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Strut-and-tie Models

How to design concrete members using strut-and-tie models in accordance with Eurocode 2

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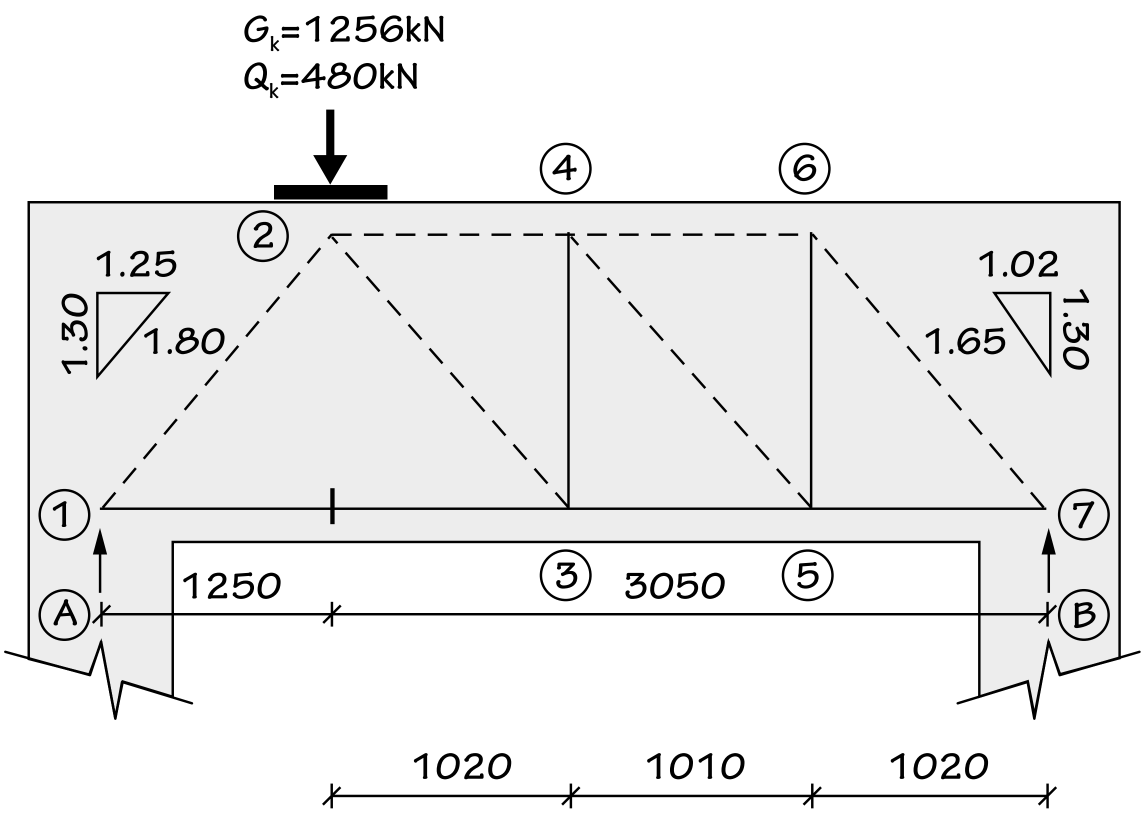

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Strut-and-tie Models

Contents

Introduction

This publication aims to explain strut-and-tie modelling (STM) to new users. It concentrates mainly on the theory but is followed by worked examples of some of the most popular applications. The real benefit of STM comes in the design and analysis of complex elements and structures and some examples are given to show the potential of the method - potential to rival finite element analysis and design.

STM

STM is a simple method which effectively expresses complex stress patterns as triangulated models. STM is based on truss analogy and can be applied to many elements of concrete structures. It is usually adopted to design non-standard elements or parts of elements of concrete structures such as pile caps, corbels, deep beams (where depth > span/3), beams with holes, connections, etc. where normal beam theory does not necessarily apply.

STM is a powerful engineering tool where the engineer stays in control. With a reasonable amount of experience, it can help design engineers provide simple engineering solutions to complex structural problems.

STM is a lower bound plastic theory which means it is safe providing that:

- Equilibrium is satisfied.
- The structure has adequate ductility for the assumed struts and ties to develop.
- Struts and ties are proportioned to resist their design forces.

Possibly due to the lack of applicable design standards, STM was not popular in the UK and its use was generally limited. However, Eurocode 2 now includes STM, allowing and perhaps encouraging its more widespread use. Even so, there is little simple guidance within Eurocode 2 or indeed elsewhere. The intention of this publication is therefore to give guidance and impart understanding of the method.

The STM design process

The design process for strut-and-tie models can be summarised into four main stages:

- Define and isolate B- and D-regions (see Figure 1.1).
- Develop a STM - a truss system to represent the stress flow through the D-region and calculate the member forces in the truss.
- Design the members of the STM - dimension and design the truss members to resist the design forces.
- Iterate to optimise the STM as necessary to minimise strain energy.

These four steps are explained in the first four sections of this publication and are then followed by examples of design. The overall process is shown by the flow chart in Chapter 7. A very simple example is shown opposite in Panel i.

Key

Within the main text, references to Eurocode 2 EN 1992-1-1[6] and other relevant texts are shown in blue arrowheads. Within the calculations references are given in the margin.

Panel i
Strut-and-tie design of a two-pile cap

Determine the amount of tension reinforcement required for a two-pile cap supporting a 500 mm square column carrying 2500 kN (ULS).

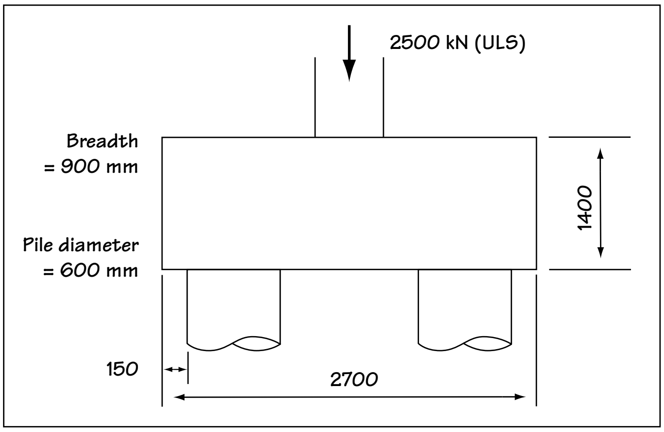


Figure i
Section

1) The whole pile cap consists of D regions. So STM is appropriate.

2) A relevant STM is easy to construct:

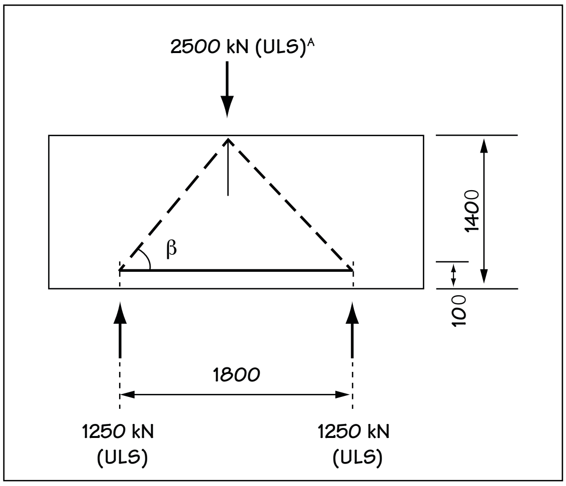


Figure ii
STM

Strut and tie forces are calculated:

Angle of strut, $\beta = \tan^{-1}(1300/900) = 55.3°$

Force per strut $= 1250/\sin 55.3° = 1520$ kN

Force in tie $= 1250 \cot 55.3° = 866$ kN

3) Design members — The area of steel in the tie:

$A_{s,reqd} \geq 866 \times 10^3/(500/1.15) \geq 1991$ mm^2

So use say 5 H25s (2455 mm^2)[B,C]

4) Iteration — This might include optimising the depth of the pile cap.

Notes:

A For clarity, the self-weight of the pile cap assumed to be included.

B Although not usually critical for pile caps in a structural grade of concrete, in a full final design the stresses around the nodes and the capacity of the struts should be checked. See Section 5.1.

C Some attention should also be given to reinforcement details, particularly anchorage which, when using strut and tie, is different to that using beam theory. See Section 5.1.

1. B- and D-regions

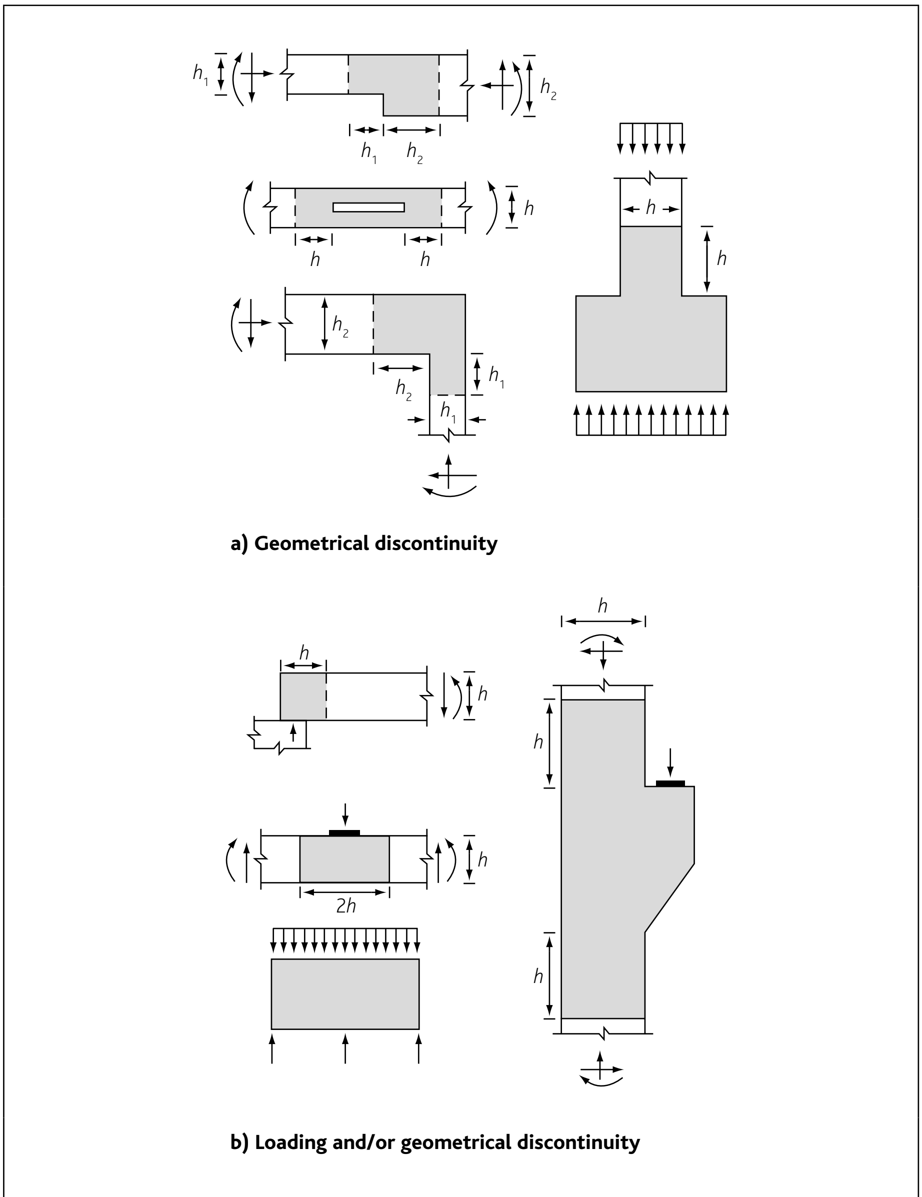
A structure can be divided into:
B (or beam or Bernoulli) regions in which plane sections remain plane and design is based on 'normal' beam theory. While Eurocode 2 allows strut-and-tie models (STM) to be used in B-regions, it is unusual to do so.

D (or discontinuity or disturbed) regions in which plane sections do not remain plane; so 'normal' beam theory may be considered inappropriate. D-regions arise as a result of discontinuities in loading or geometry and can be designed using STMs. Typical examples of D-regions include connections between beams and columns, corbels, openings in beams, deep beams and pile caps, etc. As illustrated in Figure 1.1 discontinuity regions are assumed to extend a depth or width from the discontinuity.

Figure 1.1
D-regions in structures[1]

Key

= D region



2. Developing a strut-and-tie model

2.1 STMs

Strut-and-tie models (STM) are trusses consisting of struts, ties and nodes. Figure 2.1a shows a STM for a simply supported deep beam loaded with a point load at mid-span. This is usually drawn as an idealised model as shown in Figure 2.1b where, conventionally, struts are drawn as dashed lines and ties as full lines. Either nodes or struts and ties may be numbered.

For more complex structures, the loadpath method of Schlaich and Schafer[2] or finite element analysis is useful for identifying the flow of forces. For example, see the wall loaded with a point load at its edge in Figure 2.2.

In recognition of concrete's limited ductility it is best to align struts and ties with un-cracked elastic analysis.

Figure 2.1
Strut-and-tie model for a simple deep beam

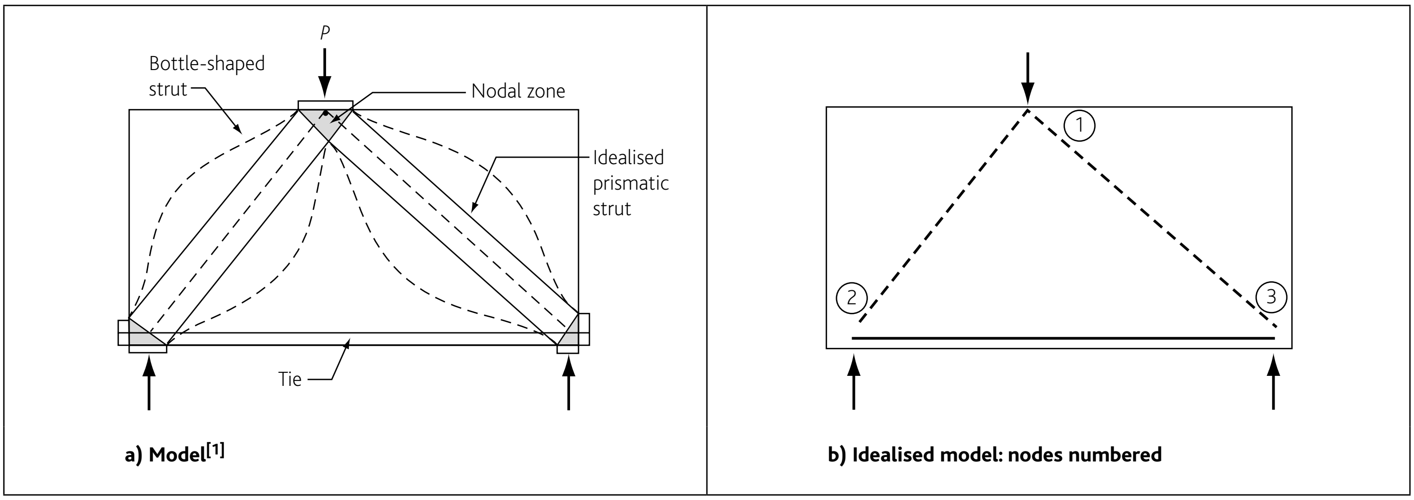
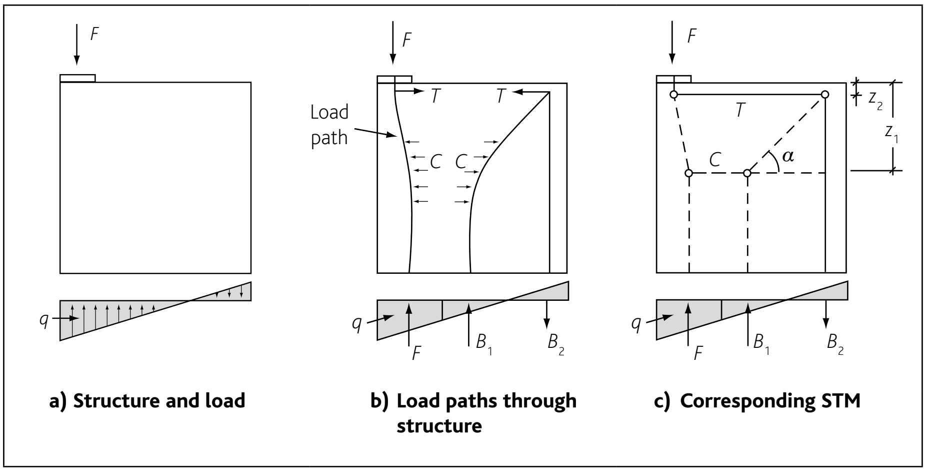


Figure 2.2
Load path method for a wall



Notes: The forces F, B_1 and B_2 are derived from the contributory areas of stress and they act through the centre of gravity of those areas.

The vertical ordinate of the horizontal strut C in Figure 2.2c can be found by either assuming the angle α is 45° or greater or alternatively by performing an elastic finite element analysis to determine the centre of gravity of the compressive stress field.

The first step in developing an STM is to draw stress paths which show the elastic flow of forces that transfer the load through the structure without crossing each other. The stress paths are replaced with polygons of forces in the STM with additional struts and ties provided as required for equilibrium. Struts should be oriented along the mean directions of principal compressive stresses but the reinforcement can generally be oriented parallel and perpendicular to the edges of the member. Tie centrelines should allow for sufficient cover and for the possibility of multiple layers of reinforcement.

The next step is to then calculate the idealised forces in the struts and ties. In simple cases this is done by using elementary trigonometry. Initially the struts and ties may be sized using rudimentary analysis and minimum allowable stresses. Iteration of the STM may prove necessary at a later stage.

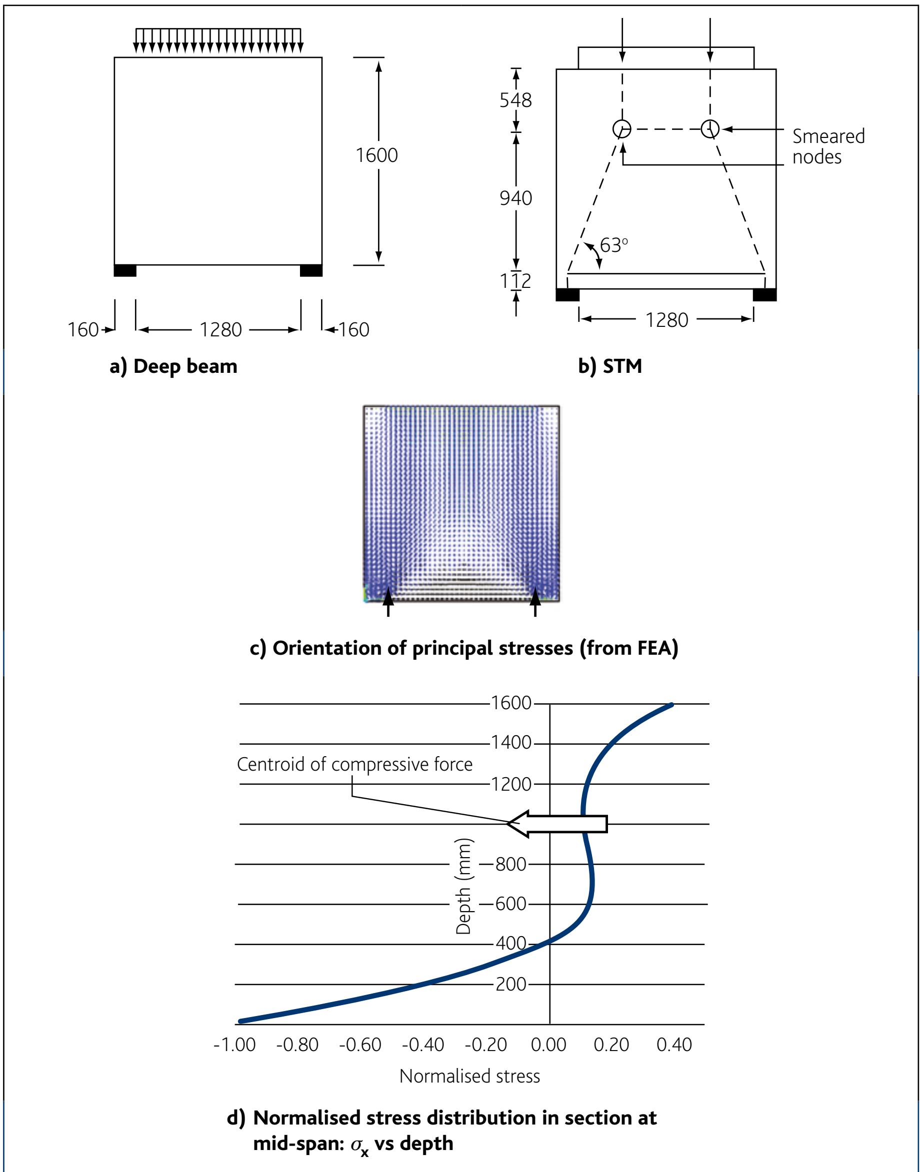
Figure 2.3 shows how elastic finite element analysis can be used to refine an STM for a deep beam. (It also illustrates that, compared to STM, it can be difficult to determine the distribution of reinforcement using elastic finite element analysis (FEA)).

Figure 2.3
Construction of STM for deep beam using load path method

Key

Direction and magnitude of compressive stress

Direction and magnitude of tensile stress



a) Deep beam

b) STM

c) Orientation of principal stresses (from FEA)

d) Normalised stress distribution in section at mid-span: σ_x vs depth

2.2 Choice of STM

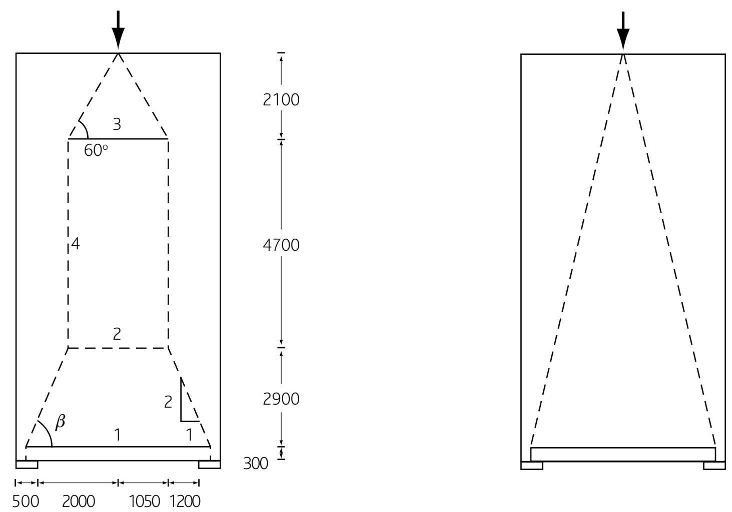
It is usually possible to develop a number of possible alternative STMs for a particular loading arrangement and doubts can arise over the best choice of model.

The orientation of the STM can be predicted with an elastic finite element analysis of the element before the concrete cracks. The orientation of the struts changes after cracking due to the change in stiffness, which occurs as the ties are activated. The orientation of the STM remains reasonably constant after cracking until the reinforcement yields, after which a further reorientation occurs as the loads increase to failure.

In many cases, acceptable STMs can be generated using a simple 2:1 dispersion rule. This is illustrated by Figure 2.4a (which gives similar results to the elastic finite element procedure illustrated in Figure 2.5c). The STM in Figure 2.4a is appropriate prior to the yielding of tie 1. Subsequent to the tie yielding, the angle β increases as the load is increased with the geometry of the STM approaching that shown in Figure 2.4b at failure.

Theoretically, STMs should be developed at the serviceability limit states (SLS) and ultimate limit states (ULS). In practice, it is usually sufficient to design the structure at the ULS using a STM that is acceptable at the SLS, such as that in Figure 2.4a. The STM in Figure 2.4b is unsuitable at the SLS since it can only develop once tie 1 has yielded (and beyond the realms of elastic finite element analysis). Therefore, crack widths would be excessive at the SLS if the reinforcement was designed using the STM shown in Figure 2.4b.

Figure 2.4
Use of 2:1 dispersion rule to distinguish between good and bad STM at the SLS



a) Good model
(akin to elastic distribution of stresses at SLS)

b) Bad model
(akin to distribution of stresses at ULS)

The 2:1 dispersion rule illustrated in Figure 2.4a, is a useful way of rejecting poorly conditioned STMs, as illustrated in Figure 2.4b. Another way of assessing that the STM in Figure 2.4b is poorly conditioned is to note that the deep beam comprises of two adjoining D-regions (top and bottom) which should each be designed individually.

As is shown in Figure 2.5 the aspect ratio of deep beams has little effect on the elastic stress distribution at the top and bottom of the beam. Model Code 90[3] and ACI 318[1] give some advice on the conditioning of STMs. Section 3.4.3 gives guidance on tie depths and lever arms.

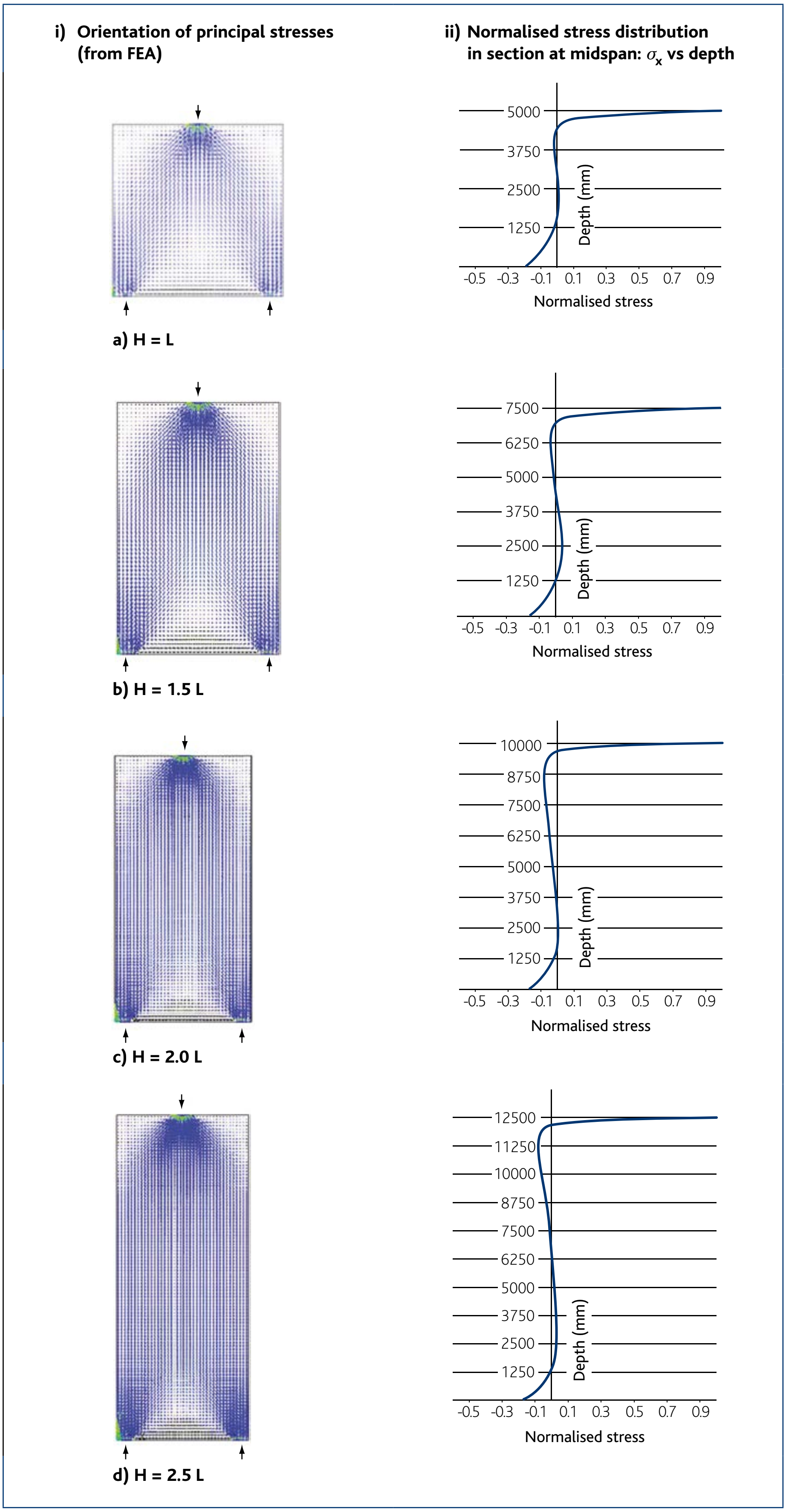
Figure 2.5

Influence of beam aspect ratio on elastic stress distribution for the same span and load

Key

Direction and magnitude of compressive stress

Direction and magnitude of tensile stress



2.3 Optimisation of STM

STM arrangements based on elastic stress fields are frequently, but not always, appropriate as they do not necessarily recognise the redistribution in stress that occurs on cracking. The best model is that which requires the least strain energy. This can be achieved by minimising the strain energy.

Strain energy = $\Sigma F_i l_i \varepsilon_{mi}$
where

F_i is the force in the i^{th} strut or tie,

l_i is the length of i^{th} member,

ε_{mi} is the mean strain in the i^{th} member.

More simply the best model usually has the shortest length of unyielded ties*.

The angle between the struts and ties should be large enough to avoid strain incompatibilities, i.e. large enough to avoid ties extending and struts shortening in almost the same direction. The minimum angle between struts and ties should not be taken as less than 35°.

It is important to remember that the strut-and-tie method is based on the lower bound theorem of plasticity and is only valid if the structure has adequate ductility for the assumed truss mechanism to develop. In line with Eurocode 2, ductility may be deemed to be satisfied through the use of Class B or C reinforcement. It is assumed that concrete has adequate ductility.

It should also be noted that STMs are kinematic, in other words separate models need to be developed for each loading arrangement.

* Unyielded ties are those where $A_{s,prov} > A_{s,req'd}$

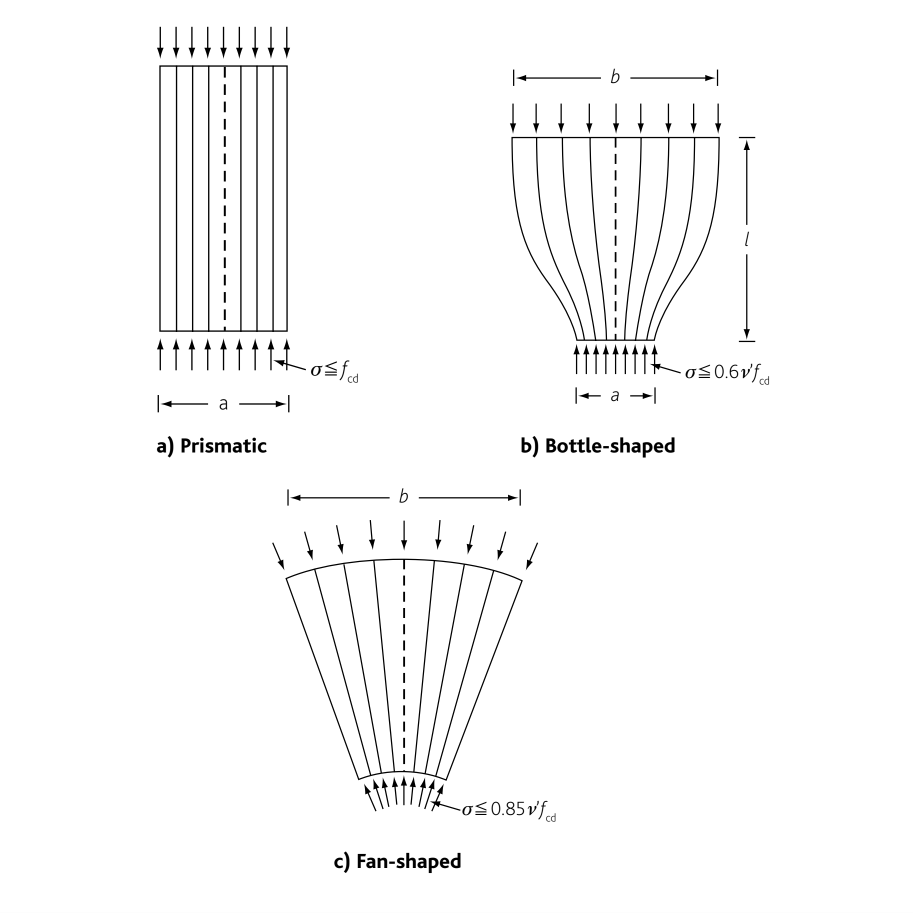
** This restriction should not be applied to a) a series of parallel struts (e.g. in the web of a slender beam) where cot $\theta \leq 2.5$ nor *b*) where the strength of the strut is related to its angle of inclination as in the Canadian Code CSA[4] A.23.3-04 which is based on the Modified Compression Field Theory of Collins et al[5]. See Section 4.1.3.

3. Design of STM members

3.1 Struts

Struts are categorised as having prismatic, bottle- or fan-shaped stress fields. Figure 3.1 shows these types of strut and their respective compressive stress fields and allowable stresses, σ_{Rd}. Prismatic stress fields typically arise in B-regions. Fan- and bottle-shaped stress fields arise in D-regions due to the dispersion of the stress paths radiating out from concentrated loads or reactions.

Figure 3.1
Types of strut.
Showing compressive stress fields and allowable stress, σ_{Rd}.



3.1.1 Axial strength of prismatic struts

Eurocode 2 defines the design concrete strength of a strut with no tensile transverse stress as f_{cd} and therefore the capacity of the strut is

$F_{Rd} = f_{cd}ta$ Exp (6.55)[6]

where

t = thickness of the element

a = width of the strut

3.1.2 Axial strength of unreinforced bottle-shaped struts

Any transverse tension reduces the compressive strength of a concrete strut to $0.6\nu' f_{cd}$.

This is the case in bottle-shaped stress fields, where transverse tensile stresses occur a distance away from the end nodes as compressive stresses change direction. Thus the compressive capacity of a bottle-shaped strut without transverse reinforcement equals:

$F_{Rd} = 0.6\nu' f_{cd} ta$ Exp (6.56)[6]

where

$\nu' = 1 - f_{ck}/250$ Exp (6.57)[6]

$f_{cd} = \alpha_{cc} f_{ck}/\gamma_c$ Exp (3.15)

where

$\alpha_{cc} = 0.85$* 3.1.6 (1) & NA

$\gamma_c = 1.5$ Table 2.1N

t = thickness of the element.

a = width of the strut (see Figures 3.2 and 3.4).

In terms of strength, a bottle-shaped strut might be considered as a relatively weak idealised prismatic strut between nodes (see Figure 2.1a). However, transverse tensile forces and stresses must be checked and where necessary, designed reinforcement must be provided (as outlined below). It should be noted that the the area (ta) and shape of a strut may be different each end of a strut; both ends may need to be checked.

3.1.3 Reinforcing bottle-shaped struts

The strength of bottle-shaped struts can be increased by the provision of transverse reinforcement which controls the transverse tensile strain in the strut**. Once adequately reinforced, the strength of the strut will then be governed by bearing stresses at the nodes (see Section 3.1.4).

Where the capacity of a strut is required to increase from $0.6v'f_{cd}$ to a maximum of $1.0v'f_{cd}$ transverse reinforcement is required. Eurocode 2 uses Expressions (6.58) and (6.59) to calculate the tensile force and hence the area of transverse reinforcement required to strengthen bottle-shaped struts which are designed as having either partial or full discontinuity as below.

3.1.3.1 Tensile force in cases of partial discontinuity ($b \leq H/2$)

Consider one of the D-regions in the strut shown in Figure 3.2 and the idealised forces on one side of it as shown in Figure 3.3d.
Moment equilibrium about point 'O' gives:

$0.5\text{F}(b-a)/4 = 0.5bT$

$T = 0.25(1-a/b)\text{F}$

$T = \text{F}(b-a)/4b$ Exp (6.58)[6]

where

T = tensile force

F = force in strut

b = available strut width

a = node width

* The UK National Annex[6a] states that α_{cc} = 0.85 for flexure and axial loading and 1.00 for other phenomena or may conservatively be taken as 0.85 for all phenomena. α_{cc} = 0.85 is used in this document but gives rise to some inconsistencies: it is consistent within the STM rules but not with shear in beams. Some sources adopt 1.00[7].

** Axially reinforced struts are feasible but are beyond the scope of Eurocode 2 and this publication.

Figure 3.2
Strut with partial discontinuity: design parameters

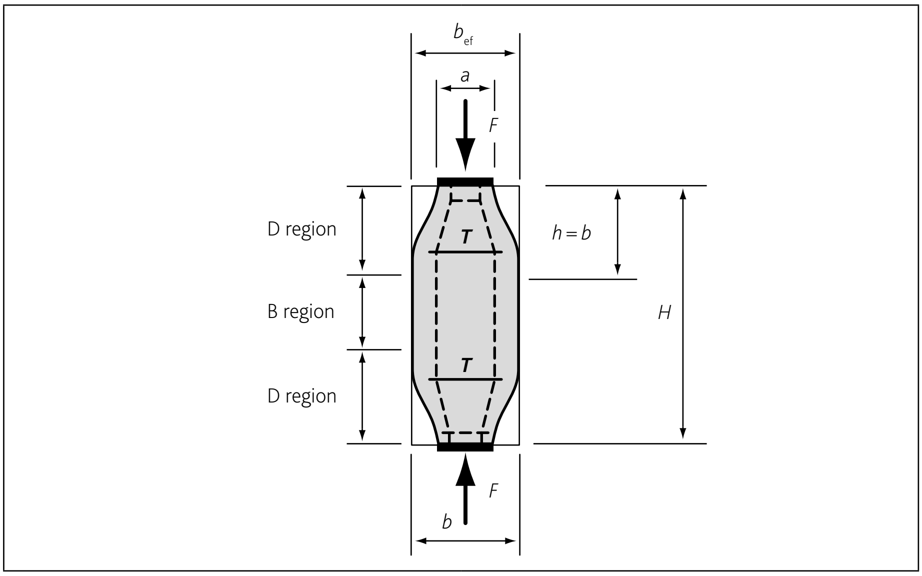
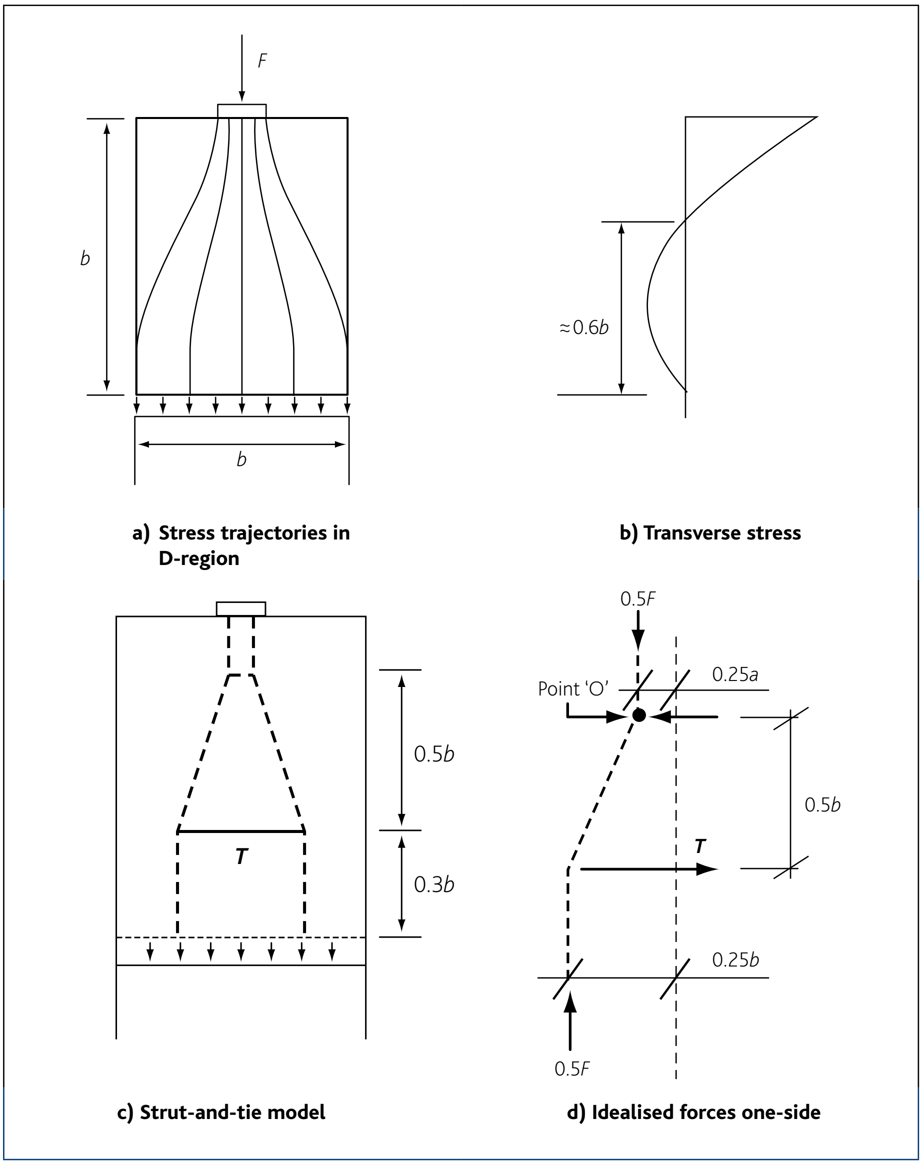


Figure 3.3
D-region in strut with partial discontinuity



a) Stress trajectories in D-region

b) Transverse stress

c) Strut-and-tie model

d) Idealised forces one-side

3.1.3.2 Tensile force in cases of full discontinuity ($b > H/2$)

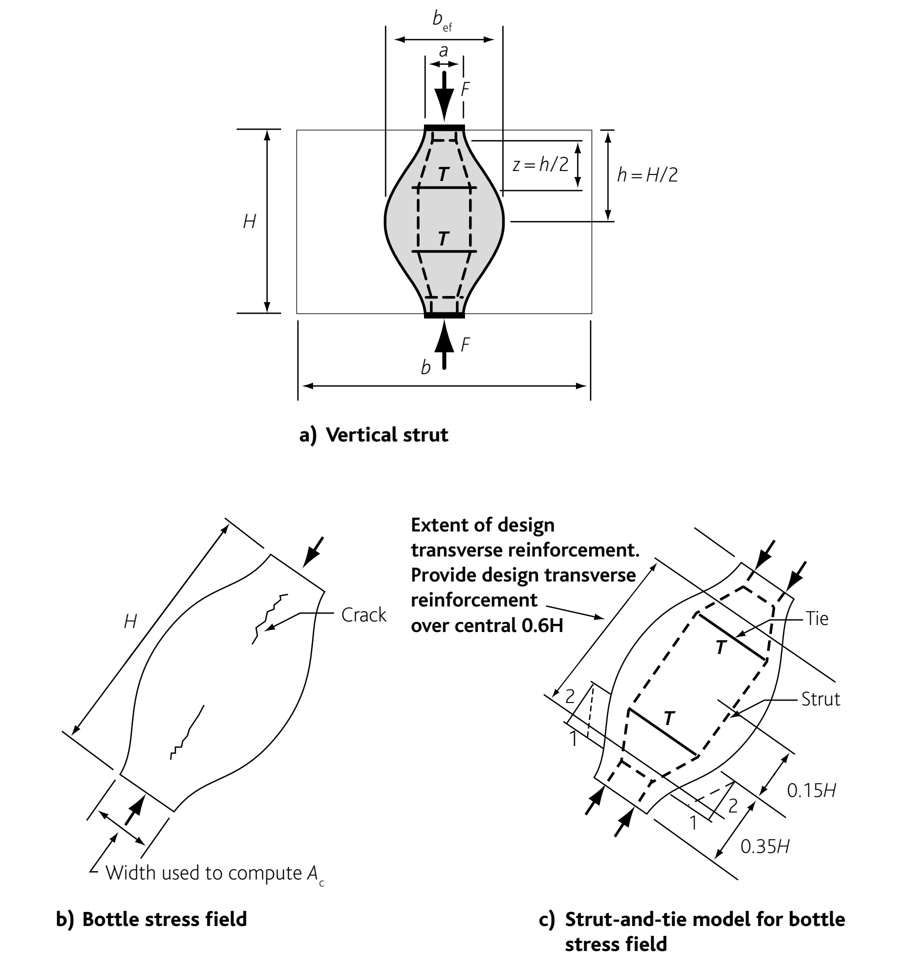
Similarly for the full discontinuity strut shown in Figure 3.4:

$T = F\,[1.0 - 0.7a\,/H)]\,/\,4$* Exp (6.59)

where

T = tensile force in each tie
F = force in strut
a = node width
H = length of strut

Figure 3.4
Full discontinuity (struts in wide elements)



3.1.3.3 Check bottle stress fields

In the case of pure bottle stress fields as illustrated in Figure 3.4 (but not fan stress fields in deep beams etc., as illustrated in Figure 3.7), transverse splitting occurs and transverse (or bursting) reinforcement is required if:

$T \geq 0.3t\,H\,f_{ctd}$

where

H = length of the strut (0.3 H = effective length of the tensile zone)
t = thickness
$f_{ctd} = \alpha_{ct}\,f_{ctk}\,/\,\gamma_c$ Exp (3.16)

* This representation of Exp (6.59) corrects a misprint in BS EN 1992-1-1 that was recognised in 2010 (Should have read 'H' not 'h'[21]).

where

α_{ct} = 1.0* 3.1.6(2) & NA

f_{ctk} = $0.7f_{ctm} = 0.21f_{ck}^{2/3}$ for $f_{ck} \leq 50$MPa Table 3.1

γ_c = 1.5

3.1.3.4 Transverse (bursting) reinforcement

Where bursting reinforcement is required, it should be provided to satisfy:

$$T = \Sigma A_{si} f_{yd} \sin \alpha_i$$

where

A_{si} = area of reinforcement in the i^{th} direction, mm^2

f_{yd} = design strength of reinforcement

= f_{yk}/γ_s

α_i = the angle the reinforcement makes to the axis of the strut.

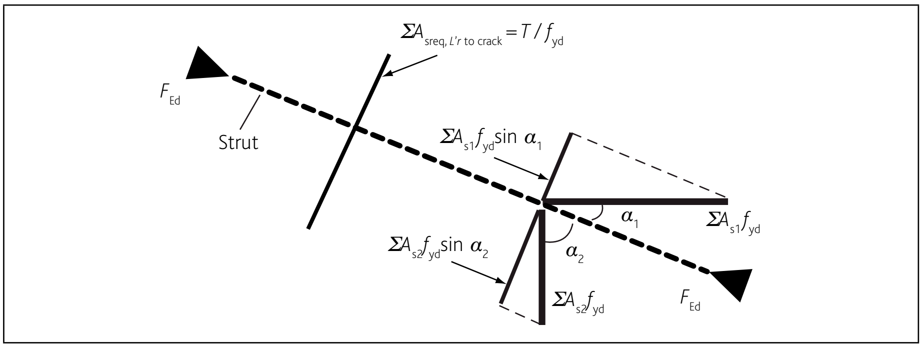
As illustrated by Figure 3.5, the reinforcement should be placed in either:

- Two orthogonal layers at angles α_1 and α_2 to the axis of the strut

or

- In one direction at an angle α_1 to the axis of the strut where $\alpha_1 \geq 40^o$.

Figure 3.5
Bursting reinforcement in two orthogonal layers, A_{s1} and A_{s2}



3.1.3.5 Orthogonal transverse reinforcement

It should be noted that where A_s is provided as orthogonal reinforcement (e.g. horizontal and vertical which is measured in terms of mm^2/m) then an additional sin γ_l needs to be considered in the trigonometry of both the area of steel and its spacing[8]. This means that in terms of mm^2/m both the vertical and the horizontal reinforcement should be numerically equal to the reinforcement required perpendicular to the strut (and potential crack).

Consider Figure 3.6 and let area of reinforcement required perpendicular to the crack = $A_{sreq,\ L'r\ to\ crack}$

where

$$\Sigma A_{sreq,\ L'r\ to\ crack} = T / f_{yd}$$

Provide vertical reinforcement say A_{sv}/s_v

Contribution of A_{sv}/s_v to $\Sigma A_{sreq,\ L'r\ to\ crack} = A_{sv} \sin \alpha_v / (s_v / \sin \alpha_v)$

$= \sin^2 \alpha_v \ A_{sv} / s_v$

* Where unreinforced, it may be prudent to adopt $\alpha_{ct,pl} = 0.8$ (see EN 1992-1-1 Cl 12.3.1)

where

A_{sv} = area of reinforcing bar in the vertical direction, mm^2

s_v = spacing of A_{sv}, mm

α_v = the angle the vertical reinforcement makes to the axis of the strut.

It will be noted that:

$A_{sreq,\,L'r\text{ to crack}}/s_{v\text{ along crack}} = \Sigma sin^2\alpha_i A_{si}/s_i$

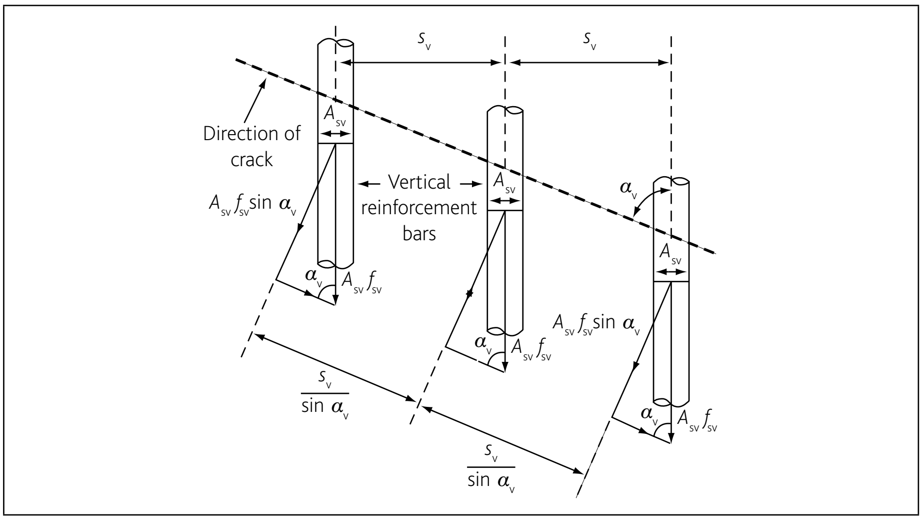
So, in the case of the same horizontal and vertical reinforcement.

$$\Sigma sin^2\alpha_i A_{si}/s_i = sin^2\alpha_v A_{sv}/s_v + cos^2\alpha_v A_{sh}/s_h = A_{sv}/s_v = A_{sh}/s_h$$

Thus, for equal horizontal and vertical reinforcement:

$A_{sv}/s_v = A_{sh}/s_h = A_{sreq'\,L'r\text{ to crack}}/s_{v\text{ along crack}}$

Figure 3.6
Trigonometry of vertical bars contribution to required reinforcement



3.1.3.6 Placement of bursting reinforcement.

The bursting reinforcement should be smeared between 0.4h and h from each loaded surface: for full discontinuity, this equates to 2A_{si} being provided in the middle 0.6H as shown in Figure 3.4c, where the transverse tension exists.*

3.1.4 Strength of struts: bearing (at nodes)

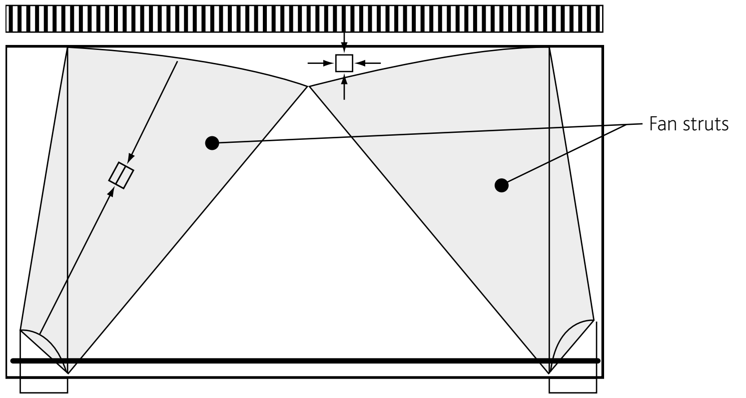
Where a bottle-shaped strut is reinforced for tensile stresses, the maximum possible strut force is then limited by the design concrete strength in bearing at each end (i.e. in bearing at the interface with the node). The allowable compressive stress at a node depends on which type it is. Types of node and their respective allowable design stresses are described in Section 3.3.

* Eurocode 2 does not give any guidance as to where the tensile reinforcement should be placed. The Designer's Guide to EN 1992-2[7] recommends that it should be placed in the central 0.6H. Schlaich and Shafer[2] indicate 0.8H. Nonetheless, the central 0.6H is recommended. However, a factor of 0.8 may be justified where this level of bursting reinforcement is provided uniformly throughout the length of the strut.

3.1.5 Fan-shaped struts

Fan-shaped stress fields typically arise at supports of deep beams supporting uniformly distributed load, as shown in Figure 3.7. The flow of internal forces in the uniformly loaded deep beam may be visualized either by strut-and-tie action or by more elaborate discontinuous stress fields.

Figure 3.7
Stress field in uniformly loaded deep beam at ULS

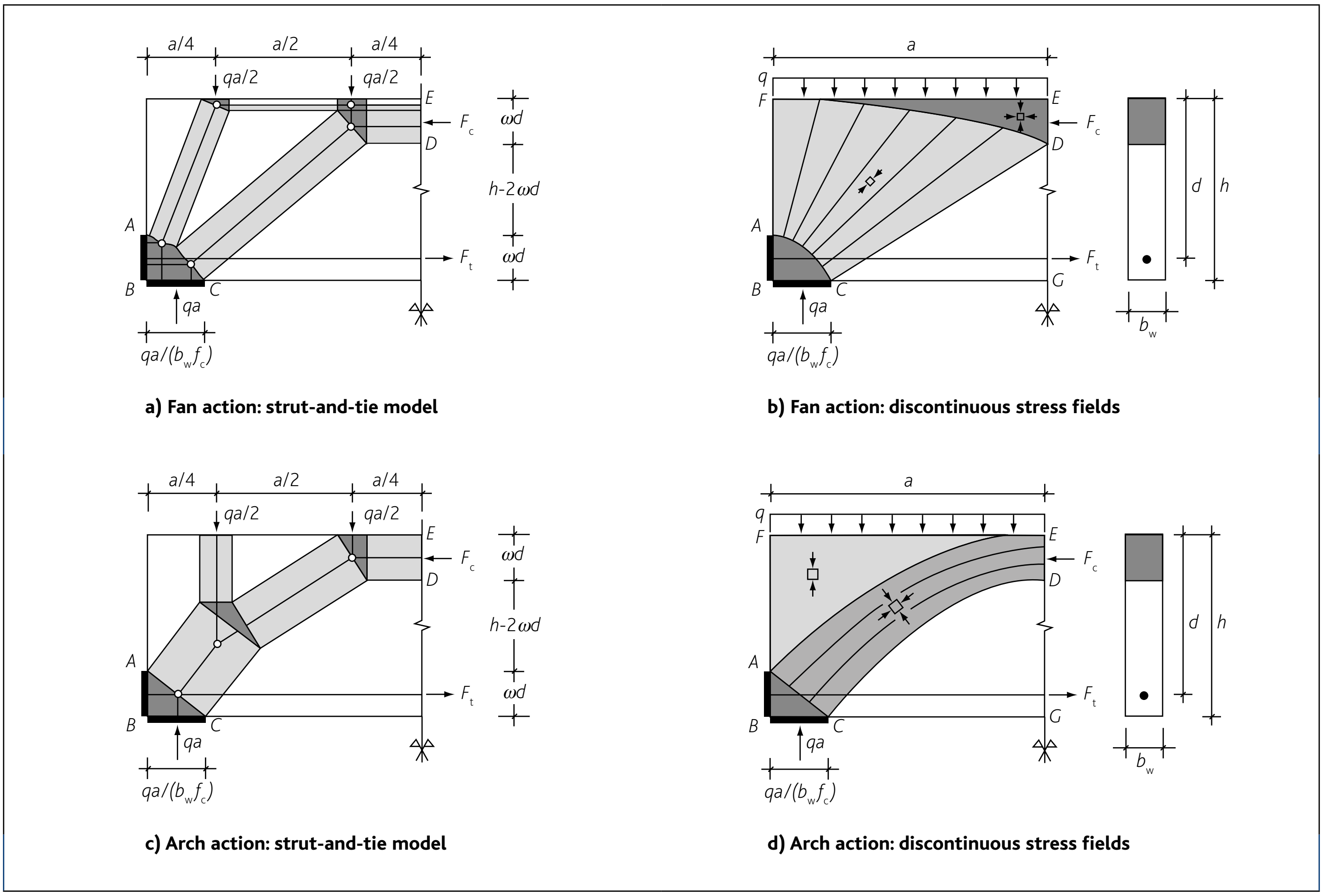


In Figure 3.8a and 3.8c the distributed load q is replaced by two statically equivalent single loads $qa/2$ which are transferred to the supports by struts, which are balanced by the support reactions and the tie force. The transition to the fan-shaped stress field shown in Figure 3.8b is achieved by subdividing the span into differential elements δa and considering infinitely thin struts carrying loads δq whose ends are bounded by the nodal zone ABC and the compression zone DEF. The fan-shaped stress field is based on the assumption that the principal transverse tensile stress in the concrete is zero.

In a similar manner to the formation of fan-shaped struts, the arch strut shown in Figure 3.8d is achieved by considering the STM in Figure 3.8c and subdividing the span into differential elements δx and considering infinitely thin struts carrying loads $q\delta x$ whose ends are bounded by the compression zone AEDC. All the stress fields shown in Figure 3.8 are statically equivalent; those in Figures 3.8b and 3.8d being most realistic.

The design strength of the concrete in the struts at the bottom CCT node (see Figure 3.10b) is $0.85\nu' f_{cd} t$. Since no transverse reinforcement is provided, a direct load transfer to the supports is required.

Figure 3.8
Uniformly loaded deep beam without transverse reinforcement[9]



a) Fan action: strut-and-tie model

b) Fan action: discontinuous stress fields

c) Arch action: strut-and-tie model

d) Arch action: discontinuous stress fields

3.2 Ties

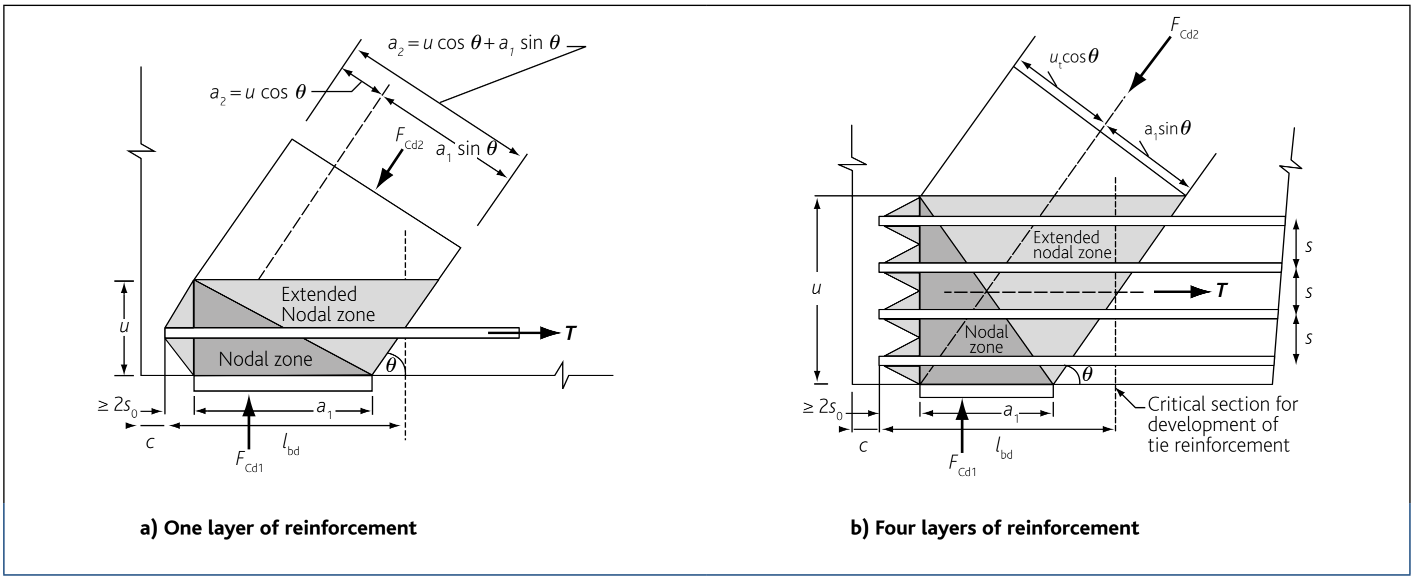
Tie forces should normally be carried by reinforcement where the area of reinforcement required:

$A_s = T/f_{yd}$

The reinforcement should have sufficient anchorage at the nodes to develop the design tensile forces. Reinforcement can be anchored with mechanical devices, standard hooks, or straight development lengths. Eurocode 2 states that reinforcement should be adequately anchored in nodes. The development length can be started from the point where the reinforcement intersects the extended nodal zone as shown in Figure 3.9.

In highly stressed concentrated nodes, it is beneficial to provide the tensile reinforcement in several layers since this increases the node dimensions as shown by comparison of Figure 3.9a with 3.9b.This also increases the capacity of the incoming struts. Using several smaller bars lessens the required anchorage lengths, but any changes in position of the centreline of the tie force should be accounted for.

Figure 3.9
Effect of reinforcement distribution on nodal zone dimension.



3.3 Nodes

Nodes are defined as regions where struts change direction or where struts and ties intersect. Nodes can be subdivided into smeared nodes and concentrated nodes.

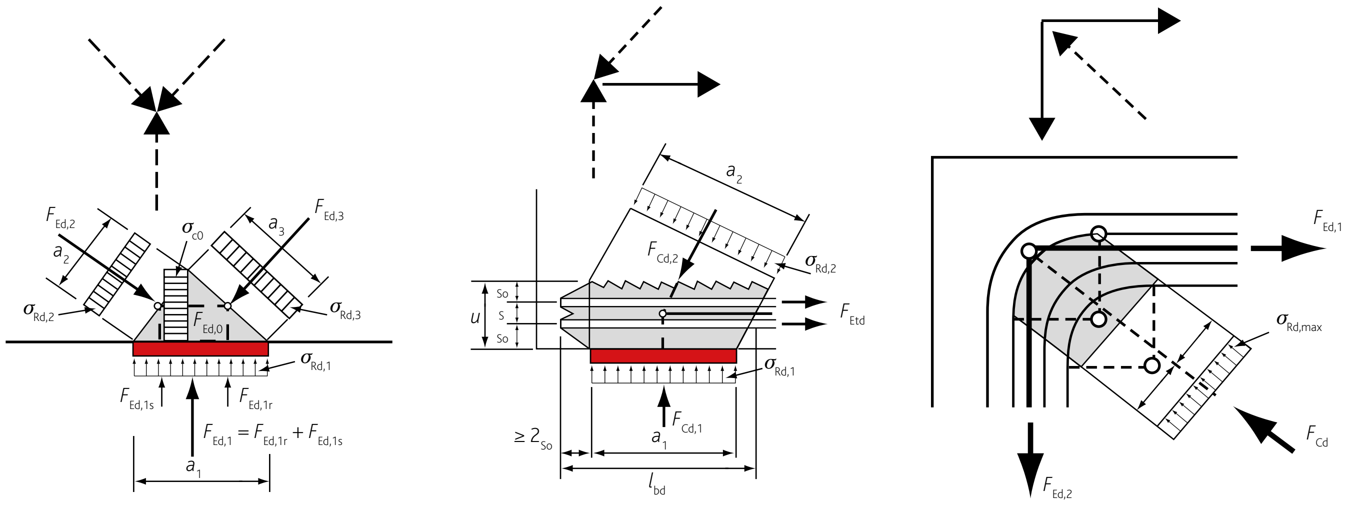
3.3.1 Smeared nodes

Smeared nodes occur in the body of a member where the orientation of a wide stress field is diverted. Examples are shown in Figure 2.3b and at either end of the tie T shown in Figure 3.3c. Most nodes in STMs are smeared (or continuous) nodes. The concrete stresses are not usually critical in smeared nodes and so are not usually checked in design.

3.3.2 Concentrated nodes

Figure 3.10 shows typical examples of concentrated nodes which arise at the intersection of concentrated struts and ties. Nodes are classified in Eurocode 2 as CCC (three compressive struts), CCT (two compressive struts and one tie), and CTT (one compressive strut and two ties). In Figure 3.10b forces are transferred from the tie into the node through a combination of bearing at the back of the node and bond stresses within the extended node.

Figure 3.10
Different types of concentrated node



a) CCC Compression node

b) CCT Compression node with a tie in one direction

c) CTT Compression node with ties in two directions

3.3.2.1 Concentrated node design

Concentrated nodes are typically highly stressed and need to be carefully designed to ensure that the incoming forces can be accommodated without the concrete failing in compression.

The maximum design compresive stress $\sigma_{Rd,max}$ at a node should normally be taken from Table 3.1.

Table 3.1
Eurocode 2[6,6a] recommendations for nodal strength

Type of node			Design comprehensive strength $\sigma_{Rd,max}$	
Description	**Typical location**	**Notation**		
Compression nodes without ties or any transverse tension	Under mid-span concentrated load (see top node in Figure 2.2)	CCC	$1.0\ \nu' f_{cd}$	Exp (6.60)
Compression-compression tension node	At end supports (see bottom node in Figure 2.2)	CCT	$0.85\ \nu' f_{cd}$	Exp (6.61)
Compression-tension-tension node	At the top of the tip of a cantilever	CTT	$0.75\ \nu' f_{cd}$	Exp (6.62)
Note: For definitions of ν' and f_{cd}, see 3.1.2				

It is not usually necessary to check stresses on the back face of a concentrated CCT node. In reality, the reinforcement is anchored through a combination of bond stresses within the node and bearing at the back of the node and checked accordingly.

It should be noted that the stresses in a supporting (or supported) reinforced concrete column may overstress the nodal contact area of a supported wall or deep beam. It is therefore important to continue column bars and links into the wall, so as to distribute axial stresses. Careful consideration needs to be given when the wall and column widths are not the same.

3.4 Dimensions

The dimensions of STMs should be given to the centroid of nodes (i.e. the intersections of the assumed centrelines of actions). In the case of ties allowance must be made for cover and layers of reinforcement.

Following initial design it might be deemed necessary to make adjustments. Where critical, iteration through reanalysis and redesign is recommended.

3.4.1 Node dimensions

The dimensions of concentrated nodes (and adjacent idealised prismatic struts) need to be chosen to ensure that the stresses on the node boundaries are less than or equal to the design concrete strengths given in Table 3.1. However, proportioning nodes so that nodal stresses are reasonably high can avoid the problems of unrealistic STMs.

The dimensions of concentrated nodes may seem rather arbitrary but initially they are governed by the dimensions of bearings and ties.

If allowable stresses are exceeded it may be possible to reduce them to acceptable values by increasing the dimensions of bearing plates and ties. For instance, increasing the width of the tie in Figure 3.9 increases the inclined dimension of the node (which in turn, as discussed in Section 3.2, increases the width of the adjacent inclined strut).

3.4.2 Strut areas

Strut dimensions are governed by node dimensions. As illustrated by Figure 3.9, the width of a strut at a CCT node, a_2, is given by:

$a_2 = a_1 \sin \theta + u \cos \theta$

where

$a_1 = l_b - 2s_o$

where

l_b = length of the bearing,

s_o = axis distance to an edge

θ = the angle of the strut to the bearing and

u = width of the tie or the height of the back face of the node, which subject to the recommendations below

$= 2s_o + (n-1)s$

where

s = spacing between bars

n = number of bars.

In the analysis of forces it is beneficial for u to be as wide as possible (so long as struts and their associated bottle stresses do not overlap). It may be seen that u and l_b can be varied within practical limits to suit circumstances.

Strut areas are not always rectangular; struts within circular-pile pile caps will be eliptical.

3.4.3 Tie depths and lever arms

Eurocode 2 does not give specific guidance on the maximum depth of the tie u. However, for single span deep beams, Model Code 90[3] recommends that:

u = the bottom tie depth (see Figure 3.9)
= 0.12 x (lesser of span, L, or height h)

and

the lever arm between tie and compression chord (e.g. strut 2 in Figure 2.4a) is taken as 0.6 to 0.7 x (lesser of span, L, or height h).

By comparison, ACI 318[1] states that:

$0.5u_{tmax} > u > u_{tmax}$

where

$u_{tmax} = F/(t\sigma_{Rd\ node})$

where

$\sigma_{Rd\ node}$ = allowable design bearing stress at the bottom node.

To finalise tie depths and/or widths and lever arms at least one iteration of the STM (when the quantity and arrangement of reinforcement can be estimated) is required. The tie depth or width includes surrounding concrete which is assumed not to contribute to the axial capacity of the tie, but will undoubtedly reduce elongation at SLS.

3.5 Minimum reinforcement

Generally, a minimum area of 0.1% A_c horizontal and vertical reinforcement should be provided in each face (i.e. a total area of at least 0.2% A_c) at no greater than 300 mm centres. For deep beams, an orthoganol mesh of reinforcement should be provided. The NA to BS EN 1992-1-1[6a] requires 0.2% reinforcement to be provided on each face in each direction.

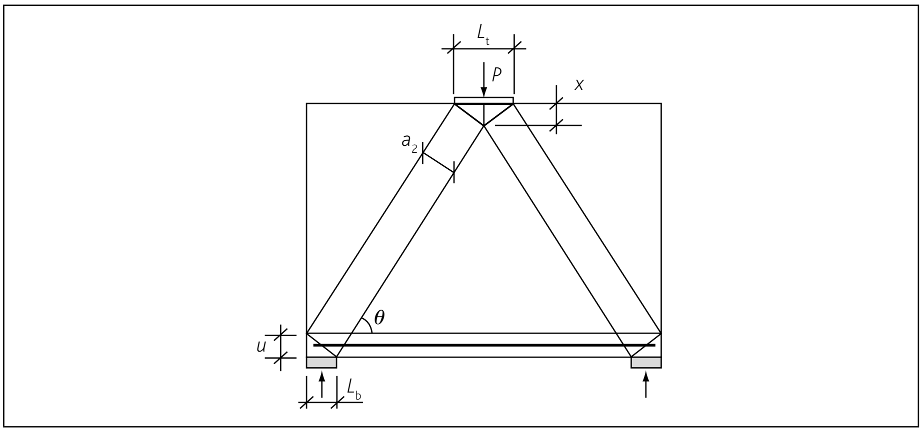
3.6 Corbels and frame corners

In the UK, corbels and frame corners (corners subject to opening or closing moments) should be designed in accordance with the guidance given in PD 6687[15] Annex B.

4. Design iteration

Consider the deep beam shown in Figure 4.1 which shows a possible strut-and-tie model.

Figure 4.1
STM for deep beam



4.1 Stresses in struts

Essentially the design of struts comes down to ensuring $\sigma_{Ed} < \sigma_{Rd,max}$ in all locations.

4.1.1 Design stresses

Here, the design stress in the strut is given by:

$\sigma_{Ed} = F/a_2t$

where

F = force in compression (In Figure 4.1 = $0.5P/\sin\theta$)

t = the beam thickness

a_2= width of the strut (could be different top and bottom):

= $a_1 \sin\theta + u \cos\theta$ (as before, see Figure 3.9)

4.1.2 Allowable stresses in struts

According to Eurocode 2 the design strength of a strut (without transverse reinforcement) is given by:

$$\sigma_{Rd,max} = 0.6\nu f_{cd}$$
$$= 0.6\,(1\text{-}f_{ck}/250)\ \alpha_{cc} f_{ck}/\gamma_c$$

Where necessary, the strength of a strut ($\sigma_{Rd,max}$) can be increased up to the stress limits of the nodes (see Section 4.2) either end by providing calculated transverse/shear reinforcement. The required area of reinforcement can be calculated by:

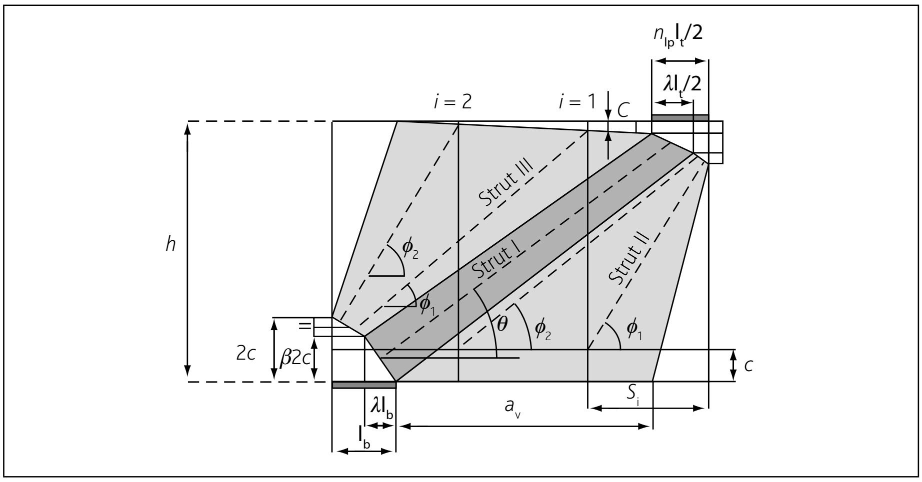
- treating the inclined strut as a bottle stress field as in Figure 3.4b and providing designed reinforcement, based on the lesser strut width, to Expressions (6.58) and (6.59) in Eurocode 2 as detailed in section 3.1.3,

  or
- developing an alternative STM (as shown in Figure 4.2) and designing shear reinforcement accordingly.

  or

- using the design equations in Eurocode 2 for shear in beams, which should always be used if a_v/d exceeds 1.5.
  Here*, no calculated beam shear reinforcement is required if the design shear stress:
  $\beta v_{Ed} \leq v_{Rdc}$
  where
  $\beta = a_v/2d$
  where
  a_v = distance between edge of load and edge of support as defined in Eurocode 2 6.2.2(6)
  d = effective depth
  v_{Rdc} is given by Eurocode 2, Exp (6.2.a)
  If required, an area $\Sigma A_{sw} = \beta V_{Ed}/f_{yd}$ should be provided within the central ¾ of the shear span (Eurocode 2, Cl 6.3.2(8)).

Figure 4.2
Alternative STM for design of shear reinforcement



4.1.3 The MCFT alternative

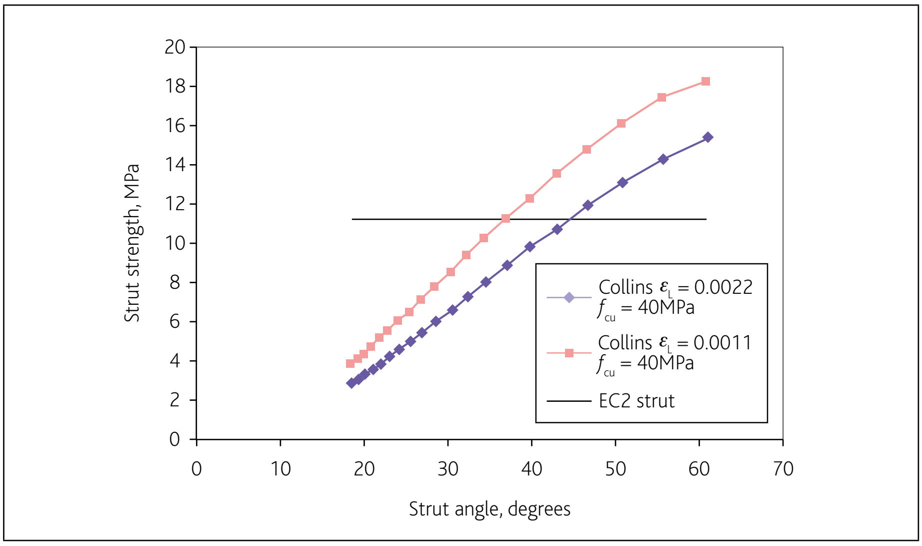
As an alternative, some references apply Collins and Mitchell's Modified Compression Field Theory[5] (MCFT) to STM. According to MCFT, the concrete strength of the strut (f_{csb}) at a CCT node should be taken as:

$f_{csb} = \phi f_{ck}/\ (0.8 + 170\varepsilon_1)$
where
ϕ = capacity reduction factor
= 0.65 in the Canadian Code CSA A.23.3-04 [4]
$\varepsilon_1 = \varepsilon_L + (\varepsilon_L + 0.002)\cot^2 \theta$
where
ε_L is the strain in the tie.

This compares to $0.6(1\text{-}f_{ck}/250)f_{cd}$ used in Eurocode 2[6]. A comparison is made in Figure 4.3 for C40/50 concrete. The differences at low strut angles should be noted.

* This verification assumes that all loads are applied within av ≤ 2.0d of a support.
In more general application, b only applies to the contribution to shear made by loads with av ≤ 2.0d.

Figure 4.3
Comparison between EC2 and MCFT design concrete strengths in strut with transverse tension for C40/50 concrete



4.2 Allowable stresses in nodes

As stated in Table 3.1 allowable stresses in nodes are as follows:

- where there is no transverse tension, i.e CCC nodes (like the top node in Figure 4.1) the design compressive strength of the concrete is given by:
  $\sigma_{Rd,max} = 1.0\ \nu f_{cd}$
- where there is a CCT node (like the bottom node in Figure 4.1), the design compressive strength of the concrete is given by:
  $\sigma_{Rd,max} = 0.85\ \nu f_{cd}$
- where there is a CTT node (typically at the top of the tip of a cantilever), the design compressive strength of the concrete is given by:
  $\sigma_{Rd,max} = 0.75\ \nu f_{cd}$

Whilst the stresses in all nodes should be checked, it will be noted that checks on or at the ends of struts serve as checks on stresses around nodes. Usually, the only additional checks to be made are on nodes with support bearings (indeed in practical design, these may be the first checks to be made).

Stresses at the bottom CCT node are usually more critical than those at the top CCC node. If bearing stresses at the ends of a strut are critical, the most straightforward way of increasing the strength of the direct strut is to increase the width of the strut at the bottom node. This is most easily achieved by increasing the width of the tie, *u*. As noted earlier, the dimensions a_1 and *u* can be chosen so that a_2 enables $\sigma_{Ed} \leq \sigma_{Rd}$ to be satisfied.

The dimensions of the top node can be calculated by limiting the bearing stress at the top node and using direct calculation (or trial and error) to find the depth of the top node (dimension x in Figure 4.1) at which the stress on a vertical section through the centre of the node equals the design strength.

4.3 Iteration

Where stresses are too great, dimensions of nodes and struts are amended and the STM is adjusted. Stresses are again checked and the process repeated until the model is considered satisfactory. As noted in Section 2.3, the best model usually has the shortest length of unyielded ties.

5. Design examples

5.1 Two-pile cap

The Concrete Centre	Project details Two-pile cap	Calculated by chg	Job no. 810
		Checked by S Alright	Sheet no. 4/1
		Client TCC	Date Dec 2014

Extend the design of the pile cap presented in Panel i (page 3) where a two-pile cap supports a 500 mm square column carrying 2500 kN (ULS) on two 600 mm diameter piles. Assume that the self-weight of the pile cap is included, f_{ck} = 30MPa and the minimum cover is 50 mm to H16 lacers.

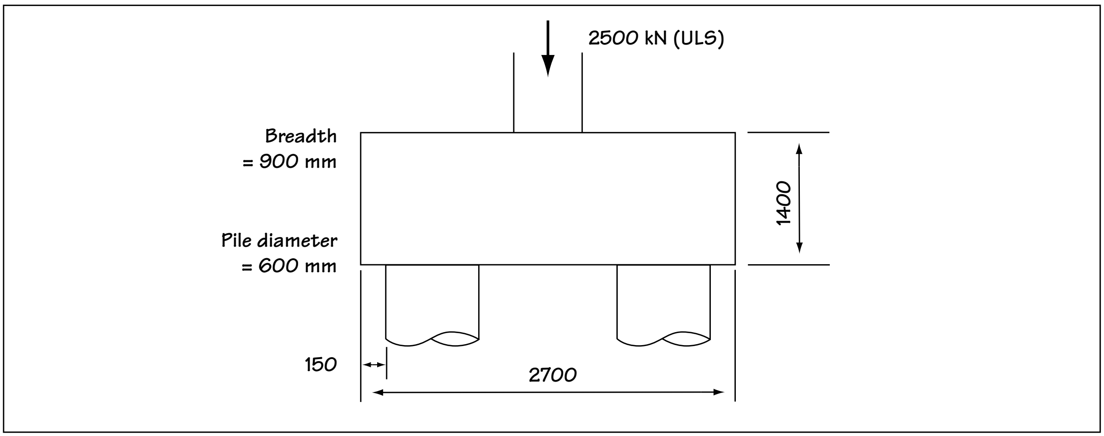


Figure 5.1: Two-pile cap

5.1.1 Define D-regions

The whole element is within h of a support or load so may be treated as a D-region.

5.1.2 Proposed STM

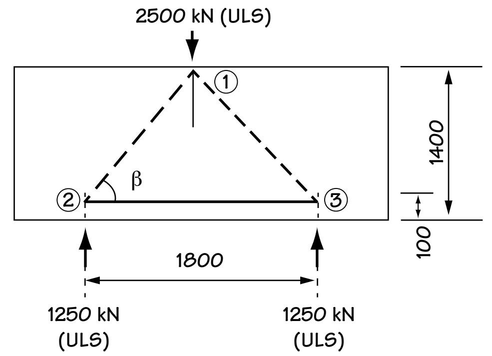


Figure 5.2: Proposed STM*

Angle of strut, β = $\tan^{-1}$(1300/900) = 55.3°
Force per strut = 1250/sin 55.3° = 1520 kN
Force in tie = 1250 cot 55.3° = 866 kN

5.1.3 Check node stresses

Check at node 1

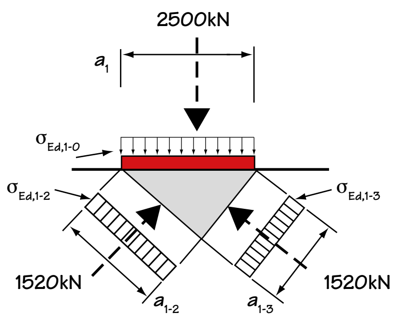


Figure 5.3: Elevation on node 1

$\sigma_{Ed,1-0}$ = 2500 x 10^3/500^2
= 10.0 MPa
$\sigma_{Ed,1-2}$ = 10.0 MPa (as above: hydrostatic pressure)

Or

a_{1-2} = (500/2)/sin 55.3°
= 304 mm
$\sigma_{Ed,1-2}$ = 1520 x 10^3/(304 x 500)
= 10.0 MPa

* In line with BS 8004[10] "to cover unavoidable variations up to 75 mm each way in the positions of individual piles, it was traditional practice to allow at least an additional 75 mm in spans. EN 1992-1-1[6] Clause 9.8.1(1) states that the "expected deviation of the pile on site should be taken into account". An allowance was considered unnecessary in this case.

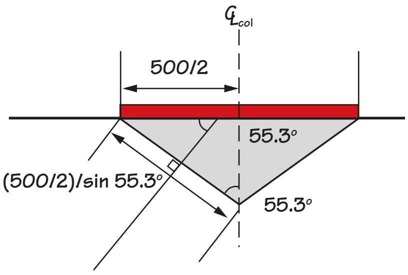


Figure 5.4: Geometry at node 1*

$\sigma_{Ed,1\text{-}3}$ = 10.0 MPa (as above)

$\sigma_{Rd,max,1}$ (for CCC node) — Exp (6.60)

= $1.0\nu' f_{cd}$

= $1.0(1\text{-}f_{ck}/250)\alpha_{cc}f_{ck}/\gamma_c$

= 1.0 x (1-30/250) x 0.85 x 30/1.5

= 0.57 x (1-30/250) x 30

= 15.0 MPa

$\sigma_{Rd,max,1} > \sigma_{Ed}$

∴ OK

Check at node 2 (and 3)

$\sigma_{Ed,2}$ = 1250 x $10^3/(\pi \times 300^2)$ = 4.4 MPa

$\sigma_{Rd,max,2}$ (for CCT node) = 0.85 x (1-30/250) x 0.85 x 30/1.5 = 12.7 MPa

$\sigma_{Rd,max} > \sigma_{Ed}$

∴ OK

5.1.4 Check struts

Check strut at node 1

$\sigma_{Ed,\ 1\text{-}2}$ = 10.0 MPa (as above)

$\sigma_{Rd,max}$ = f_{cd} (for regions with no or some compressive transverse stress)

= 0.85 x 30/1.5

= 17.0 MPa — Exp (6.55)

$\sigma_{Rd,max} > \sigma_{Ed}$

∴ OK

* The centreline of $a_{1\text{-}2}$ will not coincide with the centreline of the column unless β = 45°, rendering the STM inaccurate. This discrepancy is often disregarded.

Check strut at node 2 (and 3)

$\sigma_{Ed,\,2\text{-}1}$ = 4.4 MPa (as above)

$\sigma_{Rd,max}$ = $0.6\nu' f_{cd}$ (for cracked compression zones, i.e. with transverse tension) Exp (6.56)

= $0.6\,(1-f_{ck}/250)\alpha_{cc}f_{ck}/\gamma_c$

= 0.6 x (1-30/250) x 0.85 x 30/1.5

= 9.0 MPa

$\sigma_{Rd,max} > \sigma_{Ed}$

∴ OK

5.1.5 Tie

The area of steel in the tie:

$A_{s,reqd}$ ≥ 866 x 10^3/(500/1.15) ≥ 1991 mm^2

Noting that above 12 mm diameter, BS 8666[16] Table 1, designation H equates to Grade B500B or Grade B500C

So use say 5 H25s (2455 mm^2)*

5.1.6 Check anchorage 9.8.1 (1)

Average length available** = Pile diameter + allowance – cover

= 600 + 150 – 50

= 700 mm

Using tables[14] for anchorage of a straight fully stressed H25 in C30/37 in good bond conditions:

$l_{bd,table}$ = 900 mm (assuming α_b, available =1.0)

$l_{bd,table} > l_{bavailable}$

∴ no good***

Therefore consider in more detail, provide bends and/or design anchorage length. Usual practice is to provide tension steel with large radius bobs each end.

* Where flexural design has been used it is common UK practice to provide uniform distribution of reinforcement. However, EN 1992-1-1 Clause 9.8.1(3) suggests that "the tensile reinforcement . . . should be concentrated in the stress zones between the tops of the piles". There is evidence to suggest that bunching orthogonal reinforcement leads to a standard 4-pile cap being 15% stronger than using the same amount of uniformly distributed reinforcement[11]. The requirement for concentrating reinforcement can be interpreted in different ways but the apparent shortcoming can be alleviated by providing transverse tension and tie-back reinforcement to distribute forces from bars as indicated in Figure 5.5. For pile caps supporting structures other than bridges, there would appear to be little reason to deviate from the advice given in BS8110[12] " . . only the reinforcement within 1.5 times the pile diameter from the centre of a pile shall be considered to constitute a tension member of a truss". So in this case, 5 no. H25s distributed across a 900 mm wide pile cap section is considered satisfactory.

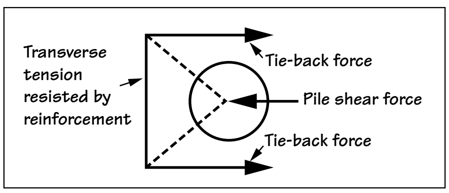


Figure 5.5: Spread of load from a pile to adjacent tie bars[7]

** In a typical CCT situation with a rectangular section for support, anchorage of bars is assumed to start in the 'extended nodal zone' – See Figure 3.9. Above piles, the 'extended nodal zone' detailed in EN 1992-1-1 Clause 9.8.1(5) might be used. Some references[13] advocate anchoring from the centreline of the pile. However, in the UK, it is usual to assume anchorage starts at the face of the pile remote from the edge of the cap as per Clause 9.8.1(1) and that is the method adopted here.

*** Note: A common mistake made by designers is to underestimate the need for anchorage of the reinforcement at supports.

Design anchorage length:	
$l_{bd} = \alpha_{lbrqd} = \alpha\ (\phi/4)(\sigma_{sd}/f_{bd})$	Exp (8.4)
where	Exp (8.3)
$\alpha = \alpha_1.\alpha_2.\alpha_3.\alpha_4.\alpha_5.$	
where:	
α_1 = 1.0 (straight bar assumed)	
$\alpha_2 = 0.7 < 1\text{-}0.15(cd\text{-}\phi)/\phi < 1.0$	
where:	
c_d = min (side cover, bottom cover or clear spacing /2)	
= say min (50 + 16 , 75, (900-66 x 2 -25)/(4 x 2))	
= min (66, 75, 93)	
= 66 mm	
ϕ = bar diameter	
= 25 mm	
$\alpha_2 = 0.75$	
α_3 = 1.0 (confinement by transverse reinforcement)	
α_4 = 1.0 (confinement by transverse reinforcement)	
$\alpha_5 = 0.7 < 1 - 0.04\rho < 1.0$	
where:	
ρ = transverse pressure, MPa	
= 4.4 MPa (as before)	
$\alpha_5 = 0.824$	
But	
$\alpha_2.\alpha_3.\alpha_5 \leq 0.7$	Exp (8.5)
$\therefore\ \alpha = 0.7$	
σ_{sd} = say (500 / 1.15) x (1991/2455) = 435 x 0.81 = 353 MPa	
$f_{bd} = 2.25\eta_1\eta_2 f_{ctk} / \gamma_m$	Exp (8.2)
where	
η_1 = 1.0 for good bond	
η_2 = 1.0 for bar diameter ≤ 32 mm.	Table 3.1
$f_{ctk} = 0.7 \times 0.3\ f_{ck}^{2/3} = 0.7 \times 0.3 \times 30^{2/3} = 2.0$ MPa	Table 2.1N
$\gamma_m = 1.5$	
f_{bd} = 2.25 x 1.0 x 1.0 x 2.0 / 1.5	
= 3.0 MPa	
l_{bdreqd} = 1.0 x (25 / 4) x (353 / 3.0)	
= 736 mm	
l_{bd} = 0.7 x 736	
= 515 mm	
$l_{bd} < l_{bavailable}$	
$\therefore$ OK	
Nonetheless provide bars bobbed each end*	
* This case is not highly loaded and it was found unnecessary in theory to resort to designed bends. However, it is traditional practice to provide bars bobbed at both ends. Later it is shown that fully stressed bars need to be checked for minimum mandrel diameter (or minimum radius) to Exp (8.1). Note that providing bobbed bars and a cover >3ϕ, (in this case 75 mm), would have attracted an additional α_1 factor of 0.7.	

5.1.7 Shear

As by inspection a_v <1.5d. So no beam shear check is necessary. Cl. 2.19 [15]
Punching shear check is inappropriate in this case.

5.1.8 Minimum reinforcement

To control cracks, provide transverse bars based on requirements for minimum steel*:

$A_{smin} = k_c k f_{ct,eff} A_{ct} / \sigma_s$ Exp (7.1N)

where

$k_c = 1.0$

$k = 0.65$

$f_{ct,eff} = f_{ct,mf} = 0.30 f_{ck}^{2/3} = 0.30 \times 30^{2/3} = 2.9$ MPa

$A_{ct} = b \times \min(2.5(h-d), (h-x)/3, h/2)$

$= 1000 \times \min(2.5(1400-1300), (1400 - \text{say } 0.3 \times 1300)/3, 1400/2)$

$= 1000 \times \min(250, 336, 700)$

$= 250000$ mm^2

$\sigma_s = f_{yk} = 500$ MPa

$A_{smin} = 1.0 \times 0.65 \times 2.9 \times 250000 / 500 = 507$ mm^2/m

Provide min H16@300 cc (670 mm^2/m)

* Note: Clause 9.8.1(3) allows, where there is no risk of tension, sides and top surfaces of pile caps to be unreinforced, e.g. in 2-, 3- and 4- pile caps. Similarly it allows the areas between concentrations of minimum reinforcement above piles to be unreinforced.

However, consideration should be given to minimum reinforcement amounts and maximum bar spacings to control cracking at the serviceability limit state (e.g. early thermal cracking) and provide ductility to the structure. Also consideration should be given to providing stability for column starters.

Normal UK practice is to provide at least nominal H16 reinforcement as lacers to extended bobbed bottom bars[13].
In this case minimum reinforcement is provided to provide a cage based on using EN1992-1-1 Exp (7.1N)

5.1.9 Commentary

It will be noted that this exhaustive design, gives in essence the same results as the outline given in Panel i in *Introduction*. Designers soon become accustomed to the speed of design and judging the criticality of needing to check struts and nodes. However, the previous worked examples highlight the need to check anchorage lengths of large and highly stressed tie bars in pile caps.

5.1.9.1 Anchorage

With regard to anchorage, had fully stressed H32s been necessary:

l_{bd} = 0.7 x (32 / 4) x (435 / 3.0)
= 812 mm

So a straight length would have been insufficient and it would have been necessary to check the minimum mandrel size and where necessary to specify a design bend radius. The following calculation is intended to show the design process:

$\phi_{m,min} \geq F_{bt}[(1/ab)+1/2\phi)]/f_{cd}$ Exp(8.1)

where

F_{bt} = the force in the bar at the start of the bend
= force in the bar – bond over straight length
Assuming uniform bond
≡ A_s x (500/1.15) x (812 - straight length before bend)/812
The distance from start of pile to start of an assumed standard 3.5ϕ radius bend on the H32:
600 + 150 – 50 – 16 – 3.5 x 32 = 572 mm

F_{bt} = 804 x (500/1.15) x (812-572)/812
= 130.9 kN

a_b = min (side cover + ϕ/2, bottom cover + ϕ/2 or clear spacing /2)
= say min (50 + 16 +16 , 75 +16, (900-66 x 2 -25)/(4 x 2))
= min (82, 91, 93)
= 82 mm

$f_{cd} = \alpha_{cc} f_{ck}/\gamma_m$
= 0.85 x 30 / 1.5
= 17.0 MPa as before.

$\phi_{m,min}$ ≥ 130.9 x 10^3 x [1/82 + 1 /(2 x 32)]/ 17.0
≥ 214 mm

Compared to standard mandrel size[6,15]: 7 x 32 = 224 mm

∴ theoretically OK

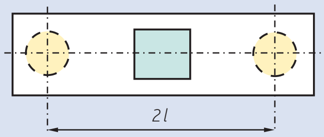
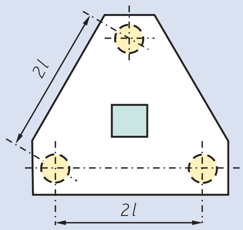
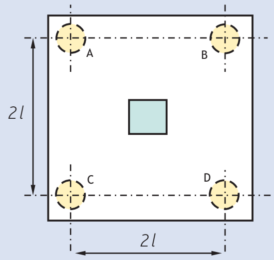
Check bob length:
Min bob length required = 812 - 572 – (π/2) x (3.5 + 0.5) x 32 = 39 mm
Compared to minimum bob of 5ϕ [16]

∴ OK

5.1.9.2 Tie forces in 2-, 3- and 4-pile pile caps

For simply supported centrally loaded 2-, 3- and 4-pile pile caps the tie force might be derived from Table 5.1.

Table 5.1
Tensile force between piles[22]

Pilecap layout	Tension force in reinforcement
	$F_t = Pl/(2d)$ where P = load in the column l = distance from column to pile (see diagram) d = effective depth
	$F_{t(AB)} = F_{t(BC)} = F_{t(AC)}$ $= 2Pl/(9d)$ where P = load in the column l = distance from column to pile (see diagram) d = effective depth
	$F_{t(AB)} = F_{t(AC)} = F_{t(BD)} = F_{t(CD)}$ $= Pl/(4d)$ Force in longitudinal and transverse direction: $F_t = Pl/(2d)$ where P = load in the column l = distance from column to pile (see diagram) d = effective depth

Notes:
- Where column size is taken into account there may be efficiences to be gained.
- It is usual to space piles at three times their diameter.

5.1.9.3 Shear

It will be noted that there is no check for shear. Although it is often done, in theory there is no need to check beam shear when using strut-and-tie. PD 6687[15] Cl 2.19 states that no beam shear check is necessary providing $a_v < 1.5d$.

Where the pile spacings exceed $3\phi_{pile}$ it is customary to carry out punching shear checks.

5.2 Deep Beam 1

Project details	Calculated by	Job no.
Deep beam 1	chg	810
	Checked by: R Vetal	Sheet no.: WE 1/1
	Client: TCC	Date: Dec 2014

The 5000 x 1500 x 450 thick beam shown in Figure 5.6 is supported on 600 x 450 thick columns at 4400 mm centres. It supports a 450 x 450 bearing plate with actions of G_k = 1256 kN and Q_k = 480 kN acting 950 from one support. Determine the reinforcement assuming C35/45 concrete and f_{yk} = 500 MPa. c_{nom} = 25 mm.

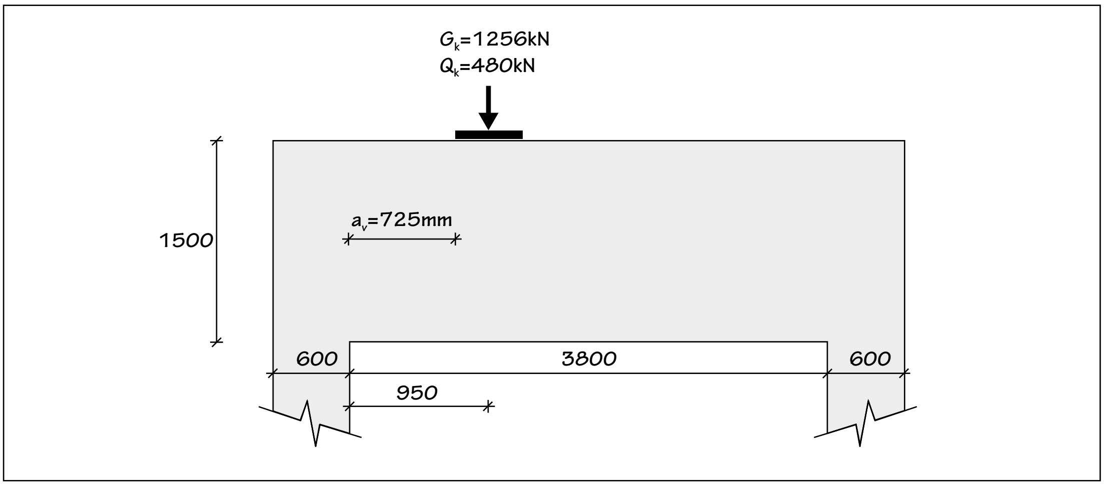


Figure 5.6: Deep Beam 1

For this design it will be sufficient to:

a) Check bearing stresses
b) Check stresses in inclined struts
c) Design ties and anchorages
d) Design bursting / distribution reinforcement.

5.2.1 Define D-regions

By inspection whole deep beam consists of D-regions.

5.2.2 Proposed STM

ULS load, F = 1256 x 1.35 + 480 x 1.5 + 5.0 x 1.5 x 0.45 x 25 x 1.35
= 2529 KN (self weight assumed to act at node 2)

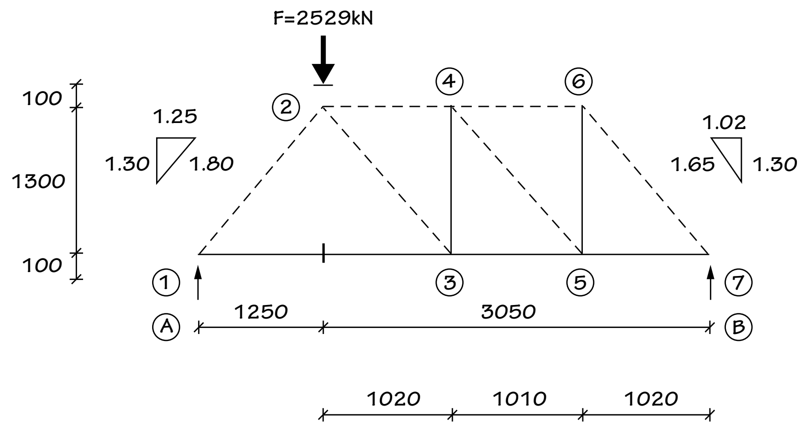


Figure 5.7: Proposed STM

Figure 5.7 shows a possible STM for the deep beam and resulting dimensions and slopes. It allows 100 mm top and 100 mm bottom to centrelines of compression strut C_{24} and tie$_{13}$ (The maximum depth of tie $T_{13} \approx 0.12h$ = 180 mm say 200 mm).

Forces:

Consider moment about *B*

R_A = 2529 x 3.05 / 4.30 = 1794 kN

$\therefore R_B$ = 2529 - 1794 = 735 kN

F_{12}	= 1794 x 1.80 / 1.30	= 2484 kN	strut
F_{13}	= 2484 x 1.25 / 1.80	= 1725 kN	tie
F_{34}	= F_{56}	= 735 kN	ties

5.2.3 Check bearing stresses

At node 2, under load *F*

σ_{Ed} = 2529 x 10^3 / (450 x 450) = 12.5 MPa

CCC node $\therefore$ σ_{Rd} = 1.0 x (1 – 35/250) x 0.85 x 35 / 1.5 = 17.1 MPa $\therefore$ OK

At node 1 at support A (see Figure 5.8)

$a_1 = 600 - c_{nom} - 2s_o$

s_o = say 12 mm link + 25/2 = 50 mm

a_1 = 600 – 25 – 2 x 50 = 475 mm

σ_{Ed} = 1794 x 10^3 / (475 x 450) = 8.39MPa — Exp (6.58)

CCT node $\therefore$ σ_{Rd} = 0.85 x (1 – 35/250) x 0.85 x 35 / 1.5 = 14.5 MPa $\therefore$ OK

At node 7 at support B — OK by inspection

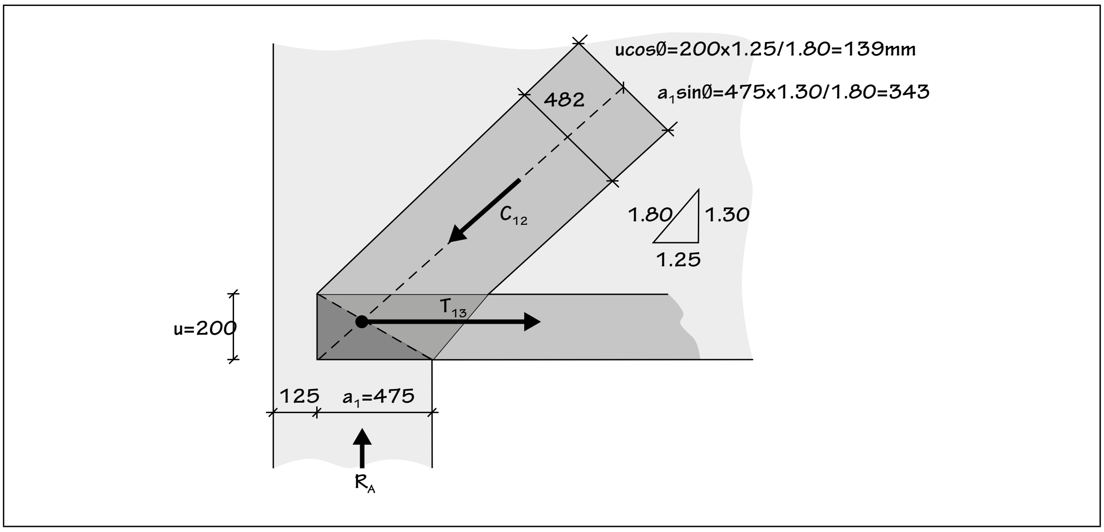


Figure 5.8: Geometry at Support A

5.2.4 Ties

a) F_{13}

F_{13} = 1725 kN

$A_{s\ req'd}$ = 1725 x 10^3 / (500/1.15) = 3968 mm^2

Try 8H25 (3928 mm^2 say OK) in two layers
i.e. 2 x 4 H25 @ 50 mm cc

Check anchorage

For H25, anchorage required assuming straight bar
in 'good' condition in C35 / 45 concrete = 790 mm

How to Detailing[14]

Average anchorage available beyond face of compression strut
= bearing + extended node - cover - u-bar diameter
= 600 + 200/2 - 26 - 16 = 655 mm ∴ no good.
∴ by inspection provide bobs at end of bars*

b) F_{34}

F_{34} = F_{56} = 735 kN

$A_{s,req'd}$ = 735 x 10^3 / (500/1.15) = 1690 mm^2 per tie
i.e. per 3.05/3m say 1690 mm^2/m

Try H16@225 both sides (1768 mm^2/m)

* Designing out the anchorage in 'good' bond conditions:-

$l_{bd} = a l_{brqd} = a(f/4)(s_{sd}/f_{bd})$

$f_{bd} = 2.25 h_1 h2 f_{ctk} / g_m = 2.25 \times 1.0 \times 1.0 \times (0.7 \times 0.3 \times 35^{2/3}) / 1.5 = 3.37$ MPa

$l_{bd} = 0.7 \times (25 / 4) \times (435 / 3.37)$

$= 570$ mm

∴ OK

5.2.5 Struts

a) Check strength of direct strut in left hand shear span

Check stress in strut 1-2 at the bottom node. Transverse reinforcement is required if the design stress in the inclined strut at the bottom node exceeds the design strength of the strut in the presence of transverse tension, i.e. if $\sigma_{Ed} > \sigma_{Rd}$.

The maximum width of the strut is given by:

a_{12} = $L_b \sin\theta + u \cos\theta$ (See Figure 5.8)
= 475 x 1.25/1.80 + 200 x 1.30/1.80
= 330 + 144 = 474 mm
σ_{Ed} = 2484 x 10^3 / (474x450) = 11.6 MPa*
σ_{Rd} = 0.6 x (1 – 35/250) x 0.85 x 35 / 1.5 = 10.23 MPa — Exp (6.56)

Therefore, calculated shear/transverse anti-bursting reinforcement is required.

Bursting forces (bottle ties)

In this case the design strength of the strut at the bottom node can be increased to the design strength of a CCT node ($\sigma_{Rd,bot} = 0.85 \times (1 - f_{ck}/250) f_{cd}$) by the provision of transverse reinforcement in accordance with expression 6.58 or 6.59 as appropriate.

Check strut 1-2

F_{12} = 2484 KN

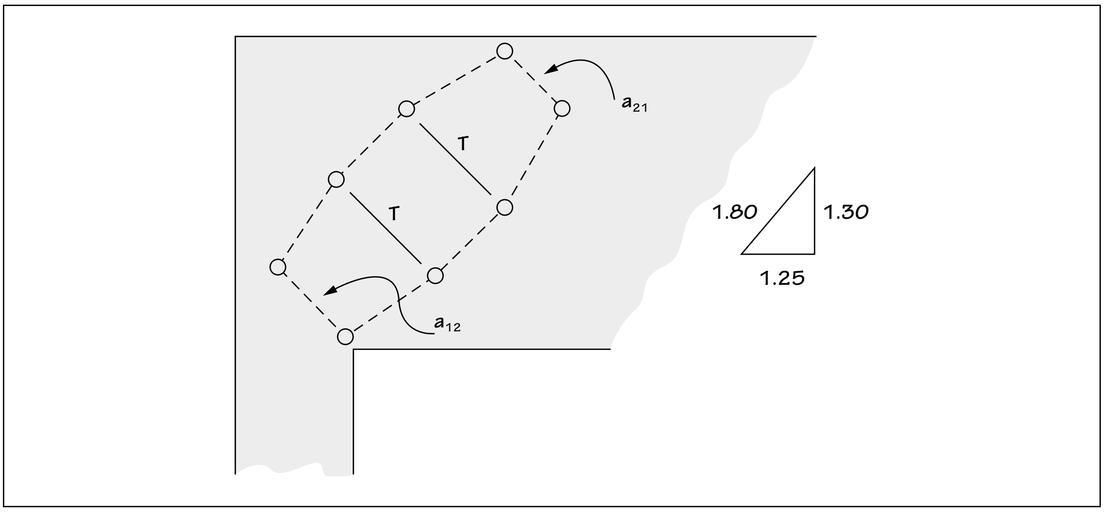


Figure 5.9: Bursting forces, *T*, in strut 1-2

* According to ACI 318 u could be increased to $u_{tmax} = F_{nt}/(t\sigma_{Rd\ node})$ = 1725 x 10^3 / (450 x 14.5) = 264 mm. a_{12} would become 545 mm and σ_{Ed} = 10.1 MPa i.e.> σ_{Rd}. This increase is considered inappropriate in this case as the *u* used was marginally greater than 0.12 h recommended by Model Code 90[3].

By inspection strut has full discontinuity

Exp (6.59) applies and at one end of the strut: — Exp (6.59)

$T = ¼ (1 – 0.7a/H) F$

where

a = width of strut at end

$= a_{21}$ or a_{12}

To maximise T (by minimising a/H) consider minimum value of a, i.e. a_{21} at node 2 (which is $<a_{12}$ at node 1, as k_1 for CCC node at node 2 >> k_1 for CCT node at node 1)

$a_{21} = F / t\ \sigma_{Rdmax}$

where

F = 2484 kN

t = 450 mm

$\sigma_{Rdmax} = k_1\ \nu' f_{cd}$ — Exp (6.60) & NA

= 1 x (1 -35/250) x 0.85 x 35 / 1.5 = 17.1 MPa

$a_{21} = 2484 \times 10^3 / (450 \times 17.1)$

= 323.6 mm

H = Strut length

= 1800

a/H = 323.6/1800 = 0.18

T = ¼ (1 – 0.7 x 0.18) x 2484

= 542.8 kN

$\therefore A_{s\ reqd} = 542.8 \times 10^3 / (500 / 1.15)$

$= 1248\ mm^2$

To be placed between 0.2H and 0.5H from the loaded surface.

i.e. 1248 mm^2 to be placed over 0.3 x 1800 = 540 mm

≡ 2311 mm^2/m over 540 mm at 1.25 in 1.30 slope

Considering both ends of the strut and singularity of the reinforcement layout, use this value throughout LHS i.e. use:

≡ 2311 mm^2/m horizontally and 2311 mm^2/m vertically*.

Try H16@ 175 (1148 mm^2/m) both ways both sides
(2296 mm^2/m both ways (say OK))

b) Struts in right hand shear span

By inspection — OK

* $A_{sreq}H$ and $A_{sreq}V$ should not be determined from vectors. See 3.1.3e)

5.2.6 Check STM

a) Tie

With reference to Figure 5.7, centreline of 8H25 coincides with assumed centreline of tie

∴ OK

b) Check compression strut 2-4

Presuming no transverse reinforcement* σ_{Rd} = 10.23 MPa as before

Depth = 1148 x 10^3 / (450 x 10.23) = 249 mm

∴ centreline 125 mm from top

Compared to 100 mm assumed. Say OK

5.2.7 Check shear

According to PD 6687 shear should be verified where a_v > 1.5d. — Cl. 2.19[15]

Where:

a_v = distance between load and support

For LHS a_v = 950 – 450 / 2 = 725 mm (see Figure 5.6)

For RHS a_v = 3800 - 950 – 450 / 2 = 2625 mm

d = effective depth = 1400 mm

For LHS, a_v < 1.5d, so no shear design required

For RHS, a_v > 1.5d, so shear design is required:

Shear design for RHS

$\beta = a_v/2d$ = 2625 / (2 x 1400) — 6.2.3(8)

= 0.94

βV_{Ed} = 0.94 735

= 691 kN

$A_{sw} \geq V_{Ed}/ f_{ywd} \times \sin \alpha$ — Exp (6.19)

= 691000 / ((500 / 1.15) x 1.0)

= 1589 mm^2 to be provided in the middle 0.75a_v

≡ 1589/(0.75 x 2.625)

= 807 mm^2/m

Try H12 in 2 legs @250 (A_{sw} = 904 mm^2/m)

But by inspection (see 5.2.8 later) not critical

* The design actually calls for adequate transverse bursting reinforcement so σ_{Rd} = 17.1 MPa giving the depth of strut 2-4 depth = 149 mm. So OK.

5.2.8 Minimum reinforcement	
In deep beams, the minimum area of horizontal and vertical reinforcement that needs to be provided in each face is $0.002A_c$ mm^2/m which equals 2h mm^2/m.	9.7.1 & NA
∴ Provide 900 mm^2/m in each face	
Provide min H16@225 b.w. EF (893 mm^2/m) (say OK)	

5.2.9 Summary of reinforcement requirements

Tie : 8H25 c/w bends at end A
Horizontal reinforcement: H16@175 EF (minimum)
Vertical reinforcement:
LHS: H16@175 EF
RHS: H16@225 EF (minimum)

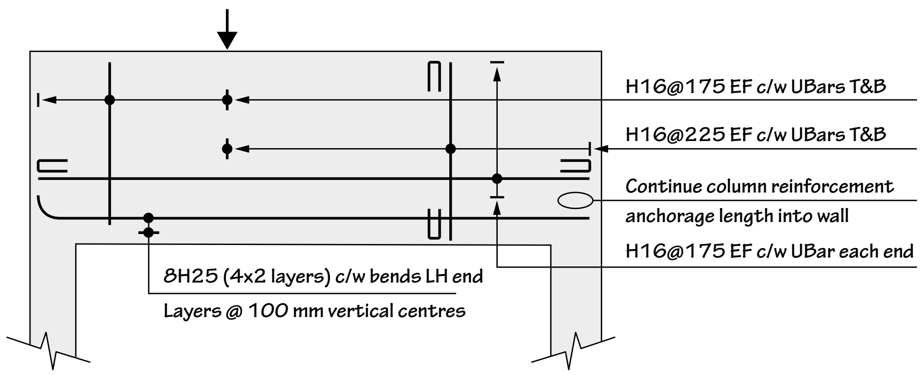


Figure 5.10: Summary of reinforcement for deep beam

5.3 Deep beam 2

Project details	Calculated by	Job no.
Deep beam 2	chg	810
	Checked by: R Vetal	Sheet no.: WE 2/1
	Client: TCC	Date: Dec 2014

A 5400 x 3000 beam 250 mm thick is supported on 400 x 250 columns. As Figure 5.11 shows it spans 5.0 m and supports actions of g_k = 75 kN/m and q_k = 32.5 kN/m at the top and bottom of the beam. Assume C25 / 30 concrete, f_{yk} = 500 MPa and c_{nom} = 25 mm

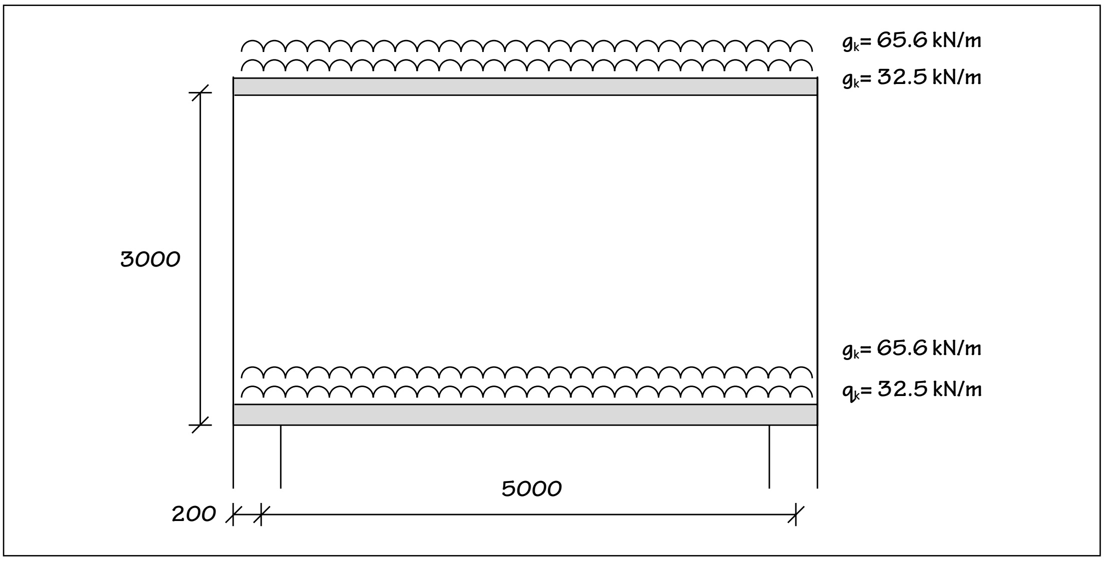


Figure 5.11: Deep Beam 2

5.3.1 Define D-regions

The whole element is within h (= 3000 mm) of a support load so may be treated as a D-region.

5.3.2 Proposed STM

Two STMs may be considered.

Fan-shaped STM

Firstly at ULS, as there is direct load transfer to the supports an STM with two fan-shaped stress fields is evident. Here, it is assumed that the principal tensile stress in the concrete is zero. The design strength of the concrete in the struts at the bottom node is $0.85v'f_{cd}t$.

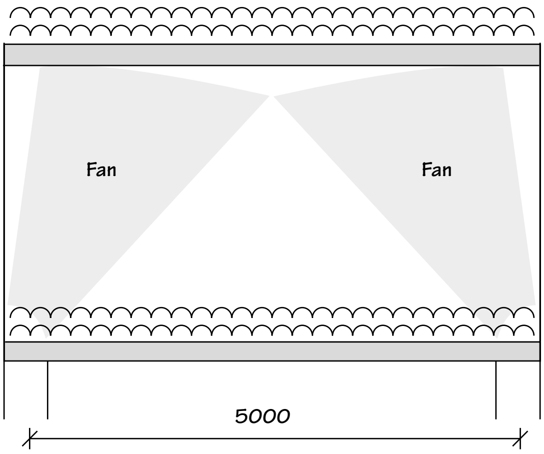
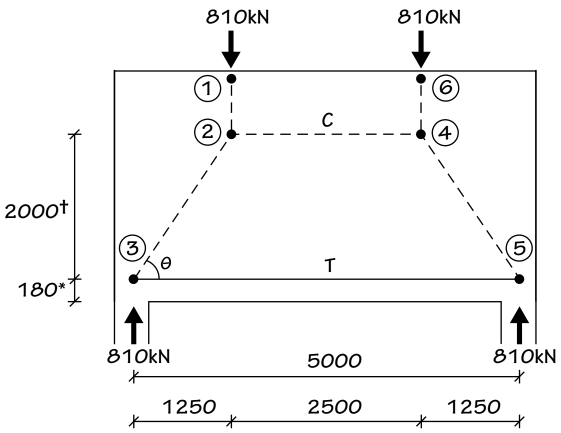


Figure 5.12: STM for ULS fan stress distribution

Bottle-shaped STM

Secondly at ULS, an STM may be constructed to determine strut-and-tie forces: see Figure 5.13. Here the UDLs top and bottom are resolved into two point loads applied at ¼ spans at the top of the wall.

ΣF = [2 x (65.6 x 1.35 + 32.5 x 1.5) + 3.0 x 0.25 x 25 x 1.35] x 5.4
= [2 x 137.3 + 25.3] x 5.4
= 1619.5 = say 2 x 810 KN



Notes: † MC90[3] gives z = 0.6-0.7 x minimum (h, L). 0.67 x 3000 = 2000 mm
* MC90 gives u ≈ 0.12 x minimum (h, L). 0.12 x 3000 = 360 mm. 180 mm to centreline

Figure 5.13: STM for design of flexural reinforcement

Check θ

tan θ = 2000/1250 = 1.6 i.e. < 2/1 ∴ OK

θ = 58°

Forces:-

C_{12} = 810 kN
Length of C_{23} = $(2000^2 + 1250^2)^{0.5}$ = 2358 mm
By trigonometry:
C_{23} = (2358 / 2000) x 810 = 955 kN
T_{35} = (1250 / 2358) x 955 = 506 kN

Choice:-

A fan-shaped stress field is appropriate for the ULS but not necessarily for the SLS where the lever arm can be determined from elastic analysis or alternatively in accordance with the recommendations of MC90 (see Section 3.4.3 or Figure 5.13). Designed reinforcement will not be required if the design bearing stress is less than $\sigma_{Rdmax} = 0.85\upsilon' f_{cd}$: in that case the design loads will be safely transmitted to the supports through the fan-shaped stress field.

Suspension reinforcement is required to transmit the bottom loading to the top of the beam. In addition, minimal horizontal reinforcement is required for crack control.

5.3.3 Check (fan) strut at node 3

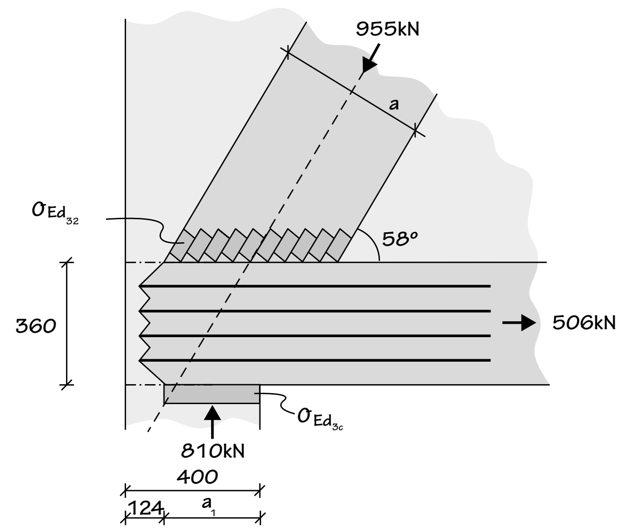


Figure 5.14: Node 3

Strut in bearing, C_{32}
For CCT Node (and fan-shaped strut)
$\sigma_{Rdmax} = 0.85\upsilon' f_{cd}$
where
$\upsilon' = 1 - f_{ck} / 250 = 1 - 25 / 250 = 0.90$
$f_{cd} = \alpha_{cc} f_{ck} / \gamma_m = 0.85 \times 25 / 1.5 = 14.2$
σ_{Rdmax} = 10.8 MPa

$\sigma_{Ed32} = F_c / ab$

where

F_c = 955 kN

a = width of strut

= $(a_{col} - c_{nom} - 2s_o) \sin 58 + u \cos 58$

= $(400 - 25 + 2 \times (25 + \text{say } 12 + 25/2)) \sin 58 + 360 \cos 58$

= $(400 - 124) \sin 58 + 360 \cos 58$

= 234 + 191 = 425 mm

b = thickness

= 250 mm

σ_{Ed32} = $955 \times 10^3 / (425 \times 250)$

= 8.99 MPa

i.e. < 10.8 MPa

$\therefore$ OK

NB: As $\sigma_{Ed32} < \sigma_{Rdmax}$ no further checks on strut 2-3 are necessary since the stress field is fan-shaped at the ULS.

5.3.4 Ties

a) Main tie

A_s required = F_t / f_{yd}

= $506 \times 10^3 / (500 / 1.15)$

= 1164 mm^2

Try 6H16 (1206 mm^2)

Check anchorage:

Assuming straight bar

$l_{bd} = \alpha l_{brqd} = \alpha\, (\phi/4)\, (\sigma_{sd}/f_{bd})$ — Exp (8.4) & (8.3)

where

α = 1.0 (assumed)*

ϕ = diameter of bar = 16 mm

σ_{sd} = 500 / 1.15 = 435 MPa

$f_{bd} = 2.25 \eta_1 f_{ctk} / \gamma_m$

= $2.25 \times 1.0 \times 1.0 \times 1.8 / 1.5$

= 2.7 MPa — Exp (8.2)

l_{bd} = $1.0 \times (16 / 4) \times (435 / 2.7)$

= 644 mm

Average length available = $400 - 25 + \cot 58° \times 360 / 2$

= 487 mm – no good

Try 8H16 (1608)

l_{bd} = 644 x 1164/1608 = 466 mm: OK

$\therefore$8H16 OK

* Conservative assumption. As in previous example, 5.1.6, a is often as low as 0.7 due to cover and transverse compression.

5.3.5 Vertical tie steel

Vertical tie steel is required to take loads from bottom level to top level.

A_s required = (65.6 x 1.35 + 32.5 x 1.5 / (500/1.15)

= 137.3 x 10^3 / (500/ 1.15)

= 315 mm^2 / m

5.3.6 Minimum areas of reinforcement

Consider as a wall — 9.6.2.1, 9.6.3.1 & NA

A_{svmin} = 0.002 A_c

= 0.002 x 1000 x 250

= 500 mm^2 / m

Vertically, say minimum area and tie steel additive. Therefore provide

315 + 500 mm^2/m = 815 mm^2/m

Consider as deep beam — 6.2.1(9)

A_{sdbmin} = 0.2% A_c each surface: i.e. require 500 mm^2/m bw EF.* — 9.7(1) & NA

∴ Use H12@225 bw EF (502 mm^2/m each way each side)

5.3.7 Summary of reinforcement required

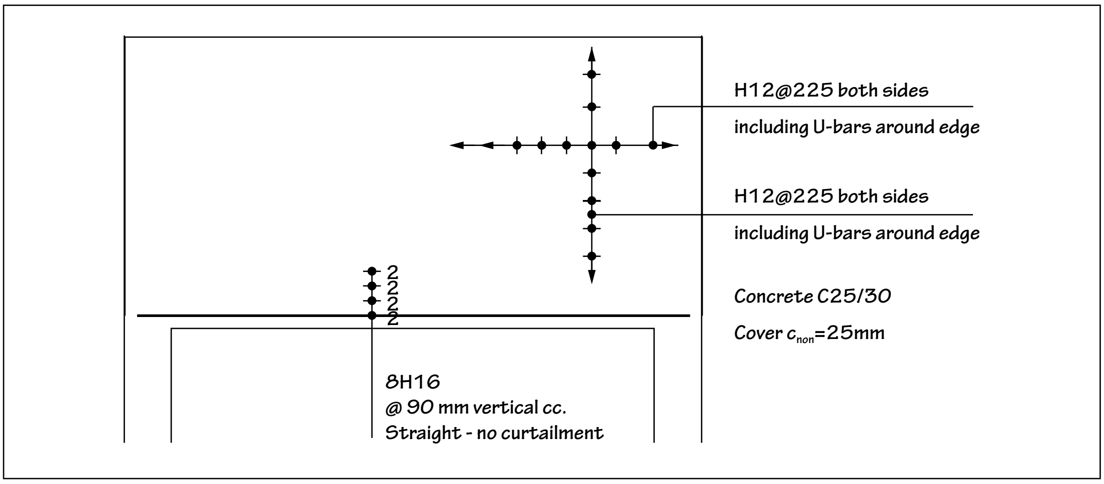


Figure 5.15: Summary of reinforcement required for deep beam 2

Note: (360/2 - 25 - 12 - 16/2) / 1.5 = 90 mm vertical centres.

* Minimum reinforcement should be provided in all cases including fan shaped stress fields. In this instance specification of Grade B or C reinforcement is considered unnecessary.

5.4 Corbel

The Concrete Centre	Project details **Corbel**	Calculated by chg	Job no. 810
		Checked by R Vetal	Sheet no. WE 3/1
		Client TCC	Date Dec 2014

Consider a corbel to carry an ultimate load of 625 kN onto a 500 x 500 column as illustrated in Figure 5.16. Assume f_{ck} = 40 MPa , f_{yk} = 500 MPa and c_{nom} = 35 mm

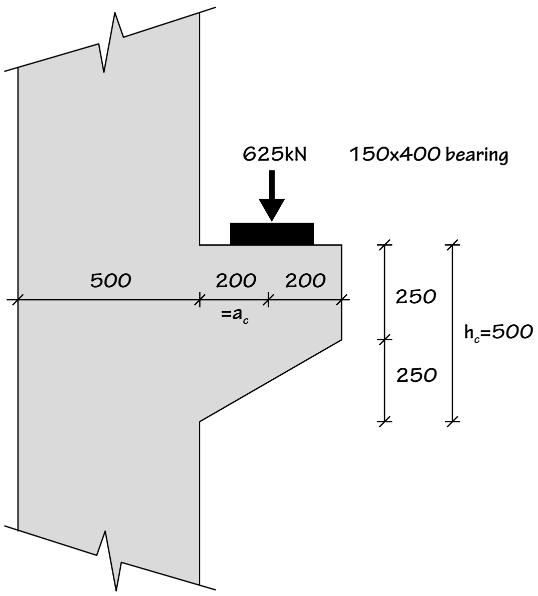


Figure 5.16: Corbel

5.4.1 Define D-region

As a_c < h_c design using strut-and-tie (rather than as cantilever).
D-regions extend 500 mm above and below corbel.

5.4.2 Proposed STM

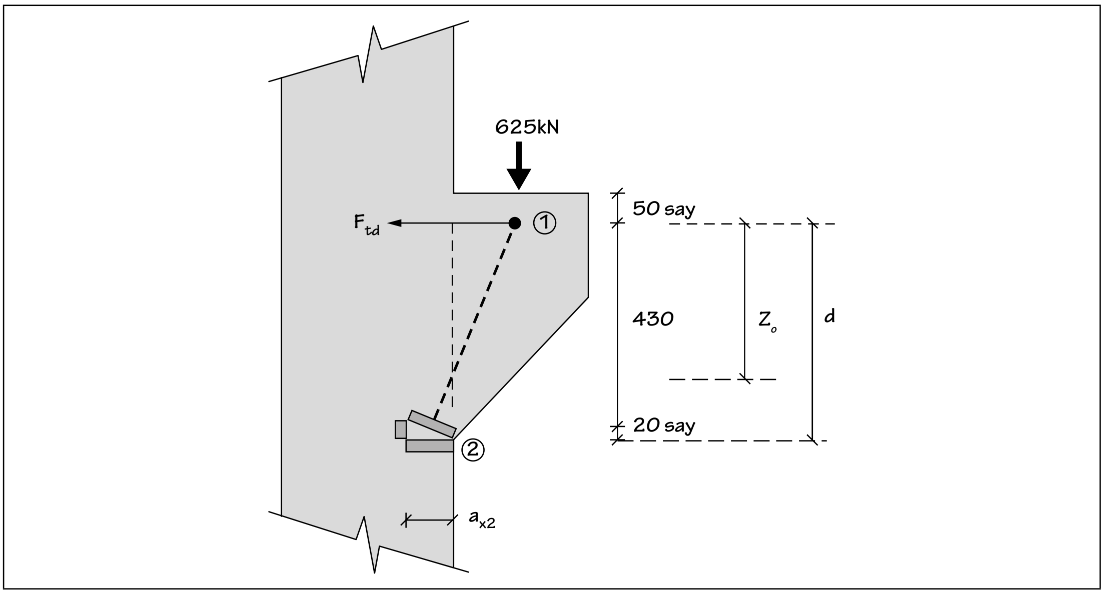


Figure 5.17: Preliminary STM*

In order to dimension the STM and calculate forces, it is advantageous to calculate the width of node 2, a_{x2}

$a_{x2} = F_{Edy} / b\ \sigma_{Rdmax}$

where

F_{Edy} = 625 KN

b = 500 mm

$\sigma_{Rdmax} = k_1 \upsilon' f_{cd}$ — PD 6687[15] B.4

where

k_1 = 1.0 for a C-C-C node assuming sufficient anti-bursting reinforcement is provided

υ' = 1 – f_{ck} / 250 = 1 – 40 / 250 = 0.84

f_{cd} = $\alpha_{cc} f_{ck} / \gamma_m$ = 0.85 x 40 / 1.5 = 22.7

σ_{Rdmax} = 1.0 x 0.84 x 22.7 = 19.1 MPa

a_{x2} = 625 x 10^3 / (500 x 19.1)

= 65.5 mm

Say 70 mm but use 35 mm to centreline of node.

a_{y2} = by similar triangles, say 70 x 235/430 = 38 mm but use 20 mm to centreline of node

∴ vertical distance = 450 - 20 = 430 mm.

∴ distance 1 – 2 = $(430^2 + (200 + 70/2)^2)^{0.5}$ = 490 mm

In order to avoid brittle failure, it is recommended that the lever arm z_o should exceed 0.75 times the effective depth d.

Here z_o = [35/(35 + 200)]. 430 = 365 mm.

∴ z_o/d = 365/450 = 0.81 - ∴ OK

* This model complies with PD 6687[15] Annex B4. In a full STM, a complementary strut extending from node 2 to a node at the inside of the radiused bend of the cantilever tie bars would be modelled. In effect this would double the load at node 2 and double dimension ax2 and provide a mirror image of the stresses shown at node 2 in Figure 5.12. Otherwise, it would have no discernible effect on the design.)

F_{12} = 625 x 490 / 430 = 712 KN
F_{td} = 712 x 235/490 = 341 kN
Unless steps are taken to avoid horizontal forces being transmitted it is considered good practice to allow an additional force of 0.2F.
i.e. 0.20 x 625 = 125
$\therefore$ F_{td} = 341 + 125 = 466 kN

5.4.3 Bearing and Node 1

Check bearing under load
σ_{Ed} = 625 x 10^3 / (400 x 150) = 10.4 MPa
Considered as a partially loaded area and assuming $A_{c0} = A_{c1}$:
$f_{Rdu} = f_{cd}$ = 22.7 MPa <u>OK</u> 6.7(2)

Check as CCT node
σ_{Rdmax} = 0.85 x 0.84 x 22.7 = 16.2 MPa <u>OK</u> Exp (6.61)

5.4.4 Check strut at node 1,

$\sigma_{Ed,2\text{-}1}$ = 712 x 10^3/(500 x $(70^2 + 38^2)^{0.5}$) = 17.8 MPa
$\sigma_{Rd,max}$ = 19.2 as before (assuming adequate transverse reinforcement) <u>OK</u>

5.4.5 Tie

$A_{sreq'd} = F_{td}/f_{yd}$
= 466 x 10^3 / (500 / 1.15)
= 1072 mm^2

<u>Try 4H20 (1256 mm^2)</u>

5.4.6 Check anchorages and radii of bends required

a) In top of corbel

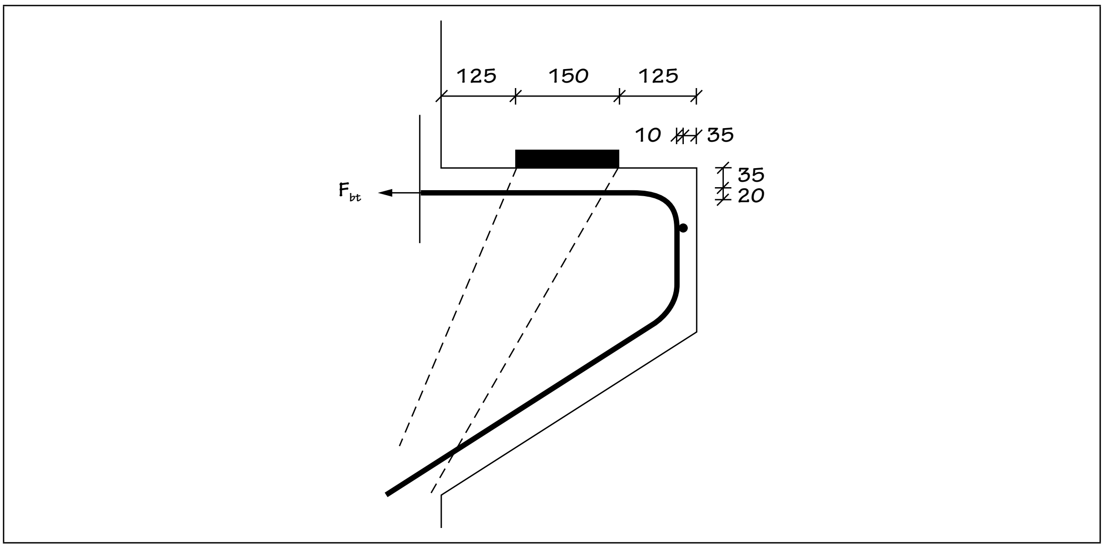


Figure 5.18: Anchorage of main tension steel

Anchorage required for H20 in C40 / 50 in 'poor' bond conditions = 820 mm both ends	Figure 8.2, PD 6687 Cl. B.4.4[15]
Find force in one bar at beginning of bend, F_{bt}:	
According to PD 6687, the straight anchorage available in the corbel is measured from the inner face of the loading plate. So assuming standard radius on bend, straight length available is:	
= 150 + 125 – 35 – 10 –20 –70 = 140 mm $\therefore F_{bt}$ = {(820 – 140)/820} x 314 x 500 / 1.5) x 1072 / 1256 = 96.6 KN	
Check mandrel diameter:	
$\phi_{mmin} \geq F_{bt} (1/a_b + 1/2\phi) / f_{cd}$ where F_{bt} = 96.6 kN a_b = half centre to centre spacing = [500 - 2 x (35 + 10 + 32) - 20] / [3 x 2] = 108/2 = 54 mm say 50 mm ϕ = bar diameter = 20 mm $f_{cd} = \alpha_{cc} f_{ck} / \gamma_m$ = 0.85 x 40 / 1.5 = 22.67 MPa $\phi_{mmin} \geq 96.6 \times 10^3 (1/50 + 1/40) / 22.67$ = 192 mm $\therefore$ radius required = 96 mm	Exp (8.1)
By inspection non-standard radius will not fit in corbel. $\therefore$ Try 4 no H20 bars with a welded transverse bar.	
Try H32 welded transverse bar:	
Capacity $F_{btd} = l_{td} \phi_t \sigma_{td} \leq F_{wd}$ where l_{td} = design length of transverse bar $= 1.16 \phi_t (f_{yd} / \sigma_{td})^{0.5} \leq l_t$ where ϕ_t = diameter of transverse bar = 32 mm σ_{td} = concrete stress $= f_{ctd} / y \leq 3 f_{cd}$ where	8.6 (2) Exp(8.8)
$f_{ctd} = \alpha_{ct} f_{ctk,0.05} / \gamma_c$ = 1 x 2.5 / 1.5 = 1.67 MPa $y = 0.015 + 0.4e^{(-0.18x)}$	3.1.6(2) Table 3.1

where $x = 2c / \phi_t + 1$ where c = nominal cover perpendicular to both bars = 35 mm $x = 3.18$, $y = 0.24$ $f_{cd} = \alpha_{cc} f_{ck} / \gamma_c$ = 0.85 x 40 / 1.5 = 22.66 MPa σ_{td} = 1.67 / 0.24 = 7.0 MPa ≤ 3 x 22.67 l_t = length of welded bar but ≤ spacing of bars to be anchored. = (500 – 2 x 87) = 326 mm ≤ 108 mm say 105 mm l_{td} = 1.16 x 32 (500 / (1.15 x 7.0)) $^{0.5}$ ≤ 105 = 293 ≤ 105 ∴ l_{td} = 105 mm F_{btd} = 105 x 32 x 7.0 ≤ F_{wd} = 0.5 x 314 x 500 / 1.15 = 23.5 KN ≤ 68.3 KN ∴ Force to be anchored F_{bt} = 95.2 – 23.5 = 71.7 kN	
Mandrel diameter required:	
$\phi_{min} = F_{bt}\ (1/a_b + 1/2\phi) / f_{cd}$ = 71.7 x 10^3 (1/50 + 1/40) / 22.67 = 142 mm diameter ∴ say standard radius, (= 70 mm,) OK, but use welded H32 welded bar in corbel.	Exp (8.1)
b) In column beyond inside reinforcement	
Anchorage required for H20 in 'good' bond conditions in C40 / 50 = 600 mm	Figure 8.2
Straight anchorage available beyond centreline of inner column bar (32 mm assumed) = 500 – (35 + 10 +32 / 2) x 2 = 378 mm ∴ bend required	PD 6687[15] B.4.4
Assume 70 mm radius Straight available = 378 – 20 – 70 = 288 mm ∴ F_{bt} = {(600 – 288)/600} x 314 x 500 / 1.15) x 1072 / 1256 = 60.6 kN	
Check mandrel diameter:	
$\phi_{mmin} \geq F_{bt}\ (1/a_b + 1/2\phi) / f_{cd}$ where	

F_{bt} = 60.6 kN
a_b = 67 mm as before
ϕ = 20 mm
f_{cd} = 22.67 MPa as before
$\phi_{mmin} \geq 60.6 \times 10^3 (1/67 + 1/40) / 22.67$
= 107 mm
∴ radius required = 53 mm
∴ standard radius bend = 3.5ϕ = 70 mm is OK

Use standard bend in column — BS 8666[16]

5.4.7 Horizontal links (bursting forces on strut)

As $a_c < 0.5h_c$ provide $0.5 \times A_{s\ req'd}$ as closed links
i.e. provide 0.5 x 1072 = 536 mm^2 in the mid 0.6 H of the strut*

∴ provide 4B10 links (8 legs ≡ 628 mm^2) — PD 6687[15] B.4.2

5.4.8 Summary of reinforced requirements

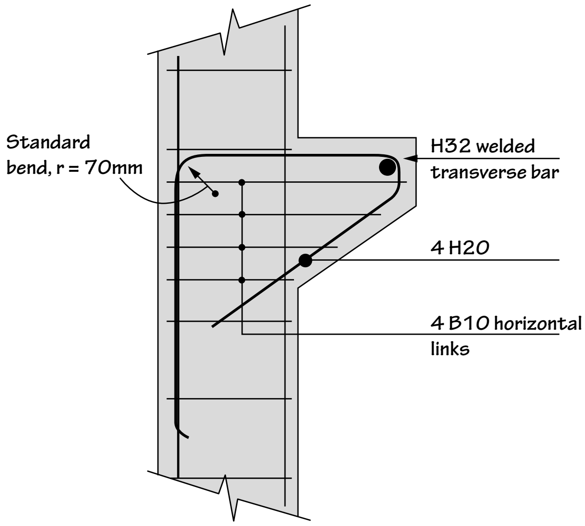


Figure 5.19: Summary of corbel reinforcement

Note:
In commercial design, it is usual to:
a) to ensure that the overall outstand of the corbel is less than 0.70 x the height.
b) to allow for construction tolerances in the position of the load.
c) in consideration of shrinkage and creep in supported precast elements, to apply a notional horizontal load of up to 20% of the vertical load (as presented).

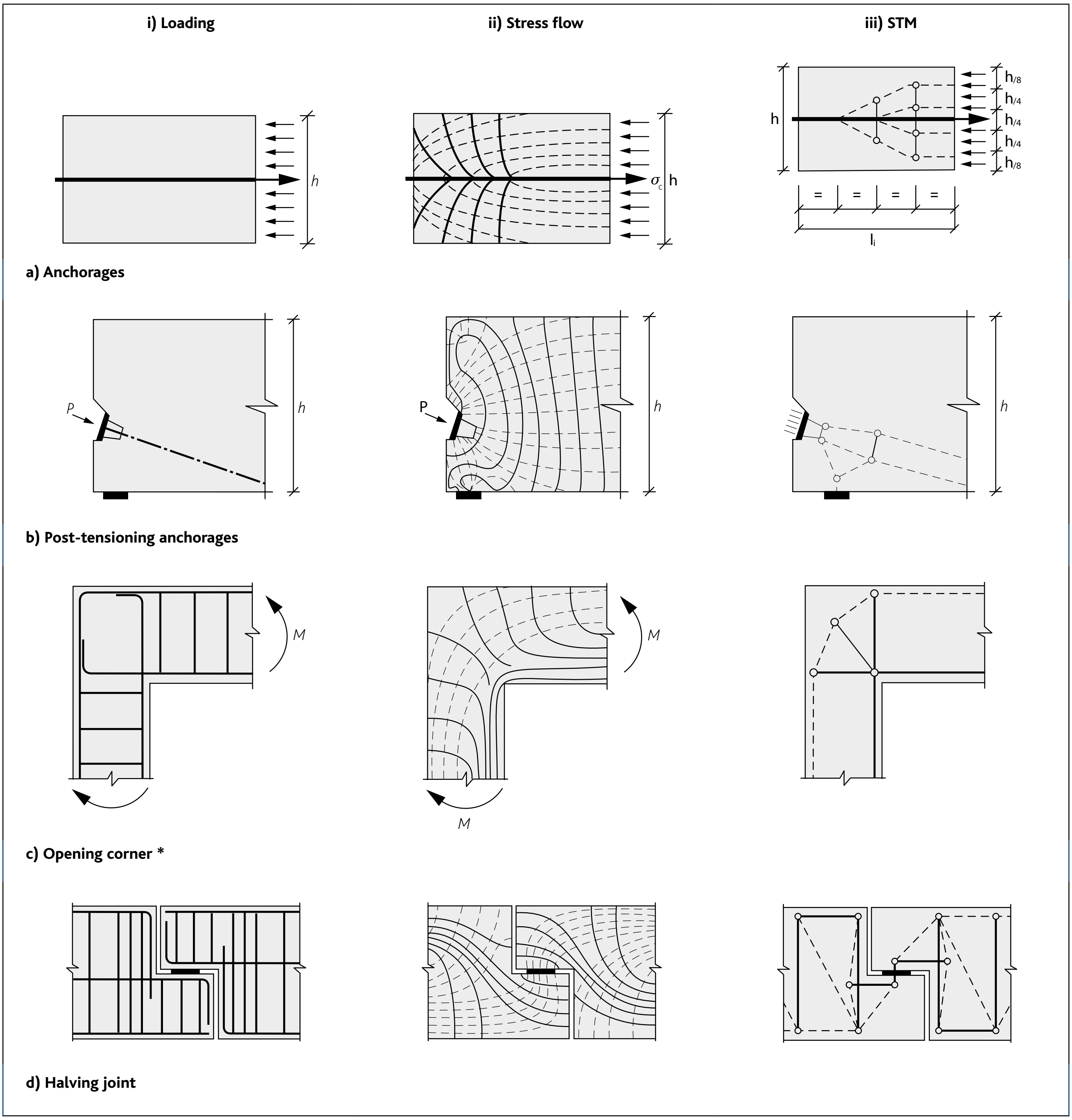
* Compared to BS 8110 which required links in the top 2/3rds of the corbel.

6. Other examples

6.1 Common examples

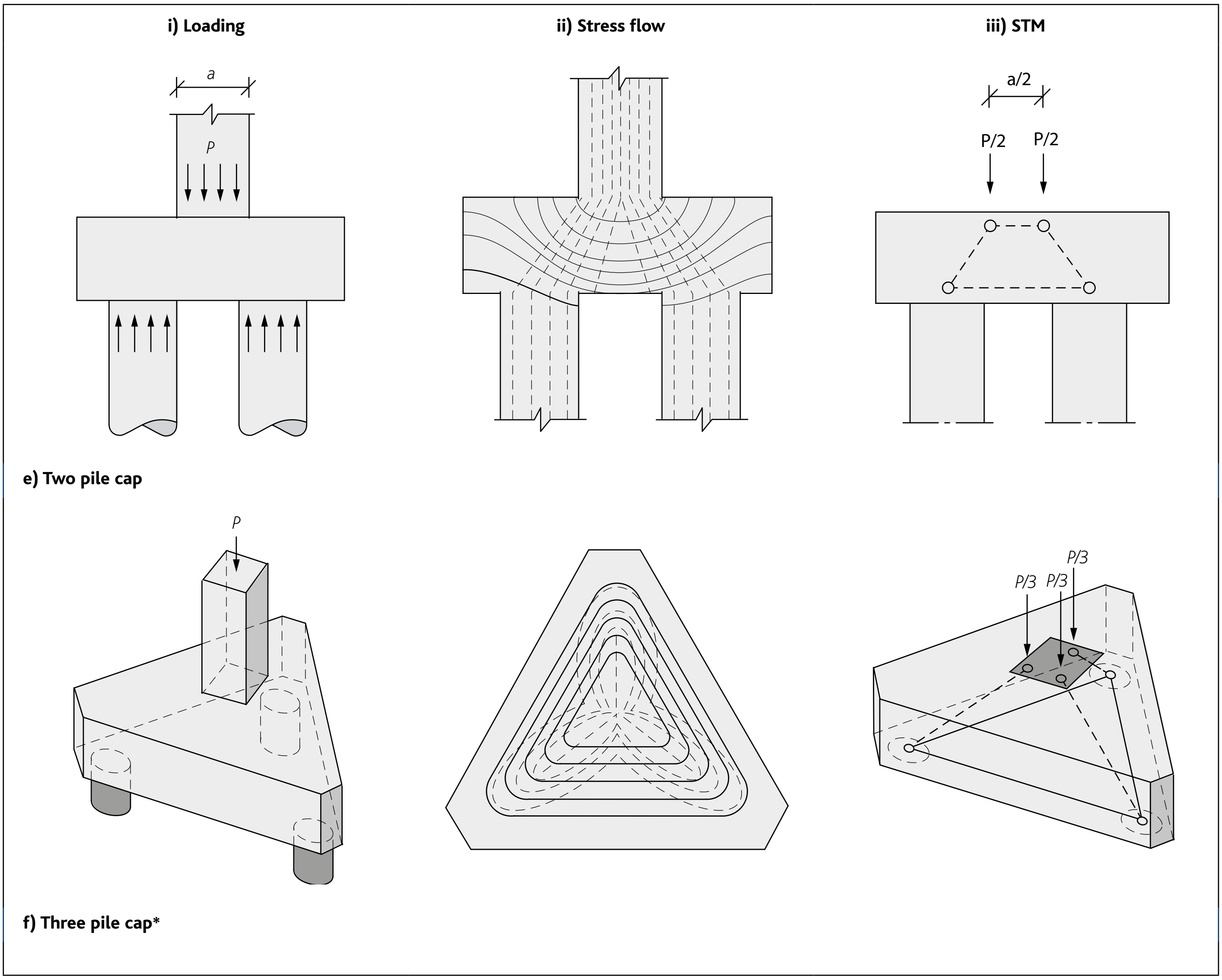
These examples show how strut-and-tie might be used to analyse and design commonly occurring discontinuities in elements or parts of structures. In each case, typical loadings, stress flows and STMs are given.

Figure 6.1a
Common examples[17]



* See also PD 6687 Figures B.2 and B.3s.

Figure 6.1b
Common examples continued [17]

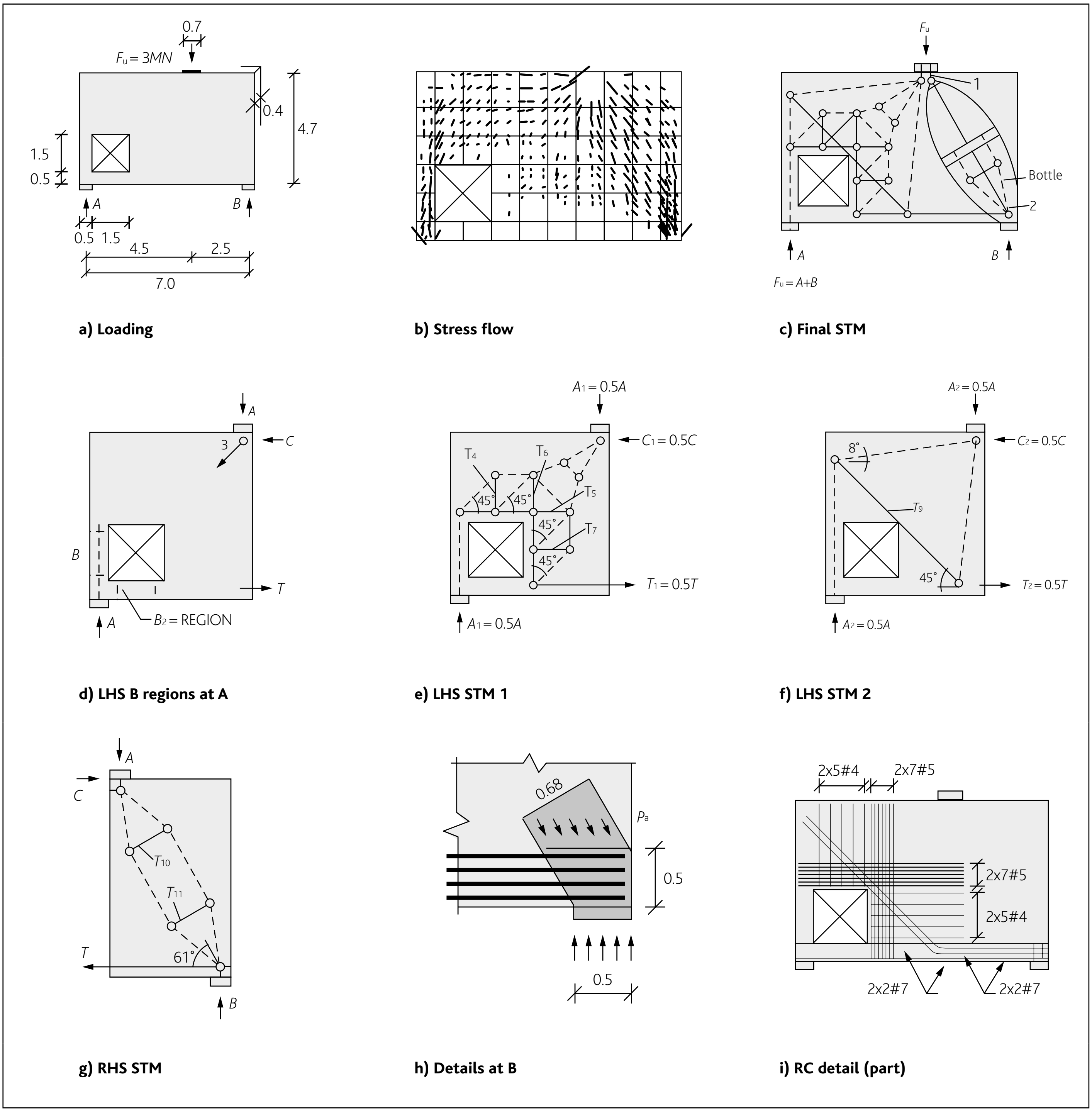


***** A study into the design of standard pile caps[19] found that there was little to choose between designing pile caps using strut-and-tie or bending theory. A basic difference is the amount of anchorage required. Also bending theory is conducive to using orthogonal reinforcement in odd numbered pile caps.

6.2 Deep beam with hole

This example illustrates how to deal with a deep beam with a significant hole. Knowing the loads and reactions, each side of the beam can be analysed in isolation. The right hand side Figure 6.2g has been treated as a simple bottle strut. The left-hand-side of the final STM is the supposition of two models Figure 6.2e and 6.2f each assumed to take 50% of the load. This gives a more realistic reinforcement arrangement and illustrates the 'art' of selecting the correct model.

Figure 6.2
Deep beam with hole[18]



6.3 Advanced examples

These examples are presented in order to illustrate the potential of STM in experienced hands. It should be understood that the stress fields would in reality be continuous rather than consisting of discrete struts and ties as shown. Modelling of the type shown is best supported by complimentary non linear* FE modelling to confirm that the assumed struts and ties are likely to develop.

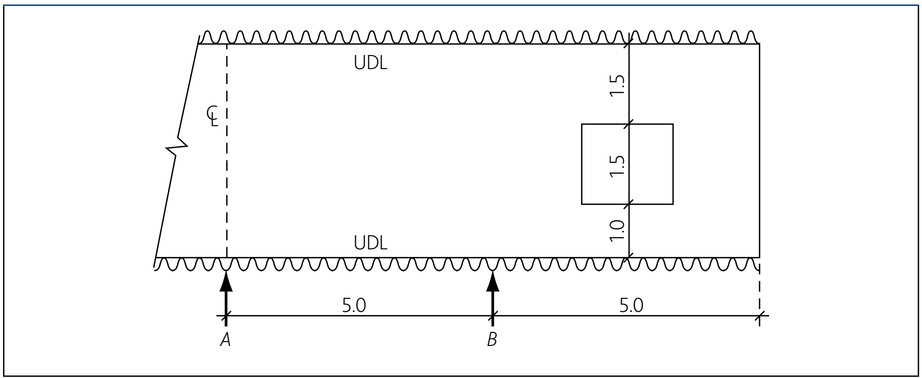
6.3.1 Cantilever deep beam with window[18]

The example consists of a 4m deep beam wall 300 mm thick that is continuous over three supports at 5m centres and with 5m cantilevers each end. The cantilever sections of the wall have a 2.0 x 1.5m window and the wall supports an ultimate UDL of 260 kN/m on its upper and lower surfaces. Due to symmetry only half the wall is analysed. f_{ck} = 30 MPa.

Initial analysis: All D-regions.

Approximate cantilever moment
at $B = wL^2/2 = 260 \times 2 \times 5^2 / 2 = 6500$ kNm
Assume lever arm = 2.75 (between centrelines above and below window)
Force = +/- 6500/2.75 = +/- 2364 kN
$A_s = 2364 \times 10^3 \times 1.15 / 500 = 5437$ mm^2
$A_c = 2364 \times 10^3 / (0.60 \times (1- 30/250) \times 0.85 \times 30/ 1.5)$
$= 2364 \times 10^3 / 8.98 = 26337$ mm^2 say 300 x 900 mm
So for initial purposes, assume that all tie members are 6000 mm^2 reinforcement and all strut members are 300 x 900.

Figure 6.3
Cantilever deep beam with window



* Elastic FE does not necessarily model cracked concrete accurately.

Using these properties the initial STM can then be drawn to scale making a judgement as to which members are likely to be in compression and which in tension.

As illustrated by Figure 6.4, this process might require a few iterations of changing properties and member configurations with a view to:

- minimising deflection
- trying to ensure that the diagonal members are in compression and that tension only occurs orthogonally
- If diagonal tension is unavoidable use area of concrete member and limit tensile strength to f_{cdt}. Some tensile capacity may be developed in the concrete but it is preferable to use reinforcement for tensile forces.

Once the system is reasonably stable then the calculated forces can be used to determine more exact member sizes, e.g. areas of ties for a range of tensions:

- 0 to 1000 kN = 1000 x $10^3/(f_{yk}/1.15)$ = 2300 mm^2 say 2500 mm^2
- 1000kN to 2000 kN = 5000 mm^2
- 2000kN to 3000 kN = 7500 mm^2

A similar procedure is used to determine the size of the concrete struts. With these new member sizes the framework can be sketched out and the forces more accurately determined. Figure 6.4 shows the iterations.

The next step is to determine the reinforcement and to check the stress in the concrete at key locations; normally this will be at the bearings or points of load application.

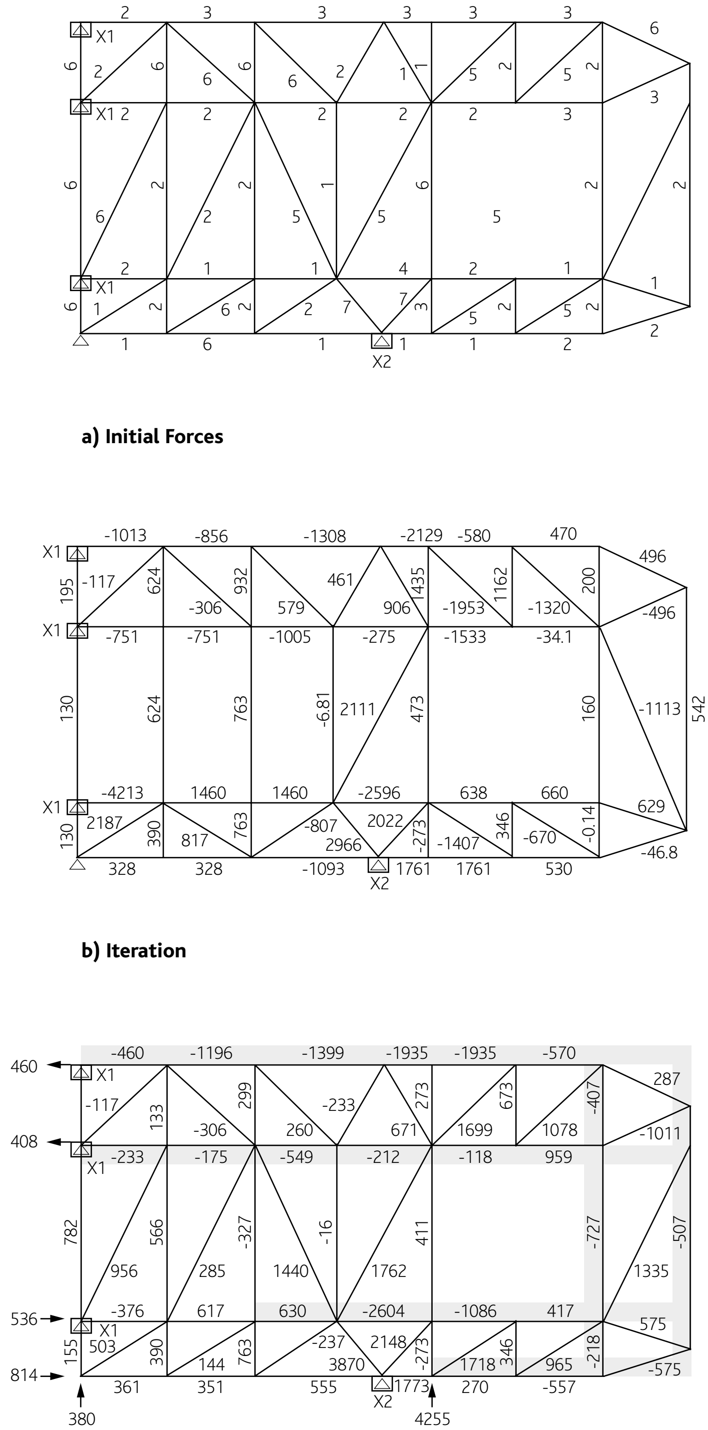
The fourth iteration (Figure 6.4c) shows the layout of the major bands of tensile reinforcement. The bars should be anchored into adjacent compression zones with anchorage lengths in accordance with Eurocode 2. In this example, to achieve an orthogonal bar arrangement, horizontal reinforcement has been provided through to the end of the cantilever. The original model would have been improved if this rectangular form had been adopted from the start.

The vertical tensions indicate the requirements for vertical reinforcement in the form of links in each zone. Elsewhere, where there are tensile forces a check should be carried out to ensure that the tensile capacity of the concrete is not exceeded. Furthermore it is advisable to use minimum reinforcement required by Eurocode 2 and possibly more.

In preference the forces in inclined ties in the top and bottom chords should be resolved into orthogonal tension steel to resist these forces.

Minimising strain energy is a key part of the solution, and it should be appreciated that it is not always good practice to fully stress the reinforcement. Extra reinforcement will reduce strain and help the serviceability condition. It is then important to use judgement.

Figure 6.4
STMs of cantilever deep beam with window



a) Initial Forces

b) Iteration

c) Fourth iteration

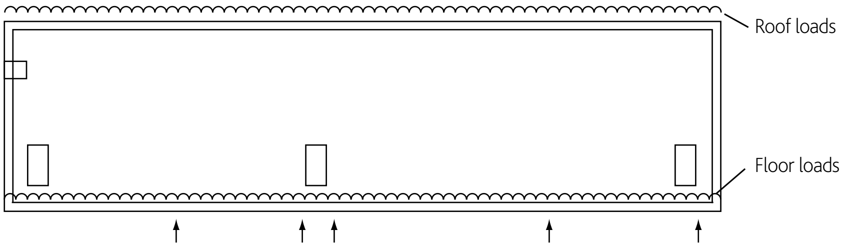
Notes:

- The use of two diagonals in each panel would have produced a clearer result. However, it is not critical in this case as only anti bursting reinforcement is required.
- The convention of using dashes to indicate struts has not been used in this Figure.

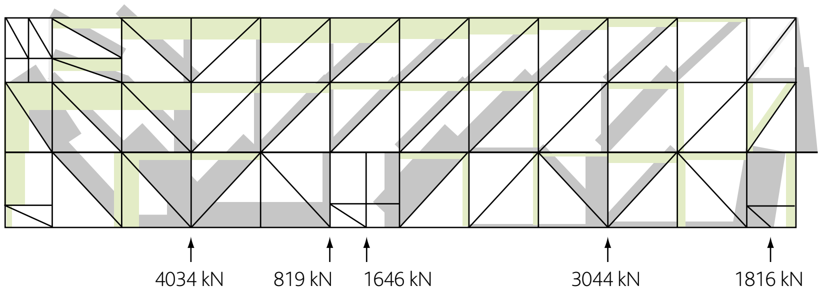
6.3.2 Wall beam

Taken from the analysis of a public building.

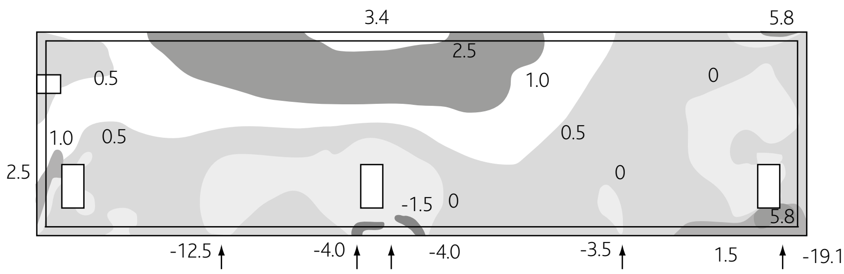
Figure 6.5
Analysis of two-storey wall beam



a) Sectional elevation on wall



b) STM: axial force distribution.
Key Tension Compression

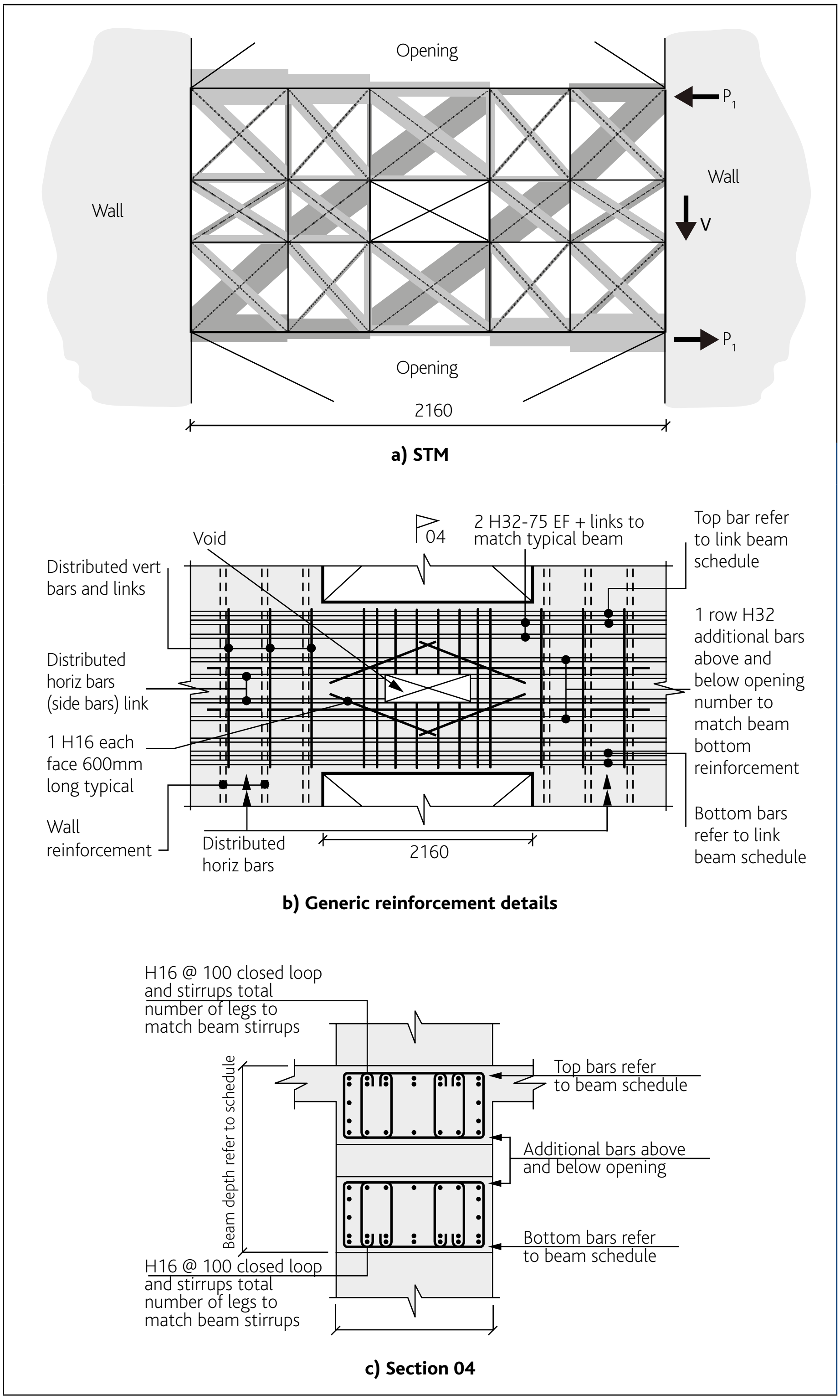


c) Principal stresses from FE analysis.

6.3.3 Coupling beam

Taken from the analysis and design of a coupling beam (with hole) within a shear wall in a 54-storey block.

Figure 6.6
Coupling beam

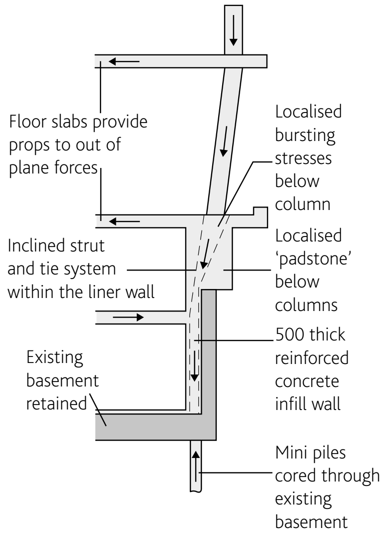


Note: Beam details: Clear span: 2160 between walls, Dimensions: 1460 deep x 1000 thick, Concrete: C70/85, Reinforcement: f_{yk} = 420 MPa. Forces (ULS): V = 6000 kN, P_1 = 7700 kN, P_2 = 10300 kN.

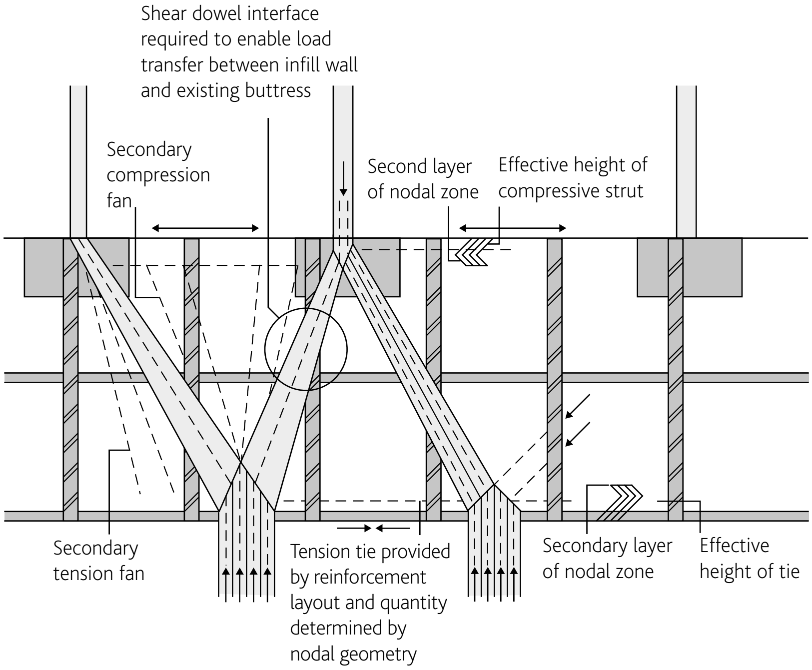
6.3.4 Basement wall

Strut-and-tie in accordance with Eurocode 2 was used on a new infill/liner wall to justify the reuse of the existing basement at No1 New York Street, Manchester[20]. "This approach… had the added benefits of easily interpretable output for the final design, allowing for the simple resolution of the reinforcement detailing". "As the design was progressed, finite element analysis was used to confirm understanding of the basement's structural behaviour. These models allowed the principle stress vectors to be seen visually and verified against the simplified strut-and-tie arrangement with due allowance made for plastic relaxation. Additional hand calculations were used to validate the design and accuracy of the modelling."

Figure 6.7
Basement at No1 New York Street, Manchester[20]



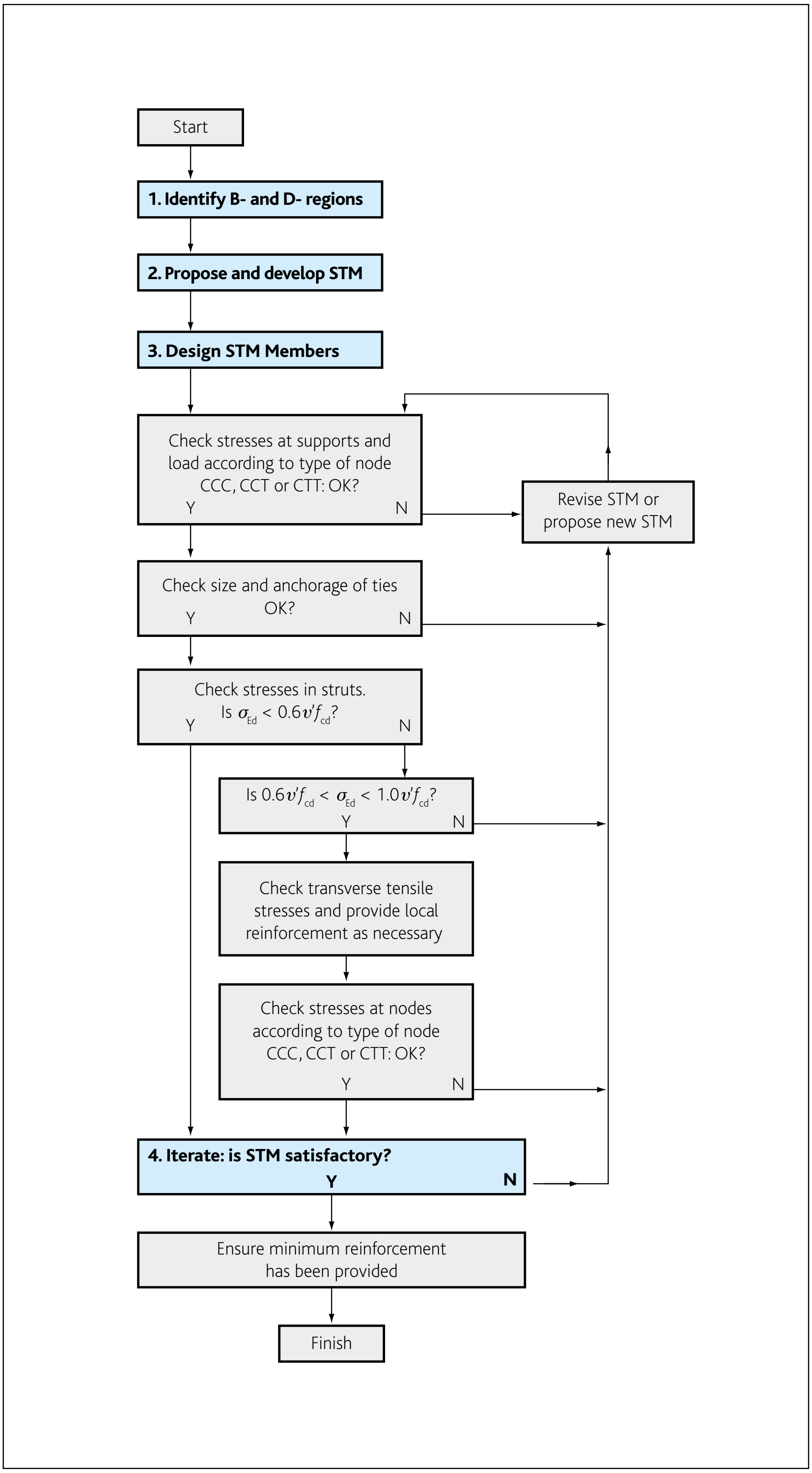
a) Principles of basement structure



b) Idealised structural model of infill basement walls

7. Flow chart

Figure 7.1
Flow chart for strut-and-tie design



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Strut-and-tie Models

This publication aims to explain strut-and-tie modelling (STM) which has become available for use under Eurocode 2.

The text gives guidance on the developing STMs and designing STM members. It provides worked examples for the common applications of pile caps, deep beams and corbels. It gives references to Eurocode 2 requirements and other relevant texts. It illustrates more advanced applications.

This introduction to STM is intended to describe how the method can be a useful tool in the analysis and design of complex reinforced concrete elements and structures.

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