DESIGN OF OPTIMAL WATER DISTRIBUTION SYSTEMS

IOAN SÂRBU
"Politehnica" University of Timişoara

1. Introduction

Distribution networks are an essential part of all water supply systems. The reliability of supply is much greater in the case of looped networks. Distribution system costs within any water supply scheme may be equal to or greater than 60 % of the entire cost of the project. Also, the energy consumed in a distribution network supplied by pumping may exceed 60 % of the total energy consumption of the system [9].

Attempts should be made to reduce the cost and energy consumption of the distribution system through optimization in analysis and design. A water distribution network that includes pumps mounted in the pipes, pressure reducing valves, and check-valves can be analyzed by several common methods such as Hardy-Cross, linear theory, and Newton-Raphson [10]. Traditionally, pipe diameters are chosen according to the average economical velocities (Hardy-Cross method) [3]. This procedure is cumbersome, uneconomical, and requires trials, seldom leading to an economical and technical optimum.

This paper develops a nonlinear model for optimal design of looped networks supplied by direct pumping from one or more node sources, which has the advantage of using not only cost criteria, but also energy consumption, consumption of scarce resources, operating expenses etc. Also, this new model considers the transitory or quadratic turbulence regime of water flow. The discharge continuity at nodes, energy conservation in loops, and energy conservation along some paths between the pump stations and the adequate "critical nodes" are considered as contraints. The nonlinear optimization model considers head losses or discharges through pipes as variables to be optimized in order to establish the optimal diameters of pipes and is coupled with a hydraulic analysis. This model can serve as guidelines to supplement existing procedures of network design.

2. Networks design optimization criteria

Optimization of distribution network diameters considers a mono- or multicriterial objective function. Cost or energy criteria may be used, simple or complex, which considers the network cost, pumping energy cost, operating expenses, included energy, consumed energy etc. These criteria can be expressed in a complex objective multicriterial function [8], with the general form:

$$F_c = \xi_1 \sum_{ij=1}^{T} (a + b D_{ij}^{\alpha}) L_{ij} + \psi \sum_{j=1}^{NP} Q_{p,j} (\sum h_{ij} + H_o)_j \qquad (1)$$

in which:

$$r_a = \frac{(1+\beta_o)^t - 1}{\beta_o (1+\beta_o)^t} \qquad (2) \qquad \xi_1 = r_a p_1 + \frac{t}{T_r} \; ; \qquad \xi_2 = r_a p_2 + \frac{t}{T_r} \qquad (3)$$

$$\psi = \frac{9.81}{\eta} (f \sigma \xi_2 + 730 r_a e \tau \sum_{1}^{12} \Phi_k) \qquad (4)$$

where: T is the number of pipes in a network; *a, b,* α –cost parameters depending on pipe material [9]; D_{ij}, L_{ij} – diameter and length of pipe *ij*; *NP* – number of pump stations; $Q_{p,j}$ – pumped discharge of pump station *j*; Σh_{ij} – sum of head losses along a path between the pump station and the critical node; H_o – geodezic and utilization component of the pumping total dynamic head; $\beta_o = 1/T_r$ – amortization part for the operation period T_r;. p_1, p_2 – repair, maintenance and periodic testing part for network pipes and pumping stations, respectively; *t* – period for which the optimization criterion expressed by the objective function is applied, having the value 1 or T_r; η – efficiency of pumping station; *f* – installation cost of unit power; σ – a factor greater than one which takes into account the installed reserve power; *e* – cost of electrical energy; $\tau = T_p/8760$ – pumping coefficient, which takes into account the effective number T_p of pumping hours per year; Φ_k – ratio between the average monthly discharge and the pumped discharge [8].

The general function (1) enables us to obtain a particular objective function by particularization of the time parameter *t* and of the other economic and energetic parameters, characteristic of the distribution system. For example, from t =1, r_a =1, e =1, f =0 the minimum energetic con-sumption criterion is obtained.

For networks supplied by pumping, the literature [1], [2], [4], [5], [11] suggests the use of *minimum annual total expenses criterion* (*CAN*), but choosing the optimal diameters obtained in this way, the networks become uneconomical at some time after construction, due to inflation.

Therefore, it is recommended the fore-mentioned criterion be subject to dynamization by using the *criterion of total updated minimum expenses* (*CTA*), the former being in fact a specific case of the latter when the investment is realised within a year; the operating expenses are the same from one year to another and the expected life-time of the distribution system is high. In particular, the use of energetical criteria different from cost criteria is recommendable. Thus, another way to approach the problem, with has a better validity in time and the homogenization of the objective function is network dimensioning according to *minimum energetic consumption* (*WT*).

3. Nonlinear model of optimization in designing distribution networks

The head loss is given by the Darcy-Weisbach functional relation:

$$h_{ij} = \frac{8}{\pi^2 g} \lambda_{ij} \frac{L_{ij}}{D_{ij}^r} Q_{ij}^2 \qquad (5)$$

where: r is an exponent having the value 5.0; g – acceleration of gravity; λ_{ij} – friction factor of pipe ij which can be calculated using the Colebrook-White formula; Q_{ij} – discharge of the pipe ij.

Equation (5) is difficult to use in the case of pipe networks and therefore it is convenient to write it similar to the Chézy-Manning formula:

$$h_{ij} = R_{ij}\, Q_{ij}^{\beta} \tag{6}$$

where: $R_{ij} = KL_{ij} / D_{ij}^{r}$ is the hydraulic resistance of pipe ij; β – exponent which has values between 1.85 and 2.

Specific consumption of energy for distribution of water w_{sd}, in kWh/m^3, is obtained by referring the hydraulic power dissipated in pipes to the sum of node discharges:

$$w_{sd} = 0.00272\,\frac{\sum\limits_{ij=1}^{T} R_{ij}\left|Q_{ij}\right|^{\beta+1}}{\sum\limits_{\substack{j=1\\ q<0}}^{N}\left|q_j^{'}\right|} \tag{7}$$

where $q_j^{'}$ is the outflow at the node j.

The nonlinear optimization model (MON) allows the optimal designing of looped networks by using one of the CAN, CTA or WT optimization criteria, expressed by the objective function (1).

If the diameter D_{ij} is expressed in relation (7) through the discharge and head losses:

$$D_{ij} = K^{\frac{1}{r}}\, Q_{ij}^{\frac{\beta}{r}}\, h_{ij}^{-\frac{1}{r}}\, L_{ij}^{\frac{1}{r}} \tag{8}$$

and in the objective function (1) is replaced the resulting expression, we have:

$$F_c = \xi_1 \sum_{ij=1}^{T}\left(a + b\,K^{\frac{\alpha}{r}}\, Q_{ij}^{\frac{\beta\,\alpha}{r}}\, h_{ij}^{-\frac{\alpha}{r}}\, L_{ij}^{\frac{\alpha}{r}}\right)L_{ij} + \psi\sum_{j=1}^{NP} Q_{p,j}\left(\sum h_{ij} + H_o\right)_j \tag{9}$$

which is limited by the following constraints:

– discharge continuity at nodes :

$$\sum_{\substack{i=1\\ i\neq j}}^{N} Q_{ij} + q_j = 0 \qquad (j = 1,\ldots, N - NP) \tag{10}$$

– energy conservation in loops:

$$\sum_{\substack{ij\in m\\ ij=1}}^{T} \varepsilon_{ij}\, h_{ij} - f_m = 0 \qquad (m = 1,\ldots, M) \tag{11}$$

– energy conservation along some paths between the pump stations and adequate "critical nodes":

$$Z_{SP,j} - \sum_{ij=1}^{NT_j} \varepsilon_{ij}\,(h_{ij} - H_{p,ij}) - Z_{o,j} = 0 \qquad (j = 1,\ldots, NP) \tag{12}$$

in which: Q_{ij} is the discharge through pipe ij, with the sign (+) when entering node j and (–) when leaving it; q_j – concentrated discharge at node j with the sign (+) for node inflow and

(–) for node outflow; h_{ij} – head loss of the pipe ij; ε_{ij} –orientation of flow trough the pipe, having the values (+1) or (-1) as the water flow sense is the same or opposite to of the path sense of the loop m and (0) value if $ij \notin m$; f_m – pressure head introduced by the potential elements of the loop m [8]; M – number of independent loops (closed-loops and pseudoloops); $Z_{SP,j}$ – available piezometric head at the pump station SP_j; $H_{p,ij}$ – pumping head of the pump mounted in the pipe ij, for the discharge Q_{ij}, approximated by parabolic interpolation on the pump curve given by points [9]; $Z_{o,j}$ – piezometric head at the critical node O_j; NT_j – pipe number of a path SP_j- O_j.

The optimization model (9)…(12) represent a nonlinear programming problem, which results in a system of non-linear equations by applying the Lagrange nondetermined coefficients method. This system will have $2T+NP$ equations with unknown variables Q_{ij}, h_{ij}, $Z_{SP,j}$, formed by:

a) $N – NP$ nodal equations (10);
b) M loop equations (11);
c) NP functional equations (12);
d) $N – NP$ energy-economy equations for nodes:

$$\sum_{\substack{i \neq j \\ i=1}}^{N} Q_{ij}^{*} = \begin{cases} -\dfrac{\psi}{A} Q_{p,j}; & \text{for pumping nodes } (j = 1,\ldots, NP) \\ 0; & \text{for other nodes } (j = NP+1,\ldots, N-NP) \end{cases} \tag{13}$$

in which:

$$Q_{ij}^{*} = Q_{ij}^{\frac{\beta\alpha}{r}} L_{ij}^{\frac{\alpha+r}{r}} h_{ij}^{-\frac{\alpha+r}{r}} \tag{14}$$

$$A = \frac{\alpha}{r} \xi_l b K^{\frac{\alpha}{r}} \tag{15}$$

e) M energy-economy equations for loops:

$$\sum_{\substack{ij \in m \\ ij=1}}^{T} H_{ij}^{*} = 0 \qquad (m = 1,\ldots, M) \tag{16}$$

in which:

$$H_{ij}^{*} = h_{ij}^{-\frac{\alpha}{r}} L_{ij}^{\frac{\alpha+r}{r}} Q_{ij}^{\frac{\beta\alpha-r}{r}} \tag{17}$$

Equations (13) can be expressed similarly with the discharge continuity equations, by giving Q_{ij}^{*} the same sign as Q_{ij}. Equations (16) are similar to the energy conservation equations in the loops, by giving H_{ij}^{*} the same sign as h_{ij}.

In order to establish an extreme of the objective function, we should specify a set of variables (Q_{ij} or h_{ij}). Thus, if the flow discharges in pipes are known, the values h_{ij} are to be determined by minimizing the objective function F_c. If only the head losses are the given values, the variables Q_{ij} are to be determined by maximizing the objective function F_c. Considering variables h_{ij} to be unknown, pipes discharges could be calculated in an variety of ways for equations (10) to be satisfied; this however affects the reliability and technical and economic-energetical conditions of the system. That is why optimization of the flow discharges in pipes must be performed accor-ding to the minimum bulk transport criterion [7], which takes into account the network reliability. In this case, computation of the optimal design of looped networks must be performed in the following stages:

– Establishment of optimal distribution for discharges through pipes, Q_{ij} [8].
– Determination of head losses through pipes and piezometric heads at the supply nodes, by solving the nonlinear equation system (11), (12), (13) using the gradients method [6].
– Computation of optimal pipes diameters D_{ij} using expression (8) and their approximation to the closest commercial values.

– A new computation of the head losses using relation (5) or (6) and the hydraulic equilibrium for pipes network using Hardy-Cross method.

If the head losses are the given values, the unknown variables Q_{ij} are to be determined by solving the equation system (10), (12), (16), and used to calculate the optimal diameters in relation (8).

The piezometric heads Z_n can be determined starting from a node of known piezometric head. The residual pressure head H_n at the node n is calculated from the relation:

$$H_n = Z_n - ZT_n \tag{18}$$

where ZT_n is the elevation head at the node n.

For an optimal design, the piezometric line of a path of NT_j pipes, situated in the same pressure zone, must represent a polygonal line which resemble as closely as possible the optimal form expressed by the equation:

$$Z_n = Z_{SP,j} - \left[1 - \left(1 - \frac{d}{\sum_{ij=1}^{NT_j} L_{ij}}\right)^{\frac{\beta\,\alpha}{\alpha+r}+1}\right] \sum_{ij=1}^{NT_j} h_{ij} \tag{19}$$

in which: Z_n is piezometric head at the node n; d – distance between the node n and the pump station SP_j.

The computer program OPNELIRA was designed [9] based on the nonlinear optimization model. It was realized in the FORTRAN 5.1 programming language, for IBM-PC compatible computers.

4. Numerical application

The looped distribution network with the topology from figure 1 is considered. It is made of cast iron and is supplied with a discharge of 0.23 m^3/s. The following data are known: pipe length L_{ij}, in m, elevation head ZT_j, in m, and necessary pressure HN_j = 24 m.

A comparative study of network dimensioning is performed using the classic model of average economical velocities (MVE), Moshnin optimization model (MOM) [1] and the nonlinear optimization model (MON) developed above.

Calculus was performed considering a transitory turbulence regime of water flow and the optimization criterion used was that of minimum energetic consumption. Results of the numerical solution performed by means of an IBM-PC PENTIUM III computer, referring to the hydraulic characteristics of the pipes (discharge, diameter, head loss, velocity) are presented in table 1. The significance of the (–) sign of discharges and head losses in table 1 is the change of flow sense in the respective pipes with respect to the initial sense considered in figure 1.

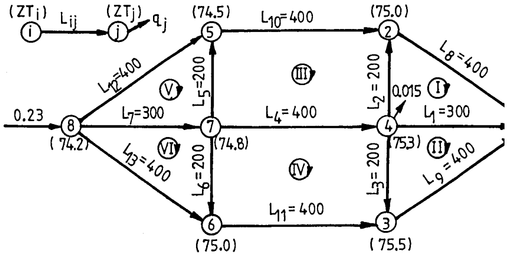


Fig. 1 Scheme of the designed distribution ne

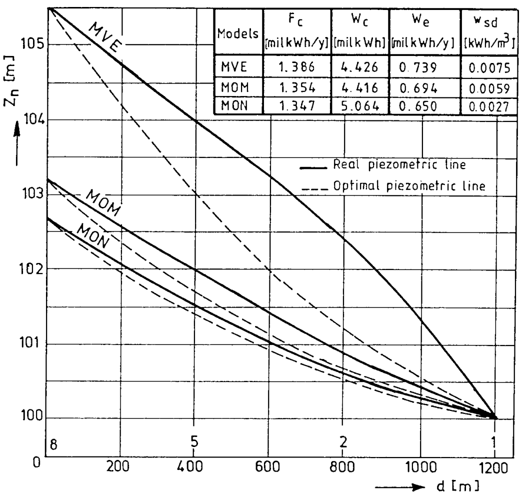


Fig. 2 Representation of piezometric lines along the path 8–5–2–1

In figure 2 there is a graphic representation, starting from the node source 8 to the critical node 1, on the path 8–5–2–1, the piezometric lines being obtained by using the three mentioned models of computa-tion, and highlighting their deviation from the optimal theoretical form. Figure 2 also includes the corresponding values of the objective function F_c, the network included energy W_c, pumping energy W_e, as well as specific energy consumption for water distribution w_{sd}.

Table 1. Hydraulic characteristics of the pipes

Pipe	L	Classic model (MVE)				Moshnin model (MOM)				Non-linear model (MON)			
i - j	[m]	Q_{ij} [m^3/s]	D_{ij} [mm]	h_{ij} [m]	V_{ij} [m/s]	Q_{ij} [m^3/s]	D_{ij} [mm]	h_{ij} [m]	V_{ij} [m/s]	Q_{ij} [m^3/s]	D_{ij} [mm]	h_{ij} [m]	V_{ij} [m/s]
0	1	2	3	4	5	6	7	8	9	10	11	12	13
4–1	300	0.00782	100	4.009	1.00	0.00786	150	0.510	0.45	0.01986	200	0.694	0.63
4–2	200	0.00512	100	1.177	0.65	–0.00174	100	–0.154	0.22	–0.00326	100	–0.498	0.42
4–3	200	0.00512	100	1.177	0.65	–0.00174	100	–0.154	0.22	–0.00327	100	–0.501	0.42
7–4	400	0.05924	300	0.963	0.84	0.04557	250	1.473	0.93	0.05452	300	0.820	0.77
7–5	200	0.00517	100	1.199	0.66	0.00097	100	0.052	0.15	–0.00117	100	–0.074	0.15
7–6	200	0.00517	100	1.199	0.66	0.00097	100	0.052	0.15	–0.00118	100	–0.075	0.15
8–7	300	0.09576	350	0.833	1.00	0.07370	300	1.104	1.04	0.07835	350	0.565	0.81
2–1	400	0.01669	150	2.886	0.94	0.01666	200	0.662	0.53	0.01066	150	1.216	0.60
3–1	400	0.01669	150	2.886	0.94	0.01666	200	0.662	0.53	0.01067	150	1.218	0.60
5–2	400	0.03538	250	0.902	0.72	0.04221	250	1.270	0.86	0.03773	300	0.404	0.53
6–3	400	0.03538	250	0.902	0.72	0.04221	250	1.270	0.86	0.03775	300	0.404	0.53
8–5	400	0.05403	250	2.051	1.10	0.06506	300	1.155	0.92	0.06271	350	0.490	0.65
8–6	400	0.05403	250	2.051	1.10	0.06506	300	1.155	0.92	0.06274	350	0.490	0.65

According to the performed study it was established that:

– there is a general increase of pipe diameters obtained by optimization models (MOM, MON) with respect to MVE, because the classical model does not take into account the minimum consumption of energy and the diversity of economical parameters;

– in comparison with the results obtained by MVE, those obtained by optimization models are more economical, a substantial reduction of specific energy consumption for water distri-bution is achieved (MOM - 21.3 %, MON - 64 %), as well as a reduction of pumping energy (MOM - 6.4 %, MON - 12 %); at the same time the objective function has also smaller values (MOM - 2.3 %, MON - 2.9 %);

– the optimal results obtained using MON are superior energetically to those offered by MOM, leading to pumping energy savings of 6.2 %;
– also, the application of MON has led to the minimum deviation from the optimal form of the piezometric line, especially to a more uniform distribution of the pumping energy, by elimination of a high level of available pressure at some nodes. The smallest value of the specific energetic consumption, namely that of 0.0027 kWh/m^3, also supports this assertion;
– reduction of the pressure in the distribution network achieved in this way, is of major practical import, contributing to the diminishing of water losses from the system.

5. Conclusions

The computer program developed in this study, a very general and practical one, offers the possibility of optimal design of water supply networks using multiple criteria of optimization and considers the transitory or quadratic turbulence regime of water flow. It has the advantage of using not only cost criteria, but also energy consumption, consumption of scarce resources, and other criteria can be expressed by simple options in the objective function (1). The optimization approach used in this study does not require calculation of derivatives. This makes the method more efficient and consequently helps the designer to get the best design of water distribution systems with fewer efforts.

The nonlinear optimization model could be applied to any looped network, when piezometric heads at pump stations must be determined. A more uniform distribution of pumping energy is achieved so that head losses and parameters of pump stations can be determined more precisely.

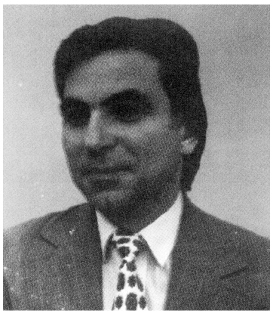
For different analyzed networks, the saving of electrical energy due to diminishing pressure losses and operation costs when applying this new optimization model, represents about 10...30 %, which is of great importance, considering the general energy issues.

References

[1] ABRAMOV, N. N.: Teoria i metodica rasceta sistem podaci i raspredelenia vodî, Stroizdat, Moskva, 1972.
[2] CENEDESE, A. – MELE, P.: Optimal design of water distribution networks, *Journal of the Hydraulics Division*, ASCE, no. HY2, 1978.
[3] CROSS, H.: Analysis of flow in network of conduits or conductors, *Bulletin no. 286*, Univ. of Illinois Engrg. Experiment Station, III, 1936.
[4] DIXIT, M. – RAO, B. V.: A simple method in design of water distribution networks, Afro-Asian Conference on Integrated Water Management in Urban Areas, Bombay, 1987.
[5] ORMSBEE, L.E. – WOOD, D.J.: Hydraulic design algorithms for pipe networks, *Journal of Hydraulic Engineering*, ASCE, no. HY2, 1986.
[6] PCHÉNITCHNY, B. – DANILINE Y.: Méthodes numériques dans les problèmes d'extremum, Edition Mir, Moscou, 1977.
[7] SÂRBU, I.: L'optimisation de la répartition des débits dans les réseaux maillés de distribution d'eau, *Buletinul Ştiinţific al U.P. Timişoara*, Tom 36, 1991.
[8] SÂRBU, I. – BORZA I.: Optimal design of water distribution networks, *Journal of Hydraulic Research*, no. 1, 1997.

[9] SÂRBU, I.: Energetical optimization of water distribution systems, Editura Academiei, Bucureşti, 1997.
[10] STEPHENSON, D.: Pipe flow analysis, *Elsevier Science Publishers B.V.*, 1984.
[11] THAWAT, W.: Least-cost design of water distribution systems, *Journal of the Hydraulics Division*, ASCE, no. HY9, 1973.

Main author presentation



Phd. eng. Ioan Sârbu is professor at "POLITEHNICA" University of Timişoara, Romania.