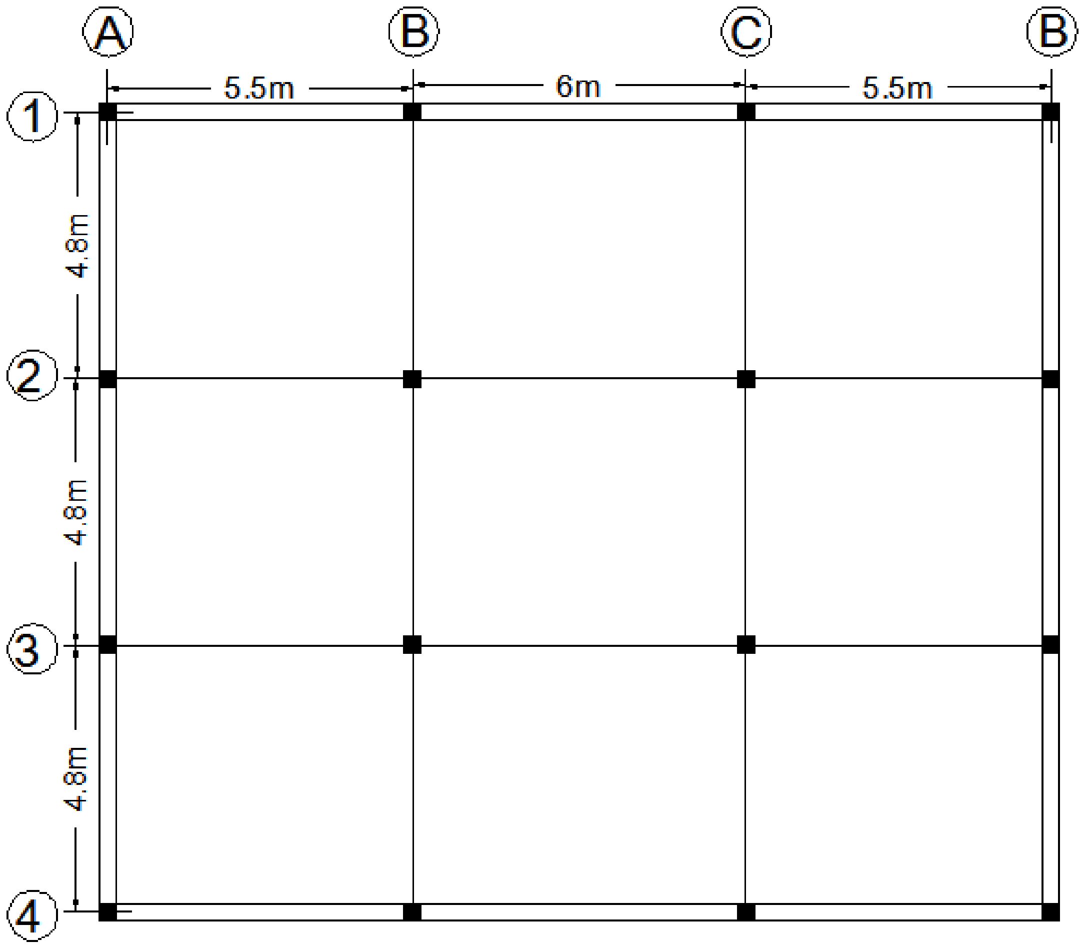
Example 3.2 Flat slabs

The following floor plan is intended to be a flat slab system. The slab thickness is 200mm thick and supports a characteristic dead load of 2.7 KN/m2 in addition to self-weight and a characteristic live load of 3 KN/m^2. The slab is provided on edge beams of b_w= 300 mm and h=400mm and all columns are 300mm by 300mm. Design the slab system using C20/25, S – 400.



Solution:

Step 1: Material property

Concrete:

$$fcd = \frac{0.85 * 20}{1.5} = 11.33Mpa$$

$$fctk, 0.05 = 1.5Mpa$$

$$fctm = 2.2Mpa$$

$$\gamma c = 1.5$$

$fck = 20Mpa, fcu = 25Mpa$

Rebar: $fyk = 400Mpa$

$$fyd = \frac{fyk}{1.15} = 347.83Mpa$$

$$\varepsilon yd = \frac{fyd}{Es} = \frac{347.83}{200} = 1.74‰$$

Step 2: Depth check for deflection

- According to ACI code minimum thickness of slabs without interior beams: without drop panels for f_{yk} = 400 Mpa
  - Exterior panels with edge beams = l_n /33

    Slab considered to have edge beam if $\alpha_f = \frac{I_b}{I_s} > 0.8$

    for this case $\alpha_f = \frac{I_b}{I_s} = \frac{20.33x10^8}{19.33x10^8} = 1.05 > 0.8 ok!$

    (see the computation of I_b and I_s on page 6)
  - Interior panels = l_n /33

    L_n is the length of the clear span in longer direction.
  - For edge panel L_n = 5500 - 300 = 5200mm
  - For edge panel L_n = 6000 - 300 = 5700mm

$$d = \frac{l_n}{33} = \frac{5700}{33} = 172.73mm$$

$$h = d + cover + \frac{\theta}{2} = 172.73 + 15 + \frac{12}{2} = 193.7mm$$ <200 *mm* **ok!**

Step 3: Loading

Dead load:

- Self-weight → 0.2 * 25 = 5 KN/m^2
- Imposed dead load → 2.7 KN/m^2

  G_k = 7.7 KN/m^2

Variable Load:

- Live load Q_k = 3 KN/m^2

Design load:

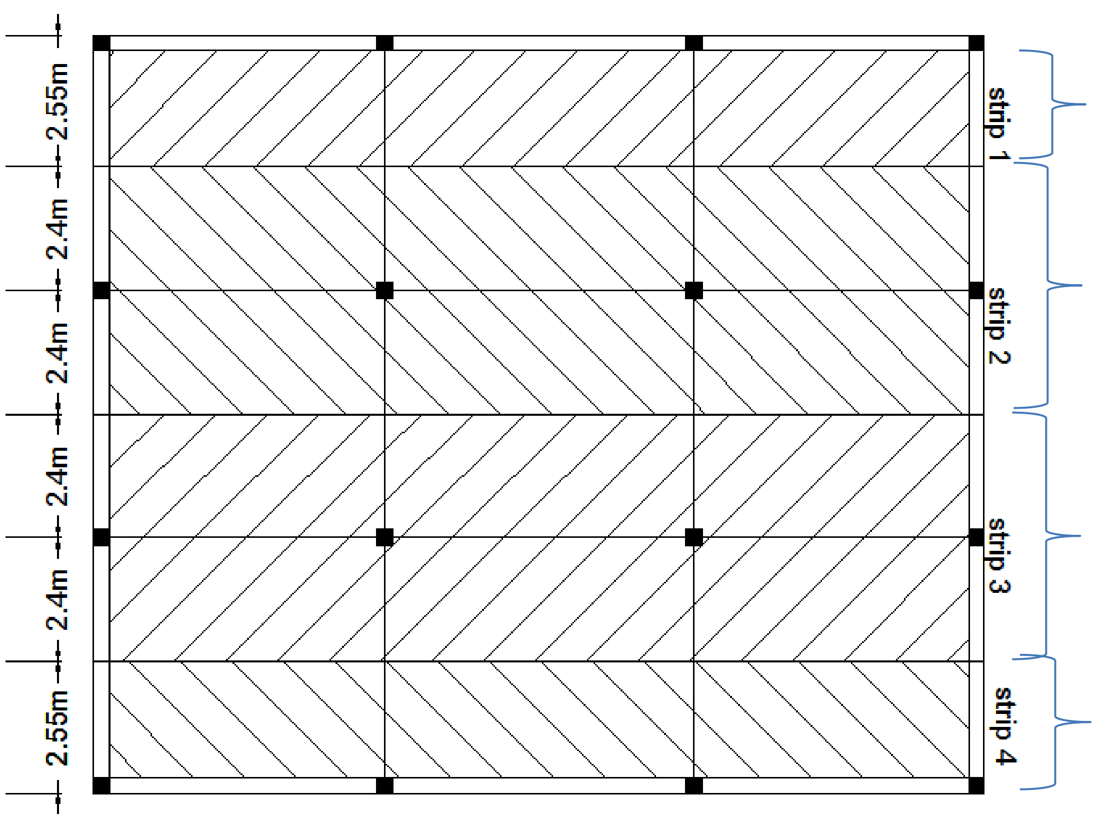
- $Gd = 1.35 * Gk = 1.35 * 7.7 = 10.39$ KN/m^2
- $Qd = 1.5 * Qk = 1.5 * 3 = 4.5$ KN/m^2
- $Therefore\ Pd$ =14.89 KN/m^2

Step 4: Limitations to direct design method

1. 3 span in each direction………… Ok!
2. $\frac{L_y}{L_x} = \frac{5.5}{4.8} = 1.15 < 2 \rightarrow and \ \frac{6}{4.8} = 1.25 < 2........Ok!$
3. Successive span length in each direction shall not differ by more than 1/3 of longer span.
   $6 - 5.5 = 0.5 < \frac{1}{3}x6 = 2 \cdots Ok!$
4. All loads must be due to gravity only …… (By assuming there are no lateral loads on our slab) Ok!
5. Factored live load must be less than twice of the factored dead load
   $\frac{LL}{DL} < 2 \Rightarrow \frac{4.5}{10.395} = 0.43 < 2 \cdots Ok!$
6. Maximum offset of column from either axis between center-line of successive columns shall not exceed 10% of the span in direction of offset …. Ok!
7. For a panel with beams between supports on all sides the relative stiffness of the beams in the two perpendicular direction given by
   $(\alpha_1.\ell_2^2)/(\alpha_2.\ell_1^2)$ Shall not be less than 0.2 or greater than 5 …. Ok!

➢ **The direct design method (DDM) can be used.**

Step 5: Analysis



- Due to symmetry strip-1 & strip-4 are similar and

  Strip -2 & strip 3 are similar.

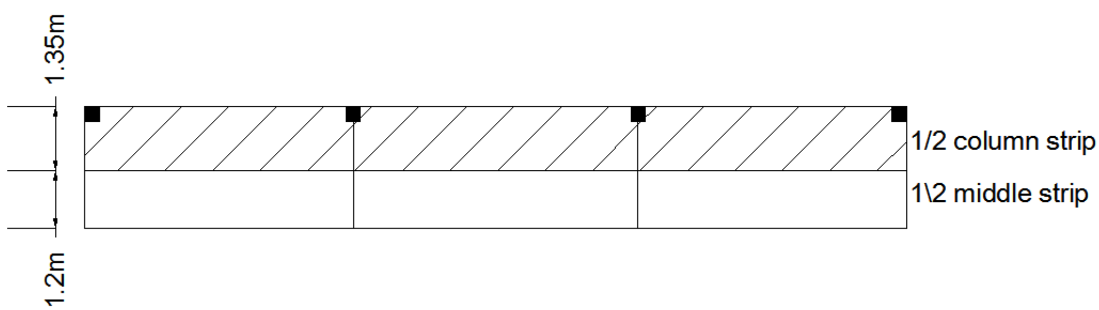
Strip -1 (along axis -1)

i. Compute l2, $\ell_2 = \frac{4.8}{2} + \frac{0.3}{2} = 2.4 + 0.15 = 2.55mm$

ii. Compute Ln, $l_n = \begin{cases} 5500 - 300 = 5200mm \ \frac{b}{n} \ axes\ A\ and\ B\ ,C\ and\ D \\ 6000 - 300 = 5700mm \frac{b}{n} \ axes\ B\ and\ C \end{cases}$

iii. Compute lx, $l_x = 4.8m$ The minimum panel dimensions

$l_x = \frac{4.8}{4} = 1.2$



iv. Compute static moments

$$M_o = \frac{P_d l_2 l_n^2}{8}$$

- Between axis A & B or C & D

$$M_0 = \frac{14.89x2.55x5.2^2}{8} = 128.34KN.m$$

- Between axis B & C

$$M_0 = \frac{14.89x2.55x5.7^2}{8} = 154.2KN.m$$

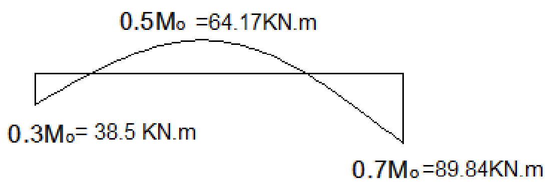
v. Longitudinal distribution of M_o

a. Between axis A & B or C & D

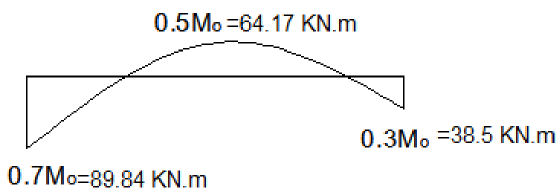
$$M_o = 128.34KN.m$$

N. B:-It is case-4, slabs without beam between interior supports and with edge beam.

- **For panel between axis A and B**

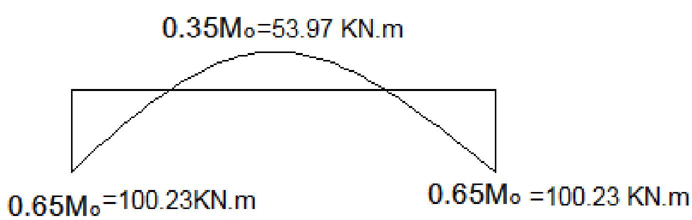


- **For panel between axis C and D**



b. Between axis B &C

$$M_o = 153.38KN.m$$

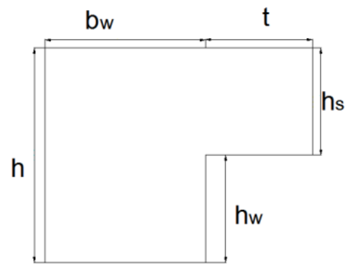


N.B Since there is a beam on the column strip some amount of the column strip moment will be resisted by the beam

$$\alpha_f = \frac{4E_{cb}\frac{I_b}{l}}{4E_{cb}\frac{I_b}{l}}$$

Since "l" is the same for both the beam and slab and $E_{cb} = E_{cs}$

$$\alpha_f = \frac{I_b}{I_s}$$



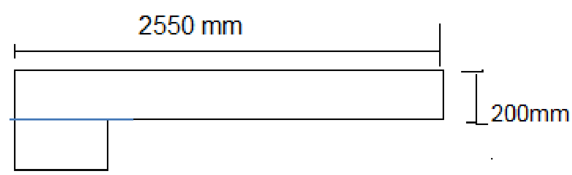
$$h = 400mm$$
$$hs = 200mm$$
$$hw = 200mm$$
$$bw = 300mm$$
$$t = \min(hw, 4hs) = 200mm$$

$$\bar{y} = \frac{(500x200)x300 + (300x200)x100}{(500x200) + (300x200)} = 225mm$$

$$I_s = \frac{1}{12}bh^3$$
$$= \frac{1}{12}x2550x200^3$$
$$= 1.7x10^9 = 1700x10^6 mm$$

$$I_b = \frac{1}{12}x300x200^3 + (300x200)(225-100)^2 + \frac{1}{12}x500x200^3 + (500x200)(175-100)^2$$

$$= 1137.5x10^6 + 895.83x10^6$$
$$= 2033.33x10^6 mm^4$$



$$I_s = \frac{1}{12}x2550x200^3 = 1700x10^6 mm^4$$

$$\alpha = \frac{I_b}{I_s} = \frac{2033.33x10^6}{1700x10^6} = 1.196$$

Calculate $\alpha_f (\frac{l_2}{l_1})$

- $\ell_2 = 4.8m$ *For panel between axis A & B or C & D*
- $\ell_1 = 6.0m$ for panel between axis B & C.

- **For panel between axis A & B and C & D**

$$\alpha_f(\frac{\ell_2}{\ell_1}) = 1.196(\frac{4.8}{5.5})$$
$$= 1.045 > 1.0$$

- **For panel between axis B & D**

$$\alpha_f(\frac{\ell_2}{\ell_1}) = 1.196(\frac{4.8}{6})$$
$$= 0.957 < 1.0$$

vi. Transverse distribution of moments (i.e. to column and middle strip)

- **For panel between axis A & B and C & D**

a. Interior negative moment beam**, M=89.84 KN.m**
   - There is intermediate beam

$$\alpha_f(\frac{\ell_2}{\ell_1}) \geq 1.0$$
$$\frac{\ell_2}{\ell_1} = \frac{4.8}{5.5} = 0.87$$

   - There is intermediate beam, 78.9 % (interpolated form table)

C.S moment $= 0.789x89.84 = 70.88KN.m$

$$(1-0.789) = 0.211$$

Half M.S moments $= 0.211x89.84 = 18.96KN.m$

b. Positive moment, **M=64.17 KN.m**

$$\alpha_f(\frac{\ell_2}{\ell_1}) \geq 1.0$$
$$\frac{\ell_2}{\ell_1} = \frac{4.8}{5.5} = 0.87$$

   - 78.9% of moment to column strip

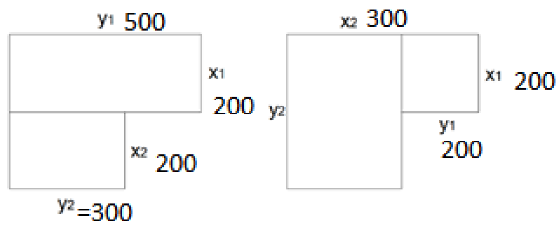
C.S moment $= 0.789x64.17 = 50.63KN.m$

Half M.S moments $= 0.211x64.17 = 13.54KN.m$

c. Exterior negative moment, **M=38.50 KN.m**
   - $\alpha_f(\frac{\ell_2}{\ell_1}) \geq 1.0$

- There is edge beam $\beta_t \neq 0$

  Calculate $\beta = \frac{c}{2I_s}$



$$c = \sum[(1-0.63\frac{x}{y})*\frac{x^3y}{3}]$$

$$N.B\ Always\ x < y$$

Case 1:

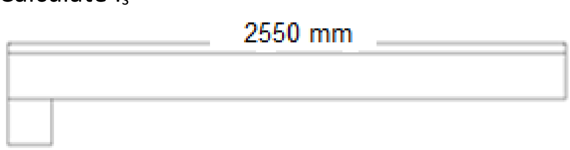
$$c_1 = [(1-0.63*\frac{0.2}{0.5})*(\frac{0.2^3*0.5}{3})]+[(1-0.63*\frac{0.2}{0.3})*(\frac{0.2^3*0.3}{3})]$$
$$= (0.748*0.00133)+(0.58+0.0008)$$
$$= 0.00097+0.00046$$
$$= 1.43*10^{-3}m^4$$

Case 2:

$$c_1 = [(1-0.63*\frac{0.2}{0.2})*(\frac{0.2^3*0.3}{3})]+[(1-0.63*\frac{0.2}{0.4})*(\frac{0.3^3*0.4}{3})]$$
$$= (0.37*0.00053)+(0.5275+0.0036)$$
$$= 0.0002+0.00140$$
$$= 2.10*10^{-3}m^4$$

$$c = \max(c_1, c_2)$$
$$c = 2.10*10^{-3}m^4$$

- Calculate I_s



$$I_s = \frac{1}{12} * b * h^3$$

$$I_s = \frac{1}{12} * 2.55 * 0.2^3$$

$$= 1.7 * 10^{-3} m^4$$

$$\beta_t = \frac{C}{2 * I_s} = \frac{2.1 * 10^{-3}}{2 * 1.7 * 10^{-3}}$$

$$= 0.618$$

Percentage of column strip moment can be determined by interpolation.

$$for\ \beta_t = 0.618, \alpha_{f1} \, l2/l1 \geq 1, and\ \frac{l2}{l1} = 0.5 \rightarrow 97.528$$

$$for\ \beta_t = 0.618, \alpha_{f1} \, l2/l1 \geq 1, and\ \frac{l2}{l1} = 1 \rightarrow 93.82$$

$$for\ \frac{l2}{l1} = 0.872\ and\ the\ values\ obtained\ above \rightarrow 94.76$$

$$C.S\ moment = 0.948 * 38.5$$

$$= 36.5 KNm$$

$$M.S\ moment = 0.052 * 38.5$$

$$= 2.00 KNm$$

- **<u>For panel between axis B & C</u>**

a) Interior negative moment, **M = 100.23 KN.m**

$$inerpoliate\ for\ \alpha_{f1} \, l2/l1 = 0, and\ \frac{l2}{l1} = 0.8 \rightarrow 75$$

$$inerpoliate\ for\ \alpha_{f1} \, l2/l1 \geq 1, and\ \frac{l2}{l1} = 0.8 \rightarrow 81$$

$$inerpoliate\ for\ \alpha_{f1} \, l2/l1 = 0.957, and\ the\ values\ obtained\ above \rightarrow 80.742$$

$$C.S\ moment = 0.8074 * 100.23$$

$$= 80.93 KNm$$

$$M.S\ moment = 0.1926 * 100.23$$

$$= 19.3 KNm$$

b) Positive moment**, M = 53.97 KN.m**

$$inerpoliate\ for\ \alpha_{f1}\ {l2}/{l1} = 0, and\ \frac{l2}{l1} = 0.8 \rightarrow 60$$

$$inerpoliate\ for\ \alpha_{f1}\ {l2}/{l1} \geq 1, and\ \frac{l2}{l1} = 0.8 \rightarrow 81$$

$$inerpoliate\ for\ \alpha_{f1}\ {l2}/{l1} = 0.957, and\ the\ values\ obtained\ above \rightarrow 80.097$$

$$C.S\ moment = 0.801 * 53.97$$
$$= 43.23 KNm$$
$$M.S\ moment = 0.194 * 53.97$$
$$= 10.74 KNm$$

Note: If $\alpha_f (\frac{l_2}{l_1}) \geq 1.0$ 85% of the column strips moment goes to the beam and 15% to the slab.

- For panel between axis A & B or C & D, 85% of the column strip moment goes to beam and 15% to the slab.
- For panel between axis B & C

>= 1.0	85%
0.957	x
0.0	0%

$$\frac{0.957 - 0.0}{x - 0} = \frac{1.0 - 0.0}{85 - 0}$$

$$x = \frac{0.957 x 85}{1} = 81.35\%$$

Therefore, 81.35% of column strip moment goes to the beam and 18.65% goes to the slab.

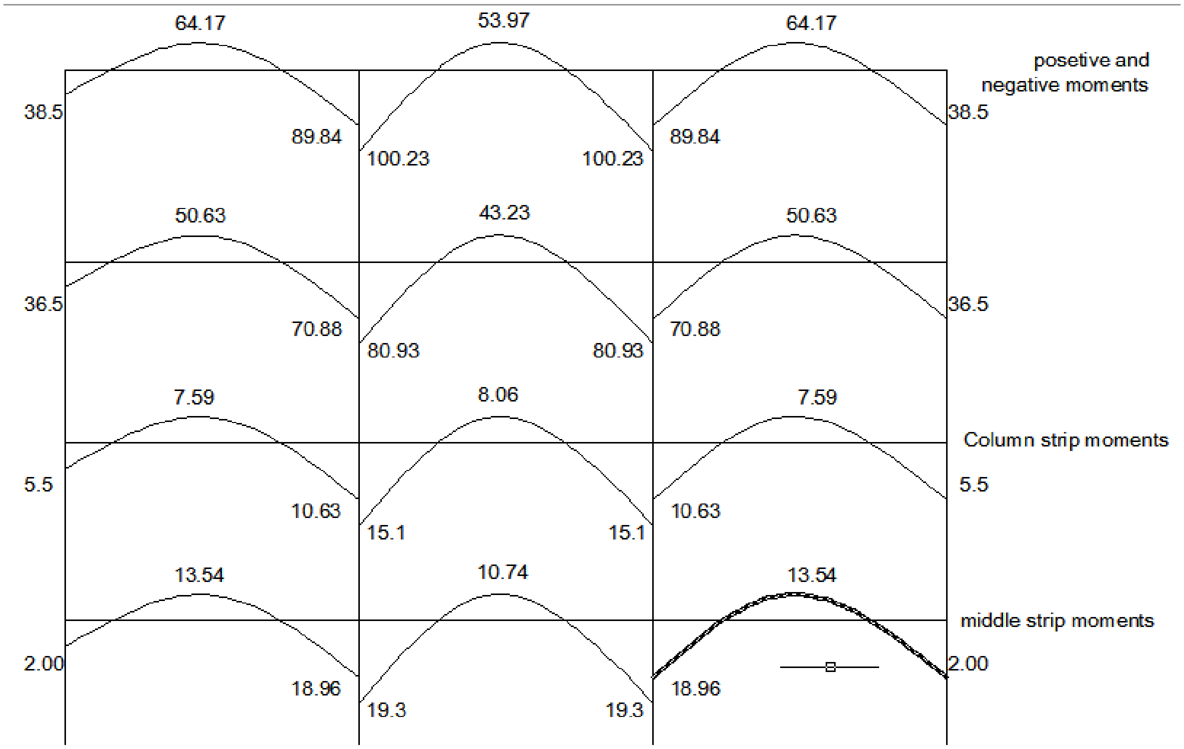
- **<u>For panel between axis A & B and C & D</u>**

Moment Location	**Value**	**To beam (85%)**	**To slab (15%)**
Exterior –ve moment	36.5	31.0	5.5
Positive moment	50.63	43.04	7.59
Interior –ve moment	70.88	60.248	10.632

- **<u>For panel between axis B & C</u>**

Moment Location	**Value**	**To beam**	**To slab**
Interior –ve moment	80.93	65.43	15
Positive moment	43.23	35.17	8.06

Moment summary

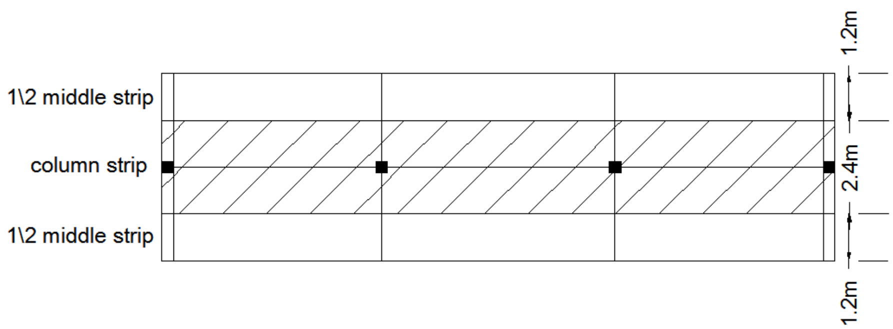


Strip 2: [along axis 2]

i) Compute l2, $l2 = 4.8/2 + 4.8/2$
$= 4.8$

ii) Compute ln,
$\ln = 5500 - 300 = 5200mm\ b/n\ axis\ A \& B\ and\ C \& D$
$= 6000 - 300 = 5700mm\ b/n\ axis\ B \& C$

iii) Compute lx, $lx = \min(l2, l\,n) = 4.8m$



$$half\ a\ weidth\ of\ the\ column\ strip = {lx}/{2} = 1.2m$$

iv) Compute the static moment, Mo.

$$M_o = \frac{P_d * l_2 * l\,n^2}{8}$$

- Between axis A & B or C & D

$$M_o = \frac{14.89 * 4.8 * 5.2^2}{8}$$
$$= 241.58 KN - m$$

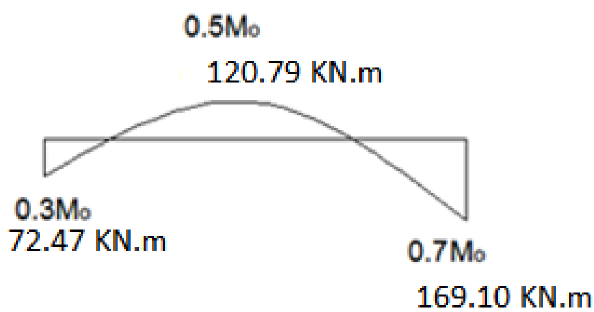
- Between axis A & B or C & D

$$M_o = \frac{14.89 * 4.8 * 5.7^2}{8}$$
$$= 290.26 KN - m$$

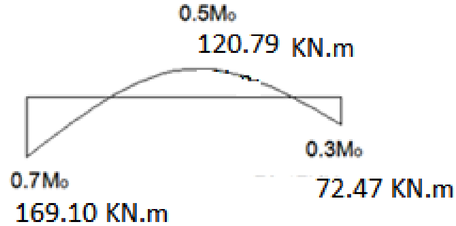
v) Longitudinal distribution of **Mo = 241.58KN-m**

a) Between axis A & B or C & D

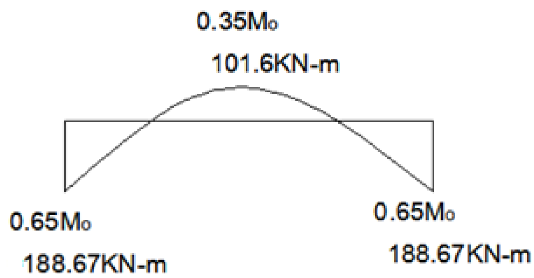
- <u>For panel between axis A and B,</u>



- <u>For panel between axis C and D</u>



b. Between axis B &C M_0 =290.26KN-m



vi) **Transverse distribution** of moments (i.e to column and middle strip)

- **For panel between axis A & B and C & D**

a) Interior negative moment**, M=169.1 KN-m**
- No intermediate beam → α=0
- 75% of moment to column strip

$$C.S\ moment = 0.75*169.1$$
$$= 126.8KNm$$
$$half\ M.S\ moment = 0.125*169.1$$
$$= 21.14KNm$$

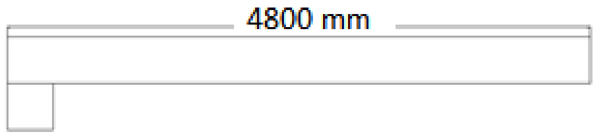
b) Positive moment, **M=120.79 KN-m**
- No intermediate beam → α=0
- 60% of moment to column strip

$$C.S\ moment = 0.6*120.79$$
$$= 72.47KNm$$
$$half\ M.S\ moment = 0.2*120.79$$
$$= 24.16KNm$$

c) Exterior negative moment, **M=72.47 KN-m**
- No intermediate beam → α=0
- There is edge beam → $\beta_t \neq 0$

calculate $\beta_t = \frac{C}{2*I_s}$

$$take\ c = 2.10*10^{-3}m^4\ (form\ previous\ calculation)$$



$$I_s = \frac{1}{12} * b * h^3$$

$$I_s = \frac{1}{12} * 4.8 * 0.2^3$$

$$= 3.2 * 10^{-3} m^4$$

$$\beta_t = \frac{C}{2 * I_s} = \frac{2.1 * 10^{-3}}{2 * 3.2 * 10^{-3}}$$

$$= 0.328$$

Percentage of column strip moment can be determined by interpolation

$$\beta_t = 0 --- 100\%$$

$$0.328 ---- x$$

$$\beta_t = 2.5 -- 75\%$$

$$x = 96.72\%$$

$$C.S\ moment = 0.9672 * 72.47$$

$$= 70.09 KNm$$

$$half\ M.S\ moment = 0.0164 * 72.47$$

$$= 1.188 KNm$$

- **<u>For panel between axis B & C</u>**

a) Interior negative moment**, M = 188.67KN-m**
   - No intermediate beam → α=0
   - 75% of moment to column strip

$$C.S\ moment = 0.75 * 188.67$$

$$= 141.5 KNm$$

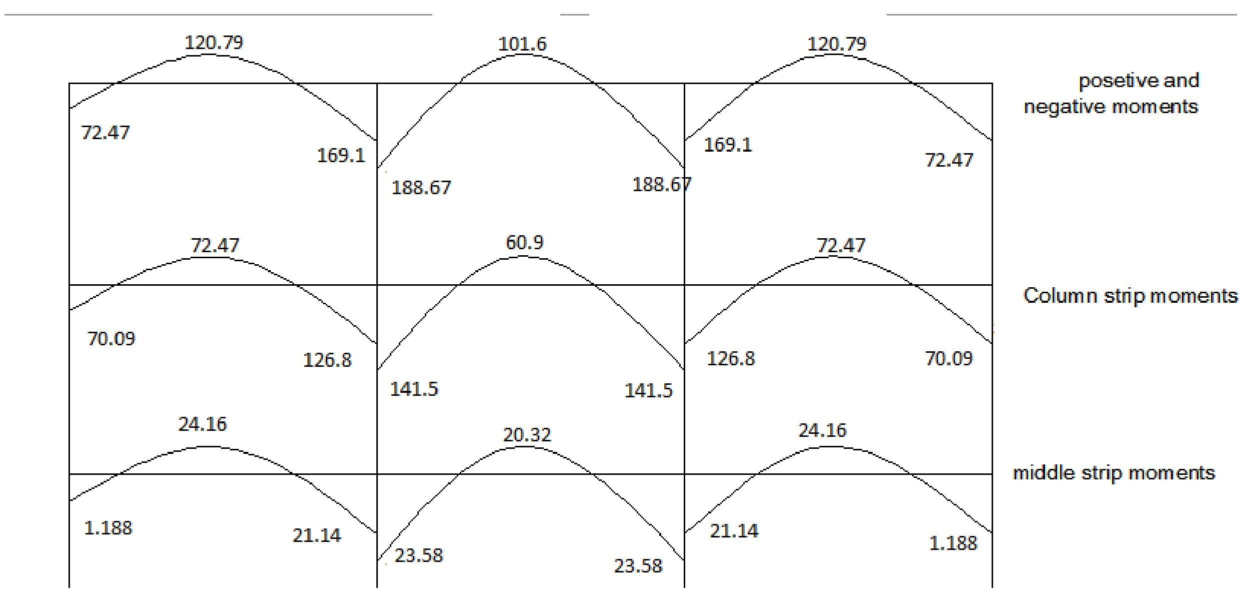
$$half\ M.S\ moment = 0.125 * 188.67$$

$$= 23.58 KNm$$

b) positive moment, **M = 101.6KN-m**
   - No intermediate beam → α=0
   - 60% of moment to column strip

$$C.S\ moment = 0.6*101.6$$
$$= 60.9KNm$$
$$half\ \ M.S\ moment = 0.2*101.6$$
$$= 20.32KNm$$

Moment summary



Step 6: Design

$$d = h - cover - \phi/2 \quad for\ the\ bottom\ layer\ bar$$
$$= 200 - 15 - 10/2$$
$$= 180mm$$

Area of minimum reinforcement

$$A_{s,\min} = \max\left(\frac{0.26 f_{ctm} b_t d}{f_{yk}}, 0.0013bd\right)$$
$$= \max\left(\frac{0.26*2.2*1000*180}{400}, 0.0013*1000*180\right)$$
$$= \max(257.4, 234)$$
$$A_{s,\min} = 257.4mm^2$$

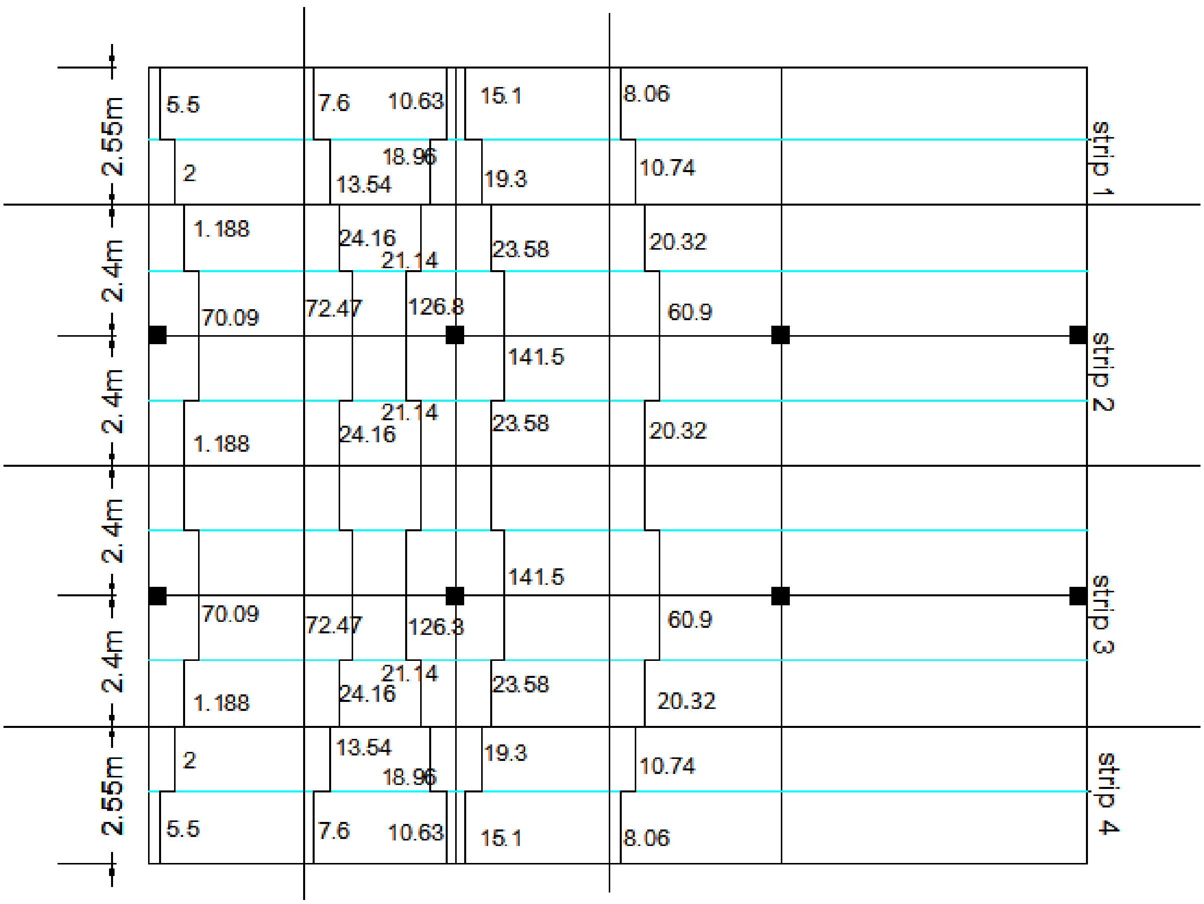
Minimum spacing:

$$S_{\min} = \max\left(\phi, aggrigate\, size + 5mm, 20mm\right)$$

Maximum spacing:

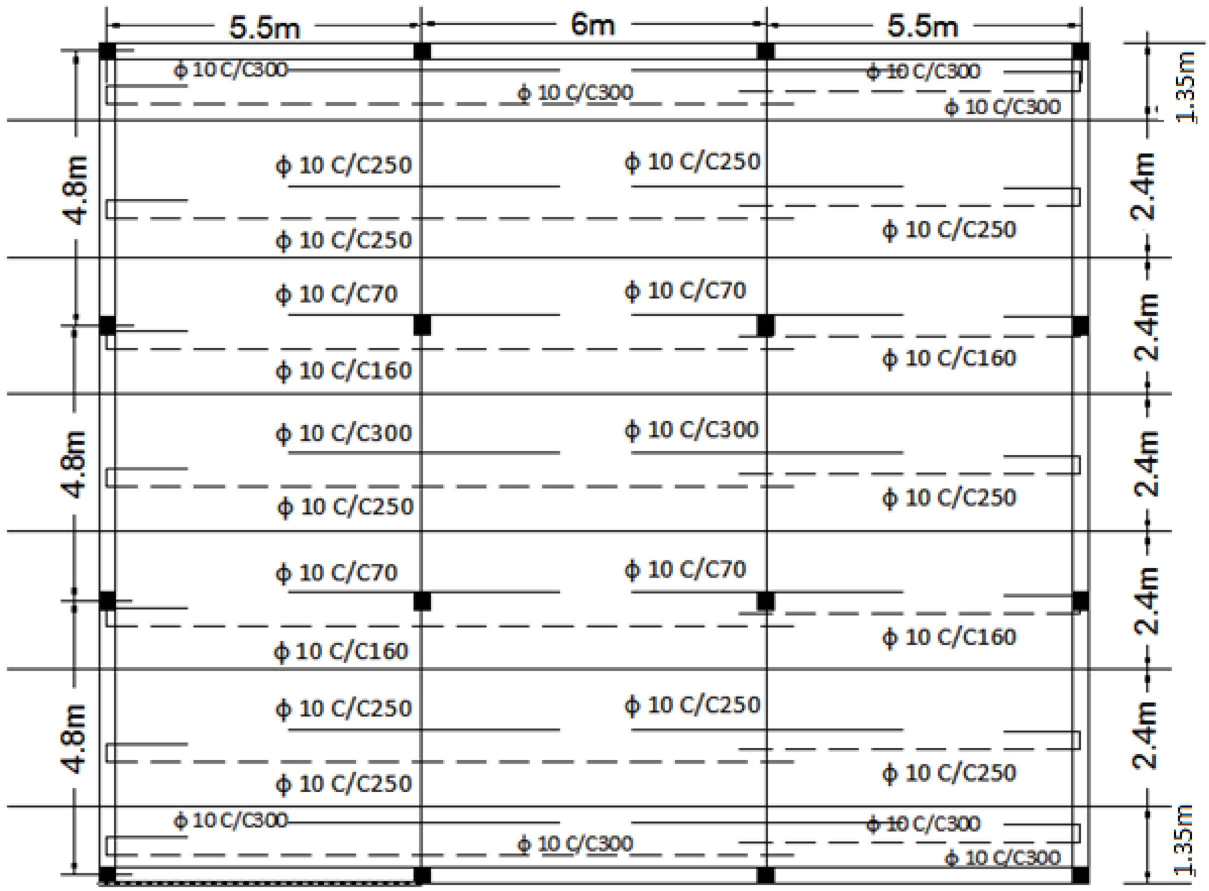
$$S_{\max} = \min\left(3h, 400mm\right),\ for\ primary\ reinforcement$$
$$S_{\max} = 400mm$$

Moment diagram (not drawn to scale)



strip	Moment	strip width	moment per meter	ω	As,req	s, req	s provided
C.S	5.5	1.35	4.07	0.011	257.4	333	use ϕ 10 C/C300
	7.59	1.35	5.62	0.015	257.4	333	use ϕ 10 C/C300
	10.632	1.35	7.88	0.0216	257.4	333	use ϕ 10 C/C300
	15.1	1.35	20.39	0.057	332.32	250	use ϕ 10 C/C300
	8.06	1.35	10.88	0.03	257.4	333	use ϕ 10 C/C300
M.S	18.96	1.2	15.80	0.044	257.4	333	use ϕ 10 C/C300
	13.54	1.2	11.28	0.317	257.4	333	use ϕ 10 C/C300
	2	1.2	1.67	0.0045	257.4	333	use ϕ 10 C/C300
	19.3	1.2	16.08	0.0446	261.9	333	use ϕ 10 C/C300
	10.74	1.2	8.95	0.024	257.4	333	use ϕ 10 C/C300
C.S	126.8	2.4	52.63	0.157	921.99	90.9	use ϕ 10 C/C90
	72.47	2.4	30.03	0.0889	521.19	166.67	use ϕ 10 C/C160
	70.09	2.4	29.20	0.083	45.5	166.67	use ϕ 10 C/C160
	141.5	2.4	58.96	0.177	1036.99	76.9	use ϕ 10 C/C70
	60.9	2.4	25.40	0.072	422.8	200	use ϕ 10 C/C200
M.S	21.14	1.2	17.54	0.0487	286.01	333	use ϕ 10 C/C300
	24.16	1.2	20.03	0.0559	327.9	250	use ϕ 10 C/C250
	1.188	1.2	0.99	0.0027	257.4	333	use ϕ 10 C/C300
	23.58	1.2	19.65	0.0559	327.9	250	use ϕ 10 C/C250
	20.32	1.2	16.93	0.047	277.39	333	use ϕ 10 C/C300

Detailing:


5.5m
6m
5.5m
4.8m
4.8m
4.8m
1.35m
2.4m
2.4m
2.4m
2.4m
2.4m
1.35m
ϕ 10 C/C300
ϕ 10 C/C300
ϕ 10 C/C300
ϕ 10 C/C300
ϕ 10 C/C250
ϕ 10 C/C250
ϕ 10 C/C250
ϕ 10 C/C250
ϕ 10 C/C70
ϕ 10 C/C70
ϕ 10 C/C160
ϕ 10 C/C160
ϕ 10 C/C300
ϕ 10 C/C300
ϕ 10 C/C250
ϕ 10 C/C250
ϕ 10 C/C70
ϕ 10 C/C70
ϕ 10 C/C160
ϕ 10 C/C160
ϕ 10 C/C250
ϕ 10 C/C250
ϕ 10 C/C250
ϕ 10 C/C250
ϕ 10 C/C300
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