MOMENT INFLUENCE COEFFICIENTS FOR CONTINUOUS POST-TENSIONED STRUCTURES

Tables are presented to simplify the computation of moments over the supports in continuous structures under post-tensioning loads. Coefficients are provided for two-span structures and for symmetric structures of three or more spans. Tendon profiles are parabolic segments. A procedure accounting for friction losses is included.

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The bending moments in a beam continuous over several supports produced by post-tensioned prestressing tendons are usually computed by the equivalent load method as presented by Moorman[1]. All the forces between the tendon and the concrete are applied to the concrete beam, in effect as an exterior load assuming the tendons to be omitted. The elastic analysis of continuous beams under these loads presents no theoretical difficulties; however, it is tedious if performed manually by moment distribution, slope deflection or similar methods. Generally, these methods involve two steps: the computation of fixed end moments; and the elastic distribution of these moments. The second step is explained in any text on structural analysis[2]. The computation of fixed end moments is simplified by various charts and tables. Formulas and graphs for a variety of conditions are presented by Parme and Paris[3] and tables for beams of constant cross section are presented by Bailey and Ferguson[4].

This paper presents tables which simplify the bending moment computations for multispan beams with typical draped parabolic profile tendons. The restrictions are that the beams must be prismatic between supports; moreover, for three or more spans, the geometry of the structure must be symmetric. Except for the two-span case, coefficients are given only for tendon geometry that is symmetric about the centerline of the structure. Within these restrictions, the coefficients are given for a range of geometry parameters which covers the designs usually encountered. The determination of moments in beams with long tendons, where friction losses must be taken into account, is also considered, both for symmetric tensioning from both ends and for tensioning from only one end. The method presented here

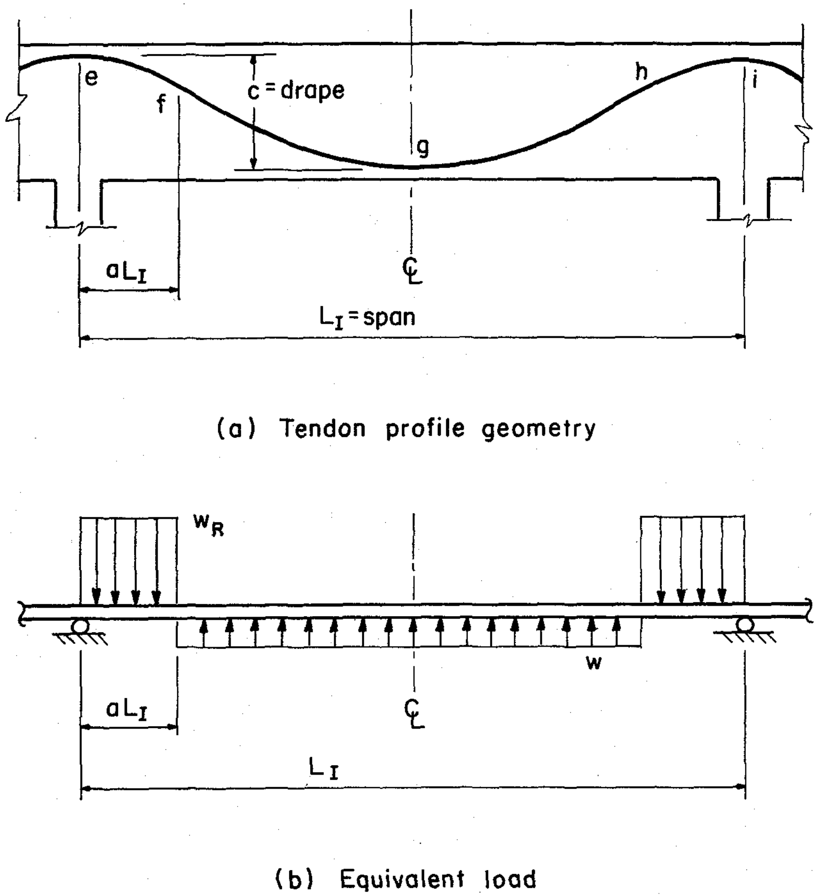
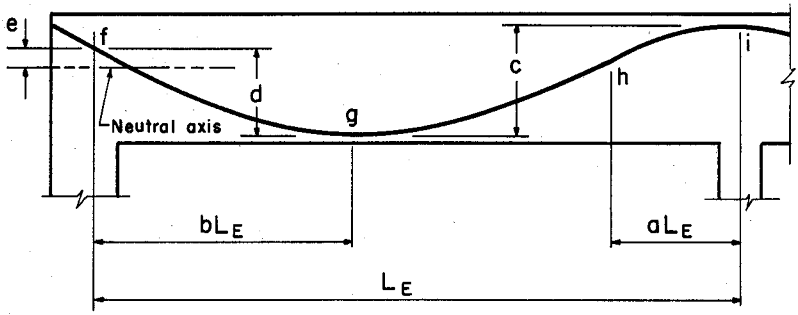


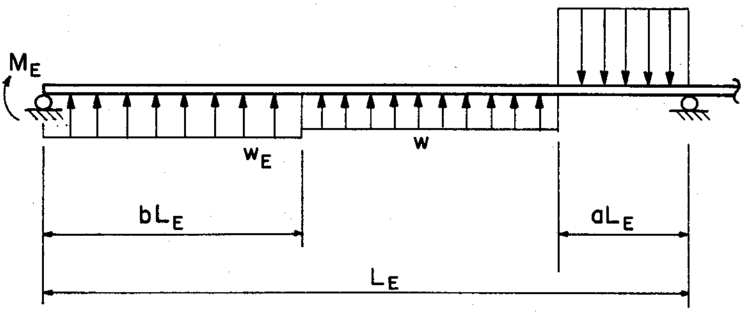
Fig. 1. Typical interior span

is not very cumbersome and should be suitable for general engineering use.

Problems beyond the scope of this paper, such as general variation of cross section or non-symmetric structures with more than two spans, can be analyzed by slope deflection methods with the fixed end moments computed using the curves developed by Parme and Paris[3]. Fixed end moments for cubic parabola tendon profiles can be computed using formulas presented by Fiesenheiser[5] and those for sine curve tendon profiles can be computed using graphs presented by Parme and Paris. The moments due to post-tensioning can also be obtained by using a general digital computer program for frame analysis, such as STRUDL[6], if it allows members of the desired shape with the equivalent loads as the applied loading. None of these



(a) Tendon profile geometry



(b) Equivalent load

Fig. 2. Typical exterior span

methods consider the continuity between the beam and its supporting columns, except for STRUDL where this effect may be taken into account.

The load balancing approach presented by Lin[7] is particularly applicable when the tendon profile in each span is one parabola, i.e., does not have a reversed curve over the supports. In this method, the equivalent load is considered to counteract a portion of the dead load, or the total dead load and a portion of the live load. For beams with reversed tendons, the method can still be used in one of two ways:

1. As an exact method, add the equivalent load to the dead load and analyze the beam under the resulting "balanced" dead load. This method is rather tedious unless a suitable computer program is available.

2. As an approximate method, replace the actual tendon profile geometry throughout the span by a single parabola. In many cases this will not be accurate enough for a final engineering analysis.

EQUIVALENT LOADS

The equivalent vertical distributed tendon load imposed on any point of the beam is computed as the product of the curvature of the tendon profile and the horizontal component of the tendon force at that point. It is usually accurate enough to consider the horizontal tendon force at each point equal to the total tendon force, particularly if the drape of the tendon profile is less than 4% of the span length.

The tendon profile in an interior span is in the shape of three parabolic segments, shown in Fig. 1(a) as ef, fgh, and hi. Segments ef and hi are the reversed parabolas. Points e, g and i are the horizontal points of the parabolas and points f and h are points of common tangency. The profile is assumed to be symmetric about the centerline of the span with its high point over the supports and low point at the span centerline.

The corresponding idealized structure, with the equivalent loads applied, is shown in Fig. 1(b). The magnitude of the upward distributed loads from the main portion of the tendon is

$$w = \frac{8Pc}{(1-2a)L_I^2} \tag{1}$$

where P is the horizontal tendon force component*. The downward load at the reversed parabola segment is

$$w_R = \frac{1-2a}{2a}\, w \tag{2}$$

However, it need not be considered separately when using the tables presented in this paper.

A typical exterior span, as shown in Fig. 2(a), has a tendon profile which consists of three parabolic segments: fg, gh, and hi. Segments fg and gh have a common horizontal low point at g; segments gh and hi have a common tangent at h; and the reversed parabola segment, hi, has a horizontal high point at i directly over the support.

The equivalent tendon load acting on the exterior span is considered in three parts, Fig. 2(b). First, the end moment

$$M_E = Pe \tag{3}$$

Second, the upward load due to the tendons in the external parabola segment where the upward prestress load is given by

$$w_E = \frac{2Pd}{b^2 L_E^2} \tag{4}$$

And, third, the load due to the remaining part of the tendons for which the upward segment of the prestress load is given by

$$w = \frac{2Pc}{(1-b)(1-b-a)L_E^2} \tag{5}$$

MOMENT INFLUENCE COEFFICIENTS

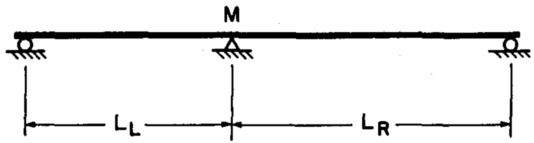
Tables I through VI are to be used in computation of beam moments at the support points. These tables are intended for beams of constant cross section over all spans, but may be used if the section changes from span to span, as explained later. Table I covers the 2-span beam for which the two spans may or may not

Notation is summarized at the end of the report.

TABLE I INFLUENCE SEGMENT COEFFICIENTS FOR 2 SPANS - - 1-ST INTERIOR SUPPORT

* NUMBERS REFER TO PER CENT OF SPAN LOADING DESCRIPTION	REVERSE CURVE	- - RATIO OF LEFT SPAN LENGTH TO RIGHT SPAN LENGTH - - 0.650	0.700	0.750	0.800	0.850	0.900	0.950	1.000
- LOAD ON ONE SPAN ONLY -									
FIRST 30 OF FIRST SPAN	00	0.003576	0.004335	0.005180	0.006111	0.007132	0.008244	0.009447	0.010743
LAST 70 OF FIRST SPAN	05	0.014459	0.017528	0.020943	0.024711	0.028839	0.033333	0.038197	0.043438
LAST 70 OF FIRST SPAN	10	0.011971	0.014511	0.017339	0.020458	0.023876	0.027596	0.031623	0.035962
LAST 70 OF FIRST SPAN	15	0.009751	0.011821	0.014124	0.016666	0.019450	0.022480	0.025761	0.029295
FIRST 40 OF FIRST SPAN	00	0.006124	0.007424	0.008871	0.010467	0.012216	0.014119	0.016180	0.018399
LAST 60 OF FIRST SPAN	05	0.012306	0.014918	0.017824	0.021031	0.024545	0.028369	0.032510	0.036970
LAST 60 OF FIRST SPAN	10	0.010173	0.012332	0.014735	0.017386	0.020290	0.023452	0.026875	0.030562
LAST 60 OF FIRST SPAN	15	0.008271	0.010027	0.011980	0.014135	0.016497	0.019067	0.021850	0.024848
FIRST 50 OF FIRST SPAN	00	0.009102	0.011034	0.013183	0.015555	0.018154	0.020982	0.024044	0.027343
LAST 50 OF FIRST SPAN	05	0.009724	0.011789	0.014085	0.016619	0.019396	0.022418	0.025690	0.029214
LAST 50 OF FIRST SPAN	10	0.007947	0.009634	0.011511	0.013582	0.015851	0.018320	0.020994	0.023874
LAST 50 OF FIRST SPAN	15	0.006362	0.007712	0.009215	0.010873	0.012689	0.014667	0.016807	0.019113
LAST 30 OF LAST SPAN	00	0.013022	0.012639	0.012278	0.011937	0.011614	0.011309	0.011019	0.010743
FIRST 70 OF LAST SPAN	05	0.052652	0.051103	0.049643	0.048264	0.046960	0.045724	0.044552	0.043438
FIRST 70 OF LAST SPAN	10	0.043590	0.042308	0.041099	0.039958	0.038878	0.037855	0.036884	0.035962
FIRST 70 OF LAST SPAN	15	0.035510	0.034465	0.033481	0.032551	0.031671	0.030837	0.030047	0.029295
LAST 40 OF LAST SPAN	00	0.022303	0.021647	0.021028	0.020444	0.019891	0.019368	0.018871	0.018400
FIRST 60 OF LAST SPAN	05	0.044812	0.043494	0.042251	0.041078	0.039967	0.038916	0.037918	0.036970
FIRST 60 OF LAST SPAN	10	0.037045	0.035955	0.034928	0.033958	0.033040	0.032171	0.031346	0.030562
FIRST 60 OF LAST SPAN	15	0.030119	0.029233	0.028398	0.027609	0.026863	0.026156	0.025485	0.024848
LAST 50 OF LAST SPAN	00	0.033143	0.032169	0.031250	0.030381	0.029560	0.028782	0.028044	0.027343
FIRST 50 OF LAST SPAN	05	0.035411	0.034370	0.033388	0.032460	0.031583	0.030752	0.029963	0.029214
FIRST 50 OF LAST SPAN	10	0.028939	0.028088	0.027285	0.026527	0.025810	0.025131	0.024487	0.023875
FIRST 50 OF LAST SPAN	15	0.023167	0.022486	0.021843	0.021237	0.020663	0.020119	0.019603	0.019113
UNIT MOMENT ON LEFT END		-0.196969	-0.205882	-0.214285	-0.222222	-0.229729	-0.236842	-0.243589	-0.250000
UNIT MOMENT ON RIGHT END		-0.303030	-0.294117	-0.285714	-0.277777	-0.270270	-0.263157	-0.256410	-0.250000
- - - APPLIED LOADS - - -									
UNIT DEAD LOAD ON 1-ST SPAN		-0.020804	-0.025220	-0.030133	-0.035555	-0.041494	-0.047960	-0.054959	-0.062500
UNIT DEAD LOAD ON 2-ND		-0.075757	-0.073529	-0.071428	-0.069444	-0.067567	-0.065789	-0.064102	-0.062500
UNIT DEAD LOAD ON BOTH SPANS		-0.096562	-0.098749	-0.101562	-0.104999	-0.109062	-0.113749	-0.119062	-0.125000

*For example, in the third line of coefficients the 70 and 10 indicate 70% and 10% of the first span.



$$M = \left[\sum w\cdot \text{coef.}\right]\cdot L_R^2 + \sum M_E\cdot \text{coef.}$$

be the same. Tables II to VI cover 3-, 4- and 5-span beams for which the geometry of the structure must be symmetric. A beam of more than five spans can be analyzed by taking the additional interior spans as equivalent to the center span of the 5-span beam. For the 3-span case coefficients are given only for the first interior support moment, and for the 4- and 5-span cases they are given for the first two interior supports. The moment over any other support is the same as the corresponding symmetric support moment if the loading is symmetric. If the loading is not symmetric, e.g., due to friction losses in a long beam tensioned from one end only, the moment is obtained by reversing the sign of the corresponding anti-symmetric component load coefficient as explained later.

Within each of the tables, the loadings considered consist of either uniformly distributed load segments or applied end moments. In the first case, the moment M is obtained by multiplying the coefficient by both the intensity of the load w and the square of the interior span length L_I; that is, the coefficients are moments for a unit load intensity applied to a structure with unit interior span length. In the second case the moment M is obtained by multiplying the coefficient by the applied end moment M_E.

The algebraic signs of all moments follow the beam convention: positive for a moment giving compression in the top fiber. The moments obtained from the tabulated influence coefficients will follow this sign convention provided the sign of the distributed load is positive if it is applied in its usual direction. That is, a distributed dead load acting downward is positive and a distributed prestress load acting upward over the major portion of the tendons is also positive. The distributed load due to the prestressing tendon is always expressed as that of the major (upward curvature) portion. The effect of the reverse curvature portion is already included in the tabulated moment coefficients.

Four types of distributed loads are considered in the tables:

1. Loads applied to the end portion of the exterior spans over segment fg denoted by bL_E in Fig. 2(a). Coefficients are given for a b of 30%, 40% and 50%.
2. Loads applied to the remaining (interior) portion of the exterior spans. The reverse curvature portion is segment hi in Fig. 2(a), denoted by aL_E in Fig. 2(a). Coefficients are tabulated for an a of 5%, 10% and 15%.
3. Loads applied to the interior spans. Here the reverse curvature segments ef and hi are denoted by aL_I in Fig. 1(a), with coefficients given for the above percentages for a. Note that the tendon profile is assumed to be symmetric within each interior span.
4. Uniform loads applied to specific spans for use in computing moments due to dead load as well as live load.

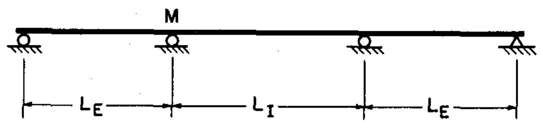
In this discussion L and I refer to span length and moment of inertia, respectively; subscripts E, I, L, R and C denote exterior, interior, left, right and center, respectively.

Coefficients for reverse curvature segment lengths, other than those tabulated, can be obtained by linear interpolation. For reverse curvatures of less than 5% the coefficient for the 0% case can be extrapolated by taking the corresponding 15% coefficient plus three times the difference

TABLE II INFLUENCE SEGMENT COEFFICIENTS FOR 3 SPANS - - 1-ST INTERIOR SUPPORT

NUMBERS REFER TO PER CENT OF SPAN — — RATIO OF EXTERIOR SPAN LENGTH TO INTERIOR SPAN LENGTH — —

LOADING DESCRIPTION	REVERSE CURVE	0.650	0.700	0.750	0.800	0.850	0.900	0.950	1.000
- - SYMMETRIC PRESTRESS - -									
END 30 OF END SPANS (SYM)	00	0.002744	0.003350	0.004028	0.004783	0.005615	0.006526	0.007519	0.008594
INNER 70 OF END SPANS	05	0.011096	0.013544	0.016289	0.019339	0.022703	0.026388	0.030402	0.034750
INNER 70 OF END SPANS	10	0.009187	0.011213	0.013485	0.016011	0.018796	0.021847	0.025170	0.028769
INNER 70 OF END SPANS	15	0.007484	0.009134	0.010985	0.013043	0.015311	0.017797	0.020504	0.023436
END 40 OF END SPANS	00	0.004700	0.005737	0.006899	0.008191	0.009616	0.011177	0.012878	0.014719
INNER 60 OF END SPANS	05	0.009444	0.011527	0.013863	0.016459	0.019322	0.022459	0.025875	0.029576
INNER 60 OF END SPANS	10	0.007807	0.009529	0.011460	0.013606	0.015973	0.018566	0.021390	0.024449
INNER 60 OF END SPANS	15	0.006347	0.007748	0.009318	0.011062	0.012987	0.015095	0.017391	0.019878
END 50 OF END SPANS	00	0.006985	0.008526	0.010253	0.012173	0.014291	0.016611	0.019137	0.021875
INNER 50 OF END SPANS	05	0.007463	0.009109	0.010955	0.013006	0.015269	0.017747	0.020447	0.023371
INNER 50 OF END SPANS	10	0.006099	0.007444	0.008953	0.010629	0.012478	0.014504	0.016710	0.019099
INNER 50 OF END SPANS	15	0.004882	0.005959	0.007167	0.008509	0.009989	0.011611	0.013377	0.015290
CENTER SPAN	05	0.049709	0.048579	0.047499	0.046467	0.045478	0.044531	0.043622	0.042749
CENTER SPAN	10	0.041860	0.040909	0.039999	0.039130	0.038297	0.037499	0.036734	0.035999
CENTER SPAN	15	0.034593	0.033806	0.033055	0.032337	0.031649	0.030989	0.030357	0.029750
UNIT MOMENTS ON THE ENDS		-0.151162	-0.159090	-0.166666	-0.173913	-0.180851	-0.187499	-0.193877	-0.200000
- ANTI-SYMMETRIC PRESTRESS -									
END 30 OF END SP.(ANTI-	00	0.005131	0.006141	0.007252	0.008462	0.009774	0.011188	0.012705	0.014324
INNER 70 OF END SP. -SYM)	05	0.020746	0.024832	0.029320	0.034215	0.039520	0.045237	0.051369	0.057917
INNER 70 OF END SP.	10	0.017175	0.020558	0.024274	0.028327	0.032719	0.037452	0.042528	0.047949
INNER 70 OF END SP.	15	0.013990	0.016745	0.019772	0.023073	0.026650	0.030506	0.034641	0.039056
END 40 OF END SP.	00	0.008787	0.010518	0.012419	0.014493	0.016740	0.019162	0.021759	0.024533
INNER 60 OF END SP.	05	0.017657	0.021134	0.024954	0.029121	0.033636	0.038501	0.043720	0.049293
INNER 60 OF END SP.	10	0.014596	0.017471	0.020629	0.024073	0.027806	0.031828	0.036142	0.040749
INNER 60 OF END SP.	15	0.011867	0.014204	0.016772	0.019572	0.022607	0.025877	0.029385	0.033131
END 50 OF END SP.	00	0.013059	0.015631	0.018457	0.021538	0.024877	0.028476	0.032336	0.036458
INNER 50 OF END SP.	05	0.013953	0.016701	0.019719	0.023012	0.026580	0.030425	0.034548	0.038953
INNER 50 OF END SP.	10	0.011402	0.013648	0.016115	0.018806	0.021721	0.024864	0.028234	0.031833
INNER 50 OF END SP.	15	0.009130	0.010928	0.012903	0.015058	0.017392	0.019908	0.022607	0.025488
UNIT MOMENTS ON THE ENDS		-0.282608	-0.291666	-0.300000	-0.307692	-0.314814	-0.321428	-0.327586	-0.333333
- - - APPLIED LOADS - - -									
UNIT DEAD LOAD ON 1-ST SPAN		-0.022908	-0.027608	-0.032812	-0.038528	-0.044764	-0.051528	-0.058827	-0.066666
UNIT DEAD LOAD ON 2-ND		-0.058139	-0.056818	-0.055555	-0.054347	-0.053191	-0.052083	-0.051020	-0.050000
UNIT DEAD LOAD ON 3-RD		0.006941	0.008120	0.009375	0.010702	0.012098	0.013560	0.015084	0.016666
UNIT DEAD LOAD ON ALL SPANS		-0.074106	-0.076306	-0.078993	-0.082173	-0.085857	-0.090052	-0.094764	-0.100000
UNIT D.L. ON SPANS 1 AND 2		-0.081048	-0.084427	-0.088368	-0.092876	-0.097956	-0.103612	-0.109848	-0.116666
UNIT D.L. ON SPANS 1 AND 3		-0.015966	-0.019488	-0.023437	-0.027826	-0.032666	-0.037968	-0.043743	-0.050000



$$M = \left[\sum w \cdot \text{coef.} \right] \cdot L_I^2 + \sum M_E \cdot \text{coef.}$$

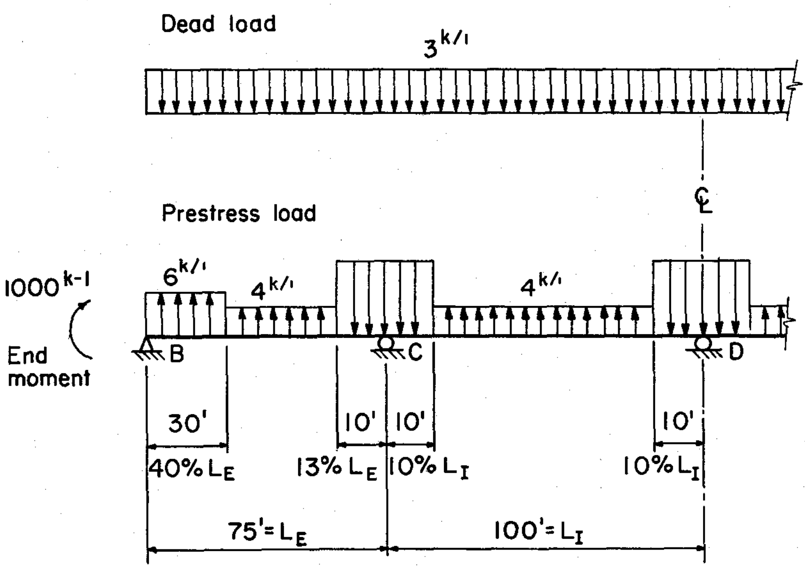


Fig. 3. 4-span structure for Example 1

between the 5% coefficient and the 10% coefficient. Similarly, the 20% case can be obtained by adding to the 5% case three times the difference between the 15% and the 10% cases. These extrapolations give results accurate to about 0.1%. The error due to linear interpolation is at most 0.3%.

Each table for the moment coefficients over a support is developed from the influence line for the moment in the beam over that support. The coefficients are obtained by computing the area under the influence line over the segment that is loaded by a constant distributed load. If the coefficient represents the effect of several load segments, then it is the sum of the area under each of the segments multiplied by the ratio of the equivalent loads.

Example 1

Consider the 4-span structure shown in Fig. 3. The moment at support C is to be computed for each of the following loadings:

a = distributed prestress load as shown

b = a 1000 k.-ft. end moment acting on both ends

c = a 3 k./ft. uniform dead load

The end to interior span ratio is 0.75. From Table III, the coefficient for the end 40% of the end span is 0.0103. The coefficient for the inner 60% with 13.3% reverse curve is interpolated as 0.0150. The coefficient for the middle spans is 0.0300. So the moment at support C due to the distributed prestress load a is

$$M = (0.0103 \times 6 + 0.0150 \times 4 + 0.0300 \times 4)100^2 = 2424 \text{ k.-ft.}$$

TABLE III INFLUENCE SEGMENT COEFFICIENTS FOR 4 SPANS - - 1-ST INTERIOR SUPPORT

NUMBERS REFER TO PER CENT OF SPAN — — RATIO OF EXTERIOR SPAN LENGTH TO INTERIOR SPAN LENGTH — —

LOADING DESCRIPTION	REVERSE CURVE	0.650	0.700	0.750	0.800	0.850	0.900	0.950	1.000
- - SYMMETRIC PRESTRESS - -									
END 30 OF END SPANS (SYM)	00	0.004215	0.005082	0.006043	0.007097	0.008247	0.009493	0.010836	0.012278
INNER 70 OF END SPANS	05	0.017041	0.020550	0.024433	0.028697	0.033345	0.038383	0.043815	0.049643
INNER 70 OF END SPANS	10	0.014108	0.017013	0.020228	0.023758	0.027606	0.031777	0.036274	0.041099
INNER 70 OF END SPANS	15	0.011493	0.013859	0.016478	0.019354	0.022489	0.025886	0.029550	0.033480
END 40 OF END SPANS	00	0.007218	0.008705	0.010349	0.012155	0.014124	0.016258	0.018559	0.021028
INNER 60 OF END SPANS	05	0.014504	0.017490	0.020795	0.024424	0.028380	0.032668	0.037290	0.042251
INNER 60 OF END SPANS	10	0.011990	0.014459	0.017191	0.020190	0.023461	0.027006	0.030827	0.034928
INNER 60 OF END SPANS	15	0.009748	0.011755	0.013977	0.016415	0.019075	0.021956	0.025063	0.028398
END 50 OF END SPANS	00	0.010727	0.012936	0.015380	0.018064	0.020990	0.024161	0.027580	0.031250
INNER 50 OF END SPANS	05	0.011461	0.013821	0.016433	0.019300	0.022426	0.025815	0.029468	0.033388
INNER 50 OF END SPANS	10	0.009366	0.011295	0.013429	0.015772	0.018327	0.021096	0.024082	0.027285
INNER 50 OF END SPANS	15	0.007498	0.009042	0.010751	0.012627	0.014672	0.016889	0.019279	0.021843
TWO MIDDLE SPANS	05	0.038169	0.036853	0.035624	0.034475	0.033398	0.032386	0.031433	0.030535
TWO MIDDLE SPANS	10	0.032142	0.031034	0.029999	0.029032	0.028124	0.027272	0.026470	0.025714
TWO MIDDLE SPANS	15	0.026562	0.025646	0.024791	0.023992	0.023242	0.022537	0.021875	0.021250
UNIT MOMENTS ON THE ENDS		-0.232142	-0.241379	-0.250000	-0.258064	-0.265625	-0.272727	-0.279411	-0.285714
- ANTI-SYMMETRIC PRESTRESS -									
END 30 OF END SP.(ANTI-	00	0.003576	0.004335	0.005180	0.006111	0.007132	0.008244	0.009447	0.010743
INNER 70 OF END SP. -SYM)	05	0.014459	0.017528	0.020943	0.024711	0.028839	0.033333	0.038197	0.043438
INNER 70 OF END SP.	10	0.011971	0.014511	0.017339	0.020458	0.023876	0.027596	0.031623	0.035962
INNER 70 OF END SP.	15	0.009746	0.011815	0.014117	0.016657	0.019439	0.022468	0.025747	0.029279
END 40 OF END SP.	00	0.006124	0.007424	0.008871	0.010467	0.012216	0.014119	0.016180	0.018399
INNER 60 OF END SP.	05	0.012306	0.014918	0.017824	0.021031	0.024545	0.028369	0.032510	0.036970
INNER 60 OF END SP.	10	0.010173	0.012332	0.014735	0.017386	0.020291	0.023452	0.026875	0.030562
INNER 60 OF END SP.	15	0.008271	0.010027	0.011980	0.014135	0.016497	0.019067	0.021850	0.024848
END 50 OF END SP.	00	0.009102	0.011034	0.013183	0.015555	0.018154	0.020982	0.024044	0.027343
INNER 50 OF END SP.	05	0.009724	0.011789	0.014085	0.016619	0.019396	0.022418	0.025690	0.029214
INNER 50 OF END SP.	10	0.007947	0.009634	0.011511	0.013582	0.015851	0.018320	0.020994	0.023874
INNER 50 OF END SP.	15	0.006367	0.007719	0.009223	0.010882	0.012700	0.014679	0.016821	0.019129
TWO MIDDLE SPANS	05	0.064772	0.062867	0.061071	0.059374	0.057770	0.056249	0.054807	0.053437
TWO MIDDLE SPANS	10	0.054545	0.052941	0.051428	0.049999	0.048648	0.047368	0.046153	0.044999
TWO MIDDLE SPANS	15	0.045106	0.043779	0.042528	0.041347	0.040229	0.039171	0.038166	0.037212
UNIT MOMENTS ON THE ENDS		-0.196969	-0.205882	-0.214285	-0.222222	-0.229729	-0.236842	-0.243589	-0.249999
- - - APPLIED LOADS - - -									
UNIT DEAD LOAD ON 1-ST SPAN		-0.022662	-0.027394	-0.032645	-0.038422	-0.044736	-0.051593	-0.059001	-0.066964
UNIT DEAD LOAD ON 2-ND		-0.060200	-0.058316	-0.056547	-0.054883	-0.053315	-0.051834	-0.050433	-0.049107
UNIT DEAD LOAD ON 3-RD		0.015557	0.015212	0.014880	0.014560	0.014252	0.013955	0.013668	0.013392
UNIT DEAD LOAD ON 4-TH		-0.001857	-0.002174	-0.002511	-0.002867	-0.003241	-0.003633	-0.004041	-0.004464
UNIT DEAD LOAD ON ALL SPANS		-0.069162	-0.072672	-0.076822	-0.081612	-0.087040	-0.093106	-0.099806	-0.107142
UNIT D.L. ON SPANS 1,2 AND 4		-0.084720	-0.087885	-0.091703	-0.096173	-0.101293	-0.107061	-0.113475	-0.120535
UNIT D.L. ON SPANS 2 AND 3		-0.044642	-0.043103	-0.041666	-0.040322	-0.039062	-0.037878	-0.036764	-0.035714
UNIT D.L. ON SPANS 1 AND 3		-0.007105	-0.012181	-0.017764	-0.023861	-0.030484	-0.037638	-0.045332	-0.053571
UNIT D.L. ON SPANS 2 AND 4		-0.062057	-0.060490	-0.059058	-0.057750	-0.056556	-0.055467	-0.054474	-0.053571

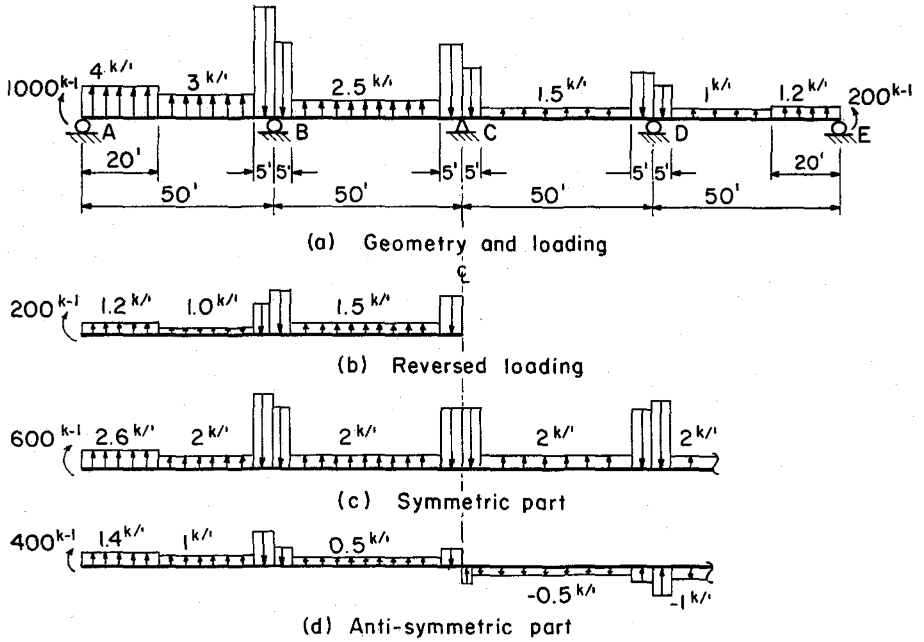


Fig. 4. 4-span structure with non-symmetric loading for Example 2

The coefficient for end moments is –0.25 so the desired moment due to loading b is $M = -250$ k.-ft. The coefficient for unit dead load on all spans is –0.0768, so the moment due to the 3 k./ft. dead load is $M = -2304$ k.-ft.

NON-SYMMETRIC LOADINGS

As pointed out previously, symmetry is not a consideration in the 2-span case so only structures of three or more spans are considered in this section. As long as a structure is symmetric, any loading can be separated into two loadings, one of which is symmetric and the other anti-symmetric. This division is usually obvious. However, if not, it can be obtained by reversing the original loading, taking half of the sum of the original and reversed loadings as the symmetric part, and half of the difference as the anti-symmetric part. The moments are then computed for both parts using the appropriate coefficients. The moments for the left half of the structure are equal to the sum of the computed moments; those for the right are equal to the difference.

Example 2

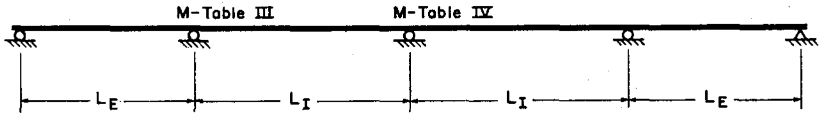
The moments at the supports of a beam of constant cross section with four equal spans are to be computed. A typical non-symmetric equivalent prestress loading as produced when tensioning long beams from one end is considered. End moments, as considered in this example, would appear only if the tendons are an-

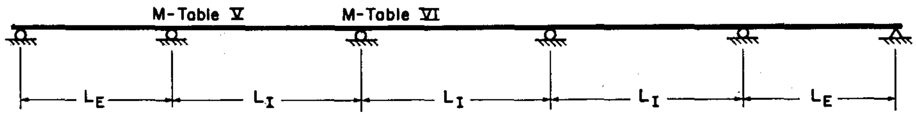
TABLE IV INFLUENCE SEGMENT COEFFICIENTS FOR 4 SPANS - - 2-ND INTERIOR SUPPORT

NUMBERS REFER TO PER CENT OF SPAN

LOADING DESCRIPTION	REVERSE CURVE	- - RATIO OF EXTERIOR SPAN LENGTH TO INTERIOR SPAN LENGTH - -							
		0.650	0.700	0.750	0.800	0.850	0.900	0.950	1.000
- - SYMMETRIC PRESTRESS - -									
END 30 OF END SPANS (SYM)	00	-0.002107	-0.002541	-0.003021	-0.003548	-0.004123	-0.004746	-0.005418	-0.006139
INNER 70 OF END SPANS	05	-0.008520	-0.010275	-0.012216	-0.014348	-0.016672	-0.019191	-0.021907	-0.024821
INNER 70 OF END SPANS	10	-0.007054	-0.008506	-0.010114	-0.011879	-0.013803	-0.015888	-0.018137	-0.020549
INNER 70 OF END SPANS	15	-0.005746	-0.006929	-0.008239	-0.009677	-0.011244	-0.012943	-0.014775	-0.016740
END 40 OF END SPANS	00	-0.003609	-0.004352	-0.005175	-0.006077	-0.007062	-0.008129	-0.009279	-0.010514
INNER 60 OF END SPANS	05	-0.007252	-0.008745	-0.010397	-0.012212	-0.014190	-0.016334	-0.018645	-0.021125
INNER 60 OF END SPANS	10	-0.005995	-0.007229	-0.008595	-0.010095	-0.011730	-0.013503	-0.015413	-0.017464
INNER 60 OF END SPANS	15	-0.004874	-0.005877	-0.006988	-0.008207	-0.009537	-0.010978	-0.012531	-0.014199
END 50 OF END SPANS	00	-0.005363	-0.006468	-0.007690	-0.009032	-0.010495	-0.012080	-0.013790	-0.015625
INNER 50 OF END SPANS	05	-0.005730	-0.006910	-0.008216	-0.009650	-0.011213	-0.012907	-0.014734	-0.016694
INNER 50 OF END SPANS	10	-0.004683	-0.005647	-0.006714	-0.007886	-0.009163	-0.010548	-0.012041	-0.013642
INNER 50 OF END SPANS	15	-0.003749	-0.004521	-0.005375	-0.006313	-0.007336	-0.008444	-0.009639	-0.010921
TWO MIDDLE SPANS	05	0.087789	0.088448	0.089062	0.089636	0.090175	0.090681	0.091157	0.091606
TWO MIDDLE SPANS	10	0.073928	0.074482	0.074999	0.075483	0.075937	0.076363	0.076764	0.077142
TWO MIDDLE SPANS	15	0.061094	0.061552	0.061979	0.062379	0.062754	0.063106	0.063437	0.063750
UNIT MOMENTS ON THE ENDS		0.116071	0.120689	0.125000	0.129032	0.132812	0.136363	0.139705	0.142857
- ANTI-SYMMETRIC PRESTRESS -									
		0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
- - - APPLIED LOADS - - -									
UNIT DEAD LOAD ON 1-ST SPAN		0.006130	0.007392	0.008789	0.010322	0.011994	0.013806	0.015760	0.017857
UNIT DEAD LOAD ON 2-ND		-0.051339	-0.051724	-0.052083	-0.052419	-0.052734	-0.053030	-0.053308	-0.053571
UNIT DEAD LOAD ON 3-RD		-0.051339	-0.051724	-0.052083	-0.052419	-0.052734	-0.053030	-0.053308	-0.053571
UNIT DEAD LOAD ON 4-TH		0.006130	0.007392	0.008789	0.010322	0.011994	0.013806	0.015760	0.017857
UNIT DEAD LOAD ON ALL SPANS		-0.090418	-0.088663	-0.086588	-0.084193	-0.081479	-0.078446	-0.075096	-0.071428
UNIT D.L. ON SPANS 1,2 AND 4		-0.039079	-0.036939	-0.034505	-0.031774	-0.028745	-0.025416	-0.021787	-0.017857
UNIT D.L. ON SPANS 2 AND 3		-0.102678	-0.103448	-0.104166	-0.104838	-0.105468	-0.106060	-0.106617	-0.107142
UNIT D.L. ON SPANS 1 AND 3		-0.045209	-0.044331	-0.043294	-0.042096	-0.040739	-0.039223	-0.037548	-0.035714
UNIT D.L. ON SPANS 2 AND 4		-0.045209	-0.044331	-0.043294	-0.042096	-0.040739	-0.039223	-0.037548	-0.035714

$$M = \left[\sum w \cdot \text{coef.} \right] \cdot L_I^2 + \sum M_E \cdot \text{coef.}$$





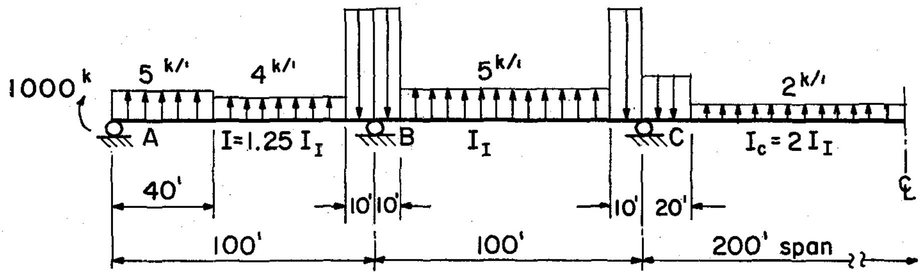


Fig. 5. 5-span structure with varying moments of inertia for Example 3

chored away from the neutral axis of the cross section, or if the beam is cantilevered. The complete loading diagram is shown in Fig. 4a. Load diagrams (b), (c) and (d) show the reversed, the symmetric portion, and the anti-symmetric portion respectively. Note that only the left half of the beam need be considered for these three loadings. Load diagram (b) was derived by folding the right part of the structure about its centerline. Load diagram (c) is half the sum of (a) and (b). Load diagram (d) can be computed either as the difference between (a) and (c) or as half the difference between (a) and (b). The moment at support B caused by the symmetric part of the loading is computed by using the coefficients in the symmetric prestress portion of Table III.

$$M_{BS} = (2.6 \times 0.02103 + 2.0 \times 0.03493 + 2.0 \times 0.02571)50^2 + 600 \times (-0.2857) = 268.5 \text{ k.-ft.}$$

The moment at support B due to the anti-symmetric prestress is

$$M_{BA} = (1.4 \times 0.01840 + 1.0 \times 0.03056 + 0.5 \times 0.04500)50^2 + 400 \times (-0.2500) = 97 \text{ k.-ft.}$$

The moment at support C due to the symmetric load (Table IV) is

$$M_{CS} = (-2.6 \times 0.01051 - 2.0 \times 0.01746 + 2.0 \times 0.07714)50^2 + 600 \times 0.1429 = 266 \text{ k.-ft.}$$

The moment at support C due to anti-symmetric prestress is zero.

$$M_{CA} = 0$$

Finally, the moments at the three supports are

left: $M_B = M_{BS} + M_{BA} = 365.5$ k.-ft.
$M_C = M_{CS} + M_{CA} = 266$ k.-ft.

right: $M_D = M_{BS} - M_{BA} = 171.5$ k.-ft.
$M_C = M_{CS} - M_{CA} = 266$ k.-ft.

SPANS WITH DIFFERENT MOMENTS OF INERTIA

Two cases of spans with different moments of inertia may be analyzed using the tables. First, the cross section of the end spans may be different (e.g. the cross sections of the spans in a 2-span beam). Second, the center span cross section of a 5-span beam may be different from the other two interior spans.

If the cross sections of the end spans differ, replace the end span ratio computation L_E/L_I by $L_E I_I / L_I I_E$ for selecting coefficients in the

TABLE V INFLUENCE SEGMENT COEFFICIENTS FOR 5 SPANS - - 1-ST INTERIOR SUPPORT

NUMBERS REFER TO PER CENT OF SPAN — - - RATIO OF EXTERIOR SPAN LENGTH TO INTERIOR SPAN LENGTH - -

LOADING DESCRIPTION	REVERSE CURVE	0.650	0.700	0.750	0.800	0.850	0.900	0.950	1.000
- - SYMMETRIC PRESTRESS - -									
END 30 OF END SPANS (SYM)	00	0.003807	0.004606	0.005493	0.006471	0.007540	0.008702	0.009958	0.011309
INNER 70 OF END SPANS	05	0.015392	0.018624	0.022212	0.026165	0.030487	0.035184	0.040262	0.045724
INNER 70 OF END SPANS	10	0.012743	0.015418	0.018389	0.021662	0.025240	0.029129	0.033333	0.037855
INNER 70 OF END SPANS	15	0.010381	0.012560	0.014980	0.017646	0.020561	0.023729	0.027154	0.030837
END 40 OF END SPANS	00	0.006520	0.007888	0.009409	0.011083	0.012914	0.014903	0.017054	0.019368
INNER 60 OF END SPANS	05	0.013100	0.015850	0.018905	0.022269	0.025947	0.029945	0.034267	0.038916
INNER 60 OF END SPANS	10	0.010829	0.013103	0.015628	0.018409	0.021450	0.024755	0.028328	0.032171
INNER 60 OF END SPANS	15	0.008805	0.010653	0.012706	0.014967	0.017439	0.020127	0.023031	0.026156
END 50 OF END SPANS	00	0.009689	0.011723	0.013982	0.016470	0.019191	0.022148	0.025344	0.028782
INNER 50 OF END SPANS	05	0.010352	0.012525	0.014939	0.017597	0.020504	0.023663	0.027078	0.030752
INNER 50 OF END SPANS	10	0.008460	0.010236	0.012208	0.014381	0.016756	0.019338	0.022129	0.025131
INNER 50 OF END SPANS	15	0.006772	0.008194	0.009773	0.011513	0.013414	0.015481	0.017716	0.020119
2-ND AND 4-TH SPANS	05	0.055161	0.053437	0.051818	0.050294	0.048857	0.047499	0.046216	0.044999
2-ND AND 4-TH SPANS	10	0.046451	0.044999	0.043636	0.042352	0.041142	0.039999	0.038918	0.037894
2-ND AND 4-TH SPANS	15	0.038387	0.037187	0.036060	0.035000	0.034000	0.033055	0.032162	0.031315
CENTER SPAN	05	-0.013790	-0.013359	-0.012954	-0.012573	-0.012214	-0.011874	-0.011554	-0.011249
CENTER SPAN	10	-0.011612	-0.011249	-0.010909	-0.010588	-0.010285	-0.009999	-0.009729	-0.009473
CENTER SPAN	15	-0.009596	-0.009296	-0.009015	-0.008750	-0.008500	-0.008263	-0.008040	-0.007828
UNIT MOMENTS ON THE ENDS		-0.209677	-0.218750	-0.227272	-0.235294	-0.242857	-0.249999	-0.256756	-0.263157
- ANTI-SYMMETRIC PRESTRESS -									
END 30 OF END SP.(ANTI-	00	0.003978	0.004806	0.005725	0.006735	0.007839	0.009037	0.010330	0.011720
INNER 70 OF END SP. -SYM)	05	0.016084	0.019433	0.023147	0.027233	0.031694	0.036538	0.041767	0.047387
INNER 70 OF END SP.	10	0.013316	0.016089	0.019164	0.022546	0.026240	0.030249	0.034579	0.039231
INNER 70 OF END SP.	15	0.010847	0.013106	0.015611	0.018366	0.021375	0.024642	0.028168	0.031958
END 40 OF END SP.	00	0.006813	0.008231	0.009805	0.011535	0.013425	0.015477	0.017692	0.020072
INNER 60 OF END SP.	05	0.013689	0.016540	0.019701	0.023178	0.026975	0.031097	0.035548	0.040331
INNER 60 OF END SP.	10	0.011316	0.013673	0.016286	0.019160	0.022300	0.025707	0.029387	0.033340
INNER 60 OF END SP.	15	0.009200	0.011116	0.013241	0.015578	0.018130	0.020901	0.023892	0.027107
END 50 OF END SP.	00	0.010124	0.012233	0.014571	0.017142	0.019951	0.023000	0.026292	0.029829
INNER 50 OF END SP.	05	0.010817	0.013070	0.015568	0.018315	0.021316	0.024574	0.028091	0.031870
INNER 50 OF END SP.	10	0.008840	0.010681	0.012722	0.014968	0.017420	0.020082	0.022956	0.026045
INNER 50 OF END SP.	15	0.007077	0.008551	0.010185	0.011983	0.013946	0.016077	0.018378	0.020851
2-ND AND 4-TH SPANS	05	0.048033	0.046467	0.044999	0.043622	0.042326	0.041105	0.039953	0.038863
2-ND AND 4-TH SPANS	10	0.040449	0.039130	0.037894	0.036734	0.035643	0.034615	0.033644	0.032727
2-ND AND 4-TH SPANS	15	0.033425	0.032335	0.031314	0.030355	0.029454	0.028604	0.027802	0.027044
UNIT MOMENTS ON THE ENDS		-0.219101	-0.228260	-0.236842	-0.244897	-0.252475	-0.259615	-0.266355	-0.272727
- - - APPLIED LOADS - - -									
UNIT DEAD LOAD ON 1-ST SPAN		-0.022644	-0.027379	-0.032633	-0.038415	-0.044734	-0.051598	-0.059013	-0.066985
UNIT DEAD LOAD ON 2-ND		-0.060347	-0.058423	-0.056618	-0.054921	-0.053323	-0.051816	-0.050391	-0.049043
UNIT DEAD LOAD ON 3-RD		0.016129	0.015625	0.015151	0.014705	0.014285	0.013888	0.013513	0.013157
UNIT DEAD LOAD ON 4-TH		-0.004168	-0.004076	-0.003987	-0.003901	-0.003818	-0.003739	-0.003662	-0.003588
UNIT DEAD LOAD ON 5-TH		0.000497	0.000582	0.000672	0.000768	0.000868	0.000973	0.001082	0.001196
UNIT DEAD LOAD ON ALL SPANS		-0.070534	-0.073671	-0.077414	-0.081764	-0.086723	-0.092291	-0.098471	-0.105263
UNIT D.L. ON SPANS 1,2 AND 4		-0.087160	-0.089879	-0.093239	-0.097238	-0.101877	-0.107154	-0.113067	-0.119617
UNIT D.L. ON SPANS 2,3 AND 5		-0.043721	-0.042216	-0.040794	-0.039447	-0.038169	-0.036953	-0.035795	-0.034688
UNIT D.L. ON SPANS 1,3 AND 5		-0.006018	-0.011171	-0.016808	-0.022941	-0.029580	-0.036736	-0.044417	-0.052631
UNIT D.L. ON SPANS 2 AND 4		-0.064516	-0.062500	-0.060606	-0.058823	-0.057142	-0.055555	-0.054054	-0.052631

tables. Then multiply each distributed load applied to the end spans by $(I_E/I_I)^2$. For the 2-span case, the ratio is $L_L I_R / L_R I_L$ and the multiplying factor is $(I_L/I_R)^2$ applied to a distributed load on the left span.

The center span section of a 5-span beam may be different only to the extent that its stiffness remains the same as for the other interior spans. That is $I_C/L_C = I_I/L_I$ where C refers to the center span and I refers to the other interior spans. Any distributed loading applied to the center span must then be multiplied by $(I_C/I_I)^2$.

Example 3

The moment at point C of the symmetric beam shown in Fig. 5 is to be computed. The end span ratio to be used is

$$100 \times I_I/100 \times 1.25\, I_I = 0.8$$

The distributed load factor $(I_E/I_I)^2$ is 1.5625. The center span stiffness requirement is satisfied since $I_C/L_C = I_I/L_I$. Its distributed load factor is 4.0. Hence, the required moment is obtained by using Table VI.

$$M_C = 0.04706 \times 1000 + (-0.00222 \times 5 \times 1.56 - 0.00368 \times 4 \times 1.56 + 0.02753 \times 5 + 0.03812 \times 2 \times 4.0)100^2 = 4070 \text{ k.-ft.}$$

Similarly, from Table V, the moment at point B is

$$M_B = 3048 \text{ k.-ft.}$$

BENDING MOMENTS BETWEEN THE SUPPORTS

Bending moments between the supports can be computed by two methods. The first method is simply to compute the moment at any point by statics using the applied equivalent loads and the computed moments at the supports. The second method, which requires considerably less computation, is to compute a primary moment which is the moment that would be present if the beam spans were free to rotate at their ends, and a secondary moment which is the moment produced by restoring beam continuity over the supports.

M'_x, the primary moment at any point x, is the horizontal component of the prestress force at that point times the eccentricity of the tendon profile from the neutral axis

$$M'_x = P_x e_x \tag{6}$$

The secondary moment is linear between the supports and, for a typical span AB

$$M''_x = M''_A\left(1 - \frac{x'}{L}\right) + M''_B \frac{x'}{L} \tag{7}$$

where x' is the distance from support point A to the point x, L is the length of span AB, M'' is the secondary moment at the point indicated by the subscript. The secondary moment at a support, as required in Equation (7), is computed by subtracting the primary moment from the total moment obtained by using the moment influence coefficients. So the total moment at any point x is obtained from Equations (6) and (7) as

$$M_x = P_x e_x + (M_A - P_A e_A) \times \left(1 - \frac{x'}{L}\right) + (M_B - P_B e_B)\frac{x'}{L} \tag{8}$$

Example 4

The moment in the first span of Example 3 is to be computed. The tendon profile is shown in Fig. 6(a), and the horizontal component of the tendon force is 1000 k. The primary moments as computed by Equation

TABLE VI INFLUENCE SEGMENT COEFFICIENTS FOR 5 SPANS - - 2-ND INTERIOR SUPPORT

NUMBERS REFER TO PER CENT OF SPAN — — RATIO OF EXTERIOR SPAN LENGTH TO INTERIOR SPAN LENGTH — —

LOADING DESCRIPTION	REVERSE CURVE	0.650	0.700	0.750	0.800	0.850	0.900	0.950	1.000
- - SYMMETRIC PRESTRESS - -									
END 30 OF END SPANS (SYM)	00	-0.000761	-0.000921	-0.001098	-0.001294	-0.001508	-0.001740	-0.001991	-0.002261
INNER 70 OF END SPANS	05	-0.003078	-0.003724	-0.004442	-0.005233	-0.006097	-0.007036	-0.008052	-0.009144
INNER 70 OF END SPANS	10	-0.002548	-0.003083	-0.003677	-0.004332	-0.005048	-0.005825	-0.006666	-0.007571
INNER 70 OF END SPANS	15	-0.002076	-0.002512	-0.002996	-0.003529	-0.004112	-0.004745	-0.005430	-0.006167
END 40 OF END SPANS	00	-0.001304	-0.001577	-0.001881	-0.002216	-0.002582	-0.002980	-0.003410	-0.003873
INNER 60 OF END SPANS	05	-0.002620	-0.003170	-0.003781	-0.004453	-0.005189	-0.005989	-0.006853	-0.007783
INNER 60 OF END SPANS	10	-0.002165	-0.002620	-0.003125	-0.003681	-0.004290	-0.004951	-0.005665	-0.006434
INNER 60 OF END SPANS	15	-0.001761	-0.002130	-0.002541	-0.002993	-0.003487	-0.004025	-0.004606	-0.005231
END 50 OF END SPANS	00	-0.001937	-0.002344	-0.002796	-0.003294	-0.003838	-0.004429	-0.005068	-0.005756
INNER 50 OF END SPANS	05	-0.002070	-0.002505	-0.002987	-0.003519	-0.004100	-0.004732	-0.005415	-0.006150
INNER 50 OF END SPANS	10	-0.001692	-0.002047	-0.002441	-0.002876	-0.003351	-0.003867	-0.004425	-0.005026
INNER 50 OF END SPANS	15	-0.001354	-0.001638	-0.001954	-0.002302	-0.002682	-0.003096	-0.003543	-0.004023
2-ND AND 4-TH SPANS	05	0.031717	0.032062	0.032386	0.032691	0.032978	0.033249	0.033506	0.033749
2-ND AND 4-TH SPANS	10	0.026709	0.026999	0.027272	0.027529	0.027771	0.027999	0.028216	0.028421
2-ND AND 4-TH SPANS	15	0.022072	0.022312	0.022537	0.022750	0.022950	0.023138	0.023317	0.023486
CENTER SPAN	05	0.045507	0.045421	0.045340	0.045264	0.045192	0.045124	0.045060	0.044999
CENTER SPAN	10	0.038322	0.038249	0.038181	0.038117	0.038057	0.037999	0.037945	0.037894
CENTER SPAN	15	0.031669	0.031609	0.031553	0.031500	0.031450	0.031402	0.031358	0.031315
UNIT MOMENTS ON THE ENDS		0.041935	0.043750	0.045454	0.047058	0.048571	0.049999	0.051351	0.052631
- ANTI-SYMMETRIC PRESTRESS -									
END 30 OF END SP.(ANTI-	00	-0.001326	-0.001602	-0.001908	-0.002245	-0.002613	-0.003012	-0.003443	-0.003906
INNER 70 OF END SP. -SYM)	05	-0.005361	-0.006477	-0.007715	-0.009077	-0.010564	-0.012179	-0.013922	-0.015795
INNER 70 OF END SP.	10	-0.004438	-0.005363	-0.006388	-0.007515	-0.008746	-0.010083	-0.011526	-0.013077
INNER 70 OF END SP.	15	-0.003615	-0.004368	-0.005203	-0.006121	-0.007124	-0.008213	-0.009388	-0.010651
END 40 OF END SP.	00	-0.002271	-0.002743	-0.003268	-0.003845	-0.004475	-0.005159	-0.005897	-0.006690
INNER 60 OF END SP.	05	-0.004563	-0.005513	-0.006567	-0.007726	-0.008991	-0.010365	-0.011849	-0.013443
INNER 60 OF END SP.	10	-0.003772	-0.004557	-0.005428	-0.006386	-0.007433	-0.008569	-0.009795	-0.011113
INNER 60 OF END SP.	15	-0.003066	-0.003705	-0.004413	-0.005192	-0.006043	-0.006967	-0.007964	-0.009035
END 50 OF END SP.	00	-0.003374	-0.004077	-0.004857	-0.005714	-0.006650	-0.007666	-0.008764	-0.009943
INNER 50 OF END SP.	05	-0.003605	-0.004356	-0.005189	-0.006105	-0.007105	-0.008191	-0.009363	-0.010623
INNER 50 OF END SP.	10	-0.002946	-0.003560	-0.004240	-0.004989	-0.005806	-0.006694	-0.007652	-0.008681
INNER 50 OF END SP.	15	-0.002359	-0.002850	-0.003395	-0.003995	-0.004649	-0.005360	-0.006127	-0.006951
2-ND AND 4-TH SPANS	05	0.055238	0.055760	0.056249	0.056709	0.057140	0.057547	0.057932	0.058295
2-ND AND 4-TH SPANS	10	0.046516	0.046956	0.047368	0.047755	0.048118	0.048461	0.048784	0.049090
2-ND AND 4-TH SPANS	15	0.038446	0.038810	0.039150	0.039470	0.039770	0.040053	0.040321	0.040574
UNIT MOMENTS ON THE ENDS		0.073033	0.076086	0.078947	0.081632	0.084158	0.086538	0.088785	0.090909
- - - APPLIED LOADS - - -									
UNIT DEAD LOAD ON 1-ST SPAN		0.006071	0.007340	0.008747	0.010295	0.011987	0.013824	0.015809	0.017942
UNIT DEAD LOAD ON 2-ND		-0.050851	-0.051358	-0.051834	-0.052280	-0.052701	-0.053098	-0.053473	-0.053827
UNIT DEAD LOAD ON 3-RD		-0.053225	-0.053125	-0.053030	-0.052941	-0.052857	-0.052777	-0.052702	-0.052631
UNIT DEAD LOAD ON 4-TH		0.013754	0.013858	0.013955	0.014045	0.014130	0.014209	0.014283	0.014354
UNIT DEAD LOAD ON 5-TH		-0.001642	-0.001980	-0.002354	-0.002765	-0.003213	-0.003699	-0.004222	-0.004784
UNIT DEAD LOAD ON ALL SPANS		-0.085893	-0.085265	-0.084517	-0.083647	-0.082655	-0.081541	-0.080305	-0.078947
UNIT D.L. ON SPANS 1,2 AND 4		-0.031024	-0.030159	-0.029131	-0.027939	-0.026584	-0.025064	-0.023380	-0.021531
UNIT D.L. ON SPANS 2,3 AND 5		-0.105719	-0.106464	-0.107219	-0.107987	-0.108772	-0.109575	-0.110398	-0.111244
UNIT D.L. ON SPANS 1,3 AND 5		-0.048796	-0.047765	-0.046638	-0.045411	-0.044083	-0.042652	-0.041116	-0.039473
UNIT D.L. ON SPANS 2 AND 4		-0.037096	-0.037500	-0.037878	-0.038235	-0.038571	-0.038888	-0.039189	-0.039473

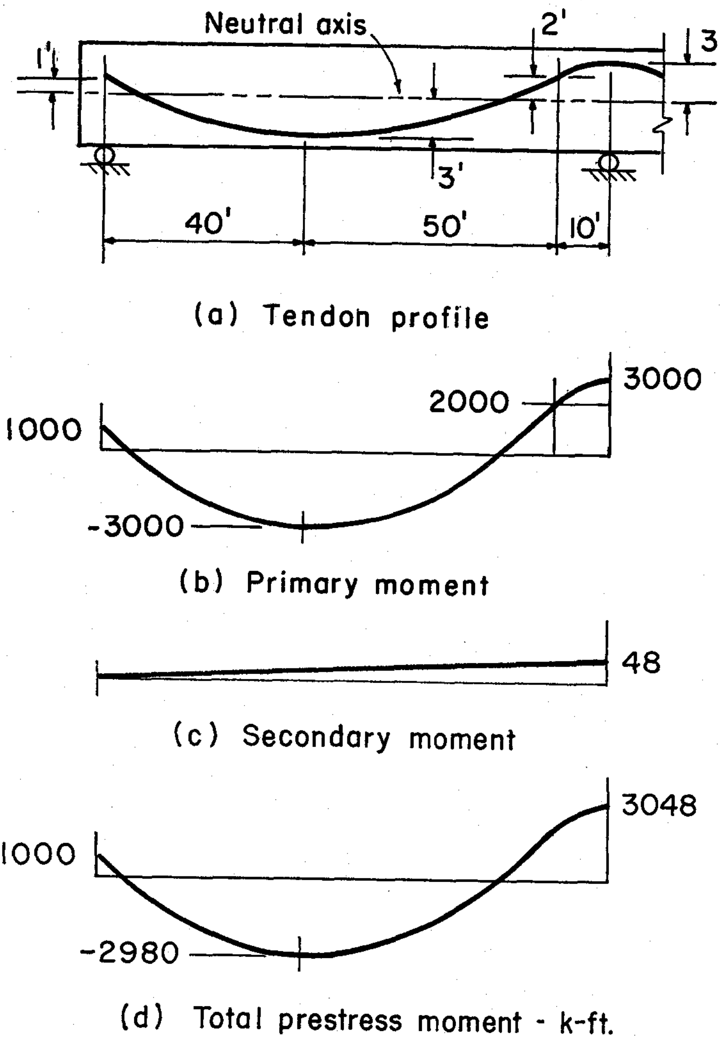


Fig. 6. Bending moments between supports for Example 4

(6) are shown in Fig. 6(b). The secondary moments at the ends are

$$M''_A = M_A - M'_A = 1000 \text{ k.-ft.} - 1000 \text{ k.-ft.} = 0 \text{ (free rotation)}$$

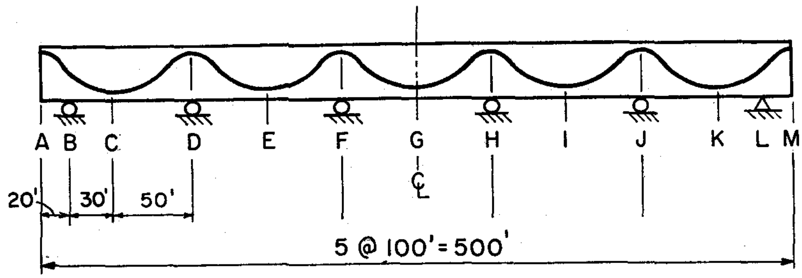
$$M''_B = M_B - M'_B = 3048 \text{ k.-ft.} - 3000 \text{ k.-ft.} = 48 \text{ k.-ft.}$$

and the secondary moment for the span as computed by Equation (7) is shown in Fig. 6(c). The sum of the primary and secondary moments gives the total moment as shown in Fig. 6(d). The moment at any point of this curve can be computed directly from Equation (8).

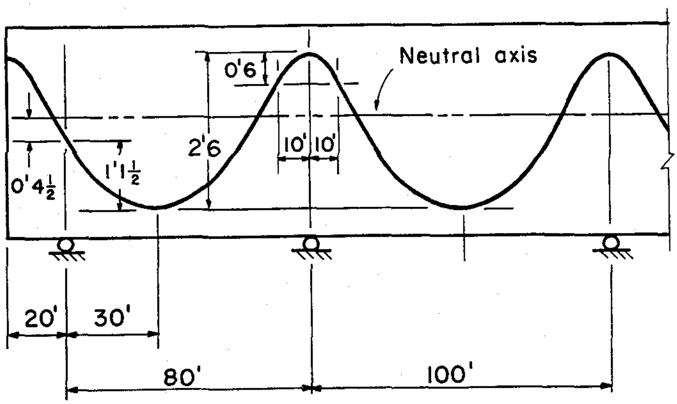
FRICTION LOSSES

The equivalent load due to prestressing, as given by Equations (1) to (5), is proportional to P, the horizontal component of the force in the prestressing tendons. However, P is not constant along the beam since it is reduced by friction losses along the tendons. A further variation of force is caused by anchor set as the load is transferred from the jacking device. Anchor set causes a reversal of friction forces in the end sections.

For short prestressing tendons the friction losses can usually be neglected provided the total angular change of the tendon profile is small. However, anchor set losses may be large. In this case both effects may be accommodated by using a re-



(a) Supports and notation



(b) Tendon geometry

Fig. 7. 5-span structure for Examples 5 and 6 (Note: practical application would call for tendon profile to be above neutral axis at supports B and L—examples illustrate computational technique only.)

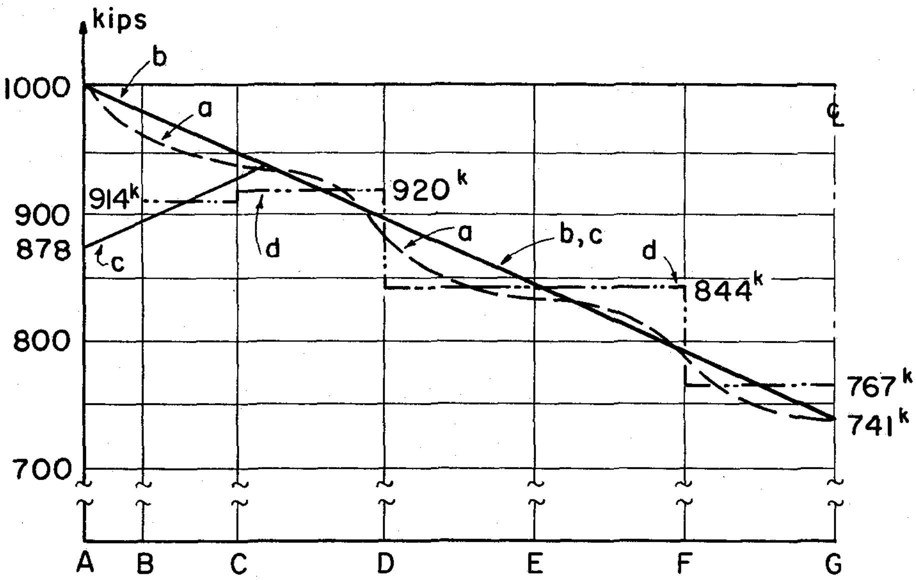


Fig. 8. Tendon force variation, jacking from both ends

duced constant value of P for the length of the beam.

For long post-tensioned tendons, the friction losses cannot be neglected in the final analysis. An ACI Building Code[8] formula gives the following value for P at any section x in the beam:

$$P_x = P_o e^{-(KL+\mu\alpha)} \quad (9)$$

If the value of $KL + \mu\alpha$ is below 0.3, in accordance with the ACI Code, Equation (9) may be replaced by

$$P_x = \frac{P_o}{1 + KL + \mu\alpha} \quad (10)$$

Equations (9) or (10) may also be used to compute friction losses through any segment of the beam[7] in which case the reference section is that end of the segment at which the tendon force P_o has already been computed. For reasonable accuracy in this case, Equation (10) should not be used if the value of $KL + \mu\alpha$ for the segment is greater than about 0.1.

The computed tendon force at various sections along the beam can now be plotted. If the slope of the tendon is large, the horizontal component can be computed by multiplying the tendon force by $(1 - \frac{1}{2}s^2)$ where s is the tendon slope. A linear approximation for the tendon force variation with distance along the beam is sufficiently accurate for most cases, and can be obtained by a straight line approximation of the plotted tendon force.

The loss of prestress force at the anchor section due to anchor set is

$$\Delta P_o = 2\sqrt{rAE\Delta L} \quad (11)$$

However, if the computed ΔP_o is greater than $2 \times (P_o - P_{min})$, where P_o is the jacking force and P_{min} is the lowest computed prestress force in the beam—either at the non-jacking end for post-tensioning from one end, or near the midpoint for post-tensioning from both ends—then

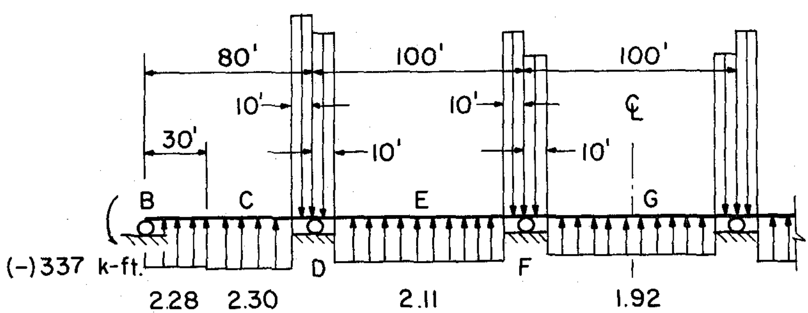
$$\Delta P_o = P_o - P_{min} + \frac{rAE\Delta L}{P_o - P_{min}} \quad (12)$$

This value will be greater than the ΔP_o computed by Equation (11). The prestress force plot can be revised to include the anchor set loss by noting that the friction losses will be reversed in the regions affected. The prestress force at the anchor will be $P_o - \Delta P_o$, and will increase with distance from the anchor at a rate of r.

Example 5

The moments over the supports in the 5-span beam shown in Fig. 7 will be computed. The jacking force is 1000 k., applied at both ends of the beam, and the friction parameters K and μ are 0.0002 and 0.25 respectively. The maximum slope of the tendon profile is 0.1 and occurs at the inflection points. The $KL + \mu\alpha$ friction loss factors are 0.027 for the 10-ft. reverse parabola sections and 0.033 for the 40-ft. parabola sections. The total loss factor from anchorage to beam centerline is 0.3 and so the tendon force at the centerline, as computed by Equation (9), is 741 k. Here the tendon force will be considered linear between the anchorage and the centerline as shown by curve b in Fig. 8. The actual tendon force along the beam, which can be computed by taking shorter segments, is plotted in Fig. 8 as curve a, but will not be used. The horizontal component of the tendon force will be taken as the force itself since the difference from a more exact calculation is at most 5 k.

For anchor set losses the anchor movement is 0.3 in., A is 5 in.[2], E is 29,000 ksi, and the rate of loss r is computed from curve b in Fig. 8 as 1.036 k./ft. From Equation (11), ΔP_o is 122 k. To accommodate this



Influence segment coefficients:

-.2353	.00980	.01745	.04235	-.01059	for support D
.0471	-.00197	-.00350	.02753	.03812	for support F

Fig. 9. Equivalent loads and coefficients from Example 5

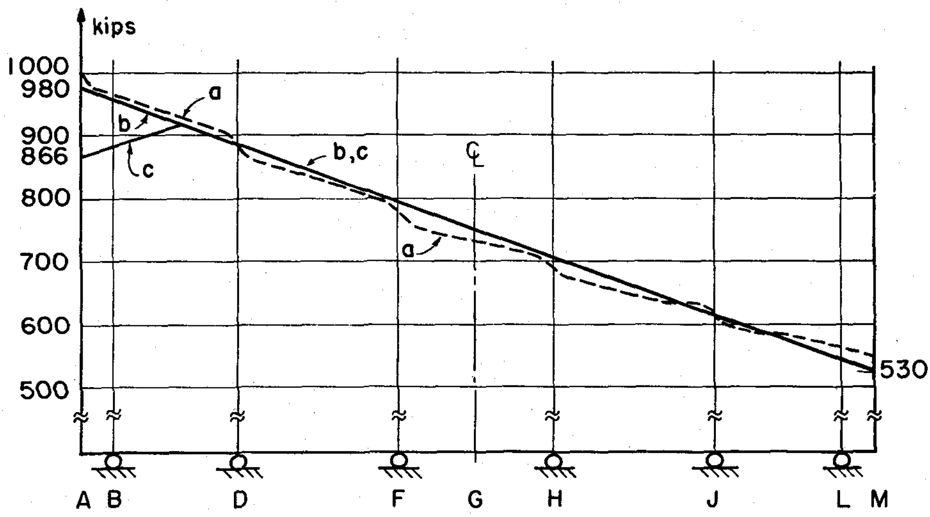


Fig. 10. Tendon force variation, jacking from one end

loss the friction forces will be reversed in the end 59 ft. as shown by curve c in Fig. 8. Finally, to use the moment influence coefficients, an average prestress force for each of the segments BC, CD, DF and FG is computed and the result is the step curve d plotted in Fig. 8.

The equivalent loads for these four segments are 2.28, 2.30, 2.11 and 1.92 k./ft. computed by equations (4), (5), (1) and (1), respectively. These loads and the equivalent end moment are shown in Fig. 9, together with the corresponding moment influence coefficients for computing the beam moments over the first and second interior supports. Summarizing the moments gives $M_D = 1394$ k.-ft. and $M_F = 1171$ k.-ft. Moments in the beam between supports can now be computed by Equation (8). Losses due to shrinkage and creep can also be accounted for by reducing all moments by appropriate loss percentages.

The accuracy of averaging the tendon forces in the segments has been studied. The difference in the computed moments using curve d in place of curve c in Fig. 8 is about 1.0 percent. Thus, this simplification does not appreciably affect the accuracy of the results.

Example 6

The beam considered in the previous example will now be re-analyzed for the case of prestressing from one end only. The geometry, jacking force, friction parameters, etc., remain the same. The prestress force is computed by applying Equation (9) to each of the parabolic segments between inflection points of the tendon profile and has been plotted as curve a in Fig. 10. Curve b is a linear approximation to be used in further computations, and gives a friction loss rate of 0.90 k./ft. The anchor set loss, based on this curve, is 114 k. and the resulting tendon force is curve c in Fig. 10. Fig. 11 shows the symmetric and the anti-symmetric portions of the tendon force and their average values over the required beam segments.

The moments over the first two interior supports due to the symmetric portion of the prestress can now be computed. The equivalent loads are the end moment and the distributed loads on the four segments

$$
\begin{aligned}
w_{BC} &= -268 \text{ k.-ft.}\\
M_E &= 1.83 \text{ k./ft.}\\
w_{CD} &= 1.88 \text{ k./ft.}\\
w_{DF} &= 1.89 \text{ k./ft.}\\
w_{FG} &= 1.89 \text{ k./ft.}
\end{aligned}
$$

The corresponding moment influence coefficients for the first two interior supports are those shown in Fig. 9. The resulting moments are:

$$
\begin{aligned}
M_{DS} &= 1170 \text{ k.-ft.}\\
M_{FS} &= 1126 \text{ k.-ft.}
\end{aligned}
$$

For the anti-symmetric portion the equivalent loads are:

$$
\begin{aligned}
M_E &= -63 \text{ k.-ft.}\\
w_{BC} &= 0.42 \text{ k./ft.}\\
w_{CD} &= 0.39 \text{ k./ft.}
\end{aligned}
$$

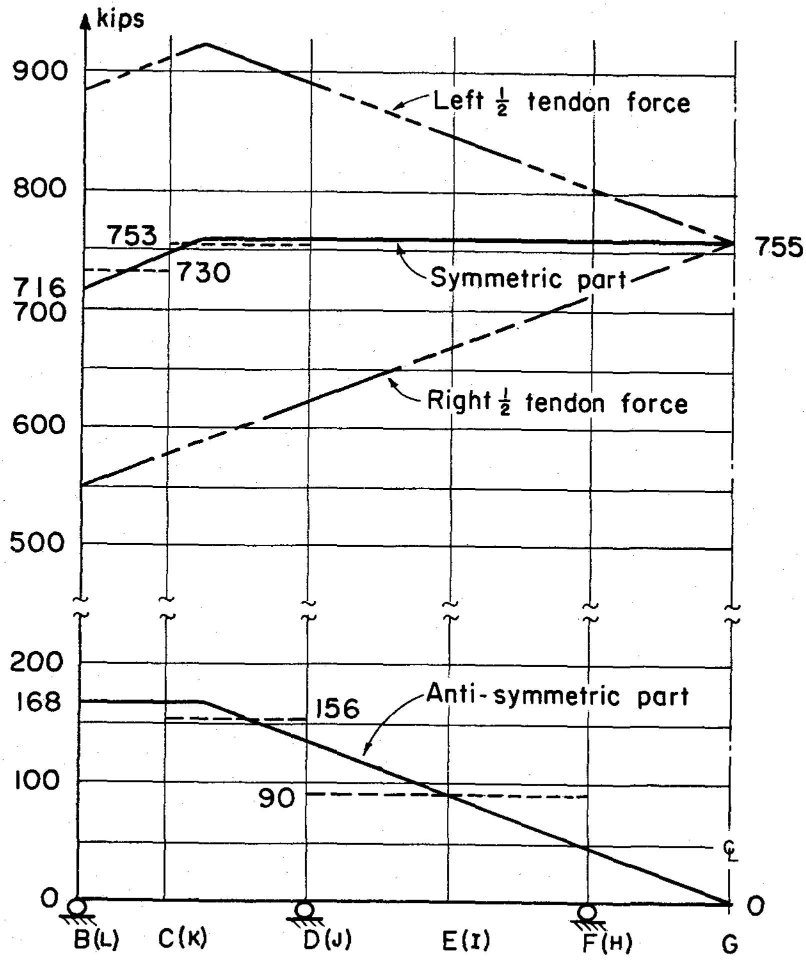


Fig. 11. Symmetric and anti-symmetric tendon forces for Example 6

Table VII. Comparison of prestressing moments (k.-ft.)

Point	Example 5(1)	Example 6(2)	Point	Example 5	Example 6
A	+ 878	+ 866	M	+ 878	+ 350
B	− 847	− 832	L	− 847	− 515
C	−1208	−1182	K	−1208	− 735
D	+1394	+1382	J	+1394	+ 958
E	− 830	− 827	I	− 830	− 653
F	+1171	+1191	H	+1171	+1061
G	− 734	− 761	G	− 734	− 761

1. Post-tensioning from both ends
2. Post-tensioning from one end only

$$w_{DF} = 0.23 \text{ k./ft.}$$
$$w_{FG} = 0$$

The corresponding moment influence coefficients at the first two interior supports are:

Support D	Support F
−0.02449	+0.08163
+0.01028	−0.00341
+0.01822	−0.00602
+0.03673	−0.04776
0	0

The resulting moments are:

$$M_{DA} = 212 \text{ k.-ft.}$$
$$M_{FA} = 65 \text{ k.-ft.}$$

Thus, the moments at the interior supports due to prestressing force in the tendons are:

$$M_D = 1382 \text{ k.-ft.}$$
$$M_F = 1191 \text{ k.-ft.}$$
$$M_H = 1061 \text{ k.-ft.}$$
$$M_J = 958 \text{ k.-ft.}$$

The total prestressing moments for the last two examples are summarized in Table VII. The reduction in prestressing moments caused by tensioning from one end only can be seen from this comparison.

NOTATION

A = cross-sectional area of the prestressing tendons

E = elastic modulus of the prestressing tendons

I_E, I_I = moment of inertia of the cross section in the span indicated

K = friction loss factor related to length

L = length of the segment over which friction loss is computed

L_E, L_I = length of the span indicated

ΔL = tendon movement at the anchor due to anchor set

M_E = end moment due to eccentricity of the tendon over the exterior support

M_x, M_A = bending moment at the point indicated

M' and M'' = primary and secondary bending moments respectively

P_o = jacking force

ΔP_o = loss of prestress force at the jacking end due to anchor set

P_{min} = lowest prestress force considering friction losses

P_x, P_A = horizontal component of the prestressing tendon force at the point indicated

a = ratio of the reverse curve length to the span length

b = ratio of the end segment length to the span length in an exterior span

c = drape of the tendon profile, high point to low point

d = drape of the tendon profile in the end segment of an exterior span

e = eccentricity of the tendon profile above the neutral axis at the exterior support

e_x, e_A, etc. = eccentricity of the tendon profile above the neutral axis at the point indicated

r = loss of prestress force per unit length of beam

s = slope of the tendon profile

w = equivalent upward distributed load over the major segment of the tendon profile

w_E = equivalent upward distributed load in the end segment of an exterior span

w_R = equivalent downward distributed load over the reverse curvature segment of the tendon profile

x' = distance from the end of a span to a point x

α = angular change of the tendon profile in the segment over which friction loss is computed

μ = friction loss factor related to angular change of the tendon profile

Subscripts:

A and S designate anti-symmetric and symmetric, respectively.

x, A, B, C, etc. designate points along the beam.

L, R, E, I and *C* designate left, right, exterior, interior and center spans, respectively.

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Discussion of this paper is invited. Please forward your comments to PCI Headquarters by May 1 to permit publication in the May-June 1972 issue of the PCI JOURNAL.