# Systems Analysis in Water-Distribution Network Design: From Theory to Practice<sup>a</sup>

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ABSTRACT: Review of the use of system-analysis techniques, and in particular optimization, to design water-distribution networks reveals that in spite of the considerable development of models in the literature they have not been accepted into practice. This lack of acceptance is present even though a competitive evaluation of the component design models has shown them to be capable of designing realistic networks. The lack of acceptance is attributed primarily to the absence of suitable packaging to make the algorithms useful in a design office environment. This evidence suggests that, from a practice point of view, there is relatively little need for further development of these component design models, other than the packaging. Reliability analysis in water-distribution network design has not yet entered practice either. In contrast to the component size problem, reliability analysis has not been accepted primarily because of a lack of reliability measure that is both comprehensive in its interpretation of reliability and computationally feasible. More research is needed in this area before reliability can be explicitly incorporated into design procedures. Development of decision support systems (DSS) incorporating optimization and classical simulation models with an interactive graphics capability is seen as being a major priority for future research in the field, particularly if optimization approaches are to become more common in design practice. These DSSs can assist the designer in including and evaluating reliability and generating alternative solutions.

## INTRODUCTION

Research on the application of systems-analysis methods, and in particular optimization procedures, to the design of water-distribution networks has been reported since the 1960s, e.g., Karmeli et al. (1968) and Schaake and Lai (1969). In recent years, three comprehensive reviews of the state of research in the field have been undertaken (Walski 1985b; Goulter 1987; Walters 1988). Now that the methodologies have been around for nearly 25 years, it is useful to examine the extent to which the techniques and approaches have been incorporated into engineering practice. Such an evaluation is the focus of this paper.

The review itself is undertaken in four major steps. First, a number of questions that define how such an evaluation might be performed are posed. The path research has actually taken as it relates to these questions is then examined. The impacts of the research on practice are then discussed. Finally, some future directions for research, as defined by fundamental theoretical considerations and the requirements of practice, are reviewed.

# CRITERIA FOR EVALUATION

Perhaps the biggest question relating to the use of systems analysis in water-supply practice is whether true "optimal" designs have ever been

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obtained for inherently complicated, but common, looped water-distribution networks, either by traditional (classical) design approaches, e.g., using simulation, or by formal optimization (systems analysis) procedures. Without being able to provide a formal proof, it is reasonable to say that such optimal designs have not been obtained. Good, and indeed "better," designs have been obtained, but there has been no guarantee of global optimality.

Before proceeding further it is wise to examine what actually constitutes as an optimal design, or, more specifically, what the objective of an optimal design is. Here the optimal design is that which meets the applied demands at least cost. Note that this definition incorporates the implied consideration of multiple loadings, failure conditions in the system components, and reliability. Reliability, however, is not explicitly analyzed, at least not in a probabilistic sense. Even with this simple but realistic definition, the problem has not been solved for global optimality.

In attempting to assess the value of optimization models in practice, it would be useful to be able to show how close the designs produced by them are to the global optimum and, presumably, how much better they are than classically designed systems. A comparison between systems-analysis-derived designs and classical designs presents no conceptual problem. However, one potential difficulty would be the variation in the "goodness" of the classically designed systems caused by ranges in experience of the designers. Such an evaluation would also be very time-consuming.

However, comparison to the true optimal is a more difficult task because the global optimum is not known. It might reasonably be asked, Should the results actually be compared to the global optimum? A new engineering approach is useful, and will probably be adopted in practice, if it gives better results than existing methodologies with similar computational effort. The comparison with classical designs would therefore appear to be the only one necessary. Although such a comparison has not been carried out, a number of optimization models have been assessed relative to each on a practical design problem (Walski et al. 1987). The most interesting result of the assessment of Walski et al. (1987) was the similarity in, rather than the difference between, the designs, both in terms of cost and in component selection. In fact, the costs of the solutions determined by the models only varied by 12%. Furthermore, the most expensive system arose because of increased levels of reliability imparted by additional storage in the system. This most expensive system, which constitutes the worst solution from the narrow definition of optimality just given, may in fact be the most desirable when all issues are considered.

From an overall evaluation perspective the similarity of the results in Walski et al. (1987) given the quite different optimization schemes in the models indicates that the optimization models are relatively robust and that the optimization technique itself is not of great importance. The fact that the models were able to be used to design a practical system also shows that they could be used in a design environment.

## **HISTORY OF RESEARCH**

This section examines how research on the development of optimization approaches for distribution network design has progressed; how that progress has reflected the requirements of design practice; and, where applicable, how this research has impacted practice. It does not purport to provide a complete summary of all research in the field. This sort of summary was

undertaken by Walski (1985a), Goulter (1987), and Walters (1988). This paper focuses on the approaches covered in those reviews plus more recent techniques that have constituted a significant advancement in the field.

The earliest models for water-distribution network design were developed for branched networks, e.g., Karmeli et al. (1968) Schaake and Lai (1969). These models did not consider appurtenances such as valves, but indicated that the impacts of these fittings in terms of hydraulic performance and cost could easily be evaluated once the formal network structure of pipe layout and pipe sizes had been determined. The approaches guaranteed a global optimum but, since they were applicable only to branched systems, were not useful in practicing engineers working in urban water-distribution network design. Other nonlinear models were also proposed around this time [e.g., Schaake and Lai (1969) and Liang (1971)], but they, too, were applicable to branched systems only, did not have computational advantages of the linear programming based approaches, and do not appear to give better results.

An indication of the relative strength of the linear programming methodologies was the use of a fundamental linear programming formulation in the first model for looped systems able to attract any theoretical and practical interest. This model, proposed by Alperovits and Shamir (1977) and corrected by Quindry et al. (1979), used additional constraints to ensure that hydraulic consistency was maintained in the loops, i.e., that the algebraic sum of head losses around a loop be zero.

A fundamental problem in the least-cost design of looped systems relative to branched systems occurs when assigning the flows in the individual pipes. In branched systems, a given demand pattern defines the flows in the pipes explicitly and uniquely. In a looped system there are an infinite number of distributions of flow in the network that can meet a specified demand pattern. The distribution of flows assumed in the pipes affects the lowest cost that can be achieved for network. Alperovits and Shamir (1977) approached this problem by employing a gradient-search approach to identify the flow pattern that permits the minimum overall cost for the distribution system to be obtained. Quindry et al. (1981) also used a two-step methodology for the design of looped networks. This use in these two models of a two-step process in an optimization framework represented a new stage in the development of the optimization models for network design. Employment of a two-step iterative process is an implicit recognition of the complexity of the network design task and the apparent inability of "one-step single-run" models to provide a solution even close to optimal for realistic networks.

All the aforementioned procedures suffered from two major but related shortcomings. They were not able to handle even moderate numbers of multiple-load cases simultaneously without significant, and apparently impractical, increases in computational requirements; and they tended to optimize redundancy (or flexibility) out of the system by designing a system that was implicitly branched (Templeman 1982). More specifically, their designs had loops, but the loops were essentially created by cross connections between branches that were composed of minimum specified pipe sizes. These minimum pipe diameters did not provide sufficient capacity or ability to provide flow by alternative supply paths should the links containing them be required for such alternative flow paths. Designs that are not implicitly branched could be obtained by taking a sufficient number of combinations of load cases and failed components. However, as noted, simultaneous consideration of such cases is not computationally practical.

The next major step in the development of design models focused on ways of incorporating multiple-load patterns into the design procedure. The approaches taken to handle the multiple-loads issue were again two-step procedures, a process consistent with the complexity of the design problem noted earlier. One of the first procedures was that of Rowell and Barnes (1982), who in their initial step developed an optimal (least cost) branched system and then designed cross connections with sufficient capacity to meet demands with any single link being broken. Morgan and Goulter (1985) proposed a two-step heuristic procedure based on a linear programming approach to address the requirement for multiple-load cases, a procedure that required an iterative interaction between the two stages of the model and was able to handle simultaneously as many load cases as there were links in the network.

Lansey and Mays (1989) reported on the development of two-step procedure for design of water distribution under multiple loading conditions. This work was an advance over the models of Rowell and Barnes (1982) and Morgan and Goulter (1985) in that it considered sizing and location of pumps, storage tanks, and valves as well as pipes. The procedure was still computationally intensive, however, requiring a large number of iterations between an optimization model and a simulation model, with gradient terms being considered at each step.

It might be argued that at this point the problem of developing models that are computationally useful in practical applications has been solved. The "Battle of the Network Models" showed that when used correctly, i.e., as tools rather than as approaches that give the absolute answer, the available models were robust and provided useful solutions for water-distribution network design, upgrading, or rehabilitation (Walski et al., 1987). However, reliability of the network was not yet able to be handled explicitly. Furthermore, the problem of the optimal layout and design of the network was not well solved.

Reliability has two dimensions that cause its explicit inclusion in optimization design models to be quite difficult. Consider network "failure" as the event in which a network is not able to provide sufficient flow or sufficient pressure to meet the demand. Under this definition, failure can occur either if a component (e.g., a pipe) fails and the system is unable to meet the demands while that component is out of service, or if the actual demands exceed either the design demand values for new systems (or the network capacity in aged networks). Although there is some relationship between improved resistance to one type of failure and improved resistance to the other, i.e., large pipes fail less often and can carry large volumes of flow, the two cases are independent for practical purposes.

This difficulty has prompted predictions that explicit consideration of network reliability in design optimization models is one of the most challenging tasks facing researchers working in the field (Goulter 1987; Walters 1988). In line with these predictions, considerable effort has been directed at the reliability question over the last few years. This work has ranged from analysis of the supply-and-demand issues using a partial-duration series approach (Biem and Hobbs 1988; Hobbs and Biem 1988; Duan and Mays 1990), "availability" definitions (Cullinane 1986); cut-set-based procedures (Shamir and Howard 1985; Mays et al. 1986; Su et al. 1987; Wagner et al. 1988; Quimpo and Shamsi 1987; Shamsi 1990), approaches combining optimal design of the pumping and distribution system (Duan et al. 1990), recognition of the uncertainties in the parameters of the system (Lansey et

al. 1989), heuristics based upon probability of nodes being isolated and demands exceeding design values (Goulter and Coals 1986; Goulter and Bouchart 1990) and ratio of expected maximum total demand to total water demanded (Fujiwara and De Silva 1990; Fujiwara and Tung 1991).

Relatively little success has been achieved in obtaining comprehensive measures of network reliability that are computationally feasible and physically realistic (Goulter 1987; Jacobs and Goulter 1988; Lansey and Basnet 1990). The measures that give good representations of reliability are computationally impractical, e.g., the 200.5 min of computer time on a Dual Cyber required by the Su et al. (1987) model to analyze a three-loop example. On the other hand, those approaches that are computationally suitable for inclusion in an optimization framework provide very poor descriptions of network performance. This observation applies to measures that are able to incorporate only one of the two failure types and even more so to measures that attempt to consider the two failure types jointly.

One of the more interesting recent approaches to the joint consideration of the two failure modes is the work of Bouchart and Goulter (1991), who explicitly recognized that demands are not in fact concentrated at nodes but are distributed along the links. This point is relevant in that it reflects how practice handles reliability of supply to customers. Improved reliability performance is often achieved in practice through the addition of valves to the links so that when a pipe failure does occur smaller lengths of main (and therefore fewer customers) have to be isolated during the failure and repair processes.

Another development on the reliability issue that appears to have value for practitioners is an approach proposed by Ormsbee and Kessler (1990). In that work, an efficient algorithm is used to identify two independent paths to each demand node. These two paths can then each be designed to carry the full demand or, alternatively, one path can be designed to carry the demand and the other some reduced, but acceptable, level of the full load, with a corresponding decrease in the cost of the system.

Increased interest in the layout of networks as well as the sizing of the components also began to occur parallel to the efforts to recognize reliability. The early design models had concentrated on finding the least-cost combination of components (pipe, pumps, etc.) able to fulfill the design under a predefined layout of pipes. Interest in the layout question arose both from the question of how layout affects cost and from the realization by a number of researchers that network reliability is in fact defined, or more specifically constrained, by the fundamental layout of the network (Goulter 1988). Networks with better shapes, i.e., with more redundancy in terms of interconnections etc., will be more reliable.

Difficulties in designing the layout for optimal reliability and cost arose from the fact that cost and reliability are both dependent on the shape of the network and that improving one tends to degrade the other. It is useful, however, to understand what an optimally reliable network looks like. Once this is known it becomes a target, but not necessarily a requirement, for the layout. Unfortunately, no such specification of layout specifically for water-distribution network reliability is available. Jacobs and Goulter (1988) noted that the optimally reliable network for a specified number of links in a set of nodes is regular, i.e., has an equal number of links incident on each node. For practical water-distribution networks this requirement is unrealistic, because nodes on the periphery of the network will have fewer links incident upon them than those nodes in the interior of the network. A

preliminary investigation of the impacts of using the regular graph target for layout of water-distribution networks was subsequently carried out by Jacobs and Goulter (1989), but the results were inconclusive.

### IMPACTS ON PRACTICE

Given the progress in system-analysis modeling for water-distribution networks outlined in the previous section, it might be anticipated that systems analysis is either already well integrated into practice or at least rapidly gaining in acceptance. Furthermore, Walski (1985a) and Goulter (1987) both predicted that within the next 10–15 years optimization design models would be in widespread use in design offices. Reality, however, is quite different; optimization models are not widely used and, even more disturbingly, show no signs of being accepted.

The question may therefore be asked as to why optimization has not enjoyed its anticipated success in water distribution design. An engineering tool or methodology will be accepted in design practice if it gives a better answer for about the same cost of design. The answer does not have to be optimal, just better than those obtained using present techniques. In examining the usefulness of optimization models in this light there are four potential reasons why the models have not been accepted: (1) The models do not work, i.e., they cannot derive solutions for practical problems or they do not given sensible solutions; (2) the solutions they provide offer no improvements over established methods and professional judgment; (3) they are too difficult to use; or (4) practitioners are not comfortable with the overall optimization approach. All four are related in some way to the ability of optimization models to derive better answers with reasonable levels of effort. In terms of point 1, the "Battle of the Network Models" (Walski et al. 1987) lends strong support to an assertion that the models do in fact work. The models do not necessarily give the answer, but they are very useful tools for getting good answers that can be refined and verified using simulation techniques.

Point 2 is more troubling. It is not yet clear that the models are able to give better solutions than established methods. This issue was raised earlier in relation to comparison of the designs produced by optimization models and established techniques. Such a comparison has not yet been undertaken and would be a very useful investigation in its own right. If the optimization solutions give even comparable results in less time and with less cost, a strong argument would be made for their use.

With reference to point 3, many of the models are very difficult to use, not because they are complex but because the input-output interfaces are very rudimentary. The models are almost invariably developed in academic environments, where the algorithm rather than the input-output interface is the most important issue. Another reason for the lack of use of these academic codes is that they are not often readily available for outside use. Once these codes are made more accessible and the input-output modes improved to the level currently available for simulation approaches, e.g., KPIPE (Wood 1980), optimization design models are likely to be more accepted in practice. Acceptance of the optimization approach will also be accelerated by the growing numbers of practitioners who are familiar with the optimization techniques themselves. On a cautionary note, it is unreasonable to expect the academic environment to develop these polished interfaces. Such development is computationally intensive and does not, in

general, fit the research requirements of university activity. The onus therefore falls upon software manufacturers to develop the interfaces.

Point 4, namely that users are not comfortable with the approach, is closely related to point 3. Many of the older engineers involved in network design have not had the opportunity to study formal optimization techniques. It could therefore be argued that they are uncomfortable with optimization design models because they do not understand the algorithms. This argument is a little misleading because many of the same engineers may not fully understand the solution algorithms, e.g., Newton-Raphson, in the simulation models. Yet these simulation models are in widespread use in practice. A more likely reason for discomfort with the overall approach is that the models are difficult to use (see point 3). The effort required to use the models, therefore, prevents practitioners from using them on a sufficiently frequent basis to become comfortable with the strengths and weaknesses of the approach. Improvements in the user interfaces will accomplish a great deal toward overcoming this problem.

## FUTURE DIRECTIONS FOR RESEARCH

Given the strengths of the design optimization models demonstrated in the "Battle of the Network Models" (Walski et al. 1987), it is reasonable to assert that no more research is needed for the design models. The last few years have only seen marginal improvements in the basic models. Furthermore, these improvements have only been obtained with sophisticated and elegant algorithms, which hold more attraction for theoreticians than for practitioners.

This assertion does not remove the need for work on better packaging for the optimization models. In the previous sections it was asserted that the interface development should be left to software specialists. Nevertheless, there is still room for more research on how to integrate simulation and optimization models in a single "decision-support system" (DDS) whereby the strength of each type of approach can be exploited. This task is not as straightforward as it seems, particularly if the reliability aspect of network design is to be recognized explicitly.

It is useful to look at the exact role that reliability will take in waterdistribution network design. Reliability, particularly in its probabilistic sense, is not currently explicitly included in design processes, not just because explicit, and generally acceptable, measures are unavailable but also because the "codified" approaches combined with the verification by simulation have worked well historically. In the opinion of the writers, reliability will become an increasingly important aspect of network design practice. There is, therefore, a great need to develop reasonable reliability measures for distribution networks. These reliability measures should have a sound theoretical basis and yet be understandable to practitioners. Such characteristics represent a major challenge given the complexity of the reliability question.

Since network reliability and cost are functions of network layout as well as component design there is also a need to develop a better understanding of what constitutes a reliable layout and the implication of that layout on the component design. A better theoretical understanding of reliability in terms of layout is required; practicing engineers are also likely to have an intuitive understanding of the issue. A problem facing reasearchers is how to quantify this understanding and match it to theoretical definitions of reliability and redundancy.

Development of an appropriate reliability measure, as described previously, would be a very useful contribution to such a DSS. Given the current lack of such a measure, the graphical component of a DSS should not just be a simple display (summary) of input and output data but must also include reliability indicators. An appropriate use of graphics in the DSS would be in assisting the design engineer in interpreting the level of reliability of a particular network design and, if necessary, providing a means of easily improving that reliability. How graphics can be employed in this context is difficult to say at this time, which is precisely the reason it is fruitful area for basic theoretical research.

Development of this DSS can also be extended to the whole question of what constitutes a good (in terms of both cost and reliability) design. Knowing the answer to this question, or at least having the mechanism, e.g. a DSS, to develop the answer for a particular network would reduce the number of iterations of the optimization and simulation models necessary to obtain a final acceptable design. The major difficulties facing the development of such a system is that there is not even an objective function statement. Furthermore, the design of complex network systems and water-distribution network design for minimum cost and maximum reliability is such a problem that pure optimization loses its value for obtaining the answer. Dubois (1983) summarizes the problem for networks well with "... many solutions quite different in nature have objective function values very close to each other."

In other words, even if an appropriate objective function could be formulated, optimizing that objective function may prevent the identification of solutions that are very similar in objective function value but quite different in general form and perhaps more desirable overall on the basis of characteristics excluded, either consciously or unconsciously, from the objective function. It might be argued that engineering judgment would be able to take the solution from the optimization and modify it to a desired solution. However, if the alternative layouts are quite different in nature, it is far less likely that tinkering with a particular solution will actually identify the true alternatives. In this context, optimization is clearly only a tool in the overall design process and not a provider of the answer. In fact, optimization becomes an initiator in the complete design process, with the real design occurring after development of the alternative optimal solution(s).

Is it reasonable to expect that such a DSS can be developed? In the opinion of the writer the answer is yes. The basis of this confidence is recent advances in the development of DSSs for layout and design of branched rural gas distribution networks (Davidson and Goulter 1991a, b). The DSS developed in that work was able to generate an alternative network design for an existing distribution system that was able to fulfill all the design requirements with a 10% saving in cost. The structure of the DSS, together with the graphical representation and interpretation of the problem, in particular the design steps taken to develop the solution, provide a model for application to water-distribution networks. The key element in that DSS structure was the division of the system into a procedural component in which those rules that are easily stated mathematically can be incorporated into the model, and a cognitive component in which a rule base is used to transfer the expert's knowledge to the design process. More applied research will be required in this area to transfer the concepts used in the gas-distribution.

bution problem to the looped water networks; it is an area that appears to have significant benefits for the practitioner as well as theoretical challenges for researchers.

## SUMMARY

A review of the application of systems analysis in water-distribution network design reveals that, although the subject has been studied by researchers, there is little or no acceptance of optimization into engineering practice. This lack of acceptance is somewhat surprising given recent work that has showed that the component design models are quite robust, versatile, and capable of handling relatively complicated design problems (Walski et al. 1987). The primary cause for the nonacceptance of these design approaches appears to be the lack of suitable packaging of the approaches for ease of use in a design environment. It also appears that there is no pressing need, other than for theoretical considerations, for further development or refinement of component design algorithms in order for them to become accepted in practice.

It is anticipated that reliability issues will become increasingly important in design of water-distribution networks. At this time the reliability question is often not addressed explicitly in the design procedures. Unlike the component design problem, the neglect of reliability is due to the lack of a network reliability measure that is both comprehensive in its interpretation of reliability and computationally feasible. The lack of such a measure is due in large part to the complexity of the reliability problem in waterdistribution networks. Although recent research efforts have been directed at those reliability questions a lot more research is needed before reliability can be explicitly considered in design models or, for that matter, the design process.

There also appears to be a need for development of decision support systems (DSS) for design of water-distribution networks. These DSSs should be able to combine optimization and simulation models to exploit the strengths of each, and to use an interactive graphical basis not just to display input and output data but also to assist in the inclusion and interpretation of reliability in the network solutions. The DSS should also be able to develop alternative solutions. Although these requirements for a DSS appear to be very demanding, the success of a DDS with similar characteristics applied to branched gas-distribution systems indicates a real potential for the development of systems for more-complex looped water-distribution networks.

From a purely theoretical point of view there is still a need for research on fundamental issues related to optimal network design in terms of cost, layout, and reliability. The relationship between layout and cost and/or reliability in particular is not well understood. A better theoretical understanding of this relationship would define targets for the practical design models (DSS) that have to address these issues.

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