Optimization of the Designed Water Distribution System Using MATLAB

Al-Amin D. Bello^{1,*}, Waheed A. Alayande², Johnson A. Otun³, Abubakar Ismail³, Umar F. Lawan¹

¹Faculty of Civil Engineering, Universiti Teknologi Malaysia, Johor Bahru, Malaysia ²National Water Resources Institute (NWRI) Mando, Kaduna State, Nigeria ³Department of Water Resources and Environmental Engineering, Ahmadu Bello University Zaria, Nigeria

Abstract This paper provides a technique of cost optimization for the proposed water distribution system before implementation. A lot of existing water distribution network analysis software lack optimization modules but ensure other essential conditions are satisfied. Dukku Town chose as a case study, it suffers from poor water distribution network which called for the system upgrade. The models developed by Alperovits and Shamir (1977) and modified by Goulter and Coal (1986) were used to accomplish the desire objectives. A programme is written in MATLAB using linear optimization components to compute the least cost possible. Changes in pipe diameters were obtained and there is slight significant decrease in total cost of pipes. A total of 7.15% reductions in the initial cost are noted, as the hydraulic properties of the entire distribution network is improved. The maximum pressure before optimization is 14.79m and after optimization increased to 15.71m, the minimum pressure on the former is 7.59m and 9.42m for the later. The same improvement is observed in the flow rate, velocities and Headloss, all falls within the designed criterion. The efficiency of the network and risk is also considered by incorporating reliability constraint. The final optimized designed water distribution network addresses the problem of the water shortage of the studied area.

Keywords Optimization, Linear programming, MATLAB, Statistical Analysis, Dukku town

1. Introduction

A water distribution system (WDS) is a hydraulic conveyance system laid on road shoulders where topology and topography are known and that transmit water from the Source to the consumers; it consists of elements such as pipes, valves, pumps, tanks and reservoirs, flow regulating and control devices [1]. Water supply system efficiency is of primary importance in designing either new water distribution networks or expanding existing ones [2]. A WDS is normally designed and operated to satisfy various customer demands over its service life. Decision makers always explore innovative and efficient strategies to reduce the huge economic requirements of designing and construction of WDS, coupled with satisfying the quantity and performance objective of the system [3]. With significant development of urbanized areas and construction of thousands of small and large-scale water supply and distribution systems in recent decades, yet few people have access to clean water and adequate sanitation. However the quality of service provided by water utilities are unsatisfactorily and the cost of new systems are expensive

[4]. In order to provide a reliable framework for system's operation, it is necessary to identify the most critical system requirement with a least cost that will enhance proper deliverance and effective operational management [5]. A lot of Research had been conducted to achieve these objectives [6-12]. Although pipe sizes and cost are the most important components of a water supply networks but other element are also considered because any water distribution network consists of three major parts and components: namely pumps, distribution storages, and distribution piping network [13]. Although in optimizing a system, the designer must take some expected and unexpected loading conditions into consideration to ensure effective and sufficient deliverance of water to end user [14].

Selection of pipe diameters from a set of commercially available diameters to form a water distribution network of least capital cost has been shown to be a difficult task [1]. The cost of maintenance and operation of a water distribution system may be considerable, but still one of the main costs is that of the pipelines. The use of optimization methods for WDS has been mostly discussed over the last decades. Optimizing WDS involves the resolution of two issues: such as design and operations [14]. Design optimization usually deals with pipes sizes while WDS operation optimization takes into account of the pump schedules [15].

In recent years a number of optimization techniques have

^{*} Corresponding author:

ask4alamin@gmail.com (Al-Amin D. Bello)

Published online at http://journal.sapub.org/ijhe

Copyright © 2015 Scientific & Academic Publishing. All Rights Reserved

been developed mainly for the cost minimization aspect of network planning, although some reliability studies and stochastic modelling of demands have been attempted [1]. Some of the earlier studies utilized linear programming [16-18], while later studies applied nonlinear programming (NP) [19-21] or chance constrained [20] to the pipe network optimization problem. Much of the recent literature has utilized genetic algorithms for the determination of low cost water distribution network design and they have been shown to have several advantages over more traditional optimization methods [22, 23]. Linear optimization methods have been widely studied for the case of determining optimal design of water distribution networks [24, 25].

2. Material and Methods

2.1. Description of the Study Area

Dukku is the headquarter of Dukku Local Government Area (LGA) of Gombe State (Fig. 1). The LGA has an Area of 3,815km² and Population of 207,190 as at 2006 census. Dukku is located at Latitude 10.82° N and Longitude 10.77° E with Elevation of 608 m above sea level.

2.2. Dukku Water Supply System

The Dukku water supply source is purely underground water source and the town draws its water supply from a network of five boreholes drilled on the floodplains of the River Abba along the Gombe – Darazo road. The first three

sets of borehole were drilled in 1975 and two additional one were drilled by the Upper Benue River Basin Development Authority. Average water discharge from the boreholes is about 320m³/hr.

The boreholes discharge water through a 15km long 300mm diameter ductile iron rising mains to the booster station near *Wuro-Tara*. The booster pumps also pump through another 15km rising mains 250mm ductile iron pipe to deliver water to Dukku. The initial design of WDS layout before optimization is shown in figure 2.



Figure 1. Dukku town



Figure 2. WDN Layout before optimization [26]

2.3. Software and Programming Tool Used

Water Distribution Network optimization analysis have broad components and complexity in terms of analysis and problem solving [27]. Therefore, a lot of programmes are developed to solved those problems and it required flexible, simple to write and easier to understand concepts. For this study MATLAB (matrix laboratory) developed by MathWork [28] was_used because it can solve many technical computational problems, especially those with matrix and vector formulations, in a fraction of the time it would take to write a program in a scalar non-interactive language as compared to C or FORTRAN [29]. EPANET water distribution network analysis software is employed to analyze the network. This solver uses the basic hydraulic principles to solve and analyze WDN [30], of which the results obtained in the solver are prepared as an input file for MATLAB to Optimized.

2.4. Optimization Procedure

This approach, which seeks to determine the pipe sizes and associated lengths so as to minimize the cost of the system while satisfying hydraulic criteria and reliability requirements, is derived from a model developed by Goulter and Coals [31] which in turn is originated from an earlier model developed by Alperovits and Shamir [16] with slight modification, as shown below.

Objective Function:

Minimization of network Cost

The objective function used to minimize the network cost for the Dukku water supply system formulated as [16, 29] given in equation 1

$$Cost(c) = \sum_{j=1}^{NI} \sum_{k=1}^{n(j)} C_{jk} X_{jk}$$
(1)

Where; C : total cost of the system (N)

- C_{ik} : cost of pipe of diameter k in link j (N/km)
- X_{jk} : length of pipe of diameter k in link j (km)
- N_L : total number of links within the system
- $n_{(j)}$: number of different pipe diameters in link j
- j : link index

k : diameter type index

Subject to the following constraints:

i. Length: the sum of the lengths of pipe in each link must equal the total length of the link where a link represents a pipe connecting two nodes directly as shown in equation 2.

$$\sum_{k=1}^{n_j} X_{jk} = L_{oj}$$
(2)

For all link j,

Where; L_{oj} : total length of link j (km)

ii. Head loss: minimum and maximum permissible head at each demand point or Node must be satisfied (i.e

Minimal and Maximal Pressures) shown in equation 3 and 4

$$H_{o} - \sum_{j=p(n)} \sum_{k=1}^{n(j)} J_{jk} \cdot X_{jk} \ge H_{n\min}$$
(3)

$$H_{o} - \sum_{j \in p(n)} \sum_{k=1}^{n(j)} J_{jk} X_{jk} \le H_{n \max}$$
 (4)

For all nodes n

Where; H_o : original head at source (m) J_{jk} : hydraulic gradient for pipe diameter k in link j (m/km)

 H_{nmin} : minimum allowable head at node n (m) H_{nmax} : maximum allowable head at node n (m) $p_{(n)}$: links in the path from source to node n

iii. Head loss: the total head loss must be equal the sum of head losses of all the pipes in series shown in equation 5.

$$\sum_{j \in p(b)} \sum_{k=1}^{n(j)} J_{jk} X_{jk} = J_{oj} L_{oj}$$
(5)

Where; J_{oj} : Total Headloss within the link j (m/km)

~ ~

iv. Non-negativity: Is assume all the pipe length are positive as shown in equation 6

$$X_{ik} \ge 0 \tag{6}$$

for all j and k

v. Reliability: an estimate of reliability is included into this constraint put by equation 7 which limit the expected (average) number of leakages or breaks in given time period in any link within the network.

...:

$$\sum_{k=1}^{n_j} R_{jk} \cdot X_{jk} \prec R_{oj} \tag{7}$$

Where;

 R_{jk} : expected number of breaks/km/year for diameter k in link j

 R_{oj} : maximum allowable number of failures per year in link j

The steps of the solution are summarized in the following points:

Step 1 of the algorithm: preparing the input file for EPANET

The designed water distribution network of Dukku were extracted and prepared as input file for EPANET and saved as Dukku.inp. The text input file contains network information such as:

- i. Junction elevations and flow demands,
- ii. Reservoir elevations,
- iii. Pipe lengths, assumed diameters, roughness, and topology properties.

iv. Hydraulic analysis options.

Input.inp file was created from EPANET command menu (Fig 3).



Figure 3. Showing Dukku Network in EPANET Interface

Step 2 of algorithm: Network hydraulic analysis by EPANET

In this step, EPANET analyzes network defined by Dukku.inp file.

Step 3 of algorithm: Output of hydraulic analysis by EPANET

In this step output of the hydraulic analysis is obtained and saved.

Step 4 and step 5 of algorithm: Prepare input file for LOP – MATLAB

The Linear Optimization Programming (LOP) required the basic input from the solver which covered the flow rate for each pipe in between two nodes, the generated Unit head loss per link and the length of each link. The LOP uses the Hazen William equation to determine the unit head loss for the alternative diameters which is in consonant with the EPANET solver which uses the same Equation in the analysis.

Objective function and constraint equations try to find best diameters for network links to reach the optimum result. Note that, in this approach, assumed unknown parameter for any link is not the pipe diameter but the lengths of the available pipe diameters.

In the study its assume that predefined available diameters for any sample link is: 100, 150, 200 and 250 mm; and associated cost and lengths for these diameter types are C_1 , C_2 , C_3 , C_4 and X_1 , X_2 , X_3 , X_4

Objective function for any sample link is:

$$\mathbf{F} = \mathbf{C}_1 * \mathbf{X}_1 + \mathbf{C}_2 * \mathbf{X}_2 + \mathbf{C}_3 * \mathbf{X}_3 + \mathbf{C}_4 * \mathbf{X}_4$$

Linear Optimization Programme (LOP) defines it as matrix of *mxn* of which C_1 , C_2 , C_3 , C_4 , C_5 , C_6 have a define value (CONSTANT) and X_1 , X_2 , X_3 , X_4 , X_5 , X_6 are the Variables subject to the Constraints

Constraints 1: LOP defines it as Equality Constraints of mxn matrix, in reference to Equation 2.0.

$$X_1 + X_2 + X_3 + X_4 = L$$

Where L is the Total length per Link

Constraints 2: LOP defines it as a Matrix of define Rows and Column and it's called Inequality Constraints, see the Equations (3.0) and (4.0), reproduced respectively as

For easier conversion to matrix form its changes to;

$$X_1h_1+X_2h_2+X_3h_3+X_4h_4 \le (H_0-H_{min}) - (X_1h_1+X_2h_2 + X_3h_3+X_4h_4) \le (H_{max}-H_0)$$

And, H_0 - $H_{min} = H$, H_{max} - H_0 = H_L

Where:

 H_0 = Head of the Reservoir (m) H_{max} = Maximum allowable head (m) in the Network-User define H_{min} = Minimum allowable head (m) in the Network-User define

H₁, h₂, h₃, h₄ = Are the respective unit head loss of the sample diameter of corresponding lengths X₁, X₂, X₃, X₄ (Determine by LOP using Hazen William Equation) in the Programme the *H* and H_L are User-define

Constraints 3: It is also called Equality constraint in LOP programme, as shown in Equation (5.0)

Is converted as; $X_1h_1+X_2h_2+X_3h_3+X_4h_4 = L_oH$

Where;

L = total length of the link in view H = Head loss generated by the EPANET

corresponding to the link in view

Constraints 4: LOP defines it as Bound constraints, Equation (6.0). This constraint makes all the variables in the forms

$$X_1 \ge 0 \qquad X_2 \ge 0 \qquad X_3 \ge 0 \qquad X_4 \ge 0$$

Constraints 5: LOP defines it as a Matrix of define Rows and Column and its called Inequality Constraints, Equation (7.0)

It is converted to matrix form in this format

$$X_1r_1 + X_2r_2 + X_3r_3 + X_4r_4 \le R$$

Where;

R = it is the expected number of break (greater than or equal to zero) per year in the whole network (user define)

 r_1,r_2,r_3,r_4 = the expected number of breaks per km per year corresponding to the variables X_1,X_2,X_3,X_4 .

The program is run, and the output result is display in a text format

Step 6, 7, 8 and 9 of algorithm: output the result obtained in MATLAB code

After the result is obtained in the text format, the respective diameters of each link is observed and the diameter with higher length is chosen or if the length is relatively close to the higher length but shifted significantly on the diameter that is higher than the initial diameter, the observed changes are then updated and the solver is re-run again. The result obtained in the solver is compare to the previous result and if all condition are satisfied then optimization done. The Condition includes

i. Are the design condition satisfied

ii. Is the flow direction constant

These procedures are maintained for all the links and updated in the Solver until all the conditions are satisfied.

3. Result and Discussion

The result shows additional 150mm and 250mm size pipes in the network. The network in the end has four different sizes of pipes (Water Distribution Network layout of the study area is updated as shown in fig. 3) unlike the initial designed that considered two different sizes, 100mm and 200mm only (fig. 2).

In other to ascertain the significant of the optimization, a statistical analysis was used and the results obtained are explained according to the desired parameters and importance.



Figure 4. WDN layout after Optimization

Cost: A statistical comparison of the cost of pipes before and after optimization using non-parametric analysis (The Populations are not normally distributed) based on Wilcoxon test type and Monte Carlo Confidence interval of 95% was computed, and the result obtained is shown in Tables 1.0 and 2.0. The null hypothesis is that, there is no difference between the cost of pipes of our sample prior to optimization of the Network and the final cost obtained after optimization of the Network. Using two related sample method, the "Positive Ranks" are have a much greater sum than the negative ones. Here, "positive" means that the costs of pipes were higher before optimization than after optimization.

 Table 1.
 Wilcoxon Signed Rank Test for Cost of the Network

	Ranks	Ν	Mean Rank	Sum of Ranks
	Negative Ranks	7a	14.57	102
Pre-Cost – Post-Cost	Positive Ranks	23b	15.78	363
	Ties	62c		
	Total	92		

a. Precost<Postcost

b. Pre-cost>Post-cost

c. Pre-cost=Post-cost

Looking at the test statistic summary, $P=0.007 < 0.05=\alpha$, it suggest that the observed data are inconsistent with the assumption that the null hypothesis is true, that the hypothesis must be rejected. However, from 95% Monte Carlo confidence interval, the lower bound was 0 while the upper bound was 0.259. This indicates that there is no much important difference between the two costs. In summary, there is not enough evidence to reject the null hypothesis.

Table 2. Detailed Statistical Test Results for Cost of the Network

Test Statistics^b

	Pre-Cost – Post-Cost
Z	-2.684ª
Asymp. Sig. (2-tailed)	0.007

a. Based on negative ranks.

b. Wilcoxon Signed Ranks Test

Pressure: A statistical comparison of the Pressures at the nodes before and after optimization indicates that the mean pressure before optimization is 12.3 while for Post optimization is 13.4. Using the null hypothesis that, "there is no difference between the two pressures before and after optimization of the water distribution network". On this basis, the alternative hypothesis is, "there is difference between the pressure after optimization and before optimization". From Table 3.0, it shows that the "Positive Rank" has a much greater sum than the negative ones. Here, "positive" means that the pressures at the Nodes were higher after optimization than before optimization and zero tie shows that the two pressures compare are not equivalent in any way.

Table 3. Wilcoxon Signed Rank Test for pressures at the Nodes

	Ranks	Ν	Mean Rank	Sum of Ranks
processro	Negative Ranks	0 ^a	.00	.00
After -	Positive Ranks	65 ^b	33.00	2145.00
pressure	Ties	0^{c}		
Belore	Total	65		

a. Pressure After < Pressure Before

b. Pressure After > Pressure Before

c. Pressure After = Pressure Before

From the test statistic summary in Table 4.0, $P=0.00 < 0.05=\alpha$, the level of significance of 0.05 (or 5%) is chosen. The p-value is less than this limit, the result is significant and it is agreed that the null hypothesis should be rejected and the alternative hypothesis-that there is a difference-is accepted

Table 4. Statistical Test Results for Pressures at Nodes

Test Statistics^b

	Pressure After – Pressure Before
Z	-7.009ª
Asymp. Sig. (2-tailed)	.000

a. Based on negative ranks.

b. Wilcoxon Signed Ranks Test

Flow rate: The population data indicate non-parametric from the test of normality which has P=0.001<0.05, Using the Wilcoxon Rank test, A statistical comparison of the Flows in the pipes before and after optimization, derived from the null hypothesis, "There is no difference between the two flow rates with respect to pre and post optimization." The result of Wilcoxon Rank test (see Table 5.0) show that the sum of the "positive rank" is higher than that of the "Negative rank", it means that the flow rate before optimization.

Table 5. Wilcoxon Signed Rank Test for Flow in Links

	Ranks	Ν	Mean Rank	Sum of Ranks
	Negative Ranks	23 ^a	36.91	849.00
After Flow – Before Flow	Positive Ranks	57 ^b	41.95	2391.00
	Ties	12 ^c		
	Total	92		

a. Flow After<Flow Before

b. Flow After>Flow Before

c. Flow After=Flow Before

Looking at the test statistic summary, $P=0.003<0.05=\alpha$, it suggest that the observed data are in consonant with the assumption that the null hypothesis is true, that the hypothesis must be rejected. This indicates that there is difference between the two flows. In summary, there is high chance to accept the alternative hypothesis, that the flow after optimization is difference with the flow before optimization.

Table 6. Statistical Test Results for Flows in Pipes

Test Statistics^t

	AfterFlow - BeforeFlow
Z	-1.698ª
Asymp. Sig. (2-tailed)	.003

Headloss: As regard to the null hypothesis regarding the Head loss before optimization has no difference with the Headloss after Optimization, The statistical rank test (see Table 7.0) shows that the sum of "Positive Rank" has a

much greater sum than the negative ones. Its means that the Headloss in the links were higher after optimization than before optimization

Table 7. Wilcoxon Signed Rank Test for Head losses in Links

	Ranks	Ν	Mean Rank	Sum of Ranks
After Flow – Before Flow	Negative Ranks	26 ^a	37.62	924.00
	Positive Ranks	54 ^b	39.87	2316.00
	Ties	12 ^c		
	Total	92		

a. Headloss After < Headloss Before

b. Headloss After > Headloss Before

c. Headloss After = Headloss Before

Observing the test statistic summary, $P=0.001<0.05=\alpha$, it suggest that the observed data are discrepant with the assumption that the null hypothesis is true, the hypothesis must be rejected. This shows that there is difference between the Headloss. In summary, the alternative hypothesis will be accepted; the Headloss after optimization is difference with the Headloss before optimization, even though the difference is insignificant.

Table 8. Statistical Test Results for Head losses in Links

Test Statistics^b

	AfterFlow - BeforeFlow
Z	-3.518 ^a
Asymp. Sig. (2-tailed)	.001

Several notes can be deduced from the analysis of result obtained after optimizing the network, this include;

- 1. Mathematically, the program using MATLAB, can give the corresponding diameter combinations for each link. Which means at each links in view an equivalent pipe can produce an optimum result.
- 2. The selected diameters and hydraulic analysis result of node and link values of the network at the end of final run of the solver. All velocity and Pressure head results are in allowable range defined by constraints
- 3. The Pressures (m) of the Post-Optimization hydraulic results of the Nodes are greater than that of the Pre-Optimization. This indicates that the pressure increases with increase in flow rate and decrease in pipe diameters within the nodes. The maximum pressure in the pre-optimization results is at node J2, of value 14.79m while Node J13 have the maximum pressure of 15.71m after optimization. The minimum pressure in the Network is at J52 of value 7.59m in the former and 9.42m for the later. This shows that the minimum pressure occurs at the same node but maximum pressure shifted to another node. The statistical comparison using two related sample method of non-parametric distribution (Wilcoxon Signed Rank Test), shows a significant difference between the pressures compared. In summary the

pressure increase makes the network more hydraulically efficient because for peak demand period, the pressure will be sufficient to accommodate the demand and the problem of shortage of supply is resolved. Hence, the minimum pressure of 9.42m achieved after optimization is sufficient.

- 4. There was a slight increase in head loss after Optimization by considering the statistical result, but from direct comparison it is observed that the maximum head loss is at Link P95 (7.62m/km) compare to the same Link before Optimization which was 21m/km, meaning that the head loss decrease three fold at P95 and is closer to the maximum Headloss of 7.7m/km of pipes ranging from 100mm to 400mm sizes. Again the minimum head loss at link P92 was 0.05m/km after optimization while before optimization it was 0.08m/km.
- 5. As optimization is aimed at reducing the total cost possible, the total cost initially computed was \$716501.868 for 100mm and 200mm pipe which decreased by 7.15% after optimization (as a direct comparison). Using the statistical analysis, the cost of optimized network decrease to some certain degree differs from the cost before optimization couple with improvement of the hydraulic parameters (properties) of the entire water distribution network.

4. Conclusions

Both the capital and maintenance cost of a water distribution network (not including operation cost) is enormous. As a result design Engineers are looking for new approach for the best design of water distribution networks in addition with the usual methods. In this research, a methodology is designed which uses an optimization procedure employing linear relationship of the model parameters (LOP) while the objective function ensured minimization of the capital cost of the pipes.

A designed Network of Dukku water distribution system is used as a case study. Instead of using an Extended period simulation (EPS) in running the solver, a single period simulation (SPS) is used due to the sources of the water which was mainly Bore hole.

Although the WDS is initially design with two types of pipes (200mm and 100mm pipes), and being that the minimum acceptable pipe size for any WDS is 100mm, the optimization is done using pipes of 100mm,150mm, 200mm and 250mm diameters. There is a total decreased of 7.15% (direct comparison) of the initial estimate of the pipes. But using statistical comparison it shows slight (minimal) significant differences of the cost before and after optimization. In terms of hydraulic properties the network is more efficient and pressures can accommodate peak demand at every part of the distribution network which solves the problems of water shortage in Dukku if implemented.

It is important to know that a node isolation approach can

be used especially in the decision making process for improving the network if a limited amount of money is available.

ACKNOWLEDGEMENTS

This study is supported by Spicks Associates a Water Resources based Consultants. The authors wish to acknowledge the help and corporation received from the staff of the Water Resources and Environmental Engineering (ABU) Zaria in conducting this research work, and all other relevant agencies that make the research possible.

REFERENCES

- [1] Akdoğan, T., 2005, Design of Water distribution system by optimization using Reliability considerations (Doctoral dissertation, Middle East Technical University).
- [2] Burgschweiger J, Gna dig B, & Marc C., 2004, Optimization Models for Operative Planning in Drinking Water Networks. ZIB-Report, Berlin-Dahlem Germany.
- [3] Simon B., 2007, Closing the loop in water supply optimization. the IET Water Event, Derceto Limited, Auckland, New Zealand.
- [4] Mahdi M. J., 2008, Performance measurement of water distribution system (WDS). Thesis-Civil Engineering, University of Toronto, Canada.
- [5] Micheal H., 2012, Critical Node Analysis for Water Distribution System using flow distribution. California Polytechnic University, USA, p. 50-145.
- [6] Jacobs, P., & Coulter, I., 1991, Estimation of maximum cut-set size for water network failure. Journal of Water Resources Planning and Management, 117(5), 588-605.
- [7] Tanyimboh, T.T., Tabesh, M. & Burrows, R., 2001, Appraisal of Source head methods for calculating Reliability of Water distribution networks, Journal of Water Resources Planning and Management, ASCE, 127(4), pp 206-213.
- [8] Kalungi, P., & Tanyinboh, T.T., 2003, Redundancy Model for Water distribution system, Reliability Engineering and system safety, 82(3), 275-286.
- [9] Klebber T. M. et al., 2003, Optimal Design of Water Distribution System by Multiobjective Evolutionary Methods. São Carlos School of Engineering, University of São Paulo, São Carlos - SP – Brazil.
- [10] Khalid A. M.H., 2008, Cost value function of water distribution Network, a reliability Based approached using MATLAB. An-najwah National University, Palestine.
- [11] Čistý M. and Bajtek Z., 2009, Optimization of the water distribution Networks with search space reduction. Slovak University of Technology, Bratislava.
- [12] Mays, L. W., 2000, Reliability analysis of water distribution system, ASCE, New York.

- [13] Zheng, Y.W., & Walski T.M., 2005. Optimizing water system improvement for a growing community. International conference of computing and control in the water industry, Exeter, UK.
- [14] Vasan A. & Simonovic S.P., 2010, Optimization of Water Distribution Network Design Using Differential Evolution. Journal of Water Resources Planning and Management, Vol. 136, No. 2, ASCE.
- [15] Deprada C (prof.), 2000, Basic concept of Optimization. Lecture note, Department of System Engineering and Automatic Control, UVA.
- [16] Alperovits, E., & Shamir, U., 1977, Design of optimal water distribution systems. Water Resources Research, 13(6), 885–900.
- [17] Quindry, G., Brill, E., & Liberman, J., 1981, Optimization of Looped water distribution systems, Journal of Environmental Engineering, ASCE, New York (NY).
- [18] Shamir, U., & Howard, C. D., 1985, Reliability and risk assessment for water supply systems. In Computer applications in water resources, pp. 1218-1228, ASCE.
- [19] Su, Y. C., Mays, L. W., Duan, N., & Lansey, K. E., 1987, Reliability-based optimization model for water distribution systems. Journal of Hydraulic Engineering, 113(12), 1539-1556.
- [20] Lansey, K. E., & Mays, L. W., 1989, Optimization model for water distribution system design. Journal of Hydraulic Engineering, 115(10), 1401-1418.
- [21] Xu, C., & Goulter, I. C., 1999, Reliability-based optimal design of water distribution networks. Journal of Water Resources Planning and Management, 125(6), 352-362.
- [22] Simpson, T. W., Booker, A. J., Ghosh, D., Giunta, A. A.,

Koch, P. N., & Yang, R. J., 2004, Approximation methods in multidisciplinary analysis and optimization: a panel discussion. *Structural and multidisciplinary optimization*, 27(5), 302-313.

- [23] Savic, D. A., & Walters, G. A., 1997, Genetic algorithms for least-cost design of water distribution networks. Journal of water resources planning and management, 123(2), 67-77.
- [24] Schaake, J. C., & Lai, D., 1969, Linear programming and dynamic programming application to water distribution network design. Hydrodynamics Laboratory, Department of Civil Engineering, Massachusetts Institute of Technology.
- [25] Walski, T. M., 1993, Practical aspects of providing reliability in water distribution systems. Reliability Engineering & System Safety, 42(1), 13-19.
- [26] Spicks Associate, 2007. Final report of Dukku water distribution system, Ahmadu Bello Way, Kaduna, Nigeria, pp.6-52.
- [27] Von Lücken, C., Barán, B., & Sotelo, A., 2004, Pump scheduling optimization using asynchronous parallel evolutionary algorithms. CLEI Electronic Journal,7(2).
- [28] Gilat, A. 2004. MATLAB: An Introduction with Applications 2nd Edition. John Wiley & Sons. ISBN 978-0-471-69420-5.
- [29] Xenophontos, C., 1998, A Beginners Guide to MATLAB, Technical Report, Department of Mathematics and Computer Science, Clarkson University.
- [30] Rossman. L., 2000, EPANET User's Manual. Cincinnati: Environmental Protection Agency.
- [31] Goulter, I. C., & Coals, A. V., 1986, Quantitative approaches to reliability assessment in pipe networks. Journal of Transportation Engineering.