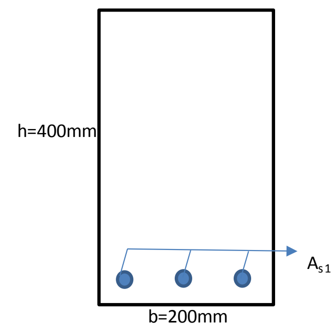
Example1.1.

For RC beam section with b/h=200/400mm, casted out of C20/25 concrete and reinforced by s-400 ϕ10c/c150. Determine the moment curvature relationship of the section?

a. 3ϕ14

b. 3ϕ24

[Use cover to longitudinal reinforcement bar 33mm]



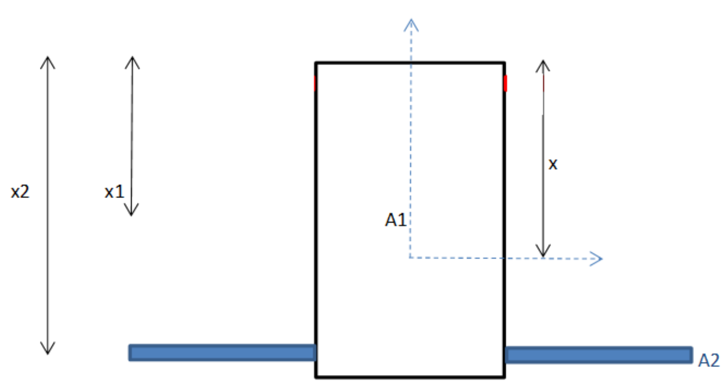
Solution **a. 3ϕ14**

Step1: Summarize the given parameters

Material	C20/25	f_{ck}=20MPa;
		f_{cd}=11.33MPa;
		f_{ctm}=2.2MPa;
		E_{cm}=30,000MPa
		ε_{c2}=2.0‰
	S-400	f_{yk}=400MPa;
		f_{yd}=347.83MPa;
		E_s=200,000MPa
		ε_y=1.74‰
	Modular ratio $n= E_s/ E_{cm}$	n= 6.67
Geometry	d=h-cover- ϕ/2=400-33-7=360mm	
	$A_{s1}=3x\pi x(7mm)^2=461.81mm^2$	

Step2: Compute the cracking moment and corresponding curvature. [M_{cr}, K_{cr}]

I. *Uncracked section properties.*



The neutral axis depth of the uncracked section

$$A_1 = b \times h = 200x400 = 80000mm^2$$

$$A_2 = (n-1) \times A_{s1} = (6.67-1) \times 461.81 = 2618.46mm^2$$

And considering the top fiber as a refrence axis

$$x_1 = \frac{h}{2} = 200mm$$

$$x_2 = d = 360mm$$

Therefore:-

$$x = \frac{\sum A_i x_i}{\sum A_i} = \frac{(A_1 \times x_1) + (A_2 \times x_2)}{(A_1 + A_2)} = 205.07mm$$

The second moment of the area of the uncracked section

$$I_1 = \left(\frac{bh^3}{12}\right) = \left(\frac{200 \times 400^3}{12}\right) = 1066666666.67mm^4$$

$$I_2 \approx 0$$

$$A_1 = b \times h = 200x400 = 80000mm^2$$

$$A_2 = (n-1) \times A_{s1} = (6.67-1) \times 461.81 = 2618.46mm^2$$

$$y_1 = x - \frac{h}{2} = 205.07 - 200 = 5.07mm$$

$$y_2 = d - x = 360 - 205.07 = 154.93mm$$

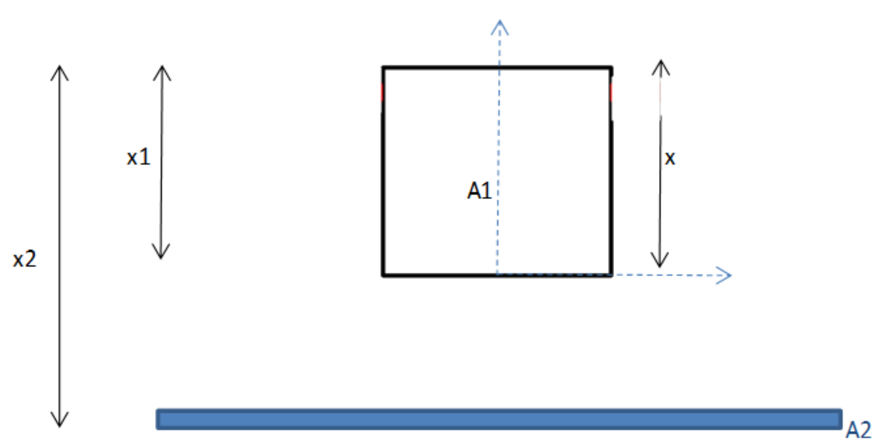
Therefore : -

$$I_I = I_1 + I_2 + (A_1 \times y_1^2) + (A_2 \times y_2^2)$$

$$I_I = 1066666666.67 + 0 + (80000 \times 5.07^2) + (2618.46 \times 154.93^2)$$

$$I_I = 1131574752.42mm^4$$

II. ***Cracked section properties.***



The neutral axis depth of the cracked section

From equilibrium of forces carried by the concrete in the compression zone and the tension force carried by the transformed concrete area in tension we have the following expression.

$$\frac{1}{2}b\left(k_x d\right)^2 = nA_{s1}(d - k_x d)$$

Dividing the above expression by bd^2 and denoting $\rho = A_{s1}/bd$ results in:

$$k_x = \frac{x}{d} = -[n\rho] + \sqrt{[n\rho]^2 + 2[n\rho]}$$

$$n = 6.67$$

$$\rho = \frac{461.81}{360 \times 200} = 0.006414$$

$$x = 0.258d = 91.023mm$$

The second moment of the area of the cracked section

$$I_1 = \left(\frac{bx^3}{12}\right) = \left(\frac{200 \times 91.023^3}{12}\right) = 12569042.224mm^4$$

$$I_2 \approx 0$$

$$A_1 = b \times x = 200x91.023 = 18204.6mm^2$$

$$A_2 = n \times A_{s1} = 6.67 \times 461.81 = 3080.27mm^2$$

$$y_1 = x - \frac{x}{2} = 45.5115mm$$

$$y_2 = d - x = 360 - 91.023 = 268.977mm$$

Therefore : -

$$I_{II} = I_1 + I_2 + (A_1 \times {y_1}^2) + (A_2 \times {y_2}^2)$$

$$I_{II} = 12569042 + 0 + (18204.6 \times 45.5115^2) + (3080.27 \times 268.977^2)$$

$$I_{II} = 273129472.51mm^4$$

III. *Compute the cracking moment.*

$$M_{cr} = \frac{f_{ctm} I_I}{y_t}$$

$$y_t = h - x = 400 - 205.07 = 194.93mm$$

Therefore

$$M_{cr} = \frac{2.2 \times 1131574752.42}{194.93} = 12.77\text{kNm}$$

IV. *Compute the curvature just before cracking.*

$$\kappa_{cr} = \frac{M_{cr}}{E_c I_I}$$

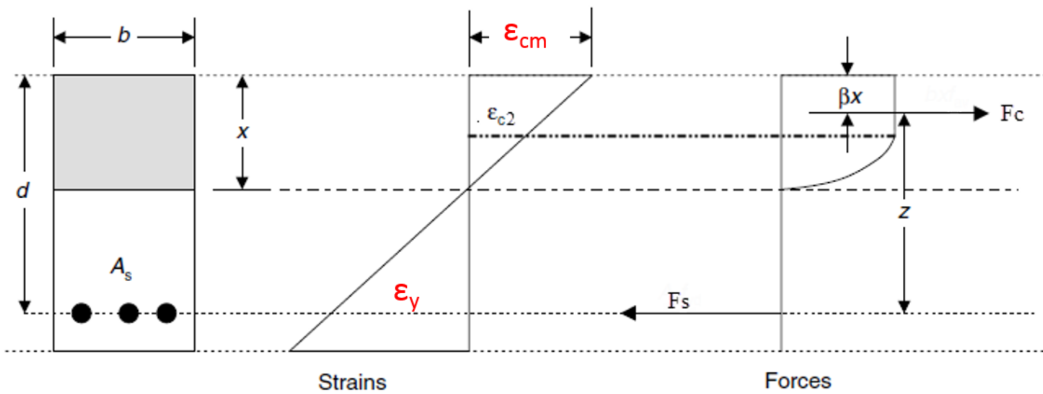
$$\kappa_{cr} = \frac{12770000Nmm}{30000\dfrac{N}{mm^2} \times 1131574752.42mm^4} = 0.3767 \times 10^{-6}\text{mm}^{-1}$$

V. <u>*Compute the curvature just after cracking.*</u>

$$\kappa_{cr} = \frac{M_{cr}}{E_c I_{II}}$$

$$\kappa_{cr} = \frac{12770000Nmm}{30000\frac{N}{mm^2} \times 273129472.51mm^4} = 1.558 \times 10^{-6} mm^{-1}$$

Step3: Compute the yielding moment and corresponding curvature. [M_y, K_y]



Assuming 0< ε_{cm} <2‰ and from force equilibrium.

$$Cc = Ts$$

$$\alpha_c f_{cd} bd = A_{s1} f_{yd}$$

$$\alpha_c = \frac{A_{s1} f_{yd}}{f_{cd} bd} = \frac{461.81 \times 347.83}{11.33 \times 200 \times 360} = 0.197$$

From the strain profile

$$k_x = \frac{\varepsilon_{cm}}{\varepsilon_{cm} + \varepsilon_y} = \frac{\varepsilon_{cm}}{\varepsilon_{cm} + 1.74}$$

From the simplified equations discussed in chapter two of RC-1

$$\alpha_c = \varepsilon_{cm} \left[\frac{6 - \varepsilon_{cm}}{12}\right] k_x = 0.197$$

From the two equations above we can solve for ε_{cm} to be 1.208 ... Assumption correct!

$$k_x = \frac{1.208}{1.208 + 1.74} = 0.410$$

$$x = d \times k_x = 360 \times 0.410 = 147.6mm$$

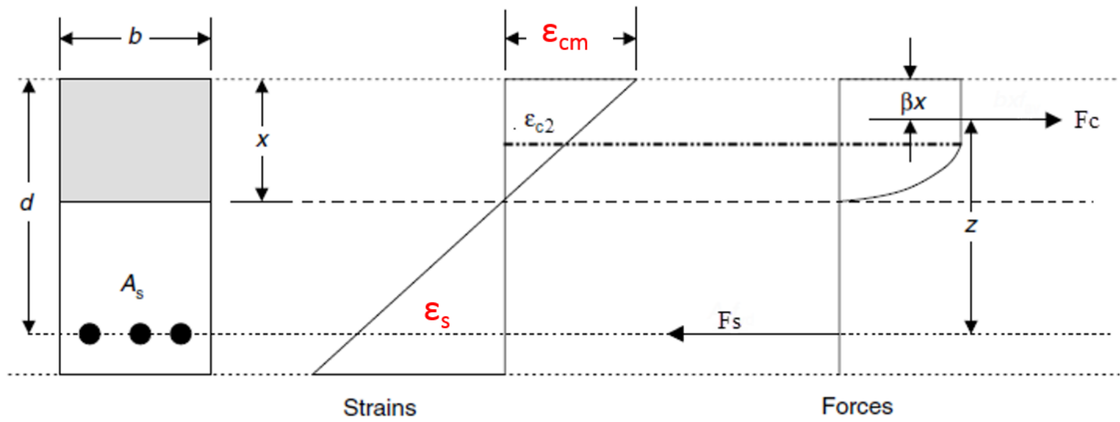
$$\beta_c = k_x \left[\frac{8 - \varepsilon_{cm}}{4(6 - \varepsilon_{cm})}\right] = 0.145$$

$$z = d(-\beta_c) = 360(1 - 0.145) = 307.8mm$$

$$M_y = A_{s1} f_{yd} z = 49.442kNm$$

$$\kappa_y = \frac{\varepsilon_{cm}}{x} = \frac{1.178 \times 10^{-3}}{145.44mm} = 8.10 \times 10^{-6} mm^{-1}$$

Step4: Compute the ultimate moment and corresponding curvature. [M_u, K_u]



Assuming a compression failure ε_{cm} =3.5‰, $\varepsilon_y < \varepsilon_s$ <25‰ and from force equilibrium.

$$Cc = Ts$$

$$\alpha_c f_{cd} bd = A_{s1} f_{yd}$$

$$\alpha_c = \frac{A_{s1} f_{yd}}{f_{cd} bd} = \frac{461.81 \times 347.83}{11.33 \times 200 \times 360} = 0.197$$

From the strain profile

$$k_x = \frac{3.5}{3.5 + \varepsilon_s}$$

From the simplified equations discussed in chapter two of RC-1

$$\alpha_c = k_x \left[\frac{3\varepsilon_{cm} - 2}{3\varepsilon_{cm}} \right] = 0.197$$

From the two equations above we can solve for ε_s to be 10.88 ... Assumption correct!

$$k_x = \frac{3.5}{3.5 + 10.88} = 0.243$$

$$x = d \times k_x = 360 \times 0.243 = 87.48mm$$

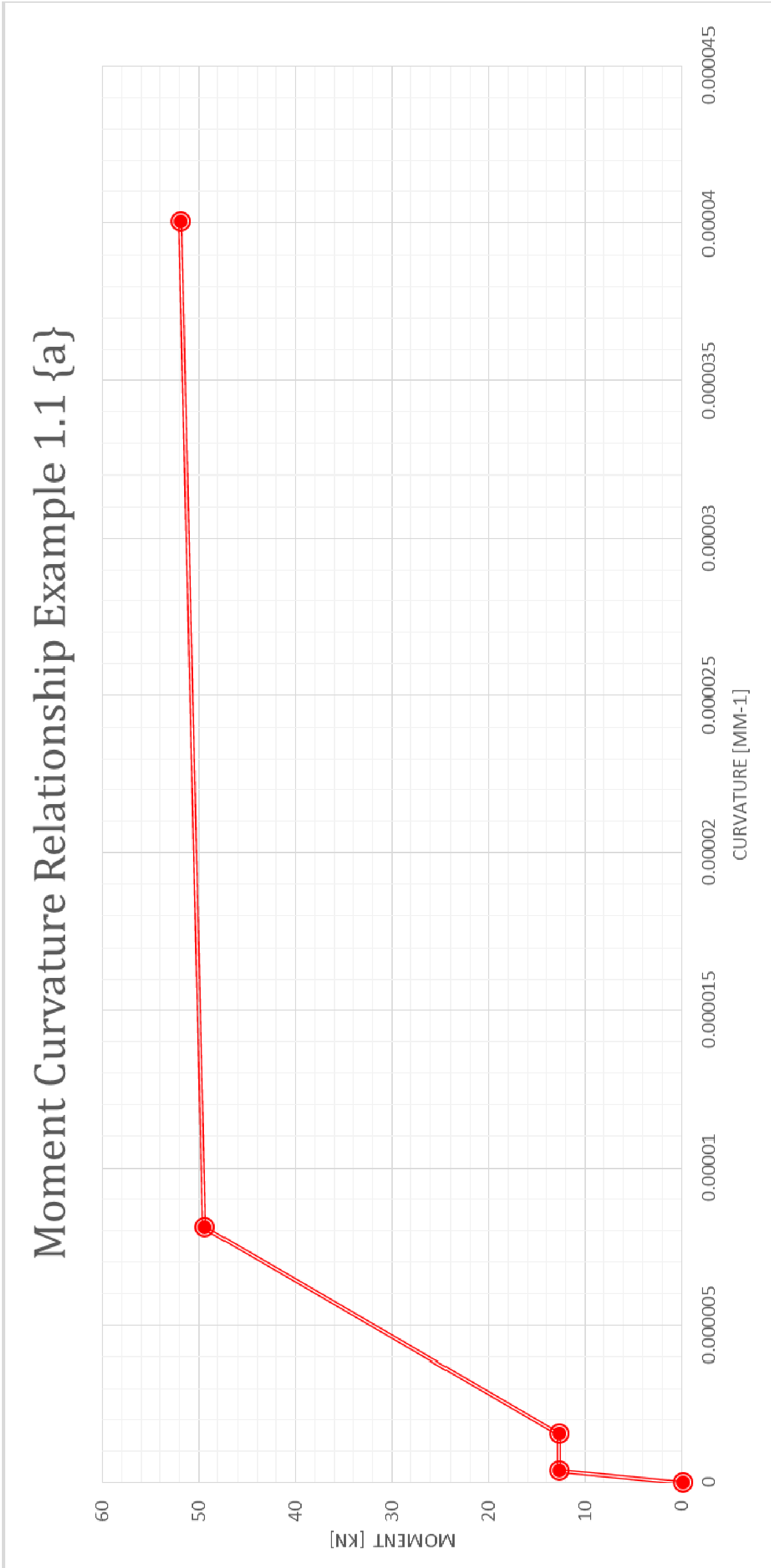
$$\beta_c = k_x \left[\frac{\varepsilon_{cm}(3\varepsilon_{cm} - 4) + 2}{2\varepsilon_{cm}(3\varepsilon_{cm} - 2)} \right] = 0.1011$$

$$z = d(-\beta_c) = 360(1 - 0.101) = 323.64mm$$

$$M_u = A_{s1} f_{yd} z = 51.99kNm$$

$$\kappa_u = \frac{\varepsilon_{cm}}{x} = \frac{3.5 \times 10^{-3}}{87.48mm} = 40.01 \times 10^{-6} mm^{-1}$$

Step5: Plot the moment vs curvature diagram.



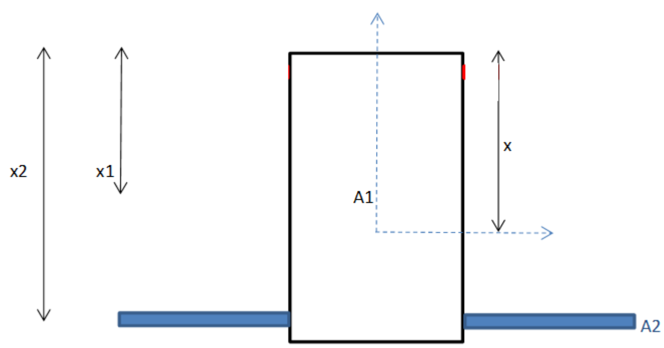
Solution b. 3ϕ24

Step1: Summarize the given parameters

Material	C20/25	f_{ck}=20MPa;
		f_{cd}=11.33MPa;
		f_{ctm}=2.2MPa;
		E_{cm}=30,000MPa
		ε_{c2}=2.0‰
	S-400	f_{yk}=400MPa;
		f_{yd}=347.83MPa;
		E_s=200,000MPa
		ε_y=1.74‰
	Modular ratio n= E_s/ E_{cm}	n= 6.67
Geometry	d=h-cover- ϕ/2=400-33-12=355mm	
	A_{s1}=3xπx(12mm)^2=1356.48mm^2	

Step2: Compute the cracking moment and corresponding curvature. [M_{cr}, K_{cr}]

I. *Uncracked section properties.*



The neutral axis depth of the uncracked section

$$A_1 = b \times h = 200x400 = 80000mm^2$$

$$A_2 = (n-1) \times A_{s1} = (6.67-1) \times 1356.48 = 7691.24mm^2$$

And considering the top fiber as a refrence axis

$$x_1 = \frac{h}{2} = 200mm$$

$$x_2 = d = 355mm$$

Therefore:-

$$x = \frac{\sum A_i x_i}{\sum A_i} = \frac{(A_1 \times x_1) + (A_2 \times x_2)}{(A_1 + A_2)} = 213.6mm$$

The second moment of the area of the uncracked section

$$I_1 = \left(\frac{bh^3}{12}\right) = \left(\frac{200\times 400^3}{12}\right) = 1066666666.67mm^4$$

$$I_2 \approx 0$$

$$A_1 = b\times h = 200x400 = 80000mm^2$$

$$A_2 = (n-1)\times A_{s1} = (6.67-1)\times 1356.48 = 7691.24mm^2$$

$$y_1 = x - \frac{h}{2} = 213.6 - 200 = 13.6mm$$

$$y_2 = d - x = 355 - 213.67 = 141.33mm$$

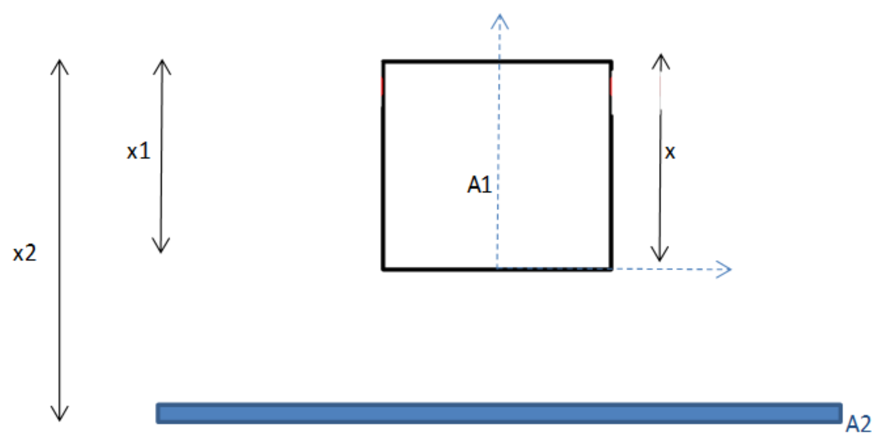
Therefore : -

$$I_I = I_1 + I_2 + (A_1\times {y_1}^2) + (A_2\times {y_2}^2)$$

$$I_I = 1066666666.67 + 0 + (80000\times 13.6^2) + (7691.24\times 141.33^2)$$

$$I_I = 1235089593.48mm^4$$

II. <u>Cracked section properties.</u>



The neutral axis depth of the cracked section

From equilibrium of forces carried by the concrete in the compression zone and the tension force carried by the transformed concrete area in tension we have the following expression.

$$\frac{1}{2}b\left(k_x d\right)^2 = nA_{s1}(d - k_x d)$$

Dividing the above expression by bd^2 and denoting $\rho = A_{s1}/bd$ results in:

$$k_x = \frac{x}{d} = -[n\rho] + \sqrt{[n\rho]^2 + 2[n\rho]}$$

$$n = 6.67$$

$$\rho = \frac{1356.48}{355\times 200} = 0.0191$$

$$x = 0.393d = 139.60mm$$

The second moment of the area of the cracked section

$$I_1 = \left(\frac{bx^3}{12}\right) = \left(\frac{200 \times 139.60^3}{12}\right) = 45342452.27mm^4$$

$$I_2 \approx 0$$

$$A_1 = b \times x = 200x139.60 = 27920mm^2$$

$$A_2 = n \times A_{s1} = 6.67 \times 1356.48 = 9047.72mm^2$$

$$y_1 = x - \frac{x}{2} = 69.8mm$$

$$y_2 = d - x = 355 - 139.60 = 215.4mm$$

Therefore : -

$$I_{II} = I_1 + I_2 + (A_1 \times y_1^{\;2}) + (A_2 \times y_2^{\;2})$$

$$I_{II} = 45342452.27 + 0 + (27920 \times 69.8^2) + (2618.46 \times 215.4^2)$$

$$I_{II} = 302858916.6mm^4$$

III. *Compute the cracking moment.*

$$M_{cr} = \frac{f_{ctm} I_I}{y_t}$$

$$y_t = h - x = 400 - 213.6 = 186.4mm$$

Therefore

$$M_{cr} = \frac{2.2 \times 1235089593.48}{186.4} = 14.58\text{kNm}$$

IV. *Compute the curvature just before cracking.*

$$\kappa_{cr} = \frac{M_{cr}}{E_c I_I}$$

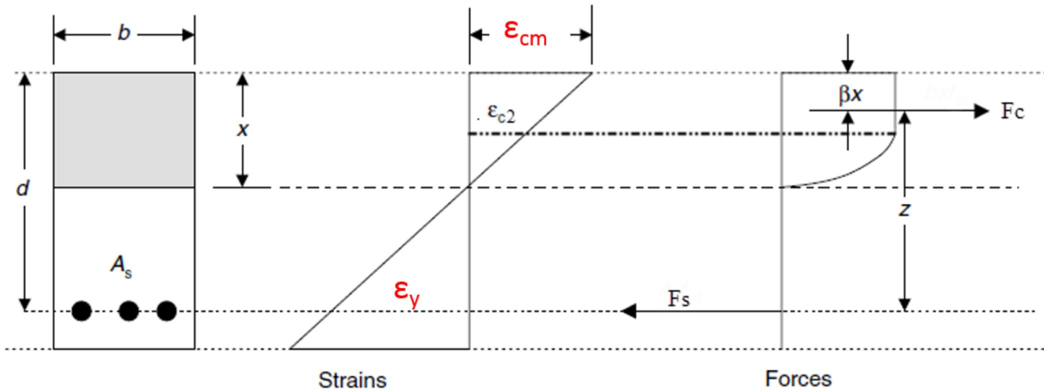
$$\kappa_{cr} = \frac{12770000Nmm}{30000\frac{N}{mm^2} \times 1235089593.48mm^4} = 0.34464 \times 10^{-6}\text{mm}^{-1}$$

V. *Compute the curvature just after cracking.*

$$\kappa_{cr} = \frac{M_{cr}}{E_c I_{II}}$$

$$\kappa_{cr} = \frac{14580000Nmm}{30000\frac{N}{mm^2} \times 302858916.6mm^4} = 1.605 \times 10^{-6}\text{mm}^{-1}$$

Step3: Compute the yielding moment and corresponding curvature. [M_y, K_y]



Assuming 2‰< ε_{cm} <3.5‰ and from force equilibrium.

$$Cc = Ts$$

$$\alpha_c f_{cd} bd = A_{s1} f_{yd}$$

$$\alpha_c = \frac{A_{s1} f_{yd}}{f_{cd} bd} = \frac{1356.48 \times 347.83}{11.33 \times 200 \times 355} = 0.587$$

From the strain profile

$$k_x = \frac{\varepsilon_{cm}}{\varepsilon_{cm} + \varepsilon_y} = \frac{\varepsilon_{cm}}{\varepsilon_{cm} + 1.74}$$

From the simplified equations discussed in chapter two of RC-1

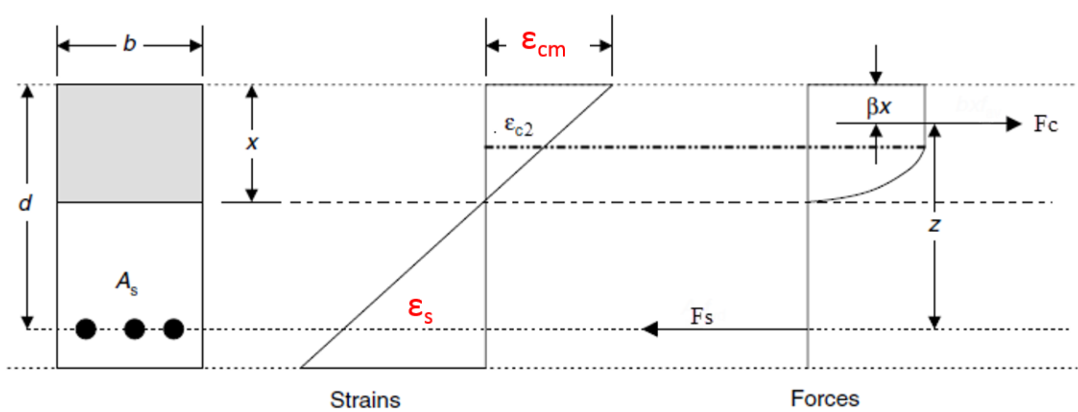
$$\alpha_c = k_x \left[\frac{3\varepsilon_{cm} - 2}{3\varepsilon_{cm}} \right] = 0.587$$

From the two equations above we can solve for ε_{cm} to be 4.08

4.08‰>3.5‰, implies that the concrete in the compression zone has crushed even before the reinforcement in the tension zone has yielded.

Hence the section has reached its ultimate moment capacity, along with the corresponding curvature, before the yielding of the reinforcement.

Step4: Compute the ultimate moment and corresponding curvature. [M_u, K_u]



Assuming a compression failure ε_{cm} =3.5‰, $\varepsilon_s < \varepsilon_y$ and from force equilibrium.

$$Cc = Ts$$

$$\alpha_c f_{cd} bd = A_{s1}\sigma_s = A_{s1}(E_s \times \varepsilon_s)$$

$$\alpha_c = \frac{A_{s1}\sigma_s}{f_{cd}bd} = \frac{1356.48 \times 200000 \times \varepsilon_s}{11.33 \times 200 \times 355} = 0.33725\varepsilon_s$$

Where ε_s is in ‰.

From the strain profile

$$k_x = \frac{3.5}{3.5 + \varepsilon_s}$$

From the simplified equations discussed in chapter two of RC-1

$$\alpha_c = k_x \left[\frac{3\varepsilon_{cm} - 2}{3\varepsilon_{cm}}\right] = 0.33725\varepsilon_s$$

From the two equations above we can solve for ε_s to be 1.636 ... Assumption correct!

$$k_x = \frac{3.5}{3.5 + 1.636} = 0.681 \text{ ... Indicates a brittle failure!}$$

$$x = d \times k_x = 355 \times 0.681 = 241.755mm$$

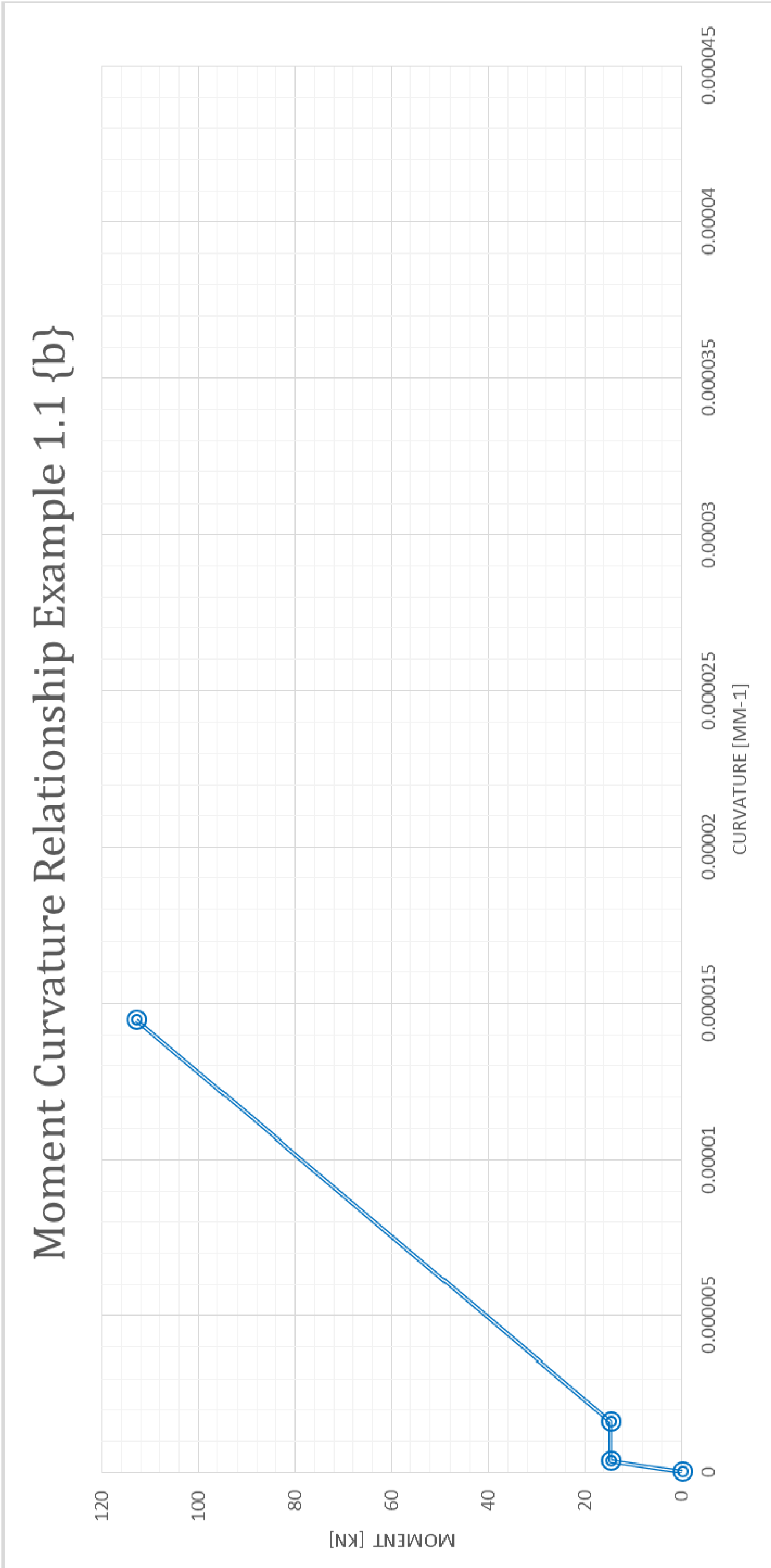
$$\beta_c = k_x \left[\frac{\varepsilon_{cm}(3\varepsilon_{cm} - 4) + 2}{2\varepsilon_{cm}(3\varepsilon_{cm} - 2)}\right] = 0.283$$

$$z = d(-\beta_c) = 355(1 - 0.101) = 254.43mm$$

$$M_u = A_{s1}(E_s \times \varepsilon_s)z = 112.93kNm$$

$$\kappa_u = \frac{\varepsilon_{cm}}{x} = \frac{3.5 \times 10^{-3}}{241.755mm} = 14.477 \times 10^{-6} mm^{-1}$$

Step5: Plot the moment vs curvature diagram.



Step6: Comparing the two diagrams.