CHAPTER 4

4. Kani Method of Analysis

4.1 Introduction

Kani method may be considered as a further simplification of moment distribution method. In analyzing using Kani method;

i. Frame analysis is carried out by solving the slope - deflection equations by successive approximations. Useful in case of side sway as well.
ii. Operation is simple, as it is carried out in a specific direction. If some error is committed, it will be eliminated in subsequent cycles if the restraining moments and distribution factors have been determined correctly.

Consider a typical member AB loaded as shown below:

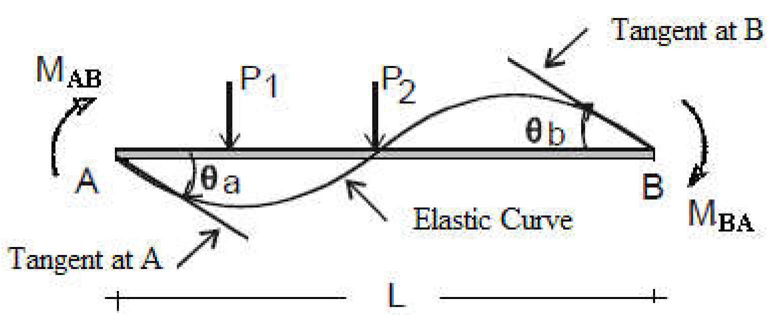

Fig 4.1 General beam element under end moments and loads

As discussed in the slope deflection method, the slope deflection equation can be written as:

$$M_{AB} = M^F_{AB} + 2M'_{AB} + M'_{BA} \tag{4.1}$$

Where:

M^F_{AB} : Fixed end moments at A due to applied loads

M'_{AB} : Rotation contribution of end A of member AB $= \frac{EI}{L}(2\theta_A)$

M'_{BA} : Rotation contribution of end B of member AB $= \frac{EI}{L}(\theta_B)$

The near and far end rotation contributions of end A of member AB are given in Eq. (4.2) below.

$$M'_{AB} = \frac{EI}{L}(2\theta_A) = 2Ek_{AB}\theta_A \quad \text{and} \quad M'_{BA} = \frac{EI}{L}(\theta_B) = Ek_{AB}\theta_B \tag{4.2}$$

Where: $k_{AB} = \frac{I_{AB}}{L_{AB}}$

Now consider a generalized joint *E* in a frame shown in the following figure where members *EB*, *EC*, *ED* and *EA* meet. It carries a moment *M*.

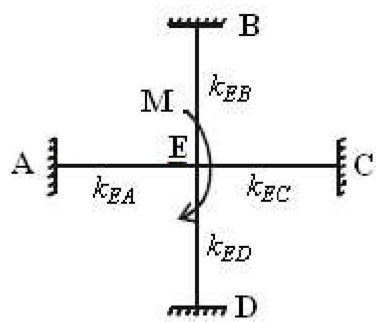

Fig 4.2

where: k_{EA}, k_{EC}, k_{EB}, k_{ED} are stiffness factors of members *EA*, *EC*, *EB* and *ED* respectively.

From equilibrium of joint *E*, $\sum M_E = 0$

$$\sum \mathrm{M}_\mathrm{E} = M_{EB} + M_{EC} + M_{ED} + M_{EA} = 0 \tag{4.3}$$

Write these moments in the form of Eq. (4.1)

$$\sum M_E = \sum M^F_{(EB+EC+ED+EA)} + 2\sum M'_{(EB+EC+ED+EA)} + \sum M'_{(BE+CE+DE+AE)} = 0 \tag{4.4}$$

Let the net FEM at joint *E* = $M^\mathrm{F}_\mathrm{E} = M^F_{EB} + M^F_{EC} + M^F_{ED} + M^F_{EA}$. This moment is also called restrained moment at joint *E*. The total near end rotation contribution from Eq. (4.4) becomes:

$$\sum M'_{(EB+EC+ED+EA)} = -\frac{1}{2}[M^\mathrm{F}_\mathrm{E} + \sum M'_{(BE+CE+DE+AE)}] = 0 \tag{4.5}$$

Upon substitution and simplification the following equation is obtained.

$$\begin{aligned}\sum M'_{(EB+EC+ED+EA)} &= -2[Ek_{EB}\theta_E + Ek_{EC}\theta_E + Ek_{ED}\theta_E + Ek_{EA}\theta_E] \\ &= -2E\theta_E(k_{EB} + k_{EC} + k_{ED} + k_{EA}] \\ &= -2E\theta_E \sum k_E\end{aligned}$$

Thus, the rotation of joint E, θ_E, can be written as follow:

$$\theta_E = -\frac{\sum M'_{(EB+EC+ED+EA)}}{2E\sum k_E} \tag{4.6}$$

where:
$\sum k_E$ =sum of the member stiffnesses framing in at joint *E*

From Eq. (4.2), the rotation contribution of end *E* of member *EB* , M'_{EB} is:

$$M'_{EB} = 2Ek_{EB}\theta_E \tag{4.7}$$

Substituting Eq. (4.6) in to Eq. (4.7) gives the following near end moment equation for member EB.

$$M'_{EB} = 2Ek_{EB}\left(\frac{-\sum M'_{(EB+EC+ED+EA)}}{2E\sum k_E}\right) = -\frac{k_{EB}}{\sum k_E}\left(\sum M'_{(EB+EC+ED+EA)}\right) \tag{4.8}$$

Combining Eqs. (4.5) and (4.8) yields the following general equation;

$$\sum M'_{EB} = -\frac{1}{2}\frac{k_{EB}}{\sum k_E}\left(M^{\mathrm{F}}_{\mathrm{E}} + \sum M'_{(BE+CE+DE+AE)}\right) \tag{4.9}$$

Thus, the rotation contribution of near end of member EB is -1/2 of the sum of the rotations contributions of far ends of members meeting at E.

Note that the sum of rotation factors of different members meeting at a joint is equal to –1/2.

- ⇒ If net fixed end moment at any joint along with sum of the far end contribution of members meeting at that joint are known then near end moment contribution can be determined.
- ⇒ If far end contributions are approximate, near end contributions will also be approximate.
- ⇒ When far end contributions are not known (as in the first cycle), they can be assumed to be zero.

Rules for calculating rotation contributions:

4.2 Without sides way

Definition: "Restrained moment at a joint is the algebraic sum of FEM's of different members meeting at that joint."

1. Sum of the restrained moment of a joint and all rotation contributions of the far ends of members meeting at that joint is multiplied by respective rotation factors to get the required near end rotation contribution. For the first cycle when far end contributions are not known, they may be taken as zero (1st approximation).
2. By repeated application of this calculation procedure and proceeding from joint to joint in an arbitrary sequence but in a specific direction, all rotation contributions are known. The process is usually stopped when end moment values converge. This normally happens after three or four cycles. But values after 2nd cycle may also be acceptable for academic.

4.3 With side sway (joint translations)

In this case in addition to rotation contribution, linear displacement contributions(Sway contributions) of columns of a particular storey are calculated after every cycle as follows:

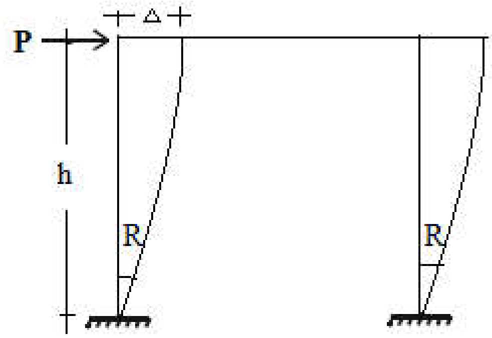

Fig 4.3 Sidesway Frame

For the first cycle

(A) → **Linear Displacement Contribution** (LDC) of a column = Linear displacement factor (LDF) of a particular column of a story multiplied by [storey moment + contributions at the ends of columns of that story]

⇒ Linear displacement factor (LDF) for columns of a story = −3/2

⇒ Linear displacement factor of a column = $\frac{-3}{2}\frac{k}{\sum k}$

Where k=stiffness of the column being considered

Σk is the sum of stiffness of all columns of that storey.

(B) → **Storey moment** = Storey shear x1/3of storey height.

(C) → **Storey shear** : It may be considered as reaction of column at horizontal beam / slab levels due to lateral loads by considering the columns of each sotrey as simply supported beams in vertical direction. "If applied load gives + R value (according to sign conversion of slope deflection method), storey shear is +ve or vice versa."

For 1st cycle with side sway

(D) Near end contribution of various members meeting at that joint = respective rotation contribution factor ×[Restrained moment + far end contributions]

Linear displacement contributions will be calculated after the end of each cycle for the columns only.

For 2nd and subsequent cycles

(E) Near end contribution of various members meeting at that joint = respective rotation contribution factor ×[Restrained moment + far end contributions+ linear displacement contribution of columns of different storeys meeting at that joint]

Rules for the calculation of final end moments (sidesway cases)

(F) → For beams, End moment = FEM + 2 near end contribution + Far end contributions.

(G) → For columns, End moment. = FEM + 2 near end contribution + Far end contribution + linear displacement contribution of that column for the latest cycle.