Chapter 4

Soil Water, Permeability & Seepage

AAiT

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General Outline

- Soil Water
- Permeability
- Seepage

1. Soil Water

- Introduction
- Hydrologic Cycle
- Subsurface Water
- Types of Soil Moisture

Soil Water

Geotechnical engineers are interested in ground water because of its effect on soil behaviour and construction opeartions.

- Ground water affects many engineering structures adversely by reducing the bearing capacity of the soil.
- Deep excavation can be difficult because of large inflow of ground water.
- The presence of water in the soil above and below the ground water table may be a controlling factor in many engineering studies and foundation design.

Soil Water cntd

Hydrologic cycle

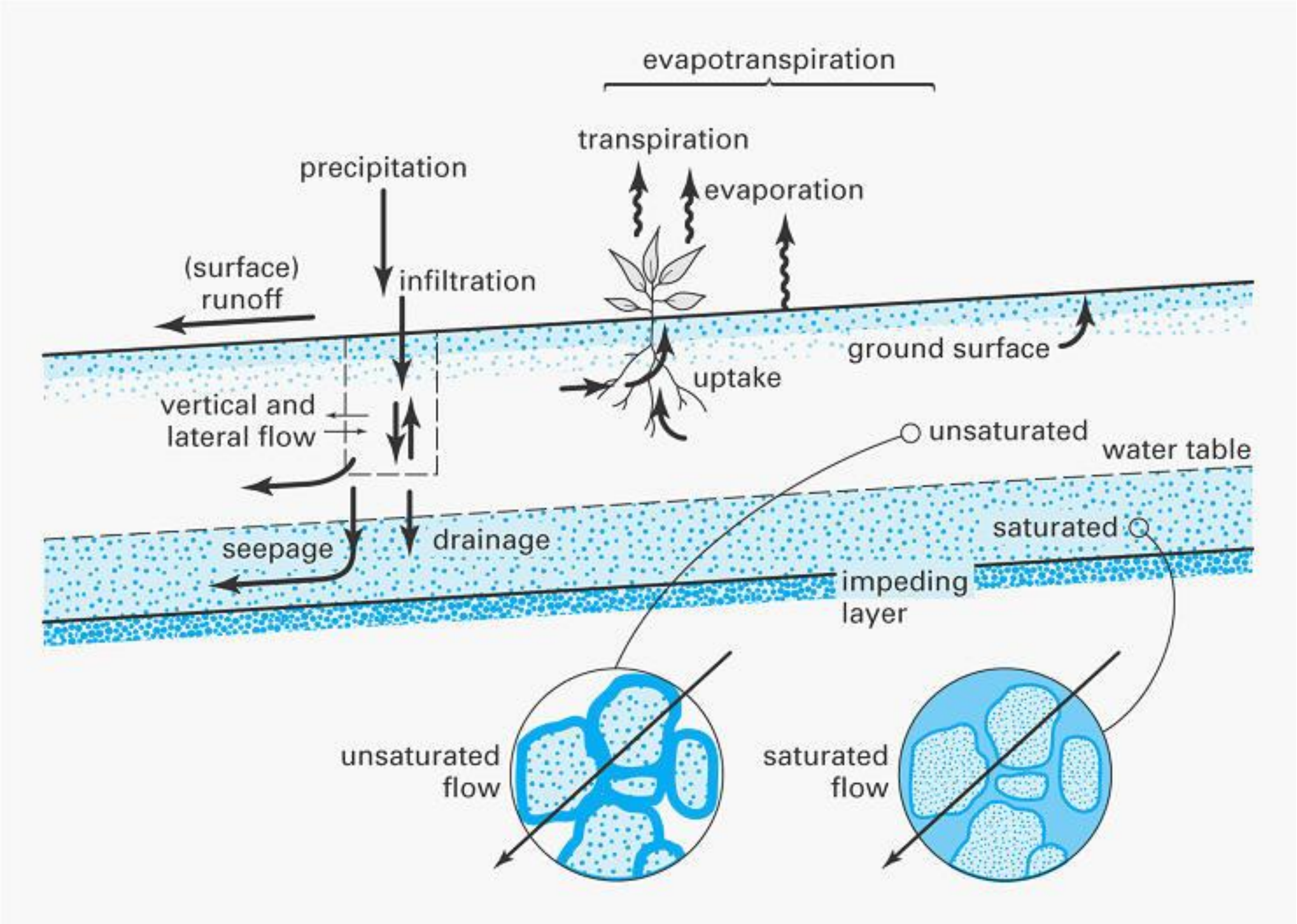
Moisture vapour in the clouds condenses under the influence of temperature changes and fall to the earth as rain, snow, hail etc.

A part of this precipitation may not reach land surface but evaporates in the air while falling or may evaporate from leaves or roofs etc.

Most of the precipitation, however, falls on the land which ends up being disposed in three ways;

- Evaporates directly from the soil
- Run off the surface (runoff)
- Soaks into the soil.

Soil Water cntd



Soil Water cntd

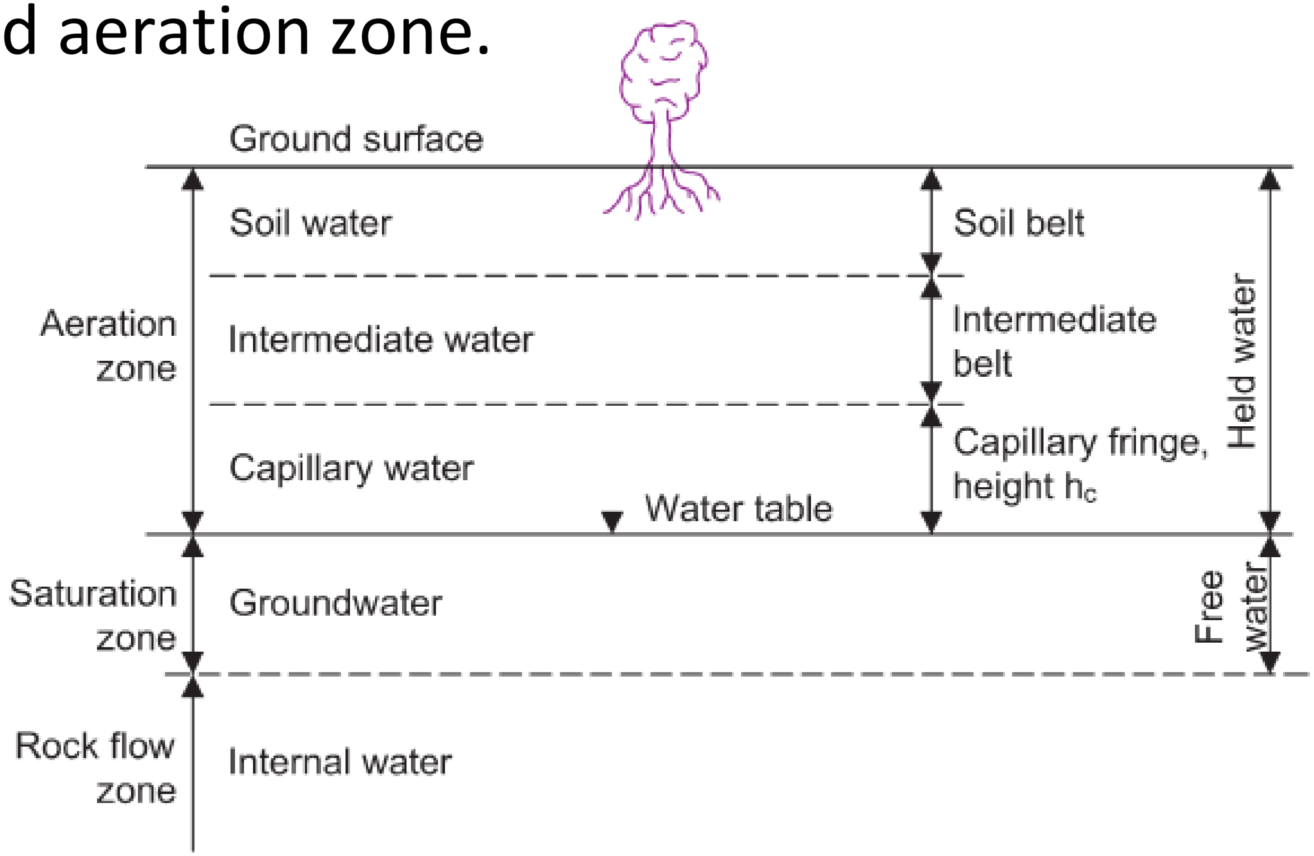
Infiltration vs Percolation vs Seepage

- When rain falls upon the ground it first of all wets the vegetation or the bare soil. When the surface cover is completely wet, subsequent rain must either penetrate the surface layers, if the surface is permeable, or run off the surface towards a stream channel if it is impermeable.
- If the surface layers are porous and have minute passages available for the passage of water droplets, the water ***infiltrates*** into the subsurface soil. Soil with vegetation growing on it is always permeable to some degree. Once infiltrating water has passed through the surface layers, it ***percolates*** downward under the influence of gravity until it reaches the zone of saturation at the phreatic surface.

Soil Water cntd

Subsurface Water

- refer to all water found beneath the Earth's surface.
- can be split into two distinct zones: saturation zone and aeration zone.

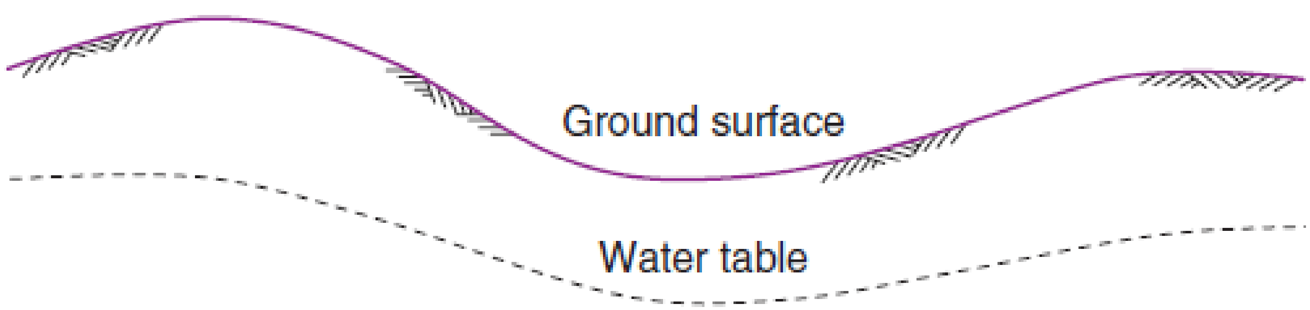


Soil Water cntd

Saturation zone: depth throughout which all the fissures & voids are filled with water under hydrostatic pressure.

The upper level of this water is known as the water table, phreatic surface or groundwater level, and water within this zone is called phreatic water or groundwater.

The water table tends to follow in a more gentle manner than the topographical features of the surface above.



Soil Water cntd

- At groundwater level, the hydrostatic pressure is zero, so another definition of water table is the level to which water will eventually rise in an unlined borehole.
- The water table is not constant but rises and falls with variations of rainfall, atmospheric pressure, temperature, etc., whilst coastal regions are affected by tides.

Ground water – the continous body of sub-surface water that fills the soil voids and fissures and is free to move under the influence of gravity.

Soil Water cntd

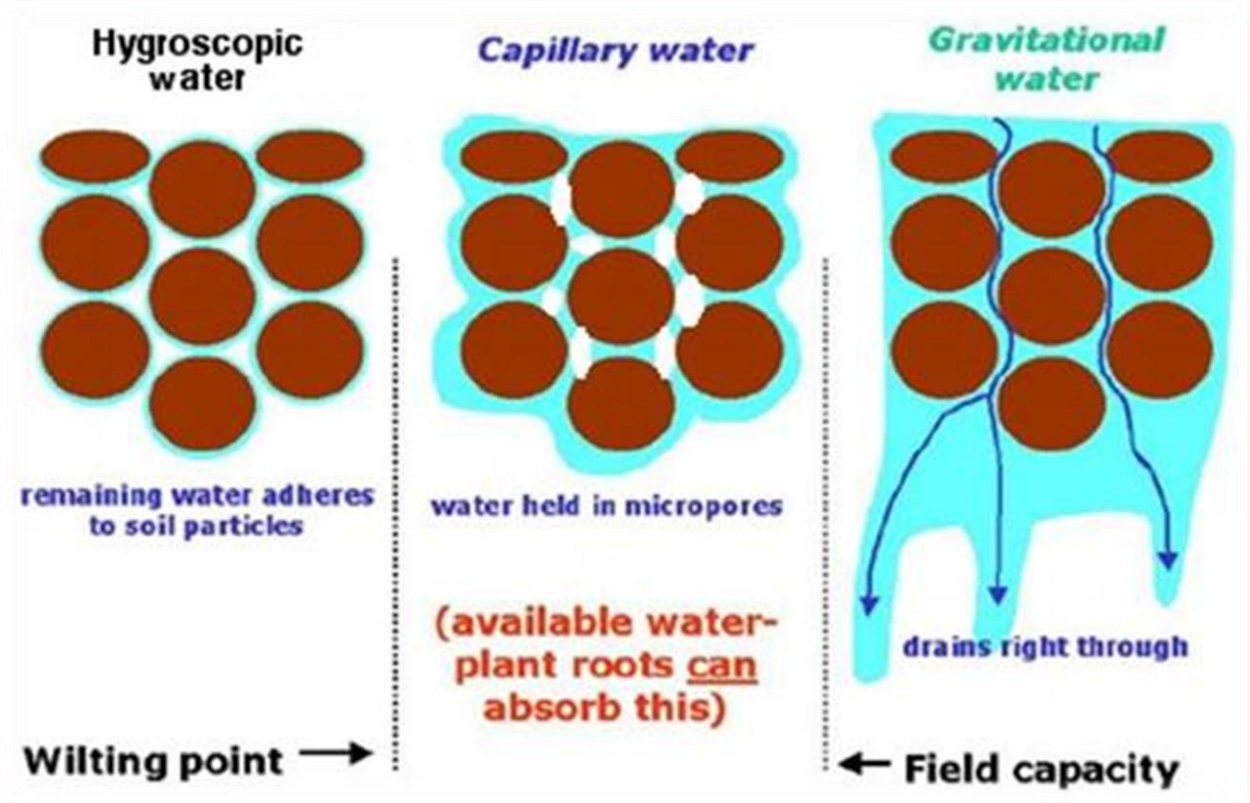
Aeration zone (Vadose zone)

- occurs between the water table and the surface
- can be split into three sections.
  - Soil belt – zone constantly affected by precipitation, evaporation and plant transpiration.
  - Intermediate belt – zone where certain amount of rainwater percolating downward to the water table is held in the soil by the action of surface tension, capillarity, abdsorption and chemical action.
  - Capillary fringe – zone where water is drawn up above the water table into the interstices of the soil or rock owing to capillarity phenomena.

Soil Water cntd

Classification of Soil Moisture

- Adsorbed
- Capillary
- Gravitational
- Perched
- Artesian



Soil Water   cntd

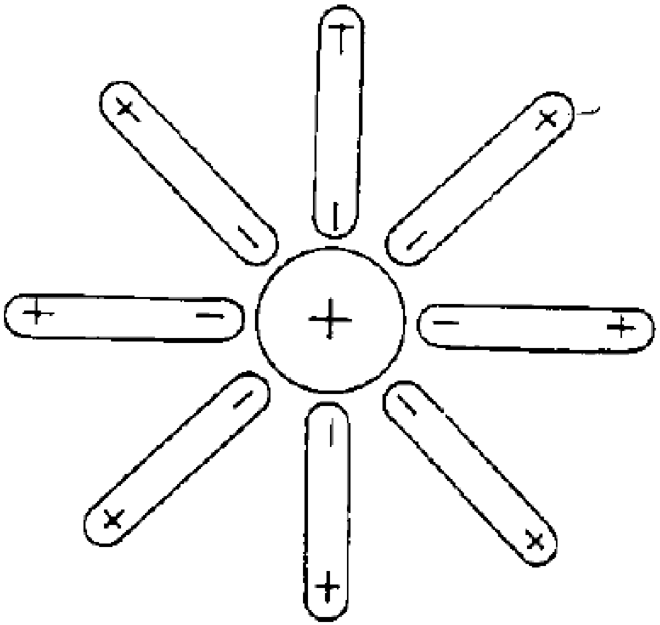
Adsorbed Water : water held on the surface of soil particles by forces of adsorption.

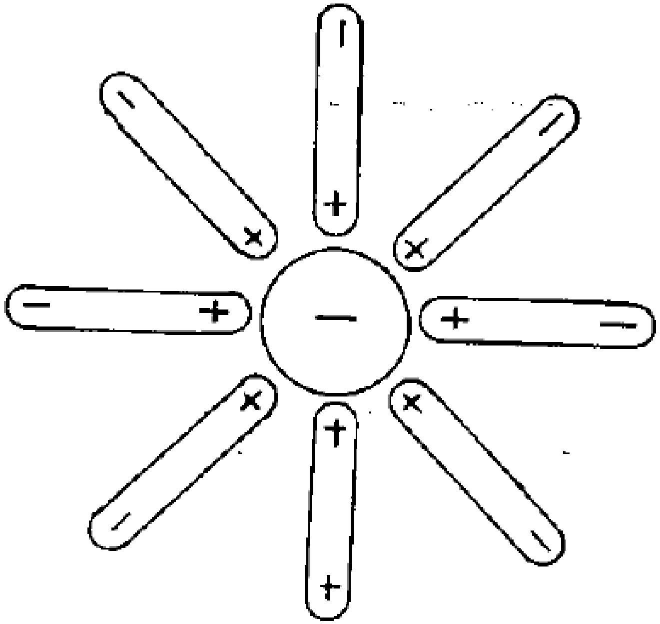
- Soil particles under natural conditions normally have net electrical charge at their surface.
- Water molecue as a single unit may be considered electrically neutral. However, its construction is such that the centers of the positive and negative charges of its individual components do not exactly coincide.

As a result, it has in effect two poles, like a small bar magnet.

Soil Water cntd

Water molecules close to electrically charged surfaces of soil particle are strongly attracted and held by the soil particle.





Soil Water cntd

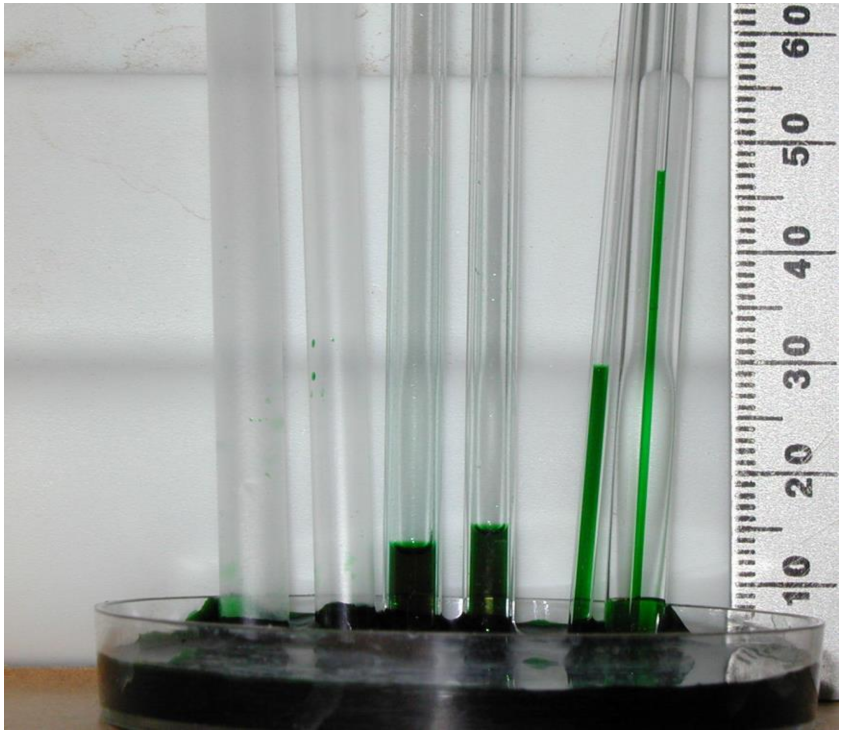
- Water adsorbed on the surfaces of soil particles is referred to as adsorbed water because of its immobility, as in bound water.
- The amount of water held by adsorption depends on specific surfaces which in turn depend on particle size, shape, and gradation.
- A relatively well graded material will normally have much greater adsorption power.
- Adsorbed water may be removed by evaporation (oven drying of soil).

Soil Water cntd

Capillary Water – water retained in a soil mass due to the capillary phenomenon which enables dry soil to draw water to elevation above the water table and enables a draining soil to retain water above the atmospheric line.

- Removed by air drying.

The movement and retention of water above the ground water table is similar in many respects to the rise and retention of water in capillary tube.



Soil Water cntd

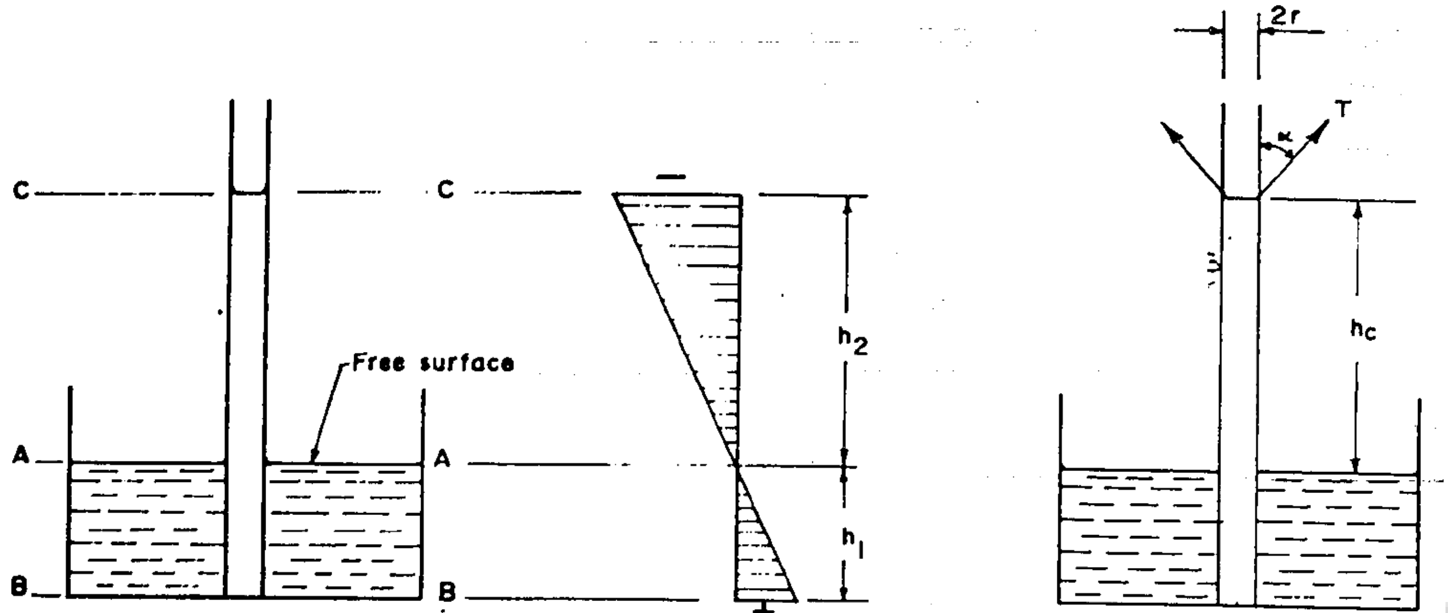
Water pressure varies linearly both below & above water table.

At level BB, the water pressure is $\gamma_w h_1$

Total pressure at level BB is $\gamma_w h_1 + P_a$ (P_a=atmospheric pressure)

The negative water pressure at level CC is $-h_2\gamma_w$.

Hence the total pressure at level CC would be $P_a - h_2\gamma_w$



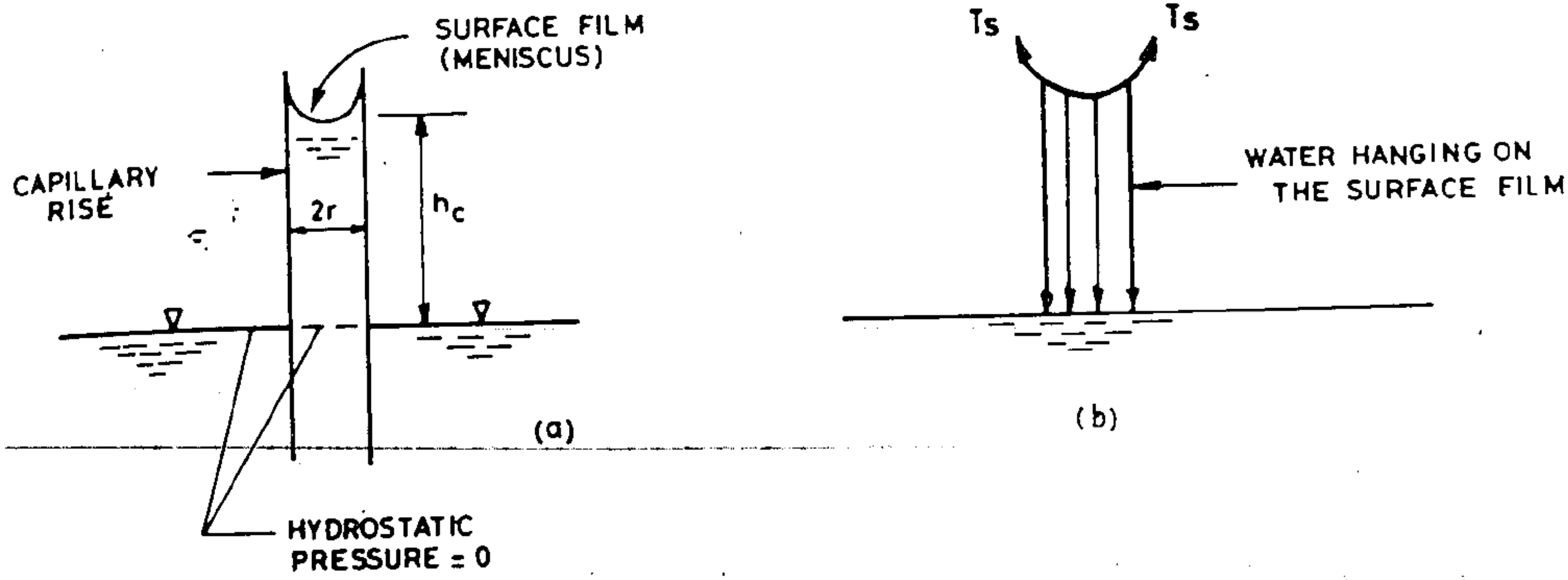
(a) Capillary rise (b) Stress in water in capillary tube (c) Height of capillary rise.

Consider the rise of water in a capillary tube.

- At the base of the column of water having a capillary rise of h_c, which has the same elevation as the free water level outside, the hydrostatic pressure is zero.
- The shearing stress around the cylindrical surface is zero and yet the water is in equilibrium. This could only happen if the mechanical properties of the upper most layer of the column of water are different from those of ordinary water.
- This upper layer called surface film or meniscus keeps the element from sinking and be visualized as a membrane from which water is hanging.

The meniscus joins the wall of the tube at an angle α called the contact angle.

The surface film is in state of two dimensional tension parallel to its surface. This is the surface tension, T_s, the magnitude of which is $0.075\ cN/cm$ at 20ºC for water.



Soil Water cntd

The total force developed along the perimeter is

$$F = 2\pi r T_s \cos\alpha$$

The capillary stress U would then be

$$U = \frac{F}{A} = \frac{2\pi r T_s \cos\alpha}{\pi r^2} = \frac{2T_s \cos\alpha}{r}$$

The maximum capillary stress occurs when $\alpha = 0$.

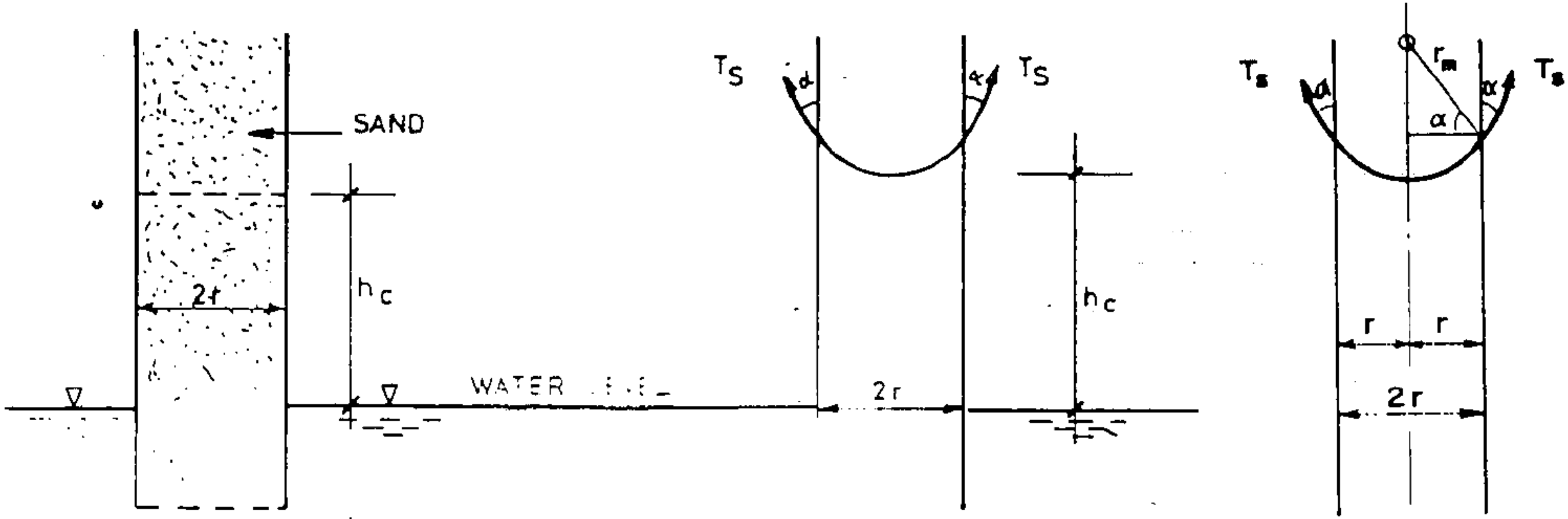
Hence

$$U_{max} = \frac{2T_s}{r}$$

Soil Water cntd

$$r = r_m \cos\alpha$$

$$U = \frac{2T_s \cos\alpha}{r_m \cos\alpha} = \frac{2T_s}{r_m}$$



Soil Water cntd

Rise of Water in Capillary Tube of Uniform Internal Diameter

When thin glass tube, open at both ends, is dipped into water, the water will rise in the tube to a certain height.

The capillary rise can easily be related to the surface tension by considering the equilibrium of capillary column.

Let

- the surface tension per unit perimeter = T_s
- The contact angle = α

Force acting upward = $2\pi r T_s \cos\alpha$

Force acting downward = $h_c \gamma_w \pi r^2$

For equilibrium condition

$$h_c \gamma_w \pi r^2 = 2\pi r T_s \cos\alpha$$

Soil Water cntd

$$h_c = \frac{2T_s \cos \alpha}{r \cdot \gamma_w}$$

For chemically clean water and clean tube, $\alpha = 0$.

Hence $h_c = \frac{2T_s}{r \cdot \gamma_w}$ where h_c is height of capillary rise.

Approximate formula for capillary rise

$$h_c = \frac{C}{e \cdot D_{10}}$$ Terezaghi & Peck

$$h_c = \frac{0.0306}{0.2D_{10}}$$ Hazen's

C=empirical constant varying between 10.0 and 50.0 /mm²)

$e = void\ ratio\ ; D_{10} = effective\ diamter\ (mm)$

Soil Water cntd

Effect of surface tension on soil mass

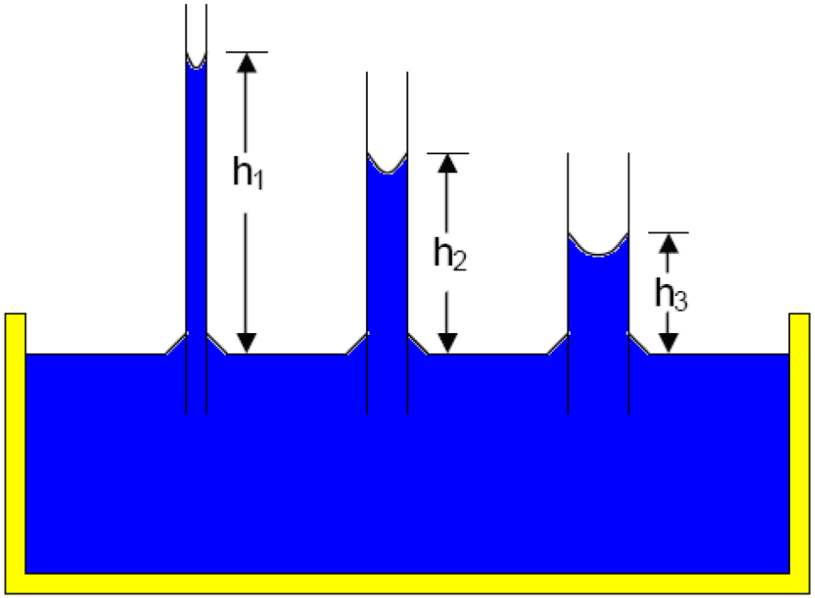
- At all points where moisture menisci touch soil particles, surface tension forces act, causing a grain-to-grain pressure within the soil and contribute to the shear and stability of the soil mass.
- This induced strength is only temporary in character and may be destroyed entirely upon full saturation of soil; since complete saturation eliminates inteface meniscie, contact pressure reduces to zero.

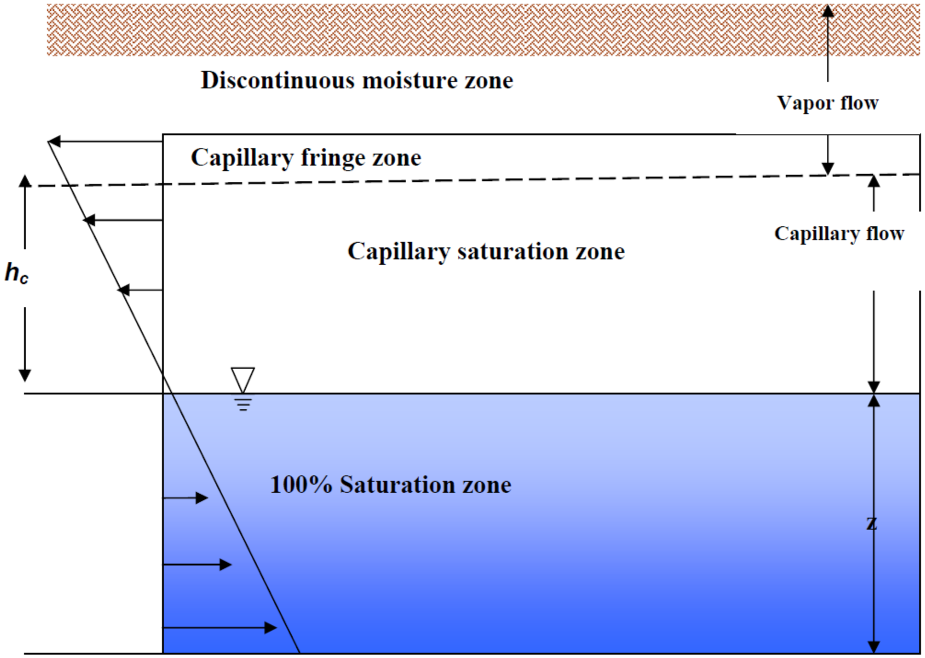
Video: What is Surface Tension - Richard Hammond's Invisible Worlds

Soil Water cntd

Factors affecting capillary rise in soils

- Positions of the ground water table
- Evaporation opportunity
- Grain size of the soil





→ capillary is more pronounced in fine grained soils than in course grained .

Video: Capillary rise with varying tube diameters

Soil Water cntd

Gravitational Water : completely free to move through or drain from soil under the influence of gravity.

The flow of gravitational water is caused by the action of gravity which tends to pull water downward to a lower elevation.

The gravitational pull acts to overcome resistance to movement or flow of water which is due to viscous drag along the side walls of pore spaces in the case of soil.

2. Permeability

- Introduction
- Darcy's Law
- Hydraulic Gradient
- Determination of Permeability
- Permeability in Stratified Soils

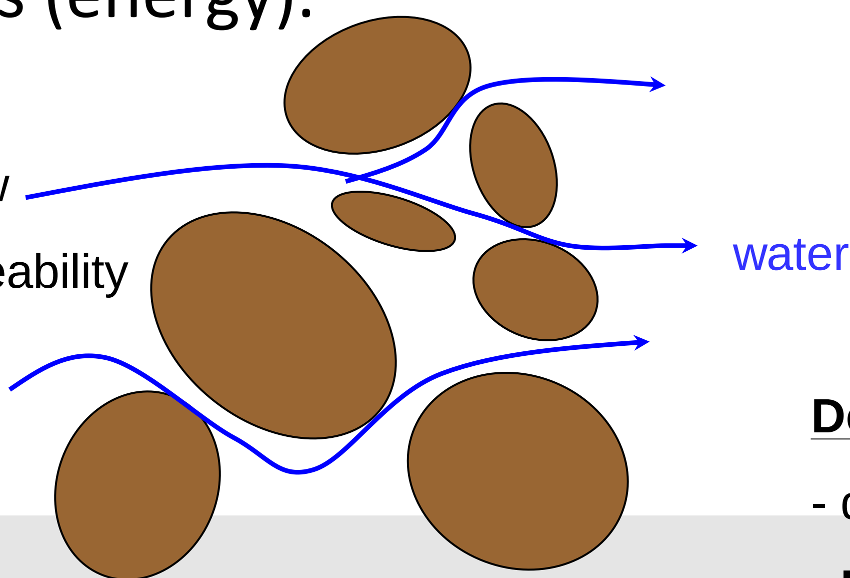
Permeability

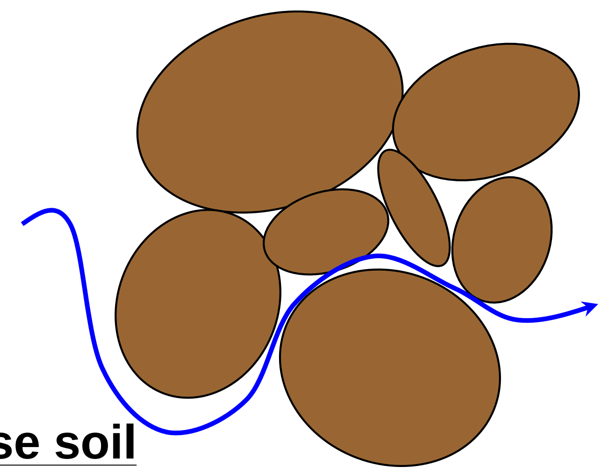
- facility with which water flows through soil.
- measure of how easily a fluid (e.g., water) can pass through a porous medium (e.g., soils).

Soil is permeable due to the existence of voids between soil grains that are interconnected and allow flow from points of higher potentials towards points of lower potentials (energy).

Loose soil

- easy to flow
- **high** permeability





Dense soil

- difficult to flow
- **low** permeability

Permeability cntd

APPLICATIONS

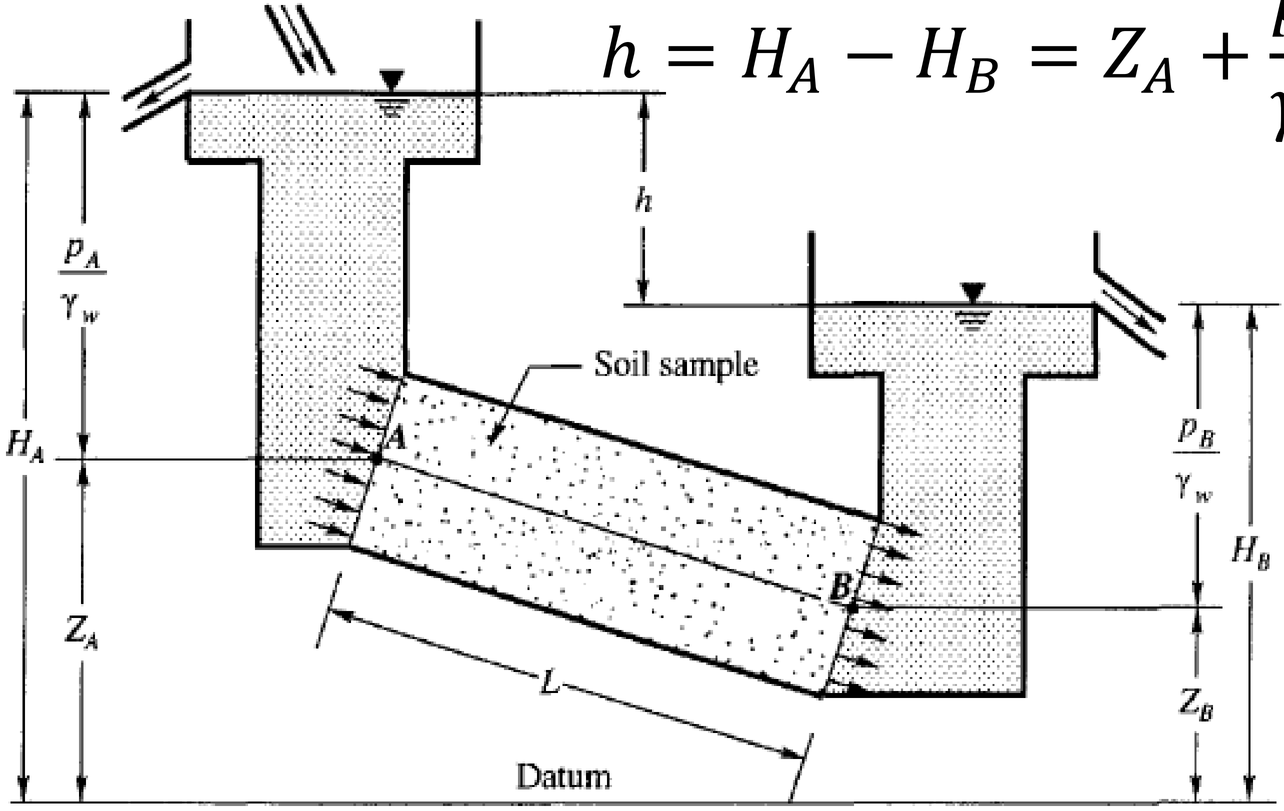
- Design of pumping systems for dewatering of excavations.
- Estimating of seepage losses through and under earth dams.
- Stability analyses of earth dams
- Stability of earth retaining structures.
- Design of clay liners and cut off walls.

Permeability cntd

Bernoulli's equation for total head: $H = Z + \frac{p}{\gamma_w} + \frac{v^2}{2g}$

In soils where velocity of flow is very low, $H = Z + \frac{p}{\gamma_w}$

$$h = H_A - H_B = Z_A + \frac{p_A}{\gamma_w} - Z_B + \frac{p_B}{\gamma_w}$$



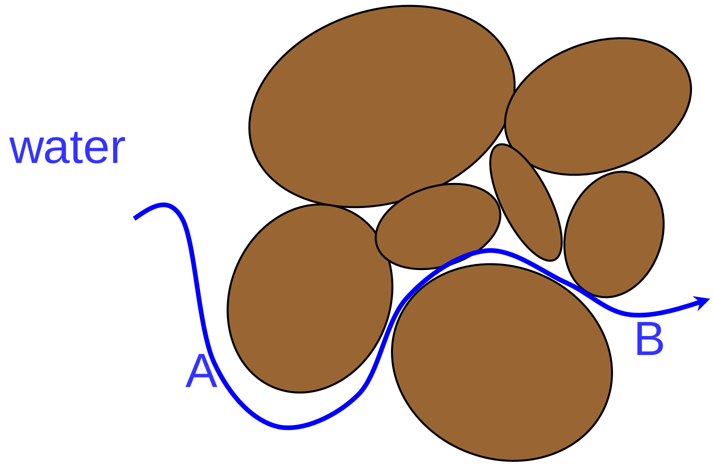
Permeability cntd

Hydraulic Gradient – loss of head per unit length of flow.

$$i = \frac{h}{L}$$

$$i = \frac{TH_A - TH_B}{l_{AB}}$$

length AB, along the stream line



Permeability cntd

Darcy‘s Law

$$v = ki$$

v=discharge velocity, which is the quantity of water flowing in unit time through a unit gross cross-sectional area at right angles to the direction of flow

k=hydraulic conductivity (coefficient of permeability)

$$q = vA = kiA$$

Darcy’s law given is true for laminar flow through the void spaces.

Permeability cntd

A criterion for investigating the range over which Darcy's law is valid can be furnished by the Reynolds number.

For flow through soils, Reynolds number R_n can be given by the relation

$$R_n = \frac{vD\rho}{\mu}$$

where v=discharge (superficial) velocity, cm/s
D=average diameter of the soil particle, cm
ρ=density of the fluid, g/cm^3
μ=coefficient of viscosity, g/(cm.s)

Permeability cntd

Coefficient of Permeability

- Depends on:
  - ❑ Fluid viscosity
  - ❑ Pore-size distribution
  - ❑ Grain-size distribution
  - ❑ Void ratio
  - ❑ Roughness of mineral particles
  - ❑ Degree of soil saturation
  - ❑ Temperature

Permeability cntd

(after Casagrande and Fadum, 1939)

k (cm/sec)	Soils type	Drainage conditions
10^1 to 10^2	Clean gravels	Good
10^1	Clean sand	Good
10^{-1} to 10^{-4}	Clean sand and gravel mixtures	Good
10^{-5}	Very fine sand	Poor
10^{-6}	Silt	Poor
10^{-7} to 10^{-9}	Clay soils	Practically impervious

Coefficient of Permeability k (m/s)

10^0 10^{-1} 10^{-2} 10^{-3} 10^{-4} 10^{-5} 10^{-6} 10^{-7} 10^{-8} 10^{-9} 10^{-10} 10^{-11}

Drainage	Good			Poor	Practically Impervious
Soil types	Clean gravel	Clean sands, clean sand and gravel mixtures	"Impervious" soils modified by effects of vegetation and weathering	Very fine sands, organic and inorganic silts, mixtures of sand silt and clay, glacial till, stratified clay deposits, etc.	"Impervious" soils, e.g., homogeneous clays below zone of weathering

Permeability cntd

Methods of determining hydraulic conductivity

- Empirical Methods
  - Kozeny-Carman / Taylor's Formula
  - Hazen's Formula
- Laboratory Methods
  - Constant Head Test
  - Falling Head Test
  - Capillary Permeability Test
  - Oedometer Test (Indirectly)
- Field Methods
  - Pumping In Test / Pumping Out Tests
  - Unconfined / Confined Aquifer
  - Constant Head / Falling Head

Permeability cntd

Empirical Methods

- Taylor's Formula

$$k = D_{50}^2 \frac{\gamma_w}{\mu} \frac{C_1 e^3}{1+e}$$

C_1 is a constant related to shape that can be obtained from laboratory experiments.

- Hazen's Formula

$$k = C \cdot D_{10}^2$$

(unit: cm/s)

where C is a constant varying between 0.4 and 1.4 where D_{10} is in mm.

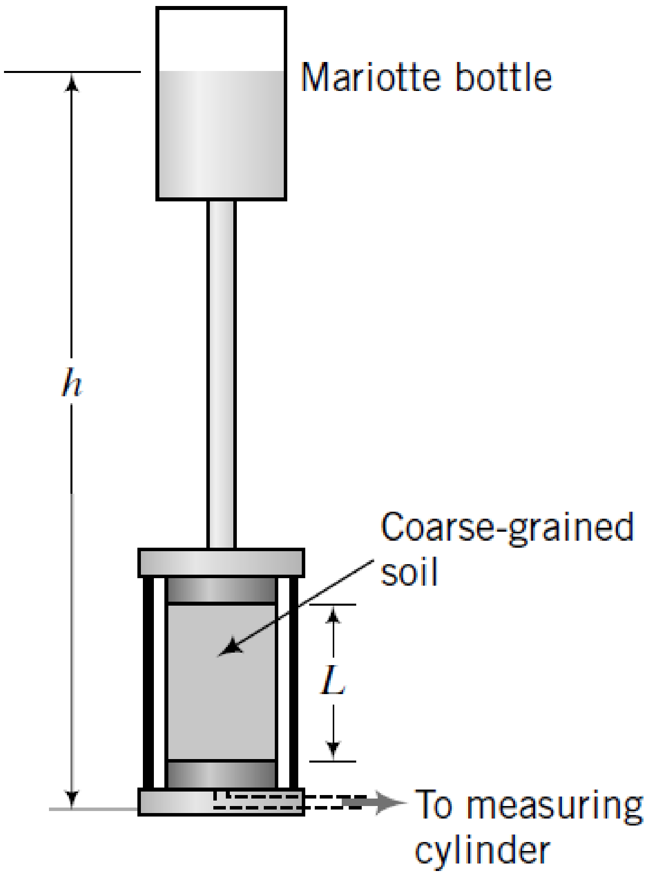
Permeability cntd

Laboratory Methods Constant Head Test

-used to determine the hydraulic conductivity of coarse-grained soils.

- Water is allowed to flow through a cylindrical sample of soil under a constant head (h).
- The outflow (Q) is collected in a graduated cylinder at a convenient duration (t).

$$\Delta H = h \;\; and \; i = \frac{\Delta H}{L} = \frac{h}{L}$$



Permeability cntd

The flow rate through the soil is $q_z = Q/t$, where Q is the total quantity of water collected in the measuring cylinder over time t.

$$k_z = \frac{q_z}{Ai} = \frac{QL}{tAh}$$

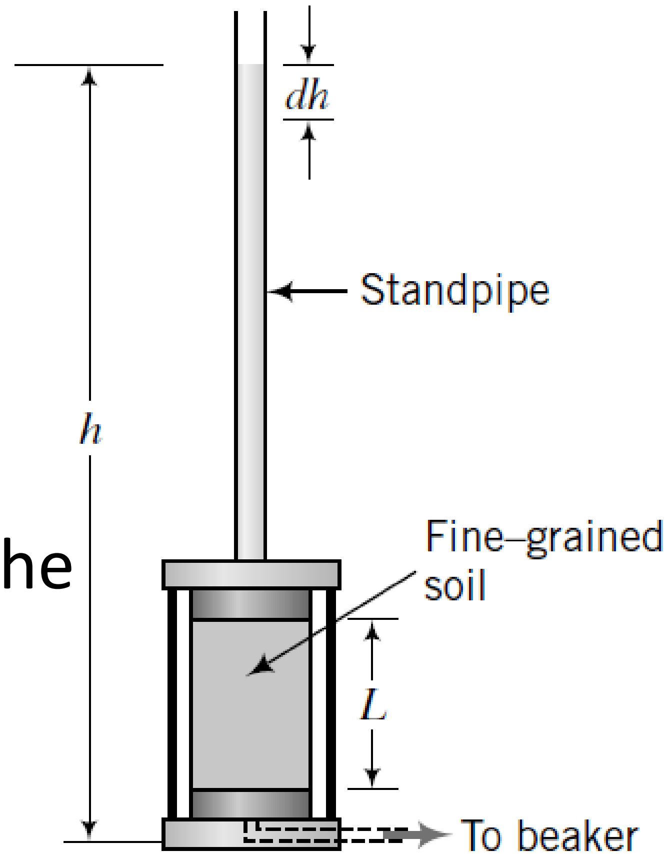
where k_z is the hydraulic conductivity in the vertical direction and A is the cross-sectional area.

Video: Constant head test

Permeability cntd

Laboratory Methods Falling Head Test

-used for fine-grained soils because the flow of water through these soils is too slow to get reasonable measurements from the constant-head test.



- A compacted soil sample or a sample extracted from the field is placed in a metal or acrylic cylinder.
- Porous stones are positioned at the top and bottom faces of the sample to prevent its disintegration and to allow water to percolate through it.

Permeability cntd

Water flows through the sample from a standpipe attached to the top of the cylinder.

The head of water (h) changes with time as flow occurs through the soil.

At different times, the head of water is recorded.

Let dh be the drop in head over a time period dt.

The velocity or rate of head loss in the tube is

$$v = -\frac{dh}{dt}$$

and the inflow of water to the soil is

$$(q_z)_{in} = av = -a\frac{dh}{dt}$$

where a is the cross-sectional area of the tube.

Permeability cntd

The outflow using Darcy's law is:

$$(q_z)_{out} = Aki = Ak\frac{h}{L}$$

where A is the cross-sectional area, L is the length of the soil sample, and h is the head of water at any time t.

The continuity condition requires that $(q_z)_{in} = (q_z)_{out}$

$$-a\frac{dh}{dt} = Ak\frac{h}{L}$$

$$\frac{Ak}{aL}\int_{t_1}^{t_2} dt = -\int_{h_1}^{h_2}\frac{dh}{h}$$

$$k = k_z = \frac{aL}{A(t_2 - t_1)}\ln\left(\frac{h_1}{h_2}\right)$$

Video: Falling head test

Permeability cntd

The viscosity of the fluid, which is a function of temperature, influences the value of k_z.

$$k_{20°\mathrm{C}} = k_{T°\mathrm{C}} \frac{\mu_{T°\mathrm{C}}}{\mu_{20°\mathrm{C}}} = k_{T°\mathrm{C}} R_T$$

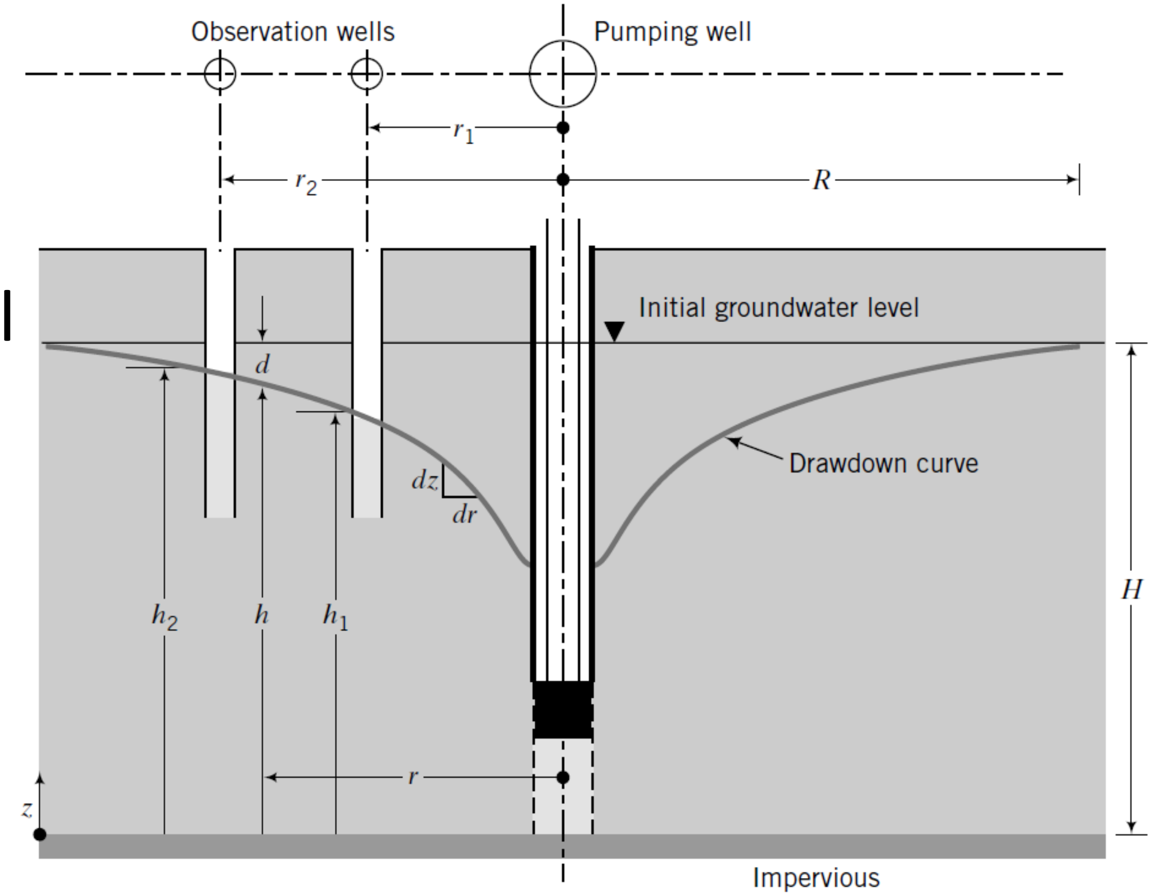
where μ is the dynamic viscosity of water, T is the temperature in °C at which the measurement was made, and is the temperature correction factor given by

$R_T = 2.42 - 0.475 \ln(T)$

Permeability cntd

Field Methods Pumping Tests

One common method of determining the hydraulic conductivity in the field is by pumping water at a constant flow rate from a well and measuring the decrease in groundwater level at observation wells.



Permeability cntd

The equation, called the simple well formula, is derived using the following assumptions.

1. The water-bearing layer (called an aquifer) is unconfined and nonleaky.
2. The pumping well penetrates through the water-bearing stratum and is perforated only at the section that is below the groundwater level.
3. The soil mass is homogeneous, isotropic, and of infinite size.
4. Darcy's law is valid.
5. Flow is radial toward the well.
6. The hydraulic gradient at any point in the water-bearing stratum is constant and is equal to the slope of groundwater surface (Dupuit's assumption).

Permeability cntd

Let dz be the drop in total head over a distance dr. Then, according to Dupuit's assumption, the hydraulic gradient is $i = \frac{dz}{dr}$

The area of flow at a radial distance *r* from the center of the pumping well is

$$A = 2\pi rz$$

where z is the thickness of an elemental volume of the pervious layer.

From Darcy's law, the flow is

$$q_z = 2\pi rzk\frac{dz}{dr}$$

Permeability cntd

Rearanging and integrating the equation between the limits r_1 and r_2 and h_1 and h_2

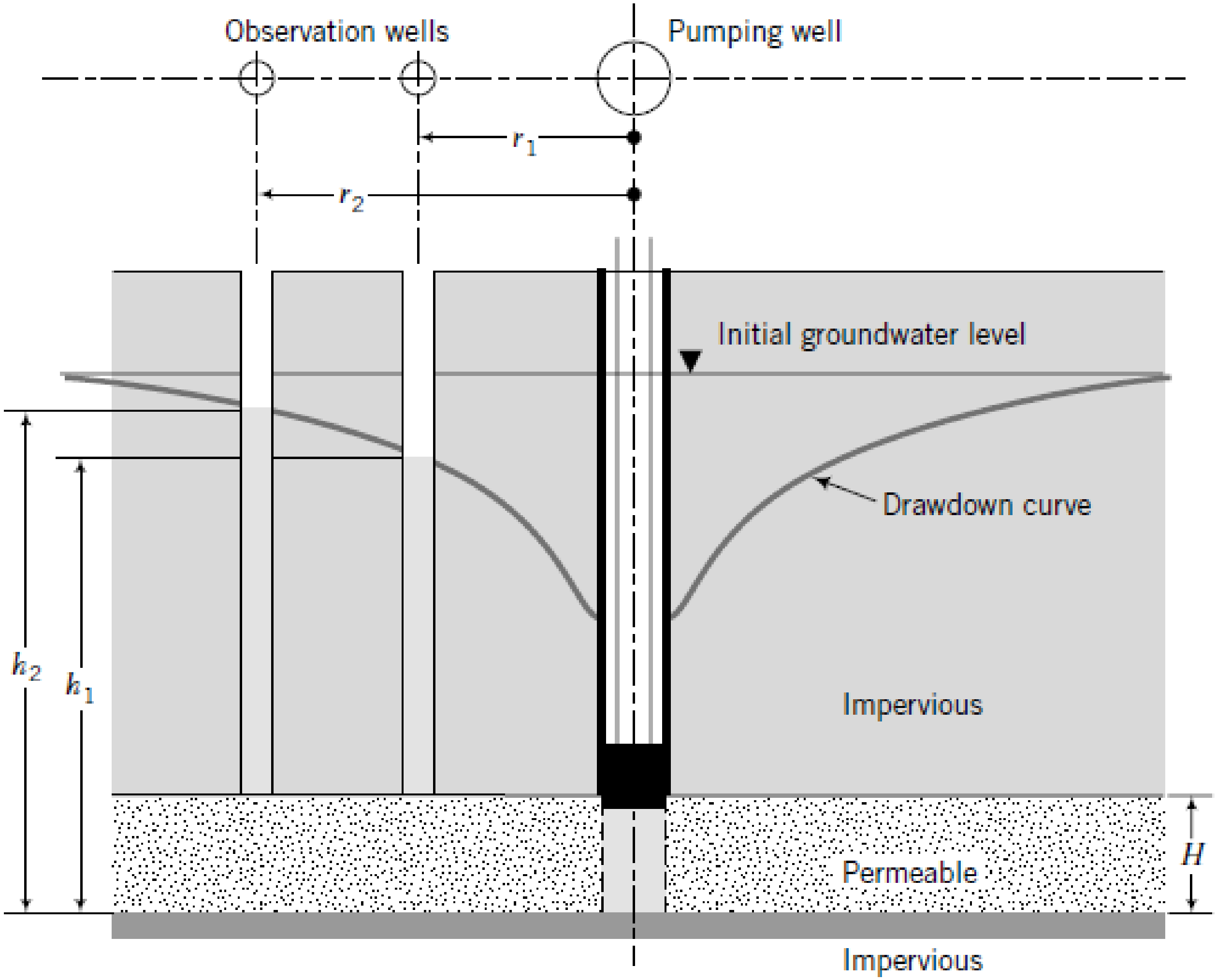
$$q_z \int_{r_1}^{r_2} \frac{dr}{r} = 2k\pi \int_{h_1}^{h_2} z dz$$

$$k = \frac{q_z \ln(r_2/r_1)}{\pi(h_2^2 - h_1^2)}$$

NB. Pumping test is only practical for coarse-grained soils.

Permeability cntd

Confined Aquifers



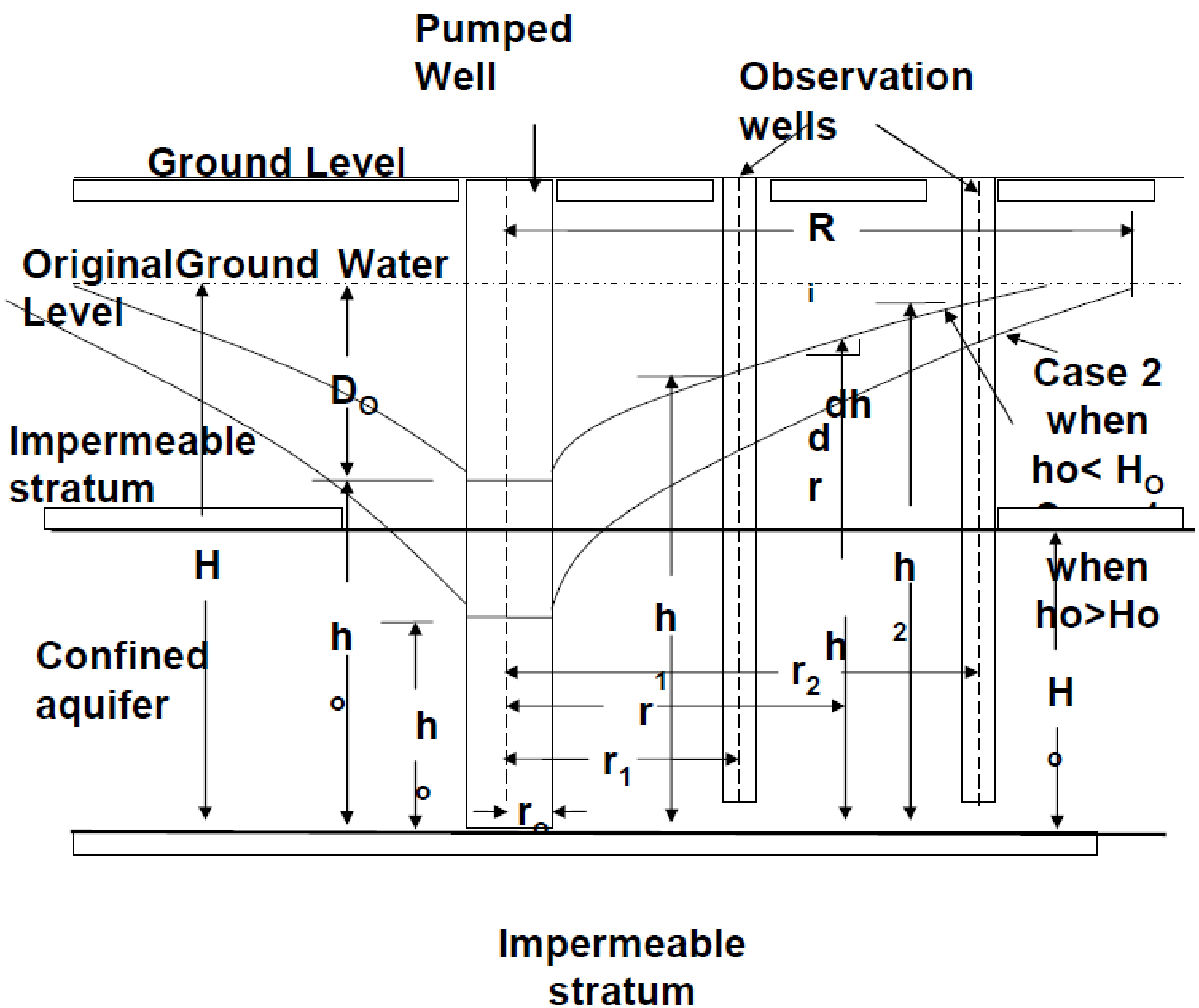
Permeability cntd

For pumping in confined aquifers, two cases might arise. They are:

Case 1: The water level in the pumped well might remain above the roof level of the aquifer at steady flow condition. i.e. $h_o > H_o$

Case 2: The water level in the pumped well might fall below the roof level of the aquifer at steady flow condition. i.e. $h_o < H_o$

Permeability cntd



Permeability cntd

Case 1: When $h_o > H_o$

$$A = 2\pi r H_o \; and \; i = dh/dr$$

$$Q = kiA = k(dh/dr)(2\pi r H_o)$$

$$\frac{dr}{r} = \frac{2\pi k H_o}{Q} dh$$

$$\int_{r_1}^{r_2} \frac{dr}{r} = \frac{2\pi k H_o}{Q} \int_{h_1}^{h_2} dh$$

$$k = \frac{Q \ln(r_2/r_1)}{2\pi H_o[h_2 - h_1]}$$

Permeability cntd

Case 2: When $h_o < H_o$

The flow pattern close to the well is similar to unconfined aquifer whereas at distances further from the well the flow is artesian.

$$k = \frac{Q \ln(r_i / r_o)}{\pi \, [2HH_o - H_o^2 - h_o^2]}$$

Radius of influence, R_i , for stabilized flow condition is given by:

$$R_i = 3000 D_o \sqrt{k} \quad (m)$$

EXERCISE 4.2.4 – BOREHOLE TEST

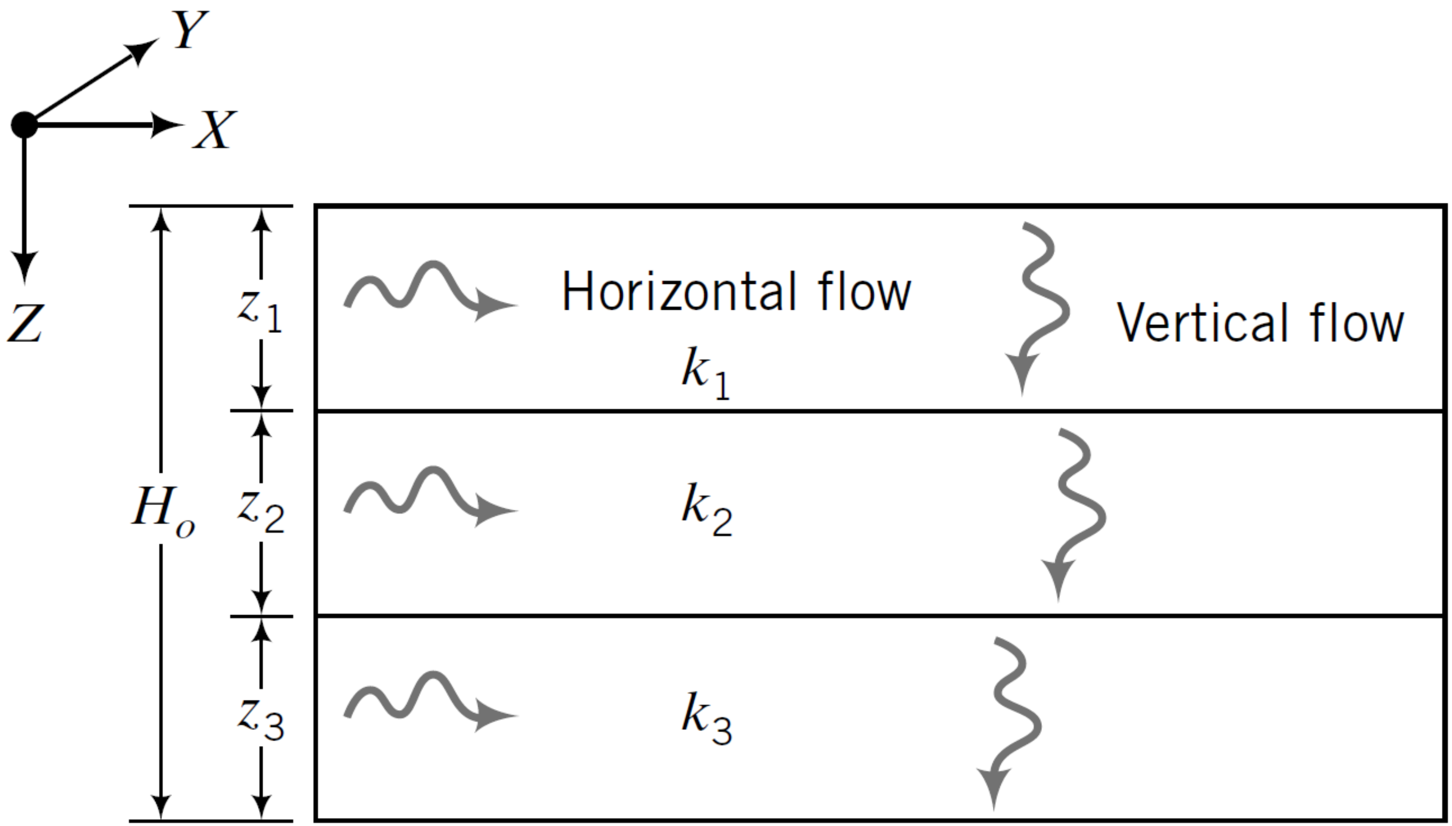
A test boring was performed at an elevation of 925 m above m.s.l where the water table was found 8 m below the ground surface. A piezometer, installed 3.2 km downstream showed the phreatic surface to be at an elevation of 907 m above msl. An aquifer of almost uniform thickness of 15 m was observed between these two points. If the quantity of flow was measured 3mm^3/s per unit width, compute the coefficient of permeability and comment on the most probable soil type using Cassagrande and Fadum's recommendation.

Permeability cntd

Permiability in Stratified Soil Layers

Flow parallel to soil layers → hydraulic gradient is the same.

Flow normal to soil layers → velocity in each layer is the same.



Permeability cntd

Flow parallel to soil layers

The flow through the soil mass as a whole is equal to the sum of the flow through each of the layers.

Consider a unit width (in the y direction) of flow

$$q_x = Av = (1 \times H_o)k_{x(eq)}i$$
$$= (1 \times z_1)k_{x1}i + (1 \times z_2)k_{x2}i + \ldots + (1 \times z_n)k_{xn}i$$

where H_o is the total thickness of the soil mass, $k_{x(eq)}$ is the equivalent permeability in the horizontal direction, $z_1\ to\ z_n$ are the thickness of the first to the nth layers, k_{x1} to k_{xn} are the horizontal hydraulic conductivities og the first to the nth layer.

$$k_{x(eq)} = \frac{1}{H_o}(z_1 k_{x1} + z_2 k_{x2} + \ \ldots + z_n k_{xn})$$

Flow normal to soil layers

The head loss in the soil mass is the sum of the head losses in each layer.

$$\Delta H = \Delta h_1 + \Delta h_2 + \ \ldots + \Delta h_n$$

where ΔH is the total head loss, Δh_1 to Δh_n are the head losses in each of the n layers..

$$k_{z(eq)} \frac{\Delta H}{H_o} = k_{z1} \frac{\Delta h_1}{z_1} = k_{z2} \frac{\Delta h_2}{2} = \ldots = k_{zn} \frac{\Delta h_n}{z_n}$$

where $k_{z(eq)}$ is the equivalent hydraulic conductivity in the vertical direction and k_{z1} to k_{zn} are the vertical hydraulic conductivities of the first to the nth layer.

$$k_{z(eq)} = \frac{H_o}{\frac{z_1}{k_{z1}} + \frac{z_2}{k_{z2}} + \ldots + \frac{z_n}{k_{zn}}}$$

Permeability cntd

Equivalent Hydraulic Conductivity

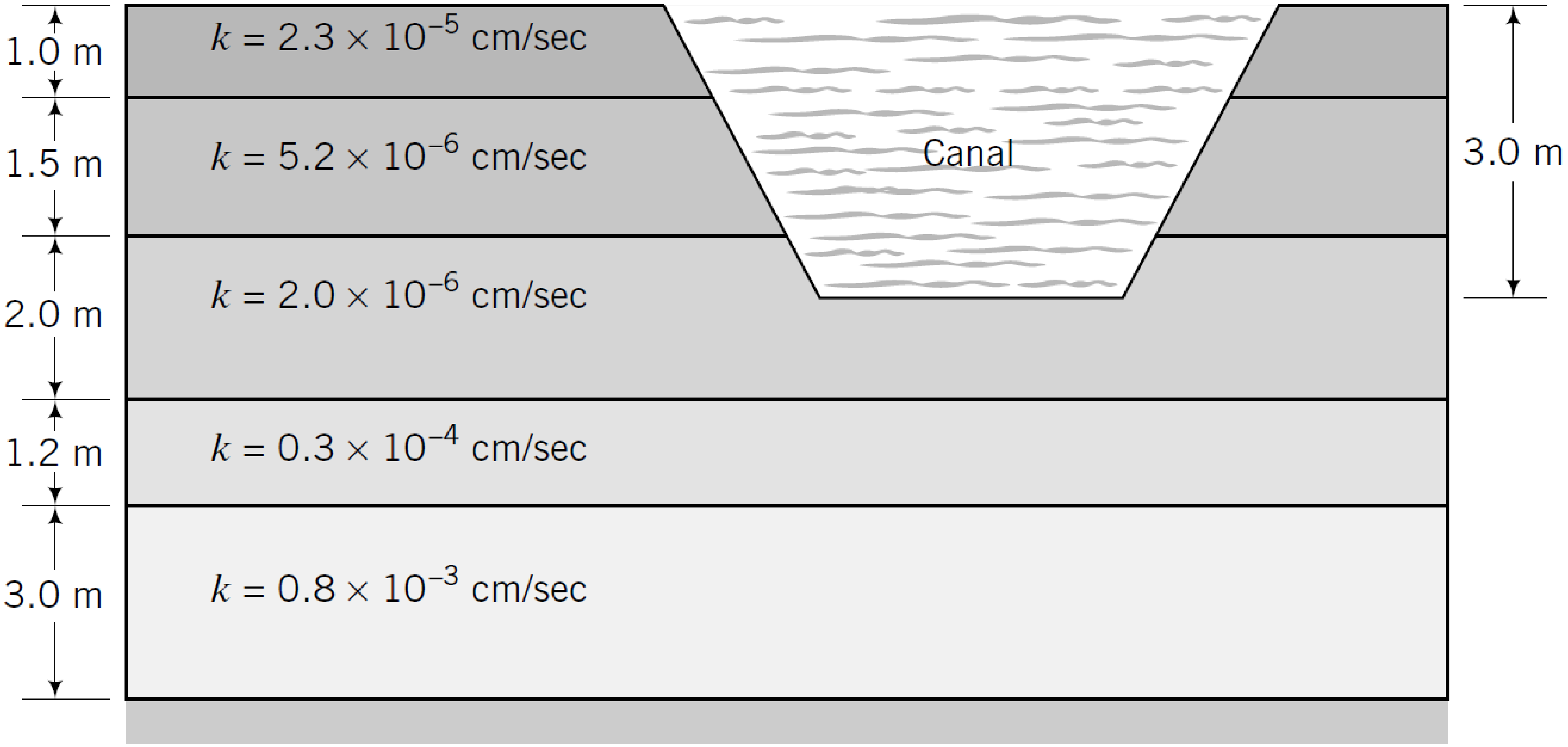
$$k_{eq} = \sqrt{k_{x(eq)}k_{z(eq)}}$$

NB. Values of $k_{z(eq)}$ are generally less than $k_{x(eq)}$ - sometimes as much as 10 times less.

EXERCISE 4.2.4 – VERTICAL & HORIZONTAL FLOWS IN LAYERED SOILS

A canal is cut into a soil with a stratigraphy shown in FIGURE. Assuming flow takes place laterally and vertically through the sides of the canal and vertically below the canal, determine the equivalent hydraulic conductivity in the horizontal and vertical directions. The vertical and horizontal hydraulic conductivities for each layer are assumed to be the same. Calculate the ratio of the equivalent horizontal hydraulic conductivity to the equivalent vertical hydraulic conductivity for flow through the sides of the canal.

Permeability cntd



3. Seepage

- Introduction
- Laplace Equation
- Flow Nets
  - Sketching (Isotropic / Anisotropic)
  - Interpretation (Flow rate, piping)
- Design of Filters

Seepage

Introduction

Seepage: flow of water through the soil pores under pressure gradient.

Flow is not one directional only and is not uniform over the entire area perpendicular to the flow.

Seepage cntd

Laplace‘s Equation

- describes the flow of water through soils.

Flow of water through soils is analogous to steady-state heat flow and flow of current in homogeneous conductors.

The popular form of Laplace’s equation for two-dimensional flow of water through soils is

$$k_x \frac{\partial^2 H}{\partial x^2} + k_z \frac{\partial^2 H}{\partial z^2} = 0$$

where H is the total head and k_x and k_z are the hydraulic conductivities in the X and Z directions.

Seepage cntd

Laplace‘s equation expresses the condition that the changes of hydraulic gradient in one direction are balanced by changes in the other directions.

The assumptions in Laplace’s equation are:

- Darcy’s law is valid.
- There is inviscid flow. This assumption means that the shear stresses are neglected.
- The soil is homogeneous and saturated.
- The soil and water are incompressible (no volume change occurs).
- Irrotational flow (vorticity) is negligible.

Seepage cntd

The last assumption leads to the following two-dimensional relationship in velocity gradients.

$$\frac{\partial v_z}{\partial z} = \frac{\partial v_x}{\partial x}$$

where v_z and v_x are the velocities in the Z and X directions, respectively.

Laplace's equation is also called the potential flow equation because the velocity head is neglected.
If the soil is an isotropic material, then $k_x = k_z$ and Laplace's equation becomes

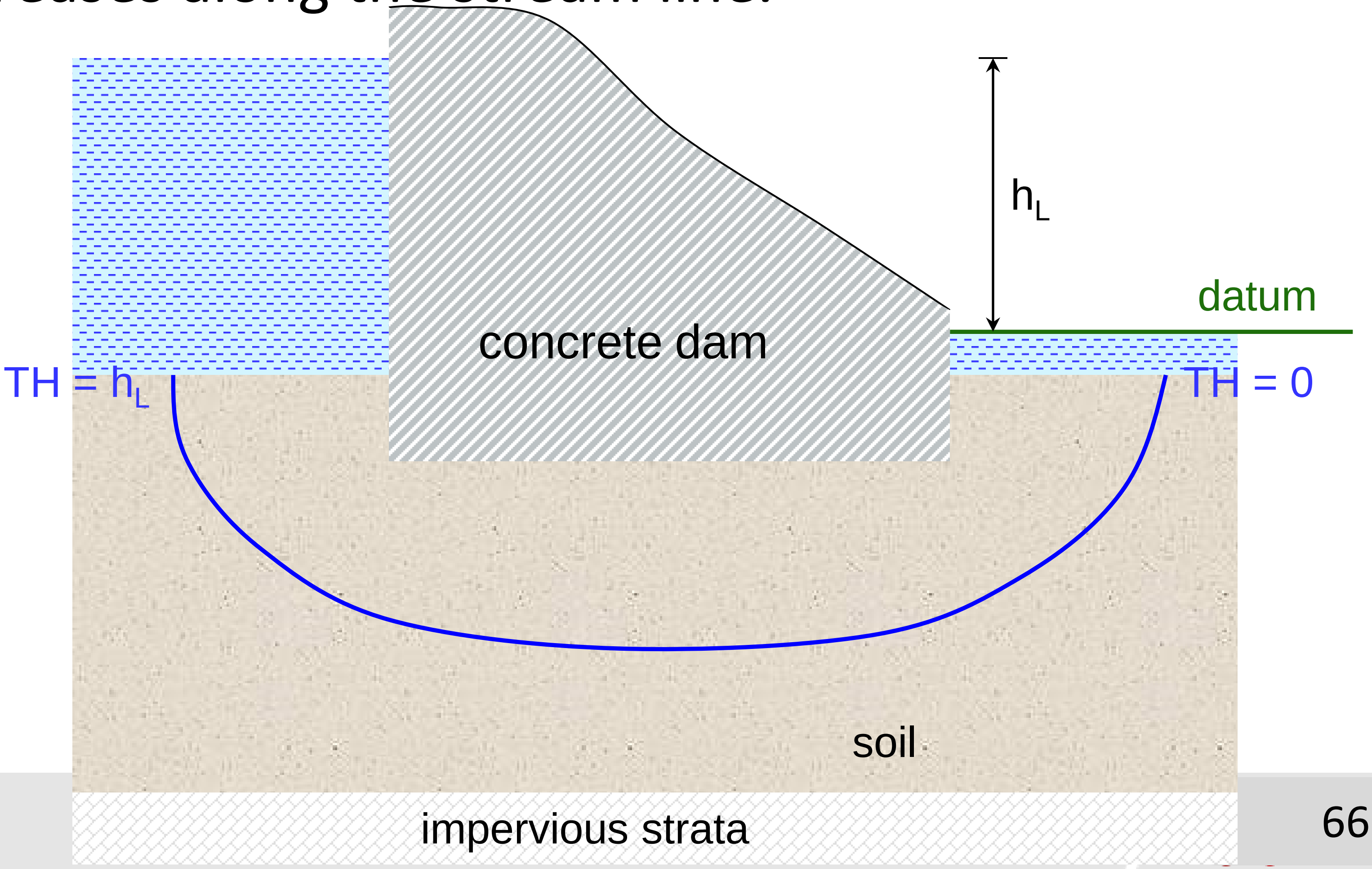
$$\frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial z^2} = 0$$

Seepage cntd

- The solution of any differential equation requires knowledge of the boundary conditions which are complex for most “real” structures. As a result, it is difficult to obtain an analytical solution or closed-form solution for these structures.
- We have to resort to approximate solutions, which we can obtain using numerical methods such as finite difference, finite element, and boundary element.
- One is an approximate method called flownet sketching; a simple and flexible method which conveys a picture of the flow regime. It is the method of choice among geotechnical engineers.

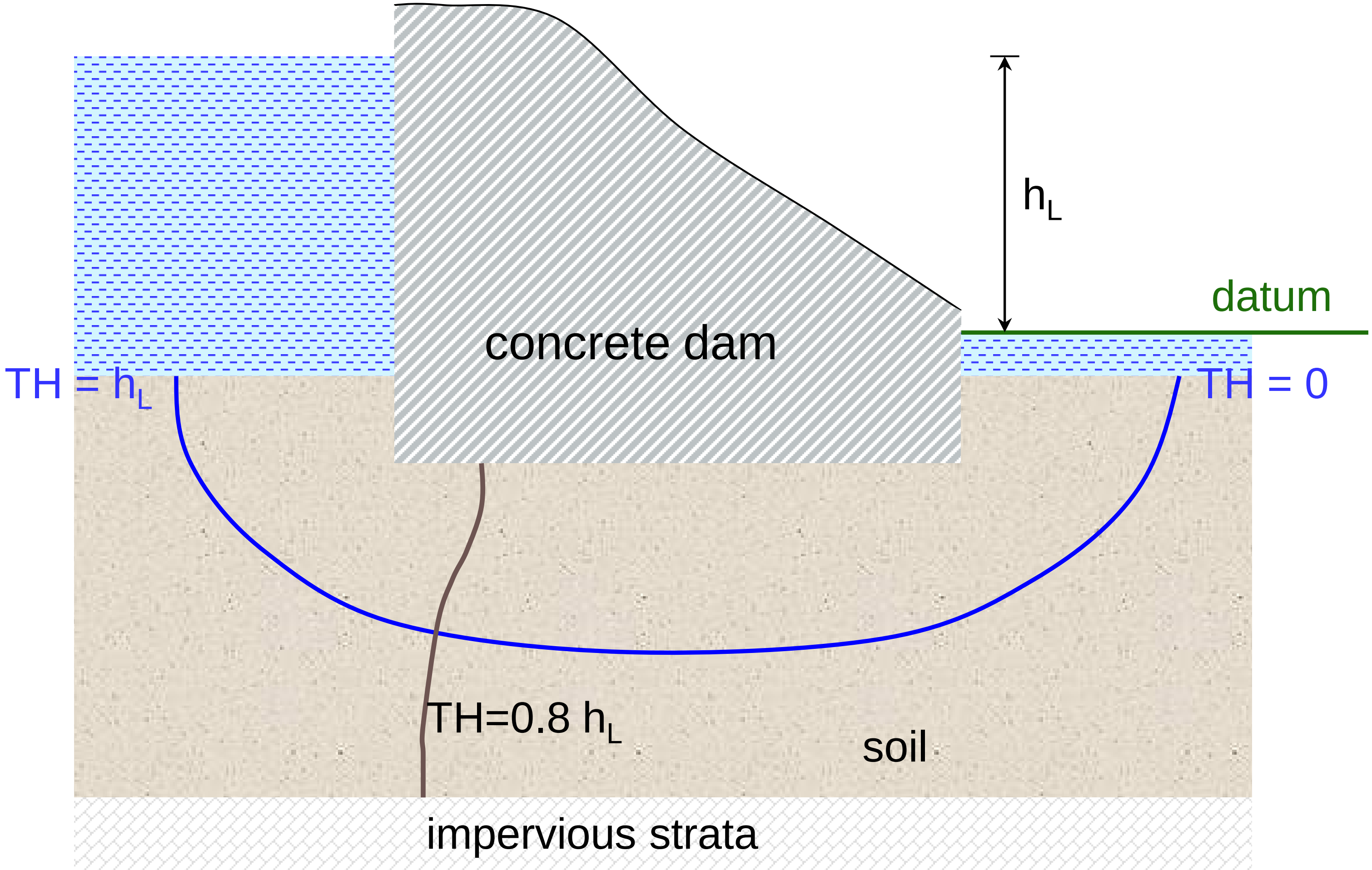
Seepage

Stream line is simply the path of a water molecule. From upstream to downstream, total head steadily decreases along the stream line.



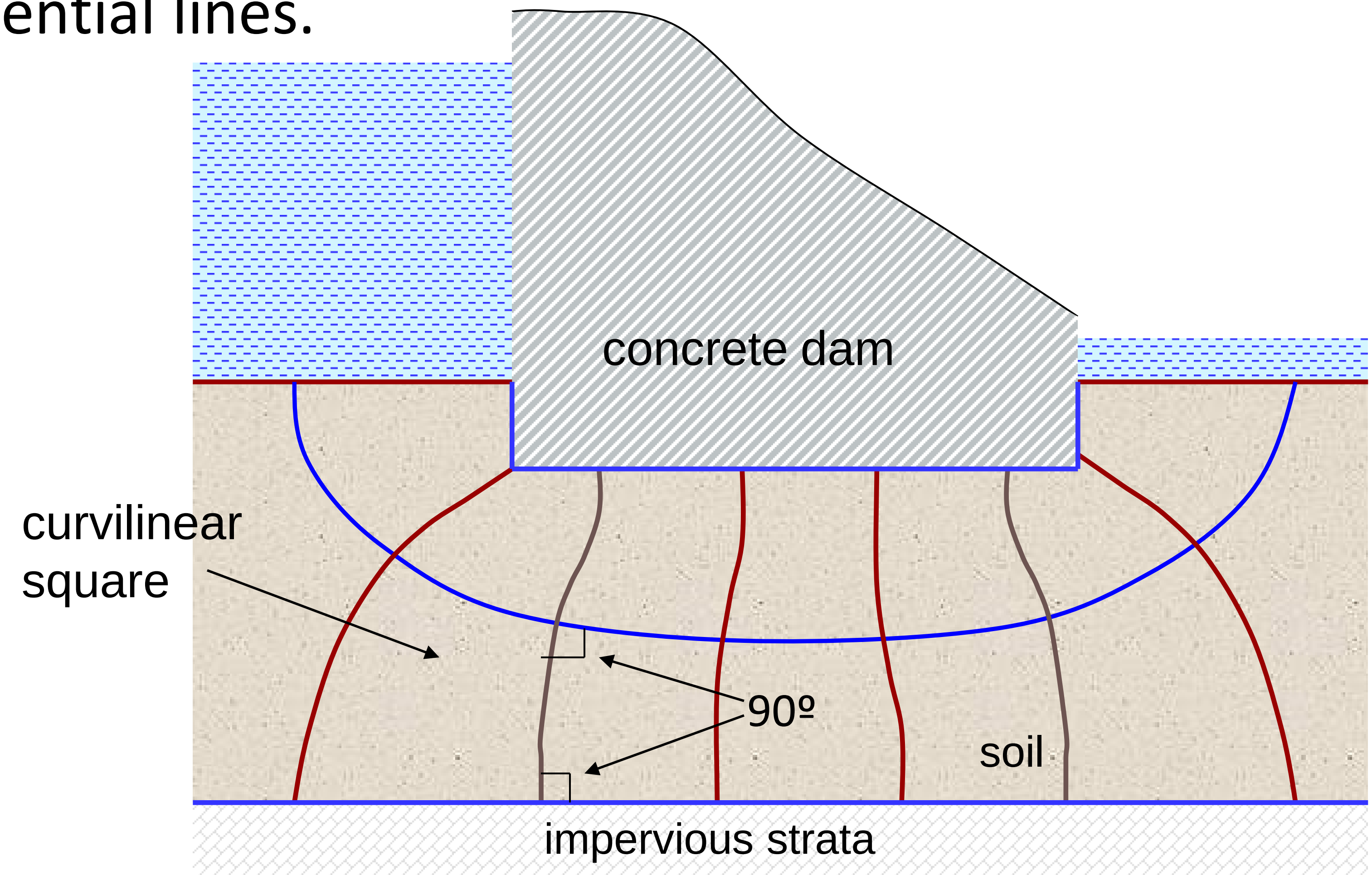
Seepage cntd

Equipotential line is simply a contour of constant total head.



Seepage cntd

Flow Net - a network of selected stream lines and equipotential lines.



Seepage cntd

A flow net must meet the following criteria:

1. The boundary conditions must be satisfied.
2. Flow lines (which represent flow paths of particles of water) must intersect equipotential lines at right angles.
3. The area between flow lines and equipotential lines must be curvilinear squares.
[A curvilinear square has the property that an inscribed circle can be drawn to touch each side of the square and continuous bisection results, in the limit, in a point.]
4. The quantity of flow through each flow channel(which area between two flow lines) is constant.

Seepage cntd

5. The head loss between each consecutive equipotential line is constant.
6. A flow line cannot intersect another flow line (i.e. flow cannot occur across flow lines.)
7. An equipotential line cannot intersect another equipotential line.
8. The velocity of flow is normal to the equipotential line.
9. Flow lines and equipotential lines are orthogonal (perpendicular) to each other.
10. The difference in head between two equipotential lines is called the potential drop or head loss.

Seepage cntd

Flow nets construction methods

1. Analytical method – based on the Laplace equation although rigorously precise, is not universally applicable in all cases because of the complexity of the problem involved.
2. Electrical analog method – extensively made use of in many important design problems.
3. Scaled model method – useful to demonstrate the fundamentals of fluid flow, but their use in other respects is limited because of the large amount of time and effort required to construct such models.
4. Graphical method – used in most of the cases in the field of soil mechanics where the estimation of seepage flows and pressures are generally required.

Seepage cntd

Graphical methods

- The graphical method developed by Forchheimer (1930) has been found to be very useful in solving complicated flow problems.
- A. Casagrande (1937) improved this method by incorporating many suggestions.
- The main drawback of this method is that a good deal of practice and aptitude are essential to produce a satisfactory flow net.
- In spite of these drawbacks, the graphical method is quite popular among engineers.

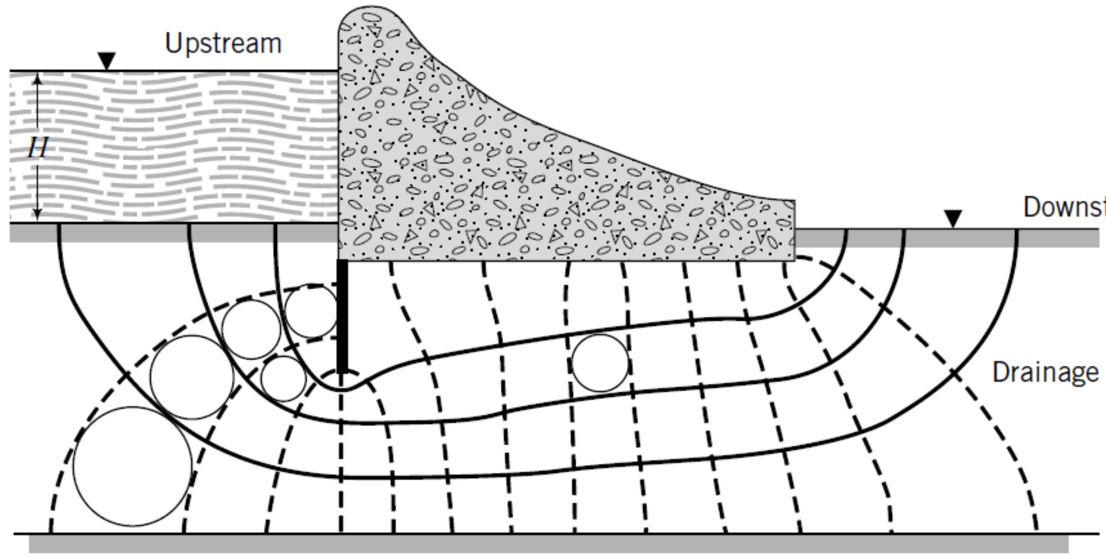
Flow Net Construction for Isotropic Soils

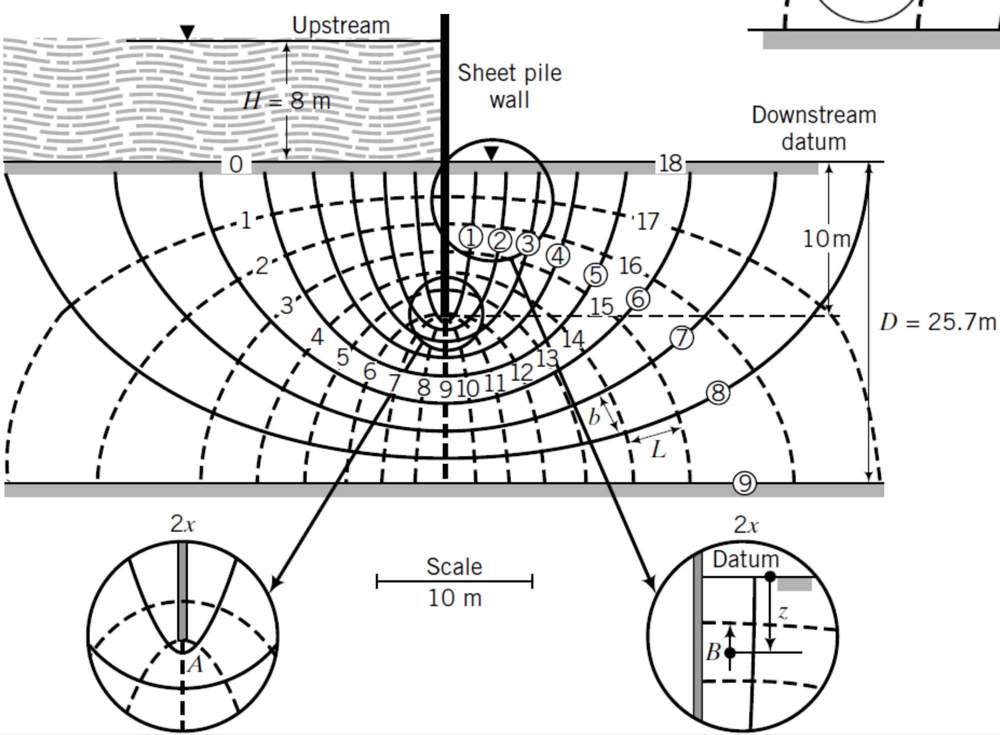
1. Draw the structure and soil mass to a suitable scale.
2. Identify impermeable and permeable boundaries.
   - The soil–impermeable boundary interfaces are flow lines because water can flow along these interfaces.
   - The soil–permeable boundary interfaces are equipotential lines because the total head is constant along these interfaces.
3. Sketch a series of flow lines (four or five) and then sketch an appropriate number of equipotential lines such that the area between a pair of flow lines and a pair of equipotential lines (cell) is approximately a curvilinear square.

Seepage cntd

- An infinite number of flow lines and equipotential lines can be drawn to satisfy Laplace's equation.
- However, only a few are required to obtain an accurate solution.
- One would have to adjust the flow lines and equipotential lines to make curvilinear squares.
- You should check that the average width and the average length of a cell are approximately equal by drawing an inscribed circle.
- You should also sketch the entire flownet before making adjustments.

Seepage cntd





Seepage cntd

Flow Net Construction for Anisotropic Soils

$$k_x \frac{\partial^2 H}{\partial x^2} + k_z \frac{\partial^2 H}{\partial z^2} = 0$$

Let $C = \sqrt{k_z/k_x}$ and $x_1 = C_x$.

$$\frac{\partial x_1}{\partial x} = C$$

$$\frac{\partial H}{\partial x} = \frac{\partial H}{\partial x_1}\frac{\partial x_1}{\partial x} = C\frac{\partial H}{\partial x_1}$$

$$\frac{\partial^2 H}{\partial x^2} = C^2 \frac{\partial^2 H}{\partial {x_1}^2}$$

Seepage cntd

Through substitution, Laplace equation becomes

$$C^2 \frac{\partial^2 H}{\partial {x_1}^2} + C^2 \frac{\partial^2 H}{\partial z^2} = 0$$

Simplifying

$$\frac{\partial^2 H}{\partial {x_1}^2} + \frac{\partial^2 H}{\partial z^2} = 0$$

Implication: for anisotropic soils we can use the procedure for fl ownet sketching described for isotropic soils by scaling the x distance by $\sqrt{k_z/k_x}$.

i.e. one must draw the structure and flow domain by multiplying the horizontal distances by $\sqrt{k_z/k_x}$.

Seepage cntd

Quantity of Seepage (Q)

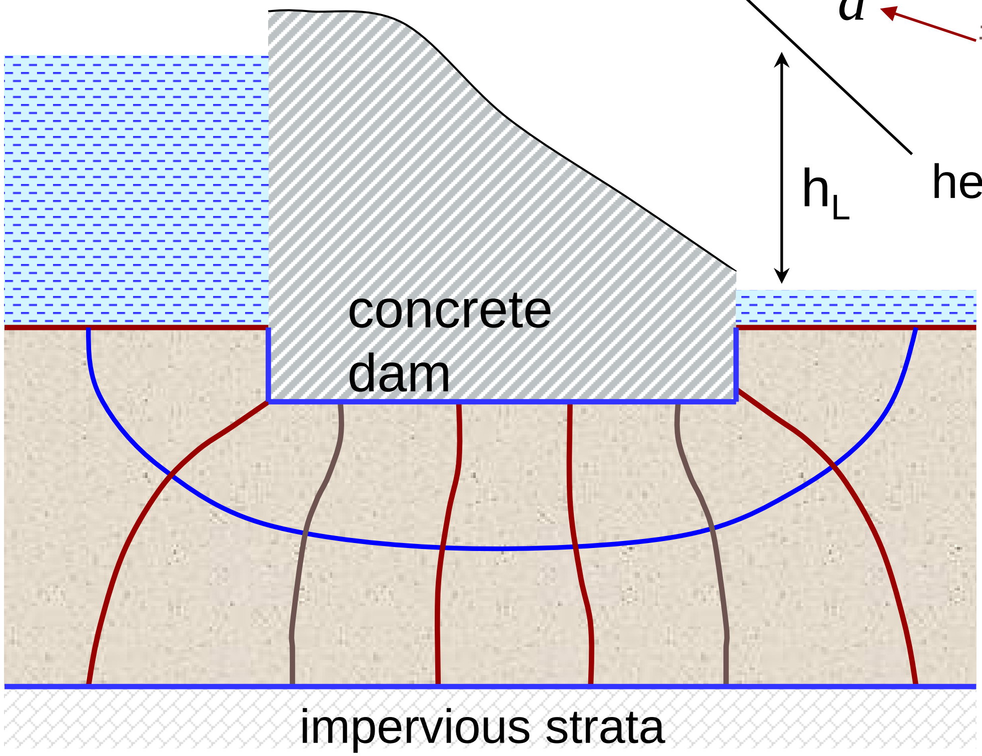
$$Q = kh_L \frac{N_f}{N_d}$$

# of flow channels

….per unit length normal to the plane

# of equipotential drops

head loss from upstream to downstream



Seepage cntd

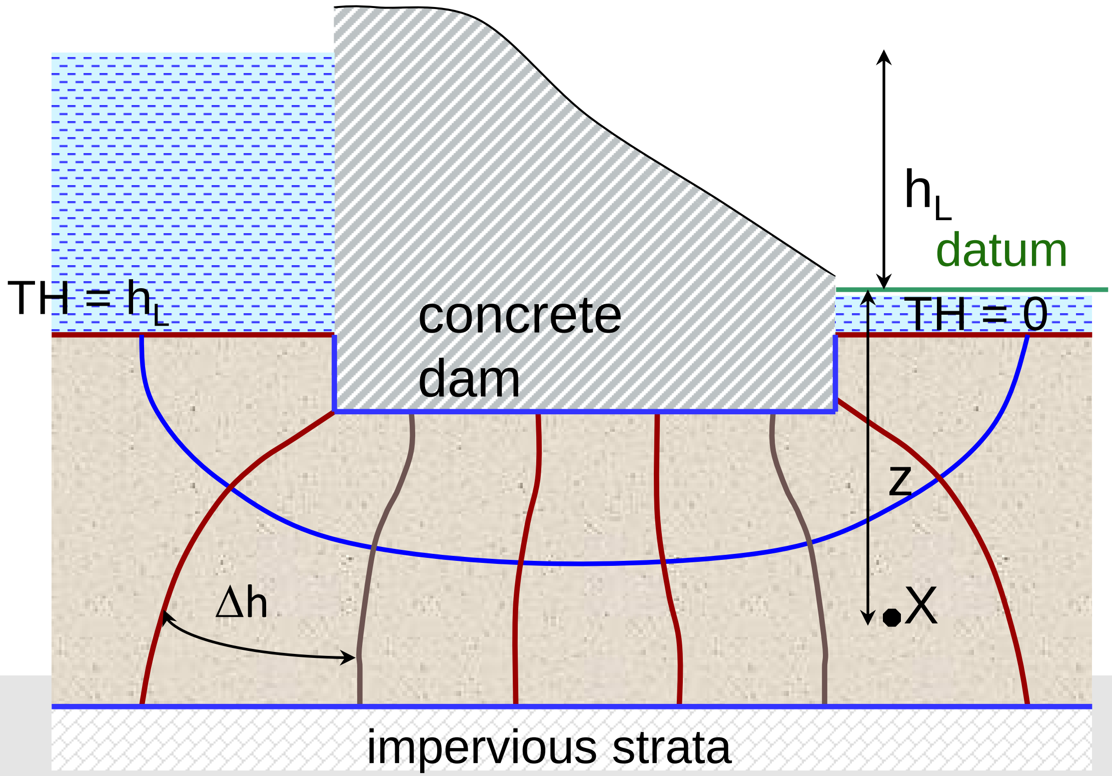
Heads at Point X

Total head = h_L - # of drops from upstream x Δh

Elevation head = -z

Pressure head = Total head – Elevation head

$$= \frac{h_L}{N_d}$$



EXERCISE 4.3.3

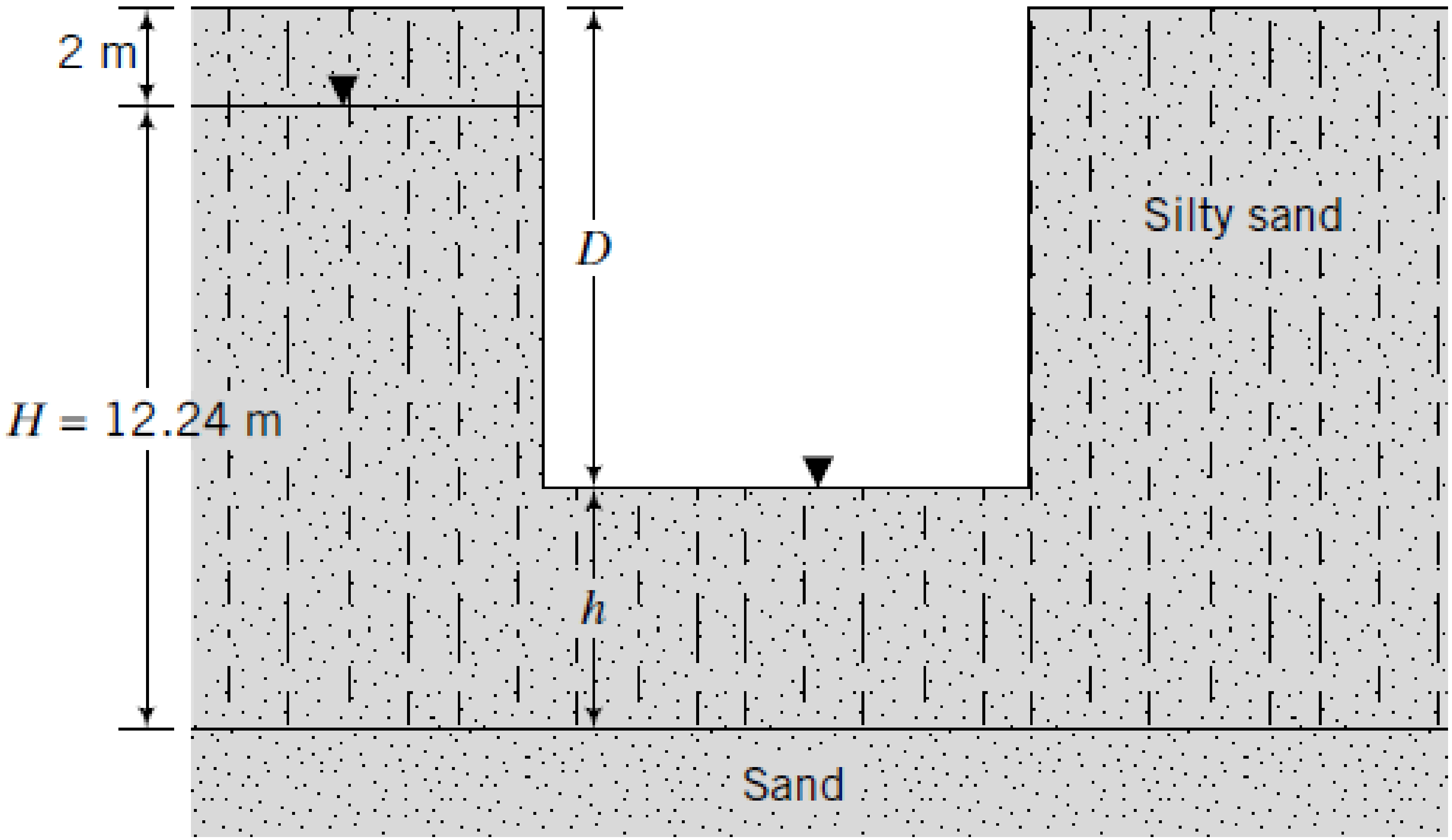
An excavation is proposed for a site consisting of a homogeneous, isotropic layer of silty clay, 14.24 m thick, above a deep deposit of sand. The groundwater is 2 m below ground level outside the excavation. The groundwater level inside the excavation is at the bottom (see Figure on next slide). The void ratio of the silty clay is 0.62 and its specific gravity is 2.7. What is the limiting depth of the excavation to avoid heaving?

Assume artesian condition is not present.

NB. Heaving will occur if $i > i_{cr}$

Seepage cntd

EXERCISE 4.3.3

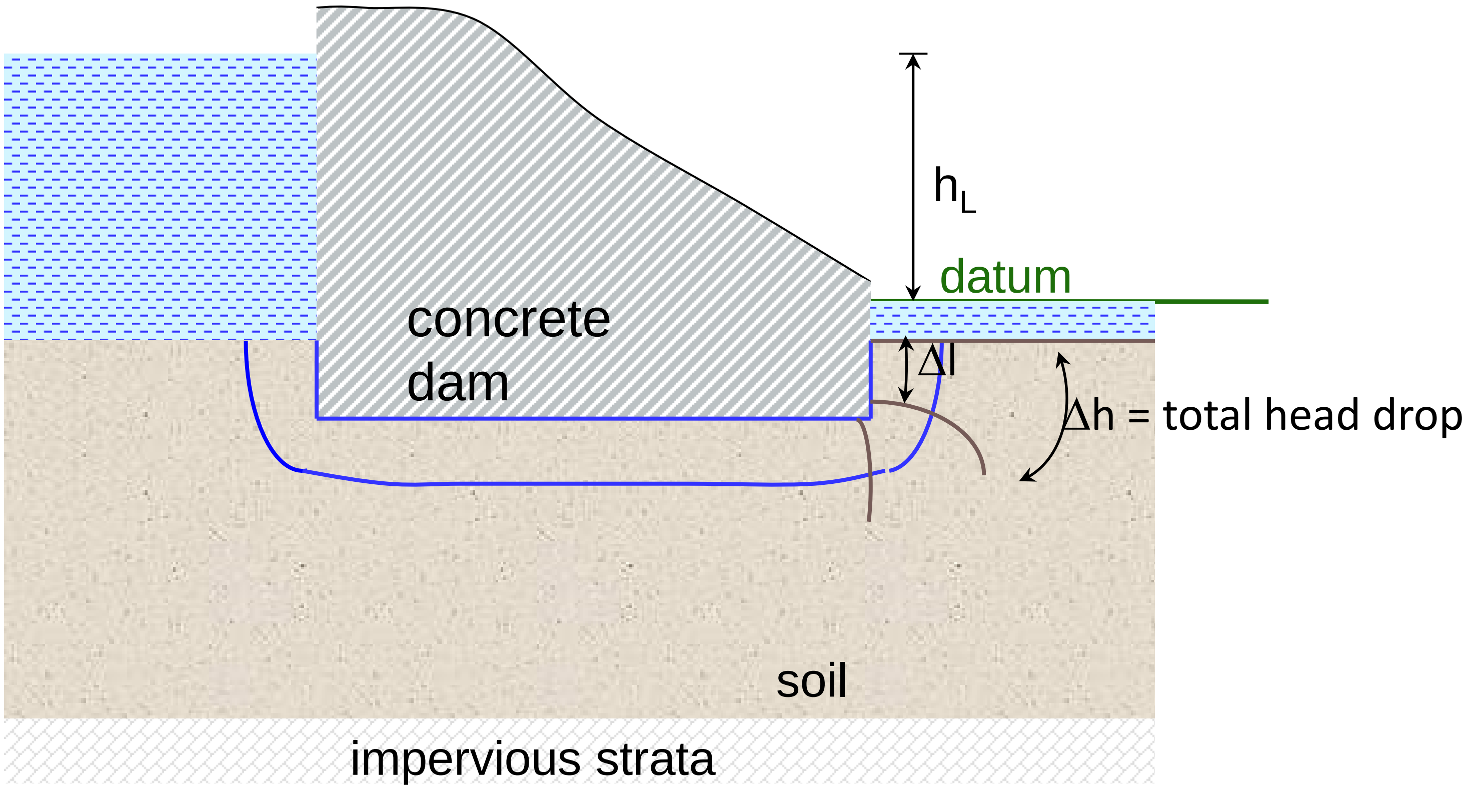


Seepage cntd

Piping in Granular Soils

At the downstream, near the dam, the exit hydraulic gradient

$$i_{exit} = \frac{\Delta h}{\Delta l}$$

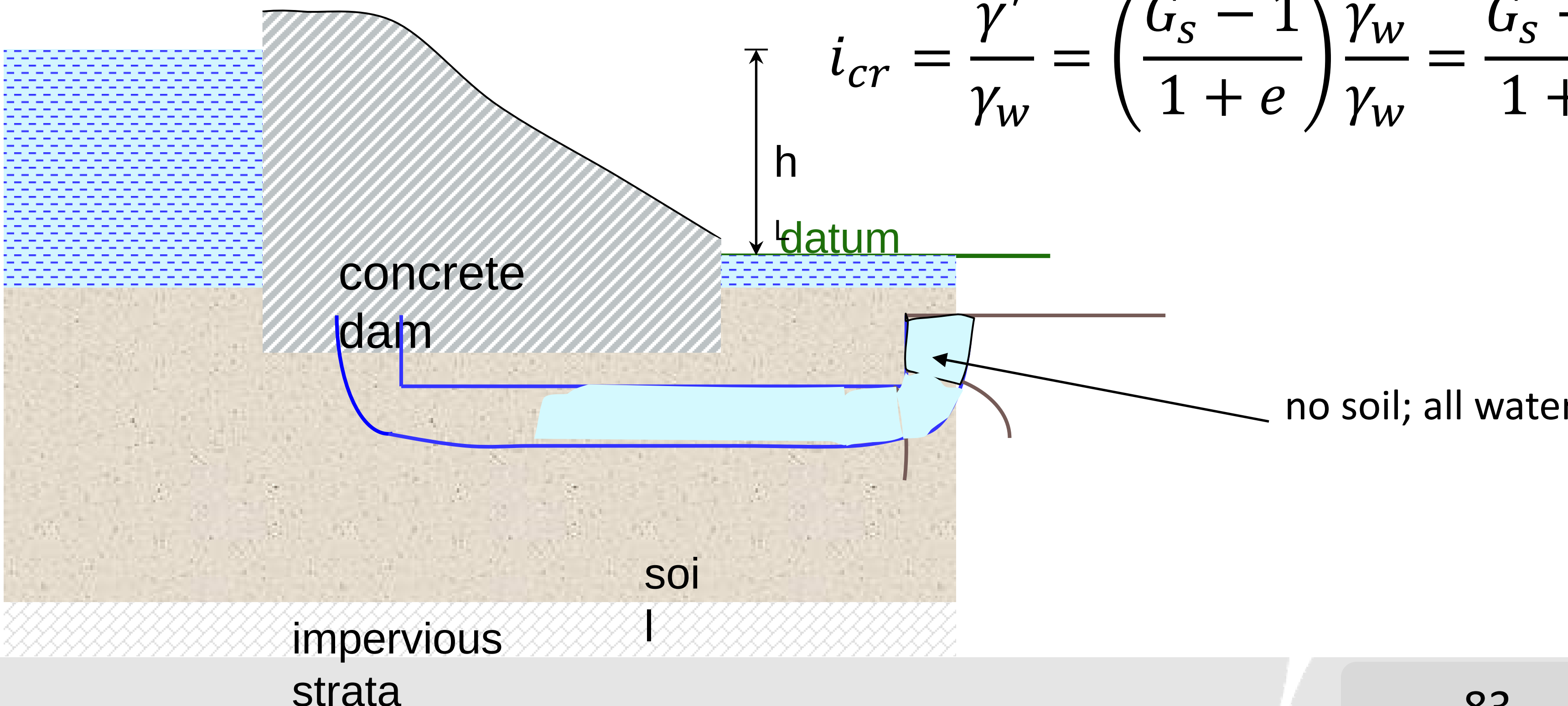


Seepage cntd

If i_{exit} exceeds the critical hydraulic gradient (i_c), firstly the soil grains at exit get washed away.

This phenomenon progresses towards the upstream, forming a free passage of water ("pipe").

$$i_{cr} = \frac{\gamma'}{\gamma_w} = \left(\frac{G_s - 1}{1 + e}\right)\frac{\gamma_w}{\gamma_w} = \frac{G_s - 1}{1 + e}$$



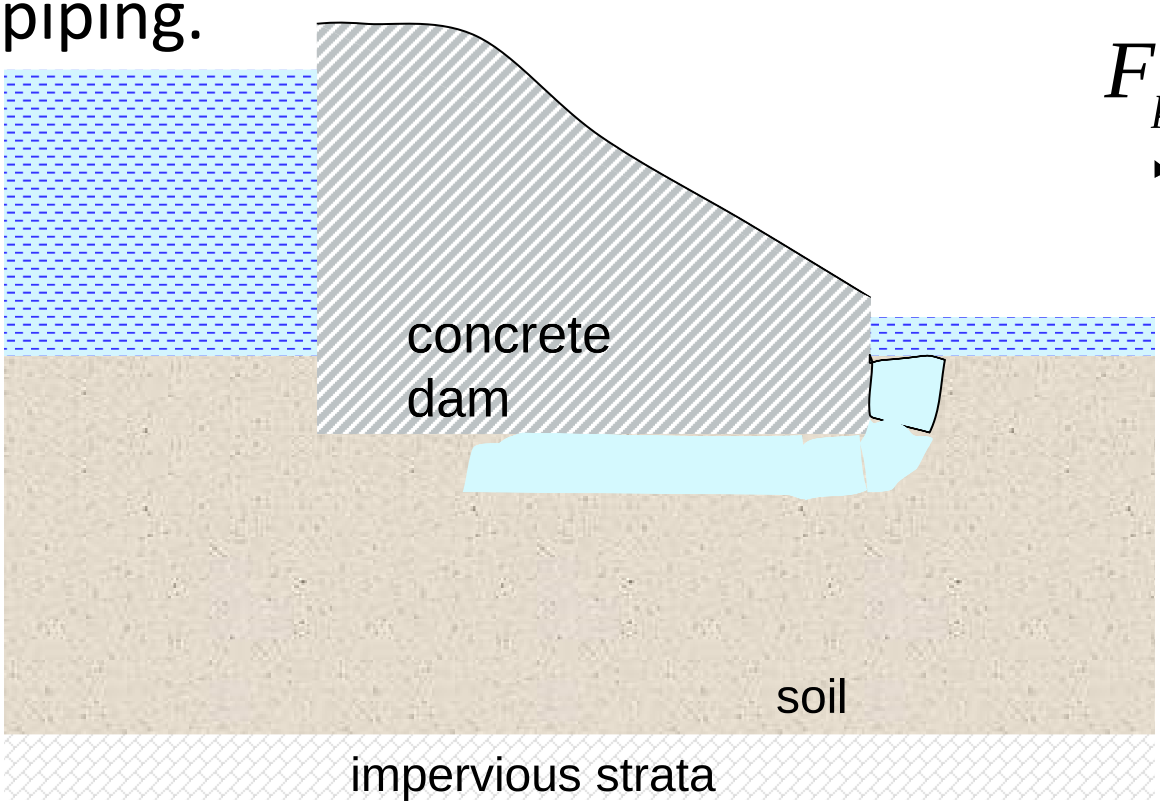
Seepage cntd

Piping is a very serious problem. It leads to downstream flooding which can result in loss of lives.

Therefore, provide adequate safety factor against piping.

$$F_{piping} = \frac{i_c}{i_{exit}}$$

typically 5-6



EXERCISE 4.3.4

A bridge pier is to be constructed in a riverbed by constructing a cofferdam (see figure in the next slide). A cofferdam is a temporary enclosure consisting of long, slender elements of steel, concrete, or timber members to support the sides of the enclosure. After construction of the cofferdam, the water within it will be pumped out.

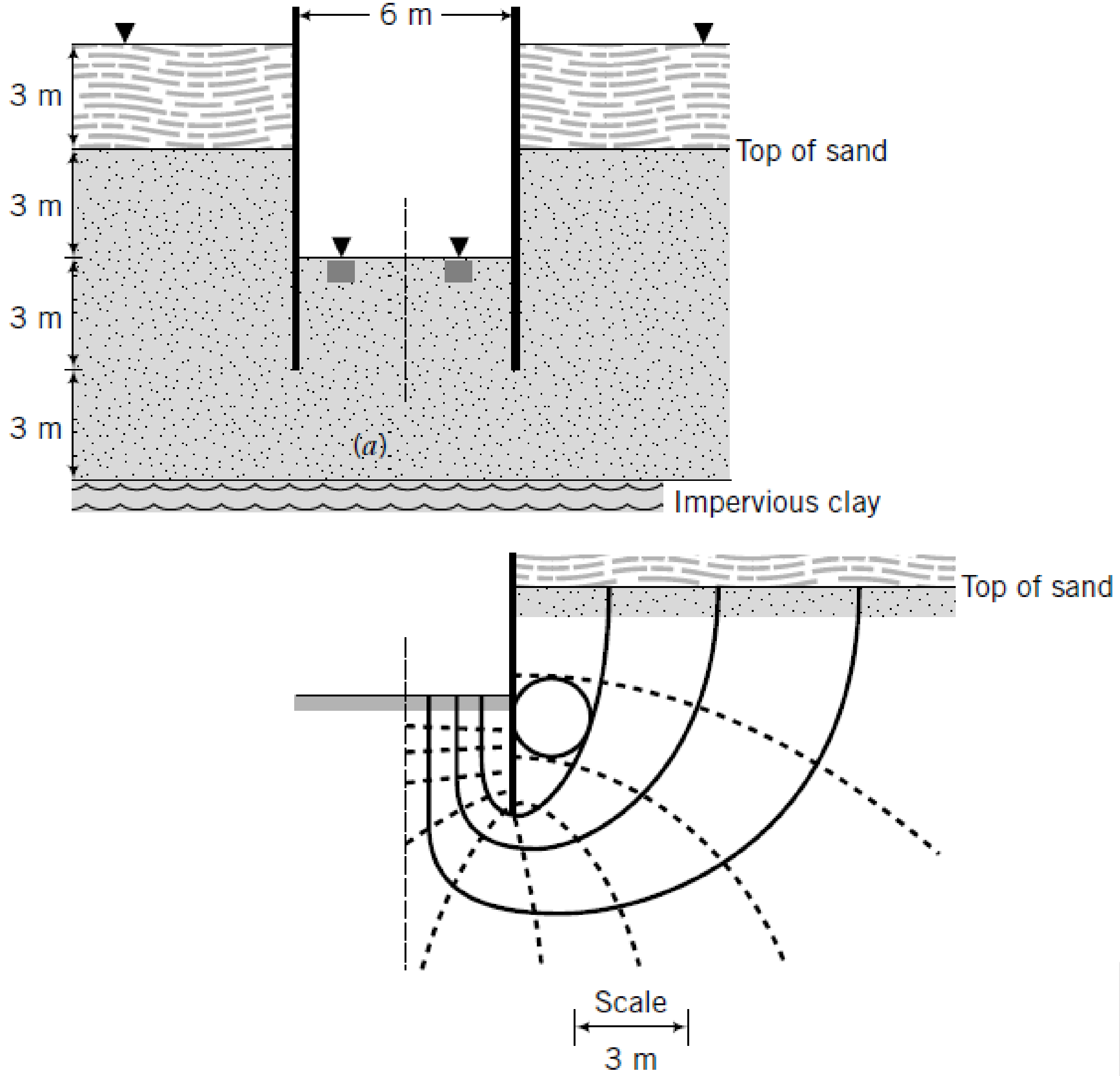
Determine

(a) the flow rate using $k = 1 \times 10^{-4}\ cm/s$ and

(b) the factor of safety against piping.

The void ratio of the sand is 0.59. There was a long delay before construction began, and a 100-mm layer of silty clay with $k = 1 \times 10^{-6}\ cm/s$ was deposited at the site. What effect would this silty clay layer have on the factor of safety against piping?

EXERCISE 4.3.4



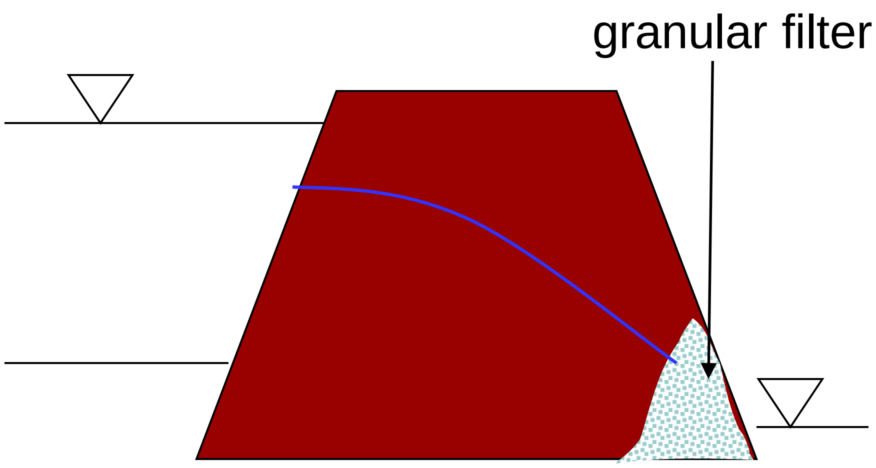
Seepage cntd

Filters

- used for facilitating drainage and preventing fines from being washed away.
- used in earth dams and retaining walls.

<u>Filter Materials:</u>

- Granular soils
- Geotextiles



<u>Two major criteria:</u>

- Retention
- Permeability

Seepage cntd

Retention Criteria

- to prevent washing out of fines

∴ Filter grains must not be too coarse

$D_{15,\ filter} < 5\ D_{85,\ soil}$

Permeability Criteria

- to facilitate drainage & thus avoid build-up of pore pressures

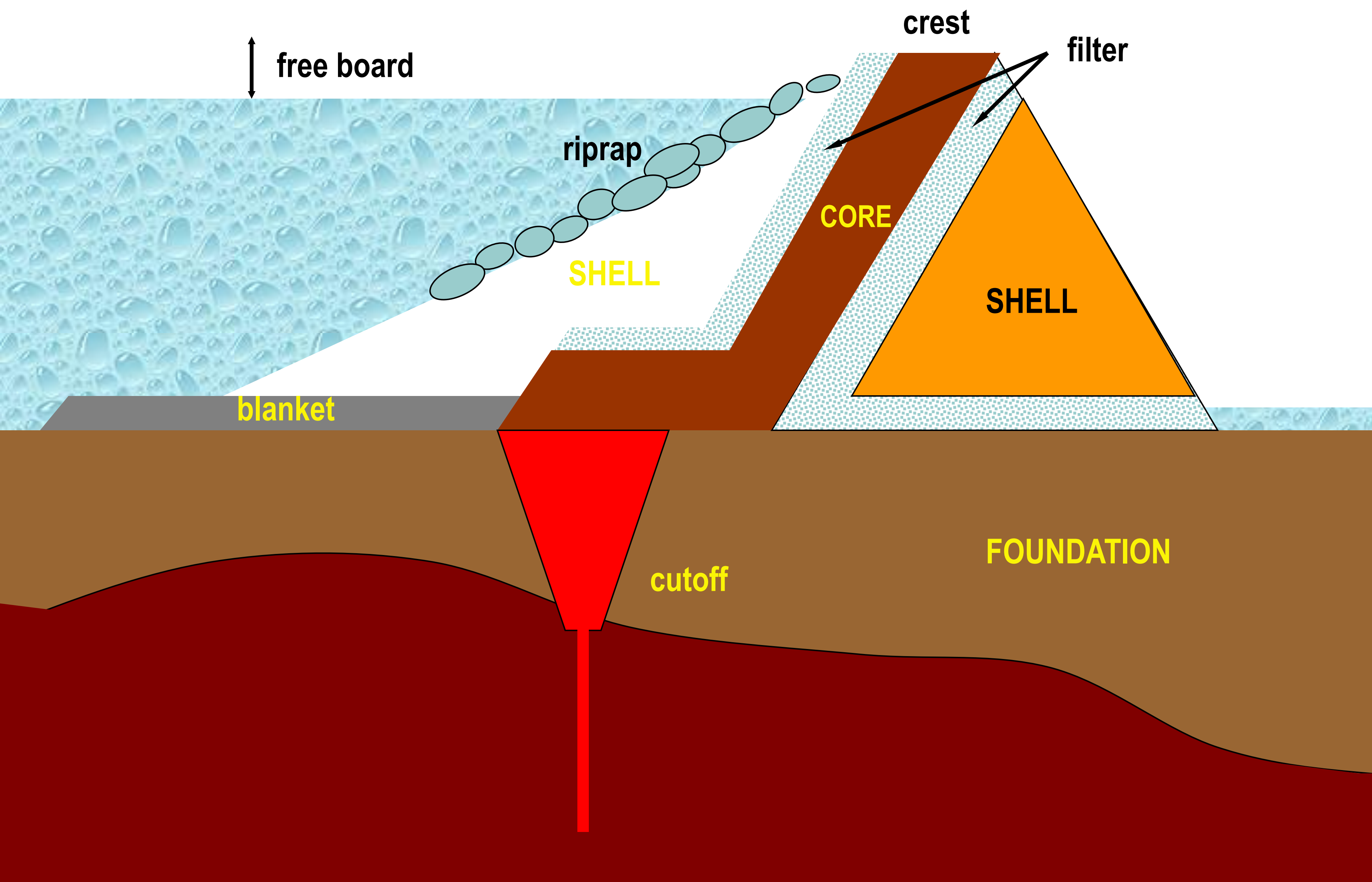
∴ Filter grains must not be too fine

$D_{15,\ filter} > 4\ D_{15,\ soil}$

average filter pore size

GSD Curves for the soil and filter must be parallel

Large Earth Dam



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Galatoma!