

DESIGN OF CONCRETE STRUCTURES

Design Aids for Eurocode 2



**BETONVERENIGING
THE CONCRETE SOCIETY
DEUTSCHER BETON-VEREIN E.V.**

Design aids for Eurocode 2,
part 1 (ENV 1992-1-1)



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Design aids for EC2

Design of concrete structures

Design aids for ENV 1992-1-1 Eurocode 2, part 1

Betonvereniging
The Concrete Society

Deutscher Beton-Verein



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Preface

The European concrete standards in practice

The German, UK and Netherlands Concrete Societies are working together on a SPRINT project for the development of supporting tools for use with the European Structural Concrete Code. The project is in three parts essentially covering:

1. An investigation of what tools the industry needs and prefers to enable it to work with the new code.
2. The development of preferred tools.
3. Publication and dissemination of the tools developed and consideration of the possible development of further aids to the use of the code.

In the first phase, the societies questioned a wide range of practitioners about their needs and preferences for design tools. It was found that, although there is considerable interest in developing information systems through computer processes, the immediate need and preference was for a traditional “hard copy” Technical Document containing information, guidance and examples of the use of the Code.

In response, the societies concentrated efforts in the second phase into the production of such a document, which this now is. During the development of the material, an important meeting was held in Amsterdam in October 1994 when the societies were able to present draft material for examination and comment and to seek views on the direction of their work. Discussion at this meeting confirmed the earlier analysis of the industry’s immediate needs and interest in the development of other information systems for the future. Comments made on the draft at and after the meeting were subsequently considered by the societies and, where appropriate, material was modified or added.

The publication of this document marks the completion of the second phase and forms part of the final phase which will concentrate on the dissemination of the information in this document. This last phase will also involve a further examination of other methods to highlight the material that has been prepared and to consider how other tools and systems may be developed to aid industry.

Finally, it must be stressed that this document is not an alternative to the European Structural Concrete Code. It is an aid to use in conjunction with the Code to help designers in their work.

March 1996

1

General information

Dr.-Ing. H.-U.Litzner, Wiesbaden: Chairman of CEN/TC250/SC2

1.1

Construction products directive and European harmonized standards for concrete structures

The European construction market was officially established in January 1993. This means that in this market, as in other areas of the economy, goods, services, people and capital are able to move freely within the European Union (EU). An important instrument in this connection is the “Construction products directive” [1], adopted by the EU-Commission in December 1988. This directive sets out the conditions under which a construction product (e.g. cement, ready-mixed concrete, reinforcement, precast element) can be imported and exported and used for its intended purposes without impediment in EU countries. This directive has been integrated into the national legislation of most EU Member States.

“Technical specifications”—i.e. harmonized European standards, or, where these are lacking, European technical approvals—are necessary for the practical application of this directive. [Figure 1.1](#) shows the European code of practice system for concrete structures that is currently being elaborated at different levels on the basis of the Directive. This standards system will quantify requirements for concept, design, detailing and execution of structures.

According to Article 6 of the directive, a construction product may move freely within the EU provided it meets certain basic requirements. These criteria, denoted in the Directive as “Essential requirements”, primarily relate, however, to the structure into which the construction product is to be incorporated. The “Essential requirements” concern:

- mechanical resistance and stability
- safety in case of fire
- hygiene, health and the environment
- safety in use
- protection against noise
- energy economy and heat retention.

This establishes the framework for further consideration.

The “Essential requirements” are only qualitatively described in the directive text. Further European documents are needed for practical application. These include the so-called “Interpretative documents”, in which the essential requirements are defined, the previously mentioned “Technical specifications” (European harmonized standards and European guidelines for technical approval), as well as regulations for the positive assessment of the conformity of a construction product (“Certification”).

1.2

Future European code of practice for concrete structures

On the basis of provisional mandates of the EU, a code of practice for concrete structures is being established by the European Committee for Standardization (abbreviated CEN) which, in the longer term, will replace national standards. Its structure is comparable to that of existing national standards systems ([Figure 1.1](#)).

It comprises:

- a safety concept relevant to any type of construction (ENV 1991–1);
- Eurocode 1 concerning actions on structures (including traffic loads in ENV 1991–3);
- codes of practice for design and execution of structures;
- construction material standards (concrete, reinforcement, prestressing steel);
- standards for the testing of construction materials (ISO or CEN standards).

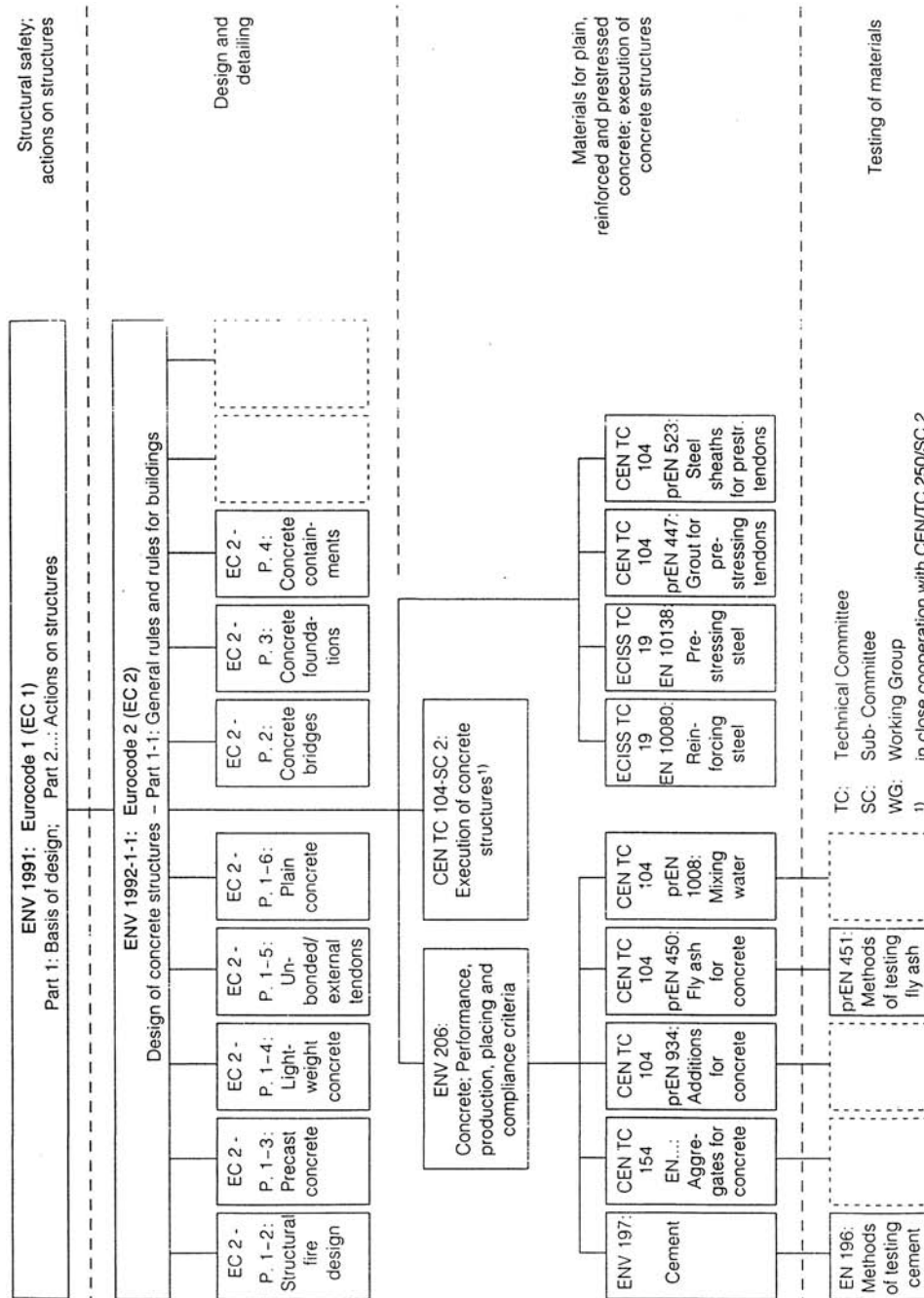


Figure 1.1 Structure of the future European harmonized standards for concrete.

From this it becomes clear that the future European standards for concrete structures are aimed at the “essential requirements”, particularly at the mechanical resistance and stability, structural fire design and safety in use, whereby the initially mentioned requirement also incorporates criteria regarding durability. This objective is also expressed in the foreword to Eurocode 2 [2] which states, among other things, the following:

“0.1 Objectives of the Eurocodes

- (1) The Structural Eurocodes comprise a group of standards for the structural and geotechnical design of buildings and civil engineering works.
- (2) They are intended to serve as reference documents for the following purposes:

- (a) As a means to prove compliance of building and civil engineering works with the essential requirements of the Construction Products Directive (CPD)
 - (b) As a framework for drawing up harmonized technical specifications for construction products.
- (3) They cover execution control only to the extent that is necessary to indicate the quality of the construction products, and the standard of the workmanship, needed to comply with the assumptions of the design rules.
- (4) Until the necessary set of harmonized technical specifications for products and for methods of testing their performance is available, some of the Structural Eurocodes cover some of these aspects in informative annexes.”

“0.2 Background to the Eurocode programme

- (1) The Commission of the European Communities (CEC) initiated the work of establishing a set of harmonized technical rules for the design of building and civil engineering works which would initially serve as an alternative to the different rules in force in the various Member States and would ultimately replace them. These technical rules became known as the ‘Structural Eurocodes’.
- (2) In 1990, after consulting their respective Member States, the CEC transferred work of further development, issue and updates of the Structural Eurocodes to CEN and the EFTA Secretariat agreed to support the CEN work.
- (3) CEN Technical Committee CEN/TC250 is responsible for all Structural Eurocodes.”

Paragraph 0.1 (2)(b) quoted above applies in particular to precast structural elements for which the CEN Technical Committee (TC) 229 is currently elaborating product standards in accordance with the 1988 Directive. These products include, for example, prestressed concrete hollow slabs and factory produced concrete masts and piles. As far as possible, the design concept is based on Eurocode 2 [2].

1.3

Safety concept relevant to any type of construction material

The outlines of the safety concept for any type of construction material in the Eurocodes are defined in the interpretative document “Mechanical resistance and stability”. [3] Based on this, ENV 1991-1 [4] explains how the satisfaction of these “Essential requirements” in accordance with the Construction products directive [1] may be verified and provides as models the ultimate limit states concept as well as serviceability limit states.

The ultimate limit states concern the danger potential associated with collapse of the structure or other forms of structural failure. Among other criteria, these include the loss of global equilibrium (transformation into a mechanism, sliding, overturning), the failure or a state before failure of parts of the structure (failure of cross-section, states of deformation, exceeding the bearing capacity), loss of stability (buckling, lateral buckling of slender beams, local buckling of plates) as well as material fatigue.

These ultimate limit states are modelled mathematically in EC2. In its chapter 4.3, the ultimate limit states are distinguished as:

- 4.3.1 ultimate limit states for bending and longitudinal force;
- 4.3.2 ultimate limit states for shear;
- 4.3.3 ultimate limit states for torsion;
- 4.3.4 ultimate limit states of punching;
- 4.3.5 ultimate limit states induced by structural deformation (buckling).

The serviceability limit states in EC2 correspond to a structural state beyond which the specified service requirements are no longer met. The corresponding models in its chapter 4.4 are:

- 4.4.2 limit states of cracking;
- 4.4.3 limit states of deformation;

as well as excessive stresses in the concrete, reinforcing or prestressing steel under serviceability conditions, which likewise can adversely affect proper functioning of a member (section 4.4.1).

1.4

Eurocode 2 for the design and execution of concrete structures

1.4.1

General

Eurocode 2 “Design of concrete structures; Part 1–1: General rules and rules for buildings” was issued as European Prestandard ENV 1992–1–1 [2] by the European Committee for Standardisation (CEN). There is no obligation to implement this Prestandard into national standard systems or to withdraw conflicting national standards.

Consequently, the first parts of the future European system of harmonized standards for concrete structures (Figure 1.1) are available in the form of ENV 1992–1–1 (EC2) and the Prestandard ENV 206 for concrete technology. The gaps, which are due to the current lack of further ENV standards, e.g. covering constituent materials for concrete, reinforcement, prestressing steel, quality control, are covered by National Application Documents (NAD). This is to enable the provisional application of the new European standards as recommended by the EU. Approval (“notification”) as a technical building regulation (guideline) by the relevant supervisory authorities has been carried out in most Member States.

1.4.2

Contents of Eurocode 2: principles and application rules: indicative numerical values

The design concept of EC2 does not differentiate between prestressed and non-prestressed structural members. Likewise, no distinction is made between full, limited or partial prestressing.

EC2 is divided into “Principles” and “Application rules”. “Principles” comprise verbally defined general requirements (e.g. regarding structural safety), to which no alternative is permitted. On the whole, these are definitions and obvious requirements which can be adopted by all EU countries. The “Application rules” are generally recognized rules (for example detailing rules) that follow the “Principles” and satisfy their requirements.

It is permissible to use alternative design rules provided that it is shown that these rules accord with the relevant “Principles” and that they are at least equivalent to those in EC2. Similar questions regarding methods have yet to be resolved. However, the principle of interchangeability of rules is generally anchored in the national codes of practice. A further characteristic of EC2 is the so-called “indicative” values, i.e. figures given as an indication (e.g. the partial factors of safety) and identified in the text by a “box”.

During an interim period, at least, they can be determined nationally by the individual EU countries. Where necessary, such modifications are given in special cases in the National Application Documents (NAD) during provisional application of EC2.

1.4.3

Essential requirements for design and execution

The essential requirements in chapter 2.1 of EC2 for design and construction stipulate among other things:

- “P(1) A structure shall be designed and constructed in such a way that:
- with acceptable probability, it will remain fit for the use for which it is required, having due regard to its intended life and its cost, and
 - with appropriate degrees of reliability, it will sustain all actions and influences likely to occur during execution and use and have adequate durability in relation to maintenance costs.”
- “P(2) A structure shall also be designed in such a way that it will not be damaged by events like explosions, impact or consequence of human errors, to an extent disproportionate to the original cause...”
- “P(4) The above requirements shall be met by the choice of suitable materials, by appropriate design and detailing and by specifying control procedures for production, construction and use as relevant to the particular project.”

With these requirements the overall framework is clearly defined into which the subsequent EC2 chapters 2.2 to 2.5 and 3 to 7 fit with their technical content (Table 1.1). Worthy of note is the fact that the durability requirement ranks high. This was one of the main reasons for the drafting of chapter 4.1 “Durability requirements” which, in the form of a sort of “checklist”, specifies the essential parameters which are to be seen in connection with durability. Attention is also drawn here to the CEN standard ENV 206 which includes important requirements for the choice of constituent materials for concrete and for the composition of concrete.

Table 1.1 Contents of Eurocode 2

| Chapter | Title |
|----------------|---------------------------------|
| 1 | Introduction |
| 2 | Basis of design |
| 2.1 | Fundamental requirements |
| 2.2 | Definitions and classifications |
| 2.3 | Design requirements |
| 2.4 | Durability |
| 2.5 | Analysis |
| 3 | Material properties |
| 4 | Section and member design |
| 4.1 | Durability requirements |
| 4.2 | Design data |
| 4.3 | Ultimate limit states |
| 4.4 | Serviceability limit states |
| 5 | Detailing provisions |
| 6 | Construction and workmanship |
| 7 | Quality control |

1.5 References

1. The Council of the European Communities: Council Directive of 21 December 1988 on the approximation of laws, regulations and administrative provisions of the Member States relating to construction products (89/106/EEC).
2. ENV 1992-1-1: 1991: Eurocode 2: Design of Concrete Structures. Part 1: General Rules and Rules for Buildings; European Prestandard. December 1991.
3. Commission of the European Communities: Interpretative Document for the Essential Requirement No. 1—Mechanical Resistance and Stability. Last version complete, July 1993.
4. ENV 1991-1-Eurocode 1: Basis of design and actions on structures. Part 1: Basis of design. Edition 1994.

Main symbols used in EC2

| | |
|--------------|--|
| A_c | Total cross-sectional area of a concrete section |
| A_{cl} | Maximum area corresponding geometrically to A_{co} , and having the same centre of gravity |
| A_{co} | Loaded area |
| $A_{ct,ext}$ | Area of concrete external to stirrups |
| $A_{c,eff}$ | Effective area of concrete in tension |
| A_k | Area enclosed within the centre-line of the idealized thin-walled cross-section including inner hollow areas |
| A_{ct} | Area of concrete within the tension zone |
| A_p | Area of a prestressing tendon or tendons |
| A_s | Area of reinforcement within the tension zone |
| A_{s2} | Area of reinforcement in the compression zone at the ultimate limit state |
| A_{sf} | Area of reinforcement across the flange of a flanged beam |
| $A_{s,min}$ | Minimum area of longitudinal tensile reinforcement |
| $A_{s,prov}$ | Area of steel provided |
| $A_{s,req}$ | Area of steel required |
| $A_{s,surf}$ | Area of surface reinforcement |
| A_{st} | Area of additional transverse reinforcement parallel to the lower face |
| A_{sv} | Area of additional transverse reinforcement perpendicular to the lower face |
| A_{sw} | Cross-sectional area of shear reinforcement |
| E_{cd} | Design value of the secant modulus of elasticity |
| $E_{c(t)}$ | Tangent modulus of elasticity of normal weight concrete at a stress of $\sigma_c=0$ and at time t |
| $E_{c(28)}$ | Tangent modulus of elasticity of normal weight concrete at a stress of $\sigma_c=0$ and at 28 days |
| E_{cm} | Secant modulus of elasticity of normal weight concrete |
| $E_{c,nom}$ | Either the mean value of E_{cm} or The corresponding design value E_{cd} |
| $E_{d,dst}$ | Design effects of destabilising actions |
| $E_{d,stb}$ | Design effects of stabilising actions |
| E_s | Modulus of elasticity of reinforcement or prestressing steel |
| F_c | Force due to the compression block at a critical section at the ultimate limit state |
| ΔF_d | Variation of the longitudinal force acting in a section of flange within distance a_c |
| F_{px} | Ultimate resisting force provided by the prestressing tendons in a cracked anchorage zone |
| $F_{sd,sup}$ | Design support reaction |
| F_s | Force in the tension reinforcement at a critical section at the ultimate limit state |
| F_s | Tensile force in longitudinal reinforcement |
| F_v | Vertical force acting on a corbel |
| $G_{d,inf}$ | Lower design value of a permanent action |
| $G_{d,sup}$ | Upper design value of a permanent action |
| G_{ind} | Indirect permanent action |
| $G_{k,inf}$ | Lower characteristic value of a permanent action |
| $G_{k,sup}$ | Upper characteristic value of a permanent action |
| $G_{k,j}$ | Characteristic values of permanent actions |
| H_c | Horizontal force acting at the bearing on a corbel |
| H_{fd} | Additional horizontal force to be considered in the design of horizontal structural elements, when taking account of imperfections |
| ΔH_j | Increase in the horizontal force acting on the floor of a frame structure, due to imperfections |

| | |
|-----------------|---|
| ΔM_{Sd} | Reduction in the design support moment for continuous beams or slabs, due to the support reaction $F_{sd,sup}$, when the support provides no restraint to rotation |
| I_b | Moment of inertia (gross section) of a beam |
| I_c | Second moment of area of a concrete section |
| I_{col} | Moment of inertia (gross section) of a column |
| $J(t, t_0)$ | Creep function at time t |
| K_1 | Reduction factor for the calculation of the second order eccentricity e_2 |
| K_2 | Coefficient, taking account of decrease in curvature ($1/r$) due to increasing axial force |
| M_{Rd} | Design resisting moment |
| M_{sd} | Design value of the applied internal bending moment |
| M_{sd1} | First order applied moment |
| N_{pd} | Prestressing force corresponding to initial value without losses |
| N_{Rd} | Resisting design axial compression force |
| N_{sd} | Design value of the applied axial force (tension or compression) |
| N_{ud} | Design ultimate capacity of the section subjected to axial load only |
| $P_{m,t}$ | Mean value of the prestressing force at time t , at any point distance x along the member |
| P_o | Initial force at the active end of the tendon immediately after stressing |
| Q_{ind} | Indirect variable action |
| $Q_{k,1}$ | Characteristic value of one of the variable actions |
| $Q_{k,i}$ | Characteristic values of the other variable actions |
| T_{Sd} | Design value of the applied torsional moment |
| V_{ccd} | Force component in the compression zone, parallel to V_{od} , of elements with variable depth |
| V_{cd} | Design shear contribution of the concrete section |
| V_{od} | Design shear force in the section, uncorrected for effects of variable section depth |
| V_{pd} | Force component due to inclined prestressing tendons |
| V_{Rd1} | Design shear resistance of a section in elements without shear reinforcement |
| V_{Rd2} | Maximum design shear force that can be carried without web failure |
| $V_{rd2,red}$ | Reduced value of V_{Rd2} , due to axial force |
| V_{Rd3} | Design shear resistance of a section, in elements with shear reinforcement |
| V_{Rds} | Total resistance to flexural and punching shear |
| V_{Sd} | Design value of the applied shear force at the ultimate limit state |
| V_{id} | Force component in the tensile zone, parallel to V_{od} , in elements with variable depth |
| V_{wd} | Contribution of shear reinforcement |
| a | Horizontal clear distance between two parallel laps |
| a_1 | Horizontal displacement of the envelope line of the tensile force (shift rule) |
| a_c | Distance between the point of application of the applied vertical load and the face of the supporting member (corbel design) |
| a_d | Design values of geometrical data |
| a_{nom} | Nominal value of geometrical data |
| a_v | Distance between points of zero and maximum moment |
| b | Overall width of a cross-section or Actual flange width in a T or L beam or Lateral concrete cover in the plane of a lap |
| b_{eff} | Effective flange width of a T or L beam |
| b_{sup} | Breadth of a support |
| b_t | Mean width of a beam in tension zone |
| b_w | Width of the web on T, I or L beams |
| $b_{w,nom}$ | Nominal web thickness |
| c | Minimum concrete cover |
| d | Effective depth of a cross-section |
| d_{crit} | Distance of critical section for punching shear from the centroid of a column |
| d_g | Largest nominal maximum aggregate size |
| e_2 | Second order eccentricity |

| | |
|------------------|--|
| e_a | Additional eccentricity covering the effects of geometrical imperfections |
| e_e | Equivalent eccentricity |
| e_o | First order eccentricity |
| e_{o1}, e_{o2} | Values of the first order eccentricity of the axial load at the ends of the member, denoted so that $ e_{o1} $ $ e_{o2} $ |
| e_{tot} | Total eccentricity |
| e_y | Eccentricity in the y-direction |
| e_z | Eccentricity in the z-direction |
| f_{bd} | Design value for ultimate bond stress |
| f_c | Compressive strength of concrete |
| f_{cd} | Design value of concrete cylinder compressive strength |
| f_{ck} | Characteristic compressive cylinder strength of concrete at 28 days |
| f_{cm} | Mean value of concrete cylinder compressive strength |
| $f_{ct,eff}$ | The tensile strength of the concrete effective at the time when cracks are expected |
| f_{ctk} | Characteristic axial tensile strength of concrete |
| $f_{ctk,0.05}$ | Lower characteristic tensile strength (5% fractile) |
| $f_{ctk,0.95}$ | Upper characteristic tensile strength (95% fractile) |
| f_{ctm} | Mean value of axial tensile strength of concrete |
| f_p | Tensile strength of prestressing steel |
| f_{pk} | Characteristic tensile strength of prestressing steel |
| $f_{p0.1}$ | 0.1% proof stress of prestressing steel |
| $f_{p0.1k}$ | Characteristic 0.1% proof-stress of prestressing steel |
| $f_{0.2k}$ | Characteristic 0.2% proof-stress of reinforcement |
| f_t | Tensile strength of reinforcement |
| f_{tk} | Characteristic tensile strength of reinforcement |
| f_y | Yield strength of reinforcement |
| f_{yd} | Design yield strength of reinforcement |
| f_{yk} | Characteristic yield strength of reinforcement |
| f_{ywd} | Design yield strength of shear reinforcement |
| h | Overall depth of a cross-section |
| h_c | Overall depth of a corbel at the face of the supporting member |
| h_f | Overall depth of a flange in T or L beams |
| h_H | Depth of an enlarged column head |
| k | Coefficient which allows for the effects of non-uniform self-equilibrating stresses |
| k_c | Stress distribution coefficient |
| k_1 | Coefficient to take account of the influence of the bond properties of bar on the crack spacing |
| k_2 | Coefficient to take account of the influence of the form of the strain distribution on the crack spacing |
| k_A or k_B | Coefficients describing the rigidity of restraint at the column ends |
| l | Length or Span or Total height of a structure in metres |
| l_{col} | Height of column measured between idealized centres of restraint |
| l_{eff} | Effective span of beams and slabs |
| l_n | Clear distance between the faces of the supports |
| l_0 | Length of span(s) between points of zero moment |
| l_{ot} | Length of a compression flange measured between lateral supports |
| l_b | Basic anchorage length for reinforcement |
| $l_{b,min}$ | Minimum anchorage length |
| $l_{b,net}$ | Required anchorage length |
| l_{ba} | Anchorage length over which the ultimate tendon force (F_{pu}) in pre-tensioned members is fully transmitted to the concrete |
| l_{bp} | Transmission length, over which the prestressing force from a pre-tensioned tendon is fully transmitted to the concrete |
| l_{bpd} | Design value for transmission length |
| l_{bpo} | Length of a neutralized zone at the ends of pre-tensioned members, in the case of sudden release |

| | |
|------------------------------|---|
| $l_{p,eff}$ | Dispersion length, over which the concrete stresses gradually disperse to a linear distribution across the section (effective transfer) |
| l_s | Necessary lap length |
| $l_{s,min}$ | Minimum lap length |
| l_x, l_y | Spans between columns on the x - and y -directions respectively |
| m_{Sdx}, m_{Sdy} | Minimum design bending moments in the x - and y -directions respectively |
| n | Number of transverse bars along anchorage length or Number of vertical continuous members acting together |
| n_1 | Number of layers with bars anchored at the same point |
| n_2 | Number of bars anchored in each layer |
| n_b | Number of bars in a bundle |
| p | Mean transverse pressure (N/mm ²) over the anchorage length |
| $1/r$ | Curvature at the critical section at the base of a model column |
| s | Spacing of stirrups |
| s_1 | Spacing of longitudinal wires in a welded mesh fabric, or in surface reinforcement |
| s_f | Spacing of reinforcing bars across the flange of flanged beams |
| s_{max} | Maximum longitudinal spacing of successive series of stirrups |
| s_{rm} | Average final crack spacing |
| s_t | Spacing of transverse wires in a welded mesh fabric or in surface reinforcement |
| s | Snow load according to Eurocode 1 |
| t | Thickness of a supporting element |
| u_k | Circumference of area A_k |
| u | Perimeter of critical section for punching shear or Perimeter of concrete cross-section |
| v_{Rd1} | Design shear resistance per unit length of the critical perimeter, for a slab without shear reinforcement |
| v_{Rd2} | Maximum design shear resistance per unit length of the critical perimeter, for a slab with shear reinforcement |
| v_{Rd3} | Design shear resistance per unit length of the critical perimeter, for a slab with shear reinforcement |
| v_{Sd} | Shear force per unit length along critical section |
| w_k | Design or characteristic crack width |
| x | Neutral axis depth |
| z | Lever arm of internal forces |
| Z_{cp} | Distance between the centre of gravity of the concrete section and the tendons |
| α | Reduction factor for concrete compressive strength or Angle of the shear reinforcement with the longitudinal reinforcement (main steel) or E_s/E_{cm} |
| α_a | A coefficient for determining the effectiveness of anchorages |
| α_1 | Coefficient for effectiveness of laps |
| α_2 | Coefficient for the calculation of the lap length of welded mesh fabrics |
| β | Coefficient taking account of the effects of eccentricity of load or Coefficient relating the average crack width to the design width or l_o/l_{col} or Shear force enhancement coefficient |
| β_1 | Coefficient taking account of the influence of the bond properties of bar on the average strain |
| β_2 | Coefficient taking account of the influence of the duration of the loading or of repeated loading on the average strain |
| β_b | Coefficient relating transmission length of prestressing tendons to concrete strength |
| γ_A | Partial safety factor for accidental actions A |
| γ_c | Partial safety factor for concrete material properties |
| $\gamma_{G,inf}$ | Partial safety factor for permanent actions, in calculating the lower design values |
| $\gamma_{G,sup}$ | Partial safety factor for permanent actions, in calculating the upper design values |
| $\gamma_{GA}, \gamma_{GA,j}$ | Partial safety factor for permanent actions, for accidental design situations |
| $\gamma_{G,j}$ | Partial safety factor for any permanent action j |
| γ_M | Partial safety factor for a material property, taking account of uncertainties in the material property itself and in the design model used |
| γ_P | Partial safety factor for actions associated with prestressing, P |

| | |
|------------------------|---|
| $\gamma_{Q,i}$ | Partial safety factor for any variable action i |
| $\gamma_{Q,1}$ | Partial safety factor for the basic most unfavourable variable action |
| γ_s | Partial safety factor for the properties of reinforcement or prestressing steel |
| Δa | Change made to nominal geometrical data for particular design purposes (e.g. assessment of effects of imperfections) |
| δ | Ratio of redistributed moment to the moment before redistribution |
| ϵ_c | Compressive strain in the concrete |
| ϵ_{c1} | Compressive strain in the concrete at the peak stress f_c |
| ϵ_{cu} | Ultimate compressive strain in the concrete |
| ϵ_{sm} | Strain in the reinforcement taking account of tension stiffening |
| ϵ_{s1} | Strain in tension reinforcement, for section analysis |
| ϵ_{s2} | Strain in compression reinforcement, for section analysis |
| ϵ_{pm} | Steel strain corresponding to $P_{m,t}$ |
| ϵ_{yd} | Design yield strain of the steel reinforcement |
| θ | Angle between the concrete struts and the longitudinal axis or Sum of angular displacement over a distance x (irrespective of direction or sign) |
| λ | Slenderness ratio |
| λ_{crit} | Critical slenderness ratio |
| μ | Coefficient of friction between the tendons and their ducts |
| ν | Angle of inclination of a structure, assumed in assessing the effects of imperfections or Efficiency factor or Coefficient relating the average design compressive stress in struts to the design value of concrete compressive strength (f_{cd}) |
| ρ_l | Equivalent longitudinal reinforcement ratio |
| ρ_{lx} | Longitudinal reinforcement ratio in x -direction |
| ρ_{ly} | Longitudinal reinforcement ratio in y -direction |
| ρ_r | Effective reinforcement ratio |
| ρ_w | Reinforcement ratio for shear reinforcement |
| σ_c | Compressive stress in the concrete |
| σ_{cu} | Compressive stress in the concrete at the ultimate compressive strain |
| σ_{cg} | Stress in the concrete adjacent to the tendons, due to self-weight and any other permanent actions |
| σ_{cpo} | Initial stress in the concrete adjacent to the tendons, due to prestress |
| $\sigma_{o,max}$ | Maximum stress applied to a tendon |
| σ_{pmo} | Stress in the tendon immediately after stressing or transfer |
| σ_{pgo} | Initial stress in the tendons due to prestress and permanent actions |
| σ_s | Stress in the tension reinforcement calculated on the basis of a cracked section |
| σ_{sr} | Stress in the tension reinforcement calculated on the basis of a cracked section under conditions of loading leading to formation of the first crack |
| τ_{Rd} | Basic shear strength of members without shear reinforcement |
| $\varphi(\infty, t_o)$ | Final value of creep coefficient |
| \emptyset | Diameter of a reinforcing bar or of a prestressing duct |
| \emptyset_n | Equivalent diameter of a bundle of reinforcing bars |
| \emptyset_s | Adjusted maximum bar diameter |
| \emptyset_s^* | Unadjusted maximum bar diameter (Table 4.11) |
| ψ | Factors defining representative values of variable actions |
| ψ_0 | Used for combination values |
| ψ_1 | Used for frequent values |
| ψ_2 | Used for quasi-permanent values |

3

Overview of flow charts

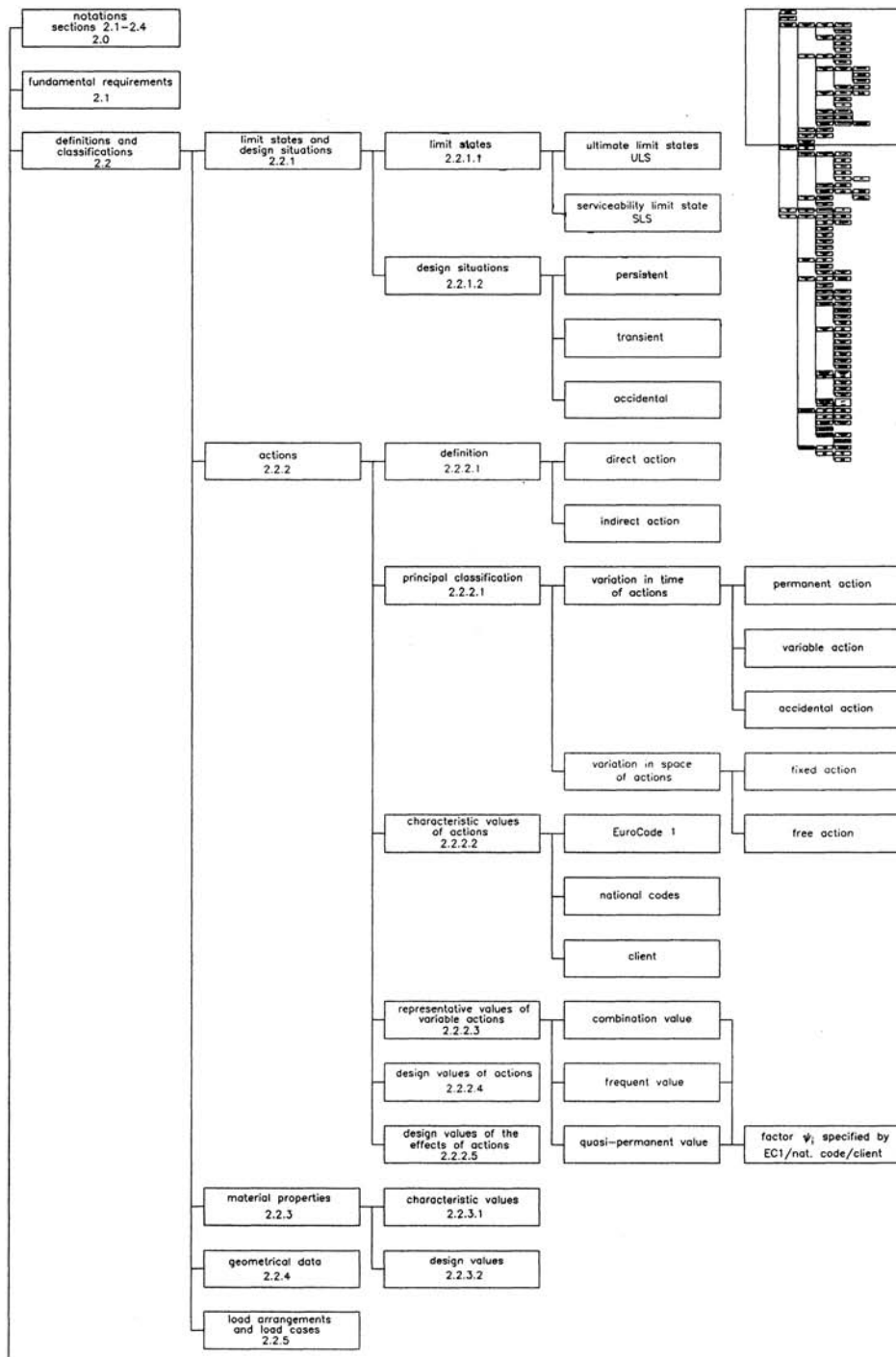
The flow charts function as a guide through Eurocode 2. The cross-references used in the flow charts therefore refer to Eurocode 2.

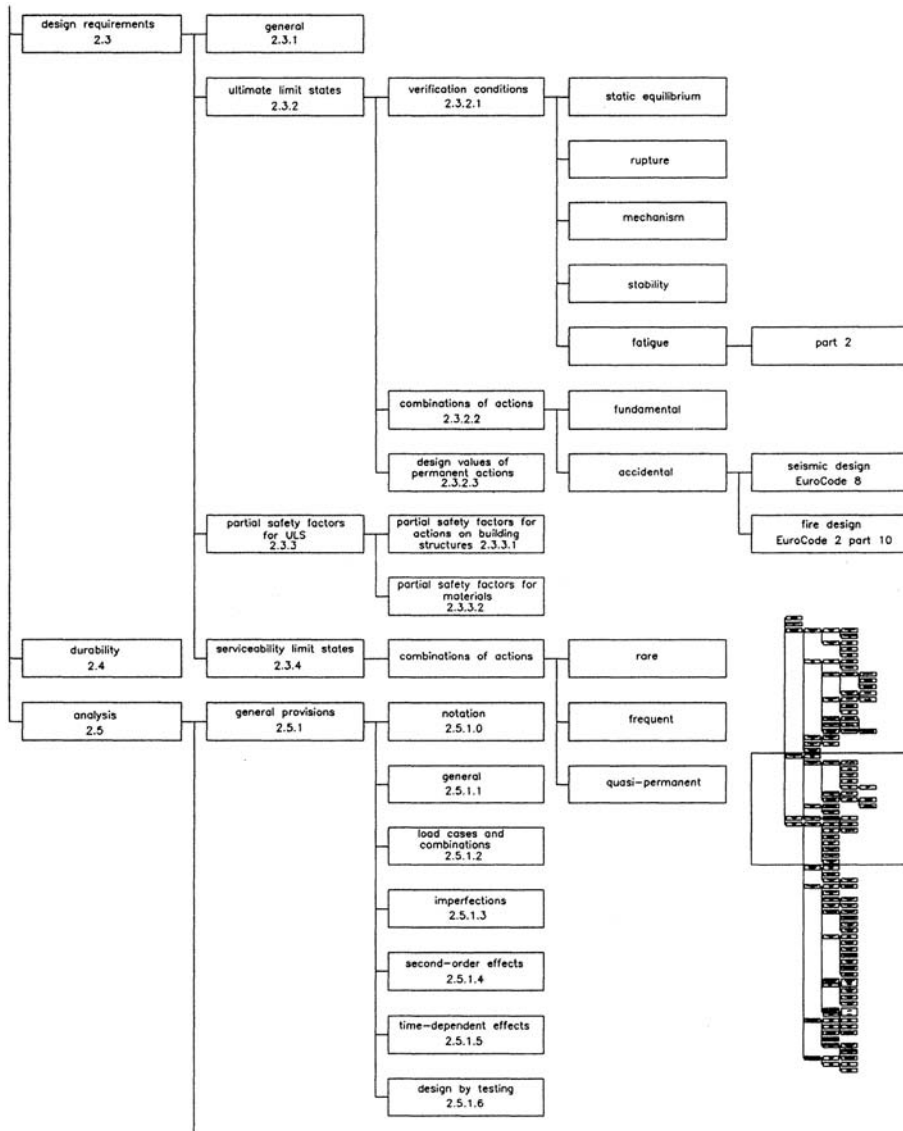
There are three main levels of flow charts.

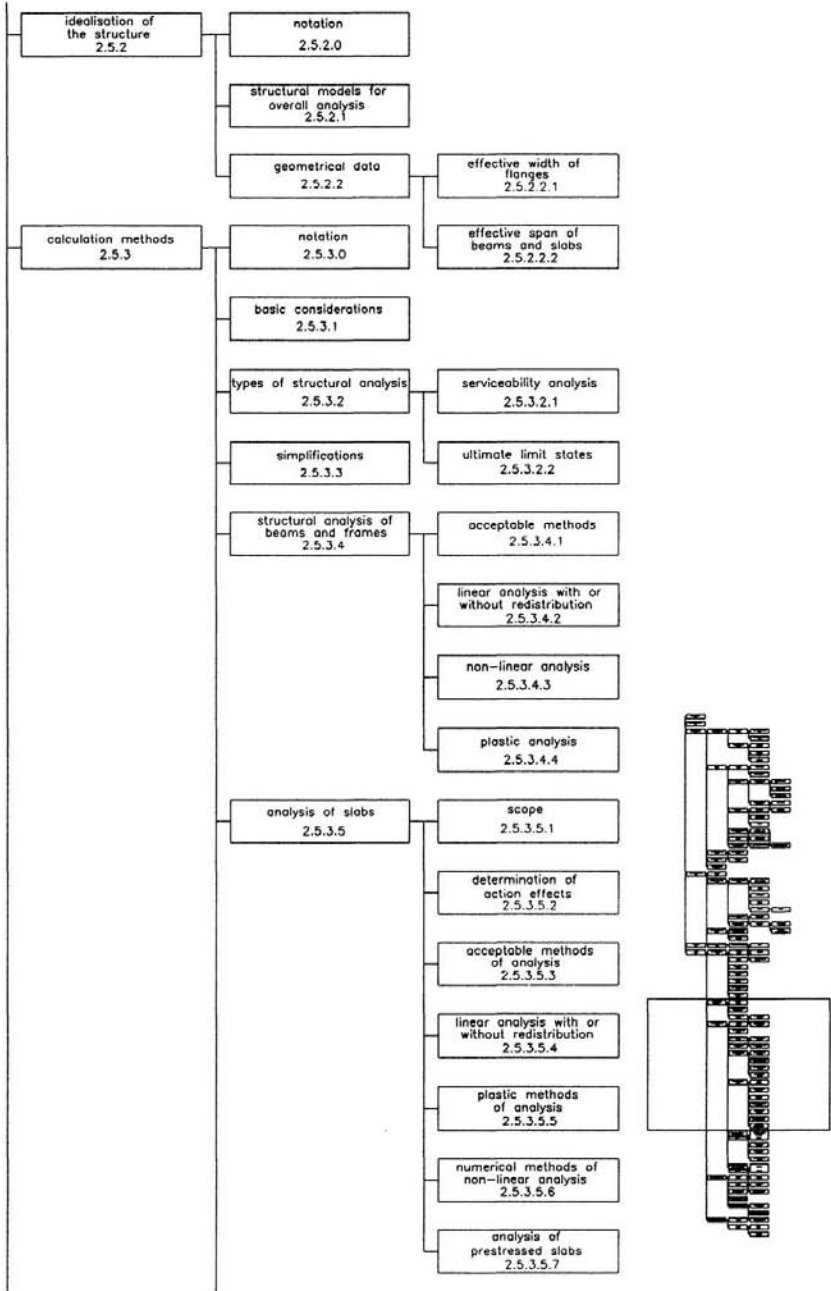
| | | | |
|----------------|--|--------------------------------------|-------|
| Level 1 | Basis of design | | 2. |
| | Flow chart 3.0 | Overview | |
| Level 2 | Section and member design | | 4. |
| | Flow chart 3.0.1 | General | |
| | Flow chart 3.0.2 | Ultimate limit states (ULS) | |
| | Flow chart 3.0.3 | Serviceability limit states (SLS) | |
| Level 3 | Detailed calculations and detailing provisions | | 4. |
| Level 3.1 | ULS | | 4.3 |
| Level 3.1.1 | Bending | | 4.3.1 |
| | Flow chart 3.1.1.1 | Bending and longitudinal force | |
| Level 3.1.2 | Shear | | 4.3.2 |
| | Flow chart 3.1.2.1 | Design method | |
| | Flow chart 3.1.2.2 | Elements with shear reinforcement | |
| Level 3.1.3 | Torsion | | 4.3.3 |
| | Flow chart 3.1.3.1 | Pure torsion | |
| | Flow chart 3.1.3.2 | Torsion, combined effects of actions | |
| | Flow chart 3.1.3.3 | Torsion and flexure | |
| | Flow chart 3.1.3.4 | Torsion and shear | |
| Level 3.1.4 | Punching | | 4.3.4 |
| | Flow chart 3.1.4.1 | Punching | |
| | Flow chart 3.1.4.2 | Punching shear reinforcement | |
| Level 3.1.5 | Buckling | | 4.3.5 |
| | Flow chart 3.1.5.1 | General guide | |
| | Flow chart 3.1.5.2 | Structure as a whole | |
| | Flow chart 3.1.5.3 | Isolated columns | |
| Level 3.2 | SLS | | 4.4 |
| Level 3.2.1 | Stresses | | 4.4.1 |
| | Flow chart 3.2.1.1 | Limitation of stresses | |
| Level 3.2.2 | Cracking | | 4.4.2 |
| | Flow chart 3.2.2.1 | Minimum reinforcement | |
| | Flow chart 3.2.2.2 | With or without calculation | |
| Level 3.2.3 | Deformations | | 4.4.3 |
| | Flow chart 3.2.3.1 | Deformation without calculation | |
| | Flow chart 3.2.3.2 | Deformation by calculation | |
| Level 3.3 | Detailing | | 5. |
| Level 3.3.1 | Anchorage | | 5.2.3 |
| | Flow chart 3.3.1.1 | General | |
| Level 3.3.2 | Splices | | 5.2.4 |
| | Flow chart 3.3.2.1 | Splices for bars or wires | |
| | Flow chart 3.3.2.2 | Splices for welded mesh fabrics | |

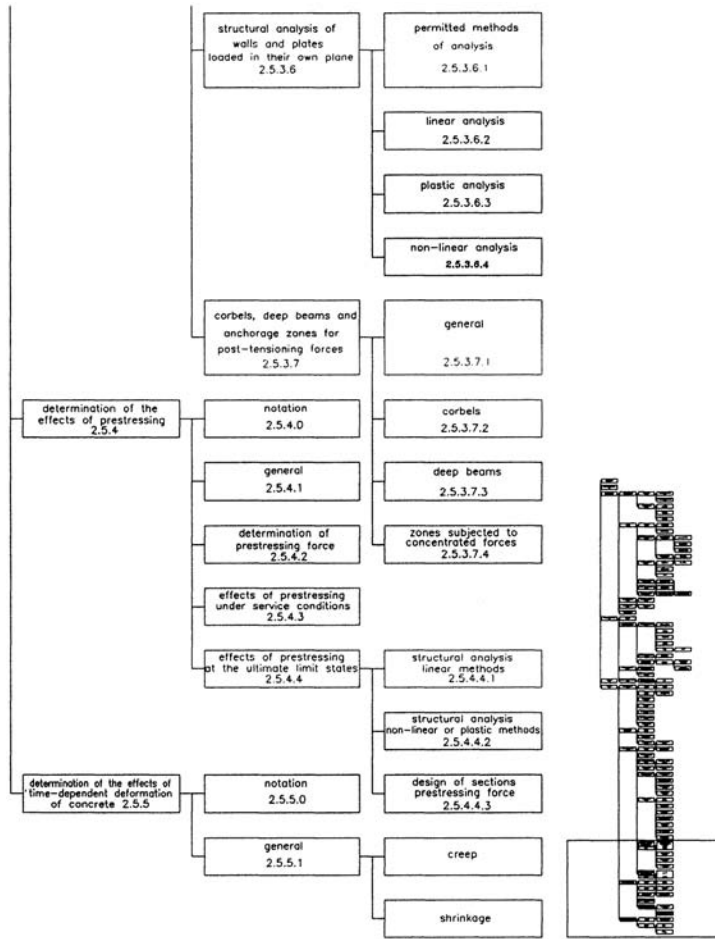
Flowchart 3.0

Basis of design: overview

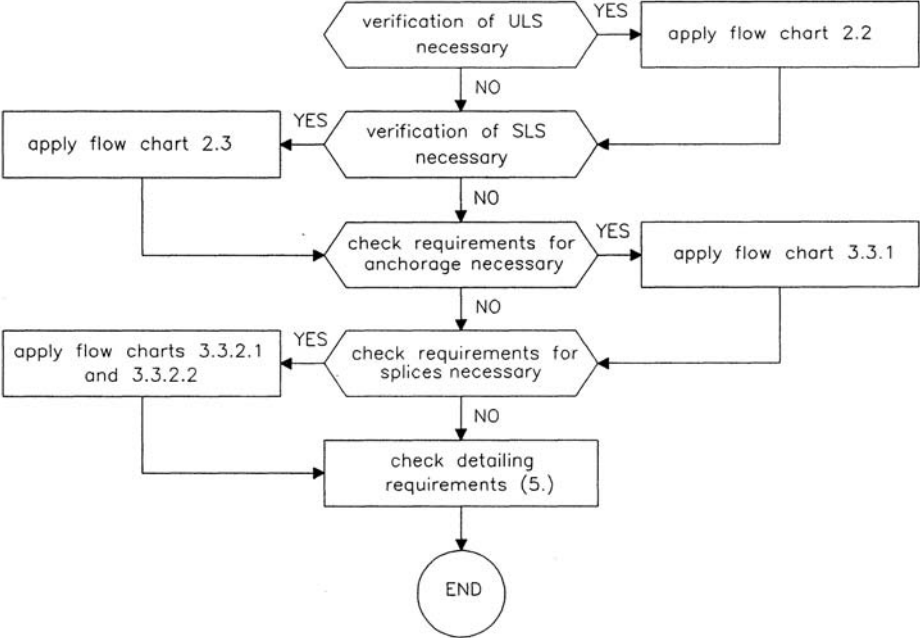






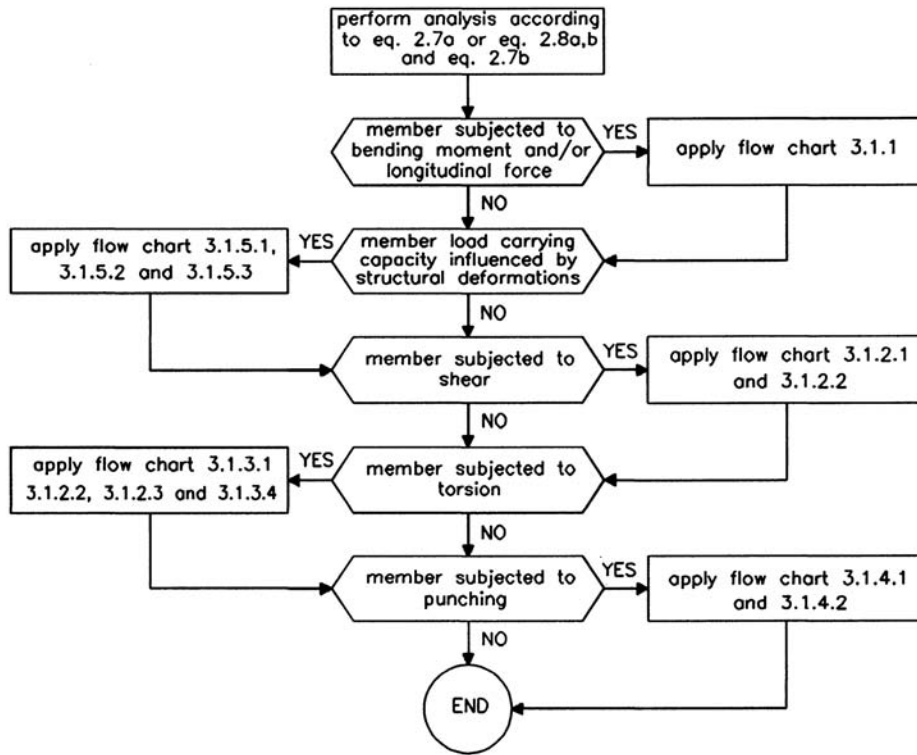


Flow chart 3.0.1
Section and member design: general



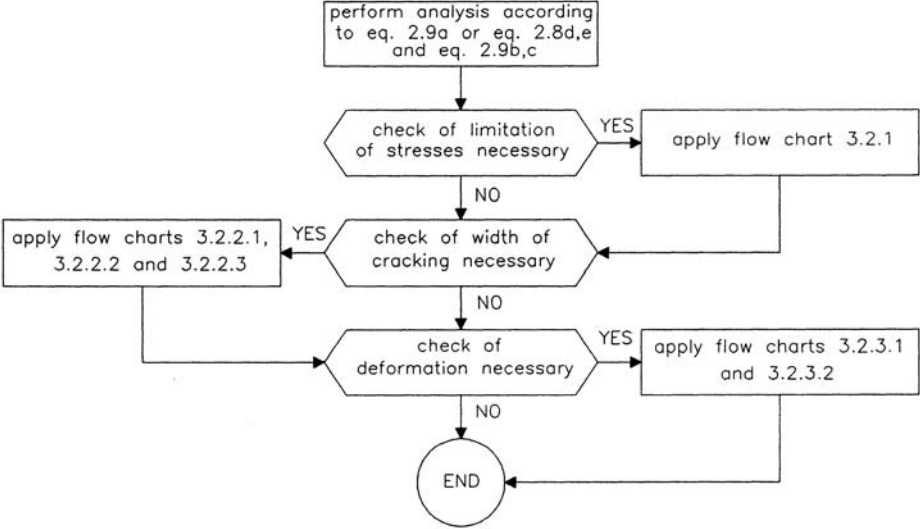
Flow chart 3.0.2

Section and member design: ultimate limit state (ULS)



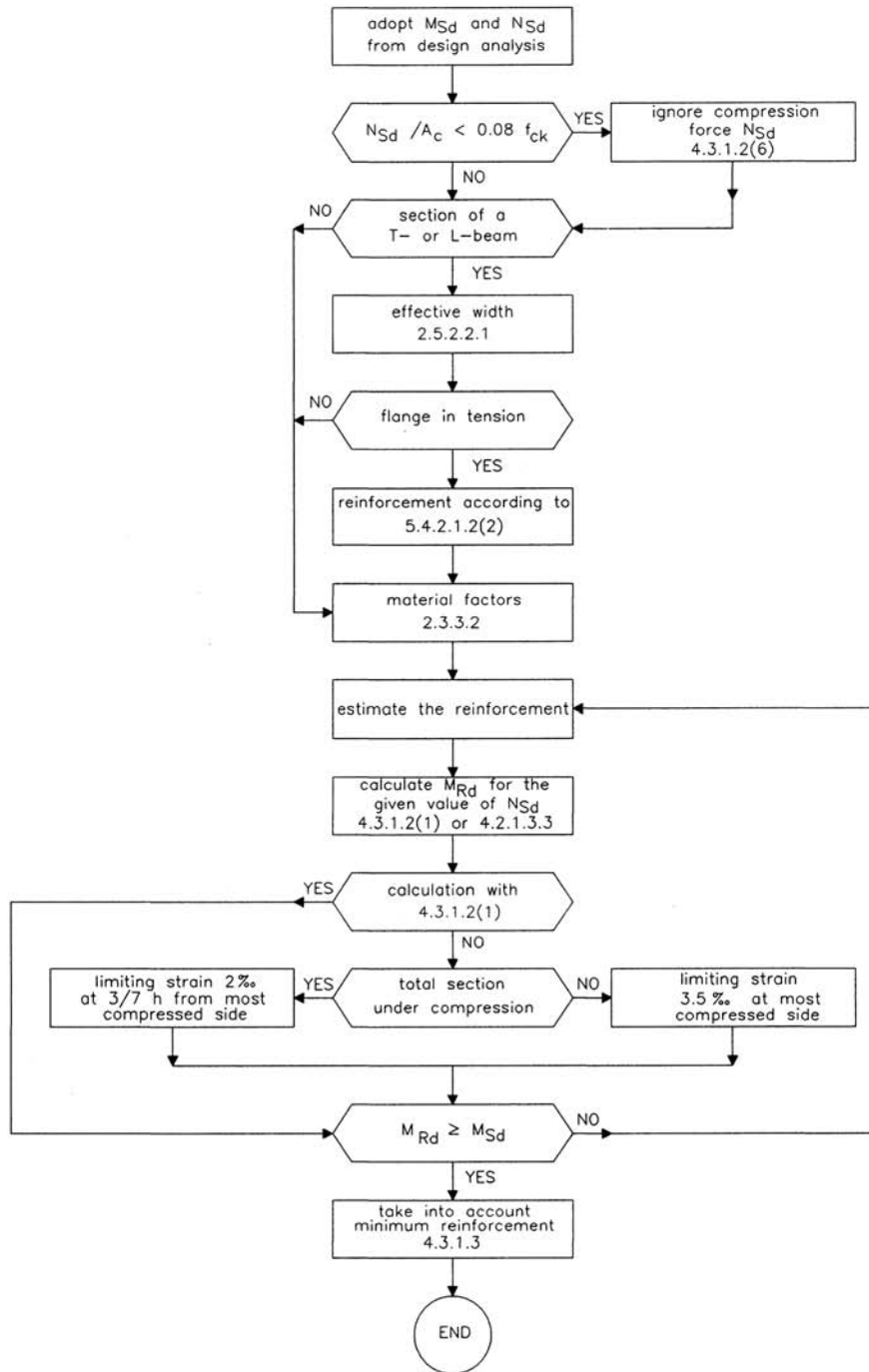
Flow chart 3.0.3

Section and member design: serviceability limit state (SLS)

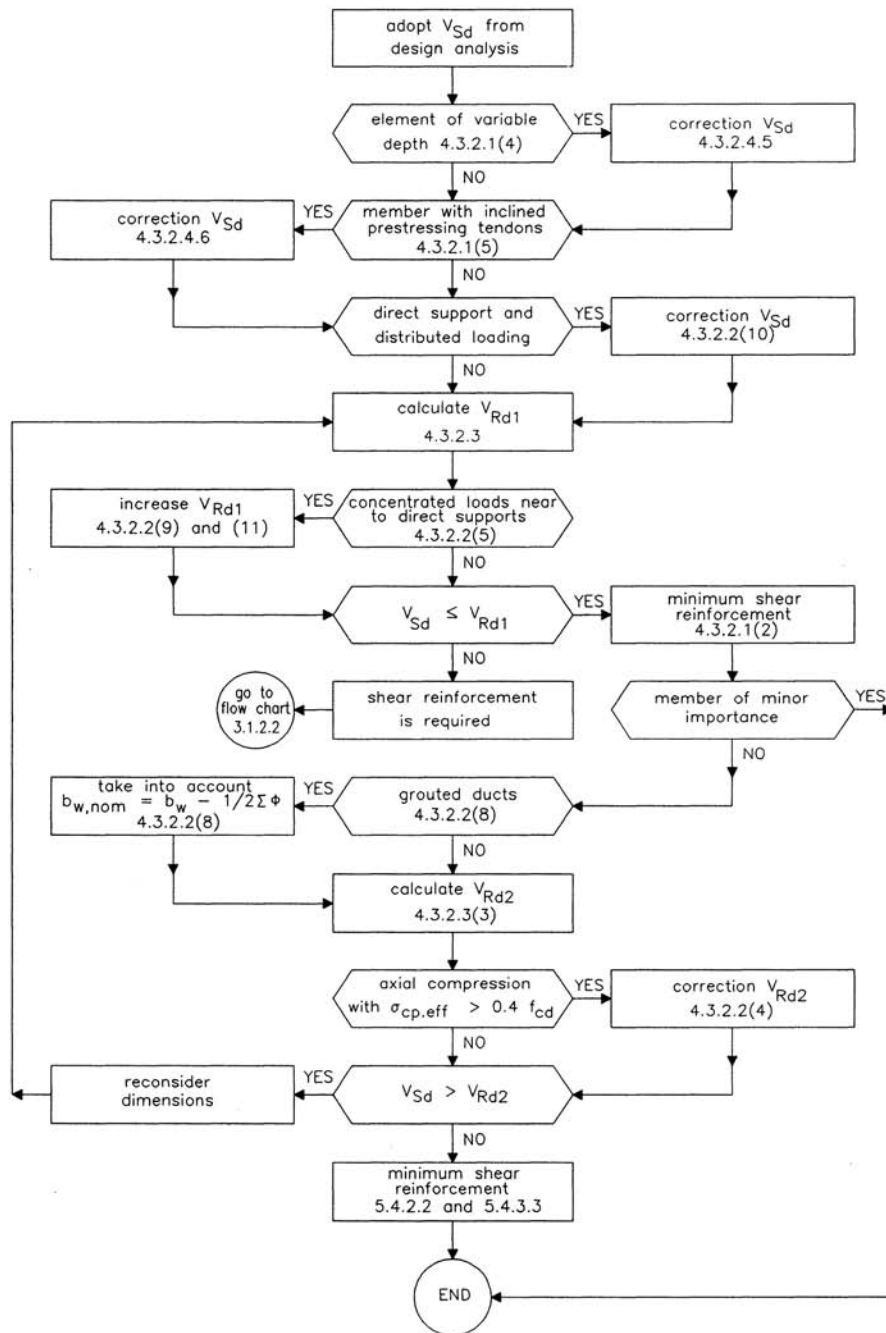


Flow chart 3.1.1.1

Bending: bending and longitudinal force

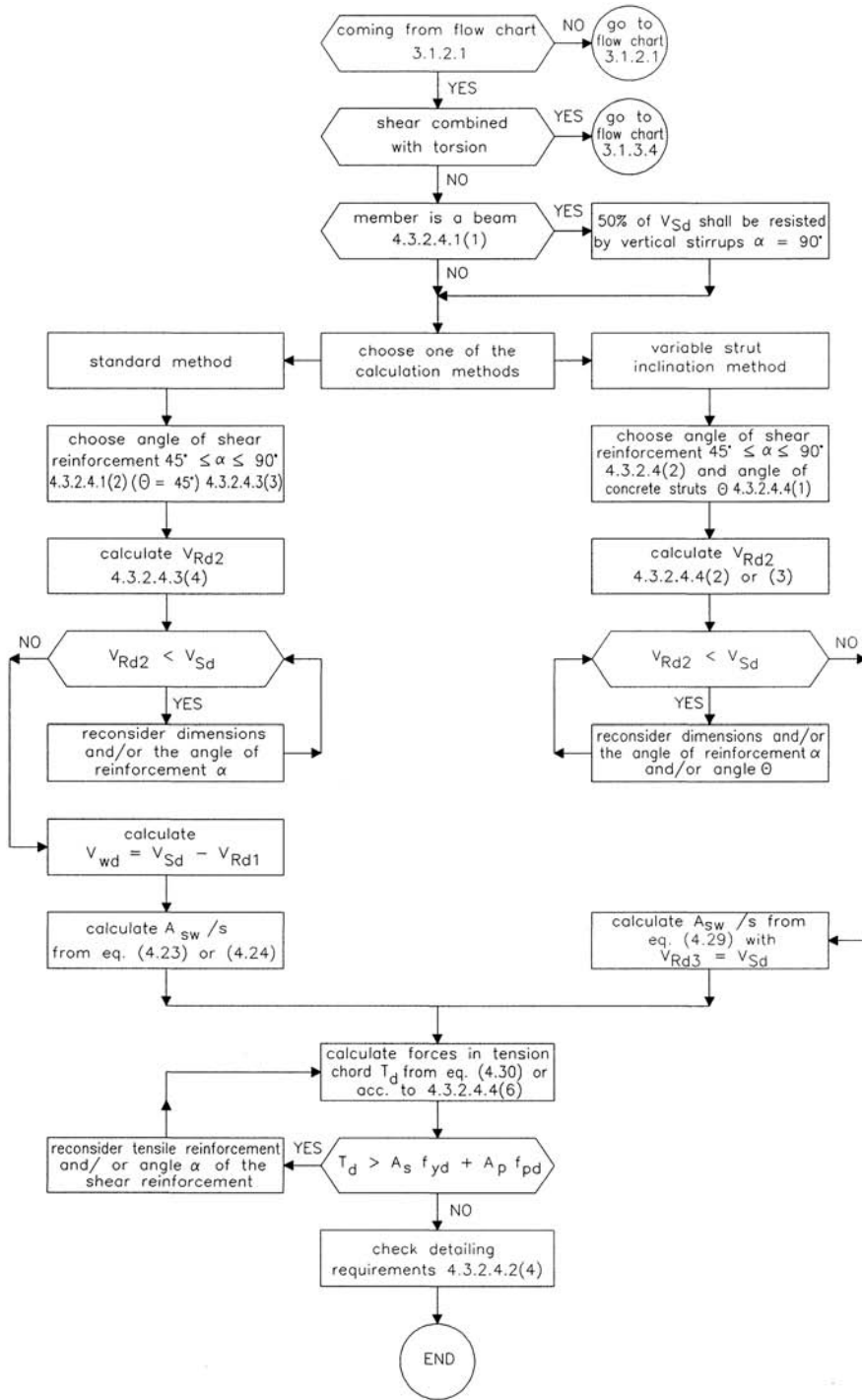


Flow chart 3.1.2.1
Shear: design method

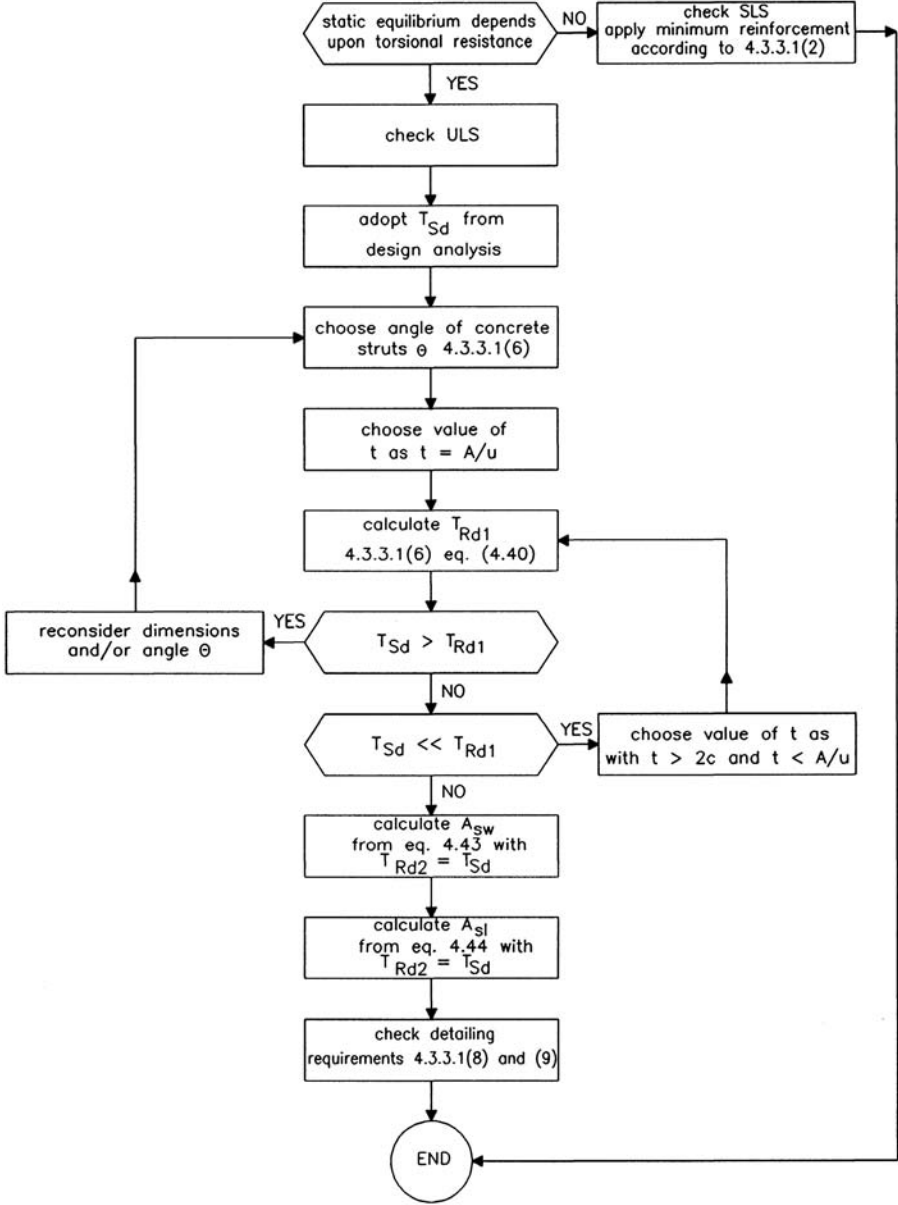


Flow chart 3.1.2.2

Shear: elements with shear reinforcement

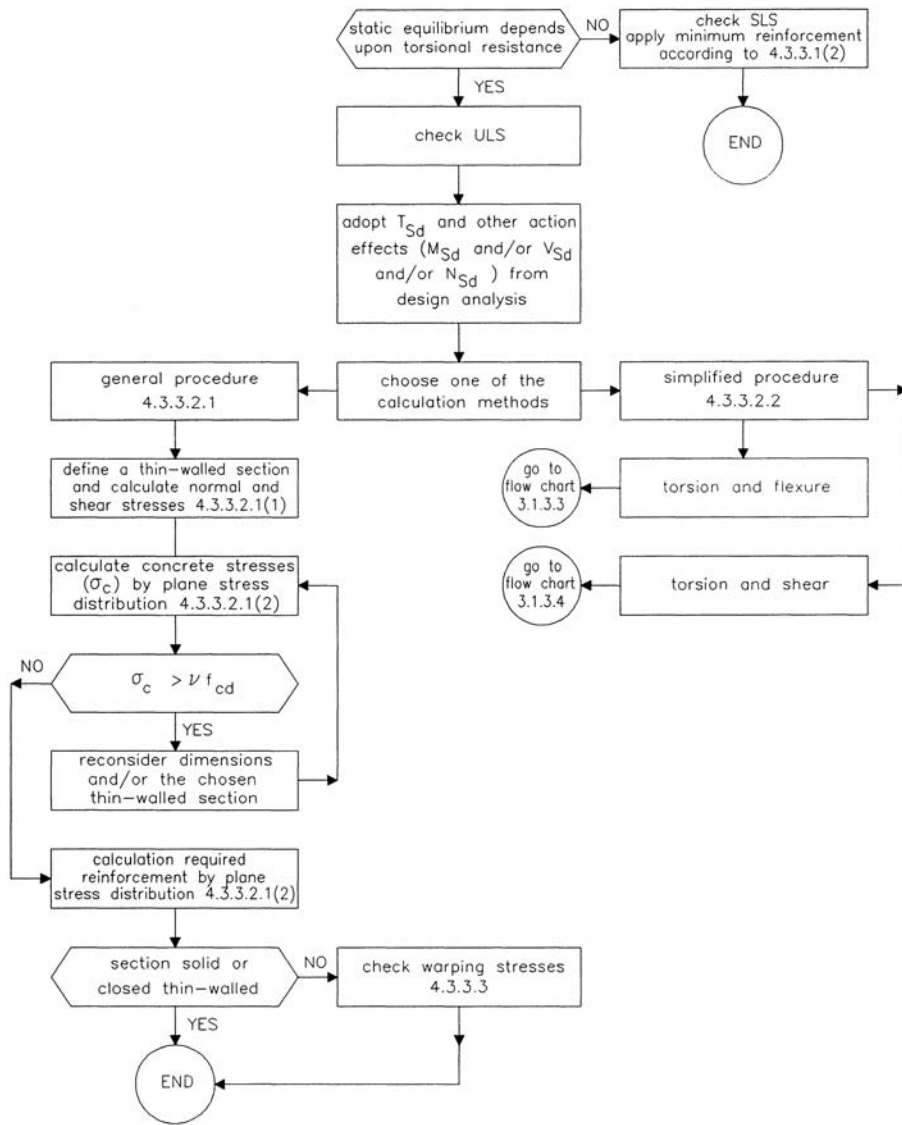


Flow chart 3.1.3.1
Torsion: pure torsion

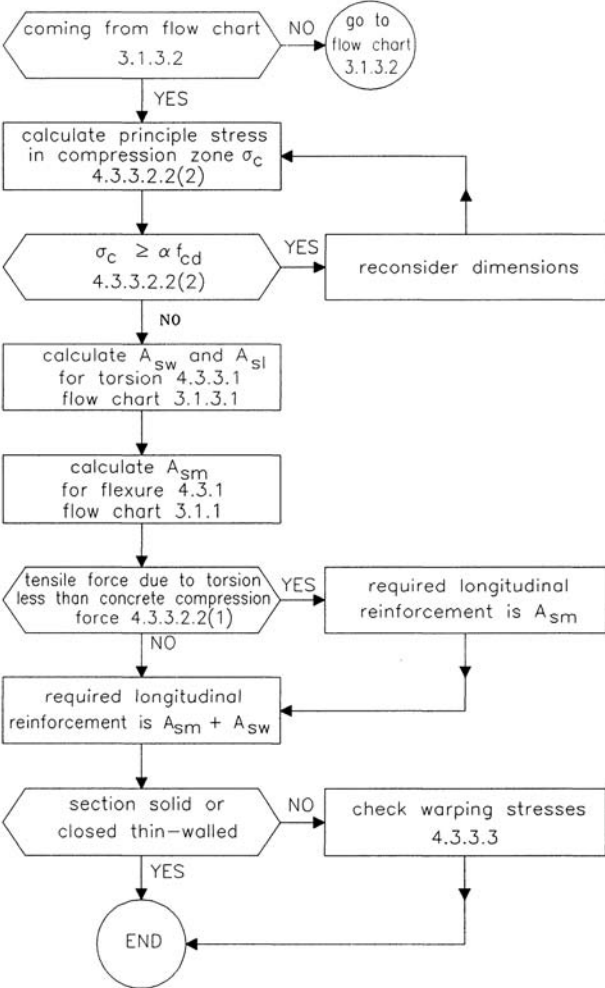


Flow chart 3.1.3.2

Torsion: torsion, combined effects of action

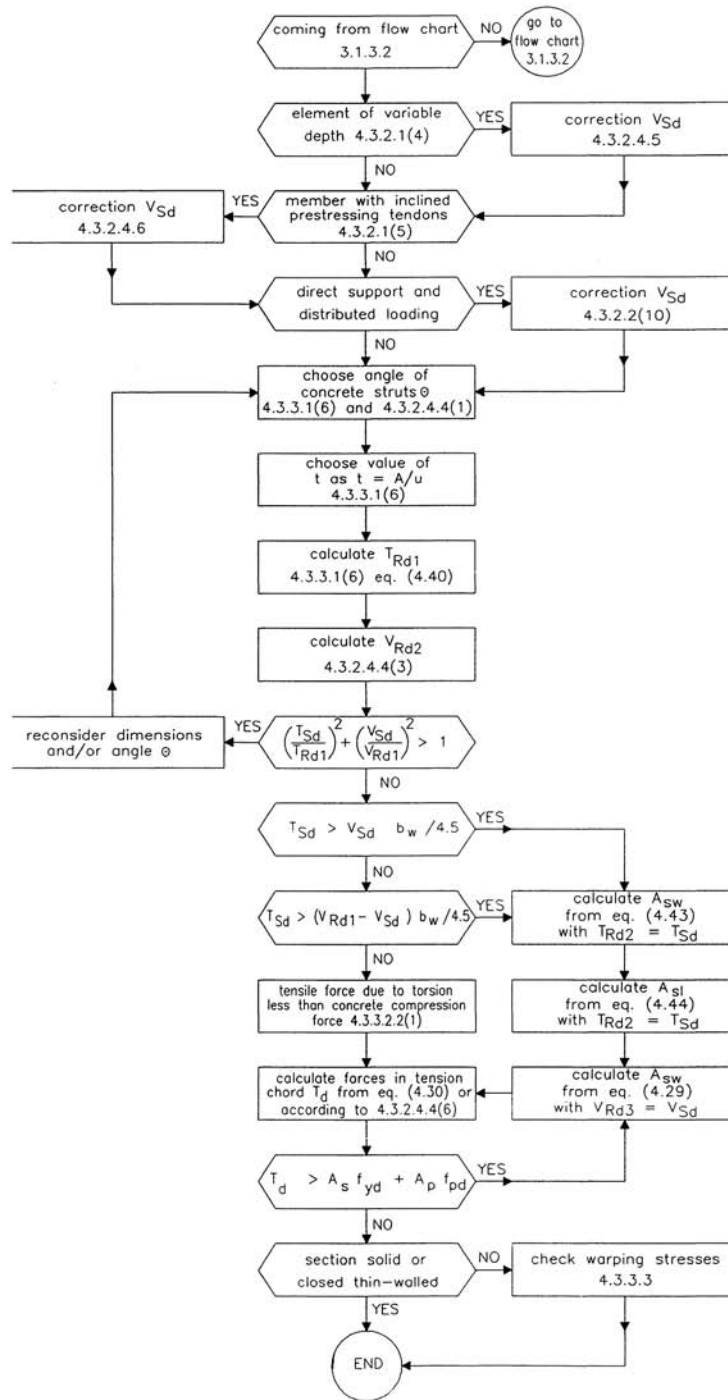


Flow chart 3.1.3.3
Torsion: torsion and flexure

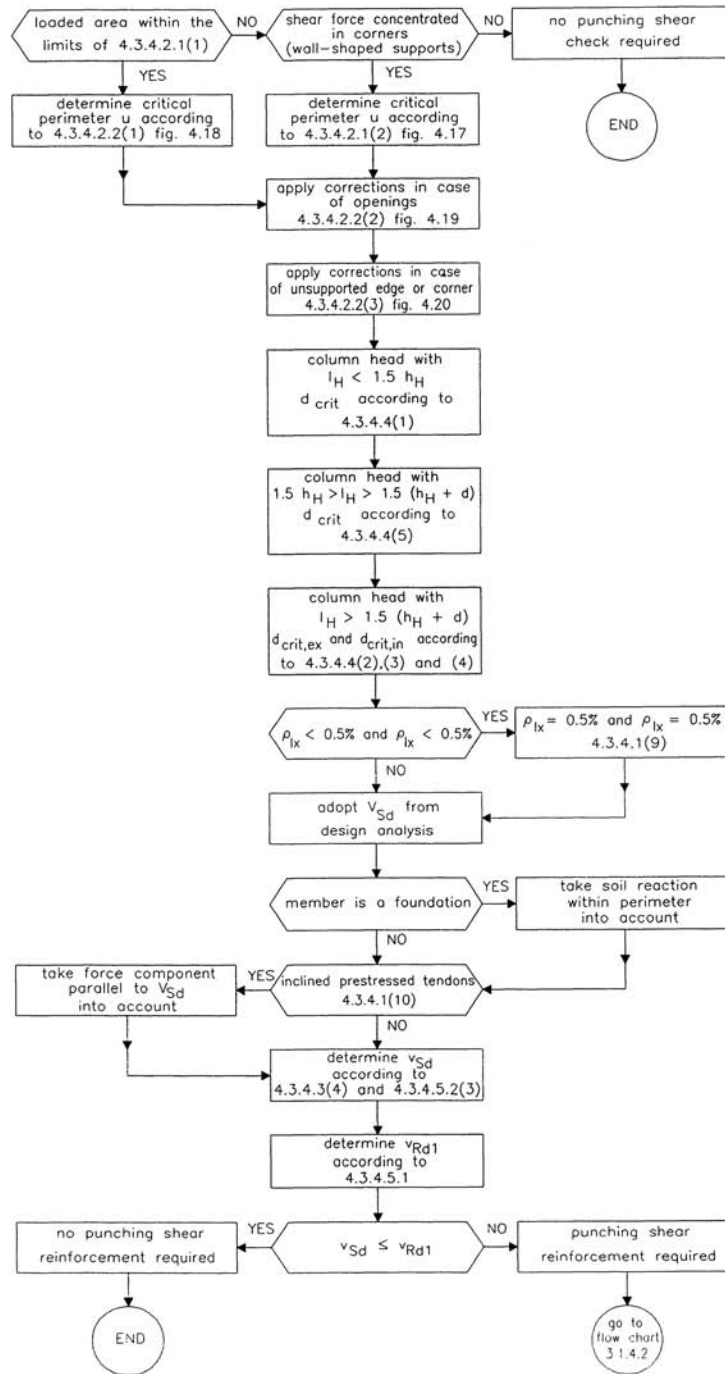


Flow chart 3.1.3.4

Torsion: torsion and shear

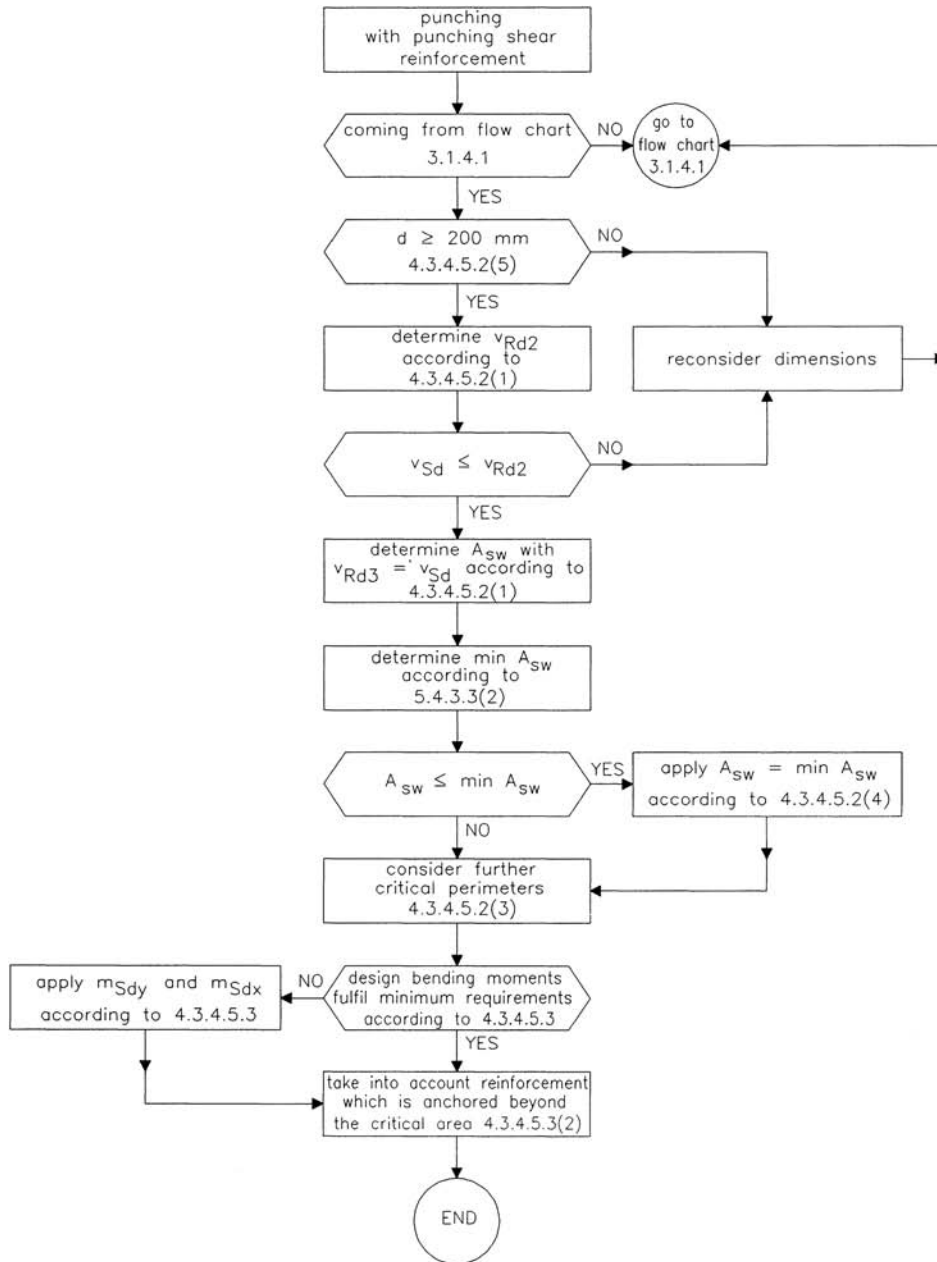


Flow chart 3.1.4.1
Punching: punching

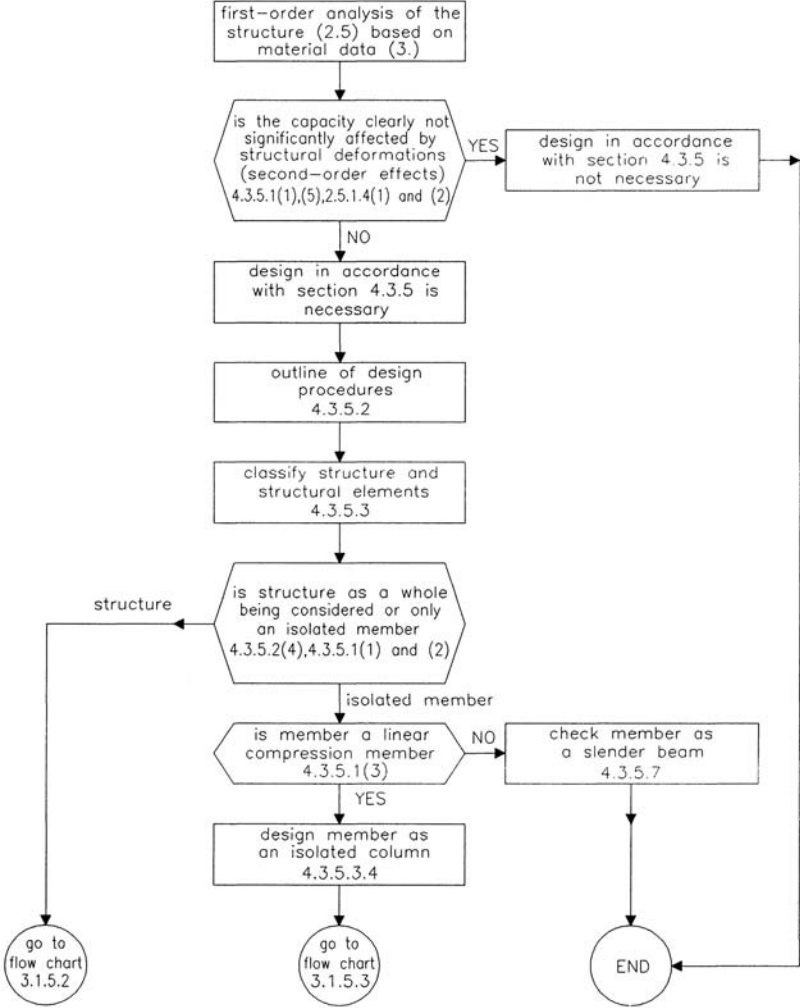


Flow chart 3.1.4.2

Punching: punching shear reinforcement

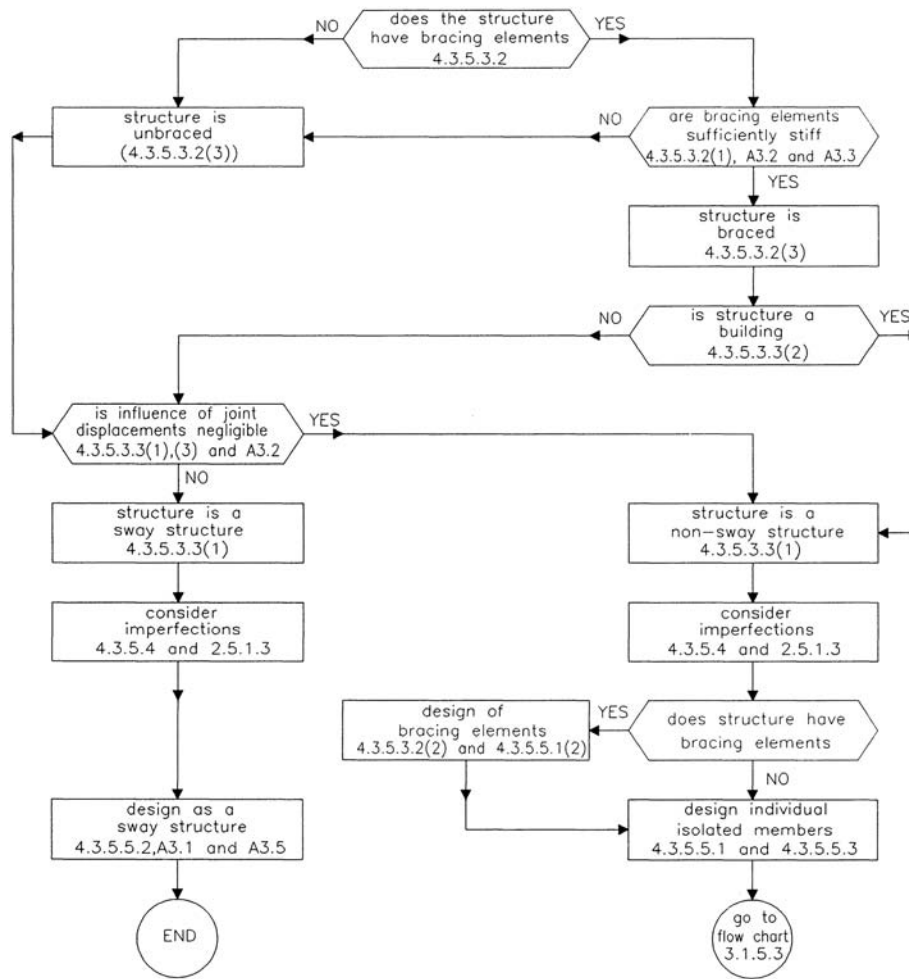


Flow chart 3.1.5.1
Buckling: general guide



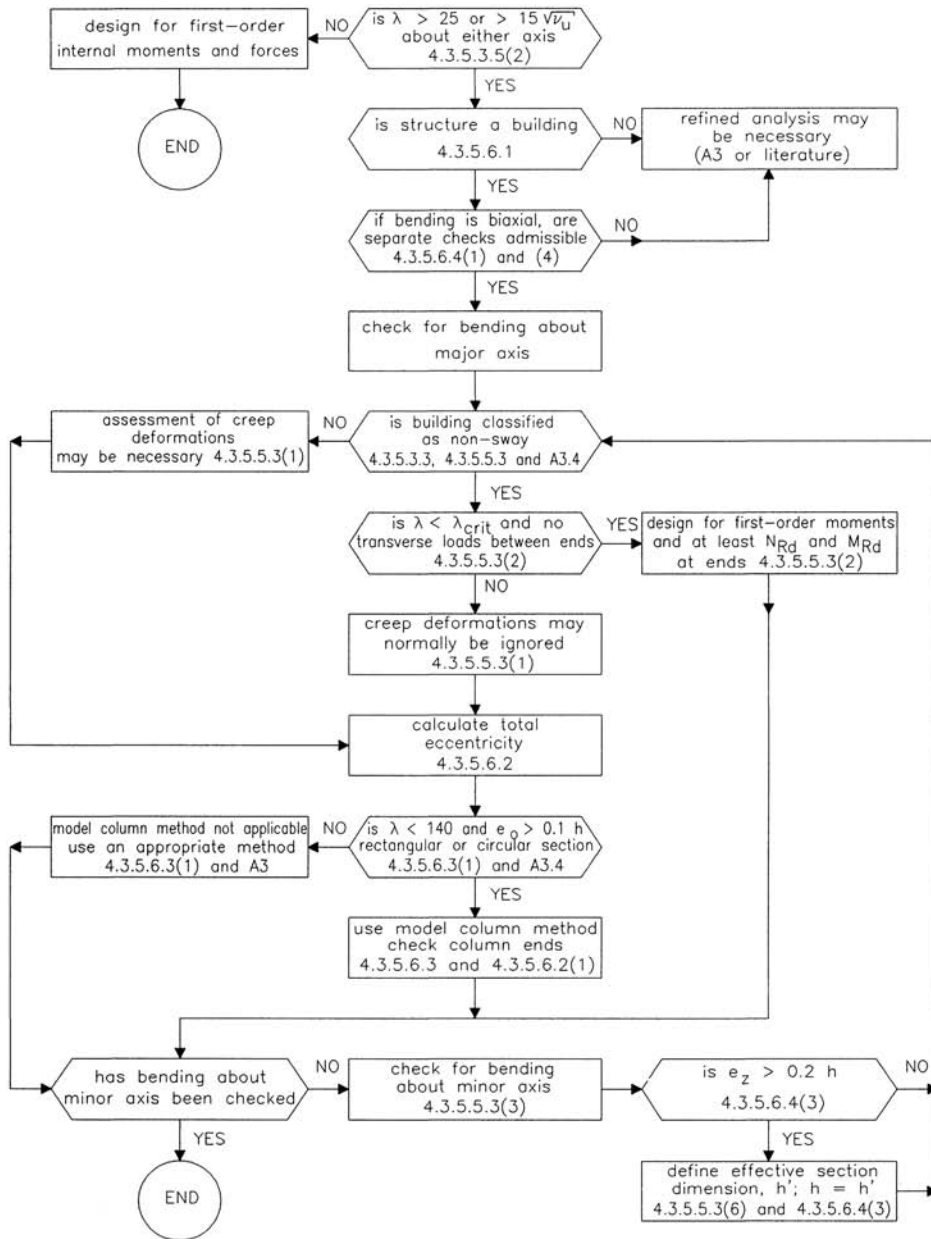
Flow chart 3.1.5.2

Buckling: structure as a whole



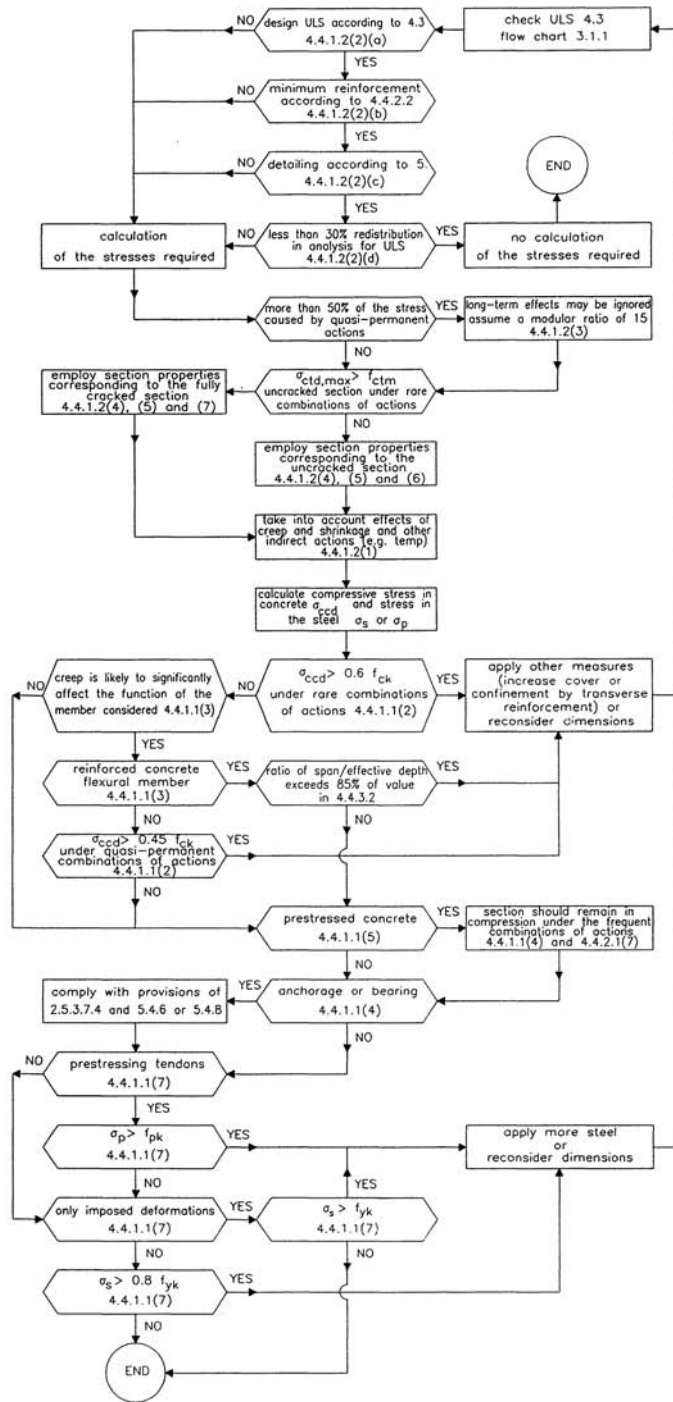
Flow chart 3.1.5.3

Buckling: isolated columns

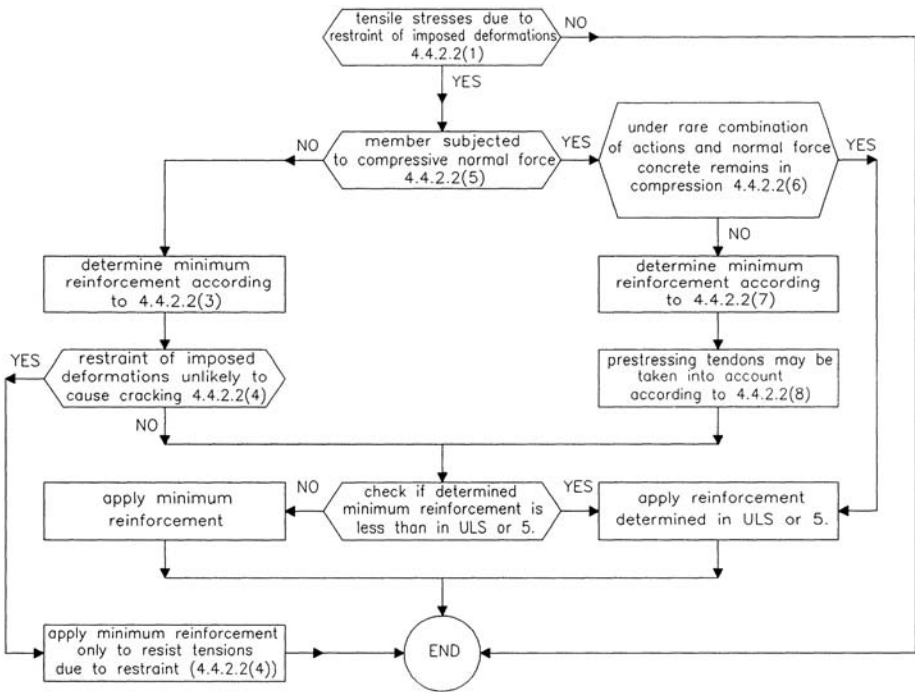


Flow chart 3.2.1.1.

Stresses: limitation of stresses

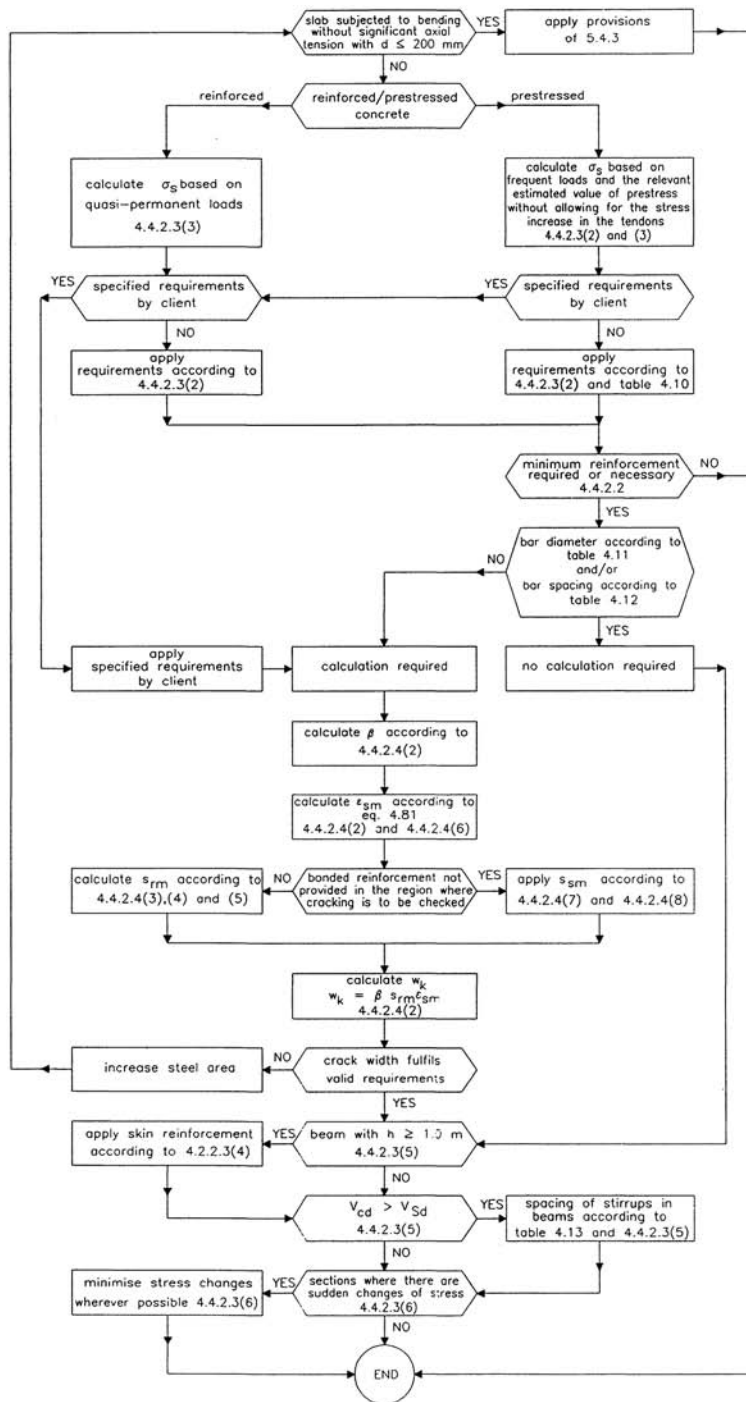


Flow chart 3.2.2.1
Cracking: minimum reinforcement



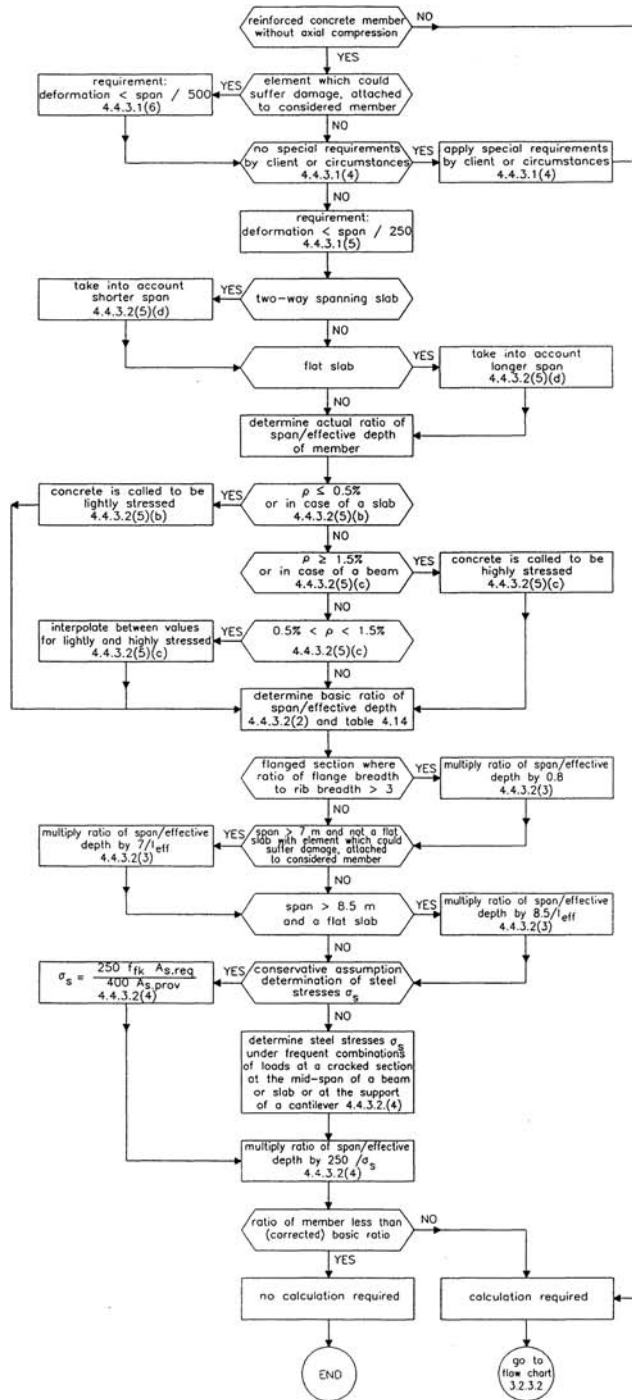
Flow chart 3.2.2.2

Cracking: with or without calculation



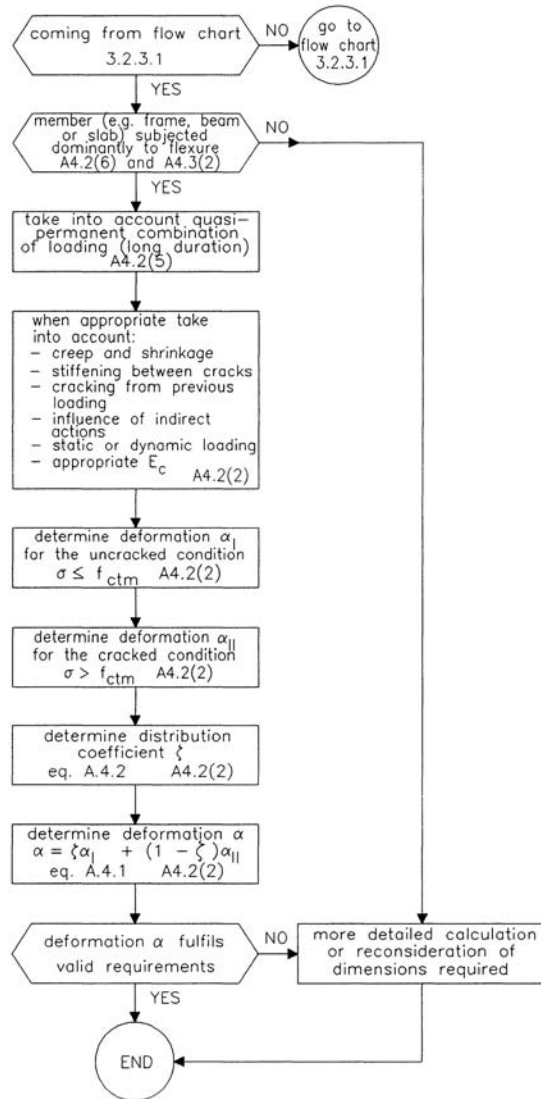
Flow chart 3.2.3.1

Deformation: deformation without calculation

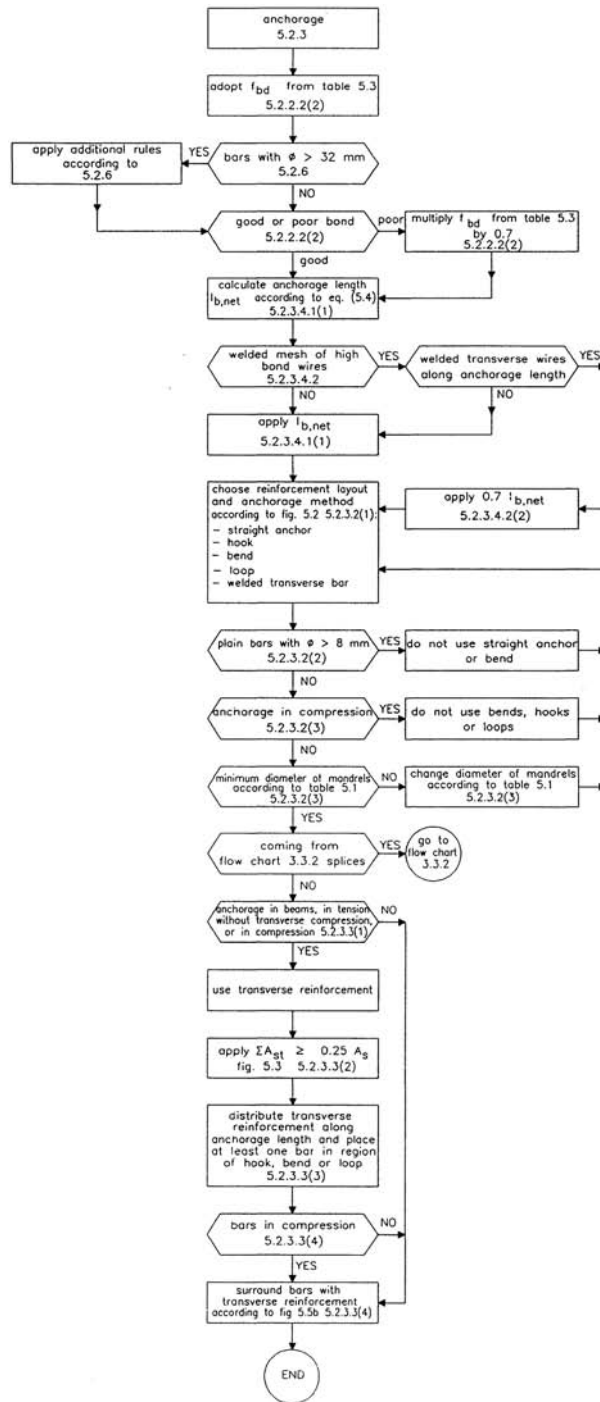


Flow chart 3.2.3.2

Deformation: deformation by calculation

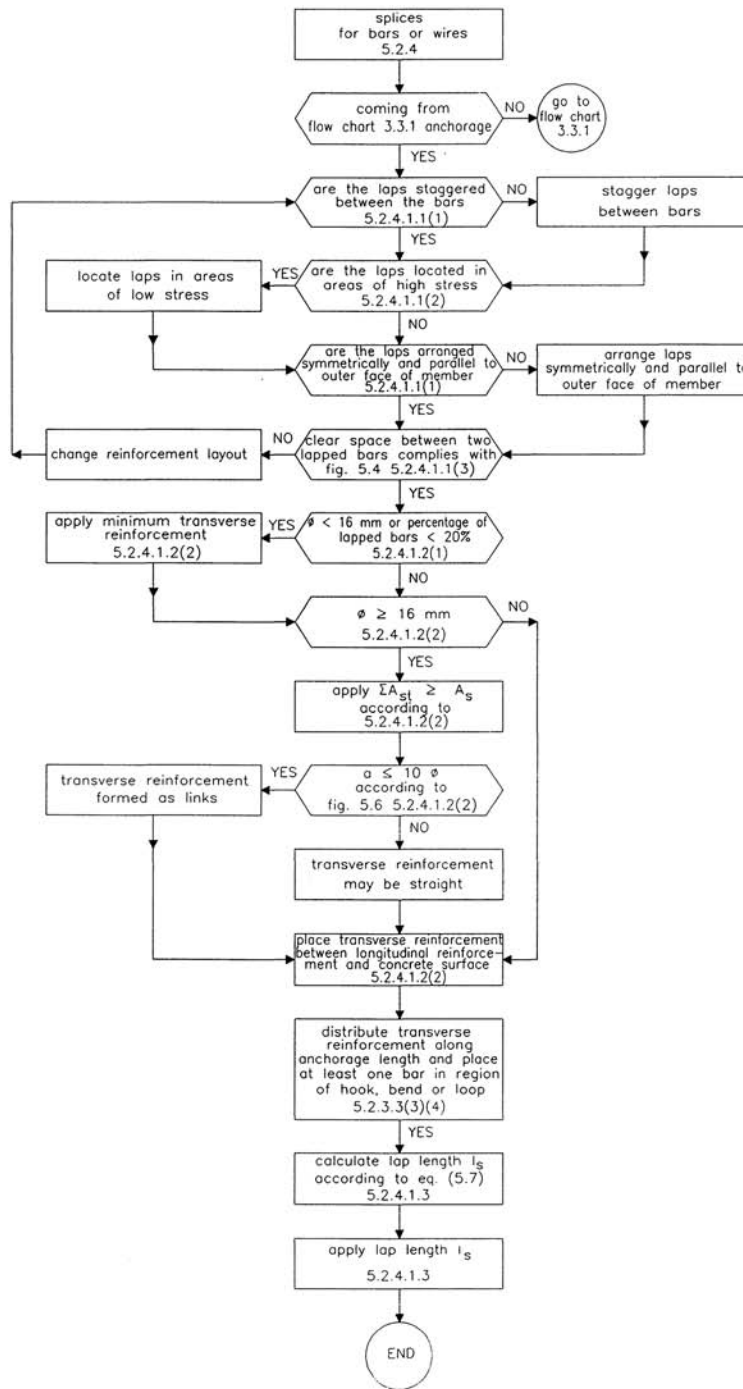


Flow chart 3.3.1.1
Anchorage: general



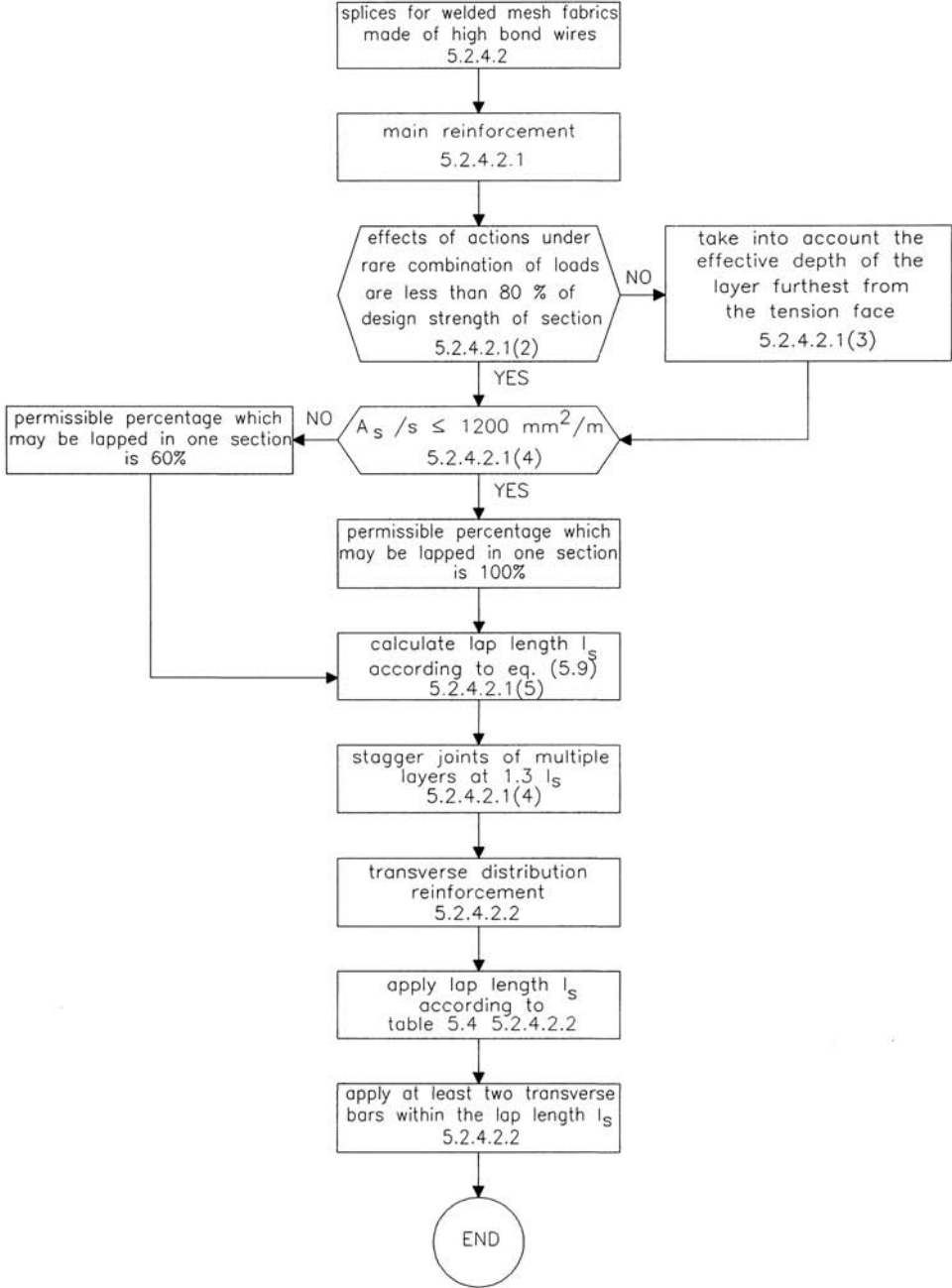
Flow chart 3.3.2.1

Splices: splices for bars or wires



Flow chart 3.3.2.2

Splices: splices for welded mesh fabrics



Design requirements

Throughout the following, the numbers on the right refer to relevant clauses of EC2 and square brackets - [] - in these references refer to relevant formulae in EC2. Please note, however, that square brackets in text indicate boxed values in the appropriate NAD.

4.1

Combinations of actions

Ultimate limit states 2.3.2.2

Fundamental combinations

$$\sum(\gamma_{G,j}G_{k,j}) + \gamma_{Q,1}Q_{k,1} + \sum_{i>1}(\gamma_{Q,i}\psi_{0,i}Q_{k,i}) \quad [2.7(a)]$$

Accidental combinations

$$\sum(\gamma_{GA,j}G_{k,j}) + A_d + \psi_{1,1}Q_{k,1} + \sum_{i>1}(\psi_{2,i}Q_{k,i}) \quad [2.7(b)]$$

$G_{k,j}$ = characteristic values of permanent actions

$Q_{k,1}$ = characteristic value of one of the variable actions

$Q_{k,i}$ = characteristic values of the other variable actions

A_d = design value (specified value) of the accidental actions

$\gamma_{G,j}$ = partial safety factors for any permanent action j

$\gamma_{GA,j}$ as $\gamma_{G,j}$ but for accidental design situations

$\gamma_{Q,i}$ = partial safety factors for any variable action i

ψ_0, ψ_1, ψ_2 = combination coefficients to determine the combination, frequent and quasi-permanent values of variable actions

In expressions [2.7(a)] and [2.7(b)], prestressing shall be introduced where relevant.

Simplified method for fundamental combinations 2.3.3.1(8)

One variable action

$$\sum(\gamma_{G,j}G_{k,j}) + [1.5] Q_{k,1} \quad [2.8(a)]$$

Two or more variable actions

$$\sum(\gamma_{G,j}G_{k,j}) + [1.35] \sum_{i \geq 1} Q_{k,i} \quad [2.8(b)]$$

whichever gives the larger value

For the boxed values, apply the values given in the appropriate NAD.

Serviceability limit states 2.3.4

Rare combinations

$$\sum G_{k,j} (+ P) + Q_{k,1} + \sum_{i>1}(\psi_{0,i}Q_{k,i}) \quad [2.9(a)]$$

Frequent combinations

$$\sum G_{k,j} (+ P) + \psi_{1,1}Q_{k,1} + \sum_{i>1}(\psi_{2,i}Q_{k,i}) \quad [2.9(b)]$$

Quasi-permanent combinations

$$\sum G_{k,j} (+ P) + \sum_{i \geq 1}(\psi_{2,i}Q_{k,i}) \quad [2.9(c)]$$

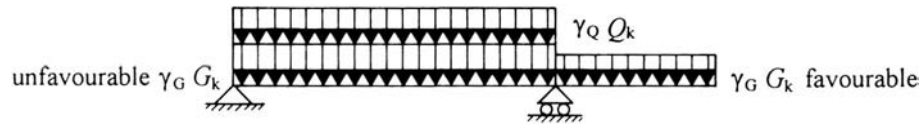


Figure 4.1 Maximum (positive) bending moment in middle of span and maximum shear at bearings of span.

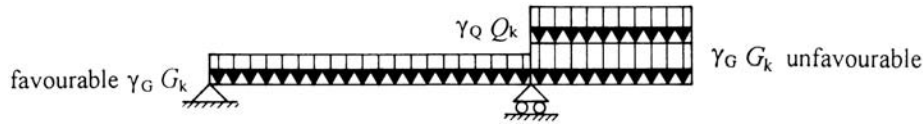


Figure 4.2 Minimum (positive or negative) bending moment in middle of span and maximum (negative) bending moment and maximum shear at bearing of cantilever.

P =prestressing force

Simplified method for rare combinations 2.3.4(6)

One variable action

$$\sum G_{k,j} + Q_{k,i} \quad [2.9(d)]$$

Two or more variable actions

$$\sum G_{k,j} + 0.9 \sum_{i \geq 1} Q_{k,i} \quad [2.9(e)]$$

whichever gives the larger value.

Permanent actions

Where the results of a verification may be very sensitive to variations of the magnitude of a permanent action from place to place in the structure, the unfavourable and the favourable parts of this action shall be considered as individual actions in ULS (2.3.2.3(3)).

For beams and slabs in buildings with cantilevers subjected to dominantly uniformly distributed loads, this requirement leads to the following decisive combinations of actions (see Figures 4.1 and 4.2):

For continuous beams and slabs in buildings without cantilevers subjected to dominantly uniformly distributed loads, it will generally be sufficient to consider only the two load cases in ULS (2.5.1.2(4)): alternate spans carrying the design variable and permanent loads ($\gamma_Q Q_k + \gamma_G G_k$), other spans carrying only the design permanent load ($\gamma_G G_k$) (2.5.1.2(4)(a)) (see Figure 4.3); any two adjacent spans carrying the design variable and permanent loads ($\gamma_Q Q_k + \gamma_G G_k$), other spans carrying only the design permanent load ($\gamma_G G_k$) (2.5.1.2(4)(b)) (see Figure 4.4).

4.2

Categories and values of imposed loads

Categories of imposed loads (Eurocode 1, part 2.1 (ENV 1991-2-1))

Areas of dwelling, offices, etc.

Category A Areas for domestic and residential activities,

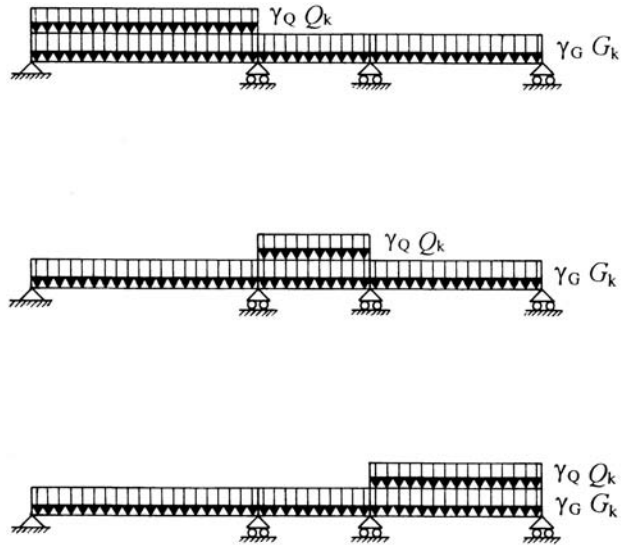


Figure 4.3 Alternate spans carrying the design variable load.

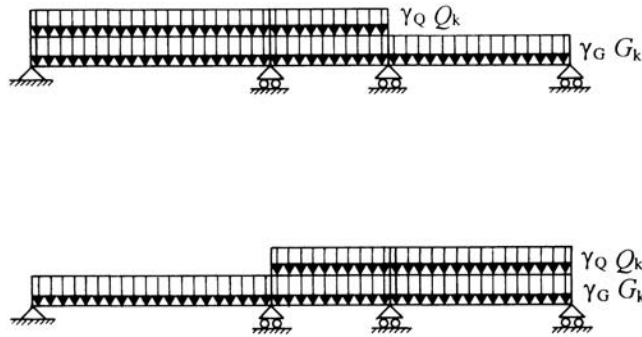


Figure 4.4 Two adjacent spans carrying the design variable load.

e.g. rooms in residential buildings and houses; rooms and wards in hospitals; bedrooms in hotels and hostels; kitchens and toilets.

Category B Office areas

Category C Areas where people may congregate (with the exception of areas defined under categories A, B, D and E)

C1 areas with tables, etc.
e.g. areas in schools, cafés, restaurants, dining halls, reading rooms, receptions, etc.

C2 areas with fixed seats,
e.g. areas in churches, theatres or cinemas, conference rooms, lecture halls, assembly halls, waiting rooms, etc.

C3 obstacle-free areas for moving people,
e.g. areas in museums, exhibition rooms, and access areas in public and administration buildings, hotels, etc.

C4 areas with possible physical activities,
e.g. dance halls, gymnasiums, stages, etc.

C5 areas susceptible to overcrowding,
e.g. in buildings for public events like concert halls, sports halls including stands, terraces and access areas, etc.

Category D Shopping areas

D1 areas in general retail shops

D2 areas in department stores,

e.g. areas in warehouses, stationery and office stores, etc.

Category E Areas susceptible to accumulation of goods, including access areas

Areas for storage including libraries. The loads defined in Table 4.1 with values of imposed loads shall be taken as minimum loads unless more appropriate loads are defined for the specific case

Garage and vehicle traffic areas

- Category F Traffic and parking areas for light vehicles (30 kN total weight and 8 seats excluding driver)
 Category G Traffic and parking areas for medium-weight vehicles (>30 kN, 60 kN total weight, on two axles)

Areas for storage and industrial activities

Roofs

- Category H Roofs not accessible except for normal maintenance, repair and cleaning
 Category I Roofs accessible with occupancy according to categories A-G
 Category K Roofs accessible for special services

Values of imposed loads

Table 4.1 Values of imposed loads (Eurocode 1, part 2.1 (ENV 1991–2–1))

| Loaded areas | q_k (kN/m ²) | Q_k (kN) |
|--|----------------------------|-----------------------------|
| Areas of dwellings, offices, etc. | | |
| Category A | general | 2.0 |
| | stairs | 3.0 |
| | balconies | 4.0 |
| Category B | 3.0 | 2.0 |
| Category C | C1 | 3.0 |
| | C2 | 4.0 |
| | C3 | 5.0 |
| | C4 | 5.0 |
| | C5 | 5.0 |
| Category D | D1 | 5.0 |
| | D2 | 5.0 |
| Category E | 6.0 | 7.0 |
| Garage and vehicle traffic areas | | |
| Category F | 2.0 | 10 |
| Category G | 5.0 | 45 |
| Areas for storage and industrial activities | to be specified | to be specified |
| Roofs | | |
| Category H | roof slope: <20° | 0.75* |
| | >40° | 0.00* |
| Category I | | according to categories A-G |
| Category K | | to be specified |

* For roof slopes between 20° and 40°, q_k may be determined by linear interpolation

4.3 ψ factors

Table 4.2 ψ factors (Eurocode 1, part 2.1 (ENV 1991–2–1))

| Loaded areas | ψ_0 | ψ_1 | ψ_2 |
|---|----------|----------|----------|
| Areas of dwelling, offices, etc. | | | |
| Category A | 0.7 | 0.5 | 0.3 |
| Category B | 0.7 | 0.5 | 0.3 |

| Loaded areas | ψ_0 | ψ_1 | ψ_2 |
|--|-----------------------------|-----------------------------|-----------------------------|
| Category C | 0.7 | 0.7 | 0.6 |
| Category D | 0.7 | 0.7 | 0.6 |
| Category E | 1.0 | 0.9 | 0.8 |
| Garage and vehicle traffic areas | | | |
| Category F | 0.7 | 0.7 | 0.6 |
| Category G | 0.7 | 0.5 | 0.3 |
| Areas for storage and industrial activities | to be specified | to be specified | to be specified |
| Roofs | | | |
| Category H | 0.0 | 0.0 | 0.0 |
| Category I | according to categories A-G | according to categories A-G | according to categories A-G |
| Category K | to be specified | to be specified | to be specified |

4.4 Partial safety factors for actions

Table 4.3 Partial safety factors for actions (Eurocode 1, part 1 (ENV 1991-1: 1993))

| Case⁽¹⁾ | Action | Symbol | Situations |
|--|--|--------------------------------|-------------------|
| P/T | A | | |
| Case A Loss of static equilibrium; strength of structural material or ground insignificant (see 9.4.1) Permanent actions: self-weight of structural and non-structural components, permanent actions caused by ground-water and free water | | | |
| - unfavourable | $\gamma_{Gsup}^{(2,4)}$ | [1.10] | [1.00] |
| - favourable | $\gamma_{Ginf}^{(2,4)}$ | [0.90] | [1.00] |
| Variable actions | | | |
| - unfavourable | γ_Q | [1.50] | [1.00] |
| Accidental actions | γ_A | | [1.00] |
| Case B ⁽⁵⁾ Failure of structure or structural elements, including those of the footing, piles, basement walls, etc., governed by strength of structural materials (see 9.4.1) | Permanent actions ⁽⁶⁾ (see above) | | |
| - unfavourable | $\gamma_{Gsup}^{(3,4)}$ | [1.35] | [1.00] |
| - favourable | $\gamma_{Ginf}^{(3,4)}$ | [1.00] | [1.00] |
| Variable actions | | | |
| - unfavourable | γ_Q | [1.50] | [1.00] |
| Accidental actions | γ_A | | [1.00] |
| Case C ⁽⁵⁾ Failure in the ground Permanent actions (see above) | | | |
| - unfavourable | $\gamma_{Gsup}^{(4)}$ | [1.00] | [1.00] |
| - favourable | $\gamma_{Ginf}^{(4)}$ | [1.00] | [1.00] |
| Variable actions | | | |
| - unfavourable | γ_Q | [1.30] | [1.00] |
| Accidental actions | γ_A | | [1.00] |
| P: Persistent situation | T: Transient situation | A: Accidental situation | |

NOTES

- The design should be separately verified for each case A, B and C as relevant.
- In this verification, the characteristic value of the unfavourable part of the permanent action is multiplied by the factor 1.1 and the favourable part by 0.9. More refined rules are given in ENV 1993 and ENV 1994.

| Case ⁽¹⁾ | Action | Symbol | Situations |
|--|----------|--------|------------|
| P/T | A | | |
| 3. In this verification, the characteristic values of all permanent actions from one source are multiplied by 1.35 if the total effect of the resulting action is unfavourable and by 1.0 if the total effect of the resulting action is favourable. | | | |
| 4. When the limit state is sensitive to variations of permanent actions, the upper and lower characteristic values of these actions should be taken according to 4.2 (3). | | | |
| 5. For cases B and C, the design ground properties may be different: see ENV 1997–1–1. | | | |
| 6. Instead of using γ_G (1.35) and γ_Q (1.50) for lateral earth pressure actions, the design ground properties may be introduced in accordance with ENV 1997 and a model factor γ_{sd} applied. | | | |

For the boxed values, apply the values given in the appropriate NAD.

Table 4.4 Partial safety factors for actions (Eurocode 2, part 1 (ENV 1992–1–1: 1991))

| | Permanent actions (γ_G) | Variable actions (γ_Q) | | Prestressing (γ_P) |
|-----------------------------------|-------------------------------------|---------------------------------|--------|--------------------------------|
| One with its characteristic value | Others with their combination value | | | |
| Favourable effect | [1.00] | - | - | [0.9] or [1.0] |
| Unfavourable effect | [1.35] | [1.50] | [1.50] | [1.2] or [1.0] |

4.5

Partial safety factors for materials

Table 4.5 Partial safety factors for materials (Eurocode 2, part 1 (ENV 1992–1–1: 1991))

| Combination | Concrete (γ_c) | Steel reinforcement or prestressing tendons (γ_s) |
|---------------------------------|----------------------------|---|
| Fundamental | [1.50] | [1.15] |
| Accidental (except earthquakes) | [1.30] | [1.00] |

For the boxed values, apply the values given in the appropriate NAD.

5

Calculation methods

5.1 Flat slabs

5.1.1 Introduction

Slabs are classified as flat slabs when they transfer loads to columns directly without any beam supports. Slabs may be solid or coffered (ribbed in two directions). Unlike two-way spanning slabs, flat slabs can fail by yield lines in either of the two orthogonal directions. Flat slabs should therefore be designed to carry the total load on the panel in each direction.

EC2 does not provide any specific guidance for the analysis of the flat slabs. The methods given are based on common practice in a number of countries in Europe. General methods of analysis include: (a) equivalent frame method; (b) use of simplified coefficients; (c) yield-line analysis; and (d) grillage analysis.

5.1.2 Equivalent frame method

The structure is divided in two orthogonal directions into frames consisting of columns and strips of slab acting as “beams”. The width of the slab to be used as “beams” is determined as follows:

For vertical loading,

when $l_y < 2l_x$,

| | | |
|-------------------------|---|----------------------|
| width in x -direction | = | $0.5(l_{x1}+l_{x2})$ |
| width in y -direction | = | $0.5(l_{x1}+l_{y2})$ |

when $l_y > 2l_x$,

| | | |
|-------------------------|---|----------------------|
| width in x -direction | = | $0.5(l_{x1}+l_{x2})$ |
| width in y -direction | = | $(l_{x1}+l_{x2})$ |

In these expressions, l_x and l_y are the shorter and longer spans respectively and l_{x1} and l_{x2} refer to the lengths of adjacent spans in x -direction. The stiffness of the “beams” for analysis should be based on the widths calculated above. When the loading is horizontal, the stiffness used in analysis should be taken as half that derived for vertical loading, to allow for uncertainties associated with the slab-column joints.

Analysis

A braced structure may be analysed using any of the standard linear elastic methods such as moment distribution method. The structure may be analysed as a whole or split into sub-frames consisting of the slab at any one level and the columns. The remote ends of the columns are normally treated as fixed unless they are obviously not.

Lateral distribution of moments

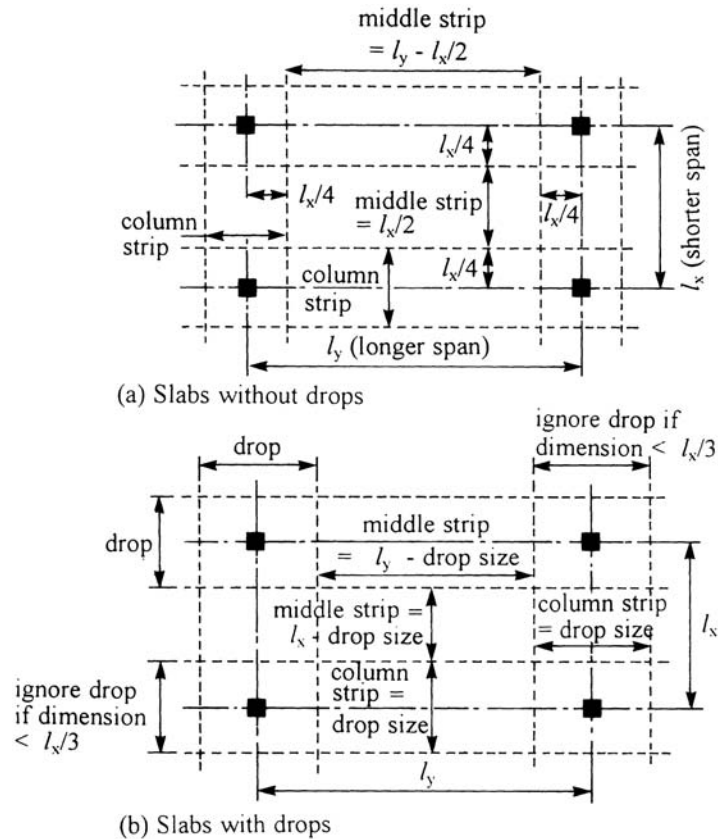


Figure 5.1: Division of slab into strips.

The slab should be divided into column and middle strips as shown in Figure 5.1. The slab bending moments obtained from analysis should be apportioned across the width of the slab as follows:

| | Column strip | Middle strip |
|------------------|--------------|--------------|
| Negative moments | 75% | 25% |
| Positive moments | 55% | 45% |

These figures are percentages of the total positive or negative moments obtained in analysis. Where the width of the column strip is taken as equal to that of a drop and thereby the width of the middle strip is increased, the design moments to be resisted by the middle strip should be increased in proportion to the increased width. The design moments in the column strip may be reduced accordingly.

Moment transfer at edge columns

The effective width to the slab through which moments are transferred between the edge (or corner) columns and slab should be calculated as shown in Figure 5.2. The maximum moment that can be transferred to the column is

$$M_{\max} = 0.167b_c d^2 f_{ck} \quad \text{for concrete grades C35/45 or less;}$$

$$M_{\max} = 0.136b_c d^2 f_{ck} \quad \text{for concrete grades C40/50 or greater.}$$

The structure should be sized so that M_{\max} is at least 50% of the moment obtained from an elastic analysis.

When the bending moment at the outer support obtained from the analysis exceeds M_{\max} , the moment at this support should be limited to M_{\max} and the moment in the span should be increased accordingly.

5.1.3 Use of simplified coefficients

Bending moments using the coefficients given below may be used for flat slabs where:

- (a) the structure consists of at least three spans; and

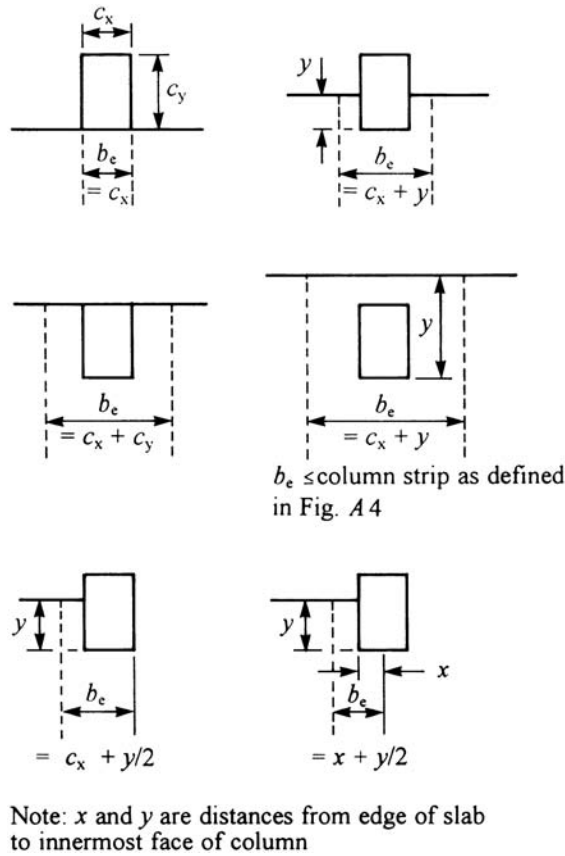


Figure 5.2

- (b) the ratio of the longest to the shortest span does not exceed 1.2; and
- (c) the loading is predominantly uniformly distributed

| At outer support | Near middle of end span | At first interior support | At middle of interior spans | At interior supports |
|------------------|-------------------------|---------------------------|-----------------------------|----------------------|
| 0 | $0.09Fl$ | $0.11Fl$ | $0.07Fl$ | $0.10Fl$ |

NOTES

l is the effective span. F is the total ultimate load on the span $= 1.35G_k + 1.5Q_k$. No redistribution should be carried out on the moments.

5.1.4 Reinforcement

Reinforcement should be sufficient to resist the minimum bending moment specified in Table 4.9 of EC2. The reinforcement required in each column and middle strip should be distributed uniformly. In slabs without drops, the reinforcement required to resist the negative moment in the column strips should be placed with 66% of the reinforcement within the middle half of the strip.

5.2 Strut-and-tie models

Strut-and-tie models may be used for structural analysis, where the assumption of linear strain distribution through the structure is not valid. This powerful plastic method is useful in a number of instances, including anchorage zones of prestressed members, members with holes, pile caps, deep beams and beam-column junctions. Typical models are shown in Figure 5.3.

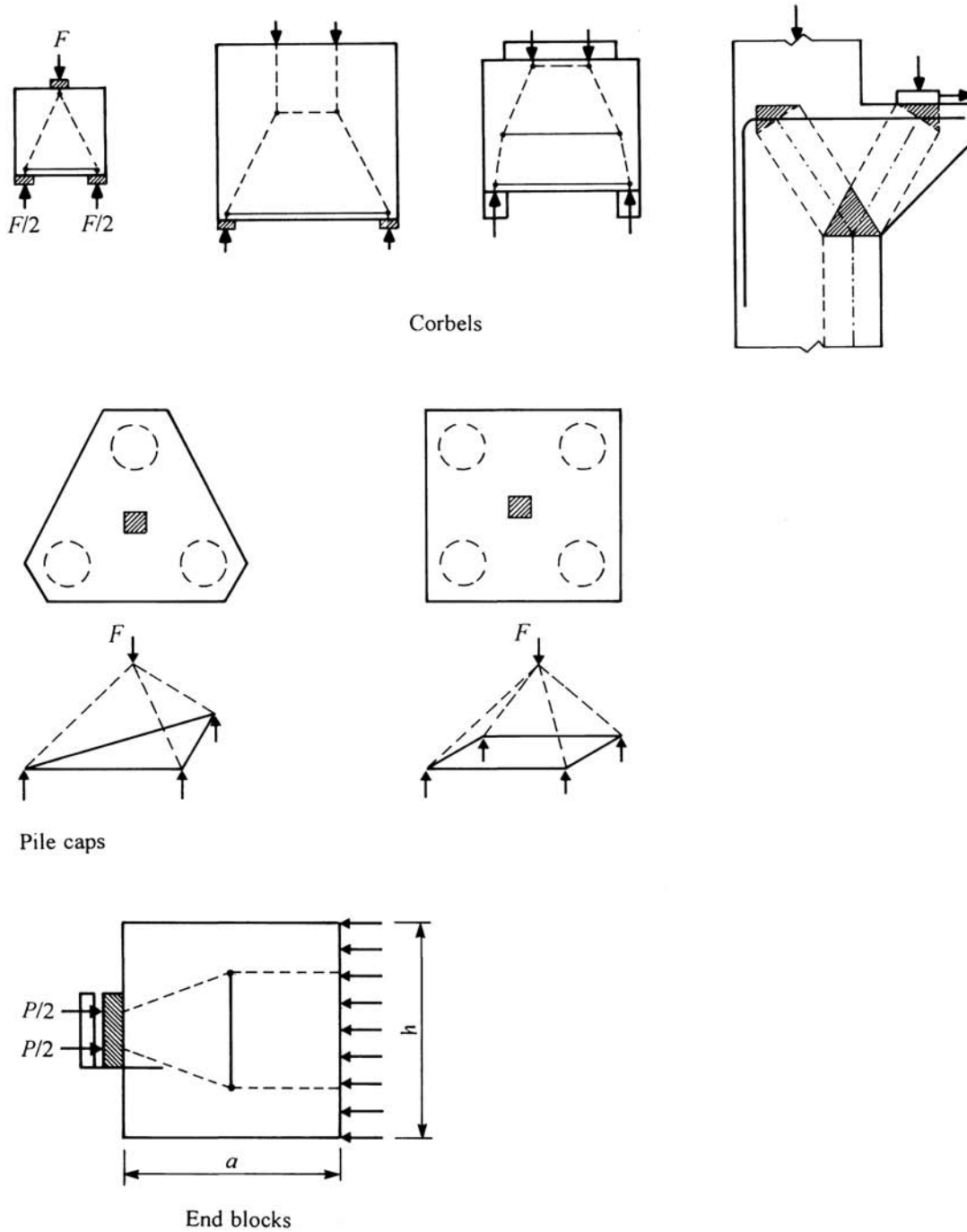


Figure 5.3 Typical strut-and-tie models.

The structure is divided into struts (concrete) and ties (reinforcement bars). The model should reflect closely the elastic stress trajectories. In general, the angle between the struts and ties should not be less than 30°. Internal stresses are calculated so that equilibrium with external loads is achieved.

Limiting permissible stresses are as follows.

- Reinforcement ties
- Struts under uniaxial stress
- Struts under triaxial stress

$$\begin{aligned}
 & f_{yd} \\
 & 0.6f_{cd} \\
 & 1.0f_{cd}
 \end{aligned}$$

6

Material properties

6.1 Concrete

Material properties of concrete (Eurocode 2, part 1 (ENV 1992–1–1: 1993))

| Strength class | f_{ck} | $f_{cm}^{(1)}$ | f_{cd} | $\alpha f_{ck}/\gamma_c^{(2)}$ | f_{ctm} | $f_{ctk\ 0.05}$ | $f_{ctk\ 0.95}$ | τ_{Rd} | $E_{cm}^{(1)}$ | $E_{cd}^{(1)}$ | $\varepsilon_{cu}^{(1)} (\%)$ | $\varepsilon_{cu}^{(2)} (\%)$ |
|----------------|----------|----------------|----------|--------------------------------|-----------|-----------------|-----------------|-------------|----------------|----------------|-------------------------------|-------------------------------|
| C12/15 | 12 | 20 | 8.0 | 6.4 | 1.6 | 1.1 | 2.0 | 0.18 | 26000 | 17300 | 3.6 | 3.5 |
| C16/20 | 16 | 24 | 10.7 | 9.1 | 1.9 | 1.3 | 2.5 | 0.22 | 27500 | 18300 | 3.5 | 3.5 |
| C20/25 | 20 | 28 | 13.3 | 11.3 | 2.2 | 1.5 | 2.9 | 0.26 | 29000 | 19300 | 3.4 | 3.5 |
| C25/30 | 25 | 33 | 16.7 | 14.2 | 2.6 | 1.8 | 3.3 | 0.30 | 30500 | 20300 | 3.3 | 3.5 |
| C30/37 | 30 | 38 | 20.0 | 17.0 | 2.9 | 2.0 | 3.8 | 0.34 | 32000 | 21300 | 3.2 | 3.5 |
| C35/45 | 35 | 43 | 23.3 | 19.8 | 3.2 | 2.2 | 4.2 | 0.37 | 33500 | 22300 | 3.1 | 3.5 |
| C40/50 | 40 | 48 | 26.7 | 22.7 | 3.5 | 2.5 | 4.6 | 0.41 | 35000 | 23300 | 3.0 | 3.5 |
| C45/55 | 45 | 53 | 30.0 | 25.5 | 3.8 | 2.7 | 4.9 | 0.44 | 36000 | 24000 | 2.9 | 3.5 |
| C50/60 | 50 | 58 | 33.3 | 28.3 | 4.1 | 2.9 | 5.3 | 0.48 | 37000 | 24700 | 2.8 | 3.5 |

NOTES

- Structural analysis of sections with a rectangular compression zone; take into account f_{cm} and E_{cm} or f_{cd} and E_{cd}
- Cross-section design

- f_{ck} = characteristic compressive cylinder strength of concrete at 28 days in N/mm²
 f_{cm} = mean value of compressive cylinder strength of concrete at 28 days in N/mm²
 = $f_{ck} + [8]$ (N/mm²)
 f_{cd} = design value of compressive cylinder strength of concrete at 28 days in N/mm²
 $\frac{\alpha f_{ck}}{\gamma_c}$ = f_{ck}/γ_c where γ_c = partial safety factor for concrete = [1.5]; if $\gamma_c > 1.5$, multiply by $1.5/\gamma_c$
 = reduced design compressive cylinder strength of concrete at 28 days in N/mm²
 α = coefficient taking account of long-term effects on the compressive cylinder strength of concrete and of unfavourable effects resulting from the way the load is applied
 = [0.85]; if $\alpha < 0.85$, multiply by $\alpha/0.85$
 γ_c = [1.5]; if $\gamma_c > 1.5$, multiply by $1.5/\gamma_c$
 f_{ctm} = mean value of the axial tensile strength of concrete at 28 days in N/mm²
 $f_{ctk\ 0.05}$ = lower characteristic axial tensile strength (5%-fractile) of concrete at 28 days in N/mm²
 = $0.7f_{ctm}$
 $f_{ctk\ 0.95}$ = upper characteristic axial tensile strength (95%-fractile) of concrete at 28 days in N/mm²
 = $1.3f_{ctm}$
 τ_{Rd} = basic design shear strength of concrete at 28 days in N/mm² = $\frac{0.25 f_{ctk\ 0.05}}{\gamma_c}$ with $\gamma_c = [1.5]$; if $\gamma_c > 1.5$, multiply by $1.5/\gamma_c$
 E_{cm} = mean value of secant modulus of elasticity of concrete in N/mm²
 = $9.5 \cdot 10^3 (f_{ck} + 8)^{1/3}$
 E_{cd} = design value of secant modulus of elasticity of concrete in N/mm² = E_{cm}/γ_c with $\gamma_c = [1.5]$; if $\gamma_c > 1.5$, multiply by $1.5/\gamma_c$
 ε_{cu} = ultimate compressive strain in the concrete in ‰

For the boxed values, apply the values given in the appropriate NAD.

6.2 Reinforcing steel

Material properties of reinforcing steel (Eurocode 2, part 1 (ENV 1992–1–1: 1993) and ENV 10080: 1994)

| Steel name | f_{tk} (N/mm ²) | f_{td} (N/mm ²) | f_{yk} (N/mm ²) | f_{yd} (N/mm ²) | ϵ_{uk} (%) |
|------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|---------------------|
| B500A | 525 | 455 | 500 | 435 | 2.5 ⁽¹⁾ |
| B500B | 540 | 470 | 500 | 435 | 5.0 |

NOTES

1. 2.0% for bars with $d=5.0$ and 5.5 mm, where d is diameter of bar in mm

- f_{tk} = characteristic tensile strength of reinforcing steel in N/mm²
 f_{td} = design tensile strength of reinforcing steel in N/mm² $f_{td} = \gamma_c f_{tk}$
 γ_s = partial safety factor for reinforcing steel = [1.15]; if $\gamma_s = 1.15$, multiply by $1.15/\gamma_s$
 f_{yk} = characteristic yield stress of reinforcing steel in N/mm²
 f_{yd} = design yield stress of reinforcing steel in N/mm² $f_{yd} = f_{yk}/\gamma_s$ with $\gamma_s = [1.15]$; if $\gamma_s = 1.15$, multiply by $1.15/\gamma_s$
 $f_{0.2k}$ = characteristic 0.2% proof-stress of reinforcing steel in N/mm²
 $f_{0.2d}$ = design 0.2% proof-stress of reinforcing steel in N/mm² $f_{0.2d} = f_{0.2k}/\gamma_s$
 ϵ_{uk} = characteristic elongation of reinforcing steel at maximum load in %
 $(f_t/f_y)_k$ = characteristic ratio of tensile strength to yield stress
 E_s = modulus of elasticity of reinforcing steel $E_s = 2 \times 10^5$ N/mm²
 Density = 7850 kg/m³.
 Coefficient of thermal expansion = $10^{-5}/^\circ\text{C}$

Bond characteristics

Ribbed bars: resulting in high bond action (as specified in EN 10080)

Plain, smooth bars: resulting in low bond action

Ductility characteristics

High ductility: $\epsilon_{uk} > [5.0]\%$ and $(f_t/f_y)_k > [1.08]$
 Normal ductility: $\epsilon_{uk} > [2.5]\%$ and $(f_t/f_y)_k > [1.05]$

For the boxed values, apply the values given in the appropriate NAD.

6.3 Prestressing steel

Material properties of prestressing steel (Eurocode 2, part 1 (ENV 1992–1–1:1993) and ENV 10138: 1994)

Wires

| Steel name | f_{pk} (N/mm ²) | f_{pd} (N/mm ²) | $f_{p0.1k}$ (N/mm ²) | $f_{p0.1d}$ (N/mm ²) | E_s (N/mm ²) | ϵ_{uk} (%) |
|------------|-------------------------------|-------------------------------|----------------------------------|----------------------------------|----------------------------|---------------------|
| Y1860C | 1860 | 1620 | 1600 | 1390 | 205000 | 3.5 |
| Y1770C | 1770 | 1540 | 1520 | 1320 | 205000 | 3.5 |
| Y1670C | 1670 | 1450 | 1440 | 1250 | 205000 | 3.5 |
| Y1570C | 1570 | 1370 | 1300 | 1130 | 205000 | 3.5 |

Strands

| Steel name | f_{pk} (N/mm ²) | f_{pd} (N/mm ²) | $f_{p0.1k}$ (N/mm ²) | $f_{p0.1d}$ (N/mm ²) | E (N/mm ²) | ϵ_{uk} (%) |
|------------|-------------------------------|-------------------------------|----------------------------------|----------------------------------|--------------------------|---------------------|
| Y2060S | 2060 | 1790 | 1770 | 1540 | 195000 | 3.5 |

| Steel name | f_{pk} (N/mm ²) | f_{pd} (N/mm ²) | $f_{p0.1k}$ (N/mm ²) | $f_{p0.1d}$ (N/mm ²) | E (N/mm ²) | ϵ_{uk} (%) |
|------------|-------------------------------|-------------------------------|----------------------------------|----------------------------------|--------------------------|---------------------|
| Y1960S | 1960 | 1700 | 1680 | 1460 | 195000 | 3.5 |
| Y1860S | 1860 | 1620 | 1600 | 1639 | 195000 | 3.5 |
| Y1770S | 1770 | 1540 | 1520 | 1250 | 195000 | 3.5 |

Bars

| Steel name | f_{pk} (N/mm ²) | f_{pd} (N/mm ²) | $f_{p0.1k}$ (N/mm ²) | $f_{p0.1d}$ (N/mm ²) | E_s (N/mm ²) | ϵ_{uk} (%) |
|------------|-------------------------------|-------------------------------|----------------------------------|----------------------------------|----------------------------|---------------------|
| Y1030 | 1030 | 900 | 830 | 720 | 205000 | 4.0 |
| Y1100 | 1100 | 960 | 900 | 780 | 205000 | 4.0 |
| Y1230 | 1230 | 1070 | 1080 | 940 | 205000 | 4.0 |

f_{pk} = characteristic tensile strength of prestressing steel in N/mm²

f_{pd} = design tensile strength of prestressing steel in N/mm² = f_{pk}/γ_s

γ_s = partial safety factor for prestressing steel=[1.15]; if γ_s 1.15, multiply by 1.15/ γ_s

$f_{p0.1k}$ = characteristic 0.1% proof-stress of prestressing steel in N/mm²

$f_{p0.1d}$ = design 0.1% proof-stress of prestressing steel in N/mm²= $f_{p0.1k}/\gamma_s$ with γ_s =[1.15]; if γ_s 1.15, multiply by 1.15/ γ_s

ϵ_{uk} = characteristic elongation of prestressing steel at maximum load in %

E_s = modulus of elasticity of reinforcement $E_s=2 * 10^5$ N/mm² (taken into account in stress-strain diagram)

Density=7850 kg/m³

Coefficient of thermal expansion= $10^{-5}/^{\circ}\text{C}$

Classes of relaxation

Class 1: for wires and strands, high relaxation

Class 2: for wires and strands, low relaxation

Class 3: for bars

For the boxed values, apply the values given in the appropriate NAD.

7

Basic design

Table 7.1 Exposure classes

| Exposure class | Examples of environmental conditions | |
|---|--------------------------------------|--|
| 1 Dry environment | | Interior of dwellings or offices |
| 2 Humid environment | (a) Without frost | Interior of buildings with high humidity, e.g. laundries Exterior components Components in non-aggressive soil and/or water |
| | (b) With frost | Exterior components exposed to frost Components in non-aggressive soil and/or water and exposed to frost Interior components where the humidity is high and exposed to frost |
| 3 Humid environment with frost and de-icing agents | | Interior and exterior components exposed to frost and de-icing agents |
| 4 Seawater environment | (a) Without frost | Components completely or partially submerged in seawater or in the splash zone Components in saturated salt air (coastal area) |
| | (b) With frost | Components partially submerged in seawater or in the splash zone and exposed to frost Components in saturated salt air and exposed to frost |
| <i>The following classes may occur alone or in combination with the above</i> | | |
| 5 Aggressive chemical environment ⁽²⁾ | (a) | Slightly aggressive chemical environment (gas, liquid or solid) Aggressive industrial atmosphere |
| | (b) | Moderately aggressive chemical environment (gas, liquid or solid) |
| | (c) | Highly aggressive chemical environment (gas, liquid or solid) |

NOTES

1. This exposure class is valid as long as, during construction, the structure or some of its components are not exposed to more severe conditions over a prolonged period
2. Chemically aggressive environments are classified in ISO 9690. The following exposure conditions may be used:
Exposure class 5a: ISO classification A1G, A1L, A1S
Exposure class 5b: ISO classification A2G, A2L, A2S
Exposure class 5c: ISO classification A3G, A3L, A3S

Table 7.2 Minimum cover requirements for normal weight concrete

| | | Exposure class according to Table 7.1 | | | | | | | | |
|---------------------------|--------------------|---|----|----|----|----|----|----|----|----|
| | | 1 | 2a | 2b | 3 | 4a | 4b | 5a | 5b | 5c |
| Minimum cover (mm) | Reinforcement | 15 | 20 | 25 | 40 | 40 | 40 | 25 | 30 | 40 |
| | Prestressing steel | 25 | 30 | 35 | 50 | 50 | 50 | 35 | 40 | 50 |

NOTES

1. For slab elements, a reduction of 5 mm may be made for exposure classes 2–5.
2. A reduction of 5 mm may be made where concrete of strength class C40/50 and above is used for reinforced concrete in exposure classes 2a–5b and for prestressed concrete in exposure classes 1–5b. However, the minimum cover should never be less than that for class 1.
3. For exposure class 5c, a protective barrier should be used to prevent direct contact with aggressive media.

Table 7.3 Durability requirements related to environmental exposure

| | Exposure class | | | | | | | | |
|--|----------------|------|------|------|------|------|--|------|-------------------|
| | 1 | 2a | 2b | 3 | 4a | 4b | 5a | 5b | 5c ⁽¹⁾ |
| <i>Maximum w/c ratio for⁽²⁾</i> | | | | | | | | | |
| Plain concrete | - | 0.70 | | | | | | | |
| Reinforced concrete | 0.65 | 0.60 | 0.55 | 0.50 | 0.55 | 0.50 | 0.55 | 0.50 | 0.45 |
| Prestressed concrete | 0.60 | 0.60 | | | | | | | |
| <i>Minimum cement content⁽²⁾ (kg/m³) for</i> | | | | | | | | | |
| Plain concrete | 150 | 200 | 300 | | | | 200 | | |
| Reinforced concrete | 260 | 280 | 280 | 300 | 300 | 300 | 280 | 300 | 300 |
| Prestressed concrete | 300 | 300 | 300 | | | | 300 | | |
| <i>Minimum air content of fresh concrete (%) for nominal maximum aggregate size of⁽³⁾</i> | | | | | | | | | |
| 32 mm | - | - | 4(4) | 4(4) | - | 4(4) | - | - | - |
| 16 mm | - | - | 5 | 5 | - | 5 | - | - | - |
| 8mm | - | - | 6 | 6 | - | 6 | - | - | - |
| Frost-resistant aggregates ⁽⁶⁾ | - | - | Yes | Yes | - | Yes | - | - | - |
| Impermeable concrete according to clause 7.3.1.5 | - | - | Yes | Yes | Yes | Yes | Yes | Yes | Yes |
| Types of cement for plain and reinforced concrete according to EN 197 | | | | | | | Sulfate-resisting cement ⁽⁵⁾ >500 mg/kg in water or >3000 mg/kg in soil | | |

NOTES

These w/c ratios and cement contents are based on cements for which there is considerable experience in many countries. However, at the time this pre-standard was drafted, experience with some of the cements standardized in EN 197 was limited to local climates in some countries. Therefore, during the life of this prestandard, particularly for exposure classes 2b, 3 and 4b, the choice of cement type and composition should follow the national standards and regulations locally in force. Alternatively, cement CEI may be used generally for prestressed concrete. Other types may be used if experience of them is available and the application is permitted by the national standards or local regulations.

1. In addition, the concrete shall be protected against direct contact with aggressive media by coatings unless such protection is considered unnecessary. 2. For minimum cement content and maximum w/c ratio in this pre-standard, only cement listed in 4.1 shall be taken into account. When pozzolanic or latent hydraulic additions are added to the mix, national standards or regulations locally

| Exposure class | | | | | | | | |
|--|----|----|---|----|----|----|----|-------------------|
| 1 | 2a | 2b | 3 | 4a | 4b | 5a | 5b | 5c ⁽¹⁾ |
| in force may state whether, and how, the minimum or maximum values may be modified. 3. With a spacing factor of the air-entrained void system <20 mm measured on the hardened concrete. 4. Applicable where the degree of saturation is high for prolonged periods of time. Other values may apply if the concrete is tested and documented to have adequate frost resistance according to the national standards or regulations locally in force. 5. The sulfate resistance of the cement shall be judged on the basis of national standards or regulations locally in force. 6. Assessed against the national standards or regulations locally in force. | | | | | | | | |

Table 7.4 Strength classes to satisfy maximum water/cement ratio requirements

| Strength class of cement | Water/cement ratio | | | | |
|--------------------------|--------------------|--------|--------|--------|--------|
| 0.65 | 0.60 | 0.55 | 0.50 | 0.45 | |
| CE 32.5 | C20/25 | C25/30 | C30/37 | C35/45 | C40/50 |
| CE 42.5 | C25/30 | C30/37 | C35/45 | C40/50 | C45/55 |

7.5

Prestressed concrete

7.5.1

Material properties

Concrete grades should be chosen to satisfy durability requirements for particular exposure conditions. In any case, the strength class for post-tensioned work should not be less than C25/30 and for pre-tensioned work not less than C30/37.

A bilinear stress-strain diagram with a horizontal branch with a value of $(0.9f_{pk})/\gamma_s$ may be used for prestressing steel, where f_{pk} is the characteristic strength of prestressing steel and $\gamma_s=1.15$.

7.5.2

Minimum number of tendons

In isolated statically determinate members a minimum number of prestressing bars/wires/ tendons, as shown below, should be provided.

| Type | Minimum number |
|---|----------------|
| Individual bars and wires | 3 |
| Bars and wires forming a strand or tendon | 7 |
| Tendons except strands | 3 |

7.5.3

Initial prestressing force

The maximum tendon force is given by $P_o=A_p (0.8f_{pk})$ or $A_p (0.9f_{p0.1k})$ whichever is less, where $f_{p0.1k}$ is the characteristic 0.1 % proof-stress of the prestressing steel and A_p is the area of prestressing steel.

The prestressing force applied to the concrete immediately after tensioning (in pre-tensioned work) or after transfer (in post-tensioned work) is given by $P_{m,0}=A_p (0.75f_{pk})$ or $A_p (0.85f_{p0.1k})$, whichever is less.

The force applied to the concrete should be calculated allowing for losses caused by: (a) friction (if applicable), short-term relaxation and elastic shortening for pre-tensioned members; and (b) duct friction, anchorage slip and elastic shortening for post-tensioned members.

7.5.4

Loss of prestress

(a) The mean effective prestressing force $P_{m,t}$, is the force at the active end of the tendon less the relevant losses, which should be calculated on the basis of experience and data relating to the materials and methods used. As a result, only the types of

losses to be taken into account are indicated below. National practice conforming to the National Application Document should be adopted.

(b) The losses to be considered are anchorage slip, elastic shortening, friction in ducts, creep of concrete, shrinkage of concrete and relaxation of steel.

7.5.5 Anchorage

(a) Pre-tensioned members

The transmission length is given by $l_{bp} = \beta_b \phi$, when ϕ is the nominal size of the tendon and β_b is obtained, in the absence of other more accurate data, from the Table below.

The design value of l_{bp} should be taken as either $0.8l_{bp}$ or $1.2l_{bp}$ whichever is more critical. The length over which the stresses across the section of concrete gradually disperse to a linear distribution may be taken as

$$l_{p,eff} = \sqrt{(l_{bpd}^2 + d^2)}$$

If the principal tensile stress at the ultimate limit state does not exceed $0.7f_{ctm}$ the anchorage is considered satisfactory. If not, the following should be satisfied.

$[(M_{sd}/z) + (V_{sd}/2)] (X/l_{bpd}) P_o A_{p0.1k} / 1.15$, where X is the distance of a section from the support.

(b) Post-tensioned members

The bearing stress behind anchorage plates caused by the force $A_p f_{ck}$ should not exceed

$$f_{Rd,u} = 0.67 f_{ck} \sqrt{A_{c1}/A_{co}} \leq 2.2 f_{ck}$$

where A_{c1} is the maximum area having the same centre of gravity and shape as the loaded area A_{co} , which it is possible to inscribe within the total area of member A_c .

Tensile stresses caused by the concentrated forces should be assessed by strut-and-tie model or other appropriate idealization and the anchorage zone should be reinforced accordingly.

Table 7.5 Minimum dimensions for fire resistance of rectangular or circular reinforced (normal weight) concrete columns

Standard fire resistance

Column width b /axis distance a (both in mm)

| Column exposed on more than one side | Column exposed on one side | |
|--------------------------------------|----------------------------|--------|
| R 30 | 150/10 | 100/10 |
| R 60 | 200/10 | 120/10 |
| R 90 | 240/35 | 140/10 |
| R 120 | 280/40 | 160/45 |
| R 180 | 360/50 | 200/60 |
| R 240 | 450/50 | 300/60 |

NOTE

The ratio of the design effect of actions in the fire to the cold resistance of the structural element is assumed to be 0.7.

Table 7.6 Minimum dimensions for fire resistance of load-bearing reinforced (normal weight concrete walls made with siliceous aggregate)

| Standard fire resistance | Wall thickness/axis distance (both in mm) | |
|--------------------------|---|--------|
| Exposed on one side | Exposed on two sides | |
| REI 30 | 120/10 | 120/10 |
| REI 60 | 130/10 | 140/10 |
| REI 90 | 140/25 | 170/25 |
| REI 120 | 160/35 | 220/35 |
| REI 180 | 210/55 | 300/55 |
| REI 240 | 270/70 | 360/70 |

NOTE

The ratio of the design effect of actions in the fire to the cold resistance of the structural element is assumed to be 0.7.

Table 7.7 Minimum dimensions for fire resistance of simply supported reinforced concrete (normal weight) beams

| Standard fire resistance (mm) | Possible combinations of the average axis distance a and the beam width b (both in mm) | | | | Web thickness b_w of I-beams (mm) |
|--|--|----------------|-----------------------------|----------------|-------------------------------------|
| | $a=25$ $b=80$ | $a=15$ $b=120$ | $a=10$ $b=160$ | $a=10$ $b=200$ | |
| R 30 | $a=25$ $b=80$ | $a=15$ $b=120$ | $a=10$ $b=160$ | $a=10$ $b=200$ | 80 |
| R 60 | $a=40$ $b=120$ | $a=35$ $b=160$ | $a=30$ $b=200$ | $a=25$ $b=300$ | 100 |
| R 90 | $a=55$ $b=150$ | $a=45$ $b=200$ | $a=45$ $b=250$ | $a=35$ $b=400$ | 100 |
| R 120 | $a=65$ $b=200$ | $a=55$ $b=240$ | $a=50$ $b=300$ | $a=45$ $b=500$ | 120 |
| R 180 | $a=80$ $b=240$ | $a=70$ $b=300$ | $a=65$ $b=400$ | $a=60$ $b=600$ | 140 |
| R 240 | $a=90$ $b=280$ | $a=80$ $b=350$ | $a=75$ $b=500$ | $a=70$ $b=700$ | 160 |
| $a_{st}=a+10$ mm (see note below) | | | $a_{st}=a$ (see note below) | | |
| a_{st} =increased axis distance of the outermost bar (tendon, wire) from the side surface of the cross-section, where steel is in a single layer | | | | | |

NOTES

- For prestressed members, the axis distances should be increased by 10 mm for prestressing bars and by 15 mm for wires or strands.
- The table applies to beams exposed to fire on three sides.
- For beams exposed to fire on all four sides, the height should at least equal the minimum dimension b_{min} in the table for the required fire resistance and its cross-sectional area should be at least $2b_{min}^2$.
- The minimum axis distance to any individual bars should not be less than that required for R 30 in the table nor less than half the average axis distance.

Table 7.8 Minimum dimensions for fire resistance of continuous reinforced concrete (normal weight) beams

| Standard fire resistance (mm) | Possible combinations of the average axis distance a and the beam width b (both in mm) | | | Web thickness b_w of I-beams (mm) |
|--|--|----------------|-----------------------------|-------------------------------------|
| | $a=12$ $b=80$ | $a=12$ $b=200$ | $a=20$ $b=200$ | |
| R 30 | $a=12$ $b=80$ | | $a=20$ $b=200$ | 80 |
| R 60 | $a=25$ $b=120$ | $a=12$ $b=200$ | $a=25$ $b=300$ | 100 |
| R 90 | $a=35$ $b=150$ | $a=45$ $b=250$ | $a=25$ $b=400$ | 100 |
| R 120 | $a=45$ $b=200$ | $a=35$ $b=300$ | $a=35$ $b=500$ | 120 |
| R 180 | $a=50$ $b=240$ | | $a=50$ $b=600$ | 140 |
| R 240 | $a=60$ $b=280$ | | $a=60$ $b=700$ | 160 |
| $a_{st}=a+10$ mm (see note below) | | | $a_{st}=a$ (see note below) | |
| a_{st} =increased axis distance of the outermost bar (tendon, wire) from the side surface of the cross-section, where steel is in a single layer | | | | |

NOTES

- For prestressed members, the axis distances should be increased by 10 mm for prestressing bars and by 5 mm for wires or strands.
- The table applies to beams exposed to fire on three sides.
- For beams exposed to fire on all four sides, the height should at least equal the minimum dimension b_{min} in the table for the required fire resistance and its cross-sectional area should be at least $2b_{min}^2$.
- The minimum axis distance to any individual bars should not be less than that required for R 30 in the table nor less than half the average axis distance.
- For R 90 and above, the top reinforcement over each intermediate support should extend at least $0.3l_{eff}$ from the centre of support, where the effective span $l_{eff}>4$ metres and $l_{eff}/h>20$, h being the beam depth. In other cases, this minimum may be reduced to $0.15l_{eff}$.
- If the above detailing requirement is not met and the moment redistribution in the analysis exceeds 15%, each span of the continuous beam should be assessed as a simply supported beam.
- In a continuous I-beam, b_w should not be less than b for a distance of $2h$ from an intermediate support unless a check for explosive spalling is carried out.
- In two-span I-beam systems with no rotational restraint at the end, with predominantly concentrated loading with M_{sd}/V_{sd} between 2.5 and 3, and with $V_{sd}>^{2/3}V_{rd2}$, the minimum width of the beam web between the concentrated loads should be: 220 mm for R 120, 400 mm for R 180 and 600 mm for R 240.

Table 7.9 Minimum dimensions for fire resistance for solid (normal weight) reinforced concrete slabs spanning one and two ways

| Standard fire resistance | Slab thickness h_s (mm) | | Average axis distance span a (mm) | |
|--------------------------|---------------------------|-----------------|-------------------------------------|----|
| | One way | Two way | | |
| | $l_y/l_x<1.5$ | $1.5<l_y/l_x<2$ | | |
| REI 30 | 60 | | 10 | 10 |
| REI 60 | 80 | | 20 | 10 |
| REI 90 | 100 | | 30 | 15 |
| REI 120 | 120 | | 40 | 20 |

| Standard fire resistance | Slab thickness h_s (mm) | Average axis distance span a (mm) | | |
|--------------------------|---------------------------|-------------------------------------|----|----|
| One way | Two way | | | |
| $l_y/l_x < 1.5$ | $1.5 < l_y/l_x < 2$ | | | |
| REI 180 | 150 | 55 | 30 | 40 |
| REI 240 | 175 | 65 | 40 | 50 |

l_x and l_y are the spans of a two-way slab (two directions at right-angles) where l_y is the longer span

NOTES

- For prestressed members, the axis distances should be increased by 10 mm for prestressing bars and by 15 mm for wires or strands.
- The minimum cover to any bar should not be less than half the average axis distance.
- The table values of axis distance for two-way slabs apply to slabs supported on all four edges. For all other support conditions, the values for one-way slabs should be used.
- The table values of slab thickness and cover for two-way slabs with $l_y/l_x < 1.5$ should be used.
- For R 90 and above, the top reinforcement over each intermediate support should extend at least $0.3l_{eff}$ from the centre of support, where the effective span $l_{eff} > 4$ metres and $l_{eff}/h > 20$, h being the beam depth. In other cases, this minimum may be reduced to $0.15l_{eff}$.
- If the above detailing requirement is not met and the moment redistribution in the analysis exceeds 15%, each span of the continuous slab should be assessed as a simply supported slab.
- Minimum top reinforcement of $0.005/A_c$ should be used over intermediate supports when the reinforcement has “normal” ductility, when there is not rotational restraint at ends of two-span slabs, and when transverse redistribution of load effects cannot occur.

Table 7.10 Minimum dimensions for fire resistance of reinforced and prestressed (normal weight) concrete slabs

| Standard fire resistance | Slab thickness h_s (mm), excluding finishes | Axis distance a (mm) |
|--------------------------|---|------------------------|
| REI 30 | 150 | 10 |
| REI 60 | 200 | 15 |
| REI 90 | 200 | 25 |
| REI 120 | 200 | 35 |
| REI 180 | 200 | 45 |
| REI 240 | 200 | 50 |

NOTES

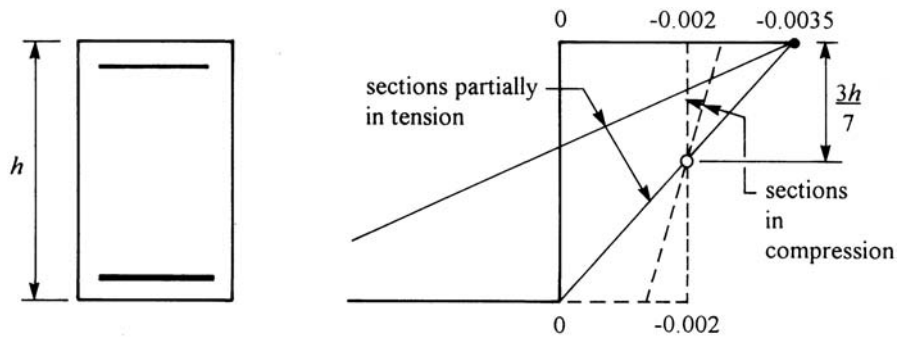
- For prestressed members, the axis distances should be increased by 10 mm for prestressing bars and by 15 mm for wires or strands.
- It is assumed that the moment redistribution in this analysis does not exceed 15%. If it does exceed 15%, the axis distances in this table should be replaced by those for one-way slabs.
- Over intermediate supports in each direction, at least 20% of the total top reinforcement calculated for cold design should extend over the full span, in the column strips.

8

Bending and longitudinal force

8.1 Conditions at failure

Figure 8.1 (taken from 4.11 in EC2) shows the strain conditions assumed at the ultimate limit state for reinforced concrete.



For cross-sections not fully in compression, the concrete is assumed to fail in compression when the strain reaches 0.0035. The strain in the tension reinforcement need not be limited where a horizontal top branch is assumed for the reinforcement stress-strain curve.

For cross-sections that are completely in compression, the strain is limited to 0.002 at a height of $3/7h$ from the most compressed face.

The strains in the reinforcement at ultimate are given by the formulae in [Table 8.1](#).

Table 8.1 Strains in reinforcement at ultimate

| | |
|-----|---|
| (A) | $x < h$ |
| | Compression reinforcement |
| | $\epsilon'_s = \frac{0.0035}{x} (x - d')$ |
| | Reinforcement near tension or least compressed face |
| (B) | $x > h$ |
| | Reinforcement near most compressed face |
| | $\epsilon_s = -\frac{0.002}{(x - 3h/7)} (x - d')$ |
| | Reinforcement near least compressed face |
| | $\epsilon_s = \frac{0.002}{(x - 3h/7)} (d - x)$ |

In general, it is satisfactory to assume that the reinforcement near to the most compressed face is yielding but there are cases when this may not be so. [Table 8.2](#) sets out the conditions for the reinforcement to be yielding, assuming a bilinear stress-strain diagram.

Table 8.2 Conditions for yield of reinforcement

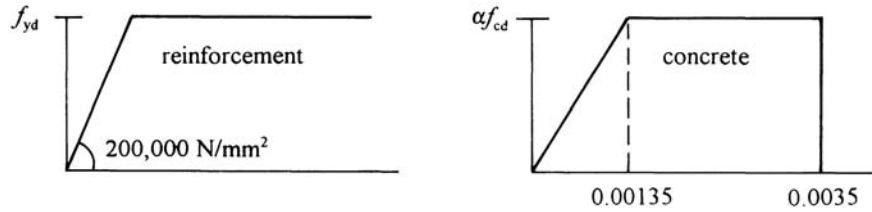
| | |
|-----|--|
| (A) | $x < h$ |
| | Compression steel |
| | $f_{yd} \leq 700(1 - d'/x)$ or $\frac{d'}{x} \leq 1 - f_{yd}/700$ |
| | Tension steel |
| | $\frac{x}{d} \leq \frac{1}{f_{yd}/700 + 1}$ |
| (B) | $x > h$ |
| | Compression steel |
| | $\frac{(1 - d'/x)}{(1 - 3h/7x)} > \frac{f_{yd}}{900}$ or $\frac{x}{h} \leq \frac{\left(\frac{3f_{yd}}{2800} - d'/h\right)}{\left(\frac{f_{yd}}{400} - 1\right)}$ |

8.2

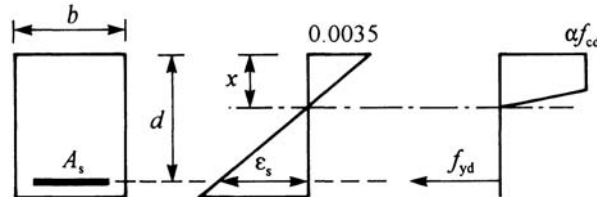
Design of rectangular sections subject to flexure only

I Derivation of equations

Stress-strain curves for reinforcement and concrete.



Conditions in section at ultimate in a singly reinforced section.



The limits to x/d will ensure that stress in steel is at yield. The average stress in compression zone is $0.807f_{cd}$. Distance from compression face to centre of concrete compression is $0.411x$.

Equilibrium of axial forces gives:

$$\frac{x}{d} = \frac{1.239 A_s f_{yd}}{\alpha f_{cd} b d}$$

Defining $\omega = \frac{A_s f_{yd}}{\alpha f_{cd} b d}$

$$\frac{x}{d} = 1.239 \omega \tag{I}$$

The lever arm, z , is given by:

$$\frac{z}{d} = 1 - 0.411 \left(\frac{x}{d} \right) \tag{IIa}$$

or

$$\frac{z}{d} = 1 - 0.5092 \omega \tag{IIb}$$

The moment is given by:

$$M = A_s f_{yd} z$$

$$\text{hence } \frac{M}{bd^2 \alpha f_{cd}} = \omega (1 - 0.5092\omega)$$

IIIa

Defining

$$m = \frac{M}{bd^2 \alpha f_{cd}} \text{ and solving for } \omega \text{ gives:}$$

IIIb

$$\omega = (1 - \sqrt{1 - 2.0368m}) / 1.0184$$

$$\text{or, approximately, } \omega = 1 - \sqrt{1 - 2m}$$

Equation 2.17 in Eurocode 2 can be rewritten to give:

$$\text{for } f_{cu} \leq 35 \quad \left(\frac{x}{d}\right)_{lim} = (\delta - 0.44) / 1.25$$

IVa

$$\text{for } f_{cu} > 35 \quad \left(\frac{x}{d}\right)_{lim} = (\delta - 0.56) / 1.25$$

IVb

From I

$$\omega_{lim} = 0.807 \left(\frac{x}{d}\right)_{lim}$$

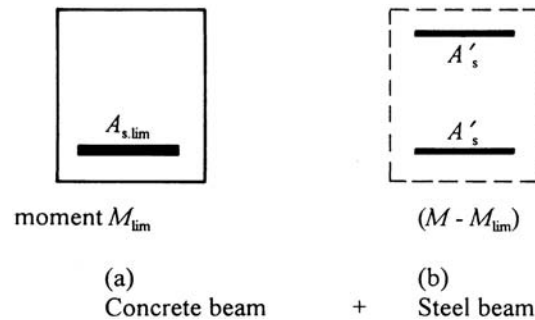
V

From III(a) and II(a)

$$m_{lim} = \left(\frac{x}{d}\right)_{lim} \left(0.411 \left(\frac{x}{d}\right)_{lim}\right)$$

VI

If $m > m_{lim}$, compression steel is needed to maintain the neutral axis at the limiting value. The moment capacity can then be calculated by assuming two superimposed sections.



The steel area required in the 'steel beam' is given by:

$$A_s = \frac{(M - M_{lim})}{f_{yd}(d - d')}$$

VIIa

(Assuming reinforcement in compression is yielding)

$$\text{or } \omega = \frac{A_s f_{yd}}{\alpha f_{cd}} = \frac{(m - m')}{(1 - d'/d)}$$

VIIb

The area of steel required for the 'concrete beam' is given by equation V.

Hence, total areas of reinforcement are given by:

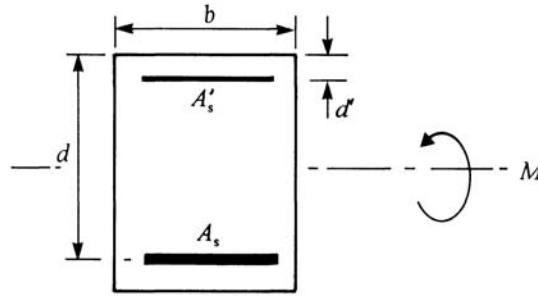
$$\omega = \frac{(m - m')}{(1 - d'/d)}$$

and

$$\omega = \omega_{lim} + \omega'$$

VIII

The procedure for using these equations directly for calculating reinforcement areas is summarized below in [Table 8.3](#).

Table 8.3 Design of rectangular beams

1. Calculate $m = \frac{M}{bd^2\alpha f_{cd}}$
2. Calculate $\left(\frac{x}{d}\right)_{lim}$:
 if $f_{cu} \leq 35$, $\left(\frac{x}{d}\right)_{lim} = (\delta - 0.44)/1.25$
 if $f_{cu} > 35$, $\left(\frac{x}{d}\right)_{lim} = (\delta - 0.56)/1.25$
3. Calculate $M_{lim} = \left(\frac{x}{d}\right)_{lim} (1 - 0.411\left(\frac{x}{d}\right)_{lim})$
4. If $m < m_{lim}$, simply reinforced beam will suffice

$$\omega = 1 - \sqrt{1 - 2m}$$

Hence calculate A_s - END

5. If $m > m_{lim}$

$$\omega' = \frac{(m - m_{lim})}{(1 - d'/d)}$$

$$\omega = 0.807\left(\frac{x}{d}\right)_{lim} + \omega'$$

Hence calculate A_s and A_s' -END

Design Tables

The equations can be presented as design tables as shown below.

Table 8.4 gives values of x/d and ω for singly reinforced beams as a function of m .

Table 8.4 Values of x/d and ω for singly reinforced beams

| $\frac{M}{bd^2\alpha f_{cd}}$ | $\frac{A_s f_{yd}}{bd\alpha f_{cd}}$ | $\frac{x}{d}$ | $\frac{M}{bd^2\alpha f_{cd}}$ | $\frac{A_s f_{yd}}{bd\alpha f_{cd}}$ | $\frac{x}{d}$ |
|-------------------------------|--------------------------------------|---------------|-------------------------------|--------------------------------------|---------------|
| 0.01 | 0.010 | 0.012 | 0.17 | 0.188 | 0.233 |
| 0.02 | 0.020 | 0.025 | 0.18 | 0.200 | 0.248 |
| 0.03 | 0.030 | 0.038 | 0.19 | 0.213 | 0.264 |
| 0.04 | 0.041 | 0.052 | 0.2 | 0.226 | 0.280 |
| 0.05 | 0.051 | 0.064 | 0.21 | 0.239 | 0.296 |
| 0.06 | 0.062 | 0.077 | 0.22 | 0.252 | 0.313 |
| 0.07 | 0.073 | 0.090 | 0.23 | 0.266 | 0.330 |
| 0.08 | 0.084 | 0.104 | 0.24 | 0.280 | 0.347 |
| 0.09 | 0.095 | 0.117 | 0.25 | 0.294 | 0.364 |
| 0.1 | 0.106 | 0.131 | 0.26 | 0.308 | 0.382 |
| 0.11 | 0.117 | 0.145 | 0.27 | 0.323 | 0.400 |
| 0.12 | 0.128 | 0.159 | 0.28 | 0.338 | 0.419 |
| 0.13 | 0.140 | 0.173 | 0.29 | 0.354 | 0.438 |
| 0.14 | 0.152 | 0.188 | 0.3 | 0.370 | 0.458 |
| 0.15 | 0.164 | 0.203 | 0.31 | 0.386 | 0.478 |
| 0.16 | 0.176 | 0.218 | 0.32 | 0.402 | 0.499 |

Table 8.5 gives $\left(\frac{x}{d}\right)_{\text{lim}}$, ω_{lim} and m_{lim} as a function of the amount of redistribution.

Table 8.5 Limiting values of $\left(\frac{x}{d}\right)$, $\frac{M}{bd^2\alpha f_{\text{ck}}}$ and $\frac{A_s f_{\text{yd}}}{bd\alpha f_{\text{ck}}}$

| Percentage redistribution | δ | $\left(\frac{x}{d}\right)_{\text{lim}}$ | | $\left(\frac{M}{bd^2\alpha f_{\text{ck}}}\right)_{\text{lim}}$ | | $\left(\frac{A_s f_{\text{yd}}}{bd\alpha f_{\text{ck}}}\right)_{\text{lim}}$ | |
|---------------------------|----------|---|----------------------|--|----------------------|--|----------------------|
| | | $f_{\text{ck}} \leq 35$ | $f_{\text{ck}} > 35$ | $f_{\text{ck}} \leq 35$ | $f_{\text{ck}} > 35$ | $f_{\text{ck}} \leq 35$ | $f_{\text{ck}} > 35$ |
| 0 | 1.00 | 0.448 | 0.352 | 0.295 | 0.243 | 0.362 | 0.284 |
| 5 | 0.95 | 0.408 | 0.312 | 0.274 | 0.220 | 0.329 | 0.252 |
| 10 | 0.90 | 0.368 | 0.272 | 0.252 | 0.195 | 0.267 | 0.220 |
| 15 | 0.85 | 0.328 | 0.232 | 0.229 | 0.169 | 0.265 | 0.187 |
| 20 | 0.80 | 0.288 | 0.192 | 0.205 | 0.143 | 0.232 | 0.155 |
| 25 | 0.75 | 0.248 | 0.152 | 0.180 | 0.115 | 0.200 | 0.123 |
| 30 | 0.70 | 0.208 | 0.112 | 0.154 | 0.086 | 0.168 | 0.090 |

Tables 8.4 and 8.5 can be used to streamline the procedure set out in Table 8.3.

Flanged beams

Since concrete in tension is ignored, the design of a flanged beam is identical to that for a rectangular beam provided that the neutral axis at failure lies within the flange.

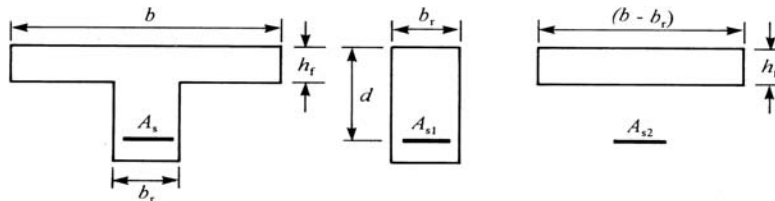
Thus the procedure for design can be:

1. Follow steps 1 to 4 in Table 8.3 using the overall flange breadth as b .

2. Calculate $\left(\frac{x}{d}\right) = \omega/0.807 > \left(\frac{x}{d}\right)_{\text{lim}}$

If $\left(\frac{x}{d}\right) < \left(\frac{h_f}{d}\right)$, design is OK. This will normally be the case.

If $\left(\frac{x}{d}\right) > \left(\frac{h_f}{d}\right)$, then further equations need to be derived. This can most easily be achieved by considering the base to be made up of two parts as shown below:



It will be assumed that the neutral axis is large enough for the whole flange to be at a stress of αf_{cd} .

Hence, by equilibrium,

$$A_{s2} = (b - b_r) h_f \alpha f_{\text{cd}} / f_{\text{yd}} \quad \text{IX}$$

$$M_2 = A_{s2} f_{\text{yd}} (d - h_f / 2) \quad \text{X}$$

The steel area required for the rectangular rib can now be obtained by using Table 8.3 to assess the reinforcement area needed for a rectangular beam of breadth b_r to support a moment of $M_1 = (M - M_2)$.

Although very unlikely to be exceeded, the limiting moment for a flanged beam where $(x/d)_{\text{lim}}$ exceeds (h_f/d) is given by:

$$M_{\text{lim}} = M_{1,\text{lim}} + b_{\text{hf}} \alpha f_{\text{cd}} (d - h_f / 2)$$

$$\left[\left(\frac{x}{d}\right)_{\text{lim}} \left[1 - 0.0411 \left(\frac{x}{d}\right)_{\text{lim}} \right] b_r d^2 + b h_f \left[d - \left(\frac{h_f}{z}\right) \right] \right] \alpha f_{\text{cd}}$$

The required steel areas can then be calculated using Equations VIIIa, XI, X and V.

The procedure for the design of flanged sections is summarized in Table 8.6.

Table 8.6 Design of flanged sections for flexure

1. Calculate $m = \frac{M}{bd\alpha f_{cd}}$
2. Follow Table 8.3 to obtain ω . Calculate $\frac{x}{d} = \frac{\omega}{0.807} \times \left(\frac{x}{d}\right)_{im}$
If $\left(\frac{x}{d}\right) \leq \left(\frac{h_f}{d}\right)$, calculate A_s from ω (END)
3. If $\left(\frac{x}{d}\right) > \left(\frac{h_f}{d}\right)$
Calculate $A_{s2} = (b - b_r)h_f \alpha f_{cd} / f_{yd}$
 $M_2 = A_{s2} f_{yd} (d - h_f / 2)$
4. Use Table 8.3 to calculate steel areas for rectangular sections of breadth b_r to resist moment of $(M - M_2)$.
5. Areas of steel = sum of those obtained from steps 3 and 4.

Minimum reinforcement

There are two provisions defining minimum areas of flexural steel. These are:

- (a) minimum for crack control 4.4.2.2.
- (b) overall minimum 5.4.2.1.1.

The formula in 4.4.2.2 is:

$$A_s \geq A_{s2} \geq k_c k f_{ct,eff} A_{ct} / \sigma_s$$

where, for bending, $k_c = 0.4$

$f_{ct,eff}$ is suggested as 3, k is 0.8 for sections with depths not greater than 300 mm and 0.5 for sections deeper than 800 mm, σ_s may be taken as f_{yk} . A_{ct} , the area of concrete in the tension zone immediately before cracking, will be $bh/2$ for rectangular sections and an approximate value for flanged beams could be taken as $0.75 b_t h$ where b_t is the breadth of the tension zone. If h is assumed to be $1.15d$, the above equation thus reduces to:

| | | |
|-----------------------|-----------|--------------------|
| for rectangular beams | h 300mm | $0.55bd/f_{yk}$ |
| | h 800mm | $0.34bd/f_{yk}$ |
| for flanged beams | h 300mm | $0.83b_t d/f_{yk}$ |
| | h 800mm | $0.55b_t d/f_{yk}$ |

Interpolation is permitted for depths between 300 and 800 mm.

Clause 5.4.2.1.1 gives:

$$A_s \geq \frac{0.6 b_t d}{f_{yk}} \geq 0.0015 b_t d$$

Assuming $f_{yk} > 400$, $0.0015 b_t d$ will govern.

It will be seen, in any case, that the rule in 5.4.2.1.1 will always govern except for shallow flanged beams and, for commonly used reinforcement, the limit of $0.0015 b_t d$ will be the controlling factor in 5.4.2.1.1. The following general rule therefore seems adequate for normal beams.

Table 8.7: Minimum tension reinforcement

- If $f_{yk} = 500 \text{ N/mm}^2$
or $f_{yk} < 500 \text{ N/mm}^2$ and beam is either rectangular or flanged with $h < 700 \text{ mm}$
then $A_s = 0.001 b_t d$
else

$$A_s \geq 0.083 - \frac{(h - 300)}{1786} \frac{b_t d}{f_{yk}} \geq 0.0015 b_t d$$

8.5**Design charts for columns (combined axial and bending)**

The following 59 charts are provided for the design of symmetrically reinforced rectangular columns and circular columns.

The charts provided are:

Charts 8.1 to 8.12: Charts for uniaxial bending of rectangular sections.

Charts are drawn for ratios of d_1/h of 0.05, 0.1, 0.15 and 0.2 for three different arrangements of reinforcement as follows.

Charts 8.1 to 8.4: Columns with the reinforcement concentrated along the edges parallel to the axis of bending.

Charts 8.5 to 8.8: Columns with the reinforcement distributed along the edges perpendicular to the axis of bending. Three bars in each face are assumed: near the corners and at the centre of the face. This represents the worst case.

Charts 8.9 to 8.12: Columns with the reinforcement distributed along all sides. Bars are assumed at the corners and at the centre of the sides.

Charts 8.13 to 8.19: Charts for circular columns. The reinforcement is assumed to consist of six bars uniformly spaced round the perimeter. This will be slightly conservative for larger numbers of bars. Curves are drawn for values of $\frac{A_s f_{yd}}{h^2 \alpha_{fd}}$ from 0 to 1.0.

Charts 8.20 to 8.59: Charts for bi-axial bending of symmetrically reinforced rectangular columns.

The bi-axial charts are used as follows:

- (1) Calculate $m_x = \frac{M_x}{bh^2 \alpha_{fd}}$
and $m_y = \frac{M_y}{hb^2 \alpha_{fd}}$
- (2) M_{max} =greater of M_x or M_y
 M_{min} =lesser of M_x or M_y
Calculate M_{min}/M_{max}
- (3) Calculate $\frac{N_{ed}}{bh \alpha_{fd}}$
- (4) Select most appropriate chart for the reinforcement arrangement, d'/h and M_{min}/M_{max} and read off $\frac{A_s f_{yd}}{bh \alpha_{fd}}$
Interpolation between charts may be necessary.

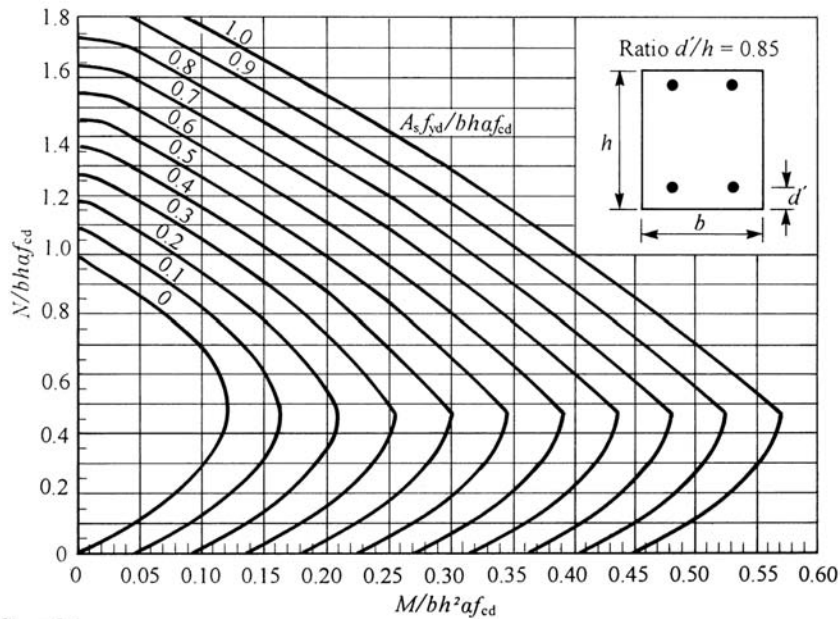


Chart 8.1

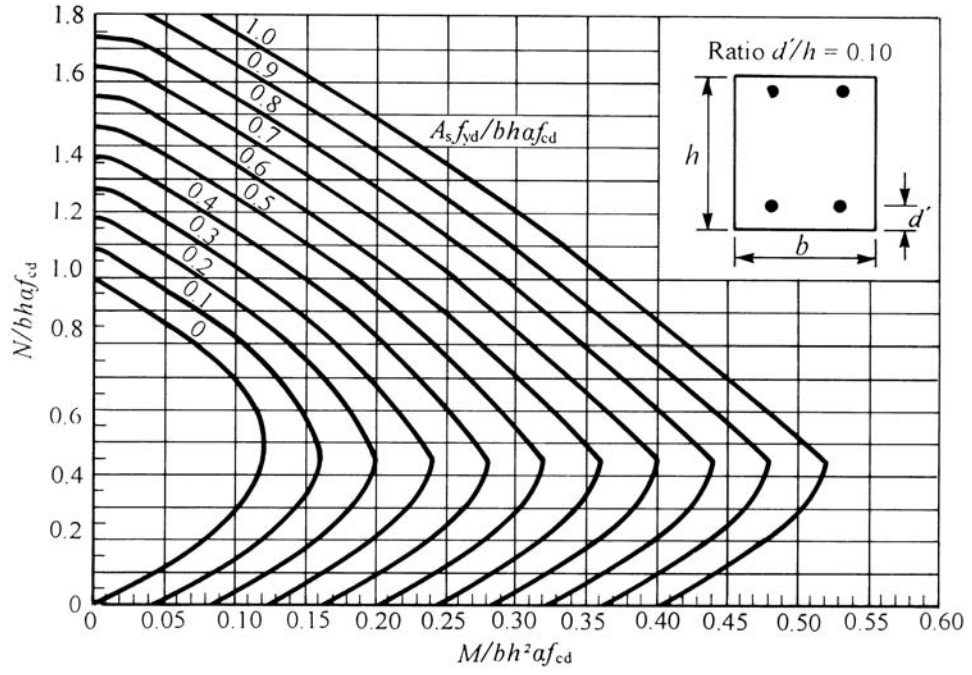


Chart 8.2

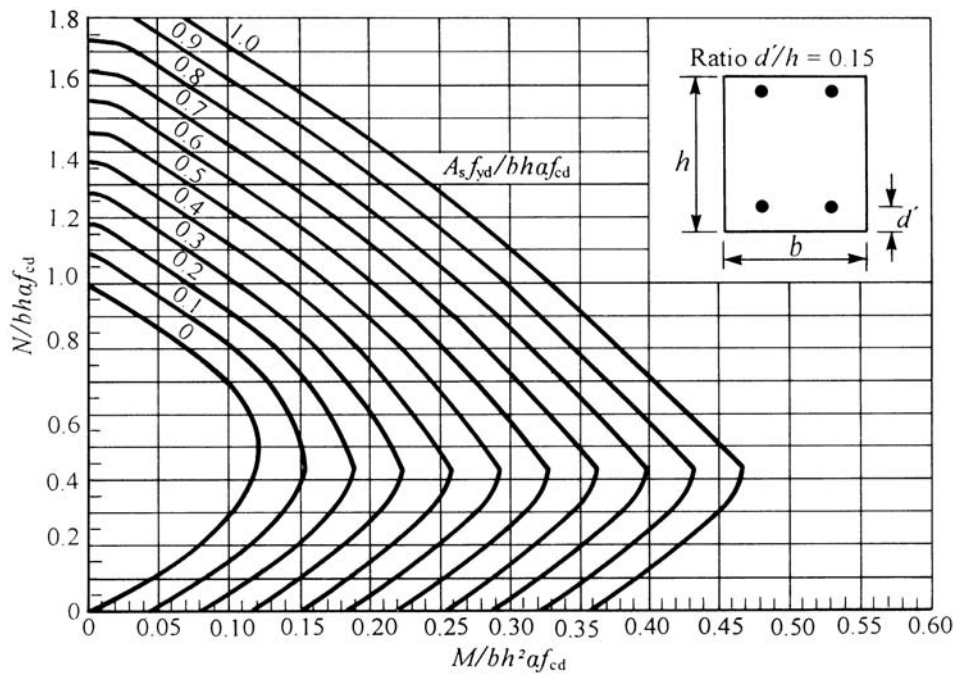


Chart 8.3

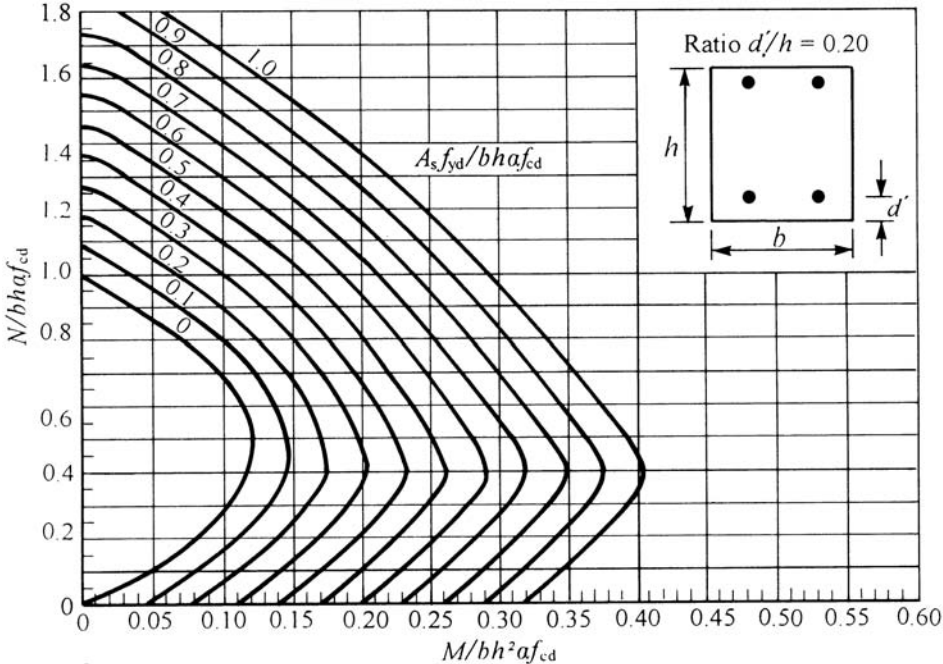


Chart 8.4

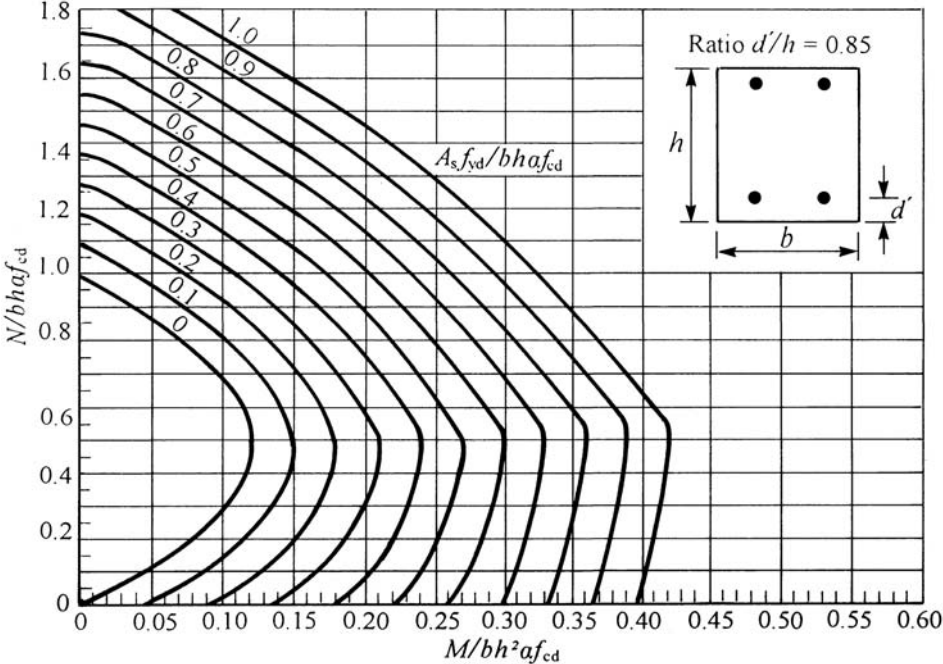


Chart 8.5

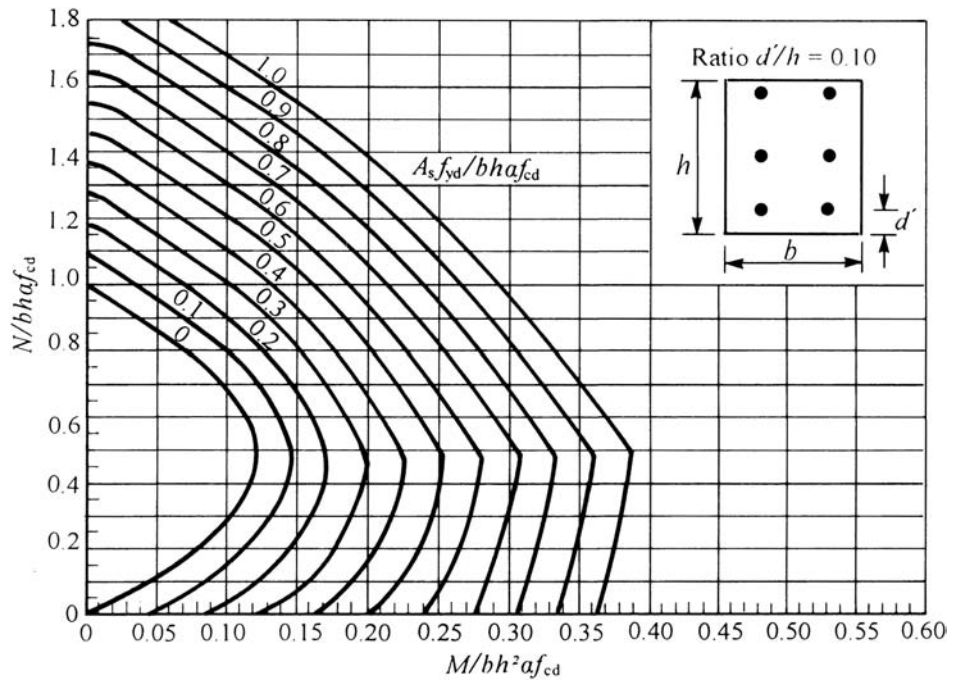


Chart 8.6

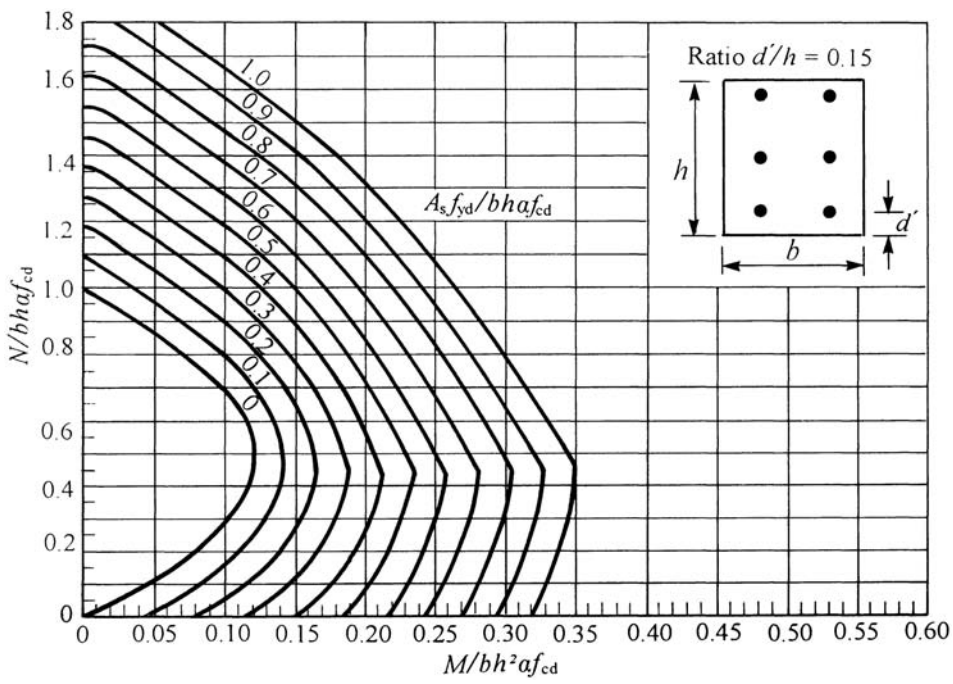


Chart 8.7

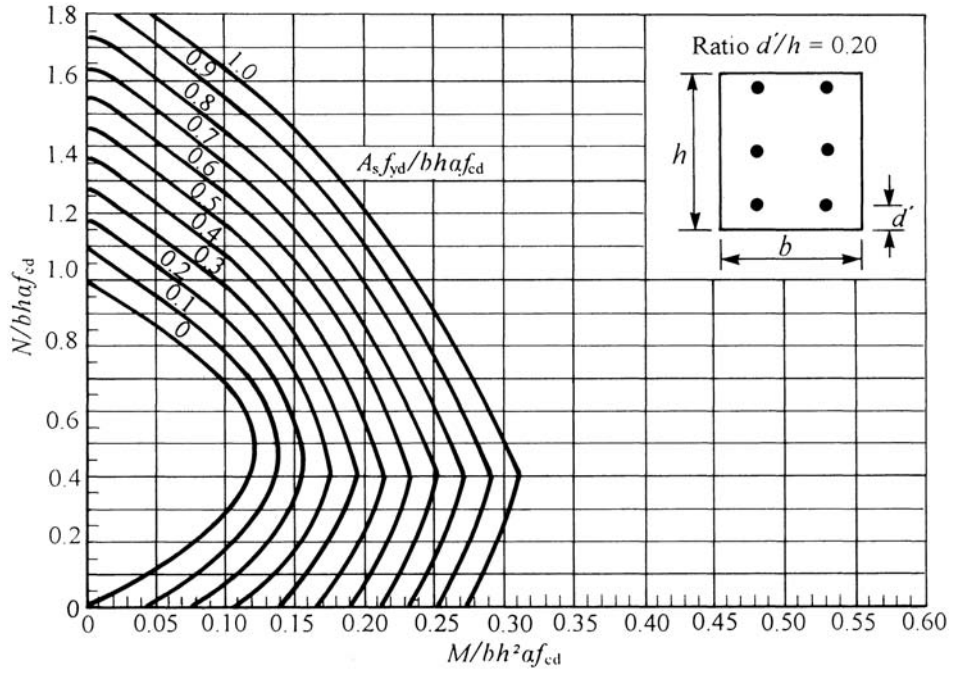


Chart 8.8

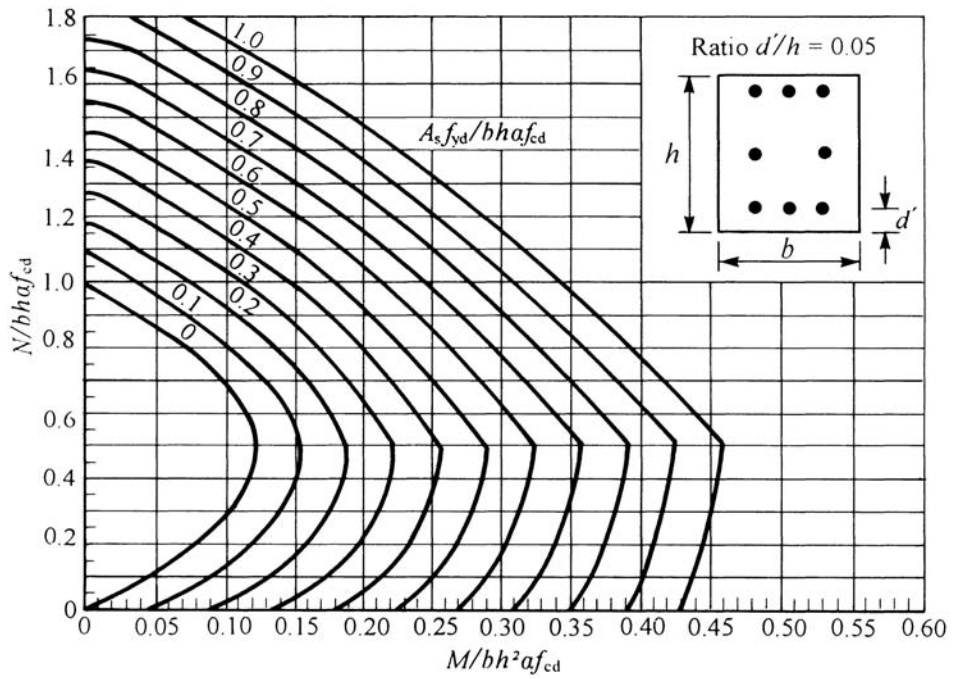


Chart 8.9

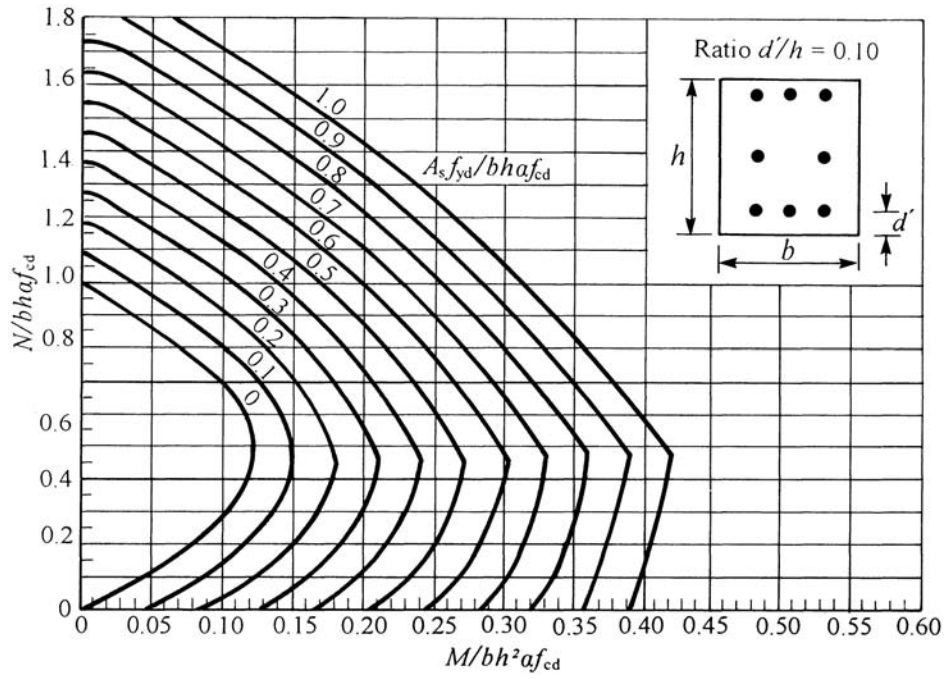


Chart 8.10

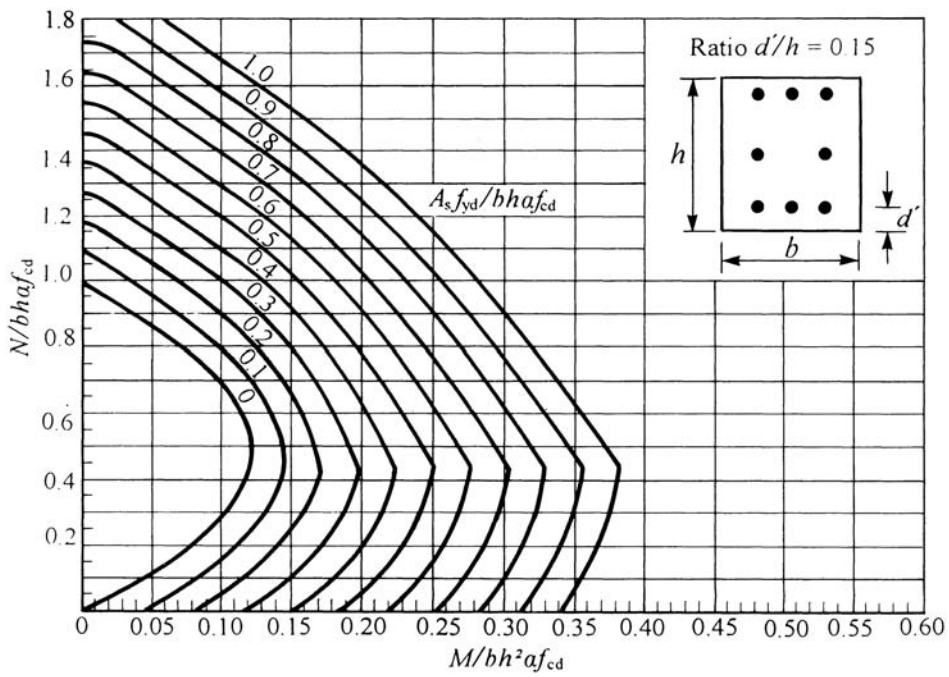


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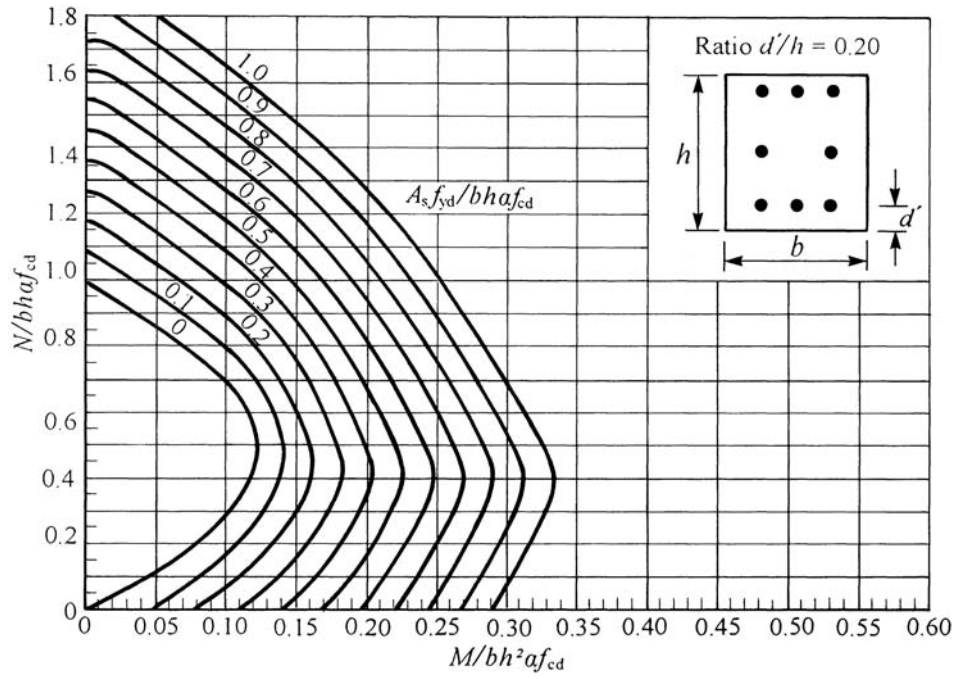


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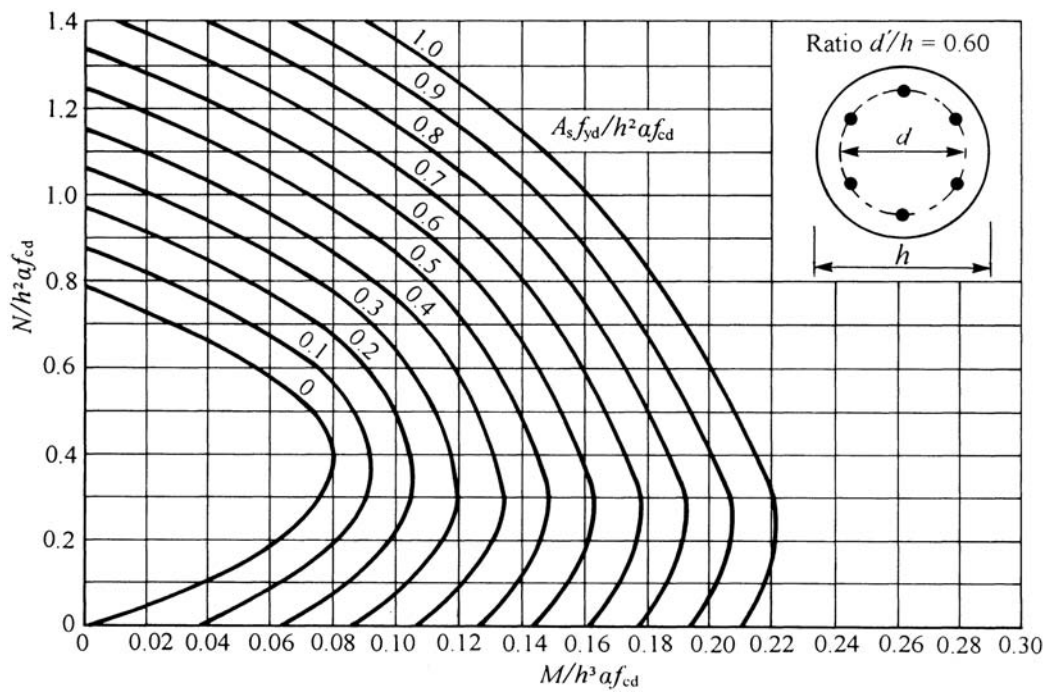


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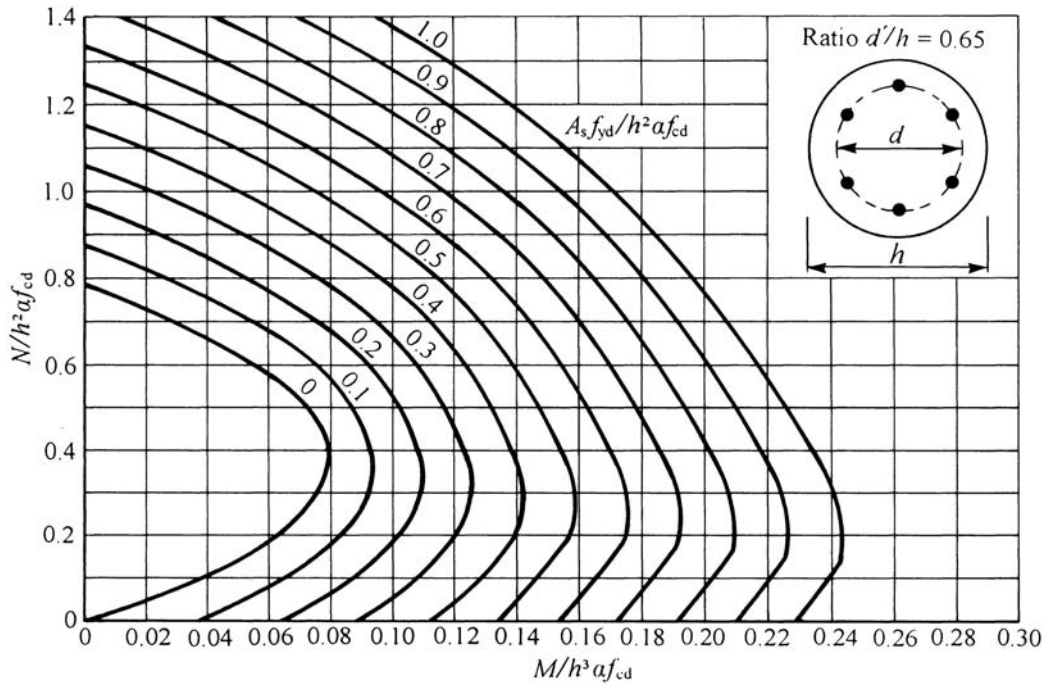


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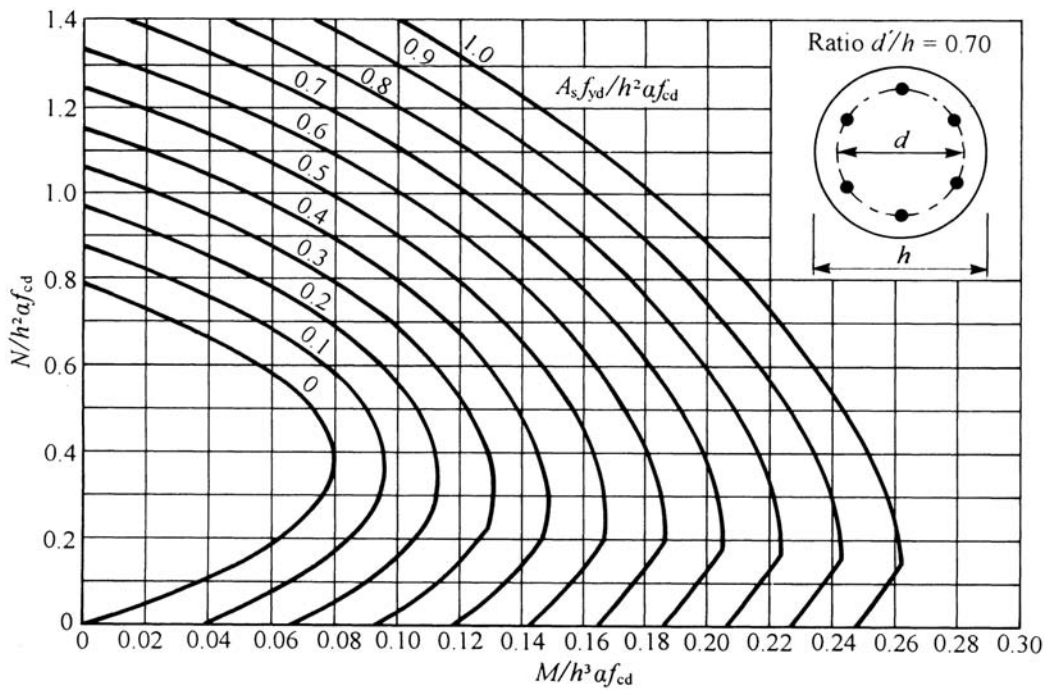


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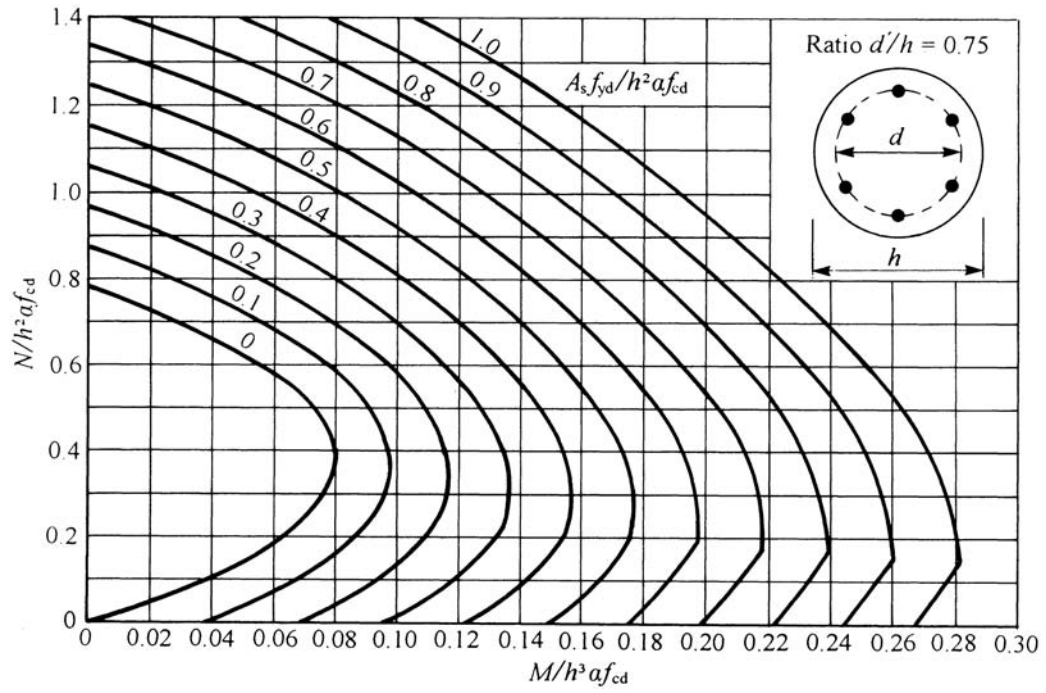


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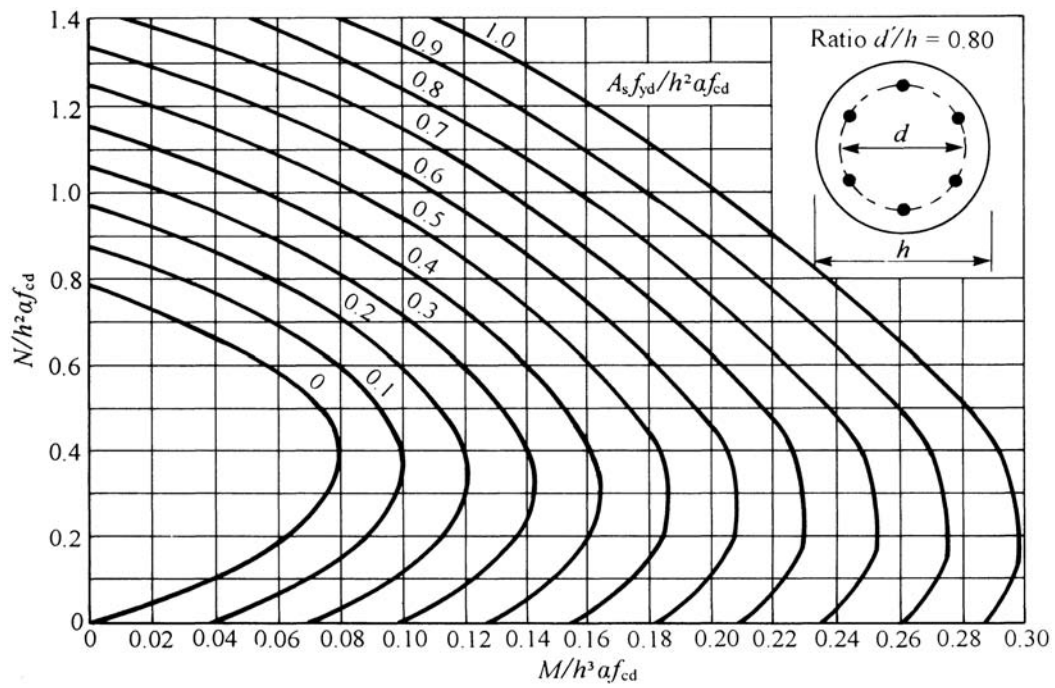


Chart 8.17

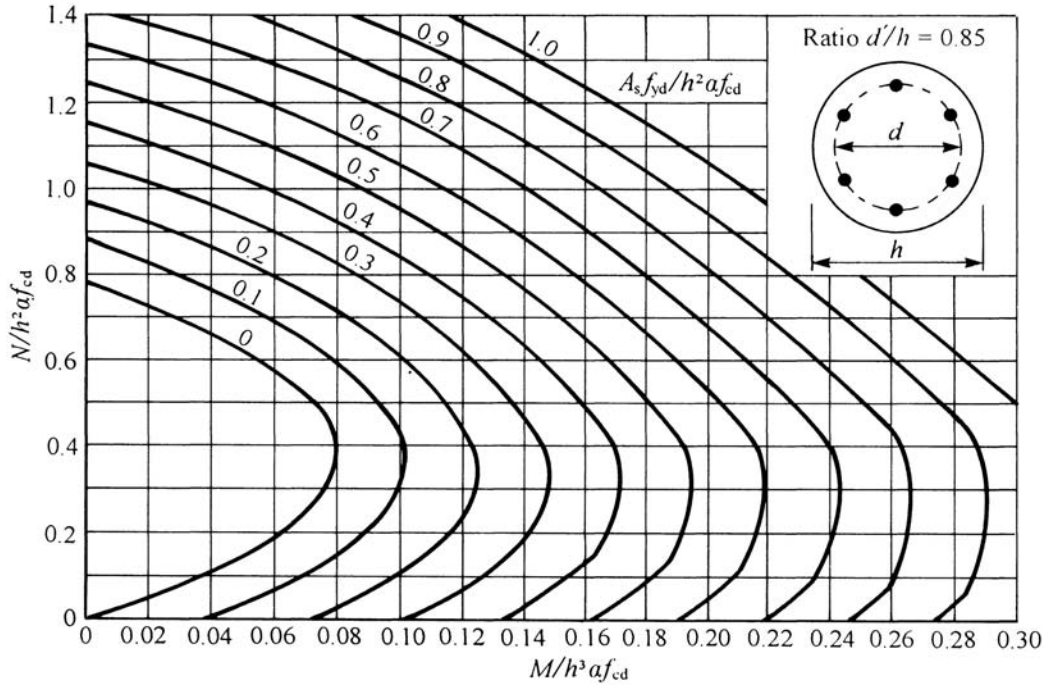


Chart 8.18

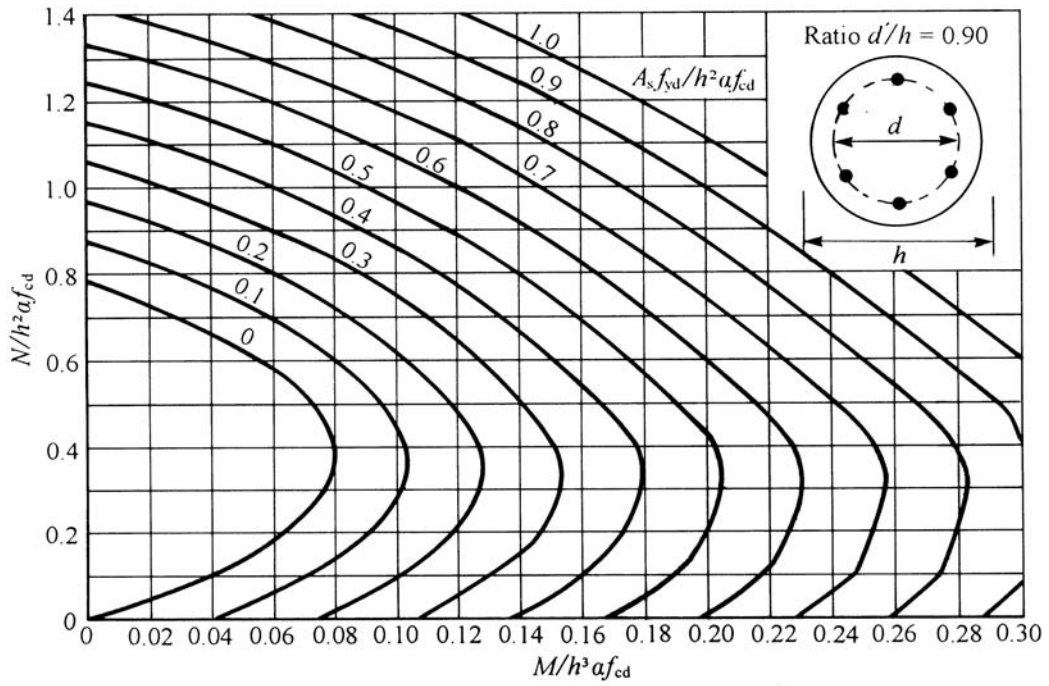


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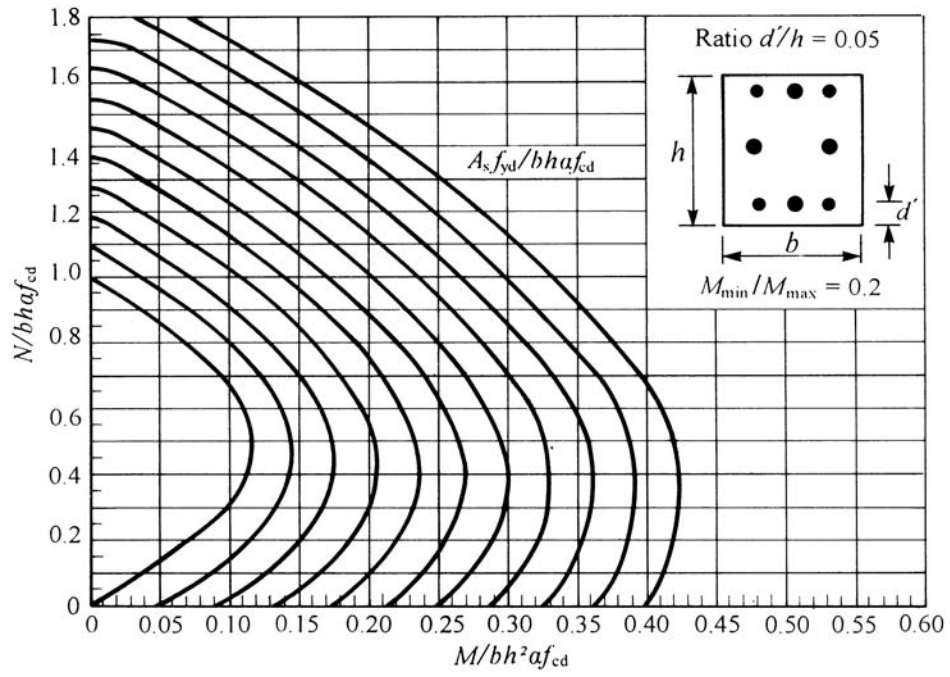


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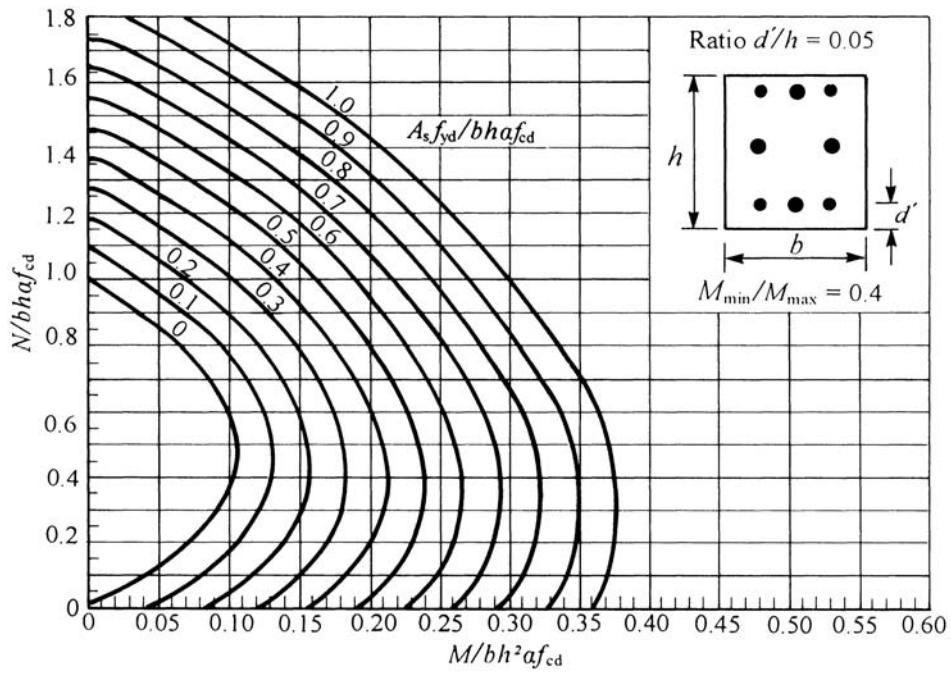


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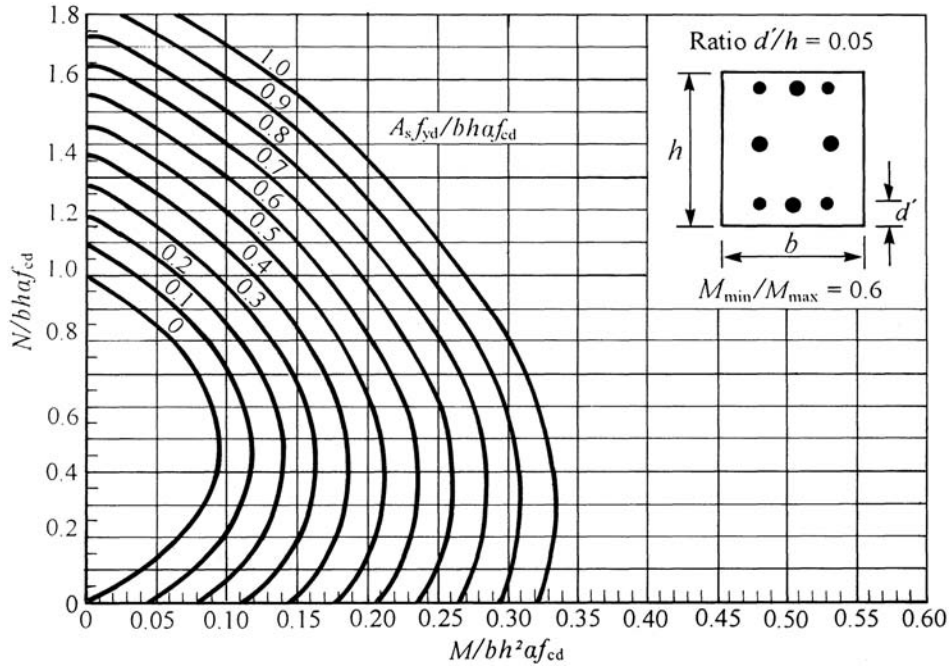


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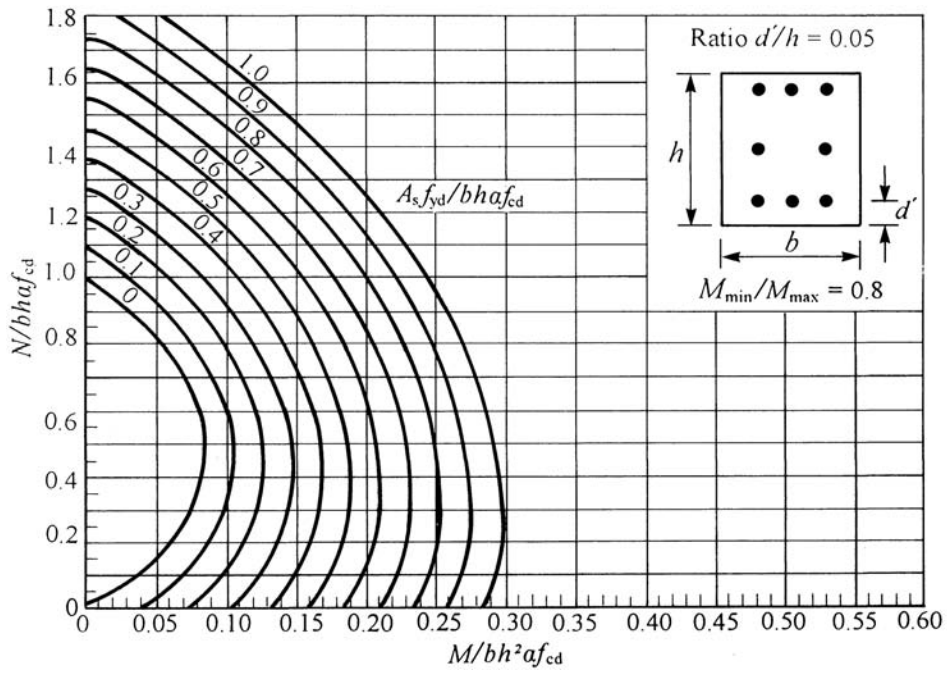


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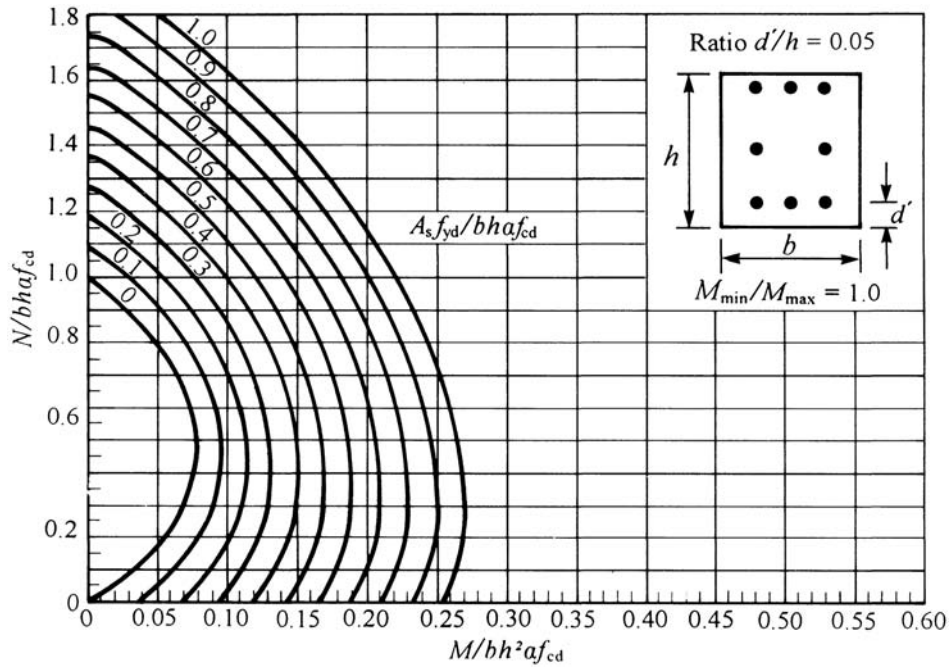


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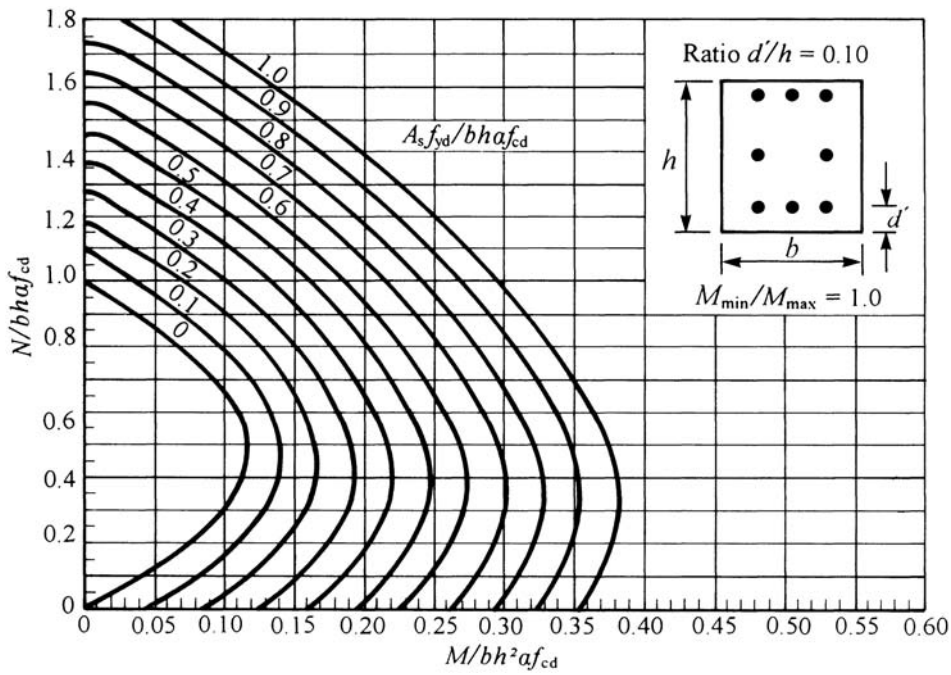


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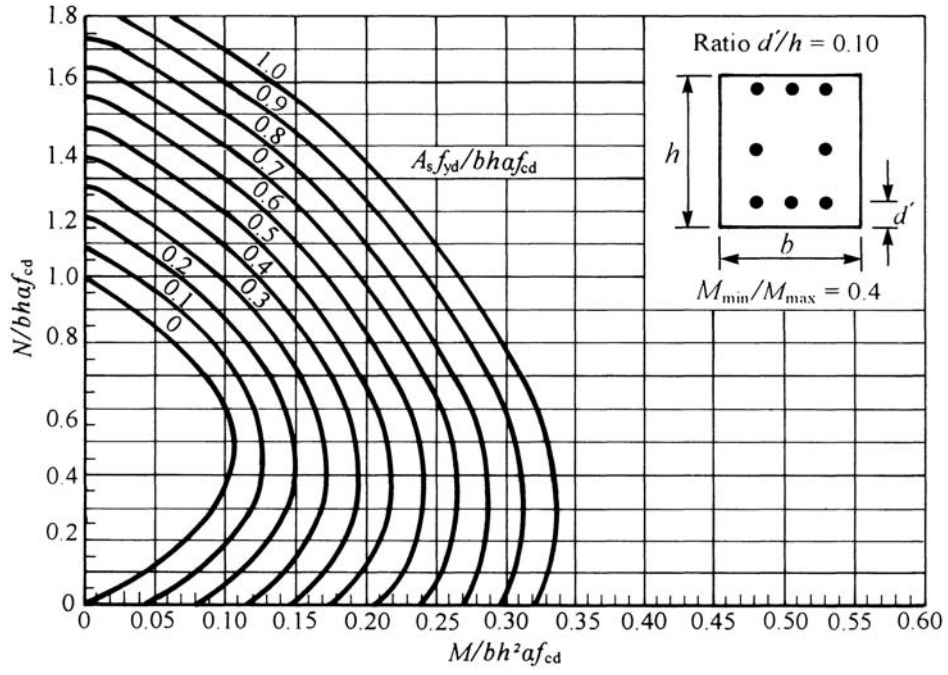


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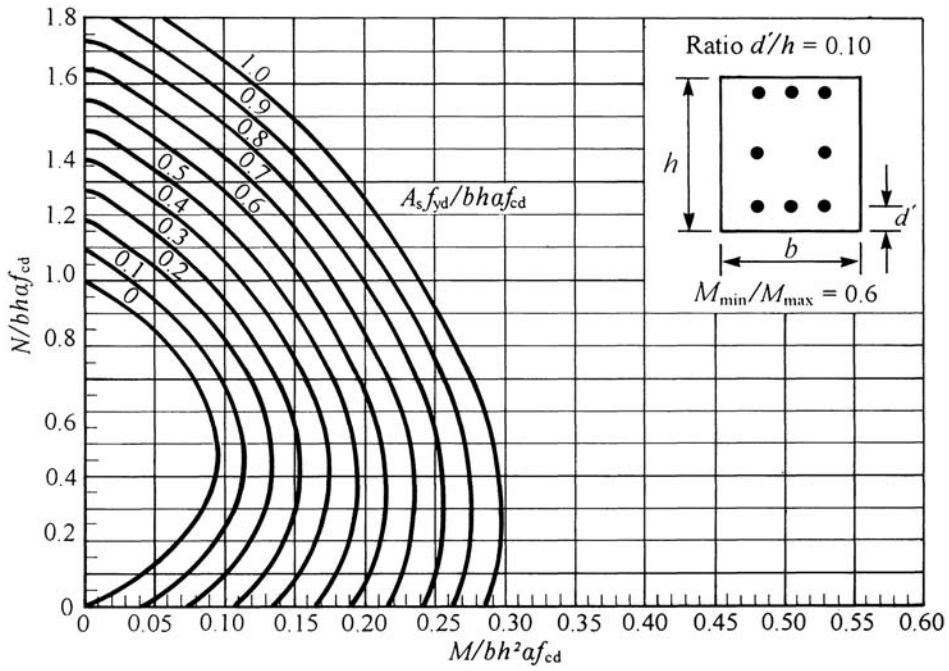


Chart 8.27

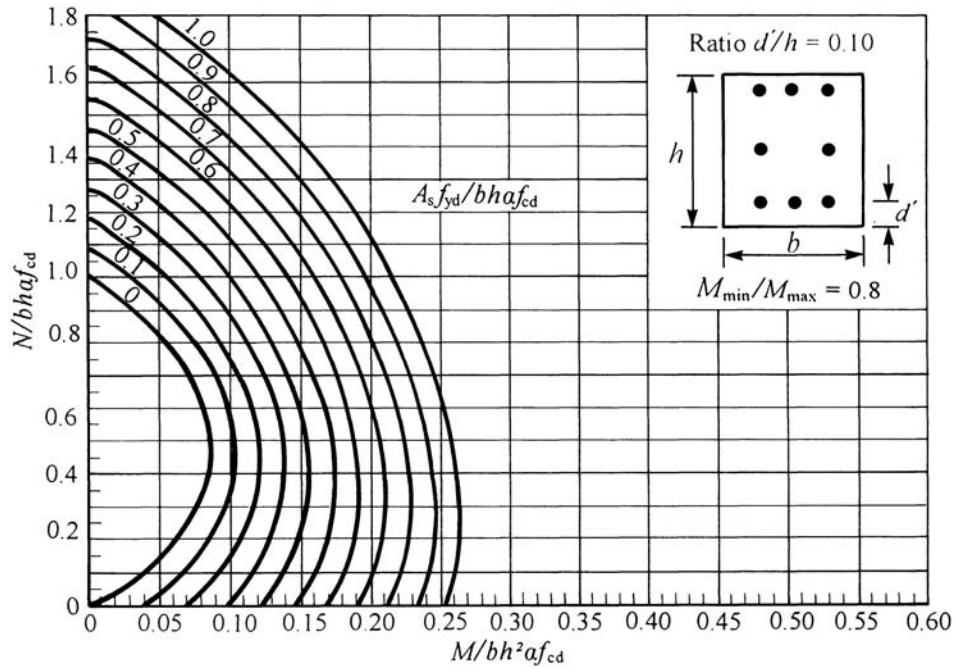


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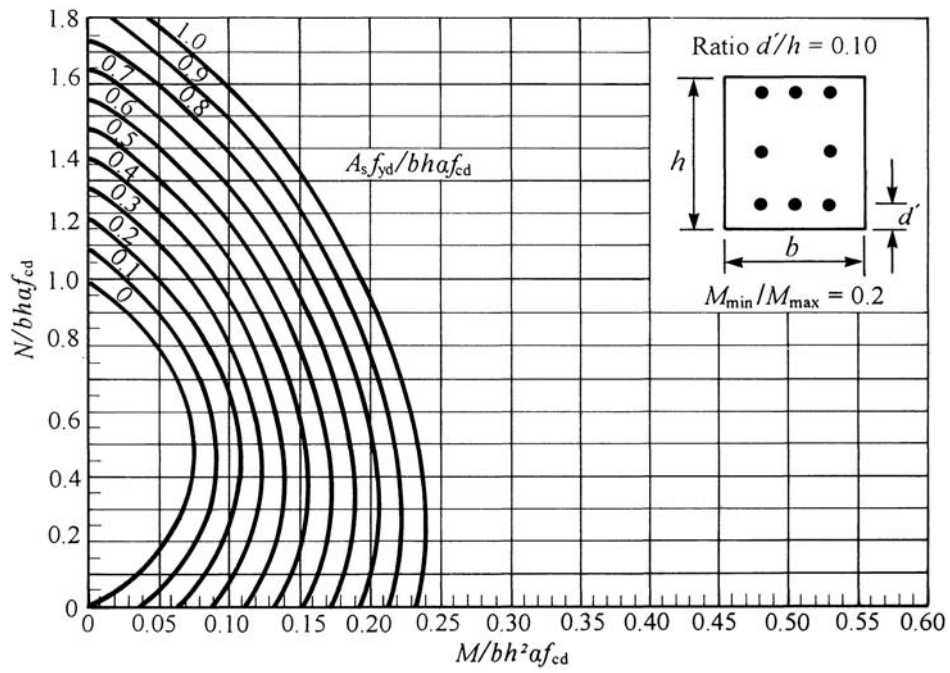


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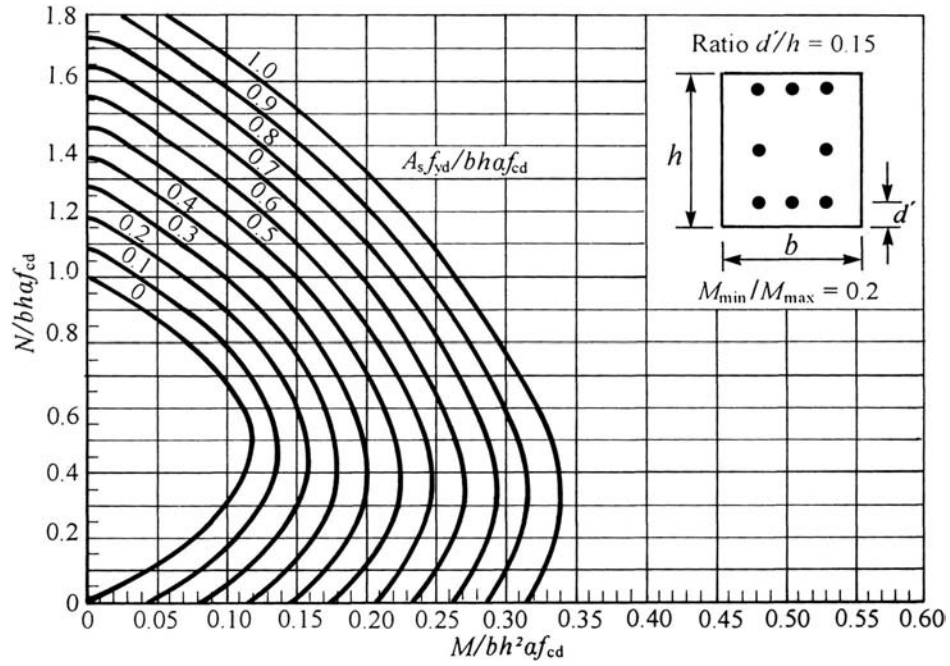


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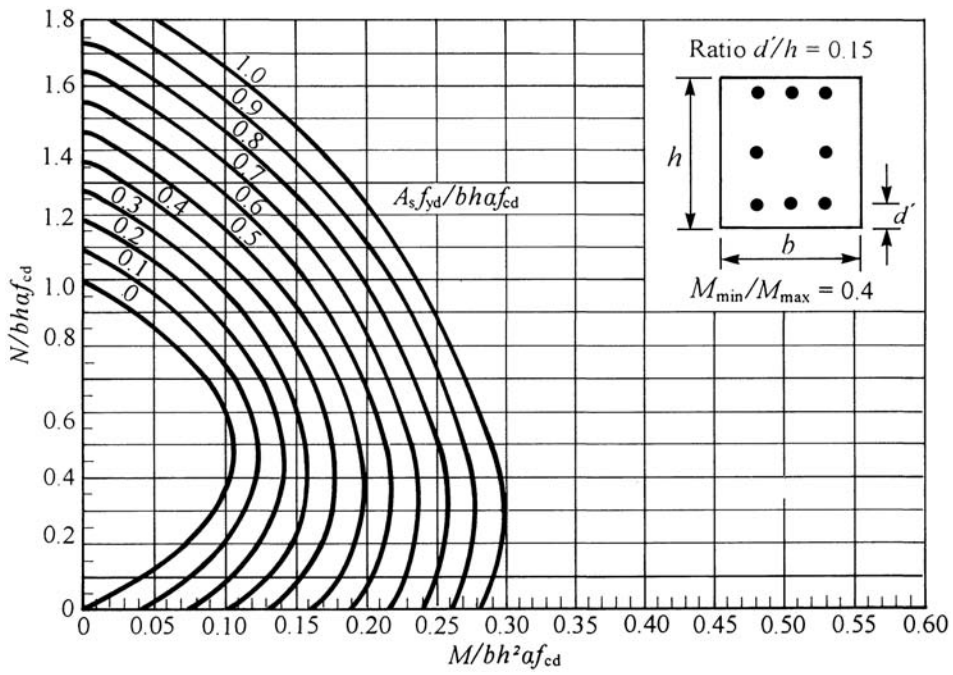


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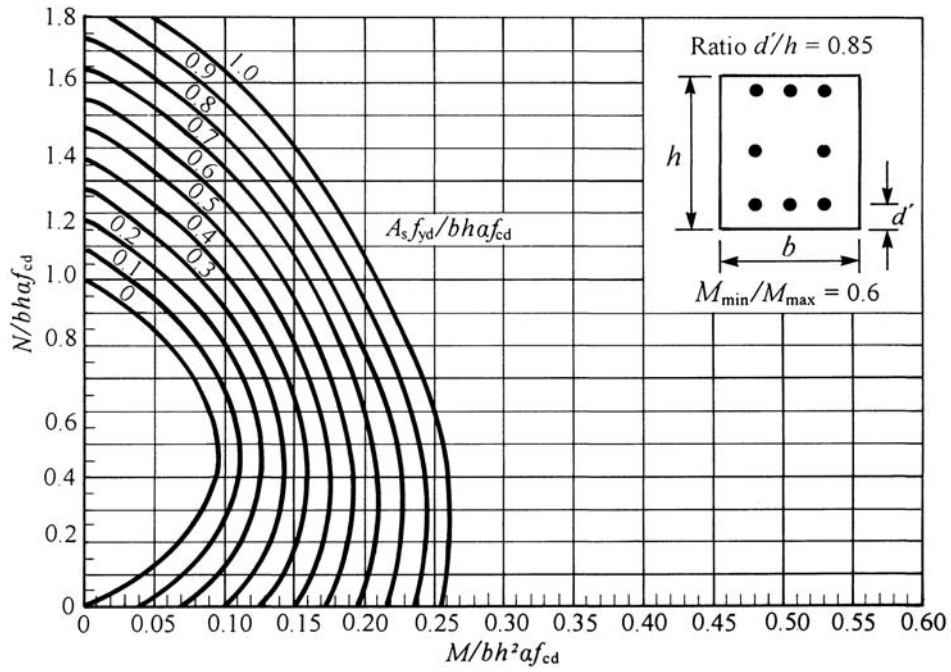


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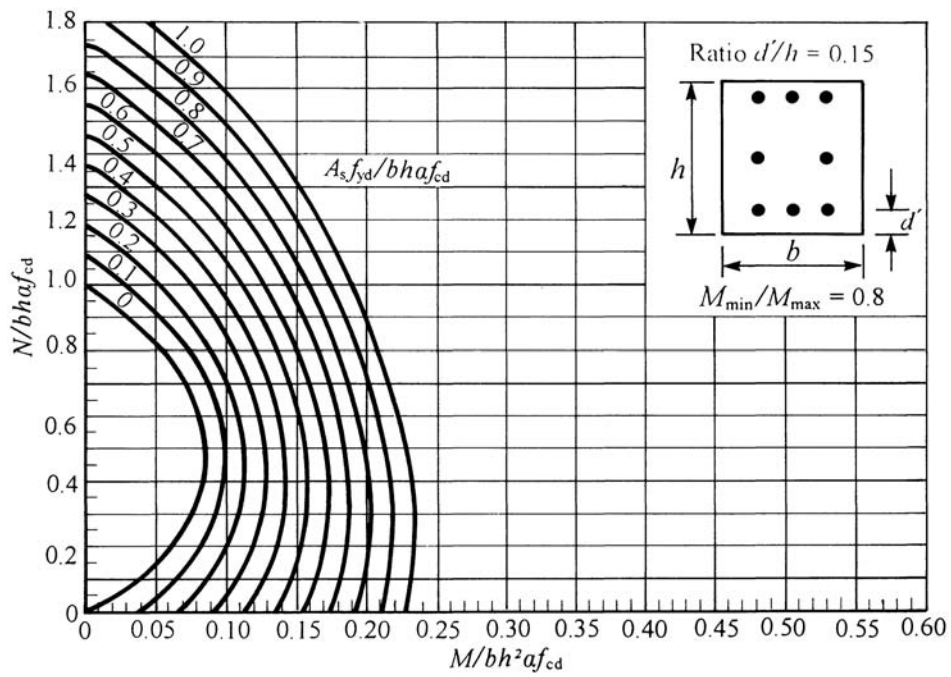


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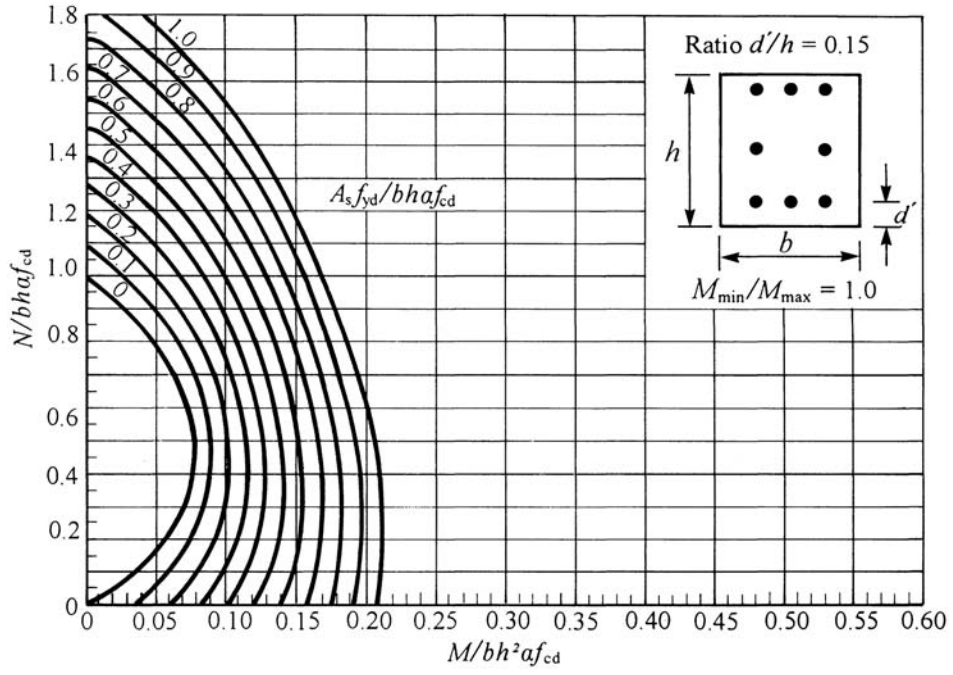


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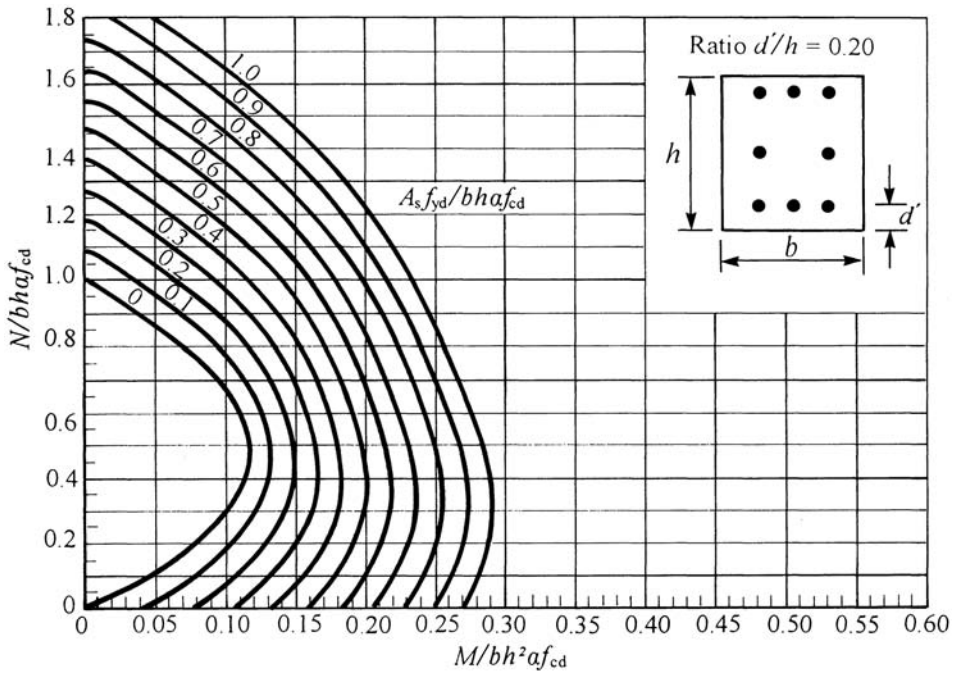


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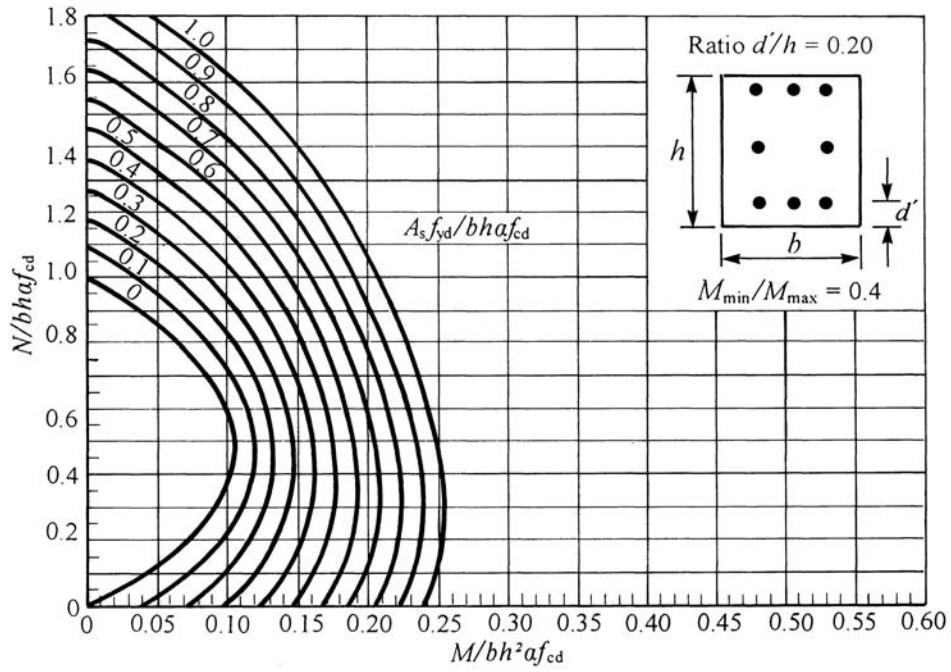


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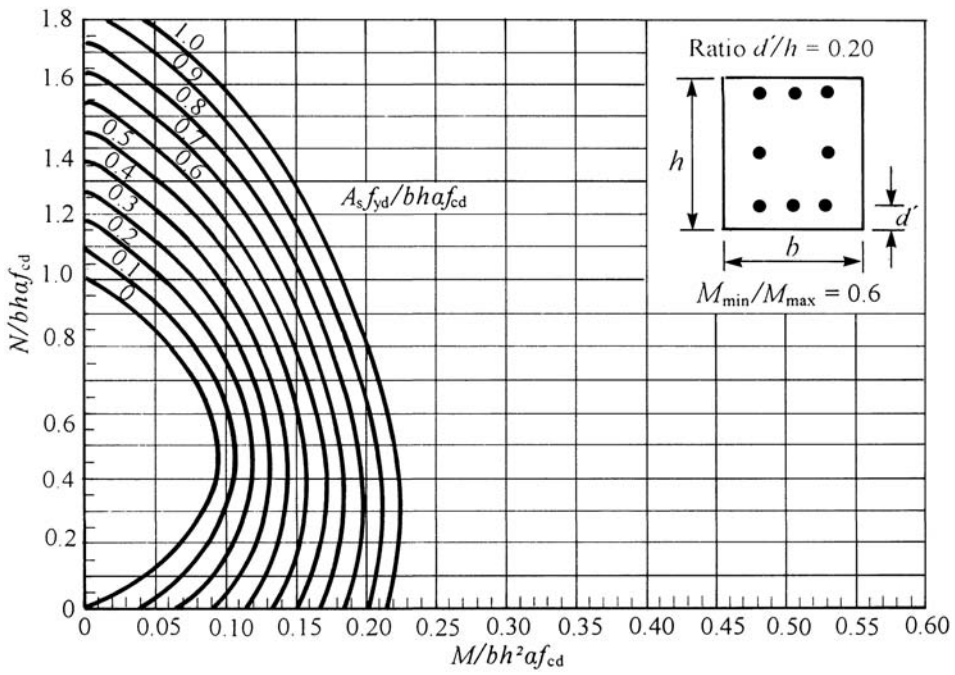


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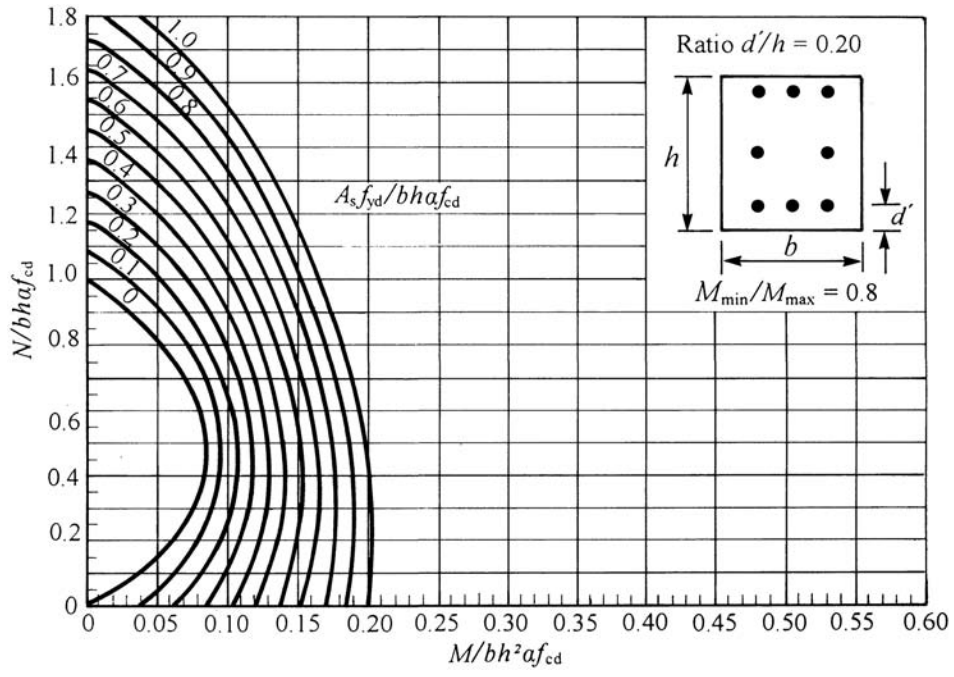


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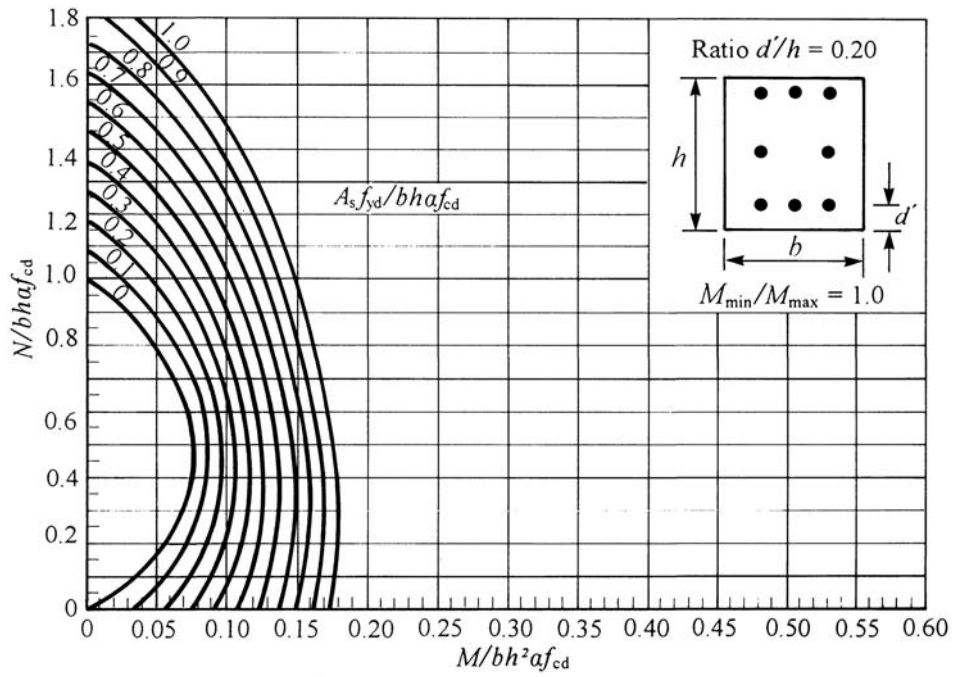


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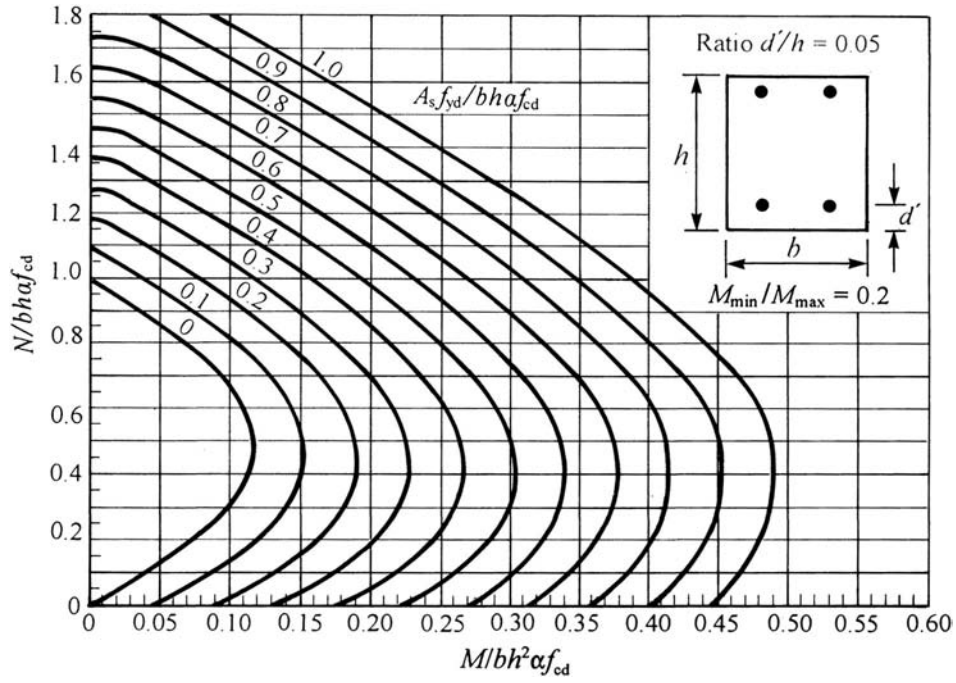


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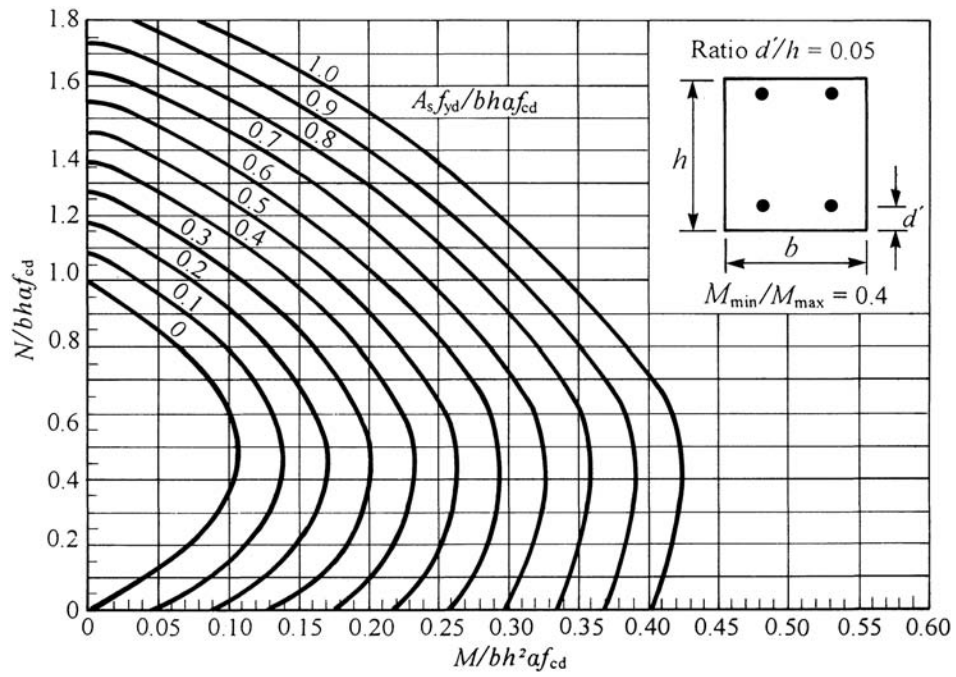


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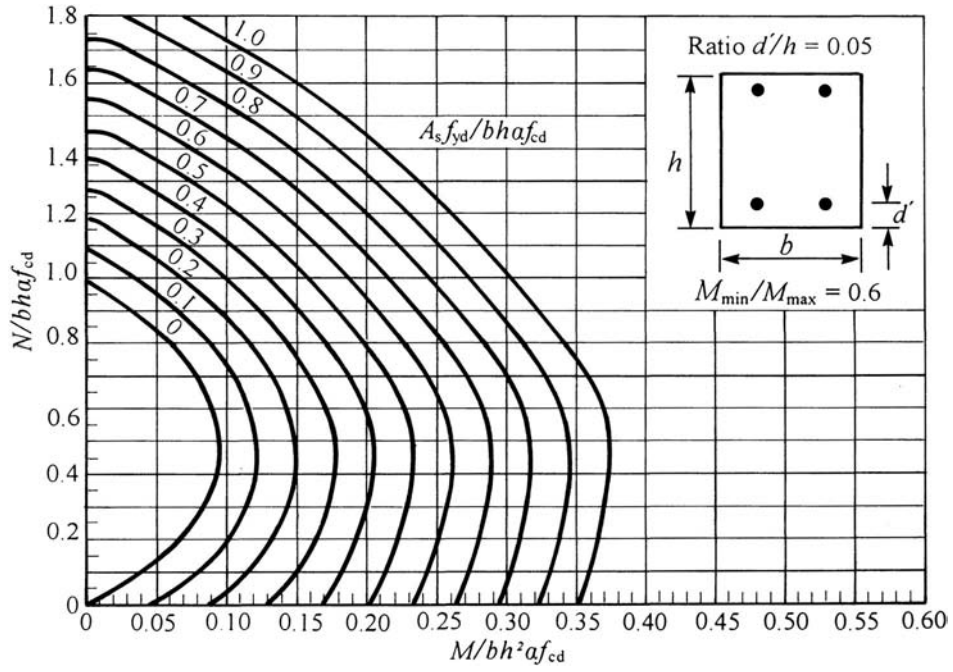


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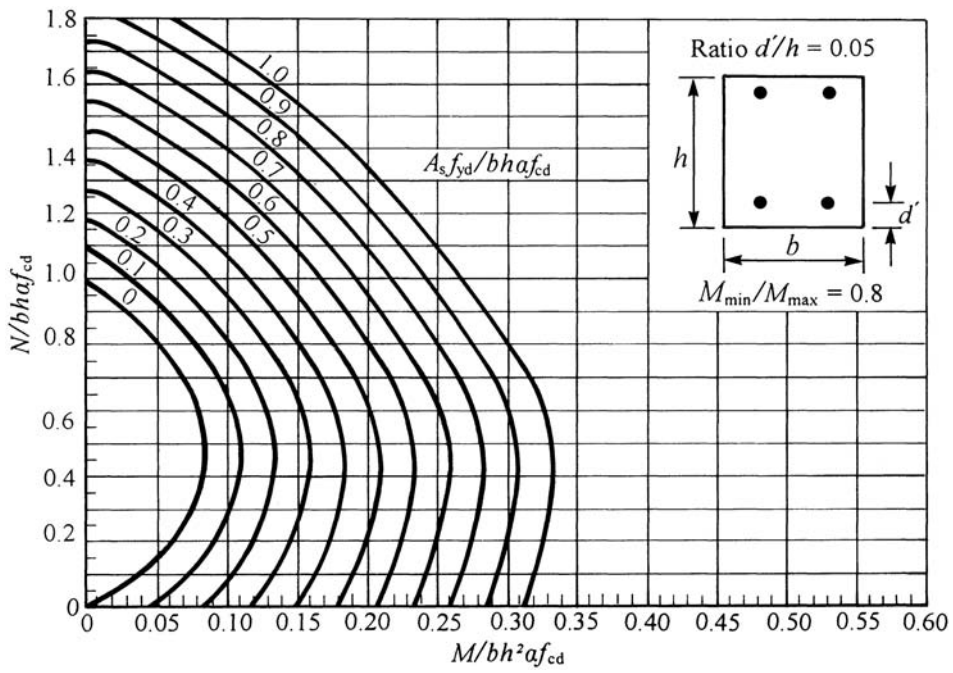


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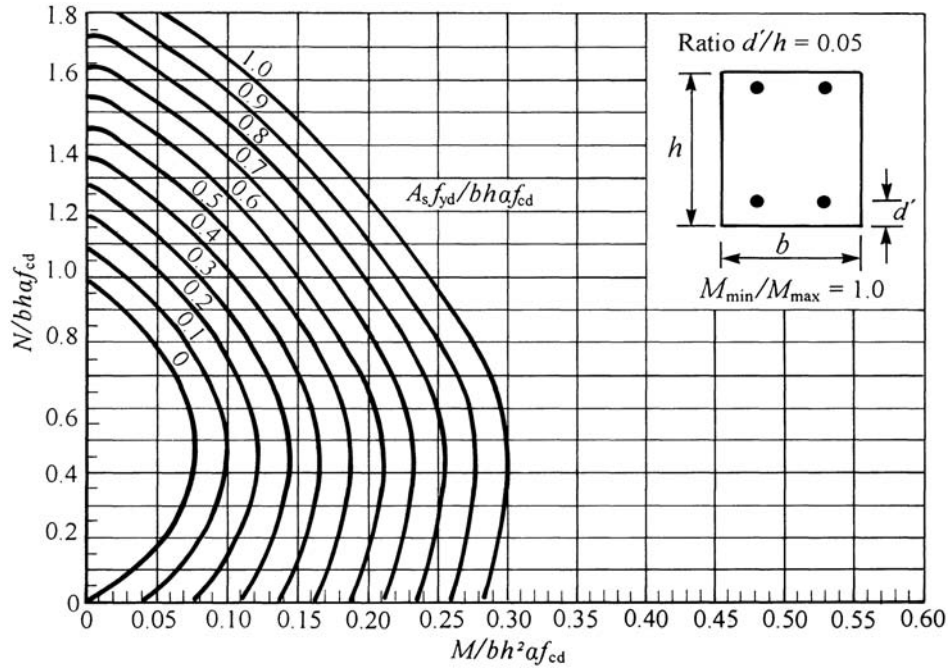


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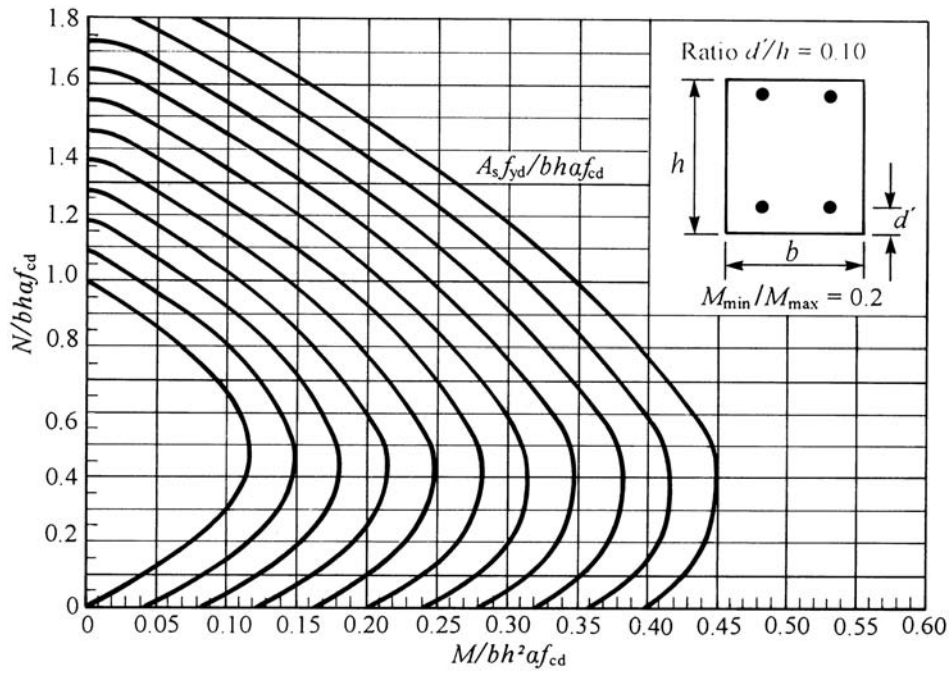


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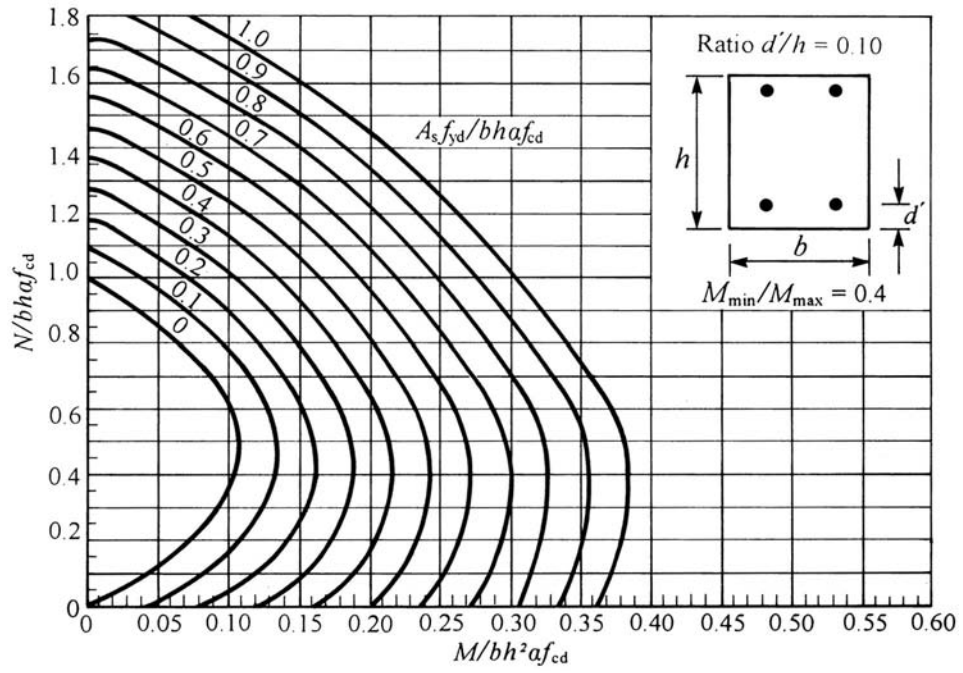


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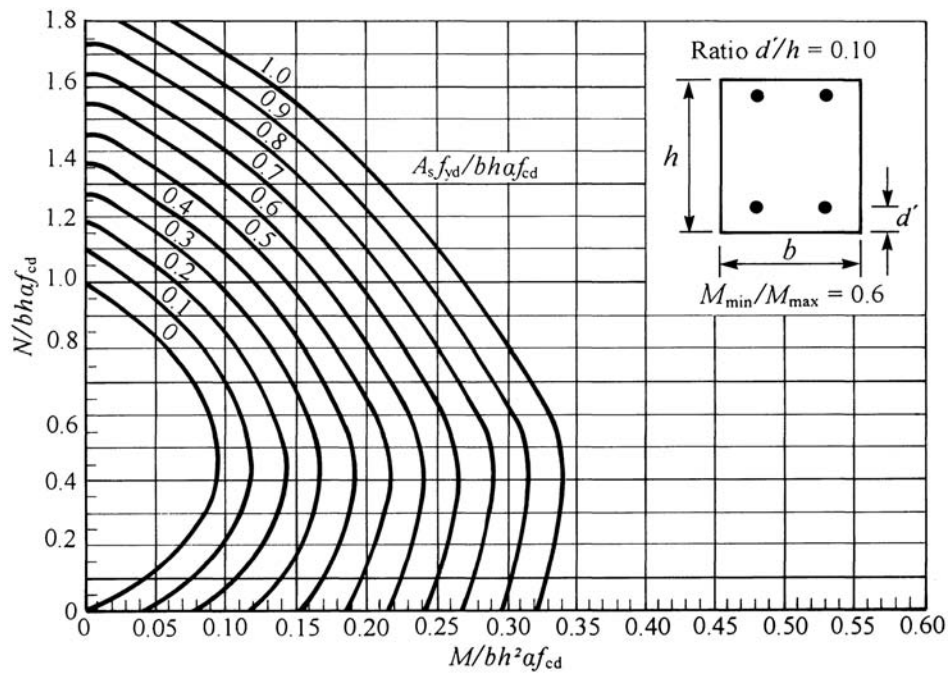


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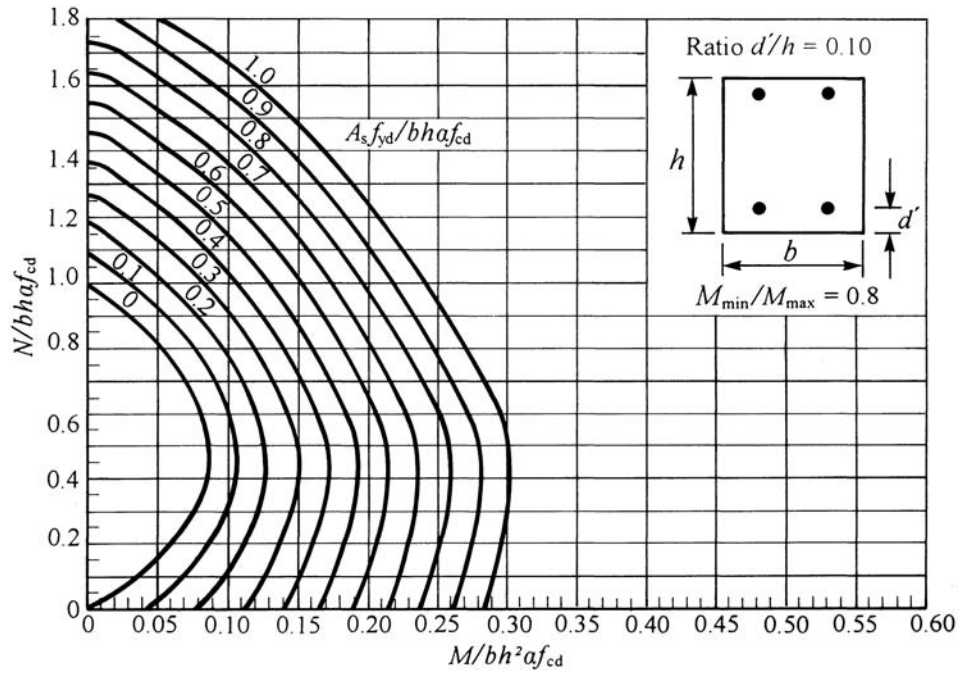


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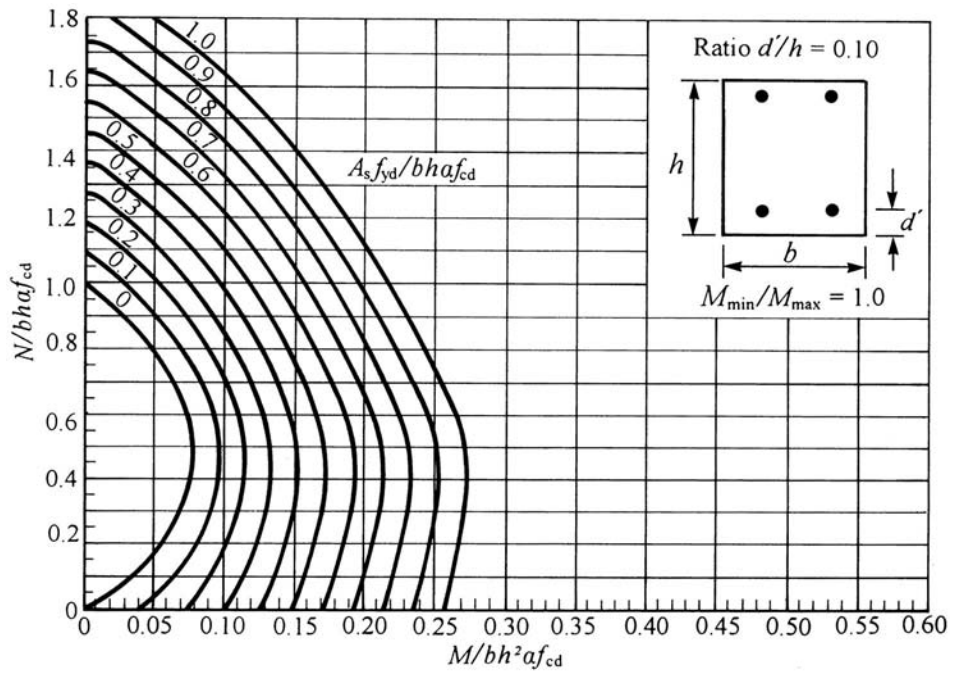


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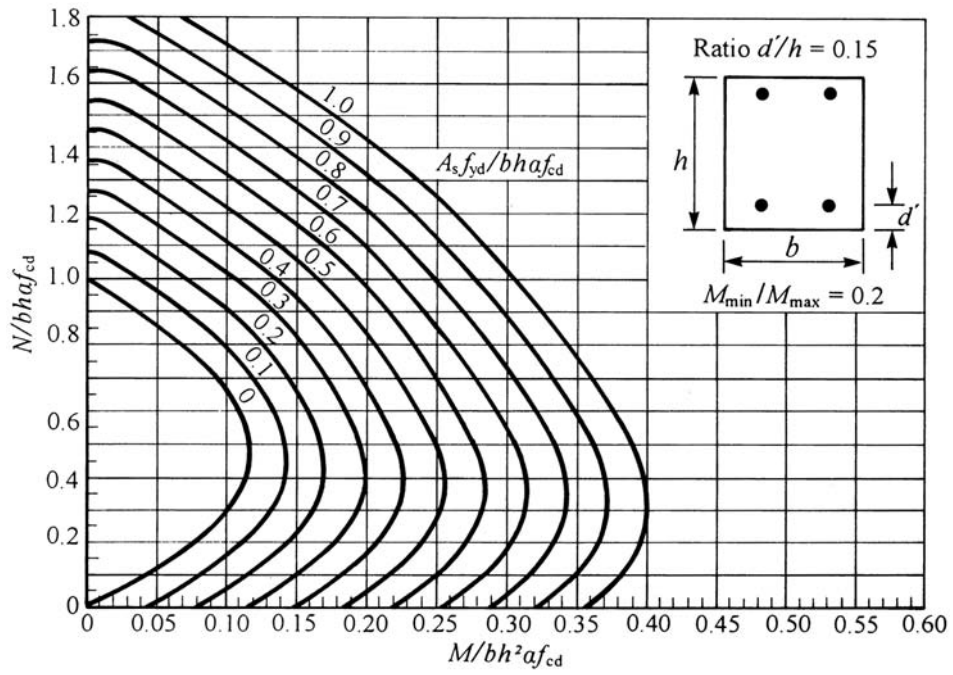


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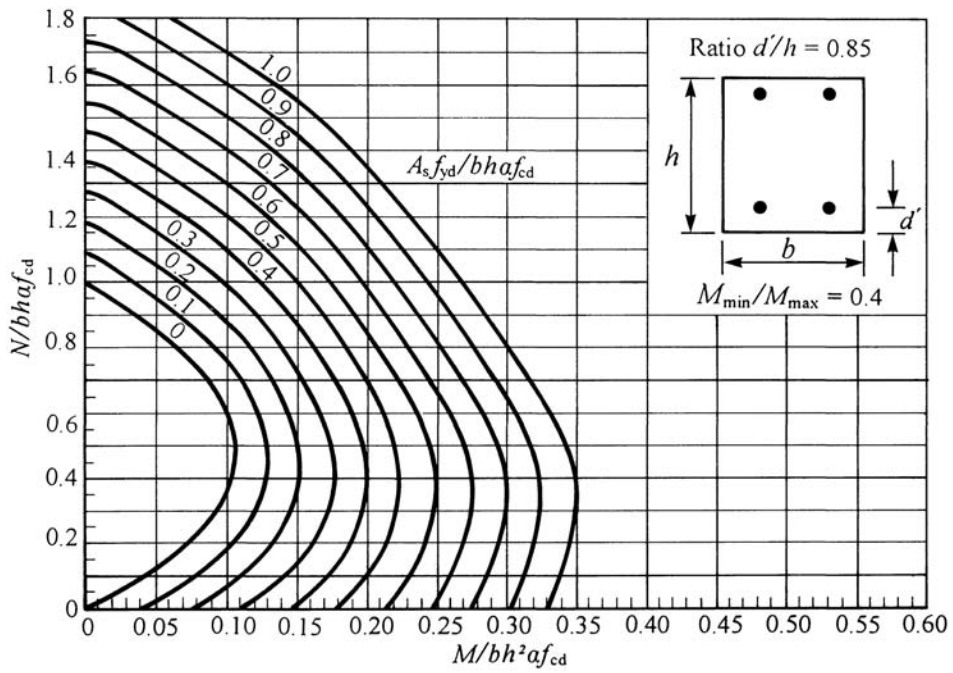


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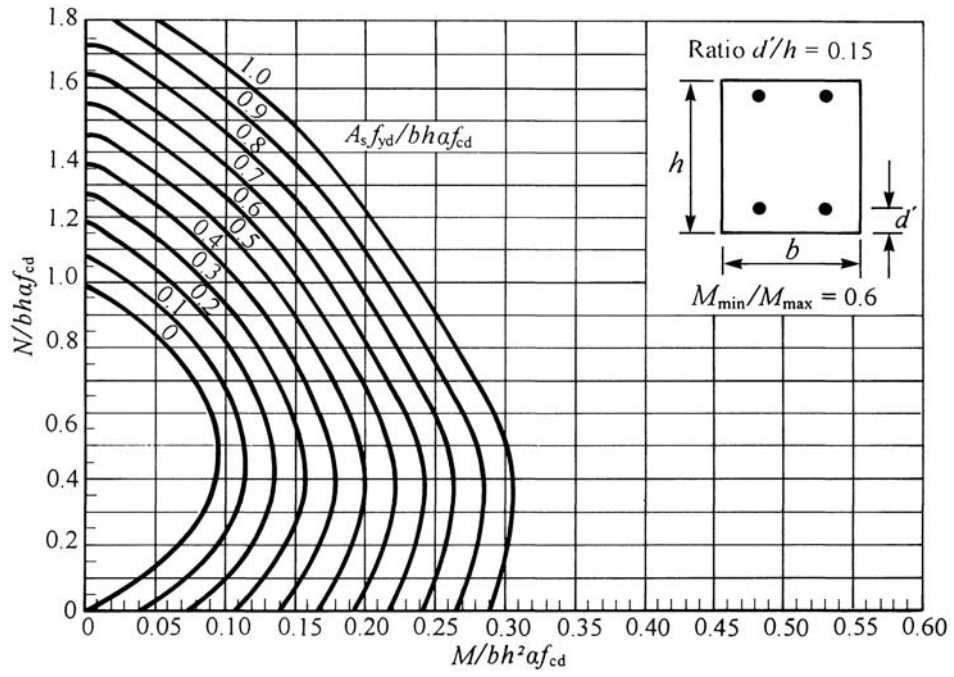


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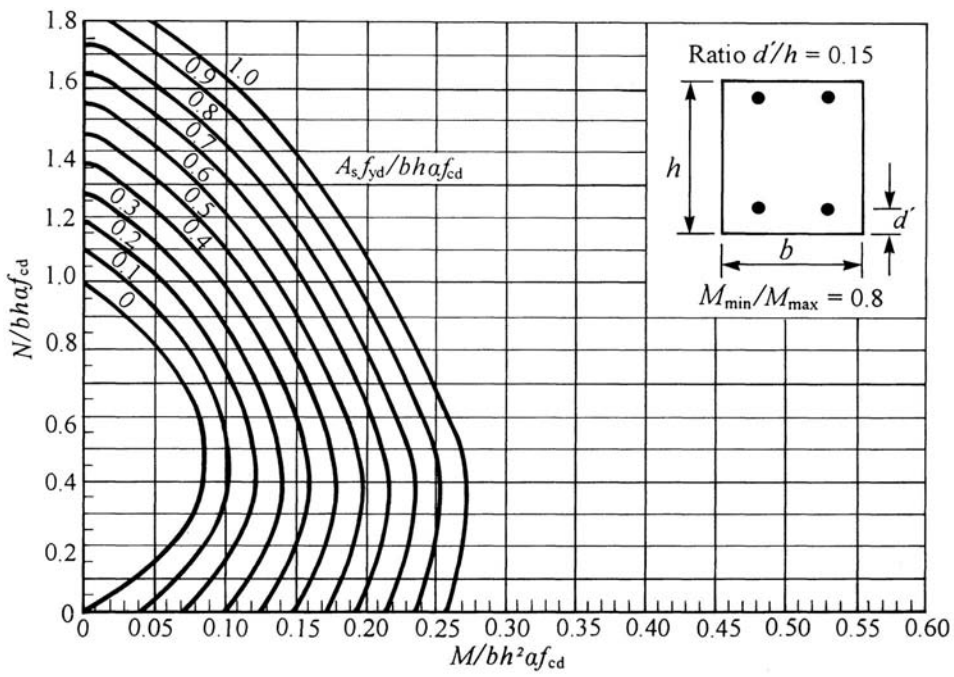


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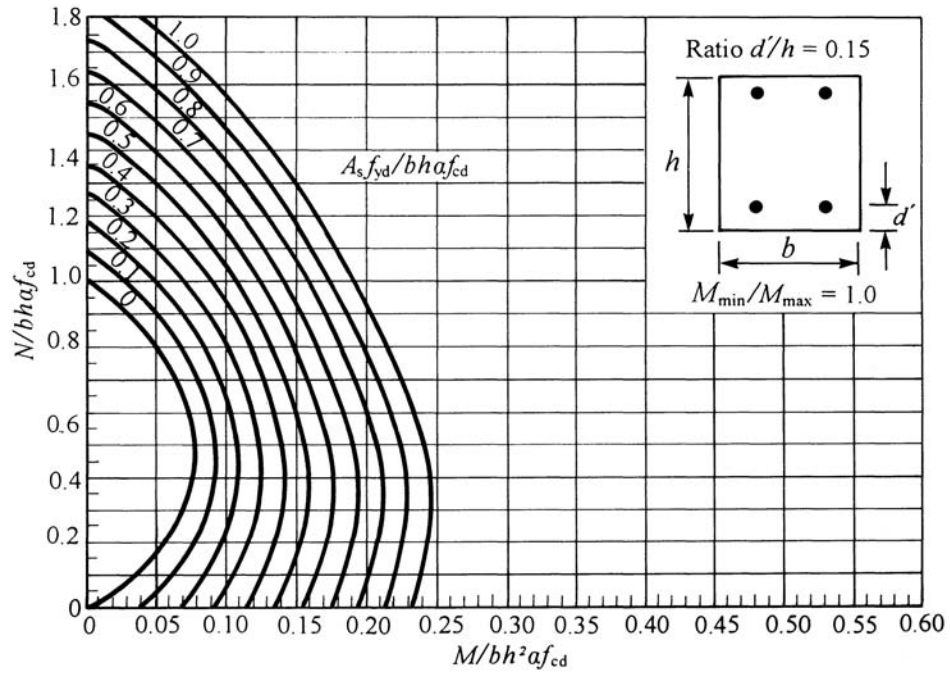


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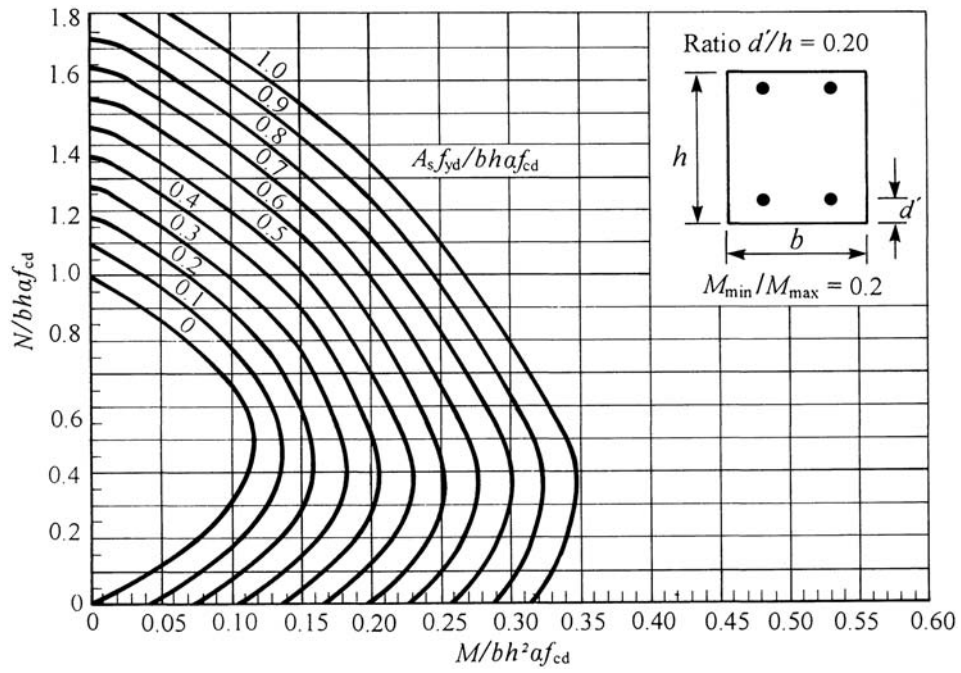


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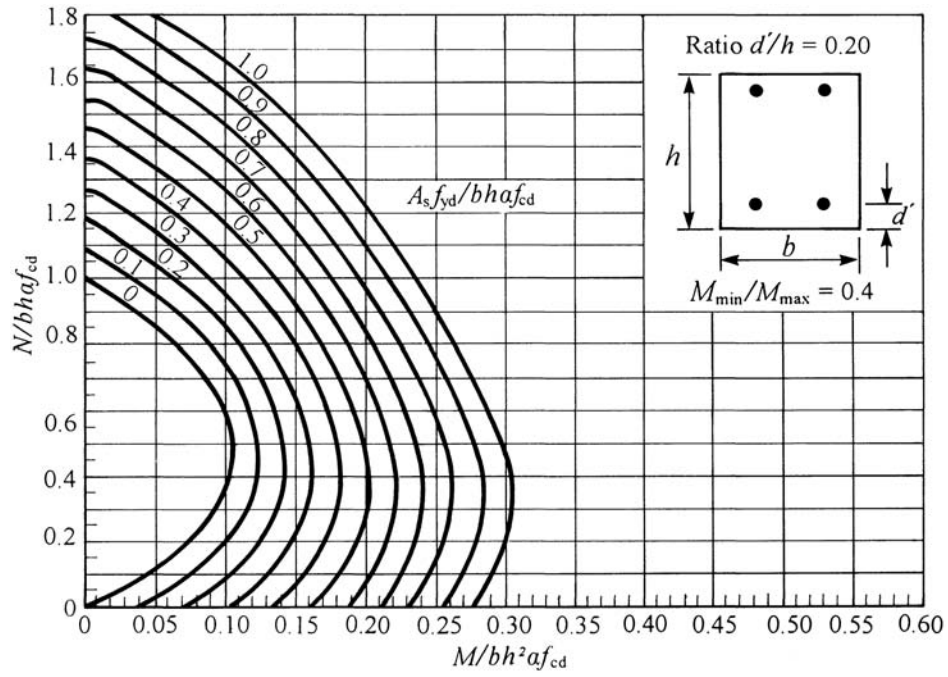


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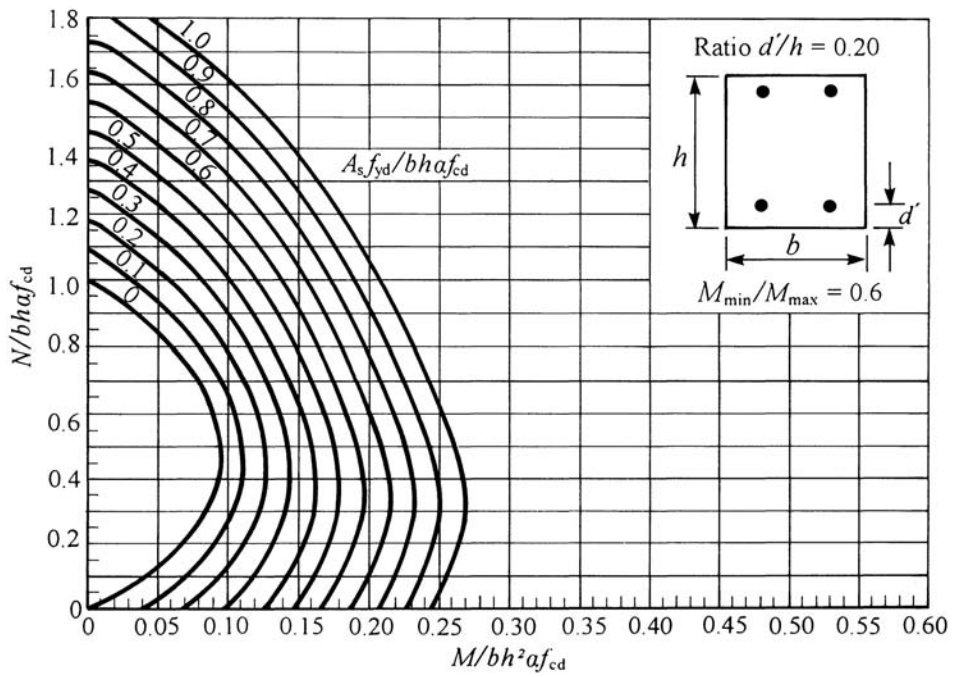


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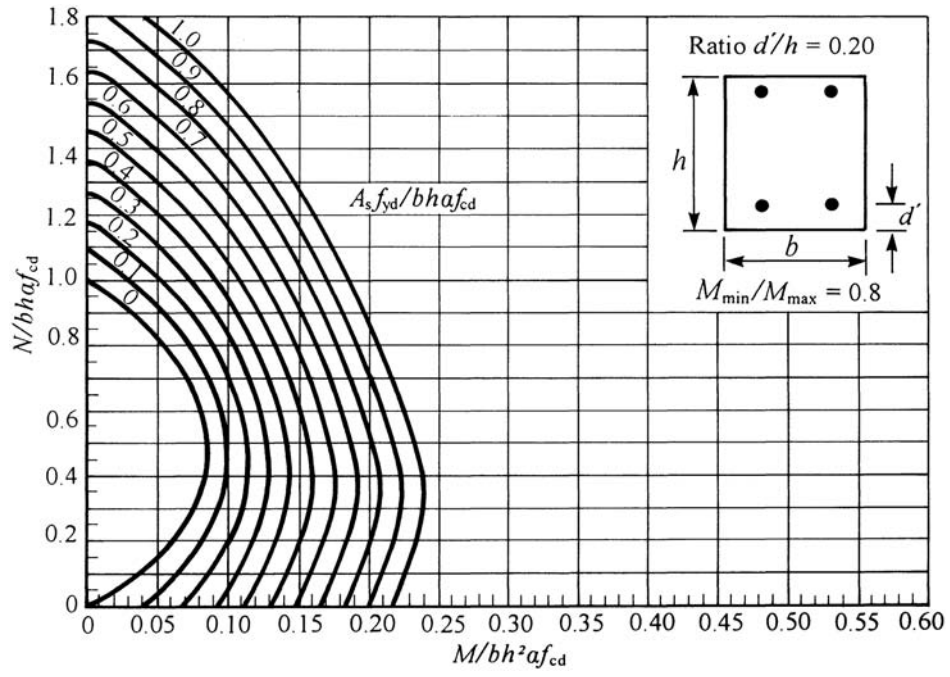


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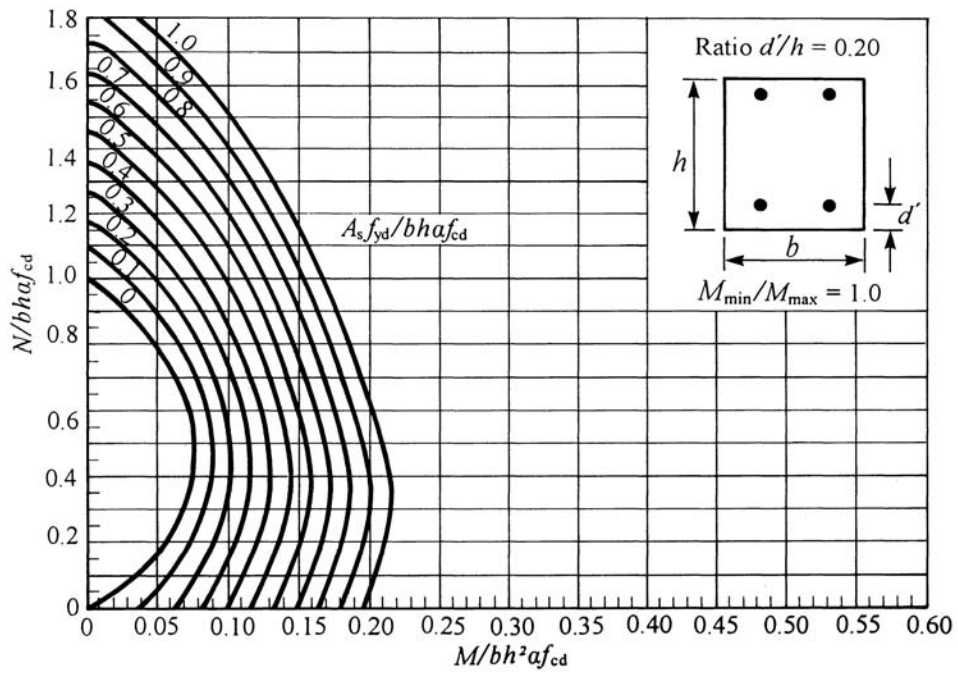


Chart 8.59

9 Shear and torsion

9.1 Shear

9.1.1 General

Elements without shear reinforcement 4.3.2.3

Requirement:

$$V_{Sd} \leq V_{Rd1} \quad \rightarrow \quad \frac{V_{Sd}}{b_w d} \leq \frac{V_{Rd1}}{b_w d} \quad 4.3.2.2(2)$$

| | |
|-----------|---|
| V_{Sd} | design shear force |
| V_{Rd1} | design shear resistance of the member without shear reinforcement |
| b_w | minimum width of the web |
| d | effective depth |

$$\frac{V_{Rd1}}{b_w d} = \beta (\tau_{Rd} k (1.2 + 40\rho_l) + 0.15\sigma_{cp}) \quad \text{according to Table 9.1.2 below}$$

4.3.2.4

Elements with shear reinforcement

Standard method

4.3.2.4.3

Requirements:

$$V_{Sd} \leq V_{Rd3} \quad 4.3.2.2(3)$$

with

$$V_{Rd3} = V_{Rd1} + V_{wd} \quad \rightarrow \quad \frac{V_{Sd}}{d} \leq \frac{V_{Rd1}}{d} + \frac{V_{wd}}{d} \quad [4.22]$$

and

$$V_{Sd} \leq V_{Rd2} \quad \rightarrow \quad \frac{V_{Sd}}{b_w d} \leq \frac{V_{Rd2}}{b_w d} \quad 4.3.2.2(4)$$

| | |
|-----------|---|
| V_{Rd3} | design shear resistance of the member with shear reinforcement |
| V_{wd} | contribution of the shear reinforcement |
| V_{Rd2} | maximum design shear force that can be carried without crushing of the notional concrete compressive struts |

$$\frac{V_{wd}}{d} = \frac{A_{sw}}{s} 0.9 \frac{f_{yk}}{\gamma_s} (1 + \cot \alpha) \sin \alpha$$

according to Table 9.1.5 below

$$\frac{V_{Rd2}}{b_w d} = \frac{1}{2} v \frac{f_{ck}}{\gamma_c} 0.9 (1 + \cot \alpha)$$

according to Table 9.1.3a below

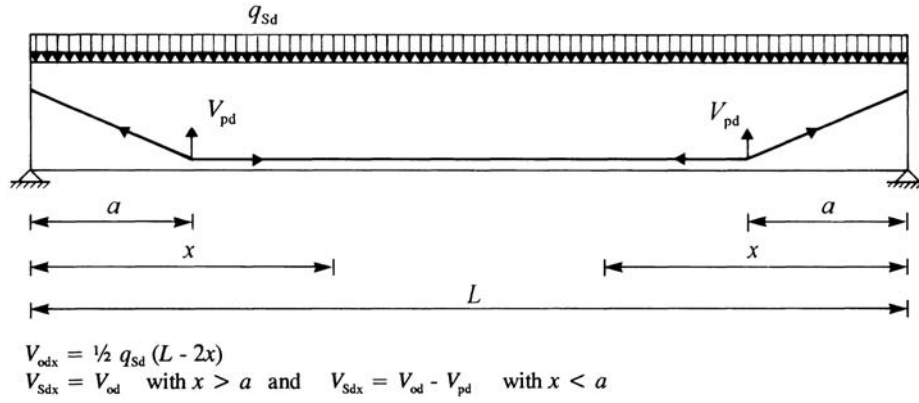


Figure 9.1 Force component (V_{pd}) of the inclined prestressed tendons, parallel to the design shear force in the section (V_{od}).

If the effective average stress in the concrete ($\sigma_{cp,eff}$) is more than 40% of the design value of the compressive cylinder strength of concrete (f_{cd}), V_{Rd2} should be reduced in accordance with the following equation:

$$V_{Rd2,red} = 1.67 V_{Rd2} \left(1 - \frac{\sigma_{cp,eff}}{f_{ck}} \gamma_c \right) \leq V_{Rd2} \quad \text{according to Table 9.1.4 below}$$

$V_{Rd2,red}$ reduced maximal design shear force that can be carried without crushing of the notional concrete compressive struts

Variable strut inclination method

4.3.2.4.4 below

Requirements:

$$V_{Sd} \leq V_{Rd3} \quad \rightarrow \quad \frac{V_{Sd}}{d} \leq \frac{V_{Rd3}}{d} \quad 4.3.2.2(3)$$

and

$$V_{Sd} \leq V_{Rd2} \quad \rightarrow \quad \frac{V_{Sd}}{b_w d} \leq \frac{V_{Rd2}}{b_w d} \quad 4.3.2.2(4)$$

$$\frac{V_{wd}}{d} = \frac{A_{sw}}{s} \frac{z}{d} \frac{f_{yw}}{\gamma_s} (1 + \cot \alpha) \sin \alpha k_{\theta}$$

according to Table 9.1.5 below

with $k_{\theta} = \frac{\cot \theta + \cot \alpha}{1 + \cot \alpha}$

$$\frac{V_{Rd2}}{b_w d} = \frac{v \frac{f_{ck}}{\gamma_c} \frac{z}{d} (\cot \theta + \cot \alpha)}{(1 + \cot^2 \theta)}$$

according to Table 9.1.3b below

If the effective average stress in the concrete ($\sigma_{cp,eff}$) is more than 40% of the design value of the compressive cylinder strength of concrete (f_{cd}), V_{Rd2} should be reduced in accordance with the following equation (4.3.2.2(4)):

$$V_{Rd2,red} = 1.67 V_{Rd2} \left(1 - \frac{\sigma_{cp,eff}}{f_{ck}} \gamma_c \right) \leq V_{Rd2}$$

according to Table 9.1.4 below

For members with inclined prestressing tendons, V_{Sd} is given by:

$$V_{Sd} = V_{od} - V_{pd} \quad [4.32]$$

V_{od} design shear force in the section

V_{pd} force component of the inclined prestressed tendons, parallel to V_{od} (see Figure 9.1)

Apply the detailing requirements according to 4.3.2.4(4)

For the notation for members subjected to shear, see Figure 9.2.

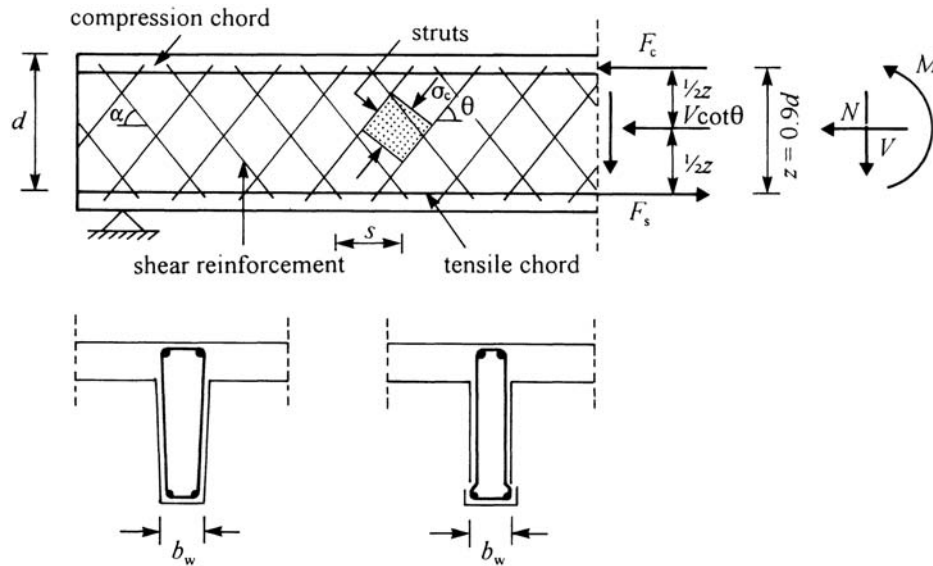


Figure 9.2 Notation for members subjected to shear

9.1.2 $\frac{V_{Rd1}}{b_w d}$

[4.17 and 4.18]

Table 9.1

| 100ρ ₁ (%) | V _{Rd1} /b _w d (with β=1.0; γ=1.5; k=1.0; σ _{cp} =0 N/mm ²) in N/mm ² per concrete class | | | | | | | | |
|---|--|--------|--------|--------|--------|--------|--------|--------|-------|
| C12/15 | C16/20 | C20/25 | C25/30 | C30/37 | C35/45 | C40/50 | C45/55 | C50/60 | |
| 0.0 | 0.216 | 0.264 | 0.312 | 0.360 | 0.408 | 0.444 | 0.492 | 0.528 | 0.576 |
| 0.1 | 0.223 | 0.273 | 0.322 | 0.372 | 0.422 | 0.459 | 0.508 | 0.546 | 0.595 |
| 0.2 | 0.230 | 0.282 | 0.333 | 0.384 | 0.435 | 0.474 | 0.525 | 0.563 | 0.614 |
| 0.3 | 0.238 | 0.290 | 0.343 | 0.396 | 0.449 | 0.488 | 0.541 | 0.581 | 0.634 |
| 0.4 | 0.245 | 0.299 | 0.354 | 0.408 | 0.462 | 0.503 | 0.558 | 0.598 | 0.653 |
| 0.5 | 0.252 | 0.308 | 0.364 | 0.420 | 0.476 | 0.518 | 0.574 | 0.616 | 0.672 |
| 0.6 | 0.259 | 0.317 | 0.374 | 0.432 | 0.490 | 0.533 | 0.590 | 0.634 | 0.691 |
| 0.7 | 0.266 | 0.326 | 0.385 | 0.444 | 0.503 | 0.548 | 0.607 | 0.651 | 0.710 |
| 0.8 | 0.274 | 0.334 | 0.395 | 0.456 | 0.517 | 0.562 | 0.623 | 0.669 | 0.730 |
| 0.9 | 0.281 | 0.343 | 0.406 | 0.468 | 0.530 | 0.577 | 0.640 | 0.686 | 0.749 |
| 1.0 | 0.288 | 0.352 | 0.416 | 0.480 | 0.544 | 0.592 | 0.656 | 0.704 | 0.768 |
| 1.1 | 0.295 | 0.361 | 0.426 | 0.492 | 0.558 | 0.607 | 0.672 | 0.722 | 0.787 |
| 1.2 | 0.302 | 0.370 | 0.437 | 0.504 | 0.571 | 0.622 | 0.689 | 0.739 | 0.806 |
| 1.3 | 0.310 | 0.378 | 0.447 | 0.516 | 0.585 | 0.636 | 0.705 | 0.757 | 0.826 |
| 1.4 | 0.317 | 0.387 | 0.458 | 0.528 | 0.598 | 0.651 | 0.722 | 0.774 | 0.845 |
| 1.5 | 0.324 | 0.396 | 0.468 | 0.540 | 0.612 | 0.666 | 0.738 | 0.792 | 0.864 |
| 1.6 | 0.331 | 0.405 | 0.478 | 0.552 | 0.626 | 0.681 | 0.754 | 0.810 | 0.883 |
| 1.7 | 0.338 | 0.414 | 0.489 | 0.564 | 0.639 | 0.696 | 0.771 | 0.827 | 0.902 |
| 1.8 | 0.346 | 0.422 | 0.499 | 0.576 | 0.653 | 0.710 | 0.787 | 0.845 | 0.922 |
| 1.9 | 0.353 | 0.431 | 0.510 | 0.588 | 0.666 | 0.725 | 0.804 | 0.862 | 0.941 |
| 2.0 | 0.360 | 0.440 | 0.520 | 0.600 | 0.680 | 0.740 | 0.820 | 0.880 | 0.960 |
| $\frac{V_{Rd1}}{b_w d} = \beta(\tau_{Rd} k(1.2 + 40\rho_1) + 0.15\sigma_{cp})$ (with β=1.0; γ=1.5; k=1.0; σ _{cp} =0 N/mm ²) in N/mm ² | | | | | | | | | |
| τ _{Rd} (N/mm ²) | 0.18 | 0.22 | 0.26 | 0.30 | 0.34 | 0.37 | 0.41 | 0.44 | 0.48 |

• If the distance x of a concentrated load is less than 2.5d from the face of the support, multiply by β=2.5d/x 5 to determine the design shear resistance of the member without shear reinforcement for the concentrated load

100ρ₁ (%) V_{Rd1}/b_wd (with β=1.0; γ=1.5; k=1.0; σ_{cp}=0 N/mm²) in N/mm² per concrete class

C12/15 C16/20 C20/25 C25/30 C30/37 C35/45 C40/50 C45/55 C50/60

- If γ_c 1.5, multiply by 1.5/γ_c
- If d<0.6 m, multiply by k=1.6-d (d in metres)
- If 100 ρ₁>2.0%, take 100ρ₁=2.0% into account
- If σ_{cp} 0 N/mm², add β*0.15σ_{cp} (compression positive)

9.1.3a

Standard method

$$\frac{V_{Rd2}}{b_w d}$$

[4.19, 4.20 and 4.25]

Table 9.2

| α (degrees) | V _{Rd2} /b _w d (with γ _c =1.5) in N/mm ² per concrete class | | | | | | | | |
|-------------|---|--------|--------|--------|--------|--------|--------|--------|-------|
| C12/15 | C 16/20 | C20/25 | C25/30 | C30/37 | C35/45 | C40/50 | C45/55 | C50/60 | |
| 90 | 2.30 | 2.98 | 3.60 | 4.31 | 4.95 | 5.51 | 6.00 | 6.75 | 7.50 |
| 85 | 2.51 | 3.24 | 3.92 | 4.69 | 5.38 | 5.99 | 6.52 | 7.34 | 8.16 |
| 80 | 2.71 | 3.50 | 4.23 | 5.07 | 5.82 | 6.48 | 7.06 | 7.94 | 8.82 |
| 75 | 2.92 | 3.77 | 4.56 | 5.47 | 6.28 | 6.99 | 7.61 | 8.56 | 9.51 |
| 70 | 3.14 | 4.06 | 4.91 | 5.88 | 6.75 | 7.52 | 8.18 | 9.21 | 10.23 |
| 65 | 3.38 | 4.36 | 5.28 | 6.32 | 7.26 | 8.08 | 8.80 | 9.90 | 11.00 |
| 60 | 3.63 | 4.69 | 5.68 | 6.80 | 7.81 | 8.70 | 9.46 | 10.65 | 11.83 |
| 55 | 3.92 | 5.06 | 6.12 | 7.33 | 8.42 | 9.37 | 10.20 | 11.48 | 12.75 |
| 50 | 4.24 | 5.47 | 6.62 | 7.93 | 9.10 | 10.14 | 11.03 | 12.41 | 13.79 |
| 45 | 4.61 | 5.95 | 7.20 | 8.63 | 9.90 | 11.03 | 12.00 | 13.50 | 15.00 |

$$\frac{V_{Rd2}}{b_w d} = \frac{1}{2} v \frac{f_{ck}}{\gamma_c} 0.9 (1 + \cot \alpha) \quad (\text{with } v = 0.7 - \frac{f_{ck}}{200} \leq 0.5 \text{ (} f_{ck} \text{ in N/mm}^2 \text{) and } \gamma_c = 1.5 \text{ in N/mm}^2)$$

For sections without designed shear reinforcement, α=90° should be taken [4.19].

- If γ_c 1.5, multiply by 1.5/γ_c

9.1.3b

Variable strut inclination method

$$\frac{V_{Rd2}}{b_w d}$$

[4.26 and 4.28]

Table 9.3

| α (degrees) | θ (degrees) | V _{Rd2} /b _w d (with γ _c = 1.5 and z/d = 0.9) in N/mm ² per concrete class | | | | | | | | |
|-------------|-------------|---|--------|--------|--------|--------|--------|--------|-------|-------|
| C12/15 | C16/20 | C20/25 | C25/30 | C30/37 | C35/45 | C40/50 | C45/55 | C50/60 | | |
| 90 | 68 | 1.60 | 2.07 | 2.50 | 3.00 | 3.44 | 3.83 | 4.17 | 4.69 | 5.21 |
| 60 | 2.00 | 2.58 | 3.12 | 3.73 | 4.29 | 4.77 | 5.20 | 5.85 | 6.50 | |
| 45 | 2.30 | 2.98 | 3.60 | 4.31 | 4.95 | 5.51 | 6.00 | 6.75 | 7.50 | |
| 30 | 2.00 | 2.58 | 3.12 | 3.73 | 4.29 | 4.77 | 5.20 | 5.85 | 6.50 | |
| 22 | 1.60 | 2.07 | 2.50 | 3.00 | 3.44 | 3.83 | 4.17 | 4.69 | 5.21 | |
| 75 | 68 | 2.66 | 3.44 | 4.16 | 4.98 | 5.72 | 6.37 | 6.93 | 7.80 | 8.67 |
| 60 | 2.92 | 3.77 | 4.56 | 5.47 | 6.28 | 6.99 | 7.61 | 8.56 | 9.51 | |
| 45 | 2.92 | 3.77 | 4.56 | 5.47 | 6.28 | 6.99 | 7.61 | 8.56 | 9.51 | |
| 30 | 2.30 | 2.98 | 3.60 | 4.31 | 4.95 | 5.51 | 6.00 | 6.75 | 7.50 | |
| 22 | 1.77 | 2.29 | 2.77 | 3.32 | 3.81 | 4.24 | 4.62 | 5.20 | 5.77 | |
| 60 | 68 | 3.89 | 5.02 | 6.07 | 7.28 | 8.35 | 9.30 | 10.12 | 11.39 | 12.65 |
| 60 | 3.99 | 5.15 | 6.24 | 7.47 | 8.57 | 9.55 | 10.39 | 11.69 | 12.99 | |
| 45 | 3.63 | 4.69 | 5.68 | 6.80 | 7.81 | 8.70 | 9.46 | 10.65 | 11.83 | |
| 30 | 2.66 | 3.44 | 4.16 | 4.98 | 5.72 | 6.37 | 6.93 | 7.79 | 8.66 | |
| 22 | 1.97 | 2.55 | 3.08 | 3.69 | 4.24 | 4.72 | 5.14 | 5.78 | 6.43 | |

| α (degrees) | θ (degrees) | $\frac{V_{Rd2}}{b_w d}$ (with $\gamma_c = 1.5$ and $\frac{z}{d} = 0.9$) in N/mm ² per concrete class | | | | | | | | |
|-----------------------|-----------------------|--|--------|--------|--------|--------|--------|--------|--------|--------|
| | | C12/15 | C16/20 | C20/25 | C25/30 | C30/37 | C35/45 | C40/50 | C45/55 | C50/60 |
| 45 | 68 | 5.56 | 7.18 | 8.69 | 10.41 | 11.95 | 13.31 | 14.48 | 16.29 | 18.10 |
| 60 | 5.45 | 7.04 | 8.52 | 10.20 | 11.71 | 13.04 | 14.20 | 15.97 | 17.75 | |
| 45 | 4.61 | 5.95 | 7.20 | 8.63 | 9.90 | 11.03 | 12.00 | 13.50 | 15.00 | |
| 30 | 3.15 | 4.07 | 4.92 | 5.89 | 6.76 | 7.53 | 8.20 | 9.22 | 10.25 | |
| 22 | 2.25 | 2.90 | 3.51 | 4.21 | 4.83 | 5.38 | 5.85 | 6.58 | 7.31 | |

$$\frac{V_{Rd2}}{b_w d} = \frac{v \frac{f_{ck}}{\gamma_c} \frac{z}{d} (\cot\theta + \cot\alpha)}{(1 + \cot^2\theta)} \quad (\text{with } v = 0.7 - \frac{f_{ck}}{200} \leq 0.5 \text{ (} f_{ck} \text{ in N/mm}^2\text{); } \gamma_c = 1.5 \text{ and } \frac{z}{d} = 0.9 \text{ in N/mm}^2)$$

- If $\gamma_c = 1.5$, multiply by $1.5/\gamma_c$
- If $z = 0.9d$, multiply by $z/(0.9d)$

9.1.4 $\frac{V_{Rd2,red}}{V_{Rd2}}$

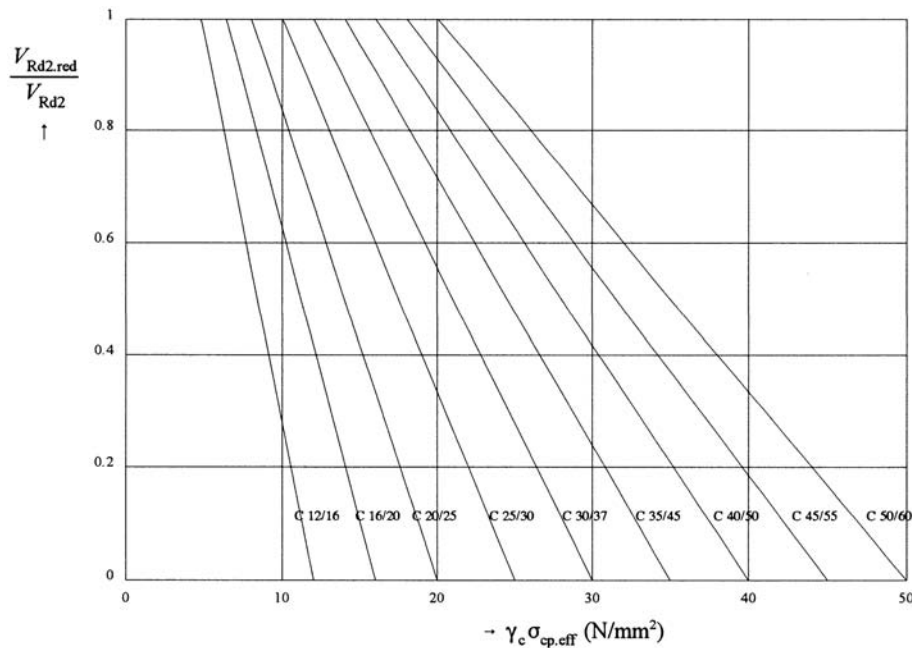
[4.15 and 4.16]

Table 9.4

$$\frac{V_{Rd2,red}}{V_{Rd2}} = 1.67 \left(1 - \frac{\sigma_{cp,eff}}{f_{ck}} \gamma_c \right) \leq 1 \quad [4.15]$$

$$\sigma_{cp,eff} = \frac{N_{Sd}}{A_c} - \frac{A_{s2} f_{yk}}{A_c \gamma_s} \quad [4.16]$$

- If $f_{yk}/\gamma_s > 400$ N/mm², take $f_{yk}/\gamma_s = 400$ N/mm² into account



9.1.5 $\frac{V_{wd}}{d}$ and $\frac{V_{Rd3}}{d}$

[4.24 and 4.29]

Table 9.5

| Stirrups with two legs | | $\frac{A_{sw}}{s}$ (mm ² /m) | $\frac{V_{wd}}{d}$ in kN/m for $\alpha =$ | | | | Stirrups with two legs | | $\frac{A_{sw}}{s}$ (mm ² /m) | $\frac{V_{wd}}{d}$ in kN/m for $\alpha =$ | | | |
|------------------------|-----|---|---|-----|-----|-----|------------------------|-----|---|---|------|------|------|
| 90° | 75° | 60° | 45° | 90° | 75° | 60° | 45° | | | | | | |
| ø 5 | 100 | 393 | 154 | 188 | 210 | 217 | ø 10 | 100 | 1571 | 615 | 753 | 840 | 870 |
| | 150 | 262 | 103 | 126 | 140 | 145 | | 150 | 1047 | 410 | 502 | 560 | 580 |
| | 200 | 196 | 77 | 94 | 105 | 109 | | 200 | 785 | 307 | 377 | 420 | 435 |
| | 250 | 157 | 61 | 75 | 84 | 87 | | 250 | 628 | 246 | 301 | 336 | 348 |
| | 300 | 131 | 51 | 63 | 70 | 72 | | 300 | 524 | 205 | 251 | 280 | 290 |
| ø 6 | 100 | 565 | 221 | 271 | 302 | 313 | ø 12 | 100 | 2262 | 886 | 1085 | 1210 | 1252 |
| | 150 | 377 | 148 | 181 | 202 | 209 | | 150 | 1508 | 590 | 723 | 806 | 835 |
| | 200 | 283 | 111 | 136 | 151 | 157 | | 200 | 1131 | 443 | 542 | 605 | 626 |
| | 250 | 226 | 88 | 108 | 121 | 125 | | 250 | 905 | 354 | 434 | 484 | 501 |
| | 300 | 188 | 74 | 90 | 101 | 104 | | 300 | 754 | 295 | 362 | 403 | 417 |
| ø 8 | 100 | 1005 | 393 | 482 | 538 | 557 | ø 16 | 100 | 4021 | 1,574 | 1928 | 2151 | 2226 |
| | 150 | 670 | 262 | 321 | 358 | 371 | | 150 | 2681 | 1,050 | 1285 | 1434 | 1484 |
| | 200 | 503 | 197 | 241 | 269 | 278 | | 200 | 2011 | 787 | 964 | 1075 | 1113 |
| | 250 | 402 | 157 | 193 | 215 | 223 | | 250 | 1608 | 630 | 771 | 860 | 891 |
| | 300 | 335 | 131 | 161 | 179 | 186 | | 300 | 1340 | 525 | 643 | 717 | 742 |

• If $\theta=45^\circ$

Standard method

$$\frac{V_{wd}}{d} = \frac{A_{sw}}{s} 0.9 \frac{f_{yk}}{\gamma_s} (1 + \cot \alpha) \sin \alpha \quad (\text{with } \gamma_s = 1.15 \text{ and } f_{yk} = 500 \text{ N/mm}^2 \text{ in N/mm}^2)$$

• If γ_s 1.15 multiply by 1.15/ γ_s

• If f_{yk} 500 N/mm², multiply by $f_{yk}/500$

• If θ 45°

Variable strut inclination method

$$\frac{V_{wd}}{d} = \frac{A_{sw}}{s} \frac{z}{d} \frac{f_{yk}}{\gamma_s} (1 + \cot \alpha) \sin \alpha k_\theta \quad (\text{with } \frac{z}{d} = 0.9)$$

k_θ according to:

| θ | α | k_θ | θ | α | k_θ | θ | α | k_θ | θ | α | k_θ |
|----------|----------|------------|----------|----------|------------|----------|----------|------------|----------|----------|------------|
| 68 | 90 | 0.404 | 60 | 90 | 0.577 | 30 | 90 | 1.732 | 22 | 90 | 2.475 |
| | 75 | 0.530 | | 75 | 0.667 | | 75 | 1.577 | | 75 | 2.163 |
| | 60 | 0.622 | | 60 | 0.732 | | 60 | 1.464 | | 60 | 1.935 |
| | 45 | 0.702 | | 45 | 0.789 | | 45 | 1.366 | | 45 | 1.738 |

$$k_\theta = \frac{\cot \theta + \cot \alpha}{1 + \cot \alpha}$$

• If z 0.9d, multiply by $z/(0.9d)$

The upper part of the Table represents the values V_{wd}/d according to equation 4.24

The values in the upper part multiplied by k_θ represent V_{Rd3}/d according to equation 4.29

9.2
Torsion

9.2.1
General

Requirements

$$T_{Sd} \leq T_{Rd1} \quad \rightarrow \quad \frac{T_{Sd}}{h^3} \leq \frac{T_{Rd1}}{h^3} \quad [4.38]$$

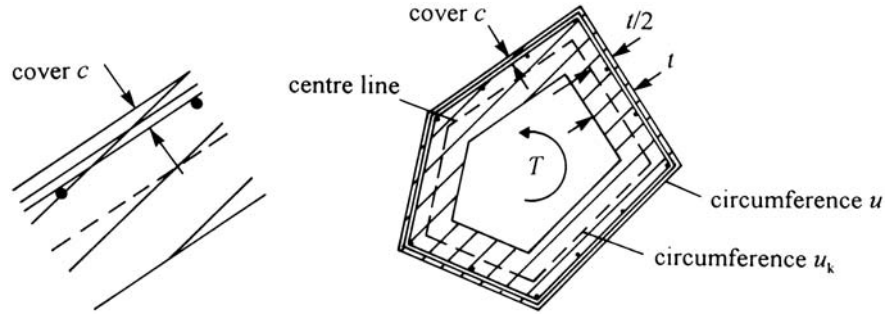


Figure 9.3 Notation used in relation to torsion

and

$$T_{Sd} \leq T_{Rd2} \quad \rightarrow \quad \frac{T_{Sd}}{h^2} \leq \frac{T_{Rd2}}{h^2}$$

or

$$\rightarrow \frac{T_{Sd}}{h^3} \leq \frac{T_{Rd2}}{h^3} \quad [4.39]$$

- T_{Sd} design torsional moment
- T_{Rd1} maximum torsional moment that can be resisted by the compressive struts in the concrete
- T_{Rd2} maximum torsional moment that can be resisted by the torsion reinforcement

$$\frac{T_{Rd1}}{h^3} = v f_{cd} \frac{t}{h} \left(\frac{b}{h} - \frac{t}{h} \right) \left(1 - \frac{t}{h} \right) k_{\theta,1}$$

with $k_{\theta,1} = \frac{2}{\cot \theta + \tan \theta}$

according to Table 9.2.2 below

$$\frac{T_{Rd2}}{h^2} = 2 \left(\frac{b}{h} - \frac{t}{h} \right) \left(1 - \frac{t}{h} \right) \frac{f_{yk}}{\gamma_s} \frac{A_{sw}}{s} k_{\theta,2}$$

with $k_{\theta,2} = \cot \theta$

according to Table 9.2.3 below

$$\frac{T_{Rd2}}{h^3} = \frac{A_{sl}}{bh} \frac{f_{yk}}{\gamma_s} \frac{b}{h} \left(\frac{b}{h} - \frac{t}{h} \right) \left(1 - \frac{t}{h} \right) \frac{1}{\frac{b}{h} - 2\frac{t}{h} + 1} k_{\theta,3}$$

with $k_{\theta,3} = \frac{1}{\cot \theta}$

according to Table 9.2.4 below

For the notations used in relation to torsion, see Figure 9.3.

9.2.2 $\frac{T_{Rd1}}{h^3}$

[4.40]

Table 9.6

| $\frac{b}{h}$ | $\frac{t}{h}$ | $\frac{T_{Rd1}}{h^3}$ in N/mm ² per concrete class | | | | | | | | |
|---------------|---------------|---|--------|--------|--------|--------|--------|--------|-------|-------|
| | | C20/25 | C25/30 | C30/37 | C35/45 | C40/50 | C45/55 | C50/60 | | |
| 1.0 | C 16/20 | | | | | | | | | |
| | 0.250 | 0.504 | 0.651 | 0.788 | 0.943 | 1.083 | 1.206 | 1.313 | 1.477 | 1.641 |
| | 0.200 | 0.459 | 0.593 | 0.717 | 0.859 | 0.986 | 1.098 | 1.195 | 1.344 | 1.493 |
| | 0.150 | 0.388 | 0.502 | 0.607 | 0.727 | 0.834 | 0.929 | 1.012 | 1.138 | 1.264 |
| | 0.100 | 0.290 | 0.375 | 0.454 | 0.543 | 0.624 | 0.695 | 0.756 | 0.851 | 0.945 |
| | 0.050 | 0.162 | 0.209 | 0.253 | 0.303 | 0.347 | 0.387 | 0.421 | 0.474 | 0.526 |

| $\frac{b}{h}$ | $\frac{t}{h}$ | $\frac{T_{Rd1}}{h^3}$ in N/mm ² per concrete class | | | | | | | | |
|---------------|---------------|---|---------|--------|--------|--------|--------|--------|--------|--------|
| | | C12/15 | C 16/20 | C20/25 | C25/30 | C30/37 | C35/45 | C40/50 | C45/55 | C50/60 |
| 0.8 | 0.222 | 0.358 | 0.462 | 0.559 | 0.670 | 0.769 | 0.856 | 0.932 | 1.049 | 1.165 |
| | 0.200 | 0.344 | 0.444 | 0.538 | 0.644 | 0.739 | 0.823 | 0.896 | 1.008 | 1.120 |
| | 0.150 | 0.297 | 0.384 | 0.464 | 0.556 | 0.638 | 0.711 | 0.774 | 0.870 | 0.967 |
| | 0.100 | 0.226 | 0.292 | 0.353 | 0.423 | 0.485 | 0.540 | 0.588 | 0.662 | 0.735 |
| | 0.050 | 0.128 | 0.165 | 0.200 | 0.239 | 0.274 | 0.305 | 0.333 | 0.374 | 0.416 |
| 0.6 | 0.188 | 0.225 | 0.291 | 0.352 | 0.422 | 0.484 | 0.539 | 0.587 | 0.660 | 0.733 |
| | 0.150 | 0.206 | 0.266 | 0.321 | 0.385 | 0.442 | 0.492 | 0.536 | 0.602 | 0.669 |
| | 0.100 | 0.161 | 0.208 | 0.252 | 0.302 | 0.347 | 0.386 | 0.420 | 0.473 | 0.525 |
| | 0.050 | 0.094 | 0.121 | 0.146 | 0.175 | 0.201 | 0.224 | 0.244 | 0.274 | 0.305 |
| 0.4 | 0.143 | 0.113 | 0.146 | 0.176 | 0.211 | 0.242 | 0.270 | 0.294 | 0.331 | 0.367 |
| | 0.100 | 0.097 | 0.125 | 0.151 | 0.181 | 0.208 | 0.232 | 0.252 | 0.284 | 0.315 |
| | 0.050 | 0.060 | 0.077 | 0.093 | 0.112 | 0.128 | 0.143 | 0.155 | 0.175 | 0.194 |
| 0.2 | 0.083 | 0.032 | 0.041 | 0.050 | 0.060 | 0.069 | 0.076 | 0.083 | 0.094 | 0.104 |
| | 0.050 | 0.026 | 0.033 | 0.040 | 0.048 | 0.055 | 0.061 | 0.067 | 0.075 | 0.083 |

$$\frac{T_{Rd1}}{h^3} = v f_{cd} \frac{t}{h} \left(\frac{b}{h} - \frac{t}{h} \right) \left(1 - \frac{t}{h} \right) k_{\theta,1} \quad \text{with } v = 0.7 \left(0.7 \frac{f_{ck}}{200} \right) \leq 0.35 (f_{ck} \text{ in N/mm}^2) ; \gamma_c = 1.5; \theta = 45^\circ$$

and so $k_{\theta,1}=1.0$ in N/mm²

For t/h any value meeting the requirements $t \leq A/u$, $t \leq$ the actual wall thickness of a hollow section, and $t \leq 2c$ may be chosen. The maximum value $T=A/u$ is given as the maximum value in the Table for each value of b/h .

- If $\gamma_c = 1.5$, multiply by $1.5/\gamma_c$
- If $\theta = 45^\circ$, multiply by $k_{\theta,1}$ according to:

| | | | | | | |
|--------------------|------|------|------|------|------|------|
| θ (degrees) | 68 | 65 | 60 | 55 | 50 | 45 |
| 22 | 25 | 30 | 35 | 40 | 45 | |
| $k_{\theta,1}$ | 0.69 | 0.77 | 0.87 | 0.94 | 0.98 | 1.00 |

$$k_{\theta,1} = 2 / (\cot\theta + \tan\theta)$$

9.2.3a $\frac{T_{Rd2}}{h^2}$ [4.43]

Table 9.7

| $\frac{b}{h}$ | $\frac{t}{h}$ | $\frac{T_{Rd2}}{h^2}$ in kN/m per configuration of stirrups with one leg | | | | | | | | | |
|---------------|---------------|--|-------|------|------|------|---------------------------------|-------|-------|------|------|
| | | Stirrups \emptyset 5-s | | | | | Stirrups \emptyset 6-s | | | | |
| | | 100 | 150 | 200 | 250 | 300 | 100 | 150 | 200 | 250 | 300 |
| | | A_{sw}/s (mm ² /m) | | | | | A_{sw}/s (mm ² /m) | | | | |
| | | 196 | 131 | 98 | 79 | 65 | 283 | 188 | 141 | 113 | 94 |
| 1.0 | 0.250 | 96.1 | 64.1 | 48.0 | 38.4 | 32.0 | 138.4 | 92.2 | 69.2 | 55.3 | 46.1 |
| | 0.200 | 109.3 | 72.9 | 54.7 | 43.7 | 36.4 | 157.4 | 105.0 | 78.7 | 63.0 | 52.5 |
| | 0.150 | 123.4 | 82.3 | 61.7 | 49.4 | 41.1 | 177.7 | 118.5 | 88.9 | 71.1 | 59.2 |
| | 0.100 | 138.4 | 92.2 | 69.2 | 55.3 | 46.1 | 199.2 | 132.8 | 99.6 | 79.7 | 66.4 |
| | 0.050 | 154.2 | 102.8 | 77.1 | 61.7 | 51.4 | 222.0 | 148.0 | 111.0 | 88.8 | 74.0 |
| 0.8 | 0.222 | 76.8 | 51.2 | 38.4 | 30.7 | 25.6 | 110.5 | 73.7 | 55.3 | 44.2 | 36.8 |
| | 0.200 | 82.0 | 54.7 | 41.0 | 32.8 | 27.3 | 118.1 | 78.7 | 59.0 | 47.2 | 39.4 |
| | 0.150 | 94.4 | 62.9 | 47.2 | 37.8 | 31.5 | 135.9 | 90.6 | 68.0 | 54.4 | 45.3 |
| | 0.100 | 107.6 | 71.7 | 53.8 | 43.0 | 35.9 | 155.0 | 103.3 | 77.5 | 62.0 | 51.7 |

| $\frac{b}{h}$ | $\frac{t}{h}$ | $\frac{T_{Rd2}}{h^2}$ in kN/m per configuration of stirrups with one leg | | | | | | | | | |
|---------------------------------|---------------|--|------|------|---------------------------------|------|-------|-------|------|------|------|
| Stirrups \emptyset 5-s | | | | | Stirrups \emptyset 6-s | | | | | | |
| 100 | 150 | 200 | 250 | 300 | 100 | 150 | 200 | 250 | 300 | | |
| A_{sw}/s (mm ² /m) | | | | | A_{sw}/s (mm ² /m) | | | | | | |
| 196 | 131 | 98 | 79 | 65 | 283 | 188 | 141 | 113 | 94 | | |
| | 0.050 | 121.7 | 81.1 | 60.9 | 48.7 | 40.6 | 175.3 | 116.8 | 87.6 | 70.1 | 58.4 |
| 0.6 | 0.188 | 57.3 | 38.2 | 28.6 | 22.9 | 19.1 | 82.4 | 55.0 | 41.2 | 33.0 | 27.5 |
| | 0.150 | 65.3 | 43.6 | 32.7 | 26.1 | 21.8 | 94.1 | 62.7 | 47.0 | 37.6 | 31.4 |
| | 0.100 | 76.9 | 51.2 | 38.4 | 30.7 | 25.6 | 110.7 | 73.8 | 55.3 | 44.3 | 36.9 |
| 0.4 | 0.050 | 89.3 | 59.5 | 44.6 | 35.7 | 29.8 | 128.5 | 85.7 | 64.3 | 51.4 | 42.8 |
| | 0.143 | 37.7 | 25.1 | 18.8 | 15.1 | 12.6 | 54.2 | 36.1 | 27.1 | 21.7 | 18.1 |
| | 0.100 | 46.1 | 30.7 | 23.1 | 18.4 | 15.4 | 66.4 | 44.3 | 33.2 | 26.6 | 22.1 |
| 0.2 | 0.050 | 56.8 | 37.9 | 28.4 | 22.7 | 18.9 | 81.8 | 54.5 | 40.9 | 32.7 | 27.3 |
| | 0.083 | 18.3 | 12.2 | 9.1 | 7.3 | 6.1 | 26.3 | 17.5 | 13.2 | 10.5 | 8.8 |
| | 0.050 | 24.3 | 16.2 | 12.2 | 9.7 | 8.1 | 35.1 | 23.4 | 17.5 | 14.0 | 11.7 |

$$\frac{T_{Rd2}}{h^2} = 2 \left(\frac{b}{h} - \frac{t}{h} \right) \left(1 - \frac{t}{h} \right) \frac{f_{yw} A_{sw}}{\gamma_s s} k_{\theta,2} \quad (\text{with } f_{yw} = 0.5 \text{ kN/mm}^2; \gamma_s = 1.15; \theta = 45^\circ \text{ and so } k_{\theta,2} = 1.0) \text{ in kN/m}$$

- If $\gamma_s = 1.15$, multiply by $1.15/\gamma_s$.
- If $f_{yw} = 0.5 \text{ kN/mm}^2$, multiply by $f_{yw}/0.5$.
- If $\theta = 45^\circ$, multiply by $k_{\theta,2} = \cot\theta$.

For t/h and θ , the same values as in Table 9.6 should be used.

9.2.3b $\frac{T_{Rd2}}{h^2}$

[4.43]

Table 9.8

| $\frac{b}{h}$ | $\frac{t}{h}$ | $\frac{T_{Rd2}}{h^2}$ in kN/m per configuration of stirrups with one leg | | | | | | | | | |
|---------------------------------|---------------|--|-------|-------|---------------------------------|-------|-------|-------|-------|-------|-------|
| Stirrups \emptyset 8-s | | | | | Stirrups \emptyset 10-s | | | | | | |
| 100 | 150 | 200 | 250 | 300 | 100 | 150 | 200 | 250 | 300 | | |
| A_{sw}/s (mm ² /m) | | | | | A_{sw}/s (mm ² /m) | | | | | | |
| 503 | 335 | 251 | 201 | 168 | 785 | 524 | 393 | 314 | 262 | | |
| 1.0 | 0.250 | 246.0 | 164.0 | 123.0 | 98.4 | 82.0 | 384.4 | 256.2 | 192.2 | 153.7 | 128.1 |
| | 0.200 | 279.9 | 186.6 | 139.9 | 112.0 | 93.3 | 437.3 | 291.5 | 218.7 | 174.9 | 145.8 |
| | 0.150 | 316.0 | 210.6 | 158.0 | 126.4 | 105.3 | 493.7 | 329.1 | 246.8 | 197.5 | 164.6 |
| | 0.100 | 354.2 | 236.1 | 177.1 | 141.7 | 118.1 | 553.5 | 369.0 | 276.7 | 221.4 | 184.5 |
| | 0.050 | 394.7 | 263.1 | 197.3 | 157.9 | 131.6 | 616.7 | 411.1 | 308.3 | 246.7 | 205.6 |
| 0.8 | 0.222 | 196.5 | 131.0 | 98.3 | 78.6 | 65.5 | 307.1 | 204.7 | 153.5 | 122.8 | 102.4 |
| | 0.200 | 209.9 | 139.9 | 105.0 | 84.0 | 70.0 | 328.0 | 218.7 | 164.0 | 131.2 | 109.3 |
| | 0.150 | 241.6 | 161.1 | 120.8 | 96.6 | 80.5 | 377.5 | 251.7 | 188.8 | 151.0 | 125.8 |
| | 0.100 | 275.5 | 183.7 | 137.8 | 110.2 | 91.8 | 430.5 | 287.0 | 215.2 | 172.2 | 143.5 |
| | 0.050 | 311.6 | 207.7 | 155.8 | 124.6 | 103.9 | 486.8 | 324.6 | 243.4 | 194.7 | 162.3 |
| 0.6 | 0.188 | 146.6 | 97.7 | 73.3 | 58.6 | 48.9 | 229.0 | 152.7 | 114.5 | 91.6 | 76.3 |
| | 0.150 | 167.3 | 111.5 | 83.6 | 66.9 | 55.8 | 261.4 | 174.2 | 130.7 | 104.5 | 87.1 |
| | 0.100 | 196.8 | 131.2 | 98.4 | 78.7 | 65.6 | 307.5 | 205.0 | 153.7 | 123.0 | 102.5 |
| | 0.050 | 228.5 | 152.3 | 114.2 | 91.4 | 76.2 | 357.0 | 238.0 | 178.5 | 142.8 | 119.0 |
| | 0.4 | 0.143 | 96.4 | 64.3 | 48.2 | 38.6 | 32.1 | 150.6 | 100.4 | 75.3 | 60.2 |
| 0.100 | | 118.1 | 78.7 | 59.0 | 47.2 | 39.4 | 184.5 | 123.0 | 92.2 | 73.8 | 61.5 |

| $\frac{b}{h}$ | $\frac{t}{h}$ | $\frac{T_{Rd2}}{h^2}$ in kN/m per configuration of stirrups with one leg | | | | | | | | | |
|---------------------------------|---------------|--|------|------|---------------------------------|------|-------|-------|-------|------|------|
| Stirrups \varnothing 8-s | | | | | Stirrups \varnothing 10-s | | | | | | |
| 100 | 150 | 200 | 250 | 300 | 100 | 150 | 200 | 250 | 300 | | |
| A_{sw}/s (mm ² /m) | | | | | A_{sw}/s (mm ² /m) | | | | | | |
| 503 | 335 | 251 | 201 | 168 | 785 | 524 | 393 | 314 | 262 | | |
| | 0.050 | 145.4 | 96.9 | 72.7 | 58.2 | 48.5 | 227.2 | 151.5 | 113.6 | 90.9 | 75.7 |
| 0.2 | 0.083 | 46.8 | 31.2 | 23.4 | 18.7 | 15.6 | 73.1 | 48.7 | 36.5 | 29.2 | 24.4 |
| | 0.050 | 62.3 | 41.5 | 31.2 | 24.9 | 20.8 | 97.4 | 64.9 | 48.7 | 38.9 | 32.5 |

$$\frac{T_{Rd2}}{h^2} = 2 \left(\frac{b}{h} - \frac{t}{h} \right) \left(1 - \frac{t}{h} \right) \frac{f_{yw}}{\gamma_s} \frac{A_{sw}}{s} k_{\theta,2} \quad (\text{with } f_{yw} = 0.5 \text{ kN/mm}^2; \gamma_s = 1.15; \theta = 45^\circ \text{ and so } k_{\theta,2} = 1.0) \text{ in kN/m}$$

- If γ_s 1.15, multiply by 1.15/ γ_s
- If f_{yw} 0.5 kN/mm², multiply by $f_{yw}/0.5$
- If θ 45°, multiply by $k_{\theta,2} = \cot\theta$

For t/h and θ , the same values as used in Table 9.6 should be used.

9.2.3c

$$\frac{T_{Rd2}}{h^2} \tag{4.43}$$

Table 9.9

| $\frac{b}{h}$ | $\frac{t}{h}$ | $\frac{T_{Rd2}}{h^2}$ in kN/m per configuration of stirrups with one leg | | | | | | | | | |
|----------------------------------|---------------|--|-------|-------|----------------------------------|-------|--------|--------|-------|-------|-------|
| Stirrups \varnothing 12-s | | | | | Stirrups \varnothing 16-s | | | | | | |
| 100 | 150 | 200 | 250 | 300 | 100 | 150 | 200 | 250 | 300 | | |
| A_{sw}/s in mm ² /m | | | | | A_{sw}/s in mm ² /m | | | | | | |
| 1131 | 754 | 565 | 452 | 377 | 2011 | 1340 | 1005 | 804 | 670 | | |
| 1.0 | 0.250 | 553.5 | 369.0 | 276.7 | 221.4 | 184.5 | 983.9 | 656.0 | 492.0 | 393.6 | 328.0 |
| | 0.200 | 629.7 | 419.8 | 314.9 | 251.9 | 209.9 | 1119.5 | 746.3 | 559.8 | 447.8 | 373.2 |
| | 0.150 | 710.9 | 473.9 | 355.5 | 284.4 | 237.0 | 1263.8 | 842.6 | 631.9 | 505.5 | 421.3 |
| | 0.100 | 797.0 | 531.3 | 398.5 | 318.8 | 265.7 | 1416.9 | 944.6 | 708.4 | 566.8 | 472.3 |
| | 0.050 | 888.0 | 592.0 | 444.0 | 355.2 | 296.0 | 1578.7 | 1052.5 | 789.3 | 631.5 | 526.2 |
| 0.8 | 0.222 | 442.2 | 294.8 | 221.1 | 176.9 | 147.4 | 786.1 | 524.1 | 393.0 | 314.4 | 262.0 |
| | 0.200 | 472.3 | 314.9 | 236.1 | 188.9 | 157.4 | 839.6 | 559.8 | 419.8 | 335.9 | 279.9 |
| | 0.150 | 543.6 | 362.4 | 271.8 | 217.5 | 181.2 | 966.5 | 644.3 | 483.2 | 386.6 | 322.2 |
| | 0.100 | 619.9 | 413.3 | 309.9 | 248.0 | 206.6 | 1102.0 | 734.7 | 551.0 | 440.8 | 367.3 |
| | 0.050 | 701.1 | 467.4 | 350.5 | 280.4 | 233.7 | 1246.3 | 830.9 | 623.2 | 498.5 | 415.4 |
| 0.6 | 0.188 | 329.8 | 219.9 | 164.9 | 131.9 | 109.9 | 586.3 | 390.8 | 293.1 | 234.5 | 195.4 |
| | 0.150 | 376.4 | 250.9 | 188.2 | 150.5 | 125.5 | 669.1 | 446.1 | 334.5 | 267.6 | 223.0 |
| | 0.100 | 442.8 | 295.2 | 221.4 | 177.1 | 147.6 | 787.2 | 524.8 | 393.6 | 314.9 | 262.4 |
| | 0.050 | 514.1 | 342.7 | 257.1 | 205.6 | 171.4 | 914.0 | 609.3 | 457.0 | 365.6 | 304.7 |
| 0.4 | 0.143 | 216.9 | 144.6 | 108.4 | 86.7 | 72.3 | 385.5 | 257.0 | 192.8 | 154.2 | 128.5 |
| | 0.100 | 265.7 | 177.1 | 132.8 | 106.3 | 88.6 | 472.3 | 314.9 | 236.1 | 188.9 | 157.4 |
| | 0.050 | 327.2 | 218.1 | 163.6 | 130.9 | 109.1 | 581.6 | 387.7 | 290.8 | 232.6 | 193.9 |
| 0.2 | 0.083 | 105.2 | 70.2 | 52.6 | 42.1 | 35.1 | 187.1 | 124.7 | 93.5 | 74.8 | 62.4 |
| | 0.050 | 140.2 | 93.5 | 70.1 | 56.1 | 46.7 | 249.3 | 166.2 | 124.6 | 99.7 | 83.1 |

$$\frac{T_{Rd2}}{h^2} = 2 \left(\frac{b}{h} - \frac{t}{h} \right) \left(1 - \frac{t}{h} \right) \frac{f_{yw}}{\gamma_s} \frac{A_{sw}}{s} k_{\theta,2} \quad (\text{with } f_{yw} = 0.5 \text{ kN/mm}^2; \gamma_s = 1.15; \theta = 45^\circ \text{ and so } k_{\theta,2} = 1.0) \text{ in kN/m}$$

- If γ_s 1.15, multiply by 1.15/ γ_s
- If f_{yw} 0.5 kN/mm², multiply by $f_{yw}/0.5$

| $\frac{b}{h}$ | $\frac{t}{h}$ | $\frac{T_{Rd2}}{h^2}$ in kN/m per configuration of stirrups with one leg | | | | | | | |
|----------------------------------|---------------|--|-----|-----------------|----------------------------------|------|------|-----|-----|
| Stirrups ø 12-s | | | | Stirrups ø 16-s | | | | | |
| 100 | 150 | 200 | 250 | 300 | 100 | 150 | 200 | 250 | 300 |
| A_{sw}/s in mm ² /m | | | | | A_{sw}/s in mm ² /m | | | | |
| 1131 | 754 | 565 | 452 | 377 | 2011 | 1340 | 1005 | 804 | 670 |

• If $\theta = 45^\circ$, multiply by $k_{\theta,2} = \cot\theta$

For t/h and θ , the same values as in Table 9.6 should be used.

9.2.4 $\frac{T_{Rd2}}{h^3}$ [4.43]

Table 9.10

| $\frac{b}{h}$ | $\frac{t}{h}$ | $\frac{T_{Rd2}}{h^3}$ (N/mm ²) | | | | | | | | | |
|---------------|---------------|--|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| | | $\frac{100A_{st}}{bh}$ (%) | | | | | | | | | |
| 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.8 | 1.0 | 1.2 | max | | |
| 1.0 | 0.250 | 0.163 | 0.326 | 0.489 | 0.653 | 0.816 | 0.979 | 1.305 | 1.631 | 1.958 | 1.641 |
| | 0.200 | 0.174 | 0.348 | 0.522 | 0.696 | 0.870 | 1.044 | 1.392 | 1.740 | | 1.493 |
| | 0.150 | 0.185 | 0.370 | 0.555 | 0.740 | 0.924 | 1.109 | 1.479 | | | 1.264 |
| | 0.100 | 0.196 | 0.392 | 0.587 | 0.783 | 0.979 | | | | | 0.945 |
| | 0.050 | 0.207 | 0.413 | 0.620 | | | | | | | 0.526 |
| 0.8 | 0.222 | 0.115 | 0.231 | 0.346 | 0.461 | 0.577 | 0.692 | 0.923 | 1.154 | 1.384 | 1.165 |
| | 0.200 | 0.119 | 0.239 | 0.358 | 0.477 | 0.597 | 0.716 | 0.955 | 1.193 | | 1.120 |
| | 0.150 | 0.128 | 0.256 | 0.385 | 0.513 | 0.641 | 0.769 | 1.025 | | | 0.967 |
| | 0.100 | 0.137 | 0.274 | 0.411 | 0.548 | 0.685 | 0.822 | | | | 0.735 |
| | 0.050 | 0.146 | 0.292 | 0.438 | | | | | | | 0.416 |
| 0.6 | 0.188 | 0.071 | 0.143 | 0.214 | 0.286 | 0.357 | 0.428 | 0.571 | 0.714 | 0.857 | 0.733 |
| | 0.150 | 0.077 | 0.154 | 0.230 | 0.307 | 0.384 | 0.461 | 0.614 | 0.768 | | 0.669 |
| | 0.100 | 0.084 | 0.168 | 0.252 | 0.336 | 0.419 | 0.503 | 0.671 | | | 0.525 |
| | 0.050 | 0.091 | 0.182 | 0.273 | 0.364 | | | | | | 0.305 |
| 0.4 | 0.143 | 0.034 | 0.069 | 0.103 | 0.138 | 0.172 | 0.207 | 0.275 | 0.344 | 0.413 | 0.367 |
| | 0.100 | 0.039 | 0.078 | 0.117 | 0.157 | 0.196 | 0.235 | 0.313 | 0.392 | | 0.315 |
| | 0.050 | 0.045 | 0.089 | 0.134 | 0.178 | 0.223 | | | | | 0.194 |
| 0.2 | 0.083 | 0.009 | 0.018 | 0.027 | 0.036 | 0.045 | 0.054 | 0.072 | 0.090 | 0.108 | 0.104 |
| | 0.050 | 0.011 | 0.023 | 0.034 | 0.045 | 0.056 | 0.068 | 0.090 | | | 0.083 |

$$\frac{T_{Rd2}}{h^3} = \frac{A_{st}}{bh} \frac{f_{yk}}{\gamma_s} \frac{b}{h} \left(\frac{b-t}{h} \right) \left(\frac{1-t}{h} \right) k_{\theta,3} \quad (\text{with } f_{yk} = 500 \text{ N/mm}^2; \gamma_s = 1.15; \theta = 45^\circ \text{ and so } k_{\theta,3} = 1.0) \text{ in N/mm}^2$$

- If $\gamma_s = 1.15$, multiply by $1.15/\gamma_s$
- If $f_{yk} = 500$, N/mm² multiply by $f_{yk}/500$
- If $\theta = 45^\circ$, multiply by $k_{\theta,3} = 1/\cot\theta$

For t/h and θ , the same values as in Table 9.6 should be used.

9.3 Combination of torsion and shear

Torsion and shear 4.3.3.2.2(3)

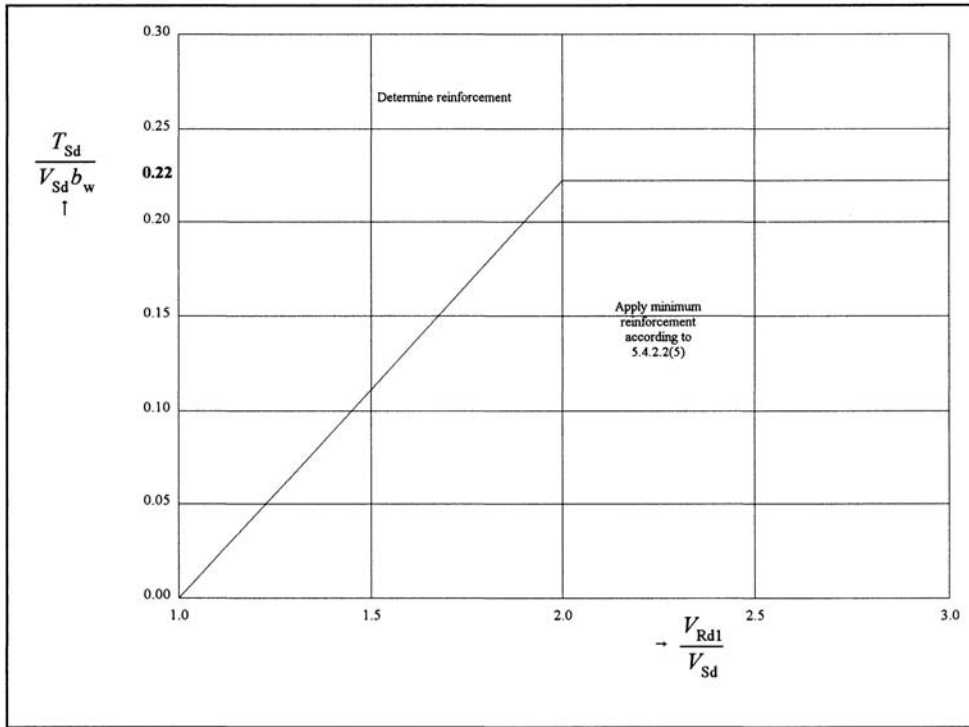


Figure 9.3

Determine T_{Rd1} according to Table 9.6, V_{Rd2} according to Table 9.7, 9.8, 9.9 or 9.10 and check whether the following condition is satisfied:

$$\left(\frac{T_{Sd}}{T_{Rd1}} \right)^2 + \left(\frac{V_{Sd}}{V_{Rd2}} \right)^2 \leq 1 \quad [4.47]$$

Torsion and shear for solid, approximately rectangular sections 4.3.3.2.2(5)

$$\frac{T_{Sd}}{V_{Sd} b_w} \leq \frac{1}{4.5} \quad \text{for} \quad \frac{V_{Rd1}}{V_{Sd}} > 2 \quad [4.48]$$

$$\frac{T_{Sd}}{V_{Sd} b_w} = \frac{1}{4.5} \left(\frac{V_{Rd1}}{V_{Sd}} - 1 \right) \quad \text{for} \quad 1 \leq \frac{V_{Rd1}}{V_{Sd}} \leq 2 \quad [4.49]$$

10 Punching

10.1 General

Punching 4.3.4

Slabs without punching shear reinforcement 4.3.4.5.1

Requirement:

$$v_{Sd} \leq v_{Rd1} \quad \rightarrow \quad \frac{v_{Sd}}{d} \leq \frac{v_{Rd1}}{d} \quad 4.3.4.3(2)$$

with:

$$\frac{v_{Sd}}{d} = \frac{V_{Sd}}{d u} \beta = \frac{V_{Sd}}{d^2} \beta = \frac{u}{d}$$

according to Table 10.2a or 10.2b

$$\frac{v_{Rd1}}{d} = \tau_{Rd} k (1.2 + 40 \rho_1)$$

according to Table 10.3

Slabs with punching shear reinforcement 4.3.4.5.2

Requirements:

$$v_{Sd} \leq v_{Rd3} \quad \rightarrow \quad \frac{v_{Sd}}{d} \leq \frac{v_{Rd3}}{d} \quad 4.3.4.3(3)$$

and

$$v_{Sd} \leq [1.6] v_{Rd1} \quad \rightarrow \quad \frac{v_{Sd}}{d} \leq [1.6] \frac{v_{Rd1}}{d} \quad [4.57]$$

with:

$$\frac{v_{Sd}}{d} = \frac{V_{Sd}}{d u} \beta = \frac{V_{Sd}}{d^2} \beta = \frac{u}{d}$$

according to Table 10.2a or 10.2b

$$\frac{v_{Rd1}}{d} = \tau_{Rd} k (1.2 + 40 \rho_1)$$

according to Table 8.3

$$\frac{v_{Rd3}}{d} = \frac{v_{Rd1}}{d} + \frac{\sum A_{sw} f_{yk} \sin \alpha}{u d \gamma_s}$$

according to Table 10.4a or 10.4b

Apply minimum punching shear reinforcement by taking [60%] of the appropriate value of Table 5.5 (EC2). 4.3.4.5.2(4)

For the boxed values, apply the values given in the appropriate NAD.

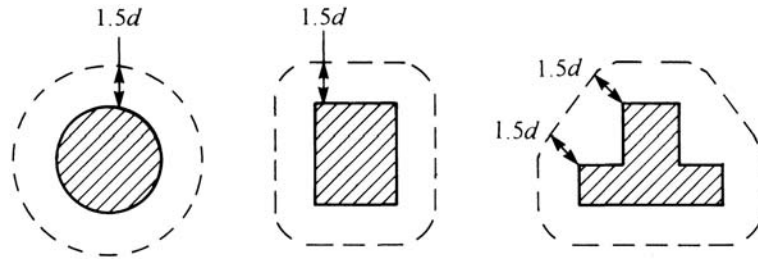


Figure 10.1 Critical perimeter round loaded areas located away from an unsupported edge.

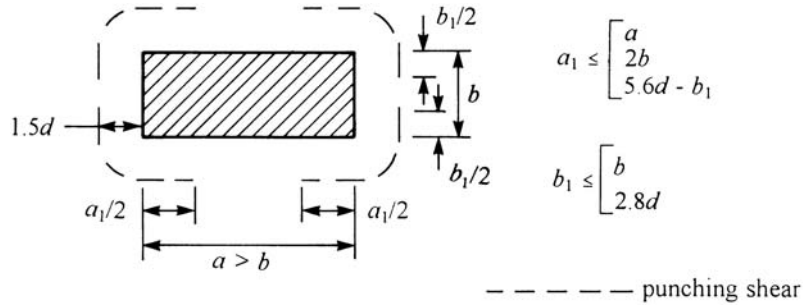


Figure 10.2 Application of punching provisions in non-standard cases.

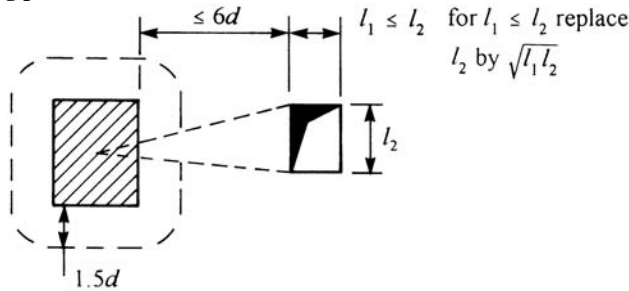


Figure 10.3 Critical perimeter near an opening.

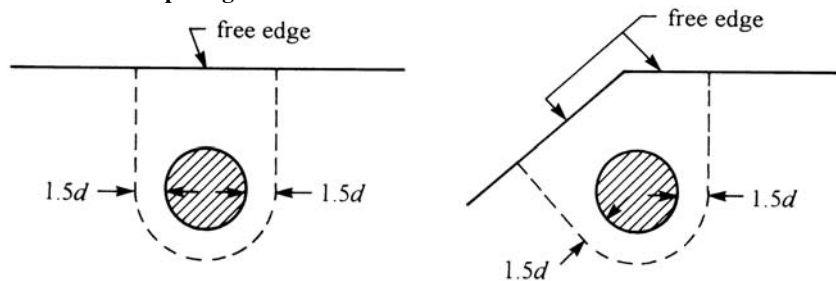


Figure 10.4 Critical sections near unsupported edges.

Loaded area, critical perimeter and critical section 4.3.4.2.1-4

The critical perimeter is defined as a perimeter surrounding the perimeter of the loaded area at a defined distance of 1.5d (Figures 10.1 and 10.2).

For a circular loaded area with diameter a, the perimeter of the loaded area is πa .

The critical perimeter for a circular loaded area located away from unsupported edges is:

$$u = \pi(a + 3d) \rightarrow u/d = \pi(a/d + 3)$$

Limiting value: $a/d \geq 3.5$

For a rectangular loaded area with dimensions a and b the perimeter of the loaded area is $2(a+b)$.

The critical perimeter for a rectangular loaded area located away from unsupported edges is:

$$u = 2(a + b) + 3\pi d \rightarrow u/d = 2(a/d + b/d) + 3\pi$$

Limiting values: $a/d + b/d \geq 5.5$ and $a/b \geq 2$

For openings, determine the critical perimeter according to Figure 10.3.

For loaded areas near or on an unsupported edge or corner, determine the critical perimeter according to Figure 10.4.

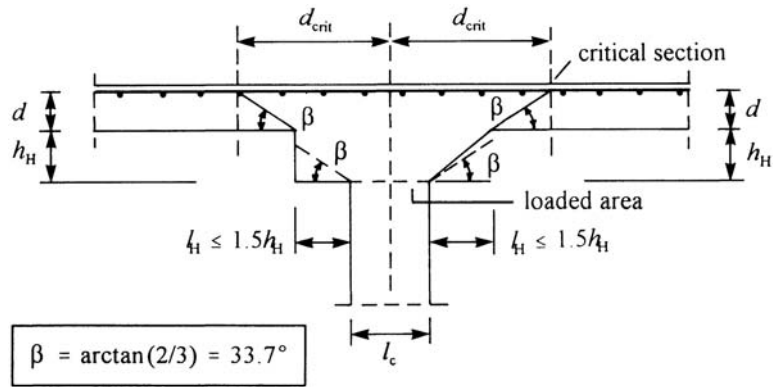


Figure 10.5 Slab with column heads where $l_H \leq 1.5h_H$.

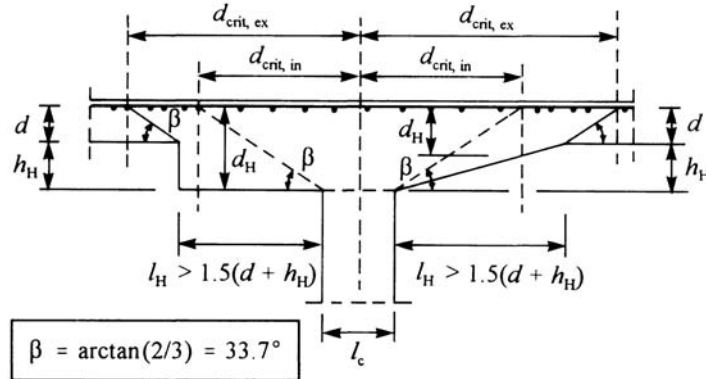


Figure 10.6 Slabs with enlarged column head where $l_H > 1.5(d+h_H)$.

For slabs with column heads where $l_H < 1.5h_H$, determine critical sections according to [Figure 10.5](#).

For slabs with enlarged column head where $l_H > 1.5(d+h_H)$, determine critical sections according to [Figure 10.6](#).

For column heads where $1.5h_H < l_H < 1.5(d+h_H)$, the distance from the centroid of the column to the critical section may be taken as:

$$d_{\text{crit}} = 1.5l_H + 1.5l_c$$

Coefficient β 4.3.4.3(4)

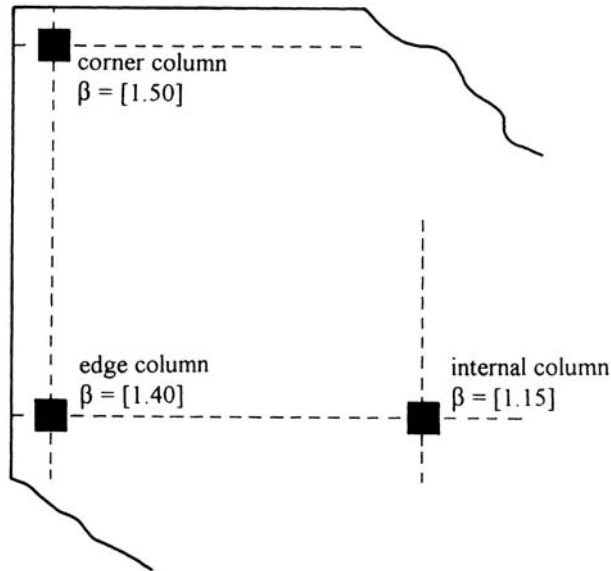


Figure 10.7 Approximate values for β .

β is a coefficient which takes account of the effects of eccentricity of loading. In cases where no eccentricity of loading is possible, β may be taken as 1.0. In other cases, the values given in Figure 10.7 may be adopted (4.3.4.3(4)). For the boxed values, apply the values given in the appropriate NAD.

Minimum design moments 4.3.4.5.3

Design slab for minimum bending moments per unit width, m_{Sdx} and m_{Sdy} in the x - and y -direction, unless structural analysis leads to higher values according to:

$$m_{Sdx} \text{ (or } m_{Sdy}) \geq nV_{Sd} \tag{4.25}$$

Take n into account according to Table 10.1 and Figure 10.8.

| Position of column | n for m_{Sdx} | | n for m_{Sdy} | | | |
|--|-------------------|-----------------|-------------------|--------|-----------------|------------|
| | Bottom | Effective width | Top | Bottom | Effective width | |
| Internal column | -0.125 | 0 | $0.30 l_y$ | -0.125 | 0 | $0.3 l_x$ |
| Edge columns, edge of slab parallel to x -axis | -0.250 | 0 | $0.15 l_y$ | -0.125 | +0.125 | (per m) |
| Edge columns, edge of slab parallel to y -axis | -0.125 | +0.125 | (per m) | -0.250 | 0 | $0.15 l_x$ |
| Corner column | -0.500 | 0 | (per m) | +0.500 | -0.500 | (per m) |

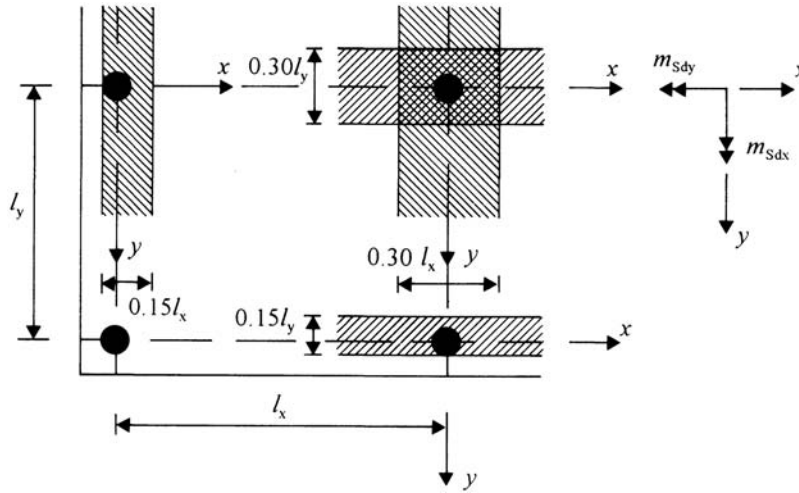
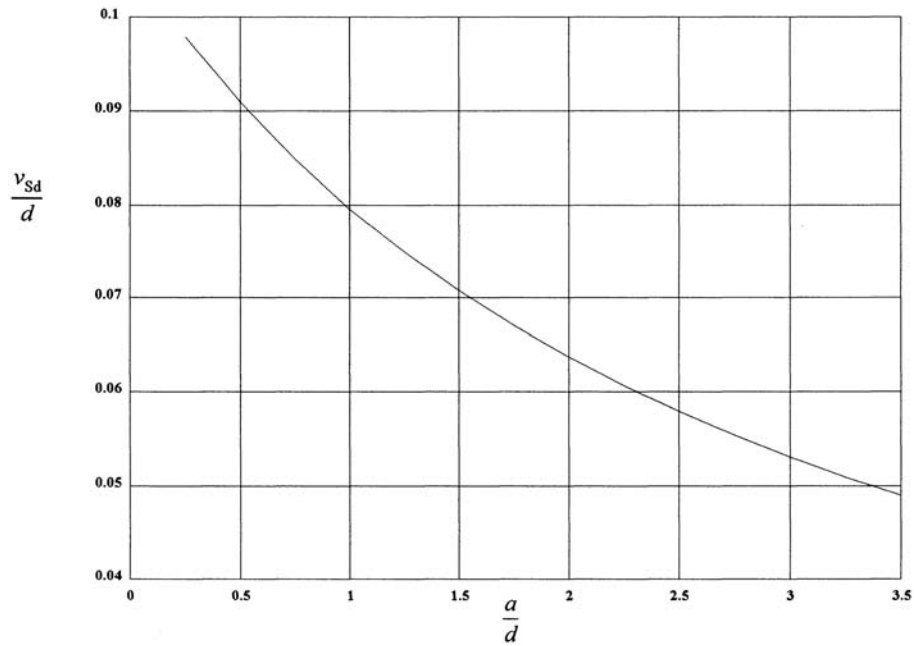


Figure 10.8 Bending moments m_{Sdx} and m_{Sdy} in slab-column joints subjected to eccentric loading, and effective width for resisting these moments.

10.2a
 $\frac{v_{Sd}}{d}$ for circular loaded areas [4.50]

Table 10.2

$\frac{v_{Sd}}{d}$ in N/mm^2 for circular loaded areas



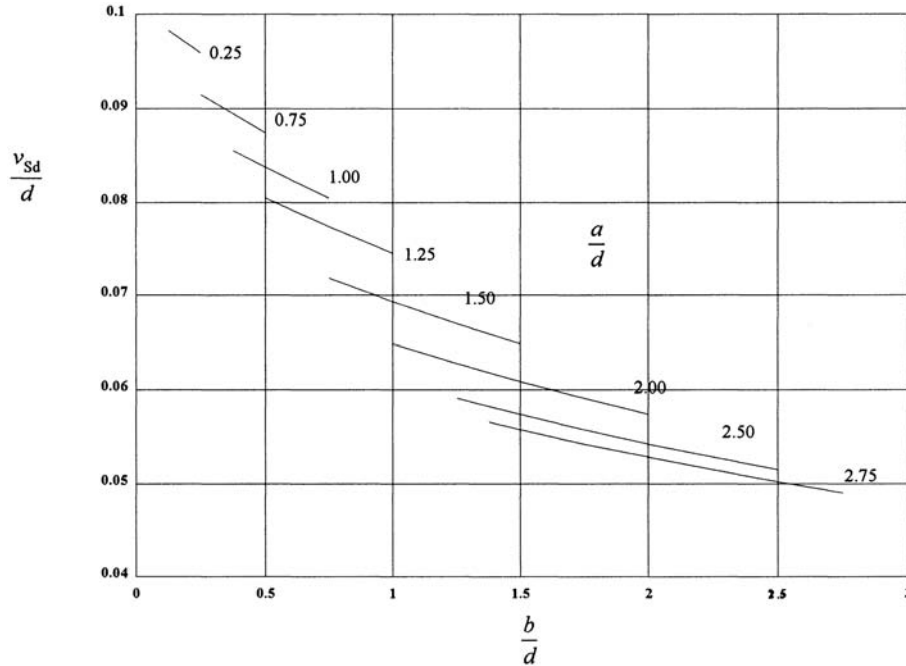
$$\frac{v_{Sd}}{d} = \frac{\frac{V_{Sd}}{d^2} \beta}{\pi \left(\frac{a}{d} + 3 \right)} \text{ with } \frac{V_{Sd}}{d^2} = 1.0 \text{ N/mm}^2 \text{ and } \beta = 1.0$$

- If $\frac{V_{Sd}}{d^2} \neq 1.0 \text{ N/mm}^2$, multiply by $\frac{V_{Sd}}{d^2}$
- If $\beta \neq 1.0$, multiply by β according to Figure 10.7

10.2b
 $\frac{v_{Sd}}{d}$ for rectangular loaded areas [4.50]

Table 10.3

$\frac{v_{Sd}}{d}$ in N/mm² for rectangular loaded areas



$$\frac{v_{Sd}}{d} = \frac{V_{Sd} \beta}{d^2} \cdot \frac{1}{2 \left(\frac{a}{d} + \frac{b}{d} \right) + 3\pi} \quad \text{with } \frac{V_{Sd}}{d^2} = 1.0 \text{ N/mm}^2 \text{ and } \beta = 1.0$$

- If $\frac{V_{Sd}}{d^2} \neq 1.0$ If N/mm², multiply by $\frac{V_{Sd}}{d^2}$
- If $\beta \neq 1.0$, multiply by β according to Figure 10.7

10.3
 $\frac{v_{Rd1}}{d}$ [4.56]

Table 10.4

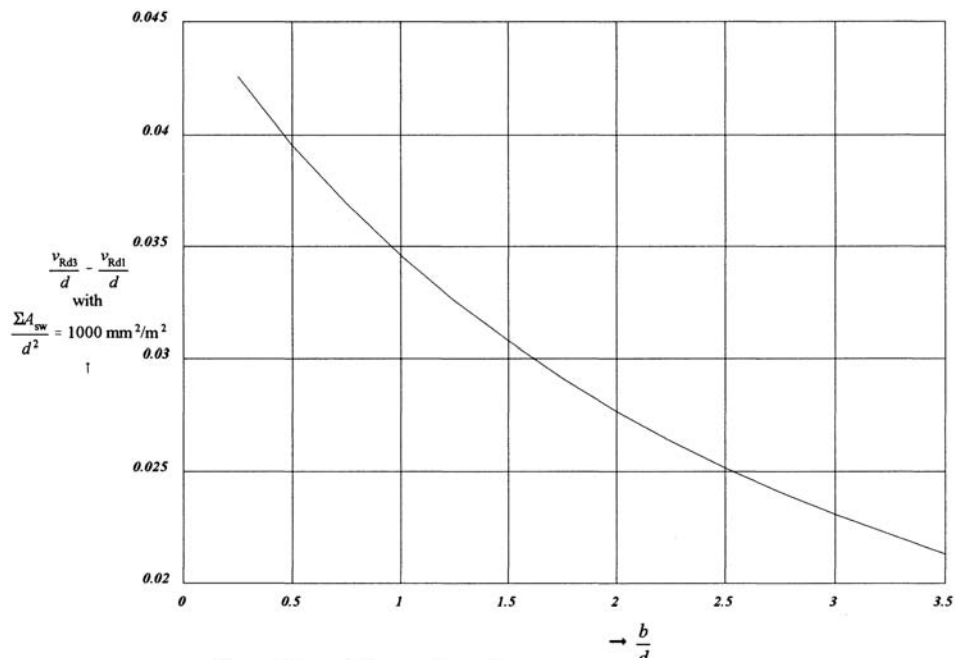
| $100\rho_1$ (%) | $\frac{v_{Rd1}}{d}$ (with $\gamma_c=1.5$ and $k=1.0$) in N/mm ² per concrete class | | | | | | | | |
|--------------------|--|--------|--------|--------|--------|--------|--------|--------|-------|
| C12/15 | C 16/20 | C20/25 | C25/30 | C30/37 | C35/45 | C40/50 | C45/55 | C50/60 | |
| 0.5 | 0.252 | 0.308 | 0.364 | 0.420 | 0.476 | 0.518 | 0.574 | 0.616 | 0.672 |
| 0.6 | 0.259 | 0.317 | 0.374 | 0.432 | 0.490 | 0.533 | 0.590 | 0.634 | 0.691 |
| 0.7 | 0.266 | 0.326 | 0.385 | 0.444 | 0.503 | 0.548 | 0.607 | 0.651 | 0.710 |
| 0.8 | 0.274 | 0.334 | 0.395 | 0.456 | 0.517 | 0.562 | 0.623 | 0.669 | 0.730 |
| 0.9 | 0.281 | 0.343 | 0.406 | 0.468 | 0.530 | 0.577 | 0.640 | 0.686 | 0.749 |
| 1.0 | 0.288 | 0.352 | 0.416 | 0.480 | 0.544 | 0.592 | 0.656 | 0.704 | 0.768 |
| 1.1 | 0.295 | 0.361 | 0.426 | 0.492 | 0.558 | 0.607 | 0.672 | 0.722 | 0.787 |
| 1.2 | 0.302 | 0.370 | 0.437 | 0.504 | 0.571 | 0.622 | 0.689 | 0.739 | 0.806 |
| 1.3 | 0.310 | 0.378 | 0.447 | 0.516 | 0.585 | 0.636 | 0.705 | 0.757 | 0.826 |
| 1.4 | 0.317 | 0.387 | 0.458 | 0.528 | 0.598 | 0.651 | 0.722 | 0.774 | 0.845 |
| 1.5 | 0.324 | 0.396 | 0.468 | 0.540 | 0.612 | 0.666 | 0.738 | 0.792 | 0.864 |

| 100ρ ₁ (%) | $\frac{v_{Rd1}}{d}$ (with γ _c =1.5 and k=1.0) in N/mm ² per concrete class | | | | | | | | |
|---|--|--------|--------|--------|--------|--------|--------|--------|------|
| C12/15 | C 16/20 | C20/25 | C25/30 | C30/37 | C35/45 | C40/50 | C45/55 | C50/60 | |
| $\frac{v_{Rd1}}{d} = \tau_{Rd} k (1.2 + 40\rho_1)$ (with γ _c = 1.5 and k = 1.0) in N/mm ² | | | | | | | | | |
| τ _{Rd} (N/mm ²) | 0.18 | 0.22 | 0.26 | 0.30 | 0.34 | 0.37 | 0.41 | 0.44 | 0.48 |

- If γ_c 1.5, multiply by 1.5/γ_c
- If d < 0.6 m, multiply by $k = 1.6 - d$ where d is in metres
- If 100ρ₁ < 0.5%, with ρ₁ = √ρ_{1x} + ρ_{1y}, apply 100ρ₁ = 0.5%
- If 100ρ₁ > 1.5 %, with ρ₁ = √ρ_{1x} + ρ_{1y}, take 100ρ₁ = 1.5% into account

10.4a
 $\frac{v_{Rd3}}{d} - \frac{v_{Rd1}}{d}$ circular loaded areas [4.58]

Table 10.5
 $\frac{v_{Rd3}}{d} - \frac{v_{Rd1}}{d}$ in N/mm² for circular loaded areas



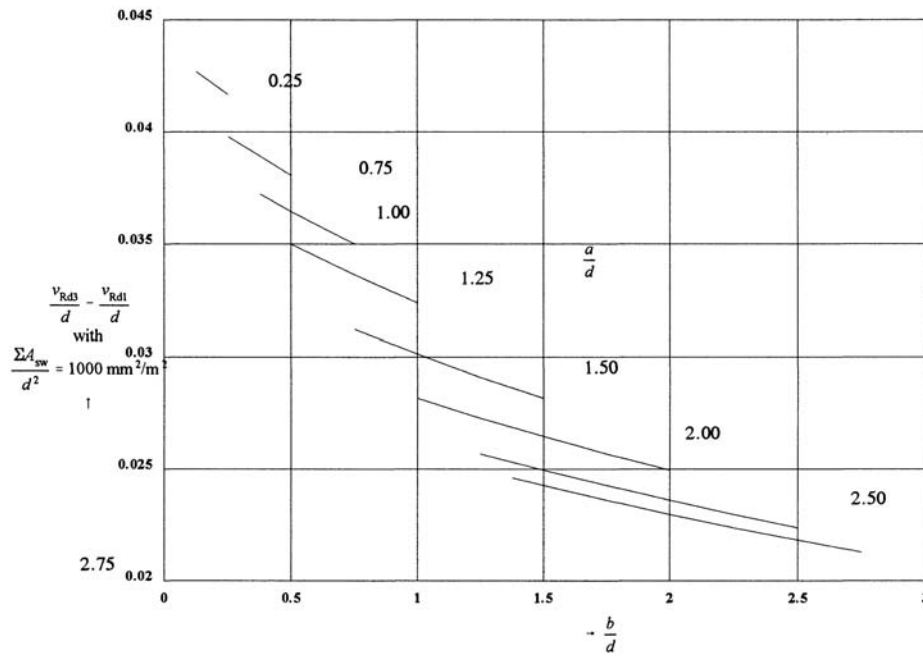
$$\frac{v_{Rd3}}{d} - \frac{v_{Rd1}}{d} = \frac{\Sigma A_{sw}}{d^2} \frac{1}{\pi \left(\frac{a}{d} + 3 \right)} \frac{f_{yw} \sin \alpha}{\gamma_s}$$

(with $\frac{\Sigma A_{sw}}{d^2} = 1000 \text{ mm}^2/\text{m}^2$; γ = 1.15; f_{ywk} = 500 N/mm²; α = 90°)

- If $\frac{\Sigma A_{sw}}{d^2} \neq 1000$ mm²/m², multiply by $\frac{\Sigma A_{sw}}{1000 d^2}$
- If γ_s 1.15, multiply by 1.15/γ_s
- If f_{ywk} 500, multiply by f_{ywk}/500
- If α 90°, multiply by sin α

10.4b
 $\frac{v_{Rd3}}{d} - \frac{v_{Rd1}}{d}$ rectangular loaded areas [4.58]

Table 10.6
 $\frac{v_{Rd3}}{d} - \frac{v_{Rd1}}{d}$ in N/mm² for rectangular loaded areas



$$\frac{V_{Rd3}}{d} - \frac{V_{Rd1}}{d} = \frac{\Sigma A_{sw}}{d^2} \frac{1}{2 \left(\frac{a}{d} + \frac{b}{d} \right) + 3\pi} \frac{f_{ywk}}{\gamma_s} \sin \alpha$$

(with $\frac{\Sigma A_{sw}}{d^2} \neq 1000 \text{ mm}^2/\text{m}^2$; $\gamma_s = 1.15$; $f_{ywk} = 500 \text{ N/mm}^2$ and $\alpha = 90^\circ$)

- If $\frac{\Sigma A_{sw}}{d^2} = 1000 \text{ mm}^2/\text{m}^2$, multiply by $\frac{A_{sw}}{1000 d^2}$
- If $\gamma_s = 1.15$, multiply by $1.15/\gamma_s$
- If $f_{ywk} = 500$, multiply by $f_{ywk}/500$
- If $\alpha = 90^\circ$, multiply by $\sin \alpha$

11

Elements with second order effects

11.1 Determination of effective length of columns

The effective length of a column depends on the stiffness of the column relative to the stiffness of the structure connected to either end of the column. The effective length may be estimated from the relation:

$$l_{\text{eff}} = \beta l_{\text{col}}$$

where β may be obtained from Figure 11.1.

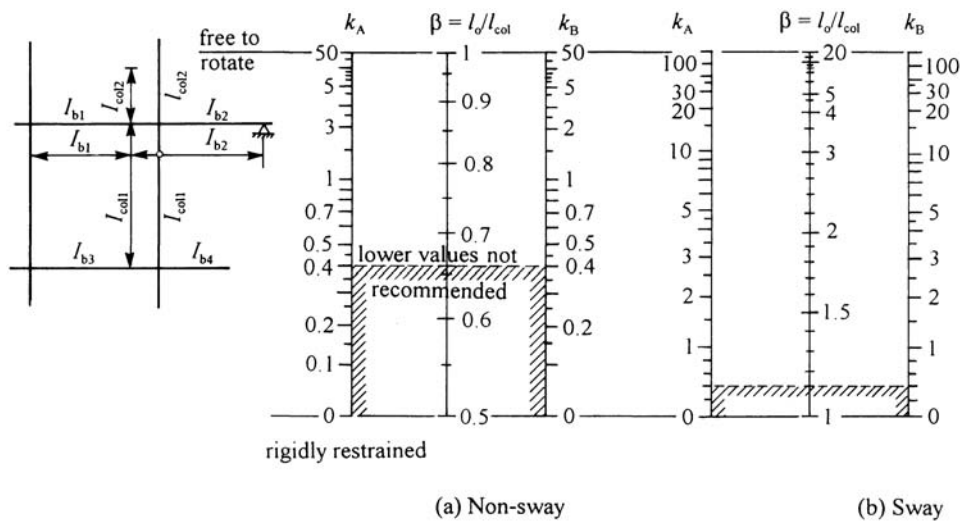


Figure 11.1: Nomogram for assessing effective lengths.

coefficients K_A and K_B denote the rigidity of restraint at the column ends:

$$K = \frac{\sum E_{cm} I_{col} / l_{col}}{\sum E_{cm} \alpha I_b / l_{\text{eff}}}$$

where

- E_{cm} = modulus of elasticity of the concrete
- I_{col}, I_b = moment of inertia (gross section) of the column or beam respectively
- l_{col} = height of the column measured between centres of restraint
- l_{eff} = effective span of the beam
- α = factor taking into account the conditions of restraint of the beam at the opposite end:
 - = 1.0 opposite end elastically or rigidly restrained
 - = 0.5 opposite end free to rotate
 - = 0 for a cantilever beam

Alternatively, for columns in braced frames, the effective height for framed structures may be taken as the lesser of:

$$l_e = l_o[0.7 + 0.05(\alpha_{c,1} + \alpha_{c,2})] < l_o$$

$$l_e = l_o[0.85 + 0.05\alpha_{c,min}] < l_o$$

The effective height for unbraced framed structures may be taken as the lesser of:

$$l_e = l_o[1.0 + 0.15(\alpha_{c,1} + \alpha_{c,2})]$$

$$l_e = l_o[2.0 + 0.3\alpha_{c,min}]$$

where

- l_e = effective height of a column in the plane of bending considered
- l_o = height between end restraints
- $\alpha_{c,1}$ = ratio of the sum of the column stiffnesses to the sum of the beam stiffnesses at the lower end of a column
- $\alpha_{c,2}$ = ratio of the sum of the column stiffnesses to the sum of the beam stiffnesses at the upper end of a column
- $\alpha_{c,min}$ = lesser of $\alpha_{c,1}$ and $\alpha_{c,2}$

Where creep may significantly affect the performance of a member (e.g. where members are not well restrained at the ends by monolithic connections), this can be allowed for by increasing the effective length by a factor:

$$\left(1 + \frac{M_{qp}}{M_{sd}}\right)^{1/2}$$

where

- M_{qp} is the moment under the quasi-permanent load
- M_{sd} is the design first order moment.

Table 11.1 Simplified assessment of β for non-sway frames

- (A) Assess K for each end of column using the following method:
- (i) $K=0.5$
 - (ii) If there is a column continuing beyond the joint, $K=K*2$
 - (iii) If there is a beam on only one side of the joint, $K=K*2$
 - (iv) If the span of the beam is more than twice the height of the columns, $K=K*1.5$
 - (v) If the beams or slabs framing into the column are shallower than the column dimension, $K=K*2$
 - (vi) If the joint nominally carries no moment (e.g. connection with a pad footing), $K=10$
- (B) Obtain β from the following:

| K for lower joint | K for upper joint | | | | | | | PIN |
|--------------------------|--------------------------|------|------|------|------|------|------|------|
| | 0.5 | 0.75 | 1.0 | 1.5 | 2 | 3 | 10 | |
| 0.5 | 0.69 | 0.70 | 0.74 | 0.75 | 0.77 | 0.8 | 0.81 | 0.84 |
| 0.75 | 0.70 | 0.74 | 0.75 | 0.77 | 0.80 | 0.81 | 0.84 | 0.85 |
| 1.0 | 0.74 | 0.75 | 0.77 | 0.80 | 0.81 | 0.84 | 0.85 | 0.86 |
| 1.5 | 0.75 | 0.77 | 0.80 | 0.81 | 0.84 | 0.85 | 0.86 | 0.90 |
| 2 | 0.77 | 0.80 | 0.81 | 0.84 | 0.85 | 0.86 | 0.90 | 0.92 |
| 3 | 0.80 | 0.81 | 0.84 | 0.85 | 0.86 | 0.90 | 0.92 | 0.95 |
| 10 | 0.81 | 0.84 | 0.85 | 0.86 | 0.90 | 0.92 | 0.95 | 0.98 |
| PIN | 0.84 | 0.85 | 0.86 | 0.90 | 0.92 | 0.95 | 0.98 | 1.00 |

Table 11.2 Model column method for isolated non-sway columns

- If $\lambda = \lambda_{crit}$, $N_{Rd} = N_{sd}$
 $M_{Rd} = N_{sd}h/20$
- If $\lambda > \lambda_{crit}$, $N_{Rd} = N_{sd}$
 $M_{Rd} = N_{sd}e_{tot} < N_{sd}h/20$
 $< N_{sd}e_{0,2}$
- e_{tot} = $e_o + e_a + e_2$
- e_o = first order eccentricity
= $0.4e_{0,1} + 0.6e_{0,2} < 0.4e_{0,2}$
- e_a = accidental eccentricity

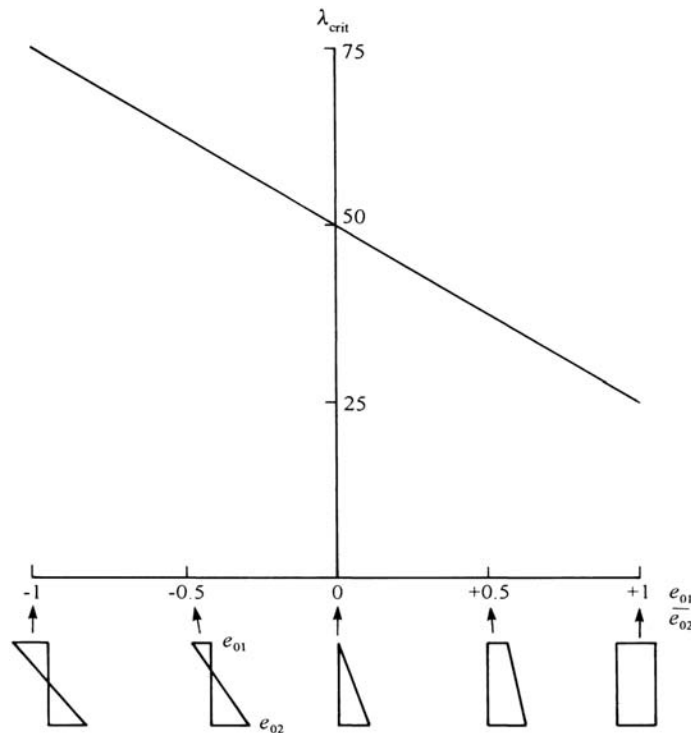


Figure 11.2 Critical slenderness ratio for isolated column.

$$v = \frac{\alpha_n}{\sqrt{100l}} \geq \frac{l}{200} = vl_0/2$$

$$\alpha_n = \sqrt{(1 + l/n)/2}$$

l = total height of structure in metres
 n = number of vertical elements acting together
 e_2 = second order eccentricity

$$= \frac{k_1 k_2 l_o^2 f_{yd}}{90,000}$$

$$k_1 = \frac{\lambda}{20} - 0.75 \text{ for } 15 < \lambda \leq 35$$

$$= 1 \text{ for } \lambda > 35$$

$$k_2 = (N_{ud} - N_{sd}) / (N_{ud} - N_{bal}) \leq 1$$

Table 11.3 Detailing requirements for columns (EC2 Clause 5.4.1)

Minimum dimensions:

- 200 mm vertical columns, cast in-situ
- 140 mm precast columns cast horizontally

Minimum area of longitudinal reinforcement

$$A_{s,min} = \frac{0.15N_{sd}}{f_{yd}} < 0.003A_c$$

Maximum area of longitudinal reinforcement

$$A_{s,max} = 0.08A_c$$

Transverse reinforcement:

Minimum diameter of links: $\frac{\text{maximum longitudinal bar size}}{4} < 6 \text{ mm}$

Maximum spacing: the smallest of:

- 12 times minimum diameter of longitudinal bars
- the least dimension of the column
- 300mm

The resulting maximum spacing should be multiplied by 0.6

- in sections immediately above or below a beam or slab over a height equal to the larger dimension of the column
 - near lapped joints where the size of the longitudinal bars exceed 14 mm
-

12

Control of cracking

It should be clearly understood that there are many causes of cracking and that only certain of these lead to cracks that will be controlled by the provisions of chapter 4.4.2 of EC2. Chapter 4.4.2 is concerned with cracks that form in hardened concrete either from restrained imposed deformations, such as shrinkage or early thermal movements, or from the effects of loads.

The fundamental principle behind the provisions of the code is as follows. Crack control is only possible where spread cracking can occur (i.e. the tensile strain is accommodated in multiple cracks, or a crack accommodates only tensile strains that arise near the crack). For this to occur, there must be sufficient reinforcement in the section to ensure that the reinforcement does not yield on first cracking. The rules for minimum reinforcement areas in 4.4.2.2 are aimed at ensuring that this requirement is met. Provided this minimum is present, crack widths can normally be controlled by simple detailing rules.

Table 12.1 Minimum areas of reinforcement

$$A_s \geq K_c K f_{ct,eff} A_c / \sigma_s$$

where:

A_c = the area of concrete in tension immediately before the formation of the first crack

$f_{ct,eff}$ = the tensile strength of the concrete effective at the time when the cracks first form. Except where the cracks can be guaranteed to form at an early age, it is suggested that the value chosen should not be less than 3 N/mm²

σ_s = the stress in the reinforcement, which may be taken as the yield strength of the reinforcement

K = a coefficient that takes account of the effects of non-linear stress distribution. See Table 12.2 for values for K

K_c = a coefficient taking account of the form of loading causing the cracks. See Table 12.2 for values of K_c

Table 12.2 Values of K and K_c

- (1) Values of K :
 - (a) Extrinsic, or external deformations imposed on a member: $K=1.0$
 - (b) Internal deformations (e.g. restrained shrinkage or temperature change):
 - for members with least dimension 300 $K=0.8$
 - for members with least dimension 800 $K=0.5$Interpolation may be used between these values
 - (2) Values of K_c :
 - (a) Pure tension $K_c=1.0$
 - (b) Pure flexure: $K_c=0.4$
 - (c) Section in compression with zero stress at least compressed fibre (under rare load combination) $K_c=0$
 - (d) Sections where the neutral axis depth calculated on the basis of a cracked section under the cracking load is less than the lesser of $h/2$ or 500 mm: $K_c=0$
 - (e) Box sections
 - Webs: $K_c=0.4$
 - Tension chords: $K_c=0.8$
 - (f) Parts of sections in tension distant from main reinforcement
 - $0.5 < K_c < 1.0$
-

To help interpolation between (a), (b), (c), Fig. 12.1 may be used. Checking the crack width requires (a) crack width criteria and (b) an estimate of the stress in the reinforcement under the quasi-permanent load. The criteria are given in Table 12.3.

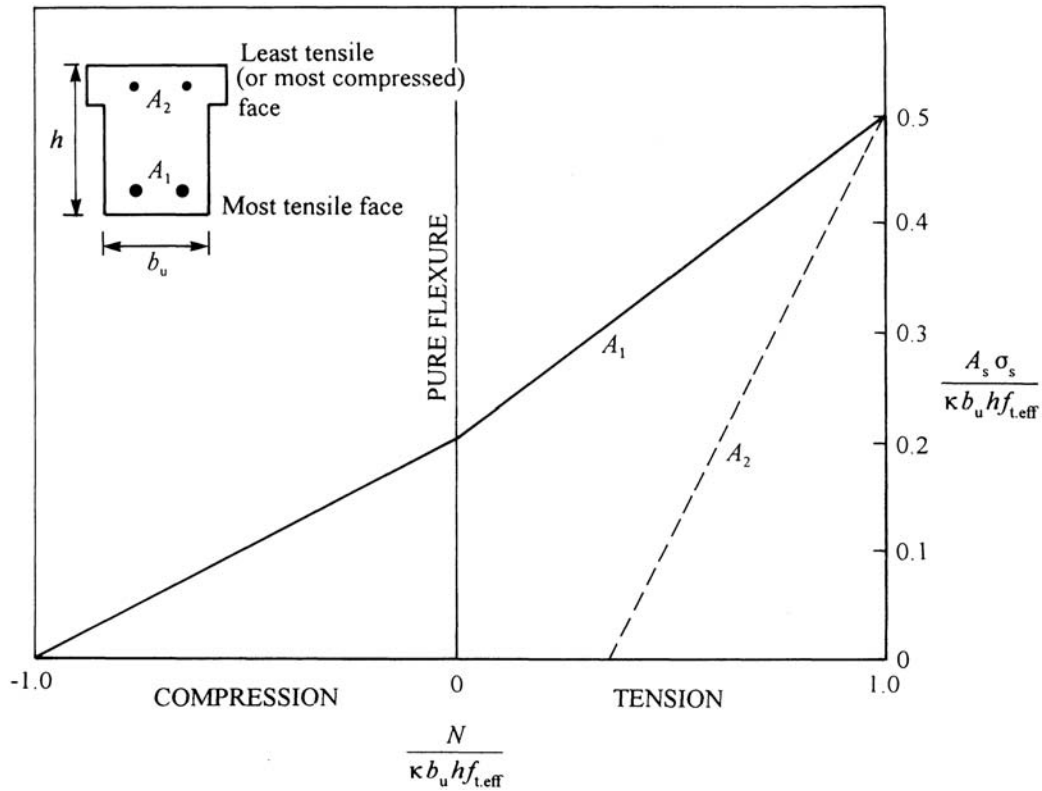
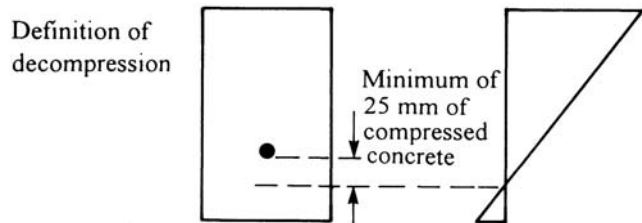


Figure 12.1 Calculation of minimum reinforcement areas.

Table 12.3: Crack width criteria

- (1) Reinforced concrete: 0.3 mm. If acceptable, a greater value may be used in exposure Class 1.
- (2) Prestressed members.

| Exposure class | Design crack width, w_k , under the frequent load combination (mm) |
|----------------|--|
| | Post-tensioned Pre-tensioned |
| 1 | 0.2 0.2 |
| 2 | 0.2 Decompression |
| 3 | Decompression or coating of the tendons and $w_k=0.2$ |
| 4 | |



The steel stress may be calculated on the basis of a cracked section under the quasi-permanent load. Creep may be allowed for by taking the modular ratio as 15. Table 12.4 and Figures 12.2 and 12.3 may be used to estimate the properties of a cracked section.

Alternatively, an approximate estimate of the stress may be obtained for reinforced concrete using the formula:

$$\sigma_s = \frac{f_{yk} \delta}{\gamma_s} \frac{M_{qp}}{M_{sd}} \frac{A_{s,req}}{A_{s,prov}}$$

where

δ = the ratio of the design ultimate moment after redistribution to the elastically calculated value under the ultimate loads

γ_s = the partial safety factor on the reinforcement (i.e. 1.15)

M_{qp} = the moment under the quasi-permanent load

M_{sd} = the design ultimate load

$A_{s,req}$ = the reinforcement area required for the ultimate limit state

$A_{s,prov}$ = the area of tension reinforcement provided

M_{qp}/M_{sd} may be taken approximately as N_{qp}/N_{sd} where N_{qp} , and N_{ad} are, respectively, the quasi-permanent and design ultimate loads on the member.

Where the stress in the reinforcement is dominantly due to imposed deformations, the value of σ_s used in Table 12.1 should be adopted.

Crack control may be achieved either by satisfying the provisions of either Table 12.5 or Table 12.6 or by direct calculation of crack widths. This is covered in Table 12.7.

Table 12.4 Neutral axis depths and moments of inertia for flanged beams (a) with $hf/d=0.2$

| ap | $br/b=1$ | | $br/b=0.5$ | | $br/b=0.4$ | | $br/b=0.3$ | | $br/b=0.2$ | |
|------|----------|----------|------------|----------|------------|----------|------------|----------|------------|----------|
| | x/d | I/bd^3 | x/d | I/bd^3 | x/d | I/bd^3 | x/d | I/bd^3 | x/d | I/bd^3 |
| 0.02 | 0.181 | 0.015 | | | | | | | | |
| 0.03 | 0.217 | 0.022 | 0.217 | 0.022 | 0.217 | 0.022 | 0.217 | 0.022 | 0.217 | 0.022 |
| 0.04 | 0.246 | 0.028 | 0.248 | 0.028 | 0.248 | 0.028 | 0.249 | 0.028 | 0.249 | 0.028 |
| 0.05 | 0.270 | 0.033 | 0.274 | 0.033 | 0.275 | 0.033 | 0.276 | 0.033 | 0.278 | 0.033 |
| 0.06 | 0.292 | 0.038 | 0.298 | 0.038 | 0.300 | 0.038 | 0.302 | 0.038 | 0.304 | 0.038 |
| 0.07 | 0.311 | 0.043 | 0.320 | 0.043 | 0.322 | 0.043 | 0.325 | 0.043 | 0.327 | 0.043 |
| 0.08 | 0.328 | 0.048 | 0.340 | 0.047 | 0.343 | 0.047 | 0.346 | 0.047 | 0.349 | 0.047 |
| 0.09 | 0.344 | 0.052 | 0.358 | 0.052 | 0.361 | 0.052 | 0.365 | 0.051 | 0.369 | 0.051 |
| 0.10 | 0.358 | 0.057 | 0.375 | 0.056 | 0.379 | 0.056 | 0.383 | 0.055 | 0.388 | 0.055 |
| 0.11 | 0.372 | 0.061 | 0.390 | 0.060 | 0.395 | 0.059 | 0.400 | 0.059 | 0.406 | 0.059 |
| 0.12 | 0.384 | 0.064 | 0.405 | 0.063 | 0.410 | 0.063 | 0.416 | 0.063 | 0.422 | 0.062 |
| 0.13 | 0.396 | 0.068 | 0.418 | 0.067 | 0.424 | 0.066 | 0.430 | 0.066 | 0.437 | 0.065 |
| 0.14 | 0.407 | 0.072 | 0.431 | 0.070 | 0.437 | 0.070 | 0.444 | 0.069 | 0.452 | 0.069 |
| 0.15 | 0.418 | 0.075 | 0.443 | 0.073 | 0.450 | 0.073 | 0.457 | 0.072 | 0.466 | 0.071 |
| 0.16 | 0.428 | 0.078 | 0.455 | 0.076 | 0.462 | 0.076 | 0.470 | 0.075 | 0.478 | 0.074 |
| 0.17 | 0.437 | 0.082 | 0.466 | 0.079 | 0.473 | 0.078 | 0.481 | 0.078 | 0.491 | 0.077 |
| 0.18 | 0.446 | 0.085 | 0.476 | 0.082 | 0.484 | 0.081 | 0.493 | 0.080 | 0.502 | 0.079 |
| 0.19 | 0.455 | 0.088 | 0.486 | 0.085 | 0.494 | 0.084 | 0.503 | 0.083 | 0.513 | 0.082 |
| 0.20 | 0.463 | 0.091 | 0.495 | 0.087 | 0.504 | 0.086 | 0.513 | 0.085 | 0.524 | 0.084 |
| 0.21 | 0.471 | 0.094 | 0.504 | 0.090 | 0.513 | 0.089 | 0.523 | 0.088 | 0.534 | 0.086 |
| 0.22 | 0.479 | 0.096 | 0.513 | 0.092 | 0.522 | 0.091 | 0.532 | 0.090 | 0.543 | 0.089 |
| 0.23 | 0.486 | 0.099 | 0.521 | 0.094 | 0.531 | 0.093 | 0.541 | 0.092 | 0.552 | 0.091 |
| 0.24 | 0.493 | 0.102 | 0.529 | 0.097 | 0.539 | 0.095 | 0.549 | 0.094 | 0.561 | 0.093 |

Table 12.4 Neutral axis depths and moments of inertia for flanged beams (b) with $hf/d=0.3$

| ap | $br/b=1$ | | $br/b=0.5$ | | $br/b=0.4$ | | $br/b=0.3$ | | $br/b=0.2$ | |
|------|----------|----------|------------|----------|------------|----------|------------|----------|------------|----------|
| | x/d | I/bd^3 | x/d | I/bd^3 | x/d | I/bd^3 | x/d | I/bd^3 | x/d | I/bd^3 |
| 0.02 | 0.181 | 0.015 | | | | | | | | |
| 0.03 | 0.217 | 0.022 | | | | | | | | |
| 0.04 | 0.246 | 0.028 | | | | | | | | |
| 0.05 | 0.270 | 0.033 | | | | | | | | |
| 0.06 | 0.292 | 0.038 | | | | | | | | |
| 0.07 | 0.311 | 0.043 | 0.311 | 0.043 | 0.311 | 0.043 | 0.311 | 0.043 | 0.311 | 0.043 |
| 0.08 | 0.328 | 0.048 | 0.328 | 0.048 | 0.329 | 0.048 | 0.329 | 0.048 | 0.329 | 0.048 |

| αp | $br/b=1$ | | $br/b=0.5$ | | $br/b=0.4$ | | $br/b=0.3$ | | $br/b=0.2$ | |
|------------|----------|----------|------------|----------|------------|----------|------------|----------|------------|----------|
| | x/d | I/bd^3 | x/d | I/bd^3 | x/d | I/bd^3 | x/d | I/bd^3 | x/d | I/bd^3 |
| 0.09 | 0.344 | 0.052 | 0.345 | 0.052 | 0.345 | 0.052 | 0.345 | 0.052 | 0.346 | 0.052 |
| 0.10 | 0.358 | 0.057 | 0.360 | 0.056 | 0.361 | 0.056 | 0.361 | 0.056 | 0.362 | 0.056 |
| 0.11 | 0.372 | 0.061 | 0.375 | 0.060 | 0.375 | 0.060 | 0.376 | 0.060 | 0.377 | 0.060 |
| 0.12 | 0.384 | 0.064 | 0.388 | 0.064 | 0.389 | 0.064 | 0.390 | 0.064 | 0.391 | 0.064 |
| 0.13 | 0.396 | 0.068 | 0.401 | 0.068 | 0.402 | 0.068 | 0.403 | 0.068 | 0.404 | 0.068 |
| 0.14 | 0.407 | 0.072 | 0.413 | 0.071 | 0.414 | 0.071 | 0.416 | 0.071 | 0.417 | 0.071 |
| 0.15 | 0.418 | 0.075 | 0.425 | 0.075 | 0.426 | 0.075 | 0.428 | 0.075 | 0.430 | 0.075 |
| 0.16 | 0.428 | 0.078 | 0.436 | 0.078 | 0.437 | 0.078 | 0.439 | 0.078 | 0.441 | 0.078 |
| 0.17 | 0.437 | 0.082 | 0.446 | 0.081 | 0.448 | 0.081 | 0.450 | 0.081 | 0.452 | 0.081 |
| 0.18 | 0.446 | 0.085 | 0.456 | 0.084 | 0.458 | 0.084 | 0.461 | 0.084 | 0.463 | 0.084 |
| 0.19 | 0.455 | 0.088 | 0.466 | 0.087 | 0.468 | 0.087 | 0.471 | 0.087 | 0.473 | 0.087 |
| 0.20 | 0.463 | 0.091 | 0.475 | 0.090 | 0.477 | 0.090 | 0.480 | 0.090 | 0.483 | 0.089 |
| 0.21 | 0.471 | 0.094 | 0.483 | 0.093 | 0.486 | 0.092 | 0.489 | 0.092 | 0.493 | 0.092 |
| 0.22 | 0.479 | 0.096 | 0.492 | 0.095 | 0.495 | 0.095 | 0.498 | 0.095 | 0.502 | 0.095 |
| 0.23 | 0.486 | 0.099 | 0.500 | 0.098 | 0.503 | 0.098 | 0.507 | 0.097 | 0.511 | 0.097 |
| 0.24 | 0.493 | 0.102 | 0.508 | 0.100 | 0.511 | 0.100 | 0.515 | 0.100 | 0.519 | 0.099 |

Table 12.4 Neutral axis depths and moments of inertia for flanged beams (c) with $hf/d=0.4$

| αp | $br/b=1$ | | $br/b=0.5$ | | $br/b=0.4$ | | $br/b=0.3$ | | $br/b=0.2$ | |
|------------|----------|----------|------------|----------|------------|----------|------------|----------|------------|----------|
| | x/d | I/bd^3 | x/d | I/bd^3 | x/d | I/bd^3 | x/d | I/bd^3 | x/d | I/bd^3 |
| 0.02 | 0.181 | 0.015 | | | | | | | | |
| 0.03 | 0.217 | 0.022 | | | | | | | | |
| 0.04 | 0.246 | 0.028 | | | | | | | | |
| 0.05 | 0.270 | 0.033 | | | | | | | | |
| 0.06 | 0.292 | 0.038 | | | | | | | | |
| 0.07 | 0.311 | 0.043 | | | | | | | | |
| 0.08 | 0.328 | 0.048 | | | | | | | | |
| 0.09 | 0.344 | 0.052 | | | | | | | | |
| 0.10 | 0.358 | 0.057 | | | | | | | | |
| 0.11 | 0.372 | 0.061 | | | | | | | | |
| 0.12 | 0.384 | 0.064 | | | | | | | | |
| 0.13 | 0.396 | 0.068 | | | | | | | | |
| 0.14 | 0.407 | 0.072 | 0.407 | 0.072 | 0.407 | 0.072 | 0.407 | 0.072 | 0.407 | 0.072 |
| 0.15 | 0.418 | 0.075 | 0.418 | 0.075 | 0.418 | 0.075 | 0.418 | 0.075 | 0.418 | 0.075 |
| 0.16 | 0.428 | 0.078 | 0.428 | 0.078 | 0.428 | 0.078 | 0.428 | 0.078 | 0.428 | 0.078 |
| 0.17 | 0.437 | 0.082 | 0.438 | 0.082 | 0.438 | 0.082 | 0.438 | 0.082 | 0.438 | 0.082 |
| 0.18 | 0.446 | 0.085 | 0.447 | 0.085 | 0.447 | 0.085 | 0.448 | 0.085 | 0.448 | 0.085 |
| 0.19 | 0.455 | 0.088 | 0.456 | 0.088 | 0.457 | 0.088 | 0.457 | 0.088 | 0.457 | 0.088 |
| 0.20 | 0.463 | 0.091 | 0.465 | 0.091 | 0.465 | 0.091 | 0.466 | 0.091 | 0.466 | 0.091 |
| 0.21 | 0.471 | 0.094 | 0.473 | 0.094 | 0.474 | 0.094 | 0.474 | 0.094 | 0.474 | 0.093 |
| 0.22 | 0.479 | 0.096 | 0.481 | 0.096 | 0.482 | 0.096 | 0.482 | 0.096 | 0.483 | 0.096 |
| 0.23 | 0.486 | 0.099 | 0.489 | 0.099 | 0.490 | 0.099 | 0.490 | 0.099 | 0.491 | 0.099 |
| 0.24 | 0.493 | 0.102 | 0.496 | 0.101 | 0.497 | 0.101 | 0.498 | 0.101 | 0.498 | 0.101 |

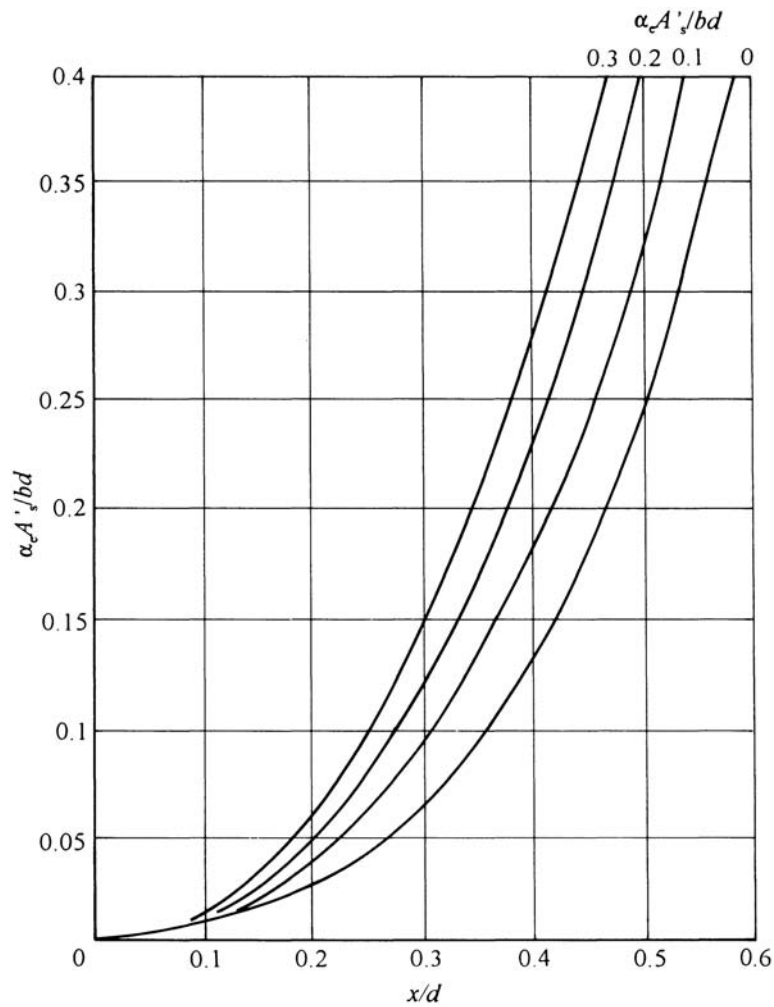


Figure 12.2 Neutral axis depths for rectangular sections.

Table 12.5 Maximum bar diameters

| Steel stress (MPa) | Maximum bar size (mm) | |
|---------------------|-----------------------|----|
| Reinforced sections | Prestressed sections | |
| 160 | 32 | 25 |
| 200 | 25 | 16 |
| 240 | 20 | 12 |
| 280 | 16 | 8 |
| 320 | 12 | 6 |
| 360 | 10 | 5 |
| 400 | 8 | 4 |
| 450 | 6 | |

For reinforced concrete, the maximum bar diameter may be modified as follows

$$\sigma_s = \sigma_s^* f_{ctm} / (2.5) h / [10(h - d)] \geq \sigma_s^* (f_{ctm} / 2.5) \text{ for restraint cracking}$$

$$\sigma_s = \sigma_s^* \frac{h}{10(h - d)} \geq \sigma_s^* \text{ for load-induced cracking}$$

where:

| | | |
|--------------|---|--|
| σ_s | = | the adjusted maximum bar diameter |
| σ_s^* | = | the maximum bar size in Table 12.5 |
| h | = | the overall depth of the section |

Table 12.6 Maximum bar spacings for high bond bars

| Steel stress (MPa) | Maximum bar spacing (mm) | | |
|--------------------|--------------------------|--------------------------------|-----|
| | Pure tension | Prestressed sections (bending) | |
| Pure flexure | | | |
| 160 | 300 | 200 | 200 |
| 200 | 250 | 150 | 150 |
| 240 | 200 | 125 | 100 |
| 280 | 150 | 75 | 50 |
| 320 | 100 | - | - |
| 360 | 50 | - | - |

Table 12.7 Crack width $w = \beta \left(50 + 0.25 k_1 k_2 \left(\frac{\sigma}{\rho_r} \right) \right) \epsilon_{sm}$

Design crack width,

- β = coefficient relating the maximum crack spacing to the average value.
= 1.7 for load-induced cracking and for restraint cracking in members with a minimum dimension greater than 800 mm
= 1.3 for sections with a minimum dimension less than 300 mm. Intermediate values may be interpolated
- σ = bar size in mm. For a mixture of bar sizes in a section, take the average
- k_1 = a coefficient that takes account of the bond properties of the bars; $k_1=0.8$ for high bond bars and 1.6 for plain bars. In the case of imposed deformations, k_1 should be replaced by $k_1 k$, with k being in accordance with Table 12.2
- k_2 = a coefficient that takes account of the form of the strain distribution
= 0.5 for bending and 1.0 for pure tension
For cases of eccentric tension or for local areas, intermediate values of k_2 should be used which can be calculated from the relation:

$$k_2 = \frac{\epsilon_1 + \epsilon_2}{2\epsilon_1}$$

where ϵ_1 is the greater and ϵ_2 the lesser tensile strain at the boundaries of the section considered, assessed on the basis of a cracked section

- ρ_r = the effective reinforcement ratio, $A_s/A_{c,eff}$, where A_c is the area of reinforcement contained within the effective tension area $A_{c,eff}$

The effective tension area is generally the area of concrete surrounding the tension reinforcement of depth equal to 2.5 times the distance from the tension face of the section to the centroid of the reinforcement (see Figure 12.4). For slabs,

ϵ_{sm} is the mean strain allowing for the effects of tension stiffening, shrinkage, etc. under the relevant load combinations, and may be calculated from the relation:

$$\epsilon_{sm} = \frac{\sigma_s}{E_s} \left[1 - \beta_1 \beta_2 \left(\frac{\sigma_{sr}}{\sigma_s} \right)^2 \right]$$

where

- σ_s = the stress in the tension reinforcement calculated on the basis of a cracked section
- σ_{sr} = the stress in the tension reinforcement calculated on the basis of a cracked section under the loading conditions causing first cracking
- β_1 = a coefficient that takes account of the bond properties of the bars
= 1.0 for high bond bars
= 0.5 for plain bars
- β_2 = a coefficient that takes account of the duration of the loading or of repeated loading
= 1.0 for a single, short-term load
= 0.5 for a sustained load or for many cycles of repeated loading

For members subjected only to intrinsic imposed deformations, σ_s may be taken as equal to σ_{sr} .

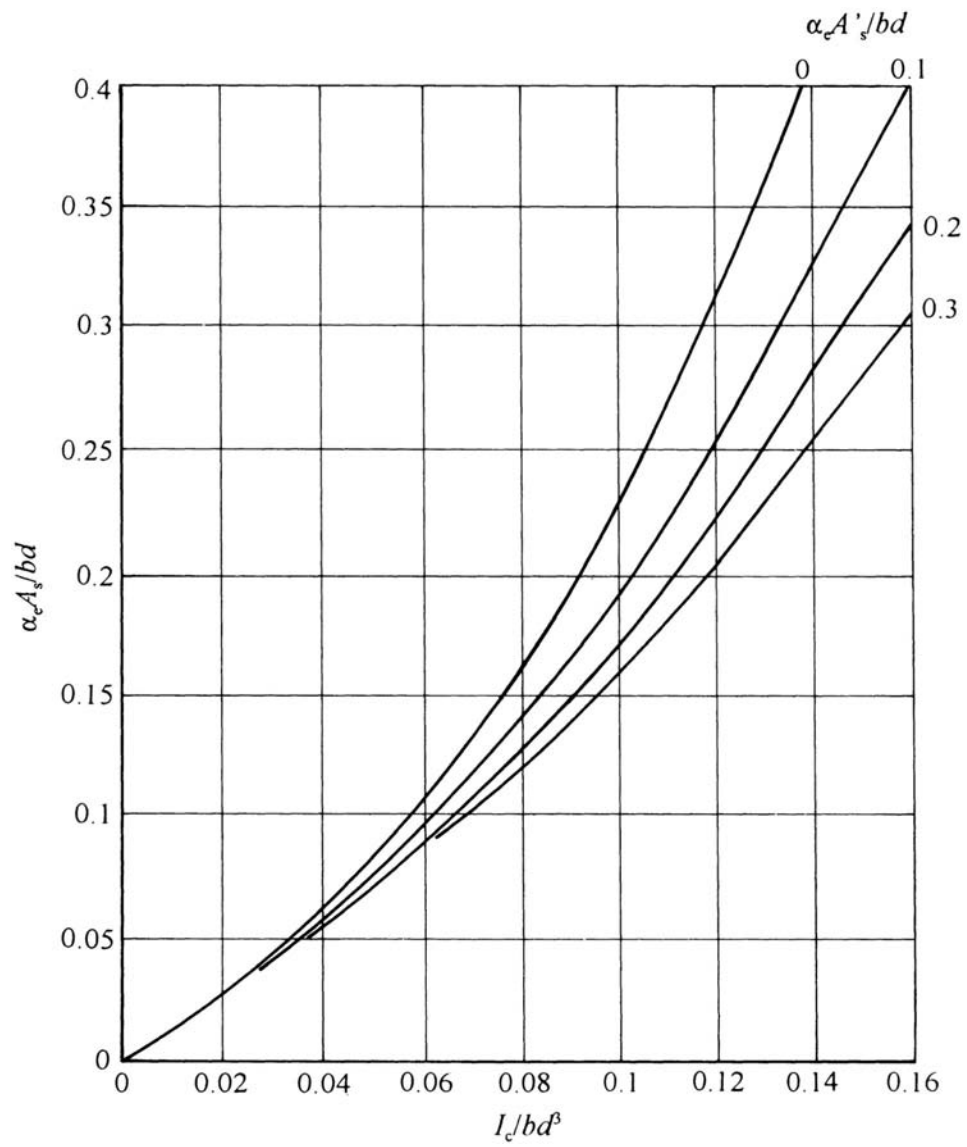


Figure 12.3 Second moments of area of rectangular sections based on a cracked transformed section

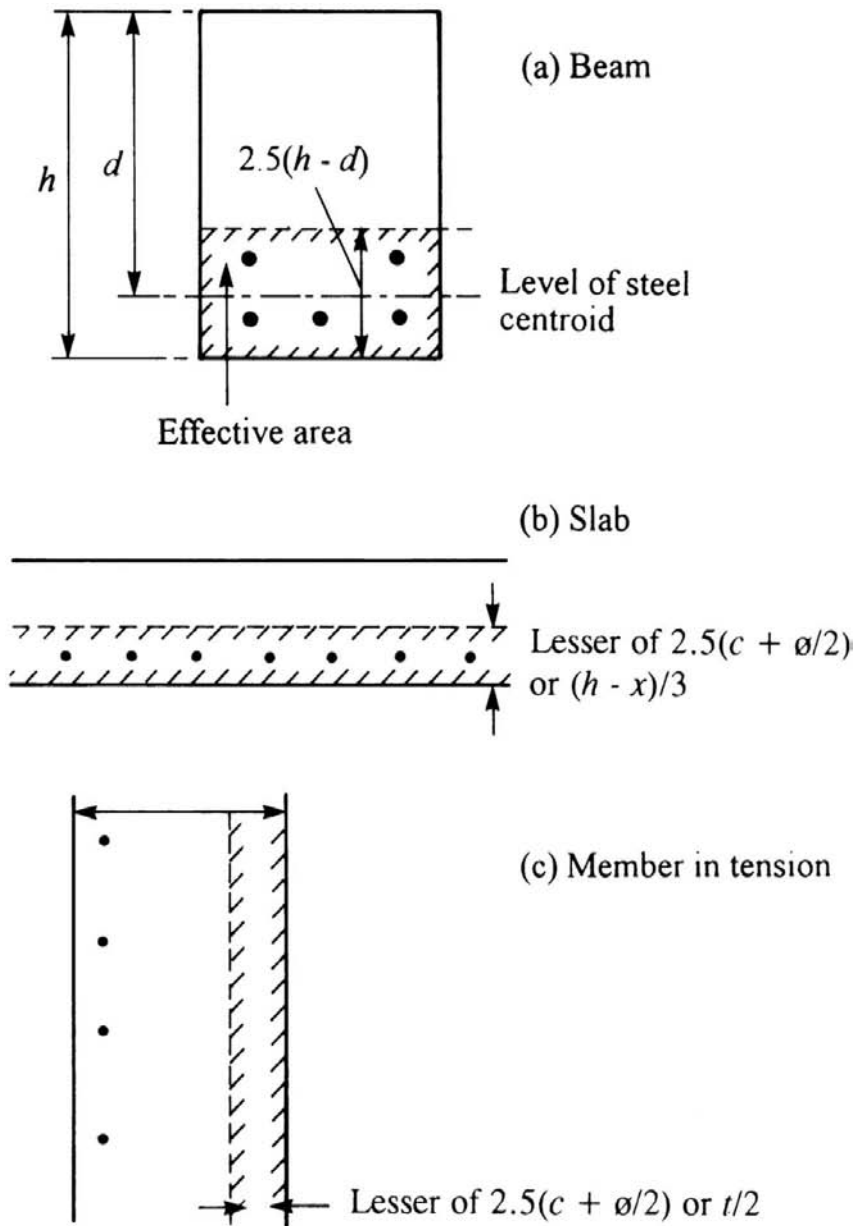


Figure 12.4 Effective area for a typical case.

13 Deflections

13.1 General

Eurocode 2 assumes that deflections will generally be checked using span/effective depth ratios, though a calculation method is also given (in Appendix 4). Owing particularly to uncertainties about the likely tensile strength of the concrete, calculation of deflection in the design stage for reinforced concrete members is likely to be very approximate. Hence direct calculation, rather than use of simple checks, is generally inappropriate.

Limits to deflection should be considered in the light of the intended function of the structure and the nature of finishes and partitions. The limits given in the code are intended only as guidance. They are: (1) Limit to overall total deflection: span/250; (2) Limit to deflection after construction of partitions and finishes where these are susceptible to damage: span/500

13.2 Ratios of span to effective depth

The span/effective depth ratios should generally ensure that these limits are met. The ratios depend upon: the nature of the structural system; the stress in the tension reinforcement; the reinforcement ratio; the geometry of the section (whether flanged or rectangular).

Figure 13.1 gives permissible ratios on the assumption that f_{yk} is 500 N/mm² and hence that the service stress at the critical section is approximately 250 N/mm². The values in Figure 13.1 should be adjusted according to those in Table 13.1. The critical section for assessing the reinforcement ratio and the steel stress is at mid-span for all members but cantilevers where the support section is used. For two-way spanning slabs supported on beams on all sides, the span/effective depth ratios should be based on the shorter span. For flat slabs, the longer span should be used.

Table 13.1 Adjustment factors for span/effective depth ratios

(1) Different levels of stress in tension reinforcement

Multiply ratios by $250/f_s$

f_s = stress under quasi-permanent load. This may be estimated approximately from:

$$f_s = \frac{f_{yk}}{\gamma_s} \frac{M_{qp}}{M_{sd}} \frac{A_{s,req}}{A_{s,prov}} \frac{1}{\delta}$$

where

γ_s = partial safety factor for reinforcement

M_{qp} = moment at critical section under the quasi-permanent load

M_{sd} = design ultimate load

$A_{s,req}$ = area of tension reinforcement required at critical section

$A_{s,prov}$ = area of tension reinforcement provided

δ = ratio of design moment after redistribution to the elastically calculated moment

(2) Flanged beams where $b_f/b < 0.3$

Multiply ratios by 0.8

(3) Long spans

(a) Members other than flat slabs with spans >7 m

Multiply by 7/span

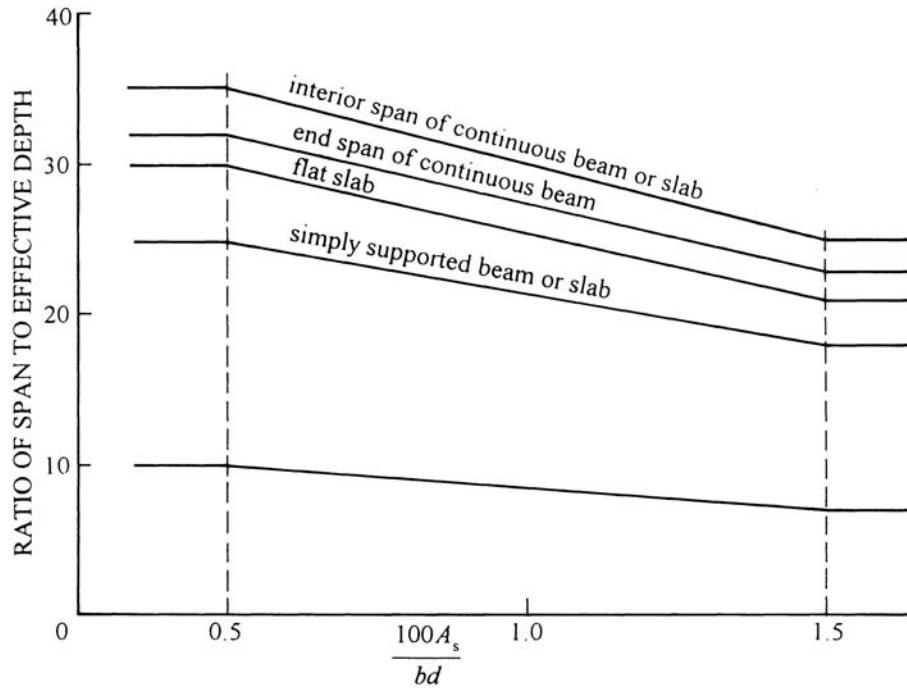


Figure 13.1 Permissible ratios of span to effective depth.

- (b) Flat slabs with spans >8.5 m
Multiply by 8.5/span
- (c) Other deflection limits: for total deflections other than span/250
Multiply by 250/k where new deflection limit is span/k

13.3 Calculation of deflection

There are two ways of approaching the calculation of deflections: one rigorous, the other more approximate. In the more rigorous approach, the curvature is calculated at a reasonable number of sections along the beam and then the deflection is calculated by numerical double integration.

The curvature may be calculated from:

$$\frac{1}{r} = \xi(1/r)_{II} + (1 - \xi)(1/r)_I$$

where

$(1/r)_I$ = the curvature calculated assuming the section is uncracked

$(1/r)_{II}$ = the curvature calculated assuming the section to be fully cracked

ξ = a distribution factor = $-\beta_1\beta_2\left(\frac{\sigma_s}{\sigma_{sr}}\right)^2$ where

β_1 = a coefficient that takes account of the bond properties of the bars

= 1 for high bond bars

= 0.5 for plain bars

β_2 = a coefficient that takes account of the duration of the loading or of repeated loading

= 1 for a single short-term loading

= 0.5 for sustained loads or many cycles of repeated loading

σ_s = the stress in the tension steel calculated on the basis of a cracked section

σ_{sr} = the stress in the tension steel calculated on the basis of a cracked section under the loading which will just cause cracking at the section being considered (Note: σ_s/σ_{sr} can be replaced by M/M_{cr} for flexure or N/N_{cr} for pure tension.)

ξ = zero for uncracked sections

The curvature may be calculated from the relation:

$$\frac{1}{r} = \frac{M}{EI_1} \text{ or } \frac{M}{EI_{II}} \quad \text{as appropriate or, for cracked sections, from the relation:}$$

$$\frac{1}{r} = \frac{\varepsilon_{sII}}{(d-x)}$$

where ε_{sII} is the strain in the reinforcement calculated on the basis of a cracked section. Values of I_{II} can be obtained from [Table 12.4](#) or [Figure 12.3](#).

The second method is to use the relation:

$$a = \xi a_1 + (1 - \xi) a_{II}$$

where

| | | |
|----------|---|--|
| a | = | the deflection |
| a_1 | = | the deflection calculated on the basis of an uncracked section |
| a_{II} | = | the deflection calculated on the basis of a cracked section |

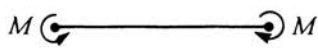
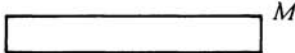
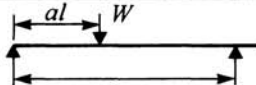



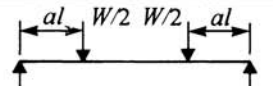
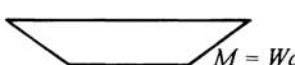
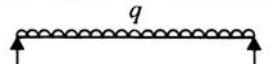


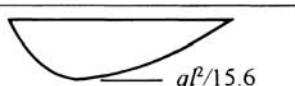
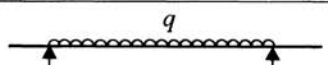
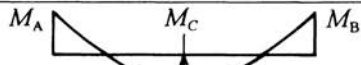
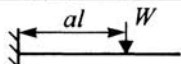

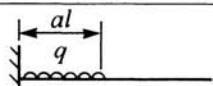
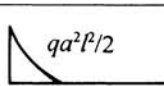
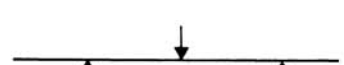

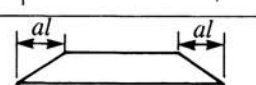
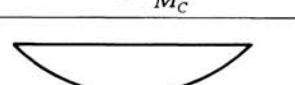
Standard elastic formulae may be used for obtaining a_1 and a_{II} , using the appropriate values of II and III.

The calculation of a_1 and a_{II} may be obtained from the relation:

$$a = \frac{kL^2M}{EI}$$

where k is a coefficient that depends on the shape of the bending moment diagram. Values for k are given in [Table 13.2](#), taken from the UK code, BS8110, Part 2.

Table 13.2 Values of κ for various bending moment diagrams

| Loading | Bending moment diagram | κ |
|---|--|--|
|  |  | 0.125 |
|  |  | $\frac{3 - 4a^2}{48(1 - a)}$ if $a = \frac{1}{2}$, $\kappa = \frac{1}{12}$ |
|  |  | 0.0625 |
|  |  | $0.125 - \frac{a^2}{6}$ |
|  |  | 0.104 |
|  |  | 0.102 |
|  |  | $\kappa = 0.104(1 - \frac{\beta}{10})$ $\beta = \frac{M_A + M_B}{M_C}$ |
|  |  | end deflection = $\frac{a(3 - a)}{6}$ load at end $\kappa = 0.333$ |
|  |  | $\frac{a(4 - a)}{12}$ if $a = l$, $\kappa = 0.25$ |
|  |  | $\kappa = 0.083(1 - \frac{\beta}{4})$ $\beta = \frac{M_A + M_B}{M_C}$ |
|  |  | $\frac{1}{80} \frac{(5 - 4a^2)^2}{3 - 4a^2}$ $(Wl^2/24)(3 - 4a^2)$ |

14 Detailing

14.1 Bond conditions

The bond conditions affect the anchorage and lap lengths. Good and poor bond conditions are illustrated in Figure 14.1.

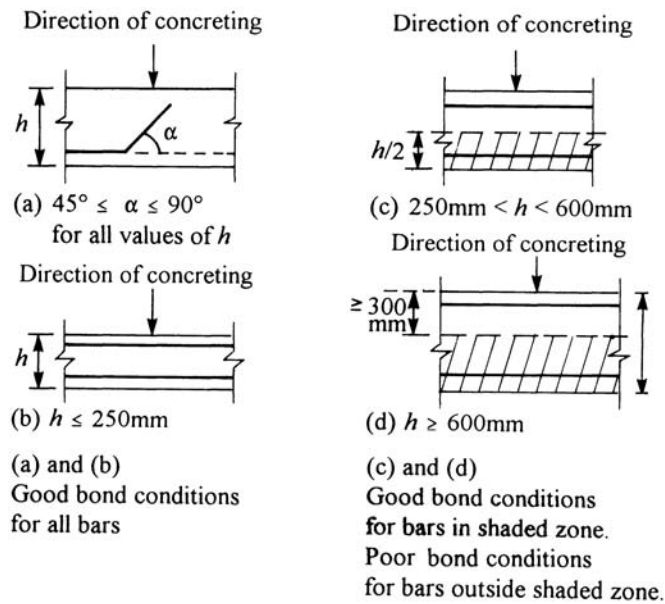


Figure 14.1 Good bond conditions.

14.2 Anchorage and lap lengths

Anchorage and lap lengths should be obtained from Table 14.1 for high-bond bars and Table 14.2 for weld mesh fabric made with high-bond bars.

14.3 Transverse reinforcement

(a) Anchorage zones

Transverse reinforcement should be provided for all anchorages in compression. In the absence of transverse compression caused by support reactions, transverse reinforcement should also be provided for anchorage in tension.

The minimum total area of transverse reinforcement required within the anchorage zone is 25% of the area of the anchored bar.

The transverse reinforcement should be evenly distributed in tension anchorages and concentrated at the ends of compression anchorages.

(b) Laps

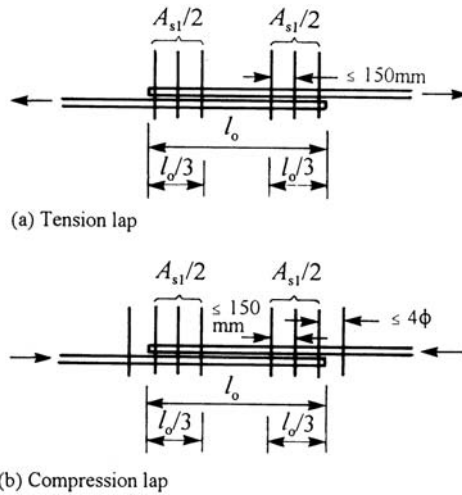


Figure 14.2 Transverse reinforcement at laps.

No special transverse reinforcement is required if the size of bars lapped is less than 16 mm or fewer than 20% of the bars in the section are lapped. When required, the transverse reinforcement should be placed as shown in Figure 14.2.

14.4

Curtailement of bars in flexural members

When a bar is curtailed in a flexural member, it should be anchored beyond the point when it is no longer required, for a length of $l_{b.net}$ or d , whichever is the greater.

In determining the location when a bar is no longer required, force in bars should be calculated taking into account (a) the bending moment and (b) the effect of truss modal for resisting shear.

A practical method for curtailment is as follows:

- (a) Determine where the bar can be curtailed based on bending moment alone; and
- (b) Anchor this bar beyond this location for a distance $l_{b.net} + a_1$, where $a_1 = 0.45d$ for beams and $1.0d$ for slabs.

This procedure is diagrammatically illustrated in Figure 14.3.

At simply supported ends, the bars should be anchored beyond the line of contact between the member and its support by

- 0.67 $l_{b.net}$ at a direct support and
- 1.00 $l_{b.net}$ at an indirect support.

This requirement is illustrated in Figure 14.4.

Table 14.1 Anchorage and lap lengths as multiples of bar size: high bond bars $f_{yk} = 500 \text{ N/mm}^2$

| Concrete strength (N/mm ²) | f_{ck} | 20 | 25 | 30 | 35 | 40 |
|---|----------|----|----|----|----|----|
| | f_{cu} | 25 | 30 | 37 | 45 | 50 |
| Anchorage straight bars compression and tension | | 48 | 40 | 37 | 33 | 29 |
| Anchorage - curved bars[4] tension | | 34 | 28 | 26 | 23 | 21 |
| Laps - compression - tension [5] | | 48 | 40 | 37 | 33 | 29 |
| Laps - tension [6] | | 67 | 57 | 52 | 46 | 41 |
| Laps - tension [7] | | 96 | 80 | 74 | 65 | 59 |

NOTES:

General

1. For bars with f_{yk} other than 500 N/mm^2 , the values should be multiplied by $(f_{yk}/500)$.

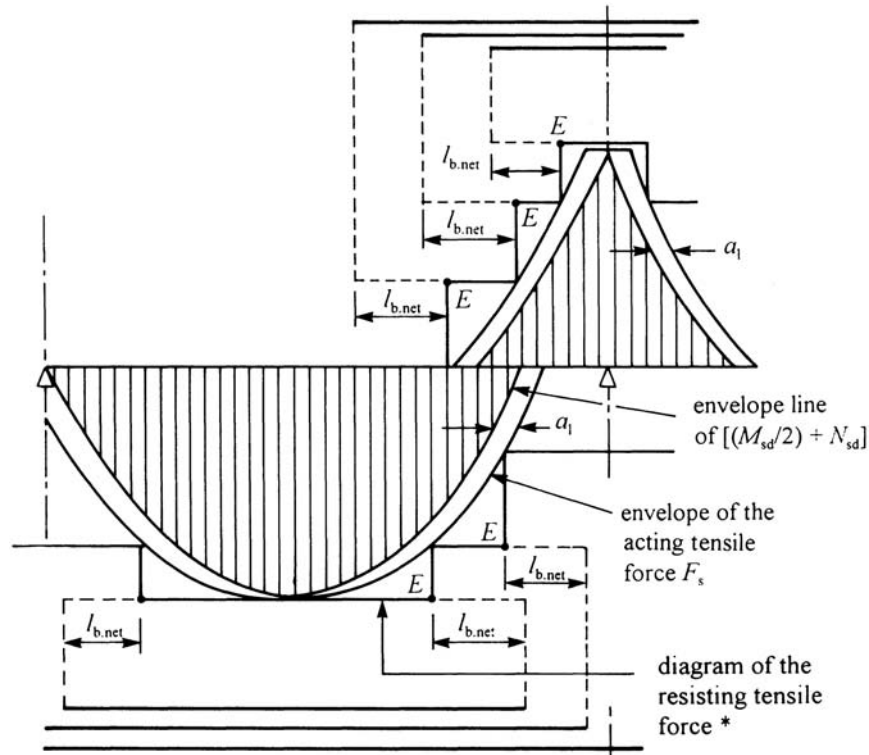


Figure 14.3 Illustration of 'shift-rule' for curtailment of bars. (* It is also permitted to use a diagram in which the resisting tensile force progressively decreases along the length $l_{b.net}$.)

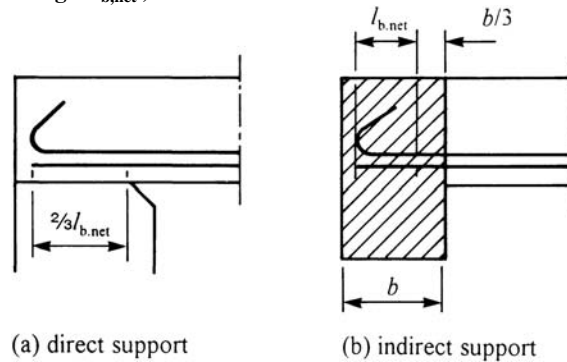


Figure 14.4 Anchorage of bottom reinforcement on end supports.

2. The values in the Table apply to (a) good bond conditions (see Fig. 14.1) and (b) bar size ≤ 32 .
3. For poor bond conditions (see Figure 14.1), the Table values should be divided by 0.7.
4. For bar size > 32 , the values should be divided by $[(132 - \phi)/100]$, where ϕ is the bar diameter in mm.

Specific conditions

5. In the anchorage region, cover perpendicular to the plane of curvature should be at least 3ϕ .
6. Proportion of bars lapped at the section $< 30\%$ and clear spacing between bars $\geq 10\phi$ and side cover to the outer bar $\geq 5\phi$.
7. Proportion of bars lapped at the section $> 30\%$ or clear spacing between bars $< 10\phi$ or side cover to the outer bar $< 5\phi$.
8. Proportion of bars lapped at the section $> 30\%$ and clear spacing between bars $< 10\phi$ or side cover to the outer bar $< 5\phi$.

Table 14.2 Anchorage and lap lengths as multiples of bar size. Welded mesh fabric made with high-bond bars with $f_{yk} = 500 \text{ N/mm}^2$

| Concrete strength $f_{ck} \text{ (N/mm}^2\text{)}$ | 20 | 25 | 30 | 35 | 40 |
|---|----|----|----|----|----|
|---|----|----|----|----|----|

| | | | | | |
|---|----|----|----|----|----|
| Basic anchorage and lap lengths (mm) | 48 | 40 | 37 | 33 | 29 |
|---|----|----|----|----|----|

Notes:

1. For bars with f_{yk} other than 500 N/mm², the values should be multiplied by $(f_{yk}/500)$.
2. Where welded transverse bars are present in the anchorage zone, the Table values for anchorage may be multiplied by 0.7.
3. The values given in the Table apply to good bond conditions and to bar sizes ≤ 32 mm.
4. For poor bond conditions, the values should be divided by 0.7.
5. For bar sizes >32 mm, the values should be divided by $[(132-\phi)/100]$, where ϕ is the diameter of the bar in mm.
6. The Table values should be multiplied by the following factors corresponding to the different (A_s/S) values. A_s is the area of the main reinforcement (mm²) bar and S is the spacing of the bars forming the main reinforcement (m).

| | | | | | |
|-------------------|------|------|------|------|------|
| A_s/S | 480 | 680 | 880 | 1080 | 1280 |
| Multiplier | 1.00 | 1.25 | 1.50 | 1.75 | 2.00 |

15

Numerical examples designed to ENV 1992-1-1

15.1 Introduction

Three types of building have been designed to Eurocode 2 (ENV 1992-1-1). Criteria for the choice of the buildings were:

- the type of structural members
- magnitude of vertical (imposed) loads
- character of the imposed loads (i.e. static or dynamic)
- the ultimate limit states to be considered (e.g. punching, fatigue).

The objectives of these calculations were to demonstrate the applicability of Eurocode 2 in practice. The main conclusion of these calculations therefore is that no basic difficulties have been observed when applying the new European Prestandard in a practical design process.

15.2 References

| | | |
|---------------------------|---|---------|
| ENV 1991-1: | Eurocode 1: Basis of design and actions on structures. Part 1: Basis of design. Edition 1994. | |
| ENV 1991-2-1: | Eurocode 1: Basis of design and actions on structures. Part 2.1: Densities, self-weight and imposed loads. Final draft April 1993. | |
| ENV 1991-2-3: Eurocode 1: | Basis of design and actions on structures. Part 2.3: Snow loads. Final draft April 1993. | |
| ENV 1991-2-4: Eurocode 1: | Basis of design and actions on structures. Part 2.4: Wind loads. Final draft April 1993. | |
| ENV 1992-1: | Eurocode 2: Design of concrete structures. Part 1: General rules and rules for buildings. Edition 1991. | EC2 |
| ENV 1992-1-2: | Eurocode 2: Design of concrete structures. Part 1-2: Structural fire design. Draft August 1994. | EC2-1.2 |
| pr ENV 1992-2: | Eurocode 2: Design of concrete structures. Part 2: Concrete bridges. Draft June 1995. | EC2-2 |
| ENV 10 080: | Steels for the reinforcement of concrete; Weldable ribbed reinforcing steel grade B500; Technical delivery conditions for bars, coils and welded fabrics. Final draft April 1994. | |
| ENV 206: | Concrete production, placing and compliance criteria. Edition 1990. | |
| DIN 15 018: | Cranes; Principles for steel structures, stress analysis. Part 1. Edition November 1984. | |

Abbreviation

| |
|---------|
| EC1-1 |
| EC1-2.1 |
| EC1-2.3 |
| EC1-2.4 |

ENV 10 080

ENV 206

15.2 References

1. Litzner, H.-U.: *Design of Concrete Structures to ENV 1992-Eurocode 2. Concrete Structures-Euro-Design Handbook*. 1st volume 1994/1996. Berlin: Ernst & Sohn 1994.
2. Deutscher Ausschluß für Stahlbeton (DAfStb): *Bemessungshilfsmittel zu Eurocode 2 Teil 1 (DIN V ENV 1992 Teil 1-1, Ausgabe 06. 92). 2. ergänzte Auflage. Heft 425 der DAfStb-Schriftenreihe*. Berlin, Köln: Beuth Verlag GmbH 1992.
3. British Cement Association: *Worked examples for the design of concrete buildings*. Crowthorne: British Cement Association 1994.
4. Deutscher Beton-Verein E.V.: *Beispiele zur Bemessung von Betontragwerken nach EC2*. Wiesbaden, Berlin: Bauverlag GmbH 1994.
5. Dieterle, H.: *Zur Bemessung quadratischer Stützenfundamente aus Stahlbeton unter zentrischer Belastung mit Hilfe von Bemessungsdiagrammen*. Heft 387 der DAfStb-Schriftenreihe 1987.
6. British Cement Association: *Concise Eurocode for the design of concrete buildings*. Crowthorne, 1993.
7. Betonvereniging: *GTB Deel 2. Grafieken en Tabellen voor Beton*. Gouda 1992.

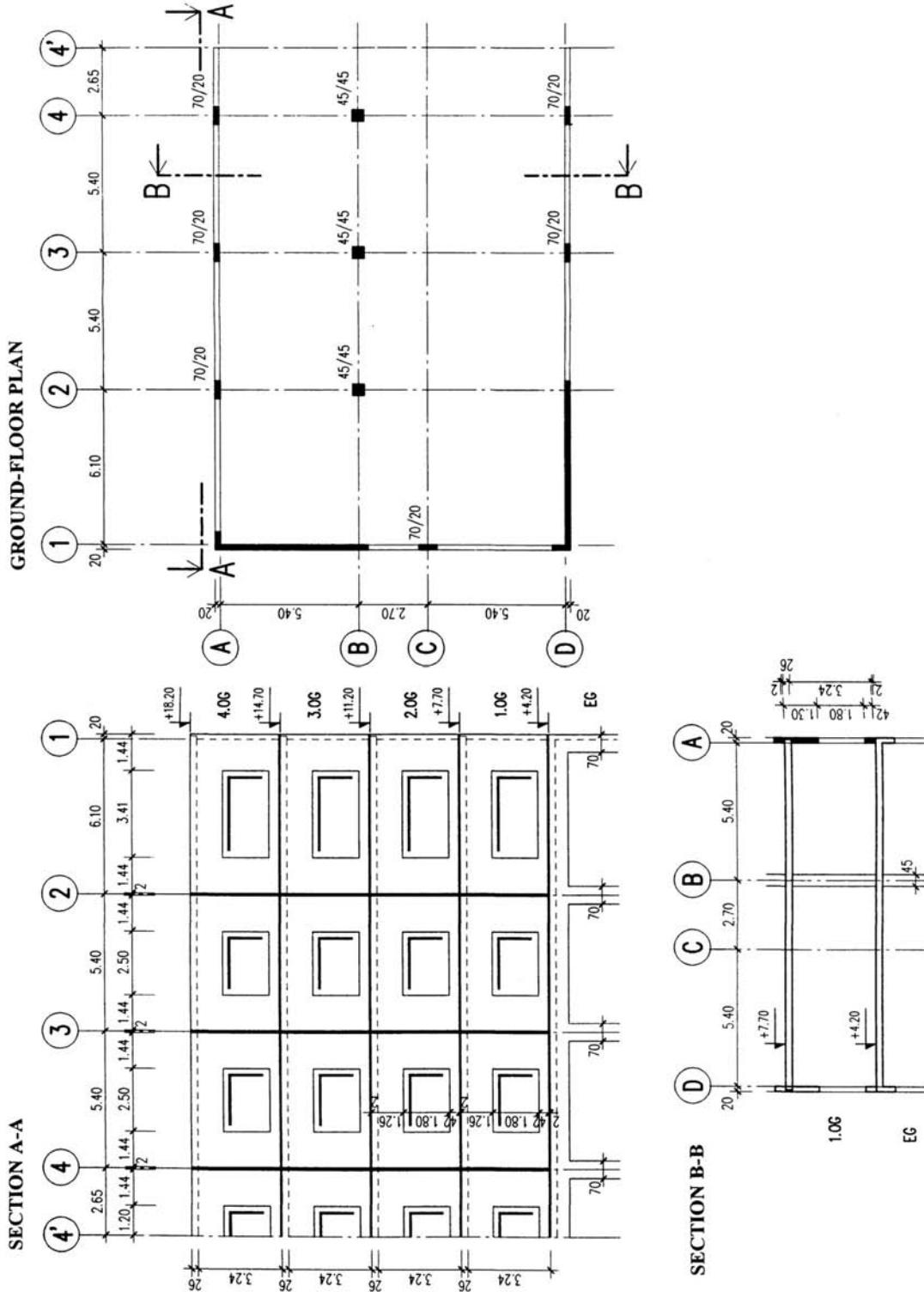
15.3
Calculation for an office building

15.3.1
Floor plan, structural details and basic data

15.3.1.1
Floor plan of an office building



15.3.1.2
Structural details of an office building



15.3.1.3

Basic data of structure, materials and loading

| | | | |
|--|---|-----------|-------------------------|
| Intended use: | Office block | | |
| Fire resistance: | 1 hour for all elements | | |
| Loading (excluding self-weight of structure): | | | |
| Flat slab: | - imposed: | Q_k | = 3kN/m ² |
| | - finished: | $G_{k,2}$ | = 1.25kN/m ² |
| Category B | | | |
| | - partitions: | $G_{k,3}$ | = 1.25kN/m ² |
| Combination factors: | | | |
| Frequent actions: | | ψ_1 | = 0.5 |
| Quasi-permanent actions: | | ψ_2 | = 0.3 |
| Exposure classes: | | | |
| Flat slab: | | | |
| Internal columns: | Class 1 (indoors) | | |
| Façade elements: | Class 2b (humid environment with frost) | | |
| Block foundation: | Class 5a (slightly aggressive chemical environment) | | |
| Subsoil conditions: | | | |
| Sand, gravel | Allowable pressure 300 kN/m ² | | |
| Materials: | | | |
| Concrete grade | C 30/37 | | |
| Steel grade | B500 | | |
| Self-weight of concrete | 25 kN/m ³ | | |

Reference

see floor plan
 EC2–1.2, 1.3
 EC1–2.1
 EC1–2.1
 EC1–1, Table 9.3, Category B
 EC2, Table 4.1
 from soil investigation
 EC2, Table 3.1; ENV 206, Table 3 and Table 20; ENV 10 080; EC1–2.1

15.3.2

Calculation of a flat slab

15.3.2.1

Actions

| | | |
|--|-------|---------------------------|
| Self-weight of slab: 0.26*25 | | = 6.50 kN/m ² |
| Finishes | | 1.25kN/m ² |
| Partitions | | 1.25 kN/m ² |
| Permanent actions: | G_k | = 9.00 kN/m ² |
| Imposed load: | Q_k | = 3.00 kN/m ² |
| Design values of actions at the ultimate limit states: | | |
| $\gamma_G G_k + \gamma_Q Q_k = 1.35 * 9.0 + 1.5 * 3.0$ | | = 16.65 kN/m ² |
| Design values of actions at the serviceability limit states: | | |
| Rare combination of actions: | | = 12.00 kN/m ² |
| Frequent combination: | | |
| $G_k + \psi_1 Q_k = 9.00 + 0.5 * 3.00$ | | = 10.50 kN/m ² |
| Quasi-permanent combination: | | |
| $G_k + \psi_2 Q_k = 9.00 + 0.3 * 3.00$ | | = 9.90 kN/m ² |

Reference

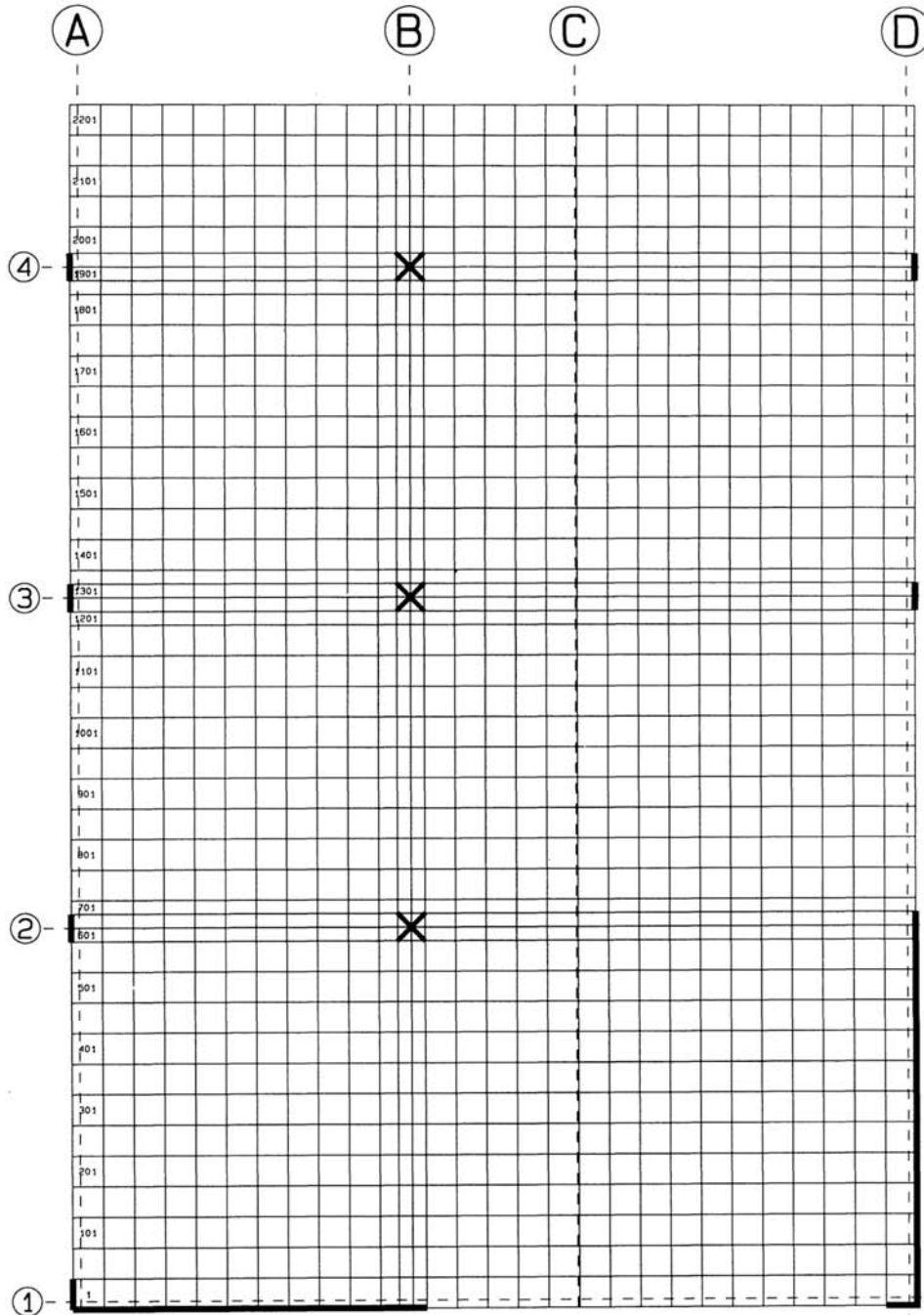
$h=0.26$ m

EC2, Equation (2.7a), fundamental combination

EC2, 2.3.4

15.3.2.2

Structural model at the ultimate limit states (finite element grid)



15.3.2.3
Design values of bending moments (example)

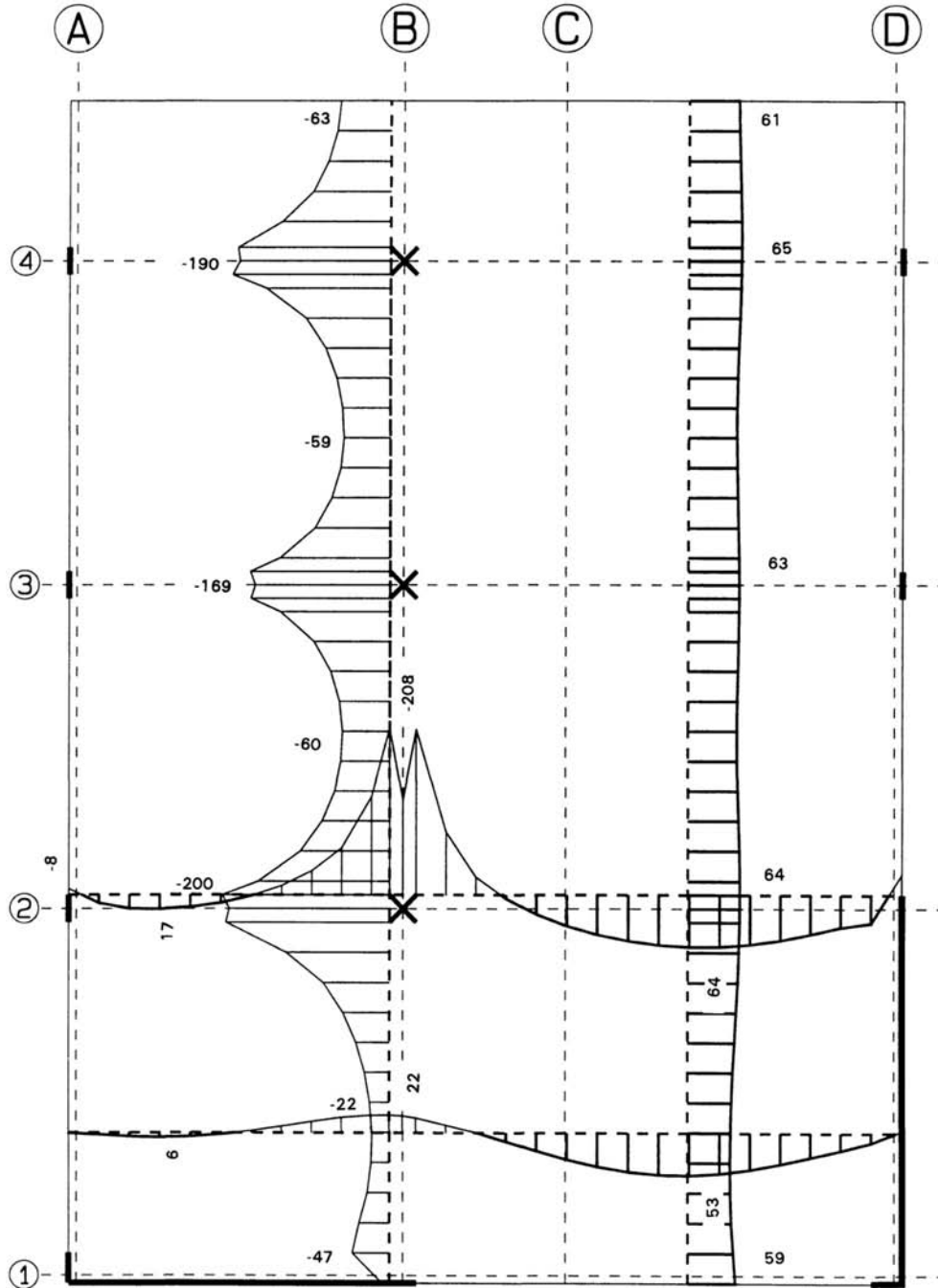


Table 15.1: Design bending moments at the ultimate limit states

| Direction | Location | Section | Kind of moment | Moment (kNm/m) | Mean value of moment (kNm/m) |
|-----------|----------|------------------|-----------------------|----------------|------------------------------|
| x | B/2 | left of axis B | support: min m_{Sd} | -184.59 | -180.72 |
| | | centre of axis B | | -175.07 | |
| | | right of axis B | | -182.50 | |
| x | B-D/2 | left of axis B | span: max m_{Sd} | 63.26 | 63.85 |
| | | centre of axis B | | 64.02 | |
| | | right of axis B | | 64.26 | |

| Direction | Location | Section | Kind of moment | Moment (kNm/m) | Mean value of moment (kNm/m) |
|-----------|----------|------------------|-------------------|----------------|------------------------------|
| y | B/2 | top of axis 2 | support: m_{Sd} | -208.26 | -204.16 |
| | | centre of axis 2 | | -200.02 | |
| | | bottom of axis 2 | | -204.21 | |
| y | B/1-2 | top of axis 2 | span: m_{Sd} | 93.14 | 92.95 |
| | | centre of axis 2 | | 93.00 | |
| | | bottom of axis 2 | | 92.71 | |

15.3.2.4 Design of bending at the ultimate limit states

Table 15.2: Design for bending

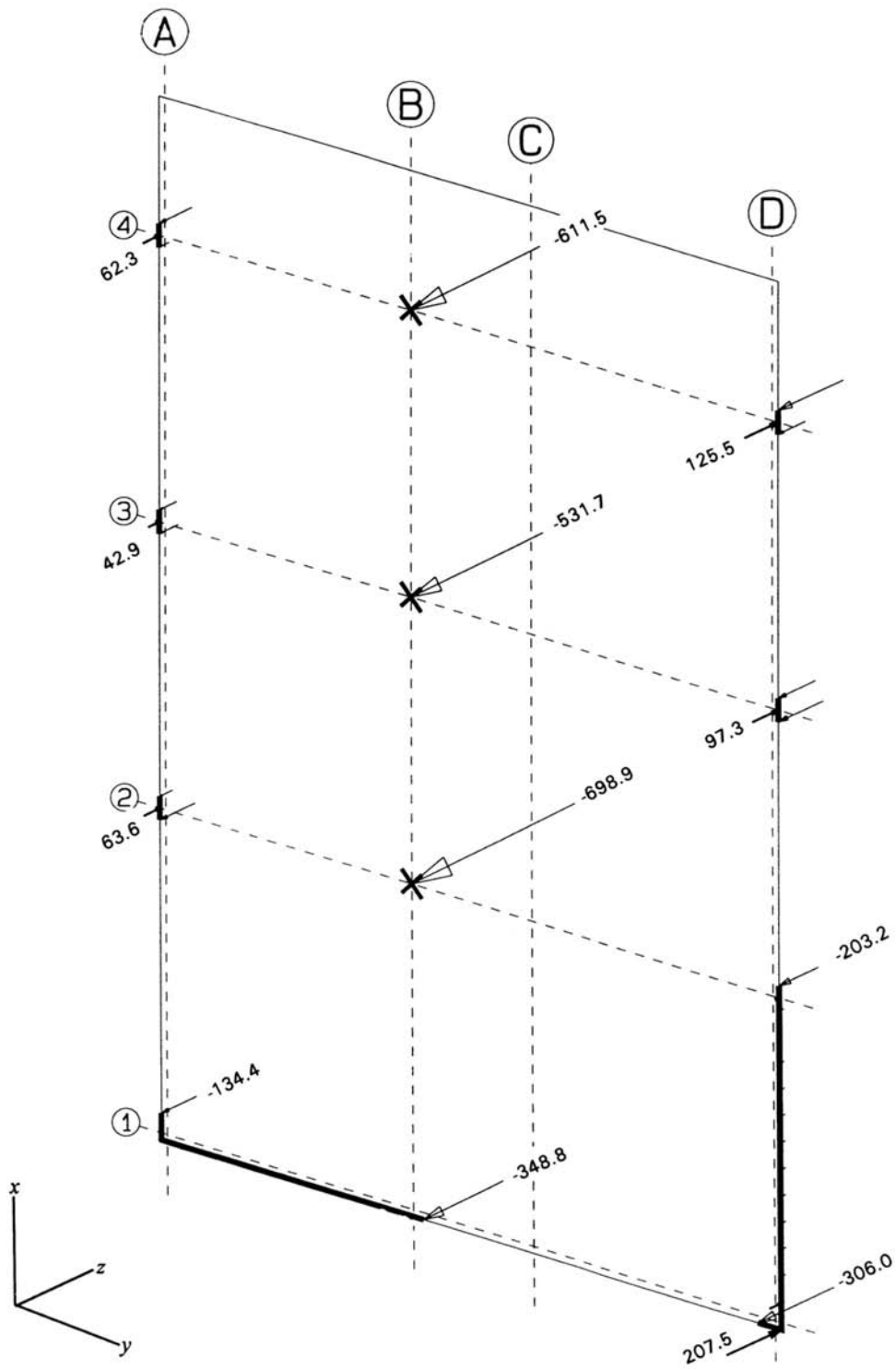
| Direction | x | | y | |
|------------------------------------|------------------------|------------------------|------------------------|------------------------|
| | B/1-3 | | 2/A-D | |
| Location | Support | Span | Support | Span |
| m_{Sd} (kN/m) | 180.72 | 63.85 | 204.16 | 92.95 |
| $d(m)$ | 0.219 | 0.224 | 0.233 | 0.235 |
| μ_{Sds} | 0.188 | 0.063 | 0.188 | 0.084 |
| ω | 0.2163 | 0.0657 | 0.2163 | 0.0888 |
| ξ | 0.3142 | 0.1069 | 0.3142 | 0.1332 |
| $A_{s, req}$ (mm ² /m) | 21.78*10 ² | 6.80*10 ² | 23.17*10 ² | 9.59*10 ² |
| Selected B 500B(S) | 2* \emptyset 14-14.0 | | 2* \emptyset 14-14.0 | |
| Selected B 500A(M) | | 2* \emptyset 7.0-100 | | 2* \emptyset 8.0-100 |
| $A_{s, prov}$ (mm ² /m) | 22.00*10 ² | 7.70*10 ² | 23.68*10 ² | 10.05*10 ² |
| f_{cd} | = | 30/1.5 | = | 20N/mm ² |
| f_{yd} | = | 500/1.15 | = | 435N/mm ² |

Calculation for supports

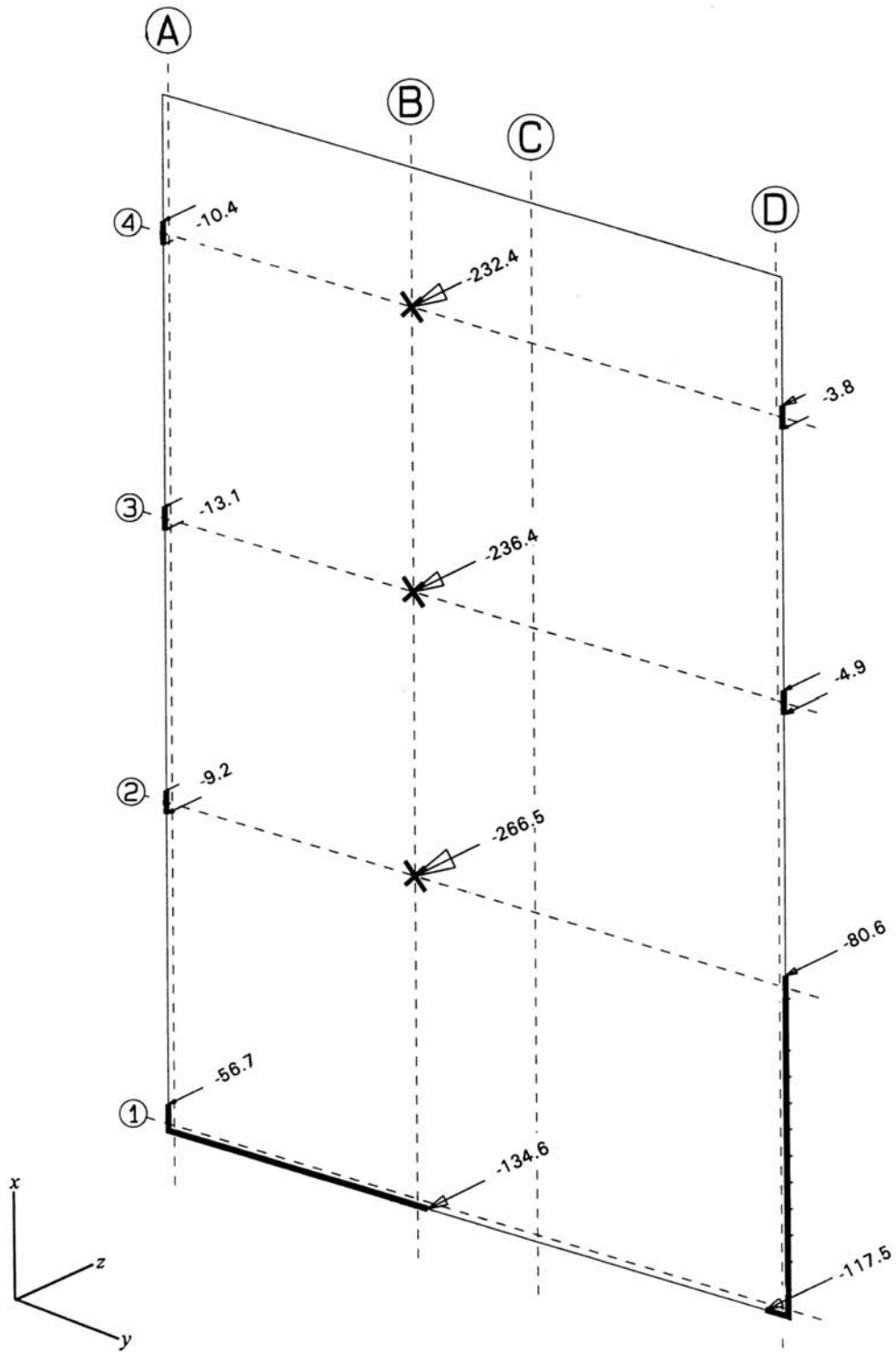
$$\begin{aligned}
 d_y &= h - (\min c + \Delta/h) - \emptyset/2 \\
 &= 0.260 - (0.015 + 0.005) - 0.014/2 &= & 0.233 \text{ m} \\
 d_x &= d_y - \emptyset = 0.233 - 0.014 &= & 0.219 \text{ m} \\
 \text{(S):} & \text{Reinforcing bars} \\
 \text{(M):} & \text{Welded mesh fabric}
 \end{aligned}$$

15.3.2.5 Ultimate limit state for punching shear

Shear forces due to permanent actions



Shear forces due to variable actions



Design for punching shear in axis B/2

| | | | | |
|--------------------|---|-------------------------|---|---------------------|
| V_{sd} | = | $698.9+266.5$ | = | 966 kN |
| d_m | = | $(0.233+0.219)/2$ | = | 0.226 m see Table 2 |
| Critical perimeter | | | | |
| u | = | $4*0.45+2\pi*1.5*0.226$ | = | 3.93m |
| Acting shear force | | | | |
| v_{sd} | = | $966*1.15/3.93$ | = | 283 kN/m |

Shear resistance of slabs without shear reinforcement

$$\begin{aligned} \rho_1 &= 0.01 \\ V_{Rd1} &= 0.34 \cdot (1.6 - 0.226) \cdot (1.2 + 40 \cdot 0.01) \cdot 0.226 \cdot 10^3 \\ &= 169 \text{ kN/m} \\ 1.6 \cdot v_{Rd1} &= 1.6 \cdot 169 = 270 \text{ kN/m} \\ \rho_1 &\text{ must be increased} \\ \text{Required } \rho_1 &= [283 / (1.6 \cdot 0.34 \cdot 1.374 \cdot 0.226 \cdot 10^3) - 1.2] / 40 = 1.19\% \\ \text{Calculation of shear reinforcement} \\ v_{Sd} - v_{Rd1} &= 283 - 169 = 114 \text{ kN/m} \\ A_{sw}^s &= 107 \cdot 10^3 \cdot 3.93 / (435 \cdot \sin 60^\circ) = 11.2 \cdot 10^2 \text{ mm}^2 \end{aligned}$$

Selected four bent-up bars ϕ 14 mm provided $4.2.1.5.3 = 12.2 \cdot 10^2 \text{ mm}^2$

Minimum shear reinforcement

$$\begin{aligned} 0.6 \cdot \min \rho_w &= 0.6 \cdot 0.11 = 0.066\% \\ \text{The critical area minus the loaded area} &= \\ A_{sw, \min} &= 4 \cdot 0.45 \cdot 1.5 \cdot 0.226 + \pi \cdot (1.5 \cdot 0.226)^2 = 0.97 \text{ m}^2 \\ &= 0.066 \cdot 10^{-2} \cdot 0.97 \cdot 10^6 / \sin 60^\circ = 7.9 \cdot 10^2 \text{ mm}^2 \\ &< 12.2 \cdot 10^2 \text{ mm}^2 \end{aligned}$$

Reference

EC2, 4.3.4
 see distribution of shear forces
 EC2, 4.3.4.2.2
 EC2, Eq.(4.50) for internal columns
 EC2, 4.3.4.5.1 see Table 2; $>0.5\%$
 EC2, Eq.(4.56)
 $<1.5\%$ Table 2: $\rho_{1x} = 1.19 \cdot 21.9 \cdot 10^3 = 26.0 \cdot 10^2 \text{ mm}^2/\text{m}$ $\rho_{1y} = 1.19 \cdot 23.3 \cdot 10^2 = 27.7 \cdot 10^2 \text{ mm}^2/\text{m}$
 EC2, Eq. (4.58)
 $\alpha = 60^\circ$
 EC2, 4.3.4.5.2(4)
 EC2, Table 5.5

Minimum design moment

$$\begin{aligned} m_{Sd, \min} &= -966 \cdot 0.125 = \ominus 121 \text{ kNm/m} \\ &< m_{Sd} \end{aligned}$$

15.3.2.6

Limitation of deflections

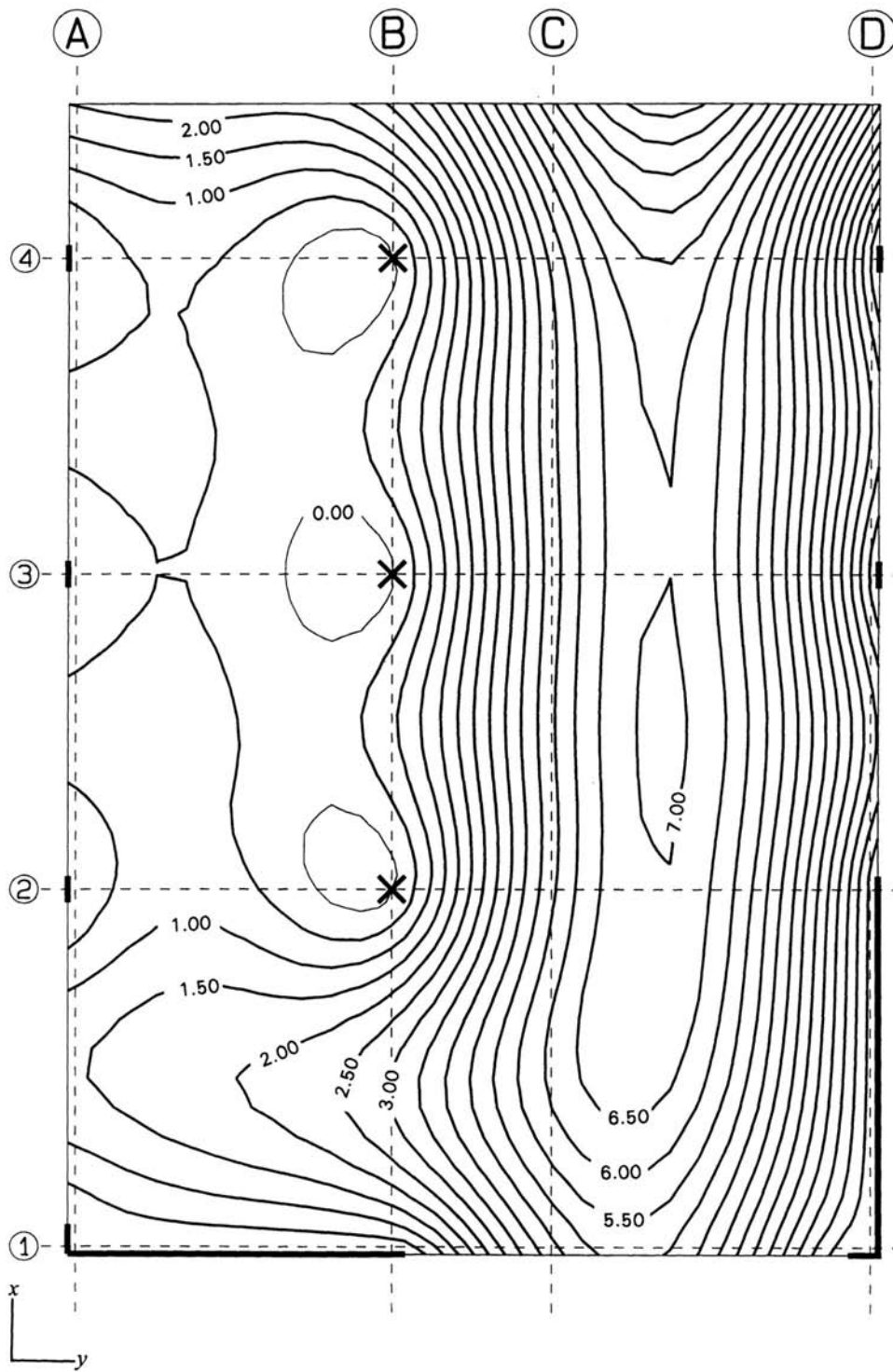
It is assumed that, with regard to deflections under quasi-permanent actions, a limiting value of 25 mm was agreed with the client. The deflection diagram for cracked cross-section shows that this requirement is met between axes 1 and 4. The deflection of the cantilever slab accounting for creep deformations is about 34 mm.

Therefore, in order to ensure proper functioning and appearance of the structure, precamber of the cantilever slab of 10 mm is suggested.

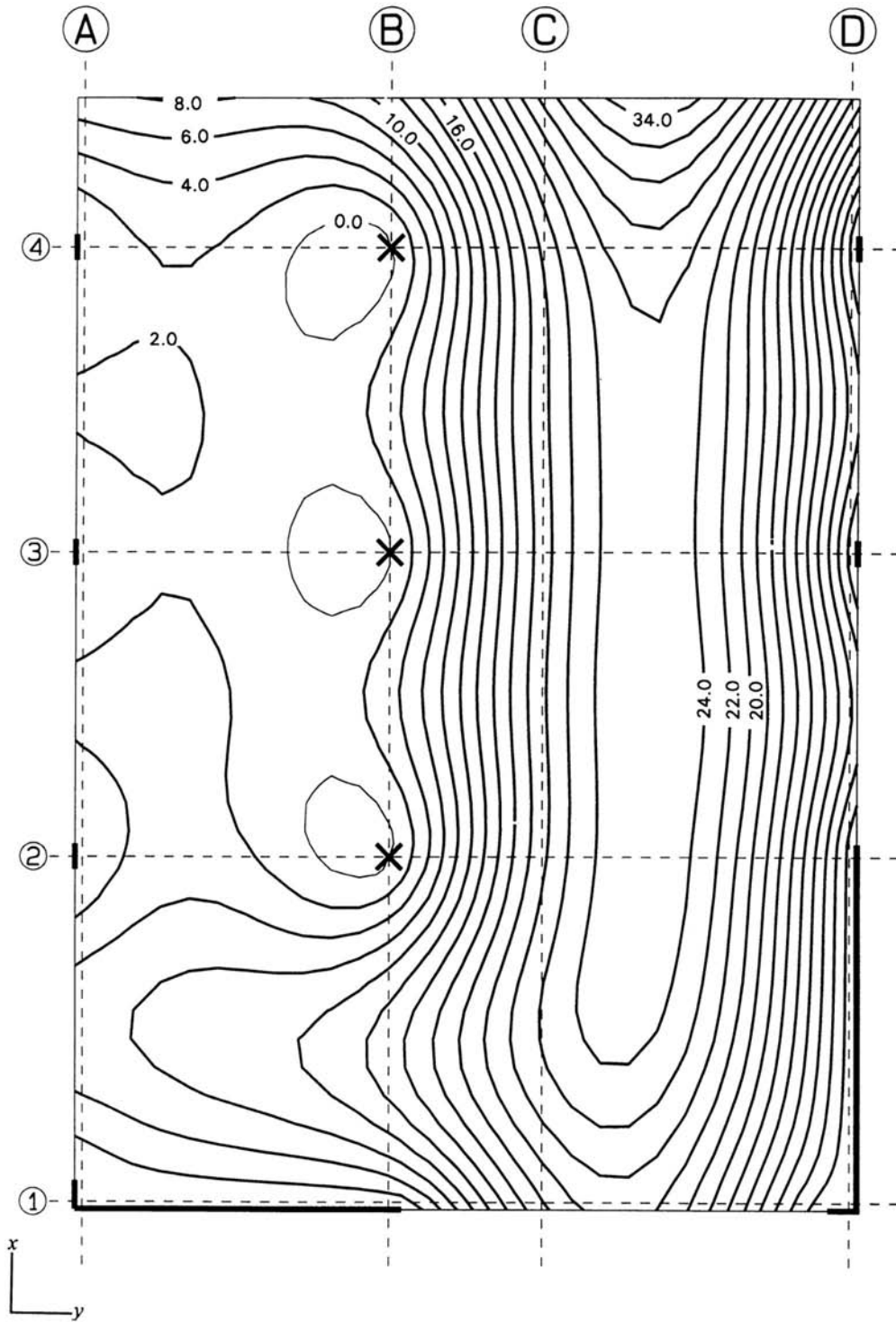
Reference

EC2, 4.3.4.5.3
 for internal columns see [Table 15.1](#) above
 EC2, 4.4.3
 EC2, 4.4.3.1P(2)
 see following deflection Figures EC2, Eq.(A.4.3)

Deformations of flat slab due to quasi-permanent actions, uncracked cross-sections assumed

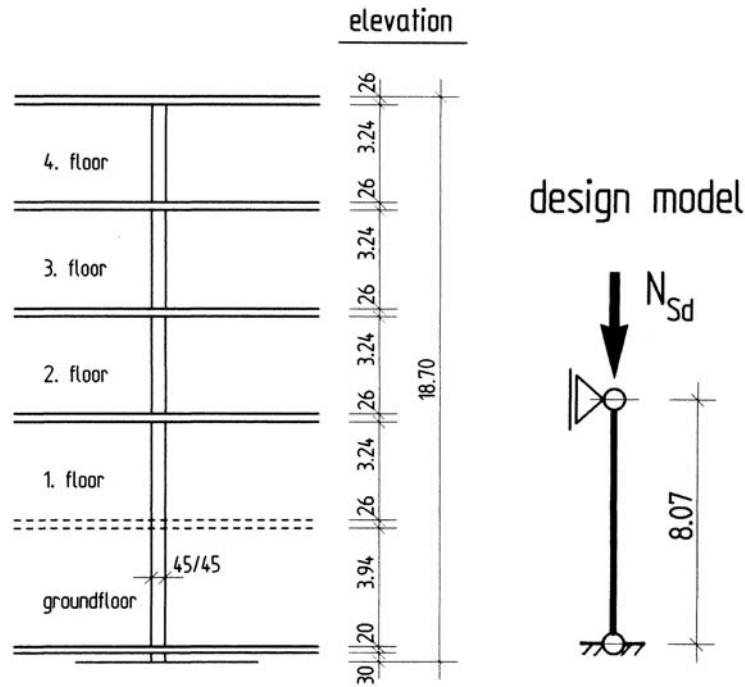


Deformations of flat slab due to quasi-permanent actions, cracked cross-sections assumed



15.3.3
Internal column

Design model of the column



The column in the ground floor/first floor in axis B/2 will be designed to EC2. The structural model is shown in the Figure above. The column is analysed on the assumption that the adjacent slab and block foundation provide no rotational restraint.

Design value of the axial force N_{sd} :

On the roof, a uniformly distributed snow load is assumed:

$$s = 0.9 \text{ kN/m}^2$$

The combination factor for this load is taken as:

$$\psi_0 = 1.0$$

$$N_{sd} = -[4 \cdot 698.9 + 3 \cdot 266.5 + 266.5 \cdot 0.9 / 3.0 + 0.45^2 \cdot 18.7 \cdot 25]$$

$$= -3800 \text{ kN}$$

Reference

see 15.3.2.5 above EC2, 2.5.3.3(3)

see 15.3.2.5 above

EC1-2.3

conservative assumption

see 15.3.2.5, Figures of shear forces

Design of the column

$$f_{cd} = 20 \text{ N/mm}^2$$

$$f_{yd} = 435 \text{ N/mm}^2$$

Creep deformations are neglected.

Additional eccentricity e_a :

$$e_a = l_0 / 400 = 8.07 / 400 = 0.02 \text{ m}$$

$$e_a / h = 0.02 / 0.45 = 0.05$$

$$l_0 / h = 8.07 / 0.45 = 18$$

$$\nu_u = -3.8 / (0.45^2 \cdot 20) = -1.0$$

From the design diagram, ω is taken as:

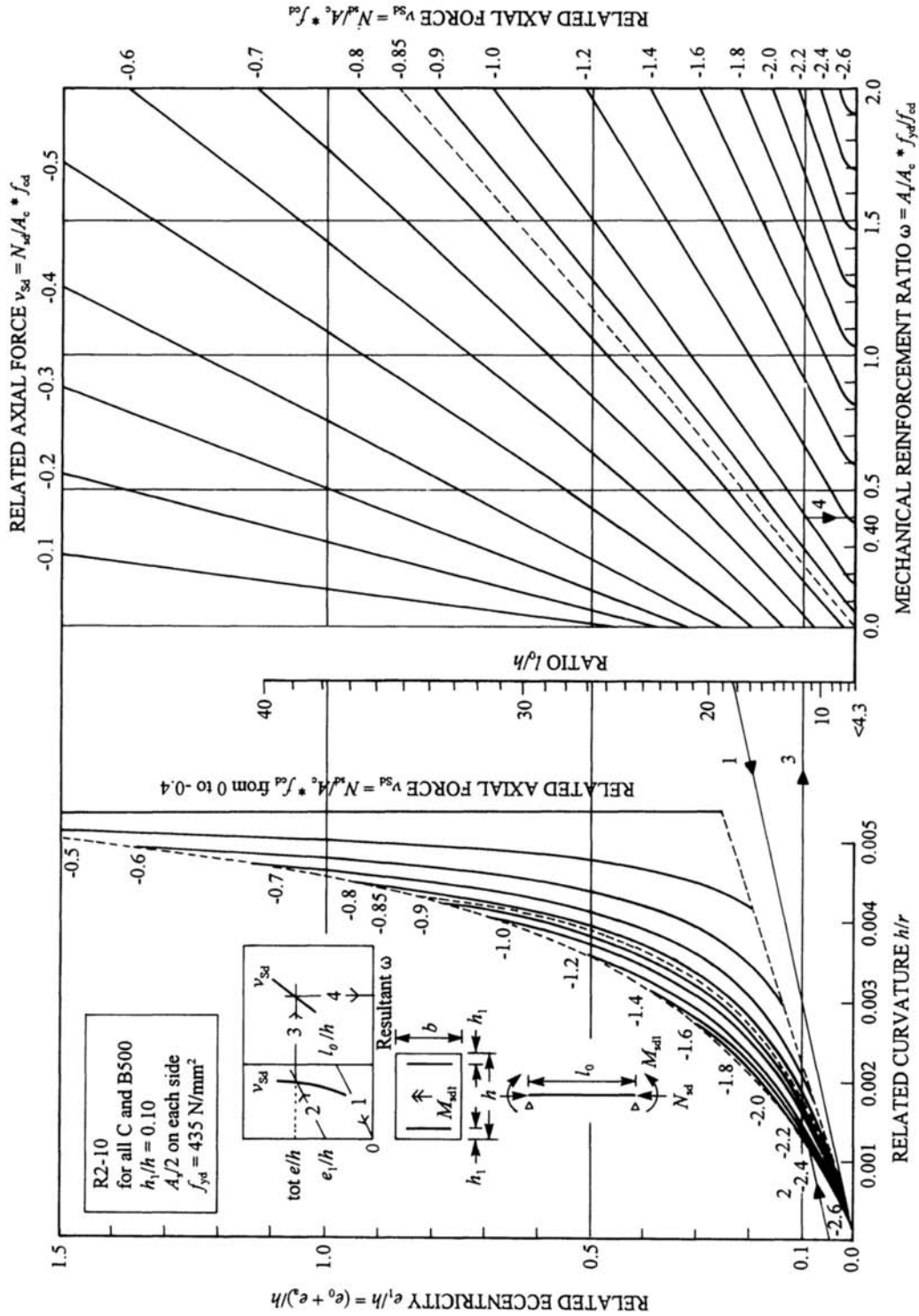
$$\omega = 0.40$$

$$A_{s,tot} = 0.40 \cdot 450^2 \cdot 20 / 435 = 37.3 \cdot 10^2 \text{ mm}^2$$

Selected eight bars ϕ 25 mm $A_{s,prov} = 39.3 \cdot 10^2 \text{ mm}^2$ Links ϕ 8 mm-300 mm

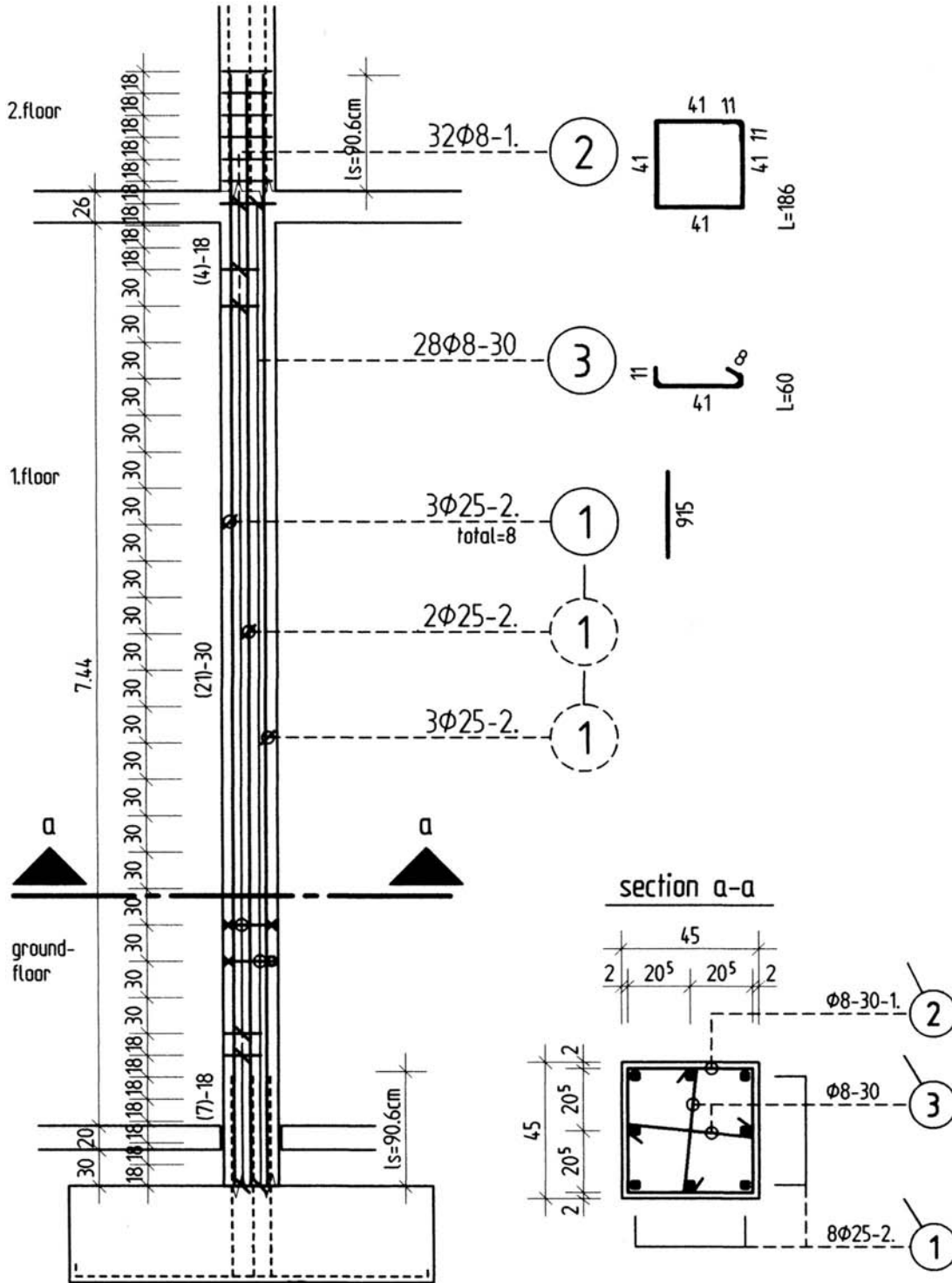
Reference

using the Figures in [2] C 30/37
 EC2, A.3.4(9)
 EC2, 4.3.5.4(3)
 EC2, 5.4.1.2.1(2)
 $A_{s,min}$ not relevant here
 Design diagram for the column



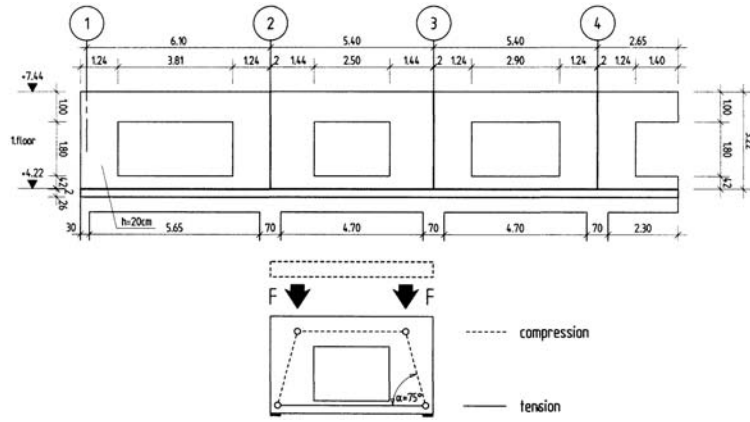
Reference [2], diagram R2-05

Detailing of reinforcement



**15.3.4
Facade element**

The facade of the building consists of precast elements (see Figure below). As an example, the element between axis 2 and 3 will be designed to EC2. As model, strut and ties are used. For simplification, the maximum shear forces, between axis 1 and 2 are considered

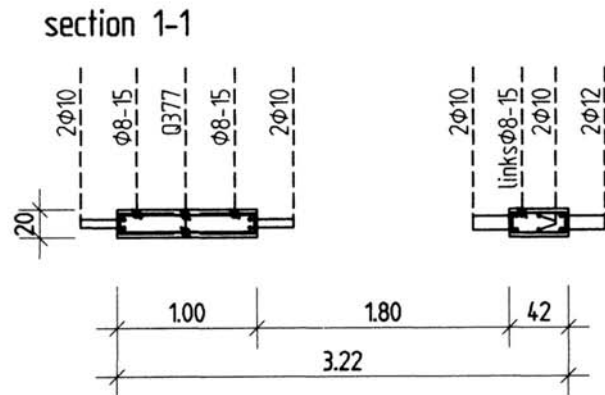
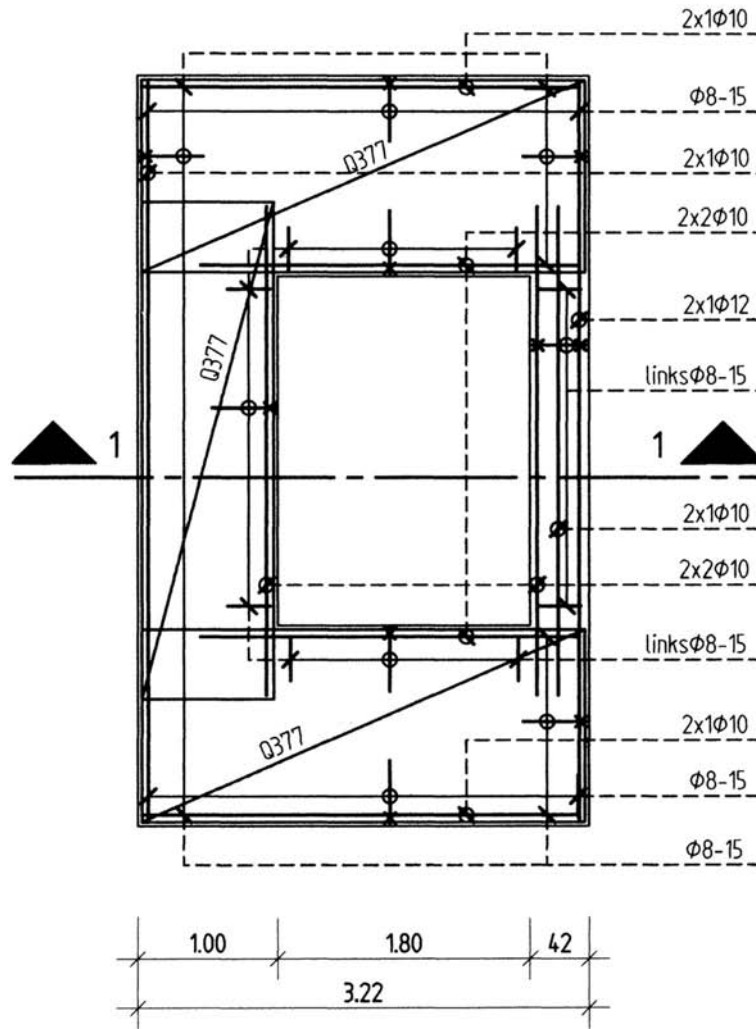


| | | | | |
|--------------------|---|---|---|-------------------------------|
| $\max F$ | = | $135+70.6$ | = | 206 kN |
| Maximum tie force: | | | | |
| $\max T$ | = | $\max F \cos \alpha$ | = | $206 \cos 75^\circ$ |
| | | | = | 79 kN |
| $A_{s,req}$ | = | $79 \cdot 10^3 / 435$ | = | $1.8 \cdot 1$ |
| $A_{s,prov}$ | = | $2 \cdot \emptyset 12 + 2 \cdot \emptyset 10$ | = | $3.8 \cdot 10^2 \text{ mm}^2$ |

Reference

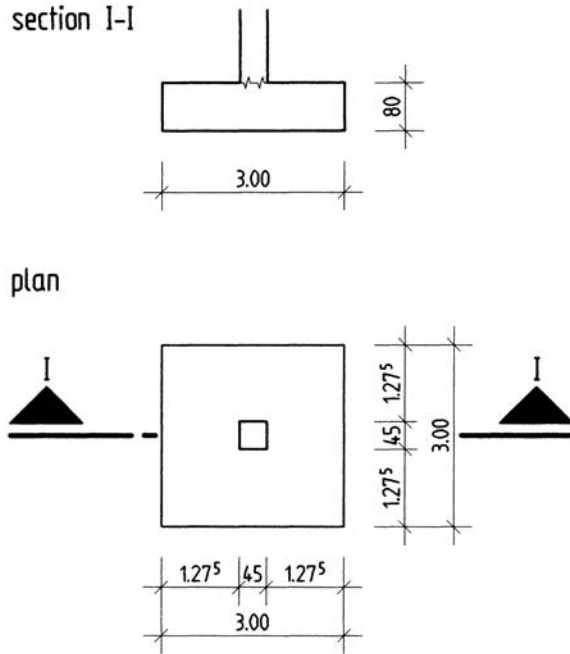
see 15.3.2.5, Figures for shear forces are considered
 see details of reinforcement

Reinforcement details



15.3.5 Block foundation

It is assumed that the foundation is subjected to an axial force N_{Sd} only which acts in the centre of gravity of the foundation slab. The axial force N_{Sd} results from the internal column in axis B/2 and is given by



$$\begin{aligned}
 N_{Sd} &= & &= & -3800 \text{ kN} \\
 \text{Design value of bending moment:} & & & & \\
 M_{sd} &= & N_{Sd} a(1-h_{col}/a)^2/8 & & \\
 &= & 3800*3.0*(1-0.45/3.0)^2/8 & &= & 1030 \text{ kNm}
 \end{aligned}$$

Reference

see 15.3.3 above
page 97 in [5]

Design for bending

Effective depth:

$$\begin{aligned}
 d_x &= & h_f - (\min c + \Delta h + \phi_x/2) & & \\
 &= & 0.800 - (0.040 + 0.010 + 0.012/2) & &= & 0.744 \text{ m} \\
 d_y &= & d_x - \phi = 0.744 - 0.012 & &= & 0.732 \text{ m} \\
 \mu_{Sds} &= & 1.030 / (3.0 * 0.732^2 * 20) & &= & 0.032 \\
 \omega &= & & &= & 0.033 \\
 A_{s,req} &= & 0.033 * 3000 * 732 * 20 / 435 & &= & 34 * 10^2 \text{ mm}^2
 \end{aligned}$$

Provided in both directions:

$$A_{s,prov} = 36 \phi 12 = 40.7 * 10^2 \text{ mm}^2$$

Design for punching shear

$$\begin{aligned}
 d_m &= & (0.744 + 0.732) / 2 & &= & 0.738 \\
 \text{Distance of the critical perimeter from the face of the column} & & & & & \\
 s &= & 1.5 * d_m = 1.5 * 0.738 & &= & 1.10 \text{ m} \\
 \text{Length of critical perimeter} & & & & & \\
 u &= & 4 * 0.45 + 2\pi * 1.10 & &= & 8.71 \text{ m} \\
 \text{Mean value of ground pressure due to} & & & & & \\
 \sigma_s &= & N_{sd} / a^2 = 3800 / 9.00 & &= & 422 \text{ kN/m}^2 \\
 \text{Area within critical perimeter} & & & & & \\
 &= & 0.45^2 + 4 * 0.45 * 1.10 + \pi * 1.10^2 & &= & 6.0 \text{ m}^2 \\
 \text{Critical force to be resisted} & & & & &
 \end{aligned}$$

$$\begin{aligned}
 V_{Sd} &= 3800 - 6.0 \cdot 422 &= 1268 \text{ kN} \\
 v_{Sd} &= 1268 / 6.0 &= 212 \text{ kN/m} \\
 \text{Design shear resistance of slabs without punching shear reinforcement:} \\
 \rho_1 &= 40.7 / (300 \cdot 73.8) &= 18\% \\
 k &= 1.6 - 0.738 &= 0.862 \\
 &&< 1.0 \\
 \tau_{Rd} &= &= 0.34 \text{ N/mm}^2 \\
 v_{Rd1} &= 1.0 \cdot 0.34 \cdot (1.2 + 40 \cdot 0.0018) \cdot 0.738 \cdot 10^3 &= 319 \text{ kN/m} \\
 &&> v_{Sd}
 \end{aligned}$$

Reference

EC2, 4.3.1

EC2, 4.1.3.3(9): min $c=40$ mm assumption: $\phi=12$ mm [1], [Table 7.1b](#)

spacing see reinforcement details below

EC2, 4.3.4

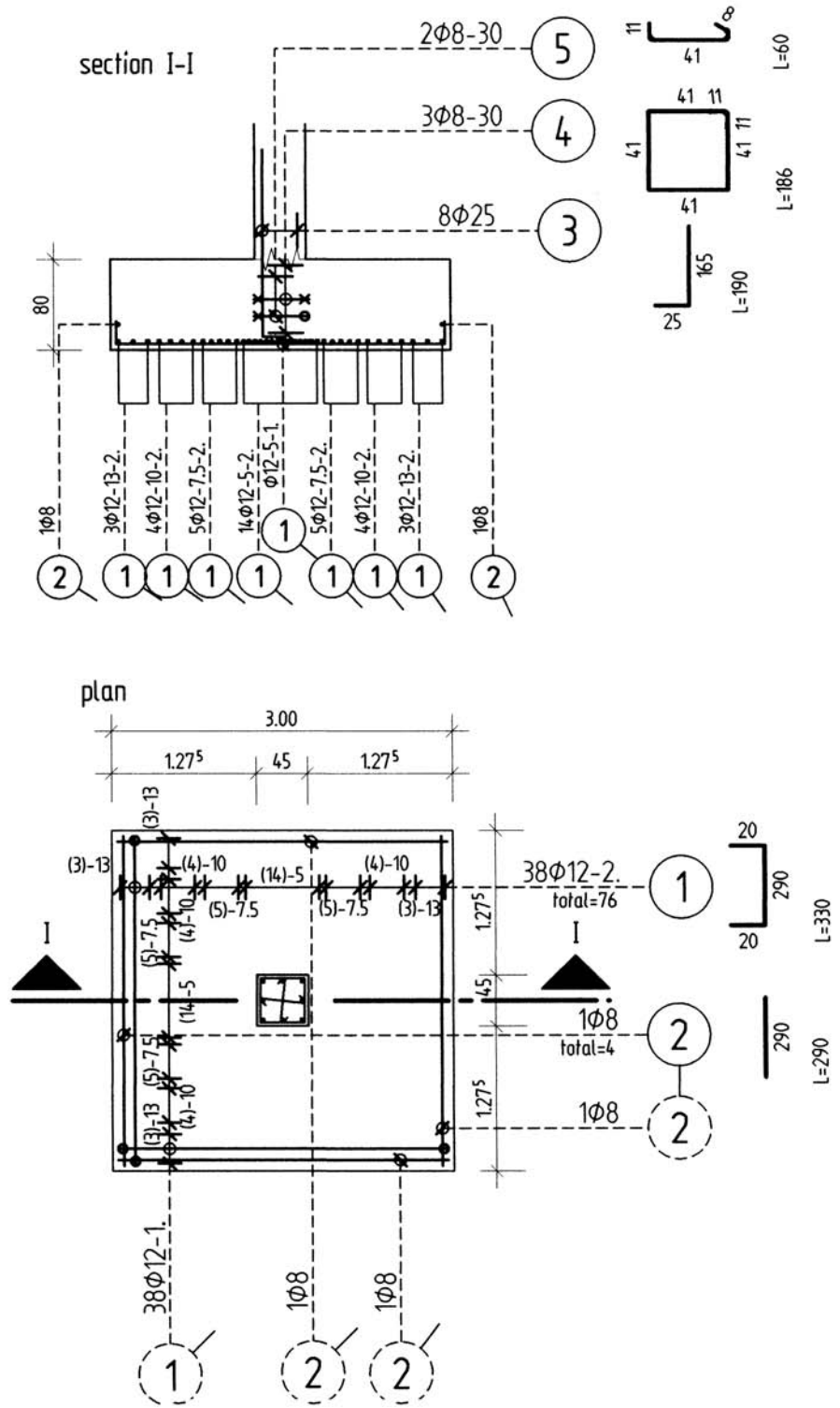
EC2, 4.3.4.2.2

EC2, 4.3.4.1(5)

EC2, Eq.(4.56)

for C 30/37

Reinforcement details



15.4
Calculation for a residential building



15.4.1.2
Basic data of structure, materials and loading

Intended use:

Fire resistance:

Loading (excluding self-weight of structure):

Residential building

1 hour for all elements

Continuous slab: -imposed: 2.0 kN/m²
-finishes: 1.5 kN/m²

Combination factors:

Serviceability limit states are not considered

Exposure classes:

Class 1 (indoor) for all members

Materials:

| | |
|-------------------------|----------------------|
| Concrete grade | C 30/37 |
| Steel grade | B500 |
| Self-weight of concrete | 25 kN/m ³ |

Reference

see floor plan

EC2–1.2, 1.3

EC1–2.1 for Category A EC1–2.1

EC2, [Table 4.1](#)

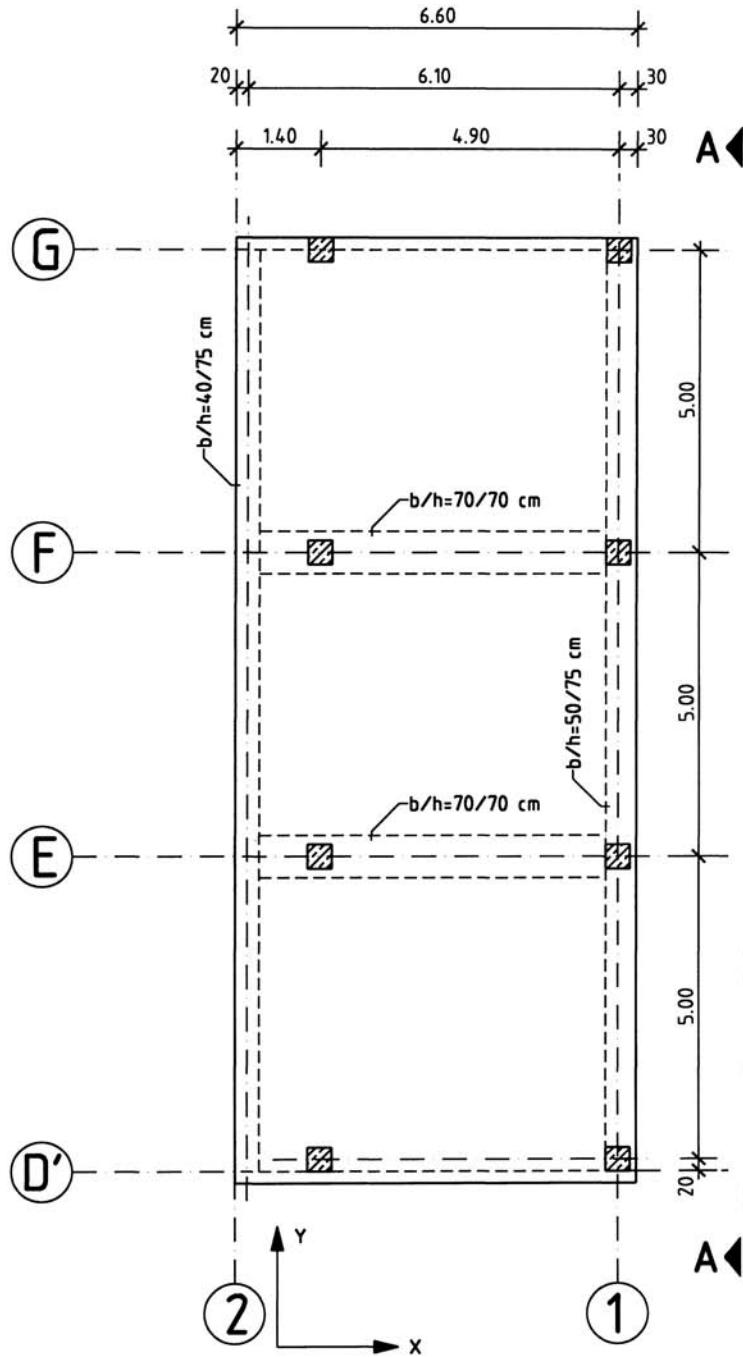
EC2, Table 3.1

ENV 10 080

EC1–2.1

15.4.2**Continuous slab (end span)****15.4.2.1****Floor span and idealization of the structure**

Floor plan of the continuous slab



Idealization of structure:

The end span of a two-way continuous slab is designed to Eurocode 2.

The effective spans are given as:

$$\begin{aligned}
 l_x &= 6.10 + 0.2/3 + 0.3/3 = 6.27\text{m} \\
 l_y &= 5.00\text{m}
 \end{aligned}$$

15.4.2.2 Limitation of deflections

Assumptions:

- admissible deflection is given by $l_y/250$
- σ_s 250 N/mm² in service conditions
- ρ_1 0.5%, i.e. concrete is considered as lightly stressed.

From Table 4.14 in Eurocode 2 with ϕ assumed as 10 mm:

| | | | | |
|-----------|---|--|---|---------|
| l_y/d | = | | = | 32 |
| d_{req} | = | 5.00/32 | = | 0.156 m |
| h_{req} | = | $d_{req} + \text{nom } c + \phi/2 = 0.156 + 0.025 + 0.005$ | = | 0.186 m |
| Selected: | | h | = | 0.25 m |

15.4.2.3 Actions

| | | | | |
|------------------------------|---|-------|---|------------------------|
| Self-weight of slab: 0.25*25 | = | | = | 6.25 kN/m ² |
| Finishes | = | | = | 1.50 kN/m ² |
| Permanent load | | G_k | = | 7.75 kN/m ² |
| Imposed load | | Q_k | = | 2.0 kN/m ² |

15.4.2.4 Structural analysis

In the present example, only the ultimate limit states are considered. The slab is analysed using the simplified yield-line method in [6], A3.2. This method is based on the following assumptions.

- high ductility reinforcement is used
- at the ultimate limit state for bending, the ratio x/d 0.25
- the spans in any one direction are approximately the same
- the loadings on the adjacent panels are approximately the same.

Loading on the panels:

| | | | | |
|----------------|---|-----------|---|-------------------------|
| $\gamma_G G_k$ | = | 1.35*7.75 | = | 10.50 kN/m ² |
| $\gamma_Q Q_k$ | = | 1.5*2.00 | = | 3.00 kN/m ² |
| | | | = | 13.50 kN/m ² |

Reference

EC2, 2.5.2.1
 EC2, Eq. (2.15)
 a_i in axis 1 and 2
 EC2, 4.4.3
 EC2, 4.4.3.1 and 4.4.3.2
 ϕ assumed as 10 mm
 EC2, 2.5
 EC2, 2.5.3.2.2
 EC2, 3.2.4.2
 [6], A3.2(1)
 $l_x = 5.0$ m for all span conditions

Design moment over the continuous edge ($l_y/l_x = 6.27/5.0 = 1.25$):

| | | | | |
|------------|---|-------------------|---|--------------|
| m_0 | = | $13.50 * 5.0^2$ | = | 337.5 kN |
| $m_{S,dx}$ | = | $-337.5 * 0.0735$ | = | -24.81 kNm/m |

Design span moment in x -direction:

| | | | | |
|------------|---|-----------------|---|--------------|
| $m_{Sd,x}$ | = | $337.5 * 0.055$ | = | +18.56 kNm/m |
|------------|---|-----------------|---|--------------|

Span moment in y -direction:

$$m_{Sd,y} = 337.5 * 0.044 = 14.85 \text{ kNm/m}$$

Maximum shear force:

$$V_{sd} = 0.52 * 13.5 * 5.0 = 35.1 \text{ kN/m}$$

15.4.2.5

Design at ultimate limit states for bending and axial force

$$f_{cd} = 30/1.5 = 20 \text{ N/mm}^2$$

$$f_{yd} = 500/1.15 = 435 \text{ N/mm}^2$$

$$d_x = 0.25 - 0.03 = 0.22 \text{ m}$$

Design of the continuous edge:

$$\mu_{Sds} = 24.81 * 10^{-3} / (1.0 * 0.22^2 * 20) = 0.026$$

$$A_{s,req} = 0.027 * 103 * 220 * 20 / 435 = 2.73 * 10^2 \text{ mm}^2/\text{m}$$

$$x/d = 0.067 < 0.25$$

Selected: Welded mesh with twin bars \varnothing 6.0 mm

Steel B 500 B - R 377 2 * \varnothing 6.0 - 150
 $A_{s,prov} = 3.77 * 10^2 \text{ mm}^2/\text{m}$

Design for the span moments:

x-direction:

$$\mu_{Sds} = 18.56 * 10^{-3} / (1.0 * 0.22^2 * 20) = 0.019$$

$$A_{s,req} = 0.020 * 103 * 220 * 20 / 435 = 2.02 * 10^2 \text{ mm}^2/\text{m}$$

y-direction:

$$\mu_{Sds} = 14.85 * 10^{-3} / (1.0 * 0.21^2 * 20) = 0.017$$

$$A_{s,req} = 0.018 * 10^3 * 210 * 20 / 435 = 1.74 * 10^2 \text{ mm}^2/\text{m}$$

Selected in *x*-direction:

Welded mesh with twin bars \varnothing 5.5 mm

Steel B 500 B - R 317 2 * \varnothing 5.5 - 150
 $A_{s,prov} = 3.17 * 10^3 \text{ mm}^2/\text{m}$
 $A_{s,prov} = 0.64 * 10^2 \text{ mm}^2/\text{m}$

Reference

[6], A3.2(1), and Table A2 three edges discontinuous, one edge continuous

[6], Eq. (A5)

three edges discontinuous, one edge continuous [6], A3.2. (2)

EC2, 4.3.1

see 15.4.2.2 above

Table 7.1 (b) in [1]

see 15.4.2.7 and 15.4.2.8.1 below

Table 7.1(b) in [1]

see 15.4.2.7 and 15.4.2.8.1 below

in *x*-direction

in *y*-direction

Additional span reinforcement in *y*-direction:

Selected: Welded mesh fabric with bars \varnothing 7.0

Steel B 500 B - R 257 \varnothing 7.0 - 150

$$A_{s,prov} = 2.57 \cdot 10^2 \text{ mm}^2/\text{m}$$

$$A_{s,prov} = 0.64 \cdot 10^2 \text{ mm}^2/\text{m}$$

Total reinforcement in y-direction:

$$A_{s,prov} = 0.64 \cdot 10^2 + 2.57 \cdot 10^2 = 3.21 \cdot 10^2 \text{ mm}^2/\text{m}$$

15.4.2.6 Design for shear

$$\begin{aligned} \tau_{Rd} &= & &= 3.21 \cdot 10^2 \text{ mm}^2/\text{m} \\ \rho_l &= 3.77 / (100 \cdot 22) &= &= 0.17\% \\ k &= 1.6 - 0.22 &= &= 1.38 \\ V_{Rd1} &= 0.34 \cdot 1.38 \cdot (1.2 + 40 \cdot 0.0017) \cdot 0.22 \cdot 10^3 &= &= 130 \text{ kN/m} \\ &&> &> V_{sd} \end{aligned}$$

Design shear resistance of compression struts:

$$\begin{aligned} V_{Rd2} &= 0.5 \cdot 0.575 \cdot 20 \cdot 0.9 \cdot 0.22 \cdot 10^3 &= &= 1138 \text{ kN/m} \\ &&> &> V_{sd} \end{aligned}$$

15.4.2.7 Minimum reinforcement for crack control

$$\begin{aligned} A_s &= k_c k f_{ct,eff} A_{ct} / \sigma_s \\ \text{where} & & & \\ k_c &= 0.4 \text{ for bending} \\ k &= 0.8 \text{ for } h \leq 300 \text{ mm} \\ f_{ct,eff} &= 3.0 \text{ N/mm}^2 \\ A_{ct} &= 0.25 / 2 \cdot 1.0 &= &= 0.125 \text{ m}^2 \\ \sigma_s &= 400 \text{ N/mm}^2 \\ A_s &= 0.4 \cdot 0.8 \cdot 3 \cdot 0.125 \cdot 10^6 / 400 &= &= 3.0 \cdot 10^2 \text{ mm}^2/\text{m} \\ A_{s,prov} &= 3.21 \cdot 10^2 \text{ mm}^2/\text{m} &> &> 3.0 \cdot 10^2 \text{ mm}^2/\text{m} \end{aligned}$$

15.4.2.8 Detailing of reinforcement

15.4.2.8.1 Minimum reinforcement areas for the avoidance of brittle failure

$$\begin{aligned} A_{s,min} &= 0.0015 \cdot 220 \cdot 10^3 &= &= 3.3 \cdot 10^2 \text{ mm}^2/\text{m} \\ A_{s,prov} &= 3.17 \cdot 10^2 + 0.64 \cdot 10^2 &= &= 3.81 \cdot 10^2 \text{ mm}^2/\text{m} \end{aligned}$$

15.4.2.8.2 Basic anchorage length

$$\begin{aligned} l_b &= 0.25 \cdot \sigma / f_{yd} / f_{bd} \text{ or} \\ l_b &= 0.25 \cdot \sigma / f_{yd} / f_{bd} \\ f_{bd} &= &= &= 3.0 \text{ N/mm}^2 \end{aligned}$$

Reference
in x-direction

in y-direction
 EC2, 4.3.2
 EC2, Table 4.8 for C30/37
 EC2, Eq. (4.18)
 EC2, Eq. (4.19)
 EC2, 4.4.2.2
 EC2, Eq. (4.78)
 from EC2, Table 4.11, column 2, for ϕ 8 mm
 EC2, 5
 EC2, 5.4.2.1.1
 EC2, Eq. (5.14) in x-direction
 EC2, 5.2.2.2
 EC2, Eq. (5.3) for twin bars
 EC2, Table 5.3, for C30/37

| Location | | ϕ (mm) | ϕ_n (mm) | l_b (mm) |
|----------|---|-------------|---------------|------------|
| Support | | 6.0 | 8.5 | 310 |
| Span | x | 5.5 | 7.8 | 290 |
| | y | 4.5 | - | 170 |

15.4.2.8.3

Anchorage at the discontinuous edges

$$\begin{aligned}
 F_s &= V_{sd} a_1 / d + N_{sd} \\
 a_1 &= d \\
 F_s &= 35.1 * 1.0 + 0 &= 35.1 \text{ kN/m} \\
 A_{s,req} &= 35.1 * 103 / 435 &= 0.81 * 10^2 \text{ mm}^2/\text{m} \\
 \text{Required anchorage length:} \\
 l_{b,net} &= 0.7 * 290 * 0.81 / 3.17 &= 52 \text{ mm} \\
 \text{Minimum values:} \\
 l_{b,min} &= 0.3 * 290 &= 87 \text{ mm} \\
 &= 10\phi = 10 * 5.5 = 10 * 5.5 &= 55 \text{ mm} \\
 &= &= 100 \text{ mm} \\
 \text{Anchorage length:} \\
 l_{b,anch} &= 2/3 * 100 &= 70 \text{ mm}
 \end{aligned}$$

15.4.2.8.4

Anchorage at the continuous edges

$$l_{b,anch} = 10\phi = 10 * 7.8 = 80 \text{ mm}$$

15.4.2.8.5

Lap lengths of mesh fabrics in y-direction

$$\begin{aligned}
 l_s &= \alpha_2 l_b A_{s,req} / A_{s,prov} \\
 \alpha_2 &= 0.4 + 64 / 800 &= 0.48 \\
 &&< 1.00 \\
 l_s &= 1.0 * 170 * 10 &= 170 \text{ mm} \\
 l_{s,min} &= 0.3 * 1.0 * 170 &= 51 \text{ mm} \\
 &= s_t &= 150 \text{ mm} \\
 &= &= 200 \text{ mm}
 \end{aligned}$$

Reference

for mesh fabric R 317

EC2, 5.4.3.2.1(5)

EC2, Eq. (5.15)

EC2, 5.4.3.2.1(1)

EC2, 5.2.3.4.1

EC2, Eq. (5.4), and 5.2.3.4.2(2)

The largest value of $l_{b,min}$ should be used

EC2, 5.4.2.1.4(3)

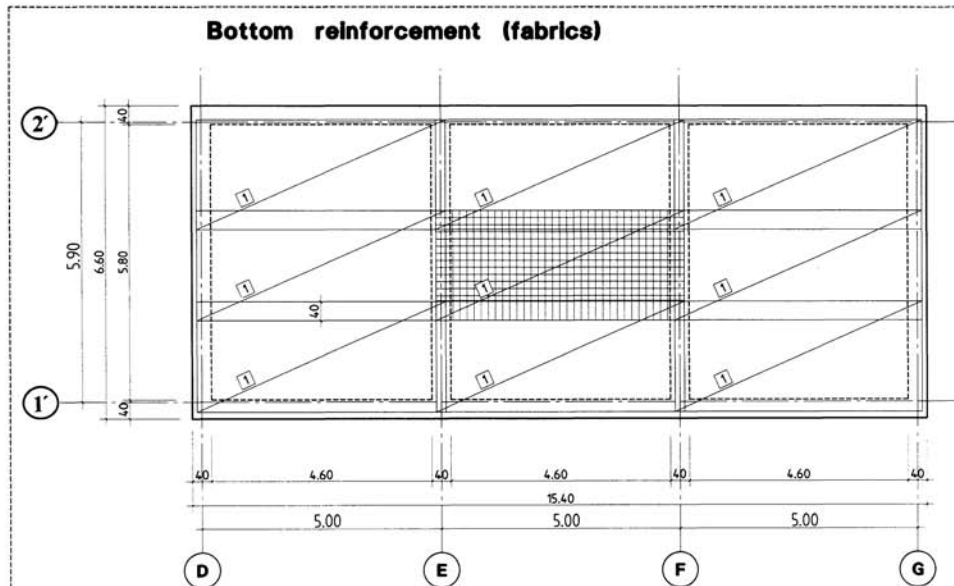
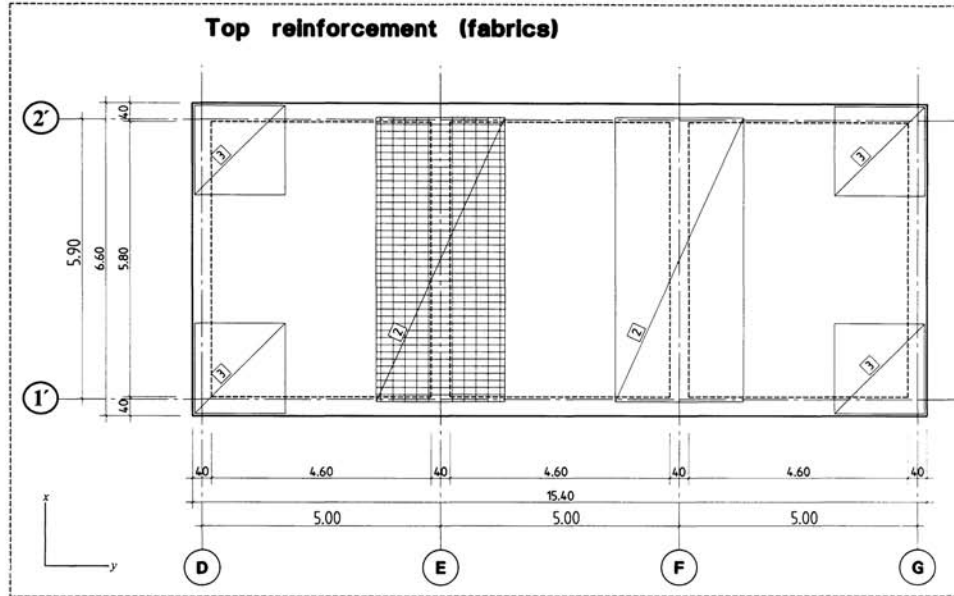
EC2, 5.4.3.2.1(5)

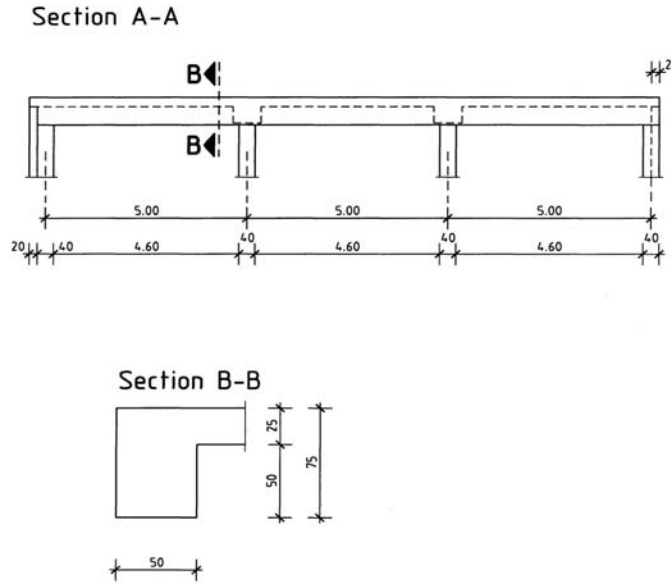
Lapping of mesh fabrics R 317 with bar diameter ϕ 4.5

EC2, Eq. (5.9)

The largest value of $l_{s,min}$ be used

Detailing of reinforcement





15.4.3 Continuous edge beam (end span)

15.4.3.1 Structural system

15.4.3.2 Actions

The beam is subjected to the following actions (see sketch below):

(a) **permanent actions**

- self-weight, $G_{k,1}$
- self-weight of parapets, $G_{k,2}$
- self-weight of supported slab, $G_{k,3}$
- self-weight of supported facade elements, $G_{k,4}$ to $G_{k,9}$
- concentrated forces due to permanent load, $G_{k,10}$ and $G_{k,11}$

(b) **variable actions**

- imposed load of the adjacent slab, $Q_{k,1}$
- variable actions transmitted by the facade elements, $Q_{k,2}$ to $Q_{k,7}$
- concentrated variable loads, $Q_{k,8}$ and $Q_{k,9}$.

Reference

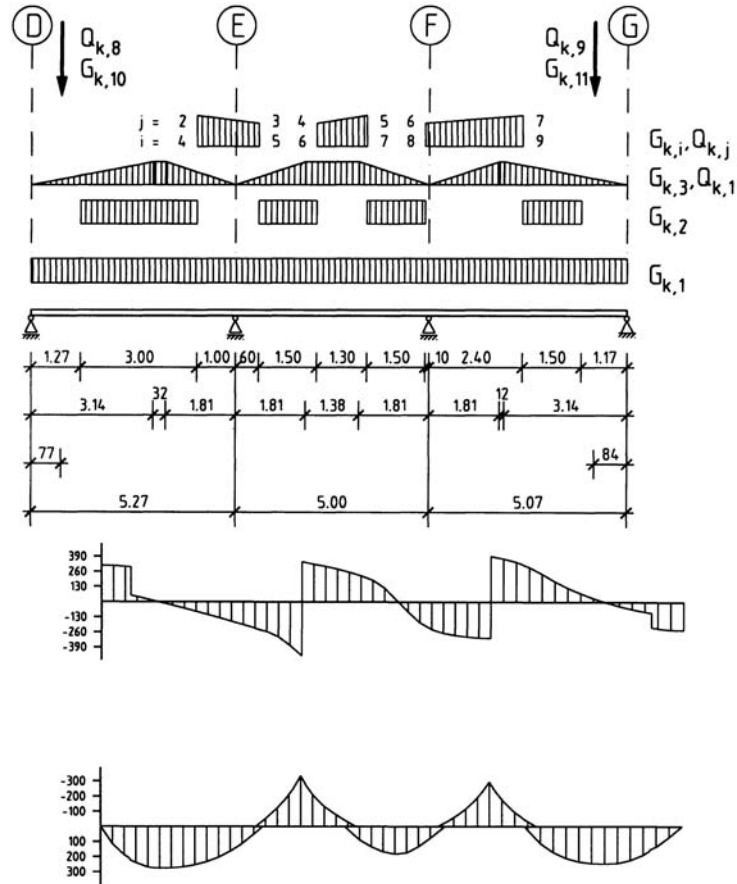
see floor plan below
see 15.4.2 above

In the following it is assumed that neither the permanent nor the variable actions are dependent upon each other.

Table 15.3: Permanent, $G_{k,j}$, and variable actions, $Q_{k,j}$, acting on the beam

| Action | Magnitude of the actions (kN/m; kN) | | | | | | | | | | | |
|--------------|-------------------------------------|------|-------|-------|--------|-------|--------|--------|-------|--------|--------|-------|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | |
| subscript i= | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | |
| subscript j= | - | - | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | |
| $G_{k,i}$ | $\gamma_o=1.0$ | 6.25 | 41.20 | 24.32 | 198.60 | 19.40 | 105.50 | 154.10 | 44.60 | 109.00 | 152.86 | 86.70 |

| Action | Magnitude of the actions (kN/m; kN) | | | | | | | | | | | |
|-----------------|-------------------------------------|-------|-------|--------|-------|--------|--------|-------|--------|--------|--------|-------|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | |
| subscript i= | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | |
| subscript j= | - | - | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | |
| $\gamma_0=1.35$ | 8.44 | 55.62 | 32.84 | 268.11 | 26.19 | 208.04 | 208.04 | 60.12 | 147.15 | 206.36 | 117.05 | |
| $Q_{k,j}$ | $\gamma_Q=1.0$ | - | - | 6.28 | 20.60 | 2.00 | 16.50 | 24.00 | 3.40 | 8.40 | 18.36 | -3.90 |
| $\gamma_Q=1.5$ | - | - | 9.42 | 30.90 | 3.00 | 24.75 | 36.00 | 5.10 | 12.60 | 27.54 | -5.85 | |



15.4.3.3 Structural analysis

(a) Linear analysis without redistribution

The action effects resulting from a linear analysis without redistribution are summarized below.

Reference

EC2, 2.5.3.4.2

Schematic shear and moment diagram

Support reactions

| Support | Support reaction (kN) due to | | | | |
|---------|------------------------------|----------------|----------------|-------------------|--------|
| | $G_{k,i}$ | $\max Q_{k,j}$ | $\min Q_{k,j}$ | $G_{k,i}+Q_{k,j}$ | |
| D | | 276.05 | 310.43 | 271.71 | 306.10 |
| E | | 717.47 | 807.24 | 711.78 | 801.55 |
| F | | 668.51 | 749.30 | 660.05 | 740.84 |
| G | | 212.23 | 226.48 | 202.75 | 217.00 |

Bending moments and shear forces in spans 1 to 3

| Span | x (m) | max V_{Sd} (kN) | min V_{Sd} (kN) | max M_{Sd} (kNm) | min M_{Sd} |
|------|-------|-------------------|-------------------|--------------------|--------------|
| 1 | 0.00 | 310.43 | 271.71 | 0.00 | 0.00 |
| | 0.77 | | | | 205.91 |
| | 1.32 | 51.09 | 42.51 | 268.41 | 233.61 |
| | 1.91 | | | 283.85 | |
| | 2.64 | -63.90 | -73.58 | 259.60 | 218.08 |
| | 3.95 | -188.03 | -209.11 | 79.70 | 44.81 |
| | 5.27 | -425.06 | -472.84 | -332.74 | -383.43 |
| 2 | 0.00 | 334.40 | 286.72 | -332.74 | -383.43 |
| | 1.25 | 221.59 | 182.91 | -23.95 | -58.02 |
| | 2.50 | 42.99 | 25.87 | 157.08 | 97.69 |
| | 2.65 | | | 159.91 | 100.12 |
| | 3.75 | -198.25 | -242.77 | 8.30 | -28.24 |
| | 5.00 | -293.13 | -342.25 | -323.75 | -374.64 |
| 3 | 0.00 | 407.05 | 366.92 | -323.77 | -374.66 |
| | 2.86 | | | 265.26 | |
| | 4.23 | | | | 166.29 |
| | 5.07 | -202.76 | -226.49 | -0.04 | - |

Reference

shear diagram
moment diagram

(b) Linear analysis with redistribution

The cross-section over support E will be designed for the design bending moment

$$M_{sd} = -333 \text{ kNm}$$

This corresponds to a distribution factor δ of

$$\delta = 332.74/383.43 = 0.867$$

15.4.3.4 Design of span 1 for bending

Design data:

| | | | |
|---|----------|---|--|
| C 30/37 | f_{cd} | = | 20 N/mm ² |
| B500 B | f_{yd} | = | 435 N/mm ² |
| effective depth | d | = | 0.71 m |
| Design of the cross-section of support E: | | | |
| b_w | = | = | 0.50 m |
| μ_{Sds} | = | = | 0.066 |
| ω | = | = | 0.070 |
| x/d | = | = | 0.139 |
| $A_{s,req}$ | = | = | 11.5 * 10 ² mm ² |
| δ_{perm} | = | = | 0.62 |
| | | < | 0.867 |

Selected 4 ϕ 20; $A_{s,prov} = 12.56 * 10^2 \text{ mm}^2$

Design for maximum span moment:

| | | | |
|-----------------|---|-------------------------|----------|
| effective width | | | |
| b_{eff} | = | 0.5 + 0.1 * 0.85 * 5.27 | = 0.95 m |
| M_{Sd} | = | 284 kNm | |

$$\begin{aligned} \mu_{s,d} &= 0.284/(0.95*0.712*20) &= 0.03 \\ \omega &= &= 0.031 \\ A_{s,req} &= 0.031*950*710*20/435 &= 9.61*10^2 \text{ mm}^2 \end{aligned}$$

Selected 2 ϕ 25; $A_{s,prov}=9.81*10^2 \text{ mm}^2$

Reference

EC2, 2.5.3.4.2 see Table above

EC2, 2.5.3.4.2(3)

for bar diameter ϕ 25

[1], Table 7.1a

permissible coefficient δ

EC2, 2.5.2.2.1 for an L-beam see Table above

[1], Table 7.1a

15.4.3.5 Design for shear

$$\begin{aligned} \max V_{sd} &= &= 473 \text{ kN} \\ \text{Design shear at the distance } d \text{ from the face of the support:} \\ V_{sd} &= \max V_{sd} - d G_{k,1} &= 467 \text{ kN} \\ \text{The variable strut-inclination method is used; assumption:} \\ \cot \theta &= &= 1.25 \\ v &= 0.7 - 30/200 &= 0.55 \\ V_{Rd2} &= 0.50 * 0.9 * 0.71 * 0.55 * 20 / 2.05 * 10^3 &= 1714 \text{ kN} \\ (A_{sw}/s)_{req} &= 467 * 10^3 / (0.9 * 0.71 * 435 * 1.25) &= 13.44 * 10^2 \text{ mm}^2/\text{m} \end{aligned}$$

Selected stirrups ϕ 12 - spacing $s=150 \text{ mm}$

$$\begin{aligned} (A_{sw}/s)_{prov} &= &= 15.07 * 10^2 \text{ mm}^2/\text{m} \\ \text{maximum spacing:} \\ V_{sd}/V_{Rd2} &= 467/1714 &= 0.273 \\ s_{max} &= &= 300 \text{ mm} \\ &> 150 \text{ mm} \\ (A_{sw}/s)_{min} &= 0.0011 * 500 * 1 * 1000 &= 5.5 * 10^2 \text{ mm}^2/\text{m} \end{aligned}$$

15.4.3.6 Control of cracking

Cracking is controlled by limiting the bar diameter ϕ . The steel stress σ_s is estimated as

$$\begin{aligned} \sigma_s &= f_{yd} A_{s,req} / A_{s,prov} (1/\gamma_F) \\ &= 435 * 9.61 / 9.81 * (1/1.5) &= 280 \text{ N/mm}^2 \end{aligned}$$

From Table 4.11 in EC2 for reinforced concrete:

$$\begin{aligned} \phi_s^* &= &= 16 \text{ mm} \\ \phi_s &= 16 * 71 / (10 * 4) &= 28 \text{ mm} \\ &> 25 \text{ mm} \end{aligned}$$

15.4.3.7 Detailing of reinforcement

Basic anchorage length

$$l_b = 0.25 \cdot 2 \cdot 20 \cdot 10^{-3} \cdot 435 / 2.8 = 1.10 \text{ m}$$

$$l_b = 0.25 \cdot 25 \cdot 10^{-3} \cdot 435 / 2.8 = 0.97 \text{ m}$$

Reference

EC2, 4.3.2

see Table above

see diagram of actions above; the opposite formula is a conservative assumption

EC2, 4.3.2.4.4

 $\theta = 40^\circ$

EC2, Eq. (4.26)

 $\alpha = 90^\circ$

EC2, 5.4.2.2(7)

EC2, Table 5.5

EC2, 4.4.2

EC2, Table 4.11

in span

EC2, 5

for $\phi = 20 \text{ mm}$ for $\phi = 25 \text{ mm}$

Anchorage of bottom reinforcement

- intermediate support

$$l_{b,net} = 10 \cdot 25 \cdot 10^{-3} = 0.25 \text{ m}$$

- end support

$$V_{Sd} = 311 \text{ kN}$$

$$\alpha_1 = 0.9 \cdot 0.71 \cdot 1.25 / 2 = 0.40 \text{ m}$$

$$F_s = 311 \cdot 0.4 / 0.71 = 175 \text{ kN}$$

$$A_{s,req} = 175 \cdot 10^3 / 435 = 4.0 \cdot 10^2 \text{ mm}^2$$

$$l_{b,net} = 1.0 \cdot 0.97 \cdot 4.0 / 9.81 = 0.396 \text{ m}$$

$$2/3 l_{b,net} = 2/3 \cdot 0.396 = 0.26 \text{ m}$$

Anchorage of the top reinforcement

$$l_{b,net} = 0.3 \cdot 1.10 = 0.33 \text{ m}$$

$$\text{or } d = 0.71 \text{ m}$$

Reference

EC2, 5.4.2.1.5

see Table above

EC2, 5.4.2.1.3(1)

EC2, 5.4.2.1.4(2)

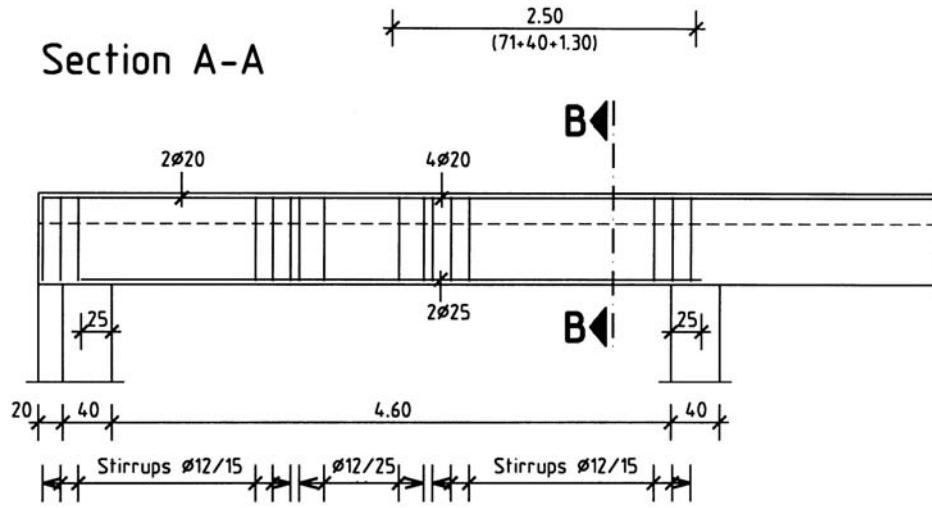
EC2, Eq. (5.4) for straight bars

EC2, Eq. (5.5)

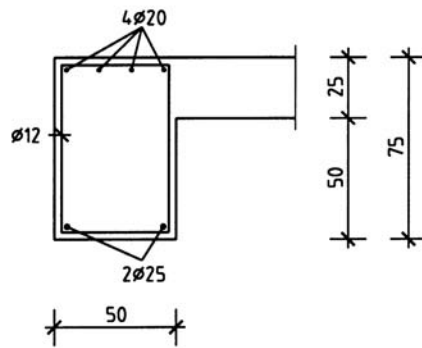
EC2, 5.4.2.1.3(2)

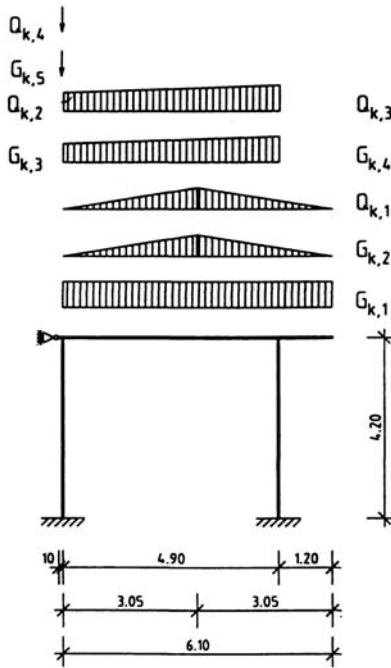
Detailing of reinforcement

Section A-A



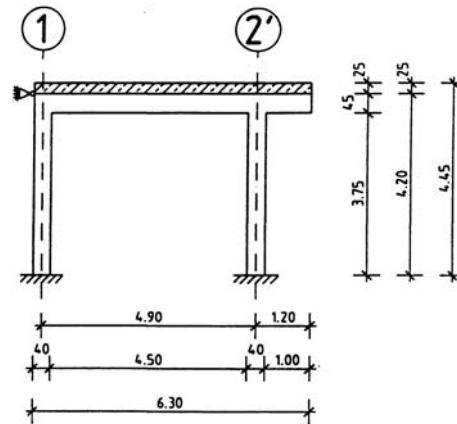
Section B-B





15.4.4
Braced transverse frame in axis E

15.4.4.1
Structural system; cross-sectional dimensions



15.4.4.2
Actions

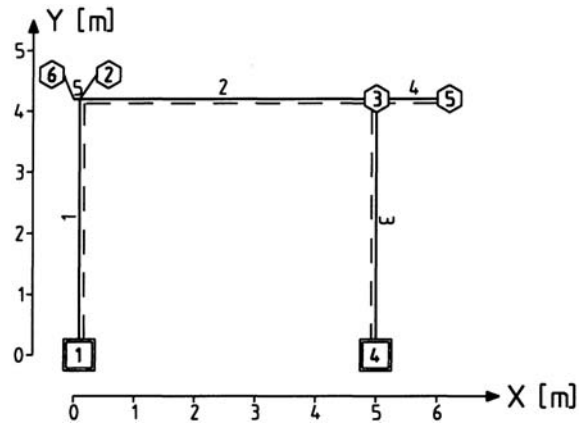
The frame is subjected to the following actions (see Figure above):

(a) **permanent actions**

- self-weight of beam, $G_{k,1}$
- self-weight of supported slab, $G_{k,2}$

Reference

see floor plan in 15.4.2.1



- self-weight of supported slab, $G_{k,3}$; $G_{k,4}$
- support reaction of continuous beam, $G_{k,5}$.

(b) **variable actions**

- imposed load of supported slab, $Q_{k,1}$
- imposed load of supported slab, $Q_{k,2}$; $Q_{k,3}$
- support reaction of continuous beam, $Q_{k,4}$.

| Action | | Magnitude of the actions (kN/m; kN) | | | | |
|-----------------|----------------|-------------------------------------|------|-------|-------|-------|
| | | 1 | 2 | 3 | 4 | 5 |
| subscript i= | | 1 | 2 | 3 | 4 | 5 |
| subscript j= | | - | 1 | 2 | 3 | 4 |
| $G_{k,i}$ | $\gamma_G=1.0$ | 7.85 | 28.1 | 30.0 | 227.0 | 540.4 |
| $\gamma_G=1.35$ | 10.60 | 37.9 | 40.5 | 306.5 | 729.5 | |
| $Q_{k,j}$ | $\gamma_Q=1.0$ | - | 7.3 | 3.0 | 70.0 | 63.2 |
| $\gamma_Q=1.5$ | - | 10.9 | 4.5 | 105.0 | 94.8 | |

It is assumed that all permanent actions and all imposed loads act simultaneously.

15.4.4.3 Structural analysis

For the purposes of structural analysis, the frame is subdivided into elements and nodes as shown below.

Reference

see 15.4.3 above
EC2, 2.5

| Node | Coordinates | | Support conditions | | |
|------|-------------|-------|--------------------|---|---|
| | x (m) | y (m) | x | y | m |
| 1 | 0.10 | 0.00 | 1 | 1 | 1 |
| 2 | 0.10 | 4.20 | 0 | 0 | 0 |
| 3 | 5.00 | 4.20 | 0 | 0 | 0 |
| 4 | 5.00 | 0.00 | 1 | 1 | 1 |
| 5 | 6.20 | 4.20 | 0 | 0 | 0 |
| 6 | 0.00 | 4.20 | 0 | 0 | 0 |

x: free in x-direction
y: free in y-direction

| | |
|------------|---------------|
| <i>m</i> : | free rotation |
| 1: | no freedom |
| 0: | freedom |

| Element left | Defined by node | | A |
|-----------------|-----------------|-------------------|-------|
| | right | (m ²) | |
| 1 | 1 | 2 | 0.160 |
| 2 | 2 | 3 | 0.677 |
| 3 | 3 | 4 | 0.160 |
| 4 | 3 | 5 | 0.677 |
| 5 | 2 | 6 | 0.677 |

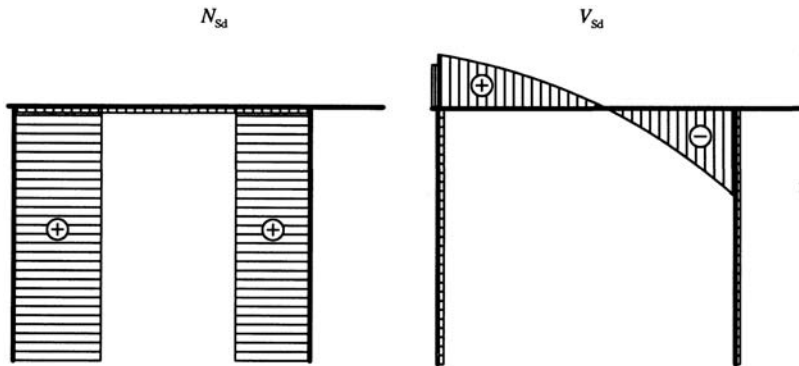
Reference

Action effects due to permanent actions

| Element No. | <i>x</i> (m) | <i>N</i> _{Sd} (kN) | <i>V</i> _{Sd} (kN) | <i>M</i> _{Sd} (kNm) |
|-------------|--------------|-----------------------------|-----------------------------|------------------------------|
| 1 | 0.00 | -1146.09 | -39.60 | -15.58 |
| 0.70 | -1146.09 | -39.60 | -43.30 | |
| 1.40 | -1146.09 | -39.60 | -71.02 | |
| 2.10 | -1146.09 | -39.60 | -98.73 | |
| 2.80 | -1146.09 | -39.60 | -126.45 | |
| 3.50 | -1146.09 | -39.60 | -154.17 | |
| 4.20 | -1146.09 | -39.60 | -181.88 | |
| 2 | 0.00 | -39.60 | 415.55 | -254.88 |
| 0.65 | -39.60 | 368.58 | | |
| 0.82 | -39.60 | 351.52 | 61.50 | |
| 1.63 | -39.60 | 243.13 | 307.13 | |
| 2.45 | -39.60 | 90.11 | 446.29 | |
| 2.85 | -39.60 | | 464.46 | |
| 3.27 | -39.60 | -106.85 | 442.29 | |
| 4.08 | -39.60 | -335.93 | 263.53 | |
| 4.70 | -39.60 | -526.38 | | |
| 4.90 | -39.60 | -593.22 | -114.13 | |
| 3 | 0.00 | -614.92 | 39.60 | -102.90 |
| 0.70 | -614.92 | 39.60 | -75.18 | |
| 1.40 | -614.92 | 39.60 | -47.46 | |
| 2.10 | -614.92 | 39.60 | -19.75 | |
| 2.60 | -614.92 | 39.60 | | |
| 2.80 | -614.92 | 39.60 | 7.97 | |
| 3.50 | -614.92 | 39.60 | 35.69 | |
| 4.20 | -614.92 | 39.60 | 63.40 | |
| 4 | 0.00 | 0.00 | 21.70 | -11.23 |
| 0.20 | 0.00 | 16.84 | -7.38 | |
| 0.40 | 0.00 | 12.48 | -4.46 | |
| 0.60 | 0.00 | 8.61 | -2.36 | |
| 0.80 | 0.00 | 5.25 | -0.98 | |
| 1.00 | 0.00 | 2.37 | -0.23 | |
| 1.20 | 0.00 | | | |
| 5 | 0.00 | 0.00 | -730.53 | 73.00 |
| 0.02 | 0.00 | -730.35 | 60.58 | |

Action effects due to permanent actions

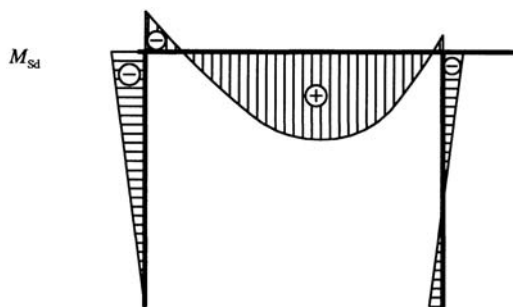
| Element No. | x (m) | N_{Sd} (kN) | V_{Sd} (kN) | M_{Sd} (kNm) |
|-------------|---------|---------------|---------------|----------------|
| 0.03 | 0.00 | -730.18 | 48.90 | |
| 0.05 | 0.00 | -730.00 | 36.49 | |
| 0.07 | 0.00 | -729.82 | 24.08 | |
| 0.08 | 0.00 | -729.65 | 12.40 | |
| 0.10 | 0.00 | -729.47 | | |


Qualitative presentation of action effects due to permanent actions
Reference
Action effects due to imposed loads

| Element | x (m) | N_{Sd} (kN) | V_{Sd} (kN) | M_{Sd} (kNm) |
|---------|---------|---------------|---------------|----------------|
| 1 | 0.00 | -208.12 | -12.32 | -8.34 |
| 0.70 | -208.12 | -12.32 | -16.97 | |
| 1.40 | -208.12 | -12.32 | -25.59 | |
| 2.10 | -208.12 | -12.32 | -34.21 | |
| 2.80 | -208.12 | -12.32 | -42.84 | |
| 3.50 | -208.12 | -12.32 | -51.46 | |
| 4.20 | -208.12 | -12.32 | -60.09 | |
| 2 | 0.00 | -12.32 | 113.35 | -69.56 |
| 0.65 | -12.32 | 105.73 | | |
| 0.82 | -12.32 | 101.64 | 19.35 | |
| 1.63 | -12.32 | 73.91 | 92.07 | |
| 2.45 | -12.32 | 30.08 | 135.64 | |
| 2.85 | -12.32 | 142.39 | | |
| 3.27 | -12.32 | -29.64 | 136.86 | |
| 4.08 | -12.32 | -101.87 | 83.97 | |
| 4.70 | -12.32 | -165.44 | | |
| 4.90 | -12.32 | -185.48 | -32.64 | |
| 3 | 0.00 | -188.04 | 12.32 | -31.62 |
| 0.70 | -188.04 | 12.32 | -23.00 | |
| 1.40 | -188.04 | 12.32 | -14.37 | |
| 2.10 | -188.04 | 12.32 | -5.75 | |
| 2.60 | -188.04 | 12.32 | | |
| 2.80 | -188.04 | 12.32 | 2.88 | |
| 3.50 | -188.04 | 12.32 | 11.50 | |
| 4.20 | -188.04 | 12.32 | 20.12 | |
| 4 | 0.00 | 0.00 | 2.56 | -1.02 |
| 0.20 | 0.00 | 1.78 | -0.59 | |

Action effects due to imposed loads

| Element | x (m) | N_{Sd} (kN) | V_{Sd} (kN) | M_{Sd} (kNm) |
|---------|---------|---------------|---------------|----------------|
| 0.40 | 0.00 | 1.14 | -0.30 | |
| 0.60 | 0.00 | 0.64 | -0.13 | |
| 0.80 | 0.00 | 0.28 | -0.04 | |
| 1.00 | 0.00 | 0.07 | 0.00 | |
| 1.20 | 0.00 | | | |
| 5 | 0.00 | 0.00 | -94.77 | 9.48 |
| 0.02 | 0.00 | -94.77 | 7.87 | |
| 0.03 | 0.00 | -94.77 | 6.35 | |
| 0.05 | 0.00 | -94.77 | 4.74 | |
| 0.07 | 0.00 | -94.77 | 3.13 | |
| 0.08 | 0.00 | -94.77 | 1.61 | |
| 0.10 | 0.00 | -94.77 | | |

**Reference****15.4.4.4****Design for the ultimate limit states****15.4.4.4.1****Basic data**

Concrete C 30/37
Steel B 500

$$f_{cd} = 20 \text{ N/mm}^2$$

$$f_{yd} = 435 \text{ N/mm}^2$$

15.4.4.4.2**Design of the beam for the ultimate limit states of bending and longitudinal force****(a) Design value of the acting bending moment M_{Sd} in node 2**

$$M_{Sd} = -254.88 - 69.56 = -324.44 \text{ kNm}$$

Bending moment at the face of the support:

$$M'_{Sd} = -324.44 + (415.55 + 113.35) * 0.2 = -219 \text{ kNm}$$

$$d = 0.645 \text{ m}$$

$$\mu_{Sds} = 0.219 / (0.7 * 0.645^2 * 20) = 0.038$$

$$\omega = 0.04$$

$$A_{s,req} = 0.04 * 700 * 645 * 20 / 435 = 8.30 * 10^2 \text{ mm}^2$$

Selected 5 ϕ 16; $A_{s,prov} = 10.1 * 10^2 \text{ mm}^2$

(b) Design in mid-span

$$\begin{aligned}
 M_{sd} &= 464.46+142.39 &= 607 \text{ kNm} \\
 d &= &= 0.645 \text{ m} \\
 \text{Effective width for a T-beam:} \\
 l_0 &= 0.7*4.90 &= 3.43 \text{ m} \\
 b_{\text{eff}} &= 0.7+0.2*3.43 &= 1.39 \\
 \mu_{Sds} &= 0.607/(1.39*0.645^2*20) &= 0.053 \\
 \omega &= &= 0.056 \\
 x/d &= &= 0.13 \\
 x &= 0.13*0.645 &= 0.084 \text{ m} \\
 & &< 0.25 \text{ m} \\
 A_{s,\text{req}} &= 0.056*1390*645*20/435 &= 23.1*10^2 \text{ mm}^2
 \end{aligned}$$

Selected 5 ϕ 20; $A_{s,\text{prov}}=25.5*10^2 \text{ mm}^2$

15.4.4.3 Design of the beam for shear

$$\begin{aligned}
 \max V_{sd} &= 593.22+185.48 &= 778.7 \text{ kN} \\
 \text{For the design, the variable strut inclination method is used.} \\
 \text{Design shear force at the distance } d \text{ from the face of the support:} \\
 V'_{sd} &= \max V_{sd} - (0.2+d) (G_{k,1} + G_{k,4} + Q_{k,3}) \\
 &= 778.7 - 0.845(10.6+260+87) &= 477 \text{ kN} \\
 \cot \theta &= 1.25; \alpha=90^\circ
 \end{aligned}$$

Reference

EC2, 4.3

see 15.4.1 above

Element No. 2

see Table in 15.4.4.3 above

EC2, 2.5.3.3(5)

[1], Table 7.1a

EC2, 2.5.2.2.1

[1], Table 7.1a

EC2, 4.3.2

EC2, 4.3.2.4.4

 $G_{k,4}$; $Q_{k,3}$ coordinates at $x=0.845 \text{ m}$ [1]

$$\begin{aligned}
 A_{sw}/s &= 477*10^3/(0.9*0.645*435*1.25) &= 15.1*10^2 \text{ mm}^2/\text{m} \\
 V_{Rd2} &= 0.7*0.9*0.645*0.55*20/2.05*10^3 &= 2180 \text{ kN} \\
 & &> V'_{sd} \\
 V'_{sd}/V'_{Rd2} &= 477/2180 &= 0.22 \\
 \text{Maximum longitudinal spacing of stirrups:} \\
 \max s_w &= 0.6*645 &= 387 \text{ mm} \\
 & &> 300 \text{ mm} \\
 \text{Maximum transverse spacing of legs:} \\
 \max s_{w,t} &= &= 300 \text{ mm}
 \end{aligned}$$

Selected shear links with four legs ϕ 12 - 300 mm
 $(A_{sw}/S)_{\text{prov}}=15.08*10^2 \text{ mm}^2/\text{m}$

In mid-span:

Shear links with two legs ϕ 12 - 300 mm
 $(A_{sw}/S)_{prov} = 7.54 \cdot 10^2 \text{ mm}^2/\text{m}$

$$\rho_w = 7.54 \cdot 10^{-4} / (0.70 \cdot 1.0 \cdot 1.0) = 0.0011 = \min \rho_w$$

15.4.4.4

Design for the ultimate limit states induced by structural deformations (buckling)

In this example, only element No. 1 is designed to EC2.

Design action effects:

$$N_{sd} = -1146.09 - 208.12 = -1354 \text{ kN}$$

Bending moment in node 1:

$$M_{Sd,1} = -15.58 - 8.34 = -24 \text{ kNm}$$

Bending moment in node 2:

$$M_{Sd,2} = -181.88 - 60.09 = -242 \text{ kNm}$$

Cross-sectional dimensions: $b/h = 400/400 \text{ mm}$

Slenderness ratio in the plane of the frame:

$$\begin{aligned} \beta &= 0.7 \\ l_0 &= 0.7 \cdot 4.20 = 2.94 \text{ m} \\ \lambda &= 2.94 / (0.289 \cdot 0.40) = 25.5 \\ \lambda_{ilm} &= 15 / v_u \\ v_u &= 1.354 / (0.42 \cdot 20) = 0.423 \\ \lambda_{lim} &= 15 / (0.423) = 23.0 \\ \lambda_{crit} &= 25 \cdot (2 - 0.018 / 0.179) = 47.5 \end{aligned}$$

Check for second order effects is not necessary.

$$M_{Rd} = N_{sd} \cdot h / 20 = 1354 \cdot 0.4 / 20 = 27.1 \text{ kNm} < 242 \text{ kNm}$$

Reference

EC2, Eq. (4.27)

EC2, 5.4.2.2(7)

EC2, 5.4.2.2(9)

EC2, Table 5.5, for C 30/37 and B 500

EC2, 4.3.5

see 15.4.4.3 above

$e_0 = 0.018 \text{ m}$

$e_0 = 0.179 \text{ m}$

EC2, 4.3.5.3.5; in the transverse direction, buckling is prevented by structural members

EC2, 4.3.5.3.5(2)

not relevant here EC2, Eq. (4.62)

EC2, Eq. (4.64)

$$d = 0.355 \text{ m}$$

Design of the column in node 2 using the tables in [2]

$$v_{sd} = -1.354 / (0.16 \cdot 20) = -0.423$$

$$\mu_{sd} = 0.242 / (0.16 \cdot 0.4 \cdot 20) = 0.20$$

$$\omega = 0.28$$

$$A_{s,tot} = 0.28 \cdot 400^2 \cdot 20 / 435 = 20.6 \cdot 10^2 \text{ mm}^2$$

| |
|--|
| <p>Selected 2*5=10 ϕ 16</p> <p>$A_{s,prov}=20.1*10^2 \text{ mm}^2$</p> |
|--|

In element 3, 4 ϕ 16 are provided on each side.

15.4.4.5 Detailing of reinforcement

15.4.4.5.1 Columns

| | | |
|----------------------------------|-----------------------|----------------------------|
| Bar diameters provided: | ϕ | = 16 mm |
| | | > 12 mm |
| Minimum reinforcement areas: | | |
| min A_s | = $0.15*1354*103/435$ | = $4.7*10^2 \text{ mm}^2$ |
| or | = $0.003*400^2$ | = $4.8*10^2 \text{ mm}^2$ |
| | | < $20.1*10^2 \text{ mm}^2$ |
| Transverse reinforcement (links) | ϕ_w | = 10 mm |
| | | > 6 mm |
| Spacing: | | |
| $s_{w,max}$ | = $12*16$ | = 192 mm |
| $0.6s_{w,max}$ | = $0.6*192$ | = 115 mm |

15.4.4.5.2 Beam

| | | |
|--|--------------------------------|---------------------------|
| Minimum reinforcement area to avoid brittle failure: | | |
| $A_{s,min}$ | = $0.0015*700*645$ | = $6.8*10^2 \text{ mm}^2$ |
| | | < $A_{s,prov}$ |
| Anchorage of bottom reinforcement | | |
| l_b | = $10 \phi=10*25*10^{-3}$ | = 0.25 m |
| Basic anchorage length of bars with $\phi=16$ mm | | |
| l_b | = $0.25* 2*16*10^{-3}*435/3.0$ | = 0.82 m |
| Lap length of the bars ϕ 16 in node 2: | | |
| l_s | = $\alpha_1 l_{b,net}$ | |
| | = $2.0*1.0*0.82*8.30/10.1$ | = 1.36 m |
| $l_{s,min}$ | = $0.3*1.0*2.0*0.82$ | = 0.50 m |

Reference

assumption

[2], page 64, Table 6.4 b:

elements 1 and 3

EC2, 5.4.1.2.1

EC2, 5.4.1.2.1(2)

EC2, 5.4.1.2.2(1)

EC2,5.4.1.2.2(3), (4) relevant here

EC2, Eq. (5.14)

supports in nodes 2 and 3 are considered as restrained

EC2, Eq. (5.3)

EC2, Table 5.3, for poor bond conditions

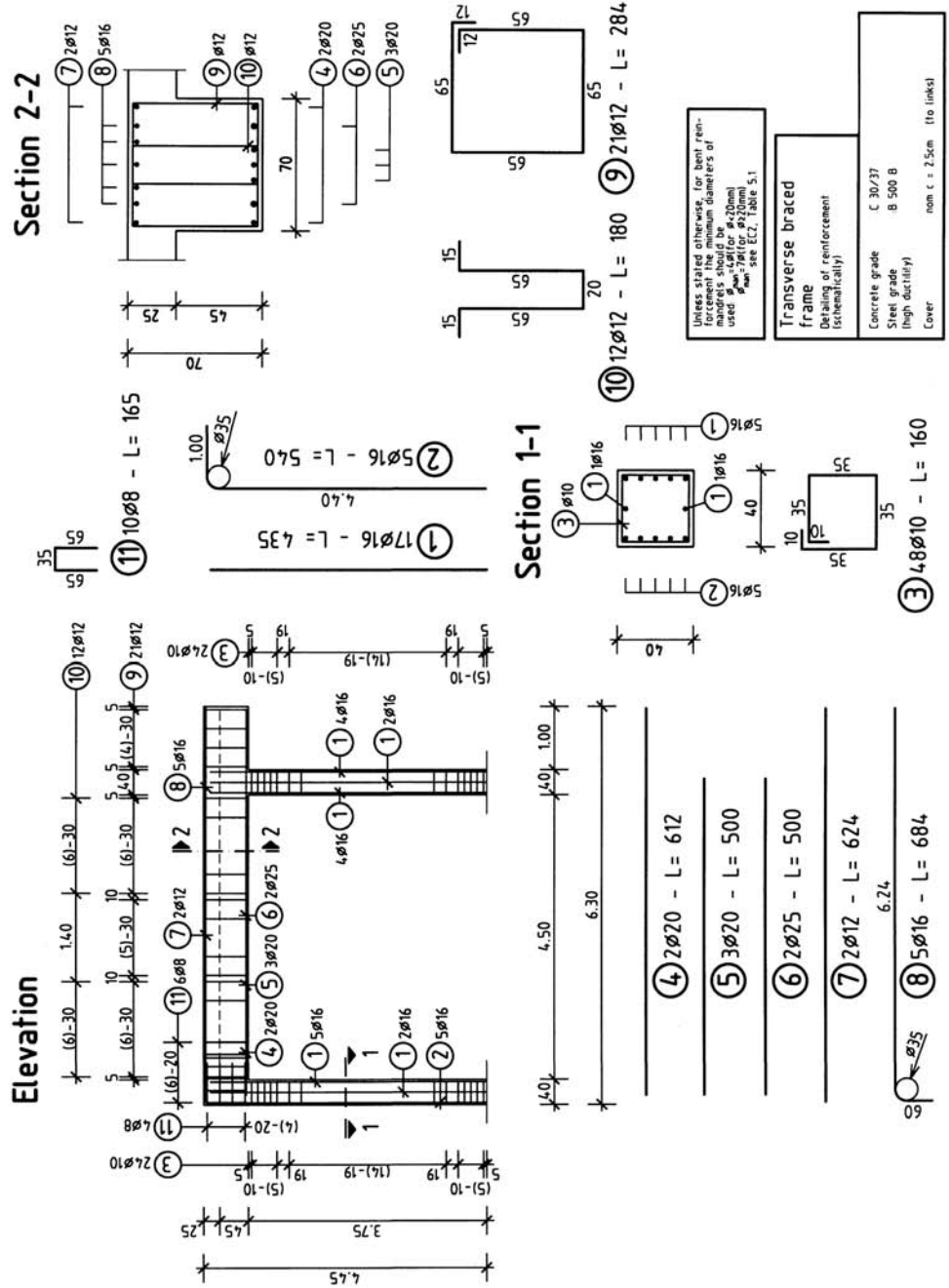
EC2, 5.2.4.1.3(1)

EC2, Eq.(5.7)

EC2, 5.2.4.1.3(1)

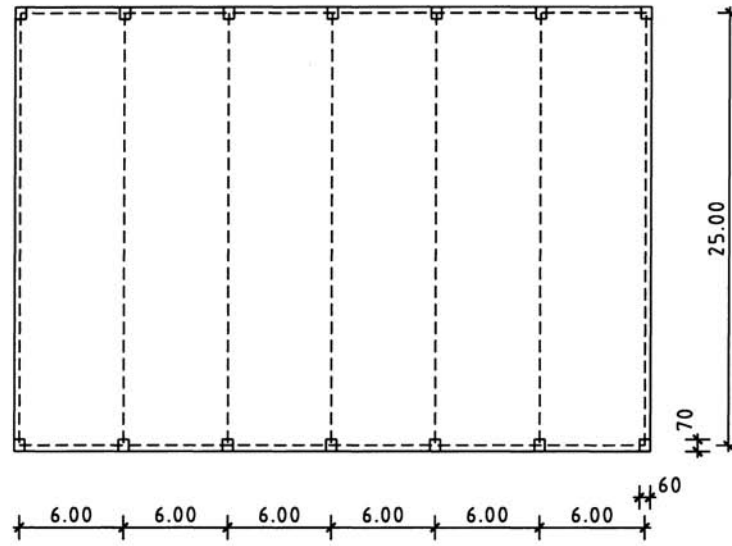
EC2, Eq. (5.8)

Calculation for a residential building

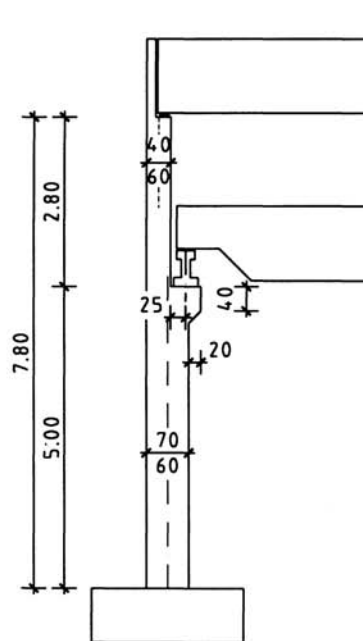


15.5.1 Floor plan; elevation

Floor plan



Elevation



Reference

15.5.2 Calculation of prestressed concrete beam

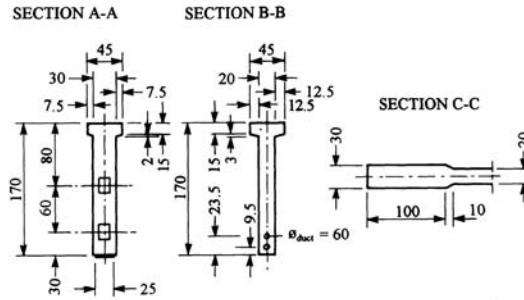
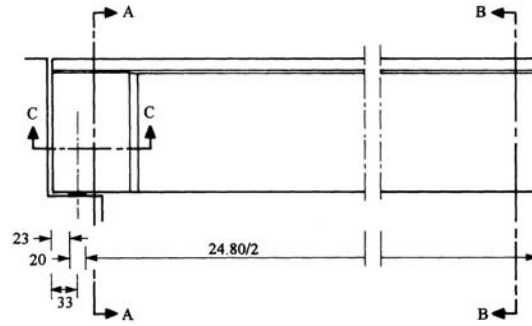
15.5.2.1 Basic data

Structural system; cross-sectional dimensions

Elevation

Exposure class:

Class 1 (indoor conditions)



Materials:

Concrete grade C 35/45
 Steel grade B 500

Tendons:
 7-wire strands $f_{p0.1} \cdot k / f_{pk} = 1500/1770 \text{ N/mm}^2$
 Modulus of elasticity $E_s = 200000 \text{ N/mm}^2$
 Relaxation class 2
 Diameter of sheathing $\phi_{duct} = 60 \text{ mm}$
 Cross-sectional area: $A_p = 7.0 \cdot 10^2 \text{ mm}^2$

Reference

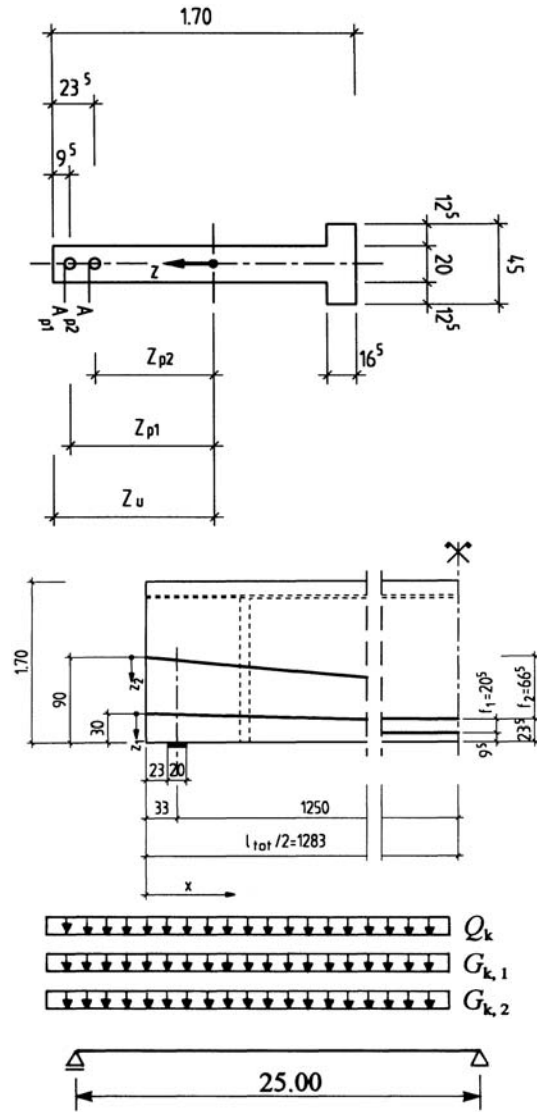
- EC2, Table 4.1
- EC2, Table 3.1 ENV 10 080
- EC2, 4.2.3.4.1(2)

Coefficient of friction: $\mu = 0.22$
 Anchorage slip $\Delta l_{sl} = 3.0 \text{ mm}$
 Unintentional displacement $k = 0.005$
 Cover to reinforcement:
 - links: $\text{nom } c_w = 25 \text{ mm}$
 - tendons: $\text{nom } c_p = 65 \text{ mm}$

Geometric data of the beam in mid-span section:

$\phi_{duct} = 60 \text{ mm}$
 $A_{p1} = A_{p2} = 7.0 \cdot 10^2 \text{ mm}^2$
 $\alpha_e = 200000/33500 = 5.97$

| Cross-section | $A_c; A_{c1} \text{ (m}^2\text{)}$ | $I_c; I_{c1} \text{ (m}^4\text{)}$ | $Z_u \text{ (m)}$ | $Z_{p1} \text{ (m)}$ | $Z_{p2} \text{ (m)}$ |
|---------------|------------------------------------|------------------------------------|-------------------|----------------------|----------------------|
| A_c | 0.381 | 0.104 | 0.933 | 0.838 | 0.698 |



| Cross-section | $A_c; A_{c1}$ (m ²) | $I_c; I_{c1}$ (m ⁴) | Z_u (m) | Z_{p1} (m) | Z_{p2} (m) |
|---------------|---------------------------------|---------------------------------|-----------|--------------|--------------|
| $A_{c,net}$ | 0.376 | 0.100 | 0.945 | 0.850 | 0.710 |
| A_{ci} | 0.406 | 0.122 | 0.927 | 0.832 | 0.692 |

Reference

EC2, 4.1.3.3

modular ratio

Tendon profile

Description of the tendon profile:

Tendon 1:

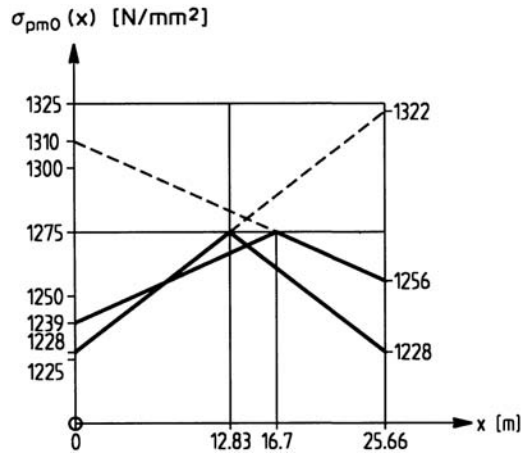
$$Z_1(x) = 4 \cdot 0.205 \cdot [x/l_{tot} - (x/l_{tot})^2]$$

Tendon 2:

$$Z_2(x) = 4 \cdot 0.665 \cdot [x/l_{tot} - (x/l_{tot})^2]$$

15.5.2.2

Actions



| | | | |
|-----------|---|---|-----------|
| $G_{k,1}$ | = | = | 9.5 kN/m |
| $G_{k,2}$ | = | = | 10.0 kN/m |
| Q_k | = | = | 4.8 kN/m |

15.5.2.3
Action effects due to $G_{k,1}$, $G_{k,2}$ and Q_k

| | | | | |
|---------------|---|---|---|----------|
| $\max M_{sd}$ | = | $[1.35 \cdot 19.5 + 1.5 \cdot 4.8] \cdot 25.02 / 8$ | = | 2620 kNm |
| $\max V_{sd}$ | = | $[1.35 \cdot 19.5 + 1.5 \cdot 4.8] \cdot 25.0 / 2$ | = | 420 kN |

Reference

- $l_{tot} = 25.66$ m
- self-weight of beam
- roofing
- snow

15.5.2.4
Action effects due to prestress

Stresses σ_{pm0} in the tendons at $t=0$ allowing for friction, anchorage slip and unintentional angular displacement
Action effects N_p , M_p and V_p due to prestressing at the serviceability limit states

| Location | Action effects at | | | | | |
|---------------|-------------------|------------|------------|-------------|------------|-------|
| | $t=0$ | | | $t=$ | | |
| | M_p (kNm) | V_p (kN) | N_p (kN) | M_p (kNm) | V_p (kN) | |
| Left support | -1727.2 | -483.6 | -117.3 | -1452.8 | -406.8 | -98.6 |
| Mid-span | -1779.4 | -1387.9 | 0 | -1505.0 | -1146.8 | 0 |
| Right support | -1738.8 | -486.9 | -118.1 | -1464.4 | -410.1 | -99.4 |

Reference

15.5.2.5
Design for the ultimate limit states for bending and longitudinal force

(a) Material data; design values of material strength

| | | | | | |
|----------|---------|---------------------|----------|--------|---------------------------|
| Concrete | C 35/45 | | f_{ck} | = | 35/1.5 N/mm ² |
| f_{cd} | = | f_{ck} / γ_c | = | 35/1.5 | = 23.33 N/mm ² |

| | | | | | |
|--------------------|-----------|----------------------|----------|-----------------|--------------------------|
| Reinforcing steel | B 500 | | f_{yk} | = | 500 N/mm ² |
| f_{yd} | = | f_{yk}/γ_s | = | 500/1.15 | = 435 N/mm ² |
| Prestressing steel | 1500/1770 | | f_{pk} | = | 1770 N/mm ² |
| f_{pd} | = | $0.9f_{pk}/\gamma_s$ | = | $0.9*1770/1.15$ | = 1385 N/mm ² |

(b) Design at mid-span

$$\max M_{Sd} = 2620 \text{ kNm}$$

Effective depth at mid-span:

$$d_m = 1.70 - [(4*2.0/26.0)*4.1 + (2*2.0/26.0)*7.7 + (14.0/26.0)*16.5]*10^{-2} = 1.58 \text{ m}$$

Related bending moment:

$$\mu_{Sds} = \max M_{Sd} / (b_f d_m^2 f_{cd}) = 2.620 / (0.45 * 1.58^2 * 35 / 1.5) = 0.10$$

with

$$h_f/d = 0.165/1.58 = 0.1$$

$$b_f/b_w = 45/20 = 2.25$$

The mechanical reinforcement ratio ω is given as:

$$\omega = 108/1000 = 0.108$$

$$A_{s,req} = (1/f_{yd})(\omega b_f d_m f_{cd} - A_p \sigma_{pd})$$

where

$$\sigma_{pd} = (\gamma_p \epsilon_{pm} + \Delta \epsilon_p) E_p \leq f_{pd} = 1385 \text{ N/mm}^2$$

A trial calculation has shown that

$$(\gamma_p * \epsilon_{pm} + \Delta \epsilon_p) > \epsilon_{p0.1k},$$

so that

$$\sigma_{pd} = f_{pd} = 1385 \text{ N/mm}^2$$

$$A_{s,req} = (1/435)(0.108 * 0.45 * 1.58 * 23.33 - 14.0 * 10^{-4} * 1385) 10^4$$

$$< 0$$

i.e., for the resistance of $\max M_{Sd}$, no reinforcement is necessary.**Reference**

EC2, 4.3.1

see 15.5.2.3 above

[2], p. 59

[2], p. 59, Table 6.3a

Minimum reinforcement area required to avoid brittle failure:

$$A_{s,min} = 0.0015 * 200 * 1580 = 4.74 * 10^2 \text{ mm}^2$$

Selected reinforcing steel B 500 6 bars ϕ 16; $A_{s,prov} = 12 * 10^2 \text{ mm}^2$

(c) Check of the pre-compressed tensile zone

It needs to be checked that the resistance of the pre-compressed tensile zone subjected to the combination of the permanent load $G_{k,1}$ and prestress is not exceeded.

Design value of bending moment due to $G_{k,1}$:

$$M_{Sd,G} = \gamma_G G_{k,1} l_{eff}^2 / 8 = 1.0 * 9.5 * 25.02^2 / 8 = 742 \text{ kNm}$$

Characteristic value of prestress:

$$P_k = \gamma_p P_{m0} = 1.0 * 1780 = 1780 \text{ kN}$$

Bending moment due to prestress:

$$M_k = \gamma_p M_p = -1.0 * 1388 = -1388 \text{ kNm}$$

The cross-section in mid-span needs to be designed for the combination of

$$N_{Sd} = -\gamma_p P_k = -1780 \text{ kN}$$

$$M_{Sd} = M_{Sd,G} + \gamma_p M_k = 742 - 1388 = -646 \text{ kNm}$$

Distance z_s of the reinforcement in the flange from the centre of gravity:

$$z_s = h - z_u - h_f / 2 = 1.70 - 0.945 - 0.165 / 2 = 0.67 \text{ m}$$

$$\begin{aligned}
 d &= h-h_f/2=1.70-0.165/2 &= & 1.60 \text{ m} \\
 \mu_{Sds} &= (0.646-1780*0.67)/(0.2*1.602*23.33) &= & 0.15 \\
 & & < & 435 \text{ N/mm}^2
 \end{aligned}$$

$$\begin{aligned}
 \omega &= 0.167; & \sigma_{sd} &= f_{yd} = 435 \text{ N/mm}^2 \\
 A_{s,req} &= (1/435) (0.167*0.2*1.6*23.33-1.78)10^4 & < & 0
 \end{aligned}$$

No reinforcement in the flange is necessary.

15.5.2.6 Design for shear

Design value of the applied shear force:

$$V_{Sd} = V_{od} - V_{pd}$$

Design value V'_{od} at a distance d from the face of the support:

$$\begin{aligned}
 V'_{od} &= (\gamma_G G_k + \gamma_Q Q_k)(l_{eff}/2 - a_L/2 - d) &= & \\
 &= (1.35*19.5 + 1.5*4.8)(12.5 - 0.1 - 1.65) &= & 361 \text{ kN}
 \end{aligned}$$

Force component V_{pd} due to the inclined tendons:

$$V_{pd} = \gamma_p \sigma_{p,m,t} A_p \tan \alpha_i$$

Reference

EC2, 5.4.2.1.1(1)

see 15.5.2.4 above

no compression reinforcement necessary [1], [Table 7.1](#)

EC2, 4.3.2.4.6

EC2, Eq. (4.32)

EC2, 4.3.2.2(10)

The stress $\sigma = p_m$, for $t =$ was calculated as:

$$\sigma_{p,m} = 1040 \text{ N/mm}^2$$

$\tan \alpha_i$ is given by:

- for tendons 1:

$$\tan \alpha_1 = 4*0.205[1/25.66 - 2(1.65+0.43)/25.66^2] = 0.0265$$

- for tendons 2:

$$\begin{aligned}
 \tan \alpha_2 &= 4*0.665[1/25.66 - 2(2.08)/25.66^2] &= & 0.0868 \\
 V_{pd} &= 0.9*1040*7*10^{-1} (0.0268+0.0868) &= & 74 \text{ kN} \\
 V_{Sd} &= 361-74 &= & 287 \text{ kN}
 \end{aligned}$$

Selected stirrups ϕ 8-200

$A_{sw}/s_w = 5.0*10^2 \text{ mm}^2/\text{m}$

Design shear resistance V_{Rd3} using the variable strut inclination method and assuming $\alpha=90^\circ$ and $\cot \theta=1.25$:

$$V_{Rd3} = 5.0*10^2*0.9*1.65*435*1.25*10^{-3} = 403.7 \text{ kN}$$

Design shear resistance of the compression struts

$$\begin{aligned}
 b_{w,\text{net}} &= b_w - \theta_{\text{duct}}/2 = 0.20 - 0.006/2 &= & 0.17 \text{ m} \\
 \nu &= 0.7 - 35/200 &= & 0.525 \\
 V_{\text{Rd}2} &= 0.17 * 0.9 * 1.65 * 0.525 * 23.33 / 2.05 * 10^3 &= & 1508 \text{ kN}
 \end{aligned}$$

Minimum shear reinforcement:

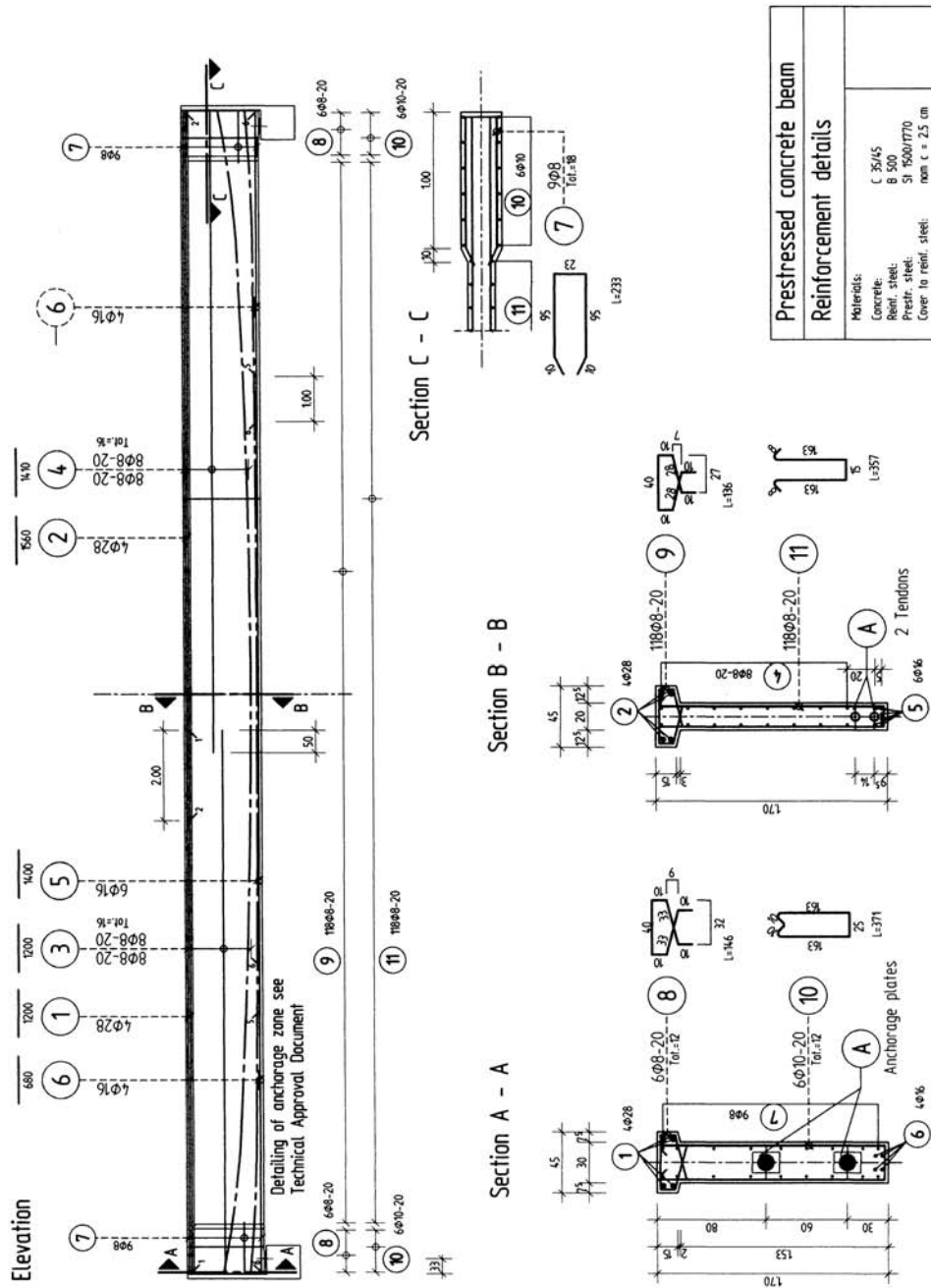
$$(A_{\text{sw}}/s_w)_{\text{min}} = 0.0011 * 200 * 1000 = 2.2 * 10^2 \text{ mm}^2/\text{m}$$

maximum longitudinal spacing $s_{w,\text{max}}$:

$$\begin{aligned}
 V_{\text{sd}}/V_{\text{Rd}2} &= 287/1508 &= & 0.19 \\
 &&& < & 0.20 \\
 s_{w,\text{max}} &= &= & 300 \text{ mm}
 \end{aligned}$$

Reference

see [15.5.2.1](#) above
 EC2, 4.3.2.4.4
 EC2, Eq. (4.26)
 EC2, 5.4.2.2(5)
 EC2, Eq. (5.16)
 EC2, 5.4.2.2(7)
 EC2, Eq. (5.17)
 Detailing of reinforcement



15.5.3 Calculation of edge column subjected to crane-induced actions

15.5.3.1 Basic data and design value of actions

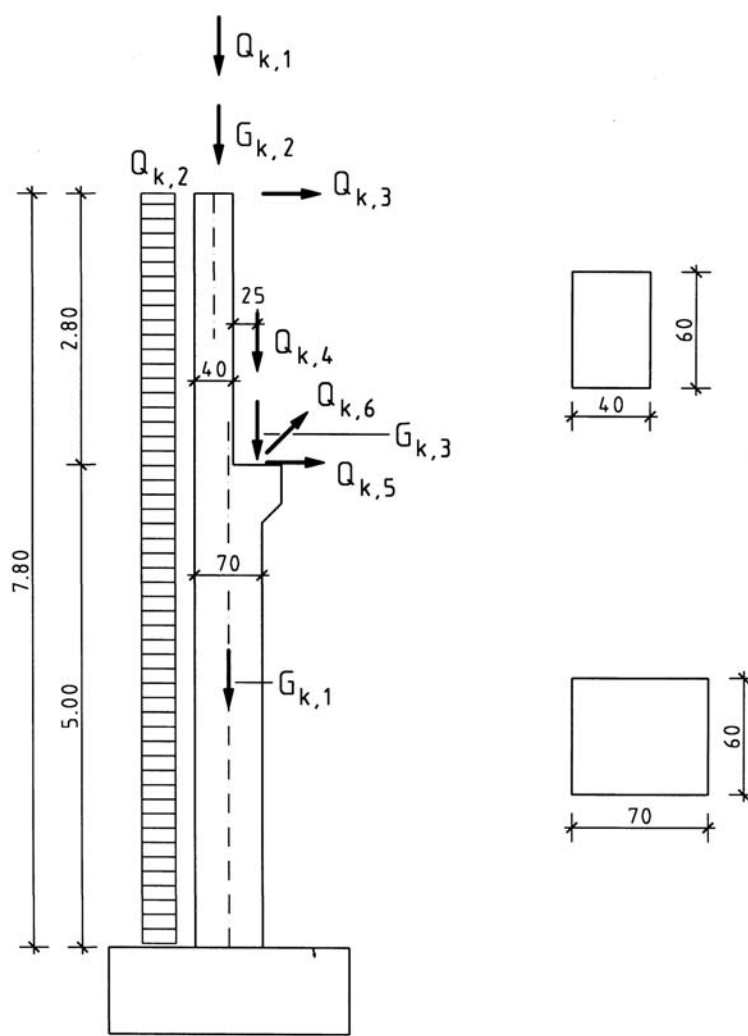
Structural system; cross-sectional dimensions

Elevation

Reference

i.e. fatigue verification to EC2-2 is performed

Exposure class:



Class 2a (humid environment without frost)

Materials:

Concrete grade

C 45/55

For the second order analysis of the column (see 15.5.3.2 below), the general stress-strain diagram acc. to Figure 4.1 in EC2 will be used. For the design of the cross-section, the parabolic-rectangular diagram will be applied.

Steel grade

B 500

For structural analysis and the design of cross-sections, the bi-linear diagram with a horizontal top branch will be used.

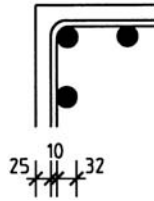
Cover to reinforcement (stirrups)

| | | | |
|-----------|---|---|-------|
| min c_w | = | = | 20 mm |
| nom c_w | = | = | 25 mm |

Actions

Permanent actions (self-weight)

| | | | |
|-----------|---|------------------------|--------------------|
| $G_{k,1}$ | = | 25.0 kN/m ³ | |
| $G_{k,2}$ | = | 244.0 kN | (prestressed beam) |



$$G_{k,3} = 42.0 \text{ kN} \quad (\text{crane girder})$$

Crane-induced variable actions

$$\begin{aligned} Q_{k,1v} &= 551.0 \text{ kN} && (\text{vertical}) \\ Q_{k,1t} &= 114.0 \text{ kN} && (\text{transverse action}) \\ Q_{k,1b} &= \pm 70.0 \text{ kN} && (\text{braking force}) \end{aligned}$$

Variable actions except crane-induced actions

$$\begin{aligned} Q_{k,2} &= 60.0 \text{ kN} && (\text{snow}) \\ Q_{k,3} &= 3.6 \text{ kN/m} && (\text{wind}) \\ Q_{k,4} &= 16.0 \text{ kN} && (\text{sliding force}) \end{aligned}$$

Combination coefficients

| | | | |
|-----------------------------|--------------|---|-----|
| - for crane-induced actions | ψ_{0c} | = | 1.0 |
| - for snow | ψ_{0s} | = | 0.6 |
| - for wind | ψ_{0w} | = | 0.6 |
| - for sliding force | ψ_{0sl} | = | 0.6 |

15.5.3.2 Design values of actions

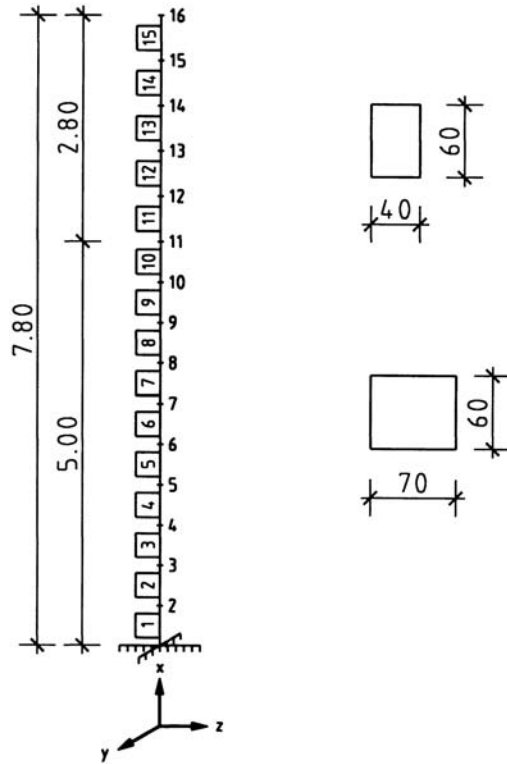
(a) Permanent actions ($\gamma_G=1.35$)

$$\begin{aligned} \gamma_G G_{k,1} &= 1.35 \cdot 25.0 &= 33.8 \text{ kN/m}^2 \\ \gamma_G G_{k,2} &= 1.35 \cdot 244.0 &= 329.4 \text{ kN} \\ \gamma_G G_{k,3} &= 1.35 \cdot 42.0 &= 56.7 \text{ kN} \end{aligned}$$

Reference

- EC2, Table 4.1
 - EC2, Table 3.1
 - EC2, 4.2.1.3.3(a), (5)
 - EC2, Eq. 4.2
 - ENV 10080
 - EC2, 4.2.2.3.2
 - EC2, Fig. 4.5
 - EC2, Table 4.2, for exposure class 2a
 - see 15.5.2.1 above
 - EC1-1, 9.4.4
 - see 15.5.2.1 above
 - EC2, 2.2.2.4
 - The combination with $\gamma_G=1.0$ is not relevant in this example
- #### (b) Variable actions ($\gamma_Q=1.50$)

$$\begin{aligned} \gamma_Q Q_{k,1v} &= 1.5 \cdot 551.0 &= 826.5 \text{ kN} \\ \gamma_Q Q_{k,1t} &= 1.5 \cdot 114.0 &= 171.0 \text{ kN} \end{aligned}$$



| | | | | |
|-------------------------------|---|----------------------------|---|----------------|
| $\gamma_Q Q_{k,1b}$ | = | $\pm 1.5 \cdot 70.0$ | = | 54.0 kN |
| $\gamma_Q \Psi_{0.2} Q_{k,2}$ | = | $1.5 \cdot 0.6 \cdot 60.0$ | = | ± 105.0 kN |
| $\gamma_Q \Psi_{0.3} Q_{k,3}$ | = | $1.5 \cdot 0.6 \cdot 3.6$ | = | 3.24 kN/m |
| $\gamma_Q \Psi_{0.4} Q_{k,4}$ | = | $1.5 \cdot 0.6 \cdot 16.0$ | = | 14.4 kN |

(c) Fundamental combination of actions

$$\begin{aligned} \Sigma(\gamma_{G,j} G_{k,j}) + \gamma_{Q,1} Q_{k,1} + \Sigma(\gamma_{Q,i} \Psi_{0,i} Q_{k,i}) \\ = 1.35(G_{k,1} + G_{k,2} + G_{k,3}) \\ + 1.5(Q_{k,1v} + Q_{k,1t} + Q_{k,1b}) \\ + 1.5 \cdot 0.6(Q_{k,2} + Q_{k,3} + Q_{k,4}) \end{aligned}$$

15.5.3.3

Design of the column for the ultimate limit states induced by structural deformations

(a) General

For the design of the column at the ultimate limit states induced by structural deformations, a rigorous computer-based second-order analysis is carried out. The design model is shown below. In this program, the steel reinforcement, $A_{s,req}$, required in the individual cross-sections is calculated automatically.

Reference

vertical crane load transverse action braking force snow wind sliding force

EC2, Eq. (2.7a); the accidental combination of actions is not considered

Crane-induced actions are the main variable actions.

EC2, 4.3.5

see 15.5.3.3 (e) below

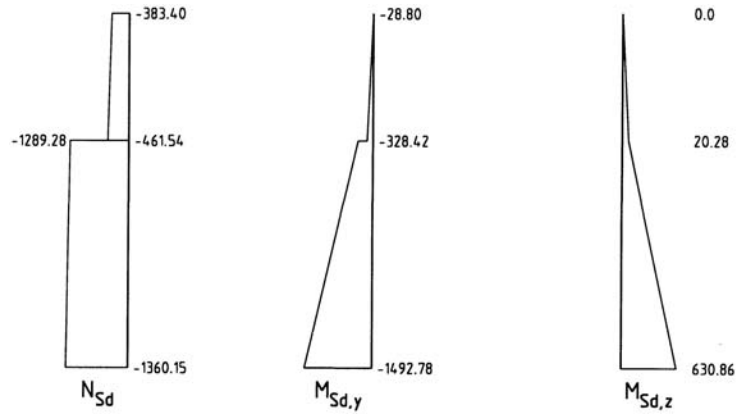
(b) Imperfections

For structural analysis, an inclination of

v = 1/200

in the direction of the theoretical failure plane is assumed.

(c) Creep



Allowance for creep deformations is made by using the simplified method and Appendix 3 proposed by QUAST in [2], i.e. to multiply the second order eccentricities e_2 by a coefficient

$$f_{\psi} = (1 + M_{Sd,c} / M_{Sd})$$

where

$M_{Sd,c}$ is the factored bending moment due to quasi-permanent actions

M_{Sd} is the bending moment due to the relevant combination of permanent and variable actions

(d) Design actions in the nodes of the design model

| Node | $F_{Sd,x}$ (kN) | $F_{Sd,y}$ (kN) | $F_{Sd,z}$ (kN) | $M_{Sd,y}$ (kNm) |
|------|-----------------|-----------------|-----------------|------------------|
| 16 | 14.4 | 0 | 383.4 | -28.8 |
| 11 | 171.0 | -105.0 | 883.2 | -204.0 |

Wind:

$$q_{Sd,x} = 3.24 \text{ kN/m}$$

Rotation due to imperfections and creep

$$v_x = -4.24 \cdot 10^{-3}$$

$$v_y = -7.57 \cdot 10^{-3}$$

(e) Summary of design results Internal forces and moments

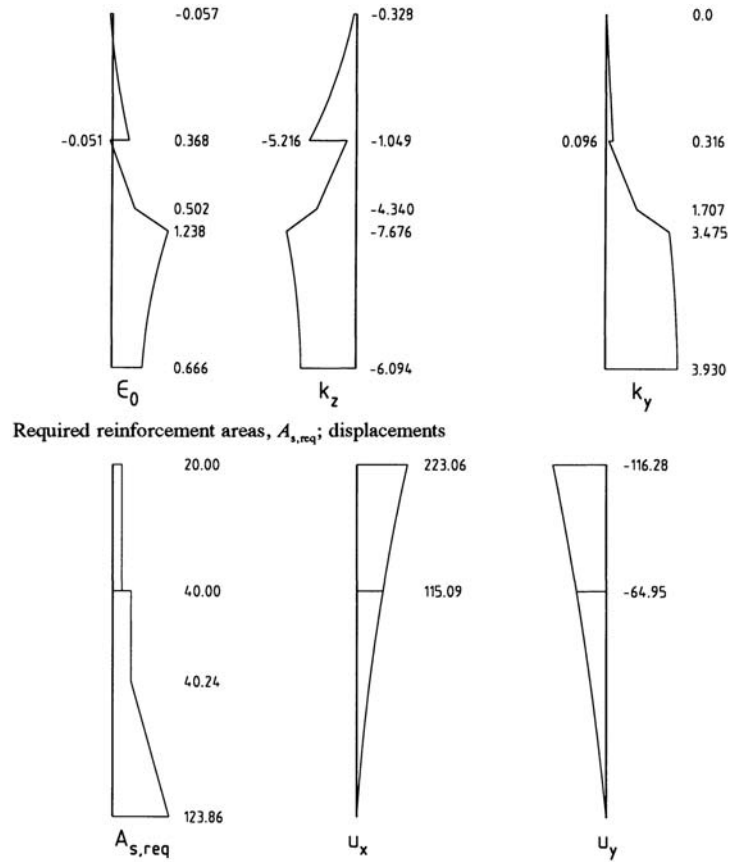
Reference

EC2, 4.3.5.4 and 2.5.1.3, Eq. (2.10)

introduced by iteration

EC2, 4.3.5.5.3

[2], p. 85, 9.4.3



Deformation and curvature

15.5.3.4 Design of the column; detailing of reinforcement

Required reinforcement area, $A_{s,req}$ at the restrained cross-section:

$$A_{s,req} = = 123.86 \cdot 10^2 \text{ mm}^2$$

Selected steel B 500 2*8=16 ø 32
 $A_{s,prov} = 128.68 \cdot 10^2 \text{ mm}^2$

Detailing of reinforcement:

see Figure below

Reference

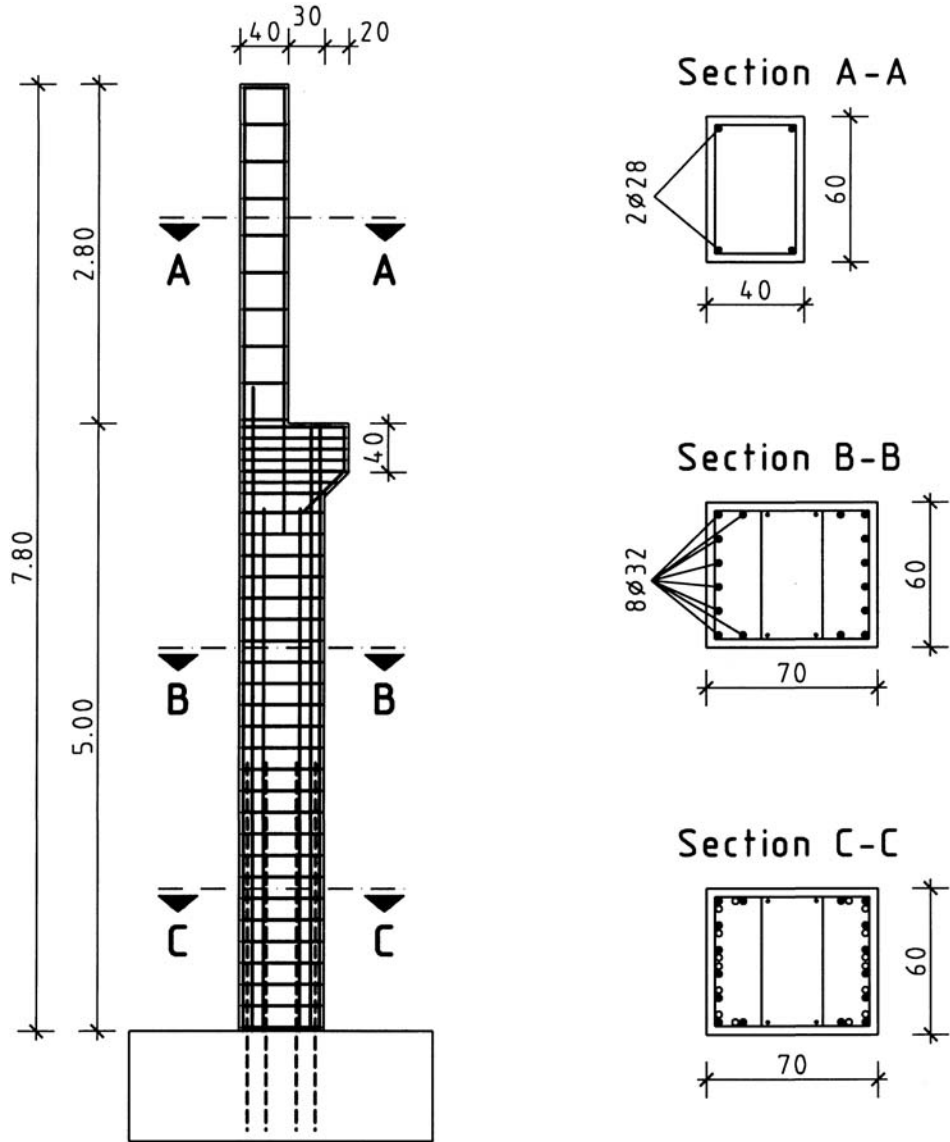
$A_{s,req}$ in $\text{mm}^2 \cdot 10^2$

see 15.5.3.3 (e), above

EC2,5

Detailing of reinforcement

Elevation



**15.5.3.5
Ultimate limit state of fatigue**

**15.5.3.5.1
General**

The edge column is subjected to crane-induced actions. It needs therefore to be checked for the ultimate limit state for fatigue. In this ultimate limit state, see 15.5.3.0 above, it shall be verified that

$$D_{sd} = \sum_i (n_i/N_i) \leq 1$$

where

- D_{sd} is the design value of the fatigue damage factor calculated using the PALMGREN-MINER summation
- n_i denotes the number of acting stress cycles associated with the stress range for steel and the actual stress levels for concrete
- N_i denotes the number of resisting stress cycles

For the above verification, the stress calculation shall be based on the assumption of cracked cross-sections neglecting the tensile strength of concrete but satisfying compatibility of strains.

The fatigue strength of reinforcing steel and concrete are given by EC2–2, 4.3.7.8 and 4.3.7.9 respectively.

15.5.3.5.2 Combination of actions

In the present example, fatigue verification will be performed under the frequent combination of actions using the partial safety factors

| | | | |
|------------------|---|------|-------------------------|
| γ_F | = | 1.0 | for actions |
| γ_{Sd} | = | 1.0 | for model uncertainties |
| $\gamma_{c,fat}$ | = | 1.5 | for concrete |
| $\gamma_{s,fat}$ | = | 1.15 | for reinforcing steel |

Therefore the relevant combination of actions is given by:

$$E_d = \sum (1.0 * G_{k,j}) + 1.0 * \psi_{1,1} (Q_{k,1v} + Q_{k,1b}) + \sum (1.0 * \psi_{2,i} Q_{k,i})$$

where

$Q_{k,1b}$ = the component of the braking force $Q_{k,1b}$ that is relevant for fatigue verification. In this example, it is assumed that
 $Q_{k,1b} = \pm 59.7$ kN

For the verification, the following combination coefficients $\psi_{1,1}$ and $\psi_{2,i}$ are assumed:

| | | | |
|--------------|---|-----|---|
| $\psi_{1,1}$ | = | 1.0 | (for crane-induced actions) |
| $\psi_{2,2}$ | = | 0 | (for snow loads) |
| $\psi_{2,3}$ | = | 0 | (for wind) |
| $\psi_{2,i}$ | = | 0 | (for all other variable actions $Q_{k,i}$) |

Design values of actions:

(a) Vertical actions

| | | | | |
|------------|---|-----------------|---|----------|
| $G_{d,j}$ | = | 25.0+244.0+42.0 | = | 311.0 kN |
| $Q_{d,1v}$ | = | | = | 551.0 kN |

Reference

EC2–2, 4.3.7

EC2–2, 4.3.7.5

EC2–2, 4.3.7.3

EC2–2, 4.3.7.2

see 15.5.3.1 above $i=1t, 2, 3, 4$

see 15.5.3.1 above

(b) Horizontal actions

| | | | | |
|------------|---|--|---|---------------|
| $Q_{d,1b}$ | = | | = | ± 59.7 kN |
| $Q_{d,4}$ | = | | = | 16.0 kN |

15.5.3.5.3 Damage factor D_{Sd}

For the calculation of the damage factor D_{Sd} , the spectrum of actions S2 in DIN 15 018 Part 1 is assumed. This approach is based on a linear relationship between actions and stresses assuming cracked cross-sections.

15.5.3.5.4**Calculation of the stress range $\Delta\sigma_s$**

A trial calculation shows that the most unfavourable stress range $\Delta\sigma_s$ occurs at the coordinate $x=3.0$ m above the level of restraint. A rigorous second order analysis leads to the following stresses:

$$\begin{aligned}\sigma_{s,\max} &= & &= & +112.2 \text{ N/mm}^2 \\ \sigma_{s,\min} &= & &= & -24.3 \text{ N/mm}^2 \\ \Delta\sigma_s &= & 112.2 - (-24.3) &= & 136.5 \text{ N/mm}^2\end{aligned}$$

Since both tensile and compressive stresses in the reinforcing steel occur, fatigue verification is necessary.

15.5.3.5.5**Calculation of the stress range $\Delta\sigma_c$**

The extreme concrete stresses occur at the level of restraint (i.e. $x=0$). A rigorous second order analysis leads to the following values:

$$\begin{aligned}\sigma_{c,\max} &= & &= & -11.3 \text{ N/mm}^2 \\ \sigma_{c,\min} &= & &= & -2.9 \text{ N/mm}^2 \\ \Delta\sigma_c &= & 11.3 - 2.9 &= & 8.4 \text{ N/mm}^2\end{aligned}$$

15.5.3.5.6**Verification of the fatigue strength of the reinforcing steel**

The fatigue requirement for reinforcing steel will be met if the following expression is satisfied:

$$\gamma_F \gamma_{Sd} \Delta\sigma_{s,\text{equ}} \leq \Delta\sigma_{Rsk}(N^*) / \gamma_{s,\text{fat}}$$

where

$$\begin{aligned}\Delta\sigma_{Rsk}(N^*) &= \text{stress range at } N^* \text{ cycles from the appropriate S-N lines} \\ \Delta\sigma_{s,\text{equ}} &= \text{the damage equivalent stress range which is the stress range of a constant stress spectrum with } N^*=10^6 \\ &\quad \text{stress cycles which results in the same damage as the spectrum of stress ranges caused by flowing traffic} \\ &\quad \text{loads} \\ \gamma_F &= 1.0 \\ \gamma_{Sd} &= 1.0 \\ \gamma_{s,\text{fat}} &= 1.15\end{aligned}$$

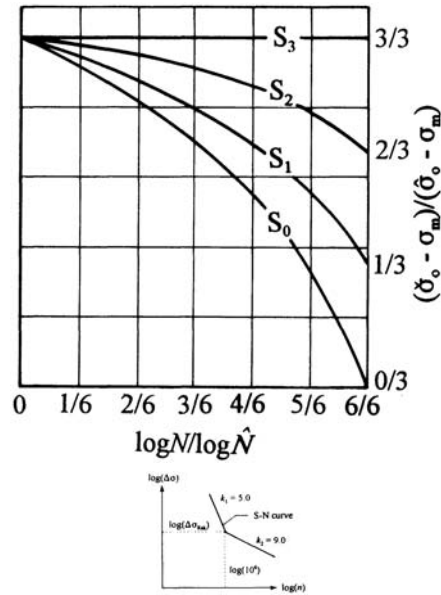
For bars with diameter, $\phi > 25$ mm, the stress range, $\Delta\sigma_{Rsk}$, at $N^*=10^6$ cycles is given as:

$$\Delta\sigma_{Rsk}(N^*) = = 195 \text{ N/mm}^2$$

Reference

for reinforcing steel
Tension Compression
EC2-2, 4.3.7.1
for concrete
Compression
Compression
EC2-2, 4.3.7.5
see 15.5.3.4.2 above
EC2-2, 4.3.7.8
The shape of the relevant S-N curve is defined by the coefficients

$$\begin{aligned}k_1 &= & &= & 5.0 \\ k_2 &= & &= & 9.0\end{aligned}$$



For $N+N^*$ cycles, the damage equivalent stress range $\Delta\sigma_{s,eq}$ is given by:

$$\Delta\sigma_{s,eq} = \alpha_p \kappa \Delta\sigma_s$$

For the calculation of the coefficients α_p and κ , the following assumptions have been used:

- spectrum of actions S2 according to DIN 15 018 Part 1

- $\Delta\sigma_s = 136.5 \text{ N/mm}^2$

- number of cycles: n

design lifetime: 50 years

working time: 10 hours/per day

one cycle/minute

$$n = 50 \cdot 365 \cdot 10 \cdot 60 \cdot 1 = 1.1 \cdot 10^7$$

From the α_p -diagram below:

$$\alpha_p = 1.0$$

Coefficient A:

$$A = \frac{\gamma_{Sd} \Delta\sigma_{s,max} \gamma_s / \Delta\sigma_{Rsk}}{1.0 \cdot 136.5 \cdot 1.15 / 195} = 0.81 < 1.0$$

From the κ -diagram below:

$$\kappa = 1.0$$

Thus:

$$\Delta\sigma_{s,eq} = 1.0 \cdot 1.0 \cdot 136.5 < 195 / 1.15 = 169.5 \text{ N/mm}^2$$

Requirements for reinforcing steel are met.

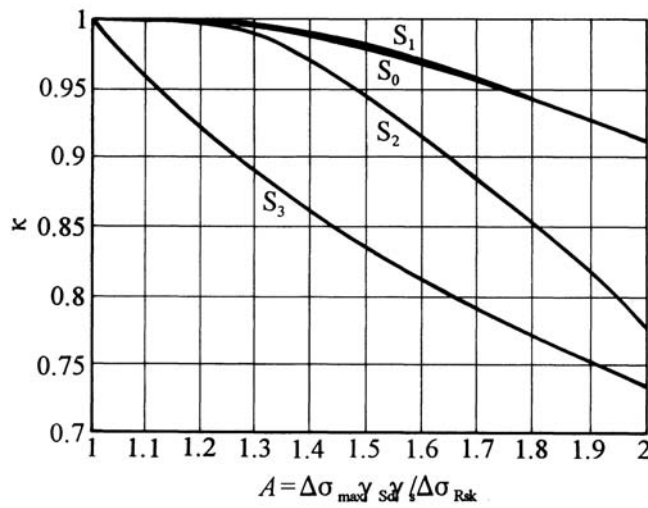
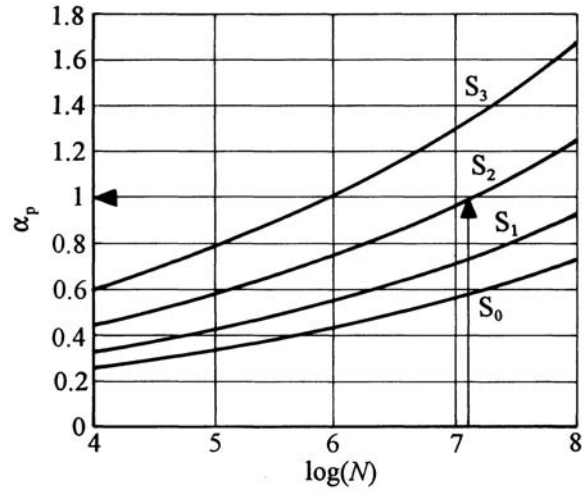
Reference

see 15.5.3.4.3 above

see 15.5.3.4.4 above

[Annex A](#) gives more details on the fatigue verification

Diagrams for reinforcing steel



15.5.3.5.7

Verification of the fatigue strength of concrete

The design fatigue strength of concrete is given by the S-N curve according to:

$$\log(N) = 14 \frac{1 - S_{cd,max}}{\sqrt{1 - R}}$$

$$S_{cd,max} = |\sigma_{c,max}| \frac{1}{f_{cd,fat}}$$

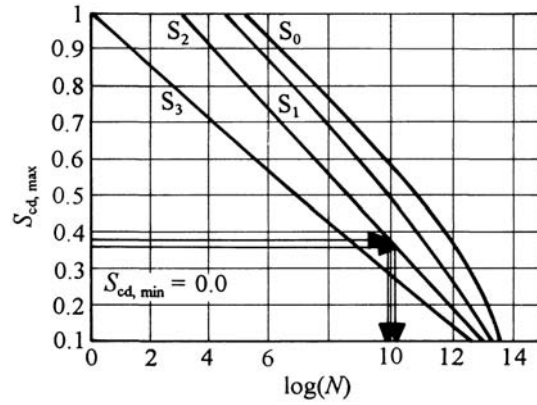
$$S_{cd,min} = |\sigma_{c,min}| \frac{1}{f_{cd,fat}}$$

$$R = \frac{S_{cd,min}}{S_{cd,max}}$$

$$f_{cd,fat} = \beta_{cc}(t) \frac{f_{ck}}{\gamma_c}$$

$$\beta_{cc} = \exp\left(s \left[1 - \sqrt{\frac{28}{t_0/t_1}} \right] \right)$$

where



N = number of cycles to failure

$\beta_{cc}(t_0)$ = coefficient which depends on the age of concrete t_0 in days when fatigue loading starts. If no information is available β_{cc} can be taken as 1.0

t_1 = 1 day

In the present case, the basic data are given by:

| | | | | |
|-------------------------|----------|----------|---|----------------------|
| Concrete strength class | C 45/55: | f_{yk} | = | 45 N/mm ² |
| $f_{cd,fat}$ | = | 45/1.5 | = | 30 N/mm ² |
| $S_{cd,max}$ | = | 11.3/30 | = | 0.38 |
| $S_{cd,min}$ | = | 2.9/30 | = | 0.10 |
| | | | | 0 |

from the diagram below:

$$N = 7.9 \cdot 10^9 > 1.1 \cdot 10^7$$

Reference

EC2-2, 4.3.7.4

see 15.5.3.4.5 above

ANNEX A

15.6

Guidance for the calculation of the equivalent stress range $\Delta\sigma_{s,eq}$ for reinforcing steel and of the S-N curve for concrete in compression using the single load level method

15.6.1

Reinforcing steel

Design value of the fatigue damage D_d , using the PALMGREN-MINER summation:

$$D_d = \sum \frac{n_i}{N_i} \tag{1}$$

where

n_i denotes the number of acting stress cycles associated with the stress range for steel and the actual stress levels for concrete

N_i denotes the number of resisting stress cycles

The shape of the S-N curve is given by:

$$\Delta\sigma^k N^* = \Delta\sigma_i^k N_i; k_j = 1 \text{ or } 2 \tag{2}$$

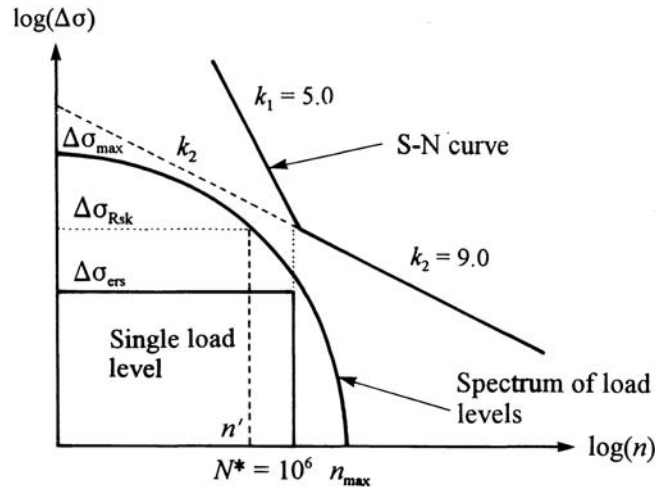


Figure A1 Graphical presentation of the design concept for reinforcing steel.

$$\Delta\sigma^* = \frac{\Delta\sigma_{Rsk}}{\gamma_s}$$

$$\Delta\sigma_i = \Delta\sigma_{max} \eta \gamma_{sd} \quad (3)$$

where

| | | |
|-------------------------|---|--|
| η | = | coefficient describing the spectrum of load levels |
| $\Delta\sigma_{max}$ | = | maximum acting stress range |
| γ_s, γ_{sd} | = | partial safety factors |

Equation 2 may be written as:

$$N_i = \left(\frac{\Delta\sigma^*}{\Delta\sigma_i} \right)^{k_i} N^* \quad (4)$$

Equations 1 and 4 lead to the following expression for E_d :

$$D_d = \sum_0^{n'} \left[\frac{n_i}{\left(\frac{\Delta\sigma^*}{\Delta\sigma_i} \right)^{k_1} N^*} \right] + \sum_{n'}^{n_{max}} \left[\frac{n_i}{\left(\frac{\Delta\sigma^*}{\Delta\sigma_i} \right)^{k_2} N^*} \right] \quad (5)$$

$$D_d = \sum_0^{n'} \left[\frac{n_i}{\left(\frac{\Delta\sigma^*}{\Delta\sigma_i} \right)^{k_1} N^*} \right] + \sum_0^{n_{max}} \left[\frac{n_i}{\left(\frac{\Delta\sigma^*}{\Delta\sigma_i} \right)^{k_2} N^*} \right] - \sum_0^{n'} \left[\frac{n_i}{\left(\frac{\Delta\sigma^*}{\Delta\sigma_i} \right)^{k_2} N^*} \right] \quad (6)$$

or

$$D_d = \frac{1}{N^*} \left(\frac{1}{\Delta\sigma^*} \right)^{k_2} \left[\sum_0^{n_{max}} (\Delta\sigma_i^{k_2} n_i) + \sigma^{*(k_2 - k_1)} \sum_0^{n'} (\Delta\sigma_i^{k_1} n_i) - \sum_0^{n'} (\Delta\sigma_i^{k_2} n_i) \right] \quad (7)$$

An equivalent single load level with N^* cycles shall satisfy the condition:

$$D_d = D_{equ} = \frac{N^*}{N_{equ}} \quad (8)$$

Using equation 2 for the S-N curve and equation 7 for the equivalent damage factor, the equivalent steel stress $\Delta\sigma_{s,equ}$ may be calculated as:

$$\text{S-N curve} \quad \Delta\sigma^{*k_2} N^* = \Delta\sigma_{s,equ}^{k_2} N_{s,equ} \quad (9)$$

$$\text{Equivalent number of cycles} \quad N_{s,equ} = \left(\frac{\Delta\sigma^*}{\Delta\sigma_{s,equ}} \right)^{k_2} N^* \quad (10)$$

Equivalent damage factor

$$D_d = D_{s, \text{equ}} = \left[\frac{\Delta \sigma_{s, \text{equ}}}{\Delta \sigma^*} \right]^{k_2} \quad (11)$$

Equivalent steel stress

$$\Delta \sigma_{s, \text{equ}} = \Delta \sigma^* \sqrt[k_2]{D_d} \quad (12)$$

From equation 7, it follows that

$$\Delta \sigma_{s, \text{equ}} = \Delta \sigma^* \sqrt[k_2]{\frac{1}{N^*} \left(\frac{1}{\Delta \sigma^*} \right)^{k_2} \left[\sum_0^{n_{\text{max}}} (\Delta \sigma_i^{k_2} n_i) + \sigma^{*(k_2 - k_1)} \sum_0^{n'} (\Delta \sigma_i^{k_1} n_i) - \sum_0^{n'} (\Delta \sigma_i^{k_2} n_i) \right]} \quad (13)$$

The equivalent steel stress $\Delta \sigma_{s, \text{equ}}$ may be expressed by:

$$\Delta \sigma_{s, \text{equ}} = \Delta \sigma_{\text{max}} \gamma_{\text{sd}} \alpha_p \kappa \quad (14)$$

The coefficients α_p and κ are defined as:

$$\alpha_p = \sqrt[k_2]{\frac{1}{N^*} \sum_0^{n_{\text{max}}} \eta_i^{k_2} n_i} \quad (15)$$

and

$$\kappa = \sqrt[k_2]{1 + \frac{\Delta \sigma^{*(k_2 - k_1)} \sum_0^{n'} (\Delta \sigma_i^{k_1} n_i) - \sum_0^{n'} (\Delta \sigma_i^{k_2} n_i)}{\sum_0^{n_{\text{max}}} (\Delta \sigma_i^{k_2} n_i) - \sum_0^{n_{\text{max}}} (\Delta \sigma_i^{k_2} n_i)}} \quad (16)$$

Format for fatigue verification:

$$\begin{aligned} \Delta \sigma_{s, \text{equ}} &\leq \frac{\Delta \sigma_{\text{Rsk}}}{\gamma_s} \\ \gamma_{\text{sd}} \Delta \sigma_{\text{max}} \alpha_p \kappa &\leq \frac{\Delta \sigma_{\text{Rsk}}}{\gamma_s} \end{aligned} \quad (17)$$

The coefficients α_p and κ may be taken from [Figure A2](#).

15.6.2 Concrete

The fatigue verification of concrete is analogous to that for steel reinforcement. However, there are differences. The fatigue requirements under cyclic loading will be met if the required lifetime (number of cycles, n_{max} is less than or equal to the number of cycles to failure (N_{equ}). In addition, the simplified S-N function given by equation 18 is used.

S-N curve of concrete:

$$\log(N) = 14 \frac{1 - S_{\text{cd, max}}}{\sqrt{1 - R}} \quad (18)$$

The calculation of the coefficients in equation 18 is based on EC2-2 (pr ENV 1992-2):

S-N curve of concrete, i.e. equation 1:

$$D_d = \sum \frac{n_i}{N_i} \quad (1)$$

Equivalent damage factor, D_{equ} :

$$D_d \leq D_{\text{equ}} = \frac{n_{\text{max}}}{N_{\text{equ}}} \quad (19)$$

Equivalent spectrum of load levels:

N_{equ} is calculated on the assumption that $D_{\text{sd}}=1.0$ for a given spectrum of load levels and for given parameters $S_{\text{cd, max}}$ and R of the relevant S-N curve for concrete.

$$D_d \leq 1$$

Verification format:

$$n_{\text{max}} \leq N_{\text{equ}} \quad (20)$$

N_{equ} should be taken from [Figure A3](#).

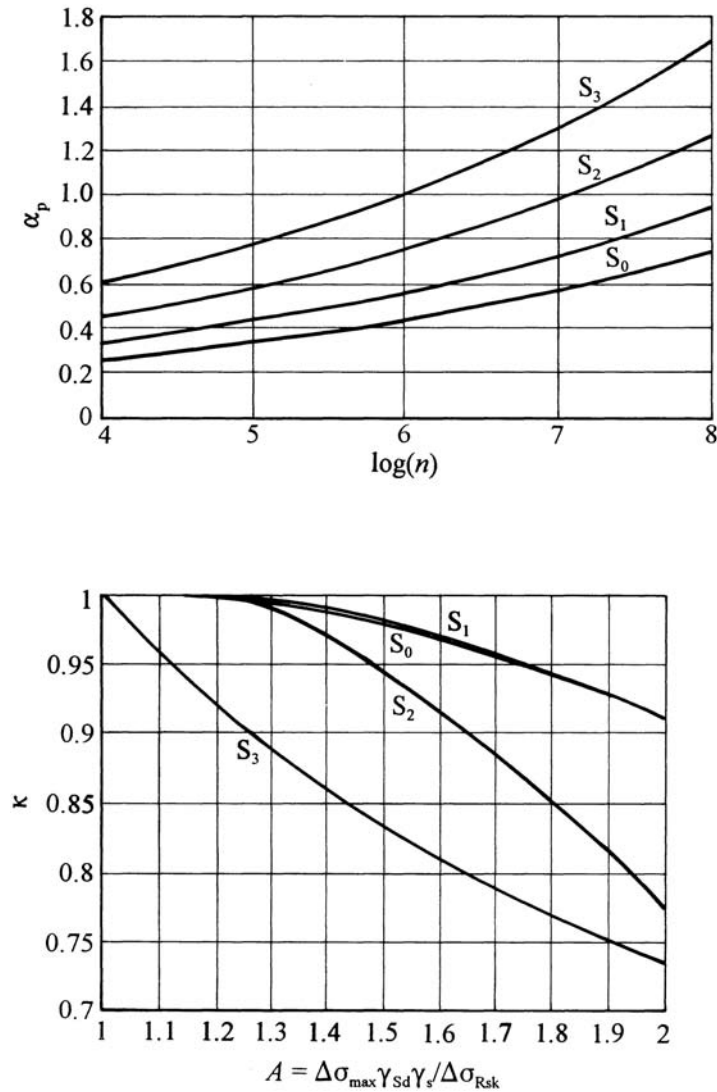


Figure A2 Coefficients for reinforcing steel, (a) α_p (b) κ .

ANNEX B

15.7

Design of purpose-made fabrics

In the present design examples, purpose-made fabrics as defined in ENV 10 080 *Steels for the reinforcement of concrete; Weldable ribbed reinforcing steel B 500* have been chosen. The graphical representation is shown in Figure B1 for the top reinforcement of a continuous slab and in Figure B2 for the respective bottom reinforcement.

Each individual fabric is characterized by a position number, i.e. ① to ③ in Figures B1 and B2. Their characteristics are described graphically in Figures B3 to B5 by means of the diameters and spacing of both the longitudinal and transverse bars. The total number of bars and their lengths lead to the total weight of the fabric.

The presentation of the fabrics corresponds to ISO 3766-1977(E) *Building and civil engineering drawings—Symbols for concrete reinforcement*, particularly clause 2.3.1. Each fabric is characterized by a rectangular frame (see, for example, Figure B2), the diagonal connected to the position number denoting the direction of the main bars. In Figure B2, the lap length ($s_l=400$ mm) of the transverse bars is also defined.

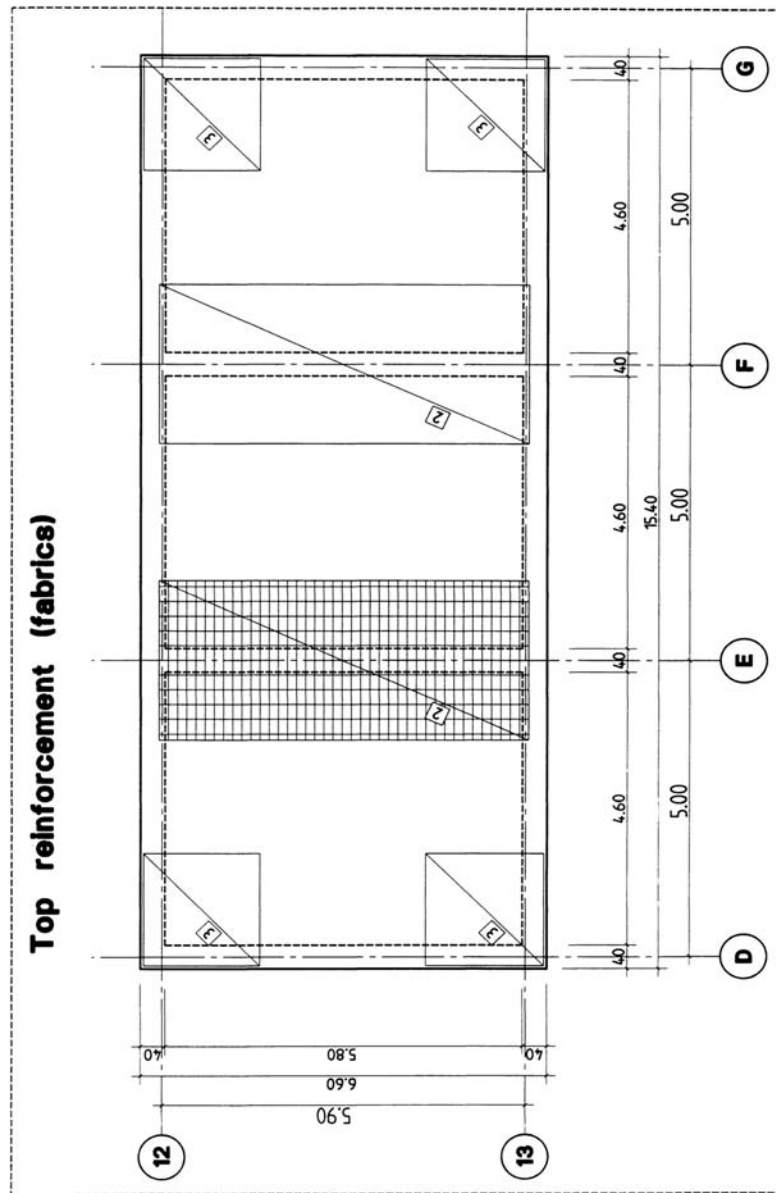
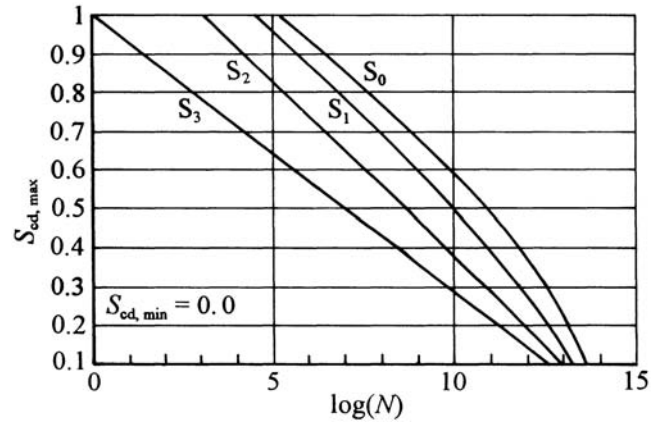


Figure A3 Relationship between N_{equ} and $S_{c,max}$ for different values of $S_{cd,min}$.

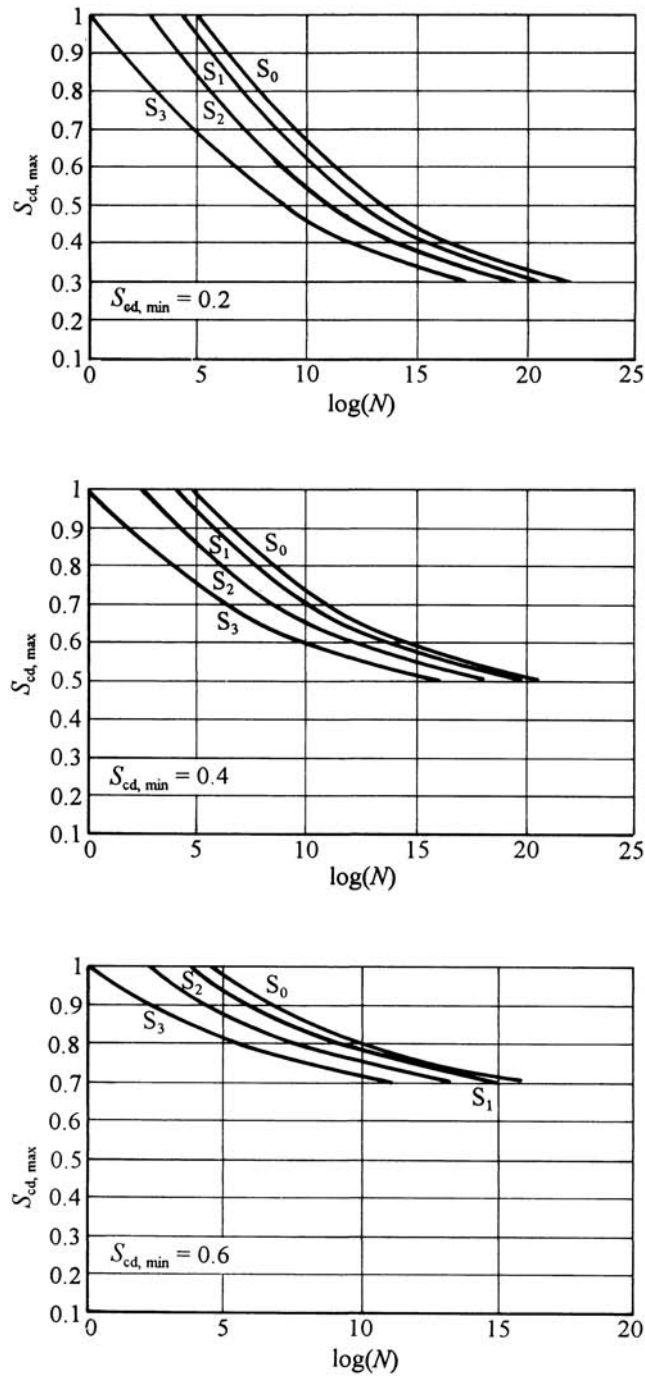


Figure B1

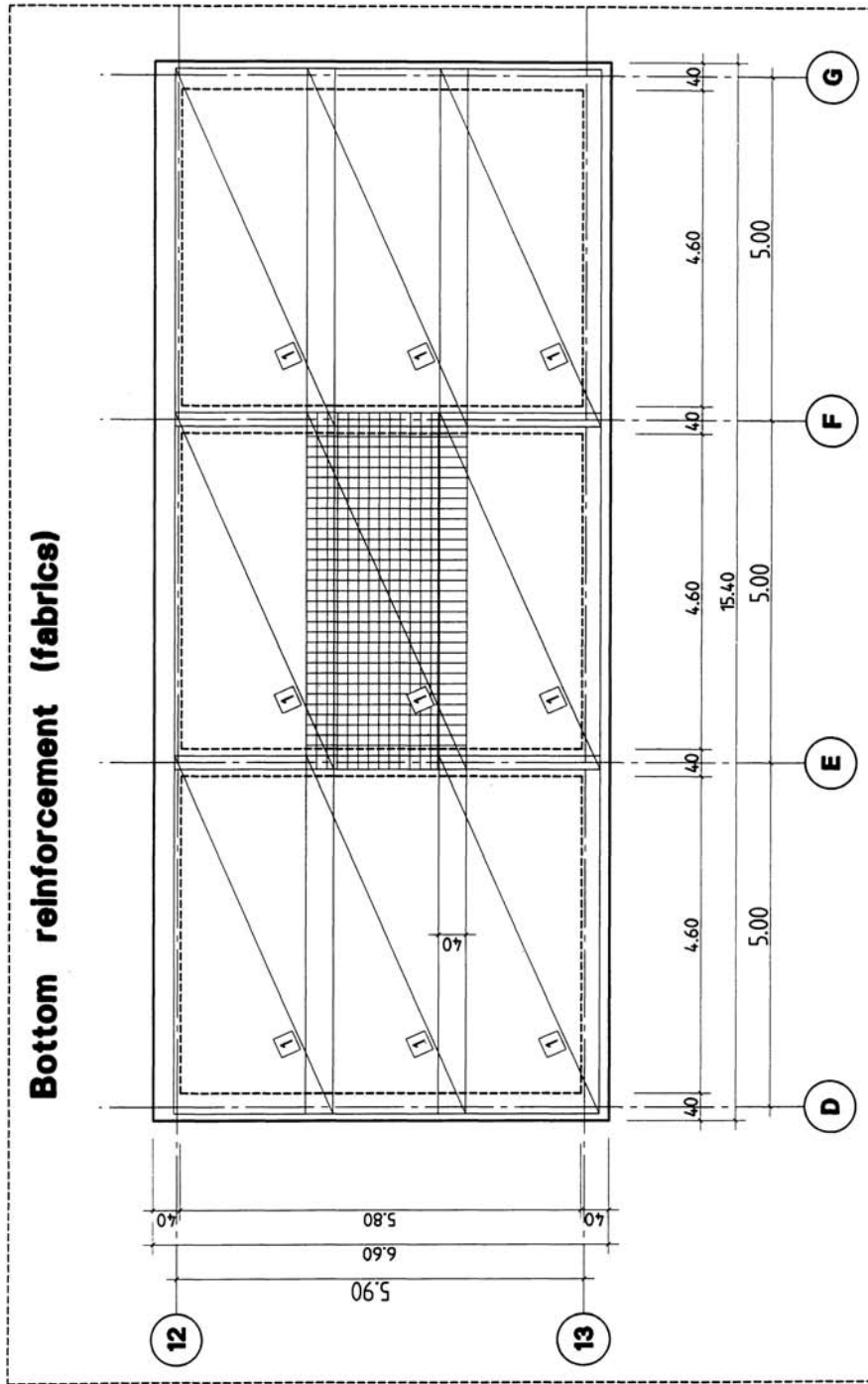
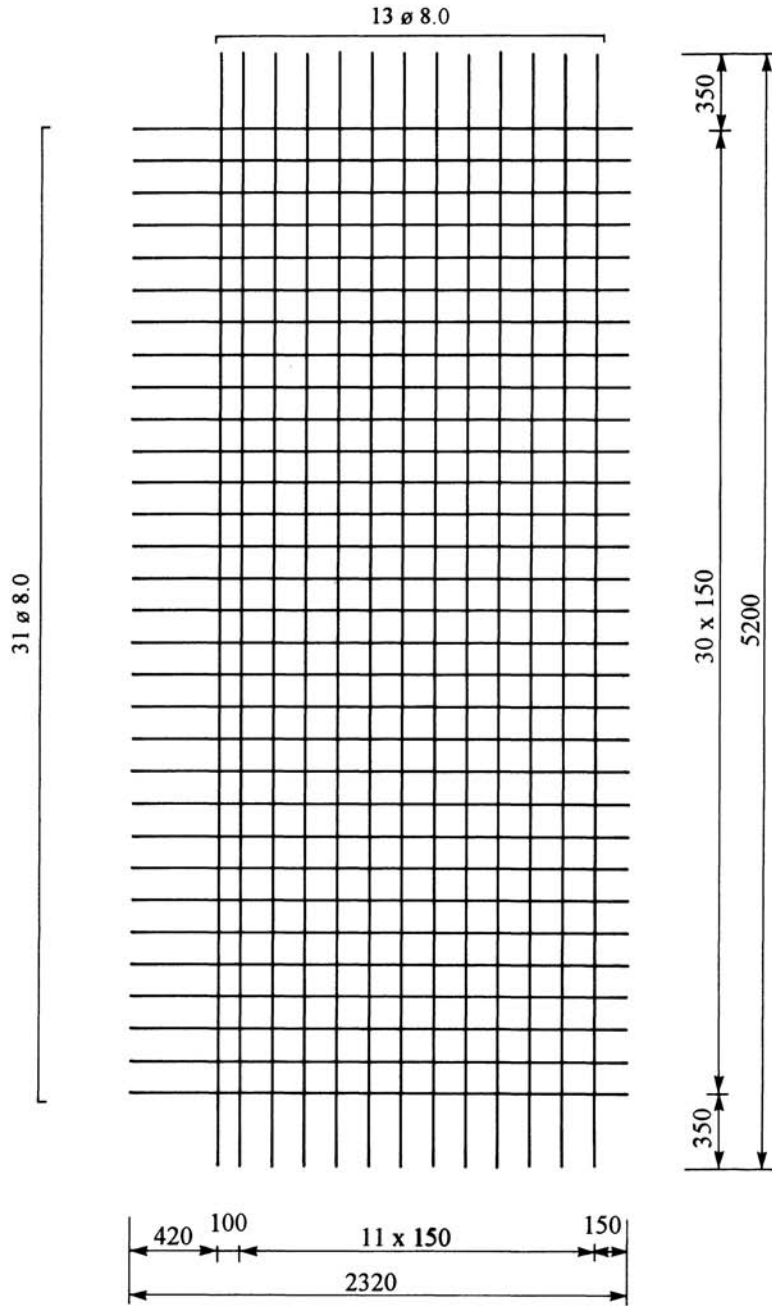
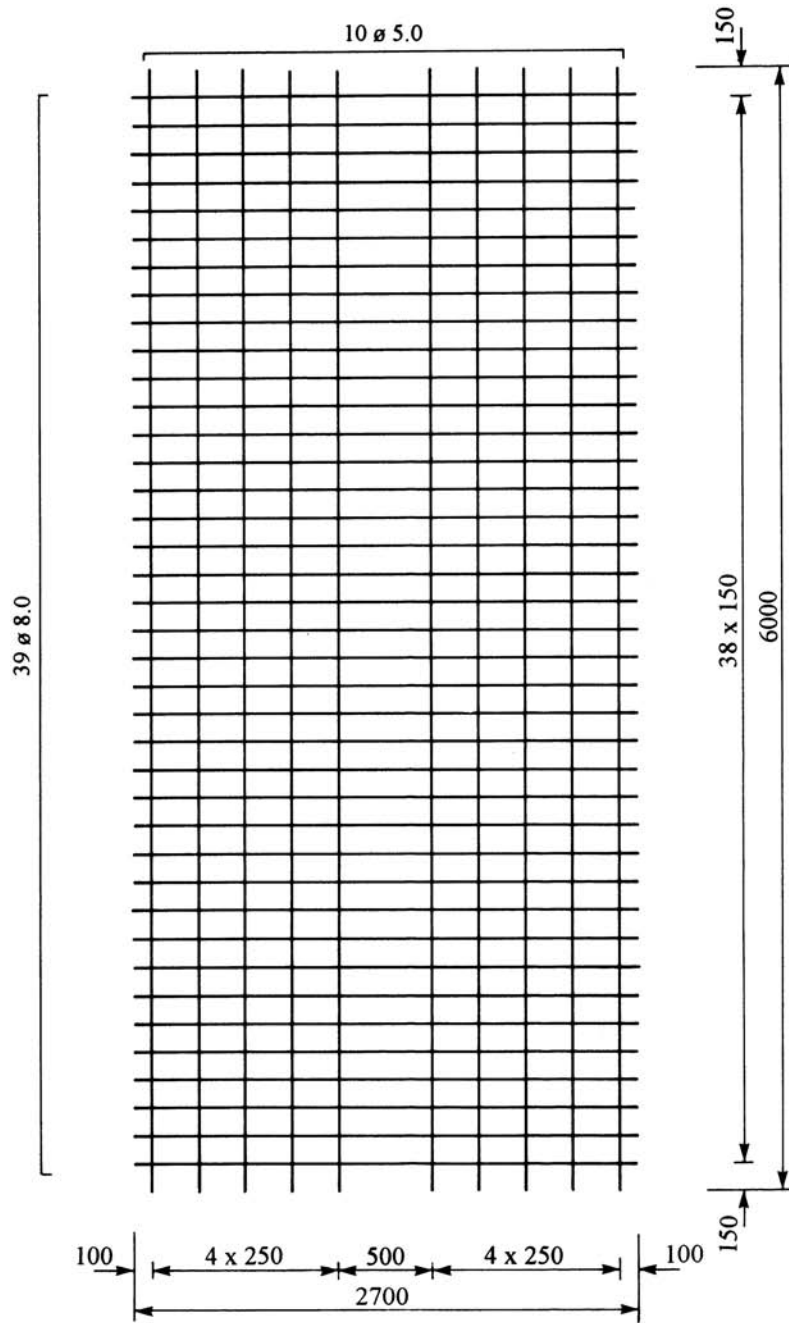


Figure B2



Longitudinal bars $13 \text{ } \varnothing 8.0 \times 5200 = 26.702 \text{ kg}$
Transverse bars $31 \text{ } \varnothing 8.0 \times 2320 = 28.408 \text{ kg}$
Total weight 58.110 kg

Figure B3

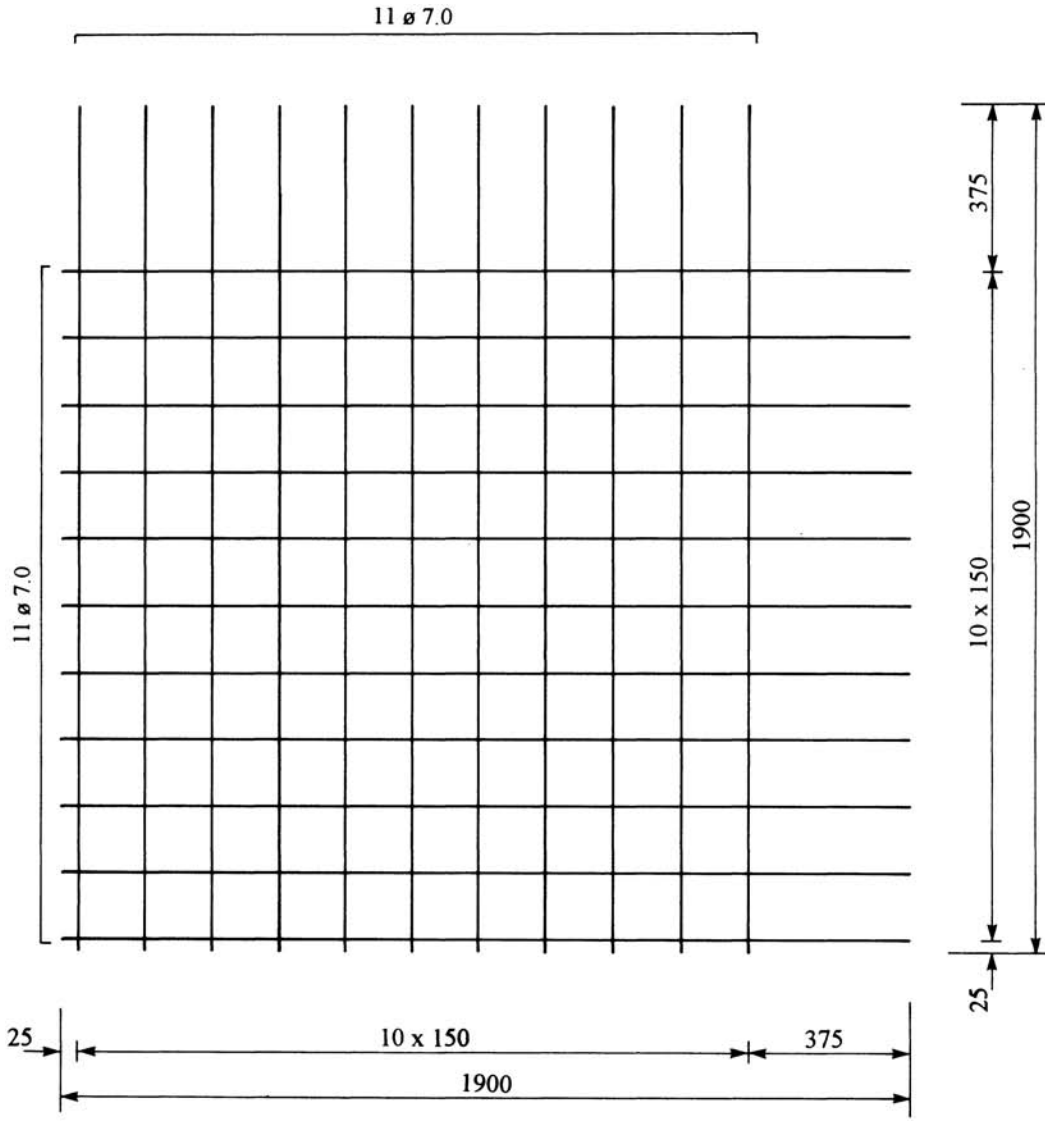


Longitudinal bars $10 \text{ } \varnothing 5.0 \times 6000 = 9.240 \text{ kg}$

Transverse bars $39 \text{ } \varnothing 8.0 \times 2700 = 41.594 \text{ kg}$

Total weight 50.834 kg

Figure B4



Longitudinal bars 11 \varnothing 7.0 x 1900 = 6.312 kg
Transverse bars 11 \varnothing 7.0 x 1900 = 6.312 kg
Total weight 12.624 kg

Figure B5

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