Available online at www.sciencedirect.com

ScienceDirect

Procedia Engineering 119 (2015) 573 – 582

Procedia Engineering

www.elsevier.com/locate/procedia

13th International Conference on Computing and Control for the Water Industry, CCWI2015

Water distribution network model building, case study: Milano, Italy

Mario Castro Gama[a,]*, Emilio Attilio Lanfranchi[b], Quan Pan[a] and Andreja Jonoski[a]

[a]UNESCO-IHE, Institute for Water Education, Westvest 7, 2611AX Delft, The Netherlands.
[b]Metropolitana Milanese S.p.A., Via del Vecchio Politecnico, 8, 20121 Milano, Italy.

Abstract

Most developments for Water Distribution Network (WDN) modeling purposes refer to optimization approaches for design, rehabilitation or operation. However, most case studies presented in the literature are rather small and represent only a small portion or simplification of a network. The city of Milano with a population of around 1.3 million inhabitants is an example of a WDN where many phenomena occur simultaneously and where the utility, Metropolitana Milanese S.p.A, (MM), needs to deal with many factors for its daily operation. Under the framework of the EU-FP7 ICeWater project, a new model for the WDN of the city of Milano has been developed with UNESCO-IHE, during the last 2.5 years. This article presents the process of model building, the challenges for the calibration process, open issues and tasks to be developed in the near future.

Keywords: Water Distribution Neworks, ICeWater project, Milano, Setup, Calibration.

---

1. Introduction

Many examples of benchmark case studies for optimization in Water Distribution Networks (WDN) have been developed in the last 50 years. The first networks to be used for such purposes were *New York* (Shake and Lai, 1967), *TwoLoops* (Alperovits and Shamir, 1977), *TwoReservoirsA* (Gessler, 1985), *Anytown* (Walski et al., 1987) and *Hanoi* (Fujiwara and Khang, 1990). Later on, other WDN have appeared as benchmarks for the understanding of problems in different locations such as, *ThreeTanks*, (van Zyl et al., 2004), *Apulian* (Giustolisi et al., 2009),

---

* Corresponding author. Tel.: +31-(0)65-271-4956
E-mail address: m.castrogama@unesco-ihe.org

doi:10.1016/j.proeng.2015.08.910

Balerma (Reca and Martınez, 2006), and with time these networks have grown in size and number of elements as *LargeNetwork* (Kang and Lansley, 2012), *One-Reservoir*, *DoubleReservoir*, *ThreeReservoir*s, *FiveReservoirs* (Zheng and Zecchin, 2014, although *FiveReservoirs* is a modification of *LargeNetwork*). Such networks models are useful to serve as a proof of concept for optimization purposes, although these lack the complexities of real systems and deal with a simple problem in water supply at a time.

Nomenclature

AMR	Automated Meter Readings
CMR	Consorcio Milanese Recherche
DB	Database
DSS	Decision Support System
EU- FP7	European Union, Seventh Framework Programme
EPANET	US Environmental Protection Agency, pressurized pipe network simulation software
GGA	Global Gradient Algorithm
ICeWater	ICT Solutions for efficient Water Resources Management
ICT	Information Communication Technologies
MM	Metropolitana Milanese S.p.A.
MOO	Multi-Objective Optimization
PMZ	Pressure Management Zone
SCADA	Supervisory Control and Data Acquisition
VSP	Variable Speed Pumps
WDN	Water Distribution Network
WSS	Water Supply System
WTN	Water Transmission Network

In the case of *Richmond* and *RichmondSkeletonized* (van Zyl et al., 2004), *Exnet* (Farmani et al., 2004), Battle of the water sensors network 1 (*BWSN1*) (Ostfeld et al., 2008), *Parete* and *Villarica* (Di Nardo and Di Natale, 2010) and *C-Town* (Ostfeld et al., 2012), these WDN correspond to approximations of real networks with a larger number of elements increasing the computational runtimes. Other known cases of networks used in the literature for modeling and optimization purposes are *Barcelona*, Spain (Cembrano et al., 1988); *City T*, China (Shihu et al., 2010); *Goiania*, Brazil (Carrijo et al., 2004); *Haifa-A*, Israel (Salomons et al., 2007); *Madrid*, Spain (Gomez et al., 2014) and *Valencia*, Spain (Martinez et al., 2007).

Several algorithms are available for simulation of real systems (Todini and Rossman, 2013), being the most common used in the literature Global Gradient Algorithm GGA) (Todini and Pilati, 1988) as implemented in EPANET 2.0 (Rossman, 2000). With the investment of utilities in Geographical Information Systems (GIS), for the management and visualization of their WDN, the number of elements in a network increases and operational rules become more complex (Savic and Banyard, 2011). However, in most of the cases, WDN simulation deals with a simplification of the real systems or with a limited number of pipes, pumps and sources.

One example of a WDN which contains many assets (e.g. pipes, pumps, tanks, valves, connections) is the system of Milano, currently operated by Metropolitana Milanese S.p.A (MM). The system provides water for around 1.3 million inhabitants and 0.7 million commuters. Currently, it is operated as a single Pressure Management Zone (PMZ), limiting the amount of energy that can be saved in a daily basis. For this reason, in 2012, MM agreed to participate in the EU-FP7 ICeWater project[†], "*ICT Solutions for efficient Water Resources Management*". ICeWater combines sophisticated ICT solution to provide real time data from the field. Thanks to

---

[†] http://www.icewater-project.eu/

real time data acquisition, an advanced Decision Support System (DSS) is being developed to achieve an efficient management of the water network. The project focus is on improvement of management of Energy use, Water Losses, Water Quality, Asset Management and Demand Forecast. A complete description of the project is presented by Fantozzi et al., (2013).

The reduction of energy consumption in Milano is performed using Multi-Objective Optimization (MOO) on a model of the system. It was the interest of the utility to have the most accurate model possible given the age of the pipes in some parts of the city dating to beginning of last century. Such model was built in EPANET, and the process of building a setup and calibration from several sources of information poses challenges for the operator MM and UNESCO-IHE. This paper focuses on the building of the WDN model, based on data collected from the utility during the last 2.5 years. The intention of the utility and the research project was to have a model as accurate as possible to simulate the operation of the system and some partial results of the calibration in the most critical zone of the system are presented.

The paper outline is as follows in section 2, the description of the city, the WDN and the need for the model set-up is presented. In Section 3, the workflow for the model construction, based on authors' experiences is presented including the challenges for the operators, the hurdles overcome during model building stages for such a large system and the methodology followed. Finally, the manuscript conclusions and final remarks are presented in Section 4.

2. Milano city and its WDN

The city has developed during many centuries in a radial distribution and it is ranked fifth by area and population in the EU (Demographia, 2015). It is located in the North of Italy, being the largest city of the country by population in its metropolitan area (including Como, Lecco and Varese), and the second largest population in a single municipality in the north of Italy (~1.3million inhabitants). Although is the only city with a population larger than a million with negative population growth rate (-0.2%), meaning people is leaving the city every year.

Topographically, the city has a gradient in the elevation from the north (~150 m.a.s.l.) to the south part of the city (~90masl), which creates a natural gradient for any constructed infrastructure. Due to high urbanization the city is densely covered by infrastructure and edifications across its surface. There is no possibility, for example to buy land without incurring in high costs. On top of that, as a comparison, the area of Milano (182 km^2) is, almost twice (1.7) the one of Paris city center (100km^2), but Milano's population density per square kilometer (~7,800 inhabitants per km^2) is only around one third of that of Paris (~22,000 inhabitants per km^2).

The WDN covers the whole city and it is a vast system where water is obtained from groundwater sources only. Water is pumped to storage units. Inside the storage units, water is treated. This subsystem is known as Water Transmission Network (WTN). After treatment, water is pumped from the storage units once again into the WDN for final consumption by customers. The system is operated as a single Pressure Management Zone (PMZ).

Data on the billing system of Milano indicates that there are approximately ~50,000 customers or connections to the Water Supply System (WSS) of MM, which does not represent individual households or industrial connection, due to the aggregation of demands in buildings and neighborhoods.

The major issue for MM is the large amount of energy used by the company for water supply, which amount for approximately €16,000,000 per year. Of this amount around 50% is spent in the pumping at the WTN, 45% at the WDN and 5% is spent for other facilities such as the control centers in different pump stations.

In ICeWater project, one of the DSS components deals with reduction of energy consumption during operation with the aid of MOO. For this two different problems are implemented: (1) a sectorization of the system by splitting the network into smaller PMZ, (2) through selection of pump schedules at every pump station or PMZ, although the DSS for such task is implemented only in an isolated pilot area at south of the city known as *Abbiategrasso*.

Only for the purposes of calibration of the model of *Abbiategrasso* PMZ, a total of 250 sensors have been installed in the last two years. Additional investments of the utility on the systematization of the pump station in the pilot zone was also favorable for the purposes of ICeWater project given that the pump station was refurbished, with the installation of 2 brand new pumps in 2013. The purpose of such investment was to be able to operate the system with the implementation of Variable Speed Pumps (VSP) instead of pump switches ON/OFF as it was until 2012. With that installation it will be possible to regulate in a better way the pressure in *Abbiategrasso*. It was the interest of the utility to improve the operational capabilities of its engineers by bringing the system to the state of the art in ICT.

3. Model Building

When building a model of any WDN it is necessary to collect as much of information from the elements and assets that compose the real system. In our case, most of the data has been integrated in GIS geo-database (DB). The IT division of MM is in charge of updating the DB as soon as new elements are identified. For example, if the construction of a new residential complex was approved by the municipality the drawings of the corresponding connections are available for MM. Since 2013, a system is being implemented for Asset Management denominated MAXIMO, so it was the responsibility of the authors to export and import its data into WDN model as soon as it was available. Most of the data was available in DB's from MM.

3.1. Data sharing

The first hurdle into the development of such model is the confidence among institutions which allows data share and clear ways for the communication among managers, operators, researchers, IT developers, hydroinformaticians and technical staff on the field to deliver information which will add value to the model. In this category contractors may be also in the loop, given that some utilities have the special requirements for sensors and equipment installation. There is also an additional factor as it is the security of the data itself. We refer here to guarantee its privacy and confidentiality for third parties.

An example of this is the data of billing and demand consumption, which is fundamental information for the commercial division of a utility, but must not become open to every partner of the project. Another example in the data sharing privacy is the extreme case of terrorism treat to the system, due to leakage of the geographical locations of certain critical assets in the WDN.

The second aspect of data sharing among the project for purposes of model building was the early stage negotiation and definition of standards of data and models. While MM has invested in Infoworks WS tools[‡] for WDN modeling purposes, it was of the interest of the project to use open software as much as possible. For this reason, the final decision was to develop together all models in EPANET. This change of philosophy increases the challenges because some features are not the same among both software packages (e.g. valves are nodes in Infoworks WS, while in EPANET are considered links), although their hydraulic solvers are similar. The advantage of such decision, after a long negotiation, is that both MM personnel and project partners, interested in information, do not require additional licenses to make models run. Also, in case that the utility decides to participate in new research projects, the data will be ready for use by others in a common data standard.

A second example of data sharing among partners was the case of communication of sensor data for model developers. The standard selected for data sharing was WaterML2.0 and SensorML2.0 (OGC, 2014a and OGC,2014b), although some adaptations were required because in both cases some variables and equipment are not available in those standards. In this way it was possible to fetch and modify sensor data raw or processed in a clear way among DSS partners, because there is a documented standard and source codes to read WaterML and SensorML file schemas.

The disadvantage of such approach is that the markup languages present a challenge to the network transfer queue and hardware involved due to the large size of files to be generated. For example, a file containing the data of measurements of pressure in a particular part of the networks with a time resolution of 1minute for the last nine

---

[‡] Commercial software produced by Innovyze (http://www.innovyze.com)

months has a size of 75Mb. If the system continues its operation for the coming ten years an user which may want to fetch the data of that particular sensor will require approximately 1.0Gb of storage (for a single sensor).

3.2. Topology corrections

One of the first challenges of the model building of Milano WDN was the large amount of elements (assets) and its transformation into model elements. In total, there were around 30 iterations among MM and UNESCO-IHE for the update of the model. In most of the cases, information that was displayed into GIS was not corresponding with the topology of a WDN. What is evident for the GIS operator of the DB at MM may not represent the hydraulic connectivity of the system. Most computer engineers lack knowledge on basic hydraulics, so every time that a new topological inconsistency was found it required update. Starting from GIS data, a complex export and import of all the features was performed. The data verification chain that was used is presented in Fig 1.

As typical errors compromising simulations are related to incorrect schematization such as: pipe location and vertex, wrong elevations, misplaced elements in the pump stations, pipe's friction factor and pipe diameters, lack of knowledge about current status of the valves, this required an expensive update. This activity revealed many geospatial inconsistencies inside GIS which were corrected to improve GIS DB and model reliability in order to reduce subsequent tasks of calibration.

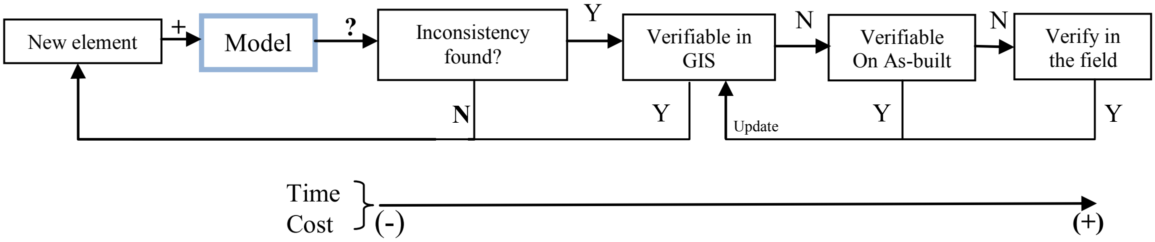

Figure 1. Process of model update element by element. More data validations required for a particular element required more time and additional costs expenses.

First as soon as a new element was identified it was added to the model, if an inconsistency was found there were three levels of verification. The first and most simple verification was to check the GIS databases to validate the inconsistency, if this one persisted, then it was verified on drawings as built and as a last resort the need to send a field engineer to verify the topology of the element was required. Fig. 1 also shows that the most expensive verifications were made on the field. For basic elements such as pipes and valves most of the time a simple verification on as-built drawing was the highest level reached; however, for vital elements such as tanks, wells and pumps (e.g. geometry, localization, pump curves and efficiencies) to guarantee a reliable modeling it was necessary to have field measurements. Such measurements were performed by external contractors and verified internally by the utility. There are in total, 26 active pump stations in Milano. And as it was confirmed some pump stations have been abandoned due to its age and low efficiency.

In an additional effort by the utility, 60% of the valves in the pilot area *Abbiategrasso* where verified one by one, proceeding from larger diameters to smaller ones. A total of 1,100 valves were checked on the field. To our knowledge such an effort has never been made in published research to match a model with a real system. As a total to February 2015, approximately 12,000 topological corrections were made in the model and were submitted for update of GIS DB of MM.

3.3. Demand consumptions

Demand consumption with its spatial distribution and variability was known from the billing data of the utility. The historical records of billing data cover the range between 2010 and 2014. The sensor data was available starting November of 2014. A customer connection does not literally transfers into a model junction, so a spatial allocation process was performed. This allocation of customer demands into model junctions was performed using shortest distance. In that case the spatial distribution of demands was solved and the magnitude of the average base demands per junction was obtained based on an historical average.

For the daily pattern of demands there are two alternatives available. The first option is a Demand Forecast performed by a project partner Consorcio Milanese Recherche (CMR) through DSS (Candelieri and Archetti, 2014). This option, allows us to update the model demand based on historical records and some seasonal factors (e.g. summer, winter, working day, weekend, holiday break), with the drawback of having a maximum resolution of 1 hour. The second alternative is the extraction of flow data from the sensor located at the exit of the pump station, from a DB developed in ICeWater by other partners of the project. The second option allows us to have a pattern based on a specific daily operation of the system but has a resolution of 1min which must be aggregated for larger time intervals to use in the model.

3.4. Different model setups required

In order to comply with the requirements of ICeWater project DSS there were several setups required for MOO purposes. For example, one of the partners deals only with the pump scheduling problem in the WTN at *Abbiategrasso* PMZ. Given that the model was setup as a whole, it was possible to extract only this portion of the system and provide it to them. Other partner, deals with the issue of background physical losses detection in the WDN of the pilot *Abbiategrasso* PMZ, and as before it was possible to extract only this portion of the system. In both cases the model, of these two subsystems, allowed the DSS developers to perform their tasks and to share data in an efficient way.

Other model setups included the different variations of demands during the year (e.g. winter demand, autumn and holiday break), the different records of energy tariffs which are negotiated in a yearly basis and models which were used with sensor data in order to perform the calibration. In general, a setup of a daily operation of 24 hours was provided applying a demand pattern based on data collected from sensors Automated Meter Readings (AMR) while typical operation of pump schedules was obtained from the central SCADA with a time resolution of 1 hour.

3.5. Milano WDN model

After finalizing all the analysis as presented above for every element available contained in the DB's of MM up until February of 2015, the final result is the network presented in Fig 2, showing the topographical gradient (Elevation [m.a.s.l.]) in EPANET user interface. The total number of elements is presented in Table 1, showing the large size of the network. To our knowledge there is no system with such a vast number of pump stations for water supply in the EU. The variation in the base demand presented in Table 1, is due to daily variation of demand during the day by the customers. The total pump installed capacity is around 3 times the average demand, proving the existing overkill and the need for an efficient operation of the system.

Table 1. Element counts in Milano WDN model.

Element type	Total count
Number of Junctions	149,639
Number of Pipes	118,950
Pumping stations	26
Booster pumps	95

Element type	Total count
Wells and well pumps	501
Storage tanks	33
Valves	36,295
Check valves	602
Total base demand (m^3/s)	7,5 ±4.2

Figure 2. Milano WDN model. Elevations of the elements of the model. Gradient from North to South.

3.6. Model Calibration requirements

In order to perform the calibration of such system additional campaigns are required. However, in the pilot area Abbiategrasso this task has been performed successfully with sensor data of December 5th of 2014. Some examples of the calibration of results of model calibration are presented. Fig 3. A-D shows observed pressures and simulated pressures in four sensors of the pilot area. In Fig. 3, E-H the same data is presented as scatter plots showing the fitness of the model to real data with bands of 5%, 7.5% and 15% of error. The lowest accuracy obtained for all sensors is for the drop on pressure which occurs during the switch of one of the pumps in the PMZ between 12:00am and 1:00 am as presented in Fig 3 A-D. It is also significant the accuracy of the trend presented by the WDN model as compared with the real system for a fine time resolution of 1 minute at times of high consumption 6:00 am to 9:00am and 8:00 pm to 10:00pm.

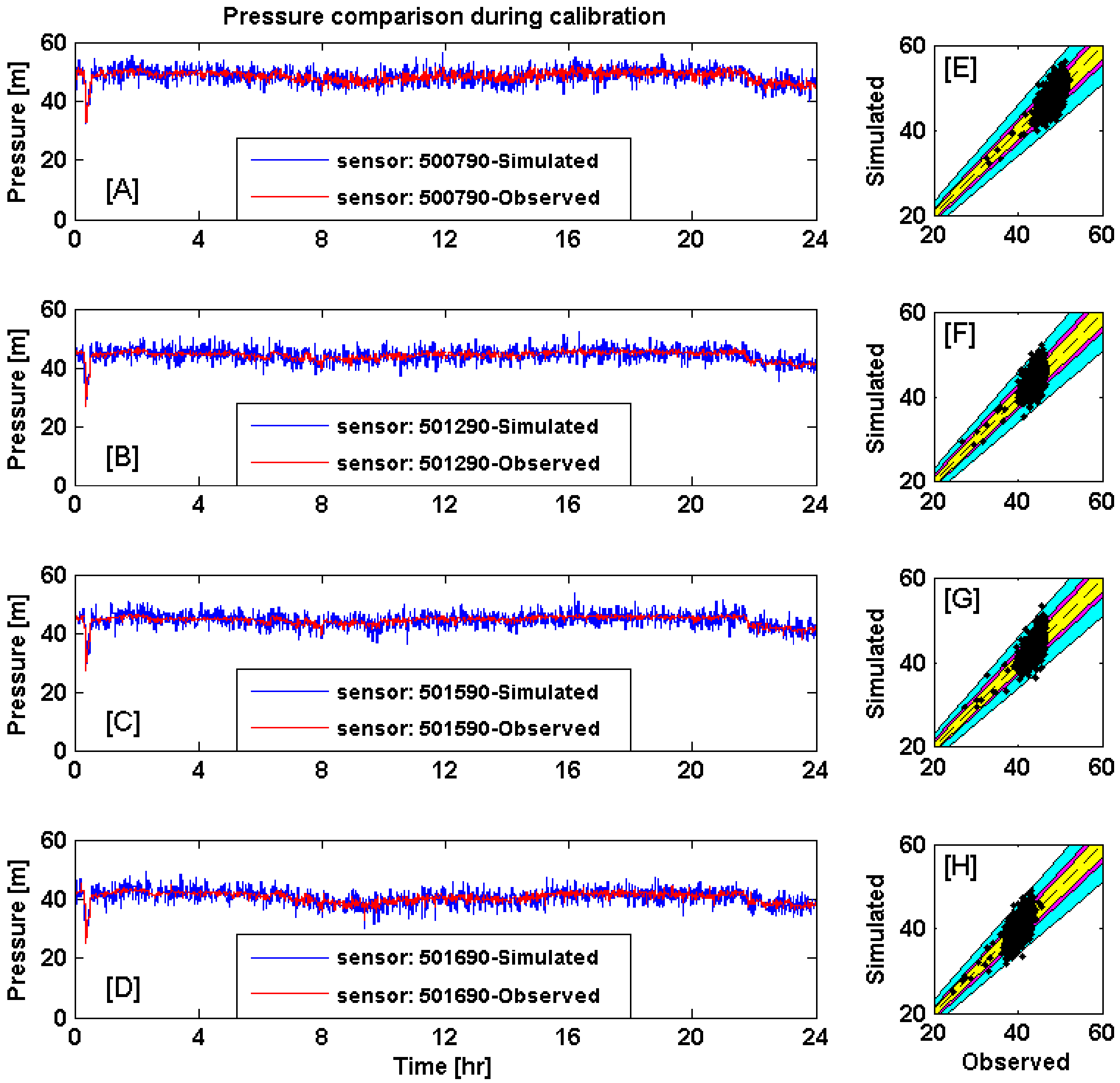

Figure 3. Results of model calibration at Abbiategrasso PMZ. Figures A-D, pressures measured with sensors and simulated pressures, during calibration period in December 2014. Figures E-H, scatter plots of the observed pressures in four sensors versus simulated pressures for the same sensors locations in the model. Error bands are included for ± 5% (yellow), ± 7.5% (magenta) and ± 15% (cyan).

Given the good results of data compilation and topological update presented here have proven to benefit MM with the reduction of model calibration tasks in Abbiategrasso, it is expected that similar results will be obtained for the full network (Fig. 2). For this, an extensive campaign for model calibration is currently under preparation by MM, extending beyond ICeWater project until 2017. A total of 25 portable data loggers have been purchased recently to develop the measurement campaigns in critical areas of the system, and to potentially identify elements such as valves which are broken or completely closed by measuring pressures at upstream and downstream locations.

4. Conclusions

The authors have presented some of the challenges and requirements for the proper construction and update of large WDN models. Some of the hurdles presented here are common in the setup of most WDN models, however the scale of the model presented here and its complexity make it an important milestone at MM for the understanding of the WSS of Milano. Having such model will increase the capacity and flexibility of the utility to test alternatives for operation of the system for efficient energy consumption through pumps scheduling and sectorization.

A proper communication channel across all dependencies in a utility is necessary for the operators. If such channel is not available, it is impossible to perform daily operations in an efficient way. One of the most important tasks for operators is to be able to apply several operational settings without the need to manipulate the real system. For this, a simulation model of the real WDN is necessary, however building and adapting a model becomes a huge task for utilities to deal with, due to the large number of elements involved and the new constructions and reparations that are constantly taking place.

Due to the need to simulate the system without model simplifications, creating the full model of the WDN of Milano has proven to be a time consuming task, but it is shown that the calibration of the pilot area has a minimum difference between measured data and simulation results. Being that the case the applicability of a large model without simplifications with all computer infrastructures available in any location of the world must be possible. In authors' view this should be the way forward into water distribution systems modeling and optimization.

An on-going task in the development of MM model set-up is the continuation of the effort made so far to integrate new parts of the system such as new buildings and complexes. A second task that must be developed, by the utility, in the near future, is to continue the field measurement campaigns for model calibration. If both are not met, the WDN model presented here will become obsolete in no time, due to the constant change in the system of topology and operational strategies.

In the near future, it is expected that this model will serve as a benchmark for MOO on WDN, due to its complex spatial distribution and heterogeneous characteristics. This model is expected to be available for joint research at the end of September of 2015, although some modifications will be made to it for security reasons.

Acknowledgements

The authors would like to thank the operators of Metropolitana Milanese S.p.A. for sharing the data, their effort, patience, dedication and support through unnumbered discussions, technical meetings, teleconferences, fieldwork and phone calls made in the last 2.5 years. Without their help the execution of the work would have been impossible and incomplete. In addition, the two PhD researchers from UNESCO-IHE would like to thank the European Commission (EC) for funding parts of their research through the EU-FP7, ICeWater project.

References

Alperovits, E., Shamir, U., 1977. Design of Optimal Water Distribution Networks. Water Resources Research 13(6), 885-900.

Candelieri, A., Archetti, F., 2014. Identifying typical urban water demand patterns for a reliable short-term forecasting - The ICeWater project approach, Procedia Engineering, 89 (C), 1004-1012.

Carrijo, I.B., et al., 2004. Operational optimization of WDS based on multiobjective genetic algorithms and operational extraction rules using data mining. In: ASCE, ed. Critical Transitions in Water and Environmental Resources Management. Salt Lake city, US; 2004. DOI: 10.1061/40737(2004)442.

Cembrano, G., et al., 1988. Optimization of a multi-reservoir water network using a conjugate gradient technique: a case study. Lecture notes in Control and Information Sciences, 1998,111,987-990. DOI: 10.1007/BFb0042283.

Demographia, 2015. http://www.demographia.com/db-worldua.pdf , Last visited May 05, 2015.

Di Nardo, A., Di Natale, M.A., 2010. Design support methodology for district metering of water supply networks. In: Water Distribution Systems Analysis 2010, Proceedings of the 12th International Conference, Tucson, Arizona, 12–15 September 2010, ASCE, pp. 870–887.

Fantozzi, M., et al., 2013. ICT for efficient water resources management: the ICeWater energy management and control approach. In: Proceedia Engineering for the 12th International Conference on Computing and Control for the Water Industry, CCWI2013. DOI: 10.1016/j.proeng.2014.02.069.

Farmani, R., Savic, D., Walters, G., 2004. EXNET Benchmark Problem for MultiObjective Optimization of Large Water Systems. In IFAC Workshop on Modelling and Control for Participatory Planning and Managing Water Systems.

Fujiwara O., Khang, D.B., 1990. A two phase decomposition method for optimal design of looped WDN. Water Resources Research, 26(4)539-549.

Gessler, J., 1985. Pipe network optimization by enumeration. In: Proceedings of Computer Applications for Water Resources. New York: ASCE; 1985,572-581.

Giustolisi, O., Laucelli, D., Colombo, A., 2009. Deterministic versus Stochastic Design of Water Distribution Networks. Journal of Water Resources Planning and Management, 135(2)117–127.

Giustolisi O., Laucelli, D., Berardi, L., Savic, D., 2012. Computationally efficient modeling method for large WN analysis. Journal of Water Resources Planning and Management, DOI:10.1061/(ASCE)HY.1943-7900.0000517.

Gomez, P., Cubillo, F., Martin, F.J., 2014. Comprehensive and efficient sectorization of distribution networks. Procedia Engineering, 70(2014)753-762. DOI: 10.1016/j.proeng.2014.02.082.

Kang, D., Lansey, K., 2012. Revisiting optimal water-distribution system design: issues and a heuristic hierarchical approach. Journal of Water Resources Planning and Management, 138(3),208-217.

Martinez, F., et al., 2007. Optimizing the operation of the Valencia water-distribution network, Journal of Hydroinformatics, 2007,9(1),65-78. DOI: 10.2166/hydro.2006.018.

OGC, 2014a WaterML 2.0. http://www.opengeospatial.org/projects/groups/waterml2.0swg. Last visted: 06 May 2015.

OGC, 2014b SensorML 2.0. http://www.opengeospatial.org/standards/sensorml. Last visted: 06 May 2015.

Ostfeld, A., et al., 2008. The battle of the water sensor networks: a design challenge for engineers and algorithms. Journal of Water Resources Planning and Management, 134(6),556-568.

Ostfeld, A., et al., 2012. Battle of the Water Calibration Networks. Journal of Water Resources Planning and Management, 138, 523-532. DOI:10.1061/(ASCE)WR.1943-5452.0000191.

Rossman, L.A., 2000. EPANET 2.0 User manual, Water Supply and Water Resources Division National Risk Management Research Laboratory Cincinnati, OH 45268. EPA/600/R-00/057.

Salomons, E., et al., 2007. Optimizing the operation of the Haifa-A water distributon network. Journal of Hydroinformatics 2007,9(1),51-63. DOI: 10.2166/hydro.2006.017.

Savić, D. A., Banyard, J.K., 2011. Water distribution systems, ICE Publishing, London. DOI:10.1680/wds.41127.

Shaake, J. Lai, D., 1969. Linear Programming and Dynamic programming applications to water distribution network design. Rep 116, Dept of Civil Engineering, MIT. Cambridge, Mass, US.

Shihu, S., et al., 2010. Power saving in water supply system with pump operation optimization. In: Proceedings of Power and Energy Engineering Conference (APPEEC). Asia-Pacific, 2010. DOI: 10.1109/APPEEC.2010.5449192.

Todini, E., Pilati, S., 1988. A gradient Algorithm for the analysis of pipe networks. Computer applications in water supply, Systems Analysis and Simulation. 1,1-20. Research Studies Press, Taunton, UK.

Todini, E., Rossman, L.A., 2012. A Unified Framework For Deriving Simultaneous Equations Algorithms For Water Distribution Networks. Journal of Hydraulic Engineering. DOI:10.1061/(ASCE)HY.1943-7900.0000703.

van Zyl, J.E., Savic, D.A., Walters, G.A., 2004. Operational Optimization of Water Distribution Systems Using a Hybrid Genetic Algorithm. Journal of Water Resources Planning and Management Division, 130(2),160–170.

Walski T.M., et al., 1987. Battle of The Network Models: Epilogue. Journal of Water Resources Planning and Management, 113(2)191-203.

Zheng, F., Zecchin, A., 2014. An efficient decomposition and dual-stage multi-objective optimization methods for WDS with multiple supply sources. DOI: 10.1016/j.envsoft.2014.01.028.