

A Workbook for Chemical Reaction Equilibria

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The motivation for this project derives from Paper 59c at the AIChE Meeting in Los Angeles, November, 2000, by V.A. Burrows and G.B. Raupp. The concept is to provide a self-contained tutorial supplement to the Elliott-Lira textbook similar in format to the kinds of workbooks that should be familiar from K-12 education. Basically, the material should be presented in whatever manner makes the students feel most comfortable. The important thing is that they learn the material.

The intent is to briefly supplement the discussion in the text, then provide example problems to illustrate in great detail how a particular analysis is performed. Exercise problems immediately following each example are designed to look very similar to the example, instantly reinforcing the principles covered in the example. The disadvantage of this approach is that it discourages the independent thought and analysis process that engineers must ultimately develop. Note, however, that this is a supplement to the existing text. The existing homework problems remain as general statements like homework problems in any text.

Clearly there is more work involved for students who pursue the workbook. There is nothing wrong with that if the workbook provides a path for students to learn the material who might not have learned it otherwise. Hence, we suggest that the workbook not be a required element for all students, but completion of the workbook might gain recognition in a small component of the overall grading such as “class participation.” Long term, it would be very interesting to study how student success correlates with completion of the workbook. We would expect a significant correlation that might depend on the interaction between overall grade in the course and completion of the workbook.

The specific chapter covered by this particular workbook is Chapter 14, Reacting Systems. More than any other chapter, we have heard suggestions that the coverage in this chapter is quite broad, with insufficient examples to thoroughly grasp the principles. Addressing this concern is quite fitting within the workbook context since providing examples is what the workbook is all about. Note that we have included complete solutions to the practice problems stated in the text in addition to the additional workbook examples. With these complete solutions, combined with the additional workbook examples, the total number of solved examples is roughly tripled. We hope that these additions will help to relieve the difficulty perceived by students with this subject matter.

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14 Reacting Systems

14.1 Reaction Coordinate

The key point of this section is for you to review mole balances and their implications. You should be familiar with the basic concepts from courses in mass and energy balances (MEBs) and introductory chemistry so we do not dwell too long on all of these basics. The following questions should flesh out a few subtleties if you missed them.

14.1.1 Example. Consider the reaction $\text{CO} + \text{H}_2\text{O} = \text{CO}_2 + \text{H}_2$.

- Identify ν_1 - ν_4 .
- If the initial moles of $\text{H}_2\text{O}:\text{CO}$ were 2:1, how would that affect the values of ν_1 - ν_4 ?
- If the reaction were rewritten as $2\text{CO} + 2\text{H}_2\text{O} = 2\text{CO}_2 + 2\text{H}_2$, how would that affect the values of ν_1 - ν_4 ?
- Reconsider the reaction $\text{CO} + \text{H}_2\text{O} = \text{CO}_2 + \text{H}_2$, assuming a stoichiometric feed this time. A “stoichiometric feed” means that the reactants are fed in the ratio of their ν coefficients. In this case, that means moles of $\text{H}_2\text{O}:\text{CO}$ of 1:1, or 2:2, etc. Compute the mole fractions of all components assuming the extent of reaction is, ξ , 0.25. Also compute the ratio of product mole fractions to reactant mole fractions.
- Reconsider the reaction $\text{CO} + \text{H}_2\text{O} = \text{CO}_2 + \text{H}_2$, assuming a feed in which the moles of H_2O and CO are 2 and 1. Compute the mole fractions of all components assuming the extent of reaction, ξ , is 0.25. Also compute the ratio of product mole fractions to reactant mole fractions. What are the total moles going out of the reactor and the moles coming in?
- The “limiting reactant” is the reactant that is depleted first, hence stopping the reaction. If the initial moles of $\text{H}_2\text{O}:\text{CO}$ were 2:1, which component would be the limiting reactant?

Solution:

- $\nu_1 = \nu_2 = -1$; $\nu_3 = \nu_4 = 1$
- The initial moles do not affect the stoichiometric coefficients. Only the mole fractions are affected.
- Everything would be multiplied by two. Note that the ΔG value would also be doubled with the implication that the K_a value would be squared, since it is $\exp(-\Delta G)$. All the mole fractions of products/reactants would also be squared because of being raised to the power of the stoichiometric coefficient.
- Making a table to keep track of the stoichiometry:

CO	1	$-\xi$
H ₂ O	1	$-\xi$
CO ₂		ξ
H ₂		ξ
Tot	2+	0

$$y_{\text{CO}} = y_{\text{H}_2\text{O}} = (1-\xi)/2 = 0.75/2 = 0.375$$

$$y_{\text{CO}_2} = y_{\text{H}_2} = \xi/2 = 0.125$$

$$\text{Products/reactants} = y_{\text{CO}_2} * y_{\text{H}_2} / (y_{\text{CO}} * y_{\text{H}_2\text{O}}) = 1/9 = 0.111$$

- Making a table to keep track of the stoichiometry:

CO	1	$-\xi$
H ₂ O	2	$-\xi$
CO ₂		ξ
H ₂		ξ
Tot	3+	0

$$y_{\text{CO}} * y_{\text{H}_2\text{O}} = (1-\xi) * (2-\xi) / 9 = 0.1458$$

$$y_{\text{CO}_2} * y_{\text{H}_2} = \xi^2 / 9 = 0.0139$$

$$\text{Products/reactants} = y_{\text{CO}_2} * y_{\text{H}_2} / (y_{\text{CO}} * y_{\text{H}_2\text{O}}) = 0.0952$$

- CO

14.1.2 Exercise. Consider the reaction $\text{N}_2 + 3\text{H}_2 = 2\text{NH}_3$.

- Identify ν_1 - ν_3 .
- If the initial moles of $\text{H}_2:\text{N}_2$ were 2:1, how would that affect the values of ν_1 - ν_3 ?
- If the reaction were rewritten as $\frac{1}{2}\text{N}_2 + \frac{3}{2}\text{H}_2 = \text{NH}_3$, how would that affect the values of ν_1 - ν_3 ?
- Reconsider the reaction $\text{N}_2 + 3\text{H}_2 = 2\text{NH}_3$, assuming a stoichiometric feed this time. A “stoichiometric feed” means that the reactants are fed in the ratio of their ν coefficients. In this case, that means moles of

H₂:N₂ of 3:1. Compute the mole fractions of all components assuming the extent of reaction is, ξ , 0.25. Also compute the ratio of product mole fractions to reactant mole fractions.

- (e) Reconsider the reaction $\text{N}_2 + 3\text{H}_2 = 2\text{NH}_3$, assuming a feed in which the moles of H₂ and N₂ are 2 and 1. Compute the mole fractions of all components assuming the extent of reaction, ξ , is 0.25. Also compute the ratio of product mole fractions to reactant mole fractions. What are the total moles going out of the reactor and the moles coming in?
- (f) The “limiting reactant” is the reactant that is depleted first, hence stopping the reaction. If the initial moles of H₂:N₂ were 2:1, which component would be the limiting reactant?

14.1.3 Exercise. SO₂ Oxidation

Air is a mixture of roughly 79%N₂ and 21%O₂ and a common feed component in oxidation reactions because it is inexpensive and readily available. One consideration is that the nitrogen in air is inert in most oxidation reactions. Suppose a feed of 30% SO₂ and 70% air is fed to an oxidation reactor at 250°C and 0.15 MPa. You may wish to refer to Ex. 14.4 for an example that includes inerts.

- (a) Write the balanced reaction for oxidation of SO₂ to SO₃.
- (b) Write the stoichiometry table and express the mole fractions of all components in terms of the extent of reaction, ξ , (cf. Ex. 14.1).
- (c) Which component is the limiting reactant?
- (d) If the extent of reaction was 100%, what would be the composition and flow rate leaving the reactor?

14.2 Equilibrium Constraint

You should recall from introductory chemistry courses that the equilibrium constant is equal to products over reactants. For example, $K_w = [\text{OH}^-][\text{H}^+]/[\text{H}_2\text{O}]$ should remind you of the decomposition of water into its ions, giving rise to the acidity and basicity that we commonly indicate by the pH. This section derives the equilibrium constant relationship from the general expression for Gibbs Energy of a mixture developed in Chapter 9. The basic idea is that the Gibbs energy is minimized at equilibrium. Taking the derivative of the Gibbs energy with respect to the extent of reaction gives the minimum Gibbs energy at the equilibrium extent.

Perhaps the most confusing point in the study of reacting systems is the way that the reference state gets buried. By assuming a reference state of the ideal gas at 1 bar, we save the trouble of computing the free energy and fugacity of the reference state. After all, the fugacity of any ideal gas at 1 bar is 1 bar. As a result, the ratio of product to reactant fugacities that comprises the equilibrium constant does not indicate anything about the reference state fugacity of 1 bar. After all, ratios divided by unity are just the same quantity as before the division. But now the reference state is invisible, it is buried in the tabulated values of G_i° for each component. To stay out of trouble, you should perform all reaction equilibrium calculations in the same pressure units as the tabulated values of G_i° , at least until you completely comprehend reference states as discussed below. For this text, that means converting all pressure units to bars.

Besides converting to bars, one essential point is that the numerical value of K_a does not depend on the process pressure. Here is a little brain teaser. Suppose we write that $K_a = (y_{\text{NH}_3}P)^2(y_{\text{N}_2}P)^{-1}(y_{\text{H}_2}P)^3$ for the ammonia synthesis reaction. Factoring out the pressure effect, we see that $K_a \sim P^2$. So raising the value of pressure should decrease the value of K_a . Right? WRONG!!!!!! This question fools about 50% of students, so don't feel too embarrassed if you missed it.¹ Eq. 14.16 shows that $K_a = \exp(-\Delta G_T^\circ/RT)$. Now it is true that ΔG_T° does depend on the reference state pressure, but the reference state pressure is a single standard value regardless of what process you are running. How then can K_a possibly be independent of process pressure and still equal to the ratio of partial pressures of products over reactants? Answer: the mole fractions must change to compensate for the increase in pressure. For the ammonia synthesis reaction, this means the mole fraction of ammonia must increase. In other words, we can increase conversion by increasing pressure. This is why industrial processes for ammonia production operate at pressures above 100 bars. Ammonia is very important for fertilizer production, by the way.

Comprehending Reference States

Practically, you can solve a lot of problems without understanding all the subtleties of reference states. So you may want to put some of these detailed concerns on the back burner. Performing all calculations in bars circumvents the reference state problem temporarily, but you should not be afraid of understanding it properly. We learned that pressure affects the Gibbs energy of ideal gases as specified in Eq. 8.16. So, assuming a different value for the reference state pressure must give a different value for G_i° in the obvious manner of Eq. 8.16. Suppose we wanted to use the ideal gas at 298.15K and 1 MPa as our reference state. Noting that 1 bar equals 0.1 MPa, the value of G_i° would simply be $G_i^\circ(1) = G_i^\circ(0.1) + [G_i^\circ(1) - G_i^\circ(0.1)] = G_i^\circ(0.1) + \ln(10)$. But wait. Applying that relation to, say, hydrogen would give a non-zero value for the element at its reference condition. It is conventional to set the reference state to zero for the elements at their reference conditions. So we need to subtract $\ln(10)$ from all compounds with H₂ in them after adding $\ln(10)$. But wait. CH₄, for example, has two H₂'s in it. So now what do we do? Would you believe... subtract $2\ln(10)$? So the value for G_f° of CH₄ would be: $-50.45 + \ln(10) - 2\ln(10) = -52.75$ kJ/mol in a table with standard state of 1MPa.

What about the C in CH₄? Elemental carbon (C) is a solid. If you think back to why Gibbs energy depends on pressure, it is because Gibbs energy includes entropy. Entropy for an ideal gas depends on pressure because it changes as $-\ln(V_2/V_1)$. Remember the particles in boxes from Chapter 3? Think about a lump of coal; it is mostly elemental carbon. If you raised the pressure in a container with a lump of coal in it from 1 bar to 1 MPa, do you think the volume of the coal would change significantly? No way, it is as hard as a rock (because it is a rock). So zero volume change at constant temperature translates into no change in Gibbs energy for solids. The value of -52.75 kJ/mol for CH₄ with a standard state of 1MPa stands. Pretty confusing? Yes, but you have really grasped the implications of reference states in general once you grasp this. These implications come up in Gibbs energy minimization (Ex. 14.11) and in the study of electrolytes (Section 14.10), so you might as well make yourself comfortable with the issue as soon as possible.

¹ Don't be too embarrassed *this time*. Fooled you once, shame on me. Fool you twice, *shame on you*.

14.2.1 Example: K_a for $\text{CO} + \text{H}_2\text{O} = \text{CO}_2 + \text{H}_2$.

- Look up the values of G_f° at 298.15K for the above reaction make a table like the one in Ex 14.2
- Compute ΔG_{298}° and K_a for this reaction.
- Assuming $f_i = y_i P$, write the equation for K_a in terms of product to reactant mole fractions and pressure.
- Assuming a stoichiometric feed and expressing the mole fractions in terms of the extent of reaction as in Ex 14.1, develop an equation relating the value of K_a to the extent of reaction.

Solution:

From Appendix E.6, most are in the "Gases" section:

	$\Delta H_{f,298}$ kJ/mol	$\Delta G_{f,298}$ kJ/mol
CO	-110.53	-137.16
CO ₂	-393.51	-394.38
H ₂	0	0
H ₂ O	-241.835	-228.614

(b) $\Delta G_{298}^\circ = -393.51 - (-110.53 - 241.835 + 0) = -28.606$ kJ/mol

$K_a = \exp[+28.606 \cdot 1000 / (8.314 \cdot 298.15)] = \exp[-0.10578] = 102,744$

(c) $K_a = (y_{\text{CO}_2} P) (y_{\text{H}_2} P) (y_{\text{H}_2\text{O}} P)^{-1} (y_{\text{CO}} P)^{-1} = (y_{\text{CO}_2} y_{\text{H}_2}) / (y_{\text{H}_2\text{O}} y_{\text{CO}})$

(d) The stoichiometric feed means that 1 mole of CO is fed for every mole of H₂O. To get mole fractions, we can assume a basis of 1 mole of feed for each. So $n_{\text{CO}} = 1 - \xi$, $n_{\text{H}_2\text{O}} = 1 - \xi$, $n_{\text{CO}_2} = \xi$, $n_{\text{H}_2} = \xi$ and $n_T = 2$.

$K_a = \xi^2 / (1 - \xi)^2$.

14.2.2 Exercise. Consider the reaction $\text{N}_2 + 3\text{H}_2 = 2\text{NH}_3$.

- Look up the values of G_f° at 298.15K for the above reaction make a table like the one in Ex 14.2
- Compute ΔG_{298}° and K_a for this reaction.
- Assuming $f_i = y_i P$, write the equation for K_a in terms of product to reactant mole fractions and pressure.
- Assuming a stoichiometric feed and expressing the mole fractions in terms of the extent of reaction as in Ex 14.1, develop an equation relating the value of K_a to the extent of reaction.

14.2.3 Exercise. Air oxidation of SO₂

Air is a common feed component in oxidation reactions because it is inexpensive and readily available. One consideration is that the nitrogen in air is inert in most oxidation reactions. Suppose a feed of 30% SO₂ and 70% air is fed to an oxidation reactor at 250°C and 0.15 MPa and oxidized to SO₃.

- Look up the values of G_f° at 298.15K for the above reaction make a table like the one in Ex 14.2
- Compute ΔG_{298}° and K_a for this reaction.
- Assuming $f_i = y_i P$, write the equation for K_a in terms of product to reactant mole fractions and pressure.
- Assuming a stoichiometric feed and expressing the mole fractions in terms of the extent of reaction as in Ex 14.1, develop an equation relating the value of K_a to the extent of reaction.

14.2.4 Related Practice Problems

Check out the solutions to Practice Problems 14.1, 14.2, and 14.5.

14.3 Reaction equilibria for ideal solutions

The ideal solution assumption simply states that $f_i = y_i P$ (Eq. 9.27) for gases or $f_i = x_i P_i^{sat}$ for liquids when the Poynting correction is negligible (Eq. 9.48). The most commonly applied form for reactions is Eq. 9.27 because most reactions are carried out at high temperatures and low pressures. Substitution of Eq. 9.27 to generate a single equation in the single unknown ξ should be straightforward for you now. All that remains is to solve the equation. Perhaps you have done this for the previous examples already. If so, you can simply write your answers to the next couple of problems.

14.3.1 Example. Equilibrium conversion for the reaction $\text{CO} + \text{H}_2\text{O} = \text{CO}_2 + \text{H}_2$.

- Solve the equation developed in the previous section for ξ of this reaction at 1 bar and 298.15K.
- Compute the equilibrium mole fractions of all components.

Solution:

(a) We left off with $K_a = \xi^2/(1-\xi)^2 = 102,744$. Cross-multiplying and rearranging gives a quadratic equation. $(1/102,744-1)\xi^2 + 2\xi - 1 = 0 = -0.99999\xi^2 + 2\xi - 1 \Rightarrow \xi = [-2 \pm (4 - 4*0.99999*1)^{1/2}] / (2*(-0.99999))$

We can eliminate the negative root because it would give negative values for the mole fractions of products. Then, $\xi = [-1 + (1.1116)^{1/2}] / 0.1116 = 0.99689$. This reaction strongly favors products at these conditions.

(b) Recalling, $n_{\text{CO}} = 1-\xi$, $n_{\text{H}_2\text{O}} = 1-\xi$, $n_{\text{CO}_2} = \xi$, $n_{\text{H}_2} = \xi$ and $n_T = 2$:

$$y_{\text{CO}} = y_{\text{H}_2\text{O}} = (1-\xi)/2 = 0.0016, y_{\text{CO}_2} = y_{\text{H}_2} = \xi/2 = 0.4984$$

14.3.2 Exercise. Consider the reaction $\text{N}_2 + 3\text{H}_2 = 2\text{NH}_3$.

- Assuming ideal solution estimates for the component fugacities, solve the equation developed in the previous section for ξ of this reaction at 1 bar and 298.15K and 5 MPa.
- Compute the equilibrium mole fractions of all components.

14.3.3 Exercise. Air oxidation of SO_2

Air is a common feed component in oxidation reactions because it is inexpensive and readily available. One consideration is that the nitrogen in air is inert in most oxidation reactions. Suppose a feed of 30% SO_2 and 70% air is oxidized to SO_3 at 250°C and 0.15 MPa.

- Assuming ideal solution estimates for the component fugacities, solve the equation developed in the previous section for ξ of this reaction at 1 bar and 298.15K and 5 MPa.
- Compute the equilibrium mole fractions of all components.

14.3.4 Related Practice Problems

Check out the solutions to Practice Problems 14.1, 14.2, and 14.5.

14.4 Temperature Effects

One of the essential relations that you must assimilate thoroughly is the van't Hoff relation, especially the part about the extent of reaction increasing with temperature for endothermic reactions and decreasing for exothermic reactions. It is straightforward to apply Eqs. 14.28 and 14.30 when the heat capacities are expressed as third order polynomials, but could you develop your own expressions analogous to Eqs. 14.28 and 14.30 if you were given different expressions for the heat capacities?.

14.4.1 Example. Temperature dependence for the reaction $\text{CO} + \text{H}_2\text{O} = \text{CO}_2 + \text{H}_2$.

(a) Compute the values of ΔG , ΔH , and K_a of this reaction at 1 bar and 400K.

(b) Compute the values of ΔG , ΔH , and K_a of this reaction at 1 bar and 500K.

Solution: This example is extremely similar to Example 14.5. Try to solve it yourself without looking at the solution below, then check your answers for errors.

	$\Delta H_{f,298}$ kJ/mol	$\Delta G_{f,298}$ kJ/mol	a	b	c	d
CO ₂	-393.51	-394.38	1.98E+01	7.34E-02	-5.60E-05	1.72E-08
H ₂	0	0	0	0	0	0
CO	-110.53	-137.16	3.09E+01	-1.29E-02	2.79E-05	-1.27E-08
H ₂ O	-241.835	-228.614	32.24	1.92E-03	1.06E-05	-3.60E-09
Δ	-41.145	-28.606	-16.17	9.364E-02	-1.083E-04	4.111E-08

(a) From WB Ex. 14.2.1, we have $\Delta G_{298}^\circ = -28.606$ and $K_a^{298} = 102,744$. Referring to the values of ΔH in Appendix E.6, $\Delta H_{298}^\circ = -393.51 + 0 - (-110.53 - 241.835) = -41.145$ kJ/mol. Similar operations yield the values for Δa - Δd in the table above. Substituting into Eq. 14.28,

$$-41.145 \cdot 1000 = J \cdot 1000 - 16.17 \cdot 298.15 + 0.09364 \cdot 298.15^2/2 - 1.083E-4 \cdot 298.15^3/3 + 4.111E-8 \cdot 298.15^4/4$$

$\Rightarrow J = -39.611$ kJ/mol. Note that we multiply by 1000 to convert from kJ/mol to J/mol.

$$\Delta H_T^\circ (\text{J/mol}) = -39,611 + 16.17 T - 0.04682 \cdot T^2 + 0.3610E-4 \cdot T^3 - 1.0278E-8 \cdot T^4$$

Substituting into Eq. 14.30,

$$-28.606 \cdot 1000 / (8.314 \cdot 298.15) = -39.611 \cdot 1000 / (8.314 \cdot 298.15) + 16.17 \cdot \ln(298.15) / 8.314 -$$

$$0.09364 \cdot 298.15 / (2 \cdot 8.314) + 1.083E-4 \cdot 298.15^2 / (6 \cdot 8.314) - 4.111E-8 \cdot 298.15^3 / (12 \cdot 8.314) + I$$

$\Rightarrow I = -5.145$

The resulting formula is,

$$\Delta G_T^\circ (\text{J/mol}) = -39,611 + 16.17 T \ln T - 0.04682 \cdot T^2 + 0.1805E-4 \cdot T^3 - 0.3426E-8 \cdot T^4 - 42.776 T$$

Evaluating at 400 K, $\Delta G_{400}^\circ = -24.392$ kJ/mol, $\Delta H_{400}^\circ = -40.634$ kJ/mol, $K_a = 1532$

(b) The beauty of developing the general formula is that computing K_a at different temperatures becomes easy.

Evaluating at 500 K, $\Delta G_{500}^\circ = -20.416$ kJ/mol, $\Delta H_{500}^\circ = -39.859$ kJ/mol, $K_a = 136$

14.4.2 Example. Consider the reaction $\text{N}_2 + 3\text{H}_2 = 2\text{NH}_3$.

(a) Compute the values of ΔG° , ΔH° , and K_a of this reaction at 100 bar and 600K.

(b) Assuming ideal solution estimates for the component fugacities and a stoichiometric feed, solve for ξ of this reaction at 600K and 10 MPa.

Solution:

(a) The procedure from Ex. 14.5 can be repeated for hand calculations, but this quickly becomes tiresome. The spreadsheet Kcalc.xls minimizes the tedium. The table below summarizes the values from the spreadsheet.

	$\Delta H_{f,298}$ kJ/mol	$\Delta G_{f,298}$ kJ/mol	a	b	c	d
NH3	-45.94	-16.4013	27.31	0.02383	1.71E-05	-1.19E-08
H2	0	0	27.14	0.00927	-1.38E-05	7.65E-09
N2	0	0	31.15	-0.01357	2.68E-05	-1.17E-08
Δ	-91.880	-32.803	-57.95	0.03341	4.877E-05	-3.496E-08

Evaluating at 600 K, $\Delta G_{600}^\circ = 31.685$ kJ/mol, $\Delta H_{600}^\circ = -102.827$ kJ/mol, $K_a = 0.00174$. This value is different from the tabulated value in Ex 14.7 for two reasons: (1) the reaction in Ex 14.7 is written as $\frac{1}{2} \text{N}_2 + \frac{3}{2} \text{H}_2 = \text{NH}_3$, so this 0.00174 should be the square of the value for the reaction in Ex 14.7 (2) the value tabulated in Ex 14.7 (ie. 0.06633) was computed by the shortcut equation discussed later. We have computed a more precise value.

Note that the *process* pressure of 100 bar has no impact on K_a , etc. This information is impertinent for this calculation. Did it confuse you? Shame on you if it did.

(b) Remember: Our reference state is in bars, so our K_a values are in units of bars. In fact, the K_a value for this particular reaction is in units of bars². We should convert from 10 MPa to 100 bars. Then,

$$y_{\text{N}_2} = (1-\xi)/(4-2\xi); y_{\text{H}_2} = (3-3\xi)/(4-2\xi); y_{\text{NH}_3} = 2\xi/(4-2\xi);$$

$K_a = (2\xi)^2(4-2\xi)^2/[(3-3\xi)^3(1-\xi)] = 16\xi^2*(2-\xi)^2/[27(1-\xi)^4] = 100^2*0.00174$. We can simplify if we multiply by 27/16 and take the square root of both sides. Then, $\xi(2-\xi) = 5.4187(1-\xi)^2$. Solving the quadratic gives: $\xi = 0.605$

14.4.3 Exercise. Air oxidation of SO2

Air is a common feed component in oxidation reactions because it is inexpensive and readily available. One consideration is that the nitrogen in air is inert in most oxidation reactions. Suppose a feed of 30% SO2 and 70% air is oxidized to SO3 at 250°C and 0.15 MPa.

- Compute the values of ΔG° , ΔH° , and K_a of this reaction at 250°C and 0.15 MPa.
- Assuming ideal solution estimates for the component fugacities, solve for ξ of this reaction at 250°C and 0.15 MPa.
- Compute the equilibrium mole fractions of all components.

14.4.4 Related Practice Problems

Check out the solution to Practice Problems 14.3 and 14.4.

14.5 Shortcut Estimation of Temperature Effects

It is quite typical for chemical processes to be operated for long periods of time at very similar conditions. With this stipulation, it is often convenient to have a simpler equation than the complete equation with all terms. The shortcut van't Hoff equation can be convenient in this way, but it is valuable to apply the shortcut equation accurately. Like any approximate equation, it is most accurate near the conditions at which it is developed. If we have sufficient data to compute the K_a value at a temperature near the process conditions, accuracy is improved by doing so. The result is a simple but reliable equation in the form of $K_a = A + B/T$.

14.5.1 Example. Temperature dependence for the reaction $\text{CO} + \text{H}_2\text{O} = \text{CO}_2 + \text{H}_2$.

- Compute the values of A , B in $K_a = A + B/T$ of this reaction at 1 bar and 400K.
- Compute the values of A , B in $K_a = A + B/T$ of this reaction at 1 bar and 500K. Compare the values of A , B to those from part (a). Also estimate the K_a value at 400K from the shortcut equation centered at 500K and compare it to the value from part (a).

Solution:

(a) Recalling at 400 K, $\Delta G_{400}^\circ = -24.392$ kJ/mol, $\Delta H_{400}^\circ = -40.634$ kJ/mol, $K_a = 1532$

$$\ln(K_a) = \ln(1532) + (+40,634/8.314)*(1/T - 1/400) = 4887/T - 12.219; \quad A = -4.885; \quad B = 4887$$

(b) Recalling at 500 K, $\Delta G_{500}^\circ = -20.416$ kJ/mol, $\Delta H_{500}^\circ = -39.859$ kJ/mol, $K_a = 136$

$$\ln(K_a) = \ln(136) + (+39,859/8.314)*(1/T - 1/500) = 4794/T - 9.588; \quad A = -4.676; \quad B = 4794$$

The value of A decreases slightly with temperature and B increases. Because these contributions are opposite in sign, their changes may cancel in large part. Considering the value of K_a at 400K estimated by the A , B from part (b), we have $K_a^{400} \sim 1494$. This is different from the value of 1532 in the third significant figure.

14.5.2 Example. Consider the reaction $\text{N}_2 + 3\text{H}_2 = 2\text{NH}_3$.

- Compute the values of A , B in $K_a = A + B/T$ of this reaction at 1 bar and 600K.
- Compute the values of A , B in $K_a = A + B/T$ of this reaction at 1 bar and 700K. Compare the values of A , B to those from part (a). Also estimate the K_a value at 600K from the shortcut equation centered at 700K and compare it to the value from part (a), and the value of 0.06633 tabulated in Ex 14.7.

Solution:

(a) Recalling at 600 K, $\Delta G_{600}^\circ = 31.685$ kJ/mol, $\Delta H_{600}^\circ = -102.827$ kJ/mol, $K_a = 0.00174$. This value is different

$$\ln(K_a) = \ln(0.00174) + (+102,827/8.314)*(1/T - 1/600) = 12368/T - 26.967; \quad A = -26.965; \quad B = 12368$$

(b) At 700 K, $\Delta G_{700}^\circ = 54.310$ kJ/mol, $\Delta H_{700}^\circ = -105.351$ kJ/mol, $K_a = 8.855\text{E-}5$

$$\ln(K_a) = \ln(8.855\text{E-}5) + (+105,351/8.314)*(1/T - 1/700) = 12672/T - 27.434; \quad A = -27.434; \quad B = 12672$$

The value of K_a at 600 K estimated from the shortcut equation centered at 700K is 0.00181. This is varying in the second significant figure from the value of 0.00174. The corresponding value from the shortcut equation centered at 298K is 0.06633 2 = 0.00440. The shortcut equation centered at 298K is inaccurate by ~100%! This is why it is important not to push the shortcut equation over wide ranges of temperature.

14.5.3 Exercise. Air oxidation of SO_2

Air is a common feed component in oxidation reactions because it is inexpensive and readily available. One consideration is that the nitrogen in air is inert in most oxidation reactions. Suppose a feed of 30% SO_2 and 70% air is oxidized to SO_3 at 250°C and 0.15 MPa.

- Compute the values of ΔG° , ΔH° , and K_a of this reaction at 250°C and 0.15 MPa.
- Assuming ideal solution estimates for the component fugacities, solve for ξ of this reaction at 250°C and 0.15 MPa.
- Compute the equilibrium mole fractions of all components.

14.5.4 Related Practice Problems

Check out the solution to Practice Problems 14.3 and 14.4.

14.6 Energy Balances for Reactions

Most reactions of industrial interest are exothermic. This means that the heat of reaction must be balanced in some way. In an adiabatic reactor, the heat of warming the reactants balances the heat of reaction. Otherwise, some form of cooling removes the heat. Perhaps the catalyst is fluidized and sent through a regeneration reactor. Perhaps cooling tubes are placed in the reactor, but this tends to be expensive. Or perhaps the reaction is conducted in roughly adiabatic stages with heat removal between stages. It is actually somewhat rare that the reactor can be perfectly characterized as isothermal. Therefore, accurate models generally require simultaneous estimation of the energy evolution implicit in ΔH as well as the equilibrium limitations implicit in ΔG .

14.6.1 Example. Heat of reaction for $\text{CO} + \text{H}_2\text{O} = \text{CO}_2 + \text{H}_2$.

The reaction of $\text{CO} + \text{H}_2\text{O} = \text{CO}_2 + \text{H}_2$ is called the water-gas shift reaction. It is important in processes designed to enrich the hydrogen content of reformer effluents. Reforming is generally considered to be the conversion of natural gas or other hydrocarbons into some form of CO and H₂. Reforming is relevant to the design of fuel cells as well as more traditional processes. We would like to estimate the number of tubes required to maintain a water-gas shift reactor at roughly 500K. The flowrate of the stoichiometric feed is to be 100 kmol/hr for both reactants combined. The heat transfer coefficient may be estimated as 250 W/m²-K and the temperature difference between the tubes and reactor is 100K. Tubes are available in 40 ft lengths and half-inch diameter. Assuming the entire reactor is to be held at the same temperature and concentration (well-mixed), how many tubes are required?

Solution:

Recalling at 500 K, $\Delta G_{500}^\circ = -20.416$ kJ/mol, $\Delta H_{500}^\circ = -39.859$ kJ/mol, $K_a = 136$

100 kmol/hr = 27.8 mol/s. The equilibrium conversion at 500K is given by $136(1-\xi)^2 = \xi^2 \Rightarrow \xi = 0.921$.

thus, the heat evolved is $0.921 * 39,859 \text{ J/mol} * 27.8 \text{ mol/s} = 1.020 \text{ MJ/s} = 1.020 \text{ MW}$. Each tube has an area of $40 \text{ ft} * 0.3048 \text{ m/ft} * 3.14 * 0.5 \text{ in} * 0.0254 \text{ m/in} = 0.486 \text{ m}^2 / \text{tube}$. $250 \text{ W/m}^2\text{-K} * 100 \text{ K} * 0.486 \text{ m}^2 / \text{tube} = 12155 \text{ W/tube}$. $1,020,000 \text{ W} / 12155 \text{ W/tube} = 84 \text{ tubes}$. There's more engineering than thermo here, but that's okay.

14.6.2 Example. Heat generation in a catalytic converter

When designing a catalytic converter for automobile exhaust, it is important to consider the heat evolution and the temperature maintained in the catalytic converter. The key reaction is $\text{CO} + \frac{1}{2} \text{O}_2 = \text{CO}_2$. Suppose 600 ppm (mole basis) of CO in 75% N₂ with remainder CO₂ enter the reactor at 600K and are completely and adiabatically converted to CO₂. What would be the temperature of the exhaust stream?

Solution:

Referring to Kcalc.xls at 600 K, $\Delta H_{600}^\circ = -283.648$ kJ/mol. The K_a value is very large, indicating complete conversion. We can estimate the C_p value as $0.75 * 30 + 0.25 * 47 = 34 \text{ J/mol-K}$. Basis: 1 mole of feed to the converter. $0.000600 \text{ moles CO} * 283,648 \text{ J/mol} = 34 * (T-600) \Rightarrow T = 605 \text{ K}$.

14.6.3 Exercise. Air oxidation of SO₂

Air is a common feed component in oxidation reactions because it is inexpensive and readily available. One consideration is that the nitrogen in air is inert in most oxidation reactions. Suppose a feed of 30% SO₂ and 70% air is oxidized to SO₃ at 250°C and 0.15 MPa. The heat transfer coefficient may be estimated as 250 W/m²-K and the temperature difference between the tubes and reactor is 100K. Tubes are available in 40 ft lengths and half-inch diameter. Assuming the entire reactor is to be held at the same temperature and concentration (well-mixed), how many tubes are required?

14.7 General Observations about Pressure Effects

The ideal solution approximation suffices for most gas phase reactions, but may become inaccurate at high pressures if K_ϕ deviates significantly from unity. As noted in the text, this is unlikely since the deviations in the numerator tend to be canceled by the deviations in the denominator. To get some feel for the conditions when K_ϕ needs to be considered, we consider the following examples.

14.7.1 Example. K_ϕ for $\text{CO} + \text{H}_2\text{O} = \text{CO}_2 + \text{H}_2$.

(a) Use the Peng-Robinson equation to estimate K_ϕ at 500K and 10 bars.

(b) Use the Peng-Robinson equation to estimate K_ϕ at 500K and 100 bars.

Solution:

(a) Running the FU option assuming equimolar compositions, we get

(b) $K_\phi = \phi_{\text{CO}_2}\phi_{\text{H}_2}/(\phi_{\text{CO}}\phi_{\text{H}_2\text{O}}) = 0.9928*1.0068/(1.006*0.9744) = 1.0197$

Note that $\phi_i = f_i/y_iP$, so we must divide the fugacities from the program appropriately.

(b) Running the FU option, we get $K_\phi = \phi_{\text{CO}_2}\phi_{\text{H}_2}/(\phi_{\text{CO}}\phi_{\text{H}_2\text{O}}) = 0.9390*1.06988/(1.0672*0.7771) = 1.3798$

Basically, the deviation from unity is dominated by the fugacity coefficient of water because it is the only component that is not above its critical temperature.

14.7.2 Example. K_ϕ for $\text{N}_2 + 3\text{H}_2 = 2\text{NH}_3$

(a) Use the Peng-Robinson equation to estimate K_ϕ at 600K and 10 bars.

(b) Use the Peng-Robinson equation to estimate K_ϕ at 600K and 100 bars.

Solution:

(a) Running the FU option assuming equimolar compositions, we get

$K_\phi = \phi_{\text{NH}_3}^2/(\phi_{\text{CO}}\phi_{\text{H}_2}^3) = 0.96812^2/(1.0368*1.0277^3) = 0.8329$

Note that $\phi_i = f_i/y_iP$, so we must divide the fugacities from the program appropriately.

(b) Running the FU option, we get $K_\phi = \phi_{\text{NH}_3}^2/(\phi_{\text{CO}}\phi_{\text{H}_2}^3) = 0.9842^2/(1.0773*1.0570^3) = 0.7016$

In this case, the numerator is subcritical, but the denominator is highly supercritical, so the asymmetry causes the deviation from unity.

14.7.3 Exercise. Air oxidation of SO_2

(a) Compute K_ϕ for the SO_2 oxidation reaction at 250°C and 0.15 MPa.

(b) Compute K_ϕ for the SO_2 oxidation reaction at 250°C and 15 MPa.

14.7.4 Related Practice Problems

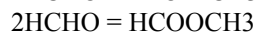
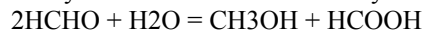
Check out the solution to Practice Problems 14.6 and 14.7.

14.8 Multireaction Equilibria

Multiple reactions frequently occur in engineering design situations, but many courses in introductory thermodynamics skip this issue. That is really unfortunate, because isomerizations, control of waste stream effluents, and many other problems generally focus on trade-offs between various multireaction outcomes. The extension is fairly straightforward when solving the reactions explicitly as in Examples 14.8 and 14.9, but the Gibbs energy minimization method of Examples 14.10 and 14.11 represents a quite different approach to the whole problem and it is worth knowing about. The Gibbs minimization method is frequently applied in preliminary designs and you should know its advantages and limitations. It tends to be much more sensitive to initial guesses and precision of the formation energies than the explicit approach. But it is very desirable to simply list a number of possible products without deriving specific equations and reaction energies during preliminary stages, especially if the preliminary calculation indicates that several of the possible products are not likely to form. Then you can avoid deriving specifics for those products if you do decide to apply the explicit method during later stages. The examples below explore the explicit method in a couple of instances and the Gibbs minimization method in a few more instances.

14.8.1 Example. Formaldehyde Decomposition

Formaldehyde and water are fed in a 2:1 molar ratio at 355°C and 1 bar, reacting to form methanol, formic acid, and methyl formate. The reactions may be written as:



(a) For the reactions at 355°C and 1 bar, derive numerical values for A and B in the expression:

$$\ln K a_i = A + B/T, \text{ where } T [=] \text{ K. (5)}$$

(b) Write expressions for the extents of both reactions in terms of the values for $K a_1$ and $K a_2$. (5)

(c) Solve for the extents of reaction and the mole fractions of all species exiting the reactor. (10)

Compo	$H_f(298)\text{kJ/mol}$	$G_f(298)\text{kJ/mol}$	$C_p^A(\text{J/mole-K})$	C_p^B	C_p^C	C_p^D
Formaldehyde	-116.0	-110.0	23.48	0.0316	2.985E-5	-2.300E-8
Methyl Formate	-352	-295	1.632	0.2700	-1.949E-4	5.702E-8
Formic Acid	-378.9	-351.2	11.71	0.1358	-8.411E-5	2.017E-8

Solution:

Substituting these values into the KaCalc.xls workbook and setting the temperature to $T = (273.15 + 355) = 628.15$,

(a) $\ln K a_1 = -17.141 + 12943/T$; $\ln K a_2 = -16.16 + 13992/T$.

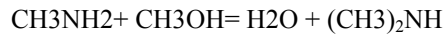
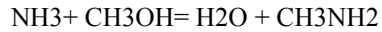
$$(b) \quad K a_1 = 17.29 = \frac{(3 - \xi_1 - \xi_2)^3}{(1 - \xi_1)^2 (1 - \xi_1 - \xi_2)^2} \frac{\xi_1^2}{(3 - \xi_1 - \xi_2)^2} = \frac{\xi_1^2 (3 - \xi_1 - \xi_2)}{4(1 - \xi_1)(1 - \xi_1 - \xi_2)^2}$$

$$K a_2 = 132.8 = \frac{(3 - \xi_1 - \xi_2)^2}{2^2 (1 - \xi_1 - \xi_2)^2} \frac{\xi_2}{(3 - \xi_1 - \xi_2)} = \frac{\xi_2 (3 - \xi_1 - \xi_2)}{4(1 - \xi_1 - \xi_2)^2}$$

(c) Typing these constraints into DualRxn spreadsheet and calling solver, $\xi_1 = 0.20$, $\xi_2 = 0.76$.

14.8.2 Exercise. Amine Synthesis By Explicit Reactions

Ammonia (NH₃) and methanol (CH₃OH) are fed in a 3:1 molar ratio at 355°C and 1 bar, reacting over a moderately selective catalyst to form water, methylamine (CH₃NH₂), and dimethylamine [(CH₃)₂NH]. The reaction may be written as:



(a) For these reactions at 355°C and 1 bar, derive numerical values for A and B in the expression:

$$\ln K_a = A + B/T, \text{ where } T [=] \text{ K. (5)}$$

(b) Write expressions for the extents of both reactions in terms of the values for K_{a1} and K_{a2} . (8)

(c) Solve for the extents of reaction and the mole fractions of all species exiting the reactor. (7)

Compo	$H_f(298)\text{J/mol}$	$G_f(298)\text{J/mol}$	$C_p(\text{J/mole-K})$
methanol	-200,940	-162,320	55.5
ammonia	-45,898	-16,401	40.2
methylamine	-22,970	32,070	68.9
dimethylamine	-18,450	68,390	96.7
water	-241,820	-228,590	34.7

14.8.3 Example. Isomers Of Xylene And Ethylbenzene

Based on a feed of 111 kmol/hr of pure ethylbenzene, all xylene isomers and ethylbenzene exit the reactor in their equilibrium proportions. Estimate the mole fractions of each component exiting the reactor at 355°C and 1 bar. The reactor operates isothermally at 355°C and 1 bar. The equilibrium distribution of all species is to be studied by Gibbs energy minimization.

- Write the atom balance constraints for all elements.(5)
- Compute the relevant G_{fi}/RT .(8)
- Solve for the mole fractions of all species exiting the reactor.(7)

Compo	C_p^A (J/mole-K)	C_p^B	C_p^C	C_p^D	$H_f(298)$ kJ/mol	$G_f(298)$ kJ/mol
o-xylene	-15.85	0.5962	-3.443E-4	7.528E-8	19.00	122.2
m-xylene	-29.17	0.6297	-3.747E-4	8.478E-8	17.25	118.9
p-xylene	-25.09	0.6042	-3.374E-4	6.820E-8	17.96	121.2
Ethylbenzene	-43.10	0.7072	-4.811E-4	1.301E-7	29.81	130.7

Solution:

a. H-bal: $10 \cdot \sum(n_i) = 10$; C-bal: $8 \cdot \sum(n_i) = 8$.

b. Typing the values into KaCalc.xls and noting in each case the formation reaction is $8C + 5H_2 = C_8H_{10}$, where C_8H_{10} refers to different isomers for different reactions:

Compo	$G_f(628.15)$ kJ/mol
o-xylene	47.150
m-xylene	46.339
p-xylene	46.506
Ethylbenzene	48.316

c. Calling the solver we have

Compo	y_i
o-xylene	0.179
m-xylene	0.416
p-xylene	0.346
Ethylbenzene	0.059

14.8.4 Exercise. Amine Synthesis by Gibbs Minimization

Based on a feed of 1:3 kmol/hr of methanol:ammonia, methylamine $[CH_3NH_2]$, dimethyl amine $[(CH_3)_2-NH]$, and trimethyl amine $[(CH_3)_3-N]$ exit the reactor along with unreacted feed components, all in their equilibrium proportions. Estimate the mole fractions of each component exiting the reactor at 355°C and 1 bar. The reactor operates isothermally at 355°C and 1 bar. The equilibrium distribution of all species is to be studied by Gibbs energy minimization.

- Write the atom balance constraints for all elements. (5)
- Assuming constant heat capacities as given below, estimate the relevant G_{fi}/RT .(8)
- Taking an initial guess of zero conversion, solve for the mole fractions of all species exiting the reactor.(7)

Compo	$H_f(298)$ J/mol	$G_f(298)$ J/mol	C_p (J/mole-K)
methanol	-200,940	-162,320	55.5
ammonia	-45,898	-16,401	40.2
methylamine	-22,970	32,070	68.9
dimethylamine	-18,450	68,390	96.7
trimethylamine	-23,849	98,910	128.3
water	-241,820	-228,590	34.7

14.8.5 Related Practice Problems

Check out the solution to Practice Problems 14.6 and 14.7.

Chapter 14 Practice Problem Solutions

(P14.1) An equimolar mixture of H₂ and CO can be obtained by the reaction of steam with coal. Compute the equilibrium compositions at 550 C based on an equimolar feed of H₂, CO, and H₂O. The reaction is H₂O + CO = H₂ + CO₂. ΔG_f° data at 550 C are given.

$$\Delta G_T = \Sigma \Delta G_f(\text{product}) - \Sigma \Delta G_f(\text{reactant}) = -8.84 \text{ kJ/mol}$$

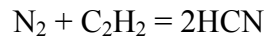
$$K_a(550^\circ\text{C}) = \exp(8840/(8.314*(550+273.15))) = 3.369$$

Compound	In	Out
H ₂ O	0.333	0.333- ξ
CO	0.333	0.333- ξ
H ₂	0.333	0.333+ ξ
CO ₂	0	ξ
Total	1	1

$$K_a = \frac{(\xi)(0.333 + \xi) * P^2}{(0.333 - \xi) * (0.333 - \xi) * P^2} = 3.369$$

$$\xi = 0.176; \text{ Ans. } y_1 = 0.157, y_2 = 0.157, y_3 = 0.509, y_4 = 0.176$$

(P14.2) One method for the production of hydrogen cyanide is by the gas-phase nitrogenation of acetylene according to the reaction below. For a stoichiometric feed at 300°C, calculate the product composition at 1 and 200 bar given $\Delta G_T^\circ = 30.08$ kJ/mole.



Solution: Two mole basis. Assume ideal gas.

$$\# \text{ of moles (N}_2) = \# \text{ of moles (C}_2\text{H}_2) = 1.0$$

Compound	In	Out
N ₂	1	1- ξ
C ₂ H ₂	1	1- ξ
HCN	0	2 ξ
Total	2	2

$$K_a = \exp(-30080/(8.314*(300+273.15))) = 0.0018$$

$$K_a = \frac{(2 * \xi)^2 * P^2}{(1 - \xi)^2 * P^2} = 0.0018, \quad \xi = 0.0208$$

$$(\text{Ans. } y_1 = 0.4896, y_2 = 0.4896, y_3 = 0.0416)$$

Chapter 14 Practice Problem Solutions

(P.14.3) Butadiene can be prepared by the gas-phase catalytic dehydrogenation of 1-Butene: $C_4H_8 = C_4H_6 + H_2$. In order to suppress side reactions, the butene is diluted with steam before it passes into the reactor.

- (a) Estimate the temperature at which the reactor must be operated in order to convert 30% of the 1-butene to 1,3-butadiene at a reactor pressure of 2 bar from a feed consisting of 12 mol of steam per mole of 1-butene.
 (b) If the initial mixture consists of 50 mol% steam and 50mol% 1-butene, how will the required temperature be affected?

ΔG_f°	600K	700K	800K	900K
C4H6	195.73	211.71	227.94	244.35
C4H8	150.92	178.78	206.89	235.35

Solution:

Compound	In	Out
C4H8	1	1- ξ
C4H6	0	ξ
H2	0	ξ
H2O	12	12
Total	13	13+ ξ

$P = 2$ bar

$$K_a = \frac{\left(\frac{\xi}{13+\xi}\right)^2 * P^2}{\left(\frac{1-\xi}{13+\xi}\right) P} = \frac{\xi^2 * P}{(13+\xi)*(1-\xi)}$$

$$K_a = 0.01933$$

Noting that $\ln K_a = -\Delta G_{tot}/RT$, we can identify the temperature by fitting a trendline to the given data.

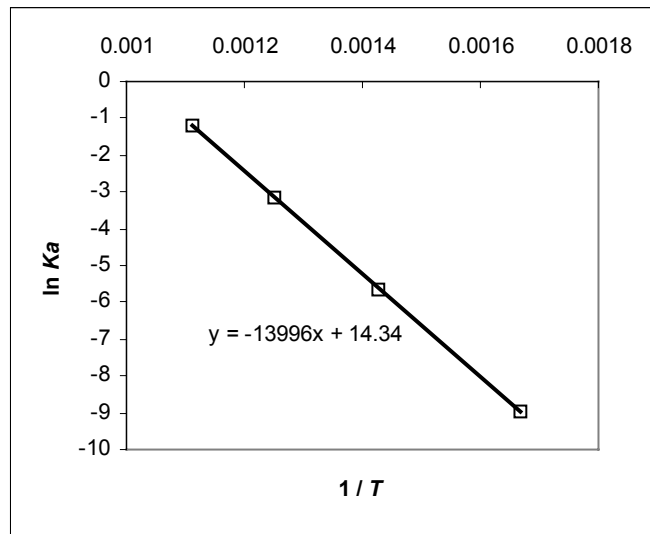
$$\Rightarrow \ln 0.01933 = -3.95,$$

substitute in the equation of straight line,

$$\Rightarrow x = -(-14.34 - 3.95)/13996.$$

$$\Rightarrow x = 0.001306 = 1/T,$$

$$\Rightarrow T = 765.22 \text{ K} = 492^\circ\text{C}$$



Compound	In	Out
C4H8	1	1- ξ
C4H6	0	ξ
H2	0	ξ
H2O	1	1
Total	2	2+ ξ

$$K_a = \frac{\xi^2 * P}{(2+\xi)*(1-\xi)} = 0.1118$$

$$\text{Similarly, } T = 846.65\text{K} = 573.6^\circ\text{C}$$

\Rightarrow We need higher T .

Chapter 14 Practice Problem Solutions

(P14.4) The standard Gibbs energy change for ethylene oxide at 298K for the reaction is -79.79 kJ/mole. This large negative value of ΔG_T° indicates that equilibrium is far to the right at 298K but what about 550K? Heat capacity expressions are given as $C_p = a + b T$.

Solution: The heat of reaction must be looked up. Referring to Apx E.6 for ethylene and the DIPPR handbook for ethylene oxide, $\Delta H_{298}^\circ = -52.6 - 52.51 = -105.1$ kJ/mol. Following Eqs. 14.28 and 14.30,

$$-105100 = J + (6.57 - 15.4 - 26.65/2) 298 + (0.1389 - 0.0937 - 0.00845/2) 298^2/2$$

$$-105100 = J - 22.155 * 298 + 0.040975 * 298^2/2 \Rightarrow J = -100317 \text{ J/mol}$$

$$-79790 / (8.314 * 298) = -100317 / (8.314 * 298) -$$

$$\frac{-79790}{8.314 * 298} = \frac{-100317}{8.314 * 298} - \frac{-22.155}{8.314} \ln 298 - \frac{0.040975}{2 * 8.314} 298 + I \Rightarrow I = -6.16$$

$$\frac{\Delta G^\circ}{8.314 * 550} = \frac{-100317}{8.314 * 550} - \frac{-22.155}{8.314} \ln 550 - \frac{0.040975}{2 * 8.314} 550 + 6.16 = 12.6$$

$\Rightarrow \Delta G^\circ = -57.7$ kJ/mol. If $T = 550$ K, increasing T will give an adverse effect on equilibrium, but the reaction is still very strongly favored and the impact will be indistinguishable.

(P14.5) The “water gas” shift is to be carried out at a specified temperature and pressure employing a feed containing only CO and H₂O. Show that the maximum equilibrium mole fraction of H₂ in the product results when the feed contains CO and H₂O in their stoichiometric proportions. Assume ideal gas behavior.

Solution: $CO + H_2O = CO_2 + H_2$

Compound	In	Out	$Ka = \frac{\xi^2}{(z - \xi)(1 - \xi)}$ $\Rightarrow \xi^2 - Ka(z - \xi)(1 - \xi) = 0$ $0 = \xi^2 - Ka[z - (1 + z)\xi + \xi^2]$
CO	z	$z - \xi$	
H ₂ O	1	$1 - \xi$	
CO ₂	0	ξ	
H ₂	0	ξ	
Total	$1 + z$	$1 + z$	

$$\xi^2(1 - Ka) + Ka(1 + z)\xi - Ka z = 0 \Rightarrow \xi = \{-Ka(1 + z) + [Ka^2(1 + z)^2 + 4z(1 - Ka)Ka]^{1/2}\} / [2(1 - Ka)]$$

$$y_{H_2} = \xi / (1 + z) = \{-Ka(1 + z) + [Ka^2(1 + z)^2 + 4z(1 - Ka)Ka]^{1/2}\} / [2(1 + z)(1 - Ka)]$$

$$y_{H_2} = \xi / (1 + z) = \{-Ka + [Ka^2 + 4z(1 - Ka)Ka / (1 + z)^2]^{1/2}\} / [2(1 - Ka)]$$

To find maximum, take derivative and set equal to zero.

$$dy/dz = 0.5[Ka^2 + 4z(1 - Ka)Ka / (1 + z)^2]^{-1/2} [4(1 - Ka)Ka / (1 + z)^2 - 8z(1 - Ka)Ka / (1 + z)^3] = 0$$

$$1 = 2z / (1 + z) \Rightarrow z = 1. \text{ QED.}$$

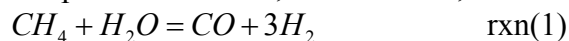
(P14.6) Assuming ideal gas behavior, estimate the equilibrium composition at 400K and 1 bar of a reactive mixture containing the three isomers of pentane. Formation data are given at 400K.

Solution: This is best solved by the Gibbs minimization method, adapting Example 14.10 and GibbsMin from the Rxns.xls workbook, we obtain the following.

	Gf(J/mole)	Gf400/RT	feed	n_i	$\log(n_i)$	y_i	$n_i(G_i/RT + \ln y_i)$
nPentane	40170	12.08	0	0.111	-0.95	0.111	1.100
iPentane	34310	10.32	1	0.648	-0.19	0.648	6.408
neoPentane	37610	11.31	0	0.240	-0.62	0.240	2.376
Tot			1	1.000			9.884
	Out	In					
C-bal	5	5					
H-bal	12	12					

Chapter 14 Practice Problem Solutions

(P14.7) One method for the manufacture of synthesis gas depends on the vapor-phase catalytic reaction of methane with steam according to the equation below. The water-gas shift reaction also is important. Bases on stoichiometric feed of methane and steam, compute the eq composition at 600K, 1300K and 1, 100 bars.



Compound	In	Out
CH4	1	1-ξ ₁
H2O	1	1-ξ ₁ -ξ ₂
CO	0	ξ ₁ -ξ ₂
H2	0	3ξ ₁ +ξ ₂
CO2	0	ξ ₂
Total	2	2+2ξ ₁

$$\Delta G_{600K} = -164.68 + 214.01 + 22.97 = 72.3kJ / mole \dots \dots \dots \text{rxn(1)}$$

$$\Delta G_{600K} = -395.14 + 214.01 + 164.68 = -16.45kJ / mole \dots \dots \dots \text{rxn(2)}$$

$$Ka_{600K,1} = \exp\left(\frac{-72300J / mole}{8.314 * 600}\right) = 5.08E - 7$$

$$Ka_{600K,2} = \exp\left(\frac{16450}{8.314 * 600}\right) = 27.05$$

$Ka_1 = \frac{(\xi_1 - \xi_2)(3\xi_1 + \xi_2)^3 * P^4}{(1 - \xi_1)(1 - \xi_1 - \xi_2) * P^2} \bigg/ \frac{(2 + 2\xi_1)^4}{(2 + 2\xi_1)^2}$ $= \frac{(\xi_1 - \xi_2)(3\xi_1 + \xi_2)^3 * P^2}{(1 - \xi_1)(1 - \xi_1 - \xi_2)(2 + 2\xi_1)^2}$ $Ka_2 = \frac{\xi_2(\xi_1 + \xi_2) * P^2}{(1 - \xi_1)(\xi_1 - \xi_2) * P^2} \bigg/ \frac{4}{4} = \frac{\xi_2(\xi_1 + \xi_2)}{(1 - \xi_1)(\xi_1 - \xi_2)}$	<p>PP 14.7</p> <p>Two simultaneous reactions:</p> <p>CH4 + H2O = CO + 3H2</p> <p>H2+CO2 = CO + H2O</p> <p>(Details of equations described in text)</p> <table border="1"> <tbody> <tr> <td>P(bars)</td> <td>100</td> </tr> <tr> <td>T(K)</td> <td>1300</td> </tr> <tr> <td>K_{a1}</td> <td>16113</td> </tr> <tr> <td>K_{a2}</td> <td>0.5400</td> </tr> <tr> <td>ξ₁</td> <td>0.4513</td> </tr> <tr> <td>ξ₂</td> <td>0.1491</td> </tr> <tr> <td>y₁</td> <td>0.1891</td> </tr> <tr> <td>y₂</td> <td>0.1377</td> </tr> <tr> <td>y₃</td> <td>0.1041</td> </tr> <tr> <td>y₄</td> <td>0.5178</td> </tr> <tr> <td>y₅</td> <td>0.0514</td> </tr> <tr> <td>nTot</td> <td>2.9025</td> </tr> </tbody> </table> <p>Objective Functions</p> <table border="1"> <tbody> <tr> <td>err1</td> <td>0.0000</td> </tr> <tr> <td>err2</td> <td>0.0000</td> </tr> </tbody> </table>	P(bars)	100	T(K)	1300	K_{a1}	16113	K_{a2}	0.5400	ξ ₁	0.4513	ξ ₂	0.1491	y₁	0.1891	y₂	0.1377	y₃	0.1041	y₄	0.5178	y₅	0.0514	nTot	2.9025	err1	0.0000	err2	0.0000
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(P14.8) Is there any danger that solid carbon will form at 550°C and 1 bar by the reaction:
 $2CO = C_s + CO_2$

	$\Delta H_{298K} \text{ kJ/mole}$	$\Delta G_{298K} \text{ kJ/mole}$	IN	Out
CO	-110.53	-137.16	2	2-2ξ
C _s	0	0	0	0 (gas)
CO ₂	-393.51	-394.38	0	ξ
Tot				2-ξ

$$\Delta G_{298K,T} = -394.38 + 2 * 137.16 = -120.06 \text{ kJ/mole}$$

$$\Delta H_{298K,T} = -393.51 + 2 * 110.53 = -172.45 \text{ kJ/mole}$$

Increasing T , adverse affect on equilibrium

$$K_{a_{298K}} = \exp\left[\frac{-\Delta G_{298K}}{RT}\right] = \exp\left(\frac{120060}{8.314 * 298.15}\right) = 1.08E21$$

Using Shortcut Van't Hoff Eq. 14.31

$$\Rightarrow \ln\left(\frac{K_a}{K_{a_{298}}}\right) = \frac{-\Delta H_{298}}{R} \left(\frac{1}{T} - \frac{1}{T_{298}}\right) = \ln\left(\frac{K_a}{1.08E21}\right) = \frac{172450}{8.314} \left(\frac{1}{823.15} - \frac{1}{298.15}\right)$$

$$\Rightarrow \ln K_a = \ln(1.08E21) - 44.4 = 4.06 \Rightarrow K_a = \exp(4.06) = 57.99$$

$$\Rightarrow K_a = \frac{y_{CO_2} * P}{(y_{CO} * P)^2} = \frac{\left(\frac{\xi}{(2-\xi)}\right)}{\left(\frac{(2-2\xi)}{(2-\xi)}\right)^2} = 57.99 = \frac{\xi * (2-\xi)}{(2-2\xi)^2}$$

$$\xi = 0.9345 \Rightarrow \text{ratio of carbon solid to feed is } \frac{0.9345}{2} = 0.46725. \Rightarrow \text{There is danger.}$$

Note: this exemplifies a very important and undesirable side reaction in many catalytic reactions know as “coking.” The carbon tends to clog the catalyst pores and substantially reduce its effectiveness. Because of this problem, fluidized catalytic crackers were developed (aka. Cat crackers). The solid catalyst particles are fluidized by the upflow of gaseous reactants. As they ultimately settle at the bottom, they are removed and recirculated through an oxidation zone that burns off the coke then recycles the catalyst to the top of the bed. This is a good example of how thermodynamics impacts reactor design.

(P14.9) Calculate the equilibrium percent conversion of ethylene oxide to ethylene glycol at 298K and 1 bar if the initial molar ratio of ethylene oxide to water is 3.

	In	Out(z_i)
EtO	3	3-ξ
Water	1	(1-ξ)
Glycol	0	ξ
Tot		4-ξ

$$K_w = P^{sat}/P = 0.0425; K_{EtO} = 1.76; K_{Gly} = 8.6E-4; y_i = z_i K_i / [K_i + L/F * (1 - K_i)]$$

$$K_a = \frac{y_{Gly} P}{y_{EtO} * y_W * P^2} = \exp\left(\frac{7824}{298.15 * 8.314}\right) = 23.52 \quad \text{and } \sum y_i = 1 \text{ are constraints, } \xi \text{ and } L/F \text{ are}$$

unknown. Guess, $\xi = 0.99$, $L/F = 1/3$ (all glycol in liquid, all EtO in vapor).

To accompany *Introductory Chemical Engineering Thermodynamics*

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Chapter 14 Practice Problem Solutions

(P14.9) Sample solution of one reaction with vle:

(Details of input equations described in text by Elliott and Lira)

P(bar)	T(K)	Ka1			
1.000	298	23.52			
	pSat(bar)	K-ratios	zFeed	yi	xi
EtO	1.76000	1.76000	0.66676	0.99961	0.56796
Water	0.04250	0.04250	0.00029	1.579E-05	0.00037
Glycol	0.00086	0.00086	0.33295	0.00037	0.43167
			1.0000	1.0000	1.0000
1-ξ	8.7055E-04				
ξ	0.99913		sum(yi-xi)	0.00000	
L/F	0.77111		ErrKa	5.583E-07	

As it turns out, the ethylene oxide is not so volatile after all and dissolves a fair amount in the liquid. The guess about the extent of conversion being high was good though. A more clever engineering approach would be to assume complete conversion and solve the simple flash. Then back out the exact conversion assuming L/F does not change.

(P14.10) Acetic acid vapor dimerizes according to $2A_1 = A_2$. Assume that no higher-order associations occur. Supposing that a value for Ka is available, and that the monomers and dimers behave as an ideal gas, derive an expression for y_{A_1} in terms of P and Ka . Then develop an expression for PV/n_0RT in terms of y_{A_1} , where n_0 is the superficial number of moles neglecting dimerization. Hint: write n_0/n_T in terms of y_{A_1} where $n_T = n_1 + n_2$.

Solution:

$$\frac{n_0}{n_T} = \frac{n_1 + 2n_2}{n_T} = y_{A_1} + 2(1 - y_{A_1}) = 2 - y_{A_1}$$

$$\frac{y_{A_2}}{y_{A_1}^2 P} = Ka \Rightarrow 1 - y_{A_1} = y_{A_1}^2 PKa \Rightarrow y_{A_1} = \frac{-1 + \sqrt{1 + 4PKa}}{2PKa}$$

$$\text{Ideal gas} \Rightarrow \frac{PV}{n_T RT} = 1 \Rightarrow \frac{PV}{n_0 RT} = \frac{n_T}{n_0} = \frac{1}{2 - y_{A_1}}$$

Note: as $Ka \rightarrow \infty$, $PV/n_0RT \rightarrow 1/2$ because the monomer is converted to dimer. Note also that PV/n_0RT is what we normally refer to as the compressibility factor, Z . This is an interesting result with regard to equations of state and phase equilibria. Since Ka is simply a function of temperature [ie. $\exp(-\Delta G/RT)$], it says that we can compute Z given a pressure and temperature. This is analogous to the pressure explicit virial equation (Section 6.4), but the form of the pressure dependence is more complex. Exploring this perspective, generalizing to density-dependent equations, and adapting to multimer-forming species and mixtures is the subject of Chapter 15. Most of the physical insight contained in Chapter 15 is contained in this simple practice problem.

14.10 Sample Tests

14.11 Test 1 (1 hr)

1. Water (H₂O) and 2-butanol (CH₃CH₂CH₂CH₂OH) are fed in a 1:10 molar ratio at 171°C and 11 bar, reacting over a catalyst to form water and 1-butene (CH₃CH₂CH=CH₂). Note that for 2-butanol:

Name	$\Delta H_{f,298}^{\circ}$ (kJ/mol)	$\Delta G_{f,298}^{\circ}$ (kJ/mol)	Constants for C_p in J/mol-K			
			a	b	c	d
2-Butanol	-292.629	-167.611	5.748816	0.4242576	-0.0002327	4.77E-08

(a) Assuming only vapor species for this reaction at 171°C and 11 bar, derive numerical values for A and B in the expression: $\ln Ka = A+B/T$, where T [=] K. (5)

(b) Would raising the temperature favor products or reactants? (3)

(c) Would raising the pressure favor products or reactants? (3)

(d) Write an expression for computing the extent of reaction given Ka . (5)

(e) Solve for the extent of reaction and the mole fractions of all species exiting the reactor at 11 bar. (4)

2. Ethanol (CH₃CH₂OH), water (H₂O) and 2-butanol (CH₃CH₂CH₂CH₂OH) are fed in a 100:1:10 molar ratio at 171°C and 11 bar, reacting over a catalyst to form ethylene (CH₂=CH₂), water, and 1-butene (CH₃CH₂CH=CH₂). For the ethanol reaction at 171°C and 11 bar, numerical values are 15.78 and -5639 for A and B in the expression: $\ln Ka = A+B/T$, where T [=] K.

(a) Write expressions for the extents of both reactions in terms of the values for Ka_1 and Ka_2 . (10)

(b) Solve for the extents of reaction and the mole fractions of all species exiting the reactor. (10)

3. Water (H₂O) and 2-butanol (CH₃CH₂CH₂CH₂OH) are fed in a 1:10 molar ratio at 171°C and 11 bar, reacting over a catalyst to form 1-butanol, water and 1-butene (CH₃CH₂CH=CH₂). The reactor operates isothermally at 171°C and 11 bar. The equilibrium distribution of all species is to be studied by Gibbs energy minimization.

a. Write the atom balance constraints for all elements. (5)

b. Estimate the relevant G_f/RT for 1-butanol, the others are given below. (8)

c. Taking an initial guess of 70% conversion of 2-butanol and 0.0001 mole fraction of 1-butanol, solve for the mole fractions of all species exiting the reactor. Note: do not use ChemCad for this. (7)

Compo	G_f/RT
1-butanol	
1-butene	28.92173
2-butanol	-28.20177
water	-60.09647

14.12 Test 2 (1 hr)

- 1.a. Ethanol and acetic acid react to form ethyl acetate and water. Evaluate the K_a at 150°C .(5)
- 1.b. If the temperature of the above reaction was increased, would the extent of reaction tend to increase or decrease? Why?(5)
- 1.c. If the above reaction was carried out at atmospheric pressure and 150°C with a stoichiometric feed, what would be the equilibrium extent of reaction? (5)
2. It has been suggested that the number of C2 hydrocarbons formed during methane pyrolysis would be insufficient to explain the observed rate of carbon deposition even if they approached their maximum equilibrium limits. To evaluate the legitimacy of this suggestion, it is necessary to determine the maximum equilibrium compositions of these species.
- (a) Perform this determination at 1 atm and 1500K considering the following species to be in mutual equilibrium: CH_4 , C_2H_6 , C_2H_4 , C_2H_2 and H_2 .(15)
- (b) Would decreasing the pressure to 40 mmHg tend to enhance the equilibrium composition of C2 species, or diminish them? Why?(5)
3. Returning to the problem of ethyl acetate synthesis from acetic acid and ethanol at 150°C , what pressure must be maintained to keep 50% of the solution as a liquid and what will be the equilibrium compositions of the phases at these compositions? You may assume ideal solution behavior for the liquid phase with the short-cut vapor pressure approximation.(20)
- (For ethyl acetate: $T_c=523.2\text{K}$, $P_c = 37.8$ bars and $\omega = 0.363$)