The diamond pattern framework of this high-rise building is used to resist loadings due to earthquake or wind.

1 Types of Structures and Loads

This chapter provides a discussion of some of the preliminary aspects of structural analysis. The phases of activity necessary to produce a structure are presented first, followed by an introduction to the basic types of structures, their components, and supports. Finally, a brief explanation is given of the various types of loads that must be considered for an appropriate analysis and design.

1–1 Introduction

A *structure* refers to a system of connected parts used to support a load. Important examples related to civil engineering include buildings, bridges, and towers; and in other branches of engineering, ship and aircraft frames, tanks, pressure vessels, mechanical systems, and electrical supporting structures are important.

When designing a structure to serve a specified function for public use, the engineer must account for its safety, esthetics, and serviceability, while taking into consideration economic and environmental constraints. Often this requires several independent studies of different solutions before final judgment can be made as to which structural form is most appropriate. This design process is both creative and technical and requires a fundamental knowledge of material properties and the laws of mechanics which govern material response. Once a preliminary design of a structure is proposed, the structure must then be *analyzed* to ensure that it has its required strength and rigidity. To analyze a structure properly, certain idealizations must be made as to how the members are supported and connected together. The loadings are determined from codes and local specifications, and the forces in the members and their displacements are found using the theory of structural analysis, which is the subject matter of this text. The results of this analysis then can be used to redesign the structure, accounting for a more accurate determination of the weight of the members and their size. Structural design, therefore, follows a series of successive approximations in which every cycle requires a structural analysis. In this book, the structural analysis is applied to civil engineering structures; however, the method of analysis described can also be used for structures related to other fields of engineering.

1–2 Classification of Structures

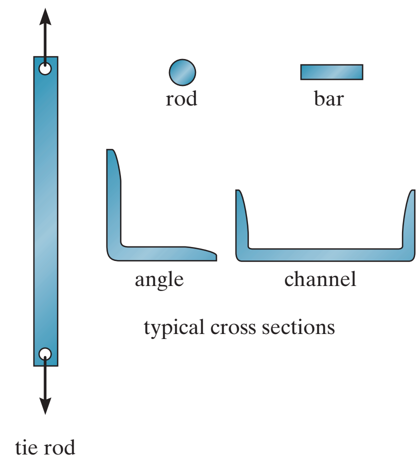


Fig. 1–1

It is important for a structural engineer to recognize the various types of elements composing a structure and to be able to classify structures as to their form and function. We will introduce some of these aspects now and expand on them at appropriate points throughout the text.

Structural Elements. Some of the more common elements from which structures are composed are as follows.

Tie Rods. Structural members subjected to a *tensile force* are often referred to as *tie rods* or *bracing struts.* Due to the nature of this load, these members are rather slender, and are often chosen from rods, bars, angles, or channels, Fig. 1–1.

Beams. Beams are usually straight horizontal members used primarily to carry vertical loads. Quite often they are classified according to the way they are supported, as indicated in Fig. 1–2. In particular, when the cross section varies the beam is referred to as tapered or haunched. Beam cross sections may also be "built up" by adding plates to their top and bottom.

Beams are primarily designed to resist bending moment; however, if they are short and carry large loads, the internal shear force may become quite large and this force may govern their design. When the material used for a beam is a metal such as steel or aluminum, the cross section is most efficient when it is shaped as shown in Fig. 1–3. Here the forces developed in the top and bottom *flanges* of the beam form the necessary couple used to resist the applied moment **M**, whereas the *web* is effective in resisting the applied shear **V**. This cross section is commonly referred to as a "wide flange," and it is normally formed as a single unit in a rolling mill in lengths up to 75 ft (23 m). If shorter lengths are needed, a cross section having tapered flanges is sometimes selected. When the beam is required to have a very large span and the loads applied are rather large, the cross section may take the form of a *plate girder.* This member is fabricated by using a large plate for the web and welding or bolting plates to its ends for flanges. The girder is often transported to the field in segments, and the segments are

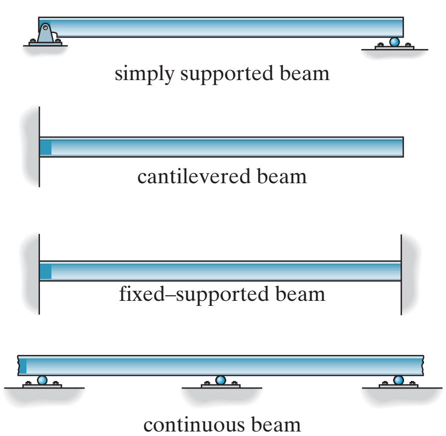


Fig. 1–2

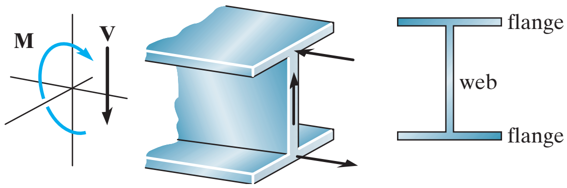


Fig. 1–3

The prestressed concrete girders are simply supported and are used for this highway bridge. Also, note the tapered beams used to support these girders.

designed to be spliced or joined together at points where the girder carries a small internal moment.

Concrete beams generally have rectangular cross sections, since it is easy to construct this form directly in the field. Because concrete is rather weak in resisting tension, steel "reinforcing rods" are cast into the beam within regions of the cross section subjected to tension. Precast concrete beams or girders are fabricated at a shop or yard in the same manner and then transported to the job site.

Beams made from timber may be sawn from a solid piece of wood or laminated. *Laminated* beams are constructed from solid sections of wood, which are fastened together using high-strength glues.

Shown are typical splice plate joints used to connect the steel girders of a highway bridge.

The steel reinforcement cage shown on the right and left is used to resist any tension that may develop in the concrete beams which will be formed around it.

Wide-flange members are often used for columns. Here is an example of a beam column.

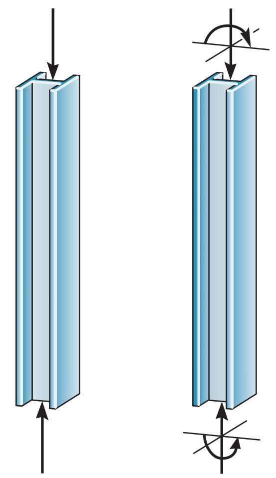


Fig. 1–4

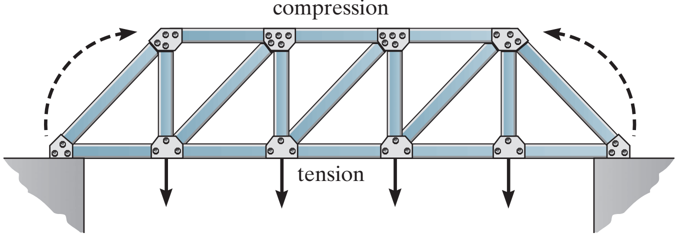
Columns. Members that are generally vertical and resist axial compressive loads are referred to as *columns,* Fig. 1–4. Tubes and wide-flange cross sections are often used for metal columns, and circular and square cross sections with reinforcing rods are used for those made of concrete. Occasionally, columns are subjected to both an axial load and a bending moment as shown in the figure. These members are referred to as *beam columns.*

Types of Structures.

The combination of structural elements and the materials from which they are composed is referred to as a *structural system.* Each system is constructed of one or more of four basic types of structures. Ranked in order of complexity of their force analysis, they are as follows.

Trusses. When the span of a structure is required to be large and its depth is not an important criterion for design, a truss may be selected. *Trusses* consist of slender elements, usually arranged in triangular fashion. *Planar trusses* are composed of members that lie in the same plane and are frequently used for bridge and roof support, whereas *space trusses* have members extending in three dimensions and are suitable for derricks and towers.

Due to the geometric arrangement of its members, loads that cause the entire truss to bend are converted into tensile or compressive forces in the members. Because of this, one of the primary advantages of a truss, compared to a beam, is that it uses less material to support a given



Loading causes bending of truss, which develops compression in top members, tension in bottom members

Fig. 1–5

load, Fig. 1–5. Also, a truss is constructed from *long and slender elements,* which can be arranged in various ways to support a load. Most often it is economically feasible to use a truss to cover spans ranging from 30 ft (9 m) to 400 ft (122 m), although trusses have been used on occasion for spans of greater lengths.

Cables and Arches. Two other forms of structures used to span long distances are the cable and the arch. *Cables* are usually flexible and carry their loads in tension. Unlike tension ties, however, the external load is not applied along the axis of the cable, and consequently the cable takes a form that has a defined sag, Fig. 1–6*a*. Cables are commonly used to support bridges and building roofs. When used for these purposes, the cable has an advantage over the beam and the truss, especially for spans that are greater than 150 ft (46 m). Because they are always in tension, cables will not become unstable and suddenly collapse, as may happen with beams or trusses. Furthermore, the truss will require added costs for construction and increased depth as the span increases. Use of cables, on the other hand, is limited only by their sag, weight, and methods of anchorage.

The *arch* achieves its strength in compression, since it has a reverse curvature to that of the cable, Fig. 1–6*b*. The arch must be rigid, however, in order to maintain its shape, and this results in secondary loadings involving shear and moment, which must be considered in its design. Arches are frequently used in bridge structures, dome roofs, and for openings in masonry walls.

cables support their loads in tension
(a)

arches support their loads in compression
(b)

Fig. 1–6

Here is an example of a steel frame that is used to support a crane rail. The frame can be assumed fixed connected at its top joints and pinned at the supports.

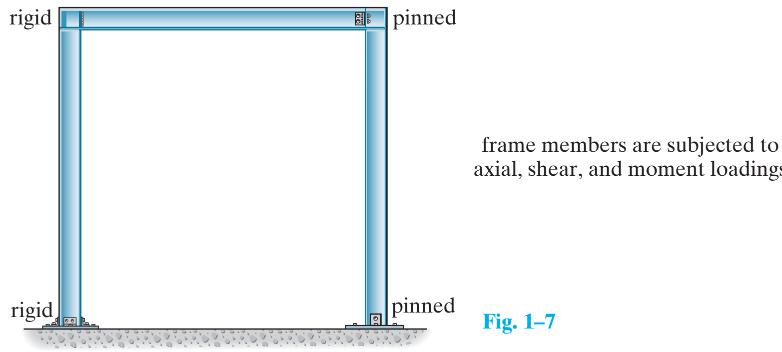


Fig. 1–7

Frames. Frames are often used in buildings and are composed of beams and columns that are either pin or fixed connected, Fig. 1–7. Like trusses, frames extend in two or three dimensions. The loading on a frame causes bending of its members, and if it has rigid joint connections, this structure is generally "indeterminate" from a standpoint of analysis. The strength of such a frame is derived from the moment interactions between the beams and the columns at the rigid joints. As a result, the economic benefits of using a frame depend on the efficiency gained in using smaller beam sizes versus increasing the size of the columns due to the "beam-column" action caused by bending at the joints.

Surface Structures. A *surface structure* is made from a material having a very small thickness compared to its other dimensions. Sometimes this material is very flexible and can take the form of a tent or air-inflated structure. In both cases the material acts as a membrane that is subjected to pure tension.

Surface structures may also be made of rigid material such as reinforced concrete. As such they may be shaped as folded plates, cylinders, or hyperbolic paraboloids, and are referred to as *thin plates* or *shells.* These structures act like cables or arches since they support loads primarily in tension or compression, with very little bending. In spite of this, plate or shell structures are generally very difficult to analyze, due to the three-dimensional geometry of their surface. Such an analysis is beyond the scope of this text and is instead covered in texts devoted entirely to this subject.

The roof of the "Georgia Dome" in Atlanta, Georgia can be considered as a thin membrane.

1–3 Loads

Once the dimensional requirements for a structure have been defined, it becomes necessary to determine the loads the structure must support. Often, it is the anticipation of the various loads that will be imposed on the structure that provides the basic type of structure that will be chosen for design. For example, high-rise structures must endure large lateral loadings caused by wind, and so shear walls and tubular frame systems are selected, whereas buildings located in areas prone to earthquakes must be designed having ductile frames and connections.

Once the structural form has been determined, the actual design begins with those elements that are subjected to the primary loads the structure is intended to carry, and proceeds in sequence to the various supporting members until the foundation is reached. Thus, a building floor slab would be designed first, followed by the supporting beams, columns, and last, the foundation footings. In order to design a structure, it is therefore necessary to first specify the loads that act on it.

The design loading for a structure is often specified in codes. In general, the structural engineer works with two types of codes: general building codes and design codes. *General building codes* specify the requirements of governmental bodies for minimum design loads on structures and minimum standards for construction. *Design codes* provide detailed technical standards and are used to establish the requirements for the actual structural design. Table 1–1 lists some of the important codes used in practice. It should be realized, however, that codes provide only a general guide for design. *The ultimate responsibility for the design lies with the structural engineer.*

TABLE 1–1 • Codes
General Building Codes
Minimum Design Loads for Buildings and Other Structures, SEI/ASCE 7-05, American Society of Civil Engineers *International Building Code*
Design Codes
Building Code Requirements for Reinforced Concrete, Am. Conc. Inst. (ACI) *Manual of Steel Construction,* American Institute of Steel Construction (AISC) *Standard Specifications for Highway Bridges,* American Association of State Highway and Transportation Officials (AASHTO) *National Design Specification for Wood Construction*, American Forest and Paper Association (AFPA) *Manual for Railway Engineering,* American Railway Engineering Association (AREA)

Since a structure is generally subjected to several types of loads, a brief discussion of these loadings will now be presented to illustrate how one must consider their effects in practice.

Dead Loads. *Dead loads* consist of the weights of the various structural members and the weights of any objects that are permanently attached to the structure. Hence, for a building, the dead loads include the weights of the columns, beams, and girders, the floor slab, roofing, walls, windows, plumbing, electrical fixtures, and other miscellaneous attachments.

In some cases, a structural dead load can be estimated satisfactorily from simple formulas based on the weights and sizes of similar structures. Through experience one can also derive a "feeling" for the magnitude of these loadings. For example, the average weight for timber buildings is $40–50\ \text{lb/ft}^2$ $(1.9–2.4\ \text{kN/m}^2)$, for steel framed buildings it is $60–75\ \text{lb/ft}^2$ $(2.9–3.6\ \text{kN/m}^2)$, and for reinforced concrete buildings it is $110–130\ \text{lb/ft}^2$ $(5.3–6.2\ \text{kN/m}^2)$. Ordinarily, though, once the materials and sizes of the various components of the structure are determined, their weights can be found from tables that list their densities.

The densities of typical materials used in construction are listed in Table 1–2, and a portion of a table listing the weights of typical building components is given in Table 1–3. Although calculation of dead loads

TABLE 1–2 • Minimum Densities for Design Loads from Materials*

	lb/ft^3	kN/m^3
Aluminum	170	26.7
Concrete, plain cinder	108	17.0
Concrete, plain stone	144	22.6
Concrete, reinforced cinder	111	17.4
Concrete, reinforced stone	150	23.6
Clay, dry	63	9.9
Clay, damp	110	17.3
Sand and gravel, dry, loose	100	15.7
Sand and gravel, wet	120	18.9
Masonry, lightweight solid concrete	105	16.5
Masonry, normal weight	135	21.2
Plywood	36	5.7
Steel, cold-drawn	492	77.3
Wood, Douglas Fir	34	5.3
Wood, Southern Pine	37	5.8
Wood, spruce	29	4.5

*Reproduced with permission from American Society of Civil Engineers *Minimum Design Loads for Buildings and Other Structures,* SEI/ASCE 7-05. Copies of this standard may be purchased from ASCE at 345 East 47th Street, New York, N.Y. 10017-2398.

TABLE 1–3 • Minimum Design Dead Loads*

Walls	psf	kN/m^2
4-in. (102 mm) clay brick	39	1.87
8-in. (203 mm) clay brick	79	3.78
12-in. (305 mm) clay brick	115	5.51
Frame Partitions and Walls		
Exterior stud walls with brick veneer	48	2.30
Windows, glass, frame and sash	8	0.38
Wood studs 2 × 4 in., (51 × 102 mm) unplastered	4	0.19
Wood studs 2 × 4 in., (51 × 102 mm) plastered one side	12	0.57
Wood studs 2 × 4 in., (51 × 102 mm) plastered two sides	20	0.96
Floor Fill		
Cinder concrete, per inch (mm)	9	0.017
Lightweight concrete, plain, per inch (mm)	8	0.015
Stone concrete, per inch (mm)	12	0.023
Ceilings		
Acoustical fiberboard	1	0.05
Plaster on tile or concrete	5	0.24
Suspended metal lath and gypsum plaster	10	0.48
Asphalt shingles	2	0.10
Fiberboard, $\frac{1}{2}$-in. (13 mm)	0.75	0.04

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based on the use of tabulated data is rather straightforward, it should be realized that in many respects these loads will have to be estimated in the initial phase of design. These estimates include nonstructural materials such as prefabricated facade panels, electrical and plumbing systems, etc. Furthermore, even if the material is specified, the unit weights of elements reported in codes may vary from those given by manufactures, and later use of the building may include some changes in dead loading. As a result, estimates of dead loadings can be in error by 15% to 20% or more.

Normally, the dead load is not large compared to the design load for simple structures such as a beam or a single-story frame; however, for multistory buildings it is important to have an accurate accounting of all the dead loads in order to properly design the columns, especially for the lower floors.

EXAMPLE 1-1

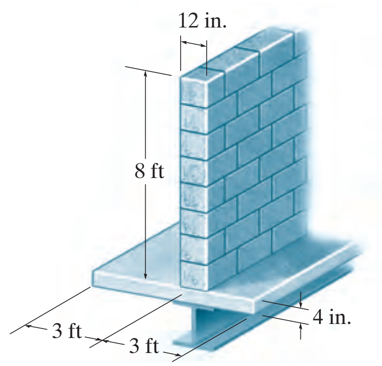


Fig. 1–8

The floor beam in Fig. 1–8 is used to support the 6-ft width of a lightweight plain concrete slab having a thickness of 4 in. The slab serves as a portion of the ceiling for the floor below, and therefore its bottom is coated with plaster. Furthermore, an 8-ft-high, 12-in.-thick lightweight solid concrete block wall is directly over the top flange of the beam. Determine the loading on the beam measured per foot of length of the beam.

Solution

Using the data in Tables 1–2 and 1–3, we have

Concrete slab:	$[8\text{ lb}/(\text{ft}^2\cdot\text{in.})](4\text{ in.})(6\text{ ft}) =$	192 lb/ft
Plaster ceiling:	$(5\text{ lb}/\text{ft}^2)(6\text{ ft}) =$	30 lb/ft
Block wall:	$(105\text{ lb}/\text{ft}^3)(8\text{ ft})(1\text{ ft}) =$	840 lb/ft
Total load		1062 lb/ft = 1.06 k/ft ***Ans.***

Here the unit k stands for "kip," which symbolizes kilopounds. Hence, 1 k = 1000 lb.

Live Loads. *Live Loads* can vary both in their magnitude and location. They may be caused by the weights of objects temporarily placed on a structure, moving vehicles, or natural forces. The minimum live loads specified in codes are determined from studying the history of their effects on existing structures. Usually, these loads include additional protection against excessive deflection or sudden overload. In Chapter 6 we will develop techniques for specifying the proper location of live loads on the structure so that they cause the greatest stress or deflection of the members. Various types of live loads will now be discussed.

Building Loads. The floors of buildings are assumed to be subjected to *uniform live loads,* which depend on the purpose for which the building is designed. These loadings are generally tabulated in local, state, or national codes. A representative sample of such *minimum live loadings,* taken from the ASCE 7-05 Standard, is shown in Table 1–4. The values are determined from a history of loading various buildings. They include some protection against the possibility of overload due to emergency situations, construction loads, and serviceability requirements due to vibration. In addition to uniform loads, some codes specify *minimum concentrated live loads,* caused by hand carts, automobiles, etc., which must also be applied anywhere to the floor system. For example, both uniform and concentrated live loads must be considered in the design of an automobile parking deck.

TABLE 1–4 • Minimum Live Loads*

Occupancy or Use	Live Load psf	Live Load kN/m²	Occupancy or Use	Live Load psf	Live Load kN/m²
Assembly areas and theaters			Residential		
Fixed seats	60	2.87	Dwellings (one- and two-family)	40	1.92
Movable seats	100	4.79	Hotels and multifamily houses		
Dance halls and ballrooms	100	4.79	Private rooms and corridors	40	1.92
Garages (passenger cars only)	50	2.40	Public rooms and corridors	100	4.79
Office buildings			Schools		
Lobbies	100	4.79	Classrooms	40	1.92
Offices	50	2.40	Corridors above first floor	80	3.83
Storage warehouse					
Light	125	6.00			
Heavy	250	11.97			

*Reproduced with permission from *Minimum Design Loads for Buildings and Other Structures*, ASCE 7-05.

For some types of buildings having very large floor areas, many codes will allow a *reduction* in the uniform live load for a *floor*, since it is unlikely that the prescribed live load will occur simultaneously throughout the entire structure at any one time. For example, ASCE 7-05 allows a reduction of live load on a member having an *influence area* $(K_{LL}A_T)$ of 400 ft^2 (37.2 m^2) or more. This reduced live load is calculated using the following equation:

$$L = L_o\left(0.25 + \frac{15}{\sqrt{K_{LL}A_T}}\right) \quad \text{(FPS units)}$$

$$L = L_o\left(0.25 + \frac{4.57}{\sqrt{K_{LL}A_T}}\right) \quad \text{(SI units)} \qquad (1\text{–}1)$$

where

L = reduced design live load per square foot or square meter of area supported by the member.

L_o = unreduced design live load per square foot or square meter of area supported by the member (see Table 1–4).

K_{LL} = live load element factor. For interior columns $K_{LL} = 4$.

A_T = tributary area in square feet or square meters.*

The reduced live load defined by Eq. 1–1 is limited to not less than 50% of L_o for members supporting one floor, or not less than 40% of L_o for members supporting more than one floor. No reduction is allowed for loads exceeding 100 lb/ft^2 (4.79 kN/m^2), or for structures used for public assembly, garages, or roofs. Example 1–2 illustrates Eq. 1–1's application.

Shown is a typical example of an interior building column. The live load office floor loading it supports can be reduced for purposes of design and analysis.

*Specific examples of the determination of tributary areas for beams and columns are given in Sec. 2–1.

EXAMPLE 1-2

A two-story office building has interior columns that are spaced 22 ft apart in two perpendicular directions. If the (flat) roof loading is $20\ \text{lb/ft}^2$, determine the reduced live load supported by a typical interior column located at ground level.

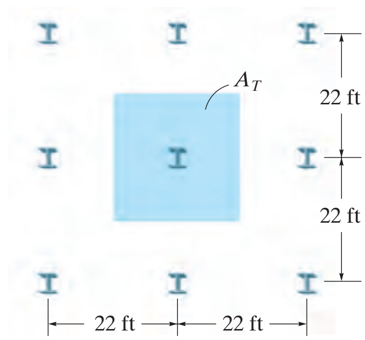


Fig. 1–9

Solution

As shown in Fig. 1–9, each interior column has a tributary area or effective loaded area of $A_T = (22\ \text{ft})(22\ \text{ft}) = 484\ \text{ft}^2$. A ground-floor column therefore supports a roof live load of

$$F_R = (20\ \text{lb/ft}^2)(484\ \text{ft}^2) = 9680\ \text{lb} = 9.68\ \text{k}$$

This load cannot be reduced, since it is not a floor load. For the second floor, the live load is taken from Table 1–4: $L_o = 50\ \text{lb/ft}^2$. Since $K_{LL} = 4$, then $4A_T = 4(484\ \text{ft}^2) = 1936\ \text{ft}^2$ and $1936\ \text{ft}^2 > 400\ \text{ft}^2$, the live load can be reduced using Eq. 1–1. Thus,

$$L = 50\left(0.25 + \frac{15}{\sqrt{1936}}\right) = 29.55\ \text{lb/ft}^2$$

The load reduction here is $(29.55/50)100\% = 59.1\% > 50\%$. O.K. Therefore

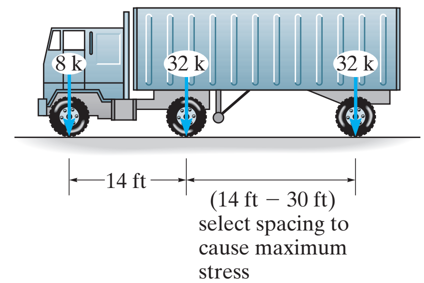
$$F_F = (29.55\ \text{lb/ft}^2)(484\ \text{ft}^2) = 14\,300\ \text{lb} = 14.3\ \text{k}$$

The total live load supported by the ground-floor column is thus

$$F = F_R + F_F = 9.68\ \text{k} + 14.3\ \text{k} = 24.0\ \text{k}$$ ***Ans.***

Highway Bridge Loads. The primary live loads on bridge spans are those due to traffic, and the heaviest vehicle loading encountered is that caused by a series of trucks. Specifications for truck loadings on highway bridges are reported in the *LRFD Bridge Design Specifications* of the American Association of State and Highway Transportation Officials (AASHTO). For two-axle trucks, these loads are designated with an H, followed by the weight of the truck in tons and another number which gives the year of the specifications in which the load was reported. For example, an H 15-44 is a 15-ton truck as reported in the 1944 specifications. H-series truck weights vary from 10 to 20 tons. However, bridges located on major highways, which carry a great deal of traffic, are often designed for two-axle trucks plus a one-axle semitrailer. These are designated as HS loadings, for example, HS 20-44. In general, a truck loading selected for design depends upon the type of bridge, its location, and the type of traffic anticipated.

The size of the "standard truck" and the distribution of its weight is also reported in the specifications. For example, the HS 20-44 loading is shown in Fig. 1–10. Although trucks are assumed to occupy 10-ft lanes, all lanes on the bridge need not be fully loaded with a row of trucks to obtain the critical load, since such a loading would be highly improbable. The details are discussed in Chapter 6.

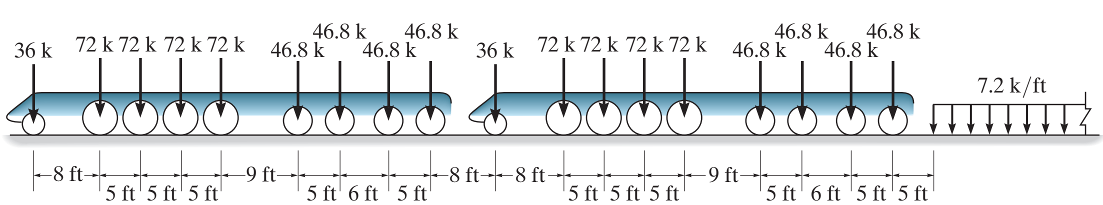


HS 20–44 loading

Fig. 1–10

The girders of this railroad bridge were designed to meet AREA specifications.

Railroad Bridge Loads. The loadings on railroad bridges are specified in the *Specifications for Steel Railway Bridges* published by the American Railroad Engineers Association (AREA). Normally, E loads, as originally devised by Theodore Cooper in 1894, are used for design. For example, a modern train having a 72-k loading on the driving axle of the engine is designated as an E-72 loading. The entire E-72 loading for design is distributed as shown in Fig. 1–11. D. B. Steinmann has since updated Cooper's load distribution and has devised a series of M loadings, which are also acceptable for design. Since train loadings involve a complicated series of concentrated forces, to simplify hand calculations, tables and graphs are sometimes used in conjunction with influence lines to obtain the critical load.



E-72 loading

Fig. 1–11

Impact Loads. Moving vehicles may bounce or sidesway as they move over a bridge, and therefore they impart an *impact* to the deck. The percentage increase of the live loads due to impact is called the *impact factor, I*. This factor is generally obtained from formulas developed from experimental evidence. For example, for highway bridges the AASHTO specifications require that

$$I = \frac{50}{L + 125} \qquad \text{but not larger than 0.3}$$

where L is the length of the span in feet that is subjected to the live load.

In some cases provisions for impact loading on the structure of a building must also be taken into account. For example, the ASCE 7-05 Standard requires the weight of elevator machinery to be increased by 100%, and the loads on any hangers used to support floors and balconies to be increased by 33%.

Hurricane winds caused this damage to a condominium in Miami, Florida.

Wind Loads. When structures block the flow of wind, the wind's kinetic energy is converted into potential energy of pressure, which causes a wind loading. The effect of wind on a structure depends upon the density and velocity of the air, the angle of incidence of the wind, the shape and stiffness of the structure, and the roughness of its surface. For design purposes, wind loadings can be treated using either a static or a dynamic approach.

For the static approach, the fluctuating pressure caused by a constantly blowing wind is approximated by a mean velocity pressure that acts on the structure. This pressure q is defined by its kinetic energy, $q = \frac{1}{2}\rho V^2$, where ρ is the density of the air and V is its velocity. According to the ASCE 7-05 Standard, this equation is modified to account for the importance of the structure, its height, and the terrain in which it is located. It is represented as

$$q_z = 0.00256 K_z K_{zt} K_d V^2 I \ (\text{lb/ft}^2)$$
$$q_z = 0.613 K_z K_{zt} K_d V^2 I \ (\text{N/m}^2) \qquad (1\text{–}2)$$

where,

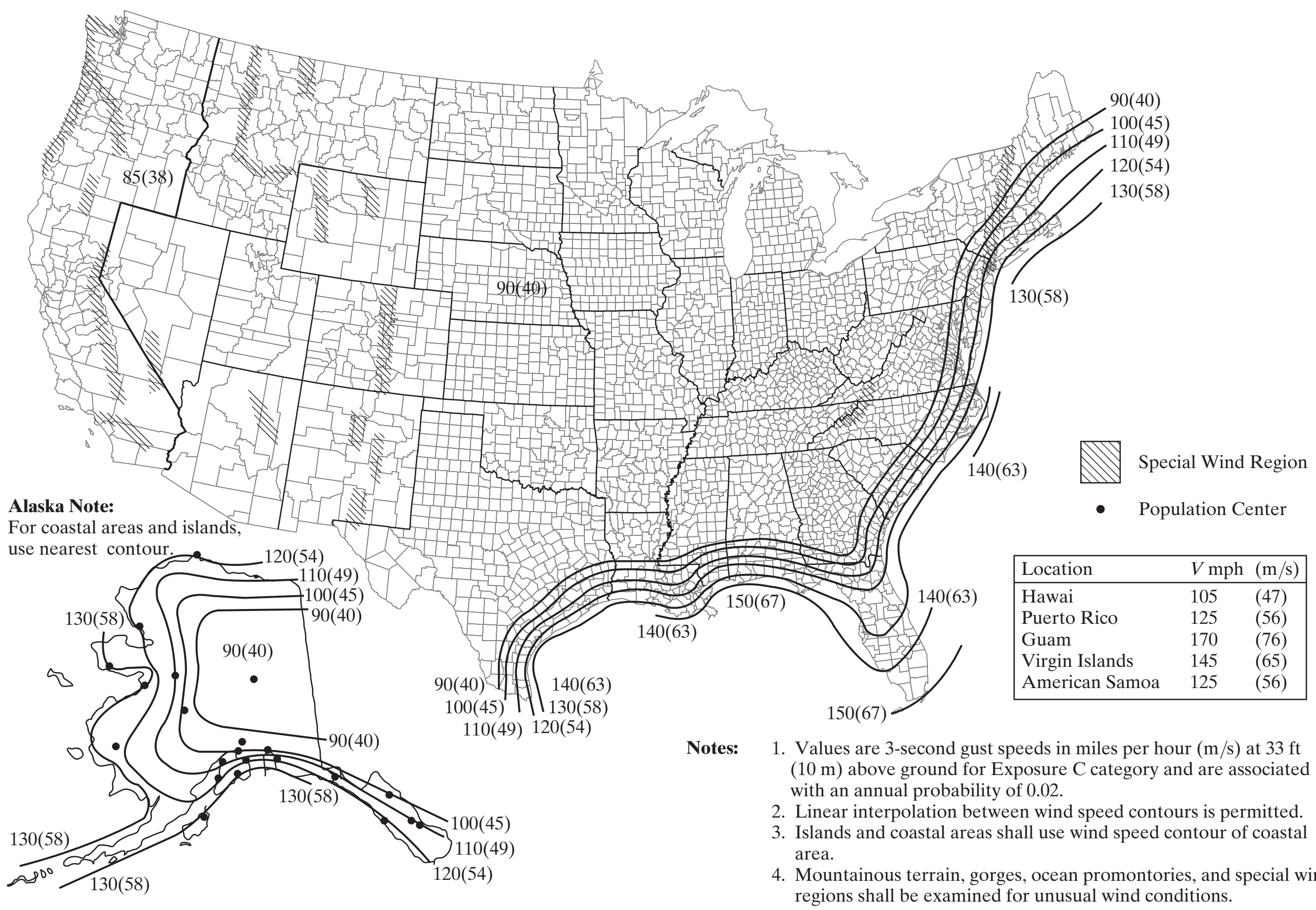
V = the velocity in mi/h (m/s) of a 3-second gust of wind measured 33 ft (10 m) above the ground during a 50-year recurrence period. Values are obtained from a wind map, shown in Fig. 1–12.

I = the importance factor that depends upon the nature of the building occupancy; for example, for buildings with a low hazard to human life, such as agriculture facilities in a non-hurricane prone region, $I = 0.87$, but for hospitals, $I = 1.15$.

K_z = the velocity pressure exposure coefficient, which is a function of height and depends upon the ground terrain. Table 1–5 lists values for a structure which is located in open terrain with scattered low-lying obstructions.

K_{zt} = a factor that accounts for wind speed increases due to hills and escarpments. For flat ground $K_{zt} = 1$.

K_d = a factor that accounts for the direction of the wind. It is used only when the structure is subjected to combinations of loads (see Sec. 1–4). For wind acting alone, $K_d = 1$.



Location	V mph	(m/s)
Hawai	105	(47)
Puerto Rico	125	(56)
Guam	170	(76)
Virgin Islands	145	(65)
American Samoa	125	(56)

Notes:
1. Values are 3-second gust speeds in miles per hour (m/s) at 33 ft (10 m) above ground for Exposure C category and are associated with an annual probability of 0.02.
2. Linear interpolation between wind speed contours is permitted.
3. Islands and coastal areas shall use wind speed contour of coastal area.
4. Mountainous terrain, gorges, ocean promontories, and special wind regions shall be examined for unusual wind conditions.

Fig. 1–12

TABLE 1–5 • Velocity Pressure Exposure Coefficient for Terrain with Low-Lying Obstructions

z ft	z m	K_z
0–15	0–4.6	0.85
20	6.1	0.90
25	7.6	0.94
30	9.1	0.98
40	12.2	1.04
50	15.2	1.09

Design Wind Pressure for Enclosed Buildings. Once the value for q_z is obtained, the design pressure can be determined from a list of relevant equations listed in the ASCE 7-05 Standard. The choice depends upon the flexibility and height of the structure, and whether the design is for the main wind-force resisting system, or for the building's components and cladding. For example, for a conservative design *wind-pressure* on nonflexible buildings of any height is determined using a two-termed equation resulting from both external and internal pressures, namely,

$$p = qGC_p - q_h(GC_{pi}) \qquad (1\text{–}3)$$

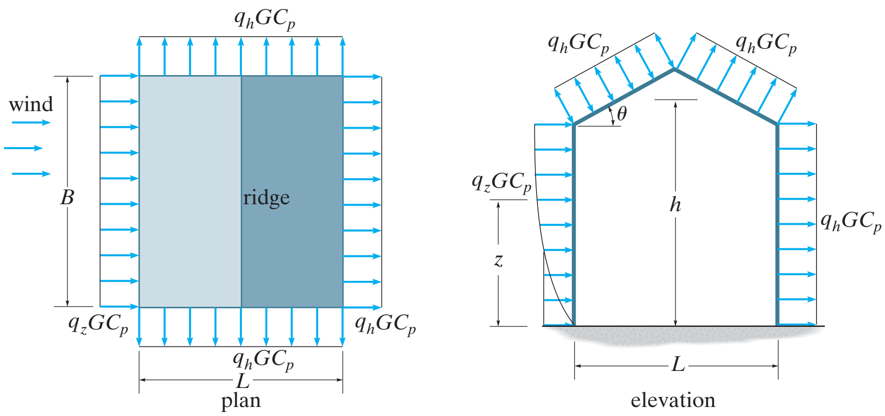
Here

$q = q_z$ for the windward wall at height z above the ground (Eq. 1–2), and $q = q_h$ for the leeward walls, side walls, and roof, where $z = h$, the mean height of the roof.

$G =$ a wind-gust effect factor, which depends upon the exposure. For example, for a rigid structure, $G = 0.85$.

$C_p =$ a wall or roof pressure coefficient determined from a table. These tabular values for the walls and a roof pitch of $\theta = 10°$ are given in Fig. 1–13. Note in the elevation view that the pressure will vary with height on the windward side of the building, whereas on the remaining sides and on the roof the pressure is assumed to be constant. Negative values indicate pressures acting away from the surface.

$(GC_{pi}) =$ the internal pressure coefficient which depends upon the type of openings in the building. For fully enclosed buildings $(GC_{pi}) = \pm 0.18$. Here the signs indicate that either positive or negative (suction) pressure can occur within the building.



Surface	L/B	C_p	Use with
Windward wall	All values	0.8	q_z
Leeward wall	0–1 2 ≥4	−0.5 −0.3 −0.2	q_h
Side walls	All values	−0.7	q_h

Wall pressure coefficients, C_p

(a)

Wind direction	Windward angle θ		Leeward angle
	h/L	10°	$\theta = 10°$
Normal to ridge	≤0.25 0.5 >1.0	−0.7 −0.9 −1.3	−0.3 −0.5 −0.7

Maximum negative roof pressure coefficients, C_p, for use with q_h

(b)

Fig. 1–13

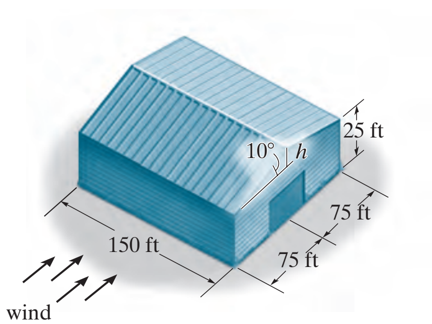
Application of Eq. 1–3 will involve calculations of wind pressures from each side of the building, with due considerations for the possibility of either positive or negative pressures acting on the building's interior.

To allow for normal and oblique wind directions, this resultant force is assumed to act either through the geometric center of the face of the sign or from a vertical line passing through the geometric center a distance of 0.2 times the average width of the sign.

For high-rise buildings or those having a shape or location that makes them wind sensitive, it is recommended that a *dynamic approach* be used to determine the wind loadings. The methodology for doing this is also outlined in the ASCE 7-05 Standard. It requires wind-tunnel tests to be performed on a scale model of the building and those surrounding it, in order to simulate the natural environment. The pressure effects of the wind on the building can be determined from pressure transducers attached to the model. Also, if the model has stiffness characteristics that are in proper scale to the building, then the dynamic deflections of the building can be determined.

EXAMPLE 1-3

The enclosed building shown in Fig. 1–14*a* is used for agricultural purposes and is located outside of Chicago, Illinois on flat terrain. When the wind is directed as shown, determine the design wind pressure acting on the roof and sides of the building using the ASCE 7-05 Specifications.



(a)

Fig. 1–14

Solution

First the velocity pressure will be determined using Eq. 1–2. From Fig. 1–12, the basic wind speed is $V = 90$ mi/h, and since the building is used for agricultural purposes, the importance factor is $I = 0.87$. Also, for flat terrain, $K_{zt} = 1$. Since only wind loading is being considered, $K_d = 1$. Therefore,

$$
\begin{aligned}
q_z &= 0.00256\, K_z K_{zt} K_d V^2 I \\
&= 0.00256\, K_z(1)(1)(90)^2(0.87) \\
&= 18.04\, K_z
\end{aligned}
$$

From Fig. 1–14*a*, $h' = 75 \tan 10° = 13.22$ ft so that the mean height of the roof is $h = 25 + 13.22/2 = 31.6$ ft. Using the values of K_z in Table 1–5, calculated values of the pressure profile are listed in the table in Fig. 1–14*b*. Note the value of K_z was determined by linear interpolation for $z = h$, i.e., $(1.04 - 0.98)/(40 - 30) = (1.04 - K_z)/(40 - 31.6)$, $K_z = 0.990$, and so $q_h = 18.04(0.990) = 17.9$ psf.

In order to apply Eq. 1–3 the gust factor is $G = 0.85$, and $(GC_{pi}) = \pm 0.18$. Thus,

$$
\begin{aligned}
p &= qGC_p - q_h(GC_{pi}) \\
&= q(0.85)C_p - 17.9(\pm 0.18) \\
&= 0.85qC_p \mp 3.21 \qquad (1)
\end{aligned}
$$

The pressure loadings are obtained from this equation using the calculated values for q_z listed in Fig. 1–14*b* in accordance with the wind-pressure profile in Fig. 1–13.

z (ft)	K_z	q_z (psf)
0–15	0.85	15.3
20	0.90	16.2
25	0.94	17.0
$h = 31.6$	0.990	17.9

(b)

Windward Wall. Here the pressure varies with height z since $q_z GC_p$ must be used. For all values of L/B, $C_p = 0.8$, so that from Eq. (1),

$$p_{0-15} = 7.19 \text{ psf} \quad \text{or} \quad 13.6 \text{ psf}$$
$$p_{20} = 7.81 \text{ psf} \quad \text{or} \quad 14.2 \text{ psf}$$
$$p_{25} = 8.35 \text{ psf} \quad \text{or} \quad 14.8 \text{ psf}$$

Leeward Wall. Here $L/B = 2(75)/150 = 1$, so that $C_p = -0.5$. Also, $q = q_h$ and so from Eq. (1),

$$p = -10.8 \text{ psf} \quad \text{or} \quad -4.40 \text{ psf}$$

Side Walls. For all values of L/B, $C_p = -0.7$, and therefore since we must use $q = q_h$ in Eq. (1), we have

$$p = -13.9 \text{ psf} \quad \text{or} \quad -7.44 \text{ psf}$$

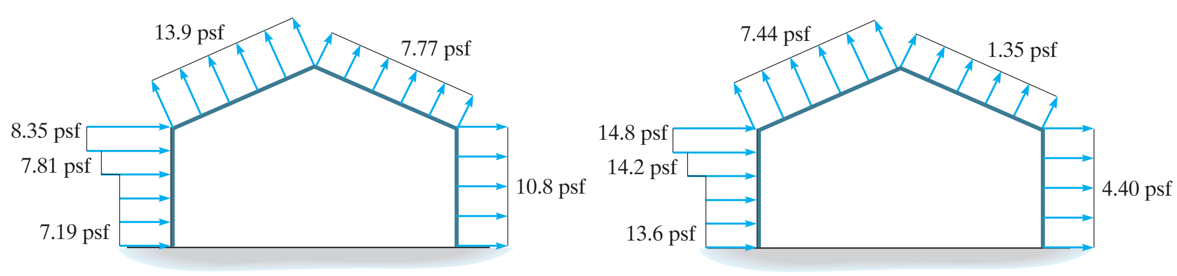
Windward Roof. Here $h/L = 31.6/2(75) = 0.211 < 0.25$, so that $C_p = -0.7$ and $q = q_h$. Thus,

$$p = -13.9 \text{ psf} \quad \text{or} \quad -7.44 \text{ psf}$$

Leeward Roof. In this case $C_p = -0.3$; therefore with $q = q_h$, we get

$$p = -7.77 \text{ psf} \quad \text{or} \quad -1.35 \text{ psf}$$

These two sets of loadings are shown on the elevation of the building, representing either positive or negative (suction) internal building pressure, Fig. 1–14*c*. The main framing structure of the building must resist these loadings as well as loadings calculated from wind blowing on the front or rear of the building.



(c)

Fig. 1–14

Design Wind Pressure for Signs. If the structure represents a sign, the wind will produce a *resultant force* acting on the face of the sign which is determined from

$$F = q_z G C_f A_f \tag{1–4}$$

Here

q_z = the velocity pressure evaluated at the height z of the centroid of A_f.

G = the wind-gust coefficient factor defined previously.

C_f = a force coefficient which depends upon the ratio of the large dimension M of the sign to the small dimension N.* Values are listed in Table 1–6.

A_f = the area of the face of the sign projected into the wind.

TABLE 1–6 • Force Coefficients for Above-Ground Solid Signs, C_f

M/N	C_f
<6	1.2
10	1.3
20	1.5
40	1.75
60	1.85

Snow Loads. In some parts of the country, roof loading due to snow can be quite severe, and therefore protection against possible failure is of primary concern. Design loadings typically depend on the building's general shape and roof geometry, wind exposure, location, its importance, and whether or not it is heated. Like wind, snow loads in the ASCE 7-05 Standard are generally determined from a zone map reporting 50-year recurrence intervals of an extreme snow depth. For example, on the relatively flat elevation throughout the mid-section of Illinois and Indiana, the ground snow loading is 20 lb/ft^2 (0.96 kN/m^2). However, for areas of Montana, specific case studies of ground snow loadings are needed due to the variable elevations throughout the state. Specifications for snow loads are covered in the ASCE 7-05 Standard, although no single code can cover all the implications of this type of loading.

*If the distance from the ground to the bottom edge of the sign is less than 0.25 times the vertical dimension, then consider the sign to extend from the ground level.

Excessive snow and ice loadings act on this roof.

If a roof is flat, defined as having a slope of less than 5%, then the pressure loading on the roof can be obtained by modifying the ground snow loading, p_g, by the following empirical formula

$$p_f = 0.7C_eC_tIp_g \tag{1–5}$$

Here

C_e = an exposure factor which depends upon the terrain. For example, for a fully exposed roof in an unobstructed area, $C_e = 0.8$, whereas if the roof is sheltered and located in the center of a large city, then $C_e = 1.3$.

C_t = a thermal factor which refers to the average temperature within the building. For unheated structures kept below freezing $C_t = 1.2$, whereas if the roof is supporting a normally heated structure, then $C_t = 1.0$.

I = the importance factor as it relates to occupancy. For example, $I = 0.8$ for agriculture and storage facilities, and $I = 1.2$ for hospitals.

If $p_g \leq 20\ \text{lb/ft}^2$ ($0.96\ \text{kN/m}^2$), then use the *largest value* for p_f, either computed from the above equation or from $p_f = Ip_g$. If $p_g > 20\ \text{lb/ft}^2$ ($0.96\ \text{kN/m}^2$), then use $p_f = I(20\ \text{lb/ft}^2)$.

EXAMPLE 1-4

The unheated storage facility shown in Fig. 1–15 is located on flat open terrain near Cario, Illinois, where the ground snow load is 15 lb/ft^2. Determine the design snow load on the roof.

Fig. 1–15

Solution

Since the roof is flat, we will use Eq. 1–5. Here, $C_e = 0.8$ due to the open area, $C_t = 1.2$ and $I = 0.8$. Thus,

$$p_f = 0.7 C_e C_t I p_g$$
$$= 0.7(0.8)(1.2)(0.8)(15 \text{ lb/ft}^2) = 8.06 \text{ lb/ft}^2$$

Since $p_g = 15 \text{ lb/ft}^2 < 20 \text{ lb/ft}^2$, then also

$$p_f = I p_g = 1.2(15 \text{ lb/ft}^2) = 18 \text{ lb/ft}^2$$

By comparison, choose

$$p_f = 18 \text{ lb/ft}^2$$ ***Ans.***

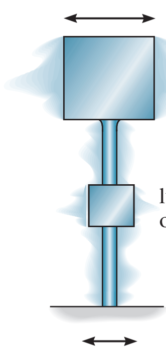


Fig. 1–16

Earthquake Loads. Earthquakes produce loadings on a structure through its interaction with the ground and its response characteristics. These loadings result from the structure's distortion caused by the ground's motion and the lateral resistance of the structure. Their magnitude depends on the amount and type of ground accelerations and the mass and stiffness of the structure. In order to provide some insight as to the nature of earthquake loads, consider the simple structural model shown in Fig. 1–16. This model may represent a single-story building, where the top block is the "lumped" mass of the roof, and the middle block is the lumped stiffness of all the building's columns. During an earthquake the ground vibrates both horizontally and vertically. The horizontal accelerations create shear forces in the column that put the block in sequential motion with the ground. If the column is *stiff* and the block has a *small* mass, the period of vibration of the block will be *short* and the block will accelerate with the same motion as the ground and undergo only slight relative displacements. For an actual structure which is designed to have large amounts of bracing and stiff connections this can be beneficial, since less stress is developed in the members. On the other hand, if the column in Fig 1–16 is very flexible and the block has a large mass, then earthquake-induced motion will cause small accelerations of the block and large relative displacements.

In practice the effects of a structure's acceleration, velocity, and displacement can be determined and represented as an *earthquake response spectrum*. Once this graph is established, the earthquake loadings can be calculated using a *dynamic analysis* based on the theory of structural dynamics. This type of analysis is gaining popularity, although it is often quite elaborate and requires the use of a computer. Even so, such an analysis becomes mandatory if the structure is large.

Some codes require that specific attention be given to earthquake design, especially in areas of the country where strong earthquakes predominate. Also, these loads should be seriously considered when designing high-rise buildings or nuclear power plants. In order to assess the importance of earthquake design consideration, one can check a seismic ground-acceleration map published in the ASCE 7-05 Standard. This map provides the peak ground acceleration caused by an earthquake measured over a 50-year period. Regions vary from low risk, such as parts of Texas, to very high risk, such as along the west coast of California.

For small structures, a *static analysis* for earthquake design may be satisfactory. This case approximates the dynamic loads by a set of externally applied *static forces* that are applied laterally to the structure. One such method for doing this is reported in the ASCE 7-05 Standard. It is based upon finding a seismic response coefficient, C_s, determined from the soil properties, the ground accelerations, and the vibrational response of the structure. This coefficient is then multiplied by the structure's total dead load W to obtain the "base shear" in the structure, The value of C_s is actually determined from

$$C_s = \frac{S_{DS}}{R/I}$$

where

S_{DS} = the spectral response acceleration for short periods of vibration.

R = a response modification factor that depends upon the ductility of the structure. Steel frame members which are highly ductile can have a value as high as 8, whereas reinforced concrete, having low ductility, can have a value as low as 3.

I = the importance factor that depends upon the use of the building. For example, $I = 1$ for agriculture and storage facilities, and $I = 1.5$ for hospitals and shelters.

With each new publication of the Standard, values of these coefficients are updated as more accurate data about earthquake response become available.

Hydrostatic and Soil Pressure. When structures are used to retain water, soil, or granular materials, the pressure developed by these loadings becomes an important criterion for their design. Examples of such types of structures include tanks, dams, ships, bulkheads, and retaining walls. Here the laws of hydrostatics and soil mechanics are applied to define the intensity of the loadings on the structure.

The design of this retaining wall requires estimating the soil pressure acting on it. Also, the gate of the lock will be subjected to hydrostatic pressure that must be considered for its design.

Other Natural Loads. Several other types of live loads may also have to be considered in the design of a structure, depending on its location or use. These include the effect of blast, temperature changes, and differential settlement of the foundation.

1–4 Structural Design

Whenever a structure is designed, it is important to give consideration to both material and load uncertainties. Material uncertainties occur due to variability in material properties, residual stress in materials, intended measurements being different from fabricated sizes, accidental loadings due to vibration or impact, and material corrosion or decay. Allowable-stress design methods include all these factors into a single factor of safety to account for their uncertainties. The many types of loads discussed previously can occur simultaneously on a structure, but it is very unlikely that the maximum of all these loads will occur at the same time. In *working-stress design* the computed elastic stress in the material must not exceed the allowable stress along with the following typical load combinations as specified by the ASCE 7-05 Standard.

- dead load
- 0.6 (dead load) + wind load
- 0.6 (dead load) + 0.7 (earthquake load)

For example, both wind and earthquake loads normally do not act simultaneously on a structure. Also, when certain loads are assumed to act in combination, the combined load can be *reduced* by a load-combination factor.

Since uncertainty can be considered using probability theory, there has been an increasing trend to separate material uncertainty from load uncertainty. This method is called *strength design* or LRFD (load and resistance factor design). In particular, ultimate strength design is based on designing the ultimate strength of critical sections in reinforced concrete, and the plastic design method is used for frames and members made from steel. To account for the uncertainty of loads, this method uses load factors applied to the loads or combinations of loads. For example, according to the ASCE 7-05 Standard, some of the load factors and combinations are

- 1.4 (dead load)
- 1.2 (dead load) + 1.6 (live load) + 0.5 (snow load)
- 1.2 (dead load) + 1.5 (earthquake load) + 0.5 (live load)

In all these cases, the combination of loads is thought to provide a maximum, yet realistic loading on the structure.

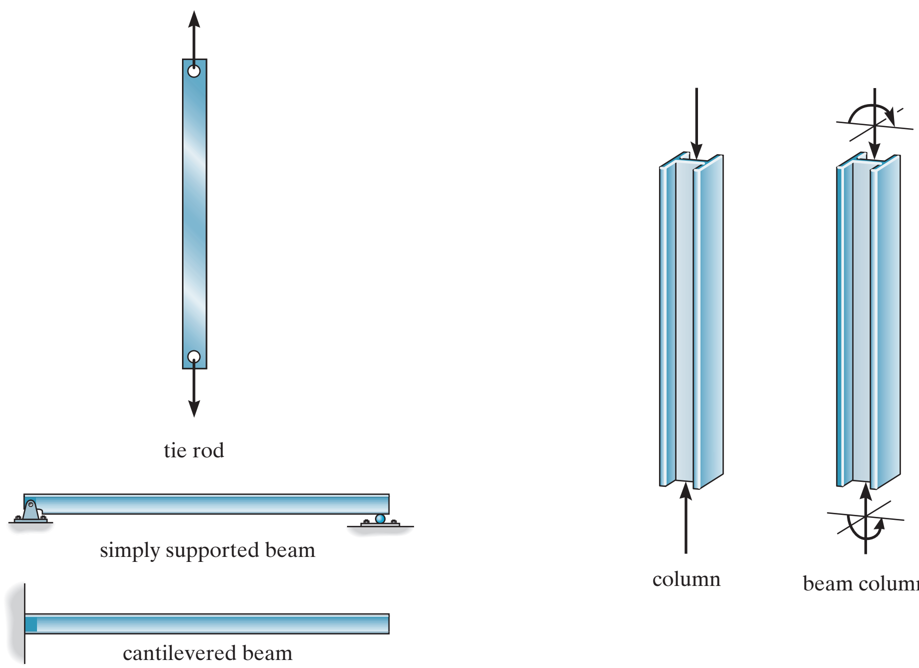
Chapter Review

The basic structural elements are:

Tie Rods—Slender members subjected to tension. Often used for bracing.

Beams—Members designed to resist bending moment. They are often fixed or pin supported and can be in the form of a steel plate girder, reinforced concrete, or laminated wood.

Columns—Members that resist axial compressive force. If the column also resists bending, it is called a *beam column*.



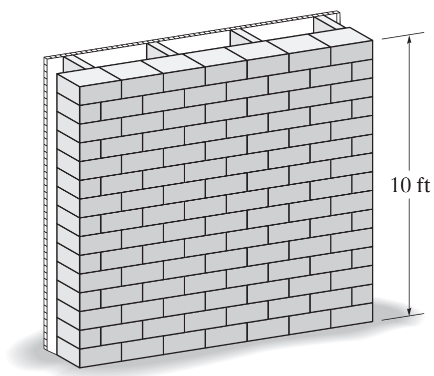
The types of structures considered in this book consist of *trusses* made from slender pin-connected members forming a series of triangles, *cables and arches,* which carry tensile and compressive loads, respectively, and *frames* composed of pin- or fixed-connected beams and columns.

Loads are specified in codes such as the ASCE 7-05 code. *Dead loads* are fixed and refer to the weights of members and materials. *Live loads* are movable and consist of uniform building floor loads, traffic and train loads on bridges, impact loads due to vehicle and machine bouncing, wind loads, snow loads, earthquake loads, and hydrostatic and soil pressure.

PROBLEMS

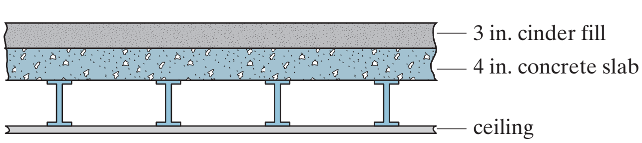
1–1. The floor of a light storage warehouse is made of 6-in.-thick cinder concrete. If the floor is a slab having a length of 10 ft and width of 8 ft, determine the resultant force caused by the dead load and that caused by the live load.

1–2. The building wall consists of 8-in. clay brick. In the interior, the wall is made from 2×4 wood studs, plastered on one side. If the wall is 10 ft high, determine the load in pounds per foot of length of wall that the wall exerts on the floor.



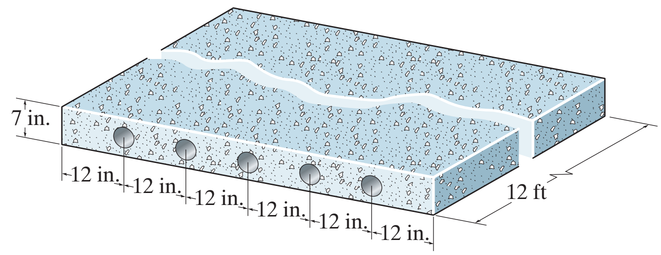
Prob. 1–2

1–3. The second floor of a light manufacturing building is constructed from a 4-in.-thick stone concrete slab with an added 3-in. cinder concrete fill as shown. If the suspended ceiling of the first floor consists of metal lath and gypsum plaster, determine the dead load for design in pounds per square foot of floor area.



Prob. 1–3

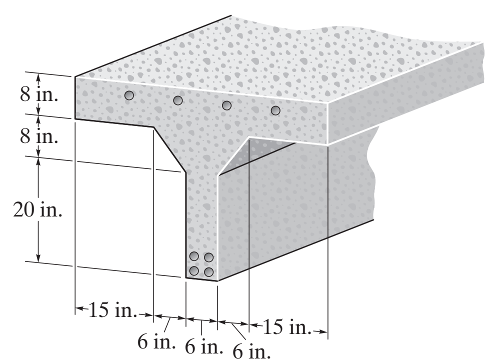
***1–4.** The hollow core panel is made from plain stone concrete. Determine the dead weight of the panel. The holes each have a diameter of 4 in.



Prob. 1–4

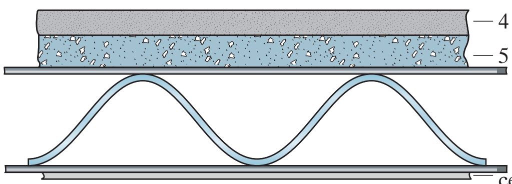
1–5. The floor of a classroom is made of 125-mm thick lightweight plain concrete. If the floor is a slab having a length of 8 m and width of 6 m, determine the resultant force caused by the dead load and the live load.

1–6. The pre-cast T-beam has the cross-section shown. Determine its weight per foot of length if it is made from reinforced stone concrete and eight $\frac{3}{4}$-in. cold-formed steel reinforcing rods.



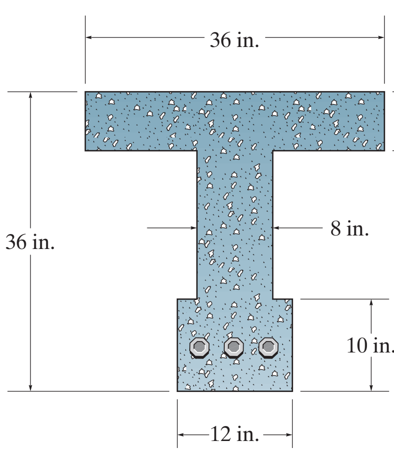
Prob. 1–6

1–7. The second floor of a light manufacturing building is constructed from a 5-in.-thick stone concrete slab with an added 4-in. cinder concrete fill as shown. If the suspended ceiling of the first floor consists of metal lath and gypsum plaster, determine the dead load for design in pounds per square foot of floor area.



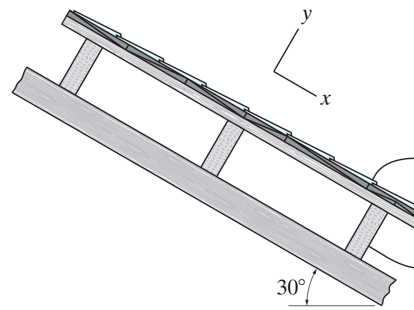
Prob. 1–7

***1–8.** The T-beam used in a heavy storage warehouse is made of concrete having a specific weight of 125 lb/ft^3. Determine the dead load per foot length of beam, and the load on the top of the beam per foot length of beam. Neglect the weight of the steel reinforcement.



Prob. 1–8

1–9. The beam supports the roof made from asphalt shingles and wood sheathing boards. If the boards have a thickness of $1\frac{1}{2}$ in. and a specific weight of 50 lb/ft^3, and the roof's angle of slope is 30°, determine the dead load of the roofing—per square foot—that is supported in the x and y directions by the purlins.



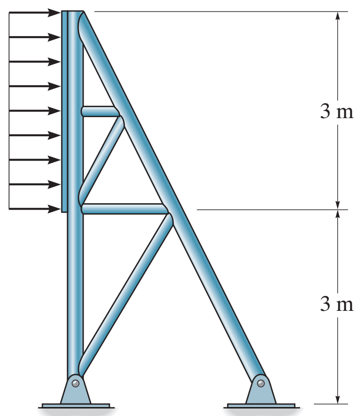
Prob. 1–9

1–10. A two-story school has interior columns that are spaced 15 ft apart in two perpendicular directions. If the loading on the flat roof is estimated to be 20 lb/ft^2, determine the reduced live load supported by a typical interior column at (a) the ground-floor level, and (b) the second-floor level.

1–11. A four-story office building has interior columns spaced 30 ft apart in two perpendicular directions. If the flat-roof loading is estimated to be 30 lb/ft^2, determine the reduced live load supported by a typical interior column located at ground level.

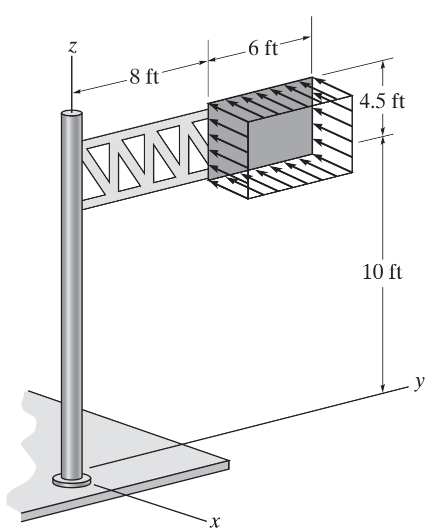
***1–12.** A three-story hotel has interior columns that are spaced 20 ft apart in two perpendicular directions. If the loading on the flat roof is estimated to be 30 lb/ft^2, determine the live load supported by a typical interior column at (a) the ground-floor level, and (b) the second-floor level.

1–13. Determine the resultant force acting on the face of the truss-supported sign if it is located near Los Angeles, California on open flat terrain. The sign has a width of 6 m and a height of 3 m as indicated. Use an importance factor of $I = 0.87$.



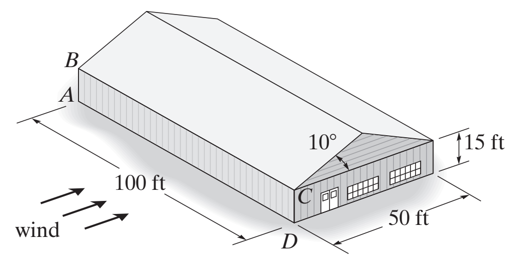
Prob. 1–13

1–14. The sign is located in Minnesota on open flat terrain. Determine the resultant force of the wind acting on its face. Use an importance factor of $I = 0.87$.



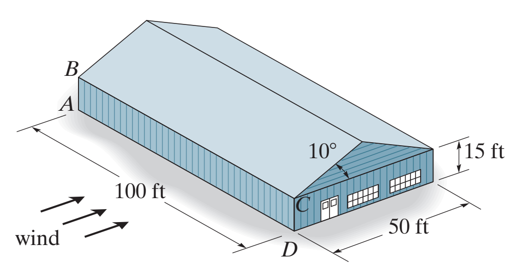
Prob. 1–14

1–15. Wind blows on the side of the fully enclosed agriculture building located on open flat terrain in Oklahoma. Determine the external pressure acting on the roof. Also, what is the internal pressure in the building which acts on the roof? Use linear interpolation to determine q_h and C_p in Figure 1–13.



Prob. 1–15

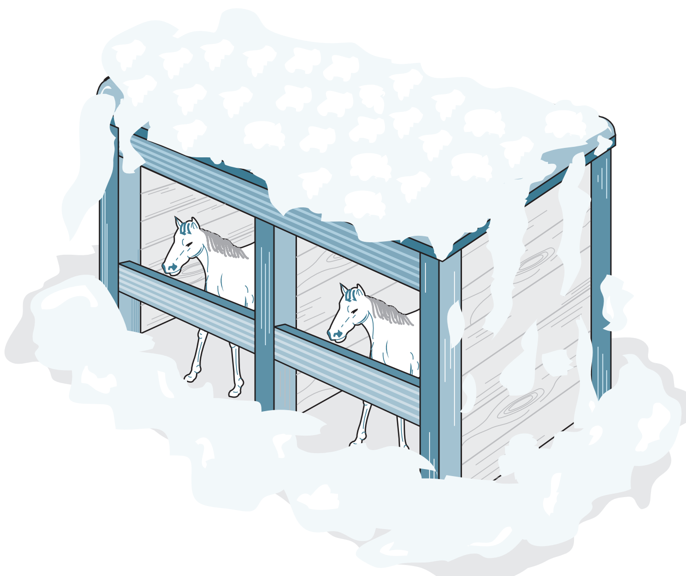
***1–16.** Wind blows on the side of the fully enclosed agriculture building located on open flat terrain in Oklahoma. Determine the external pressure acting over the windward wall, the leeward wall, and the side walls. Also, what is the internal pressure in the building which acts on the walls? Use linear interpolation to determine q_h.



Prob. 1–16

1–17. The horse stall has a flat roof with a slope of 80 mm/m. It is located in an open field where the ground snow load is 1.20 kN/m^2. Determine the snow load that is required to design the roof of the stall.

1–18. The horse stall has a flat roof with a slope of 80 mm/m. It is located in an open field where the ground snow load is 0.72 kN/m^2. Determine the snow load that is required to design the roof of the stall.



Probs. 1–17/1–18

1–19. A hospital located in Chicago, Illinois, has a flat roof, where the ground snow load is 25 lb/ft^2. Determine the design snow load on the roof of the hospital.

Oftentimes the elements of a structure, like the beams and girders of this building frame, are connected together in a manner whereby the analysis can be considered statically determinate.

2 Analysis of Statically Determinate Structures

In this chapter we will direct our attention to the most common form of structure that the engineer will have to analyze, and that is one that lies in a plane and is subjected to a force system that lies in the same plane. We begin by discussing the importance of choosing an appropriate analytical model for a structure so that the forces in the structure may be determined with reasonable accuracy. Then the criteria necessary for structural stability are discussed. Finally, the analysis of statically determinate, planar, pin-connected structures is presented.

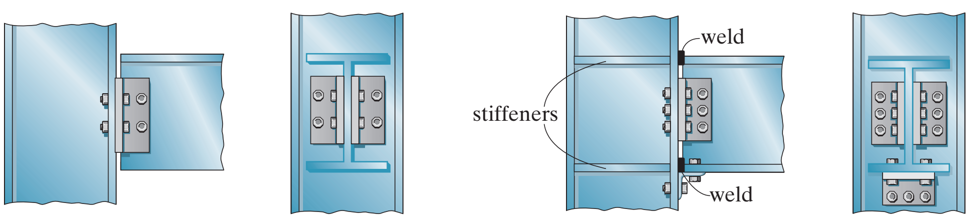
2–1 Idealized Structure

In the real sense an exact analysis of a structure can never be carried out, since estimates always have to be made of the loadings and the strength of the materials composing the structure. Furthermore, points of application for the loadings must also be estimated. It is important, therefore, that the structural engineer develop the ability to model or idealize a structure so that he or she can perform a practical force analysis of the members. In this section we will develop the basic techniques necessary to do this.

The deck of this concrete bridge is made so that one section can be conidered roller supported on the other section.

Support Connections. Structural members are joined together in various ways depending on the intent of the designer. The three types of joints most often specified are the pin connection, the roller support, and the fixed joint. A pin-connected joint and a roller support allow some freedom for slight rotation, whereas a fixed joint allows no relative rotation between the connected members and is consequently more expensive to fabricate. Examples of these joints, fashioned in metal and concrete, are shown in Figs. 2–1 and 2–2, respectively. For most timber structures, the members are assumed to be pin connected, since bolting or nailing them will not sufficiently restrain them from rotating with respect to each other.

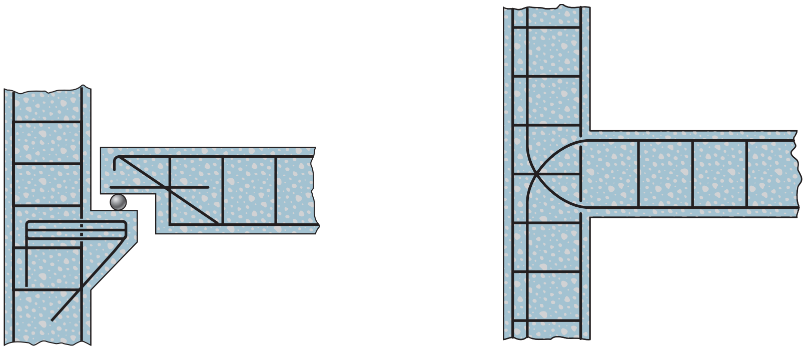
Idealized models used in structural analysis that represent pinned and fixed supports and pin-connected and fixed-connected joints are shown in Figs. 2–3*a* and 2–3*b*. In reality, however, all connections exhibit some stiffness toward joint rotations, owing to friction and material behavior. In this case a more appropriate model for a support or joint might be that shown in Fig. 2–3*c*. If the torsional spring constant $k = 0$, the joint is a pin, and if $k \rightarrow \infty$, the joint is fixed.



typical "pin-supported" connection (metal)
(a)

typical "fixed-supported" connection (metal)
(b)

Fig. 2–1



typical "roller-supported" connection (concrete)
(a)

typical "fixed-supported" connection (concrete)
(b)

Fig. 2–2

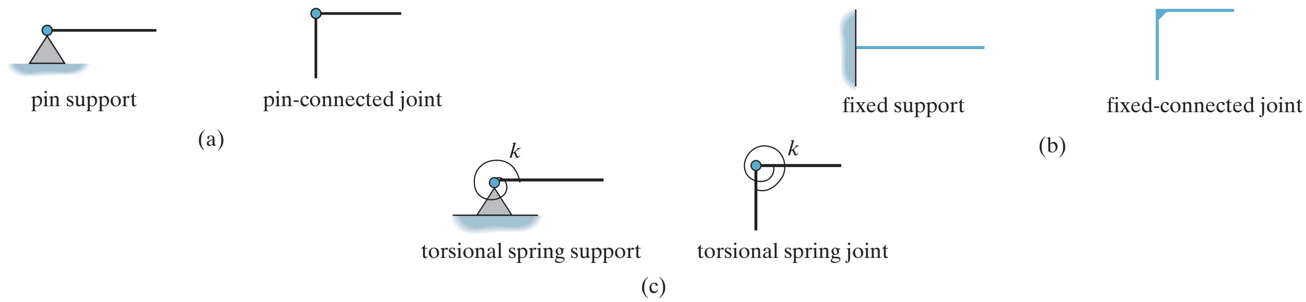
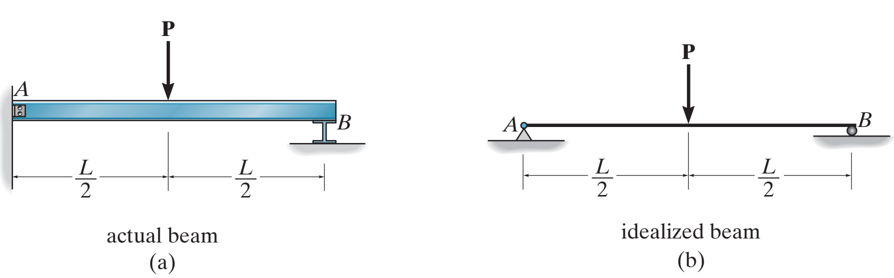


Fig. 2–3

When selecting a particular model for each support or joint, the engineer must be aware of how the assumptions will affect the actual performance of the member and whether the assumptions are reasonable for the structural design. For example, consider the beam shown in Fig. 2–4*a*, which is used to support a concentrated load **P**. The angle connection at support A is like that in Fig. 2–1*a* and can therefore be idealized as a typical pin support. Furthermore, the support at B provides an approximate point of smooth contact and so it can be idealized as a roller. The beam's thickness can be neglected since it is small in comparison to the beam's length, and therefore the idealized model of the beam is as shown in Fig. 2–4*b*. The analysis of the loadings in this beam should give results that closely approximate the loadings in the actual beam. To show that the model is appropriate, consider a specific case of a beam made of steel with $P = 8$ k (8000 lb) and $L = 20$ ft. One of the major simplifications made here was assuming the support at A to be a pin. Design of the beam using standard code procedures* indicates that a W 10 × 19 would be adequate for supporting the load. Using one of the deflection methods of Chapter 8, the rotation at the "pin" support can be calculated as $\theta = 0.0103$ rad $= 0.59°$. From Fig. 2–4*c*, such a rotation only moves the top or bottom flange a distance of $\Delta = \theta r = (0.0103 \text{ rad})(5.12 \text{ in.}) = 0.0528$ in.! This *small amount* would certainly be accommodated by the connection fabricated as shown in Fig. 2–1*a*, and therefore the pin serves as an appropriate model.

The two girders and floor beam are assumed to be pin connected to this column.



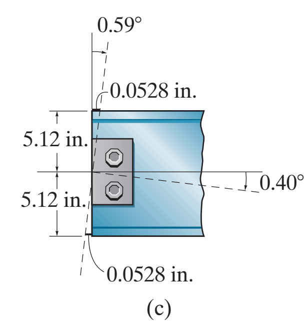


Fig. 2–4

*Codes such as the *Manual of Steel Construction*, American Institute of Steel Construction.

A typical rocker support used for a bridge girder.

Rollers and associated bearing pads are used to support the prestressed concrete girders of a highway bridge.

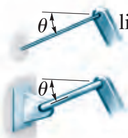
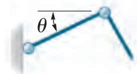
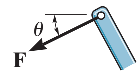
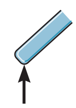
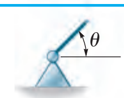
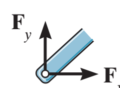
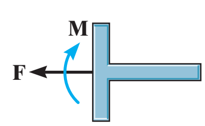
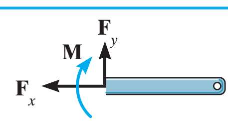
Other types of connections most commonly encountered on coplanar structures are given in Table 2–1. It is important to be able to recognize the symbols for these connections and the kinds of reactions they exert on their attached members. This can easily be done by noting how the connection *prevents* any degree of freedom or displacement of the member. In particular, the support will develop a *force* on the member if it *prevents translation* of the member, and it will develop a *moment* if it *prevents rotation* of the member. For example, a member in contact with a smooth surface (3) is prevented from translating only in one direction, which is perpendicular or normal to the surface. Hence, the surface exerts only a *normal force* **F** on the member in this direction. The magnitude of this force represents *one unknown*. Also note that the member is free to rotate on the surface, so that a moment cannot be developed by the surface on the member. As another example, the fixed support (7) prevents *both* translation and rotation of a member at the point of connection. Therefore, this type of support exerts two force components and a moment on the member. The "curl" of the moment lies in the plane of the page, since rotation is prevented in that plane. Hence, there are *three unknowns* at a fixed support.

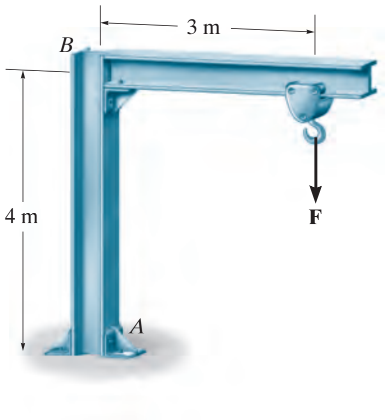
In reality, all supports actually exert *distributed surface loads* on their contacting members. The concentrated forces and moments shown in Table 2–1 represent the *resultants* of these load distributions. This representation is, of course, an idealization; however, it is used here since the surface area over which the distributed load acts is considerably *smaller* than the *total* surface area of the connecting members.

The short link is used to connect the two girders of the highway bridge and allow for thermal expansion of the deck.

Typical pin used to support the steel girder of a railroad bridge.

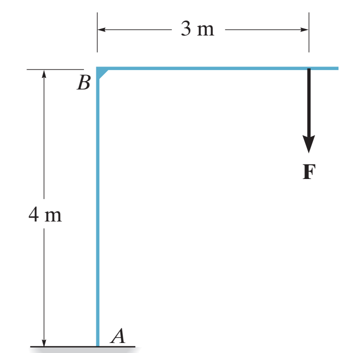
TABLE 2–1 • Supports for Coplanar Structures

Type of Connection	Idealized Symbol	Reaction	Number of Unknowns
(1)  light cable, weightless link			One unknown. The reaction is a force that acts in the direction of the cable or link.
(2)  rollers, rocker			One unknown. The reaction is a force that acts perpendicular to the surface at the point of contact.
(3)  smooth contacting surface			One unknown. The reaction is a force that acts perpendicular to the surface at the point of contact.
(4)  smooth pin-connected collar			One unknown. The reaction is a force that acts perpendicular to the surface at the point of contact.
(5)  smooth pin or hinge			Two unknowns. The reactions are two force components.
(6)  slider  fixed-connected collar			Two unknowns. The reactions are a force and a moment.
(7)  fixed support			Three unknowns. The reactions are the moment and the two force components.



actual structure

(a)



idealized structure

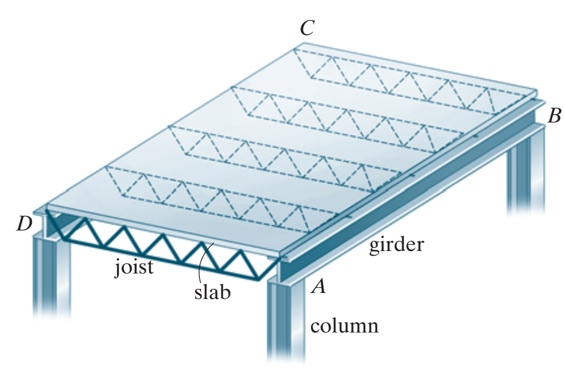
(b)

Fig. 2–5

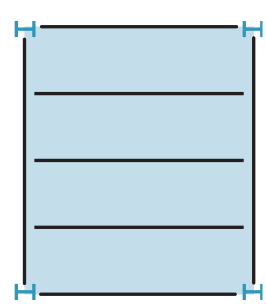
Idealized Structure. Having stated the various ways in which the connections on a structure can be idealized, we are now ready to discuss some of the techniques used to represent various structural systems by idealized models.

As a first example, consider the jib crane and trolley in Fig. 2–5*a*. For the structural analysis we can neglect the thickness of the two main members and will assume that the joint at *B* is fabricated to be rigid. Furthermore, the support connection at *A* can be modeled as a fixed support and the details of the trolley excluded. Thus, the members of the idealized structure are represented by two connected lines, and the load on the hook is represented by a single concentrated force **F**, Fig. 2–5*b*. This idealized structure shown here as a *line drawing* can now be used for applying the principles of structural analysis, which will eventually lead to the design of its two main members.

Beams and girders are often used to support building floors. In particular, a *girder* is the main load-carrying element of the floor, whereas the smaller elements having a shorter span and connected to the girders are called *beams*. Often the loads that are applied to a beam or girder are transmitted to it by the floor that is supported by the beam or girder. Again, it is important to be able to appropriately idealize the system as a series of models, which can be used to determine, to a close approximation, the forces acting in the members. Consider, for example, the framing used to support a typical floor slab in a building, Fig. 2–6*a*. Here the slab is supported by *floor joists* located at even intervals, and these in turn are supported by the two side girders *AB* and *CD*. For analysis it is reasonable to assume that the joints are pin and/or roller connected to the girders and that the girders are pin and/or roller connected to the columns. The top view of the structural framing plan for this system is shown in Fig. 2–6*b*. In this "graphic" scheme, notice that the "lines" representing the joists do not touch the girders and the lines for the girders do not touch the columns. This symbolizes pin- and/ or roller-supported connections. On the other hand, if the framing plan is intended



(a)



idealized framing plan

(b)

Fig. 2–6

to represent fixed-connected members, such as those that are welded instead of simple bolted connections, then the lines for the beams or girders would touch the columns as in Fig. 2–7. Similarly, a fixed-connected overhanging beam would be represented in top view as shown in Fig. 2–8. If reinforced concrete construction is used, the beams and girders are represented by double lines. These systems are generally all fixed connected and therefore the members are drawn to the supports. For example, the structural graphic for the cast-in-place reinforced concrete system in Fig. 2–9*a* is shown in top view in Fig. 2–9*b*. The lines for the beams are dashed because they are below the slab.

Structural graphics and idealizations for timber structures are similar to those made of metal. For example, the structural system shown in Fig. 2–10*a* represents beam-wall construction, whereby the roof deck is supported by wood joists, which deliver the load to a masonry wall. The joists can be assumed to be simply supported on the wall, so that the idealized framing plan would be like that shown in Fig. 2–10*b*.

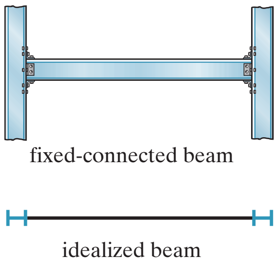


Fig. 2–7

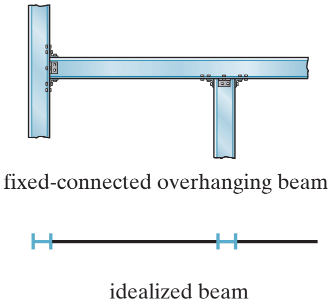


Fig. 2–8

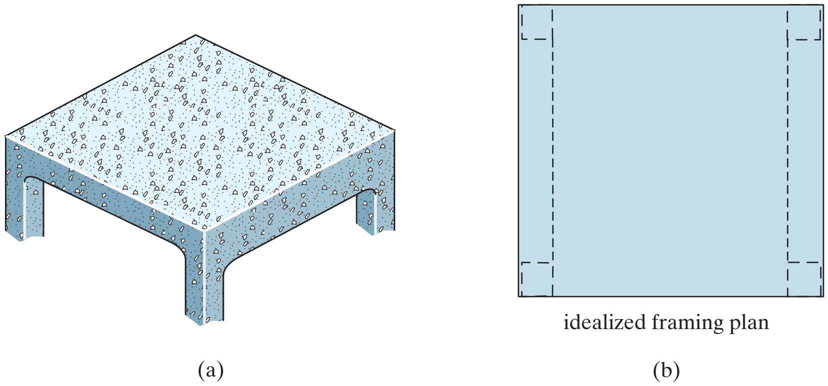


Fig. 2–9

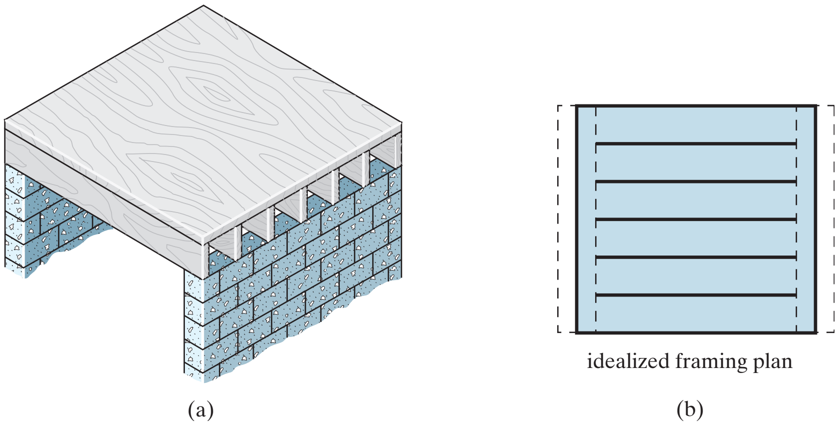


Fig. 2–10

The structural framework of this building consists of concrete floor joists, which were formed on site using metal pans. These joists are simply supported on the girders, which in turn are simply supported on the columns.

Tributary Loadings. When flat surfaces such as walls, floors, or roofs are supported by a structural frame, it is necessary to determine how the load on these surfaces is transmitted to the various structural elements used for their support. There are generally two ways in which this can be done. The choice depends on the geometry of the structural system, the material from which it is made, and the method of its construction.

One-Way System. A slab or deck that is supported such that it delivers its load to the supporting members by one-way action, is often referred to as a *one-way slab*. To illustrate the method of load transmission, consider the framing system shown in Fig. 2–11*a* where the beams AB, CD, and EF rest on the girders AE and BF. If a uniform load of 100 lb/ft^2 is placed on the slab, then the center beam CD is assumed to support the load acting on the *tributary area* shown dark shaded on the structural framing plan in Fig. 2–11*b*. Member CD is therefore subjected to a *linear* distribution of load of $(100\text{ lb/ft}^2)(5\text{ ft}) = 500\text{ lb/ft}$, shown on the idealized beam in Fig. 2–11*c*. The reactions on this beam (2500 lb) would then be applied to the center of the girders AE (and BF), shown idealized in Fig. 2–11*d*. Using this same concept, do you see how the remaining portion of the slab loading is transmitted to the ends of the girder as 1250 lb?

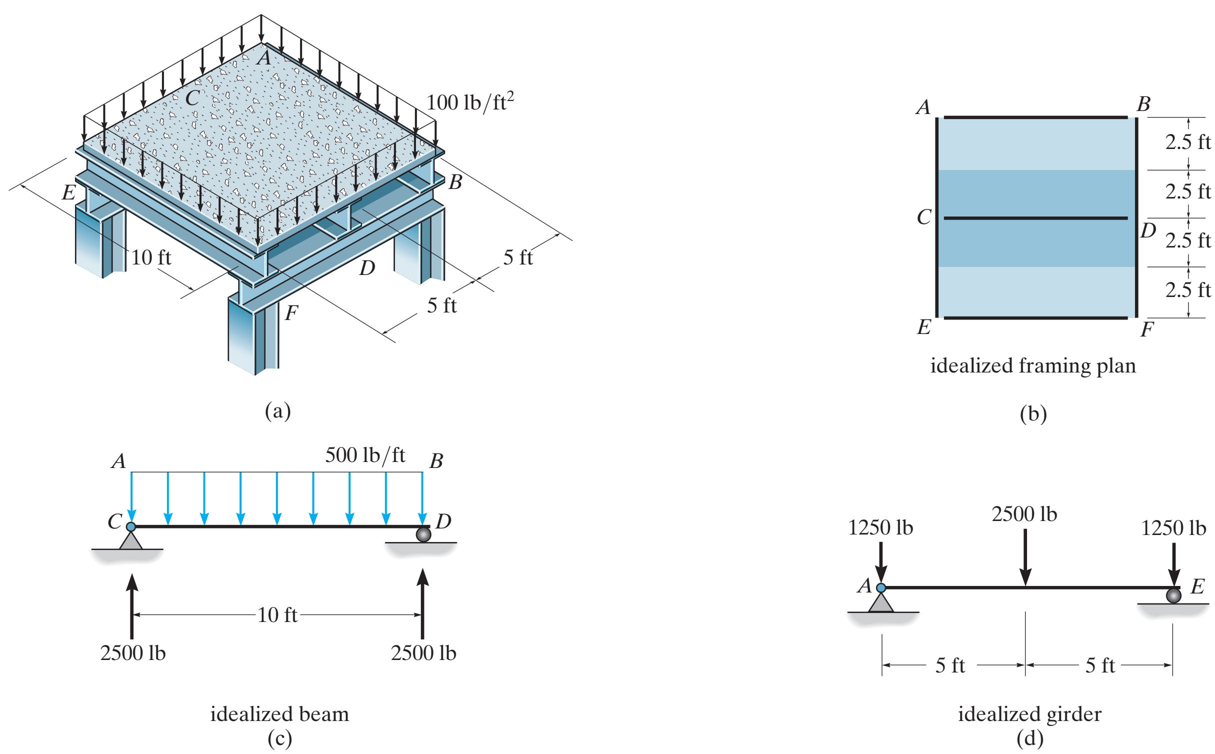


Fig. 2–11

An example of one-way slab construction of a steel frame building having a poured concrete floor on a corrugated metal deck. The load on the floor is considered to be transmitted to the beams, not the girders.

For some floor systems the beams and girders are connected to the columns at the *same elevation,* as in Fig. 2–12*a*. If this is the case, the slab can in some cases also be considered a "one-way slab." For example, if the slab is reinforced concrete with reinforcement in *only one direction,* or the concrete is poured on a corrugated metal deck, as in the above photo, then one-way action of load transmission can be assumed. On the other hand, if the slab is flat on top and bottom and is reinforced in *two directions,* then consideration must be given to the *possibility* of the load being transmitted to the supporting members from either one or two directions. For example, consider the slab and framing plan in Fig. 2–12*b*. According to the ASCE 7-05 Standard, if $L_2 \geq L_1$ *and if the span ratio* $(L_2/L_1) \geq 1.5$, *the slab will behave as a one-way slab,* since as L_1 becomes smaller, the beams AB, CD, and EF provide the greater stiffness to carry the load.

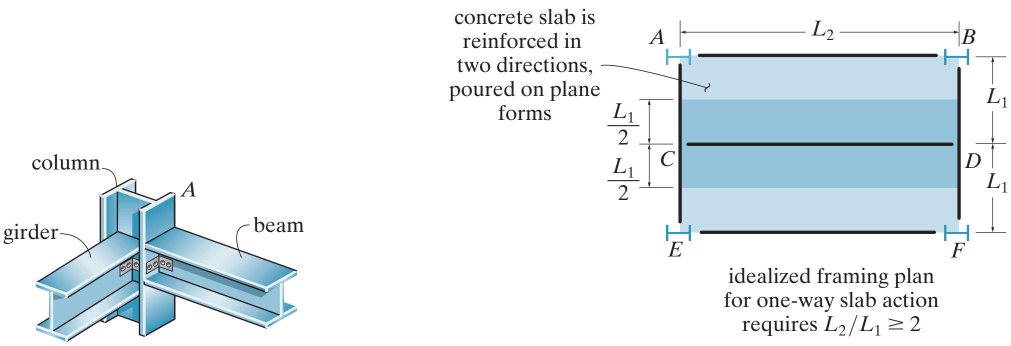


Fig. 2–12

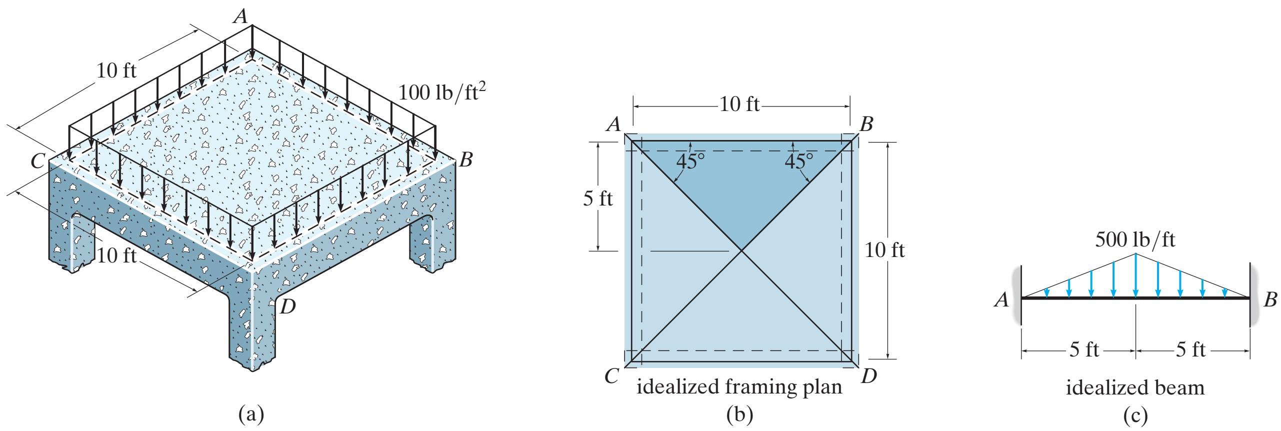


Fig. 2–13

Two-Way System. If, according to the ASCE 7-05 Standard the support ratio in Fig. 2–12*b* is $(L_2/L_1) < 1.5$, the load is assumed to be delivered to the supporting beams and girders in two directions. When this is the case the slab is referred to as a *two-way slab*. To show one method of treating this case, consider the square reinforced concrete slab in Fig. 2–13*a*, which is supported by four 10-ft-long edge beams, AB, BD, DC, and CA. Here $L_2/L_1 = 1$. Due to two-way slab action, the assumed *tributary area* for beam AB is shown dark shaded in Fig. 2–13*b*. This area is determined by constructing diagonal 45° lines as shown. Hence if a uniform load of 100 lb/ft² is applied to the slab, a peak intensity of $(100\ \text{lb/ft}^2)(5\ \text{ft}) = 500\ \text{lb/ft}$ will be applied to the center of beam AB, resulting in a *triangular* load distribution shown in Fig. 2–13*c*. For other geometries that cause two-way action, a similar procedure can be used. For example, if $L_2/L_1 = 1.5$ it is then necessary to construct 45° lines that intersect as shown in Fig. 2–14*a*. A 100-lb/ft² loading placed on the slab will then produce *trapezoidal* and *triangular* distributed loads on members AB and AC, Fig. 2–14*b* and 2–14*c*, respectively.

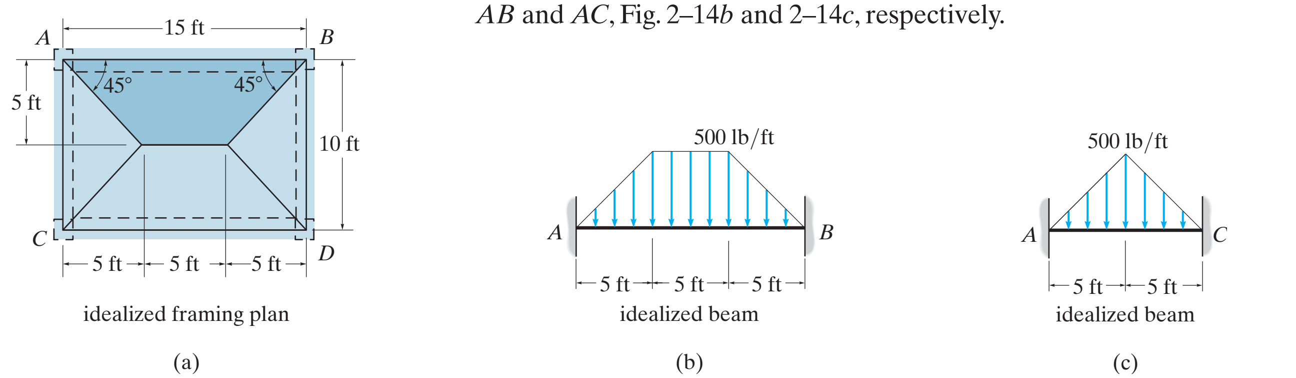


Fig. 2–14

The ability to reduce an actual structure to an idealized form, as shown by these examples, can only be gained by experience. To provide practice at doing this, the example problems and the problems for solution throughout this book are presented in somewhat realistic form, and the associated problem statements aid in explaining how the connections and supports can be modeled by those listed in Table 2–1. In engineering practice, if it becomes doubtful as to how to model a structure or transfer the loads to the members, it is best to consider *several* idealized structures and loadings and then design the actual structure so that it can resist the loadings in all the idealized models.

EXAMPLE 2-1

The floor of a classroom is supported by the bar joists shown in Fig. 2–15*a*. Each joist is 15 ft long and they are spaced 2.5 ft on centers. The floor itself is made from lightweight concrete that is 4 in. thick. Neglect the weight of the joists and the corrugated metal deck, and determine the load that acts along each joist.

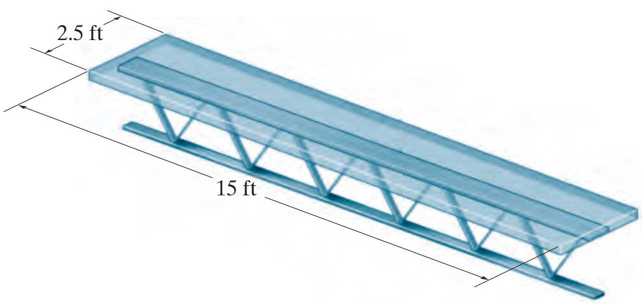
(a)

Solution

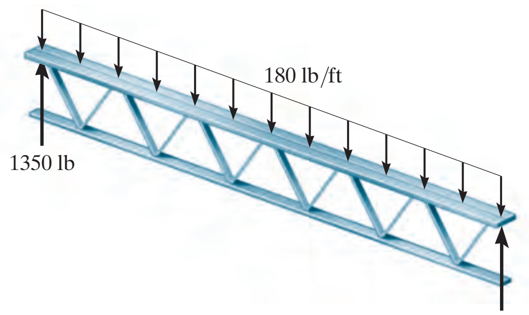
The dead load on the floor is due to the weight of the concrete slab. From Table 1–3 for 4 in. of lightweight concrete it is $(4)(8\text{ lb/ft}^2) = 32\text{ lb/ft}^2$. From Table 1–4, the live load for a classroom is 40 lb/ft^2. Thus the total floor load is $32\text{ lb/ft}^2 + 40\text{ lb/ft}^2 = 72\text{ lb/ft}^2$. For the floor system, $L_1 = 2.5$ ft and $L_2 = 15$ ft. Since $L_2/L_1 > 1.5$ the concrete slab is treated as a one-way slab. The tributary area for each joist is shown in Fig. 2–15*b*. Therefore the uniform load along its length is

$$w = 72\text{ lb/ft}^2(2.5\text{ ft}) = 180\text{ lb/ft}$$

This loading and the end reactions on each joist are shown in Fig. 2–15*c*.



(b)

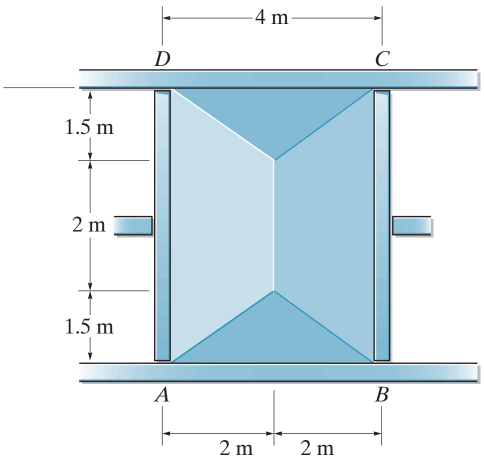


(c)

Fig. 2–15

EXAMPLE 2-2

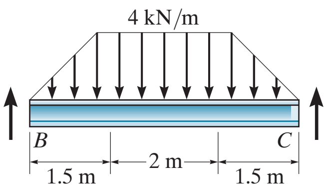
The flat roof of the steel-frame building in Fig. 2–16*a* is intended to support a total load of $2\ \text{kN/m}^2$ over its surface. If the span of beams *AD* and *BC* is 5 m and the space between them (*AB* and *DC*) is 3 m, determine the roof load within region *ABCD* that is transmitted to beam *BC*.



(a)

Solution

In this case $L_1 = 5$ m and $L_2 = 4$ m. Since $L_2/L_1 = 1.25 < 1.5$, we have two-way slab action. The tributary loading is shown in Fig. 2–16*b*, where the shaded trapezoidal area of loading is transmitted to member *BC*. The peak intensity of this loading is $(2\ \text{kN/m}^2)(2\ \text{m}) = 4\ \text{kN/m}$. As a result, the distribution of load along *BC* is shown in Fig. 2–16*b*. This process of tributary load transmission should *also* be calculated for the two square regions to the right of *BC* in Fig. 2–16*a*, and this additional load should then be placed on *BC*.



(b)

Fig. 2–16

2–2 Principle of Superposition

The principle of superposition forms the basis for much of the theory of structural analysis. It may be stated as follows: *The total displacement or internal loadings (stress) at a point in a structure subjected to several external loadings can be determined by adding together the displacements or internal loadings (stress) caused by each of the external loads acting separately*. For this statement to be valid it is necessary that a *linear relationship* exist among the loads, stresses, and displacements.

Two requirements must be imposed for the principle of superposition to apply:

1. The material must behave in a linear-elastic manner, so that Hooke's law is valid, and therefore the load will be proportional to displacement.
2. The geometry of the structure must not undergo significant change when the loads are applied, i.e., small displacement theory applies. Large displacements will significantly change the position and orientation of the loads. An example would be a cantilevered thin rod subjected to a force at its end.

Throughout this text, these two requirements will be satisfied. Here only linear-elastic material behavior occurs; and the displacements produced by the loads will not significantly change the directions of applied loadings nor the dimensions used to compute the moments of forces.

The "shear walls" on the sides of this building are used to strengthen the structure when it is subjected to large hurricane wind loadings applied to the front or back of the building.

2–3 Equations of Equilibrium

It may be recalled from statics that a structure or one of its members is in equilibrium when it maintains a balance of force and moment. In general this requires that the force and moment equations of equilibrium be satisfied along three independent axes, namely,

$$\begin{aligned} \Sigma F_x &= 0 \quad \Sigma F_y = 0 \quad \Sigma F_z = 0 \\ \Sigma M_x &= 0 \quad \Sigma M_y = 0 \quad \Sigma M_z = 0 \end{aligned} \tag{2–1}$$

The principal load-carrying portions of most structures, however, lie in a single plane, and since the loads are also coplanar, the above requirements for equilibrium reduce to

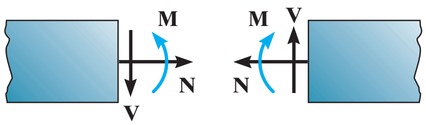
$$\begin{aligned} \Sigma F_x &= 0 \\ \Sigma F_y &= 0 \\ \Sigma M_O &= 0 \end{aligned} \tag{2–2}$$

Here ΣF_x and ΣF_y represent, respectively, the algebraic sums of the x and y components of all the forces acting on the structure or one of its members, and ΣM_O represents the algebraic sum of the moments of these force components about an axis perpendicular to the x–y plane (the z axis) and passing through point O.

Whenever these equations are applied, *it is first necessary to draw a free-body diagram of the structure or its members*. If a member is selected, it must be *isolated* from its supports and surroundings and its outlined shape drawn. All the forces and couple moments must be shown that act *on the member*. In this regard, the types of reactions at the supports can be determined using Table 2–1. Also, recall that forces common to two members act with equal magnitudes but opposite directions on the respective free-body diagrams of the members.

If the *internal loadings* at a specified point in a member are to be determined, the *method of sections* must be used. This requires that a "cut" or section be made perpendicular to the axis of the member at the point where the internal loading is to be determined. A free-body diagram of either segment of the "cut" member is isolated and the internal loads are then determined from the equations of equilibrium applied to the segment. In general, the internal loadings acting at the cut section of the member will consist of a normal force **N**, shear force **V**, and bending moment **M**, as shown in Fig. 2–17.

We will cover the principles of statics that are used to determine the external reactions on structures in Sec. 2–5. Internal loadings in structural members will be discussed in Chapter 4.



internal loadings

Fig. 2–17

2–4 Determinacy and Stability

Before starting the force analysis of a structure, it is necessary to establish the determinacy and stability of the structure.

Determinacy. The equilibrium equations provide both the *necessary and sufficient* conditions for equilibrium. When all the forces in a structure can be determined strictly from these equations, the structure is referred to as *statically determinate*. Structures having more unknown forces than available equilibrium equations are called *statically indeterminate*. As a general rule, a structure can be identified as being either statically determinate or statically indeterminate by drawing free-body diagrams of all its members, or selective parts of its members, and then comparing the total number of unknown reactive force and moment components with the total number of available equilibrium equations.* For a coplanar structure there are at most *three* equilibrium equations for each part, so that if there is a total of n parts and r force and moment reaction components, we have

$$\begin{aligned} r &= 3n, \text{ statically determinate} \\ r &> 3n, \text{ statically indeterminate} \end{aligned} \qquad (2\text{–}3)$$

In particular, if a structure is *statically indeterminate,* the additional equations needed to solve for the unknown reactions are obtained by relating the applied loads and reactions to the displacement or slope at different points on the structure. These equations, which are referred to as *compatibility equations,* must be equal in number to the *degree of indeterminacy* of the structure. Compatibility equations involve the geometric and physical properties of the structure and will be discussed further in Chapter 10.

We will now consider some examples to show how to classify the determinacy of a structure. The first example considers beams; the second example, pin-connected structures; and in the third we will discuss frame structures. Classification of trusses will be considered in Chapter 3.

*Drawing the free-body diagrams is not strictly necessary, since a "mental count" of the number of unknowns can also be made and compared with the number of equilibrium equations.

EXAMPLE 2-3

Classify each of the beams shown in Fig. 2–18*a* through 2–18*d* as statically determinate or statically indeterminate. If statically indeterminate, report the number of degrees of indeterminacy. The beams are subjected to external loadings that are assumed to be known and can act anywhere on the beams.

Solution

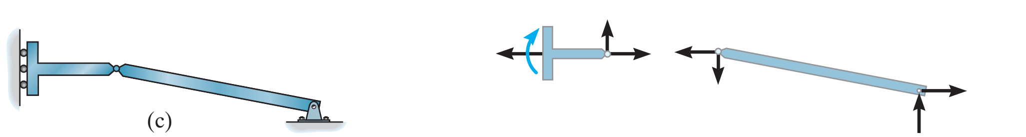
Compound beams, i.e., those in Fig. 2–18*c* and 2–18*d*, which are composed of pin-connected members must be disassembled. Note that in these cases, the unknown reactive forces acting between each member must be shown in equal but opposite pairs. The free-body diagrams of each member are shown. Applying $r = 3n$ or $r > 3n$, the resulting classifications are indicated.

(a)

$r = 3, n = 1, 3 = 3(1)$ Statically determinate ***Ans.***

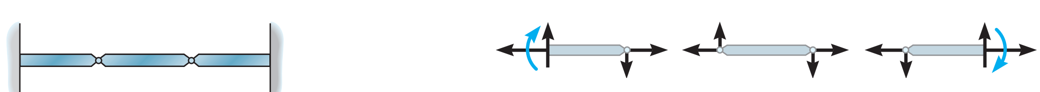
(b)

$r = 5, n = 1, 5 > 3(1)$ Statically indeterminate to the second degree ***Ans.***



(c)

$r = 6, n = 2, 6 = 3(2)$ Statically determinate ***Ans.***



(d)

$r = 10, n = 3, 10 > 3(3)$ Statically indeterminate to the first degree ***Ans.***

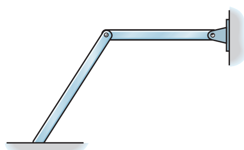
Fig. 2–18

EXAMPLE 2-4

Classify each of the pin-connected structures shown in Fig. 2–19*a* through 2–19*d* as statically determinate or statically indeterminate. If statically indeterminate, report the number of degrees of indeterminacy. The structures are subjected to arbitrary external loadings that are assumed to be known and can act anywhere on the structures.

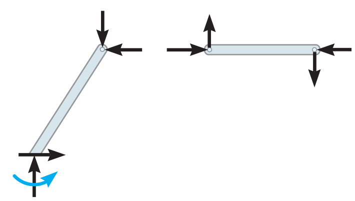
Solution

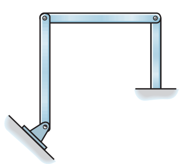
Classification of pin-connected structures is similar to that of beams. The free-body diagrams of the members are shown. Applying $r = 3n$ or $r > 3n$, the resulting classifications are indicated.



(a)

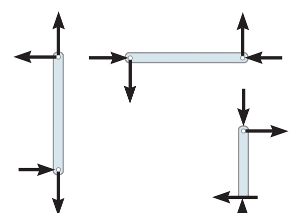
$r = 7,\ n = 2,\ 7 > 6$
Statically indeterminate to the first degree **Ans.**

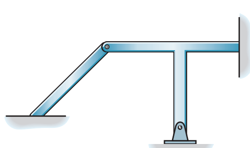




(b)

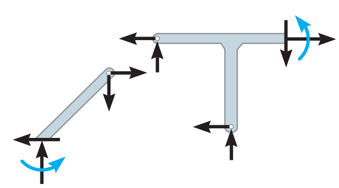
$r = 9,\ n = 3,\ 9 = 9,$
Statically determinate **Ans.**

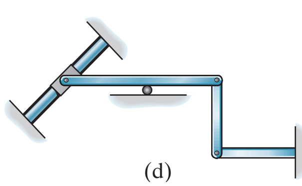




(c)

$r = 10,\ n = 2,\ 10 > 6,$
Statically indeterminate to the fourth degree **Ans.**





(d)

$r = 9,\ n = 3,\ 9 = 9,$
Statically determinate **Ans.**

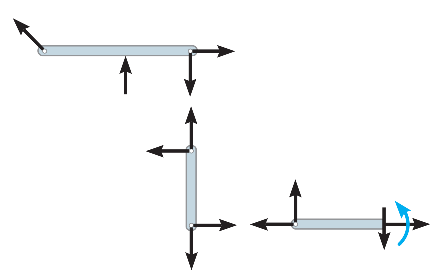


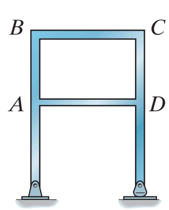
Fig. 2–19

EXAMPLE 2-5

Classify each of the frames shown in Fig. 2–20*a* and 2–20*b* as statically determinate or statically indeterminate. If statically indeterminate, report the number of degrees of indeterminacy. The frames are subjected to external loadings that are assumed to be known and can act anywhere on the frames.

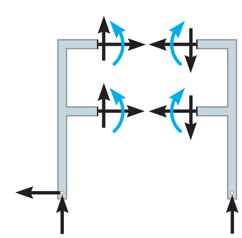
Solution

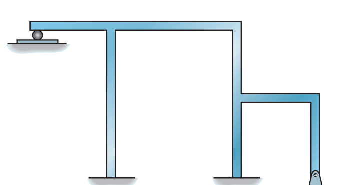
Unlike the beams and pin-connected structures of the previous examples, frame structures consist of members that are connected together by rigid joints. Sometimes the members form internal loops as in Fig. 2–20*a*. Here *ABCD* forms a closed loop. In order to classify these structures, it is necessary to use the method of sections and "cut" the loop apart. The free-body diagrams of the sectioned parts are drawn and the frame can then be classified. Notice that only *one section* through the loop is required, since once the unknowns at the section are determined, the internal forces at any point in the members can then be found using the method of sections and the equations of equilibrium. The frame in Fig. 2–20*b* has no closed loops and so it does not need to be sectioned between the supports when finding its determinancy. The resulting classifications are indicated in Fig. 2–20*a* and 2–20*b*.



(a)

$r = 9$, $n = 2$, $9 > 6$,
Statically indeterminate to the third degree ***Ans.***





(b)

(This frame has no closed loops.)

$r = 9$, $n = 1$, $9 > 3$,
Statically indeterminate to the sixth degree ***Ans.***

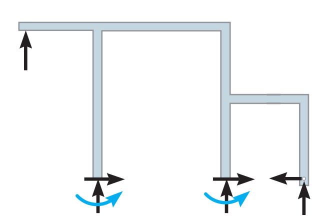


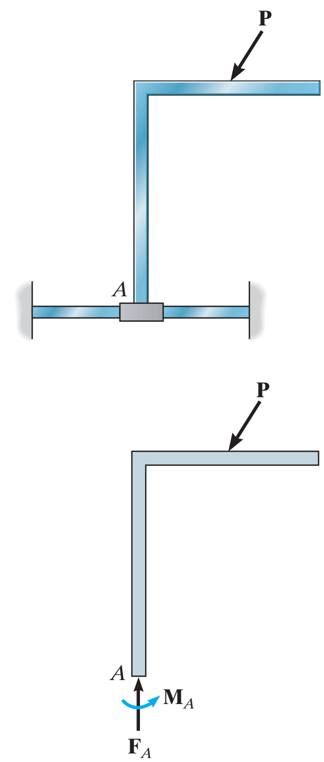
Fig. 2–20

Stability. To ensure the equilibrium of a structure or its members, it is not only necessary to satisfy the equations of equilibrium, but the members must also be properly held or constrained by their supports. Two situations may occur where the conditions for proper constraint have not been met.

Partial Constraints. In some cases a structure or one of its members may have *fewer* reactive forces than equations of equilibrium that must be satisfied. The structure then becomes only *partially constrained*. For example, consider the member shown in Fig. 2–21 with its corresponding free-body diagram. Here the equation $\Sigma F_x = 0$ will not be satisfied for the loading conditions and therefore the member will be unstable.

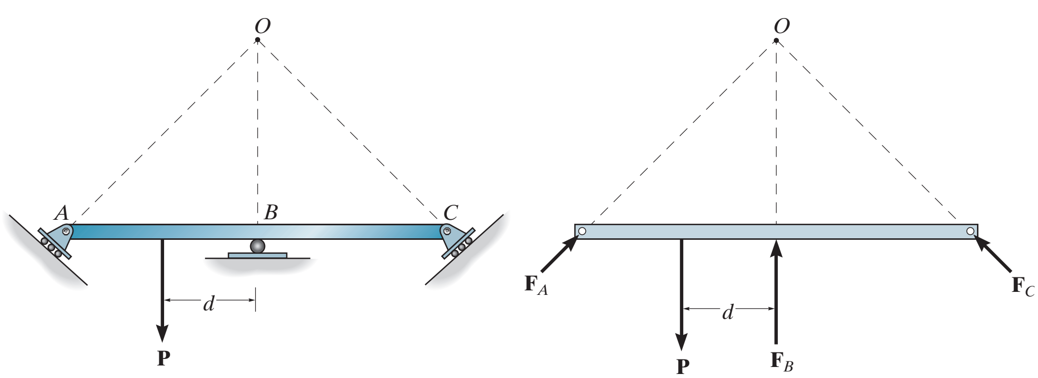
Improper Constraints. In some cases there may be as many unknown forces as there are equations of equilibrium; however, *instability* or movement of a structure or its members can develop because of *improper constraining* by the supports. This can occur if all the *support reactions are concurrent* at a point. An example of this is shown in Fig. 2–22. From the free-body diagram of the beam it is seen that the summation of moments about point O will *not* be equal to zero $(Pd \neq 0)$; thus rotation about point O will take place.

Another way in which improper constraining leads to instability occurs when the *reactive forces* are all *parallel*. An example of this case is shown in Fig. 2–23. Here when an inclined force **P** is applied, the summation of forces in the horizontal direction will not equal zero.



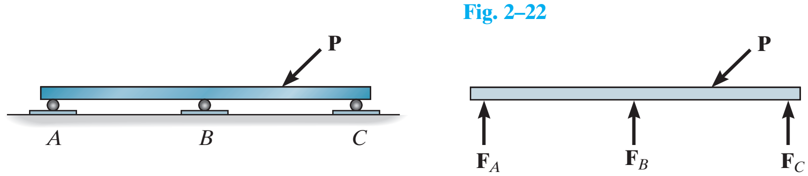
partial constraints

Fig. 2–21



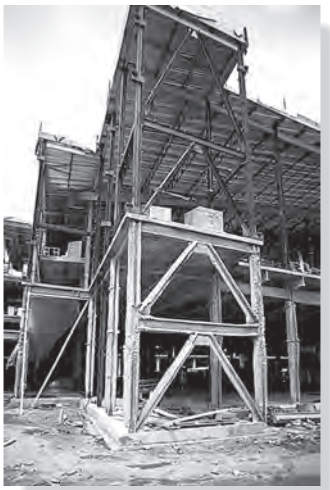
concurrent reactions

Fig. 2–22



parallel reactions

Fig. 2–23



The K-bracing on this frame provides stability, that is lateral support from wind and vertical support of the floor girders. Notice the use of concrete grout, which is applied to insulate the steel to keep it from losing its strength in the event of a fire.

In general, then, a structure will be geometrically unstable—that is, it will move slightly or collapse—if there are fewer reactive forces than equations of equilibrium; or if there are enough reactions, instability will occur if the lines of action of the reactive forces intersect at a common point or are parallel to one another. If the structure consists of several members or components, local instability of one or several of these members can generally be determined *by inspection*. If the members form a collapsible mechanism, the structure will be unstable. We will now formalize these statements for a *coplanar structure* having n members or components with r unknown reactions. Since three equilibrium equations are available for each member or component, we have

$r < 3n$	unstable	(2–4)
$r \geq 3n$	unstable if member reactions are concurrent or parallel or some of the components form a collapsible mechanism	

If the structure is unstable, *it does not matter* if it is statically determinate or indeterminate. In all cases such types of structures must be avoided in practice.

The following examples illustrate how structures or their members can be classified as stable or unstable. Structures in the form of a truss will be discussed in Chapter 3.

EXAMPLE 2-6

Classify each of the structures in Fig. 2–24*a* through 2–24*e* as stable or unstable. The structures are subjected to arbitrary external loads that are assumed to be known.

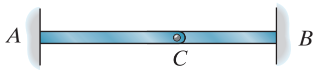
Solution

The structures are classified as indicated.

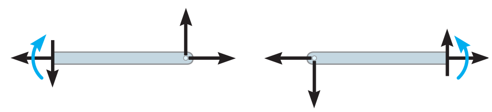
(a)

Fig. 2–24

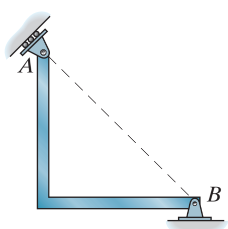
The member is *stable* since the reactions are nonconcurrent and nonparallel. It is also statically determinate. ***Ans.***



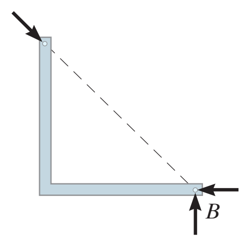
(b)



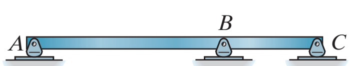
The compound beam is *stable*. It is also indeterminate to the second degree. ***Ans.***



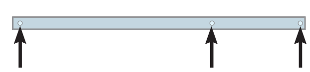
(c)



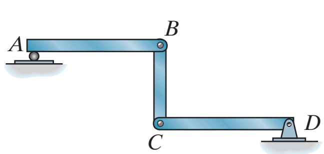
The member is *unstable* since the three reactions are concurrent at B. ***Ans.***



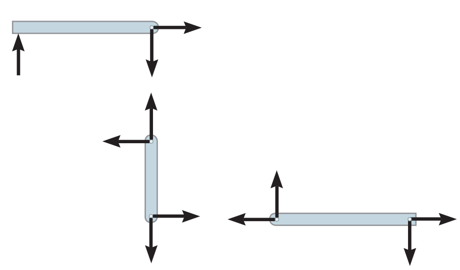
(d)



The beam is *unstable* since the three reactions are all parallel. ***Ans.***



(e)



The structure is *unstable* since $r = 7$, $n = 3$, so that, by Eq. 2–4, $r < 3n$, $7 < 9$. Also, this can be seen by inspection, since AB can move horizontally without restraint. ***Ans.***

2–5 Application of the Equations of Equilibrium

Occasionally, the members of a structure are connected together in such a way that the joints can be assumed as pins. Building frames and trusses are typical examples that are often constructed in this manner. Provided a pin-connected coplanar structure is properly constrained and contains no more supports or members than are necessary to prevent collapse, the forces acting at the joints and supports can be determined by applying the three equations of equilibrium $(\Sigma F_x = 0, \Sigma F_y = 0, \Sigma M_O = 0)$ to each member. Understandably, once the forces at the joints are obtained, the size of the members, connections, and supports can then be determined on the basis of design code specifications.

To illustrate the method of force analysis, consider the three-member frame shown in Fig. 2–25*a*, which is subjected to loads $\mathbf{P}_1$ and $\mathbf{P}_2$. The free-body diagrams of each member are shown in Fig. 2–25*b*. In total there are nine unknowns; however, nine equations of equilibrium can be written, three for each member, so the problem is *statically determinate*. For the actual solution it is *also* possible, and sometimes convenient, to consider a portion of the frame or its entirety when applying some of these nine equations. For example, a free-body diagram of the entire frame is shown in Fig. 2–25*c*. One could determine the three reactions $\mathbf{A}_x$, $\mathbf{A}_y$, and $\mathbf{C}_x$ on this "rigid" pin-connected system, then analyze *any two* of its members, Fig. 2–25*b*, to obtain the other six unknowns. Furthermore, the answers can be checked in part by applying the three equations of equilibrium to the remaining "third" member. To summarize, this problem can be solved by writing *at most* nine equilibrium equations using free-body diagrams of any members and/or combinations of connected members. Any more than nine equations written would *not* be unique from the original nine and would only serve to check the results.

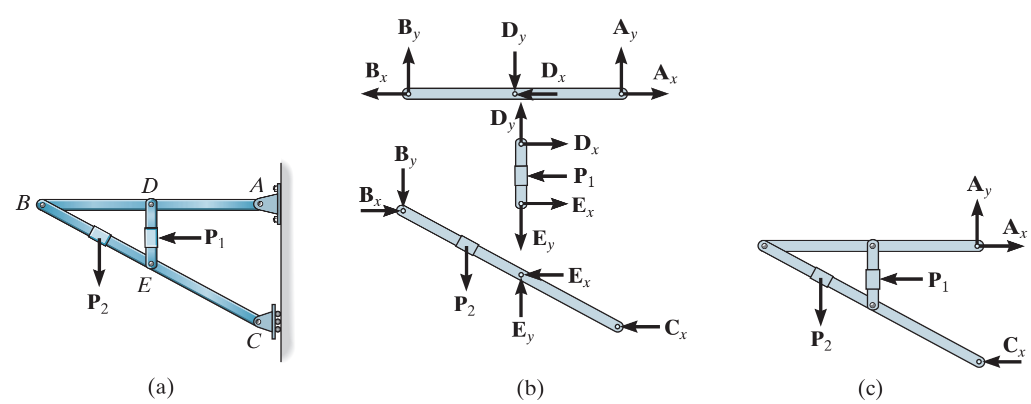


Fig. 2–25

Consider now the two-member frame shown in Fig. 2–26*a*. Here the free-body diagrams of the members reveal six unknowns, Fig. 2–26*b*; however, six equilibrium equations, three for each member, can be written, so again the problem is statically determinate. As in the previous case, a free-body diagram of the entire frame can also be used for part of the analysis, Fig. 2–26*c*. Although, as shown, the frame has a tendency to collapse without its supports, by rotating about the pin at B, this will not happen since the force system acting on it must still hold it in equilibrium. Hence, if so desired, all six unknowns can be determined by applying the three equilibrium equations to the entire frame, Fig. 2–26*c*, and also to either one of its members.

The above two examples illustrate that if a structure is properly supported and contains no more supports or members than are necessary to prevent collapse, the frame becomes statically determinate, and so the unknown forces at the supports and connections can be determined from the equations of equilibrium applied to each member. Also, if the structure remains *rigid* (noncollapsible) when the supports are removed (Fig. 2–25*c*), all three support reactions can be determined by applying the three equilibrium equations to the entire structure. However, if the structure appears to be nonrigid (collapsible) after removing the supports (Fig. 2–26*c*), it must be dismembered and equilibrium of the individual members must be considered in order to obtain enough equations to determine *all* the support reactions.

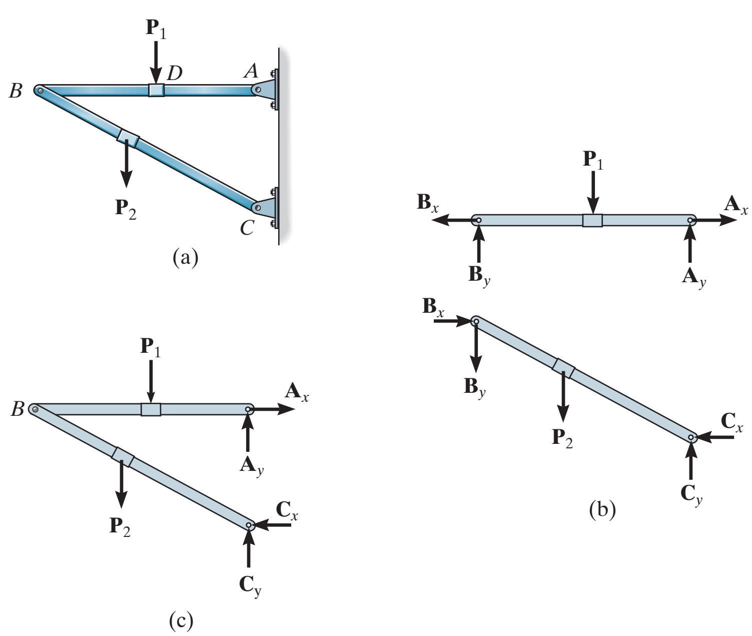


Fig. 2–26

PROCEDURE FOR ANALYSIS

The following procedure provides a method for determining the *joint reactions* for structures composed of pin-connected members.

Free-Body Diagrams

- Disassemble the structure and draw a free-body diagram of each member. Also, it may be convenient to supplement a member free-body diagram with a free-body diagram of the *entire structure*. Some or all of the support reactions can then be determined using this diagram.
- Recall that reactive forces common to two members act with equal magnitudes but opposite directions on the respective free-body diagrams of the members.
- All two-force members should be identified. These members, regardless of their shape, have no external loads on them, and therefore their free-body diagrams are represented with equal but opposite collinear forces acting on their ends.
- In many cases it is possible to tell by inspection the proper arrowhead sense of direction of an unknown force or couple moment; however, if this seems difficult, the directional sense can be assumed.

Equations of Equilibrium

- Count the total number of unknowns to make sure that an equivalent number of equilibrium equations can be written for solution. Except for two-force members, recall that in general three equilibrium equations can be written for each member.
- Many times, the solution for the unknowns will be straightforward if the moment equation $\Sigma M_O = 0$ is applied about a point (O) that lies at the intersection of the lines of action of as many unknown forces as possible.
- When applying the force equations $\Sigma F_x = 0$ and $\Sigma F_y = 0$, orient the x and y axes along lines that will provide the simplest reduction of the forces into their x and y components.
- If the solution of the equilibrium equations yields a *negative* magnitude for an unknown force or couple moment, it indicates that its arrowhead sense of direction is *opposite* to that which was assumed on the free-body diagram.

EXAMPLE 2-7

Determine the reactions on the beam shown in Fig. 2–27*a*.

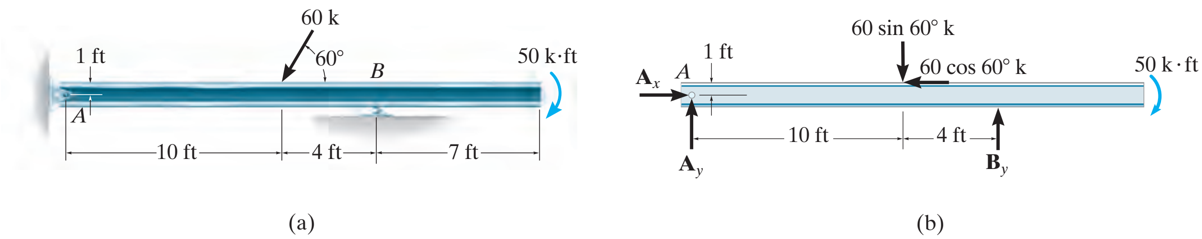


Fig. 2–27

Solution

Free-Body Diagram. As shown in Fig. 2–27*b*, the 60-k force is resolved into *x* and *y* components. Furthermore, the 7-ft dimension line is not needed since a couple moment is a *free vector* and can therefore act anywhere on the beam for the purpose of computing the external reactions.

Equations of Equilibrium. Applying Eqs. 2–2 in a sequence, using previously calculated results, we have

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad A_x - 60 \cos 60° = 0 \quad A_x = 30.0 \text{ k}$ ***Ans.***

$\circlearrowleft + \Sigma M_A = 0; \quad -60 \sin 60°(10) + 60 \cos 60°(1) + B_y(14) - 50 = 0 \quad B_y = 38.5 \text{ k}$ ***Ans.***

$+\uparrow \Sigma F_y = 0; \quad -60 \sin 60° + 38.5 + A_y = 0 \quad A_y = 13.4 \text{ k}$ ***Ans.***

EXAMPLE 2-8

Determine the reactions on the beam in Fig. 2–28*a*.

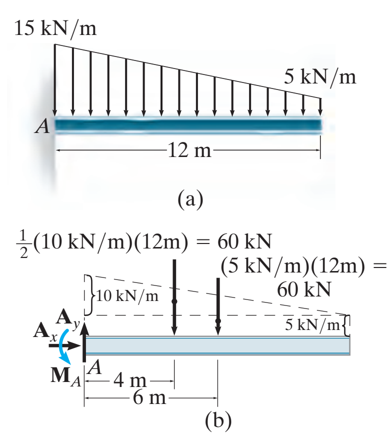


Fig. 2–28

Solution

Free-Body Diagram. As shown in Fig. 2–28*b*, the trapezoidal distributed loading is segmented into a triangular and uniform load. The *areas* under the triangle and rectangle represent the *resultant* forces. These forces act through the centroid of their corresponding areas.

Equations of Equilibrium

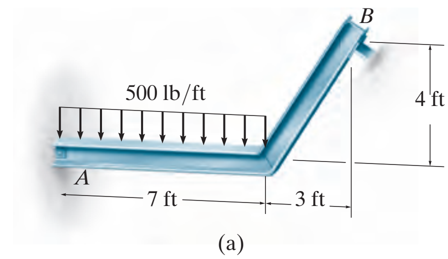
$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad A_x = 0$ ***Ans.***

$+\uparrow \Sigma F_y = 0; \quad A_y - 60 - 60 = 0 \quad A_y = 120 \text{ kN}$ ***Ans.***

$\circlearrowleft + \Sigma M_A = 0; \quad -60(4) - 60(6) + M_A = 0 \quad M_A = 600 \text{ kN} \cdot \text{m}$ ***Ans.***

EXAMPLE 2-9

Determine the reactions on the beam in Fig. 2–29*a*. Assume A is a pin and the support at B is a roller (smooth surface).

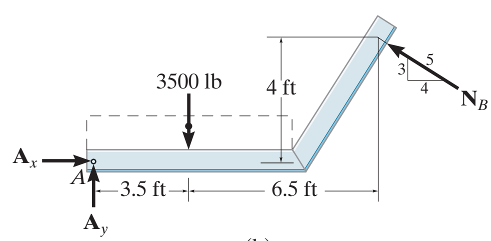


(a)

Fig. 2–29

Solution

Free-Body Diagram. As shown in Fig. 2–29*b*, the support ("roller") at B exerts a *normal force* on the beam at its point of contact. The line of action of this force is defined by the 3–4–5 triangle.



(b)

Equations of Equilibrium. Resolving $\mathbf{N}_B$ into x and y components and summing moments about A yields a direct solution for N_B. Why? Using this result, we can then obtain A_x and A_y.

$$\circlearrowleft + \Sigma M_A = 0; \quad -3500(3.5) + \left(\tfrac{4}{5}\right)N_B(4) + \left(\tfrac{3}{5}\right)N_B(10) = 0 \qquad \textit{Ans.}$$

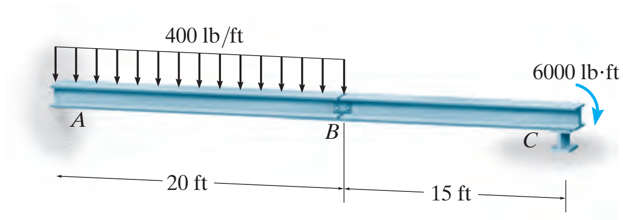
$$N_B = 1331.5 \text{ lb} = 1.33 \text{ k}$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad A_x - \tfrac{4}{5}(1331.5) = 0 \qquad A_x = 1.07 \text{ k} \qquad \textit{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \quad A_y - 3500 + \tfrac{3}{5}(1331.5) = 0 \qquad A_y = 2.70 \text{ k} \qquad \textit{Ans.}$$

EXAMPLE 2-10

The compound beam in Fig. 2–30*a* is fixed at *A*. Determine the reactions at *A*, *B*, and *C*. Assume that the connection at *B* is a pin and *C* is a roller.

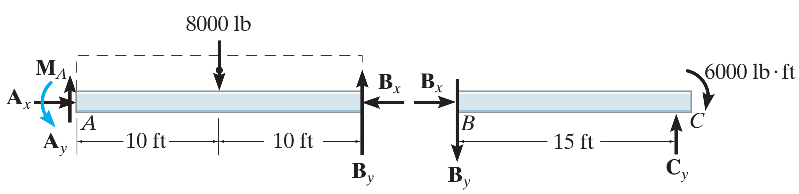


(a)

Fig. 2–30

Solution

Free-Body Diagrams. The free-body diagram of each segment is shown in Fig. 2–30*b*. Why is this problem statically determinate?



(b)

Equations of Equilibrium. There are six unknowns. Applying the six equations of equilibrium, using previously calculated results, we have

Segment BC:

$\circlearrowleft + \Sigma M_C = 0;\quad -6000 + B_y(15) = 0 \qquad B_y = 400\text{ lb}$ ***Ans.***

$+\uparrow \Sigma F_y = 0;\quad -400 + C_y = 0 \qquad C_y = 400\text{ lb}$ ***Ans.***

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad B_x = 0$ ***Ans.***

Segment AB:

$\circlearrowleft + \Sigma M_A = 0;\quad M_A - 8000(10) + 400(20) = 0$

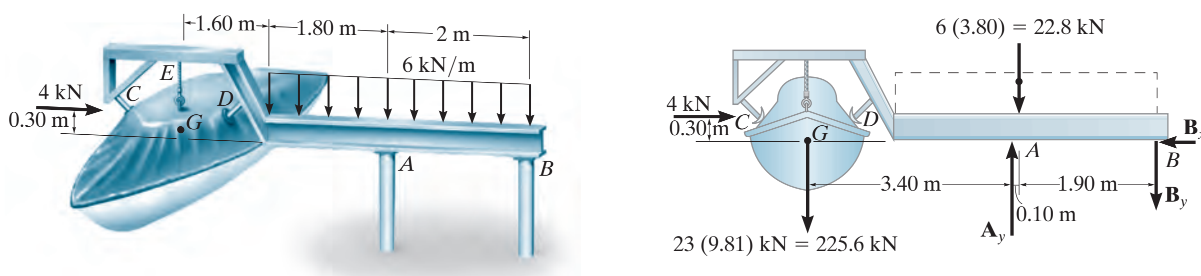
$M_A = 72.0\text{ k}\cdot\text{ft}$ ***Ans.***

$+\uparrow \Sigma F_y = 0;\quad A_y - 8000 + 400 = 0 \qquad A_y = 7.60\text{ k}$ ***Ans.***

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad A_x - 0 = 0 \qquad A_x = 0$ ***Ans.***

EXAMPLE 2-11

The side girder shown in the photo supports the boat and deck. An idealized model of this girder is shown in Fig. 2–31*a*, where it can be assumed *A* is a roller and *B* is a pin. Using a local code the anticipated deck loading transmitted to the girder is 6 kN/m. Wind exerts a *resultant* horizontal force of 4 kN as shown, and the mass of the boat that is supported by the girder is 23 Mg. The boat's mass center is at *G*. Determine the reactions at the supports.



(a) (b)

Fig. 2–31

Solution

Free-Body Diagram. Here we will consider the boat and girder as a single system, Fig. 2–31*b*. As shown, the distributed loading has been replaced by its resultant.

Equations of Equilibrium. Applying Eqs. 2–2 in sequence, using previously calculated results, we have

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \quad 4 - B_x = 0$$

$$B_x = 4 \text{ kN} \qquad \textit{Ans.}$$

$$\circlearrowleft + \Sigma M_B = 0; \quad 22.8(1.90) - A_y(2) + 225.6(5.40) - 4(0.30) = 0$$

$$A_y = 630.2 \text{ kN} = 630 \text{ kN} \qquad \textit{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \quad 630.2 - B_y - 22.8 - 225.6 = 0$$

$$B_y = 382 \text{ kN} \qquad \textit{Ans.}$$

Note: If the girder alone had been considered for this analysis then the normal forces at the shoes *C* and *D* would have to first be calculated using a free-body diagram of the boat. (These forces exist if the cable pulls the boat snug against them.) Equal but opposite normal forces along with the cable force at *E* would then act on the girder when its free-body diagram is considered. The same results would have been obtained; however, by considering the boat-girder system, these normal forces and the cable force become internal and do not have to be considered.

EXAMPLE 2-12

Determine the horizontal and vertical components of reaction at the pins A, B, and C of the two-member frame shown in Fig. 2–32*a*.

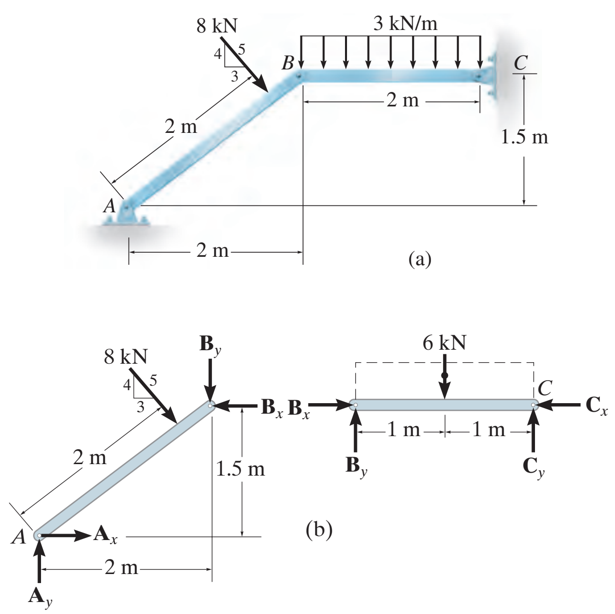


Fig. 2–32

Solution

Free-Body Diagrams. The free-body diagram of each member is shown in Fig. 2–32*b*.

Equations of Equilibrium. Applying the six equations of equilibrium in the following sequence allows a direct solution for each of the six unknowns.

Member BC:

$\circlearrowleft +\Sigma M_C = 0; \quad -B_y(2) + 6(1) = 0 \qquad B_y = 3 \text{ kN}$ ***Ans.***

Member AB:

$\circlearrowleft +\Sigma M_A = 0; \quad -8(2) - 3(2) + B_x(1.5) = 0 \qquad B_x = 14.7 \text{ kN}$ ***Ans.***

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad A_x + \frac{3}{5}(8) - 14.7 = 0 \qquad A_x = 9.87 \text{ kN}$ ***Ans.***

$+\uparrow \Sigma F_y = 0; \quad A_y - \frac{4}{5}(8) - 3 = 0 \qquad A_y = 9.40 \text{ kN}$ ***Ans.***

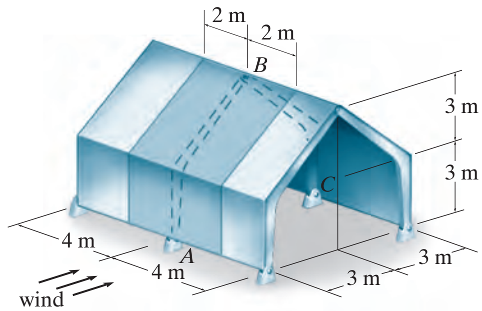
Member BC:

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 14.7 - C_x = 0 \qquad C_x = 14.7 \text{ kN}$ ***Ans.***

$+\uparrow \Sigma F_y = 0; \quad 3 - 6 + C_y = 0 \qquad C_y = 3 \text{ kN}$ ***Ans.***

EXAMPLE 2-13

The side of the building in Fig. 2–33*a* is subjected to a wind loading that creates a uniform *normal* pressure of 15 kPa on the windward side and a suction pressure of 5 kPa on the leeward side. Determine the horizontal and vertical components of reaction at the pin connections *A*, *B*, and *C* of the supporting gable arch.

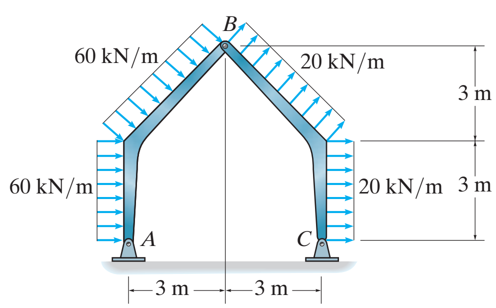


(a)

Fig. 2–33

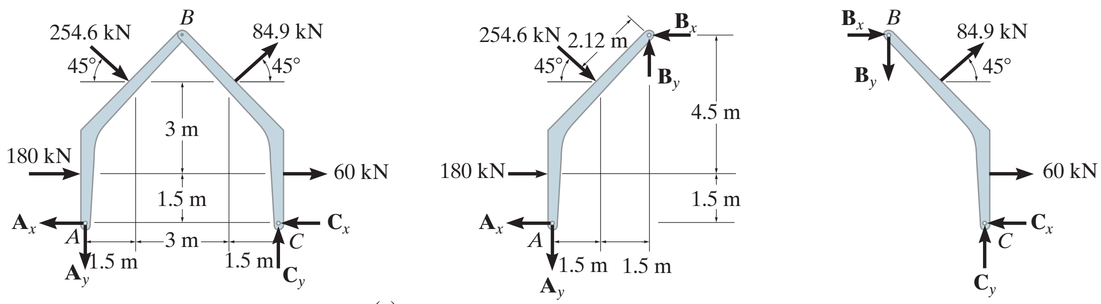
Solution

Since the loading is evenly distributed, the central gable arch supports a loading acting on the walls and roof of the dark-shaded tributary area. This represents a uniform distributed load of $(15\ \text{kN/m}^2)(4\ \text{m}) = 60\ \text{kN/m}$ on the windward side and $(5\ \text{kN/m}^2)(4\ \text{m}) = 20\ \text{kN/m}$ on the suction side, Fig. 2–33*b*.



(b)

Free-Body Diagrams. Simplifying the distributed loadings, the free-body diagrams of the entire frame and each of its parts are shown in Fig. 2–33*c*.



(c)

Equations of Equilibrium. Simultaneous solution of equations is avoided by applying the equilibrium equations in the following sequence using previously computed results.*

Entire Frame:

$$\circlearrowleft + \Sigma M_A = 0; \; -(180 + 60)(1.5) - (254.6 + 84.9)\cos 45°(4.5)$$
$$- (254.6 \sin 45°)(1.5) + (84.9 \sin 45°)(4.5) + C_y(6) = 0$$
$$C_y = 240.0 \text{ kN} \qquad \textit{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \quad -A_y - 254.6 \sin 45° + 84.9 \sin 45° + 240.0 = 0$$
$$A_y = 120.0 \text{ kN} \qquad \textit{Ans.}$$

Member *AB*:

$$\circlearrowleft + \Sigma M_B = 0; \quad -A_x(6) + 120.0(3) + 180(4.5) + 254.6(2.12) = 0$$
$$A_x = 285.0 \text{ kN} \qquad \textit{Ans.}$$
$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad -285.0 + 180 + 254.6 \cos 45° - B_x = 0$$
$$B_x = 75.0 \text{ kN} \qquad \textit{Ans.}$$
$$+\uparrow \Sigma F_y = 0; \quad -120.0 - 254.6 \sin 45° + B_y = 0$$
$$B_y = 300.0 \text{ kN} \qquad \textit{Ans.}$$

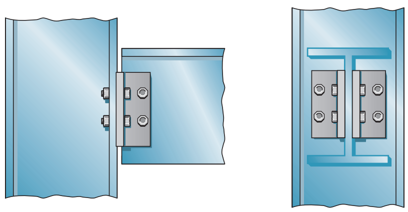
Member *CB*:

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad -C_x + 60 + 84.9 \cos 45° + 75.0 = 0$$
$$C_x = 195.0 \text{ kN} \qquad \textit{Ans.}$$

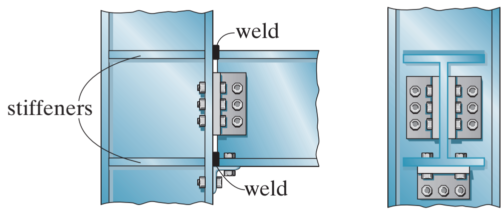
*The problem can also be solved by applying the six equations of equilibrium only to the two members. If this is done, it is best to first sum moments about point *A* on member *AB*, then point *C* on member *CB*. By doing this one obtains two equations to be solved simultaneously for B_x and B_y.

Chapter Review

Supports—Structural members are often assumed to be pin connected if only slight relative rotation can occur between them, and fixed connected if no rotation is possible.

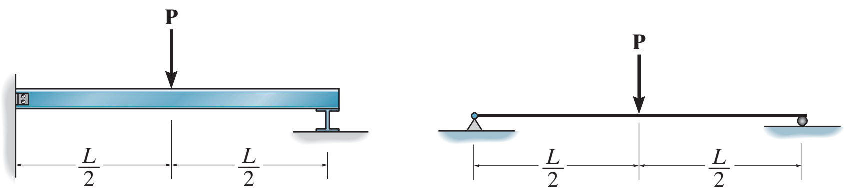


typical "pin-supported" connection (metal)

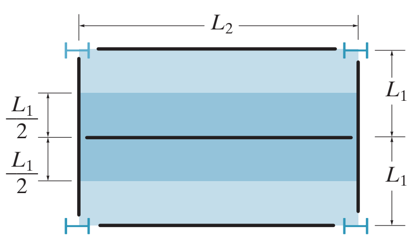


typical "fixed-supported" connection (metal)

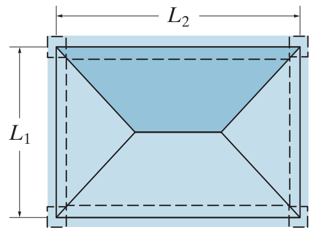
Idealized Structures—By making assumptions about the supports and connections as being either hinged, pinned, or fixed, the members can then be represented as lines, so that we can establish an idealized model that can be used for analysis.



The tributary loadings on slabs can be determined by first classifying the slab as a one-way or two-way slab. As a general rule, if L_2 is the largest dimension, and $L_2/L_1 \geq 1.5$, the slab will behave as a one-way slab. If $L_2/L_1 < 1.5$, the slab will behave as a two-way slab.



one-way slab action
requires $L_2/L_1 \geq 1.5$



two-way slab action
requires $L_2/L_1 < 1.5$

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Principle of Superposition—Either the loads or displacements can be added together provided the material behaves elastically and only small displacements of the structure occur.

Equilibrium—Statically determinate structures can be analyzed by disassembling them and applying the equations of equilibrium to each member. The analysis of a statically determinate structure requires first drawing the free-body diagrams of all the members, and then applying the equations of equilibrium to each member.

$$\sum F_x = 0$$
$$\sum F_y = 0$$
$$\sum M_O = 0$$

The number of equations of equilibrium for all n members of a structure is $3n$. If the structure has r reactions, then the structure is *statically determinate* if

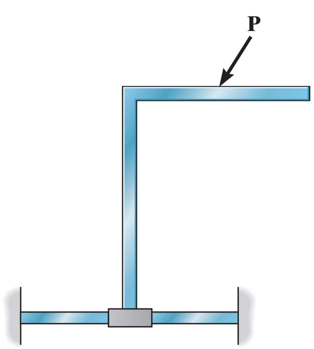
$$r = 3n$$

and *statically indeterminate* if

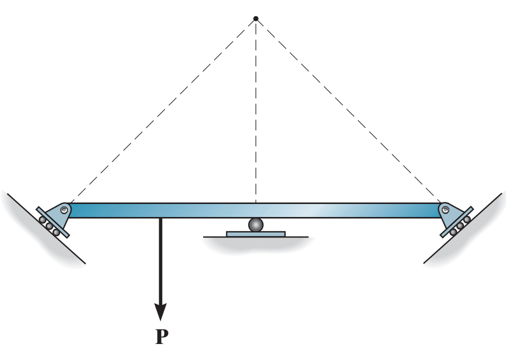
$$r > 3n$$

The additional number of equations required for the solution refers to the degree of indeterminacy.

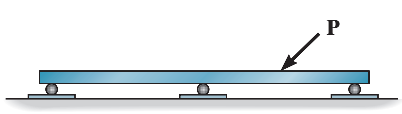
Stability—If there are fewer reactions than equations of equilibrium, then the structure will be unstable because it is partially constrained. Instability due to improper constraints can also occur if the lines of action of the reactions are concurrent at a point or parallel to one another.



partial constraint



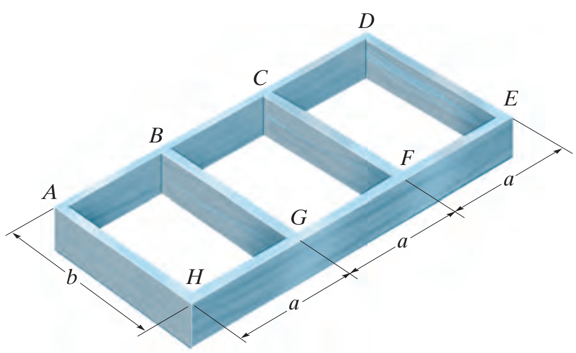
concurrent reactions



parallel reactions

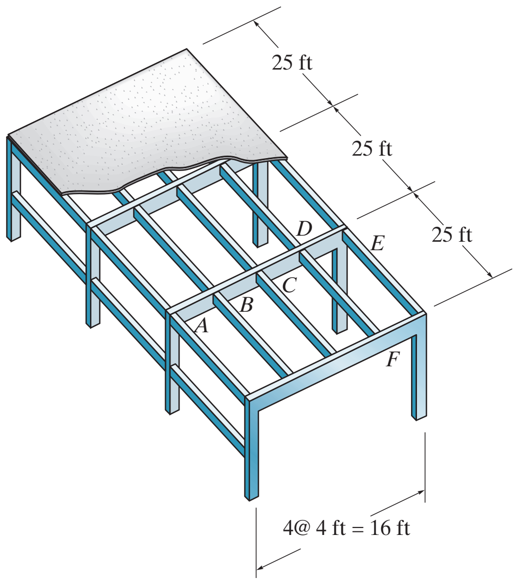
PROBLEMS

2–1. The frame is used to support a wood deck (not shown) that is to be subjected to a uniform load of 130 lb/ft^2. Sketch the loading that acts along members BG and $ABCD$. Take $b = 10$ ft, $a = 5$ ft.



Prob. 2–1

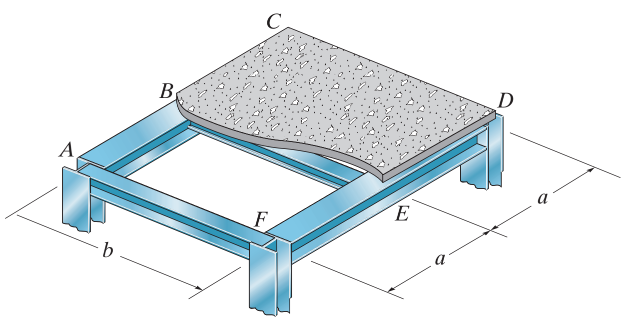
2–2. The roof deck of the single story building is subjected to a dead plus live load of 125 lb/ft^2. If the purlins are spaced 4 ft and the bents are spaced 25 ft apart, determine the distributed loading that acts along the purlin DF, and the loadings that act on the bent at A, B, C, D, and E.



Prob. 2–2

2–3. The steel framework is used to support the 4-in. reinforced lightweight concrete slab that carries a uniform live loading of 500 lb/ft^2. Sketch the loading that acts along members BE and FD. Set $b = 10$ ft, $a = 7.5$ ft. *Hint:* See Table 1–3.

***2–4.** Solve Prob. 2–3, with $b = 12$ ft, $a = 4$ ft.

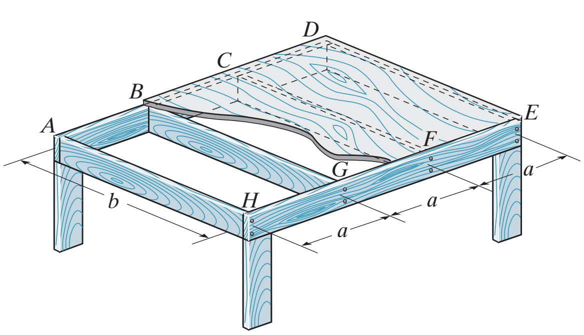


Probs. 2–3/2–4

2–5. The frame is used to support the wood deck in a residential dwelling. Sketch the loading that acts along members BG and $ABCD$. Set $b = 10$ ft, $a = 5$ ft. *Hint:* See Table 1–4.

2–6. Solve Prob. 2–5 if $b = 8$ ft, $a = 8$ ft.

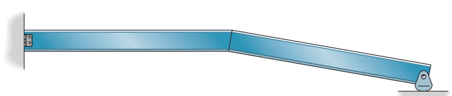
2–7. Solve Prob. 2–5 if $b = 15$ ft, $a = 10$ ft.



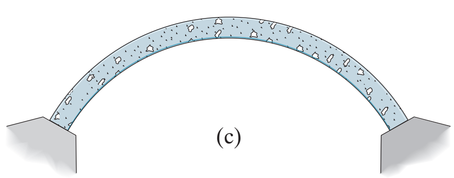
Probs. 2–5/2–6/2–7

***2–8.** Classify each of the structures as statically determinate, statically indeterminate, stable, or unstable. If indeterminate, specify the degree of indeterminacy.

(a)



(b)

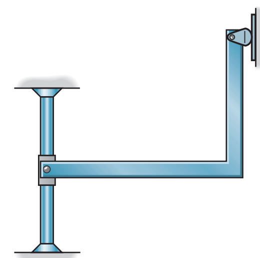


(c)

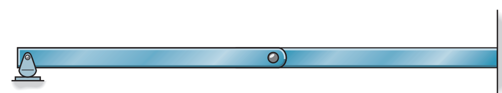
(d)

Prob. 2–8

2–9. Classify each of the structures as statically determinate, statically indeterminate, or unstable. If indeterminate, specify the degree of indeterminacy.



(a)



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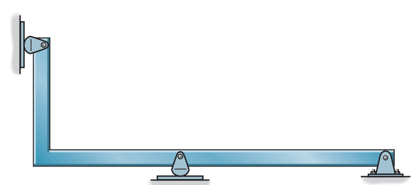

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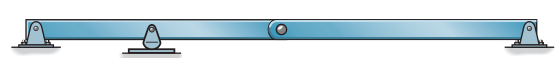
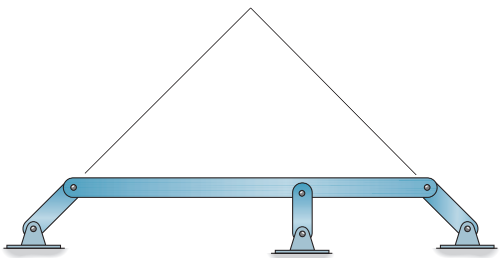


(d)

Prob. 2–9

2–10. Classify each of the structures as statically determinate, statically indeterminate, or unstable. If indeterminate, specify the degree of indeterminacy.



(a)

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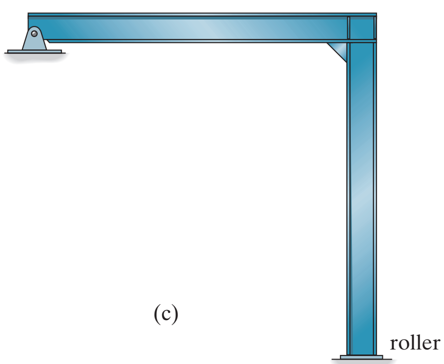

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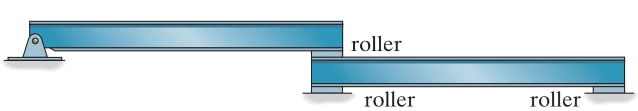
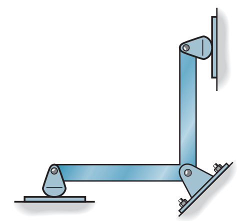


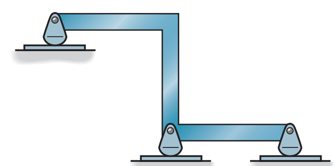
(d)

Prob. 2–10

2–11. Classify each of the structures as statically determinate, statically indeterminate, or unstable. If indeterminate, specify the degree of indeterminacy.



(a)



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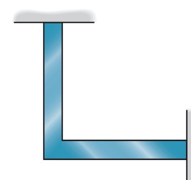

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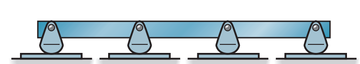
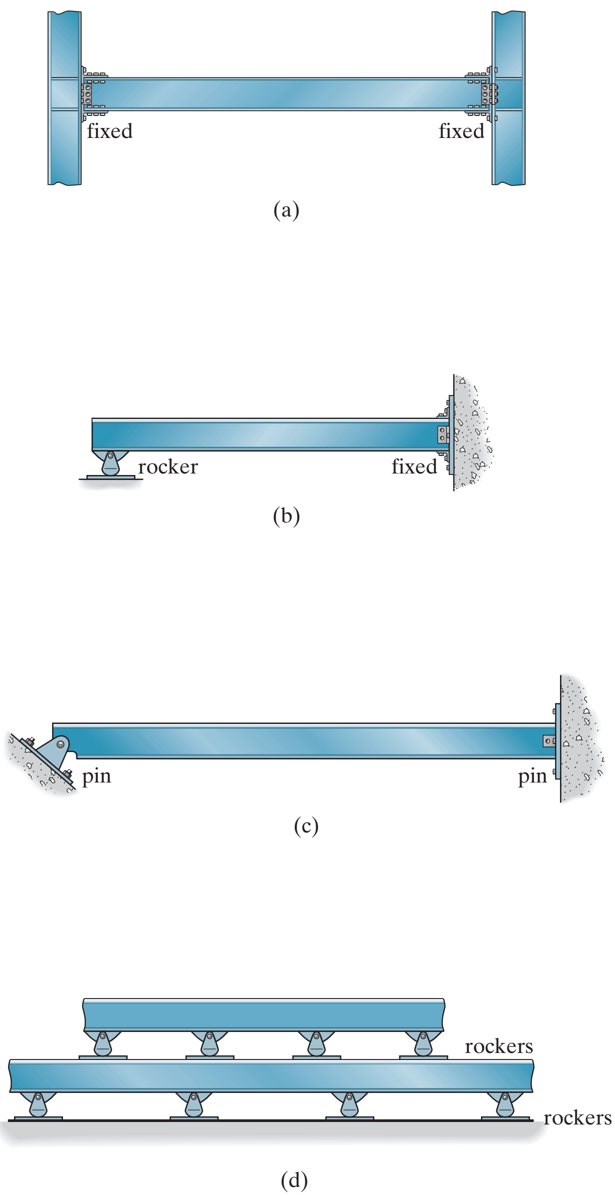


(d)

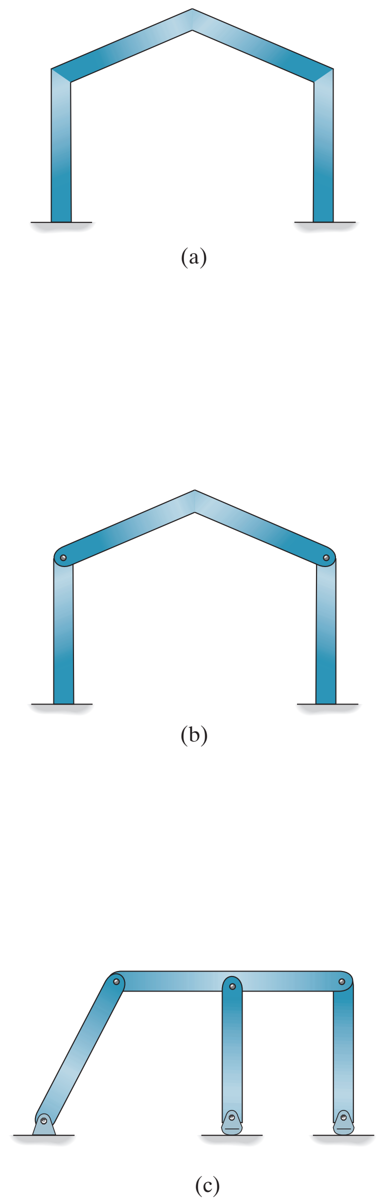
Prob. 2–11

***2–12.** Classify each of the structures as statically determinate, statically indeterminate, or unstable. If indeterminate, specify the degree of indeterminacy. The supports or connections are to be assumed as stated.



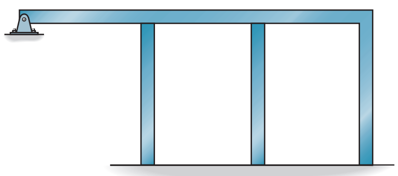
Prob. 2–12

2–13. Classify each of the structures as statically determinate statically indeterminate, or unstable. If indeterminate, specify the degree of indeterminacy.

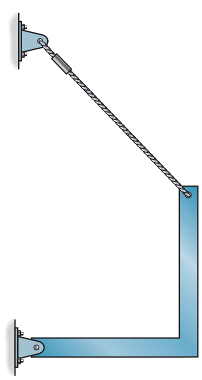


Prob. 2–13

2–14. Classify each of the frames as statically determinate, statically indeterminate, or unstable. If indeterminate, specify the degree of indeterminacy. All internal joints are fixed connected.



(a)



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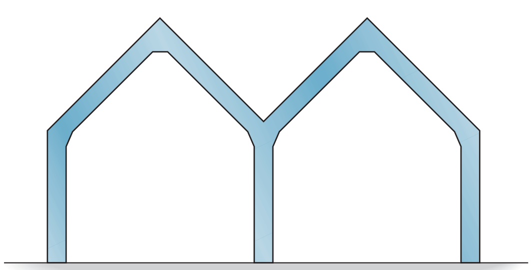

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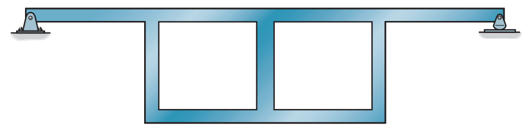

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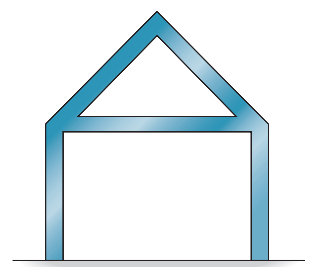
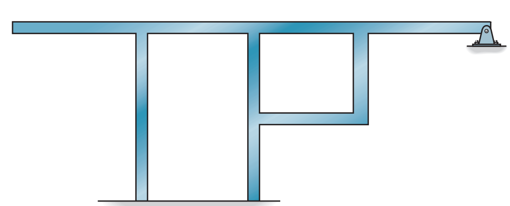


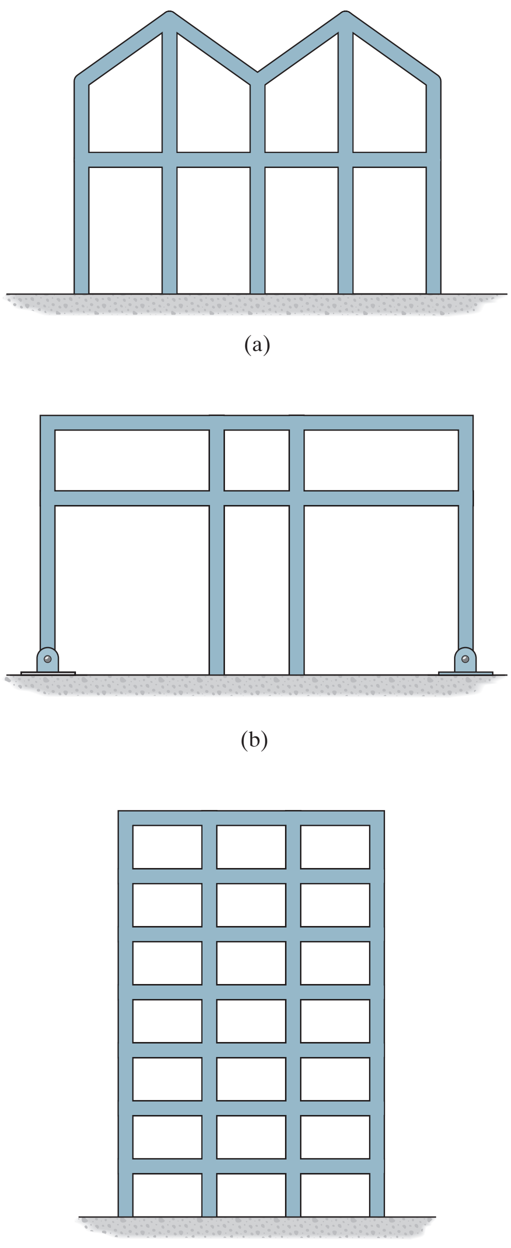
(e)



(f)

Prob. 2–14

2–15. Determine the degree to which the frames are statically indeterminate. All internal joints are fixed connected.



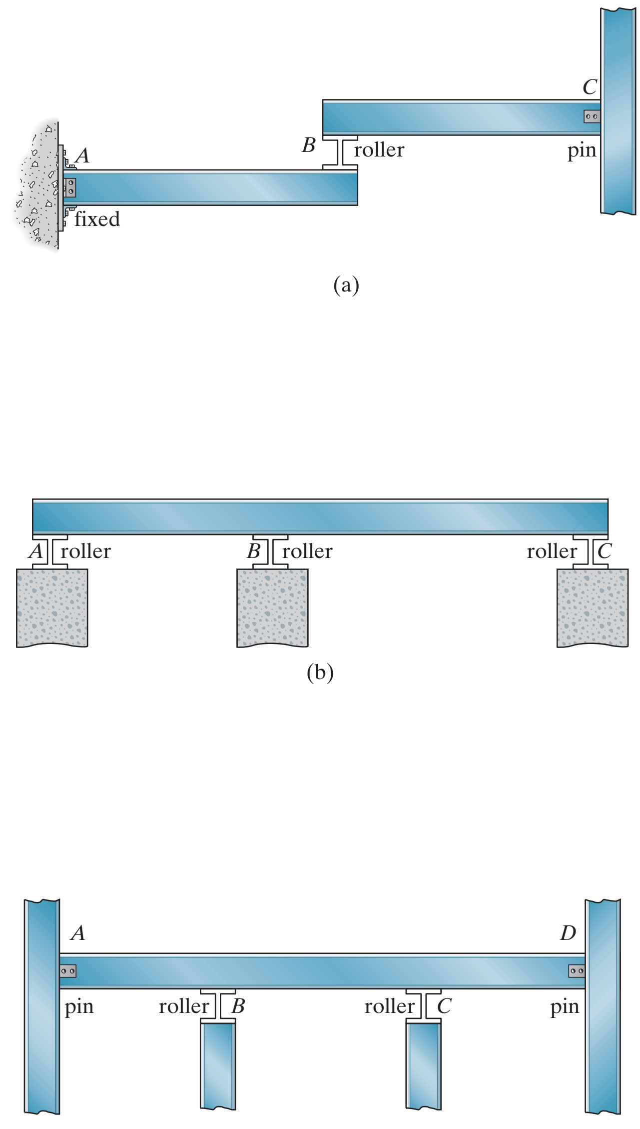
(a)

(b)

(c)

Prob. 2–15

***2–16.** Classify each of the structures as statically determinate, statically indeterminate, or unstable. If indeterminate, specify the degree of indeterminacy. The supports or connections are to be assumed as stated.



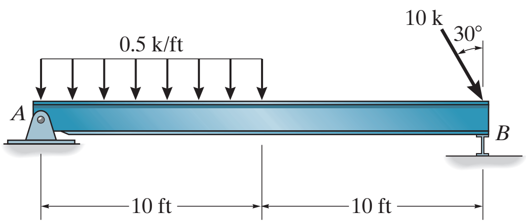
(a)

(b)

(c)

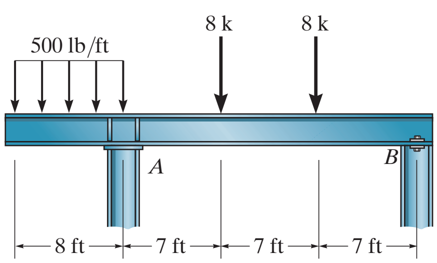
Prob. 2–16

2–17. Determine the reactions on the beam. The support at *B* can be assumed to be a roller. Neglect the thickness of the beam.



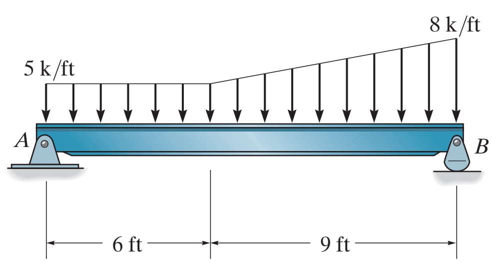
Prob. 2–17

2–18. Determine the reactions at the supports *A* and *B*. Assume *A* is a roller and *B* is a pin.



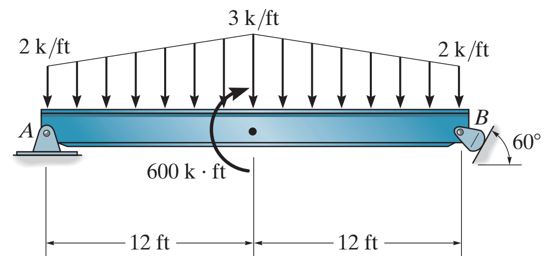
Prob. 2–18

2–19. Determine the reactions on the beam.



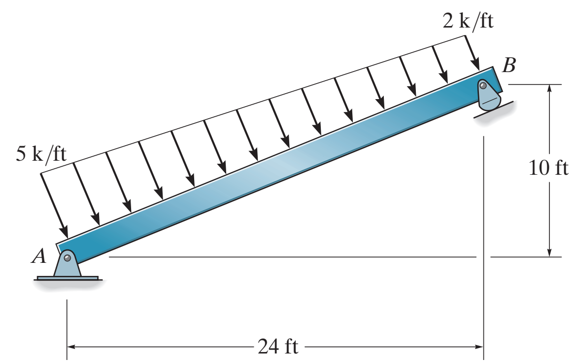
Prob. 2–19

***2–20.** Determine the reactions on the beam.



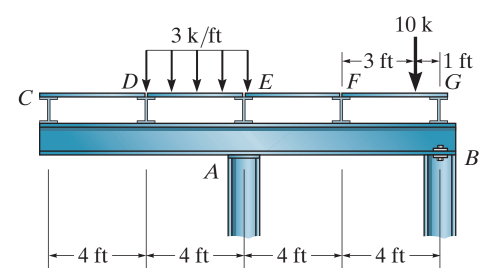
Prob. 2–20

2–21. Determine the reactions on the beam.



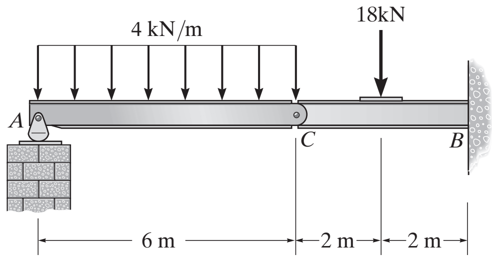
Prob. 2–21

2–22. Determine the reactions at the supports *A* and *B*. The floor decks *CD*, *DE*, *EF*, and *FG* transmit their loads to the girder on smooth supports. Assume *A* is a roller and *B* is a pin.



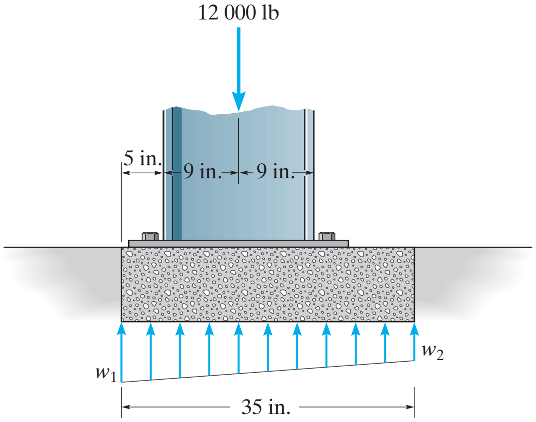
Prob. 2–22

2–23. Determine the reactions at the supports *A* and *B* of the compound beam. There is a pin at *C*.



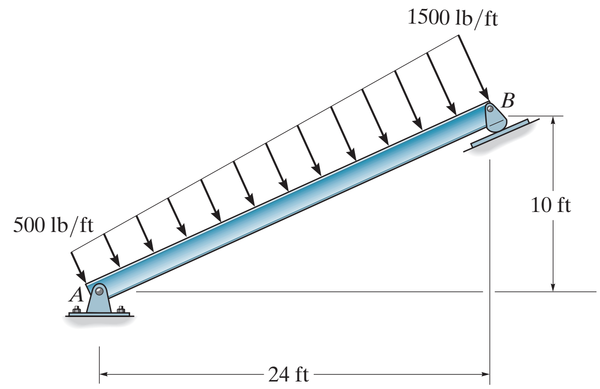
Prob. 2–23

***2–24.** The pad footing is used to support the load of 12 000 lb. Determine the intensities w_1 and w_2 of the distributed loading acting on the base of the footing for equilibrium.



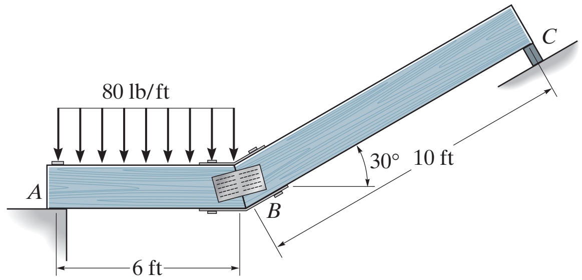
Prob. 2–24

2–25. Determine the reactions on the beam.



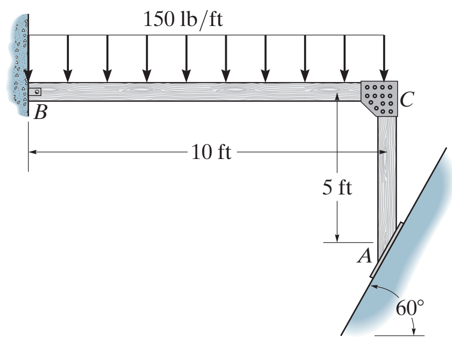
Prob. 2–25

2–26. Determine the reactions at the smooth support *C* and pinned support *A*. Assume the connection at *B* is fixed connected.



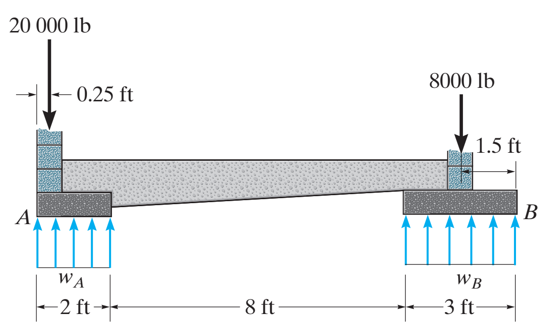
Prob. 2–26

2–27. Determine the reactions at the smooth support A and pin support B. The connection at C is fixed.



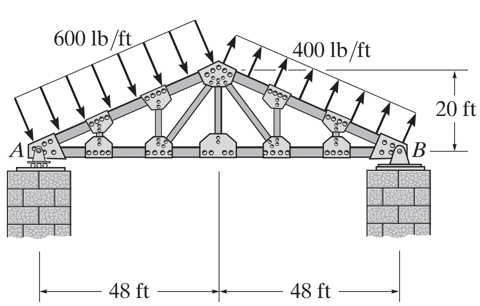
Prob. 2–27

***2–28.** The cantilever footing is used to support a wall near its edge A so that it causes a uniform soil pressure under the footing. Determine the uniform distribution loads, w_A and w_B, measured in lb/ft at pads A and B, necessary to support the wall forces of 8000 lb and 20 000 lb.



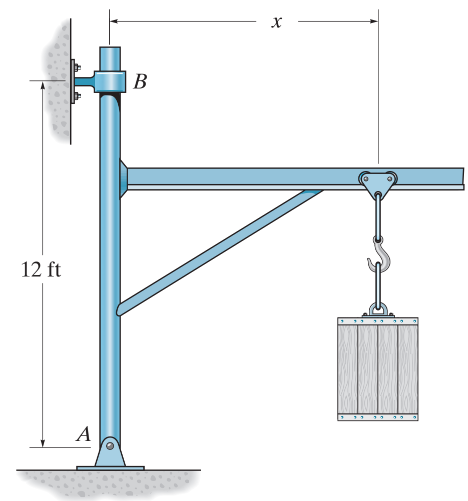
Prob. 2–28

2–29. Determine the reactions at the truss supports A and B. The distributed loading is caused by wind.



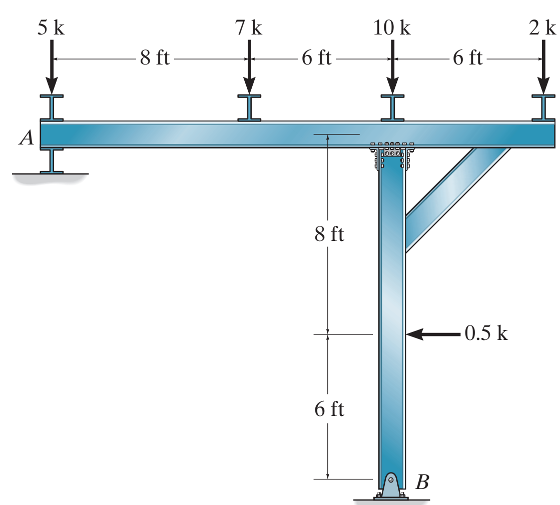
Prob. 2–29

2–30. The jib crane is pin-connected at A and supported by a smooth collar at B. Determine the roller placement x of the 5000-lb load so that it gives the maximum and minimum reactions at the supports. Calculate these reactions in each case. Neglect the weight of the crane. Require $4\text{ ft} \le x \le 10\text{ ft}$.



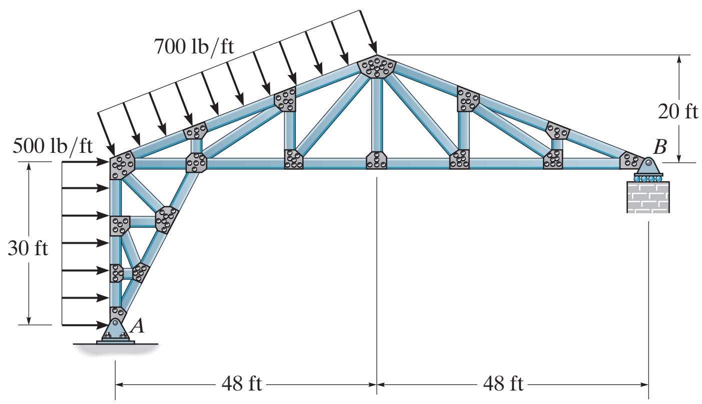
Prob. 2–30

2–31. Determine the reactions at the supports A and B of the frame. Assume that the support at A is a roller.



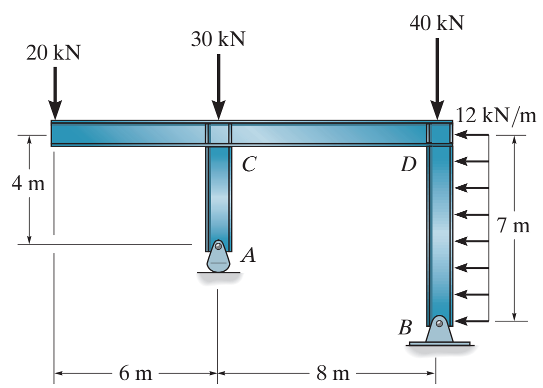
Prob. 2–31

*2–32. Determine the reactions at the truss supports A and B. The distributed loading is caused by wind pressure.



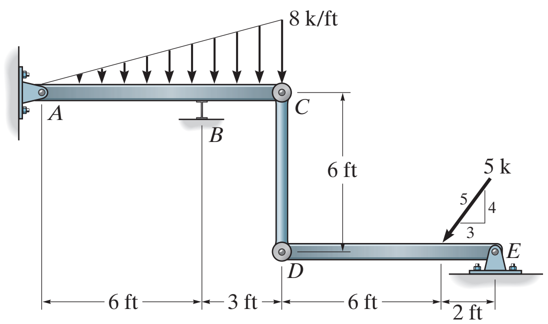
Prob. 2–32

2–33. Determine the horizontal and vertical components of reaction at the supports A and B. The joints at C and D are fixed connections.



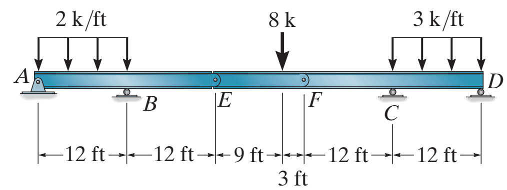
Prob. 2–33

2–34. Determine the reactions at the supports A, B, and E. Assume the bearing support at B is a roller.



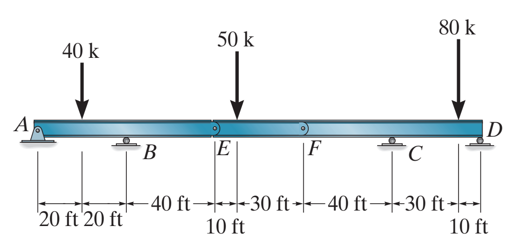
Prob. 2–34

2–35. Determine the reactions at the supports A, B, C, and D.



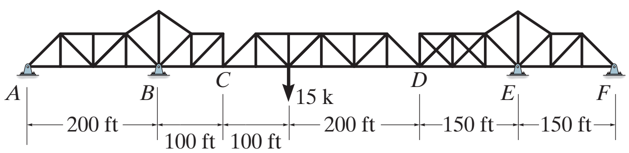
Prob. 2–35

*2–36. Determine the reactions at the supports for the compound beam. There are pins at A, E, and F.



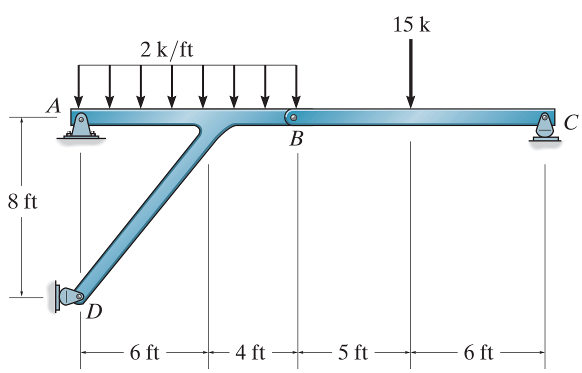
Prob. 2–36

2–37. The construction features of a cantilever truss bridge are shown in the figure. Here it can be seen that the center truss *CD* is suspended by the cantilever arms *ABC* and *DEF*. *C* and *D* are pins. Determine the vertical reactions at the supports *A*, *B*, *E*, and *F* if a 15-k load is applied to the center truss.



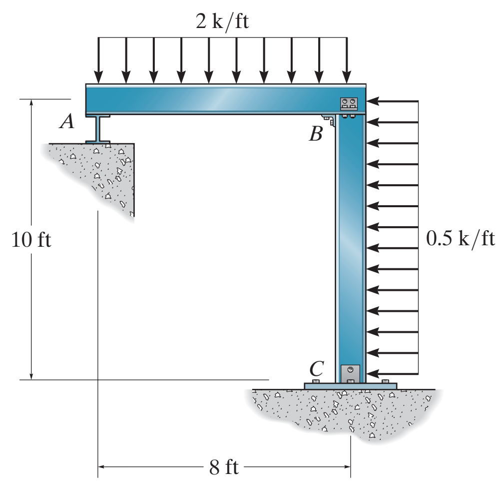
Prob. 2–37

2–38. Determine the reactions at the supports *A*, *C*, and *D*. *B* is pinned.



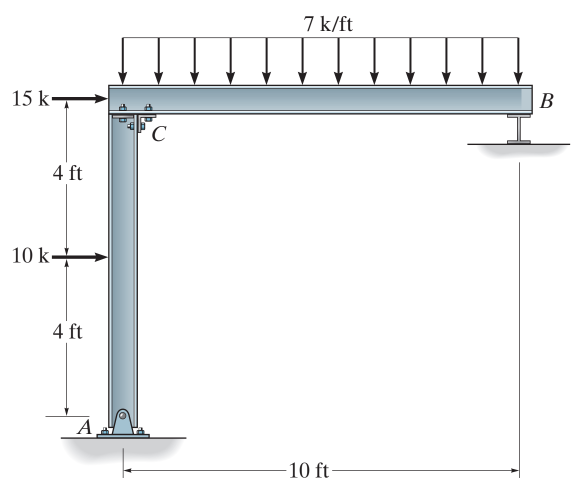
Prob. 2–38

2–39. Determine the reactions at the supports *A* and *C*. Assume the support at *A* is a roller, *B* is a fixed-connected joint, and *C* is a pin.



Prob. 2–39

***2–40.** Determine the reactions at the supports *A* and *B*. Assume the support at *B* is a roller. *C* is a fixed-connected joint.



Prob. 2–40

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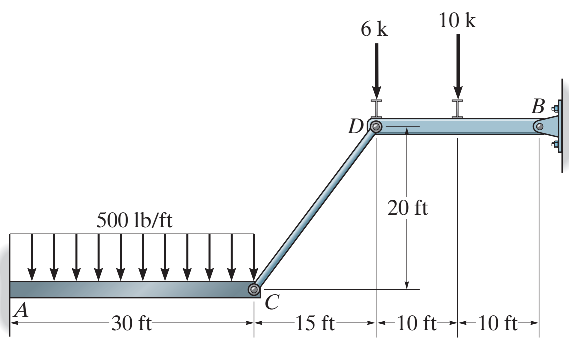
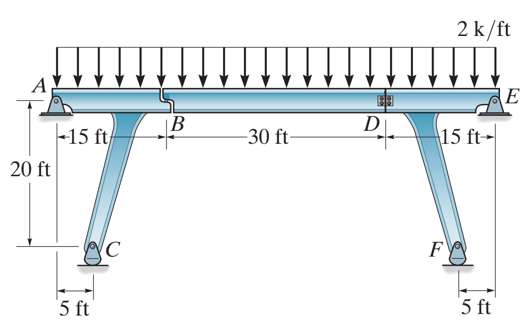
2–41. Determine the reactions at the supports A and B.



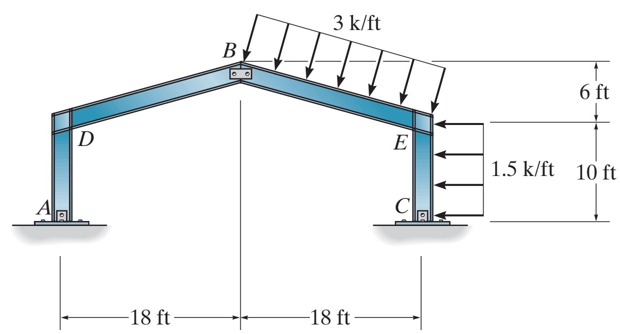
Prob. 2–41

2–42. The bridge frame consists of three segments which can be considered pinned at A, D, and E, rocker supported at C and F, and roller supported at B. Determine the horizontal and vertical components of reaction at all these supports due to the loading shown.



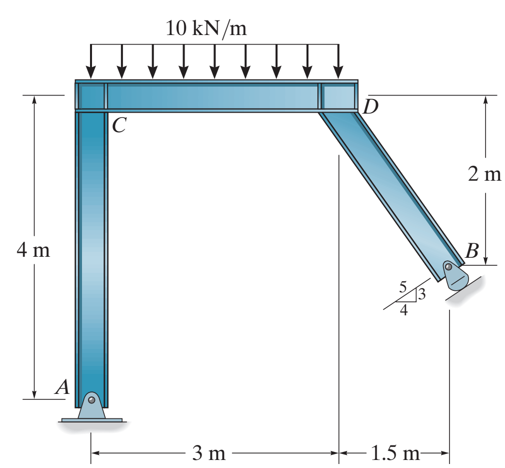
Prob. 2–42

2–43. Determine the horizontal and vertical components at A, B, and C. Assume the frame is pin connected at these points. The joints at D and E are fixed connected.



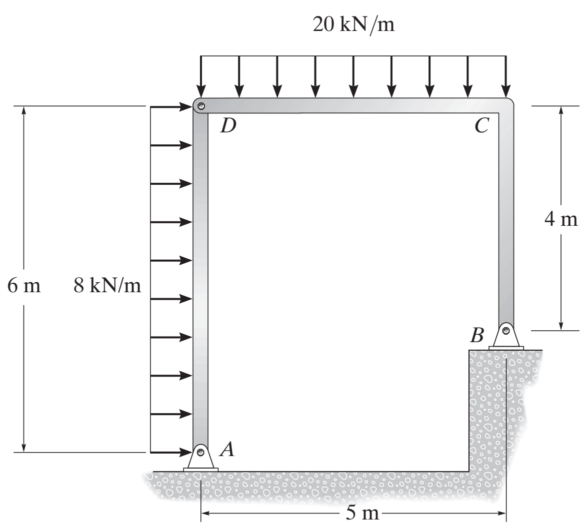
Prob. 2–43

***2–44.** Determine the reactions at the supports A and B. The joints C and D are fixed connected.



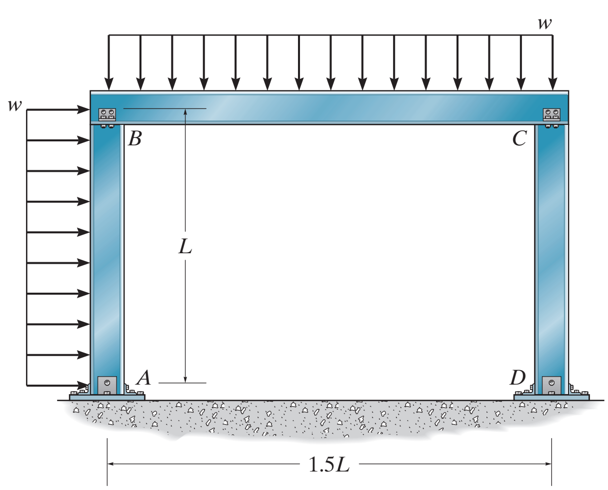
Prob. 2–44

2–45. Determine the horizontal and vertical components of reaction at the supports *A* and *B*.



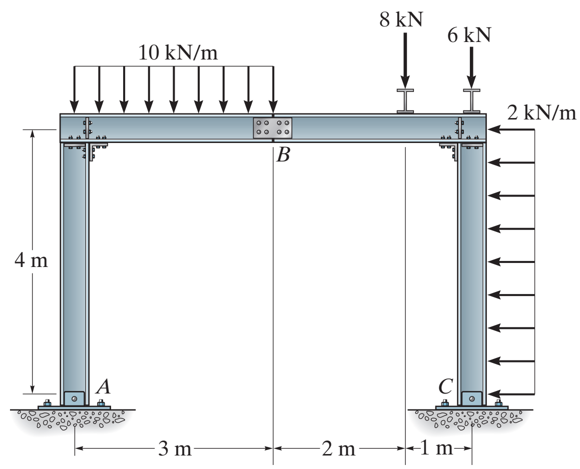
Prob. 2–45

2–46. Determine the reactions at the supports *A* and *D*. Assume *A* is fixed and *B* and *C* and *D* are pins.



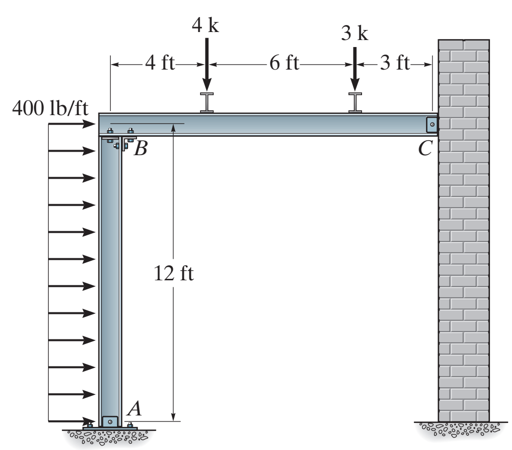
Prob. 2–46

2–47. Determine the reactions at the supports *A* and *C*. The frame is pin connected at *A*, *B*, and *C* and the two joints are fixed connected.



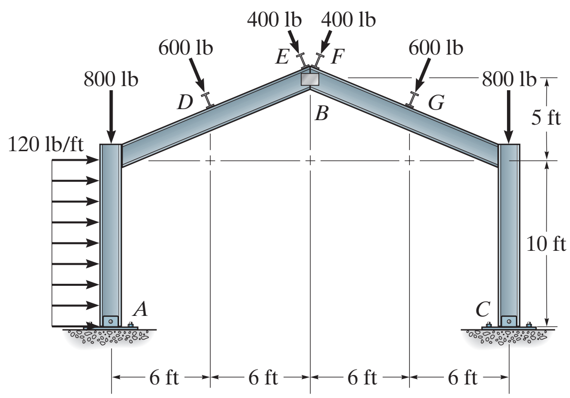
Prob. 2–47

***2–48.** Determine the horizontal and vertical components of force at the connections *A*, *B*, and *C*. Assume each of these connections is a pin.



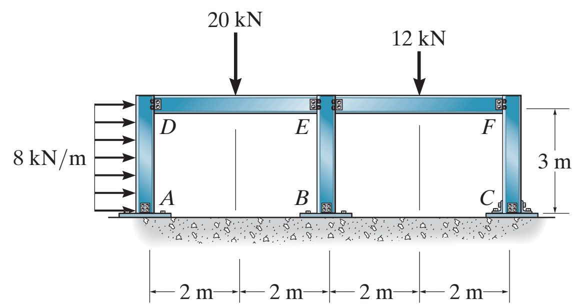
Prob. 2–48

2–49. Determine the horizontal and vertical reactions at the connections *A* and *C* of the gable frame. Assume that *A*, *B*, and *C* are pin connections. The purlin loads such as *D* and *E* are applied perpendicular to the center line of each girder.



Prob. 2–49

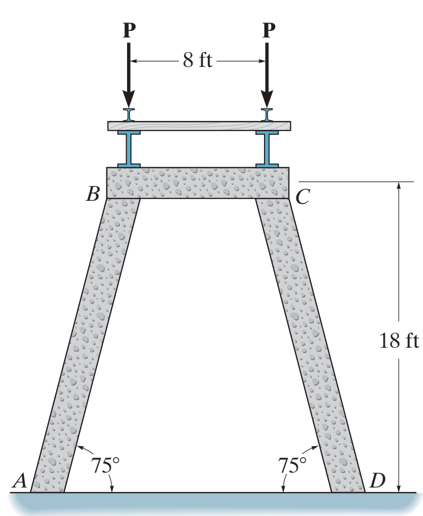
2–50. Determine the horizontal and vertical components of reaction at the supports *A*, *B*, and *C*. Assume the frame is pin connected at *A*, *B*, *D*, *E*, and *F*, and there is a fixed connected joint at *C*.



Prob. 2–50

PROJECT PROBLEM

2–1P. The railroad trestle bridge shown in the photo is supported by reinforced concrete bents. Assume the two simply supported side girders, track bed, and two rails have a weight of 0.5 k/ft and the load imposed by a train is 7.2 k/ft (see Fig. 1–11). Each girder is 20 ft long. Apply the load over the entire bridge and determine the compressive force in the columns of each bent. For the analysis assume all joints are pin connected and neglect the weight of the bent. Are these realistic assumptions?



Project Prob. 2–1P