C^\infty(M)}{\otimes} \Omega^1(M)$). There exists also the *spin connection* on the spinor bundle S

$$\nabla^S : \Gamma(S) \to \Omega^1(M) \underset{C^\infty(M)}{\otimes} \Gamma(S)$$

satisfying the Leibniz rule for $\psi \in \Gamma(S)$ and $f \in C^\infty(M)$:

$$\nabla^S(\psi f) = \nabla^S(\psi) f + \psi \otimes df$$
$$\nabla^S(\gamma(\alpha)\psi) = \gamma(\nabla^g(\alpha))\,\psi + \gamma(\alpha)\nabla^S(\psi)$$

where γ is the spin representation. The Dirac operator on $\mathcal{H} = L^2(M,S)$ is then defined as

$$D := \gamma \circ \nabla^S .$$

If $\psi \in \Gamma(S)$, we have (making the f acting on the left in $\mathcal{H}$)

$$\begin{aligned} D(f\psi) &= \gamma\left(\nabla^S(\psi f)\right) \\ &= \gamma\left(\nabla^S(\psi) f + \psi \otimes df\right) \\ &= \gamma\left(\nabla^S(\psi)\right) f + \gamma(\psi \otimes df) \\ &= fD(\psi) + \gamma(df)\,\psi \end{aligned}$$

and therefore $[D,f](\psi) = fD(\psi) + \gamma(df)\,\psi - fD(\psi) = \gamma(df)\,\psi$, that is

$$[D,f] = \gamma(df)\,.$$

In the standard case where $M = \mathbb{R}^n$ and $S = \mathbb{R}^n \times V$, V being a Cl_n-module of spinors ($Cl_n = Cl(\mathbb{R}^n, g_{Euclid})$), we have seen that D is a differential operator with constant coefficients taking its values in V.

$$D = \sum_{k=1}^{k=n} \gamma^\mu \frac{\partial}{\partial x^\mu}$$

with the constant matrices $\gamma^\mu \in \mathcal{L}(V)$ satisfying the anti-commutation relations

$$\{\gamma^\mu, \gamma^\nu\} = -2\delta^{\mu\nu}$$

The fundamental point is that the γ^μ are associated with the basic 1-forms dx^μ through the isomorphism

$$c : \mathcal{C} = \Lambda^*(M) \to gr(Cl(TM))$$

$$[D,f] = \gamma(df) = c(df)$$

and $\|[D,f]\|$ is the norm of the Clifford action of *df* on the space of spinors $L^2(M,S)$. But

$$\begin{aligned}\|c(df)\|^2 &= \underset{x\in M}{Sup} g_x^{-1}\left(d\overline{f}(x), df(x)\right)\\ &= \underset{x\in M}{Sup} g_x\left(grad_x\overline{f}, grad_x f\right)\\ &= \|grad(f)\|_\infty^2\ .\end{aligned}$$

Whence the definition:

$$d(p,q) = Sup\left\{|f(p) - f(q)| : f \in \mathcal{A}, \|[D,f]\| \leq 1\right\}\ .$$

In this reinterpretation, *ds* corresponds to *the propagator of the Dirac operator D*. As an operator acting on the Hilbert space $\mathcal{H}$, D is an unbounded self-adjoint operator such that $[D,f]$ is bounded $\forall f \in \mathcal{A}$ and such that its resolvent $(D - \lambda I)^{-1}$ is compact $\forall \lambda \notin Sp(D)$ (which corresponds to the fact that *ds* is infinitesimal) and the trace $Trace\left(e^{-D^2}\right)$ is *finite*. In terms of the operator $G = [F,x^\mu]^* g_{\mu\nu}[F,x^\nu]$, we have $G = D^{-2}$.

7 Noncommutative Spectral Geometry

Basing himself on several examples, Alain Connes arrived at the following concept of NC geometry.

In the classical commutative case, $\mathcal{A} = C^{\infty}(M)$ is the commutative algebra of "coordinates" on M represented in the Hilbert space $\mathcal{H} = L^2(M,S)$ by pointwise multiplication[11] and ds is a symbol non commuting with the $f \in \mathcal{A}$ and satisfying the commutation relations $\left[\left[f, ds^{-1}\right], g\right] = 0, \forall f, g \in \mathcal{A}$.

Any specific geometry is defined through the representation $ds = D^{-1}$ of ds by means of a Dirac operator $D = \gamma^{\mu}\nabla_{\mu}$. The differential $df = [D, f]$ is then the Clifford multiplication by the gradient ∇f and its norm in $\mathcal{H}$ is the Lipschitz norm of f: $\|[D, f]\| = \underset{x \in M}{Sup}\|\nabla f\|$.

These results can be taken as a definition in the general case. The geometry is defined by a *spectral triple* $(\mathcal{A}, \mathcal{H}, D)$ where $\mathcal{A}$ is a NC C^*-algebra with a representation in an Hilbert $\mathcal{H}$ and D is an unbounded self-adjoint operator on $\mathcal{H}$ such that $ds = D^{-1}$ and more generally the resolvent $(D - \lambda I)^{-1}$, $\lambda \notin \mathbb{R}$, is compact, and at the same time all $[D, a]$ are bounded $\forall a \in \mathcal{A}$ (there is a tension between these two last conditions).[12] As Connes (2000) emphasizes

> It is precisely this lack of commutativity between the line element and the coordinates on a space [between ds and the $a \in \mathcal{A}$] that will provide the measurement of distance.

The new definition of differentials are then $da = [D, a]\ \forall a \in \mathcal{A}$.

8 Yang-Mills Theory of a NC Coupling Between Two Points and Higgs Mechanism

A striking example of pure NC physics is given by Connes' interpretation of the Higgs phenomenon. In the Standard Model, the Higgs mechanism was an *ad hoc* device used for confering a mass to gauge bosons. It lacked any geometrical interpretation. One of the deepest achievement of the NC framework has been to show that Higgs fields correspond effectively to gauge bosons, but for a *discrete* NC geometry.

8.1 Symmetry Breaking and Classical Higgs Mechanism

Let us first recall the classical Higgs mechanism. Consider e.g. a φ^4 theory for two scalar real fields φ_1 and φ_2. The Lagrangian is

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[11] If $f \in \mathcal{A}$ and $\xi \in \mathcal{H}$, $(f\xi)(x) = f(x)\xi(x)$.

[12] Let λ_n be the eigenvalues of D ($\lambda_n \in \mathbb{R}$ since D is self-adjoint). $|\lambda_n| = \mu_n(D)$ and as $(D - \lambda I)^{-1}$ is compact, $|\lambda_n| \xrightarrow[n \to \infty]{} \infty$.

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \varphi_1 \partial^\mu \varphi_1 + \partial_\mu \varphi_2 \partial^\mu \varphi_2\right) - V\left(\varphi_1^2 + \varphi_2^2\right)$$

with the quartic potential

$$V\left(\varphi_1^2 + \varphi_2^2\right) = \frac{1}{2}\mu^2\left(\varphi_1^2 + \varphi_2^2\right) + \frac{1}{4}|\lambda|\left(\varphi_1^2 + \varphi_2^2\right)^2$$

It is by construction $SO(2)$-invariant.

For $\mu^2 > 0$ the minimum of V (the quantum vacuum) is non degenerate: $\varphi_0 = (0,0)$ and the Lagrangian $\mathcal{L}_{os}$ of small oscillations in the neighborhood of φ_0 is the sum of 2 Lagrangians of the form:

$$\mathcal{L}_{os} = \frac{1}{2}\left(\partial_\mu \psi \partial^\mu \psi\right) - \frac{1}{2}\mu^2 \psi^2$$

describing particles of mass μ^2.

But for $\mu^2 < 0$ the situation becomes completely different. Indeed the potential V has a full circle (an $SO(2)$-orbit) of minima

$$\varphi_0^2 = -\frac{\mu^2}{|\lambda|} = v^2$$

and the vacuum state is highly *degenerate*.

One must therefore *break the symmetry* to choose a vacuum state. Let us take for instance $\varphi_0 = \begin{bmatrix} v \\ 0 \end{bmatrix}$ and translate the situation to φ_0:

$$\varphi = \begin{bmatrix} \varphi_1 \\ \varphi_2 \end{bmatrix} = \begin{bmatrix} v \\ 0 \end{bmatrix} + \begin{bmatrix} \xi \\ \eta \end{bmatrix} .$$

The oscillation Lagrangian at φ_0 becomes

$$\mathcal{L}_{os} = \frac{1}{2}\left(\partial_\mu \eta \partial^\mu \eta + 2\mu^2 \eta^2\right) + \frac{1}{2}\left(\partial_\mu \xi \partial^\mu \xi\right)$$

and describes two particles:

1. A particle η of mass $m = \sqrt{2}\,|\mu|$, which corresponds to radial oscillations.
2. A particule ξ of mass $m = 0$, which connects vacuum states. ξ is the *Goldstone boson*.

As is well known, the Higgs mechanism consists in using a cooperation between gauge bosons and Goldstone bosons to confer a mass to gauge bosons. Let $\varphi = \frac{1}{\sqrt{2}}\left(\varphi_1 + i\varphi_2\right)$ be the scalar complex field associated to φ_1 and φ_2. Its Lagrangian is

$$\mathcal{L} = \partial_\mu \overline{\varphi} \partial^\mu \varphi - \mu^2 |\varphi|^2 - |\lambda| |\varphi|^4 .$$

It is trivially invariant by the global internal symmetry $\varphi \to e^{i\theta}\varphi$. If we *localize* the global symmetry using transformations $\varphi(x) \to e^{iq\alpha(x)}\varphi(x)$ and take into account the coupling with an electro-magnetic field deriving from the vector potential A_μ, we get

$$\mathcal{L} = \nabla_\mu \overline{\varphi} \nabla^\mu \varphi - \mu^2 |\varphi|^2 - |\lambda| |\varphi|^4 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

where ∇ is the covariant derivative

$$\nabla_\mu = \partial_\mu + iqA_\mu$$

and F the force field

$$F_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu .$$

The Lagrangian remains invariant if we balance the localization of the global internal symmetry with a change of gauge

$$A_\mu \to A'_\mu = A_\mu - \partial_\mu \alpha(x).$$

For $\mu^2 > 0$, $\varphi_0 = 0$ is a minimum of $V(\varphi)$, the vacuum is non degenerate, and we get 2 scalar particles φ and $\overline{\varphi}$ and a photon A_μ.

For $\mu^2 < 0$, the vacuum is degenerate and there is a spontaneous symmetry breaking. We have $|\varphi_0|^2 = -\frac{\mu^2}{2|\lambda|} = \frac{v^2}{2}$. If we take $\varphi_0 = \frac{v}{\sqrt{2}}$ and write

$$\varphi = \varphi' + \varphi_0 = \frac{1}{\sqrt{2}}(v + \eta + i\xi) \approx \frac{1}{\sqrt{2}} e^{i\frac{\xi}{v}}(v + \eta) \text{ for } \xi \text{ and } \eta \text{ small,}$$

we get for the Lagrangian of oscillations:

$$\mathcal{L}_{os} = \frac{1}{2}\left(\partial_\mu \eta \partial^\mu \eta + 2\mu^2 \eta^2\right) + \frac{1}{2}\left(\partial_\mu \xi \partial^\mu \xi\right) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + qvA_\mu \left(\partial_\mu \xi\right) + \frac{q^2 v^2}{2} A_\mu A^\mu .$$

1. The field η (radial oscillations) has mass $m = \sqrt{2}|\mu|$.
2. The boson A_μ acquires a mass due to the term $A_\mu A^\mu$ and interacts with the Goldstone boson ξ.

The terms containing the gauge boson A_μ and the Goldstone boson ξ write

$$\frac{q^2 v^2}{2}\left(A_\mu + \frac{1}{qv}\partial_\mu \xi\right)\left(A^\mu + \frac{1}{qv}\partial^\mu \xi\right)$$

and are therefore generated by the gauge change

$$\alpha = \frac{\xi}{qv}$$
$$A_\mu \to A_\mu + \partial_\mu \alpha .$$

We see that we can use the gauge transformations

$$A_\mu \to A'_\mu = A_\mu + \frac{1}{qv}\partial^\mu \xi$$

for *fixing* the vacuum state. The transformation corresponds to the phase rotation of the scalar field

$$\varphi \to \varphi' = e^{-i\frac{\xi}{v}}\varphi = \frac{v+\eta}{\sqrt{2}}\ .$$

In this new gauge where the Goldstone boson ξ disappears, the vector particule A'_μ acquires a mass qv. The Lagrangian writes now

$$\mathcal{L}_{os} = \frac{1}{2}\left(\partial_\mu \eta \partial^\mu \eta + 2\mu^2\eta^2\right) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{q^2v^2}{2}A'_\mu A'^\mu\ .$$

The Goldstone boson connecting the degenerate vacuum states is in some sense "captured" by the gauge boson and transformed into mass.

8.2 NC Yang-Mills Theory of Two Points and Higgs Phenomenon

The NC equivalent of this description is the following. It shows that Higgs mechanism is actually the standard Yang-Mills formalism applied to a purely discrete NC geometry.

Let $\mathcal{A} = \mathcal{C}(Y) = \mathbb{C}\oplus\mathbb{C}$ be the C^*-algebra of the space Y composed of two points a and b. Its elements $f = \begin{bmatrix} f(a) & 0 \\ 0 & f(b)\end{bmatrix}$ act through multiplication on the Hilbert space $\mathcal{H} = \mathcal{H}_a \oplus \mathcal{H}_b$. We take for Dirac operator an operator of the form

$$D = \begin{bmatrix} 0 & M^* = D^- \\ M = D^+ & 0 \end{bmatrix}$$

and introduce the "chirality" $\gamma = \begin{bmatrix} 1 & 0 \\ 0 & -1\end{bmatrix}$ (the γ_5 of the standard Dirac theory). In this discrete situation we define *df* as

$$df = [D,f] = \Delta f \begin{bmatrix} 0 & M^* \\ -M & 0\end{bmatrix}$$

with $\Delta f = f(b) - f(a)$. Therefore

$$\|[D,f]\| = |\Delta f|\,\lambda$$

where $\lambda = \|M\|$ is the greatest eigenvalue of M.

If we apply now the formula for the distance, we find:

$$\begin{aligned} d(a,b) &= Sup\{|f(a)-f(b)| : f\in\mathcal{A}, \|[D,f]\|\leq 1\} \\ &= Sup\{|f(a)-f(b)| : f\in\mathcal{A}, |f(a)-f(b)|\lambda\leq 1\} \\ &= \frac{1}{\lambda} \end{aligned}$$

and we see that the distance $\frac{1}{\lambda}$ between the two points a and b has a *spectral* content and is measured by the Dirac operator.

To interpret differential calculus in this context, we consider the two idempotents (projectors) e and $1-e$ defined by

$$\begin{aligned} e(a) &= 1, e(b)=0 \\ (1-e)(a) &= 0, (1-e)(b)=1 \ . \end{aligned}$$

Every $f\in\mathcal{A}$ writes $f = f(a)e + f(b)(1-e)$, and therefore

$$\begin{aligned} df &= f(a)de + f(b)d(1-e) \\ &= (f(a)-f(b))\,de \\ &= -(\Delta f)de \\ &= -(\Delta f)ede + (\Delta f)(1-e)d(1-e) \end{aligned}$$

This shows that ede and $(1-e)d(1-e) = -(1-e)de$ provide a natural basis of the space of 1-forms $\Omega^1\mathcal{A}$. Let

$$\begin{aligned} \omega &= \lambda ede + \mu(1-e)d(1-e) \\ &= \lambda ede - \mu(1-e)de \end{aligned}$$

be a 1-form. ω is represented by

$$\omega = (\lambda e - \mu(1-e))\,[D,e] \ .$$

But on $\mathcal{H}$ $[D,e] = -\begin{bmatrix} 0 & M^* \\ -M & 0 \end{bmatrix}$ and therefore

$$\omega = \begin{bmatrix} 0 & -\lambda M^* \\ -\mu M & 0 \end{bmatrix} \ .$$

Let us now construct the *Yang-Mills theory* corresponding to this situation. A vector potential V – a connection in the sense of gauge theories – is a self-adjoint 1-form and has the form

$$\begin{aligned} V &= -\overline{\varphi} e de + \varphi(1-e)de \\ &= \begin{bmatrix} 0 & \overline{\varphi} M^* \\ \varphi M & 0 \end{bmatrix}. \end{aligned}$$

Its curvature is the 2-form

$$\theta = dV + V \wedge V$$

and an easy computation gives

$$\theta = -\left(\varphi + \overline{\varphi} + \varphi\overline{\varphi}\right) \begin{bmatrix} -M^*M & 0 \\ 0 & -MM^* \end{bmatrix}.$$

The Yang-Mills *action* is the integral of the curvature 2-form, that is the *trace* of θ:

$$YM(V) = Trace\left(\theta^2\right) .$$

But as $\varphi + \overline{\varphi} + \varphi\overline{\varphi} = |\varphi + 1|^2 - 1$ and

$$Trace\left(\begin{bmatrix} -M^*M & 0 \\ 0 & -MM^* \end{bmatrix}^2\right) = 2Trace\left((M^*M)^2\right)$$

we get

$$YM(V) = 2\left(|\varphi + 1|^2 - 1\right)^2 Trace\left((M^*M)^2\right).$$

8.3 Higgs Mechanism

This Yang-Mills action manifests *a pure Higgs phenomenon of symmetry breaking*. The minimum of $YM(V)$ is reached everywhere on the circle $|\varphi + 1|^2 = 1$ (degeneracy) and the gauge group $\mathcal{U} = U(1) \times U(1)$ of the unitary elements of $\mathcal{A}$ acts on it by

$$V \to uVu^* + udu^*$$

where $u = \begin{bmatrix} u_1 & 0 \\ 0 & u_2 \end{bmatrix}$ with $u_1, u_2 \in U(1)$.

The field φ is a Higgs bosonic field corresponding to a gauge connection on a NC space of two points. If $\psi \in \mathcal{H}$ represents a fermionic state, the fermionic action is $I_D(V, \psi) = \langle \psi, (D+V)\psi \rangle$ with

$$D + V = \begin{bmatrix} 0 & (1+\overline{\varphi})M^* \\ (1+\varphi)M & 0 \end{bmatrix}.$$

The complete action coupling the fermion ψ with the Higgs boson φ is therefore

$$YM(V) + I_D(V, \psi).$$

9 The NC Derivation of the Glashow-Weinberg-Salam Standard Model (Connes-Lott)

A remarkable achievement of this NC approach of Yang-Mills theories is given by Connes-Lott's NC derivation of the Glashow-Weinberg-Salam Standard Model. This derivation was possible because, as was emphasized by Martin et al. (1997, p. 5), it ties

> the properties of continuous spacetime with the intrinsic discreteness stemming from the chiral structure of the Standard Model.

9.1 Gauge Theory and NCG

It is easy to reinterpret in the NC framework classical gauge theories where M is a spin manifold, $\mathcal{A} = \mathcal{C}^\infty(M)$, D is the Dirac operator and $\mathcal{H} = L^2(M,S)$ is the space of L^2 sections of the spinor bundle S. $Diff(M) = Aut(\mathcal{A}) = Aut(\mathcal{C}^\infty(M))$ is the relativity group (the gauge group) of the theory: a diffeomorphism $\varphi \in Diff(M)$ is identified with the $*$-automorphism $\alpha \in Aut(\mathcal{A})$ s.t. $\alpha(f)(x) = f\left(\varphi^{-1}(x)\right)$. The main problem is to reconcile QFT with GR, that is non abelian gauge theories which are non commutative at the level of their *internal* space of quantum variables with the geometry of the *external* space–time M with its group of diffeomorphism $Diff(M)$. The NC solution is an extraordinary principled one since it links the standard "inner" non commutativity of quantum internal degrees of freedom with the new "outer" non commutativity of the external space.

9.1.1 Inner Automorphisms and Internal Symmetries

The key fact is that, in the NC framework, there exists in $Aut(\mathcal{A})$ the normal subgroup $Inn(\mathcal{A})$ of *inner automorphisms* acting by conjugation $a \rightarrow uau^{-1}$. $Inn(\mathcal{A})$ is trivial in the commutative case and constitutes one of the main feature of the NC case. As Alain Connes (1996) emphasized:

> Amazingly, in this description the group of gauge transformation of the matter fields arises spontaneously as a normal subgroup of the generalized diffeomorphism group $Aut(\mathcal{A})$. It is the *non commutativity* of the algebra $\mathcal{A}$ which gives for free the group of gauge transformations of matter fields as a (normal) subgroup of the group of diffeomorphisms.

In $Inn(\mathcal{A})$ there exists in particular the *unitary* group $\mathcal{U}(\mathcal{A})$ of unitary elements $u^* = u^{-1}$ acting by $\alpha_u(a) = uau^*$.

9.1.2 Connections and Vector Potentials

In the NC framework we can easily reformulate standard Yang-Mills theories. For that we need the concepts of a connection and of a vector potential.

Let $\mathcal{E}$ be a finite projective (right) $\mathcal{A}$-module. A connection ∇ on $\mathcal{E}$ is a collection of morphisms (for every p)

$$\nabla : \mathcal{E} \otimes_{\mathcal{A}} \Omega^{p}(\mathcal{A}) \to \mathcal{E} \otimes_{\mathcal{A}} \Omega^{p+1}(\mathcal{A})$$

satisfying for every $\omega \in \mathcal{E} \otimes_{\mathcal{A}} \Omega^{p}(\mathcal{A})$ and every $\rho \in \Omega^{q}(\mathcal{A})$ the Leibniz rule in $\mathcal{E} \otimes_{\mathcal{A}} \Omega^{p+q+1}(\mathcal{A})$

$$\nabla(\omega \otimes \rho) = \nabla(\omega) \otimes \rho + (-1)^{p} \omega \otimes d\rho$$

where we use $\Omega^{p+1}(\mathcal{A}) \otimes_{\mathcal{A}} \Omega^{q}(\mathcal{A}) = \Omega^{p}(\mathcal{A}) \otimes_{\mathcal{A}} \Omega^{q+1}(\mathcal{A})$.

∇ is determined by its restriction to $\Omega^{1}(\mathcal{A})$

$$\nabla : \mathcal{E} \otimes_{\mathcal{A}} \Omega^{0}(\mathcal{A}) = \mathcal{E} \to \mathcal{E} \otimes_{\mathcal{A}} \Omega^{1}(\mathcal{A})$$

satisfying $\nabla(\xi a) = \nabla(\xi) a + \xi \otimes da$ for $\xi \in \mathcal{E}$ and $a \in \mathcal{A}$.

The curvature θ of ∇ is given by $\nabla^{2} : \mathcal{E} \to \mathcal{E} \otimes_{\mathcal{A}} \Omega^{2}(\mathcal{A})$. As

$$\begin{aligned} \nabla^{2}(\xi a) &= \nabla(\nabla(\xi) a + \xi \otimes da) \\ &= \nabla^{2}(\xi) a - \nabla(\xi) \otimes da + \nabla(\xi) \otimes da + \xi \otimes d^{2}a \\ &= \nabla^{2}(\xi) a\,, \end{aligned}$$

∇^{2} is $\mathcal{A}$-linear. And as $\mathcal{E}$ is a projective $\mathcal{A}$-module,

$$\theta = \nabla^{2} \in End_{\mathcal{A}}\mathcal{E} \otimes_{\mathcal{A}} \Omega^{2}(\mathcal{A}) = M(\mathcal{A}) \otimes_{\mathcal{A}} \Omega^{2}(\mathcal{A})$$

is a matrix with elements in $\Omega^{2}(\mathcal{A})$.

Now, ∇ defines a connection $[\nabla, \bullet]$ on $End_{\mathcal{A}}\mathcal{E}$ by

$$\begin{aligned} [\nabla, \bullet] : End_{\mathcal{A}}\mathcal{E} \otimes_{\mathcal{A}} \Omega^{p}(\mathcal{A}) &\to End_{\mathcal{A}}\mathcal{E} \otimes_{\mathcal{A}} \Omega^{p+1}(\mathcal{A}) \\ \alpha &\mapsto [\nabla, \alpha] = \nabla \circ \alpha - \alpha \circ \nabla \end{aligned}$$

and the curvature θ satisfies the *Bianchi identity* $[\nabla, \theta] = 0$.

A vector potential A is a self-adjoint operator interpreting a 1-form

$$A = \sum_{j} a_{j}[D, b_{j}]$$

and the force is the curvature 2-form

$$\theta = dA + A^{2}.$$

The unitary group $\mathcal{U}(\mathcal{A})$ acts by gauge transformations on A and its 2-form curvature θ

$$\begin{aligned} A &\to uAu^* + udu^* = uAu^* + u[D,u^*] \\ \theta &\to u\theta u^*. \end{aligned}$$

9.2 Axioms for Geometry

There are characteristic properties of classical (commutative) and NC geometries which can be used to axiomatize them.

1. (Classical and NC geometry). $ds = D^{-1}$ is an infinitesimal of order $\frac{1}{n}$ (n is the dimension)[13] and for any $a \in \mathcal{A}$ integration is given by $Tr_{Dix}\left(a\,|D|^{-n}\right)$ (which is well defined and $\neq 0$ since $|D|^{-n}$ is an infinitesimal of order 1). One can normalize the integral dividing by $V = Tr_{Dix}\left(|D|^{-n}\right)$.
2. (Classical geometry). Universal commutation relations: $[[D,a],b] = 0$, $\forall a,b \in \mathcal{A}$. So (Jones and Moscovici, 1997)

   > while *ds* no longer commutes with the coordinates, the algebra they generate does satisfy non trivial commutation relations.

3. (Classical and NC geometry). $a \in \mathcal{A}$ is "smooth" in the sense that a and $[D,a]$ belong to the intersection of the domains of the functionals δ^m where $\delta(T) = [|D|, T]$ for every operator T on $\mathcal{H}$.
4. (Classical geometry). If the dimension n is *even* there exists a $\widetilde{\gamma}$ interpreting a n-form $c \in Z_n(\mathcal{A},\mathcal{A})$ associated to orientation and chirality (the γ^5 of Dirac), $\widetilde{\gamma}$ being of the form $a_0[D,a_1]\dots[D,a_n]$ and s.t. $\widetilde{\gamma} = \widetilde{\gamma}^*$ (self-adjointness), $\widetilde{\gamma}^2 = 1$, $\{\widetilde{\gamma}, D\} = 0$ (anti-commutation relation) and $[\widetilde{\gamma}, a] = 0$, $\forall a \in \mathcal{A}$ (commutation relations). $\widetilde{\gamma}$ decomposes D into two parts $D = D^+ + D^-$ where $D^+ = (1-p)Dp$ with $p = \frac{1+\widetilde{\gamma}}{2}$. If e is a self-adjoint ($e = e^*$) idempotent ($e^2 = e$) of $\mathcal{A}$ (i.e. a projector), eD^+e is a Fredholm operator from the subspace $ep\mathcal{H}$ to the subspace $e(1-p)\mathcal{H}$. This can be extended to the projectors of $e \in M_q(\mathcal{A})$ defining finite projective left $\mathcal{A}$-modules $\mathcal{E} = \mathcal{A}^N e$ (if $\xi \in \mathcal{E}$ then $\xi e = \xi$) with the $\mathcal{A}$-valued inner product $(\xi,\eta) = \sum_{i=1}^{i=N} \xi_i \eta_i^*$. If n is odd we ask only that there exists such an n-form c interpreted by 1: $a_0[D,a_1]\dots[D,a_n] = 1$.
5. (Classical and NC geometry). $\mathcal{H}_\infty = \bigcap_m Domain(D^m)$ is finite and projective as $\mathcal{A}$ -module and $\langle a\xi,\eta\rangle = Tr_{Dix} a(\xi,\eta)\,ds^n$ ((ξ,η) being the scalar product of

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[13] In the NC framework, *ds* and *dx* are completely different sort of entities. *dx* is the differential of a coordinate and *ds* doesn't commute with it. In the classical case, the order of *ds* as an infinitesimal is not 1 but the dimension of M. As we will see later, the Hilbert-Einstein action is the NC integral of ds^{n-2}.

$\mathcal{H}$ and Tr_{Dix} the Dixmier trace of infinitesimals of order 1) define an Hermitian structure on $\mathcal{H}_\infty$.

6. (Classical geometry). One can define an *index pairing* of D with $K_0(\mathcal{A})$ and an *intersection form* on $K_0(\mathcal{A})$[14]. If $[\mathcal{E}] \in K_0(\mathcal{A})$ is defined by the projector e, we consider the scalar product $\langle IndD, e\rangle$ which is an integer. We define therefore $\langle IndD, e\rangle : K_0(\mathcal{A}) \to \mathbb{Z}$. As $\mathcal{A}$ is commutative, we can take the multiplication $m : \mathcal{A} \otimes \mathcal{A} \to \mathcal{A}$ given by $m(a \otimes b) = ab$ which induces $m_0 : K_0(\mathcal{A}) \otimes K_0(\mathcal{A}) \to K_0(\mathcal{A})$. Composing with $IndD$ we get the intersection form
$$\begin{aligned} \langle IndD, m_0\rangle \ &: \ K_0(\mathcal{A}) \otimes K_0(\mathcal{A}) \to \mathbb{Z} \\ &(e,a) \to \langle IndD, m_0(e \otimes a)\rangle \,. \end{aligned}$$
Poincaré duality: the intersection form is invertible.
7. *Real structure* (Classical geometry). There exists an anti-linear isometry (charge conjugation) $J : \mathcal{H} \to \mathcal{H}$ which combines charge conjugation and complex conjugation and gives the $*$-involution by algebraic conjugation: $JaJ^{-1} = a^* \ \forall a \in \mathcal{A}$, and s.t. $J^2 = \varepsilon$, $JD = \varepsilon' DJ$, and $J\gamma = \varepsilon'' \gamma J$ with ε, ε', $\varepsilon'' = \pm 1$ depending of the dimension $n \; mod 8$:

n	0	1	2	3	4	5	6	7
ε	1	1	-1	-1	-1	-1	1	1
ε'	1	-1	1	1	1	-1	1	1
ε''	1		-1		1		-1	

In the classical case (M smooth compact manifold of dimension n), Connes proved that these axioms define a unique Riemannian spin geometry whose geodesic distance and the spin structure are those defined by D. Moreover, the value of the Dixmier trace $Tr_{Dix} ds^{n-2}$ is the *Einstein-Hilbert action functional*:

$$Tr_{Dix} ds^{n-2} = c_n \int_M R\sqrt{g}\, d^n x = c_n \int_M R dv$$

with dv the volume form $dv = \sqrt{g}\, d^n x$ and $c_n = \frac{n-2}{12}(4\pi)^{-\frac{n}{2}}\Gamma\left(\frac{n}{2}+1\right)^{-1} 2^{\left[\frac{n}{2}\right]}$. $Tr_{Dix} ds^{n-2}$ is well defined and $\neq 0$ since ds is an infinitesimal of order $\frac{n-2}{n} < 1$). For $n = 4$, $c_4 = \frac{1}{6}(4\pi)^{-2}\Gamma(3)^{-1} 2^2 = \frac{1}{48\pi^2}$.

In the NC case the characteristic properties (2), (6), (7) must be modified to take into account the NC:

7^{NC}. *Real structure* (NC geometry). In the noncommutative case, the axiom $JaJ^{-1} = a^*$ is transformed into the following axiom saying that the conjugation by J of the involution defines the *opposed* multiplication of $\mathcal{A}$. Let $b^0 = Jb^* J^{-1}$, then $\left[a, b^0\right] = 0$, $\forall a, b \in \mathcal{A}$. By means of this real structure, the Hilbert space $\mathcal{H}$

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[14] Remember that $K_0(\mathcal{A}) = \pi_1(GL_\infty(\mathcal{A}))$ classifies the finite projective $\mathcal{A}$-modules and that $K_1(\mathcal{A}) = \pi_0(GL_\infty(\mathcal{A}))$ is the group of connected components of $GL_\infty(\mathcal{A})$.

becomes not only a (left) $\mathcal{A}$-module through the representation of $\mathcal{A}$ into $\mathcal{L}(\mathcal{H})$ but also a $\mathcal{A}\otimes\mathcal{A}^{\circ}$-module (where $\mathcal{A}^{\circ}$ is the opposed algebra of $\mathcal{A}$) or a (left-right) $\mathcal{A}$-bimodule through $\left(a\otimes b^{0}\right)\xi = aJb^{*}J^{-1}\xi$ or $a\xi b = aJb^{*}J^{-1}\xi$ for every $\xi\in\mathcal{H}$.

2^{NC}. The universal commutation relations $[[D,f],g]=0, \forall f,g\in\mathcal{A}$ become in the NC case $[[D,a],b^{\circ}]=0, \forall a,b\in\mathcal{A}$ (which is equivalent to $[[D,b^{\circ}],a]=0, \forall a,b\in\mathcal{A}$ since a and b° commute by 7^{NC}).

6^{NC}. K-theory can be easily generalized to the NC case. We consider finite projective $\mathcal{A}$-modules $\mathcal{E}$, that is direct factors of free $\mathcal{A}$-modules $\mathcal{A}^{N}$. They are characterized by a projection $\pi:\mathcal{A}^{N}\rightarrow\mathcal{E}$ admitting a section $s:\mathcal{E}\rightarrow\mathcal{A}^{N}$ $(\pi\circ s = Id_{\mathcal{E}})$. $K_0(\mathcal{A})$ classifies them. The structure of $\mathcal{A}\otimes\mathcal{A}^{\circ}$-module induced by the real structure J allows to define the intersection form by $(e,a)\rightarrow\langle IndD, e\otimes a^{\circ}\rangle$ with $e\otimes a^{\circ}$ considered as an element of $K_0(\mathcal{A}\otimes\mathcal{A}^{\circ})$.

One of the fundamental aspects of the NC case is that inner automorphisms $\alpha_u(a) = uau^{*}$, $u\in\mathcal{U}(\mathcal{A})$ act upon the Dirac operator D via NC gauge connections (vector potentials) A

$$\begin{aligned}&\widetilde{D} = D + A + JAJ^{-1} \text{ with}\\&A = u[D,u^{*}]\ .\end{aligned}$$

the equivalence between D and $\widetilde{D}$ being given by $\widetilde{D} = UDU^{-1}$ with $U = uJuJ^{-1} = u(u^{*})^{\circ}$.

9.3 The Crucial Discovery of a Structural Link Between "External" Metric and "Internal" Gauge Transformations

One can generalize these transformations of metrics to gauge connections A of the form $A=\sum a_i[D,b_i]$ which can be interpreted as *internal perturbations of the metric* or as *internal fluctuations of the spectral geometry* induced by the internal degrees of freedom of gauge transformations. This coupling between metric and gauge transformations is what is needed *for coupling gravity with QFT*. In the commutative case, this coupling *vanishes* since $U = uu^{*} = 1$ and therefore $\widetilde{D} = D$. The vanishing $A + JAJ^{-1} = 0$ comes from the fact that A is self-adjoint and that, due to its special form $A = a[D,b]$, we have $JAJ^{-1} = -A^{*}$. Indeed, since $[D,b^{*}] = -[D,b]^{*}$

$$\begin{aligned}JAJ^{-1} &= Ja[D,b]J^{-1} = JaJ^{-1}J[D,b]J^{-1} = a^{*}[D,b^{*}]\\&= -a^{*}[D,b]^{*} = -(a[D,b])^{*} = -A^{*}\end{aligned}$$

So the coupling between the "external" metric afforded by the Dirac operator and the internal quantum degrees of freedom is a purely NC effect which constitutes a breakthrough for the unification of GR and QFT in a "good" theory of Quantum Gravity (QG).

9.4 Generating the Standard Model (Connes-Lott)

Before concluding this paper with some remarks on QG, let us recall that the first main interest of NC geometry in physics was generated by the possibility of coupling classical gauge theories with purely NC such theories. This led to the NC interpretation of Higgs fields. Connes' main result is:

Connes' theorem. The Glashow-Weinberg-Salam Standard Model (SM) can be entirely reconstructed from the NC C^*-algebra

$$\mathcal{A} = \mathcal{C}^\infty(M) \otimes (\mathbb{C} \oplus \mathbb{H} \oplus M^3(\mathbb{C}))$$

where the "internal" algebra $\mathbb{C} \oplus \mathbb{H} \oplus M^3(\mathbb{C})$ has for unitary group the symmetry group

$$U(1) \times SU(2) \times SU(3).$$

The first step is to construct the toy model which is the product $\mathcal{C}^\infty(M) \otimes (\mathbb{C} \oplus \mathbb{C})$ of the classical Dirac fermionic model $(\mathcal{A}_1, \mathcal{H}_1, D_1, \gamma_5)$ and the previously explained, purely NC, 2-points model $(\mathcal{A}_2, \mathcal{H}_2, D_2, \gamma)$ with $D_2 = \begin{bmatrix} 0 & M^* \\ M & 0 \end{bmatrix}$:

$$\begin{cases} \mathcal{A} = \mathcal{A}_1 \otimes \mathcal{A}_2 \\ \mathcal{H} = \mathcal{H}_1 \oplus \mathcal{H}_2 \\ D = D_1 \otimes 1 + \gamma_5 \otimes D_2 \ . \end{cases}$$

The second step is to complexify the model and to show that it enables to derive the complete GWS Lagrangian.

The key idea is to take the product of a 4-dimensional spin manifold M with a finite NC geometry $(\mathcal{A}_F, \mathcal{H}_F, D_F)$ of dimension 0 where $\mathcal{H}_F$ is the Hilbert space with basis the generations of fermions: quarks, leptons. The particule/antiparticule duality decomposes $\mathcal{H}_F$ into $\mathcal{H}_F = \mathcal{H}_F^+ \oplus \mathcal{H}_F^-$, each $\mathcal{H}_F^\pm$ decomposes into $\mathcal{H}_F^\pm = \mathcal{H}_l^\pm \oplus \mathcal{H}_q^\pm$ ($l =$ lepton and $q =$ quark), and chirality decomposes the $\mathcal{H}_p^\pm$ ($p =$ particule) into $\mathcal{H}_{pL}^\pm \oplus \mathcal{H}_{pR}^\pm$ ($L =$ left, $R =$ right). The four quarks are u_L, u_R, d_L, d_R ($u =$ up, $d =$ down) with three colours (12 quarks for each generation) and the three leptons are e_L, ν_L, e_R, the total being of $2(12+3) = 30$ fermions for each generation.

The real structure J is given for $\mathcal{H}_F = \mathcal{H}_F^+ \oplus \mathcal{H}_F^-$ by $J\begin{pmatrix} \xi \\ \overline{\eta} \end{pmatrix} = \begin{pmatrix} \eta \\ \overline{\xi} \end{pmatrix}$ that is, if $\xi = \sum_i \lambda_i p_i$ and $\overline{\eta} = \sum_j \mu_j \overline{p_j}$,

$$J\left(\sum_i \lambda_i p_i + \sum_j \mu_j \overline{p_j}\right) = \left(\sum_j \overline{\mu_j} p_j + \sum_i \overline{\lambda_i} \overline{p_i}\right).$$

The action of the internal algebra $\mathcal{A}_F = \mathbb{C} \oplus \mathbb{H} \oplus M^3(\mathbb{C})$ is defined in the following way. Let $a = (\lambda, q, m) \in \mathcal{A}_F$, $\lambda \in \mathbb{C}$ being a complex scalar acting upon $\mathbb{C}^2$ as the diagonal quaternion $\begin{pmatrix} \lambda & 0 \\ 0 & \overline{\lambda} \end{pmatrix}$, $q = \alpha + \beta j \in \mathbb{H}$ being a quaternion written as $\begin{pmatrix} \alpha & \beta \\ -\overline{\beta} & \overline{\alpha} \end{pmatrix}$, $j = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, and $m \in M^3(\mathbb{C})$ being a 3×3 complex matrix. The element $a = (\lambda, q, m)$ acts on quarks, independently of color, via $au_R = \lambda u_R$, $au_L = \alpha u_L - \overline{\beta} d_L$, $ad_R = \overline{\lambda} d_R$, $ad_L = \beta u_L + \overline{\alpha} d_L$, that is as

$$(\lambda, q, m) \begin{pmatrix} u_L \\ d_L \\ u_R \\ d_R \end{pmatrix} = \begin{pmatrix} \alpha & -\overline{\beta} & 0 & 0 \\ \beta & \overline{\alpha} & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \overline{\lambda} \end{pmatrix} \begin{pmatrix} u_L \\ d_L \\ u_R \\ d_R \end{pmatrix} = \begin{pmatrix} \alpha u_L - \overline{\beta} d_L \\ \beta u_L + \overline{\alpha} d_L \\ \lambda u_R \\ \overline{\lambda} d_R \end{pmatrix}$$

(the pair (u_R, d_R) can be considered as an element of $\mathbb{C} \oplus \mathbb{C}$, while (u_L, d_L) can be considered as an element of $\mathbb{C}^2$). It acts on leptons via $ae_R = \overline{\lambda} e_R$, $ae_L = \beta \nu_L + \overline{\alpha} e_L$, $a\nu_L = \alpha \nu_L - \overline{\beta} e_L$, that is as

$$(\lambda, q, m) \begin{pmatrix} e_R \\ \nu_L \\ e_L \end{pmatrix} = \begin{pmatrix} \overline{\lambda} & 0 & 0 \\ 0 & \alpha & -\overline{\beta} \\ 0 & \beta & \overline{\alpha} \end{pmatrix} \begin{pmatrix} e_R \\ \nu_L \\ e_L \end{pmatrix} = \begin{pmatrix} \overline{\lambda} e_R \\ \alpha \nu_L - \overline{\beta} e_L \\ \beta \nu_L + \overline{\alpha} e_L \end{pmatrix}.$$

It acts on anti-particules via $a\overline{l} = \lambda \overline{l}$ for antileptons and via $a\overline{q} = m\overline{q}$ for antiquarks where m acts upon color.

The internal Dirac operator D_F is given by the matrix of Yukawa coupling $D_F = \begin{pmatrix} Y & 0 \\ 0 & \overline{Y} \end{pmatrix}$ where $Y = (Y_q \otimes 1_3) \oplus Y_l$ (the $\otimes 1_3$ comes from the three generations of fermions) with

$$Y_q = \begin{array}{c} \\ u_L \\ d_L \\ u_R \\ d_R \end{array} \begin{array}{cccc} u_L & d_L & u_R & d_R \\ \end{array} \hspace{-14em} \begin{array}{c} \\ \left(\begin{array}{cccc} 0 & 0 & M_u & 0 \\ 0 & 0 & 0 & M_d \\ M_u^* & 0 & 0 & 0 \\ 0 & M_d^* & 0 & 0 \end{array} \right) \end{array}$$

and

$$Y_l = \begin{array}{c|ccc} & e_R & \nu_L & e_L \\ \hline e_R & 0 & 0 & M_l \\ \nu_L & 0 & 0 & 0 \\ e_L & M_l^* & 0 & 0 \end{array}$$

where (Connes, 1996) M_u, M_d, and M_l are matrices “which encode both the masses of the Fermions and their mixing properties”.

Chirality is given by $\gamma_F(p_R) = p_R$ and $\gamma_F(p_L) = -p_L$ (p being any particule or anti-particule).

Connes and Lott then take the product of this internal model of the fermionic sector with a classical gauge model for the bosonic sector:

$$\begin{cases} \mathcal{A} = C^\infty(M) \otimes \mathcal{A}_F = (C^\infty(M) \otimes \mathbb{C}) \oplus (C^\infty(M) \otimes \mathbb{H}) \oplus \left(C^\infty(M) \otimes M^3(\mathbb{C})\right) \\ \mathcal{H} = L^2(M,S) \otimes \mathcal{H}_F = L^2(M, S \otimes \mathcal{H}_F) \\ D = (D_M \otimes 1) \oplus (\gamma_5 \otimes D_F). \end{cases}$$

The extraordinary "tour de force" is that this model, which is rather simple at the conceptual level (a product of two models, respectively fermionic and bosonic, which takes into account only the known fundamental properties of these two sectors), is in fact extremely complex and generates SM in a *principled* way. Computations are very intricate (see Kastler papers in the bibliography). One has to compute first vector potentials of the form $A = \sum_i a_i [D, a'_i]$, $a_i, a'_i \in \mathcal{A}$ which induce fluctuations of the metric. As D is a sum of two terms, it is also the case for A. Its discrete part comes from $\gamma_5 \otimes D_F$ and generates the Higgs bosons. Let $a_i(x) = (\lambda_i(x), q_i(x), m_i(x))$. The term $\sum_i a_i [\gamma_5 \otimes D_F, a'_i]$ yields γ_5 tensored by matrices of the form:

- Quark sector:

$$\begin{pmatrix} 0 & 0 & M_u \varphi_1 & M_u \varphi_2 \\ 0 & 0 & -M_d \overline{\varphi_2} & M_d \overline{\varphi_1} \\ M_u^* \varphi'_1 & M_d^* \varphi'_2 & 0 & 0 \\ -M_u^* \overline{\varphi'_2} & M_d^* \overline{\varphi'_1} & 0 & 0 \end{pmatrix}$$

with

$$\begin{cases} \varphi_1 = \sum_i \lambda_i (\alpha'_i - \lambda'_i) \\ \varphi_2 = \sum_i \lambda_i \beta'_i \\ \varphi'_1 = \sum_i \alpha_i (\lambda'_i - \alpha'_i) + \beta_i \overline{\beta'_i} \\ \varphi'_2 = \sum_i \beta_i \left(\overline{\lambda'_i} - \overline{\alpha'_i}\right) - \alpha_i \beta'_i . \end{cases}$$

- Lepton sector:

$$\begin{pmatrix} 0 & -M_d \overline{\varphi_2} & M_d \overline{\varphi_1} \\ M_d^* \varphi'_2 & 0 & 0 \\ M_d^* \overline{\varphi'_1} & 0 & 0 \end{pmatrix}.$$

Let $q = \varphi_1 + \varphi_2 j$ and $q' = \varphi'_1 + \varphi'_2 j$ be the quaternionic fields so defined. As $A = A^*$, we have $q' = q^*$. The $\mathbb{H}$-valued field $q(x)$ is the *Higgs doublet*.

The second part of the vector potential A comes from $D_M \otimes 1$ and generates the gauge bosons. The terms $\sum_i a_i [D_M \otimes 1, a'_i]$ yield

- The $U(1)$ gauge field $\Lambda = \sum_i \lambda_i d\lambda'_i$.
- The $SU(2)$ gauge field $Q = \sum_i q_i dq'_i$.
- The $U(3)$ gauge field $V = \sum_i m_i dm'_i$.

The computation of the fluctuations of the metric $A+JAJ^{-1}$ gives:

- Quark sector:

$$\begin{array}{c} \\ u_L \\ d_L \\ u_R \\ d_R \end{array}\begin{array}{c} \begin{array}{cccc} u_L \qquad & \qquad d_L \qquad & \qquad u_R \qquad & \qquad d_R \end{array} \\ \begin{pmatrix} Q_{11}1_3+V & Q_{12}1_3 & 0 & 0 \\ Q_{21}1_3 & Q_{22}1_3+V & 0 & 0 \\ 0 & 0 & \Lambda 1_3+V & 0 \\ 0 & 0 & 0 & -\Lambda 1_3+V \end{pmatrix} \end{array}$$

which is a 12 × 12 matrix since V is 3×3.

- Lepton sector:

$$\begin{array}{c} \\ e_R \\ \nu_L \\ e_L \end{array}\begin{array}{c} \begin{array}{ccc} e_R \qquad & \qquad \nu_L \qquad & \qquad e_L \end{array} \\ \begin{pmatrix} -2\Lambda & 0 & 0 \\ 0 & Q_{11}-\Lambda & Q_{12} \\ 0 & Q_{21} & Q_{22}-\Lambda \end{pmatrix} \end{array}$$

One can suppose moreover that $TraceV=\Lambda$, that is $V=V'+\frac{1}{3}\Lambda$ with V' traceless, which gives the correct hypercharges.

The crowning of the computation is that the total (bosonic + fermionic) action

$$Tr_{Dix}\theta^2 ds^4+\left\langle\left(D+A+JAJ^{-1}\right)\psi,\psi\right\rangle = YM(A)+\langle D_A\psi,\psi\rangle$$

(where $\theta=dA+A^2$ is the curvature of the connection A) enables to derive *the complete GWS Lagrangian*

$$\mathcal{L}=\mathcal{L}_G+\mathcal{L}_f+\mathcal{L}_\varphi+\mathcal{L}_Y+\mathcal{L}_V\ .$$

1. $\mathcal{L}_G$ is the Lagrangian of the gauge bosons

$$\begin{aligned} \mathcal{L}_G &= \frac{1}{4}\left(G_{\mu\nu a}G_a^{\mu\nu}\right)+\frac{1}{4}\left(F_{\mu\nu}F^{\mu\nu}\right) \\ G_{\mu\nu a} &= \partial_\mu W_{\nu a}-\partial_\nu W_{\mu a}+g\varepsilon_{abc}W_{\mu b}W_{\nu c}, \\ &\quad \text{with } W_{\mu a} \text{ a } SU(2) \text{ gauge field (weakisospin)} \\ F_{\mu\nu} &= \partial_\mu B_\nu-\partial_\nu B_\mu,\ \text{with } B_\mu \text{ a } SU(1) \text{ gauge field.} \end{aligned}$$

2. $\mathcal{L}_f$ is the fermionic kinetic term

$$\begin{aligned} \mathcal{L}_f = -\sum \overline{f_L}\gamma^\mu\left(\partial_\mu+ig\frac{\tau_a}{2}W_{\mu a}+ig'\frac{Y_L}{2}B_\mu\right)f_L + \\ \overline{f_R}\gamma^\mu\left(\partial_\mu+ig'\frac{Y_R}{2}B_\mu\right)f_R \end{aligned}$$

with $f_L=\begin{bmatrix}\nu_L \\ e_L\end{bmatrix}$ left fermion fields of hypercharge $Y_L=-1$ and $f_R=(e_R)$ right fermion fields of hypercharge $Y_R=-2$.

3. $\mathcal{L}_\varphi$ is the Higgs kinetic term

$$\mathcal{L}_\varphi = -\left|\left(\partial_\mu + ig\frac{\tau_a}{2}W_{\mu a} + i\frac{g'}{2}B_\mu\right)\varphi\right|^2$$

with $\varphi = \begin{bmatrix}\varphi_1\\ \varphi_2\end{bmatrix}$ a $SU(2)$ pair of scalar complex fields of hypercharge $Y_\varphi = 1$.

4. $\mathcal{L}_Y$ is a Yukawa coupling between the Higgs fields and the fermions

$$\mathcal{L}_Y = -\sum\left(H_{ff'}\left(\overline{f_L}.\varphi\right)f'_R + H^*_{ff'}\overline{f'_R}\left(\varphi^+.f_L\right)\right)$$

where $H_{ff'}$ is a coupling matrix.

5. $\mathcal{L}_V$ is the Lagrangian of the self-interaction of the Higgs fields

$$\mathcal{L}_V = \mu^2\left(\varphi^+\varphi\right) - \frac{1}{2}\lambda\left(\varphi^+\varphi\right)^2 \ with\ \lambda > 0.$$

10 Quantum Gravity, Fluctuating Background Geometry, and Spectral Invariance (Connes-Chamseddine)

10.1 Quantum Field Theory and General Relativity

As we have already emphasized, Alain Connes realized a new breakthrough in Quantum Gravity by coupling such models with General Relativity. In NCG, QG can be thought of in a principled way because it becomes possible to introduce in the model of QFT the gravitational Einstein-Hilbert action as a direct consequence of the specific invariance of spectral geometry, namely *spectral invariance*. As Alain Connes (1996) explains:

> However this [the previous NC deduction of the SM] requires the definition of the curvature and is still in the spirit of gauge theories. (...) One should consider the internal gauge symmetries as part of the diffeomorphism group of the non commutative geometry, and the gauge bosons as the internal fluctuations of the metric. It follows then that the action functional should be of a purely gravitational nature. We state the principle of spectral invariance, stronger than the invariance under diffeomorphisms, which requires that the action functional only depends on the spectral properties of $D = ds^{-1}$ in $\mathcal{H}$.

The general strategy for coupling a Yang-Mills-Higgs gauge theory with the Einstein-Hilbert action is to find a C^*-algebra $\mathcal{A}$ s.t. the normal subgroup $Inn(\mathcal{A})$ of inner automorphisms is the gauge group and the quotient group $Out(\mathcal{A}) = Aut(\mathcal{A})/Inn(\mathcal{A})$ of "external" automorphisms plays the role of *Diff*(M) in a gravitational theory. Indeed, in the classical setting we have principal bundles $P \longrightarrow M$ with a structural group G acting upon the fibers and an exact sequence

$$Id \to \mathcal{G} \to Aut(P) \to Diff(M) \to Id$$

where $\mathcal{G} = C^{\infty}(M, G)$ is the gauge group. The non abelian character of these gauge theories comes solely from the non commutativity of the group of *internal* symmetries G. The total symmetry group $Aut(P)$ of the theory is the *semidirect* product $\mathfrak{G}$ of $Diff(M)$ and $\mathcal{G} = C^{\infty}(M, G)$. If we want to geometrize the theory completely, we would have to find a generalized space X s.t. $Aut(X) = \mathfrak{G}$.

> If such a space would exist, then we would have some chance to actually geometrize completely the theory, namely to be able to say that it's pure gravity on the space X. (Connes, 2000)

But this is impossible if X is a manifold since a theorem of John Mather proves that in that case the group $Diff(X)$ would be simple (without normal subgroup) and could'nt therefore be a semidirect product. *But it is possible with a NC space* $(\mathcal{A}, \mathcal{H}, D)$. For then (Iochum et al., 1996)

> the metric 'fluctuates', that is, it picks up additional degrees of freedom from the internal space, the Yang-Mills connection and the Higgs scalar. (...) In physicist's language, the spectral triplet is the Dirac action of a multiplet of dynamical fermions in a background field. This background field is a fluctuating metric, consisting of so far adynamical bosons of spin 0,1 and 2.

If we find a NC geometry $\mathcal{A}$ with $Inn(\mathcal{A}) \simeq \mathcal{G}$, a correct spectral triple and apply the spectral action, then gravity will correspond to $Out(\mathcal{A}) = Aut(\mathcal{A})/Inn(\mathcal{A})$. As was emphasized by Martin et al. (1997):

> The strength of Connes' conception is that gauge theories are thereby deeply connected to the underlying geometry, on the same footing as gravity. The distinction between gravitational and gauge theories boils down to the difference between outer and inner automorphisms.

Jones and Moscovici (1997) add that this implies that

> Connes' spectral approach gains the ability to reach below the Planck scale and attempt to decipher the fine structure of space–time.

So, just as GR extends the Galilean or Minkowskian invariance into diffeomorphism invariance, NCG extends both diffeomorphism invariance and gauge invariance into a larger invariance, the spectral invariance.

The philosophically significant content of the NC point of view must be emphasized. We already saw that in GR the metric of M is no longer a background structure (but the differentiable structure of M remains a background) while in QFT the metric of M is still a background structure. In the NC framework the metric is no longer a background structure, as in GR, but in addition it is a quantum fluctuating structure.

10.2 The Spectral Action and the Eigenvalues of the Dirac Operator as Dynamical Variables for General Relativity

The key device is the bosonic spectral action

$$Trace\left(\phi\left(\frac{D^2}{\Lambda^2}\right)\right) = Trace\left(\phi\left(\frac{|D|}{\Lambda}\right)\right)$$

where Λ is a cut-off of the order of the inverse of Planck length and ϕ a smooth approximation of the characteristic function $\chi_{[0,1]}$ of the unit interval. $D^2 = (D_M \otimes 1 + \gamma_5 \otimes D_F)^2$ is computed using Lichnerowicz' formula $D^2 = \Delta^S + \frac{1}{4}R$. As this action counts the number $N(\Lambda)$ of eigenvalues of D in the interval $[-\Lambda, \Lambda]$, the key idea is, as formulated by Giovanni Landi and Carlo Rovelli (1997),

> to consider the eigenvalues of the Dirac operator as dynamical variables for general relativity.

This formulation highlights the physical and transcendental significance of the NC framework: *since the distance is defined through the Dirac operator D, the spectral properties of D can be used in order to modify the metric*. The eigenvalues are spectral invariants and are therefore, in the classical case, automatically $Diff(M)$ invariant.

> Thus the general idea is to describe spacetime geometry by giving the eigen-frequencies of the spinors that can live on that spacetime. [...] The Dirac operator D encodes the full information about the spacetime geometry in a way usable for describing gravitational dynamics. (Landi-Rovelli (1997): the quotation concerns D_M acting on the Hilbert space of spinor fields on M.)

This crucial point has also been well explained by Steven Carlip (2001, p. 47). As we have seen in the Introduction, in GR points of space–time lose any physical meaning so that GR obervables must be radically non-local. This is the case with the eingenvalues of D which

> provide a nice set of non local, diffeomorphism-invariant obervables.

They yield

> the first good candidates for a (nearly) complete set of diffeomorphism-invariant observables.

Let us look at $N(\Lambda)$ for $\Lambda \to \infty$. $N(\Lambda)$ is a step function which encodes a lot of information and can be written as a sum of a mean value and a fluctuation (oscillatory) term $N(\Lambda) = \langle N(\Lambda)\rangle + N_{osc}(\Lambda)$ where the oscillatory part $N_{osc}(\Lambda)$ is random. The mean part $\langle N(\Lambda)\rangle$ can be computed using a semi-classical approximation and a heat equation expansion. A wonderful computation shows that for $n = 4$ the asymptotic expansion of the spectral action is

$$Trace\left(\phi\left(\frac{D^2}{\Lambda^2}\right)\right)=\Lambda^4 f_0 a_0\left(D^2\right)+\Lambda^2 f_2 a_2\left(D^2\right)+f_4 a_4\left(D^2\right)+O\left(\Lambda^{-2}\right)$$

where

- $f_0=\int_{\mathbb{R}}\phi\left(u\right)u du$, $f_2=\int_{\mathbb{R}}\phi\left(u\right)du$, $f_4=\phi\left(0\right)$
- $a_j\left(D^2\right)=\int_M a_j\left(x,D^2\right)dv$ ($dv=\sqrt{g}d^4x$)
- $a_0\left(x,D^2\right)=\frac{1}{(4\pi)^2}Trace_x\left(1\right)$
- $a_2\left(x,D^2\right)=\frac{1}{(4\pi)^2}Trace_x\left(\frac{1}{6}s1-E\right)$
- $a_4\left(x,D^2\right)=\frac{1}{360(4\pi)^2}Trace_x\left(5s^2 1-2r^2 1+2R^2 1-60sE+180E^2+30R^{\nabla}_{\mu\nu}R^{\nabla\mu\nu}\right)$
- R is the curvature tensor of M and $R^2=R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}$
- r is the Ricci tensor of M and $r^2=r_{\mu\nu}r^{\mu\nu}$
- s is the scalar curvature of M
- E and $R^{\nabla}_{\mu\nu}$ come from Lichnerowicz' formula.

Let

$$\mathcal{E}=C^{\infty}(M,S\otimes\mathcal{H}_F)=C^{\infty}(M,S)\otimes_{C^{\infty}(M)}C^{\infty}(M,\mathcal{H}_F).$$

The connection on $\mathcal{E}$ is

$$\nabla=\nabla^S\otimes Id_{C^{\infty}(M,\mathcal{H}_F)}+Id_{C^{\infty}(M,S)}\otimes\nabla^F$$

and $R^{\nabla}_{\mu\nu}$ is the curvature 2-tensor of this total connection ∇. If $D=ic^{\mu}\nabla_{\mu}+\varphi$ with $c^{\mu}=\gamma^{\mu}\otimes Id_{C^{\infty}(M,\mathcal{H}_F)}$, then $D^2=\Delta+E$, with

$$\begin{cases}\Delta=-g^{\mu\nu}\left(\nabla_{\mu}\nabla_{\nu}-\Gamma^{\alpha}_{\mu\nu}\nabla_{\alpha}\right)\\ E=\frac{1}{4}s1-\frac{1}{2}c\left(R^F\right)+ic^{\mu}\left[\nabla_{\mu},\varphi\right]+\varphi^2\\ c\left(R^F\right)=-\gamma^{\mu}\gamma^{\nu}\otimes R^F_{\mu\nu}\ (R^F=\text{curvature of }\nabla^F).\end{cases}$$

The asymptotic expansion of the spectral action is dominated by the first two terms which identify with the Einstein-Hilbert action with a cosmological term. The later can be eliminated by a change of ϕ.

11 Conclusion

We have seen how NCG reformulated on a new basis the mathematical interpretation of the categories of physical objectivity. Let us summarize its main steps.

1. The primitive fact, namely how phenomena are given, is constituted by the NC C^*-algebra $\mathcal{A}$ of observables. What is physically observable and measurable are the *spectral* properties of the observables of $\mathcal{A}$ interpreted as operators on an Hilbert space $\mathcal{H}$. Spectral data are physically more primitive than geometrical ones and physical geometry must be reconstructed from the outset as a spectral geometry. Classical geometrical transcendental aesthetics determines the first transcendental

moment of "Phoronomy". As was already shown in Petitot (1991a), this was already converted into a spectral moment in Quantum Mechanics. Now in NCG this moment becomes a *geometrical-spectral* moment. We can speak of a "spectral phoronomy".

2. Differential calculus and infinitesimals, which determine the second transcendental moment, namely that of "Dynamics", are entirely interpreted anew from the formula $da = [D, a]$.

3. As in GR, metric is "promoted" from the "Phoronomy" moment (where it acts as a background structure) to the "Mechanics" moment (where it becomes a physical field) while, conversely, the "Mechanics" moment is "demoted" to the "Phoronomy" moment (forces are absorbed in a larger relativity principle). This transcendental chiasm provides the philosophical interpretation for the elimination of metric as background structure. In NCG this is expressed by the constitutive role of the Dirac operator D in the definition of metric. D is a physical operator and in that sense metric is "physicalized". But at the same time, differentials are defined by $da = [D, a]$ and in the classical case the eigenvalues of D are $\mathit{Diff}(M)$ invariant, that is, the metric still belongs to the moments of "Phoronmy" and "Dynamics".

4. This deep recasting of the mathematical "construction" of transcendental moments of physical objectivity has many important consequences. Let me focus here on two of them.

1. The possibility of deriving the whole complexity of the Standard Model from an empirical nucleus via the product of a classical spin geometry with a NC discrete geometry generating Higgs fields.
2. The possibility of defining a spectral action unifying a QFT à la Yang-Mills with GR via the eigenvalues of the Dirac operator used as dynamical variables for the metric.

We see that, after having been applied to symplectic mechanics, general relativity, non-abelian gauge theories and string theories, a correctly generalized and "historicized" transcendentalism is able to support the conceptual breakthrough brought about by Noncommutative Geometry.

Addendum. In a forthcoming book, Alain Connes, Ali Chamseddine and Matilde Marcolli show how their previous results can be strongly improved and yield a derivation of the standard model minimally coupled to gravity (Einstein-Hilbert action) with massive neutrinos, neutrino mixing, Weinberg angle, and Higgs mass (of the order of 170 GeV). This new achievement is quite astonishing.

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Bibliography

Abraham, R., Marsden, J. 1978. *Foundations of Mechanics*, Benjamin Cummings, New-York, Reading.

Arnold, V., 1976, *Méthodes mathématiques de la mécanique classique*, Moscou, Mir.

Arnowitt, R., Deser, S., Misner, C. W., 1962. "The Dynamics of General Relativity", *Gravitation: an introduction to current research*, L. Witten (ed.), New York, Wiley, 227–265. ArXiv:gr-qc/0405109.

Baez, J. (ed), 1994. *Knots and Quantum Gravity*, Oxford, Clarendon Press.

Brading, K., Castellani, E. (eds), 2003. *Symmetries in Physics. Philosophical Reflections*, Cambridge, Cambridge University Press.

Carlip, S., 2001. "Quantum Gravity: a Progress Report", *Report on Progress in Physics*, 64, ArXiv:gr-qc/0108040v1.

Chamseddine, A., Connes, A., 1996. "Universal formulas for noncommutative geometry actions", *Phys. Rev. Letters*, 77, 24, 4868–4871.

Connes, A., 1990. *Géométrie non commutative*, Paris, InterEditions.

Connes, A., 1996. "Gravity Coupled with Matter and the Foundation of Noncommutative Geometry", *Commun. Math. Phys.*, 182, 155–176. ArXiv:hep-th/9603053.

Connes, A., Kreimer, D., 1998. *Hopf Algebras, Renormalization and Noncommutative Geometry*, ArXiv:hep-th/9808042 v1.

Connes, A., Kreimer, D., 1999. "Renormalization in quantum field theory and the Riemann-Hilbert problem", *J.High Energy Phys.*, 09, 024 (1999). ArXiv:hep-th/9909126.

Connes, A., 2000. *Noncommutative Geometry Year 2000*, ArXiv:math.QA/0011193 v1.

Connes, A., 2000. *A Short Survey of Noncommutative Geometry*, ArXiv:hep-th/0003006 v1.

Connes, A., Marcolli, M., 2008. *Noncommutative Geometry, Quantum Fields and Motives* (forthcoming).

Friedman, M., 1985. "Kant's Theory of Geometry", *The Philosophical Review*, XCIV, 4, 455–506.

Friedman, M., 1999. *Dynamics of Reason*, Stanford, CA, CSLI Publications.

Grünbaum, A., 1973. *Philosophical Problems of Space and Time*, Dordrecht Boston MA, Reidel.

Hilbert, D., 1915. "Die Grundlagen der Physik (Erste Mitteilung)", *Königliche Gesellschaft der Wissenschaften zu Göttingen*, Mathematisch-physikalische Klasse, Nachrichten, 395–407.

Iochum, B., Kastler, D., Schücker, T., 1996. *On the universal Chamseddine-Connes action. I. Details of the action computation* , hep-th/9607158.

Itzykson, C., Zuber, J.B., 1985. *Quantum Field Theory*, Singapour, Mc Graw-Hill.

Jones, V., Moscovici, H., 1997. "Review of *Noncommutative Geometry* by Alain Connes", *Notice of the AMS*, 44, 7, 792–799.

Kaku, M., 1988. *Introduction to Superstrings*, New York, Springer.

Kant, I., 1786. *Metaphysische Anfangsgründe der Naturwissenschaft*, Kants gesammelte Schriften, Band IV, Preussische Akademie der Wissenschaften, Berlin, Georg Reimer, 1911.

Kastler, D., Schücker, T., 1994. *The Standard Model à la Connes-Lott*, hep-th/9412185.

Kastler, D., 1995. "The Dirac operator and gravitation", *Commun. Math. Phys.*, 166, 633–643.

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Kastler, D., [NCG]. *Noncommutative geometry and basic physics*, Centre de Physique Théorique, Marseille-Luminy.

Landi, G., Rovelli, C., 1997. "General Relativity in Terms of Dirac Eigenvalues", *Phys. Rev. Lett.*, 78, 3051–3054 (see also on the Web "Gravity from Dirac Eigenvalues").

Le Bellac, M., 1988. *Des phénomènes critiques aux champs de jauge*, Paris, InterEditions – C.N.R.S.

Majer, U., Sauer, T., 2004. "Hilbert's 'World Equations' and His Vision of a Unified Science", arXiv:physics/0405110v1.

Manin, Y. I., 1988. *Gauge Field Theory and Complex Geometry*, Berlin/New York, Springer.

Marsden, J., 1974. *Applications of Global Analysis in mathematical Physics*, Berkeley, Publish or Perish.

Martin, C.P., Gracia-Bondia, J.M., Várilly, J.C., 1997. "The Standard Model as a noncommutative geometry: the low energy regime", hep-th/9605001 v2.

Misner, C.W., Thorne, K.S., Wheeler, J.A., 1973. *Gravitation*, San Francisco CA, Freeman.

Petitot, J., 1990a. "Logique transcendantale, Synthétique a priori et Herméneutique mathématique des Objectivités", *Fundamenta Scientiæ*, (special issue in honor of Ludovico Geymonat), 10, 1, 57–84.

Petitot, J., 1990b, "Logique transcendantale et Herméneutique mathématique : le problème de l'unité formelle et de la dynamique historique des objectivités scientifiques", *Il pensiero di Giulio Preti nella cultura filosofica del novecento*, 155–172, Franco Angeli, Milan.

Petitot, J., 1991a. *La Philosophie transcendantale et le problème de l'Objectivité*, Entretiens du Centre Sèvres, (F. Marty ed.), Paris, Editions Osiris.

Petitot, J., 1991b. "Idéalités mathématiques et Réalité objective. Approche transcendantale", *Hommage à Jean-Toussaint Desanti*, (G. Granel ed.), 213–282, Editions TER, Mauvezin.

Petitot, J., 1992a. "Actuality of Transcendental Aesthetics for Modern Physics", *1830–1930 : A Century of Geometry*, (L. Boi, D. Flament, J.-M. Salanskis eds.), Berlin/New York, Springer.

Petitot, J., 1992b. "Continu et Objectivité. La bimodalité objective du continu et le platonisme transcendantal", *Le Labyrinthe du Continu*, (J.-M. Salanskis, H. Sinaceur eds.), 239–263, Paris, Springer.

Petitot, J., 1994. "Esthétique transcendantale et physique mathématique", *Neukantianismus. Perspektiven und Probleme* (E.W. Orth, H. Holzhey Hrsg.), 187–213, Würzburg, Königshausen & Neumann.

Petitot, J., 1995. "Pour un platonisme transcendantal", *L'objectivité mathématique. Platonisme et structures formelles* (M. Panza, J-M. Salanskis eds.), 147–178, Paris, Masson.

Petitot, J., 1997. "Objectivité faible et Philosophie transcendantale", *Physique et Réalité*, in honor of B. d'Espagnat, (M. Bitbol, S. Laugier eds.), Paris, Diderot Editeur, 201–236.

Petitot, J., 2002. "Mathematical Physics and Formalized Epistemology", *Quantum Mechanics, Mathematics, Cognition and Action* (M. Mügur-Schächter, A. van der Merwe eds.), Dordrecht, Kluwer, 73–102.

Petitot, J., 2006. "Per un platonismo trascendentale", *Matematica e Filosofia*, PRISTEM/Storia 14–15, (S. Albeverio, F. Minazzi eds.), University Luigi Bocconi, Milan, 97–143.

PQG, 1988. *Physique quantique et géométrie* (Colloque André Lichnerowicz, D. Bernard, Y. Choquet-Bruhat eds.), Paris, Hermann.

Quigg, C., 1983. *Gauge Theories of the Strong, Weak, and Electromagnetic Interactions*, Reading MA, Benjamin-Cummings.

Seiberg, N., Witten, E., 1999. "String theory and noncommutative geometry", *J. High Energy Physics*, JHEP09(1999)032.

Souriau, J.M., 1975. *Géométrie symplectique et physique mathématique*, Coll. Internat. du C.N.R.S., 237, Paris.

Weinstein, A., 1977. *Lectures on Symplectic Manifolds*, C.B.M.S., Conf. Series, Am. Math. Soc., 29, Providence RI.

Weyl, H., 1922. *Space – Time – Matter*, New York, Dover.

Part III

Debate About the Relevance of Transcendental Epistemology for Modern Physics: Transcendentalism, Empiricism and Realism

Can Empiricism Leave Its Realism Behind? Toward a Dialogue with Transcendentalists[1]

Bas C. van Fraassen

Abstract Today's empiricism and transcendentalism both reject metaphysics, but each appears sometimes to the other as actually engaging in the rejected metaphysics. From an empiricist standpoint, transcendentalism seems to grant too much to the knowing subject whereas from a transcendentalist standpoint, empiricism seems to concede too much to realism. The challenge posed for empiricism is to explain how it could make sense, within an empiricist stance, to say that *there could be* things that are not describable (in our language in use) and hence not knowable, let alone known. The remaining 'common sense' realism, that I acknowledge in response, can – I submit – be clearly distinguished from any metaphysical version vulnerable to transcendentalist critique.

Empiricists today and transcendental idealists today have two things in common. The first is that we see around us philosophers engaged in the sort of metaphysics that, in our view, was over and done with after Kant's critique. The second is our positions seem hard to formulate without appearing – at least to the other – to land squarely inside that rejected metaphysics.

But the differences between us are not so clear. "Realism" is an accordion word on which many a pretty melody is played. Are the current metaphysical realisms the same as the sorts of metaphysics done away with by Kant's critique? In my view, empiricism must at least distance itself from the analytic metaphysics now much in vogue. So I will begin by examining a sustained recent attempt in analytic philosophy to classify realist and non-realist positions. I will argue that an aspirant non-realist should escape this classification. The escape I outline may still look quite realist to transcendentalist eyes, but I shall argue that the remaining realism is not metaphysical.

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M. Bitbol et al. (eds.), *Constituting Objectivity*,

1 Can There Be a Real Non-realism in Realist Eyes?

When different philosophical traditions ostensibly confront the same problem, they see a different range of alternative possible responses. Once you go along, you are limited to a single 'menu' for all possible philosophical positions, nicely exemplified by the sort David Lewis displayed in his *Philosophical Papers*.[2] So what are the options for realism and non-realism in the frame of analytic metaphysics? I will draw on Lewis' student Mary Kathryn McGowan's insightful *Realism or Non-realism; Undecidable in Theory, Decidable in Practice*.[3]

1.1 The Philosophical Menu and the Triviality Problem

McGowan classifies an array of philosophical positions as united by common basis but divided into "realist" and "non-realist".[4] What all the positions have in common is that they recognize a common set of "mandatory" problems and take as inescapable a certain type of solution for those problems. These mandatory problems, to be addressed by any philosophy worth its salt, seem to concern something that in the neo-Kantian tradition is discussed under the heading of "constitution".

1. The first and most fundamental problem is to provide a basis for *similarity judgments*.

Specifically, what are the truth conditions or assertability conditions for such statements as "A and B are more similar to each other than to C" or "A is similar to B with respect to color but not with respect to height"?

2. Putnam's paradox brings out the second mandatory problem, which is to allow for the falsity of at least some ideal theories.
3. The third mandatory problem is a traditional one: to formulate a coherent inductive rule or policy.

I will leave the second aside here.[5] After presenting the first and third problem, I'll begin without contesting the basic presuppositions, but then attempt to strike out beyond them.

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[2] David Lewis, *Philosophical Papers*, vol. I (Oxford: Oxford University Press, 1983), pp. x–xi. See also the footnotes there.

[3] McGowan 1996; See further her 2001 and 2003.

[4] McGowan's conclusions are in sympathy with what I shall argue below. According to McGowan, we can opt for one such position as she surveys, but only on the basis of a value judgment. What the enterprise of metaphysics is must presumably be conceived in terms of the criteria of choice: metaphysics' *telos* is to produce *valuable* systems in the appropriate sense. Compare Finch, 2003 for a related conception of metaphysics.

[5] Putnam, 1976; Lewis, 1984. I have addressed this subject at length in my 1997, so will omit discussion here.

1.2 The Essential Mandatory Problem: Similarity

Suppose we begin naively by defining ‘similar’ as ‘having some properties in common’, and allow any properties at all to count in this. Unfortunately it seems that any two things have some properties in common! There is always some description, even if perhaps contrived, which fits both. So by this naive criterion any two things are similar, *e basta!*

Consider a statement of form “A is similar to B [with respect to …].” What are its truth conditions? Suppose we reply:

> “A is similar to B” is true if and only if A and B have some properties in common.
>
> “A is similar to B with respect to color” is true if and only if A and B have some color properties in common.

And so forth. Even in the second form, the reply is untenable. For any two familiar things we can describe some color property they share, and for the unfamiliar, we can count *having no color* as a color property too. The result would be that all the former are similar to each other with respect to color, and color-dissimilar only to the latter. The descriptions drawn on for this result may be baroque from our point of view, even gerrymandered.[6] But what does nature or the WORLD care about what looks baroque to us?[7]

If in practice we dismiss cases with a simple “that is a shared property, but not a relevant one” then that shows that in our own reactions we readily discern a sense of *privileging* seemingly involved in a serious judgment of similarity. David Lewis honored this acknowledgement by postulating that some classifications are naturally privileged: there are *natural classes*, which ‘carve nature at the joints’ in the ancient phrase. This is the paradigm *realist* solution in McGowan’s taxonomy of positions.

Call “structure-defining” those properties – whatever they may be – that count in assessments of similarity. The form of solution which provides the common basis for all the philosophical positions McGowan considers is then:

> Properties are divided into two classes, the *privileged* and the *unprivileged*. The privileged are the *structure-defining properties*.
>
> “A is similar to B” is true if and only if A and B have some privileged property in common.
>
> “A is similar to B with respect to X” is true if and only if A and B have some privileged X-property in common.

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[6] Gerrymandered color similarities would include “both are grue,” “both are warm colors,” or “both have colors loved by the Emperor”. But in fact the color spectrum is such that if two objects are colored at all, there is some part of that spectrum which they both instantiate somewhere, so here at least gerrymandering is not required.

[7] I use Putnam’s somewhat disrespectful capitalized “WORLD”; see further my discussion of the question “Does the world exist?” in Chapter 1 of my 2002.

Although thus any feasible philosophical position must acknowledge such a division between 'privileged' and 'unprivileged' properties, they will differ on the principle of division. It is in this respect that the positions can be classified as realist or non-realist.

1.3 Goodman's Grue and the 'New Problem of Induction'

Adapting Nelson Goodman's famous example, define: an object is *grue* if it is observed before the year AD 3000 and green, or not observed before the year AD 3000 and blue. Similarly, an object is *bleen* if it is observed before the year AD 3000 and blue, or not observed before the year AD 3000 and green.

Emeralds are green, we say; but all observations of emeralds so far, on which we could draw for evidence, were before 3000. It is equally true that all emeralds observed so far have been grue. If induction is to be based on the data from observation, why infer that all emeralds are green, rather than that all are grue? The latter conclusion would of course entail that any emeralds brought to light from AD 3000 on will be blue, and somehow we do not believe that.

While the 'grue' example is in itself only an objection to the naïve sort of numerical induction that Bacon already descried, it can be generalized to a problem for any ampliative rule of inference on the basis of gathered data, when those data are redescribed in some 'gruesome' way.[8] But this inductive ambiguity would trivialize the entire enterprise of induction.

Once again the conclusion here is that any feasible philosophy will acknowledge such a 'privileging' distinction among properties. The only way to save the idea of induction is to insist that properties expressed by such contrived 'gruesome' predicates *do not have the same status* as those that should occur in description of the data fed to inductive rules. This is once more a 'privileging' solution. Nelson Goodman's own response took exactly that form, with predicates divided into 'projectable' and 'un-projectable', associated with historical 'entrenchment'. This is a prime example of what McGowan calls *non-realist*. Goodman's privileging principle refers to actual inductive practice and entrenchment of predicates in actual human use, unlike Lewis' division into natural and non-natural sets, which purports to refer to a distinction 'in nature' rather than in human history.

The putatively inescapable form for any solution links the mandatory problems, so the danger to be averted is the same in each. The basic problem is thus aptly labeled the *Triviality Problem*: in each case something taken to be non-trivial is trivialized. When we construct a representation, we display a certain structure. The trivializing arguments then come along and appears to tell us:

> Whatever structure you display, we can regard the WORLD as having that structure.

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[8] This apt use of 'gruesome' is due to Catherine Elgin, I believe.

The realist metaphysician's reply is that some discernible structure in the world is privileged in some fashion; not just any displayed structure could really be the structure of the WORLD. To stop the trivializing arguments we must show a way of privileging certain properties as alone 'structure-defining'.

2 Dissolving the Triviality Problem

McGowan distinguishes two main options with respect to privilege, realist and non-realist. Each position accepts the priviliged/unprivileged distinction but will be characterized in the first place by how it takes the privileging to come about. *Realists* hold that there is here a division in nature. *Non-realists* are precisely those who hold that the division is wholly or partly "interest-relative": that is, dependent in some way on us, our interests, values, goals, practice, history, taxonomies, concepts, acts of classification or the like.[9]

All the positions examined, both realist and non-realist, share the form of solution. They differ on what makes the privileged properties privileged. All agree as to the need for such a distinction in view of the mandatory problems. *What if the need is illusory*? Then they are all barking up the wrong tree.

My argument that the need is indeed illusory will indicate a possible parting of the ways. Going one way, we stay solidly within the proffered menu of 'realist' and 'non-realist' solutions as there conceived. But that road leads to an unreal non-realism which defeats itself: we find that all the canvassed positions are realisms after all. Going the other way we hope to escape the metaphysical trap.

2.1 The Problem of 'Objective' Similarity

The privileging solution does indeed provide a *form* for the truth conditions of statements of similarity. In any particular case we'll see an instance of this form. But what is ignored is that these statements *may not have context-independent truth-conditions at all*. In differing contexts in which a judgment of similarity occurs the speaker and audience may be "privileging" something else, depending on contextual factors.

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[9] Details: it is possible for all structure defining properties to supervene on a small family of privileged properties – a *base*. There may be several bases and distinct, though possibly overlapping, families of structure defining properties that supervene on those bases. In these terms, several options are open to the non-realists. The most obvious non-realist tack may be to identify a 'subjective' base (a base selected in terms of human interests, goals, or practices). But that is not the only non-realist option. The selection of bases could be entirely through objective constraints, while the selection of supervening privileged properties be laden with subjective factors.

That purposes, preoccupations, or salient features play a role in actual examples springs to the eye: for a metal worker a hammer and a wooden mallet are not similar, for the non-specialist they are.

If we are asked to consider an example out of the blue, or in a philosophy seminar, this may not be obvious, and our judgments of similarity and difference may have an air of *context-independent privileging*. But this we should take as deceptive. In the absence of definite practical concerns, we are likely to judge similarity with respect to familiar purposes or considerations that automatically *come to mind if we don't pay attention to what we have in mind*. The idea that there is a context-independent 'privileging' which selects only certain properties as significant structure-defining ones has no support in the phenomena of everyday discourse.

At first blush this reflection establishes that there is no 'objective' similarity, if 'objective' is meant in the sense of independence of contextual factors such as use, value, practice, history … (i.e. the sense of being not 'interest-relative', in our present terminology). Does this simply put is squarely in the class of McGowan's 'non-realists'? No – not precisely. Her non-realists offer a basis for context-independent relations of similarity! The form of solution to the mandatory problems that she describes involves context-independent privileging. The range of alternatives is thereby artificially limited.

2.2 The Problem of Grue-Some Predicaments

It is not easy to find a principled distinction between "grue" and "green". As Goodman pointed out, while "grue" and "bleen" are new predicates defined in terms of the familiar "green" and "blue", it only takes a little logical dexterity to define "green" and "blue" in terms of "grue" and "bleen".

The typical initial reaction to use of grue–bleen rather than green–blue descriptions, that there is something fundamentally wrong with it, seems simply mistaken to me. We can imagine circumstances in which, for certain purposes, the use of grue–bleen sorts of descriptions would be advantageous. An example is this: a broker in precious stones foresees a sharp change in demand from green to blue stones as of the year 3000. During the year 2999 he asks his sales manager to draw up a list of all mines expected to be good sources of stones which are either observed (dug up) before 3000 and green or not so observed (dug up after 3000) and blue. For brevity, he says "grue stones", and there is a practical advantage in adopting this terminology. To generalize: every describable property is such that its cognizance is especially advantageous in some circumstances relative to some purpose.

Can we still explain the common reactions in philosophy seminars? Easily, I think. Individual 'rightness' judgment is always in terms of circumstances and purposes. That is so even in first-blush reactions in a seminar, but then – as I said above – these circumstances and purposes are the ones that automatically and tacitly come to mind *when we pay no attention to what we have in mind*. (That is de rigueur in the academic context – one of those quaint features that sets academia truly apart.)

Could this point be parlayed into a 'non-realist' or 'realist' position on privilege? Turning the relative usefulness into a division of properties generally, we can only single out the humanly describable properties. That certainly will not separate the "grue"-like from the "green"-like. So it will not solve that supposedly mandatory problem. However, that does not really matter. For once we see our tendency to classify "grue" as *unheimlich* as due only to variable contextual factors, that 'mandatory' problem dissolves.

What if some describable properties enjoy no such advantage at all, and some less so than others? Perhaps only certain properties actually need to be taken cognizance of in our history, or perhaps some need this much more frequently than others. That could be because we are carbon-based rational beings or because only certain forms of culture actually develop in history or because we live only in certain types of environments.[10] But that is just the sense in which e.g. hammers might be called privileged among tools. In human history as a whole hammers have been much more useful than micrometers or telephones – though the latter might of course win in the long run. Taking cognizance of a property, or using a certain predicate, may be useful in just the same way that use of a hammer may be useful.

Such an ordinary statement, as that taking cognizance of a certain property is useful in some situations for some purposes, does not land us in a metaphysical position.

2.3 The So-called 'New Riddle of Induction'

The grue/green riddle provided the impetus for Goodman's new riddle of induction. That induction is not univocal (in that the same evidence can point equally clearly to each of several mutually opposed hypotheses) was not exactly a new point. The 'grue' example brings out graphically one feature shared by any form of inference of that sort. With the alternative of describing one's evidence either in terms of the usual sorts of properties or in terms of their grue- and bleen-like variants, even the most strictly framed extrapolation rule will lead to ambiguous or even contradictory results. A truly neat encapsulation of that old chestnut!

Goodman attempted to solve this problem by designating certain predicates as 'projectable' – accounted for in terms of historical 'entrenchment'. That is certainly a 'privileging' sort of solution. As we saw, it is a privileging sort of solution of the sort McGowan classifies as *non-realist* because it refers to our history; we may well take it as the paradigm for this class. *But is it right; and is the problem mandatory*?

Goodman's solution to the riddle has the same *form* as a realist solution. But all it does is show how a certain besetting ambiguity in the idea of an ampliative inductive

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[10] Some sorts of realists might have a better candidate, e.g. that certain things about us, some of our needs, purposes, features, or even living conditions are *essential* to us; but this would also require realism about modalities or essences, hence would not provide a common ground for all parties.

rule could be partly or wholly removed. But is this a mandatory problem? Philosophy since Hume is permeated with the idea that there really are canons of right reasoning which give a recipe for induction. To avoid confusion, let's use 'induction' with a small letter "i" to denote just any moves at all to opinions beyond what we have so far. The philosophers' stone in this area is not that, but the idea of *Induction*, with a capital "I", which is a rationally compelling form of inference, that will reliably if not certainly lead to true conclusions beyond its premises. That there is induction is beyond doubt; that there is such a thing as Induction is at best controversial and dubitable.

To show that there is nothing mandatory about this form of solution or this take on the so-called Problem of Induction we need only mention De Finetti's and similar Bayesian or probabilist views. These hold that the proper and rational thing to do is to start where we are, i.e. with our actual prior opinions, and update them by conditionalizing on the evidence received. If that is so then following an ampliative rule of Induction is not rational! Ampliative rules – any form of Induction worthy of that name – are shown to make one vulnerable to Dutch books, or other forms of incoherence.[11] It is therefore not Goodman's solution, but the problem he offers it for, that I would classify as based on a mistake.

3 Will the Real Non-realism Please Stand Up?

The felt need for a metaphysically privileged status disappears once we realize that our judgments of similarity are contextual. McGowan's reply is that this solves the triviality problem for similarity by privileging a certain relation to us and our human contexts, so it is also privileging solution – and she calls this sort 'non-realist'.

3.1 Self-Defeating Non-realism

Since we are part of the world, and in given contexts bestow a privileged status on certain properties, those properties are privileged because they bear that relation to us and our interests, purposes, etc. Our privileging bestows a privileged status on certain properties. So the non-realist, though purveying some sort of relativism, is in fact also a metaphysician.

The non-realist option is thus presented as offering a relativized version of the realist option. Realist views are views to the effect that *what the world is like* is, in all but trivial or easily accommodated ways, *independent of our practice, values,*

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[11] I am not a Bayesian; but from my own (liberal probabilist) point of view the problem of induction is also based on a mistake; in fact, more than one. See my 2000.

goals, … that they are not ‘interest-relative’. The realist maintains that there are independent constraints which select the structure-defining properties. The positions classified here as viable non-realist positions offer interest-relative surrogates for those objective constraints. To put it briefly, those properties are privileged *which bear a certain relation, call it R, to us*.

But here we encounter a crucial ambiguity. Pay careful attention to the use of the first person pronouns “we”, and “us” in this division of philosophical positions. Could these be everywhere replaced by common noun clauses such as “human beings”, “the human race”, “rational animals”, “conscious organisms, not necessarily carbon based”, etc.? When a position amounts to something of the form:

> Those properties are *privileged* which bear a certain relation, call it R, to the human species [carbon based calculating organisms, featherless bipeds, Earthlings, …].[12]

Then it is indeed quite clearly **a realist position**!

So construed, those so-called ‘interest-relative’ constraints are then not ‘subjective’ at all, in any non-trivial sense. They are constraints on the properties’ relations to *a certain part of the universe* (the human race, or the conscious organisms, etc.). Hence they are as much a matter of fact – if a little curiously chosen – as any proffered by the so-called realists. For properties are then *privileged* if they bear relation R to certain kinds of beings (and/or the events in which those beings are involved) in a presumably small region or perhaps widely dispersed family of small regions of space–time. The privilege may then be linked to the fact that these beings have a certain kind of complexity, qualia, or some other currently trendy feature. Be that as it may, it remains that this is privileging by facts about what the world is like. For it is a fact that light affects the anthropoid in the way it does. So how can we see the division as non-arbitrary, or the resultant sort of position as non-realist?

It is not surprising if the regular sorts of realists find this astonishing. Recall that in McGowan’s terms, the privileged properties are structure-defining. Hence this sort of realist view (misleadingly called ‘non-realist’) would have a remarkable consequence for world history. The emergence of the relevant sorts of beings, or the relevant sort of complexity, would then have had the effect of changing the universe from a ‘property chaos’ to a finely structured whole[13]:

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[12] For my argument to go through R must be describable without the use of indexicals such as “we” and “our”, I agree. If it is meant to rule out relations describable ‘in the space of reasons’, i.e. by norm or value laden terms, then I want to point out that these relations can be construed in two ways as well. A sociologist can describe what is regarded as valuable to a given community in language which is not itself value-laden, without implying for example that in his/her opinion it is indeed valuable for that community. If on the contrary this sociologist forms the judgment that such and such a practice ***is*** valuable to that community, then his “valuable” expresses a value judgment of his own, and he could have said “valuable according to us”.

[13] A greater difficulty: how could the position be properly stated at all? Since the attribution of complexity could in this context only be attribution of complexity under some description, not under a description in terms of structure defining properties. But this difficulty (see McGowan, p. 48) belongs exactly to the kind of problems McGowan takes up for her non-realists.

> In the beginning the WORLD was without form and void, and darkness was upon the face of the deep … But eventually the spirit of the anthropoid stirred near the face of the waters and took notice that there was light. Thus was the light separated from the darkness….

McGowan's non-realists are realists, but of an egregiously implausible sort.

How could fundamental structure of the universe derive from features of such an accidental, late, limited, and indeed ontologically miniscule fragment of it as the human race? Why not the rat race or mouse race or the species inhabiting some other region of the cosmos? Or in fact, why should the cognitive or practical interests of a given species count in some special way at all? Wouldn't the properties instantiated only at the microphysical or at the global-cosmological level be more reasonably invoked? Of course, we are not sure what those are; but the fact that we know better what humans are like does not count here. There is no rhyme nor reason to it. Hence no matter how hard such a non-realist strives s/he can at best, staying within this context, arrive at a possibly consistent and coherent but egregiously implausible position.

3.2 The Indexical Epistemic Turn

The frequent use of first-person pronouns in formulating "non-realist" ideas should alert us to elements of unacknowledged rhetoric supporting the presentation. Recall that we found this self-defeating option by reading the terms "we" and "us" as everywhere replaceable by such non-indexical terms as "the human race". Suppose we do not hold to this. Then we can propose an alternative construal of what McGowan points to with the term "non-realist". Suppose we take this use of "we", "us", "our" indexical language seriously. That means: we take seriously the following view:

> The very discourse of 'privileging', 'base', 'classifying', 'structure-defining' is to be understood both as context-dependent and as first-person discourse not equivalent to anything formulated without the use of indexicals.

This entails that this discourse cannot be translated into anything sayable by means of what Quine calls "eternal sentences". If we suppose that, our supposition will at once pre-empt the move:

> Premise: *relation to us = relation to a certain independently specifiable part of the universe*,
>
> **therefore**
>
> Conclusion: the so-called interest-relative constraints are simply one sort of factual constraint.

The premise would still be true, in one sense: although the two phrases "privileged by us" and "privileged by human beings" do not have the same meaning, they could have the same extension – and do, if we are precisely the human beings.

Any aspect at all of the world can also be specified in a context-independent way.[14] It does not follow that sentences of the form "… is thusly related to us" will be in all respects equivalent to, and can everywhere be replaced by ones of form "… is thusly related to X", where the place of "X" is filled by some context-independent, non-indexical phrase. The conclusion of the above argument is prey to an equivocation: true innocuously and false in what it insinuates.

3.3 Privileging by First-Person Value Judgment

Have we now arrived at the possibility for a radical alternative to metaphysical realism, an *anti-realism* worthy of the name? The response we can now give to all those 'mandatory' philosophical problems amounts simply to this:

> The distinctions we draw are drawn in terms important to us, and the result is that triviality-to-us is averted, no more. But that is enough.

Saying this, we do not share the presuppositions concerning similarity, induction, and success in theorizing that created those mandatory problems in the above form. But is this a viable position, and is it stable enough not to collapse into what McGowan classified as non-realism?

Privileging relative to us involves value judgment as well as factual judgment. If in a certain context I say that a hammer is more similar to a wrench than to a T-junction, it is clear that I am counting as relevant not the shape but the function, and that is because in this context, it is the function (as a tool) that is important to me. If in another context I need something heavy as a paperweight I might say that a hammer is more similar to a bottle than to a feather – for there it is weight that is important to me. So the requisite selection of relevant versus irrelevant properties for the similarity judgment is a value judgment made in the first person, for it expresses what is important to me in this particular case, relative to the purpose at hand.

In realist eyes, however, our valuings and opinings are both intelligible precisely as attempts to track what reality is like: *the privileged properties are the important ones; blessed are they who discern them!* Science is successful precisely when its theories – contingently but actually – reveal that privileged structure. In a word, realist epistemology succeeds because it makes its home in realist metaphysics. What does our new anti-realist have to offer in response?

This anti-realist must respond that any privileged status which a property has it receives in our value judgments. It is true that forming opinions that go beyond our

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[14] I am not asserting that all statements are equivalent to 'eternal' context-independent sentences. The distinction to be made concerns the sense of equivalence. The sentence "I am sitting" is not everywhere replaceable, salva veritate, with "BvF is sitting": "If I had forgotten my name then I would still know that I am sitting, but not that BvF is sitting" – that is also true. Yet "I am sitting" and "BvF is sitting" must of course have the same truth-value, since I am BvF – in that sense my being sitting can also be described without the use of indexicals.

evidence typically involves selective attention to some properties rather than others – but the rationality of this practice will consist in its coherence (including coherence with the rest of our practices).

Clearly the two antagonists are not answering the same question. The realist wants to postulate some fact about the world that explains the success of our practices, and this fact must be something that makes them successful, not simply a generalization of that success. Thus the realist thus wishes to assert *conditions for the success* of the practice. But the anti-realist will understand such an assertion as merely extending a practice for which we would like to understand the *conditions of rationality*.

3.4 That This Anti-realism Is Viable and Distinct

The danger in disputation with metaphysics is that one ends up positioning oneself on their turf, and ends up looking like – even becoming – a metaphysician oneself. But the alternative I have introduced as the 'indexical turn' is not a variant on any item in the metaphysician's menu.

What would drive an argument to the contrary is an unreal dilemma. To begin we all agree: there are mountains, trees, rocks, people of various sorts, light, water, lightning and storms, …. So the world is divided up into entities of different sorts.

Fine, but now the metaphysician asks: what accounts for, explains, or grounds this, or any other, such division into sorts? McGowan, following Lewis perhaps, sees a dilemma: either (a) the world or nature itself is structured in a way that has nothing to do with us or our language or thinking, desiring, etc., or else (b) it is structured by us, by our language or thinking, desiring, etc. Above, I pointed out an ambiguity. If we look closely, we discern an implicit assumption when this is seen as an exhaustive bipole dilemma. That is the assumption that the "us" indexical language is not essential – "us" and "our" are replaceable by "humans" and "human". If we use the words of option (b) but with the indexicality of "us" and "our" given full significance, we are outside the dilemma, and off the menu so to speak.[15]

No wonder our nouns and adjectives speak loudly of what is pertinent to our tasks – our language grew up in praxis. In that sense we furnished the grounds for

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[15] Recall a famous saying by Hilary Putnam: "the mind and the world jointly make up the mind and the world". This too can be understood in two ways. Putnam is not so naïve, of course: it is clear that he resists a naïve reading of these words: "If one must use metaphorical language, then let the metaphor be this: the mind and the world jointly make up the mind and the world" (Hilary Putnam, *Reason, Truth, and History*, p. xi. Cited and discussed further in his 1987, Preface, p. 1). In the first lecture in his 1987 he rejects subjective idealism; his aim is to show that common sense realism and conceptual relativism are compatible. I'd like to think that I'm showing something similar, although I would resist the name "conceptual relativism" as misleading, and also insist that any attempt to express the view in third person non-indexical language is self-defeating.

how our descriptions are structured. Even the words "world" and "nature" and "thing" belong to the vocabulary that grew up with us in this way. Therefore all we can really say is that there is structure in our descriptions of what we describe – nothing new is added if we replace "what we describe" by "world" or "nature". The divisions marked in our descriptions – and what other divisions are you asking about? – are those which are important to us. This importance is entirely summarized by saying 'they are ours' – 'a poor thing but our own'.[16]

I don't know how to show a determined fly the way out of the fly-bottle. Or how to exorcize bewitchment by a picture. Except just to carry on, and glory in speaking with the vulgar.

In our escape from the metaphysical menu of realists and non-realists, we chose the indexical epistemic turn – perhaps, if it were not hubris, I could have called it a sort of Wittgensteinian turn. Real anti-realism must be a position that can only be expressed in the first person. (Preferably the first person plural ….) Traditional philosophical problems may look very different after the indexical epistemic turn. They can now sometimes appear in their proper "1st person" form, and the solution is then strictly a "1st person" form of solution. The real anti-realism is not any view that could masquerade as a quasi-scientific theory of organisms participating in nature. At the risk of being read once more through the metaphysicians' glasses, I'll venture this:

> It is a view of how things are *for us*, of what it is to exist in this world *in the way we are there*.

But it is not a view in the sense of a theory, it is a view in the sense of a way of seeing, an attitude in which to approach the questions we find.

4 Remaining Realism

Metaphysical realism must be distinguished from 'being realistic', as Cora Diamond calls it, and (if that is not the same) from 'common sense realism'. But is the realism that empiricism retains not a metaphysics? I shall attempt to imagine here how such a challenge could come from the side of transcendental idealism.

4.1 A Challenge

Perhaps the above anti-realist view is stated against a background – within a presupposition – *of a thoroughly realist-conceived world picture*. Let's summarize this anti-realist view thus: the properties that count in a specific similarity or non-

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[16] I suppose this catch-phrase comes from "An ill-favoured thing, sir, but mine own" (Shakespeare, *As You Like It*. Act v. Sc. 4.).

similarity judgments are precisely those which are important for us, important for the very reasons for which we are making that judgment. Count how? Count as determining whether the judgment is correct in that context – and that amounts to, surely, whether *what it means there* is true.

> CHALLENGE: You do not seem to have gone beyond the sort of realism you wanted to escape. Whether *what is meant by that judgment in that context is true*: is that not a context-independent fact? Would *that* not equally be the case or not the case, even if there were no such context and no such judgment?
>
> If so, the supposedly anti-realist view is formulated within a presupposed realist-conceived world picture, and is unintelligible apart from that.

I want to concede a bit to the challenge, but then point out a crucial difference that is not taken into account here.

The phrase "what is meant by that judgment in that context" is held up as standing for something that does not depend on the context. To adapt an example due to David Kaplan: imagine that there are a number of ropes lying on the ground. I pick up an end, saying "This is the same rope as" (picking up another end in my other hand) "that one". I express my judgment in a sentence which is context-dependent, since it involves the demonstratives "this" and "that". But this situation can also be described quite impersonally: clearly there are just two ends of ropes on which there are hands. If those ends were ends of the same rope then my judgment – which took a distinctly context-dependent form – was true in its context, and otherwise it was not true in that context. So now we have stated the truth conditions for the indexical utterance in a context-independent way.

We can put it like this: *the meaning* (what Kaplan called the "character") of the indexical sentence is something sensitive to context, but *what is meant* is something that can be stated by means of a context-independent sentence, as in fact I just did. What is meant we can identify through the statement's truth-conditions in that context.

Can we now apply this so nicely illustrated point to the judgment "This is important to me", made in a context in which we point to the properties that count in a specific similarity or non-similarity judgment?

There is an important dissimilarity with the above examples. To play its role here, this utterance must *express a value judgment*, not *make an (autobiographical) statement*. We may indeed add that what I say is correct (in the sense that it does accurately and correctly express my pertinent valuation) if and only if *the thing is important by BvF's evaluation of the matter*. To that extent the cases are similar. But the statement that *the thing is important by BvF's evaluation of the matter*, is a statement of fact, not an expression of value or opinion. It is precisely what a third person reporting on the situation would say. It can be said using a context-independent sentence. In addition, in that context, having said what I said (and provided I accept "BvF" as denoting myself) *I must endorse* the third-person statement.[17]

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[17] The point about forced endorsement is also just a point of logic, but at the level of pragmatics. Specifically, I would be pragmatically incoherent if I acknowledged myself to be BvF, expressed my evaluation of something as important, and denied that it was important by BvF's evaluation of the matter. Such logical points do not tell for or against any philosophical position.

But none of this removes the crucial difference between the two. For in saying "This is important to me" my speech act has the role of expressing a valuation, not just of stating a biographical fact.

To see how this removes the sting from the above Challenge, let's scrutinize the Challenge one part at a time:

1. *Whether what is meant by that judgment in that context is true*: is that not a context-independent fact?
2. Would *that* not equally be the case or not the case, even if there were no such context and no such judgment?[18]
3. If so, the anti-realist view is formulated within a presupposed realist-conceived world picture, and is unintelligible apart from that.

The judgment was made in a certain context, and if there had been no such context then there would not have been such a judgment either. But in Part 2, the word "*that*" refers not to the judgment-utterance, but to *what is meant by that judgment in that context*. Would that have been true nevertheless?

At first blush, looking only to our simpler examples, the answer is *Yes*. That the two rope ends belong to the same rope, for example, does not require that the speaker or anyone else says or realizes or thinks, just then, that this is so. But we have to look beyond such simple cases.

Since our crucial case is one in which the context-dependent sentence must express an evaluation – rather than state an autobiographical fact – we need to ask whether expression of an evaluation has a closer context-dependence than a statement to the effect that one has or endorses that value. Take our initial example: I judge in a certain context that a hammer is more similar to a wrench than to a T-junction. It is clear that I am counting as relevant not the shape but the function, and that is because in this context, it is the function (as a tool) that is important to me. In other contexts, other aspects – such as shape – would be important to the purpose at hand. So does it make sense at all to ask whether the expressed evaluation would have been correct if there had been no such context as this one? Would that not be like asking

> [I]f you had not needed a tool, what would you have needed? Something that would still have favored some aspects that hammers and wrenches share, but a hammer does not share with a T-junction?

But who knows and who cares? What I would have needed if I had not been in this situation has nothing to do with whether my evaluation of what is important here is correct here. The question, which makes perfect sense in the ropes example, namely whether what is meant would have been true even if this context had not occurred, makes no sense at all when asked about the expression of this value judgment.

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[18] About "would" in this line: the Yes-answer to (2) simply grants a logical point, to the effect that one sentence does not logically imply another one.

4.2 A Further Challenge

I am aware that I kept to a way of speaking that seems simply to take it for granted that properties as well as things are there, prior to the sort of privileging that goes on when we make e.g. similarity judgments.

> CHALLENGE 2. If a property has privilege of some sort bestowed on it in a given context by value judgments made there, it must be a property that already was there to be noticed. So with respect to the reality of properties, the view advanced is realist.

The first part of this statement is fair enough – I can't value something that I don't notice. But what of the gloss put on this by the second statement?[19] The question is *whether it makes sense* to ask what things are like independently of our judgments:

> CHALLENGE 3. It is rather a matter of questioning any approach at all that neglects the historical and pragmatic conditions of developing knowledge, while presupposing that the objects of knowledge have their determinations independently of the fact that they are represented and of how they are represented.[20]

We have to tread very lightly here, since the transcendentalist position today is not simply Kantianism.[21] Transcendentalists, like empiricists, usually regard what they challenge in other philosophical positions as *not making sense* rather than as *asserting something intelligible but false*. So Challenge 3 should not be read to imply the opposite of

> (A) "the objects of knowledge have their determinations independently of the fact that they are represented and of how they are represented".

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[19] If meant as an imputation of platonism, it is another case of an ordinary innocuous statement read through a metaphysician's glasses. But if a transcendentalist were to say this, the charge would not be platonism.

[20] «Il s'agit bien plutôt de mettre en question les approches qui négligent la considération des conditions historiques et pragmatiques du développement de la connaissance en présupposant que les déterminations de l'objet de la connaissance sont indépendantes du fait qu'il soit représenté, et de la manière dont il peut l'être.» (Isabelle Peschard)

[21] To call the challenge here *transcendental idealist*, and the view opposed, *transcendental realist*, would be in accord with Kant' use of the term: "In the transcendental aesthetic we proved that everything intuited in space and time, all objects of a possible experience, are nothing but phenomena, that is, mere representations; and that these, as presented to us – as extended bodies, or as series of changes – have no self-subsistent existence apart from human thought. This doctrine I call Transcendental Idealism.* The realist in the transcendental sense regards these modifications of our sensibility, these mere representations, as things subsisting in themselves." (Kant, *Critique of Pure Reason*, Meiklejohn tr.; *Transcendental Dialectic* Book II, Chapter II. 'The Antinomy of Pure Reason'; Section VI. 'Transcendental Idealism as the Key to the Solution of Pure Cosmological Dialectic', initial paragraph.) At * Kant has a footnote: "I have elsewhere termed this theory formal idealism, to distinguish it from material idealism, which doubts or denies the existence of external things. To avoid ambiguity, it seems advisable in many cases to employ this term instead of that mentioned in the text."

That opposite would be something like

> (Not-A) "what the objects we know are like depends on the fact that we represent them and how we represent them".

For if that opposite made sense then so would the opposite of this opposite, i.e. the very idea whose intelligibility is challenged. So we must take it that the transcendentalist would rather be questioning the very sense of any discussion of whether (A) or (Not-A) is the case.

4.3 Disarming the Challenge

Instead of countering this challenge, I will support it, and show how an empiricist can agree to what I take to be the implied view. This requires me to make the challenge precise in one particular way; it is possible of course that it is not how a transcendental idealist would make it precise. But my reading of it may provide a basis for discussion.

The objects of our knowledge are objects we represent, or are able to represent, and I'll assume we can give verbal representations, that is, descriptions in our language in use. The ideas of dependence and independence I'll take to be expressible roughly and informally by talk of what would be the case if we were not in a position to give such descriptions. If that is accepted then the above transcendentalist challenge appears to imply (but even this is once again deceptive):

> It does not make sense to say that **there could be** things that are not describable (in our language in use) and hence not knowable, and thus also certainly not known.

I can imagine an attempt to reduce this implication to absurdity, which would proceed as follows. Consider the sparrows: we can describe them, and can say (truly or falsely) that there are sparrows, by means of the sentence "There are sparrows". Now suppose for a moment (something that I will deny below), that

> Instances of Tarski's celebrated equivalence schema
>
> 'A' is true if and only if A
>
> (which is the same as
>
> 'A' is a true sentence if and only if A)
>
> are tautologies in the same sense that any logical truth such as
>
> A if and only if (A or A)
>
> is a tautology.

In that case

> 0. 'There are sparrows' is a true sentence
>
> is tautologically equivalent to
>
> 1. there are sparrows

and so 0 and 1 can be substituted for each other in any context in the same language.[22] So then we could infer that the sentence

2. If we had no such word as 'sparrow', then 'There are sparrows' would not be a sentence

which implies

3. If we had no such word as 'sparrow', then 'There are sparrows' would not be a true sentence

therefore also implies

4. If we had no such word as 'sparrow', then it would not be the case that there are sparrows.

But this conclusion is absurd. What went wrong?

We know what went wrong. Instances of Tarski's schemas are not tautologies in the sense of standard logical truths: they are *pragmatic tautologies*. A language *user* cannot deny, can only assent to, an instance of Tarski's schema in the user's *own* language.[23] So, given that we do have the word "sparrow", we cannot but assent to line 0 above. What would be quite egregious is to infer from this reflection that[24]:

5. If we had no such word as 'sparrow', then we could deny the equivalence of " 'There are sparrows' is a true sentence" and "There are sparrows"
6. If we had no such word as 'sparrow', then we could deny one of the sentences " 'There are sparrows' is a true sentence" and "There are sparrows" while asserting the other.

These are just plain silly: if we had no such word as 'sparrow' then we could not deny or assert any sentence containing that word at all!

But now let's add that in fact we do have the word "sparrow". Therefore we are in a position to say

7. There are sparrows, and even if we did not have the word "sparrow" there would still be sparrows

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[22] That includes modal and other intensional contexts, but not hyper-intensional contexts, nor occurrence inside quotation marks, and (important for our argument here!) not always in context-dependent or indexical discourse.

[23] See my 1997. I am leaving out some niceties here. Given that the language may have truth value gaps, the schema cannot be read as a material equivalence, but only as indicating that each side can be validly inferred from the other.

[24] I do not accuse anyone of actually making such inferences. Everyone knows that from 'if A then B' you cannot infer 'if not A then not B'. But in this case there is the somewhat subtle distinction between a logical tautology and a pragmatic tautology to be brought into the light of day.

where the latter counterfactual is based solely on the reflection that

The sentence “There are sparrows” does not logically imply “we have the word ‘sparrow’ ”[25]

which is an innocuous logical point, with no philosophical or practical implications. We should also note of course that the following is correct:

Under no conditions could we assert “There are sparrows but we do not have the word ‘sparrow’ ”[26]

for the assertion in question would be as much a *pragmatic contradiction* as the statement “There are no statements”. You cannot use a word and simultaneously refuse to acknowledge it as belonging to your own language. (You could *mention* it; that is different.)

Perhaps this is again a Wittgensteinian point. In the words of some commentators (more or less), there is no point of view we can have external to our own language.[27] If you want to specify truth conditions for the statements in your own language in use, you will be doing so in your own language in use – don’t even think of avoiding *this* circularity.[28]

So where are with respect to the transcendental realist? What I wrote in line 7 sounds like one of the transcendental realist’s favorite reflections, and so threaten a dangerously metaphysical commitment:

8. There are sparrows, and even if we did not have the word “sparrow” there would still be sparrows.

But we have seen how we can assent to it, with no metaphysical implications at all – after we make clear the pragmatics of language use, the examples have no bite, they are innocuous and support no metaphysical position at all. What fails to make sense is not anything like 7, but rather the gloss put on it when read through metaphysicians’ eyes.

So what does this mean for Challenge 3? A philosophical question so often collapses into one a logical question and a factual question. Are there objects for which we have no representation? There are numbers for which there are no numerals, and asteroids that have not been, and may never be, given names. But numbers

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[25] Nor does the one logically imply the other if we add in all the information we can pertinently keep fixed in this context.

[26] To see the parallel, note that this amounts to: the sentence ‘There are sparrows’ pragmatically implies ‘we have the word “sparrow”, where X pragmatically implies Y if the language user cannot under any conditions assert X while denying Y.

[27] Crary and Read, *The New Wittgenstein*, Introduction. Cave! Such an informal statement invites realist construal: if it makes sense to deny that there are points of view we can take external to our own language, then the denial of this denial (the assertion that we can) also makes sense. That is not intended here.

[28] Or of going beyond the near-triviality of Tarski’s schema when it comes to truth – a fortiori. I take Kant’s argument, that there cannot be a criterion of truth that is both material and general, to support this point. See *Critique of Pure Reason*, Second Part. ‘Transcendental Logic’; Introduction. ‘Idea of a Transcendental Logic’.II. Of Transcendental Logic; III. Of the Division of General Logic into Analytic and Dialectic, A58, B 83.

and asteroids are not things that fall under no heading at all in our language. Any attempt to give an example of something that does would have to be done in our language and thus self-defeating. But a logical distinction remains, as it does for the example "There are no statements". No one could express a truth by saying or writing that, but what it means does not violate logic – and this reflection has no metaphysical implications.

4.4 My Remaining Realism

Transcendental idealism got off the ground with Kant's critique of the seventeenth century metaphysics continued in his own century by Wolff and by himself when young. Empiricism today, trying to make good on the promise of logical empiricism, which got its start in neo-Kantian circles, must accept that critique as well-taken (even against its own seventeenth century historical roots). But how shall we go on from here?

In stating his or her own philosophical view the philosopher uses a form of discourse that s/he trusts for that role. Philosophy is carried on in dialogue, not in oblivion of other views. So the form of discourse used purports to be a common basis for participants in the dialogues in which that view is proposed and defended. When that purported common access is missing, then we have warrant for that famous bit of abuse: *philosophy is the art of speaking nonsense in language expressly designed for that purpose*.

The common basis that I assume is language in which reference is unproblematic to trees and mountains, people and books, to lightning and car crashes, as well as to the processes of aging, burning, and flooding. These are all observable, and I emphasize reference; the descriptive terms we use are often theory-laden or culturally parochial; that does not to me make reference problematic. In the language I trust I say for example that there were mountains long before us.[29] To this I could indeed add that it is as much part of the concept of mountain that it is an old geological feature, as it is part of the concept of electron to be electrically charged. But whatever is meant by such additions is not to be something that contradicts the original assertions – nor to take away my assertion of them as literal factual truths. Our reflections *about* our language and concepts cannot, on pain of incoherence on our own part, contradict what we assert *in* our language.

This is not to set our discourse beyond critique. To see philosophy as always conducted in a trusted language-in-use does not imply seeing it as oblivious of the language it lives in, nor as resistant to language-change. But in this, as elsewhere, we

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[29] There would have been mountains even if conscious beings had never existed. On my view of counterfactuals this is true simply because the statement "there are mountains", even if supplemented with other information that we hold fixed in this context, does not logically imply "conscious beings exist". Realists of a different sort – realists with respect to modality – will quarrel with both transcendental idealists and empiricists on this issue, insisting on truth-values for counterfactuals that are not determined by such logical and contextual relations.

always start from where we are. We can't step out of where we are into a presupposition-less discourse any more than into a view from nowhere. This trusting start, by us mariners who repair our boat on the high seas, is precisely what is not made explicit when we state our positions. To some extent, the lack of explicitness is inevitable. If we do pay attention to our own language in use, and begin to make its features more explicit, we still do that in a part of this very discourse. Thus we continue to rely on its trustworthiness implicitly in that very act. We can bring only so much of what we are doing to light, for this bringing to light is, after all, a part of what we are doing.

Description and assessment of both scientific activity and scientific product, as I characterize them, are carried out within that common sense realist discourse – the same that I designated above as the trusted basic discourse in which I formulate my views on constructive empiricism, voluntarism in epistemology, and the like. Is this realism? A kind of realism, certainly, but I think not one that involves us in noxious metaphysics.

References

Crary, Alice and Rupert Read (eds.) *The New Wittgenstein*. New York: Routledge, 2000.

Finch, Alicia "Review of B. van Fraassen, *The Empirical Stance*", *American Catholic Philosophical Quarterly* 77 (2003), 302–307.

Lewis, David Kellogg, *Philosophical Papers*, vol. I. Oxford: Oxford University Press, 1983.

Lewis, David Kellogg, "Putnam's Paradox", *Australasian Journal of Philosophy* 62 (1984), 221–236.

McGowan, Mary Kathryn *Realism or Non-realism; Undecidable in Theory, Decidable in Practice*. Dissertation, Princeton University, 1996.

McGowan, Mary Kathryn "Privileging Properties", *Philosophical Studies* 105 (2001), 1–23.

McGowan, Mary Kathryn "Realism, Reference and Grue: Why Metaphysical Realism Cannot Solve the Grue Paradox," *American Philosophical Quarterly* 40 (2003), 47–57.

Putnam, Hilary "Realism and reason" (1976); reprinted pp. 123–140 in his *Meaning and the Moral Sciences*. New York: Routledge, 1978.

Putnam, Hilary *The Many Faces of Realism*. LaSalle, IL: Open Court, 1987.

van Fraassen, Bas "Putnam's Paradox: Metaphysical Realism Revamped and Evaded", pp. 17–42 in *Philosophical Perspectives*, vol. 11. Boston, MA: Blackwell, 1997.

van Fraassen, Bas "The False Hopes of Traditional Epistemology", *Philosophy and Phenomenological Research* 60 (2000), 253–280.

van Fraassen, Bas *The Empirical Stance*. New Haven, CT: Yale University Press, 2002.

A Physicist's Approach to Kant

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Abstract Since Kant's time considerable developments in physics greatly modified the set of the conceivable word views that are compatible with what we factually know. And this, in turn, was bound to induce substantial changes as regards the relationship between Kantism and physics and the degree of compatibility of the former with the latter. The main changes are examined. As could be expected, it is found that several significant aspects of Kantism, including both arguments in its favor and consequences derived from it, cannot be kept in their original form. On the other hand it turns out that quantum physics as well as the outcomes of recent physical experiments yield strong support to two of its most essential features, the ideality of space (or space-time as now we would preferably say) and the (correlated) fact that, far from being independently existing out there, phenomena are essentially representations in our mind.

Why is it that women sitting at a tea table all see a teapot standing on it and that later, if asked, all of them agree that they then saw one? Upholders of various philosophical doctrines do not all give the same answer to this question. Adepts of "transcendental realism" (to use Kant's terminology) – that is, "men-in-the-street", the overwhelming majority of scientists and you and I when we de not philosophize – just simply answer: "because a teapot was really there, period". Vienna Circle positivists and very strict neo-Kantians point out that the question is a metaphysical one and that metaphysical questions have no answer. As for Kantians, most of them, I guess, would at least agree with Schrödinger that the realist's answer is flawed. Erwin Schrödinger was a very great physicist, one of the main founders of quantum mechanics. But at heart he was a philosopher, and in fact one of the very few philosophers who condescended to cast a look at this question, which may be termed "the intersubjective agreement puzzle". His contention went as follows. He first noted that the realist's answer obviously rests on the postulated similarity between the real world R and the mental picture each one of the participants (the

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involved women) has in her head. And he pointed out "Whoever thinks this way forgets that *R* is not observed."[1] Of course, Schrödinger had there in mind the fact that to ask "why" is to ask for a cause and that, according to Kantism, causes operate only from phenomena to phenomena. It is true that, at some places Kant himself mentioned what he called the "transcendental object" and claimed that it is the ultimate cause of all that appears to us. But he mentioned it in the singular and immediately stated that this object is forever unknown from us; so, clearly, we are not supposed to identify the teapot with it.

A closely related difficulty that readers of Kant have to cope with concerns Kant's theory of the ideality of space. Indeed, while, as a rule, philosophers take this notion to be hardly objectionable (they got accustomed to it to the extent that, not unfrequently, they just forget to mention it when they report on Kantism) most scientists and scientifically oriented people are utterly taken aback by it. "How that – they ask. – Is the distance between us and Sirius, or the fact that Great Britain is an island, just a representation in our heads? In a sense – they grant after reading Berkeley – maybe it is. After all, we are unable to prove it isn't. But then – they say, falling back on the teapot riddle – how is it that we all agree about contingent facts such as these ones?". The most inquiring among them turn to *Transcendental aesthetics* in the hope of finding a clue, but it cannot be said that they find one matching their thirst. True, they read there that space is "pure intuition", that is, pure form; and that the form of the phenomena – what orders them according to given relationships – is something fully general, a priori present in our mind and ready to apply to any phenomenon in particular. They go so far as to grant that, in a way, this might explain that all of us have, or seem to have, the same general spatial notions. That we understand one another when we speak of circles and triangles. But they fail to grasp in what way it explains that we all agree about our seeing, or not seeing, a teapot on a table. Whereas, of course, if space and objects are considered existing as they are, quite independently of the existence of the human minds, and are knowable by us, the explanation is obvious: we see a teapot there just because a teapot *is* there.

Since this "intersubjective agreement riddle" reveals an aspect of Kantianism that makes physicists uneasy it may constitute a good starting point for entering the "Kant and the physicists" dialectic. In fact, Kant himself seems to have considered the problem nonexistent; I mean, he, apparently, took its solution to be trivial to the extent of not even calling for any explicit examination. If we try to, nevertheless, find out by ourselves his reason for this attitude we may conjecture that, after all, it still was, somehow but implicitly, linked to his notion of a "transcendental object" considered to be the "purely intelligible" cause of phenomena in general. We may remark however that for an explanation along such lines to really hold good concerning contingent facts it is necessary that (contrary, as it seems, to some of Kant's own assertions) these contingent facts should have some sort of one-to-one correspondence with features within the transcendental object.

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[1] E. Schrödinger, *Mind and Matter*, Cambridge University Press, 1958.

I mean: for both you and I to have the impression that the table and the teapot are roughly at the same place it seems necessary that the transcendental object (alias: reality per se) should itself be endowed with some sort of contingent features that, even though not imbedded in space–time and not knowable, still are the causes of our contingent perceptions.

In theory such an idea is, at least, tenable, in the sense that it is not at variance with any law of logic. At first sight however, as an explanation of the intersubjective agreement it looks far fetched and, what is worse, hopelessly vague. It is understandable that many classical physicists preferred the above-stated simple "realist" explanation: "we see them at the same place because they *are* at the same place". I think therefore that if physics had just continued along the conceptual lines fixed up by Galileo, Newton, d'Alembert, Laplace and others, physicists, with but few exceptions, would have gone on considering space to be real rather than ideal and the notion we have of it to be an adequate one, faithfully mirroring reality as it really is.

The reason why they could rationally have taken up this standpoint notwithstanding Kant's objections to it becomes even clearer when we consider in detail the nature of the said objections. Indeed, concerning space Kant's argument went as follows. For me to be able – he pointed out – to refer my sensations to something external to me (that is, something situated at some place other than where I am); and, correlatively, for me to be able to picture to myself the things as being "outside there" and lying next to one another, and hence as being, not only different but also situated at different places, it is necessary that I should already have some idea of space. Hence this idea cannot be experimentally derived from observed relationships between external phenomena. Now, the argument looks convincing enough but is it really? I consider that, at least to us, who are informed of the evolution of species, it is not, for it ignores the power living beings have of learning by apprenticeship. Clearly neural systems are able to "take notice" of which associations of small gestures and ideas have been useful and which proved inadequate, and progressively favor the first ones. We, who have good reasons (which, admittedly, Kant had not!) for believing in evolution, may we discard the assumption that a "real space" always existed, and that, within living beings, a similar process of trial and error gave rise to the progressive discovery of it? I mean: may we convincingly do so just by means of sound reasoning? As for me, I don't think we can.

On the other hand, it is a fact that physics did *not* continue along the conceptual lines that had proved so fruitful during its first 2 centuries. New discoveries forced physicists to gradually weaken the rule Descartes had stated, according to which inanimated nature is to be ultimately described exclusively by means of the familiar concepts this author took to be innate. Admittedly, at first this evolution also seemed to speak against Kant's standpoint. Indeed, the discovery of (special and general) relativity definitely showed that, contrary to one of Kant's basic axiom, the a priori forms of human sensibility are not, at least concerning space and time, the right concepts with the help of which physics is to be constructed if it is to account for the observed data. But then quantum physics came into being and this changed very much the whole picture. Not, of course, that it restored Kantism in its original form. There could be no question of salvaging Euclidean space and Newtonian universal time as a priori – and therefore ultimate – framework elements of physics! But still, quantum mechanics sort of comforted Kant's conceptions on such matters, in that it cast serious doubts, if not on the ultimate reality of space itself, at least on the pertinacy of the view that at any time, all the physical objects we are able to discourse on are, by themselves, at definite places in space. It is true that, by demonstrating the inappropriateness of the notion that light is an aether wave, relativity had, in a sense, already paved the way for this rejection, for it is only when waves are viewed as being the motion of some stuff (aether for example) that the idea of strict localization in space remains clear (localization being then attributed to each one of the elements of the said stuff). Being simultaneously present at an infinity of places, fields (electromagnetic and others) assumed to exist by themselves independently of any support obviously do not fall within this category. But, as we shall presently see, quantum mechanics strongly suggested going much farther along this line.

The point is that in elementary quantum mechanics the wave function notion is the basic one, that when viewed as *per se* entities, one-particle wave functions are just as incompatible with locality as fields are (with the additional difficulty that measurements "reduce them at a distance"), and that the case of multiparticle wave functions is even worse since speaking of their value at a given point simply makes no sense. Unfortunately, for reasons that cannot be here entered into, more sophisticated descriptive tools that can be substituted to the wave function fare, in this respect, no better. And in fact it is not only on the notion of localized, per se existing objects that quantum mechanics throws discredit. In reality it does so concerning the general notion of *individual* such objects. It thereby forces clear-sighted quantum physicists to take up a position comparable to Kant's one in that both he and they deny that we can gain genuine knowledge of what reality-per-se really is (this at least is true regarding the contingent aspects of Reality; the question concerning its law-like structures – known as the "structural realism" question – will be examined below).

It may therefore be claimed that – on the basis of arguments grounded on physical data unknown in Kant's time and radically differing from Kant's purely philosophical ones – quantum mechanics sort of leads us towards a conception of the relationship between mind and the world that has rather striking similarities with the Kantian one and seems therefore to back it. And this impression is still reinforced when it is observed that in the process of doing so quantum physics happens to greatly alleviate the main difficulty we noted Kantianism met with (at least in the eyes of a physicist), namely the one of accounting for our intersubjective agreement concerning the (contingent) localization of an object.

To see how this goes let us state again the difficulty in terms of the teapot example. If Alice is a conventional realist she judges that the teapot is really on the table and that Betty therefore also sees it there. Consequently she may predict that when they come to meet their memories of the event (the records, say, in their notebooks) will coincide. Whereas, on the contrary, if, along the line of Kantism, she does not believe the objects really exist "per se", quite independently of us, she has no obvious reason to conjecture that also Betty sees a teapot on that table. Nothing therefore prompts her to make the prediction in question so that, when the two, later, compare their records the observation that they coincide must appear a fortuitous coincidence. And it then seems miraculous that such coincidences should take place over and over again.

In the foregoing I sketched a reasoning by means of which a Kantian might try to overcome such a difficulty but I also noted that it looks quite vague and artificial. It is therefore worthwhile to observe that in quantum physics the difficulty may be said to vanish, in the sense that the basic quantum laws do predict the intersubjective agreement in question. To show they do, let us imagine replacing the teapot by the pointer of a measuring instrument and assume this pointer may only take one out of two possible positions U and D; and assume further that Alice and Betty both look at this pointer after the instrument was made to interact with some quantum system initially prepared in one of the states the physicists call "quantum superpositions of the two eigenstates corresponding to U and D". In such circumstances what Alice will see is not predetermined. The formalism just yields the probability for her to have the impression of seeing the pointer at, say, position U. Same of course concerning Betty. But the crucial point is that it yields a probability equal to zero for Alice to see it at U *and* Betty to see it at D, which indeed shows that intersubjective agreement holds good in this case. Note moreover the most significant point that to get this result there is no need to postulate that the pointer itself "is really" at some definite place independently of whether or not it is looked at. Indeed, the formalism even forbids that this postulate should be made. To repeat: it just yields the probabilities that, when observed, the pointer will be seen to lie either at one well-defined place or at another well-defined place. But in compensation, so to speak, it provides us, as we saw, with the certainty that Alice and Betty will both see the pointer lying at the *same* place. Admittedly, the difficulty Schrödinger pointed to is, thereby, removed only at the price of taking for granted the predictive laws of quantum mechanics, and some may judge that such predictive laws should, in turn, be somehow explained. On the other hand, everybody agrees that, to avoid infinite regress, the quest for explanation must be stopped at some stage, and it is commonly held that an explanation is valid if it is grounded on sufficiently general laws. Since the predictive quantum laws seem to be quite universal it may be considered that the condition is met and that, consequently, the above stated explanation is a genuine one as it stands. And, in fact, this conclusion is all the more gratifying as (contrary to what is the case in classical physics in which realist interpretations are straightforward) to interpret quantum physics as being a description of "what really exists out there quite independently of us" proves immensely difficult.

The latter point clearly speaks in favor of the idea that the perceived objects are not of the nature of objects per se, and this inference is further backed up by the, relatively recent, discovery of *physical nonseparability*. Thanks to the Bell theorem, physical nonseparability, alias *nonlocality* could be experimentally established, quite independently of the quantum mechanical axioms, and it shows that if events took place quite independently of any knowledge we have about them there should exist, between some of them, correlation effects that seem practically impossible to interpret otherwise than by assuming influences between them whose magnitude would be independent of the distance. Since this conclusion is at variance with relativity theory when the latter is also interpreted realistically – and since relativity theory is experimentally well established – we must therefore admit, very much in line with

Kant's views, that the perceived events – hence also the perceived objects – are but "for-us". That they are products of an "objectivization process".

Under these conditions it may at first sight look surprising that, when quantum mechanics first appeared, prominent philosophers and physicists who wrote on Kantism considered that this new theory struck a severe blow to the latter. Heisenberg, in particular, explained to his students that Kant's approach had to be radically reconsidered. Cassirer expressed similar views. It is worthwhile to examine the reasons they had of judging that way.

The task is not very easy for, in the texts of such authors that are meant to enumerate which basic conceptions quantum mechanics forces us to give up, criticisms that do have a bearing on Kant's views often happen to be intimately intermingled with ones that merely concern the classical approach. And, moreover, among those that were explicitly addressed to Kantism some were stated in very general terms, leaving room for ambiguity. On the whole, however, we may consider that the occurrence of all these, generally pertinent, reservations was due to the fact that, in parallel with his rejection of transcendental realism, Kant had built up what, apparently, he took to be a workable substitute to it: a substitute which, presumably, he thought would make it possible to discard radical, Berkeley-like, versions of idealism. However that may be, in the course of the nineteenth century most philosophers and apparently many of the scientists who cared to reflect on the nature of science had found this substitute conceptually reasonable and had adopted it (or, perhaps, reinvented it). In short, it amounted to alleviate the loss of transcendental realism by justifying the use of, at least, a *universal objectivist language*, a justification that, finally, enabled scientists and laymen alike to quietly think, argue, behave etc. *as if* transcendental realism were true. Now, as we shall presently check in detail, the advent of quantum mechanics did indeed strike severe blows to this Kantian construction, which of course justifies the reservations then expressed about it and Kantism in general.

One such blow had to do with the combination of causality and space–time description with the help of which the universal objective language substituted to realism had been built up. Language refers to experience. It uses words, most of which designate concepts and, among them, abstract concepts such as substance and causality (called by Kant "categories"). And as for experience, it makes use of such notions as those of (directly or indirectly observable) events, which are intrinsically space–time localized. A universal objectivist language is therefore one in which the said concepts – among which, causality – are susceptible of being applied to everything that might conceivably be observed, that is, essentially, to all events. According to Kant, human imagination then applies them through the procedure that Kant called "schematism". In particular, the scheme of causality consisted, he stated, in the succession of events in time inasmuch as this succession follows from a rule (as we know, Kant identified causality with Leibniz' *Principle of Sufficient Reason*, that is, with determinism). Now, in quantum mechanics there is indeed a rule governing time evolution, and this rule is quite a strict, deterministic one. It is represented by the unitary evolution operator (in nonrelativistic quantum mechanics, by the, more familiar, time-dependent Schrödinger equation). But this rule does not bear on a succession of conceivably observable events. It merely yields – at any given time – the probabilities we have of observing such and such event were we to perform the appropriate manipulations. Moreover, the formal structure of quantum mechanics is such that even just to assume the existence of some (hitherto unknown) succession rule of the type that Kant had in mind is incompatible with it. This is probably what Cassirer meant when he wrote that schematism got finally limited by the advent of quantum mechanics as we may not any more combine causality and spatio-temporal description.[2] And, without explicitly referring to Kant, Bohr expressed similar views when, concerning complementarity, he explained that the latter concept entailed strict incompatibility between spatio-temporal description of phenomena and dynamical conservation laws.

The just reviewed incompatibility between quantum mechanics and Kantianism proper is not the only one in existence. In fact, another one was pointed out, at a rather early stage, in particular by Cassirer (*loc.cit.*). It differs from the former one in that it does not concern the impossibility of combining space–time description with causality but, more radically in a way, the impossibility of reconciling quantum mechanics, independently of causality problems, just simply with phenomenal space–time descriptions. Or at least with the type of space–time descriptions (involving positions and velocities) that, according to Kant, should in principle be possible. It has to do with what Kant called the "law of complete determination". Any object, Kant stated (in the *Critique of Pure Reason*) is subject to this law, according to which: "If all possible predicates are taken together with their contradictory opposites then one of each pair of contradictory opposites must belong to it (the object)". It is clear from his writings that Kant tightly linked this law with his notion of (empirical) reality. And the incompatibility alluded to consists in the fact that the said law is obviously at variance with Heisenberg's uncertainty relationships. Incidentally, it is interesting to note that it is also inconsistent with the best known of the so-called "ontologically interpretable" models meant to reproduce the observable predictions from quantum mechanics, namely the one originally sketched by Louis de Broglie, later greatly developed by David Bohm and commonly known as "the Bohm model". Or, at least, it is so as long as, in Kant's spirit, we identify "predicate" with "what should, in principle, be measurable". For, the momentum $\boldsymbol{p}$ of a particle is a predicate in this sense and still, in the model, it is in general not true, concerning some still unobserved particle, that its momentum $\boldsymbol{p}$ either *has* value $\boldsymbol{a}$ or *has not* value $\boldsymbol{a}$. In fact, a statement of this type is correct concerning another physical quantity, π, which, in the model, possesses indeed all the features that, in classical physics, characterise momenta, but π is not observable.

Of course, "orthodox" Kantianism has to face other, better known, difficulties, that add to the two just explained ones. The one concerning relativity theory was

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[2] E. Cassirer, *Determinism and Indeterminism in Modern Physics*, New Haven, Yale University Press, 1956, quoted by C. Chevalley, *Objectivité et intersubjectivité chez Bohr*, Epistémologiques, **1**, p. 315 (2000).

already alluded to above. It does not just boil down to the necessity of substituting one concept to two other ones (space–time to both Euclidean space and universal time) a change that, from a philosophical point of view, might be considered inconsiderable. In fact the change in question *is* philosophically significant, essentially because it forces Kantianists to give up the most basic tenet of *Transcendental Aesthetics*, namely the principle that human knowledge about things must necessarily fit in the mould of the a priori forms of human sensibility. Another well known mismatch between Kantism and contemporary physics stems from the – above mentioned – fact that Kant believed in strict – that is, deterministic – causality whereas the basic quantum mechanical laws are intrinsically indeterminist. It remains true however that, even though they are quite serious, the two last difficulties, taken by themselves, could be overcome: The one about sensibility by substituting to the mould of human sensibility the mould of human understanding (as neo-Kantians did). And the one about determinism either by substituting – as a requirement – statistical determinism to strict determinism or by substituting the Bohm model (which is determinist) to conventional quantum mechanics. On the other hand, when a would-be Kantianist considers these difficulties together with the two first described ones, he/she may well feel worried. Clearly, this accounts for the feeling that, as above noted, many people had at the time: the feeling that the advent of quantum mechanics bore quite a severe blow to Kantism.

Now then, when all is said and done what opinion should we have concerning the degree of matching or mismatch between Kantism and quantum theory? To try and answer this question we must remember that, at the time when this theory appeared, scientists and philosophers alike had realized that giving up all ontological prejudices, keeping very close to experiment and operationally definable notions, was by far the best procedure for generating reliable knowledge. At the same time however, classical physics had brilliantly developed and conveyed with it a world view implicitly but quite strongly centered on realism. And Kantism had been taken to offer an ingenious way of reconciling these two apparently somewhat opposite features by making it possible to go on reasoning in terms of objectivist realism while keeping quite radically away from ontology. In short, Kantism was essentially considered to be a conception tightly linked with classical physics and making the latter philosophically acceptable. When it became clear that classical physics utterly failed to account for such data as, for example, atomic spectra people immediately realized that in this respect its "Kantian" version fared no better than its naive "ontological" one. And this clearly is what explains the trouble that, as noted above, was expressed upon the advent of quantum mechanics by philosophers and physicists accustomed to the said "Kantian version".

However, all this, now, is an old story. It is true that contemporary physics forces us to give up, or at least considerably weaken, schematism and drop the "complete determination" law, both of which were significant, although non central, elements of Kant's thinking. But it more than compensates this blow by practically compelling us to adopt the idea that was, in fact, at the very core of Kantism and constitutes its truly original contribution to philosophical thinking, to wit, the view that things and events, far from being elements of a "reality per se", are just phenomena, that is, elements of our experience.

We are therefore finally led, by arguments that are altogether different from those Kant used, to a conception that partakes of Kantism, not, admittedly, as regards details but concerning what, to anybody interested in the question of possible world views, appears as, by far, its most essential feature. Under these conditions it would of course be appropriate to investigate which other features of Kantism physics incites us to keep and which ones it suggests we should drop. However, it turns out that concerning them the indications from physics are far from being as compelling as those are that concern the above considered issue.

Hence, I shall just – and briefly – consider but one such question, merely touched upon above. It concerns the "reality-per-se" (or "thing-in itself") concept and, more precisely, the question whether or not the said reality should be taken to have features. A reader of the *Critique of Pure Reason* may well have the feeling that Kant himself was not completely clear on the issue. True, in general, as noted above, he characterized "reality-per-se" (alias "the transcendental object") as being fundamentally unique, which seems somehow to imply it has no features. But in the Preface to the Second Edition he explained that even though we cannot know the objects as things-in-themselves, still we can think of them as such (and he even pointed out that it is necessary for us to do so since otherwise we would get to the absurd conclusion that there are phenomena [or appearances] without there being anything that appears). Clearly at that place he mentioned the said objects in the plural, which seems to suggest that reality-in-itself has, after all, some sort of (admittedly unknowable) features, or structures. Now, to some scientifically oriented minds this idea looks attractive. The point is that all scientists are deeply convinced that what they find has to do with a deep reality of some sort. Of course, many of them realize that their findings cannot be interpreted as constituting faithful descriptions of what reality-per-se really is. And quantum physicists are especially aware of this fact since the basic quantum laws, far from being descriptive, are, when all is said and done, merely predictive of observations. But still, they strongly dislike the idea that such laws should be of *exclusively* human origin; that the whole content of science should finally boiled down to mere words. They feel that there must be some reason for the fact that the, most general, observational predictive laws of quantum physics, which apply to all sorts of domains, always yield correct predictions (even in cases in which these predictions seem to defy common sense!). Indeed, along these lines they might, at first sight, incline toward a form of structural realism according to which the great, mathematically expressed, physical laws – such as the Maxwell equations – faithfully describe structures of reality-per-se. However, what we saw above concerning nonlocality seems to imply that, then, relativity theory would be violated. Since it could be shown that nonlocality does not make it possible to send superluminal messages this most unwelcome consequence can – fortunately! – be avoided. But it is possible to do so only at the price of not considering Maxwell's equations – and the great physical laws in general – to be faithful descriptions of "reality as it really is". Hence I think structural realism should be substituted with a conception according to which, even though reality-per-se is still structured, its structures are not knowable as they really are. They act as hidden sources of the physical laws we know.

Hence, to put it all in a nutshell, the situation seems to me to be as follows. On the one hand present day physical knowledge prohibits us from keeping Kantism in its integrality. We must give up, not only (as has been known for a long time) the notions of Euclidean space and universal time but also schematism, the "complete determination" law and, as it seems, the view that reality-per-se is totally structureless. But on the other hand, the same present-day physical knowledge unexpectedly incites us to adhere to a modified form of Kantism that preserves and enhances those among its features that, a priori, take most scientists aback. The main one among these is, by far, Kant's basic idea that objects, events etc. are mere phenomena and not in the least elements of a "reality-per-se". Indeed, quantum mechanics and experimental discoveries related to it have confirmed this Kantian view to such an extent that it has now become very difficult for a well-informed scientist to go on believing that science will ever completely lift the veil of appearances and describe "Reality as it really is".[3] Philosophers will hardly believe it but in the mind of most scientists this, still nowadays, comes as a most unwelcome surprise and a conceptual revolution quite comparable in its magnitude to the one Copernic subjected astronomy to (Kant hit on the proper image!). On the other hand, should the list of the features of Kantism that should be kept be limited to just his "Copernician revolution"? Personally, I do not think so. For the above stated reasons I consider that, contrary to the claims of neo-Kantians and other radical idealists, also the reality-per-se notion is a most significant one. I am quite convinced that it has to be kept.

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[3] Admittedly this last sentence may raise some surprise for what it states happens not to be a widely popularized truth. Let me therefore comment a little about it. In fact, not only in the articles they write for the general public but also in their properly scientific papers, physicists currently state that electrons "exist" in atoms, that such and such types of quarks have been found to "exist" and so on. This language is not improper (after all, Kant himself often used such words as "real" and "reality" to refer to phenomena, that is, to mere human representations), but it is misleading nevertheless, in that it strongly conveys the idea that such objects may be believed to exist in some absolute sense, whereas, in standard quantum physics, this, in the last analysis, is not a tenable hypothesis. True, also classical physics may be conceived of as only referring to phenomena in a Kantian sense; and this is actually the way Kant interpreted it. But the main conceptual difference between classical and quantum physics is that the basic laws and principles of the former are of such a form as to be interpretable (naively, a kantian would say) as referring to absolutely existing entities. Not so concerning those of quantum physics.

Structural Realism and Abductive-Transcendental Arguments

Holger Lyre

Abstract The paper deals with an attempt to present an "abductive-transcendental" argument in favour of a particular version of structural realism (SR), dubbed Intermediate SR. In the first part of the paper the general structure of transcendental arguments is scrutinized with a close view on Kant's original version and the prospect of their abductive variation. Then the role of symmetries in modern physics, especially symmetries without real instantiations and in particular gauge symmetries is discussed. This is combined with a presentation of SR as a promising current version of scientific realism. The discussion is supported by various arguments from gauge theories in modern physics. Intermediate SR, a realist position about all and only structurally derivable entities located between the extremes of Epistemic and Ontic SR turns out as the best fit to our current fundamental gauge physics and this finally leads to an abductive-transcendental reasoning concerning this position.

1 Transcendental and Abductive-Transcendental Arguments

It seems to be a clear lesson from the history of modern physics, that the Kantian program – taken literally – is wrong-headed. Kant's way of deriving Newtonian physics in his *Metaphysical Foundations of Natural Science* (1786) seems evidence enough that his way of deriving the fundamental laws of physics is fatally flawed. Why, then, should one be interested in the Kantian program at all? Indeed a reason could be that if we strip off the inclinations of armchair philosophy from Kant's program, there might remain a core which could be still of value in modern science.

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This core, the core of the whole Kantian enterprise, survives in the structure and nature of transcendental arguments on which we will focus in the following.

Kant introduces transcendental arguments as arguments referring to the very preconditions of experience. In Critique B 25 (Kant A 1781/B 1787) he writes:

> *I call all cognition transcendental that is occupied not so much with objects but rather with our mode of cognition of these objects insofar as this is to be possible a priori.* (p. 149)

And in B 80 we find:

> *…that not every a priori cognition must be called transcendental, but only that by means of which we cognize that and how certain representations (intuitions or conceptions) are applied entirely a priori, or are possible (i.e., the possibility of cognition or its use a priori).* (p. 196)

Hence, transcendental knowledge is meta-knowledge about how a priori knowledge is used. As such, transcendental knowledge is itself a priori knowledge. We must therefore first clarify our understanding of the a priori. One of the difficulties with Kant is that in his formulations he invites us to misinterpret the way in which his account leads to a subject-relatedness of knowledge. We may very well understand his talk about "modes of cognition" and "representations" as talk about the particular human cognitive machinery. And Kant endorses such an understanding for instance in passages like A 26–27 about the a priori character of space:

> *We can therefore speak of space, extended objects, and so on, only from the human standpoint … For we cannot judge at all whether the intuitions of other thinking beings are bound to the same conditions that limit our intuition and that are universally valid for us.* (p. 160)

From a general epistemological perspective, however, it is certainly not necessary to restrict the a priori forms of cognition especially to humans. On the contrary: in B 72 for instance Kant, too, adds that

> *It is also not necessary for us to limit the kind of intuition in space and time to the sensibility of human beings; it may well be that all finite thinking beings must necessarily agree with human beings in this regard (though we cannot decide this)…* (p. 191)

Kant's point is in fact far better made if we think of the preconditions of experience – manifested in the a priori forms of cognition – as entirely general methodological preconditions. Call this the *generality claim*. It is the claim that the results of the transcendental method aren't at all bound to human experience or cognition in particular, but rather to experience or cognition in general. And that they therefore apply to any cognitive system or agent with "empirical competence."

While it is perhaps not entirely clear whether Kant himself definitely subscribes to the generality claim, he nevertheless subscribes to a further feature of his enterprise, which is in fact a natural consequence of the generality claim. This feature is the view of the a priori as being no part of the cognitive machinery itself, or, to put it bluntly, the view that the a priori forms of cognition are not themselves instantiated in terms of the mind's hardware. Surely, Kant falls back on a whole variety of capacities of the mind such as intuition, understanding, faculty of judgement or the like to spell out his aprioristic approach, but simultaneously he explicitly denies that ideas a priori – neither pure intuitions nor categories – are innate (see for instance the "Streitschrift gegen Eberhard", 1790). Speaking in more modern terms, the a priori forms are not subject to empirical cognitive science, as also pointed out in the Critique B 78:

> *[Transcendental logic…] "has no empirical principles, thus it draws nothing from psychology (as one has occasionally been persuaded), which therefore has no influence at all on the canon of the understanding. It is a proven doctrine, and everything in it must be completely a priori."* (p. 195)

Thus, Kant subscribes to a mind-dependency of our knowledge in a rather indirect sense only. What he intends is a *methodological understanding of the a priori*: It is, for instance, a methodological requirement to bring the manifold of sense data into a temporal one after another, a spatial side by side, or a causal connectedness. It is the general structure of a transcendental argument from which the justification of the doctrine of such a methodological a priori can be drawn.

What, now, is the general structure of a transcendental argument? An informal version following Kant would be that the possibility of experience (E) demands the very preconditions of experience (PE) to hold. By reading this in the sense that the fact of E implies PE, one would make the argument an instance of modus ponens:

$$\begin{array}{l} E \\ E -> PE \\ \hline PE \end{array}$$

The major drawback of this reading, however, is that this doesn't render transcendental arguments as synthetic a priori, for neither the first nor the second premiss of the above conclusion is synthetic a priori. The first premiss, the mere fact that E, is contingent and thus synthetic a posteriori, whereas the second premiss represents an analytic statement. Obviously, the transcendental argument structure cannot be captured in a strict logical way, but should perhaps rather be reconstructed as an inference to the best explanation: the existence and validity of PE is the most plausible explanation for E.

Of course, the reconstruction of transcendental arguments as inferences to the best explanation heavily undermines Kant's own far more rigorous understanding of his enterprise. For it is Kant's special claim that synthetic judgements a priori are accompanied by necessity and generality. But it is exactly this demand, which should better be weakened in view of a modern revised use of transcendental arguments, and indeed in two senses: As the best explanation for E, PE is only very likely, but not necessary. Further on, the contingency of E as a premiss indicates that the conclusion doesn't yield in any possible world, but only in E-worlds, worlds in which experience takes place. We might therefore introduce the term *abductive-transcendental* argument for any inference to the most plausible and obvious preconditions of experience.

As a further digression from Kant's original approach, mention should be made that the introduction of the thing-in-itself isn't a necessary consequence of the transcendentally motivated methodological a priori. It is widely believed that perhaps the most urgent reason for Kant to introduce the thing-in-itself doesn't lie in his transcendental epistemology as such, but rather in the overall architecture of his philosophical system – in particular his account of free will beyond the dichotomy of compatibilism and non-compatibilism. Kant saw it as an explanation how we can have strict causal determinism in the realm of theoretical philosophy, where we are dealing with the things as phenomena, and at the same time the freedom of will within the realm of practical philosophy, where we are bound to the idea of absolute freedom.

To sum up: neither the general structure of an abductive-transcendental argument nor the methodological nature of the a priori depends on

- The existence of a particular, perhaps innate structure of our cognitive ability
- Necessity in the strong sense of being the case in all possible worlds
- The existence of epistemically inaccessible things-in-themselves

Remarkably, perhaps, such an understanding of a methodological a priori is in principle also in tune with a naturalistic picture of the world. Transcendental arguments impose constraints on the constitution of the knowledgeable world and possess insofar a certain normative character. Construed as abductive-transcendental arguments, however, that is as inferences to the best explanation, such normativity doesn't come equipped with absolute necessity.

2 Physical Symmetries Without Real Instantiations

From a transcendental meta-perspective about science we now shift to modern science directly, in particular to the role of symmetries. Symmetries are the hallmark of modern physics – and they come in many different varieties: we distinguish continuous from discrete, global from local, active from passive, geometrical from dynamical, and so on. Given the preeminent role symmetries play in contemporary physics, it is more than natural to ask for a deeper reason for their omnipresence. One such reason can be seen in the quite general characterization which can be given to symmetries as an overall *topos*: they seem to provide a formal device for the general interplay between change – captured in terms of symmetry transformations – and persistence – the symmetry invariants. Let us follow this general theme in some more detail.

Given any domain D (usually the state space of the physical system), a symmetry of D consists in a set of one-to-one mappings, the symmetry transformations, of D onto itself, such that the structure of D is preserved. The structure of D can be understood as a set of relations imposed on D. It can then easily be seen that the symmetry transformations of D form a group and simultaneously exemplify equivalence relations and, hence, induce a partitioning of D into equivalence classes. It becomes clear from such a characterization that symmetries can be used as tools to filter out specific structural aspects. Moreover, since the set of all symmetries of a domain D is sufficient to characterize the entire structure of D and if structural aspects provide the relevant aspects of a given domain, symmetries may very well work as filters for relevance.

Consider the simple example of the (cyclic) group of rotations of a square. We get four rotations around 90°, 180° and 270° as well as the identity mapping with 0° or 360° respectively. These four rotations R_1 to R_4 form a group G, that is any combined transformation $R_i \times R_j$ can be mapped onto one of the elements of G. G is the group of isometries of the square, that is the group of actions which preserve and characterize the "relevant structure" of the square as a geometrical figure.

A further distinction, not yet mentioned, is definitely important for modern physics: the distinction between symmetry transformations *with or without real instantiations*. The example just given is obviously a symmetry for which real instantiations in terms of rotations of squared corporeal objects in the world can be considered. Quite generally, space–time transformations instantiated in terms of rotations, translations or boosts of reference frames are paradigmatic cases of symmetries in physics with real instantiations. By way of contrast, symmetries without real instantiations include for instance scale transformations, coordinate transformations or, what will be our particular concern, gauge transformations.

One might in fact wonder why symmetries without real instantiations are of any value in physics at all. Apparently, such symmetries reflect a redundancy in the description of a system. Striving for non-redundancy such symmetries seem, on a rigorous level, to be superfluous. At the same time, however, the merit of symmetries about superfluous descriptive structure may nevertheless lie in their way of highlighting certain relevant structural invariants. In this sense symmetries without real instantiations fall precisely under the class of "filters for relevance."

Gauge theories in modern physics, that is theories which are characterised by their respective gauge symmetries, are most prominently displayed in the Yang-Mills theories of the Standard Model and represent the most important case of an application of symmetries as filters for relevance. Within the Lagrangian approach, the common mathematical framework of the practising high energy physicist, a gauge theory is mainly represented by a Lagrangian L built out of three terms: the kinetic part L_{kin}, the gauge field Lagrangian L_f and the interaction term L_{int}. In Standard Model gauge theories, L shows a symmetry under both global and local gauge transformations, that is transformations of the Dirac wave spinor which represents the fundamental fermionic matter fields and which is governed by L_{kin}. The requirement to formulate a respective gauge theory in the most general gauge covariant representation, that is to represent the theory in such a way that no dependency from the obviously non-instantiable gauge transformations exists (both global and local), is usually known as the gauge principle. This requirement is fulfilled on the Lagrangian level if L_{kin} is enlarged to $L_{kin} + L_{int}$, where L_{int} includes an inhomogeneous term – basically the gradient of the phase of the wave function – needed to satisfy the gauge symmetry requirement. As such, L_{kin} has no further physical significance (as sometimes mistakenly stated in parts of the textbook literature), the structure of L_{kin}, however, does of course reflect the particular gauge symmetry. Technically speaking, L_{kin} mainly includes the connection field of a principal fiber bundle, the appropriate geometrical structure to describe a Lagrangian gauge theory. The connection has values in the Lie algebra of the Lie structure group of the bundle, the physically considered gauge group (for a detailed presentation see for instance Lyre, 2004a, b).

The gauge symmetry group not only determines the connection, physically identified as the gauge potential, but also the derivative of the potential, the gauge field tensor (out of which L_f is built). While the gauge potential isn't a directly observable, i.e. measurable, quantity, the gauge field strength is. Whether a particular gauge field is realized in nature is of course an empirical question, its mathematical characterization can however simply be given by the gauge group under which the gauge tensor, technically the curvature tensor of the bundle, is preserved. Due to the Standard Model, the groups U(1), SU(2) and SU(3) refer to gauge fields with real instantiations. Again, the field tensors are invariants under the respective gauge symmetries, meaning that the gauge transformations themselves have no physical instantiations. In principle only the gauge invariants allow for a realistic interpretation (though this does not necessarily mean that the gauge invariance of a quantity is sufficient for its observability).

3 Two Strands of Structural Realism

It is now time to introduce Structural Realism (SR). Broadly construed, SR is the doctrine that scientific realists should be committed to the structural content of our best theories rather than its content in terms of object-like entities. The account has its forerunners in such eminent thinkers like Poincaré, Eddington, Weyl on the physicists side as well as Cassirer, Schlick, Russell and Quine on the side of philosophy. The recent debate decomposes into two strands. John Worrall (1989) recommended SR as an appropriate means to make capital out of the prospects of realism, notably its straightforward explanation of the success of science without making it a miracle (the "no miracles argument"), while simultaneously escaping the two most dominant anti-realistic arguments: the pessimistic meta-induction (PMI) and theory underdetermination (TUD). Let us call this the Worrall-type SR, based on arguments from the general philosophy of science, in particular arguments about the pros and cons of scientific realism. The second strand was mainly inaugurated by the work of Steven French (1998) and James Ladyman (1998; French and Ladyman, 2003) and here the idea is that arguments in favour of SR are derived directly from an ontological analysis of the particular sciences, notably modern physics. Call this the French–Ladyman-type SR, to which we will focus in a moment.

But let us, ever so briefly, first consider Worrall-type SR. Confronted with PMI, SR proponents point out that the structural aspects show a far greater continuity through theory change than the notorious turn overs of theoretical terms referring to supposed object-like entities. A nice little example can be given from the early history of gauge theories. Classical electromagnetism shows a well-known gauge freedom in the potential-formulation under a group isomorphic to the real numbers. Quantum electrodynamics, the historical the successor, is properly understood as a

U(1) gauge theory with an underlying fiber bundle structure, where the potentials represent connection fields which again take on values in the reals, now considered as the Lie algebra of the bundle's structure group, the gauge group U(1). The group and invariance structure of the former theory has thus been embedded into the larger mathematical structure of the successor theory.

Surely not every progress in science shows such an elegant retention of structure through theory change. In order to avoid the force of PMI, one has to come up with an at least half-convincing reconstruction of the history of scientific progress in terms of an entirely structural characterization of theories. And this, of course, is an open research project, albeit certain steps in this direction are in fact already done (one such step is the work of Scheibe (1997/1999) about theory reduction within a fairly structural approach). Nevertheless, it seems quite reasonable to try to circumvent the anti-realistic threat of PMI from a structuralist perspective, and likewise the threat of TUD. By not committing us to an ontology of objects, SR proponents claim, we are able to escape the usual underdetermination scenarios. For there is a far greater similarity between theory alternatives or rivalling theoretical frameworks regarding their structural content as opposed to their entity content. Of course it is not clear whether TUD as a general threat is always limited to rival entities as opposed to structure. But given the fact that convincing cases of TUD (i.e. no fancy artificial or pathological examples) within the practising sciences are seldom to be found anyhow, a convincing case of structural incompatibility of two otherwise empirically indistinguishable theories is indeed not in sight.

For TUD, too, a nice gauge theoretic example can be given: Gauge theories comprise so-called holonomy and non-trivial topological effects. Regarding the former the Aharonov-Bohm (AB) effect is a case at hand. In this type of effect the gauge field seems to interact non-locally with the matter field wave function, thereby violating both the principle of local action and the concept of point-like interaction. In order to avoid such non-locality an explanation in terms of gauge potentials can be given at the metaphysical price of giving up separability and observability of the underlying entities while at the same time retaining local action and point-like interaction. A third interpretation is also feasible in terms of holonomies or Wilson loops as basic entities (that is loop integrals of the potential), which violate separability and point-like interaction, but are gauge invariant and, hence, observable. The metaphysical question about the underlying basic entities of gauge theories (fields, potentials or holonomies) is thus underdetermined by empirical evidence. SR, however, has a straightforward response at hand: it is precisely the gauge group (as well as the entire fiber bundle structure) which is invariant in all three rivalling interpretations. The anti-realistic threat of TUD vanishes once our realistic commitment sticks with the group structure only (cf. Lyre, 2004a, b).

But, as mentioned, such "indirect" Worrall-type evidence for SR isn't the only game in town. A more direct proof for SR's metaphysical picture of the world would be far more significant. This is exactly the more recent strategy of the French–Ladyman-type of arguments in favour of SR. And although it is certainly not possible to read off one's metaphysics from physics in a naïve manner, it might very well be possible to draw certain consequences from an ontological analysis.

One such consequence could be the "metaphysical underdetermination" ascribed to quantum theory by French (1998). He argues that quantum theory is plagued by an underdetermination of a very special kind which affects even the core metaphysical concept of individuality itself – more precisely the question whether quantum particle indistinguishability leaves us with non-individuals or rather some form of primitive thisness or haecceity. And indeed a rather similar scenario applies to the metaphysics of spacetime points, where diffeomorphism invariance plays the analogous role to permutation invariance in quantum theory (cf. Stachel, 2002).

In view of the above mentioned TUD scenario of gauge theories we may as well state that insofar as the notions of locality and separability are underdetermined in their application and hence the spatiotemporal nature of our physics' most fundamental entities as well as the doctrine of Humean supervenience are left undecided, again a type of underdetermination occurred which affects genuine and deep metaphysical concepts – and which therefore suggests in a more direct way to give up any realistic commitment according object-like entities and to stick with the symmetry structure alone. So besides quantum theory and general relativity we also find French–Ladyman-type of arguments for SR in the field of gauge theories. Indeed, an increasing community of authors within recent years has joined the new strand of SR including Tian Yu Cao, Mauro Dorato, Michael Esfeld, Michael Redhead, Dean Rickles, Simon Saunders and Howard Stein, to mention just a few.

4 Intermediate Structural Realism

James Ladyman (1998) introduced another distinction between two kinds or variants of SR: an epistemic (ESR) and an ontic version (OSR). His idea was to combine the French–Ladyman-type of arguments for SR in particular with OSR. There is, however, no immediate reason for this parallel. While it may be the case that Worrall-type SR is more naturally in tune with ESR – the view that structures as well as the bearers of the structure exist, but that such bearers are epistemically inaccessible to us – no such immediate connection between French–Ladyman-type arguments and OSR – the radical view that, as the slogan goes, "structures is all there is" – exists. It may instead very well be the case that authors who prefer to consider French–Ladyman-type of arguments for SR, will nevertheless reject OSR.

Michael Esfeld (2004), for instance, argues for a version of SR he calls moderate SR. But at the same time, Esfeld joins company with the majority of both defenders and critics of SR who feel uneasy – to say the least – with the radical idea of OSR, which seems to suggest that if it is only structures that exist and if structures are really just sets of relations, but that such relations need certain relata as their very constituents, then the doctrine of OSR seems to be a doctrine about relations without relata – and that is a logical inconsistency. Instead, what is quite possible and according to Esfeld appropriate to quantum theory (with its tensor product structure of Hilbert state spaces which lead to correlations and entangled states) is a pure ontology of relations, such that the relata exist but are individuated only through their structural and, hence, relational properties. Esfeld therefore seeks to reject the idea of intrinsic properties on the fundamental level.

A further difficulty, however, arises here as well. As we have seen above, the overall *topos* of symmetries is their handling of the quite general concepts of change and persistence. This is in particular true for the symmetries of physical state spaces such as the Hilbert space of a quantum system. The symmetries of the state space highlight its structural features as invariants under the symmetry transformations. Such invariants, however, comprise intrinsic properties of physical object-systems. But Esfeld's moderate SR focuses exclusively on state-dependent properties connected with the self-adjoint operators on a Hilbert space such as spatial position or particular spin directions. Inevitably, however, via its symmetries, any state space will at the same time also comprise state-independent, intrinsic properties – on the fundamental level all and those properties which are given by the invariants under the Poincaré and various gauge groups, i.e. mass, spin and the various charges.

It is therefore implausible to seek for a *pure* metaphysics of relations – and this is all the more so if we are in search for a structurally declared metaphysics build on symmetries. State-independent, intrinsic properties are a necessary consequence of this view. And this is particularly important in case of the invariants of a state space under symmetry transformations without real instantiations such as gauge transformations. In this case *a fortiori* only the symmetry invariants can have a certain physical significance.

Nevertheless, two further limiting remarks about the nature of the symmetry invariance properties are in order. First, such properties are, as we may say, still structurally defined or derived. That is, their ontological status is secondary in contrast to the symmetry structure which is primary. And second: Invariance properties are not sufficient for an individuation of objects. They merely allow for the determination of object classes – no essentialism has been invoked. Let us call the view about SR thus developed "Intermediate SR" – a position between ESR and OSR. Its connection to ESR is that the relata are not denied, its OSR connection is that the relata are, however, only structurally defined in the above sense.

5 Structural Realism and Abductive-Transcendental Analysis

It has sometimes been claimed in the literature that ESR is a variant of Kantianism (cf. French and Ladyman, 2003). Taken literally, this claim is of course nonsense, but a certain family resemblance stems from the fact that ESR denies access to the intrinsic natures of the things – and this echoes the Kantian thing-in-itself. But Kant's program has of course far wider scope not invoked by ESR. It is a merging of transcendental idealism and empirical realism and, according to the former, the Kantian view refers to structures as transcendental structures of knowledge or cognition (from which the transcendental subject can be determined), whereas according to the latter it shows no realistic preference for structures over objects.

Moreover, SR, as a branch of scientific realism, concerns the status of theoretical terms rather than common-sense reality, whereas Kant considers reality *in toto*. So the claim should not be that any version of SR is literally a variant of Kantianism, but nevertheless the attempt shall now be made to collect the previous results together and to sketch the structure of an abductive-transcendental argument in favour of SR, more particularly ISR.

The first premiss is about the notion of experience: *Experience means to recognize the change of the variable in relation to the permanent or persistent*. Take this as an analytic statement about experience and, hence, empirical science. Experience would be impossible in both a world of absolute change or absolute persistence.

This relates to the second premiss: *Symmetry is the general mathematical tool to analyze the interplay between change and persistence*. Any empirical science must allow, at its bottom level, a law-like description of the world in terms of symmetry structures.

Thirdly: *ISR is the minimal doctrine satisfying a realist commitment to empirical science*. This is of course not to say that ISR is the only reasonable realist doctrine (which would be an absurd armchair verdict), but that it is minimal as regards a science of structurally defined entities.

From this it follows that *ISR is most plausibly the core of any realist position of fundamental empirical science*. Here again the conclusion only presents ISR as a core position which must perhaps eventually be embedded in a wider position in the sense that more than structurally defined entities constituted by relational properties and intrinsic invariance properties are taken into account. The assumption, however, is that Occam's razor (a further tacit premiss in the above argument) cuts in favour of ISR. Hence, as an inference from the most basic premisses about empirical science to the best realist explanation of its success today, ISR turns out the most plausible and natural candidate.

References

M. Esfeld (2004): *Quantum Entanglement and a Metaphysics of Relations*. Studies in History and Philosophy of Modern Physics, 35(4):601–617.

S. French (1998): *On the Withering Away of Physical Objects*. In: Interpreting Bodies: Classical and Quantum Objects in Modern Physics, ed. E. Castellani. Princeton, NJ: Princeton University Press, pp. 93–113.

S. French and J. Ladyman (2003): *Remodelling Structural Realism: Quantum Physics and the Metaphysics of Structure*. Synthese, 136(1):31–56.

I. Kant. *Kritik der reinen Vernunft*. Riga, 1781 (B edition 1787). [Kant quotes and pages refer to: P. Guyer and A. W. Wood (eds.), *I. Kant: Critique of Pure Reason*. Cambridge University Press, 1998].

I. Kant. *Metaphysische Anfangsgrunde der Naturwissenschaft*. Riga, 1786.

I. Kant. *Über eine Entdeckung nach der alle neue Kritik der reinen Vernunft durch eine ältere entbehrlich gemacht werden soll ("Streitschrift gegen Eberhard")*. 1790.

J. Ladyman (1998): *What Is Structural Realism*? Studies in History and Philosophy of Science, 29(3):409–424.

H. Lyre (2004a): *Lokale Symmetrien und Wirklichkeit*. Mentis, Paderborn.

H. Lyre (2004b): *Holism and Structuralism in U(1) Gauge Theory*. Studies in History and Philosophy of Modern Physics 35(4):643–670.

E. Scheibe (1997/1999): Die Reduktion physikalischer Theorien. 2 volumes, Springer, Berlin.

J. Stachel (2002): *'The Relations Between Things' Versus 'The Things Between Relations': The Deeper Meaning of the Hole Argument*. In: D. Malament (ed.), Reading Natural Philosophy: Essays in the History and Philosophy of Science and Mathematics. Open Court, La Salle, IL, pp. 231–266.

J. Worrall (1989): *Structural Realism: The Best of Both Worlds*? Dialectica, 43:99–124.

Provisional Knowledge

Paul Teller

Abstract Physics, and science generally, rarely function according to the mechanist tradition of founding all scientific knowledge on “shaped matter in motion” of the physical parts of a system. Rather we employ a vast range of explanatory strategies a great many of which work in terms of “stripping detail” when detail is not relevant to the problem at hand. Most of these strategies involve some level of idealization, inaccuracy, or distortion, which raises the worry: When accounts in science involve distortion, how can they count as knowledge? This problem motivates reconstruing knowledge, and in particular its requirement of (unqualified) truth in its content component, in terms of the kinds of standards that require something less than perfect precision and accuracy, in analogy to the context and interest dependent standards that we apply for representational accuracy of things such as maps and pictures. The paper concludes with consideration of possible connections with pragmatism and with ways of thinking about “independent reality”.

1 Introduction

The expression, “provisional knowledge” would appear to be a complete oxymoron. If something *is* knowledge, it is justified and true – what could be provisional about the knowledge itself? The qualifier, “provisional”, could only be understood as applying to ones attitude towards a knowledge claim, expressing caution as to whether what is claimed to be knowledge will really prove to be so. It may transpire that one had made a mistake, in which case there never was any knowledge, provisional or otherwise.

Or so it would seem. I will provide a critique of a foundationalist attitude towards knowledge in science, one that will then require us to rethink the nature of such knowledge for which, in turn, the qualification of “provisional” will prove to be exactly appropriate.

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2 An Empiricist Rational for Physics-Fundamentalism

The following sketch may be no better than a caricature. But if successful, as with any successful caricature, it will make manifest important features, the examination of which will help us vividly to see some of empiricism's shortcomings.[1]

From the corpuscularian philosophy and the clock metaphor of the scientific revolution we have inherited the entrenched idea that the behavior, indeed everything about objects, is to be explained in terms of their parts, and, ultimately, the shaped matter in motion of the parts. I will use *mechanism* broadly for this attitude. As science has developed we have evolved how we think of these "parts" and how their operation collectively constitutes the behavior of the larger wholes. But thinking of the behavior of the whole in terms of the (successors of) "shaped matter in motion of the parts" has remained entrenched. Henceforth I will use "shaped matter in motion" with the quotes as shorthand for contemporary theorizing about the ultimate constituents of matter.

I suspect that the following line of argument has implicitly encouraged the view that all factual knowledge ultimately concerns only "shaped matter in motion". Here is a three-line version of the argument:

(a) Our only source of information about what is external to us is via perception of "shaped matter in motion".
(b) "Shaped matter in motion" can only carry information about "shaped matter in motion".
(c) So all knowledge of the external world is knowledge of "shaped matter in motion".

In a little more detail: By factual knowledge I will understand all knowledge except definitions, combinatorial facts, and generally facts about structure abstracted from concrete realizations, viz, mathematics broadly. Henceforth restriction to the factual will be understood. Now, as good empiricists we assume that all factual knowledge comes through our senses and hence comes via (impressions of) physical things such as trees and (bodies of) people and their observable properties and behavior: shapes, sizes, configurations, color….[2] That is, any difference in what can be known must be "underwritten" by a difference in sensory input and so a difference in the perceived (macroscopic) physical configuration. Variations in the conclusions we draw from experience must correspond to variations in the perceptual evidence. (Note well: This presupposes a unique function from "evidence" to rational – that is justified – conclusions.)

The upshot is that whatever we know must be in causal connection with the physical processes that immediately affect our senses. Assuming further that what is causally connected with these exemplars of the physical fairly counts as physical we conclude that *all* objects of factual knowledge count as physical. The argument concludes by applying the basic precept of mechanism: Ultimately all understanding

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[1] See Quine (1981), p. 98 for a statement of the argument I am about to sketch.

[2] Color and other so-called secondary properties provide a problem for this list. See note 3.

of physical phenomena proceeds in terms of the "shaped matter in motion" of the parts. So the objects of all factual knowledge are, ultimately, knowledge about "shaped matter in motion".

By *fundamental physics* I understand the science of the ultimate constituents of matter and their behavior. Then, since the objects of factual knowledge involve only "shaped matter in motion" and fundamental physics is the ultimate science of that subject, we conclude.

Physics-fundamentalism: Fundamental physics is the ultimate source for understanding all natural phenomena.

3 Critical Evaluation

Short of this sort of extreme empiricism, I don't see how physics-fundamentalism gets its credentials. But it only has to be explicitly stated for flaws to leap out at us.

To begin with, we contribute to perceptual phenomena. For example, our color phenomenology is a physiological construct of our trichromatic color visual system. Limiting artifacts such as color metamers and color constancy show that the system is no mirror of nature.[3] For a second example, the problems of constitution and of indeterminate spatial and temporal boundaries show that both our perception and our conception of discrete physical objects are idealizations that we impose on or build into experience. These examples multiply: broadly, perceptual phenomena are inexact in the sense that we refine their content with more exact scientific accounts of perception.

Further, we contribute by what we do with what we perceive, that is we contribute to the theoretical phenomena.[4] By theoretical phenomena I mean any phenomena, in a very broad sense, that do not count as perceptual: For example, the advance of the perihelion of Mercury, phase transitions as from liquid to gas, and beta decay.

Kant's a priori synthetic contribution to (largely Newtonian) theoretical phenomena, of course, no longer receive serious consideration. But the late nineteenth and early twentieth centuries saw proposal for a relativized, "dynamical" conception of constitutive a priori principles. Citing just one of many expositions, Friedman (2001) suggests that in Newtonian mechanics the three laws of motion function as constitutive a priori principles that characterize the notion of an inertial frame required for the empirical content of the law of gravitation. In the general theory of relativity Einstein revises the constitutive principles by using the equivalence principle to found empirical content for his law of the metrical field.

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[3] Practitioners of the scientific revolution were aware that secondary qualities pose a problem for the corpuscularian philosophy. Their response was to treat secondary qualities in terms of primary qualities. Commentators from Berkeley forward have noted that the same difficulties that apply to secondary qualities likewise apply to the primary qualities.

[4] Above I noted that, as described, empiricism assumed a unique function from perceptual evidence to rational conclusions. The collapse of epistemic foundationalism amounts to rejection of that assumption. I here have no space to review this important grist for the Kantian mill.

We can say much more broadly that theoretical phenomena are instituted through the (successful) models that we devise, while here leaving open the extent to which the structure of such modeling might be further articulated as suggested by Friedman and others. Hooke's law provides a simple example of what I have in mind. To what does it apply? What counts as a spring? Whenever the restoring force for a deformed material can be expressed as a Taylor series there is a linear first term. We count a material as a spring just in case, for ranges in which we are interested, the restoring force is well approximated by the linear first term of the Taylor expansion, that is exactly if the material satisfies Hooke's law. More generally, laws, or as Giere recommends calling them, (model building) principles, function as tools in constructing models that can be brought into agreement, closely enough for our purposes, with perceptual or other, theoretical, phenomena (Giere, 1999, Chapter 5; 2006, pp. 69–71)

Moving on from the point that we contribute to both the perceptual and theoretical phenomena, in the mechanist tradition explanation is to run in terms of "shaped matter in motion". But physics – indeed, *most* of science – employs a vast range of alternative kinds of explanatory accounts. Many can be broadly characterized as "detail-stripping" accounts, that is ones that work without, or even in virtue of ignoring, the details of "shaped matter in motion". I will illustrate:

To begin in an Aristotelian spirit, consider functional accounts. In understanding how a clock works we want to understand how the pendulum and gears interact to produced the signature regular motion. We want only relevant macroscopic facts – the details of microscopic configuration would positively get in the way. (Putnam, 1975, pp. 295–298) Ironically, this consideration is implicit in, but almost always ignored by, the mechanist tradition.

The example generalizes to the idea of functional explanation (Cummins, 1975), as illustrated with computer programs and wiring diagrams for radios. Suppose that we want to understand some complex disposition of a composite object. We analyze the object into relevant parts and characterize the relevant simpler dispositions of these parts. We then analyze the disposition of the whole as the organized, arranged, or programmed deployment of the dispositions of the parts. As in the special case of explaining the functioning of a clock, the microscopic details of how the parts get their simpler dispositions are irrelevant: The account stands however the component dispositions are realized.

Functional accounts overlap with many others. For example, Batterman (2002) discusses what he calls "asymptotic reasoning" on which we ignore – or strip the account of – details by using the description provided by the ideal limit of some parameter, as in treatment of critical phenomena such as transitions from solid to liquid to gas. Such limiting techniques bring to the fore the relevant relational structure of the whole, for example self-similarity in renormalization group treatments. Such accounts facilitate us in ignoring, for example, exactly which molecule touches off the process of condensation of a droplet, something that is never relevant to the kind of understanding we want when we ask why or how condensation occurs. We can also see the idea of limits as detail stripping in cases such as recovering Newtonian descriptions from special relativity by letting v/c – > 0.

So doing enables us to ignore the details of the finite value of the velocity of light and its relativistic repercussions.

For another kind of example, statistical mechanics uses a probability distribution over the states of motion of individual molecules precisely to strip an account of detailed information about individual particle behavior, which details are irrelevant to the facts about thermal phenomena.

Broadly, physics, and the rest of science, uses a wide range of techniques of idealization and approximation, such as a fixed external potential or treating interacting particles as interacting only with their average field. Such modeling techniques omit detail, simplify, and outright distort in ways irrelevant to solution of the present problem; or, while relevant, deploy departures that one can live with and that substantially facilitate problem solution. Idealization can be thought of generally as detail-stripping insofar as idealization omits characteristics or distorts in ways that avoid complications that impede solving the problem to hand.[5]

In a great many cases the accounts stand on their own, not needing any "foundation" from a more fundamental theory to provide solutions for the problems they were designed to address. For example, the accomplishments of fluid mechanics of the eighteenth and nineteenth centuries stand as solutions to the problems that they addressed without any need for retrospective "foundational" support. Where a more fundamental theory does add detail to such theories, usually what is added is just that: further detail.

Moreover, when a "more fundamental" theory does play a role in understanding a "less fundamental" theory, the account makes heavy use of further idealization. Often one requires exogenous supplementation and the resulting account does not conform to the mechanist paradigm of spelling out blow-by-blow details of the "shaped matter in motion". For example in the relation between thermodynamics and statistical mechanics the use of a distribution over the motions of individual constituents not only strips the account of the details of the state of motion of the individual constituents but also requires a material assumption in addition to the "foundational theory" in the appeal to an initial distribution. More broadly, there is no uniquely "correct" way to break a composite into parts, an important consideration in treating correlation based phenomena such as phase transitions, superconductivity, and superfluidity. Choices of state and phase spaces descriptions are best made with a view to solving specific problems (Auyang, 1998, pp. 57 and passim.)

4 Problems with Science as a Source of Truths

The foregoing problems with traditional fundamentalism themselves face difficulties. They are at odds with the tradition according to which science is a source of unqualified truths. I will characterize our traditional conception of knowledge as

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[5] See Auyang (1998) for a survey and analysis of systematic methods of idealization that are used across the sciences.

involving a justificatory component and a content component that requires representation of things as they are. Ordinarily we take the content component to be truth, in turn understood as something that is not graded, not context dependent, not qualified in any way. I will encapsulate this attitude towards truth by speaking of "unqualified truth". We now face the following problem: When science produces only idealizations it fails to provide unqualified truths, and so fails to provide knowledge. As I have already sketched, and will further sketch below, usually – if not always – this is the best that physics has to offer. Must we, after all, count physics as epistemically deeply impoverished? The problem generally infects much of science.

To plumb the depths of this problem it will be instructive critically to examine a contemporary reason for expecting unqualified truths from physics and elsewhere in science. It is claimed that the world exhibits exact natural kinds and quantities, that we can identify and label these, and so conclude that we can use the terms that refer to the exact kinds and quantities to formulate unqualified, exceptionless, general truths. But, in fact, we never succeed in the claimed exact reference fixing. The paradigmatic example of mass will have to suffice. The Newtonian era took 'mass' to refer to a completely determinate quantity. In special relativity mass splits into rest and relativistic mass, and relativistically mass and energy are introconvertible. In a special case – gravitational energy – it isn't even exactly localizable. In quantum field theories the status of mass is further complicated by the fact that it functions as a renormalization parameter: We substitute (imperfect) observational values for what clearly is not accurately described in the theory (Teller, 1995, Chapter 7). It's a real stretch to think that anywhere in this history has 'mass' been univocally attached to some one determinate "feature of nature".

The travails of mass exemplify a general characteristic of physics as it is actually practiced: Broadly, the detailed-stripped accounts are inaccurate. I won't assume that they always are. For example Batterman (2002) might argue that since the stripped details are irrelevant to the question at hand, no inaccuracy is involved. But when we examine the way in which the details are stripped in practice, often inaccuracy results – to be sure, inaccuracy that is harmless to the levels of accuracy required by the problem to hand.

Traditionally these complaints are brushed aside: Fundamentalists characterize the detail stripped accounts as "useful fictions", to be "made honest" by micro-accounts. But we never have the reductions that would be needed for such a resuscitation of unqualified truth in physics, and even if we did, they would fail to reinstate unqualified truth because all of our current "fundamental" theories are themselves highly idealized. The standard model of the constituents of mater is so-called for good reason. Its quanta are an artifact of an idealized flat (or symmetrically curved) space–time. It suffers the internal inconsistency of Haag's problem, it does not incorporate gravity. Conversely, we don't know how to quantize our best theory of gravitation (Teller, 2004, pp. 433, 435–437).

Elsewhere I will argue that all knowledge in science – indeed, all knowledge of any kind – exhibits some combination of imprecision and inaccuracy.[6] In particular, no Theory of Everything (TOE) is in sight, and as all advocates acknowledge, any TOE will be of no help for most of the knowledge that we value. A sober survey will reveal that most, if not all, that we now count as knowledge in science is in some way imprecise and/or inaccurate. Insofar as our objective is to interpret the knowledge provided by our current science, or any remotely like it with respect to the imitations it now exhibits, we must come to terms with this situation.

5 Reconsidering Representational Success for Statements

I take these problems to indicate that we need to reexamine the way in which we evaluate statements for success in their objective of representing things the way they are.

Truth is the form of representational success that we attribute to statements. We understand the kind of success involved in truth to be something that is not graded, not context dependent, not qualified in any way: what earlier I characterized as unqualified truth. Unqualified truth as the proffered form of representational success for statements contrasts with what counts as representational success for analog representations such as maps and pictures. These also achieve success in representing things the way they are. But their success is always something graded, admitting of degrees; and what counts as a satisfactory degree of success is always context dependent.

I suggest that truth as we commonly understand it is a kind of idealization that abstracts away from the qualifying features characterizing success for analog representations, and that the problems that I have described[7] are an artifact of neglecting the circumstance that truth involves such an idealization. This approach will in turn suggest an attractive revision for how to think about both truth and knowledge that, among other things, will provide the basis for resolving the enumerated difficulties.

6 Knowledge Is Provisional

Conventionally we understand knowledge as something known to be true, true in turn understood as unqualifiedly true in the sense explained above.[8] On this reading we know much less that we thought in science: Most of physics fails of complete

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[6] Teller (in preparation a, in preparation b). For a start, see Teller (2004, 2005).

[7] As well many other problems. See (Teller, in preparation a).

[8] At least in our traditional philosophical usage. Ordinary usage would appear to conform to the characterization that I will propose.

accuracy.[9] I propose to turn this line of thought on its head. In the spirit of Kant's attitude towards Hume, let us instead start from the conviction that a great deal of what we have in physics *does* count as knowledge. Given the foregoing, we can retain this conviction only by reconfiguring what we take to be knowledge, in particular, how we understand its appeal to truth in its content component.

To achieve this reconfiguration let us understand the content component of knowledge as *advanced as adequate*. We will understand "adequate" as context dependent, more specifically as precise, and accurate enough to meet current interests, demands, or standards. We then make the corresponding adjustments in the justification component of knowledge – not justified as (unqualifiedly) true but justified as meeting the standards in play.

In effect I am proposing that we understand "known" as "known well enough". This works smoothly for knowledge how: I know how to ski well enough to safely traverse that slope, I know linear algebra well enough to apply it to this problem. However we are here concerned not with knowledge how but knowledge that. If truth is understood as unqualified, perhaps we can still have an epistemic reading of "known well enough", in the sense of justified well enough to meet current standards of justification for knowledge. But if knowledge's content component is understood as requiring unqualified truth, there will be no application of "well enough" to the content component as required to come to terms with the ubiquitous inexactness of epistemic achievements in physics and elsewhere in science.

We avoid this impasse by reinterpreting how we understand truth as an evaluation for statements by taking it to work in something like the way the corresponding evaluations work for analog representations. Truth constitutes representational success for statements. We conventionally understand this in an absolute and context independent matter. But if we understand truth as operating more like representational success for, e.g., maps, "known well enough" will have the needed application to the content as well as to the justification component of knowledge. If we in this way reconfigure knowledge as what is known well enough – as advanced as both adequately justified and adequate in content – we reinstate a vast range of expertise in physics, and throughout science, as knowledge in its own right. This is the reconfiguration of knowledge that I intend when I speak of knowledge as *provisional*.

Traditional knowledge requires unqualified truth, which (completely determinately specified) objects, or kinds of objects, have which (completely determinately specified) properties – e.g., values of a quantity in a precisely specified interval. Provisional knowledge, as I have characterized it, does not provide that kind of representation of an independent world. What is required is not unqualified truth but levels of precision and accuracy that are good enough for us. Does knowledge in this sense count as objective and as pertaining to an independent reality?

A robust sense of objectivity is not here in question. We set the standards. But then it is up to nature to determine whether, or when, a representation meets these standards.

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[9] Elsewhere I will contend that when we do speak of truth in science, it has been purchased by compromising precision (Teller, in preparation b).

Knowledge that p, in the sense that I am recommending, is objective in the same way that the accuracy of a map is objective, no matter that we set the standards of accuracy required.

But there is still a worry: A map is something that *we* construct. We cannot, logically cannot, hold a map up against an unrepresented world to see how well it fits. We can only hold it up "against" other representations, perceptual or descriptive. Where, for representation with maps or statements, does an independent reality come in?

Truth, unqualified truth, was supposed to side step that limitation. It was thought that we could identify unique objects and properties, attach these to terms in our language and then use them to formulate unqualified truths. The scheme of exact denotation and truth might have circumvented many needs for a Kantian approach. But this scheme does not work. In the indefinitely complex world we are never able with perfect determinacy to pick out precise objects and properties.[10] We can always compare what we have with a new level of representation from the point of view of which the prior level can be judged to have failings of precision and/or accuracy. But short of a unique limit of inquiry, we have no absolute standard for comparison. In the present scheme any claimed imprecision and inaccuracy are, of course, understood not in relation to comparison with some unrepresented reality, nor in relation to any standard offered by an ephemeral unique Peircian limit of inquiry. Evaluation with respect to precision and accuracy are always understood in terms of the ubiquitous potential for constructing an alternative representational platform from the point of view of which the prior representational tools can be understood as (relatively) imprecise and/or inaccurate.

Human knowledge is no exact mirror of nature but is always a clouded image. We can often sharpen the image but, as a matter of contingent fact, owing to the vast complexity of nature, we can never bring it into perfect focus.

7 Provisional Knowledge, Truth, and Pragmatism

When we reinterpret knowledge as that which is known well enough, and where we took knowledge to require truth, being known well enough will require what is known to be true enough, the "enough" clearly to be understood as relative to current interests and concerns. Can we say more about how to understand "true enough"?

What is it for a map to be accurate enough? A map is accurate enough for our needs when the metrical relations that it represents correspond closely enough for our needs to those we experience when we are finding our way around. Oh! That works only for maps intended for use in judging distances and directions. If we are using a subway map we will want the topological relations to correspond to the connections between the stations as we find them. If the map gets wrong the ordering of two very close stations, and we don't mind the few minutes walk, the map will

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[10] It is the contingent fact of our representational limitations that absolutely enforces the need for contemporary dynamical Kantian methods.

be topologically accurate enough for our current needs. To summarize with a familiar phrase, for a map to be accurate enough is for it to work well enough when we use it in our endeavors.

This conclusion for maps suggests thinking of being true enough as working well enough, which in turn suggests that the view I am developing may prove to be a form of pragmatism. The few words I have said provide no more than a hint of a connection with pragmatism, but even this hint may be enough for the many who dismiss pragmatism likewise to dismiss the present ideas. Thus it is important for me to explain why the standard objections to pragmatism have no clear-cut application when knowledge and truth are reinterpreted as being known well enough and as being true enough.

One objection is that pragmatism is an epistemic account of truth: Lots of things have been found to work – and so to be assertable with warrant – but are not true. However, finding that a representation has so far functioned well for us is just "induction" writ large, and so concerns the justification component of knowledge, not the content component, not the component that concerns truth. The content component will correspond to working in fact, whether we have been misled so far nor not, in the sense that my computer may be in working order though I have not found it to work – being a computer dunce, I have not been able to get it to work for me. Critics also reject working in fact as an account of truth. Lots of representations that systematically and objectively work very well for us are not true as traditionally understood, for example when there are systematic compensating errors. This objection is fatal when one understands truth in the traditional, unqualified way. But when we understand truth in terms of a qualified analog of unqualified truth the objection, at the very least, no longer clearly applies. When, as I have urged, representational success for statements requires sufficient precision and accuracy for the standards in play, and so for our present needs, there is room for error relative to more exact standards that might reflect new needs. Thus truth understood as "true enough" has room for inaccuracies and the objection then, at best, is not clearly correct. Maps work when they help us get around, to places we want to go and to the standards of precision that we have set for the job. 'True' applies appropriately to a statement when, in analogy, it "helps" us get around.

We must say much more here. Words don't work by themselves. For 'true' to apply the range of relevant tasks must not be too narrow. We must develop, not just gesticulate as I have done here, at the map analogy. Pragmatism is a program, an approach, not a one-liner. What we have accomplished here is to see that one requires the present attitude to address the familiar objection to pragmatism.

8 A Metaphysical Metaphor

Having acknowledged the need for details for a substantive pragmatist account, let's return to my conclusion that any such account needs to be without semantic foundations, without any fixed representational anchor.

I see no intellectual obligation to address the absence of any representational foundation, but addressing it is intellectually permissible. We are always free to adumbrate the idea of provisional knowledge with a model or way of thinking about it that address the felt need to relate what we know to an independent, unrepresented world. I stress that I offer the following suggestion not as a piece of metaphysics but as a metaphor that one will find attractive according to personal taste.

Think of the world independent of any representation – the noumena, if you will – as a constituting a system of potentiality to yield various representational responses when probed with various representational tools. The metaphor is to think of the world independent of any specific representation as a kind of Aristotelian "prime matter", as "pure potentiality" that is actualized by representational activity. The idea is not to claim (or deny) that any thing distant in time or space from any representing agent exists only in potentiality but has no actuality. So claiming would be actively to represent as merely potential things that, were we to be present, we would represent as actual. Where and when no representation is going on it is representation, not things, that are potential. Rather, the metaphor is a way of acknowledging, with Kant, that there is no thought of any object except of the object as represented, but differing from Kant by noting that there is an unlimited range of representational tools that might be applied, and supplementing views of a of a neo-Kantian relativized or dynamic a priori by noting that a great deal of the flexibility in our representational tools arises from their limitations with regard both to precision and to accuracy. The world is exceedingly complex, and creatures with radically different representational, in particular with different perceptual and theoretical, powers from those that we possess might well apply representational tools that are strikingly different from ours. The metaphor suggests that what is actual involves an indefinitely complex potentiality to be represented in this way when probed with these representational tools, in that way when probed with those representational tools, all with complete consistency since the differing phenomena, in the Kantian sense, are always relative to the conceptual framework that has been applied.

In admitting this much of a role to representing agents do we retain a robust sense of objectivity in our representational activity? When we actively apply various representational tools, various specific responses or realizations occur. Such circumstances about what realizations are possible, what occurs when we probe with specific representational forms, are objective in the sense of being independent of us. That our representational tools are inexact in no way comprises such objective circumstances.

Again, to think in terms of a kind of ultimate potentiality for representation provides no more than a metaphor the substantive content of which was wonderfully expressed by Locke:

> He that will not set himself proudly at the top of all things; but will consider the Immensity of this Fabrick, and the great variety, that is to be found in this little and inconsiderable part of it, which he has to do with, may be apt to think, that in other Mansions of it, there may be other, and different intelligent Beings, of whose Faculties, he has as little Knowledge or Apprehension, as a Worm shut up in one drawer of a Cabinet, hath of the Senses or Understanding of a Man. (Essay: book II, Chapter II, 3)

9 Provisional Knowledge

When I qualify knowledge as provisional, in the sense that I intend, I am not expressing epistemic caution, that is *not* saying that what I *think* is knowledge might not be so. On any familiar conception of knowledge, if such a proviso should be activated, it would transpire that what was provisionally advanced as knowledge was no such thing.

Rather the reservation conveyed by ‘provisional’ concerns the standards of precision and accuracy that we put in play in the proffered knowledge. When these standards become more exact, new instances of knowledge must meet the more exacting standards. The prior, provisional knowledge continues to count as knowledge relative to the prior standards: It is the standards that are provisional. We say that knowledge is provisional precisely to acknowledge the eventuality that more exacting standards of precision and accuracy may come into play. In this sense (and possible exceptions in mathematics aside) humanly accessible knowledge is in science, and I believe always, provisional.

References

Auyang, Sunny (1998) *Foundations of Complex-Systems Theories in Economics, Evolutionary Biology and Statistical Physics*. Cambridge: Cambridge University Press.

Batterman, Robert (2002) *The Devil in the Details: Asymptotic Reasoning in Explanation, Reduction and Emergence*. Oxford: Oxford University Press.

Cummins, Robert (1975) “Functional Analysis.” *Journal of Philosophy*, 72, 741–764.

Friedman, Michael (2001) *The Dynamics of Reason*. Stanford, CA: CSLA Publications

Giere, Ronald (1999) *Science Without Laws*. Chicago, IL: University of Chicago Press.

Giere, Ronald (2006) *Scientific Perspectivism*. Chicago, IL: University of Chicago Press.

Putnam, Hilary (1975) ‘Philosophy and Our Mental Life’ in *Philosophical Papers, Vol. 2*. Cambridge: Cambridge University Press, pp. 291–303.

Quine, Willard Van Orman (1981) *Theories and Things*. Cambridge: Harvard University Press.

Teller, Paul (1995) *An Interpretive Introduction to Quantum Field Theory*. Princeton, NJ: Princeton University Press.

Teller, Paul (2004) “How We Dapple the World.” *Philosophy of Science*, 425–447.

Teller, Paul (in preparation a) “The truth Idealization.”

Teller, Paul (in preparation b) “Some Dirty Little Secrets About Truth.”

Bibliography

Abraham, R. and Marsden, J. (1978), *Foundations of Mechanics*, New York/Reading, MA: Benjamin Cummings.

Allison, H. (1994), "Causality and Causal Laws in Kant. A Critique of Michael Friedman", in: P. Parrini (ed.), *Kant and Contemporary Epistemology*, Dordrecht, The Netherlands: Kluwer.

Allison, H. (2004), *Kant's Transcendental Idealism. Revised and Enlarged Edition*, New Haven, CT/London: Yale University Press.

Arnold, V. (1976), *Méthodes mathématiques de la mécanique classique*, Moscow: Mir.

Arnowitt, R., Deser, S. and Misner, C. W. (1962), "The Dynamics of General Relativity", in: L. Witten (ed.), *Gravitation: An Introduction to Current Research*, New York: Wiley.

Aspect, A., Grangier, P. and Roger, G. (1982), "Experimental Realization of Einstein-Podolsky-Rosen-Bohm *Gedankenexperiment*. A New Violation of Bell's Inequalities", *Physical Review Letters*, 49, 91–94.

Auyang, S. (1995), *How Is Quantum Field Theory Possible*?, New York: Oxford University Press.

Auyang, S. (1998), *Foundations of Complex-Systems Theories in Economics, Evolutionary Biology and Statistical Physics*, Cambridge: Cambridge University Press.

Bacciagaluppi, G. (2005), "The Role of Decoherence in Quantum Mechanics", in: E. N. Zalta (ed.), *The Stanford Encyclopedia of Philosophy*. http://plato.stanford.edu/archives/sum2005/entries/qm-decoherence/

Baez, J. (ed.) (1994), *Knots and Quantum Gravity*, Oxford: Clarendon.

Balashov, Y. (1992), "Transcendental Background to the Anthropic Reasoning in Cosmology", *Man & World*, 25, 115–132.

Balashov, Y. (1994), "Uniformitarianism in Cosmology: Background and Philosophical Implications of the Steady-State Theory", *Studies in History and Philosophy of Science*, 25B, 933–958.

Balashov, Y. (2002), "Laws of Physics and the Universe", in: Y. Balashov and V. Vizgin (eds.), *Einstein Studies in Russia*, Boston, MA/Basel, Switzerland/Berlin: Birkhäuser.

Barrett, J. (1999), *The Quantum Mechanics of Minds and Worlds*, Oxford: Oxford University Press.

Barrow, J. D. and Tipler, F. J. (1986), *The Anthropic Cosmological Principle*, Oxford: Clarendon.

Batterman, R. (2002), *The Devil in the Details: Asymptotic Reasoning in Explanation, Reduction and Emergence*, Oxford: Oxford University Press.

Beck, L. W. (ed.) (1974), *Kant's Theory of Knowledge*, Dordrecht, The Netherlands: Reidel.

Bell, J. S. (1964), "On Einstein-Podolsky-Rosen paradox", *Physics*, 1, 195–200. Reprinted in: J. A. Wheeler and W. H. Zurek (eds.), *Quantum Theory and Measurement*, Princeton, NJ: Princeton University Press, 1983.

Bell, J. S. (1990), "Against 'Measurement' ", *Physics World*, 8, 33–40.

Bergson, H. (1938), *La pensée et le mouvant*, Paris: Presses Universitaires de France.

Berkson, W. (1974), *Fields of Force: The Development of a World View from Faraday to Einstein*, London: Routledge & Kegan Paul.

Bertet, P., Osnaghi, S., Rauschenbeutel, A., Nogues, G., Auffèves, A., Brune, M., Raimond, J. M. and Haroche, S. (2001), "A Complementarity Experiment with an Interferometer at the Quantum-Classical Border", *Nature*, 411, 166–170.

Bird, G. (2003), "Carnap"s Internal and External Questions", in: T. Bonk (ed.), *Language, Truth and Knowledge. Contributions to the Philosophy of R. Carnap*, Dordrecht, The Netherlands: Kluwer.

Bitbol, M. (1996a), *Schrödinger's Philosophy of Quantum Mechanics*, Dordrecht, The Netherlands: Kluwer.

Bitbol, M. (1996b), *Mécanique quantique, une introduction philosophique*, Paris: Flammarion.

Bitbol, M. (1998a), *L'aveuglante proximité du réel, anti-réalisme & quasi-réalisme en physique*, Paris: Flammarion.

Bitbol, M. (1998b), "Some Steps Towards a Transcendental Deduction of Quantum Mechanics", *Philosophia Naturalis*, 35, 253–280.

Bitbol, M. (2000a), "Relations, Synthèses, Arrière-Plans; sur la philosophie transcendantale et la physique moderne", *Archives de Philosophie*, 63, 595–620.

Bitbol, M. (2000b), "Arguments transcendantaux en physique moderne", in: S. Chauvier and F. Capeillères (eds.), *La querelle des arguments transcendantaux. Revue philosophique universitaire de Caen*, 35, 81–101.

Bitbol, M. (2000c), *Physique et philosophie de l'esprit*, Paris: Flammarion.

Bitbol, M. (2005), "Anticiper l'unité: une méthode de connaissance", in: M. Cazenave (ed.), *Unité du monde, unité de l'être*, Paris: Dervy.

Bohm, D. (1957), *Causality and Chance in Modern Physics*, London: Routledge & Kegan Paul.

Bohr, N. (1913), "On the Constitution of Atoms and Molecules", BCW 2, 161–233.

Bohr, N. (1918–1922), "On the Quantum Theory of Line Spectra", BCW 3, 67–184.

Bohr, N. (1921), "On the Application of Quantum Theory to Atomic Problems", in BCW 3, 364–380.

Bohr, N. (1922), *The Theory of Spectra and Atomic Constitution*, Cambridge: Cambridge University Press.

Bohr, N. (1927), "The Quantum Postulate and the Recent Development of Atomic Theory", in: Bohr (1934).

Bohr, N. (1929), "Introductory Survey", in: Bohr (1934), 1–24.

Bohr, N. (1934), *Atomic Theory and the Description of Nature*, Cambridge: Cambridge University Press.

Bohr, N. (1935), "Can Quantum-Mechanical Description of Reality Be Considered Complete?", *Physical Review*, 48, 696–702.

Bohr, N. (1936), "Analysis and Synthesis in Science", *International Encyclopedia of Unified Science* I, No. 1, Chicago, IL: Chicago University Press.

Bohr, N. (1937), "Causality and Complementarity", in: Bohr (1998), 83–91.

Bohr, N. (1948), "On the notions of causality and complementarity", *Dialectica*, 2, 312–319.

Bohr, N. (1949), "Discussion with Einstein on Epistemological Problems in Atomic Physics", in: Bohr (1958a), 32–66.

Bohr, N. (1954), "The Unity of Knowledge", in: *Atomic Physics and Human Knowledge*, New York: Wiley (1958a).

Bohr, N. (1956), "Mathematics and Natural Philosophy", in: Bohr (1998), 164–169.

Bohr, N. (1958a), *Atomic Physics and Human Knowledge*, New York: Wiley.

Bohr, N. (1958b), "Quantum Physics and Philosophy. Causality and Complementarity", in: Bohr (1963), 1–7.

Bohr, N. (1963), *Essays 1958–1962 on Atomic Physics and Human Knowledge*, New York: Wiley.

Bohr, N. (1972), *Bohr's Collected Works* (BCW), L. Rosenfeld, J. Rud Nielsen, E. Rüdinger and F. Aaserud (eds.), Amsterdam: North-Holland; New York: American Elsevier.

Bohr, N. (1985), *Naturbeskrivelse og menneskelig erkendelse*, Köbenhavn: Rhodos

Bohr, N. (1986), *Niels Bohr's Collected Works*, Vol. VI, Amsterdam: North-Holland.

Bohr, N. (1998), *Causality and Complementarity. The Philosophical Writings of Niels Bohr*, Vol. IV, J. Faye and H. Folse (eds.), Woodbridge, CT: Ox Bow.

Bohr, N., Kramers, H. and Slater, J. (1924), "The Quantum Theory of Radiation", in: BCW 5, 99–118.

Bondi, H. (1960), *Cosmology*, Cambridge: Cambridge University Press.

Bondi, H. and Gold, T. (1948), "The Steady-State Theory of the Expanding Universe", *Monthly Notices of the Royal Astronomical Society*, 108, 252–270.

Boniolo, G. (2007), *On Scientific Representations. From Kant to a New Philosophy of Science*, Houndmills: Palgrave Macmillan.

Boole, G. (1952), "On the Theory of Probabilities", in: G. Boole (ed.), *Studies in Logic and Probability*, Chicago, IL: Open Court.

Born, M. (1926a), "Zur Quantenmechanik der Stoßvorgänge", *Z. f. Physik*, 37, 863–867. English translation in: Wheeler and Zurek (1983).

Born, M. (1926b), "Quantenmechanik der Stoßvorgänge", *Z. f. Physik*, 38, 803–827.

Born, M. (1998), 'Physical Reality', in: E. Castellani (ed.), *Interpreting Bodies. Classical and Quantum Objects in Modern Physics*, Princeton, NJ: Princeton University Press.

Boscovich, R. J. (1763) [1966], *A Theory of Natural Philosophy*, Cambridge, MA: MIT Press.

Bostrom, N. (2002), *Anthropic Bias: Observation Selection Effects in Science and Philosophy*, New York: Routledge.

Bothe, W. and Geiger, H. (1925), "Über das Wesen des Comptoneffekts; ein experimenteller Beitrag zur Theorie der Strahlung", *Zeitschrift für Physik*, 32, 639–663.

Bouwmeester, D., Ekert, A. K. and Zeilinger, A. (eds.) (2001), *The Physics of Quantum Information*, Berlin: Springer.

Brading, K. (2002), "Which Symmetry? Noether, Weyl, and the Conservation of Charge", *Studies in History and Philosophy of Modern Physics*, 33, 3–22.

Brading, K. and Castellani, E. (eds.) (2003), *Symmetries in Physics. Philosophical Reflections*, Cambridge: Cambridge University Press.

Brading, K. and Ryckman, T. (to appear), "Causality as a Condition of Possible Experience: Hilbert, General Relativity, and the Philosophical Background of Noether's Theorems".

Bricker, P. and Hughes, R. I. G. (eds.) (1990), *Philosophical Perspectives on Newtonian Science*, Cambridge, MA: MIT Press.

Brittan, G. (1978), *Kant's Theory of Science*, Princeton, NJ: Princeton University Press.

Brittan, G. (1994), "Kant and the Quantum theory", in: P. Parrini (ed.), *Kant and Contemporary Epistemology*, Dordrecht, The Netherlands: Kluwer.

Brock, S. (2003), *Niels Bohr's Philosophy of Quantum Physics*, Berlin: Logos.

Brown, H. I. (1979), *Perception, Theory and Commitment. The New Philosophy of Science*, Chicago, IL: The University of Chicago Press.

Brune, M., Hagley, E., Dreyer, J., Maître, X., Maali, A., Wunderlich, C., Raimond, J. M. and Haroche, S. (1996), "Observing the Progressive Decoherence of the Meter in a Quantum Measurement", *Physical Reviews Letters*, 77, 4887.

Bruner, J. S. (1983), *In Search of Mind*, New York: Harper & Row.

Brylinski, J.-L. (1993), "Loop Spaces, Characteristic Classes, and Geometric Quantization", *Progress in Mathematics*, 107, Boston, MA: Birkhäuser.

Bub, J. (1997), *Interpreting the Quantum World*, Cambridge: Cambridge University Press.

Buchdahl, G. (1971), "The Conception of Lawlikeness in Kant's Philosophy of Science", *Synthese*, 23, 24–46.

Buchholz, D. and Haag, R. (2000), "The Quest for Understanding in Relativistic Quantum Physics", *Journal of Mathematical Physics*, 41, 3674–3697.

Burge, T. (2003), "Logic and Analyticity", in: H.-J. Glock, K. Glüer and G. Keil (eds.), *Fifty Years of Quine's 'Two Dogmas', Grazer Philosophische Studien*, 66, 199–249.

Busch, P., Lahti, P. and Mittelstaedt, P. (1996), *The Quantum Theory of Measurement*, Heidelberg, Germany: Springer.

Butterfield, J. and Isham, C. (1999), “On the Emergence of Time in Quantum Gravity”, in: J. Butterfield (ed.), *The Arguments of Time*, Oxford: Oxford University Press.

Butts, R. (ed.) (1986), *Kant’s Philosophy of Physical Science*, Dordrecht, The Netherlands: Reidel.

Cao, T. Y. (1996), *Conceptual Developments of 20th Century Field Theories*, Cambridge: Cambridge University Press.

Cao, T. Y. and Schweber, S. S. (1993), “The Conceptual Foundations and the Philosophical Aspects of Renormalization Theory”, *Synthese*, 97, 33–108.

Carlip, S. (2001), “Quantum Gravity: A Progress Report”, *Report on Progress in Physics*, 64, ArXiv:gr-qc/0108040v1.

Carnap, R. (1923), “Über die Aufgabe der Physik”, *Kant-Studien*, 28, 90–107.

Carnap, R. (1928), *Der logische Aufbau der Welt: Scheinprobleme in der Philosophie*, Berlin: Weltkreis-Verlag; English translation in: *The Logical Structure of the World: Pseudoproblems in Philosophy*, Berkeley/Los Angeles, CA: University of California Press, 1967.

Carnap, R. (1932), “Überwindung der Metaphysik durch logische Analyse der Sprache” (*Erkenntnis*, 2), “The Elimination of Metaphysics through Logical Analysis of Language”, in: A. Ayer (ed.), *Logical Positivism*, New York: Free Press, 1959.

Carnap, R. (1934), *Logische Syntax der Sprache*, Wien: Springer; English ed., *The Logical Syntax of Language*, London: Routledge & Kegan Paul, 1937.

Carnap, R. (1950), “Empiricism, Semantics, and Ontology”, *Revue internationale de philosophie*, 4, 20–40.

Carnap, R. (1956), *Meaning and Necessity. A Study in Semantics and Modal Logic*, Chicago, IL: University of Chicago Press.

Carnap, R. (1963), “Carnap’s Intellectual Autobiography”, in: P. A. Schilpp (ed.), *The Philosophy of Rudolf Carnap*, La Salle, IL: Open Court.

Carnap, R. (1966), *Philosophical Foundations of Physics*, New York: Basic Books.

Carson, E. and Huber, R. (eds.) (2006), *Intuition and the Axiomatic Method*, Dordrecht, The Netherlands: Springer.

Cartwright, N. (1983), *How the Laws of Physics Lie*, Oxford: Clarendon.

Cartwright, N. (1989), *Nature’s Capacities and Their Measurement*, Oxford: Clarendon.

Cartwright, N. (1999), *The Dappled World*, Cambridge: Cambridge University Press.

Cassirer, E. (1907), “Kant und die moderne Mathematik”, *Kantstudien*, 12, 1–49.

Cassirer, E. (1918), *Kant Leben und Lehre*, English translation in: *Kant’s Life and Thought*, New Haven, CT: Yale University, 1981.

Cassirer, E. (1921), *Zur Einsteinschen Relativitätstheorie: Erkenntnistheoretische Betrachtungen*, Berlin: Bruno Cassirer. Translation by W. Swabey and M. Swabey, in: *Substance and Function and Einstein’s Theory of Relativity*, 1923. Reprint edition, New York: Dover, 1953.

Cassirer, E. (1923–1929), *Philosophie der Symbolischen Formen*, Berlin: Bruno Cassirer.

Cassirer, E. (1936), *Ziele und Wege der Wirklichkeitserkenntnis*, Hamburg, Germany: Felix Meiner, 1998.

Cassirer, E. (1944), “The Concept of Group and the Theory of Perception”, *Philosophy and Phenomenological Research*, 5, 1–36.

Cassirer, E. (1956), *Determinism and Indeterminism in Modern Physics: Historical and Systematic Studies of the Problem of Causality*, New Haven, CT: Yale University Press.

Cassirer, E. (1987), “Determinismus und Indeterminismus in der modernen Physik”, in: *Zur Modernen Physik*, Darmstadt, Germany: Wissenschaftliche Buchgesellschaft.

Cassirer, E. (1995), “Das Symbolproblem und seine Stellung im System der Philosophie”, in: E. Cassirer (ed.), *Symbol, Technik, Sprache. Aufsätze aus den Jahren 1927–1933*, Hamburg, Germany: Meiner, pp. 1–21.

Castagnino, M., Catren, G. and Ferraro, R. (2002), “Time Asymmetries in Quantum Cosmology and Boundary Conditions to the Wheeler-DeWitt Equation”, Classical and quantum gravity, 19, 4729–4746.

Catren, G. (2008), “On Classical and Quantum Objectivity”, *Foundations of Physics*, doi:10.1007/s10701-008-9216-2.

Chamseddine, A. and Connes, A. (1996), “Universal Formulas for Noncommutative Geometry Actions”, *Physical Reviews Letters*, 77, 4868–4871.

Chevalley, C. (1991), “*Le dessin et la couleur* (introduction) and Glossaire”, in: *Niels Bohr: Physique atomique et connaissance humaine*, Paris: Gallimard.

Chevalley, C. (1994), “Niels Bohr’s Words and the Atlantis of Kantianism”, in: J. Faye and H. Folse (eds.), *Niels Bohr and Contemporary Philosophy*, Dordrecht, The Netherlands: Kluwer.

Chevalley, C. (1995a), “Philosophy and the Birth of Quantum Theory”, in: K. Gavrogly et al. (eds.), *Physics, Philosophy and the Scientific Community*, Dordrecht, The Netherlands: Kluwer, 1995.

Chevalley, C. (1995b), “On Objectivity as Intersubjective Agreement”, in: Krüger and Falkenburg (eds.), *Physik, Philosophie und die Einheit der Wissenschaften*, Heidelberg, Germany: Spektrum.

Coffa, A. J. (1991), *The Semantic Tradition from Kant to Carnap. To the Vienna Station*, Cambridge: Cambridge University Press.

Cohen, H. (1999), *Le principe de la méthode infinitésimale et son histoire*, Paris: J. Vrin.

Cohen, H. (2000), *Commentaire de la « Critique de la raison pure » de Kant*, Paris: Les Editions du Cerf.

Cohen, H. (2001), *La théorie kantienne de l’expérience*, Paris: Les Editions du Cerf.

Coleman, R. and Korté, H. (2001), “Hermann Weyl: Mathematician, Physicist, Philosopher”, in: E. Scholz (ed.), *Hermann Weyl’s ‘Raum-Zeit-Materie’ and a General Introduction to His Scientific Work*, Basel, Switzerland: Birkhäuser.

Connes, A. (1990), *Géométrie Non Commutative*, Paris: InterEditions.

Connes, A. (1996), “Gravity Coupled with Matter and the Foundation of Noncommutative Geometry”, *Communications in Mathematical Physics*, 182, 155–176. ArXiv:hep-th/9603053.

Connes, A. (2000a), Noncommutative Geometry Year 2000, ArXiv:math.QA/0011193 v1.

Connes, A. (2000b), “A Short Survey of Noncommutative Geometry”, ArXiv:hep-th/0003006 v1.

Connes, A. and Kreimer, D. (1998), “Hopf Algebras, Renormalization and Noncommutative Geometry”, ArXiv:hep-th/9808042 v1.

Connes, A. and Kreimer, D. (1999), “Renormalization in Quantum Field Theory and the Riemann-Hilbert problem”, ArXiv:hep-th/9909126.

Connes, A. and Marcolli, M. (2007), *Noncommutative Geometry, Quantum Fields and Motives* American Mathematical Society.

Crary, A. and Read, R. (eds.) (2000), *The New Wittgenstein*, New York: Routledge.

Creath, R. (1991), “Every Dogma Has Its Day”, *Erkenntnis*, 35, 347–389.

Cummins, R. (1975), “Functional Analysis”, *Journal of Philosophy*, 72, 741–764.

Cushing, J. T. (1989), “A Background Essay”, in: J. T. Cushing and E. McMullin (eds.), *Philosophical Consequences of Quantum Theory: Reflections on Bell’s Theorem*, Notre Dame, IN: University of Notre Dame Press.

Darrigol, O. (1992), *From C-numbers to Q-numbers. The Classical Analogy in the History of Quantum Theory*, Berkeley, CA: University of California Press.

Descartes, R. (1644), *Principia Philosophiæ*, Amsterdam: Elsevier.

Dirac, P. A. M. (1958), *Principles of Quantum Mechanics*, Oxford: Oxford University Press.

DiSalle, R. (1988), *Space, Time, and Inertia in the Foundations of Newtonian Physics*, Doctoral Dissertation, University of Chicago.

DiSalle, R. (1991), “Conventionalism and the Origins of the Inertial Frame Concept, *PSA 1990*, 2, 139–147.

DiSalle, R. (2002), “Reconsidering Ernst Mach on Space, Time, and Motion”, in: D. Malament (ed.), *Reading Natural Philosophy: Essays in the History and Philosophy of Science and Mathematics*, Chicago, IL: Open Court, pp. 167–191.

DiSalle, R. (2006), *Understanding Space-Time: The Philosophical Development of Physics from Newton to Einstein*, Cambridge: Cambridge University Press.

Dosch, H. G., Müller, V. F. and Sieroka, N. (2005), *Quantum Field Theory in a Semiotic Perspective*, Berlin/Heidelberg, The Netherlands: Springer.

Dreyer, J. (1997), *Atomes de Rydberg et cavités: Observation et décohérence dans une mesure quantique*, Thèse de doctorat de l'Université Paris VI.

Eddington, A. (1923), *The Mathematical Theory of Relativity*, Cambridge: Cambridge University Press.

Edwards, J. (2000), *Substance, Force, and the Possibility of Knowledge: On Kant's Philosophy of Material Nature*, Berkeley, CA: University of California Press.

Einstein, A. (1905a), "Über einen die Erzeugung und Verwandlung des Lichts betreffenden heuristischen Gesichtspunkt", *Annalen der Physik*, 17, 132–148.

Einstein, A. (1905b), "Zur Elektrodynamik bewegter Körper", *Annalen der Physik*, 17, 891–921. Translated by G. Jeffrey and W. Perrett (1923) as "On the Electrodynamics of Moving Bodies", in: Lorentz et al. (eds.), *The Principle of Relativity: A Collection of Original Memoirs on the Special and General Theories of Relativity*, London: Methuen.

Einstein, A. (1917), "Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie", *Königlich Preussiche Akademie der Wissenschaften* (Berlin). *Sitzungsberichte*, 142–152.

Einstein, A. (1921), *Geometrie und Erfahrung. Erweiterte Fassung des Festvortrages gehalten an der Preussischen Akademie der Wissenschaft zu Berlin am 27. Januar 1921*, Berlin: Springer. Translated by G. Jeffrey and W. Perrett (1923) as "Geometry and Experience", in: *Sidelights on Relativity*, London: Methuen.

Einstein, A. (1925), "Nichteuklidische Geometrie und Physik", *Die neue Rundschau*, 36, 16–20.

Einstein, A. (1948), "Quantemmechanik und Wirklichkeit", *Dialectica*, 2, 320–324.

Einstein, A. (1949), "Autobiographical Notes", in: Schilpp (1949), 1–94.

Einstein, A. (1953), "Remarques préliminaires sur les concepts fondamentaux", in: G. André (ed.), *Louis de Broglie: Physicien et penseur*, Paris: Albin Michel.

Einstein, A. et al. (1923), *The Principle of Relativity*, London: Methuen.

Einstein, A., Podolsky, B. and Rosen, N. (1935), "Can Quantum Mechanical Description of Physical Reality Be Considered Complete?", *Physical Review*, 47, 777–780.

Elitzur, S. (1975), "Impossibility of Spontaneously Breaking Local Gauge Theories", *Physical Review*, D12, 3978–3982.

Espagnat (d'), B. (1994), *Le réel voilé*, Paris: Fayard.

Everett III, H. (1957), "Relative State Formulation of Quantum Mechanics", *Reviews of Modern Physics*, 29, 454–462.

Faddeev, L. D. and Slavnov, A. A. (1991), *Gauge Fields: An Introduction to Quantum Theory*, Redwood City, CA: Addison-Wesley.

Falkenburg, B. (1996), "The Analysis of Particle Tracks. A Case for Trust in the Unity of Physics", *Studies in History and Philosophy of Modern Physics*, 27, 337–371.

Falkenburg, B. (1997), "Incommensurability and Measurement", *Theoria*, 12, 467–491.

Falkenburg, B. (1998), "Bohr's Principles of Unifying Quantum Disunities", *Philosophia Naturalis*, 35, 195–120.

Falkenburg, B. (2000), *Kants Kosmologie. Die wissenschaftliche Revolution der Naturphilosophie im 18. Jahrhundert*, Frankfurt, Germany: Klostermann.

Falkenburg, B. (2002), "Symbol and Intuition in Modern Physics", in: M. Ferrari and I.-O. Stamatescu (eds.), *Symbol and Physical Knowledge*, Heidelberg, Germany: Springer.

Falkenburg, B. (2006), "Functions of Intuition in Quantum Physics", in: Carson and Huber (2006).

Falkenburg, B. (2007), *Particle Metaphysics. A Critical Account of Subatomic Reality*, New York: Springer.

Finch, A. (2003), "Review of B. van Fraassen, *The Empirical Stance" American Catholic Philosophical Quarterly*, 77, 302–307.

Fine, A. (1984), "The Natural Ontological Attitude", in: J. Leplin (ed.), *Scientific Realism*, Berkeley/Los Angeles, CA: University of California Press, pp. 83–107.

Folina, J. (1992), *Poincaré and the Philosophy of Mathematics*, New York: Macmillan.

Frank, P. (1941), *Modern Science and Its Philosophy*, Cambridge, MA: Harvard University Press.

Franklin, A. (1986), *The Neglect of Experiment*, Cambridge: Cambridge University Press.

French, S. (2003), "Scribbling on the Blank Sheet: Eddington's Structuralist Conception of Objects", *Studies in History and Philosophy of Modern Physics*, 34, 227–259.

French, S. and Krause, D. (2006), *Identity in Physics: A Historical, Philosophical, Formal Analysis*, New York: Oxford University Press.

French, S. and Ladyman, J. (2003), "Remodelling Structural Realism: Quantum Physics and the Metaphysics of Structure", and "The Dissolution of Objects: A Reply to Cao", *Synthese*, 136, 31–56, 73–77.

Friedman, M. (1983), *Foundations of Space-Time Theories*, Princeton, NJ: Princeton University Press.

Friedman, M. (1985), "Kant's Theory of Geometry", *The Philosophical Review*, XCIV, 4, 455–506.

Friedman, M. (1990), "Kant and Newton: Why Gravity is Essential to Matter", in: Bricker and Hughes (1990), pp. 185–202.

Friedman, M. (1992a), *Kant and the Exact Sciences*, Cambridge, MA: Harvard University Press.

Friedman, M. (1992b), "Causal Laws and the Foundations of Natural science", in: P. Guyer (ed.), *The Cambridge Companion to Kant*, Cambridge: Cambridge University Press.

Friedman, M. (1994), "Kant and the Twentieth Century", in: P. Parrini (ed.), *Kant and Contemporary Epistemology*, Dordrecht, The Netherlands: Kluwer.

Friedman, M. (1997a), "Descartes on the Real Existence of Matter", *Topoi*, 16, 1–10.

Friedman, M. (1997b), "Helmholtz's *Zeichentheorie* and Schlick's *Allgemeine Erkenntnislehre*: Early Logical Empiricism and its Nineteenth-Century Background", *Philosophical Topics*, 25, 19–50.

Friedman, M. (1999), *Reconsidering Logical Positivism*, Cambridge: Cambridge University Press.

Friedman, M. (2000a), "Geometry, Construction, and Intuition in Kant and His Successors", in: G. Scher and R. Tieszen (eds.), *Between Logic and Intuition: Essays in Honor of Charles Parsons*, Cambridge: Cambridge University Press.

Friedman, M. (2000b), "Transcendental Philosophy and A Priori Knowledge: A Neo-Kantian Perspective", in: P. Boghossian and C. Peacocke (eds.), *New Essays on A Priori*, Oxford: Clarendon.

Friedman, M. (2000c), *A Parting of the Ways. Carnap, Cassirer, and Heidegger*, Chicago, IL: Open Court.

Friedman, M. (2001), *Dynamics of Reason. The 1999 Kant Lectures at Stanford University*, Stanford, CA: CSLI Publications.

Friedman, M. (2002), "Geometry as a Branch of Physics: Background and Context for Einstein's 'Geometry and Experience", in: D. Malament (ed.), *Reading Natural Philosophy: Essays in the History and Philosophy of Science and Mathematics*, Chicago, IL: Open Court, pp. 193–229.

Friedman, M. (trans. and ed.) (2004), *Immanuel Kant: Metaphysical Foundations of Natural Science*, Cambridge: Cambridge University Press.

Friedman, M. (2008), "Descartes and Galileo: Copernicanism and the Metaphysical Foundations of Physics", in: J. Broughton and J. Carriero (eds.), *A Companion to Descartes*, Oxford: Blackwell (forthcoming).

Gale, G. (1992), "Rationalist Programmes in Early Modern Cosmology", *The Astronomy Quarterly*, 8, 4–21.

Gale, G. (1999), "Milne, Bondi and the 'Second Way' to Cosmology", in: H. Goenner, J. Renn, J. Ritter and T. Sauer (eds.), *The Expanding Worlds of General Relativity*, Boston, MA/Basel, Switzerland/Berlin: Birkhäuser.

Gale, G. and Shanks, N. (1996), "Methodology and the Birth of Modern Cosmological Inquiry", *Studies in the History and Philosophy of Modern Physics*, 27B, 279–296.

Gale, G. and Urani, J. (1993), "Philosophical Midwifery and the Birth-pangs of Modern Cosmology", *American Journal of Physics*, 61, 66–73.

Gawedzki, K. and Kupiainen, A. (1985), "Gross-Neveu Model Through Convergent Perturbation Expansion", *Communications in Mathematical Physics*, 102, 1–30.

Ghirardi, G. C., Rimini, A. and Weber, T. (1986), "Unified Dynamics for Microscopic and Macroscopic Systems", *Physical Review*, D34, 470–491.

Gibson, J. J. (1979), *The Ecological Approach to Visual Perception*, New York: Houghton Miflin.

Giere, R. (1988), *Explaining Science. A Cognitive Approach*, Chicago, IL: Chicago University Press.

Giere, R. (1999), *Science Without Laws*, Chicago, IL: University of Chicago Press.

Giere, R. (2006a), "Perspectival Pluralism" Scientific Pluralism, volume XIX, in: S. Kellert, H. Longino and C. K. Waters (eds.), *Minnesota Studies in the Philosophy of Science*, Minneapolis, MN: University of Minnesota Press.

Giere, R. (2006b), *Scientific Perspectivism*, Chicago, IL: University of Chicago Press.

Giulini, D., Joos, E., Kiefer, C., Kupsch, J., Stamatescu, I.-O. and Zeh, H. D. (1996), *Decoherence and the Appearance of a Classical World in Quantum Theory*, Berlin: Springer.

Glimm, J. and Jaffe, A. (1987), *Quantum Physics*, New York: Springer.

Gloy, K. (1976), *Die Kantische Theorie der Naturwissenschaft*, Berlin: de Gruyter.

Goldstein, H. (1981), *Classical Mechanics*, Cambridge, MA: Addison-Wesley.

Gómez, R. (1987), "Resonancias kantianas en el pensamiento de Bohr", *Revista Latinoamericana de Filosofía*, XIII, 3–23.

Grünbaum, A. (1973), *Philosophical Problems of Space and Time*, Dordrecht, The Netherlands: Reidel.

Guyer, P. (1990), "Reason and Reflective Judgment: Kant on the Significance of Systematicity", *Nous*, 24, 17–43.

Hacking, I. (1983), *Representing and Intervening*, Cambridge: Cambridge University Press.

Hagley, E., Maître, X., Wunderlich, C., Brune, M., Raimond, J. M. and Haroche, S. (1997), "Generation of Einstein-Podolsky-Rosen pairs of atoms", *Physical Reviews Letters*, 79, 1–5.

Hall, A. R. and M. B. Hall (eds.) (1962), *Unpublished Scientific Papers of Isaac Newton*, Cambridge: Cambridge University Press.

Harré R. (1986), *Varieties of Realism. A Rationale for the Natural Sciences*, Oxford: Blackwell.

Harré R. (1990), "Exploring the Human Umwelt", in: R. Bhaskar (ed.), *Harré and His Critics*, Oxford: Blackwell.

Harré R. (1998), "Recovering the Experiment", *Philosophy*, 73, 353–777.

Hartle, J. B. and Hawking, S. W. (1983), "Wave Function of the Universe", *Physical Review*, D28, 2960–2975.

Hartmann, S. (2001), "Effective Field Theories, Reductionism and Scientific Explanation", *Studies in History and Philosophy of Modern Physics*, 32, 267–304.

Hatfield, G. (1990), *The Natural and the Normative: Theories of Spatial Perception from Kant to Helmholtz*, Cambridge, MA: MIT Press.

Heelan, P. (2004), "The Phenomenological Role of Consciousness in Measurement", *Mind and Matter*, 2, 61–84.

Heisenberg, W. (1927), "Über den Anschaulichen Inhalt der quantentheoretischen Kinematik und Mekanik", *Zeitschrift für Physik*, 43, 197.

Heisenberg, W. (1930a), *Physikalische Prinzipien der Quantentheorie*, Leipzig, Germany: Hirzel.

Heisenberg, W. (1930b), *The Physical Principles of the Quantum Theory*, Chicago, IL: The University of Chicago Press.

Heisenberg, W. (1939), "Gedanken der antiken Naturphilosophie in der modernen Physik", *Die Antike*, 13, 118–124.

Hermann, G. (1996), *Les fondements philosophiques de la mécanique quantique*, Paris: Vrin.

Hermann, G. (1999), "The Foundations of Quantum Mechanics in the Philosophy of Nature", *Harvard Review of Philosophy*, 7, 35–44.

Hertz, H. (1894), *Die Prinzipien der Mechanik*, Leipzig, Germany: Barth.

Hilbert, D. (1900), "Mathematische Probleme", *Göttinger Nachrichten. Mathematisch-Physikalische Klasse*, 253–297. English translation in: *Bulletin of the American Mathematical Society*, 8 (1902), 437–479.

Hilbert, D. (1916), "Die Grundlagen der Physik (Erste Mitteilung)", *Göttinger Nachrichten. Mathematisch-Physikalische Klasse*, 395–407.

Hilbert, D. (1917), "Die Grundlagen der Physik (Zweite Mitteilung)", *Göttinger Nachrichten. Mathematisch-Physikalische Klasse*, 53–76.

Hilbert, D. (1918), “Axiomatisches Denken”, *Mathematische Annalen*, 78, 405–415. English translation in: W. Ewald (ed.), *From Kant to Hilbert: A Source Book in the Foundations of Mathematics*, Vol. II, Oxford: Clarendon, 1996, pp. 1105–1115.

Hilbert, D. (1924), “Die Grundlagen der Physik”, reprinted in *Hilbertiana – Fünf Aufsätze von David Hilbert*, Darmstadt, Germany: Wissenschaftliche Buchgesellschaft, 1964, pp. 47–78.

Hilbert, D. (1930), “Naturerkennen und Logik”, *Die Naturwissenschaften*, 18, 959–963. English translation in: W. Ewald (ed.), op.cit., Vol. II., pp. 1157–1165.

Hilgevoord, J. and Uffink, J. (2006), “The Uncertainty Principle”, in: E. N. Zalta (ed.), *The Stanford Encyclopedia of Philosophy*.

Honner, J. (1987), *The Description of Nature. Niels Bohr and the Philosophy of Quantum Physics*, Oxford: Clarendon.

Hoppe, H. (1969), *Kants Theorie der Physik*, Frankfurt, Germany: Klostermann.

Howard, D. (1994), “Einstein, Kant, and the Origins of Logical Empiricism”, in: W. Salmon and G. Wolters (eds.), *Logic, Language, and the Structure of Scientific Theories. Proceedings of the Carnap-Reichenbach Centennial, University of Konstanz, 21–24 May, 1991*, Pittsburgh, PA/Konstanz, Germany: University of Pittsburgh Press/Universitätsverlag Konstanz.

Hoyle, F. (1948), “A New Model for the Expanding Universe”, *Monthly Notices of the Royal Astronomical Society*, 108, 372–382.

Hughes, R. I. G. (1989), *Quantum Mechanics, Its Structure and Interpretation*, Cambridge, MA: Harvard University Press.

Hume, D. (1978), *A Treatise of Human Nature*, Oxford: Clarendon.

Humphreys, P. (1997), “How Properties Emerge”, *Philosophy of Science*, 64, 1–17.

Husserl, E. (1910), *“Philosophie als strenge Wissenschaft”, Logos*, I, 1910–1911, 289–341. Translation by Q. Lauer as “Philosophy as Rigorous Science”, in: E. Husserl, *Phenomenology and the Crisis of Philosophy*, New York: Harper & Row, 1965.

Husserl, E. (1929), *“Formale und transzendentale Logik: Versuch einer Kritik der logischen Vernunft”, Jahrbuch für Philosophie und phänomenologische Forschung*, 10, 1–298. Translation by Dorion Cairns as *Formal and Transcendental Logic*, The Hague, The Netherlands: Martinus Nijhoff, 1969.

Husserl, E. (1962), *Die Krisis der europäischen Wissenschaften und die tranzendentale Phänomenologie. Eine Einleitung in die phänomenologische Philosophie*. W. Biemel (hrsg). 2. Auflage. Haag, Martins Nijhoff, 29–30. Parts originally published in Philosophia, 1 (1936). Translated by D. Carr as *The Crisis of European Sciences and Transcendental Phenomenology*, Evanston, IL: Northwestern University Press, 1970.

Ihmig, K.-N. (1999), “Ernst Cassirer and the Structural Conception of Objects in Modern Science: The Importance of the ‘Erlanger Program’ ”, *Science in Context*, 12, 513–529.

Iochum, B., Kastler, D. and Schücker, T. (1996), “On the Universal Chamseddine-Connes Action. I. Details of the action computation”, hep-th/9607158.

Itzkoff, S. W. (1997), *Ernst Cassirer: Scientific Knowledge and the Concept of Man*, Notre Dame, IN: University of Notre Dame Press.

Itzykson, C. and Zuber, J.-B. (1980), *Quantum Field Theory*, New York: McGraw-Hill.

Janiak, A. (ed.) (2004), *Isaac Newton: Philosophical Writings*, Cambridge: Cambridge University Press.

Jones, V. and Moscovici, H. (1997), “Review of Noncommutative Geometry by Alain Connes”, *Notice of the AMS*, 44, 792–799.

Joos E., Zeh, H.-D., Kiefer, C., Giulini, D., Kupsch, J. and Stamatescu, I.-O. (2003), *Decoherence and the Appearance of a Classical World in Quantum Theory*, IInd ed., New York/Heidelberg, Germany: Springer.

Jost, R. (1965), *The General Theory of Quantized Fields*, Providence, RI: American Mathematical Society.

Kaku, M. (1988), *Introduction to Superstrings*, New York: Springer.

Kant, I. (1770), “On the Form and Principles of the Sensible and the Intelligible World”, in: *Theoretical Philosophy, 1755–1770*, in: H. Allison, R. Brandt, P. Guyer, R. Meerbote, C. D. Parsons, H. Robinson, J. B. Schneewind and A. W. Wood (eds.), *The Cambridge Edition of the Works of Immanuel Kant in Translation*, Cambridge: Cambridge University Press.

Kant, I. (1781), *Critique of Pure Reason*, Edited by P. Guyer and A. W. Wood, in: H. Allison, R. Brandt, P. Guyer, R. Meerbote, C. D. Parsons, H. Robinson, J. B. Schneewind and A. W. Wood (eds.), *The Cambridge Edition of the Works of Immanuel Kant in Translation*, Cambridge: Cambridge University Press, 1999.

Kant, I. (1783), "Prolegomena to Any Future Metaphysics That Will Be Able to Come Forward as Science", in: *Theoretical Philosophy after 1781*, H. Allison, R. Brandt, P. Guyer, R. Meerbote, C. D. Parsons, H. Robinson, J. B. Schneewind and A. W. Wood (eds.), *The Cambridge Edition of the Works of Immanuel Kant in Translation*, Cambridge: Cambridge University Press, 2002.

Kant, I. (1786), *Metaphysical Foundations of Natural Science*, Translated by M. Friedman, in: *Theoretical Philosophy After 1781*, H. Allison, R. Brandt, P. Guyer, R. Meerbote, C. D. Parsons, H. Robinson, J. B. Schneewind and A. W. Wood (eds.), *The Cambridge Edition of the Works of Immanuel Kant in Translation*, Cambridge: Cambridge University Press, 2002.

Kant, I. (1790), *Critique of the Power of Judgment*, Guyer, P. and Matthews, E. (trans.), Guyer, P. (ed.), in: H. Allison, R. Brandt, P. Guyer, R. Meerbote, C. D. Parsons, H. Robinson, J. B. Schneewind and A. W. Wood (eds.), *The Cambridge Edition of the Works of Immanuel Kant in Translation*, Cambridge: Cambridge University Press, 2000.

Kant, I. (1800), *Logik*, English translation in: R. S. Hartman and W. Schwarz (eds.), *Logic*, New York: Dover, 1974.

Kant, I. (1902), *Kants gesammelte Schriften*, Königlichen Preußischen (Deutschen) Berlin: Akademie der Wissenschaften.

Kant, I. (1957), "Metaphysische Anfangsgründe der Naturwissenschaft. Vorrede", in: *Werke*, Band V, Darmstadt, Germany: Wissenschaftliche Buchgesellschaft.

Kant, I. (1986), *Briefwechsel*, Hamburg, Germany: Felix Meiner Verlag.

Kastler, D. (1995), "The Dirac Operator and Gravitation", *Communications in Mathematical Physics*, 166, 633–643.

Kastler, D. and Schücker, T. (1994), "The Standard Model à la Connes-Lott", hep-th/9412185.

Kemp, S. N. (1923), *A Commentary to Kant's 'Critique of Pure Reason'*, London: Humanities, 1950.

Kent, A. (1990), "Against Many-World Interpretation", *International Journal of Modern Physics*, A5, 1745–1762.

Kerszberg, P. (1999), *Kant et la Nature*, Paris: Belles-Lettres.

Kitcher, P. (1994), "The Unity of Science and the Unity of Nature", in: P. Parrini (ed.), *Kant and Contemporary Epistemology*, Dordrecht, The Netherlands: Kluwer.

Kitcher, P. (1998), "Projecting the Order of Nature", in: P. Kitcher (ed.), *Kant's Critique of Pure Reason. Critical Essays*, Lanham, MD: Rowan & Littlefield.

Kostant, B. (1970), "Quantization and Unitary Representations", *Lecture Notes in Mathematics*, Vol. 170, Berlin/Heidelberg, Germany/New York: Springer.

Koyré, A. (1965), *Newtonian Studies*, Cambridge, MA: Harvard University Press.

Kragh, H. (1996), *Cosmology and Controversy: The Historical Development of Two Theories of the Universe*, Princeton, NJ: Princeton University Press.

Krantz, D. H., Duncan, R., Suppes, L. P. and Tversky, A. (1971), *Foundations of Measurement*, Vol. I, San Diego, CA: Academic.

Krieger, L. (1965), "Kant and the Crisis of Natural Law", *Journal of the History of Ideas*, 26, 191–210.

Kuhn, T. S. (1962), *The Structure of Scientific Revolutions*, Chicago, IL: University of Chicago Press.

Kuhn, T. S. (1983), "Rationality and Theory Choice", *Journal of Philosophy*, 80, 563–570.

Ladyman, J. (1998), "What Is Structural Realism?", *Studies in History and Philosophy of Science*, 29, 409–424.

Lanczos, C. (1986), *The Variational Principles of Mechanics*, New York: Dover.

Landi, G. and Rovelli, C. (1997), "General Relativity in Terms of Dirac Eigenvalues", *Physical Review Letters*, 78, 3051–3054.

Laugwitz, D. (1958), "Über eine Vermutung von Hermann Weyl zum Raumproblem", *Archiv der Mathematik*, 9, 128–133.

Laywine, A. (1993), Kant's Early Metaphysics and the Origins of the Critical Philosophy, Atascadero, CA: Ridgeview.

Le Bellac, M. (1988), *Des phénomènes critiques aux champs de jauge*, Paris: InterEditions – C.N.R.S.

Leibniz, G. W. and Clarke, S. (1715/16), *The Leibniz-Clarke Correspondence*, Edited with introduction and notes by H. G. Alexander. Manchester: Manchester University Press, 1956.

Leggett, A. J. (1987), "Reflections on the Quantum Measurement Paradox", in: B. J. Hiley and F. D. Peat (eds.), *Quantum Implications: Essays in Honour of David Bohm*, London: Routledge & Kegan Paul.

Lewis, D. K. (1983), *Philosophical Papers*, Vol. I, Oxford: Oxford University Press.

Lewis, D. K. (1984), "Putnam's Paradox", *Australasian Journal of Philosophy* 62, 221–236.

Libermann, P. and Marle, C.-M. (1987), *Symplectic Geometry and Analytical Mechanics*, Dordrecht, The Netherlands: Reidel.

Locke, J. (1689), *An Essay Concerning Human Understanding*, Oxford: Clarendon, 1975.

Mach, E. (1989), *The Science of Mechanics. Account of Its Development*, La Salle, IL: Open Court.

Maddy, P. (2000), "Naturalism and the *A Priori*", in: P. Bogohassian and C. Peacocke (eds.), *New Essays on the A Priori*, New York: Oxford University Press, pp. 92–116.

Majer, U. and Sauer, T. (2004), "Hilbert's 'World Equations' and His Vision of a Unified Science", arXiv:physics/0405110v1.

Makey, G. (1963), *The Mathematical Foundations of Quantum Mechanics*, New York: Benjamin.

Manin, Y. I. (1988), *Gauge Field Theory and Complex Geometry*, Berlin/New York: Springer.

Marsden, J. (1974), *Applications of Global Analysis in Mathematical Physics*, Berkeley, CA: Publish or Perish.

Marsden, J. and Ratiu, T. S. (1999), *Introduction to Mechanics and Symmetry*, New York: Springer.

Martin, C. P., Gracia-Bondia, J. M. and Varilly, J. C. (1997), "The Standard Model as a Noncommutative Geometry: The Low Energy Regime", hep-th/9605001 v2.

Mates, B. (1986), *The Philosophy of Leibniz*, Oxford: Oxford University Press.

Maxwell, J. C. (1873), *Treatise on Electricity and Magnetism*, New York: Dover, 1952.

McGabe, G. (2005), "The Structure and Interpretation of Cosmology: Part II. The Concept of Creation in Inflation and Quantum Cosmology", *Studies in History and Philosophy of Modern Physics*, 36, 67–102.

McGowan, M. K. (1996), *Realism or Non-Realism; Undecidable in Theory, Decidable in Practice*, Dissertation, Princeton University.

McGowan, M. K. (2001), "Privileging Properties", *Philosophical Studies*, 105, 1–23.

McGowan, M. K. (2003), "Realism, Reference and Grue: Why Metaphysical Realism Cannot Solve the Grue Paradox", *American Philosophical Quarterly*, 40, 47–57.

McMullin, E. (1993), "Indifference Principle and Anthropic Principle in Cosmology", *Studies in the History and Philosophy of Science, 24*, 359–389.

McMullin, E. (1994), "Long Ago and Far Away: Cosmology as Extrapolation", in: R. Fuller, (ed.), *Bang: The Evolving Cosmos*, Lanham, MD: University Press of America.

Menn, S. (1998), *Descartes and Augustine*, Cambridge: Cambridge University Press.

Merleau P. J. (1965), *Cosmologie du XX Siècle*, Paris: Gallimard.

Michotte, A. (1963), *The Perception of Causality*, London: Methuen.

Milne, E. A. (1948), *Kinematic Relativity*, Oxford: Clarendon.

Misner, C. W., Thorne, K. S., Wheeler, J. A. (1973), *Gravitation*, San Francisco, CA: Freeman.

Mittelstaedt, P. (1994), "The Constitution of Objects in Kant's Philosophy and in Modern Physics", in: P. Parrini (ed.), *Kant and Contemporary Epistemology*, Dordrecht, The Netherlands: Kluwer.

Mittelstaedt, P. (1995), "Constitution of Objects in Classical Mechanics and in Quantum Mechanics", *International Journal of Theoretical Physics*, 34, 1615–1626.

Mittelstaedt, P. (1998), *The Interpretation of Quantum Mechanics and the Measurement Process*, Cambridge: Cambridge University Press.

Mittelstaedt, P. (2006a), "The Intuitiveness and Truth of Modern Physics", in: Carson and Huber (2006), pp. 251–266.

Mittelsteadt, P. (2006b), "Einstein's Objections Against Quantum Mechanics", *Physics and Philosophy*, http://hdl.handle.net/2003/22995

Mohrhoff, U. (2001), "Two Theories of Decoherence", arXiv: quant-ph/0108002.

Montvay, I. and Münster, G. (1994), *Quantum Fields on a Lattice*, Cambridge: Cambridge University Press.

Mudroch, V. (1987), *Kants Theorie der physikalischen Gesetze*, Berlin: de Gruyter.

Munitz, M. (1962), "The Logic of Cosmology", *The British Journal for the Philosophy of Science*, 13, 34–50.

Murdoch, D. (1987), *Niels Bohr's Philosophy of Physics*, Cambridge: Cambridge University Press.

Nagel, E. (1961), *The Structure of Science. Problems in the Logic of Scientific Explanation*, New York: Harcour, Brace & World.

Newton, I. (1687), *Philosophiae Naturalis Principia Mathematica*, Londinum. Quoted after the new english translation in: *The Principia. Mathematical Principles of Natural Philosophy*. Translated by I. Bernard Cohen and Anne Whitman with the assistance of Julia Budenz. Berkeley, CA/London: University of California Press, 1999.

Newton, I. (1730), *Opticks*, London (4th edn.). Reprint with a Preface by I. B. Cohen. New York: Dover, 1979.

Noguès, G., Rauschenbeutel, A., Osnaghi, S., Brune, M., Raimond, J. M. and Haroche, S. (1999), "Seeing a Single Photon Without Destroying It", *Nature*, 400, 239–242.

North, J. (1965), *The Measure of the Universe. A History of Modern Cosmology*, Oxford: Clarendon.

Norton, J. (1985), "What Was Einstein's Principle of Equivalence?", *Studies in History and Philosophy of Science*, 16, 203–246. Reprinted in: D. Howard and J. Stachel (eds.) (1989), *Einstein and the History of General Relativity*, Boston, MA: Birkhäuser.

Nozick, R. (1998), "Invariance and Objectivity", *Proceedings and Addresses of the American Philosophical Association*, 72, 21–48.

Omnès, R. (1994), *The Interpretation of Quantum Mechanics*, Princeton, NJ: Princeton University Press.

O'Shea, J. R. (1997), "The Needs of Understanding: Kant on Empirical Laws and Regulative Ideals", *International Journal of Philosophical Studies*, 5, 216–254.

Osnaghi, S., Bertet, P., Auffèves, A., Maioli, P., Brune, M., Raimond, J. M. and Haroche, S. (2001), "Coherent Control of an Atom Collision in a Cavity", *Physical Review Letters*, 87, 037902–037905.

Park, J. L. (1973), "The Self-Contradictory Foundations of Formalistic Quantum Measurement Theories", *International Journal of Theoretical Physics*, 8, 211–218.

Parrini, P. (1976), *Linguaggio e teoria. Due saggi di analisi filosofica*, Florence, Italy: La Nuova Italia.

Parrini, P. (1983), *Empirismo logico e convenzionalismo*, Milano, Italy: Angeli.

Parrini, P. (1994), "With Carnap Beyond Carnap. Metaphysics, Science, and the Realist/Instrumentalism Controversy", in: W. Salmon and G. Wolters (eds.), *Logic, Language, and the Structure of Scientific Theories. Proceedings of the Carnap-Reichenbach Centennial, University of Konstanz, 21–24 May, 1991*, Pittsburgh, PA/Konstanz, Germany: University of Pittsburgh Press/Universitätsverlag Konstanz.

Parrini, P. (1995), *Knowledge and Reality. An Essay in Positive Philosophy*, Dordrecht, The Netherlands: Kluwer.

Parrini, P. (2002a), "Realism and Anti-Realism from an Epistemological Point of View", in: M. Marsonet (ed.), *The Problem of Realism*, Bodmin, Cornwall: Ashgate.

Parrini, P. (2002b), *L'empirismo logico. Aspetti storici e prospettive teoriche*, Roma: Carocci.

Parrini, P. (2003a), "Reason and Perception. In Defense of a Non-Linguistic Version of Empiricism", in: P. Parrini, M. Salmon and W. Salmon (eds.), *Logical Empiricism. Historical and Contemporary Perspectives*, Pittsburgh, PA: University of Pittsburgh Press.

Parrini, P. (2003b), "On the Formation of Logical Empiricism", in: F. Stadler (ed.), *The Vienna Circle and Logical Empiricism. Re-Evaluation and Future Perspectives*, Dordrecht, The Netherlands: Kluwer.

Parrini, P. (2004), "The 'Dogma' of Analyticity Fifty Years After", in: M. Marsonet and M. Benzi (eds.), *Logic and Metaphysics*, Genoa, Italy: Name Edizioni.

Patrides, C. A. (ed.), (1980), *The Cambridge Platonists*, Cambridge: Cambridge University Press.

Pauli, W. Jr. (1921). *Relativitätstheorie*, in *Encyklopädie der mathematischen Wissenschaften*, 19. Leipzig, Germany: B. G. Teubner. Translated as *The Theory of Relativity*, New York: Pergamon, 1958.

Pearson, K. (1892), *The Grammar of Science*, London: J.M. Dent, 1943.

Peirce, C. S. (1991), Notes concerning the "Architecture of Theories", MS 956, in: H. Pape (ed.), *Charles S. Peirce. Naturordnung und Zeichenprozess*, Frankfurt am Main, Germany: Suhrkamp.

Peirce, C. S. (1998), *The Essential Peirce—Volume II*, Bloomington, IN: Indiana University Press.

Peres, A. (1995), *Quantum Theory, Concepts and Methods*, Dordrecht, The Netherlands: Kluwer.

Petitot, J. (1990a), "Logique transcendantale, Synthétique a priori et Herméneutique mathématique des Objectivités", *Fundamenta Scientiæ* (special issue in honor of Ludovico Geymonat), 10, 57–84.

Petitot, J. (1990b), "Logique transcendantale et Herméneutique mathématique: le problème de l'unité formelle et de la dynamique historique des objectivités scientifiques", in: *Il pensiero di Giulio Preti nella cultura filosofica del novecento*, Milan, Italy: Franco Angeli.

Petitot, J. (1991a), *La Philosophie transcendantale et le problème de l'objectivité*, in: F. Marty (ed.), *Entretiens du Centre Sèvres*, Paris: Editions Osiris.

Petitot, J. (1991b), "Idéalités mathématiques et Réalité objective. Approche transcendantale", in: G. Granel (ed.), *Hommage à Jean-Toussaint Desanti*, Mauvezin, France: Editions TER.

Petitot, J. (1992a), "Actuality of Transcendental Aesthetics for Modern Physics", in: L. Boi, D. Flament and J.-M. Salanskis (eds.), *1830–1930 A Century of Geometry*, Berlin/New York: Springer.

Petitot, J. (1992b), "Continu et Objectivité. La bimodalité objective du continu et le platonisme transcendantal", in: J.-M. Salanskis and H. Sinaceur (eds.), Le *Labyrinthe du Continu*, Paris: Springer.

Petitot, J. (1994), "Esthétique transcendantale et physique mathématique", in: E. W. Orth and H. Holzhey (eds.), *Neukantianismus. Perspektiven und Probleme*, Wurzburg, Germany: Königshausen & Neumann.

Petitot, J. (1995), "Pour un platonisme transcendantal", in: M. Panza and J.-M. Salanskis (eds.), *L'objectivité mathématique, Platonisme et structures formelles*, Paris: Masson.

Petitot, J. (1997), "Objectivité faible et Philosophie transcendantale", in: M. Bitbol and S. Laugier (eds.), *Physique et Réalité, in honor of B. d'Espagnat*, Paris: Frontières-Diderot Editeur.

Petitot, J. (2002), "Mathematical Physics and Formalized Epistemology", in: M. Mugur- Schächter and A. van der Merwe (eds.), Dordrecht, The Netherlands: Kluwer.

Petitot, J. (2006), "Per un platonismo trascendentale", in: S. Albeverio and F. Minazzi (eds.), *Matematica e Filosofia*, PRISTEM/Storia, Milan, Italy: University of Luigi Bocconi, Milan.

Petruccioli, S. (1993), *Atoms, Metaphors and Paradoxes. Niels Bohr and the Construction of a New Physics*, Cambridge: Cambridge University Press.

Pickering, A. (1984), *Constructing quarks. A sociological history of particle physics*, Chicago, IL: Chicago University Press.

Piron, C. (1976), *Foundations of Quantum Physics*, Reading, MA: Benjamin.

Pitowsky, I. (1994), "George Boole's 'Conditions of Possible Experience' and the Quantum Puzzle", *British Journal for the Philosophy of Science*, 45, 95–125.

Plaass, P. (1994), *Kant's Theory of Natural Science*, Dordrecht, The Netherlands: Reidel.

Planck, M. (1908), “Die Einheit des physikalischen Weltbildes”, in: *Vorträge und Erinnerungen*, Darmstadt, Germany: Wissenschaftliche Buchgesellschaft, 1965.

Planck, M. (1910), “Zur Theorie der Wärmestrahlung”, in: *Physikalische Abhandlungen und Vorträge*, Braunschweig, Germany: Vieweg, 1958, Vol. II, pp. 237–247.

Planck, M. (1944), *Wege zur physikalischen Erkenntnis. Reden und Vorträge*, Leipzig, Germany: S. Hirzel.

Poincaré, H. (1902), *La Science et l'Hypothèse*, Paris: Flammarion. Translated as *Science and Hypothesis* in Poincaré (1913b).

Poincaré, H. (1905), *La Valeur de la Science*, Paris: Flammarion.

Poincaré, H. (1908), *Science et Méthode*, Paris: Flammarion.

Poincaré, H. (1913a), *Dernières Pensées*, Paris: Flammarion. Translated by J. Buldoc (1963) as *Mathematics and Science: Last Essays*, New York: Dover.

Poincaré, H. (1913b), *The Foundations of Science*, G. Halstead, Lancaster (trans. and ed.): Science Press.

Polanyi, J. (1986), “Reaction Dynamics”, *Science*, 234, 673.

Pollok, K. (2001), *Kants Metaphysische Anfangsgründe der Naturwissenschaft: Ein kritischer Kommentar*, Hamburg, Germany: F. Meiner.

Popper, K. (1972), *Objective Knowledge*, Oxford: Clarendon.

Posy, C. (1992), “Kant's Mathematical Realism”, in: C. J. Posy (ed.), *Kant's Philosophy of Mathematics*, Dordrecht, The Netherlands: Kluwer.

Pringe, H. (2007), *Critique of the Quantum Power of Judgment. A Transcendental Foundation of Quantum Objectivity*, Berlin: de Gruyter.

Putnam, H. (1975), “Philosophy and Our Mental Life”, in: *Philosophical Papers, Vol. 2*, Cambridge: Cambridge University Press, pp. 291–303.

Putnam, H. (1976), “Realism and Reason”, in: H. Putnam (ed.), *Meaning and the Moral Sciences*, New York: Routledge, 1978.

Putnam, H. (1983), “Why There Isn't a Ready-Made World”, *Realism and Reason, Philosophical papers*, Vol. III, Cambridge: Cambridge University Press.

Putnam, H. (1987), *The Many Faces of Realism*, LaSalle, IL: Open Court.

Quigg, C. (1983), *Gauge Theories of the Strong, Weak, and Electromag-netic Interactions*, Reading, MA: Benjamin-Cummings.

Quine, W. V. (1951a), “Two Dogmas of Empiricism. In: From a Logical Point of View”, Harvard: Harvard University Press, 1953.

Quine, W. V. (1951b), “On Carnap's Views on Ontology”, in: W. V. Quine (ed.), *The Ways of Paradox and Other Essays*, Cambridge, MA: Harvard University Press, 1976.

Quine, W. V. (1953), *From a Logical Point of View. Logico-philosophical Essays*, Cambridge, MA: Harvard University Press.

Quine, W. V (1981), *Theories and Things*, Cambridge: Harvard University Press.

Quine, W. V. (1986), “Reply to Geoffrey Hellman”, in: L. E. Hahn and P. A. Schilpp (eds.), *The Philosophy of W. V. Quine*, La Salle, IL: Open Court.

Raimond, J. M., Brune, M. and Haroche, S. (1997), “Reversible Decoherence of a Mesoscopic Superposition of Field States”, *Physics Review Letters*, 79, 1964–1967.

Raimond, J. M., Brune, M. and Haroche, S. (2001), “Manipulating Quantum Entanglement with Atoms and Photons in a Cavity”, *Review of Modern Physics*, 73, 565–582.

Redhead, M. (1995), *From Physics to Metaphysics*, Cambridge: Cambridge University Press.

Reichenbach, H. (1920), *Relativitätstheorie und Erkenntnis apriori* (Berlin: Springer); English translation in: *The Theory of Relativity and A Priori Knowledge*, Berkeley/Los Angeles, CA: University of California Press, 1965.

Reichenbach, H. (1922), “Der Gegenwärtige Stand der Relativitätsdiskussion” (*Logos*, 10), English translation, “The Present State of the Discussion on Relativity”, in: H. Reichenbach (ed.), *Selected Writings (1909–1953)*, Vol. II, Dordrecht, The Netherlands: Reidel, 1978.

Reichenbach, H. (1924), *Axiomatik der relativistischen Raum-Zeit-Lehre* (Braunschweig: Vieweg); English translation in: *Axiomatization of the Theory of Relativity*, Berkeley/Los Angeles, CA: University of California Press, 1969.

Reichenbach, H. (1928), *Philosophie der Raum-Zeit-Lehre* (Berlin/Leipzig, Germany: Walter de Gruyter), English translation in: *The Philosophy of Space and Time*, New York: Dover Publications, 1958.

Reichenbach, H. (1936), "L'empirisme logistique et la désagrégation de l'*a priori*", in: *Actes du Congrès International de Philosophie Scientifique*, Paris: HermannReichenbach, H. (1965), *The Theory of Relativity and A Priori Knowledge*, Berkeley, CA: University of California Press.

Reichenbach, H. (1956), *The Direction of Time*, San Diego, CA: University of California Press.

Reichenbach, H. (1957), *The Philosophy of Space and Time*, Translated by M. Reichenbach and J. Freund, New York: Dover.

Robinson, D. (1992), "Renormalization and the Effective Field Theory Programme", in: Hull, D., Forbes, M. and Okruhlik, K. (eds.), *Proceedings of the 1992 Biennial Meeting of the Philosophy of Science Association*, Vol. I, East Lansing: Philosophy of Science Association, pp. 393–403.

Rorty, R. (1998), "Truth and Progress", in: *Philosophical Papers* Vol. III, Cambridge: Cambridge University Press.

Roush, S. (2003), "Copernicus, Kant, and the Anthropic Cosmological Principles", *Studies in History and Philosophy of Modern Physics*, 34, 5–35.

Rovelli, C. (1996), "Relational Quantum Mechanics", *International Journal of Theoretical Physics*, 35, 1637–1678.

Rowe, D. E. (2001), "Einstein Meets Hilbert: At the Crossroads of Physics and Mathematics", *Physics in Perspective*, 3, 379–424.

Rudolph, E. (1994), "Substance as Function: Ernst Cassirer's Interpretation of Leibniz as Criticism of Kant", in: E. Rudolph and I. O. Stamatescu (eds.), *Philosophy, Mathematics and Modern Physics. A dialogue*, Heidelberg, Germany: Springer.

Russell, B. (1927), *The Analysis of Matter*, London: Kegan Paul.

Ryckman, T. (1991), "*Conditio sine qua non? Zuordnung* in the early epistemologies of Cassirer and Schlick", *Synthese*, 88, 57–95.

Ryckman, T. (1999), "Einstein, Cassirer, and General Covariance – Then and Now", *Science in Context*, 12, 585–619.

Ryckman, T. (2005), *The Reign of Relativity: Philosophy in Physics 1915–1925*, New York: Oxford University Press.

Sauer, T. (1999), "The Relativity of Discovery: Hilbert's First Note on the Foundations of Physics", *Archive for the History of Exact Sciences*, 53, 529–575.

Saunders, S. (1993a), "Decoherence, Relative States and Evolutionary Adaptation", *Foundations of Physics*, 23, 1553–1585.

Saunders, S. (1993b), "To What Physics Corresponds", in: *Correspondence, Invariance, and Heuristics; Essays in Honour of Heinz Post*, in: S. French and H. Kaminga (eds.), Dordrecht, The Netherlands: Kluwer.

Scaravelli, L. (1968), *Scritti kantiani*, Firenze, Italy: La Nuova Italia.

Schäfer, L. (1966), *Kants Metaphysik der Natur*, Berlin: de Gruyter.

Scheibe, E. (2001), *Between Rationalism and Empiricism*, Selected Papers in Philosophy of Physics, Edited by B. Falkenburg, New York: Springer.

Schilpp, P. A. (ed.) (1949), *Albert Einstein: Philosopher – Scientist*, Evanston, IL: Library of Living Philosophers.

Schlick, M. (1918), *Allgemeine Erkenntnislehre* (Berlin: Julius Springer, Berlin); English translation in: *General Theory of Knowledge*, Wien/New York: Springer, 1974.

Schlick, M. (1921), "Kritizistische oder empiristiche Deutung der neuen Physik?" (*Kant-Studien*, 26), English translation: "Critical or Empiricist Interpretation of Modern Physics?", in: M. Schlick, *Philosophical Papers*, Vol. I, Dordrecht, The Netherlands: Reidel, 1979.

Schlick, M. (1922). *"Die Relativitätstheorie in der Philosophie", Verhandlungen der Gesellschaft Deutscher Naturforscher und Ärzte*, Leipzig, Germany, 58–69.

Schlick, M. (1931), "Die kausalität in der gegenwartigen Physik", *Die Naturwissenschaften*, 19, 145–162; English translation "Causality in Contemporary Physics", in: M. Schlick (ed.), *Philosophical Papers*, Vol. II, Dordrecht, The Netherlands: Reidel, 1979.

Schlick, M. (1932), “Positivismus und Realismus” (*Erkenntnis*, 3), English translation “Positivism and Realism”, in: M. Schlick (ed.), *Philosophical Papers*, Vol. II, Dordrecht, The Netherlands: Reidel, 1979.

Schmitz-Rigal, C. (2002), *Die Kunst des offenen Wissens*, Hamburg, Germany: Meiner. Cassirer-Forschungen, Vol. VII.

Scholz, E. (1994), “Hermann Weyl’s Contribution to Geometry, 1917–1923”, in: S. Chikara, S. Mitsuo and J. W. Dauben (eds.), *The Intersection of History and Mathematics*—, Basel, Switzerland/Boston, MA/Berlin: Birkhäuser Verlag, pp. 203–230.

Schönfeld, M. (2000), *The Philosophy of the Young Kant*, Oxford: Oxford University Press.

Schrödinger, E. (1935a), “Die gegenwärtige Situation in der Quantenmechanik”, *Die Naturwissenschaften*, 23, 807–812, 823–828, 844–849. Translated into English in: J. A. Wheeler and W. H. Zurek (eds.), *Quantum Theory and Measurement*, Princeton, NJ: Princeton University Press, 1983. Translated into French in: E. Schrödinger, *Physique quantique et représentation du monde*, Paris: Points-Seuil, 1992.

Schrödinger, E. (1935b), “Discussion of Probability Relations Between Separated Systems”, *Proceedings of the Cambridge Philosophical Society*, 31, 555–563.

Schrödinger, E. (1995), *The Interpretation of Quantum Mechanics*, M. Bitbol (ed.) Woodbridge, CT: Ox Bow.

Schuhmann, K. and Schuhmann, E. (eds.) (1994). *Edmund Husserl Briefwechsel*, Vol. VII, Dordrecht, The Netherlands/Boston, MA/London: Kluwer.

Schwinger, J. (2003), *Quantum Mechanics, A Symbolism of Measurements*, Berlin/ Heidelberg, Germany: Springer.

Shimony, A. (1984), “Contextual Hidden Variables Theories and Bell’s Inequalities”, *British Journal for the Philosophy of Science*, 35, 25–45.

Shimony, A. (2005), “Bell’s Theorem”, in: E. N. Zalta (ed.), *The Stanford Encyclopedia of Philosophy*.

http://plato.stanford.edu/archives/sum2005/entries/bell-theorem

Simons, P. (1987), *Parts. A Study in Ontology*, Oxford: Clarendon.

Souriau, J.-M. (1975), *Géométrie symplectique et physique mathématique*, Paris: Collection Internationale du C.N.R.S. 237.

Souriau, J.-M. (1997), *Structure of Dynamical Systems. A Symplectic View of Physics*, Cambridge, MA: Birkhäuser Boston.

Stachel, J. (1980), “Einstein and the Rigidly Rotating Disk”, in: A. Held (ed.), *General Relativity and Gravitation*, New York: Plenum. Reprinted as “The Rigidly Rotating Disk as the ‘Missing Link’ in the History of General Relativity, in: D. Howard and J. Stachel (eds.) (1989). *Einstein and the History of General Relativity*, Boston, MA: Birkhäuser.

Stamatescu, I.-O. (1995), “Anschauung und wissenschaftliche Erkenntnis”, in: E. Rudolph and B.-O. Küppers (eds.), *Kulturkritik nach Ernst Cassirer*, Hamburg, Germany: Felix Meiner Verlag.

Stamatescu, I.-O. (2002), “On the Role and Character of Symbols in Modern Physical Theories”, in: M. Ferrari and I.-O. Stamatescu (eds.), *Symbol and Physical Knowledge*, Heidelberg, Germany: Springer.

Stein, H. (1967), “Newtonian Space-Time”, *Texas Quarterly*, 10, 174–200.

Stein, H. (2002), “Newton’s Metaphysics”, in: I. B. Cohen and G. E. Smith (eds.), *The Cambridge Companion to Newton*, Cambridge: Cambridge University Press.

Stöltzner, M. (2000), “Le principe de moindre action et les trois ordres de la téléologie formelle dans la Physique”, *Archives de Philosophie*, 63, 621–655.

Stöltzner, M. (2003), “The Least Action Principle as the Logical Empiricist’s Shibboleth”, *Studies in History and Philosophy of Modern Physics*, 34, 285–318.

Stöltzner, M. (2005), “Drei Ordnungen formaler Teleologie. Ansichten des Prinzips der kleinsten Wirkung”, in: M. Stöltzner and P. Weingartner (eds.), *Formale Teleologie und Kausalität*, Paderborn, Germany: Mentis, pp. 199–241.

Strawson, P. F. (1966). *The Bounds of Sense: An Essay on Kant’s Critique of Pure Reason*, London: Methuen.

Streater, R. F. and Wightman, A. S. (1964), *PCT, Spin and Statistics, and all that*, New York: Benjamin.

Stroud, B. (1987), *The Significance of Philosophical Scepticism*, Oxford: Clarendon.

Sudarshan, E. C. G. and Mukunda, N. (1974), *Classical Dynamics: A Modern Perspective*, NewYork: Willy.

Teller, P. (1981), "The Projection Postulate and Bohr's Interpretation of Quantum Mechanics", in: P. Asquith and R. Giere (eds.), *PSA 1980: Proceedings of the 1980 Biennial Meeting of the Philosophy of Science Association. Vol. II*, East Lansing, MI: Philosophy of Science Foundation, Michigan State University.

Teller, P. (1995), *An Interpretive Introduction to Quantum Field Theory*, Princeton, NJ: Princeton University Press.

Teller, P. (2004), "How We Dapple the World", *Philosophy of Science*, 425–447.

Teller, P. (in preparation a), "The Truth Idealization".

Teller, P. (in preparation b), "Some Dirty Little Secrets About Truth".

Toulmin, R. (1953), *The Philosophy of Science: An Introduction*, London: Hutchinson.

Tuschling, B. (1971), *Metaphysische and transzendentale Dynamik in Kants opus postumum*, Berlin: de Gruyter.

Vaidman, L. (2002), "The Many-Worlds Interpretation of Quantum Mechanics", in: E. N. Zalta (ed.), *The Stanford Encyclopedia of Philosophy*. http://plato.stanford.edu/archives/sum2002/entries/qm-manyworlds

Vallentyne, P. (1988), "Explicating Lawhood", *Philosophy of Science*, 55, 598–613.

Van Fraassen, B. (1980), *The Scientific Image*, Oxford: Oxford University Press.

Van Fraassen, B. (1991), *Quantum Mechanics, An Empiricist View*, Oxford: Oxford University Press.

Van Fraassen, B. (1997), "Putnam's Paradox: Metaphysical Realism Revamped and Evaded", in: *Philosophical Perspectives*, Vol. XI, Boston, MA: Blackwell.

Van Fraassen, B. (2000), "The False Hopes of Traditional Epistemology", *Philosophy and Phenomenological Research*, 60, 253–280.

Van Fraassen, B. (2002), *The Empirical Stance*, New Haven, CT: Yale University Press.

Veltman, M. (2003), *Facts and Mysteries in Elementary Particle Physics*, Singapore: World Scientific.

Vilenkin, A. (2004), "Anthropic Predictions: The Case of the Cosmological Constant", http://www.arxiv.org/abs/astro-ph/0407586

Von Goethe, J. W. (1949), "Zur Naturwissenschaft im Allgemeinen", HA, Bd.13, Hamburg, Germany: Chr. Wegner Verlag.

Von Helmholtz, H. (1886), "Über die physikalische Bedeutung des Princips der kleinsten Wirkung", in: *Wissenschaftliche Abhandlungen*, Leipzig, Germany, 1895, Vol. III.

Von Helmholtz, H. (1892), *Handbuch der physiologischen Optik*, Hamburg, Germany: Voss.

Von Helmholtz, H. (1921), "Die Tatsachen in der Wahrnehmung", in: P. Hertz and M. Schlick (eds.), *Hermann von Helmholtz, Schriften zur Erkenntnistheorie*, Berlin: Springer.

Von Helmholtz, H. (1971), "Die Tatsachen der Wahrnehmung", Rektoratsrede, in: *Philosophische Vorträge und Aufsätze*, Berlin: Akademie-Verlag.

Von Laue, M. (1921), *Die Relativitätstheorie. Bd.II. Die Allgemeine Relativitätstheorie and Einsteins Lehre der Schwerkraft*, Braunschweig, Germany: Vieweg.

Von Neumann, J. (1932), *Mathematical Foundations of Quantum Mechanics*, Princeton, NJ: Princeton University Press, 1955.

Von Weizsäcker, C. F. (1971), *Die Einheit der Natur*, München, Germany/Wien: Hanser.

Von Weiszäcker, C. F. (1980), *The Unity of Nature*, New York: Farrar-Straus-Giroux.

Von Weizsäcker, C. F. (1985), *Der Aufbau der Physik*, München, Wien: Hanser.

Vuillemin J. (1955), *Physique et métaphysique kantiennes*, Paris: Presses Universitaires de France.

Walford, D. and Meerbote, R. (eds.) (1992), *Immanuel Kant: Theoretical Philosophy, 1755–1770*, Cambridge: Cambridge University Press.

Warren, D. (2001), *Reality and Impenetrability in Kant's Philosophy of Nature*, New York: Routledge.

Watkins, E. (ed.) (2001), *Kant and the Exact Sciences*, Oxford: Oxford University Press.

Watkins, E. (2005), *Kant and the Metaphysics of Causality*, Cambridge: Cambridge University Press.

Watson, W. H. (1938), *On Understanding Physics*, Cambridge: Cambridge University Press.

Weitzenböck, R. (1920), *"Über die Wirkungsfunktion in der Weyl'schen Physik". Akademie der Wissenschaften in Wien. Sitzungsberichte, Abteilung IIa. Mathematisch-naturwissenschaftliche Klasse*, 129, 683–708.

Weyl, H. (1976), *Die Philosophie der Mathematik und der Naturwissenschaften*, München, Germany: Oldenbourg.

Weyl, H. (1918a), *Das Kontinnum: Kritische Untersuchungen über die Grundlagen der Analysis*. Leipzig, Germany: Veit. Translated by S. Pollard and T. Bowl as *The Continuum: A Critical Examination of the Foundations of Analysis* (1994), New York: Dover.

Weyl, H. (1918b), *Raum-Zeit-Materie*, Berlin: J. Springer.

Weyl, H. (1918c), *"Gravitation und Elektrizität", Preußische Akademie der Wissenschaften (Berlin) Sitzungsberichte. Physikalisch-Mathematische Klasse*, 465–480, Reprinted in Weyl (1968), 2, 29–42.

Weyl, H. (1918d), *"Reine Infinitesimalgeometrie", Mathematische Zeitschrift*, 2, 384–411, Reprinted in Weyl (1968), 2, 1–28.

Weyl, H. (1921a), *Raum-Zeit-Materie*, 4th ed. Berlin: J. Springer. Translated by H. L. Brose as *Space-Time-Matter*, London: Methuen, 1922; reprinted, New York: Dover, 1953.

Weyl, H. (1921b), *"Über die physikalischen Grundlagen der erweiterten Relativitätstheorie". Physikalische Zeitschrift*, 22, 473–480. Reprinted in Weyl (1968), 2, 99–112.

Weyl, H. (1921c), *"Feld und Materie", Annalen der Physik*, 65, 541–563. Reprinted in Weyl (1968), 2, 237–259.

Weyl, H. (1922), *Space – Time – Matter*, New York: Dover.

Weyl, H. (1923a), *Raum-Zeit-Materie*, 5th edn, Berlin: J. Springer.

Weyl, H. (1923b), *Mathematische Analyse des Raumproblems: Vorlesungen gehalten in Barcelona und Madrid*, Berlin: J. Springer.

Weyl, H. (1926), *"Philosophie der Mathematik und Naturwissenschaft"*, in: A. Baeumler and M. Schröter (eds.), *Handbuch der Philosophie, Abteilung II*, Munich, Germany/Berlin: R. Oldenbourg Verlag. Revised and augmented English edition, based on the translation of O. Helmer, *Philosophy of Mathematics and Natural Science*, Princeton, NJ: Princeton University Press, 1949.

Weyl, H. (1929), "Elektron und Gravitation", *Zeitschrift für Physik*, 56, 330–352. Reprinted in Weyl (1968), 3, 245–267.

Weyl, H. (1931), *"Geometrie und Physik", Die Naturwissenschaften*, 18, 49–58. Reprinted in Weyl (1968), 3, 336–345.

Weyl, H. (1949), "Wissenschaft als symbolische Konstruktion des Menschen", *Eranos-Jahrbuch*, 375–431. Reprinted in *Gesammelte Abhandlungen*, K. Chandrasekharan (ed.), Berlin: Springer, 1968.

Weyl, H. (1952), *Symmetry*, Princeton, NJ: Princeton University Press.

Weyl, H. (1953), "Über den Symbolismus der Mathematik und mathematischen Physik", *Studium generale*, 6, 219–228. Reprinted in *Gesammelte Abhandlungen*, K. Chandrasekharan (ed.), Berlin, 1968.

Weyl, H. (1955), *"Erkenntnis und Besinnung" (Vortrag, gehalten an der Universität Lausanne im Mai 1954). Studia Philosophia* (Jahrbuch der Schwiezerischen Philosophischen Gesellschaft), 15, 153–171. Reprinted in Weyl (1968), 4, 631–649.

Weyl, H. (1968), *Gesammelte Abhandlungen*, Bd. 1–4, in: S. Chandrasekharan (hrsg.), Berlin/Heidelberg, Germany/New York: Springer.

Wheeler, J. A. (1957), "Assessment of Everett's 'Relative State' Formulation of Quantum Theory", *Reviews of Modern Physics*, 29, 463–465.

Wheeler, J. A. and Zurek, W. H. (eds.) (1983), *Quantum Theory and Measurement*, Princeton, NJ: Princeton University Press.

Whitehead, A. N. (1925), *Science and the Modern World*, New York: Macmillan.

Whitehead, A. N. (1978), *Process and Reality*, New York: Free Press.

Wigner, E. (1961), "Remarks on the Mind-Body Question", in: I. J. Good (ed.), *The Scientist Speculates*, London: Heinemann.

Wigner, E. (1964), "Symmetry and Conservation Laws", *Proceedings of the National Academy of Sciences USA*, 51, 956–965. Reprinted in *Philosophical Reflections and Syntheses*. Heidelberg, Germany: Springer (1997).

Wilson, K. G. (1974), "Confinement of Quarks", *Physical Review*, D10, 2445–2459.

Wismann, H. and Stamatescu, I.-O. (1994), "Questions Concerning Theory and Experience and the Role of Mathematics in Physical Science", in: E. Rudolph and I.-O. Stamatescu (Eds.), *Philosophy, Mathematics and Modern Physics – A Dialogue*, Heidelberg, Germany: Springer.

Wittgenstein, L. (1979), *On Certainty*, D. Paul and G. E. M. Anscombe (trans.), Oxford: Blackwell.

Woodhouse, N. (1992), *Geometric Quantization*, Oxford: Oxford University Press.

Zurek, W. H. (1982), "Environment-Induced Superselection Rules", *Physical Review*, D26, 1862–1880.

Zurek, W. H. (2003), "Decoherence, Einselection, and the Quantum Origins of the Classical", *Review of Modern Physics*, 75, 715–775.

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