Optimization of Water Distribution Networks Using Integer Linear Programming

Article *in* Journal of Hydraulic Engineering · May 2006

DOI: 10.1061/(ASCE)0733-9429(2006)132:5(501)

Hossein Mohammad Vali Samani
Shahid Chamran University of Ahvaz

OPTIMIZATION OF WATER DISTRIBUTION NETWORKS USING INTEGER LINEAR PROGRAMMING

Hossein M. V. Samani[1]   Alireza Mottaghi[2]

ABSTRACT

In this study, the optimum design of municipal water distribution networks is determined by the branch and bound integer linear programming technique. The hydraulic and optimization analyses are linked through an iterative procedure to develop a model which enables us to design a water distribution system that satisfies all required constraints with a minimum total cost. The constraints include pipe sizes, which are limited to the commercially available sizes, reservoir heights, pipe flow velocities and nodal pressures.

INTRODUCTION

In order to design a municipal water distribution network many combinations of pipe sizes and pressure generating facilities may be selected. The design alternative that has a minimum total cost is the one, which must be seeked. Expenses of a water distribution system consists of expenses of pipes and their installations and expenses of pressure generating facilities such as reservoirs and pumps. Utilization of large size pipes decreases head losses and consequently less costs for pressure generating facilities will be required and vice versa for the case of utilizing small size pipes. However, the designer should select the combination of pipe sizes and pressure generating facilities that have the least total cost. Important design constraints consist of commercially available pipe sizes, flow velocities and nodal pressures.

A wide large variety of techniques have been proposed in the literature for optimal design of municipal pipe networks. The bulk transport function has been used as an objective function in the optimization analysis. Strictly, this will not yield the optimum design, since the economy of scale is not introduced (Stephenson, 1984). Dixit and Rao (1987) have used a method in which only the cost of pipes is minimized. They have chosen the possible critical paths between the source and the critical node using a maximum allowable hydraulic grade line.

The common methods of optimization are the linear programming. Dantzig (1963), Gupta (1969), Lai and Schaake (1969), Gupta and Hassan (1972), Alperovits and Shamir (1977), Quindry etal (1981), and Bhave and Sonak (1992) used the linear programming. Walski (1987), Ormsbee (1989), and Samani and Naeeini (1996) used the nonlinear programming. Walski etal (1990) developed the WADISO pipe network design program in which the

[1]Associate Professor, Civil Engineering Department, Faculty of Engineering, Shahid Chamran University.

[2]Graduate Student, Civil Engineering Department, Faculty of Engineering, Shahid Chamran University.

hydraulic analysis was linked to the linear programming for optimization analysis. Morgan and Goulter (1985) linked a Hardy-Cross network hydraulic analysis with linear programming. Zick (1991) employed the GIS and AUTO-CAD and linked them with the WADISO computer program. Taher etal (1996) developed a computer program in which a nonlinear programming is used for hydraulic analysis of the network and linear programming is employed for the optimization analysis and coupled them with the GIS.

In this study, the hydraulic analysis of the network is based on continuity at nodes and Darcy Weisbach or Hazen-Williams formulas and the optimization analysis is performed by the integer linear programming. The hydraulic and optimization analyses are linked in an iterative procedure to obtain the optimal design.

HYDRAULIC ANALYSIS

The hydraulic analysis is performed by employing a network solver using the combined continuity and head loss equations in terms of heads as indicated below:

$$\sum_{i=1}^{NJ} \frac{(H_i - H_j)}{(K_{ij})^{1/n}} \left| H_i - H_j \right|^{1/n - 1} + q_j = 0 \qquad (1)$$

In this equation:

$$K_{ij} = \frac{8 \; f_{ij} \; L_{ij}}{\Pi^2 \; D_{ij}^5}$$ for Darcy Weisbach head-loss relation,

$$K_{ij} = \frac{10.7 \; L_{ij}}{C_{ij}^{1.85} \; D_{ij}^{4.87}}$$ for Hazen-Williams head-loss relation,

f_{ij} = Darcy Weisbach friction factor,

C_{ij} = Hazen-Williams coefficient,

H_i = head at node i of pipe ij,

H_j = head at node j of pipe ij,

L_{ij} = length of pipe ij,

D_{ij} = diameter of pipe ij,

q_i = consumption discharge at node j,

NJ = number of junctions in the network,

n = 2 for Darcy Weisbach relation,

n = 1.8 for Hazen-Williams relation

Colebrook formula is used to calculate the friction factor f in cases where Darcy-Weisbach relation is to be employed. Equation (1) is modified as required for networks including booster pumps, pressure reducing valves or check valves. Applying equation (1) for all nodal points in the network results in a nonlinear system of equations. This system of equations is solved by the Newton-Raphson method.

TOTAL COST OBJECTIVE FUNCTION

The total cost of a municipal water distribution network can be introduced as:

$$F = f(D_N) + g(H_K) \tag{2}$$

Where $f(D_N)$ represents the expenses of pipes and their installations which is a function of pipe diameters D_N, and $g(H_K)$ indicates the expenses of pressure generating facilities (reservoirs and pumps) which is a function of reservoir height or pump total dynamic head. These functions can be shown as follows:

$$f(D_N) = \sum_{N=1}^{NP} L_N \, CP_N \tag{3}$$

$$g(H_K) = \sum_{K=1}^{NR} CR_K \tag{4}$$

Where:

L_N = length of pipe number N,
N = subscript representing pipe number in the network,
CP_N = unit length cost of pipe N which is a function of pipe diameter D_N,
CR_K = cost of pressure generating facility number K, which is a function of H_K (reservoir height or pump total dynamic head),
NP = number of pipes in the network,
NR = number of pressure generating facilities in the network,

Substituting equations (3) and (4) in equation (2) results in:

$$F(D_N, H_K) = \sum_{N=1}^{NP} L_N \, CP_N(D_N) + \sum_{K=1}^{NR} CR_K(H_K) \tag{5}$$

Multiplying terms of the first and second summations of equation (5) by zero-unity variables such as X_{NJ} and Y_{KM}, respectively and adding for all commercially available pipes and reservoirs or pumps yields:

$$F(D_N, H_K) = \sum_{J=1}^{NPA} \sum_{N=1}^{NP} L_{NJ}\, CP_{NJ}(D_N)\, X_{NJ} + \sum_{M=1}^{NRA} \sum_{K=1}^{NR} CR_{KM}(H_K)\, Y_{KM} \qquad (6)$$

Where:

NPA = number of commercially available pipe sizes,
NRA = number of reservoirs or pumps,

When X_{NJ} in equation (6) is equal to zero means that the corresponding commercial pipe size is not included and vice versa when it is equal to unity. Y_{KM} has a similar definition but it is for reservoirs and pumps.

CONSTRAINTS

Equation (6) should include all commercially available pipe sizes for every pipe in the network which means that the number of the terms of the pipe costs will be equal to the number of pipes of the network multiplied by the number of commercially available pipe sizes. The final solution will include only one size for every branch in the network, in other words, no more than one of the variables X_{NJ} for a given N can be equal to unity. Therefore, the following constraint should be considered for every branch in the network:

$$\sum_{J=1}^{NPA} X_{NJ} = 1 \qquad (7)$$

Similarly, the constraint for pressure generating facilities may be introduced as:

$$\sum_{M=1}^{NRA} Y_{KM} = 1 \qquad (8)$$

PRESSURE CONSTRAINT

In order to define pressure constraints a reference node should be selected first. The reference node may be one of the nodes representing the location of a pressure generating facility. Applying the energy equation between the reference node and any node, namely as i, results in:

$$\frac{P_i}{\gamma} = H_R - \Delta Z_{R-i} - \sum_{I=NHR}^{L} h_{f\,I} \tag{9}$$

Where

$\frac{P_i}{\gamma}$ = pressure head at node i,

ΔZ_{R-i} = elevation difference of the reference node and node i,

H_R = head at the reference node,

$\sum_{I=NHR}^{L} h_{f\,I}$ = summation of head-losses of the path starts from the reference node and ends at node i,

NHR = number of the pipe connected to the reference node in the path R-i,

L = number of pipes of the path R-i

Other pressure constraints are upper and lower limits, which can be expressed as:

$$\frac{P_i}{\gamma} \le \frac{P_{\max}}{\gamma} \tag{10}$$

$$\frac{P_i}{\gamma} \ge \frac{P_{\min}}{\gamma} \tag{11}$$

Substituting equation (9) in equation (10) and (11) results in:

$$H_R - \Delta Z_{R-i} - \sum_{I=NHR}^{L} h_{f\,I} \le \frac{P_{\max}}{\gamma} \tag{12}$$

$$H_R - \Delta Z_{R-i} - \sum_{I=NHR}^{L} h_{f\,I} \ge \frac{P_{\min}}{\gamma} \tag{13}$$

In a similar manner to that which was done for the objective function, the zero-unity variables can be applied for equations (12) and (13) to result in:

$$\sum_{M=1}^{NRA} H_{RM}\, Y_{RM} \;-\; \Delta Z_{R-i} \;-\; \sum_{J-1}^{NPA}\;\sum_{I=NHR}^{L} h_{f\,IJ}\, X_{IJ} \;\leq\; \frac{P_{\max}}{\gamma} \tag{14}$$

$$\sum_{M=1}^{NRA} H_{RM}\, Y_{RM} \;-\; \Delta Z_{R-i} \;-\; \sum_{J-1}^{NPA}\;\sum_{I=NHR}^{L} h_{f\,IJ}\, X_{IJ} \;\geq\; \frac{P_{\min}}{\gamma} \tag{15}$$

VELOCITY CONSTRAINTS

Flow velocity constraints can be introduced as:

$$V_{\min} \;\leq\; V_N \;\leq\; V_{\max} \tag{16}$$

Where $V_{\min}$ and $V_{\max}$ are minimum and maximum allowable flow velocities in pipes, respectively. Again, applying the zero-unity variables and substituting for the flow velocity of equation (16) in terms of flow discharge Q_N and pipe diameter D_N gives:

$$V_{\min} \;\leq\; \sum_{J=1}^{NPA} \frac{Q_N}{\frac{\Pi}{4} D_{NJ}^2} X_{NJ} \;\leq\; V_{\max} \tag{17}$$

OPTIMIZATION ANALYSIS BY INTEGER LINEAR PROGRMMING

The objective function given in equation (6) subject to the constraints depicted by equations (7),(8),(14),(15) and (17) should be minimized in order to get the optimum design which has the least total cost. In this typical optimization problem, the unknowns are the zero-unity variables, which are integers. Thus, the integer linear programming can be employed. **LINDO** computer program which utilizes the binary branch and bound method of integer linear programming has been employed in this study to find the optimum design solution . This program solves for X_{NJ} and Y_{KM} variables by such a way that the total cost will be minimized. Based on the determined zero-unity variables the corresponding pipes and pressure generating facilities are obtained.

COUPLED HYDRAULIC AND OPTIMIZATION ANALYSIS

It can be noticed from the explained optimization equations that the pipe flow discharges in the water supply network should be known in order to be able to perform the optimization analysis. In branched networks, the pipe flow discharges can be determined initially just by applying the continuity equation at nodes. But, in looped networks, the pipe flow discharges cannot be determined initially, since they are functions of the pipe sizes and energy generating facility heads that are unknown too. Therefore, an iterative procedure is proposed. The proposed iterative coupled hydraulic and optimization algorithm can be summarized as:

1) Assume pipe diameters and reservoir heights and energy generating facility heads (reservoir heights and pump heads) using commercially available ones.
2) Perform the hydraulic analysis to get the nodal heads, pipe flow discharges and head-losses.
3) Substitute the flow discharges determined through step (2) in pressure and velocity constraint equations (14), (15), and (17).
4) Utilizing the integer linear programming solver for the objective function [equation (6)] subject to the constraints [equations (7), (8), (14), (15) and (17)] to solve for X_{NJ} and Y_{KM} and determine corresponding pipe sizes and energy generating facility heads.
5) Compare resulted pipe sizes and energy generating facility heads with the assumed ones. If the differences are less than a certain defined tolerance then the problem is solved, otherwise, use the resulted pipes and heads as new assumed ones and repeat the procedure starting from step (2), until convergence is achieved.

COMPUTATIONAL RESULTS

Example 1.

A pipeline connected to a reservoir is illustrated in Fig.1. It is desired to determine the pipe size and reservoir height, which yields a minimum total cost.

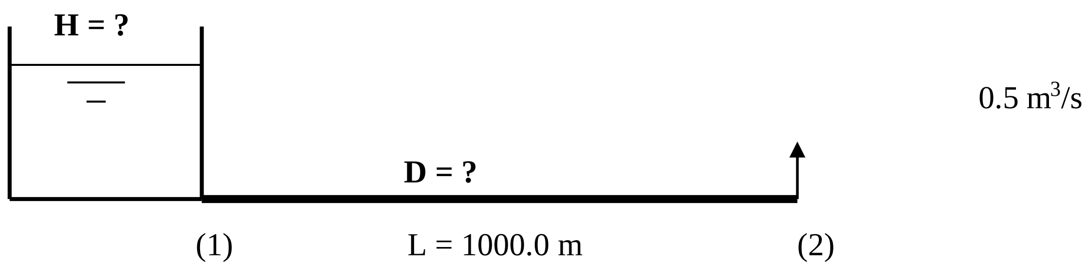

Fig.1 Pipeline of Example 1

The Hazen-Williams coefficient of the pipeline is 130. Commercially available pipes and reservoirs and constraints are given in Tables 1 through 3.

Table1 Commercially available pipes

Diameter (mm)	Unit length cost (Iranian rial)	Zero-unity variable
400	520000	X_{11}
500	580000	X_{12}
600	640000	X_{13}
700	700000	X_{14}

Table 2 Commercially available reservoirs

Reservoir height (m)	Cost (Iranian rial)	Zero-unity variable
35	39000000	Y_{11}
40	48000000	Y_{12}
45	56000000	Y_{13}

Table 3 Velocity and Pressure constraints

	Allowable velocity (m/s)	Allowable Pressure head (m)
Minimum	0.3	20
Maximum	2.5	60

This example indicates a branched network. Thus, the pipe flow discharge is known and the solution will not require iterations. Using equation (6) to obtain the total cost objective function in terms of the zero-unity variables results in:

$$F = 520000 \times 1000\, X_{11} + 580000 \times 1000\, X_{12} + 640000 \times 1000\, X_{13} + 700000 \times 1000\, X_{14} + 39000000\, Y_{11} + 48000000\, Y_{12} + 56000000\, Y_{13} \qquad (18)$$

subject to the constraints obtained by equations (14), (15), (17), (7) and (8), respectively as:

$$35Y_{11} + 40Y_{12} + 45Y_{13} - 31.8X_{11} - 10.78X_{12} - 4.47X_{13} - 2.13X_{14} \leq 60.0 \qquad (19)$$

$$35Y_{11} + 40Y_{12} + 45Y_{13} - 31.8X_{11} - 10.78X_{12} - 4.47X_{13} - 2.13X_{14} \geq 20.0 \qquad (20)$$

$$0.3 \leq \frac{0.5}{\frac{\Pi}{4}(0.4)^2}X_{11} + \frac{0.5}{\frac{\Pi}{4}(0.5)^2}X_{12} + \frac{0.5}{\frac{\Pi}{4}(0.6)^2}X_{13} + \frac{0.5}{\frac{\Pi}{4}(0.7)^2}X_{14} \leq 2.5 \qquad (21)$$

$$X_{11} + X_{12} + X_{13} + X_{14} = 1 \qquad (22)$$

$$Y_{11} + Y_{12} + Y_{13} = 1 \qquad (23)$$

Applying the integer linear programming solver for the above-mentioned problem results in:

$$X_{11} = 0,\ X_{12} = 0,\ X_{13} = 1,\ X_{14} = 0,$$
$$Y_{11} = 1,\quad Y_{12} = 0\ ,\ Y_{13} = 0$$

Hence, the corresponding pipe size and reservoir height will be:

D = 600 mm
H_R = 35 m

Substituting the above results in equation (18) gives the minimum total cost of the network as:

F = 6.79×10^{8} Iranian rials

It should be mentioned that the developed computer program is written so that it does all calculations of the coefficients to set up the problem for the optimization and hydraulic analysis.

Example 2

The network illustrated in Fig.2 consists of three pipes and one reservoir in a looped system. It is desired to determine the optimum pipe sizes and reservoir height.

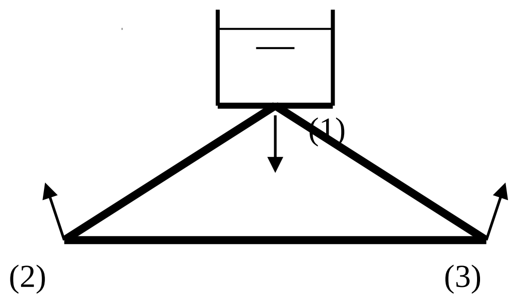

Fig.2 Network of Example 2

The input data of the network are given in Tables 4 through 8.

Table 4 Pipe data

Pipe I	Pipe j	Hazen-Williams coefficient	Pipe length (m)
1	2	130	300
1	3	130	300
2	3	130	450

Table 5 Nodal data

Node number	Elevation (m)	Nodal consumption discharge (l/s)
1	0	180
2	0	10
3	0	8

Table 6 Velocity and pressure head constraints

	Allowable velocity (m/s)	Allowable pressure head (m)
Minimum	0.3	20.0
Maximum	2.0	60.0

Table7 Commercially available pipes and their unit length cost

Diameter (mm)	Unit length cost (Iranian rial)
80	960000
100	1020000
150	1110000
200	1200000
250	1290000
300	1380000

Table 8 Commercially available reservoir heights and costs

Reservoir height (m)	Cost (Iranian rial)
20	12000000
25	21000000
30	30000000
35	39000000
40	48000000
45	56000000
50	64000000
60	80000000

The results obtained by the developed model are given in Tables 9 and 10.

Table 9 Results of Example 2

Nodes	Diameter (mm)	Discharge (l/s)	Head-loss (m)	Flow velocity (m/s)

i	j				
1	2	250	98.01	4.516	2.0
1	3	250	81.98	3.244	1.67
3	2	80	1.99	1.272	0.39

Table 9 Results of Example 2 (continued)

Node	Z (m)	Head (m)	Pressure head (m)
1	0	25.0	25.0
2	0	20.84	20.84
3	0	21.76	21.76

As it is noticed the optimum pipe sizes and reservoir height are:

$D_1 =$ 250 mm, $D_2 =$ 250 mm, $D_3 =$ 80 mm, $H_R =$ 25 m.

The optimum total cost of the network is:

$F = 4.23 \times 10^8$ Iranian rials

Example 3

In order to show the capability of the developed model, a network that consists of two reservoirs, fiftteen pipes, a pump and a check valve as illustrated in Fig.3 is considered.

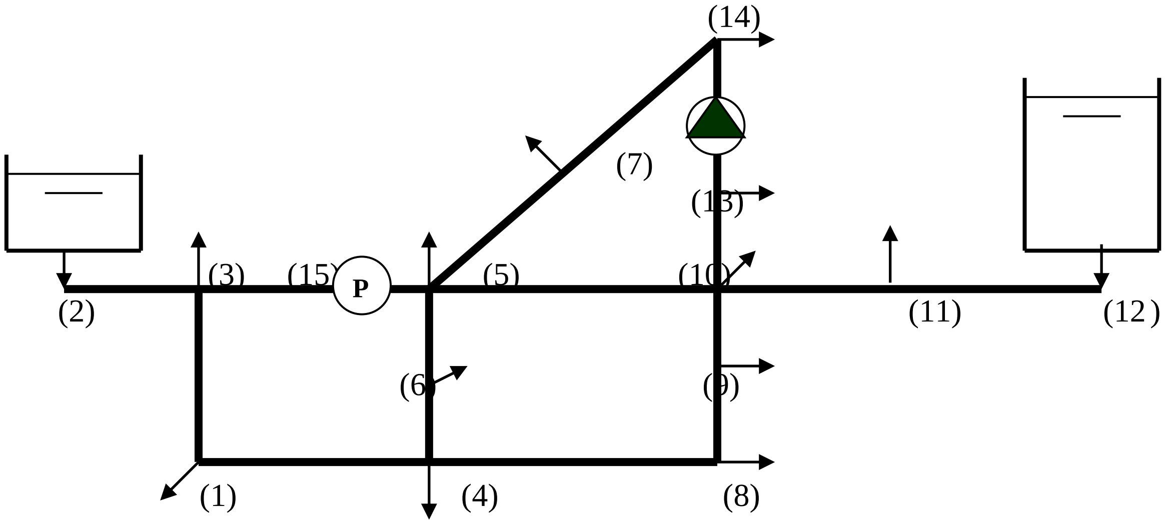

Fig.3 Network of Example 3

The input data of the network are given in Tables 11 through 15.

Table 11 Pipe data

Pipe i	Pipe j	Hazen-Williams coefficient	Pipe length (m)
1	3	130.0	1200.0
1	4	130.0	903.0
2	3	130.0	700.0
3	15	130.0	750.0
4	6	95.0	600.0
4	8	100.0	1800.0
5	6	95.0	600.0
5	7	100.0	1500.0
5	10	130.0	1800.0
5	15	PUMP	
7	14	100.0	1200.0
8	9	100.0	600.0
9	10	100.0	600.0
10	11	130.0	2100.0
10	13	90.0	1200.0
11	12	130.0	2200.0
13	14	90.0	1100.0

Table 12 Nodal data

Node number	Elevation Z (m)	Nodal consumption discharge (l/s)
1	345.0	25.0
2	410.0	0.0
3	365.0	6.0
4	330.0	12.0
5	335.0	12.0
6	328.0	20.0
7	340.0	3.0
8	340.0	30.0
9	335.0	30.0
10	330.0	12.0
11	380.0	6.0
12	400.0	0.0
13	338.0	6.0
14	338.0	3.0
15	335.0	0.0

Table 13 Velocity and pressure head constraints

	Allowable velocity (m/s)	Allowable pressure head (m)
Minimum	0.1	30.0
Maximum	3.0	90.0

Table 14 Commercially available pipe sizes and their unit length cost

Diameter (mm)	Unit length cost (Iranian rial)
150	370000.0
200	400000.0
250	430000.0
300	460000.0
350	490000.0
400	520000.0
450	550000.0
500	580000.0
550	610000.0
600	640000.0

Table 15 Commercially available reservoir heights and costs

Reservoir height (m)	Cost (Iranian rial)
10	20000000.0
20	25000000.0
30	30000000.0
40	48000000.0
50	64000000.0
60	80000000.0
70	96000000.0
80	112000000.0

The results calculated by the developed computer program are given in Tables 16 and 17.

Table 16 Results of Example 3

Nodes i	j	Diameter (mm)	Hazen-Williams coefficient	Length (m)	Discharge (l/s)	Head-loss (m)	Velocity (m/s)
1	3	300	130.0	1200.0	-206.0	29.4	2.92
1	4	450	130.0	903.0	181.0	2.46	1.14
2	3	250	130.0	700.0	130.6	17.94	2.66
3	15	200	130.0	750.0	-81.37	23.69	2.59
4	6	350	95.0	600.0	106.8	3.69/	1.11
4	8	400	100.0	1800.0	61.9	1.91	0.49
5	6	300	95.0	600.0	-86.94	5.33	1.23
5	7	200	100.0	1500.0	5.34	0.47	0.17
5	10	150	130.0	1800.0	-11.5	6.21	0.65
5	14	150	100.0	1200.0	2.12	0.31	0.12

8 9	300	100.0	600.0	31.8	0.76	0.45
9 10	150	100.0	600.0	1.94	0.13	0.11
10 11	150	130.0	2100.0	-31.4	46.14	1.78
10 13	150	90.0	1200.0	9.72	6.04	0.55
11 12	200	130.0	2200.0	-37.37	16.46	1.19
13 14	150	90.0	1100.0	3.89	0.96	0.22
5 15		PUMP		81.37	64.57	

Table 17 Results of Example 3 (continued)

Node	Consumption (l/s)	Elevation (m)	Total head (m)	Pressure head (m)
1	25.0	345.0	372.16	27.16
2	-130.60	379.5	419.50	40.00
3	6.0	365.0	401.56	36.56
4	12.0	330.0	369.70	39.70
5	12.0	335.0	360.68	25.68
6	20.0	328.0	366.01	38.01
7	3.0	339.0	360.21	21.21
8	30.0	340.0	367.78	27.78
9	30.0	335.0	367.03	32.03
10	12.0	330.0	366.90	36.90
11	6.0	380.0	413.04	33.04
12	-37.37	369.5	429.50	60.00
13	6.0	338.0	360.86	22.86
14	6.0	338.0	359.90	21.90
15	0.0	335.0	425.25	90.25

The optimum pipe sizes are depicted in Table 16 and the optimum reservoir heights are given in Table 17 as pressure heads at nodes 2 and 12. The calculated optimum total cost of the design is:

$F = 8.079 \times 10^9$ Iranian rials

which includes the pump cost (8.0×10^8 Iranian rials)

CONCLUSION

The total cost objective function and constraints of a municipal pipe network are normally nonlinear. Optimization methods using nonlinear programming are very common to solve such problems. But, there is no guarantee to obtain the global optimum by using the nonlinear programming. Optimization methods utilizing linear programming are capable to determine the global optimum directly and very fast. In this study, the nonlinear objective function and the constraints are linearised by using the zero-unity variables. Therefore, the unknowns will be only the zero-unity variables, which are integers. This enables us to use the integer linear programming for solving optimization problems of municipal systems in which obtaining the global optimum is guaranteed. The proposed method has also the advantage of being able to determine the discrete characteristics of the decision variables in real cases (i.e., commercially available pipe sizes) while nonlinear optimization methods do not have such capability and they obtain the variables in a continuous domain which is mostly not realistic. In addition, the proposed method was found very fast in term of convergence.

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APPENDIX II. NOTATION

The following symbols are used in this paper:

C_{ij} = Hazen-Williams coefficient,

CP_N = unit length cost of pipe N which is a function of pipe diameter D_N ,

CR_K = cost of pressure generating facility number K,

D_{ij} = diameter of pipe ij,

F = total cost of a municipal water distribution network,

f_{ij} = Darcy Weisbach friction factor,

$f(D_N)$ = expenses of pipes and their installations,

$g(H_K)$ = expenses of pressure generating facilities (reservoirs and pumps),

H_i = head at node i of pipe ij,

H_j = head at node j of pipe ij,

H_R = head at the reference node,

L = number of pipes of the path R-I,

L_{ij} = length of pipe ij,

L_N = length of pipe number N,

N = subscript representing pipe number in the network,

NJ = number of junctions in the network,

NP = number of pipes in the network,

NR = number of pressure generating facilities in the network,

NHR = number of the pipe connected to the reference node in the path R-i,

n = a constant equals to 2 for Darcy Weisbach relation and equals to 1.8 for 1.8 for Hazen-Williams relation,

$\frac{P_i}{\gamma}$ = pressure head at node i,

$P_{\min}$ = minimum allowable pressure at nodes,

$P_{\max}$ = maximum allowable pressure at nodes,

q_i = consumption discharge at node j,

$V_{\min}$ = minimum allowable flow velocity in pipes,

$V_{\max}$ = maximum allowable flow velocity in pipes,

X_{NJ} = zero-unity variable for pipes,

Y_{KM} = zero-unity variable for reservoirs or pumps,

ΔZ_{R-i} = elevation difference of the reference node and node i,