Chapter Two Examples

Example 2.1 - Continuous beam

A continues beam shown below supports a design dead load of 14.5 KN/m including self-weight and a variable design load of 29 KN/m. Take cover 25 mm (Cover to longitudinal bar).

- ✓ **Use section dimension of**
  b_w =250 mm D=450 mm
- ✓ **Material**
  C20/25 concrete and S400 rebar

➢ **Design the beam**

a. Without moment redistribution
b. With 20% moment redistribution

SOLUTION A. . WITHOUT MOMENT REDISTRIBUTION

Step 1: Sectional and material property

$$f_{cd} = \frac{\alpha_{cc} f_{ck}}{\gamma c}, \quad \alpha_{cc} \text{Should be taken between 0.8 and 1.} \quad \text{Take } \alpha_{cc} = 0.85$$

$$f_{cd} = \frac{0.85 * 20}{1.5} = 11.33\ mpa.$$

$$f_{yd} = \frac{f_{yk}}{\gamma s} = \frac{400}{1.15} = 347.8\ mpa.$$

Summarized: **Material**	C20/25	f_{ck} = 20 Mpa
		f_{cd}=11.33 Mpa
		f_{ctm}=2.2 Mpa
		E_{cm}=30,000Mpa

S-400

ε_{c2}=2.0%
f_{yk}= 400 Mpa
F_{yd}= 347.83 Mpa
E_s= 200,000 Mpa
ε_y=1.74%

Geometry Using cover of 25 mm, stirrup of 8 mm and main bar of 20 mm,

$$d' = 25 + 8 + \frac{20}{2} = 43 \text{ mm}$$

$$d = 450 - 43 = 407\ mm$$

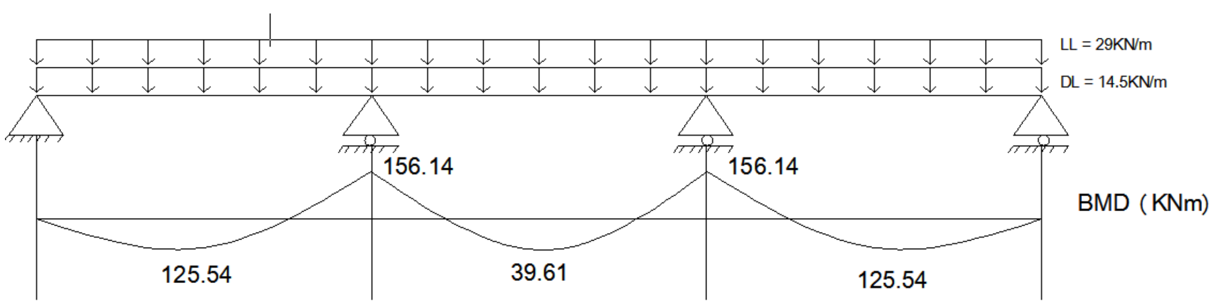
Step 2: Loading [design loads given]

Design Dead load [G_d] = 14.5 KN/m
Design Live load [Q_d] =29 KN/m

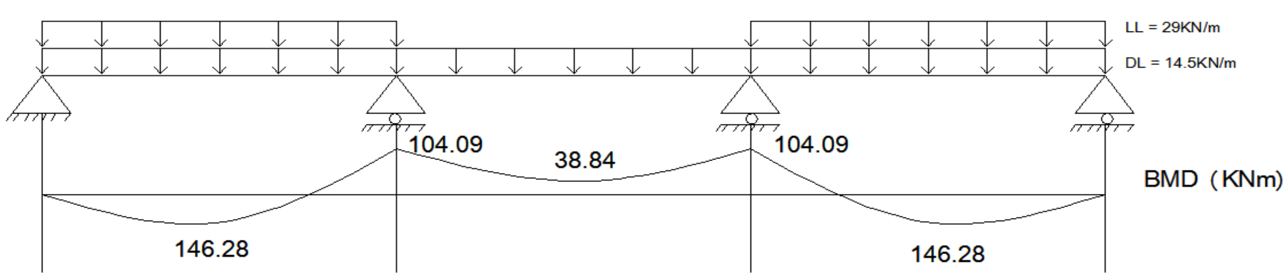
Step 3: Analysis [Using moment envelope]

- *Load arrangements and bending moment diagrams*

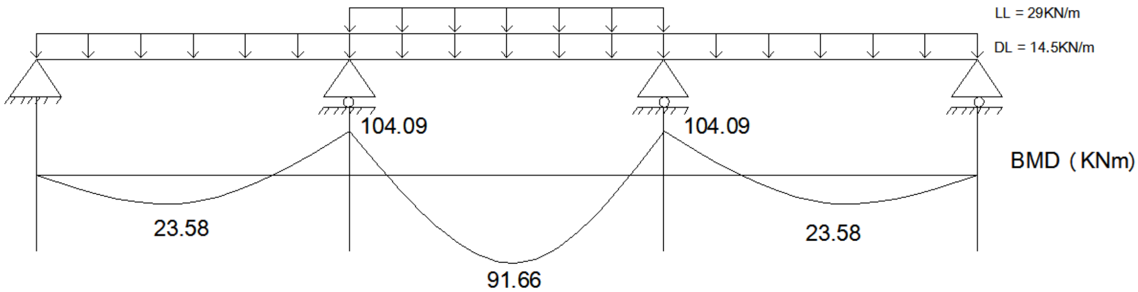
Case one: when the whole section is loaded



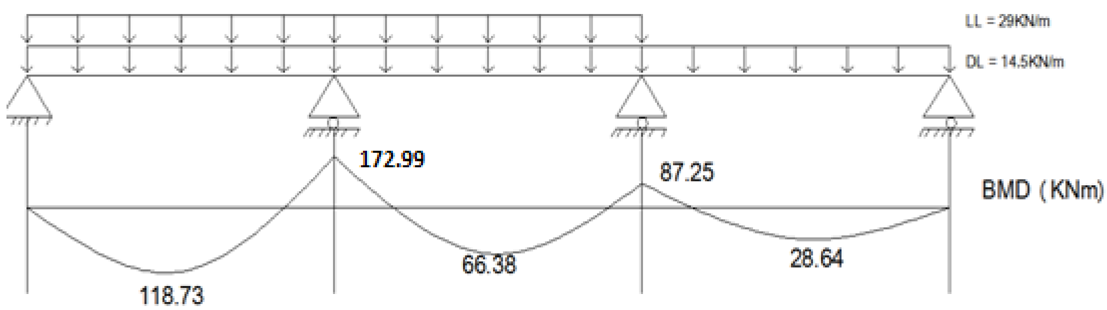
Case two: To get maximum span moment in span AB and CD



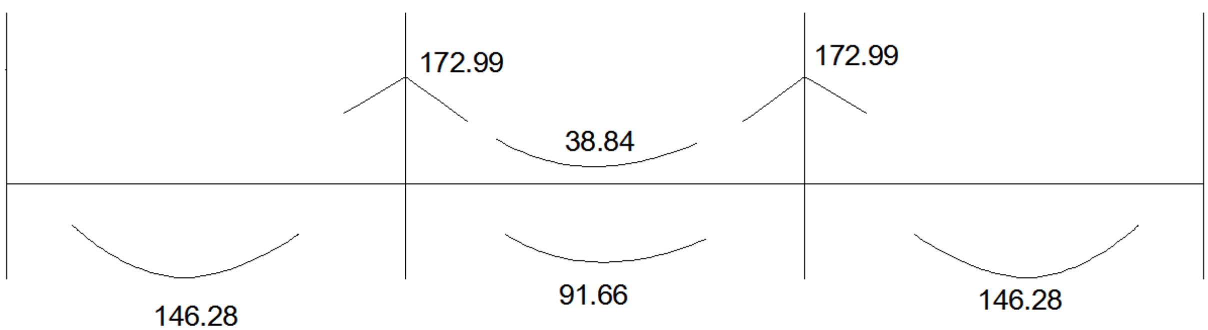
Case three: **To get maximum span moment in span BC**



Case four: **To get maximum moment at support B and C**



- ***Moment envelope*** *(The maximum moments) for 0% redistribution*



Step 4: Design [Using design chart]

a) Support B and C (-ve moment)

$$M_{sds} = 172.99\ KNm$$

$$b = 250 \quad d = 450 - 25 - 8 - \frac{20}{2} = 407mm$$

$$\mu_{sd} = \frac{M_{sd}}{f_{cd}bd^2} = \frac{172.99 * 10^6}{11.33 * 250 * 407^2} = 0.369 > \mu_{sd,lim} = 0.295$$
$$= 0.295 \quad \textbf{use Double reinforced}$$

$$K_{z,lim} = 0.814 \ldots\ldots\ldots read\ from\ the\ chart\ using\ \mu_{sd} = \mu_{sd,lim} = 0.295$$

$$M_{sd,lim} = \mu_{sd,lim} f_{cd} bd^2 = 0.295 * 11.33 * 250 * 407^2 = 138.414\ KNm$$

$$Z = K_{z,lim} * d = 0.814 * 407 = 331.298\ mm$$

$$A_{s1} = \frac{M_{sd,lim}}{Zf_{yd}} + \frac{M_{sd,s} - M_{sd,lim}}{f_{yd}(d - d_{2)}} = \frac{138.414 * 10^6}{347.8 * 331.298} + \frac{(172.99 - 138.414) * 10^6}{347.8 * (407 - 43)}$$
$$= 1474.28\ mm^2$$

$\boldsymbol{use\ 5\varnothing 20}$

- *Check the number of bars that can be placed on the single row.*

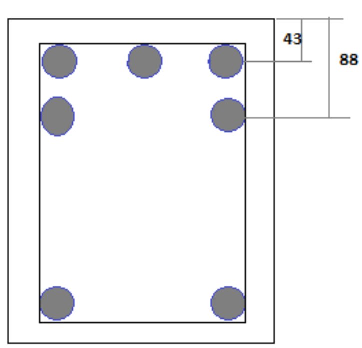
Setting one 45 mm gap to insert a vibrator and making the other gaps equal to 25 mm

$$20n + 45 + 25(n - 2) = 250 - 2 * 25 - 2 * 8$$

$$20n + 45 + 25n - 50 = 184$$

$$n = 4.2$$

A maximum of 4 bars can be supported. Therefore, arrange the bars in to 2 rows.



$$d'_{new} = Y = \frac{\sum AiYi}{\sum Ai} = \frac{3*314*43+2*314*88}{5*314} = 61\ mm$$

$$so\ d = 450 - 61 = 389\ mm$$

$$\mu_{sd} = \frac{M_{sd}}{f_{cd}bd^2} = \frac{172.99*10^6}{11.33*250*389^2} = 0.403 > \mu_{sd,lim} = 0.295 \quad \boldsymbol{Double\ reinforced}$$

$$K_{z,lim} = 0.814 \ldots from\ page\ 4$$

$$M_{sd,lim} = \mu_{sd,lim} f_{cd} bd^2 = 0.295*11.33*250*389^2 = 126.48\ KNm$$

$$Z = K_{z,lim}*d = 0.814*389 = 316.646\ mm$$

$$A_{s1} = \frac{M_{sd,lim}}{Zf_{yd}} + \frac{M_{sd,s} - M_{sd,lim}}{f_{yd}(d-d_{2)}} = \frac{126.48*10^6}{347.8*316.646} + \frac{(172.99-126.442)*10^6}{347.8*(389-43)} = 1534.84\ mm^2$$

$$\boldsymbol{use\ 5\varnothing 20}$$

- *Compression reinforcement design*
  - Check if the reinforcement has yielded

$$\frac{d_2}{d} = \frac{43}{389} = 0.1 \qquad \varepsilon_{s2} = 2.6‰\ \ (read\ from\ chart)$$

$$\varepsilon_{s2} = 2.6‰ > \varepsilon_{yd}\ use\ f_{yd} = 347.826$$

  - Calculate the stress in the concrete at the level of compression reinforcement to avoid double counting of area.

$\varepsilon_{cs2} = 2.6‰ \geq 2‰$, Therefore, we take

$$\varepsilon_c = 3.5‰\ \ and\ \sigma_{cd,s2} = 11.33\ mpa$$

$$A_{s2=\frac{1}{(\sigma_{s2}-\sigma_{cd,s2})}(\frac{M_{sds}-M_{sd,lim}}{d-d_2})} = \frac{1}{(347.826-11.33)}(\frac{(172.99-138.44)*10^6}{(389-43)} = 399.48\ mm^2$$

$$\boldsymbol{use\ 2\varnothing 20}$$

b) Span AB and CD (+ve moment)

$$M_{sds} = 146.28\ KNm$$

$$b = 250 \quad d = 450 - 25 - 8 - \frac{20}{2} = 407mm$$

$$\mu_{sd} = \frac{M_{sd}}{f_{cd}bd^2} = \frac{146.28 * 10^6}{11.33 * 250 * 407^2} = 0.3117 > \mu_{sd,lim} = 0.295 \quad \textbf{Double reinforced}$$

$$K_{z,lim} = 0.814 \ldots\ldots from\ page\ 4$$

$$M_{sd,lim} = \mu_{sd,lim} f_{cd} bd^2 = 0.295 * 11.33 * 250 * 407^2 = 138.414\ KNm$$

$$Z = K_{z,lim} * d = 0.814 * 407 = 331.298\ mm$$

$$A_{s1} = \frac{M_{sd,lim}}{Zf_{yd}} + \frac{M_{sd,s} - M_{sd,lim}}{f_{yd}(d - d_{2)}} = \frac{138.414 * 10^6}{347.8 * 331.298} + \frac{(146.28 - 138.414) * 10^6}{347.8 * (407 - 43)} = 1263.378\ mm^2$$

$$use\ 5⌀20$$

$$d'_{new} = Y = \frac{\sum AiYi}{\sum Ai} = \frac{3 * 314 * 43 + 2 * 314 * 88}{5 * 314} = 61\ mm$$

$$so\ d = 450 - 61 = 389\ mm$$

$$\mu_{sd} = \frac{M_{sd}}{f_{cd}bd^2} = \frac{146.28 * 10^6}{11.33 * 250 * 389^2} = 0.34128 > \mu_{sd,lim} = 0.295 \quad Double\ reinforced$$

$$K_{z,lim} = 0.814 \ldots\ldots from\ page\ 4$$

$$M_{sd,lim} = \mu_{sd,lim} f_{cd} bd^2 = 0.295 * 11.33 * 250 * 389^2 = 126.442\ KNm$$

$$Z = K_{z,lim} * d = 0.814 * 389 = 316.646\ mm$$

$$A_{s1} = \frac{M_{sd,lim}}{Zf_{yd}} + \frac{M_{sd,s} - M_{sd,lim}}{f_{yd}(d - d_{2)}} = \frac{126.442 * 10^6}{347.8 * 316.646} + \frac{(146.28 - 126.442) * 10^6}{347.8 * (389 - 43)} = 1312.972mm^2$$

$$use\ 5⌀20$$

- *Compression reinforcement design*
  - Check if the reinforcement has yielded

$$\frac{d_2}{d} = \frac{43}{389} = 0.1 \qquad \varepsilon_{s2} = 2.6‰\ \ (read\ from\ chart)$$

$$\varepsilon_{s2} = 2.6‰ > \varepsilon_{yd}\ use\ f_{yd} = 347.826$$

- Calculate the stress in the concrete at the level of compression reinforcement to avoid double counting of area.

$\varepsilon_{cs2} = 2.6‰ \geq 2‰$, Therefore, we take

$\varepsilon_c = 3.5‰ \text{ and } \sigma_{cd,s2} = 11.33 \, mpa$

$$A_{s2}=\frac{1}{(\sigma_{s2}-\sigma_{cd,s2})}\left(\frac{M_{sds}-M_{sd,lim}}{d-d_2}\right) = \frac{1}{(347.826-11.33)}\left(\frac{(146.28-138.44)*10^6}{(389-43)}\right) = 170.07 \, mm^2$$

$$\boldsymbol{use\ 2\varnothing 20}$$

c) *Span BC (+ ve moment)*

M_{sd}=91.66 KN m

$$\mu sd,s = \frac{Msd,s}{fcd*b*d^2} = \frac{91.66*10^6}{11.33*250*407^2} = 0.195 < \mu sd,lim = 0.295$$

- Singly reinforced section

Kz = 0.89 (read from chart)

$$As1 = \frac{1}{fyd} * \frac{Msd,s}{z}$$

$$As1 = \frac{1}{347.8} * \frac{91.66*10^6}{0.89*407}$$

$$As1 = 727.5 \, mm^2$$

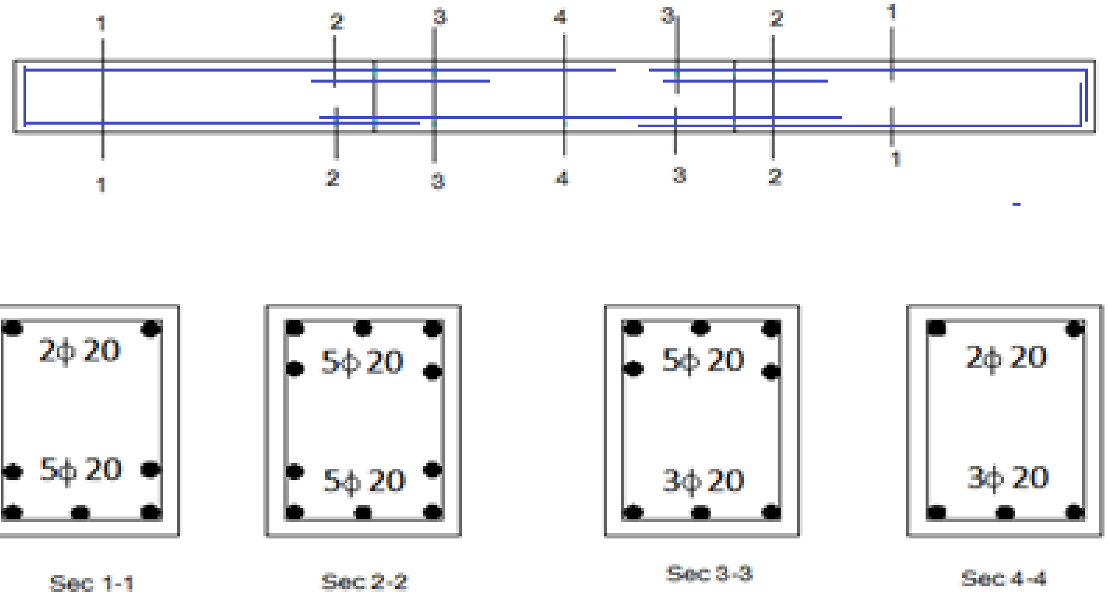
Use 3ϕ20

d) *Span BC (- ve moment)*

M_{sd}=38.84 KN m

Use 2 ϕ 20, *minimum reinforcements since the moment is small.*

Step 5: Detailing



SOLUTION B.. WITH 20% MOMENT REDISTRIBUTION

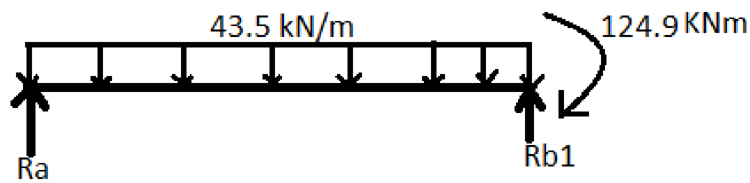
Step 6: Analyze the beam with 20% redistribution

a. Span moment (span AB and CD)

Increase support moment at B and C by 20%

$$m_b = m_c = 104.9 + 0.2 * 104.9 = 124.91\ KNm < 172.99\ KNm\ \ OK!$$

-To find the span moment in AB



From equilibrium

$$\sum M_a = 0$$

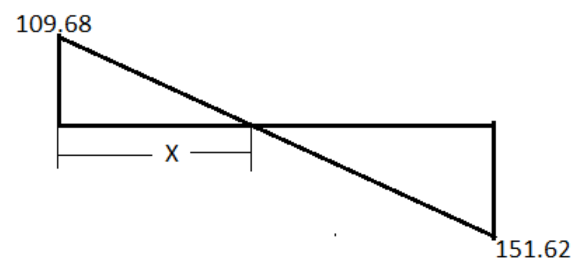
$$= \frac{-43.5 * 6^2}{2} - 124.91 + R_{b1} * 6 = 0$$

$$R_{b1} = 151.32KN$$

$$\sum F_y = 0$$

$$R_a + R_{b1} = 43.5 * 6$$

$$R_A = 109.68KN$$



From similarity of triangle

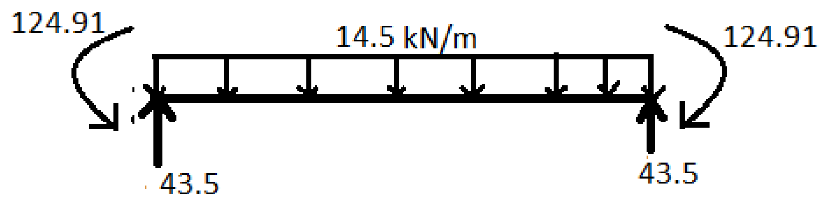
$$\frac{109.68}{x} = \frac{151.32}{6-x}$$

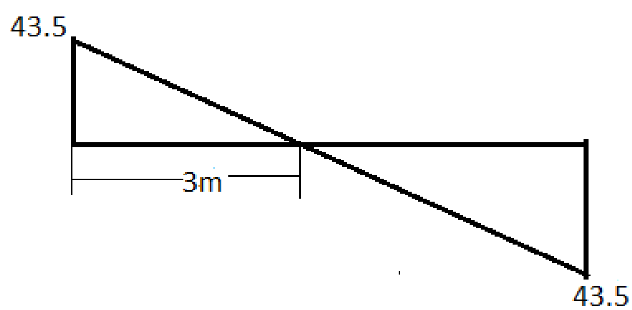
x =2.52m

$M_{max,span}$ = 0.5 x 109.68 x 2.52=**138.13 KNM < 146.28KNm**

N.B If we reduce the span moment by more than 6% the support moment will increase by more than 20%.

-Find the redistributed span moment in span BC



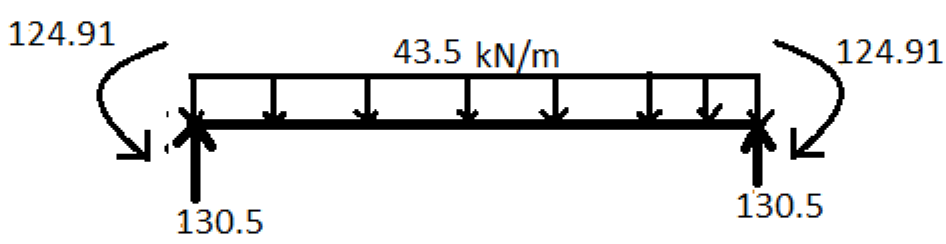


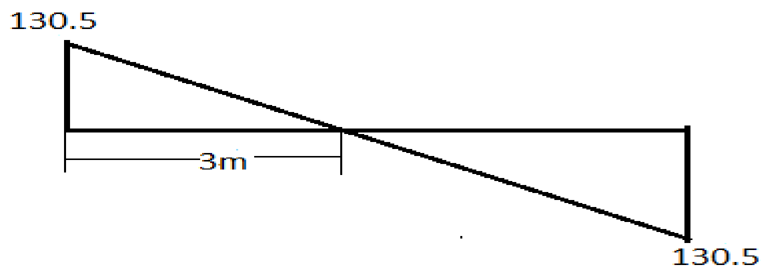
$$Mmax, span = (0.5 * 43.5 * 3) - 124.91 = -59.66\ KNM$$

b. <u>Span Moment In BC</u>

-increase support moment at B and C by 20%

M_b=M_c=124.91 KN.m < 172.99 KNM………OK!



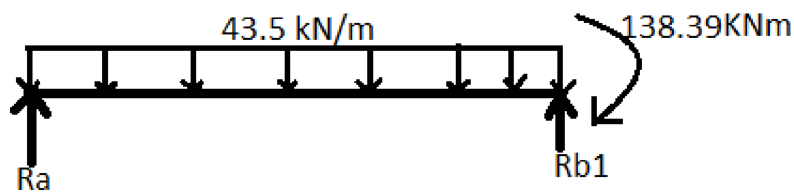


$$Mmax, span = (0.5 * 3 * 130.5) - 124.91 = \mathbf{70.84\ KNm} < 91.66\ KNm$$

B) <u>Support moment at B and C</u>

Reduce the support moment by 20% 0.8*172.99=138.39 KNm

To find the redistributed moment in AB



From equilibrium

$$\sum M_a = 0$$

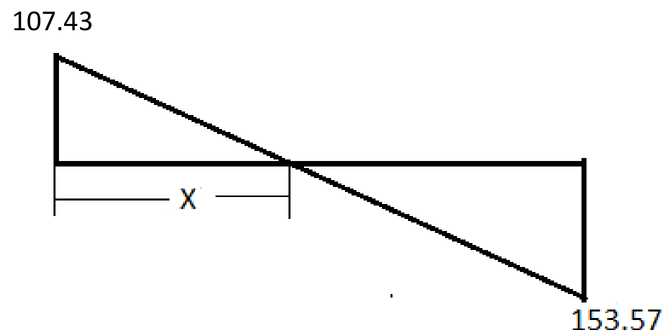
$$= \frac{-43.5 * 6^2}{2} - 136.39 + R_{b1} * 6 = 0$$

$$R_{b1} = 153.57KN$$

$$\sum F_y = 0$$

$$R_a + R_{b1} = 43.5 * 6$$

$$R_A = 107.43KN$$



From similarity of triangle

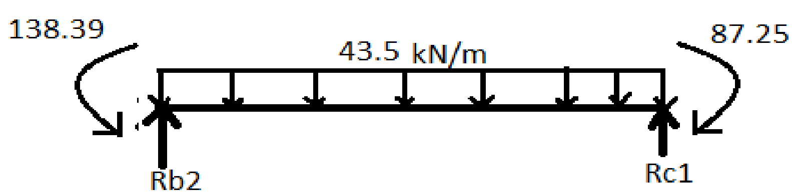
$$\frac{107.43}{x} = \frac{153.57}{6-x}$$

X=2.47m

$M_{max,span}$ = 0.5 x 107.43 x 2.47=**132.66 KN m** < 146.28KNm (maximum elastic moment)

<138.13 (maximum redistributed span moment)

- Find the redistributed moment in span BC



From equilibrium

$$\sum M_b = 0$$

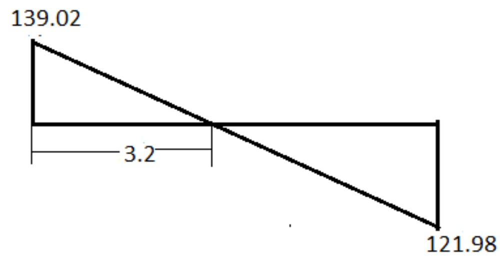
$$= \frac{-43.5*6^2}{2} + 138.39 - 87.25 + R_{c1}*6 = 0$$

$$R_{c1} = 121.98KN$$

$$\sum F_y = 0$$

$$R_a + R_{b1} = 43.5 * 6$$

$$R_A = 139.02KN$$

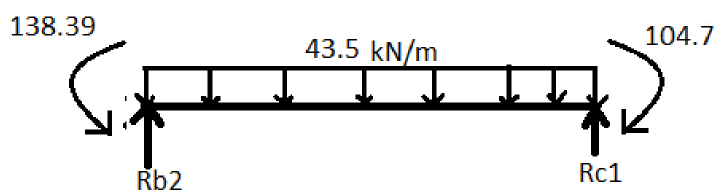


$$Mmax, span = 0.5 * 3.2 * 139.02 - 138.39 = 84.4\ kNm$$

$$Mmax, span = 84.4 > 70.84\ \ldots\ldots NOT\ OK!$$

- Let us increase the support moment at c by 20%

M_C =1.2*87.25=104.7 KNm < 138.39 KNm…….OK!



From equilibrium

$$\sum M_b = 0$$

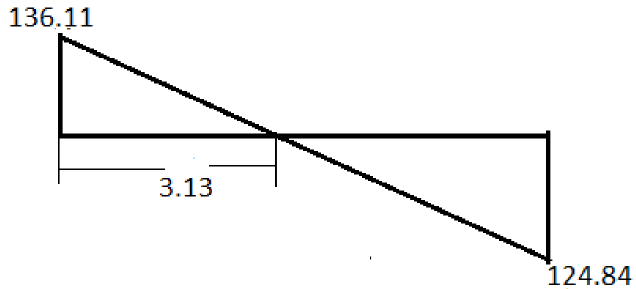
$$= \frac{-43.5 * 6^2}{2} + 138.39 - 104.7 + R_{c1} * 6 = 0$$

$$R_{c1} = 124.89KN$$

$$\sum F_y = 0$$

$$R_a + R_{b1} = 43.5 * 6$$

$$R_A = 136.11KN$$

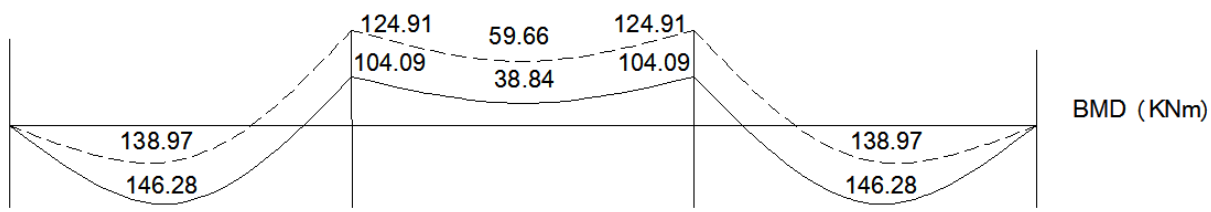


$$Mmax, span = 0.5 * 3.13 * 136.11 - 138.39 = 74.62\ kNm$$

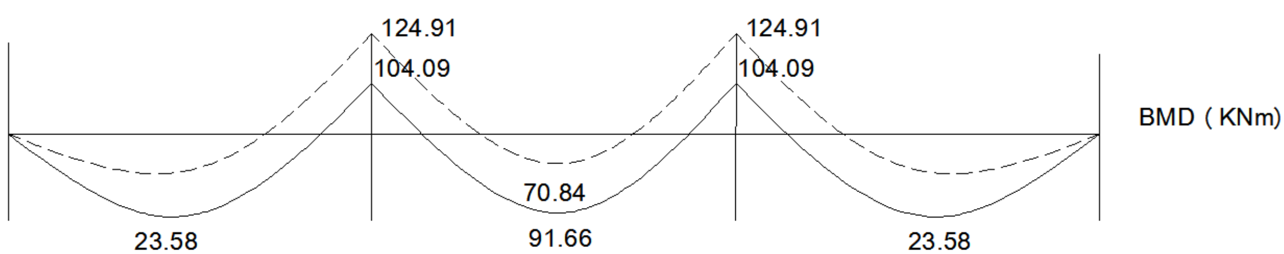
$$Mmax, span = 74.64\ is\ fairly\ close\ to\ 70.84 \ldots\ldots..OK!$$

Step 7: Bending Moment diagram after redistribution

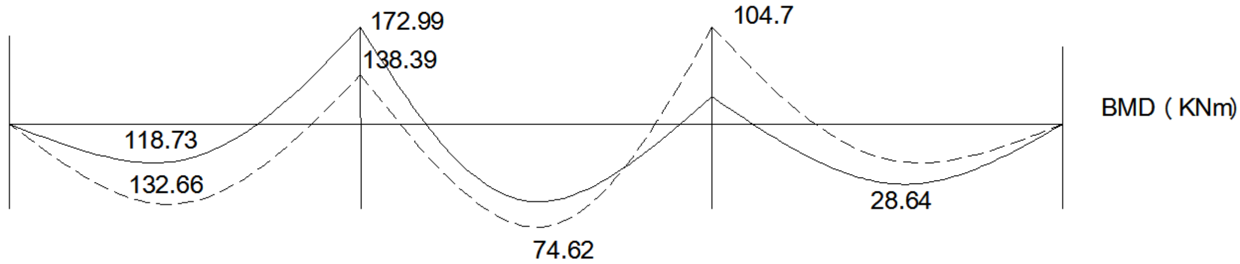
- *Max moment of span AB and CD*



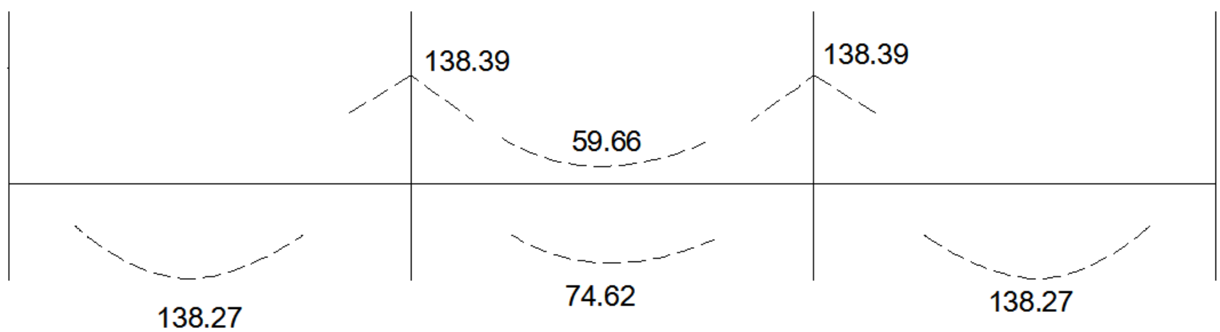
- *Bending moment of span BC*



- *Max Moment at support B*



- *Moment envelope*



Step 8 : Design (for 20% moment redistribution)

a. Support (B and C)and span (AB and CD)

$$M_{sds} = 138.39\ KNm$$

$$b = 250 \quad d = 450 - 25 - 8 - \frac{20}{2} = 407mm$$

$$\mu_{sd} = \frac{M_{sd}}{f_{cd}bd^2} = \frac{138.44 * 10^6}{11.33 * 250 * 407^2} = 0.295 > \ \mu_{sd,lim} = \mathbf{0.205} \quad Double\ reinforced$$

$$K_{z,lim} = 0.88$$

$$M_{sd,lim} = \mu_{sd,lim} f_{cd} bd^2 = 0.205 * 11.33 * 250 * 407^2 = 96.21\ KNm$$

$$Z = K_{z,lim} * d = 0.88 * 407 = 358.16\ mm$$

$$A_{s1} = \frac{M_{sd,lim}}{Zf_{yd}} + \frac{M_{sd,s} - M_{sd,lim}}{f_{yd}(d - d_{2)}} = \frac{96.21 * 10^6}{347.8 * 358.16} + \frac{(138.39 - 96.21) * 10^6}{347.8 * (407 - 43)} = 1105.8\ mm^2$$

$$\boldsymbol{use\ 4\varnothing 20}$$

N.B: By redistributing the moments we are able to reduce the number of bars by one. For this reason we are able to place the bars on one row, which is easier for workmanship.

- *Compression reinforcement design*
  - Check if the reinforcement has yielded

$$\frac{d_2}{d} = \frac{43}{389} = 0.1 \qquad \varepsilon_{s2} = 2.6‰\ \ (read\ from\ chart)$$

$$\varepsilon_{s2} = 2.6‰\ > \varepsilon_{yd}\ use\ f_{yd} = 347.826$$

  - Calculate the stress in the concrete at the level of compression reinforcement to avoid double counting of area.

$\varepsilon_{cs2} = 2.6‰ \geq 2‰$, Therefore, we take

$$\varepsilon_c = 3.5‰\ \ and\ \sigma_{cd,s2} = 11.33\ mpa$$

$$A_{s2=\frac{1}{(\sigma_{s2}-\sigma_{cd,s2})}(\frac{M_{sds}-M_{sd,lim}}{d-d_2})} = \frac{1}{(347.826 - 11.33)}(\frac{(138.44 - 96.21) * 10^6}{(407 - 43)} = 344.547\ mm^2$$

$$\boldsymbol{use\ 2\varnothing 20}$$

b. <u>Span moment BC (+ve moment)</u>

$$\text{Msd} = 74.62 \text{ KNm}$$

$$\mu sd,s = \frac{Msd,s}{fcd*b*d^2} = \frac{74.62*10^6}{11.33*250*407^2} = 0.159 < \mu sd,lim = 0.205$$

- Singly reinforced section

$$\text{Kz} = 0.91 \text{ (read from chart)}$$

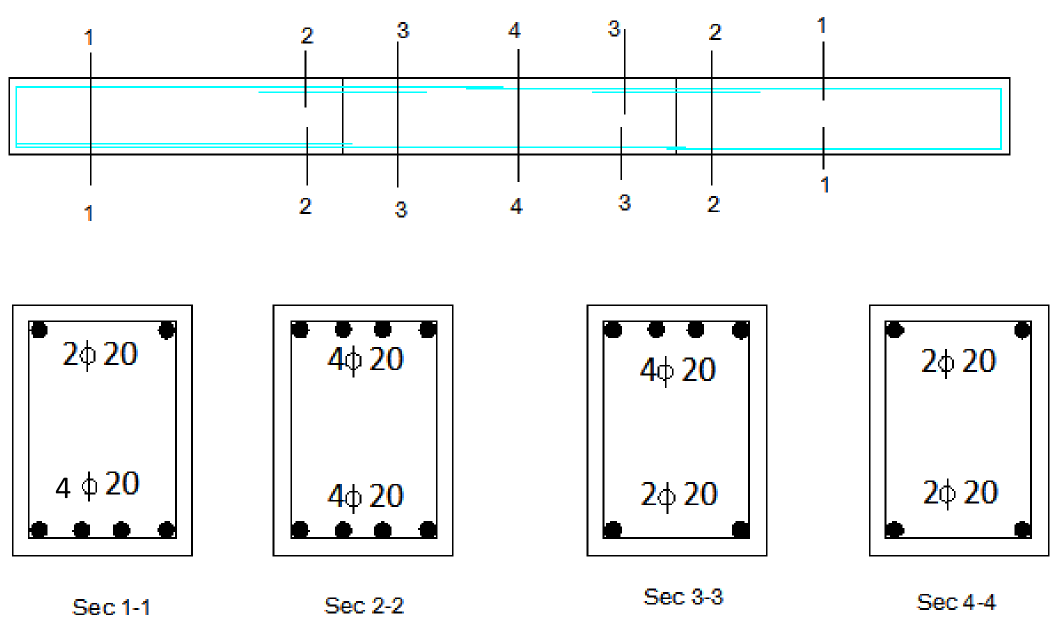
$$As1 = \frac{1}{fyd} * \frac{Msd,s}{z}$$

$$As1 = \frac{1}{347.8} * \frac{74.62 * 10^6}{0.91 * 407}$$

$$As1 = 579.23\ mm^2$$

Use 2ϕ20

Step 9: Detailing (for 20% moment redistribution)



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